

USE OF DISCRIMINANT ANALYSIS FOR SELECTING
STUDENTS FOR NINTH GRADE ALGEBRA OR
GENERAL MATHEMATICS.

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ABSTRACT.

This thesis by the use of discriminant analysis attempted to discover existing psychological and intellectual factors that characterized algebra students from general mathematics students. Corollary to this purpose, was the intent to find out if the student's self-concept and self-expectations are important determinants for success in algebra or general mathematics.

The research population consisted of Grade IX students of metropolitan Montreal. The subjects were 285 boys and girls in the algebra group and 144 boys and girls in the general mathematics group.

The criterion in this study was the successful completion of the ninth grade provincial algebra examination or general mathematics final course examination.

The predictor variables may be classified into three general areas:

- (1) Acquired skills measured by the Grade VII arithmetic marks.
- (2) Aptitudes were measured by (a) scores from the Otis Self Administering test of Mental Ability, (b) scores achieved on the Academic Promise Tests.
- (3) Personal information was measured by (a) the academic self-concept scale which was constructed by Brookover, (b) the self-expectations inventory developed by Binder.

The statistical data for the discriminant function analysis was computed from canned programs developed by Cooley and Lohnes, and modified by Veldman.

The results of the study supported the hypothesis that the students in general mathematics and algebra have measurably different aptitudes and self-concept. The three highest variables that discriminated between the two groups, in both the male and female sample, were the Grade 7 mathematics marks, the Portland Prognostic Test and the self-concept score. The self-expectations was the lowest.

The validation sample found that the total number of hits in the male and female sample was 90%.

In essence, then, the predictive effectiveness achieved by the total sample battery of tests appears to be quite satisfactory.

DISCRIMINANT ANALYSIS FOR SELECTING STUDENTS FOR MATHEMATICS

BY

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CHAPTER I

INTRODUCTION

Owing to a combination of societal, political and economic factors, the high school population in Montreal is rapidly becoming more heterogeneous in academic ability. The trend which results in greater student retention falls in line with the three-fold educational goal which was viewed by the Parent Report: "...to afford everyone the opportunity to learn; to make available to each the type of education best suited to his aptitudes and interests; to prepare the individual for life in society."¹

The Parent Report and the new philosophical orientations in education have acted as a catalyst to diversify the mathematics curriculum. Dr. D. Robitaille, the Mathematics Supervisor for the Montreal Catholic School Commission, initiated a programme consisting of three streams or levels of instruction in mathematics.

- (1) Enriched mathematics for students with superior mathematical ability.
- (2) Regular mathematics for students with average and above average mathematical ability.

1. Report of the Royal Commission of Inquiry on Education in the Province of Quebec. (1963) Vol. 1. p. 75.

- (3) General mathematics for the non-mathematically oriented student.

The first two categories are college-oriented and is a necessary pre-requisite for most faculties for college admission while the third is a terminal course. For the purposes of this study, the enriched and regular mathematics courses will be termed "algebra".

This tri-level programme is presently in practise at the eighth and ninth grade levels.

Because of this diversification of the curriculum, and other developments complicating the life of the student, every effort should be made to help him clarify the relevant aspects of important decisions. The Guidance Department feels that by helping students to assess some of their personal qualities and to compare themselves with groups of their peers, they will be better able to make an appropriate choice of their mathematics program. This help is especially important when one considers the serious career implications of the decision, since the algebra credit is mandatory for college admission and other post-secondary education in Quebec Province.

The need for predicting success in mathematics has been demonstrated by research dating back to the 1920's. A review of the research indicates that initially cognitive variables were used for predicting elementary algebra ability. Binder, however, observes that despite

continuing efforts to improve aptitude tests as predictors of achievement, the obtained coefficient appears to have reached an asymptote in the area of .50.²

Several researchers have tried to improve predictability by including various non-cognitive variables.

Contemporary psychologists, Maslow³ (1954), Snyggs and Combs⁴ (1959) and Wylie⁵ (1961) consider self-concept to be the basic source of human motivation. Mead⁶ (1934) feels that the basis of our behaviour is an outcome of the society in which we live. We tend to play roles. Sarbin⁷ (1954) feels that this is a manifestation of the interaction of self and role. Sarbin has explicitly linked role theory with self-theory, and feels that one's self-expectations are also the result of an

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2. D. Binder, Relationships among self-expectations, self-concept, and academic achievement. Unpublished doctoral dissertation. (University of Wisconsin. 1965.)
 3. A.H. Maslow, Motivation and Personality. (New York: Harper) (1954.)
 4. D. Snyggs and A.W. Combs, Individual Behaviour. (New York: Harper and Row, Inc.) (1959)
 5. R. Wylie, The Self-Concept. Lincoln, Nebraska: (University of Nebraska Press, 1961.)
 6. G.H. Mead, Mind, Self and Society. (Chicago: University of Chicago Press, 1934.)
 7. T.R. Sarbin, Role Theory. In G. Lindzey (Ed.) Handbook of Social Psychology. Vol. 1. (Reading, Massachusetts: Addison-Wesley, 1954.)

interaction between the organism and stimulus of objects and events. Roth⁸ (1959), Brookover⁹ (1965), Binder¹⁰ (1965), Jones¹¹ (1966), Hayes¹² (1967) seem to feel that there is a significant relation between self-concept, self-expectations and academic achievement. Assuming that classroom learning is of a social and psychological nature, a measure for each of self-concept and self-expectations will be investigated in this study as the non-cognitive variables affecting mathematical achievement.

This experiment is an attempt to determine if the student's self-concept and self-expectations are important determinants of the mathematics program selected by grade nine students at John F. Kennedy High School, Montreal, Quebec.

It has been well recognized by psychologists

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8. R.M. Roth, "The Role of Self-Concept in Achievement." Journal of Experimental Education. Vol. 27. (June, 1959) p. 265-281.
 9. W.B. Brookover, "Self-Concept of Ability and School Achievement." Cooperative Research Project. (1962, 1965)
 10. D.M. Binder, Relationships among self-expectations, self-concept and academic achievement. (Unpublished dissertation, University of Wisconsin, 1965.)
 11. J.G. Jones, Relationships among identity development and intellectual and nonintellectual factors. (Unpublished dissertation, University of Wisconsin, 1966.)
 12. E.J. Hayes, Relationships between self-concept of arithmetic and arithmetic achievement. (Unpublished dissertation, Michigan State University, 1967.)

and educators that when predicting academic success it is important to consider the variables of intelligence, aptitude and past achievement. While it is not the intention of this study to deal with prediction per se, this aspect is, of course, implied. It is therefore most important that these variables also be considered by the study.

The design used in the present investigation is a replica of a study made by Ivanoff, DeWayne and Praem, for streaming students to algebra or general mathematics.¹³ It is hoped that a discriminant analysis of the above information will generate results which will enable the student to identify himself with the group he most resembles. It is hoped that this will help to provide information which would result in wise and appropriate student decisions regarding their mathematic program options.

STATEMENT OF THE PROBLEM

This investigation using the discriminant analysis technique seeks (a) to discover existing psychological or intellectual factors for discriminating students in the algebra and general mathematics programme, and thereby (b)

13. J.M. Ivanoff, E.T. DeWayne, O. Praem, "The Use of Discriminant analysis for selecting students for ninth-grade algebra or general mathematics." The Mathematics Teacher. (May, 1965) p. 412-416.

to provide information to assist the student in choosing the best mathematics program for him. If a student knows what group he most resembles, with regard to intelligence, aptitude, achievement, self-concept and self-expectations, this will enhance his decision-making ability. The criterion in this study will be the successful completion of the ninth grade algebra or general mathematics. An attempt will then be made to determine if the variables chosen for this research differentiate between the two groups.

The variables to be considered are both cognitive and non-cognitive in nature. The literature is replete with statistical proof that intelligence and aptitude correlate positively with mathematical achievement. However Jersild¹⁴ (1958), Reeder¹⁵ (1955), Stevens¹⁶ (1956), Staines¹⁷ (1958), Brookover (1965) and Binder (1965) have demonstrated that the student's self-concept and self-expectations are also associated with academic achievement.

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14. A.T. Jersild, In Search of Self. (New York: Bureau of Publications, Teachers College, Columbia University, 1952.)
 15. T.A. Reeder, "A Study of some relationships between level of self-concept, academic achievement, and classroom adjustment." Dissertation Abstract. XV (1955) p. 2472.
 16. P.H. Stevens, "An investigation of the relationship between certain aspects of self-concept behaviour and students' academic achievement." Dissertation Abstract. XVI (1956) p. 2531-2532.
 17. J.W. Staines, "The self-picture as a factor in the classroom." British Journal of Educational Psychology. Vol. XXVIII, (June, 1958) p. 97-111.

The term, "student", implies a role different from other roles that individuals may assume. Sarbin (1954) felt that individuals not only define their roles in terms of expectations, but they may also tend to define their ability to succeed in these roles. Hence the predictor variables may be classified into three general areas: (1)acquired skills, (2)aptitudes and (3)personal factors.

(1) Measures of acquired skills will be the Grade VII arithmetic marks. Grade VII marks will be used as this was the last year that all students were subjected to the same curriculum and a common examination.

(2) Measures of aptitudes will be: (a)Scores from the Otis Self Administering test of Mental Ability, (b)Scores achieved on the Academic Promise Tests. Turnbull says, "...the tests should have real utility as relatively brief measures of general academic development and promise and should be of supplementary help in sectioning and placement."¹⁸ The four individual tests which comprise the Academic Promise Test are: Verbal, Numerical, Abstract Reasoning, and Language Usage. The Verbal and Language Usage tests provide a measure of broad competence with verbal materials; and the Non-Verbal Reasoning ability is represented by a combination of Numerical and Abstract

18. W.M. Turnbull, The Sixth Mental Measurement Year Book.
(New Jersey: The Gryphon Press, 1965) pp. 1001.

Reasoning test scores. (c) The Portland Prognostic Tests for mathematics. Hoyt¹⁹ feels that the Portland tests can effectively predict success in algebra.

(3) Personal information refers to:

(a) Data concerning the student's self-concept (S.C.) and Self-Expectations (S.E.). The academic self-concept scale has been constructed by Brookover. It consists of nine multiple-choice questions devised to explore the student's concept of his academic abilities. Student responses to the questions are scored on a five point scale. Responses representing the low self-concept answer are weighted one and responses representing the high self-concept answers are weighted five.

(b) The Self-Expectations (S.E.) inventory was developed by Binder²⁰ (1965). It tries to measure the expectations an individual holds for himself in the role of a student.

PURPOSE OF THE STUDY

(1) Recent research has established a relationship between accurate self-concept and personal adjustment. One of the roles of the counsellor is to assist the student to greater self-knowledge and thus to an accurate self-concept.

19. C.J. Hoyt, The Sixth Mental Measurement Year Book.
p, 588.

20. D. Binder, p. 171-172.

One of the means the counsellor frequently uses to improve self-perception is the interpretation of standardized tests. Parker²¹ (1957), Froehlich and Moser²² (1954), felt that students remembered best their highest tested aptitude, but were less affected by results at the lower end. Similar results are reported in studies by Herman and Ziegler²³ (1961), Berdie²⁴ (1954), and Singer and Stefflre²⁵ (1954). Each of these authors sees the problem as resistance towards a change in self-concept. Peckens and Bennett²⁶ carried out an experiment which seemed to suggest that this was rather due to a lack of

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21. J.J. Parker, "A Study of immediate recall, delayed recall, and distortion of objective test data interpreted in counselling." Dissertation Abstract. (1957) 17, p. 305-306.
 22. C.P. Froehlich and W.E. Moser, "Do Counselees remember test scores?" Journal of Counselling Psychology. (1954) p. 149-152.
 23. L. Herman and M. Ziegler, "The effectiveness of interpreting freshman counselling -- test scores to parents in a group situation". Personel and Guidance Journal. (1961) p. 143-149.
 24. R. Berdie, "Changes in self-ratings as a method of evaluating counselling." Journal of Counselling Psychology. (1954) p. 49-54.
 25. S. Singer and B. Stefflre, "Analysis of The Self-estimate in the evaluation of counselling." Journal of Counselling Psychology. (1954) p. 252.
 26. R.G. Peckens and L.M. Bennett, "A Study of the effectiveness of the secondary school counsellor in test interpretation." The School Counsellor. (Jan. 1968) p. 203-208.

proper communication during test interpretation. Goslin and Glass²⁷ feel that the process involved in arriving at a conception of one's ability is extremely complex. They feel that test scores, if interpreted correctly, play a modest role in shaping one's self-conceptions. Erickson²⁸ attests to the comprehensiveness, but also feels that the self-concept is developmental. He reported that parents and teachers exert an important influence on the self-concept and learning behaviour of students.

It would appear that most researchers agree in varying degrees that feedback of test results helps towards more realistic self-evaluation and wiser decisions. This need to help the student to compare himself with his fellow classmates in his grade level seems imperative for two reasons:

- (a) There is a trend now to gear examinations and teaching to the unique needs and abilities of each class. Ironically, all students are subject to the same provincial examination at the end of ninth grade.
- (b) The student is faced with a choice of algebra or general mathematics.

27. D.A. Goslin and D.C. Glass, "The Social effects of standardized Testing." Sociology of Education. (Spring, 1967) p. 130.

28. L.H. Erickson, "Certain ability factors and their effect of arithmetic achievement." The Arithmetic Teacher. Vol. 5, No. 6. (1958) p. 287-293.

The results of this study, if used with other relevant information, could enable the student to make wiser decisions, prevent traumatic experiences, frustration and loss of time.

(2) The second purpose of the study would be to determine the importance of self-concept and self-expectations factors in the differentiation of the algebra and general mathematic students.

White, Anderson and Cryder found that the self-concept tends to modify and mature as the individual progresses from early adolescence to middle adolescence²⁹. Piers and Harris³⁰ found in a longitudinal study that there were fluctuations of self-concept between the third and tenth grades. Rogers³¹ supports the developmental nature of the self-concept, and feels that counselling facilitates change.

29. W.F. White, H.E. Anderson and H. Cryder, "The emerging self-concept in relation to select variables of secondary school students." Journal of Social Psychology. (1967) p. 81.

30. E. Piers and D.B. Harris, "Age and other correlates of self-concept in children." Journal of Educational Psychology. Vol. 55. (1964) p. 91-95.

31. C.R. Rogers, Client-centered therapy. (Chicago: Houghton-Mifflin, 1951.)

(3) Ivanoff, DeWayne and Praem³² feel that this prediction scheme has a two-fold purpose. Primarily it has the discriminating power of separating potentially successful algebra students from general mathematics students. Secondly, it provides the students with an appraisal of their marginal situation, and thus brings out the need for real effort if they are to succeed in an algebra program. This of course is subject to yearly cross-validation and to other variables.

BASIC ASSUMPTIONS.

Since this study implies prediction, the following basic assumptions have been postulated:

- (1) That the students in algebra and general mathematics differ significantly on a number of cognitive and personal variables. The variables considered by this study include:
 - (a) Acquired skills.
 - (b) Aptitudes.
 - (c) Personal factors -- Academic self-concept and self-expectations.
- (2) That the Grade Seven mathematic mark was a measure of success in mathematic skills.
- (3) That the Portland Prognostic Algebra Test and the Academic Promise Tests are satisfactory measures of mathematic abilities and aptitudes.

32. J.M. Ivanoff, E. DeWayne and O. Praem, p. 415.

(4) That the scores on the self-concept and self-expectations inventory reflect true measures of these variables.

(5) That the individual has developed a set of expectations for his own behaviour in the role of student. These expectations influence the way the individual behaves in the role of student.

(6) That the individual's self is defined in part by a self-concept. This is a definition of who and what he is. The self-concept exerts a regulatory effect on behaviour. The pattern of life is largely that of an individual's living out his self-concept.

(7) That the discriminant analysis technique is a valid method of discriminating successful algebra students from general mathematic students.

DEFINITION OF TERMS.

(a) Enriched, Regular or General Mathematics.

Enriched mathematics is algebra of a more abstract nature than regular algebra. Both the enriched and the regular math have greater academic rigor than the general mathematics programme. Successful completion of the algebra courses provide a student with credits acceptable for college entrance; completion of the general math program does not. The General mathematics programme, however, is an acceptable credit towards the Provincial High School Leaving Diploma.

(b) A pass in the Montreal Catholic School Board is a mark of 50% or more on the first two terms or final examinations.

(c) Self-concept is a personal perceptual framework that includes the individual's conscious attitudes towards, and understandings of himself, his experiences and his capacities in relation to his surrounding world as motivation for behaviour and learning. Self-concept of ability. This is a set of self-referent statements about academic ability which an individual in the student role accepts or does not accept as definitions of his ability to achieve academically according to the assessment used in this study.

(d) Self-expectations. These are evaluative standard or "shoulds" for the incumbents of the position "student" as assessed in this study. They constitute limits on behaviour and may be held not only by others but by the role incumbent himself. Those expectations held by the individual in a specific role constitute his own "shoulds" and will be termed "self-expectations".

(e) Academic aptitude refers to those competencies specifically related to academic success in terms of measured potentialities. Functionally, aptitude is defined as the student's score on the Academic Promise Tests.

LIMITATIONS OF THE STUDY.

This investigation is confined to John F. Kennedy High School which is situated in the North-east end of Montreal. The school population is predominantly Italian and belongs to the low or middle socio-economic group. The conclusions and implications will be limited by these considerations.

SUMMARY.

This chapter provided an introduction to the problem under investigation. Briefly, the investigator selected pertinent cognitive and non-cognitive variables and by means of the discriminant analysis technique sought to obtain results which might enable the student to identify himself with the algebra or general mathematics group. The assumptions basic to the research design were presented, and finally, the key terms and concepts used in this investigation were defined.

The remainder of the study is organized as follows: Chapter II reviews the pertinent literature. Chapter III deals with the research design and statistical methodology. The analysis of the data and statistical results are presented in Chapter IV. Chapter V contains a general summary of the investigation, discussion, and conclusions; also, implications for further study are presented in this chapter.

CHAPTER II

REVIEW OF RELATED LITERATURE.

A review of the literature has revealed several studies predicting achievement in high school algebra. Most of the studies initially were conducted to determine the effectiveness of a single predictor, the most popular being intelligence. Ross and Hooks reported a survey of prediction studies using intelligence as the single predictor and found that coefficients of correlation ranged between .12 and .69 with a median of .48³³. A correlation of .48 indicates that prediction is only 12% better than chance.

Later prognostic algebra tests were considered by many experts to be better predictors than intelligence tests, since they included background in mathematics in addition to intelligence in mathematics. Segel reported a review of the literature on prediction which indicated that predictors of achievement in algebra ranged as follows: first, special algebra aptitudes; second, arithmetic tests; and third, intelligence tests³⁴. Guiler also reported a review of several studies where the Iowa Algebra Aptitude

33. C.C. Ross and N.T. Hooks, "How shall we predict high school achievement?" Journal of Educational Research, XXII. (October, 1930) p. 191.

34. D. Segel, "Measurement of aptitude in special fields." Review of Educational Research. XI. (February 1941) p. 42-56.

Test was one of the predictor variables, and the Breslich Survey test was the criterion. He discovered that the coefficient correlations ranged from .33 to .77³⁵.

A great deal of variation in the prognostic studies in algebra is due to (a) the types of instruments employed for measuring achievement, (b) the type of instruments used for predicting achievement, (c) the predictive efficiency of the various instruments.

The most effective prediction equation was produced by Guilford, Hoepfner and Peterson who found that combinations of factor-test scores, especially the symbolic and evaluation factors (most of them dealing with the products of relations and implications), discriminated between above-median algebra students and general mathematics students with a very high degree of accuracy³⁶. There appears to be a lack of follow-up study, which could be primarily due to the tremendous work involved. Secondly, Guilford's experiment was on the traditional curricula, whereas the curricula in most schools have absorbed the "new" mathematics.

35. W.S. Guiler, "Forecasting achievement in elementary algebra." Journal of Educational Research. Vol. 38. No. 1. (September, 1944) p. 30.

36. A. Guilford, R. Hoepfner and A. Peterson, "Predicting achievement in ninth-grade mathematics from measures of intellectual-aptitude factors." Educational and Psychological Measurement. p. 659.

Scandura notes that our findings in mathematics achievement are marked by variation and attributes this to a need for the identification of "a large number of unspecified, but crucial 'ideal' competencies which underlie mathematical behaviour."³⁷ He also feels that we still have to learn how the inherent capacities of learners and their previously acquired knowledge interact with new input to produce mathematical learning and performance.³⁸

Since there has been a large number of investigations in predicting mathematical success in the junior high school, the review will be presented in tabulated form. Table I will cover studies that have cognitive variables and Table II will deal with studies that cover non-cognitive variables.

Recently, most psychologists and educators seem to have realized that academic achievement is closely linked with non-cognitive variables. The review would indicate that several types of non-cognitive variables have been added to the prediction equation with varying results. Cattell discovered that the prediction level of educational achievement from ability tests could be doubled in accuracy

37. J.M. Scandura, "Research in Psychomathematics." The Mathematics Teacher. (October, 1968) p. 591.

38. Ibid. p. 591.

by adding personality source traits and trebled by adding objectively measured dynamic traits³⁹.

This study will differ from other prediction studies in that it will consider the expectations an individual holds for himself in the role of student. It will also consider his self-concept of ability as one of the variables for achievement in mathematics. Hence, it seems necessary also to review the literature in the areas of role theory and self-concept theory in relation to academic achievement.

39. R.B. Cattell, A.P. Sealy and A.B. Sweeney, "What can personality and motivation source trait measurements add to the prediction of school achievement?" British Journal of Educational Psychology. Vol. XXXVI. (Nov. 1966) p. 280.

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39. R.B. Cattell, A.P. Sealy and A.B. Sweeney, "What can personality and motivation source trait measurements add to the prediction of school achievement?" British Journal of Educational Psychology. Vol. XXXVI. (Nov. 1966) p. 280.

A. TABLE I.

A REVIEW OF RELATED LITERATURE ON COGNITIVE VARIABLES FOR SUCCESS IN
ELEMENTARY MATHEMATICS.

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Bright ⁴⁰	1921	How to predict mathematical achievement in high school by means of a Terman Test.	<p><u>Procedure:</u> The Terman Test was correlated with mathematical achievement.</p> <p><u>Result:</u> The correlation coefficient was .50.</p>
Jordan ⁴¹	1922	To find out which of the four standardized tests had the highest correlation with mathematical achievement.	<p><u>Procedure:</u> The army Alpha Test was correlated with mathematical achievement.</p> <p><u>Result:</u> Correlation coefficient .37.</p> <p><u>Procedure:</u> The Miller Test was correlated with mathematics.</p> <p><u>Result:</u> Correlation Coefficient .34.</p> <p><u>Procedure:</u> Terman Test was correlated with mathematical achievement.</p> <p><u>Result:</u> Correlation Coefficient .34.</p>

40. I.J. Bright, "The Intelligence Examination for High School Freshmen." Journal of Educational Research. IV. (June 1921) p. 41-55.

41. A.M. Jordan, "Correlations of Four Intelligence Tests with Grades." Journal of Educational Psychology. XIII. (Sept. 1922) p. 419-429.

TABLE I (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Jordan (Cont'd.)			<u>Procedure:</u> The Otis Test was correlated with mathematics. <u>Result:</u> Correlation Coefficient .28.
Grover ⁴²	1932	Develop a multiple regression equation to predict ninth grade algebra.	<u>Procedure:</u> (a)An achievement Test was used as the dependent variable. (b)An Algebra prognosis test and an intelligence test were used as independent variables. <u>Result:</u> A coefficient correlation of .65 was obtained.
Dickter ⁴³	1933	(1)What are the comparative prognostic values of the intelligence quotient, teachers' marks, and a prognostic test? (2)What is the best criterion for predicting achievement in algebra?	<u>Procedure:</u> (a)Breslich Algebra Survey test was used as the criterion. (b)A combination of the following variables was used as predictor variables. (1)Roger's Test of Algebra Ability, (2)Arithmetic grades, (3)Otis Intelligence test. <u>Result:</u> A coefficient correlation of .74 was obtained.

42. C.C. Grover, "Results of an Experiment in Predicting Success in Two Oakland Junior High Schools." Journal of Educational Psychology. XXIII. (April 1932) p. 313.

43. M.R. Dickter, "Predicting Algebraic Ability." School Review. XLI. (October 1933) p. 605.

TABLE I (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Ayers ⁴⁴	1934	(1)What are the comparative predictive values of certain measures available before instruction is begun in algebra? (2)What is the best combination of these measures in predicting algebraic success?	<u>Procedure:</u> The criterion variable is grades in algebra. The predictor variables used were (a)The South Pasadena prognostic test. (b)8A Mechanics Test. (c)8A Reasoning Test. (d)Teacher Estimate. (e) Intelligence. <u>Result:</u> The best combination of the measures above in predicting algebraic success is the combination of the South Pasadena Prognostic Test, 8A Reasoning Test and the Teacher Estimate of Mathematical ability. The correlation coefficient with the criterion was .70.
Dunn ⁴⁵	1937	What is the influence of the teacher factor in ninth grade algebra?	<u>Procedure:</u> The Douglas Standard Algebra Survey test was used as the criterion. The predictor variables were - (1)Orleans Algebra Prognosis Test. (2)General Achievement by Stanford. (3)Achievement in Arithmetic by Stanford. (4)The Terman Group test of mental ability.

44. G.H. Ayers, "Predicting success in algebra." School and Society. Vol. 39. No. 993. (Jan. 1934) p. 17 & 18.

45. W.H. Dunn, "The Influence of the Teacher Factor in Predicting Success in Ninth Grade Algebra." Journal of Educational Research. XXX (April 1937) p. 581.

TABLE I (continued.)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Dunn (Cont'd.)			<p><u>Results:</u> (1)Coefficient correlation of .44.</p> <p>(2)A careful study of the results indicate that the Teacher factor is very influential in determining the size of the correlation between achievement in algebra and the various predictive criteria taken separately and in combinations.</p>
Kellar ⁴⁶	1937	Predict ninth grade algebra.	<p><u>Procedure:</u> The Algebra Survey Test was used as the criterion. The following tests were used as predictor variables.</p> <p>(1)Algebra Computation.</p> <p>(2)Ability to do Arithmetic problems.</p> <p>(3)Memory.</p> <p>(4)Intelligence.</p> <p><u>Results:</u> The coefficient correlation is .81.</p>

46. W.R. Kellar, "The Relative Contribution of Certain Factors to Individual Differences in Algebraic Problem Solving Ability." Journal of Experimental Education. VIII (September, 1939) p. 26-35.

TABLE I (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Morris ⁴⁷	1940	Forecasting achievement in algebra.	Procedure: (a) Teachers' marks in Algebra as the criterion of achievement. (b) 3 Algebra aptitude tests. (1) Iowa Algebra Aptitude Test. (2) Lee Test of Algebraic Ability. (3) Orleans Algebra Prognosis test. Result: Correlation of .64.
Clifton ⁴⁸	1940	Predict ninth grade algebra.	Procedure: <u>Criterion</u> - Grades in Algebra. <u>Predictor Variables</u> - 1. Reading, 2. Arithmetic Reasoning, 3. Dictation, 4. Intelligence. Result: Correlation coefficient is .57.
Layton ⁴⁹	1941	An attempt to predict a pupil's chances of success in algebra, with	Procedure: <u>Criterion</u> - Algebra Survey Test. <u>Predictor Variables</u> - (1) Intelligence by Otis.

47. L.M. Morris, "Prognosis in First Year Algebra." Ohio Schools. XVIII. (January 1940) p. 20-22.

48. L.L. Clifton, "Prediction of High School Marks in Elementary Algebra." Journal of Experimental Education. VIII. (June, 1940) p. 411.

49. R.B. Layton, "Study of Prognosis in High School Algebra." Journal of Educational Research. XXXIV. (April, 1941) p. 604.

TABLE I (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Layton (Cont'd.)		various criteria and various combinations.	(2)8th Grade Arithmetic grades. (3)Achievement Test in Arithmetic by Stanford. (4)Algebra Prognostic Test, by Lee. <u>Result:</u> The correlation coefficient is .76.
Guiler ⁵⁰	1944	(a)To discover the comparative value of certain standardized measures of algebraic aptitude, of arithmetical achievement, and of initial algebraic achievement in forecasting success in elementary algebra. (b)The extent to which this success can be forecasted from various combinations of these three predictive measures.	<u>Criterion</u> - Breslich Algebra Survey Test. <u>Predictor Variables</u> - (1)Iowa Algebra Aptitude Test. (2)Arithmetic Computation. (3)Algebra Prognosis Test. <u>Result:</u> Coefficient correlation .85.

50. W. Guiler, "Forecasting Achievement in Elementary Algebra." Journal of Educational Research. XXXVIII. (September, 1944) p. 25-33.

TABLE I (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Kraft ⁵¹ Betz ⁵² Eagle ⁵³	1946 1948 1948	To investigate the value of the Iowa Algebra Aptitude Test. Is reading comprehension and general vocabulary correlated highly with mathematical ability?	<u>Procedure:</u> Scores on this test were used to classify pupils into "surely", "maybe" and "no" groups. <u>Result:</u> No evidence that improvement in general reading comprehension or general vocabulary would increase proficiency in 8th and 9th grade mathematics.
Fay ⁵⁴	1950	Is reading ability indicative of mathematical ability?	<u>Procedure:</u> Mathematical achievement is correlated with reading ability for Grade 6. <u>Result:</u> Those students who were superior in reading skills had no higher achievement in arithmetic than those who were inferior.

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51. Ora Kraft, "Methods used in the selection of Pupils for the Study of Algebra and Geometry in Cleveland." The Math Teacher. VI. XLIX. (May 1946) p. 236-239.
52. W. Betz, "Functional Competence in Mathematics -- Its meaning and attainment." The Mathematics Teacher. 41: (May 1948) p. 195-205.
53. Edwin Eagle, "The Relationships of Certain Reading Abilities to success in Mathematics." The Mathematics Teacher. Vol. XLI. p. 175-179.
54. L.C. Fay, "The Relationship between Specific Reading Skill and Selected Areas of Sixth Grade Achievement." Journal of Educational Research. Vol. 43. p. 541-547.

TABLE I (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Rosilda ⁵⁵	1951	Explores the relationship between I.Q. and achievement in algebra.	<p><u>Procedures:</u> Criterion - Cooperative Elementary Algebra Test. Predictor - California Test of Mental Maturity. <u>Result:</u> Correlation Coefficient .42.</p>
Shaw ⁵⁶	1956	Prediction of success in elementary algebra.	<p><u>Procedure:</u> Criterion - Algebra Survey Test. Predictor Variables - (1) Intelligence. (2) Algebra Aptitude Test. (3) Reading Test. <u>Results:</u> The correlation coefficient with mathematical achievement is .77. The Iowa Algebra Aptitude Test, the Otis I.Q. and the Iowa Reading Test are, in descending order, good indicators of group succession algebra. A greater degree of success of prediction is achieved by using the combined scores of the 3 tests.</p>

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55. M. Rosilda, "Is an I.Q. an Index to Algebra Ability?" Journal of Educational Research. Vol. 44. (January, 1951) p. 391-393.
56. G.S. Shaw, "Prediction of Success in Elementary Algebra." The Mathematics Teacher. XLIX. (March, 1956) p. 173.

TABLE I (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Frost ⁵⁷	1956	Find out factors which will predict success in Beginning Algebra.	<u>Criterion</u> - Marks in Algebra. <u>Predictor Variables</u> - (a) Otis Intelligence Score. <u>Result:</u> A correlation coefficient of .51.
Wrigley ⁵⁸	1958	To investigate the structure of mathematical ability in grammar and technical school by means of factor analysis.	<u>Procedure:</u> Factor analysis. <u>Result:</u> (1) There is a close connection between mathematical and general ability; high intelligence is the most important single factor for success in math. (2) In addition, there exists a clearly identifiable math group factor. The different branches of math are linked together more closely than they would be if a general factor only were in operation. (3) When the influence of general ability is eliminated, verbal ability has little connection with math ability.

57. N.F. Frost and L.G. Brands, "Factors for Predicting Success in Beginning Algebra." California Journal of Educational Research. VII. (1956) p. 79-83.

58. J. Wrigley, "The Factorial Nature of Ability in Elementary Mathematics." British Journal of Educational Psychology. Vol. 28. (Feb. 1958) p. 61.

TABLE I (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Dinkel ⁵⁹	1959	Prognosis for success in algebra.	<p><u>Procedure:</u> Criterion - Seattle Algebra Achievement Test and the Cooperative Elementary Algebra Test were taken.</p> <p>Predictors - (1) Orleans Algebra Prognosis Test.</p> <p>(2) I.Q. - California Test of Mental Maturity.</p> <p>(3) Arithmetic Achievements - previous teachers' grades.</p> <p>(4) Arithmetic Competency (Cooperative Math Test for Grades 7, 8, and 9.</p> <p>(5) Author made test.</p> <p><u>Result:</u> Correlation coefficient .86.</p>
Guilford ⁶⁰	1965	(1) To see whether composites of measures of structure - of intellect	<p><u>General Results:</u></p> <p>(1) The 9 standard predictors versus the 20 factor predictors, just</p>

59. R.E. Dinkel, "Prognosis for studying algebra." The Arithmetic Teacher. VI (December 1959) p. 318.

60. J.P. Guilford and P.R. Merrifield, "Predicting achievement in ninth-grade mathematics from measures of intellectual-aptitude factors." Educational and Psychological Measurement. Vol. XXV. No. 3. (1965) p. 659-681.

TABLE I (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Guilford, (Cont'd.)		<p>factors would yield high degree of prediction of achievement in courses in grade mathematics, compared to 3 test batteries of the traditional, standard type of academic aptitude tests;</p> <p>(2)Whether such measures would contribute predictive value over and above that available from standard test batteries;</p> <p>(3)To see how well a battery of factor tests can discriminate between successful students in general mathematics and in algebra;</p> <p>(4)and which factors are most relevant for predictions of success within courses and between courses.</p>	<p>about break even, The standard predictors apparently do better in predicting achievement of basic mathematic students, the factor battery apparently does better in the regular algebra group.</p> <p>(2)Batteries of factor scores were better predictors of achievement than 2 of the standard - test combinations especially in the prediction of achievement in algebra.</p> <p>(3)A composite of 13 factor scores gave increased prediction when added to each of the three standard test combinations, significantly the algebra courses.</p> <p>(4)Combinations of factor - test scores discriminated between successful (above-median) algebra students and general mathematic students with an accuracy close to 90%.</p> <p>(5)With only predictors that gave statistically significant contribu-</p>

TABLE I (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Guilford (Cont'd.)			tions to prediction of achievement, some 12 different factors were found relevant. Most of these factors are symbolic; very few were cognition factors and quite a number were evaluation factors. Most of them dealt with products of relations and implications. ⁶¹
Collins ⁶²	1967	To determine the predictive value of eight variables in predicting successful performance in mathematics for seventh grade students.	<p><u>Procedure:</u> Criterion - Grades in Algebra. The Predictor Variables were -</p> <ul style="list-style-type: none"> (1) Arithmetic Problem solving and concepts. (2) Numerical Reasoning. (3) Arithmetic Computation Test. (4) Reading Test. (5) Language Usage Test. <p><u>Results:</u> The best single predictor of success in seventh grade mathematics was Arithmetic problem solving and concepts.</p>

61. Ibid. p. 681.

62. Collins, F.E., "A Study of the relative importance of certain factors in prediction of successful performance in seventh grade mathematics." Dissertative Abstract. (1967) p. 118.A.

B. TABLE II.

REVIEW OF RELATED LITERATURE ON PSYCHOLOGICAL, SOCIOLOGICAL AND INTELLECTUAL FACTORS FOR SUCCESS IN MATHEMATICS.

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Cattell ⁶³	1945	Explore possibilities in the relation of personality traits to mathematical and verbal abilities.	<u>Procedure:</u> Factor Analysis. <u>Result:</u> Mathematical ability is related to intelligence, character integration and general education. Mathematicians are generally low in dominance and more introspective.
Pitts ⁶⁴	1952	(1)To determine the inter-relationship of 4 variables: functional competence in mathematics, reading grade levels, mental ability expressed in terms of Ganna I.Q.'s and chronological age, (2)and to determine the	<u>Procedure:</u> Criterion - Test devised by Davis to measure functional competence. The predictor variables were: (1)The Iowa Silent Reading Test. (2)Otis I.Q. <u>Result:</u> All three sets of measures correlate negatively with age.

63. R.B. Cattell, "Personality traits associated with abilities II: With Verbal and Mathematical Abilities." The Journal of Educational Psychology. (April 1945)

64. R.J. Pitts, "Relationship between functional competence in mathematics and reading grade levels, mental ability, and age." The Journal of Educational Psychology. (Dec. 1952) p. 491.

TABLE II (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Pitts (Cont'd.)		relationship of each variable with the functional competence score when other variables are partialled out.	
Stephens ⁶⁵	1960	(1)Are the attitudes of the group of high achievers significantly different from those of the groups of lower achievers? (2)Will a measure of attitudes towards arithmetic aid in selection of students for accelerated classes.	<u>Procedure:</u> Dalton Math attitude scale correlated with accelerated regular and remedial groups. <u>Result:</u> Highly significant difference between accelerated and regular groups. The difference between regular and remedial was not significant.
Dale ⁶⁶ Lewis Robey J.J. Cody	1966	Identify low-academic and average-academic ninth grade male students.	<u>Procedure:</u> (1)Achievement as measured by the Iowa Tests of Education Development. (2)Intelligence as measured by

65. L. Stephens, "Comparison of Attitudes & Achievement Among Junior High School Mathematics Classes." The Arithmetic Teacher. (November 1960) p. 351-356.

66. Dale, Lewis & Robey, "Differential Diagnosis of low and average academic ninth grade male students." The Journal of Experimental Education. (Summer 1966) Vol. 34. No. 4. p. 40-43.

TABLE II (Continued.)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Dale Lewis Robey J.J. Cody (Cont'd.)			<p>the Otis Quick Scoring Mental Ability Test.</p> <p>(3) Aptitude as measured by the Differential Aptitude Tests.</p> <p>(4) Personality as measured by the California Test of Personality.</p> <p>(5) Socio-economic status as measured by the Warner Index of Social Status.</p> <p>(6) Behaviour as measured by the Behaviour Preference record.</p> <p>(7) Study methods as measured by the California Study Methods Survey.</p> <p><u>Results:</u> In general the low-academic student differed from the average-academic student in the broad areas of academic skills, in skills that required a great deal of organization and abstract thinking, and in personal attributes which seem conducive to effective social conduct in a democratic society.</p>
Duncan ⁶⁷	1960	Prediction of 8th grade algebra achievement.	<p><u>Procedure:</u> Used 21 variables and found that the following contributed most to the prediction of eighth</p>

67. R.L. Duncan, "The Prediction of Success in Eighth Grade Algebra." Unpublished Doctoral Dissertation. University of Oklahoma, (1960)

TABLE II (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Duncan (Cont'd.)			<p>grade algebra achievement:</p> <p>(1) I.Q.</p> <p>(2) Interest scores in science and literature.</p> <p>(3) Scores on the Orleans Algebra Prognostic Test.</p> <p>(4) Grade Placement in arithmetic computation.</p> <p><u>Result:</u> The multiple correlation was .76.</p>
Lambert ⁶⁸	1960	To find out the relation between mathematical ability and masculinity.	<p><u>Procedure:</u> Male and female mathematic major students were given an arithmetic test and the MF scale of the M.M.P.I.</p> <p><u>Result:</u> There was no correlation found between arithmetic proficiency and masculinity of interest pattern for either sex.</p>

68. P. Lambert, "Mathematical Ability and Masculinity." The Arithmetic Teacher.
(1960) p. 21.

TABLE II (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Aiken ⁶⁹	1961	(1) Find relations between selected intellectual and non intellectual variables and math attitudes and, (2) The contributions of these attitudes to the prediction of achievement in mathematics.	<u>Procedure:</u> The Math Attitude Scale, D.A.T. Numerical Ability, Verbal Reasoning Test, the Cooperative Math Pretest, the Minnesota Counseling Inventory, and an adaptation of the Intensive Personal Data Sheet were administered to college freshmen in math courses to determine in a limited way the etiology of math attitudes. <u>Result:</u> Regression and correlation analyses of the intercorrelations of these measures and their relations with grades in High School and college math courses supported to a modest extent the supposition that direct experiences in relation to math contribute to math attitudes.
Messler ⁷⁰	1961	An investigation of pupil age and achievement in eighth-grade algebra.	<u>Procedure:</u> The criteria were the Otis I.Q. score and the score on the California Arithmetic Test. Two groups were selected for a year's

69. L.R. Aiken, "The Effect of Attitudes on Performance in Mathematics." Journal of Educational Psychology. (1961) Vol. 52. No. 1. p. 24.

70. D.L. Messler, "A study of pupil age and achievement in eighth-grade algebra." The Mathematics Teacher. (November, 1961) p. 561-564.

TABLE II (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Messler (Cont'd.)			study. One group consisted of eighth-grade students and the other group consisted of ninth-grade students, and they were subjected to the same program. <u>Result:</u> The test results indicate that age was not detrimental to the achievement in elementary algebra.
Leonhardt ⁷¹	1962	This study was designed to describe the relationship of selected factors to achievement in mathematics.	<u>Procedure:</u> The factors considered in this study were teacher preparation, years of service, mathematics course offerings, elementary education, size of school. <u>Result:</u> The Subject matter preparation of the teacher is the important factor for achievement in mathematics.

71. E.A. Leonhardt, "An analysis of selected factors in relation to high and low achievement in mathematics." Dissertation abstract. (1962) University of Nebraska. p. 3689.

TABLE II (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Cattell ⁷² Sealy Sweney	1965	What can personality and motivation source trait measurements add to the prediction of school achievement?	<u>Procedure:</u> (1) Intelligence (Fluid-culture fair, and crystallized traditional test.) (2) 14 primary Personality factors (on the H.S.P.Q.) (3) 15 dynamic structure factors (School Motivation Analysis Test.) <u>Results:</u> The prediction of level of educational achievement from ability tests could be doubled in accuracy by adding personality source traits and trebled by adding objectively measured dynamic traits.
Hayes ⁷³	1966	Test the relationship between arithmetic self-concept & arithmetic achievement as indicated by the results of standardized achievement tests and teacher evaluation of	<u>Procedure:</u> (1) Teacher assigned arithmetic grades. (2) Achievement test. (3) Arithmetic self-concept test. (4) Social-status students. <u>Results:</u> Arithmetic self-concept is significantly and positively related to arithmetic achievement as indicated

72. R.B. Cattell, A.P. Sealy and A.B. Sweney, "What can personality and motivation source trait measurements add to the prediction of school achievement?" British Journal of Education Psychology. (Nov. 1966) Vol. XXXVI. p. 280-295.

73. E.J. Hayes, "Relationships between self-concept of arithmetic ability and arithmetic achievement in a selected group of sixth grade students." Dissertation Michigan State University. (1967)

TABLE II (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Hayes (Cont'd.)		arithmetic competency in the ongoing classroom milieu.	by both arithmetic achievement test results and teacher-assigned arithmetic grades.
Chanski ⁷⁴	1966	To determine the relationship between anxiety, intelligence and achievement in ninth grade algebra.	<p><u>Procedure:</u> The children's Manifest Anxiety Scale, the Primary Mental Abilities test were administered to 23 boys and girls. This was correlated with a teacher-made algebra test.</p> <p><u>Result:</u> Anxiety and achievement were negatively correlated in girls, and numerical aptitude and achievement were negatively correlated in boys but positively correlated in girls. Anxiety and aptitude influence achievement differently in boys and girls.</p>

74. N.M. Chanski, "Anxiety, Intelligence and Achievement in Algebra." The Journal of Educational Research. Vol. 60. No. 2. (Oct. 1966) p. 90 & 91.

TABLE II (Continued)

<u>AUTHOR</u>	<u>DATE</u>	<u>PURPOSE</u>	<u>PROCEDURE & RESULTS</u>
Johnson ⁷⁵	1966	<p>This study was designed to determine</p> <p>(1)The interaction of groups of variables involving "classroom procedures" and "student variables."</p> <p>(2)The unique contribution of each for mathematics achievement.</p> <p>"Classroom procedures", included homework, drill, tests, special assignments, and tutoring; the other "student variables" included student attitude toward mathematics, teachers' estimate of this attitude, and teachers' estimate of how well students would do on a mathematics achievement test.</p>	<p><u>Procedures:</u> Correlational procedures were used.</p> <p>The class was used as unit of analysis. They were divided into groups of high, middle and low intelligence on the basis of Raven scores, with I.Q. cutting points roughly at 117 and 100.</p> <p>The tests given were:</p> <p>(1)The Raven Progressive Matrices Test I.Q.</p> <p>(2)The STEP mathematics test.</p> <p>(3)A student attitude questionnaire.</p> <p>(4)Teacher's estimate of students' attitudes towards mathematics.</p> <p>(5)Teachers' predictions of students' achievements.</p> <p>(6)Inventory to assess pertinent classroom practices.</p> <p><u>Result:</u> Intelligence as measured by the Raven, student attitude, and homework proved to be the most closely related to mathematics achievement, usually in that order.</p>

75. S.A. Johnson, "Some selected classroom variables and their relationship to mathematics achievement in central Minnesota and the greater London area." Dissertation Abstract. (1966) p. 140A.

RELATION OF SELF-CONCEPT AND SELF-EXPECTATIONS FOR
MATHEMATIC ACHIEVEMENT.

In recent years a number of research projects have been conducted to explore what aspects of the student's affective make-up influence the learning process. Many of the current research projects reflect the growing interest of educational psychologists in the phenomenon of self-concept as an influence on behaviour, and, in particular, the behaviour underlying academic achievement in the role of a student.

James (1890) made us aware of the individual's social self. Mead (1934) further developed the idea of role theory. He felt that we are actors who assume roles which are the outcome of influences of social demands and normative forces upon individuals. Sargent (1951) feels that roles have ingredients of cultural, personal and situational determination. Role is never wholly cultural, wholly personal or wholly situational. He feels that the individual takes an active part in defining his situation and uses as referents the social positions of himself and others.

Sarbin (1954) conceptualizes a position as a system of role expectations and uses a self-and-role-in-inter-action thesis. He accepts the inter-action of role and self, but, in addition, he feels that there is a higher

level of complexity where there is structure within the individual and structure within the environment. How an individual perceives his role or his self-concept forms the basis for the implementation of his own expectations for his role enactment. Expressed in simple terms, this would include the ideas, perceptions and beliefs that form the image of himself that the individual has created.

Among the theorists who assign importance to the self-concept are Rogers⁷⁶, Snygg and Combs⁷⁷, and Lecky⁷⁸. They postulate that self-concept is the conscious core of an individual's behaviour that mediates behaviour by serving as an intermediary between perceptions and behaviour. For example, Lecky (1945), stresses preservation of the self-concept and relates this variable to school behaviour. He believes that the child who sees himself as weak, bad or stupid will behave in ways that will tend to perpetuate this picture, and will refuse to accept the idea that he is good, strong, or clever. Snygg and Combs indicate that the self-concept is a shorthand description of the innumerable discrete perceptions of self an individual possesses. Snygg and Combs also feel that the self-

76. C.R. Rogers, Client-centered Therapy. (Chicago: Houghton, Mifflin Co., 1951)

77. D. Snygg and H.W. Combs, Individual Behaviour. (New York: Harper and Row, Inc., 1959)

78. P. Lecky, Self-Consistency: A Theory of Personality. (New York: Island Press. 1945)

concept is a stable organization. They term this stability a "fixation" and submit two factors which contribute to it: (a) inertia of the organization itself which tends to resist disintegration, and (b) selection of perceptions which are meaningful to the individual and which have derived their meanings from the relation they bear to the phenomenal self already in existence. Snygg and Combs feel that the self-concept has therefore an important bearing on learning as it determines the capacity for intelligent behaviour and the development of abilities. Lecky (1945), who feels that the focal point of behaviour is consistency, expressed a similar concept:

"Any value entering this system (of organization of self evaluation) which is inconsistent with the individual's valuation of himself cannot be assimilated: it meets with resistance and is likely, unless the general reorganization occurs, to be rejected."⁷⁹

This would imply that self-concept would be of significant importance in academic achievement.

Lecky also tells us that the way an individual is treated by parents and by others who play a significant role in his life is an important determinant in the development of the self-concept since the introjection of the values of others plays a significant role in the unfolding of the self.

79. P. Lecky. p. 153.

Sarbin (1954) moreover, has also linked the self-concept to academic achievement:

Even such a prosaic, although important topic as the prediction of college achievement is subject to analysis by means of the concepts heretofore outlined. Included in the college student's role expectations are certain actions such as going to classes, organizing abstract material, using the library, etc., and certain qualities such as friendly, cooperative, good natured, etc. If the actions and qualities which comprise this role are congruent with the self-concept of a particular person, then there is a high probability that he will perform according to the role expectations of the professor and other members of the college community.⁸⁰

In a longitudinal study of self-concept of ability and academic achievement, Brookover and associates obtained a significant correlation coefficient between mathematic self-concept and mathematic grades in junior high school students: $r = .57$ for boys and girls.⁸¹

Hayes found in his study of sixth grade students that there was a significant positive correlation between arithmetic achievement test results and arithmetic self-concept: $r = .62$. There was also a significant positive correlation between teacher-assigned arithmetic grades and arithmetic self-concept: $r = .71$.⁸²

80. T.R. Sarbin. p. 250-251.

81. W.B. Brookover, A. Patterson and A. Thomas. Self-Concept of Ability and School Achievement. Cooperative Research Project.

82. Hayes. p. ii.

Binder, in her study of ninth and twelfth grade boys and girls, found the self-expectations and self-concepts of their academic abilities were significantly related to grade-point-averages. Moreover, SE and SCA correlated significantly with each other.⁸³

On the other hand, Jones, in his study of twelfth grade students, found that self-concept contributed significantly to the prediction of academic achievement of both boys and girls, but the SE variable did not contribute significantly in the female sample.⁸⁴

In view of this discussion, it seems relevant to explore the relationship between a limited aspect of the self-concept and theoretically relevant variables. In the present investigation, an attempt will be made to find out if the self-concept of academic ability and expectations in the role of a student increases the prediction for mathematical achievement. Since previous academic achievement and aptitudes are considered important, they will be included in the prediction scheme.

83. Binder. p. 123.

84. Jones. p. 53.

SUMMARY.

The intent of this chapter has been to review the literature on

- (a) The relation of cognitive variables for success in elementary mathematics.
- (b) The relation of psychological, sociological, and intellectual factors for success in mathematics.
- (c) The relation of self-concept and self-expectations for mathematic achievement.

There seems to be general agreement that mathematical achievement is positively related in varying degrees to cognitive variables.

Moreover, recent empirical studies have demonstrated the pertinency of self-concept to academic achievement.

This study is concerned both with cognitive variables and the non-cognitive variables of self-expectations in a student role and his self-concept of ability and their relations to academic achievement.

The design and methodology used to investigate this problem will be discussed in Chapter III.

CHAPTER III

DESIGN AND METHODOLOGY

The design and methods used in this study are described under six main headings: (1)Sample; (2)Operational definitions of key terms; (3)Selection of the variables; (4)Selection and description of the instruments used in this study; (5)Procedures for collecting data; (6)Treatment of data.

1. THE SAMPLE.

The ninth grade population of John F. Kennedy High School of the Montreal Catholic School Commission provided the subjects for this study. Test data was obtainable for 429 students.

The students in the study were grouped into two sections - general mathematics and algebra. The students were originally placed in these two groups in Grade 8 according to achievement or desire, by the principals of the school. The sample included 144 boys and girls in the general mathematics group and 285 boys and girls in the algebra group.

The ninth grade was chosen for this study because the algebra program terminates at this level. The student in Grade 9 writes the provincial examination in

algebra; success in this examination provides a credit for college admission. Hence, if a student decides to select algebra as one of his options, his objective is to pass the Grade 9 algebra provincial examination.

The students in this sample live in metropolitan Montreal.

2. OPERATIONAL DEFINITIONS OF KEY TERMS.

The operational definitions of the terms pertinent to this study are:

Mathematic Achievement -- At the Grade 9 level mathematic achievement in algebra or general mathematics was used as the dependent variable. An average of 50% or more in the provincial examination was required for inclusion in this study.

The Grade 7 mathematic achievement was the grade earned by each student in the June examination.

Self-expectations -- This concept was measured by the Self Expectation Inventory (SEI), which is composed of items relating to behaviour that are known to be relevant to student scholastic achievement. (Binder, 1965).

Self-Concept of Ability -- This variable was defined as the student's concept of his own academic ability (SC). It was measured by a scale composed of items relating to academic achievement. (Brookover, et al., 1962).

3. SELECTION OF THE VARIABLES.

The variables included in this study were chosen for practical as well as theoretical reasons: relationship of the dependent variable to the independent variables as indicated by related research, and the accessibility of data related to the specific variables.

Research cited in Chapter II suggests that there are several factors that appear to be related to mathematic achievement of junior high school students. The variables used in this study were: (a)Acquired skills i.e. Grade 7 achievement in mathematics. (b)Intelligence Quotients. (c)Scholastic Aptitudes. (d)Algebraic Aptitudes. (e)General self-concept of abilities. (f)Self-expectations in the role of a student. These variables were defined in Chapter I.

4. SELECTION AND DESCRIPTION OF THE INSTRUMENTS USED.

The dependent variable used in this study is defined in terms of whether the student successfully completed the ninth grade algebra, or whether he completed the general mathematics course on the first and second term average.

Using the distinction of algebra versus general mathematics, an attempt was made to determine whether or not the predictor variables differentiated between the two groups.

The independent or predictor variables are:

(a) Mathematic achievement scores in Grade VII arithmetic.

A review of the research indicates that past achievement is a valuable predictive measure.

In this study, the Grade 7 arithmetic results in June were used because a common examination was administered to all students in our commission.

Marks were obtained from the permanent school records which are housed in the respective elementary schools involved in this study.

(b) Intelligence Quotient rating by Otis. This was administered by the Guidance Counsellor in the fall season.

Alfred Yates felt that though the evidence concerning the validity of the test for predictive purposes is somewhat limited, nevertheless, it has value, "...if it is required, for example, to segregate pupils into relatively homogeneous groups for teaching, administrative, or research purposes, tests of this kind can afford a satisfactory, rough classification. They would be especially advantageous in circumstances in which little is known about the pupils' previous attainments or at the outset of a new type of course, when knowledge of previous attainment is not necessarily predictive of the likelihood of success."⁸⁵ The Otis I.Q. is also part of the regular

85. A. Yates, (Senior Research Officer, National Foundation for Educational Research, London, England) Fifth Mental Measurement Year Book. p. 499.

school testing program in our commission.

(c) Scholastic aptitude was obtained from the Academic Promise Tests. Four separate scores have been derived: Abstract Reasoning (AR), Numerical (N), Verbal (V), and Language Usage (LU). In addition, the first two scores (AR+N) are summed to provide a measure of nonverbal reasoning ability and the last two (V+LU) to afford a measure of competence with verbal materials. Finally, a total score is derived from all four parts. Turnball says, "these tests should prove to be a quick and economical way to identify talented students and those who need remedial help."⁸⁶ He further adds, "both reliability and validity coefficients are satisfactory, and the test scores should be helpful for pupil sectioning and placement."⁸⁷

(d) Algebraic Aptitude scores were obtained from the Portland Prognostic Tests for Mathematics. The validity for predicting success in first year algebra was .68. Hoyt considers this predictive validity coefficient substantial for prognostic tests.⁸⁸ The internal consistency reliability coefficient is .92. Hoyt also found that the predictive coefficient increased when previous teachers' evaluations were considered.⁸⁹

86. W.M. Turnball, (Executive Vice President Educational Testing Service, Princeton, New Jersey). Sixth Mental Measurement Year Book. p. 999.

87. Ibid. p. 1001.

88. C.J. Hoyt, (Professor of Educational Psychology, University of Minnesota, Minneapolis, Minnesota). Sixth Mental Measurement Year Book. p. 877.

89. Ibid. p. 878.

(e) General self-concept of ability. For this purpose an inventory constructed by Brookover and his research staff at Michigan State University was used. Brookover was concerned with the perceptions individuals had of their abilities to succeed in academic pursuits and the relation of these perceptions to the actual academic achievements of these individuals.⁹⁰ Brookover et al. (1962) obtained internal consistency reliability coefficients for the SC using Hoyt's analysis of variance. The obtained coefficients were .82 and .77 for males and females, respectively. Since a stability test of reliability was not done by Brookover, Binder (1965) chose to make this reliability test prior to employing the SC in her study.⁹¹ She did a test-retest stability study and obtained coefficients of .95 for males and .91 for females. There was a week interval between testing.

(f) The Self-Expectations in the role of a student was measured by an inventory constructed by Binder (1965). She obtained stability coefficients of .93 and .85 for twelfth-grade boys and girls, respectively. She used a cross-validation process to arrive at the final 39 items used in her instrument. Of these items, 26 were validated for boys of both the ninth and twelfth-grade levels, and

90. Brookover,

91. Binder, p. 89.

21 for girls of both grade levels. Eight items were common to both boys and girls. The boys' and girls' items were incorporated into one instrument and separate scoring keys were developed. (See Appendix)

Each of the subjects was asked to mark each item according to how strongly he felt the item applied or did not apply to him in the student role. The response to each item was made in terms of a four-point continuum ranging from, "Yes, I feel that this is what I expect of myself." to, "No, I feel that this is not what I expect of myself." Each item, responded to and scored in this way, was correlated with the GPA. The items which correlated significantly and positively with the GPA were considered positive items, and those which correlated significantly and negatively were considered negative items for the purposes of scoring the responses on the ET. For the positive statements, the number responses were scored as answered (i.e., 1=1, 2=2, etc.). For the negative items, the scoring system was reversed and the number responses 1, 2, 3, and 4 were represented by the score values 4, 3, 2, and 1 respectively. By summing all the items scored in this way, a total score was derived. Theoretically, the students obtaining the higher total scores on the ET should also present higher academic achievement indices. Conversely, the students with the

lower scores on the ET should present evidence of lower academic achievement as measured by the GPA.

5. PROCEDURES FOR COLLECTION OF DATA.

John F. Kennedy High School is housed in four separate buildings and has two separate administrative staffs. The three sections involved in this project were: (1) John F. Kennedy Girls' High School.

(2) John F. Kennedy Boys' High School.

(3) John F. Kennedy Girls' Annex.

In the fall all the Grade nine students were administered the Otis Intelligence Test, as part of the regular school testing program. This test was administered by the guidance counsellor of the school.-

The rest of the testing involved in this project was done in the spring. In each of the three schools, all the students were called together as a group in the cafeteria and given a general overview of the purpose of this testing and details concerning the tasks they were being asked to perform. Materials were distributed one at a time, and detailed instructions were given for the completion of each instrument as it was presented. For the self-concept and self-expectation inventories, no indication was made to the students as to what constituted a "correct" answer to questions contained in those instruments.

The requirement of individual effort was stressed at all administrations. The students were guaranteed anonymity and assured that all the data collected would be treated in a professional and confidential manner. They appeared to be favourably impressed on being given the opportunity of helping themselves and future students by sharing in this research.

The testing was carried out in two sessions on two successive mornings with a break of fifteen minutes for recess.

6. TREATMENT OF THE DATA.

Since the criterion in this study, whether the student is in the algebra or general mathematics group, is considered a dichotomious variable, Fisher's⁹² discriminant analysis was used. Fisher originally developed the discriminant function for ascertaining appropriate weights for a series of variables yielding maximum separation into two groups, each of which is assumed to be normally distributed. Excellent and complete reviews of the development and diverse applications of this statistic have been authored by Tatsuoka and Tiedeman⁹³,

92. R.A. Fisher, "The Use of Multiple Measurement in Taxonomic Problems." Annals of Eugenics. VII (1936) p. 179-188.

93. M. Tatsuoka and D.V. Tiedeman, "Discriminant Analysis"-Review of Educational Research. (1954) 24. p. 402-420.

Norton⁹⁴, Cramer and Bock⁹⁵, and Cooley and Lohnes⁹⁶.

For this study, the attempt was made by discriminant analysis to see whether the algebra group could be differentiated from the general mathematics group on the basis of the selected independent variables.

The discriminant function technique exhausts all linear information about group separation provided by the n-variable data and in turn transforms this information into the discriminant - function space.

The discriminant function also helps to determine the relative importance of each variable in a battery and, hence, identify those variables that are contributing importantly to the inter-group variation and those variables that are not.

A two-group discriminant analysis, involving the Otis I.Q. score, Academic Promise tests; two inventories namely the self-concept and self-expectations inventory; the grade 7 arithmetic score and the Portland Prognostic Mathematic Test were used.

The findings of this study indicated a definite sex difference. Hence, the sample was divided by sex.

94. D.W. Norton, "Developments in Analysis of Variance and Designs of Experiments." Review of Educational Research. (1963) 33. p. 490-500.

95. E.M. Cramer and R.D. Bock, "Multivariate Analysis." Review of Educational Research. (1966) 36. p. 604-617.

96. W. Cooley and P.R. Lohnes, Multivariate Procedures for the Behaviour Sciences. (New York: John Wiley and Sons, Inc., 1962)

To provide for a cross-validation study, some students were randomly pulled out before the finding of the discriminant function. The actual group membership status for each student was compared with the predicted group assignment, and the number of hits and misses was computed and compared.

SUMMARY.

Description of the sample source, and the description of the instruments used to collect the data in this study was presented. Procedures employed for data collection and statistical treatment of the data were described.

The presentation and analysis of the data are contained in Chapter IV. That chapter also includes the investigator's interpretation of the results of the statistical analysis of the data.

CHAPTER IV

ANALYSIS OF THE RESEARCH DATA.

Chapter IV is a report on the analysis of the research data based upon the methodological and statistical procedures outlined in Chapter III. The primary purpose of the present study was to find out if students taking algebra could be differentiated from general mathematics students. A secondary problem which is related to the first, is: what are the existing factors responsible for scholastic success in algebra and general mathematics? The third phase involves a cross-validation sample. How well does the discriminant space classify students as belonging to algebra or general mathematic groups?

The statistical data for the discriminant function analysis was computed from canned programs developed by Cooley and Lohnes. The direct factoring of $W^{-1}A$ and the internal computation of correlations between original variables and discriminant functions was done by Veldman. The data was coded on I.B.M. cards prior to computer analysis.

FINDINGS

Question I.

The first question was answered by a simultaneous comparison of the differences of variable mean values between the two groups. (Wilk's Lambda.) The present investigation yielded an index of discrimination. This index, tested for significance, yielded a highly significant F ratio ($F = 28.232$; df 9. and 91.; $p = 0.000$). This indicated that there were significant differences between the centroids of the variables for the general mathematic and algebra groups.

Question II.

The principle advantage of the use of the discriminant function technique is that it ascertains what variables account for the major share of discrimination between the two groups.

The identification of the variables, the mean scores, and a one-way analysis of variance between the groups on each variable for the males are shown in Table III; and for the females in Table IV. The differences between the algebra and general mathematics groups for both males and females is quite significant in the one-way analyses of variance.

It is apparent from Tables III and IV that in

TABLE III

SUMMARY OF THE DATA FOR MALES.

Numerical Variable	Symbol	Means General Math	Means Algebra	F Ratio One-Way Analysis of Variance	p.	Correlations Between Original Variable and Discriminant Function
1. Self-Expectations.	X ₁	41.1	45.9	4.38	0.04	0.24
2. Self-Concept.	X ₂	24.3	28.6	25.50	0.00	0.53
3. Otis I.Q.	X ₃	94.95	111.95	56.73	0.00	0.70
4. Abstract Reasoning.	X ₄	41.15	67.85	23.12	0.00	0.51
5. Numerical Reasoning.	X ₅	51.75	86.32	119.80	0.00	0.86
6. Verbal Reasoning.	X ₆	29.75	63.96	48.57	0.00	0.67
7. Language Usage.	X ₇	31.65	71.00	81.74	0.00	0.78
8. Grade 7 Math Mark.	X ₈	56.7	86.64	198.63	0.00	0.95
9. Portland Prognostic Algebra Test.	X ₉	71.9	94.06	68.07	0.00	0.74

TABLE IV

SUMMARY OF THE DATA FOR FEMALES.

Numerical Variable	Symbol	Means General Math	Means Algebra	F Ratio One-Way Analysis of Variance	p.	Correlations Between Original Variable and Discriminant Function
1. Self-Expectations.	X ₁	45.16	47.99	5.74	0.02	0.19
2. Self-Concept.	X ₂	23.27	27.22	70.16	0.00	0.60
3. Otis I.Q.	X ₃	92.97	102.48	75.20	0.00	0.62
4. Abstract Reasoning.	X ₄	33.03	51.05	34.20	0.00	0.44
5. Numerical Reasoning.	X ₅	38.36	67.56	142.31	0.00	0.78
6. Verbal Reasoning.	X ₆	23.87	45.42	73.44	0.00	0.61
7. Language Usage.	X ₇	41.82	67.38	97.26	0.00	0.68
8. Grade 7 Math Mark.	X ₈	61.15	83.43	209.75	0.00	0.88
9. Portland Prognostic Algebra Test.	X ₉	47.96	80.89	193.62	0.00	0.86

both the male and female sample, the Grade 7 mathematics mark accounts for the greatest difference between the algebra and general mathematics groups; and the four highest variables for both groups are Grade 7 mathematics marks, Portland Prognostic Test, Numerical Reasoning and Language Usage Test. Conversely, the Self-Expectation score accounted for the smallest difference between the general mathematics and algebra groups.

In both the male and female sample, the algebra group is maximally separated from the general mathematics group in the sense that the ratio of the between-groups variance to the within-groups variance is maximized.

The general form of the discriminant function was:

$$V = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5$$

where the x's are the measurements of the different characteristics and the a's are the weights which will produce the maximum distinction between the two groups of students. Substitution of the appropriate values yield, upon solution, the discriminant function:

The discriminant function for males is:

$$V = +0.1679X_1 - 0.4839X_2 - 0.0585X_3 - 0.0407X_4 + 0.2705X_5 \\ + 0.0623X_6 + 0.0890X_7 + 0.7866X_8 + 0.1700X_9.$$

The discriminant function for females is:

$$V = -0.1584X_1 + 0.7841X_2 - 0.1080X_3 - 0.0475X_4 + 0.0528X_5 \\ + 0.0424X_6 + 0.1230X_7 + 0.5137X_8 + 0.2501X_9.$$

- X_1 = the best estimate of a student's score on Self-Expectations.
- X_2 = the student's score on Self-Concept.
- X_3 = the student's intelligence quotient determined by the Otis Test of Mental Ability.
- X_4 = the student's score on Abstract Reasoning.
- X_5 = the student's score on Numerical Reasoning.
- X_6 = the student's score on Verbal Reasoning.
- X_7 = the student's score on Language Usage.
- X_8 = the student's final Grade 7 mathematics mark.
- X_9 = the student's score on the Portland Prognostic Algebra Test.

In Tables V and VI, the discriminant weights for each variable for males and females are respectively shown.

The discriminant function and the correlation between groups shows that the Grade 7 mathematic mark accounts for the greatest difference between the general mathematics and algebra groups, since the correlation for males is 0.95 and the discriminant weight is 0.79. The correlation and discriminant weight for females is 0.88 and 0.51 respectively. The second highest variable that accounts for the difference between the general mathematic and algebra groups is the self-concept score --- for males the correlation is 0.53 but the discriminant weight is 0.48. It appears to be greater for females as the

TABLE VCONTRIBUTIONS OF EACH VARIABLE TO THE TOTAL
DISCRIMINATING POWER OF THE BATTERY FOR MALES.

Battery Number	Variable	Discriminant Score Weight
1. Acquired Skill.	Grade 7 mathematic mark.	0.7866
2. Aptitudes.	a. Otis (I.Q.)	0.0585
	b. Academic Promise Tests.	
	(i) Abstract Reasoning.	0.0407
	(ii) Numerical Ability.	0.2705
	(iii) Verbal Reasoning.	0.0623
	(iv) Language Usage.	0.0890
	c. Portland Prognostic Test for Mathematics.	0.1700
3. Personal Information.	a. Self-Concept of Ability Scale.	0.4839
	b. Self-Expectations Inventory.	0.1679

TABLE VICONTRIBUTIONS OF EACH VARIABLE TO THE TOTAL
DISCRIMINATING POWER OF THE BATTERY FOR FEMALES.

Battery Number	Variable	Discriminant Score Weight
1. Acquired Skill.	Grade 7 mathematic mark.	0.5137
2. Aptitudes.	a. Otis (I.Q.)	0.1080
	b. Academic Promise Tests.	
	(i) Abstract Reasoning.	0.0475
	(ii) Numerical Ability.	0.0528
	(iii) Verbal Reasoning.	0.0424
	(iv) Language Usage.	0.1230
	c. Portland Prognostic Test for Mathematics.	0.2501
3. Personal Information.	a. Self-Concept of Ability Scale.	0.7841
	b. Self-Expectations Inventory.	0.1584

correlation between the self-concept score and the discriminant function is .60 but the discriminant weight is 0.78.

Question III.

The validation phase of this study involves the prediction of group membership of a random sample of students from known groups of algebra-general mathematics students in an effort to determine the usefulness of the computed discriminant equation. Ten percent of the original sample of 116 male students and 313 female students was randomly selected for purposes of cross-validating the discriminant equations computed in the construction phase of the study. This was done before the discriminant function was computed. A discriminant score for each student of the validation sample was computed by the mathematical substitution of his raw scores into the two predetermined discriminant equations according to sex. Each student's discriminant score defined a point in two-dimensional space which was then compared with the original algebra and general mathematics centroids for the male and female samples. This comparison resulted in two probability statements regarding resemblance in a group for all individuals on the validation sample. That is whether the individual resembled the successful student in general mathematics or the successful student in algebra.

TABLE VII

ACTUAL VS. PREDICTED GROUP: CROSS VALIDATION SAMPLE.
TOTAL SAMPLE VARIABLES FOR MALES.

Predicted Group Identity	Hits	Misses	Total	% Hits
General Math	5		5	100%
Algebra	8	2	10	80%
TOTAL	13	2	15	90%

TABLE VIII

ACTUAL VS. PREDICTED GROUP: CROSS VALIDATION SAMPLE.
TOTAL SAMPLE VARIABLES FOR FEMALES.

Predicted Group Identity	Hits	Misses	Total	% Hits
General Math	11	1	12	91%
Algebra	17	2	19	89%
TOTAL	28	3	31	90%

A measure of the stability of the discriminating power of the computed discriminant equations can be noted in Table VII for males and in Table VIII for females. These tables present the results obtained from classifying the validating sample according to the construction sample. It was found that for the male general mathematic group the hits were 100% whereas, in the female sample, the accuracy of prediction was 91%. For the algebra group the hits were 80% for males and 89% for females.

In essence, then, the predictive effectiveness achieved by the total sample battery of tests appears to be quite satisfactory.

SUMMARY.

The data concerning the three primary questions were analyzed in terms of four subgroups: (a) algebra boys; (b) general mathematic boys; (c) algebra girls; and (d) general mathematic girls. Analysis was made by means of the discriminant analysis technique. The first question concerned the validity of differentiating between the algebra and general mathematic groups for both males and females. The results of the study support the hypothesis that the students in general mathematics and algebra have skills, measurably different aptitudes, and self-concept.

The second question was to find out what were the underlying variables that discriminated between the two groups. In both the male and female sample, the Grade 7 mathematic mark and the self-concept score were the two highest variables, and the self-expectations was the lowest.

The third question concerned itself with validation. It was found that the total number of hits in the male and female sample was 90%.

The results of the analysis of the data pertinent to these three questions, as presented, have led to the formulation of several conclusions. These conclusions and their implications are presented in the following chapter.

CHAPTER V

SUMMARY, FINDINGS AND CONCLUSIONS:

DISCUSSION AND RECOMMENDATIONS.

The final chapter of this study is organized into five main divisions: 1)Restatement of the Problem; 2)Review of Design and Procedures; 3)Summary of Findings; 4)Conclusions; 5)Discussion and Recommendations.

PROBLEM.

The purpose of the present study was to find out if algebra students could be differentiated from general mathematics students. Subordinate purposes were: 1)to identify the existing psychological or intellectual factors responsible for scholastic success in algebra and general mathematics, with special interest in the self-concept and self-expectations of the individual; and 2) to provide information which will assist the student in choosing the mathematics program best suited to himself.

REVIEW OF DESIGN AND PROCEDURES.

The subjects in this study were 116 male and 313 female ninth grade students. All the students attend John F. Kennedy High School in Montreal, Quebec. There

was a total of 91 students in the algebra male group and a total of 25 students in the general mathematic male group. There was a total of 119 students in the general mathematics female group and a total of 194 females in the algebra group.

The dependent variable or criterion used in this study was successful completion of the ninth grade provincial algebra examination, or the general mathematics final course examination. The basis upon which discrimination was attempted was the predictor variables which focused around acquired skill, aptitudes and personal information. The Grade 7 mathematics mark measured acquired skill, and aptitudes were measured by the Academic Promise Tests, the Portland Prognostic Test for Mathematics and the Otis I.Q. Personal information consisted of two non-cognitive variables - the self-concept of ability and the student's expectations of himself.

A discriminant analysis involving these nine predictor variables was used to assess the differences between the groups.

Wilk's Lambda was computed. This measured the discrimination of the two groups.

Wilk's Lambda for males = 0.26

Wilk's Lambda for females = 0.45.

This was tested for significance and was followed by a chi-square test for the discriminant function.

After obtaining parameters and the data-card format from the CCDS Subroutine, the C and W matrices were zeroed, since they held raw and deviation cross-products accumulated over groups. As each group of subjects was input from data cards punched in the usual manner with scores for variables across each subject's card, the sum of scores on each variable was stored in matrix S and a within-group matrix was computed and added to W. Raw cross-products were added to matrix C. Raw sums of squares were saved in Q for the later univariate analyses, and the covariance, among and within-group matrices were computed. $W^{-1}A$ was then computed and submitted to subroutine AEVS. Discriminant score weights were computed and output. This was followed by the computation of correlations between original variables and discriminant functions.

Group centroids were then computed and univariate analyses of variance were computed for each of the original variables for comparison purposes. Finally, discriminant scores were computed for each subject.

In order to test the stability of the discriminant function, a cross-validation step was performed.

SUMMARY OF FINDINGS.

Hypothesis I stated that there were no differences between the algebra group and the general mathematics group for either boys or girls.

Result: Hypothesis rejected.

The findings indicated that there were significant differences between the two groups for each sex. There was a highly significant F ratio for males ($F = 28.232$; $df\ 9\ and\ 91$; $p = 0.000$) and for females ($F = 36.981$; $df\ 9\ and\ 272$; $p = 0.000$). The results of the univariate analyses indicate that all variables considered for both sexes separate the groups adequately by themselves.

The variables listed in order of their significance for males on the univariate F tests were Grade 7 mathematics, Numerical Reasoning, Language Usage, Portland Prognostic Algebra Test, Otis I.Q., Verbal Reasoning, Self-Concept, Abstract Reasoning and Self-Expectations.

The order of significance on the univariate F - tests for females - was Grade 7 mathematics, Portland Prognostic Algebra Test, Numerical Reasoning, Language Usage, Otis I.Q., Verbal Reasoning, Self-Concept, Abstract Reasoning and Self-Expectations.

The contributions of the nine variables to the composite discriminant function, according to heirarchial order, were the Grade 7 mathematics marks, the self-concept, Numerical Ability and the Portland Prognostic Test. In the female sample the order of contribution was very similar to that of the male sample, because the weights, according to heirarchial order, were the self-concept,

Grade 7 mathematic mark, Portland Prognostic Test for Mathematics and the student's self-expectations.

On the basis of this information, a discriminant function was calculated. The discriminant equation for males was:

$$V = +0.1679X_1 - 0.4839X_2 - 0.0585X_3 - 0.0407X_4 + 0.2705X_5 \\ + 0.0623X_6 + 0.0890X_7 + 0.7866X_8 + 0.1700X_9.$$

The discriminant equation for females was:

$$V = -0.1584X_1 + 0.7841X_2 - 0.1080X_3 - 0.0475X_4 + 0.0528X_5 \\ + 0.0424X_6 + 0.1230X_7 + 0.5137X_8 + 0.2501X_9.$$

It would appear that the self-concept contributes a great deal to the composite discriminant function as it has the highest single weight for females and the second highest single weight for males.

- X_1 = The best estimate of a student's score on Self-Expectations.
- X_2 = The student's score on Self-Concept.
- X_3 = The student's intelligence quotient determined by the Otis Test of Mental Ability.
- X_4 = The student's score on Abstract Reasoning.
- X_5 = The student's score on Numerical Reasoning.
- X_6 = The student's score on Verbal Reasoning.
- X_7 = The student's score on Language Usage.
- X_8 = The student's final Grade 7 mathematics mark.
- X_9 = The student's score on the Portland Prognostic Algebra Test.

CONCLUSIONS.

The literature related to the present study suggested that there was a significant relationship between cognitive variables and academic achievement. Past research has also suggested that non-cognitive variables such as academic self-concept and self-expectations influence academic achievement.

(1) The results of the study appear to lend empirical support to the theory that a relationship exists between self-concept and mathematics achievement. One's self-expectations, however, did not contribute much to mathematic achievement. Perhaps this was due to the fact that the student population involved was an upwardly mobile immigrant group which apparently fosters high self-expectations.

(2) This research also supports other studies which have shown that Grade 7 achievement marks and the Portland Prognostic Algebra Test account for many of the differences between the algebra and general mathematic groups. This was expected as it bears directly on the problem at hand.

(3) The self-concept alone is far from the best group separator, but when it is included with other predictor variables, it has the highest weighting for females and the second highest weighting for males. This implies

that self-concept does contribute to mathematic achievement.

(4) The Numerical Ability and Language sub-scores of the Academic Promise Tests contribute to the differences between the two groups in univariate analysis and in the composite discriminant function.

(5) The intelligence quotient does not contribute significantly to the discriminant function although it does discriminate between the two groups with a significant univariate F - test. This appears to suggest that the contributions which an intelligence quotient makes to the discriminant function has already been accounted for by other achievement variables.

DISCUSSION AND RECOMMENDATIONS.

It appears that the use of a few discriminant variables of an intellectual nature and the use of the self-concept permit meaningful comparisons which indicate whether the student resembles the algebra or general mathematic group. Fisher proposed the discriminant function as a solution to the problem of using information from a number of correlated variables to classify an unclassified object into one of two groups to which it must belong. It takes into consideration the combined effects of a number of variables.

Since there is an overlapping component in this scheme, the students who fall within this marginal area should be made aware of this. One possible solution would be to make special provision for such students to complete the algebra program in three years instead of the usual two years. Hopefully, this would build up the student's confidence and his self-concept in mathematics.

Since this study was conducted in a rather unique community which was composed chiefly of recent immigrants from Italy, there are limitations intrinsic to its findings. No effort was made to establish the sample as a representative one of the general population. Hence it should be replicated in other areas and schools to determine what discriminant formula would be applicable to other populations. The findings of the study should be updated each year due to demographic changes within the school population.

This technique makes the student aware of the group he is "most like", and this results in wiser decision-making. Knowledge of group membership should give the hesitant but potential algebra student added reassurance. On the other hand, it should provide a "break" for those students who "best fit" the general mathematics program but who may be rushing headlong into something fruitless by taking the algebra course. It must be

remembered though that this prediction scheme does not take into account other important achievement variables such as study habits, personality characteristics, and probable measurement error.

This formula should not be used alone but it should be used in conjunction with other factors including teacher estimate of the student's ability, educational objectives, competitive spirit, workload and emotional maturity.

APPENDIX ATHE SELF-CONCEPT OF ABILITY,
AND SCORING INSTRUCTIONS

The Self-Concept of Ability Scale.

The scale is scored in the following manner:

- (1) The answer to each question is
converted as follows:

1 - 4

2 - 3

3 - 2

4 - 1

- (2) The inverted scores are totalled, and
this sum is the S's score on the scale.

INTRODUCTION: This is part of a study to find out what high school students think about themselves and their school work. You can help us to better understand people your age by answering the following questions as honestly as you can.

The questions are of the multiple-choice type with which you are familiar. Please read carefully the directions before you answer. If you have any questions, please raise your hand and someone will help you.

DIRECTIONS: Below are eight questions. Circle the letter in front of the statement which best answers each question.
Make sure that you answer every question.

1. Where do you think you would rank in your class in high school?
 1. among the best
 2. above average
 3. average
 4. below average
 5. among the poorest

2. In order to become a doctor, lawyer, or university professor, work beyond four years of college is necessary. How likely do you think it is that you could complete such advanced work?
 1. very likely
 2. somewhat likely
 3. not sure either way
 4. unlikely
 5. most unlikely

3. Do you think you have the ability to complete college?
 1. yes, definitely
 2. yes, probably
 3. not sure either way
 4. probably not
 5. no

4. How do you rate yourself in school ability compared with those in your class at school?
 1. I am among the best
 2. I am above average
 3. I am average
 4. I am below average
 5. I am among the poorest
5. Forget for a moment how others grade your work. In your own opinion how good do you think your work is?
 1. my work is excellent
 2. my work is good
 3. my work is average
 4. my work is below average
 5. my work is much below average
6. What kind of grades do you think you are capable of getting?
 1. mostly A's
 2. mostly B's
 3. mostly C's
 4. mostly D's
 5. mostly F's
7. Where do you think you would rank in your class in college?
 1. among the best
 2. above average
 3. average
 4. below average
 5. among the poorest

8. How do you rate yourself in school ability compared with your close friends?

1. I am the best
2. I am above average
3. I am average
4. I am below average
5. I am the poorest

SELF EXPECTATIONS INVENTORY

NAME _____ DATE _____

M _____ F _____ GRADE _____ SCHOOL _____

DIRECTIONS: This instrument is part of a study being carried out in high school to learn more about high school students. Listed below are 39 statements. Go through them quickly, without spending too much time on any single one. Answer the statements in order, without skipping. Feel free to answer exactly how you feel for the school will not see your answers.

This is not a test. There are no right or wrong answers. As you read the items, consider each one carefully and then mark each statement in the left margin according to how strongly you feel that it applies to you, or does not apply to you. Please mark every one. Write in the number (1, 2, 3, or 4) to stand for the following answers:

- 1.- Yes, I feel that this is what I expect of myself.
 - 2.- Yes, I feel that this is probably what I expect of myself.
 - 3.- No, I feel that this is probably not what I expect of myself.
 - 4.- No, I feel that this is not what I expect of myself.
-

AS A STUDENT, I EXPECT MYSELF TO:

- _____ 1. do good work even in classes I don't like.
- _____ 2. ask my teachers for help with my schoolwork.
- _____ 3. spend as much time as possible with my boy-friend (girlfriend).
- _____ 4. attend school regularly.
- _____ 5. be active in organizing student activities.
- _____ 6. get as much out of my classes as possible.
- _____ 7. check and recheck my homework before turning it in.
- _____ 8. listen carefully to class discussions.
- _____ 9. do homework even if not interested in it.
- _____ 10. behave as my teachers expect me to behave.
- _____ 11. do my schoolwork ahead of time.
- _____ 12. do all the schoolwork assigned.
- _____ 13. know all about the newest and most popular records.
- _____ 14. want good grades.
- _____ 15. do the best I can in school.
- _____ 16. keep up with the latest in teen-age fads.
- _____ 17. attend non-athletic functions at school.
- _____ 18. obey school rules only when I am watched.
- _____ 19. work hard even in classes I don't like.
- _____ 20. study hard so as to be able to get into college.
- _____ 21. do more than the minimum of schoolwork expected of me.
- _____ 22. set the pace in the latest style.

- _____23. spend much of my out-of-school time studying.
- _____24. hand my homework in on time.
- _____25. really keep busy with my schoolwork.
- _____26. hand in near perfect homework.
- _____27. put forth my best efforts at all times.
- _____28. do the work necessary to "get by".
- _____29. work hard.
- _____30. do my schoolwork before becoming involved with other activities.
- _____31. take school seriously.
- _____32. be hard to get to know.
- _____33. take advantage of any educational opportunity offered to me.
- _____34. pay attention in class.
- _____35. work hard for what I get.
- _____36. do my schoolwork independently.
- _____37. work well with kids in my classes.
- _____38. consider good grades important.
- _____39. stick with a problem until it is solved.

APPENDIX B

THE SELF-EXPECTATION INVENTORY
AND SCORING INSTRUCTIONS

The Self-Expectation Inventory

The scale is scored in the following manner:

- (1) The scale is scored differently according to the sex of the S.
- (2) For the males, the following questions are scored:

1-	14-
2-	15-
3+	16+
4-	17-
5-	18-
6-	19-
7-	20-
8-	21-
9-	22+
10-	23-
11-	24-
12-	25-
13+	26-

- (3) The answers to questions which have a (-) following them are inverted as

follows:

1	-	4
2	-	3
3	-	2
4	-	1

The answers to questions with (+) following them are not inverted.

- (4) The converted scores and the (+) scores are totaled, and this sum becomes the S's score.

(5) For the females, the rules for converting (-) questions and not converting (+) scores are identical with those for the males.

(6) The questions scored for females are:

19-	26-	33-
20-	27-	34-
21-	28+	35-
22+	29-	36-
23-	30-	37-
24-	31-	38-
25-	32+	39-

(7) The converted scores and (+) scores are summed, and this total becomes the S's score.

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