

EFFECT OF <sup>3</sup>ANGLE OF INCLINATION AND OF CROSSFLOW  
ON FLOW, HEAT AND MASS TRANSFER  
FOR A LAMINAR IMPINGING SLOT JET

by



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# ABSTRACT

The influence of angle of impingement and of presence of a crossflow on the flow, heat and mass transfer of a laminar semi-confined slot jet impinging on an isothermal wet surface was studied numerically. The digital computer simulation model was obtained by solving the governing Navier Stokes equations in their primitive form (velocity and pressure) together with the continuity, energy and species equations using a hybrid upwind and central finite difference scheme based on the control volume integration approach. Fluid property variations with both temperature and composition were taken into account.

The parameters studied include nozzle-to-plate spacing, effect of the presence of a crossflow, ratio of the Reynolds numbers of the crossflow and the jet flow and angle of impingement.

The presence of a crossflow was found to reduce the stagnation point and average heat and mass transfer rates below what would be expected if crossflow were not present. For moderate crossflow magnitude (with respect to the jet) the heat and mass transfer rates on the plate are those of an impinging jet weakened by the crossflow, while for large values of the crossflow magnitude they are those of channel flows along a plate. Impingement exists in the former case but not in the latter. For a given crossflow magnitude and a given nozzle-to-plate spacing there exists an optimum angle (directed slightly against the crossflow direction) for which the heat and mass transfer rates are maximum.

### RÉSUMÉ

On a étudié numériquement l'influence de l'angle d'inclinaison et de la présence d'un vent transversal sur les caractéristiques de l'écoulement et des transferts de chaleur et de masse d'un jet d'air laminaire semi-confiné issu d'une buse de section rectangulaire et impactant sur une surface humide maintenue à température constante. La simulation du problème sur l'ordinateur fut obtenue en résolvant simultanément les équations de Navier Stokes sous leur forme primitive (vitesse et pression) et les équations de conservation de la masse, de l'énergie et des espèces, tout en tenant compte des modifications des propriétés physiques du fluide dues aux variations de sa température et de sa composition (fraction de vapeur d'eau).

L'étude a porté sur les paramètres suivants: espacement entre la sortie de la buse et la plaque d'impact, effet de la présence d'un vent transversal, rapport des nombres de Reynolds respectifs de l'écoulement transversal et du jet, et angle d'inclinaison de la buse.

Pour les nombreux cas étudiés on a analysé les champs de vitesse et la distribution de la pression, du cisaillement, du transfert de chaleur (nombre de Nusselt) et du transfert de masse (nombre de Sherwood) le long de la plaque.

Les resultats obtenus confirment que la presence d'un vent transversal réduit les taux locaux et moyens de transfert de masse et de chaleur du jet impactant. Pour de faibles écoulements transversaux, les coefficients de transfert sur la plaque sont ceux d'un jet impactant affaibli et dévié; pour de forts écoulements transversaux on obtient par contre des coefficients de transfert du type de ceux que produit un écoulement d'air parallèle à une plaque. Le jet impacte effectivement sur la plaque dans le premier cas, alors qu'il est emporté par le vent transversal et n'impacte pas dans le second.

L'étude de l'influence de l'inclinaison de la buse a montré que pour un écartement donné de cette buse vis-a-vis de la plaque et pour un vent transversal donné, il existe un angle d'inclinaison optimum pour lequel les coefficients de transfert de masse et de chaleur sont maximums.

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# NOMENCLATURE

|                      |                                                                                                                                                                                                           |
|----------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| A                    | control volume cross sectional area                                                                                                                                                                       |
| $a_e, a_w, a_n, a_s$ | see Equation (4.10)                                                                                                                                                                                       |
| $A_p$                | see Equation (4.19)                                                                                                                                                                                       |
| $C_e, C_w, C_n, C_s$ | convective flux, $\rho u A$ ; see Equation (4.8)                                                                                                                                                          |
| $C_{f,}$             | skin friction coefficient, $\tau / \frac{1}{2} \rho_j U^2$                                                                                                                                                |
| $C_p$                | specific heat capacity of fluid mixture at constant pressure, J/kg. $^{\circ}$ K                                                                                                                          |
| D                    | jet nozzle width, m                                                                                                                                                                                       |
| $D_e, D_w, D_n, D_s$ | diffusive flux, $\Gamma A / \Delta x$ ; see Equation (4.9)                                                                                                                                                |
| $DU_p, DV_p$         | gradient of velocity w.r.t. pressure change from iteration to iteration; see Equations (4.21) and (4.22)                                                                                                  |
| F                    | flux at control volume boundaries; see Equations (4.1) to (4.5)                                                                                                                                           |
| FACTOR               | residual normalization factor                                                                                                                                                                             |
| g                    | gravitational acceleration, 9.807 m/s $^2$                                                                                                                                                                |
| h                    | heat transfer coefficient, $Q / (T_j - T_p)$ , W/m $^2$ . $^{\circ}$ K                                                                                                                                    |
| H                    | nozzle-to-impingement-plate spacing, m                                                                                                                                                                    |
| $J_i$                | molar flux of i relative to mass average velocity, mole/m $^2$ .s                                                                                                                                         |
| $j_i$                | mass flux of i relative to mass average velocity, kg/m $^2$ .s                                                                                                                                            |
| k                    | mass transfer coefficient, $W(1-m_p) / (m_j - m_p)$ , kg/m $^2$ .s                                                                                                                                        |
| l                    | space coordinate, see Figure 1.1                                                                                                                                                                          |
| L                    | nozzle-to-plate spacing measured on inclined jet axis, $H / \cos \theta$ , see Figure 2.1                                                                                                                 |
| m                    | mass fraction of water vapour, mass of water vapour / total mass of mixture                                                                                                                               |
| M                    | ratio of crossflow volumetric flow rate to jet volumetric flow rate; also equal to the ratio of the corresponding mass flow rates, and to the ratio of the Reynolds numbers of the crossflow and the jet. |

|                 |                                                                                                                                               |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| $m_s$           | mass fraction of water vapour at jet nozzle exit.                                                                                             |
| $m_p$           | mass fraction of water vapour along impingement plate.                                                                                        |
| $MW$            | molecular weight                                                                                                                              |
| $\overline{MW}$ | molecular weight of fluid mixture; $1/\overline{MW} = M_A/MW_A + M_B/MW_B$ , kg/kilomole                                                      |
| $n_A, n_B$      | mass fluxes of species A and B respectively along impingement plate, kg./m <sup>2</sup> .s                                                    |
| $NI, NJ$        | size of grid mesh in x- and y-direction respectively.                                                                                         |
| $Nu$            | Nusselt number, $hD/\lambda_j$                                                                                                                |
| $P$             | pressure, N/m <sup>2</sup>                                                                                                                    |
| $P_{sat}$       | saturated vapour pressure of water vapour, N/m <sup>2</sup>                                                                                   |
| $\Delta P$      | difference between the local and ambient pressure.<br>In Chapter 4: pressure correction term from iteration to iteration, $P = P' + \Delta P$ |
| $Pe$            | local Peclet number $\rho u \Delta x / \Gamma$                                                                                                |
| $Pr$            | Prandtl number, $C_p \mu / \lambda$                                                                                                           |
| $Q$             | heat flux along impingement plate in positive x-direction, W/m <sup>2</sup>                                                                   |
| $q$             | energy flux relative to mass average velocity, J/m <sup>2</sup> .s                                                                            |
| $R$             | half width of nozzle, $D/2$ , m                                                                                                               |
| $r$             | space coordinate; see Figure 1.1                                                                                                              |
| $Re$            | Reynolds number, $\rho_j U D / \mu_j$                                                                                                         |
| $RESID$         | normalized residual; see Equation (5.6)                                                                                                       |
| $S_\phi$        | source term                                                                                                                                   |
| $S_{\phi, vol}$ | integral source term; see Table 4.1                                                                                                           |
| $S_{false}$     | false source term; see Equation (4.19)                                                                                                        |
| $Sc$            | Schmidt number, $\mu / (\rho \delta)$                                                                                                         |
| $Sh$            | Sherwood number, $kD / \delta_j \rho_j$                                                                                                       |

|                                |                                                                                                 |
|--------------------------------|-------------------------------------------------------------------------------------------------|
| St                             | Stanton number, $h/(\rho_j c_{p_j} \bar{U})$                                                    |
| Su                             | component of linearized source term; see Equation (4.20)                                        |
| T                              | thermodynamic temperature, °K                                                                   |
| U                              | velocity vector                                                                                 |
| $\bar{U}$                      | average velocity at the exit of the jet nozzle, m/s                                             |
| u                              | velocity component in the x-direction, m/s                                                      |
| $u_{suc}$                      | suction velocity along impingement plate in positive x-direction, m/s                           |
| URF                            | under-relaxation factor; see Section 5.5.1                                                      |
| $\bar{V}$                      | average velocity at entry of crossflow, m/s                                                     |
| v                              | velocity component in the y-direction                                                           |
| V                              | volume of control volume, 1.0 uv                                                                |
| W                              | mass flux of water vapour along impingement plate in positive x-direction, kg/m <sup>2</sup> .s |
| $x_A$                          | molar concentration of water vapour, mole/m <sup>2</sup> .s                                     |
| x                              | space coordinate, m; see Figure 1.1                                                             |
| y                              | space coordinate, m; see Figure 1.1                                                             |
| $y_0$                          | coordinate of jet centerline on exit plane.                                                     |
| $\Delta x, \Delta y, \Delta l$ | size of control volume; see Figure 4.1                                                          |
| $y_R$                          | half width of nozzle aperture, $R/\cos\theta$                                                   |

#### Greek letters

|           |                                                      |
|-----------|------------------------------------------------------|
| $\Gamma$  | exchange coefficient; see Table 3.1                  |
| $\delta$  | mass diffusivity of fluid mixture, m <sup>2</sup> /s |
| $\lambda$ | thermal conductivity of fluid mixture, W/m.°K        |

|             |                                                                                           |
|-------------|-------------------------------------------------------------------------------------------|
| $\mu$       | dynamic viscosity of fluid mixture, kg/m.s                                                |
| $\rho$      | mass density of fluid mixture, kg/m <sup>3</sup>                                          |
| $\tau$      | shear stress along impingement plate, $\mu \partial v / \partial y _p$ , N/m <sup>2</sup> |
| $\phi$      | variable; see Table 3.1                                                                   |
| $\phi_{AB}$ | weighting factor for fluid properties                                                     |
| $\theta$    | angle of impingement (inclination of nozzle)                                              |

## Subscripts

|         |                                                                                            |
|---------|--------------------------------------------------------------------------------------------|
| A       | water vapour                                                                               |
| B       | dry air                                                                                    |
| j       | at nozzle exit                                                                             |
| p       | along impingement plate                                                                    |
| e,w,n,s | east, west, north and south side boundaries of control volume; see Figures (4.1) and (4.2) |
| E,W,N,S | see Figures (4.1) and (4.2)                                                                |
| P       | center of control volume                                                                   |

## CHAPTER 1

INTRODUCTION

Impinging air jets of various sizes and geometrical configurations are widely used in a large number of industrial processes involving heating, cooling, and drying of surfaces because of the high heat and mass transfer rates they can yield. Applications include annealing of foil, tempering and toughening of glass, manufacturing of prestressed automobile windshields, cooling of leading edges of turbine blades and electronic components. In the paper industry, for example, some typical applications are: drying of tissue on Yankee dryers, drying of veneer with hot air or superheated steam, drying of newsprint at relatively low moisture content by combining impinging jets with a suction applied through the wet web.

Extensive work has been previously done by many researchers to investigate flow configuration and heat/mass transfer under round and slot jets impinging on permeable or impermeable surfaces. For a long time this work has been concerned with the study of only one single transport process (i.e. either heat or mass transfer), but studies reporting simultaneous heat and mass transfer results have been published recently.

One of the important variables, the effect of which has not yet been studied in the case of a semi-confined laminar impinging jet involving both heat and mass transfer is the angle of impingement. It may be noted that industrial devices seldom entail a single jet, but rather comprise one or several rows of jets with the consequence that each jet impinges in a crossflow created by the neighbouring ones.

The present study is concerned with simultaneous heat and mass transfer under a semi-confined slot jet impinging at a range of angles on an impermeable plate over which a crossflow is also imposed. By "semi-confined" it is intended that the jet is partially confined by a surface parallel to the impingement plate and situated at the level of the nozzle exit.

The computational method presented by Gosman et al. (G7) is employed to develop a digital computer simulation model for a steady, laminar, relatively dry and hot slot air jet discharging into a cross-flowing air stream and impinging on a cooler wet flat plate. The configuration is sketched in Figure 1.1. The simulation is obtained by solving iteratively the governing equations of motion in their primitive form along with the energy and conservation of species equations using a hybrid central and upwind differencing scheme based on the control volume integration approach. A non-uniform grid is imposed on the flow field and a line-by-line method is used to generate a tri-diagonal matrix which is then solved numerically. The entire domain is swept for each variable at a time and then iterated over all variables.

A brief review of the related literature is given in Chapter 2.

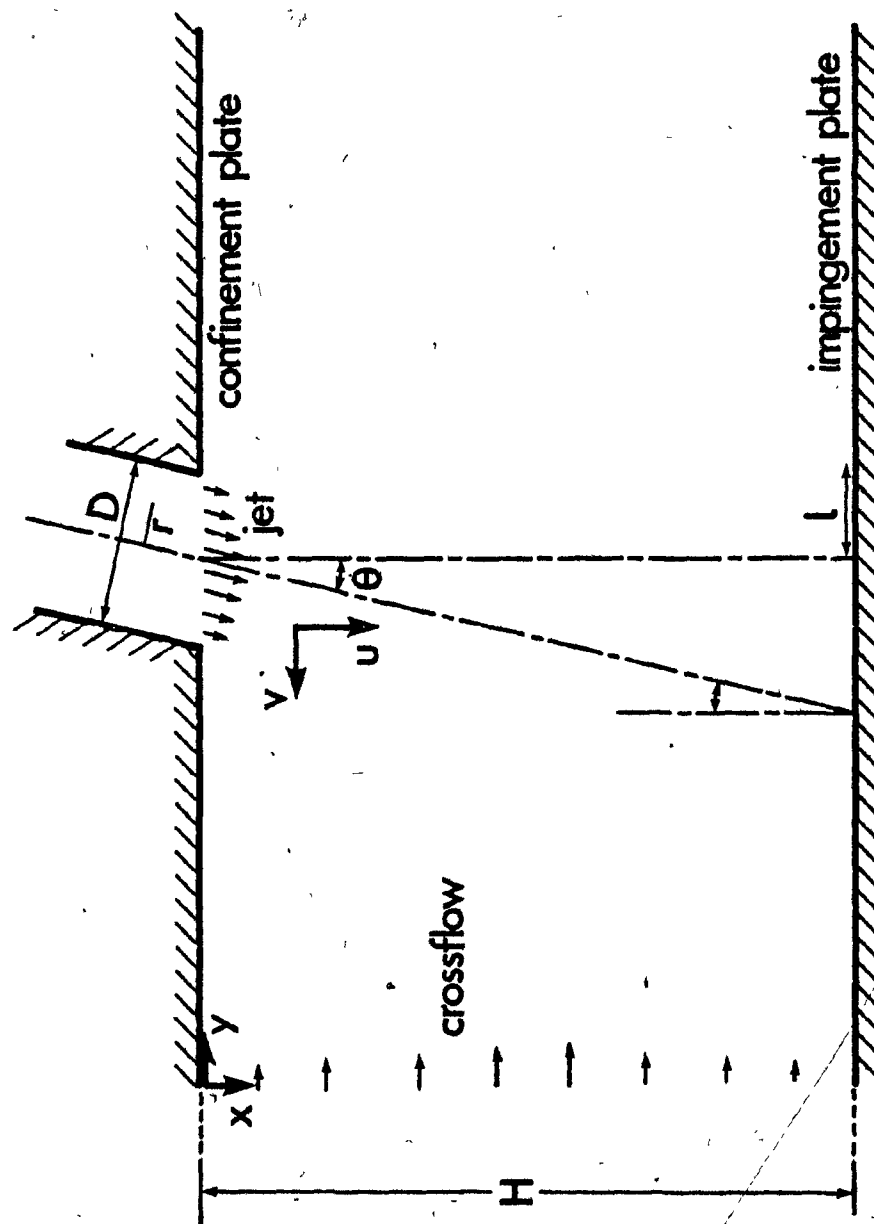


Figure 1.1 : Geometrical configuration of the jet and crossflow system.

The theoretical development which leads to the finite difference equations is presented in Chapter 3 while the numerical and computational procedures are discussed in Chapters 4 and 5 respectively. The numerical results for flow, heat and mass transfer characteristics are discussed in Chapter 6. The parameters which have been studied include nozzle-to-plate spacing, effect of the presence of crossflow (by comparison with cases without crossflow), ratio of the crossflow mass flow-rate to the jet mass flow-rate, and angle of impingement for inclined jets impinging in the crossflow direction (downstream) as well as for jets impinging against the crossflow direction (upstream).

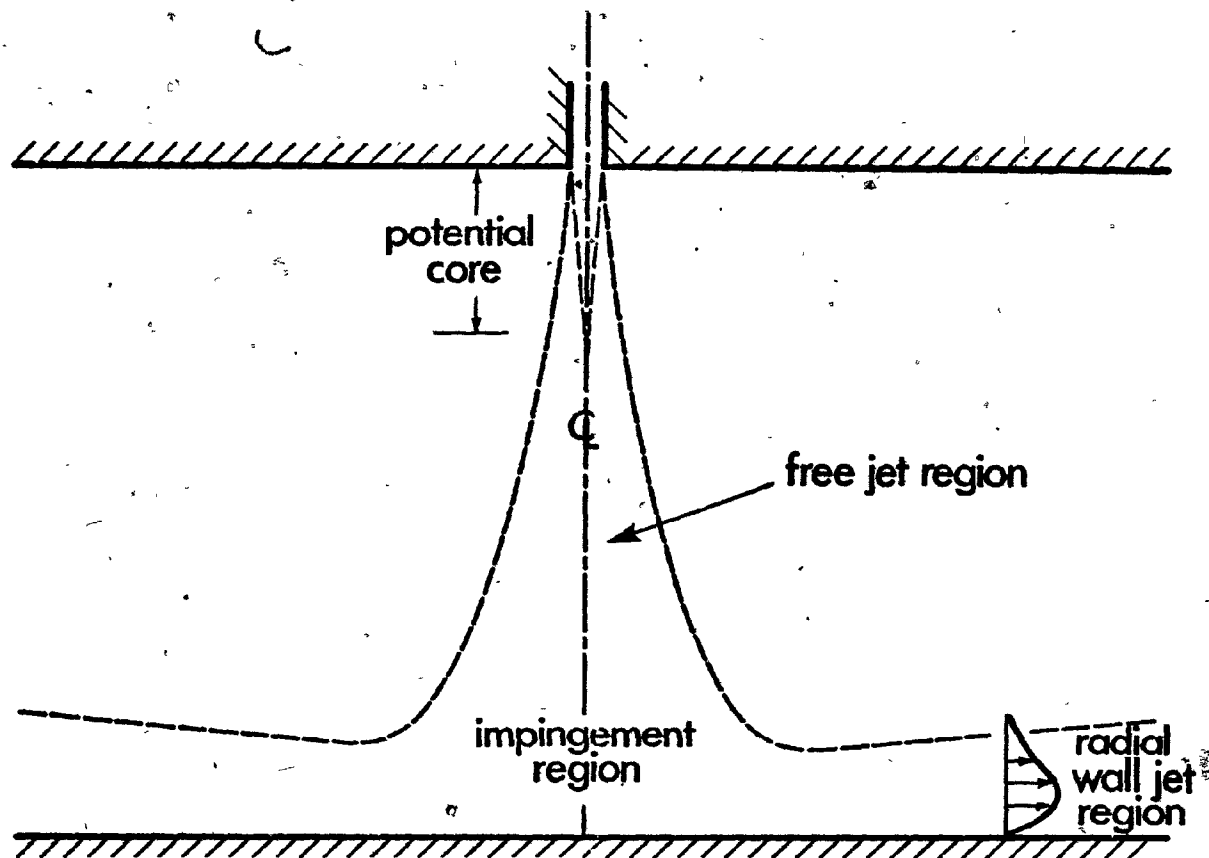


Figure 1.2 : Flow characteristics of an impinging jet.

## CHAPTER 2

### REVIEW OF RELATED LITERATURE

#### 2.1 INTRODUCTION

Many publications have appeared in recent years on the fluid mechanics and heat and mass transfer aspects of both laminar and turbulent impingement flows. These studies, both numerical and experimental, were concerned with various types of jet velocity profiles and of confinement configurations. A number of publications have also examined the flow and heat transfer characteristics of multiple impinging jets. Parameters such as nozzle-to-plate spacing, presence of confining walls, motion of the impingement surface, temperature dependent properties, presence of swirl, and application of suction or blowing have been considered. A few investigators have studied the effect of non-normal impingement, but they all were only concerned with turbulent flows, and no previous data or results have been found for the case of a laminar impinging jet discharging in a crossflow at various angles. In view of the large number of parameters governing flow, heat and mass transfer under impinging jets, it is not surprising that a serious disparity exists between the results and correlations of various workers. Obot (01) in particular has analyzed the extent of disagreement. A number of extensive reviews on various aspects of impinging jets have appeared in recent years (G1, M1, M3).

The reader is referred to these review articles for general knowledge and comprehension of the phenomena related to impinging jets. Only the more relevant references of direct interest to this study will be mentioned in the present thesis.

It may be noted that most of the numerous publications in impingement transport are concerned with axisymmetric round jets. The case of slot jets has been studied to a lesser extent, but, although the behaviours of these two types of jets are not quite identical, the trends of the transport phenomena and the influence of many parameters are similar; consequently, work on round jets as well as slot jets will be cited here.

## 2.2 IMPINGING JETS

### 2.2.1 Flow Characteristics

The flow patterns of jets impinging on a solid surface have been discussed by many investigators (G1, M1, D1); four distinct flow regions can be characterized. These are shown on Figure 1.2 and are described as follows:

(1) Region I is the region of flow establishment. It extends from the nozzle exit to the apex of the potential core. The so-called potential core is the central portion of the flow in which velocity remains equal to nozzle exit velocity.

(2) Region II is a region of established flow in the direction of the jet beyond the peak of the potential core; it is characterized by a diminution of the centerline jet velocity and by a spreading of the jet in the transverse direction. This region is often referred to as the free jet region.

(3) Region III is the region where the jet is deflected from the axial

direction due to the presence of the impingement surface. It is usually called the impingement flow region.

(4) Region IV is the region of lateral flow along the impingement surface and outside the stagnation zone. This region is known as the wall jet region, where the flow increases in thickness as the boundary layer builds up along the solid surface.

The preceding characterization of flow regions is generally valid for an unconfined jet. In the present study, as noted in Chapter 1, the flow is confined between two parallel plates (the impingement surface and the upper confining plate of equal size), and this induces a region of recirculating flow between the confining surfaces (H3, S2, V2). Most industrial applications of impinging jets for heating, cooling or drying do involve the presence of a confining wall of some kind.

Of fundamental importance in the study of impinging jets (or any jet) is the distinction between laminar and turbulent jets. For a round jet, for example, Vickers (V1) states that the critical Reynolds number (based on nozzle diameter and nozzle exit velocity) which distinguishes laminar jet flows from turbulent jet flows is about 1000. On the other hand, McNaughton reports four jet patterns for free jets (M2), namely:

- (1) The dissipated laminar jet,  $Re < 300$ . In this case, the viscous forces are large compared to the inertial forces, and the jet diffuses rapidly into the surrounding fluid.
- (2) Fully laminar jets,  $300 < Re < 1000$ . In this case there is no noticeable diffusion of the jet into the surrounding fluid.

(3) A transition or semi-turbulent jet,  $1000 < Re < 3000$ .

(4) A fully turbulent jet,  $Re > 3000$ .

These results were confirmed by Cederwell (C1) who also found that jets are turbulent for  $Re > 3000$ . Although this discussion relating jet patterns to  $Re$  range is based on studies of circular jets, corresponding criteria appear to apply also to jets issuing from slots. Gardon and Akfirat (G6) indicate that slot jets are turbulent for  $Re = 2000$  (where  $Re$  is based on the slot width).

A theoretical study reported by Levey (L3) indicated that the effect of the impingement surface was not felt by the jet further than one slot width from the surface; this study was for an idealized case, i.e. incompressible flow with no account for viscous effects or turbulent mixing. Other studies (G1, G2) and particularly the experimental work of Tani and Komatsu (T3) confirmed that no effect of the impingement surface on the jet was observed beyond about two jet diameters from the plate. Consequently, an impinging jet can be considered to behave like a free jet except in the immediate vicinity of the impingement surface.

Results of some previous studies on flow in the four characteristic regions are described below.

#### Region of flow establishment (Region I)

Near the nozzle exit the jet is decelerated by the tangential shear stress while the surrounding fluid is correspondingly accelerated. This interaction produces a zone of mixing at the jet flow periphery

which grows in width in the downstream direction, leaving a potential core within which the fluid properties and the velocity remain relatively unchanged from their values at the nozzle exit. Eventually, at some position downstream, the core is dissipated as a result of the mixing action.

Experimental data reported by Albertson (A1) suggest that the sole force producing deceleration of the jet and acceleration of the surrounding fluid is tangential shear within the mixing region. Then, if viscous action has no influence on the mixing process, the mean flow should be dynamically similar under all conditions. Thus a similar velocity profile must characterize every section in the diffusion zone. As a matter of fact Albertson's experimental data follow the general trend of the Gaussian distribution and he writes for each value of  $x$ :

$$\frac{u(y)}{u_c} = \text{EXP}\left(\frac{-y^2}{2s^2}\right) \quad (2.1)$$

where:

$x$  = distance along the axis

$y$  = transverse distance from the jet axis

$u(y)$  = velocity at distance  $y$  from axis

$u_c$  = velocity on the jet centerline

$s$  = standard or root-mean-square deviation (the value of  $y$  for which  $u = 0.606 u_c$ ).

This equation is merely a result of a curve fit and has no theoretical basis. The condition of dynamic similarity requires that at all cross sections, regardless of the efflux velocity,

$$\frac{s}{x} = K \quad (2.2)$$

where the value of  $K$  must be determined experimentally for a slot jet. Combining this analysis of the velocity field in the potential core (Region I) with a similar analysis in Region II and with his experimental data, Albertson states that the potential core length for slot jets extends 5.2 slot widths. He investigated air flowing through slots of widths 0.635, 0.159, and 0.079 cm for a range of Reynolds numbers.

In an appendix to reference (A1), Baines examined the results of Albertson more closely and concluded that the Reynolds number has an effect on the potential core length. For a round jet he recommended values of 5 nozzle diameters for  $Re = 14\ 000$ , and 7 nozzle diameters for  $Re = 100\ 000$ .

The length of this potential core has been measured by several other researchers. Hrycak et al. (H4) found that the length of the potential core is dependent upon jet Reynolds number. He reported lengths of 15 jet diameters at  $Re = 500$  for round jets. This length increases with Reynolds number to a maximum of about 20 diameters for  $Re = 1000$ . As  $Re$  increases further, small scale turbulence is established and the potential core length decreases sharply, becoming relatively constant at a length of about 6 to 7 diameters for Reynolds numbers larger than 10 000. Other researchers (S1, D1, N1, S7) have reported the length of the potential cores of axisymmetric turbulent jets to extend from 4.0 to 7.5 nozzle diameters.

Since from a heat transfer standpoint core length is important, it appears in view of the disparity of these results that additional work in this area is required.

Region of established flow (Region II)

For free slot jets Albertson et al. (A1) showed that the decay of centerline axial velocity is inversely proportional to square root of the dimensionless distance from the jet nozzle exit. With the definition for K given in Equation (2.2), Albertson writes:

$$\frac{u_c(x)}{u_c(0)} = \sqrt{\frac{1}{\sqrt{\pi} K}} \left( \frac{x}{D} \right)^{-1/2} \quad \text{or} \quad \frac{u_c(x)}{u_c(0)} = \sqrt{C} \left( \frac{x}{D} \right)^{-1/2}$$

where:

$u_c(x)$  = centerline velocity at distance x from nozzle exit.

$D$  = nozzle width

$C$  = potential core length

For free circular jets Albertson (A1), Gauntner et al. (G1) and Donaldson et al. (D1) found that the decay of the centerline velocity is inversely proportional to dimensionless distance from the nozzle exit. If  $K'$  is the value of K — as defined in Equation (2.2) — determined for a round jet, one writes:

$$\frac{u_c(x)}{u_c(0)} = \frac{1}{2K'} \left( \frac{x}{D} \right)^{-1} \quad \text{or} \quad \frac{u_c(x)}{u_c(0)} = C' \left( \frac{x}{D} \right)^{-1}$$

where:

$D'$  = nozzle diameter

$C'$  = potential core length

For laminar impinging jets, Saad (S2) and Huang (H2) have observed in their numerical studies that rate of spread and axial velocity decay

are dependent on shape of the velocity profile at the nozzle exit. Unlike the case of a parabolic inlet jet profile, there is a strong and immediate interaction between the jet flow and the surrounding fluid for a flat (plug) velocity profile. In this latter case, axial momentum just inside the free streamline is an order of magnitude greater than that in the entrained fluid just outside the free streamline. Accordingly, the high momentum fluid of the jet begins to diffuse immediately into the weaker flow field into which it is discharging, causing a considerably higher rate of spread of the jet. For the same reasons, the axial velocity decay is also faster for a jet issuing with a plug profile than for one issuing with a parabolic velocity profile.

In some applications, and particularly in the present study, the impingement surface is placed inside the potential core region. In this case the free jet region will be vanishingly small.

#### Region of deflection (Region III)

As the jet flow approaches the impingement surface, it is rapidly decelerated and pressure increases. If the impingement surface is impermeable and non-evaporative, the velocity is zero and the pressure attains a local maximum at the stagnation point. Over the impingement surface, the axial flow is deflected i.e., axial momentum is transformed into transverse momentum. Flow is of the boundary layer type with the influence of viscosity restricted to a thin layer near the impingement surface.

Beltaos et al. (B4) in a study of shear stress along the impingement plate in the case of turbulent axisymmetric jets, found that it increases almost linearly with radial distance  $r$  from the stagnation point to a maximum value, after which it decreases with further increase in  $r$ . Similar results have been found numerically by Saad (S2). Saad (S2) and Li (L2) found that shear stress on the plate, expressed nondimensionally as skin friction coefficient,  $C_f$ , does not depend on square root of the Reynolds number as is predicted by boundary layer theory for a flat plate boundary layer. They found, nevertheless, that  $C_f$  decreases as jet Reynolds number increases.

Evaporation along the impingement plate increases the mass flow-rate of fluid over the plate. As the evaporation rate increases, the "effective local Reynolds number" increases at constant jet Reynolds number, causing  $C_f$  to decrease (L2). In addition to this  $Re$  effect, evaporation also decreases the axial gradient of the radial velocities, thereby further lowering  $C_f$ . In his computer simulation of axisymmetric round jet Li (L2) also found that the application of suction through the plate leads to an increase in  $C_f$ .

#### Wall Jet Region (Region IV)

Glauert (G3) divided this flow region into two parts:

- (1) an inner layer where the effect of the wall is felt by the fluid flow.
- (2) an outer layer which is characterized by the features of a free flow.

The boundary between these two layers is taken as the position of the

maximum velocity.

In the case of round jets, Poreh et al. (P3) and Glauert (G3) showed that this maximum velocity can be expressed as a function of the radial distance  $r$  as follows:

$$V_{\max} = \frac{\text{Constant}}{r^n}$$

where  $n$  ranges from 1.1 to 1.143 according to various investigators.

Martin (M1) and Donaldson et al. (D1) further report that :

$$V_{\max} / u_j = B r^{-n}$$

where  $n$  takes the same values as in the previous equation, and where  $B$  is a function of  $H/D$  only (S1).

### 2.2.2 Oblique Impingement

It has been seen in the previous paragraph that the existence of four distinct flow regions have been established in the case of a jet impinging normally on a flat surface, and that a fifth region (region of recirculating flow) also exists when the jet is confined. The same regions can be expected to exist in the case of oblique impingement (B5). However, while for normal impingement the flow is entirely symmetric, this is not so for oblique impingement.

Beltaos (B6) measured the wall pressure distributions in the impingement zone (Region III) for slot turbulent jets impinging with various angles. Operating at  $L/D$  ratios ranging from 45.5 to 68.2, he found that the stagnation point, which corresponds to the location

of maximum wall pressure, does not coincide with the intersection of the wall and the jet centerline but is shifted by a distance,  $s$ , (see Figure 2.1). This shift is caused by the tendency of the stagnation streamline to intersect the wall at right angles. With their experimental data obtained for  $L/D = 20, 30$  and  $40$ , Schauer and Eustis (S10) evaluated the eccentricity of the stagnation point and proposed the equation,

$$\frac{S}{L} = 0.154 \tan \theta$$

with notations as indicated on Figure 2.1.

For the case,  $\theta = 60^\circ$ , Beltaos (B6) reports that the pressure excess with respect to ambient mean pressure is seen to take negative values on the side of negative  $x$  (Figure 2.1). This occurrence has also been observed by other investigators (K1, S10) and is believed to be the result of a vortex formation in the entrainment field. When  $\theta$  becomes sufficiently large, the boundaries of the free jet and of the wall jet get so close to each other on the side of negative  $x$  that some of the fluid in the upper layers of the wall jet is entrained in the free jet, thus completing a circulatory motion and forming a vortex. At the same time, the vortex is located sufficiently close to the wall so as to impress some low pressures on it (B6).

### 2.2.3 Deflection by a crossflow

Although several investigators have studied various aspects of

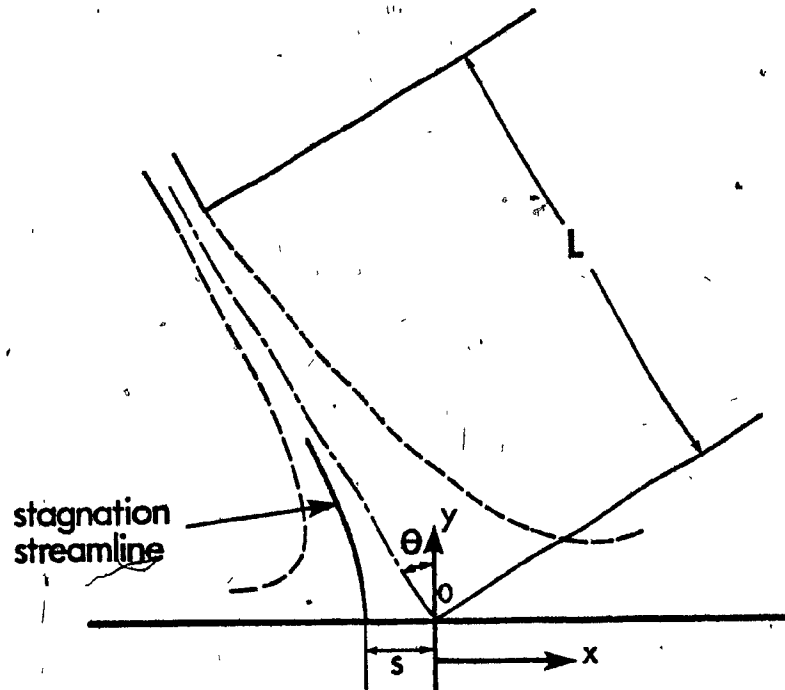


Figure 2.1 : Definition sketch for oblique impingement.

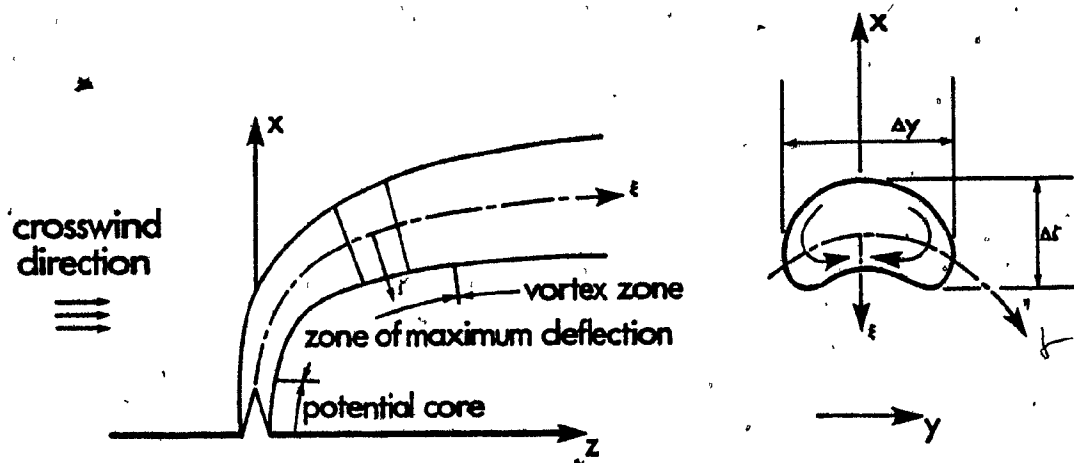


Figure 2.2 : Pattern of a free jet deflected by a crosswind.

impingement heat transfer for a turbulent jet in a cross flow, only one publication (83), as will be seen further, reports observations of the flow pattern characteristics for such impinging jets. However, because of the applications of free jets in the areas of fluid waste disposal, dispersion of effluents from chimney stacks, and fan and wing combinations used by V/STOL aircraft, many researchers have studied the flow of turbulent jet flows deflected by a crosswind. Since their conclusions are of interest to the present study, they will be briefly summed up here.

Careful observation made by Pratte and Baines (P4) of a jet of nonbuoyant smoke discharging normal to a crossflow, lead to the delineation of three flow zones (see Figure 2.2):

(1) The potential core still exists. In this potential zone the streamlines are essentially parallel to the jet direction, and the crossflow is affected in the same manner as if a solid cylinder were present. The small entrainment velocity moves fluid toward the jet at the sides and a separation occurs just behind the maximum width. In the wake, the mean velocity is much smaller than in either jet or crossflow and two attached vortices can be seen, exactly as found in flow around a cylinder at low Reynolds number. At the end of the potential zone the eddies are of comparable size to the jet and appear to be strong enough to add to the jet entrainment. Downstream of the jet and attached eddies, the crossflow recloses, forming a decelerated but relatively steady motion.

(2) Zone of maximum deflection: this zone is characterized by the deflection of the jet and by a rapid decrease of mean velocity. Just

beyond the potential zone the cross and jet flows mix rapidly because the volume of fluid entrained increases with the effective jet diameter. The action of the crossflow momentum causes the jet to bend over. Turbulent shear acts as in the simple jet (i.e. without crossflow) to entrain the surrounding air, as do the attached eddies which grow and intensify to occupy a large part of the jet cross-section as a pair of vortices. The mean velocity decreases much more rapidly than in the case of a simple jet (K2). In a relatively short length the jet is moving in a direction only a few degrees different from the crossflow direction (see Figure 2.2), and no longitudinal relative motion between the two flows can be detected.

(3) In the third region, referred to by Pratte and Douglas as the vortex zone, the two turbulent vortices are carried along at the crossflow velocity. The profile continues to rise at a slow and decreasing rate, the vortices increasing in size along with the jet cross section, but decreasing in angular velocity. It would be expected that in an infinite crossflow, the turbulent and vortex motions would be dissipated by viscosity, so that the jet would approach the conditions of its surroundings. An indication of the length of the vortex zone is given by observation of its continued existence 1000 jet diameters downstream of the source in a wind tunnel.

Pratte's results are in good accordance with what has also been found by Keffer and Baines (K2) and by Platten and Keffer (P5) who plotted the loci of maximum velocity obtained for a jet discharging at right angles into the main stream. They report that the most marked result of increasing the velocity ratio (i.e. the ratio of the jet velocity to the crossflow velocity)

is that the trajectories experience a less abrupt deflection upon entering the crossflow. One would expect the absolute jet penetration to depend upon the initial momentum flux of the jet relative to the crosswind, and this is confirmed by the higher values of  $x/D$  reached by the high velocity ratios within the curvilinear zone in Platten's experiments (P5).

The only study of the flow pattern of an impinging jet in a crossflow that this literature research could uncover is the one published by Bouchez and Goldstein (B3). Flow visualization has been performed for turbulent jets in a wind tunnel using a fog generated by dropping carbon dioxide pellets (dry ice) into a pressure vessel filled with hot water. The authors show interesting pictures for  $H/D = 6$  and 12. They found that when the jet impinges violently on the opposite wall, there is a strong turbulent recirculation zone upstream of the impingement area. This vortex is associated with a noticeable underpressure in this area. Just downstream of this underpressure, an overpressure is present and is in effect the signature of the stagnation point. Due to rapid lateral spreading of this zone, there is a sidewall effect when the tunnel walls interfere with the flow. This occurs only a few diameters downstream of the stagnation point and causes an uplift of this area of the jet. As the blowing rate mass flux ratio (of the jet to crossflow) decreases, the recirculation zone diminishes and finally disappears as does the underpressure.

It may be noted here that all the publications reported in this section are concerned with turbulent jets. An extensive literature research failed to uncover any previous study of the flow field characteristics of an impinging laminar jet in a crossflow.

## 2.3 IMPINGEMENT HEAT AND MASS TRANSFER

### 2.3.1 Analytical Studies

#### 2.3.1.1 Various theoretical approaches

Two categories of theoretical approaches have been adopted by most researchers to predict the laminar impinging jet heat or mass transfer characteristics: (1) inviscid flow - boundary layer solution matching schemes, and (2) iterative finite difference procedures to solve the full elliptic governing equations.

(1) Inviscid flow - boundary layer solution matching schemes: This approach is used near the stagnation region, based on the assumption that in the free jet region (as defined in paragraph 2.2.1) the flow is inviscid and can be represented by an appropriate potential flow solution, and that in the stagnation region a laminar flow exists. The potential flow solution is used to provide the free stream velocity for computation of the wall boundary layer flow. The governing boundary layer equations are transformed into convenient forms by using suitable similarity variables, and the resulting differential equations can be solved by finite difference methods, hence allowing prediction of the stagnation region heat transfer. The main drawback of this method, besides its inability to predict the effect of turbulence, is the uncertainty as to the size of the laminar boundary layer flow. Measured values were found to exceed predicted values, sometimes by a large amount, both for circular and slot jets (H5).

Wall jet heat transfer predictions, although mostly limited to slot jets, have been found to agree with experimental results (H5). The

theoretical approaches employ consideration of two flow layers: an inner layer along the wall and an outer layer assumed to behave as a free jet. Both integral and finite difference solutions, often complicated, have been obtained by various investigators.

(2) Iterative finite difference procedure: The full elliptic partial differential equations of conservation of mass, momentum, and energy are transformed into sets of finite difference equations which are then solved iteratively. This method, like the previous one, is not satisfactory to handle turbulent flow problems. For laminar jet flows, however, this approach provides solutions which are in excellent agreement with experimental measurements, as has been shown by Saad et al. (S11) and by van Heiningen et al. (V2)

#### 2.3.1.2 Parameters governing heat and mass transfer

(i) Jet velocity profile at nozzle exit: Sparrow and Lee (S7) studied the effect of jet exit conditions and showed that, for a non-uniform initial velocity profile of the jet, the streamwise velocity gradient at the stagnation point was relatively constant over a large range of nozzle-to-plate spacings. For a uniform (flat) initial velocity profile, however, there were significant changes in velocity gradient for different spacings. From the relationship between stagnation point heat or mass transfer and streamwise velocity gradient, the behaviour of the stagnation point heat transfer can be inferred. Scholtz and Trass (S4, S5) derived a potential flow solution for the impinging region and then used it as the boundary condition for the boundary layer along the wall. They found that, for equal Reynolds number, mass transfer at the stagnation point in the case of a uniform initial velocity is less than half that for the non-uniform jet under the conditions tested:  $0.5 < H/D < 6$  and  $570 < Re < 1970$ .

Similar results were also found by van Heiningen et al. (V2) who

solved numerically the full Navier - Stokes equations, in their stream function-vorticity formulation, coupled with the energy equation for an impinging slot jet. They found that for  $H/D = 4$  and Reynolds number between 100 and 950, the stagnation heat transfer rate for an initially parabolic velocity was 1.5 to 2 times the value for an initially flat velocity profile. Van Heiningen explained his results by noting that the jet contracts slightly below the nozzle for a parabolic profile, while for a flat profile the jet expands continuously. Consequently the free streamline is significantly closer to the impingement plate for the parabolic velocity profile owing to its higher momentum and lower rate of spreading, leading thereby to steeper velocity and temperature gradients near the impingement plate.

Saad (S2) in his simulation of flow and heat transfer characteristics of a laminar semi-confined impinging round jet further confirmed these results.

(ii) Nozzle-to-plate spacing: Although the rate of heat transfer must decrease for sufficiently large values of nozzle-to-plate spacings, for small values of  $H/D$  heat transfer has been reported by several researchers to be insensitive to nozzle-to-plate spacing. Sparrow and Lee (S7), for example, formulated this conclusion for  $H/D$  around 0.25 and 1.5. Saad (S11) in his simulation of an axisymmetric round jet found that, over the range  $1.5 < H/D < 12$ , there is no perceptible decrease at  $Re = 950$  and only 15% decrease at  $Re = 450$ .

In his simulation of simultaneous heat and mass transfer for a round axisymmetric impinging jet, Li (L2) found that transfer rates decreased with increasing nozzle-to-plate spacing for  $H/D > 6$ . He associated this result with a decrease in the limiting radial velocity gradient.

(iii) Jet confinement: When the impingement and confinement surfaces are

sufficiently large, a complete recirculation bubble attached to the confinement plate is established. Such a recirculation bubble influences the spreading of the jet and the flow pattern of the deflected jet. As is reported by van Heiningen et al. (V2), this feature of the flow influences the heat transfer rate distribution along the impingement surface. As for just the stagnation zone however, by comparing his own results for semi-confined jets with the experimental results for unconfined jets of Scholtz and Trass (S4) and Gardon and Akfirat (G4), Saad (S2) showed that the presence of a confinement plate has only a minor influence on the stagnation point heat transfer for  $2 < H/D < 4$  and  $450 < Re < 2500$ , regardless of the shape of the initial jet velocity profile. Van Heiningen's results for the stagnation region are consistent with Saad's conclusion.

This observation is in good agreement with the statement of Giralt et al. (G2) that the stagnation flow region can be considered to start at a distance around one nozzle diameter from the impingement plate. It is the velocity profile at this particular location that determines transfer rates in the stagnation zone. Hence, provided that the effect of the confinement plate - induced recirculation flow is not sufficiently strong to affect the free streamlines (i.e. region II of the flow field), the effect of partial confinement is not felt in the impingement region.

(iv) Temperature - sensitive fluid properties: Van Heiningen et al. (V2) presented the influence of variable properties (viscosity, thermal conductivity, etc.) on the Stanton number. They allowed viscosity to vary with temperature according to the Van Driest interpolation formula. It was found that the non-dimensional temperature gradient at the wall was affected by the change from constant to variable fluid properties. However, away from the stagnation point

the heat transfer coefficient in form of Stanton number based on properties at the nozzle was found to be hardly affected by the change in variable properties. Van Heiningen noted that this was in agreement with the theoretical solutions of the influence of variable properties on two-dimensional stagnation flow and laminar boundary-layer flow over a flat plate without a pressure gradient.

(v) Suction and blowing: Saad (S2), van Heiningen (V2) and Huang (H2) investigated the effect of suction on impingement heat transfer for round and slot jets; in Saad's case, for example, the suction velocity was 1% of the jet velocity. They all found that suction enhances the transfer rates by about a constant absolute increment over the entire region from the stagnation point out to the wall jet region. Skin friction was likewise found to increase with the application of suction.

On the other hand blowing leads to a lowering in local heat transfer coefficients over the entire impingement surface by an equivalent constant absolute increment. The enhancement of the heat transfer rate due to suction has been observed experimentally by Obot (O1) for a turbulent round jet.

(vi) Presence of evaporative mass flux: This is the case of simultaneous heat and mass transfer. No computer simulation of this case was available until the recent study of Li (L2). He found that the presence of an evaporative mass flux increases the various boundary layer thicknesses and reduces both heat and mass transfer rates. He observed that this reduction was almost independent of the jet Reynolds number, but was dependent on the magnitude of the axial velocities over the impingement surface.

(vii) Presence of a swirl: The study of Huang (H2) seems to have been the

only attempt to investigate numerically the effect of swirl on laminar impingement transfer rates. Huang solved the full Navier-Stokes equations together with the energy equation in their primitive form using a hybrid central and upwind difference scheme. He observed that swirl enhances the rate of spread of the jet causing, under certain conditions, development of a toroidal recirculation region next to the impingement plate. This closed recirculation flow in the stagnation region effectively insulates the plate from the jet flow, causing a substantial decrease in stagnation region heat transfer. Furthermore, the stagnation point Stanton number was found to be insensitive to variation in the nozzle-to-plate spacing for  $1.5 < H/D < 4$ . Since introduction of a swirl generally results in lowered average heat transfer rates ( $H_2$ ), it is evident that swirl should be avoided in all practical applications except when a less peaked transfer rate profile is desired.

(viii) Reynolds Number: Initial attempts to predict heat transfer from an impinging jet to a plane surface analytically were limited to the neighbourhood of the stagnation point. Most investigators assumed laminar conditions to exist in the impingement region, even for turbulent jets. This assumption is supported by Gardon and Akfirat (G4, G6) who observed experimentally the existence of secondary peaks in the local heat transfer distribution along the plate for small  $H/D$  ratios, and who ascribed these secondary peaks to a transition from laminar to turbulent boundary layers. With this laminar flow assumption, a  $Re^{1/2}$  dependence for the Nusselt number was found by most researchers. Kezios (K3), for example, reports the following expression for the stagnation point heat transfer:

$$Nu_0 = 0.475 \sqrt{Re_{D/2}}$$

where the characteristic length is  $D/2$  (nozzle radius).

It is however appropriate to remark here that Wolfstein (W1) noted that the observed dependence of Nusselt number on Reynolds number, as indicated by the exponent on the latter varying from 0.56 to 0.87, suggests existence of turbulence even near the stagnation point. The effect of free-stream turbulence should therefore be considered, and one of the major reasons for the disagreement between theory and measurements may be neglect of this influence.

(ix) Fluid properties (Schmidt or Prandtl number): Many heat and mass transfer correlations in terms of Reynolds, Prandtl and Schmidt numbers have been established. Waltz (W2), using the results originally derived by Eckert, obtained the expression:

$$Nu = 0.44 Pr^{0.36} Re_r^{0.5}$$

for the local values in the axisymmetric flow, where  $Re_r$  is the local Reynolds number at distance  $r$  from the stagnation point. For the average values he found:

$$\overline{Nu} = 0.88 Pr^{0.36} Re_r^{0.5}$$

Metzger (M4), using Eckert's wedge flow solution, obtained for slot jets the following equation for local values of Nusselt number based on distance from the stagnation point:

$$Nu = 0.57 Pr^{0.37} Re_r^{0.5}$$

and for the average values of Nusselt number he found:

$$Nu = 1.14 Pr^{0.37} Re_r^{0.5}$$

Wolfstein (W1) found a  $Pr^{0.333}$  dependence for the stagnation Nusselt number, in the range  $0.7 \leq Pr \leq 10$ .

The inability of such formulas to account directly for the effect of the nozzle-to-plate spacing must be considered their principal weakness (H5).

The problem of mass transfer in laminar radial wall jets has been studied analytically by Scholtz and Trass (T6) using the von Karman - Polhausen approach which assumes polynomial forms for the velocity and concentration profiles. Assuming  $Sc > 1$  they found that the mass transfer coefficient for the laminar wall jet was dependent on  $(x/R)^{-1.25}$  and  $Re^{0.75}$ . These results received experimental confirmation by the work of Kapur and McLeod (K4) who used a laser holography technique to measure local sublimation rates. Scholtz and Trass (S4, S5) later obtained a solution for mass transfer in the stagnation region. For a non-uniform laminar round jet with  $1 < Sc < 10$ , they gave the following correlation for the stagnation point Sherwood number:

$$Sh = 1.6484 Re^{1/2} Sc^{0.361}$$

This correlation has been found to be in good agreement with experimental results for the mass transfer coefficients in the case of sublimation of naphthalene into air ( $Sc = 2.45$ ) (S5).

### 2.3.2 Experimental Studies

2.3.2.1 Introduction: Whereas numerical investigations on impinging jets were mostly concerned with laminar jets, almost all experimental investigations have been for turbulent jets.

Hrycak et al. (H5) surveyed the correlations proposed by various researchers and noted that a comparison between results of heated jets impinging on room-temperature surfaces and room-temperature jets impinging on heated surfaces is difficult because of fundamentally different boundary conditions. In addition heat transfer rates were measured in different ways by the many researchers. The various methods include use of metal rod calorimeters, steam calorimeters, thermocouples, and heat transfer gages. The areas over which these measurements were made varied in size, and what some investigators called local values were in fact at best average values (H5). As for mass transfer rates measurements, methods such as laser holographic interferometry, weight loss determination of humid materials, and sublimation of naphthalene were used.

In a critical review of the many different analytical and experimental studies, Obot (01) has found large quantitative differences between the results. However, the experiments reported in the present review of more limited scope reveal in general the same trends.

#### 2.3.2.2 Parameters governing heat and mass transfer

(i) Nozzle-to-plate spacing: Huang (H1) investigated both stagnation and average heat transfer from a hot turbulent jet to a surface normal to the jet flow with various nozzle diameters and nozzle-to-plate spacings.

( In his investigation the heat transfer coefficient was found to be independent of spacing for  $H/D < 6$ . In contrast, major changes were found by Gardon and Cobonpue (G5) who present results of tests of cool air impinging from a circular jet onto an isothermal hot surface. These authors provide results for variation of heat transfer rates with radial distance from the stagnation point, for a nozzle exit Reynolds number of 28 000. They obtained, for large nozzle-to-plate spacings, smooth bell shaped profiles with the maximum at the stagnation point. As nozzle-to-plate spacing was reduced, they obtained secondary peaks in the heat transfer rate profiles. At  $H/D = 6$  an anular hump begins to develop and grows into a well defined secondary peak as  $H/D$  decreases further. When nozzle-to-plate spacing is less than the potential core length, they found that, at the stagnation point, the central peak was replaced by a central minimum in heat transfer. For values of  $H/D < 0.5$ , the impingement of a free jet gives way to a wall jet and the heat transfer-rate increases again.

( Chia et al. (G2), who studied mass transfer under turbulent axisymmetric impinging jets at  $Re = 34\ 000$ , report the same type of radial distribution of mass transfer-rate at various nozzle-to-plate spacings. The radial variation at  $H/D = 1.2$ , in particular, shows two pairs of peaks at values of  $x/R$  of 1.2 and 3.9, in contrast with the bell shaped distribution found at  $H/D = 8.0$ . Similar peaks were also observed by Gardon et Akfirat (G4). The inner peaks were attributed to a thinning of the accelerating boundary layer flow when the nozzle-to-plate spacings are small; the formation of the outer peaks was attributed to flow turbulence,

specifically to transition from laminar to turbulent flow.

The experimental investigations mentioned above have been carried out with relatively high Reynolds numbers. Of great importance for the present study are the results of Kapur and McLeod (K4) whose Reynolds number is of the same order of magnitude as that used in the present computer simulation. They report that the mass transfer coefficients near the impingement point for  $Re = 1340$  and nozzle-to-plate distances of 0.5, 1 and 1.5 diameters, are found to be independent of  $H/D$ , both at the stagnation point and in the region close to it, as are those in the wall jet region more remote from that point. These results are in agreement with those of Gardon and Akfirat (G6) who found that for  $Re$  less than 2000 and nozzle-to-plate distances of 0.5 to 5.0 diameters, the stagnation point Nusselt numbers depend on the Reynolds number only.

The large variety of experimental conditions chosen by the various researchers, as well as the complexity of the problem, make it difficult to draw precise quantitative conclusions as to the effect of the nozzle-to-plate spacing on heat and mass transfer. However, the general trends have been established clearly enough and could be summed up as follows: For very small  $H/D$  ratios (up to 2 or 3) some researchers find no effect whereas other investigators find a small diminution of the stagnation point heat transfer when  $H/D$  increases. For larger values of  $H/D$ , the local heat transfer coefficient at the stagnation region increases with spacing, reaches a maximum value for a spacing corresponding to the potential core length (usually between 6.0 and 8.0), and then decreases for higher spacings.

Among the many published correlations, the one of Gardon and Akfirat (G6)

could be cited. They report  $St$  to be proportional to  $(H/D)^{-0.62}$  for  $H/D > 16$ . For a round jet, Gardon and Cobonpue (G5) found  $St$  to be proportional to  $(H/D)^{-1}$  for  $H/D > 20$ .

(ii) Nozzle design: The nozzle shape influences the impingement heat or mass transfer because it determines the jet velocity profile at the nozzle exit, the turbulence level and the length of the potential core. Metzger (M4), who compared heat transfer for a slot jet and a circular jet when the same target area, flow rate, and nozzle area were considered, found that the round jet gave a heat transfer rate 8 percent higher than the slot jet. It was noted however that the shapes of the areas cooled were quite different, and it was concluded that the shape of the area to be cooled (or dried) is of primary importance in deciding which type of jet to use.

(iii) Fluid properties (Schmidt and Prandtl number): Chamberlain (C2) used room-temperature turbulent air jets impinging on a segmented, Invar, steam-heated surface. Nozzle exit Reynolds numbers to 67 000, and nozzle-to-plate spacings to 50 nozzle diameters were investigated. He reported the following correlation for the local stagnation point Nusselt number:

$$Nu_0 = 1.16 Re_D^{0.447} Pr^{0.333} \quad \text{for } H/D \leq 7$$

$$Nu_0 = 0.384 Re_a^{0.569} Pr^{0.333} \quad \text{for } H/D \geq 7$$

where  $Re_a$  is the Reynolds number based on arrival velocity. For locations along the plate away from the stagnation point, the correlation

$$\frac{h_r}{h_0} = \exp \left[ -1.56 \left( \frac{r}{H} \right)^{0.75} \right]$$

was suggested; here  $h_r$  is the local coefficient at a distance  $r$  from the stagnation point.

Nakatogawa et al. (N1), found good agreement between the correlation

$$Nu_D = 0.535 Re_D^{0.5} Pr^{0.4}$$

and their experimental values for heat transfer at the stagnation point for various Reynolds numbers (from  $Re = 10^4$  to  $Re = 10^5$ ) for spacings lower than the potential core.

(iv) Angle of impingement: Very few investigators have studied the problem of heat and mass transfer under an oblique impinging jet. In 1954, Perry (P1) investigated heat transfer from a hot round oblique jet to a plane surface. The impingement plate was water cooled and he measured the heat flux with a calorimeter of 0.65 inch diameter placed at its center, from the temperature rise of a metered flow of water. The calorimeter diameter was used as the characteristic length in  $Nu$  and  $Re$  for all the correlated results. This was so because the heat flow was measured only over the area of the calorimeter and also temperature and velocity values for the correlations were obtained as mean figures over the area of this instrument. Perry found that changing the angle of jet impingement from 15 to 90 degrees increased the stagnation heat transfer coefficient by nearly 100 percent for any temperature and velocity.

The nozzle-to-plate spacing was at least 8 nozzle diameters, and his test results were correlated by :

$$Nu = K Re^{0.7} Pr^{0.33}$$

where the value of K is dependent on the angle as follows:

| Angle of Impingement (deg.) | $K = Nu Re^{-0.7} Pr^{-0.33}$ |
|-----------------------------|-------------------------------|
| 90                          | 0.1810                        |
| 75                          | 0.1745                        |
| 60                          | 0.1579                        |
| 45                          | 0.1422                        |
| 30                          | 0.1224                        |
| 15                          | 0.1037                        |

Nothing else seems to have appeared on this matter until 1972 when Korger and Krizek (K5) published results for mass transfer under an oblique impinging round turbulent jet. Their investigation technique was based on sublimation of naphthalene. They found that the maximum mass transfer location is displaced in the direction of the jet nozzle for an inclined jet (i.e. away from the intersection between the jet axis and the plate) by an amount  $\epsilon$

$$\epsilon = (14 + 0.15 L) \tan \alpha$$

where L (nozzle-to-plate separation along axis of the jet) and  $\epsilon$  are expressed in mm, and where  $\alpha$  is the angle of impingement. This formula was found to be independent of the jet velocity for  $10 < u_j < 40$  m/sec.

From the potential flow analysis of Schah (S8) it is found by Kroger and Krizek that the stagnation point for inclined round jets coincides closely with the maximum mass transfer point.

The authors also give the mass ratio of the two dividing air streams after impingement:

$$\frac{q_1}{q_2} = \frac{1 + \cos \alpha}{1 - \cos \alpha}$$

This equation has also been reported by Beltaos assuming that the ratio behaves as that of a potential jet (B6).

Korger and Krizek reported that the maximum mass transfer value decreases when the angle is decreased under  $90^\circ$ . Studying the mass transfer distribution along the plate, they found that when the jet comes from the left side, for example, the values on the right side of the maximum transfer point are higher than on the left side. This is because most of the jet stream flows on the right side.

Figures for the average mass transfer coefficients are shown, and the results indicate that for  $30^\circ < \alpha < 90^\circ$  the average mass transfer values are sensibly the same as the one for normal impingement ( $\alpha = 90^\circ$ ). It is interesting to note that Perry (P1) made the same type of conclusion concerning the average heat transfer coefficient for round jets impinging at angles between 15 and 90 degrees.

(v) Presence of crossflow: Metzger and Korstad (M5) studied experimentally the effects of a cross-flowing air stream on the heat transfer characteristics of a single line of circular air jets. The crossflow, perpendicular to the line of jets, is characterized by the ratio of the crossflow to jet mass flow-rates,  $M$ . The general effect of  $H/D$  in the downstream region is found to be similar to the behaviour previously reported for jet flow alone, but the

crossflow increases and complicates it. The upstream propagation distance of the jets is relatively short, even for moderate crossflow rates. As a consequence, the heat transfer-rates immediately upstream of the nozzle exit plane are reduced significantly below what would be expected if crossflow were not present. Correspondingly, the average heat transfer-rates, over regions immediately downstream are increased significantly above what would be expected if crossflow were not present, despite the fact that the impingement angle for the deflected jet must be less than 90 degrees. Further, they found that on an overall basis for a symmetrical region extending ten nozzle diameters both upstream and downstream of the nozzle exit plane, the presence of crossflow always reduces the average heat transfer below what would be expected without crossflow.

Sparrow et al. (s9) report measurements of the local heat transfer coefficients resulting from impingement of a turbulent round jet which interacts with a crossflow.  $H/D$  ratios were between 3 and 12, and the ratio of the jet to crossflow mass velocity ( $M' = (\rho u)_j / (\rho u)_\infty$ ) was varied from 4 to 12. The deflection of the jet by the crossflow was found to be small for mass velocity ratios  $M' > 8$ , and for these mass velocities the impingement point heat transfer coefficient achieves a maximum value at separation distances  $H/D = 5$  or 6. Again, this result is comparable to what has been found by prior researchers in the absence of crossflow. For a mass velocity ratio  $M' = 4$ , the maximum is attained at smaller separation distances owing to the substantial deflections experienced by the jet at larger  $H/D$ . The influence of the presence of a semi-confinement plate was limited to cases where  $H/D > 4$  with the largest effects at the lowest mass velocity ratio.

Bouchez and Goldstein (B3), who studied experimentally the impingement

cooling from a circular jet in a crossflow, found that heat transfer coefficients are lower than those reported in the literature for an impinging jet in the absence of crossflow.

Nagpal (N2) studied experimentally the effect of spent-flow in perforated-plate air impingement heat transfer. The spent-flow is in fact the crossflow coming from neighbouring jets and interfering with any particular jet. Nagpal correlated the magnitude of the local heat transfer rates with the geometry of the perforated plate and the air flow rate. From his results and analysis he provided a basis for engineering design of multi-jet air - impingement cooling or heating devices. Most of his results concerning heat transfer rates are highly conditioned by the particular configuration of the perforated plates he used (free flow area ratio, etc.). The mean heat transfer rates, for example, are averaged over areas containing several plate perforations and thus several jets and could therefore not be compared with the ones obtained in the present study. However it is of importance that his results on the mean heat transfer conclude that there exists an optimum plate spacing at which the average heat transfer is maximum. This optimum value of  $H/D$  is a function of the plate configuration (dimension and number of orifices, free-flow area ratio) among other things.

## CHAPTER 3

MATHEMATICAL FORMULATION3.1 INTRODUCTION

The governing equations for the flow of a variable-property binary mixture issuing from a rectangular jet are presented in Section 3.2. In Section 3.3 all these equations are converted to a unique standard form so that a single solution algorithm can be applied to all. The boundary conditions associated with this set of elliptic partial differential equations are established in Section 3.4. Finally, the regression equations to account for variation with temperature and composition of physical properties of the binary air-water vapour mixture are presented in Section 3.5.

3.2 GOVERNING EQUATIONS

Application of the laws of conservation of mass, momentum, energy and species to multicomponent Newtonian fluid flows yield the following vector equations (B2):

$$\text{mass: } \frac{D}{Dt} \rho + \rho (\nabla \cdot \mathbf{U}) = 0$$

$$\text{momentum: } \rho \frac{D}{Dt} \mathbf{U} = - \nabla P - \nabla \cdot \underline{\underline{\tau}} + \rho \mathbf{g} \quad (3.1)$$

$$\begin{aligned} \text{energy: } \rho C_p \frac{DT}{Dt} = & -\nabla \cdot q - \underline{\tau} : \nabla U + \sum_{i=1}^n (j_i \cdot \dot{q}_i) \\ & + \left( \frac{\partial \ln p}{\partial \ln T} \right)_{P, X_i} \frac{DP}{Dt} + \sum_{i=1}^n \bar{H}_i (\nabla \cdot J_i - R_i) \end{aligned}$$

$$\text{species } i: \frac{D}{Dt} \rho_i + \rho_i (\nabla \cdot U) = -\nabla \cdot j_i + r_i \quad (3.2)$$

where:  $\rho$  = bulk density of the fluid

$\rho_i$  = mass of  $i$  per unit volume

$P$  = hydrodynamic pressure

$\underline{\tau}$  = viscous stress tensor

$J_i$  = molar flux of  $i$  relative to mass average velocity

$j_i$  = mass flux of  $i$  relative to mass average velocity

$q$  = energy flux per unit area, relative to mass average velocity

$\bar{H}_i$  = partial molar enthalpy of  $i$

$R_i$  = molar rate of production of species  $i$

$n_i$  = mass flux with respect to stationary coordinates

$r_i$  = mass rate of production of species  $i$ .

The above equations, which are general for the description of Newtonian fluids, constitute the set of governing equations in the present study.

Because of the particular geometry of the system, these equations will be used in two-dimensional cartesian coordinates. The fluid flow is assumed to be Newtonian and steady. All  $\partial/\partial t$  terms are then eliminated.

Further simplifications and assumptions made in individual equations are listed below.

### Momentum Equations

Gravity is supposed to be acting in the positive x-direction.

### Energy Equation

(i) The gas flow velocities involved in this study are low.

Consequently, the pressure work  $\left(\frac{\partial \ln p}{\partial \ln T}\right)_{p, x_i} \frac{DP}{Dt}$ , and the viscous dissipation  $\mu U^2$  are very small with respect to the other terms in the energy equation, and hence are neglected.

(ii) The diffusion-thermo effect is assumed to be negligible, and therefore the heat flux vector for a multicomponent fluid is given by:

$$\mathbf{q} = -\lambda \nabla T + \sum_{i=1}^n \bar{H}_i \mathbf{J}_i \quad (3.3)$$

### Species Equation

(i) The fluid is chemically inert and is an ideal mixture of water vapor (A) and dry air (B). Thus,  $R_i = 0$ .

(ii) The thermal-diffusion effect is assumed to be negligible, and therefore the mass flux formulation reduces to Fick's first law:

$$\mathbf{j}_i = -\rho \delta_{AB} \nabla m_i \quad (3.4)$$

where  $m_i$  is the mass fraction of species  $i$ , defined as mass of  $i$  per unit mass of the mixture.

Since the fluid properties are temperature dependent, it is more convenient to choose temperature as the dependent variable in the energy equation. Applying the caloric equation of state, enthalpy terms in the energy equation can be expressed in terms of temperature,  $T$ , as:

$$J_i \cdot \nabla \bar{H}_i = j_i \cdot \nabla h_i$$

$$\text{and } h_i = h_{i,\text{ref}} + \int_{T_{\text{ref}}}^T C_{p_i} dT \quad (3.5)$$

where  $h_{i,\text{ref}}$  is the reference enthalpy, and the integral term in Equation (3.5) represents the sensible enthalpy per unit mass.

Further, for a two-component system of A and B, the diffusion coefficients are equal, i.e.  $\delta_{AB} = \delta_{BA} = \delta$ , and thus  $j_A = -j_B$ . (3.6)

By substituting the flux equations (3.3) and (3.4) into the governing equations (3.2), and by using equations (3.5) and (3.6), one obtains the following set of simplified conservation equations:

mass:

$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0$$

momentum:

$$x - \text{component: } \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} - \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} \right) + \rho g_x$$

y - component:  $\rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial P}{\partial y} - \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right)$

energy:

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \delta \rho \left\{ \frac{\partial m_A}{\partial x} - \frac{\partial}{\partial x} \left[ (C_{pA} - C_{pB}) T \right] + \frac{\partial m_A}{\partial y} \frac{\partial}{\partial y} \left[ (C_{pA} - C_{pB}) T \right] \right\}$$

species:

$$\frac{\partial}{\partial x} (\rho m_A u) + \frac{\partial}{\partial y} (\rho m_A v) = \frac{\partial}{\partial x} \left( \rho \delta \frac{\partial m_A}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho \delta \frac{\partial m_A}{\partial y} \right) \quad (3.7)$$

The viscous stress tensor for a Newtonian fluid can be expressed as follows (B2)::

$$\begin{aligned} \tau_{xx} &= -\mu \left\{ 2 \frac{\partial u}{\partial x} - \frac{2}{3} (\nabla \cdot \mathbf{U}) \right\} \\ \tau_{yy} &= -\mu \left\{ 2 \frac{\partial v}{\partial y} - \frac{2}{3} (\nabla \cdot \mathbf{U}) \right\} \\ \tau_{xy} = \tau_{yx} &= -\mu \left\{ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\} \end{aligned} \quad (3.8)$$

where

$$\nabla \cdot \mathbf{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

which, by using the conservation of mass equation, yields:

$$\nabla \cdot \mathbf{U} = -\frac{1}{\rho} \left\{ u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} \right\}$$

### 3.3 GENERAL TRANSPORT EQUATION

The set of five conservation equations (3.7) is the set of governing equations to be solved numerically. Following the method of Gosman et al. (G7, C5) we transform these five equations into one standard form so that they can all be treated by the same solution algorithm.

This standard form, also called "general transport equation", is given by equation (3.10) in rectangular coordinates:

$$\frac{\partial}{\partial x} (\rho u \phi) + \frac{\partial}{\partial y} (\rho v \phi) - \frac{\partial}{\partial x} \left( \Gamma \frac{\partial \phi}{\partial x} \right) - \frac{\partial}{\partial y} \left( \Gamma \frac{\partial \phi}{\partial y} \right) = S_{\phi} \quad (3.10)$$

|← convection terms →| |← diffusion terms →| + source term

Equation (3.10) describes the transport of a quantity represented as the variable  $\phi$ . The first two terms account for convective transport of  $\phi$  due to bulk motion of fluid. The following two terms represent diffusional transport of  $\phi$ . The term on the right hand side, the "source term", accounts for any sources of transport of  $\phi$  other than by convection and diffusion.

#### Conservation of Mass:

The equation of conservation of mass can be written in the form of

the general transport equation where  $\phi = 1$ ,  $r = 0$  and  $S_\phi = 0$ . However this equation will not be solved directly but will be used to generate an equation for pressure, as will be shown further.

Conservation of Momentum:

x - component: Substitution of the viscous stress equation into the x - component conservation of momentum equation gives:

$$\rho \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = - \frac{\partial P}{\partial x} + \frac{4}{3} \mu \frac{\partial^2 u}{\partial x^2} + \frac{1}{3} \mu \frac{\partial^2 v}{\partial x \partial y} + \mu \frac{\partial^2 u}{\partial y^2} + \rho g_x \quad (3.11)$$

The continuity equation multiplied by u gives:

$$u \frac{\partial}{\partial x} (\rho u) + u \frac{\partial}{\partial y} (\rho v) = 0 \quad (3.12)$$

Adding equations (3.11) and (3.12):

$$\begin{aligned} \rho u \frac{\partial u}{\partial x} + u \frac{\partial}{\partial x} (\rho u) + v \frac{\partial u}{\partial y} + u \frac{\partial}{\partial y} (\rho v) = & - \frac{\partial P}{\partial x} + \frac{4}{3} \mu \frac{\partial^2 u}{\partial x^2} + \frac{1}{3} \mu \frac{\partial^2 v}{\partial x \partial y} \\ & + \mu \frac{\partial^2 u}{\partial y^2} + \rho g_x \end{aligned}$$

and rearranging:

$$\frac{\partial}{\partial x} (\rho u u) + \frac{\partial}{\partial y} (\rho v u) - \frac{\partial}{\partial x} \left[ \mu \frac{\partial u}{\partial x} \right] - \frac{\partial}{\partial y} \left[ \mu \frac{\partial u}{\partial y} \right] = - \frac{\partial P}{\partial x} + \frac{1}{3} \mu \frac{\partial^2 u}{\partial x^2} + \frac{1}{3} \mu \frac{\partial^2 v}{\partial x \partial y} + \rho g_x$$

which is in the form of the general transport equation with:

$$\begin{cases} \phi = u \\ \Gamma = \mu \\ S_\phi = - \frac{\partial P}{\partial x} + \frac{1}{3} \mu \frac{\partial}{\partial x} (\nabla \cdot \mathbf{U}) + \rho g_x \end{cases}$$

y - component: Substitution of the viscous stress equations into the y - component of the momentum equation gives:

$$\rho \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = - \frac{\partial P}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \frac{1}{3} \mu \frac{\partial^2 u}{\partial y \partial x} + \frac{4}{3} \mu \frac{\partial^2 v}{\partial y^2} \quad (3.13)$$

The continuity equation multiplied by v is:

$$v \frac{\partial}{\partial x} (\rho u) + v \frac{\partial}{\partial y} (\rho v) = 0 \quad (3.14)$$

Adding equations (3.13) and (3.14) gives:

$$\rho u \frac{\partial v}{\partial x} + v \frac{\partial}{\partial x} (\rho u) + \rho v \frac{\partial v}{\partial y} + v \frac{\partial}{\partial y} (\rho v) = - \frac{\partial P}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \frac{1}{3} \mu \frac{\partial^2 u}{\partial y \partial x} + \frac{4}{3} \mu \frac{\partial^2 v}{\partial y^2}$$

and by rearranging:

$$\frac{\partial}{\partial x} (\rho u v) + \frac{\partial}{\partial y} (\rho v v) - \frac{\partial}{\partial x} \left[ \mu \frac{\partial v}{\partial x} \right] - \frac{\partial}{\partial y} \left[ \mu \frac{\partial v}{\partial y} \right] = - \frac{\partial p}{\partial y} + \frac{1}{3} \mu \frac{\partial}{\partial y} \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right]$$

which is in the form of the general transport equation with:

$$\begin{cases} \phi = v \\ \Gamma = \mu \\ S_\phi = - \frac{\partial p}{\partial y} + \frac{1}{3} \mu \frac{\partial}{\partial y} (\nabla \cdot \mathbf{U}) \end{cases}$$

#### Conservation of Energy:

Since specific heat capacities of dry air and water vapour are only weak functions of temperature, their variation within a small control volume is negligible. This simplification eliminates the  $\partial C_p$  terms in the equation of energy conservation (3.7) and gives:

$$\begin{aligned} \rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} &= \frac{1}{C_p} \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{1}{C_p} \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) \\ &+ \delta \rho \frac{C_{pA} - C_{pB}}{C_p} \left[ \frac{\partial m_A}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial m_A}{\partial y} \frac{\partial T}{\partial y} \right] \end{aligned} \quad (3.15)$$

Multiplying the continuity equation by  $T$  gives:

$$T \frac{\partial}{\partial x} (\rho u) + T \frac{\partial}{\partial y} (\rho v) = 0 \quad (3.16)$$

Adding Equations (3.15) and (3.16) yields:

$$\begin{aligned} \rho u \frac{\partial T}{\partial x} + T \frac{\partial}{\partial x} (\rho u) + \rho v \frac{\partial T}{\partial y} + T \frac{\partial}{\partial y} (\rho v) &= \frac{1}{C_p} \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{1}{C_p} \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) \\ &+ \delta \rho \frac{C_{pA} - C_{pB}}{C_p} \left[ \frac{\partial m_A}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial m_A}{\partial y} \frac{\partial T}{\partial y} \right] \end{aligned} \quad (3.17)$$

Combining the terms of the left hand side, and noting that:

$$\begin{cases} \frac{1}{C_p} \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) = \frac{\partial}{\partial x} \left( \frac{\lambda}{C_p} \frac{\partial T}{\partial x} \right) - \lambda \frac{\partial T}{\partial x} \frac{\partial}{\partial x} \left( \frac{1}{C_p} \right) \\ \frac{1}{C_p} \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( \frac{\lambda}{C_p} \frac{\partial T}{\partial y} \right) - \lambda \frac{\partial T}{\partial y} \frac{\partial}{\partial y} \left( \frac{1}{C_p} \right) \end{cases}$$

enables us to rewrite Equation (3.17) in the following form:

$$\begin{aligned} \frac{\partial}{\partial x} (\rho u T) + \frac{\partial}{\partial y} (\rho v T) - \frac{\partial}{\partial x} \left[ \frac{\lambda}{C_p} \frac{\partial T}{\partial x} \right] - \frac{\partial}{\partial y} \left[ \frac{\lambda}{C_p} \frac{\partial T}{\partial y} \right] &= - \lambda \frac{\partial T}{\partial x} \frac{\partial}{\partial x} \left( \frac{1}{C_p} \right) \\ &- \lambda \frac{\partial T}{\partial y} \frac{\partial}{\partial y} \left( \frac{1}{C_p} \right) + \delta \rho \frac{C_{pA} - C_{pB}}{C_p} \left[ \frac{\partial m_A}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial m_A}{\partial y} \frac{\partial T}{\partial y} \right] \end{aligned}$$

which is in the form of the general transport equation with:

$$\left\{ \begin{array}{l} \phi = T \\ \Gamma = \lambda / C_p \\ S_\phi = \delta \rho \frac{C_{pA} - C_{pB}}{C_p} \left[ \frac{\partial m_A}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial m_A}{\partial y} \frac{\partial T}{\partial y} \right] - \lambda \left[ \frac{\partial T}{\partial x} \frac{\partial}{\partial x} \left( \frac{1}{C_p} \right) + \frac{\partial T}{\partial y} \frac{\partial}{\partial y} \left( \frac{1}{C_p} \right) \right] \end{array} \right.$$

### Conservation of Species:

Rearranging the terms of the species equation transforms it into the general form of the transport equation:

$$\frac{\partial}{\partial x} (\rho u m_A) + \frac{\partial}{\partial y} (\rho v m_A) - \frac{\partial}{\partial x} \left[ \rho \delta \frac{\partial m_A}{\partial x} \right] - \frac{\partial}{\partial y} \left[ \rho \delta \frac{\partial m_A}{\partial y} \right] = 0$$

with

$$\left\{ \begin{array}{l} \phi = m_A \\ \Gamma = \delta \rho \\ S_\phi = 0 \end{array} \right.$$

A summary of all the governing equations in the standard form is given in Table 3.1.

### 3.4 BOUNDARY CONDITIONS

With the four governing equations, listed on Table 3.1, must be associated the boundary conditions at the five boundaries of the flow field, indicated on Figure (3.1). The conditions at these five boundaries are detailed below.

Table 3.1

General Form of the Transport Equation

$$\frac{\partial}{\partial x} (\rho u \phi) + \frac{\partial}{\partial y} (\rho v \phi) - \frac{\partial}{\partial x} \left( \Gamma \frac{\partial \phi}{\partial x} \right) - \frac{\partial}{\partial y} \left( \Gamma \frac{\partial \phi}{\partial y} \right) = S_\phi$$

| Equation   | $\phi$ | $\Gamma$        | $S_\phi$                                                                                                                                                                                                                                                                                                                                                                               |
|------------|--------|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x-momentum | $u$    | $\mu$           | $-\frac{\partial P}{\partial x} + \frac{1}{3} \mu \frac{\partial}{\partial x} (\nabla \cdot \mathbf{U}) + \rho g_x$                                                                                                                                                                                                                                                                    |
| y momentum | $v$    | $\mu$           | $-\frac{\partial P}{\partial y} + \frac{1}{3} \mu \frac{\partial}{\partial y} (\nabla \cdot \mathbf{U})$                                                                                                                                                                                                                                                                               |
| Energy     | $T$    | $\lambda / C_p$ | $\delta \rho \frac{C_{pA}}{C_p} \left[ \frac{\partial m_A}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial m_A}{\partial y} \frac{\partial T}{\partial y} \right] - \lambda \left[ \frac{\partial T}{\partial x} \frac{\partial}{\partial x} \left( \frac{1}{C_p} \right) + \frac{\partial T}{\partial y} \frac{\partial}{\partial y} \left( \frac{1}{C_p} \right) \right]$ |
| Species    | $m_A$  | $\delta \rho$   | 0                                                                                                                                                                                                                                                                                                                                                                                      |

Boundary I

Nozzle exit: Fully developed parabolic profile

$$U = \frac{3}{2} \bar{U} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

where the mean velocity  $\bar{U}$  is fixed by the choice of the Reynolds number:  $\bar{U} = \mu Re / \rho D$ , and since for  $x = 0$  we have  $r = |y - y_0| \cos \theta$  (see Figure 3.1) where

$y_0$  = ordinate of jet centerline on exit plane

and  $y_R = R / \cos \theta$

the velocity at the nozzle exit can also be written:

$$U = \frac{3}{2} \bar{U} \left[ 1 - \left( \frac{|y - y_0| \cos \theta}{y_R \cos \theta} \right)^2 \right]$$

so that the boundary conditions are finally:

$$\begin{cases} u = \frac{3}{2} \bar{U} \left[ 1 - \left( \frac{y - y_0}{y_R} \right)^2 \right] \cos \theta \\ v = \frac{3}{2} \bar{U} \left[ 1 - \left( \frac{y - y_0}{y_R} \right)^2 \right] \sin \theta \end{cases}$$

$T$  = prescribed value  $T_j$

$m_A$  = prescribed value  $m_j$

Boundary II

Plate confinement: no-slip, non-evaporative, impermeable, isothermal

wall at the temperature of the inlet jet, i.e.

$$u = 0; \quad v = 0; \quad T = T_j; \quad \partial m_A / \partial x = 0$$

### Boundary III

Impingement plate: evaporative, permeable, isothermal wall with a thin film of liquid water (i.e. free unbound moisture) at the surface, where uniform suction may be applied.

The y-component of the velocity along this plate is  $v = 0$ .

For the normal component an analysis of the mass transport phenomena in the vicinity of the plate is required. The resultant velocity along the impingement plate is a vector, sum of the evaporation velocity and of the prescribed suction velocity which is superimposed on it.

Assuming negligible solubility of dry air in water, the evaporation velocity can be derived starting from Fick's first law in terms of  $n_A$ , the mass flux relative to stationary coordinates:

$$n_A = - m_A ( n_A + n_B ) - \rho \delta_{AB} \nabla m_A \quad (3.18)$$

where  $\delta_{AB}$  is the mass diffusivity in  $m^2 s^{-1}$

$m_A$  is the mass fraction of A ( $m_A = \rho_A / \rho$ )

$n_A$  and  $n_B$  are the mass fluxes of species A and B, respectively, in  $kg \cdot m^{-2} \cdot s^{-1}$ .

This equation shows that the diffusion flux  $n_A$  relative to stationary coordinates is the resultant of two vector quantities: the vector

$$m_A ( n_A + n_B )$$

which is the mass flux of A resulting from bulk motion of the fluid, and the vector

$$j_A = - \rho \delta_{AB} \nabla m_A$$

which is the molar flux of A resulting from diffusion superimposed on the bulk flow.

In our case, where the evaporating system has attained a steady state, there is a net motion of A away from the evaporating surface. In the vicinity of this surface the bulk flow can be neglected since there is no tangential slip ( $v \neq 0$ ) and since the present calculation of the evaporation velocity does not involve suction, which will be accounted for subsequently by adding the term  $u_{\text{suc}}$  to the velocity. As the flux of the non-evaporating component B at evaporating surface is zero, Equation (3.18) with  $n_B = 0$  gives:

$$n_A = - \frac{\rho \delta}{1 - m_A} \nabla m_A$$

According to our assumptions the evaporation is normal to the plate, so that we can solve for  $n_{A,x}$ :

$$n_{A,x} = - \frac{\rho \delta}{1 - m_A} \frac{\partial m_A}{\partial x}$$

and since  $n_{A,x} = \rho_A u_A$  where  $u_A$  is the velocity of species A, noting that  $\rho/\rho_A = 1/m_A$  we can write:

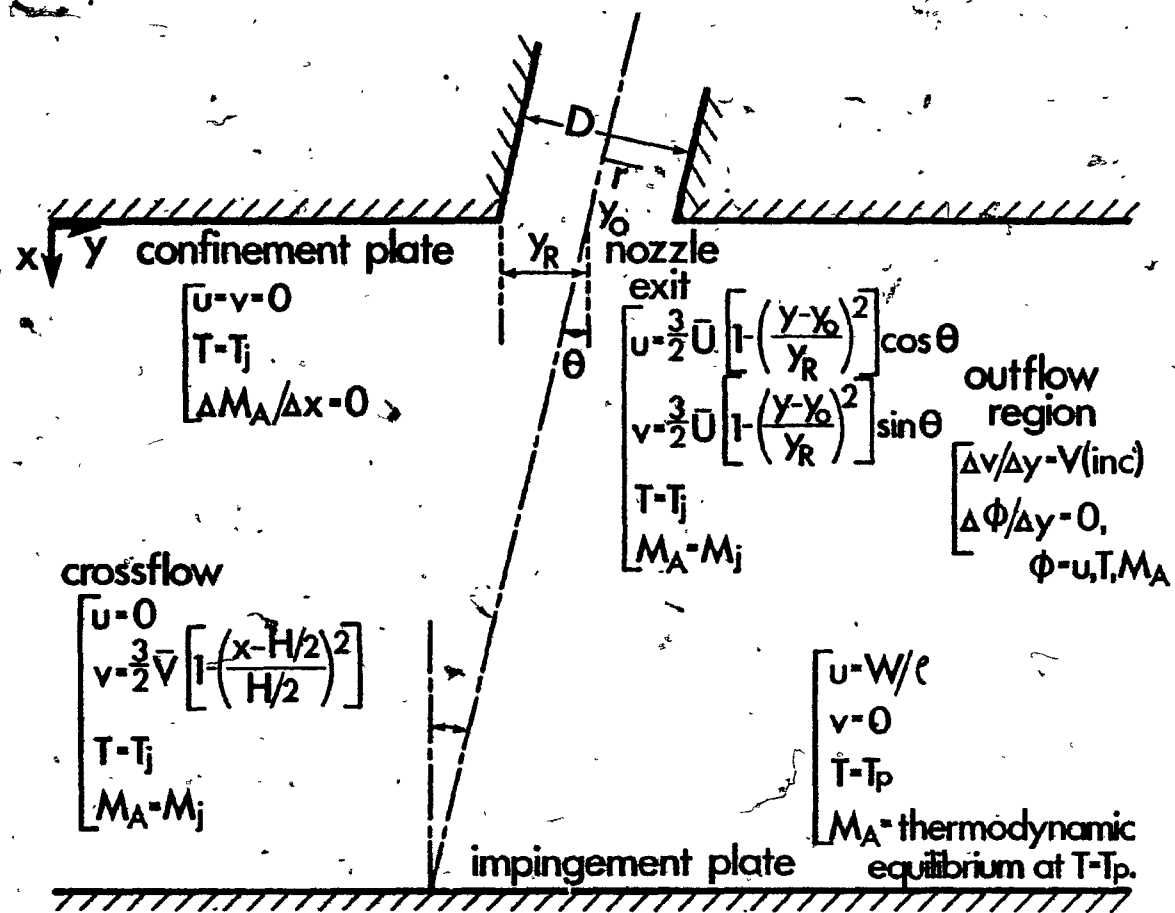


Figure 3.1 : Boundary conditions.

$$u = \frac{-\delta}{m_A (1 - m_A)} \frac{\partial m_A}{\partial x}$$

Noting that  $u = m_A u_A + m_B u_B$ , but with  $u_B = 0$  in the present case, and adding the term for suction finally yields the following boundary condition for the velocity in this region:

$$u = - \frac{\delta}{(1 - m_A)} \frac{\partial m_A}{\partial x} + u_{suc}$$

Note that since the mass transfer rate is unknown before the problem is solved, being part of the solution to the problem, the  $u$  component boundary condition along the impingement plate is of an iterative nature.

Although provision has been made in the last equation to incorporate the influence of suction, this parameter has not been investigated because of time limitations. Therefore for the present study  $u_{suc} = 0$ .

The boundary condition for the mass fraction of water vapour along the plate is:

$$m_A = \frac{P_{sat} M_A}{(P_{sat}) M_A + (P - P_{sat}) M_B}$$

This boundary condition is derived by assuming existence of thermodynamic equilibrium between liquid water and water vapour. Also Raoult's law which assumes ideal solution and no differences between fugacities and pressures, is used. These are reasonable

assumptions in the present case because the solubility of air in water under the temperature range investigated is negligible (H1), and because the operating pressure is low.

#### Boundary IV:

##### Outflow region.

The two different cases of impingement in the absence and in the presence of crossflow must be examined separately.

(i) In the absence of crossflow: In this case there are of course two outflow regions, situated at the open extremities of the impingement flow field (see Figure 1.2). Because of entrainment of fluid by the confined jet, a recirculation zone is formed between the confinement and impingement plates. Two flow patterns are possible at this boundary. Depending on the size of the impingement and confining plates (the two being in all cases of the same size) this recirculation bubble may either be completely enclosed, for wide plates, or cut by the outflow boundary in the case of narrow plates. In the first case, fluid flows out at all points across this boundary. In the second case, however, there are regions of inflow and outflow adjacent, respectively, to the confinement and impingement surfaces.

The manner in which the  $y$ -direction velocity,  $v$ , is specified is valid for both the above cases. Before a converged solution is obtained, the mass residues for each control volume can be relatively large. Within each iterative cycle, as the computation marches towards the boundary, the resulting  $v$ -velocity profile generally leads to an overall mass accumulation inside the physical system. In order to fulfill

the overall mass balance, a correction must be made at each iteration so that the overall mass inflow equals the overall mass outflow. This can be done by writing:

$$v(J=1) = v(J=2) + \frac{1}{2}v_{inc} \quad (3.19)$$

$$v(J=NJ) = v(J=NJ-1) + \frac{1}{2}v_{inc}$$

In these equations,  $v(J=1)$  and  $v(J=2)$  are the  $v$ -components of the velocity along and immediately inside the outflow boundary respectively for the left side. And  $v(J=NJ)$  and  $v(J=NJ-1)$  are the  $v$ -components of the velocity along and immediately inside the outflow boundary respectively for the right side. The term  $v_{inc}$  is the correction term determined by:

$$v_{inc} \int_{\text{Bound. IV}} \rho dA = \int_{\text{Bound. I, III}} \rho u dA - \int_{\text{Bound. IV}} \rho v^* dA$$

where  $v^*$  is taken one grid point inside Boundary IV, i.e. stands for  $v(J=2)$  and  $v(J=NJ-1)$  on the left and right hand side of the flow field respectively.

Note that the right hand side of this equation represents the net mass accumulation inside the physical boundaries. Equations (3.19) mean that the shape of the velocity profile calculated in each iteration is retained, but scaled to satisfy the overall mass balance. The corrected value of  $v$  at Boundary IV will affect the adjacent points and the effect, in general, carries into the entire flow field. This correction is applicable regardless of whether or not an inflow occurs at this boundary.

The boundary conditions for  $u$ ,  $T$ , and  $m_A$  are established by specifying no more variation in the  $y$ -direction for these variables at Boundary IV, i.e.  $\partial/\partial y = 0$ . This boundary condition is correct when there is only outflow at this boundary, but in the regions where inflow occurs it is somehow paradoxical as it implies determination of upstream conditions from downstream conditions. Van Heiningen (V2) who studied various possible boundary conditions at the outflow boundary for a semi-confined laminar slot jet came to the conclusion that the impingement region flow field and heat transfer are insensitive to variations in specifications of boundary conditions at the outflow, provided that this flow boundary is sufficiently distant from the impingement region. Saad (S2) and Huang (H2) have used successfully the equivalent condition of zero radial variation at this boundary in their numerical studies of round jet impingement. Li (L2) also used this formulation for the outflow boundary condition in his laminar round jet simulation with an impingement plate significantly smaller than that of the present study. He showed that using a larger impingement plate (i.e. having the outflow boundary conditions specified further from the impingement region) has a noticeable effect only over the outer part of the plate, whereas the impingement region results are unaffected.

In view of this analysis, the  $\partial/\partial y = 0$  boundary condition was adopted over the entire Boundary IV.

(ii) In the presence of crossflow: In this case the outflow region is located on the right hand side of the domain of interest, whereas the left hand side, the source of the crossflow, constitutes Boundary V ( see Figure 3.1 )

As in subsection (i), two flow patterns are possible, i.e. with the recirculation being totally or only partially enclosed. Again, a correction must be made at each iteration in order to fulfill the overall mass balance. This is done by writing:

$$v(J=NJ) = v(J=NJ-1) + v_{inc}$$

where the correction term  $v_{inc}$  is determined by:

$$v_{inc} \int_{\text{Bound. IV}} \rho dA = \int_{\text{Bound. I, III}} \rho u dA + \int_{\text{Bound. V}} \rho v dA - \int_{\text{Bound. IV}} \rho v^* dA$$

Following the discussion given in subsection (i), the same boundary condition is taken for  $u$ ,  $T$ , and  $m_A$ , namely  $\frac{\partial}{\partial y} = 0$ .

#### Boundary V:

Crossflow entry.

The crossflow, of temperature and water vapour content identical to that of the inlet jet, enters parallel to the confinement and impingement plates.

The crossflow is specified to have a fully developed velocity profile:

$$V = \frac{3}{2} \bar{V} \left[ 1 - \left( \frac{|x - H/2|}{H/2} \right)^2 \right]$$

The magnitude of the crossflow is defined by the ratio,  $M$ , of the crossflow volumetric flow rate to the jet volumetric flow rate:

$$M = \frac{\bar{V} H}{U D}$$

where  $\bar{V}$  is the mean velocity of the crossflow at its entry. As the crossflow and the jet flow are <sup>of</sup> the same fluid (air in this case) at the same temperature, and at essentially the same pressure, the density and viscosity are also effectively the same. Thus  $M$  may also be regarded as the mass flow rate ratio between the crossflow and the jet flow, or as the corresponding ratio of the Reynolds numbers of the crossflow and the jet flow.

Hence Boundary  $V$  is defined by:

$$\begin{cases} u = 0 \\ v = \frac{3}{2} \bar{V} \left[ 1 - \left( \frac{x - H/2}{H/2} \right)^2 \right] \\ T = T_j \\ m_A = m_j \end{cases}$$

Note that Boundary  $V$  does not exist in the limiting case, included in this study, where there is no crossflow.

### 3.5 FLUID PROPERTY EQUATIONS

The fluid throughout the flow field is assumed to be an ideal mixture of water vapour (species A) and dry air (species B).

The effect of variable temperature and composition on all fluid properties is considered. Because the pressure variations encountered in the present study are small, the effect of pressure on all fluid properties, except density and mass diffusivity, is assumed to be negligible. Second order polynomials were derived from available data to express the dependence of each fluid property of interest on temperature. These polynomials were used by Li (L2) and gave satisfactory results.

### 3.5.1 Density

Within the temperature range of 300°K to 600°K and approximately  $10^5$  N/m<sup>2</sup> pressure (1 atm), the compressibility factor for air is sufficiently close to one that the ideal gas law is applicable, giving:

$$\rho = P \overline{MW} / RT$$

where  $R$  = gas constant = 8 314.2 J/kilo-gmole °K

$\overline{MW}$  = the mixture molecular weight, expressed as (B2)

$$1/\overline{MW} = m_A/MW_A + m_B/MW_B$$

### 3.5.2 Viscosity

#### (i) Water Vapour

Based on their own experimental data, Thomas and Jackson (T1) proposed the following equation for the dynamic viscosity of water vapour:

$$\mu_A = (0.99354 \text{ E-6}) + (0.27067 \text{ E-7})T + (0.88906 \text{ E-11})T^2$$

where  $\mu$  = dynamic viscosity, kg/m.s

$T$  = temperature, °K

All their data were taken between 500°K and 800°K, and the average deviation of data from the equation is 1.88%. Comparison of results from the equation with other existing data (H6, B1) for temperatures as low as 273°K is also satisfactory.

#### (ii) Dry Air

Data for viscosity of dry air reported in reference (H6) are fitted to a second degree function of temperature using the least

squares method. The resulting equation is:

$$\mu_B = (0.25641 \text{ E-5}) + (0.60198 \text{ E-7})T - (0.23723)T^2$$

Comparison of results with data from other sources (B1) is satisfactory (L2); the average deviation being only 1.4% over the temperature range 350°K to 450°K.

(iii) Mixtures

The variation of viscosity of binary mixtures with composition is usually not linear, and mixture viscosities are often larger than the values predicted by mole-fraction average. At low pressures, Reid and Sherwood (R2) recommend use of Wilke (W3) estimation method for mixtures. For binary mixtures of dry air and water vapour, the Wilke formulation is:

$$\mu = \mu_A / [1 + (x_B/x_A) \phi_{AB}] + \mu_B / [1 + (x_A/x_B) \phi_{BA}]$$

where  $x$  = mole-fraction

$$\phi_{AB} = [1 + 1.126 \sqrt{\mu_A/\mu_B}]^2 / 3.6022$$

$$\phi_{BA} = [1 + 0.888 \sqrt{\mu_B/\mu_A}]^2 / 4.5674$$

For reduced temperature of approximately 2, which is within the temperature range under investigation, viscosities of dry air and water vapour are independent of pressure up to about  $10^6 \text{ N/m}^2$  (B1).

### 3.5.3 Specific Heat Capacity

(i) Water Vapour

Crotogino (C3) recommends the following regression equation for

the specific heat of water vapour:

$$C_{pA} = (0.18241 \text{ E-4}) + (0.21976)T + (0.23487 \text{ E-3})T^2$$

where  $C_p$  = specific heat capacity J/kg. $^{\circ}$ K

#### (ii) Dry Air

A least squares fit of the data given in reference (H6) for the specific heat of dry air between 300 $^{\circ}$ K and 600 $^{\circ}$ K gives:

$$C_{pB} = (0.10256 \text{ E-4}) - (0.17317)T + (0.36146 \text{ E-3})T^2$$

Comparison of the results with those tabulated in the CRC Handbook (B1) shows negligible difference between the experimental data and the least squares equation.

#### (iii) Mixture

The specific heat of the binary mixture is assumed to be a mass-fraction average of the individual species (B2):

$$C_p = C_{pA} m_A + C_{pB} m_B$$

### 3.5.4 Thermal Conductivity

#### (i) Water Vapour

A least squares fit of the tabulated data in the CRC Handbook (B1) to a second degree function of temperature, gives:

$$\lambda_A = (0.86868 \text{ E-2}) + (0.11318 \text{ E-4})T + (0.85793 \text{ E-7})T^2$$

where  $\lambda$  = thermal conductivity, W/m °K

Comparison with data from other sources (H6) in the temperature range of 373°K to 600°K shows average and maximum deviation of approximately 2% and 4% respectively (L2).

(ii) Dry Air

Thermal conductivity data for dry air is reported in reference (B1).

A least squares fit of these data gives:

$$\lambda_B = (0.38793 \text{ E-3}) + (0.95425 \text{ E-4})T - (0.30699 \text{ E-7})T^2$$

(iii) Mixture

Reid and Sherwood (R2) recommend the Lindsay and Bromley (L1) equation for the thermal conductivity of a mixture of polar and non-polar gases:

$$\lambda = \lambda_A / \{1 + (x_B/x_A)\phi_{AB}\} + \lambda_B / \{1 + (x_A/x_B)\phi_{BA}\}$$

where

$$\phi_{AB} = \frac{1}{4} \left[ 1 + \sqrt{1.428 \frac{\mu_A}{\mu_B} \frac{1 + 410/T}{1 + 120/T}} \right]^2 \frac{1 + 155/T}{1 + 410/T}$$

$$\phi_{BA} = \frac{1}{4} \left[ 1 + \sqrt{\frac{1}{1.428} \frac{\mu_B}{\mu_A} \frac{1 + 120/T}{1 + 410/T}} \right]^2 \frac{1 + 155/T}{1 + 120/T}$$

The Lindsay-Bromley equation is estimated by Cheung to have an average error of 2.1% with a maximum error of 6.9% (R2).

### 3.5.5 Mass Diffusivity

The variation of mass diffusivity of a mixture of air and water vapour with temperature and pressure can be estimated by using the following well known equation:

$$P\delta = CT^n$$

where  $\delta$  = mass diffusivity,  $m^2/s$ .

Various researchers suggest different values for  $n$ , which range from 1.5 to 2.3 in the literature (B2, T<sub>2</sub>). Careful review of the literature shows that  $n$  is calculated using experimental data for low temperature ranges only (L4). Little experimental work on diffusivity has been reported for even moderately elevated temperatures. The only notable high temperature study measurements of the diffusivity of the air-water vapour system was made by Klibanov (K6). The temperature covered ranges from 300°K to over 1500°K. The experimental data were correlated by Crotoigino (C3) giving:

$$\delta = 0.2794 \text{ E-4} - 0.1306 \text{ E-6}T + 0.4016 \text{ E-9}T^2 \text{ for } T \leq 521^\circ\text{K}$$

$$\delta = -0.1057 \text{ E-3} + 0.335 \text{ E-6}T \text{ for } T \geq 521^\circ\text{K}$$

### 3.5.6 Auxiliary Equations and Data

#### Saturated Vapour Pressure of Water Vapour

Its evaluation is necessary in order to specify the boundary condition for  $m_A$  along the impingement plate. Data tabulated in steam tables (B1) are fitted to an Antoine type of equation giving:

$$\ln P_{\text{sat}} = 23.365 - \frac{3919.9}{T + 42.062}$$

with  $P_{\text{sat}}$  = saturated vapour pressure of water vapour, N/m<sup>2</sup>

### Molecular Weights

The molecular weight of water vapour and of dry air,  $MW_A$  and  $MW_B$  respectively, are taken to be 18.02 and 28.97 g/gmole (T2).

## CHAPTER 4

NUMERICAL FORMULATION4.1 FINITE DIFFERENCE EQUATIONS

In this chapter the finite difference form of the general transport equation is established. There are many ways in which sets of algebraic equations can be constructed so as to simulate, in major respects, the behaviour of partial differential equations; but some ways are better than others in accuracy of simulation, convenience in use, and tendency to promote convergence in iterative solution procedures. Huang (H2) reviewed the various solution procedures which could be applied to solve problems of the present type and gave an extensive review of the literature. In the present study the finite difference equations are derived using the control volume integration technique (G7). Thus the general transport equation is integrated over a control volume.

The field of interest will be covered by a rectangular grid network. At each nodal point (intersection of the grid lines) the properties of the fluid (density, viscosity) are defined. Since these properties depend on the temperature, pressure and composition of the fluid, these variables ( $T$ ,  $P$ ,  $m_A$ ) are also defined at the same point. Because all scalar quantities are calculated at grid line intersections, this type of grid is referred to as the scalar grid. The remaining variables are the two components of the velocity vector. It would also be possible to define them at the same grid points, but since the velocity is a result

of pressure gradient, it is highly preferable to calculate velocity components at locations between the nodal points used for pressure. These velocity components are then calculated at points midway between the scalar nodes; these points are on the boundaries of the scalar control volumes.

One of the advantages of using such a "staggered" grid system is that it simplifies computation of mass flux into and out of the scalar grid control volume, as the velocity vectors  $u$  and  $v$  are located at the control volume boundary itself (Figure 4.1). The control volume of a node is the space which surrounds the node and is bounded midway between neighbouring nodes. It is important to note that, if the control volume is defined for  $\phi$ , then  $\phi$  should be at the center of the control volume. Hence, the control volumes for  $u$  and  $v$  are not the same as those for the scalar variables.

Two different approaches may be used to specify the physical boundaries, i.e. for the physical boundaries to coincide with the scalar grid lines or with the boundaries of the scalar grid control volumes. Since the boundary conditions for velocity are usually well specified at the physical boundaries, and since the specification of the physical properties at the wall boundaries is difficult and ambiguous, the physical boundaries are therefore chosen to coincide with the scalar control volume boundaries.

Having specified the variables and the control volume, one can integrate the transport equation over the corresponding volume, inside which the scalar properties are assumed constant. Thus:

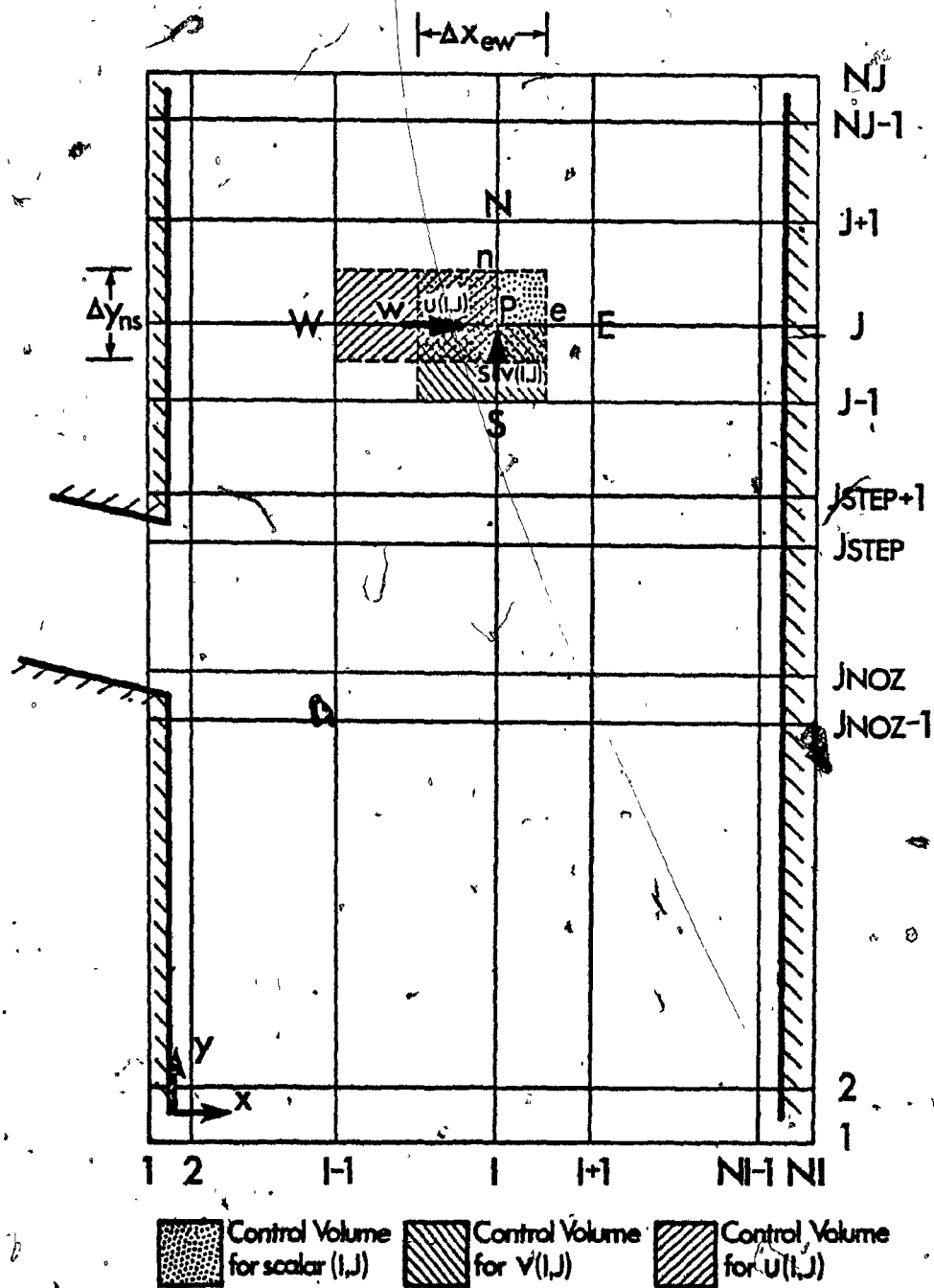


Figure 4.1 : Finite difference mesh.

$$\iiint_V \left[ \frac{\partial}{\partial x} (\rho u \phi) + \frac{\partial}{\partial y} (\rho v \phi) - \frac{\partial}{\partial x} \left( \Gamma \frac{\partial \phi}{\partial x} \right) - \frac{\partial}{\partial y} \left( \Gamma \frac{\partial \phi}{\partial y} \right) \right] dV = \iint_V S_\phi dV$$

using Gauss' theorem gives then:

$$\int_s^n \left[ \rho u \phi - \Gamma \frac{\partial \phi}{\partial x} \right] dy \Big|_{\text{boundary surface}} + \int_w^e \left[ \rho v \phi - \Gamma \frac{\partial \phi}{\partial y} \right] dx \Big|_{\text{boundary surface}} - \iint_V S_\phi dV = 0$$

that is:

$$\int_s^n \left[ (\rho u \phi - \Gamma \frac{\partial \phi}{\partial x})_e - (\rho u \phi - \Gamma \frac{\partial \phi}{\partial x})_w \right] dy + \int_w^e \left[ (\rho v \phi - \Gamma \frac{\partial \phi}{\partial y})_n - (\rho v \phi - \Gamma \frac{\partial \phi}{\partial y})_s \right] dx - \iint_V S_\phi dV = 0$$

or in short:

$$F_e - F_w + F_n - F_s - \iint_V S_\phi dV = 0 \quad (4.1)$$

where the subscripts e, w, n, s indicate that the corresponding quantities are evaluated at the east, west, north and south boundaries respectively,

and:

$$F_e = \int_s^n (\rho u \phi - \Gamma \frac{\partial \phi}{\partial x})_e dy \quad (4.2)$$

$$F_w = \int_s^n (\rho u \phi - \Gamma \frac{\partial \phi}{\partial x})_w dy \quad (4.3)$$

$$F_n = \int_w^e (\rho v \phi - \Gamma \frac{\partial \phi}{\partial y})_n dx \quad (4.4)$$

$$F_s = \int_w^e \left( \rho v \phi - \Gamma \frac{\partial \phi}{\partial y} \right)_s dx \quad (4.5)$$

$(F_e - F_w)$  represents the net flux of  $\phi$  in the x-direction through the control volume, and  $(F_n - F_s)$  represents the net flux in the y-direction. All geometric quantities and locations relative to an arbitrary scalar control volume are shown on Figure 4.1.

Equation (4.1) can be re-written under the more compact form

$$\sum_i F_i = S_{\phi, vol} \quad (4.6)$$

where  $\sum_i$  denotes summing over n, s, e, and w with appropriate signs,

$$\text{and } S_{\phi, vol} = \iiint_V S_{\phi} dV$$

By assuming that transport across the control volume boundary is one-dimensional, flux expressions derived from one-dimensional flow theory can be used (C5), and approximate solutions for the  $F$  terms can be obtained (C5, S12). For the particular case of  $F_w$  this solution takes the following form:

$$F_w = \rho_w u_w A_w \left[ f_w \phi_w + (1 - f_w) \phi_p \right] \quad (4.7)$$

$$\text{where } f_w = \text{EXP}(Pe)_w / [\text{EXP}(Pe)_w - 1]$$

$$(Pe)_w = \rho_w u_w \Delta x_{ew} / \Gamma_w$$

$$A_w = \Delta y_{ns} \quad 1.0 \quad (\text{area of the boundary perpendicular to x-direction})$$

$$\rho_w = (\rho_w + \rho_p) / 2$$

$$\Gamma_w = (\Gamma_w + \Gamma_p) / 2$$

Note that the points P and W, and the intervals  $\Delta x_{ew}$  and  $\Delta y_{ns}$  as shown on Figure 4.1 correspond to the particular case of transport of a scalar variable; thus, when  $\phi$  is a scalar (temperature, T, for example) the subscripts of Equation (4.7) correspond to points indicated on Figure 4.1. But if  $\phi$  is a vector quantity, the figure will not be the same. For  $\phi = u$ , for example, the control volume to be considered is translated to the west since control volumes for vector and scalar variables do not coincide. Hence, when Equation (4.7) applies to the u-velocity, the subscripts represent points indicated on Figure 4.2. Similarly, when  $\phi = v$  the domain of interest is shifted to the south and the points corresponding to the subscripts of Equation (4.7) have to be determined with respect to the v-velocity control volume of integration.

The calculation of exponentials by digital computers is relatively slow. However, by using Spalding's method (S12) to express exponentials in a simple algebraic form, computer time can be reduced. For any x,

$$\frac{e^x}{e^x - 1} \approx \begin{cases} \frac{1}{2} \left( 1 + \frac{2}{x} \right) & \text{if } |x| < 2 \\ 1 & \text{if } x > 2 \\ 0 & \text{if } x < -2 \end{cases}$$

Using this approximation and letting  $a_w = \rho_w u_w A_w f_w$ , and  $C_w = \rho_w u_w A_w$  Equation (4.7) becomes:

$$F_w = a_w (\phi_w - \phi_p) + C_w \phi_p \quad (4.8)$$

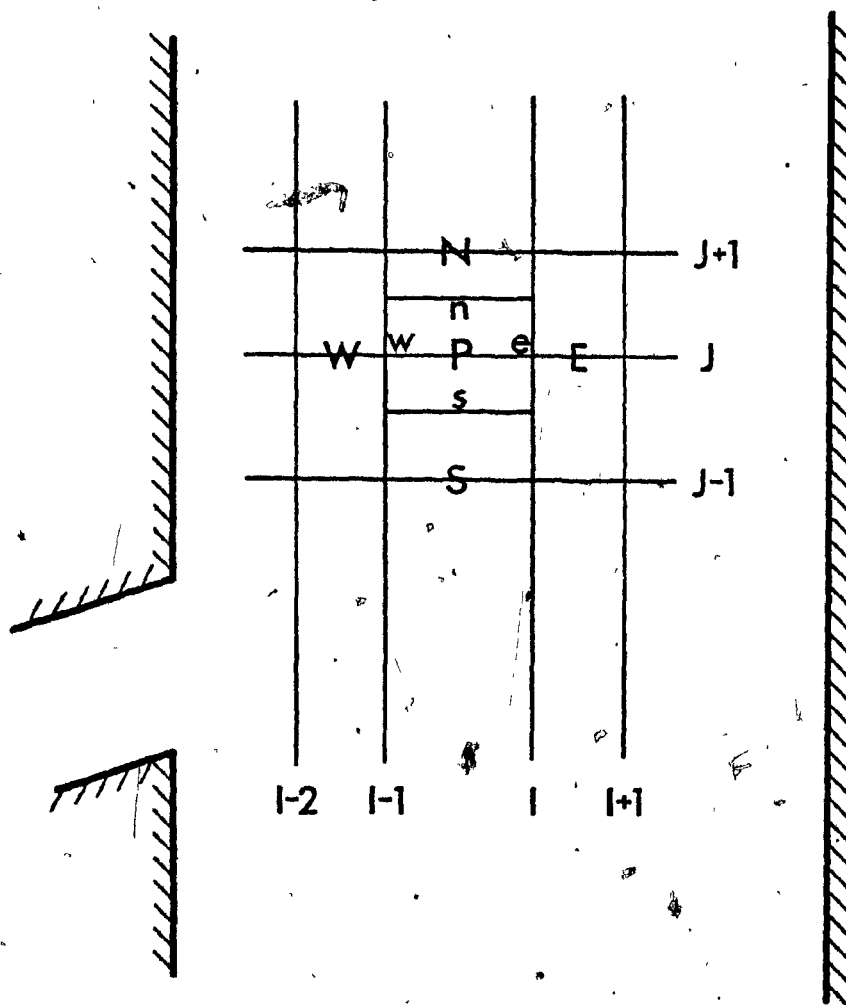


Figure 4.2 : Geometrical subscripts for a  $u$ -velocity control volume.

where

$$a_w \approx \begin{cases} C_w \left( \frac{1}{2} + (Pe)_w^{-1} \right) & \text{when } |(Pe)_w| < 2 \\ C_w & \text{when } (Pe)_w > 2 \\ 0 & \text{when } (Pe)_w < -2 \end{cases}$$

Further, letting  $D_w = r_w A_w / \Delta x_{ew}$ ,  $(Pe)_w$  becomes  $C_w / D_w$  and the coefficient  $a_w$  becomes:

$$a_w \approx \begin{cases} C_w/2 + D_w & \text{when } |C_w/2| < D_w \\ C_w & \text{when } C_w/2 > D_w \\ 0 & \text{when } C_w/2 < -D_w \end{cases} \quad (4.9)$$

The various expressions of  $a_w$  in Equation (4.9) can be expressed conveniently as:

$$a_w = \max ( |C_w/2|, D_w ) + \frac{1}{2} C_w \quad (4.10)$$

It should be recalled that, by definition, the  $C$  and  $D$  terms represent convective transfer and transfer due to diffusion, respectively. The parameter  $Pe$ , the local Peclet number is the ratio of convective to diffusive transfer rates. One can then see that, according to Equation (4.8) the total flux across the boundaries into or out of the control volume is a weighted sum of the convective and diffusive fluxes. For a local Peclet number between  $-2$  and  $2$  both convective and diffusive

transfers are of comparable magnitude. However, as the convective transfer increases and becomes dominant, the diffusive transfer can be neglected. Thus, for large  $|C_w|$ , the flux  $F_w$  across the west side boundary of the control volume becomes either  $C_w \phi_w$  or  $C_w \phi_p$  depending on the direction of the flow.

Equations (4.8) and (4.9) may be regarded as a hybrid of central and upwind-difference schemes (C5) in that they reduce to the former when  $|Pe|$  is less than 2 but yield the latter for large values of  $|Pe|$ . Recalling the conclusion of Spalding (S12) that the upwind-difference scheme is preferable to the central-difference scheme when  $|Pe|$  is greater than 2, it can be seen that the present hybrid scheme has the advantages of being more accurate over a wide range of Peclet numbers than either of its components (R3), and of yielding a diagonally-dominant matrix of coefficients for all  $Pe$ .

Applying the same principle for the other  $F$  terms and summing over  $n, s, e$  and  $w$  yields:

$$\sum_i F_i = \sum_i a_i \phi_i + \phi_p \sum_i C_i - \phi_p \sum_i a_i \quad (4.11)$$

from definition of the  $C$  terms, it appears that the term  $\sum_i C_i$  on the right hand side of Equation (4.11) represents the total mass accumulation in the control volume. From the continuity equation it can be equated to zero. Therefore:

$$\sum_i F_i = \sum_i a_i \phi_i - \phi_p \sum_i a_i \quad (4.12)$$

Substituting into the original transport equation (in the form of Equation (4.6)) yields the following equation:

$$\sum_i a_i \phi_i - \phi_P \sum_i a_i = S_{\phi, vol}$$

where the subscript  $i$  stands for e, n, w and s, and  $S_{\phi, vol} = \iiint_V S_{\phi} dV$  is the integral of the source term over the control volume.

#### 4.2 CALCULATION OF THE SOURCE TERMS

The evaluation of  $S_{\phi, vol}$  for  $u$ ,  $v$ ,  $m$  and  $T$  is necessary. Each of them are obtained by integrating the corresponding  $S_{\phi}$ 's over the control volume, and the final form is expressed in a finite difference form.

##### 4.2.1 x-component of velocity vector, $u$

$$S_{\phi, vol}^u = \iiint_V S_u dx dy \quad 1.0$$

$$S_{\phi, vol}^u = 1.0 \left\{ \iiint \left[ -\frac{\partial P}{\partial x} + \frac{1}{3} u \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \rho g \right] dx dy \right\}$$

and with the equation of conservation of mass:

$$S_{\phi, vol}^u = 1.0 \left\{ \iiint \left[ -\frac{\partial P}{\partial x} - \frac{1}{3} \frac{u}{\rho} \frac{\partial}{\partial x} \left( u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} \right) + \rho g \right] dx dy \right\}$$

$$S_{\phi, vol}^u = 1.0 \left\{ \int \left[ \int - \frac{\partial P}{\partial x} dx \right] dy - \frac{1}{3} \frac{\mu}{\rho} \int \left[ \int \frac{\partial}{\partial x} \left( u \frac{\partial \rho}{\partial x} \right) dx \right] dy \right. \\ \left. + \frac{1}{3} \frac{\mu}{\rho} \int \left[ \int \frac{\partial}{\partial x} \left( v \frac{\partial \rho}{\partial y} \right) dx \right] dy + \iint \rho g dx dy \right.$$

$$S_{\phi, vol}^u = \int (P|_e^w) dy + \frac{1}{3} \frac{\mu}{\rho} \int \left( u \frac{\partial \rho}{\partial x} \right) |_e^w dy - \frac{1}{3} \frac{\mu}{\rho} \int \left( v \frac{\partial \rho}{\partial y} \right) |_e^w dy \\ + (\rho g) |_p \iint dx dy \quad 1.0$$

$$S_{\phi, vol}^u = 1.0 \left\{ P|_e^w \Delta y_{ns} + \frac{1}{3} \left[ \frac{\mu}{\rho} \left( u \frac{\Delta \rho}{\Delta x} + v \frac{\Delta \rho}{\Delta y} \right) \right] |_e^w \Delta y_{ns} + (\rho g) |_p \Delta x_{ew} \Delta y_{ns} \right\} \quad (4.14)$$

note that the subscripts e, w, n, s and P are relative to the control volume of u.

#### 4.2.2 y-component of velocity vector, v.

$$S_{\phi, vol}^v = 1.0 \iint_V S_v dx dy$$

$$S_{\phi, vol}^v = 1.0 \iint \left\{ - \frac{\partial P}{\partial y} - \frac{1}{3} \mu \frac{\partial}{\partial y} \left[ \frac{1}{\rho} \left( u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} \right) \right] \right\} dx dy$$

$$S_{\phi, vol}^v = 1.0 \left\{ \int \left[ \int - \frac{\partial P}{\partial y} dy \right] dx - \int \left[ \int \frac{1}{3} \mu \frac{\partial}{\partial y} \left( \frac{1}{\rho} u \frac{\partial \rho}{\partial x} + \frac{1}{\rho} v \frac{\partial \rho}{\partial y} \right) dy \right] dx \right\}$$

$$S_{\phi, vol}^V = 1.0 \left\{ \int P \Big|_n^s dx + \int \left[ \frac{1}{3} \mu \left( \frac{1}{\rho} u \frac{\partial \rho}{\partial x} + \frac{1}{\rho} v \frac{\partial \rho}{\partial y} \right) \Big|_n^s \right] dx \right\}$$

$$S_{\phi, vol}^V = 1.0 \left\{ P \Big|_n^s \Delta x_{ew} + \frac{1}{3} \left( \frac{\mu}{\rho} \left( u \frac{\Delta \rho}{\Delta x} + v \frac{\Delta \rho}{\Delta y} \right) \right) \Big|_n^s \Delta x_{ew} \right\} \quad (4.15)$$

note that the subscripts e, w, n, s are relative to the control volume of v.

#### 4.2.3 Energy equation

$$S_{\phi, vol}^T = 1.0 \iint_V S_T dx dy$$

$$S_{\phi, vol}^T = 1.0 \iint \left\{ \delta \rho \frac{C_{pA} - C_{pB}}{C_p} \left[ \frac{\partial m_A}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial m_A}{\partial y} \frac{\partial T}{\partial y} \right] - \lambda \left[ \frac{\partial T}{\partial x} \frac{\partial}{\partial x} \left( \frac{1}{C_p} \right) + \frac{\partial T}{\partial y} \frac{\partial}{\partial y} \left( \frac{1}{C_p} \right) \right] \right\} dx dy$$

$$S_{\phi, vol}^T = 1.0 \left\{ \delta \rho \frac{C_{pA} - C_{pB}}{C_p} \left[ \frac{\Delta m_A}{\Delta x} \frac{\Delta T}{\Delta x} + \frac{\Delta m_A}{\Delta y} \frac{\Delta T}{\Delta y} \right] - \lambda \left[ \frac{\Delta T}{\Delta x} \frac{\Delta}{\Delta x} \left( \frac{1}{C_p} \right) + \frac{\Delta T}{\Delta y} \frac{\Delta}{\Delta y} \left( \frac{1}{C_p} \right) \right] \right\} \Delta x \Delta y \quad (4.16)$$

#### 4.2.4 Species Equation

$$S_{\phi, vol}^{m_A} = \iint_V S_m dy dx = 0 \quad (4.17)$$

#### 4.2.5 Modification of source term for stability

The finite difference equations derived in the previous section constitute a set of non-linear algebraic equations to be solved iteratively. The stability of the iterative solution scheme must then be examined.

Consider a set of algebraic equations of the form

$$x_i = \sum_{j \neq i} (a_{ij} x_j) + b_i$$

where the  $x$ 's are any unknown variable vectors. If the equations are linear, i.e.  $a$ 's and  $b$ 's are constants, the use of algebraic equations as substitution formulae in iterative procedures is stable and yields convergent solutions when

$$\left. \begin{array}{l} \sum_{j \neq i} |a_{ij}| < 1 \quad \text{for each } i \\ \text{and } \sum_{j \neq i} |a_{ij}| < 1 \quad \text{for at least one } i \end{array} \right\} \quad (4.18)$$

For non-linear equations, these conditions are often sufficient, although they may not be the necessary condition for convergence (C5).

In order that our system of equations satisfies these conditions, and also to avoid numerical problems when the coupling coefficients are zero during an iteration, we will introduce a false source term,  $S_{\text{false}}$ , into Equation (4.13) as follows:

$$S_{\text{false}} = \max(\text{net mass outflow from control volume}, 0.0) (\phi_p' - \phi_p)$$

$$\text{or } S_{\text{false}} = \max(\sum_i C_i, 0.0) (\phi_p' - \phi_p)$$

where the superscript prime indicates values from the previous iteration.

Note that  $S_{\text{false}}$  will not affect the final results for converged solutions.

$$\phi_p \sum_i a_i = \sum_i a_i \phi_i - S_{\phi, \text{vol}} + \max(\sum_i C_i, 0.0) (\phi_p' - \phi_p)$$

$$\text{or } \phi_p \left\{ \sum_i a_i + \max(\sum_i C_i, 0.0) \right\} = \sum_i a_i \phi_i - S_{\phi, \text{vol}} + \phi_p' \max(\sum_i C_i, 0.0)$$

$$\text{or } \phi_p A_p = \sum_i a_i \phi_i + S_u \quad (4.19)$$

$$\text{where } A_p = \sum_i a_i + \max(\sum_i C_i, 0.0)$$

$$S_u = -S_{\phi, \text{vol}} + \max(\sum_i C_i, 0.0) \phi_p' \quad (4.20)$$

and since  $\max(\sum_i C_i, 0.0)$  can be only positive or equal to zero, the conditions in (4.18) are satisfied.

The finite difference form of  $S_u$  for the variables are summarized in Table 4.1.

### 4.3 PRESSURE CALCULATION

Although there is no simple means of estimating the pressure terms required for solution of the momentum equations, this difficulty may be overcome by making use of the continuity equation. The net accumulation of mass inside each control volume is calculated and the local pressure is adjusted at successive iterations so that velocities are consistent with conservation of mass in that control volume. This procedure is carried out by: (1), defining a new dependent variable  $\Delta P$ , the pressure correction term, such that  $P = P' + \Delta P$  where  $P'$  is an estimated value of pressure resulting from the previous iteration, and  $P$  is the actual local pressure. (2), substituting  $(P' + \Delta P)$  for  $P'$  in the source term of the u-momentum equation. (3), subtracting the u-equation from the u'-equation. This u'-equation, although it belongs to the same iteration as the u-equation, results from the value of pressure of the previous iteration, i.e.  $P'$ .

$$a_p(u_p - u_p') = \sum_i a_i(u_i - u_i') + 1.0 (\Delta P_w - \Delta P_e) \Delta y_{ns}$$

For approximately converged results,  $u_i \approx u_i'$  for  $i = e, w, s, n$ , one can for simplicity assume  $u_i - u_i' = 0$ . Hence:

$$a_p(u_p - u_p') = 1.0 \Delta y_{ns} (\Delta P_w - \Delta P_e)$$

$$\text{or } u_p = u_p' + (\Delta P_w - \Delta P_e) \frac{\Delta y_{ns}}{a_p} \quad (4.21)$$

$$\text{where } \frac{\Delta y_{ns}}{a_p} = 1.0 (\Delta y_{ns}) / a_p$$

From its definition,  $DU_p$  can be visualised as the derivative of velocity with respect to the pressure change from iteration to iteration.

In a similar manner one obtains:

$$v_p = v_p' + (\Delta P_s - \Delta P_n) \cdot DV_p \quad (4.22)$$

where  $DV_p = 1.0 (\Delta x_{ew}) / \delta_p$

Similar expressions could be obtained for all surrounding nodes, and noting that the nodal points for pressure lie on the control volume boundaries of  $u$  and  $v$

$$\begin{aligned} u_E &= u_E' + (\Delta P_p - \Delta P_E) DU_E \\ u_W &= u_W' + (\Delta P_W - \Delta P_p) DU_W \\ v_N &= v_N' + (\Delta P_p - \Delta P_N) DV_N \\ v_S &= v_S' + (\Delta P_S - \Delta P_p) DV_S \end{aligned} \quad (4.23)$$

Now, from continuity equation:

$$(\rho u A)_W + (\rho u A)_S - (\rho u A)_E - (\rho u A)_N = 0 \quad (4.24)$$

where  $A$  stands for the cross sectional areas of the control volume boundaries. Substitution of Equation (4.23) into Equation (4.24) gives:

$$\begin{aligned}
& (\rho u^* A)_W + (\rho v^* A)_S - (\rho u^* A)_E - (\rho v^* A)_N \\
& + (\rho A DU)_W (\Delta P_W - \Delta P_P) + (\rho A DV)_S (\Delta P_S - \Delta P_P) \\
& + (\rho A DU)_E (\Delta P_E - \Delta P_P) + (\rho A DV)_N (\Delta P_N - \Delta P_P) = 0
\end{aligned}$$

Rearranging,

$$\begin{aligned}
& \left[ (\rho A DU)_E + (\rho A DU)_W + (\rho A DV)_N + (\rho A DV)_S \right] \Delta P_P \\
& = (\rho A DU)_E \Delta P_E + (\rho A DU)_W \Delta P_W + (\rho A DV)_N \Delta P_N + (\rho A DV)_S \Delta P_S - \sum_i C_i
\end{aligned}$$

where  $\sum_i C_i$  is the net accumulation of mass.

At this point one can see that this equation for  $\Delta P$  also fits the general form of the algebraic Equation (4.19):

$$\phi_P (A_P) = \sum_i a_i \phi_i + Su$$

where  $\phi = \Delta P$ ,  $a = \rho A DU$ ,  $A_P = \sum_i a_i$ , and  $Su = - \sum_i C_i$

This enables determination of  $\Delta P$  using the same solution algorithm as the one used for the other variables. After solving for  $\Delta P$ , one can then update the velocity components by Equations (4.21) and (4.22). In the present simulation study it is rather the pressure difference that is of interest; this pressure difference is calculated with reference to the jet nozzle pressure.

Table 4.1  
Definition of Su

$$\phi_p \left\{ \sum_i a_i + \max \left( \sum_i C_i, 0.0 \right) \right\} = \sum_i a_i \phi_i + Su$$

where  $Su = \phi_p \max \left( \sum_i C_i, 0.0 \right) - S_{\phi, vol}$

| $\phi$ | $S_{\phi, vol}$                                                                                                                                                                                                                                                                                                                                                                                       |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| u      | $P_e^w \Delta y_{ns} + \frac{1}{3} \left[ \frac{\mu}{\rho} \left( u \frac{\Delta \rho}{\Delta x} + v \frac{\Delta \rho}{\Delta y} \right) \right] \Big _e^w \Delta y_{ns} + (\rho g) \Big _p \Delta x_{ew} \Delta y_{ns}$                                                                                                                                                                             |
| v      | $P_n^s \Delta x_{ew} + \frac{1}{3} \left[ \frac{\mu}{\rho} \left( u \frac{\Delta \rho}{\Delta x} + v \frac{\Delta \rho}{\Delta y} \right) \right] \Big _n^s \Delta x_{ew}$                                                                                                                                                                                                                            |
| T      | $\delta p \frac{C_{pA} - C_{pB}}{C_p} \left[ \frac{\Delta m_A}{\Delta x} \frac{\Delta T}{\Delta x} + \frac{\Delta m_A}{\Delta y} \frac{\Delta T}{\Delta y} \right] \Delta x \Delta y$<br>$- \lambda \left[ \frac{\Delta T}{\Delta x} \frac{\Delta}{\Delta x} \left( \frac{1}{C_p} \right) + \frac{\Delta T}{\Delta y} \frac{\Delta}{\Delta y} \left( \frac{1}{C_p} \right) \right] \Delta x \Delta y$ |
| $m_A$  | 0                                                                                                                                                                                                                                                                                                                                                                                                     |

#### 4.4 CLOSURE

The computational procedure used for solving the set of algebraic equations derived in this chapter is described and discussed in Chapter 5.

## CHAPTER 5

COMPUTATIONAL PROCEDURE5.1 INTRODUCTION

Implementation of a finite difference scheme requires discretization of the domain of interest. This discretization is achieved by selection of a grid which covers the whole domain. This grid distribution and its effect on accuracy of the solution are described in Section 5.2.

Finite difference methods always result in sets of algebraic equations, and when these are tridiagonal in nature, as in the present problem, solution is simpler than for a general set of algebraic equations. It will be seen in Section 5.3 that the iterative solution procedure used in the present study is based on the tridiagonal matrix algorithm (TDMA). The TDMA is described in Section 5.4. Section 5.5 is a discussion of convergence criteria and of the use of under-relaxation factors, while an outline of the computer program completes the chapter as Section 5.6.

5.2 GRID DESIGN

Size and distribution of the grid are of fundamental importance

in influencing not only accuracy of solution but also the convergence characteristics. In the present simulation the grids used are non-uniform, i.e. the spacing between grid lines is not constant. Thus the grid is finer in areas where gradients of the variables are large, and vice versa. Although the grid distribution is not the same for all runs, it is fixed at some predetermined pattern throughout each computation. Due to large differences in the flow field between the various cases studied, use of a single grid pattern throughout the present study would have been inappropriate and inefficient. As gradients of the flow field variables and of heat and mass transfer coefficients are high in the impingement region, a fine grid distribution is required here. Since location of the impingement region is a function of angle of impingement and strength of the crossflow, the grids used were adjusted according to these variables. Consequently, more than ten different grids were employed in the present study. However these grids are all of the same type, differing only by shifting the finer grid spacing zone to follow the impingement region.

Velocity, temperature and composition boundary layers build up along the impingement and confinement surfaces. Here the relatively large gradients of these variables require use of fine grid spacing. This is particularly important for the impingement plate along which heat and mass transfer are to be calculated. For example, the equation used for the case of heat transfer is

$$h A (T_j - T_p) = A k \left( \frac{dT_p}{dx} \right)_{x=x_p}$$

where  $A$  represents the area under consideration. And it is clear that the quantity  $(dT/dx)_{x=x_p}$  cannot be determined properly if the grid line adjacent to the plate is not placed sufficiently close. Numerical tests were made to ensure that further increases in fineness of the grid spacings adjacent to the impingement plate have no effect on the computed transfer rates. The grid spacing in the  $x$ -direction is thus small near the plate and larger in the central region. It may be mentioned here that the crossflow entrance velocity profile would be more accurately defined if the grid were also small in the central region, but multiplication of the grid points leads to a tremendous increase in computer time; there is a tradeoff between these two considerations. In view of the small nozzle-to-plate spacings considered here, the  $x$ -direction resolution chosen for the present study is however sufficient to define adequately the crossflow parabolic entrance velocity profile.

As to the grid spacing in the  $y$ -direction, it must be small in the area covering the nozzle so that the inlet jet profile is adequately defined, and in the impingement region to ensure accurate calculation of the stagnation point transfer rates. In the upstream region occupied by the crossflow, as well as in the downstream region near the outflow boundary, rates of change of the variables are small and a wider resolution has consequently been used.

A  $20 \times 40$  grid system was used in the case of a jet without crossflow, and  $20 \times 50$  grids were used in all computer runs involving the effect of a crossflow. A typical grid spacing distribution is listed in Appendix B

as part of the program output.

It has been mentioned earlier that better accuracy can be obtained through higher resolution, i.e. by using smaller grid line spacings. It is appropriate to recall here that this statement is subject to the following limitation: when a numerical scheme such as the one considered here is used on a digital computer, reducing the grid spacing will improve accuracy only up to a certain point after which smaller spacing yet will in fact reduce accuracy. This limitation is due to the fact that digital computers have a finite word length, i.e. that the resolution of numbers by the computer is finite. In computation of derivatives of a function, two numbers corresponding to the values of the function on two successive grid lines are subtracted and the result is divided by the grid spacing. Hence it is clear that in the case of small gradients and of very small spacing between the grid lines, a fixed round-off error eventually causes an increased error in the results.

Another factor, also related to the grid distribution, which affects solution accuracy is control volume size. As discussed in Section 4.1, the hybrid central and upwind differences method is based on the value of the Peclet number,  $Pe$ , which is computed locally at each grid point as follows:

$$Pe = \frac{\rho u \delta l}{\Gamma}$$

where  $\delta l$  is the length of the control volume. Hence the value of  $Pe$  depends not only on local fluid properties, but also on the size  $\delta l$  of the control volume, which in turn depends on grid spacing.

Clearly, it is possible to design a grid pattern such that the diffusion terms are neglected in the whole flow field, thus leading to unacceptable solutions. Since the velocity field is not known before computation, it is not possible to determine the local Peclet number and then design the grid distribution accordingly. A trial and error method must be used.

The grid spacing distribution used in the present study and shown in Appendix B is the result of taking all factors discussed above into consideration.

### 5.3 SOLUTION PROCEDURE

The finite difference equations derived in Chapter 4 constitute a set of strongly coupled non-linear algebraic equations. Basically the solution to these equations is obtained by determining values of the variables at each grid point from values at neighbouring points by means of an iterative procedure.

For a  $NI \times NJ$  grid mesh where  $NI$  and  $NJ$  are the numbers of grid lines in the  $x$ - and  $y$ -directions respectively, there are  $(NI-2) \cdot (NJ-2)$  algebraic equations for each of the five variables  $u$ ,  $v$ ,  $\Delta P$ ,  $T$ ,  $m_A$ , making a total of  $5 (NI-2) \cdot (NJ-2)$  simultaneous equations aside from the equations for the physical properties of the fluid which are functions of  $T$  and  $m_A$ . In the case of the present  $20 \times 50$  grid this makes 4320 equations, and if one were to solve for the unknowns using a classical matrix inversion method, the size of the matrix would be enormous and a tremendous amount of computer time would be required.

In the present case it is possible to simplify the problem by reducing the number of simultaneous equations to be solved from  $5(NI-2) (NJ-2)$  to  $(NI-2) (NJ-2)$ . This is done by solving separately for each of the five variables using the iterative cycle of steps listed below:

1. Guess the values of all variables,  $u, v, P, T, m_A$ .
2. Calculate the corresponding properties of the fluid (density, viscosity, etc.)
3. Assemble the coefficients of the  $u$ -momentum equation according to the guessed values, then solve for  $u$ .
4. Assemble the coefficients of the  $v$ -momentum equation according to the guessed values, then solve for  $v$ .
5. Calculate  $\Delta P$ , then  $P$  and update the  $u$  and  $v$  according to Equations (4.21) and (4.22).
6. Assemble the coefficients of the energy equation according to the guessed values, then solve for  $T$ .
7. Similarly for  $m_A$ .
8. Treat the modified variables as improved guesses and repeat steps 2 to 8 until the specified convergence criteria are satisfied.

Steps 2 to 7 constitute an iteration. In steps 3, 4, 5, 6, and 7, only the equations for individual  $\phi$ 's need be solved simultaneously. The method chosen to solve each time this set of  $(NI-2) \cdot (NJ-2)$  simultaneous equations uses the tridiagonal matrix algorithm (TDMA), whereby a set of equations each with exactly three unknowns in a particular order (except the first and last ones which have only two unknowns) may be solved sequentially.

In the present study the marching direction goes from confinement plate to impingement plate. This marching procedure is clearly preferable when crossflow is not present as information on the jet inflow to the flow field is provided at the beginning of the computation. In the case where a crossflow is present an alternative marching procedure (from crossflow entry to outflow boundary) could be used since it is the main-flow direction, but trials with this procedure did not result in any substantial computer time saving.

Values at grid nodes along a grid line  $J$  are taken to be unknown (values at  $W$ ,  $P$  and  $E$  for each point  $P$ ), but values at each  $N$  and  $S$  neighbour, i.e. variable along grid lines  $J-1$  and  $J+1$  are taken as known using the most recent values. The TDMA is then applied to this grid line by recasting the algebraic equations as presented in Equation (4.19) into the following form:

$$a_p \phi_p = (a_n \phi_N + a_s \phi_S + Su) + a_w \phi_W + a_e \phi_E$$

with the underlined quantities assumed unknown. This results in equations with three unknowns, except for nodes adjacent to boundaries where only two unknowns remain after specification of boundary conditions. Hence for each grid line there is  $(NI-2)$  equations with  $(NI-2)$  unknowns which can be solved by the TDMA as described in the next section.

It may be noted here that it is not necessary to obtain a fully converged solution for the matrix since the other variables have only approximate values. Therefore a limited number of sweeps from south to north is sufficient. Following the recommendation of Gosman et al. (G7),

only three sweeps were performed at each iteration for  $u$ ,  $v$ ,  $T$  and  $m_A$ , while five sweeps were used for  $\Delta P$ .

#### 5.4 TRIDIAGONAL MATRIX

The problem having been transformed into one represented by a tridiagonal matrix, the solution is obtained by iterating over the entire flow field and over the several variables. The TDMA will be rapidly presented here and the reader is referred to reference (C4) for more details relative to its practical application.

As seen in Section (5.3), the equations at hand are of the form:

$$a_p \phi_p = (a_n \phi_n + a_s \phi_s + Su) + a_w \phi_w + a_e \phi_e \quad (5.1)$$

where  $\phi_p$ ,  $\phi_w$  and  $\phi_e$  are the dependent variables while all other terms are assumed constant.

Letting

$$\begin{aligned} a &= a_e \\ b &= a_w \\ c &= a_n \phi_n + a_s \phi_s + Su \\ d &= a_p \end{aligned}$$

Equation (5.1) can be rewritten for each control volume as:

$$-b\phi_w + d\phi_p - a\phi_e = c$$

And for all control volumes in the same line one gets:

$$-b_2 \phi_1 + d_2 \phi_2 - a_2 \phi_3 = c_2$$

$$-b_3 \phi_2 + d_3 \phi_3 - a_3 \phi_4 = c_3$$

$$-b_i \phi_{i-1} + d_i \phi_i - a_i \phi_{i+1} = c_i$$

$$-b_{NI-1} \phi_{NI-2} + d_{NI-1} \phi_{NI-1} - a_{NI-1} \phi_{NI} = c_{NI-1}$$

(5.3)

or in matrix form:

$$\begin{bmatrix} d_2 & -a_2 & & & & \\ -b_3 & d_3 & -a_3 & & & \\ & -b_4 & d_4 & -a_4 & & \\ & & & & & \\ & & & -b_i & d_i & -a_i \\ & & & & & & \\ & & & & & -b_{NI-2} & d_{NI-2} & -a_{NI-2} \\ & & & & & & -b_{NI-1} & d_{NI-1} \end{bmatrix} \begin{bmatrix} \phi_2 \\ \phi_3 \\ \phi_4 \\ \vdots \\ \phi_i \\ \vdots \\ \phi_{NI-2} \\ \phi_{NI-1} \end{bmatrix} = \begin{bmatrix} c_2 + b_2 \phi_1 \\ c_3 \\ c_4 \\ \vdots \\ c_i \\ \vdots \\ c_{NI-2} \\ c_{NI-1} + a_{NI-1} \phi_{NI} \end{bmatrix}$$

which can be abbreviated as:  $[A][\phi] = [C]$ 

We know that  $A$  can be factored into two  $(NI-1) \cdot (NI-1)$  matrices,  $L$  and  $U$ , such that  $L$  and  $U$  are unique. So:

$$[A] = [L][U]$$

$$\text{and } [A] [\phi] = [L] [U] [\phi]$$

$$\text{Recalling that, } [A] [\phi] = [C]$$

$$\text{and letting } [U] [\phi] = [C'] \quad (5.4)$$

where  $C'$  is a column matrix with elements  $C_2, C_3, \dots, C_{NI-1}$ , yields:

$$[L] [C'] = [C] \quad (5.5)$$

Defining  $C_1 = \phi_1$ , all the  $C_i$ 's are given by Equation (5.5) and one can solve for the  $\phi$ 's using Equation (5.4).

## 5.5 UNDER-RELAXATION FACTORS AND CONVERGENCE CRITERIA

### 5.5.1 Under-relaxation factors

Large changes in the tridiagonal matrix coefficients from iteration to iteration might lead to divergence. To cope with this drawback the so-called under-relaxation method is used. If  $\phi_{p,k-1}$  and  $\phi_{p,k}$  are the values of  $\phi_p$  computed in the (k-1)-th and in the k-th iteration respectively, then the value actually used in the following iteration, say  $\phi_{p,k'}$ , is given by:

$$\phi_{p,k'} = \text{URF } \phi_{p,k} + (1 - \text{URF}) \phi_{p,k-1}$$

where URF is the under-relaxation factor which is a real number between 0 and 1.

### 5.5.2 Convergence Criteria

As seen earlier the numerical procedure entails two iterative loops. The first one scans the seven dependent variables, the second one repeatedly treats the flow field for individual variables. It has been seen that the second iteration loop does not require a converged solution. Therefore, only the criterion for the first iteration loop need be defined.

The iterative procedure is considered a converged solution if the absolute normalized residuals (Equation 5.6) for  $u$ ,  $v$ ,  $\Delta P$ ,  $T$  and  $m_A$  are all less than a prescribed small number  $CC$ , the convergence criterion. Thus:

$$RESID = \sum_{\substack{\text{all} \\ \text{nodes}}} |\sum_i a_i \phi_i + S_u - A_p \phi_p| / FACTOR \leq CC \quad (5.6)$$

where  $FACTOR$ , the normalization factor, will be interpreted later in this section. The term  $RESID$  as defined in Equation (5.6) is the difference between the left and right hand sides of the governing equations (4.19); hence for exact solutions  $RESID = 0$ . For the equations for  $u$  and  $v$ ,  $FACTOR$  equals the total inflow of jet and crossflow momentum; physically this means that the convergence criterion implies that the total accumulation of momentum inside the physical system has to be less than a prescribed percentage of the momentum inflow. Similarly, for  $\Delta P$ ,  $FACTOR$  represents the total inflow of mass through the jet and crossflow. For  $T$  and  $m_A$ ,  $FACTOR$  represents the product of total volumetric inflow and a characteristic difference in  $T$  or  $m_A$ , respectively

After testing the effect of a range of values of convergence criterion, CC, the value of this important computational parameter was set at 0.015. Figure (5.1) shows the values taken by velocity at a monitoring point in the flow field ( $I=18$ ,  $J=20$ ) along the iterative computation process. It is clearly evident that a large value for CC would give quite inaccurate results, while a lower value would result in an increase in computing time without yielding a discernable improvement in the velocity results. The transfer rate values are subject to the same behaviour. Choice of  $CC = 0.015$  was therefore clearly a good choice for the convergence criterion.

#### 5.6 OUTLINE OF THE COMPUTER PROGRAM

Figure (5.2) is a flow diagram showing the computational procedure. Figure (5.3) describes Subroutine CALCU which solves for the u-momentum equation; solution of all other equations follows identical steps, and Subroutines CALCV, CALCP, CALCT and CALCW are similar to Subroutine CALCU.

The grid distribution is set up by Subroutine GRID. Subroutine INIT computes subsequently all the geometric quantities relative to each control volume and stores them in arrays for subsequent reference. All variables are then initialized, and fluid properties ( $\rho$ ,  $\mu$ ,  $C_p$ ,  $\lambda$  and  $\delta$ ) are calculated by Subroutine PROPS and stored in arrays. The non-iterative boundary conditions are then prescribed, and a first output is printed describing the flow field as initialized.

One complete iteration is then performed by calling sequentially the subprograms CALCU, CALCV, CALCP, CALCT and CALCW which update  $u$ ,  $v$ ,  $\Delta P$ ,  $T$  and  $m_A$  values throughout the grid. Based on these updated values, the arrays containing the fluid properties are then modified by Subroutine PROPS. After each of these iterations several tests are

performed: if either the computer time spent or the number of iterations performed exceeds the prescribed values, the iteration procedure terminates automatically. Also the residuals RESID are checked for convergence after each iteration. If the RESID for the five equations ( $u$ ,  $v$ ,  $\Delta P$ ,  $T$  and  $m_A$ ) are all less than the convergence criterion, CC, the program is assumed to have reached the converged solution. The local transport coefficients are then calculated along the impingement plate from gradients at the wall of the  $v$ ,  $T$  and  $m_A$  distributions.

The line-by-line solution of the  $u$ -equation is performed in subprogram CALCU which contains loops for the computation of the convective terms  $C$  and of the diffusive terms  $D$ ; from these terms CALCU establishes the TDMA coefficients,  $a$ , according to Equation (4.10), and finally computes the source terms  $S_u$  for all nodes. Iterative boundary conditions are prescribed in subprogram MODU. After calculation of the  $u$ -residual, CALCU applies the under-relaxation factors and calls subroutine LISOLV. LISOLV performs the line-by-line solution procedure using the TDMA. The maximum number of sweeps (traverses) to be performed is designated by NSWPU in the flow diagram of Figure 5.3.

The complete program is listed in Appendix A, while a typical output listing will be found in Appendix B. Additional information on some aspects of this program can be found in reference (G7).

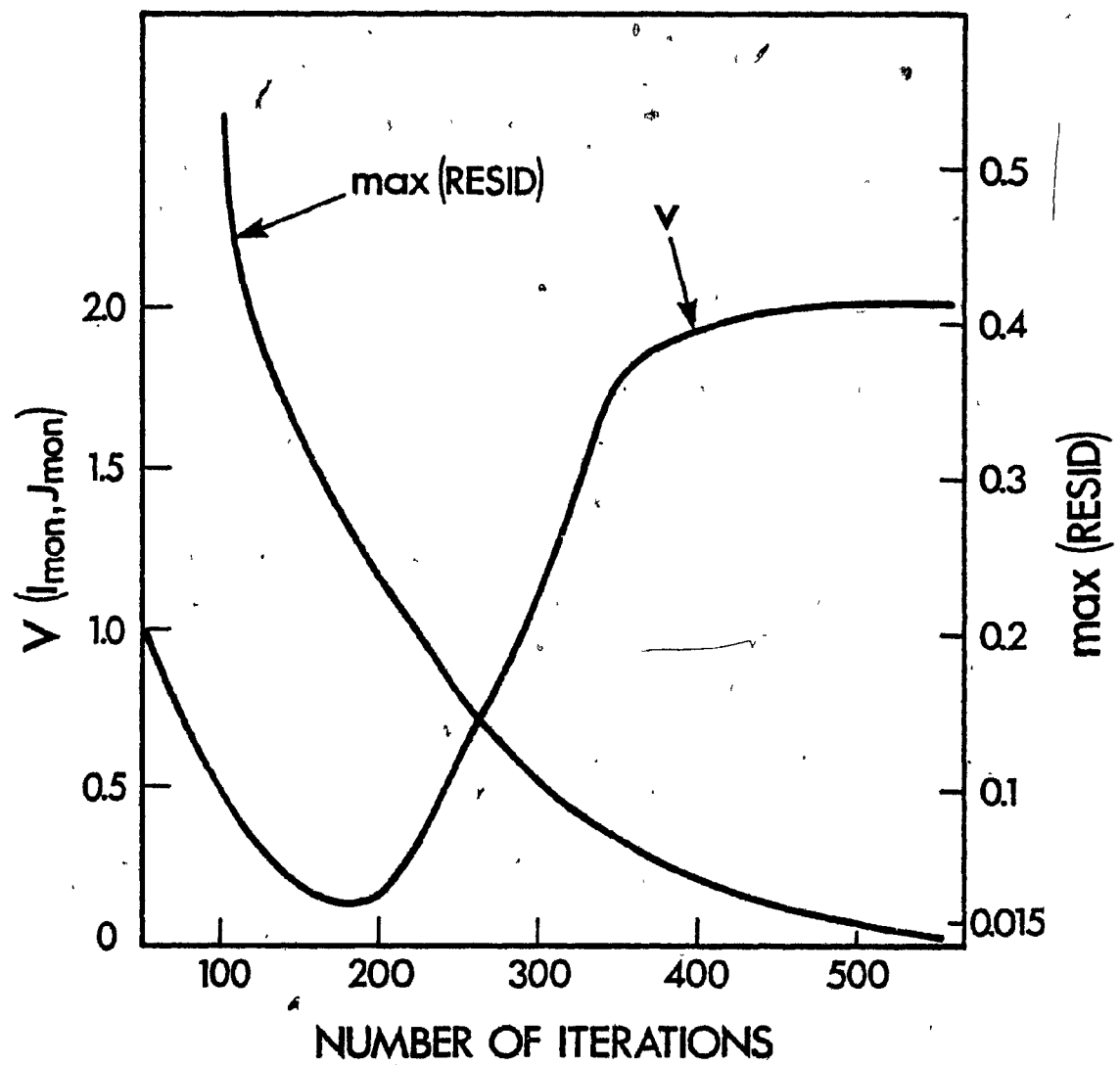


Figure 5.1 : Numerical variation of v-velocity result at monitoring point throughout the iteration cycle.

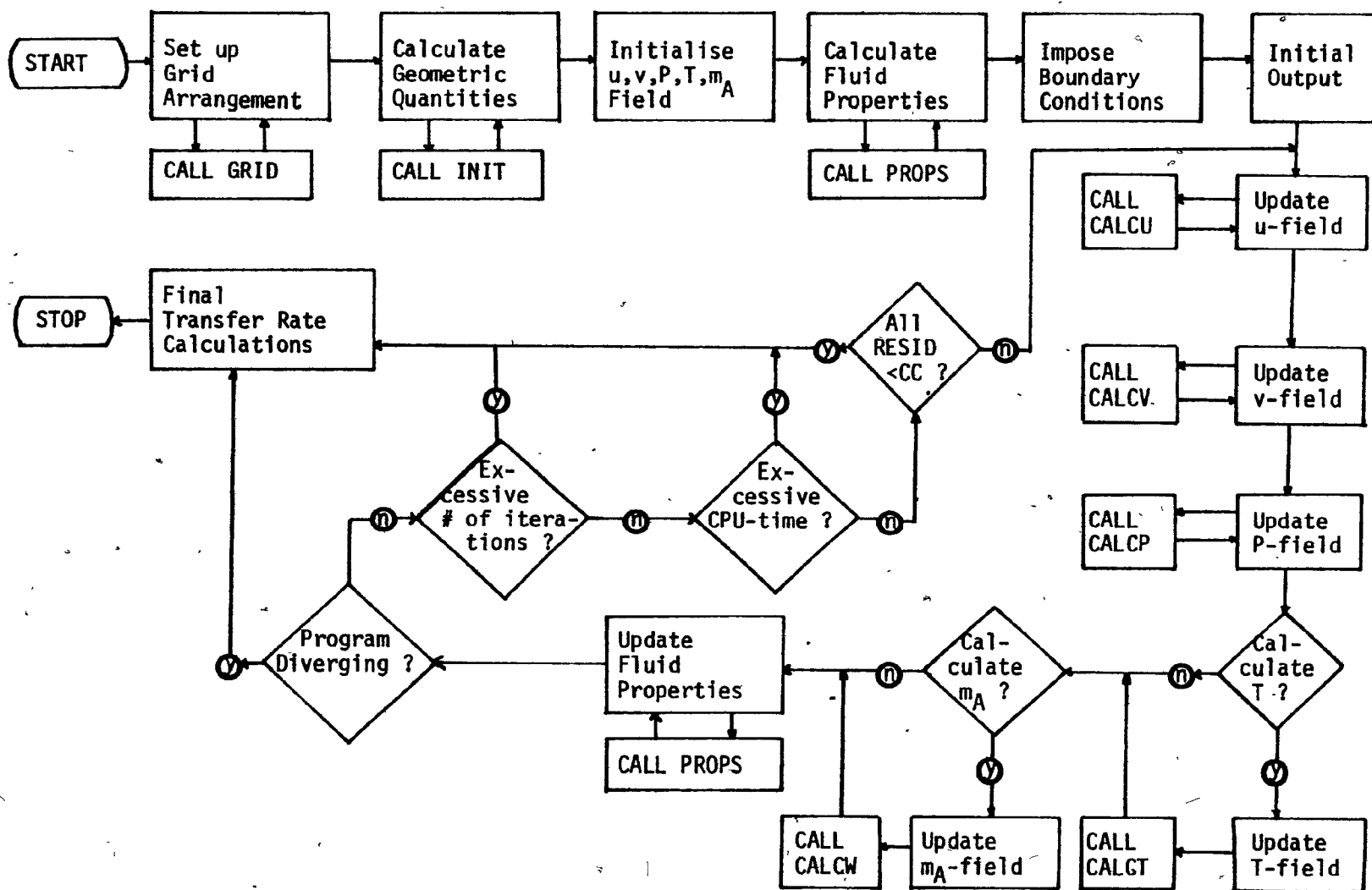


FIGURE 5.2 Computational Flow Diagram: Main Program

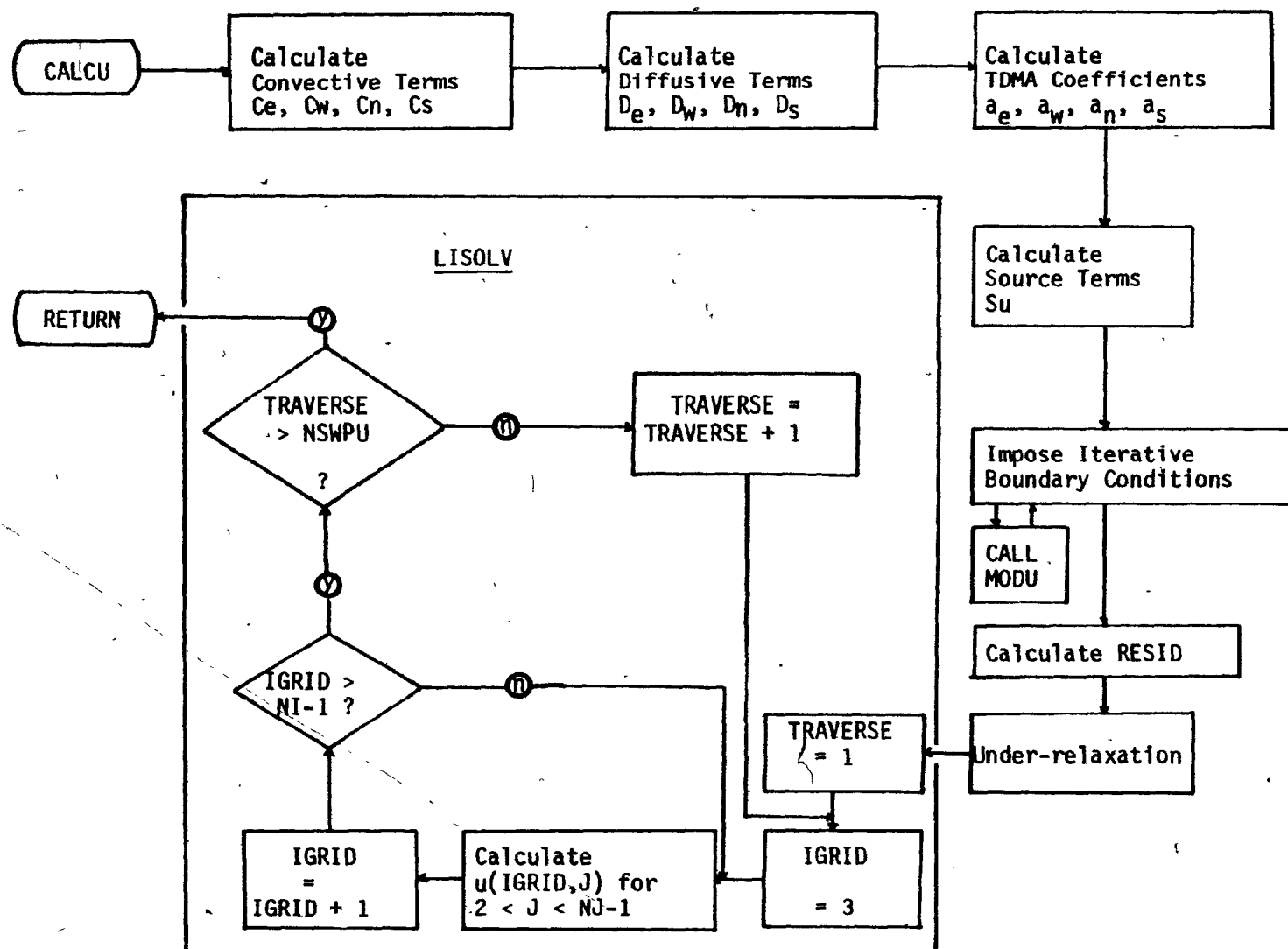


FIGURE 5.3 Computational Flow Diagram: CALCU

## CHAPTER 6

### RESULTS AND DISCUSSION

#### 6.1 RANGE OF PARAMETERS

Using the finite difference equations derived in Chapter 4 and the numerical procedures described in Chapter 5, a simulation of the flow, heat transfer and mass transfer characteristics was performed for a laminar semi-confined rectangular jet discharging perpendicularly or with an inclination in a crossflow and impinging on an isothermal wetted surface. Simulation runs of an identical jet impinging normally but without crossflow provide comparison between crossflow and non-crossflow cases. The results presented in this chapter include parameters of nozzle-to-plate spacing, effect of presence of a crossflow, ratio of crossflow to jet Reynolds numbers, and angle of impingement. For the latter variable the range of variation included jets inclined towards both the crossflow upstream and downstream direction (negative and positive angles, respectively).

The ranges of the parameters used in the present study are given in Table 6.1. A serial listing of all computer runs is given in Appendix C. A nozzle width of 3.2 mm and a fully developed parabolic velocity profile were selected as the base case for the numerical study. The Reynolds number of the jet at the nozzle exit is 1000, based on nozzle width, and is constant throughout. For all runs the impingement surface

Table 6.1Values of Parameters used in the Numerical Experiments

| Ratio of Reynolds<br>numbers of cross-<br>flow to jet | Nozzle-to-plate<br>spacing | Angle of inclination<br>of nozzle |
|-------------------------------------------------------|----------------------------|-----------------------------------|
| M                                                     | H/D                        | $\theta$                          |
| 0.01                                                  | 2                          | -45°                              |
| 0.1                                                   | 3                          | -30°                              |
| 0.167                                                 | 4                          | -10°                              |
| 0.25                                                  |                            | -5°                               |
| 0.4                                                   |                            | 0°                                |
| 0.5                                                   |                            | 5°                                |
| 0.7                                                   |                            | 10°                               |
| 1.0                                                   |                            | 30°                               |
| 2.0                                                   |                            |                                   |

was maintained at a temperature of 350°K, while the jet and the crossflow temperature was 450°K. The confinement plate temperature was also 450°K. The impingement surface temperature gives a mass fraction of water vapour along the plate,  $m_p$ , equal to 0.302. This value varies slightly between 0.3020 and 0.3023 from run to run due to its dependence on the local pressure which varies very slightly. The jet pressure at the nozzle exit is  $1.013 \times 10^5 \text{ N/m}^2$  (1 atm) and its water content for all runs is 0.1 mass fraction water. The crossflow enters parallel to the confinement and impingement plates with a fully developed velocity profile, but its entrance temperature and humidity are constant (flat temperature and humidity profiles). The crossflow entry (or southern edge of the plate) is situated at a distance 6.4 D from the center of the nozzle aperture.

The magnitude of the crossflow is defined by the ratio, M, of the crossflow volumetric flow-rate to the jet volumetric flow-rate:  $M = \bar{V}_H / \bar{U}D$ . Figure (6.21) on which the pressure difference between the crossflow entry pressure and the ambient pressure (or nozzle pressure) is plotted versus M shows that the overpressure due to the crossflow on this side is negligible with respect to the ambient pressure the order of magnitude of which is  $10^5$ . As the crossflow and the jet flow are of the same fluid (air) at the same temperature and water vapour content, and essentially the same pressure, the density and viscosity are also effectively the same. Thus M may also be regarded as the mass flow-rate ratio between the crossflow and the jet flow, or as the corresponding ratio of the Reynolds numbers of the crossflow to the jet flow.

The length of the plates on the outflow side was varied between 6.1 D and 8 D in order to have longer plates when the nozzle inclination angle has a large positive value ( $\theta = 30^\circ$  e.g.) and when the crossflow magnitude is high (e.g.  $M = 1.0$  or  $2.0$ ). This was done so that the transfer rate peaks would not be too close to the edge of the plates, which could have introduced an undesired interaction with the outflow boundary conditions. In retrospect this provision was in fact not necessary for the small H/D ratios studied here, but would be needed for study of larger values of H/D.

For cases without crossflow, the conclusions of Li (L2) concerning influence of plate lengths on transfer rates were used to reduce computer time. Li found that the effect of extending plate length from 4.7 D to 18 D is felt only over the outer parts of the plate, whereas the impingement region results are unaffected. Thus a length of 4.75 D was chosen in the present study for runs without crossflow. Only a few additional runs have been done with a larger plate (total length of 12.5 D) in order to compare the average value of the transfer coefficients with results obtained with crossflow.

Results in the literature are generally presented in non-dimensional form. In fact the governing equations themselves are commonly transformed into appropriate non-dimensional form by using suitable characteristic parameter values (e.g. Saad (S2), van Heiningen et al. (V2)). But choosing correct scaling parameters is difficult or even impossible when temperature and composition dependent physical property variation is allowed. Consequently, in the present study the governing equations have been solved in their dimensional form, and the dimensional results have

been converted subsequently into a non-dimensional form for presentation. Thus the lengths and distances are normalized with respect to the nozzle width, the jet flow conditions are expressed in terms of Reynolds number, while the heat and mass transfer results are presented in terms of Nusselt and Sherwood numbers respectively.

## 6.2 FLOW CHARACTERISTICS

### 6.2.1 Centerline Velocity Profile

#### 6.2.1.1 Without Crossflow:

In the case of the runs performed without crossflow we can state, in view of the results recorded in Section 2.2.1 for the potential core length, that for the close nozzle-to-plate spacings of the present study we are dealing with potential core type of flow up to the stagnation region adjacent to the impingement surface. This expectation is confirmed by the results of the present investigation presented in Figure 6.1 as decay of centerline axial velocity. The three solid lines show this velocity profile for the case without crossflow, for values of  $H/D = 2, 3$ , and  $4$ . As is typical for this type of flow, the  $u$ -velocity decreases only slightly with axial distance from the nozzle exit until the flow is about one nozzle width from the evaporating surface. Thereafter, axial velocity decreases rapidly to reach the value of the evaporation velocity at the impingement surface, which is not discernably different from zero on the scale of Figure 6.1. This evaporation velocity is directed away from the impingement surface and

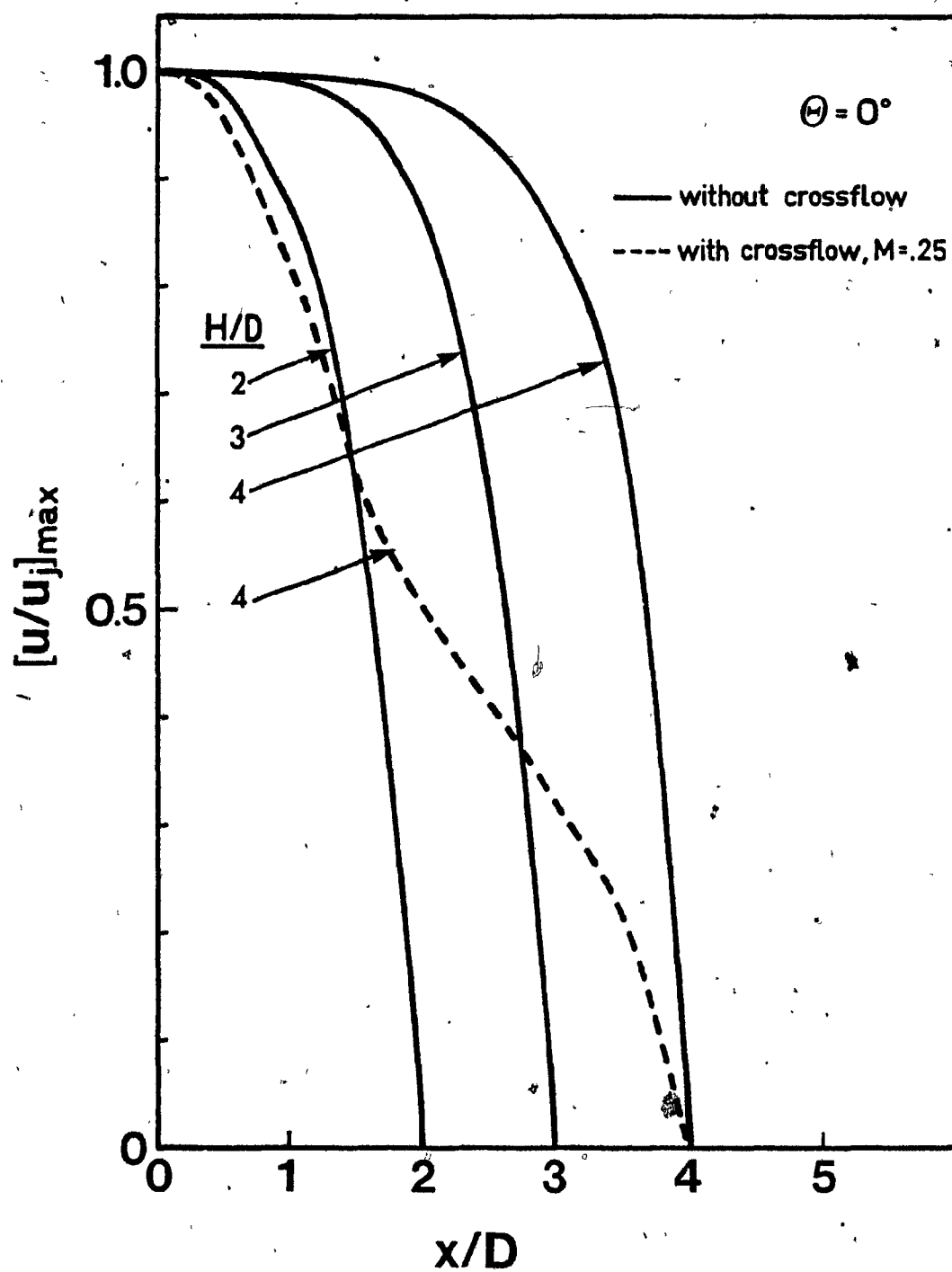


Figure 6.1 : Centerline axial velocity decay.

is thus negative with the sign conventions taken here. However, as this evaporation velocity is three orders of magnitude smaller than the jet velocity,  $(u/u_j)_{\max}$  seems on Figure 6.1 to reach zero, as it would were the plate non-evaporative. Negative values for  $u$ -velocity in the vicinity of the plate are of course printed on the computer outputs.

The axial velocity decay behaviour is consistent with the findings of previous investigators, including Giralt (G2), Saad (S2), Huang (H2) and Li (L2), who showed that, over a wide range of nozzle-to-plate spacing and Reynolds number, the centerline axial velocity of a round jet (both semi-confined and unconfined) is affected by the presence of the impingement plate at only about one nozzle diameter from the plate. This fact is further confirmed by the observation that the three curves for  $H/D = 2, 3$ , and 4 collapse effectively into a single curve before the effect of the impingement plate is sensed. This sudden decrease of centerline axial velocity near the impingement region is of course associated with the correspondingly large transformation of  $x$ -direction momentum into  $y$ -direction momentum.

#### 6.2.1.2 With Crossflow:

As it is the  $u$ -component of velocity which produces the desirable impingement characteristics of the flow field, it is of interest to note the effect of crossflow on the  $u$ -velocity. Comparison on Figure 6.1 of the axial decay profiles of  $u_{\max}$  for  $H/D = 4$ , with and without crossflow, indicates that the  $u_{\max}$  velocity profile is altered drastically by a crossflow of Reynolds number only 25% of that of the jet.

Since the jet trajectory is now deflected by the crossflow, the loci of maximum velocity are no longer on the nozzle centerline as is the case without crossflow. Thus the values of  $(u/u_j)_{\max}$  on this and subsequent graphs in the presence of crossflow represent the decay of the maximum  $u$ -component velocity situated along the central streamline of the deflected jet.

Because of the large change in the profile caused by a value of  $M$  as small as 0.25, a set of such profiles for a range of 4 values of  $M$  is shown on Figure 6.2. Although it might appear from Figure 6.2 that there is an anomalously abrupt change in profile between  $M=10\%$  and the comparative profile shown without crossflow, it should be realised that a profile for the case of crossflow with  $M = 0$  (not shown on Figure 6.2) would be a curve located much closer to that for  $M = 10\%$  than to the profile shown without crossflow. This effect of crossflow is due to interaction between the jet and crossflow starting right from the point where the jet discharges into the crossflow. Consequently, whereas  $u_{\max}$  decreases scarcely perceptibly (by ca. 1%) one nozzle width from the nozzle exit in the absence of crossflow,  $u_{\max}$  decreases by 15-20% over the same distance for  $M$  in the range 10-25%. Likewise, only 35-40% of the drop in  $u_{\max}$  occurs over the last one nozzle width distance above the evaporating surface, as compared to 85% of the drop in  $u_{\max}$  over this same region in the absence of crossflow.

Figure 6.2 shows that the greater the crossflow, the more rapidly the  $u_{\max}$  velocity decreases. The direction of this trend was to be expected since it is apparent that for very large values of  $M$ , the profile of Figure 6.2 would approach the limit of a vertical line; i.e.  $u_{\max}$  dropping almost to zero for a small value of  $x$  approaching zero.

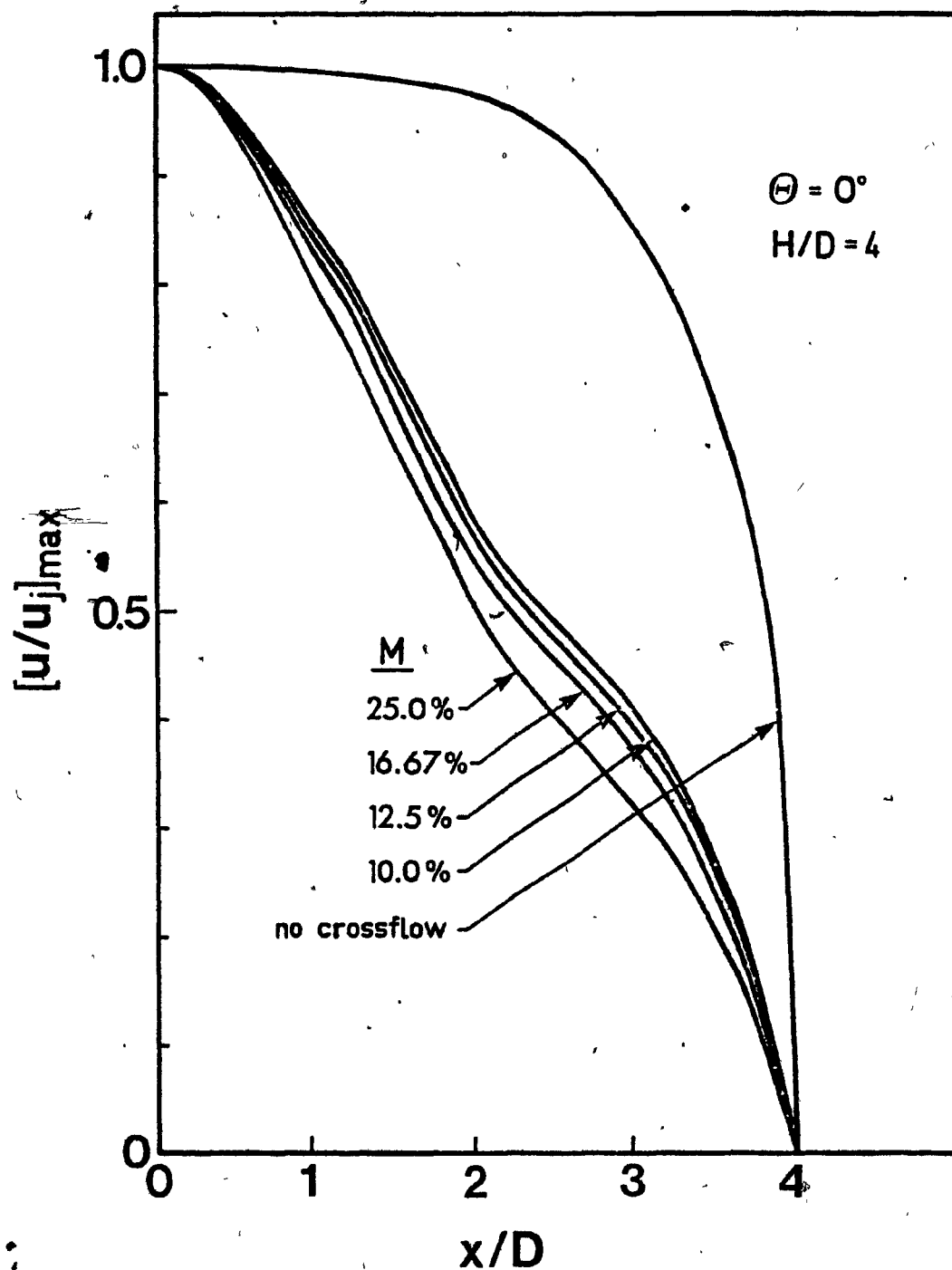


Figure 6.2 : Decay of jet maximum u-velocity for various crossflow magnitudes.

This limiting case corresponds to a strong crossflow affected imperceptibly by a negligible jet flow.

In view of the evidence from the flow field results that the effect of crossflow is to cause a sharp decrease of the  $u$ -velocity component which is the essence of the impingement process, one could already anticipate that the heat and mass transfer rates at the evaporating surface would be lower when a crossflow is present.

On Figure 6.3 velocity decay profiles are shown for the cases of an orthogonal nozzle and of a nozzle inclined at an angle of  $-10^\circ$  (i.e. directed slightly against the crossflow direction). In both cases a crossflow of  $M = 10\%$  is present. Since  $-10^\circ$  is a rather small inclination angle, the two profiles of  $u_{\max}$  are not greatly different. However, by reference to the curves for  $u_{\max}$  it is none the less clear that for  $\theta = -10^\circ$  the  $u_{\max}$  velocity decays less sharply than for  $\theta = 0^\circ$ . In other words, at a constant crossflow ( $M = 10\%$ ), a change in  $\theta$  from  $0^\circ$  to  $-10^\circ$  shifts the  $u$ -velocity profile somewhat closer to the profile without crossflow. Such a trend in the flow field would be expected to be accompanied by an increase in transfer rates at the impingement surface, as will indeed be confirmed subsequently. Decay of the  $u$ -velocity is due to transformation of the jet  $u$ -momentum into positive  $y$ -momentum by action of the crossflow. This transformation is seen as deflection of the jet towards the positive  $y$ -direction, and the onset of this deflection will obviously be retarded and less significant if the jet leaves the nozzle with a negative inclination angle.

Figure 6.3 also shows the decay of the total velocity in terms of

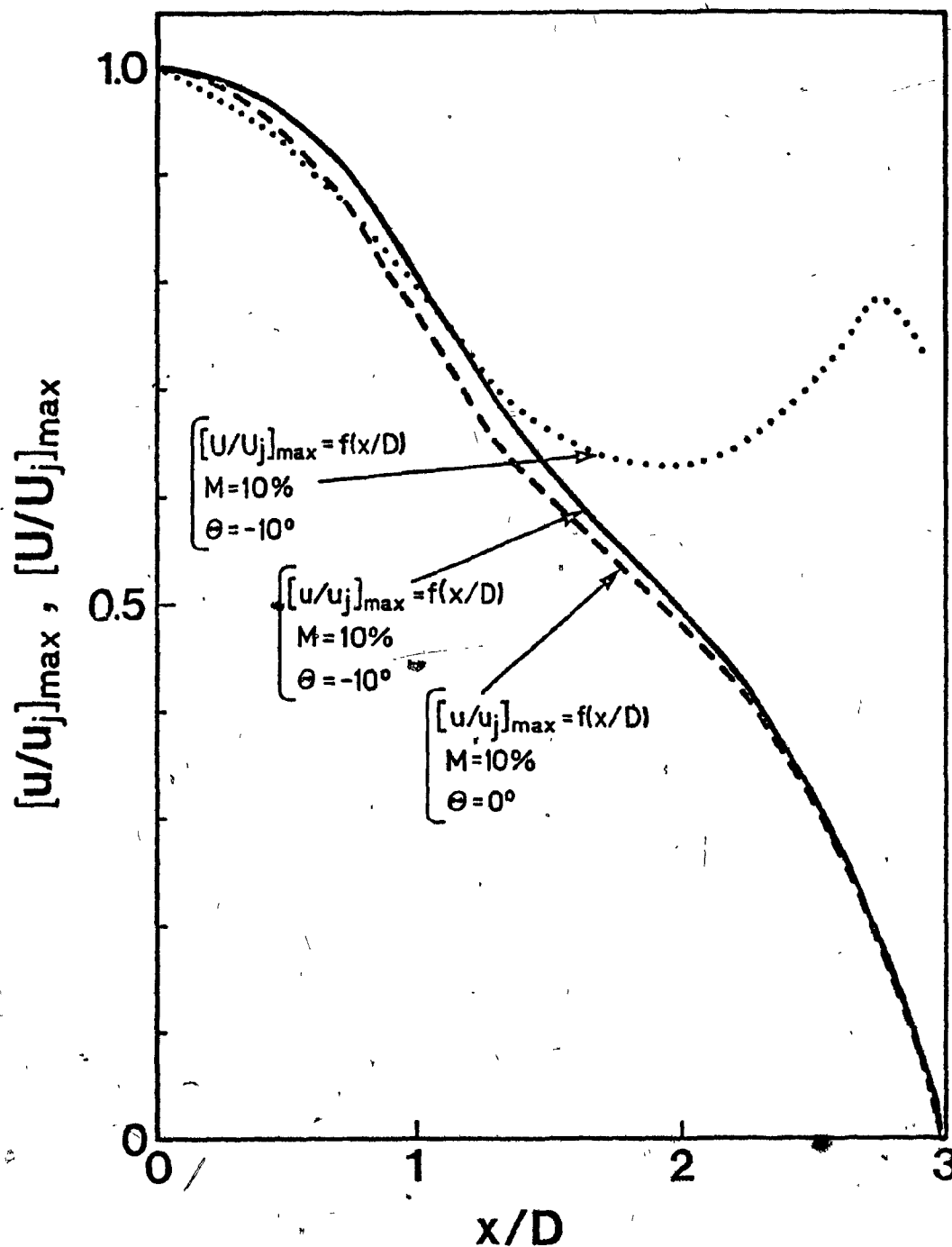


Figure 6.3 : Decay of maximum velocity for jets discharging in a crossflow at different angles.

the ratio  $(U/U_j)_{\max}$ . One can see that, for small distances away from the nozzle aperture, the decay of the total velocity normalized with the jet inlet total velocity is greater than the decay of the u-component.

This is due to the fact that  $U$  also entails the v-component of velocity which, at the beginning, is directed against the crossflow direction and which will be affected more than all other flow characteristics by the crossflow since the first crossflow effect felt by the jet is the impediment of the jet stream progression in the negative y-direction. This total velocity,  $U$ , is not as relevant to the impingement process as is the u-component because it is not directed perpendicular to the plate. Up to halfway to the impingement surface the profile for decay of  $U$  is similar to that for  $u_{\max}$ . Thereafter the profile for  $U$  rises to a maximum quite close to the impingement surface. This maximum is of course that of the wall jet. Beyond the wall jet maximum, the  $U$  velocity profile drops very sharply as the impingement surface is approached.

#### 6.2.2 Flow Field Analysis:

The complete flow field, from impingement to confinement plate, and from crossflow inlet to total flow outlet was mapped for two cases of inclined impingement into a crossflow. This was done through an additional subroutine, SPEED, which calculated total velocity,  $U$ , and its angle with the x-axis at all grid nodes. The results of this computation for  $H/D = 3$  and for a crossflow magnitude  $M = 0.1$  for jets impinging at inclination angles of  $-10^\circ$  and  $+30^\circ$  are as presented on Figures 6.4 and 6.5. On these figures the velocity vector is represented

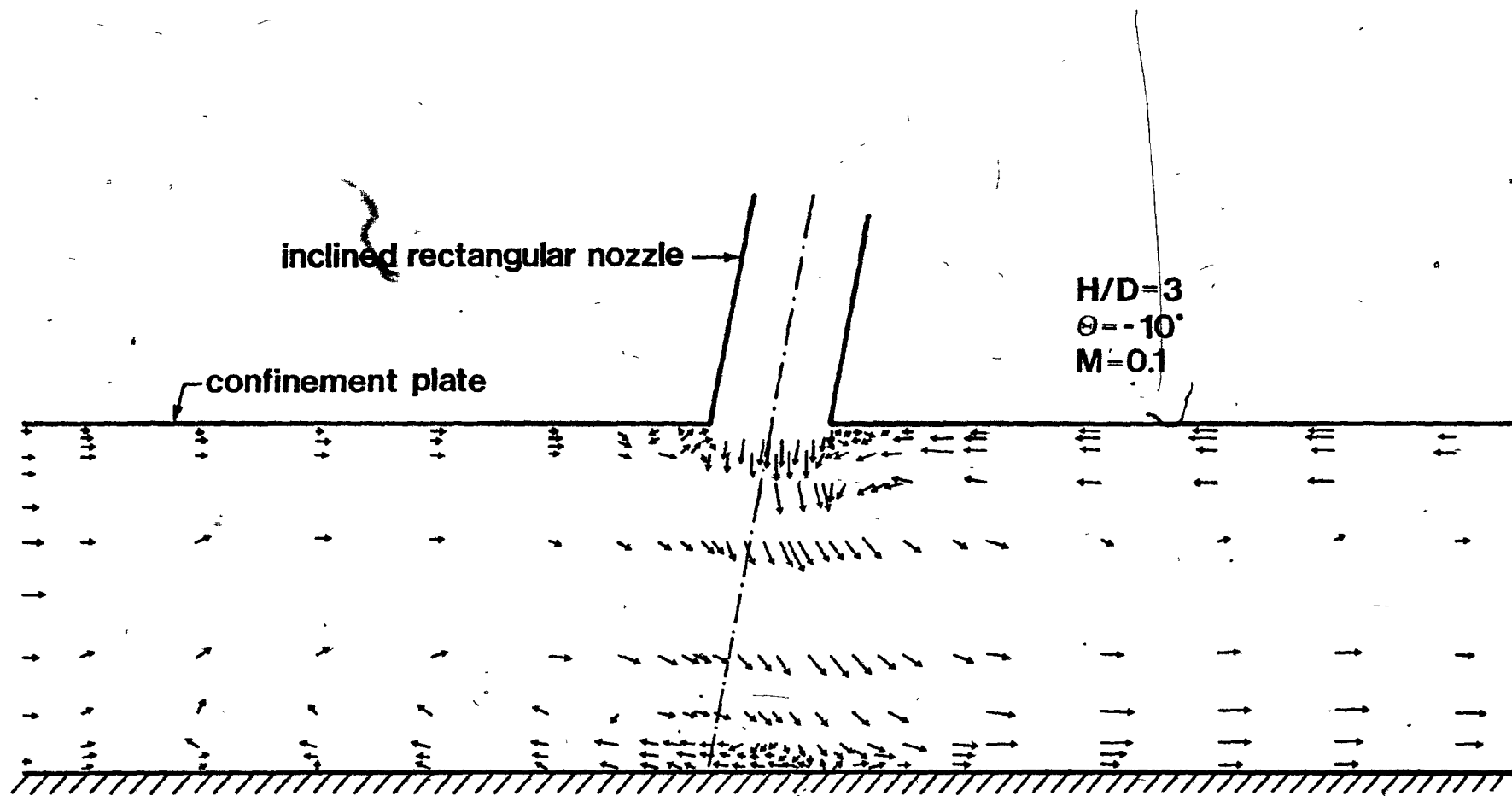


Figure 6.4 : Velocity field for an impinging jet discharging  
in a crossflow at an angle  $\theta = -10^\circ$ .

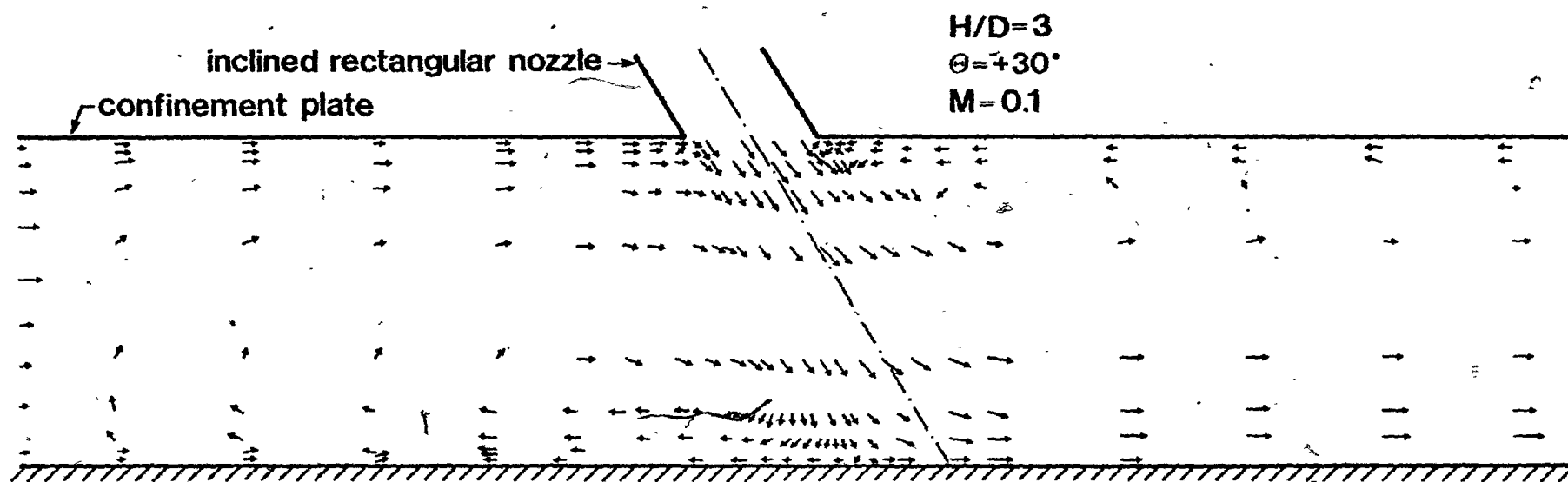


Figure 6.5 : Velocity field for an impinging jet discharging  
in a crossflow at an angle  $\theta = 30^\circ$ .

by arrows drawn with the appropriate angle, and of length approximating the actual magnitude of velocity.

Figure 6.4 (for  $\theta = -10^\circ$ ) shows clearly the deflection of the jet from the nozzle centerline by interaction with the crossflow. At impingement the jet is split into two large recirculating flows. The bulk of the jet goes in the downstream direction of the crossflow, while the smaller part forms a wall-jet, about 1D thick, in the direction opposing the crossflow. With the relatively low intensity of crossflow for the case shown on Figure 6.4, i.e.  $M = 0.1$ , the recirculation bubble on this side can be seen to extend about 3/4 the way to the crossflow entry. Such a recirculation bubble on the crossflow inlet side was also observed by Bouchez and Goldstein (B7) in their flow visualization for turbulent jets in a wind tunnel. They found a strong turbulent recirculation zone upstream of the impingement area. They also observed that as the flow rate ratio, jet to crossflow, decreases, the recirculation zone diminishes and finally disappears. Although the flow fields for  $M = 1.0$  and  $2.0$ , have not been plotted, this last observation of Bouchez is none the less confirmed as values of  $v$ -velocity in this region of the flow field in the present study show only positive values when  $M = 1.0$  or  $2.0$ . It is also of interest to note that on this side of the flow field, just adjacent to the nozzle, there is another recirculation bubble, but a very small one, due to the fact that the jet at the nozzle exit is directed somewhat against the crossflow stream direction.

The outflow side of the flow field is filled with a large recirculation bubble of the type discussed in Section 3.4. This recirculating flow,

is cut by the outflow boundary (Boundary IV) so that there is actually outflow over most of this flow boundary but with inflow for a region about  $1D$  wide adjacent to the confinement plate. This inflow is a reflection of the pumping or entrainment power of the jet. This inflow stream flows along the confinement plate in the negative  $y$ -direction to a point near the nozzle aperture where it becomes entrained by the jet; the junction of these two streams creates a pair of small vortices near the confinement plate.

The stagnation point is carried downstream by the crossflow. For the conditions of Figure 6.4 the stagnation point can be seen to be about  $0.5D$  downstream from the point on the impingement surface which is directly beneath the nozzle centerline.

Figure 6.5 shows the same general type of velocity pattern for  $\theta = +30^\circ$ . One difference from Figure 6.4 is that the stagnation point is of course shifted further downstream than for the case of  $\theta = -10^\circ$ , being located now about  $1D$  down from the point directly beneath the nozzle centerline. Intuition might have suggested that, with the jet inclined at an angle of  $30^\circ$  towards the crossflow direction, a recirculation bubble on the crossflow entry side might not have been formed. However, even though the fraction of the jet flow which recirculates upstream against the crossflow is greatly reduced with  $\theta = +30^\circ$ , it is still sufficiently strong to produce a recirculating flow extending most of the distance to the crossflow entry region. On the other hand, the recirculation bubble observed near the nozzle aperture on the crossflow entry side of the flow field

for  $\theta = -10^\circ$  is not present when  $\theta = +30^\circ$  due to the smooth merging of the crossflow and the jet.

Because of the evaporation occurring at the impingement surface in the present study, the velocity at this surface must in all cases be a vector directed perpendicularly into the flow field. However, within the limits of scale necessary for the representations of Figures 6.4 and 6.5, the closest velocity vectors to the impingement surface are still sufficiently far into the flow that their direction reflects the air flow field rather than the evaporative boundary condition. It has been mentioned in Subsection 6.2.1.1 that this evaporation velocity is three orders of magnitude smaller than the jet velocity.

### 6.2.3 Shear Stress Analysis:

It is instructive to analyse the effect of  $\theta$  and  $M$  on the lateral distribution of shear stress along the impingement surface by reference first to the case without crossflow and for  $\theta = 0^\circ$ .

When there is no crossflow and  $\theta = 0^\circ$ , the flow field is symmetric with respect to the nozzle axis. Starting from this axis ( $l = 0.0$ ) the  $v$ -component velocity is zero at the stagnation point, but increases very sharply with  $|l|$  under the influence of the pressure gradient, then falls off because of viscous action and of the increase of the cross-sectional area occupied by the spreading stream. This flow pattern results in the shear stress distribution along the impingement surface shown on Figure 6.6. For negative values of  $l$ , the fluid flows in the negative  $y$ -direction, which leads to negative values for  $\tau$ , so for convenience  $|\tau|$  has been plotted. As expected in view of the findings of previous researchers, the influence of nozzle-to-plate spacing is very little for the range of small values of  $H/D$  studied here, which

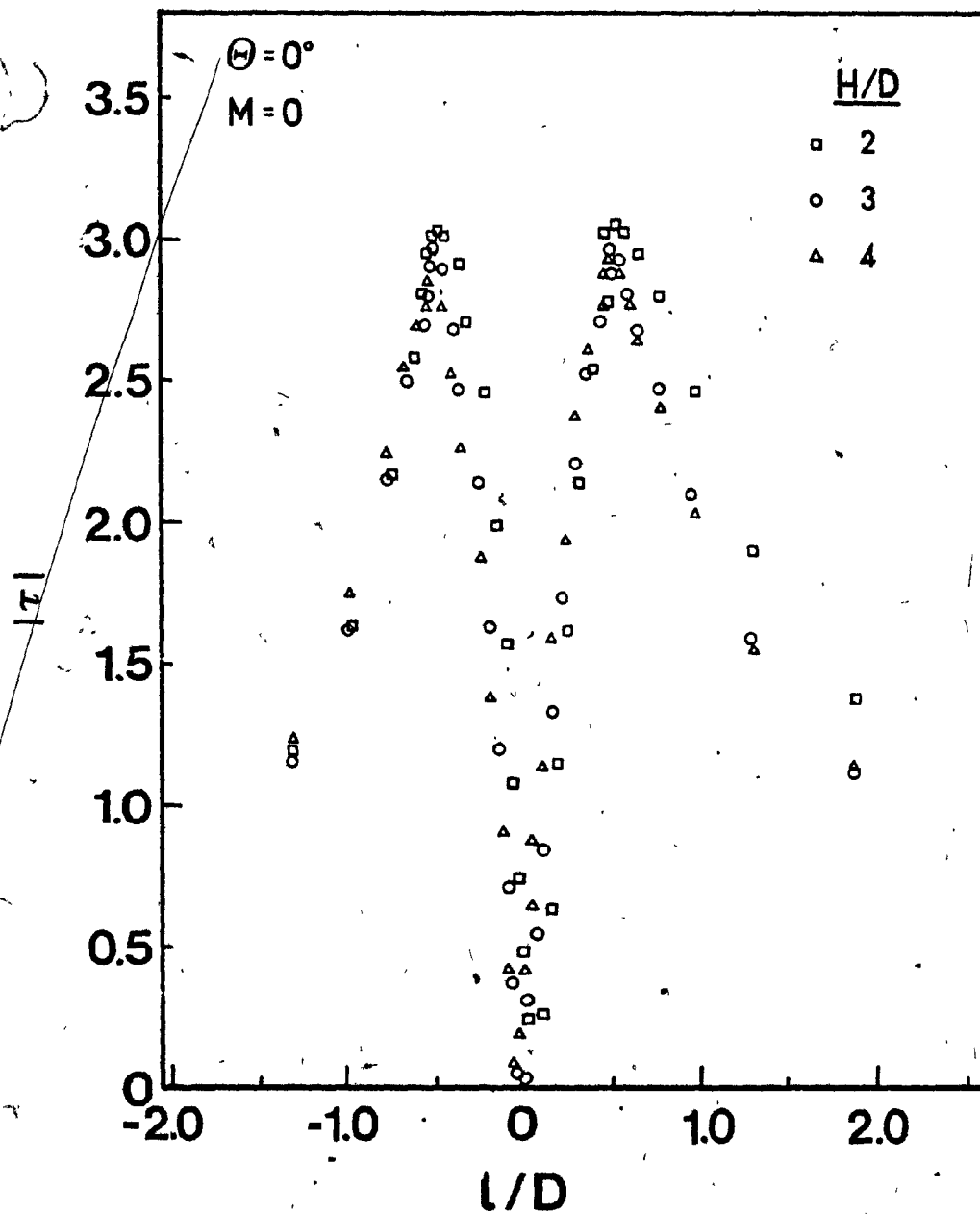


Figure 6.6 : Shear stress along the plate for impingement without crossflow at various  $H/D$  ratios.

correspond to location of the impingement surface within the potential core of the jet. Thus over the range from  $H/D = 2$  to  $H/D = 4$  there is a reduction in the maximum value of shear stress of only 6.6%.

Figure 6.7 shows the difference in the shear stress distribution between the case without crossflow and that of a jet discharging into a crossflow of magnitude  $M = 0.1$ . The presence of even this small amount of crossflow sharply reduces the shear stress on the impingement surface. The maximum value of  $\tau$  is reduced by two thirds. This result which shows that the crosswind general tendency is to smother the jet properties and characteristics by reducing all gradients at the impingement surface will be further confirmed by the results obtained for transfer rates. This effect for shear stress as shown by Figure 6.7 is consistent with that shown on Figure 6.2 for the  $u$ -component velocity. Figure 6.7 shows also clearly the existence of a stagnation point (where the shear stress becomes zero) in the case where  $M = 0.1$  and  $H/D = 4$ ; it is shifted downstream by the action of the crossflow, as was already noted in the flow field plots, Figures 6.4 and 6.5. The stagnation point corresponds also to a maximum of the pressure along the plate; such a maximum for  $P$  is shown on Figure 6.10 for a similar case ( $M = 0.1$  and  $H/D = 3$ ).

As the crossflow of Figure 6.7 is quite small ( $M=0.1$ ), the shear stress distribution along the impingement surface for a jet impinging normally has been plotted on Figure 6.8 for larger values of crossflow. The two quite different types of flow behaviour, depending on the magnitude of the crossflow, are evident on Figure 6.8. For small

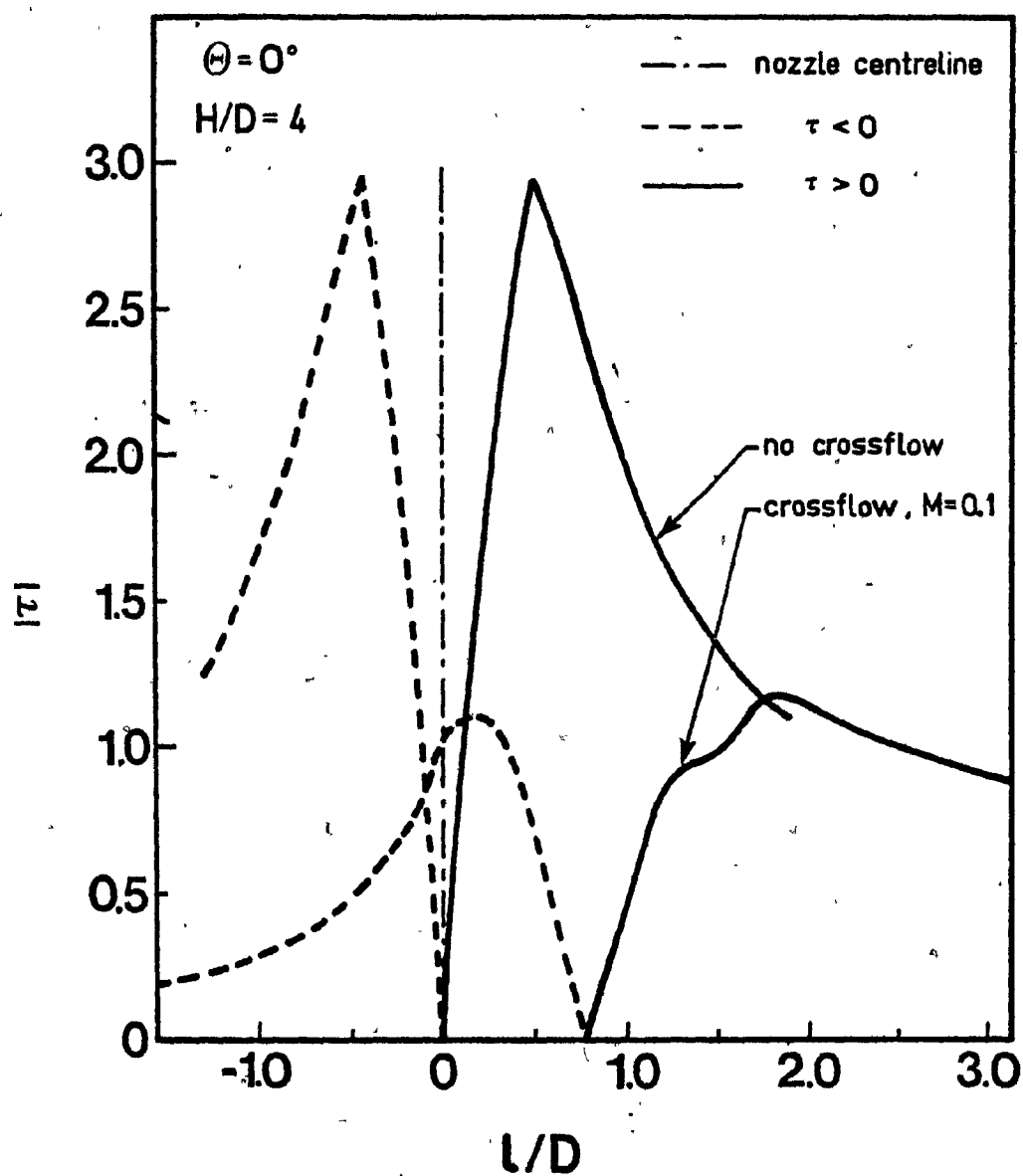


Figure 6.7 : Comparison of shear stress distribution along the plate for impingement with and without crossflow.

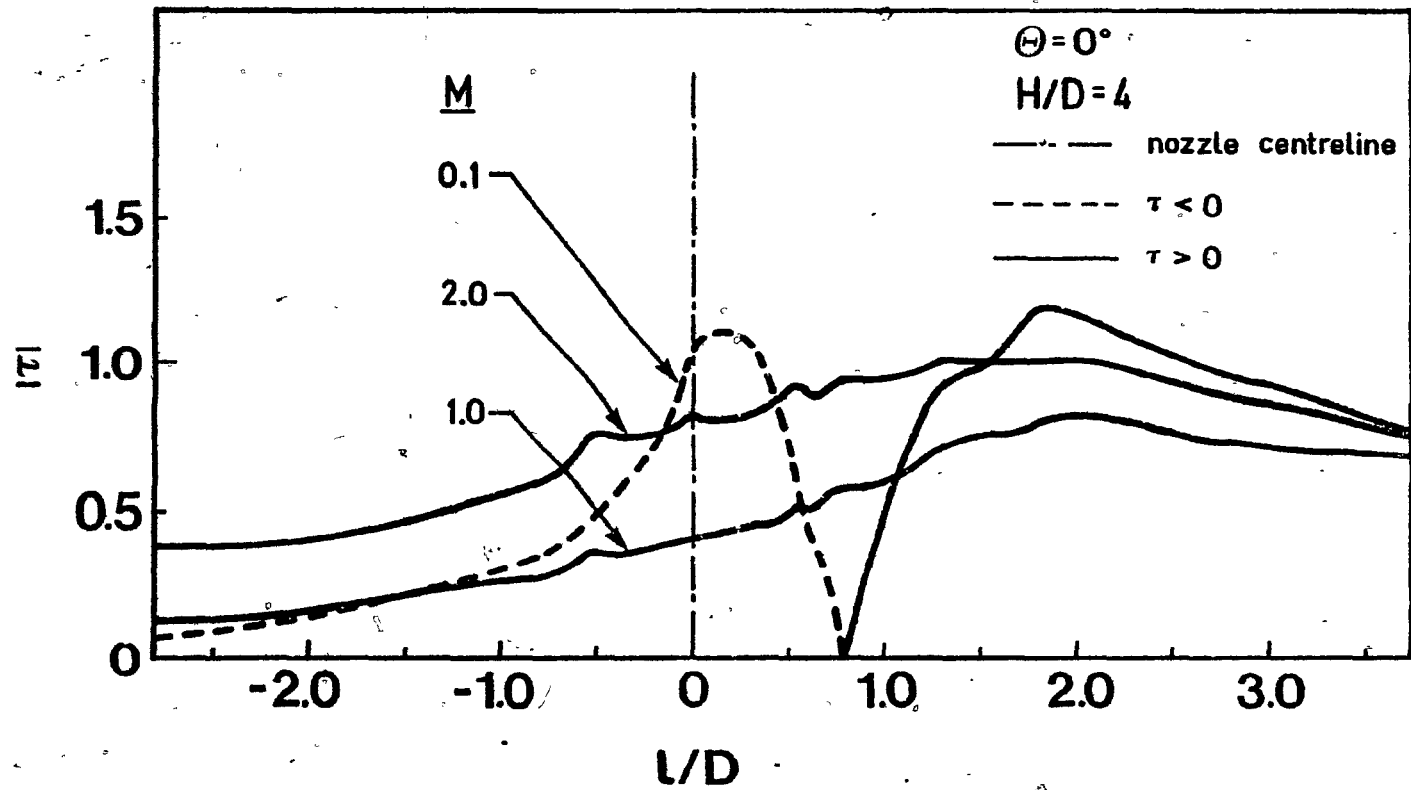


Figure 6.8 : Influence of crossflow magnitude on shear stress along impingement plate.

values of  $M$ , as represented by  $M = 0.1$ , the jet effectively impinges on the plate, there is a stagnation point, and the flow field, although strongly affected by the presence of the crossflow, is still dominated by the jet. But for large values of  $M$ , represented by  $M = 1.0$  and  $2.0$ , the shear stress at the evaporating surface is always positive, i.e. there is no stagnation point. The profile of  $\tau$  near the crossflow entry approaches that for pure channel flow. However, from about  $1/D = -2.0$  the shear stress increases significantly due to the additional flow provided by the jet. Shear stress reaches a maximum on the downstream side of the jet inlet at about  $1/D = 2$ , i.e. at about the same position as for the case of  $M = 0.1$ , although the maximum  $\tau$  is now less even though the total flow is significantly larger. Clearly, no impingement occurs in this case. This absence of impingement is confirmed by the absence of a peak in pressure profile along the evaporating surface when  $M = 1.0$  or  $2.0$ . This pressure profile is similar to that shown on Figure 6.10 for  $M = 2.0$  and  $\theta = +30^\circ$ . Thus one may say that for large values of  $M$  the flow field is dominated by the crossflow, with the crossflow sweeping away the jet and preventing it from even impinging on the transfer surface. With such a flow behaviour the system is expected to yield very poor transfer rates for large values of  $M$ , as will be confirmed by the heat and mass transfer results presented subsequently.

The effect on shear stress profile of varying angle of inclination of the jet nozzle for a flow field which is crossflow-dominated ( $M = 2.0$ ) is shown on Figure 6.9. Moreover, comparison of the curves for  $\theta = 0^\circ$  and  $M = 2.0$  on Figure 6.9 ( $H/D = 3$ ) and on Figure 6.8 ( $H/D = 4$ ) also indicates the magnitude of the effect of  $H/D$  on shear stress in the range of close nozzle-to-plate spacing. Shear stress must of course

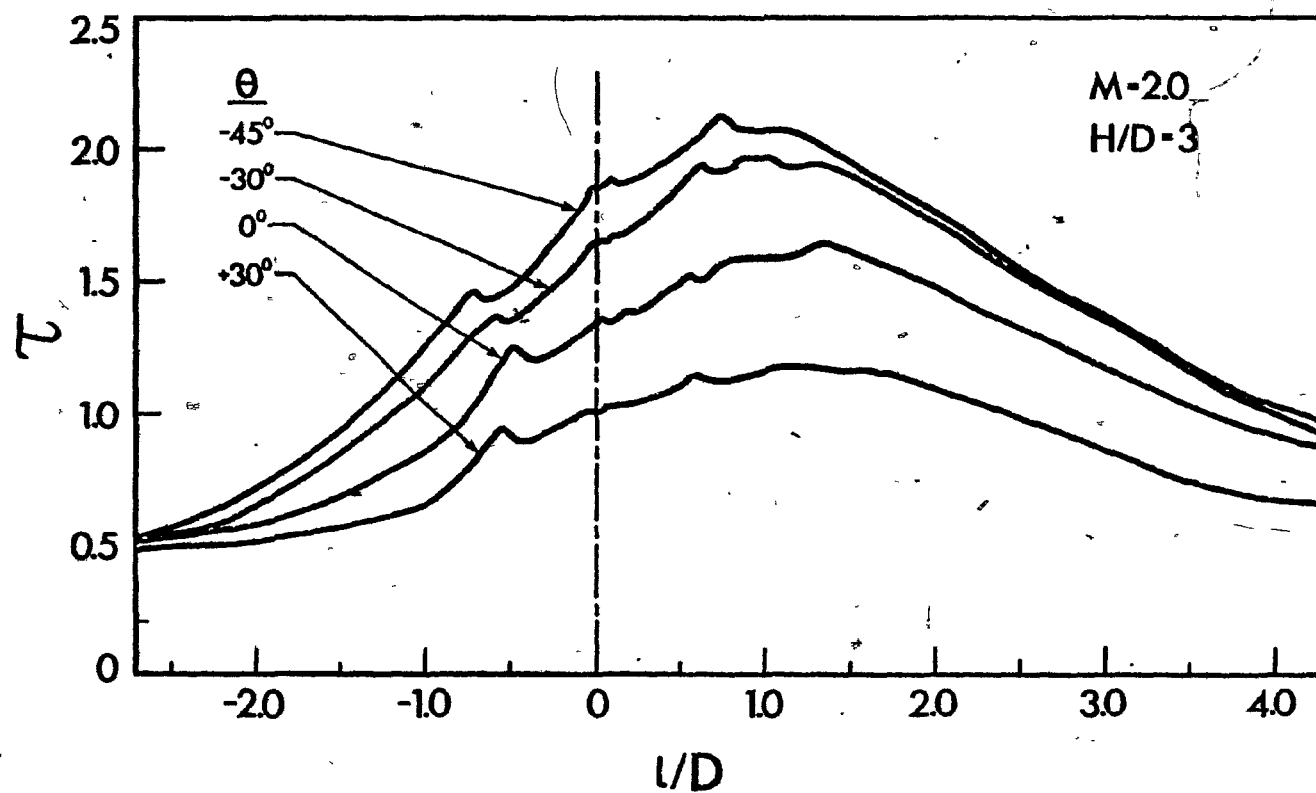


Figure 6.9 : Influence of nozzle inclination angle on shear stress along impingement plate in the case of strong crossflow.

increase as  $H/D$  decreases. In this connection it is useful to recall the definition of crossflow magnitude, i.e.  $M = H\bar{V}/D\bar{U}$ . As  $\bar{U}$  and  $D$  are held constant, the crossflow mean velocity will be increased if the  $H/D$  ratio is decreased for a constant value of  $M$ . With the no-slip boundary condition for the  $v$ -component velocity at the evaporative surface, the velocity gradient here will also be higher when  $H/D$  is smaller, resulting in a higher shear stress at the surface. Comparison of the curves for  $M = 2.0$ ,  $\theta = 0^\circ$  on Figures 6.8 and 6.9 indicates that the peak in shear stress, which is the signature of the influence of the jet on the crossflow is increased by about 60% (i.e. from 1.0 to 1.6) as  $H/D$  is decreased from 4.0 to 3.0. These calculations quantify the intuitive conclusion that the further the jet nozzle is spaced from the evaporating surface, the more the system is crossflow-dominated.

Comparison of the various curves on Figure 6.9 indicates the significant fact that the shear stress is steadily increased as the jet nozzle is rotated in the direction from positive through negative angles of inclination. As can be seen by reference to the plots of the flow field for  $\theta = -10^\circ$  and  $\theta = +30^\circ$ , Figures 6.4 and 6.5, the more the nozzle is rotated in the direction of negative angles, the more of the jet flow which is directed upstream against the crossflow and towards the impingement surface, and hence the larger the  $v$ -component velocities subsequently in the region beneath and immediately downstream from the nozzle. The result is a higher gradient in  $v$ -velocity near the plate, causing higher shear stress.

The small secondary peaks evident on these curves have their

origin in the computational procedure, and seem to be associated with the relatively fine computational grid near the nozzle edges (around gridlines JNOZ and JSTEP) which is required for an accurate definition of these edges. Indeed, these small computational maxima correspond exactly to  $|1/D| = 0.5$  when  $\theta = 0^\circ$  and go slightly further away from this point as  $\theta$  is increased to  $+30^\circ$  and  $+45^\circ$ , corresponding to the slight shift in the location of the nozzle edges which are located at  $1 = \pm D/\cos\theta$ .

The pressure profiles along the evaporative surface, shown as Figure 6.10, have already been referred to in connection with discussion of the shear stress profiles. The three typical profiles shown illustrate the progression from the condition without crossflow, to a small crossflow ( $M = 0.1$ ) for which there is still impingement and a stagnation point at the pressure maximum, to the case of a totally crossflow-dominated system ( $M = 2.0$ ) in which there is no pressure maximum or stagnation point, but instead a continuous pressure loss in the direction of the controlling crossflow.

### 6.3 HEAT TRANSFER RESULTS

#### 6.3.I Without Crossflow

Profiles of heat transfer coefficient at the impingement surface are plotted in terms of local Nusselt number on Figure 6.II for  $H/D$  ratios of 2, 3 and 4. These curves, bell shaped around the stagnation point as expected show clearly that, within this range of  $H/D$  ratios, nozzle-to-plate spacing has negligible influence on the local Nusselt number as the variation at the peak is only 3%. This confirms the conclusion of Huang (HI) for turbulent jets and of Kapur and McLeod (K4)

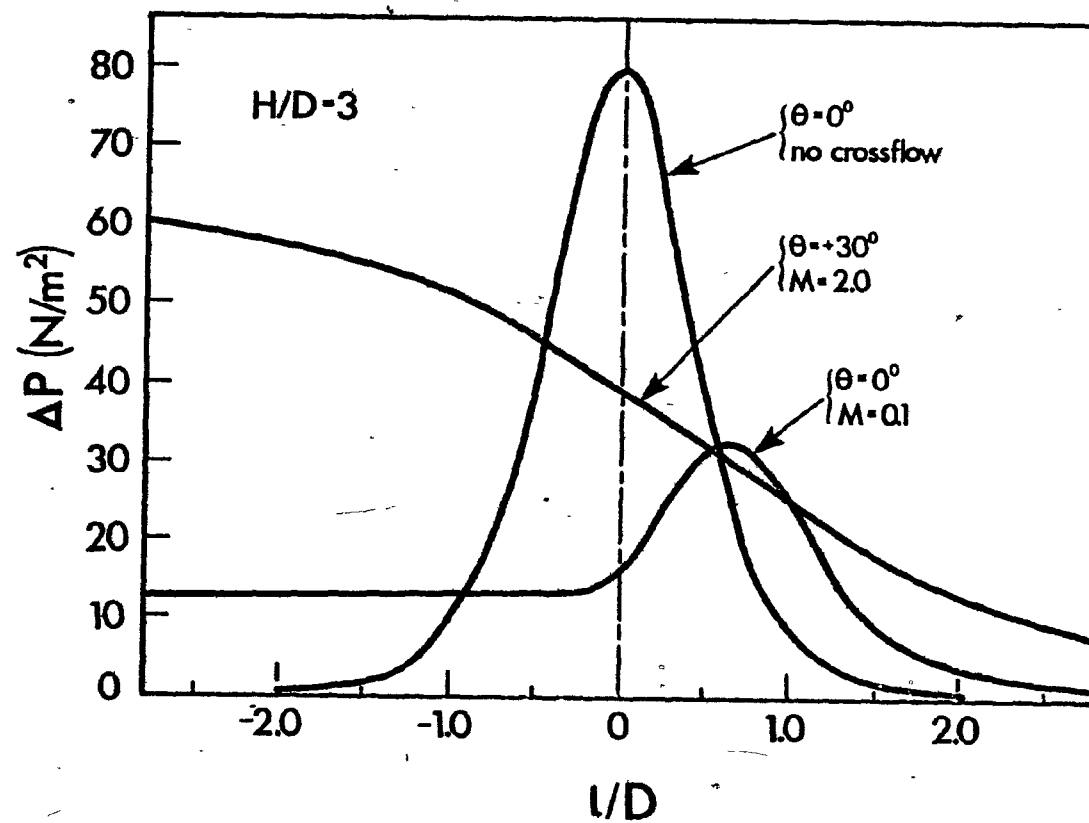


Figure 6.10 : Pressure distribution along impingement plate.

and Gardon and Akfirat (G6) who operated with Reynolds numbers of comparable magnitude to the one chosen here. This finding with respect to heat transfer is consistent also with the small variation in shear stress noted in Section 6.2.3.

The stagnation point Nusselt numbers are somewhat lower than those found by van Heiningen (V2) who also studied heat transfer under a rectangular impinging jet. His results, expressed dimensionlessly as  $Nu/\sqrt{Re}$ , gave  $Nu/\sqrt{Re} = 0.81$  at the stagnation point for  $Re = 950$  and  $H/D = 4$  in the case of a parabolic velocity profile for the jet at the nozzle exit. For  $H/D = 4$  and  $Re = 1000$  the value found in the present study is  $Nu/\sqrt{Re} = 0.7$  at the stagnation point. This difference is accounted for by the simultaneous mass transfer which takes place here but not in van Heiningen's study. The reduction in heat transfer caused by linkage with a mass transfer flux in the opposing direction was studied numerically by Li (L2) for the case of a round axisymmetric impinging jet.

### 6.3.2 Effect of Crossflow

As the domain of interest of the present study is evaporation under impinging jets, most of the curves presented in the following sections are limited, for convenience of format, to the central area of the flowfield where interaction between jet and crossflow is felt. Consequently it is important to note that the extent of the region shown in the remaining figures of Chapter 6 is less than the total extent of the flow system. Thus results are usually shown over the prime interest range of  $H/D$  of about  $\pm 3$ , although the crossflow entry is at  $H/D = -6.4$  and the flow exits at a value of  $H/D$  in the range from 6.1 to 8. However, for completeness, one profile is presented which covers the entire system, from crossflow inlet to the flow exit. As this complete profile is for mass transfer rate (Figure 6.26), it is discussed

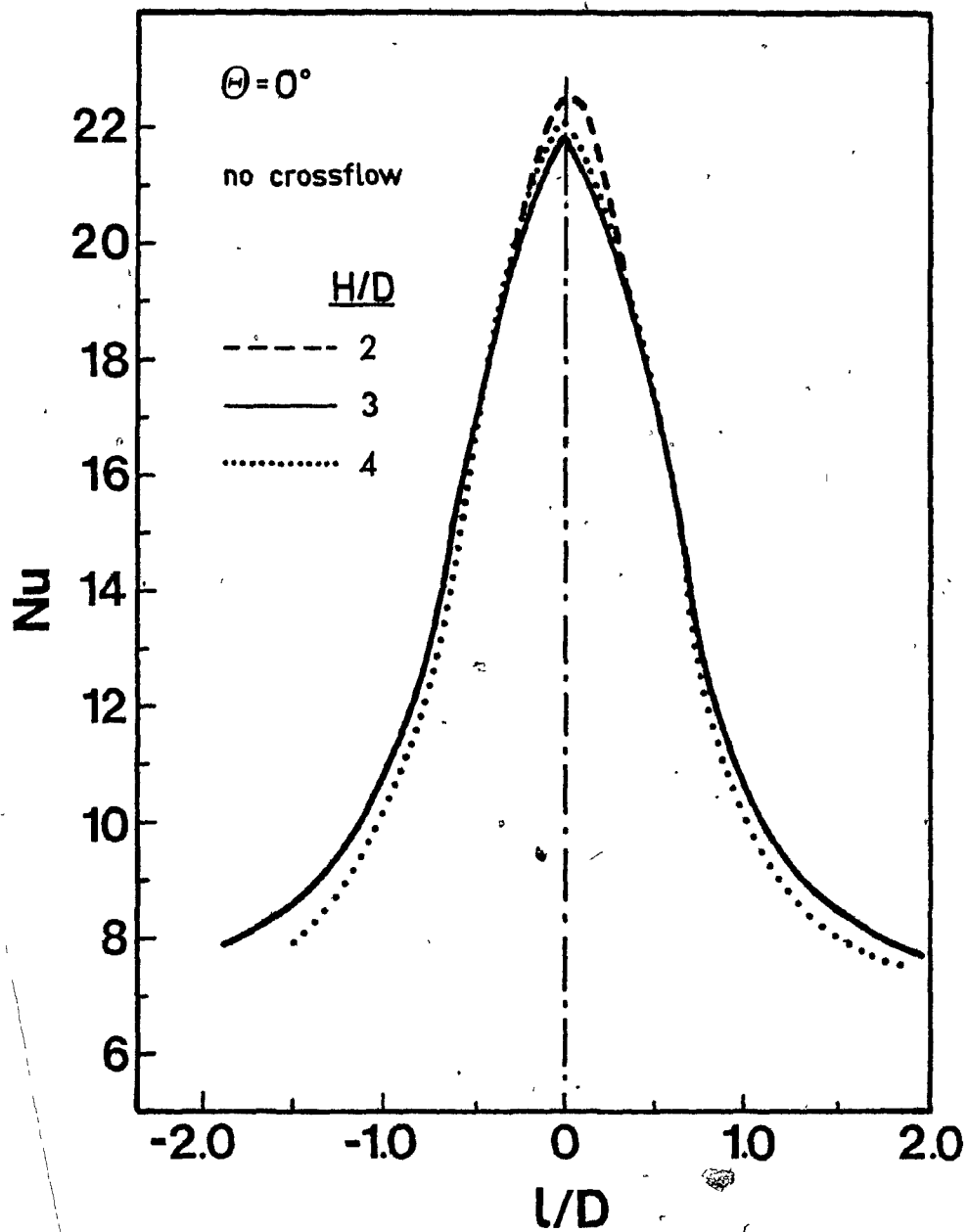


Figure 6.11 : Influence of H/D on heat transfer for a jet impinging normally without crossflow.

subsequently. The trends noted there for mass transfer would by analogy apply also to heat transfer.

#### 6.3.2.1 Influence of Crossflow Ratio

On Figure 6.12 the local Nusselt number distribution is shown for a wide range of crossflow ratios,  $M$ , covering both the jet-dominated and crossflow-dominated regimes. The basic difference in the heat transfer profiles between flow regimes of these types as illustrated by Figure 6.12 parallels the corresponding differences in shear stress profiles already noted on Figure 6.8. For the spacing of  $H/D = 4$  on Figures 6.12 and 6.13, the profiles for values of  $M$  of 0.01, 0.1, 0.167 and 0.25 are for jet-dominated flow, while the profiles for  $M = 1.0$  and 2.0 clearly correspond to crossflow-dominated flow. These profiles with crossflow may be compared to those of Figure 6.11 without crossflow, as the fact that the plate widths are not the same does not have a significant effect in the central region of interest. Such a comparison indicates that the stagnation point heat transfer is sharply decreased by the crossflow, i.e. it is reduced by 36% for only  $M = 0.01$ , and by 50% when  $M = 0.25$ . As expected the stagnation point is shifted in the downstream direction by crossflow, and this shift increases for increasing values of  $M$ . The curves with crossflow are of course not symmetric with respect to the stagnation point, as the mass flow-rate is greater on the downstream than on the upstream side of the stagnation point.

Metzger and Korstad (M5) claimed that the upstream propagation distance of a jet along the impingement surface is relatively short and that, as a consequence, the heat transfer rates immediately upstream of the nozzle exit plane are reduced significantly below what would be expected if crossflow were not present. However, this effect on the upstream side may be viewed simply as a consequence of the simultaneous reduction

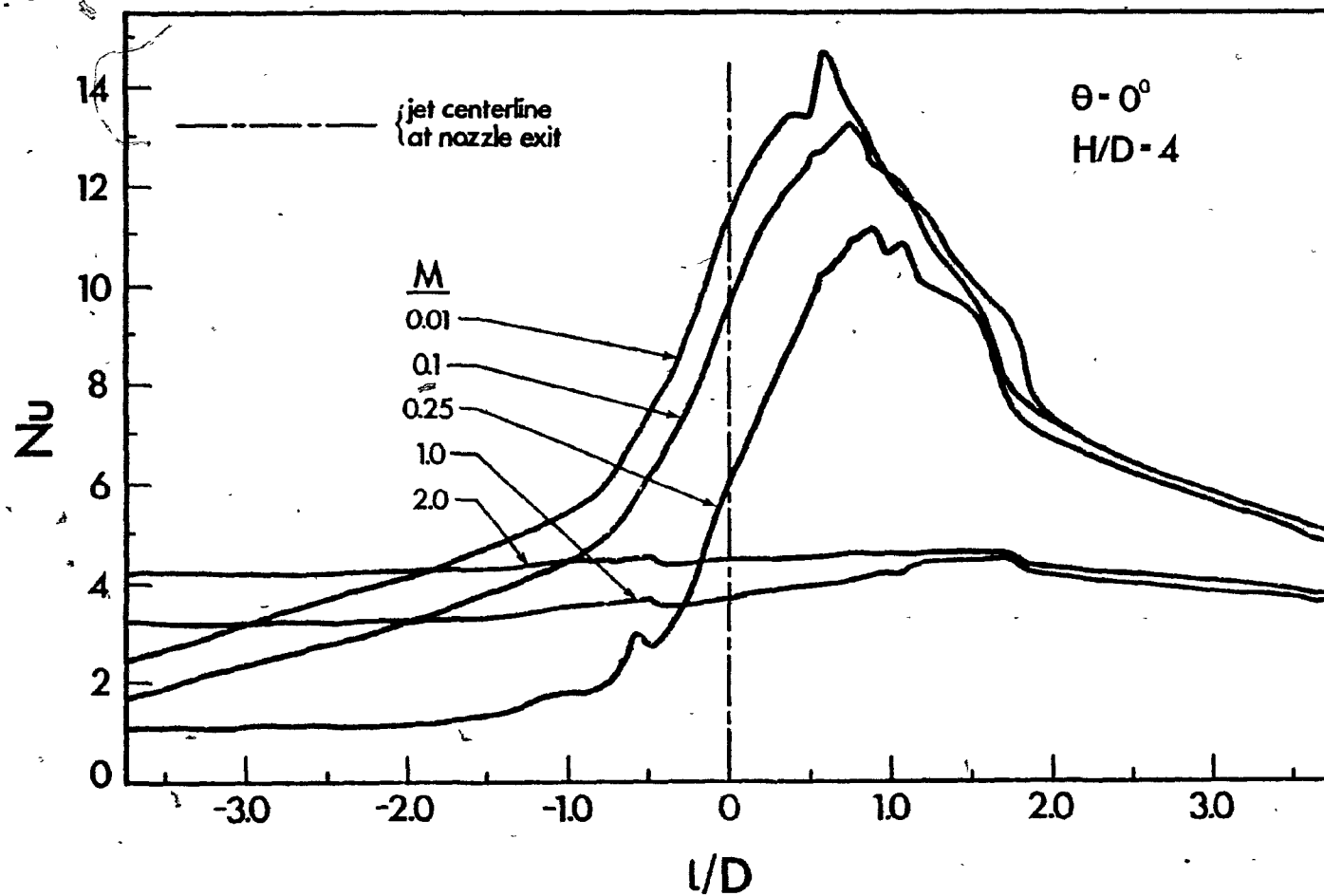


Figure 6.12 : Comparison of the Nusselt number distribution along the plate between jet-dominated and crossflow-dominated systems for a perpendicular nozzle.

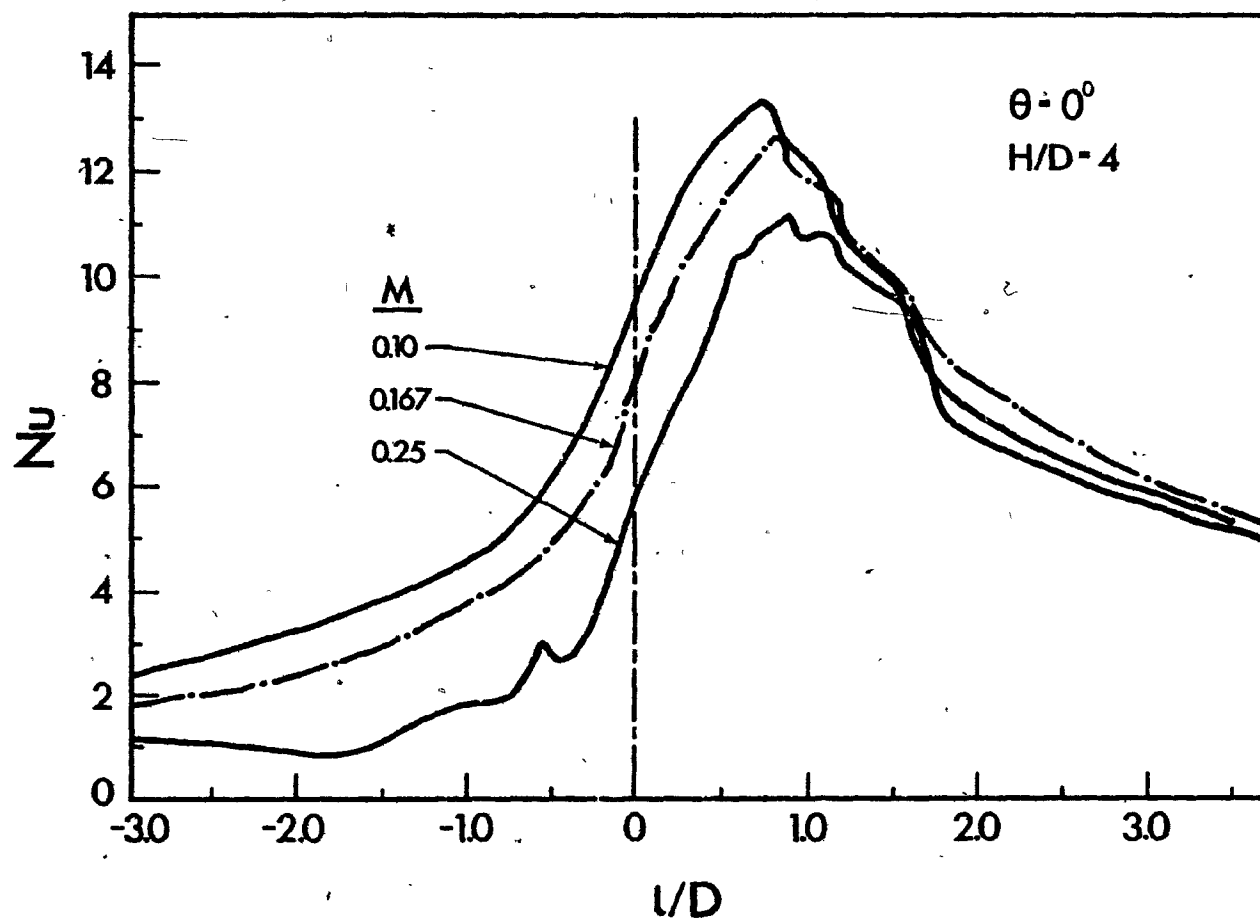


Figure 6.13 : Heat transfer for jet-dominated systems.

in the entire profile and its shift downstream which appear clearly when comparing Figures 6.11 and 6.12. On the other hand, the present results disagree with Metzger's further conclusion that average heat transfer over regions immediately downstream is increased above that without crossflow. The average Nusselt number calculated over different regions of the impingement plate are presented on Table 6.2. It appears clearly that for all the regions considered on Table 6.2 the average heat transfer rate with crossflow is less than that without crossflow. It seems more logical anyway to speak in terms of Nusselt numbers averaged over regions starting from the stagnation point (instead of the nozzle exit plane) or extending on both sides of the stagnation point (see  $Nu_4$  and  $Nu_5$  on Table 6.2). With such definitions it appears clearly that the effect of the crossflow is to reduce the average heat transfer coefficients. On an overall basis for a symmetrical region extending ten nozzle widths both upstream and downstream of the nozzle exit plane, Metzger did however conclude that the presence of crossflow always reduces the average heat transfer below that without crossflow (M5). Although the results of the present study contradict Metzger's statement concerning heat transfer in the near downstream region, they nonetheless confirm the latter conclusion of Metzger et al.

For large values of  $M$ , ( $M = 1.0$  or  $M = 2.0$ ), Figure 6.12 shows that the heat transfer profiles are totally different, as could be expected from the shear stress profiles for large and small crossflow ratios shown on Figure 6.8. For this range of  $M$  and  $H/D$  ratios we are no longer dealing with impinging jets, as the shear stress results showed that for these conditions there is no stagnation point. Thus the local Nusselt number distribution corresponds to that for channel flow but modified somewhat by the addition of the jet flow. The maxima of the curves are not due to impingement, but only to the increase in flow-rate through the channel that is associated

| M            | Nu <sub>1</sub> | Nu <sub>2</sub> | Nu <sub>3</sub> | Nu <sub>4</sub> | Nu <sub>5</sub> |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.01         | 13.3            | 11.7            | 9.8             | 8.2             | 6.6             |
| 0.1          | 12.0            | 11.0            | 9.3             | 7.5             | 6.0             |
| 0.25         | 9.2             | 9.3             | 8.2             | 6.7             | 4.5             |
| no crossflow | 16.8            | 12.5            | 10.7            | 10.7            | 9.3             |

Nu<sub>1</sub> : Nusselt number averaged on the region of the impingement plate extending on a one nozzle width-length downstream of the nozzle exit plane.

Nu<sub>2</sub> : Nusselt number averaged on the region of the impingement plate extending on a two nozzle width-length downstream of the nozzle exit plane.

Nu<sub>3</sub> : Nusselt number averaged on a region of the impingement plate extending on a three nozzle width-length downstream of the nozzle exit plane

Nu<sub>4</sub> : Nusselt number averaged on the region of the plate extending on a three nozzle width-length downstream of the stagnation point plane.

Nu<sub>5</sub> : Nusselt number averaged over a symmetrical region extending four nozzle widths both upstream and downstream of the nozzle exit plane.

Table 6.2 : Average Nusselt numbers.

with the fluid added from the jet nozzle.

Although heat transfer decreased sharply with increasing crossflow in the low range of  $M$  (i.e. at least up to  $M = 0.25$  for  $H/D = 4$ ), it is useful to note that in the high range of  $M$  (i.e. from 1.0 to 2.0), heat transfer increases with  $M$ . The initial decrease of  $Nu$  with increasing  $M$  corresponds to destruction of the beneficial impingement flow pattern by crossflow, while the later increase of  $Nu$  with  $M$  simply corresponds to normal channel flow behaviour. This reversal of the effect of  $M$  on  $Nu$  is illustrated on Figure 6.14 for one value of  $H/D$  and  $\theta$ . The value of  $M$  which marks the transition between jet-dominated and crossflow-dominated flow could be taken as the value of  $M$  which gives the minimum value of  $Nu_{max}$  on Figure 6.14. This limiting value of  $M$  would decrease as  $H/D$  increases, and would as well be a function of the nozzle inclination angle.

#### 6.3.2.2 Influence of $H/D$

The influence of nozzle-to-plate spacing over the small range of  $H/D$  investigated is shown on Figure 6.15. The influence of  $H/D$  on heat transfer profile, very small without crossflow, becomes quite noticeable with even a small crossflow. For  $\theta = 0^\circ$ , the stagnation point heat transfer coefficient is reduced by 19% by an increase in  $H/D$  from 2 to 4 with  $M = 0.1$ , whereas the corresponding reduction without crossflow was only 3%. The increased sensitivity of heat transfer profile to  $H/D$  is a natural consequence as  $H/D$  measures the distance over which the crossflow acts on the jet. For the same reason the maximum  $Nu$  at the stagnation point is also deflected increasingly downstream. At any value of  $M$ , increasing  $H/D$  will eventually lead to a crossflow-dominated flow.

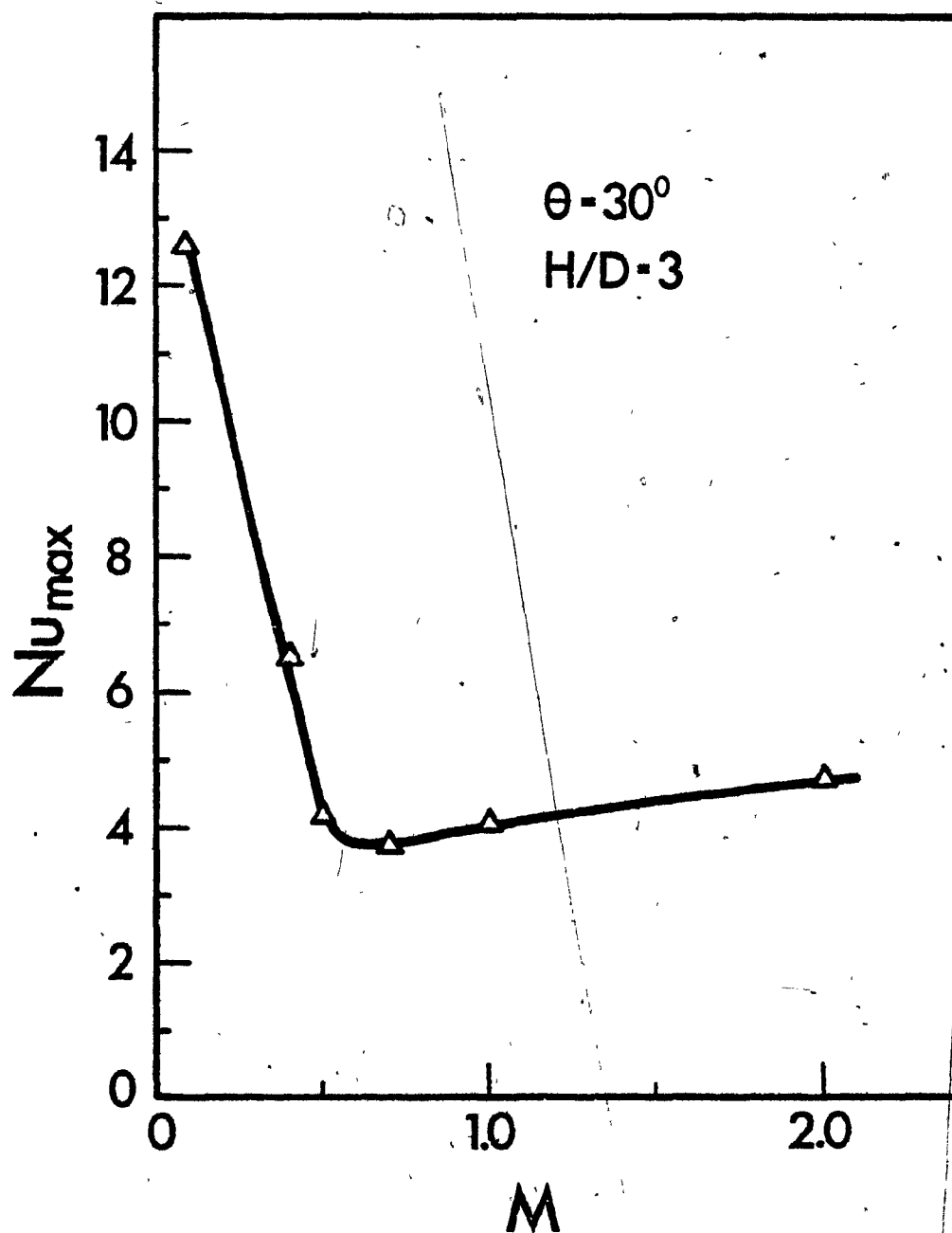


Figure 6.14 : Maximum local Nusselt number as a function of  $M$ .

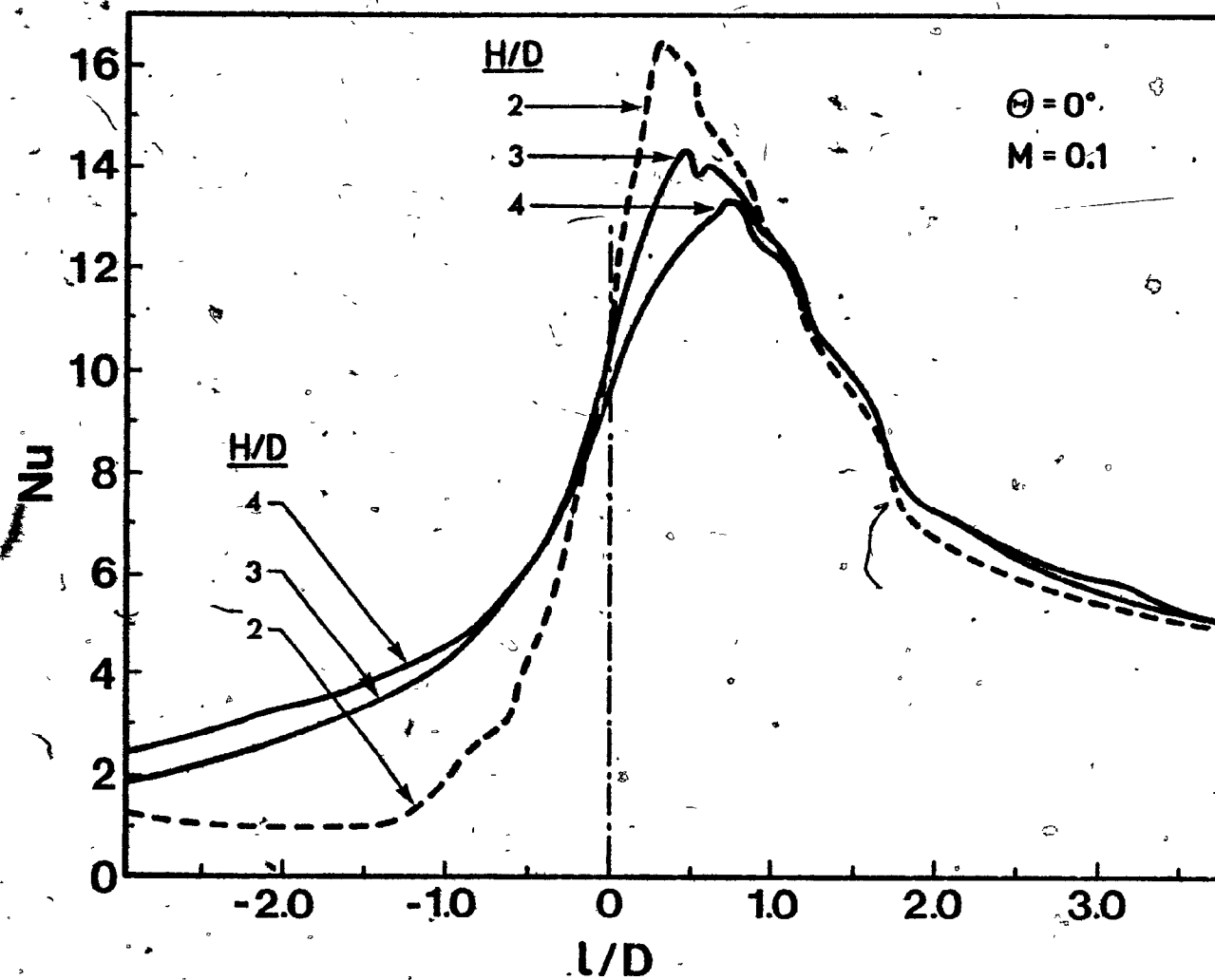


Figure 6.15 : Influence of  $H/D$  on local heat transfer for a jet in a crossflow.

### 6.3.3 Effect of Angle of Inclination

Since it has been established in the previous sections that for a particular nozzle-to-plate spacing two quite different types of flow behaviour can be encountered depending on the value of  $M$ , it is useful to review the influence of angle of impingement separately for each case. Subsections 6.3.3.1 and 6.3.3.2 deal, respectively, with small and large values of the ratio  $M$ .

#### 6.3.3.1 Effect of Angle for moderate $M$ ratios

Angles varying from  $-45^\circ$  to  $30^\circ$  have been investigated for  $M = 0.1$  and  $H/D = 3$ . From the curve for  $M = 0.1$  it may be seen on Figure 6.16 that a distinct maximum in  $Nu_{max}$  occurs at a negative angle situated between  $-30^\circ$  and  $-15^\circ$ . To interpret this fact it is convenient to start from large positive angles (i.e. jet directed in the same direction as the crossflow) and to see how the flowfield is modified as the angle decreases, takes the value zero (right angle jet) and finally increases in the direction opposed to the crossflow.

Figure 6.17 shows the local Nusselt number distribution obtained along the plate for  $\theta = 30^\circ$ ,  $\theta = 0^\circ$  and  $\theta = -10^\circ$ . For  $\theta = 30^\circ$  the jet is more easily taken away by the crossflow because of the compatibility of nozzle inclination with crossflow direction. The jet is thus strongly deflected (as shown by the shifting of the peak in local Nusselt number). As the nozzle is inclined more in the direction of the crossflow, the conversion of  $x$ -momentum into  $y$ -momentum occurs further from the impingement surface, hence the resulting reduction in heat transfer rates. However, the shear stress results obtained for this run show that there is still a stagnation point where  $\tau = 0$ , and that the position of the heat transfer maximum is at the stagnation point. As  $\theta$  decreases from  $30^\circ$  to  $0^\circ$  and to  $-10^\circ$  it is clear from Figure 6.17 that the deflection of the jet is smaller and that higher values of maximum heat transfer rates result from

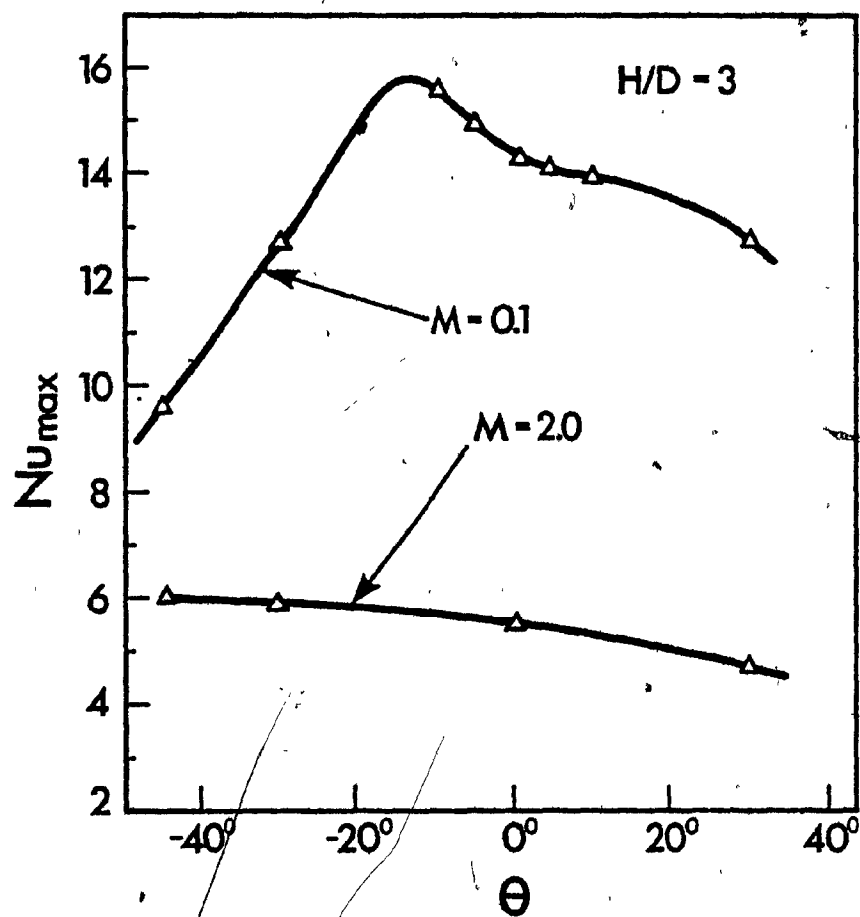


Figure 6.16 : Maximum local Nusselt number as a function of nozzle inclination angle.

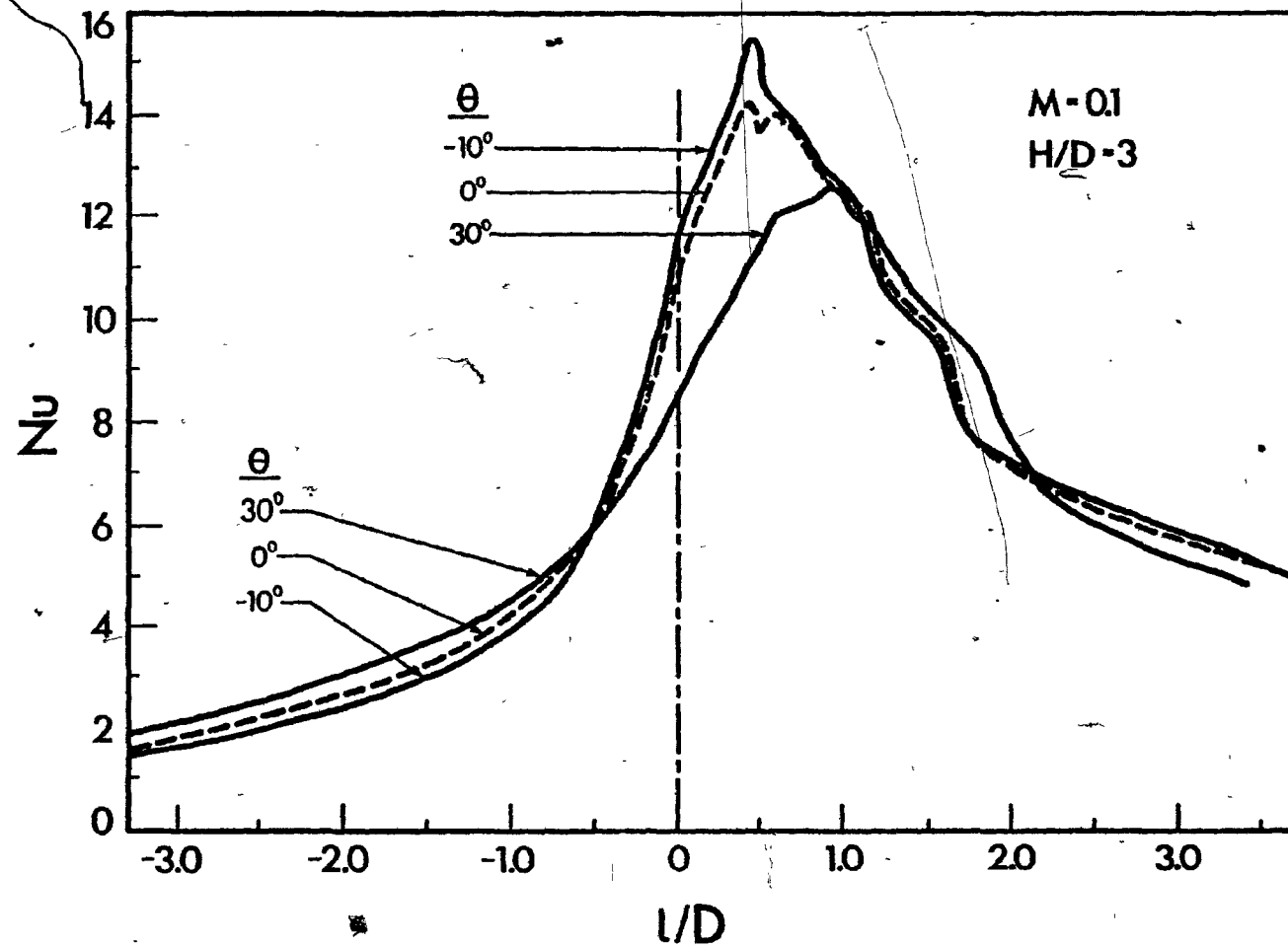


Figure 6.17 : Influence of angle of inclination on local Nusselt number for small values of  $M$ .

the increasingly vigorous impingement on the plate.

If the nozzle is inclined more strongly yet against the crossflow, e.g. for  $\theta = -30^\circ$  or  $-45^\circ$ , then the dominant effect becomes a loss of momentum of the jet in this collision-type process where the two flows are directed against one another. From Figure 6.18 it can be seen that both the maximum and the average values of heat transfer decrease sharply for increasingly negative angles of inclination from  $-10^\circ$  to  $-30^\circ$  to  $-45^\circ$ .

As a conclusion it appears that, for a given crossflow magnitude and nozzle-to-plate spacing, there exists an optimum angle (directed slightly against the crossflow direction) for which heat transfer is maximum. It will be seen in Section 6.4 that this conclusion is also valid for the mass transfer rates. This observation can be important in industrial practise where a crossflow (caused for example by a neighbouring nozzle) cannot be eliminated, since it appears that the decrease in transfer coefficients caused by the crossflow can be reduced by choice of the optimum nozzle angle.

#### 6.3.3.2 Effect of the Angle for large M ratios

In this case, as seen earlier, there is no stagnation point and hence no real impingement. Figure 6.19 shows that the heat transfer distribution is quite flat for all inclination angles. The heat transfer rates do increase significantly as the angle of inclination is changed from  $30^\circ$  to  $0^\circ$ , and continues to increase slightly as the angle is changed to  $-30^\circ$  and  $-45^\circ$ . As there is no impingement, this increase is simply due to the augmentation of the crossflow by the nozzle flow in the region beneath and downstream from the location of the nozzle. This behaviour is consistent with the discussion of Section 6.2.3 and with the shear stress results presented on Figure 6.9.

For a fixed angle,  $\theta = 30^\circ$ , and for a nozzle-to-plate spacing

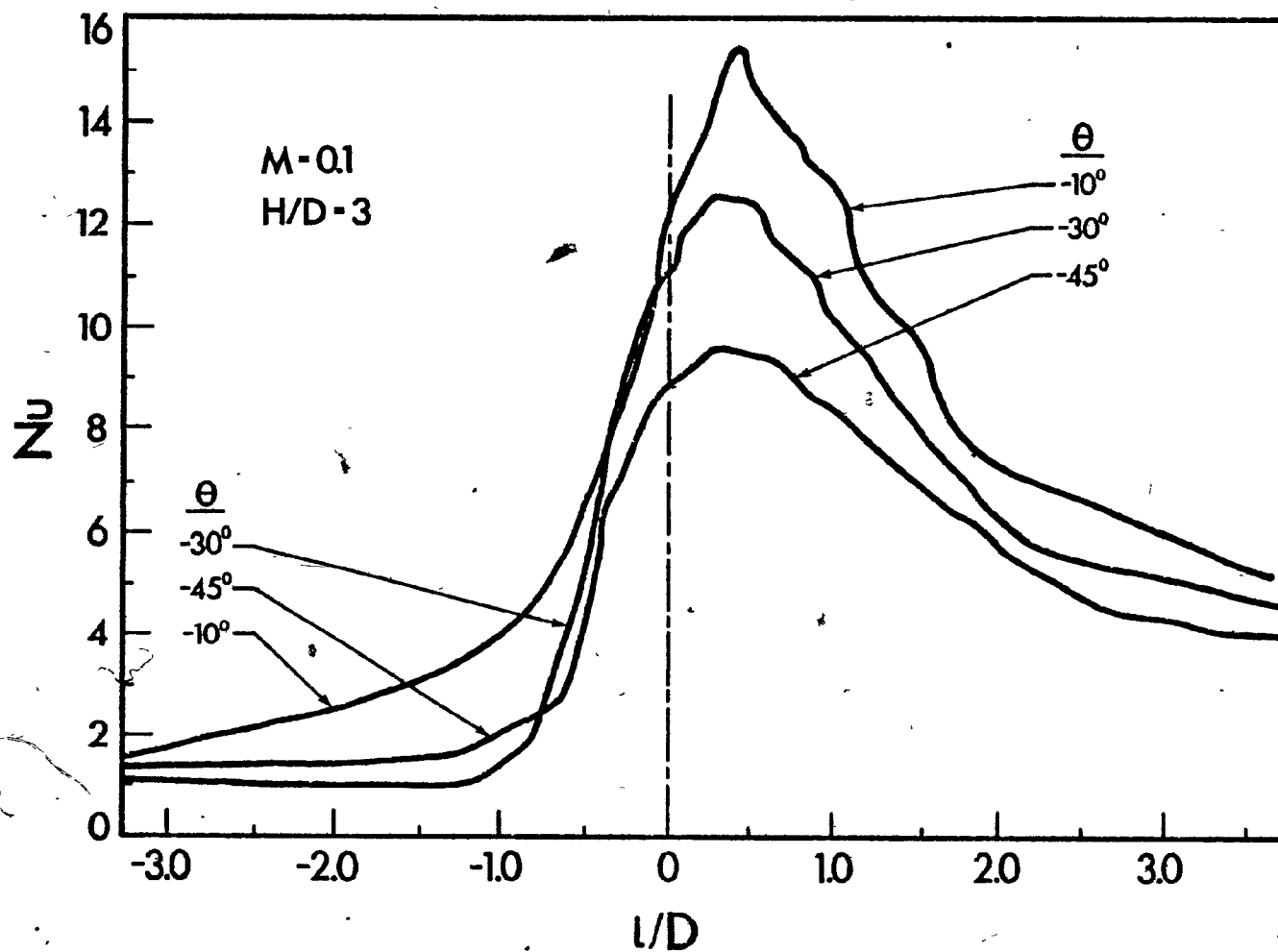


Figure 6.18 : Influence of  $\theta$  on local Nusselt number for small values of  $M$ .

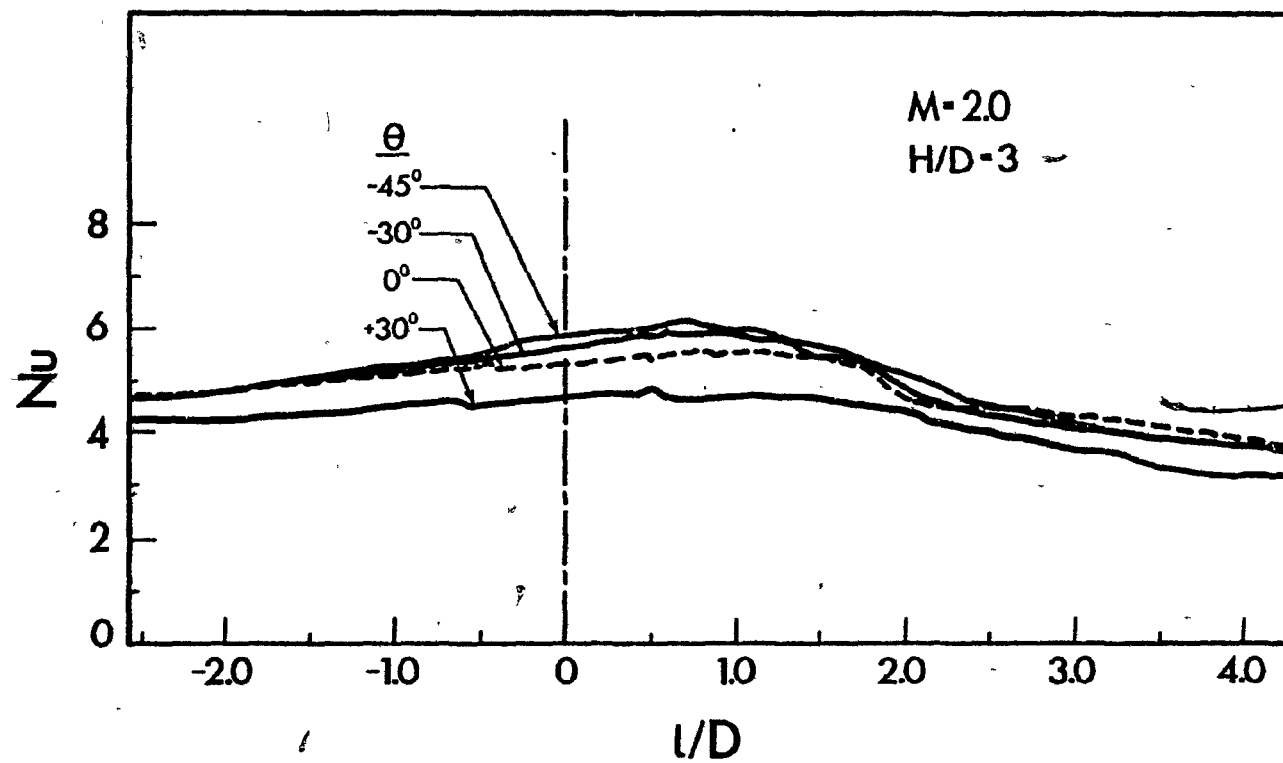


Figure 6.19 : Influence of  $\theta$  on local Nusselt number for large values of  $M$ .

of 3, several crossflow magnitudes were tested and the representation of these results on Figure 6.20 again indicates the transition between jet-dominated systems and crossflow-dominated systems. It appears that for  $M = 2.0, 1.0$  and  $0.7$  the system is dominated by the crossflow whereas it is dominated by the jet for  $M = 0.1$  and  $M = 0.4$ . The case for  $M = 0.5$  appears intermediate. As the existence of a stagnation point where  $\tau = 0$  could be taken as a criterion for terming a system as a jet-dominated impingement flow, it is of interest to note that there is a stagnation point for the cases of  $M = 0.1, 0.4$ , and  $0.5$ , but not for  $M = 0.7, 1.0$  and  $2.0$ .

#### 6.4, MASS TRANSFER RESULTS

Figures 6.22 through 6.24 show that mass transfer is affected in exactly the same manner as heat transfer by the presence of a crossflow. As in Section 6.3, it appears here that, depending on the value of  $M$ , the mass transfer can be dominated by the jet or by the crossflow. The variation of maximum local Sherwood number as a function of  $M$ , shown on Figure 6.25, parallels exactly what was observed for local Nusselt number on Figure 6.14, i.e. the stagnation point Sherwood number decreases very sharply with  $M$  for jet-dominated flows, whereas the maximum Sherwood number increases slightly with  $M$  for large crossflow magnitudes simply because of the augmentation of crossflow by the nozzle flow.

Figure 6.26 shows the influence of  $H/D$  on the mass transfer coefficient but shown this time on the entire impingement plate from crossflow entry to the outflow boundary. The central portion parallels the results obtained for heat transfer (Figure 6.15). On Figure 6.26 the dimensional mass transfer coefficient,  $k$ , was represented instead of the dimensionless Sherwood number, as this choice permits the case for pure

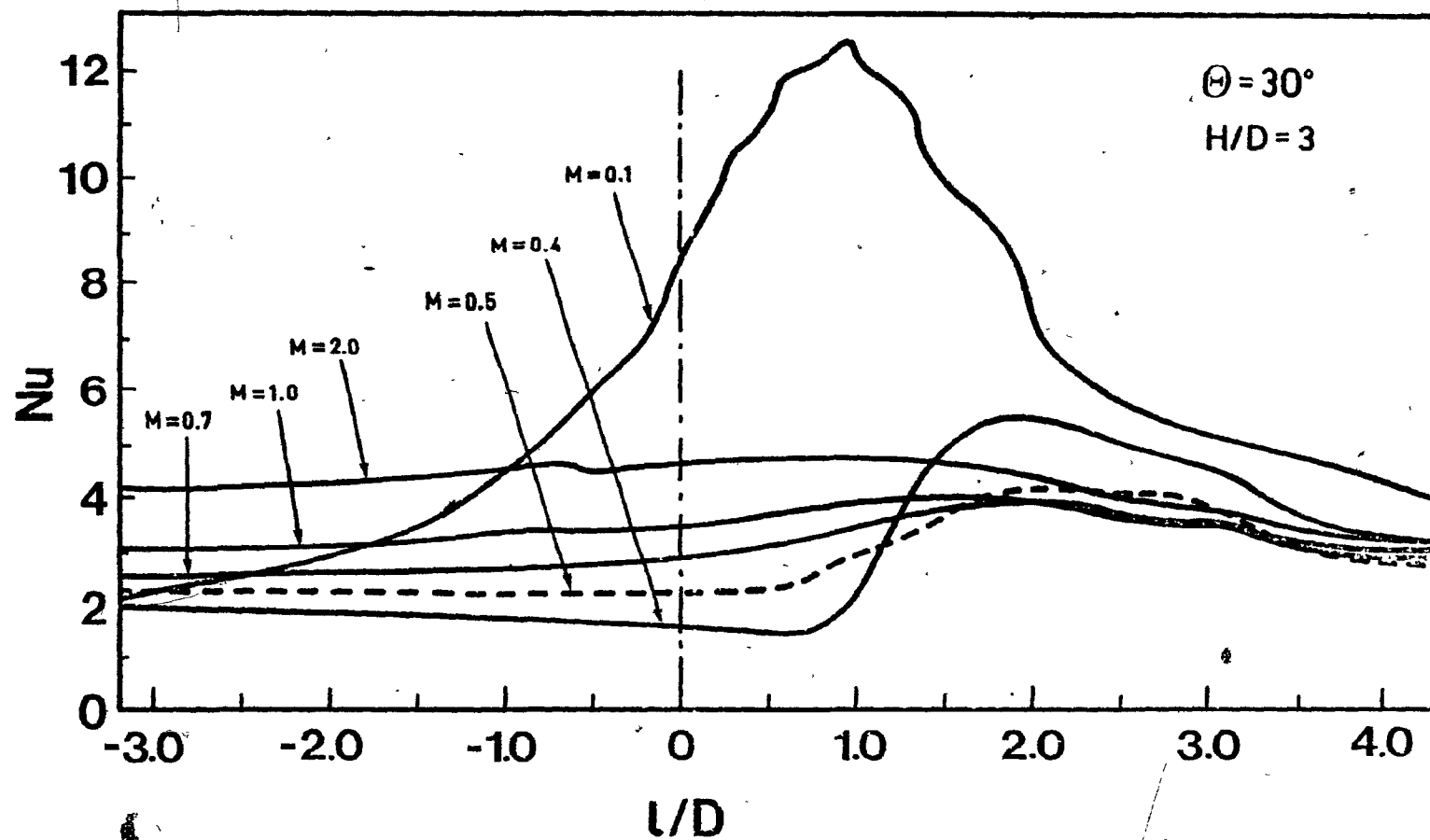


Figure 6.20 : Influence of  $M$  on local Nusselt number for  $\theta = 30^\circ$  and  $H/D = 3$ .

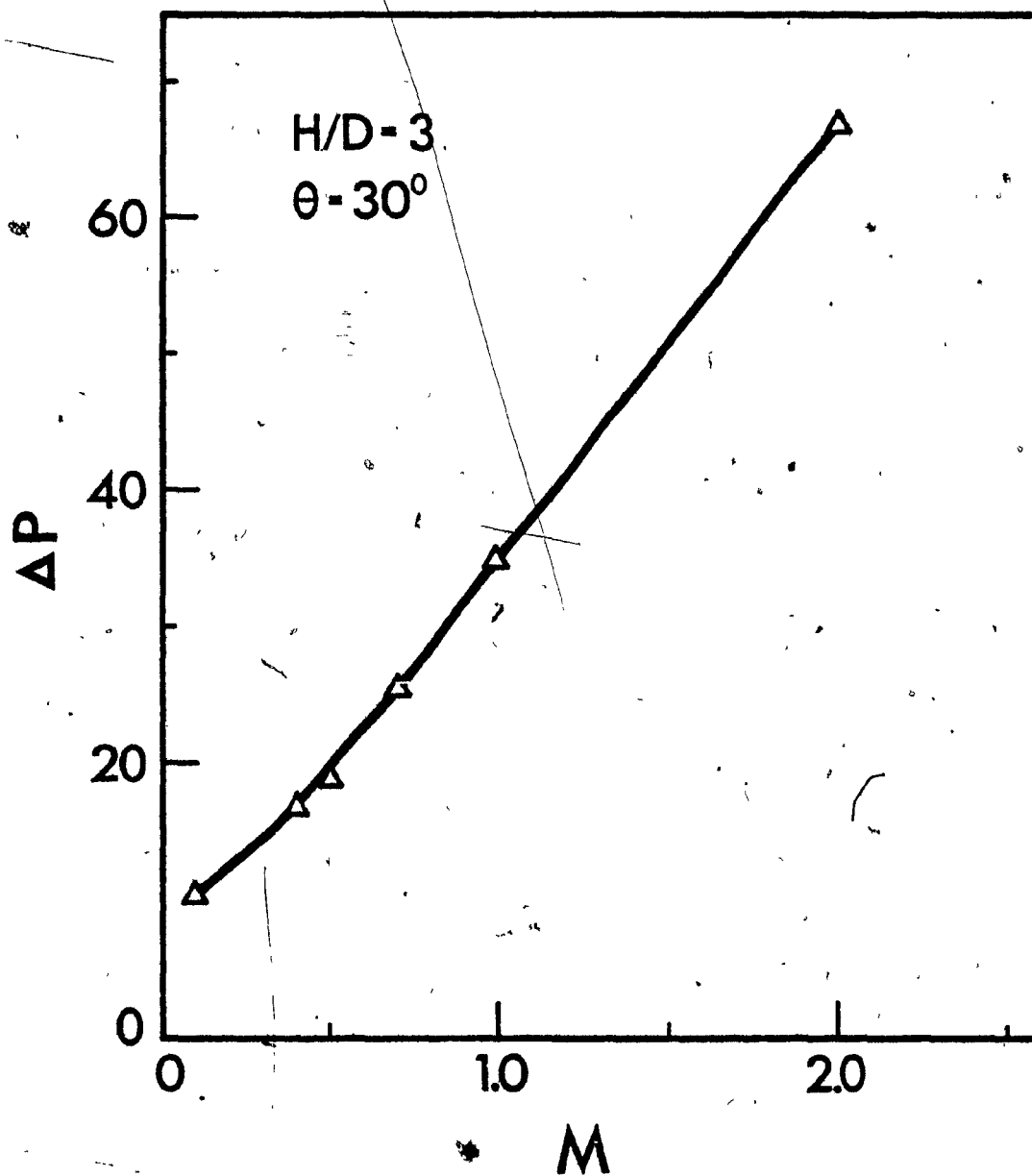


Figure 6.21 : Crossflow entry pressure versus crossflow magnitude,  $M$ .

crossflow to be included for comparison. As Sherwood and Nusselt number require use of a characteristic dimension, and as for impingement flows  $D$  is the appropriate dimension, it is evident that representation of the limiting case of pure crossflow cannot be in terms of such a dimensionless parameter. Equally, while  $H$  is appropriate as a characteristic dimension for crossflow-dominated flows, it would not be for approaching the other limiting case of pure jet flow.

The dashed line curve, representing pure crossflow, is typical of channel flows, i.e.  $k$  steadily decreases from its entrance maximum value corresponding to the absence of thermal and concentration boundary layers at the entrance. It may be recalled that in the present study the temperature and humidity profiles at the crossflow entry are flat, but that the flow enters with a fully developed laminar velocity profile. The thermal and concentration boundary layers build up rapidly downstream of the crossflow entry, leading to stabilization of  $k$  at a slowly decreasing value around  $0.01 \text{ kg/m}^2\text{s}$ . From the edge of the plate up to  $1/D = -4$  it is clear that the jet does not influence the mass transfer as these curves are all close to the pure crossflow case. For the same value of  $M$ , the smaller the plate spacing, the higher the crossflow velocity, and hence the further the flow proceeds from the crossflow entry before feeling the presence of the jet flow. Thus the position at which the mass transfer coefficient begins to be affected by the jet moves progressively back from values of  $1/D$  of about  $-1.5$  to  $-3.5$  to  $-4.5$  as  $H/D$  is increased, respectively, from 2 to 3 to 4. Figure 6.15 showed that the trends were similar for local heat transfer distribution, but the location of the curves is clarified by the inclusion on Figure 6.26 of the pure crossflow case. As noted at the beginning of Section 6.3.2.1, these conclusions concerning entrance effect

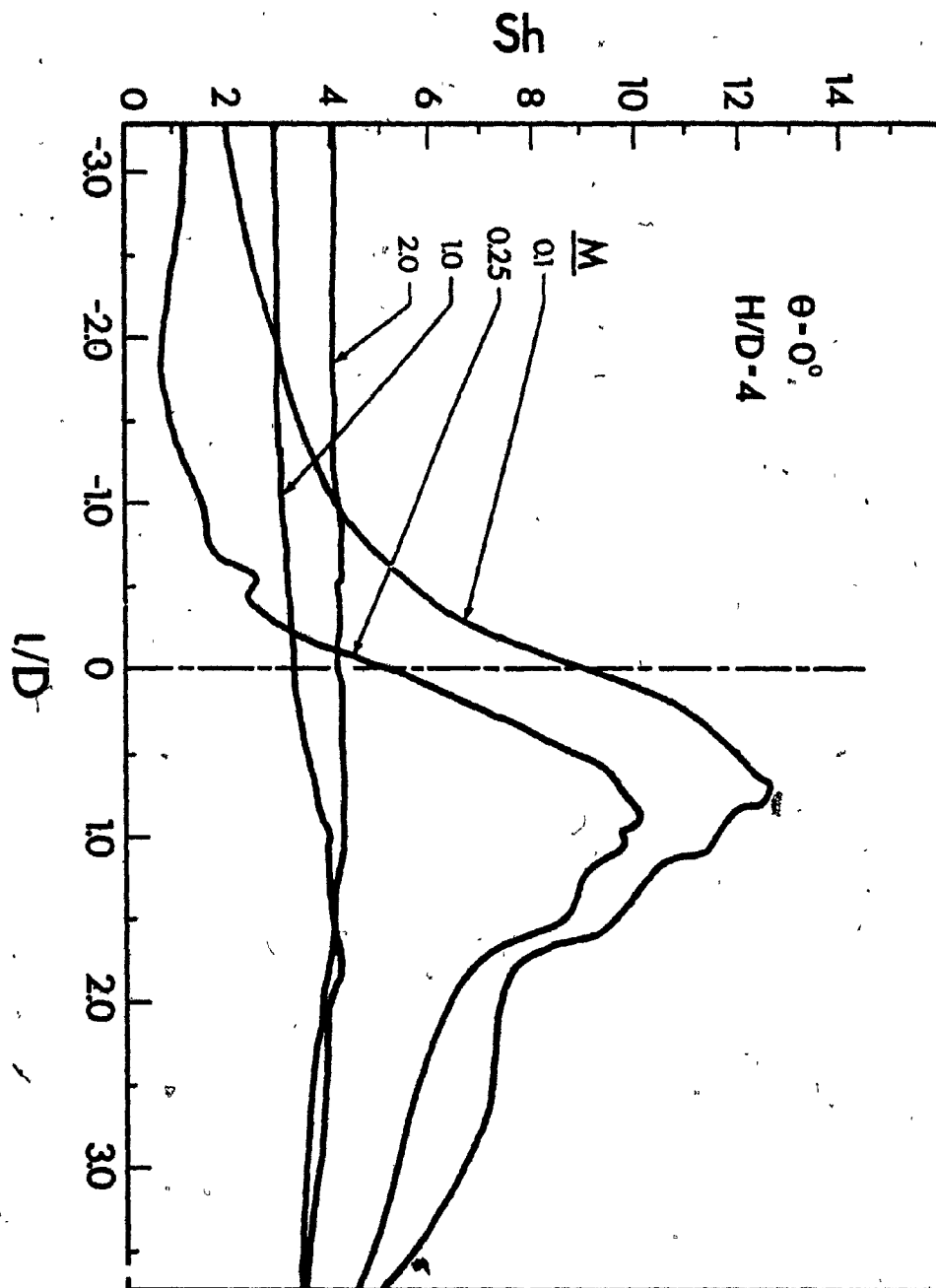


Figure 6.22 : Influence of crossflow magnitude,  $M$ , on local mass transfer.

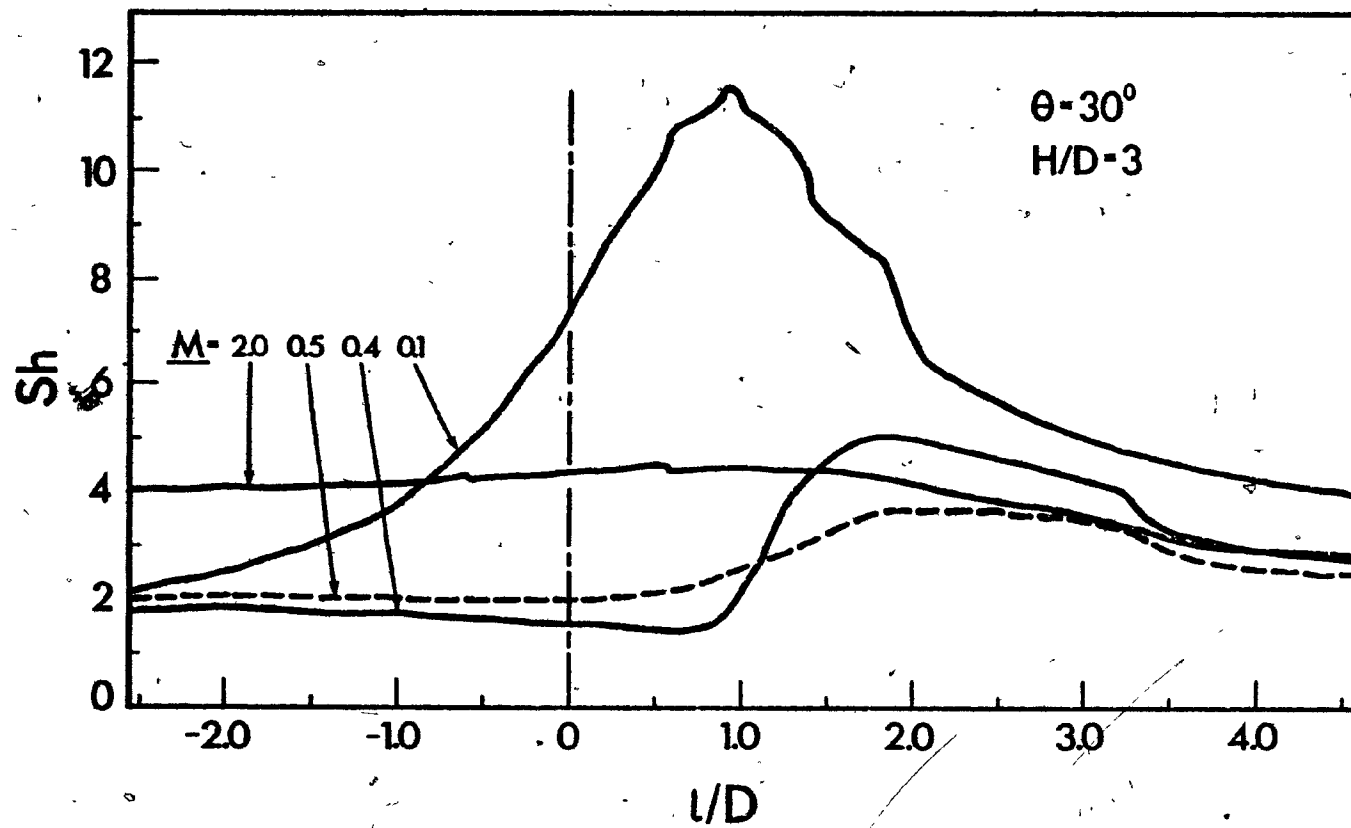


Figure 6.23 : Influence of crossflow magnitude,  $M$ , on local mass transfer.

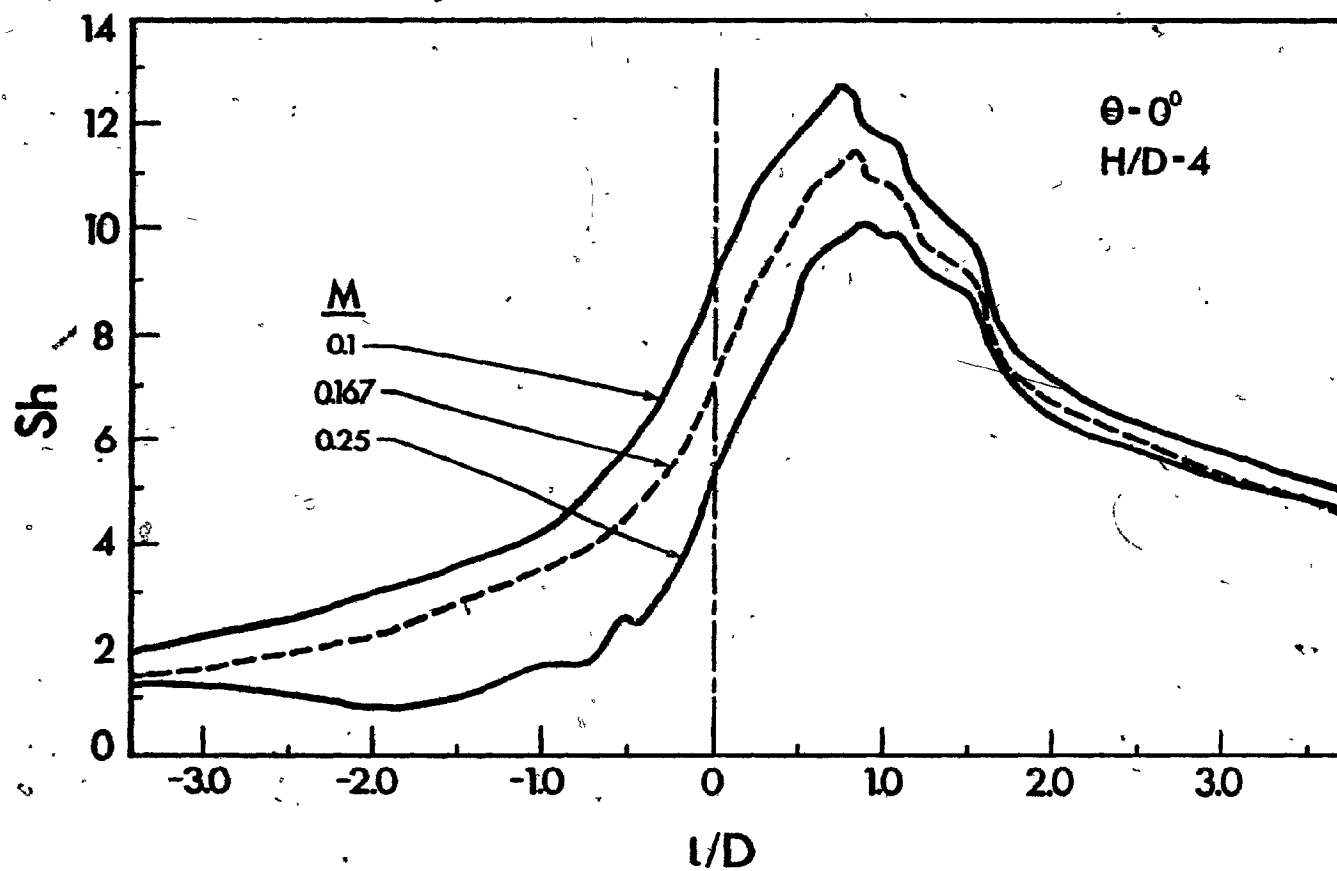


Figure 6.24 : Local mass transfer for jet-dominated systems.

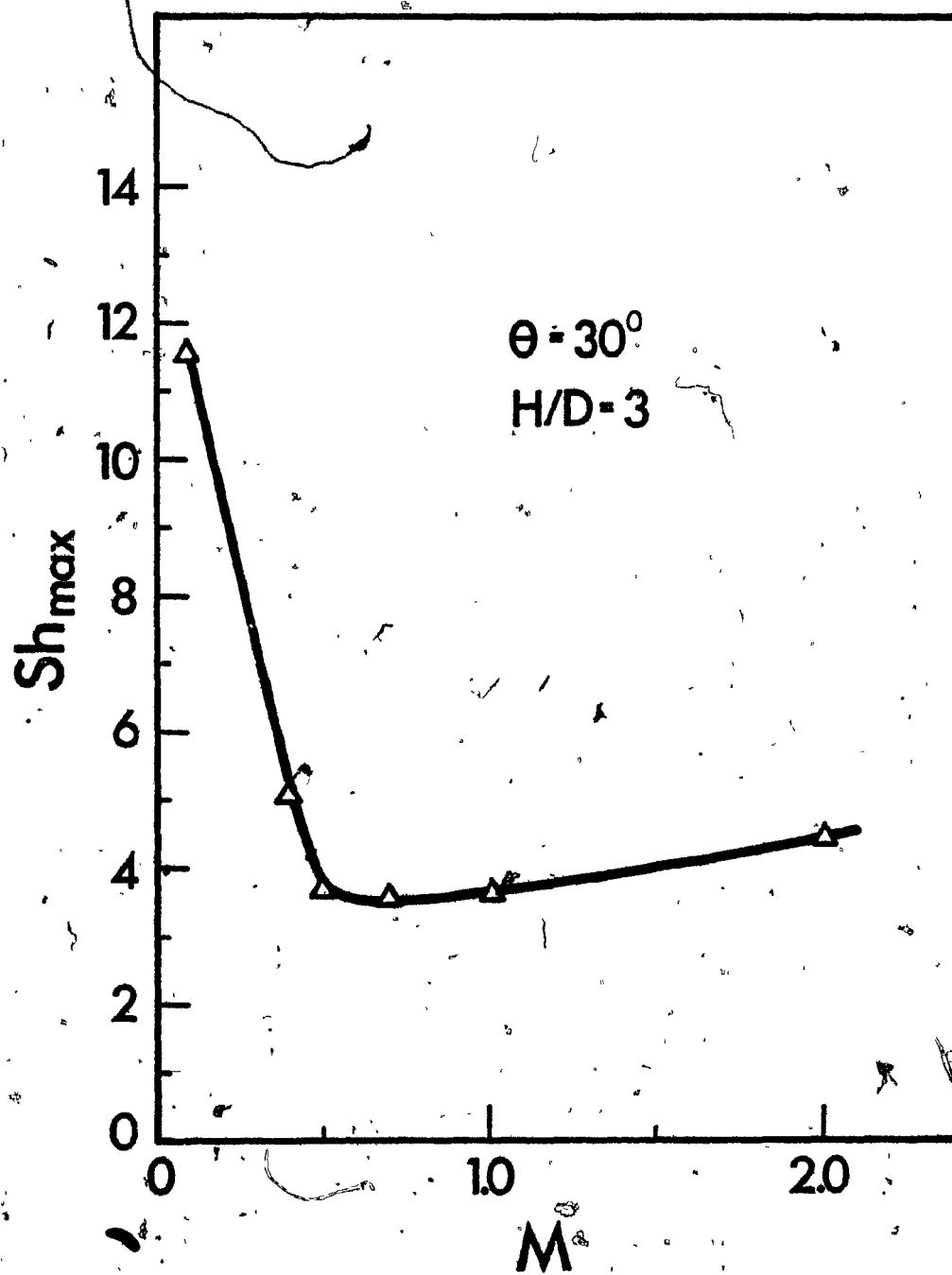


Figure 6.25 : Maximum local Sherwood number as a function of  $M$ .

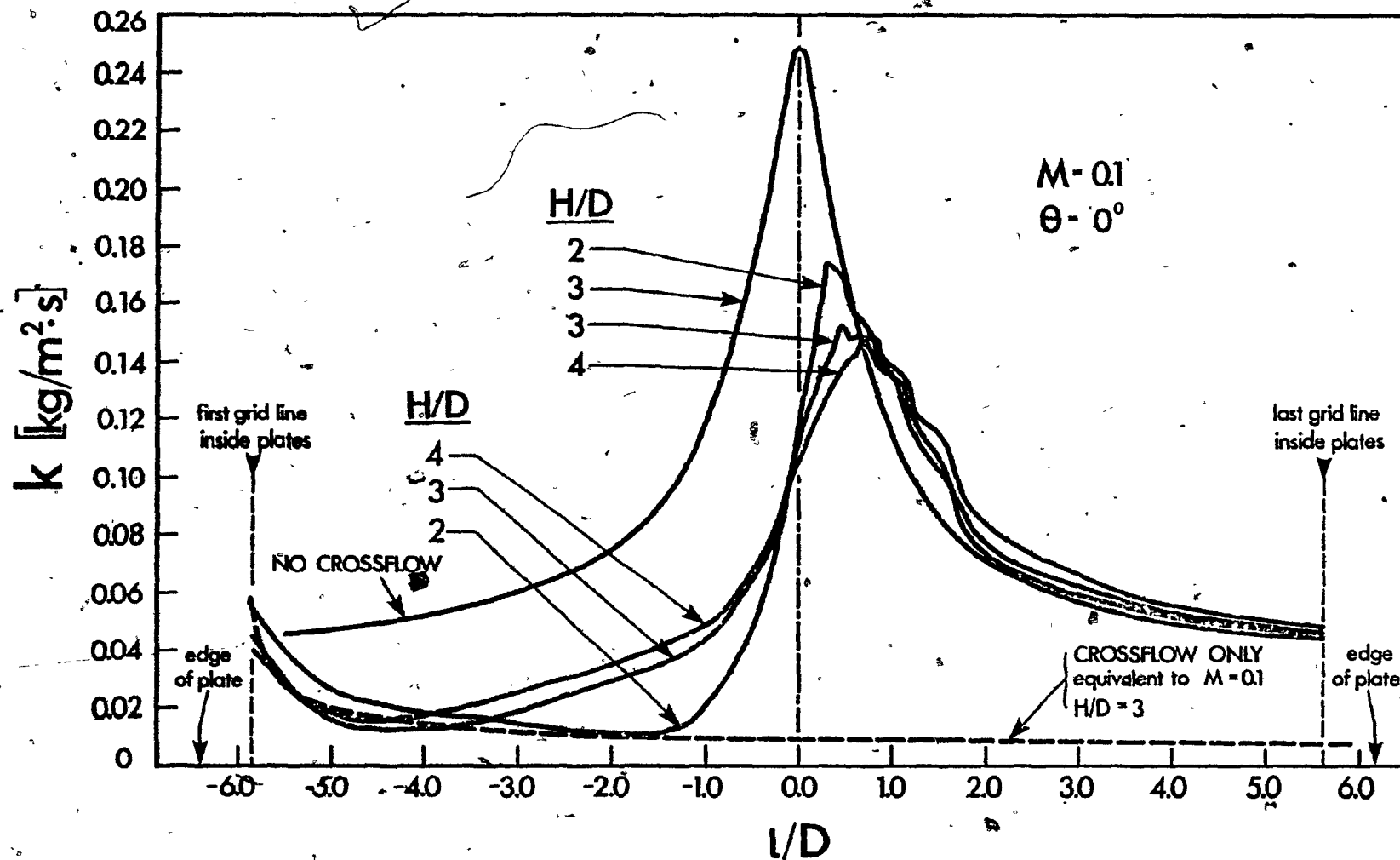


Figure 6.26 : Effect of  $H/D$  on local mass transfer for jets in a crossflow. Comparison with non-crossflow case.

on transfer rates at the impingement surface are equally applicable to both mass and heat transfer.

As to the influence of nozzle inclination on mass transfer, Figures 6.27, 6.28 and 6.29 suggest exactly the same comments as for heat transfer, i.e. for a given crossflow magnitude and nozzle-to-plate spacing there exists an optimum angle for which the mass transfer is maximum, and this optimum angle is for the nozzle inclined slightly against the crossflow.

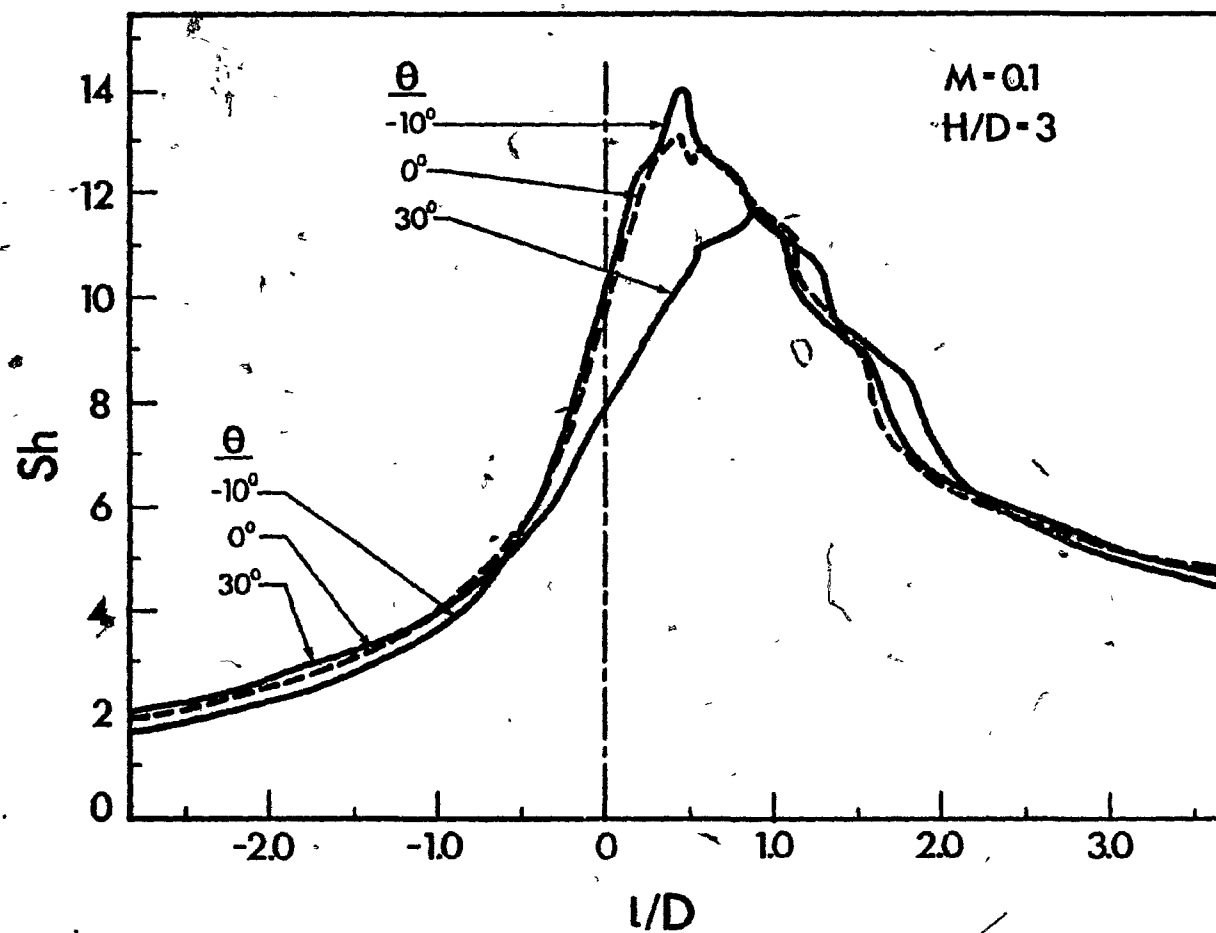


Figure 6.27 : Influence of  $\theta$  on local Sherwood number for small values of  $M$ .

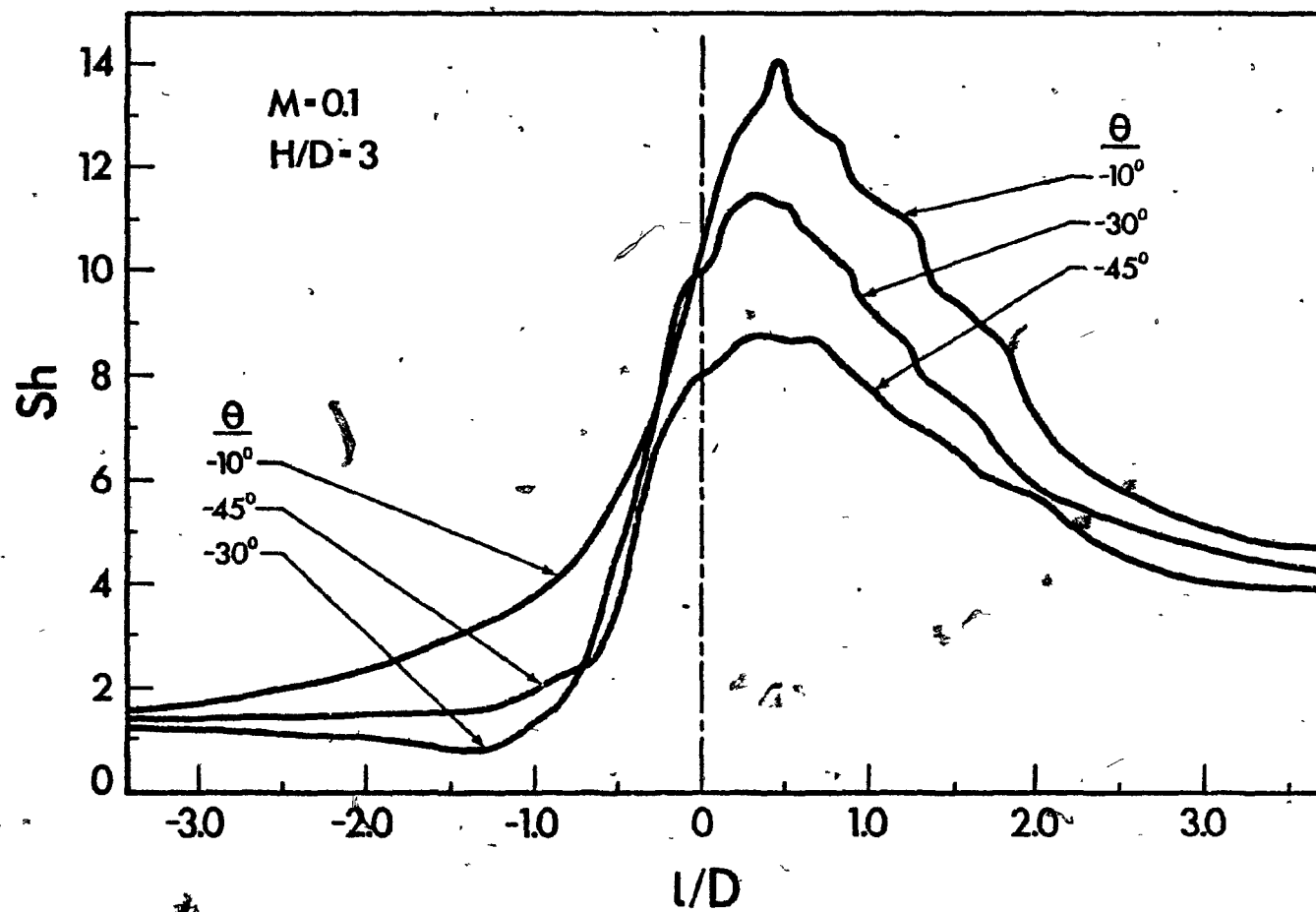


Figure 6.28 : Influence of  $\theta$  on local Sherwood number for small values of  $M$ .

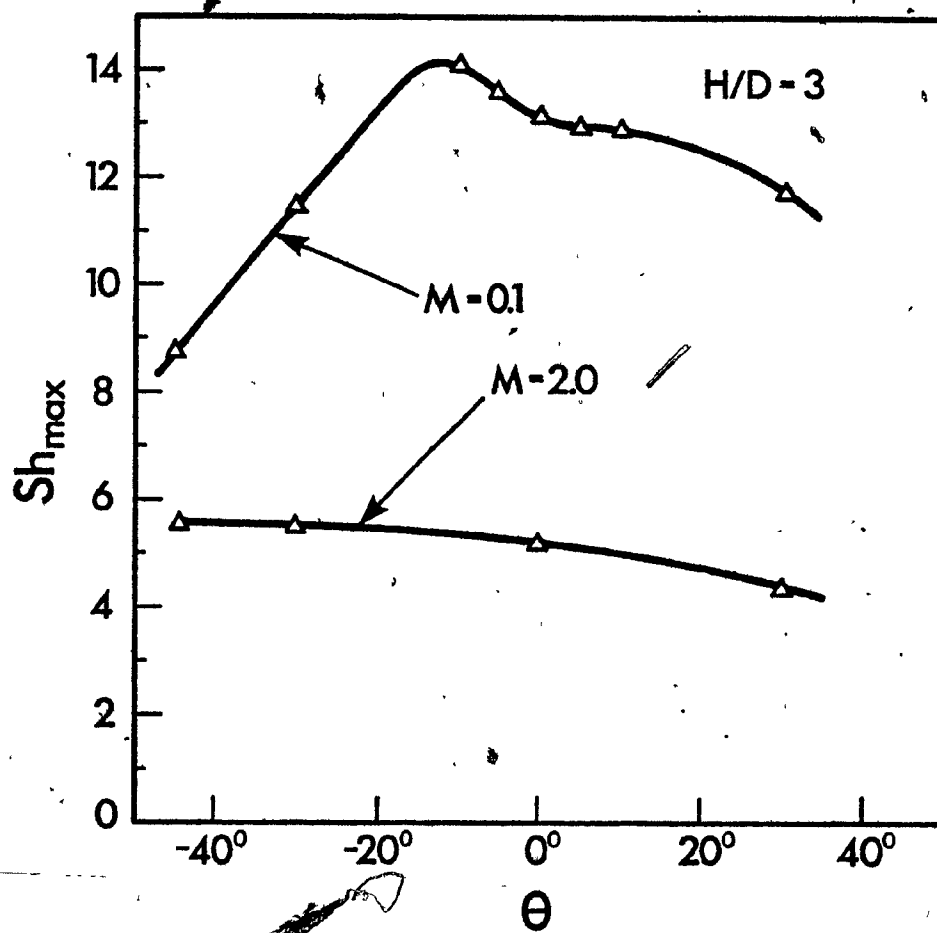


Figure 6.29 : Maximum local Sherwood number as a function of  $\theta$ .

## CHAPTER 7

## SUMMARY AND CONCLUSIONS

(i) Numerical solutions for the effect of crossflow and of nozzle inclination on the flow, heat and mass transfer characteristics of a semi-confined rectangular laminar jet impinging on a wetted isothermal flat surface have been obtained by solving iteratively the full governing equations. These equations are the Navier Stokes Equations coupled with the continuity, energy and species equations. The primitive variables are the velocity components, the pressure, the temperature and the mass fraction of water vapour. Allowance is made in the computer program for temperature and composition dependent fluid property variations. The crossflow magnitude is defined by the ratio  $M$  of the crossflow volumetric flow-rate to the jet volumetric flow-rate. The nozzle inclination angle  $\theta$  is counted negative when the jet is directed against the crossflow and positive when it is directed in the crossflow direction. The ranges of the parameters investigated are:

$$2 < H_0/D < 4$$

$$0.01 < M < 2.0$$

$$-45^\circ < \theta < 30^\circ$$

The nozzle width is 0.0032 m, the jet characteristics at the exit of the nozzle are:

$$T_j = 450^\circ\text{K}$$

$$m_j = 0.1$$

$$Re = 1000$$

The impingement plate was maintained at a temperature  $T_p = 350 \text{ K}$  ( $m_p = 0.302$ ).

The present results compare favourably with the experimental data of Bouchez and Goldstein (B7) and of Metzger and Korstad (M5).

(ii) For an orthogonal nozzle the velocity field analysis showed that for moderate values of  $M$  the jet is split into two parts at impingement: the main part of the jet flows in the crossflow direction whereas the second stream flows in the opposite direction along the plate. The interaction of this secondary stream with the crossflow creates a recirculation zone upstream of the impingement. As the crossflow magnitude,  $M$ , increases this recirculation zone diminishes and finally disappears.

(iii) In accordance with these observations the shear stress analysis along the plate shows that for small values of  $M$  the jet actually impinges on the plate whereas it is taken away by the crossflow and does not impinge for large values of  $M$ . The value of  $M$  which delimitates these two behaviours is a function of  $H/D$  and of the nozzle inclination.

(iv) The presence of a crossflow always reduces the stagnation point and average heat transfer rates below what would be expected if crossflow were not present.

(v) The influence of nozzle-to-plate spacing, which is negligible within the range  $2 < H/D < 4$  when there is no crossflow, becomes significant in presence of crossflow since  $H/D$  measures the distance over which the crossflow acts on the jet. Increasing the nozzle-to-plate spacing leads to more significant deflection of the jet and lower stagnation point transfer rates.

(vi) For moderate values of  $M$ , the heat and mass transfer rates on the plate are the ones of an impinging jet deflected and weakened by a crossflow. The larger the crossflow, the more effect on the jet; consequently the transfer rates are decreased as  $M$  increases within a range of values which permit impingement.

(vii) For large values of  $M$ , the heat and mass transfer rates on the plate are the ones of a channel flow (although influenced by the jet). Consequently the higher the crossflow, the higher the transfer rates. But, due to the absence of impingement, these transfer rates are much lower than in the case of moderate crossflow.

(viii) For a given crossflow magnitude and a given nozzle-to-plate spacing there exists an optimum angle (directed slightly against the crossflow direction) for which the heat and mass transfers are maximum. It appears hence that in practical cases where the presence of a crossflow cannot be eliminated, the decay in transfer coefficients caused by this crossflow can be limited by the choice of the appropriate impingement angle.

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APPENDIX A

LISTING OF COMPUTER PROGRAM

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MAIN

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PAGE 0001

SUBROUTINE CONTRO

MAIN0001  
MAIN0002  
MAIN0003  
CHAIN0004  
CHAIN0005

PROGRAMME D'ORDINATEUR POUR LE CALCUL DES  
TRANSFERTS DE MASSE ET DE CHALEUR SOUS UN  
JET RECTANGULAIRE IMPACTANT AVEC DIFFERENTS  
ANGLES SUR UNE PLAQUE HUMIDE

A COMPUTER PROGRAM FOR THE CALCULATION OF  
HEAT AND MASS TRANSFER OF SLOT JETS  
IMPINGING ON A PLATE

CHAIN0008  
CHAIN0009

CHAIN0012  
CHAIN0013  
CHAIN0014  
MAIN0015

0001  
0002  
0003  
0004

COMMON NITER  
COMMON SPLIT(200)  
COMMON CNIV(200)  
COMMON HEDU(9), HEDV(9), HEDPI(9), HEDT(9), HEDW(9), HEDVIS(9),  
HEDDEN(9), HEDCP(9), HEDDIF(9), HEDCON(9)

0005  
0006  
0007  
0008  
0009

DIMENSION HPUNCH(9)  
REAL NU  
LOGICAL PUNIN, PUNOUT  
LOGICAL CALW, CALT, FLAT  
COMMON

MAIN0019  
MAIN0020  
MAIN0021  
MAIN0022  
MAIN0023

1/UVEL/RESORU, NSWPU, NRFU, DXEPU(22), DXPWU(22), SEWU(22)  
1/VVEL/RESORV, NSWPV, URUV, DYNPV(22), DYPV(22), SNSV(22), YCV(22)  
1/PCOR/RESORM, NSWPU, URUV, DU(20,20), DV(20,20), IPREF, JPREF  
1/TEMP/RESORT, NSWPT, URFT  
1/SPEC/RESORW, NSWPW, URFW

0010

COMMON  
1/ALL/IT, JT, NI, NJ, NIM1, NJM1, GREAT  
1/VAR/U(20,20), V(20,20), P(20,20), PP(20,20), T(20,20), W(20,20)  
1/GEOM/X(22), Y(22), DXEP(22), DXPW(22), DYNP(22),  
2 DYP(22), SNS(22), SEW(22), XU(22), YV(22)  
1/PGM/PUNIN, PUNOUT, TIMAX, MAXIT, SORMAX, INOPRI

MAIN0027  
MAIN0028  
MAIN0029  
MAIN0030

0011

COMMON  
1/FLUPR/RHOA, RHOB, RATID, CPA, CPB, CPDE,  
2 DEN(20,20), VIS(20,20), CPP(20,20), DIFF(20,20), COND(20,20)  
1/FLUPR2/CONDA1, CONDA2, CONDA3, CONDB1, CONDB2, CONDB3,  
2VISA1, VISA2, VISA3, VISB1, VISB2, VISB3, DIFFU,  
3CPA1, CPA2, CPA3, CPB1, CPB2, CPB3

MAIN0034  
MAIN0035  
MAIN0036

0012

COMMON /MOLEWT/MTA, WT8  
1/PROB1/RE, TIN, PIN, TWALL, WIN, FLOWIN, FLOWEV,  
2 ISTEP, JSTEP, ISTP1, JSTP1, ISTH1, JSTH1  
3 JNOZ, JNOZP1, JNOZM1  
1/COEF/AP(20,20), AN(20,20), AS(20,20), AE(20,20), AW(20,20),  
2 SU(20,20), SP(20,20)

MAIN0041  
MAIN0042  
MAIN0043

0013  
0014  
0015

COMMON /SUCTION/SUCR, SUCR  
COMMON/ANGLE/THETA  
COMMON/CROSS/PCNTCR, DELTIN, DELWIN

MAIN0046

CHAPTER 0 0 0 0 0 0 0 0 PRELIMINARIES 0 0 0 0 0 0 0 0 MAIN0047

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C-----DEPENDENT VARIABLE SELECTION
0016 CALL CLOCK1
0017 READ(5,010) HEDU,HEDV,HEDP,HEDT,HEDW,HEDVIS,HEDDEN,HEDCP,HEDDIF
      1, HEDCON
0018 010 FORMAT(9A4)
CHAPTER 1 1 1 1 1 PARAMETERS AND CONTROL INDICES 1 1 1 1 1
C
0019 NITER=0
0020 FLAT=.FALSE.
0021 CALT=.TRUE.
0022 CALW=.TRUE.
0023 INPROP=2
0024 CALL GRID(JNOZ,JSTEP,RSMALL,HORATO)
0025 NIMI=NI-1
0026 NJMI=NJ-1
0027 ISTEP=ISTEP+1
0028 ISTM=ISTEP-1
0029 JSTEP=JSTEP+1
0030 JSTM=JSTEP-1
0031 JNOZP1 = JNOZ+1
0032 JNOZM1 = JNOZ-1
0033 CALL INIT
0034 IMON=NIMI-1
0035 JMON=JO
CHAPTER 2 2 2 2 2 INITIAL OPERATIONS 2 2 2 2 2 2 2 2 2
C
0036 GAS CONSTANT. R = 8.3142E+3 J/KILO-GMOLE-K
0037 GAS=8.3142E+3
0038 ATEMP=(TIN+TWALL)*0.5
0039 CPB=CPB1+CPB2*ATEMP+CPB3*ATEMP*ATEMP
0040 RHOA=WTA/GAS
0041 RHOB=WTB/GAS
0042 RENO = SORT(RE)
0043 XAJ=WIN/(WIN+(1.-WIN)/RATIO)
0044 OJET=2.*RSMALL
0045 DJP=HORATO*OJET
0046 IF(.NOT.PUNIN) GOTO 208
C-----FIELD INITIALIZATION
0047 READ(5,10) HPUNCH
0048 READ(5,600) ((U(I,J),I=1,NI),J=1,NJ)
0049 READ(5,600) ((V(I,J),I=1,NI),J=2,NJ)
0050 READ(5,600) ((P(I,J),I=1,NI),J=1,NJ)
0051 READ(5,600) ((T(I,J),I=1,NI),J=1,NJ)
0052 READ(5,600) ((W(I,J),I=1,NI),J=1,NJ)
0053 GOTO 210
0054 208 CONTINUE
0055 DO 200 I=1,NI
0056 DO 200 J=1,NJ
0057 U(I,J)=0.0

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0058      V(I,J)=0.0
0059      P(I,J)=0.0
0060      W(I,J)=WIN
0061      T(I,J)=TIN
0062      200 CONTINUE
0063      211 CONTINUE
0064      DO 206 I=1,NI
0065      DO 206 J=1,NJ
0066      IF(.NOT.CAL T) T(I,J)=ATEMP
0067      IF(.NOT.CAL W) W(I,J)=WIN
0068      PP(I,J)=0.0
0069      DU(I,J)=0.0
0070      DV(I,J)=0.0
0071      SU(I,J)=0.0
0072      SP(I,J)=0.0
0073      206 CONTINUE
0074      C----- FLUID PROPERTIES - DEN,VIS,DIFF,CPP,COND -- CALCULATIONS
0075      CALL PROPS
0076      C----- CROSS FLOW DEFINITION
0077      UIN=V(5(1,1)*RE/DEN(1,1)/RSMALL*0.5
0078      VIN=PCNTCR*UIN/HDRATO
0079      DO 212 I=2,NIMI
0080      U(I,1)=0.0
0081      HRATIO=ABS(X(I)-DJP/2.)/DJP/2.
0082      V(I,2)=3./2.*VIN*(1.-HRATIO*HRATIO)-
0083      P(I,1)=0.0
0084      T(I,1)=TIN-DELTIN
0085      W(I,1)=WIN-DELWIN
0086      212 CONTINUE
0087      C----- SPECIFY CONFINEMENT PLATE CONDITIONS
0088      DO 201 J=1,NJ
0089      T(1,J)=TIN
0090      U(2,J)=0.0
0091      IF(J.EQ.1) GO TO 201
0092      V(1,J)=0.0
0093      201 CONTINUE
0094      C----- SPECIFY IMPINGMENT PLATE CONDITIONS
0095      PSTT=PST(TWALL)
0096      WWALL=PSTT/(PSTT+(PIN-PSTT)*RATIO)
0097      IF(.NOT.CAL W) WWALL=WIN
0098      DO 207 J=1,NJ
0099      T(NI,J)=TWALL
0100      W(NI,J)=WWALL
0101      IF(J.EQ.1) GO TO 207
0102      V(NI,J)=0.0
0103      207 CONTINUE
0104      C----- CALCULATE RESIDUAL SOURCE-NORMALIZATION FACTORS-----
0105      FLOJET=0.0
0106      XMOJET=0.0
0107      YMOJET=0.0
0108      FLOCRS=0.0
0109      YMOCRS=0.0

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MAIN0100  
 MAIN0101  
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MAIN0145



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0152      IF(CALW) RESORW=RESORW/SNORMW
0153      IF(CALT) RESORT=RESORT/SNORMT
0154      WRITE(6,311) NITER,RESORU,RESORV,RESORW,RESORT,RESORW
1          U(IMON,JMON),V(IMON,JMON),P(IMON,JMON),T(IMON,JMON)
2          W(IMON,JMON),SPLIT(NITER),CNIV(NITER)
      IF(MOD(NITER,INDPRI).NE.0) GO TO 301
      CALL PRINT(1.1,NI,NJ,IT,JT,XU,Y,U,HEDU)
      CALL PRINT(1.2,NI,NJ,IT,JT,X,YV,V,HEDV)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,P,HEDP)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,T,HEDT)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,W,HEDW)
      WRITE(6,310) IMON,JMON
0161      CONTINUE
0162
C-----
301      TERMINATION TESTS
      SORCE=AMAX1(RESORW,RESORU,RESORV,RESORT,RESORW)
0163      IF(NITER.EQ.MAXIT) GO TO 302
C-----
      TIMER
      CALL CLOCK2(ETIME)
0165      IF(ETIME/60.GT.TIMAX) GO TO 302
0166
C
      IF(RESORT.LT.SORMAX) GO TO 303
      GO TO 304
0167
303      CALT=.TRUE.
      IF(MOD(NITER,10).NE.1) CALT=.FALSE.
0168
304      IF(RESORW.LT.SORMAX) GO TO 305
      GO TO 306
0169
305      CALW=.TRUE.
      IF(MOD(NITER,10).NE.1) CALW=.FALSE.
0170
306      CONTINUE
      IF(SORCE.GT.SORMAX) GO TO 300
0171
      IF(NITER.LE.10) GOTO 300
0172
302      CONTINUE
0173
C-----
CHAPTER 4 4 4 4 4 4 FINAL OPERATIONS AND OUTPUT 4 4 4 4 4
C-----
      FLOUT=0.0
      DO 421 I=2,NIMI
0174
0175      ARDENJ=0.5*(DEN(I,NJ)+DEN(I,NJM1))*SEW(I)*0.01
0176
0177      FLOUT=FLOUT+ARDENJ*V(I,NJ)
0178
421      CONTINUE
      ACCUM=FLOWEV+FLOWIN-FLOUT
      WRITE(6,420) FLOWEV,FLOUT,ACCUM,PUNOUT
C-----
      CALL PRINT(1.1,NI,NJ,IT,JT,XU,Y,U,HEDU)
      CALL PRINT(1.2,NI,NJ,IT,JT,X,YV,V,HEDV)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,P,HEDP)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,T,HEDT)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,W,HEDW)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,VIS,HEDVIS)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,DEN,HEDDEN)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,CPP,HEDCP)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,DIFF,HEDDIF)
      CALL PRINT(1.1,NI,NJ,IT,JT,X,Y,COND,HEDCON)
      IF(.NOT.PUNOUT) GOTO 410

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MAIN0198
MAIN0199
MAIN0200
MAIN0201
MAIN0203
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MAIN0221
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MAIN0230
MAIN0231
MAIN0232
MAIN0233
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MAIN0235
MAIN0236
MAIN0237
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MAIN0239
MAIN0240
MAIN0241
MAIN0242

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0197      WRITE(7,600) ((U(1,J),I=1,N1),J=1,NJ)      MAIN0243
0198      WRITE(7,600) ((V(1,J),I=1,N1),J=2,NJ)      MAIN0244
0199      WRITE(7,600) ((P(1,J),I=1,N1),J=1,NJ)      MAIN0245
0200      WRITE(7,600) ((T(1,J),I=1,N1),J=1,NJ)      MAIN0246
0201      WRITE(7,600) ((W(1,J),I=1,N1),J=1,NJ)      MAIN0247
0202      410 CONTINUE                                MAIN0248
C-----CALCULATIONS OF SHEAR-STRESS COEFF, NUSSELT NO, STANTON NO, SHERWOMAIN0249
0203      DZN=XU(N1)-X(N1M1)                          MAIN0250
0204      DDZN=XU(N1)-X(N1-2)                          MAIN0251
0205      DELT=TWALL-TIN                               MAIN0252
0206      IF(ABS(DELT-0.),LE.1.E-8) DELT=1.            MAIN0253
0207      ATEMP=0.5*(TIN+TWALL)                        MAIN0254
0208      CPA=CPA1+CPA2*ATEMP+CPA3*ATEMP*ATEMP        MAIN0255
0209      IF(ABS(DELT-0.),LE.1.E-8) TERM=1.            MAIN0256
0210      WRITE(6,402)                                  MAIN0257
0211      DO 401 J=2,NJM1                              MAIN0258
0212      ACPP=0.5*(CPP(2,2)+CPP(N1,J))                MAIN0259
0213      ADEN=0.5*(DEN(2,2)+DEN(N1,J))                MAIN0260
0214      ADIFF=0.5*(DIFF(N1,J)+DIFF(2,2))             MAIN0261
0215      ACOND=0.5*(COND(2,2)+COND(N1,J))             MAIN0262
0216      APRESS=0.5*(P(N1,J)+PIN)                     MAIN0263
C-----MASS TRANSFER CALCULATIONS                   MAIN0264
0217      WP=W(N1,J)                                  MAIN0265
0218      XAP=WP/(WP+(1.-WP)/RATIO)                     MAIN0266
0219      WDIFF=ALOG((1.-XAJ)/(1.-XAP))                 MAIN0267
0220      IF(ABS(WDIFF-0.0),LE.1.E-8) WDIFF=1.0        MAIN0268
0221      WFLUX=U(N1,J)*DEN(N1,J)                      MAIN0269
0222      HMI=-WFLUX/WDIFF/MTA                         MAIN0270
0223      SH1=HMI*DJET/ADIFF*(GAS*ATEMP/APRESS)        MAIN0271
0224      HM=-WFLUX*(1.-W(N1,J))/(W(N1,J)-W(2,2))    MAIN0272
0225      SH=HM*DJET/DIFF(2,2)/DEN(2,2)                MAIN0273
C-----HEAT TRANSFER CALCULATIONS                   MAIN0274
0226      IF(.NOT.CALW) WFLUX=1.E-4                   MAIN0275
0227      HFLUX=(T(N1-2,J)-TWALL)/DDZN                MAIN0276
0228      HFLUX=CONO(N1,J)*HFLUX                       MAIN0277
0229      H=-HFLUX/DELT                                MAIN0278
0230      NU=H*DJET/COND(2,2)                          MAIN0279
0231      ST=H/(DEN(2,2)*CPP(2,2)*UIN)                 MAIN0280
0232      YI=Y(M)-DJET*TAN(THETA)                     MAIN0281
0233      YY=(Y(J)-YI)/DJET                            MAIN0282
0234      WRITE(6,403) J,YY,WDIFF,WFLUX,HMI,SH1,HM,SH,HFLUX,H,NU,ST MAIN0283
0235      IF((J-1)/5*5.EQ.(J-1)) WRITE(6,422)         MAIN0284
0236      401 CONTINUE                                MAIN0285
C-----SHEAR STRESS CALCULATIONS                   MAIN0286
0237      WRITE(6,425)                                  MAIN0287
0238      DD 407 J=3,NJM1                              MAIN0288
0239      YYY=YV(J)/DJET                               MAIN0289
0240      TAU=V IS(N1,J)*V(N1M1,J)/DZN                 MAIN0290
0241      CF=TAU*2./(DEN(1,2)*UIN*UIN)                 MAIN0291
0242      ACF=0.5*(CF*RENO)                             MAIN0292
0243      WRITE(6,426) J,YYY,TAU,CF,ACF                MAIN0293
0244      IF((J-2)/5*5.EQ.(J-2)) WRITE(6,422)         MAIN0294
0245      407 CONTINUE                                MAIN0295

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C-----AVERAGE TRANSPORT PROPERTIES CALCULATIONS
0246 WRITE (6,424)
0247 DO 430 J=2,NJMI
0248 PR=CPP(NI,J)/VIS(NI,J)/COND(NI,J)
0249 SC=VIS(NI,J)/(DEN(NI,J)*DIFF(NI,J))
0250 PR=0.5*(PR+PRANDL)
0251 SC=0.5*(SC+SCHMIT)
0252 AVIS=0.5*(VIS(NI,J)+VIS(2,2))
0253 ADEN=0.5*(DEN(NI,J)+DEN(2,2))
0254 ACP=0.5*(CPP(NI,J)+CPP(2,2))
0255 ADIFF=0.5*(DIFF(NI,J)+DIFF(2,2))
0256 ACOND=0.5*(COND(NI,J)+COND(2,2))
0257 WRITE (6,423) J,PR,SC,AVIS,ADEN,ACP,ADIFF,ACOND
0258 IF((J-1)/5*.EQ.(J-1)) WRITE (6,422)
0259 430 CONTINUE
0260 STOP

C-----FORMAT STATEMENTS
0261 210 FORMAT (10,9X,'PULJET',///,10X,'SIMULTANEOUS HEAT & MASS TRANSFER
UNDER AN IMPINGING JET',///,5X,'SCALAR GRID SYSTEM: X-, Y-
2DIRECTION',/)
0262 220 FORMAT (///,15X,'JET INLET CONDITIONS',/,
2 20X,'REYNOLDS NUMBER',T40,IPE11.3,/,
3 20X,'AVERAGE VELOCITY',T40,IPE11.3,/,
4 20X,'TEMPERATURE',T40,IPE11.3,/,
5 20X,'PRESSURE',T40,IPE11.3,/,
6 20X,'MASS FRACTION OF A',T40,IPE11.3,/,
7 20X,'MOLE FRACTION, A',T40,0PE11.3)
0263 221 FORMAT (///,20X,'PRANDTL NO',T40,IPE11.3,/,
2 20X,'SCHMIDT NO',T40,IPE11.3)
0264 222 FORMAT (///,8X,'CROSS FLOW INLET CONDITIONS',/,
2 20X,'REYNOLDS NUMBER',T40,IPE11.3,/,
3 20X,'AVERAGE VELOCITY',T40,IPE11.3,/,
4 20X,'TEMPERATURE',T40,IPE11.3,/,
5 20X,'PRESSURE',T40,IPE11.3,/,
6 20X,'MASS FRACTION OF A',T40,IPE11.3)
0265 223 FORMAT (///,18X,'CROSS MASS FLOW-RATE IS',F8.2,1X,'% OF JET MASS FL
LOW RATE')
0266 230 FORMAT (///,15X,'IMPINGEMENT WALL CONDITIONS',/,
2 20X,'TEMPERATURE',T40,IPE11.3,/,20X,'USUC/UIN',T40,0PE11.3)
0267 232 FORMAT (///,15X,'GEOMETRY OF SYSTEM',/,20X,'JET RADIUS',
2 T40,IPE11.3,/,20X,'H/D RATIO',T40,IPE11.3,/,
3 20X,'NOZZLE FROM J',T40,I11,/,29X,'TO J',T40,I11,
4 //,20X,'IMPINGEMENT ANGLE',T40,0PE11.6)
0268 250 FORMAT (///,15X,'PROGRAM CONDITIONS',/,
2 20X,'MAX ALLOW. ITER',T40,I11,/,
3 20X,'MAX ALLOW. TIME',T40,IPE11.3,/,
4 20X,'MAX ALLOW. RESI',T40,IPE11.3)
0269 260 FORMAT (///,15X,'JET MASS FLOW IN',T35,IPE15.6,/,
2 13X,'JET X-MOMENTUM IN',T35,IPE15.6,/,
3 13X,'JET Y-MOMENTUM IN',T35,IPE15.6)
0270 261 FORMAT (///,8X,'CROSS-FLOW MASS FLOW IN',T35,IPE15.6,/,
2 6X,'CROSS-FLOW Y-MOMENTUM IN',T35,IPE15.6,/,
3 15X,'VARIABLE FIELD INITIALISED AS :')

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MAIN0296  
 MAIN0297  
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 MAIN0301  
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 MAIN0333  
 MAIN0334  
 MAIN0335

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0271      280 FORMAT(1H,/,8(1PE15.7))
0272      310 FORMAT(1H1.4HITER,4X,95HI-----ABSOLUTE RESIDUAL SOURCE SUMS---MAIN0338
          1-----1      1--FIELD VALUES AT MONITORING LOCATION(,12,1H,,12,
          21H),3H--1,6X,*SPLIT RATE*/2X,2HNO,6X,4HUMOM,6X,4HVMOM,6X,4HMASS,6X
          3,4HENER,6X,4HSPEC,11X,1HU,9X,1HV,9X,1HP,9X,1HT,9X,1HW,7X,5HALPHA,
          44X,4HVINC,/)
0273      311 FORMAT(1H,13,1X,1P5E10,3,6X,1P5E10,3,2X,0PF6,4,1X,1PE10,3)
0274      402 FORMAT(1H1,/,10X,*TRANSFER COEFFICIENTS',/,1X,1 J      R/D      WDIFFMAIN0344
          2      *FLUX',8X,*HMI',8X,*SH1',9X,*HM',9X,*SH',
          36X,*HFLUX',10X,*H',9X,*NU',9X,*ST',
          4/,130('---'))
          403 FORMAT(' ',12,F8.5,1X,F8.5,9(1PE11,3))
0275      420 FORMAT(//,5X,*OVERALL MASS EVAPOTATION =',1PE15.6,/,
0276      25X,*OVERALL MASS OUTFLOW =',1PE15.6,/,5X,*NET MASS ACCUMUL. =',
          3 1PE15.6,/,5X,*OUTPUT PUNCH =',L3,/)
          422 FORMAT(' ')
0277      423 FORMAT(' ',13,1X,8(1PE11,4))
0278      424 FORMAT(1H1,20X,*AVERAGE TRANSPORT PROPERTIES',/,
          1      3X,*J',9X,*PR',9X,*SC',8X,*VIS',8X,*DEN',9X,*CP',7X
          2      *DIFF',10X,*K',/,2X,81('---'))
0280      425 FORMAT(//,3X,*J',7X,*VV/D',8X,*TAU',9X,*CF',3X,*CF*RED/2',/,2X,
          2      46('---'))
          426 FORMAT(' ',13,1X,8(1PE11,4))
0281      600 FORMAT(10Z8)
0282      602 FORMAT(' ',15X,5X,9A4,/)
0283      END
0284

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MAIN0362

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*OPTIONS IN EFFECT* NOTERN,ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT* NAME = MAIN      . LINECNT =      56
*STATISTICS*      SOURCE STATEMENTS =      284,PROGRAM SIZE =      11276
*STATISTICS*      NO DIAGNOSTICS GENERATED

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0001      C---BLOCK DATA-----DATA0001
0002      BLOCK DATA              DATA0002
                                COMMON DATA0003
                                1/UVEL/RESORU,NSWPV,URFU,DXEPU(22),DXPNU(22),SEWU(22)
                                1/VVEL/RESORV,NSWPV,URFV,DYNPV(22),DYPSV(22),SNSV(22),YCV(22)
                                1/PCOR/RESORM,NSWPP,URFP,OU(20,20),DV(20,20),IPREF,JPREF
                                1/TEMP/RESORT,NSWPT,URFT
                                1/SPEC/RESORW,NSWPW,URFW
                                0003      COMMON/MOLEWT/MTA,MTB
                                1/FLUPR2/CONDA1,CONDA2,CONDA3,CONDB1,CONDB2,CONDB3,
                                2VISA1,VISA2,VISA3,VISB1,VISB2,VISB3,DIFFU,
                                3CPA1,CPA2,CPA3,CPB1,CPB2,CPB3
                                1/PROB1/RE,TIN,PEN,TWALL,WIN,FLOWIN,FLOWEV,
                                2      ISTEP,JSTEP,ISTP1,JSTP1,ISTM1,JSTM1
                                3      JNOZ,JNOZP1,JNOZM1
                                0004      COMMON
                                1/ALL/IT,JT,NT,NJ,NIMP,NJM1,GREAT
                                1/VAPP/ANTON1,ANTON2,ANTON3
                                1/PGM/PUNIN,PUNOUT,TIMAX,MAXIT,SORMAX,INDPRI
                                0005      COMMON/DIFFUS/DIFF1,DIFF2,DIFF3,DIFF4,DIFF5
                                0006      COMMON/SUCTON/USUC,SUCR
                                0007      COMMON/ANGLE/THETA
                                0008      COMMON/CROSS/PCNTCR,DELTIN,DELWIN
                                0009      LOGICAL PUNIN,PUNOUT
                                DATA0021
                                C---LENGTH      M      DATA0022
                                C---VELOCITY      M/S      DATA0023
                                C---TEMP      K      DATA0024
                                C---SPECIES      MASS FRACTION OF SPECIES A      DATA0025
                                C---PRESSURE      N/M**2      DATA0026
                                C---DENSITY      KG/M**3      DATA0027
                                C---DIFFUSITY      M**2/S      DATA0028
                                C---SPECI.HEAT      J/KG/K      DATA0029
                                C---THERM.CONDU      W/M/K      DATA0030
                                C---VISCOSITY      KG/M/S      DATA0031
                                C---MOLE.WT      G/MOLE      DATA0032
                                C---M      METER      DATA0033
                                C---KG      KILOGRAM      DATA0034
                                C---G      GRAM      DATA0035
                                C---S      SEC.      DATA0036
                                C---N      NEWTON      DATA0037
                                C---J      JOULE      DATA0038
                                C---W      WATT      DATA0039
                                C-----DATA0040
                                0010      PROGRAM- AND PRINTOUT-CONTROL DATA
                                0011      DATA ISTEP,GREAT/2,1,E30/
                                DATA NSWPV,NSWPP,NSWPT,NSWPW/3,3,5,3,3/
                                6      ,IT,JT,IPREF,JPREF/20,50,2,24/
                                3      ,URFU,URFV,URFP,URFT,URFW/0.2,0.25,0.5,0.9,0.9/
                                4      ,PUNIN,PUNOUT/,TRUE,/,TRUE,/
                                5      ,MAXIT,TIMAX,SORMAX,INDPRI/200,900,0.0,15,101/
                                C-----DATA0048
                                C-----PHYSICAL DATA      DATA0049
                                BINARY MIXTURE OF WATER VAP (A) AND AIR (B)      DATA0050
                                0012      DATA MTA,MTB/18.02,28.97/      DATA0051

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0013      1 ,ANTON1,ANTON2,ANTON3/23.365186,3919.884,-42.061625/
0014      DATA VISA1,VISA2,VISA3/0.99354E-6,0.27067E-7,0.88906E-11/
0015      DATA VISB1,VISB2,VISB3/0.25641E-8,0.60198E-7,-0.23723E-10/
0016      DATA CPA1,CPA2,CPA3/0.18241E4,0.21976,0.23487E-3/
0017      DATA CPB1,CPB2,CPB3/0.10256E4,-0.17317,0.36146E-3/
0018      DATA CONDA1,CONDA2,CONDA3/0.86868E-2,0.11318E-4,0.85793E-7/
0019      DATA CONDB1,CONDB2,CONDB3/0.38793E-3,0.95425E-4,-0.30699E-7/
0020      DATA DIFF1,DIFF2,DIFF3/0.2794E-4,-0.1306E-6,0.4016E-9/
          DATA DIFF4,DIFF5/-0.1057E-3,0.335E-6/
    
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DATA0052
DATA0053
DATA0054
DATA0055
DATA0056
DATA0057
DATA0058
DATA0059
DATA0060
DATA0061
DATA0062
DATA0063
DATA0064
    
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0021      C-----PROBLEM SPECIFICATION
0022      DATA PIN/101325.0/
0023      DATA SUCR/0.0/
0024      DATA THETA/0.00/
0025      DATA RE,TIN,TWALL,WIN/1000.,450.,350.,0.1/
0026      DATA PCNTCR,DELTIN,DELTIN/0.10,0.00,0.00/
          END
    
```

DATA0066

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*OPTIONS IN EFFECT* NOTERM,ID,EBCDIC,SOURCE,NOLIST,NOECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT* NAME = BLK DATA, LINECNT = 56
*STATISTICS* NO DIAGNOSTICS GENERATED
    
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0001 C--GRID-----GRID0001
0002 C--GRID DISTRIBUTION CALCULATIONS-----GRID0002
      SUBROUTINE GRID(JNDZ,JSTEP,RSMALL,HDRATO)
      COMMON
      1/AL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
      1/GEOM/X(22),Y(22),DXEP(22),DXPW(22),DYNP(22),
      2/DYP5(22),SNS(22),SEW(22),XU(22),YV(22)
      COMMON/ANGLE/THETA
      DIMENSION ODX(19)
      C-----GRID 20*50
      DATA CDX/1.,2.,4.,8.,16.,32.,64.,128.,64.,32.,16.,8.,4.,2.,1./
      2 0.5,0.5,0.25,0.18/
      NI=20
      NJ=20
      NIM1=NI-1
      JNDZ=14
      JSTEP=33
      RSMALL=1.6E-03
      HDRATO=3.
      DY=2.0E-03
      Y(1)=-0.5*DY
      DO 6 I=2,NJ
      6 Y(I)=Y(I-1)+DY
      SUM=0.0
      DJP=HDRATO*2.*RSMALL
      DO 30 I=1,NIM1
      30 SUM=SUM+DDX(I)
      SUM = SUM-0.5*(DDX(1)+DDX(NIM1))
      DX=0JP/SUM
      X(1)=-DDX(1)*DX*0.5
      DO 40 I=2,NI
      40 X(I)=X(I-1)+DX*DDX(I-1)
      RETURN
      END

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GRID0004  
GRID0005

GRID0011  
GRID0012

GRID0016

GRID0035  
GRID0036

GRID0038

GRID0040  
GRID0041  
GRID0042  
GRID0043  
GRID0044  
GRID0045

\*OPTIONS IN EFFECT\* NOTERN, ID, EBCDIC, SOURCE, NOLIST, NODECK, LOAD, NOMAP, NOTEST  
 \*OPTIONS IN EFFECT\* NAME = GRID LINECNT = 56  
 \*STATISTICS\* SOURCE STATEMENTS = 27, PROGRAM SIZE = 864  
 \*STATISTICS\* NO DIAGNOSTICS GENERATED

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C--INIT
C--GEOMETRICAL QUANTITIES CALCULATIONS -----
0001 SUBROUTINE INIT
C
CHAPTER 0 0 0 0 0 0 0 0 0 PRELIMINARIES 0 0 0 0 0 0 0
C
COMMON
1/LEVEL/RESORV,NSWPV,URFU,DXPWI(22),DXPWU(22),SEWU(22)
1/VVEL/RESORV,NSWPV,URFV,DYNPV(22),DYPSV(22),SNSV(22),YCV(22)
1/ALL/IT,JT,NI,NJ,NMI,NJMI,GREAT
1/PROB/RE,TIN,PIN,TWALL,WIN,FLOWIN,FLOWEV,
2 STEP,JSTEP,ISTPI,ISTPI,ISTMI,JSTMI
3 JNOZ,JNOZPI,JNOZMI
COMMON
1/GEOM/X(22),Y(22),DXEP(22),DXPW(22),DYNP(22),
2 DYP(22),SNS(22),SEW(22),XU(22),YV(22)
C
CHAPTER 1 1 1 1 1 CALCULATE GEOMETRICAL QUANTITIES 1 1 1 1 1
C
DXPW(1)=0.0
DXEP(NI)=0.0
DO 101 I=1,NMI
DXEP(I)=X(I+1)-X(I)
101 DXPW(I+1)=DXEP(I)
DYP(1)=0.0
DYNP(NJ)=0.0
DO 102 J=1,NJMI
DYNP(J)=Y(J+1)-Y(J)
102 DYPS(J+1)=DYNP(J)
SEW(1)=0.0
SEW(NI)=0.0
DO 103 I=2,NMI
SEW(I)=0.5*(DXEP(I)+DXPW(I))
SNS(1)=0.0
SNS(NJ)=0.0
DO 104 J=2,NJMI
SNS(J)=0.5*(DYNP(J)+DYPS(J))
XU(1)=0.0
DO 105 I=2,NMI
XU(I)=0.5*(X(I)+X(I-1))
DXPWU(1)=0.0
DXPWU(2)=0.0
DXEPU(1)=0.0
DXEPU(NI)=0.0
DO 106 I=2,NMI
DXEPU(I)=XU(I+1)-XU(I)
106 DXPWU(I+1)=DXEPU(I)
SEWU(1)=0.0
SEWU(2)=0.0
SEWU(NI)=0.0
DO 107 I=3,NMI
SEWU(I)=0.5*(DXEPU(I)+DXPWU(I))
YV(1)=0.0
DO 108 J=2,NJ

```

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INIT0001
INIT0002
INIT0003
INIT0004
INIT0005
INIT0006
INIT0007
INIT0010
INIT0011
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INIT0040
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INIT0049
INIT0050
INIT0051
INIT0052
INIT0053
INIT0054
INIT0055
INIT0057

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INIT

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23/48/10

PAGE 0002

```

0039      YV(J) = 0.5*(Y(J)+Y(J-1))
0040      YCV(J) = 0.5*(YV(J)+YV(J-1))
0041      DYPSV(1)=0.0
0042      DYPSV(2)=0.0
0043      DYNPV(NJ)=0.0
0044      DO 109 J=2,NJMI
0045      DYNPV(J)=YV(J+1)-YV(J)
0046      109 DYPSV(J+1)=DYNPV(J)
0047      SNSV(1)=0.0
0048      SNSV(2)=0.0
0049      SNSV(NJ)=0.0
0050      DO 110 J=3,NJMI
0051      110 SNSV(J)=0.5*(DYNPV(J)+DYPSV(J))
C
0052      RETURN
0053      END

```

```

INIT0061
INIT0062
INIT0063
INIT0064
INIT0065
INIT0066
INIT0067
INIT0068
INIT0069
INIT0070
INIT0071
INIT0072
INIT0073
INIT0074

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*OPTIONS IN EFFECT* NOTERM, ID, EBCDIC, SOURCE, NOLIST, NODECK, LOAD, NMAP, NOTEST
*OPTIONS IN EFFECT* NAME = INIT, LINECT = 56
*STATISTICS* SOURCE STATEMENTS = 53, PROGRAM SIZE = 1286
*STATISTICS* NO DIAGNOSTICS GENERATED

```

[illegible]

PROP0001  
-PROP0002  
PROP0003  
PROP0004  
PROP0005  
PROP0006  
PROP0007  
  
PROP0009  
  
PROP0011  
PROP0012  
PROP0013  
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PROP0015  
PROP0016  
  
PROP0018  
PROP0019  
PROP0020  
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PROP0023  
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PROP0026  
PROP0027  
PROP0028  
PROP0029  
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PROP0031  
PROP0032  
PROP0033  
  
PROP0034  
PROP0035  
PROP0036  
PROP0037  
PROP0038  
PROP0039  
PROP0040  
PROP0041  
PROP0042  
PROP0043  
PROP0044  
PROP0045  
PROP0046  
PROP0047  
PROP0048  
PROP0049

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PROPS

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14/58/32

PAGE 0002

```

0033      PHI8A=(1.+1./CONST)*(1.+1./CONST)*A/(4.*C)
0039      COND(I,J)=CONDA/(1.+PHI8A*YB/YA)+CONDB/(1.+PHI8A*YA/YB)
0041      CPP(I,J)=W(I,J)*CPA+(1.-W(I,J))*CPB
0042      100 CONTINUE
0043      VIS(I,NJ)=VIS(I,NJM1)
0044      DEN(I,NJ)=DEN(I,NJM1)
0045      DIFF(I,NJ)=DIFF(I,NJM1)
0046      CPP(I,NJ)=CPP(I,NJM1)
0047      COND(I,NJ)=COND(I,NJM1)
0048      110 CONTINUE
0049      RETURN
0049      END

```

```

PROP0050
PROP0051
PROP0052
PROP0053
PROP0059
PROP0060
PROP0061
PROP0062
PROP0063
PROP0064
PROP0065
PROP0066

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*OPTIONS IN EFFECT* NOTERM,ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT* NAME = PROPS , LINECNT = 56
*STATISTICS* SOURCE STATEMENTS = 49, PROGRAM SIZE = 1902
*STATISTICS* NO DIAGNOSTICS GENERATED

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FORTRAN IV G1 RELEASE 2.0

MAIN

DATE = 79037

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PAGE 0001

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0001 C-----CALCU-----CALCU001
C-----SOLUTION OF U EQUATION-----CALCU002
SUBROUTINE CALCU-----CALCU003
C-----CHAPTER 0 0 0 0 0 0 0 0 0 PRELIMINARIES 0 0 0 0 0 0 0 0 0 0-----CALCU005
C-----CALCU006
0002 COMMON NITER
0003 COMMON SPLIT(200)
0004 COMMON CNIV(200)
0005 COMMON HEDU(9), HEDV(9), HEDP(9), HEDT(9), HEDW(9), HEDVIS(9),
0006 HEDDEN(9), HEDCP(9), HEDDIF(9), HEDCON(9)
COMMON
1/VEL/RESORU, NSWPU, URFU, DXPU(22), DXPW(22), SEWU(22)
1/PCOR/RESORM, NSWPP, URFP, DU(20,20), DV(20,20), IPREF, JPREF
1/ALL/IT, JT, NI, NJ, NIM1, NJM1, GREAT
1/VAR/U(20,20), V(20,20), P(20,20), PP(20,20), T(20,20), W(20,20)
COMMON
1/GEOM/X(22), Y(22), DXEP(22), DXPW(22), DYNP(22),
2 DYP(22), SNS(22), SEW(22), XU(22), YV(22)
1/FLUPR/RHOA, RHOB, RATID, CPA, CPB, CPDEL,
2 DEN(20,20), VIS(20,20), CPP(20,20), DIFF(20,20), COND(20,20)
COMMON
1/PROB/RE, TIN, PIN, TWALL, WIN, FLOWIN, FLOWEV,
2 ISTEP, JSTEP, ISTEP1, JSTEP1, ISTM1, JSTM1
3 JNOZ, JNOZP1, JNOZM1
1/COEF/AP(20,20), AN(20,20), AS(20,20), AE(20,20), AW(20,20),
2 SU(20,20), SP(20,20)
GRAVITY=9.807
C-----CHAPTER 1 1 1 1 1 1 ASSEMBLY OF COEFFICIENTS 1 1 1 1 1 1 1 1 1 1-----CALCU022
C-----CALCU023
C-----CALCU024
C-----CALCU025
C-----CALCU026
0010 DO 101 J=2, NJM1
0011 DELUE = 0.0
0012 AREAWE = SNS(J)*0.01
0013 DO 100 I=3, NIM1
0014 DENIJ=DEN(I,J)
C-----COMPUTE AREAS AND VOLUME
0015 AREAN = DXPW(I)*0.01
0016 AREAS = DXPW(I)*0.01
0017 VOL = AREAWE*DXPW(I)
C-----CALCULATE CONVECTION COEFFICIENTS
0018 GN=0.5*(DEN(I,J+1)+DENIJ)*V(I,J+1)
0019 GNW=0.5*(DEN(I-1,J)+DEN(I-1,J+1))*V(I-1,J+1)
0020 GS=0.5*(DEN(I,J-1)+DENIJ)*V(I,J)
0021 GSW=0.5*(DEN(I-1,J)+DEN(I-1,J-1))*V(I-1,J)
0022 GE=0.5*(DEN(I+1,J)+DENIJ)*U(I+1,J)
0023 GP=0.5*(DENIJ+DEN(I-1,J))*U(I,J)
0024 GW=0.5*(DEN(I-1,J)+DEN(I-2,J))*U(I-1,J)
0025 CN=0.5*(GN+GNW)*AREAN
0026 CS=0.5*(GS+GSW)*AREAS
0027 CE=0.5*(GE+GP)*AREAWE
0028 CW=0.5*(GP+GW)*AREAW
C-----CALCULATE DIFFUSION COEFFICIENTS
0029 VISN=0.25*(VIS(I,J)+VIS(I,J+1)+VIS(I-1,J)+VIS(I-1,J+1))

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0030      VISS=0.25*(VIS(I,J)+VIS(I,J-1)+VIS(I-1,J)+VIS(I-1,J-1))
0031      DN=VISN*AREAN/DYNP(J)
0032      DS=VISS*AREAS/DYPS(J)
0033      DE=VIS(I,J)*AREAW/DXEPW(I)
0034      DW=VIS(I-1,J)*AREAW/DXPW(I)
C----- CALCULATE COEFFICIENTS OF SOURCE TERMS
0035      SMP=CN-CS+CE-CW
0036      CP=AMAX1(0.0,SMP)
0037      CPQ=CP
C----- ASSEMBLE MAIN COEFFICIENTS
0038      AN(I,J)=AMAX1((ABS(0.5*CN),DN)-0.5*CN
0039      AS(I,J)=AMAX1((ABS(0.5*CS),DS)+0.5*CS
0040      AE(I,J)=AMAX1((ABS(0.5*CE),DE)-0.5*CE
0041      AW(I,J)=AMAX1((ABS(0.5*CW),DW)+0.5*CW
0042      DU(I,J)=AREAEW
0043      SU(I,J)=CPQ*U(I,J)+DU(I,J)*(P(I-1,J)-P(I,J))
0044      SP(I,J)=-CP
0045      DRDXE = 0.5*((DEN(I+1,J)+DEN(I,J))-(DEN(I,J)+DEN(I-1,J)))/SEW(I)
0046      DROYE = 0.5*((DEN(I,J+1)+DEN(I,J))-(DEN(I,J)+DEN(I,J-1)))/SNS(J)
0047      DELUE = DELUE
0048      DELUE=0.5*(U(I+1,J)+U(I-1,J))*DRDXE+0.5*(V(I,J+1)+V(I,J))*DROYE
0049      DELUE = DELUE*VIS(I,J)/DEN(I,J)
0050      ALPHA=0.3333*(DELUE-DELUE)*0.01
0051      SU(I,J) = SU(I,J)+ALPHA*SNS(J)
0052      SU(I,J)=SU(I,J)+SNS(J)*DXPW(I)*GRAVITY*0.5
C-----
0053      100 CONTINUE
0054      101 CONTINUE
C
C CHAPTER 2 2 2 2 2 2 2 2 PROBLEM MODIFICATIONS 2 2 2 2 2 2
C
0055      CALL MODU
C
C CHAPTER 3 FINAL COEFF. ASSEMBLY AND RESIDUAL SOURCE CALCULATION 3 3
C
0056      RESORU=0.0
0057      DO 300 I=3,NIM1
0058      DO 301 J=2,NJM1
0059      AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)
0060      DU(I,J)=DU(I,J)/AP(I,J)
0061      RESOR=AN(I,J)+U(I,J+1)+AS(I,J)+U(I,J-1)+AE(I,J)+U(I+1,J)
C-----
0062      +AW(I,J)+U(I-1,J)-AP(I,J)+U(I,J)+SU(I,J)
0063      VOL = SNS(J)*DXPW(I)*0.01
0064      SORVOL=GREAT*VOL
0065      IF (-SP(I,J).GT.0.5*SORVOL) RESOR=RESOR/SORVOL
C----- UNDER-RELAXATION
0066      AP(I,J)=AP(I,J)/URFU
0067      SU(I,J)=SU(I,J)+(1.-URFU)*AP(I,J)+U(I,J)
0068      DU(I,J)=DU(I,J)*URFU
0069      301 CONTINUE
0070      300 CONTINUE
C
C CHAPTER 4 4 4 4 4 4 4 4 SOLUTION OF DIFFERENCE EQUATION 4 4 4 4 4 4

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CALCU049  
 CALCU050  
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 CALCUI103  
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 CALCUI107  
 CALCUI108  
 CALCUI109  
 CALCUI110  
 CALCUI111  
 CALCUI112

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CALCU

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0071  
0072  
0073C 400 CALL LOSILV(3,2,NI,NJ,IT,JT,U,NSWPU,0.)  
RETURN  
END

CALCUL13

CALCUL15  
CALCUL16

30

\*OPTIONS IN EFFECT\* NOTERM, ID, EBCDIC, SOURCE, NDLIST, NODECK, LOAD, NOMAP, NOTEST  
\*OPTIONS IN EFFECT\* NAME = CALCU, LINECNT = 56  
\*STATISTICS\* SOURCE STATEMENTS = 73, PROGRAM SIZE = 2570  
\*STATISTICS\* NO DIAGNOSTICS GENERATED

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MAIN

DATE = 79037

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PAGE 0001

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0001      C---CALCV-----
0001      C---SOLUTION OF V EQUATION-----
0001      SUBROUTINE CALCV
0001      C
0001      CHAPTER 0 0 0 0 0 0 0 0 0 PRELIMINARIES 0 0 0 0 0 0 0 0
0001      C
0002      COMMON NITER
0003      COMMON
0003      1/VVEL/RESORV,NSWPV,URFV,DYNPV(22),DYPSV(22),SNSV(22),YCV(22)
0003      1/PCOR/RESORM,NSWPP,URFP,DU(20,20),DV(20,20),IPREF,JPREF
0003      1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
0003      1/GEOM/X(22),Y(22),DXEP(22),DXPW(22),DYNP(22),
0003      2    DYPS(22),SNS(22),SEW(22),XU(22),YV(22)
0004      COMMON
0004      1/VAR/U(20,20),V(20,20),P(20,20),PP(20,20),T(20,20),W(20,20)
0004      1/FLUPR/RHOA,RHOB,RATIO,CPA,CPB,CPDEL,
0004      2    DEN(20,20),VIS(20,20),CPP(20,20),DIFF(20,20),COND(20,20)
0004      1/PROB/RE,TIN,PIN,TWALL,WIN,FLOWIN,FLOWEV,
0004      2    ISTEP,JSTEP,ISTP1,JSTP1,ISTM1,JSTM1
0004      3    JNOZ,JNOZP1,JNOZM1
0005      COMMON
0005      1/COEF/AP(20,20),AN(20,20),AS(20,20),AE(20,20),AW(20,20),
0005      2    SU(20,20),SP(20,20)
0005      C
0005      CHAPTER 1 1 1 1 1 1 ASSEMBLY OF COEFFICIENTS 1 1 1 1 1 1
0005      C
0006      DO 100 I=2,NIM1
0007      DELUN = 0.0
0008      DO 101 J = 3,NJM1
0009      AREAEW = DYPS(J)*0.01
0010      DENIJ=DEN(I,J)
0011      C----- COMPUTE AREAS AND VOLUME
0011      AREAN = SEW(I)*0.01
0012      AREAS = SEW(I)*0.01
0013      VOL = AREAEW*SEW(I)
0014      C----- CALCULATE CONVECTION COEFFICIENTS
0014      GN=0.5*(DEN(I,J+1)+DENIJ)*V(I,J+1)
0015      GP=0.5*(DENIJ+DEN(I,J-1))*V(I,J)
0016      GS=0.5*(DEN(I,J-1)+DEN(I,J-2))*V(I,J-1)
0017      GE=0.5*(DEN(I+1,J)+DENIJ)*U(I+1,J)
0018      GSE=0.5*(DEN(I,J-1)+DEN(I+1,J-1))*U(I+1,J-1)
0019      GW=0.5*(DENIJ+DEN(I-1,J))*U(I,J)
0020      GSW=0.5*(DEN(I,J-1)+DEN(I-1,J-1))*U(I,J-1)
0021      CN=0.5*(GN+GP)*AREAN
0022      CS=0.5*(GP+GS)*AREAS
0023      CE=0.5*(GE+GSE)*AREAEW
0024      CW=0.5*(GW+GSW)*AREAEW
0025      C----- CALCULATE DIFFUSION COEFFICIENTS
0025      VISE=0.25*(VIS(I,J)+VIS(I+1,J)+VIS(I,J-1)+VIS(I+1,J-1))
0026      VISW=0.25*(VIS(I,J)+VIS(I-1,J)+VIS(I,J-1)+VIS(I-1,J-1))
0027      DN=VIS(I,J)*AREAN/DYNPV(J)
0028      DS=VIS(I,J-1)*AREAS/DYPSV(J)
0029      DE=VISE*AREAEW/DXEP(I)
0030      DW=VISW*AREAEW/DXPW(I)

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CALCV001
CALCV002
CALCV003
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CALCV005
CALCV006
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CALCV116

McGill University Computing Centre

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CALCV

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\*STATISTICS\* NO DIAGNOSTICS GENERATED

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MAIN

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PAGE 0001

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C---CALCP-----
C---SOLUTION OF P EQUATION-----
SUBROUTINE CALCP
C
CHAPTER 0 0 0 0 0 0 0 0 0 PRELIMINARIES 0 0 0 0 0 0 0 0
C
COMMON NITER
COMMON
1/PCOR/RESQRM,NSWPP,URFP,DU(20,20),DV(20,20),IPREF,JPREF
1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
1/VAR/U(20,20),V(20,20),P(20,20),PP(20,20),T(20,20),W(20,20)
1/GEON/X(22),Y(22),DXEP(22),DXPW(22),DYNP(22),
2  DYP(22),SNS(22),SEW(22),XU(22),YV(22)
COMMON
1/FLUPR/RHOA,RHOB,RATIO,CPA,CPB,CPDEL,
2  DEN(20,20),VIS(20,20),CPP(20,20),DIFF(20,20),COND(20,20)
1/COEF/AP(20,20),AN(20,20),AS(20,20),AE(20,20),AW(20,20),
2  SU(20,20),SP(20,20)
C
CHAPTER 1 1 1 1 1 1 ASSEMBLY OF COEFFICIENTS 1 1 1 1 1 1
C
RESORM=0.
DO 101 J=2,NJM1
AREAEM = SNS(J)+0.01
DO 100 I=2,NIM1
DENIJ=DEN(I,J)
PP(I,J)=0.0
C-----COMPUTE AREAS AND VOLUME
AREAN = SEW(I)*0.01
AREAS = SEW(I)*0.01
VOL = AREAEM*SEW(I)
C-----CALCULATE COEFFICIENTS
DENN=0.5*(DENIJ+DEN(I,J+1))
DENS=0.5*(DENIJ+DEN(I,J-1))
DENE=0.5*(DENIJ+DEN(I+1,J))
DENW=0.5*(DENIJ+DEN(I-1,J))
AN(I,J)=DENN*AREAN*DV(I,J+1)
AS(I,J)=DENS*AREAS*DV(I,J)
AE(I,J)=DENE*AREAEM*DU(I+1,J)
AW(I,J)=DENW*AREAEM*DU(I,J)
C-----CALCULATE SOURCE TERMS
CN=DENN*V(I,J+1)*AREAN
CS=DENS*V(I,J)*AREAS
CE=DENE*U(I+1,J)*AREAEM
CW=DENW*U(I,J)*AREAEM
SMP=CN-CS+CE-CW
SP(I,J)=0.0
SU(I,J)=-SMP
C-----COMPUTE SUM OF ABSOLUTE MASS SOURCES
RESORM=RESORM+ABS(SMP)
100 CONTINUE
101 CONTINUE
C
CHAPTER 2 2 2 2 2 2 2 2 2 PROBLEM MODIFICATIONS 2 2 2 2 2 2 2 2

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CALCP001
CALCP002
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C      CALL MOOP
C      CHAPTER 3 3 3 3 3 FINAL COEFFICIENT ASSEMBLY 3 3 3 3 3 3
C      DO 300 I=2,NIM1
0032      DO 301 J=2,NJM1
0033      301 AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)
0034      300 CONTINUE
C      CHAPTER 4 4 4 4 4 SOLUTION OF DIFFERENCE EQUATIONS 4 4 4 4 4
C      400 CALL LOSILV(2,2,NI,NJ,IT,JT,PP,NSWPP,0.)
0036      C      CHAPTER 5 5 5 5 CORRECT VELOCITIES AND PRESSURE 5 5 5 5 5 5
C      C-----VELOCITIES
C      C-----PRESSURES (WITH PROVISION FOR UNDER-RELAXATION)
0037      PPREF=PP(IPREF,JPREF)
0038      DO 500 I=2,NIM1
0039      DO 501 J=2,NJM1
0040      IF(I.NE.2) U(I,J)=U(I,J)+DU(I,J)*(PP(I-1,J)-PP(I,J))
0041      V(I,J)=V(I,J)+DV(I,J)*(PP(I,J-1)-PP(I,J))
0042      P(I,J)=P(I,J)+URFP*(PP(I,J)-PPREF)
0043      501 CONTINUE
0044      500 CONTINUE
0045      RETURN
0046      END

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CALCP054
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CALCP079
CALCP080
CALCP081

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*OPTIONS IN EFFECT* NOTERM,ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT* NAME = CALCP , LINECNT = 56
*STATISTICS* SOURCE STATEMENTS = 46, PROGRAM SIZE = 1572
*STATISTICS* NO DIAGNOSTICS GENERATED

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0001 C--CALCT-----CALCT001
C--SOLUTION OF TEMPERATURE EQUATION-----CALCT002
SUBROUTINE CALCTCALCT003
CCHAPTER 0 0 0 0 0 0 0 PRELIMINARIES 0 0 0 0 0 0 0CALCT004
CCALCT005
COMMON NITERCALCT006
COMMONCALCT007
1/TEMP/RESORT,NSWPT,URFTCALCT008
1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREATCALCT009
1/VAR/UC(20,20),V(20,20),P(20,20),PP(20,20),T(20,20),W(20,20)
1/GEOM/X(22),Y(22),DXEP(22),DXPW(22),DYNP(22),
2 DYP(22),SNS(22),SEW(22),XU(22),YV(22)
0004 COMMONCALCT013
1/FLUPR/RHOA,RHOB,RATIO,CPA,CPB,CPDEL,CALCT014
2 DEN(20,20),VIS(20,20),CPP(20,20),DIFF(20,20),COND(20,20)
1/PROB1/RE,TIN,PIN,TWALL,WIN,FLOWIN,FLOWEV,CALCT016
2 ISTEP,JSTEP,ISTP1,JSTP1,ISTM1,JSTM1,CALCT017
3 JNOZ,JNOZP1,JNOZM1
0005 COMMONCALCT018
1/COEF/AP(20,20),AN(20,20),AS(20,20),AE(20,20),AW(20,20),
2 SU(20,20),SP(20,20)
0006 COMMON/HEAT/CPDEL(20,20)
CHAPTER 1 1 1 1 1 1 ASSEMBLY OF COEFFICIENTS 1 1 1 1 1 1
C
DO=101 J=2,NJM1
0008 AREAEW = SNS(J)*0.01
0009 DO 100 I=2,NIM1
0010 DENIJ=DEN(I,J)
C-----COMPUTE AREAS VOLUME
0011 AREAN = SEW(I)*0.01
0012 AREAS = SEW(I)*0.01
0013 VOL=AREAEW*SEW(I)
0014 CPDEL=CPPDEL(I,J)
C-----CALCULATE CONVECTION COEFFICIENTS
0015 GN=0.5*(DENIJ+DEN(I,J+1))*V(I,J+1)
0016 GS=0.5*(DENIJ+DEN(I,J-1))*V(I,J)
0017 GE=0.5*(DENIJ+DEN(I+1,J))*U(I+1,J)
0018 GW=0.5*(DENIJ+DEN(I-1,J))*U(I,J)
0019 CN=GN*AREAN
0020 CS=GS*AREAS
0021 CE=GE*AREAEW
0022 CW=GW*AREAEW
C-----CALCULATE DIFFUSION COEFFICIENTS
0023 GAM=COND(I,J)/CPP(I,J)
0024 GAMN=0.5*(GAM+COND(I,J+1)/CPP(I,J+1))
0025 GAMS=0.5*(GAM+COND(I,J-1)/CPP(I,J-1))
0026 GAME=0.5*(GAM+COND(I+1,J)/CPP(I+1,J))
0027 GAMW=0.5*(GAM+COND(I-1,J)/CPP(I-1,J))
0028 DN=GAMN*AREAN/DYNP(J)
0029 DS=GAMS*AREAS/DYP(J)
0030 DE=GAME*AREAEW/DXEP(I)
0031 DW=GAMW*AREAEW/DXPW(I)
CALCT022
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0032 C-----SOURCE TERMS
0033 SMP=CN-CS+CE-CW
0034 CP=AMAX1(0.0,SMP)
      CPO=CP
0035 C-----ASSEMBLE MAIN COEFFICIENTS
0036 AN(I,J)=AMAX1(ABS(0.5*CN),DN)-0.5*CN
0037 AS(I,J)=AMAX1(ABS(0.5*CS),DS)+0.5*CS
0038 AE(I,J)=AMAX1(ABS(0.5*CE),DE)-0.5*CE
0039 AW(I,J)=AMAX1(ABS(0.5*CW),DW)+0.5*CW
0040 SU(I,J)=CPO*T(I,J)
0041 DELZ=DXEP(I)
0042 IF(I.EQ.NIMI) DELZ=DELZ*0.5
0043 DTZ1=(T(I+1,J)-T(I,J))/DELZ
0044 DTZ2=(T(I,J)-T(I,J-1))/DXPW(I)
0045 DTR1=(T(I,J+1)-T(I,J))/DYNP(J)
0046 DTR2=(T(I,J)-T(I,J-1))/DYPS(J)
0047 DWZ1=(W(I+1,J)-W(I,J))/DELZ
0048 DWZ2=(W(I,J)-W(I,J-1))/DXPW(I)
0049 DWR1=(W(I,J+1)-W(I,J))/DYNP(J)
0050 DWR2=(W(I,J)-W(I,J-1))/DYPS(J)
0051 DTZ=0.5*(DTZ1+DTZ2)
0052 DTR=0.5*(DTR1+DTR2)
0053 DWZ=0.5*(DWZ1+DWZ2)
0054 DWR=0.5*(DWR1+DWR2)
0055 WTERM=(DWZ*DTZ+DWR*DTR)*VOL*DENIJ*DIFF(I,J)*CPDEL/CPPI(I,J)
0056 DCPZ1=(CPP(I,J)-CPP(I+1,J))/(CPP(I+1,J)*CPP(I,J)*DELZ)
0057 DCPZ2=(CPP(I-1,J)-CPP(I,J))/(CPP(I,J)*CPP(I-1,J)*DXPW(I))
0058 DCPR1=(CPP(I,J)-CPP(I,J+1))/(CPP(I,J)*CPP(I,J+1)*DYNP(J))
0059 DCPR2=(CPP(I,J-1)-CPP(I,J))/(CPP(I,J)*CPP(I,J-1)*DYPS(J))
0060 DCPZ=0.5*(DCPZ1+DCPZ2)
0061 DCPR=0.5*(DCPR1+DCPR2)
0062 CPTERM=(DCPR*DTR+DCPZ*DTZ)*VOL*COND(I,J)
0063 SU(I,J)=SU(I,J)+WTERM-CPTERM
0064 SP(I,J)=-CP
0065 100 CONTINUE
      101 CONTINUE
0066 C
      CHAPTER 2 2 2 2 2 2 PROBLEM MODIFICATIONS 2 2 2 2 2 2
      C
      CALL MODT
0067 CHAPTER 3 FINAL COEFFICIENT ASSEMBLY AND RESIDUAL SOURCE CALCULATION 3
0068 C
      RESORT=0.0
      DO 300 I=2,NIMI
0069 DO 301 J=2,NJMI
0070 AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)
0071 RESOR=AN(I,J)*T(I,J+1)+AS(I,J)*T(I,J-1)+AE(I,J)*T(I+1,J)
      +AW(I,J)*T(I-1,J)-AP(I,J)*T(I,J)+SU(I,J)
0072 VOL = SNS(J)*SEW(I)*0.01
0073 SORVOL=GREAT*VOL
0074 IF(-SP(I,J).GT.0.5*SORVOL) RESOR=RESOR/SORVOL
0075 RESORT=RESORT+ABS(RESOR)
      C-----UNDER-RELAXATION
0076 AP(I,J)=AP(I,J)/URFT

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CALCT053
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CALCT106

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0077  
0078  
0079

SU(I,J)=SU(I,J)+(1.0-URFT)\*AP(I,J)\*T(I,J)  
301 CONTINUE  
300 CONTINUE

CALCT107  
CALCT108  
CALCT109  
CALCT110  
CALCT111  
CALCT112

0080  
0081  
0082

C CHAPTER 4 4 4 4 4 SOLUTION OF DIFFERENCE EQUATIONS 4 4 4 4 4  
C 400 CALL LOSILV(2,2,NI,NJ,IT,JT,T,NSWPT,0.)  
RETURN  
END

CALCT114  
CALCT115

\*OPTIONS IN EFFECT\* NOTERM, ID, EBCDIC, SOURCE, NDLIST, NODECK, LOAD, NOMAP, NOTEST  
\*OPTIONS IN EFFECT\* NAME = CALCT, LINECNT = 56  
\*STATISTICS\* SOURCE STATEMENTS = 82, PROGRAM SIZE = 2818  
\*STATISTICS\* NO DIAGNOSTICS GENERATED

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0001 C--CALCW-----CALCW001
C--SOLUTION OF SPECIES EQUATION-----CALCW002
SUBROUTINE CALCW-----CALCW003
C-----CALCW004
C CHAPTER 0 0 0 0 0 0 0 0 PRELIMINARIES 0 0 0 0 0 0 0 0 CALCW005
C-----CALCW006
COMMON NITER
COMMON
1/SPEC/RESORW,NSWPW,URFW CALCW007
1/ALL/IT,JT,NJ,NIM1,NJM1,GREAT CALCW008
1/VAR/U(20,20),V(20,20),P(20,20),PP(20,20),T(20,20),W(20,20) CALCW009
1/GEOM/X(22),Y(22),DXEP(22),DXPW(22),DYNP(22),
2 DYP(22),SNS(22),SEW(22),XU(22),YV(22)
COMMON
1/FLUPR/RHOA,RHOB,RATIO,CPA,CPB,CPDEL, CALCW013
2 DEN(20,20),VIS(20,20),CPP(20,20),DIFF(20,20),COND(20,20) CALCW014
1/PROB/RE,TIN,PIN,TWALL,WIN,FLOWIN,FLOWEV, CALCW015
2 ISTEP,JSTEP,ISTPI,JSTPI,ISTMI,JSTMI CALCW017
3 JNOZ,JNOZPI,JNOZMI
COMMON
1/COEF/AP(20,20),AN(20,20),AS(20,20),AE(20,20),AW(20,20), CALCW018
2 SU(20,20),SP(20,20)
C-----CALCW021
C CHAPTER 1 1 1 1 1 1 ASSEMBLY OF COEFFICIENTS 1 1 1 1 1 1 CALCW022
C-----CALCW023
DO 101 J=2,NJM1
AREAEW = SNS(J)*0.01
DO 100 I=2,NIM1
DENIJ=DEN(I,J)
C-----COMPUTE AREAS VOLUME
AREAAN = SEW(I)*0.01
AREAS = SEW(I)*0.01
VOL=AREAEW*SEW(I)
C-----CALCULATE CONVECTION COEFFICIENTS
GN=0.5*(DENIJ+DEN(I,J+1))*V(I,J+1)
GS=0.5*(DENIJ+DEN(I,J-1))*V(I,J)
GE=0.5*(DENIJ+DEN(I+1,J))*U(I+1,J)
GW=0.5*(DENIJ+DEN(I-1,J))*U(I,J)
CN=GN*AREAN
CS=GS*AREAS
CE=GE*AREAEW
CW=GW*AREAEW
C-----CALCULATE DIFFUSION COEFFICIENTS
GAM=DIFF(I,J)*DENIJ
GAMN=0.5*(GAM+DIFF(I,J+1)*DEN(I,J+1))
GAMS=0.5*(GAM+DIFF(I,J-1)*DEN(I,J-1))
GAME=0.5*(GAM+DIFF(I+1,J)*DEN(I+1,J))
GAMW=0.5*(GAM+DIFF(I-1,J)*DEN(I-1,J))
DN=GAMN*AREAN/DYNP(J)
DS=GAMS*AREAS/DYP(22)
DE=GAME*AREAEW/DXEP(I)
DW=GAMW*AREAEW/DXPW(I)
C-----SOURCE TERMS
SMP=CN-CS+CE-CW

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CALCW001
CALCW002
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0031      CP=AMAX1(0.0,SMP)
0032      CPO=CP
C-----ASSEMBLE MAIN COEFFICIENTS
0033      AN(I,J)=AMAX1(ABS(0.5*CN),DN)-0.5*CN
0034      AS(I,J)=AMAX1(ABS(0.5*CS),DS)+0.5*CS
0035      AE(I,J)=AMAX1(ABS(0.5*CE),DE)-0.5*CE
0036      AW(I,J)=AMAX1(ABS(0.5*CW),DW)+0.5*CW
0037      SU(I,J)=CPO*W(I,J)
0038      SP(I,J)=-CP
0039      100 CONTINUE
0040      101 CONTINUE
C
C CHAPTER 2 2 2 2 2 2 PROBLEM MODIFICATIONS 2 2 2 2 2 2
C
0041      CALL MODW
C CHAPTER 3 FINAL COEFFICIENT ASSEMBLY AND RESIDUAL SOURCE CALCULATION 3
C
0042      RESORW=0.0
0043      DO 300 I=2,NIM1
0044      DO 301 J=2,NJM1
0045      AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)
0046      RESOR=AN(I,J)*W(I,J+1)+AS(I,J)*W(I,J-1)+AE(I,J)*W(I+1,J)
        +AW(I,J)*W(I-1,J)-AP(I,J)*W(I,J)+SU(I,J)
0047      VOL = SEW(I)*SNS(J)*0.01
0048      SORVOL=GREAT*VOL
0049      IF(-SP(I,J).GT.0.5*SORVOL) RESOR=RESOR/SORVOL
0050      RESORW=RESORW+ABS(RESOR)
C-----UNDER-RELAXATION
0051      AP(I,J)=AP(I,J)/URFW
0052      SU(I,J)=SU(I,J)+((1.0-URFW)*AP(I,J)*W(I,J)
0053      301 CONTINUE
0054      300 CONTINUE
C
0055      400 CALL LOSILV(2.2,NI,NJ,IT,JT,W,NSWPW,0.)
C CHAPTER 4 4 4 4 4 SOLUTION OF DIFFERENCE EQUATIONS 4 4 4 4 4
C
0056      RETURN
0057      END

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CALCW054  
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\*OPTIONS IN EFFECT\* NOTERM,ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST  
 \*OPTIONS IN EFFECT\* NAME = CALCW , LINECNT = 56  
 \*STATISTICS\* SOURCE STATEMENTS = 57, PROGRAM SIZE = 1952  
 \*STATISTICS\* NO DIAGNOSTICS GENERATED

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0001 C PROMOD-----MOD00001
C--ITERATIVE BOUNDARY CONDITIONS CALCULATIONS-----MOD00002
SUBROUTINE PROMODMOD00003
CMOD00004
CHAPTER 0 0 0 0 0 0 0 0 PRELIMINARIES 0 0 0 0 0 0 0 0MOD00005
C
0002 COMMON NITER
0003 COMMON SPLIT(200)
0004 COMMON CNIV(200)
0005 COMMON
1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREATMOD00007
1/VAR/U(20,20),V(20,20),P(20,20),PP(20,20),T(20,20),W(20,20)MOD00008
1/GEOM/X(22),Y(22),DXEP(22),DXPW(22),DYNP(22),
2 DYP(22),SNS(22),SEW(22),XU(22),YV(22)
1/PGM/PUNIN,PUNOUT,TIMAX,MAXIT,SORMAX,INDPRI
0006 COMMONMOD00012
1/FLUPR/RHOA,RHOB,RATIO,CPA,CPB,CPDEL,MOD00013
2 DEN(20,20),VIS(20,20),CPP(20,20),DIFF(20,20),COND(20,20)MOD00014
1/PROB1/RE,TIN,PIN,TWALL,WIN,FLOWIN,FLOWEV,MOD00016
2 ISTEP,JSTEP,ISTPI,JSTPI,ISTM1,JSTM1MOD00017
3 JNDZ,JNDZP1,JNDZM1
1/COEF/AP(20,20),AN(20,20),AS(20,20),AE(20,20),AW(20,20),
2 SU(20,20),SP(20,20)
0007 COMMONMOD00020
1/VVEL/RESORV,NSWPV,URFV,DYNPV(22),DYPV(22),SNSV(22),YCV(22)
0008 COMMON /SUCTION/USUC,SUCRMOD00022
0009 COMMON/ANGLE/THETA
C
CHAPTER 2 2 2 2 2 2 2 2 U MOMENTUM 2 2 2 2 2 2 2 2MOD00023
0010 ENTRY MODUMOD00024
C-----CONFINEMENT WALLMOD00025
C-----IMPINGEMENT WALLMOD00026
FLOWEV=0.0MOD00027
0011 DO 210 J=2,NJM1MOD00028
0012 U(NI,J)=-DIFF(NI,J)*(W(NI,J)-W(NIM1,J))/(XU(NI)-X(NIM1))MOD00029
0013 2 / (1.-W(NI,J))MOD00030
U(NI,J)=U(NI,J)+USUCMOD00031
0014 FLOWEV=FLOWEV-DEN(NI,J)*SNS(J)*0.01*U(NI,J)MOD00032
0015 210 CONTINUEMOD00034
0016 U(NI,1)=0.0
0017 C-----OUTFLOW REGIONMOD00037
DO 212 I=2,NI
0018 212 U(I,NJ)=U(I,NJM1)MOD00038
0019 RETURNMOD00041
0020 C-----IMPINGEMENT WALLMOD00042
CHAPTER 3 3 3 3 3 3 3 3 V MOMENTUM 3 3 3 3 3 3 3 3MOD00043
C
0021 ENTRY MODVMOD00044
C-----CONFINEMENT WALLMOD00045
DO 305 J=2,NJMOD00046
0022 AW(2,J)=0.0MOD00047
0023 SP(2,J)=SP(2,J)-0.5*(VIS(2,J)+VIS(2,J-1))/X(2)*DYP(J)*0.01
0024 305 CONTINUE
0025 C-----IMPINGEMENT WALLMOD000053

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0026      XPP=XU(NI)-X(NIMI)
0027      DO 311 J=2,NJ
0028      AE(NIMI,J)=0.0
0029      SP(NIMI,J)=SP(NIMI,J)-0.5*(VIS(NIMI,J)+VIS(NIMI,J-1))/XPP
      I*DYPS(J)*0.01
0030      311 CONTINUE
      C-----OUTFLOW REGION
0031      ARDENT=0.0
0032      FLOWL = 0.0
0033      FLOWR = 0.0
0034      DO 312 I=2,NIMI
0035      V(I,NJ)=V(I,NJM1)
0036      ARDENR = 0.5*(DEN(I,NJ)+DEN(I,NJM1))*SEW(I)*0.01
0037      ARDENT=ARDENT+ARDENR
0038      FLOWR = FLOWR+ARDENR*V(I,NJ)
0039      312 CONTINUE
0040      FLOW=FLOWR
0041      SPLIT(NITER)=1.234
0042      VINCL = (FLOWIN+FLOWEV-FLOW)/ARDENT
0043      CNIV(NITER)=VINCL
0044      DO 313 I=2,NIMI
0045      V(I,NJ)=V(I,NJ)+VINCL
0046      313 CONTINUE
0047      RETURN
      C
      C CHAPTER 5 5 5 5 5 5 5 THERMAL ENERGY 5 5 5 5 5 5 5
      C
0048      ENTRY MODT
      C-----CONFINEMENT WALL
0049      XP=X(ISTEP)-XU(ISTEP)
0050      DO 501 J=2,NJ
0051      TMULT = -COND(I,J)/(CPP(I,J)*XP)
0052      TAREA = TMULT*SNS(J)*0.01
0053      SU(ISTEP,J) = SU(ISTEP,J)+TAREA*TIN
0054      SP(ISTEP,J) = SP(ISTEP,J)+TAREA
0055      AW(ISTEP,J) = 0.0
0056      501 CONTINUE
      C-----IMPINGEMENT WALL
0057      XP=XU(NI)-X(NIMI)
0058      DO 510 J=2,NJ
0059      TMULT=-COND(NI,J)/(CPP(NI,J)*XP)
0060      TAREA = TMULT*SNS(J)*0.01
0061      SU(NIMI,J)=SU(NIMI,J)-TAREA*TWALL
0062      SP(NIMI,J)=SP(NIMI,J)+TAREA
0063      510 AE(NIMI,J)=0.0
      C-----OUTFLOW REGION
0064      DO 511 I=2,NIMI
0065      T(I,NJ)=T(I,NJM1)
0066      511 RETURN
      C
      C
0067      ENTRY MODW
      C-----CONFINEMENT WALL
0068      DO 605 J=2,NJ

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MOD00048

MOD00055

MOD00056

MOD00058

MOD00060

MOD00061

MOD00072

MOD00073

MOD00074

MOD00075

MOD00076

MOD00077

MOD00078

MOD00085

MOD00086

MOD00087

MOD00088

MOD00090

MOD00091

MOD00092

MOD00094

MOD00095

MOD00098

MOD00099

MOD00100

MOD00101

MOD00102

MOD00103

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0069      W(1,J) = W(2,J)
0070      605 AW(2,J) = 0.0
          C-----IMPINGEMENT WALL
0071      XP=XU(NI)-X(NIMI)
0072      DO 610 J=2,NJ
0073      PSTT=PST(T(NI,J))
0074      W(NI,J)=PSTT/(PSTT+(P(NIMI,J)+PIN-PSTT)*RATIO)
0075      TMULT=-DIFF(NI,J)*DEN(NI,J)/XP
0076      TAREA = TMULT*SNS(J)*0.01
0077      SU(NIMI,J)=SU(NIMI,J)-TAREA*W(NI,J)
0078      SP(NIMI,J)=SP(NIMI,J)+TAREA
0079      AE(NIMI,J)=0.0
0080      610 CONTINUE
          C-----OUTFLOW REGION
0081      DO 613 I=2,NI
0082      613 W(I,NJ)=W(I,NJMI)
0083      RETURN
0084      END

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MOD00107
MOD00108
MOD00109
MOD00110
MOD00111
MOD00112

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MOD00114
MOD00115
MOD00116
MOD00117
MOD00119
MOD00120
MOD00123
MOD00124
MOD00125

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*OPTIONS IN EFFECT* NOTERM,TD,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT* NAME = PROMOD , LINECNT = 56
*STATISTICS* SOURCE STATEMENTS = 84,PROGRAM SIZE = 3244
*STATISTICS* NO DIAGNOSTICS GENERATED

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LISOLV

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\*OPTIONS IN EFFECT\* - NOTERM, ID, EBCDIC, SOURCE, NOLIST, NODCK, LOAD, NOMAP, NOTEST  
 \*OPTIONS IN EFFECT\* NAME = LISOLV \* LINECNT = 56  
 \*STATISTICS\* SOURCE STATEMENTS = 40, PROGRAM SIZE = 1700  
 \*STATISTICS\* NO DIAGNOSTICS GENERATED

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0001      C---LISOLV-----LISOLV01
      C---TOMA OF LINE-BY-LINE SOLUTION-----LISOLV02
      SUBROUTINE LOSILV(I,ISTART,JSTART,NI,NJ,IT,JT,PHI,NSW,CAGE)
      CHAPTER 0 0 0 0 0 0 0 0 0 PRELIMINARIES 0 0 0 0 0 0 0 0 LISOLV04
      C-----LISOLV05
0002      DIMENSION PHI(IT,JT),A(21),B(21),C(21),D(21)
0003      COMMON NITER
0004      COMMON
      1/COEF/AP(20,20),AN(20,20),AS(20,20),AE(20,20),AW(20,20),
      2 SU(20,20),SP(20,20)
0005      LOGICAL SKIP
0006      LOGICAL LEND
0007      SKIP=.FALSE.
0008      LEND=.FALSE.
0009      NIMI=NI-1
0010      NJMI=NJ-1
0011      JSTM1=JSTART-1
      C
      C
0012      A(JSTM1)=0.0
      C-----
0013      COMMENCE W-E SWEEP
0014      DO 100 I=ISTART,NIMI
      C(JSTM1)=PHI(I,JSTM1)
      C-----
0015      COMMENCE S-N TRAVERSE
      DO 101 J=JSTART,NJMI
      C-----ASSEMBLE TOMA COEFFICIENTS
0016      A(J)=AN(I,J)
0017      B(J)=AS(I,J)
0018      C(J)=AE(I,J)*PHI(I+1,J)+AW(I,J)*PHI(I-1,J)+SU(I,J)
0019      D(J)=AP(I,J)
0020      IF((D(J)-B(J)*A(J-1)).EQ.0.) SKIP=.TRUE.
0021      IF(.NOT.SKIP) GO TO 50
0022      WRITE(6,199) SKIP,I,J
0023      199 FORMAT(5X,'SKIP =',L2,'ON I=',I2,',J=',I2)
0024      LEND=.TRUE.
0025      GO TO 108
      C-----CALCULATE COEFFICIENTS OF RECURRENCE FORMULA
0026      50 TERM=1./((D(J)-B(J)*A(J-1)))
0027      A(J)=A(J)*TERM
0028      C(J)=(C(J)+B(J)*C(J-1))*TERM
0029      IF(.NOT.LEND) GO TO 108
0030      A(J-1)=A(J)
0031      C(J-1)=C(J)
0032      LEND=.FALSE.
0033      108 SKIP=.FALSE.
0034      101 CONTINUE
      C-----OBTAIN NEW PHIS
0035      DO 102 JJ=JSTART,NJMI
0036      J=NJ+JSTM1-JJ
0037      102 PHI(I,J)=A(J)*PHI(I,J+1)+C(J)
0038      100 CONTINUE
0039      RETURN
0040      END

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LISOLV07

LISOLV10  
 LISOLV11  
 LISOLV12  
 LISOLV13  
 LISOLV14  
 LISOLV15  
 LISOLV16  
 LISOLV17  
 LISOLV18  
 LISOLV19  
 LISOLV20  
 LISOLV21  
 LISOLV22  
 LISOLV23  
 LISOLV24  
 LISOLV25

LISOLV26

LISOLV28

LISOLV30  
 LISOLV31  
 LISOLV32  
 LISOLV33  
 LISOLV34  
 LISOLV35  
 LISOLV36

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LOSILV

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\*OPTIONS IN EFFECT\* NOTERM, ID, EBCDIC, SOURCE, NOLIST, NODECK, LOAD, NOMAP, NOTEST

\*OPTIONS IN EFFECT\* NAME = LOSILV, LINECNT = 56

\*STATISTICS\* SOURCE STATEMENTS = 40, PROGRAM SIZE = 1708

\*STATISTICS\* NO DIAGNOSTICS GENERATED

30

```

C-- PST -----PST00001
C--SATURATED VAPOR PRESSURE CALCULATION -----PST00002
0001      FUNCTION PST(TEMP) PST00003
0002      COMMON /VAPP/ANTON1,ANTON2,ANTON3 PST00004
0003      PST=ANTON1-ANTON2/(TEMP+ANTON3) PST00005
0004      PST=EXP(PST) PST00006
0005      RETURN PST00007
0006      END PST00008
    
```

```

*OPTIONS IN EFFECT* NOTERM,IO,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT* NAME = PST, LINECNT = 56
*STATISTICS* SOURCE STATEMENTS = 6, PROGRAM SIZE = 342
*STATISTICS* NO DIAGNOSTICS GENERATED
    
```

```

C--PRINT-----PRINT001
C--PRINTING SUBPROGRAM-----PRINT002
SUBROUTINE PH1(ISTART,JSTART,NI,NJ,IT,UT,X,Y,PHI,HEAD)
DIMENSION PHI(11,JT),X(11),Y(JT),HEAD(9),STORE(50)
105 FORMAT(4H I=,2X,'1',9X,'2',9X,'3',9X,'4',9X,'5',9X,'6',9X,'7',9X
1 8',9X,'9',8X,'10',8X,'11',10X,'Y =')
0004 106 FORMAT(4H I=,12',8X,'13',8X,'14',8X,'15',8X,'16',8X,'17',8X,
1 18',8X,'19',8X,'20',8X,'21',8X,'22',10X,'Y =')
0005 107 FORMAT(4H I=,23',8X,'24',8X,'25',8X,'26',10X,'Y =')
0006 165 FORMAT(4H I=,2X,'3',9X,'4',9X,'5',9X,'6',9X,'7',9X,'8',
1 9X,'9',8X,'10',8X,'11',8X,'12',8X,'13',10X,'Y =')
0007 166 FORMAT(4H I=,14',8X,'15',8X,'16',8X,'17',8X,'18',8X,'19',
1 8X,'Y =')
0008 NIM1=NI-1
0009 NJM1=NJ-1
0010 ISKIP=1
0011 JSKIP=1
0012 WRITE(6,110)HEAD
0013 ISTA = ISTART-1
0014 100 CONTINUE
0015 ISTA = ISTA+1
0016 IEND = ISTA+10
0017 IF(ISTA.EQ.14) IEND=NIM1
0018 IEND=MIN0(NI,IEND)
0019 IF(ISTA.EQ.1) WRITE(6,105)
0020 IF(ISTA.EQ.3) WRITE(6,165)
0021 IF(ISTA.EQ.12) WRITE(6,106)
0022 IF(ISTA.EQ.14) WRITE(6,166)
0023 IF(ISTA.EQ.23) WRITE(6,107)
0024 WRITE(6,112)
0025 IF(ISTA.EQ.3) GO TO 44
0026 IF(ISTA.EQ.14) GO TO 44
0027 DO 101 JJ=JSTART,NJ,JSKIP
0028 J=JSTART+NJ-JJ
0029 DO 120 I=ISTA,IEND
0030 A=PHI(I,J)
0031 IF(ABS(A).LT.1.E-20) A=0.0
0032 120 STORE(I)=A
0033 101 WRITE(6,113) J,(STORE(I),I=ISTA,IEND,ISKIP),Y(J)
0034 WRITE(6,115)
0035 WRITE(6,114) (X(I),I=ISTA,IEND,ISKIP)
0036 IF(IEND.LT.NI)GO TO 100
0037 RETURN
0038 44 DO 161 JJ=JSTART,NJM1,JSKIP
0039 J=JSTART+NJM1-JJ
0040 DO 126 I=ISTA,IEND
0041 A=PHI(I,J)
0042 IF(ABS(A).LT.1.E-20) A=0.0
0043 126 STORE(I)=A
0044 161 WRITE(6,113) J,(STORE(I),I=ISTA,IEND,ISKIP),Y(J)
0045 WRITE(6,115)
0046 WRITE(6,114) (X(I),I=ISTA,IEND,ISKIP)
0047 IF(IEND.LT.NIM1) GO TO 100
0048 RETURN

```

PRINT011  
PRINT012  
PRINT013

PRINT015

PRINT018

PRINT021

PRINT022  
PRINT023  
PRINT024  
PRINT025  
PRINT026  
PRINT027  
PRINT028

PRINT029  
PRINT031  
PRINT032

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```
0049      110 FORMAT(1H1,///.1X,20(2H*-).7X,9A4,7X,20(2H-#))
0050      112 FORMAT(3H  J)
0051      113 FORMAT(1H 13,1P11E10.3,0PF10.4)
0052      114 FORMAT(4H X= ,11F10.7)
0053      115 FORMAT(' ')
0054      END
```

```
PRINT033
PRINT034
```

PRINT037

```
*OPTIONS IN EFFECT* NOTERN, ID, EBCDIC, SOURCE, NDLIST, NODECK, LOAD, NOMAP, NOTEST
*OPTIONS IN EFFECT* NAME = PRINT , LINECNT = 56
*STATISTICS* SOURCE STATEMENTS = 54, PROGRAM SIZE = 2446
*STATISTICS* NO DIAGNOSTICS GENERATED
```

```
*STATISTICS* NO DIAGNOSTICS THIS STEP
```

F64-LEVEL LINKAGE EDITOR OPTIONS SPECIFIED SIZE=(95K,14K)  
VARIABLE OPTIONS USED - SIZE=(97280,14336)  
SYSPRINT DEFAULT BLOCKING USED 1 - 1  
\*\*\*\*\*MAIN DOES NOT EXIST BUT HAS BEEN ADDED TO DATA SET.  
AUTHORIZATION CODE IS 0.

**APPENDIX B**

**LISTING OF COMPUTER OUTPUT**

## PULJET

## SIMULTANEOUS HEAT &amp; MASS TRANSFER UNDER AN IMPINGING JET

SCALAR GRID SYSTEM: X- Y- DIRECTION

|                |               |               |               |               |               |               |               |
|----------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| -1.2537872E-05 | 1.2537872E-05 | 6.2689360E-05 | 1.6299234E-04 | 3.6359811E-04 | 7.6481001E-04 | 1.5672338E-03 | 3.1720814E-03 |
| 6.3817762E-03  | 7.9866238E-03 | 8.7890476E-03 | 9.1902576E-03 | 9.3908608E-03 | 9.4911605E-03 | 9.5413104E-03 | 9.5663853E-03 |
| 9.5789209E-03  | 9.5914565E-03 | 9.5977224E-03 | 9.6022338E-03 |               |               |               |               |
| -1.5873094E-03 | 1.5873094E-03 | 4.7619268E-03 | 7.9365447E-03 | 1.1111163E-02 | 1.4285780E-02 | 1.6153201E-02 | 1.7251685E-02 |
| 1.7897852E-02  | 1.8277947E-02 | 1.8501531E-02 | 1.8633053E-02 | 1.8710416E-02 | 1.8787779E-02 | 1.8942505E-02 | 1.9174606E-02 |
| 1.9406702E-02  | 1.9638799E-02 | 1.9870896E-02 | 2.0025626E-02 | 2.0180356E-02 | 2.0257719E-02 | 2.0335082E-02 | 2.0412445E-02 |
| 2.0489808E-02  | 2.0567171E-02 | 2.0721901E-02 | 2.0876631E-02 | 2.1108728E-02 | 2.1340825E-02 | 2.1572921E-02 | 2.1805018E-02 |
| 2.1959748E-02  | 2.2037111E-02 | 2.2191841E-02 | 2.2389177E-02 | 2.2586513E-02 | 2.2783849E-02 | 2.2981185E-02 | 2.3178522E-02 |
| 2.3544680E-02  | 2.3910839E-02 | 2.4276998E-02 | 2.5210708E-02 | 2.6144419E-02 | 2.9319037E-02 | 3.2493655E-02 | 3.5668273E-02 |
| 3.8842890E-02  | 4.2017508E-02 |               |               |               |               |               |               |

JET INLET CONDITIONS  
 REYNOLDS NUMBER 1.000E+03  
 AVERAGE VELOCITY 9.879E+00  
 TEMPERATURE 4.500E+02  
 PRESSURE 1.013E+05  
 MASS FRACTION OF A 1.000E-01  
 MULE FRACTION, A 0.152E+00

PRANDTL NO 6.928E-01  
 SCHMIDT NO 6.261E-01

CROSS FLOW INLET CONDITIONS  
 REYNOLDS NUMBER 1.667E+02  
 AVERAGE VELOCITY 4.803E-01  
 TEMPERATURE 4.500E+02  
 PRESSURE 0.0  
 MASS FRACTION OF A 1.000E-01

CROSS MASS FLOW-RATE IS 10.00 % OF JET MASS FLOW RATE

IMPINGEMENT WALL CONDITIONS  
 TEMPERATURE 3.500E+02  
 USUC/UM 0.0

GEOMETRY OF SYSTEM  
 JET RADIUS 1.600E-03  
 H/D RATIO 3.000E+00  
 NOZZLE FROM J 14  
 TO J 33  
 IMPINGEMENT ANGLE 0.174532

PROGRAM CONDITIONS

MAX ALLOW. ITER 200  
MAX ALLOW. TIME 9.000E+02  
MAX ALLOW. RESI 1.500E-02

JET MASS FLOW IN 2.331540E-04  
JET X-MOMENTUM IN 2.731954E-03  
JET Y-MOMENTUM IN 8.418890E-05

CROSS-FLOW MASS FLOW IN 3.474274E-05  
CROSS-FLOW Y-MOMENTUM IN 1.677235E-05

VARIABLE FIELD INITIALISED AS :  
MESSAGE DE PERFORATION:DATA# 163

| ITER<br>NO | UMOM      | VMOM      | ABSOLUTE<br>MASS | RESIDUAL<br>ENER | SOURCE<br>SPEE | SUMS | U          | V          | P         | T         | W         | ALPHA  | SPLIT RATE<br>VINC |
|------------|-----------|-----------|------------------|------------------|----------------|------|------------|------------|-----------|-----------|-----------|--------|--------------------|
| 1          | 1.774E-02 | 1.236E-02 | 2.542E-02        | 1.060E-02        | 8.866E-03      |      | -3.421E-02 | -4.167E-01 | 7.227E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 6.054E-03          |
| 2          | 2.072E-02 | 1.274E-02 | 2.800E-02        | 8.908E-03        | 7.817E-03      |      | -3.359E-02 | -4.167E-01 | 7.186E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 3.198E-03          |
| 3          | 1.918E-02 | 1.210E-02 | 2.653E-02        | 8.908E-03        | 7.817E-03      |      | -3.345E-02 | -4.168E-01 | 7.166E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.312E-03          |
| 4          | 1.819E-02 | 1.191E-02 | 2.671E-02        | 8.908E-03        | 7.817E-03      |      | -3.346E-02 | -4.169E-01 | 7.147E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 3.937E-03          |
| 5          | 1.759E-02 | 1.188E-02 | 2.683E-02        | 8.908E-03        | 7.817E-03      |      | -3.346E-02 | -4.169E-01 | 7.128E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.129E-03          |
| 6          | 1.718E-02 | 1.188E-02 | 2.674E-02        | 8.908E-03        | 7.817E-03      |      | -3.347E-02 | -4.170E-01 | 7.108E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.129E-03          |
| 7          | 1.687E-02 | 1.187E-02 | 2.663E-02        | 8.908E-03        | 7.817E-03      |      | -3.347E-02 | -4.170E-01 | 7.089E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.126E-03          |
| 8          | 1.671E-02 | 1.180E-02 | 2.644E-02        | 8.908E-03        | 7.817E-03      |      | -3.348E-02 | -4.171E-01 | 7.068E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.126E-03          |
| 9          | 1.663E-02 | 1.171E-02 | 2.638E-02        | 8.908E-03        | 7.817E-03      |      | -3.349E-02 | -4.171E-01 | 7.047E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.123E-03          |
| 10         | 1.657E-02 | 1.165E-02 | 2.616E-02        | 8.908E-03        | 7.817E-03      |      | -3.349E-02 | -4.172E-01 | 7.026E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.123E-03          |
| 11         | 1.651E-02 | 1.166E-02 | 2.593E-02        | 8.908E-03        | 7.817E-03      |      | -3.350E-02 | -4.172E-01 | 7.004E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.129E-03          |
| 12         | 1.642E-02 | 1.171E-02 | 2.570E-02        | 7.959E-03        | 7.122E-03      |      | -3.350E-02 | -4.173E-01 | 6.981E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.126E-03          |
| 13         | 1.638E-02 | 1.177E-02 | 2.549E-02        | 7.959E-03        | 7.122E-03      |      | -3.339E-02 | -4.173E-01 | 6.955E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.272E-03          |
| 14         | 1.637E-02 | 1.192E-02 | 2.530E-02        | 7.959E-03        | 7.122E-03      |      | -3.340E-02 | -4.174E-01 | 6.931E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 3.983E-03          |
| 15         | 1.638E-02 | 1.204E-02 | 2.478E-02        | 7.959E-03        | 7.122E-03      |      | -3.340E-02 | -4.174E-01 | 6.907E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.123E-03          |
| 16         | 1.623E-02 | 1.218E-02 | 2.428E-02        | 7.959E-03        | 7.122E-03      |      | -3.341E-02 | -4.174E-01 | 6.884E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.123E-03          |
| 17         | 1.615E-02 | 1.234E-02 | 2.370E-02        | 7.959E-03        | 7.122E-03      |      | -3.341E-02 | -4.174E-01 | 6.861E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.123E-03          |
| 18         | 1.613E-02 | 1.251E-02 | 2.308E-02        | 7.959E-03        | 7.122E-03      |      | -3.342E-02 | -4.175E-01 | 6.839E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.123E-03          |
| 19         | 1.613E-02 | 1.267E-02 | 2.242E-02        | 7.959E-03        | 7.122E-03      |      | -3.342E-02 | -4.175E-01 | 6.817E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.126E-03          |
| 20         | 1.613E-02 | 1.284E-02 | 2.175E-02        | 7.959E-03        | 7.122E-03      |      | -3.343E-02 | -4.175E-01 | 6.795E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.123E-03          |
| 21         | 1.613E-02 | 1.300E-02 | 2.105E-02        | 7.959E-03        | 7.122E-03      |      | -3.343E-02 | -4.175E-01 | 6.774E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.123E-03          |
| 22         | 1.614E-02 | 1.316E-02 | 2.031E-02        | 7.351E-03        | 6.518E-03      |      | -3.344E-02 | -4.175E-01 | 6.753E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.123E-03          |
| 23         | 1.619E-02 | 1.330E-02 | 1.963E-02        | 7.351E-03        | 6.518E-03      |      | -3.335E-02 | -4.175E-01 | 6.731E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.250E-03          |
| 24         | 1.645E-02 | 1.343E-02 | 1.903E-02        | 7.351E-03        | 6.518E-03      |      | -3.335E-02 | -4.175E-01 | 6.711E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.002E-03          |
| 25         | 1.636E-02 | 1.351E-02 | 1.814E-02        | 7.351E-03        | 6.518E-03      |      | -3.336E-02 | -4.175E-01 | 6.691E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.119E-03          |
| 26         | 1.631E-02 | 1.359E-02 | 1.725E-02        | 7.351E-03        | 6.518E-03      |      | -3.336E-02 | -4.175E-01 | 6.674E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.110E-03          |
| 27         | 1.630E-02 | 1.365E-02 | 1.635E-02        | 7.351E-03        | 6.518E-03      |      | -3.336E-02 | -4.175E-01 | 6.657E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.113E-03          |
| 28         | 1.631E-02 | 1.367E-02 | 1.543E-02        | 7.351E-03        | 6.518E-03      |      | -3.337E-02 | -4.175E-01 | 6.642E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.123E-03          |
| 29         | 1.633E-02 | 1.367E-02 | 1.450E-02        | 7.351E-03        | 6.518E-03      |      | -3.337E-02 | -4.175E-01 | 6.628E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.110E-03          |
| 30         | 1.635E-02 | 1.364E-02 | 1.355E-02        | 7.351E-03        | 6.518E-03      |      | -3.337E-02 | -4.175E-01 | 6.615E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.113E-03          |
| 31         | 1.636E-02 | 1.357E-02 | 1.259E-02        | 7.351E-03        | 6.518E-03      |      | -3.338E-02 | -4.175E-01 | 6.603E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.110E-03          |
| 32         | 1.638E-02 | 1.340E-02 | 1.163E-02        | 7.159E-03        | 6.129E-03      |      | -3.338E-02 | -4.174E-01 | 6.593E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.116E-03          |
| 33         | 1.639E-02 | 1.337E-02 | 1.081E-02        | 7.159E-03        | 6.129E-03      |      | -3.331E-02 | -4.174E-01 | 6.581E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.217E-03          |
| 34         | 1.649E-02 | 1.327E-02 | 1.013E-02        | 7.159E-03        | 6.129E-03      |      | -3.331E-02 | -4.174E-01 | 6.574E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.100E-03          |
| 35         | 1.637E-02 | 1.312E-02 | 9.750E-03        | 7.159E-03        | 6.129E-03      |      | -3.331E-02 | -4.174E-01 | 6.565E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.048E-03          |
| 36         | 1.627E-02 | 1.296E-02 | 9.372E-03        | 7.159E-03        | 6.129E-03      |      | -3.331E-02 | -4.173E-01 | 6.559E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.109E-03          |
| 37         | 1.619E-02 | 1.281E-02 | 9.214E-03        | 7.159E-03        | 6.129E-03      |      | -3.331E-02 | -4.173E-01 | 6.556E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.106E-03          |
| 38         | 1.611E-02 | 1.263E-02 | 9.150E-03        | 7.159E-03        | 6.129E-03      |      | -3.332E-02 | -4.172E-01 | 6.552E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.106E-03          |
| 39         | 1.603E-02 | 1.245E-02 | 9.204E-03        | 7.159E-03        | 6.129E-03      |      | -3.332E-02 | -4.172E-01 | 6.549E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.106E-03          |
| 40         | 1.594E-02 | 1.224E-02 | 9.377E-03        | 7.159E-03        | 6.129E-03      |      | -3.332E-02 | -4.171E-01 | 6.548E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.106E-03          |
| 41         | 1.580E-02 | 1.200E-02 | 9.643E-03        | 7.159E-03        | 6.129E-03      |      | -3.332E-02 | -4.171E-01 | 6.548E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.109E-03          |
| 42         | 1.568E-02 | 1.176E-02 | 9.969E-03        | 7.148E-03        | 5.770E-03      |      | -3.332E-02 | -4.170E-01 | 6.549E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.106E-03          |
| 43         | 1.558E-02 | 1.149E-02 | 1.041E-02        | 7.148E-03        | 5.770E-03      |      | -3.328E-02 | -4.170E-01 | 6.549E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.207E-03          |
| 44         | 1.557E-02 | 1.121E-02 | 1.074E-02        | 7.148E-03        | 5.770E-03      |      | -3.328E-02 | -4.169E-01 | 6.550E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.096E-03          |
| 45         | 1.532E-02 | 1.094E-02 | 1.107E-02        | 7.148E-03        | 5.770E-03      |      | -3.328E-02 | -4.169E-01 | 6.553E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.093E-03          |
| 46         | 1.509E-02 | 1.065E-02 | 1.139E-02        | 7.148E-03        | 5.770E-03      |      | -3.328E-02 | -4.168E-01 | 6.557E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.100E-03          |
| 47         | 1.493E-02 | 1.035E-02 | 1.176E-02        | 7.148E-03        | 5.770E-03      |      | -3.327E-02 | -4.168E-01 | 6.562E+00 | 3.529E+02 | 2.965E-01 | 1.2340 | 4.100E-03          |

OVERALL MASS EVAPUTATION = -6.293779E-06  
OVERALL MASS OUTFLOW = 2.741902E-04  
NET MASS ACCUMUL. = 0.0

OUTPUT PUNCH = T



```

45 9.527E-02-7.483E-02-8.085E-02-5.193E-02-3.720E-02-3.173E-02-2.913E-02-2.817E-02-2.798E-02 2.614E-02
44 6.873E-01 2.475E-01 4.980E-02-1.170E-02-2.897E-02-3.394E-02-3.594E-02-3.643E-02-3.643E-02 2.521E-02
43 1.040E+00 4.017E-01 8.716E-02-1.149E-02-3.353E-02-3.831E-02-3.983E-02-4.000E-02-3.993E-02 2.428E-02
42 1.376E+00 6.144E-01 2.103E-01 4.007E-02-1.663E-02-3.423E-02-4.174E-02-4.393E-02-4.416E-02 2.391E-02
41 1.533E+00 7.016E-01 2.574E-01 6.055E-02-9.597E-03-3.233E-02-4.225E-02-4.526E-02-4.563E-02 2.354E-02
40 1.702E+00 7.766E-01 2.695E-01 5.409E-02-1.493E-02-3.542E-02-4.390E-02-4.629E-02-4.653E-02 2.318E-02
39 1.824E+00 8.635E-01 3.329E-01 8.648E-02-2.773E-03-3.171E-02-4.433E-02-4.819E-02-4.867E-02 2.298E-02
38 1.890E+00 9.133E-01 3.654E-01 1.014E-01 2.174E-03-3.044E-02-4.474E-02-4.913E-02-4.969E-02 2.278E-02
37 1.944E+00 9.695E-01 4.146E-01 1.144E-01 4.712E-03-3.034E-02-4.550E-02-5.010E-02-5.069E-02 2.259E-02
36 1.975E+00 1.003E+00 3.868E-01 1.017E-01 1.044E-03-3.115E-02-4.515E-02-4.937E-02-4.989E-02 2.239E-02
35 1.956E+00 9.014E-01 3.219E-01 8.158E-02-4.023E-03-3.177E-02-4.388E-02-4.757E-02-4.802E-02 2.219E-02
34 1.883E+00 8.520E-01 3.153E-01 8.686E-02 6.711E-04-2.910E-02-4.263E-02-4.698E-02-4.756E-02 2.204E-02
33 1.886E+00 8.869E-01 3.269E-01 7.068E-02-1.191E-02-3.517E-02-4.431E-02-4.665E-02-4.687E-02 2.196E-02
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31 1.699E+00 7.764E-01 2.779E-01 6.561E-02-0.186E-03-3.200E-02-4.239E-02-4.555E-02-4.593E-02 2.157E-02
30 1.504E+00 6.645E-01 2.138E-01 3.664E-02-1.891E-02-3.542E-02-4.227E-02-4.418E-02-4.436E-02 2.134E-02
29 1.232E+00 5.138E-01 1.350E-01 4.276E-03-2.972E-02-3.839E-02-4.161E-02-4.230E-02-4.229E-02 2.111E-02
28 9.495E-01 3.668E-01 7.221E-02-1.550E-02-3.459E-02-3.890E-02-4.035E-02-4.055E-02-4.048E-02 2.088E-02
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26 5.887E-01 1.635E-01 2.417E-02-5.291E-02-4.647E-02-4.163E-02-3.886E-02-3.764E-02-3.736E-02 2.057E-02
25 5.024E-01 1.092E-01 6.050E-02-7.068E-02-5.246E-02-4.333E-02-3.861E-02-3.676E-02-3.638E-02 2.049E-02
24 4.213E-01 5.669E-02-9.028E-02-8.177E-02-5.581E-02-4.402E-02-3.806E-02-3.579E-02-3.535E-02 2.041E-02
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12-2.682E-01-2.498E-01-1.121E-01-4.190E-02-2.623E-02-2.278E-02-2.157E-02-2.124E-02-2.117E-02 1.863E-02
11-2.745E-01-2.494E-01-1.049E-01-4.388E-02-2.764E-02-2.306E-02-2.113E-02-2.050E-02-2.038E-02 1.850E-02
10-2.762E-01-2.377E-01-9.339E-02-4.156E-02-2.642E-02-2.188E-02-1.992E-02-1.927E-02-1.915E-02 1.828E-02
9-2.613E-01-2.068E-01-7.455E-02-3.435E-02-2.291E-02-1.952E-02-1.805E-02-1.757E-02-1.748E-02 1.790E-02
8-2.241E-01-1.585E-01-5.501E-02-2.674E-02-1.900E-02-1.673E-02-1.576E-02-1.544E-02-1.538E-02 1.725E-02
7-1.798E-01-1.071E-01-3.923E-02-2.062E-02-1.548E-02-1.397E-02-1.331E-02-1.309E-02-1.305E-02 1.615E-02
6-1.725E-01-7.865E-02-3.047E-02-1.640E-02-1.237E-02-1.116E-02-1.063E-02-1.046E-02-1.043E-02 1.429E-02
5-1.820E-01-6.975E-02-2.622E-02-1.323E-02-9.476E-03-8.352E-03-7.869E-03-7.712E-03-7.685E-03 1.111E-02
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3-1.751E-01-3.705E-02-1.259E-02-7.228E-03-5.913E-03-5.559E-03-5.416E-03-5.374E-03-5.367E-03 4.762E-03
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1 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -1.587E-03

```

X= 0.0089897 0.0092906 0.0094410 0.0095162 0.0095538 0.0095727 0.0095852 0.0095946 0.0096000

[illegible]

|    |           |           |           |           |           |           |           |           |     |           |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----|-----------|
| 44 | 7.292E+00 | 6.819E+00 | 5.228E+00 | 3.398E+00 | 2.120E+00 | 1.384E+00 | 5.834E-01 | 1.582E-01 | 0.0 | 2.474E-02 |
| 43 | 5.909E+00 | 5.513E+00 | 4.418E+00 | 3.022E+00 | 1.943E+00 | 1.288E+00 | 5.516E-01 | 1.510E-01 | 0.0 | 2.409E-02 |
| 42 | 4.986E+00 | 4.564E+00 | 3.630E+00 | 2.484E+00 | 1.597E+00 | 1.059E+00 | 4.531E-01 | 1.240E-01 | 0.0 | 2.373E-02 |
| 41 | 3.978E+00 | 3.509E+00 | 2.706E+00 | 1.812E+00 | 1.151E+00 | 7.571E-01 | 3.215E-01 | 8.757E-02 | 0.0 | 2.336E-02 |
| 40 | 3.115E+00 | 2.580E+00 | 1.927E+00 | 1.304E+00 | 8.393E-01 | 5.569E-01 | 2.385E-01 | 2.532E-02 | 0.0 | 2.308E-02 |
| 39 | 2.487E+00 | 1.899E+00 | 1.304E+00 | 8.450E-01 | 5.343E-01 | 3.514E-01 | 1.490E-01 | 4.058E-02 | 0.0 | 2.288E-02 |
| 38 | 1.849E+00 | 1.194E+00 | 6.341E-01 | 3.343E-01 | 1.906E-01 | 1.192E-01 | 4.790E-02 | 1.265E-02 | 0.0 | 2.269E-02 |
| 37 | 1.211E+00 | 4.792E-01 | 1.302E-01 | 2.300E-01 | 1.785E-01 | 1.268E-01 | 5.811E-02 | 1.648E-02 | 0.0 | 2.249E-02 |
| 36 | 5.736E-01 | 3.173E-01 | 8.558E-01 | 7.486E-01 | 5.183E-01 | 3.537E-01 | 1.558E-01 | 4.330E-02 | 0.0 | 2.229E-02 |
| 35 | 4.448E-02 | 9.871E-01 | 1.402E+00 | 1.143E+00 | 7.799E-01 | 5.298E-01 | 2.324E-01 | 6.445E-02 | 0.0 | 2.211E-02 |
| 34 | 4.461E-01 | 1.397E+00 | 1.744E+00 | 1.404E+00 | 9.645E-01 | 6.589E-01 | 2.911E-01 | 8.106E-02 | 0.0 | 2.200E-02 |
| 33 | 8.374E-01 | 1.826E+00 | 2.131E+00 | 1.657E+00 | 1.110E+00 | 7.479E-01 | 3.250E-01 | 8.967E-02 | 0.0 | 2.188E-02 |
| 32 | 1.465E+00 | 2.519E+00 | 2.738E+00 | 2.080E+00 | 1.386E+00 | 9.317E-01 | 4.044E-01 | 1.115E-01 | 0.0 | 2.169E-02 |
| 31 | 2.177E+00 | 3.274E+00 | 3.372E+00 | 2.528E+00 | 1.682E+00 | 1.132E+00 | 4.915E-01 | 1.356E-01 | 0.0 | 2.146E-02 |
| 30 | 2.825E+00 | 3.955E+00 | 3.898E+00 | 2.863E+00 | 1.889E+00 | 1.266E+00 | 5.474E-01 | 1.507E-01 | 0.0 | 2.122E-02 |
| 29 | 3.379E+00 | 4.525E+00 | 4.285E+00 | 3.070E+00 | 2.000E+00 | 1.332E+00 | 5.725E-01 | 1.570E-01 | 0.0 | 2.099E-02 |
| 28 | 3.753E+00 | 4.892E+00 | 4.498E+00 | 3.166E+00 | 2.047E+00 | 1.360E+00 | 5.827E-01 | 1.596E-01 | 0.0 | 2.080E-02 |
| 27 | 4.005E+00 | 5.133E+00 | 4.600E+00 | 3.164E+00 | 2.018E+00 | 1.331E+00 | 5.659E-01 | 1.542E-01 | 0.0 | 2.064E-02 |
| 26 | 4.169E+00 | 5.272E+00 | 4.639E+00 | 3.145E+00 | 1.991E+00 | 1.308E+00 | 5.542E-01 | 1.508E-01 | 0.0 | 2.053E-02 |
| 25 | 4.271E+00 | 5.354E+00 | 4.640E+00 | 3.108E+00 | 1.955E+00 | 1.281E+00 | 5.411E-01 | 1.470E-01 | 0.0 | 2.045E-02 |
| 24 | 4.364E+00 | 5.424E+00 | 4.634E+00 | 3.055E+00 | 1.908E+00 | 1.246E+00 | 5.245E-01 | 1.421E-01 | 0.0 | 2.037E-02 |
| 23 | 4.451E+00 | 5.482E+00 | 4.603E+00 | 2.990E+00 | 1.854E+00 | 1.206E+00 | 5.061E-01 | 1.369E-01 | 0.0 | 2.030E-02 |
| 22 | 4.532E+00 | 5.527E+00 | 4.553E+00 | 2.913E+00 | 1.794E+00 | 1.163E+00 | 4.865E-01 | 1.313E-01 | 0.0 | 2.022E-02 |
| 21 | 4.641E+00 | 5.572E+00 | 4.435E+00 | 2.778E+00 | 1.698E+00 | 1.099E+00 | 4.583E-01 | 1.236E-01 | 0.0 | 2.010E-02 |
| 20 | 4.759E+00 | 5.584E+00 | 4.262E+00 | 2.602E+00 | 1.569E+00 | 1.007E+00 | 4.168E-01 | 1.118E-01 | 0.0 | 1.995E-02 |
| 19 | 4.872E+00 | 5.543E+00 | 3.998E+00 | 2.383E+00 | 1.425E+00 | 9.115E-01 | 3.762E-01 | 1.008E-01 | 0.0 | 1.975E-02 |
| 18 | 4.956E+00 | 5.439E+00 | 3.696E+00 | 2.146E+00 | 1.265E+00 | 8.039E-01 | 3.294E-01 | 8.784E-02 | 0.0 | 1.952E-02 |
| 17 | 5.001E+00 | 5.297E+00 | 3.405E+00 | 1.938E+00 | 1.133E+00 | 7.176E-01 | 2.930E-01 | 7.799E-02 | 0.0 | 1.929E-02 |
| 16 | 5.017E+00 | 5.131E+00 | 3.137E+00 | 1.758E+00 | 1.023E+00 | 6.463E-01 | 2.634E-01 | 7.002E-02 | 0.0 | 1.906E-02 |
| 15 | 5.012E+00 | 4.994E+00 | 2.942E+00 | 1.619E+00 | 9.322E-01 | 5.856E-01 | 2.372E-01 | 6.281E-02 | 0.0 | 1.887E-02 |
| 14 | 5.004E+00 | 4.908E+00 | 2.834E+00 | 1.542E+00 | 8.814E-01 | 5.512E-01 | 2.221E-01 | 5.861E-02 | 0.0 | 1.875E-02 |
| 13 | 4.998E+00 | 4.843E+00 | 2.759E+00 | 1.502E+00 | 8.623E-01 | 5.410E-01 | 2.190E-01 | 5.797E-02 | 0.0 | 1.867E-02 |
| 12 | 4.988E+00 | 4.744E+00 | 2.663E+00 | 1.460E+00 | 8.440E-01 | 5.318E-01 | 2.163E-01 | 5.743E-02 | 0.0 | 1.857E-02 |
| 11 | 4.969E+00 | 4.568E+00 | 2.520E+00 | 1.385E+00 | 8.022E-01 | 5.060E-01 | 2.059E-01 | 5.470E-02 | 0.0 | 1.839E-02 |
| 10 | 4.923E+00 | 4.270E+00 | 2.314E+00 | 1.266E+00 | 7.315E-01 | 4.609E-01 | 1.873E-01 | 4.972E-02 | 0.0 | 1.809E-02 |
| 9  | 4.819E+00 | 3.812E+00 | 2.042E+00 | 1.113E+00 | 6.421E-01 | 4.043E-01 | 1.642E-01 | 4.355E-02 | 0.0 | 1.757E-02 |
| 8  | 4.611E+00 | 3.209E+00 | 1.718E+00 | 9.368E-01 | 5.411E-01 | 3.408E-01 | 1.385E-01 | 3.674E-02 | 0.0 | 1.670E-02 |
| 7  | 4.224E+00 | 2.542E+00 | 1.355E+00 | 7.392E-01 | 4.267E-01 | 2.686E-01 | 1.091E-01 | 2.892E-02 | 0.0 | 1.522E-02 |
| 6  | 3.414E+00 | 1.743E+00 | 8.910E-01 | 4.751E-01 | 2.705E-01 | 1.689E-01 | 6.805E-02 | 1.794E-02 | 0.0 | 1.270E-02 |
| 5  | 2.227E+00 | 8.380E-01 | 3.502E-01 | 1.631E-01 | 8.521E-02 | 5.065E-02 | 1.930E-02 | 4.901E-03 | 0.0 | 9.524E-03 |
| 4  | 7.265E-01 | 2.142E-02 | 1.072E-01 | 8.455E-02 | 5.657E-02 | 3.813E-02 | 1.654E-02 | 4.565E-03 | 0.0 | 6.349E-03 |
| 3  | 7.044E-01 | 5.272E-01 | 3.253E-01 | 1.898E-01 | 1.126E-01 | 7.172E-02 | 2.942E-02 | 7.853E-03 | 0.0 | 3.175E-03 |
| 2  | 4.681E-01 | 4.657E-01 | 4.645E-01 | 4.638E-01 | 4.635E-01 | 4.633E-01 | 4.632E-01 | 4.631E-01 | 0.0 | 0.0       |

X= 0.0091903 0.0093909 0.0094912 0.0095413 0.0095664 0.0095789 0.0095915 0.0095977 0.0096022

| J  | 1   | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          | 10         | 11         | Y      |
|----|-----|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|--------|
| 50 | 0.0 | 0.0        | 0.0        | 0.0        | 0.0        | 0.0        | 0.0        | 0.0        | 0.0        | 0.0        | 0.0        | 0.0420 |
| 49 | 0.0 | -1.359E+01 | -1.359E+01 | -1.359E+01 | -1.359E+01 | -1.359E+01 | -1.358E+01 | -1.357E+01 | -1.364E+01 | -1.384E+01 | -1.401E+01 | 0.0388 |
| 48 | 0.0 | -1.372E+01 | -1.372E+01 | -1.372E+01 | -1.372E+01 | -1.372E+01 | -1.370E+01 | -1.369E+01 | -1.351E+01 | -1.343E+01 | -1.344E+01 | 0.0357 |
| 47 | 0.0 | -1.390E+01 | -1.390E+01 | -1.390E+01 | -1.390E+01 | -1.389E+01 | -1.385E+01 | -1.383E+01 | -1.316E+01 | -1.249E+01 | -1.228E+01 | 0.0325 |
| 46 | 0.0 | -1.461E+01 | -1.461E+01 | -1.462E+01 | -1.462E+01 | -1.464E+01 | -1.454E+01 | -1.450E+01 | -1.184E+01 | -9.373E+00 | -8.530E+00 | 0.0293 |
| 45 | 0.0 | -1.881E+01 | -1.882E+01 | -1.882E+01 | -1.883E+01 | -1.895E+01 | -1.953E+01 | -1.870E+01 | -7.961E+00 | -1.544E+00 | -1.068E+00 | 0.0261 |
| 44 | 0.0 | -2.167E+01 | -2.166E+01 | -2.167E+01 | -2.182E+01 | -2.241E+01 | -2.420E+01 | -2.195E+01 | -5.516E+00 | -4.123E+00 | -8.651E+00 | 0.0252 |
| 43 | 0.0 | -2.201E+01 | -2.199E+01 | -2.200E+01 | -2.209E+01 | -2.247E+01 | -2.399E+01 | -2.174E+01 | -2.725E+00 | -8.752E+00 | -1.478E+01 | 0.0243 |
| 42 | 0.0 | -2.149E+01 | -2.148E+01 | -2.146E+01 | -2.149E+01 | -2.172E+01 | -2.240E+01 | -2.082E+01 | -1.431E+01 | -1.097E+01 | -1.788E+01 | 0.0239 |
| 41 | 0.0 | -2.126E+01 | -2.125E+01 | -2.124E+01 | -2.125E+01 | -2.128E+01 | -2.108E+01 | -1.927E+01 | -1.283E+01 | -1.281E+01 | -2.042E+01 | 0.0235 |
| 40 | 0.0 | -3.105E+01 | -3.104E+01 | -3.104E+01 | -3.106E+01 | -3.106E+01 | -3.018E+01 | -1.709E+01 | 6.302E+01  | 1.355E+01  | 2.161E+01  | 0.0232 |
| 39 | 0.0 | -2.077E+01 | -2.076E+01 | -2.076E+01 | -2.077E+01 | -2.114E+01 | -2.062E+01 | -1.592E+01 | 1.104E+01  | 1.402E+01  | 2.219E+01  | 0.0230 |
| 38 | 0.0 | -2.061E+01 | -2.060E+01 | -2.060E+01 | -2.060E+01 | -2.124E+01 | -2.170E+01 | -1.463E+01 | 1.493E+01  | 1.421E+01  | 2.242E+01  | 0.0228 |
| 37 | 0.0 | -2.036E+01 | -2.036E+01 | -2.037E+01 | -2.029E+01 | -2.036E+01 | -2.197E+01 | -1.182E+01 | 2.017E+01  | 1.386E+01  | 2.181E+01  | 0.0224 |
| 36 | 0.0 | -2.004E+01 | -2.006E+01 | -2.006E+01 | -1.987E+01 | -1.849E+01 | -1.024E+01 | -1.020E+01 | 2.080E+01  | 1.323E+01  | 2.066E+01  | 0.0222 |
| 35 | 0.0 | -2.015E+01 | -2.008E+01 | -1.994E+01 | -1.964E+01 | -1.885E+01 | -1.650E+01 | -8.294E+00 | 2.002E+00  | 1.598E+01  | 1.998E+01  | 0.0220 |
| 34 | 0.0 | -2.051E+01 | -2.042E+01 | -2.028E+01 | -1.981E+01 | -1.880E+01 | -1.668E+01 | -8.066E+00 | 2.390E+00  | 1.171E+01  | 1.837E+01  | 0.0218 |
| 33 | 0.0 | -2.002E+01 | -1.994E+01 | -1.974E+01 | -1.912E+01 | -1.739E+01 | -1.407E+01 | -4.850E+00 | 2.419E+00  | 1.042E+01  | 1.835E+01  | 0.0216 |
| 32 | 0.0 | -1.790E+01 | -1.784E+01 | -1.764E+01 | -1.700E+01 | -1.521E+01 | -1.121E+01 | -2.963E+00 | 2.462E+00  | 9.053E+00  | 1.417E+01  | 0.0213 |
| 31 | 0.0 | -1.426E+01 | -1.425E+01 | -1.414E+01 | -1.366E+01 | -1.201E+01 | -8.179E+00 | -1.204E+00 | 2.551E+00  | 7.738E+00  | 1.188E+01  | 0.0211 |
| 30 | 0.0 | -9.400E+00 | -9.54E+00  | -9.710E+00 | -9.574E+00 | -8.420E+00 | -5.314E+00 | 8.698E+00  | 2.571E+00  | 6.566E+00  | 9.725E+00  | 0.0209 |
| 29 | 0.0 | -6.735E+00 | -6.808E+00 | -6.877E+00 | -6.808E+00 | -5.953E+00 | -3.748E+00 | 1.033E+00  | 2.733E+00  | 5.964E+00  | 8.431E+00  | 0.0207 |
| 28 | 0.0 | -3.738E+00 | -3.872E+00 | -4.027E+00 | -4.104E+00 | -3.659E+00 | -1.766E+00 | 1.596E+00  | 2.800E+00  | 5.497E+00  | 7.425E+00  | 0.0206 |
| 27 | 0.0 | -2.485E+00 | -2.546E+00 | -2.670E+00 | -2.790E+00 | -2.512E+00 | -8.936E+01 | 1.984E+00  | 2.908E     |            |            |        |

|    |           |           |           |           |           |           |           |           |     |            |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----|------------|
| 45 | 1.874E+00 | 2.085E+00 | 2.130E+00 | 2.135E+00 | 2.134E+00 | 2.133E+00 | 2.131E+00 | 2.130E+00 | 0.0 | 2.614E-02  |
| 44 | 1.009E+01 | 1.041E+01 | 1.045E+01 | 1.046E+01 | 1.046E+01 | 1.046E+01 | 1.047E+01 | 1.047E+01 | 0.0 | 2.521E-02  |
| 43 | 1.574E+01 | 1.755E+01 | 1.767E+01 | 1.770E+01 | 1.771E+01 | 1.771E+01 | 1.771E+01 | 1.771E+01 | 0.0 | 2.428E-02  |
| 42 | 2.043E+01 | 2.117E+01 | 2.129E+01 | 2.133E+01 | 2.134E+01 | 2.135E+01 | 2.135E+01 | 2.136E+01 | 0.0 | 2.391E-02  |
| 41 | 2.431E+01 | 2.411E+01 | 2.424E+01 | 2.427E+01 | 2.428E+01 | 2.429E+01 | 2.430E+01 | 2.430E+01 | 0.0 | 2.354E-02  |
| 40 | 2.481E+01 | 2.579E+01 | 2.599E+01 | 2.603E+01 | 2.605E+01 | 2.606E+01 | 2.606E+01 | 2.607E+01 | 0.0 | 2.318E-02  |
| 39 | 2.552E+01 | 2.656E+01 | 2.678E+01 | 2.682E+01 | 2.684E+01 | 2.685E+01 | 2.686E+01 | 2.687E+01 | 0.0 | 2.298E-02  |
| 38 | 2.583E+01 | 2.691E+01 | 2.714E+01 | 2.719E+01 | 2.721E+01 | 2.722E+01 | 2.724E+01 | 2.724E+01 | 0.0 | 2.278E-02  |
| 37 | 2.570E+01 | 2.678E+01 | 2.702E+01 | 2.708E+01 | 2.710E+01 | 2.712E+01 | 2.713E+01 | 2.714E+01 | 0.0 | 2.259E-02  |
| 36 | 2.515E+01 | 2.622E+01 | 2.650E+01 | 2.655E+01 | 2.658E+01 | 2.659E+01 | 2.660E+01 | 2.661E+01 | 0.0 | 2.239E-02  |
| 35 | 2.422E+01 | 2.536E+01 | 2.560E+01 | 2.565E+01 | 2.567E+01 | 2.567E+01 | 2.568E+01 | 2.569E+01 | 0.0 | 2.219E-02  |
| 34 | 2.332E+01 | 2.442E+01 | 2.462E+01 | 2.465E+01 | 2.467E+01 | 2.468E+01 | 2.469E+01 | 2.469E+01 | 0.0 | 2.204E-02  |
| 33 | 2.272E+01 | 2.372E+01 | 2.392E+01 | 2.398E+01 | 2.400E+01 | 2.401E+01 | 2.402E+01 | 2.403E+01 | 0.0 | 2.196E-02  |
| 32 | 2.151E+01 | 2.253E+01 | 2.274E+01 | 2.278E+01 | 2.280E+01 | 2.281E+01 | 2.282E+01 | 2.282E+01 | 0.0 | 2.181E-02  |
| 31 | 1.940E+01 | 2.039E+01 | 2.058E+01 | 2.063E+01 | 2.064E+01 | 2.065E+01 | 2.066E+01 | 2.066E+01 | 0.0 | 2.157E-02  |
| 30 | 1.690E+01 | 1.780E+01 | 1.798E+01 | 1.803E+01 | 1.804E+01 | 1.805E+01 | 1.805E+01 | 1.806E+01 | 0.0 | 2.134E-02  |
| 29 | 1.430E+01 | 1.511E+01 | 1.529E+01 | 1.533E+01 | 1.534E+01 | 1.535E+01 | 1.535E+01 | 1.535E+01 | 0.0 | 2.111E-02  |
| 28 | 1.186E+01 | 1.260E+01 | 1.274E+01 | 1.279E+01 | 1.280E+01 | 1.280E+01 | 1.280E+01 | 1.280E+01 | 0.0 | 2.088E-02  |
| 27 | 1.024E+01 | 1.091E+01 | 1.110E+01 | 1.115E+01 | 1.116E+01 | 1.116E+01 | 1.116E+01 | 1.115E+01 | 0.0 | 2.072E-02  |
| 26 | 9.020E+00 | 9.635E+00 | 9.761E+00 | 9.780E+00 | 9.782E+00 | 9.782E+00 | 9.781E+00 | 9.779E+00 | 0.0 | 2.057E-02  |
| 25 | 8.315E+00 | 8.900E+00 | 9.109E+00 | 9.159E+00 | 9.164E+00 | 9.163E+00 | 9.160E+00 | 9.158E+00 | 0.0 | 2.049E-02  |
| 24 | 7.828E+00 | 8.382E+00 | 8.559E+00 | 8.594E+00 | 8.596E+00 | 8.594E+00 | 8.590E+00 | 8.588E+00 | 0.0 | 2.041E-02  |
| 23 | 7.391E+00 | 7.908E+00 | 8.064E+00 | 8.092E+00 | 8.092E+00 | 8.089E+00 | 8.085E+00 | 8.082E+00 | 0.0 | 2.034E-02  |
| 22 | 7.000E+00 | 7.479E+00 | 7.627E+00 | 7.650E+00 | 7.648E+00 | 7.645E+00 | 7.640E+00 | 7.636E+00 | 0.0 | 2.026E-02  |
| 21 | 6.659E+00 | 7.102E+00 | 7.251E+00 | 7.267E+00 | 7.262E+00 | 7.257E+00 | 7.252E+00 | 7.249E+00 | 0.0 | 2.018E-02  |
| 20 | 6.232E+00 | 6.523E+00 | 6.570E+00 | 6.576E+00 | 6.571E+00 | 6.567E+00 | 6.562E+00 | 6.558E+00 | 0.0 | 2.003E-02  |
| 19 | 5.812E+00 | 6.058E+00 | 6.116E+00 | 6.115E+00 | 6.108E+00 | 6.103E+00 | 6.099E+00 | 6.096E+00 | 0.0 | 1.987E-02  |
| 18 | 5.447E+00 | 5.552E+00 | 5.561E+00 | 5.558E+00 | 5.552E+00 | 5.548E+00 | 5.544E+00 | 5.541E+00 | 0.0 | 1.964E-02  |
| 17 | 5.193E+00 | 5.256E+00 | 5.261E+00 | 5.254E+00 | 5.249E+00 | 5.245E+00 | 5.242E+00 | 5.240E+00 | 0.0 | 1.941E-02  |
| 16 | 5.049E+00 | 5.077E+00 | 5.075E+00 | 5.068E+00 | 5.063E+00 | 5.060E+00 | 5.057E+00 | 5.056E+00 | 0.0 | 1.917E-02  |
| 15 | 4.981E+00 | 4.962E+00 | 4.957E+00 | 4.954E+00 | 4.950E+00 | 4.948E+00 | 4.945E+00 | 4.943E+00 | 0.0 | 1.894E-02  |
| 14 | 4.972E+00 | 4.980E+00 | 4.972E+00 | 4.968E+00 | 4.964E+00 | 4.962E+00 | 4.959E+00 | 4.957E+00 | 0.0 | 1.879E-02  |
| 13 | 4.973E+00 | 5.051E+00 | 5.027E+00 | 5.007E+00 | 5.001E+00 | 4.998E+00 | 4.997E+00 | 4.996E+00 | 0.0 | 1.871E-02  |
| 12 | 4.968E+00 | 5.045E+00 | 5.028E+00 | 5.016E+00 | 5.011E+00 | 5.010E+00 | 5.009E+00 | 5.009E+00 | 0.0 | 1.863E-02  |
| 11 | 4.960E+00 | 4.999E+00 | 4.997E+00 | 4.994E+00 | 4.991E+00 | 4.989E+00 | 4.988E+00 | 4.987E+00 | 0.0 | 1.850E-02  |
| 10 | 4.958E+00 | 4.964E+00 | 4.960E+00 | 4.956E+00 | 4.953E+00 | 4.952E+00 | 4.950E+00 | 4.950E+00 | 0.0 | 1.828E-02  |
| 9  | 4.968E+00 | 4.951E+00 | 4.943E+00 | 4.939E+00 | 4.937E+00 | 4.936E+00 | 4.935E+00 | 4.935E+00 | 0.0 | 1.790E-02  |
| 8  | 4.984E+00 | 4.967E+00 | 4.961E+00 | 4.958E+00 | 4.957E+00 | 4.956E+00 | 4.955E+00 | 4.955E+00 | 0.0 | 1.725E-02  |
| 7  | 4.980E+00 | 4.970E+00 | 4.967E+00 | 4.965E+00 | 4.964E+00 | 4.964E+00 | 4.964E+00 | 4.963E+00 | 0.0 | 1.615E-02  |
| 6  | 4.933E+00 | 4.934E+00 | 4.933E+00 | 4.932E+00 | 4.932E+00 | 4.931E+00 | 4.931E+00 | 4.931E+00 | 0.0 | 1.429E-02  |
| 5  | 5.214E+00 | 5.219E+00 | 5.218E+00 | 5.217E+00 | 5.217E+00 | 5.217E+00 | 5.216E+00 | 5.216E+00 | 0.0 | 1.111E-02  |
| 4  | 6.189E+00 | 6.195E+00 | 6.194E+00 | 6.193E+00 | 6.193E+00 | 6.193E+00 | 6.192E+00 | 6.192E+00 | 0.0 | 7.937E-03  |
| 3  | 7.144E+00 | 7.147E+00 | 7.146E+00 | 7.146E+00 | 7.145E+00 | 7.145E+00 | 7.145E+00 | 7.145E+00 | 0.0 | 4.762E-03  |
| 2  | 7.647E+00 | 7.650E+00 | 7.653E+00 | 7.654E+00 | 7.654E+00 | 7.655E+00 | 7.655E+00 | 7.655E+00 | 0.0 | 1.587E-03  |
| 1  | 0.0       | 0.0       | 0.0       | 0.0       | 0.0       | 0.0       | 0.0       | 0.0       | 0.0 | -1.587E-03 |

X= 0.0091503 0.0093409 0.0094912 0.0095413 0.0095664 0.0095789 0.0095915 0.0095977 0.0096022



|    |            |            |           |           |           |           |           |           |           |           |
|----|------------|------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 45 | 4.4373E+02 | 4.1033E+02 | 3.831E+02 | 3.680E+02 | 3.503E+02 | 3.354E+02 | 3.526E+02 | 3.507E+02 | 3.500E+02 | 2.614E+02 |
| 44 | 4.410E+02  | 4.225E+02  | 3.927E+02 | 3.735E+02 | 3.635E+02 | 3.585E+02 | 3.534E+02 | 3.509E+02 | 3.500E+02 | 2.521E+02 |
| 43 | 4.420E+02  | 4.267E+02  | 3.966E+02 | 3.758E+02 | 3.648E+02 | 3.593E+02 | 3.538E+02 | 3.510E+02 | 3.500E+02 | 2.428E+02 |
| 42 | 4.417E+02  | 4.303E+02  | 4.007E+02 | 3.786E+02 | 3.665E+02 | 3.603E+02 | 3.542E+02 | 3.511E+02 | 3.500E+02 | 2.391E+02 |
| 41 | 4.413E+02  | 4.314E+02  | 4.021E+02 | 3.795E+02 | 3.670E+02 | 3.607E+02 | 3.543E+02 | 3.511E+02 | 3.500E+02 | 2.354E+02 |
| 40 | 4.409E+02  | 4.322E+02  | 4.031E+02 | 3.801E+02 | 3.674E+02 | 3.609E+02 | 3.544E+02 | 3.512E+02 | 3.500E+02 | 2.318E+02 |
| 39 | 4.406E+02  | 4.332E+02  | 4.049E+02 | 3.814E+02 | 3.682E+02 | 3.614E+02 | 3.546E+02 | 3.512E+02 | 3.500E+02 | 2.298E+02 |
| 38 | 4.403E+02  | 4.336E+02  | 4.058E+02 | 3.820E+02 | 3.686E+02 | 3.617E+02 | 3.547E+02 | 3.513E+02 | 3.500E+02 | 2.278E+02 |
| 37 | 4.400E+02  | 4.340E+02  | 4.067E+02 | 3.827E+02 | 3.690E+02 | 3.619E+02 | 3.548E+02 | 3.513E+02 | 3.500E+02 | 2.259E+02 |
| 36 | 4.396E+02  | 4.335E+02  | 4.060E+02 | 3.822E+02 | 3.687E+02 | 3.617E+02 | 3.548E+02 | 3.513E+02 | 3.500E+02 | 2.235E+02 |
| 35 | 4.391E+02  | 4.320E+02  | 4.042E+02 | 3.810E+02 | 3.680E+02 | 3.613E+02 | 3.546E+02 | 3.512E+02 | 3.500E+02 | 2.219E+02 |
| 34 | 4.387E+02  | 4.314E+02  | 4.036E+02 | 3.807E+02 | 3.678E+02 | 3.612E+02 | 3.545E+02 | 3.512E+02 | 3.500E+02 | 2.204E+02 |
| 33 | 4.385E+02  | 4.312E+02  | 4.032E+02 | 3.803E+02 | 3.675E+02 | 3.610E+02 | 3.545E+02 | 3.512E+02 | 3.500E+02 | 2.196E+02 |
| 32 | 4.381E+02  | 4.308E+02  | 4.030E+02 | 3.802E+02 | 3.675E+02 | 3.610E+02 | 3.544E+02 | 3.512E+02 | 3.500E+02 | 2.161E+02 |
| 31 | 4.375E+02  | 4.300E+02  | 4.021E+02 | 3.797E+02 | 3.672E+02 | 3.608E+02 | 3.544E+02 | 3.512E+02 | 3.500E+02 | 2.157E+02 |
| 30 | 4.369E+02  | 4.287E+02  | 4.008E+02 | 3.788E+02 | 3.666E+02 | 3.604E+02 | 3.542E+02 | 3.511E+02 | 3.500E+02 | 2.134E+02 |
| 29 | 4.363E+02  | 4.270E+02  | 3.989E+02 | 3.775E+02 | 3.659E+02 | 3.600E+02 | 3.540E+02 | 3.511E+02 | 3.500E+02 | 2.111E+02 |
| 28 | 4.359E+02  | 4.254E+02  | 3.971E+02 | 3.764E+02 | 3.652E+02 | 3.595E+02 | 3.539E+02 | 3.510E+02 | 3.500E+02 | 2.088E+02 |
| 27 | 4.355E+02  | 4.239E+02  | 3.954E+02 | 3.752E+02 | 3.645E+02 | 3.591E+02 | 3.537E+02 | 3.510E+02 | 3.500E+02 | 2.072E+02 |
| 26 | 4.353E+02  | 4.227E+02  | 3.941E+02 | 3.744E+02 | 3.640E+02 | 3.588E+02 | 3.536E+02 | 3.509E+02 | 3.500E+02 | 2.057E+02 |
| 25 | 4.351E+02  | 4.218E+02  | 3.931E+02 | 3.738E+02 | 3.637E+02 | 3.586E+02 | 3.535E+02 | 3.509E+02 | 3.500E+02 | 2.049E+02 |
| 24 | 4.349E+02  | 4.209E+02  | 3.921E+02 | 3.731E+02 | 3.633E+02 | 3.583E+02 | 3.534E+02 | 3.509E+02 | 3.500E+02 | 2.041E+02 |
| 23 | 4.347E+02  | 4.199E+02  | 3.910E+02 | 3.724E+02 | 3.629E+02 | 3.581E+02 | 3.533E+02 | 3.509E+02 | 3.500E+02 | 2.034E+02 |
| 22 | 4.345E+02  | 4.189E+02  | 3.899E+02 | 3.710E+02 | 3.625E+02 | 3.578E+02 | 3.532E+02 | 3.508E+02 | 3.500E+02 | 2.026E+02 |
| 21 | 4.340E+02  | 4.172E+02  | 3.883E+02 | 3.708E+02 | 3.619E+02 | 3.575E+02 | 3.530E+02 | 3.508E+02 | 3.500E+02 | 2.018E+02 |
| 20 | 4.334E+02  | 4.151E+02  | 3.863E+02 | 3.697E+02 | 3.613E+02 | 3.571E+02 | 3.529E+02 | 3.508E+02 | 3.500E+02 | 2.003E+02 |
| 19 | 4.324E+02  | 4.123E+02  | 3.840E+02 | 3.684E+02 | 3.605E+02 | 3.566E+02 | 3.527E+02 | 3.507E+02 | 3.500E+02 | 1.987E+02 |
| 18 | 4.31       |            |           |           |           |           |           |           |           |           |

X= 0.0091903 0.0093909 0.0094912 0.0095413 0.0095864 0.0095789 0.0095915 0.0095977 0.0096022



|    |           |           |           |           |           |           |           |           |           |           |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 45 | 1.273E-01 | 1.820E-01 | 2.357E-01 | 2.660E-01 | 2.814E-01 | 2.892E-01 | 2.970E-01 | 3.008E-01 | 3.022E-01 | 2.614E-02 |
| 44 | 1.177E-01 | 1.582E-01 | 2.173E-01 | 2.552E-01 | 2.752E-01 | 2.852E-01 | 2.953E-01 | 3.004E-01 | 3.022E-01 | 2.521E-02 |
| 43 | 1.170E-01 | 1.496E-01 | 2.097E-01 | 2.507E-01 | 2.726E-01 | 2.836E-01 | 2.946E-01 | 3.002E-01 | 3.022E-01 | 2.428E-02 |
| 42 | 1.176E-01 | 1.428E-01 | 2.018E-01 | 2.455E-01 | 2.694E-01 | 2.816E-01 | 2.938E-01 | 3.000E-01 | 3.022E-01 | 2.391E-02 |
| 41 | 1.183E-01 | 1.405E-01 | 1.991E-01 | 2.437E-01 | 2.683E-01 | 2.809E-01 | 2.936E-01 | 2.999E-01 | 3.022E-01 | 2.354E-02 |
| 40 | 1.191E-01 | 1.387E-01 | 1.970E-01 | 2.425E-01 | 2.677E-01 | 2.805E-01 | 2.934E-01 | 2.998E-01 | 3.022E-01 | 2.316E-02 |
| 39 | 1.199E-01 | 1.367E-01 | 1.935E-01 | 2.400E-01 | 2.661E-01 | 2.795E-01 | 2.930E-01 | 2.997E-01 | 3.022E-01 | 2.298E-02 |
| 38 | 1.205E-01 | 1.358E-01 | 1.919E-01 | 2.388E-01 | 2.654E-01 | 2.790E-01 | 2.928E-01 | 2.997E-01 | 3.022E-01 | 2.278E-02 |
| 37 | 1.212E-01 | 1.351E-01 | 1.901E-01 | 2.375E-01 | 2.646E-01 | 2.786E-01 | 2.926E-01 | 2.996E-01 | 3.022E-01 | 2.259E-02 |
| 36 | 1.220E-01 | 1.360E-01 | 1.916E-01 | 2.386E-01 | 2.652E-01 | 2.789E-01 | 2.927E-01 | 2.997E-01 | 3.022E-01 | 2.239E-02 |
| 35 | 1.231E-01 | 1.392E-01 | 1.951E-01 | 2.408E-01 | 2.666E-01 | 2.798E-01 | 2.931E-01 | 2.998E-01 | 3.022E-01 | 2.219E-02 |
| 34 | 1.239E-01 | 1.405E-01 | 1.963E-01 | 2.415E-01 | 2.669E-01 | 2.800E-01 | 2.932E-01 | 2.998E-01 | 3.022E-01 | 2.204E-02 |
| 33 | 1.243E-01 | 1.408E-01 | 1.970E-01 | 2.422E-01 | 2.675E-01 | 2.803E-01 | 2.933E-01 | 2.998E-01 | 3.022E-01 | 2.196E-02 |
| 32 | 1.252E-01 | 1.415E-01 | 1.975E-01 | 2.425E-01 | 2.676E-01 | 2.804E-01 | 2.933E-01 | 2.998E-01 | 3.022E-01 | 2.181E-02 |
| 31 | 1.264E-01 | 1.434E-01 | 1.992E-01 | 2.435E-01 | 2.682E-01 | 2.808E-01 | 2.935E-01 | 2.999E-01 | 3.022E-01 | 2.157E-02 |
| 30 | 1.276E-01 | 1.460E-01 | 2.020E-01 | 2.444E-01 | 2.693E-01 | 2.815E-01 | 2.938E-01 | 3.000E-01 | 3.022E-01 | 2.134E-02 |
| 29 | 1.288E-01 | 1.496E-01 | 2.058E-01 | 2.479E-01 | 2.708E-01 | 2.825E-01 | 2.942E-01 | 3.001E-01 | 3.022E-01 | 2.111E-02 |
| 28 | 1.298E-01 | 1.530E-01 | 2.094E-01 | 2.502E-01 | 2.722E-01 | 2.833E-01 | 2.946E-01 | 3.002E-01 | 3.022E-01 | 2.088E-02 |
| 27 | 1.305E-01 | 1.560E-01 | 2.127E-01 | 2.524E-01 | 2.735E-01 | 2.842E-01 | 2.949E-01 | 3.003E-01 | 3.022E-01 | 2.072E-02 |
| 26 | 1.310E-01 | 1.584E-01 | 2.153E-01 | 2.540E-01 | 2.745E-01 | 2.848E-01 | 2.952E-01 | 3.003E-01 | 3.022E-01 | 2.057E-02 |
| 25 | 1.313E-01 | 1.601E-01 | 2.172E-01 | 2.553E-01 | 2.752E-01 | 2.853E-01 | 2.953E-01 | 3.004E-01 | 3.022E-01 | 2.049E-02 |
| 24 | 1.317E-01 | 1.619E-01 | 2.192E-01 | 2.565E-01 | 2.760E-01 | 2.857E-01 | 2.955E-01 | 3.004E-01 | 3.022E-01 | 2.041E-02 |
| 23 | 1.321E-01 | 1.639E-01 | 2.213E-01 | 2.578E-01 | 2.767E-01 | 2.862E-01 | 2.957E-01 | 3.005E-01 | 3.022E-01 | 2.034E-02 |
| 22 | 1.327E-01 | 1.660E-01 | 2.234E-01 | 2.591E-01 | 2.775E-01 | 2.867E-01 | 2.959E-01 | 3.006E-01 | 3.022E-01 | 2.026E-02 |
| 21 | 1.334E-01 | 1.693E-01 | 2.266E-01 | 2.610E-01 | 2.786E-01 | 2.874E-01 | 2.962E-01 | 3.006E-01 | 3.022E-01 | 2.018E-02 |
| 20 | 1.351E-01 | 1.737E-01 | 2.305E-01 | 2.633E-01 | 2.799E-01 | 2.882E-01 | 2.965E-01 | 3.007E-01 | 3.022E-01 | 2.003E-02 |
| 19 | 1.371E-01 | 1.791E-01 | 2.349E-01 | 2.657E-01 | 2.813E-01 | 2.891E-01 | 2.969E-01 | 3.008E-01 | 3.022E-01 | 1.987E-02 |
| 18 | 1.397E-01 | 1.852E-01 | 2.395E-01 | 2.682E-01 | 2.827E-01 | 2.900E-01 | 2.973E-01 | 3.009E-01 | 3.022E-01 | 1.964E-02 |
| 17 | 1.424E-01 | 1.910E-01 | 2.433E-01 | 2.703E-01 | 2.839E-01 | 2.907E-01 | 2.976E-01 | 3.010E-01 | 3.022E-01 | 1.941E-02 |
| 16 | 1.451E-01 | 1.962E-01 | 2.465E-01 | 2.721E-01 | 2.849E-01 | 2.914E-01 | 2.978E-01 | 3.011E-01 | 3.022E-01 | 1.917E-02 |
| 15 | 1.473E-01 | 2.001E-01 | 2.488E-01 | 2.734E-01 | 2.857E-01 | 2.918E-01 | 2.980E-01 | 3.011E-01 | 3.022E-01 | 1.894E-02 |
| 14 | 1.484E-01 | 2.022E-01 | 2.500E-01 | 2.740E-01 | 2.861E-01 | 2.921E-01 | 2.981E-01 | 3.011E-01 | 3.022E-01 | 1.879E-02 |
| 13 | 1.490E-01 | 2.034E-01 | 2.506E-01 | 2.743E-01 | 2.862E-01 | 2.922E-01 | 2.982E-01 | 3.012E-01 | 3.022E-01 | 1.871E-02 |
| 12 | 1.499E-01 | 2.053E-01 | 2.515E-01 | 2.748E-01 | 2.865E-01 | 2.923E-01 | 2.982E-01 | 3.012E-01 | 3.022E-01 | 1.863E-02 |
| 11 | 1.518E-01 | 2.086E-01 | 2.532E-01 | 2.757E-01 | 2.870E-01 | 2.927E-01 | 2.984E-01 | 3.012E-01 | 3.022E-01 | 1.850E-02 |
| 10 | 1.555E-01 | 2.141E-01 | 2.560E-01 | 2.773E-01 | 2.880E-01 | 2.933E-01 | 2.986E-01 | 3.013E-01 | 3.022E-01 | 1.828E-02 |
| 9  | 1.619E-01 | 2.218E-01 | 2.600E-01 | 2.795E-01 | 2.892E-01 | 2.941E-01 | 2.989E-01 | 3.014E-01 | 3.022E-01 | 1.790E-02 |
| 8  | 1.722E-01 | 2.313E-01 | 2.650E-01 | 2.822E-01 | 2.908E-01 | 2.950E-01 | 2.993E-01 | 3.015E-01 | 3.022E-01 | 1.725E-02 |
| 7  | 1.868E-01 | 2.417E-01 | 2.706E-01 | 2.852E-01 | 2.925E-01 | 2.961E-01 | 2.998E-01 | 3.016E-01 | 3.022E-01 | 1.615E-02 |
| 6  | 2.071E-01 | 2.535E-01 | 2.769E-01 | 2.884E-01 | 2.944E-01 | 2.973E-01 | 3.003E-01 | 3.017E-01 | 3.022E-01 | 1.429E-02 |
| 5  | 2.303E-01 | 2.681E-01 | 2.835E-01 | 2.922E-01 | 2.965E-01 | 2.986E-01 | 3.008E-01 | 3.018E-01 | 3.022E-01 | 1.111E-02 |
| 4  | 2.543E-01 | 2.786E-01 | 2.900E-01 | 2.957E-01 | 2.985E-01 | 2.999E-01 | 3.013E-01 | 3.020E-01 | 3.022E-01 | 7.937E-03 |
| 3  | 2.578E-01 | 2.777E-01 | 2.892E-01 | 2.952E-01 | 2.982E-01 | 2.997E-01 | 3.012E-01 | 3.020E-01 | 3.022E-01 | 4.762E-03 |
| 2  | 2.062E-01 | 2.417E-01 | 2.672E-01 | 2.824E-01 | 2.906E-01 | 2.948E-01 | 2.992E-01 | 3.014E-01 | 3.022E-01 | 1.587E-03 |
| 1  | 1.000E-01 | 1.000E-01 | 1.000E-01 | 1.000E-01 | 1.000E-01 | 1.000E-01 | 1.000E-01 | 1.000E-01 | 3.022E-01 | 1.587E-03 |

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|    |           |           |           |           |           |           |           |           |           |           |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 50 | 2.251E-05 | 2.072E-05 | 1.897E-05 | 1.801E-05 | 1.753E-05 | 1.729E-05 | 1.705E-05 | 1.693E-05 | 1.689E-05 | 2.614E-02 |
| 49 | 2.282E-05 | 2.151E-05 | 1.958E-05 | 1.836E-05 | 1.773E-05 | 1.742E-05 | 1.710E-05 | 1.695E-05 | 1.689E-05 | 2.521E-02 |
| 48 | 2.263E-05 | 2.179E-05 | 1.983E-05 | 1.850E-05 | 1.781E-05 | 1.747E-05 | 1.712E-05 | 1.695E-05 | 1.689E-05 | 2.428E-02 |
| 47 | 2.281E-05 | 2.203E-05 | 2.009E-05 | 1.868E-05 | 1.792E-05 | 1.753E-05 | 1.715E-05 | 1.696E-05 | 1.689E-05 | 2.391E-02 |
| 46 | 2.279E-05 | 2.210E-05 | 2.018E-05 | 1.873E-05 | 1.795E-05 | 1.755E-05 | 1.716E-05 | 1.696E-05 | 1.689E-05 | 2.354E-02 |
| 45 | 2.276E-05 | 2.216E-05 | 2.025E-05 | 1.877E-05 | 1.797E-05 | 1.757E-05 | 1.716E-05 | 1.696E-05 | 1.689E-05 | 2.318E-02 |
| 44 | 2.274E-05 | 2.222E-05 | 2.030E-05 | 1.885E-05 | 1.802E-05 | 1.760E-05 | 1.718E-05 | 1.697E-05 | 1.689E-05 | 2.298E-02 |
| 43 | 2.272E-05 | 2.225E-05 | 2.042E-05 | 1.889E-05 | 1.804E-05 | 1.761E-05 | 1.718E-05 | 1.697E-05 | 1.689E-05 | 2.275E-02 |
| 42 | 2.270E-05 | 2.227E-05 | 2.048E-05 | 1.894E-05 | 1.807E-05 | 1.763E-05 | 1.719E-05 | 1.697E-05 | 1.689E-05 | 2.259E-02 |
| 41 | 2.267E-05 | 2.224E-05 | 2.043E-05 | 1.890E-05 | 1.805E-05 | 1.762E-05 | 1.719E-05 | 1.697E-05 | 1.689E-05 | 2.239E-02 |
| 40 | 2.264E-05 | 2.214E-05 | 2.031E-05 | 1.883E-05 | 1.801E-05 | 1.759E-05 | 1.717E-05 | 1.697E-05 | 1.689E-05 | 2.219E-02 |
| 39 | 2.261E-05 | 2.210E-05 | 2.028E-05 | 1.881E-05 | 1.800E-05 | 1.758E-05 | 1.717E-05 | 1.697E-05 | 1.689E-05 | 2.204E-02 |
| 38 | 2.260E-05 | 2.209E-05 | 2.025E-05 | 1.878E-05 | 1.798E-05 | 1.757E-05 | 1.717E-05 | 1.696E-05 | 1.689E-05 | 2.196E-02 |
| 37 | 2.257E-05 | 2.206E-05 | 2.023E-05 | 1.878E-05 | 1.798E-05 | 1.757E-05 | 1.717E-05 | 1.696E-05 | 1.689E-05 | 2.181E-02 |
| 36 | 2.253E-05 | 2.201E-05 | 2.018E-05 | 1.875E-05 | 1.796E-05 | 1.756E-05 | 1.716E-05 | 1.696E-05 | 1.689E-05 | 2.157E-02 |
| 35 | 2.249E-05 | 2.192E-05 | 2.009E-05 | 1.869E-05 | 1.792E-05 | 1.754E-05 | 1.715E-05 | 1.696E-05 | 1.689E-05 | 2.134E-02 |
| 34 | 2.245E-05 | 2.181E-05 | 1.997E-05 | 1.860E-05 | 1.787E-05 | 1.751E-05 | 1.714E-05 | 1.696E-05 | 1.689E-05 | 2.111E-02 |
| 33 | 2.242E-05 | 2.170E-05 | 1.985E-05 | 1.853E-05 | 1.783E-05 | 1.748E-05 | 1.713E-05 | 1.695E-05 | 1.689E-05 | 2.088E-02 |
| 32 | 2.240E-05 | 2.160E-05 | 1.974E-05 | 1.846E-05 | 1.779E-05 | 1.745E-05 | 1.712E-05 | 1.695E-05 | 1.689E-05 | 2.072E-02 |
| 31 | 2.238E-05 | 2.152E-05 | 1.960E-05 | 1.841E-05 | 1.776E-05 | 1.743E-05 | 1.711E-05 | 1.695E-05 | 1.689E-05 | 2.057E-02 |
| 30 | 2.237E-05 | 2.146E-05 | 1.950E-05 | 1.837E-05 | 1.774E-05 | 1.742E-05 | 1.710E-05 | 1.695E-05 | 1.689E-05 | 2.049E-02 |
| 29 | 2.236E-05 | 2.140E-05 | 1.953E-05 | 1.833E-05 | 1.771E-05 | 1.741E-05 | 1.710E-05 | 1.695E-05 | 1.689E-05 | 2.041E-02 |
| 28 | 2.234E-05 | 2.134E-05 | 1.946E-05 | 1.829E-05 | 1.769E-05 | 1.739E-05 | 1.709E-05 | 1.694E-05 | 1.689E-05 | 2.034E-02 |
| 27 | 2.233E-05 | 2.127E-05 | 1.939E-05 | 1.825E-05 | 1.766E-05 | 1.737E-05 | 1.709E-05 | 1.694E-05 | 1.689E-05 | 2.026E-02 |
| 26 | 2.230E-05 | 2.116E-05 | 1.929E-05 | 1.819E-05 | 1.763E-05 | 1.735E-05 | 1.708E-05 | 1.694E-05 | 1.689E-05 | 2.018E-02 |
| 25 | 2.225E-05 | 2.102E-05 | 1.916E-05 | 1.811E-05 | 1.759E-05 | 1.733E-05 | 1.707E-05 | 1.694E-05 | 1.689E-05 | 2.003E-02 |
| 24 | 2.219E-05 | 2.084E-05 | 1.902E-05 | 1.803E-05 | 1.754E-05 | 1.730E-05 | 1.706E-05 | 1.693E-05 | 1.689E-05 | 1.987E-02 |
| 23 | 2.210E-05 | 2.064E-05 | 1.887E-05 | 1.795E-05 | 1.750E-05 | 1.727E-05 | 1.704E-05 | 1.693E-05 | 1.689E-05 | 1.964E-02 |
| 22 | 2.202E-05 | 2.045E-05 | 1.875E-05 | 1.789E-05 | 1.746E-05 | 1.725E-05 | 1.704E-05 | 1.693E-05 | 1.689E-05 | 1.941E-02 |
| 21 | 2.193E-05 | 2.027E-05 | 1.864E-05 | 1.783E-05 | 1.743E-05 | 1.723E-05 | 1.703E-05 | 1.693E-05 | 1.689E-05 | 1.917E-02 |
| 20 | 2.185E-05 | 2.014E-05 | 1.857E-05 | 1.779E-05 | 1.740E-05 | 1.721E-05 | 1.702E-05 | 1.693E-05 | 1.689E-05 | 1.894E-02 |
| 19 | 2.181E-05 | 2.008E-05 | 1.853E-05 | 1.777E-05 | 1.739E-05 | 1.720E-05 | 1.702E-05 | 1.692E-05 | 1.689E-05 | 1.879E-02 |
| 18 | 2.179E-05 | 2.004E-05 | 1.851E-05 | 1.776E-05 | 1.739E-05 | 1.720E-05 | 1.702E-05 | 1.692E-05 | 1.689E-05 | 1.871E-02 |
| 17 | 2.176E-05 | 1.998E-05 | 1.848E-05 | 1.775E-05 | 1.738E-05 | 1.720E-05 | 1.702E-05 | 1.692E-05 | 1.689E-05 | 1.863E-02 |
| 16 | 2.169E-05 | 1.987E-05 | 1.843E-05 | 1.772E-05 | 1.736E-05 | 1.719E-05 | 1.701E-05 | 1.692E-05 | 1.689E-05 | 1.850E-02 |
| 15 | 2.158E-05 | 1.970E-05 | 1.834E-05 | 1.767E-05 | 1.734E-05 | 1.717E-05 | 1.700E-05 | 1.692E-05 | 1.689E-05 | 1.838E-02 |
| 14 | 2.138E-05 | 1.946E-05 | 1.822E-05 | 1.761E-05 | 1.730E-05 | 1.715E-05 | 1.699E-05 | 1.692E-05 | 1.689E-05 | 1.790E-02 |
| 13 | 2.106E-05 | 1.915E-05 | 1.807E-05 | 1.752E-05 | 1.725E-05 | 1.712E-05 | 1.698E-05 | 1.692E-05 | 1.689E-05 | 1.725E-02 |
| 12 | 2.061E-05 | 1.882E-05 | 1.789E-05 | 1.743E-05 | 1.720E-05 | 1.708E-05 | 1.697E-05 | 1.691E-05 | 1.689E-05 | 1.615E-02 |
| 11 | 1.999E-05 | 1.845E-05 | 1.770E-05 | 1.732E-05 | 1.714E-05 | 1.705E-05 | 1.695E-05 | 1.691E-05 | 1.689E-05 | 1.429E-02 |
| 10 | 1.922E-05 | 1.805E-05 | 1.749E-05 | 1.721E-05 | 1.707E-05 | 1.701E-05 | 1.694E-05 | 1.690E-05 | 1.689E-05 | 1.111E-02 |
| 9  | 1.847E-05 | 1.766E-05 | 1.729E-05 | 1.710E-05 | 1.701E-05 | 1.697E-05 | 1.692E-05 | 1.690E-05 | 1.689E-05 | 7.937E-03 |
| 8  | 1.813E-05 | 1.752E-05 | 1.721E-05 | 1.707E-05 | 1.699E-05 | 1.696E-05 | 1.692E-05 | 1.690E-05 | 1.689E-05 | 4.762E-03 |
| 7  | 1.943E-05 | 1.802E-05 | 1.711E-05 | 1.666E-05 | 1.649E-05 | 1.647E-05 | 1.644E-05 | 1.679E-05 | 1.689E-05 | 1.587E-03 |
| 6  | 2.338E-05 | 2.338E-05 | 2.338E-05 | 2.338E-05 | 2.338E-05 | 2.338E-05 | 2.338E-05 | 2.338E-05 | 2.338E-05 | 1.587E-03 |

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|    |           |           |           |           |           |           |           |           |           |           |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 45 | 7.495E-01 | 7.748E-01 | 8.041E-01 | 8.260E-01 | 8.369E-01 | 8.425E-01 | 8.483E-01 | 8.512E-01 | 8.522E-01 | 2.614E-02 |
| 44 | 7.459E-01 | 7.625E-01 | 7.943E-01 | 8.183E-01 | 8.322E-01 | 8.395E-01 | 8.470E-01 | 8.509E-01 | 8.522E-01 | 2.521E-02 |
| 43 | 7.438E-01 | 7.585E-01 | 7.898E-01 | 8.153E-01 | 8.303E-01 | 8.383E-01 | 8.466E-01 | 8.508E-01 | 8.522E-01 | 2.428E-02 |
| 42 | 7.462E-01 | 7.552E-01 | 7.850E-01 | 8.117E-01 | 8.260E-01 | 8.368E-01 | 8.460E-01 | 8.507E-01 | 8.522E-01 | 2.391E-02 |
| 41 | 7.465E-01 | 7.543E-01 | 7.839E-01 | 8.106E-01 | 8.273E-01 | 8.363E-01 | 8.458E-01 | 8.507E-01 | 8.523E-01 | 2.354E-02 |
| 40 | 7.468E-01 | 7.535E-01 | 7.824E-01 | 8.098E-01 | 8.268E-01 | 8.361E-01 | 8.457E-01 | 8.507E-01 | 8.523E-01 | 2.318E-02 |
| 39 | 7.471E-01 | 7.527E-01 | 7.804E-01 | 8.081E-01 | 8.257E-01 | 8.353E-01 | 8.454E-01 | 8.506E-01 | 8.523E-01 | 2.298E-02 |
| 38 | 7.473E-01 | 7.524E-01 | 7.795E-01 | 8.073E-01 | 8.251E-01 | 8.349E-01 | 8.452E-01 | 8.505E-01 | 8.523E-01 | 2.278E-02 |
| 37 | 7.476E-01 | 7.520E-01 | 7.784E-01 | 8.063E-01 | 8.245E-01 | 8.345E-01 | 8.450E-01 | 8.505E-01 | 8.523E-01 | 2.259E-02 |
| 36 | 7.479E-01 | 7.524E-01 | 7.791E-01 | 8.070E-01 | 8.249E-01 | 8.348E-01 | 8.451E-01 | 8.505E-01 | 8.523E-01 | 2.239E-02 |
| 35 | 7.483E-01 | 7.538E-01 | 7.811E-01 | 8.085E-01 | 8.259E-01 | 8.354E-01 | 8.454E-01 | 8.506E-01 | 8.523E-01 | 2.219E-02 |
| 34 | 7.487E-01 | 7.543E-01 | 7.817E-01 | 8.089E-01 | 8.261E-01 | 8.355E-01 | 8.455E-01 | 8.506E-01 | 8.523E-01 | 2.204E-02 |
| 33 | 7.488E-01 | 7.544E-01 | 7.822E-01 | 8.094E-01 | 8.265E-01 | 8.358E-01 | 8.456E-01 | 8.506E-01 | 8.523E-01 | 2.190E-02 |
| 32 | 7.491E-01 | 7.547E-01 | 7.825E-01 | 8.095E-01 | 8.266E-01 | 8.358E-01 | 8.456E-01 | 8.506E-01 | 8.523E-01 | 2.181E-02 |
| 31 | 7.496E-01 | 7.555E-01 | 7.833E-01 | 8.101E-01 | 8.269E-01 | 8.361E-01 | 8.457E-01 | 8.506E-01 | 8.523E-01 | 2.157E-02 |
| 30 | 7.501E-01 | 7.565E-01 | 7.848E-01 | 8.113E-01 | 8.277E-01 | 8.366E-01 | 8.458E-01 | 8.506E-01 | 8.523E-01 | 2.134E-02 |
| 29 | 7.506E-01 | 7.580E-01 | 7.869E-01 | 8.129E-01 | 8.287E-01 | 8.372E-01 | 8.461E-01 | 8.507E-01 | 8.522E-01 | 2.111E-02 |
| 28 | 7.509E-01 | 7.595E-01 | 7.889E-01 | 8.144E-01 | 8.297E-01 | 8.378E-01 | 8.464E-01 | 8.507E-01 | 8.522E-01 | 2.088E-02 |
| 27 | 7.512E-01 | 7.609E-01 | 7.908E-01 | 8.159E-01 | 8.306E-01 | 8.385E-01 | 8.466E-01 | 8.508E-01 | 8.522E-01 | 2.072E-02 |
| 26 | 7.514E-01 | 7.620E-01 | 7.923E-01 | 8.170E-01 | 8.313E-01 | 8.389E-01 | 8.468E-01 | 8.508E-01 | 8.522E-01 | 2.057E-02 |
| 25 | 7.515E-01 | 7.628E-01 | 7.935E-01 | 8.178E-01 | 8.318E-01 | 8.392E-01 | 8.469E-01 | 8.509E-01 | 8.522E-01 | 2.045E-02 |
| 24 | 7.517E-01 | 7.637E-01 | 7.947E-01 | 8.187E-01 | 8.324E-01 | 8.396E-01 | 8.471E-01 | 8.509E-01 | 8.522E-01 | 2.041E-02 |
| 23 | 7.519E-01 | 7.647E-01 | 7.960E-01 | 8.196E-01 | 8.329E-01 | 8.400E-01 | 8.472E-01 | 8.509E-01 | 8.522E-01 | 2.034E-02 |
| 22 | 7.521E-01 | 7.657E-01 | 7.973E-01 | 8.205E-01 | 8.335E-01 | 8.403E-01 | 8.474E-01 | 8.510E-01 | 8.522E-01 | 2.026E-02 |
| 21 | 7.524E-01 | 7.673E-01 | 7.993E-01 | 8.218E-01 | 8.343E-01 | 8.408E-01 | 8.476E-01 | 8.510E-01 | 8.522E-01 | 2.018E-02 |
| 20 | 7.529E-01 | 7.695E-01 | 8.018E-01 | 8.234E-01 | 8.353E-01 | 8.415E-01 | 8.478E-01 | 8.511E-01 | 8.522E-01 | 2.003E-02 |
| 19 | 7.537E-01 | 7.722E-01 | 8.047E-01 | 8.252E-01 | 8.364E-01 | 8.422E-01 | 8.481E-01 | 8.512E-01 | 8.522E-01 | 1.987E-02 |
| 18 | 7.547E-01 | 7.755E-01 | 8.077E-01 | 8.271E-01 | 8.375E-01 | 8.429E-01 | 8.484E-01 | 8.512E-01 | 8.522E-01 | 1.964E-02 |
| 17 | 7.559E-01 | 7.787E-01 | 8.103E-01 | 8.286E-01 | 8.384E-01 | 8.435E-01 | 8.487E-01 | 8.513E-01 | 8.522E-01 | 1.941E-02 |
| 16 | 7.570E-01 | 7.817E-01 | 8.124E-01 | 8.299E-01 | 8.392E-01 | 8.440E-01 | 8.489E-01 | 8.514E-01 | 8.522E-01 | 1.917E-02 |
| 15 | 7.581E-01 | 7.840E-01 | 8.140E-01 | 8.308E-01 | 8.397E-01 | 8.443E-01 | 8.490E-01 | 8.514E-01 | 8.522E-01 | 1.894E-02 |
| 14 | 7.588E-01 | 7.852E-01 | 8.149E-01 | 8.313E-01 | 8.400E-01 | 8.445E-01 | 8.491E-01 | 8.514E-01 | 8.522E-01 | 1.879E-02 |
| 13 | 7.592E-01 | 7.859E-01 | 8.152E-01 | 8.315E-01 | 8.401E-01 | 8.446E-01 | 8.491E-01 | 8.514E-01 | 8.522E-01 | 1.871E-02 |
| 12 | 7.598E-01 | 7.869E-01 | 8.157E-01 | 8.317E-01 | 8.402E-01 | 8.446E-01 | 8.491E-01 | 8.514E-01 | 8.522E-01 | 1.863E-02 |
| 11 | 7.607E-01 | 7.886E-01 | 8.167E-01 | 8.323E-01 | 8.406E-01 | 8.449E-01 | 8.492E-01 | 8.515E-01 | 8.522E-01 | 1.850E-02 |
| 10 | 7.622E-01 | 7.915E-01 | 8.183E-01 | 8.333E-01 | 8.412E-01 | 8.452E-01 | 8.494E-01 | 8.515E-01 | 8.522E-01 | 1.828E-02 |
| 9  | 7.647E-01 | 7.958E-01 | 8.208E-01 | 8.347E-01 | 8.420E-01 | 8.458E-01 | 8.496E-01 | 8.516E-01 | 8.522E-01 | 1.790E-02 |
| 8  | 7.684E-01 | 8.015E-01 | 8.241E-01 | 8.366E-01 | 8.431E-01 | 8.465E-01 | 8.499E-01 | 8.516E-01 | 8.522E-01 | 1.725E-02 |
| 7  | 7.734E-01 | 8.080E-01 | 8.279E-01 | 8.388E-01 | 8.444E-01 | 8.473E-01 | 8.503E-01 | 8.517E-01 | 8.522E-01 | 1.615E-02 |
| 6  | 7.856E-01 | 8.155E-01 | 8.324E-01 | 8.413E-01 | 8.459E-01 | 8.483E-01 | 8.506E-01 | 8.518E-01 | 8.522E-01 | 1.429E-02 |
| 5  | 7.984E-01 | 8.240E-01 | 8.372E-01 | 8.440E-01 | 8.475E-01 | 8.493E-01 | 8.511E-01 | 8.519E-01 | 8.522E-01 | 1.111E-02 |
| 4  | 8.133E-01 | 8.327E-01 | 8.420E-01 | 8.467E-01 | 8.491E-01 | 8.503E-01 | 8.515E-01 | 8.521E-01 | 8.522E-01 | 7.937E-03 |
| 3  | 8.281E-01 | 8.417E-01 | 8.470E-01 | 8.493E-01 | 8.505E-01 | 8.511E-01 | 8.518E-01 | 8.522E-01 | 8.522E-01 | 4.762E-03 |
| 2  | 8.137E-01 | 8.517E-01 | 8.773E-01 | 8.888E-01 | 8.904E-01 | 8.865E-01 | 8.710E-01 | 8.594E-01 | 8.522E-01 | 1.587E-03 |
| 1  | 7.396E-01 | 7.396E-01 | 7.396E-01 | 7.396E-01 | 7.396E-01 | 7.396E-01 | 7.396E-01 | 7.396E-01 | 7.396E-01 | 1.587E-03 |

X= 0.0091503 0.0093909 0.0094912 0.0095413 0.0095664 0.0095789 0.0095915 0.0095977 0.0096022



|    |           |           |           |           |           |           |           |           |           |           |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 45 | 1.135E+03 | 1.185E+03 | 1.233E+03 | 1.260E+03 | 1.273E+03 | 1.280E+03 | 1.286E+03 | 1.290E+03 | 1.291E+03 | 2.614E-02 |
| 44 | 1.126E+03 | 1.164E+03 | 1.217E+03 | 1.250E+03 | 1.268E+03 | 1.276E+03 | 1.285E+03 | 1.289E+03 | 1.291E+03 | 2.521E-02 |
| 43 | 1.126E+03 | 1.156E+03 | 1.210E+03 | 1.246E+03 | 1.265E+03 | 1.275E+03 | 1.284E+03 | 1.289E+03 | 1.291E+03 | 2.428E-02 |
| 42 | 1.126E+03 | 1.150E+03 | 1.203E+03 | 1.242E+03 | 1.263E+03 | 1.273E+03 | 1.284E+03 | 1.289E+03 | 1.291E+03 | 2.391E-02 |
| 41 | 1.127E+03 | 1.147E+03 | 1.201E+03 | 1.240E+03 | 1.262E+03 | 1.273E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.354E-02 |
| 40 | 1.128E+03 | 1.146E+03 | 1.199E+03 | 1.239E+03 | 1.261E+03 | 1.272E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.318E-02 |
| 39 | 1.128E+03 | 1.144E+03 | 1.196E+03 | 1.237E+03 | 1.260E+03 | 1.271E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.298E-02 |
| 38 | 1.129E+03 | 1.143E+03 | 1.194E+03 | 1.236E+03 | 1.259E+03 | 1.271E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.278E-02 |
| 37 | 1.130E+03 | 1.142E+03 | 1.192E+03 | 1.235E+03 | 1.258E+03 | 1.271E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.259E-02 |
| 36 | 1.130E+03 | 1.143E+03 | 1.194E+03 | 1.236E+03 | 1.259E+03 | 1.271E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.239E-02 |
| 35 | 1.131E+03 | 1.146E+03 | 1.197E+03 | 1.238E+03 | 1.260E+03 | 1.272E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.219E-02 |
| 34 | 1.132E+03 | 1.147E+03 | 1.198E+03 | 1.238E+03 | 1.260E+03 | 1.272E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.204E-02 |
| 33 | 1.132E+03 | 1.148E+03 | 1.199E+03 | 1.239E+03 | 1.261E+03 | 1.272E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.196E-02 |
| 32 | 1.133E+03 | 1.148E+03 | 1.199E+03 | 1.239E+03 | 1.261E+03 | 1.272E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.181E-02 |
| 31 | 1.134E+03 | 1.150E+03 | 1.201E+03 | 1.240E+03 | 1.262E+03 | 1.272E+03 | 1.283E+03 | 1.289E+03 | 1.291E+03 | 2.157E-02 |
| 30 | 1.135E+03 | 1.152E+03 | 1.203E+03 | 1.242E+03 | 1.263E+03 | 1.273E+03 | 1.284E+03 | 1.289E+03 | 1.291E+03 | 2.134E-02 |
| 29 | 1.137E+03 | 1.156E+03 | 1.207E+03 | 1.244E+03 | 1.264E+03 | 1.274E+03 | 1.284E+03 | 1.289E+03 | 1.291E+03 | 2.111E-02 |
| 28 | 1.137E+03 | 1.159E+03 | 1.210E+03 | 1.246E+03 | 1.265E+03 | 1.275E+03 | 1.284E+03 | 1.289E+03 | 1.291E+03 | 2.088E-02 |
| 27 | 1.138E+03 | 1.162E+03 | 1.213E+03 | 1.248E+03 | 1.266E+03 | 1.275E+03 | 1.285E+03 | 1.289E+03 | 1.291E+03 | 2.072E-02 |
| 26 | 1.139E+03 | 1.164E+03 | 1.215E+03 | 1.249E+03 | 1.267E+03 | 1.276E+03 | 1.285E+03 | 1.289E+03 | 1.291E+03 | 2.057E-02 |
| 25 | 1.139E+03 | 1.165E+03 | 1.217E+03 | 1.250E+03 | 1.268E+03 | 1.276E+03 | 1.285E+03 | 1.289E+03 | 1.291E+03 | 2.049E-02 |
| 24 | 1.139E+03 | 1.167E+03 | 1.218E+03 | 1.251E+03 | 1.268E+03 | 1.277E+03 | 1.285E+03 | 1.289E+03 | 1.291E+03 | 2.041E-02 |
| 23 | 1.140E+03 | 1.169E+03 | 1.220E+03 | 1.253E+03 | 1.269E+03 | 1.277E+03 | 1.285E+03 | 1.289E+03 | 1.291E+03 | 2.034E-02 |
| 22 | 1.140E+03 | 1.171E+03 | 1.222E+03 | 1.254E+03 | 1.270E+03 | 1.278E+03 | 1.285E+03 | 1.289E+03 | 1.291E+03 | 2.026E-02 |
| 21 | 1.141E+03 | 1.174E+03 | 1.225E+03 | 1.256E+03 | 1.271E+03 | 1.278E+03 | 1.286E+03 | 1.290E+03 | 1.291E+03 | 2.018E-02 |
| 20 | 1.142E+03 | 1.178E+03 | 1.229E+03 | 1.257E+03 | 1.272E+03 | 1.279E+03 | 1.286E+03 | 1.290E+03 | 1.291E+03 | 2.003E-02 |
| 19 | 1.144E+03 | 1.183E+03 | 1.232E+03 | 1.259E+03 | 1.273E+03 | 1.280E+03 | 1.286E+03 | 1.290E+03 | 1.291E+03 | 1.987E-02 |
| 18 | 1.147E+03 | 1.188E+03 | 1.236E+03 | 1.262E+03 | 1.274E+03 | 1.280E+03 | 1.287E+03 | 1.290E+03 | 1.291E+03 | 1.964E-02 |
| 17 | 1.149E+03 | 1.193E+03 | 1.240E+03 | 1.263E+03 | 1.275E+03 | 1.281E+03 | 1.287E+03 | 1.290E+03 | 1.291E+03 | 1.941E-02 |
| 16 | 1.152E+03 | 1.198E+03 | 1.243E+03 | 1.265E+03 | 1.276E+03 | 1.282E+03 | 1.287E+03 | 1.290E+03 | 1.291E+03 | 1.917E-02 |
| 15 | 1.154E+03 | 1.202E+03 | 1.245E+03 | 1.266E+03 | 1.277E+03 | 1.282E+03 | 1.287E+03 | 1.290E+03 | 1.291E+03 | 1.894E-02 |
| 14 | 1.155E+03 | 1.203E+03 | 1.246E+03 | 1.267E+03 | 1.277E+03 | 1.282E+03 | 1.287E+03 | 1.290E+03 | 1.291E+03 | 1.879E-02 |
| 13 | 1.155E+03 | 1.204E+03 | 1.246E+03 | 1.267E+03 | 1.277E+03 | 1.282E+03 | 1.287E+03 | 1.290E+03 | 1.291E+03 | 1.871E-02 |
| 12 | 1.156E+03 | 1.206E+03 | 1.247E+03 | 1.267E+03 | 1.277E+03 | 1.282E+03 | 1.287E+03 | 1.290E+03 | 1.291E+03 | 1.863E-02 |
| 11 | 1.158E+03 | 1.209E+03 | 1.248E+03 | 1.268E+03 | 1.278E+03 | 1.283E+03 | 1.288E+03 | 1.290E+03 | 1.291E+03 | 1.850E-02 |
| 10 | 1.161E+03 | 1.214E+03 | 1.251E+03 | 1.269E+03 | 1.279E+03 | 1.283E+03 | 1.288E+03 | 1.290E+03 | 1.291E+03 | 1.828E-02 |
| 9  | 1.167E+03 | 1.221E+03 | 1.254E+03 | 1.271E+03 | 1.280E+03 | 1.284E+03 | 1.288E+03 | 1.290E+03 | 1.291E+03 | 1.790E-02 |
| 8  | 1.176E+03 | 1.229E+03 | 1.259E+03 | 1.274E+03 | 1.281E+03 | 1.285E+03 | 1.288E+03 | 1.290E+03 | 1.291E+03 | 1.725E-02 |
| 7  | 1.190E+03 | 1.238E+03 | 1.264E+03 | 1.276E+03 | 1.283E+03 | 1.286E+03 | 1.289E+03 | 1.290E+03 | 1.291E+03 | 1.615E-02 |
| 6  | 1.208E+03 | 1.249E+03 | 1.269E+03 | 1.279E+03 | 1.284E+03 | 1.287E+03 | 1.289E+03 | 1.290E+03 | 1.291E+03 | 1.429E-02 |
| 5  | 1.228E+03 | 1.260E+03 | 1.275E+03 | 1.282E+03 | 1.286E+03 | 1.288E+03 | 1.290E+03 | 1.291E+03 | 1.291E+03 | 1.111E-02 |
| 4  | 1.250E+03 | 1.271E+03 | 1.280E+03 | 1.285E+03 | 1.288E+03 | 1.289E+03 | 1.290E+03 | 1.291E+03 | 1.291E+03 | 7.937E-03 |
| 3  | 1.252E+03 | 1.269E+03 | 1.280E+03 | 1.285E+03 | 1.287E+03 | 1.289E+03 | 1.290E+03 | 1.291E+03 | 1.291E+03 | 4.762E-03 |
| 2  | 1.206E+03 | 1.237E+03 | 1.258E+03 | 1.272E+03 | 1.279E+03 | 1.283E+03 | 1.287E+03 | 1.290E+03 | 1.291E+03 | 1.587E-03 |
| 1  | 1.110E+03 | 1.110E+03 | 1.110E+03 | 1.110E+03 | 1.110E+03 | 1.110E+03 | 1.110E+03 | 1.110E+03 | 1.110E+03 | 1.587E-03 |

X= 0.0091903 0.0093909 0.0094912 0.0095413 0.0095664 0.0095789 0.0095915 0.0095977 0.0096022



|    |           |           |           |           |           |           |           |           |           |           |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 45 | 4.762E-05 | 4.197E-05 | 3.685E-05 | 3.426E-05 | 3.301E-05 | 3.241E-05 | 3.182E-05 | 3.153E-05 | 3.143E-05 | 2.614E-02 |
| 44 | 4.862E-05 | 4.444E-05 | 3.858E-05 | 3.519E-05 | 3.353E-05 | 3.273E-05 | 3.194E-05 | 3.156E-05 | 3.143E-05 | 2.521E-02 |
| 43 | 4.867E-05 | 4.533E-05 | 3.930E-05 | 3.557E-05 | 3.374E-05 | 3.286E-05 | 3.199E-05 | 3.157E-05 | 3.143E-05 | 2.428E-02 |
| 42 | 4.859E-05 | 4.609E-05 | 4.008E-05 | 3.605E-05 | 3.401E-05 | 3.302E-05 | 3.206E-05 | 3.159E-05 | 3.143E-05 | 2.391E-02 |
| 41 | 4.852E-05 | 4.632E-05 | 4.034E-05 | 3.620E-05 | 3.410E-05 | 3.307E-05 | 3.208E-05 | 3.159E-05 | 3.143E-05 | 2.354E-02 |
| 40 | 4.843E-05 | 4.651E-05 | 4.054E-05 | 3.631E-05 | 3.415E-05 | 3.310E-05 | 3.209E-05 | 3.159E-05 | 3.143E-05 | 2.318E-02 |
| 39 | 4.834E-05 | 4.671E-05 | 4.085E-05 | 3.654E-05 | 3.429E-05 | 3.319E-05 | 3.212E-05 | 3.160E-05 | 3.143E-05 | 2.298E-02 |
| 38 | 4.828E-05 | 4.680E-05 | 4.106E-05 | 3.665E-05 | 3.435E-05 | 3.323E-05 | 3.214E-05 | 3.161E-05 | 3.143E-05 | 2.278E-02 |
| 37 | 4.820E-05 | 4.689E-05 | 4.125E-05 | 3.677E-05 | 3.442E-05 | 3.327E-05 | 3.215E-05 | 3.161E-05 | 3.143E-05 | 2.256E-02 |
| 36 | 4.812E-05 | 4.679E-05 | 4.110E-05 | 3.668E-05 | 3.437E-05 | 3.324E-05 | 3.214E-05 | 3.161E-05 | 3.143E-05 | 2.239E-02 |
| 35 | 4.801E-05 | 4.645E-05 | 4.075E-05 | 3.648E-05 | 3.425E-05 | 3.317E-05 | 3.211E-05 | 3.160E-05 | 3.143E-05 | 2.219E-02 |
| 34 | 4.792E-05 | 4.632E-05 | 4.064E-05 | 3.642E-05 | 3.423E-05 | 3.315E-05 | 3.211E-05 | 3.160E-05 | 3.143E-05 | 2.204E-02 |
| 33 | 4.787E-05 | 4.628E-05 | 4.057E-05 | 3.635E-05 | 3.418E-05 | 3.312E-05 | 3.210E-05 | 3.160E-05 | 3.143E-05 | 2.196E-02 |
| 32 | 4.778E-05 | 4.621E-05 | 4.051E-05 | 3.633E-05 | 3.417E-05 | 3.312E-05 | 3.210E-05 | 3.160E-05 | 3.143E-05 | 2.181E-02 |
| 31 | 4.766E-05 | 4.602E-05 | 4.036E-05 | 3.624E-05 | 3.413E-05 | 3.309E-05 | 3.209E-05 | 3.159E-05 | 3.143E-05 | 2.157E-02 |
| 30 | 4.753E-05 | 4.576E-05 | 4.009E-05 | 3.608E-05 | 3.403E-05 | 3.304E-05 | 3.206E-05 | 3.159E-05 | 3.143E-05 | 2.134E-02 |
| 29 | 4.740E-05 | 4.540E-05 | 3.973E-05 | 3.586E-05 | 3.391E-05 | 3.296E-05 | 3.204E-05 | 3.158E-05 | 3.143E-05 | 2.111E-02 |
| 28 | 4.731E-05 | 4.504E-05 | 3.940E-05 | 3.567E-05 | 3.380E-05 | 3.289E-05 | 3.201E-05 | 3.158E-05 | 3.143E-05 | 2.088E-02 |
| 27 | 4.723E-05 | 4.473E-05 | 3.908E-05 | 3.547E-05 | 3.369E-05 | 3.283E-05 | 3.198E-05 | 3.157E-05 | 3.143E-05 | 2.072E-02 |
| 26 | 4.717E-05 | 4.448E-05 | 3.884E-05 | 3.533E-05 | 3.361E-05 | 3.278E-05 | 3.196E-05 | 3.157E-05 | 3.143E-05 | 2.057E-02 |
| 25 | 4.714E-05 | 4.431E-05 | 3.866E-05 | 3.523E-05 | 3.355E-05 | 3.274E-05 | 3.195E-05 | 3.156E-05 | 3.143E-05 | 2.049E-02 |
| 24 | 4.710E-05 | 4.411E-05 | 3.847E-05 | 3.512E-05 | 3.349E-05 | 3.270E-05 | 3.193E-05 | 3.156E-05 | 3.143E-05 | 2.041E-02 |
| 23 | 4.706E-05 | 4.391E-05 | 3.827E-05 | 3.500E-05 | 3.343E-05 | 3.267E-05 | 3.192E-05 | 3.155E-05 | 3.143E-05 | 2.034E-02 |
| 22 | 4.700E-05 | 4.369E-05 | 3.807E-05 | 3.489E-05 | 3.337E-05 | 3.263E-05 | 3.190E-05 | 3.155E-05 | 3.143E-05 | 2.026E-02 |
| 21 | 4.690E-05 | 4.336E-05 | 3.777E-05 | 3.473E-05 | 3.328E-05 | 3.257E-05 | 3.188E-05 | 3.154E-05 | 3.143E-05 | 2.018E-02 |
| 20 | 4.676E-05 | 4.292E-05 | 3.741E-05 | 3.454E-05 | 3.317E-05 | 3.251E-05 | 3.186E-05 | 3.154E-05 | 3.143E-05 | 2.003E-02 |
| 19 | 4.656E-05 | 4.237E-05 | 3.700E-05 | 3.433E-05 | 3.305E-05 | 3.244E-05 | 3.183E-05 | 3.153E-05 | 3.143E-05 | 1.987E-02 |
| 18 | 4.630E-05 | 4.174E-05 | 3.659E-05 | 3.411E-05 | 3.294E-05 | 3.236E-05 | 3.180E-05 | 3.152E-05 | 3.143E-05 | 1.964E-02 |
| 17 | 4.602E-05 | 4.117E-05 | 3.624E-05 | 3.393E-05 | 3.284E-05 | 3.230E-05 | 3.178E-05 | 3.152E-05 | 3.143E-05 | 1.941E-02 |
| 16 | 4.573E-05 | 4.064E-05 | 3.595E-05 | 3.379E-05 | 3.276E-05 | 3.225E-05 | 3.176E-05 | 3.151E-05 | 3.143E-05 | 1.917E-02 |
| 15 | 4.550E-05 | 4.025E-05 | 3.574E-05 | 3.368E-05 | 3.270E-05 | 3.222E-05 | 3.174E-05 | 3.151E-05 | 3.143E-05 | 1.894E-02 |
| 14 | 4.536E-05 | 4.004E-05 | 3.564E-05 | 3.362E-05 | 3.267E-05 | 3.220E-05 | 3.173E-05 | 3.151E-05 | 3.143E-05 | 1.879E-02 |
| 13 | 4.527E-05 | 3.992E-05 | 3.559E-05 | 3.360E-05 | 3.266E-05 | 3.219E-05 | 3.173E-05 | 3.151E-05 | 3.143E-05 | 1.871E-02 |
| 12 | 4.516E-05 | 3.975E-05 | 3.552E-05 | 3.357E-05 | 3.264E-05 | 3.218E-05 | 3.173E-05 | 3.150E-05 | 3.143E-05 | 1.863E-02 |
| 11 | 4.496E-05 | 3.945E-05 | 3.539E-05 | 3.350E-05 | 3.260E-05 | 3.216E-05 | 3.172E-05 | 3.150E-05 | 3.143E-05 | 1.850E-02 |
| 10 | 4.460E-05 | 3.896E-05 | 3.516E-05 | 3.339E-05 | 3.253E-05 | 3.212E-05 | 3.170E-05 | 3.150E-05 | 3.143E-05 | 1.828E-02 |
| 9  | 4.400E-05 | 3.826E-05 | 3.484E-05 | 3.322E-05 | 3.244E-05 | 3.206E-05 | 3.168E-05 | 3.149E-05 | 3.143E-05 | 1.790E-02 |
| 8  | 4.305E-05 | 3.740E-05 | 3.443E-05 | 3.301E-05 | 3.232E-05 | 3.199E-05 | 3.165E-05 | 3.148E-05 | 3.143E-05 | 1.725E-02 |
| 7  | 4.169E-05 | 3.648E-05 | 3.397E-05 | 3.277E-05 | 3.219E-05 | 3.190E-05 | 3.162E-05 | 3.148E-05 | 3.143E-05 | 1.615E-02 |
| 6  | 3.980E-05 | 3.546E-05 | 3.346E-05 | 3.251E-05 | 3.204E-05 | 3.181E-05 | 3.158E-05 | 3.147E-05 | 3.143E-05 | 1.429E-02 |
| 5  | 3.763E-05 | 3.440E-05 | 3.293E-05 | 3.223E-05 | 3.188E-05 | 3.171E-05 | 3.154E-05 | 3.145E-05 | 3.143E-05 | 1.111E-02 |
| 4  | 3.562E-05 | 3.339E-05 | 3.242E-05 | 3.196E-05 | 3.173E-05 | 3.161E-05 | 3.150E-05 | 3.144E-05 | 3.143E-05 | 7.937E-03 |
| 3  | 3.436E-05 | 3.280E-05 | 3.212E-05 | 3.180E-05 | 3.164E-05 | 3.156E-05 | 3.148E-05 | 3.144E-05 | 3.143E-05 | 4.762E-03 |
| 2  | 3.730E-05 | 3.320E-05 | 3.086E-05 | 2.983E-05 | 2.954E-05 | 2.964E-05 | 3.039E-05 | 3.101E-05 | 3.143E-05 | 1.587E-03 |
| 1  | 5.049E-05 | 5.049E-05 | 5.049E-05 | 5.049E-05 | 5.049E-05 | 5.049E-05 | 5.049E-05 | 5.049E-05 | 5.049E-05 | 1.587E-03 |

X= 0.0091503 0.0093909 0.0094912 0.0095413 0.0095664 0.0095789 0.0095915 0.0095977 0.0096022



|    |           |           |           |           |           |           |           |           |           |           |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 43 | 3.653E-02 | 3.440E-02 | 3.269E-02 | 3.076E-02 | 3.008E-02 | 2.974E-02 | 2.939E-02 | 2.922E-02 | 2.916E-02 | 2.914E-02 |
| 44 | 3.687E-02 | 3.538E-02 | 3.291E-02 | 3.125E-02 | 3.017E-02 | 2.992E-02 | 2.947E-02 | 2.924E-02 | 2.916E-02 | 2.914E-02 |
| 45 | 3.688E-02 | 3.571E-02 | 3.324E-02 | 3.145E-02 | 3.048E-02 | 2.999E-02 | 2.950E-02 | 2.925E-02 | 2.916E-02 | 2.914E-02 |
| 42 | 3.686E-02 | 3.599E-02 | 3.359E-02 | 3.169E-02 | 3.063E-02 | 3.008E-02 | 2.954E-02 | 2.926E-02 | 2.916E-02 | 2.914E-02 |
| 41 | 3.683E-02 | 3.608E-02 | 3.371E-02 | 3.177E-02 | 3.068E-02 | 3.011E-02 | 2.955E-02 | 2.926E-02 | 2.916E-02 | 2.914E-02 |
| 40 | 3.680E-02 | 3.614E-02 | 3.379E-02 | 3.182E-02 | 3.070E-02 | 3.013E-02 | 2.956E-02 | 2.927E-02 | 2.916E-02 | 2.914E-02 |
| 39 | 3.677E-02 | 3.622E-02 | 3.394E-02 | 3.194E-02 | 3.078E-02 | 3.018E-02 | 2.957E-02 | 2.927E-02 | 2.916E-02 | 2.914E-02 |
| 38 | 3.675E-02 | 3.625E-02 | 3.401E-02 | 3.199E-02 | 3.081E-02 | 3.020E-02 | 2.958E-02 | 2.927E-02 | 2.916E-02 | 2.914E-02 |
| 37 | 3.673E-02 | 3.628E-02 | 3.410E-02 | 3.205E-02 | 3.085E-02 | 3.022E-02 | 2.959E-02 | 2.928E-02 | 2.916E-02 | 2.914E-02 |
| 36 | 3.670E-02 | 3.624E-02 | 3.403E-02 | 3.201E-02 | 3.082E-02 | 3.021E-02 | 2.959E-02 | 2.927E-02 | 2.916E-02 | 2.914E-02 |
| 35 | 3.666E-02 | 3.612E-02 | 3.388E-02 | 3.190E-02 | 3.076E-02 | 3.017E-02 | 2.957E-02 | 2.927E-02 | 2.916E-02 | 2.914E-02 |
| 34 | 3.663E-02 | 3.608E-02 | 3.384E-02 | 3.188E-02 | 3.074E-02 | 3.016E-02 | 2.957E-02 | 2.927E-02 | 2.916E-02 | 2.914E-02 |
| 33 | 3.662E-02 | 3.606E-02 | 3.380E-02 | 3.184E-02 | 3.072E-02 | 3.014E-02 | 2.956E-02 | 2.927E-02 | 2.916E-02 | 2.914E-02 |
| 32 | 3.659E-02 | 3.604E-02 | 3.374E-02 | 3.183E-02 | 3.071E-02 | 3.014E-02 | 2.956E-02 | 2.927E-02 | 2.916E-02 | 2.914E-02 |
| 31 | 3.655E-02 | 3.597E-02 | 3.371E-02 | 3.179E-02 | 3.069E-02 | 3.012E-02 | 2.955E-02 | 2.927E-02 | 2.916E-02 | 2.914E-02 |
| 30 | 3.650E-02 | 3.587E-02 | 3.359E-02 | 3.171E-02 | 3.064E-02 | 3.009E-02 | 2.954E-02 | 2.926E-02 | 2.916E-02 | 2.914E-02 |
| 29 | 3.646E-02 | 3.574E-02 | 3.343E-02 | 3.159E-02 | 3.057E-02 | 3.005E-02 | 2.952E-02 | 2.926E-02 | 2.916E-02 | 2.914E-02 |
| 28 | 3.642E-02 | 3.561E-02 | 3.328E-02 | 3.149E-02 | 3.051E-02 | 3.001E-02 | 2.951E-02 | 2.925E-02 | 2.916E-02 | 2.914E-02 |
| 27 | 3.640E-02 | 3.549E-02 | 3.314E-02 | 3.139E-02 | 3.045E-02 | 2.997E-02 | 2.949E-02 | 2.925E-02 | 2.916E-02 | 2.914E-02 |
| 26 | 3.638E-02 | 3.539E-02 | 3.303E-02 | 3.132E-02 | 3.041E-02 | 2.995E-02 | 2.948E-02 | 2.925E-02 | 2.916E-02 | 2.914E-02 |
| 25 | 3.636E-02 | 3.533E-02 | 3.295E-02 | 3.127E-02 | 3.038E-02 | 2.992E-02 | 2.947E-02 | 2.924E-02 | 2.916E-02 | 2.914E-02 |
| 24 | 3.635E-02 | 3.525E-02 | 3.286E-02 | 3.121E-02 | 3.034E-02 | 2.990E-02 | 2.946E-02 | 2.924E-02 | 2.916E-02 | 2.914E-02 |
| 23 | 3.633E-02 | 3.517E-02 | 3.277E-02 | 3.115E-02 | 3.031E-02 | 2.988E-02 | 2.945E-02 | 2.924E-02 | 2.916E-02 | 2.914E-02 |
| 22 | 3.632E-02 | 3.509E-02 | 3.267E-02 | 3.109E-02 | 3.027E-02 | 2.986E-02 | 2.944E-02 | 2.924E-02 | 2.916E-02 | 2.914E-02 |
| 21 | 3.628E-02 | 3.496E-02 | 3.253E-02 | 3.101E-02 | 3.022E-02 | 2.983E-02 | 2.943E-02 | 2.923E-02 | 2.916E-02 | 2.914E-02 |
| 20 | 3.623E-02 | 3.478E-02 | 3.236E-02 | 3.091E-02 | 3.016E-02 | 2.979E-02 | 2.942E-02 | 2.923E-02 | 2.916E-02 | 2.914E-02 |
| 19 | 3.616E-02 | 3.456E-02 | 3.216E-02 | 3.080E-02 | 3.010E-02 | 2.975E-02 | 2.940E-02 | 2.923E-02 | 2.916E-02 | 2.914E-02 |
| 18 | 3.607E-02 | 3.430E-02 | 3.196E-02 | 3.068E-02 | 3.003E-02 | 2.971E-02 | 2.938E-02 | 2.922E-02 | 2.916E-02 | 2.914E-02 |
| 17 | 3.597E-02 | 3.406E-02 | 3.176E-02 | 3.058E-02 | 2.998E-02 | 2.968E-02 | 2.937E-02 | 2.922E-02 | 2.916E-02 | 2.914E-02 |
| 16 | 3.586E-02 | 3.383E-02 | 3.164E-02 | 3.051E-02 | 2.993E-02 | 2.965E-02 | 2.936E-02 | 2.921E-02 | 2.916E-02 | 2.914E-02 |
| 15 | 3.578E-02 | 3.366E-02 | 3.153E-02 | 3.045E-02 | 2.990E-02 | 2.962E-02 | 2.935E-02 | 2.921E-02 | 2.916E-02 | 2.914E-02 |
| 14 | 3.573E-02 | 3.357E-02 | 3.148E-02 | 3.042E-02 | 2.988E-02 | 2.961E-02 | 2.934E-02 | 2.921E-02 | 2.916E-02 | 2.914E-02 |
| 13 | 3.569E-02 | 3.352E-02 | 3.146E-02 | 3.041E-02 | 2.988E-02 | 2.961E-02 | 2.934E-02 | 2.921E-02 | 2.916E-02 | 2.914E-02 |
| 12 | 3.565E-02 | 3.344E-02 | 3.142E-02 | 3.039E-02 | 2.987E-02 | 2.960E-02 | 2.934E-02 | 2.921E-02 | 2.916E-02 | 2.914E-02 |
| 11 | 3.558E-02 | 3.331E-02 | 3.135E-02 | 3.035E-02 | 2.984E-02 | 2.959E-02 | 2.933E-02 | 2.921E-02 | 2.916E-02 | 2.914E-02 |
| 10 | 3.544E-02 | 3.308E-02 | 3.123E-02 | 3.028E-02 | 2.981E-02 | 2.957E-02 | 2.933E-02 | 2.921E-02 | 2.916E-02 | 2.914E-02 |
| 9  | 3.521E-02 | 3.276E-02 | 3.106E-02 | 3.019E-02 | 2.975E-02 | 2.953E-02 | 2.931E-02 | 2.920E-02 | 2.916E-02 | 2.914E-02 |
| 8  | 3.484E-02 | 3.235E-02 | 3.085E-02 | 3.007E-02 | 2.968E-02 | 2.949E-02 | 2.929E-02 | 2.920E-02 | 2.916E-02 | 2.914E-02 |
| 7  | 3.429E-02 | 3.190E-02 | 3.060E-02 | 2.994E-02 | 2.961E-02 | 2.944E-02 | 2.927E-02 | 2.919E-02 | 2.916E-02 | 2.914E-02 |
| 6  | 3.345E-02 | 3.138E-02 | 3.032E-02 | 2.979E-02 | 2.952E-02 | 2.939E-02 | 2.925E-02 | 2.919E-02 | 2.916E-02 | 2.914E-02 |
| 5  | 3.247E-02 | 3.083E-02 | 3.003E-02 | 2.963E-02 | 2.943E-02 | 2.933E-02 | 2.923E-02 | 2.918E-02 | 2.916E-02 | 2.914E-02 |
| 4  | 3.145E-02 | 3.028E-02 | 2.974E-02 | 2.947E-02 | 2.934E-02 | 2.927E-02 | 2.921E-02 | 2.917E-02 | 2.916E-02 | 2.914E-02 |
| 3  | 3.086E-02 | 3.000E-02 | 2.959E-02 | 2.940E-02 | 2.930E-02 | 2.925E-02 | 2.920E-02 | 2.917E-02 | 2.916E-02 | 2.914E-02 |
| 2  | 3.243E-02 | 3.041E-02 | 2.910E-02 | 2.847E-02 | 2.826E-02 | 2.828E-02 | 2.865E-02 | 2.896E-02 | 2.916E-02 | 2.914E-02 |
| 1  | 3.746E-02 | 3.746E-02 | 3.746E-02 | 3.746E-02 | 3.746E-02 | 3.746E-02 | 3.746E-02 | 3.746E-02 | 3.746E-02 | 3.746E-02 |

x= 0.0091903 0.0093909 0.0094912 0.0095413 0.0095664 0.0095769 0.0095915 0.0095977 0.0096022

# TRANSFER COEFFICIENTS

| J          | R/D     | WDIFF     | WFLUX     | HM1       | SH1       | HM        | SH         | MFLUX      | H          | NU         | ST |
|------------|---------|-----------|-----------|-----------|-----------|-----------|------------|------------|------------|------------|----|
| 2-5.32970  | 0.36410 | 1.000E-04 | 2.067E-03 | 5.100E+00 | 4.749E-02 | 4.076E+00 | -2.388E+04 | -2.388E+02 | -2.040E+01 | -2.942E-02 |    |
| 3-4.33703  | 0.36410 | 1.000E-04 | 6.971E-04 | 1.788E+00 | 1.602E-02 | 1.375E+00 | 1.287E+03  | 1.287E+01  | 1.099E+00  | 1.585E-03  |    |
| 4-3.34556  | 0.36411 | 1.000E-04 | 6.535E-04 | 1.676E+00 | 1.501E-02 | 1.289E+00 | 1.741E+03  | 1.741E+01  | 1.487E+00  | 2.144E-03  |    |
| 5-2.35350  | 0.36411 | 1.000E-04 | 9.981E-04 | 2.560E+00 | 2.293E-02 | 1.968E+00 | 2.606E+03  | 2.606E+01  | 2.227E+00  | 3.211E-03  |    |
| 6-1.36143  | 0.36412 | 1.000E-04 | 1.354E-03 | 3.473E+00 | 3.111E-02 | 2.670E+00 | 3.504E+03  | 3.504E+01  | 2.993E+00  | 4.317E-03  |    |
| 7-0.77780  | 0.36412 | 1.000E-04 | 1.695E-03 | 4.346E+00 | 3.893E-02 | 3.342E+00 | 4.357E+03  | 4.357E+01  | 3.722E+00  | 5.368E-03  |    |
| 8-0.43458  | 0.36412 | 1.000E-04 | 1.997E-03 | 5.122E+00 | 4.588E-02 | 3.938E+00 | 5.112E+03  | 5.112E+01  | 4.367E+00  | 6.298E-03  |    |
| 9-0.21266  | 0.36412 | 1.000E-04 | 2.270E-03 | 5.822E+00 | 5.216E-02 | 4.477E+00 | 5.772E+03  | 5.772E+01  | 4.931E+00  | 7.111E-03  |    |
| 10-0.11388 | 0.36412 | 1.000E-04 | 2.487E-03 | 6.377E+00 | 5.713E-02 | 4.904E+00 | 6.282E+03  | 6.282E+01  | 5.367E+00  | 7.740E-03  |    |
| 11-0.04401 | 0.36412 | 1.000E-04 | 2.646E-03 | 6.790E+00 | 6.083E-02 | 5.221E+00 | 6.648E+03  | 6.648E+01  | 5.679E+00  | 8.190E-03  |    |
| 12-0.00291 | 0.36412 | 1.000E-04 | 2.750E-03 | 7.052E+00 | 6.318E-02 | 5.423E+00 | 6.872E+03  | 6.872E+01  | 5.871E+00  | 8.466E-03  |    |
| 13-0.02127 | 0.36412 | 1.000E-04 | 2.797E-03 | 7.174E+00 | 6.427E-02 | 5.517E+00 | 6.965E+03  | 6.965E+01  | 5.951E+00  | 8.581E-03  |    |
| 14-0.04545 | 0.36412 | 1.000E-04 | 2.845E-03 | 7.246E+00 | 6.492E-02 | 5.572E+00 | 7.018E+03  | 7.018E+01  | 5.996E+00  | 8.646E-03  |    |
| 15-0.09380 | 0.36412 | 1.000E-04 | 2.895E-03 | 7.425E+00 | 6.652E-02 | 5.710E+00 | 7.188E+03  | 7.188E+01  | 6.141E+00  | 8.850E-03  |    |
| 16-0.16633 | 0.36412 | 1.000E-04 | 3.029E-03 | 7.768E+00 | 6.959E-02 | 5.973E+00 | 7.527E+03  | 7.527E+01  | 6.430E+00  | 9.273E-03  |    |
| 17-0.23886 | 0.36411 | 1.000E-04 | 3.203E-03 | 8.215E+00 | 7.360E-02 | 6.318E+00 | 7.974E+03  | 7.974E+01  | 6.812E+00  | 9.824E-03  |    |
| 18-0.31139 | 0.36411 | 1.000E-04 | 3.412E-03 | 8.749E+00 | 7.838E-02 | 6.728E+00 | 8.507E+03  | 8.507E+01  | 7.268E+00  | 1.048E-02  |    |
| 19-0.38392 | 0.36411 | 1.000E-04 | 3.664E-03 | 9.397E+00 | 8.418E-02 | 7.226E+00 | 9.158E+03  | 9.158E+01  | 7.824E+00  | 1.128E-02  |    |
| 20-0.43227 | 0.36411 | 1.000E-04 | 3.908E-03 | 1.002E+01 | 8.979E-02 | 7.707E+00 | 9.784E+03  | 9.784E+01  | 8.359E+00  | 1.205E-02  |    |
| 21-0.48063 | 0.36410 | 1.000E-04 | 4.138E-03 | 1.061E+01 | 9.503E-02 | 8.157E+00 | 1.037E+04  | 1.037E+02  | 8.859E+00  | 1.278E-02  |    |
| 22-0.50480 | 0.36410 | 1.000E-04 | 4.324E-03 | 1.109E+01 | 9.935E-02 | 8.528E+00 | 1.085E+04  | 1.085E+02  | 9.269E+00  | 1.337E-02  |    |
| 23-0.52898 | 0.36409 | 1.000E-04 | 4.456E-03 | 1.143E+01 | 1.024E-01 | 8.789E+00 | 1.119E+04  | 1.119E+02  | 9.557E+00  | 1.378E-02  |    |
| 24-0.55315 | 0.36409 | 1.000E-04 | 4.591E-03 | 1.177E+01 | 1.055E-01 | 9.055E+00 | 1.153E+04  | 1.153E+02  | 9.850E+00  | 1.420E-02  |    |
| 25-0.57733 | 0.36409 | 1.000E-04 | 4.724E-03 | 1.212E+01 | 1.086E-01 | 9.320E+00 | 1.187E+04  | 1.187E+02  | 1.014E+01  | 1.462E-02  |    |
| 26-0.60151 | 0.36408 | 1.000E-04 | 4.855E-03 | 1.245E+01 | 1.115E-01 | 9.571E+00 | 1.220E+04  | 1.220E+02  | 1.042E+01  | 1.502E-02  |    |
| 27-0.64986 | 0.36407 | 1.000E-04 | 5.019E-03 | 1.287E+01 | 1.153E-01 | 9.899E+00 | 1.262E+04  | 1.262E+02  | 1.078E+01  | 1.555E-02  |    |
| 28-0.69821 | 0.36406 | 1.000E-04 | 5.259E-03 | 1.349E+01 | 1.208E-01 | 1.037E+01 | 1.323E+04  | 1.323E+02  | 1.130E+01  | 1.630E-02  |    |
| 29-0.77074 | 0.36404 | 1.000E-04 | 5.494E-03 | 1.409E+01 | 1.262E-01 | 1.084E+01 | 1.381E+04  | 1.381E+02  | 1.180E+01  | 1.702E-02  |    |
| 30-0.84327 | 0.36403 | 1.000E-04 | 5.744E-03 | 1.478E+01 | 1.324E-01 | 1.137E+01 | 1.449E+04  | 1.449E+02  | 1.238E+01  | 1.785E-02  |    |
| 31-0.91580 | 0.36401 | 1.000E-04 | 5.967E-03 | 1.530E+01 | 1.371E-01 | 1.177E+01 | 1.498E+04  | 1.498E+02  | 1.280E+01  | 1.840E-02  |    |
| 32-0.98833 | 0.36399 | 1.000E-04 | 6.072E-03 | 1.557E+01 | 1.395E-01 | 1.198E+01 | 1.522E+04  | 1.522E+02  | 1.301E+01  | 1.876E-02  |    |
| 33-1.03669 | 0.36398 | 1.000E-04 | 6.090E-03 | 1.562E+01 | 1.399E-01 | 1.201E+01 | 1.526E+04  | 1.526E+02  | 1.304E+01  | 1.880E-02  |    |
| 34-1.06086 | 0.36398 | 1.000E-04 | 6.180E-03 | 1.585E+01 | 1.420E-01 | 1.219E+01 | 1.551E+04  | 1.551E+02  | 1.325E+01  | 1.911E-02  |    |
| 35-1.10921 | 0.36397 | 1.000E-04 | 6.240E-03 | 1.600E+01 | 1.434E-01 | 1.231E+01 | 1.566E+04  | 1.566E+02  | 1.338E+01  | 1.929E-02  |    |
| 36-1.17088 | 0.36397 | 1.000E-04 | 6.483E-03 | 1.663E+01 | 1.493E-01 | 1.279E+01 | 1.629E+04  | 1.629E+02  | 1.391E+01  | 2.006E-02  |    |
| 37-1.23255 | 0.36396 | 1.000E-04 | 6.586E-03 | 1.685E+01 | 1.513E-01 | 1.299E+01 | 1.653E+04  | 1.653E+02  | 1.413E+01  | 2.037E-02  |    |
| 38-1.29422 | 0.36396 | 1.000E-04 | 6.457E-03 | 1.656E+01 | 1.484E-01 | 1.273E+01 | 1.616E+04  | 1.616E+02  | 1.381E+01  | 1.991E-02  |    |
| 39-1.35589 | 0.36397 | 1.000E-04 | 6.324E-03 | 1.622E+01 | 1.453E-01 | 1.247E+01 | 1.582E+04  | 1.582E+02  | 1.352E+01  | 1.949E-02  |    |
| 40-1.41755 | 0.36397 | 1.000E-04 | 6.047E-03 | 1.551E+01 | 1.389E-01 | 1.193E+01 | 1.509E+04  | 1.509E+02  | 1.289E+01  | 1.859E-02  |    |
| 41-1.53198 | 0.36398 | 1.000E-04 | 5.929E-03 | 1.520E+01 | 1.362E-01 | 1.169E+01 | 1.482E+04  | 1.482E+02  | 1.266E+01  | 1.825E-02  |    |
| 42-1.64640 | 0.36400 | 1.000E-04 | 5.738E-03 | 1.472E+01 | 1.318E-01 | 1.132E+01 | 1.433E+04  | 1.433E+02  | 1.224E+01  | 1.766E-02  |    |
| 43-1.76083 | 0.36403 | 1.000E-04 | 5.187E-03 | 1.330E+01 | 1.192E-01 | 1.023E+01 | 1.289E+04  | 1.289E+02  | 1.101E+01  | 1.588E-02  |    |
| 44-2.05261 | 0.36408 | 1.000E-04 | 4.732E-03 | 1.214E+01 | 1.087E-01 | 9.333E+00 | 1.174E+04  | 1.174E+02  | 1.003E+01  | 1.447E-02  |    |
| 45-2.34440 | 0.36414 | 1.000E-04 | 3.634E-03 | 9.319E+00 | 8.348E-02 | 7.166E+00 | 8.893E+03  | 8.893E+01  | 7.598E+00  | 1.096E-02  |    |
| 46-3.33646 | 0.36421 | 1.000E-04 | 2.824E-03 | 7.241E+00 | 6.487E-02 | 5.568E+00 | 6.886E+03  | 6.886E+01  | 5.883E+00  | 8.483E-03  |    |
| 47-4.32853 | 0.36424 | 1.000E-04 | 2.308E-03 | 5.919E+00 | 5.302E-02 | 4.551E+00 | 5.632E+03  | 5.632E+01  | 4.812E+00  | 6.939E-03  |    |
| 48-5.32060 | 0.36424 | 1.000E-04 | 2.005E-03 | 5.142E+00 | 4.606E-02 | 3.953E+00 | 4.901E+03  | 4.901E+01  | 4.187E+00  | 6.038E-03  |    |
| 49-6.31267 | 0.36425 | 1.000E-04 | 1.866E-03 | 4.838E+00 | 4.334E-02 | 3.720E+00 | 4.636E+03  | 4.636E+01  | 3.961E+00  | 5.712E-03  |    |

| J  | YV7D       | TAU        | CF         | CF*REQ72   |
|----|------------|------------|------------|------------|
| 3  | 9.9207E-01 | 5.8858E-02 | 1.6336E-03 | 2.5830E-02 |
| 4  | 1.9841E+00 | 3.4215E-02 | 9.4964E-04 | 1.5015E-02 |
| 5  | 2.9762E+00 | 3.6727E-02 | 1.0194E-03 | 1.6118E-02 |
| 6  | 3.9683E+00 | 1.3449E-01 | 3.7327E-03 | 5.9019E-02 |
| 7  | 4.7561E+00 | 2.1677E-01 | 6.0166E-03 | 9.5131E-02 |
| 8  | 5.2195E+00 | 2.7536E-01 | 7.6428E-03 | 1.2084E-01 |
| 9  | 5.4921E+00 | 3.2641E-01 | 9.0596E-03 | 1.4324E-01 |
| 10 | 5.6325E+00 | 3.7264E-01 | 1.0343E-02 | 1.6353E-01 |
| 11 | 5.7468E+00 | 4.0994E-01 | 1.1378E-02 | 1.7990E-01 |
| 12 | 5.8023E+00 | 4.3040E-01 | 1.1946E-02 | 1.8888E-01 |
| 13 | 5.8346E+00 | 4.3443E-01 | 1.2058E-02 | 1.9065E-01 |
| 14 | 5.8591E+00 | 4.3923E-01 | 1.2191E-02 | 1.9276E-01 |
| 15 | 5.8954E+00 | 4.7077E-01 | 1.3066E-02 | 2.0600E-01 |
| 16 | 5.9558E+00 | 5.2475E-01 | 1.4565E-02 | 2.3029E-01 |
| 17 | 6.0283E+00 | 5.8448E-01 | 1.6222E-02 | 2.5650E-01 |
| 18 | 6.1009E+00 | 6.5835E-01 | 1.8273E-02 | 2.8892E-01 |
| 19 | 6.1734E+00 | 7.5519E-01 | 2.0960E-02 | 3.3141E-01 |
| 20 | 6.2338E+00 | 8.3792E-01 | 2.3257E-02 | 3.6772E-01 |
| 21 | 6.2822E+00 | 9.2600E-01 | 2.5701E-02 | 4.0638E-01 |
| 22 | 6.3184E+00 | 9.8410E-01 | 2.7314E-02 | 4.3187E-01 |
| 23 | 6.3426E+00 | 1.0258E+00 | 2.8472E-02 | 4.5018E-01 |
| 24 | 6.3668E+00 | 1.0653E+00 | 2.9569E-02 | 4.6752E-01 |
| 25 | 6.3910E+00 | 1.1015E+00 | 3.0572E-02 | 4.8339E-01 |
| 26 | 6.4152E+00 | 1.1299E+00 | 3.1360E-02 | 4.9584E-01 |
| 27 | 6.4514E+00 | 1.1560E+00 | 3.2085E-02 | 5.0731E-01 |
| 28 | 6.4998E+00 | 1.1961E+00 | 3.3198E-02 | 5.2490E-01 |
| 29 | 6.5602E+00 | 1.1767E+00 | 3.2661E-02 | 5.1642E-01 |
| 30 | 6.6327E+00 | 1.1292E+00 | 3.1340E-02 | 4.9553E-01 |
| 31 | 6.7053E+00 | 1.0162E+00 | 2.8206E-02 | 4.4597E-01 |
| 32 | 6.7778E+00 | 8.3557E-01 | 2.3192E-02 | 3.6669E-01 |
| 33 | 6.8382E+00 | 6.7204E-01 | 1.8653E-02 | 2.9492E-01 |
| 34 | 6.8745E+00 | 6.0754E-01 | 1.6862E-02 | 2.6662E-01 |
| 35 | 6.9108E+00 | 4.8305E-01 | 1.3407E-02 | 2.1198E-01 |
| 36 | 6.9658E+00 | 3.2451E-01 | 9.0070E-03 | 1.4241E-01 |
| 37 | 7.0275E+00 | 1.2354E-01 | 3.4289E-03 | 5.4216E-02 |
| 38 | 7.0891E+00 | 9.4790E-02 | 2.6309E-03 | 4.1598E-02 |
| 39 | 7.1504E+00 | 3.0416E-01 | 8.4419E-03 | 1.3348E-01 |
| 40 | 7.2127E+00 | 4.8955E-01 | 1.3588E-02 | 2.1484E-01 |
| 41 | 7.3005E+00 | 6.5632E-01 | 1.8216E-02 | 2.8803E-01 |
| 42 | 7.4146E+00 | 9.2916E-01 | 2.5789E-02 | 4.0776E-01 |
| 43 | 7.5293E+00 | 1.1315E+00 | 3.1405E-02 | 4.9656E-01 |
| 44 | 7.7325E+00 | 1.1858E+00 | 3.2911E-02 | 5.2037E-01 |
| 45 | 8.0242E+00 | 1.3328E+00 | 3.6991E-02 | 5.8488E-01 |
| 46 | 8.6602E+00 | 9.5875E-01 | 2.6611E-02 | 4.2075E-01 |
| 47 | 9.6582E+00 | 7.0104E-01 | 1.9458E-02 | 3.0765E-01 |
| 48 | 1.5650E+01 | 5.3044E-01 | 1.4723E-02 | 2.3278E-01 |
| 49 | 1.1642E+01 | 4.4955E-01 | 1.2479E-02 | 1.9730E-01 |

[illegible]

# APPENDIX C

## Code Listing of Computer Runs

| Run code | H/D | M      | $\theta$ (°) |
|----------|-----|--------|--------------|
| 127      | 4   | 0.1    | 0            |
| 128      | 4   | 0.125  | 0            |
| 130      | 4   | 0.167  | 0            |
| 133      | 4   | 0.25   | 0            |
| 135      | 3   | 0.1    | 0            |
| 136      | 3   | 0.125  | 0            |
| 141      | 3   | 0.167  | 0            |
| 145      | 3   | 0      | 0            |
| 146      | 4   | 0      | 0            |
| 147      | 2   | 0      | 0            |
| 149      | 2   | 0.1    | 0            |
| 152      | 2   | 0.125  | 0            |
| 159      | 3   | 0.1    | 5            |
| 165      | 3   | 0.1    | 10           |
| 167      | 3   | 0.1    | -5           |
| 172      | 3   | 0.1    | -10          |
| 179      | 3   | 0.1    | 30           |
| 203      | 3   | 1.0    | -30          |
| 208      | 4   | 1.0    | 0            |
| 209      | 3   | 2.0    | -30          |
| 213      | 3   | no jet |              |
| 214      | 4   | 2.0    | 0            |
| 218      | 3   | 10     | 30           |
| 223      | 3   | 2      | 30           |
| 226      | 3   | 0.7    | 30           |
| 229      | 3   | 0.5    | 30           |
| 232      | 3   | 0.5    | 30           |
| 234      | 3   | 2.0    | -45          |
| 236      | 3   | 0.4    | 30           |
| 246      | 3   | 0.1    | -45          |
| 250      | 4   | 0      | 0            |