Effects of Price and Geological Uncertainty over the Life of Mine and Ultimate Pit Limit

By

© Maria Fernanda Del Castillo

A thesis submitted to McGill University in partial fulfillment of the requirements for the degree of MEng

Department of Mining and Materials Engineering,

McGill University, Montreal,

Quebec, Canada.

March, 2014

Table of Contents

Abstract

Mining operations are highly affected by risk, where commodity price and geology can be considered as the most relevant ones, as they will define the pit's design and the operation's ability to meet production plans. Considering these uncertainties at an early stage is key for project success, and doing so by including managerial flexibility to annually re-evaluate the project allows a more realistic approach, and increases the operation's ability to react timely to change.

The current study presents a real-option based evaluation approach to assess the effects of commodity price and geological uncertainty over the performance of a mining operation and the potential life of mine and consecutive pit limit modifications that may occur. The case study presented shows the value of flexibility when dealing with stochastic scenarios, in contrast with conventional evaluation methods. It was found that the stochastic method better assesses the project's potential to expand and provide useful information that traditional methods ignore. A Geometric Brownian motion with Poisson jumps model is used to forecast price, and direct block simulation is used to model geological uncertainty. The model was optimized using the commercial software ILOG-Cplex. Results show that including the option to stop mining, or expand increase the operation's value, but only if expensive infrastructure relocations are prevented, which makes it necessary to study these options at an initial stage, as they allow management to prepare for changes and provides a clearer image of the project's real potential.

Resume

Les opérations minières sont hautement affectées par les risques. Le prix des métaux et la géologie sont considérés comme ayant le plus d'impact puisqu'ils définissent le design de la mine et la capacité des opérations à atteindre les plans de production. La planification sous incertitude est utilisée pour créer un projet performant sous ces risques de sorte que les objectifs de production seront encore atteints. À cette fin, considérer de la flexibilité dans les opérations et dans la gestion ajoute un bénéfice important afin de répondre profitablement aux changements.

Cette thèse présente une approche basée sur l'analyse par options réelles pour évaluer les effets de l'incertitude des prix des métaux et de la géologie sur les performances des opérations minières, la durée potentielle de la vie de la mine et les modifications de la taille définitive du site d'excavation. L'étude de cas présentée montre la valeur qu'ajoute la flexibilité lorsque des simulations de géologie sont considérées, par opposition aux méthodes conventionnelles. Il a été constaté que la méthode présentée permet une meilleure évaluation du potentiel d'expansion du projet et fournie des informations importantes que les methodes traditionnelles ignorent. Les résultats montrent qu'inclure l'option d'arrêt ou d'expansion de la mine à chaque année augmente la valeur des opérations, mais seulement si les dépenses de relocalisation d'infrastructures importantes sont prévues. Ceux-ci rendent nécessaire l'étude de ces options

dans une étape initiale puisqu’ils permettent aux gestionnaires de se préparer aux changements et produisent une meilleure vue du potentiel du projet.

Acknowledgments

First, I would like to thank my supervisor Professor Roussos Dimitrakopoulos, for his valuable guidance during the last years, and his support in finishing this thesis.

I would also like to thank the organizations and companies who funded this research: Canada Research Chairs (Tier I) in Sustainable Mineral Resource Development and Optimization under Uncertainty, and NSERC Collaborative Research and Development Grant CRDPJ 411270 -10, entitled Developing new global stochastic optimization and high-order stochastic models for optimizing mining complexes with uncertainty; industry collaborators: AngloGold Ashanti, Barrick Gold, BHP Billiton, De Beers, Newmont Mining and Vale, and NSERC Discovery Grant 239019.

Finally, I want to thank my family for their constant support and encouragement, and my friends here in the COSMO lab, whose help was invaluable and essential for the fulfillment of this research.

Contributions of Authors

The author of this thesis is the primary author. Professor Roussos G. Dimitrakopoulos was the supervisor of the author's Mater of Engineering degree and is included as co-author of the paper produced from this thesis, submitted to Mining Technology, Transactions of the Institutions of Mining and metallurgy: Section A.

List of Tables

List of Figures

1 Introduction

Mining reserves are function of both internal and external variables, most of which are highly uncertain. This fact makes mining projects some of the riskiest investments in the market, and at the same time, one of the most profitable. This risk can be divided into two groups: internal uncertainties and external uncertainties. The first ones consider all variables which are intrinsic to the project, productivity, metallurgical recovery, and most importantly, geology. On the other hand, external risks affect all projects equally, such as market volatility, price uncertainty and exchange rates. The complex combination uncertainties make it impossible for mine planners to determine strategic design variables such as production capacity and life of mine with certainty at the planning stage of the project. In response, later adjustments to the mine plan take place according to the context of the project, both technically and economically; however, these adjustments are not considered in the initial evaluation processes, which undervalues the project and prevents decision-makers to prepare in advance and take full advantage of opportunities.

Currently, the traditional way to deal with these uncertainties is by assuming the worst case scenario (usually chosen conservatively) and optimizing the project as if everything was known and stable, which leads to obvious deviations from target and highly suboptimal mine plans. This procedure creates robust designs which reduce risk, as they allow the project to perform faced with most of the unfavorable scenarios, but, robustness is expensive, and it doesn't provide any information on what will actually happen in the future. In other words, it is a heavy static response to a continuously changing industry. An alternative approach is to consider the 'stochastic condition' of the variables, which provides probabilistic information of the project's performance. This stochastic mine planning facilitates designing flexible operations, allowing the planning process to include decision-maker's reactions to future conditions and simultaneously optimize the timing of the reaction's costs. As stated in Sabour and Dimitrakopoulos (2010), an important aspect of stochastic planning, is that it helps answering questions such as "what is the probability that the mine closes early?" or "what is the probability of expanding the operation and when?" Probabilistic approaches provide a tree of possible outcomes and, with them, a range of potential project values which constitute a better representation of the project's actual performance. However, because of the correlation complexity of the variables that affect the mining industry, there is still much work to be done in order to build a user-comprehensive and computationally efficient stochastic model that accounts for multiple sources of uncertainty.

The goal of open pit mine planning is to first determine an optimal extraction schedule that maximizes project value and subsequently, to decide where/when to stop mining. The latter defines the ultimate pit limit of the deposit and the infrastructure location for the operation. The current study focuses on the second stage of the mine planning process, assuming that the initial stage has already been optimized. Continuing the work of Sabour and Dimitrakopoulos (2010), this study will center in creating a model that considers metal price and geological uncertainties to determine the life of mine (LOM) of a mining operation and with this, a stochastically-defined ultimate pit limit. To do this, it is necessary first to fully understand the effects that these variables have over the project, and at the same time, consider the correlation that exists between them.

2 Approach to Uncertainty Modeling

2.1 Geological Uncertainty

Because of the high costs associated with exploration, the limited information obtained from composites and flawed sampling systems, the geology of the deposit is highly uncertain, being one of the main sources of risk in a mining operation (Godoy and Dimitrakopoulos, 2004). Thus, many efforts have been put in developing a method that allows the integration of this uncertainty into the design and evaluation of the project at acceptable time limits. One of the developed methods is conditional simulation; to date this method has been successfully implemented in various projects, and will be used in the study described in Section 4.

The limitation of conventional approaches can be summarized in two parts: first, estimations don't account for the data's local variability, and second, they don't consider that the processes of mine design and production scheduling are non-linear transfer processes, and therefore assume that the optimum estimation of each component will provide the optimum solution for the whole. In contrast, conditional simulation can be used to generate equally probable representations of the orebody, which respect the spatial correlation and variability of the deposit, providing a probabilistic assessment over a group of blocks (Ravenscroft, 1992; Dowd 1994, 1997; Groovaerts, 1997).

Godoy and Dimitrakopoulos (2004) state that geological uncertainty is the major contributor to not meeting project expectations; in their study, the authors demonstrate that including this uncertainty significantly reduces the deviation from production targets, and at the same time, increases the total value of the project. The importance of geological uncertainty was first accounted for by Journel (1988), who introduced the term of stochastic simulation for modeling spatial uncertainty to include the geological risk of the deposit into the mine planning process. To date, this method has been successfully implemented in various projects, providing a probabilistic approach of the metal content of each block by generating multiple equiprobable simulations of the deposit. Further applications of the method in comparison with the deterministic assessment can be found in Ramazan and Dimitrakopoulos (2012), Albor and Dimitrakopoulos (2009, 2010) and Dimitrakopoulos and Grieco (2009).

Among the different stochastic simulation techniques, an efficient and straightforward method to generate multiple equiprobable representations of a deposit is Direct Block Simulation (DBSim), thoroughly described in Godoy (2003). DBSim is a step forward from generalized sequential Gaussian simulation (GSGS) described in Luo (1998), which mixes the upside characteristics of LU decomposition method (Davis, 1987), with the qualities of the well-known sequential Gaussian simulation (SGS). The first method (LU), it is capable of simulating simultaneously and in a fast way a group of nodes; however, it is a computationally expensive method as the decomposition of the covariance matrix requires $O(n^3)$ computations for a 'n x n' matrix. On the other hand, SGS has the upside of being easily implemented, but can turn to be very slow as the number of nodes (n) to simulate increase.

DBSim method divides the volume to be simulated into groups of nodes, generally accordant with the selective mining unit (SMU) defined by the operation (for details in how to choose the optimal group size, refer to Dimitrakopoulos and Luo, 2004). Subsequently, each of the groups is visited following a random path, as in SGS, and inside each group, the internal nodes are simulated by LU decomposition, which in these conditions is a fast and feasible method given the reduced size of the groups. The main difference between this DBSim and the previous GSGS, is that in this case, once the internal nodes of a group are simulated, they are averaged and only this value is stored, liberating the memory of storing each individual node, which at the end, would be averaged up anyway once the re-blocking process takes place. Because of this memory liberation, DBSim becomes computationally inexpensive and simple to implement. It’s important to note that, once the nodes of a group are averaged, we are left with a block value which must be used to condition subsequent blocks being simulated, so the covariance of block to block and block to node support are needed. This is a straightforward step; however, it is an important difference between this method and its predecessors.

Together with this, DBSim can easily be extended to the simulation of polymetalic deposits, by using the minimum/maximum autocorrelation factors (MAF). In this case however, a double storage of the data must be done: once the internal nodes of a block are simulated, they must be back transformed to their original coordinates, and subsequently averaged and stored outside of the simulation process, and on the other hand, the nodes in the simulation space must also be averaged (in their transformed coordinates) to be used to condition the remaining simulation. Clear examples of this process can be seen in Benndorf and Dimitrakopoulos (2007) and Boucher and Dimitrakopoulos (2009, 2012), as its implementation is outside the scope of this study.

2.2 Market Uncertainty

In addition to geological uncertainty, despite the fact that market risk is widely acknowledged, for simplification purposes, projects are traditionally evaluated assuming certainty in the price trend. Regardless, price shifts have a decisive effect over the project, and although they can't be controlled, it is possible to increase the flexibility of the project in order to be prepared to react timely to them, and overall assess the probability of alternative outcomes that can mean greater or smaller profit (Dixit and Pindyck, 1994).

Generally, a fall in the commodity price will make the ore content of the final pit be less valuable and also, cause that less material will be profitable to extract (some ore blocks are now considered as waste), triggering the final pit limit to shrink and the life of mine to decrease. In other words, the operation may close early due to a drop in price. Similarly, if the commodity price rises, the life of mine will likely increase, as there is new material that now is profitable to extract that wasn't with the previous price (waste is considered as ore after the rise in price). This ore/waste relation is mainly defined by the cut-off grade, which is function of price, as well as operational capacities and costs, and there has been a lot of work done to optimize its value in order to obtain desired targets, mainly maximize the NPV, as it is the link between geological and price variability. Details on cut-off strategies may be found in Asad (2007) and Asad and Dimitrakopoulos (2012).

If the operation, however, is not prepared for these expansions or contractions, decision-makers will not be able to take full advantage of positive scenarios, or protect the project form negative ones. Potential problems that prevent the project from benefiting from opportunities may be caused for example, because infrastructure is placed in strategic spots where the pit could expand, requiring great relocation costs. On the other hand, it may be hard for decision makers to hedge from unfavorable scenarios if contracts and leases are done over too-long term, reducing their managerial flexibility. From here, it is apparent that flexibility is highly valuable and efforts must therefore be concentrated on including it in the project's engineering design as well as in its strategic planning.

McCarthy and Monkhouse (2003) state that not considering the commodity price's uncertainty and the managerial flexibility in the evaluation process results in underestimations of the optimal LOM, that lead to plants with extra capacity, with higher initial investments, and a loss of capital in general. The authors also clarify that this can be handled by using real options valuation approach; in their paper, the option of re-opening or permanently closing a stand-by copper mine is evaluated. Similarly, Moel and Tufano (2002) use real options to study the operational state dynamics of a group of 285 North American gold mines (open pit and underground operations), defining when to exercise and when to hold on to options of temporary or permanent closing, subject to uncertain commodity prices.

To account for the effects of market uncertainty over the project's value, real option valuation (ROV) has shown to provide successful alternative results. This method, developed as an extension of financial options into investment projects, complements the NPV and addresses many of the limitations of the DCF analysis (Lee and Strang, 2003; Samis et al. 2006). In a standard real options model, the underlying state variable (in this case the commodity price) is formulated as a stochastic process, enabling the examination of the uncertain behavior of the variable (Shibata, 2006). With this, the model is capable of quantifying the value of flexibility as a response to the uncertainty, and integrates it into the project by considering the value of decision-making along the LOM.

This valuation method has been successfully implemented in various industries, with many applications in mining to consider, principally, the metal price and exchange rate uncertainties. Brennan and Schwartz (1985) evaluated the option of closing a mining operation early. Amram and Kulatilaka (1999), present a general overview or RO with an example to develop and operate a gold mine; both proving that considering the option's value provides a more reliable image of the project's performance. Sabour and Wood (2009) and Dimitrakopoulos and Sabour (2007) consider commodity price uncertainty and demonstrate that ROV method incorporates the ability of the project's management to react to change based on new information; in their study, the authors show that ROV provides substantial improvements in the mine planning and evaluating process compared to traditional methods. Samis et al. (2006) use RO based on forward contracts for copper to consider the commodity's market variability and obtain better information to select a project to invest in. Sabour and Dimitrakopoulos (2010) use RO to incorporate the option of expanding, closing early, or stopping a mining project, subject to price variability, and with this, they define a stochastic method to calculate the ultimate pit limit of the project, showing that traditional methods consistently underestimate the size of the ultimate pit. Other examples can be seen in Mardones (1993), Samis and Poulin (1998), Cardin et al. (2008), Cortazar et al. (2008), Sabour and Poulin (2010), among others.

To incorporate price uncertainty in project evaluation, however, it is necessary first to generate a reliable stochastic model that expresses this uncertainty. An extensive description of econometrics and price forecasting models based on random walks is given in Dixit and Pindick (1994) and Campbell et al. (1996). Newer and more sophisticated models are also available, which examine the use of neural networks (Mingming and Jinliang, 2012), wavelets (Jammazi and Aloui, 2012), auto-regressive models such as ARIMA, AARMA, ARMAX, GARCH, etc. (Meade, 2010; Tan et al. 2010), trend stationary processes and random walk simulations with diffusion jumps (Shafie and Topal, 2010), among others.

The inclusion of this 'jump' component to complement the variable's modeling has proven to generate a better representation of price behaviors when applied to energy and commodities such as gold and copper, without the requirement of extensive assumptions as input (Shafie and Topal, 2010; Blanco and Soronow, 2001), and as such, it will also be incorporated in the following study. This was first noticed by Merton (1976), who derives an option pricing formula that continues the work of Black and Scholes (1973), and considers stock returns as a mixture of a continuous behavior with a jump-Poisson process, both of which only depend on the current price, respecting the Markov properties of the models. Oldfield et al. (1977) introduced empirical data to support the idea of modeling stock returns as a combination of a continuous process with discrete jumps. Even though these studies are focused on stock returns, they can as well be applied for commodity price forecasting, as they also behave as market derivatives. This was carried out by Mendez and Lamothe (2009), who modeled copper price by incorporating Gaussian Poisson exponential stochastic processes (or 'jumps') to the usual mean reverting process, and by Blanco and Soronow (2001), who incorporated a jump-diffusion process to a geometric mean reverting process to forecast energy prices.

To prepare the operation for uncertainty it is necessary to have an idea of the range of values that can be obtained; in the case of market risk, this is done by forecasting models that examine the uncertainty. Studies that look to integrate market variability into the valuation process classically model price and stock returns as random walks, with different corrections and sophistications according to the characteristics of the asset being evaluated. Dixit and Pindyck (1994) state that precious metals' behavior, such as gold or platinum, are better represented by geometric Brownian motions (GBM) shown in Eq. (1), whereas base metals such as copper or lead follow a mean reverting process (MRP) resented in Eq.

(2), which means that they have a cyclical behavior, and tend to shift back to a long term price.

$$dx = \mu \, x \, dt + \sigma \, x \, dz \tag{1}$$

$$dx = \eta \, (\bar{x} - x) \, dt + \sigma \, dz \tag{2}$$

In Eq. (1), μ represents the drift or trend of the price, and σ the variability, both of which depend on the initial price x. In Eq.(2), the main difference in the model is given by η, which is the price's reversion speed towards the long-term price.

Even though these methods may seem simplistic compared to newer financial models, they allow for an adequate representation with a small number of parameters required, thus are easy to interpret and calibrate from market data and reduces the likelihood of model errors. For these reasons, these models have been used for decades, and are the basis of newer methods (Blanco et al., 2001; Shafiee and Topal, 2010).

Although these models perform competitively compared to far more complex ones, they do, however, have some important limitations, such as underestimating extreme price changes, or jumps (Blanco and Soronow, 2001). The mathematical formulation to incorporate these jumps is by adding a Poisson diffusion process into the random walk, or Geometric Brownian Motion in this case, and is presented in Eq. (3). The regressive model is also given in Eq. (4).

$$dx = \mu \, x \, dt + \sigma \, x \, dz + dq \tag{3}$$

$$x_t - x_{t-1} = \mu \cdot \Delta t \cdot x_{t-1} + \left(\sigma^2 \cdot \sqrt{\Delta t}\right) \cdot x_{t-1} \cdot \epsilon_{t1} + \eta \cdot (\kappa + \delta \epsilon_{t2}) \cdot x_{t-1} \tag{4}$$

Mathematically, as presented in Eq. (3), this jump is integrated into the model just by adding an extra diffusion term (dq) in the previous random walk models. In the regressive form shown in Eq. (4):

- $\eta = \begin{cases} 0 & with\,\mathrm{h}\,probability \quad 1 - \lambda dt \\ 1 & with\,\mathrm{h}\,probability \quad \lambda dt \end{cases}$

- $\lambda = \lambda_u + \lambda_d$, total frequency of jump (up and down)

Here, η is a binary variable that takes the value of 0 if there is no jump and 1 if there is a jump; λ represents the total frequency of jumps per year, and κ represents the average of the percentual sizes of these jumps. All this values can be obtained by inspection of the historical data. To simulate these jumps, if $\eta = 1$, a uniform random variable $u \sim U[0,1]$ is used where if $u < (\lambda_u/\lambda)$ the price jump is upwards, and if $u > (1 - \lambda_u/\lambda) = (\lambda_d/\lambda)$, the jump is considered to go downwards.

The forecasting model described in Eq.(4) will be used in the current case study, to incorporate gold price's uncertainty in the valuation process. Details on the implementation and parameters used will be given in Section 4.2.2.

2.3 Integrating Uncertainties

Because of the existence of these two strong sources of uncertainty (price and geology) where one highly affects the other, it is necessary to create a joint model that considers both variables simultaneously, as they will mostly define the overall performance of the operation. Some authors have already attempted to generate mine planning approaches that include both sources of uncertainty, Sabour et al. (2008) refer to price, exchange rate and geological uncertainty, and develop a process to rank the simulated mine designs, in order to select the most favorable one. Meagher et al. (2009) include price and exchange rate variability, as well as geological simulations for pushback design by using minimum cut algorithm; applying their methodology to a copper open pit mine, their results show an increase in project value of 10 to 50% along the life of the mine. Another approach is proposed by Dimitrakopoulos et al. (2007), who considered price and geological uncertainties, and define a “Minimum Acceptable Return” on investment to rank and decide over different schedules, considering for each option the range between the design’s capacity of hedging from downside risk and taking advantage of upside potential. Other examples can be seen in Sabour and Poulin (2006), Musingwini et al. (2007); Dimitrakopoulos and Sabour (2007).

The objective of this study is to asses the variability range of an operation’s size, and its corresponding life of mine, given the different technical and economical uncertainties that govern the context of the project. With this, we wish to find a straightforward way of allowing the operation to seize the opportunities that arise for expanding the mine, given the probability this occurring, and at the same time, prevent unnecessary expenses due to late decision-making of stop mining early, or relocating infrastructure. We wish to answer questions such as ‘Where should important infrastructure be located to avoid future relocations?’, and ‘What is the probability of expanding or contracting from the operation’s original planned design?’

The current study will consider the two main sources of risk: geological and commodity price uncertainty, to assess the optimal ultimate pit limits of a mining project and obtain a quantitative method to calculate the potential value of further expansions and with this, the operation's actual value and life of mine (LOM). To account for this uncertainty, conditional simulation will be used in the case of geology, to generate multiple representations of the deposit, and a random walk with Poisson-exponential jump model will be applied to generate a price forecasting model that accounts for price variability.

The next section of this thesis starts by describing the existing methodologies to model the different types of uncertainty, and finishes by explaining the proposed method to assess the potential of pit expansion options. After this, Section 4 provides a case study used to show the benefits of the method, and further explain the implementation of the proposed method. Finally, Section 5 presents the conclusions and implications of this study and the guidelines for future research.

3 Proposed Uncertainty-Modeling Methodology

To allow for the joint consideration of market and geological uncertainties in the mine planning process, a three-step methodology is proposed. The first step consists of creating an analytical review of the project, such as its costing structure, financing requirements, flexibility opportunities, etc., thus generating a base case scenario. The second step looks to both create flexibility in the engineering design of the project at hand, and model the uncertainties acting over the project by generating stochastic price paths and orebody models. Finally, the third step consists of developing a flexible mine planning evaluation model considering an annual re-evaluation of the mine's operational state by integrating managerial flexibility and the option of re-deciding the destination of extracted blocks at each period, according to the 'current' circumstances, both technical and economic. This last step can be also represented as the value of keeping the 'option of closing', instead of exercising it on a previous period.

Step 1: Project Review and Operational Assumptions

To create the base case model, it is first necessary to clearly specify the costing model (CAPEX and OPEX), the initial mining and processing capacities of the operation, the metal price (assumed constant) and processing recovery, etc. in order to design an optimal schedule for the estimated orebody model. Additionally, other financial data is required such as the operation's required continuing expenses, the depreciation method, taxes, financing agreements, etc.

With this information, the mining schedule is generated, which provides the 'base case' ultimate pit limit, as well as initial project value. This corresponds to the conventional pit design and project evaluation, and will be used as point of comparison for the subsequent stochastic analysis.

For simplicity, the schedule generated in this step is kept constant over the whole study, dividing the deposit into fixed 'extracted blocks per period', with changing value depending on the geology (grade) and the commodity price, so the subsequent optimization analysis will be carried out to re-define the destination of the blocks within each period, according to the changing scenarios.

Step 2: Stochastic Modeling and Creation of Flexibility

The second step is divided in two sections: firstly, the creation and inclusion of flexibility options into the project's design, in order to make the operation more responsive in case the expected context changes; and secondly, the generation of multiple equiprobable orebody simulations to account for geological uncertainty, and the forecasting of metal price to account for market variability. The last is done by using a price model chosen depending on the commodities involved and the historical information available (as mentioned in Section 2.2).

Flexibility is included in the mine design by modifying the pit design and ultimate pit limit defined by the schedule generated in the base case. As the study looks to assess the possibility of the operation expanding or contracting from its original design, avoiding infrastructure replacements and other arrangements, there must be a mining sequence available in case the operation decides to expand. To do this, the initial ultimate pit is removed from the orebody model, and the commodity price is increased over its regular value, in order to generate subsequent nested pits that contain the blocks considered in the initial pit limit (Whittle, 1999). Finally, as there is no common rock between the initial base-case and the expanded schedules, they can be combined to create an 'expanded ultimate pit limit'. This keeps the base case's schedule, pushback design and ultimate pit limit unchanged, and at the same time, defines an optimized mining sequence in case the operation decides to expand. Depending on the deposit, this process may be carried out to allow for different pit expansion.

Just as in the base-case schedule, the scheduled expansions are also considered fixed over all the analysis. It must be noted that keeping the schedule fixed no matter what scenario is being evaluated is a strong limitation of the model, as the designed schedule is only optimal for the context in which it was created, and a change in geology or price would signify in scheduling changes. However, it will be used as an initial step, and further studies on this subject are proposed as future research.

Step 3: Flexible Expansion Evaluation

To evaluate a mine plan and assess the possibility of expansion, the 'expanded schedule' generated in the previous step is run over multiple equiprobable scenarions, where a scenario consists on one of the orebody simulations and a price path forecasted, which account for ore grade and commodity price uncertainties respectively. This way, for each period, simulation and current metal price, it is possible to obtain the annual revenue of the base case, as well as for each scenario.

Together with this, managerial flexibility is considered by re-defining the destination of each block in every period: mill if rock grade is over cut-off and waste dump if it is not. This decision is taken by maximizing the project value based on a changing cut-off grade, which depends on the current selling price as well as the grade-tonnage curve of the orebody model. All the optimization work is done with the optimization tool CPLEX from IBM (ILOG CPLEX v12.1 User's Manual for CPLEX, 2010).

For this study, the project evaluation is done in an annual basis, so the decision to extract or stop mining is revised at the end of each year, considering the capital and operational expenses on the first case and the closing costs on the second. The actual project evaluation is done as a financial American option by considering the flexibility of being able to close (or exercise the option to stop mining) at any time, incurring in the corresponding costs (Hull, 1997). This means that for a given scenario the project is evaluated backwards, from the ultimate (expanded) pit limit, when the operation stops mining and the LOM is reached, along the whole forecasted price path. The evaluation is done sequentialy for consecutively decreasing closing years and, by optimizing with the objective of maximizing the value of the mining operation, it is possible to obtain the probability of being operational or closed at time t, what provides a reliable range of feasible project values.

The previous methodology is represented in Fig. 1, where Mine 'X' is evaluated from its ultimate pit limit, achieved by year 'N' (where N is the maximum LOM, or $\max LOM$ including all the expansions available), until 'today', for decreasing values of N. Every year, the operation has the probability of expanding (p_E), or stopping mining ($p_C = 1 - p_E$), and the project value is calculated as the maximum between the sunk costs, and the total project value of closing on year 't' ($TPV_{C(t)}$), for $t = \{0, ..., N\}$. This way, the evaluation process looks backwards, one year at a time, checking if the project's value would increase if the operation closed earlier (i.e. if $TPV_{C(t-1)} > TPV_{C(t)}$), considering the ore available and the time value of money.

With this analysis, the LOM distribution is obtained by taking the argument of the project's maximum value, which, assuming that the current time is '$t*$', the orebody is simulation 'S' and the price path is forecast 'p', is represented in equation 5.

$$\mathrm{LOM}(t^*, S, p) = \underbrace{\operatorname{argmax}}_{t^* \leq T \leq \mathrm{maxLOM}} \left(\sum_{t=t^*}^{T-1} V_t + V_{C(T)}\right) = \underbrace{\operatorname{argmax}}_{t^* \leq T \leq \mathrm{maxLOM}} \left(\mathrm{TPV}_{C(T)}\right) \quad (5)$$

Where:

$$V_{E(t)} = Value\ of\ extracting\ in\ "t" = f\left(Sim, Price, Ore_{t,S}, CAPEX_{t,S}, OPEX_{t,S}\right)$$

$$V_{C(T)} = Value\ of\ closing\ at\ period\ "T" = f\left(\sum_{t=0}^{T} Ore_{t,S} \,\middle|\, Sim, Price\right)$$

$$TPV_{C(t)} = Total\ Project\ Value\ if\ Mining\ stops\ in\ period\ "T"$$

For example, if the operation decides to stop mining at period T ($\mathrm{LOM} = \mathrm{T}$, with $\mathrm{T} > \mathrm{t}^*$), this means that (i) there is an overall positive value for continuing extracting, and (ii) that the maximum profit (considering the costs) is obtained by operating until the end of year ‘T’ (i.e. $TPV_{C(T)} > TPV_{C(t)},\ \forall t = t^*, \ldots, N$), even if at any given time ‘t’ extracting is temporarily not profitable (with $t^* < t < T$). This means that even if the costs of extracting on a given year are higher than the revenues obtained, the operation should continue if there is a subsequent period that presents a ‘minimum profit’, which can be user defined, but must be high enough to pay for the costs and the previous year’s losses. That is, to continue the operation past a current moment ‘t^*’, the following condition must be met:

$$\left(\sum_{t\,=\,t^*+1}^{T-1} V_t\right) + V_{C(T)} \geq \text{Minimum Profit}, \qquad \forall\ T = \{t^* + 1, .., \text{maxLOM}\} \qquad (6)$$

If this condition is satisfied for any ‘T’ ($T = \{t^* + 1, .., \text{maxLOM}\}$), then the operation ‘expands’ to the next period (‘$t^* + 1$’), if not, it means that no combination of future extractions will increase the value of the project, and so the operation should stop mining in that period. In any case, the reclamation costs are considered to be incurred in time ‘T’, and are calculated depending on the cumulative ore production up to that year (Sabour and Dimitrakopoulos, 2010). To obtain the ultimate pit limit and LOM probability distribution (i.e. the probability to stop mining at each year, together with the range of project values), this process is repeated for all simulations and for thousands of price paths. This practice provides useful information by periodically allowing new external information to be included and used in the planning process. For example, if the mine is in operation on a given year, it is useful to know what is the probability of obtaining a higher project value if the production continues (and for how long should it continue), in order to plan for infrastructure and equipment arrangements ahead of time.

It is assumed that the decision to stop mining is irreversible, and that if the operation decides to continue in production, this decision is maintained until the next year's re-evaluation (i.e. if production continues, the whole tonnage considered in that period must be extracted). Metal price is also assumed constant throughout one period. This way, an optimum LOM which maximizes the project value is obtained for each simulated price path and simulated orebody.

To assess the influence of each individual variable in the performance of the operation and the ultimate pit limit, the case study will perform separate analysis for each variable: first, considering multiple orebody realizations over a constant price path; second, evaluating the performance of the basic estimated orebody model over multiple stochastic price paths, and finally a joint evaluation that integrates both uncertainties.

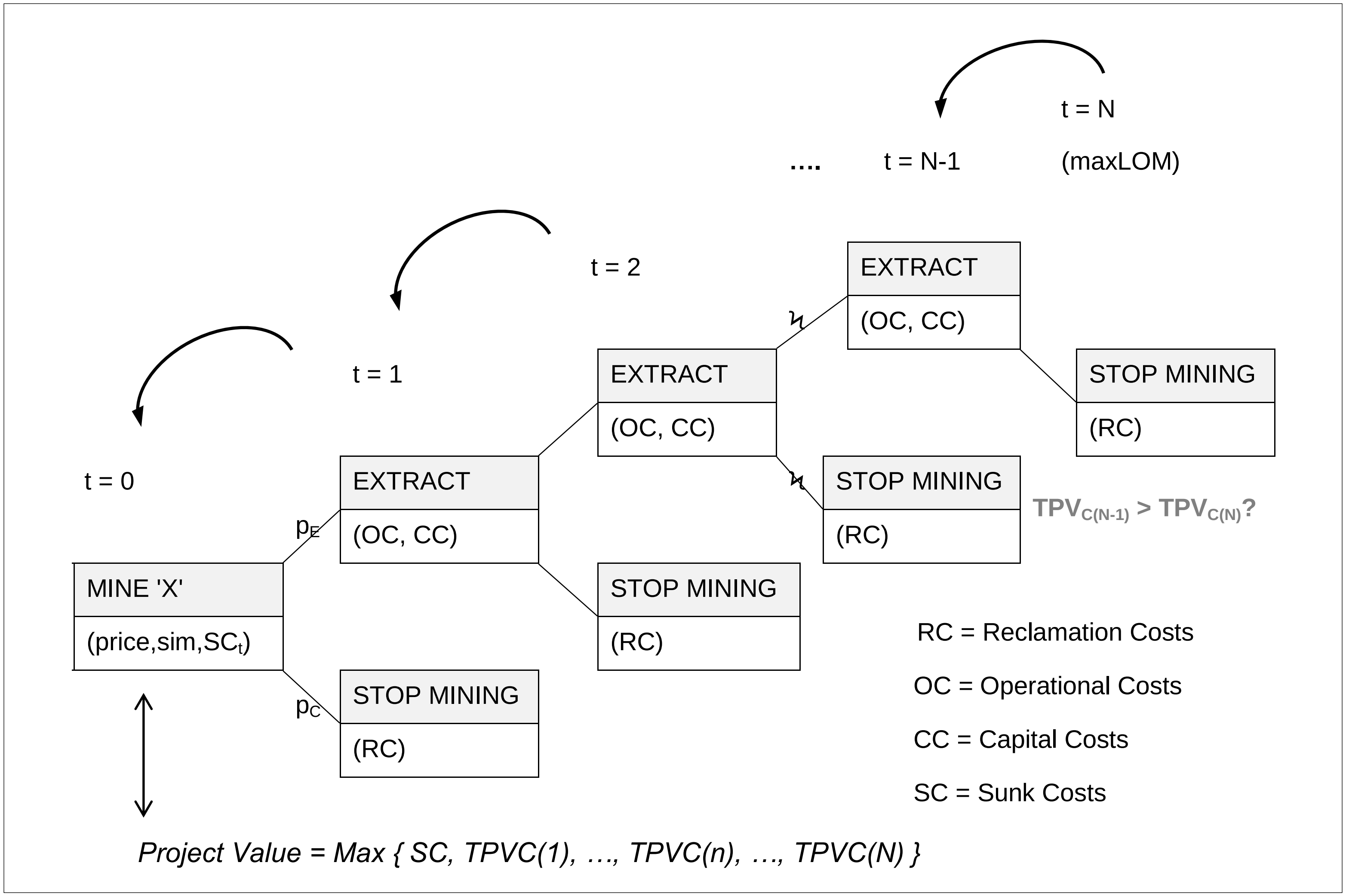


Figure 1. Evolution of the proposed stochastic mine planning evaluation model

4 Case Study

The proposed methodology is demonstrated with an open pit gold mine to show how these concepts can be applied in a real world problem, and the advantages that the information obtained can provide for the strategic decision-making and mine planning processes. The gold mine being evaluated has an extraction capacity of 15Mtpa, and a processing capacity of 5Mtpa. The operation doesn't consider stockpiling, so at each period the extracted material is either taken to the mill to be processed at a cost of 13 US$/t, or to the waste dump, at only a transportation cost; in any case, material is extracted and transported at a cost of 1.8 US$/t. Additionally, it is assumed that the mining targets of each period are met, and that the mill will be fed until the rock's grade is lower than the cut-off grade of that period. For this case study, the block destination is defined by using optimum cut-off grades based on the work of Asad (2007), who starts from Lane's formulas and maximizes the NPV according to the mine and mine-mill capacity relation ($\mathrm{g_m}$ and $\mathrm{g_{cm}}$ respectively, below). Here, the maximum cut-off value between them is selected, and corresponds to the minimum grade that a block must have to be transported to the mill instead of to the waste dump. The equations used to define these cut-offs are shown next.

$$g_m = \frac{Processing\ Cost}{(Selling\ Price - Refining\ Cost) \cdot Recovery} \tag{7}$$

$$g_{cm} = \left(\frac{Proc. Cap.}{Mining\ Cap.} - \frac{Ore(k^*)}{Total\ Rock}\right) \cdot \frac{g(k^*+1) - g(k^*)}{\left(\frac{Ore(k^*+1) - Ore(k^*)}{Total\ Rock}\right)} + g(k^*) \tag{8}$$

Calculating g_m for each period from equation (3) is straightforward. However, to obtain g_{cm}, the grade-tonnage curve is used (see Fig.4 as reference). Let $k^* \in K,\ K = \{1,...,k\}$ be a certain section defined by two consecutive cut-offs ('x' axis in Fig. 4); in the previous equation, $g(k^*)$ and $g(k^*+1)$ correspond to the lowest grades (cut-off grades) of two consecutive sections, and $Ore(k^*)$ and $Ore(k^*+1)$ correspond to the total amount of ore available with those corresponding cut-offs (Asad, 2007; Asad and Dimitrakopoulos, 2012a).

From the previous equations we can see that g_m provides the minimum cut-off grade for the operation to profit from the processing of the rock (in order to cover all the corresponding expenses), and g_{cm} makes sure that, given the amount of ore available for a given cut-off grade, the capacities are met but not exceeded. This way, if for example the price is high, g_{cm} will probably define the cutoff, as the tonnage of profitable rock may exceed the mill's capacity, and if the price is low, g_m will define the cut-off, as it may be not profitable to process most of the rock, even if there is mill capacity left.

The operational expenses, together with the initial investment and the present value of the continuing capital costs (equipment, infrastructure and closing costs) are shown in Table 1. For simplicity, it is assumed that the project is fully founded by its owners, and the depreciation is done linearly over 5 year period.

Table 1. Case study's cost structure

Cost Structure			
OPEX	Mining	US$/t rock	1.8
	Processing	US$/t ore	13.0
	Marketing	US$/oz.	5.0
CAPEX	Initial Investment	MUS$	350
	Infrastructure	MUS$ (r = 8%)	70
	Maintenance		
	Equipment		
	Closure		
Tax over Revenue		%	18

4.1 Base Case

For the base case, there is no uncertainty taken into account, and the project is evaluated in the conventional way of static discounted cash flow. An initial gold price of 700 US$/oz. is considered, which increases linearly to 900 US$/oz. with an annual growth of 50 US$/year. The deposit is discretized in eleven thousand blocks of 15x15x10 meters, and the base-case orebody model is assumed perfectly known (and referred to as "E-type"), and is obtained by averaging the grades of 20 conditional simulations of the orebody. This averaging generates a smoothed representation of the data that generally presents errors related to an overestimation of the amount of ore and an underestimation of the ore's grade, or vice-versa (for further details on effects of estimated vs. simulated orebody models see Albor and Dimitrakopoulos, 2009). To define the destination of each block (waste dump or mill), the cut-off grade is calculated for every period, as explained in Eq. (3) and (4), according to the gold's current selling price and the mine's global grade-tonnage curve obtained by the schedule designed using the Milawa NPV Algorithm of the Whittle software (Whittle 2009), generated by optimizing over the mentioned price trend. In this case, the average cut-off grade is of 0.39ppm, that results in a total of 53.2 million tons of ore; this and other base case assumptions are presented in Table 2.

Table 2. Base Case data and operational assumptions

Base Case		
Price of Au	US$/oz.	700 - 900
Reserves	Mt	53.3
Extraction Rate	Mtpa	15.0
Processing Rate	Mtpa	5.0
Discount Rate	%	8
Recovery	%	90
Average grade of ore	ppm	1.20
Average cut-off grade	ppm	0.39
NPV	**MUS$**	**89.2**

With this information, a base cash flow is elaborated, which presents a net present value (NPV) of 89.2M USD$ for the initial pit limit, which presented an optimum LOM of 11 years according to the base case schedule generated by Milawa algorithm, included in the Whittle Optimization Software.

To create and include flexibility in the mine design, a revenue factor from 0.3 to 3 was used to increase the metal price and schedule further extractions past the ultimate pit limit. This was carried out by exporting the blocks from the initial pit, and re-scheduling the remaining ore using again the Milawa NPV Algorithm in Whittle software. Figure 2, shows a cross-section of the deposit with the initial ultimate pit limit, and the flexibility of possible expansions, presented as two further stages. This scheduling increases the operation's life up to 15 and 16 years respectively, however, according to this static net present value evolution, they are not profitable to extract, reducing the project's value by 2.4% and 2.5% (exact values are presented in Table 3). Even though these value reductions may seem marginal, they do cause the rejection of the expansions with the traditional scheduling process, and leave the ultimate pit limit at the 11-year design.

The previous discounted cash flow (DCF) results show that there is a 100% probability of the mine operating until year 11 and a 0% chance of expanding any further. If the variable's evolution was perfectly known, this would be the case; however, the mine's context will certainly change, and decision-makers will adapt the operation accordingly. This is why the main evaluation advantage of real options is that, in comparison to the static DCF valuation, they do consider this dynamic decision-making process, quantify it, and include it in the project's value,

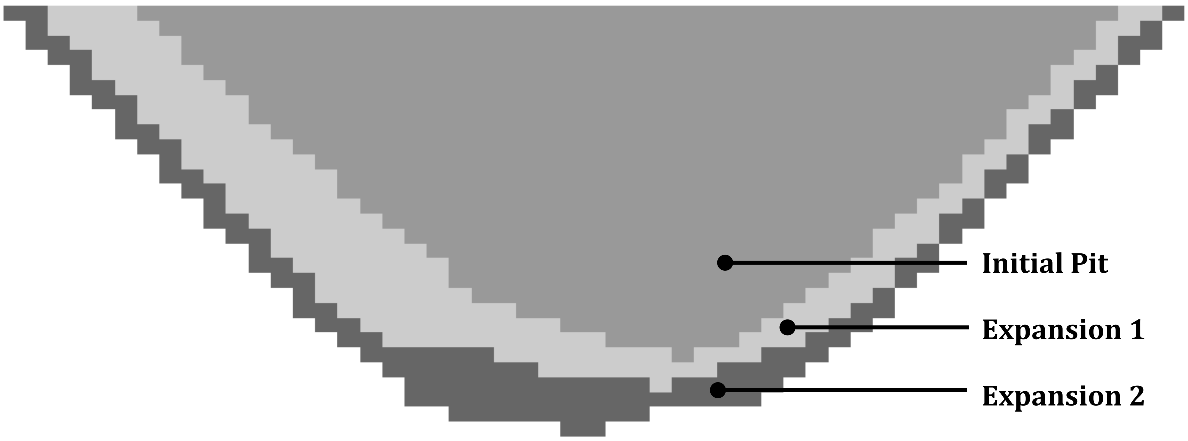


Figure 2. Cross section of the deposit pro the initial pit and the available expansions

Table 3. Economic evaluation results for the base case analysis

Base Case (DCF)	
Initial Ultimate Pit (LOM=11)	$ 89,238,244
Value Expansion 1 (LOM=15)	-$ 2,120,700
Value Expansion 2 (LOM=16)	-$ 2,250,100

instead of ignoring it and assuming nothing will change from the moment of evaluation until the end of the project's life.

4.2 Stochastic Case

4.2.1 Case of Stochastic Geology

To account for the stochasticity of the grade in the orebody, Direct Block Simulation was used to create 20 equiprobable representations of the orebody (Boucher and Dimitrakopoulos, 2009). It has been shown that after a certain number of simulated orebodies, results converge to a given distribution (Albor and Dimitrakopoulos, 2009), and considering the computational intensity of generating these simulations, given the data available and the variability perceived in this deposit, 20 simulations is an acceptable number to express the uncertainty. What means that adding more realizations will not greatly affect the distribution of the grades, which will ultimately define the probability of a given block of being considered ore or waste for a given cut-off grade.

Just as in the base case, the cut-offs are calculated for each of the orebody simulations by using Lane's formulas and are re-estimated in every period. This means that the destination of each block is re-defined subject to the geology that is encountered in the orebody simulation, and managerial flexibility is considered by having the option to expand the operation past its initial limits if the conditions are favorable (this will be referred to as the flexible case 'FC').

The difference between the initial base case and the simulations is presented in Fig. 3a for ore quantity and 3b for NPV. The simulated models present in average 14% less ore than the base case, but at the same time, more than a 21% increase in the NPV. This happens because the smoothness of the estimated model (E-type in graphs) causes the deposit to contain more medium grade blocks, and in this case, it increases the total ore tonnage (as the block's grade distribution is mostly over the selected cut-off grade), what causes extra processing costs without the benefit of more metal being produced. Simulations present higher grade dispersion with extreme values, making ore blocks more profitable to process, as there are fewer blocks with higher grade (more revenue with fewer costs).

This if further presented in Fig. 4, where it is possible to see the grade tonnage curve for each of the 20 simulations, as well as for the estimated model. This graph shows, for different cut-off grades, the amount of ore available (over that given cut-off), as well as the average grade of the ore considered. Thus, for higher cut-offs, there is less tonnage available, but the average grade increases. The 20 simulations tend to have a similar behavior, with a maximum variability of about 25 thousand tons, and a maximum grade variability of 0.5ppm.

However, there is a high difference between the simulations and the estimated model, with almost a 75 thousand ounce difference of gold and 1ppm difference in average grade. We can see that the E-type model tends to overestimate the tonnage for low to medium cut-off grades (between 0.6 and 1.3ppm), but strongly

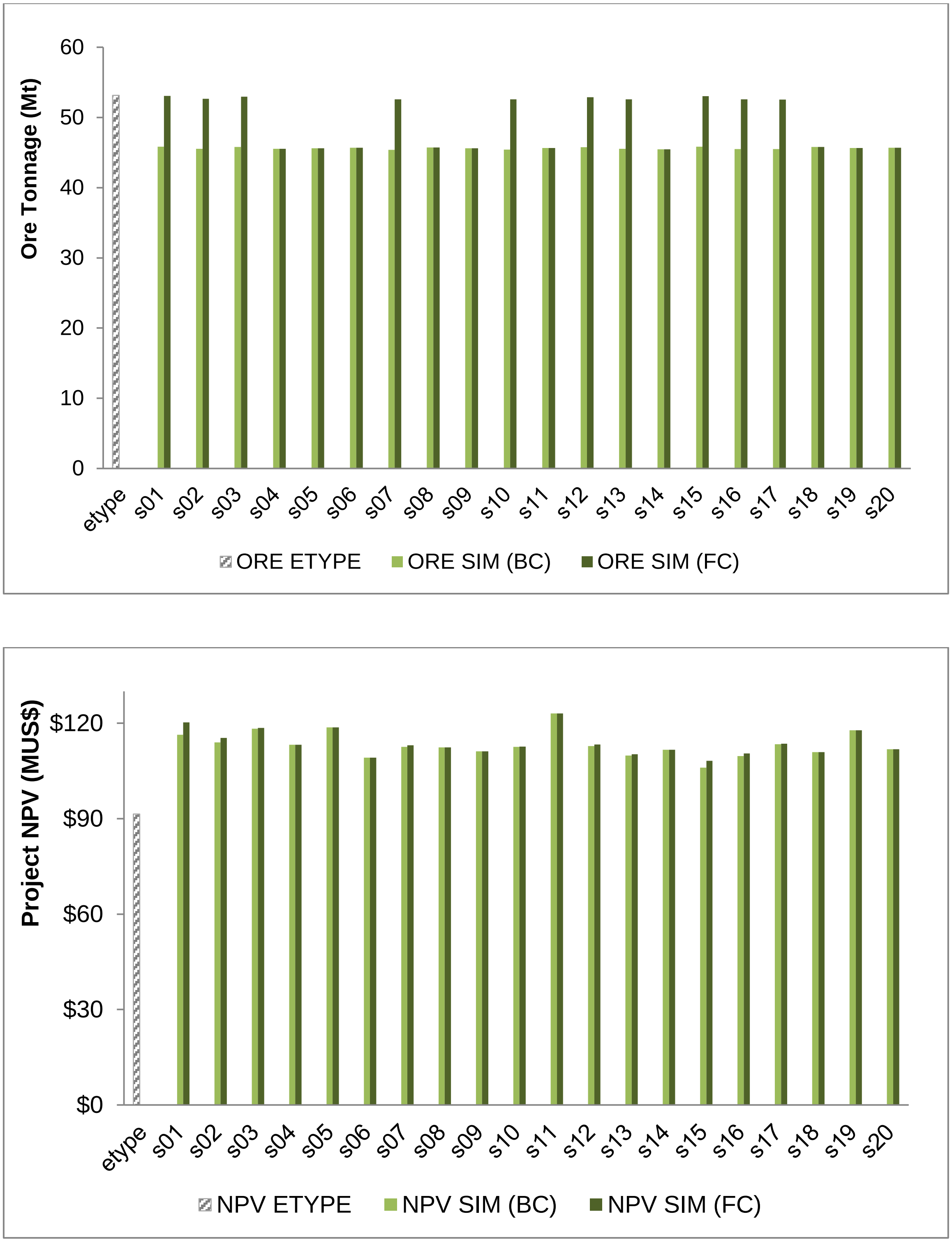


Figure 3. Results of the base case and stochastic orebody simulations over the (a) Ore tonnage and (b) NPV

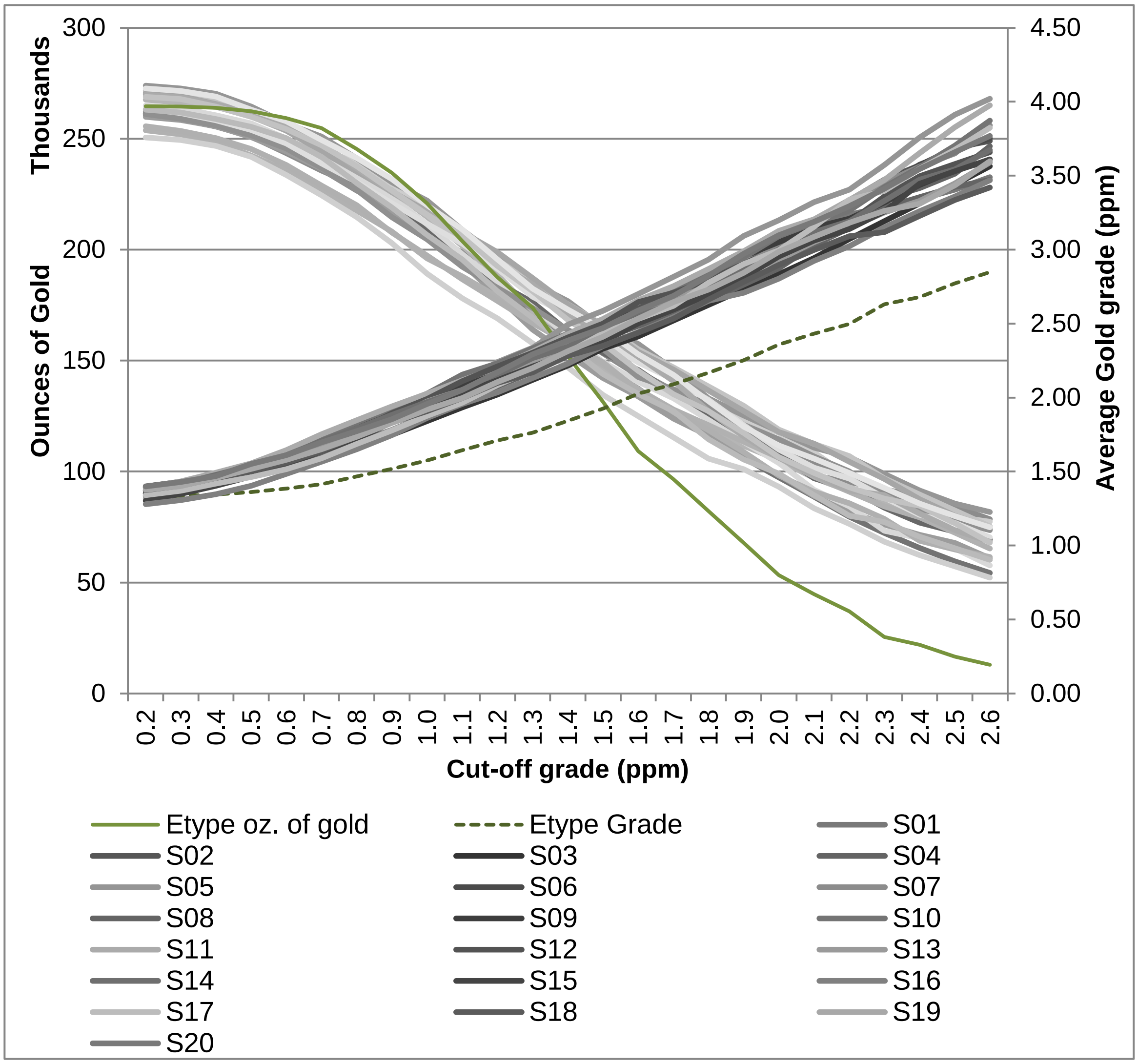


Figure 4. Grade tonnage curve for the average E-type model, and for the 20 orebody simulations for different cut-off grades

underestimates the tonnage of high grade material (over a cut-off grade of 1.5ppm). In summary, as explained before, the estimated model overestimates medium grade tonnage, but at the same time, underestimates the average grade of the ore available.

The evaluation is done considering the initial ultimate pit, or base case (BC) design of 11 years of LOM, and the expanded schedule of up to 16 years. Results show that, if the flexible option is considered, 50% of the time the operation will decide to expand up to the 15-year LOM pit limit. As mentioned earlier, just by considering geological simulations, project value is in average, increased in a 21%. If together with this the option to expand is included, this difference increases to a 22% compared to the initial evaluation (as presented in Table 4). Even though the increased value due purely to the expansion may seem marginal, longer projects allow for new opportunities, and usually have a better social acceptance. These results are presented in the right-most columns of Fig. 3a and 3b, under the label of 'FC'. In this case, the second stage expansion is not profitable for any of the simulations.

For clarification purposes, the ROV of the geological uncertainty case was calculated by considering the optimal state of each simulation, considering that decision-makers have free will to close the operation early or keep extracting past the initial limits.

Table 5 details the annual operational and capital costs, the ore production tonnage and average grade for the initial pit limit, the two possible expansions for the estimated model ('E-TYPE'), and the 50% exceedance probability value of the simulated orebodies (‘SIM P50’), what means that half of the simulations present exceed this value. The reclamation costs are specified in the final column, which are calculated proportionally to the cumulative ore tonnage extracted, and correspond to the costs that the project must incur only on the year that they decide to stop mining and close the operation. This table also shows that the first year of any of the two expansions requires extra capital expenses, mainly for mining works, accesses, scaling and support.

Table 4. Economic evaluation results for the uncertain geology case

ROV - GEOLOGY CASE	
Initial Ultimate Pit (LOM=11)	$ 113,281,947
Value Expansion 1 (LOM=15)	$ 506,450
Value Expansion 2 (LOM=16)	$ -

A summary of the project’s cash flow for each of the cases studied is presented in Table 6. The first section shows the base case’s results, and the second case presents the results of the stochastic geology case just described. Even though the NPV case presents higher revenues, the costs are also higher. This is caused for two reasons: first, the overestimation of ore tonnage in the estimated model depicted in Fig. 3a which make the processing costs to peak (OPEX), and second, because this static evaluation does not consider the possibility that the project’s value may actually increase if the operation closes later or earlier than

Table 5. Operational data of the initial pit and available expansions for the estimated model and simulated models

	YEAR	ORE (Mt)		GRADE (ppm)		OPEX (US$/t)		CAPEX (MUS$)		RECLAMATION (MUS$)	
		E-TYPE	SIM (P50)	E-TYPE	SIM (P50)	E-TYPE	SIM (P50)	E-TYPE	SIM (P50)	E-TYPE	SIM (P50)
Initial Ultimate Pit	0							350.00	350.00		
	1	4.83	4.17	1.34	1.51	90.79	82.23	15.60	14.55	0.80	0.70
	2	4.78	4.23	1.44	1.64	90.20	83.04	7.80	7.30	1.60	1.40
	3	4.95	3.90	0.97	1.14	83.13	69.33	3.73	3.31	2.43	2.05
	4	4.99	4.05	1.04	1.18	76.67	64.41	9.09	8.72	3.26	2.73
	5	4.99	4.40	1.27	1.41	90.39	82.62	5.96	5.72	4.09	3.46
	6	4.97	4.17	1.30	1.50	92.64	82.13	5.49	5.16	4.92	4.15
	7	4.96	4.22	1.01	1.14	80.13	70.35	3.42	3.13	5.75	4.85
	8	4.87	4.51	1.51	1.60	91.50	86.83	9.45	9.31	6.56	5.61
	9	4.96	4.03	0.96	1.11	92.29	80.09	6.36	5.99	7.39	6.28
	10	3.85	3.63	1.46	1.53	77.92	74.97	5.04	4.95	8.03	6.88
	11	4.99	4.43	1.18	1.29	91.03	83.67	4.44	4.22	8.86	7.62
Exp. 1	12	1.20	1.07	1.20	1.31	41.88	40.21	**17.37**	**17.32**	9.06	7.80
	13	2.92	2.58	1.08	1.18	65.47	61.04	5.54	5.41	9.55	8.23
	14	1.24	1.03	0.93	1.08	43.33	40.52	4.00	3.91	9.75	8.40
	15	2.94	2.46	0.96	1.11	65.64	59.41	3.21	3.12	10.24	8.81
Exp. 2	16	0.15	0.14	0.94	1.06	9.68	9.45	**3.21**	**3.09**	10.27	8.83

Table 6. Cash flow summary for the four stages of the analysis

Cash Flow (MUS$)	(1) NPV - BASE CASE		
	Init Pit	**Exp 1**	**Exp 2**
Revenue	1,790.08	2,006.40	2,016.08
OPEX	956.68	1,173.00	1,182.68
CAPEX	432.88	463.01	466.22
NPV	**89.24**	87.12	84.87

Cash Flow (MUS$)	(2) ROV - GEOLOGY		
	Init Pit	**Exp 1**	**Exp 2**
Revenue	1,750.59	1,851.19	1,960.86
OPEX	858.47	959.07	1,068.74
CAPEX	428.74	458.48	461.60
NPV	113.28	**113.78**	111.21

Cash Flow (MUS$)	(3) ROV - PRICE		
	Init Pit	**Exp 1**	**Exp 2**
Revenue	1,342.58	1,819.96	1,898.21
OPEX	854.35	967.77	968.36
CAPEX	421.51	447.83	448.56
NPV	123.92	140.22	**145.95**

Cash Flow (MUS$)	(4) ROV - GEO & PRICE		
	Init Pit	**Exp 1**	**Exp 2**
Revenue	1,265.09	1,741.89	1,820.30
OPEX	774.96	887.80	888.43
CAPEX	424.60	444.89	445.62
NPV	135.64	158.39	**158.58**

expected, depending on the turn of events. This last possibility is also considered in cases 3 and 4 in Table 6, which will be described in the following sections.

4.2.2 Case of Stochastic Market

The following case looks at the effect of gold price variability over the project's evaluation, and the influence this has over the operation's size and potential expansions. In this case, the orebody is represented by the estimated model, so the only difference from the base case is the inclusion of a stochastic price forecasting model.

Historical data can be used to obtain the required parameters of the formulation. Figure 5 shows the monthly gold price from January 1990 to December 2012, where the positive drift as well as the occasional jumps can be clearly perceived. For this case study, gold prices where simulated using a Geometric Brownian Motion with Poisson exponential jump diffusion model, in order to include the sudden extreme changes in price that have been seen recently in gold's price behavior. The parameters for the model where obtained by maximum likelihood over the past 15 years of price data, presenting a volatility of 13.8%, and a drift of 2.8%. In this case, we define a jump as a change of price in two consecutive periods of more than 3 standard deviations along the 10 year period. If a jump is found, its value is removed from the time series and the standard deviation is re-calculated to study the presence of more jumps. In this case, this process is done three times, what gives a frequency of 0.1 jumps per year, with a jump size of 10% (difference in size between the moment after the jump and its previous

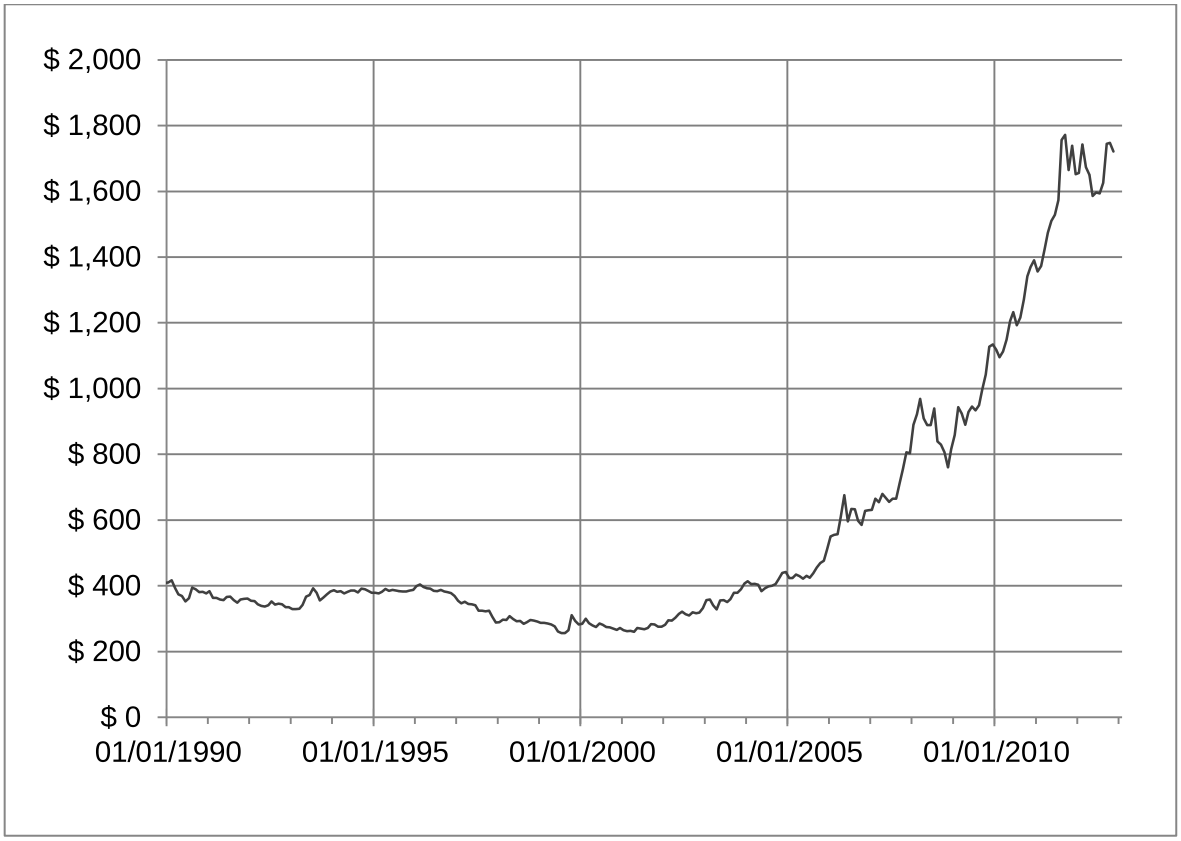


Figure 5. Historical monthly gold price data from January 1990 to December 2012

period), and a jump volatility of 15% (difference in size between different jumps). In this case, the direction of the jump used for the simulation was 50% probability upwards and 50% probability downwards.

For clarification purposes, one price simulation is a price path or vector of annual gold prices, which starts form ‘today’ at the current initial price of 700 US$/oz. (as in the base case), and evolves stochastically until time ‘T’, which corresponds to the maximum life of mine considering all expansions available. This means that all simulations share the same starting point of 700 US$/oz. and after spread out proportionally to time, according to the model and its parameters. Figure 6 presents 10 price paths that show the price evolution along the periods, where it is possible to see the effect of the added “jumps” into the model. Price forecast 1 (PF 1) presents an upwards jump along year 10, and has a favorable trend overall. PF 2 shows an upwards jump along year 1, and a downwards jump during year 8, what reduces the overall variability of that particular path. Finally, PF 3 shows a downwards jump along year 2, and steady variability from there on, providing a negative scenario to evaluate. With this, it can be seen that the forecasting model developed will provide an overall analysis of the possible price behaviors along the evaluation period.

The previous stochastic price model is applied over the operation, and the project is evaluated as explained in Section 3, assuming that the schedule is fixed for the initial pit limit as well as for the expansions. In this case, 20,000 price simulations where generated; as the computational cost of generating each forecast is marginal, and given the high variability of price, more simulations will provide a

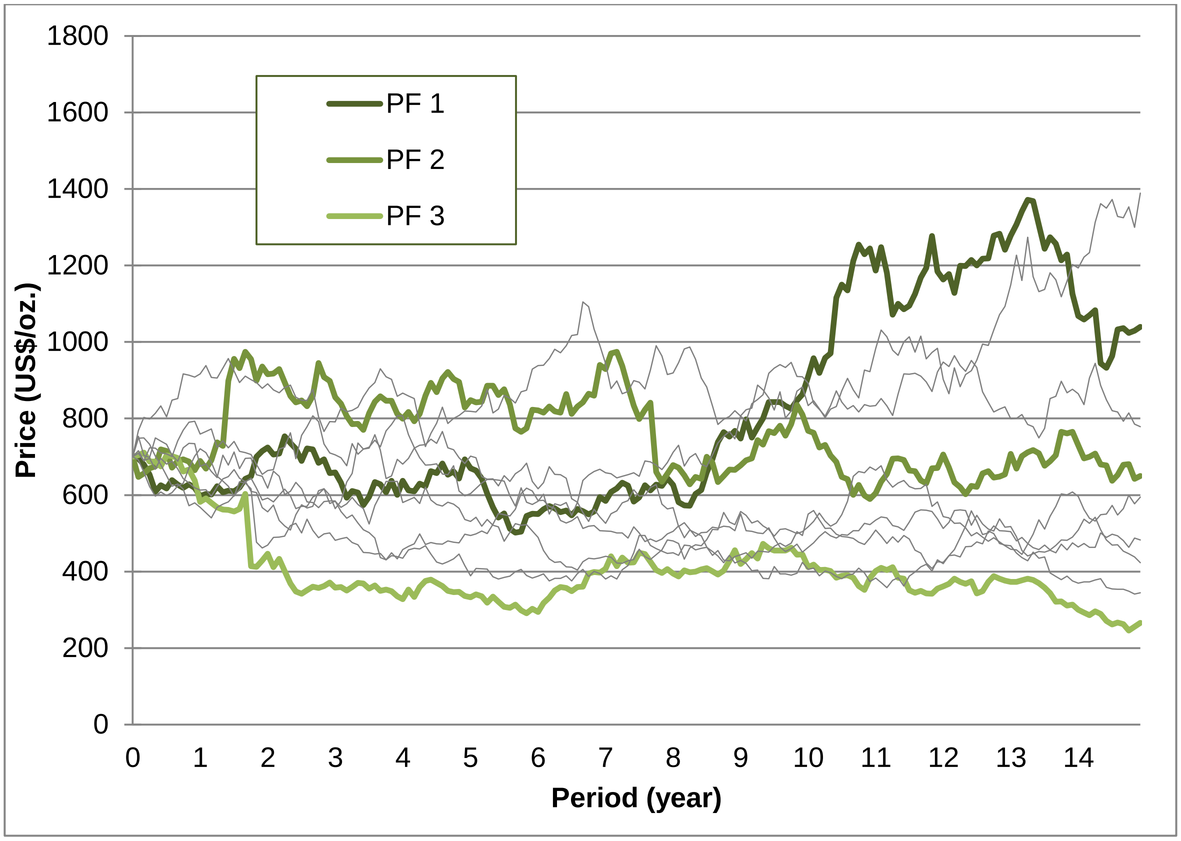


Figure 6. Example of 10 price forecasts generated with parameters obtained from historical data.

better range to represent the commodity price's distribution, which after 16 years (time span of the forecasts), varies considerably. However, it was seen that results already converge to a distribution with half the number of forecasts.

The summary of the obtained cash flow for this case is presented in the third case of Table 6, showing that having the flexibility to expand until year 15 can increase the project's value in more than a 13%, and almost 18% if the second expansion is considered. This suggests that according to the described conditions, both expansions would be feasible and profitable for the project.

It must be noted that these values consider having the flexibility to expand until year 15 or 16, what doesn't mean that the mine will actually be operating invariantly until years 15 or 16. It does mean that management can decide to stop mining whenever is considered optimal, and also expand past the initial pit's limit. This is clearly presented in Fig. 7, where the left axis shows the mine's probability of being in operation from year to year (with curves labelled 'Prob. Open'), and the right axis shows the frequency of optimal life of mine for each of the 20,000 price simulations (labelled 'LOM Freq.'). This is shown for the current case of price uncertainty ('E-TYPE'), and for comparison, Fig. 7 also includes the base case's annual probability of being operational, labelled 'Prob. Open (BC)'. This figure shows that there is a 10% chance that the mine closes beforetime by year 8 due to the price fluctuations, a 45% probability of the mine expanding to 15 years and a 5% to 16 years, and only a 30% chance of closing as expected by year 11. In contrast, the base case shows an absolute 100% - 0% probability shift

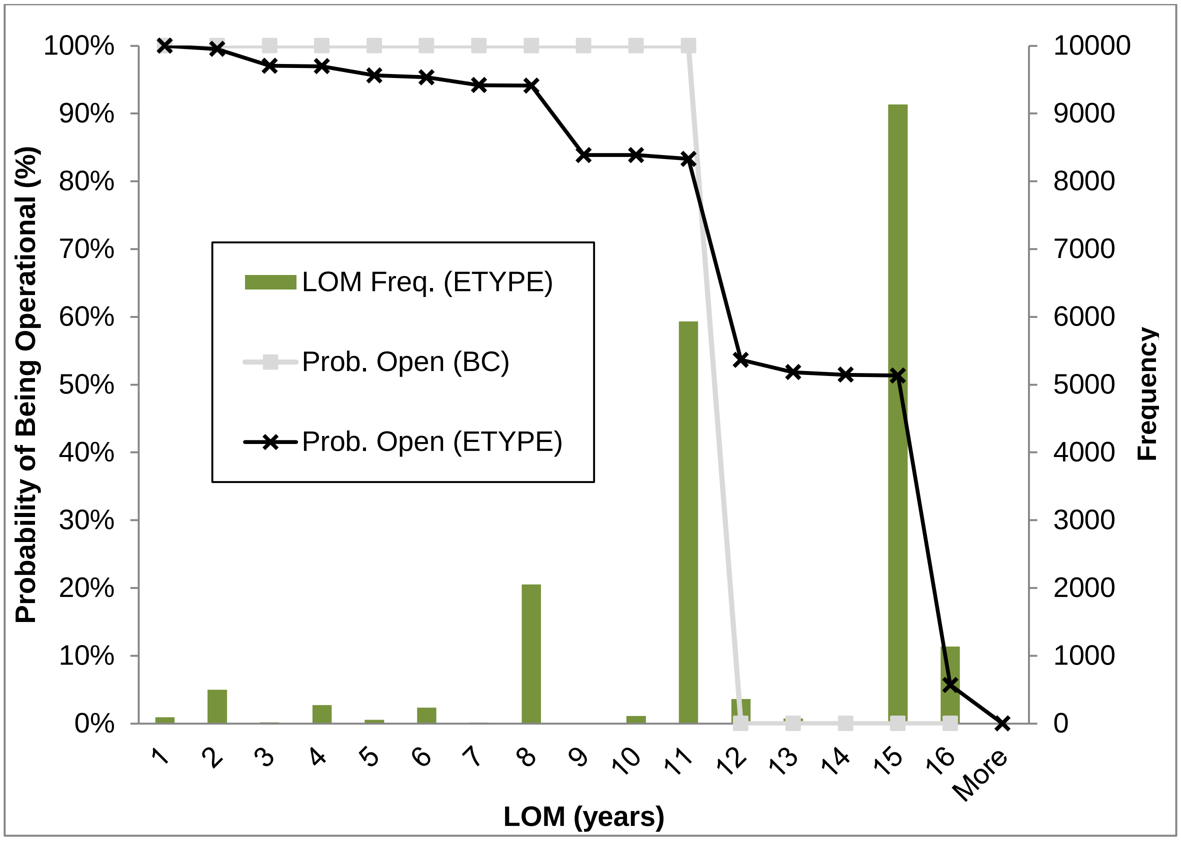


Figure 7. Probability of the mine being open and frequency of LOM considering uncertain price for the estimated model and stochastically simulated models

of being open by year 11, where none of the expansion stages available are of any interest.

An advantage of ROV is that it allows obtaining result distributions, such as the one presented in Fig. 7, which can't be obtained by the traditional evaluation methods. The information provided here can be highly valuable for the strategic decision-making process, allowing resolutions such as making sure that the operation has enough flexibility and contractual freedom to review the option of closing by year 8 if the price scenario is unfavorable, but also, that the plant and any other fixed infrastructure is located away from the 16-year LOM pit limit, to avoid relocations in case gold price increases and the operation chooses to expand. In both cases there may be some additional costs involved, such as extra transportation if the plant is further, or less-favorable contract negotiations if the lease terms are shorter, however these costs are marginal compared to the probable profits obtained or the losses prevented. All these may be considered as the cost of having (and maintaining) the 'option of closing', which is quantified and included in the evaluation process.

4.2.3 Joint Stochastic Model

This final step combines geological uncertainty with the stochastic price model in order to create a global stochastic evaluation model. In this case, the procedure is the same as the one described in Section 3, where the expanded schedule is applied over each of the orebody realizations and price paths to obtain different project values and expected LOMs. In order to obtain comparable results with the previous case of market uncertainty, the 20 simulated orebody models are evaluated over the same 20,000 price paths generated in step 4.2.2, i.e. for each estimated model evaluation, there are 20 other evaluations done over the same price path - one for each of the orebody simulations.

This case's cash flow is summarized in the final case of Table 6, where the first and second expansions have potential of increasing the project's value in a 16% and 17% respectively over the initial ultimate pit's value. The actual LOM frequency and the operation's probability of being open are presented in the right-most column of Fig. 8, where it shows that the probability of expanding to 15 and 16 years is even higher than in case 4.2.2 (46% and 6% respectively); however, the overall conclusions are the same.

Figure 9 presents the independent value of each of the individual stages, and for each of the evaluation cases, showing that there is an important value difference between considering or not considering the different uncertainty. However, the ROV of the three stochastic cases agree in the same thing: that there is an important profit to be made if the option of delaying or advancing the mine closure is considered given the underlying uncertainties, a fact that goes unnoticed by the traditional DCF method presented in the leftmost column of Fig. 9. So by accounting for uncertainty, it is possible to obtain higher rewards for an operation that presents less risk.

It is also noted from Fig. 9 that, if the price is considered stochastic (cases 3 and 4), the value difference between taking or not taking into account the geological uncertainty reduces from the initial pit to the 1st and 2nd expansions. This is

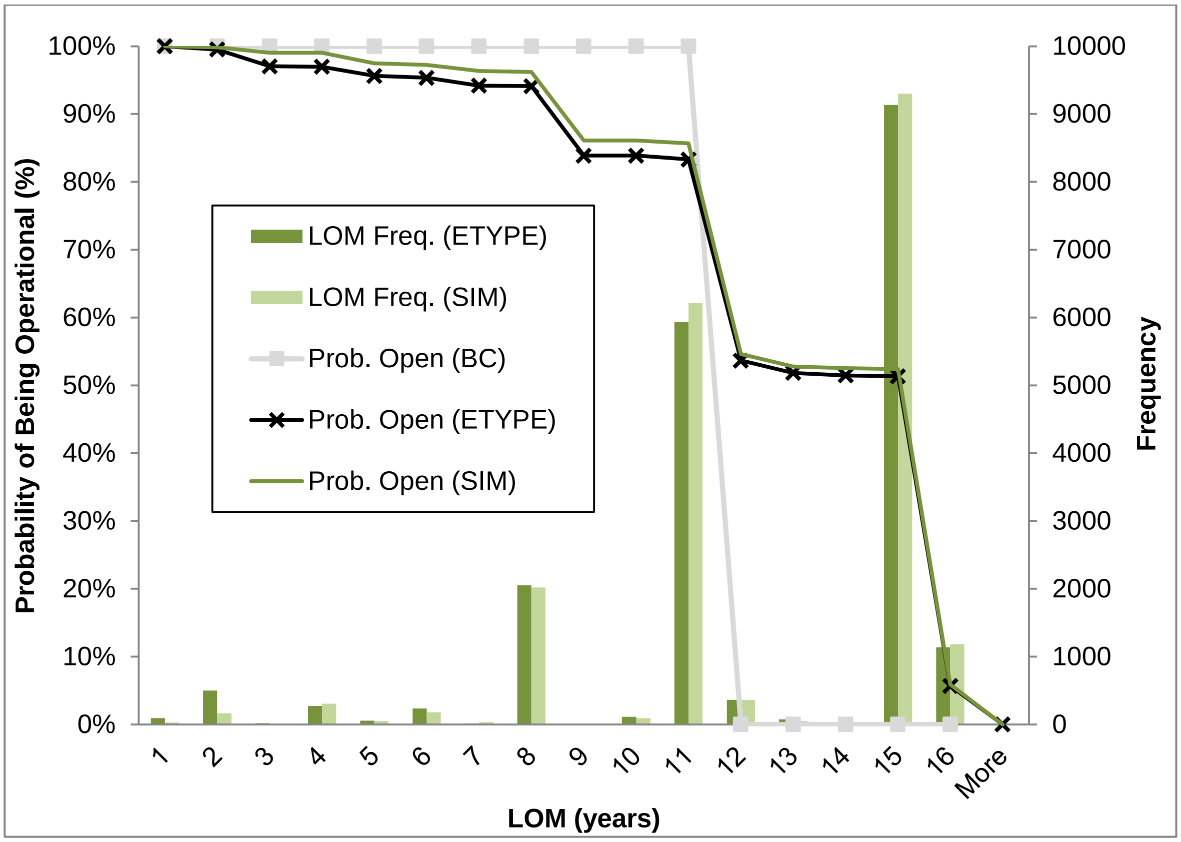
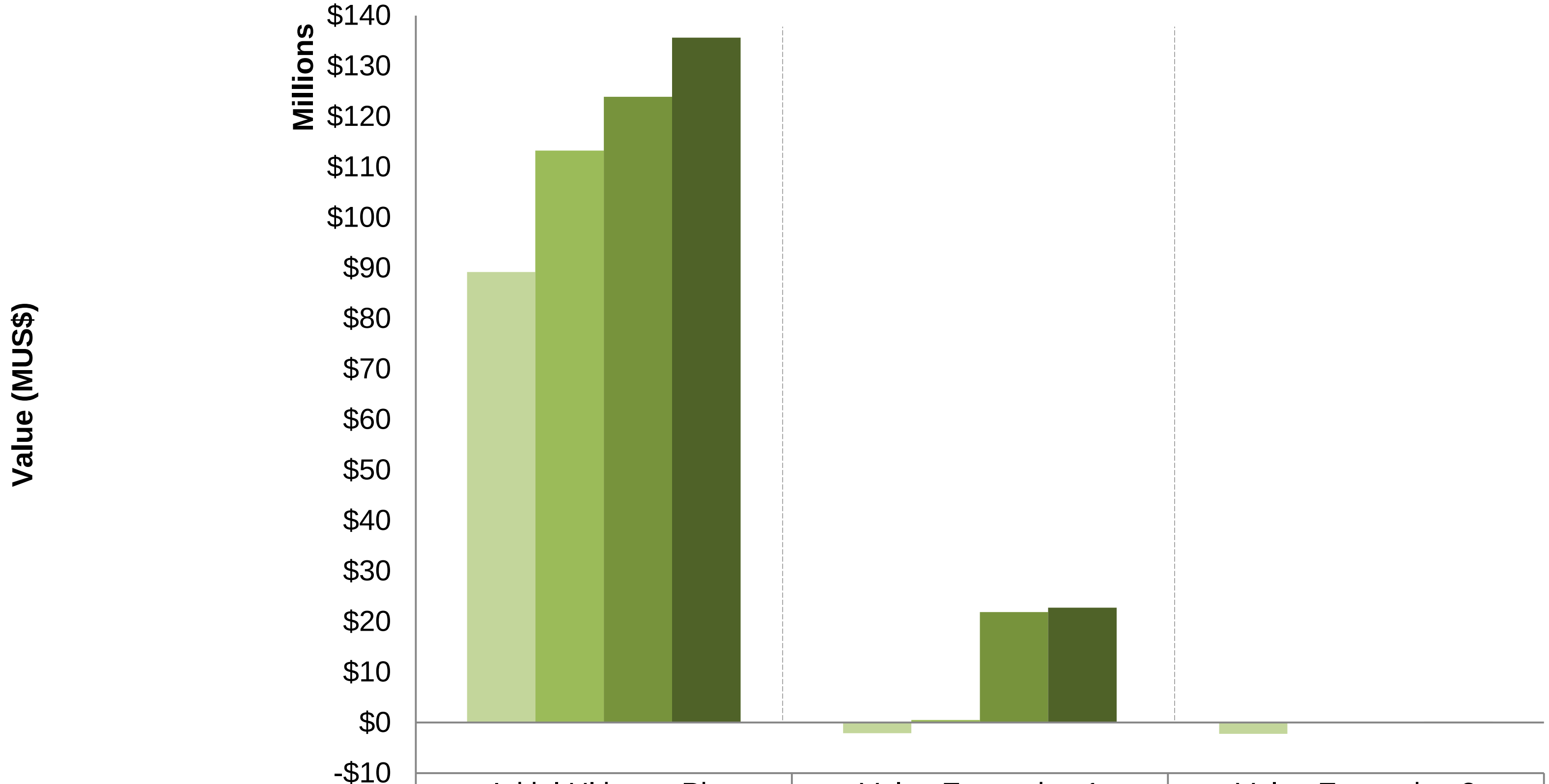


Figure 8. Probability of the mine being open and frequency of LOM considering (i) uncertain price (dark grey) and (ii) price and geology (light grey) for the estimated and stochastically simulated models

expected, given that if the mine decides to expand, there has been a favorable economic and technical development of the uncertainties, with probably high prices, what causes more material to be classified as potential ore in both the estimated model and the simulated models, making the ‘smoothness’ of the estimated model less damaging to the overall project value. However, this only occurs if the prices are ‘high enough’ to make the low grade blocks of the simulations profitable to process.

For a more detailed evolution of the project’s performance, Fig. 10 shows a risk analysis of the project’s annual cash flow for the initial pit and the two following expansions, for cases (3) and (4), considering only price uncertainty in the first case (‘E-TYPE’), and geology as well as price in the second (‘SIM’). In the figure, ‘Stopped Mining’ represents the annual probability of the operation having decided to stop the operation, which after that is irreversibly closed. ‘Negative CF’ shows the probability of having a negative net cash flow on a particular period, which implies that even though there is negative net cash flow, the overall project value is higher if the mine is operating during that period, what suggests that extracting may cover some of the closing costs, or that subsequent years will pay for that year’s losses. For example, years 13 and 15 show that if the operation is open, extracting those years will generate positive net cash flows in almost every case, what would pay for the losses incurred on the previous years (12 and 14), where the probability of having negative cash flow is almost the same as the one of profit. This risk analysis helps managers to have a better understanding of the possible behavior of the operation, and to detect the crucial periods where strategic decisions (such as expand or stop mining) will have to be taken.

From Fig. 10 it is possible to see that even though the estimated model with price uncertainty case described in Section 4.2.2 behaves similarly to the simulations analyzed in this section, there is a consistent gap between the two cases, where the estimated model has a constantly lower probability of annual positive cash flow and a higher probability of a negative one. Together with this, the estimated model presents a higher probability of closing in comparison with the simulated orebodies. The difference, however, diminishes past the initial pit limit as the favorable price scenario shadows the consequences of smoothing over the project's evaluation.



	Initial Ultimate Pit (LOM=11)	Value Expansion 1 (LOM=15)	Value Expansion 2 (LOM=16)
(1) BASE CASE	$89,238,168	-$2,120,700	-$2,250,100
(2) ROV - GEOLOGY CASE	$113,281,871	$506,456	$-
(3) ROV - PRICE CASE	$123,919,763	$21,876,089	$156,221
(4) ROV - JOINT CASE	$135,639,184	$22,751,901	$191,568

Figure 9. Value of individual expansions for each of the four stage analysis

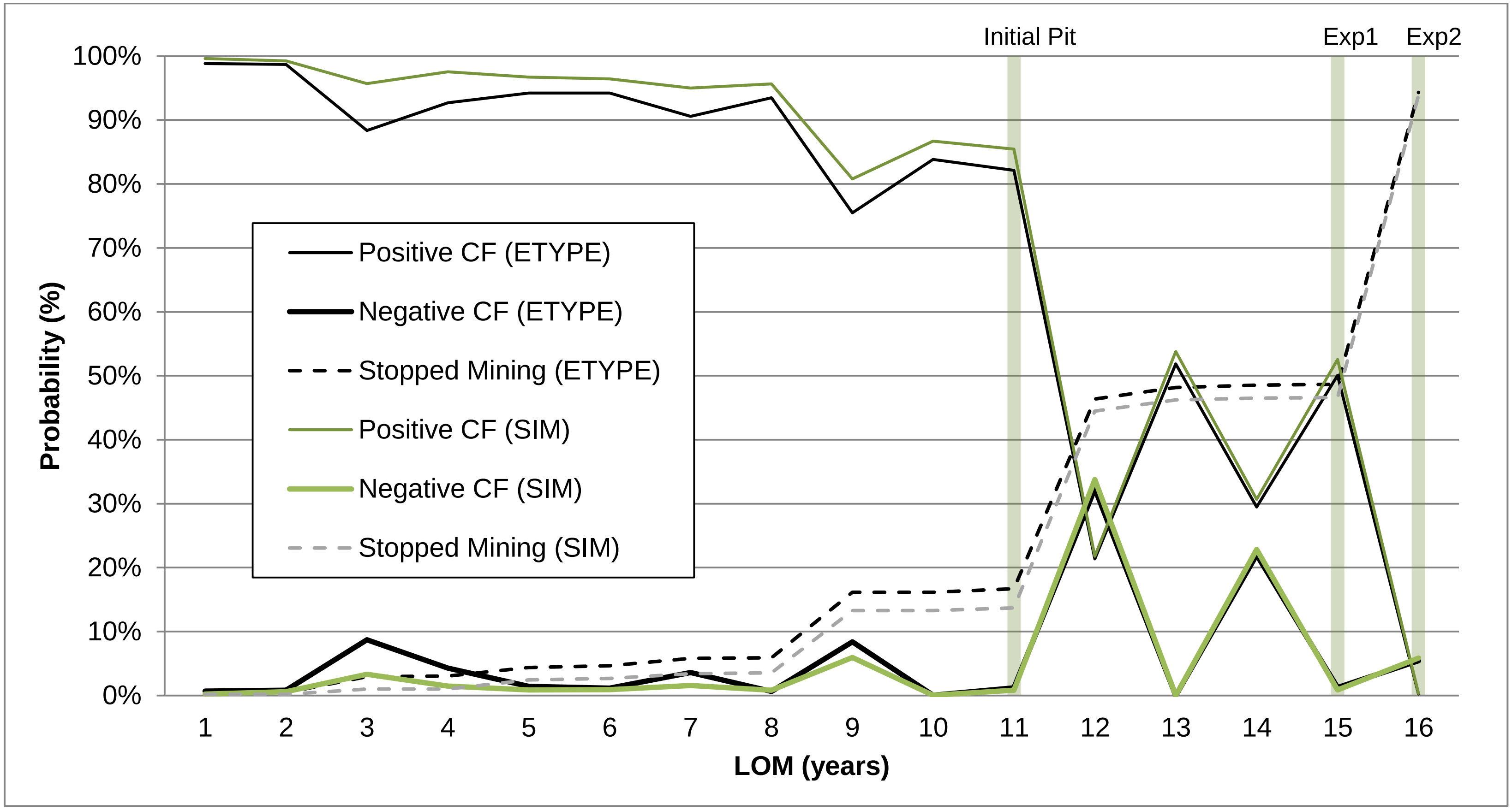


Figure 10. Risk analysis of the annual cash flows for the initial pit and two expansions, considering uncertain price for the estimated and stochastically simulated orebodies

5 Conclusions and Future Work

The current study analyzed the efficiency of the traditional evaluation methods to assess the performance of a mining operation under uncertain geological and price scenarios, and provided an alternative real option-based method that includes the option of expanding or contracting the initial ultimate pit limit, subject to these uncertainties.

A case study on an open pit gold mine is used to describe the methodology and show the benefits of the method. Experimental results indicate that traditional methods tend to underestimate the size of the final pit, ignoring possibly profitable expansions, as documented only for geological uncertainty (Albor and Dimitrakopoulos, 2010), and that considering uncertainties in the evaluation model and allowing for the operation to react to these uncertainties greatly increases the project value.

The proposed methodology accounts for the value that management and decision-makers generate by taking advantage of opportunities and hedging from unfavorable scenarios which are unknown at initial stages of the project. It was shown that real options are able to include this flexibility value in the global evaluation model. Together with this, it was shown that uncertainty-based analysis provide probabilistic results, which help decision makers be prepared to react to the continuously changing context of a mine project.

This analysis can be considered as a comprehensible way of including different sources of uncertainty in project evaluation and design, allowing to approximate the potential pit limit modifications at an early planning stage of the project, defining infrastructure-free zones and providing highly valuable information to decision makers so that slight adjustments can be done to the design at no or very low cost, in order to facilitate the execution of these flexibilities, and prevent huge losses in the future.

Even though the actual metal price, or the exact ore grade can’t be known with certainty, accounting for their uncertainty helps analyze the possible range of outcomes that the project might have when faced to the certain changes in context. To do this, direct block simulation proved to be an effective method to represent the geological uncertainty of the deposit, and the geometric Brownian motion with Poisson jumps provided a reasonable range of market values to evaluate the effect of price uncertainty in the initial mine design.

However, there are some important limitations to the method: mainly assuming that the schedule is fixed over all the evaluation stages, as, subject to price and geological changes, the schedule ceases to be optimal, what limits the applicability of the study to only asses the performance of one given schedule. Another limitation is the relevance that the inputs and parameters fed to the model have over the final value, which characterize the different uncertainties considered in the analysis. Together with this, processing costs are also assumen constant along the whole optimization process, which is not realistic in the cases of stochastic geology, as grades fluctuate, and with that, the costs of processing the material also change.

Further studies should focus on creating a stochastic integer programing model to generate an uncertainty-based schedule that includes price and geological uncertainty, in order to expand the applicability of the previous methodology. Together with this, the following step of this study should be to include the different complexities that mine operations contain, such as multi-pits, multiple products and/or multiple processing methods, as well as stockpiling options.

References

Albor, F. and Dimitrakopoulos, R., 2009. Stochastic mine design optimization based on simulated annealing: pit limits, production schedules, multiple orebody scenarios and sensitivity analysis, Mining Technology, 118(2), pp. 79-90.

Albor, F. and Dimitrakopoulos, R., 2010. An algorithmic approach to pushback design based on stochastic programming: Method, application and comparisons. IMM Trans, Mining Technology, 119(2), pp. 68-83.

Amram, M. and Kulatilaka, N., 1999. Book Real options: managing strategic investment in an uncertain world, pp. 240-246.

Asad, M.W., 2007. Optimum cut-off grade policy for open pit mining operations through net present value algorithm considering metal price and cost escalation. Engineering Computations, 24(7), pp. 723-736.

Asad, M.W. and Dimitrakopoulos, R., 2012a. Optimal production scale of open pit mining operations with uncertain metal supply and long-term stockpiles, Resources Policy, 37(1), pp. 81-89.

Asad, M.W. and Dimitrakopoulos, R., 2012b. Implementing a parametric maximum flow algorithm for optimum open pit mine design under uncertain supply and demand, Journal of Operational Research Society, 13(1), doi:10.1057/jors.2012.26

Benndorf and Dimitrakopoulos, 2007. New efficient simulation of large orebodies. Orebody Modelling and Strategic Mine Planning, Spectrum Series 12 2nd edition. The Australasian Institute of Mining and Metallurgy, Melbourne, pp. 103-111.

Black, F., and Scholes, M., 1973. The Pricing of Options and Corporate Liabilities. Journal of Political Economy, pp. 637-659.

Blanco, C., Choi, S. and Soronow, D., 2001. Energy price processes used for derivatives pricing and risk management. Commodities Now, pp. 74–80.

Blanco, C. and Soronow, D., 2001. Jump diffusion processes-energy price processes used for derivatives pricing and risk management. Commodities Now, pp. 83–87.

Boucher, A. and Dimitrakopoulos, R., 2009. Block-support simulation of multiple correlated variables. Mathematical Geosciences 41(2), pp. 215-237.

Boucher A, and Dimitrakopoulos R., 2012. Multivariate block-support simulation of the Yandi iron ore deposit, Western Australia. Mathematical Geosciences 44(4), pp. 449-468.

Brennan, M. and Schwartz, E., 1985. Evaluating Natural Resource Investments. Journal of Business, 58(2), pp.135-157.

Campbell, J., Lo, A. and MacKinlay, C., 1996. The Econometrics of Financial Markets. Princeton University Press, Princeton.

Cardin, M., de Neufville, R., and Kazakidis, V., 2008. Process to improve expected value of mining operations. Mining Technology, 117(2), pp. 65-70.

Clifford A. Ball and Torous W., 1983. A Simplified Jump Process for Common Stock Returns. Journal of Financial and Quantitative Analysis, 18, pp. 53-65.

Cortazar, G., Gravet, M. and Urzua, J., 2008. The valuation of multidimensional American real options using the LSM simulation method. Computers & Operations Research, 35(1), pp.113-129.

Davis, M. W., 1987. Production of conditional simulations via the LU triangular decomposition of the covariance matrix: Mathematical Geology, 19(2), p9. 91–98.

De Neufville, R., and Scholtes, S., 2011. Flexibility in engineering design. MIT Press.

Dimitrakopoulos, R., 2011. Stochastic optimization for strategic mine planning: A decade of developments. J Mining Science 84(2), pp. 138-150

Dimitrakopoulos, R., Farrelly, C.T. and Godoy, M.C., 2002. Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design, Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology, pp. 111.

Dimitrakopoulos, R. and Grieco, N., 2009. Stope design and geological uncertainty: Quantification of risk in conventional designs and a probabilistic alternative. Journal of Mining Science, 45(2), pp. 152-163.

Dimitrakopoulos, R., Luo, X., 2004. Generalized sequential Gaussian simulation on group size v and screen-effect approximations for large field simulations. Math. Geol. 36(5), pp. 567–591

Dimitrakopoulos, R. and Sabour, S.A., 2007. Evaluating mine plans under uncertainty: Can the real options make a difference? Resources Policy, 32(3), pp.116-125.

Dimitrakopoulos, R., Martinez, L. and Ramazan, S., 2007. A Maximum Upside /Minimum Downside Approach to the Traditional Optimization of Open Pit Mine Design. Journal of Mining Science, 43(1), pp.73-82.

Dixit, A., and Pindyck, R., 1994. Investment Under Uncertainty. Princeton University Press, Princeton.

Godoy, M., 2003. The effective management of geological risk in long-term production scheduling of open pit mines: Unpublished PhD thesis, The University of Queensland, Brisbane, Qld., pp. 256.

Godoy, M.C. and DImitrakopoulos, R., 2004. Managing risk and waste mining in long-term production scheduling. SME Transactions 316, pp. 43-50.

Godoy M, Dimitrakopoulos R (2011) A risk quantification framework for strategic risk management in mine design: Method and application at a gold mine. Journal of Mining Science 84(2).

Goovaerts, P., 1997. Geostatistics for Natural Resources Evaluation, New York, Oxford University Press.

Hull, J., 1997. Options, Futures, and Other Derivatives, 3rd ed. Upper Saddle River, NJ: Prentice Hall.

Jammazi, R. and Aloui, C., 2012. Crude oil price forecasting: Experimental evidence from wavelet decomposition and neural network modeling, Energy Economics, 34(3), pp. 828-84.

Journel, A.G., 1988. Fundamentals of geostatistics in five lessons. Stanford Center for Reservoir Forecasting, Applied Earth Sciences Department.

Lee, E. and Strang, D., 2003. Valuation techniques used in the mining industry Part three: Real option valuation approach. Mining Engineering, 55(12), pp. 8-9.

Lemelin, B., Sabour, S.A. and Poulin, R., 2007. A flexible mine production model based on stochastic price simulations: application at Reglan mine, Canada. Mining Technology 116(4), pp. 158-166.

Leite, A., and Dimitrakopoulos, R., 2007. Stochastic optimization model for open pit mine planning: application and risk analysis at copper deposit.

Luo, X., 1998, Spatiotemporal stochastic models for earth science and engineering applications: Unpublished PhD thesis, McGill University, Montreal, pp. 177.

Mardones, J., 1993. Option valuation of real assets: application to a copper mine with operating flexibility. Resources Policy 19, pp. 51–65.

McCarthy, J. and Monkhouse, P., 2003. To open or not to open - or what to do with a closed copper mine. Journal of Applied Corporate Finance 15, pp. 56-66.

Meade, N., 2010. Oil prices — Brownian motion or mean reversion? A study using a one year ahead density forecast criterion, Energy Economics, 32(6), pp. 1485-1498.

Meagher, C., Sabour, S. and Dimitrakopoulos, R., 2009 Pushback Design of Open Pit Mines Under Geological and Market Uncertainties. In: Proceedings, Orebody Modelling and Strategic Mine Planning 2009, AusIMM, pp. 297-304.

Mendez, M. and Lamothe, P., 2002. Comodity Prices Modeling using Gaussian Poisson-Exponential Stochastic Processes, A Practical Implementation in the case of Copper. Proceedings 23rd European Conference on Modelling and Simulation.

Merton, R.C., 1976. Option pricing when underlying stock returns are discontinuous, Journal of Financial Economics, Volume 3, Issues 1–2, pp. 125-144.

Mingming, T. and Jinliang, Z., 2012. A multiple adaptive wavelet recurrent neural network model to analyze crude oil prices, Journal of Economics and Business, 64(4), pp. 275-286.

Moel, A. and Tufano, P., 2002. When are real options exercised? An empirical study of mine closing. The Review of Financial Studies 15(1), pp. 35-64.

Musingwini, C., Minnitt, R. and Woodhall, M., 2007. Technical operating flexibility in the analysis of mine layouts and schedules. The Journal of the Southern African Institute of Mining and Metallurgy, 107(February), pp.129-136.

Oldfield, G., Rogalski, R. and Jarrow, R., 1977. An Autoregressive Jump Process for Common Stock Returns. Journal of Financial Economics, 5(3), pp. 389-418.

Ramazan, S. and Dimitrakopoulos, R., 2013. Production scheduling with uncertain supply: A new solution to the open pit mining problem. Optimization and Engineering 14(2), pp. 361–380.

Ravenscroft P., 1992. Risk analysis for mine scheduling by conditional simulation. Transcripts Inst. Min. Metall. 101, pp. 104-108.

Rendu, J. M., 2008. An Introduction to Cut-off Grade Estimation, SME.

Sabour, S.A. and Dimitrakopoulos, R., 2010. Will the Mine Expand? Re-thinking Beyond Ultimate Pit Limits and Uncertain Markets, COSMO Day 2010 Proceedings, pp 244-267.

Sabour, S.A. and Poulin, R., 2006. Valuing Real Capital Investments Using The Least- Squares Monte Carlo Method. The Engineering Economist, 51(2), pp.141-160.

Sabour, S.A. and Poulin, R., 2010. Mine Expansion Decisions Under Uncertainty. International Journal of Mining, Reclamation and Environment, 24(4), pp. 340-349.

Sabour, S.A. and Wood, G., 2009. Modelling financial risk in open pit mine projects: implications for strategic decision-making. The Journal of the Southern African Institute of Mining and metallurgy, 109(March), pp. 169-175.

Samis, M., Davis, G., Laughton, D. and Poulin, R., 2006. Valuing uncertain asset cash flows when there are no options: a real options approach. Resources Policy, 30(4), pp. 285–298.

Samis, M. and Poulin, R., 1998. Valuing management flexibility: a basis to compare the standard DCF and MAP frameworks. CIM Bull 91 (1019), pp. 69–74.

Shafiee, S. and Topal, E., 2010. An overview of global gold market and gold price forecasting. Resources Policy, 35(3), pp. 178-189.

Shibata, T., 2006. The impacts of uncertainties in a real options model under incomplete information. European Journal of Operational Research, 187, pp. 1368–1379.

Tan, Z., Zhang, J., Wang, J. and Xu, J., 2010. Day-ahead electricity price forecasting using wavelet transform combined with ARIMA and GARCH models. Applied Energy, 87(11), pp. 3606-3610.

Whittle, J., 1999. A decade of open pit mine planning and optimization - The craft of turning algorithms into packages. Proceedings of APCOM'99: 28 International Symposium, Colorado Sch. of Mines, Golden, Colorado USA, pp. 15-24.

Whittle, J., 2009. Whittle Consulting global optimization software. Melbourne, Australia. http://www.whittleconsulting.com.au/.