Automatic Scoring Up of Mensural Music Using Perfect Mensurations, 1300–1550

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Abstract

Music written in mensural notation—that is, most polyphonic music from the mid-thirteenth century to the sixteenth century—was written in separate parts rather than in score format. In order to study counterpoint in mensural music the parts must be aligned into a full score, a process referred to as “scoring up”. Scoring up involves transcribing the original notation and correctly aligning notes from different parts on the page, for which it is necessary to know the sounding duration of the notes. The difficulty mensural notation presents for scoring up is that the duration of individual note symbols (breve, semibreve, etc.) in perfect (triple) mensurations is not absolute, but rather context dependent. Because of this problem, it previously has not been possible to automate the process of scoring up parts. In this thesis, I present the first automatic scoring-up tool for music written in mensural notation between 1300 and 1550.

In mensural notation, every note symbol has a default value given by the *mensuration*: the metrical relation between the value of one note and that of the next smaller metrical level. This metrical relation can be either *imperfect* (i.e., duple) or *perfect* (i.e., triple). In the case of perfect mensuration, the default value of a note can be changed by its context (i.e., by the notes preceding and following it). Based on the principles of imperfection and alteration outlined by Franco of Cologne in his treatise *Ars cantus mensurabilis* (ca. 1280), I implement a system that solves the main issues of triple meter: identifying when a *perfect* note should preserve its value or be *imperfected* (i.e., worth two thirds of its original value), and when a note should keep its original duration or be *altered* (i.e., be twice as long). Additionally, the tool is able to perform the following tasks: identify the functionality of dots—that is, distinguishing between *dots of division* (i.e., dots that separate notes in perfect groups) and *dots of augmentation* (i.e., dots that add to a given note one half of its value)—despite their identical appearance; deal with hemiola coloration; and handle perfect mensuration at different note-levels (semibreve, breve, and long) in more than one note-level at a time. The performance of the scoring-up tool is tested on a set of fourteenth- and fifteenth-century pieces encoded in MEI (Music Encoding Initiative), a symbolic format that supports the encoding of mensural notation.

Résumé

La musique écrite en notation mensurelle — à savoir, la plupart des pièces polyphoniques écrites du milieu du treizième siècle jusqu'au seizième siècle — a été transcrite en parties séparées plutôt qu'en une seule partition. Afin d'étudier le contrepoint dans la musique mensurelle, il faut donc aligner les parties séparées dans une même partition, un processus appelé la « mise en partition » (*scoring up*). Ce processus implique la transcription de la notation originale et l'alignement exact sur la page des notes provenant des différentes parties. Il est donc nécessaire de connaître la durée de chaque note. La difficulté qui se présente dans la mise en partition de la notation mensurelle est que les durées des différentes notes (brève, semibrève, etc.) dans les mensurations ternaires (parfaites) ne sont pas absolues, mais dépendent du contexte. À cause de cela, à ce jour, il n'y a pas de méthode d'automatisation pour le processus de mise en partition. Dans cette thèse, nous allons présenter le premier processus automatique pour la mise en partition de la musique écrite entre les années 1300 et 1550.

Dans la musique mensurelle, chaque note a une valeur de base donnée par la *mensuration* : la relation métrique entre la valeur d'une note et celle du prochain niveau métrique plus petit. Cette relation métrique peut soit être *imparfaite* (binaire) ou *parfaite* (ternaire). Dans le cas de la mensuration parfaite, la valeur de base d'une note peut changer selon le contexte (c'est-à-dire selon les notes qui précèdent et suivent). En nous basant sur les principes de l'imperfection et de l'altération exposés par Franco de Cologne dans son traité *Ars cantus mensurabilis* (vers 1280), nous avons implémenté un outil qui résout les principaux problèmes de la division ternaire : identifier si une note *parfaite* doit préserver sa valeur ou devenir une note *imparfaite* (c'est-à-dire raccourcie d'un tiers) ; discerner si une note doit préserver sa durée originale ou être *altérée* (c'est-à-dire doubler sa durée originale).

De plus, l'outil accomplit les tâches suivantes : identifier la fonction des points, c'est-à-dire distinguer, malgré leur apparence identique, les *points de division* (les points qui séparent les notes en groupes parfaits) et les *points d'augmentation* (les points qui ajoutent à une note donnée la moitié de sa durée originale) ; prendre en compte la coloration des hémioles ; traiter la mensuration parfaite appliquée à des

valeurs de notes variées (semibrève, brève et longue) dans plusieurs niveaux de notes simultanément. L'efficacité de l'outil de mise en partition est mesurée en utilisant une collection de pièces des quatorzième et quinzième siècles encodées en format MEI (*Music Encoding Initiative*), un format symbolique qui permet l'encodage de la notation mensurelle.

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My heartfelt thanks are extended to my husband for his patience, understanding, love, and even editorial work in this thesis. I would also like to thank my parents, my sister, and my parents-in-law for all their love, support, and understanding in this time.

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Chapter 1 Introduction

Mensural notation is the system of music notation used in polyphonic vocal music between the middle of the thirteenth century and the end of the sixteenth century (Apel 1953, xxii). It introduced the use of different note shapes to represent different (relative) note values, getting one step closer to the current Western music notation system. This form of rhythmic notation made it easier to coordinate performances of polyphony written in parts rather than in score. Most music in mensural notation is presented in separate parts, with each voice located on a different section of the page or in separate books—namely, partbooks—which only contain the parts specific to that voice for a series of compositions (Boorman 2017).

In order to study counterpoint (i.e., the relation between the voices) in mensural music the pieces must be presented in full score. Scoring up (i.e., assembling the individual parts in score form) involves the correct vertical alignment of the notes from the different parts, for which it is necessary to know the durational value of the notes beforehand. The difficulty mensural notation presents, however, is that durational values in triple (or perfect) mensurations are not absolute, but rather context dependent. Because of these problems, no automatic scoring-up process has been developed yet for mensural music.

The goal of this thesis is to provide an automatic scoring-up tool for music written in mensural notation that deals with the context-dependent nature of the system. For this purpose, a literature review regarding the evolution of mensural notation and of the principles behind its interpretation will be conducted. From this literature review I will extract a set of rules regarding the interpretation of the note values based on their context. This set of rules will then be reformulated to allow them to be implemented in a programming language.

With respect to the terminology used in this thesis, the following issue has to be considered: in standard Western music notation, the term “note value” means the name, the shape, and the length of a note; in mensural notation, however, the “note value” refers to the name or shape of a note, but not to its length. For this reason, from now on, I will refer to the length of a note as its “durational value” and to the name of a note (which also denotes its shape) as its “note value”.

1.1 Brief History of Early Western Music Notation

The beginnings of what would eventually become the common Western music notation system can be traced back to the ninth-century notation used in early medieval plainchant (Parrish 1978, 3–5; Hiley and Szendrei 2017), a monophonic (i.e., single, unaccompanied, melody) sacred form of vocal music used in the Christian liturgy. Plainchant was written in *neumes*, which are symbols that indicate the melodic contour. Two basic neumes, the *virga* (represented by a stroke) and the *punctum* (represented by a dot), were used for single notes; all other neumes consisted of two or more notes joined together (Parrish 1978, 5). These neumes were located above the syllables of the text of a chant. Until the eleventh century neumes provided melodic-contour information (i.e., changes in the direction of the melody), but they did not provide absolute pitch information or even relative pitch information by specifying the size of the intervals. In the eleventh century, with the introduction of the staff by Guido d’Arezzo (ca. 1030), neumes began to convey pitch information (Hiley and Szendrei 2017). Figure 1-1 shows an example of this using neumes in square notation. According to Parrish (1978, 3, 5), once neumes reached the stage of square notation on a staff (in the late twelfth century), the notation for plainchant reached a stable state. On the other hand, the notation of polyphonic music continued to evolve from the symbols of square notation into the mensural system of the late Middle Ages (Parrish 1978, 3).

In twelfth-century polyphony written in modal rhythm, rhythmic information was conveyed by the square symbols themselves, specifically by the configuration of ligatures (these are the neumes that consist of two or more notes joined together), as pointed out in Garlandia’s *De musica mensurabili*.[1] Around the mid-thirteenth century the use of different note shapes to represent different note values was introduced (Parrish 1978, 108–9), allowing for rhythm to be conveyed in a way other than ligature arrangement. This marked the beginning of the mensural notation system. As pointed out by Kelly (2015, 125–6), no new symbols were introduced by the new system; the

[1] According to recent scholarship Garlandia was not the author, but rather the editor, of this treatise (Baltzer 2017).

notes used to represent the different note values were the already existing virga and punctum: the virga was used to represent a long note value and the punctum to represent a short one, renamed as *long* and *breve*, respectively. At the same time the use of distinct note shapes was introduced, the score arrangement, present already in the modal rhythmic system, was abandoned in favor of a separate arrangement of the voices (Bent 2017; Scholes, Nagley, and Grier 2017; Apel 1953, 271).

Figure 1-1: Square notation in staff (CDN-Hsmu, fol. 2r).[2]

In the new mensural system, the durational value of the note was context dependent: the long could be either *perfect* (equal to three breves) or *imperfect* (equal to two breves) and its actual durational value depended on the notes preceding and

[2] The chant shown is *Aspiciens a longe ecce video* from the *Salzinnes Antiphonal*. Image source: *Cantus Ultimus* project (https://cantus.simssa.ca/manuscript/133/?folio=002r).

following it, and the breve could be either *regular* or *altered* (twice its regular duration) based on the notes surrounding it. Franco of Cologne outlined a set of principles in his *Ars cantus mensurabilis* (ca. 1280) regarding the interpretation of these two note types based on the context; these principles are known as *principles of imperfection and alteration*, which were used to keep the structure of the music in triple meter. In the fourteenth century, Franco's principles of imperfection and alteration extended to other note levels. The main innovation in this period was the introduction of duple meter, which was not considered in Franco. Other features introduced include colored notes and dots of augmentation (analogous to dotted notes in common Western music notation). As found in Rastall (2010, 67), all these innovations marked the distinction between the mensural music of the old period (Franco's period), known as *Ars antiqua* (old art, which includes modal and early mensural notation), and the new period, known as *Ars nova* (new art).[3] In the early fifteenth century the note heads stopped being filled in. Mensural notation in the Renaissance using these hollow notes was called *white mensural notation* (in opposition to the *black mensural notation* that had been in use before); although the notes appeared different, the principles governing white mensural notation were the same as those used in Ars nova black notation. In the late fifteenth century, there was a trend towards notation simplification (Bent 2017). In the early sixteenth century, most music was in duple meter, and there was less complex use of triple meter: around the turn of the century, musicians increasingly often placed a dot after a note that was to be perfect, even when the older practice would not require one given the context; at the same time, the principle of alteration gradually fell into disuse (Bent 2017; Chew and Rastall 2017). In the seventeenth century, features from instrumental music were introduced into vocal polyphony, such as the barline, slur, and beams (Chew and Rastall 2017). The seventeenth century saw once again the adoption of the score format in vocal music (Chew and Rastall 2017; Scholes, Nagley, and Grier 2017; Apel 1953, 271).

[3] The name *Ars nova* was used by the proponents of this *new art*, as can be seen in Vitry's *Ars nova* treatise and Muris' *Ars novae musicae*; and the contemporary defenders of the older practice gave the *old art* the name of *Ars antiqua* (or, *Ars vetus*) as can be seen in Jacques of Liège *Speculum musicae*.

1.2 Project Overview

The goal of this thesis is to provide the first attempt at the automatic scoring up of mensural music. I am focusing on music written after the thirteenth century, since the principles governing Ars nova and white mensural music are nearly identical. The main issue that the scoring-up implementation has to solve is determining the duration of a mensural note shape based on its context. The algorithms for solving this problem are, in great part, built upon Franco's principles of imperfection and alteration. Thus, a secondary goal of this thesis is to test the completeness of Franco's system in a set of Ars nova and white notation pieces.

The scoring-up tool requires as input machine-readable files that encode mensural pieces as they appear in the sources; that is, files that encode the music as separate voices and in mensural notation. There are very few symbolic music formats which can represent mensural notation. The one chosen in this thesis is the Music Encoding Initiative (MEI).

I built a database of MEI files that encode mensural music from the fourteenth and fifteenth centuries to test the scoring-up tool. Given that most music is encoded in modern values, I used a tool called the Mensural MEI Translator to obtain the mensural encoding of these works.[4] The Mensural MEI Translator takes the encoded modern transcription of a mensural piece and translates it back to mensural values which are encoded into a (mensural) MEI file.

The result of the scoring-up is a single MEI file that encodes all the voices of the mensural piece and includes the contextual durational value for all their notes. Therefore, when rendered in MEI's engraving software (Verovio), it is presented as a score with its notes aligned according to their encoded durational values.

In the future, the scoring-up tool could be part of an Optical Music Recognition (OMR) workflow, for which the input MEI files of the scoring-up tool would come out of the results of performing OMR on the mensural music sources.

[4] I developed the Mensural MEI Translator for the Measuring Polyphony Project with Karen Desmond (https://github.com/DDMAL/CMN-MEI_to_MensuralMEI_Translator).

1.3 Thesis Organization

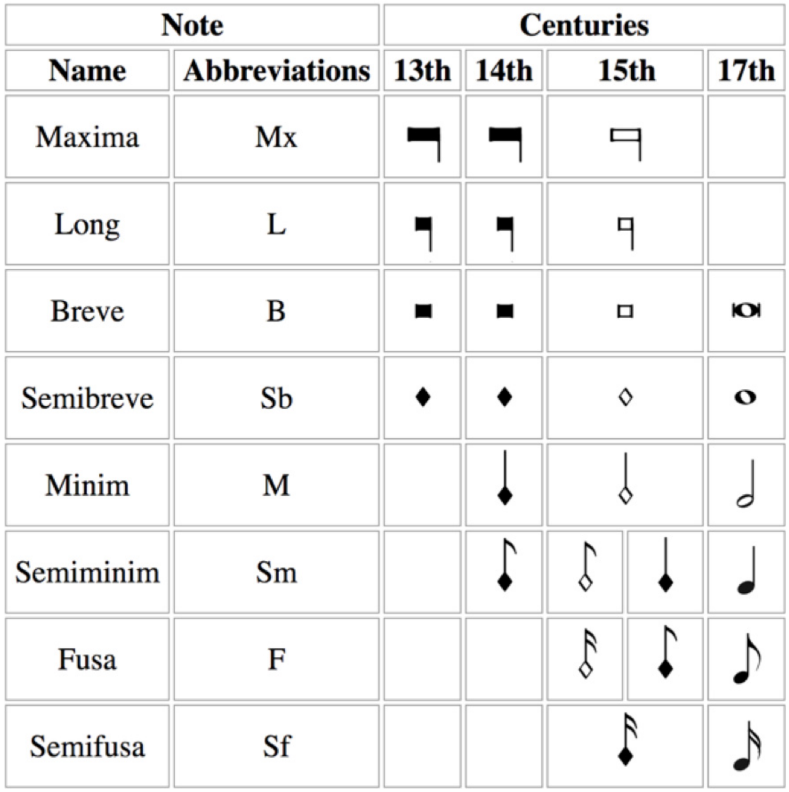
This thesis consists of five chapters. Chapter 1 (the current chapter) serves as an introduction. Chapter 2 includes an explanation of the mensural notation system, followed by a detailed history of mensural notation. This last section includes the principles outlined by Franco in his *Ars cantus mensurabilis* (ca. 1280), and a description of the expansions of the system during the period following Franco. And the third section focuses on the computer encoding of this notation by presenting a summary of the different symbolic formats that have been used to encode mensural notation, giving emphasis to the encoding systems currently available. Chapter 3 presents the design of the automatic scoring-up tool, including a description of the input files and of the operations the tool performs on them to obtain the durational value of each of the notes, allowing for the vertical alignment of them within a score. Chapter 4 consists of the description of the generation of the dataset, the evaluation method, the results of the experiment, and further discussion regarding possible methods to handle the sources of error in the scoring-up output. Finally, Chapter 5 contains the concluding remarks of this thesis.

Chapter 2 Background

This chapter presents a literature review on mensural notation, its history, and the modern technologies that support its computer encoding. The chapter is divided into four sections. Section 2.1 presents the basics of mensural notation: it introduces the type of notes used in the system, the concept of mensuration, the two types of modifications based on context, and other non-context-related features also used to modify a note's durational value. Section 2.2 contains a detailed history of mensural notation; the principles regarding the interpretation of the notes based on the context are outlined within this section. Finally, Section 2.3 presents various computer encoding systems for mensural notation.

2.1 Mensural Notation

As the predecessor of common Western music notation (CMN), mensural notation already included features that are present in CMN. In mensural notation pitch was already represented in the same way as in CMN, by making use of staff lines and clefs (Music Encoding Initiative 2017a). Furthermore, mensural notation uses different note shapes to represent different note values (Parrish 1978, 108), which is still the case in CMN. Table 2-1 shows the different note shapes used during the mensural notation period, and how they evolved through this period until they became the CMN note shapes in use since the seventeenth century.



Note		Centuries			
Name	Abbreviations	13th	14th	15th	17th
Maxima	Mx				
Long	L				
Breve	B				
Semibreve	Sb				
Minim	M				
Semiminim	Sm				
Fusa	F				
Semifusa	Sf				

Table 2-1: Note shapes used in mensural notation (from the thirteenth to the end of the sixteenth century) and in common Western music notation (since the seventeenth century)

Even though different symbols were used for different note values, one symbol could represent different durations. In CMN the note shape is enough to represent the durational value of a note. In mensural notation, while the note shape is necessary to determine a note's durational value, it is not sufficient. The relative duration of a note depends, also, on mensuration and context. This was a trait inherited from the *modal notation system* that preceded mensural notation.

Mensuration is the name given to the metrical relation between the value of one note and that of the next smaller degree (Apel 1953, 96). There are four types of mensuration, which describe the relation of the notes at different levels (see Table 2-2). The *modus major* (or *maximodus*) indicates the relation between the maxima and the long, the *modus minor* (or *modus*) indicates the relation between the long and the breve, *tempus* indicates the relation between the breve and the semibreve, and *prolation* indicates the relation between the semibreve and the minim. These relations can be either triple or duple, which can be denoted by using the adjectives "perfect" or “imperfect”, respectively, with the appropriate mensuration type: *perfect* or *imperfect modus major*, *perfect* or *imperfect modus minor*, and *perfect* or *imperfect tempus*. In the case of prolation, however, the adjectives used to denote the triple or duple durational value of the semibreve with respect to the minim are “major” and “minor”, respectively. Despite the different adjectives used for the different mensurations, a note with a triple durational value is called *perfect* and a note with a duple durational value is called *imperfect*. Thus, in ***perfect*** *tempus and* ***major*** *prolation*, the breve and the semibreve are both perfect (i.e., both of them have triple durational value).

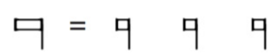
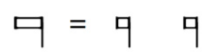
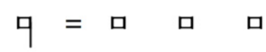
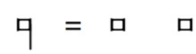
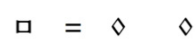
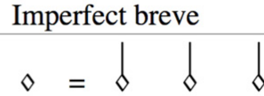
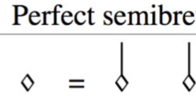
Mensuration	Defined Relation	Mensuration Value	Relative Duration
Modus Major	Maxima - Long	Perfect	 Perfect maxima
		Imperfect	 Imperfect maxima
Modus Minor	Long - Breve	Perfect	 Perfect longa
		Imperfect	 Imperfect longa
Tempus	Breve - Semibreve	Perfect	 Perfect breve
		Imperfect	 Imperfect breve
Prolation	Semibreve - Minim	Major	 Perfect semibreve
		Minor	 Imperfect semibreve

Table 2-2: Mensuration defined at the different note levels

Even though all notes have a default perfect or imperfect durational value given by the mensuration, in the case of triple meter (i.e., perfect mensuration) a note's durational value could be modified by its context (i.e., by the notes preceding and following it). Thus, a note can be modified in the following two ways:

- *Imperfection*, which means that a perfect note loses one third of its durational value, becoming imperfect.
- *Alteration*, which consists in making a note twice as long as usual. Only the note values that are one level below a perfect note value can be altered. For example: in perfect tempus, the semibreve can be altered (as the semibreve is the next note level below the breve, which, in this case, is perfect); but none of the other note values can be altered. In the case of perfect tempus and major prolation, both semibreves and minims can be altered (since the breve and the semibreve are perfect), but none of the other note values can be altered.

The *principles of imperfection and alteration* were formalized by Franco for the long-to-breve relation (these principles are listed in Section 2.2.3). According to Parrish (1978, 110), "the logic behind these principles is that the underlying movement of the music is by a series of perfections, each of which is three breves long". As an example of these principles, take for instance the sequence of longs shown in Figure 2-1a. All these longs must be perfect in order to "move in a series of perfections". Now, change the second long of the sequence to a breve as shown in Figure 2-1b. While the longs following the breve are still perfect, in order to keep the movement of the music in a series of perfections, the first long and the breve must form a perfect group (i.e., be equivalent to three breves). In order for this to happen, the first long must be imperfected, reducing its durational value from three to two breves. The imperfect long and the following breve are, then, equivalent to three breves, forming a perfection. Now, consider the case shown in Figure 2-1c, with two breves in place of a long. In this case, imperfecting the first long of the sequence would not achieve a perfection before the next long. Here is where the alteration principle is used, by doubling the durational value of the second breve in the sequence, the two breves between the longs form a perfection, and the first long is left perfect. The same principles of perfection, imperfection, and alteration, have applied to the other three note levels since the French Ars nova; see Section 2.2.3 for an explicit account of these principles.

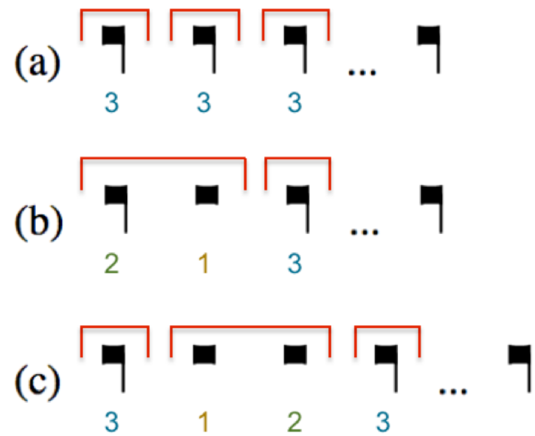


Figure 2-1: Examples of the principles of perfection (a), imperfection (b), and alteration (c). The numbers below the notes indicate the durational values in terms of the breve, while the brackets above mark the groups of notes that form a perfection, which illustrates the use of the principles of imperfection and alteration as a way to keep the structure of the music in triple meter.

The durational value of the notes in mensural notation, as one can see, is context dependent. The same note shape can have different durational values (e.g., perfect, imperfect, and altered) in the same composition, or even in the same phrase. This is why the note shape, even though necessary, is not enough to specify the durational value of a note. In addition to context, there are other features that can modify locally the durational value given to a note by the mensuration. These features are: dots and coloration.

There are two types of dots: *dots of augmentation* and *dots of division*. The dot of augmentation can only be used on imperfect notes according to the mensuration. It has the same functionality as a CMN dotted note, in the sense that it adds half its durational value to a binary (i.e., imperfect) note. This changes the imperfect durational value of the note, as given by the mensuration, to a perfect durational value at that particular instance.

The dot of division is used only in case of perfect mensuration. This dot is used to "divide" a sequence of notes to indicate which notes should be grouped to form perfections. As an example, let us consider the sequence of notes given in Figure 2-1b.

In this sequence, the first long and the following breve form a perfection; but if a dot is placed between these two notes (Figure 2-2 bottom), the first long is separated from the rest of the notes indicating that it forms a perfection by itself. This changes the interpretation of the notes in the sequence, as now the long following the breve must be imperfected (to keep the underlying movement of the music as a series of perfections) instead of the long preceding it. This particular dot of division is known as a “dot of perfection”, given its use to keep a perfect note from being imperfected. The interpretation of the notes in Figure 2-1c also changes if using a dot between the two breves; instead of an alteration, the dot of division implies that both longs should be imperfected, as each of them should form a perfection with the breve next to them (see Figure 2-3).

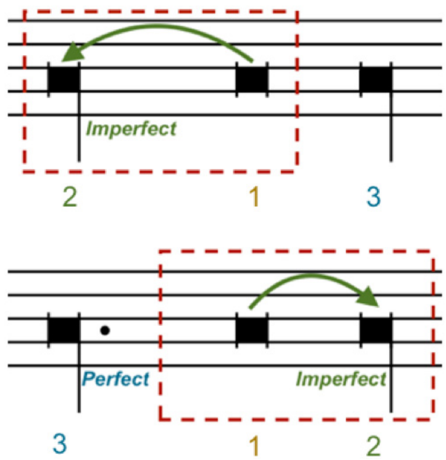


Figure 2-2: Example of the use of a dot of division in the sequence of notes of Figure 2-1b

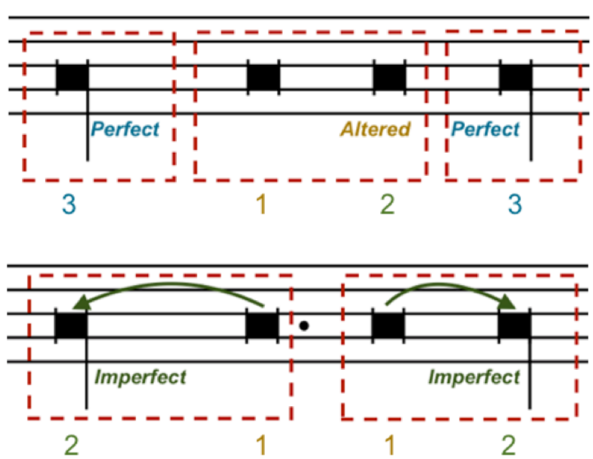


Figure 2-3: Example of the use of a dot of division in the sequence of notes of Figure 2-1c

Another way to modify the durational value of a note was by writing it in a different color, a mechanism known as *coloration* (Apel 1953, 126). In the fourteenth century, when regular notes were black, the colored notes were written in red ink. Later, in the mid-fifteenth century, with the regular use of hollow notes in white mensural notation, colored notes were filled-in in black. Regardless of the color used, the effect of coloration on the durational value of a note was to reduce it by $1/3$.[5] This means $colored\ note\ = 2/3 \times uncolored\ note$ as seen in Figure 2-4.

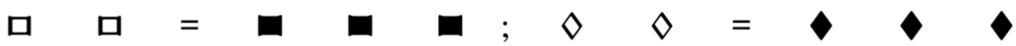


Figure 2-4: Relation between colored and uncolored notes (Apel 1953, 126).

Coloration was introduced in the fourteenth century in perfect mensurations as a way to change the durational value of a note from perfect to imperfect. Later on, it was used in imperfect mensurations, where it also reduced the note by a third; but in this case, the same action had a different effect since the notes already had an imperfect durational value—namely, it generated new note values. Regardless of the mensuration, the durational value of a colored note is always binary; this means

The type of coloration used in perfect mensuration is called *hemiola* (Greek for "containing one and a half"), which stands for the 3:2 relation between colored and uncolored notes established by this type of coloration (Figure 2-4). This type of coloration was used in perfect tempus, where three colored breves took the place of two uncolored breves (*hemiola temporis*), and in major prolation, where three colored semibreves took the place of two uncolored semibreves (*hemiola prolationis*). By substituting the notes in Figure 2-4 with smaller note values, it becomes clear that, while the relation between the colored and uncolored notes at the level of the hemiola coloration—that is, at the level of the breve in hemiola temporis, or the level of the semibreve in hemiola prolationis—is given by the 3:2 ratio, the relation for smaller notes is defined by a 1:1 ratio. This is illustrated in Figure 2-5 for the case of hemiola temporis.

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[5] In more complex fourteenth-century pieces, red coloration can be used in other ways, but I will not discuss these here.

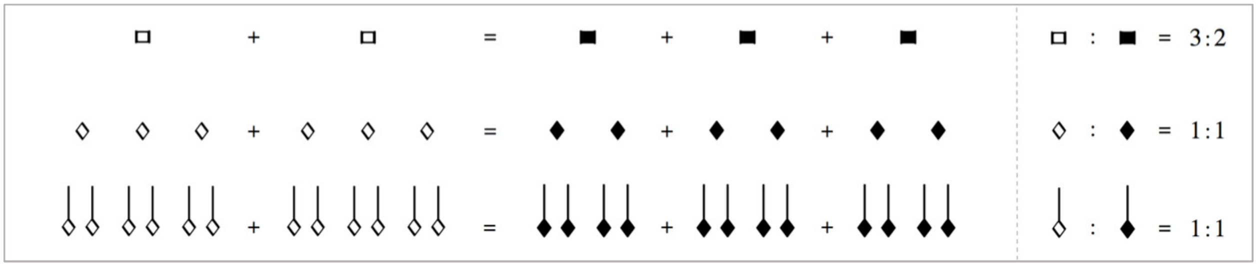


Figure 2-5: Relation between colored and uncolored notes in hemiola temporis (Apel 1953, 131)

On the other hand, in imperfect mensuration, all colored notes, even the ones with small values, are in a 3:2 relation with respect to their uncolored counterparts (Figure 2-6). A special case of coloration in imperfect mensuration is called *minor color*, which consist of a colored semibreve followed by a colored minim. The interpretation of this combination of colored notes is identical to the dotted rhythm expressed by an augmented minim followed by a semiminim (Figure 2-7).

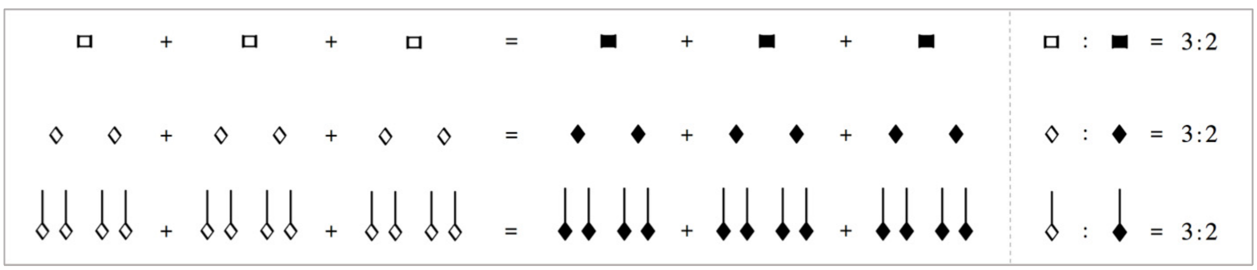


Figure 2-6: Relation between colored and uncolored notes in imperfect mensuration (Apel 1953, 131)

Figure 2-7: Minor color (Apel 1953, 128)

Besides these local changes in the durational values of the notes (due to context, or to the use of a dot of augmentation, or the use of coloration), there can also be global changes caused by a change in mensuration or the use of proportions. A change in mensuration changes the relation between the note values to the new relation indicated by the new mensuration sign. Proportions, on the other hand, consist of “the diminution or augmentation of the metrical values by a given ratio” (Apel 1953, 145); they are represented by a pair of numbers that indicate this ratio or by a single number. The interaction between proportions and mensuration is a complex topic out of the scope of this thesis; for more details on this matter, consult *Mensuration and Proportion Signs* (Busse Berger 1993).

2.2 Detailed History of Mensural Notation

Mensural notation developed from an earlier notation system used during the High Middle Ages in Paris called modal notation. This new mensural system originated with the introduction of the long and the breve to represent the long and short values of modal notation, respectively. The principles regarding the interpretation of these two notes were inherited from the old modal system, and were written out by Franco of Cologne. From then on, mensural notation underwent a series of changes that gave name to its different stages (Figure 2-8), starting with the Franconian notation of the Ars antiqua, followed by the changes of the Ars nova which, after a minor stylistic change, culminated in the white mensural notation system.

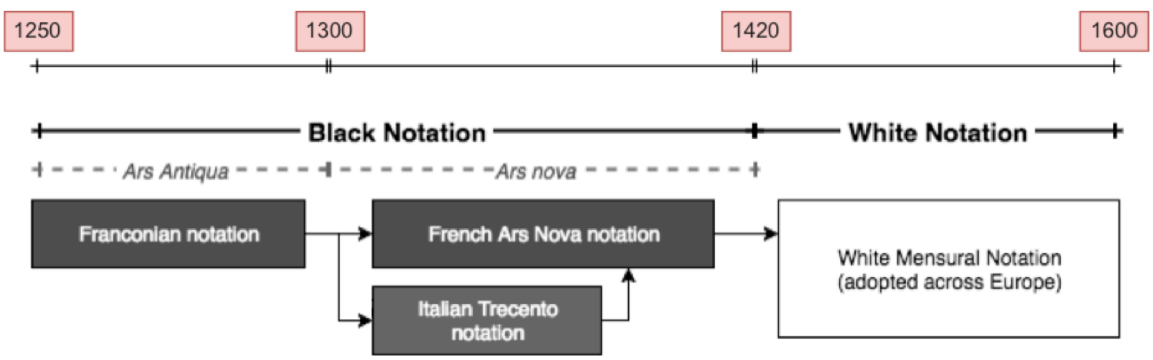


Figure 2-8: Evolution of mensural notation

2.2.1 Modal Notation

Modal notation is the first system of Western music notation to coherently record rhythm, and not just pitch information. It was developed by the Notre Dame School—the name given by modern scholars to musicians active in Paris between ca. 1150–1250 (Roesner 2017a, 2017b). As presented in Garlandia's *De musica mensurabili*, modal notation represented rhythm by conventional arrangements of ligatures. A *ligature* was a single glyph used to represent two or more notes to be performed sequentially. The most common ligature was the *binaria* (Figure 2-9), which consisted of two notes: the first one being short and the second one long. The *ternaria*, a three-note ligature, was also frequently used (Figure 2-10).

Figure 2-9: Two-note ligature: Binarias

Figure 2-10: Three-note ligatures: Ternarias

Different configurations of ligatures conveyed different rhythms, called *modal rhythms*; the six modal rhythms, together with their ligature configurations, are described in *De musica mensurabili* (Strunk and Treitler 1998, 2:113–6). In order to perform a passage with the appropriate rhythm, one has to determine first in which mode is that passage written in, which could be deduced by the arrangement of ligatures in that phrase. Table 2-3 shows the six rhythmic modes, together with their basic rhythmic unit and ligature arrangement.

According to Parrish (1978, 82), modal notation is used in various styles of the compositions from the Notre Dame school, "but it is at is clearest and simplest in the clausula".[6] A clausula was a polyphonic setting of a short chant melisma (i.e., a passage in plainchant in which several notes were sung to a single syllable of text). The tenor contained the melismatic chant melody, and the incipit of the text (i.e., the first word) to indicate the excerpt of the chant used. The second (upper) voice, called the *duplum*, had regular arrangements of ligatures showing a specific modal rhythm (see Figure 2-11) (Parrish 1978, 81–2).

[6] Some of the genres of compositions from the Notre Dame School include: conductus, organum, clausula, and early motet.

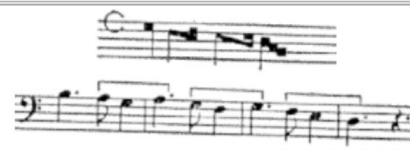
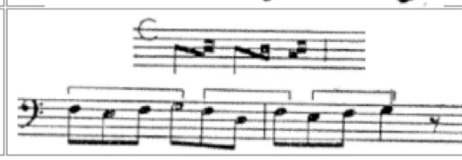
Modes	Rhythmic Unit	Rhythmic Pattern	Ligature Configuration	Examples
Mode 1	♩ ♪ Long - Short	L S L S L S L	A ternaria followed by a series of binarias 3-2-2	
Mode 2	♪ ♩ Short - Long	S L S L S L S	A series of binarias followed by a ternaria 2-2-3	
Mode 3	♩. ♪ ♩ Long - Short - Short (altered)	L S S L S S L	A single note followed by one or more ternarias 1-3-3	
Mode 4	♪ ♩ ♩. Short - Short (altered) - Long	S S L S S L S	A series of ternarias ending in a single note 3-3-1	
Mode 5	♩. ♩. Long - Long	L L L L ... L	Either a series of single notes 1, 1, 1 Or a series of ternarias 3, 3, 3	
Mode 6	♪ ♪ ♪ Short - Short - Short	S S S S ... S	A quaternaria followed by a series of ternarias 4-3-3	

Table 2-3: The six modal rhythms represented by different arrangements of ligatures. The different rhythms consisted on different patterns of short (represented by an "S") and long (represented by an "L") note values. A modification (i.e., alteration) of the short value is represented by an underlined "S". The example images were obtained from Parrish (1978, 76)

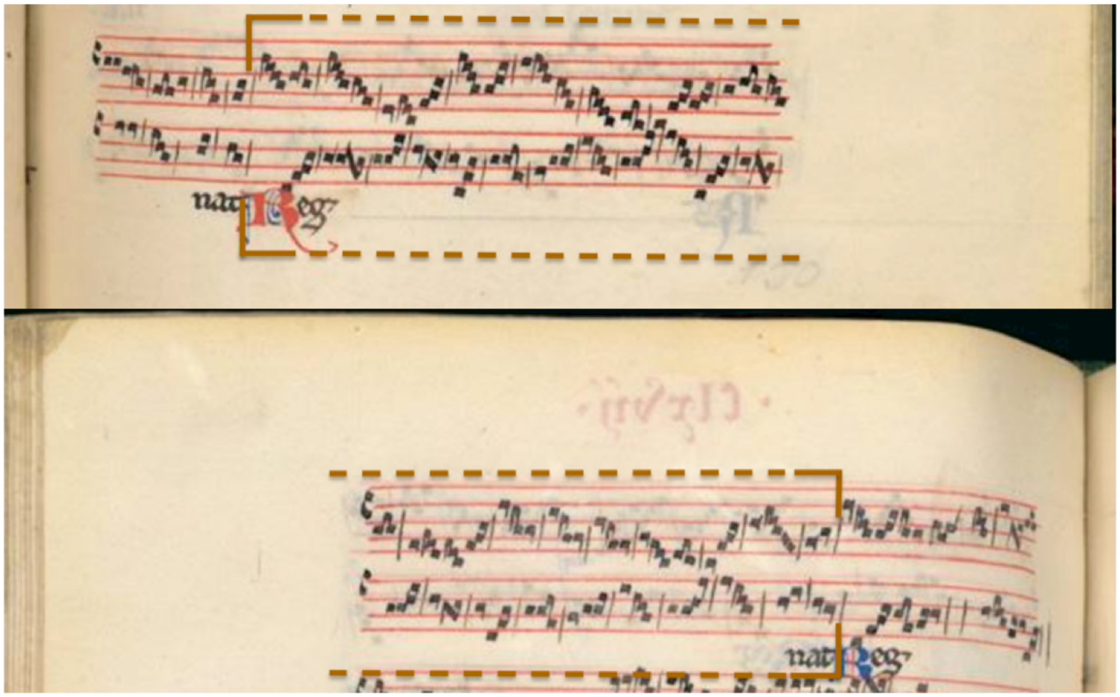


Figure 2-11: Regnat clausula (I-Fl, fol. 150r–150v)[7]

2.2.2 The Transition to Mensural Notation: The Motet and its Implications

The motet genre originated around 1225 by the addition of a full text to the duplum of a clausula, now also called *motetus* (Sanders and Lefferts 2017; Apel 1953, 81–2). Sometimes a third voice was added, called the *triplum*, also with its own text. Two changes accompanied the appearance of this new style: the transition from melismatic to syllabic notation in the texted voices, and the abandonment of the score arrangement, which would not reappear as the standard format for vocal music until the seventeenth century (Apel 1953, 271).

With syllabic notation, the assignment of one note to each syllable on the texted voices forced the breakup of the ligatures into individual notes. The rhythmic modes were still used, but they could no longer be represented by the arrangement of ligatures. Thus, different note shapes started being used to represent the short and long notes of modal rhythm.[8] The two distinct notes introduced were the punctum, now known as the

[7] Image obtained from the Biblioteca Medicea Laurenziana (http://teca.bmlonline.it). The two folios that contain the clausula can be found in the following link:
http://teca.bmlonline.it/ImageViewer/servlet/ImageViewer?idr=TECA0000342136#page/308/mode/2up.
[8] The earliest motets, the ones from the Notre Dame school, did not make use of different note shapes yet. But the modal rhythm of the *motetus* voice could be determined by checking the *duplum* of the

breve ■, and the virga, now known as the long ▜, for the short and long notes, respectively. The long had a time relation to the breve of either two to one (i.e., *imperfect*) or three to one (i.e., *perfect*) depending on the mode. The *imperfect* durational value of a long note was used for rhythms corresponding to mode 1 and 2; while the *perfect* durational value of the long was used for rhythms corresponding to modes 3 and 5. Thus, the modal rhythms from Table 2-3 were now expressed by individual notes as shown in Table 2-4. Principles of perfection, imperfection, and alteration can be inferred from Table 2-4 (e.g., mode 5 shows that long before long is perfect, modes 1 and 2 show that a long is imperfected by a following or preceding breve, and mode 3 exemplifies alteration).

Modes	**Individual Notes**							
Mode 1	▜ 2	■ 1	…	▜ 2	■ 1			
Mode 2	■ 1	▜ 2	…	■ 1	▜ 2			
Mode 3	▜ 3	■ 1	■ 2	…	▜ 3	■ 1	■ 2	
Mode 4	■ 1	■ 2	▜ 3	…	■ 1	■ 2	▜ 3	
Mode 5	▜ 3	…	▜ 3					
Mode 6	■ 1	…	■ 1					

Table 2-4: The six modal rhythms expressed by the breve and the long. The length of the note is given by the number below it, which represents the number of breves equivalent to the note.

Besides the long and the breve, the semibreve appears as a new type of note; its shape is taken from the *currentes* (diamond shapes of Figure 2-10) used in modal

original clausula (if it survives), since these motets were identical to the clausulae they were based on in everything but the use of text and single notes in the upper voices (Cumming 2000).

notation. However, unlike the long and the breve, the semibreve never appears isolated but in groups of two or three which are equivalent to a single breve (Apel 1953, 295).

2.2.3 Mensural Notation: Ars Antiqua

All the changes previously presented were summarized by Franco of Cologne in his treatise *Ars cantus mensurabilis*. It was previously thought that Franco's treatise was written around 1260 (Parrish 1978, 108) and, thus, the latter half of the thirteenth century is known as the period of Franconian notation. Recent scholarship places the date of the treatise around 1280, which places the theoretical writings of Johannes de Garlandia (c1240), Magister Lambertus (before 1279), and the St Emmeram anonymus (1279) before 1280 (Bent 2017). Even though these treatises also mentioned the changes leading up to the true mensural notation system, Franco's *Ars cantus mensurabilis* is still considered "the first clear and coherent exposition of the principles underlying that practice [the musical practice of the mid-thirteenth century]" by recent scholarship (Strunk and Treitler 1998, 2: 227).

According to the English translation of Franco's treatise found in Strunk and Treitler (1998, 227–245), Franco identifies three species of single notes (long, breve, and semibreve) and indicates the different durational values they can assume. The long has three varieties: perfect, imperfect, and duplex. The perfect long was worth three breves. The imperfect long was worth two breves and was used in combination with a preceding or following breve (as in modes 1 and 2); the imperfect long could not stand alone as only triple meter was allowed at the level of the long in the Ars antiqua. Both the perfect and imperfect long used the same figure  . The duplex long represents two longs combined into a single figure  . The breve, represented by  , has two varieties: proper (i.e., regular value) and altered. (i.e., two times a proper breve). And the semibreve can be either minor or major (i.e., twice as long). Franco's *principles of imperfection and alteration* regarding the interpretation of the long and the breve are as follows (Strunk and Treitler 1998, 2: 229–232):

1. If a long follows a long, then the first long is perfect. Apel (1953, 108) refers to this principle as “similis ante similem perfecta” (Latin for “similar before similar is perfect”).

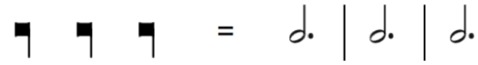


Figure 2-12: Modern transcription of the long, considering the breve as quarter note.

But if there are breves following the long, the following cases arise:

2. If a single breve follows the long, the long is imperfect (as in mode 1), unless the long is followed by a stroke (called *division of the mode* or *sign of perfection*); in this case, the first long is perfect, and the breve makes the following long imperfect. The stroke was later substituted by a dot (ca. 1280–1320), called *dot of division* or *dot of perfection* (Rastall 2010, 61, 81).

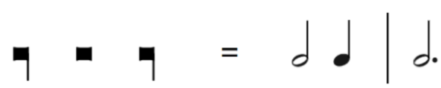


Figure 2-13: Modern transcription of a sequence with a single breve between longs.

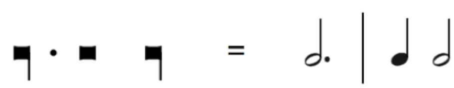


Figure 2-14: Change in the interpretation of the sequence of Figure 2-13 by the use of a dot of perfection

3. If only two breves follow the long, then the long is perfect (unless a single breve precedes it, which would correspond to the interpretation of mode 2). The first breve is considered a proper breve and the second one is considered an altered breve (as in mode 3). But if the *division of the mode* stroke (or dot of division) is placed between the two breves, then the first and second longs are imperfect, while the breves are both proper; this case is unusual.

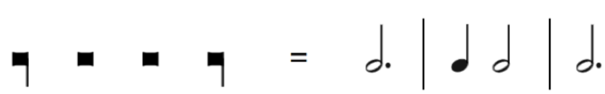


Figure 2-15: Modern transcription of a sequence with two breves between longs

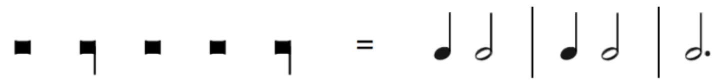


Figure 2-16: Same sequence as in Figure 2-15, but the first long is preceded by a breve which forces its imperfection, changing the interpretation of just this one note from the sequence L B B L in Figure 2-15

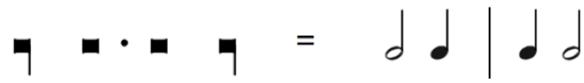


Figure 2-17: Change in the interpretation of the sequence in Figure 2-15 by the use of a dot of division

4. If only three breves stand between two longs, the first long is perfect and the three breves are proper. Again, an exception to this interpretation can be made if using a *division of the mode* stroke (or dot of division) after the first breve. In this case, the first long is made imperfect by the first breve and, with respect to the remaining two breves, the first one is proper and the last one is altered.

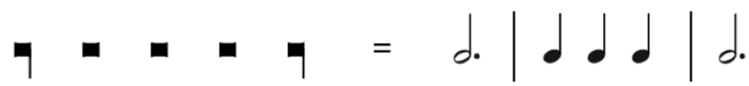


Figure 2-18: Modern transcription of a sequence with three breves between longs

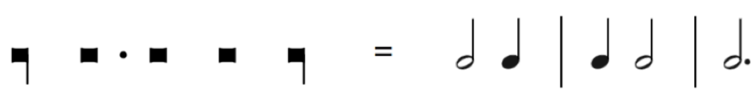


Figure 2-19: Change in the interpretation of the sequence in Figure 2-18 by the use of a dot of division

5. If there are more than three breves between two longs, then the first long is always made imperfect by the first breve that follows it (FiguresFigure 2-20, Figure 2-22, and Figure 2-24), unless the *sign of perfection* (or dot of perfection) separates the long from the breves (Figures Figure 2-21, Figure 2-23, and Figure 2-25). The remaining breves are grouped together in perfections (i.e., three breves); all of them are considered proper. If one breve is left out of these groups, it imperfects the following long (Figure 2-21 and Figure 2-22). If two are left over, then the first one is considered proper and the last one altered (Figure 2-23 and Figure 2-24).

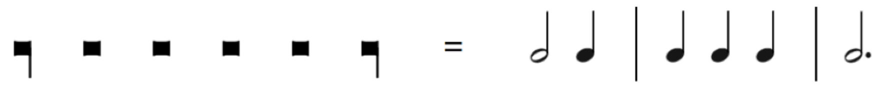


Figure 2-20: Modern transcription of a sequence of 4 (also for 7, 10, 13, etc.) breves between longs

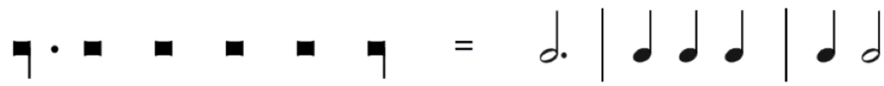


Figure 2-21: Change in interpretation of the sequence in Figure 2-20 by the use of a dot of perfection

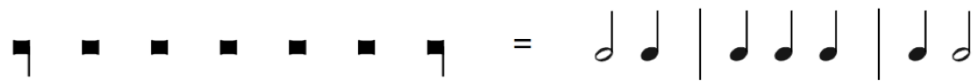


Figure 2-22: Modern transcription of a sequence of 5 (also for 8, 11, 14, etc.) breves between longs

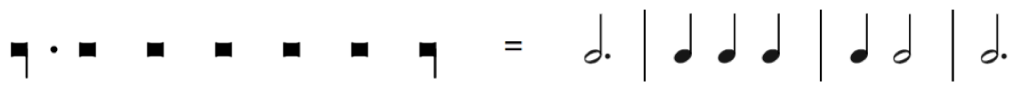


Figure 2-23: Change in interpretation of the sequence in Figure 2-22 by the use of a dot of perfection

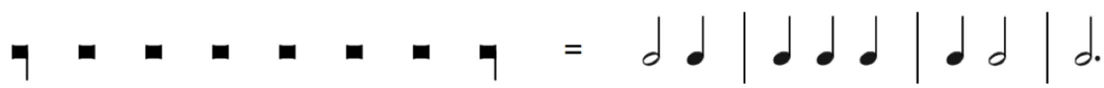


Figure 2-24: Modern transcription of a sequence of 6 (also for 9, 12, 15, etc.) breves between longs

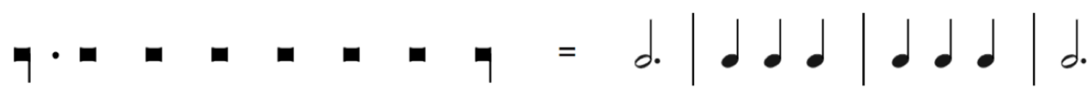


Figure 2-25: Change in interpretation of the sequence in Figure 2-24 by the use of a dot of perfection

With respect to the semibreve, Franco declared that a proper breve had a ternary division and thus, it could be substituted by either three minor semibreves (these are three semibreves of equal duration), or by a minor-major pair of semibreves (this is a pair of semibreves in which the second one doubles the durational value of the first one). “There is no provision as yet for the breve to be imperfected by the semibreve, or for the semibreve to stand alone: the breve-semibreve relationship was not at that stage analogous to that of the long-breve” (Bent 2017). According to Jacobus de Liège in *Speculum musicae*, larger groups of semibreves equivalent to the breve were introduced by Petrus de Cruce (Coussemaker 1864, II: 401, translated in Apel 1953, 318). There is controversy regarding how to interpret these groups of more than three semibreves per breve (Apel 1953, 320). These large groups of semibreves introduced the idea of smaller note values, even though they were not notated by distinct note shapes yet.

There are rests for all the note (durational) values outlined by Franco: perfect long, imperfect long (which has the same duration as an altered breve), proper breve, major semibreve, and minor semibreve. Franco represents them as vertical strokes; the number of spaces in the staff covered by the stroke is the same as the number of breves in the rest (Figure 2-26). Thus, a perfect long rest covers three spaces, an imperfect long covers two, a proper breve covers one, a major semibreve covers 2/3, and the minor semibreve rest covers 1/3 of a space.

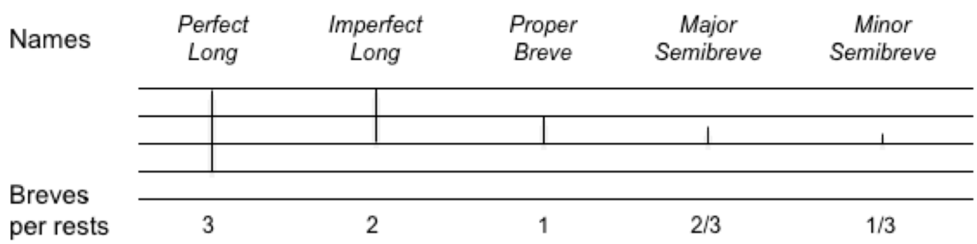


Figure 2-26: Franco's rests

2.2.4 Mensural Notation: Ars Nova[9]

Several changes distinguish the French music notation of the fourteenth century, known as *Ars nova* (Latin for "new art"), from the notation of Franco and his contemporaries, *Ars antiqua* (Latin for "old art"). The first theoretical formulations of the Ars nova are found in Philippe de Vitry's *Ars nova* and Johannes de Muris' *Ars novae musicae* from the early 1320s. The starting point of both is the teachings of Franco (Bent 2017). One of the innovations of the Ars nova is the introduction of the minim as the new note shape with a value smaller than the semibreve. Thus, the four levels of note values—long, breve, semibreve, and minim—are visually distinct. The names "modus", "tempus", and "prolation" started being used to indicate the relation at the different note levels: long-breve, breve-semibreve, and semibreve-minim, respectively;

[9] This section describes the French Ars nova notation system. While different notations were used in various countries (e.g., Italian Trecento notation and English notation) I am focusing on French notation since this is the one that continues into the Renaissance. For information on Italian Trecento notation, see Marchettus de Padua's *Pomerium* (1318), part of it is translated in Strunk and Treitler (1998, 2:141–51), or consult Parrish (1978, 166–82). For English notation, see Walter Odington's *De speculatione musicae* part vi, translated by Jay A. Huff (1973), or the short two-page summary provided by Rastall (2010, 83–4).

and the term “mensurations” was used to refer to these relations. While Franco only considered triple mensuration, in the Ars nova duple mensuration was also considered; thus, the modus, tempus, and prolation could be either binary or ternary. The four combinations of tempus and prolation were defined by Vitry as the “quatre prolacions” and were indicated by different signs (Table 2-5), although these signs were not commonly used until the fifteenth century. The modus of the music could be deduced by the type of long rests used (i.e., perfect-long rests indicated perfect modus, imperfect-long rests indicated imperfect modus); over time, perfect modus became less and less common in the compositions of the French Ars nova (Rastall 2010, 75).

Sign	Mensuration		Value of the notes [B, Sb]
	Modern Name	Vitry's Name	
⨀	perfect tempus and major prolation	major perfect time	*[3,3]*
○	perfect tempus and minor prolation	medium perfect time	*[3,2]*
C with dot	imperfect tempus and major prolation	major imperfect time	*[2,3]*
C	imperfect tempus and minor prolation	minor imperfect time	*[2,2]*

Table 2-5: The four prolacions introduced by Vitry, and their modern names and signs

With respect to the “maximodus” mensuration, Muris’ *Ars novae musicae* includes a triplex long to, as Apel (1953, 144) says, “satisfy the theoretical urge for completeness and symmetry”, since there is already a duplex long. But the triplex long is not found in the actual sources of Ars nova music, except in the form of rests (Figure 2-28 and Figure 2-29). As indicated by Rastall (2010, 75), the maximodus was a mensuration of theoretical rather than practical significance.

Franco’s principles of imperfection and alteration (listed in Section 2.2.3), describing the interpretation of the long and the breve in perfect modus, were extended to the other two note levels: the breve and semibreve (in case of perfect tempus), and the semibreve and minim (in case of major prolation). In other words, the rules regarding the imperfection of the long now applied to the breve and the semibreve; and the rules regarding the alteration of the breve applied to the semibreve and the minim. In addition to the extension of the principles of imperfection and alteration to the new note levels, other innovations from this period are: the use of red notes, the use of partial imperfection, and the use of dots of addition besides the dot of division.

The rests introduced by Franco for the long and breve were still used, the semibreve rest became a short vertical bar suspended from a staff line, and the minim rest was the same but placed upon a staff line (Figure 2-27). These rests, like the Franconian rests, had a fixed (perfect or imperfect) durational value given by the mensuration.

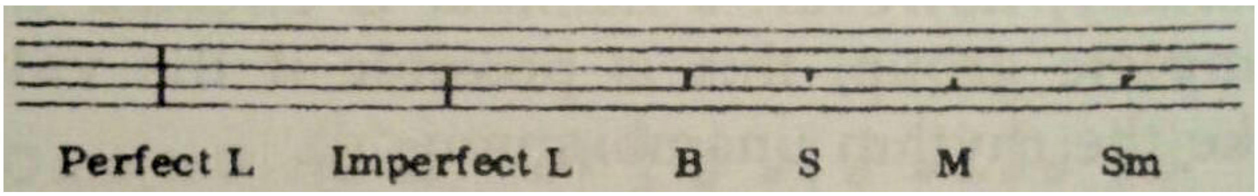


Figure 2-27: Ars nova rests (Rastall 2010, 80)

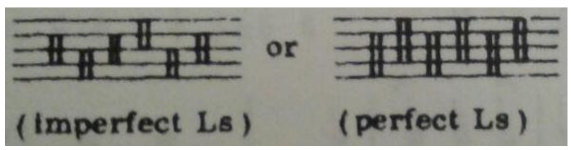


Figure 2-28: Duple-long rests (Rastall 2010, 80)

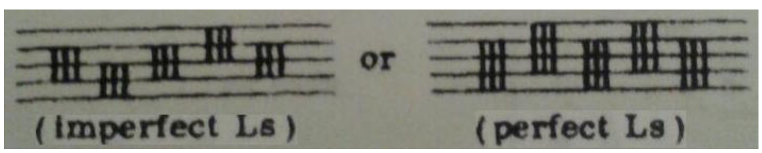


Figure 2-29: Triple-long rests (Rastall 2010, 80)

2.2.5 White Mensural Notation

The mensural notation systems mentioned above made use of black notes. Around the middle of the fifteenth century the custom of filling in the note heads with black ink became infrequent for the longer notes (see Table 2-1). This change from black to white notes, marks the historical distinction between *black mensural notation* (1250–1420) and *white mensural notation* (1420–1600). The reason of this change is a matter of debate. Many theorists relate the use of black or white notes to the surface the music was written on, relating black notation to parchment and white notation to paper. Brian Trowell has ascribed the use of white notation to the roughness of the paper (compared to the parchment) for which “the quill nibs could no longer splay out easily to produce the typical black lozenge [diamond shape]” (Robertson and Stevens 1960, 2:67).[10] But there are counterexamples to the parchment-black and paper-white relations. Rastall (2010, 102) suggests another type of relation: the use of black or white notation based on the size of the manuscript. For large choirs, he indicates that black notation is more suitable as it is more legible from a distance. At the same time, a large choir requires the manuscript to have a bigger size which, in turn, favors the use of a parchment surface—large paper leaves would tear apart, especially subjected to regular use by a choir. On the other hand, he points out that a small source required the use of a finely-cut pen for the text, and white notes allowed the use of the same pen to write the music as well. While these small manuscripts can be written on paper—there is no concern about the paper leaf breaking apart due to its size—some of them are written in parchment (e.g., the Mellon and Cordiforme chansonniers) as it is a better surface for the detailed illuminations (i.e., decorations) of these manuscripts.

Whatever the reason for this change was, the principles behind white notation were the same principles that ruled the black French Ars nova notation (Parrish 1978, 142; Rastall 2010, 102). While regular note values were written now in white notes, black notes were used for coloration, substituting for the red notes from the Ars nova. This limited the use of smaller note values since now the colored M, Sm, and F were the same in appearance as the non-colored Sm, F, and Sf (see Table 2-1).

For white notation, there is also a set of rules clearly outlined by the modern musicologist Willi Apel. Apel's set of rules regarding the interpretation of a note based on the context are essentially the principles of imperfection and alteration outlined by Franco (except they are written in terms of the breve and semibreve instead of the long and breve). Apel, however, includes a few additional rules and remarks (Apel 1953, 107–20):

- Imperfection a.p.p. always takes precedence over imperfection a.p.a.
    - Imperfection a.p.p. stands for *imperfection a parte post*, which refers to the imperfection of a note by the notes following it (e.g., the first long in Figure 2-13)

[10] For more information, see Boone (1990).

    - Imperfection a.p.a. stands for *imperfection a parte ante*, which refers to the imperfection of a note by the notes preceding it (e.g., the last long in Figure 2-14)

  This rule is implicit in Franco's principles since the only cases in which imperfection a.p.a. is favored over imperfection a.p.p. (Figure 2-14 and Figure 2-21) require the use of a dot of perfection to eliminate the possibility of imperfection a.p.p. at all.

- Rests cannot be altered nor imperfected

  This rule is also implicit in Franco's and in the discussion of the rests used in the Ars nova, where it was stated that rests have a fixed durational value. Thus, rests are not modified (i.e., being imperfected or altered) by context.

- If a breve is followed by two semibreve rests, the scribe places the two rests on different lines of the staff if the first is meant to imperfect the preceding breve and the second one belongs to the following perfection; otherwise, they belong to the same perfection and the breve remains perfect.

- In the late fifteenth century, the principle of alteration gradually fell into disuse (1953, 114) and a sequence consisting of two semibreves between breves was understood to imply imperfection even without a dot of division (1953, 114), contrary to the old practice (compare this to the Figure 2-15 and Figure 2-17 in Franco's principles regarding two breves between longs). Nevertheless, the new interpretation never superseded the old one completely (1953, 114), and there are instances of two semibreves between breves that call for imperfection and others that call for alteration. In this regard, Apel points out a few hints on how to discard one of the two interpretations:

    - To discard alteration:

        - The last semibreve is substituted by smaller note values (e.g., two minims, or a dotted minim and a semiminim)

        - The last semibreve is a rest

- To discard imperfection:
  - The last breve is followed by another breve; otherwise, by considering the former as imperfect, it would conflict with the "similis ante similem perfecta" rule (Figure 2-12).

Apel also points out that if the semibreves are written in ligature they call for alteration (1953, 114).

The trend in the late fifteenth century was towards the simplification of notation; together with the falling into disuse of the principle of alteration, musicians increasingly often placed a dot after a note that was to be perfect even when the old practice would not have required one (Bent 2017). In the early sixteenth century, music was predominantly written in duple meter with dots of augmentation to indicate ternary durational values when needed (Chew and Rastall 2017).

The period of mensural notation on which this work is focused includes both black and white mensural notation, from the moment in which the French Ars nova principles were well developed (ca. 1330) until the end of white mensural notation (ca. 1600), given that the principles ruling both French Ars nova and white mensural notation are nearly identical.[11]

2.3 Historical Review of Encodings of Mensural Notation

Since the beginning of the modern computing age, in the 1960s, there has been interest in using computational power for symbolic representations of music, including mensural music (Roland, Hankinson, and Pugin 2014). While there are projects with large databases of mensural music encoded in modern values (e.g., the Josquin Research Project), I am focusing on encodings that preserve the original mensural values.

In the early 1970s Thomas Hall, from Princeton University, developed FASTCODE for encoding white mensural notation from the fifteenth and sixteenth centuries, encoding hundreds of works from Renaissance composers (Selfridge-Field

[11] Recent research by Karen Desmond dates the beginnings of the Ars nova in the 1330s. Karen Desmond, personal communication, 11 August 2017. Desmond has a monograph on the *ars nova* forthcoming with Cambridge University Press (2018).

1997, 171, 587). The code supported note values from the semifusa to the perfect maxima, and was able to represent changes in mensuration within a single voice (Selfridge-Field 1997, 587). Another related work that appeared during the Seventies was that of Norbert Böker-Heil. Böker-Heil was the leader of the music research group at the Staatliches Institut für Musikforschung from the late 1970s until 1993; he worked on many computer-assisted music printing projects (Selfridge-Field 1997, 587). His work on the Renaissance Tenorlied resulted in the publication of the three-volume series “Das Tenorlied, Mehrstimmige Lieder in deutschen Quellen 1450–1580”,[12] considered by the RISM (Répertoire International des Sources Musicales) as part of their special volumes.

In 1980, Lynn Trowbridge wrote an extension for DARMS[13] (Digital Alternate Representation of Musical Scores) that supported the encoding of mensural notation (Selfridge-Field 1997, 207), called the *Linear Music Input Language* (LMIL), which was devised for the task of encoding a large repertoire of fifteenth-century chansons. The LMIL, unlike other extensions of DARMS, was never implemented into a software application, but made use of punch cards to encode the music (Trowbridge 1985). The DARMS-LMIL was able to encode mensural signs and used rhythmic codes to represent mensural note values (encoding both the note shape and the durational value), including colored notes (Selfridge-Field 1997, 208–10).

In 1984, a software for encoding medieval music notation (i.e., square and mensural notation) was developed by John Stinson and Brian Parish (Stinson and Stoessel 2014). *Scribe* was developed to facilitate the study of an early fourteenth-century Dominican chant manuscript by making it searchable; the program was then extended to cover mensural notation from the fourteenth century (Stinson and Stoessel 2014). In particular, it supports common neumes (e.g., virga, podatus, clivis, etc.), ligatures, and black, white, and colored mensural notation (Selfridge-Field 1997, 604). Scribe included an integrated database that can be searched for text, note shape, and pitch data. Scribe was designed for the DOS platform and, although it is still usable using an emulator, there are current efforts to transform Scribe data into a more modern

[12] The Tenorlied: Multi-voice Songs from German Sources 1450–1580.
[13] The original design of this language was created by Stefan Bauer-Mengelberg in 1963.

format that ensures its availability across the Internet. Under this idea of preservation and Internet distribution, Stinson and Stoessel (2014) reviewed different music notation encoding standards that followed an XML design. Their final selections were two current encoding standards: the Corpus Mensurabilis Musicae Electronicum (CMME) and the Music Encoding Initiative (MEI). CMME was developed specifically for white mensural notation; MEI is a flexible format that can be adapted to many different types of notation, which was the reason why they settled for it. The new format that Stoessel and his group are currently developing is called MEI NeoScribe, which is a module compatible with existing features of the Shared, Neumes, and Mensural notation MEI modules (Stinson and Stoessel 2014).

The **CMME project** was initiated in 1999 by Theodor Dumitrescu for his undergraduate thesis in Computer Science at Princeton University (Dumitrescu and van Berchum 2017). It is a scholarly initiative to computerize the editing of mensural music to offer free online access to editions of Renaissance music. From 2000 to 2004 the project was dormant; it was reopened by Dumitrescu in 2005 at the Centre d'Études Supérieures de la Renaissance, at Tours, France. During this period, the grammar system developed by Dumitrescu in 1999 was now converted to XML and tailored to encompass the major characteristics of the mensural notation system used during the Renaissance, giving rise to the cmme.xml format. The website, also initially developed in 1999, was modified to allow the user not only to explore music editions, but also to access a network of contextual information (i.e., metadata which, in this case, consists of a list of all contemporary manuscript sources, with all their individual compositions and composers). The developments of 2005 also resulted in the introduction of a graphical transcription tool. In September 2006, the project was moved to Utrecht, and its development continued under the guidance of Karl Kügle. In December 2006, CMME saw its first public release with the publication of *A Choirbook for Henry VIII and His Sisters*, an edition done by Dumitrescu. Later on, other editorial projects were added, such as the *Occo Codex* (in 2008) or *The Other Josquin: Music Excluded from the New Josquin Edition* (in 2011), which was developed in collaboration with the Josquin

Research Project (JRP). Currently, however, the CMME project is again in a dormant state.[14]

Compared to physical editions, the CMME digital editions are searchable, analyzable, configurable, and cost free (Dumitrescu and van Berchum 2017). The editions are searchable and analyzable because they are encoded in a symbolic format. The configurable part refers to the fact that the layout of the edition can be dynamically changed, as well as the variants shown, which frees editors from making presentation decisions and allows them to focus on interpreting the musical text (Dumitrescu and van Berchum 2017). And, finally, they are free of printing costs. The cmme.xml format is designed to encode mensuration signs, proportions, coloration, different note shapes and their imperfect / perfect / altered durational values, early and modern texting styles, and finally variant readings and critical notes.

The **Music Encoding Initiative** (MEI) is a community-based, open-source effort to define the best practices for encoding a wide range of musical documents in a machine-readable structure, bringing together scholars from different disciplines such as Music Theory, Musicology, Music Librarianship, and Music Technology (Hankinson, Roland, and Fujinaga 2011). This initiative was started in 1999 by Perry Roland, from University of Virginia, inspired on a similar project for the digital representation of text documents, the Text Encoding Initiative (TEI) (Crawford and Lewis 2016). MEI is a term that refers to both the research community and the symbolic format developed by Roland (Hankinson, Roland, and Fujinaga 2011).

MEI provides support for the encoding of different kinds of musical documents through a set of twenty-three core modules (Hankinson, Roland, and Fujinaga 2011). Because of this, MEI is able to encode critical editions, detailed metadata, and different music notation systems, including mensural notation. Even though these modules already provide support for the encoding of diverse material, MEI allows the user to extend its feature set, since one can select the modules useful for a particular project, and make adjustments and additions to them (Crawford and Lewis 2016).

[14] Marnix van Berchum, e-mail message in the MEI mailing list: “[MEI-L] Sample Encoding of Mensural Notation”, 14 January 2017.

Module Name	Module content
MEI	MEI infrastructure
Shared	Shared components
Header	Common metadata
CMN	Common music notation
Mensural	Mensural music notation
Neumes	Neume notation
Analysis	Analysis and interpretation
CMNOrnaments	CMN ornamentation
Corpus	Metadata for music corpora
Critapp	Critical apparatus
Edittrans	Scholarly editions and interpretations
Facsimile	Facsimile documents
Figtable	Figures and tables
Harmony	Harmonic analysis
Linkalign	Temporal linking and alignment
Lyrics	Lyrics
MIDI	MIDI-like structures
Namesdates	Names and dates
Performance	Recorded performances
Ptrref	Pointers and references
Tablature	Basic tablature
Text	Narrative textual content
Usersymbols	Graphics, shapes and symbols

Table 2-6: MEI Core modules (Hankinson, Roland, and Fujinaga 2011)

Software for specific encoding tasks in MEI has gradually become available; all this software is free and most of it is in the form of software libraries. Some examples are LibMEI, a C++ library for reading and writing MEI files, and SibMEI, a plugin for exporting Sibelius files into MEI; both of them were developed by Andrew Hankinson at McGill University. Stand-alone programs with graphical user interfaces include MEISE (MEI Score Editor), a score editor for viewing and editing Common Music Notation MEI files, and MerMEId (Metadata Editor and Repository for MEI Data), tool for editing comprehensive metadata in MEI files. Laurent Pugin developed Verovio, a library for engraving MEI music notation into SVG, for displaying encoded music in a web browser (Crawford and Lewis 2016). Pugin also developed Aruspix, an optical music recognition (OMR) software for mensural notation designed for sixteenth- and seventeenth-century music printed with movable typefaces.[15]

[15] http://www.aruspix.net/

As part of the core modules of MEI, the Mensural module provides support for mensural notation. This module is in constant development, driven by the discussions of the MEI-Mensural Interest Group, started in 2015 and chaired by Giuliano Di Bacco, director of the Center for the History of Music Theory and Literature at Indiana University. Currently, according to the Mensural MEI module guidelines, it has support for encoding mensuration, notes from the semifusa to the maxima, the perfect / imperfect / altered durational value of the notes, dots of division, dots of augmentation, ligatures, and proportions.[16] A more detailed description of the Mensural module of MEI will be given in the next chapter (Section 3.1).

In this thesis I am using MEI, encoding mensural music according to the Mensural MEI module. While both the recent formats of CMME and MEI allow for mensural music to be written in score format, my inclination for the latter is based on its flexibility and precedent of use with OMR. The latter is important for this project because the long-term goal of the scoring-up tool is to be part of an OMR workflow that returns the mensural piece in score format. The OMR process would retrieve the note shapes and mensuration of each voice, which could then be used by the scoring-up tool to determine the contextual durational values of the notes. Once these values have been derived, Verovio can be used to render the mensural piece in score format.

In this chapter, I presented the background information needed for the development of the scoring-up tool and some information potentially useful for future work regarding the OMR of mensural music sources. The first two sections of the chapter focus on the mensural notation system itself; the first section summarizes the basics of the system (the features that need to be handled by the scoring-up tool), while the second one provides the historical context for the development of the mensural system. The third section is a literature review of the various encodings for mensural notation. I conclude with a discussion of MEI, the format chosen to encode both the input and output of the scoring-up tool.

---

[16] Music Encoding Initiative. "MEI Guidelines, Version 3.0.0: 5 Mensural Notation." http://music-encoding.org/documentation/3.0.0/mensural/.

Chapter 3 Implementation

Given the separate-parts arrangement of mensural music, it is useful to copy the piece in score format to study the relation between the voices in a mensural piece. Nonetheless, scoring-up the parts of a piece requires one to know the durational value of the notes, which is context dependent. This chapter describes the implementation of an automatic scoring-up tool for combining the different parts of a mensural piece. This implementation is based on the principles outlined in Section 2.2 regarding the interpretation of the notes based on the context. The automatic scoring-up tool works with MEI files that were encoded according to the schema of the Mensural MEI module.

The implementation consists of two modules in a pipeline architecture: the merge module and the duration finder module (Figure 3-1). The **merge module** takes as input individual Mensural MEI files for each of the voices of a piece. These files only encode clef type and position, pitch, and note-shape information,[17] plus the mensuration of the voice.[18] The merge module outputs a single Mensural MEI file, which contains the same information as the individual files, but for all the voices instead of just a single part. The **duration finder module** takes this Mensural MEI file and determines the durational value (i.e., perfect, imperfect, or altered) of the notes from each voice based on the context.[19] A description of the encoding of mensural notation in these Mensural MEI files is given in Section 3.1, and the specific details of each of the individual modules that work with these files are presented in Section 3.2 and Section 3.3.

[17] These types of information are the typical output of automatic transcription systems, such as optical music recognition systems.
[18] The mensuration of the voice can also be retrieved by optical music recognition systems in the case of white mensural pieces, since it is indicated in the manuscript by the symbols shown in Table 2-5. For most fourteenth-century pieces in black notation, the mensuration information must be provided by the user, since the mensuration signs, although developed in the fourteenth century, were not commonly used until the fifteenth century.
[19] The order of the modules is interchangeable. Using the merge module first, though, allows us to visualize the default alignment of the notes (this is, the alignment of the notes without considering the context-dependent nature of their durational values) and how this alignment is changed afterwards when the duration finder module is used.

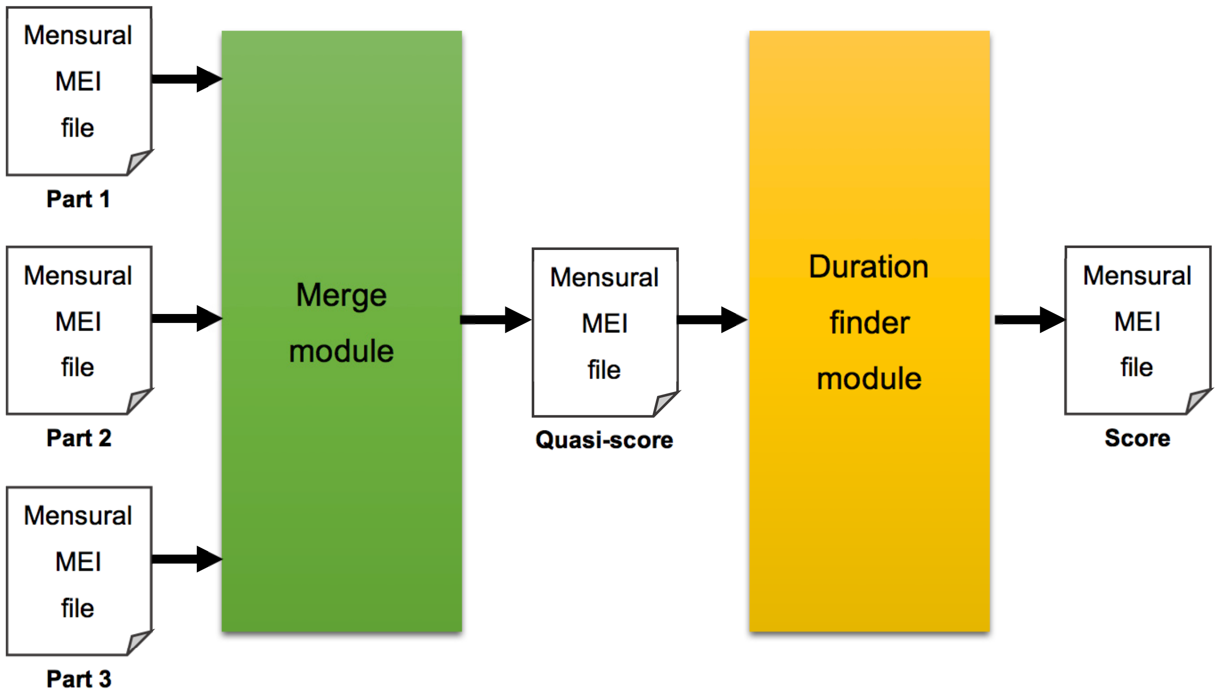


Figure 3-1: Architecture of the Scoring-Up tool

3.1 Mensural Notation in MEI

Before describing how mensural music is encoded in MEI format, I will briefly explain how music, in general, is encoded in this format. MEI is an XML-based file format, and thus has a hierarchical encoding structure (see Figure 3-2) (Music Encoding Initiative 2017b). Elements are its core objects; they are represented by tags, which consist of a name enclosed in angle brackets. For example, a note object is represented by the *<note>* tag. Attributes are used within an element to define properties of a particular object, represented as key-value pairs, taking the form key="value". Continuing with the example, a *<note>* object for representing middle C (i.e., C4) should be encoded as *<note pname="c" oct ="4"/>*. In running text, attributes are referred by adding a preceding '@' character to the attribute name (e.g., @pname represents the attribute 'pname') (Music Encoding Initiative 2017b).

```
<beam>
    <note pname="f" oct="4" dur="8"/>
    <note pname="c" oct="4" dur="8"/>
</beam>
```

Figure 3-2: Example of the hierarchical structure of MEI. The image at the right represent the notes encoded at the left (clef is provided just as reference). (Music Encoding Initiative 2017b)

The General MEI Document Structure (Music Encoding Initiative 2017c) specifies that an MEI document should always include a root <mei> element with two branching structures: <meiHead> and <music> (Figure 3-3). The <meiHead> structure contains elements that encode the **metadata of the document**, including information regarding authorship, encoding standards, and provenance. The most basic elements encoded under <meiHead> are shown in Figure 3-4. The <music> structure, on the other hand, contains information regarding the encoded music itself. One of the descendants of the <music> element is the <score> element (see Figure 3-5 to see its exact placement), which itself includes two structures: <scoreDef> and <section>; the former encodes the **metadata of the music** encoded within the latter. The <scoreDef> element includes <staffDef> elements that define the general characteristics of each of the voices contained in the score, characteristics such as the name of the voice (@label), its clef (@clef.shape and @clef.line), its meter (or mensuration, in the case of mensural notation), and also an @n attribute to identify the voice by a number. The actual music for each voice is encoded in the <staff> elements contained within the <section> element. These <staff> elements also include an @n attribute. Both the <staff> and the <staffDef> elements encoding the same voice (its notes and its metadata) must share the same value for the @n attribute. (Music Encoding Initiative 2017c)

```
<mei>
    <meiHead/>
    <music/>
</mei>
```

Figure 3-3: The root element <mei> of every MEI document and its two sub-elements <meiHead> and <music>, for encoding metadata and music content of the document, respectively

```
<mei>
    <meiHead>
        <fileDesc>
            <titleStmt>
                <title></title>
            </titleStmt>
            <pubStmt/>
        </fileDesc>
    </meiHead>
    <music/>
</mei>
```

Figure 3-4: Basic structure of the <meiHead> element

```
<mei>
    <meiHead/>
    <music>
        <body>
            <mdiv>
                <score>
                    <scoreDef>
                        <staffGrp>
                            <staffDef/>
                            ...
                            <staffDef/>
                        </staffGrp>
                    </scoreDef>
                    <section>
                        <staff>
                            <layer>
                                <!-- MUSIC WITHIN THE STAFF -->
                            </layer>
                        </staff>
                        ...
                        <staff>
                            <layer>
                                <!-- MUSIC WITHIN THE STAFF -->
                            </layer>
                        </staff>
                    </section>
                </score>
            </mdiv>
        </body>
    </music>
</mei>
```

Figure 3-5: Basic structure of the <music> element in case of encoding a score[20]

MEI provides support for encoding mensural notation through the Mensural MEI module. Some of the features of mensural notation are also shared by other systems of musical notation (e.g., the use of notes and rests, note pitches, distinct note shapes, etc.); thus, there are elements and attributes from the Shared module (Table 2-6) that are also useful when encoding mensural music.

[20] The <measure> element, which is used in CMN inside the <section> element as a container of the <staff> elements, is not included in the context of mensural notation, in which measures are not used. In the case of a Mensural MEI file, the <staff> elements can be used as direct children of the <section> element.

According to MEI’s tutorial for encoding music (Music Encoding Initiative 2017d), notes and rests are encoded with the <note> and <rest> elements, respectively. In the case of notes, pitch information is encoded within the note element using the following attributes: @pname that encodes the pitch within an octave, @oct that encodes the octave number, and @accid that encodes the accidentals. For both notes and rests, the name (or shape) of the note is encoded using the @dur attribute (Music Encoding Initiative 2017d). Since different notational systems use different notes, the values of the @dur attribute are different in each system of music notation. When the Mensural MEI module is included, @dur can take the values shown in Table 3-1 (Music Encoding Initiative 2017a).

@dur values	Mensural notes
"maxima"	
"longa"	
"brevis"	
"semibrevis"	
"minima"	
"semiminima"	
"fusa"	
"semifusa"	

Table 3-1: Values of the @dur attribute to represent the different mensural note shapes

As an example of encoding notes in mensural notation, consider the following sequence of notes and rests (Figure 3-6):

Figure 3-6: Display of the music encoded in Figure 3-7

The encoding of these notes is shown in the next image (Figure 3-7):

```
<staff n="1">
    <layer>
        <note dur="longa" pname="c" oct="5"/>
        <rest dur="brevis"/>
        <note dur="brevis" pname="e" oct="5"/>
        <note dur="semibrevis" pname="c" oct="5"/>
        <note dur="minima" pname="b" oct="4"/>
        <note dur="minima" pname="a" oct="4"/>
        <note dur="semibrevis" pname="g" oct="4"/>
        <note dur="brevis" pname="c" oct="4"/>
        <rest dur="semibrevis"/>
        <note dur="semibrevis" pname="a" oct="3"/>
        <note dur="brevis" pname="c" oct="4"/>
        <note dur="longa" pname="c" oct="4"/>
    </layer>
</staff>
```

Figure 3-7: MEI code for the sequence of notes in Figure 3-6

There are other features specific to mensural notation that need to be encoded, such as mensuration, dots of division or augmentation, and the quality (i.e., perfect, imperfect, or altered) of the notes. According to the guidelines, mensuration is supported by the Mensural MEI module through the following four attributes: @modusmaior, @modusminor, @tempus, and @prolatio; each of these attributes can have a value of either “3” or “2”, indicating triple or duple mensuration, respectively. This set of attributes can be defined within the <staffDef> element for each of the individual voices of the piece (see Figure 3-8); this is useful if the mensuration of each voice is constant (i.e., there is no change in mensuration within the voice throughout the piece). They can also be encoded in the element <mensur> (see Figure 3-9), which can be used at any place within the <staff> element, which is useful for encoding a change in mensuration within a voice.

```
<scoreDef>
    <staffGrp>
        <staffDef n="1" label="triplum" modusmaior="2" modusminor="2" tempus="3" prolatio="3"/>
        <staffDef n="2" label="duplum" modusmaior="2" modusminor="2" tempus="3" prolatio="2"/>
        <staffDef n="3" label="tenor" modusmaior="2" modusminor="3" tempus="2" prolatio="2"/>
    </staffGrp>
</scoreDef>
```

Figure 3-8: The <staffDef> elements define the mensuration for three voices, labeled as “triplum”, “duplum”, and “tenor”. The first voice (@n=“1”), the triplum, is in perfect tempus major prolation (the maximodus and the modus are both imperfect). The second voice (@n=“2”), the duplum, is in perfect tempus and minor prolation (again with imperfect maximodus and modus). The third voice (@n=“3”), the tenor, is in imperfect tempus minor prolation, but it has triple meter at the level of the long since it is in perfect modus.

```
<section>
    <staff n="1">
        <layer>
            <note ... />
            ...
            <note ... />
            <mensur modusmaior="2" modusminor="2" tempus="2" prolatio="2"/>
            <note ... />
            ...
            <note ... />
        </layer>
    </staff>
    <staff n="2">...
    </staff>
    <staff n="3">...
    </staff>
</section>
```

Figure 3-9: Continuation of the code in Figure 3-8. The mensuration of the first voice (@n="1") changes from perfect tempus and major prolation, defined in the <staffDef n="1"> element in Figure 3-8, to imperfect tempus and minor prolation, as defined within the <mensur> element (orange box).

For indicating the use of a dot in the source, one can use the Shared element <dot>. To indicate the functionality of the dot, augmentation or division, one can use the @form attribute with the values “aug” or “div”, respectively. Thus, the encoding of a dot of augmentation is <dot form=“aug”/> and the encoding of a dot of division is <dot form=“div”/>.

When an encoded piece is rendered in Verovio, all the notes are assumed to have the durational value defined by the mensuration (e.g., the breves are all assumed to be 3 semibreves long when @tempus=“3” and the semibreves are all assumed to be 2 minims long when @prolatio=“2”). This default duration can be modified by making use of the attributes @num and @numbase. This is useful for encoding imperfection, alteration, and augmentation. The pair of attributes @num and @numbase define a ratio by which the default durational value of a note is multiplied; this ratio is given by *(value of @numbase) / (value of @num)*.[21] One way to use these attributes to encode imperfection, alteration, and augmentation, as used in Verovio MEI encoding (Verovio 2017), is shown in Table 3-2.

Modification	**Default Durational Value of the Note**	**Modified Durational Value of the Note**	**Value of the Attributes**	
			@num	**@numbase**
Imperfection	Perfect	Imperfect	"3"	"2"
Augmentation[22]	Imperfect	Perfect	"2"	"3"
Alteration	Perfect or Imperfect	Altered (i.e., twice its default duration)	"1"	"2"

Table 3-2: Values of the @num and @numbase attributes to encode "imperfections", "augmentations", and "alterations". This is based on the implementation by Verovio (2017)

As an example of the use of this pair of attributes in the encoding of modifications to a note's durational value, consider the following sequence in perfect tempus: . This sequence requires the alteration of the second semibreve; thus, its encoding, with the appropriate durational value for each note, would be as shown in Figure 3-10:

[21] Although it may seem odd for the @numbase to be the numerator of the ratio, note that @num and @numbase are meant to indicate a change in the number of notes fitting in a particular unit of time. Consider the two positive integers num and numbase, the statements @num = num and @numbase = numbase are meant to represent "num notes in the time of numbase", which increases (or reduces) each note's durational value by a factor of (numbase / num).

[22] Augmentation can only happen in the presence of a dot of augmentation, which acts in the same way as the modern dotted note.

```
<staff n="1">
    <layer>
        <note dur="brevis"/>
        <note dur="semibrevis"/>
        <note dur="semibrevis" num="1" numbase="2"/>
        <note dur="brevis"/>
    </layer>
</staff>
```

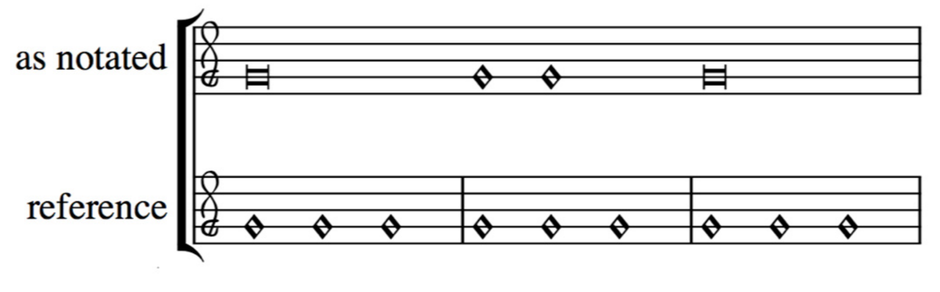


Figure 3-10: Example of the encoding of an alteration. At the top, the code corresponding to the first staff of the image, with the label "as notated". For simplicity, pitch-related information has been omitted from the code. At the bottom, an image with the "as notated" staff and a "reference" staff included to visualize the length of the notes in the "as notated" staff. The "reference" staff is barred by the breve's default value.

On the other hand, if the same sequence had a dot of division between the two semibreves, the two breves at the ends would be imperfected by the semibreve next to them. Thus, the sequence would be encoded as shown in Figure 3-11.

```
<staff n="1">
    <layer>
        <note dur="brevis" num="3" numbase="2"/>
        <note dur="semibrevis"/>
        <dot form="div"/>
        <note dur="semibrevis"/>
        <note dur="brevis" num="3" numbase="2"/>
    </layer>
</staff>
```

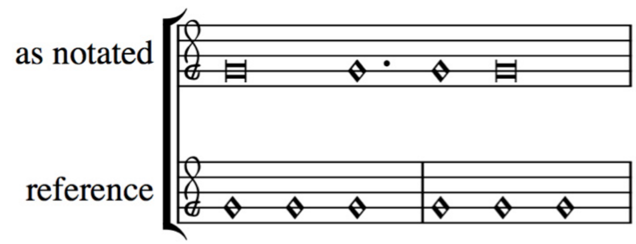


Figure 3-11: Example of the encoding of an imperfection and the use of a dot of division. At the top, the code corresponding to the first staff of the image, with the label "as notated". For simplicity, pitch-related information has been omitted from the code. At the bottom, an image with the "as notated" staff and a "reference" staff included to visualize the length of the notes in the "as notated" staff. The "reference" staff is barred by the breve's default value.

The example above showed how to change the durational value of a perfect note to an imperfect value. The reverse can also be achieved by means of a dot of augmentation. As an example, consider the following sequence in imperfect tempus: □ · ◊ . In this case, the dot following the breve is an augmentation dot and, thus, it changes the durational value of the breve from imperfect to perfect. The encoding of the sequence would be as shown in Figure 3-12.

```
<staff n="1">
    <layer>
        <note dur="brevis" num="2" numbase="3"/>
        <dot form="aug"/>
        <note dur="semibrevis"/>
    </layer>
</staff>
```

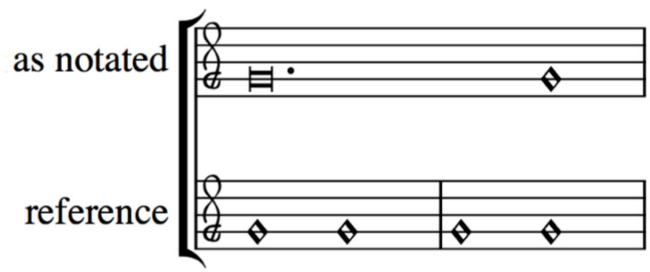


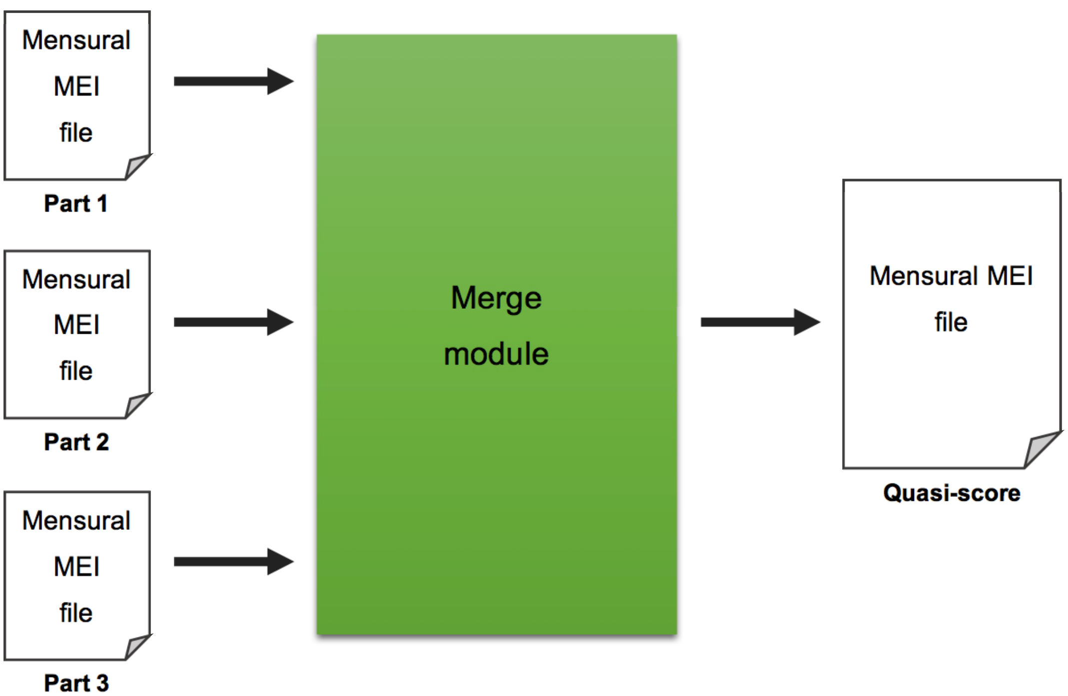
Figure 3-12: Example of the encoding of an augmentation and the use of a dot of augmentation. At the top, the code corresponding to the first staff of the image, with the label "as notated". For simplicity, pitch-related information has been omitted from the code. At the bottom, an image with the "as notated" staff and a "reference" staff included to visualize the length of the notes in the "as notated" staff. The "reference" staff is barred by the breve's default value.

MEI also provides support for encoding the graphical symbol for proportions. The fraction representing a proportion can be encoded within the <staffDef> or the <mensur> elements, by using @num and @numbase to indicate its numerator and denominator, respectively (see Figure 3-13) (Music Encoding Initiative 2017a). Coloration is not included yet in the official Mensural MEI documentation, although colored notes have already been represented in Verovio by using @colored="true", to change the color of the notes, and a pair of @num and @numbase values, encoding the effect of the coloration on the note's durational value (see Figure 3-13).

```
<mensur tempus="3" prolatio="2" num="3" />
...
<!-- coloration = hemiola -->
<note pname="d" dur="semibrevis" oct="4" colored="true" num="2" numbase="3" />
<dot/>
<note pname="c" dur="minima" oct="4" colored="true"  />
<note pname="a" dur="semibrevis" oct="3" colored="true"  />
<note pname="b" dur="semibrevis" oct="3" colored="true"  />
<note pname="g" dur="brevis" oct="3" colored="true" num="3" numbase="2" />
<barLine/>
```

Figure 3-13: Example of proportion and coloration (Verovio)

3.2 Merge Module



The merge module takes the Mensural MEI files that encode each of the individual voices of a piece and merges them into a single Mensural MEI file that encodes the piece as a whole. Each input file encodes one part of the piece using a <score> element with a single pair of <staffDef> and <staff> elements to encode the voice metadata and musical content, respectively (Figure 3-14). The voice metadata encoded in the <staffDef> element must include the mensuration of the voice (it can also include the clef type and position, and the label of the voice). The music content encoded within the <staff> element in the input files is limited to pitch and note-shape information (i.e., there is no information regarding the actual durational value of the notes). The merge module takes the voices encoded in these input files and stacks them one on top of the other, keeping the content of each of the voices intact. Thus, when rendering the output of the Mensural MEI file in Verovio, the music is presented as a pseudoscore, with the voices placed one below the other, but without any vertical alignment, since this would require knowing the durational value of each of the notes.

```
<mei>
    <meiHead/>
    <music>
        <body>
            <mdiv>
                <score>
                    <scoreDef>
                        <staffGrp>
                            <staffDef n="1" label="" modusmaior=""
                             modusminor="" tempus="" prolatio=""/>
                        </staffGrp>
                    </scoreDef>
                    <section>
                        <staff n="1"/>
                    </section>
                </score>
            </mdiv>
        </body>
    </music>
</mei>
```

Figure 3-14: General structure of the input file. Since it encodes a single voice, it contains a single pair of <staffDef> and <staff> elements, for encoding the voice's metadata and musical content, respectively.

In order to stack the voices, the merge module performs two actions:

1. In order to include the metadata for each voice, the Python program takes the <staffDef> element of each of the input files and adds it as a child of the <staffGrp> element within <scoreDef> in the output file (Figure 3-16).
2. In order to include the music content of each voice, the program takes the <staff> element of each of the input files and adds it as a child of the <section> element in the output file (Figure 3-15).

If the durational values of the notes were fixed rather than context dependent (as is the case in common Western music notation) the output file obtained by the merge module would complete the task of scoring up the parts.

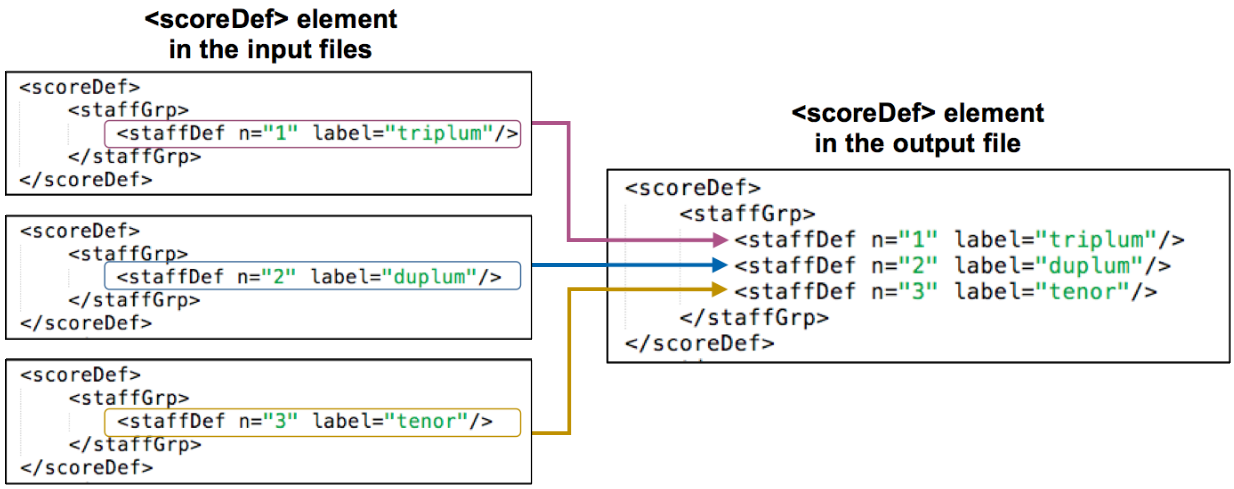


Figure 3-16: Example of how the merge module deals with the metadata of each voice for a three-voice piece. The metadata of each voice is encoded within the <staffDef> element of each input file (for simplicity, only the @n and @label attributes of the voices are shown). The merge module adds each of these <staffDef> elements as a child of the <staffGrp> element within the <scoreDef> (score metadata) element.

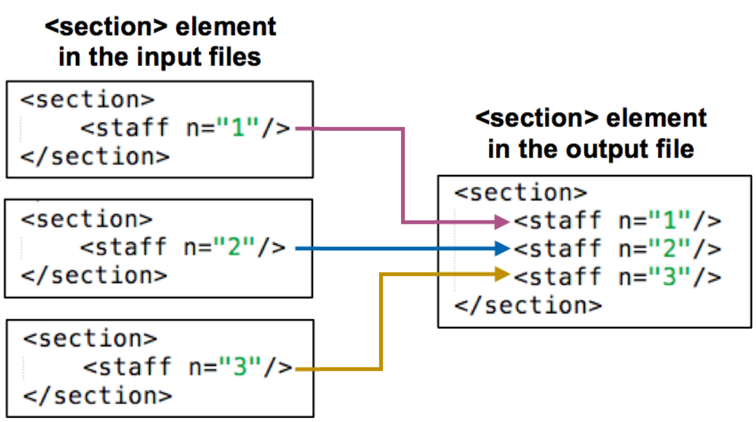
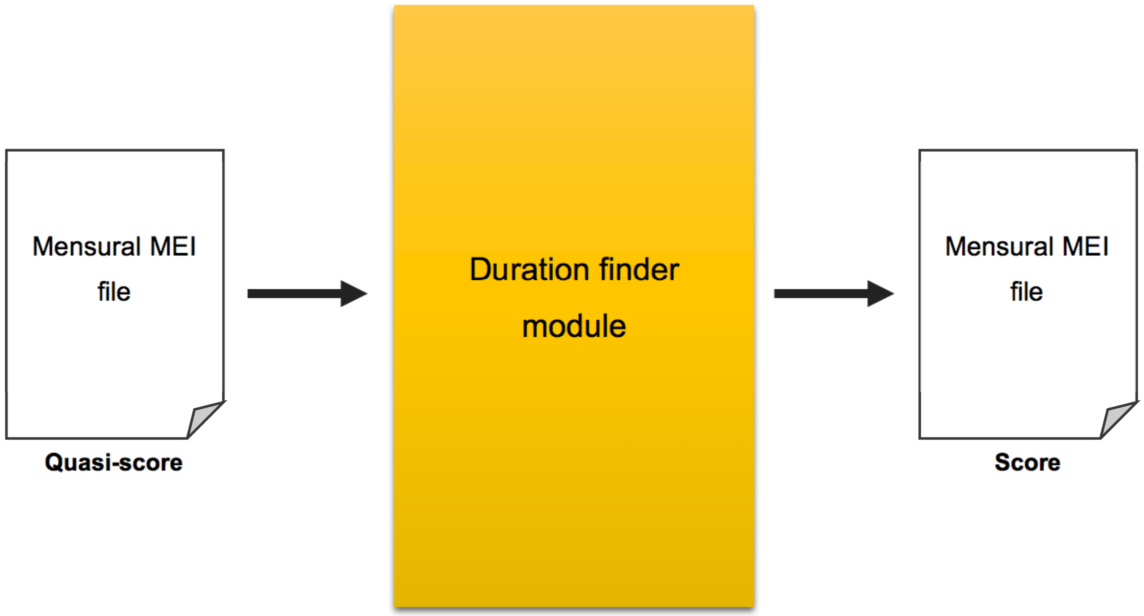


Figure 3-15: Example of how the merge module deals with the musical content of each voice voice for a three-voice piece. The musical content (i.e., notes and rests) of each voice is encoded within the <staff> element of each input file. The merge module adds each of these <staff> elements as a child of the <section> element.

3.3 Duration Finder Module



The duration finder module manages the context-dependent nature of mensural notation. It takes the Mensural MEI file obtained from the merge module and determines, for each voice, if the durational value of each note should be kept as the mensuration indicates or if it should be modified by the context, according to the principles listed in Section 2.2.3. If a note should be modified, it encodes the modification by using the attributes @num and @numbase with the values described in Table 3-2. The result is a Mensural MEI file that encodes both the note-shape and note-duration information (besides pitch information) and, when rendered in Verovio, it presents the parts of a mensural piece in a score format.

The main part of this module consists of the implementation of Franco's principles of imperfection and alteration, outlined in Section 2.2.3, since these are the basis of the interpretation of mensural notation. The implementation of these principles is presented in Section 3.3.1. In addition, other mensural notation features needed to be included in the duration finder module to deal with the specificities introduced by the Ars nova. As seen in Section 2.2.4, the Ars nova introduced coloration, dots of augmentation, and the use of perfect mensuration at other note levels besides the long. The implementation of coloration is summarized in Section 3.3.2. The identification of the functionality of the dots (i.e., division, perfection, and augmentation) is given in Section 3.3.3. And, finally, Section 3.3.4 deals with an aspect raised by the introduction of perfect mensuration at other note levels besides the one treated by Franco (i.e., longa level); that is, the use of perfect mensuration at multiple note levels simultaneously.

3.3.1 Implementation of the Rules

Franco's principles (Section 2.2.3) regarding the interpretation of the long and the breve (in what would come to be known as perfect modus) are written in terms of the number of breves between two longs. The implementation of these principles is, therefore, divided into three steps or functions: the first one segments the voice into such sequences,[23] the second one counts the notes between the boundaries of the sequence in terms of breves (in the case of perfect modus), semibreves (for perfect tempus), or minims (for major prolation), and the third function modifies a note (or notes) in the sequence according to that number by including the appropriate @num and @numbase values (given in Table 3-2). These functions were applied to the three note levels in a bottom-up fashion, proceeding first at the semibreve-minim level, then at the breve-semibreve level, and, finally, at the long-breve level (more details are given in Section 3.3.4). For simplicity, however, the examples and explanations of these three functions, detailed in the following three sections, will be given in terms of the long-breve level, just as in Franco's principles.

3.3.1.1 Function to Delimit Subsequences

This first function delineates the sequences of notes that have the form of those pointed out by Franco. This is necessary to create the building blocks with which I am going to work. The first step in dividing the musical content M of a voice into subsequences is to determine the note used as delimiter for these sequences. The *delimiter note* is the note that is in perfect mensuration (divided into three of the next lowest value; e.g., in perfect modus the delimiter note is the long). In the case of perfect mensuration at multiple note levels, this process is performed for each level, see Section 3.3.4 for more details.

[23] In the case of perfect modus, these are sequences delimited by longs; in perfect tempus, the sequences are delimited by breves; and in case of major prolation, the delimiter of the sequences are semibreves.

Once the delimiter note is set, a delimited (or bounded) sequence of notes may be defined. A *delimited sequence* $S = (s_i)_{i=1}^{k}$ is defined as a subsequence of consecutive elements (notes and rests) of M (the notes in a voice), such that s_1 and s_k are either uncolored notes (or rests) with a note value higher or equal to the delimiter note or colored notes with any value, and $s_2, \ldots, s_{k-1}$ are uncolored elements (notes or rests) with a value lower than the value of the delimiter note. The *start note* s_1 and the *end note* s_k of the sequence are called the *boundaries* (or *boundary notes*) of the sequence, and the notes between these boundaries are the *middle notes* of the sequence. Table 3-3 shows the notes used as delimiters and the notes used as boundaries for the different types of perfect mensuration.

Mensuration	**Delimiter note**	**Boundary notes**
Perfect modus	Long	Uncolored longs and maximas or any colored note
Perfect tempus	Breve	Uncolored breves, longs, and maximas or any colored note
Major prolation	Semibreve	Uncolored semibreves, breves, longs, and maximas or any colored note

Table 3-3: Delimiter and boundary notes to define the delimited sequences for the different types of perfect mensuration

As an example of how to locate the delimited sequences S that divide a voice, let us consider $M = (m_1, \ldots, m_{15})$ to be the first fifteen elements (notes and rests) of a voice written in perfect modus, as shown in Figure 3-17. Given the mensuration, the long is used as the delimiter note to define the delimited sequences S to divide M. This means that the sequences $(s_i)_{i=1}^{k}$ can have either an uncolored long or maxima, or any colored note at the s_1 and s_k boundaries, but the notes $s_2, \ldots, s_{k-1}$ must be uncolored notes shorter than a long (i.e., uncolored breves, semibreves, minims, etc.). Thus, the musical content M of the voice is divided into the following sequences delimited by the long (shown by the blue lines in Figure 3-17): (m_1, m_2, m_3), $(m_3, m_4, m_5, m_6, m_7, m_8)$, (m_8, m_9), (m_9, m_{10}, m_{11}), (m_{11}, m_{12}), (m_{12}, m_{13}), and (m_{13}, m_{14}, m_{15}).

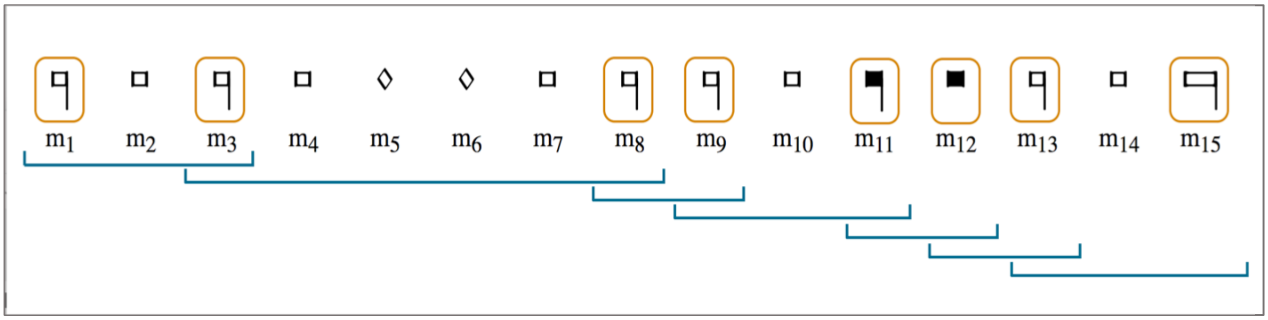


Figure 3-17: Example of a fifteen-note excerpt divided into sequences delimited by longs. The brown boxes indicate the notes that act as boundaries for each of the delimited sequences, and the blue brackets encompass all the notes that are part of each of these delimited sequences.

In two special cases the start note or the end note of a delimited sequence are assigned a value of $None$; the first case is when the first note of a voice is a note (or rest) that has a value lower than the delimiter, and the second case is in the presence of a dot of division. In the former case, the first delimited sequence is defined by $(s_i)_{i=1}^{k}$ with start note $s_1 = None$ and $s_2, \dots, s_k$ being the first $k-1$ consecutive notes of the voice; thus, s_k is the first uncolored note (or rest) of the voice with a value higher or equal to the delimiter note. As an example of this case, consider $M = (m_1, m_2, m_3, m_4, m_5, m_6)$ to be the first six notes of a voice in perfect modus, as shown in Figure 3-18; the excerpt M gets divided into the following sequences delimited by longs: $(None, m_1, m_2)$, (m_2, m_3, m_4, m_5), and (m_5, m_6).

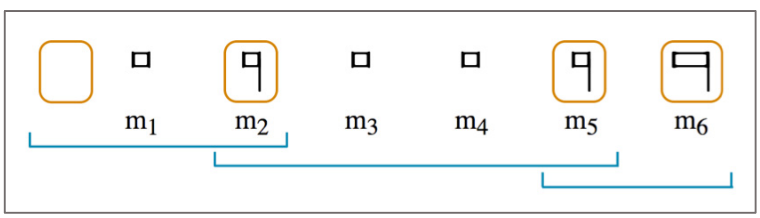


Figure 3-18: Example of a six-note excerpt that begins with a note shorter than the note used as delimiter (the long). The yellow boxes indicate the notes that act as boundaries for the delimited sequences, and the blue lines encompass all the notes that are part of each of these sequences.

The other case where a boundary note is set to $None$ is in the presence of a dot of division (Figure 3-19). Let us consider the following sequence delimited by longs $S = (s_1, \dots, s_i, s_{i+1}, \dots, s_k)$ with a dot of division between the notes s_i and s_{i+1}. This sequence is then divided into two sequences that are also delimited by longs: $(s_1, \dots, s_i, None)$ and

$(None, s_{i+1}, \ldots, s_k)$; in the former, the end note of the sequence is the one set to $None$, while in the latter it is the start note which is set to $None$.

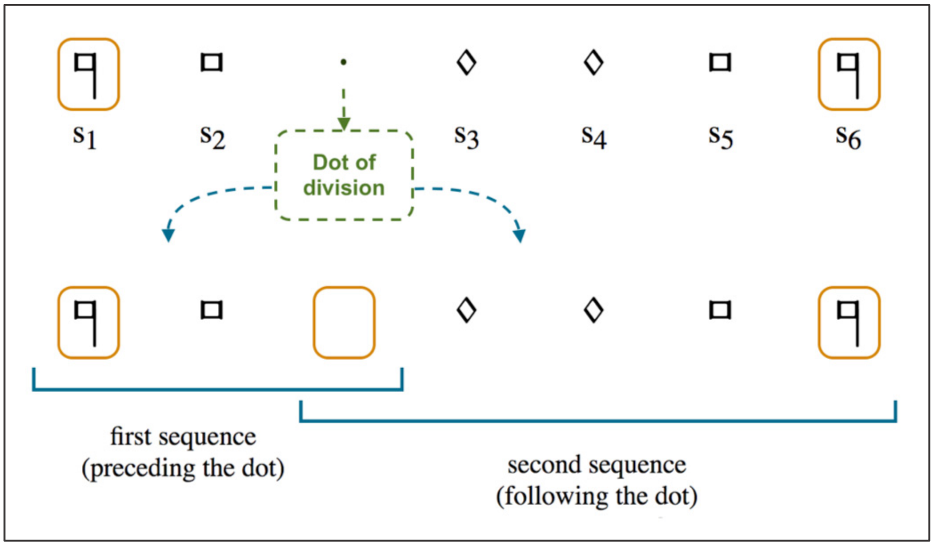


Figure 3-19: Example of the division of a sequence delimited by longs into two subsequences, also delimited by longs, due to the presence of a dot of division. The start note of the first sequence is the same as the start note of the larger sequence, but its end note is now set to None. The start note of the second sequence is None, and its end note coincides with the end note of the larger sequence.

3.3.1.2 Function to Determine the Number of Notes Between the Boundaries of the Subsequence in Terms of a Given Unit

This function counts the notes between the boundaries of the delimited sequences, found by the *function to delimit subsequences*, in terms of a given unit. In the case of perfect modus, the unit used is the breve; in perfect tempus, the unit is the semibreve; and in major prolation, the unit is the minim. The result of this function (the number of breves, semibreves, or minims) is needed by the *function to apply modifications* (Section 3.3.1.3) to determine whether a note should be imperfected or altered based on Franco's principles.

Figure 3-20 shows an example of the result of this function for a sequence delimited by longs (this is, a sequence that came from a voice written in perfect modus); the function returns the number of breves that fit between the boundaries (in this case, 4). Figure 3-20 also illustrates how this number is computed. This function performs three steps to determine the number of notes between boundaries in terms of a given unit. First, it determines the value in minims of each of the notes between the boundaries of the sequence, which is given by:

- The default value (given in Table 3-4, below)
- Or, in case the note has already been modified by any of the features treated in Section 3.3.3 (the presence of an augmentation dot) and Section 3.3.4 (a modification by context when sequences delimited by a lower note-level were considered), the value in minims is given by the product of the default value of the note and the ratio *(value of @numbase) / (value of @num)* that encodes the modification. Figure 3-20 and Figure 3-21 show examples of each of these cases.

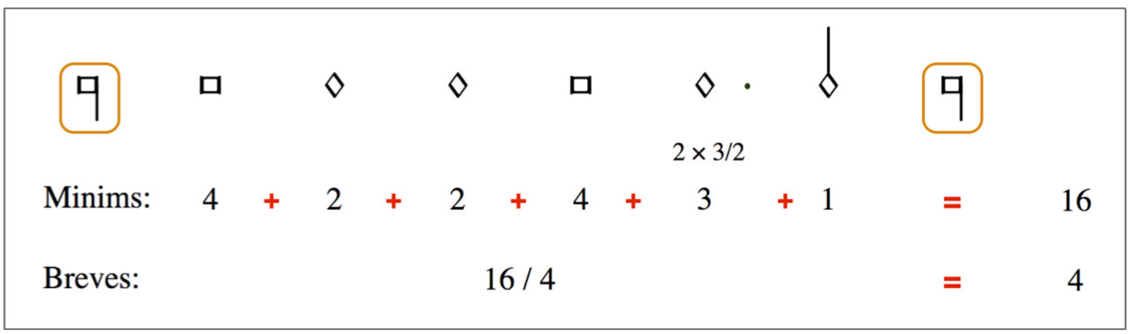


Figure 3-20: Counting the notes, in terms of breves, between the boundaries (yellow boxes) of a sequence delimited by longs, written in perfect modus. Notice that the value of the last semibreve has been modified by 3/2 from the default due to the presence of a dot of augmentation (section 3.3.3).

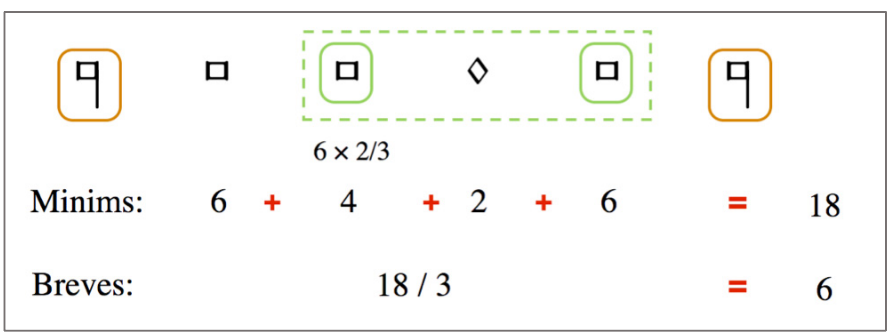


Figure 3-21: Counting the notes, in terms of breves, between the boundaries (yellow boxes) of a sequence delimited by longs, written in perfect modus and perfect tempus. Notice that the value of the second breve has been modified by 2/3 from the default since the semibreve following it imperfects it. The imperfect quality of the breve is determined when processing sequences delimited by lower note-levels (section 3.3.4); in this case, when processing sequences delimited by breves (specifically, when processing the sequence in the green box).

@dur values		Default Value (in minims)
"maxima"		*value of @modusmaior × value of @modusminor × value of @tempus × value of @prolatio*
"longa"		*value of @modusminor × value of @tempus × value of @prolatio*
"brevis"		*value of @tempus × value of @prolatio*
"semibrevis"		*value of @prolatio*
"minima"		1
"semiminima"		1/2
"fusa"		1/4
"semifusa"		1/8

Table 3-4: Value in minims of the different mensural notes

Second, after finding out the value in terms of minims of each of the notes between the boundaries of the sequence, the function sums these values to get the total number of minims between the boundaries. And third, with this number, it obtains the final result:

- In case of perfect modus, it obtains the number of breves between the boundaries by $\frac{number\ of\ minims}{breve\ default\ value}$ (with $breve\ default\ value = \ tempus * prolatio$).

- In case of perfect tempus, it obtains the number of semibreves between the boundaries by $\frac{number\ of\ minims}{semibreve\ default\ value}$ (with $semibreve\ default\ value = prolatio$).
- And in case of major prolation it uses the $number\ of\ minims$ between the boundaries.

The result is then sent to the *function to apply modifications*.

3.3.1.3 Function to Apply Modifications

This function implements Franco's principles of imperfection and alteration. It uses the number of breves, semibreves, or minims calculated by the previous function to determine the modification (imperfection or alteration) to be applied according to Franco, and it encodes the modification within the appropriate <note> element by adding the corresponding @num and @numbase values (Table 3-2):

- For imperfection: @num="3" and @numbase= "2"
- For alteration: @num="1" and @numbase="2"

More important than how imperfection and alteration are encoded is the implementation of the principles that allow us to detect when they occur, so the rest of this section will expand on this. As seen in Section 2.2.3, the principles outlined by Franco take a sequence of notes delimited by longs and then perform the following actions:

1. Arrange the notes into perfect groupings (i.e., groups of three breves)
2. Determine the number of notes left out of these groupings
3. And, based on this number, modify the durational value of a note (or notes) by either imperfecting one or both of the longs at the boundaries of the sequence or altering the last breve.

Thus, I reformulated these principles in terms of a "modulo 3" operation as shown in Table 3-5.[24] Even though the table is written for the case of perfect modus, the same

[24] Modulo 3 returns the remainder of the division of a number by 3. Therefore, the only possible results are 0, 1, and 2.

applies to the other perfect mensurations. For perfect tempus, *n* represents the number of semibreves between the boundaries and *p* stands for the number of groups of three semibreves. For major prolation, *n* represents the number of minims between the boundaries and *p* stands for the number of groups of three minims. A set of examples of sequences delimited by longs is given for each of the three possible values of *n* indicated in Table 3-5 (see Table 3-6).

Number of breves between the boundaries (n)	**Number of perfect groups of breves (p)**	**General Interpretation**	**Alternative Interpretation**	**Reference Rule**
n = 3 p + 1	p >= 0	Imperfection a.p.p.	Imperfection a.p.a.	2 and 5 (Figure 2-13, Figure 2-14, Figure 2-20, and Figure 2-21)
n = 3 p + 2	p = 0 (n = 2)	Alteration	Imperfection a.p.p. and Imperfection a.p.a.	3 (Figure 2-15 to Figure 2-17)
	p > 0	Imperfection a.p.p. and Imperfection a.p.a.	Alteration	5 (Figure 2-22 and Figure 2-23)
n = 3 p	p = 0 (n = 0)	Start note remains perfect	-	1 (Figure 2-12)
	p = 1 (n = 3)		Imperfection a.p.p. and Alteration	4 (Figure 2-18 and Figure 2-19)
	p > 1	Imperfection a.p.p. and Alteration	(start note remains perfect)	5 (Figure 2-24 and Figure 2-25)

Table 3-5: Implementation of Franco's rules regarding the interpretation of longs and breves as a modulo 3 operation.

Key: "imperfection a.p.p." = imperfection by the following notes; "imperfection a.p.a." = imperfection by the preceding notes.

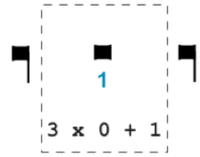
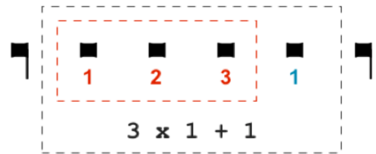
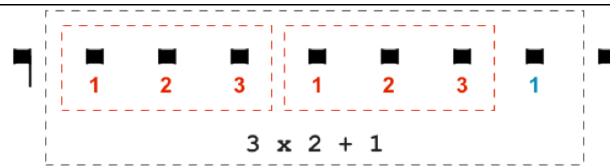
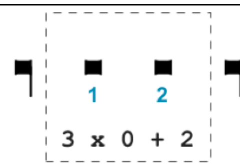
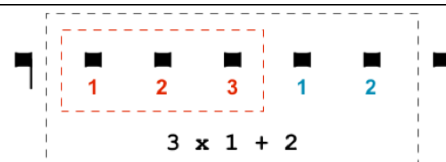
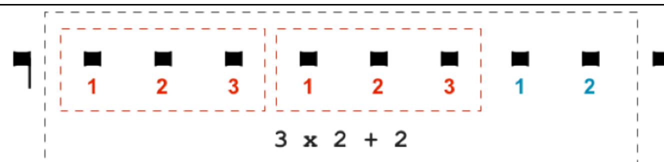
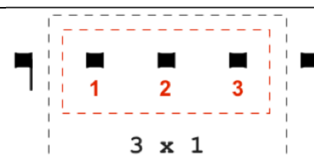
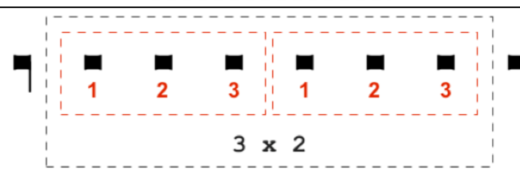
Case	Value of p	Value of n	Example
n = 3 p + 1	p = 0	n = 1	
	p = 1	n = 4	
	p = 2	n = 7	
n = 3 p + 2	p = 0	n = 2	
	p = 1	n = 5	
	p = 2	n = 8	
n = 3 p	p = 0	n = 0	
	p = 1	n = 3	
	p = 2	n = 6	

Table 3-6: Examples of sequences of notes delimited by longs for each of the three possible values of n shown in Table 3-5 (these are n = 3p + 1, n = 3p + 2, and n = 3p). The red boxes arrange the notes in groups of three breves and the remaining breves are shown in blue numbers. The variable "p" represents the number of red boxes and "n" stands for the total number of notes between the boundaries, which is equal to the notes in red boxes (3 x p) plus the remaining notes in blue numbers (which are 1, 2, or no notes).

According to Franco, the different sequences of notes outlined by the first pair of columns in Table 3-5 would be interpreted according to the “general interpretation” column of the table; if there is a dot of perfection (rows 1, 3, and 5) or a dot of division (rows 2 and 4), then the interpretation of the sequence of notes is changed to the one specified by the “alternative interpretation” column. Besides the presence of dots, there are other conditions that force an “alternative interpretation” of the sequence by forbidding one of the aspects of the “general interpretation”—this is, by forbidding either one of the two forms of imperfection, forbidding alteration, or even not allowing a note to keep its default, perfect, durational value. All these conditions are summarized in Table 3-7.

Interpretation	Conditions that prevent an interpretation	
	Related to the note	Condition of the note
Imperfection a.p.p.	Start note (s_1)	None
		It is a rest[25]
		It has a value higher than that of the delimiter note (e.g., in perfect modus this happens when s_1 is a maxima instead of a long)
		It has a dot (i.e., dot of perfection)
		It has already been modified by the previous notes (i.e., imperfection a.p.a.)
Imperfection a.p.a.	End note (s_k)	None
		It is a rest
		It has a value higher than that of the delimiter note (e.g., in perfect modus this happens when s_k is a maxima instead of a long)
		It has a dot (i.e., dot of perfection)
		It is followed by a note of the same type (e.g., in perfect modus this happens when a long is followed by another long, then the first long must be perfect).
Alteration	Last note of the middle notes (s_{k-1})	It is a rest
		It is smaller than supposed (e.g., in perfect modus this happens when the last breve is substituted by smaller note values)
Remain Perfect	Start note (s_1)	There is a dot of division
		It has already been modified by the previous notes (i.e., imperfection a.p.a.)

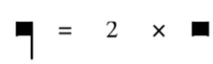
Table 3-7: Conditions that prevent imperfection, either of the start note (imperfection a.p.p.) or of the end note (imperfection a.p.a.) of the sequence, alteration, or that do not allow the start note to keep its perfect durational value.

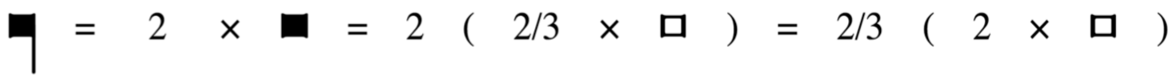
[25] Given the fact that rests have a fixed durational value, they cannot be imperfected or altered.

3.3.2 Implementation of Coloration

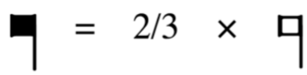
While there are plenty of pieces from the Ars nova that do not include coloration, almost all pieces from the fifteenth century that involve triple meter also involve some kind of coloration (either *hemiola* or *minor color*). Thus, in order to be able to test the current implementation on pieces from the fifteenth century, I included the handling of *hemiola coloration*. Minor color, the other main form of coloration of the period, was not included since the rhythm expressed by it can also be represented by uncolored notes (Figure 2-7); on the other hand, it is impossible to represent hemiola rhythm using uncolored notes, since the principles outlined on Section 2.2.3 would be in conflict.

As seen in Section 2.1, hemiola coloration can be defined at the level of the breve (hemiola temporis) or at the level of the semibreve (hemiola prolationis), since these are the two note levels in which perfect mensuration is generally used in fifteenth century music. The first step in the implementation consists of determining the level of the hemiola coloration. This is done by going through each of the note-levels with perfect mensuration, from top to bottom (i.e., first longs, then breves, and finally semibreves), until a colored note is found. Therefore, in the implementation, the level of hemiola coloration is defined as the largest colored note whose uncolored durational value, according to the mensuration, would be perfect. Once the level of the hemiola coloration is determined, one can calculate the durational value of all other colored notes based on Apel's description of coloration (see the end of Section 2.1).

The effect of coloration on a note depends on the level of that note with respect to the level of the hemiola coloration. If the colored note is at a lower level than the hemiola coloration's level, coloration has no effect on the note's durational value, as seen in Figure 2-5. If the colored note is at the hemiola coloration's level, then its value is given by the equation $colored\ note\ = 2/3 \times uncolored\ note$. If the colored note is at a higher level than the hemiola coloration's level, the note is also reduced to 2/3 of its original (uncolored) durational value; this is because of the way the search for the level of the hemiola is implemented. As an example, let us consider the durational value of a colored long in the case of hemiola temporis, which would be  due to the imperfect quality of colored notes, as pointed out by Apel (end of Section 2.1). Given that the $colored\ note\ = 2/3 \times uncolored\ note$ for the breve, the following is true:



The implementation for searching for the level of the hemiola, from top to bottom, implies that all the notes above the hemiola's level are either imperfect or uncolored. This means that a colored note at a higher level would have an imperfect durational value when uncolored; thus, in this example , therefore:



The effect of hemiola coloration is encoded in the @num and @numbase attributes. Therefore, for the colored notes and rests at a level higher or equal to the hemiola coloration level, the attributes @num = "2" and @numbase = "3" are added within the <note> or <rest> element; and for the colored notes at a lower level, no further treatment is required.

3.3.3 Identification of the Functionality of the Dots

The dots have three functionalities: augmentation, perfection, and division. The function of a dot of augmentation is the same as a modern dotted note, and its use is restricted to imperfect notes. With respect to the other two dots, Franco's principles indicate when are they used. From these principles, I inferred the following:

- In order to change the interpretation of a group of notes that involves imperfection a.p.p. (e.g., when a long is followed by one, four, five, or six breves), a dot is placed after the start note of the sequence (Figure 2-14, Figure 2-21, Figure 2-23, and Figure 2-25). This dot separates the start note from the rest of the notes of the sequence, indicating that it forms a perfection by itself and, thus, eliminating the possibility of an imperfection a.p.p. For this reason, this dot is known as a "dot of perfection".
- In order to change the interpretation of a group of notes that involves the start note being perfect (e.g., when a long is followed by two or three breves), a dot of division is placed after the first breve (Figure 2-17 and Figure 2-19). This dot separates these notes (the start note and the following breve) from the rest of the sequence, indicating that they should form a perfection by themselves and forcing an imperfection a.p.p.

From these remarks and from certain assumptions I made, the identification of the dot functionality is based on four premises:

- **Premise 1:** A dot of perfection comes directly after the start note of the sequence (i.e., it is zero units away from the start note).
- **Premise 2:** A dot of division is one unit away from the start note (where the unit is a breve in case of perfect modus, a semibreve in case of perfect tempus and a minim in case of major prolation).
- **Premise 3:** There can only be one dot of division (or perfection) in a sequence.
- **Premise 4:** In the presence of more than one dot in a sequence, the first dot is the only candidate for being the dot of division (or perfection).[26]

Premises 1 and 2 are based on inferences I drew from Franco's principles. Premises 3 and 4 are my hypotheses regarding the case in which more than one dot is present within a delimited sequence. Although this case was not that common, in the vast majority of the cases where it happened, all the dots were dots of augmentation. Out of approximately twenty pieces I looked at, there was only one in which this was not the case.[27] In this instance, the first dot of the delimited sequence was a dot of division which was followed by dots of augmentation. Therefore, when more than one dot is present within a delimited sequence, Premise 3 indicates that only one of these dots can be a dot of division (or perfection) and Premise 4 indicates that the dot of division can only be the first one, and consider all following dots as dots of augmentation (or dots of perfection operating at a smaller note level).[28]

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[26] Unlike Ars antiqua pieces, in which dots of division were used one after another in the same sequence to separate groups of semibreves equivalent to the breve, in Ars nova the dot is just used to divide the sequence in a different way from the default, for which it only needs to appear once in the sequence.

[27] Du Fay's chanson *Craindre vous vueil* (tenor).

[28] As will be seen on the experiment section, premise 4 does not hold in a small part of the dataset (in two instances). Premise 3 also requires an exhaustive search for counterexamples. The reason for taking on these two premises is to identify between dots of augmentation and dots of division. Apel does not provide an explicit set of rules for this, pointing out that the dot's function is "generally apparent from the musical context" (Apel 1953, 117). However, he does state that "a note of the next smaller species **must always** follow [a dot of augmentation] which provides the other half of the increase" and that "sometimes… this note does not directly follow the [dot], but that it is separated from it by notes of greater value" (Apel 1953, 117). Since I found a counterexample for this (two consecutive dotted semibreves) in

The implementation consists of finding the first dotted note in the delimited sequence $(s_i)_{i=1}^{k}$ and then determining the distance of that dot from the start note of the sequence (s_1). If the distance is 0, then the dot is a dot of perfection. If the distance is greater than 0, there are three possible scenarios. For simplicity, the scenarios are described in terms of sequences delimited by longs and, therefore, the distance is given in terms of the breve:

- The dot is more than one breve away from the start note. In this case, based on the premises outlined above, the only option is for this dot to be a dot of augmentation.[29]

- The dot is one breve away. In this case, the dot could be a dot of division, but it could also be a dot of augmentation. Thus, to determine the functionality of the dot, look at the second part of the sequence (i.e., the one following the dot). Consider that the dot is between s_j and s_{j+1}. The middle notes of the sequence can be divided into two parts: $(s_2, \ldots, s_j)$, which is the one preceding the dot, and $(s_{j+1}, \ldots, s_{k-1})$, which is the one following the dot. It has been established that the $\sum_{i=2}^{j} s_i$ is equal to one breve. Now, the sum of the notes after the dot $\sum_{i=j+1}^{k-1} s_i$ has two possibilities: being an integer or not. If it is an integer, then the dot is a dot of division since it is dividing the sequence $(s_i)_{i=1}^{k}$ into two parts with an integer number of breves, each of which would have an effect on each of the boundary notes. On the other hand, if it is not an integer, the dot is considered an augmentation dot, whose fractional part would be completed by the additional $\frac{s_j}{2}$ value generated by the augmentation dot.[30]

- The dot is less than one breve away. In this case, based on the premises outlined above, the only option is for this dot to be an augmentation dot.[31]

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one of the pieces I looked at, I decided to devise a new mechanism for determining the functionality of a dot. But, as mentioned before, counterexamples also exist for the devised set of premises.

[29] Or a dot that has already been identified as a dot of perfection when processing the sequences delimited by a note at a lower note-level (Section 3.3.4).

[30] Same as note 29.

[31] Same as note 29.

Any dot in the sequence after the first one is labeled as a dot of augmentation. Once the functionality of a dot has been determined, it is encoded within the <dot> element by using either @form = “aug”, for a dot of augmentation, or @form = “div”, for a dot of division or perfection. In the case of a dot of augmentation, its effect on the preceding note is also encoded by adding the attributes @num = “2” and @numbase = “3” within the corresponding <note> element. In the case of a dot of division, the delimited sequence $(s_i)_{i=1}^{k}$ is separated into two sequences of the form $(s_1, \dots, s_j, None)$ and $(None, s_{j+1}, \dots, s_k)$, given that the dot is placed between the s_j and s_{j+1} notes; each of these two sequences is processed separately according to the procedures presented in Section 3.3.1 (i.e., determining the number of notes between the boundaries of the sequence in terms of a given unit—this unit can be a breve, semibreve, or minim—and modifying a note or notes in the sequence according to this number). In case of a dot of perfection, no further treatment is needed.

3.3.4 Handling of Simultaneous Triple Meter at Different Note Levels

In the case where perfect mensuration is present at multiple note levels at a time, it is possible to encounter sequences delimited by short perfect notes embedded within a sequence delimited by larger perfect notes. To illustrate this, consider the sequence delimited by longs shown in Figure 3-22, with perfect mensuration at the level of the long, breve, and semibreve. The blue, green, and orange boxes of the figure surround the middle notes of the sequences delimited by longs, breves, and semibreves, respectively. As one can see, the sequence delimited by longs contains sequences delimited by breves, which, in turn, contain sequences delimited by semibreves. The numbers in orange indicate the number of minims within the boundaries of the sequence delimited by semibreves (orange box), the numbers in green indicate the number of semibreves within the boundaries of the sequences delimited by breves (green boxes), and the numbers in blue indicate the number of breves between the boundaries of the sequence delimited by longs (blue box). The arrows coming out from the boxes towards a note indicate the effect (i.e., modification) that the number of notes within that box has on the note indicated by the arrow (e.g., the two semibreves in the last green box cause the alteration of the latter of them, and the seven breves in the blue box imperfect the first long of the sequence).

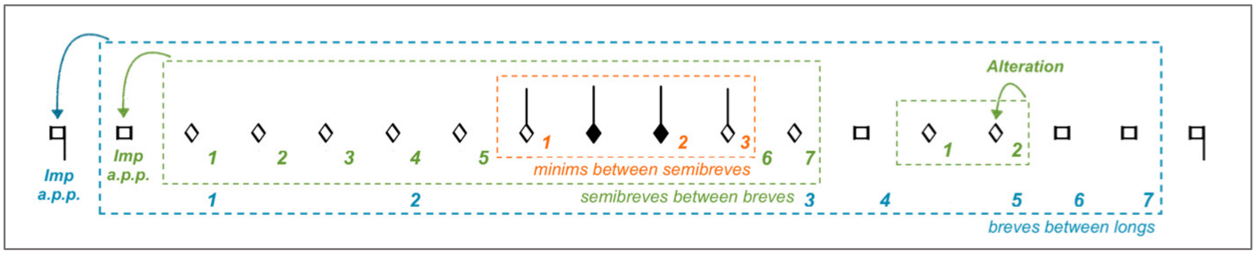

Figure 3-22: Example of sequences delimited by short notes embedded within sequences delimited by larger notes for the case of perfect modus, perfect tempus, and major prolation.

Since there was no explicit statement regarding how to deal with this situation in the theoretical sources, I devised a simple method to do so: the implementation defines and processes the sequences delimited by short notes first, and then proceeds to sequences delimited by larger notes. Because of this, it is possible to encounter notes within the boundaries of a delimited sequence that already contain a pair of @num and @numbase attributes, since these were encoded when a sequence delimited by a lower note-level was processed. For example, the first breve of Figure 3-22 has been imperfected due to the seven semibreves following it (green box); therefore, when being considered as part of the notes between the boundaries of a sequence delimited by longs (blue box), it already contains a @num and @numbase attributes that change its default durational value.

In this chapter, I presented the implementation details behind the scoring-up tool. I started by presenting the structure of the input MEI files that encode each of the individual voices of a mensural piece. The contents of these files are limited to capturing the mensuration, the pitch, and the note shape information of each voice. After a brief description on the process of merging these files into a single one, I proceeded to explain the core of the scoring-up implementation: the duration finder module. The duration finder module deals with the context-dependent nature of mensural notation in perfect mensurations. In this section, I presented the implementation of Franco's principles of imperfection and alteration, together with the methods developed for identifying the functionality of the dots (division or augmentation), interpreting notes in hemiola coloration, and handling perfect mensuration at multiple note levels. The result of the scoring-up implementation is a single MEI file that encodes all the voices of the piece in mensural notation and includes the contextual durational value of each note.

When the resulting MEI file is rendered in Verovio (the music notation engraving library designed for MEI), it is displayed as a score. Verovio aligns the notes vertically according to the mensuration (@modusminor, @tempus, and @prolatio), note shape (@dur), and modification values (@num and @numbase) encoded within the scored-up MEI file. The methods developed in this chapter will be put into practice in the following chapter, where I will score up pieces from the fourteenth and fifteenth centuries.

Chapter 4 Experiment

In this chapter, I evaluate the performance of the scoring-up tool described in Chapter 3. Although the scoring-up implementation is meant to work with Ars nova and white notation pieces written in any mensuration, compositions completely written in duple meter are not tested because there are no context-dependent issues. Therefore, the scoring-up tool was only tested using Ars nova and white notation pieces with triple meter. This restricts the timespan of the dataset to music written in the fourteenth and fifteenth centuries, since mensural music prior to this period belongs to the Ars antiqua tradition and compositions dating afterwards are, for the most part, in duple meter. The list of 19 pieces in the dataset is shown in Table 4-1. As described at the beginning of Chapter 3, the dataset consists of the Mensural MEI files that encode the mensuration, pitch, and note-shape information of each of the individual voices of the pieces from Table 4-1. In order to evaluate the performance of the scoring-up tool, I compared the Mensural MEI file resulting from the scoring-up against a ground-truth Mensural MEI score, which encoded the correct durational values for each of the notes. Since there was no existing repertoire encoded in Mensural MEI at the beginning of this work, I had to generate both the ground truth and the input data. The dataset creation methodology is presented in Section 4.1. Section 4.2 explains the method I used to evaluate the quality of the output of the scoring-up tool. Section 4.3 shows the results regarding the performance of the scoring-up in the dataset. Section 4.4 presents the analysis of the results by discussing the types of errors (i.e., misinterpretation of notes) in the output of the scoring-up. Finally, Section 4.5 presents possible strategies to handle these errors in the future.

4.1 Dataset Creation Methodology

When I began this work, there was not only a lack of music encoded in Mensural MEI, but also there was no way to encode the dataset into Mensural MEI files other than hand-coding them, since the other methods for encoding music notation (score editors and OMR) either do not support MEI or are not designed for this repertoire. There is no score editor for mensural notation that can store or export the music into a Mensural MEI file, and *Aruspix*, the most highly regarded OMR software for mensural notation, is designed for printed music from the sixteenth and seventeenth centuries, a timeframe which is not suitable for this work.

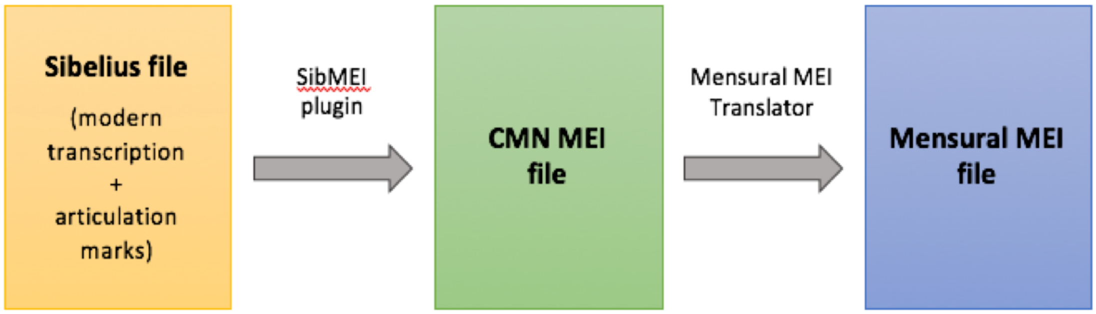

Figure 4-1: Three-stage process that takes modern transcriptions of mensural pieces and translates them back into mensural values encoded within a Mensural MEI file. This process was used to obtain the ground truth and, by further processing, the input data for this work.

In order to generate the ground truth and input data for the scoring-up tool, I used another tool I developed alongside this work, called the Mensural MEI Translator.[32] It was developed in the context of the *Measuring Polyphony Project,*[33] an ongoing project focused on music from the Ars antiqua and Ars nova. The Mensural MEI Translator is the last part of a three-stage process that takes modern transcriptions of mensural pieces (i.e., the pieces transcribed into modern values and in a score format) and translates them back into mensural values encoded within a Mensural MEI file. The resulting Mensural MEI file includes not only pitch and note-shape information for every note, but also the durational information, which is retrieved from the modern transcription of the piece. The three-stage process in which the Mensural MEI Translator is involved consists of: 1) entering a modern transcription into Sibelius and adding articulation marks to account for mensural notation specificities that are not regularly included within a modern transcription (e.g., the use of staccato marks to represent dots of division and the use of stopped-note marks to distinguish an altered note from an imperfect, larger, note), 2) using the SibMEI plugin to obtain the CMN MEI file that encodes this edited modern transcription, and 3) using the Mensural MEI Translator script to transform this CMN MEI into a Mensural MEI file. Assuming that the

[32] https://github.com/DDMAL/CMN-MEI_to_MensuralMEI_Translator.

[33] ‘Measuring Polyphony: Digitally Mediated Access to the Music of the Middle Ages’ is a project currently under development, led by Karen Desmond at Brandeis University. Desmond presented on the project in a paper entitled “Measuring Polyphony: a project to encode the semantics of the context-based (and under-prescriptive) notation of late medieval music” at the Digital Humanities (DH) 2017 Annual Conference in Montréal, Canada, August 2017.

modern transcription is correct, the Mensural MEI file obtained by the Translator encodes the correct durational value for each of the individual notes in the piece. This means it can serve as a ground truth against which to compare the results of the scoring-up tool.

Piece Code	Short Title	Composer	Source of the Modern Transcription	Source of the Mensural Piece
Iv001	Bona	Vitry	Karen Desmond, Measuring Polyphony Project (checked against the manuscript I-IV)	
Iv002	Cum venerint	Anonymous		
Iv003	Decens	Vitry		
Iv004	De touz	Machaut		
Iv005	Dieux	Anonymous		
Iv006	Durement	Vitry		
Iv007	Hugo	Vitry		
Iv008	Post misse	Anonymous		
Duf16002	Ce moys de may	Du Fay	CPDL[34]	Edition of Du Fay Chansons from GB-Ob in original notation by Ross W. Duffin
Duf22518	Je ne suy plus	Du Fay	CPDL[35]	
Duf3007.2	Caindre vous vueil	Du Fay	JRP	
Duf3025	Bon jour, bon mois	Du Fay	JRP	
Duf3069	Resvelons nous	Du Fay	JRP	
Ock3008	La despourveue	Ockeghem	JRP	US-Wc
Ock3009_Dijon	L'autre d'antan	Ockeghem	JRP	F-Dm
Ock3009_Mellon	L'autre d'antan	Ockeghem	JRP	US-NHub
Ock3012	Ma maistresse (first part)	Ockeghem	JRP	D-W
Ock3016	Presque transi	Ockeghem	JRP	US-Wc
Ock3027	Permanente vierge	Ockeghem	JRP	F-Dm

Table 4-1: Fourteenth- and fifteenth-century pieces used in the dataset.

Key: CPDL = Choral Public Domain Library;
JRP = Josquin Research Project.

[34] CPDL #16002. Edited by Brian Russell (submitted 2008-02-15).
[35] CPDL #22518. Edited by Renato Calcaterra (submitted 2010-10-27).

For the fourteenth century, I chose pieces from the Ars nova repertoire of the Measuring Polyphony Project (the first eight pieces in Table 4-1).[36] These pieces, all of them motets, were already encoded as Mensural MEI files, so no preprocessing was needed in this case. For the fifteenth century, the Mensural MEI files were obtained by converting modern transcriptions of the pieces back into mensural notation using the process outlined in Figure 4-1. For this purpose, I obtained modern transcriptions in MusicXML and MIDI formats from different projects such as the *Josquin Research Project* (JRP) and the *Choral Public Domain Library* (CPDL); these file formats were imported into Sibelius and checked against the manuscript for further editing (i.e., the addition of the articulation marks).[37] Given the amount of time invested into the edition of the transcriptions, and because some of them needed more than just the addition of the articulation marks to be fully faithful to their manuscript sources, I only used chansons, since these are short compositions. I chose triple meter chansons by Du Fay and Ockeghem to represent the beginning and middle of the fifteenth-century music, respectively. The mensuration of all these fourteenth- and fifteenth-century pieces is shown in Table 4-2. A few pieces included partial imperfection; since this feature is currently out of the scope of the scoring-up tool, the notes that were partially imperfected were substituted by either an imperfect or perfect note (depending on the case) for the purpose of this experiment.[38]

Once I obtained the ground-truth Mensural MEI files for each of the pieces in Table 4-1, I used them to generate the input data for the scoring-up tool. For this purpose, I developed a script that performed two actions: removing the attributes @num and @numbase from each of the <note> elements in the file (thus, removing the information regarding the perfect / imperfect / altered quality of the note) and separating the voices into different Mensural MEI files.

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[36] The ones that, at the current stage of the project, have been double checked against a manuscript source and include all the corresponding articulation marks.

[37] Du Fay's pieces were checked against a clean and error-free edition of manuscript sources (Ross Duffin's edition of Du Fay Chansons in original notation [Duffin 1983]), while Ockeghem's were checked against the original manuscripts. For the most part the manuscript sources used for Ockeghem pieces were error free, except for an extra note in the cantus of Ock3008, a repeated group of notes in the cantus of Ock3012, and two notes missed in the cantus of Ock3027. These errors were removed, so that both Ockeghem's and Du Fay's pieces are based on sources of the same quality.

[38] There are four instances of partial imperfection in the motetus of *Iv004*, one in the tenor of *Duf22518*, and one in the contra voice of *Duf3025*.

<table>
<tr><th rowspan="2">Piece Code</th><th colspan="3">Voice 1</th><th colspan="3">Voice 2</th><th colspan="3">Voice 3</th><th colspan="3">Voice 4</th></tr>
<tr><th>L</th><th>B</th><th>Sb</th><th>L</th><th>B</th><th>Sb</th><th>L</th><th>B</th><th>Sb</th><th>L</th><th>B</th><th>Sb</th></tr>
<tr><td>Iv001</td><td>3</td><td>2</td><td>3</td><td>3</td><td>2</td><td>3</td><td>2</td><td>2</td><td>NA</td><td colspan="3" rowspan="2">NA</td></tr>
<tr><td>Iv002</td><td>3</td><td>2</td><td>3</td><td>3</td><td>2</td><td>3</td><td>3</td><td>2</td><td>NA</td></tr>
<tr><td>Iv004</td><td>2</td><td>2</td><td>3</td><td>2</td><td>2</td><td>3</td><td>2</td><td>2</td><td>NA</td><td>2</td><td>2</td><td>NA</td></tr>
<tr><td>Iv003</td><td>3</td><td>3</td><td>2</td><td>2</td><td>3</td><td>2</td><td>3</td><td>3</td><td>2</td><td colspan="3" rowspan="4">NA</td></tr>
<tr><td>Iv005</td><td>2</td><td>2</td><td>3</td><td>2</td><td>2</td><td>3</td><td>2</td><td>2</td><td>NA</td></tr>
<tr><td>Iv006</td><td>2</td><td>2</td><td>3</td><td>2</td><td>2</td><td>3</td><td>2</td><td>2</td><td>NA</td></tr>
<tr><td>Iv007</td><td>3</td><td>2</td><td>3</td><td>3</td><td>2</td><td>3</td><td>3</td><td>2</td><td>NA</td></tr>
<tr><td>Iv008</td><td>2</td><td>2</td><td>2</td><td>3</td><td>2</td><td>2</td><td>3</td><td>2</td><td>2</td><td>3</td><td>2</td><td>2</td></tr>
<tr><td>Duf16002</td><td rowspan="2">2</td><td rowspan="2">2</td><td rowspan="2">3</td><td rowspan="2">2</td><td rowspan="2">2</td><td rowspan="2">3</td><td rowspan="2">2</td><td rowspan="2">2</td><td rowspan="2">3</td><td colspan="3" rowspan="13">NA</td></tr>
<tr><td>Duf22518</td></tr>
<tr><td>Duf3007.2</td><td rowspan="11">2</td><td rowspan="11">3</td><td rowspan="11">2</td><td rowspan="11">2</td><td rowspan="11">3</td><td rowspan="11">2</td><td rowspan="11">2</td><td rowspan="11">3</td><td rowspan="11">2</td></tr>
<tr><td>Duf3025</td></tr>
<tr><td>Duf3069</td></tr>
<tr><td>Ock3008</td></tr>
<tr><td>Ock3009_Dijon</td></tr>
<tr><td>Ock3009_Mellon</td></tr>
<tr><td>Ock3012</td></tr>
<tr><td>Ock3016</td></tr>
<tr><td>Ock3027</td></tr>
</table>

Table 4-2: Mensuration values for each voice of the pieces in Table 4-1. The numbers in the columns labeled as "L", "B", and "Sb" represent the perfect or imperfect mensuration at the level of the long (i.e., the modus), breve (i.e., the tempus), and semibreve (i.e., the prolation), respectively. Perfect mensurations are indicated by the number "3" and imperfect by the number "2", the value NA in the prolation of some voices indicates that there are no semibreves or minims in that voice and, thus, the definition of the mensuration at the level of the semibreve is "Not Applicable".

4.2 Scoring-Up Evaluation

In order to evaluate the performance of the tool in the dataset, I determined its accuracy in identifying which note values should or should not be modified (from their default value given by the mensuration) from the set of all modifiable notes in a piece (or voice). As explained in Section 2.1, in mensural notation the set of modifiable notes is limited to: notes that are subject to be modified by context (these are perfect notes—

since they are subject to imperfection—and notes that are subject to alteration), notes that are subject to augmentation (i.e., dotted notes that are not perfect), and colored notes. Table 4-3 shows these categories together with the requirements for a note to belong to each of them, since not every note is subject to a particular form of modification (e.g., a semibreve between breves in perfect tempus is not a candidate for alteration). In the *perfect notes* category, which implies notes that are subject to imperfection, colored notes are not included, even though coloration can cause imperfection; this is because the perfect category is meant for notes that are imperfected *due to context*. With respect to the *notes subject to augmentation* category, determining whether a note is regular or augmented is equivalent to determine whether a dot is acting as a dot of division or a dot of augmentation. Thus, evaluating the accuracy of the tool in identifying whether a note subject to augmentation is supposed to be augmented or not is a method of evaluating the performance of the "identification of the functionality of the dots" function presented in Section 3.3.3. Given the requirements shown in the second column of Table 4-3, the four categories of modifiable notes are mutually exclusive.

<table>
<tr><th colspan="2">Categories of Modifiable Notes (M.N.)</th><th>Requirements for belonging to an M.N. category</th><th>Quality of a note in an M.N. category</th></tr>
<tr><td rowspan="4">Notes subject to modification by context</td><td rowspan="2">Perfect notes (subject to imperfection)</td><td rowspan="2">Non-colored note</td><td>Perfect</td></tr>
<tr><td>Imperfect</td></tr>
<tr><td rowspan="2">Notes subject to alteration</td><td rowspan="2">• Penultimate note in a sequence that has either 3p+2 (with p >= 0) or 3p (with p>=1) notes between the boundaries.
• Notes not substituted by smaller note values.</td><td>Regular</td></tr>
<tr><td>Altered</td></tr>
<tr><td colspan="2" rowspan="2">Notes subject to augmentation</td><td rowspan="2">Notes followed by a dot, except for perfect notes.</td><td>Regular</td></tr>
<tr><td>Augmented</td></tr>
<tr><td colspan="2" rowspan="2">Colored notes</td><td rowspan="2">Being colored</td><td>2/3 of uncolored note durational value</td></tr>
<tr><td>Same as uncolored note durational value</td></tr>
</table>

Table 4-3: Categories of modifiable notes on a mensural piece.

Key: M.N. = modifiable notes;
Quality = the way in which the note is modifiable.

The evaluation of the performance of the scoring-up is based on the number of mislabeled notes, which are those notes whose durational value does not match the one in the ground truth. Regarding the non-context-related categories of Table 4-3, the performance of the scoring-up tool was computed for each voice as follows:

- Accuracy in determining the regular or augmented durational value of a note subject to augmentation:

$$1 - \frac{\text{Notes mislabeled as "regular" or "augmented"}}{\text{notes subject to augmentation}}$$

- Accuracy in determining the durational value of a colored note:

$$1 - \frac{\text{colored notes with a wrong durational value}}{\text{colored notes}}$$

The evaluation of the scoring-up performance in determining the "perfect/imperfect" and "regular/altered" quality of a note was evaluated for each perfect mensuration in each voice, and it was computed as follows:

$$1 - \frac{\text{notes mislabeled as "perfect" or "imperfect"} + \text{notes mislabeled as "regular" or "altered"}}{\text{notes subject to modification by context}}$$

The global accuracy of each piece was determined by:

$$1 - \frac{\text{all mislabeled notes}}{\text{all modifiable notes}}$$

In the following sections, I present the results regarding the accuracy of the tool to identify which notes should and should not be modified from the set of all the modifiable notes and an analysis of the different situations in which the tool failed in performing this task, discussing the implications of these mislabeled notes and possible methods to handle these types of errors in the future.

4.3 Results

The accuracy results of the scoring up in each piece are shown in Table 4-4. Although certain information can be inferred from the table (e.g., all colored notes were assigned the correct durational value except in *Ock3009_Dijon*, the only pieces in which the functionality of the dot was misinterpreted were *Duf16002* and *Duf22518*, the lower voices tend to have a high accuracy compared to the upper ones, etc.), more detailed information can be obtained by analyzing the context in which the errors occurred (see Section 4.4).

Piece	Voice	Note Level	Perfect/Imperfect and Regular/Altered	Colored	Regular/Augmented	Accuracy per Piece
Iv001	1	L - B	85.71%	NA	NA	99.17%
		Sb - M	100%			
	2	L - B	96.00%	NA	100%	
		Sb - M	100%			
	3	NA	NA	NA	NA	
Iv002	1	L - B	100%	NA	NA	98.88%
		Sb - M	100%			
	2	L - B	92.59%	NA	NA	
		Sb - M	100%			
	3	L - B	100%	NA	NA	
Iv003	1	Sb - M	100%	NA	NA	100%
	2	Sb - M	100%	NA	NA	
	3	NA	NA	NA	100%	
	4	NA	NA	NA	100%	
Iv004	1	L - B	100%	NA	100%	94.12%
		B - Sb	94.67%			
	2	B - Sb	85.45%	NA	100%	
	3	L - B	100%	NA	NA	
		B - Sb	100%			
Iv005	1	Sb - M	100%	NA	NA	100%
	2	Sb - M	100%	NA	NA	
	3	NA	NA	NA	NA	
Iv006	1	Sb - M	100%	NA	NA	100%
	2	Sb - M	100%	NA	100%	
	3	NA	NA	NA	NA	
Iv007	1	L - B	78.57%	NA	NA	98.95%
		Sb - M	100%			
	2	L - B	100%	NA	100%	
		Sb - M	100%			
	3	L - B	100%	NA	NA	
Iv008	1	NA	NA	NA	100%	99.04%
	2	L - B	100%	NA	NA	
	3	L - B	100%	NA	NA	
	4	L - B	100%	NA	NA	

Duf16002	1	Sb - M	97.14%	100%	80.00%	97.04%
	2	Sb - M	97.73%	100%	71.43%	
	3	Sb - M	100%	100%	100%	
Duf22518	1	Sb - M	95.24%	NA	87.50%	94.94%
	2	Sb - M	100%	100%	100%	
	3	Sb - M	89.47%	100%	100%	
Duf3007.2	1	B - Sb	100%	100%	100%	100%
	2	B - Sb	100%	NA	100%	
	3	B - Sb	100%	NA	100%	
Duf3025	1	B - Sb	92.00%	NA	100%	98.46%
	2	B - Sb	100%	100%	100%	
	3	B - Sb	100%	100%	100%	
Duf3069	1	B - Sb	100%	NA	100%	100%
	2	B - Sb	100%	NA	NA	
	3	B - Sb	100%	NA	NA	
Ock3008	1	B - Sb	100%	NA	100%	97.59%
	2	B - Sb	90.00%	100%	100%	
	3	B - Sb	100%	100%	100%	
Ock3009 (Dijon)	1	B - Sb	88.00%	90.00%	100%	94.12%
	2	B - Sb	100%	100%	100%	
	3	B - Sb	85.00%	100%	100%	
Ock3009 (Mellon)	1	B - Sb	100%	100%	100%	100%
	2	B - Sb	100%	100%	100%	
	3	B - Sb	100%	100%	100%	
Ock3012	1	B - Sb	50.00%	NA	100%	94.03%
	2	B - Sb	100%	NA	100%	
	3	B - Sb	80.00%	NA	100%	
Ock3016	1	B - Sb	75.00%	100%	100%	90.57%
	2	B - Sb	52.94%	100%	100%	
	3	B - Sb	95.00%	100%	100%	
Ock3027	1	B - Sb	100%	NA	100%	100%
	2	B - Sb	100%	NA	100%	
	3	B - Sb	100%	100%	100%	
	4	B - Sb	100%	100%	100%	
	5	B - Sb	100%	NA	100%	

Table 4-4: Accuracy of the scoring-up tool

4.4 Analysis of Errors

The number of notes in the scoring-up file whose durational value (or quality) did not match the ground-truth file is shown in the far-right column of Table 4-5. These 55 notes (out of 2866 modifiable notes) were mislabeled either as perfect, imperfect, or altered by the scoring-up tool. These errors are grouped by the context in which they occurred, these are the types of error shown in the first two columns of Table 4-5. The third column indicates the number of instances of each type of error found in the corpus; the fourth column indicates how many notes are mislabeled by a single instance of a particular type of error.

<table>
<tr><th colspan="2">Types of Error (T.E.)</th><th>Instances of a T.E.</th><th>Mislabeled notes per instance</th><th>Total of mislabeled notes</th></tr>
<tr><td rowspan="3">Errors in the Sources</td><td>NoDot A</td><td>8</td><td>2</td><td>16</td></tr>
<tr><td>NoDot B</td><td>3</td><td>3</td><td>9</td></tr>
<tr><td>Colored</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td rowspan="5">Errors in the Experiment</td><td>DotAlt</td><td>3</td><td>2</td><td>6</td></tr>
<tr><td>MisplacedDot</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>HemiolaGroup</td><td>1</td><td>2</td><td>2</td></tr>
<tr><td>LastNote</td><td>7</td><td>1</td><td>7</td></tr>
<tr><td>RestLines</td><td>2</td><td>2</td><td>4</td></tr>
<tr><td rowspan="2">Errors caused by Incompleteness of the principles</td><td rowspan="2">OnsetAt3</td><td rowspan="2">5</td><td>2 in 4 instances</td><td>8</td></tr>
<tr><td>1 in another</td><td>1</td></tr>
<tr><td></td><td></td><td></td><td></td><td>55</td></tr>
</table>

Table 4-5: Types of errors in the scoring-up output

Each type of error and its effects are described below, while potential strategies for correcting these errors are presented in Section 4.5.

4.4.1 Errors in the Sources

There were two types of errors present in the sources: the absence of dots (*NoDots*) and a mistakenly colored note (*colored*). The analysis of these types of errors is presented below.

NoDot: Absence of a dot of perfection or a dot of division within a sequence of notes

In eleven instances, the interpretation of the scoring-up for sequences that had a particular number of notes within the boundaries did not coincide with the interpretation given to them in the ground truth. The contexts in which these errors occurred, classified as *NoDot A* and *NoDot B* (Table 4-6), can be avoided by the use of a dot of division or a dot of perfection as described by Franco,[39] as will be shown in the following two sections (NoDot A and NoDot B).

Context of the Error	Notes between boundaries (n)	Instances	Number of perfect groups (p)	Instances
NoDot A	n = 3p	8	p = 1	1
			p > 1	7
NoDot B	n = 3p + 2	3	p = 0	3
			p > 0	0

Table 4-6: The two types of context in which there was an error due to the absence of a dot of division or a dot of perfection

NoDot A: For sequences with n notes between the boundaries such that n = 3p with p > 0

This error was present in three different pieces: *Iv002* (in a sequence with 6 breves between longs), *Iv004* (in six sequences with 9 semibreves between breves), and *Ock3008* (in a sequence with 3 semibreves between breves). According to Franco, in situations like the three listed above, one should use a dot (a dot of division in the case of the sequence with n = 3 and a dot of perfection in the case of the sequences with n = 6 or 9) in order to convey an interpretation that differs from the default. For the eight instances in which the scoring-up interpretation of these sequences was incorrect, the ground truth interpretation (this is, the interpretation of the sequence according to

[39] Although in some instances the absence of the dot is a typographical error, in others the grouping of the notes is conveyed by the proximity of the notes.

the modern transcription) differed from the default, although in the piece no dot is used to indicate this.

Figure 4-2 and Figure 4-3 illustrate the case for n = 3p with p = 1. Figure 4-2 shows the correct interpretation of the sequence of notes in the red box, and Figure 4-3 shows the same sequence as interpreted by the scoring-up tool (according to Franco's rules). As shown in Figure 4-3, the absence of a dot of perfection in the sequence causes the mislabeling of two notes: the first breve, which is considered perfect instead of imperfect; and the last semibreve, which is considered regular instead of altered. Besides the mislabeling, all notes that lie between the two mislabeled ones have been shifted by one semibreve (compare to Figure 4-2). This shift results in a dissonance in the tenor voice, in which the G sounds against an A in the top voice, and over a D in the bottom voice (which is an illegal fourth). In the same manuscript, there are other instances in which the same intended interpretation (red box of Figure 4-2) for this type of sequence is clearly indicated with a dot of division, as can be seen in the gray dotted box in the contra voice in Figure 4-2.

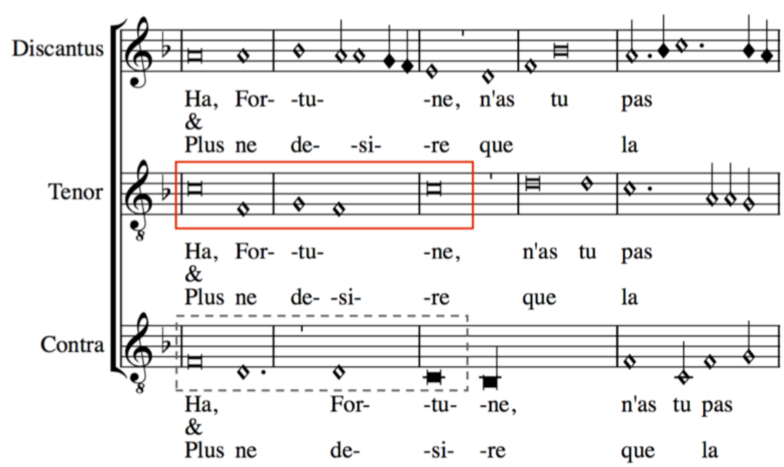

Figure 4-2: Ground truth interpretation of Ock3008 (tenor voice) based on a modern transcription. Barlines are used merely to facilitate the observation of the perfect groupings.[40]

[40] The semibreve rests are in the top line of the staff, this is the line in which Verovio places the semibreve rests by default.

Figure 4-3: Incorrect interpretation from scoring-up tool of Ock3008 (tenor voice). As in Figure 4-2, barlines are used merely to facilitate the observation of the perfect groupings.

Figure 4-4 and Figure 4-5 illustrate the case for n = 3p with p > 1 (i.e. there are the equivalent of 9 semibreves between the two breves in the sequence). Figure 4-4 shows the correct interpretation of the sequence of notes in the red box, and Figure 4-5 shows how the same sequence is interpreted by the scoring-up tool. As shown in Figure 4-5, the absence of a dot of division in the sequence causes the mislabeling of two notes: the first breve, which is considered imperfect instead of perfect; and the last semibreve, which is mislabeled as altered. There is also a shift of one semibreve between the mislabeled notes. It is clear this interpretation is incorrect since rests must not coincide in a hocket passage, which is the case with the minim rests located after the second barline. Yet again, in the same manuscript there are other instances in which the intended rhythm (red box in Figure 4-4) is clearly expressed by the use of dots of perfection (see gray dotted box in Figure 4-4).

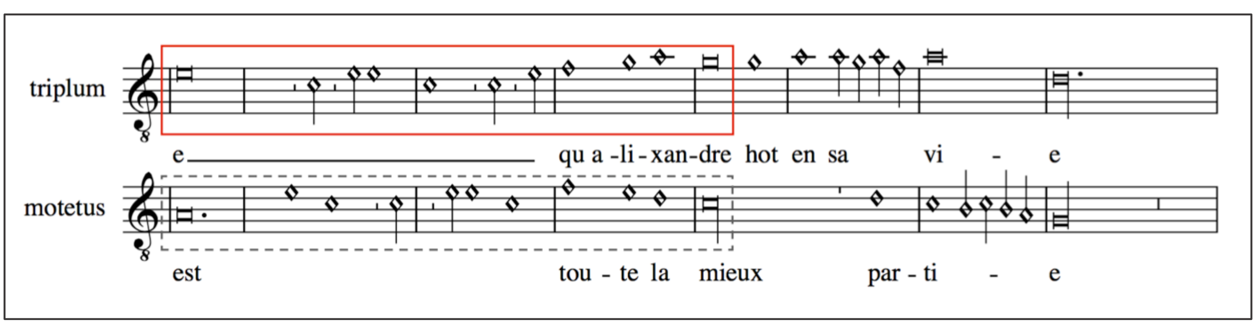

Figure 4-4: Ground truth interpretation of Iv004 (triplum) based on a modern transcription. Barlines are used to facilitate the observation of the perfect groupings.

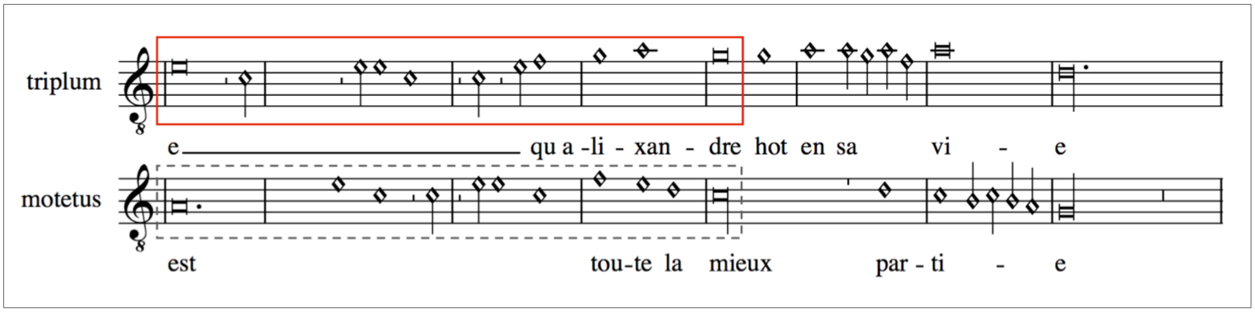

Figure 4-5: Incorrect interpretation from scoring-up tool of Iv004 (triplum).

NoDot B: For sequences with n notes between the boundaries such that n = 3p + 2 with p >= 0

This error was present in two pieces: *Ock3009* based on Dijon (in two sequences with 2 semibreves between breves) and *Ock3016* (in one sequence of the same type). As in the case of **NoDot A**, the output of the scoring-up when dealing with such sequences did not match the ground truth due to the absence of a dot of division. Figure 4-6 shows the correct interpretation of the sequence of notes in the red box, as written in the modern transcription of the piece. Figure 4-7 shows the interpretation returned by the scoring-up for the same sequence. The absence of a dot of division caused the mislabeling of three notes by the scoring-up tool: the first breve in the sequence, the last semibreve of the sequence, and the last semibreve of the following sequence (dotted red circle). The mislabeling shifts the notes between the first pair of mislabeled notes by one semibreve, but it also shifts every single note for the rest of the piece by a whole breve, which, in the case of *Ock3009*, is especially consequential since the excerpt shown in Figure 4-7 is taken from the middle of the piece. In *Mellon* (the other manuscript containing this piece) the scribe places a dot between the two semibreves of the sequence, avoiding any misinterpretation (see Figure 4-8 and Figure 4-9).

Figure 4-6: Ground truth interpretation of Ock3009 (contra voice) as found in Dijon based on a modern transcription.

Figure 4-7: Incorrect interpretation from scoring-up tool of Ock3009 (contra voice) as found in Dijon.

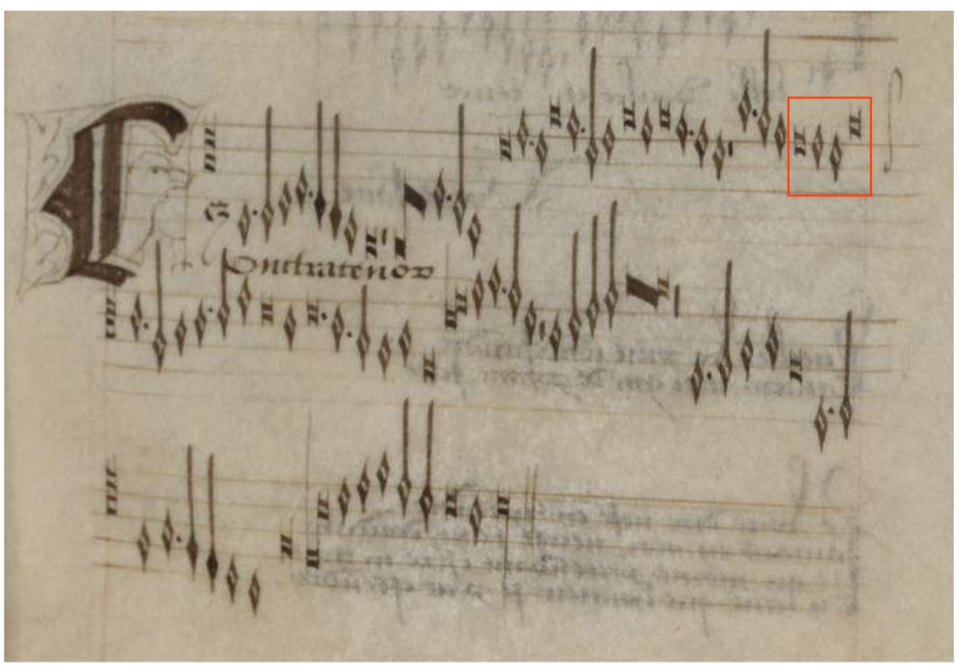

Figure 4-8: Contra from Ock3009 as found in Dijon. The red box corresponds to the sequence of notes in Figure 4-6.

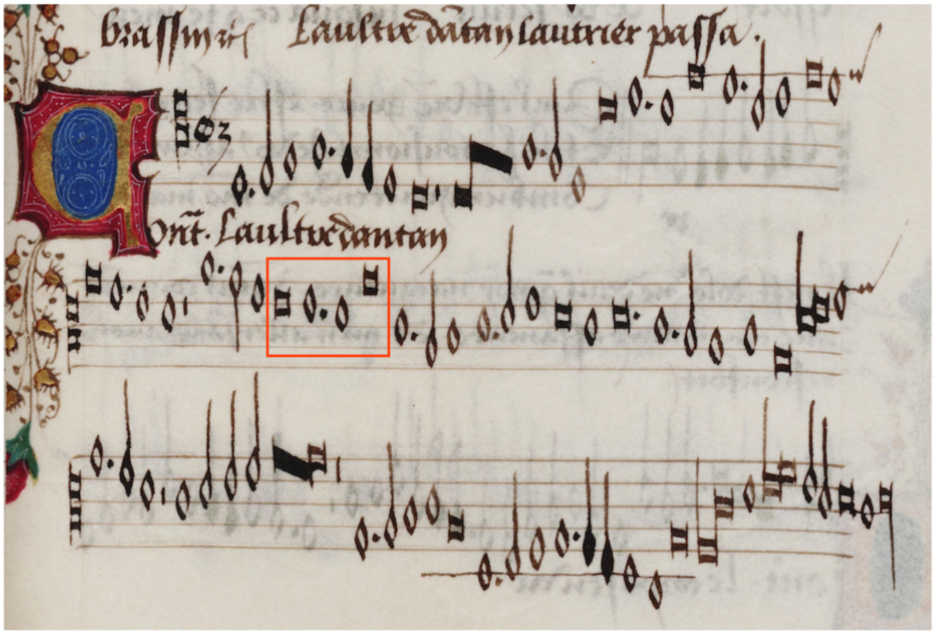

Figure 4-9: Contra from Ock3009 according to Mellon. The red box corresponds to the sequence of notes in Figure 4-6.

The case of $n = 3p + 2$ with $p > 0$ is also subject to misinterpretations from the scoring-up due to the absence of a dot of perfection. But in the corpus, both instances in which these sequences were misinterpreted fall into a different category, RestLines, and they will be discussed in that context.

Colored: Mistakenly colored note

All colored notes were assigned the correct durational value, except for one mistakenly colored note in one of the manuscript sources of *Ock3009*. In *Dijon*, *Ock3009* has an extra colored breve in the tenor (red box in Figure 4-10); the coloration of this breve leads the scoring-up tool to mistakenly consider it as imperfect. This mistake is not present in *Mellon*, where the same breve is uncolored (red box in Figure 4-11) and its interpretation by the scoring-up tool is correct in this case.

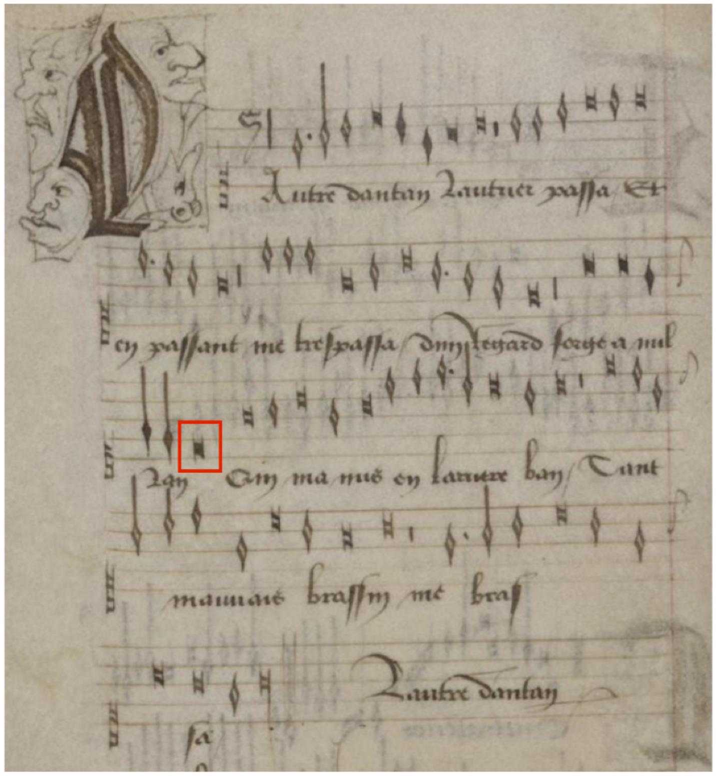

Figure 4-10: Tenor voice in Ock3009 according to Dijon.

Figure 4-11: Tenor voice in Ock3009 according to Mellon.

4.4.2 Errors in the Experiment

The errors in the experiment are associated with errors in the scoring-up implementation itself or problems with the dataset. Regarding the scoring-up tool, there were no errors related to the implementation of Franco's principles of imperfection and alteration, but two of the assumptions I made to identify the functionality of a dot and an assumption regarding hemiola coloration were not applicable to all pieces. Regarding the problems with the dataset, information regarding the position in the staff of the semibreve and minim rests was not included in the files.

4.4.2.1 Errors in the Scoring-up Implementation

There are four kinds of errors in the scoring-up implementation: two related to dots (*DotAlt* and *MisplacedDot*), one related to hemiola coloration (*HemiolaGroup*), and one related to the interpretation of the last note of the voice (*LastNote*).

DotAlt: Presence of a "dot of alteration"

In *Duf22518* and in *Duf16002*, the placement of the dot of perfection is not compliant with one of the premises that the "identification of the functionality of the dots" function is built upon (Section 3.3.3). There is one instance in *Duf22518* (and two in *Duf16002*) in which the dot is placed two minims before the end note of the sequence (Figure 4-12). This dot belongs to a different type of dot of division from the ones considered so far. Apel (1953, 115–16) briefly talks about three kinds of dots of division: dot of perfection, dot of imperfection, and dot of alteration. The dots considered in Franco are the dot of perfection and the dot of imperfection (the one generically called "dot of division"). The dot found in *Duf22518* (and in *Duf16002*) is an instance of a "dot of alteration"; it is placed two minims away from the end of the sequence to separate these last two notes from the ones preceding them, and to clarify that they form a perfection of their own (which requires the alteration of the last minim of the pair).

Since the current implementation does not handle dots of alteration, the scoring-up interpretation of the dot is as a dot of augmentation, which leads to the mistaken interpretation shown in Figure 4-13. The interpretation conveyed by the dot of alteration in Figure 4-12 can also be conveyed by the use of a dot of perfection. Actually, Apel gives two examples in which a dot of alteration also acts as a dot of perfection or as a dot of imperfection (Figure 4-14 and Figure 4-15, respectively). Thus, the presence of a dot of alteration does not suggest that Franco's rules are somehow incomplete, because the same interpretation can be conveyed by either one of the two dots considered by Franco. Moreover, shortly after describing the "dot of perfection", "dot of imperfection", and "dot of alteration", Apel discards the terms altogether, and thereafter merely refers to "dots of division" in general. However, given that dots of alteration are actually used in these examples, maintaining these three distinct categories of the dot of division is useful for the scoring-up implementation.

Figure 4-12: Ground truth interpretation of Duf22518 (cantus) based on a modern transcription.

Figure 4-13: Incorrect interpretation from scoring-up tool of Duf22518 (cantus). The misalignment of the notes in the cantus with respect to the lower voices can be seen through the barlines (compare to Figure 4-12).

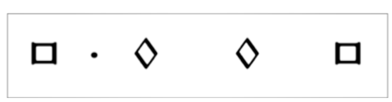

Figure 4-14: Dot of division that acts both as a dot of perfection and a dot of alteration

Figure 4-15: Dot of division that acts both as a dot of imperfection and a dot of alteration

MisplacedDot: Presence of an unnecessary and misplaced dot of perfection

There was one instance in which the unnecessary use of a dot of perfection produced a misinterpretation of the sequence of notes. This instance happened in *Duf16002* in the passage shown in Figure 4-16. Between the semibreves there are three minims, which, by default, imply that the first semibreve should be kept as perfect (there is no necessity to use a dot to indicate this). If a dot of perfection were placed after the first semibreve, the interpretation would still be correct according to the scoring-up implementation. But the dot of perfection is located far away from the start note of the sequence and this leads to its misinterpretation as a dot of augmentation by the scoring-up tool, as shown in Figure 4-17.

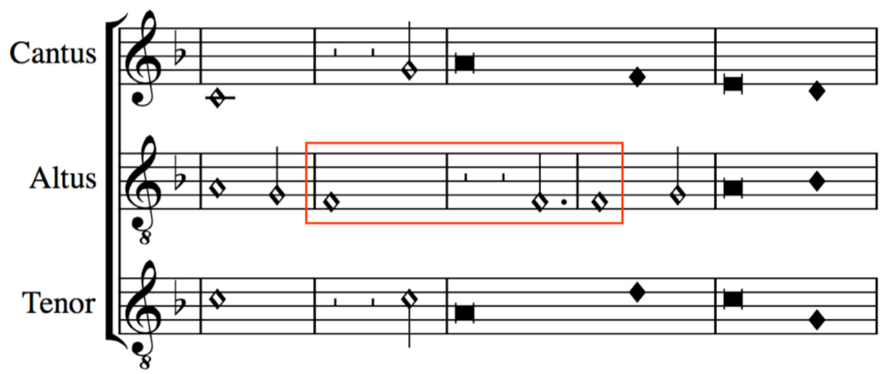

Figure 4-16: Ground truth interpretation of Duf16002 (altus) based on a modern transcription.

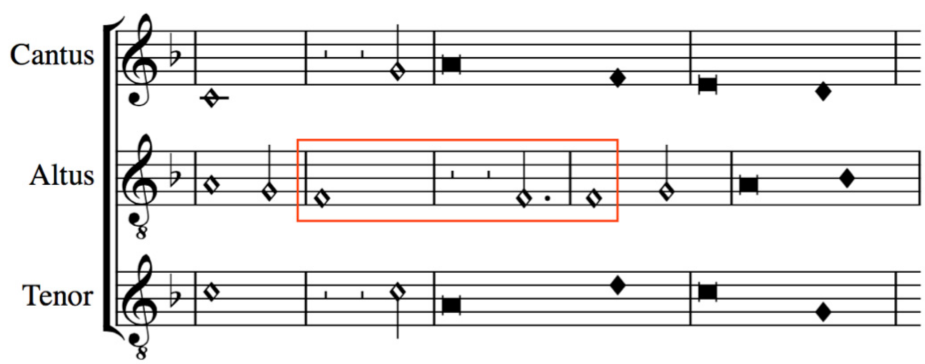

Figure 4-17: Incorrect interpretation from scoring-up tool of Duf16002 (altus). The misalignment of the notes in the altus with respect to the other voices can be seen through the barlines (compare to Figure 4-16).

HemiolaGroup: Incomplete hemiola group

The implementation of the interpretation of the durational value of a colored note (Section 3.3.2) was error-free, since all colored notes (except for the mistake in Dijon) were assigned the correct durational value. Nonetheless, the mislabeling of an uncolored note was caused by an assumption regarding hemiola coloration. Figure 4-18 shows a complete hemiola group in both the discantus and tenor—the dotted blue boxes contain three colored (imperfect) breves in the place of two uncolored (perfect) breves. This is not the case in the contra, where one single long is colored (blue box). Although the colored long was interpreted correctly, the fact that there is no complete hemiola group caused the mislabeling of one of the notes that follows the colored long—namely, the last semibreve shown in the contra in Figure 4-18 was considered as altered by the scoring-up. Therefore, the error was caused by the implicit assumption that all colored notes are part of a complete hemiola coloration group.

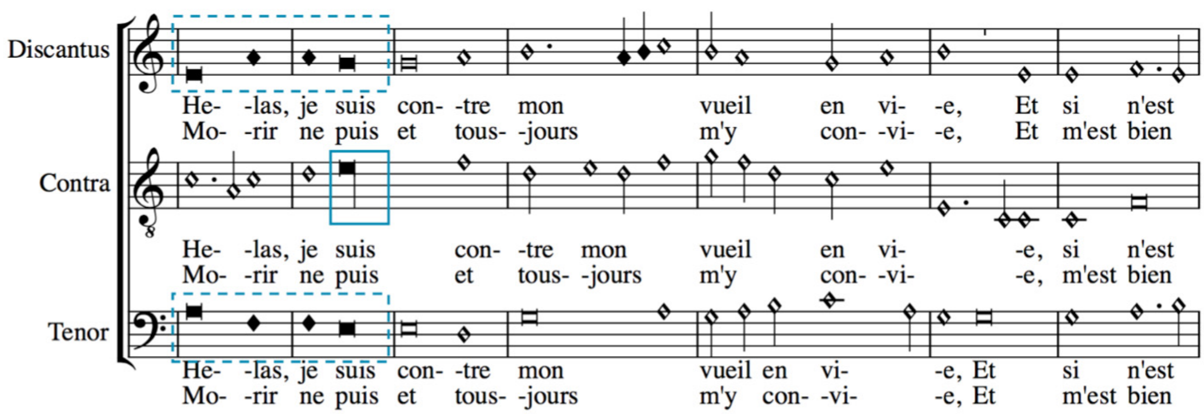

Figure 4-18: Ground truth interpretation of Ock3016 based on a modern transcription.

LastNote: Last note on a voice

The mislabeling of the last note of a voice happened in several pieces. This is the case of *Iv001*, in which the scoring-up tool interpreted all notes correctly except for the last note of the two upper voices. In the interpretation of this piece by the scoring-up tool (Figure 4-19), all the voices reach the last note at the same time, but the tenor ends sooner. This is because the longs in each voice are interpreted according to the mensuration of the voice (the two upper voices are in perfect modus and the tenor is in imperfect modus). But, in the contemporaneous musical practice, the last long at the end of each voice on a piece was not meant to have a fixed duration; it was interpreted simply as the last note of the voice. All voices were meant to come together at the beginning of their last note, which is what happens in this case, so the exact duration of the last note is not a problem.

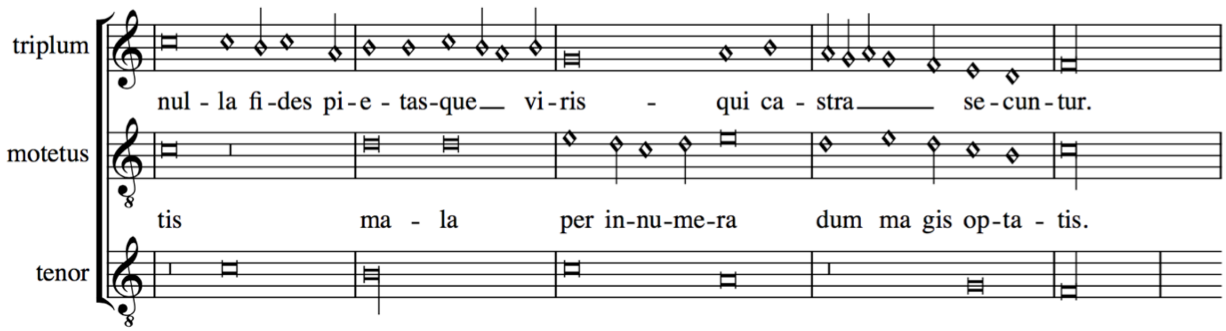

Figure 4-19: Interpretation from scoring-up tool of Iv001's ending. Even though the last note of all voices is reached at the same time, the tenor ends sooner than other voices given that the last notes are interpreted according to the mensuration (perfect modus in triplum and motetus, and imperfect modus in the tenor).

4.4.2.2 Errors in the Dataset

A problem in the dataset is that I did not encode which staff line the semibreve and minim rests are attached to (nor did I implement a way to deal with this situation within the scoring-up tool). The errors raised by this issue (*RestLines*) are detailed below.

RestLines: Start note followed by two rests placed on the same or on different staff lines

This type of error happens in two pieces: *Duf3025* and *Duf22518*. In the cantus part of *Duf3025*, in a sequence with five semibreves between two breves, the first breve is followed by two semibreve rests located on the same staff line. The correct interpretation of this sequence of notes, as indicated in the ground-truth file, is given in Figure 4-20. Since this is a case of n = 3p + 2 with p > 0 semibreves between breves (see Table 3-5), the scoring-up tool produces the interpretation shown in Figure 4-21. Even though the intended rhythm (Figure 4-20) could be conveyed by the use of a dot of perfection, it is not necessary to include such a dot in this context since, as indicated by Apel (1953, 111), two rests on the same staff line belong to the same perfection.

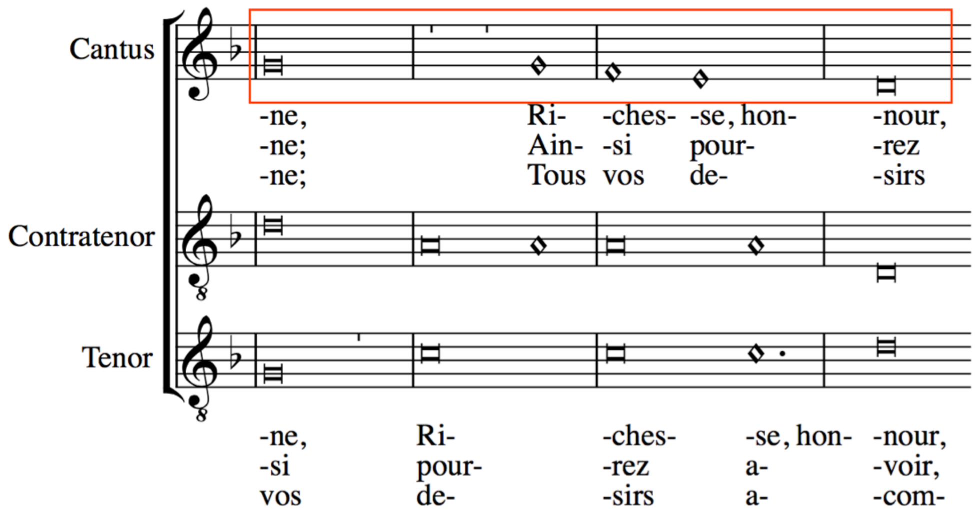

Figure 4-20: Ground truth interpretation of Duf3025 (cantus) based on a modern transcription.

Figure 4-21: Incorrect interpretation from scoring-up tool of Duf3025 (cantus).

4.4.3 Errors Caused by Incompleteness of Franco's Principles

There was one type of error (*OnsetAt3*) that the scoring-up implementation could not handle, given that it was not covered by the principles of imperfection and alteration outlined by Franco.

OnsetAt3: The onset of a perfect note falls on the third beat of a perfect mensural group

The implementation was built upon the assumption that a perfect note falls on the first beat of a perfect mensuration, or on the second beat if it is imperfected by the previous notes (i.e., in the case of imperfection a.p.a). A few instances, however, presented the case where the note landed on the third beat of the mensuration. Most of these examples were found in the mid-fifteenth century pieces, specifically in Ock3012 and Ock3016. It is also found in *Iv007*.

Figure 4-22 shows an excerpt of *Ock3012* according to the ground truth, with a breve (red box) starting at the third beat of the tempus. Figure 4-23 shows the interpretation of the same excerpt according to the scoring-up tool. As one can see in Figure 4-23, since there are three semibreves between the two breves of the contra, the scoring-up tool interprets the first breve as perfect and thus, the onset of the last breve (red box) falls on the first beat of the tempus. The modern transcription (Figure 4-22), on the other hand, interprets the first breve as imperfect and that guarantees that the last breve (red box) lands on the third beat.

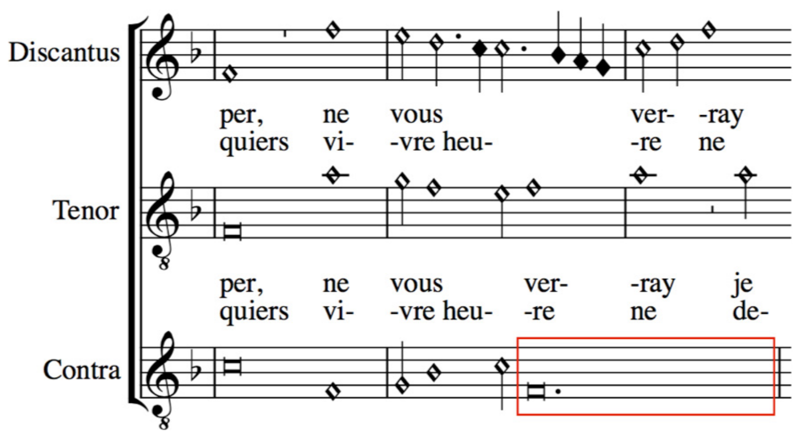

Figure 4-22: Ground truth interpretation of Ock3012 (contra voice) based on a modern transcription.

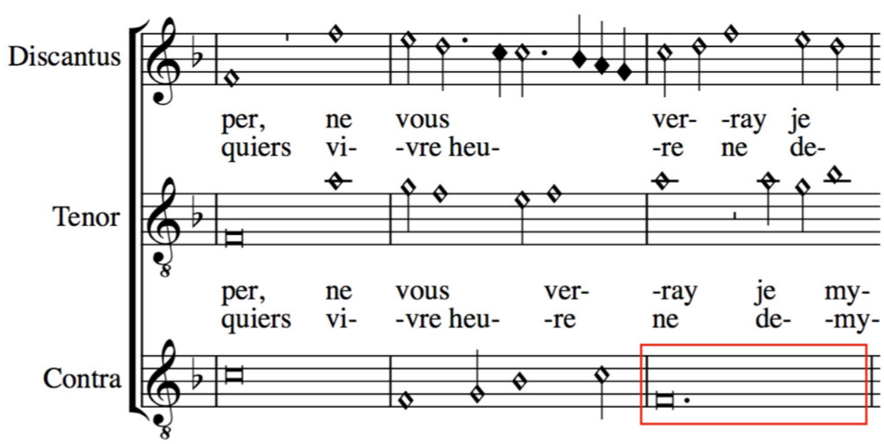

Figure 4-23: Incorrect interpretation from scoring-up tool of Ock3012 (contra voice).

The other instance in *Ock3012* of an onset at the third beat of the tempus is shown in Figure 4-24. The interpretation of the same passage by the scoring-up tool is given in Figure 4-25. The interpretation of the passage is the same in both the ground truth and the scoring-up output until reaching the last semibreve (yellow dotted box) of the delimited sequence. The scoring-up alters this semibreve, according to Franco's principle of alteration, and this mislabeling caused the onset of the breve to fall on the first beat of the tempus.

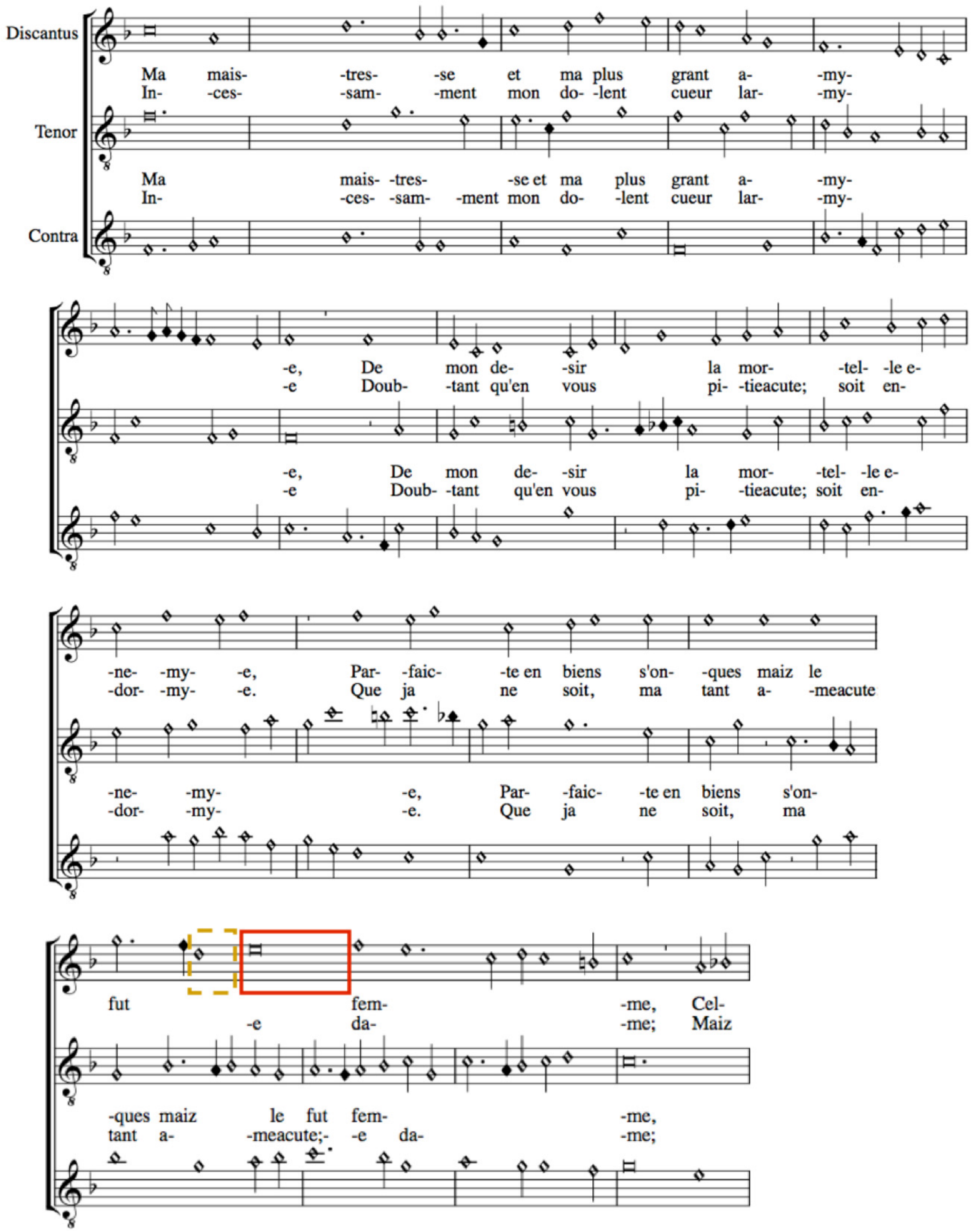

Figure 4-24: Ground truth interpretation of Ock3012 (discantus) based on a modern transcription.

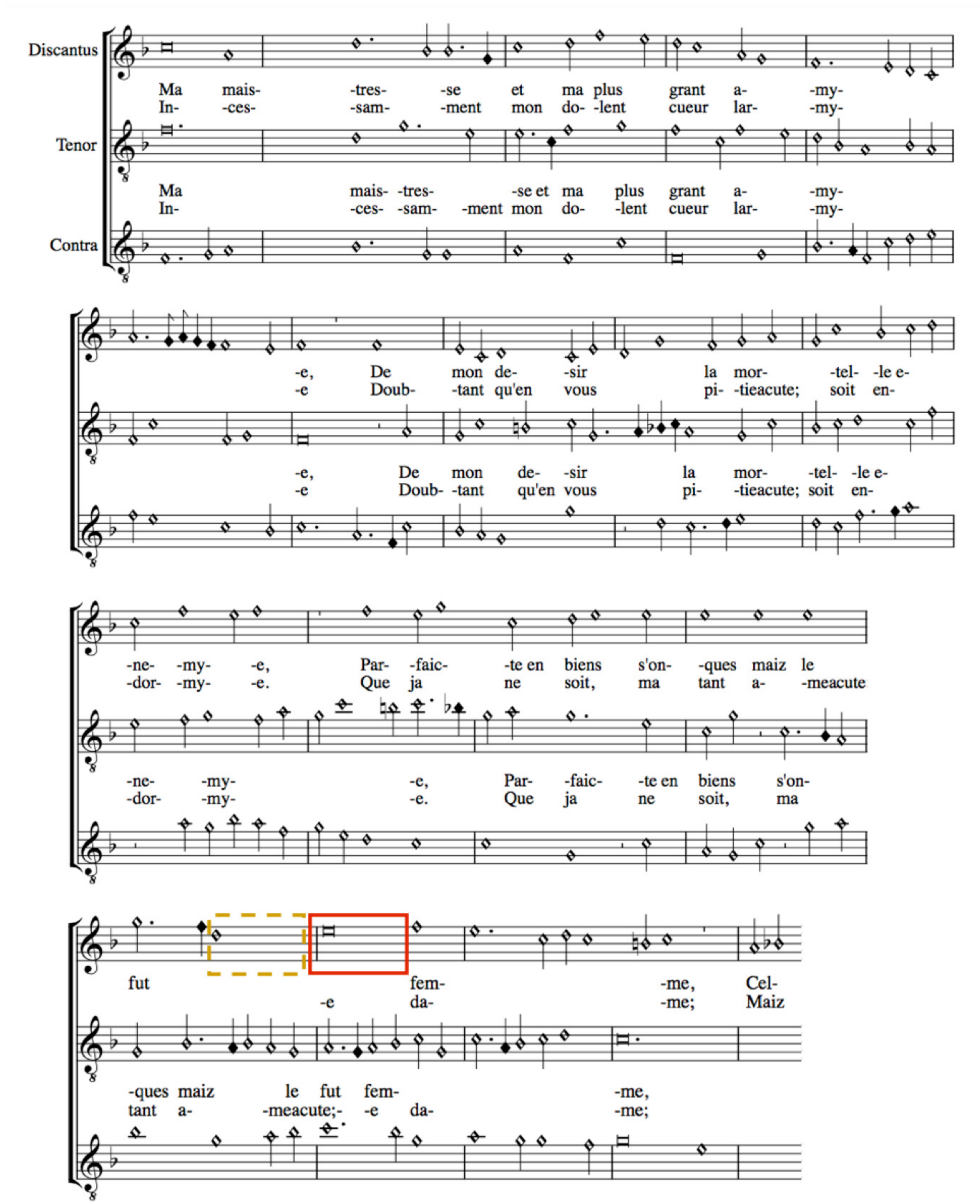

Figure 4-25: Incorrect interpretation from scoring-up tool of Ock3012 (discantus).

In *Ock3016*, there is another instance of a note, a colored breve, starting on the third beat of the tempus (Figure 4-26). The breve was interpreted correctly as imperfect given that it is colored, but the notes around it were misinterpreted given that it falls into the same OnsetAt3 category. The semibreve preceding the colored breve was mislabeled as altered by the scoring-up tool (yellow dotted box in Figure 4-27). Analyzing the results in both *Ock3016* and *Ock3012*, all the mistakes in these pieces were related to semibreves mislabeled as altered or breves mislabeled as perfect.

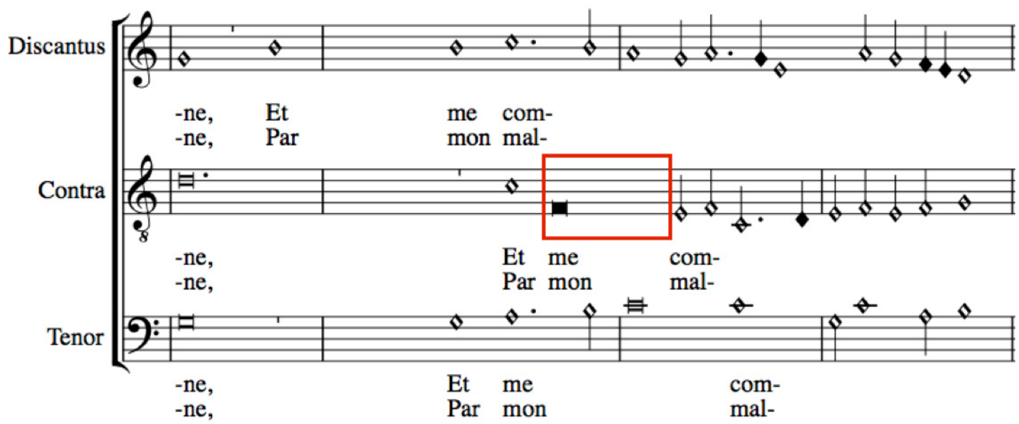

Figure 4-26: Ground truth interpretation of Ock3016 (contra voice) based on a modern transcription.

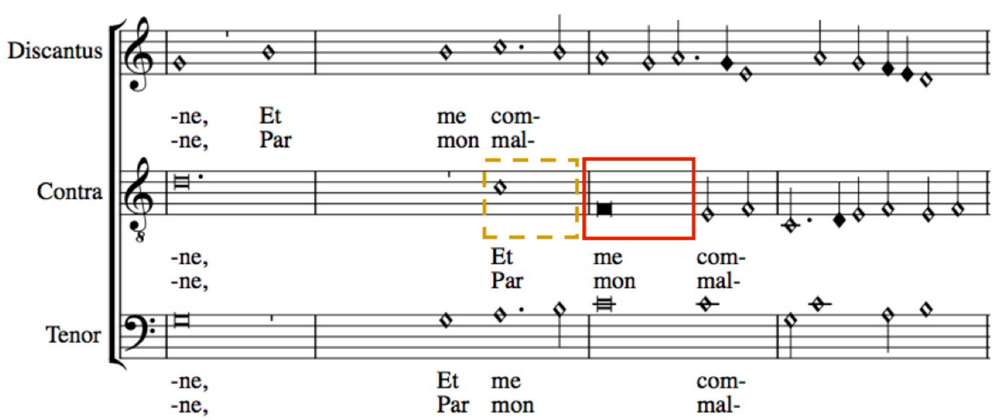

Figure 4-27: Incorrect interpretation from scoring-up tool of Ock3016 (contra voice).

Given the types of mislabeling (the scoring-up was wrong when considering notes as perfect and when considering notes as altered, which is something proper to triple meter), I ran the tool on these pieces again but treated the tempus as imperfect instead of perfect. The only notes modifiable by context that were mislabeled now were the breves that were followed by another breve.[41] Given the implementation details behind the scoring-up, these results reveal:

1. All breves in these pieces are considered imperfect except when:
    a. They are followed by another breve, as the new errors indicate.
    b. They are followed by a dot. There are breves followed by a dot in these pieces (e.g., see the breve in the red square in Figure 4-22). These breves were not mislabeled by the scoring-up when the pieces were considered as written in imperfect tempus, because they were understood as dots of augmentation (instead of the dots of perfection that they actually are).
2. There is no alteration in the pieces. Otherwise, these notes would have been mislabeled when considering imperfect tempus.

All these observations are in agreement with mid-to-late fifteenth century practice—the principle of alteration starts falling into disuse, dots are used to mark perfect notes even in contexts where their use is considered unnecessary according to the old principles, and context becomes less important. The two pieces are clearly written in triple meter (the breve rests are perfect and the breves followed by another breve are perfect as well), but the larger context seems less important here than in the older practice.

The OnsetAt3 error is also found in the triplum of the Ars nova *Iv007* motet. The mensuration given in Table 4-2 indicates that all voices are written in perfect modus. I ran the piece through the scoring-up tool a second time, but this time considering the triplum as written in imperfect modus while the other voices were kept in perfect modus. In this case, the number of mislabeled notes was reduced to zero. Considering that the upper voice is running in duple meter against the triple meter of the lower voices is

[41] Other, non-context-related, mislabeled notes were: breve rests and colored breves; these errors were expected since the piece was originally written in perfect, rather than in imperfect, mensuration.

possible with this repertoire since Vitry (the composer of this motet) played a lot with having different mensurations in different voices.[42]

4.5 Potential Strategies for Correcting Errors

Below I discuss the possible ways to handle these errors according to their type. Section 4.5.1 deals with the errors in the sources, Section 4.5.2 with the errors in the experiment, and Section 4.5.3 presents how to expand the scoring-up implementation to cover the cases that lie outside of Franco's principles of imperfection and alteration.

4.5.1 Errors in the Sources

From the previous discussion, the intended interpretation of sequences that fall within the NoDot category can be achieved following Franco's rules by the use of the appropriate dot. Thus, the presence of errors of *NoDot* type does not imply that Franco's system is incomplete. Nonetheless, since these cases do happen, as pointed out by Apel (Section 2.2.5) and by the evidence, a way to handle them needs to be included within the scoring-up implementation, especially since they cause shifts in the vertical sonorities that can endure for long periods of time (as in $n = 3p$ when p has a large value) or even until the end of the piece (as in $n = 3p + 2$). I have already implemented some methods to determine which of the two possible interpretations of a sequence of notes does not work (Table 3-7), but these were not enough to avoid the NoDot type errors completely. When transcribing a piece into modern values, in cases where two interpretations of a sequence of notes are possible, the usual method to determine which is the best option of the two is by examining the notes in the other voices and determining which of the two options produces the least amount of dissonance. According to Table 4-4, the voice with the highest level of accuracy generally corresponds to the voice written in long note values or to the voice written in imperfect mensurations; therefore, one could compare the two conflicting interpretations of a sequence of notes against this one voice (provided that none of the aspects shown in Table 3-7 were enough to discard one of them). From the results obtained from the corpus and based on Apel's rule of "imperfection a.p.p. is always preferable to

[42] Karen Desmond, personal communication, 15 July 2017.

imperfection a.p.a.”, this analysis of vertical sonorities may not be necessary in the case of n = 3p + 1.

4.5.2 Errors in the Experiment

It will be important to include support for handling all the kinds of misinterpretations related to dots that were found (i.e., support for *DotAlt*, *MisplacedDot*, and even *RestLines*), since not all manuscripts are well preserved and many have spots (e.g., spilled ink) that can be confused with actual dots, so correctly handling all types of dots will reduce the number of errors when scoring up these manuscripts. In dealing with *DotAlt* and *MisplacedDot* kinds of misinterpretations, I propose:

- Consider both ends of the sequence when looking for a dot of division.
- Avoid restricting the search for dots of perfection and dots of imperfection to dots that are just 0 or 1 notes away from the start note (e.g., 0 or 1 breves away from the first long in a sequence written in perfect modus).
- And, given the large number of dots of perfection and dots of imperfection compared to the number of dots of alteration (there were only three instances of dots of alteration in the whole corpus), consider examining the left end of the sequence and then the right end when searching for dots of division.

Here I am proposing a variant of the method described in Section 3.3.3 based on the points outlined above:

1. Look at the left end and find the first dotted note. Check if the dot is an integer number of notes away from the start note (instead of determining if it is either 0 or 1 note away from it, as is described in Section 3.3.3). If it is an integer number of notes away, the dot is a candidate to be a dot of division. Otherwise, it would be a dot of augmentation or a dot of perfection at a smaller note level (as described in Section 3.3.3), and it is necessary to check the other end of the sequence for a dot of alteration.
2. If there is a candidate for a dot of division in the left end of the sequence, proceed as described in Section 3.3.3 and check if the notes following this dot amount to an integer number or not. If they do, it is a dot of division. Otherwise, it

is a dot of augmentation or a dot of perfection at a smaller note level and the other end of the sequence must be checked.

3. When looking at the right end for a dot of alteration (provided that the possibility of a dot of division in the left end has been ruled out), determine whether or not the dot is two notes away from the end note. In the case it is (e.g., the dot is two minims away from the end note of a sequence in major prolation, or it is two breves away from the end note of a sequence in perfect modus), it is a candidate to be an alteration dot. Otherwise, it is either an augmentation dot or a dot of perfection at a smaller note level.
4. If there is a candidate for a dot of alteration, proceed in a similar fashion as described in step 4 and check if the number of notes preceding this dot is an integer number and decide based upon this.
5. Finally, any dot following a dot of division or preceding a dot of alteration is labeled as a dot of augmentation (or a dot of perfection at a smaller note level).

Regarding the *RestLines* error, the staff line position of the rests needs to be encoded within the <rest> element. In discussions with the MEI group, the attribute @loc came up as the ideal candidate for this task. The attribute @loc can have integer values that indicate the position of the element in the staff (e.g., @loc = "0" corresponds to the first staff line, @loc="2" corresponds to the second line, and @loc="1" to the space between those two lines). When two semibreve rests follow a breve (or two minim rests follow a semibreve, as it is the case in *Duf22518*), one can determine whether the rests belong to the same or different perfections by evaluating whether the value of the @loc attribute in each of the <rest> elements are the same. The same value in the @loc attribute of both <rest> elements would leave the start note perfect (having the same effect as a dot of perfection) and different values would lead to an imperfection a.p.p. (having the same effect as a dot of division).

4.5.3 Errors Caused by Incompleteness of Franco's Principles

The observations raised by the OnsetAt3 type of error drives us to think about methods to handle the late-fifteenth, and probably also mid-fifteenth, century practice. The proposed method to deal with this repertoire consists of:

1. Only enforce the “similis ante similem perfecta” principle (rule 1 in section 2.2.3), and consider the breve as imperfect otherwise. This is:
    - If n = 0, keep the breve perfect.
    - And if n > 0, the breve is imperfect.
2. Keep considering the different functionality of the dots of division, especially:
    a. Dot of perfection, since this is the only way to mark as perfect a breve that is not followed by another breve.
    b. Dot of alteration, since this is the only way to mark an alteration given that the default interpretation is no-alteration.

The scoring-up tool is highly accurate in determining when a note’s durational value should be modified from the default value. Out of more than 2000 notes subject to modification, the scoring-up tool predicted the correct durational value of 97% of them, although a single error could affect the alignment of the notes for a long segment of the piece. Because of this, it is necessary to deal with these sources of error in the future. Besides determining the durational values of the notes in music written in triple meter, which is a problem that has not been dealt with until now, using this tool provided insights regarding some of the pieces in the dataset. For instance, I was able to verify that since the mid-fifteenth century notes started being interpreted in a more note-to-note fashion while the larger context became less important. I was also able to evaluate how a piece would be interpreted considering other types of mensuration, which led to valuable insights about one of Vitry’s motets (Hugo). The fact that the principles of imperfection and alteration can automatically be applied to a piece and experiment with the mensuration can be used to test hypotheses regarding a particular piece or a repertoire belonging to a particular time period, style, or composer.

Chapter 5 Conclusions

This thesis presents the first approach to the automatic scoring up of mensural music written after the thirteenth century. Mensural music is typically characterized by the presentation of the sources in separate parts (with the voices on different areas of the page or even in different books) and by the context-dependent nature of the durational value of its notes.

The scoring-up implementation takes a set of Mensural MEI files that encode the mensuration, the pitch, and the note-shape information of each of the voices of a mensural piece; it determines the durational value of each note based on the context and the given mensuration; and, finally, it produces a single Mensural MEI file that, when rendered, shows the piece with the voices lined up in score format. The lining up of the voices is based on the durational values assigned to the notes by the scoring-up implementation. The process by which the scoring-up determines the durational value of the notes depending on the context is based on the principles of imperfection and alteration outlined by Franco of Cologne in his *Ars Cantus Mensurabilis* (ca. 1280) (Strunk and Treitler 1998, 2:116–35), regarded as the first clear exposition of the principles of the mensural notation system, and on Willi Apel's *The Notation of Polyphonic Music* (Apel 1953). Although Franco's principles were written for music of the thirteenth century, they are the basis for the interpretation of mensural music in general. In addition to these principles, the implementation also considers coloration (specifically, hemiola coloration) and the different types of dots (i.e., division and augmentation), since coloration and dots of augmentation were features introduced to mensural notation during the fourteenth century.

At the start of this thesis, there was no suitable dataset. As part of a different but related project, I developed a tool called the Mensural MEI Translator, that took modern transcriptions of mensural pieces (compliant with certain rules to account for mensural notation specificities) encoded as CMN MEI files and translated them back into mensural values, encoding the piece in a Mensural MEI file. The Mensural MEI files obtained from this process contained pitch, note-shape (or name), and note value (i.e., duration) information; thus, they served as ground truth against which I evaluated the performance of the scoring-up implementation. The input data was obtained from the same files by removing the note value information. In this manner, I encoded eight works from the fourteenth century, five from the beginning of the fifteenth century, and six from the mid-fifteenth century.

The output of the scoring-up tool was compared against the ground truth. The accuracy of the implementation in determining the correct value of a note whose value was subject to modification by the context, the presence of a dot, or coloration, was approximately 97% on average. Although very few notes were assigned a wrong durational value, in some cases one or two wrong values shifted the vertical alignment of the voices for passages of variable length, sometimes until the end of the piece. In analyzing the errors, eight different types were found, most of them related to the absence or location of a dot.

One of the goals of the scoring-up implementation was to test the completeness of Franco's system of rules. Although sometimes the interpretation of a passage differed from the ground truth, the correct interpretation could always be achieved within the framework of Franco's rules (by the addition of a dot in the appropriate place). This was true for most of the pieces in the dataset from the fourteenth and the beginning of the fifteenth century. However, in the mid-fifteenth century repertoire, I found two pieces that were not covered by Franco's principle of alteration. One possible reason for this is that as we get closer to the sixteenth century, duple meter starts to dominate, context becomes less important, and alteration is used less frequently. This seems to be supported by the fact that in both pieces, which are written in perfect tempus, all breves were imperfect except when followed by another breve or when dotted, and there was not a single instance of alteration. However, to reach more conclusive results in this matter, a larger dataset should be used in the future.

5.1 Future Work

Future work on improving the performance of the scoring-up tool includes:

- Implementing additional ways to interpret dots, providing support for dots of alteration and for other positions than the usual ones for dots of imperfection and dots of perfection

- Including information within the Mensural MEI files regarding the staff line on which semibreve and minim rests lie, since the position of two consecutive rests following a perfect note indicates whether they belong to the same perfection or not, and include support within the scoring-up tool to handle this information
- In case two interpretations of a sequence of notes are viable, using the lower voice (since it achieved the highest accuracy in general) to determine which of the two options causes the least amount of dissonance. This would be a project on its own since it would imply analyzing the vertical sonorities, defining what dissonance is, looking at beat position, etc.

Moreover, future work involves increasing the scope of the scoring-up implementation by including support for late fifteenth-century pieces, in which context becomes less important. The details of the proposed method are given in Section 4.5.3. In addition, now that regular imperfection is working, support for partial imperfection could be added (i.e., the imperfection of part of a note). Other features that need to be handled by the scoring-up tool are ligatures and half-coloration. After dealing with the issues listed above and improving the performance of the script in a given mensuration, the next step would be to include support for changes in mensuration within a voice or voices.

5.2 Contributions

The most significant contribution of this thesis is the reformulation of Franco's principles of imperfection and alteration in a manner that can be expressed as a computer program, making it possible to automate the application of these principles to determine the durational value of every note in triple mensurations. Along with this, I have provided the design and implementation of the first automatic scoring-up tool for mensural notation.[43] Given a piece that conforms to Franco's principles, the output of the scoring-up is a Mensural MEI file that encodes the piece as a score with all notes correctly lined up. This score format representation of the music allows us to study the contrapuntal relationships between the voices, previously obscured by the separate arrangement of the parts in the original source. The methods regarding the identification

[43] The source code is freely available online at https://github.com/ELVIS-Project/scoring-up.

of the functionality of the dots and the handling of simultaneous triple meter at multiple note-levels are original contributions of this work, since no explicit statement regarding the treatment of these two features was found in Apel (1953).[44] An important feature of this thesis is that the success (or failure) of the scoring-up tool for different repertoires in mensural notation actually reveals changes in the ways composers, copyists, and scribes used the notation, and sometimes it reveals traits about individual works (showing that one voice of a motet might be in duple meter, for example). In the field of computational musicology, other side contributions that came out of this work are the ground-truth Mensural MEI files I generated that encode the pieces as scores written in the original mensural values. Potential future work includes the integration of the scoring-up tool into an Optical Music Recognition (OMR) workflow, since a prior OMR stage would eliminate the need to hand-code the input files of the scoring-up tool, which would allow for the automatic encoding of mensural music into scores with minimum human intervention.

[44] With respect to simultaneous triple meter, Apel (1953, 122–3) does present the case of perfect tempus and major prolation. However, unlike the case of triple meter at a single note level (perfect tempus with minor prolation, pp. 107–20), he does not provide explicit rules for determining the note durational values in this mensuration. Instead, he discusses this case with examples, which he indicates are more of theoretical than practical importance. With respect to identification of dot functionality, see note 28 on this thesis.

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