THE DESIGN OF A DUAL FREQUENCY SHORT WAVE ANTENNA

Randolph Douglas Fairley
3015 Sherbrooke St., West
Montreal, Que.

M. Eng. Thesis | Elect. Eng.

Submitted April 17, 1950.

TABLE OF CONTENTS

PREFACE

In many respects, dual frequency antennas present problems not associated with those operating at a single frequency. The various matching devices and adjusting networks required to align an antenna of the type described herein must be designed to operate properly at two frequencies. The dual frequency antennas installed at Sackville, N. B. by the Canadian Broadcasting Corporation operate at frequencies differing by 15% to 20%. (Thus, no harmonic relationship exists). When first assigned the task of alleviating the operating difficulties encountered in using the antennas as originally designed, very little technical information in a practical form was available. The method used to overcome these difficulties and the development of various design formulas for dual frequency networks are contained in this thesis.

The author wishes to express his appreciation to Mr. J. E. Hayes for his guidance and assistance, and to other members of the C. B. C. staff, particularly Mr. W. G. Roxburgh.

Montreal, Que.,
April 17, 1950.

R. D. Fairley

INTRODUCTION

Broadside arrays employing tiers of horizontal colinear elements have attained considerable prominence in the short-wave broadcasting field. One may, in fact, cite many of their characteristics as standards by which other radiating systems are judged. As conventionally designed, however, the broadside array is used for single frequency operation only. If consideration be given to the number of frequency bands which must be used to provide optimum broadcast service throughout the cyclical variations in ionospheric density, a very costly set of arrays is indicated. Primarily to reduce this cost, one is led to investigate the possibility of modifying the design of the broadside array to permit operation in two of the adjacent frequency bands allocated to short-wave broadcasting.

This modification has been carried out by at least three broadcasting organizations. Although such is the case, very few design details have appeared in technical periodicals. Some papers have been published[(1)(2)] but have treated the dual frequency array rather superficially. One fact that is apparent, however, is that all three installations employ a different method of impedance matching. Upon this matching of the antenna impedance to the transmission line at each frequency, hinges the successful operation of the array. It so happens that the degree of mismatch normally encountered is such that voltage breakdown will occur unless corrective measures are used at an appropriate location. Conventional impedance matching devices employed in the usual way are unsatisfactory.

Before going into detail regarding one method that has been employed successfully, it is in order that the principal characteristics of the horizontal broadside array be reviewed so that other factors in the design for two-frequency operation may be dealt with. Initially, let us consider one of the more common designs for short-wave broadcasting which makes use of 32 half-wavelength elements. It is denoted in one system of nomenclature as an H/4/4/1/R array. The designation indicates that the elements are <u>horizontal</u>; that there are <u>four colinear elements</u> in each of <u>four tiers</u>; that the lowest element is <u>one wavelength above ground</u>; and that there are two sets or curtains of these elements, one acting as a <u>reflector</u>. The tiers are spaced a half-wavelength vertically; the two curtains are spaced a quarter-wavelength horizontally. This array has a gain of approximately 20 decibels over a half-wavelength dipole in free space. Figure 1 shows plan and elevation views of an H/4/4/1/R array with a graphic explanation of some of the more common terms used in describing this type of radiating system.

A strict mathematical analysis of the operation of a broadside array having two identical curtains is a laborious procedure. For engineering purposes, sufficiently accurate results are obtained by assuming that the radiated power is confined to a quadrant of a sphere.(3) The azimuthal angle, ϕ, then varies between the limits of 0° and 180°, and the zenith angle, θ, from 0° to 90°. The mathematics is further simplified by calculating the horizontal and vertical components of the radiation pattern separately. The equations(3) and patterns for an H/4/4/1/R array are shown as Figures 2 and 3.

The horizontal broadside array usually has a minimum of four colinear elements in each tier. For the purpose of applying power to the array, it is subdivided into "bays". A bay contains a pair of elements of each tier, vertically in line, plus those in the other curtain immediately behind these elements. (See Figure 1). To produce the vertical pattern shown in Figure 2, the four tiers must be fed in phase. With a tier spacing of a half-wavelength, this is accomplished very simply by a transposition of the transmission line between successive tiers. To produce the horizontal pattern shown in Figure 3, the two bays must also be fed in phase. The radiated beam may be steered or slewed a few degrees off centre azimuthally by arranging a phase difference in the power fed to each bay. Figure 4 shows the result of a 90° phase delay in the feed to one bay of an H/4/4/1/R array. A 90° phase advance will slew the beam to the other side of centre. The array can also be made to radiate in the reverse direction by interchanging the roles of the two curtains.

It will be noted from the equations given in Figure 3 that the horizontal pattern is independent of the number of tiers in the array. Thus, an H/4/2/1/R array has the same horizontal directivity as the H/4/4/1/R array. However, the patterns in the vertical plane are considerably different as is shown by a comparison of Figures 3 and 5. The magnitude of the minor lobe is less serious in short-wave broadcasting than it would be in the medium frequency range. Since considerable fading occurs even with an essentially single-lobed pattern due to multiple reflections of the radiated wave, any additional fading as a result of a large minor lobe would not, conceivably, have as noticeable an effect. The gain of an H/4/2/1/R array is approximately 17 decibels. As a result, this array is in relatively common use where economy has been a major factor to consider.

The design of the dual frequency arrays at Sackville, N. B. for the International Service of the Canadian Broadcasting Corporation was prepared by the Transmission and Development Department of the C. B. C. One dual frequency array was designed primarily for the frequencies 9.63 Mc/s and 11.72 Mc/s; Another for the frequencies 15.19 Mc/s and 17.82 Mc/s. The arrays are basically of the H/4/2/1/R type. The height above ground of the first tier was made one wavelength at the lower frequency. The tier spacing was a half-wavelength also at the lower frequency. The two curtains of elements were separated a quarter-wavelength at the higher frequency. The choice of element length was considered in relation to the impedance matching device that was used.

The input impedance of a full-wave dipole in free space has been shown by King and Blake(4) to be approximately 2000 ohms for a conductor having a diameter-to-wavelength ratio of 3×10^{-4}. If, for the moment, the mutual impedance between the various elements in the array be neglected, then the input impedance of a bay having four tiers would be about 500 ohms, since there are effectively four full-wave elements in parallel. The elements in the reflector curtain are parasitically excited. In practice this is found to be a reasonably accurate approach to the input impedance of each bay of a broadside array. However, the existence of appreciable mutual impedance is immediately apparent at frequencies off resonance if a comparison be made between the curves of King and Blake[loc.cit.] for a dipole and that for an element of an H/4/2/1/R array (Figure 10). The input impedance of each bay also varies with slewing as a result of the mutual impedance. In addition, it has been found in practice that the input impedance is modified somewhat when the functions of the two curtains are interchanged electrically. Considering that the curtains are as identical as physically possible, this may seem rather surprising. The difference is probably due to the fact that in one case the transmission line is directly in the beam radiated by the array. Thus, if the array has two slew positions in both the forward and reverse directions, a total of six antenna impedances must be matched. More will be said about this problem in a later section.

In the initial design, the elements were fed from a point mid-way between the two tiers by transmission lines of equal length thus satisfying the requirement of feeding the tiers in phase at both frequencies. Since the tier spacing is a half-wave length at the lower frequency, the feeders to the elements themselves were approximately a quarter-wavelength long at the same frequency and ~~less~~ more at the higher frequency. It so happens that this results in an impedance mismatch at the common feed point sufficient to cause voltage breakdown. To prevent this, an impedance matching device consisting of a section of line of specific length and characteristic impedance, was inserted between the junction point of the element feeders and the bay feeder. For simplicity in construction and adjustment, a four-wire line section was used for matching as shown in Figures 6 and 7. The length of the section and its effective characteristic impedance are seen to be readily changed. It should be noted that since this device must improve the impedance match at two frequencies, it must be adjusted as a compromise.

The design equations for this method of matching(5) show that to obtain even an approximate match for two impedances, the reactive components of these impedances must be of the same type. This stipulated that the elements of this array must resonate either above or below both frequencies. In the initial design, the elements were resonated about 8% below the lower operating frequency. A photograph of the array with the original matching section installed is shown as Figure 7.

However, operating experience with this dual frequency array had shown that the original design did not provide a satisfactorily low value of standing wave ratio on the transmission lines for all conditions of reversing and slewing. When it was necessary to use certain beam directions, stubs on the transmission lines had to be readjusted to bring the line impedance within the matching range of the transmitters. Further, the antenna impedance itself was such as to produce critical requirements with respect to matching. Insulators used to support the original matching section at the central feeding point in the array were unable to withstand the high voltage involved. This condition was improved somewhat by using a self-supporting matching section, but the array remained critical with respect to wind and icing conditions to an unsatisfactory degree. Consequently, this antenna was again the subject of study and experiment.

PRELIMINARY INVESTIGATION

Preliminary investigation showed that the fundamental problem was to find a means of reducing the degree of impedance mismatch at the junction of the element feeders and the bay feeders at both operating frequencies. The fact that the tiers must be fed in phase at both frequencies, produces the requirement that the element feeders be of equal length. In the original design, as noted previously, the element feeders were approximately a quarter-wavelength long at the lower frequency. Considering the impedance transforming property of a section of transmission line, there was a possibility that the length of the element feeders could be so chosen that a satisfactory match would occur at both frequencies.

The element impedance at each frequency could only be estimated so an accurate analysis was impossible. The estimated value of the element impedance, however, was thought to be sufficiently accurate to indicate whether or not this method of matching was worthy of consideration. Accordingly, the standing wave ratio that would result on the bay feeders with various lengths of the element feeders was calculated for both frequencies. This calculation was made as follows:

At the lower frequency, let the element impedance be denoted by Z_{oo}. Then, from basic transmission line theory, the impedance at the input of the element feeder, Z_s, is given by,

$$Z_s = Z_o \frac{Z_{oo} + jZ_o \tan L}{Z_o + jZ_{oo} \tan L} \quad \text{(For a line with negligible loss)}$$

where Z_o is the characteristic impedance of the element feeder;
and L is the length of the element feeder in degrees.

If we assume that the impedances of the elements of each tier are equal for the purpose of calculation, then the input impedances of the element feeders are equal, since they are of equal length. The result of these two equal impedances in parallel forms the terminating impedance of the bay feeders, Z_r.

Writing Z_s as $Z_s \underline{/\theta}$,

Then, $Z_r = \frac{Z_s}{2} \underline{/\theta}$

When the characteristic impedance of the element feeders is identical with the characteristic impedance of the bay feeders, the standing wave ratio on the bay feeders can be calculated from,

$$\text{S.W.R.} = \frac{1 + \left|\frac{Z_r - Z_o}{Z_r + Z_o}\right|}{1 - \left|\frac{Z_r - Z_o}{Z_r + Z_o}\right|}$$

The standing wave ratio was calculated for various values of L. These calculations can be carried out very simply with the Smith Transmission Line Calculator.(6)

Similarly, Z_r and the S.W.R. for the higher frequency was calculated. The S.W.R. at both frequencies was plotted against the element feeder length at the lower frequency. The lowest value of S.W.R. at which the curves intersected was approximately 3:1. The element feeder length as a function of the lower frequency for this S.W.R. was 0.4125 wavelength. If this compromise match could be obtained in practice, it would be quite satisfactory. This suggested more accurate investigation.

THE USE OF A SCALE MODEL

The basis of the entire matching scheme is an accurate knowledge of the element impedance at each frequency. It is very difficult to make the necessary measurements on an actual array to obtain this for several reasons. Firstly, it is practically impossible to make S.W.R. measurements on the vertical section of the bay feeders due to the physical difficulties involved and the doubtful accuracy of measurements made under such circumstances. Secondly, measurements made on the horizontal portion of the bay feeders are inherently inaccurate due to the existence of a switch at the junction of the horizontal and vertical sections. This switch, which serves to interchange the functions of the two curtains, has appreciable capacity. The effect of this capacity on the S.W.R. depends upon the impedance of the line at the point where the switch is inserted. This capacity could be measured and a correction applied to obtain the true S.W.R. produced by the load, but it introduces a possible source of error. For these reasons, and particularly to enable trial-and-error adjustments to be more easily made, a model of one of the dual frequency arrays was constructed.

A model will have properties similar to the full-scale prototype if certain conditions involving dimensions, frequency, and material constants are satisfied.[7] An antenna of sufficiently small size would result if the elements of the model were resonated at approximately 200 Mc/s. This produced a model whose overall dimensions were 12 feet long by 8 feet high. A transmitter having a power output of approximately 25 watts was used in the experiments.

The model was first constructed as a reproduction to scale of the 15.19/17.82 Mc/s array. As for the original antenna, the elements were resonated 8% below the desired lower operating frequency. The elements were made 30.75 inches long which, with a factor of 0.94 to compensate for end effects, were expected to resonate at approximately 185 Mc/s. Other dimensions were scaled from the original antenna.

MEASUREMENTS ON THE MODEL ANTENNA

The first measurements made were to determine the element impedance as a function of frequency. The impedances were calculated from measurements of the S.W.R. on the vertical section of the bay feeders and the distance between the nearest E_{min} point and the common element feed point. With this information, the impedance at the junction point, Z_r, can be calculated by means of the Smith Transmission Line Calculator or from the following formula, derived from basic transmission line equations,

$$Z_r = Z_o \frac{Q(\tan s + \cot s) - j(Q^2 - 1)}{\tan s + Q^2 \cot s}$$

where Z_r is the load impedance;
Z_o is the characteristic impedance of the line;
Q is the standing wave ratio = E_{max}/E_{min};
s is the distance in degrees from the E_{min} point to the load.

As previously shown, assuming equal element impedances, Z_r is equal to one half the input impedance of a single element feeder, Z_s. Knowing the length of the element feeder, the element impedance, Z_{oo}, may be calculated from the formula,

$$Z_{oo} = Z_o \frac{2Z_r - jZ_o \tan L}{Z_o - j2Z_r \tan L}$$

where L is the length of the element feeder in degrees.

For the determination of the S.W.R. an electrostatically coupled type of instrument was used. Circuit and construction details of this unit appear as Figures 8 and 9. Since the germanium diode used in the measuring circuit has a non-linear voltage versus current characteristic, it was necessary to calibrate the instrument for accurate measurement of S.W.R.

In the measurement of the antenna impedance, each value of S.W.R. and distance from an E_{min} point to the load was repeated at least three times to ensure as great an accuracy as possible. A graph of the measured element impedance of an H/4/2/1/R array versus frequency is shown as Figure 10. It should be noted that at each frequency at which measurements are to be made, it is essential that the reflector curtain be in proper adjustment; that equal power be fed to each bay; and that each bay be fed in phase. There was one particular inherent source of error in these measurements -- the spacing of the lowest element from ground, the tier spacing, and the separation between curtains, were actually only correct for an H/4/2/1/R array at one frequency. However, this error was not thought to be as great as other possible sources of error such as is probably in the measurements at the frequencies used.

In order to determine the length of the element feeders that would produce the best possible impedance match with the elements resonant below both operating frequencies, the S.W.R. on the bay feeders as a function of the element feeder length was calculated for both frequencies using the measured value of element impedance.(p.8) The results were rather unsatisfactory indicating that the minimum S.W.R. that could be obtained at the two frequencies was approximately 5:1. This was appreciably different from the preliminary calculations which had indicated that a value of about 3:1 could be obtained. The error was in the estimated value of element impedance used in the preliminary calculations. The value of 5:1 was deemed unsatisfactory so further tests were conducted.

It will be noticed in Figure 10 that the reactance component of the element impedance is zero at approximately 193 Mc/s. If the operating frequencies are chosen on either side of this frequency so that the magnitudes of the element impedance at both frequencies are more nearly equal, a better compromise match can be obtained. The S.W.R. on the bay feeders as a function of the element feeder length was calculated for two frequencies chosen so that the frequency at which the element reactance was zero was approximately the mean. The resulting graph is shown as Figure 11. It will be noticed that with an element feeder length of 0.4275 wavelength at the lower frequency, a S.W.R. of 3.4:1 can be obtained at both frequencies. This length of element feeder was substituted in the array and the resulting S.W.R. on the bay feeders was in close agreement. (See Figure 11A for feeder construction details.)

The measurements so far had been for an array designed to operate at frequencies having a ratio of 17.82/15.19. It was also necessary to check if this system of matching would be satisfactory for the frequency ratio 11.72/9.63, this ratio being greater than that previously indicated by about 5%. Using element impedances from Figure 10 for appropriate frequencies and calculating the resulting S.W.R. as a function of the element feeder length as before, it was found that the lowest value of S.W.R. that could be obtained was 4.2:1. The graph is shown as Figure 12. This is not as good a match as for the previous case. However, taking other things into account, it was considered satisfactory. Though not specifically stated previously, it is implied that the S.W.R. will be further reduced by conventional methods on the horizontal section of the bay feeders. The matching section in the array is required principally to reduce the peak voltage on the line to a value below the corona point.

One very important result of having one frequency on each side of the resonant frequency of the elements became apparent when conventional stubs were used to match the line to the model array. The antenna has six modes of operation at each frequency; forward with zero, positive, or negative slewing; and reverse with zero, positive, or negative slewing. Initially one would expect that the matching problems would be identical for the respectively similar forward and reverse modes. However, this is not the case as stated previously. When matching the antenna to the line, there are six different S.W.R.'s, and, what is more important under conditions actually encountered, the E_{min} points are widely separated. This is important since the matching must be a compromise for the six impedances.

With the antenna resonant below either operating frequency, the E_{min} points for positive and negative slewing respectively, lie on either side of the E_{min} point for zero slewing. However, with the elements resonant at the mean frequency, the E_{min} points for positive and negative slewing are practically superimposed. This, in effect, halved the separation between E_{min} points and allowed a much lower S.W.R. to be obtained as a compromise. Further investigation showed that if the S.W.R. on the bay feeders could be adjusted to 1:1 with zero slewing, the shift of E_{min} points with slewing would be practically negligible. This indicated that for optimum results conjugate stubbing should be used on the bay feeders rather than a simple stub adjusted as a compromise for the six impedances at <u>each</u> frequency as formerly employed.

A low S.W.R. is essential if an array is to be slewed. Firstly, it ensures equal power distribution to the two halves of the array; and, secondly, since phase shift and line length are not linearly dependent if standing waves exist, it is difficult to obtain the proper angle of slew when an extra section of line is used to introduce the required phase difference.

This completed the tests on the models.

MATCHING ON THE FULL SCALE ARRAY

In order to produce the results obtained experimentally on the model, some modification to the existing arrays was necessary. Some changes were easily made; others were considered impractical at the time. For instance, a basic requirement was that the elements must be shortened in order for them to be resonant at the mean frequency. Since it was impractical to move the poles used to carry the feeder lines to the elements, it was necessary to separate the two halves of the array. This modifies the radiation pattern somewhat over and above the change introduced by shortening the elements. A representative set of patterns, both horizontal and vertical, for the various frequencies and modes of operation, appear as Figures 13 to 18 inclusive.

Further, tests on the model had indicated that conjugate stubbing on the bay feeders was desirable.

Unfortunately, the bay feeders were not sufficiently long to permit this to be done. Lengthening these feeders was not considered practical at that time since it involved moving some of the necessary switchgear associated with the array. However, even though the matching on the bay feeders was a compromise for the six operating modes at each frequency, the final results were completely satisfactory.

The experiments on the model had indicated that the length of the element feeders should be 0.4275 wavelength (at the lower frequency) for the 15/17 Mc/s. array and 0.3975 for the 9/11 Mc/s. array. For both arrays, it was necessary to lengthen these feeders by 10% to obtain equal S.W.R.'s on the bay feeders at both frequencies. Part of this change, at least, was undoubtedly due to having to separate the two halves of the array with the resulting change in antenna impedance. The method used to install the rather long element feeders is shown in Figures 19 and 20.

Comparison of Results

At the transmitting site, there are four dual frequency antennas: two operating in the 15 Mc/s. and 17 Mc/s. bands, and two in the 9 Mc/s. and 11 Mc/s. bands. Tabled below are the S.W.R.'s on bay feeders both on the model and on the full scale arrays after adjustment of the length of the element feeders.

		Experimental	Actual
15/17 Mc/s. array	- No.1	3.4	3.7
" " "	- No.2		4.0
9/11 Mc/s. "	- No.1	4.2	4.0
" " "	- No.2		4.2

The S.W.R. on the bay feeders was reduced to approximately 2:1 in all cases by means of a simple stub adjusted as a compromise for the six operating modes at each frequency. On the main transmission line feeding each antenna, a combination of simple and conjugate stubbing was employed. The final S.W.R.'s on the lines after all adjustments were completed averaged 1.4:1, the maximum in any case being 1.8:1.

Although the adjustment of the stubs has been treated very lightly thus far, let it not be thought that it is unimportant. A study of Figure 20 will show the number and variety of stubs used to place a dual frequency array of this type in working order. It is the purpose of the remainder of this thesis to derive design equations for the various stubs indicated in Figure 21 ~~20~~.

CONJUGATE PHASING STUBS

To obtain an essentially unindirectional pattern; i.e., one having a high ratio of forward field to reverse field, it is necessary to obtain the proper phase relationship between the fields radiated by the two curtains. One method of doing this is illustrated in Figure 22. By adjusting the shorting bar at X, the phase of the field radiated by the element GH can be adjusted so that it acts as a reflector to the field radiated by EF by virtue of cancellation and reinforcement. The position of the shorting bar varies with frequency. In order that the reflector will be in proper adjustment at two frequencies simultaneously, a particular type of stubbing is required.

Consider Figure 23. Suppose that A is the required position of a shorting bar for a frequency, f_1, and that B is the required position for a frequency, f_2. If a shorting bar be placed at C, where the distance AC is a half-wavelength at f_1, the shorting bar effectively appears at A -- at the frequency, f_1, Further, if a stub be placed at A so that at the frequency, f_2, the impedance at A has the same value as if a shorting bar were at B, the reflector can be adjusted at the two frequencies. The stub at A will have no effect at the frequency, f_1, since a short circuit appears at this point.

This method of adjusting the reflecting elements of a dual frequency antenna is believed to have originated with the Columbia Broadcasting Corporation.

To facilitate the alignment of the dual frequency arrays, charts were prepared from equations derived on the basis of the above method of operation.

If a shorting bar were at B, the admittance at A looking towards B is given by,

$$-jY_o \cot \phi$$

where ϕ is the distance in degrees from A to B.

At the frequency, f_2, the impedance at A looking towards B is due to two shorted line sections in parallel: (1) the line from A to C; and (2) the stub, θ^o long at A. The admittance of this combination is given by,

$$-jY_o \cot \left(\frac{f2}{f_1} \times 180^o\right) - jY_o \cot \theta$$

This expression must equal $-jY_o \cot \phi$. Therefore,

$$\cot \left(\frac{f2}{f_1} \times 180^o\right) + \cot \theta = \cot \phi$$

and,

$$\cot \theta = \cot \phi - \cot \left(\frac{f2}{f_1} \times 180^o\right)$$

<u>CASE I -- $f_1 > f_2$</u>

Then, $$\frac{f2}{f_1} \times 180^o < 180^o$$

and cot $\left(\frac{f2}{f_1} \times 180^o\right)$ is negative, for $\frac{f2}{f_1} > \frac{1}{2}$

<u>CASE II -- $f_1 < f_2$</u>

Then, $$\frac{f2}{f_1} \times 180^o > 180^o$$

and $\cot\left(\frac{f_2}{f_1} \times 180^\circ\right)$ is positive, for $\frac{f_2}{f_1} < \frac{3}{2}$

It is necessary to consider both cases since either frequency may require a shorting bar nearer the reflecting element than the other. The position of these shorting bars for minimum radiation to the rear is determined experimentally.

For the frequencies 15.19 and 17.82 Mc. the equations become,

CASE I $\theta = \cot^{-1} (\cot \phi + 2.00)$

CASE II $\theta = \cot^{-1} (\cot \phi - 1.64)$

For the frequencies 9.63 and 11.72 Mc. the equations become,

CASE I $\theta = \cot^{-1} (\cot \phi + 1.60)$

CASE II $\theta = \cot^{-1} (\cot \phi - 1.23)$

Graphs of θ versus ϕ for the two cases at each pair of frequencies appear as Figures 24 and 25.

CONJUGATE MATCHING STUBS

A load impedance which is a function of frequency can be matched to a transmission line at two frequencies simultaneously providing certain boundary conditions, which will be defined later, are satisfied. One method of accomplishment is to use a simple matching stub at one frequency and a conjugate stub at the other. Stubs for matching at a single frequency are in common use. Charts which give the length and position of the stub based on the standing wave pattern are readily available.[8] Conjugate stubs, however, are not so widely used. Prior to this writing no charts were known to exist, perhaps because the preparation of such a chart is very tedious.

The original patent on conjugate stubs is held by Andrew Alford.[9] The design equations given in his patent, however, are rather complex and in the following pages somewhat simpler equations are derived. In addition, design equations for a type of conjugate stub not considered in his patent are presented.

The principle behind the various types of conjugate stubs is essentially the same. From basic transmission line theory, the input impedance of a lossless section of transmission line an odd integral multiple of a quarter-wavelength long, terminated in a short circuit, is infinite. This characteristic is also exhibited by a section of line an integral multiple of a half-wavelength long, terminated in an open circuit. What is also true is that the impedance at any point along either of these line sections is infinite.

Because their impedance is infinite, these line sections will not effect the impedance of a transmission line to which they may be shunt-connected. In practice, an infinite impedance cannot be obtained but it is sufficiently high as to have a negligible effect.

This antiresonant property is naturally confined to certain definite frequencies. (We shall only concern ourselves with fundamental antiresonant frequency.) At a frequency off resonance a finite impedance will be placed across the line. The magnitude of this impedance can be controlled by the proportioning of the sections of the conjugate stub and can thus be used for matching at frequencies off resonance.

The design equations of two different types of conjugate stubs will be derived. One is a type mentioned in Andrew Alford's patent; the other is colloquially known as the "V" stub because of its configuration.

CONVENTIONAL CONJUGATE STUBS

Figure 26 shows the general form of the conjugate stubs considered by Andrew Alford. The two sections of the stub may be both shorted, both open, or one of each type with either the shorted or open one nearer the load. All types are essentially equivalent electrically. The type specifically shown in Figure 26 will be considered in detail. It provides one of the smaller assemblies and is relatively easy to adjust.

From fundamental transmission line theory, the impedance at any point on a lossless line is repeated every half-wavelength. Thus, in Figure 26, if $d = 180^{\circ}$ and $M + N = 90^{\circ}$, the impedance of the line will not be effected as noted previously (except between P_1 and P_2). This will apply at the frequency, f_1, say. At another frequency, f_2, d will no longer equal 180° and $M + N$ will no longer equal 90°. The distance, d, is fixed by the requirements at f_1, but the proportions of M and N may be changed to secure an impedance match at f_2. That is, at f_2,

$$d = \frac{f_2}{f_1} \times 180^{\circ}$$

and,

$$M + N = \frac{f_2}{f_1} \times 90^{\circ}$$

For the line to be properly matched at the point, P_1, the input admittance of the line with the stubs attached must be Y_o. Hence,

$$Y_o = Y_{os} + Y_1$$

where Y_o is the characteristic admittance of the line;
Y_{os} is the admittance of the stub at P_1;
Y_1 is the admittance of the line at P_1 before the stub was attached.

Then,

$$Y_1 = Y_o - Y_{os}$$

The admittance, Y_2, at the point, P_2, that will produce an admittance, Y_1, at P_1, after being transformed by a line length of d°, is given by,

$$Y_2 = Y_o \frac{(Y_o - Y_{os}) - jY_o \tan d}{Y_o - j(Y_o - Y_{os}) \tan d}$$

Now the admittance at P_2 consists of the stub, Y_{ss}, plus the admittance of the line before the stub was attached, Y_3.

Then,

$$Y_2 = Y_{ss} + Y_3$$

There are now two expressions for Y_2. Equating these,

$$Y_{ss} + Y_3 = Y_o \frac{(Y_o - Y_{os}) - jY_o \tan d}{Y_o - j(Y_o - Y_{os}) \tan d}$$

and,

$$Y_3 = Y_o \frac{(Y_o - Y_{os}) - jY_o \tan d}{Y_o - j(Y_o - Y_{os}) \tan d} - Y_{ss}$$

Again from fundamental transmission line theory for a lossless line,

$$Y_{os} = jY_o \tan M$$

$$Y_{ss} = -jY_o \cot N$$

where M and N are the length of the line sections in degrees.

Substituting,

$$Y_3 = Y_o \frac{Y_o - jY_o \tan M - jY_o \tan d}{Y_o - j(Y_o - jY_o \tan M) \tan d} + jY_o \cot N$$

Collecting real and imaginary components and dividing numerator and denominator by tan d,

$$Y_3 = Y_o \frac{\cot d + \cot N - j(1 + \tan M \cot d + \tan M \cot N - \cot N \cot d)}{\cot d - \tan M - j}$$

Converting to an impedance and normalizing,

$$z_3 = \frac{\cot d - \tan M - j}{\cot d + \cot N - j(1 + \tan M \cot d + \tan M \cot N - \cot N \cot d)}$$

$$\text{where } z_3 = \frac{Z_3}{Z_o} = \frac{1}{Y_3 Z_o}$$

For any frequency ratio, d and (M + N) can be calculated. Then if values of M are assumed, hence N determined, z_3 can be calculated. For results of reasonable accuracy, the Smith Transmission Line Calculator can be used to determine both the S.W.R. that will produce this impedance and its position relative to a voltage minimum or maximum. Assumption of various values of M and N will give sufficient data for the drawing of a matching chart.

The S.W.R. and the location of z_3 may be determined more accurately by formulas. Consider the following:

By definition,

$$\text{S.W.R.} = \frac{E_{max}}{E_{min}} = Q$$

From basic theory, it is also given by,

$$Q = \frac{1 + |K|}{1 - |K|}$$

where K is the reflection coefficient of the load and is given by,

$$K = \frac{Z_R - Z_o}{Z_R + Z_o}$$

where Z_R is the load impedance. Hence,

$$Q = \frac{1 + \left|\frac{z_3 - 1}{z_3 + 1}\right|}{1 - \left|\frac{z_3 - 1}{z_3 + 1}\right|}$$

To calculate the distance, s, from a point of minimum voltage, E_{MIN}, toward the load, consider the following:

In general, considering negligible losses,

$$Z_s = Z_o \frac{Z_r + jZ_o \tan \theta}{Z_o + jZ_r \tan \theta}$$

where Z_s is the input impedance of the line;
Z_r is the terminating impedance;
Z_o is the characteristic impedance of the line;
θ is the length of the line in electrical degrees.

The impedance at the E_{MIN} point is a pure resistance whose value is Z_o/Q.[10]

Taking this value as the input impedance at an E_{MIN} point and taking $Z_r = Z_3$ at a point s^o toward the load from this E_{MIN} point, there is obtained,

$$\frac{Z_o}{Q} = Z_o \frac{Z_3 + jZ_o \tan s}{Z_o + jZ_3 \tan s}$$

Again writing $z_3 = Z_3/Z_o$ and simplifying,

$$\frac{1}{Q} = \frac{z_3 + j\tan s}{1 + jz_3 \tan s}$$

From which, after writing $z_3 = r_3 + jx_3$, and equating real components,

$$s = \tan^{-1} \frac{1 - Qr_3}{x_3}$$

In the general solution there are two cases to consider:

I. f_1 greater than f_2.

II. f_1 less than f_2.

Since making the stubs conjugate at the higher frequency (Case I) results in physically smaller stubs and greater ease of adjustment at lower S.W.R.'s, it will be the only one considered here.

One dual frequency array was designed to operate principally at the frequencies 9.63 and 11.72 Mc/s; another at the frequencies 15.19 and 17.82 Mc/s.

For the frequency ratio $\frac{9.63}{11.72}$, the equation for z_3 becomes,

$$z_3 = \frac{1.60 + \tan M + j}{1.60 - \cot N + j(1 + 1.60 \cot N + \tan M \cot N - 1.60 \tan M)}$$

And, for the ratio $\frac{15.19}{17.82}$,

$$z_3 = \frac{2.00 + \tan M + j}{2.00 - \cot N + j(1 + 2.00 \cot N + \tan M \cot N - 2.00 \tan M)}$$

Charts for matching at these frequencies appear as Figures 27 and 28.

A more general chart with frequency ratio as a parameter appears as Figure 29. The ratios used were

0.7, 0.8 and 0.9.

The lowest S.W.R. that can be reduced to unity is that which requires $M = 0$. The remaining stub at P_2 is then a common shorted stub whose length is $(f_2/f_1) \times 90^o$.

The S.W.R. on the transmission line can be reduced to unity providing the susceptance required is not greater than,

$$Y_o \cot\left(\frac{f_2}{f_1} \times 90^o\right)$$

THE "V" - TYPE CONJUGATE STUB

As far as is known, the conjugate stub in this form was first suggested and used by J. E. Hayes of the Canadian Broadcasting Corporation. Desiring to prepare matching charts for this type also, design equations were derived.

Consider Figure 30. The total length of the stub, M + N, is equal to a quarter-wavelength at the frequency at which the stub is required to have a negligible effect, f_1, say. At another frequency, f_2, the susceptance, (considering negligible losses), placed across the line to be matched will depend on the proportions of M and N.

The point of attachment of the stub and the susceptance required to remove a standing wave can be calculated as for a single frequency stub. The formulas derived below are perhaps simpler than found in the average text in terms of practical application.

From basic theory,

$$Y_R = Y_o \frac{Y_s - JY_o \tan \theta}{Y_o - JY_s \tan \theta}$$

where Y_R is the admittance of the load;
Y_s is the input admittance;
Y_o is the characteristic admittance;
θ is the distance in electrical degrees from Y_s to Y_R, measured toward Y_R.

As indicated previously, the impedance at an E_{MIN} point on a transmission line is a pure resistance of magnitude Z_o/Q, where Q is the standing wave ratio. Taking an E_{MIN} point as a reference point; i.e., Y_s is the admittance at an E_{MIN} point,

$$Y_R = Y_o \frac{QY_o - JY_o \tan \theta}{Y_o - JQY_o \tan \theta}$$

In order to remove a standing wave, the admittance at the point of attachment of the stub, after its installation, must be Y_o. Since the stub can only add susceptance (neglecting losses) the conductance component of the line admittance at the stubbing point must be Y_o. Denoting the susceptance component of the line admittance at this point by B, then,

$$Y_R = Y_o + JB$$

Inserting this value in the general equation,

$$Y_o \frac{QY_o - JY_o \tan \theta}{Y_o - JQY_o \tan \theta} = Y_o + JB$$

Solving this equation for $\tan\theta$ and B, there is obtained,

$$\tan\theta = \pm\sqrt{1/Q}$$

$$B = \pm Y_0 Q^{-\frac{1}{2}}(Q - 1)$$

The stub is, of course, required to have a susceptance of -B, when the characteristic admittances of the transmission line and the stub are identical, as is generally the case. Then, the normalized susceptance of the stub must be,

$$b = \mp Q^{-\frac{1}{2}}(Q - 1)$$

where $b = B/Y_0$

After determining the value of the susceptance required, there remains to be calculated the proportions of M and N to produce this susceptance.

Again, consider Figure 30.

Now,

$$Y_{ss} = -jY_0 \cot N$$

and

$$Y_{os} = jY_0 \tan M$$

If the required normalized susceptance is b, then,

$$b = \tan M - \cot N$$

and

$$\cot N = \tan M - b$$

If at f_1, $M + N = 90^\circ$, then at f_2,

$$M + N = (f_2/f_1) \times 90^\circ = \theta \quad \text{(say)}$$

Then,

$$\cot N = \cot (\theta - M)$$

From Trigonometry this can be resolved to,

$$\cot N = \frac{1 + \tan \theta \tan M}{\tan \theta - \tan M}$$

There are now two expressions for cot N. Equating these,

$$\tan M - b = \frac{1 + \tan \theta \tan M}{\tan \theta - \tan M}$$

This can be expressed as the quadratic equation,

$$\tan^2 M - b \tan M + b \tan \theta + 1 = 0$$

and,

$$\tan M = \frac{b \pm \sqrt{b^2 - 4(b \tan \theta + 1)}}{2}$$

It is now necessary to establish the boundary conditions.

Again, there are two cases to consider:

I. $f_1 > f_2$

II. $f_2 > f_1$

And, again, only Case I need be considered to provide a satisfactory solution to the normal matching problem.

CASE I $f_1 > f_2$

Since $\theta = (f_2/f_1) \times 90^\circ$, θ is less than 90°. It can be shown that if M + N is less than 90°, cot N is greater than tan M. Since both M and N are in the first quadrant, both tan M and cot N are positive. Therefore, the equation derived previously,

$$b = \tan M - \cot N$$

indicates that b must be negative.

Since b is negative and since tan M must be positive, only the positive root of the expression under the radical sign in the equation for tan M is applicable. Therefore, the equation for tan M can be rewritten,

$$\tan M = \frac{-b + \sqrt{b^2 + 4(b \tan \theta - 1)}}{2}$$

The smallest value of b that will satisfy the above conditions is given by,

$$b = \sqrt{b^2 + 4(b \tan \theta - 1)}$$

from which,

$$b = \cot \theta$$

Summarizing: For any two frequencies, f_1 and f_2 a standing wave ratio existing on a transmission line can be reduced to unity providing the required normalized susceptance is not less than cot $(\frac{f_2}{f_1} \times 90^\circ)$ where f_1 is greater than f_2.

A matching chart for the frequency 15.19 Mc., the stub being conjugate 17.82 Mc., appears as Figure 31.

A more general chart was drawn by using the ratios of the two frequencies as a parameter. The values for which a matching chart was drawn were 0.7, 0.8 and 0.9, giving values of θ of 63°, 72° and 81°. This chart is reproduced as Figure 32.

SIMULTANEOUS OPERATION

Although it has been arranged that the antenna and its associated transmission line will accept power at two frequencies, there remains the problem of connecting both transmitters to the line so that they may be used simultaneously. The basic requirement is that a relatively small amount of power be fed from one transmitter into the other. Cross-modulation and spurious frequency generation usually provide the criteria for successful operation. The maximum permissable magnitude of each of the above effects is limited not only by good engineering practice but also by National and International broadcasting regulations.

The general method of providing for the simultaneous operation of two transmitters into a single antenna is shown diagrammatically in Figure 33. The principal component of this system is the rejection filter.

At the frequencies used for short-wave broadcasting, filters composed of sections of transmission line are practicable. One of the simplest methods using line sections as filter elements is shown in Figure 34, and has been patented by P. S. Carter.(11) The operation of this arrangement is easily understood.

Consider the line sections at the point, A, and assume lossless lines. The $\frac{\lambda 2}{2}$ stub produces a short circuit at the point, A, -- at the frequency, f_2. Since A is $\frac{\lambda 2}{4}$ from the junction, B, a theoretically infinite impedance appears at B looking towards A. Thus, an infinitely small amount of power from Transmitter No. 2 will be accepted by the line from Transmitter No. 1. The open stub at A makes the overall length of the line section $\frac{3\lambda_1}{4}$ which, being antiresonant, results in the filter having a negligible effect on the line impedance at the frequency, f_1. The open stub does not effect the impedance at A, -- at the frequency, f_2, -- since a short-circuit appears at this point.

The filter at C operates similarly to prevent power from Transmitter No. 1 being fed to Transmitter No. 2.

Since some loss does exist in the filter sections, a finite amount of power will propagate through the filter. No equations from which accurate filter characteristics could be determined were presented in Carter's patent. In order to calculate the rejection ratio both at resonance and at frequencies slightly off resonance, it is necessary to consider general transmission line theory without the simplification of disregarding losses. It has been shown[12] that the input impedance of a resonant short-circuited line section is given by,

$$Z_s = Z_o \tanh \alpha L$$

$$\approx Z_o \alpha L \quad \text{(for small values of } \alpha L)$$

where Z_s is the input impedance;
Z_o is the characteristic impedance;
L is the length of the line section in meters;
and α is the attenuation constant of the line in nepers per meter.

The attenuation constant, for copper conductors, may be calculated from the formula,

$$\alpha = \frac{1455 \times 10^{-16}\sqrt{f}}{r \log_{10} d/r} \text{ nepers per meter}$$

where f is the frequency in cycles per second;
r is the radius of the conductor in meters;
and d is the spacing between conductors in meters;

From Figure 34, the impedance, Zs , in parallel with the impedance, Z_T, forms the termination for the quarter-wavelength section of line from the junction point of the two transmitter output lines. The impedance, Z_T, is a function of the impedance looking back into the power amplifier stage of a transmitter, the tank circuit of which is not tuned to the frequency being fed back. Because of its non-linearity, no attempt was made to calculate this impedance. However, if two impedances be placed in parallel, the magnitude of the resulting impedance is obviously smaller than either of the initial impedances. Therefore, since the rejection ratio is inversely proportional to the magnitude of the terminating impedance of the Quarter-wavelength line, at least a minimum value of rejection will be calculated if Z_T be neglected. Since Z_s is extremely small compared to Z_o, the error involved in making this simplification would be expected to be rather small. (See Figure 35.)

The problem is then reduced to finding the input impedance of a section of transmission line three-quarters of a wavelength long, terminated in a short-circuit. For this section, $\beta L = \frac{3\pi}{2}$, where β is the phase constant of the line in radians per meter. The impedance of an anti-resonant line section has been shown to be(12)

$$Z = Z_o \coth \alpha L$$

$$\approx Z_o / \alpha L \quad (\text{if } \alpha L \text{ is small})$$

It has also been shown (12) that the impedance of an antiresonant line section slightly off resonance is given by,

$$Z = Z_o \frac{1 + j\,\delta\alpha L}{\alpha L + j\,\delta}$$

where δ is the angular frequency deviation.

For small values of αL and δ, $\delta\alpha L$ is very much less than unity and the magnitude of Z is given very closely by,

$$|Z| = \frac{Z_o}{\sqrt{(\alpha L)^2 + \delta^2}}$$

The angular frequency deviation, δ, is related to the frequency by the equation,

$$\delta = \frac{2\pi L}{v}(f - f_o)$$

where f_o is the resonant frequency in cycles per second;
f is the impressed frequency;
v is the velocity of propagation in the line section in meters/sec.

At any frequency, the rejection ratio, T! is given by,

$$T' = \frac{|Z|}{|Z_L|}$$

where Z_L is the impedance at the junction point looking toward the antenna.

If $Z_L = Z_o$, as is the case for a properly matched line, and if it is assumed that this equality holds over the frequency band under investigation,

$$T' = \frac{1}{\sqrt{(\alpha L)^2 + \delta^2}}$$

$$= \frac{1/L}{\sqrt{\alpha^2 + \left(\frac{2\pi\Delta f}{v}\right)^2}}$$

where $\Delta f = f - f_o$

Now, $L = \frac{3v}{4v_o} \lambda_o$

where λ_o is the free space wavelength of f_o;
v_o is the velocity of propagation in free space.

$$\therefore \quad T' = \frac{4/3\ f_o}{\sqrt{(\alpha v)^2 + (2\pi\Delta f)^2}}$$

This is the rejection ratio for a $\frac{3}{4}$ - wavelength line section. If this line section were connected across a transmission line feeding an antenna, $1/T'$ of the power being propagated would be accepted and dissipated in the stub. For the case originally considered, (Figure 34) a transmission line leading to a transmitter was connected 1/3 of the way back from the input end of the $\frac{3}{4}$ - wavelength stub. To a very close approximation, 1/3 of the power accepted by the line section will be dissipated in the first $\frac{1}{4}$ - wavelength. Hence, only 2/3 of the input power can be fed to the transmitter and therefore the rejection ratio should be increased by 50%; i.e., $T = 1.5\ T'$.

$$\therefore \quad T = \frac{2f_o}{\sqrt{(\alpha v)^2 + (2\pi\Delta f)^2}}$$

Plots of this equation expressed in decibels for the 31, 25, 19 and 16 meter short-wave broadcasting bands with filters resonant at the center of each band, are shown as Figures 35 to 38 inclusive. Since Δf is relatively small, and further since α is proportional to the square root of the frequency, α was calculated at the mean frequency in each band only, and assumed constant throughout the band.

A rejection filter will naturally upset the impedance match at the other operating frequency. This mismatch may be corrected as shown in Figure 34 or by means of conventional stubs. Of these two methods, the former is to be preferred since it avoids the possibility of a voltage maximum occurring between the filter and the matching stub with the accompanying adverse effects if the voltage is sufficiently high.

Measurements of the filter characteristics were made as indicated diagrammatically in Figure 39. The filter was first adjusted at the desired frequency for a minimum reading on the meter, M. The ratio of the power transmitted to the antenna, to the power transmitted through the filter provides a measure of the rejection ratio. Unfortunately the measurement of the power to the antenna could not be carried out conveniently. However, the power output of the transmitter was known to be 50 KW to within about 5%, which allowed the making of measurements with reasonable accuracy. A comparison between calculated and measured rejection ratios for a specific case is shown as Figure 35.

In the region of maximum attenuation, agreement between measured and calculated values does not appear too good. However, it was found that the coupling existing between the two lines to the junction point, B, (Figure 34), prevented greater measured isolation than approximately 45 db.

This, then, gives the rejection ratio of the filter. However, there is no indication as yet as to how much of the return power will be accepted by a transmitter which is substituted for the measuring circuit. As noted previously this transmitter is not tuned to the frequency of the power being fed back. On this premis, one would expect the impedance looking back into the transmitter to be largely reactive and hence would have a large reflection coefficient. This would result in only a small fraction of the power passed by the filter being available to cause cross-modulation or the generation of spurious frequencies. Tests indicated that the cross-modulation signal was an estimated 20 to 30 db below the noise level of the transmitter when 100 watts was fed back to the transmitter. (If the filter is correctly resonated, only about 1 to 2 watts are fed back when the power used is 50 KW.)

By means of a communications-type receiver fitted with a signal strength meter and situated about 2½ miles from the antenna, spurious frequencies were found to be down between 70 and 80 db. This test was also conducted with 100 watts fed back to the transmitter. Incidentally, this power level is the maximum encountered when the filter is resonated at the center of either the 31 or 25 meter band and the operating frequency is at the extreme edge.

Measurements on the 19 and 16 meter bands have not been conducted as yet.

SUMMARY

It should be mentioned that dual frequency arrays using the system of impedance matching described in this thesis have been is use for almost two years. The instability previously apparent with ice loading and/or high winds was no longer in existence. Standing wave ratios on certain short feeder lines of the arrays were reduced from approximately 15:1 to about 4:1. This type of matching also permitted the attainment of much lower standing wave ratios on the main transmission lines feeding the arrays than previously possible. Prior to the modifications, transmitters were unable to be fully loaded under certain conditions of reversing and slewing without changing the stubbing on the transmission lines.

As originally constructed, the dual frequency arrays could be operated at one or the other of the design frequencies but not at both simultaneously. As a further refinement, isolating networks were installed to permit this simultaneous operation. One of the dual frequency arrays thus equipped was tested for a period of four months. The test was conducted during the winter, this season of the year producing the severest demand on the stability of transmission line networks. The test was completely successful.

One is thus led to the conclusion that dual frequency arrays of this type, properly adjusted, are definitely comparable in performance to much more costly arrays.

BIBLIOGRAPHY

1.	N.B.C.'s International Broadcasting System	- Raymond F. Guy	- R.C.A. Review July, 1941.
2.	C.B.S. Goes to Latin America	- A.B. Chamberlain	- Electronics July, 1941.
3.	Antenna Design,* pp. 51 - 55	- J.E. Hayes	
4.	Self-Impedance of a Symmetrical Antenna	- King and Blake	- Proc. I.R.E. July, 1942.
5.	Antenna Design,* Table III	- J.E. Hayes	
6.	Transmission Line Calculator	- P.H. Smith	- Electronics January, 1944.
7.	Standards on Antennas, Methods of Testing, p. 8		- I.R.E., 1948.
8.	Reference Data for Radio Engineers		Federal Telephone and Radio Corp.
9.	U. S. Patent No. 2,147,807		
10.	Radio Engineers' Handbook, p. 184	- F.E. Terman	- McGraw-Hill
11.	U. S. Patent No. 2,128,400		
12.	Theory and Application of Microwaves, p. 177	- Bronwell and Beam	- McGraw-Hill

* Being a set of notes prepared for a course given by the Extension Department, McGill University.

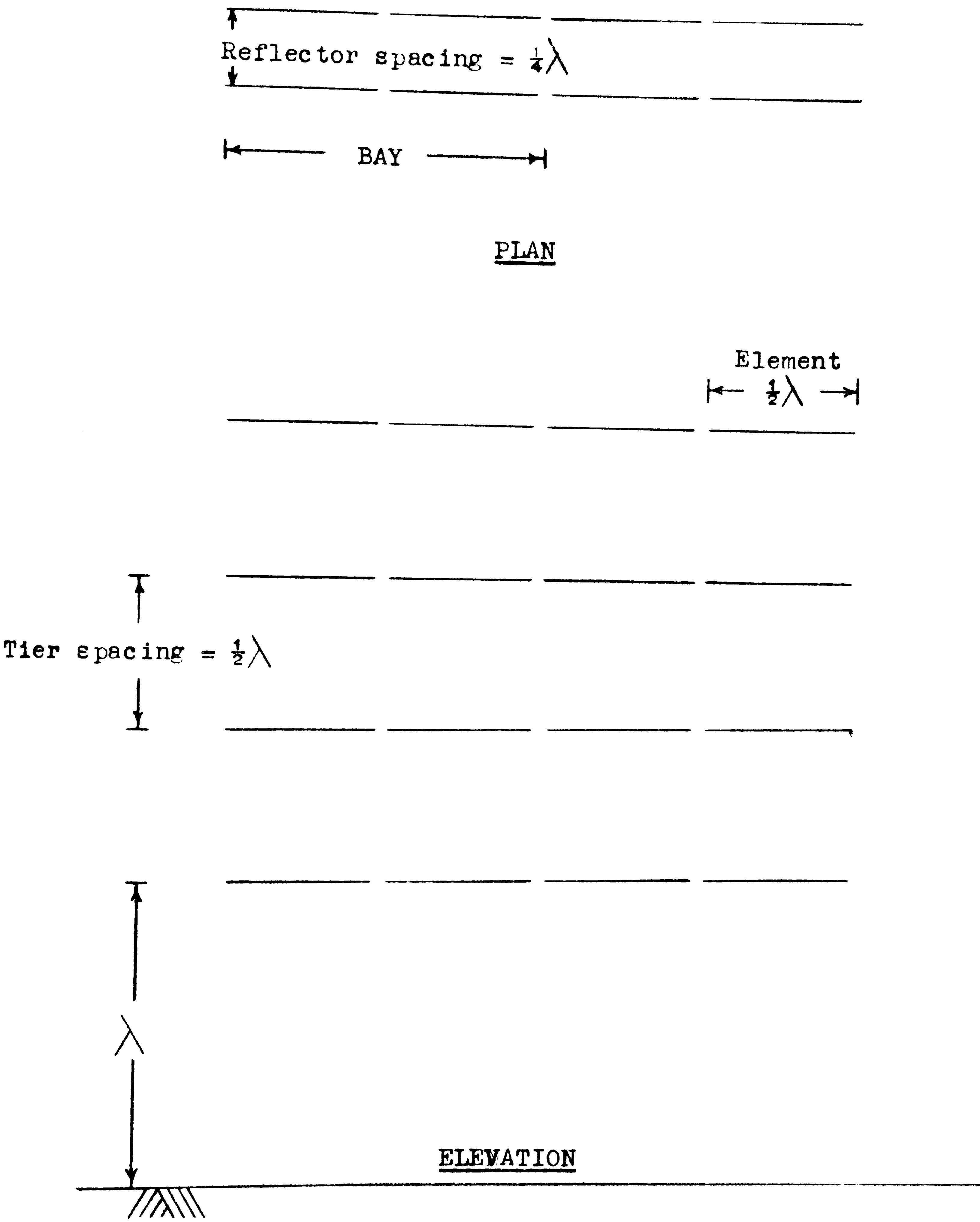
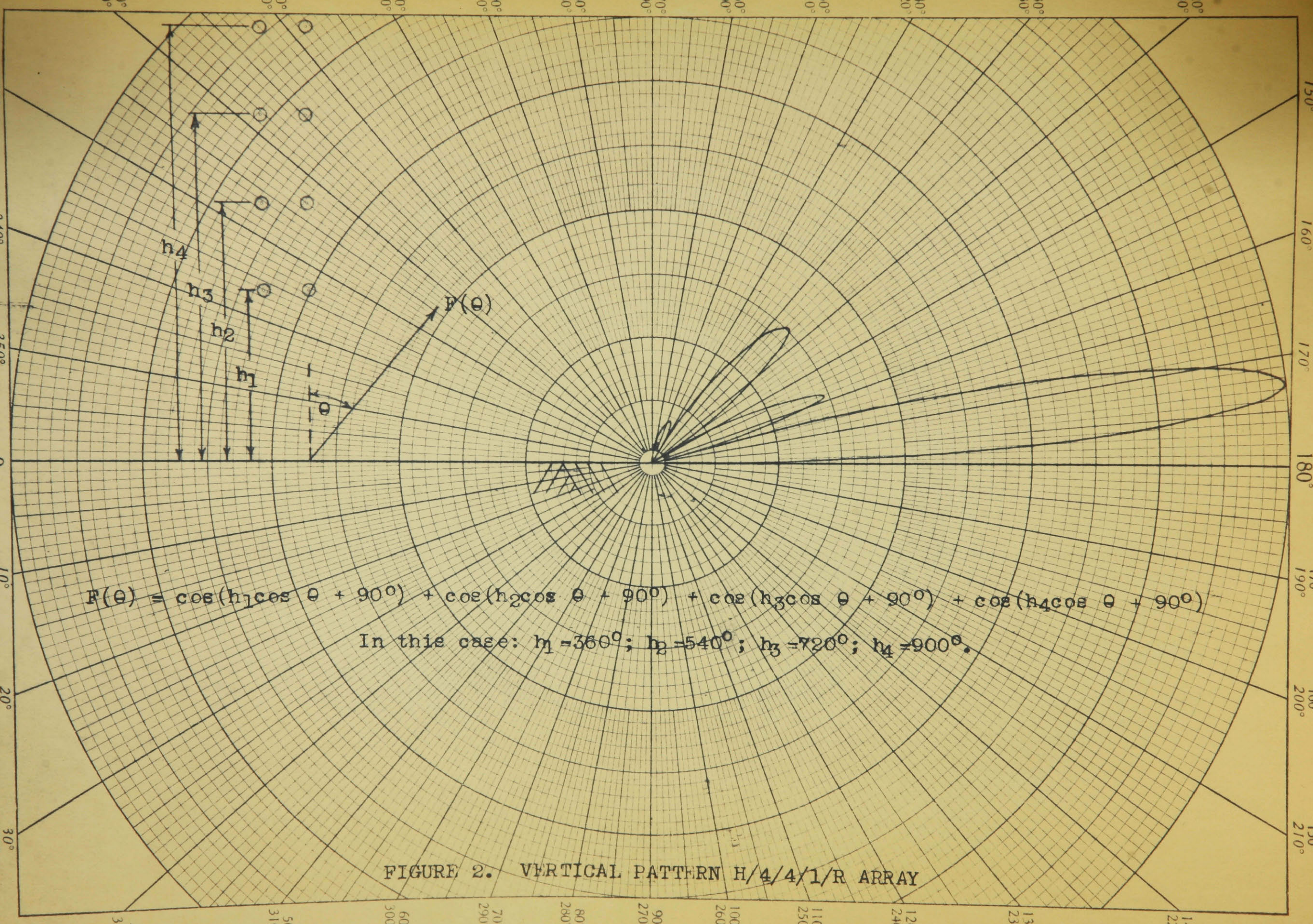


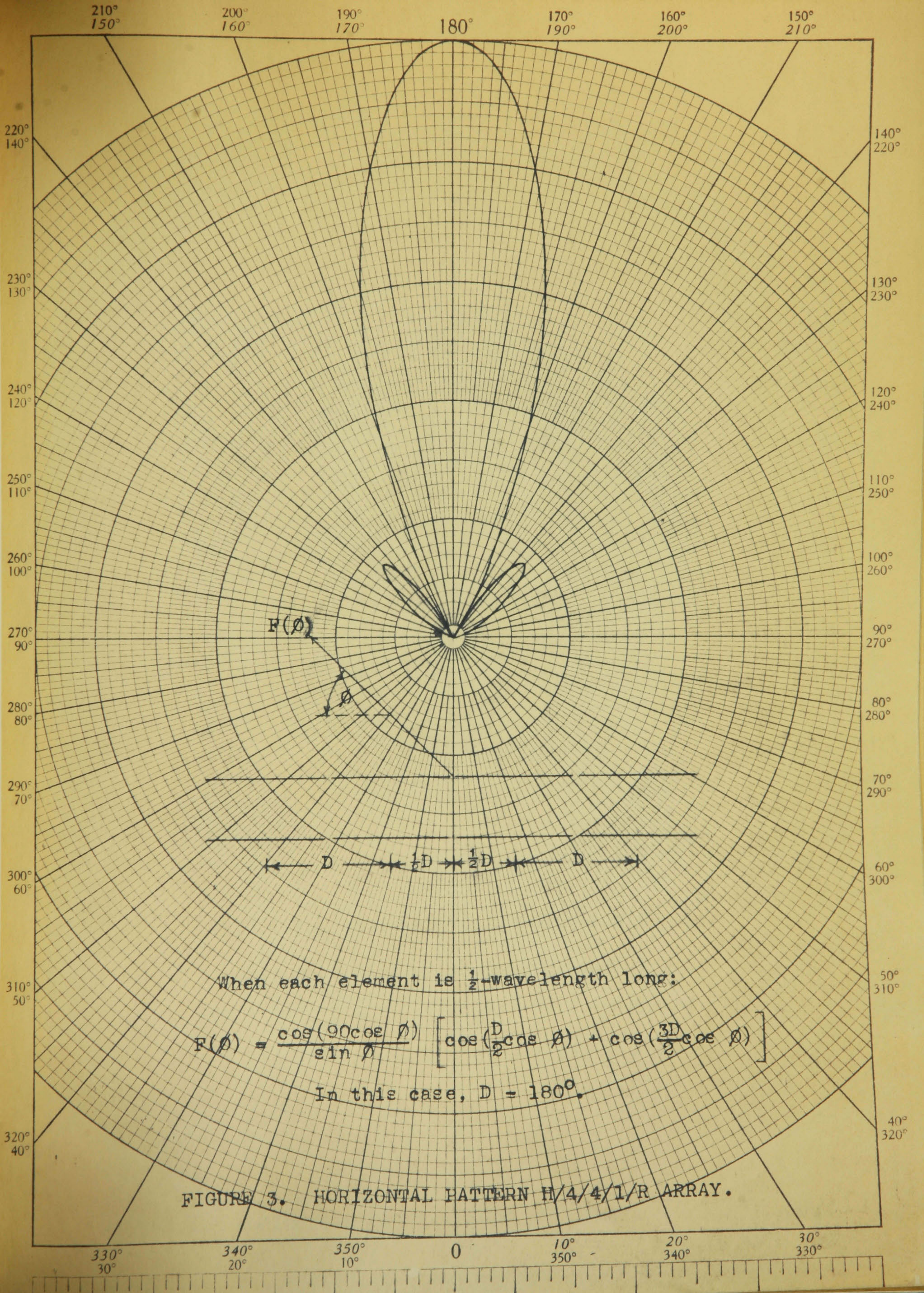
FIGURE 1. H/4/4/1/R ARRAY



$$F(\theta) = \cos(h_1\cos\theta + 90^\circ) + \cos(h_2\cos\theta + 90^\circ) + \cos(h_3\cos\theta + 90^\circ) + \cos(h_4\cos\theta + 90^\circ)$$

In this case: $h_1 = 360^\circ$; $h_2 = 540^\circ$; $h_3 = 720^\circ$; $h_4 = 900^\circ$.

FIGURE 2. VERTICAL PATTERN H/4/4/1/R ARRAY



When each element is ½-wavelength long:

$$F(\emptyset) = \frac{\cos(90\cos\emptyset)}{\sin\emptyset}\left[\cos\left(\frac{D}{2}\cos\emptyset\right) + \cos\left(\frac{3D}{2}\cos\emptyset\right)\right]$$

In this case, D = 180°.

FIGURE 3. HORIZONTAL PATTERN H/4/4/1/R ARRAY.

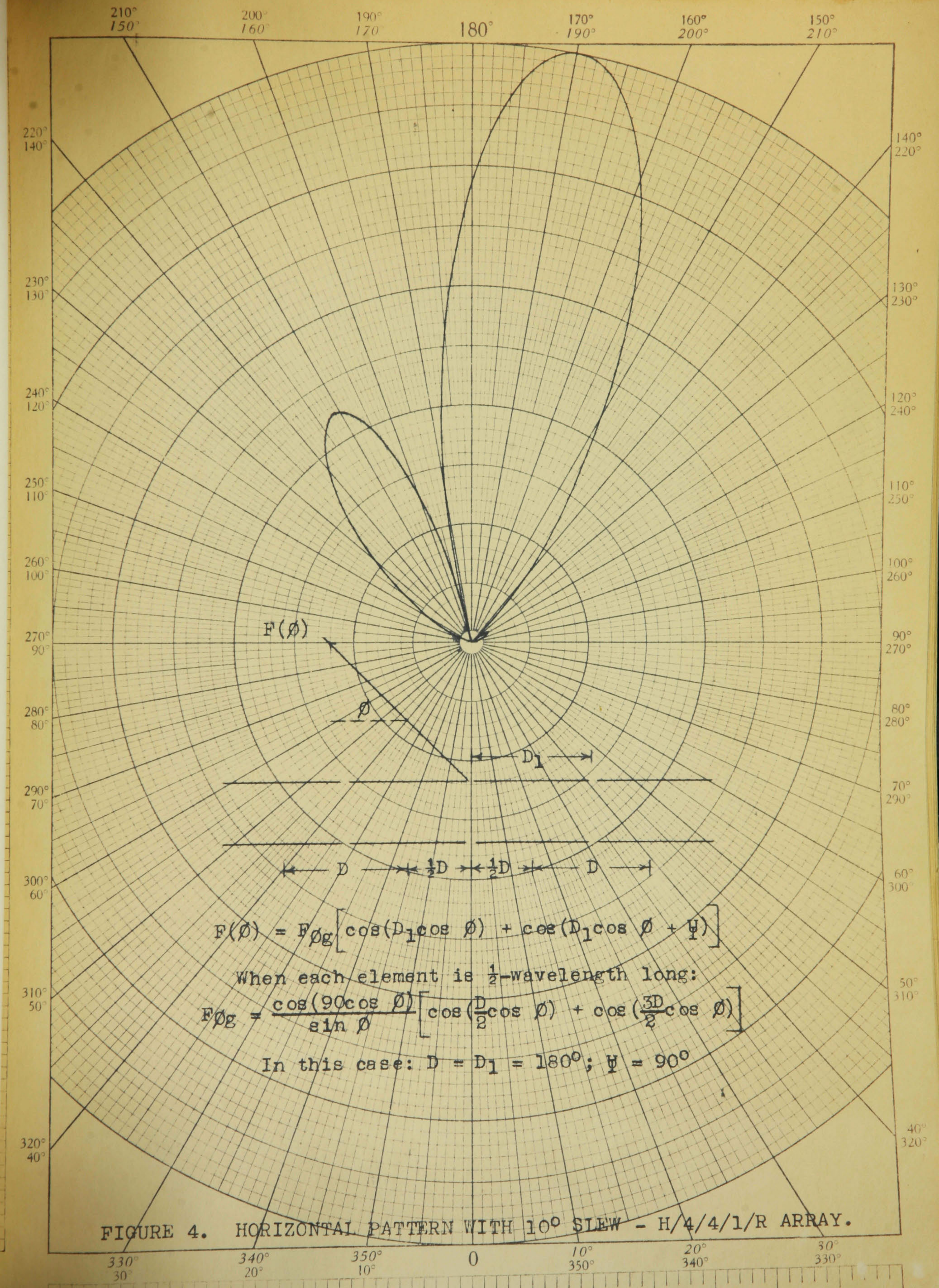
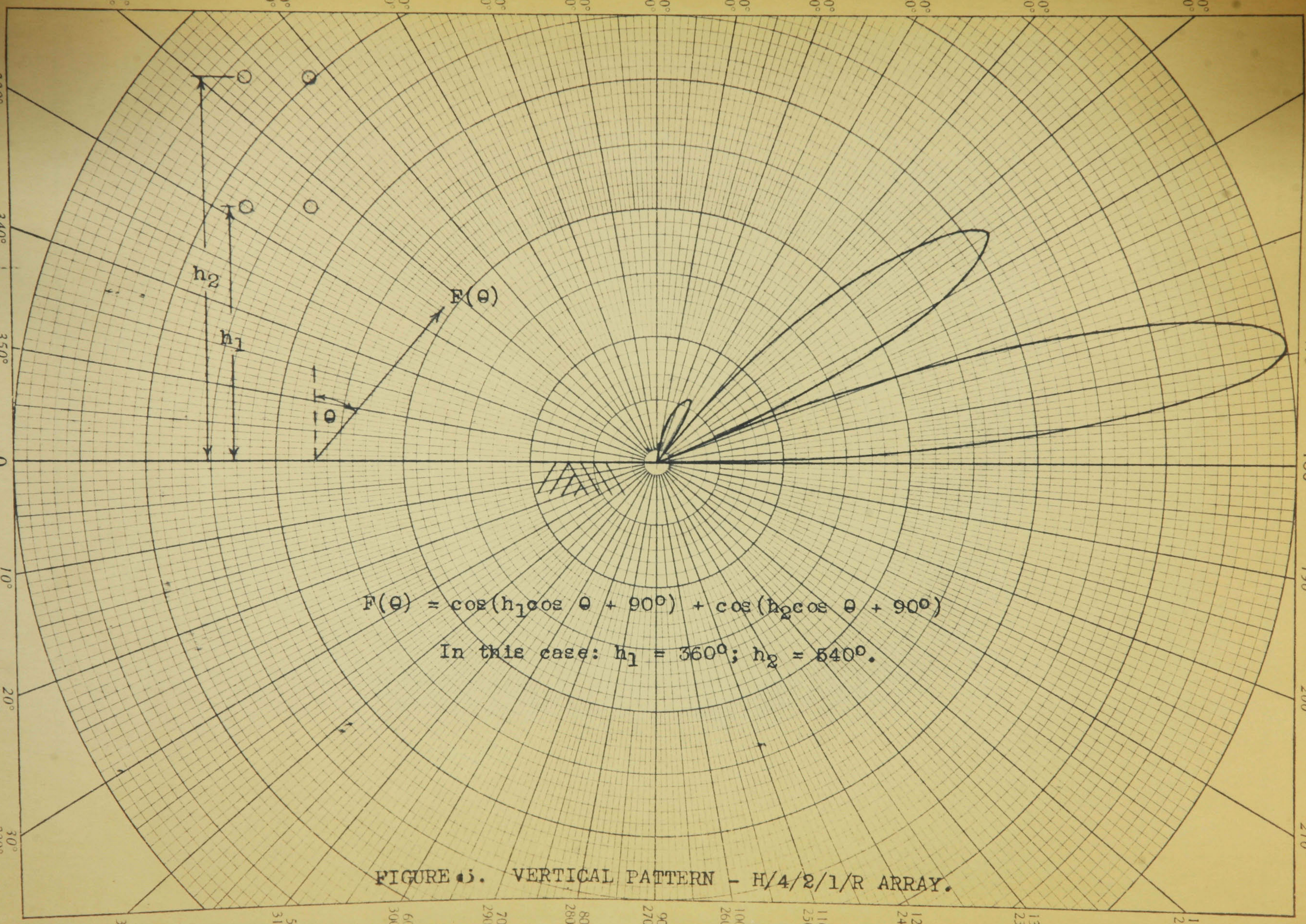


FIGURE 4. HORIZONTAL PATTERN WITH 10° SLEW - H/4/4/1/R ARRAY.



$$F(\theta) = \cos(h_1 \cos\theta + 90^\circ) + \cos(h_2 \cos\theta + 90^\circ)$$

In this case: $h_1 = 360^\circ$; $h_2 = 540^\circ$.

FIGURE 5. VERTICAL PATTERN - H/4/2/1/R ARRAY.

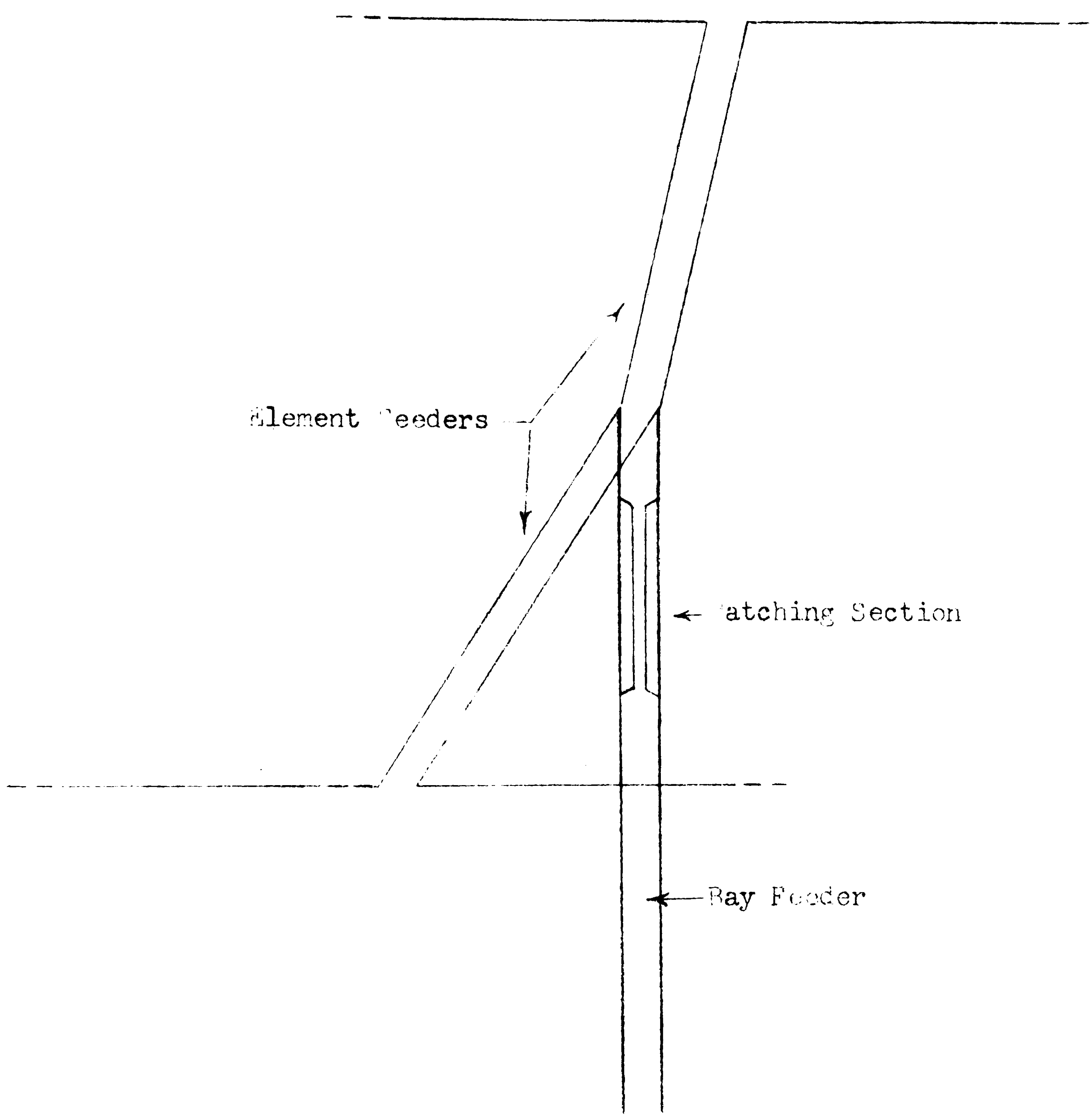
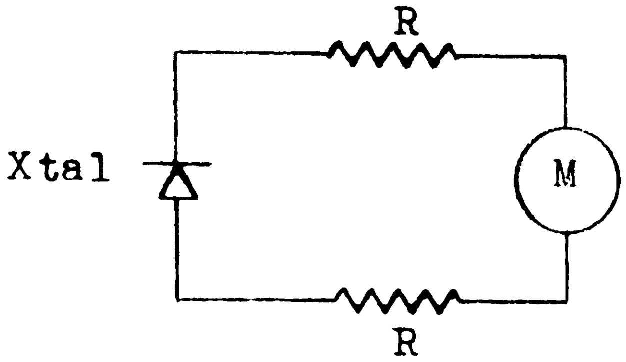


FIGURE 6.

FIGURE 7.



Xtal - Germanium diode, 1N34
R - 4700 ohms
M - 1 ma. meter.

FIGURE 8. CIRCUIT OF STANDING WAVE INDICATOR.

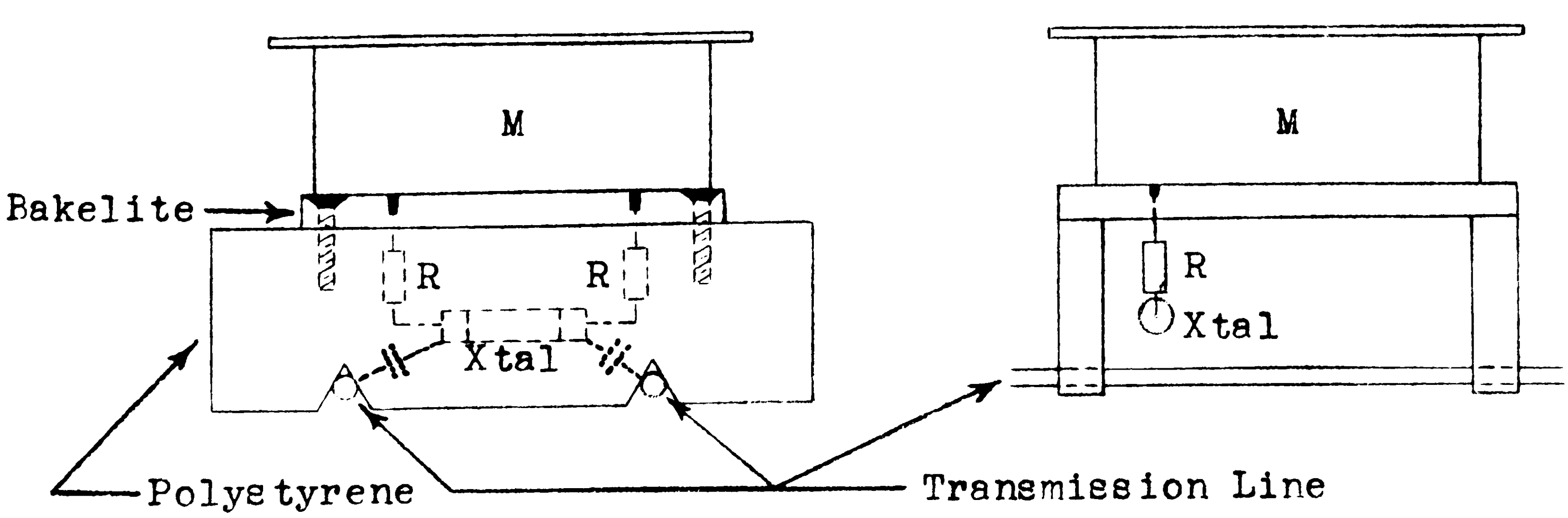
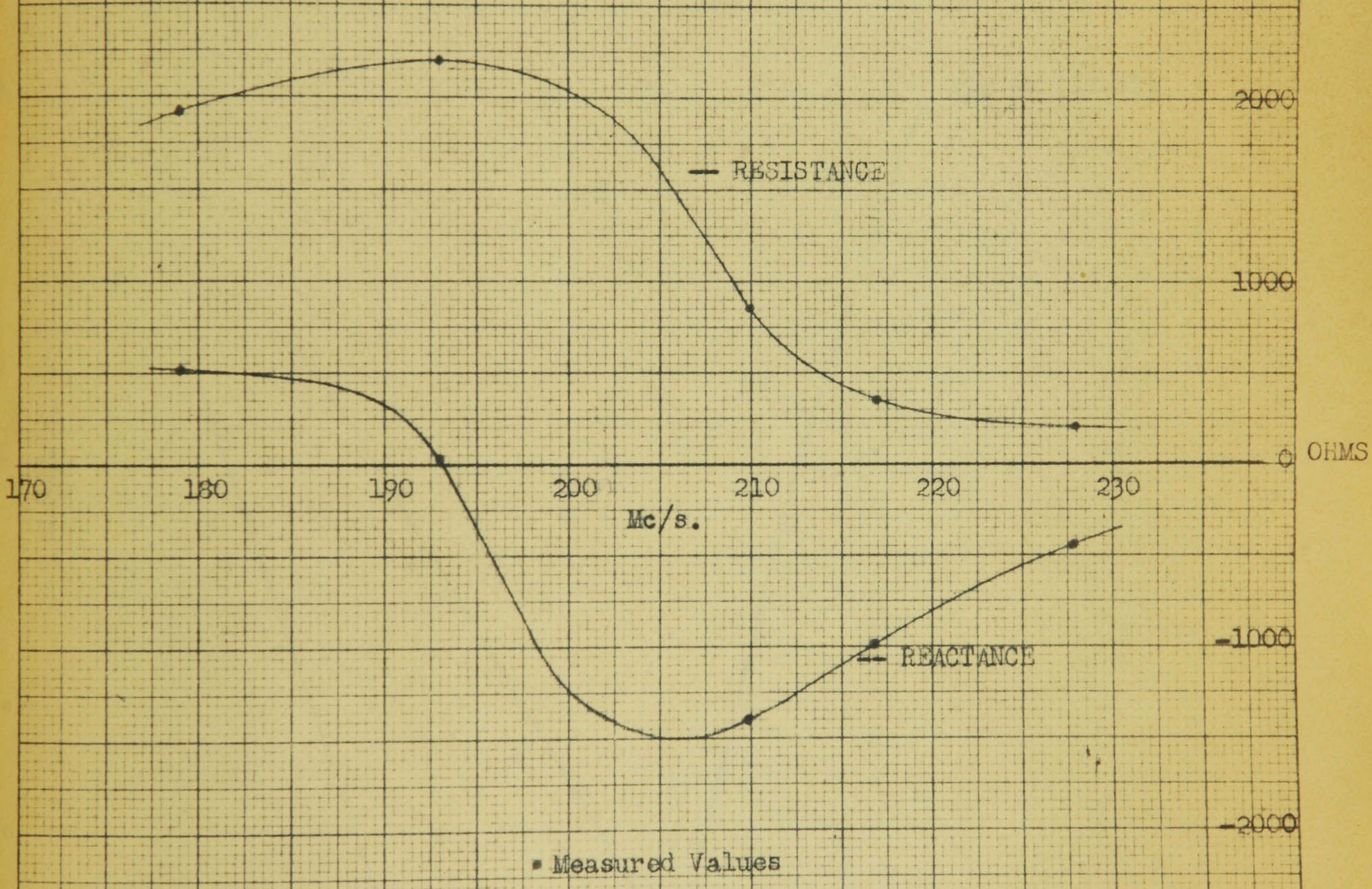


FIGURE 9. CONSTRUCTION DETAILS OF STANDING WAVE INDICATOR.



INPUT IMPEDANCE OF ELEMENTS
OF H 4/2/1 R ARRAY

Element length - 30.75"

FIGURE 10.

5/6/47

R.D.F.

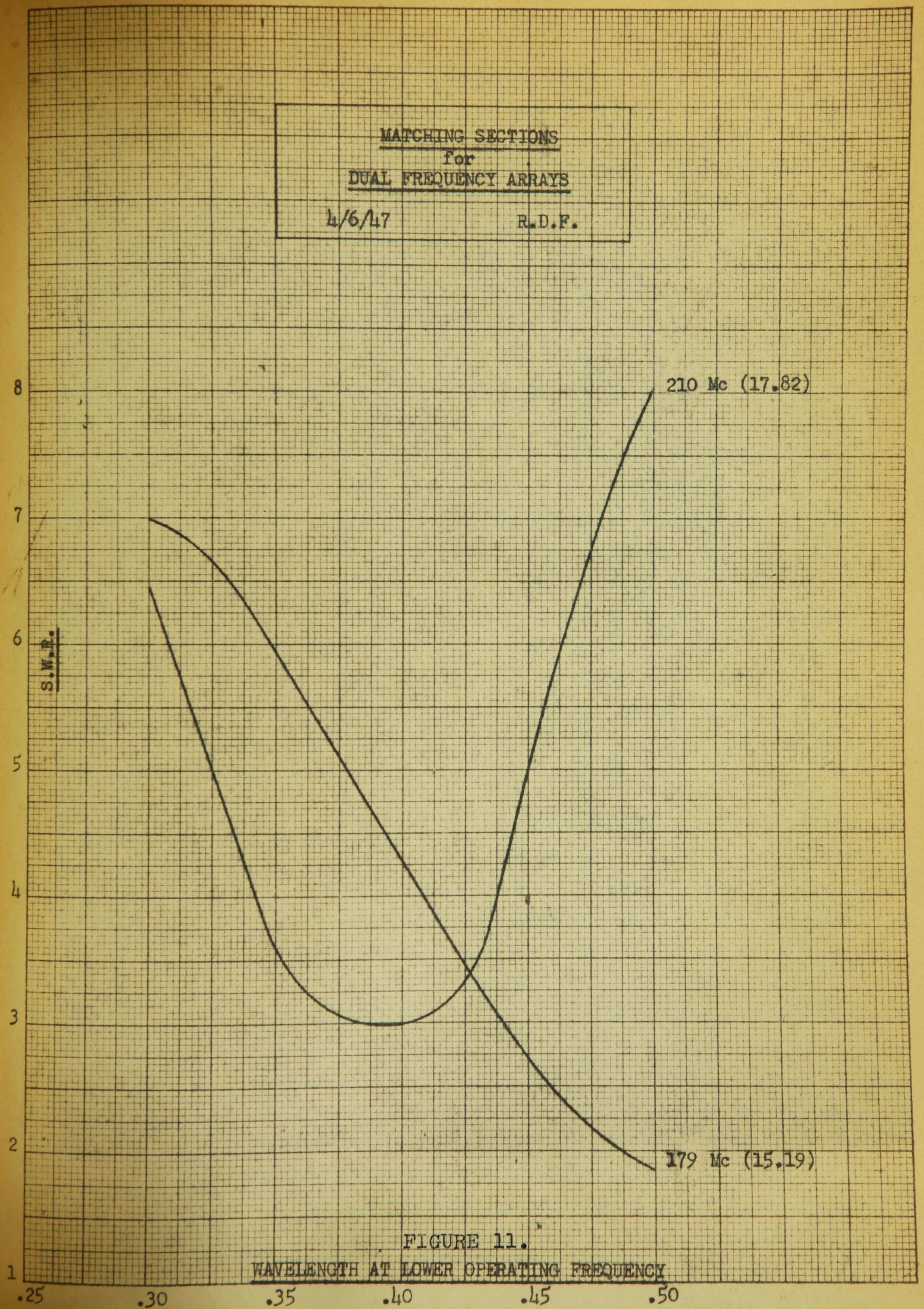


FIGURE 11.
WAVELENGTH AT LOWER OPERATING FREQUENCY

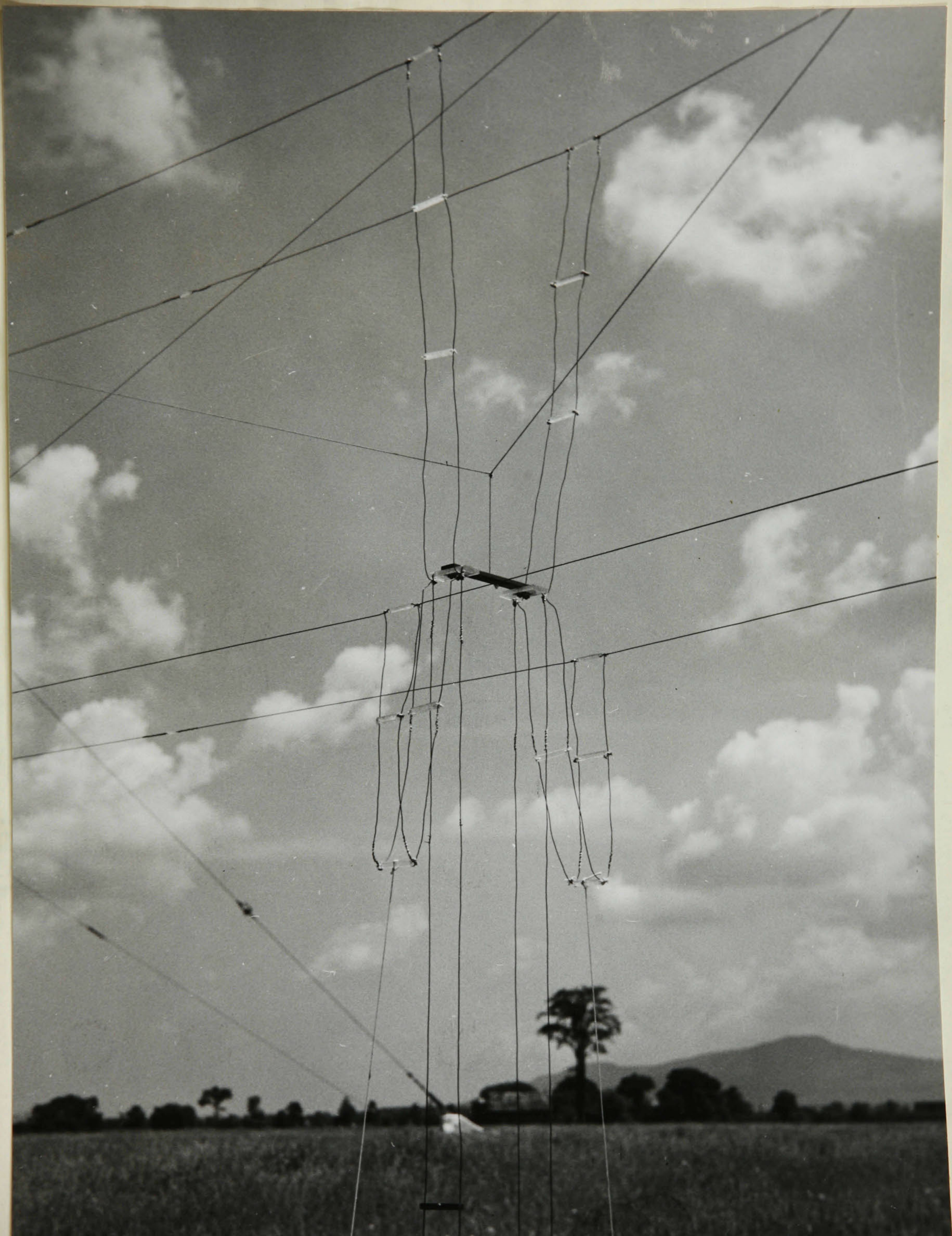


FIGURE 11A

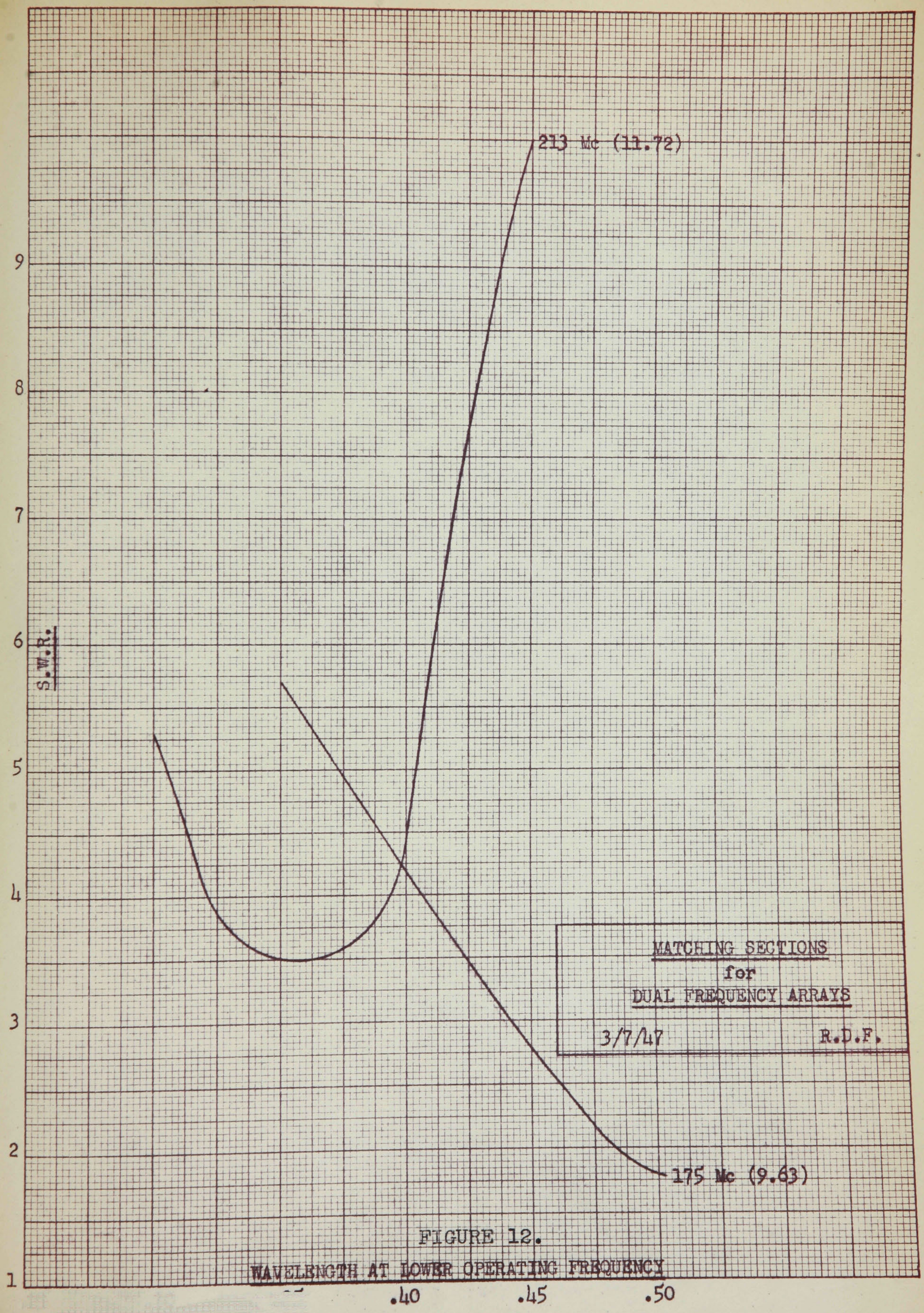

213 Mc (11.72)
175 Mc (9.63)
S.W.R.
1
2
3
4
5
6
7
8
9
.40
.45
.50
MATCHING SECTIONS
for
DUAL FREQUENCY ARRAYS
3/7/47
R.D.F.
WAVELENGTH AT LOWER OPERATING FREQUENCY

FIGURE 12.

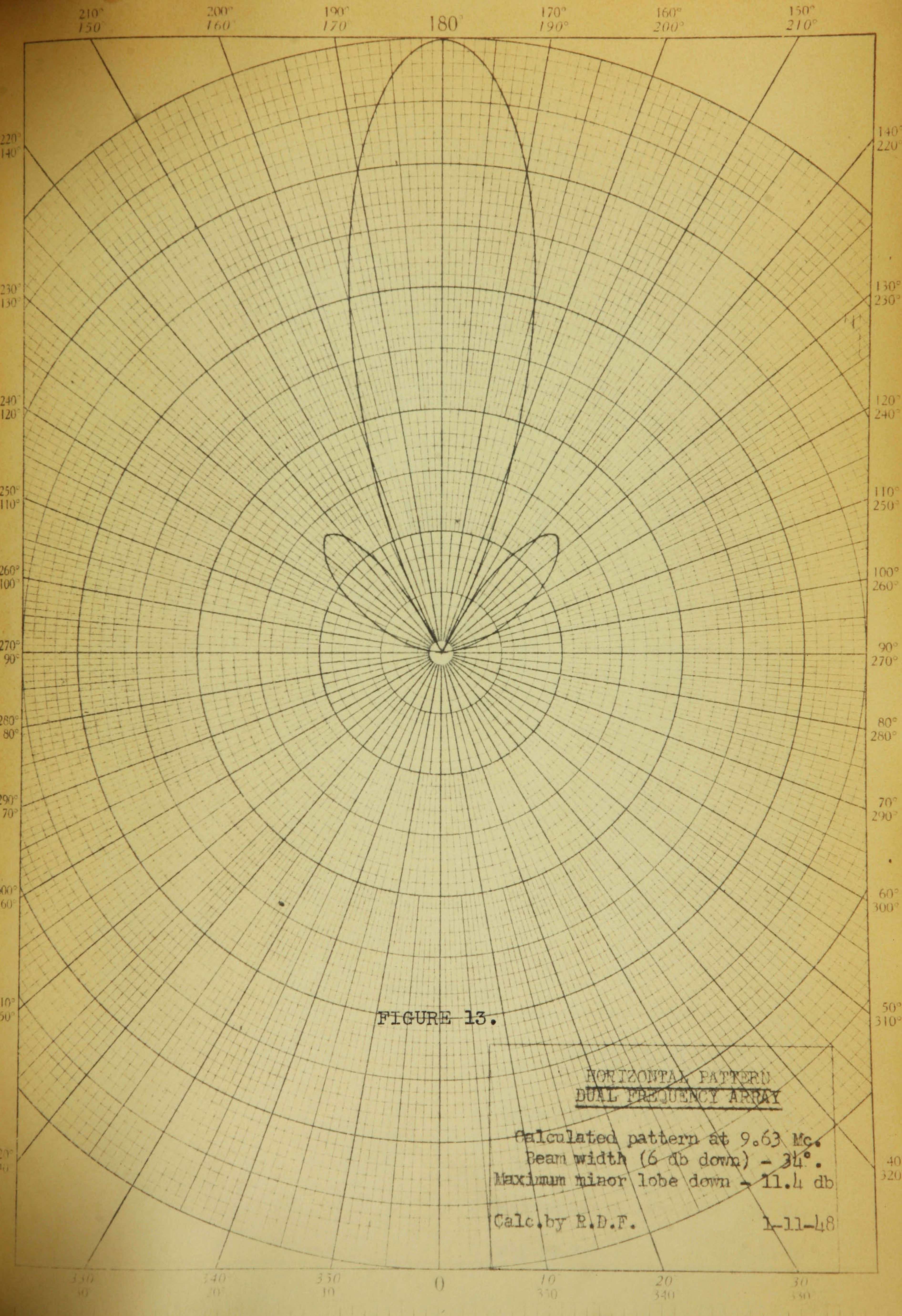


FIGURE 13.

HORIZONTAL PATTERN
DUAL FREQUENCY ARRAY

Calculated pattern at 9.63 Mc.
Beam width (6 db down) - 34°.
Maximum minor lobe down - 11.4 db

Calc. by R.D.F. 1-11-48

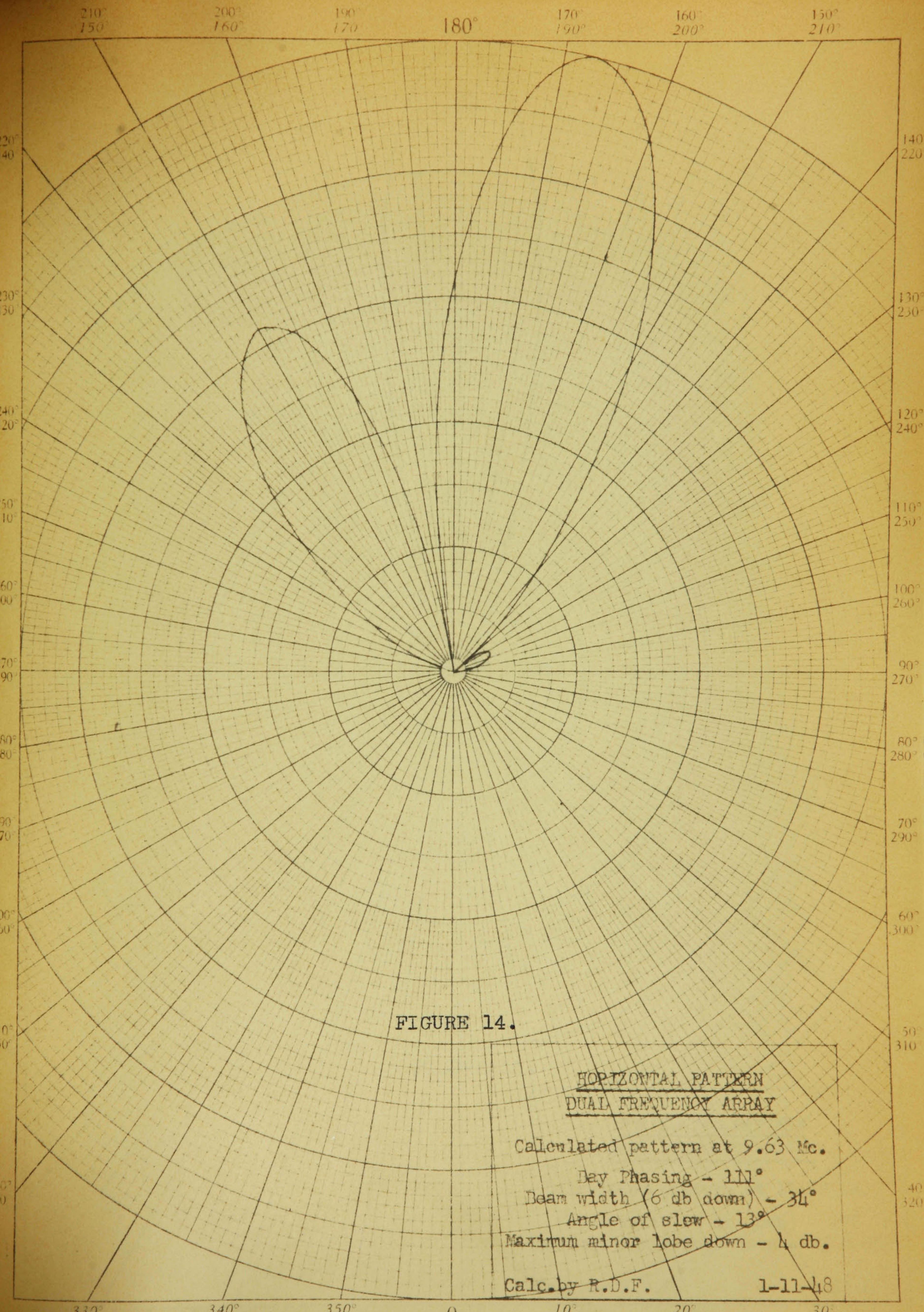
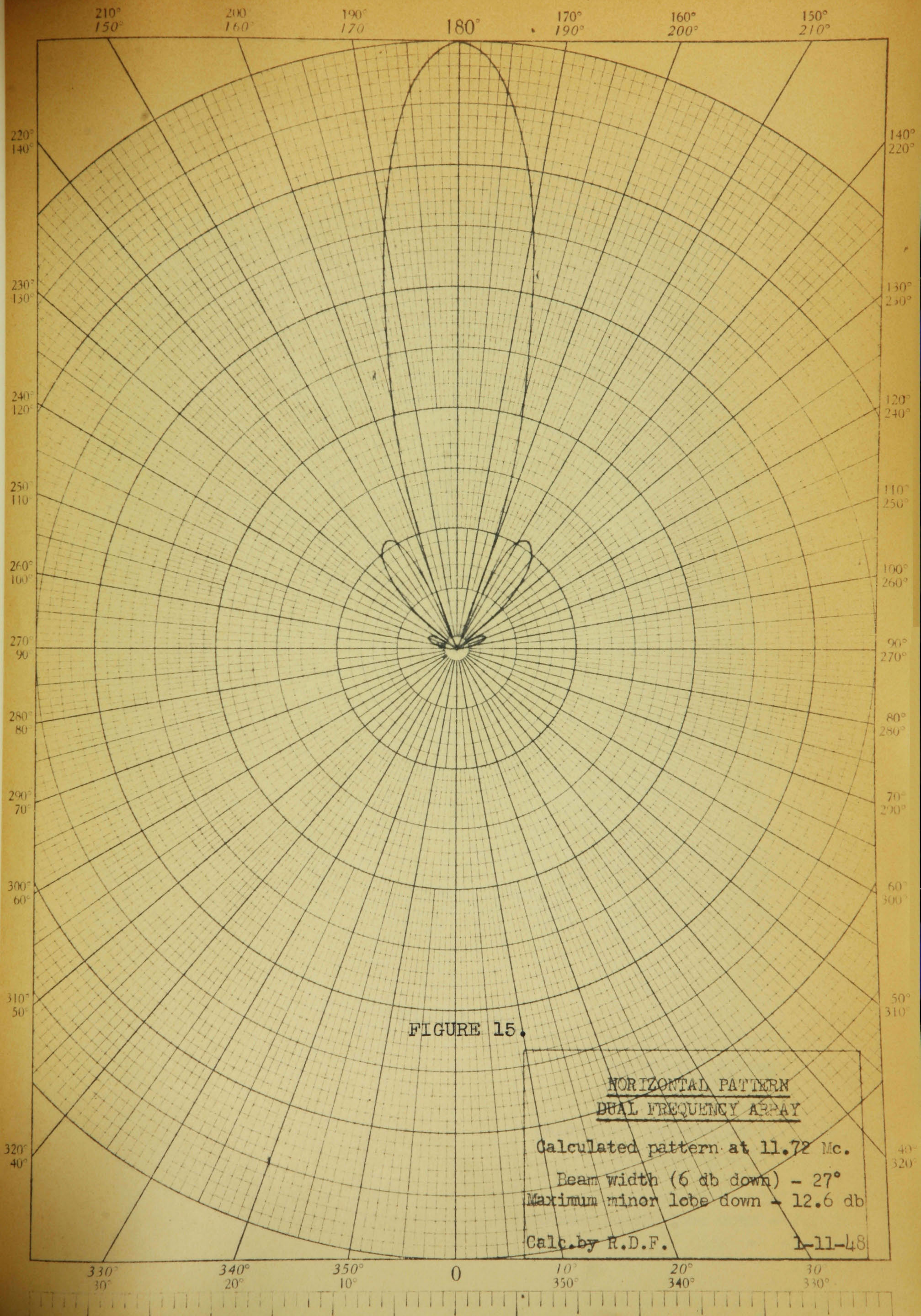
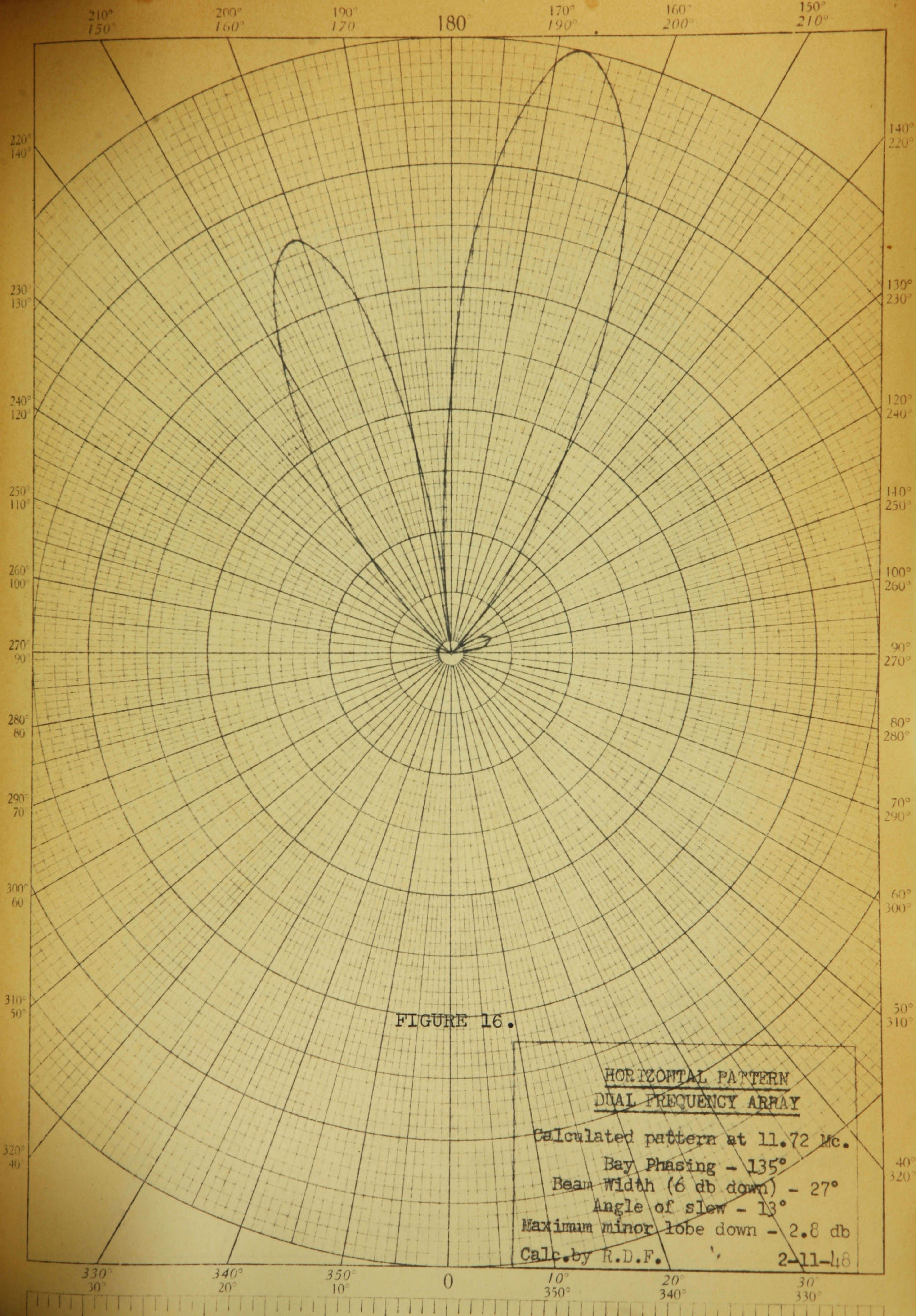

180°
0
HORIZONTAL PATTERN
DUAL FREQUENCY ARRAY
Calculated pattern at 9.63 Mc.
Day Phasing - 111°
Beam width (6 db down) - 34°
Angle of slew - 13°
Maximum minor lobe down - 4 db.
Calc. by R.D.F.
1-11-48

FIGURE 14.


180°
0
FIGURE 15.
HORIZONTAL PATTERN
DUAL FREQUENCY ARRAY
Calculated pattern at 11.72 Mc.
Beam width (6 db down) - 27°
Maximum minor lobe down - 12.6 db
Calc. by R.D.F.
1-11-48



180
HORIZONTAL PATTERN
DUAL FREQUENCY ARRAY
Calculated pattern at 11.72 Mc.
Bay Phasing - 135°
Beam Width (6 db down) - 27°
Angle of slew - 13°
Maximum minor lobe down - 2.8 db
Calc. by R.D.F.
2-11-48

FIGURE 16.

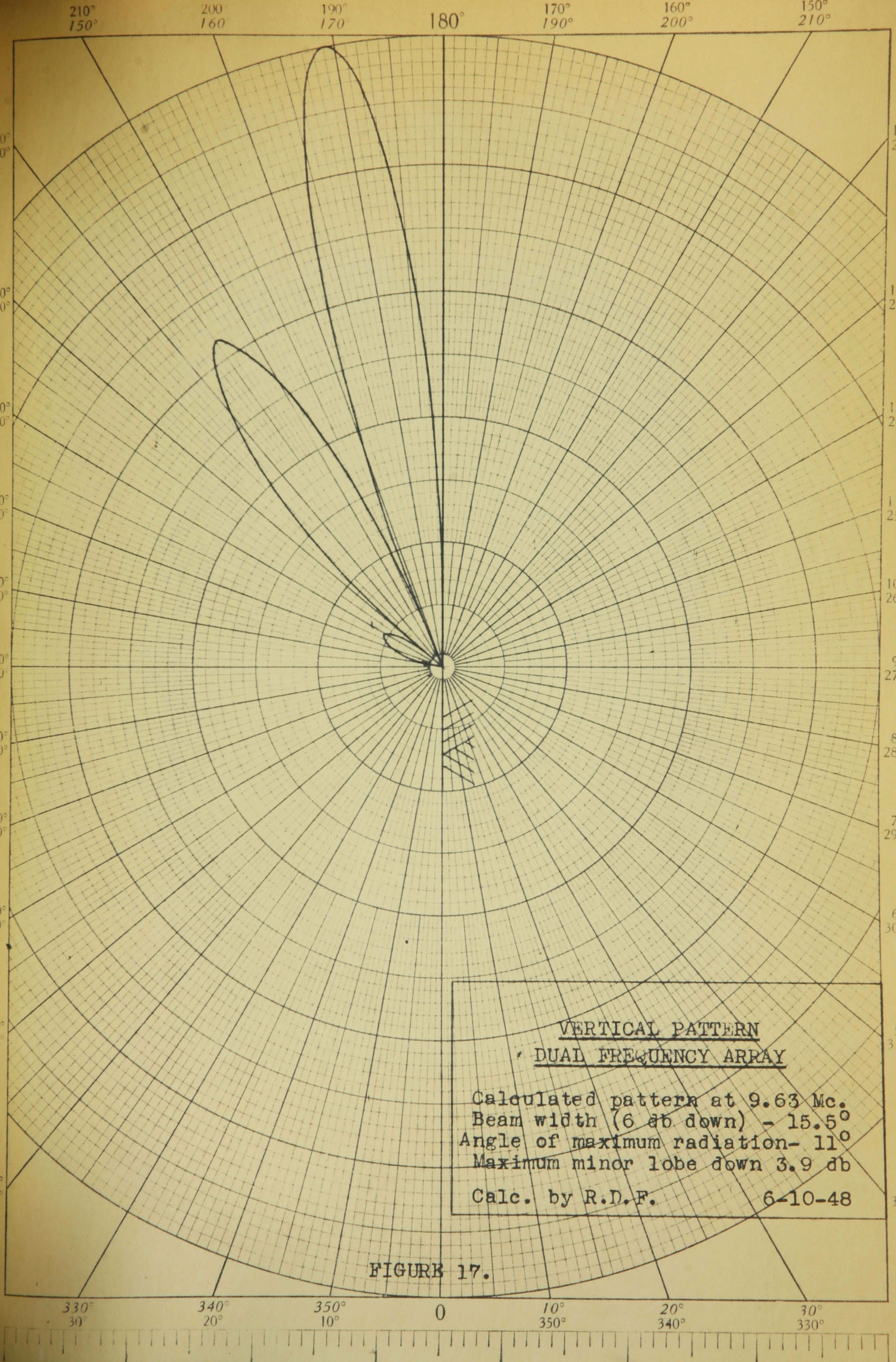

VERTICAL PATTERN
DUAL FREQUENCY ARRAY
Calculated pattern at 9.63 Mc.
Beam width (6 db down) - 15.5°
Angle of maximum radiation- 11°
Maximum minor lobe down 3.9 db
Calc. by R.D.F. 6-10-48

FIGURE 17.

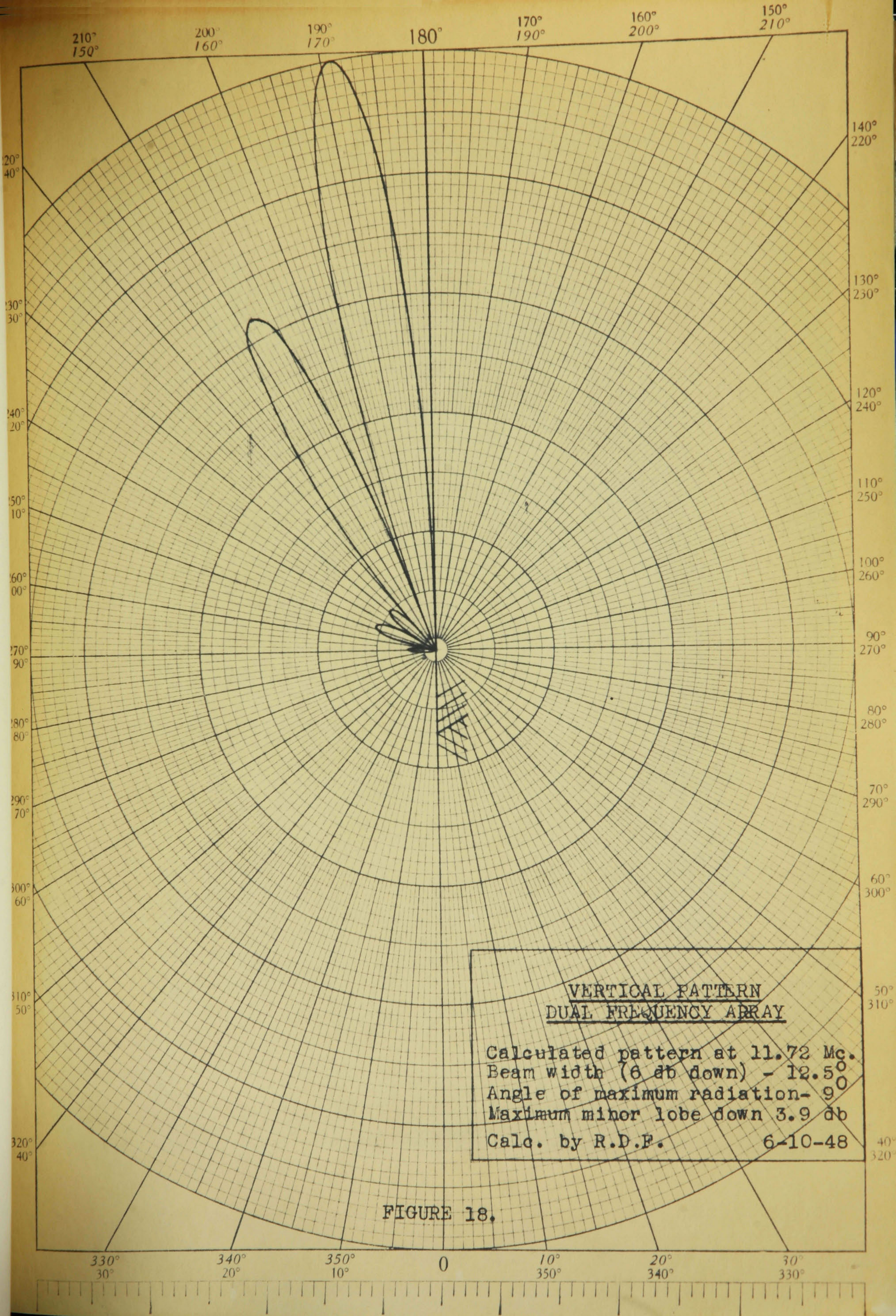

VERTICAL PATTERN
DUAL FREQUENCY ARRAY
Calculated pattern at 11.72 Mc.
Beam width (6 db down) - 12.5°
Angle of maximum radiation- 9°
Maximum minor lobe down 3.9 db
Calc. by R.D.F. 6-10-48

FIGURE 18.

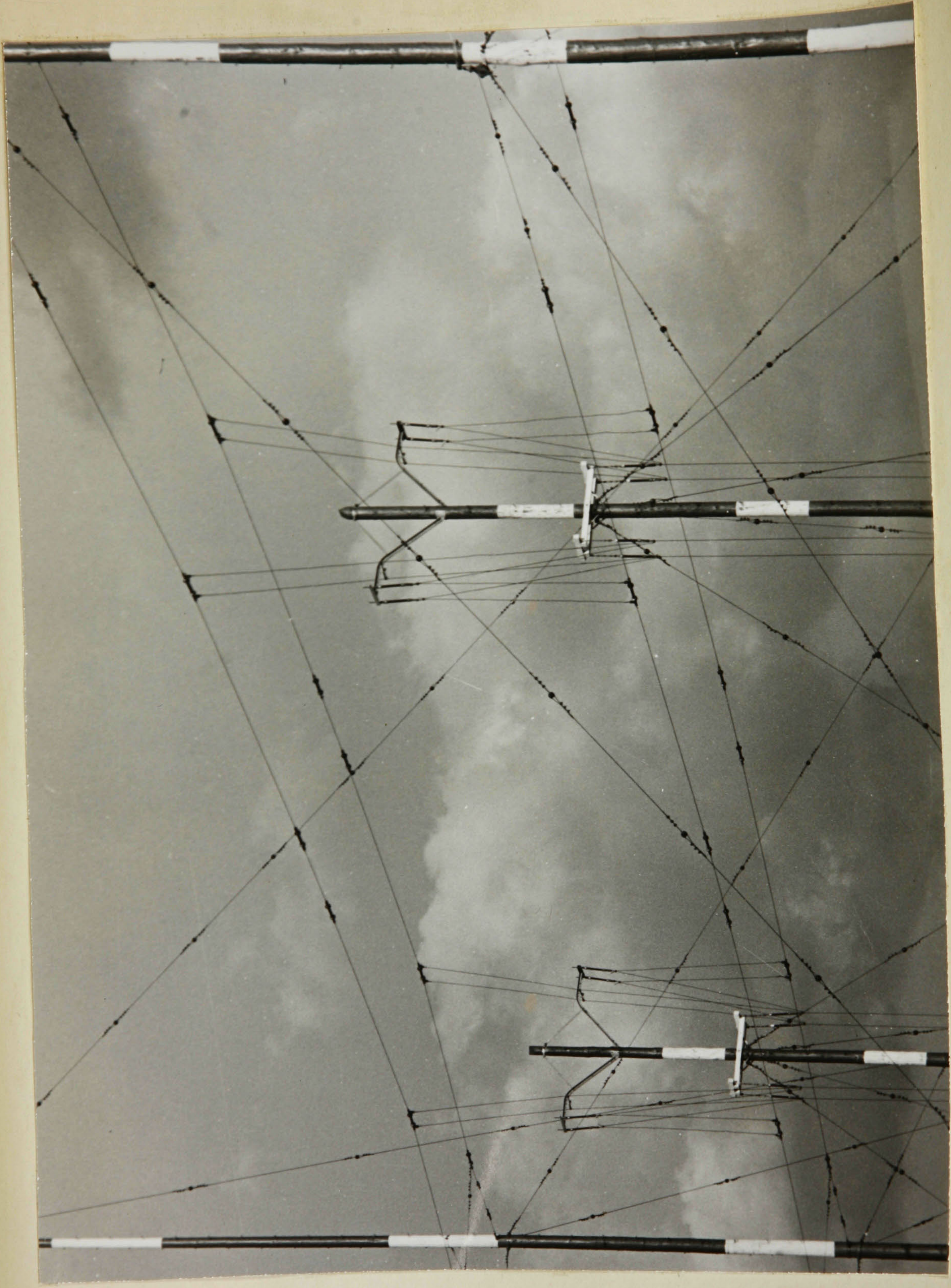


FIGURE 19.

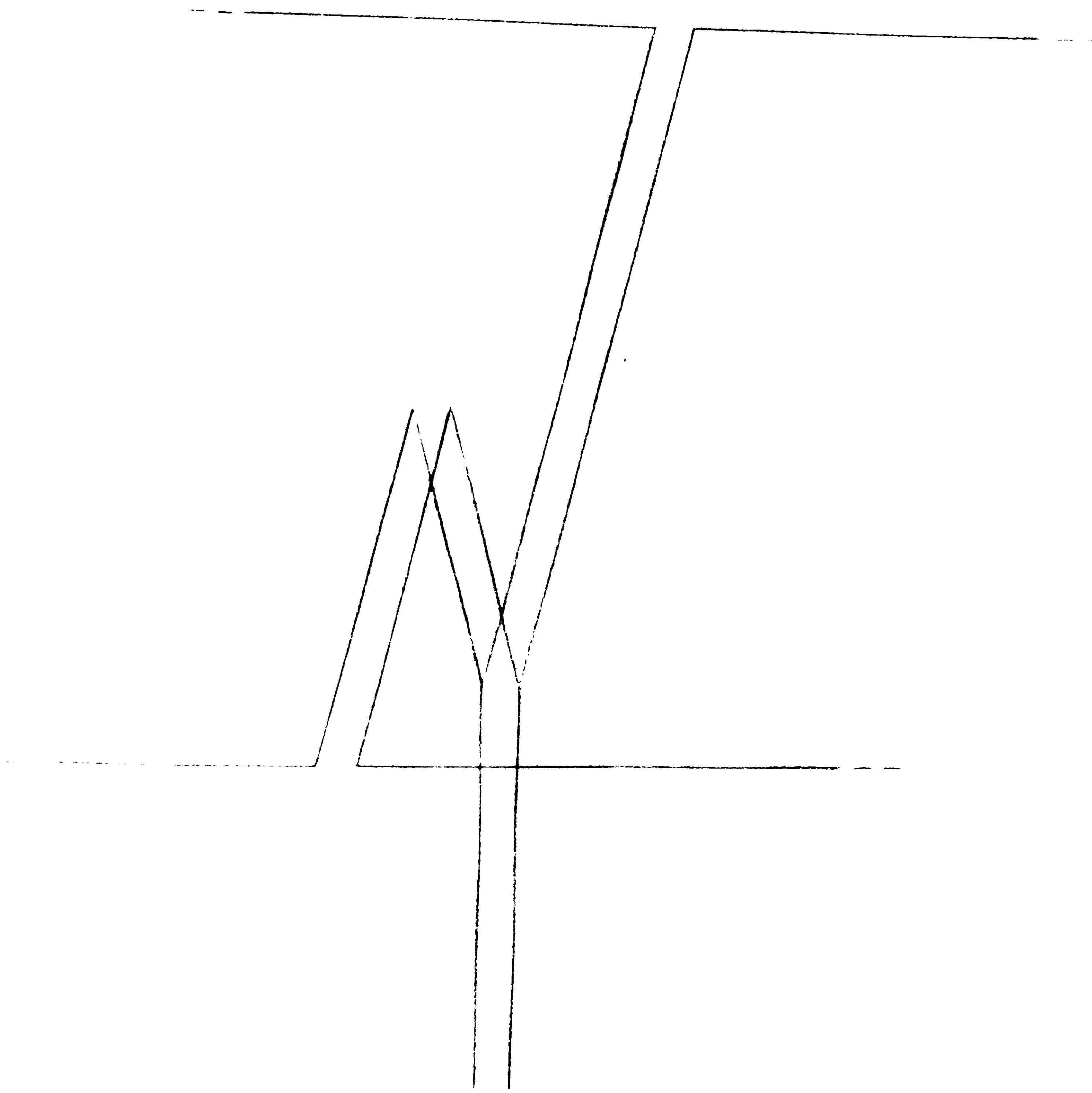


FIGURE 20.

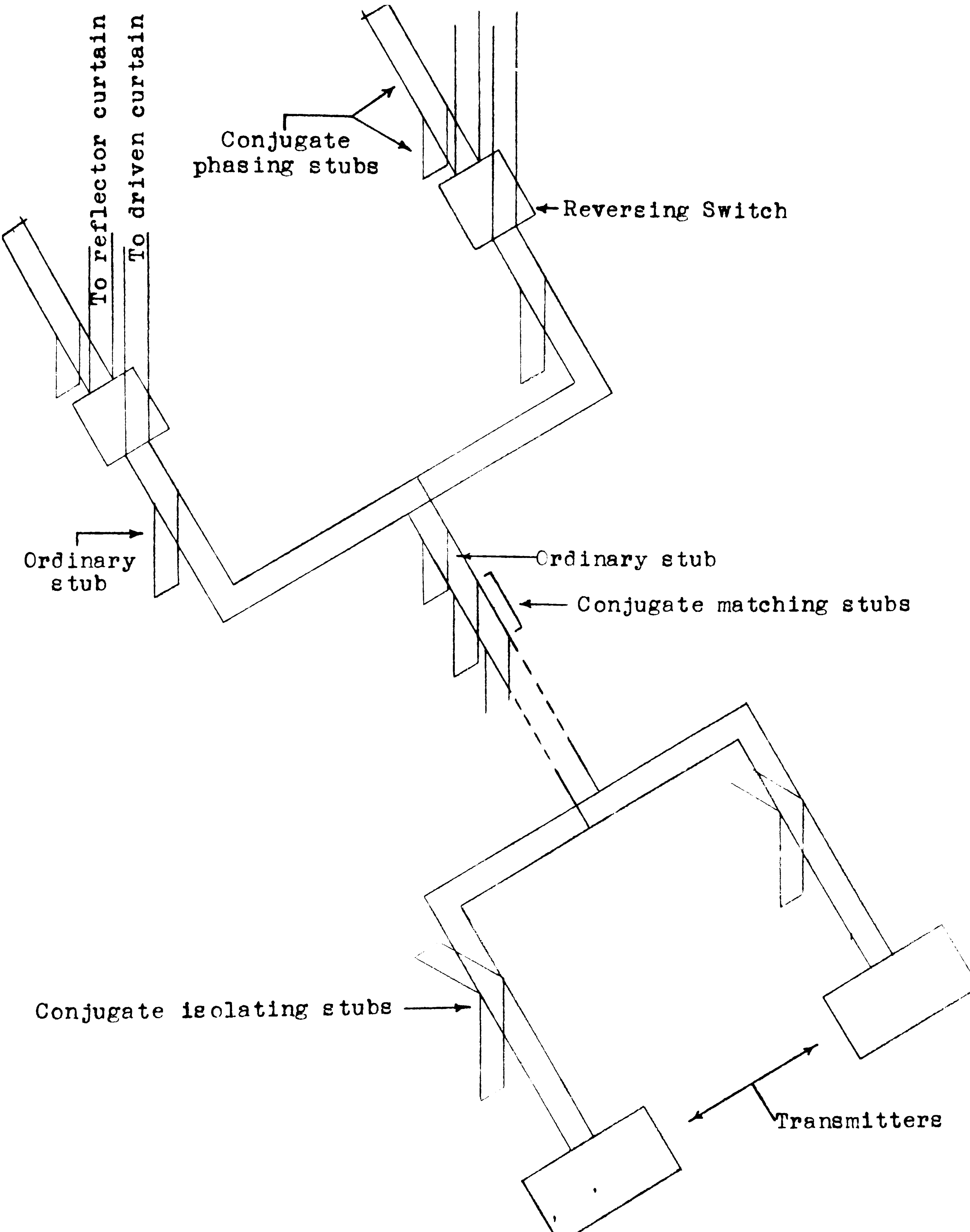

To reflector curtain
To driven curtain
Conjugate phasing stubs
Reversing Switch
Ordinary stub
Ordinary stub
Conjugate matching stubs
Conjugate isolating stubs
Transmitters

FIGURE 21.

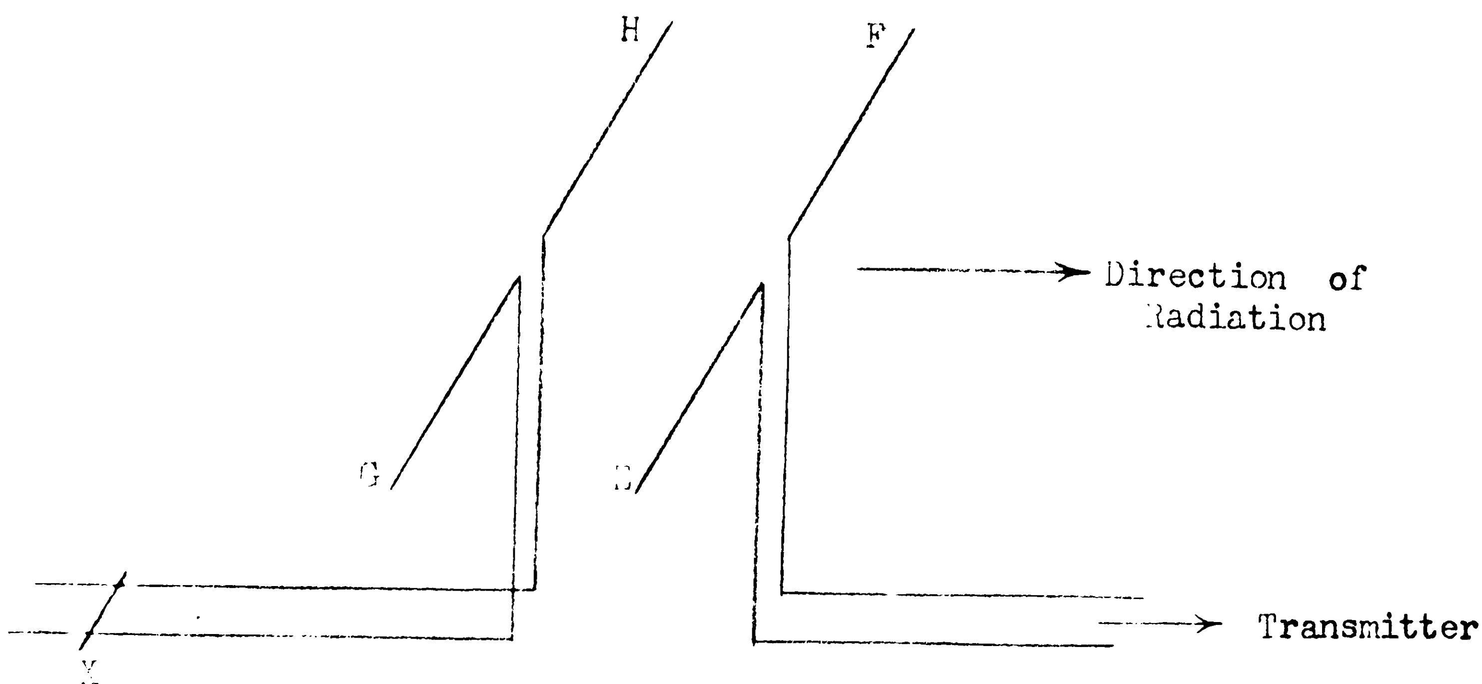


FIGURE 22.

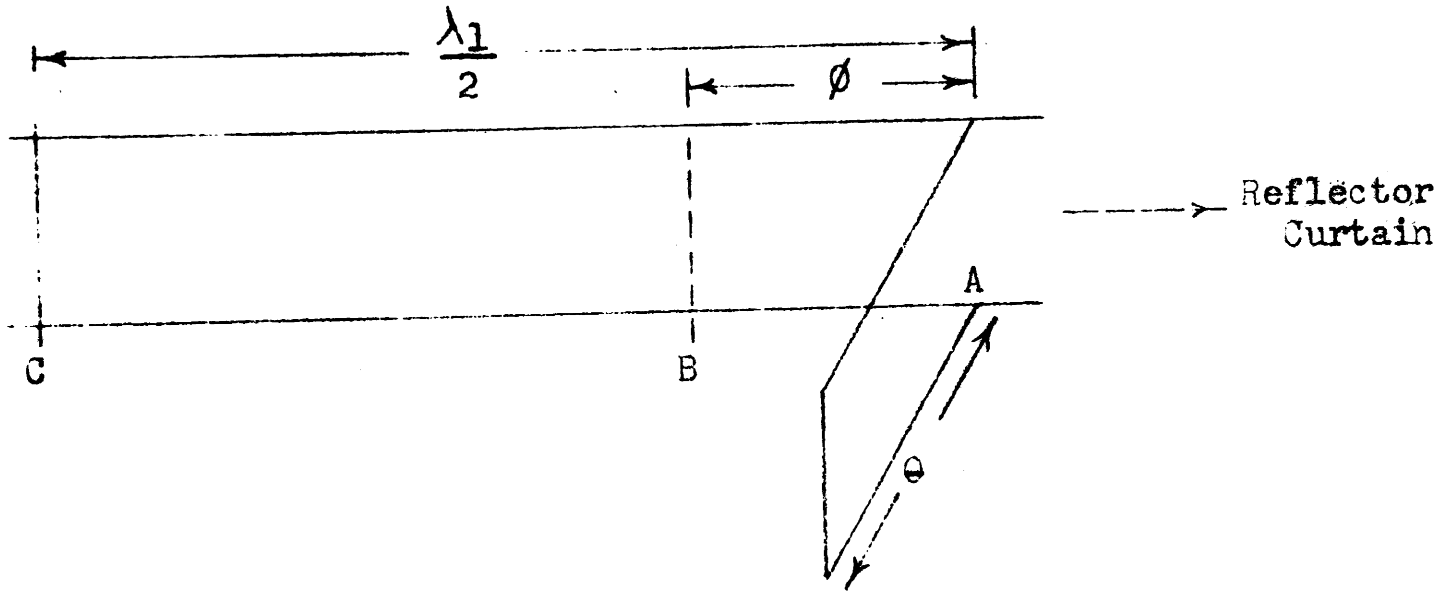


FIGURE 23.

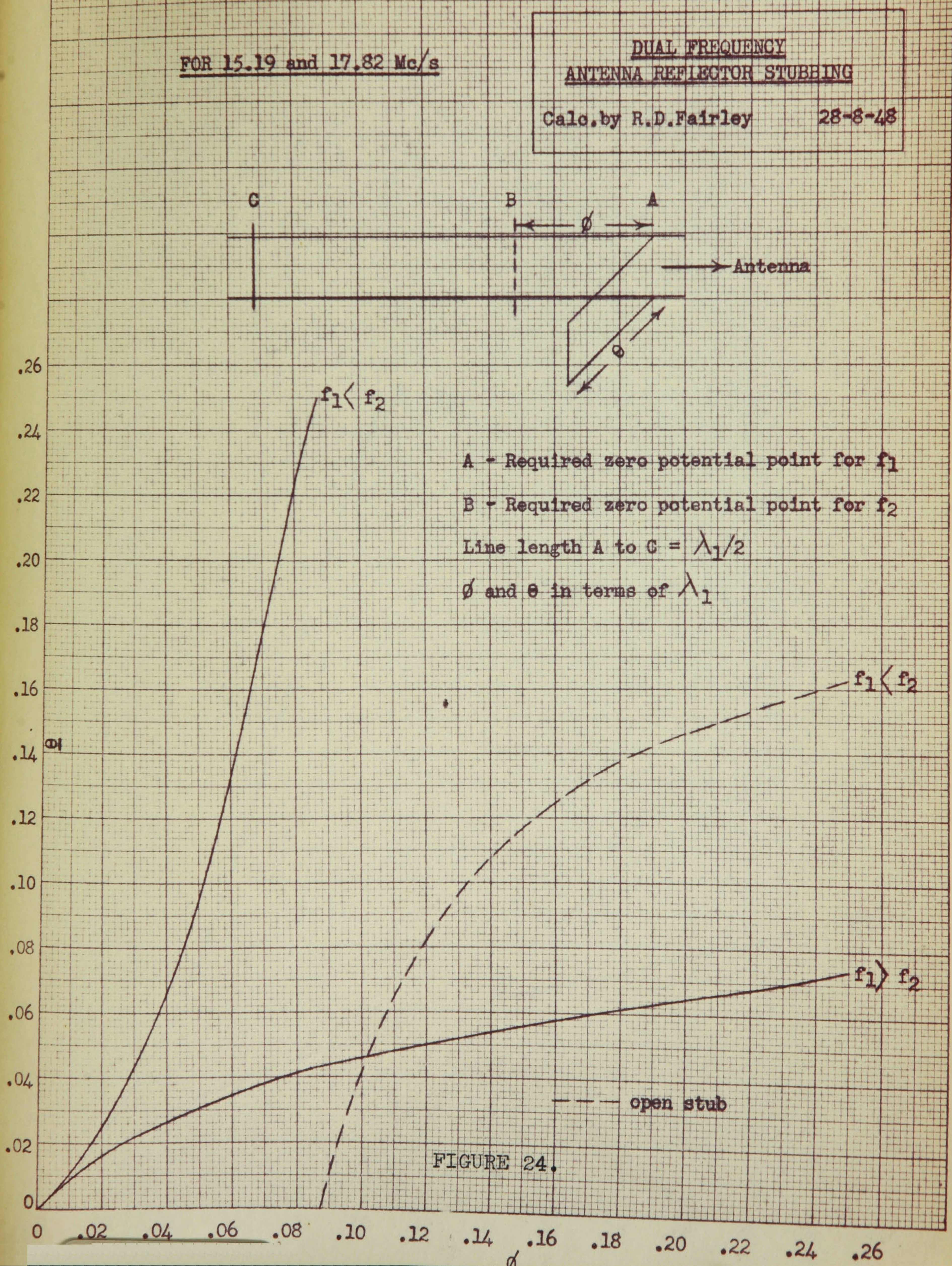
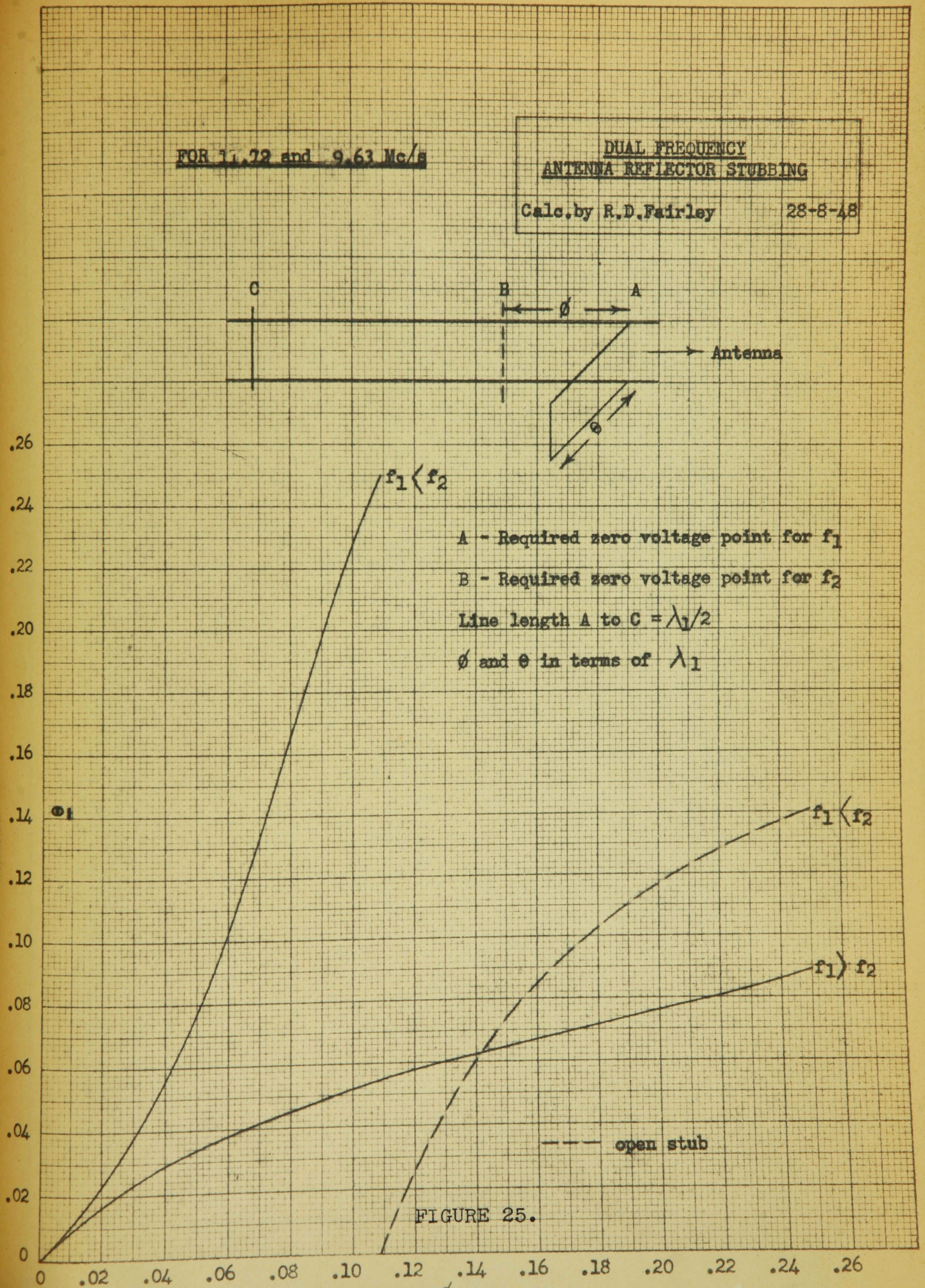

FOR 15.19 and 17.82 Mc/s
DUAL FREQUENCY
ANTENNA REFLECTOR STUBBING
Calc.by R.D.Fairley 28-8-48
C
B
A
Ø
Antenna
θ
A - Required zero potential point for f_1
B - Required zero potential point for f_2
Line length A to C = $\lambda_1/2$
Ø and θ in terms of λ_1
$f_1 < f_2$
$f_1 < f_2$
$f_1 > f_2$
open stub
θ
Ø
0
.02
.04
.06
.08
.10
.12
.14
.16
.18
.20
.22
.24
.26

FIGURE 24.



FOR 11.72 and 9.63 Mc/s
DUAL FREQUENCY
ANTENNA REFLECTOR STUBBING
Calc.by R.D.Fairley
28-8-48
C
B
A
ø
Antenna
θ
$f_1 < f_2$
A - Required zero voltage point for f_1
B - Required zero voltage point for f_2
Line length A to C = $\lambda_1/2$
ø and θ in terms of λ_1
θ
$f_1 < f_2$
$f_1 > f_2$
open stub
0
.02
.04
.06
.08
.10
.12
.14
.16
.18
.20
.22
.24
.26

FIGURE 25.

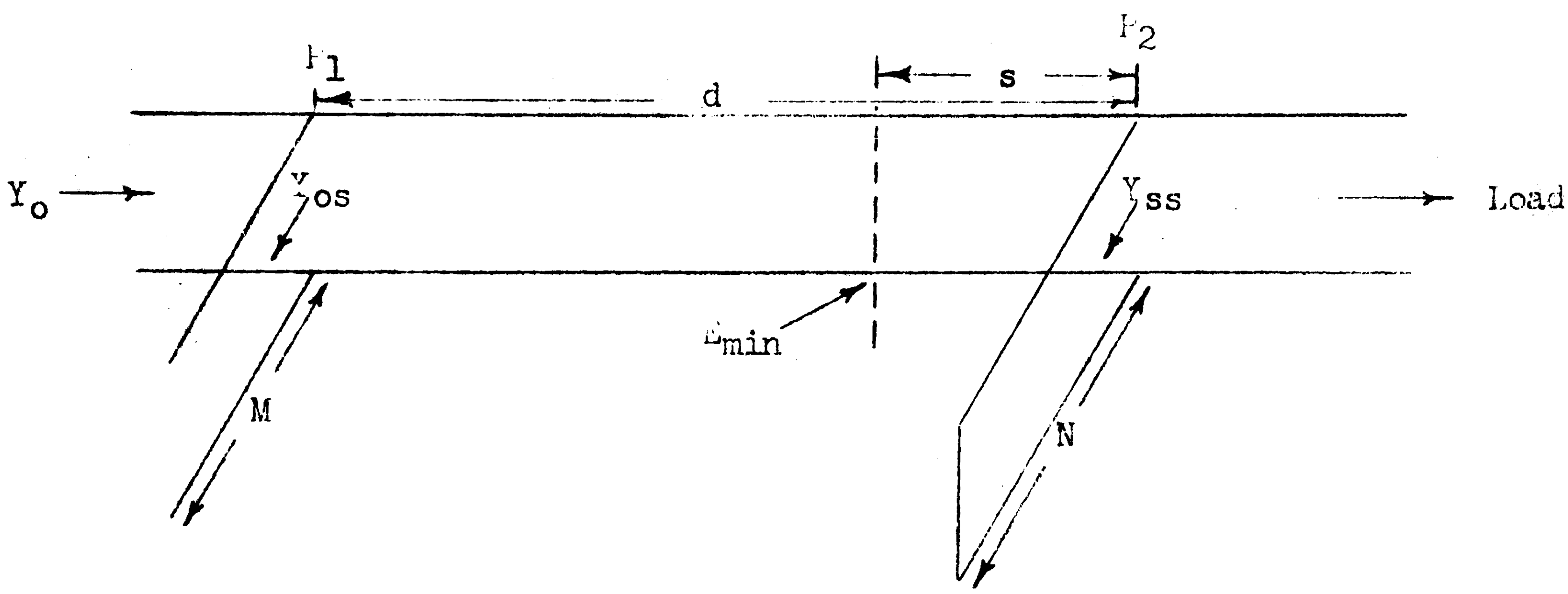
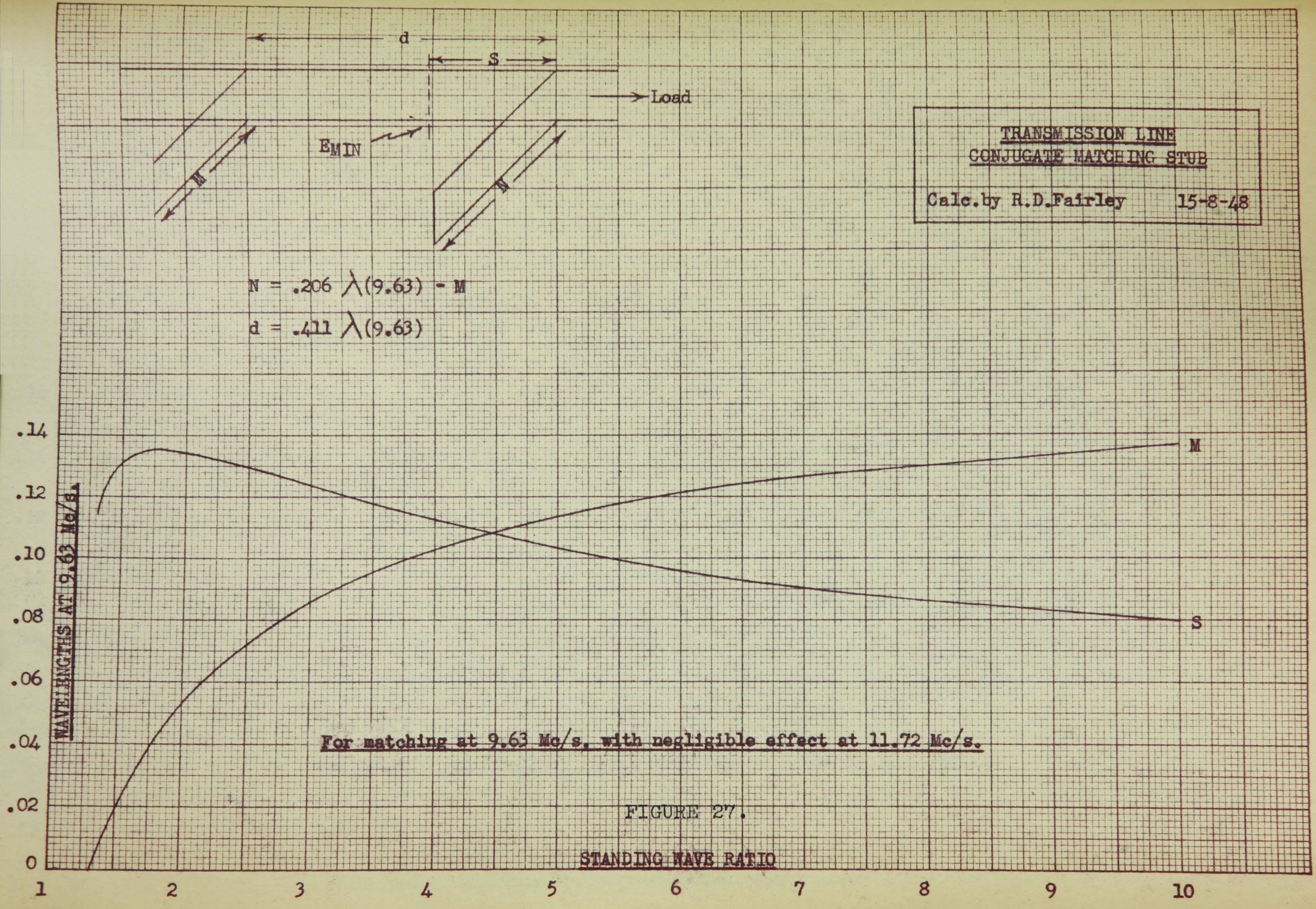


FIGURE 26.



d
S
Load
E_{MIN}
M
N
TRANSMISSION LINE
CONJUGATE MATCHING STUB
Calc.by R.D.Fairley 15-8-48
N = .206 λ(9.63) - M
d = .411 λ(9.63)
WAVELENGTHS AT 9.63 Mc/s.
.14
.12
.10
.08
.06
.04
.02
0
M
S
For matching at 9.63 Mc/s. with negligible effect at 11.72 Mc/s.
STANDING WAVE RATIO
1
2
3
4
5
6
7
8
9
10

FIGURE 27.

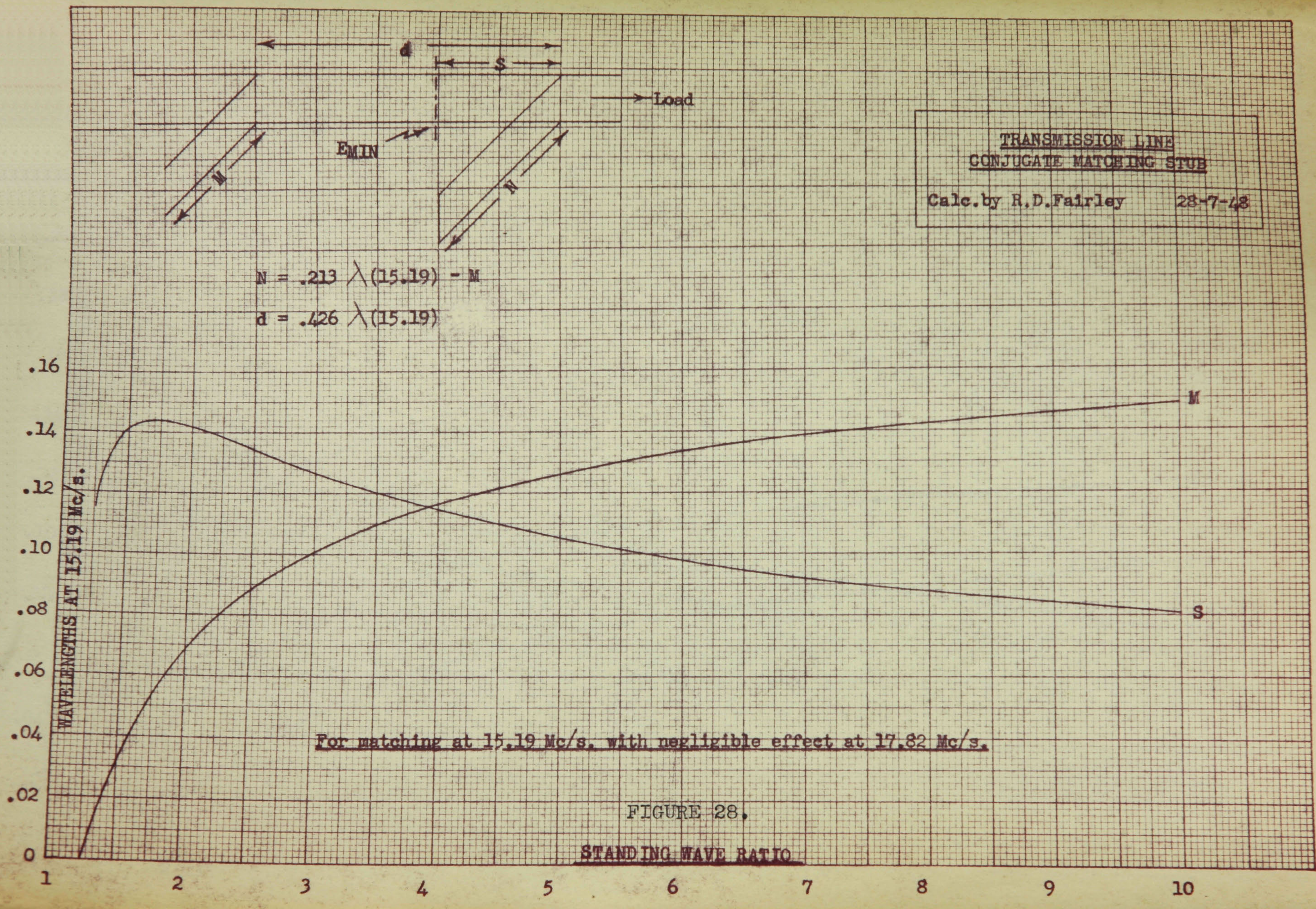
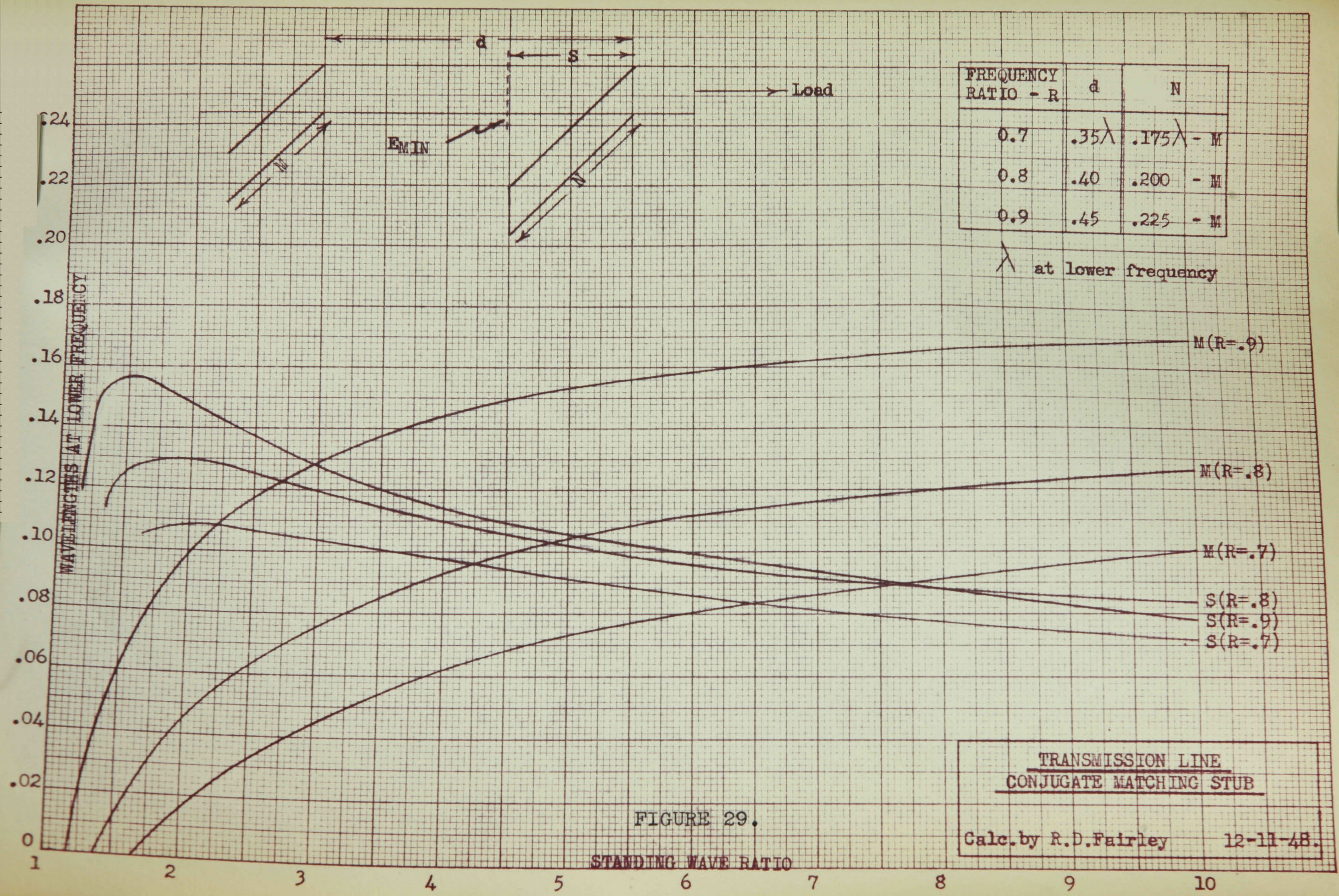

d
S
Load
EMIN
M
N
TRANSMISSION LINE
CONJUGATE MATCHING STUB
Calc. by R.D.Fairley 28-7-48
N = .213 λ(15.19) - M
d = .426 λ(15.19)
.16
.14
.12
.10
.08
.06
.04
.02
0
WAVELENGTHS AT 15.19 Mc/s.
M
S
For matching at 15.19 Mc/s. with negligible effect at 17.82 Mc/s.
STANDING WAVE RATIO
1
2
3
4
5
6
7
8
9
10

FIGURE 28.



FREQUENCY RATIO - R	d	N
0.7	.35λ	.175λ - M
0.8	.40	.200 - M
0.9	.45	.225 - M

λ at lower frequency

FIGURE 29.

TRANSMISSION LINE
CONJUGATE MATCHING STUB

Calc. by R.D.Fairley 12-11-48.

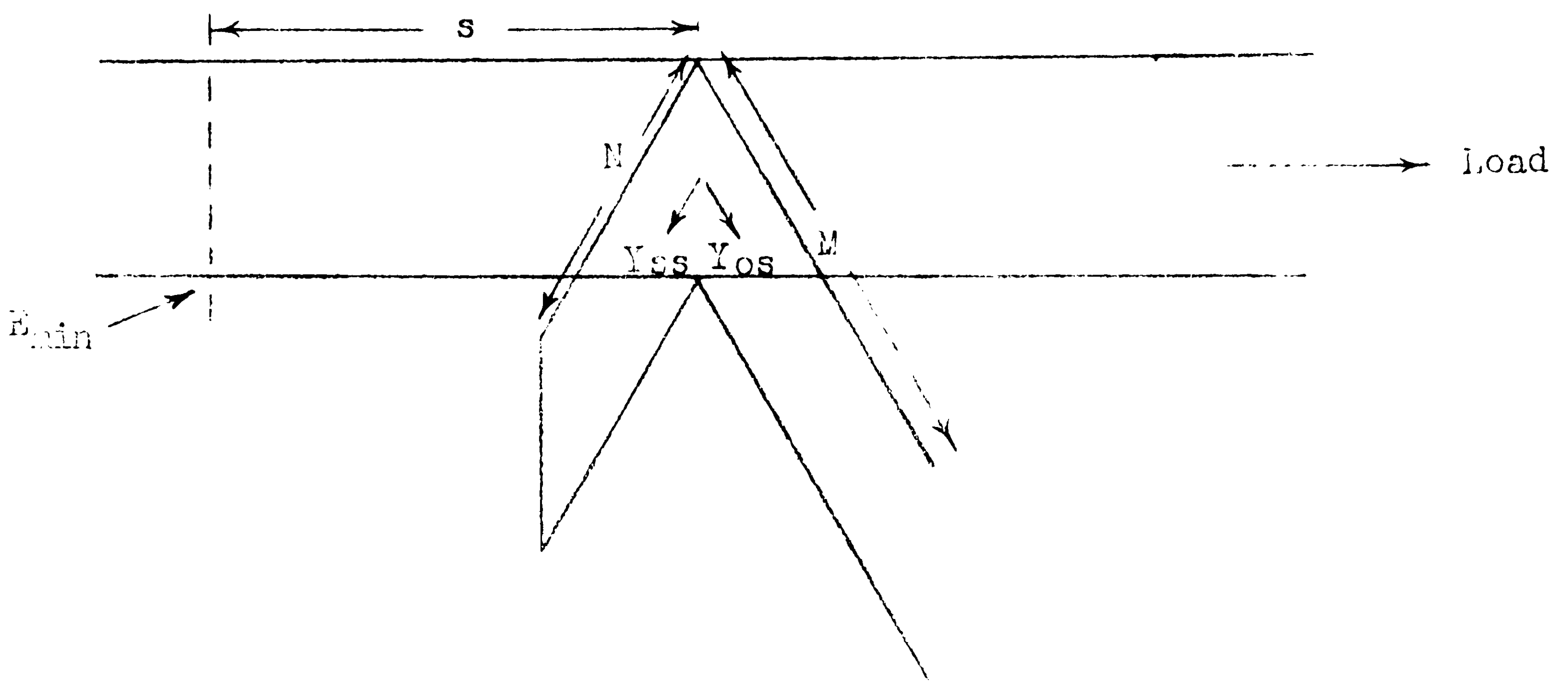


FIGURE 30.

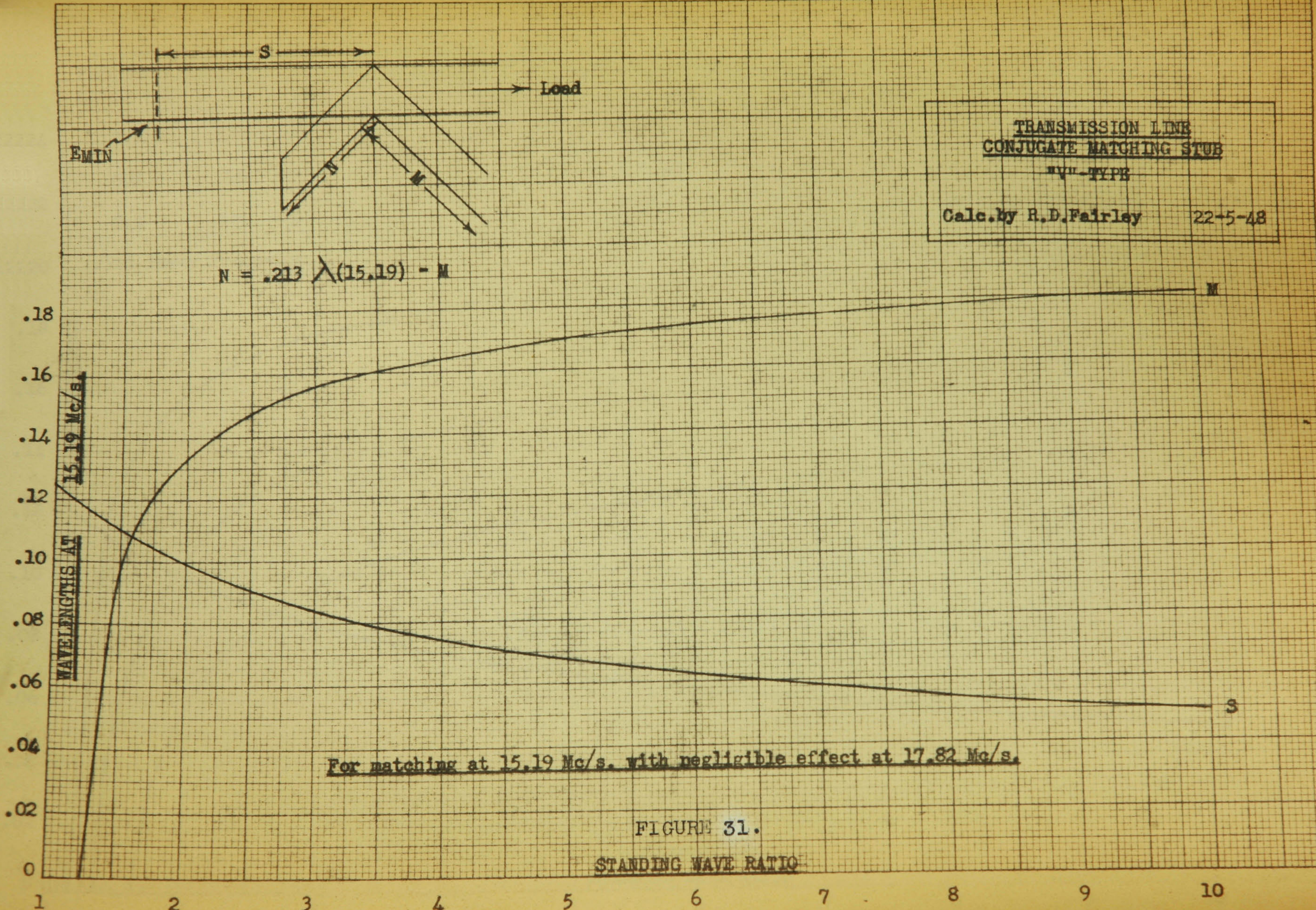

TRANSMISSION LINE
CONJUGATE MATCHING STUB
"V"-TYPE
Calc.by R.D.Fairley 22-5-48
S
Load
EMIN
N
M
N = .213 λ(15.19) - M
WAVELENGTHS AT 15.19 Mc/s.
.18
.16
.14
.12
.10
.08
.06
.04
.02
0
1
2
3
4
5
6
7
8
9
10
M
S
For matching at 15.19 Mc/s. with negligible effect at 17.82 Mc/s.
STANDING WAVE RATIO

FIGURE 31.

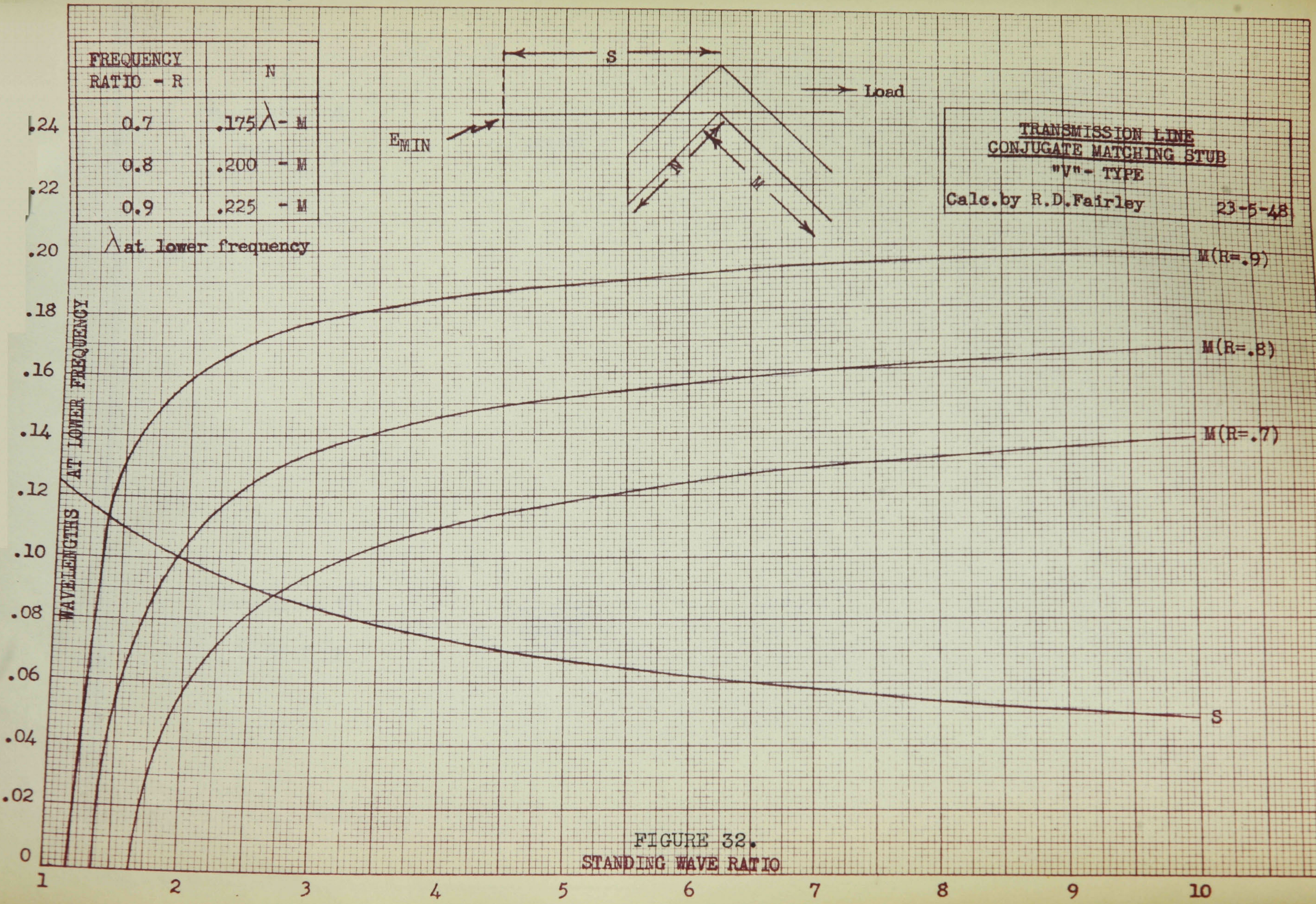

FREQUENCY RATIO - R
N
0.7
.175 λ - M
0.8
.200 - M
0.9
.225 - M
λ at lower frequency
S
E MIN
Load
N
M
TRANSMISSION LINE
CONJUGATE MATCHING STUB
"V"- TYPE
Calc.by R.D.Fairley
23-5-48
M(R=.9)
M(R=.8)
M(R=.7)
S
WAVELENGTHS AT LOWER FREQUENCY
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
0
1
2
3
4
5
6
7
8
9
10
STANDING WAVE RATIO

FIGURE 32.

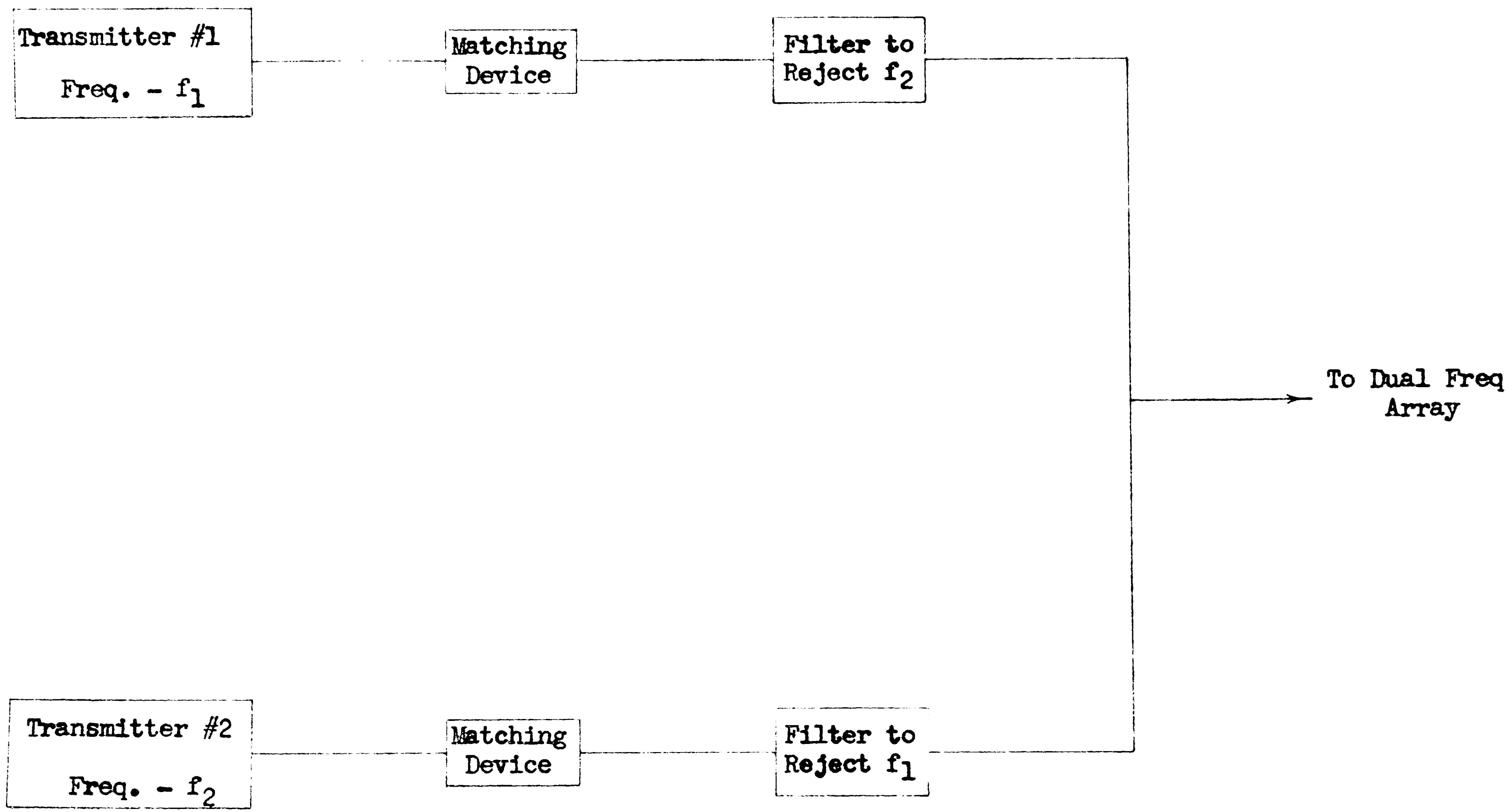
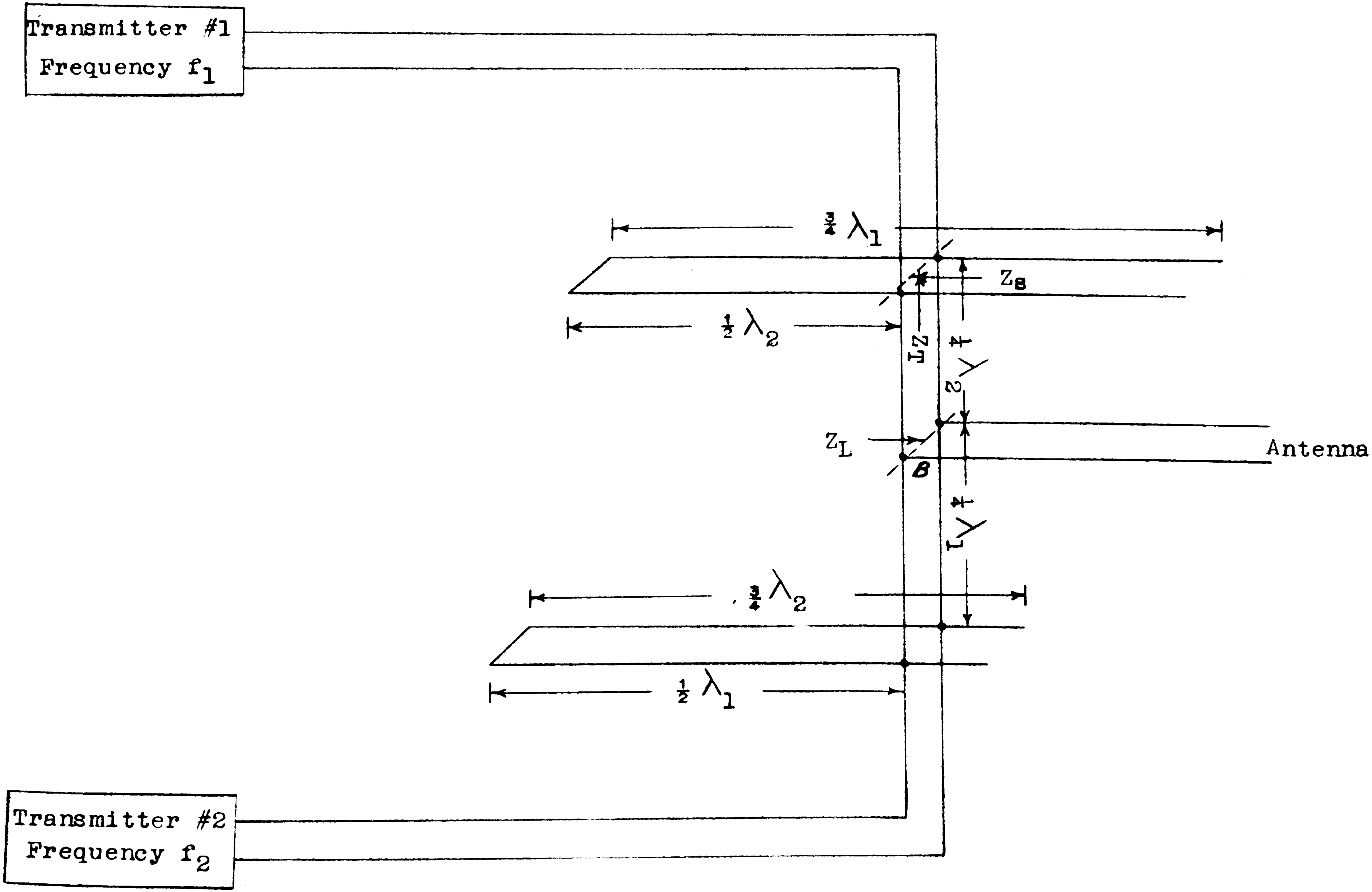

Transmitter #1
Freq. - f_1
Matching Device
Filter to Reject f_2
Transmitter #2
Freq. - f_2
Matching Device
Filter to Reject f_1
To Dual Freq. Array

FIGURE 33.



Transmitter #1
Frequency f1
Transmitter #2
Frequency f2
Antenna
ZL
ZT
Zs
B

Figure 34.

REJECTION CHARACTERISTIC OF RESONANT LINE SECTION

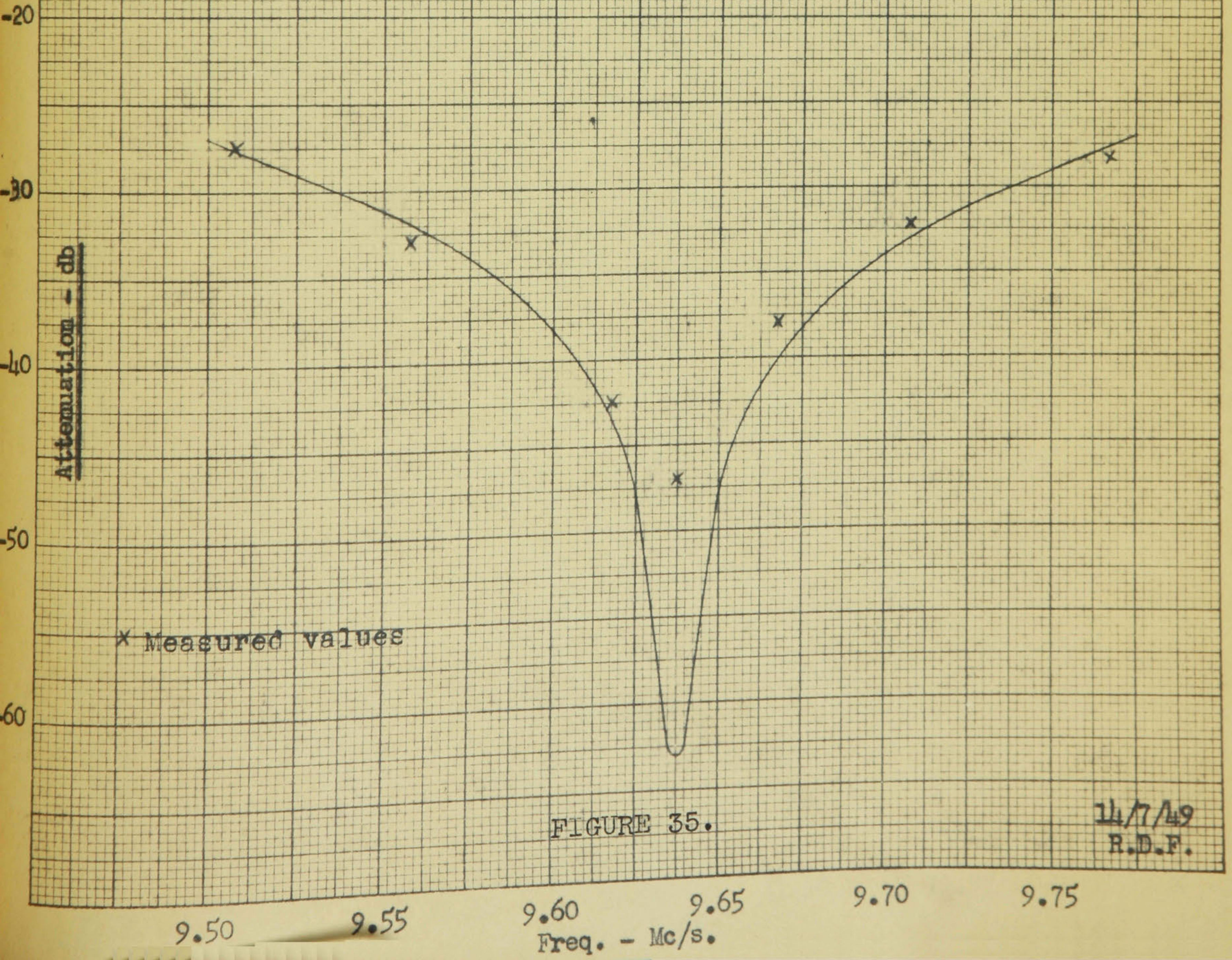
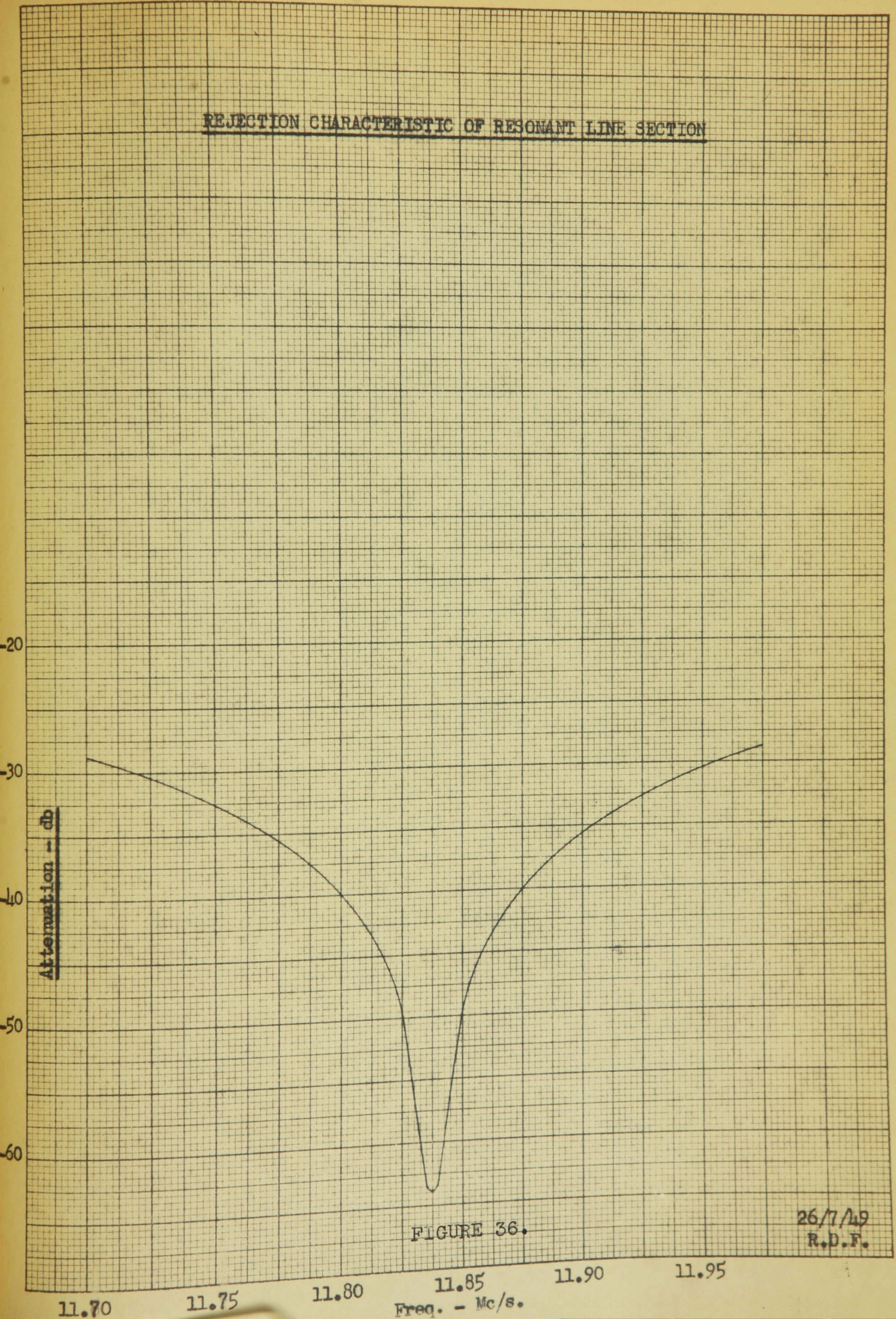


FIGURE 35.

14/7/49
R.D.F.



REJECTION CHARACTERISTIC OF RESONANT LINE SECTION
-20
-30
-40
-50
-60
Attenuation - db
11.70
11.75
11.80
11.85
11.90
11.95
Freq. - Mc/s.
26/7/49
R.D.F.

FIGURE 36.

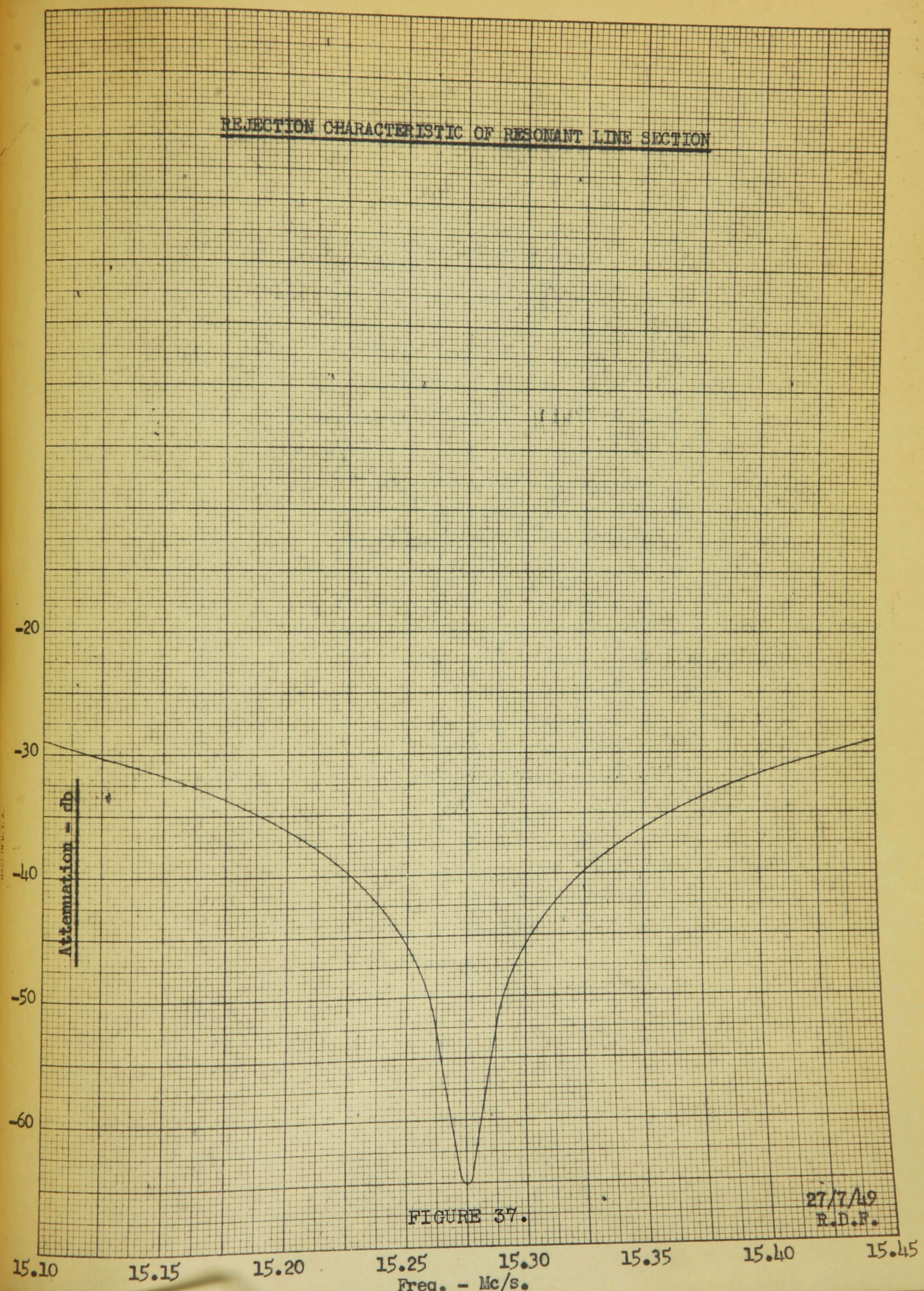


FIGURE 37.

27/7/49
R.D.F.

REJECTION CHARACTERISTIC OF RESONANT LINE SECTION

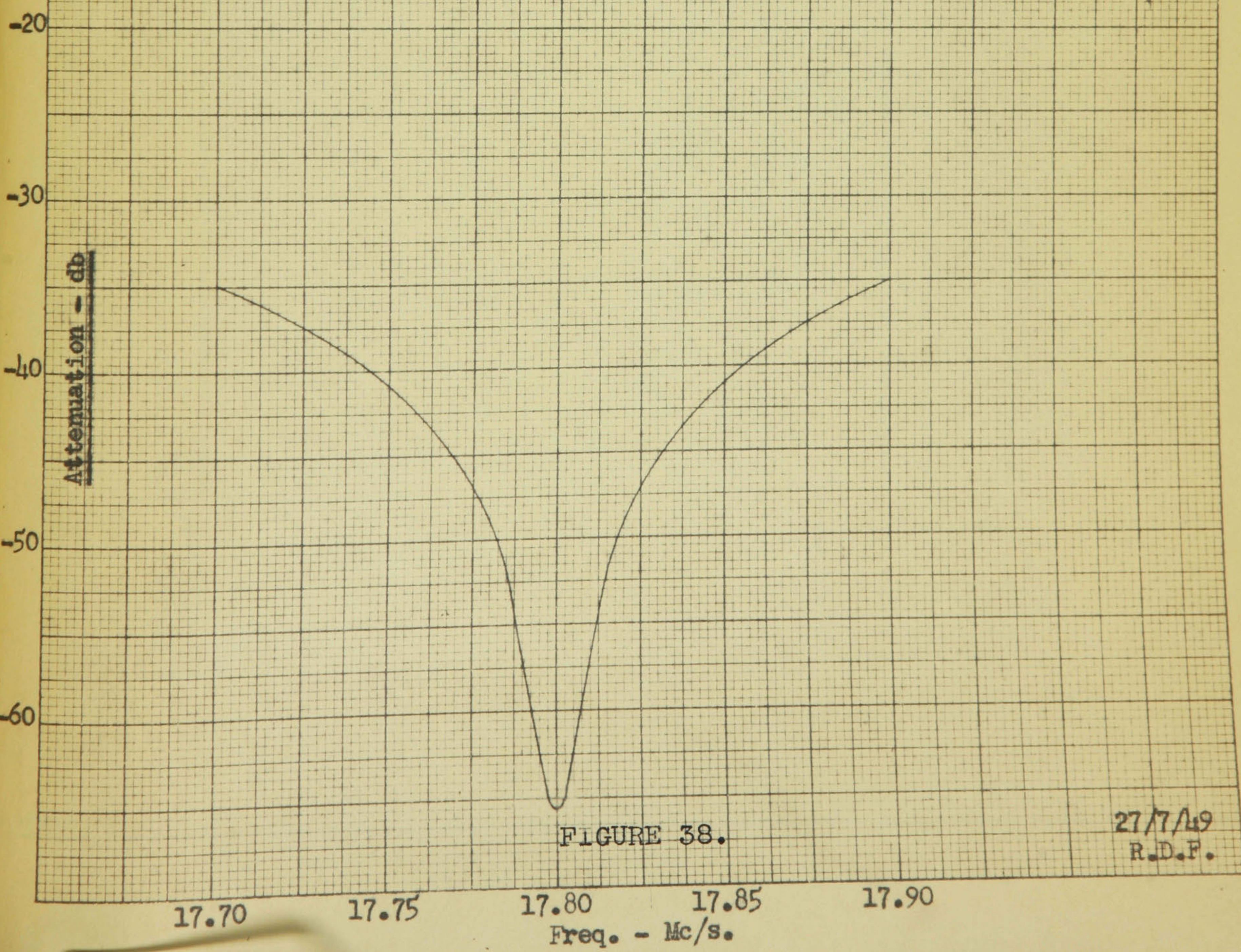


FIGURE 38.

27/7/49
R.D.F.

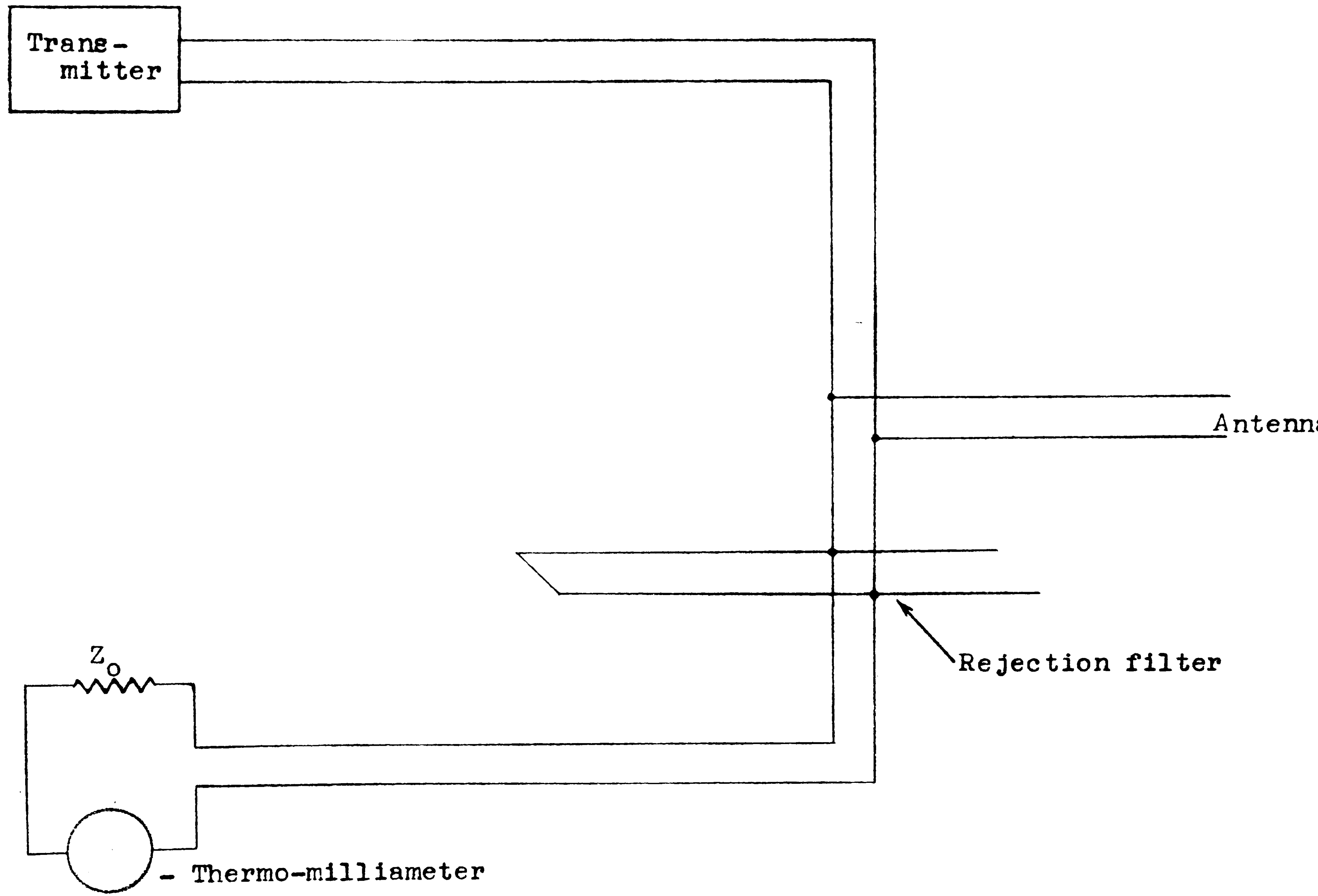

Trans-
mitter
Antenna
Rejection filter
Z_o
Thermo-milliameter

FIGURE 39.