Corporate portfolio management for sustainable healthy returns in the minerals industry

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Table of Contents

List of Figures

List of Tables

List of Abbreviations

AHP	Analytic Hierarchy Process
CDS	Credit Default Swap
ES	Expected Shortfall
EVT	Extreme value theory
MCDM	Multi-Criteria Decision-Making
MCS	Monte Carlo Simulation
NPV	Net Present Value
OECD	Organization for Economic Co-operation and Development
PG	Product Group
PROMETHEE	Preference Ranking Organization Method for Enrichment Evaluations
RL	Return Level
ROI	Return on Investment
TODIM	Acronym for Multi-Criteria Decision Making [in Portuguese]
TOPSIS	Technique for an Order of Preference by Similarity to Ideal Solution
VaR	Value at Risk

Abstract

The minerals industry has recently encountered significant price fluctuations leading to more business risk and unexpected overall returns on capital fund invested. This situation forces mining corporations to find new decision-making processes to improve productivity and efficiency in allocation or prioritization of business-related spending, including sustaining and working capital projects. This research aims to propose new portfolio management strategies to be used at the senior management level of global mining companies. Given that decision-making processes regarding a portfolio require risk management and diversification components, the main emphasis is on managing the trade-offs between risks and returns. Therefore, the effect of business unit performance of a project initiator, the country stability where the project will be implemented, the commodity market behavior under unexpected extreme events were reviewed and developed in the proposed portfolio optimization models. Quantification techniques were explored for portfolio optimization with operational performance, commodity market behavior, and international and country risks for extreme events. In addition, the phenomena affecting risk quantification such as reproduction of relationships between portfolio elements and reaction to unexpected cases were further embedded in the decision-making process. Risk indicators to be generated were used in the optimization process to maximize the return from a corporate portfolio while considering the risk-taking capacity of a global mining company. Ultimately, this research contributes to the development of effective and efficient portfolio management approaches, including prioritization of a weighted decision-making criterion in optimization models, such that mining stakeholders will benefit from optimal returns at an acceptable risk.

Résumé

L’industrie minière a connu de graves turbulences de prix ces dernières années. Ce qui oblige les entreprises minières à améliorer leurs processus de prise de décision de manière à inclure la productivité et l'efficacité de l'allocation ou de la priorisation des dépenses liées aux investissements, y compris les projets de remise à neuf et de maintien de fond de roulement. Cette recherche a pour objectif de proposer de nouvelles stratégies de gestion de portefeuille à utiliser par les cadres supérieurs des sociétés minières mondiales. Étant donné que les processus décisionnels concernant un portefeuille nécessitent des composants de gestion du risque et de diversification, l'accent a été mis sur la gestion du compromis entre risque et rendement. Les impacts de performance de l’unité d’affaire initiateur d’un projet à approuver, la stabilité du pays dans lequel le projet doit être implanter et le comportement des marches de commodité sous pression évènements extrêmes inopinés ont été revues et développés dans ce travail de recherche. En outre, les techniques d’évaluation de gestion optimale de portefeuilles liées à la performance opérationnelle, le comportement des commodités, les risques internationaux, la stabilité des pays pour des évènements extrêmes ont été explorées dans cette étude. En plus les phénomènes affectant la quantification ainsi que la relation entre les éléments de portefeuille et leurs impacts ont été inclus dans le processus de prise de décisions. Les indicateurs de risque à générer ont été utilisés dans le processus d’optimisation afin de maximiser le rendement du portefeuille de la société, ceci en tenant compte de la capacité de prise de risque d’une société minière mondiale. En fin de compte, ces travaux de recherche contribuent à la mise au point d’approches de gestion de portefeuille efficaces et efficientes, incluant la priorisation de l’importance des critères de prise de décision, dans les modelés d’optimisation de manière à ce que les parties prenantes des entreprises minières bénéficient d’un retour sur investissement optimal à un niveau de risque acceptable.

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I would also like to thank people from near and far who supported me during the realization of this thesis.

Finally, I take this opportunity to express my deep gratitude to my wife Francine, my daughters Tinaig and Maelys, and my son Anthony, who through their patience and understanding have allowed me to carry out these lengthy studies. This thesis is fully dedicated to my wonderful family. I would not be where I am without your full support and love.

Contributions of Authors

The author of this thesis is the primary author. This thesis comprises seven chapters written by the author. Chapter 1 introduces this research and highlights the main objectives with key contributions to original knowledge. Chapter 2 presents a review of the literature. Chapter 3 is an original peer-reviewed paper[1] published in 2019 in Mineral Economics and co-authored by Prof. Mustafa Kumral. The studies conducted in Chapters 4, 5, and 6 are conceived by the primary author and will be reviewed and revised for publication in peer-reviewed journals. The author conducted all data analyses and MATLAB programming of the proposed optimization models.

[1]Njike, A. N., & Kumral, M. (2019). Mining corporate portfolio optimization model with company's operational performance level and international risk. *Mineral Economics*, *32*(3), 307-315.

Chapter 1: Introduction

1.1 Sustainable healthy returns in the minerals industry

A fundamental principle in capital investment is that the decision to make a massive capital investment is often associated with the ability to quickly obtain an acceptable return on investment (ROI). Mining companies typically rush toward early production after intensive investment in the mine development to take advantage of high commodity prices (Rahmanpour & Osanloo, 2015). This early production helps companies benefit from uninflated construction costs and high profit margins.

Over the last 10 years, commodity price index has fluctuated dramatically (Figure 1.1). The index dropped almost 56% between 2011 and 2016. The same trend was experienced for the metal price index, with almost same drop of 56% (Figure 1.2). For a typical metal such as iron ore, a more drastic price drop occurred for the same period (Figure 1.3). The iron ore price has varied from a high value of US$188.90 per metric tonne in 2011 to a low value of US$38.54 per metric tonne in 2015 (IndexMundi, 2019). Compared to a decade ago, current commodity prices are low because supply exceeds demand (Ives, 2016). These low commodity prices combined with high operating costs mean that mineral resources companies do not consistently achieve high profit margins (Raffaini, 2016). High-cost producers are running out of cash and suspending operations, and low-cost producers are working to overcome this challenging market, though they face increased competition.

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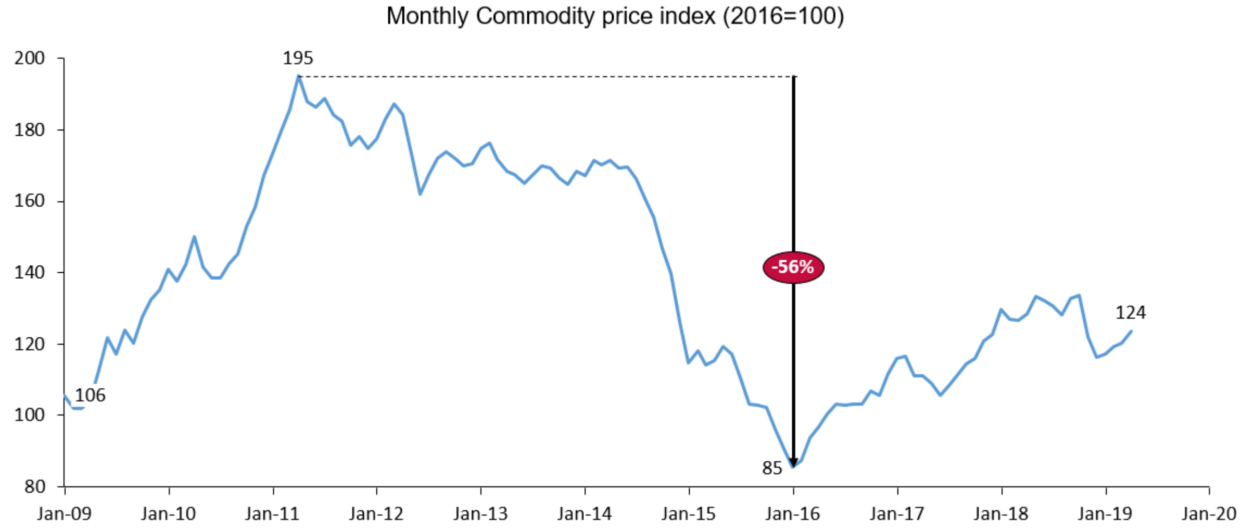


Figure 1.1: Monthly commodity price index (IndexMundi, 2019)

The commodity price index is represented by the weighted average of commodity prices based on spot or future prices. This fixed-weight index has three main categories, namely:

1- Metals: e.g., Base metals and precious metals
2- Energy: e.g., natural gas, propane, gasoline, oil or coal
3- Agriculture: e.g., Grains, Softs and Livestock

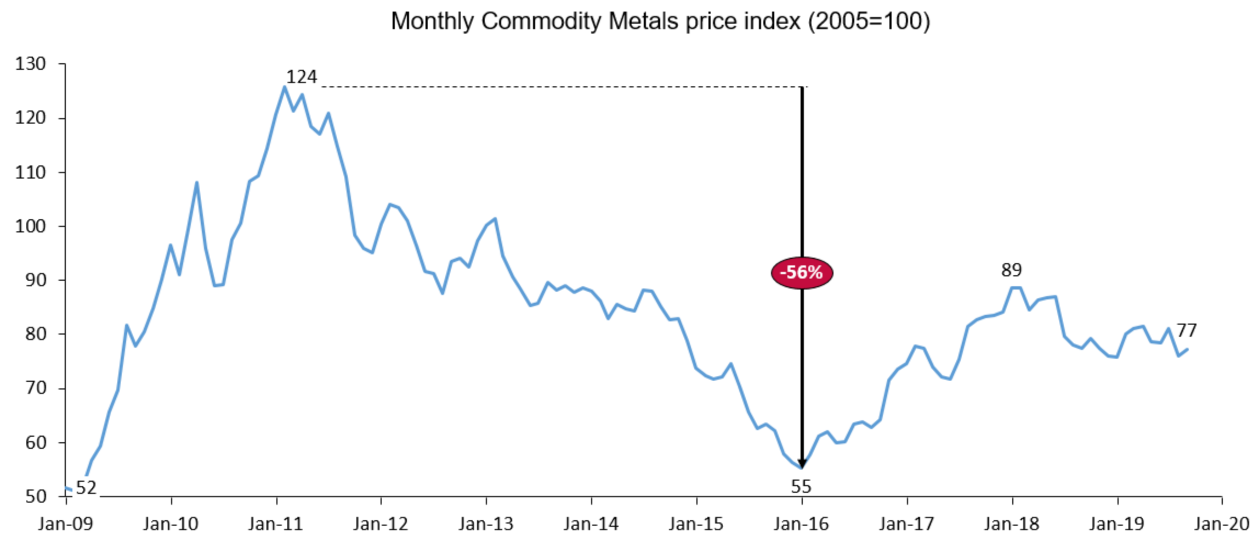


Figure 1.2: Monthly commodity metals price index (IndexMundi, 2019)

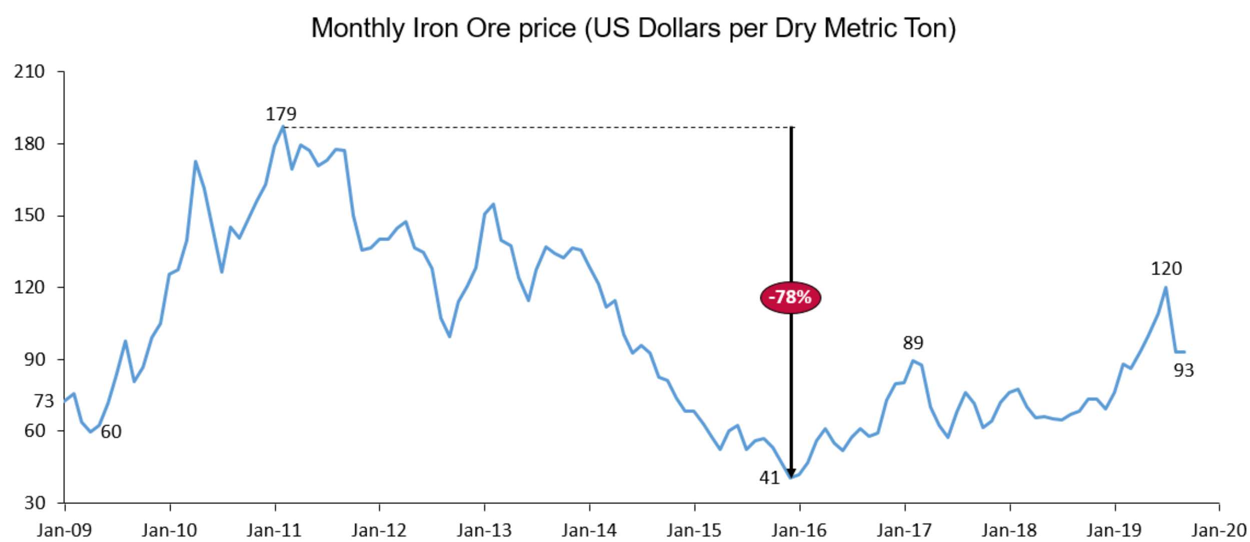


Figure 1.3: Monthly commodity iron ore price index (IndexMundi, 2019)

For company decision-makers, the challenge is to improve the effectiveness of current assets/resources/projects and better manage the risk to allocate capital investment to the most valuable initiatives. This challenge also deals with making decisions over time, which translates to a dynamic optimization problem: a decision made at one time affects future possibilities. It is critical to take this effect into account when making a decision today. The dynamic optimization model should consider the following characteristics of the minerals industry:

- Highly competitive, cyclical, and globalized
- Complex spot and future markets
- Ample room for speculation
- Tendency to be oligopolistic
- Highly sensitive to environmental sustainability and local communities
- Specific management and policy problems such as resource curse and the Dutch disease
- Significant technical, financial, political, environmental, and safety risks
- High sensitivity to economics of emerging countries
- Specific financing models

Considering these key economic, social, and environmental drivers, a sequence of effective decisions needs to be made to achieve the objective function. This will allow mining companies to obtain a safe (acceptable risk) and stable optimal ROI—also defined as a sustainable healthy return—which is a social, political environmental and financial thresholds where the companies' profitability and key stakeholders' benefit will always be greater than the company's closure value for continuing growth during any given period.

International corporations diversify their activities with regard to commodity type, sales agreements (e.g., spot or future markets), and production and sales in different regions of the world. This increases the resilience of corporations against extreme events or unexpected occurrences and generates sustainable healthy returns. For example, if the price of one product decreases, the corporation can yield sustainable returns with other commodities. Similarly, if one operation stops due to unexpected political events, the corporation will continue to see returns from operations elsewhere the world.

In addition to the diversification strategy, mine-specific improvements can increase the value of an operation. For example, replacement by new technology or new management/organization strategies or harnessing new operations research and computer science technologies for mine planning, design, and operation can provide new value to an operation.

Generating sustainable healthy returns depends upon risk diversification or mine-specific improvements, as well as also project portfolio management. The life of a specific mine is finite (Van Zyl, 2007), but the life of a global mining corporation is theoretically infinite: once one mine is closed, a new mine can be opened.

A global mining corporation holds a project portfolio containing groups of projects at different levels of development: licensing/permitting and exploration, development, production, and closure (IMF, 2016a) (Figure 1.1). Given that startup of a new mine takes several years, corporate decision-makers must manage the portfolio such that risks are mitigated and healthy returns are sustained.

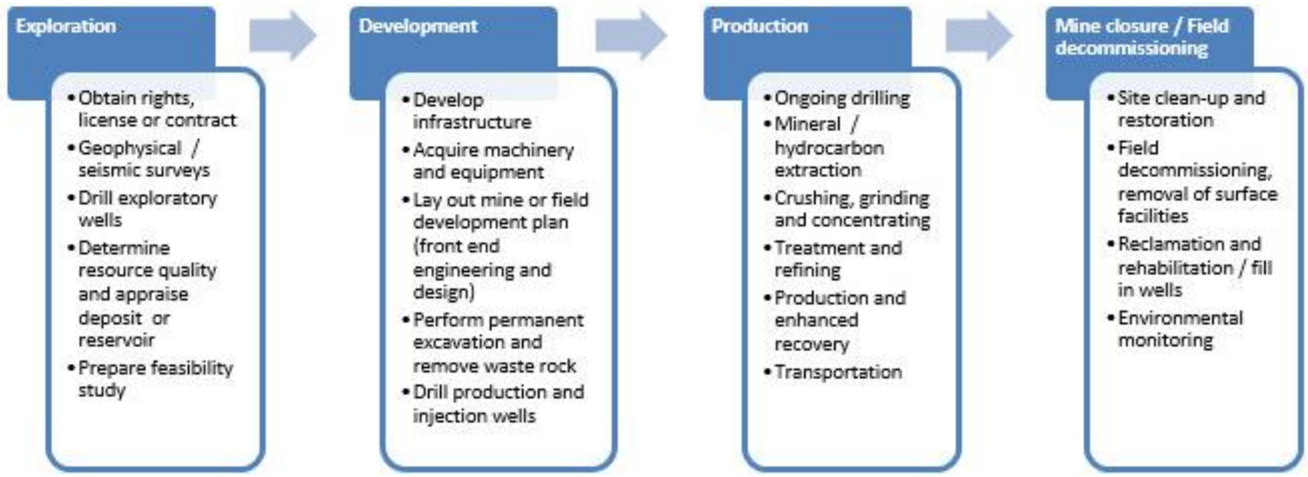


Figure 1.4: Stages in the life of a resource project (IMF, 2016b)

To sustain healthy returns, decision-makers must determine if projects: (a) proceed through the stages in Figure 1.1, (b) are taken out of the portfolio (e.g., stop exploration if there are no promising outcomes), or (c) are added to the portfolio through mergers or acquisition. The portfolio is dynamic, changing qualitatively and quantitatively over time, and it requires a complex decision-making process. Since the healthy returns also highly depend on the risks associated with the geographical location of the mining project, the sustainability of these returns depends on local communities and the indigenous people close to the mining zone. In search of social license to operate, various mining specialist teams will move across the indigenous lands at every stage of mine development, namely: geologists, surveyors, engineers, construction workers, maintainers and operators. Despite this diverse expertise, there is often a lack of an adequately consistent, solid strategy. The indigenous community on the other hand remains more consistent such that provide positive and negative feedback about the mining project. Thus, these feedbacks are ingrained in the communal memory. This is why a representative of the mining company negotiating a “single issue” will often comment during the process that the community keep complicating the conversation or adding issues or returning to the past. This is simply a reflection of the depth and interconnectedness of these accumulated narratives of their mining life experiences dating back to the first days of survey and exploration. Hence, the indigenous community and the mine's "license to operate" heavily affect the proposed corporate portfolio optimization models.

1.2 Indigenous communities and social license to operate

In the mineral industries, social risk or risk associated with social license to operate is seen as one of the most significant risk sources. Despite the major mining companies' several decades of implementing global communities and social responsibilities, community relations remain the single greatest threat to any project located on or in close geographic range of the indigenous population lands. What follows is an attempt to explain why the potential for conflict with indigenous communities remains a critical area of strategic focus within the industry. The challenge of ensuring the wellbeing of the communities and sustaining the healthy returns of the mining project seems to be a difficult task. The fact that interactions with indigenous communities will play out differently compounds the problem. For example, a mining company can achieve a productive community collaboration around an operation in one location, but fail and forcibly a similar operation in another location. The cultural, political, economic, geographic, historical and environmental variables are so specific to each indigenous community that although the lessons from one experience could relate to a new operation elsewhere, there is no guarantee for success. This uniqueness remains the heart of the sense of fragility and endangerment of indigenous communities. It drives the determination to preserve and protect a way of life for future generations, and affects the license to operate a mine. These social drivers heavily influence the sustainable healthy return of the mining companies.

1.3 Knowledge gap

In the mining industry, the current corporate project portfolio allocation approaches do not consider historical operations performance of the project initiators in the decision making process. Furthermore, country risk, which is a significant concern for multinational mining companies, in the capital allocation decision was not investigated. In addition, the effect of the poor consideration of commodity market behaviors under extreme events in the mining investment and divestment decisions was not sufficiently focused in the mining research. In this regard, this research proposes the integration in a single mining project portfolio model, the operations performance, the country stability and the commodity market behaviors.

1.4 Research objectives

This research project has two main objectives. The first is to develop portfolio optimization models with key sustainability and economics drivers of mining companies, integrating processes and synergies between operational performance, country stability, and commodity market behavior with capital investment decision. These models are practical, relevant aids to decision-makers in selecting profitable mining product groups (PGs)/commodity classes or mining projects with the dual achievement of maximum ROI and minimum risk. The second objective is to develop investment and divestment strategies to deal with uncertainties associated with project location and commodity type so that the corporation will be resilient to catastrophic events such as poor market conditions. Furthermore, as a part of the risk management strategy, the relationship between a country's stability and unexpected events within a portfolio is embedded in the investment decision-making process. Multi-national mining companies are not only producers but also players in the markets: many commodities are traded in derivative markets to assist mining company in hedging some of their risks. Therefore, this research investigates the effects of commodity market behavior on the capital investment decision.

This project uses the analytic hierarchy process (AHP) and technique for order of preference by similarity to ideal solution (TOPSIS) to prioritize the key decision-making PGs preference under the same portfolio. The Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE), an additional MCDM alternatives ranking method, is used to validate the TOPSIS results. Evaluating operational performance will identify key aspects of the effectiveness of current and future developments of mining projects or commodity classes under the same portfolio. The overview of the portfolio will include a cost-effective mining scheme and procedures for rational evaluation of uncertain projects (Montiel, et al., 2016). Knowing that every orebody is different, optimization model development takes into account specific conditions at each mine. Considering the high sunk costs of the mining industry and assuming the large scale of mines requires continual injection of capital, this research will contribute to improving companies' capacities through guiding decisions to invest or improve. To minimize the risk to invest and maximize profitability, decision-making procedures associated with capital investments in a risk environment are proposed.

This research solves a portfolio optimization problem that helps mining companies make realistic optimal investment decisions to maximize ROI and minimize risk, which facilitates achieving or sustaining their full potential. The developed portfolio optimization models help mining corporations to effectively and efficiently allocate capital funds at the right time to the right PG. The proposed portfolio optimization models help mining corporations respond to high risk exposure business and invest capital funds for safe and stable profitability.

The input data used throughout this research are based on expert judgment, experience, and current top mining corporation trends, which are available on the internet. However, all data in this research are hypothetical. Any assumption in this thesis does not refer to any specific mining company.

1.5 Original contribution to knowledge

Unlike traditional net present value (NPV) or ROI methodologies, this thesis develops optimization models that consider criteria such as the operational performance of the initiator of the capital fund approval request and risk associated with the geographical location of the investment. These additional criteria are key quantitative elements in the decision-making process for capital allocation to a PG/business unit. By considering commodity market behavior during extreme event conditions, more realistic capital allocation can be achieved within the corporate portfolio of multiple PGs. Contrary to current mining corporate portfolio management trends, funds are allocated with the basic principle of the non-utilization of all available capital funds.

This research solves the optimization problem for corporate portfolio management in market conditions at high or low commodity prices with investment and divestment decisions in specific countries. The developed models include multiple PGs within the same corporate portfolio and provide more realistic capital expenditure decisions in addition to key economics drivers.

This study provides the mining industry with improved knowledge and understanding in modeling capital portfolio allocation with enterprise-wide risk management. Given that the mining industry has a low credit rating because of high risks, this research produces mining risk management practices that can achieve a sustainable financial benefit for the company and associated communities. More specifically, mining corporations will have new tools for investment or divestment decisions, shareholder strategies, merger and acquisition negotiations, public offerings, environmental permits and management, taxation and royalty policies, the sustainability of local economies, and evaluation of business performance. The research output facilitates the development of mining engineering strategies against the backdrop of technical, political, environmental, unexpected, and extreme events. It introduces more realistic project valuation methods that consider the cyclical and volatile nature of commodity prices and the mining business in general.

Financial institutions will have new approaches to assess mining projects and more opportunities to fund projects. Given that mining corporate portfolios include licensing/exploration, development, production, and closure projects that are diversified in terms of geography, commodity type, environmental sensitivities, and integrated value chain, coordinating risk among different types of projects/processes allows determining the right investment and operation strategies with the weighting of key optimization criteria.

Mining companies run in a dynamic operational mode in the face of many uncertainties and the random behavior of currency prices, stocks, commodity prices, and resources. Integration of these random variables, which characterize the stochastic environment, into this project generates four optimization models across the value chain of a global mineral resource company.

1. The first portfolio optimization model simulates the operational performance of the business unit and country risk related to the investment.
2. The second optimization model incorporates operational performance, country risk, commodity market behavior, and investment/divestment decisions for extreme events—

characterized as a drastic drop in the commodity price or an unusual turnover rate within a very short period due to a catastrophic failure of a key asset within one of the top commodity producers. An unpredicted change into government policy within the world's top three biggest economies is also considered in this model.

3. The third model is an integrated optimization model that includes production efficiency across the operations value stream, operating cost, country stability, and unexpected events with a direct relationship between countries and unexpected events.
4. The last model reinforces the importance of criteria weights in the capital allocation fund with the inclusion of AHP, TOPSIS and PROMETHEE methodologies in the weightage criteria, alternatives ranking and portfolio decision-making process. The validation of the alternatives ranking results also provide a more realistic capital allocation fund within the whole portfolio.

1.6 Outline of the thesis

Chapter 1 provides a brief introduction to this research with objectives and contribution to original knowledge.

Chapter 2 is a literature review of capital budgeting decision methods, portfolio management, risk management, and dynamic optimization, highlighting Monte Carlo simulation and multi-criteria decision-making (MCDM).

Chapter 3 proposes a new portfolio management strategy that helps mining corporations improve decision-making processes associated with capital allocation to proposed projects within a turbulent environment. The proposed approach considers operational performance when prioritizing business-related spending on capital projects. The problem is formulated as the minimization of risk at the desired ROI under the constraints of the operational performance requirement of the project initiator (i.e., PG initiating the project). Results show that, in addition to the NPV criteria, the more diverse the portfolio, the greater the potential increase in the corporate portfolio ROI. Further, as the performance of the PG increases, so too does the number of approved projects at the corporate level.

Chapter 4 proposes a portfolio optimization model—solved using MATLAB programming—to address the global economic downturn coupled with the current uncertain atmosphere surrounding the prices of the commodities, both of which have negatively affected the growth of the mining industry. The model will help mining corporations improve decision-making procedures under extreme events so as to consider the global economic downturn in the investment or divestment of a portion of the corporate portfolio. This chapter proposes new portfolio management strategies to be used at the executive level of global mining companies. It identifies the most valuable PG in which to invest and the low-value PG to divest at the efficient frontier.

Chapter 5 develops a model to help a mining corporation face the high pressure from their intensive capital funds requirement. Combining commodity market behavior with country stability and a correlation with unexpected events heavily influences the distribution of capital funds within the corporate portfolio. In this chapter, the quantification of the country risk is discussed in detail along with different approaches. This chapter highlights the fact that, to be more realistic, the investment or divestment decision-making process should always consider the current operational performance of the capital fund initiator, the commodity market behavior, and the country stability in addition to the traditional financial criteria of NPV, internal rate of return, ROI, profitability index, and payback period.

Chapter 6 compares and weights all the decision-making criteria used in the previous chapters. The decision to invest in a project has typically been based on known financial decision criteria, without considering a systematic approach to weight them. The AHP decides the weight of each criterion and the TOPSIS and PROMETHEE II techniques are used to rank the projects. They also serve benchmarking each other. . The preferred alternative is selected and used to allocate capital funds among multiple PGs of an existing corporate portfolio.

Chapter 7 provides the conclusions and proposes future work.

Chapter 2: Literature review

2.1 Investment committees in mining organizations

Multi-national mining organizations often comprise multiple product groups (PGs), which in turn comprise multiple business units. At each level, investment committees play a prominent role in making investment decisions. Investment committee members are very often the most senior executives at that level. They follow an established decision-making process centered on the nature of the investment, the impact on safety/people, and the financial profitability.

The level of approval associated with each investment committee is defined by the amount of capital funds required. For a relatively small investment (generally less than US$2 MM), final approval is usually granted at the business unit level. For larger investments (e.g., US$2–5 MM), the final decision is often made at the PG level. For investments greater than US$5 MM, the ultimate decision-making is often done at the corporate level, though approval at the business unit and PG levels is required before moving the project to the level above.

The fundamental financial question decision-makers need to answer before approving a project is how quickly they can get the capital funds back if the project is approved and implemented as scheduled. Capital budgeting decisions use financial analysis tools (Andrew, 2016) such as net present value (NPV), payback period, profitability index, internal rate of return, return on investment (ROI), and accounting rate of return (Sekhar, 2018). All of these methods use a "snapshot" of the current knowledge of the condition to provide the analysis. In reality, outcomes are only valid if compared to benchmark financial data.

Further, project implementation must proceed through multiple critical milestones that involve several known and unknown criteria that could invalidate the initial hypothesis behind a given financial analysis. These criteria include:

- the capability of the project owner (their operational performance);
- the country risk (defined in section 2.3) where the project will be implemented,
- the commodity market behavior; and
- potential extreme events.

These criteria could heavily affect the investment decision-making process. Unfortunately, in many mining organizations, they are not consistently quantified and embedded in corporate portfolio management.

2.2 Mining corporate portfolio management

Mining companies typically have a project portfolio for each stage in the life of the resource project: licensing/exploration, development, production, and closure (see Figure 1.1). Production of mineral resources and supply requires creation of a value chain, which has a series of sequential processes (e.g., drilling, blasting, loading, hauling, blending, concentrating, railing, and ship-loading) to convert raw materials—varying qualitatively and quantitatively—to an intermediate product in a controlled fashion. Mineral resource companies with a sophisticated portfolio structure encounter serious risks associated with investment strategy, organization, portfolio management, and operational decision-making. The mining industry is known to be capital-intensive due to the high level of associated risks (Rudenno, 2012), yet current portfolio management methodologies, technologies, applications, and standards are not adequate.

Corporate portfolio management is defined as the centralized management of a group of programs, projects, and operations to achieve corporate strategic objectives. One of the key challenges is to adequately allocate resources to achieve the business plan in alignment with the overall corporate strategy (PMI, 2013). Mining corporations target a combination of multiple commodity-markets, thus mining corporate portfolio management is also known as management of multi-diversified mining programs/projects issues from mergers or acquisitions (Nippa, et al., 2011).

Effective portfolio management is a formalized and standardized process with facilitation thinking, including integration analysis of all individual strategic business units (Pidun, et al., 2011). Three key elements of effective corporate portfolio management for corporate-level decision-making should be considered in any business analysis (Pidun, et al., 2011):

1. market-based view, defined as the market attractiveness and competitive position;
2. value-based view, defined as the current value and anticipated financial returns; and
3. resource-based view (also called the parenting advantage), defined as value creation by the parent company.

Key performance indicators of the corporate portfolio are measured against corporate goals; this is built around value creation, the balance along with the cash generation versus cash use, the growth versus the profitability, and the risk versus the ROI (Pidun, et al., 2011). Bowman and Ambrosini (2000) defined two concepts related to value creation:

1. The "use value" is a subjective and individualist term referring to customers' perceptions of the quality of the product in relation to their needs (Bowman & Ambrosini, 2000).
2. The "exchange value" is the monetary gains realized when goods are exchanged at a given point in time (Velamuri, 2013).

A company is considered to generate cash (to be profitable) when it makes more money than is used for investment and for payment of all the other business costs (Times, 2016). Profitability permits company growth. Hafid (2016) demonstrated a strong correlation between growth and profitability. From the perspective of company shareholders, profitability is a measure of how effectively resources are deployed and turned into economic value. A financial return is measured as a ratio of outputs (revenues) to inputs (costs). To increase returns, companies can take on more risk—variously defined as the combination of threat and opportunity spread around expected negative or positive returns (Ulrich Hommel, 2012) or the effect of uncertainty on a desired or expected result (Luko, 2013). It is therefore a fundamental premise of good corporate management to understand how to effectively manage risk.

2.3 Risk management

Mining corporation portfolio management involves several quantitative and qualitative attributes associated with the project, the business unit (originators of the project), and the country where the project will be implemented. These attributes are assigned numerical values so that they can be incorporated into a unified risk management framework. Risk models are developed using various methods:

- variance,
- semi-variance,
- mean absolute deviation,
- variance with skewness,
- microeconomic risk analysis,
- probabilistic absolute deviation, and

- probabilistic mean-variance.

In all portfolio management activities, it is critical to know how objectives can best be met with a certain degree of confidence (Iverson, 2013). ISO 31000 (2009) defines risk management as "the systematic application of management policies, procedures and practices to the tasks of establishing the context, identifying, analyzing, assessing, treating, monitoring and communicating." Portfolio risk management requires identifying key risks faced by the portfolio and how best the identified risks can be analyzed, evaluated, treated, monitored, and reviewed (Purdy, 2010). For any portfolio risk management, Iverson (2013) considered fund, strategy, implementation, and review as the four key decision levels associated with multiple decision types, which are aggregated into key areas of risk:

- Governance risk is associated with the lack of effective oversight and decision-making processes (Tarantino, 2008).
- Asset allocation risk is associated with the choice of asset class mix (Gibson, 1996).
- Timing risk is associated with deviation from a future prediction relative to the strategic asset allocation (Frenkel, et al., 2004).
- Structural risk is associated with interrelations of portfolio elements that may not function as initially envisaged (Fight, 2005).
- Manager risk is related to the implementation of asset allocations and classifications.
- Implementation risk is associated with implementation of the investment decisions.
- Monitoring risk is associated with the performance review system.

Given that these key risk categories are interrelated, the difficulty is to continuously make the right decisions for a diversified project portfolio.

The risk categories defined by Iverson (2103) are valid for portfolio management within the same country or political, social, and economic environment. For global mining companies, decision-making related to a dynamic optimization problem must account for additional risks – country, sovereign, political, social and environmental risks. Wagner (2012) categorized firm-specific and country-specific political risks. These two categories are also included in the transactional risk, defined as the combination of country, sovereign, political, economic, financial, environmental, and social risks that need to be considered when engaging in an international investment. The combination of risks associated with payment and investment in a foreign country is also referred to as the country risk denotes any changes in a business environment that can adversely harm the financial value of assets in a foreign country (Herring & School, 1986). For international transactions, Wagner (2012) also defines three main categories of the risk of investing in a country:

1. Basic country risk is the likelihood that a foreign country may not fulfill its obligation and liabilities towards the lender. This risk is associated with specific characteristics of a country including money and fiscal polies of its central bank.
2. Sovereign risk is the likelihood that a foreign central bank of the host country will amend its regulations in order to reduce or invalidate the value of foreign exchange policies in relation to its contracts. It is a risk for a country to default on its commercial debts' obligation and liabilities.
3. Political risk is a likelihood that a country will assert control for natural resources management, policies or ownership for strategic reasons. It is mainly associated with resource nationalism issues. It is encountered when political and social development in the host country impacts the value of foreign investment or the repayment of cross border lending (Campisi & Caprioni, 2016).

In addition to these three international risks, community, indigenous groups as well as international Non-Governmental Organization can create other risks, such as social and environmental risks.

- Social risk is the likelihood that the community relations affect, disrupt or even stop mining operation. Social risk is seen as one of the top risk types in the mining industry. This special risk needs specific expertise, which is highly demanded in the industry.
- Environmental risk is the likelihood that business activities will adversely affect the environment and the living organisms.

Country risk, social risk and environmental risk play a prominent role in mining investments. The country stability and location where the investment will made determines these risks. As well as occurrences in global financial markets, country stability could also be correlated with the number of unexpected events (e.g., earthquake, flood etc.) associated with the country. These extreme cases may also have serious impacts on the mining portfolio management.

2.4 Extreme value theory (EVT)

In the financial engineering context, an extreme event refers to sudden and large turmoil in the financial markets and characterized generally by extreme or abnormal price fluctuation. In this research, the extreme events associated with financial market extremes are considered. These extreme events could also be associated with natural disasters, which is beyond this research. With the significant market instability, multiple studies using extreme value theory model the extreme events. Gilli and Këllezi (2006) applied the extreme value theory to assess the probability of extreme events and, therefore, the impact in the financial risk portfolio. The estimation of extreme quantile measurement of the financial risk corresponds to the measure of the value-at-risk, the expected shortfall and the Return level.

2.4.1 Value at Risk (VaR)

The VaR can be used to measure the maximum potential loss in a specific period due to the market risk. It is defined as the necessary amount to offset the corporate portfolio's loss over a specific period (Gilli & Këllezi, 2006).

$$VaR = F^{-1}(1-p),$$

Where F^{-1} denotes the quantile function characterizing the inverse of the distribution function F at the p^{th}quantile of F.

2.4.2 Expected Shortfall (ES)

An expected shortfall is utilized as a risk measurement tool. In EVT, it is the expected loss quantity surpassing VaR. In other words, it represents the value of an investment positioned in the worst-case scenario, which is a 100% loss of the investment. It is the average of losses greater or equal than Value at Risk (Rocco, 2014).

ES estimates the size of the corporate portfolio's loss exceeding the VaR.

$$ES = E(X|X > VaR)$$

Where *X* represents the random variable.

2.4.3 Return Level (RL)

RL is the measure of the maximum corporate portfolio's loss.

$$R_l^t = H^{-1}(1 - \frac{1}{t})$$

Where R_l^t denotes the return level value expected to be exceeded in one out of t period of length l. H is the distribution function of the observed maxima in a consecutive non-overlapping period.

The investment banks use the extreme value theory to evaluate: (a) the expected loss defined as the known loss that can arise when undertaking a specific business. (b) the unexpected loss, which is the uncommon and predictable loss the investment bankers can absorb in the normal circumstances. (c) the stress loss, or the loss associated with improbable and possible extreme scenarios, in which the investment bankers could still afford (Embrechts, et al., 1999). To offset the credit risk, the investors usually transferred the credit exposure between the two parties (Lender and borrower or buyer and seller); this type of contract or financial derivative is called credit default swap (CDS).

Longin (2016) defines an extreme event as an extreme abnormal statistical market behavior of a commodity price due to either:

1. an unpredicted change in government policy within the world's five largest economies (the United States of America, China, Japan, Germany, and the United Kingdom) or
2. a catastrophic failure within one of the world's five largest mining organizations by revenue (Glencore, Rio Tinto, BHP, Vale S.A., and Jiangxi Copper Corporation Limited).

Some technical unexpected events lead to financial unexpected events. An example of the latter is the catastrophic Vale tailings dam failure in Brazil in 2019, which initiated a disequilibrium in the iron ore/ pellet supply versus demand, thus, the sudden increase of iron ore price. As a result, the iron ore price during the first half of 2019 increased by 60% (TradingEconomics, 2019). Modern techniques consider these extreme unexpected events in portfolio optimization models.

2.5 Modeling a mining corporate portfolio

As noted above, a global mining company holds a portfolio containing projects in various stages (see Figure 1.1). This portfolio has the following dynamic characteristics:

- new licenses can be obtained,
- new projects can be added through mergers and acquisitions,
- projects move among stages,
- existing mines can have expansion, replacement, or new technology projects, and
- projects are removed from the portfolio through sells, abandonment, or closure.

All these characteristics affect the decision-making approval process, which must proceed such that the solution of the company's objective function will ensure a positive financial threshold related to sustainable healthy returns. A corporate portfolio management model is considered to be successful when it routinely picks the best projects and abandons inappropriate projects (Rad & Levin, 2006).

Xue et al. (2014) proposed a portfolio management model that can achieve both the trade-offs between returns and risk tolerance through the NPV and the operational premium of the underlying oil and gas projects. Although the performance of the project is included in the model proposed by Zhen et al. (2014), a limitation resides in the use of deterministic indicators such as NPV, which do not illustrate the practical relevance of dynamic operations management. Lopes and Almeida (2013) proposed a multi-attribute decision model to aid selecting mineral resources projects, including the stochastic and multi-objective nature of the decision context, an assessment of project synergies, and the influence of synergies on decision-makers' preferences. Quantification of project performance is not included in the Lopes and Almeida model. The present research will propose a new approach to aid selecting mining projects that will include not only quantification of project performance, but also the performance of current assets/resources in combination with project risk.

A literature review and critical analysis by Choi, et al. (2016) looked at enterprise risk management models given certain critical risk factors. Model misspecification occurs when the investor has a specific model in mind that might be mis-specified. Hence, the true model (which the investor cannot detect) lies in a set of alternative models (obtained by perturbing the reference model), which are statically close to the reference model. Olson and Wu (2017) used a set of stochastic functions (Corless, et al., 1993) to represent and model optimization processes. A similar approach was developed by Fouque, et al. (2017) using perturbation methods to drive volatility with two factors, one on a fast timescale (singular perturbation problem) and one on a slow timescale (regular perturbation problem). These studies did not embed the previous, current, and future state of the variables related to the approval decision-making process of portfolio optimization.

Behboodi, et al. (2016) looked at portfolio optimization from the power system perspective of optimal renewable energy asset integration given demand response resources. Using numerical methods, they obtained the optimal mixtures of renewable generation and demand response resources given a fixed portfolio of conventional generation assets, wind patterns, and energy use. Their model incorporated production, uncertainty, and emission costs, as well as capacity expansion and mothballing costs and considers wind variability and demand response impacts to determine the hourly price of electricity delivery. This study focused on supply versus demand with no consideration of the performance of the project initiator (i.e., the business unit initiating the project).

Solares, et al. (2019) provided a portfolio optimization method to handle uncertainty with confidence intervals. They represented uncertainty surrounding a portfolio with its volatility, modeling the risk and subjectivity of the decision-maker in terms of significant, probabilistic confidence intervals. They performed optimization using a decomposition algorithm, which deals with the second stage of portfolio optimization that begins with beliefs about future outcomes and ends with the choice of objects and the proportion of resources allocated to each of them. Although this study embedded future uncertainty in the optimization model, evaluation of extreme events in terms of the performance of the project initiator was not considered.

Saglam and Benson (2019) used the Markowitz (1952) mean-variance model for a single- and multi-period portfolio optimization with cone constraints and discrete decisions to provide investors with a balance between risk and ROI to solve the single period and multi-period portfolio. Their model incorporated transaction costs, conditional value-at-risk (VaR) constraints, diversification-by-sector constraints, and buy-in thresholds. They focused on maximizing the ROI for an accepted risk. The single and multi-period portfolio optimization is classified as a mixed-integer nonlinear programming problem solved as mixed-integer second-order cone programming. Although the objective function of their model is similar to what is being studied in this thesis research, the previous state variable combined with the extreme events was not considered.

Lejeune and Shen (2016) proposed models from a derived new mixed-integer linear program that were either equivalent reformulations or inner approximations of multi-objective probabilistically constrained programs. A balanced weighting objective between reliability, cost, or revenue was demonstrated. A downsize risk measurement was conducted for the optimal investment decision. Integrating specific risk such as country risk in their optimization model could have provided a more practical view of the decision-making process.

Kaplan, et al. (2018) solved a stochastic model for portfolio loss over a given time period by estimating the economic capital using Monte Carlo simulation (MCS) methods. The economic capital was characterized by the mean-adjusted VaR to evaluate the level of capital needed to protect against unexpected losses. Samples were analyzed separately using cumulative distributions and quantile deviations from the mean to estimate the economic capital, which can lead to much smaller variance. Nevertheless, no particular optimization of a portfolio was demonstrated.

Forsyth, et al. (2019) also use MCS methods for optimal controls to evaluate performance metrics. They compared portfolios with high and low risk of depleting savings that should be gained from the portfolio and analyzed management of this risk through life cycle optimal dynamic asset allocation—including the accumulation and dissimulation phases—by formulating the asset allocation strategy as an optimal stochastic control problem. Several objective functions were tested and compared. They focused on the risk of portfolio depletion at the terminal date using such measures as conditional VaR and the probability of ruin and secondarily through the formulation of Hamilton-Jacobi-Bellman.

Kleinknecht and Ng (2016) also used MCS methods in their analysis of the efficiency of empirical, parametric, and simulation-based and conditional VaR optimized portfolios on the regulatory capital requirements. They observed that the parametric and empirical distribution assumptions generated similar results: neither clearly outperformed the other. Their research recognized an alternative assumption, that is, to generate future ROIs with MCS. They used dynamic conditional correlation forecasting and MCS to reduce the risk for a financial institution by creating more robust market portfolios. Their results indicated that portfolios optimized with a multivariate dynamic conditional correlation simulation approach reduce the capital requirements by approximately 11%. Furthermore, they showed how the population-based incremental-learning algorithm can be used to solve constraint optimization problems. Simply stated, learning by the population can be similar to learning by the population of the PG or business units requesting approval for each project from the portfolio. This learning, characterized by the operational performance of the project initiator, is included in the current research and affects the approval decision-making process in the mining corporation portfolio.

The multi-objectives optimization model follows standard steps to solve an optimization problem (Figure 2.1). These are often used to solve the corporate mining portfolio optimization problem.

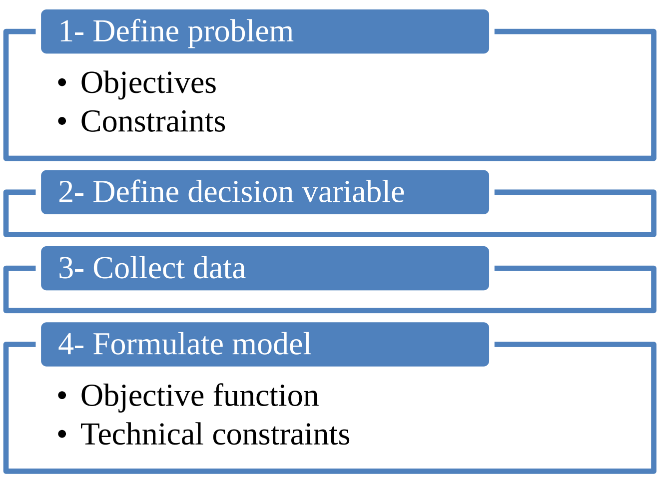


Figure 2.1: Steps to formulate the multi-objectives optimization problem using the proposed model

For the analysis of a wide variety of portfolio optimization problems, linear and nonlinear programming models are used. Although, the linear programming is very powerful for the analysis of more linear phenomena with the first-order approximation, there are some limitation when the phenomena are not linear. After the first-order approximation, the next level of complexity is mainly, the second-order approximation, where the selection of optimal corporate portfolio is illustrated by the mean-variance portfolio models of uncertainty including the maximization of the returns and the minimization of the risk. This type of nonlinear programming is the so-called quadratic programming.

2.6 Quadratic programming

A corporate portfolio quadratic programming is defined as a corporate portfolio optimization problem where the quadratic objective function is to minimize the risk and maximize the returns. The quadratic objective function are quadratic in a finite number of decision variable subject to a finite number of linear inequality or equality constraints. Mathematically, the quadratic function is defined as follows (Chang, et al., 2000):

$$Minimize\text{:}\ \ \frac{1}{2}x^T Vx + R^T x + S = \frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n} v_{kj}x_k x_j + \sum_{j=1}^{n} R_j x_j\ + S$$

$$Subject\ to\text{:}\qquad Ax \le b \quad and \quad A_{eq}x = b_{eq}$$

Where:

$x = (x_1, x_2, \dots, x_n)^T$, is the vector of the decision variables

$R = \begin{pmatrix} R_1 \\ R_2 \\ . \\ . \\ . \\ R_n \end{pmatrix}$, $R \,\epsilon\, \mathbb{R}^n$ is an $n - dimensional\ vector$

$\mathbf{V} = \begin{bmatrix} v_{1,1} & v_{1,2} & \cdots & v_{1,n} \\ v_{2,1} & v_{2,2} & \cdots & v_{2,n} \\ \dots & \dots & \dots & \dots \\ v_{n,1} & v_{n,2} & \cdots & v_{n,n} \end{bmatrix}$, $V \,\epsilon\, \mathbb{R}^{n\times n}$ is an $nxn - dimensional\ real\ symmetric\ matrix.$

$S \in \mathbb{R}, A \in \mathbb{R}^{m\times n},\ A_{eq} \in \mathbb{R}^{l\times n}, b \in \mathbb{R}^{m}, b_{eq} \in \mathbb{R}^{l}$, n is the number of decision variables, m and l are respectively the number of inequality and equality constraints.

A review of the multi-criteria used in this decision-making process in the quadratic function is required, especially the integration of the criteria weights in the decision making.

2.7 Multi-criteria decision making

In their overview of multi-criteria decision-making (MCDM) methods, Pavan and Todeschini (2009) illustrated that the optimal solution only exists for one criterion. The solution is optimal when it balances the positive outcome between gains and losses of all criteria. Alali and Tolga (2019) aimed to adapt a well-known MCDM method, TODIM, to the portfolio allocation process. TODIM relies on prospect theory, which explains the asymmetrical response of individuals during decision-making in the face of risk associated with losses on the same level as gains at a higher absolute value. This difference in perception of gains and losses by the decision maker can be factored in TODIM using an attenuation factor. Alali and Tolga (2019) first created a criteria matrix for specific periods in historical data. Criteria include short-, mid-, and long-term standard deviation, returns, and correlations. The matrix was then transformed and normalized and compared to alternatives before weighted portfolio allocation was carried out. Saaty (2008) developed a structured technique for analyzing and organizing decisions. This technique was called the analytic hierarchy process (AHP). Another multi-criteria decision analysis method called ELECTRE was proposed by Roy (1968). He suggested two main stages to apply ELECTRE. The first is to build outranking relations to compare each pair of actions in a comprehensive way. The second is the elaborations of ranking or sorting recommendations from the first stage.

Tavana, et al. (2015) studied a fuzzy hybrid project portfolio selection method using data envelopment analysis, the technique for order of preference by similarity to ideal solution (TOPSIS), and integer programming to solve the issue of project selection and resource allocation. Their proposed model comprises three stages: preparatory, project evaluation, and portfolio selection. Each stage is composed of several steps and procedures. Data envelopment analysis is used for initial screening, TOPSIS is used to rank projects, and linear integer programming is used to select the most suitable project portfolio according to organizational objectives in a fuzzy environment. The distinguishing feature of this study is that they incorporated the coordination of portfolio optimization with organizational objectives and missions. Linear integer programming is a maximization objective for the attribute scores from TOPSIS. An additional MCDM methodology the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) is used to compare the results with the TOPSIS. The PROMETHEE method was first proposed by Brans and Mareschal (2005)

In addition to variables associated with these multi-criteria in the portfolio optimization structure (defined in Figure 2.2), understanding the impact of uncertainty and risk associated with the investment that needs to be made is required using techniques such as MCS (Dhaundiyal, et al., 2019).

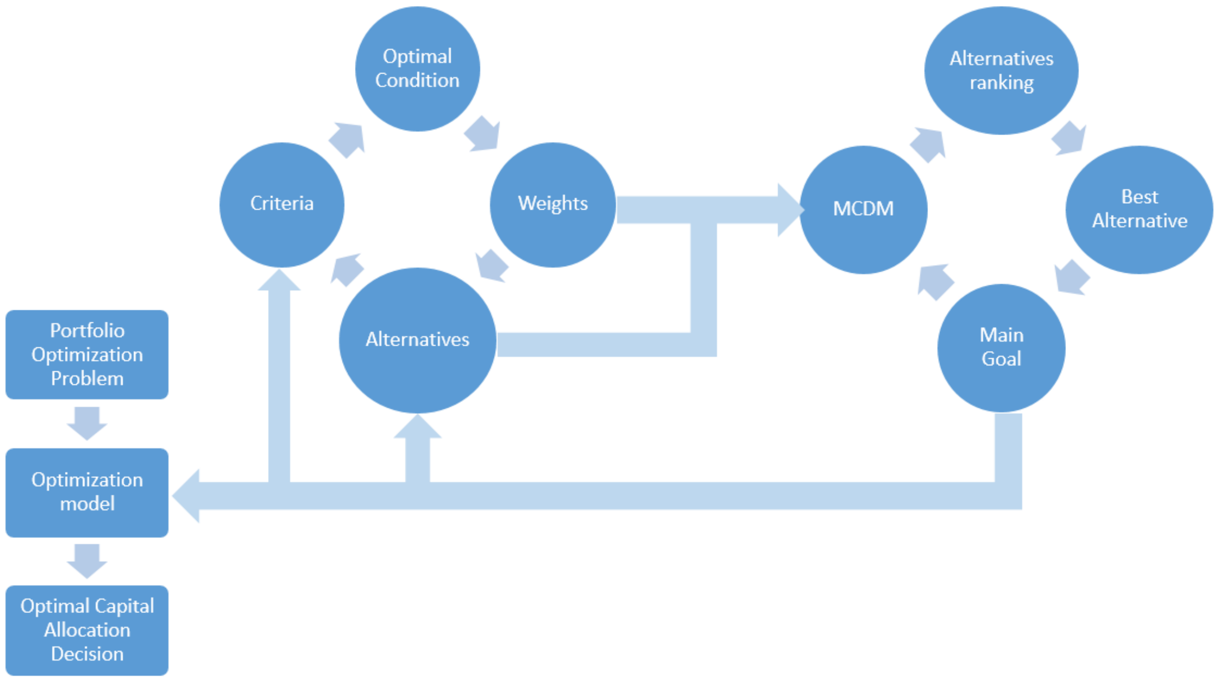


Figure 2.2: MCDM process combined with the portfolio optimization structure

2.8 Monte Carlo simulation

MCS is a computerized mathematical technique that accounts for country, project implementation, and business unit risk in quantitative analysis and decision-making (Boyle, 1977). It quantifies risk related to a project. The value of the risk associated with each investment is factored into the quadratic portfolio management model. With the assessment of the impact of the risk, MCS shows extreme events possibilities with all possible outcomes of the decision to be made under uncertainty. Specifically, Markov Chain MC sampling with Bayesian inference facilitates developing effective sampling algorithms and diagnosing convergence in mining portfolio management (Greenland, 2001). With a solid understanding of the uncertainty and risk impact in the corporate portfolio, project approval requests usually come from different business units located in the same or different countries, the business unit or country where the project will be implemented remain the key variables with a direct impact on the approval decision-making process. In MCS, fitting distribution to an uncertain data and reproduction between uncertain variables are two main aspects to be dealt with. There are many software offering MCS implementation (e.g., @Risk, CrystalBall and ModelRisk). Regarding the distribution fitting, Akaike Information, Chi Square, Kolmogorov-Smirnov, Andersen-Darling, Bayesian Information Criteria can be used. Regarding the reproduction of correlations, if the correlations are linear, Kendall's tau and if the correlations are non-linear, copulas can be utilized.

2.9 Correlation

A correlation is a statistic that measures the degree to which two random variables move in relation to each other. The correlation coefficient has a value between -1 and 1; both represent perfectly negative correlation (perfectly moving in the opposite direction) and perfectly positive correlation (perfectly moving in the same direction) (Embrechts, 2002). A value of zero implies there is no relationship at all between the two variables. Correlations may be non-linear. For example, the correlation between high gold and high copper prices may be different from the correlation between low gold and low copper prices. In this case, the Pearson or Spearman correlation coefficient cannot be used. If the variables are dependent, they are also correlated. However, if they are correlated, they are not necessarily dependent. Dependence is generally defined as any statistical relationship between two variables (Mari, 2001). However, the term correlation is a more general term used for any type of statistical relationship including dependence (Embrechts at. al., 2002 and Mari, & Kotz, 2001). Correlations between variables may change over time during the implementation of a project.

2.10 Variables related to project implementation

The variables related to the project implementation phase are linked to four key project attributes (cost, duration, scope, and quality) developed through three steps: conceptual level for authorization, semi-detailed level for continuation, and detailed level for re-evaluation (PMI, 2013). The level of detail associated with each attribute affects the accuracy of the optimal decision related to project approval. The essential question for project implementation is: does the operational performance of the project initiator have weight that is more positive on the approval decision-making process? This operational performance variable also affects the business unit variable.

2.11 Variables related to the business unit

Rad and Levin (2006) defined three variables related to the business unit that guide project prioritization according to three attributes.

1. Financial attributes determine the financial attractiveness of the investment and are defined by the ROI and payback period. They include:
   - internal rate of return,
   - NPV of earnings,
   - benefit/cost ratio,
   - expected commercialization value,
   - time to break even,
   - the discounted cash flow of the income from deliverables,
   - total cost as percentage of the total available funds, and
   - relationship to the total expected value of the portfolio.

2. Strategic attributes determine the strategic attractiveness of the deliverable and are defined by the competitive edge, time to market, and the utility. They include:
    - morale, prestige, reputation, customer relations, and productivity benefits of the deliverable to the company, and
    - strategic importance, utility of the deliverable, and probability of success of the business venture using the deliverable.
3. Funding category constraints are defined by the proportion of funding, project population distribution, and continuous pipeline to deliver within the portfolio. They include:
    - The limited number of projects in each business unit,
    - percentage limit of project funds in relation to total corporate funds,
    - continuous delivery of projects in each business unit,
    - pipeline population issues, and
    - staggered delivery dates.

2.12 New variables introduced in this research

This research will develop two additional sets of variables. The first set is related to operational performance and quantifies the achievement of the business unit's full potential of the project originator objectives. A project approval request from a low-performance business unit will have less chance to be approved compared to a high-performance business unit. The second set of variables are the international-related variables: the country risk index subject to the location where the project will be implemented. Country risk classifications from several organizations, including the Organization for Economic Co-operation and Development (OECD) (2016) are developed from a qualitative and quantitative risk assessment model. The relationship between qualitative and quantitative country risk is elaborated in Chapter 5. These improved business unit-related variables have four attributes: financial, strategic, funding category constraints, and full potential level. They are mainly manifested by multi-national mining companies as they deal with a broadly diversified type of commodity from different countries. While they endeavor to limit their risk exposure by only doing business in countries with stable political, social, and economic environments, there is no assurance that the environments will remain stable. The inclusion of country-related variables in this new corporate portfolio management model will help to obtain an optimal and realistic capital investment decision for multi-national mining companies.

The following chapter illustrates the application of the operational performance and international risk variables in the mining corporate portfolio optimization model.

Chapter 3: Mining corporate portfolio optimization model incorporating operational performance and country risk

3.1 Introduction

In the last decade, mineral resource companies have focused on investing in the development of mines and rushing to take advantage of high commodity prices, beginning production early while permanent field development is being planned and full facilities are being built (Rahmanpour & Osanloo, 2015). This early production helps companies benefit from early cash flow, uninflated construction costs, and high profit margins (Rahmanpour & Osanloo, 2016). Minsky (2008) also developed a financial instability hypothesis where in good times, companies undertake high risk through new projects. Too many new projects associated with profit greed trigger the next crises.

The current top mining companies have multiple product groups (PGs) with different Chief Executive Officers, all competing for the same global capital expenditure (CAPEX) funds. For example, a top mining corporation has several groups with annual CAPEX of multi-billion dollars distributed among PGs. A large amount of this investment is allocated to sustaining capital, which highlights the fact that it is mainly for existing mines or assets. New projects can be construction or purchase of a new mine/facility, capacity expansion of an existing mine, or replacement equipment to adopt new technologies.

From 2011 to 2016, the commodity price index dropped by 41% (Canada, 2016) and for a specific mineral commodity such as iron ore, the market price dropped by 68% (IndexMundi, 2019). Mineral resources companies are no longer achieving high profit margins because of the combination of high operating costs and low commodity prices (Raffaini, 2016). This leads to reluctance to quickly move from development to early production. Also, mining investments are irreversible and incur a significant amount of risk associated with fluctuations in global mineral markets. Collan, et al. (2017) illustrate the effect of the market price on the metal mining asset portfolio valuation in terms of net present value (NPV) and real option valuation of the assets.

Considering the internal competition of project approval between different business units/PGs within the same mining corporation, this chapter demonstrates that, on top of the market price impact on the asset valuation, the criteria of operational performance of business unit/PG–project initiator in the initial investment decision are necessary and also impact the decision to invest or not. Studies related to approval for internal sustaining capital projects have not yet been done with additional criteria of project initiator performance in previous corporate mining project portfolio valuations.

Commodity prices are low compared to 2012. There are a large number of projects and the supply exceeds the demand (Ives, 2016). High-cost producers are faced with cash problems and they must suspend or cease their operations whereas low-cost producers can overcome these challenges. As with many other business, the minerals industry is cyclical. It is very difficult task to quantify the magnitude of a cycle. Humphreys (2018) illustrates the impact of the evolution of the cycle on companies' investment strategies. The drop in commodity prices increases competition among low-cost producers. Furthermore, the complexity of international economics and globalization has led to additional challenges to the investment strategy of mining companies (Wirth, et al., 2013). For mining company decision-makers, the challenge is how to improve the effectiveness of their current assets/resources/projects and better manage the risk to allocate capital investments to the most valuable initiatives (Bowman & Ambrosini, 2000).

This challenge can be formulated as a decision-making problem, which translates into an intertemporal optimization problem (Chang, et al., 2000), namely, a quadratic portfolio management model. This model refers to a constrained nonlinear program with a quadratic objective function where all constraints are linear. The decision variables for the defined problem are the fractions of the fund invested in each project. Decisions made in any period affect future possibilities. Therefore, there is a need to take into account this effect when a decision is made.

The quadratic optimization model developed in this chapter will contain operational performance and country risk (Herring & School, 1986). A PG is defined as a combination of multiple business units associated with mining and processing operations. To obtain approval for a new project, the PG's previous performance will be a criterion. Performance attributes include for example, an excellent safety record, whether production quality and quantity targets were met, the budget was properly managed, and resources were effectively utilized.

Considering these key economic drivers, a sequence of effective decisions needs to be made to maximize utility expressed in the objective function. This approach will give decision-makers the right fraction of available CAPEX to invest in each PG in order to minimize the risk subject to a specified minimum expected rate of return defined by the corporate group as the economic threshold profitability limit of the corporate group (Xue, et al., 2014). Corporate groups are very often divided into multiple PGs with respect to commodities such as gold, oil sands, nickel, diamond, coal, aluminum, iron ore, industrial minerals, and copper.

Many mining organizations use the NPV as the critical criterion to evaluate the project from exploration to production stages (see Figure 1.1). However, for capital allocation at the corporate level, there are also risks associated with the operational performance of the PG and the location of the business unit initiating the project (project initiator). (Xu, et al., 2014) developed an improved portfolio model for project selection where an applicable technique beyond the NPV is presented for capital budgeting. Nevertheless, the current operational performance of the project initiator was not considered.

In the mining industry, the additional capital expense has always been considered as a critical element of operational performance (Xue, et al., 2014). Before buying a new asset, it is critical to optimize the asset in possession, therefore consideration of current operational performance in the approval of a new project proposed by the PG is required.

This chapter considers a single corporate office managing a portfolio of programs/projects from different PGs, defined as a set of companies producing a group of products in a specific geographic location. All projects submitted to the corporate office for approval will be considered to already meet the NPV criterion within the PG. The additional corporate criteria will be the operational performance and the country risk associated with the project initiator. Figure 3.1 illustrates the stages of project approval.

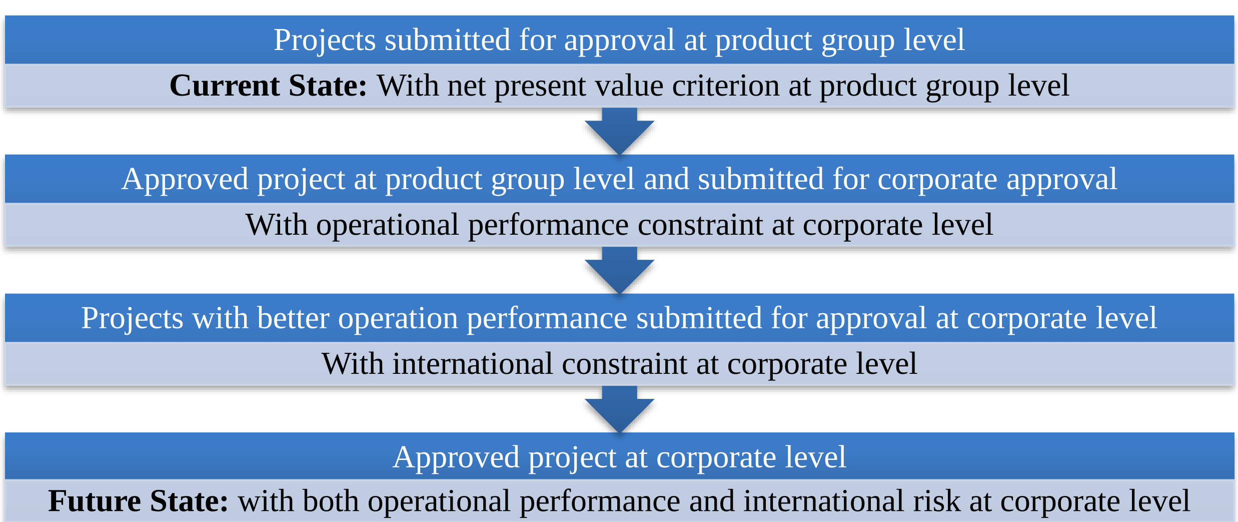


Figure 3.1: Approval stages from product group to corporate level

Bhappu and Guzman (1995) illustrated the fact that discounted cash flow techniques do not allow for consideration of premium values on project valuations. Typically mining and processing operations throughput/effectiveness are assumed to be constant in the evaluation of mining projects. However, this throughput/ effectiveness is correlated to the operational performance of the business unit requesting project approval. The originality of this chapter rests on proposing an optimization model for project portfolio management in the mining industry with operational performance as one of the key criteria for project approval.

The ultimate goal of corporate portfolio management office is to maximize returns on investments (ROIs) and minimize risk at the corporate level. There is a need to understand how to split the available fund among different projects from different business units. The difficult decision to split the available fund among projects from a different country and different commodity illustrates a class of constrained nonlinear programs. This represents a double-objectives optimization model. The quadratic programming technique is used to obtain an efficient solution to this nonlinear problem.

3.2. Formulation of the quadratic portfolio model

A quadratic portfolio management model refers to a constrained nonlinear program, which includes a quadratic objective function and all linear constraints. The decision variables for the defined problem are the fractions of the fund invested in each project. The project approval condition is "Yes or No", and these responses are associated with capital investment available or not available for the project.

Considering (p) projects are submitted for approval, (x_i) is the fraction of the portfolio invested in project i.

The main constraint forces the allocation of all of the fund:

$$\sum_{i=1}^{p} x_i = 1 \tag{3.1}$$

The achievement of full business potential is assumed independent of the overall risk associated with the investment. Applying the probability definition to achieve two independent events together, the overall probability will be the product of the current operational performance ratio of success related to the investment on each project (Xie, 2017). Assuming that the project risk (β_i) and the performance of the PG (α_i) are independent; the specified minimum expected rate of return R_e can be expressed as:

$$\sum_{i=1}^{p} \alpha_i \, (1 - \beta_i) \, x_i \geq R_e \tag{3.2}$$

Where:

α_i represents the current operational performance ratio of the PG requesting the project approval. This ratio illustrates the current operational performance of the PG where the project will be implemented (Taylor, 1986).

β_i represents the risk of investing in project P_i associated with x_i, the fraction of the corporate portfolio invested in the business unit i; initiator of project i.

$(1 - \beta_i)$ represents the probability of success without the PG's operational performance being associated with the investment in specific project i.

Combining this with the project initiator performance will reinforce the decision-making to approve reject a project;

$\alpha_i(1 - \beta_i)$ represents the overall probability of success associated with the investment on specific project i.

Equation 3.2 illustrates the consideration of operational performance as a criterion in the approval process. In most cases, project approval does not depend on PG performance. In this research, we add a criterion related to PG performance to reduce the overall risk for investment in a project. Thus, more accurate decisions related to project investment can be made. The additional consideration of PG performance will affect the approval level. What is proposed in this study is project approval, not project implementation. Nevertheless, in the most practical case of the project approval committee, the implementation phase is treated separately, and the performance of the implementer is not part of the approval equation. In most cases, by removing the current performance of the project initiator in the approval process, companies face the risk of choosing the wrong projects.

If there is a high performing group inside a company, there is less risk in project implementation; nevertheless, this is not considered at the approval stage. This new approach includes the operational performance criteria at the early stage of approval. Then, resolving the risk at the time of the initial investment decision is more realistic and help companies with billions of dollars in annual sustaining capital investment in their approval process. The approval of sustaining capital expenditures of each PG is fulfilled on an annual basis.

The difficulty is to model the variability of the ROI. The variance is the average squared deviation from the mean of multiple independent variables. In this case, the variance of the return will be used to measure the variability of return. Assuming all projects are uncorrelated (i.e., they vary independently), the variance of the overall corporate return would be the sum of the variances of each project. However, for a more workable/realistic model, it is more likely that these projects will interact. Thus, there are covariances relating movement in the different types of project (Rardin, 2016). These covariances are estimated as:

$$v_{i,j} \triangleq covariance\ between\ projects\ i\ and\ j$$

The variance of the overall portfolio return = $\sum_{i=1}^{p} Var(x_i)$ where $Var(x_i)$ is the variance of the return of project i.

In this chapter, we consider that mining projects are highly correlated through commodity type or operation location (Clemen, 2000). This portfolio model clearly represents a quadratic program with a quadratic objective function. The optimization model thus includes the covariance relating movement in each of the projects to be financed. Given a series of *p* returns for *p* projects, the covariance between projects *i* and *j* can be calculated as follows (Rardin, 2016):

$$v_{i,j} = \frac{1}{n}\sum_{t=1}^{n} d_{ti}d_{tj} - \frac{1}{n^2}\left[\sum_{t=1}^{n} d_{ti}\right]\left[\sum_{t=1}^{n} d_{tj}\right] \tag{3.3}$$

Where: d_{ti} is the value of the project *i* in period *t*. In this chapter, using the future cash flows for 5 years for each project, the covariance among the projects is calculated. Likewise, the past performance of the PG could have been used to calculate covariance.

The variance of the overall portfolio return $= \sum_{i=1}^{n} \sum_{j=1}^{n} v_{i,j} x_i x_j$ (3.4)

The matrix V of $v_{i,j}$ is given by:

$$\mathbf{V} = \begin{bmatrix} v_{1,1} & v_{1,2} & \cdots & v_{1,n} \\ v_{2,1} & v_{2,2} & \cdots & v_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ v_{n,1} & v_{n,2} & \cdots & v_{n,n} \end{bmatrix} \quad (3.5)$$

Markowitz (1952) developed a quantitative framework for the selection of a portfolio. Using the standard Markowitz mean-variance approach (Markowitz, 1959), the unconstrained portfolio optimization problem is to minimize the variance $x.\boldsymbol{V}.x$:

Minimize $x.\boldsymbol{V}.x = \sum_{i=1}^{p}\sum_{j=1}^{p} v_{i,j}\, x_i\, x_j$ (3.6)

Subject to all funds allocated for investment will be spent.

$$\sum_{i=1}^{p} x_i = 1 \quad (3.7)$$

$$x_i \geq 0 \quad \forall x_i, i = 1, \ldots, n \quad (3.8)$$

To ensure that the fraction of the corporate portfolio allocated to a specific project will be sufficient to implement the project, an additional constraint of the objective function needs to be included for project approval.

$$x_i . C_{po} \geq P_i \quad \forall i \quad (3.9)$$

Where:

x_i represents the value of the fraction of the corporate portfolio function for project i.

P_i represents the requested approval amount for project i.

C_{po} represents the overall available fund for the corporate portfolio.

If this project approval condition is not met, the project will not be approved, thus no capital investment will be made related to the specific project. This portfolio model represents a quadratic program with a quadratic objective function $f(x)$

$$\sum_j c_j\, x_j + \sum_i \sum_j v_{i,j}\, x_i x_j \tag{3.10}$$

Where:

c is a real-valued, n-dimensional vector of coefficient $c_j = 0$ for the quadratic portfolio model in this chapter.

v is the $n \times n$ dimensional real matrix of covariance $v_{i,j}$ between project i and j.

3.3. Application

In this section, a typical international mining corporate portfolio is considered containing five PGs, each with 45 projects in different geographical locations requesting approval from the corporate office. This means a total number of 225 projects around the world seeking approval during the same period. The rate of project return is a random variable with an expected value of $(1 - \beta_i)$. The problem is to find what fraction x_i to invest in each project to minimize the risk subjected to a predefined minimum expected rate of return. The matrix that defines these optimization problems is dense. In this practical case, we are solving a portfolio optimization problem using an interior-point quadratic programming algorithm.

Let us denote $v_{i,j}$ the covariance matrix of rates of project returns. This classical mean-variance model consists of minimizing the portfolio risk associated with each of the 225 projects by maximizing the expected portfolio return with the PG performance of the project originator. The portfolio risk is measured by:

$$v_{1,1}.(x_1)^2 + v_{1,2}.x_1.x_2 + \cdots + v_{1,225}.x_1.x_{225} + \cdots + v_{2,2}.(x_2)^2 + \cdots + v_{225,225}.(x_{225})^2 \quad (3.11)$$

Equation 3.11 minimizes the total variance (risk) associated with the portfolio, subject to the following two constraints.

1. The sum of the expected project return $\alpha_i.(1-\beta_i).x_i$ should be larger than a minimal rate of portfolio return R_e that the shareholder's desire. This is measured by:

$$\alpha_1.(1-\beta_1).x_1 + \alpha_2.(1-\beta_2).x_2 + \cdots + \alpha_{225}.(1-\beta_{225}).x_{225} \geq R_e = 0.2\% \quad (3.12)$$

Equation 3.12 ensures that the portfolio has an expected return of R_e.

(α_i) represents the current operational performance ratio of the PG requesting the project approval. This chapter considers an equal operational performance level for the five PGs. Thus, for equal weight distribution, the result gives 20% of the corporate current operational performance.

(β_i) represents the risk of investing in project P_i associated with (x_i) the fraction of the corporate portfolio.

$(1-\beta_i)$ represents the probability of success associated with the investment on a specific project.

2. The sum of investment fractions (percentages) should add up to unity.

$$x_1 + x_2 + \cdots + x_{225} = 1 \quad (3.13)$$

Equation 3.14 ensures that the proportions add to one. These fractions should be positive and less than 1.

$$x_1, x_2, \dots x_{225} \geq 0 \tag{3.14}$$

Since the objective to minimize portfolio risk is quadratic and the constraints are linear, the resulting optimization problem is a quadratic optimization problem. This formulation is a simple quadratic nonlinear programming problem for which computationally effective algorithms exist. In practice, there is less difficulty in calculating the optimal solution for any particular dataset.

Using the standard Markowitz mean-variance approach (Markowitz, 2008), the portfolio optimization problem is:

$$\begin{aligned} \min\ & v_{1,1}\ .(x_1)^2 + v_{1,2}.x_1.x_2 + \cdots + v_{1,225}.x_1.x_{225} + \cdots + v_{2,2}.(x_2)^2 + \cdots \\ & + v_{225,225}.(x_{225})^2 \end{aligned} \tag{3.15}$$

The above formulation can also be expressed in terms of the correlation ρ_{ij} between projects i and j with $-1 \leq \rho_{ij} \leq +1$ standard deviations σ_i, σ_j of returns for these projects.

$$v_{ij} = \rho_{ij} \cdot \sigma_i, \sigma_j \tag{3.16}$$

Assuming a rate of return range between –0.008 and 0.004 with a desired return, $R_e = 0.2\%$,

$$rr_{min} = -0.008 \leq rates\ of\ return \leq +0.004 = rr_{max} \tag{3.17}$$

MATLAB software was used to solve this portfolio optimization model for an elapsed time of 0.0347 seconds to run the quadratic program.

Three scenarios were considered:

1. allocation of the project with no constraint,

2. allocation of projects with one constraint (operational performance level), and
3. allocation of projects with two constraints (operational performance level and country risk).

3.3.1 Unconstrained allocation

Figure 3.2 illustrates the minimum number of approved projects (11 out of 225). PG 2 received 38% of the available CAPEX with only two projects; 24% of the investment was allocated to PG 1 and 19% to PG 5, each with three projects; 13% was allocated to PG 3; and 6% was allocated to PG 4. The solution of the objective function of this unconstrained portfolio optimization provides a minimum risk of 0.019%.

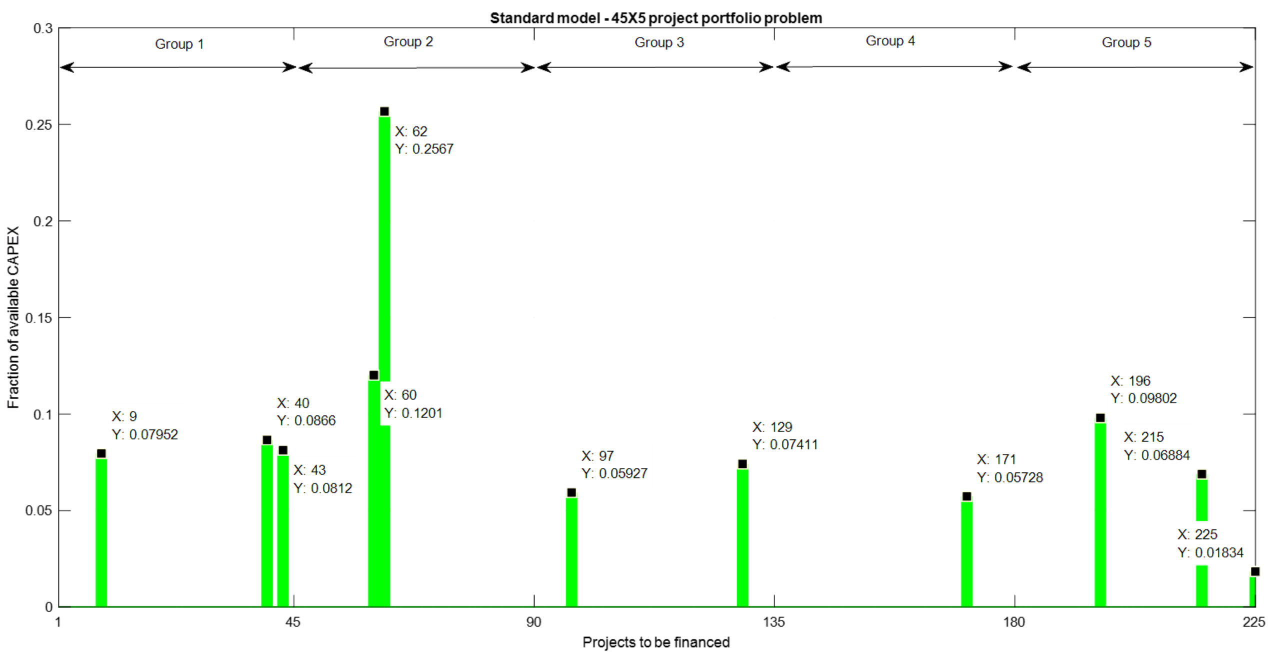


Figure 3.2: Standard unconstrained portfolio optimization

Table 3.1 illustrates that by assuming that the same operational performance is not affecting the decision for project approval, for standard unconstrained portfolio optimization, PGs 1 and 3 get one more approved project than PGs 2 and 3 and two more approved projects than PG 4.

Table 3.1: Unconstrained allocation with minimum risk

Product group #	Fraction of CAPEX (%)	Approved project (#)
1	24	3
2	38	2
3	13	2
4	06	1
5	19	3

3.3.2 Allocation of projects based on operational performance constraint

By using the same dataset with the corporate constraint associated with the operational performance level of each of the five PGs and assuming each PG has an equal level of operational performance level weighted at 20% of overall performance, the corporate office will impose an equally available CAPEX (20%) to be invested in each PG. Figure 3.3 illustrates a higher number of approved projects (13 out of 225) than the unconstrained allocation.

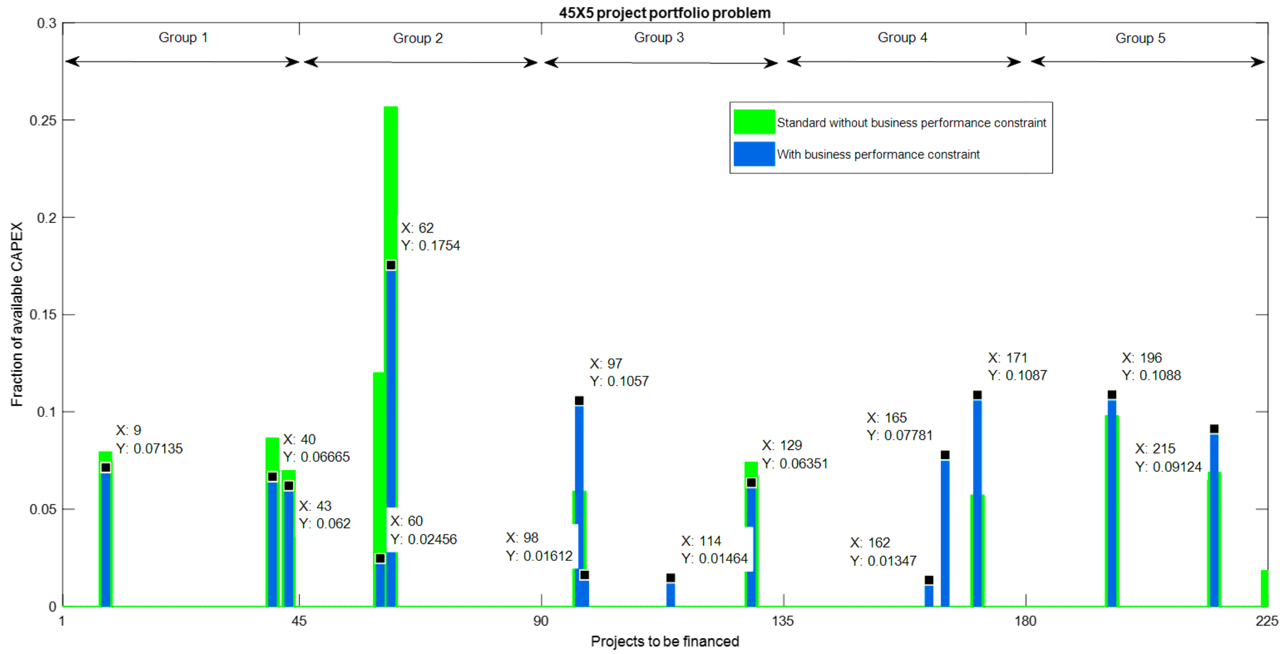


Figure 3.3: Constraint allocation based on operational performance level

By considering the operational performance constraint, two more projects received approval with a small increase in acceptable of 0.02% associated with the new constraint allocation (Table 3.2). PG 4 has two more approved projects than in the unconstrained allocation model. The portfolio is more evenly distributed across the five PGs; this illustrates the diversification and the potential increase of the corporate portfolio return.

Table 3.2: Project allocation based on operational performance constraint

	No constraint			With operational performance constraint		
Product group #	**CAPEX (%)**	**No. approved projects**		**CAPEX (%)**	**No. approved projects**	**Project variance (#)**
1	24	3		20	3	0
2	38	2		20	2	0
3	13	2		20	3	+1
4	06	1		20	3	+2
5	19	3		20	2	–1

3.3.3 Allocation of projects with operational performance level and country risk constraints

The request associated with the project approval will randomly change over time due to changes in a business environment that can adversely erode the financial value of the business unit in the country of the project initiator; this is also called country risk (Rahmanpour & Osanloo, 2015). Assuming that this risk will automatically impact the request for project approval, let us consider a randomly generated dataset for a project that required corporate approval. The solution to the objective function is illustrated in Figure 3.4 with the same constraint of proportional distribution of CAPEX across the five PGs. The reshuffle of CAPEX allocation and the number of the project approved per PG is evident. Four projects from PGs 1, 3, 4, and 5 receive 62% of the CAPEX, while the remaining seven approved projects receive 38% of the CAPEX. PG 2 is the only group with three approved projects while the other PG receives only two approved projects at an optimal solution equal to an acceptable minimum risk of 0.3%. Two projects are at the maximum return in PG 3 and 4.

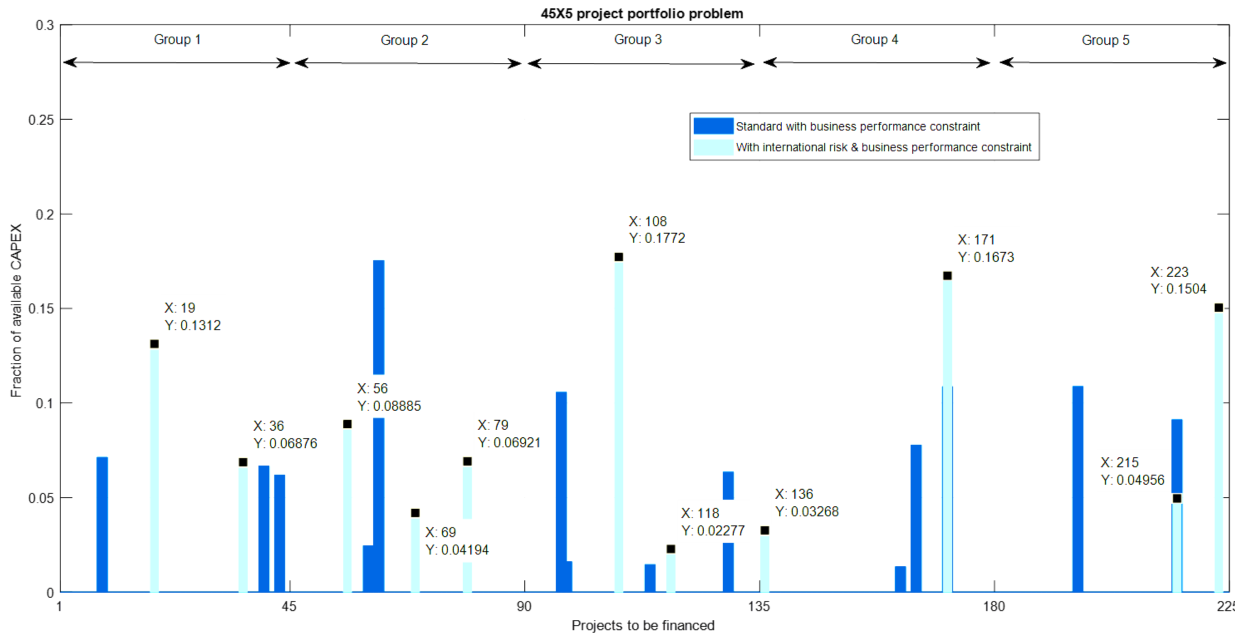


Figure 3.4: Project allocation with operational performance level and country risk constraints

We consider the same 45×5 desired number of project approvals, with the means and standard deviations of returns of –0.1–0.4 and 0.08–0.6, respectively.

$$mr_{min} = -0.1 \leq mean\_return \ \leq 0.4 = mr_{max};$$

$$mean\_return = mr_{min} + (mr_{max} - mr_{min}) \times rand(225{,}1);$$

$$sd_{min} \ = 0.1 \leq standard\ deviation\_return \ \leq 0.6 = sd_{max};$$

$$standard\ deviation\_return = sd_{min} + (sd_{max} - sd_{min}) \times rand(225{,}1);$$

Additional constraints increase the risk at the accepted level. Nevertheless, a more evenly distributed CAPEX allocation results than was seen in the single constraint allocation of projects (Table 3.3). This imposed more diversification and then a potential increase of the corporate portfolio return toward the optimal value at an acceptable minimum risk. These results clearly show that combining the operational performance of PG with country risk could significantly affect the approval of any requested project at the corporate level. This will provide more realistic decision-making that will help to minimize the risk associated with capital allocation.

Table 3.3. Allocation based on operational performance and country risk constraints

	With operational performance constraint			**With operational performance and country risk constraints**	
Product group #	**CAPEX (%)**	**No. approved projects**		**Approved project (#)**	**Project variance (#)**
1	20	3		2	–1
2	20	2		3	+1
3	20	3		2	–1
4	20	3		2	–1
5	20	2		2	–1

3.4. Conclusion

In this chapter, we considered a typical (but hypothetical) international mining company with five PGs located in multiple geographical locations, with a problem to allocate the right capital investment to the right project for the optimal return at the minimum risk. This research incorporates the current operational performance of the PG and random risk associated with the country where the project will be implemented, as well as the standard risk minimization that is traditionally used. Beyond the traditional NPV or discounted cash flow project/asset valuation, the criteria of operational performance of business unit/PG/project initiator in the initial investment decision are necessary and also impact the decision to invest or not.

The new study provides a new mining projects portfolio valuation approach with an additional criterion of a project initiator performance for internal sustaining capital projects within the multi-national mining corporation. Adding country risk and operational performance constraints show that the more you impose a diversified portfolio with correlated projects, the more you potentially increase the corporate portfolio return with a slight increase of the minimum acceptable risk. As the performance of a PG increases, the chance of approval of the proposed projects also increases. The objective function is formulated as the minimization of the risk of the desired return with correlated projects. Although this problem was solved with two additional constraints, further studies will be done to simultaneously obtain the optimal portfolio return at the minimum risk including the quantification of correlations between projects. Also, we intend to extend the research if there is need for two separate portfolio optimization strategies for good (boom) and bad times (bust).

Chapter 4: Mining portfolio optimization under extreme events – Investment/divestment decisions with commodity market behavior

4.1 Introduction

In today's globalized mining industry, economic uncertainties and risks associated with tariffs and technical aspects of operations/assets render investment and divestment decisions more difficult. Risks associated with the instability in global mineral markets are very high due to the significant and irreversible capital investment needed up front (Collan, et al., 2017). Investment and divestment decisions become more difficult in the face of extreme events such as drastic variability in commodity market behavior in very short periods, environmental disasters such as catastrophic failure of Vale tailings dam in Brazil January 2019, and tariffs imposed by countries. A term usually used to describe natural disasters (e.g., earthquakes, flood, droughts, and hurricanes) that influence social, environmental, urban, and technical systems, extreme events in the mining corporation context can be defined as sudden financial or natural events that prevent sustainable healthy returns. Minerals investments are negatively impacted by the escalation in sudden changes on taxation rules, legislation, permitting, and regulations by governments (Baker, et al., 2016), which also induces variability and unpredictability in the commodity market. In particular, international companies are sensitive to changes in taxation and mining legislation from host governments.

Corporations are often divided into multiple product groups (PGs). For illustration purposes in this chapter, a hypothetical mining corporation is divided into five PGs categorized by commodity: iron ore (PG 1), copper (PG 2), gold (PG 3), aluminum (PG 4), and lithium (PG 5). A strong price fluctuation is imposed for different periods, reflecting real-world data. For example, iron ore with

62% Fe content reached an all-time high of US$188.9/tonne in 2011 and a low of US$38.54/t in 2015 (TradingEconomics, 2019). The iron ore market price dropped 80% between 2011 and 2015 and then increased 155% in 2019 compared to 2015 (IndexMundi, 2019). The copper price decreased 56% between 2011 and 2016, from an all-time record high of US$10.09/kg to US$4.27/kg in 2016 (TradingEconomics, 2019). The aluminum price was US$2774/t in 2011 versus US$1,450/t in 2016 (InfoMine, 2019). Gold prices have ranged from a low of US$1,058/oz in 2015 to US$1,354/oz in 2016 (Goldprice, 2019). Lithium was US$1,460/tonne in 2005 and US$16,500/t in 2018 (Metalary, 2019). The price trends above illustrate a clear disparity in the price behavior of these five commodities.

The sharp declines in commodity prices above took 4–5 years, which might suggest that these are not extreme events. However, the development of a greenfield mining project typically takes 5–10 years. Expansion, technology, or replacement projects take 1–3 years. Should these projects take place during a market decline, the company will be heavily impacted because high sink costs, labor relations, and local community concerns limit their flexibility to adapt to the changes.

The price volatility of mineral commodity markets tends to follow a cyclical pattern; investment strategies of companies are based on their perception of the evolution of cycle (Humphreys, 2018). Mining corporations with multiple commodities need to make the right decisions regarding their investment or divestment of each of their PGs during specific market conditions characterized by extreme events. Gkillas (2018) investigated the impact of restricting transactions due to capital control under extreme events within a new framework of intervention policy. Njike and Kumral (2019) highlighted the fact that the decision to invest or divest in a PG primarily depends on the operational performance and country risk of the PG requesting the capital expenses.

This chapter includes the concept of extreme events in the investment/divestment decision-making process of the commodity associated with the PG. The ongoing challenge is to decide where/when to invest or divest with a sudden change in the commodity market behavior. The decision-making will not be based only on the operational performance and country risk, but also on the capability to cease the opportunity on a certain PG from the market condition characterized as an extreme event related to a specific commodity.

This challenge can be formulated as a decision-making problem during constrained portfolio optimization. In other words, extreme events can be incorporated into portfolio optimization using an "efficient frontier" with a standard mean-variance portfolio optimization model (Chang, Meade, Beasley, & Sharaiha, 2000). Bielstein (2019) defined a mean-variance portfolio optimization model using forward-looking return estimates. Several portfolio optimization models for mineral investment have been developed without an extreme event criterion. For example, Njike and Kumral (2019) developed a corporate portfolio optimization model based on country risk and the current operational performance of the PG that lacked consideration of extreme events associated with market downturns.

This chapter describes the development of a mean-variance optimization model under extreme events characterized by a sudden change in commodity market behavior. Commodity behavior and the market price are considered the key economic drivers for investment decisions. The model will assist decision-makers to allocate the fraction of the capital fund to each PG that minimizes the risk subject to a specified minimum expected rate of return defined by the corporate group as the economic threshold of profitability (Xue, et al., 2014). This additional criterion will also help corporate groups take advantage of market opportunities, for example, high product demand because the top producer is unable to supply the expected production volume due to a catastrophic failure (e.g., Vale dam failure).

4.2 Problem definition

A typical international mining corporate portfolio is considered with five PGs spread across different geographical locations. Each PG is defined as a commodity class denoted by n. The corporate group will generally benefit from an increase in commodity prices; this is also defined as a long only fund (Mateus, 2019). Given that the long only strategy is predicated on the positive performance of the commodity price, the PG allocation fund is assumed to be long only with no borrowing or leverage. Njike and Kumral (2019) defined an optimization model incorporating the current operational performance of the PG and random country risk associated with the country where the project will be implemented as well as standard risk minimization. In addition to this corporate portfolio optimization model, the current chapter adds the impact of extreme events characterized by a commodity market downturn in the decision-making process to invest or divest a PG. Based on the price performance of each commodity, the following constraints are set.

Let us consider the cost to trade (ct_i) as the transactional cost to invest or divest PG i. The ct_i could vary by PG or by geographical location. PGs rarely perform at the same level due to the behavior of their commodity and different demands from different governments. Hence, in the application of five PGs/commodity classes, it is assumed that in this corporate portfolio model, the transactional cost for the top three performing PGs is lower than the transactional cost for the bottom two performing PGs.

$$ct_i, ct_{i+1}, ct_{i+2} < ct_{i+3}, ct_{i+4} \tag{4.1}$$

For each PG evaluated for investment/divestment, $\pm x_i$ is the portfolio weight in PG i.

$+x_i$ represents the value of the total investment share of the corporate portfolio fund for PG i.

$-x_i$ represents the value of the total divestment share of the corporate portfolio fund for PG i.

The investment/divestment value share ($\pm x_i$) of the portfolio associated with each PG will not be greater than the associated Cmv_i representing the commodity price performance of PG i.

$$x_i \leq Cmv_i \tag{4.2}$$

The main constraint forces the portfolio absolute weight of each PG to be non-negative with the sum equal 1:

$$0 \leq |x_i| \leq 1 \tag{4.3}$$

$$\sum_1^n |x_i| = 1 \tag{4.4}$$

In a practical way, the key problem in this portfolio optimization model is to know the required investment/divestment value share opportunity related to each commodity class during extreme events. From the commodity market analysis, either the commodity class receives full investment/divestment, or it receives a partial investment/divestment due to its geographical location or maturity level (Greenfield or Brownfield).

The ultimate goal of corporate portfolio management office is to maximize returns and minimize risk at the corporate level as well as satisfying shareholders' expectations. It is necessary to understand the right amount to invest or divest among different PG under country risk, past performance, and extreme events. The decision to sell (divest) or buy (invest) shares associated with each PG from different geographical locations and different commodities represents a multi-objective optimization model during extreme events. The main contribution of this chapter is the incorporation of divestment decision and extreme events into the portfolio optimization. The improved optimization model builds upon the mining corporate portfolio optimization developed for a normal market condition by Njike and Kumral (2019). The solution of the optimization model will reinforce the investment decision at the optimum level (Fathi, 1989).

4.3 Application

The five PGs representing five commodities are in different geographical locations and are exposed to different types of risks, including extreme events. The PG allocation fund is assumed to be long as described above, with no investment strategy of using borrowed capital as a funding source. Optimal portfolios are those with the highest expected return for an accepted risk level, based on operational performance of each of the commodity class. To explore these portfolios, the constraints related to Equations 4.1–4.4 are used with the following specifications:

- 100% of the portfolio can be allocated to the iron ore PG due to its very high return on investment (ROI) with superior operational performance.
- As much as 85% of the portfolio can be allocated to copper and gold PGs due to high ROI with high operational performance.
- No more than 35% of the portfolio can be allocated to aluminum and lithium PGs due to low ROI and poor operational performance.
- The transactional cost to trade the top three commodity classes (iron ore, copper, and gold) is assumed to be 0.1% of the difference in their value.

- Due to the new sustainability strategy, the transactional cost to trade aluminum and lithium is two times higher than the cost to trade other commodities; this means a 0.2% difference in value.
- A typical average percentage of the portfolio of each PG conduct its operations (collect cash or sell inventory) in a specific time period can be more than 14%.

To solve this problem, a basic mean-variance portfolio optimization problem is set and gradually the constraints on the problem are initiated to reach a solution. Each PG is assumed to have a tradable PG with a real-time price. The initial portfolio with holdings in each PG has a total of US$4900 MM, along with an additional cash position of US$11 MM. These basic quantities and costs to trade are set up in the following variables:

- **PG** represents the product group names in cell array PG,
- **Price** represents current prices in the vector Price,
- **Holdings** represent current portfolio holdings in the vector Holding, and
- **UnitCost** represents the transaction costs in the vector UnitCost.

A blotter is set to track prices, holdings, portfolio weights in dataset object.

- **InitPort** is the new blotter results from computing the initial portfolio weights.
- **InitHolding** represents the new portfolio holdings in the vector InitHolding.
- **Capital** represents the maximum capital allocation fund:

$$Capital = USD\ 4900\ MM$$

- **LB** represents the lower boundary of each product group:

$$LB = [50,50,50,50,50]$$

- **UP** represents the upper boundary of each product group:

$$UB = [Capital, Capital, Capital, Capital, Capital]$$

- **TimeEnd** represents the time range of the optimization model:

$$TimeEnd\ =\ 600\ time\ period$$

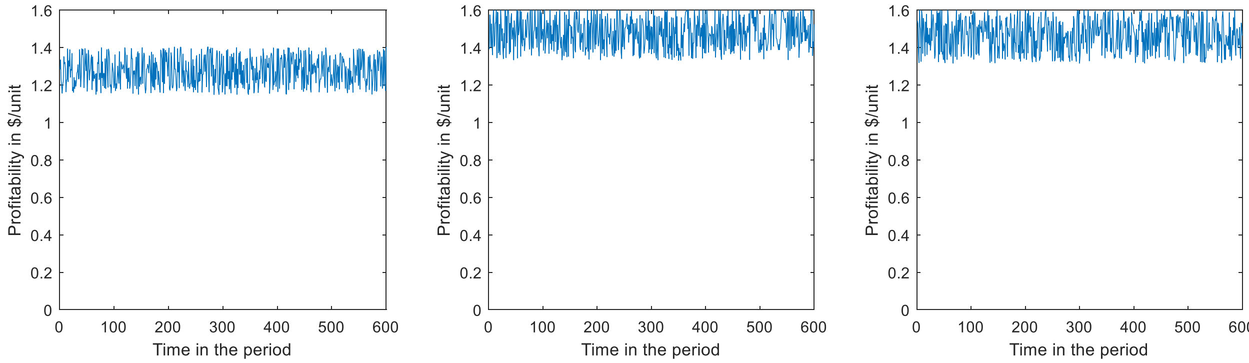
- **Length of extreme event** is assumed to be 30 time periods.
- **Prob_Of_Event_Per_Day** represents the probability of extreme events per day, assuming this value is related to the historical market behavior of each of the commodity classes:

$$Prob\_Of\_Event\_Per\_Day = [0, 0.001, 0.002, 0.005, 0.01]$$

To solve this portfolio optimization problem, two options were considered: 1) no divestment possibility with only capital allocation for potential investment and 2) both investment and divestment possibilities.

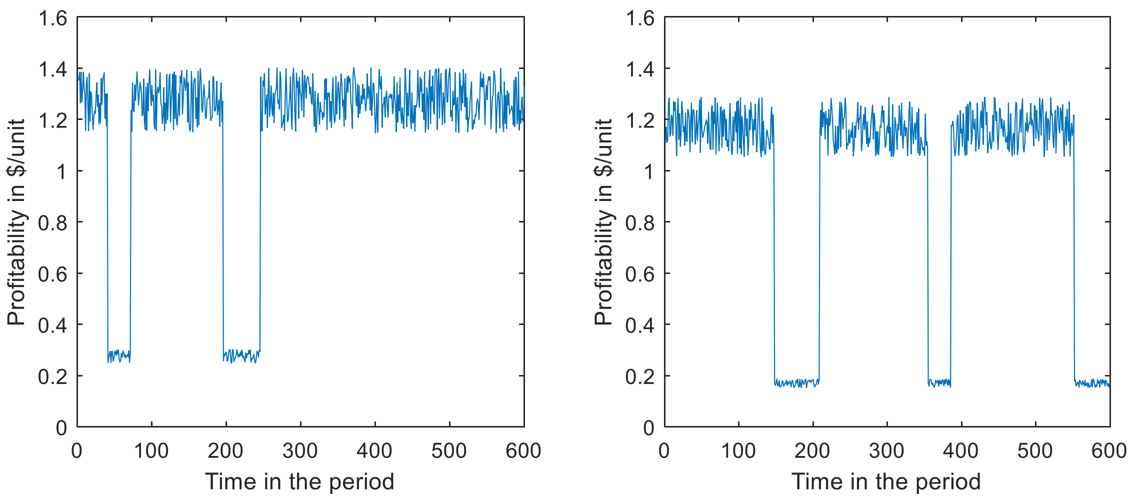
4.3.1 Product group investment with no divestment possibility

This problem was solved by MATLAB using codes developed by the author (Appendix 1). More than 600 time periods were simulated for the five PGs. PGs 1–3 remain profitable at extreme events. The variability of profitability is lower for PGs 1 and 2 (US$0.95–1.10/unit) than PG 3 (US$1.05–1.25 USD/unit) (Figure 4.1). The low operational performance of PGs 4 and 5 adds volatility and instability to the profitability during extreme events (US$0.14–1.2/unit and US$0.45–1.60/unit, respectively) (Figure 4.2).



a) b) c)

Figure 4.1: Profitability of product groups a) 1, b) 2, and c) 3 during the 600 days optimization during extreme events



a) b)

Figure 4.2: Profitability of product groups a) 4 and b) 5 during the 600-time period optimization model during extreme events

Figure 4.2 illustrates the fact that the probability of extreme events to occur is higher for PGs 4 and 5 than PGs 1, 2 and 3. This is also linked with the input data used in the MCS. Using the probability of extreme events based on the past data and future prediction reinforces the real-life extreme events in the MCS. PGs 4 and 5 can only receive the capital allocation at the lowest boundary due to this lack of profitability at extreme events (Table 4.1). The highest capital allocation is to PG 1 mainly due to the stable operating performance and very high ROI. Due to the high profitability of PG 3 (Figure 4.1c), 6.6% of the capital fund is allocated to it. Only 1.7% on the capital fund is allocated to PG 2. Hence, during extreme events, 98.0% of the capital fund is distributed between PGs 1, 2, and 3. Given that iron ore industry experienced the highest level of price reductions, a PG managing to survive can be seen as highly resilient.

Table 4.1: Capital fund allocated to each product group (commodity class)

Product Group	**Capital Allocation US$ MM**	**Capital Allocation (%)**
1 (iron ore)	4,393.708	89.7
2 (copper)	84.632	1.7
3 (gold)	321.660	6.6
4 (aluminum)	50.000	1.0
5 (lithium)	50.000	1.0
Total	**4,900.000**	**100.0**

4.3.2 Product group investment with potential divestment

Considering both investment and divestment, the cost to trade needs to be included in the overall portfolio, along with the weight of each commodity class. Assuming the same constraints as above, Table 4.2 illustrates the assumed commodity price, portfolio holding, weight, and additional cost to trade for investment or divestment.

Table 4.2. Commodity price, portfolio holding, weight, and cost to trade for five product groups

Product Group	Price (US$/share)	InitHolding (Portfolio holding)	InitPort (Weight)	Cost to Trade (UnitCost_% US$/share)
1	82	32,078,000	0.54	1/10
2	2,000	127,870	0.05	1/10
3	56	23,283,000	0.27	1/10
4	1,800	81,680	0.03	1/5
5	31.1	18,212,000	0.12	1/5

$PGMean$ represents the mean of the total annual returns of the PGs, based on five years of data on each commodity class:

$$PG_jMean = \frac{1}{n}\left(\sum_{i=1}^{n} PG_jReturn_i\right)$$

Where $PG_jReturn_i$ is the return of PG j, i is the sample number, and n the total number of samples.

$$PGMean = \begin{bmatrix} 0.05 \\ 0.1 \\ 0.12 \\ 0.18 \\ 0.15 \end{bmatrix}$$

The optimization model will include the covariance relating the movement in each of the projects to be financed. $PGCovar$ represents the covariance of the total annual returns of the PG. Given a series of *p* returns for *p* projects, the covariance between projects *i* and *j* can be calculated as follows (Rardin, 2016):

$$PGCovar_{i,j} = \frac{1}{n}\sum_{t=1}^{n} PGReturn_{ti}PGReturn_{tj} - \frac{1}{n^2}\left[\sum_{t=1}^{n} PGReturn_{ti}\right]\left[\sum_{t=1}^{n} PGReturn_{tj}\right]$$

Where $PGReturn_{ti}$ is the value of the return of PG i in period t. In this application, using the returns for five years for each PG, the covariance among the PG returns is as follows:

$$PGCovar = \begin{bmatrix} 0.0064 & 0.00408 & 0.00192 & 0 & 0.00164 \\ 0.00408 & 0.0289 & 0.0204 & 0.0119 & 0.0125 \\ 0.00192 & 0.0204 & 0.0576 & 0.0336 & 0.0478 \\ 0 & 0.0119 & 0.0336 & 0.1225 & 0.0895 \\ 0.00164 & 0.0125 & 0.0478 & 0.0895 & 0.1154 \end{bmatrix}$$

Without the cost to trade, the portfolio object defined by p is called the gross portfolio return. The plot with 600 points (i.e., 600 days of the optimization model as defined above) to obtain the efficient frontier (Kim, et al., 2015) illustrates the (gross) efficient portfolio returns (Figure 4.3). With no consideration for cost to trade in the optimization, the efficient portfolio returns fall between approximately 6 and 13% per year, which is within the positive expected return target.

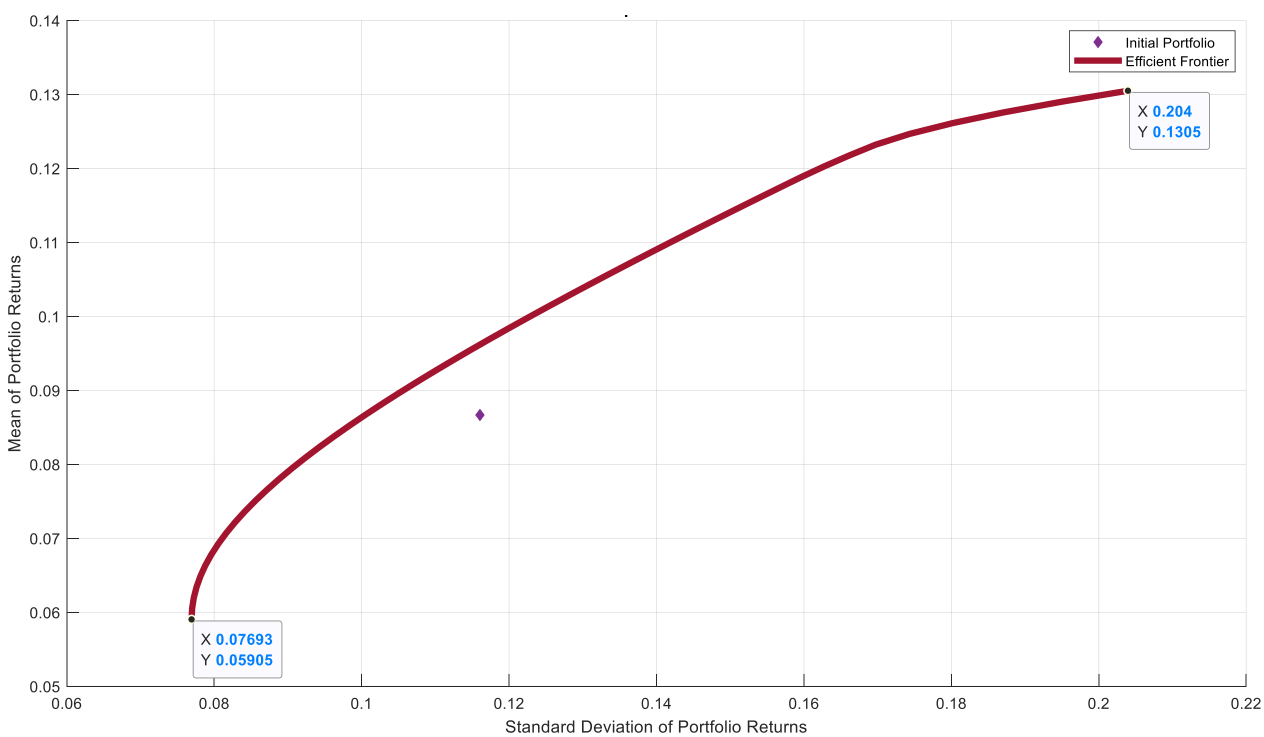


Figure 4.3. Product group allocation portfolio at the efficient frontier

4.3.3 Impact of the cost to trade on the portfolio return

Adding the additional cost to trade into the portfolio optimization model, a new variable q is defined, representing the net portfolio return. Using the method to calculate annualized portfolio returns outlined by Kim, et al. (2015) in MATLAB, it is evident that there is no impact of the cost to trade on the initial portfolio return, but the minimum and maximum efficient portfolio returns are lower (Table 4.3). The cost to trade (i.e., the difference between gross and net portfolio returns) ranges from 0.08 to 0.15%. These values represent the probability to get from the current portfolio to the efficient portfolio at the endpoints of the efficient frontier. The maximum PG return (18%, maximum value of PGMean) is greater than the maximum efficient portfolio return (13%, Figure 4.3) due to the multiple constraints on the operational performance of each commodity class, thus on the allocation of capital among the five PGs.

Table 4.3: Annualized gross versus net portfolio returns

	Gross(%)	**Net (%)**
Initial Portfolio Return	8.67	8.67
Minimum Efficient Portfolio Return	5.90	5.82
Maximum Efficient Portfolio Return	13.05	12.90

4.3.4 Portfolio on the efficient frontier at a specified return level of 0.4

Let us define two scenarios with specified return and risk levels. Assuming an average return of 0.4, a new portfolio object q^* is defined to obtain the portfolio on the efficient frontier that is at

40% of the minimum (5.82%) and maximum (12.90%) net returns. The target portfolio has a new net return of 8.65% and a risk of 10.15%.

$$q* = \begin{bmatrix} 0.5364 \\ 0.2235 \\ 0.0839 \\ 0.1562 \\ 0 \end{bmatrix}$$

4.3.5 Portfolio on the efficient frontier at risk levels of 10, 14, and 18%

Let us move from an initial target risk (*TR*) of 10% to a moderate (14%) and aggressive (18%) target risk to obtain a new efficient frontier, q^{**}:

$$TR = [\,0.10;\ 0.14;\ 0.18\,]$$

$$q** = \begin{bmatrix} 0.5476 & 0.2699 & 0.1500 \\ 0.2191 & 0.3607 & 0.2209 \\ 0.0823 & 0.1172 & 0.2791 \\ 0.1510 & 0.2522 & 0.3500 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix}$$

The investment and divestment to shift from the initial to the moderate portfolio is obtained by averaging the investment and divestment for portfolio *i* in the same time period:

$\sum(I_i + D_i)/2$ =53.07%

Where

I_i represents the investment in PG i.

D_i represents the divesment in PG i.

This result is greater than the moderate target risk of 14%. Thus, using this constrained value in the new estimated efficient portfolio with investment and divestment, the optimal results are displayed in Table 4.4.

Table 4.4: Commodity price, initial portfolio holding and weight, cost to trade, and portfolio, investment, and divestment weight

PG	Price (US$/Share)	InitHolding (Portfolio holding)	InitPort (Initial Weight)	Cost to Trade (UnitCost _%US$/share)	Port (Weight)	Invest (Weight)	Divest (Weight)
1	82	32,077,591	0.53644	1/10	0.41999	0	0.11645
2	2,000	127,871	0.05216	1/10	0.052156	0	0
3	56	23,283,308	0.26591	1/10	0.26591	0	0
4	1,800	81,680	0.02998	1/5	0.16998	0.14	0
5	31	18,212,353	0.11551	1/5	0.091958	0	0.023554

The investment and divestment values in Table 4.4 represent the changes in portfolio weights that are converted into changes in portfolio holdings to determine transactional decisions regarding the investment in or divestment of a specific PG.

$$Total\ Cost\ to\ trade\ =\sum_{i=1}^{5}(Price_i * \ InitHolding_i) * \sum_{i=1}^{5}(UnitCost_i * (Invest_i\ +\ Divest_i))$$

With the cost to trade evaluated from MATLAB (Appendix 2) at US$2.175 M, there is sufficient cash (US$11 M) set aside to pay the cost to trade.

By computing the portfolio holdings and fraction of the fund to invest and divest, the final optimization results are calculated. Table 4.5 contains proposed transactions to move from the initial portfolio of 10% to a moderate-risk portfolio of 14%. To achieve the suggested optimal transactions, there is a need to divest 13.6 M shares of PG 1 and 7.3 M shares of PG 5, and to invest an equivalent of 747 K shares value of PG 4.

Table 4.5: Commodity price, initial portfolio holding and weight, portfolio weight, and investment and divestment share

PG	Price (US$/Share)	InitHolding (Portfolio holding)	InitPort (Initial Weight)	Port (Weight)	Final Holding (Portfolio holding)	Invest (# of Share)	Divest (# of Share)
1	82	32,077,591	0.53644	0.41999	25,114,000	0	6,963,200
2	2,000	127,871	0.05216	0.052156	127,870	0	0
3	56	23,283,308	0.26591	0.26591	23,283,000	0	0
4	1,800	81,680	0.02998	0.16998	463,060	381,380	0
5	31.1	18,212,353	0.11551	0.091958	14,499,000	0	3,713,700

Figure 4.4 illustrates the proposed trade to move from the initial to moderate risk portfolio. It set the highest expected return for the moderate risk. To the efficient portfolio and initial portfolio optimization problem, it adds the location of the moderate-risk (14%) to the efficient frontier for the highest efficient portfolio return of 10.21% per year. This return is an increase compared to the initial sub-optimal return of 8.7% at 11.6% risk. The final efficient portfolio return risk reinforces the fact that higher risk yields higher total returns.

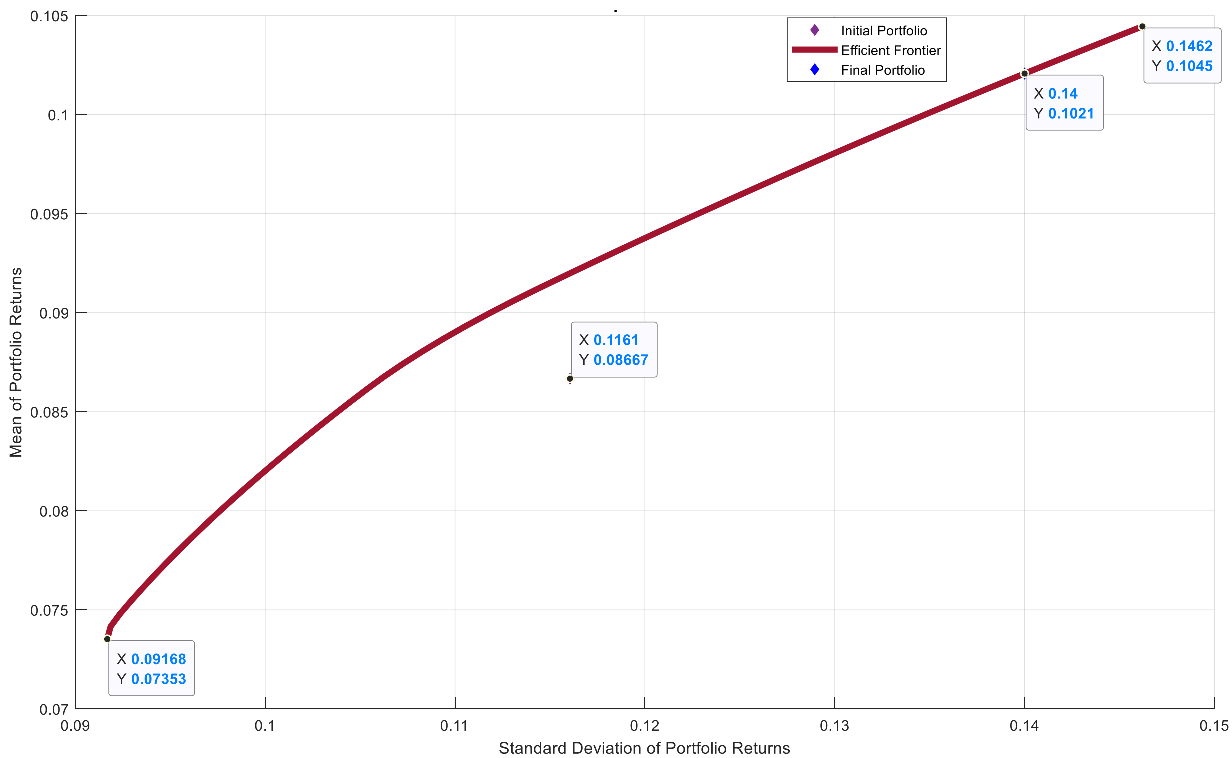


Figure 4.4: Moderate-risk (14%) product group allocation portfolio at the efficient frontier

With investment in PG 4 and divestment in PGs 1 and 5, the new capital allocation is represented by the value of new holdings described in Table 4.5. Table 4.6 compares this result with capital allocation with no option of divestment. When investment and divestment possibilities are considered, PG 1 (iron ore) retains the highest capital allocation. Nevertheless, the divestment fund of one portion of PG 1 has been re-allocated to PGs 2–5, which have seen an increase in their capital allocation fund. PG 3 (gold) retains the second largest capital fund. This methodology also allows positioning PG 4 (aluminum) and 5 (lithium) as the third and fourth largest capital fund; PG 2 (copper) became the PG with the smallest capital fund.

Table 4.6: Comparison of capital allocation without and with divestment option

Product Group	Capital Allocation - No divestment possibility (US$ MM)	Capital Allocation - With investment and divestment possibilities (US$ MM)
1	4393.708	2059.380
2	84.632	255.742
3	321.660	1303.865
4	50.000	833.508
5	50.000	450.908

These results show that in a portfolio optimization model, PGs with superior operational performance do not always yield the efficient portfolio return at an extreme event. Including commodity market behavior with investment and divestment possibilities would produce a more realistic, balanced portfolio for a better return at a moderate risk.

4.4 Conclusion

In this chapter, we considered a typical mining corporation with five PGs located in multiple geographical locations. The problem was to invest or divest capital funds from one or more PGs in extreme events conditions at the efficient portfolio return with acceptable risk. A portfolio optimization model was proposed, with commodity market behavior at the efficient frontier: the higher the probability of extreme events, the more the profitability fluctuates. Model outcomes highlight the impact of commodity market behavior in the decision-making process to invest or divest a fraction of a portfolio.

Including the PG total return prices behavior in the mineral portfolio optimization model will help corporate groups to seize more market opportunity in their investment strategy. This chapter highlights the fact that at the extreme turnover level, there is no need to rush to divest a PG and it does not necessarily provide the highest return at the lowest risk; knowledge of the commodity market behavior at an efficient frontier provides a more efficient portfolio return at moderate risk. It also highlights the fact that a portfolio optimization model with operational performance criteria at an extreme turnover level without commodity market behavior criteria is less efficient than the same model with commodity market behavior criteria. Although this chapter provides a more practical portfolio optimization model with knowledge of the PG total return price behavior, it assumes a constant extreme turnover level that could be defined in future research as a random variable, similar to the simulated PG total return price characterizing commodity market behavior.

Chapter 5: Mining portfolio optimization including country stability, operational performance, and commodity market behavior

5.1 Introduction

In this chapter, we consider the same corporate portfolio as Chapters 3 and 4 with the same available capital fund. The key criteria of the optimization problem (operational performance, country risk, and commodity market behavior at extreme events) and investment/divestment decisions are combined in this new portfolio optimization model. Maximizing the return on investment (ROI) and minimizing the risk remain the main objective function of the new model. Chapter 5 also considers the relationship between country stability and probability of unexpected events. Country risk is characterized by a percentage of the number of extreme events within a period. Monte Carlo simulation (MCS) is used to measure the impact of the uncertainty of the risk on the portfolio. In addition, the results of the simulated future commodity behavior are characterized by a non-parametric distribution of a probability density function: the kernel distribution function. Due to the asymmetry of the density function, a skewed normal distribution is used to characterize the cost function between periods. The combined impact of operational performance, country stability, and commodity market behavior provides a different distribution of capital allocation within the portfolio. The decision-making process is heavily affected by the local community relations and the country in which the mine operates.

5.2 Communities and the country stability

The investment risks due to tensions and challenges between mining companies and indigenous communities are likely to intensify in frequency as we head into the 2020s. As life-of-mine is set to expire at any number of large existing assets, mining companies are investing by expanding exploration in search of alternative new volume and/or seeking approval to expand the boundaries of existing mines into nearby, previously undeveloped, tracts. Technological progress and enhanced mining techniques mean that areas that were once considered too difficult or dangerous to develop are now viable. In the case of the quest for new deposits – the mining industry is, of necessity, extending exploration into more and more remote geographies. These areas are often characterized by remote locations such as: Amazon, Arctic, Mongolia, Western Australian, and Canadian Northern Territories. These isolated and extreme geographies frequently coincide with the remaining traditional lands of indigenous peoples. As mining pushes exploration to the geographic margins there is more and more marginalized and isolated populations. This means that from the earliest stages of exploration, communication and negotiation with local indigenous communities is necessary and critical. It also means that for indigenous populations already often forced off vast tracts of traditional lands by a wide range of historical forces – both manmade and natural - a great deal is at stake in their quest to protect their land, culture and environment. Hence, a high social risk in future investment for mining companies.

When a mining company is looking to invest into new deposits in close proximity, a number of challenges can emerge. Community generational differences are common. When the history of relationships with the company and its benefits to the community are strong and positive, the indigenous elders are more likely to be open to extending existing current agreements to new land access. Even with the old conflicts underlying the current effective agreements, the elders will often feel a sense of pride in what has been achieved for the community and look for opportunities to build on that foundation. However, younger generations may see this as their opportunity to "sit at the table" and redefine the terms and conditions for greater advantage. In some cases this can also lead to attempts to reopen historical contracts covering the current operation. An added complexity can be the divide between resident community members and those who have migrated to urban centers. Urban migration from traditional lands generally increases with the development of a successful mining operation as younger people seek higher education and employment opportunities off-land. Economic benefits from mine-related income for the community also serve to promote this mobility trend. The disruptive impact of tensions and differences around priorities and goals between stakeholders within the indigenous community adds another layer of complexity to achieving extended land access for a second life of mine. These inter-generational and resident and off-land distinctions are equally relevant to new mine deals. The traditional resident community members and the mining company will each face their own distinct problems when "off-land" members seek a voice in negotiations and access to community benefits from the mine development. The resident community population can swell once a mine development is announced. This creates a range of socio-political issues for the community and families. Simultaneously, it adds complexity to finding solutions addressing the widely varying agendas within the community.

No matter how far into the future, when there is a possible desire to invest by expanding a mining operation, the negotiations should always be key to the current mine operations and the community relationships as these will set the context for success or failure of any future investment associated with the expansion. Considering the high level of turn over and rotation rates of mining senior leaders common at remote mine sites, there is always a risk of losing sight of the long-term strategy for maximizing productivity and general the integrity of the mine while avoiding or minimizing conflict with community. It is a given that the community will bring a far deeper historical perspective to the conversations than the company representatives. Out of this evolves the common occurrence of demands for retroactive accountability and compensation for past and unresolved grievances as pre-condition to any new agreement. The history, the notion of time – past, present and future are understood and valued quite differently across indigenous cultures and this will impact what is valued, and how it is valued by a community.

It would be naïve and foolish to suggest that economic opportunity is not a critical consideration for indigenous communities in brokering a deal with a mining company but it is also true that in indigenous communities there is generally an essential intangible economy that operates at the level of symbolic value. Corporations often don't understand that this alterative economy is a powerful basis in community decision-making. While it can be a barrier it can also provide unexpected opportunities for collaboration. When serious conflict arises between the community and company, in the majority of cases this alternative intangible economy can provide insight on the root cause and the way forward. Misunderstandings and missed opportunities of this kind abound today and continue to lead to strikes and blockades. In extreme cases these failed moments can lead to a mine closure, or abandonment of a development project effecting losses in high capital investment for the mining corporation.

Country stability denotes the investment risk of a country. It is the likelihood to have to loss the investment made in a particular country due to political, economic or technological instabilities. With the globalization and expansion of financial market internationalization, there is a growing interest in assessing investment risk associated to country. Multiple organizations have developed rating tools quantifying the country risk in a single rating score that could facilitate the comparison of all countries in the world. Although the assignment of this rating score is different between agencies, Alexe, et al. (2003) demonstrated that in spite of the different analysis approaches, there is a strong correlation between the country rating score within the main financial services agencies such as Euromoney, Standard & Poor's and Moody's. Two approaches are developed to measure the country risk: the country rating scores and the CDS spread.

5.3 Country risk rating

Euromoney (2019) developed a methodology to assess the country risk value for 186 countries, by combining the scores from consensus experts, the scores on the accessibility of sovereign borrowers to the capital market, and the debt indicators from World Bank and International Monetary Fund (IMF). This assessment provides a 90% weight for a qualitative model from expert opinion, which is based on the current position of the country and a 10% weight for the basic quantitative value, which is based on the sovereign debt indicator. The 90% weight for the qualitative model is then split into four categories with 35% weight allocation to political risk rating, 35% weight for economic risk rating, 10% weight for structural risk rating and 10% weight for the international market. The assessment methodology developed by Euromoney (2019) is presented as follows:

5.3.1 Political risk rating

The assessment of political risk has six categories with a measure ranging from 10 to 0: (1) The Corruption score, where the highest value means the country as the lowest corruption. (2) The Government's non-payments score has the lowest number for an extremely high risk of interference from the government. (3) The Government stability score has the highest value for extremely high government stability and the lowest, the nonfunctioning government. (4) The Information access score has the highest value for completely unrestricted and reliable data and the lowest value for the totally restricted and unreliable data. (5) The Institutional risk score has the highest value for the extremely efficient and totally independent institution. (6) The Regulatory and policy environment score represents how well a government consistently implements its regulations and policies.

5.3.2 Economic risk rating related to a specific country

The rating of economic risk has five categories with a score ranging from 10 to 0: (1) The Employment score means that the highest refers to no risk to employment, the lowest is the risk to the economy. (2) The Economic outlook score denotes the likelihood of a catastrophic recession - The higher the number is the highest is the likelihood to have unprecedented growth. (3) The Monetary policy score denotes the credibility and effective implementation of the monetary policy. (4) The Government finances score denotes the robustness of a country's fiscal strength. (5) Bank stability score denotes a perfectly functioning banking system.

5.3.3 Structural risk rating

The rating of the structural risk has four categories with a score ranging from 10 to 0.

(1) The demographics score denotes the demographic balances on economic and political stability. (2) The labor market score denotes a good functioning labor environment. (3) The hard infrastructure score denotes the well-maintained country's physical infrastructure. (4) The soft infrastructure score denotes the balanced capacity of skilled labor force with good functioning social institutions.

In this study, country stability is the combination of the country risk, political risk and sovereign risk into a single country rating score. Other financial and economic services such as Euler Hermes, Economist Intelligence Unit and GCR Country Risk Scores have developed a similar calculation of the country rating. The main issue with these scoring schemes is how these scores will be translated into the project evaluation. The scoring based approaches are useful for a general overview; however, for a micro level specific project evaluation, it may not be really useful because it is very difficult to link the discount rate, specifically risk premium. In addition to the Country's rating methodology discussed above, the approach based on the Credit Default Swap (CDS) spreads is illustrated in the following section.

5.4 Credit Default Swap

The CDS spread can be used to represent a country's default risk that helps the investors to speculate on the likelihood of a country to default to repay its debt obligations. This usually take places when, in normal circumstances, a country borrows more than the capacity of its earning power. Multiple factors are considered in the calculation of the country's default risk (Damodaran, 2019a):

5.4.1 Degree of indebtedness factor

This is characterized by how much the country owes to investors and to its citizens. The country's Gross Domestic Product (GDP) is used to scale the debt of the country. This is represented as the percentage of GDP. Due to the additional commitment to citizens, the degree of indebtedness provides does not coverer the full default's risk as it only considered the debt level.

5.4.2 Social service commitments factor

The social service commitment represents the obligations of the country to its citizens with regard to pension's payment of health care coverage. Depending on the level of commitments, this factor could heavily affect the default risk.

5.4.3 Revenues to government's factor

The amount of tax that a government can collect from its citizens or from the companies operating in the country helps to increase the capacity of the country to meet its debt obligation.

5.4.4 Stability of revenue factor

The stability of revenue factor is characterized by diversified economies with more consumption and taxing income in the country's economy. The taxes are generated by income tax, sales and value-added tax.

5.4.5 Political decision factor

The default likelihood for a country can be associated with the political decisions that reflect the pressure on political leaders.

5.4.6 Other entity factors

This factor characterizes default risk of a country in relation to default of its main political and economic continental partners. For example, being a member of a union such as the European Union, African Union, North American Union and Union of South American Nations adds the default risk of its members as well as the benefits.

Damodaran (2019a) demonstrates that default spreads measurement is related to country rating and the country's default risk. Investors can buy protection against default through the payment of the spread specified as a percentage of notional value. The measure of the country default risk is represented by the value of the credit default swaps. For both debt and bond investment, country default risk is the most suitable measurement. Nevertheless, for those investing in equity, the notion of country equity risk premium is more suitable.

Damodaran (2019a) defines three approaches to evaluate country risk premium:

(1) Default spread: It represents the charge that investors paid for buying bonds. It is evaluated in three different ways. (a) The current default spread on the CDS market: It also represents the yields on bonds from the reference country with the default-free bond. (b) The average spread on bond: It is estimated by using the spreads from CDS categorized by Country rating/score defined in this section, and then average the spreads rating in the same class. (c) The imputed spread: It is considered for emerging market countries where the bonds are not denominated in US dollars or in a default-free rate currency.

$$DS_{country} = SBY_{country} - RFR_{country}$$

Where DS denotes the default spread. SBY denotes the sovereign bond yield, and RFR denotes the risk free rate of the country.

(2) Relative equity market standard deviations: This is the measure of the volatility of the market, it represents the variation in stock prices. The relative risk is obtained by scaling the standard deviation between two markets/countries. Then, the relative standard deviation for a country is provided by the following formula (Damodaran, 2019a):

$$RSD_{country1} = \frac{SD_{country1}}{SD_{country2}}$$

Where RSD denotes the relative standard deviation, and SD is the standard deviation for a specific country.

Assuming that the risk premium for Country 2 is available through historical data, and there is a linear relationship between equity market standard deviation and equity risk premium, the equity risk premium for Country 1 is provided by the formula below (Damodaran, 2019a).

$$ERP_{country1} = RP_{country2} \times RSD_{country1}$$

Where ERP is the equity risk premium for a specific country.

Assuming that the equity risk premium of Country 2 is the reference market point, the country risk premium for Country 1 isolated from the risk premium for Country 2 is provided by the following formula:

$$RP_{country1} = ERP_{country1} - ERP_{country2}$$

(3) Default spreads are combined with relative standard deviations: This approach considers both the volatility of the equity market and the volatility of the bond market that has been used for the spread's estimation. The country risk premium is then calculated as follows (Damodaran, 2019a):

$$RP_{country} = DS_{country} \cdot (\frac{SD_{equity}}{SD_{country\ bond}})$$

Where $RP_{country}$ denotes the country risk premium, $DS_{country}$ is the country default spread, SD_{equity} is the volatility/standard deviation in the country equity, and $SD_{country\ bond}$ is the volatility/standard deviation in the country bond.

The total equity risk premium of a country is then formulated by:

$$ERP_{country} = DS_{country} \cdot \left(\frac{SD_{equity}}{SD_{country\ bond}}\right) + ERP_{mature\ market/country\ premium)}$$

5.5 Application of the country risk premium valuation

For the case study developed in this research, five mining countries are considered: the United States of America (USA), Canada, Australia, Mongolia and South Africa. Similar to the methodology provided by Damodaran (2019a), the calculation of each of the country risk premium is described in four steps:

Step 1: Estimate the mature market risk premium

The rating agencies announced the equity risk premium in annual basis. From the Standard & Poor's 500 index data (Standard&Poor's, 2019), the implied equity risk premium for the current year is evaluated at 5.96%. Assuming the USA is the mature market premium, ERP_{USA} denotes the USA equity risk:

$$ERP_{USA} = 5.96\%$$

Step 2: Estimate the default spread for each country

To convert the country rating into to default spread, a rating equivalent in basis points needs to be considered. The equivalent rating in updated default spread in basis points is then provided in table 5.1 (Damodaran, 2019b).

Table 5.1: Equivalent default spread in basis points

S&P Rating	***Default spread in basis points***
A+	79
A	96
A-	135
AA+	45
AA	56
AA-	68
AAA	0
B+	508
B	621
B-	734
BB+	282
BB	339
BB-	406
BBB+	180
BBB	215
BBB-	248
C	1800
CCC+	846
CCC	1016
CCC-	1128
NR	NA

Based on the sovereign rating from S&P 500 (Standard&Poor's, 2019), using the average of CDS spreads and USA bond spreads by rating class, the rating-based default spread by country is illustrated in Table 5.2.

Table 5.2: Default spread for the each country

Country	**S&P's rating**	**Rating-based Default Spread**
Australia	AAA	**0.00%**
Canada	AAA	**0.00%**
Mongolia	B-	**7.34%**
South Africa	BBB-	**2.48%**
United States	AAA	**0.00%**

Step 3: Estimate the country risk premium from the default spread

To estimate the country risk premium, the calculation of the volatility/standard deviation of the equity market relative to the volatility of the bond market has been calculated. Using the S&P data, the standard deviation for the bond market bond is given by 13.68% and the standard deviation for the equity market is 11.12%; hence, the conversion of the default spread into a risk premium is calculated and illustrated in table 5.3.

$$RP_{country} = DS_{country} \cdot \left(\frac{13.68\%}{11.12\%}\right)$$

Table 5.3: Country risk premium

Country	S&P's rating	Rating-based Default Spread	Country Risk Premium
Australia	AAA	0.00%	**0.00%**
Canada	AAA	0.00%	**0.00%**
Mongolia	B-	7.34%	**9.03%**
South Africa	BBB-	2.48%	**3.06%**
United States	AAA	0.00%	**0.00%**

Step 4: Estimate the total equity risk premium

Considering the initial maturity market premium of 5.96%, the total equity risk premium can be calculated as

$ERP_{country} = RP_{country} + ERP_{mature\ market/country\ premium})$,

Where the values are given in Table 5.4.

Table 5.4: Total equity risk premium for each country

Country	S&P's rating	Rating-based Default Spread	Country Risk Premium	Total Equity Risk Premium
Australia	AAA	0.00%	0.00%	**5.96%**
Canada	AAA	0.00%	0.00%	**5.96%**
Mongolia	B-	7.34%	9.03%	**14.99%**
South Africa	BBB-	2.48%	3.06%	**9.02%**
United States	AAA	0.00%	0.00%	**5.96%**

The evaluation of country risk premium represents the characterization of the country stability. In addition to country stability's variable, previous operational performance and commodity market behaviors discussed in Chapters 3 and 4 are considered in the proposed model with the following methodology.

In portfolio management, the country risk can be incorporated through the discount rate into the project evaluation. Damodaran (2013) proposed an approach how to incorporate the county risk in the discount rate. Risk premium portrays the request of the investors undertaking average risk for their investment. This premium reflects the level of risk aversion of an investor or riskiness level of the average risk of an investment. As known, the discount rate has two components as risk-free rate and risk premium. Regarding country risk premium, there are three ways to quantify this risk: government bond spread issued in the US dollar, the CDS spread and the spread based on the value announced by the credit rating agencies.

5.6 Methodology

The modeling involves optimization with a loop across combinations of state variables through time. This is surrounded by a second optimization loop, which improves the long-term action of the first optimizer by changing the cost function weighting. The assumed pricing simulations are run first and the results are binned. The binned results give us the kernel distribution of expected future prices. (Bowman & Azzalini, 1997) define a kernel distribution $K(\cdot)$ as a characteristic of a smoothness of a density curve, where $\hat{f}_h(x)$ is the kernel density estimator defined in Equation 5.1, x_i is the random sample, x is the real value, h is the bandwidth, and s is the sample size.

$$\hat{f}_h(x) = \frac{1}{sh}\sum_{i=1}^{s} K\left(\frac{x-x_i}{h}\right) \tag{5.1}$$

The MATLAB optimizer function (described in Appendix 2) keeps certain paths over time by binning capital results and recording the combination of investments that reach that location. These bins and distributions are used in the next time stage to generate a new set of correlated data for the simulation to run. The final distribution of capital is then used as the cost function for a second optimizer, which adjusts the first optimizer's weighting function.

Assuming that the probability of extreme events per time-period is associated with each country, the simulation of extreme events is considered to highly negatively deviate from the normal condition. This is then characterized as a skewed normal distribution, defined as a shape parameter characterizing the deviation from a non-normal to a normal distribution (Gupta, et al., 2004).

5.7 Application

Consider a mining corporation with five product groups (PGs) related to five commodities where the operational performance combines the production efficiency and operating cost. PGs 1–5 are iron ore, copper, gold, aluminum, and lithium. The more productive an operation, the better managed are operating costs and the more capital is available to invest (Njike & Kumral, 2019): the capital invested is a function of the production efficiency and operating cost of each PG. The proposed optimization model takes into account the stochastic nature of production efficiency, commodity price, operating costs, and stability of the country where the project is implemented.

Knowing that mining industry also faces difficulty retaining employees, particularly in remote locations, the following assumptions are used:

- Production efficiency and operating cost are correlated with the capital invested and some random variation due to employee turnover. The production efficiency value is defined from the overall operations value stream not just for a single production line embedded in the operations. The overall operations value stream mapping allows identifying the value-added and non-value-added activities. To be efficient, the overall production needs to have the least non-value-added activities with more efficiency in the planning and execution of value-added activities.
- The operating cost function is a skewed normal distribution and correlated to the commodity price.
- The number of extreme events per time period is associated with each country.

The main objective is to define the right amount of capital to be allocated from the available capital (US$4,900 MM) to invest in all PGs with an accepted corporate risk tolerance of 0.05.

5.7.1 Capital allocation: two countries with stability correlated to number of extreme events

The five PGs are distributed in two countries, with 20 simulations per capital, PG, and country.

$Np = 5$ represents the number of PGs

$Nc = 2$ represents the number of countries

$N = 20$ represents the number of simulations per capital, PG, and country

$IM = zeros(Np * Nc, 1)$ represents the investment product "matrix", the current capital invested in PG and country

$PGnum = repmat([1{:}Np], 1, Nc)$ represents the PG number

The country stability is characterized by the country risk array (CRA).

$CRA = [1{:}Nc]/100$. This will be the percent of extreme events per regulated period.

After scaling the five commodity prices, a total of 22,800 simulations were run with the MATLAB function (Appendix 2). Results of the MCS with commodity price per unit show that the lowest selling price per unit is allocated to PG 1, while the highest price per unit is allocated to PG 5 (Figure 5.1). PGs 3 and 4 have similar commodity price ranges.

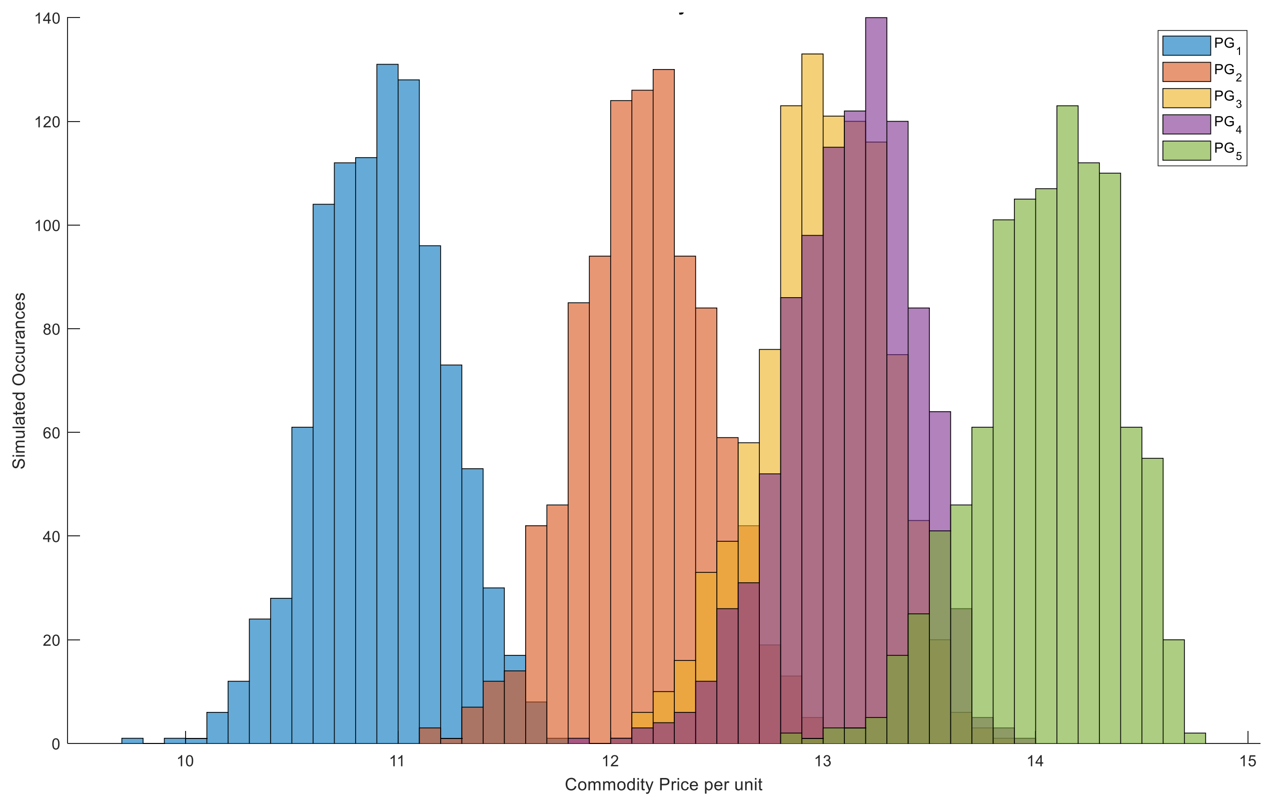


Figure 5.1: Commodity price per unit from Monte Carlo simulation

Commodity prices overlap among the PGs: the top commodity price per unit of PG 1 is the lowest price for PG 2. The top commodity price per unit of PG 2 is the lowest price for PGs 3 and 4, and the top commodity prices per unit for PGs 3 and 4 overlap with the low price range of PG 5.

The selling price of the same commodity in country 1 can be different than the selling price in country 2. Table 5.5 illustrates a reduction of weight for PG 5 in country 2, with the highest selling price per unit allocated to PG 5.

Table 5.5: Optimized weighting by product group and by country

Product Group	*Country 1*	*Country 2*
1 (iron ore)	1	1
2 (copper)	0.9525	1
3 (gold)	1	1
4 (aluminum)	0.73875	1
5 (lithium)	1	0.99406

The optimized investment in each country by PG during a time period is obtained with MATLAB code. The MATLAB optimizer function includes the weighting of the cost and the distribution of capital investment funds for the five PGs in country 1 and 2. Country 1 is less risky for investment, mainly for PGs 1 and 2 (Figure 5.2). PG 4 received the least amount of capital.

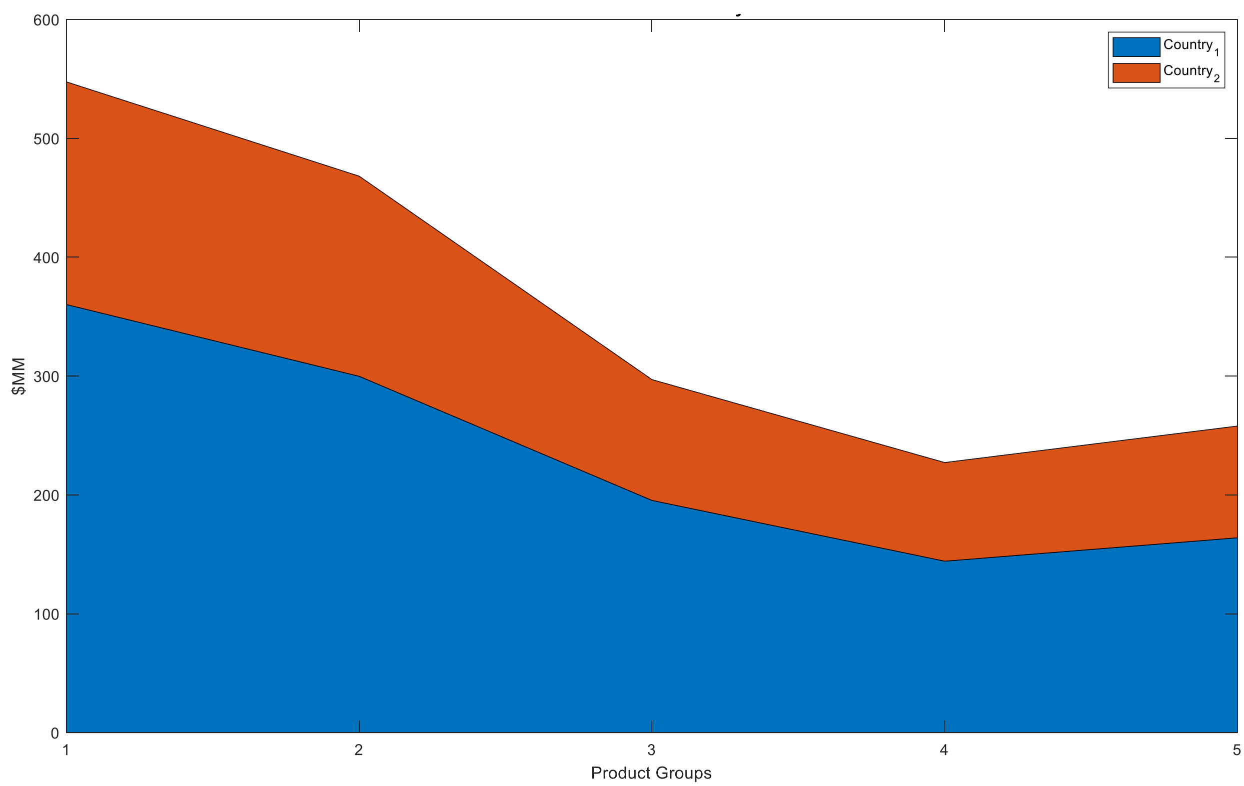


Figure 5.2: Allocation of country investment by product group

As stated earlier, the higher the operational performance, the higher the capital investment. PGs 1 and 2 are the most affected by the high investment of capital due to their high operating performance. This result reinforces the initial hypothesis of better operational performance positively affecting the investment weight in the portfolio. It also reinforces the result obtained in Chapter 3 regarding the impact of operational performance on capital investment and is reflected in Table 5.6 where 56% of the allocated capital fund is allocated to PGs 1 and 2. The better the operational performance, the easier the capital fund approval. In the capital portfolio market, more capital funding is always available for a portfolio achieving the best financial performance.

Table 5.6: Capital allocation distribution by product group and country

Product Group	*Country 1 (US$ MM)*	*Country 2 (US$ MM)*
1	360.21	187.31
2	299.81	168.39
3	195.39	101.66
4	144.27	82.983
5	163.97	93.99
Total	1,163.65	634.333

The highest selling price per unit is associated with PG 5 (Figure 5.1), yet the most capital is allocated to PG 1 (Table 5.6). The total capital fund allocated is US$1,798 MM, which is less that the available fund of US$4,900 MM. Hence, there is no need to invest all available capital; there is an optimal level at which investing should stop. The remaining capital can be used for future investments in the corporate portfolio.

5.7.2 Capital allocation: five countries with their respective country risk

Currently, investments of the top two mining companies (BHP and Rio Tinto) are distributed among five countries: the USA, Australia, Canada, South Africa, and Mongolia. Applying the methodology above (Appendix 3) to these countries while maintaining the same objective function—minimize the risk and maximize the ROI—yields a different pattern of optimal return and risk for each of the PGs within each country (Figure 5.3). Mongolia has the lowest risks and returns, due to ongoing project implementation phase there. The USA, Canada, and South Africa show similar returns, with the lowest risks in Canada and highest in South Africa. PG 5 has the highest risk commodity (2.86%) with the maximum return (9.58%). Although the risk in South Africa is 2.15%, investment in PG 4 will provide a good return (8.36%). The USA and Canada also have good returns of 8.36 and 8.48%, respectively, for risks of 1.93 and 2.03%, respectively. PGs 2 and 3 are more profitable in Australia, with respective risks of 1.05 and 1.51% and respective returns of 6.71 and 8.36%. PG 1 has the least return and risk at 5.97% and 0.08%, respectively, with no optimum level of investment for the five countries.

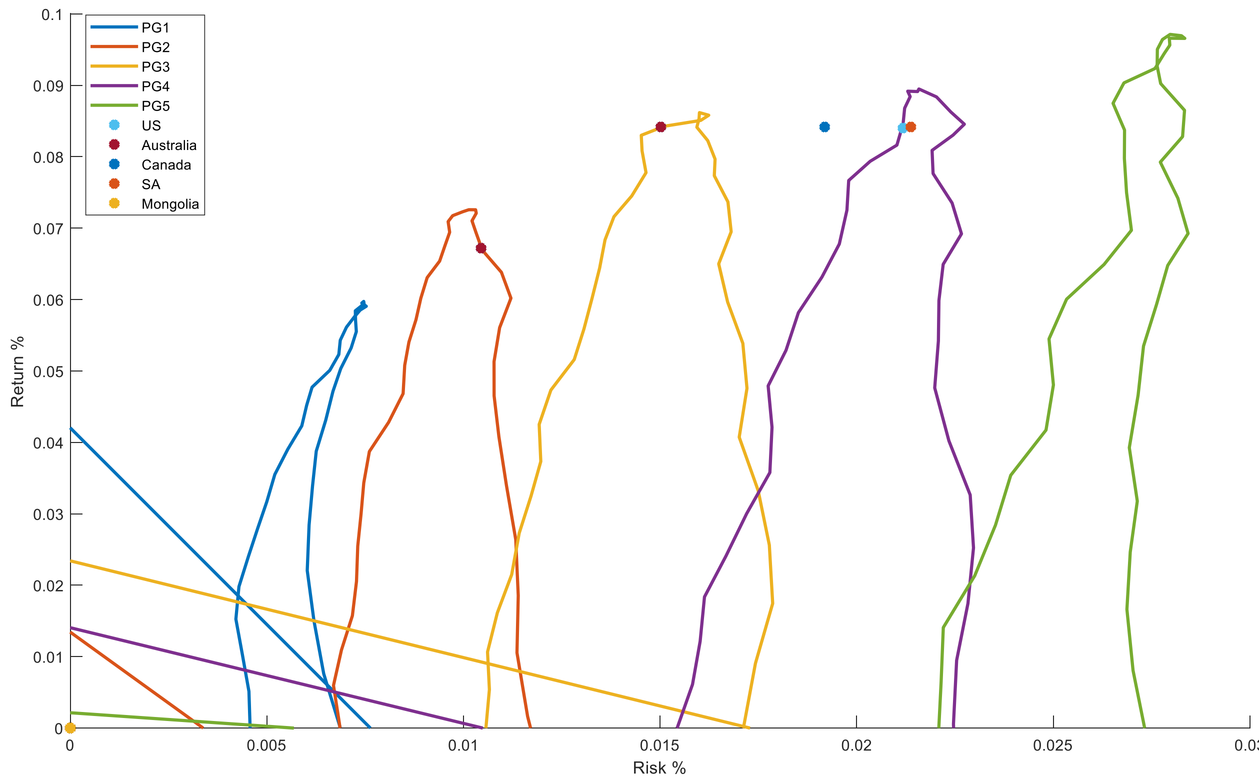


Figure 5.3: Optimal return and risk for five products groups (PGs) and countries

PG 3 will receive capital funds in four of five countries (Table 5.3). Australia will welcome investment for PGs 2 and 3, and the USA will receive investment for PGs 2, 3, and 4. No capital investment will go to PG 1 or to Mongolia.

Table 5.7: Result of the capital allocation distribution by product group and by country

Product Group	*USA*	*Australia*	*Canada*	*SA*	*Mongolia*
1	0	0	0	0	0
2	539.00	539.00	0	0	0
3	543.21	541.11	539.53	539.00	0
4	581.11	0	0	0	0
5	0	0	0	0	0

The overall expected return is US$301.5 MM and the overall expected risk of losing money for the corporate portfolio investment during the time period is US$61.3 MM. The total capital allocated is US$3,822 MM, which is less that the available capital fund of US$4,900 MM. Similar to the two-country scenario, there is no need to invest the full amount of capital. Instead, there is an optimal level above which investing more funds is wasteful. The remaining capital funds could be used for future investments in one or more countries with more profitable commodity and low risk.

5.8 Conclusion

This chapter illustrates more realistic capital allocation in a corporate portfolio, including in the same optimization model the production efficiency, commodity behavior, and country stability. Stochastic simulation of commodity price behavior combined with country risk yields more practical information in the resolution of the portfolio optimization model.

The optimization model embeds the future state of the commodity with the known parameters regarding the current performance of the company and the country risk correlated to the extreme events occurring in the geographical location. These criteria affect the distribution of the capital fund in the country of investment, contrary to traditional capital investment based on capital budgeting decisions methods such as net present value, payback period, internal rate of return, ROI, or profitability index. Although the results of the portfolio optimization problem include criteria such as profitability, operational performance, extreme events, country risks, and sustainable return, the prioritization and evaluation of their weight in the decision-making for capital allocation still needs to be done.

Chapter 6: Project selection: Decision-making and prioritization through AHP, TOPSIS and PROMETHEE

6.1 Introduction

In the previous chapters, we approached the project selection problem through portfolio optimization models. The problem was formulated with the dual objective of minimizing risk and maximizing returns under the constraints of operational performance, country risk, and unexpected events. In this chapter, the same problem will be addressed from a different perspective using multi-criteria decision-making (MCDM).

The portfolio selection and evaluation problem has been reviewed in multiple studies. Traditionally, the primary goal of portfolio management has been to maintain a sustainable healthy return. Past performance is a typical criterion for many decision-makers in the minerals industry. Nevertheless, with the competitive global market, making the right investment decision at the right time with the right weighted criteria go hand in hand. The MCDM evaluates the performance of alternatives based on the distance from the ideal solution: the preferred alternative has the shortest distance from the ideal solution. Among the many methods of MCDM, this thesis investigates the analytic hierarchy process (AHP), technique for order of preference by similarity to ideal solution (TOPSIS), and preference ranking organization method for enrichment of evaluations (PROMETHEE). AHP uses the hierarchical principle while TOPSIS uses the distance principle and PROMETHEE uses the outranking principle. In AHP method, the criteria weights are determined, and then the alternatives are ranked. In TOPSIS method, the criteria weights are set and then the rating of the alternatives are determined. In PROMETHEE method, a partial preorder is set up and a complete order is made. In AHP method the criteria value are not necessary, while with TOPSIS method a min max optimization is set for each of the criteria. This reduces the subjectivity of the decision making using TOPSIS method. In addition, in PROMETHEE method, the positive and negative outranking flow are calculated for each alternative, and the alternative with higher flow is preferred.

6.2 Analytic Hierarchy Process

The AHP is a systematic process to incorporate factors such as logic, experience, knowledge, expertise, and optimization into decision-making. Originally designed by Saaty (1990), it simplifies the multi-criteria problem into a three-tiered hierarchical structure (Figure 6.1) with four process stages (Figure 6.2).

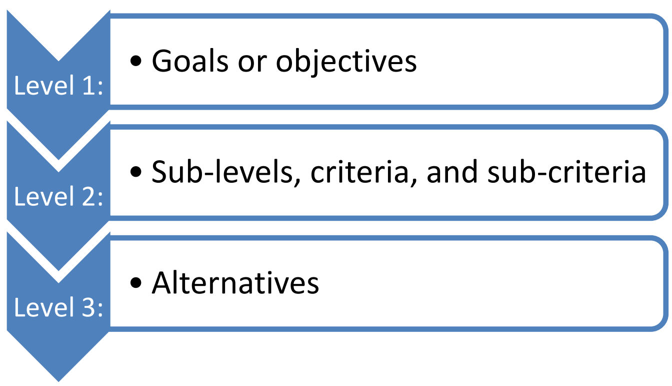


Figure 6.1: The three levels of the analytic hierarchy process

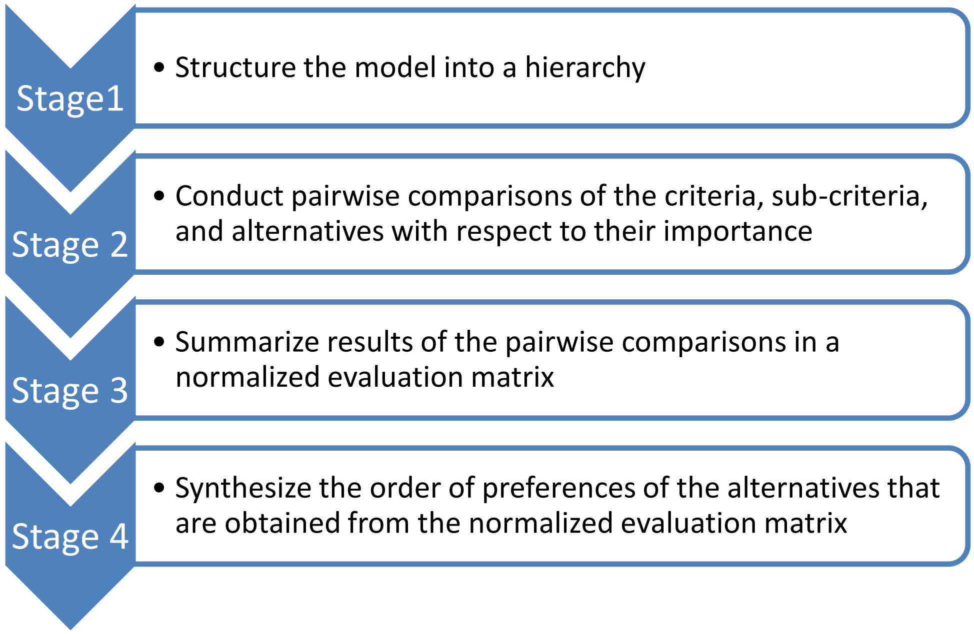


Figure 6.2: The four process stages of the analytic hierarchy process

The following criteria were ordered using AHP to obtain their respective weights to incorporate into mining portfolio management.

Sustainable healthy return: During the life of the operations, a consistent positive trend is observed in benefits for the corporate business, the employees working in the product group (PG), and the community/country where the PG is operating. This includes, social, country and environmental risks described in the following sections.

Social risk: This is the risk that a mining company will cause social disruption to local and indigenous communities.

Country risk: This is the risk to assets and investments that an international mining corporation operating in a country assumes stemming from political, legal, and taxation instabilities. In other words, it is the risk associated with effects of policies of the different countries with respect to corporate taxes, appropriation, and royalties.

Environmental risk: This is the risk that a mining operation will cause environmental damage, including risks to human health, land, soil, water, and air. The environmental effects can cause fatalities and health problems of living organisms at local, regional, or global levels.

Risk of unexpected event: The probability of an unexpected event is extremely low, but the effects can be dramatic. For example, a very sharp price decline in a short time or a natural disaster such a flood or earthquake will greatly affecting a mining operation.

Operational performance: This criterion is derived from the production efficiency of the PG in the country of operation. The past performance of the PG is also associated with the workforce.

Profitability: This criterion is defined as the net present value and the return on investment related to the investment that needs to be made at the PG level.

6.2.1 Stage 1: Develop hierarchical structure

Level 1: The main goal is to minimize risk and maximize the corporate portfolio return.

Level 2: The criteria are social risk, country risk, environmental risk, risk of unexpected event, operational performance, and profitability.

Level 3: The alternatives are PGs for iron ore, copper, gold, aluminum, and lithium.

6.2.2 Stage 2: Conduct pairwise comparison of the criteria, sub-criteria, and alternatives

The scale of relative importance is shown in Table 6.1. The following examples illustrate application pairwise comparison (row element/column element) defined by (Saaty, 1990).

- Social risk is extremely important with respect to country risk, unexpected event and profitability - hence the number 9
- Social risk has a very strong importance with respect to operations performance - hence the number 7
- Social risk is of moderate to strong importance with respect to environmental risk -hence the number in-between 3 and 5 is 4
- Country risk is 2 with respect to unexpected event: it is of equal (1) to moderate importance (3).
- Environmental risk is 8 with respect to unexpected event: it is of very strong (7) to extreme importance (9).
- Environmental risk is of very strong importance (7) with respect to country risk.

Table 6.1: Scale of the relative importance

1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2, 4, 6, 8	Intermediate values between the above
1/3, 1/5, 1/7, 1/9	Values for inverse comparison

6.2.3 Stage 3: Create evaluation matrix

Based on the mining expert judgement with multiple surveys performed in different geographical mining operations from Australia, Canada, United States, Mongolia and South Africa, Table 6.2 provides the outcome of the pairwise comparison of all criteria and alternatives. Results from the pairwise comparison matrix are then normalized, such that the sum of each row or column is 1 (Table 6.3). The weighted sum of the criteria weights is then calculated (Table 6.4).

Table 6.2: Pairwise comparison matrix

	Unexpected event	*Country risk*	*Environ. risk*	*Social Risk*	*Operation perf.*	*Profitability*
Unexpected event	1	1/2	1/8	1/9	1/5	1/4
Country risk	2	1	1/7	1/9	½	1/3
Environ. risk	8	7	1	¼	3	7
Social risk	9	9	4	1	7	9
Operation perf.	5	2	1/3	1/7	1	1/2
Profitability	4	3	1/7	1/9	2	1
	29.0	**22.5**	**5.7**	**1.7**	**13.7**	**18.1**

Table 6.3: Normalized pairwise comparison matrix

	Unexpected event	*Country risk*	*Environ. risk*	*Social Risk*	*Operation perf.*	*Profitability*	***Criteria weights***
Unexpected event	0.034	0.022	0.022	0.064	0.015	0.014	**0.029**
Country risk	0.069	0.044	0.025	0.064	0.036	0.018	**0.043**
Environ. risk	0.276	0.311	0.174	0.145	0.219	0.387	**0.252**
Social risk	0.310	0.400	0.696	0.579	0.511	0.498	**0.499**
Operation perf.	0.172	0.089	0.058	0.083	0.073	0.028	**0.084**
Profitability	0.138	0.133	0.025	0.064	0.146	0.055	**0.094**
Total	**1**	**1**	**1**	**1**	**1**	**1**	**1**

Table 6.4: Weighted sum value matrix

	Unexpected event	*Country risk*	*Environ. risk*	*Social risk*	*Operation perf.*	*Profitability*	***Weighted sum***
Unexpected event	0.028	0.021	0.031	0.055	0.017	0.023	**0.177**
Country risk	0.057	0.043	0.036	0.055	0.042	0.031	**0.265**
Environmental risk	0.228	0.300	0.252	0.125	0.251	0.655	**1.812**
Social risk	0.257	0.386	1.008	0.499	0.586	0.843	**3.580**
Operation perf.	0.143	0.086	0.084	0.071	0.084	0.047	**0.514**
Profitability	0.114	0.129	0.036	0.055	0.168	0.094	**0.596**

Table 6.5: Weighted sum value matrix as a fraction of criteria weights - Lambda

	Unexpected Event	*Country risk*	*Environ. risk*	*Social risk*	*Operation perf.*	*Profitability*	*Weighted sum*	***Lambda***
Unexpected event	0.029	0.021	0.031	0.055	0.017	0.023	0.177	6.206
Country risk	0.057	0.043	0.036	0.055	0.042	0.031	0.265	6.163
Environ. risk	0.228	0.301	0.252	0.125	0.251	0.655	1.812	7.192
Social risk	0.257	0.386	1.008	0.499	0.587	0.843	3.580	7.172
Operation perf.	0.143	0.086	0.084	0.071	0.084	0.047	0.514	6.140
Profitability	0.114	0.129	0.036	0.055	0.168	0.094	0.596	6.361

6.2.4 Stage 4: Order alternatives relative to evaluation matrix

The consistency index (*C.I.*) is calculated from the lambda max—defined as the priority vector or the Eigen vector of the matrix—and the number of attributes being evaluated ($n = 6$). The lambda max is 6.439.

C.I. = (6.439 – 6)/(6 – 1) = 0.088

Then, the consistency ratio is calculated as the *C.I.* divided by 1.24, which is the random index (*RI*) of the randomly generated pairwise matrix for six attributes (Table 6.6) (Laeven & Stadje, 2014). Thus, the consistency ratio is 0.088/1.24 = 0.071. Since this value is less than the consistency standard of 0.10 (Saaty, 2008), the matrix is reasonably consistent (i.e., it meets the standard criteria for consistency). It is then possible to proceed with the process of decision-making using the criteria weight matrix in Table 6.7. These weights from the AHP methodology are then used to inform the TOPSIS.

Table 6.6: Random index table

n	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
RI	0.00	0.00	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Table 6.7: Criteria weight matrix

Attributes	***Criteria weight***	***Ranking***
Unexpected event	0.029	6
Country risk	0.043	5
Environmental risk	0.252	2
Social risk	0.499	1
Operation performance	0.084	4
Profitability	0.094	3

6.3 Technique for Order of Preference by Similarity to Ideal Solution

The TOPSIS was developed by Hwang and Yoon (1981), who proposed six main stages to implement the methodology (Figure 6.3):

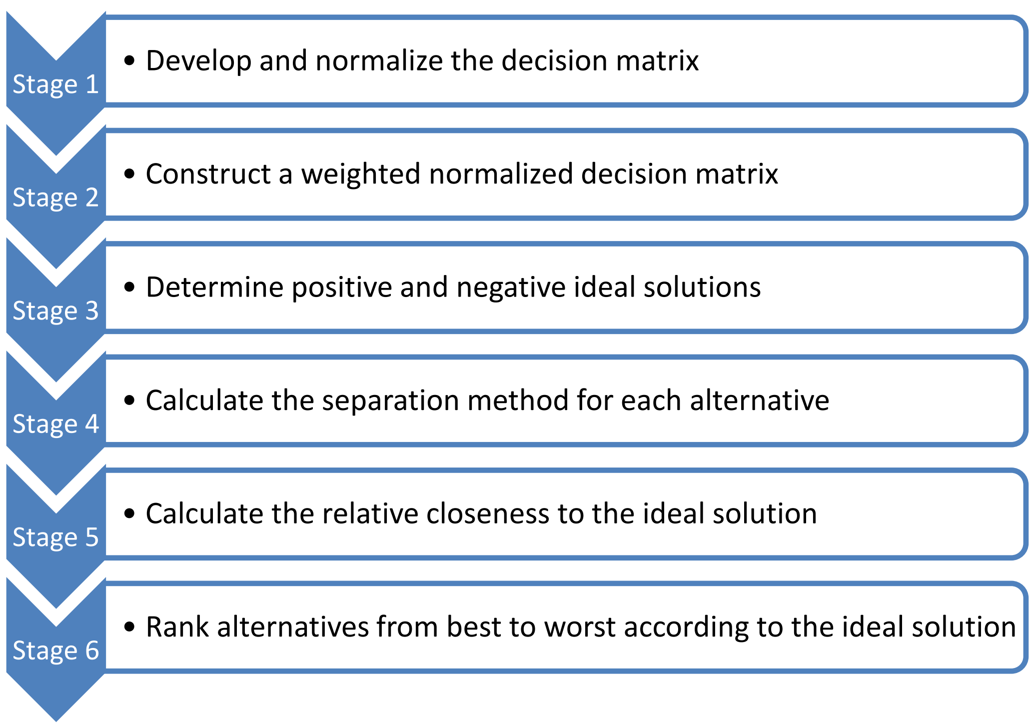


Figure 6.3: Description of six stages related to TOPSIS

6.3.1 Stage 1: Create the decision matrix

Table 6.8 provides a decision matrix of the five commodity classes against the six criteria.

Table 6.8: Decision matrix

	Unexpected event	*Country risk*	*Environ. risk*	*Social risk*	*Operation perf.*	*Profitability*
Iron	8	5	3	6	7	9
Copper	7	4	8	8	8	8
Gold	5	2	4	2	4	6
Aluminum	3	6	1	5	5	4
Lithium	9	1	2	1	6	7
$\sqrt{\sum_{i=1}^{m} x_{ij}^2}$	15.1	9.1	9.7	11.4	13.8	15.7

6.3.2 Stage 2: Calculate the normalized decision matrix

To easily express the decision, the decision matrix is normalized (Table 6.9). The normalization formula of (Behzadian, et al., 2012) was chosen for this study. The criteria weight from the AHP analysis in Table 6.5 is then applied.

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

Table 6.9: Normalized decision matrix

	Unexpected event	*Country risk*	*Environ. risk*	*Social risk*	*Operation perf.*	*Profitability*
Iron	0.530	0.552	0.309	0.526	0.508	0.574
Copper	0.464	0.442	0.825	0.702	0.580	0.510
Gold	0.199	0.663	0.103	0.439	0.363	0.255
Aluminum	0.331	0.221	0.413	0.175	0.290	0.383
Lithium	0.596	0.110	0.206	0.088	0.435	0.446

6.3.3 Stage 3: Calculate weighted normalized decision matrix

The weighted normalized decision matrix (Table 6.10) consists of the weighted normalized values illustrated in the following formula (Velasquez & Hester, 2013):

$$v_{ij} = w_j n_{ij}$$

Table 6.10: Weighted normalized decision matrix

	Unexpected event	*Country risk*	*Environ. risk*	*Social risk*	*Operation perf.*	*Profitability*
Iron	0.0151	0.0237	0.0780	0.2627	0.0426	0.0537
Copper	0.0132	0.0190	0.2079	0.3502	0.0486	0.0478
Gold	0.0057	0.0284	0.0260	0.2189	0.0304	0.0239
Aluminum	0.0095	0.0095	0.1040	0.0876	0.0243	0.0358
Lithium	0.0170	0.0047	0.0520	0.0438	0.0365	0.0418

6.3.4 Stage 4: determining the positive and negative ideal solutions

The ideal positive solution (A^+) maximizes the benefit criteria and minimizes the cost criteria, whereas the opposite is true for the negative ideal solution (A^-), calculated using the following formulae (Tavana, et al., 2015):

$$A^+ = (v_1^+, v_2^+, \dots, v_n^+) = \left(\left(max v_{ij} \middle| j \in I\right), \left(min v_{ij} \middle| j \in I\right)\right)$$

$$A^- = (v_1^-, v_2^-, \dots, v_n^-) = \left(\left(min v_{ij} \middle| j \in I\right), \left(max v_{ij} \middle| j \in I\right)\right)$$

Where *i* is associated with the benefit criteria and *j* with the cost criteria. The ideal positive ideal solution is the lowest value in the *j–th* column and vice versa for the negative ideal solution. The results of these negative and positive ideal solutions are illustrated in Table 6.11.

Table 6.11: Positive and negative ideal solutions

	Unexpected event	*Country risk*	*Environ. Risk*	*Social risk*	*Operation perf.*	*Profitability*
Iron	0.0151	0.0237	0.0780	0.2627	0.0426	0.0537
Copper	0.0132	0.0190	0.2079	0.3502	0.0486	0.0478
Gold	0.0057	0.0284	0.0260	0.2189	0.0304	0.0239
Aluminum	0.0095	0.0095	0.1040	0.0876	0.0243	0.0358
Lithium	0.0170	0.0047	0.0520	0.0438	0.0365	0.0418
A^+	**0.0057**	**0.0047**	**0.0260**	**0.0438**	**0.0486**	**0.0537**
A^-	**0.0170**	**0.0284**	**0.2079**	**0.3502**	**0.0243**	**0.0239**

6.3.5 Stage 5: Calculate the separation measures from the positive and negative ideal solutions

To calculate the distance between the best and worst conditions and the target alternative, the formula of the n-dimensional Euclidean metric (Aruldoss, et al., 2013) is applied (Table 6.12):

$$d_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}$$

Table 6.12: Separation measures from the positive and negative ideal solutions

	Unexpected event	*Country risk*	*Environ. risk*	*Social risk*	*Operation perf.*	*Profitability*	d_i^+	d_i^-
Iron	0.0151	0.0237	0.0780	0.2627	0.0426	0.0537	0.2260	0.1606
Copper	0.0132	0.0190	0.2079	0.3502	0.0486	0.0478	0.3568	0.0356
Gold	0.0057	0.0284	0.0260	0.2189	0.0304	0.0239	0.1801	0.2248
Aluminum	0.0095	0.0095	0.1040	0.0876	0.0243	0.0358	0.0946	0.2835
Lithium	0.0170	0.0047	0.0520	0.0438	0.0365	0.0418	0.0331	0.3453
A^+	**0.0057**	**0.0047**	**0.0260**	**0.0438**	**0.0486**	**0.0537**		
A^-	**0.0170**	**0.0284**	**0.2079**	**0.3502**	**0.0243**	**0.0239**		

6.3.6 Stage 6: Calculate the relative closeness to the ideal solution

The relative closeness (R_i) of the alternatives is calculated using the following formula. It is always between zero and one (Table 6.13). $R_i = \frac{d_i^-}{d_i^- + d_i^+}$

Table 6.13. Relative closeness to ideal solutions

	R_i	***Rank***	***% of total portfolio capital fund***	***Capital allocation fund per commodity***
Iron	0.3890	**4**	**0.153**	**0.747**
Copper	0.1262	**5**	**0.033**	**0.163**
Gold	0.5086	**3**	**0.204**	**0.999**
Aluminum	0.7657	**2**	**0.275**	**1.349**
Lithium	0.9150	**1**	**0.335**	**1.642**
			1	**4.9**

The results in Table 6.13 shows that lithium and aluminum are the more prominent PGs, with 61% of the capital fund allocated to their growth. Although iron and copper were the most profitable financially, the weighted criteria drastically influenced the corporate decision to invest in these two PGs: they became the least desired commodity class in which to invest.

6.4 Sensitivity Analysis

To ensure the consistency of this final TOPSIS decision, let's apply a sensitivity analysis to the TOPSIS decision making process. The impact of the any change on the criteria weight to the final decision ranking can be visualized in table 6.4.1, 6.4.2 and figure 6.4.

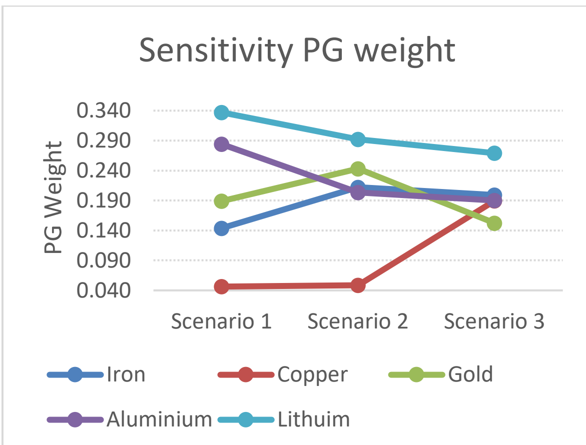
Table 6.4.1 Weightage criteria scenarios

	Extreme event	*Country risk*	*Environmental risk*	*Social risk*	*Operation performance*	*Profitability*
Scenario 1	0.029	0.043	0.252	0.499	0.084	*0.094*
Scenario 2	0.043	0.252	0.499	0.084	0.094	*0.029*
Scenario 3	0.094	0.029	0.043	0.252	0.499	*0.084*

Table 6.4.2 PG Weight scenarios

	Iron	*Copper*	*Gold*	*Aluminum*	*Lithium*
Scenario 1	0.144	0.046	0.189	0.284	0.337
Scenario 2	0.212	0.049	0.243	0.204	0.292
Scenario 3	0.199	0.190	0.152	0.190	0.269
Consistency Ratio	0.036	0.082	0.046	0.051	0.035

Table 6.4.1 illustrates the weight criteria changes scenarios. The impact of changing on criteria to final alternative rank is illustrated in figure 6.4.



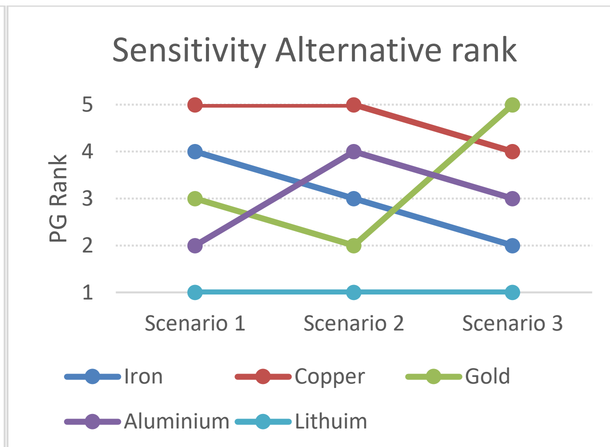


Figure 6.4: Impact of criteria change on PG weight and rank

By making a gradual change/permutation on values of each criterion, PG1(Lithium) still stay to the first rank. The rank reversal occurs to the second, third and fourth ranks. A change is also observed on PG3 (Gold) moving from third to fifth rank.

As illustrated in table 6.4.2, an estimated consistency ratio (CR) of different PG weights vary within a range from 0.03 to 0.08. With a CR of less than 10%, it can be concluded that the overall final decision from TOPSIS is consistent and reliable.

In addition to the sensitivity analysis, to validate the result of the alternatives ranking obtained from TOPSIS method, an additional analysis is done with PROMETHEE method.

6.5 Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE)

The PROMETHEE method was introduced by Brans and Mareschal (2005). This method allows the ranking of a finite number of alternatives based on a finite number of criteria. Multiple version of PROMETHEE method have been developed so far. This study considers the two of them: The PROMETHEE I for a partial ranking and PROMETHEE II for a complete ranking of alternatives It reviews the PROMETHEE I method and uses the PROMETHEE II to obtain a complete ranking from the best to the worst alternatives. Similar to TOPSIS, a pair-wise comparison of alternatives for each criteria occurs. From AHP's result, the criteria weighting through aggregation will provide a complete ranking of alternatives from the net outranking flow. The PROMETHEE method simplifies the multi criteria decision making problem into six main stage (Figure 6.5):

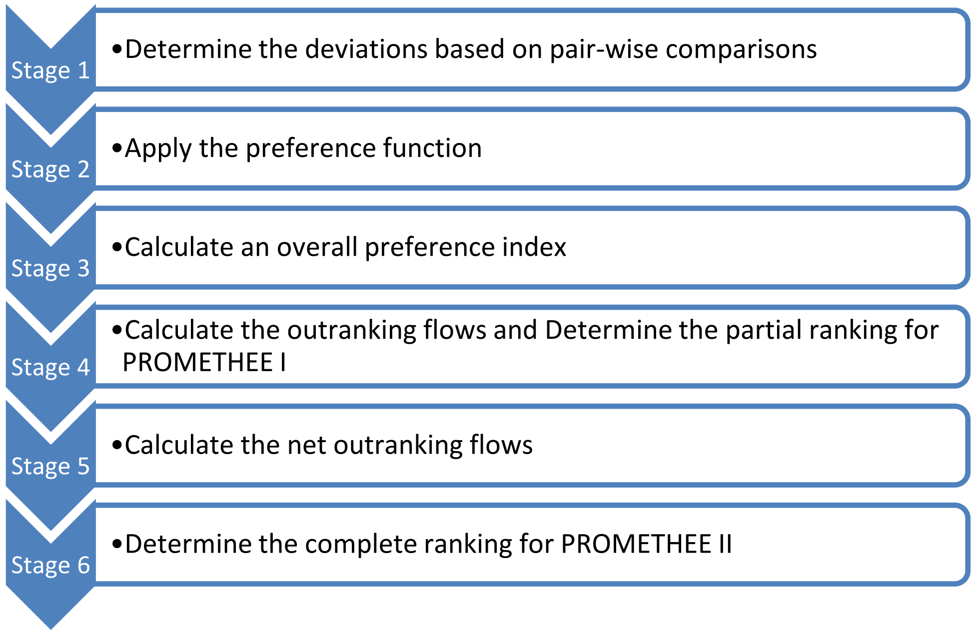


Figure 6.5: The six process stage of PROMETHEE

6.4.1 Stage 1: Determine deviations based on pair-wise comparisons

$d_j(a,b) = g_j(a) - g_j(b)$

Where $d_j(a,b)$ represents the difference between the evaluation of alternatives a and b on each criterion.

For similar reference point when comparing PROMETHEE with TOPSIS method, the normalized decision table defined in section 6.3.2 has been considered (Table 6.14 is identical to Table 6.9)

Table 6.14: Normalized decision matrix

	Unexpected event	*Country risk*	*Environ. Risk*	*Social Risk*	*Operation perf.*	*Profitability*
Iron	0.530	0.552	0.309	0.526	0.508	0.574
Copper	0.464	0.442	0.825	0.702	0.580	0.510
Gold	0.331	0.221	0.413	0.175	0.290	0.383
Aluminum	0.199	0.663	0.103	0.439	0.363	0.255
Lithium	0.596	0.110	0.206	0.088	0.435	0.446

The difference between the evaluation of alternatives a and b on each criterion is provided in table 6.15.

Table 6.15: Deviations based on pair-comparison

	Unexpected event	*Country risk*	*Environ. Risk*	*Social risk*	*Operation perf.*	*Profitability*
D(PG1-PG2)	0.066	0.110	-0.516	-0.175	-0.073	0.064
D(PG1-PG3)	0.331	-0.110	0.206	0.088	0.145	0.319
D(PG1-PG4)	0.199	0.331	-0.103	0.351	0.218	0.191
D(PG1-PG5)	-0.066	0.442	0.103	0.439	0.073	0.128
D(PG2-PG1)	-0.066	-0.110	0.516	0.175	0.073	-0.064
D(PG2-PG3)	0.265	-0.221	0.722	0.263	0.218	0.255
D(PG2-PG4)	0.132	0.221	0.413	0.526	0.290	0.128
D(PG2-PG5)	-0.132	0.331	0.619	0.614	0.145	0.064
D(PG3-PG1)	-0.331	0.110	-0.206	-0.088	-0.145	-0.319
D(PG3-PG2)	-0.265	0.221	-0.722	-0.263	-0.218	-0.255
D(PG3-PG4)	-0.132	0.442	-0.309	0.263	0.073	-0.128
D(PG3-PG5)	-0.397	0.552	-0.103	0.351	-0.073	-0.191
D(PG4-PG1)	-0.199	-0.331	0.103	-0.351	-0.218	-0.191
D(PG4-PG2)	-0.132	-0.221	-0.413	-0.526	-0.290	-0.128
D(PG4-PG3)	0.132	-0.442	0.309	-0.263	-0.073	0.128
D(PG4-PG5)	-0.265	0.110	0.206	0.088	-0.145	-0.064
D(PG5-PG1)	0.066	-0.442	-0.103	-0.439	-0.073	-0.128
D(PG5-PG2)	0.132	-0.331	-0.619	-0.614	-0.145	-0.064
D(PG5-PG3)	0.397	-0.552	0.103	-0.351	0.073	0.191
D(PG5-PG4)	0.265	-0.110	-0.206	-0.088	0.145	0.064

6.5.2 Stage 2: Apply the preference function

Each criterion ranking the alternatives has an assigned preference function; which translates the positive or negative difference value of the criterion between two alternatives, hence a preference degree ranging from 0 to 1.

$P_j(a, b) = F_j[d_j(a, b)]\ j = 1, \ldots, k.$

Where $P_j(a, b)$ represents the preference for alternative “a” to “b “ in each criterion, as a function of $d_j(a, b)$ and $F_j[\,]$ denotes the preference function, typically using 6 classes of preference functions:

6.5.2.1 Class 1: Usual criterion

Alternatives “a” and “b” are viewed as indifferent when they are equal. Then, the preference degree is zero. When they are not equal, a strict preference represents the smallest difference in value and the preference degree is one.

6.5.2.2 Class 2: Quasi criterion

When alternatives “a” and “b” are indifferent within the defined range, a preference degree is zero. Beyond the defined range, the preference degree of one.

6.5.2.3 Class 3: Criterion with linear preference

This is an extension to the usual criterion. When alternative “a” and “b” are indifferent, a preference degree is zero. Beyond a defined threshold, when the intensity of the preference increases linearly, the preference is strict and the reference degree is one.

6.5.2.4 Class 4: Level criterion

This is an similar to the quasi criterion. When alternatives “a” and “b” are indifferent within the specified range, a preference degree is zero. Beyond the first range, a second range is defined; then, a weak preference is provided and the preference degree is 0.5. Over the second range, there is a stringent preference and the preference degree is one.

6.5.2.5 Class 5: Criterion with linear preference and indifference area

This is a combination of criterion with linear preference (class 3) and Quasi criterion (class 2). When alternatives “a “ and “b” are indifferent, the preference degree is zero . Beyond a defined range, the preference increases progressively or linearly to the defined threshold and the preference degree is one.

6.5.2.6 Class 6: Gaussian criteria

This is similar to the criterion with linear preference (class 3). When one of the criterion is of the Gaussian class and there is an increase deviation between alternatives a and b with a nonlinear relationship, a value of sigma represents a distance between the origin and the inflection point. Then the preference degree will vary between zero and one. When there is indifference between alternatives "a" and "b", the preference degree is zero, and when the difference between alternatives "a" and "b" is very large, the preference degree is one.

With this defined class of preference function, the preference degree ranges from zero to one. This is illustrated in the table 6.16.

6.5.3 Stage 3: Calculate the overall preference index.

$\pi(a, b) = \sum_{j=1}^{k} P_j\ (a, b) w_j$

Where $\pi(a, b)$ of alternatives "a" over "b" is defined as the weighted sum $p(a, b)$ of each criterion, and w_j is the weight associated with criterion j. Its value varies from 0 to 1.

Table 6.17 provides the results of the overall preference index between two alternatives.

Table 6.16: Difference value of the criterion between alternatives

	Unexpected Event	Country risk	Environ. Risk	Social risk	Operation perf.	Profitability
P(PG1-PG2)	0.066	0.110	0.000	0.000	0.000	0.064
P(PG1-PG3)	0.331	0.000	0.206	0.088	0.145	0.319
P(PG1-PG4)	0.199	0.331	0.000	0.351	0.218	0.191
P(PG1-PG5)	0.000	0.442	0.103	0.439	0.073	0.128
P(PG2-PG1)	0.000	0.000	0.516	0.175	0.073	0.000
P(PG2-PG3)	0.265	0.000	0.722	0.263	0.218	0.255
P(PG2-PG4)	0.132	0.221	0.413	0.526	0.290	0.128
P(PG2-PG5)	0.000	0.331	0.619	0.614	0.145	0.064
P(PG3-PG1)	0.000	0.110	0.000	0.000	0.000	0.000
P(PG3-PG2)	0.000	0.221	0.000	0.000	0.000	0.000
P(PG3-PG4)	0.000	0.442	0.000	0.263	0.073	0.000
P(PG3-PG5)	0.000	0.552	0.000	0.351	0.000	0.000
P(PG4-PG1)	0.000	0.000	0.103	0.000	0.000	0.000
P(PG4-PG2)	0.000	0.000	0.000	0.000	0.000	0.000
P(PG4-PG3)	0.132	0.000	0.309	0.000	0.000	0.128
P(PG4-PG5)	0.000	0.110	0.206	0.088	0.000	0.000
P(PG5-PG1)	0.066	0.000	0.000	0.000	0.000	0.000
P(PG5-PG2)	0.132	0.000	0.000	0.000	0.000	0.000
P(PG5-PG3)	0.397	0.000	0.103	0.000	0.073	0.191
P(PG5-PG4)	0.265	0.000	0.000	0.000	0.145	0.064

Table 6.17: Overall preference index evaluation

	Unexpected event	*Country risk*	*Environ. risk*	*Social risk*	*Operation perf.*	*Profitability*	$\pi(a,b) = \sum_{j=1}^{k} P_j(a,b)w_j$
Weights	0.030	0.040	0.190	0.496	0.097	0.147	
*W_j*P(PG1-PG2)*	0.002	0.004	0.000	0.000	0.000	0.009	**0.016**
*Wj*P(PG1-PG3)*	0.010	0.000	0.039	0.044	0.014	0.047	**0.153**
*Wj*P(PG1-PG4)*	0.006	0.013	0.000	0.174	0.021	0.028	**0.242**
*Wj*P(PG1-PG5)*	0.000	0.018	0.020	0.218	0.007	0.019	**0.281**
*Wj*P(PG2-PG1)*	0.000	0.000	0.098	0.087	0.007	0.000	**0.192**
*Wj*P(PG2-PG3)*	0.008	0.000	0.137	0.131	0.021	0.037	**0.334**
*Wj*P(PG2-PG4)*	0.004	0.009	0.078	0.261	0.028	0.019	**0.399**
*Wj*P(PG2-PG5)*	0.000	0.013	0.118	0.305	0.014	0.009	**0.459**
*Wj*P(PG3-PG1)*	0.000	0.004	0.000	0.000	0.000	0.000	**0.004**
*Wj*P(PG3-PG2)*	0.000	0.009	0.000	0.000	0.000	0.000	**0.009**
*Wj*P(PG3-PG4)*	0.000	0.018	0.000	0.131	0.007	0.000	**0.155**
*Wj*P(PG3-PG5)*	0.000	0.022	0.000	0.174	0.000	0.000	**0.196**
*Wj*P(PG4-PG1)*	0.000	0.000	0.020	0.000	0.000	0.000	**0.020**
*Wj*P(PG4-PG2)*	0.000	0.000	0.000	0.000	0.000	0.000	**0.000**
*Wj*P(PG4-PG3)*	0.004	0.000	0.059	0.000	0.000	0.019	**0.082**
*Wj*P(PG4-PG5)*	0.000	0.004	0.039	0.044	0.000	0.000	**0.087**
*Wj*P(PG5-PG1)*	0.002	0.000	0.000	0.000	0.000	0.000	**0.002**
*Wj*P(PG5-PG2)*	0.004	0.000	0.000	0.000	0.000	0.000	**0.004**
*Wj*P(PG5-PG3)*	0.012	0.000	0.020	0.000	0.007	0.028	**0.067**
*Wj*P(PG5-PG4)*	0.008	0.000	0.000	0.000	0.014	0.009	**0.031**

6.5.4 Stage 4: Calculate outranking flows

For PROMETHEE I

$\emptyset_1^+(a) = \sum_x \pi(a,x)$ and $\emptyset_1^-(a) = \sum_x \pi(x,a)$

Where $\emptyset_1^+(a)$ and $\emptyset_1^-(a)$ respectively, represent the leaving/positive outranking flow and the entering/negative outranking flow for each alternative.

Table 6.18 provides the leaving and entering outranking flows for PROMETHEE I

Table 6.18: Outranking flows for PROMETHEE I

Aggregated Preference Function	*Iron*	*Copper*	*Aluminum*	*Gold*	*Lithium*	$\emptyset_1^+$ *Leaving flow*
Iron(PG1)		0.016	0.153	0.242	0.281	**0.69237**
Copper(PG2)	0.192		0.334	0.399	0.459	**1.38460**
Gold(PG3)	0.0044	0.009		0.155	0.196	**0.36491**
Aluminum(PG4)	0.020	0.0000	0.082		0.087	**0.18835**
Lithium(PG5)	0.002	0.0039	0.067	0.031		**0.10378**
$\emptyset_1^-$ *Entering flow*	**0.21818**	**0.02862**	**0.63585**	**0.82824**	**1.02312**	

A comparison of alternatives and elimination of incomparable situation is done to obtain the partial ranking for PROMETHEE I. three possible options are denoted.

- Alternative a is preferred over or outranks alternative b, $a\boldsymbol{P}b$.

$$a\boldsymbol{P}b\ if:\ \emptyset_1^+(a) > \emptyset_1^+(b)\ and\ \emptyset_1^-(a) < \emptyset_1^-(b);\ or$$

$$\emptyset_1^+(a) > \emptyset_1^+(b)\ and\ \emptyset_1^-(a) = \emptyset_1^-(b);$$

$$\emptyset_1^+(a) = \emptyset_1^+(b)\ and\ \emptyset_1^-(a) < \emptyset_1^-(b).$$

- Indifference situation, $a\boldsymbol{I}b$

$$aIb\ if: \emptyset_1^+(a) = \emptyset_1^+(b)\ and\ \emptyset_1^-(a) = \emptyset_1^-(b)$$

- Incomparable situation, $a\boldsymbol{R}b$

$$aRb\ if:\ \emptyset_1^+(a) > \emptyset_1^+(b)\ and\ \emptyset_1^-(a) > \emptyset_1^-(b);\ or$$

$$\emptyset_1^+(a) < \emptyset_1^+(b)\ and\ \emptyset_1^-(a) < \emptyset_1^-(b).$$

The alternatives are compared (Table 6.19) to eliminate incomparable situation (Table 6.20) and define the partial ranking (Figure 6.5)

Table 6.19: Comparison of alternatives

	$\emptyset_1^+$ *Leaving flow*	$\emptyset_1^-$ *Entering flow*
Iron(PG1)	0.69237	0.2182
Copper(PG2)	1.38460	0.0286
Gold(PG3)	0.36491	0.6358
Aluminum(PG4)	0.18835	0.8282
Lithium(PG5)	0.10378	1.0231

Based on the possible options defined in the comparison of alternatives equations, the solution of the condition equations are provided in table 6.20.

Table 6.20: Elimination of incomparable situation

Copper(PG2)	**P**	Iron(PG1)
Copper(PG2)	**P**	Gold(PG3)
Gold(PG3)	**P**	Aluminum(PG4)
Aluminum(PG4)	**P**	Lithium(PG5)
Iron(PG1)	**P**	Lithium(PG5)
Iron(PG1)	**P**	Gold(PG3)
Copper(PG2)	**P**	Aluminum(PG4)
Gold(PG3)	**P**	Lithium(PG5)
Iron(PG1)	**P**	Aluminum(PG4)
Copper(PG2)	**P**	Lithium(PG5)

Figure 6.6 illustrates the comparable situation between all PGs, and provides the ranking between alternatives. The arrows between PGs illustrate the comparable situation. If there is no arrow between two alternatives, this means there is no information allowing the ranking to be done; nevertheless, in this case there is information allowing the ranking between alternatives (Figure 6.6).

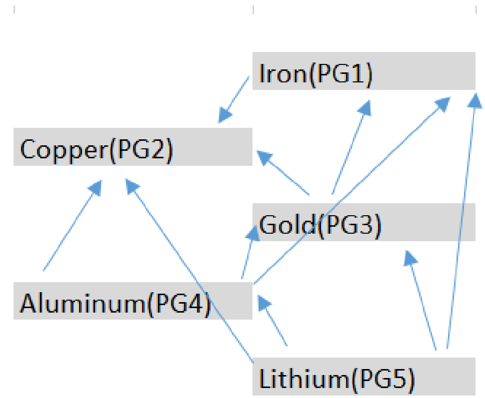


Figure 6.6: Ranking between alternatives

Assuming a partial ranking of alternatives, Section 5.2 provides a full complete ranking with PROMETHEE II methodology

For PROMETHEE II

$\emptyset_2^+(a) = \frac{1}{n-1}\sum_x \pi(a,x)$ and $\emptyset_2^-(a) = \frac{1}{n-1}\sum_x \pi(x,a)$

Where $\emptyset_2^+(a)$ and $\emptyset_2^-(a)$ respectively, denote the leaving/positive outranking flow and the entering/negative outranking flow for each alternative.

Table 6.21 provides the leaving and entering outranking flows for PROMETHEE II

Table 6.21: Outranking flows for PROMETHEE II

Aggregated Preference Function	*Iron*	*Copper*	*Gold*	*Aluminum*	*Lithium*	$\emptyset_2^+$ *Leaving flow*
Iron(PG1)		0.016	0.153	0.242	0.281	**0.17309**
Copper(PG2)	0.192		0.334	0.399	0.459	**0.34615**
Gold(PG3)	0.0044	0.009		0.155	0.196	**0.09123**
Aluminum(PG4)	0.020	0.0000	0.082		0.087	**0.04709**
Lithium(PG5)	0.002	0.0039	0.067	0.031		**0.02595**
$\emptyset_2^-$ *Entering flow*	**0.0545**	**0.0072**	**0.1590**	**0.2071**	**0.2558**	

6.5.5 Stage 5: Calculate the net outranking flows for PROMETHEE II

The difference between the entering and leaving outranking flows determines the net outranking flows.

$$\emptyset_2(a) = \emptyset_2^+(a) - \emptyset_2^-(a),$$

Where $\emptyset_2(a)$ denotes the net outranking flow for each alternative. The value of net outranking flow for each alternative is illustrated in table 6.22.

Table 6.22: Net outranking flows

	$\emptyset_2^-$ *Entering flow*	$\emptyset_2^+$ *Leaving flow*	*Net Outranking flow value* $\emptyset(a)$
Iron(PG1)	0.05454	0.17309	-0.11855
Copper(PG2)	0.00715	0.34615	-0.33900
GoldPG3)	0.15896	0.09123	0.06774
Aluminum(PG4)	0.20706	0.04709	0.15997
Lithium(PG5)	0.25578	0.02595	0.22983

6.5.6 Stage 6: Determine the complete ranking for PROMETHEE II

From table 6.22, the complete ranking (table 6.23) of all considered alternatives depends on the value of net outranking flow.

Table 6.23: Complete ranking for PROMETHEE II

	$\emptyset_2^-$ *Entering flow*	$\emptyset_2^+$ *Leaving flow*	*Net Outranking flow value* $\emptyset(a)$	*Complete Ranking for PROMETHEE II*
Iron(PG1)	0.00523	0.01474	-0.00951	4
Copper(PG2)	0.00062	0.03143	-0.03081	5
Gold(PG3)	0.01380	0.00828	0.00552	3
Aluminum(PG4)	0.01799	0.00448	0.01352	2
Lithium(PG5)	0.02320	0.00191	0.02129	1

6.6 Comparison of complete ranking from PROMETHEE and TOPSIS

Table 6.24 provides the comparison of the ranking obtained from PROMETHEE II with the one from TOPSIS.

Table 6.24: Comparison between TOPSIS and PROMETHEE II ranking

	Complete Ranking for PROMETHEE II	*Previous Ranking from TOPSIS*
Iron(PG1)	4	4
Copper(PG2)	5	5
Gold(PG3)	3	3
Aluminum(PG4)	2	2
Lithium(PG5)	1	1

TOPSIS and PROMETHEE II methods provide the same alternatives ranking results. This allows to validate the results obtained in section 6.3. Hence, the validation of our decision making process regarding the five alternatives (PG1, PG2, PG3, PG4 and PG5).

6.7 Conclusion

This chapter illustrates four complementary methods: AHP, TOPSIS and PROMETHEE I and II. Using the AHP as an input to the TOPSIS analysis reinforces the practicality and facilitates realistic analysis of corporate capital allocation. The PROMETHEE II method validates the results of TOPSIS. The TOPSIS ranking of all alternatives is very similar to the one from PROMETHEE II. The weightage of key criteria is fundamental in the decision-making process. Comparing the results in this chapter with those in Chapters 3–5 highlights the importance of characterizing criteria and their respective weights as an initial step in decision-making related to corporate portfolio management.

Although the results in this chapter can help improve the capital allocation decision, the overall available capital fund needs to be fully allocated, which could lead to inefficient capital fund use. This could be mitigated by combining the weightage criteria from the AHP methodology with the portfolio optimization model defined in Chapter 5. Then the remaining available fund can be allocated to future and more effective initiatives in the corporation.

Chapter 7: Conclusions and future work

This research aimed to develop a new portfolio management strategy for mining corporations such that—in addition to traditional financial criteria—the operational performance, country risk, extreme events, unexpected events, country stability, and commodity market behavior are all considered in the investment or divestment decision-making process. For a multinational mining company, this research proposes two resource allocation management approaches in tandem with the growth strategy. The first approach is based on the portfolio optimization model and the second approach is based on MCDM models. All data and assumptions in this thesis are hypothetical.

The thesis first presented a new portfolio management strategy to allocate the right capital investment to the right project for maximum returns at minimum risk. This new model helps to improve decision-making processes associated with capital allocation in a corporate portfolio where operational performance and country risks are included among the decision-making criteria. The proposed approach improves the prioritization of capital expenditure projects. The portfolio optimization was formulated under the constraints of country risk and operational performance requirements of the project initiator product group (PG). Results showed that approval process is easier for the project initiator with good operational performance. They also showed that diversification of the portfolio is a better way to increase the portfolio return with a slight risk increase for the projects in the portfolio. The results of combined country risk and operational performance criteria show that a more diversified portfolio with similar projects potentially increases the corporate portfolio return with a slight increase in the minimum acceptable risk. As the performance of a PG increases, the chance of approval of the proposed projects also increases.

The second section of this research solved a portfolio optimization problem under extreme events for investment or divestment decision-making. In addition to operational performance and country risk, commodity market behavior at the extreme events was considered. Results showed that the commodity market behavior affects the investment or divestment decision. The PG total return price behavior in the mineral portfolio optimization model provides an opportunity to seize more market opportunity in the investment strategy of the corporation. This model clearly illustrates that rushing to divest a PG at the extreme turnover level does not necessarily provide the highest return at the lowest risk. A more efficient portfolio return at moderate risk is obtained with knowledge of the commodity market behavior at an efficient frontier. A portfolio optimization model with operational performance criteria at extreme events without the consideration of the commodity market behavior criteria is less efficient than a portfolio optimization model with operational performance and commodity market behavior criteria.

This research also highlights the impact of the combined country stability and commodity market behavior and operational performance on distribution of the capital fund in the country of investment. The utilization of the country risk values combined with previous operational performance—including production efficiency and commodity price—affects the distribution of the capital fund to allocate to each commodity class in the multiple countries of investment.

The final chapter reinforces the importance of criteria weights and characteristics in the decision-making process related to the capital allocation fund. It also illustrates the complementarity of two multi-criteria decision making methodologies: AHP and TOPSIS. The outcome of AHP is the input to the TOPSIS analysis. The outcome of this TOPSIS analysis provides a more realistic capital allocation, and the PROMETHEE validates the alternatives ranking results from TOPSIS.

Three future research directions have been identified. (1) The development of simultaneous Capital allocation strategy in two separate mode: Good and bad period. (2) The quantification of correlation between projects and extreme turnover level. (3) The effect of criteria weight at the initial phase of suggested portfolio optimization model. The inclusion of criteria weights from the MCDM methodology with operational performance, country risk, unexpected events, and commodity market behavior. This could improve the overall portfolio optimization models, the efficiency of capital allocation through the right criteria weight, and the effective utilization of the available capital fund.

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Appendices

Appendix 1: MATLAB Program1

```
clc; clear; close all;
 %We need to define a total amount of capital that need to be allocated
global Capital
Capital=4900; %$MM
%Now we need to define the product groups that we are working with and their boundaries.
LB=[50,50,50,50,50];
UB=[Capital,Capital,Capital,Capital,Capital];
%Now lets determine a time range for this optimization
global TimeEnd
TimeEnd=600;%time period
  %From here we are missing the optimization equation/function and the market data for the
optimization
%Lets assume we have some financial model which predicts future prices of the commodities for
the time range given
global Cost
Cost=GenerateNewCostFunc();
%And we can begin the optimization function
InitialGuess=[1000,1000,1000,1000,1000];
A=[1,1,1,1,1];
B=Capital;
Aeq=[];
Beq=[];
 res=fmincon(@ObjFunc,InitialGuess,A,B,Aeq,Beq,LB,UB);

%results
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(1)
title('Product Groups 1 - 5')
for i=1:5
subplot(2,3,i)
plot([1:TimeEnd],Cost(i,:))
xlabel('Time in the period')
ylabel('Profitability in $/unit')
end
fprintf('The optimization resulted in')
```

```
for i=1:5
fprintf('\n%.3f $MM in product group %i',res(i),i)
end

%functions
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function X=PerfOps(Y)
   %Gives some arbitrary operations efficiency (non linear) in units/dollard/time period
   X = [1/365,0.85/365,0.85/365,0.35/365,0.35/368].*(Y./1000).^-0.1;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function Value=ObjFunc(Y)
   global Capital
   global Cost
   Value = -(Capital-sum(Y)+sum(Y.*PerfOps(Y).*sum(Cost,2)'));
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function Limit=CapitalConstraint(Y)
   global Capital
   Limit = -sum(Y)+Capital; %>=0
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function C=GenerateNewCostFunc()
   global TimeEnd
   %This creates a random variable for selling price from 0.9 to 1.1 $/unit
   C=zeros(5,TimeEnd);
   for i=1:5
      C(i,:)=0.9+rand(TimeEnd,1)*0.2;
   end
  %Now we can define in our model some extreme event or probability of some extreme events
occuring
   Length_of_event=30;%time period
   Prob_Of_Event_Per_Day=[0,0.001,0.002,0.005,0.01];

%Goes through and applies the extreme events
   for i=1:5
      EE=rand(TimeEnd,1)>(1-Prob_Of_Event_Per_Day(i));
```

```
        EEDays=EE;
        for j=1:Length_of_event
            rolled=circshift(EE,j);
            EEDays(j:end)=EEDays(j:end)+rolled(j:end);
        end
        C(i,:)=((EEDays==0)+rand(1)/2)'.*C(i,:);
    end
end
```

Appendix 2: MATLAB Program2

```
clc; clear;
warning('off','all')
 % Profitability will be tracked according to the following assumptions. Each country and product has a capital to be invested, production volume is a function of capital invested and operating cost is also based on the total capital invested. This optimization will take into account stochastic nature of products, prices, costs, and country stability. It is initially assumed that operational performance follow an equation of capital invested plus some random normal error variation.
 %Costs will be a function of previous time prices and will be adjusted time period by time period based on a skewed normal distribution.
 %Countries will be given a probability of extreme event/ per time period. It is assumed all invested capital in an extreme event is lost (for cost function).
 %We need to define a total amount of capital that will be allocated
Capital=4900; %$MM
RiskCI = 0.95; %and our tolerance
%Now we need to define the product groups that we are working with
Nc = 2;% Number of Countries
Np = 5; % Number of products
N = 20; %Number of simulations per Capital, Product Group, and Country
IM  = zeros(Np*Nc,1); %Investment Product "Matrix" the current capital invested in product and country
PGnum = repmat((1:Np),1,Nc); %Product Group Number
CRA = (1:Nc)/100; %Country Risk Array This will be the percent of extreme event per time period
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 %Now lets determine a time range for this optimization
TimeEnd=3;%time period
CostHist = zeros(Np,1,TimeEnd+2);
CostHist(:,1,1)=[11 12 13 13.5 14]; %two time period ago prices
CostHist(:,1,2)=[10.9 12.2 13 13 14.1]; %one time period ago prices
CostHist = repmat(CostHist,1,10000,1);

for t = 1:TimeEnd
   for P = 1:Np
      for Nsim = 1:10000
         CostHist(P,Nsim,t+2)=ProductMarket([CostHist(P,Nsim,t+1),CostHist(P,Nsim,t+0)],P);
      end
      priceKernel{P,t}=fitdist(CostHist(P,:,t+2)',"kernel");
      priceHist(P,:,t)=random(priceKernel{P,t},1000,1);
   end
```

```
end

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Modeling will involve running monte-carlo simulation of an optimization loop accross
combinations of state variables through time. This will be surrounded by a second optimization
loop which improves the long term action of the first optimizer by changing the cost function
weighting
% First we run the assumed pricing simulations and bin the results. The binnned results will give
us a kernal distribution of expected future prices
 %Starting the optimizer
load('OptPreRun.mat')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% To increase simulations speed
%[Weights,profitDistribution,iter]=SecondOptimization(priceHist,CRA,PGnum,Capital,RiskCI,
Nc,Np,N,TimeEnd);%
%[investment,Profit]=FinalInvest(priceHist,PGnum,t,Weights,Capital,Np,Nc);
%save('OptPreRun.mat')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 %The stochastic optimizer keeps correlations of the dependencies of certain paths over time by
binning capital results and recording what combination of investments reach that location. These
bins and distributions can be used in the next time stage to generate a new set of correlated data
for the simulation to run. The final distribution of capital is then used as the cost function for a
second optimizer which adjusts the first optimizers' weighting function. This optimization follows
loosely the idea of ergodic descent.
 %Plotting the results and doing analysis
for i = 1:Nc
Cname{i}=['Country_',int2str(i)];
end
for i = 1:Np
PGname{i}=['PG_',int2str(i)];
end
disp("The optimized weighting for the optimizer function is below")
array2table(reshape(Weights,[Np,Nc]),'VariableNames',Cname,'RowNames',PGname)
profitDistribution=profitDistribution*-1;%Corrects for the negative in the objective
%Profit= Profit*-1;%corrects for the negative in the objective
Eall=mean(profitDistribution); EallCI=mean(profitDistribution(N*N*(1-RiskCI):end));
EposCI=mean(profitDistribution(profitDistribution(N*N*(1-RiskCI):end)>0));
EnegCI=mean(profitDistribution(profitDistribution(N*N*(1-RiskCI):end)<0));
CI=RiskCI; Nsimall=iter*N*N*TimeEnd;
```

```
disp("The optimized investment for the current time periodis below")
array2table(reshape(investment,[Np,Nc]),'VariableNames',Cname,'RowNames',PGname)

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(1)
area(reshape(investment,Np,Nc))
xlabel('Product Groups')
ylabel('$MM')
title('Stacked Area for PGs and Country Investment')
legend(Cname)
xticks([1:5])

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(2)
hold on
for i = 1:5
histogram(priceHist(i,:,1))
end
xlabel("Commodity Price per unit")
ylabel("Simulated Occurances")
title('Commodity Price')
legend(PGname)

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(3)
hold on
clear CostEX
for j = [20:20:800]
   for i = 1:5
      CostEX(j/20,i)=CostOfProduction(j,i)*ProductionQuantity(j,i);
   end
end
   for i = 1:5
   plot([20:20:800],CostEX(:,i),'Linewidth',4);
   end
xlabel("Invested Amount")
ylabel("$MM/mo")
title('Operational Performance')
legend(PGname)
sprintf(['The overall expected value of distribution is: %.3f$MM \n' ...
```

```
        'The expected overall value of distribution with CI of %.2f is: %.1d$MM \n' ...
        'The expected positive return value of distribution with CI of %.2f is: %.2d$MM \n' ...
        'The expected negative return value of distribution with CI of %.2f is: %.2d$MM \n' ...
        'A total of %i simulations were run
overall'],Eall,CI,EallCI,CI,EposCI,CI,EnegCI,Nsimall)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function
[investment,FinalProfitBreakDown]=FinalInvest(priceHist,PGnum,t,Weights,Capital,Np,Nc)
UB = ones(1,Np*Nc)*Capital;
A=ones(1,Np*Nc);
B=Capital;
Aeq=[];
Beq=[];
options = optimoptions("patternsearch","InitialMeshSize",896,"MeshTolerance",1E-
2,"Display","off");%,"Display","iter","PlotFcn",@psplotbestf);
LB = zeros(1,Np*Nc);%its assumed once capital is invested in country and product it is "stuck"
InitialGuess=UB;
for i = 1:100
[investment(i,:),~,profit(i,:)]=NestedObjFunc(zeros(1,Np*Nc),InitialGuess,A,B,Aeq,Beq,LB,UB
,options,priceHist,PGnum,t,Weights,Capital,Np,Nc);
end
investment=mean(investment);
FinalProfitBreakDown=mean(profit);
 end
function ValueDistribution =
FirstOptimizer(CRA,PGnum,Capital,Nc,Np,N,TimeEnd,Weights,priceHist)
%For time = 0 we start with a stagnant amount of money, with "invested prior" = 0 as well
NewCapitalList=ones(N*N,1)*Capital;
tempres = ones(Np*Nc,N*N);
 %We will define the edges of the binned capital
[~,CAPedges] = histcounts(NewCapitalList);
%and then generate new investment profiles for each boundary
for j = 1:length(CAPedges)
res{j} = ones(N,Np*Nc)/10;
end

%We can then begin the simulations
for t = 1:TimeEnd
   [~,CAPedges] = histcounts(NewCapitalList);%note edges are k+1
```

```
    %First we make the kernel distributions of the capital distributions
    curCapitalDist{t} = fitdist(NewCapitalList,"kernel");

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Now we generate N random starting capital positions based on the previous time distribution
    curCapitalList=random(curCapitalDist{t},N,1);
        for i = 1:N
        curCapital=curCapitalList(i);
        [~,bin_Index]=min(abs(CAPedges-curCapital));
        %Determine which bin the starting capital belongs to and generate new portfolio profiles
based on that bin's correlations.
        if isempty(res{bin_Index})==1
            IM=zeros(N,Np*Nc);
        elseif length(res{bin_Index})==1 || length(res{bin_Index})==1
            IM = Repmat(res{bin_Index},N/length(res{bin_Index}),1);
        else
            %This generates a correlation matrix and applies a guassian guess structure for the
profiles. Originally, a multivariable kernel was going to be used, however, MATLAB does not have
a good way of getting random guesses for those distributions.
            Rho = corr(res{bin_Index},'rows','complete');
            Rho(isnan(Rho)) = 0;
            Rho(Rho>1)=1;
            Rho(Rho<-1)=-1;
            if sum(sum(Rho))==0
                Rho = diag(ones(1,Np*Nc));
            end
              try
                u = copularnd('Gaussian',Rho,N);
                [~,ndim]=size(u);
                for k = 1:ndim
                    IM(:,k)=max(ksdensity(res{bin_Index}(:,k),u(:,k),'function','icdf'),0);
                end
            catch
                disp("Had to reset correlation matrix");
                IM=zeros(N,ndim);
            end
         end

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %Now we define the starting position of the optimizer
```

```
        UB = ones(1,Np*Nc)*curCapital;
        A=ones(1,Np*Nc);
        B=curCapital;
        Aeq=[];
        Beq=[];
        %we are using a large patternsearch to account for randominity
        options = optimoptions("patternsearch","InitialMeshSize",1000,"MeshTolerance",1E-
2,"Display","off");%,"Display","iter","PlotFcn",@psplotbestf);

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %parallel for loop
        parfor j = ((i-1)*N+1):((i-1)*N+N)
        pos = j-(i-1)*N;
        LB = IM(pos,:);%its assumed once capital is invested in country and product it is "stuck"
        InitialGuess=UB;
        %parallel computing requires strange nested objects, once we get an answer we can organize
the data
[tempres(:,j),NewCapitalList(j),~]=NestedObjFunc(CRA,InitialGuess,A,B,Aeq,Beq,LB,UB,opti
ons,priceHist,PGnum,t,Weights,curCapital,Np,Nc);
        end
    end
    disp(t)
    [~,CAPedges] = histcounts(NewCapitalList);%note edges are k+1
    res=cell(length(CAPedges),1);%reset our portfolio distributions
    for i = 1:N*N
    [~,bin_Index]=min(abs(CAPedges-NewCapitalList(i)));
    %reorganize the portfolios based on thier performance bin
    res{bin_Index}=[res{bin_Index};tempres(:,i)'];
    end
end
%returns the final distribution of capital profiles
ValueDistribution = NewCapitalList;
end

%functions
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [Weights,
profitDist,iter]=SecondOptimization(priceHist,CRA,PGnum,Capital,RiskCI,Nc,Np,N,TimeEnd)
profitDist=zeros(1,N*N);
%this optimization loop adjusts the weighting of the firsts
```

```
Weights = ones(1,Np*Nc);
LB = zeros(1,Np*Nc);
UB = ones(1,Np*Nc);
InitialGuess=UB;
options = optimoptions("patternsearch","InitialMeshSize",0.38,"MeshTolerance",1E-
3,"Display","iter");%,"Display","iter","PlotFcn",@psplotbestf);
[Weights,~,~,Output] = patternsearch(@OuterObjFunc,InitialGuess,[],[],[],[],LB,UB,[],options);
iter=Output.iterations;

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%This function is the obj function for adjusting weighting
function Value=OuterObjFunc(Y)
    Weights = Y;
    valDist = FirstOptimizer(CRA,PGnum,Capital,Nc,Np,N,TimeEnd,Weights,priceHist);
    %sorting the profit after n time period
    profitDist=sort(valDist-Capital);
    %removing the bottom x% of results to redefine risk confidence interval
    profit=profitDist(N*N*(1-RiskCI):end);
   %the very last distributions optimized for max epected value for profit and losses. THESE CAN
ABSOLUTELY BE WEIGHTED IF WE CONSIDER THAT 1$ MADE =/= 1S LOST.
    Value =sum(sum(abs(profit(profit<0))))-sum(sum(abs(profit(profit>=0)))); %minimum of the
negative
  end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function
[res,NewCap,profit]=NestedObjFunc(CRA,InitialGuess,A,B,Aeq,Beq,LB,UB,options,priceHist,
PGnum,t,Weights,curCapital,Np,Nc)
%runs the patternsearch on the expected returns based on the portfolio profiles. Extreme events
are not considered in the first optimizer.
res=patternsearch(@ObjFunc,InitialGuess,A,B,Aeq,Beq,LB,UB,[],options);

%This checks if an extreme event did in fact occur and adjusts the portfolio to account for those
losses
for i= 1:Nc
    if rand(1)<CRA(i)
        res((i-1)*Np+1:i*Np)=0;
    end
end
```

```
%we now actually, evaluate a random outcome based on the optimizers best guess.
profit = EvalFunc(res);
NewCap=sum(profit)+curCapital;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   function Value=ObjFunc(Y)
      %this loop optimizes for weighted return
      Units=ProductionQuantity(Y);
      valDist = Weights.*(mean(Units.*priceHist(PGnum,:,t)')-CostOfProduction(Y,Units));
      Value =-sum(valDist); %minimum of the negative
   end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   function Value=EvalFunc(Y)
      %does a simple single evaluation of profit.
      Units=ProductionQuantity(Y);
      valDist = Units.*priceHist(PGnum,randi(1000),t)'-CostOfProduction(Y,Units);
      Value = valDist; %actual value
   end
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  function Costs = CostOfProduction(Invested,Units)
   %Lets design an x^2 function with the minimums to be at 300+30*PG# in $MM plus a random
variability
   costperunit = 10 + ((Invested-300-30*PGnum)/100).^2;
   RandomFactor =normrnd(1,0.2,1,Np*Nc);
   Costs = costperunit.*RandomFactor.*Units;
   %Costs is in MM$/time period
   end

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   function Units = ProductionQuantity(Invested)
   %We will design production in (MM)units to be a S shape curve. We will assume that rising
product number gives less units per MM$ invested.
   Units =(smf(Invested,[0 50])/10 + smf(Invested,[50
2000])*5).*(normrnd(1,0.1./(max(Invested.*PGnum/500,ones(1,Np*Nc)))))-0.2*PGnum;
   Units =max(Units,Units.*0);
   %The units are (MM)units/time period
   end
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function ProductPrice = ProductMarket(priceHist,PGnum)
   %Here we assume that the larger the product group number the larger the selling price and
we will also assume more negative skew to simulate more "extreme" events
   ProductPrice = (0.25*priceHist(2)+0.75*priceHist(1))+pearsrnd(0,0.3,-PGnum/10,3);
end
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   function CostsPerUnit = CostOfProduction(Invested,PGnum)
   %%This only exist for the graph
   CostsPerUnit = (10 + ((Invested-300-30*PGnum)/100).^2)*normrnd(1,0.2);
   End

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   function Units = ProductionQuantity(Invested,PGnum)
   %This only exists for the graph
   Units =(smf(Invested,[0 50])/10 + smf(Invested,[50
2000])*5).*(normrnd(1,0.1./(max(Invested.*PGnum/500))))-0.2*PGnum;
   Units =max(Units,Units.*0);
   %The units are (MM)units/time period
   End
```

Appendix 3: MATLAB program3

```
clc; clear;close all
 % Profitability will be tracked according to the following assumptions. Each country and product has a capital to be invested, production is a function of capital invested and operating cost is also based on the total capital invested.
%Once capital is invested it stays there.
 %This optimization will take into account stochastic nature of products, prices, costs adjusts risk based on country
%Its assumed that production efficiency and operating costs follow an equation of capital invested plus some random normal error variation while selling price for products follow a heavy tail function based on last price.
 %We need to define a total amount of capital that be allocated
Capital=4900; %$MM
%Now we need to define the product groups that we are working with
Nc = 5;% Number of Countries
Np = 5; % Number of products

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Now lets determine a time range for this optimization
TimeEnd=3;%time periods
riskTol=0.05;

%First we run the assumed pricing simulations and bin the results. The binnned results will give us a kernal distribution of expected future prices
CostHist = zeros(Np,1,TimeEnd+2);
CostHist(:,1,1)=[11 12 13 13.5 14]; %two time period ago prices
CostHist(:,1,2)=[10.9 12.2 13 13 14.1]; %one time period ago prices
CostHist = repmat(CostHist,1,10000,1);
for t = 1:TimeEnd
   for P = 1:Np
      for Nsim = 1:10000
         CostHist(P,Nsim,t+2)=ProductMarket([CostHist(P,Nsim,t+1),CostHist(P,Nsim,t+0)],P);
      end
      priceKernel{P,t}=fitdist(CostHist(P,:,t+2)',"kernel");
   end
end
close all;
for t=1:TimeEnd
    for j=1:Np
   j;
```

```
    E=zeros(100,1);
    R=zeros(100,1);
    tempKernel=priceKernel{j,t};
        parfor i=1:100
            inv=i*10;
            [E(i),R(i)]=ExpRetRisk(inv,1000,j,tempKernel);
        end
    x=10:10:1000;
    Efit{j,t}=fit(x',E,'smooth');
    Rfit{j,t}=fit(x',R,'smooth');

    if t==1
    figure(4)
    hold on
    plot(Rfit{j,t}(x)'./x,Efit{j,t}(x)'./x,'LineWidth',2)
    ylim([0,.2])
    xlim([0,0.05])
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%defining the countries and running the optimizer
CountryNames=["US","Australia","Canada","SA","Mongolia"];
CountryRisk=[0.00,0.00,5.00,7.63,9.0]/100/12;
[weight,value,riskval]=optloop(TimeEnd,Capital,Nc*Np+1,Efit,Rfit,riskTol);
for j=1:Nc
     PGCweight(j,:)=weight(1,2+(j-1)*5:1+j*5);
end
PGCweight=sort(PGCweight,'descend');
PGCweight(PGCweight<0.001)=0;
for j = 1:Nc
    for i =1:Np
            if PGCweight(j,i)<0.001
                Return(j,i)=0;
                Risk(j,i)=0;
            else
                valRe(j,i)=Efit{i,1}(PGCweight(j,i)*Capital);
                valRi(j,i)=Rfit{i,1}(PGCweight(j,i)*Capital);
        Return(j,i) =max(0,valRe(j,i)/(PGCweight(j,i)*Capital));
        Risk(j,i) =max(0,valRi(j,i)/(PGCweight(j,i)*Capital)+CountryRisk(j));
```

```
        end
    end
end
figure(4)
xlabel('Risk %')
ylabel('Return %')
plot(Risk',Return','*','linewidth',4)
legend('PG1','PG2','PG3','PG4','PG5','US','Australia','Canada','SA','Mongolia')
title('Risk vs Return Graph (Note these are a function of investment)')

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Plotting the results and doing analysis
for i = 1:Np
CNames{i}=char(CountryNames(i));
end
for i = 1:Np
PGname{i}=['Product_Group_',int2str(i)];
end
disp("The amount invested in each product  and country is below")
array2table(PGCweight'*Capital,'VariableNames',CNames,'RowNames',PGname)
 Eall=sum(sum(Return(:).*PGCweight(:)*Capital));
Rall=sum(sum(Risk(:).*PGCweight(:)*Capital));
 disp(sprintf("The overall expected return this time period is: %.1f $MM\nThe overall expected
VaR this time period is: %.1f $MM\n",Eall,Rall))

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [weight,Capital,riskval]=optloop(timeEnd,CapitalStart,nvars,Efit,Rfit,riskTol)
Capital(1)=CapitalStart;
Aeq=ones(1,nvars);
beq=1;
A=[];
b=[];
lb(1,:)=zeros(1,nvars);
for t=1:timeEnd
ub=ones(1,nvars)*1000/Capital(t);
ub(1)=1;
 options=optimoptions('patternsearch','UseCompletePoll',true,'InitialMeshSize',0.055)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
weight(t,:)=patternsearch(@fun,ones(1,nvars)/nvars,A,b,Aeq,beq,lb(t,:),ub,@nonlcon,options);
```

```
Capital(t+1)=Capital(t)-fun(weight(t,:));
lb(t+1,:)=zeros(1,nvars)+weight(t,:)*Capital(t)/Capital(t+1);
lb(t+1,1)=0;
riskval(t)=nonlcon(weight(t,:))+riskTol;
end

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function value=fun(x)
value=0;
for i=2:length(x)
value = value + Efit{mod(i-1,5)+1,t}(x(i)*Capital(t));
end
value=-value;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [C,Ceq] = nonlcon(x)
risk=0;
for i=2:length(x)
risk = risk + Rfit{mod(i-1,5)+1,t}(x(i)*Capital(t))/Capital(t);
end
C=risk-riskTol;
Ceq=0;
end
 end

  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [E,R] = ExpRetRisk(invested,trials,PGroup,kernel)
%simulate a return based on an investment in a product group
    for i=1:trials
        Ret(i)=SimReturn(invested,PGroup,kernel);
    end
    E = mean(Ret);
    R =std(Ret);
end

  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function Return = SimReturn(invested,PGroup,kernel)
%does a single simulation of a stochastic production process and product group selling price
    Units =max(0,((smf(invested,[0 50]) + smf(invested,[50 1000])*7).* ...
```

```
        (normrnd(1,0.1./(max(invested.*PGroup/500,1))))...
        -0.2*PGroup));

    costperunit = 2 +((invested-300-80*PGroup)/60)^2;
    RandomFactor =normrnd(1,0.05);
    Costs = costperunit*RandomFactor*Units;
    Price= random(kernel,1,1);
    Return = Units*Price-Costs;
end

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function ProductPrice = ProductMarket(priceHist,PGroup)
    %Here we assume that the larger the product group number the larger the selling price and we will also assume more negative skew to simulate more "extreme" events
    ProductPrice = (0.25*priceHist(2)+0.75*priceHist(1))*pearsrnd(1,0.05*PGroup,-PGroup/4,3);
end
```