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ᶜALĪ QUSHJĪ AND REGIOMONTANUS: ECCENTRIC TRANSFORMATIONS AND COPERNICAN REVOLUTIONS

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In 1973, Noel Swerdlow presented a new and significant reconstruction of how Copernicus arrived at the heliocentric theory.[1] This reconstruction was based upon several bits of newly-interpreted information, most importantly a set of notes in Copernicus's hand contained in an Uppsala University manuscript. These notes provided compelling evidence that Copernicus had transformed Ptolemy's epicyclic models of the planets into eccentric models as a first step in developing a Sun-centred astronomy.[2] But this transformation depended upon a general proposition that one could indeed convert all the epicycle models into eccentric ones. Curiously, Ptolemy denied this, claiming in Book XII of the *Almagest* that this was possible only for the outer planets (Mars, Jupiter, and Saturn) but not the inner ones (Mercury and Venus). From a modern perspective this seems odd, and it is not entirely clear why Ptolemy could not see that the epicycles of the inner planets, with a proper consideration of speeds, could also be converted into eccentrics. Indeed, Ptolemy's modern translator Gerald Toomer says: "I do not understand why Ptolemy does not recognize this."[3]

Be that as it may, it would seem that no one else recognized this until the fifteenth century. Swerdlow found what he believed to be the source for the propositions Copernicus needed to begin his conversions, namely Book XII, Chapters 1 and 2 of Regiomontanus's *Epitome of the Almagest*.[4] In Chapter 2, Regiomontanus gives a brief sketch and proof of the crucial theory for the inner planets, which would allow Copernicus to convert all the planets from epicyclic to eccentric models. Though Copernicus is sparing in his references and nowhere cites Regiomontanus for these propositions, his use of the *Epitome* is well-documented, and there would seem to have been no other European source that he could have depended upon.[5]

Whatever subsequent use was made of them, Regiomontanus's own motivation for including these propositions at the beginning of Book XII has remained unclear. Swerdlow himself signalled this when he stated: "For some reason the eccentric model must have caught Regiomontanus's attention…."[6] And Michael H. Shank has recently remarked that "We do not yet know specifically what, apart from his compulsive thoroughness, motivated Regiomontanus to explore the eccentric models of the second anomaly".[7] What is especially odd about Regiomontanus's interest is that it is apparently so unprecedented. Neither in Europe nor in the Islamic world does this eccentric alternative alluded to by Ptolemy seem to have generated much interest. And the motivation to extend this alternative to the lower planets, after being rejected by the great authority himself, is even more puzzling. Finally, there is the odd way in which Regiomontanus presents the two propositions. He himself gives no motivation — he just presents them. There is no mention of Ptolemy, no

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statement that Ptolemy was wrong, no explanation of why Ptolemy made his mistake, no claim of credit.

One possibility is that Regiomontanus does not claim credit because he was not in fact the originator of the proposition. Indeed, it would seem, based on evidence presented in the sequel, that an older contemporary of Regiomontanus named ᶜAlī Qushjī may well have been the discoverer of this crucial proposition and that Regiomontanus learned of it either while in Italy or through the intermediation of Cardinal Bessarion, who had originally suggested to Regiomontanus and his collaborator Georg Peurbach that they write the *Epitome*.[8]

Most readers of this journal will be acquainted with Regiomontanus and Peurbach, and perhaps even Bessarion, but ᶜAlī Qushjī is most likely an unknown figure. This is regrettable since he is, at least in my opinion, one of the major figures in astronomy of the fifteenth century.

ᶜAlī Qushjī was a son of a falconer,[9] but not just any falconer. His father worked at the Samarqand court of Ulugh Beg, a grandson of Tīmūr Lang (Tamerlane: 1336–1405). Ulugh Beg was governor of Transoxiana and Turkestan from 1409 until 1447, at which time he briefly became the supreme Timurid ruler until he was killed by order of his son in 1449. A major patron of the arts and sciences, in particular the mathematical sciences, Ulugh Beg attracted to Samarqand a wide array of scientists who taught at the *madrasa* (school) and worked at the impressive astronomical observatory.[10] It was in this environment that Qushjī received his education under such luminaries as Jamshīd al-Kāshī (d. *c*. 1429), Qāīḍzāde al-Rūmī (d. after 1440), and Ulugh Beg himself. After the deaths of Kāshī and Rūmī, Qushjī may have assumed a primary role at the observatory, whose main product was the *Zīj* (astronomical handbook with tables) of Ulugh Beg. When Ulugh Beg was assassinated, Qushjī was forced to seek patronage at a number of courts in Central Asia and Persia. One of his most important works during this later period was his commentary on Naṣīr al-Dīn al-Ṭūsī's theological work, the *Tajrīd al-ᶜAqā'id*. His renown spread to the Ottoman conqueror of Constantinople, Mehmet II, who invited him to Constantinople where he became a professor of the mathematical sciences at two *madrasa*s. Although he spent only two or three years in Constantinople before his death in 1474, Qushjī's influence continued in Ottoman circles for centuries as a result of his writings and the activities of his students.[11]

One thing that seems to have been emphasized in the Samarqand School was the importance of the mathematical sciences. Biographical accounts of Qāḍīzāde al-Rūmī, for example, highlight his difficulties with his teacher al-Sayyid al-Sharīf al-Jurjānī, who thought his student overly interested in mathematics at the expense of philosophy.[12] Kāshī is also noted for his embrace of the mathematical sciences, as we can see from his letters to his father,[13] and Ulugh Beg, like a number of Mongol/Turkic rulers, was predisposed to support the mathematical sciences; in addition, he himself became proficient in them.[14] It was in this atmosphere that the young ᶜAlī Qushjī was raised, and this seems to have had a profound effect upon his intellectual outlook. In his commentary on Ṭūsī's *Tajrīd*, for example, he makes the rather startling case that astronomy should dispense with its dependence upon Aristotelian physics.[15] Even more surprising, he there claims that since there are no good observational proofs for the Earth's motion and since he does not wish to depend upon Aristotle's natural philosophical arguments, the Earth's rotation is a possibility.[16]

It is with this background in mind that we can now turn to his proof that eccentric models could be used for the two lower planets. The motivation for dealing with this problem seems to have arisen in the context of his work on a Mercury model that could serve as an alternative to Ptolemy's.[17] Qushjī was in a long line of Islamic astronomers who objected to the irregular (i.e. non-uniformly rotating) motions contained in several of Ptolemy's planetary models and who had not infrequently proposed alternative models.[18] In the course of his presentation, Qushjī remarks that Ptolemy had mentioned that it was not possible to assign an eccentric as a substitute for Mercury's epicycle to represent the second anomaly, i.e. that having to do with the planet's relationship with the Sun. This was because observation showed that the time between the fastest motion and mean motion was always greater than between mean motion and least motion, a situation Ptolemy contended could be represented by an epicyclic hypothesis (in which the epicycle rotation at the apex was in the same direction as its deferent) but not by an eccentric hypothesis.[19] But Qushjī dismisses this contention, saying "the situation is not as stated by Ptolemy". He then claims that he has a "geometrical proof" but that it would not be appropriate to present it in this treatise on Mercury.[20] And indeed a several-page excursus in a four- or five-folio work would not have been appropriate.

The geometrical proof forecast in the Mercury treatise is clearly contained in the text edited and translated below. But the context is somewhat different. In the Mercury treatise, as we have seen, Qushjī refers to *Almagest* IX.5 in which Ptolemy denied that an eccentric hypothesis could account for the asymmetrical times in the second anomaly of the five retrograding planets. In this treatise, however, the focus is on XII.1, where Ptolemy actually presented just such a model for the upper planets (though without making an explicit connection to IX.5) but denied it for the lower ones. One might speculate that Qushjī had come upon his proposition while experimenting with different models for Mercury and, contra-Ptolemy, had tried to substitute an eccentric for the epicycle, which might explain why he was interested in the asymmetries of time in Mercury's second anomaly. But by the time he came to publish his proof, Qushjī perhaps noticed that Ptolemy in XII.1 had implicitly contradicted his statement in IX.5 as far as the upper planets were concerned and all that remained was to show that the epicycle models of the lower planets could be transformed into eccentrics as well.

Qushjī would seem to be claiming at least some priority for his discovery. He states that "most" of the experts have agreed with Ptolemy in denying that eccentrics could be used to replace epicycles for the lower planets, citing in particular Quṭb al-Dīn al-Shīrāzī (1236–1311). But the fact that he does not say "all" experts could be interpreted as meaning that someone may have questioned Ptolemy on this point. At any rate, from what Qushjī says in the Mercury treatise, he wants to take credit

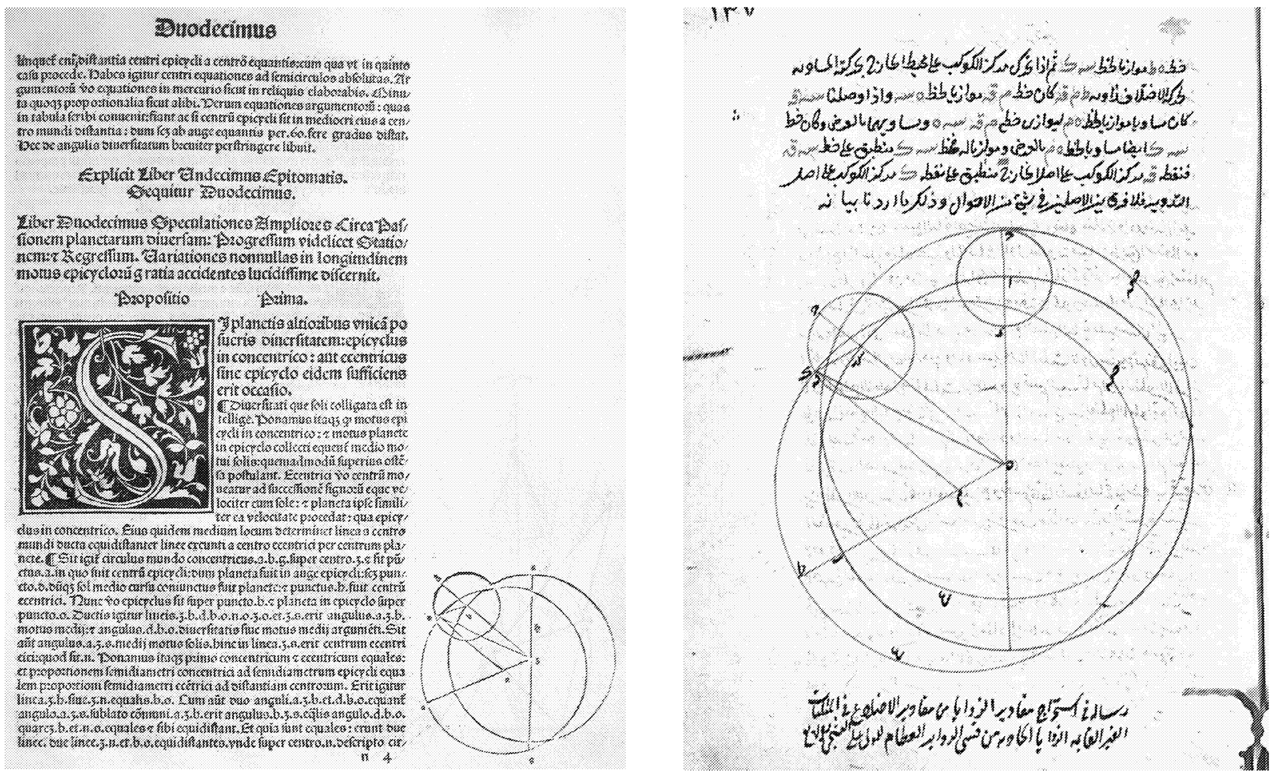


FIG. 1. Comparison of diagrams of Regiomontanus and Qushjī.[28] (*Left*) J. Regiomontanus and G. Peurbach, *Epytoma Joannis de monte regio In almagestum ptolemaei* (Venice, 1496), n4r, and (*right*) ʿAlī Qushjī, *Fī anna aṣl al-khārij*…, Carullah MS 2060, f. 137a. Reproductions courtesy of the History of Science Collections, University of Oklahoma Libraries, and of the Süleymaniye Library, Istanbul, respectively.

at least for the geometric proof.

Unfortunately, none of the three extant manuscripts provides a date of composition, but we can give an approximate date based upon other evidence. It would seem reasonable to date it to a time shortly after the Mercury treatise. Since that work cannot be precisely dated either, we can only assign both to within a certain range. Obviously the Mercury treatise was written before Ulugh Beg's assassination in 1449, since it is dedicated to him. Saliba makes a good argument for dating it to sometime in the 1420s, after Qushjī returned to Samarqand from a period of exile brought on by court intrigue.[21] And İhsan Fazlıoğlu, on the basis of other evidence, has further refined the date to *c.* 1428.[22] So we would not be too far amiss to assign a date of *c.* 1430 for this treatise on the eccentric hypothesis.

How much further did Qushjī or his students go with his discovery? Swerdlow has stated that "Copernicus's derivation of his theory rests upon the eccentric model of the second anomaly and therefore upon these two propositions in the *Epitome*. In this way Regiomontanus provided the foundation of Copernicus's great discovery. It is even possible that, had Regiomontanus not written his detailed description of the eccentric model, Copernicus would have never developed the heliocentric theory".[23] Swerdlow goes on to claim that "While I do not believe that Regiomontanus ever advocated the heliocentric theory, he was, through these two propositions, virtually handing it to any taker".[24] Can we say the same for Qushjī? Since research has just begun into the legacy of ᶜAlī Qushjī, in particular into the Istanbul circle of scientists that he helped initiate, we can only speculate. But it is certainly of considerable interest that Qushjī, like Copernicus, was open to the possibility of the Earth's rotation based upon a new, non-Aristotelian physics.[25]

Inevitably these sorts of discoveries raise anew the question of transmission of late (post-1200) Islamic astronomy to the West. Because of the paucity of research from Europeanists, we do not as yet have a great deal of knowledge of how and under what circumstances this and other products of Islamic science might have been received in the period after the translation movements of twelfth-century Spain and Sicily.[26] But the mounting number of 'coincidences' between early modern European astronomy and late medieval Islamic astronomy can only be held to be 'parallel' developments if one accepts the increasingly implausible idea that somehow the 500-year tradition of non-Ptolemaic astronomy in Islam was recapitulated in Europe in scrupulous detail in a 50-year span in the last part of the fifteenth century.[27]

TRANSLATION AND TEXT

The edited Arabic text presented below is based upon the three extant manuscripts. There are few textual problems. My comments are given as endnotes to the English translation.

The following are the manuscripts, *sigla*, and abbreviations that have been used:

C   Istanbul, Süleymaniye Library, Carullah MS 2060, ff. 136a–137a

H   Bursa, Yazma Library, Hüseyin Çelebi MS 751, ff. 124a–125a

L	Istanbul, Süleymaniye Library, Lâleli MS 3743, f. 60a
[	Separates reading in edition from any variant
:	Separates variant and manuscript *sigla*
+	What follows is an addition to the text
C_{ab}	Above the line in C
C_{mr}	In the margin of C
C_{un}	Under the line in C

Treatise on the Eccentric Hypothesis[29] Being Possible for the Two Lower [Planets] Just as for the Others By Master ᶜAlī Qushjī

In the name of God, the beneficent, the merciful. In Him is my trust.

The author of the *Almagest* held that the eccentric hypothesis is possible for the three [planets] that can be at all elongations from the Sun but that it was not possible for the two lower planets since this hypothesis results in all elongations whereas these two do not become elongated from the Sun except by a small amount.[30] So only the epicyclic hypothesis is possible for them. Most experts have agreed with him on that, including our master, the most learned author of the *Tuḥfa*.[31] But perhaps they came to this conclusion at first glance when they thought that the middle of direct and of retrograde motion, according to the eccentric hypothesis, would need to be at the apogee and perigee, whose positions on the orb are in opposition. They believed that according to the eccentric hypothesis the planet, since it is in conjunction with the mean Sun at the middle of direct motion, would be in opposition to [the Sun] at the middle of its retrogradation, and vice versa. So in going from the middle of direct motion to the middle of retrogradation, it will undergo all elongations from the Sun, which is contrary to the epicyclic hypothesis; in the [latter] case, the middle of the direct motion and of the retrogradation occur at the apex and the [epicyclic] perigee, their positions on the orb being the same.[32]

Thus according to the epicyclic hypothesis, the two lower planets will be in conjunction with the mean Sun both at the middle of direct motion and at the middle of retrogradation. So they will not experience all elongations from the mean Sun in going from the middle of direct motion to the middle of retrogradation; rather they do not become elongated from it except in the amount that is dictated by the epicycle radius.

But the situation is not as they have believed. For the mean motion according to the eccentric hypothesis could rather proceed sequentially in the amount of the sum of the Sun's mean motion and the motion of anomaly, while the motion of the eccentric is counter-sequential in the amount of the motion of anomaly.[33] So in the amount by which the eccentric causes the planet's centre to elongate from the mean Sun counter-sequentially, the eccentric's deferent by its motion sequentially will restore it. So the elongation between the planet's centre and the mean Sun will end up being none other than the amount of the equation as it was according to the epicyclic hypothesis, [whereby] it will be elongated from it only by the amount of

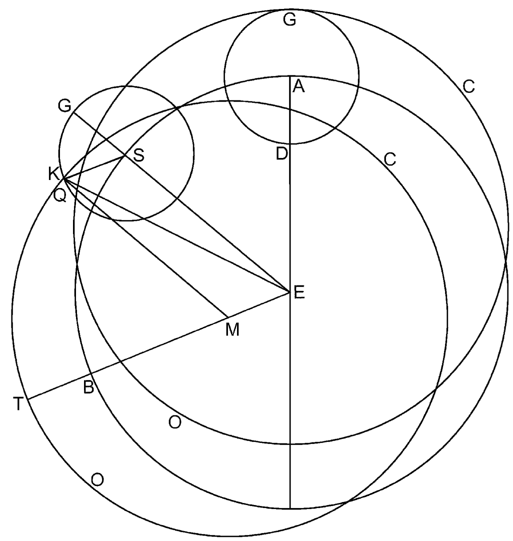


FIG. 2

the equation. So the equation at each instant according to the two hypotheses is the same.[34] Thus the centre of the two lower planets will only become elongated from the mean Sun by the same amount according to each of the two hypotheses.

In order to prove this: let circle *AB* with centre *E* be the equator[35] of both the deferent of the eccentric and of the epicycle; circle *GD* the equator of the epicycle; circle *CO* about centre *M* the equator of the eccentric. Let us take the planet to be in the middle of direct motion at the apogee of the eccentric [according to the eccentric hypothesis or] at the apex of the epicycle according to the epicyclic hypothesis. Then let the centre of the epicycle move through angle *AES* by the mean motion[36] and the centre of the planet through angle *GSK* by the motion of anomaly. We join *EK*. We will show that the centre of the planet according to the eccentric hypothesis is also at point *K*. This is so since if the eccentric apogee moves by the motion of the eccentric's deferent with a motion equal to the sum of the mean motion and anomaly through angle *AET*, then angle *SEB* is its excess over the mean and is equal to angle *GSK*, which is the anomaly. Therefore line *ET* is parallel to line *SK*. Then when the centre of the planet moves on the circumference of the eccentric with [the eccentric's] motion through angle *TMQ*, which is equal to the motion of anomaly, line *MQ* will be parallel to line *ES*.[37] When we join *SQ*, it will be equal and parallel to line *EM* since lines *MQ* and *SE* are parallel and are equal by assumption. Line *SK* is also equal to line *EM* by assumption and is parallel to it. So line *SK* is coincident with line *SQ*. Therefore point *Q* is the centre of the planet according to the eccentric hypothesis and coincident with point *K*, which is the centre of the planet according to the epicyclic hypothesis. So there is no difference between the two hypotheses in any particular. That is what we sought to prove.[38]

(C:136a; H:124a; L:60a)

رسالة في أنّ أصل الخارج يمكن في السفليين
كما في غيرهما للمولى علي القشجي[1]

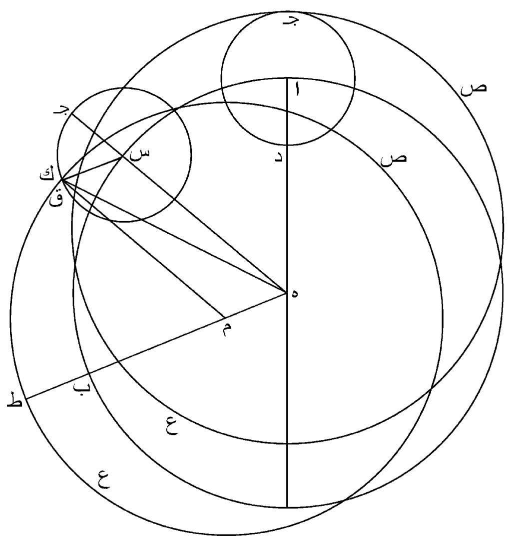
(C:136b; H:124b)

بسم الله الرحمن الرحيم وبه ثقتي[2]

ذهب صاحب المجسطي إلى أن أصل الخارج إنّما يمكن في الثلاثة[3] التي يبعد عن الشمس كلّ البعد ولا يمكن في السفليين لاقتضاء هذا الأصل كلّ الأبعاد وهما[4] لا يبعدان عن الشمس إلا بقدر[5] يسير ففيهما لا يمكن إلاّ أصل التدوير ووافقه على ذلك أكثر المحققين ومنهم مولانا العلاّمة صاحب التحفة ولعلّهم إنّما[6] حكموا بذلك لمّا رأوا في بادئ أنظارهم أنّ وسطي الاستقامة والرجعة على أصل الخارج إنّما يكونان عند الأوج والحضيض وموضعاهما على الفلك متقابلان وظنّوا أنّ الكوكب[7] على أصل الخارج إن كان في وسط الاستقامة مقارناً لوسط الشمس فهو عند وسط[8] الرجعة يصير مقابلاً له وبالعكس فيحصل له في وصوله من وسط الاستقامة إلى وسط الرجعة جميع الأبعاد من الشمس بخلاف أصل التدوير فإنّ وسطي الاستقامة والرجعة منه إنّما يكونان[9] عند الذروة والحضيض وموضعاهما على الفلك واحد .

فالسفليان على أصل التدوير يقارنان وسط الشمس عند وسطي الاستقامة والرجعة معاً فلا يحصل لهما في وصولهما من[10] وسط الاستقامة إلى وسط الرجعة جميع الأبعاد من وسط الشمس بل لا يبعدان عنه إلاّ بقدر ما يقتضيه نصف قطر التدوير .

وليس الأمر كما ظنّوا فإنّ حركة الوسط على أصل الخارج إنّما يتعرّض إلى التوالي بمقدار مجموع حركتي وسط[11] الشمس والاختلاف وحركة الخارج إلى



خلاف التوالي بمقدار حركة الاختلاف فبمقدار ما يبعد الخارج مركز[12] الكوكب عن وسط الشمس إلى خلاف التوالي يردّه حامل الخارج بحركته على التوالي ولا يبقى البعد[13] بين مركز الكوكب ووسط الشمس إلاّ بمقدار التعديل كما أنّه على أصل[14] التدوير لا يبعد عنه إلاّ بمقدار التعديل والتعديل في كلّ حين على الأصلين واحد فمركز السفليين على كلا الأصلين /(H:125a) لا يبعدان عن[15] وسط الشمس إلا بمقدار[16] واحد .

وليكن لبيان ذلك دائرة اب على مركز ه منطقة حاملي الخارج والتدوير معاً ودائرة جد منطقة التدوير ودائرة صع على مركز م منطقة الخارج المركز ولنفرض الكوكب في وسط الاستقامة على أوج الخارج وذروة التدوير على أصل التدوير[17] ثم ليتحرك مركز التدوير بحركة الوسط زاوية اهس ومركز الكوكب بحركة الاختلاف زاوية[18] جسك[19] ونصل ه ك ونبيّن أنّ مركز الكوكب على أصل الخارج أيضاً على نقطة ك وذلك لأنّ أوج الخارج إذا تحرّك بحركة حامل الخارج حركة مساوية لمجموع حركتي الوسط والاختلاف زاوية

اهط كان(!) زاوية سهب فضلها على الوسط مساوية لزاوية جسك الاختلافية فكان /(C:137a) خط هط موازياً لخط سك ثم إذا تحرّك مركز الكوكب على محيط الخارج بحركته المساوية لحركة الاختلاف زاوية طمق كان خط مق موازياً لخط هس وإذا وصلنا سق كان مساوياً موازياً لخط هم لتوازي[20] خطي مق سه وتساويهما بالفرض[21] وكان خط سك أيضاً مساوياً لخط هم بالفرض[22] وموازياً له فخط سك منطبق على خط سق فنقطة ق مركز الكوكب على أصل الخارج منطبق على نقطة ك مركز الكوكب على أصل التدوير فلا فرق بين الأصلين في شيء من الأحوال وذلك ما أردنا بيانه[23] .

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[1] القشجي] القوشجي : H (بدون نقط) = +رحمه له تع : C = +عليه الرحمه : H. [2] بسم الله الرحمن الرحيم وبه ثقتي] بسم (؟) الله سبحانه : L. [3] الثلاثة] الثلثة : C, H. [4] وهما] هما : L. [5] بقدر] +~~يسير~~ : C. [6] إنما] نما : C_{un}. [7] الكوكب] الكواكب : H. [8] وسط] C_{mr}. [9] يكونان] يكون : H (الياء بدون نقط). [10] من] H_{un}. [11] وسط] وسطى : L. [12] مركز] +فلكى(؟) : L. [13] البعد] +~~عن ذلك~~ : C. [14] أصل] +~~الخارج~~ : H. [15] عن] +(كلمة مشطوبة غير مقروءة) : L. [16] بمقدار] +(عدة كلمات مشطوبة غير مقروءة) : L. [17] التدوير] الخارج : H = والتدوير (الياء بدون نقط) : H_{ab}. [18] اهس ومركز الكوكب بحركة الاختلاف زاوية] C_{mr}. [19] جسك] د(؟) س ~~ج~~ ك : C = د(؟) س ك : L. [20] لتوازي] ليوازى : C. [21] بالفرض] بالعرض : C, H, L. [22] بالفرض] بالعرض : C, H, L. [23] أردنا بيانه] اردناه والحمد لله والصلوه على محمد واله احمعين : H (بدون نقط). (في ورقة 132a من مخطوطة H نجد الشكل الموجود في النصّ مكرّر مع الجملة التالية : «هذا الشكل في رسالة كتبها المولى علي القوشجي لبيان أنّ أصل الخارج يمكن في السفليين كما في غيرهما» (بدون نقط).

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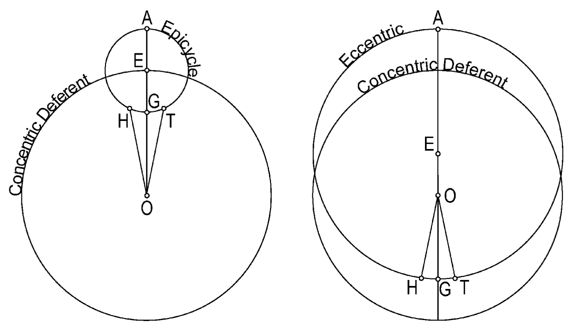
2. Recently B. R. Goldstein ("Copernicus and the origin of his heliocentric system", *Journal for the history of astronomy*, xxxiii (2002), 219–35) has argued that Copernicus's *motivation* for the heliocentric system arose from his insistence upon the distance–period relationship for the planets and a rejection of Ptolemy's nesting hypothesis, since the latter violated the former for the Sun, Venus, and Mercury. If true, this would make Swerdlow's reconstruction problematic since it depends upon Copernicus's reaching his decision in favour of heliocentrism only after rejecting a proto-Tychonic model that resulted in an intersection of orbs (specifically those of Mars and the Sun), something not allowed in most of ancient and medieval cosmology. For Swerdlow, Copernicus's initial motivation for exploring alternative models had to do with Ptolemy's violations of uniform, circular motion, thus placing him firmly within the tradition of Islamic theoretical astronomy (*hay'a*). Goldstein's position, though plausible, has little, if any, textual support from the *Commentariolus*, depending instead upon the much later *De revolutionibus*. A straightforward reading of the introduction to the *Commentariolus*, as well as consideration of the considerable space Copernicus devotes in that work to his alternative *mathematical* models, would seem to indicate that the "equant problem" was foremost in his mind around 1510. But whatever the motivation, Goldstein agrees that the mathematical transformation from a geocentric to heliocentric cosmology would still rely, as argued by Swerdlow, upon the propositions found in Book XII of Regiomontanus's *Epitome*, which is the subject of the current article.
3. G. J. Toomer, *Ptolemy's Almagest* (New York and Berlin, 1984), 555 (n. 2).
4. Though completed in 1463, the *Epitome* was first printed in 1496 in Venice, after Regiomontanus's death. For details, see Swerdlow and Neugebauer, *op. cit*. (ref. 1), 51. Swerdlow has translated Book XII.1–2 in his *op. cit*. (ref. 1), 472–5.
5. For Copernicus's references to his sources, or lack thereof, see Swerdlow, *op. cit*. (ref. 1), 437.
6. Swerdlow, *op. cit*. (ref. 1), 471.
7. Shank, *op. cit*. (ref. 1), 185.
8. On Bessarion's role in encouraging the writing of the *Epitome*, see Swerdlow and Neugebauer, *op. cit.* (ref. 1), 50–51. It is worth noting that Bessarion was originally from the Black Sea town of Trebizond, which fell to the Ottomans in 1461.
9. Whence the name Qushjī: kuş (Ottoman: qūsh) is the Turkish word for falcon or hawk; kuşçu (Ottoman: Qūshjī) is falconer.
10. For accounts of the Samarqand school and observatory, see: A. Sayılı, *The observatory in Islam* (Ankara, 1960), 260–89; E. S. Kennedy, "The heritage of Ulugh Beg", in *idem*, *Astronomy and astrology in the medieval Islamic world* (Aldershot and Brookfield, VT, 1998), XI; İ. Fazlıoğlu,

"Osmanlı felsefe-biliminin arkaplanı: Semerkand matematik-astronomi okulu", *Dîvân ilmî araştırmalar*, xiv/1 (2003), 1–66; and G. Saliba, "Reform of Ptolemaic astronomy at the court of Ulugh Beg", in *Studies in the history of the exact sciences in honour of David Pingree*, ed. by C. Burnett *et al*. (Leiden, 2004), 810–24.

11. For an account of Qushjī's life, see İ. Fazlıoğlu, "Ali Kuşçu", in *Yaşamları ve yapıtlarıyla Osmanlılar ansiklopedisi*, ed. by E. Çakıroğlu (Istanbul, 1999), i, 216–19 and *idem*, "Qūshjī", in *Biographical encyclopaedia of astronomers*, ed. by T. Hockey (Springer/Kluwer, forthcoming).
12. F. J. Ragep, "Ḳāḍī-zāde Rūmī", *The encyclopaedia of Islam* (Leiden, 2004), xii, 502.
13. E. S. Kennedy, "A letter of Jamshīd al-Kāshī to his father: Scientific research and personalities at a fifteenth century court", *Orientalia*, xxix (1960), 191–213; reprinted in E. S. Kennedy *et al.*, *Studies in the Islamic exact sciences* (Beirut, 1983), 722–44. *Cf.* M. Bagheri, "A newly found letter of Al-Kāshī on scientific life in Samarkand", *Historia mathematica*, xxiv (1997), 241–56.
14. E. S. Kennedy, "Ulugh Beg as scientist", in *idem*, *Astronomy and astrology in the medieval Islamic world* (ref. 10), X.
15. F. J. Ragep, "Freeing astronomy from philosophy: An aspect of Islamic influence on science", *Osiris*, xvi (2001), 49–71 (espec. 61–63).
16. *Ibid*. and F. J. Ragep, "Ṭūsī and Copernicus: The Earth's motion in context", *Science in context*, xiv (2001), 145–63 (espec. 156–7).
17. This work has been edited and translated by G. Saliba, "Al-Qushjī's reform of the Ptolemaic model for Mercury", *Arabic sciences and philosophy: A historical journal*, iii (1993), 161–203.
18. For an overview, see G. Saliba, "Arabic planetary theories after the eleventh century AD", in *Encyclopedia of the history of Arabic science*, ed. by R. Rashed (3 vols, London and New York, 1996), i, 58–127.
19. *Cf.* Toomer, *op. cit.* (ref. 3), ix.5, 442 and n. 38. For an informed discussion of this passage, see O. Neugebauer, *A history of ancient mathematical astronomy* (3 vols, New York, 1975), i, 149–50.
20. Saliba, *op. cit*. (ref. 17), 172 (English translation), 194 (Arabic text); I have slightly modified Saliba's translation.
21. Saliba, *op. cit*. (ref. 17), 166.
22. Fazlıoğlu, *op. cit*. (ref. 11).
23. Swerdlow, *op. cit*. (ref. 1), 472.
24. *Ibid*., 475–6 (n. 8).
25. In Ragep, *op. cit*. (ref. 16), I present evidence indicating a possible connection between Copernicus and his Islamic predecessors (including Qushjī) regarding the question of the Earth's rotation.
26. A possible transmission through Italy has been advanced (Swerdlow and Neugebauer, *op. cit*. (ref. 1), 47–48, 55). But the role of Bessarion in transmitting materials to Peurbach and Regiomontanus cannot be discounted.
27. For an elaboration of this point, see F. J. Ragep, "Copernicus and his Islamic predecessors: Some historical remarks", *Filozofski vestnik*, xxv (2004), 125–42.
28. Needless to say, the similarity in orientation of the two diagrams is striking. If one accepts that this is a case of borrowing, one might speculate that the additional epicycle and eccentric in the initial positions in Qushjī's diagram have been removed for visual simplification. One caveat: the Latin lettering does not correspond to the Arabic.
29. Hypothesis translates *aṣl*, which in turn was used to translate the Greek ὑπόθεσις. Both in Greek and in Arabic, the meaning is 'basis', especially that upon which something else is constructed. There is no implication of the modern sense of hypothesis, i.e. a tentative theory that needs to be verified. *Cf*. Toomer, *op. cit*. (ref. 3), 23–24.
30. See xii.1 of the *Almagest* (Toomer, *op. cit*. (ref. 3), 555).
31. The reference is to *al-Tuḥfa al-Shāhiyya* (The royal gift) by Quṭb al-Dīn al-Shīrāzī (1236–1311). The passage in question occurs in bk. ii, ch. 8. The work as a whole has not been edited or printed, but

this chapter has been edited and translated by Robert Morrison and will appear in a forthcoming issue of the *Journal for the history of Arabic science*.

32. Qushjī evidently has the following diagram in mind, which is an adaptation of a diagram that one may find, among other places, in *al-Tadhkira fī ᶜilm al-hay'a* by Naṣīr al-Dīn al-Ṭūsī (1201–74). (See F. J. Ragep, *Naṣīr al-Dīn al-Ṭūsī's memoir on astronomy* (2 vols, New York, 1993), i, 138–9.) In the *Almagest* (xii.1), Ptolemy had used a single diagram for both the epicyclic and the eccentric models, but Ṭūsī has split them into two separate representations.



A: apogee; *E*: centre of epicycle/eccentric; *G*: epicyclic perigee/perigee; *HT*: retrograde arc; *O*: centre of world.

Qushjī speculates that the reason Ptolemy had denied the possibility of an eccentric for the lower planets was because he thought that, like the upper planets, they would thereupon undergo retrograde motion at opposition to the Sun, which is contrary to observation. Looking at the above diagram, one can see that, indeed, at first glance one might think that the epicyclic model would allow for both the middle of direct motion and retrogradation to occur at conjunction whereas the eccentric model would require the two to occur 180° apart. This would be the case if one were to assign the same motions to the eccentric models of both the upper and lower planets, namely the concentric deferent moving (west to east) with the motion of the mean Sun while the eccentric moves (east to west) with the motion of the epicyclic anomaly. But as Qushjī shows below, one can adjust the motions appropriately so that an eccentric model will work for the lower planets.

33. For the eccentric model of the upper planets, the mean motion (or motion of centre) is sequential, i.e. in the order of the zodiacal signs, and equal to the Sun's mean motion, while the eccentric motion is counter-sequential and equal to the motion of anomaly (see O. Pedersen, *A survey of the Almagest* (Odense, 1974), 339–40). For the eccentric model of the lower planets that Qushjī describes here, the mean motion (or motion of centre) must be sequential and equal to the sum of the Sun's mean motion and the motion of anomaly; as with the upper planets, the eccentric here moves counter-sequentially and its motion is equal to the motion of anomaly. (See below and figure in text.)

34. Referring to the text figure, the equation in the epicyclic model is $\angle KES$; in the eccentric model, $\angle QES$.

35. Equator translates "minṭaqa", which can be used for both an equator on the surface of a sphere and also a parallel "inner" equator, as it is here; see Ragep, *op. cit.* (ref. 32), ii, 414, 437–8.

36. One would normally expect "motion of the centre", i.e. of the centre of the epicycle. Perhaps Qushjī felt that since he was dealing with both models simultaneously, "mean motion" was more appropriate.

37. This is so since $\angle SET = \angle QMT = \angle GSK$ = motion of anomaly.

38. The accompanying diagram occurs in all three manuscripts with essentially the same lettering and orientation. (MS L, however, is missing line *EK*.) It is also reproduced a second time in MS H, f. 132a with the following header: "This figure is in the treatise written by master ᶜAlī Qūshjī to prove that the eccentric hypothesis is possible for the lower planets as it is for the others."