Persian-Arabic Seventeen-Tone Temperament: A Microtonal Extension to the Heptatonic Scale

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Abstract

The prevalence of twelve-tone equal temperament as the fundamental basis for music worldwide can be traced back to the perspective of Western classical music, where the need arose to select a limited set of musical intervals to assign to the keyboard. However, this temperament does not encompass the rich tonal intricacies found in traditional Persian-Arabic music. In this study, I delve into the distinctive qualities of the second, third, sixth, and seventh degrees of a heptatonic scale, which in the Persian-Arabic tradition allow for *major*, *minor*, and *neutral* variations. This unique approach yields a pitch system comprised of seventeen tones, traditionally represented by seventeen Abjadic letters in early treatises of the Systematist School.

The focus of this thesis lies in exploring the various sizes of stepwise intervals in Persian dastgāhi music, achieved through ratios derived from the natural harmonic series. I present these intervals on a *bubble diagram* that encompasses the seventeen tones of the system, which is explainable with different practical and theoretical pitch set suggestions. Additionally, I shed light on eight renowned tetrachord genera of the Persian dastgāhi music, providing the approximated sizes of their intervals based on the aforementioned musical ratios. I also demonstrate the alignment of these tetrachords, forming the main scales for each distinct modal category.

The results of this study intriguingly highlight that the Persian-Arabic seventeen-tone temperament can be viewed as an *extension* of the Western twelve-tone system, offering expanded modal possibilities. By bridging the gap between these two distinct musical traditions, I aim to enrich our understanding of music's diverse expressions and pave the way for further exploration and cross-cultural musical exchange.

Résumé

La prédominance du tempérament égal à douze notes comme base fondamentale de la musique dans le monde peut être retracée à la perspective de la musique classique occidentale, où le besoin s'est fait sentir de sélectionner un ensemble limité d'intervalles musicaux à assigner au clavier. Cependant, ce tempérament n'englobe pas les riches subtilités tonales que l'on trouve dans la musique traditionnelle arabo-persane. Dans cette étude, je me penche sur les qualités distinctives des deuxième, troisième, sixième, et septième degrés d'une échelle heptatonique qui, dans la tradition arabo-persane, permet des variations *majeures*, *mineures*, et *neutres*. Cette approche unique permet d'obtenir un système de hauteurs composé de dix-sept notes, traditionnellement représentés par dix-sept lettres abjadiques dans les premiers traités de l'école systématiste.

L'objectif de cette thèse est d'explorer les différentes tailles d'intervalles progressifs dans la musique dastgāhi persane, obtenues grâce à des ratios dérivés de la série des harmoniques naturelles. Je présente ces intervalles sur un *diagramme à bulles* qui englobe les dix-sept notes du système, explicable avec différentes suggestions pratiques et théoriques d'ensembles de notes. De plus, je mets en lumière huit genres de tétracordes renommés de la musique dastgāhi persane, en fournissant les tailles approximatives de leurs intervalles sur la base des rapports musicaux susmentionnés. Je démontre également l'alignement de ces tétracordes, qui forment les échelles principales pour chaque catégorie modale distincte.

Les résultats de cette étude soulignent de manière intrigante que la musique arabo-persanne à dix-sept notes peut être considéré comme une extension du système occidental à douze notes, offrant des possibilités modales étendues. En comblant le fossé entre ces deux traditions musicales distinctes, je vise à enrichir notre compréhension des diverses expressions de la musique et à ouvrir la voie à une exploration plus approfondie et à un échange musical interculturel.

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1. An introduction to Persian seventeen-tone temperament[1]

You may have heard of *maqām*[2] and its contemporary Persian version, *dastgāh*, before. Explaining this complex thousand-year-old musical system, which belongs to a wide and diverse geographical region centred on Greater Persia and the Arab-Turk countries, would take hours of time and hundreds of pages of writing, and such an explanation is not the purpose of this thesis. But briefly, it can be defined as a collection of melodic figures, which are classified in a specific order into distinct modal categories and are used mostly for improvisational purposes. This music is tonal and heptatonic, but not limited to twelve-tone equal temperament. The tuning and temperament of maqāmi/dastgāhi music is the main concern of my research. The exact size of musical intervals in this system and their scientific explanation by music theory have been debated and disputed for a long time.

Twelve-tone equal temperament, which today is the basis of the vast majority of music in the world, arose from the point of view of Western classical music and resulted from the necessity of choosing a limited number of musical intervals to assign to the keyboard. According to the interval structure of the diatonic scale on the standard Western keyboard's twelve keys, the second, third, sixth, and seventh degrees of a heptatonic scale are only allowed to be either *major* or *minor*. However, many of the beautiful intervals of maqāmi/dastgāhi music, with similarly logical mathematical/acoustical derivations (based on numerical ratios from the natural harmonic series) remain ignored by this way of thinking. While some of these can be roughly approximated as tempered major or minor intervals in the Western system, others fall in a range between major and

[1] I preferred the word *temperament* because of the approximations and adjustments that have been always applied to this intonation.

[2] (Plural: maqāmāt), Also known as muqam, makam, magham, mugam, maquam, moqom, etc.

minor and require the attribution of an autonomous quality, which can be called *neutral*. A part of my thesis will be dedicated to defining these neutral intervals, their size, their representatives in the natural harmonic series, and their place in the maqāmi/dastgāhi heptatonic scales.

Interestingly, the presence and usefulness of various neutral intervals are traceable even in the music of some Western countries before the dominance of the keyboard and the twelve-tone temperament. Remarkable research has been done on the popularization of the keyboard and its soft colonization of many musical cultures, from Turkish (Yarman 2008), Azerbaijani, and Armenian traditional music (During 2019) to Scandinavian, Norwegian, and Swiss folk music (Wey 2020), and even to Blues (Kubik 1999). The fact is that the lack of neutral intervals on the keyboard has caused many musical ambiguities and destructive transformations in the melodic modes of these musical cultures as well as many others. In this thesis, I will investigate Persian-Arabic music: a musical culture that has not succumbed to the twelve-tone major-minor simplifications of the keyboard. It should be mentioned that although they both have roots in Persian-Arabic maqāmi/dastgāhi music, I have excluded Turkish music from the scope of this research because of the changes that were made in the music theory of this country after the westernization policies of new Turkey at the beginning of the twentieth century, and also Azerbaijani music because of the Russification policies during the Soviet Union era.

Many of the topics that I will cover in this research (which is a continuation of a previous research project about non-Western notation methods in Persia) are being studied for the first time. Due to the severe lack of sources related to my thesis, the results of this research have the potential to become a useful and concrete resource to introduce Persian-Arabic music theory to Western audiences and can be the basis of further in-depth research in the future.

The confrontation of the seventeen-tone and the twelve-tone systems

The turn of the twentieth century was the beginning of significant developments in the musical styles and genres of the West, while in Persia it was the beginning of full exposure to Western music and instruments. When King Nāsser-al-Din Shāh (1831–96) visited Paris in 1873, he admired the French military bands that welcomed him. On his return to Iran, the King asked his ambassador to France to hire French musicians to reform his military orchestras with Western standards. The French bandmaster Alfred Jean-Baptiste Lemaire was assigned to accomplish this task and started reorganizing military bands for the Persian Court. He was also asked to provide a transcription of the Persian modal system, which is based on an oral transmission tradition. As a result, he published the first account of the Persian *dastgāh* system, in 1900, entitled *Avâz et Tèsnîf Persans.*[3]

He explains his observations in a short description as follows: "The Persian scales consist of seventeen notes, the eighteenth being the octave of the first [note], so it has five notes more than our [European] scale, which explains why most Persian tunes cannot be played on our wind instruments or on the piano. They could only be faithfully rendered on the violin or other bowed instruments [...]"[4] (Lemaire 1900; Mohammadi 2017, 53-57 and 184-185). This short text illustrates the problem that arises from the confrontation of Persian seventeen-tone temperament with Western twelve-tone temperament, the central topic that I am going to explore in this research.

[3] Avâz is non-metric internal section of the modes in Persian dastgāhi music, and Tèsnîf is ballad.

[4] The text in French:

La gamme persane comprend dix-sept notes, la dix-huitième étant l'octave de la 1er, elle a donc cinq notes de plus que notre gamme, ce qui explique que la plupart des airs persans ne peuvent être joués ni sur nos instruments à vent, ni sur le piano. Ils ne pourraient être fidèlement rendus que sur le violon ou les autres instruments à archet [...].

In simple words, the conclusion of most of the music theorists from long ago until now, after observing the intervals used by music practitioners and comparing them with the existing theory, end up to the fact that at least *one other quality*, in addition to major and minor, is required for the second and third degrees of a tetrachord, or in a broader view, for the second, third, sixth, and seventh degrees of an octave, for performing all Persian and Arabic melodies. Examples of this could be seen in the treatises of the first post-Islamic trend of Persian-Arabic music theory, the Scholastic School,[5] dating back to the tenth century. For instance, in his treatise on music, Abu-Nasr Fārābi[6] (870 – 950) talks about *three* different states of the third interval (major, minor, and neutral), in a brief section called "The Kindred Powers in Melodic Lines."[7] He introduces the mentioned frets on oud and assign them to specific ratios, and never explains where the numbers in numerator and denominator come from. The major third that he presents is a Pythagorean ditone (81:64), whereas his minor and neutral thirds are two of the practical frets (called Persian and Zalzal middle frets respectively) used by the practitioners at the time (Fārābi n.d., 10-12; 1996, 87-90). I will elaborate on this in the following sections.

A similar conclusion is reached in a paragraph in one of the music treatises of Mehdi-Qoli Hedāyat’s (1863–1955), the prominent Iranian contemporary figure and a great elaborator of the second post-Islamic music theory trend, the so-called Systematist School,[8] at the beginning of a section titled “Thirty-ninth part: The approximation of the intervals by Europeans”: “The early scholars [of Systematist School], according to ear, used to divide a whole tone into *three parts*. But Europeans, after they made the piano and the other instruments alike, faced problems. Given

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5 مکتب مُدرّسی

6 ابونصر فارابی

7 قوه‌های متجانس در الحان اصلی

8 مکتب مُنتَظَمیّه

that the vast number of keys was an obstacle, a contraption should have been made. They ended up with [equal] temperament and deviation from the natural harmonic series and divided a whole tone into two parts or the perfect fourth into five equal parts”[9] (Hedāyat 1938, first section 134).[10]

On the Western side, Hermann Helmholtz, who is apparently Hedāyat’s source of acoustical information, raises a fundamental question in his *Lehre von den Tonempfindungen:* “To what purpose do we conclude our diatonic scale with the following singularly unequal arrangement of intervals: 1, 1, ½, 1, 1, 1, ½ ? […] The old scale of five tones appears to have avoided semitones as being too close. But when two such intervals already appear in the scale, why not introduce more?” (Helmholtz 1895, 280).

Helmholtz’s proposal leads inevitably to the conception of a twelve-note chromatic scale containing the heptatonic scale as a subset. Based on this idea, I designed a diagram of seventeen bubbles, each presenting one of the seventeen tones of the Persian-Arabic temperament. And I compared it to the twelve tones of the Western equal temperament, which is demonstrated in the same way with twelve bubbles (Figure 1-1). The seventeen-tone diagram includes neutral versions of the second, third, sixth, and seventh. The equal-tempered twelve-tone Western tritone is eliminated in the seventeen-tone diagram and replaced by two separate tones (neutral fourth and neutral fifth). It does not seem that such an arrangement of the seventeen tones of the Persian-

[9] The text in Persian:

گوشه سی‌ونهم – در تعدیل ابعاد نزد اروپائیان
قدما، طنینی را به‌حکومت گوش، به سه قسمت تقسیم کرده‌اند. اروپائیان پس از آنکه پیانو و امثال آنرا ساختند، به‌اشکالات برخوردند، چه کثرت مضراب در پیانو مزاحم عمل شد، تدبیری می‌بایست کرد. به تعدیل ابعاد و انحراف از سلسله طبیعی نغمات قائل شدند و طنینی را به دو قسمت، یا ذوالاربع را به پنج بعد مساوی تقسیم کردند.

[10] Interestingly, a few western twenty-century musicologists come to a similar conclusion. As an example, George Secor, American musicologist, in his article describes the significance of the seventeen-tone system with these sentences: “I now realize that, had music history taken a different turn during the later Middle Ages, it is plausible that we would now be using 17-ET instead of 12-ET. Furthermore, if we were now in the position of evaluating 12-ET as a possible alternative to 17-ET in the search for new tonal resources, we would probably dismiss 12-ET just as readily, declaring it to be melodically and harmonically bland and crude” (Secor 2006, 55).

Arabic temperament into seven scale degrees has been proposed in other sources. Rather, a few of the theorists or prominent practitioners of different times have suggested different pitch value sets, expressing tunings in either ratios or cents. I will elaborate on their suggestions and the methods that they used to obtain these intervals later in this section, but the point is that a comparison of all the different versions leads clearly to the same arrangement demonstrated in my Figure 1-1 diagram.

As you see, for both twelve-tone and seventeen-tone temperaments, the tones are categorized in seven separated groups, representing the seven scale degrees, and are vertically aligned to each other. As I mentioned above, for the second, third, sixth, and seventh scale degrees, there are two possibilities (major and minor) in twelve-tone temperament, and three possibilities (major, neutral, minor) in seventeen-tone temperament.

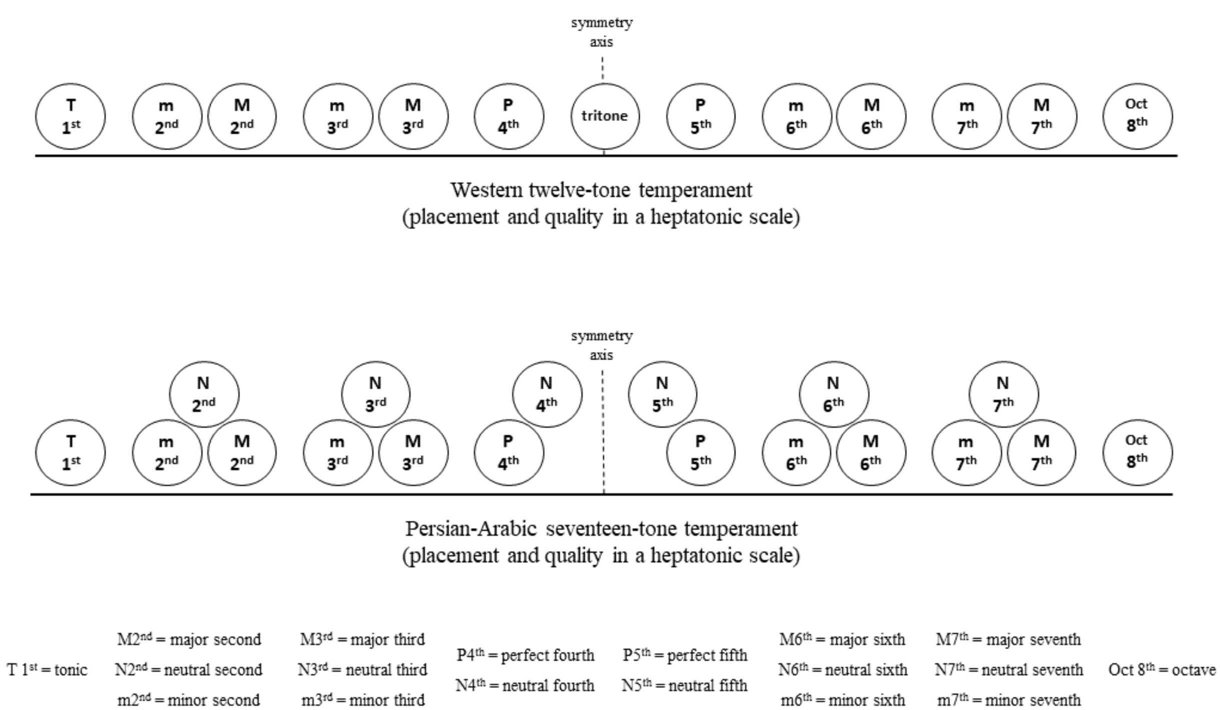

Figure 1-1: A comparison between the twelve-tone and seventeen-tone temperaments on the bubble diagram.

The beauty of the seventeen-tone bubble diagram is that you can imagine the heptatonic scale as two tetrachords separated by a whole tone. This view corresponds to the traditional Ancient Greek definition of disjunct tetrachords. Despite that the seventeen tones in this temperament are not equally distributed, we can still consider an axis of symmetry exactly at the middle of the octave, and see the two tetrachords as inversions of each other. As is demonstrated on the twelve-tone diagram, the symmetry axis coincides with the position of the tritone interval.

The origin of the tritone interval is the eleventh partial of the harmonic series, which is represented with the ratio of 11:8 (551.32 cents) if brought to the span of an octave. Needless to say, the twelve-tone equal-tempered render of this interval has no other choice than being either an F (500 cants) or an F-sharp (600 cents), which is the most deviated version on an interval compared to its real value. In contrast, in the seventeen-tone diagram, instead of the equal-tempered tritone, two separated neutral fourth and neutral fifth intervals are located symmetrically on opposite sides of the axis of symmetry. The bubble-tone specified with Neutral fourth on the diagram could be undoubtedly a more accurate approximation of the natural tritone interval. Inside each bubble, the quality of the intervals is shown with abbreviated letters: P is used for a Perfect interval, M for Major, m for minor, and N for Neutral, and obviously, the numbers indicate the scale degree.

In the seventeen-tone temperament, the idea of the inversion of the intervals within the span of an octave still works properly. The inversion of the major intervals are the minor intervals from the opposite side, and vice versa (e.g., major third to minor sixth). The inversion of a neutral interval will be a neutral interval from the opposite side (e.g., neutral second to neutral seventh). The relation of the intervals and their inversions are shown in a seventeen-tone bubble diagram in Figure 1-2.

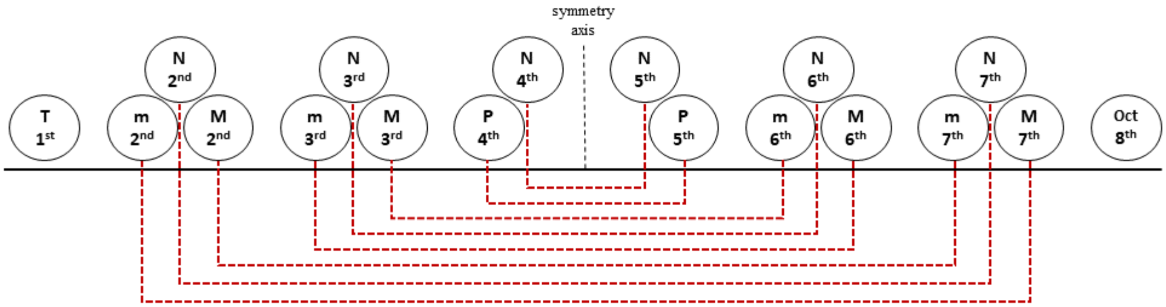

Figure 1-2: The intervals and their inversions in seventeen-tone temperament.

Theoretical and practical findings show that assigning fixed pitches to the seventeen tones of the Persian-Arabic scale is not an ideal approach, given that the exact value of some intervals might change depending on the melodic mode. In the next chapter I will talk about the notion of different prime numbers of the natural harmonic series and that how the value of the tones differs when approached through different prime numbers.

Hedāyat believes that a fifty-three-tone equal temperament (53-ET) encompasses all the needed intervals of the Persian-Arabic scale (Hedāyat 1938, third section, 20). Seemingly, Hedāyat's source of inspiration is a section in Helmholtz's treatise: "If we desire to produce a scale in almost *precisely just intonation*, which will allow of an indefinite power of modulation without having recourse to enharmonic changes, we can effect our purpose by the division of the Octave into fifty-three exactly equal parts [(53-ET)]" (Helmholtz 1895, 328). This assumption seems to be accurate and applicable through the idea of *nine-fold division of the whole tone* (the so-called 9-comma system), in which the size of a comma is approximately 22.6 cents. The problem with this system is that the large number of tones within an octave invokes many unused intervals for each mode.

The seventeen Abjadic tone-letters and different pitch suggestions

It seems that it was not until the thirteenth century that prominent music theorists of the Systematist School (in Persian: *Maktab-e Montzamiyeh*) such as Safi-al-Din Ormavi[11] (1216–94) and Qutb-al-Din Mahmud Shirāzi[12] (1236–1311) came up with the consensus of approximating musical intervals to seventeen tones in the span of an octave. Before that, different scholars suggested many different ratios or frets on oud, tanbur, etc. to demonstrate the possible intervals. I want to dedicate the beginning of this section of my thesis to a brief summary of my previous research about the seventeen Abjadic tone-letters and notation.

One of the unique methods of notation after Islam in Greater Persia was the alphabetic Abjadic notation. This method was not designed for precise transcription of musical pieces and apparently was never widely adopted by musicians, but was mostly used to practice simple melodies for oud players. The Persian-Arabic Abjad alphabet was widely employed after the Muslim conquest of Persia and the fall of the Sasanian Empire in the seventh century. At this time, a new influx of Arabic vocabulary entered the Persian language. Abjadic script is written from right to left in a cursive style and includes 28 letters. The Persian alphabet consists of these 28 Abjadic teller and four additional letters that are variations of four of Abjadic letters (32 letters in total). The order of appearance of the letters in Persian alphabet is also different than the Arabic Abjadic letters.

In the Abjadic method of notation, the pitches are shown with different letters in a horizontal line, beneath which the time value of each note is shown in another parallel line of

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11 صفی‌الدّین اُرموی

12 قطب‌الدّین محمود شیرازی

numbers. Abjadic notation is relative, in the sense that the size of each of the tones is considered in its relation to the first tone-letter A (الف), irrespective of the fixed pitches of the tones. This means that for transcription purposes, once the first tone-letter (A) takes any arbitrary pitch, the corresponding pitch of the other tone-letters will be determined, as their relationship with the first tone is already fixed.

The approach to using alphabetic Abjadic letters in Systematist School treatises has a special feature, which is the consideration of the numerical values of the letters in matching them with the number of each of the seventeen notes in an octave. Starting from the first Abjadic letter, A (Alef), each next letter takes the value of an integer number: A=1, B=2, J=3, D=4, h=5, V=6, Z=7, H=8, T=9, and Y=10. The values of the next eight letters are 20, 30, 40, 50, 60, 70, 80, 90, and the next nine letters: 100, 200, 300, 400, 500, 600, 700, 800, 900, with the twenty-eighth and last letter representing 1,000. I avoid mentioning the letters after Y, because they have no application in Abjadic notation and also for less confusion. You can find the full collection of Abjadic letters in the appendix section (Table 0-1).

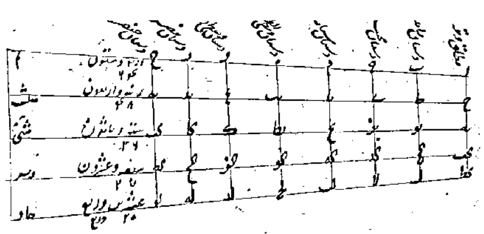
Instead of using the first seventeen letters of the Abjadic script, the scholars of the Systematist School preferred to use the first ten letters for the first ten tones. And then for the next eight tones (the eighteenth being the octave of the first letter), each tone is a two-digit combination of a Y (with a numeral value of 10) with one of the first nine letters (A, B, J, etc.). In that sense the seventeen tone-letters (plus an eighteenth for the octave) are respectively: A = 1, B = 2, J = 3, D = 4, h = 5, V = 6, Z = 7, H = 8, T = 9, Y = 10, YA = 11, YB = 12, YJ = 13, YD = 14, Yh = 15, YV = 16, YZ = 17, and YH = 18 (one octave above A). It should be mentioned that the complete number of the Abjadic tone-letters are twice to cover a full two-octave range of a traditional instrument like oud.[13]

Below in Figure 1-3, I have shown the place of the seventeen tone-letters on my bubble diagram, categorizing them into the seven scale degrees of a heptatonic scale, as well as a transcription into Western letter names by assigning the first Abjadic tone-letter (A) to the pitch C. It should be mentioned that the quartertone accidentals —the quarter-flat (𝄳) and the quarter-sharp (𝄲) signs— that are used for transcription of the neutral intervals do not necessarily mean that those notes are rendered in twenty-four-equal temperament with fixed pitches, but rather, as will be explained, they are used to indicate the approximate place of neutral intervals, which may vary in size according to each mode or tetrachord genus.

[13] The entire collection of the tone-letters is as follows, which are shown on the fretboard of an oud diagram:

A B J D h V Z H T Y YA YB YJ YD Yh YV YZ / YH YT K KA KB KJ KD Kh KV KZ KH KT L LA LB LJ LD / Lh

ا ب جـ د هـ و ز ح ط ى يا يب يجـ يد يه يو يز / يح يط ك كا كب كج كد كه كو كز كح كط ل لا لب لج لد / له



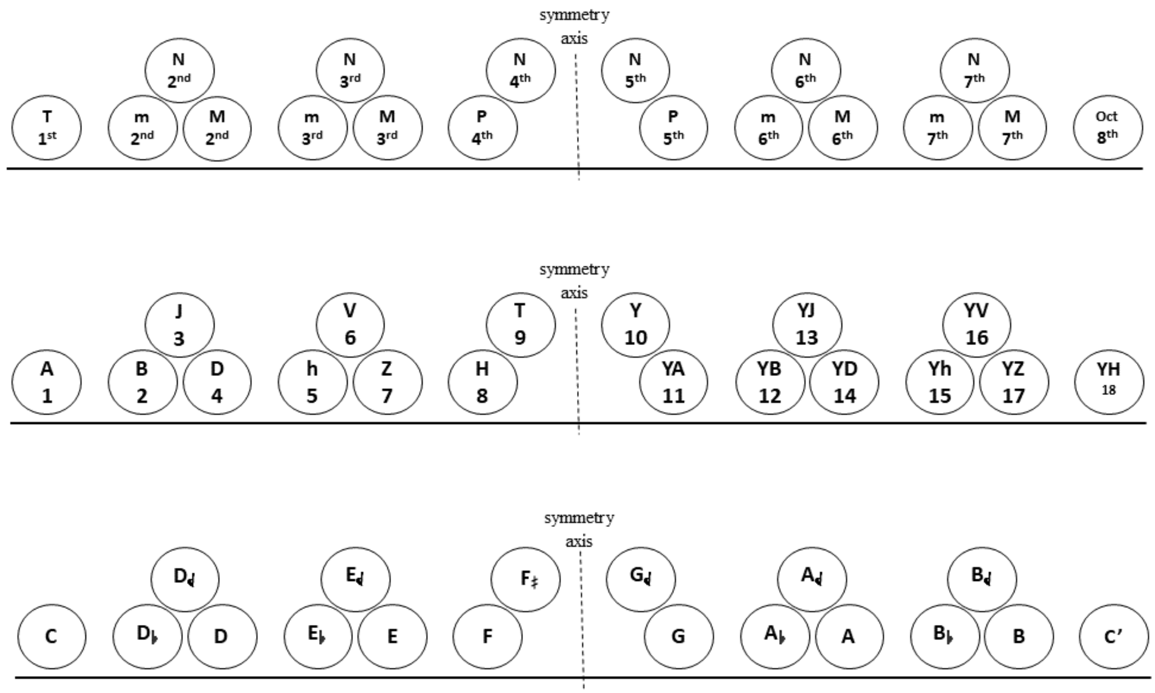


Figure 1-3: The place of seventeen Abjadic tone-letters, their title in a heptatonic scale, and their equivalent notes on the bubble diagram, considering the first tone-letter (A) transcribed as a C.

As mentioned earlier and will be elaborated in the next chapter, assigning fixed pitches to the seventeen tones is not an ideal approach, because the intervals that are used in each mode may differ in their precise tuning. Nevertheless, some musicians and scholars have tried since the thirteenth century to make a comprehensive pitch collection for this system. Hedāyat, in the third section of his treatise, *Majma'-al Adwār*[14] (published in 1938), collects and compares some of the different pitch values given for the seventeen tones in different sources in separate tables. I will present these pitch value tables in the following pages and compare them with later musicians' suggestions. In each table you can find the Abjadic tone letters and their transliterations, the degree and quality of each pitch in a heptatonic scale, the ratios of the tones (as proposed from different

[14] مجمع‌الادوار

points of view), and the length of the string needed to produce that pitch value based on an octave of 600 to 300 millimeters.[15] I have added a column showing pitch values as well as the stepwise interval comparing each pitch to the previous one in the logarithmic unit of cents, which is not mentioned in Hedāyat's tables.[16] I also expanded this collection by incorporating an additional column dedicated to the transcription of the tones into Western letter names, which facilitates the recognition and comparison of the notes.

The first table that he introduces[17] (Hedāyat 1938, third section 16) is based on the method that the theorists of the Systematist School approach the intervals.[18] He assumes the ratio of 13:12 (138.57 cents) for the tone-letter J as a generative ratio to derive the majority of neutral intervals in the system. This assumption is based on the old dispute between the practitioners of traditional music and the music theorists about the exact sizes of the neutral intervals. Given that the Pythagorean-based values sounded unrelated to the practical intervals, some scholars suggested this ratio as correspondent to the size of the mojannab fret that the practitioners utilized (Wright 1978, 36-43). It is remarkable that when you convert the ratios to the logarithmic unit of cents, the stepwise intervals are nothing but these three values: 90.22 cents, 48.35 cents, or 65.34 cents (Table 1-1). You can also find the original table in the appendix section (Figure 0-1).

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[15] There are two more columns in Hedāyats' original tables dedicated to the pitch and frequency values in decimals that I excluded for less confusion.

[16] Cents were introduced as early as 1885 by Alexander Ellis in his Helmholtz translations, though it may very well be that Hedāyat did not have access to Ellis's work.

[17] The title of the table in Persian: جدول ابعاد، به‌تقدیر صاحب ادوار

[18] An important disclaimer here is that there are different interpretations of the Systematist School pitch system based on the Pythagorean 3-limit system in other secondary sources. The common narrative is that a whole tone (204 cents) is divided to two Pythagorean limma (90 cents) and a Pythagorean comma (24 cents). Thus, the size of the larger neutral second in these treatises is assumed to be 180 cents (90 + 90), and the smaller neutral tone is approximately 114 cents (90 + 24), which is completely disassociated from the practical values. As shown in chapter three, these values later are applied as the basis of the modern Turkish music theory. Some scholars attribute the 13-limit values of the neutral intervals (13:12) to Ibn-e Sina (980-1037) and not the Systematist School scholars.

The numeral value of the Abjadic tone-letter	Abjadic tone-letters	Transliteration of Abjadic tone-letters	The degree and quality of the tone-letters in a heptatonic scale	Transcription into Western letter names (the first Abjadic letter being C)	Pitch ratios in fractions (according to Hedāyat)	String lengths in millimeters (the open string being 600 mm)	Values in cents	Values comparing to the previous pitch in cents
1	الف	A	root	C	1:1	600	00	
2	ب	B	minor 2nd	D♭	256:243	569.5*	90.22	90.22
3	جـ	J	neutral 2nd	D𝄳	13:12	553.9	138.57	48.35
4	د	D	major 2nd	D	9:8	533.3	203.91	65.34
5	هـ	H	minor 3rd	E♭	32:27	506.2	294.13	90.22
6	و	V	neutral 3rd	E𝄳	39:32	492.3	342.48	48.35
7	ز	Z	major 3rd	E	81:64	447.1	407.82	65.34
8	ح	H	perfect 4th	F	4:3	450	498.04	90.22
9	ط	T	neutral 4th	F𝄲	1024:729	427.1	588.27	90.22
10	ى	Y	neutral 5th	G𝄳	13:9	415.14	636.61	48.35
11	يا	YA	perfect 5th	G	3:2	400	701.95	65.34
12	يب	YB	minor 6th	A♭	128:81	379.6	792.18	90.22
13	يجـ	YJ	neutral 6th	A𝄳	13:8	369.3	840.52	48.35
14	يد	YD	major 6th	A	27:16	355.5	905.86	65.34
15	يه	Yh	minor 7th	B♭	16:9	337.5	996.09	90.22
16	يو	YV	neutral 7th	B𝄳	4096:2187	320.3	1086.31	90.22
17	يز	YZ	major 7th	B	52:27	311.5	1134.66	48.35
18 (1)	يح	YH	Octave	C'	2:1	300	1200	65.34

Table 1-1: Table of the pitch values of the seventeen tones for theorists of the Systematist School according to Mehdi-Qoli Hedāyat (Hedāyat, 1938, third section, 16).

** Starred numbers are corrected in this table, taking the vertical row of interval ratios from the original table as accurate.*

Hedāyat also provides another set of contemporary interval ratios for the seventeen tone-letters (Hedāyat 1938, third section, 20), suggested by Dr. Mehdi Khān Solhi (Montazam-al

Hokamā)[19] after he and two other prominent musicians at the time, Darvish Khān (1872–1926) and Ali-Naqi Vaziri (1886–1979), applied the Pythagorean and Didymean intervals on their instruments, and found them completely irrelevant to practical fretting in Persian music (Hedāyat 1938, third section, 12-13; Daemi Milani and Asadi 2021, 7-8). Except for the tone-letter H, all the other values correspond exactly to the pitches already presented in another table a few pages earlier (third section, 17), called "The table of Pythagorean intervals with semitone values of 25:24."[20] As the title implies, this is a system with the semitone size of a minor chroma (a just chromatic semitone, 25:24 = 70.67 cents). This conflicts with the first Pythagorean intervals that he presents, in which the size of a semitone was a limma (a Pythagorean minor second, 256:243 = 90.22 cents).[21] Looking at the last column, we realize that, except for the intervals between the letters V and H (43.00 cents), three constant values are repeated again: 70.67 cents, 62.65 cents, and 90.22 cents (Table 1-2). You can also find the original table in the appendix section (Figure 0-2).

[19] The title of the table in Persian: جدول ابعاد فعلی، به‌تشخیص دکتر مهدی‌خان صلحی

[20] The title of the table in Persian: جدول ابعاد فیثاغورثی، در سلسله‌ی بقایای ۲۵:۲۴

[21] It is also significant that the former ratio of 25:24 is a 5-limit ratio (= $\frac{5^2}{2^3\times 3}$), whereas the latter, 256:243, is obviously a 3-limit ratio (= $\frac{2^8}{3^5}$).

The numeral value of the Abjadic tone-letter	Abjadic tone-letters	Transliteration of Abjadic tone-letters	The degree and quality of the tone-letters in a heptatonic scale	Transcription into Western letter names (the first Abjadic letter being C)	Pitch ratios in fractions	String lengths in millimeters (the open string being 600 mm)	Values in cents	Values comparing to the previous pitch in cents
1	الف	A	root	C	1:1	600	0.00	
2	ب	B	minor 2nd	D♭	25:24	576	70.67	70.67
3	جـ	J	neutral 2nd	D𝄳	27:25	555	133.23	62.56
4	د	D	major 2nd	D	9:8	533.3	203.91	70.67
5	هـ	H	minor 3rd	E♭	32:27	506.25*	294.13	90.22
6	و	V	neutral 3rd	E𝄳	243:200	493.8*	337.14	43.00
7	ز	Z	major 3rd	E	81:64	474	407.82	70.68
8	ح	H	perfect 4th	F	4:3	450	498.04	90.22
9	ط	T	neutral 4th	F𝄲	25:18	432	568.71	70.67
10	ی	Y	neutral 5th	G𝄳	36:25	416.6*	631.28	62.57
11	یا	YA	perfect 5th	G	3:2	400	701.95	70.67
12	یب	YB	minor 6th	A♭	25:16	384	772.62	70.67
13	یجـ	YJ	neutral 6th	A𝄳	81:50	370.3	835.19	62.57
14	ید	YD	major 6th	A	27:16	355.5	905.86	70.67
15	یه	Yh	minor 7th	B♭	16:9	337.5*	996.08	90.22
16	یو	YV	neutral 7th	B𝄳	50:27	324	1066.76	70.68
17	یز	YZ	major 7th	B	48:25	312.5	1129.32	62.57
18 (1)	یح	YH	Octave	C'	2:1	300	1200	70.67

Table 1-2: Table of the pitch values of the seventeen tones in the Persian contemporary system, Dr. Mehdi Solhi's suggestion (Hedāyat, 1938, third section 20).

** Starred numbers are corrected in this table, taking the vertical row of interval ratios from the original table as accurate.*

Another reliable pitch collection is the one described in the PhD dissertation of Hormoz Farhat (1928–2021), a prominent Persian-American composer and ethnomusicologist. In the introduction of his thesis, after introducing the five different stepwise intervals in Persian music without mentioning ratios, fractions, or string lengths, he presents an approximation of the seventeen tones in cents (Farhat 1965, 17). In this case, it is not surprising that the comparative intervals do not exceed three constant rounded values: 90 cents, 45 cents, and 70 cents, since he builds his system on these approximate interval sizes from the beginning (Table 1-3).

The numeral value of the Abjadic tone-letter	Abjadic tone-letters	Transliteration of Abjadic tone-letters	The degree and quality of the tone-letters in a heptatonic scale	Transcription into Western letter names (the first Abjadic letter being C)	Values in cents	Values comparing to the previous pitch in cents
1	الف	A	root	C	00	
2	ب	B	minor 2nd	D♭	90	90
3	جـ	J	neutral 2nd	D𝄳	135	45
4	د	D	major 2nd	D	205	70
5	هـ	H	minor 3rd	E♭	295	90
6	و	V	neutral 3rd	E𝄳	340	45
7	ز	Z	major 3rd	E	410	70
8	ح	H	perfect 4th	F	500	90
9	ط	T	neutral 4th	F𝄲	565	65
10	ی	Y	neutral 5th	G𝄳	630	65
11	یا	YA	perfect 5th	G	700	70
12	یب	YB	minor 6th	A♭	790	90
13	یجـ	YJ	neutral 6th	A𝄳	835	45
14	ید	YD	major 6th	A	905	70
15	یه	Yh	minor 7th	B♭	995	90
16	یو	YV	neutral 7th	B𝄳	1040	45
17	یز	YZ	major 7th	B	1110	70*
18 (1)	یح	YH	Octave	C'	1200	90

Table 1-3: Table of the pitch values of the seventeen tones in the Persian contemporary system, accordiong to Hormoz Farhat (Farhat 1965, 17).

** The starred number is corrected in this table. It is written as 90 cents in the original text, which conflicts with the neighboring numbers in the column to the left.*

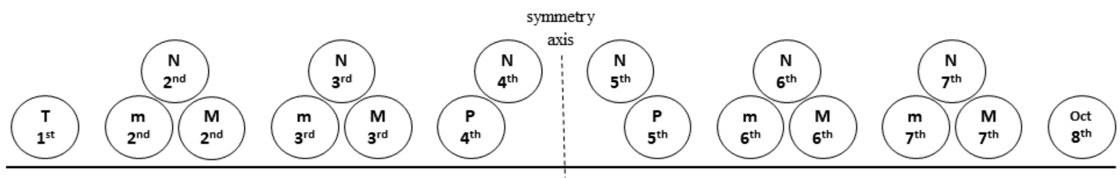
There is also the table pitch suggestions of Majid Kiāni (b. 1941) a prominent santur player which you can find below (Table 1-4).

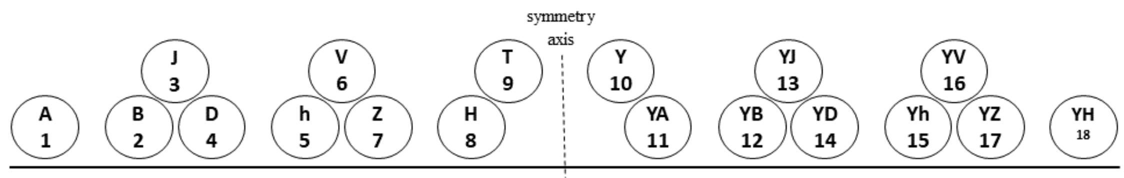
scale degree (in seventeen-tone system)	Abjadic tone-letters	transliteration of Abjadic tone-letters	position and quality in a heptatonic scale	Transcription into Western letter names (the first Abjadic letter being C)	Pitch ratios in fractions	Values in cents	Values comparing to the previous pitch in cents
1	الف	A	root	C	1:1	0	
2	ب	B	minor 2nd	D♭	256:243	90.22	90.22
3	جـ	J	neutral 2nd	D𝄳	88:81	143.49	53.27
4	د	D	major 2nd	D	9:8	203.91	60.42
5	هـ	H	minor 3rd	E♭	32:27	294.13	90.22
6	و	V	neutral 3rd	E𝄳	27:22	354.54	60.42
7	ز	Z	major 3rd	E	81:64	407.82	53.27
8	ح	H	perfect 4th	F	4:3	498.04	90.22
9	ط	T	neutral 4th	F‡	243:176	558.45	60.42
10	ی	Y	neutral 5th	G𝄳	352:243	641.54	83.10
11	یا	YA	perfect 5th	G	3:2	701.95	60.42
12	یب	YB	minor 6th	A♭	128:81	792.18	90.22
13	یجـ	YJ	neutral 6th	A𝄳	132:81	845.45	53.27
14	ید	YD	major 6th	A	27:16	905.86	60.42
15	یه	Yh	minor 7th	B♭	16:9	996.09	90.22
16	یو	YV	neutral 7th	B𝄳	81:44	1056.50	60.42
17	یز	YZ	major 7th	B	243:128	1109.77	53.27
18 (1)	یح	YH	Octave	C'	2/1	1200	90.22

Table 1-4: Table of the pitch values of the seventeen tones in Persian current system (Accordiong to Majid Kiāni) (Kiani 1992, 192).

He recognizes the ratio of 88:81 (143.49 cents) for the neutral second interval (J) and the ratio of 27:22 (354.54 cents) for the neutral third interval (V), both derived from the eleventh harmonic partial and the ratio of 11:8. Again, a brief look at the last column shows that, except for the interval between the letters T and Y (83.10 cents), just three constant interval sizes are repeated: 90.22 cents, 53.27 cents, and 60.42 cents (Kiani 1992, 189-192).

By making a comparison between these pitch collections[22] significant points are revealed. The repetition of the same values for the perfect fifth and perfect fourth intervals legitimize the idea of these intervals as the anchor points of the system. The fifth (YA) and fourth (H), along with the tonic and octave, are the framing tones of the two disjunct tetrachords that have formed the system. Let us look at the other intervals that I ordered vertically under each Abjadic tone-letter on the bubble diagram. The values are rounded to the nearest cent to conserve space (Table 1-5).





	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Abjadic tone-letters	A	B	J	D	h	V	Z	H	T	Y	YA	YB	YJ	YD	Yh	YV	YZ	YH
	الف	ب	جـ	د	هـ	و	ز	ح	ط	ی	یا	یب	یجـ	ید	یه	یو	یز	یح
Systematist (after Hedāyat)	0	90	**138**	204	294	**342**	408	498	588	637	702	792	841	906	996	1086	1133	1200
Hedāyat/ Solhi	0	70	**133**	204	294	**337**	408	498	569	631	702	772	835	906	996	1066	1129	1200
Farhat	0	90	**135**	205	295	**340**	410	500	565	630	700	790	835	905	995	1040	1110	1200
Kiāni	0	90	**143**	204	294	**354**	408	498	558	641	702	792	845	906	996	1056	1110	1200

Table 1-5: A comparison between different pitch collection values in cents according to different theorists and musicians.

[22] Some other mere mathematical-based pitch suggestions are excluded due to the unrealistic results (See Ghanbari et al. 2022).

We already discussed the two ways of approaching the minor second (B), one with the Pythagorean limma (256:243 ≃ 90 cents) and the other with a just chromatic semitone (25:24 ≃ 70 cents). Apparently, the former value is more acceptable for most of the musicians. It goes without saying that the pitch value of major second interval (D) is agreed by all to be a Pythagorean whole tone (9:8 ≃ 204 cents). Regarding the next scale degree, for the major third interval (Z), it seems that everyone is happy with the value of the Pythagorean ditone (81:64 ≃ 408 cents), and for the minor third (h), there is the consensus on the value of Pythagorean minor third (32:27 ≃ 294 cents). The place of the perfect fourth (H) and perfect fifth (YA) are already fixed (4:3 ≃ 498 cents, and 3:2 ≃ 702 cents). I skip elaborating on the intervals larger than a perfect fifth and prefer to see the intervals of the sixth and seventh scale degrees as the members of the second tetrachord forming the octave which can be understood as the inversions of the third and second intervals (which of course may not cover all cases).

Getting back to the second scale degree, you can see that the value of the neutral second or so-called *mojannab*[23] (J) is slightly different in each point of view. Hedāyat believes that the size of the mojannab interval in the thirteenth-century treatises of Systematist School is 13:12 (≃ 138 cents) (Hedāyat 1938, third section, 131), whereas he reports the ratio of 27:25 (≃ 133 cents) for what the musicians at the time suggested for this fret on the instrument setar;[24] both closely correspond to the definition of the mojannab as a two-third-tone interval. For Farhat, the value of this interval should be approximately 135 cents, which matches Hedāyat's interpretation. Kiāni, however, suggests the ratio of 88:81 (≃ 143 cents) for the mojannab fret, which is still close to the Systematist School theorists' definition, and never reaches the value of a three-quartertone interval

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[23] مجنّب

[24] سه‌تار

(150 cents). This fact reinforces Farhat's idea of considering two approximate sizes of neutral tones: the small neutral tone (n $\simeq$ 135 cents) and the large neutral tone (N $\simeq$ 160 cents) (Farhat 1965, 16). In practice, usually these two intervals are counterparts of each other to make a Pythagorean minor third interval with the value of approximately 295 cents.

In the third scale degree, however, the place of the neutral third interval (V) slightly tends more toward the exact middle between the minor and major thirds. That is because the melodic function of the neutral third interval is different than the neutral second interval, and they are used in distinct modes. The neutral third usually functions as the centric point of the mode, whereas the neutral second is generally a passing tone.

According to Hedāyat, the value of the neutral third tone (V), for the theorists of Systematist School, is presented by the ratio of 39:32 ($\simeq$ 342 cents), which is a 13-limit interval (39:32 = $\frac{3\times13}{2^5}$), whilst the practical fret that he recognizes is a 5-limit ratio of 243:200 (= $\frac{3^5}{2^3\times5^2}$ $\simeq$ 337 cents). For Farhat and his approximations, a neutral third is 45 cents higher than his minor third (= 340 cents), and for Kiāni the neutral third is exactly the Zalzalian third with its 11-limit ratio of 27:22 (= $\frac{3^3}{2\times11}$ $\simeq$ 354 cents). In the section about Fārābi and his three qualities of the interval of a third, we will see that the quality of an implied harmony is changed by changing the quality of the third between major, minor, and neutral.

The existence of different values for the neutral intervals in the second and third scale degrees can dispel any doubt that different musicians and theorists have taken different musical modes as the basis of their tuning suggestions. Furthermore, changing the centric point of a mode can affect the quality of the neutral intervals. In that sense, it could be possible that these theorists have considered a mode performed from a different scale degree, with the value of the neutral intervals changed consequently.

Interpretation of the seventeen tones on a lattice of intervals[25]

For a better understanding and analysis of the three sets of suggested ratios for the seventeen tones and its geometry, we can plot the ratios in a lattice of intervals[26]. What is obvious is that each set can be arranged using only two intervals; a Pythagorean fifth combined with a generating ratio which differs for each set. For the Systematist set, only the perfect fifth and the ratio 13:8 (841 cents), the thirteenth harmonic brought within the span of an octave, are used (Table 1-1). For Solhi's collection, it is the perfect fifth and the ratio 25:16 (773 cents), the twenty-fifth harmonic (fifth harmonic of the fifth harmonic) again brought into the range of an octave (Table 1-2). There is no single factor of 5 in Kiāni's collection; he only applies the square of 5 (25). And for Kiāni's set, the perfect fifth and the ratio of 11:8 (551 cents) are combined (Table 1-4).

Let us parse out the Systematists' ratio collection in a lattice. As you see in the table below (Figure 1-4), the horizontal row is dedicated to the ratios obtained with the Pythagorean perfect fifth (3:2), and the vertical row shows the ratios obtained via the generative ratio of the thirteenth harmonic (13:8) (Figure 1-4). Another way of conceptualizing these relationships is to replace the 13:8 with the ratio of the neutral second (mojannab) interval (13:12), which is the ratio for the note D-quarter-flat. As a result, the top row will be shifted one position to the right, so it becomes centered around C (Figure 1-5).

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[25] Thanks to Prof. Jonathan Wild, the evaluator of this thesis, for suggesting this section and advising on the arrangement of the ratios in each lattice.

[26] Lattices have been used by many theorists of just intonation, and bear similarities to the well-known Tonnetz used by nineteenth-century German theorists. One of the key figures in developing lattices for extended just intonation is Ben Johnston (Johnson 2006).

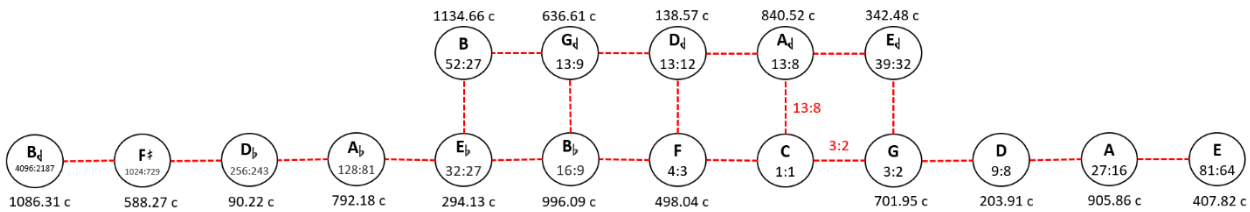

Figure 1-4: The lattice of seventeen tones of the Systematist School,
with the generative ratios of 13:8 and 3:2.

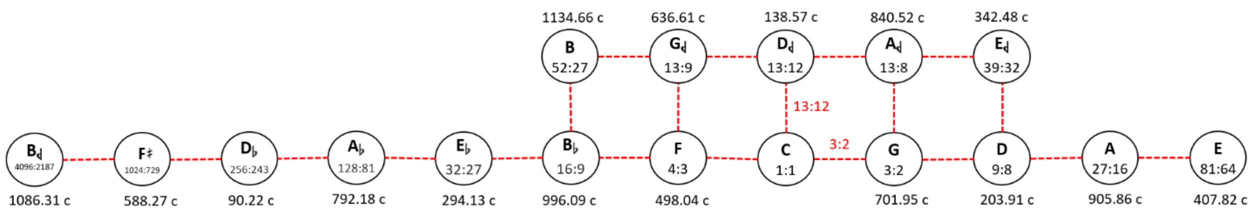

Figure 1-5: The lattice of seventeen tones of the Systematist School,
with the generative ratios of 13:12 and 3:2.

Studying the lattice of Solhi /Hedāyat's ratio set is also intriguing. The horizontal row of 3:2 perfect fifths is combined with additional series of fifths arranged a 25:16 above and below. The lattice would be symmetrical if the note B were Pythagorean (at the right end of the central row rather than below G), making E-quarter-flat less isolated in the right corner of the lattice[27] (Figure 1-6).

[27] From this diagram we also see that there are three sizes of "perfect fifth": (1) the 3:2 connections within each row; (2) the wrap-around between E and B (it would be between B and G-quarter-flat if we repositioned B in the central row instead), identical to that between A-flat and E-flat—this is a ratio of 1,024:675 (722 cents); and (3) the wrap-around between E-quarter-flat in the bottom row and B-quarter-flat in the top row, a ratio of 10,000:6,561 or approximately 730 cents.

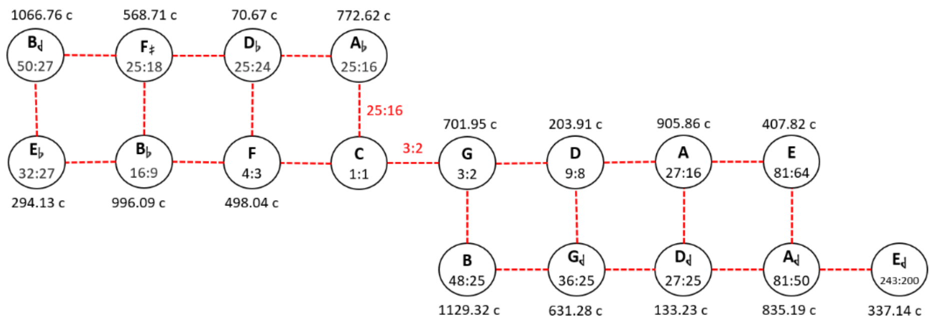

Figure 1-6: The lattice of seventeen tones of Solhi/Hedayat's set,
with the generative ratios of 25:16 and 3:2.

Kiāni's diagram is symmetrically distributed, with added rows on opposite sides of the Pythagorean row separated by 11:8. We can see that only the six neutral intervals are obtained from the generative ratio of 11:8 (Figure 1-7). Unlike the Systematists' set, here the eleven-limit ratio of the neutral second interval (12:11) is not considered for D-quarter-flat—rather, the D-quarter-flat comes from the 11:8 above A-flat.

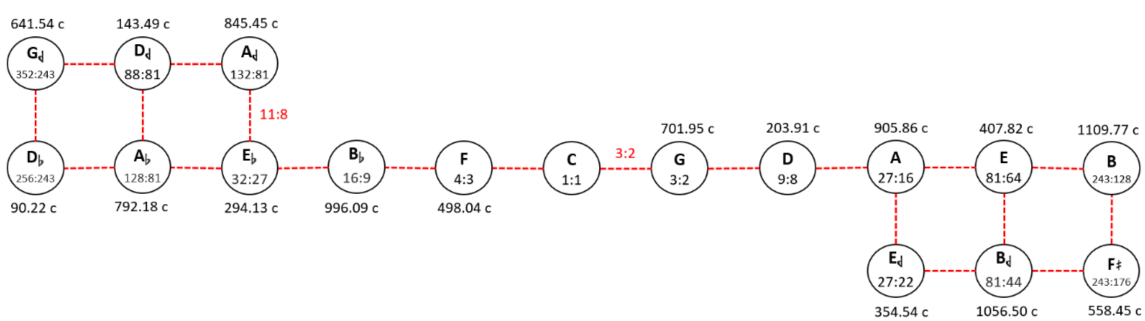

Figure 1-7: The lattice of seventeen tones of Kiani's set,
with the generative ratios of 11:8 and 3:2.

2. Reconceptualizing dastgāhi intervals with different harmonic prime-number limits

The previous section showed the significant disagreements on the size of neutral intervals between musicians and theorists at different times. This issue arose as a result of examining different musical modes, considering different tonic centers, or just because of the difference in the sonic taste of each musician. Those who are more concerned with musical practice, such as Fārābi, suggest several different ratio possibilities for the neutral intervals of each scale degree (Farhat 1965, 11), while those who are more focused on Greek- and Pythagorean-oriented music theory assign fixed pitches that are mostly contradictory to the practical intervals.

What is certain today for those familiar with physics and acoustics is that the intervals with simpler ratios that come directly from the natural harmonic series are more consonant and more acceptable to the ear. Reconceiving intervals in terms of simpler proportions is nothing new. For example, as will be explained in more detail, when Zarlino proposes a simple ratio of 5:4 for the major third instead of the previously used ratio, which was the Pythagorean ditone (81:64), instead of relying on numerical and unrealistic calculations, he proposes a simpler ratio directly from the lower harmonic partials. What happened at that time was that the concept of consonance of the major third interval, which is a fundamental basis of Western classical music and harmony, was incompatible with the complex Pythagorean ratio of 81:64 and had to be fundamentally changed.

This issue is a good introduction to the topic of different "limits of the natural harmonic series" that is raised by the American composer and musicologist Harry Partch (1901–1974). In his book, *Genesis of a Music*, he talks about the barriers imposed by twelve-tone equal temperament and the ratios from the harmonic series that are implied in it (Partch 1949, 109).

To explain these intervals in the current system, he notices a big obstacle that he calls the "limit" of the natural harmonic series. Technically, the problem is that the tone system of the western classical music is made of the ratio collection that are based on the prime number "5" from the natural harmonic series, which represents the "major third" interval. This system itself is a developed version of the previous Pythagorean tone system which was based on the prime number "3" from the natural harmonic series, representing the "perfect fifth" interval, after Zarlino challenges the Pythagorean major third interval and tries to re-define the major third as a consonant interval by revising the musical ratio that represents it!

Prior to Zarlino, the major third interval was considered dissonant, as it was seen as a combination of two whole tones, a so-called *ditone,* the ratio of which being 81:64. This ratio is obtained with the multiplication of the ratio of a whole-tone to itself (9:8 × 9:8) which is itself resulted of the multiplications of the prime numbers 3 and 2 from the harmonic series, which is the reason that Harry Partch calls it a 3-limit ratio.

Zarlino notices this barrier and based on the general rule of preference for simpler ratios as consonances by the human brain, re-introduces the major third interval with a so-called 5-limit ratio of 5:4. As Partch puts it, “In the Pythagorean ratio 81:64 both numbers are multiples of 3 or under, yet because of their excessive largeness the ear certainly prefers 5:4 for this approximate degree, even though it involves a prime number higher than 3” (ibid., 115).

Like Zarlino before him, Partch proposes that the intervals used to explain intervals are *inadequate*, since a Renaissance-era "5-limit" tone system does not have enough capacity to include the ratios from the world of quartertones that Partch talks about. He solves his problem, like Zarlino, by advocating for a higher limit which allows a different selection of ratios, many of which may offer a simpler ratio, even though higher prime numbers are used.

It seems that another revision of this kind is needed for a system to cover the intervals and ratios of Persian and Arabic music. Obviously, the Western classical music theory, being confined to a 5-limit portion of the natural harmonic series, is inadequate in the theoretical explanation of the intervals used in Persian-Arabic music using Western theory and may cause objections and ambiguities. Studying a complex musical system necessitates a complex infrastructure.

It seems that Hedāyat also encounters a similar problem of inconsistency of theoretical system to explain the practical intervals and explains his findings like this: "Abd al-Qādir [Marāqi[28]] believes that the maestros of that time did not accept the [ancient] Greek intervals and manipulate them. I decided to be more precise on the common intervals, especially since an instrument that was re-fretted with [Systematist School] Adwār intervals did not sound pleasant. […] I compared the frets. The octave, perfect fourth and fifth, and the whole tone, were in perfect health. […] But it became clear that Iranians after the period of Abd al-Qādir, or even during his time, slightly changed the intervals"[29] (Hedāyat 1938, third section 12-13; Daemi Milani and Asadi 2021, 7-8). This passage clearly shows that the values for the neutral intervals obtained by the pre-existing Pythagorean tuning system do not sound pleasant or acceptable to the practitioners.

My purpose in writing this section is to get back to the origins of music theory and start rethinking musical ratios. If going beyond the Pythagorean 3-limit tuning system and considering a higher 5-limit system makes it easier to explain the consonance of intervals like the major third,

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[28] عبدالقادر مراغی
(died 1435), a prominent musician of the second Persian-Arabic music theory trend, Systematist School

[29] The text in Persian:

عبدالقادر [مراغی] می گوید که اساتید زمان، ابعاد یونانی [باستان] را نمی‌پسندند و در آن‌ها تصرّفاتی می کنند. لازم دانستم که در ابعاد معموله دقّتی بشود، خصوص که سازی را دادم به نسبت ادواری [مکتب منتظمیه] پرده بستند و مطبوع نیفتاد. [از درویش‌خان و دکتر مهدی‌خان صلحی و علی‌نقی‌خان وزیری خواهش کردم ساز خود را به‌دقّت کوک کنند، سپس] پرده‌ها را سنجیدم. ذوالکلّ، ذوالاربع، ذوالخمس، و طنینی اوّل، در نهایت صحّت بود. [خصوص در ساز صلحی، پس از تکرار امتحان]، مسلّم شد که ایرانیان پس از دوره عبدالقادر، یا در دنباله همان دوره، ابعاد را اندکی تغییر داده‌اند.

why not expand our theoretical range to 7, 11, or 13 limits to facilitate explaining the complex intervals of Persian and Arabic music?

This is an innovative approach in the sense that most of the previous theoretical research on this music has been based on the Pythagorean tuning system which does not go further than the second and third harmonic partials and their multiplications, which for some intervals leads to mathematical results incompatible with the reality of the practical intervals. On the other hand, when the ratios of higher limits have been discussed in western or Ancient Greek treatises, it has been only occasionally, briefly, and without mentioning their application in the Persian and Arabic musical modes. For instance, when Ptolemy talks about the *equal* tetrachord genus, which includes 11-limit ratios,[30] he describes the mode as *foreign and rustic*, but *exceptionally gentle*, because of the special character of its melody, and because of the orderliness of the division (Barker 1990, 312 and 350; Helmholtz 1895, 264).

It seems that, through these phrases and the ratios he presents, he is trying to mathematically and verbally describe a tetrachord genus which probably has existed at his time but was unfamiliar for their ear, without knowing that this mode was a famous tetrachord in Persian and Arabic music. Later in this thesis I will elaborate on this tetrachord, and its approximated T-3/4T-3/4T values in Fārābi's words.

Therefore, in the following pages, I will go through Partch's notion of harmonic prime-number limits from 3 to 13, and I will study the similarity between the ratios obtained from the higher limits and the intervals found in Persian and Arabic musical modes.

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[30] In which the order of the ratios is 10:9 11:10 12:11

3-limit Pythagorean tuning:
Perfect fifth, perfect fourth, whole tone, limma

Any discussion about musical intervals and string division always starts from the ratio of a "pure" perfect fifth, 3:2. This simple ratio initiates the basis of the Pythagorean tuning system. The ratio of 3:2 is the first musical ratio to be introduced, not only because the second and third partials are the first harmonics of the natural harmonic series, but also because it is the most consonant interval and the easiest to tune by ear. In the Pythagorean tuning system, 2 and 3 are the basis, and all the other intervals are obtained by multiplication or division of these two prime numbers. This fundamental 3:2 interval has been undoubtedly favored in Persian and Arabic treatises as well, and it is entitled *zi-al-khoms.*[31] In the language of Abjadic tone-letters, a perfect fifth would be the interval between the letter A and the letter YA (A-YA).[32] The logarithmic value of a perfect fifth interval is 701.96 cents, which is usually presented with the rounded value of 702 cents.

The next interval that is obtained in this system is the counterpart of a perfect fifth to complete an octave; the perfect fourth interval with the ratio of 4:3. It goes without saying that this interval is obtainable by subtracting a perfect fifth from an octave, which in the language of numbers means dividing a 2:1 by a 3:2.[33] This interval is even more significant than a perfect fifth in Persian-Arabic music theory, because of the tetrachord-based nature of this music. The perfect

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[31] ذی‌الخُمس

[32] الف–یا

[33] $\frac{2:1}{3:2} = 4:3$

fourth interval is called *zi-al-arba'*[34] in Persian-Arabic texts, which is the interval between the letter A and the letter H (A-H).[35] The value of a perfect fourth is approximately 498 cents.

The other basic interval that is obtainable in this system is a whole tone with a ratio of 9:8. This ratio is also obviously reducible to the prime numbers 3 and 2. This ratio is obtainable either by subtracting an octave from an interval of two perfect fifths,[36] or by subtracting a perfect fourth from a perfect fifth.[37] In Persian and Arabic treatises this interval is entitled *tanini*,[38] which is the interval between the letters A and D (A-D).[39] The logarithmic value of a Pythagorean whole tone is 203.91 cents, which is normally rounded to 204 cents. This value corresponds to nine commas in a 53-ET (nine-fold division) system.

Finally, the Pythagorean diatonic semitone or limma, with the ratio of 256:243 (not of course a simple ratio), is the next favorite interval in the Persian-Arabic music texts. It is addressed given the title of *baqieh*,[40] meaning *remainder*, since it is obtained from the difference between three octaves and five perfect fifths (or two whole tones and a perfect fourth).[41] A limma, in Abjadic terms, is the interval between the letters A and B (A-B).[42] The logarithmic value of this interval is 90.22 cents, and it is one of the two acceptable sizes of a minor second in Persian-Arabic

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34 ذی‌الاربع

35 الف–ح

36 $\frac{3:2 \times 3:2}{2:1}$

37 $\frac{3:2}{4:3}$

38 طنینی

39 الف–د

40 بقیه

41 $\frac{(2:1)^3}{(3:2)^5}$

42 الف–ب

music theory. This value is approximately equal to four commas in a 53-ET (nine-fold division of the tone) system.

All the intervals mentioned so far are based upon a Pythagorean 3-limit tuning system. There are plenty of other 3-limit intervals in the Pythagorean tuning system, some of which were later rejected and replaced by simpler and more consonant musical ratios. For instance, the Pythagorean major third (81:64) and minor third (32:27) intervals were later replaced by the simple ratios of 5:4 and 6:5 in Western classical music theory.

5-limit just intonation tuning:
Major third, minor third, chromatic semitone

According to the Pythagorean tuning system, major and minor thirds were considered *dissonant* because of their complex numerical ratios. The Pythagorean major third is a ditone with a ratio of 81:64,[43] and the Pythagorean minor third is a semiditone with a ratio of 32:27,[44] until some other theorists including Zarlino present a newer definition of the thirds using simpler ratios from the harmonic series and considering them as consonant intervals. For Zarlino, the ratio of a major third is 5:4, and a minor third, with a ratio of 6:5, is its counterpart to complete a perfect fifth (Rivera 1995; Helmholtz 1895, 312-313).[45]

The harmonic foundation of Western classical music theory is based on the idea of the consonance of the major and minor thirds, redefined in 5-limit tuning as the simple ratios of 5:4

[43] 81:64 = $(9:8)^2$

[44] $\frac{2^5}{3^3}$

[45] $\frac{3:2}{5:4}$ = 6:5

and 6:5. Hugo Riemann, at the beginning of his article, raises Zarlino's notion of a six-part division of the string (the so-called *Senarius numerus*), which succeeds, as he puts it, the older Pythagorean idea of a four-part division of a string, or *Quaternarius numerus* (Riemann 1992, 113; Wienpahl 1959, 41). According to Zarlino, all the consonant intervals are extracted from the first three prime numbers: 2 (octave), 3 (perfect fifth), and 5 (major third), while the other intervals are obtained by a combination of the simple ratios that these three numbers make to each other. Riemann sees the solution of the "riddle of the nature of harmony" in Zarlino's senary division. But simultaneously, it seems that he suspects the perfection of this idea, and mentions a more mature system featuring a *twelve-part division of the string*. He writes "The *Messeltheorie*[46] of the Arabs in the fourteenth century extended the series of numbers to twelve in order to prove the consonance of the sixth" (Riemann 1992, 97-98). Obviously, the first novel ratio that comes to mind after dividing a string into twelve equal parts, is 12:11, which belongs to the family of *undecimal* intervals emerging from 11-limit tuning, which I will cover in the following sections.

It is interesting to note that most of the Western music theorists of the eighteenth and nineteenth centuries accept Zarlino's Senario idea and ignore the upper limits of the harmonic series in their theoretical systems. Jean-Philippe Rameau, in his *Traité de I'harmonie*, skips the seventh division of the string and moves on to the eighth but does not go further, since "[…] number 7, which cannot give a pleasant interval (as is evident to connoisseurs), has been replaced by number 8; the latter directly follows 7, is twice one of the numbers contained in the Senario, and forms a triple octave with 1" (Rameau 1971, book one 6-7). Moritz Hauptman, in his *Die*

[46] "Messel Theory" seems to be an arbitrary title that Riemann has chosen for the Persian-Arabic music theory, after studying the music section of the encyclopedia, entitled "Dorrat-al Tāj," of the Persian philosopher Qutb-al-Din Mahmud Shirāzi (1236–1311). This topic is a recurrent theme in Persian-Arabic music theory treatises, and it has to do with the epimoric ratios with the value of (n+1)/n. Riemann seems to draw some misconceptions about his idea of consonance of the third and sixth out of this system (Riemann, 1992: 114-115).

Natur der Harmonik, talks about arithmetical series up to number 16, but he believes that the numbers 7, 11, 13, and 14 “certainly do not correspond to the true intonation” (Hauptmann 1893, xxxv), since the notes representing them are either *too flat* or *too sharp*. But alternatively, starting from C as the fundamental tone, when he approaches the note associated with the number 11 which has an ambiguous character between F and F#, he suggests that the divisions must be continued up to the numbers 21 and 22, so then we will have respectively two separate F and F# notes (ibid., xxxvi). Riemann, when introducing the notion of overtone and undertone series in his *Vereinfachte Harmonielehre*, elaborates on the string divisions beyond 1/6 (or the multiplications beyond 6), and states that the smaller major and minor second intervals —as well as “an abundance of intervals which our note-system ignores”— appear in this region. He refers to the same numbers 7, 11, 13, and 14, which he thinks are *too flat* in the overtone series, or *too sharp* in the undertone series. He believes that the ear fails to understand these numbers and “refuses to recognize the intervals formed by them with their neighboring notes and [with] their fundamental note” (Riemann 1893, 4-5).

Insisting on the prime number 5 limit and disregarding the higher partials in the natural harmonic series could be a result of the limitations that the keyboard has caused. In this regard, Harry Partch writes, “Practice, in a cappella singing, for example, reveals proved advances beyond the 5 limits, whereas theory, in dealing with musical resources and in the building of instruments of fixed intonation, studiously excludes any higher prime number” (Partch 1949, 109). He believes that the consensus of a 5-limit system in Western classical music is because it was expedient in the building and tuning of fretted and keyboard instruments and because its demands on notation were less complex; for these and no other primary reasons, it prevailed in practical music (ibid., 119).

In Persian and Arabic music theory treatises of the thirteenth-century Systematist School, the major and minor thirds are obviously presented with their old Pythagorean values. In Hedāyat's interpretations of these treatises, the major third interval is addressed as z*i-al-salās akbar,*[47] which in Abjadic letters would be the interval between A and Z (A-Z),[48] and the minor third is called z*i-al-salās asqar,*[49] the interval between the Abjadic letters A and h (A-h).[50]

Let us quickly compare the logarithmic values of the major and minor third intervals, both in their Pythagorean 3-limit and newer 5-limit just intonation definitions. A Pythagorean major third (81:64) is 407.82 cents, and a Pythagorean minor third (32:27) is 294.13 cents, whereas a just major third (5:4) is 386.31 cents, and a just minor third (6:5) is 315.64 cents. As you see, a just major third is smaller than a Pythagorean major third, while, a Just minor third is larger than a Pythagorean minor third.

47 ذی‌الثّلاث اکبر

48 الف–ز

49 ذی‌الثّلاث اصغر

50 الف–هـ

7-limit septimal tuning:
Plus tone (augmented second)

If you have studied strict style counterpoint, you probably know that according to Western classical rules, an augmented second interval should be avoided, because it is *too large* to be accepted by the Western ear. also, because it is considered to be a chromatic interval and not a diatonic one (Fux 1965, 27; Schubert and Neidhöfer 2006, 49; Wason 1985, 51). This is while for a listener familiar with Persian and Arabic intervals not only is the augmented second interval acceptable and pleasant, it is also an inseparable part of many musical modes. It should be considered, however, that we are judging this interval in the context of twelve-tone equal temperament, which gives us an approximation.

Farhat, in his PhD dissertation, talks about an interval larger than a whole tone, called *plus tone (or plus second)*, with an approximated value of 270 cents (Farhat 1965, 16), which is approximately equal to twelve commas in a 53-ET (nine-fold division) system. This interval could pleasantly match with the simple 7-limit ratio of 7:6 from the natural harmonic series, which is equal to 267 cents. This interval is called *tanini-e mostazād* [51] in the thirteenth century Systematist school treatises (see Wright 1978).

A plus tone (P) usually is completed with a small neutral tone (n) to reach a major third interval. If the neutral tone is considered between the Abjadic letters A and J, then a plus tone appears between the letters J and Z, to reach the major third interval of A-Z (Figure 2-1).[52] It should be mentioned that Farhat's proposed values for the stepwise intervals are all approximations, like

[51] طنینی مستزاد

[52] الف–ز

the way Aristoxenus and Fārābi present the sizes of the intervals of their genera collection (which will be explained in the next chapter), hence do not correspondent to one specific tuning system.

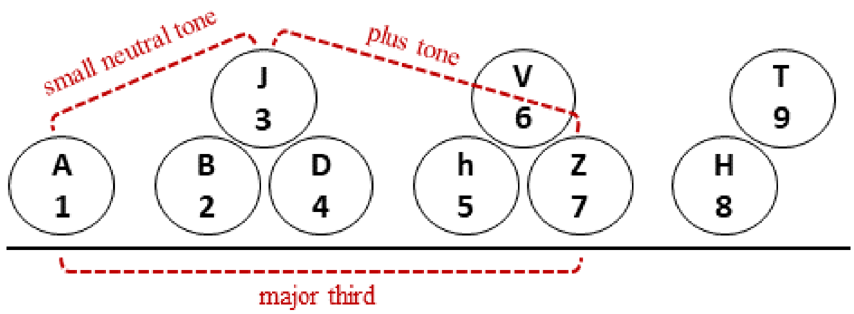

Figure 2-1: A plus tone and a small neutral tone interval, as counterparts of each other to reach a major third.

Needless to say, the augmented second interval in the context of twelve equal-tone temperament is more than 30 cents larger than the plus tone, which could be one of the reasons that it is recognized as unacceptable for the western ear! This is while the just intonation version of this interval is presented with the ratio of 75:64 ($\frac{3\times5^2}{2^6}$ = 274.58 cents), which is pretty close to the value of its 7-limit ratio and a clearly defined interval in a 5-limit tuning system.

Also, the number 7 was called and demonstrated to be consonant by Marin Mersenne (1588 – 1648) (Partch 1949, 90) in the middle of the seventeenth century, and thereafter by Tartini and Euler. Partch believes that the reason for ignoring the seventh partial by the Western theorists, though it is sometime "vaguely implied", was "lethargy and the piano" (ibid, 120). The presence of the 7-limit or so-called septimal intervals, more specifically the ratio of 7:6, is traceable in the history of Greek music theory, at least from the time of Ptolemy in the second century (ibid., 91; Barker 1990, 349-358).

11-limit undecimal tuning, 3/4-tone interval and Harry Partch: Neutral third, larger neutral tone

The world of 11-limit tuning and the so-called *undecimal* intervals seems to be interesting and understandable for musicians who are proponents of twelve-tone equal temperament. A combination of 3-limit and 11-limit ratios closely fit within a twenty-four-tone equal temperament,[53] which is the binary division of the twelve-equal-tone system. As an amazing example, consider the size of the epimoric 11-limit ratio 12:11[54] and its logarithmic value (150.64 cents) and compare it to its approximated size in the 24-ET system (150 cents). This is less than the difference between the size of the 3-limit ratio of a perfect fifth interval, 3:2 (701.96 cents) and its approximated value in 12-ET (700 cents), the interval that is the closest to its 12-ET equivalent.

The significance of the 11-limit ratios and their correspondence to the 24-ET system drew the attention of some of the twentieth-century Western musicologists and scholars who sought to get over the boundaries and constraints of the keyboard and 12-ET system, among whom Harry Partch (1901–1974), the American composer, music theorist, and builder of unique instruments, is the most significant. He seems to be familiar with Hermann von Helmholtz (1821– 94) and his unrivaled treatise about sound and music, *On the Sensations of Tone* (Helmholtz 1895). Inspired by that book, he raises some arguments about the 5-limit origin of the intervals used in Western classical music and the unwillingness of Western musicians to go higher in the harmonic series, as a result of ignorance or lethargy that already mentioned in the previous section (Partch 1949, 125).

[53] Any additions of 5 or 7 to these ratios are likely to move them far from twenty-four-tone equal temperament.

[54] In Greek musical writings, the undecimal ratio 12:11 is called *ephendekatos*, based on the name of the number 11 (*hendeka*) (Barker 1990, 503).

In his treatise *Genesis of a Music,* Partch talks about the tonality diamond (Partch 1949, 159), and the fact that it could be much more extended and diversified if the ratios up to the 11-limit were incorporated into it. Figure 2-2 is an expanded version of his previously presented 5-limit diamond (ibid., 110). As you may notice, the lines that have form the shape of the diamond are either solid or dotted. He calls for categorizing the intervals in two groups, one called *Otonalities* appearing between solid lines from left to right (which reveals the “Numerary Nexus” of that set), and the latter called *Utonalities* between dotted lines from right to left (which reveals the different possible “Udentities”). For instance, in the first left-to-right row with solid lines from the top (read from the left corner up to the top), the Numerary Nexus is 7, and its Odentities are 8, 9, 10, 11, 12, and 7, which respectively form the ratios 8:7, 9:7, 10:7, 11:7, 12:7, and 7:7. As Partch describes, “Each line of six ratios is a condensed representation of tonality ingredients; these ingredients hold the potentiality of the maximal consonance that can be achieved by the Identities 1-3-5-7-9-11” (ibid., 160).

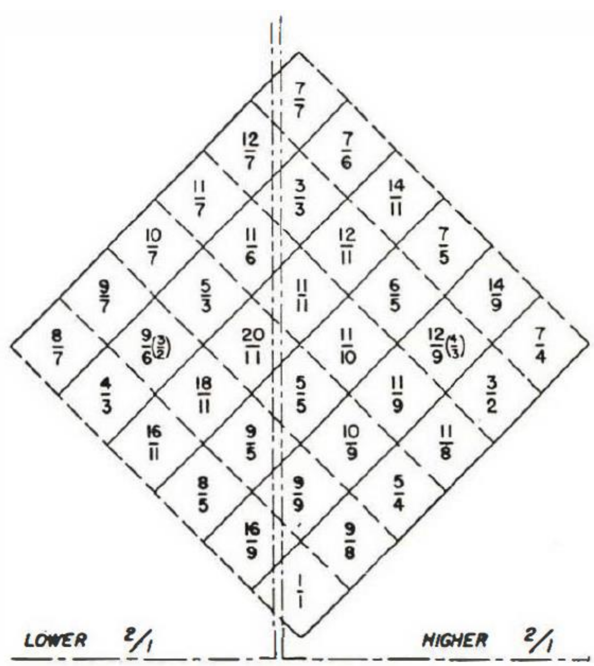

Figure 2-2: Harry Partch's 11-limit Tonality Diamond.

The interval ratio from the generating 1:1 tonic to the eleventh partial is 11:8, the so-called undecimal tritone. If we subtract the undecimal tritone (11:8) from a perfect fifth (3:2), the result is the previously mentioned neutral second with the ratio of 12:11[55] (= 150.64 cents). And if we subtract a perfect fourth (4:3) from the undecimal tritone (11:8), the resulting interval will be an undecimal quartertone with the ratio of 33:32[56] (= 53.27 cents). These values are very close to their approximated representatives in 24-ET, with the sizes of quartertone (50 cents) and three-quarter-tone (150 cents).

The significance of the eleven-limit intervals seems to be attention-catching even for theorists of Ancient Greek music. The second-century music theorist, Claudius Ptolemy (100–170), talks about a special tetrachord genus with three nearly equal ratios 10:9, 11:10, and 12:11 (10:9 × 11:10 × 12:11 = 4:3) (Baker 1990, 311). Alexander Ellis (1814–1890), the translator of Helmholtz's *On the Sensations of Tone* into English, mentions this tetrachord genus and describes it as playing the consecutive 9th, 10th, 11th, and 12th harmonics of a selected note on a horn or trumpet (Helmholtz 1895, 264). He also mentions the correspondence of the 11-limit intervals to the 24-ET system and its application in Arabic and Persian music. I will elaborate on this in the chapter about Aristoxenus, Fārābi, and their approximations.

If we consider the interval with the ratio of 12:11 (150.64 cents) as the smaller 11-limit neutral tone, then the larger 11-limit neutral tone will be the ratio of 11:10 (165 cents). According to the common opinion of many Persian and Arab practical musicians, the size of the neutral interval, specifically when it is the second or seventh scale degree from the finalis, cannot be

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55 $\frac{3:2}{11:8}$ = 12:11

56 $\frac{11:8}{4:3}$ = 33:32

exactly three quartertones. That is why rendering the dastgāhi-maqāmi modes in 24-ET is not widely accepted.

Farhat, describes two approximated sizes of the neutral-tone interval; the larger one of 160 cents (abbreviated with the letter N) which is approximately equal to seven commas in a 53-ET (nine-fold division) system. This interval could be an acceptable approximation for the 11-limit ratio 11:10 or 165 cents (Farhat 1965, 16). Practically, this larger neutral tone (N) is either completed with a smaller neutral tone (n) to reach a minor third interval.

Another remarkably important 11-limit interval is the neutral third, which, unlike the neutral second, usually has a lesser tendency towards the finalis and could be acceptably approximated with its quartertone value of roughly 350 cents. The simplest ratio from the harmonic series that corresponds to this size is 11:9 (347.41 cents), which combines with Zalzal's middle fret with the 11-limit ratio of 27:22 (354.55 cents), to complete a perfect fifth (11:9 × 27:22 = 3:2). As you may notice, the ratio of 27:22 is also an 11-limit interval, which increases the significance of the Zalzal's fret, which practically has been used by musicians since at least the time of Fārābi (the tenth century). I will explain this ratio in detail in the chapter about Fārābi and his approximations.

13-limit tridecimal tuning, 2/3-tone interval:
Smaller neutral tone

Unlike the world of 11-limit and quartertone intervals, the ratios from the 13-limit prime number roots, the so-called *tridecimal* intervals, do not seem to be properly recognized and accepted by the Western world or even Ancient Greek theorists. However, you can find brief mention of the ratio of 13:12 in Helmholtz's pitch table, as the "interval between 12th and 13th

harmonics" (Helmholtz 1895, 454). Also, Partch presents an even further expanded tonality diamond embodying the 13-limit ratios (Partch 1949, 454).

Meanwhile the ratio of 13:12 has been practically used as a neutral tone (the so-called *mojannab*[57]) since at least the tenth century in Persian and Arabic music. This ratio with the logarithmic value of 138.57 cents effortlessly corresponds to the approximated size of a 2/3-tone interval. As I mentioned before, Farhat raises the idea of two sizes of neutral tone intervals, of which the approximated value of the smaller one (135 cents) is very close to the 13-limit interval of 13:12 (Farhat 1965, 16). Farhat's small neutral tone is approximately equal to six commas in a 53-ET (nine-fold division) system.

The combination of a small neutral tone (n) with a large neutral tone (N) completes a minor third interval. Again, it should be mentioned that Farhat's values are approximations and do not fit in one specific tuning system. If the small neutral tone is considered between the Abjadic letters A and J, and a large neutral tone between the letters J and h, then the resulting minor third will be the interval A-h[58] (Figure 2-3). In the last chapter I will elaborate on Farhat's stepwise intervals and their application in the modes.

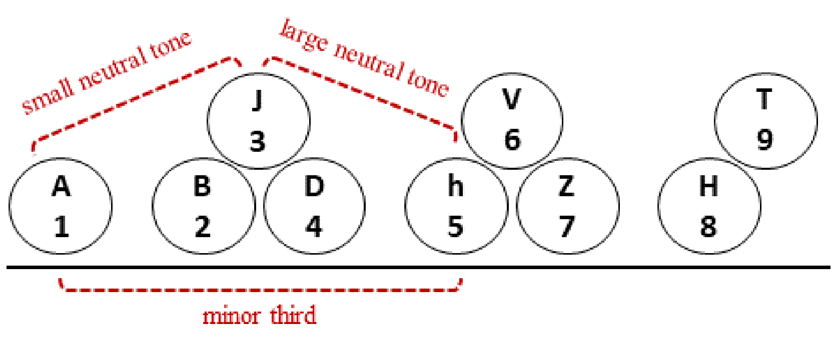

Figure 2-3: A small neutral tone and a large neutral tone as counterparts of each other to reach a minor third.

57 مجنّب

58 الف–هـ

We can summarize all the intervals mentioned in this chapter based on ratios from different prime-number limits (namely 2, 3, 5, 7, 11, and 13) as follows (Figure 2-4). It goes without saying that the ratios of the perfect fifth (3:2), perfect fourth (4:3), whole tone (9:8), and Pythagorean limma (256:243) are 3-limit intervals. The major third (5:4) and minor third (6:5), and the small minor second (25:24) are 5-limit intervals. The only 7-limit interval here is the plus tone (7:6). The large neutral tone (11:10), the equal-tempered three-quartertone (12:11), and the harmonic neutral third (11:9) are the 11-limit ratios. And finally, the small neutral tone (13:12) is 13-limit.

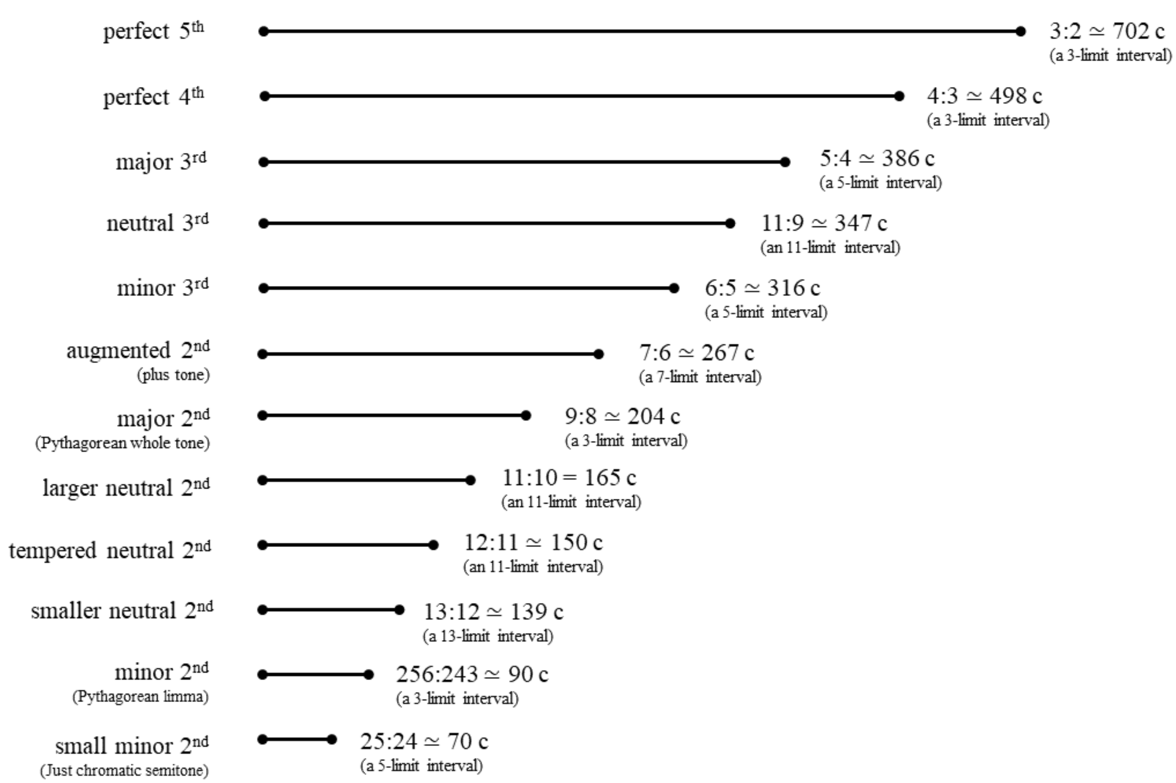

Figure 2-4: The important intervals, smaller than a perfect fifth, used in Persian and Arabic music, obtained through the simplest possible ratios from different limits of the prime numbers of the natural harmonic series.

3. Approximations in twenty-four equal quartertone temperament (24-ET)

Aristoxenus, Fārābi, and 24-ET

Today, approximation is an integral part of music. When we play piano or study tonal harmony and counterpoint on the staff, we are dealing with a rough approximation of just intonation musical intervals in the context of a twelve-tone equal temperament. The deviation of the approximated intervals on piano from their natural values in just intonation could be as small as two cents for a perfect fourth or fifth[59] or as large as fifteen cents for a minor third or a major sixth.[60]

In the modal system of Persian and Arabic music, achieving an acceptable ratio or, in some cases, approximated values for the neutral second and third intervals (and their inversions, neutral seventh and sixth intervals), as used in practical music-making, has always been a topic of debate. From the musicians of the so-called Scholastic Persian-Arabic music theory School[61], such as Fārābi (870–950), who tries to provide a logical theoretical definition for Zalzal’s practical neutral third—which is located in the middle of a major third and minor third—to the theoreticians of Systematist School,[62] such as Ormavi (1216–94) and Shirāzi (1236–1311), who inconclusively try to define the neutral intervals in the context of the Pythagorean tuning system and present a mere

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[59] Since the equal-tempered perfect 4th (500 cents) and perfect 5th (700 cents) are each two cents deviated from their natural version in harmonic series (respectively 498 cents and 502 cents).

[60] An equal-tempered minor third (300 cents) and major sixth (900 cents) are more than fifteen cents drifted than their 5-limit equivalent ratio in just tuning system (315.64 cents and 884.36 cents).

[61] مکتب مُدرّسی

[62] مکتب مُنتَظَمیّه

theoretical approximation for the seventeen-tone temperament, to more recent musicologists, such as Hedāyat (1863–1955) who suggests a different collection of approximated ratios based on the consensus of the prominent practitioners at the time, or the European-trained Vaziri (1886–1979) who renders the intervals in the twenty-four-equal-quartertone system, etc. (see Daemi Milani 2022). But among the aforementioned early theorists, the approach that Fārābi takes seems most similar to the approximation of intervals on the piano based on the equal division of the span of an octave.

We know that Fārābi[63] was well-informed about ancient Greek music theory, specifically because he restates some of Aristoxenus' ideas, although without mentioning his name in the text (Laywine n.d., 2). Fārābi, in his music treatise, *Great Book of Music*, starts his discussion with the thought-provoking idea that the sensory perception of a human being is *approximate and not precise*. On the basis of this logic and after a few lines of elaboration, he calls his audience for leniency and forbearance to accept that the *limma*[64]—which was known until then by its Pythagorean value, 256:243— is the same as the semitone and equals a half of a tone. Then, based on this approximation he takes the semitone as a unit and divides the span of an octave into twelve equal parts. As he puts "If, then, we take semitone as a unit, the octave is twelve semitones, by which, reckoning the fifth turns out to be seven [units], the fourth [turns out to be] five [units], and

[63] Abu-Nasr Mohammad Fārābi (870–950) —the so-called "Second Teacher" after Aristotle— was a Persian philosopher and music theorist who was born in Farab (one of the thirty-four provinces of Afghanistan today, located in the north of the country) in Khorasan and died in Damascus (the capital of Syria today). Like many other scholars of that time, Fārābi preferred to write his treatises in Arabic (the *lingua franca* of the Empire and the language of the patron princes) so that they could be published more easily in the lands of the Islamic territory (see Farhat 1965, 4).

[64] فضله

the whole tone [would be] two [units]”[65] (Fārābi 1967, 148; n.d., 13-14). This is exactly the definition of the twelve-tone equal temperament that we use today.

However, there is a serious problem here since he wants to present different tetrachord genera, and by defining the semitone as the unit, only the first genus, the diatonic tetrachord, is explainable. That is why he starts to divide a tone into smaller sections, i.e., quarter tones, third tones, eighth tones, and twelfth tones, to be able to explain the other types of tetrachord genera encompassing intervals other than tones and semitones. The method he uses for his divisions is a bit vague until the final paragraph of this section when he elaborates on his 144 equal temperament. “Let us, then, take the octave as measured by the number 144. In that case, the fourth will be 60, and the fifth will be 84. By that reckoning, the first genus from among the first four will be: 24, 24, 12. The second will be: 24, 18, 18. The third will be: 30, 18, 12. The fourth will be: 36, 12, 12. The first of the second set of four will be: 48, 6, 6. The second will be: 44, 8, 8. The third will be: 42, 9, 9. The fourth will be: 20, 20, 20” (Fārābi 1967, 16; Barkeshli 1978, 151-153). This is exactly the way that Aristoxenus implicitly approximates the tetrachord genera.

Fārābi’s smallest unit is computable by equally dividing a semitone into another twelve parts (12×12 = 144). In modern words, if we translate this smallest unit into frequency ratios, it would be the 12th root of the 12th root of the number 2 representing the octave ($\sqrt[12]{\sqrt[12]{2}} = \sqrt[144]{2}$). In the following table, I compare Fārābi’s approximated intervals with their value in cents (Table 3-1). As you see, although Fārābi’s unit is larger than a cent, it works better for both binary and

[65] The text in Arabic:

فإذا فرضنا الفضله واحداً، كان البُعد ذوالكلّ اثنى عشر، وبذلك المقادير يصير ذوالخمسه سبعهً، و ذوالربعه خمسهً و بُعد العوده اثنَين

ternary divisions of a tone, since it gives integer numbers for all the intervals that he wants to demonstrate.[66]

interval name	the value relative to the semitone	units in Fārābi's 144-ET system	units in cents
octave	12 semitones	144	1200
fifth	7 semitones	84	700
fourth	5 semitones	60	500
whole-tone	2 semitones	24	200
half-tone	1 semitone	12	100
third-tone		8	66.66
quarter-tone		6	50
sixth-tone		4	33.33
eighth-tone		3	25
twelfth-tone *		2	16.66

Table 3-1: A comparison between the intervals in Fārābi's units and cents.

* The twelfth-tone is the smallest division that Fārābi uses to explain his tetrachord genera, although it is two times bigger than the smallest unit that he considers in his system, a twelfth of a semitone.

We can summarize Fārābi's eight genera and compare the intervals between the four tones of the tetrachords, measured in Fārābi's units, Aristoxenus's units, and cents (Table 3-2). It seems that six of the tetrachords are the same as Aristoxenus' genera, the second type belongs to the world of Persian and Arabic music, and the last one, in which all the intervals are equal, is raised only "for the sake of considering all the possibilities" (Fārābi n.d., 15; Barker 1990, 347-349).

[66] Dr. Mehdi Barkeshli (1912 - 1988) introduces an innovative logarithmic system centered around a unit called "Fārāb" named in honor of Fārābi. This unit, quantifies at 0.00209, marked a pioneering advancement in logarithmic systems, pre-dating even Félix Savart's (1791–1841) system (Barkeshli 1978, 195-196).

An important point to consider here is that despite the Greek-based consensus of the arrangement of the tetrachord intervals that places the largest interval at the top of the tetrachord and the smallest one at the bottom, Fārābi considers the possibility of inverting the order of the intervals of the tetrachord genera. That is why, for instance, his first tetrachord, which according to Barker, is the same as Aristoxenus' tense diatonic genus, starts with a whole tone at the bottom and ends with a semitone at the top (Laywine n.d., 3 and 21).

	Tetrachord genus (T=tone)	Size of the intervals in Fārābi's units (perfect fourth = 60)	Size of the intervals in Aristoxenus' units (perfect fourth = 30)	Size of the intervals in cents (perfect fourth = 500)
1	T T 1/2T	24 24 12	12 12 6 tense diatonic	200 200 100
2	T 3/4T 3/4T	24 18 18	(not included)	200 150 150
3	(T+1/4T) 3/4T 1/2T	30 18 12	15 9 6 soft diatonic	250 150 100
4	(T+1/2T) 1/2T 1/2T	36 12 12	18 6 6 chromatic	300 100 100
5	2T 1/4T 1/4T	48 6 6	24 3 3 enharmonic	400 50 50
6	(T+5/6T) 1/3T 1/3T	44 8 8	22 4 4 soft chromatic	366.66 66.66 66.66
7	(T+3/4T) 3/8T 3/8T	42 9 9	21 4.5 4.5 hemiolic chromatic	350 75 75
8	(3/4T+1/12T) (3/4T+1/12T) (3/4T+1/12T)	20 20 20	(not included)	166.6 166.6 166.6 (abstract)

Table 3-2: Fārābi's eight tetrachord genera.

Fārābi’s three types of a third

A useful hint that gives us an idea of where Fārābi’s second tetrachord genus has come from is the argument he raises a few pages earlier, under a section titled “The Kindred Powers in Melodic Lines”[67] (Fārābi 1967, 131; n.d., 10), after he shows all possible frets with their ratios on an oud. He talks about three distinct tetrachord qualities “three relations of kinship”[68] (ibid., 136; n.d., 11) that you can make by changing the quality of a *third scale degree* —in which the third scale degree could be a major third, a minor third, or a *neutral* third. The framing notes of all three tetrachord qualities are the open string[69] (1:1) and a perfect fourth (4:3) assigned to the little finger.[70] Regarding the two insider notes of the tetrachords, the first one is always a whole tone (9:8) above the open string, played with the index finger,[71] while the second one could have three possibilities: a *Pythagorean major third* (81:64 = 9:8×9:8) played with the ring finger,[72] a *Zalzal’s neutral third* [73] (27:22 = 9:8×12:11), or a *Persian minor third* (81:68 = 9:8×18:17),[74] the latter both played with the middle finger.[75]

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[67] Title in Arabic:
القُوى المتجانسه فى اصول الالحان

[68] المتجانسات الثّلاث

[69] مطلق

[70] خنصر

[71] سبابه

[72] بنصر

[73] takes its name from Mansur Zalzal, a famous eighth-century oud player (Farhat 1965, 11; Farmer 1929, 118-119; Wright 1978, 31)

[74] Fārābi, in his collection of the thirds, also shows another value close to the Persian middle fret, entitled “next middle fret (مجنّب الوسطى)” (32:27) which perfectly fits as a Pythagorean minor third.

[75] وسطى

This corresponds closely with the threefold third scale degree of my tone-bubble diagram of the Persian-Arabic seventeen-tone temperament, introduced in Chapter 1 and reproduced with added annotations in Figure 3-1. This diagram is my own creation, based on a consideration of the seventeen Abjadic tone-letters in the Persian-Arabic theoretical treatises of the thirteenth century Systematist School, the practical fretting of traditional instruments, and different early and modern ratio suggestions. In Figure 3-1, you can see the place of Fārābi's aforementioned *three relations of kinship* on my diagram.

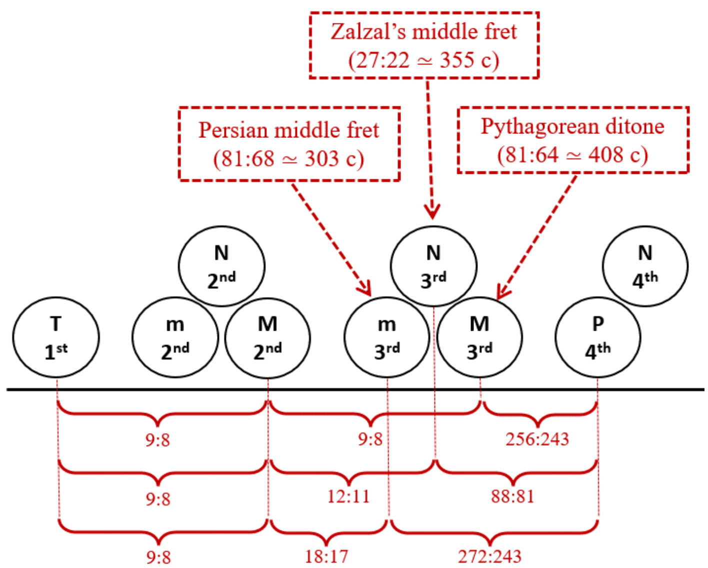

Figure 3-1: Fārābi's three approximated types of a third interval.

If we consider Aristoxenus' *tense diatonic* (T-T-½T) as an approximation for either 9:8-9:8-256:243 or 9:8-10:9-16:15 tetrachord genera (Barker 1990, 73), Fārābi's second genus (T-¾T-¾T) could be seen as an approximation for either the genus with Zalzal's middle fret 9:8-

12:11-88:81 (Fārābi n.d., 16) or Ptolemy's *equal*[76] genus, 10:9-11:10-12:11 (Barker 1990, 350; Helmholtz 1895, 264). On the other hand, Zalzal's neutral third (27:22 = 9:8×12:11) and the harmonic neutral third (11:9 = 10:9×11:10) in Ptolemy's equal genus, are counterparts of each other to complete a perfect fifth. I have shown this relationship in the following diagram (Figure 3-2).

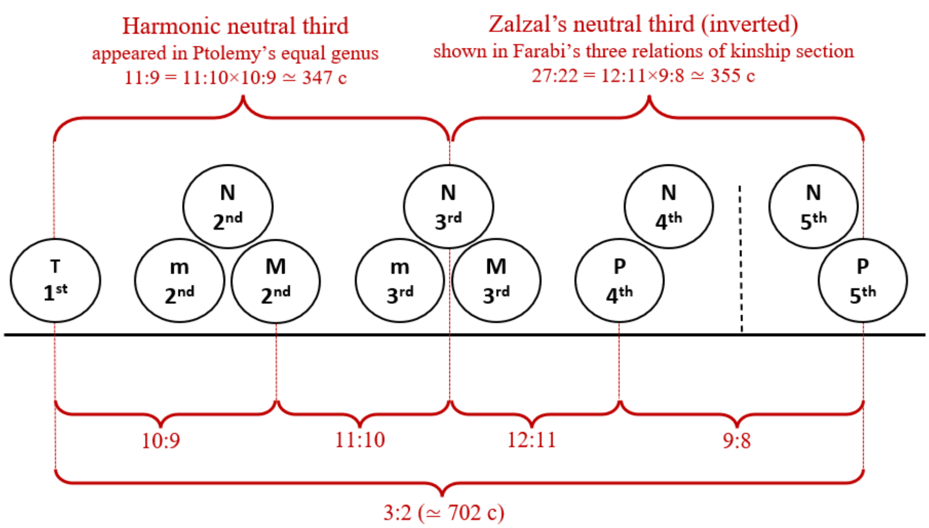

Figure 3-2: Harmonic neutral third and Zalzal's neutral third (inverted) adding to a perfect fifth.

In the context of twenty-four-tone equal temperament and Fārābi's approximations, along with the tone (major second) and semitone (minor second) intervals, the *three-quarter-tone* (neutral second) interval is an essential building block for Persian and Arabic musical modes and is widely in use. That is why Fārābi priorities his second (T-¾T-¾T) genus and puts it after the

[76] Also called *Homalon* genus; Ptolemy describes the character of this genus as "foreign and rustic, but exceptionally gentle, and the more so as our hearing becomes trained to it, so that it would not be proper to overlook it, both because of the special character of its melody, and because of the orderliness of the division. […] When a melody is played in this genus by itself, it gives no offensive shock to the hearing" (Barker 1990, 312).

tense diatonic genus (T-T-½T).[77] It is remarkably interesting that Fārābi, by considering the agenda of *"empirical adequacy"* of music theory (see Laywine n.d., 10-11), obtains the 11-limit ratio of 27:22 for Zalzal's neutral 3rd to explain a significant type of genus from a musical culture (different than Greek music) to which he was exposed geographically.

By generalizing Fārābi's three approximated values of the third interval —namely, minor third, neutral third, and major third— to the other scale degrees, and considering three tempered sizes of 1/2-tone, 3/4-tone, and tone as the building blocks of the bigger system of the seventeen-tone Persian-Arabic scale, the following collection of intervals expressed in cents is obtained (Figure 3-3). These three mentioned intervals, as well as a larger one with the rounded size of 5/4-tone form the four[78] basic stepwise intervals of the Persian and Arabic musical system (see Farhat 1965, 16; Lucas 2019, 152). In the Table 3-3, you can see a collection of the four stepwise intervals with their titles and abbreviations in the old treatises of Systematist School, as well as their approximated sizes and values. In the last chapter, I will elaborate on these intervals and their more accurate values in Farhat's terminology.

Practically nothing more than these seventeen tones is needed today to explain all the melodies of Persian and Arabic music. Although in some cases this quartertone-quality approximation is not favored by professional musicians, it is the tuning system that is most used by practitioners, and it is at least a decent way of showing the neutral intervals as autonomous qualities different than major and minor.

[77] Fārābi's second tetrachord genus (T-¾T-¾T) and its inverted version (¾T-¾T-T) are both the tempered version of very famous tetrachords in Persian and Arabic music (see the last chapter).

[78] The number of the stepwise intervals is recognized to be "five" in Farhat's view (see the last chapter).

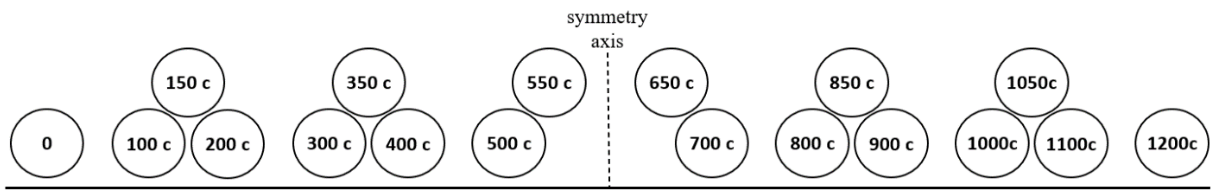

Figure 3-3: The implementation of Fārābi's quartertone approximation on the seventeen tones of the Persian-Arabic system.

Referring again to the section where Fārābi introduces the eight tetrachord genera (see Table 3-2 above), he distinguishes the first four tetrachords and describes them as “the ones that can be specified on the instrument [sc. the oud]” (Fārābi n.d., book two 15). If we take a deeper look at the sizes of the intervals inside these four genera, we can find the rough approximation of all four types of Persian-Arabic stepwise intervals including the so-called *plus tone* with a size of 250 cents. It seems that by choosing three of the Greek tetrachord genera that were compatible with the practical reality of music, and by adding the widely-in-use (T-¾T-¾T) Persian-Arabic genus as the second priority after the diatonic genus, Fārābi tries to present a collection of four tetrachord genera that are enough to explain musical practice.

Interval type	Title of the interval (in Systematist School)	abbreviation (Systematist’s)	Approximated size	approximated value in 24-ET
Semitone	baqieh (بقیه)	b (ب)	1/2-tone	100 cents
neutral tone	mojannab (مجنّب)	j (جـ)	3/4-tone	150 cents
whole tone	tanini (طنینی)	T (ط)	tone	200 cents
augmented second	tanini-e mostazād (طنینیِ مُستزاد)	h (هـ)	5/4-tone	250 cents

Table 3-3: The approximation of four stepwise intervals in Persian-Arabic music.

Mikhā'il Mashāqa, Alinaqi Vaziri, and 24-ET

We cannot talk about twenty-four-quartertone temperament without mentioning Mikhā'il Mashāqa[79] (1800–88), the well-known Lebanese musicologist and historian. Many Western scholars introduce him as the founder of the twenty-four equal quarter-tone scale. However, a deeper investigation of his treatise, *Essay on the Art of Music*, reveals that the twenty-four-quartertone temperament was already known at that time, and that in fact Mashāqa rejects this temperament because of the mathematical error he found in it (Maalouf 2003, 837). He believes that while Arab music theorists divided the octave into twenty-four equal quartertones, music in practice does not correspond to this division, given that the music actually performed did not 'embody any fixed quarter-tones' (ibid., 480). This statement is understandable because, as I explained earlier, the exact place of the fixed pitches of the system does not precisely correspond to any equal temperament, and furthermore, the value of the neutral intervals can be different depending on the musical mode.

On the Persian side, it was Alinaqi Vaziri (1886-1979), a composer and a prominent tar player, who re-introduced Persian traditional music in the context of the twenty-four-quartertone system in the twentieth century. He was educated in Europe and was familiar with Western Classical music. Vaziri is known for his ideas on the application of Western harmony to musical compositions within Persian modes. Although his notion of reduction of the neutral interval to the size of three quartertones was widely rejected by many practitioners, his inventory of *sori*[80] (quarter-sharp) and *koron*[81] (quarter-flat) accidentals are still used in the notation of Persian traditional music.

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79 میخائیل مشّاقه

80 سُری (𝄲)

81 کُرُن (𝄳)

The choice of the two words that Vaziri uses for his quarter-tone accidentals shows that he was aware of the numeral value of the Abjadic letters, since the summation of the numeral values of both the words sori[82] and koron[83] end up with the number 270 which is equal to his first name's numeral value.[84] But he never mentions the seventeen Abjadic tone-letters and the thirteenth-century theoretical period of the Systematist School. In this regard, his dedicated student and colleague, Ruh-allāh khāleqi[85] (1906-1965), in defense of Vaziri's quartertone accidentals and rejection of Hedāyat's Abjadic notation method that was introduced around the same time, writes: "In my opinion, his effort […] was in vain, because the beauty of the notation is that it is international; it should not be changed in such a way that it gets a personal aspect. Learning this [international] notation is not a difficult task, and we can also notate our own [Persian] music with the same notation without needing to invent a new one, and for some special frets and intonations of Persian music, Vaziri's accidentals have been established and publicized for years" (Daemi Milani and Asadi 2021, 6; khāleqi 2016, 434).

In fact, the idea of using 24-ET for documenting Persian and Arabic music in the twentieth century was the re-introduction of Fārābi's equal temperament in Western terms. Despite this system facilitating the notation and transcription of traditional music, with a tiny manipulation of the accidentals and intonation and articulation signs, its fundamental problem is that not every neutral interval fits into an approximated value in the grid of quartertones. The roughness of the approximation has sometimes caused ambiguities and disputes over certain musical modes. In the

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[82] The numeral value of the word *sori*: S(س)=60 + R(ر)=200 + Y(ی)=10: in total = 270

[83] The numeral value of the word *koron*: K(ك)=20 + R(ر)=200 + N(ن)=50: in total = 270

[84] The numeral value of the word *Ali-Naqi*: A'(ع)=70 + L(ل)=30 + Y(ی)=10 + N(ن)=50 + Q(ق)=100 + Y(ی)=10: in total = 270

[85] روح‌الله خالقی

case of Vaziri's modes for instance, for a mode called *bayāt-e Esfahān*[86] that contains a tetrachord genus with a whole tone surrounded by two neutral tones with different sizes (j-T-J) (see the last chapter), since the two neutral are rounded to a single size of three quartertones, and also because of the influence of the harmonic minor scale which to some extent approximately corresponds to this mode, a new 24-ET version of that mode is generated and used in parallel of the original mode, which is called *old bayāt-e Esfahān*[87] today.

86 بیات اصفهان

87 بیات اصفهان قدیم

Extinction of the neutral intervals as a result of Westernization, keyboard instruments, and politics

To conclude the topic of approximation, I want to draw your attention to a remarkably interesting historical-political issue that has affected the quality of the modes containing neutral intervals in the countries that used to be related to the maqām/dastgāh musical tradition, which broadly speaking includes the Mediterranean and Near East regions. The gradual change and eventual extinction in some of the countries of this region of the neutral intervals with their nearly quartertone, or as Jean During calls them, *Zalzalian*[88] values, due to changes in the sonic environment and some cultural policies (including the domination of Western musical culture and the emergence of the keyboard) is an important topic worth mentioning here (During, 2019: 218).

In simple terms, the problem is that in several Inner Asian countries such as Azerbaijan, Uzbekistan, and Tajikistan (a geographical region called Transoxiana), the neutral intervals of the maqām/dastgāh modes lost their characteristics during the last century and became rounded off to the adjacent semitones. Studying the early pre-Soviet audio recordings of the musical modes, the intervals between the instrument's frets in early pictures, and the preferred intonation that some of the master vocalists and instrumentalists of the previous generation still use today, all show that the neutral intervals used to be applied with their original sizes.

Of course, the most problematic neutral interval in terms of conflicts with the nature of major and minor harmony is the *neutral third*. As During puts it, "A survey of the modal scales used all over Inner Asia shows a clear-cut between the oriental Turkic and the Persian-Arabic musical cultures: shortly speaking, on one side we find quasi-regular chromatic scales, on the other, in addition to intervals of tone or semi-tones, we find the famous neutral tone and [neutral]

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[88] زلزلی

third" (During, 2009). Among the different arguments that are raised about the causes for this separation, a cultural-political reason attracts attention. During the first five decades of the Soviet period, it was prohibited to use the modes containing neutral intervals, which were the most contradictory to the musical standards which were dictated by Moscow. The tendency towards a Turkish musical culture, due to linguistic reasons, that was somewhat already separated from its antecedent Ottoman traditions after Westernization in the early nineteenth century is another significant reason.

The imitation of maqāmic musical techniques and ornamentations on common keyboard instruments like accordion or garmon in Azerbaijani music has led to the creation of more-or-less convincing twelve-tone tempered renditions of some of the traditional maqām/dastgāh modes. As an example, a tetrachord with a neutral second in it (e.g., D, E-half-flat, F, G), is changed to a new 12-ET tempered tetrachord on the keyboard, with an altered minor/major second (D, Eb/E, F, G). In other words, the neutral second interval is alternatively induced by an indecisive fluctuation between major and minor seconds.[89] It gets more critical for the modes containing a neutral third: take for example a pentachord with a neutral third scale degree (e.g., C, D, E-half-flat, F, G). As a workaround, in the 12-ET keyboard version of these modes, a back-and-forth between minor and major thirds is supposed to conceptually induce the feeling of a neutral third (C, D, Eb/E, F, G), which is an impractical and impossible expectation.[90]

Consequently, the imitation of those new keyboard-oriented modes by other accompanying string instruments has caused the gradual shift of the frets toward the 12-ET values.

[89] The Azerbaijani version of the mode Shur clearly demonstrates these two different feelings of suspension at the second scale degree, which resolves to the tonic or finalis.

[90] The Azerbaijani version of the mode Segāh is a significant example, in which the quality of the mode tends to sound more like a major mode and never succeeds in conveying the neutral quality of the original mode.

Even for the unfretted string instrumentalists, the place of the equal-tempered values is internalized and most of younger musicians do not use the original intervals.

There is no doubt that the exclusion of the neutral intervals from the maqām/dastgāh modes has caused some admirable polyphonic enhancements —namely, the emergence of new musical styles, like symphonic mugam and jazz mugam— but on the other side, it has sacrificed some of the modal parts of this enriched musical tradition. That is why a new tendency of returning to the basics and reviving the pre-Soviet musical traditions has been initiated by some curious Azerbaijani musicians and musicologists.[91]

A similar problem has happened in Turkish modern music as well, which is one of the direct inheritors of the traditional maqām musical culture. Turkish modern music theory was among the things affected by Mustafā Kemal AtaTürk's (1881–1938) policies of Westernization and cultural separation from the Ottoman tradition (Yarman 2008). The new theoretical system is correspondent to the system of nine-fold division of whole tone,[92] while it is based on the Pythagorean-driven theory of Systematist School. This so-called modern system was agreed upon and proposed by different prominent contemporary Turkish music theorists, like Rauf Yektā (1871 – 1935), Mehmet Suphi Ezgi (1869 – 1962), Sadettin Arel (1880 – 1955), etc.

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[91] Surprisingly, another genre affected by the absence of the neutral third interval in equal-tempered tuning is the *Blues*. In the African-driven aspect of the Blues.music, introduced through black slaves, many of whom had familiarity with Arabic-Islamic musical traditions, the neutral third interval holds significance. It corresponds to the second step in the African equal heptatonic scale (See Hijleh 2016; Kubic 2008 and 1999). However, replicating this interval on instruments like keyboards and guitars has led to unclear harmonies and misunderstandings. In my opinion, even the emergence of the *Hendrix chord*, serving as a tonic, could be a consequence of the lack of neutral third interval in the equal temperament.

[92] Which can result in 53- or 55-tone equal temperament, depending on whether the diatonic semitone is 4/9ths of a whole tone (4 commas) or 5/9ths of a whole tone (5 commas). The former, 53-ET, is an excellent Pythagorean approximation (which is intended in this research), and the latter, 55-ET, is an approximation to 1/6-comma meantone.

In this system, by the introduction of the microtonal unit of a comma (≃ 22.6 cents), the previously-mentioned five stepwise intervals of the Persian-Arabic music got drifted to their mere theoretical Systematist values: semitone or baqieh (b) as four commas (≃ 90 cents), small mojannab (j) as five commas (≃ 114 cents), large moJannab (J) as eight commas (≃ 180 cents), whole tone or tanini (T) as nine commas (≃ 204 cents), and augmented second or tanini-e mostazād (h) as twelve commas (≃ 270 cents) (Mohafez 2014, 36-37; Signell, 1986, 22-23). As can be seen, by avoiding the sizes of six commas (≃ 135 cents) and seven commas (≃ 160 cents), the common effort seemingly was to avoid the so-called Zalzalian intervals, which would implicitly recall the traditional middle eastern identity.

It is to be expected that the previously discussed gap between theory and musical practice appears here as well. In at least one of the fundamental maqāmic modes, called Bayāti,[93] the practitioners always apply the neutral second interval with its almost three-quartertone value and do not respect the dictated modern Pythagorean-based nine-comma rules.

[93] بیاتی

4. Tetrachord genera of the Persian dastgāh system

Numerous research endeavors delve into the dastgāh and maqām modal collections, predominantly centering on distinct mode categorizations, variations in melodic arrangements, and the intricacies of melodic syntax. When we study Persian traditional music, the first basic question that arises is *what historical period* are we talking about? Do we mean earlier *maqāmi* music, which was commonplace in Persia until around the seventeenth century, with its own theory, repertoire, and terminology? Or are we talking about the contemporary period and *dastgāhi* music, which is gradually emerged roughly from the early nineteenth century in Iran, based on melodic figures, ornamentations, and special qualities, with its different terminology?

The historical-political gap in Persian music theory should be considered here, to better understand the reason and quality of the gradual change from maqāmi to dastgāhi music. The Persian-Arabic music theory, whose sublime example is evident in the treatises of the Systematist School, had become very complex and mature by the sixteenth century, under the support of statesmen in different periods. Until the emergence of the Safavid dynasty and the major political-religious change from Sunni to Shiite, when the equation changes fundamentally, apparently due to the change of attitude of the Muslim rulers against music.

One of the important political goals of the Safavid kings was to institutionalize the ascendancy of Shiite religion in Iran, and therefore they supported the clerics and extremists who considered art, music, and dance to be essentially *harām*[94] and tried to destroy them. Therefore, especially during the reign of Shāh Tahmāsp I (1514 – 1576), confiscation of musical instruments

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[94] حرام

and breaking them, and even cutting off the hands of musicians became common.[95] It seems that in this era, due to these circumstances, many musicians leave their homeland and disperse to the surrounding countries, and eventually, the theoretical part of Persian music gradually disappears from the sixteenth to the beginning of the twentieth century (Farhat 1965, 5-6 and 121; Mash'hun 1994, 276-277; Meysami 2014, 185-188; Mohafez 2011, 230).

The Persian dastgāhi music that is common in Iran today, is basically a collection of famous melodic figures that were compiled bit by bit by various musicians, with a focus on the music of the center of Iran, mainly through oral transmission, roughly since the late eighteenth century, which formed during Qajar period, and finally transcribed by musicologists like Hedāyat, Vaziri, etc., under the art-patronizing policies during the Pahlavi era. I have to remind again that elaboration on the details of melodic progression in Persian dastgāhi music is not the purpose of the current thesis.

In this section, I will touch on the most frequently recurring distinct tetrachord genera and their application in current Persian dastgāh music, *intentionally at a very shallow level*, with a quick mention of their equivalent interval values in the earlier maqāmi system. The approach that I take has roots, historically speaking, in the ancient Greek music theory tradition for describing their tetrachord genera, which was obviously followed and replicated by later Persian and Arab

[95] This structural hostility and enmity of Shiite towards art and music can be seen even today, after the renewed ascendency of Shiite clerics in 1979, in the daily behavior of the Islamic Republic’s agents with artists in Iran.

scholars. The tetrachord genus usually is addressed with the term *dāng*[96] in Persian dastgāhi music, *zi-al-arba'* [97] in earlier maqāmi music, and *çeşni*[98] (meaning *spice)* in Turkish music.

For each tetrachord genus, I will explain the arrangement of the intervals both on the seventeen-tone bubble diagram and in staff notation using the quarter flat and sharp accidentals, with the disclaimer that they do not necessarily mean the exact quartertone intervals, but rather they approximately show the neutral tones with varied sizes. It should be mentioned that the title of the modes and maqāms often differs between the early treatises and the current system. My choice for titling the modes is based on the current names that are common in contemporary dastgāhi music, but I will also compare them with their older names in Systematist School treatises.

Considering that the modal progressions in the dastgāh system is much more complicated than a single scale, it would take hundreds of pages to explain the details of all the modal figures and ornamentations. But briefly, for the whole system, modal progressions can be seen as the stacking of tetrachord genera, which, based upon the Ancient Greek music theory tradition, can be *disjunct* or *conjunct* to each other.

For each tetrachord genus, I have tried to show the arrangement of the intervals, usually on the first half of the bubble diagram with either framing tone-letters A (الف) or H (ح) as the finalis of the mode. In some cases, I had to consider the whole one-octave span of the bubble diagram to demonstrate the exact position of the genus in the scale. A more logical way of explaining these modes is by studying a wider range of the fundamental scale that embodies at least two main

[96] دانگ

[97] ذی‌الاربع

[98] چاشنی

tetrachords, with the finalis at the middle.[99] I showed this by highlighting the place of the core genus in a more extended span of the fundamental scale. The two additional accidentals that I have used to approximately indicate the neutral intervals in the staff notation are the quarter-flat (𝄳) and the quarter-sharp (𝄲) signs.

Regarding the size of stepwise intervals, I prefer to be loyal to Farhat's values and approximations, which properly correspond to the practical intervals that are used today.[100] As he summarizes, there are five different stepwise intervals in Persian contemporary traditional music: the semitone (or Pythagorean limma) with the value of 90 cents (m), the small neutral tone with the approximated value of 135 cents (n), the large neutral tone with the approximated value of 160 cents (N), the (Pythagorean) whole tone with the value of 204 cents (M), and the plus tone with the approximated value of 270 cents (P) (Farhat 1965, 16).

It worth mentioning that all these five stepwise intervals are properly explainable in the context of fifty-three-tone equal temperament (53-ET) or the nine-comma (nine-fold) division of the tone that Hedāyat talks about (Hedāyat 1938, third section, 20):[101] semitone (m) as four commas (≃ 90 cents), small neutral tone (n) as six commas (≃ 135 cents), large neutral tone (N) as seven commas (≃ 158 cents), whole tone (M) as nine commas (≃ 204 cents), and augmented second (P) as twelve commas (≃ 271 cents).

[99] Similar to the tradition that was used by Ancient Greek theorists, putting the central tone *mese* at the middle and two or more tetrachords around it.

[100] It's important to note that certain scholars may hold differing opinions regarding Farhat's approximations and his arrangement of stepwise intervals.

[101] Of course, not with the Systematist Pythagorean-based or the modern Turkish music theory values for the large and small neutral tones, which was discussed in the previous chapter.

In comparison, the stepwise interval explained in the early music theory treatises of the Systematist School includes four types having different titles and defined entirely by Pythagorean calculations: a semitone called *baqieh* (b), a whole-tone called *tanini* (T), and a neutral-tone called *majonnab* (j), as well as an additional augmented second called *tanini-e mostazād* (h). In the following table you can see the collection of five stepwise intervals and compare the terminology and abbreviations that Farhat uses with the ones used in the music theory treatises of the thirteenth century Systematist School. Furthermore, you can see the sizes of the five stepwise intervals in fifty-three equal temperament or the so-called nine-comma system, as well as the closest simple ratio (not for the Pythagorean limma obviously) from the natural harmonic series that could represent the values of the intervals (Table 4-1).

Farhat's interval title	Farhat's abbreviation	Systematist School's interval title	Systematist School's abbreviation	Farhat's approximated value	Sizes in 53-ET or 9-comma system (a comma ≃ 22.6 cents)	Closest ratio in the natural harmonic series
semitone	m	baqieh (بقیه)	b (ب)	90 cents	4 commas	256:243 (3-limit)
smaller neutral tone	n	[small] mojannab (مجنّب [کوچك])	j (جـ)	135 cents	**6 commas** **	13:12 (13-limit)
Larger neutral tone	N	[large] moJannab (مجنّب [بزرگ]) *	J (جــــ)	160 cents	**7 commas** **	11:10 (11-limit)
whole tone	M	tanini (طنینی)	T (ط)	204 cents	9 commas	9:8 (3-limit)
plus tone (augmented second)	P	tanini-e mostazād (طنینیِ مُستزاد)	h (هـ)	270 cents	12 commas	7:6 (7-limit)

Table 4-1: Five different sizes of stepwise intervals in Persian dastgāh music.

** There is no differentiation between the larger and smaller neutral tone (mojannab) intervals for the abbreviated letter used in Systematist School treatises. In this table and later to explain the tetrachord genera, I use the uppercase J to show the larger mojannab and the lowercase j for the smaller mojannab intervals, corresponding to Farhat's uppercase and lowercase n/N.*

*** As mentioned earlier, the sizes of small and large neutral tones were considered to be respectively **5 commas** (≃ 114 cents) and **8 commas** (≃ 180 cents) in Pythagorean-based Systematist School treatises, and later in modern Turkish music theory, which are not correspondent to the values of the practical intervals.*

In total, I recognized eight distinct tetrachord genera in current Persian dastgāhi music, considering two different sizes of the neutral-tone interval.[102] In Hedāyat's view, there are seven types of tetrachords recognized by the theorists of the Systematist School, given that there is no differentiation between two sizes of neural-tone (mojannab) interval for the abbreviated letter used in those treatises (Hedāyat 1938, third section 23). Each of these seven genera, each has a descriptive title that addresses the type and order of the intervals it encompasses, and they are categorized into three different sections of so-called tense genera,[103] soft genera,[104] and a mixed genus[105] (see Figure 0-3 in the appendix section). I have collected all eight different types of tetrachord genera of the Persian dastgāhi music, with the titles by which they are known today, as well as their earlier titles in the treatises of Systematist School that Hedāyat names in the following Table 4-2, but first will introduce them individually. It is noticeable that traditionally, there is a supplementary section in early music theory treatises, after the introduction of the tetrachord genera, about another collection dedicated to the *pentachords*, by which the cycles of the octave are completed. I will not touch on the pentachord genera in the current research.

[102] Darioush Talaei declares this number to be four, probably after ignoring the difference between the two sizes of neutral-tone interval (Talaei 1993, 23).

[103] اجناس قوی

[104] اجناس لیّن

[105] جنس مختلط

Māhur (M-M-m)

There is, undoubtedly, no genus more understandable for Western listeners among the Persian tetrachord collection than Māhur, because of the familiar order and sizes of its intervals. Despite the different modal functioning of the tones, the main scale of dastgāh-e Māhur,[106] of which this genus is the core tetrachord, is similar to the Western major scale (Farhat 1965, 89). From low to high, a set of two whole tones and a semitone at the end forms this tetrachord genus (M-M-m). The equivalent of this genus in early music treatises of Systematist School is called *Ushshāq*,[107] with the tone arrangement of tanini-tanini-baqieh (T-T-b).[108] As demonstrated in the previous chapter, an equal-tempered version of this genus (T-T-1/2T) takes the first place in Fārābi's genera collection. Below, you can see this tetrachord genus on the bubble diagram, with the letters A and H as the framing tones of the tetrachord (Figure 4-1). This tetrachord genus is also the basis of another dastgāh called Rāst-Panjgāh[109] (ibid., 100).

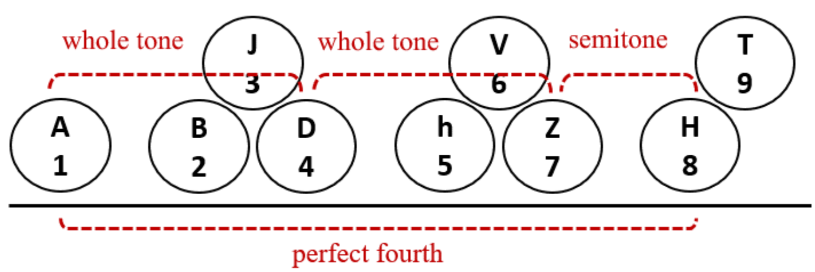

Figure 4-1: The arrangement of the stepwise intervals in the Māhur tetrachord genus (M-M-m).

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106 دستگاه ماهور

107 عشّاق

108 From right to left: طنینی–طنینی–بقیه (ط–ط–ب)

109 دستگاه راست‌پنجگاه —Do not confuse this with *maqām Rāst* in early Persian and current Arabic music, which has a neutral third interval.

Rāje' (m-M-M)

The genus of *Rāje'*[110] is usually seen as the upper disjunct tetrachord of the subordinate scales of at least two important dastgāhs, Shur and Homayun, and it is not as famous as the other main tetrachord genera in Persian dastgāh music that are named here. The arrangement of the intervals in this genus reverses the order of the Māhur genus, yielding (from low to high) a semitone followed by two whole tones (m-M-M), similar to the first tetrachord of the Western Phrygian mode. The equivalent of this genus in music treatises of Systematist School is called *Busalik,*[111] with the tone arrangement of baqieh-tanini-tanini (b-T-T).[112] Below, you can see the arrangement of the intervals in this tetrachord genus on the bubble diagram, again between the framing tone-letters A and H (Figure 4-2).

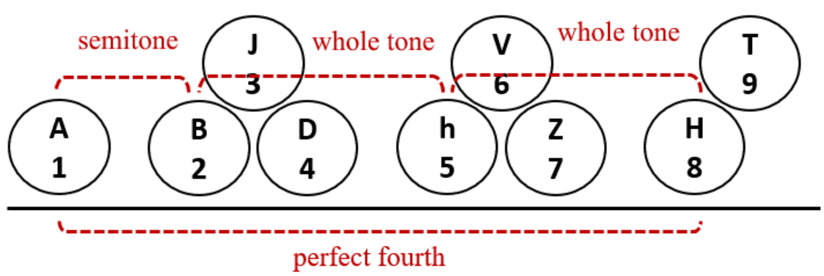

Figure 4-2:The arrangement of the stepwise intervals in the Rāje' tetrachord genus (m-M-M).

[110] راجع

[111] بوسليك

[112] From right to left: بقيه–طنينى–طنينى (ب–ط–ط)

Navā (M-m-M)

This genus, which is the lower tetrachord of the main scale of a mode called dastgāh-e Navā[113] (ibid., 81), consists of a whole tone followed by a semitone and another whole tone (M-m-M). This genus is traceable in the Systematist music theory treatises with the same title and this arrangement of the tones: tanini-baqieh-tanini (T-b-T).[114] Below, you can see the arrangement of the intervals in this genus, on the bubble diagram, between the framing tone-letters of the tetrachord, A and H (Figure 4-3).

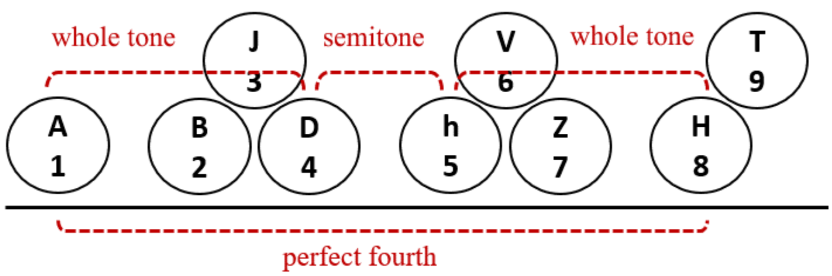

Figure 4-3: The arrangement of the stepwise intervals in the Navā tetrachord genus (M-m-M).

Shur (n-N-M)

With no exaggeration, Shur is the most fundamental and favorite tetrachord genus in Persian traditional music. This genus, from low to high consists of two consecutive neutral tones and a whole tone. The equivalent of this tetrachord genus in the Systematist music theory treatises

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113 دستگاه نوا

114 From right to left: طنینی–بقیه–طنینی (ط–ب–ط)

is called *Nowruz,*[115] in which the order of the intervals is mojannab-moJannab-tanini (j-J-T).[116] Again, I should remind here that there is no differentiation between smaller and larger neutral intervals for the abbreviated letter used in the Systematist School treatises, and both sizes are addressed with the general term of *mojannab* and abbreviated as j. Here, according to Farhat's terminology, I use the uppercase J to show the larger neutral tone (Farhat's N), and the lowercase j to show the small neutral tone (Farhat's n), allowing the more accurate description of the tones of this genus as n-N-M. In the following bubble diagram, you can see the arrangement of the intervals of the tetrachord that is extended between the tone-letters A and H (Figure 4-4).

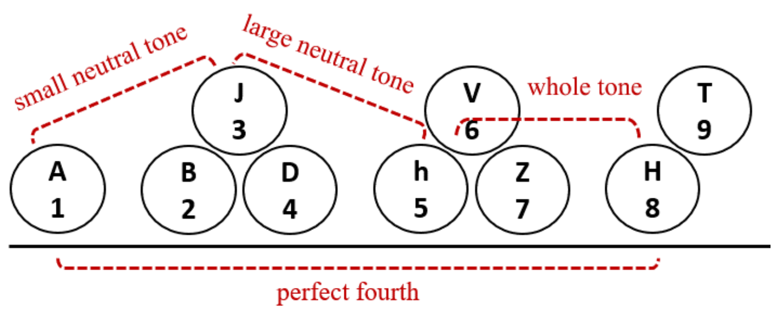

Figure 4-4: The arrangement of the stepwise intervals in the Shur tetrachord genus (n-N-M).

This genus is the core tetrachord of a widely extended collection of notes that forms the main scale of the most significant modal progression category, called dastgāh-e Shur[117] (Farhat 1965, 27), the so-called *mother of the dastgāhs* (Nettl 1992, 93-136). Almost similar to the definition of the modes of a scale in western music, the scale of Shur encompasses some other

115 نوروز

116 From right to left: مجنب کوچک-مجنب‌بزرگ-طنینی (جـ-جــ-ط)

117 دستگاه شور

modes with different finalis and distinct melodic functions, namely āvāz-e Abu-Atā, āvāz-e Bayāt-e Tork, āvāz-e Afshāri, and āvāz-e Dashti.[118] It seems that Farhat ignores this common categorization, and presents these modes as separate dastgāhs.

The place of the neutral interval on the second scale degree, and its tendency towards the finalis of the mode (given that the first neutral tone on the left is smaller than the second one), makes this mode loosely comparable to the Western Phrygian mode, which has a flat second scale degree. This tetrachord genus also appears frequently in different melodic figures of most of the seven dastgāhs of Persian music, sometimes in the main scale of the dastgāh as the counter-tetrachord to complete the octave, like in dastgāh-e Homāyun, or dastgāh-e Navā, and sometimes in the smaller melodic modes in each modal category.

Bayāt-e Tork (M-N-n)

If you consider an inverted order of the intervals in the Shur tetrachord genus and put the high-end tone as the finalis, the resulted genus will be called Bayāt-e Tork, which is the core tetrachord of the main scale of an āvāz with the same name[119] (Farhat 1965, 43). According to Farhat and some other theorists, the placement order of the neutral intervals is the same as in Shur scale (small below large). But, considering the undeniable similarity of this tetrachord to Ptolemy's Homalon (equal) genus —in which the *exceptionally gentle* (Barker 1990, 312) order of the

118 آواز ابوعطا، آواز بیات ترک، آواز افشاری، آواز دشتی

119 آواز بیات تُرک

intervals in ratios is 10:9-11:10-12:11[120]— I call for considering this arrangement of the notes as a flipped version of a Shur genus, in which the order of the neutral tones is *inverted*. Below, the place and order of the tones of the genus of Bayāt-e Tork is visible (Figure 4-5), with its starting note being the tone-letter A and the finalis being H.

Using Farhat's symbols, this arrangement could be abbreviated with the arrangement of the letters M-N-n. The equivalent of this tetrachord in the treatises of Systematist School, having the interval order of tanini-moJannab-mojannab (T-J-j) [121] is called *Rāst*.[122] In some other texts, this tetrachord is addressed also with the title of *Rohāb*[123] (Zeyn-al-din 2009). The significance of this mode is such that Fārābi puts it, as was shown in the previous chapter, in a quartertone tempered version (T-3/4T-3/4T) as the second of his collection of tetrachord genera.

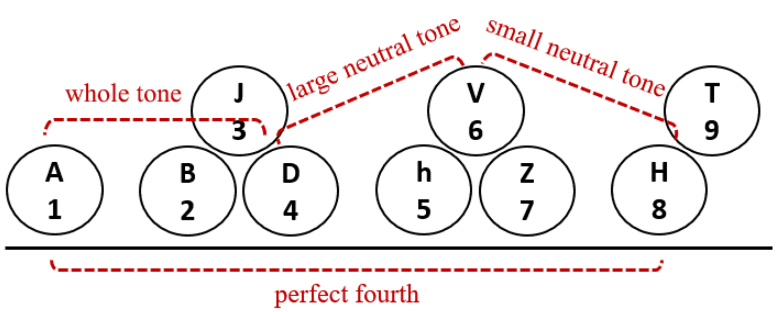

Figure 4-5: The arrangement of the stepwise intervals in the Bayāt-e Tork tetrachord genus (M-N-n).

[120] Of course, by putting the sizes of these ratios in cents, we will get different values (182.4 – 165 - 150 cents) than the approximated sizes that Farhat calls for (204 – 160 - 135 cents). But, if you consider the large-to-small order of the intervals in both tetrachords, and if you listen to the audible version of them, it is largely acceptable to consider them as approximately identical!

[121] From right to left: طنینی–مجنب‌بزرگ–مجنب کوچك (ط–جـــ–جـ)

[122] راست

[123] رهاب

Segāh (N-M–n)

Perhaps It can be said that Segāh[124] has the strangest tone arrangement and the most unique sound among the other tetrachord genera. Theoretically, the fundamental scale of the dastgāh-e Segāh[125] (Farhat 1965, 51), of which this genus is the core tetrachord, is taken as equal to the scale of dastgāh-e Shur. The difference is that the finalis of the mode, as well as the whole tetrachord genus with the same order of the tones, is *shifted up to a neutral third* above the finalis of the Shur. I have shown the Segāh tetrachord genus on the bubble diagram below with the assumption that the finals is the tone-letter V (Figure 4-6). Starting from this note, the order of the stepwise intervals in this genus will be large neutral tone – whole tone – small neutral tone (N-M-n). The equivalent of this genus in the Systematist treatises, although without any differentiation between large and small mojannab intervals as mentioned earlier, is called *Irāq*,[126] in which the arrangement of the intervals is declared to be moJannab-tanini-mojannab (J-T-j).[127]

A direct leap from the tone-letter A to the finalis of the genus (tone-letter V) is a very common in the melodic progression of this dastgāh. The value of the neutral third interval (A-V) is, according to a few narratives, different from a summation of a neutral tone and a whole tone, which makes the scale of this dastgāh different from that of Shur. The size of the neutral third interval seems to be more correspondent to the harmonic ratio of 11:9 (347.41 cents) or Zalzal's

[124] The word *Segāh* is made of two Persian words of *Se* (meaning the number three) and *Gāh* (mening the place), which implicitly addresses the third fret or the position of the third finger on a string instrument. A transformed version of this word is also found, mostly as the title of the mode *Sikah*, in Arabic and Turkish treatises.

[125] دستگاه سه گاه

[126] عراق

[127] From right to left: بزرگ (جـ–ط–جـــ) کوچك–طنینی–مجنب‌مجنب

middle fret with the ratio of 27:22 (354.55 cents), which is places roughly in the midway between the major and minor thirds.

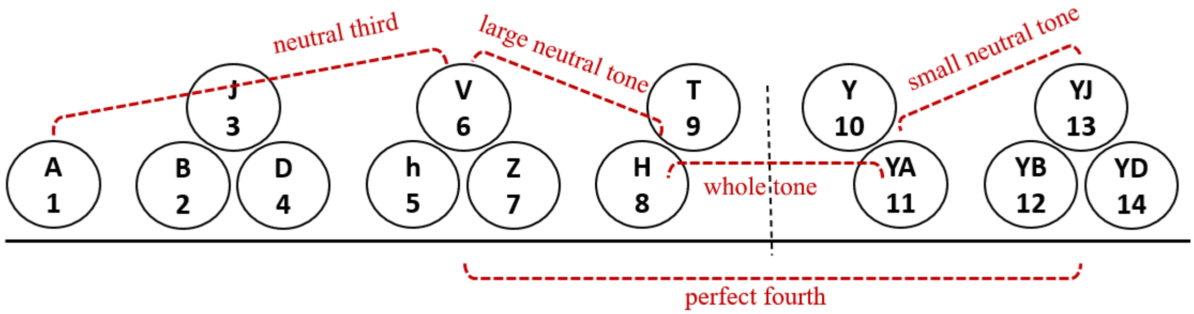

Figure 4-6: The arrangement of the stepwise intervals in Segāh tetrachord genus (N-M-n).

Bayāt-e Esfahān (n-M-N)

Similar to the harmonic minor scale, but with an undeniable accentuation on the sixth scale degree with its neutral quality —this is a decent way to describe the main scale of a Persian traditional mode called āvāz-e Bayāt-e Esfahān[128] (Farhat 1965, 76). However, this is the delineation of the newer version of this mode, after the influence of western classical music standard, which makes it categorizable under the scale of another dastgāh called *Homayun*. Musicologists at the same time talk about an older version of this mode, with a same title, consisting of, from low to high, a small neutral tone followed by a whole tone and a large neutral tone (n-M-N). Considering the place of the smaller and larger neutral tones and the whole tone in between, this tetrachord could be analyzed as an inverted version of the *Segāh* genus (N-M-n). Given that there is no segregation between the smaller and larger mojannab in early texts, the

[128] آواز بیات اصفهان

previously-mentioned *Irāq* genus could be considered as a Systematist equivalent of this arrangement of the intervals, moJannab-tanini-mojannab (J-T-j).[129]

This tetrachord genus is also used in a famous melodic figure under the dastgāh-e Segāh, called gusheh-ie Mokhālef[130] (ibid., 54). According to the order of the appearance of the melodic figure in dastgāh-e Segāh, Mokhālef (n-M-N) is placed as the secondary conjunct tetrachord, which is shown on the bubble diagram below, with the tone-letter YA as the lower end tone and the YH as the higher end of the tetrachord and also the finalis of the mode (Figure 4-7).

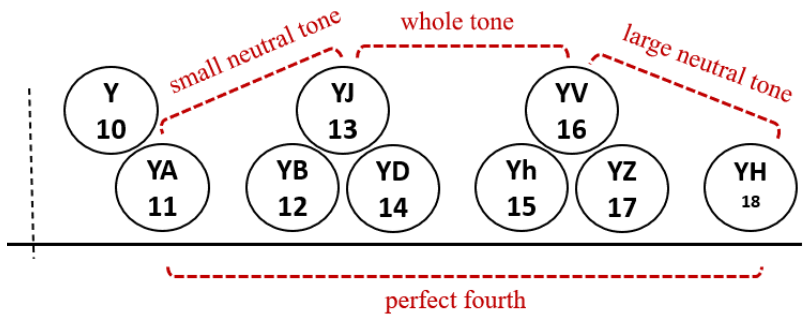

Figure 4-7:The arrangement of the stepwise intervals in the Bayāt-e Esfahān tetrachord genus (n-M-N).

Chahārgāh (n-P-m)

The tetrachord of *Chahārgāh,*[131] with its distinctly characteristic plus tone, exists in the main scale of at least two dastgāhs. The scale of dastgāh-e Chahārgāh[132] (ibid., 10) consists of the

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129 From right to left: مجنب‌بزرگ–طنینی–مجنب کوچك (جـــ–ط–جـ)

130 گوشه مُخالف سه گاه

131 The word *Chahārgāh* is made of two Persian words of *Chahār* (meaning the number four) and *Gāh* (mening the place), which implicitly addresses the fourth fret or the position of the fourth finger on a string instrument. A transformed version of this word is also found, mostly as the title of the mode *Jiharkah* in Arabic, and *Çârgâh* in Turkish treatises.

132 دستگاه چهارگاه

conjunct combination of two of the same genus, and this tetrachord also appears in the main scale of another scale called dastgāh-e Homāyun (ibid., 65),[133] which normally encompasses a conjunct combination of the genera of Chahārgāh and Shur (n-N-M). The genus of Chahārgāh, from the low end, consists of a small neutral tone, followed by a plus-tone (augmented second) and a semitone at the high end (n-P-m). It seems that the Systematist equivalent of this genus only exists in Qutb-al-Din Shirāzi's treatise, Dorrat-al Tāj,[134] with the title of *Hijāz,*[135] (see Figure 0-4 in the appendix section) in which the order of the intervals is mojannab-taninie mostazād-baqieh (j-h-b).[136] The arrangement of these intervals is shown in the first half of the following bubble diagram, with the tone-letter A as the finalis (Figure 4-8).

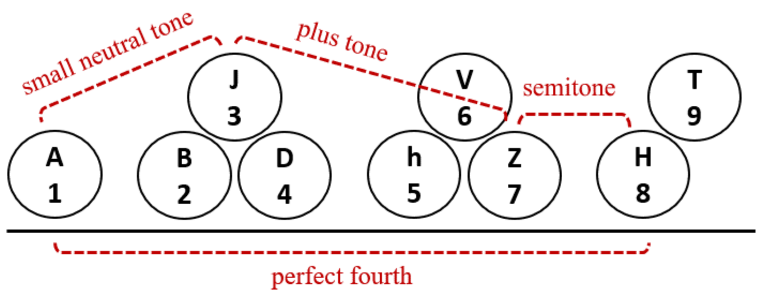

Figure 4-8: The arrangement of the stepwise intervals in Chahārgāh tetrachord genus (n-P-m).

This concludes my survey of the eight distinct tetrachord genera of Persian dastgāhi music and their details, summarized in the following table (Table 4-2). It should be mentioned that this

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133 دستگاه همایون

134 He seems to be the only Systematist theorist who talks about the stepwise interval of augmented second, which he calls tanini-e mostazād (طنینی مُستزاد) and abbreviates with the letter h (هـ). This is the same interval that Farhat labels it as plus-tone (P).

135 حجاز

136 From right to left: مجنب کوچک–طنینی مستزاد–بقیه (جـ–هـ–ب)

theoretical categorization of tetrachord genera has nothing to do with the modal classification of the seven dastgāhs of Persian music, which is based instead on the similarities of the melodic figures and their modal progression in the context of the Radif of Persian music.

Genus's title today's common terminology	Intervals (Farhat)	Abbreviations (Farhat)	Approximated values in cents (Farhat)	Genus' category (Systematist according to Hedāyat)	Genus's title (Systematist)	Intervals Systematist according to Hedāyat	Abbreviations (systematist)	Genus' tones in Abjadic letters (Systematist)
Māhur (ماهور)	whole tone-whole tone-semitone	M-M-m	204 204 90	tense genera (اجناس قوی)	Ushshāq (عشاق)	tanini-tanini-baqieh	T-T-b (ط–ط–ب)	A-D-Z-H (الف–د–ز–ح)
Rāje' (راجع)	semitone-whole tone-whole tone	m-M-M	90 204 204		Busalik (بوسليك)	baqieh-tanini-tanini	b-T-T (ب–ط–ط)	A-B-h-H (الف–ب–هـ–ح)
Navā (نَوا)	whole tone-semitone-whole tone	M-m-M	204 90 204		Navā (نوا)	tanini-baqieh-tanini	T-b-T (ط–ب–ط)	A-D-h-H (الف–د–هـ–ح)
Shur (شور)	small neutral tone-large neutral tone-whole tone	n-N-M	135 160 204	Soft Genera (اجناس لیّن)	Nowruz (نوروز)	mojannab-moJannab-tanini	j-J-T (جـ–جـــ–ط)	A-J-h-H (الف–جـ–هـ–ح)
Bayāt-e Tork (بیات تُرك)	whole tone-large neutral tone-small neutral tone	M-N-n *	204 160 135		Rāst (راست)	tanini-moJannab-mojannab	T-J-j (ط–جـــ–جـ)	A-D-V-H (الف–د–و–ح)
Segāh (سه گاه)	large neutral tone-whole tone-small neutral tone	N-M-n	160 204 135		Irāq (عراق)	moJannab-tanini-mojannab	J-T-j (جـــ–ط–جـ)	A-J-V-H (الف–جـ–و–ح) **
Bayāt-e Esfahān (بیات اصفهان)	small neutral tone-whole Tone-large neutral tone	n-M-N	135 204 160			mojannab-tanini-moJannab	j-T-J (جـ–ط–جـــ)	A-J-V-H (الف–جـ–و–ح) **
Chahārgāh (چهارگاه)	small neutral tone-plus tone-semitone	n-P-m	135 270 90	mixed genus (جنس مختلط)	Hijāz (حجاز)	mojannab-taninie mostazād-baqieh	j-h-b (جـ–هـ–ب)	A-J-Z-H (الف–جـ–ز–ح)

Table 4-2: The collection of different distinct tetrachord genera in Persian dastgāh music.

* The order of the neutral intervals that I used for this tetrachord genus is inverted compared to Farhat's suggestion
** All the Abjadic tone-letters are transposed to be starting with an A

As mentioned earlier, the rendering of Persian and Arabic intervals in the twenty-four-tone equal temperament has always been criticized by traditional musicians and string instrument players. But the fact is that for instruments with fixed pitches, such as santur[137] or piano, in practice, to avoid repeated tunings for different melodic modes, the quartertone approximations of neutral intervals is mostly acceptable to many. As a personal example, myself, as a quartertone accordion player, for the maximum capability of performing different melodic modes of Persian music on an instrument that can be tuned *only once*, there is no choice but to accept the quartertone values.

This brings us to think about the equal-tempered values of the tetrachord genera in a 24-ET system. It goes without saying that the size of the whole tone and semitone will be rounded to 100 and 200 cents respectively, and the sizes of the both small and large neutral tones will be rounded to a single size of 150 cents, which is exactly correspondent o the ratio of 12:11. For the plus tone, the closest 24-ET interval will be the size of 250 cents. If you compare the size of the intervals in the Table 4-3 below with the table of Fārābi's and Aristoxenus' tetrachord genera in the previous section (Table 3-2), you can find some similarities. Except for the Māhur and Bayāt-e Tork genera which are exactly Fārābi's first and second tetrachords, you can find the sizes of 3/4 -tone (150 cents) and 5/4-tone (250 cents) in both tables. One thing to notice is that because of the rounding of the neutral tones to one size, practically the tetrachord genera of Segāh and Bayāt-e Esfahān become identical.

137 سنتور

Genus's title (today's common terminology)	24-ET-rendered size of the intervals of the genus (T = tone)	24-ET-rendered size of the intervals in cents (perfect fourth = 500)	Abbreviations (systematist School)
Māhur (ماهور)	T T 1/2T	200 200 100	T-T-b (ط–ط–ب)
Rāje' (راجع)	1/2T T T	100 200 200	b-T-T (ب–ط–ط)
Navā (نَوا)	T 1/2T T	200 100 200	T-b-T (ط–ب–ط)
Shur (شور)	3/4T 3/4T T	150 150 200	j-j-T (جـ–جـ–ط)
Bayāt-e Tork (بیات تُرک)	T 3/4T 3/4T	200 150 150	T-j-j (ط–جـ–جـ)
Segāh (سه گاه)	3/4T T 3/4T	150 200 150	j-T-j (جـ–ط–جـ)
Bayāt-e Esfahān (بیات اصفهان)	3/4T T 3/4T	150 200 150	j-T-j (جـ–ط–جـ)
Chahārgāh (چهارگاه)	3/4T 5/4T 1/2T	150 250 100	j-h-b (جـ–هـ–ب)

Table 4-3: The rendered sizes of the tetrachord genera intervals in a twenty-four-tone equal temperament (24-ET).

Persian scales as stacks of tetrachord genera

A quick glance on the above table reveals that three of these tetrachord genera are the inverted version of each other, in the sense that, generally speaking, both order of the arrangement of the intervals and the place of the finalis tone is inverted. More specifically, the genus of Rāje' (m-M-M) is the inverted version of Māhur (M-M-m), the genus of Bayāt-e Tork (M-N-n) is the inverted version of Shur (n-N-M), and the genu“ of Bayāt-e Esfahān (n-M-N) is the inverted version of the genus of Segāh (N-M-n). For the remaining two, obviously the genus of Navā (M-m-M) could not have an inverted version, because of the cemetery of the intervals in it. And the inverted version of the genus of Chahārgāh (n-P-m) is not considered pleasant and is not found among the melodic figures.

Among different early to modern elaborations on the Persian dastgāh system and the complex collection of modal figures in it, I found the notion of analyzing each modal category as the stack of two or more tetrachords very convincing and applicable. In this view, which has roots in Ancient Greek music theory, each distinct main scale consists of either a conjunct or disjunct combination of at least two same or different tetrachord genera, which are arranged around a modal center tone, which in early Greek music texts would be called *mese* (see Barker 1990, 11-18). If we consider the finalis in Persian dastgāh music as the equivalent of mese, there are still some other important tones for each modal category, like *shahed*, *ist*, *moteqayyer*, etc.[138] (Farhat 1965, 19-26), that I ignore here explaining to simplify the subject.

Although the modal progression in each category could sometimes be deviated from the main scale and modulate to other modal categories, there is always a return progression back to the main scale. In some cases, one of the core tetrachord genera of the scale stays the same while the other changes depending on the direction and the quality of the progression. Sometimes, changing the quality of the connection between the two tetrachords from conjunct to disjunct produces a different scale. In the following images (Figures 4-9 to 4-17), you can see and compare different main scales of the Persian dastgah system on the staff, with the constituent tetrachords and the quality of their combination indicated.

For the core tetrachord genus of each main scale, the intervals of the tetrachord are highlighted above the note collection. The abbreviated letters of the intervals, in Farhat's notation, is shown under each four-note collection of the tetrachord genera. For some of the scales, there are multiple versions of the connections of the tetrachords, which are demonstrated in different staffs.

[138] شاهد، ایست، متغیّر، و غیره

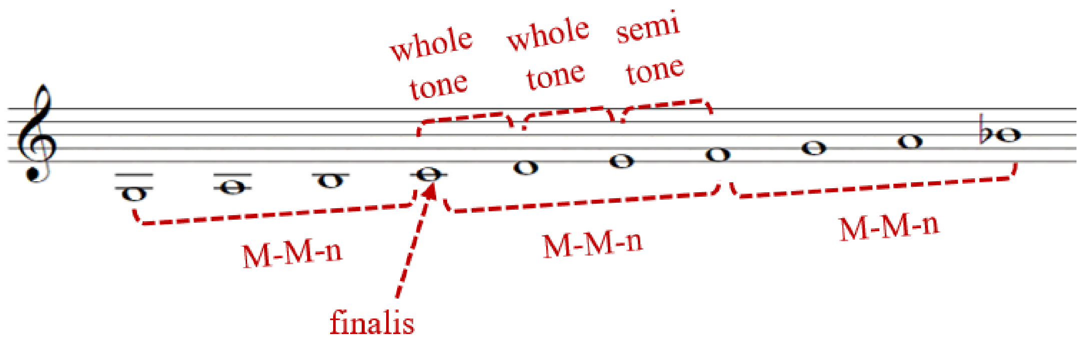


Figure 4-9: The main scale of dastgāh-e Māhur.
The center of the mode (finalis) is considered to be a C (Farhat 1965,89 and 100).

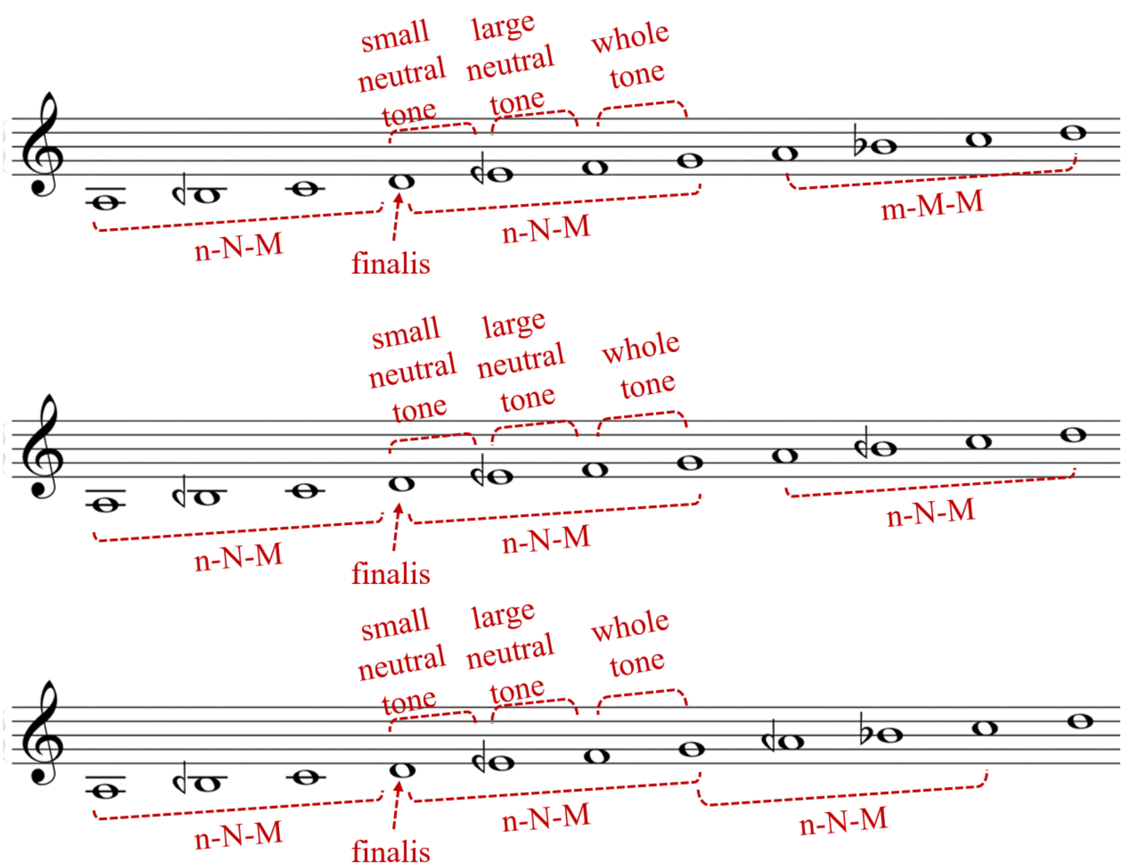


Figure 4-10: Three different versions of the main scale of dastgah-e Shur.
The center of the mode (finalis) is considered to be a D.

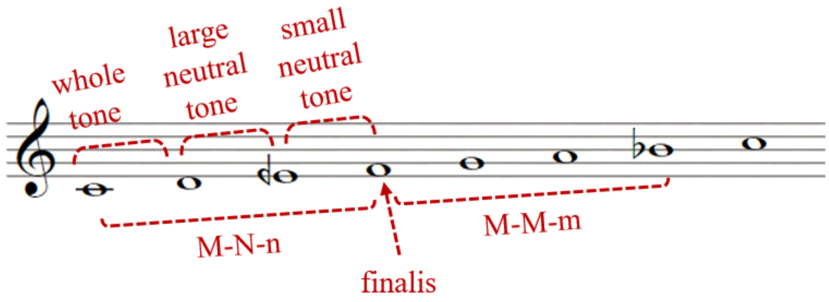


Figure 4-11: The main scale of āvāz-e Bayāt-e Tork.
The center of the mode (finalis) is considered to be an F.

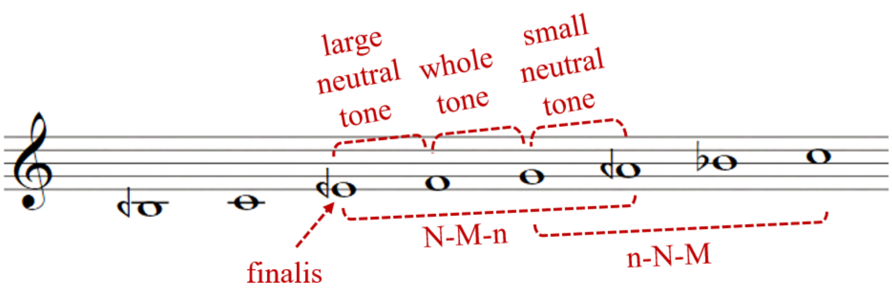


Figure 4-12: The main scale of dastgāh-e Segāh.
The center of the mode (finalis) is considered to be an E-quarter-flat.

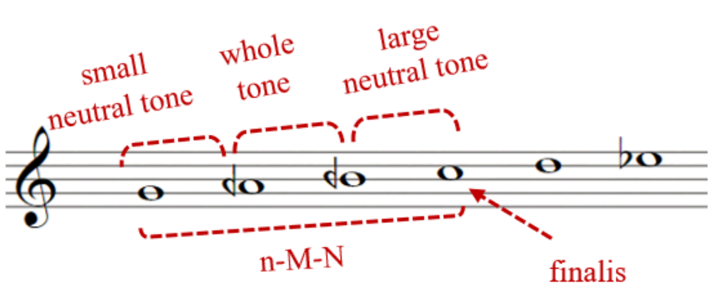


Figure 4-13: The main scale of gusheh-ie Mokhālef-e Segāh.
The center of the mode (finalis) is considered to be a C.

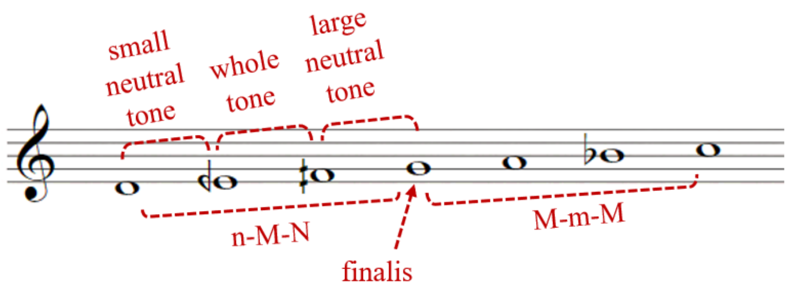


Figure 4-14: The main scale of āvāz-e Bayāt-e Esfahān.
The center of the mode (finalis) is considered to be a G.

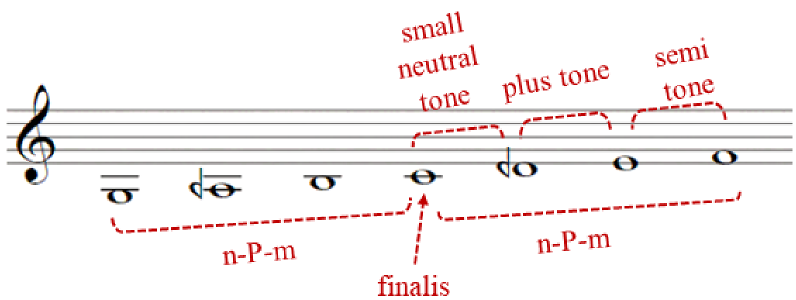


Figure 4-15: The main scale of dastgāh-e Chahārgāh.
The center of the mode (finalis) is considered to be a C.

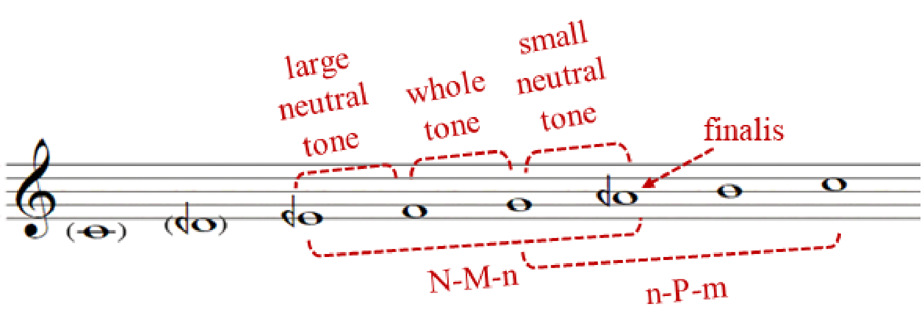


Figure 4-16: The main scale of gusheh-ie Mokhālef-e Chahārgāh.
The center of the mode (finalis) is considered to be an A-quarter-flat.

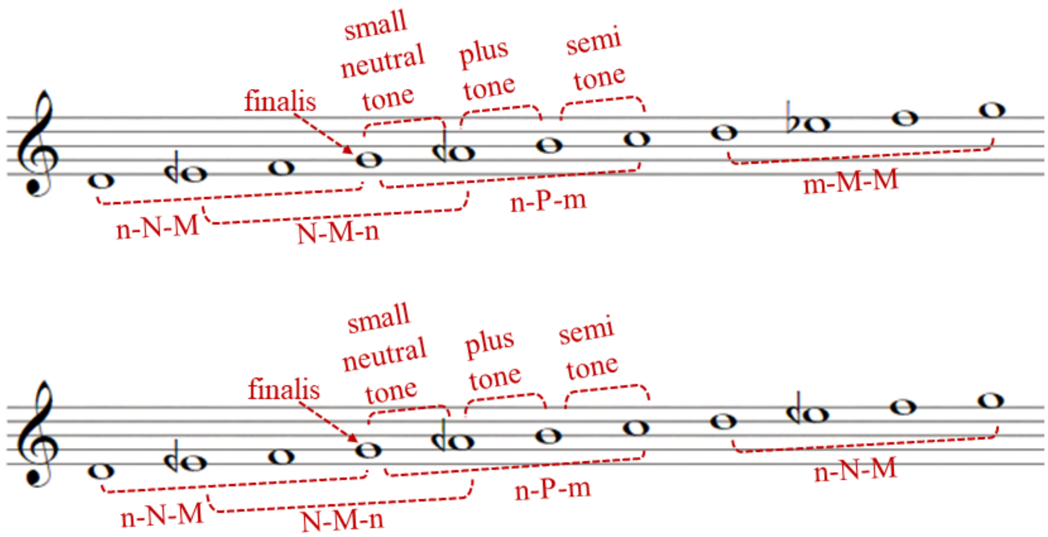


Figure 4-17: Two different versions of the main scale of dastgāh-e Homāyun.
The center of the mode (finalis) is considered to be a G.

(The tetrachord of N-M-n is indicated on the staff because it is a very common melodic figure in this dastgāh)

Summary and conclusion

The theory of Persian dastgāhi music, specifically focusing on the tetrachord genera of the main scales and stepwise intervals, has been largely overlooked in recent times. With the exception of Mehdi-Qoli Hedāyat's *Majma'-al Adwār*, there are no other contemporary music treatises that delve into the traditional Persian music theory in the manner of the thirteenth-century Systematist school, through the explanation of the Persian-Arabic seventeen-note temperament and Abjadic tone-letters. In this thesis, I sought to delve into the historical roots of the stepwise intervals of Persian dastgāhi music, following the definitions of Hormoz Farhat. I based my exploration on the ratios of different prime number limits found in the natural harmonic series. This endeavor was underpinned by the assumption that these intervals inherently encompass a degree of fluctuation. Subsequently, I redefined the main tetrachord genera using these ratios. According to Farhat, in addition to semitone (m) and whole tone (M), three more stepwise intervals are found in Persian music, namely large neutral tone (N), small neutral tone (n), and plus tone (P), with which eight different pleasant tetrachord genera with distinct characteristics are recognizable. I demonstrated these intervals and the tetrachord genera on my seventeen-tone bubble diagram.

Several notable findings emerged from this study. Firstly, exploring the mentioned intervals through a single tuning system may not yield satisfactory results. Instead, I propose considering a collection of ratios from different prime limits of the harmonic series that align closely with the values of the stepwise intervals. In addition to the Pythagorean-oriented ratios of 9:8 ($\simeq$ 204 cents) for whole tone and 256:243 ($\simeq$ 90 cents) for semitone, I found the simple epimoric ratios of 11:10 (= 165 cents), 13:12 ($\simeq$ 139 cents), and 7:6 ($\simeq$ 267 cents), closely approximating the values of the large neutral tone, the small neutral tone, and the plus tone respectively. Moreover, studying the possibility of re-defining the *major third* interval with the just intonation ratio of 5:4 (≃ 386 cents), as a practically preferred value over the theoretical size of the Pythagorean ditone (≃ 408 cents), could be an interesting idea for further studies.

Furthermore, I elaborated on the rounded values of the Persian dastgāhi intervals in twenty-four-tone equal temperament (24-ET), tracing its origins back to Fārābi's approximations in his treatise, *Great Book of Music*, which were rejected by most of the practitioners. However, I showed that acceptable approximations for these intervals, as highlighted by Helmholtz and subsequently re-proposed by Hedāyat, can be achieved in a fifty-three-tone equal temperament (53-ET). This system is founded on a nine-fold division of a whole tone, resulting in a comma of approximately 22.6 cents in size. Notably, the stepwise intervals can be expressed as four commas (≃ 90 cents) for the semitone, six commas (≃ 135 cents) for the small neutral tone, seven commas (≃ 160 cents) for the large neutral tone, nine commas (≃ 204 cents) for the whole tone, and twelve commas (≃ 270 cents) for the plus tone —aligning seamlessly with Farhat's approximations.

Taking the next step in my research, I propose an exploration of the collection of *pentachord genera* using the same method applied in my thesis to tetrachord genera. The treatises of the Systematist School indicate the existence of thirteen pleasant pentachord genera. However, I believe there could be even more, considering the greater number of permutations resulting from the two sizes of the neutral tone and other stepwise intervals. Furthermore, Hedāyat's treatise offered a wealth of intriguing ideas, such as the application of Riemannian theory of harmonic function and related triads through seventeen Abjadic tone-letters, including neutral triads in the system. These topics as well as the governmental explicit or implicit suppression of the neutral intervals in some countries hold significant potential for future research endeavors.

Appendix

	Abjadic letter	pronunciation	transliteration	numeral value
1	ا	ʾalif	A	1
2	ب	bāʾ	B	2
3	ج	jīm	J	3
4	د	dāl	D	4
5	ه	hāʾ	h	5
6	و	wāw	V	6
7	ز	zāy/zayn	Z	7
8	ح	ḥāʾ	H	8
9	ط	ṭāʾ	T	9
10	ی	yāʾ	Y	10
11	ك	kāf	k	20
12	ل	lām	l	30
13	م	mīm	m	40
14	ن	nūn	n	50
15	س	sīn	s	60
16	ع	ʿayn	ʿ	70
17	ف	fāʾ	f	80
18	ص	ṣād	ṣ	90
19	ق	qāf	q	100
20	ر	rāʾ	r	200
21	ش	shīn	sh	300
22	ت	tāʾ	t	400
23	ث	thāʾ	th	500
24	خ	khāʾ	kh	600
25	ذ	dhāl	dh	700
26	ض	ḍād	ḍ	800
27	ظ	ẓāʾ	ẓ	900
28	غ	ghayn	gh	1000

Table 0-1: Table of 28 Persian-Arabic Abjadic letters and their numeral values.

(۱۶)

نوبت سوم — جدول ابعاد بتقدیر صاحب ادوار — در ابعاد

نغمات	اجزای وتر	نسبت کسری	نسبت اعشاری	مقدار [illegible]	میلیمتر
ا	ا م	۱/۱	۱٫۰۰۰۰	۱٫۰۰۰۰۰	۶۰۰
ب	۹/۸ وم	۲۴۳/۲۵۶	۰٫۹۴۹۲	۱٫۰۵۴۵	۵۶۹٫۵
ج	۹/۸ وم	۱۲/۱۳	۰٫۹۲۳۱	۱٫۰۸۳۳	۵۵۳٫۹
د	۸/۹ آم	۸/۹	۰٫۸۸۸۸	۱٫۱۲۵۰	۵۳۳٫۳
ه	۹/۸ حم	۲۷/۳۲	۰٫۸۴۳۷	۱٫۱۸۵۱	۵۰۶٫۳
و	۱۲/۱۳ دم	۳۲/۳۹	[illegible]	۱٫۲۱۸۸	۴۹۲٫۳۴
ز	۸/۹ دم	۶۴/۸۱	۰٫۷۹۰۱	۱٫۲۶۵۶	۴۷۴٫۱
ح	۳/۴ آم	۳/۴	۰٫۷۵۰۰	۱٫۳۳۳۳	۴۵۰٫۰
ط	۳/۴ بم	۷۲۹/۱۰۲۴	۰٫۷۱۱۹	۱٫۴۰۴۶	۴۲۷٫۱
ی	۳/۴ جم	۹/۱۳	۰٫۶۹۲۳	۱٫۴۴۴۴	۴۱۵٫۴
یا	۲/۳ آم	۲/۳	۰٫۶۶۶۶	۱٫۵۰۰۰	۴۰۰
یب	۲/۳ بم	۸۱/۱۲۸	۰٫۶۳۲۸	۱٫۵۸۰۲	۳۷۹٫۶
یج	۳/۴ دم	۸/۱۳	۰٫۶۱۵۴	۱٫۶۲۵۰	۳۶۹٫۳
ید	۲/۳ دم	۱۶/۲۷	۰٫۵۹۲۶	۱٫۶۸۷۵	۳۵۵٫۵
یه	۳/۴ حم	۹/۱۶	۰٫۵۶۲۵	۱٫۷۷۷۷	۳۳۷٫۵
یو	۳/۴ طم	۲۱۸۷/۴۰۹۶	۰٫۵۳۳۹	۱٫۸۷۲۹	۳۲۰٫۳
یز	۳/۴ یم	۲۷/۵۲	۰٫۵۱۹۲	۱٫۹۲۵۲	۳۱۱٫۵
یح	۱/۲ آم	۱/۲	۰٫۵۰۰۰	۲٫۰۰۰۰	۳۰۰٫۰

[illegible]

Figure 0-1: Table of the pitch values of the seventeen tones for theorists of the Systematist School according to Mehdi-Qoli Hedāyat (Hedāyat, 1938, third section, 16).

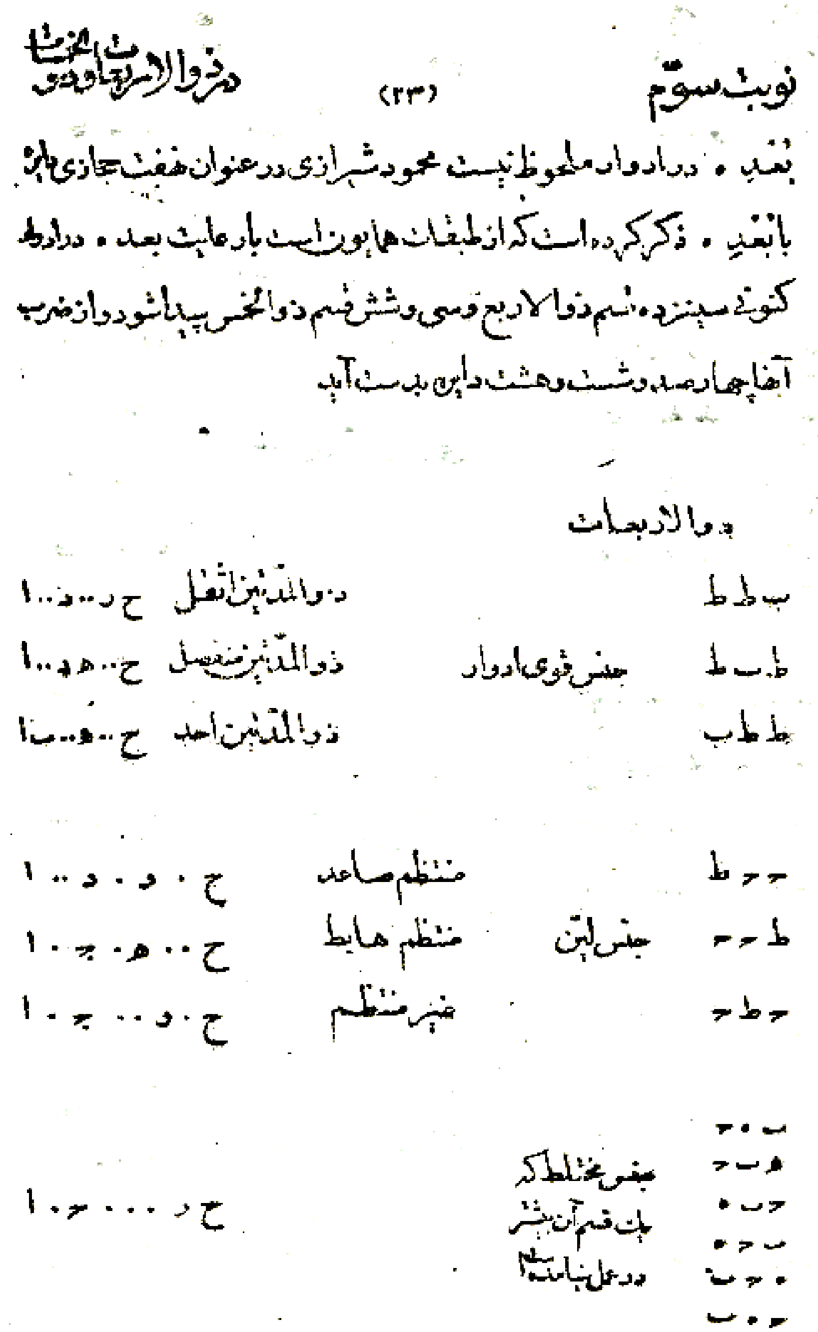
(۴)

نوبت سوم جدول ابعاد فعلی بتشخیص دکتر مهدی خان صلحی — دوازده ام

نغمات	نسبت کوک	نسبت طولی	نسبت اهتزاز	وتر به ملیمتر
ا	۱/۱	۱٫۰۰۰۰	۱٫۰۰۰۰	۶۰۰
ب	۲۴/۲۵	۰٫۹۶۰۰	۱٫۰۴۱۴	۵۷۶
ج	۲۵/۲۷	۰٫۹۲۵۰	۱٫۰۸۰	۵۵۵
د	۸/۹	۰٫۸۸۸۸	۱٫۱۲۵۰	۵۳۳٫۳
ه	۲۷/۳۲	۰٫۸۴۴۱	۱٫۱۸۵۱	۵۰۶٫۵
و	۲۰۰/۲۴۳	۰٫۸۲۰۸	۱٫۲۱۵۰	۴۹۲٫۵
ز	۶۴/۸۱	۰٫۷۹۰۰	۱٫۲۶۵۶	۴۷۴
ح	۳/۴	۰٫۷۵۰۰	۱٫۳۳۳۳	۴۵۰
ط	۱۸/۲۵	۰٫۷۲۰۰	۱٫۳۸۸۸	۴۳۲
ی	۲۵/۳۶	۰٫۶۹۴۳	۱٫۴۴۰۰	۴۱۶٫۵
یا	۲/۳	۰٫۶۶۶۶	۱٫۵۰۰۰	۴۰۰
یب	۱۶/۲۵	۰٫۶۴۰۰	۱٫۵۶۲۵	۳۸۴
یج	۵۰/۸۱	۰٫۶۱۷۱	۱٫۶۲۰۰	۳۷۰٫۳
ید	۱۶/۲۷	۰٫۵۹۲۵	۱٫۶۹۳۷	۳۵۵٫۵
یه	۹/۱۶	۰٫۵۶۲۵	۱٫۷۷۷۷	۳۳۷
یو	۲۷/۵۰	۰٫۵۴۰۱	۱٫۸۵۱۸	۳۲۴
یز	۲۵/۴۸	۰٫۵۲۰۸	۱٫۹۲۰۰	۳۱۲٫۵
یح	۱/۲	۰٫۵۰۰۰	۲٫۰۰۰۰	۳۰۰

[illegible]

Figure 0-2: Table of the pitch values of the seventeen tones in the Persian contemporary system, Dr. Mehdi Solhi 's suggestion (Hedāyat, 1938, third section 20).



نوبت سوم (۲۳) در ذوات الاربع و ذوات الخمس

بُعد • در ادوار ملحوظ نیست محمود شیرازی در عنوان هفت مجازی بالبعد • ذکر کرده است که از طبقات ها بوزن است با رعایت بعد • در ادوار کنونی سیزده قسم ذوالاربع و سی و شش قسم ذوالخمس پیدا شود و از ضرب آنها چهارصد و شصت و هشت دایره بدست آید

ذوالاربعات

ب ط ط		ذوالمدتین اثقل	ح ر .. د .. ا
ط ب ط	جنس قوی ادوار	ذوالمدتین منفصل	ح .. هـ و .. ا
ط ط ب		ذوالمدتین احد	ح .. و .. ب ا
ج ج ط		منتظم صاعد	ح • و • د .. ا
ط ج ج	جنس لین	منتظم هابط	ح .. هـ • ج • ا
ج ط ج		غیر منتظم	ح • د .. ج • ا
ب • ج			
هـ ب ج	جنس مختلط که		
ج ب •	یک قسم آن بیشتر		ح ر ... ج • ا
ب ج •	در عمل نیامده است		
• ج ب			
ج • ب			

Figure 0-3: The collection of tetrachord genera in the Systematist School treatises according to Mehdi-Qoli Hedāyat (Hedāyat, 1938, third section, 23).

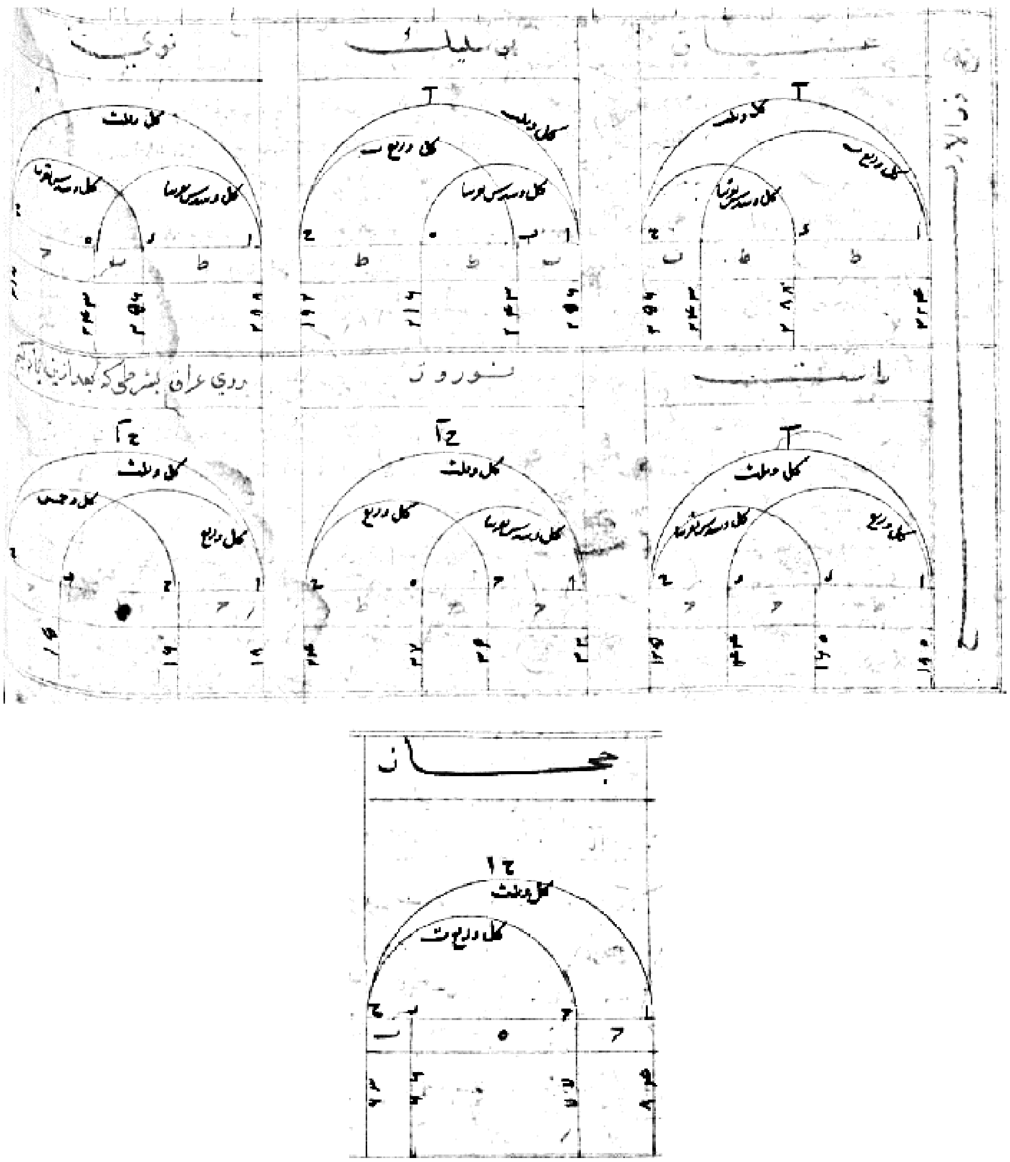


Figure 0-4: The collection of tetrachord genera in Qutb-al-Din Shirāzi's treatise, Dorrat-al Tāj (MS 1043, Malek library, Tehran).

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