Structural Engineering

Y I E L D L I N E S,
a Numerical Yield Line Analysis Program

USER'S MANUAL

by Dominique Bauer

version 1.0
November 1986

Structural Engineering Series Report No. 86-3

Department of Civil Engineering
and Applied Mechanics

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à Max et Geneviève,
Philippe et Johanne,
et Micheline,
que j'aime beaucoup

Abstract

A numerical method based on the virtual work method of the yield line theory is described, and a program is presented which uses this method to analyse plate structures and which can treat features such as orthotropy and skewness, point loads, line loads, uniformly distributed loads, fans, etc. The program also includes procedures for the optimization of the yield line mechanisms.

Since the method presented is entirely numerical, it allows the yield line analysis of plates with complex shapes and complex loading patterns, for any assumed mechanism.

Organization of the Manual

Chapter 1 presents the theory and the algorithms on which is based Yield Lines, a Numerical Yield Line Analysis Program (YL, for short).

Chapter 2 discussed in detail the input data, which must be carefully prepared by the user.

Chapter 3 describes the main features of the program. Screen displays and error messages are explained. Also, system requirements and the Yield Lines diskette are described.

Appendix A presents several examples of yield line problems with the required input data and the numerical solution using Yield Lines.

Acknowledgements

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Introduction

The yield line method is a simple and efficient method to calculate the plastic collapse load of flat, relatively thin, plates of rigid-perfectly plastic material when transversely loaded in bending. The method was developed largely by Johansen [1] and since then, it has been applied successfully to both concrete and steel plates [2,3,4].

A numerical method based on the yield line theory has recently been developed [5]. The method differs from the conventional yield line method in that it does not use a direct algebraic description of the problem but rather it uses analytical geometry, vector algebra, and the specific dimensions of the problem on hand to arrive at the solution.

The method presented is general and since it is entirely numerical, it can be applied to plates of arbitrary shape which can be assumed to form any arbitrary yield line mechanism. When implemented on computer, the method has the further advantage of requiring no algebraic manipulations and thus it is not limited by the complexity of the yield line pattern and by the resulting complexity of the algebra, as is sometimes the case with the conventional yield line method.

While straightforward and tractable by hand, the calculations involved in the numerical solution are lengthy and computer implementation is necessary for practical use. Yield Lines, a Numerical Yield Line Analysis Program has been written based on the numerical method. Yield Lines (YL, for short) can analyse simple yield line problems, as well as complex ones.

Chapter 1 Theory

The theory and the algorithms on which the computer program Yield Lines is based are presented in this chapter.

Yield Line Theory

The yield line theory is briefly reviewed below. However, a basic understanding of the yield line theory is assumed in the following discussion and the reader is referred to standard texts on the subject (see for example References 1 and 2).

The yield line method is based on the kinematic theorem of the plastic theory of structures and gives an upper bound solution for the collapse load of a plate.

In the yield line method, a plastic collapse mechanism of the plate is assumed consisting of undeformed plate segments connected by plastic hinge lines, called yield lines. The mechanism must be kinematically admissible over the whole plate and at the boundaries. The bending moment distribution is not considered and, in general, the equilibrium conditions are not verified.

There are two solution approaches in the yield line theory: the virtual work method and the so-called equilibrium method. Both methods lead to identical upper bound solutions, and it has been demonstrated that both methods represent in fact the same solution, but with a different approach [2]. The virtual work method is simpler in principle and is used for the numerical method presented herein. The virtual work method is outlined below.

In this method, a plastic collapse mechanism is assumed for a given plate and loading, and the collapse load P is found by equating the work done by the external loads on the plate, E, to the internal work dissipated by the yield lines, D, during a small motion of the assumed collapse mechanism, viz,

$$E = D \tag{1}$$

i.e.

$$\sum_{i=1..\text{no. of loads}} \lambda P_i \delta_i = \sum_{j=1..\text{no. of yield lines}} \gamma m_{p,j} \theta_j \ell_j \tag{2}$$

or, in analysis problems,

$$\lambda = \frac{\sum_{j=1..\text{no. of yield lines}} \gamma m_{p,j} \theta_j \ell_j}{\sum_{i=1..\text{no. loads}} P_i \delta_i} \tag{3}$$

alternatively, in design problems,

$$\gamma = \frac{\sum_{i=1..no.\ of\ loads} \lambda P_i \delta_i}{\sum_{j=1..no.\ of\ yield\ lines} m_{pj} \theta_j \ell_j} \tag{4}$$

λP_i's (i = 1 .. no. of loads) are the applied load, each acting through a virtual displacement δ_i. The P_i's may be thought of as the characteristic loads and, for convenience, are usually given values of unity or related to unity. λ is then the load factor. The loads are specified completely by the value of λ, and can be referred to collectively as the set of loads λ. The minimum load factor which would cause plastic collapse is termed the collapse or yield load factor λ_{min}.

γm_{pj}'s are the plastic moment resistances per unit length, θ_j's are the rotations, and ℓ_j's are the lengths of every yield line in the assumed mechanism. The m_{pj}'s may be thought of as the characteristic plastic moment resistances per unit length and, for convenience, are usually given values of unity or related to unity. γ is then the plastic-moment-resistance/unit-length factor, in short the moment-resistance factor. The plastic moment resistances / unit length are specified completely by the value of γ, and can be referred to collectively as the set of plastic moment resistances / unit length γ. The maximum moment-resistance factor for which plastic collapse would occur is termed the collapse moment-resistance factor γ_{max}.

In analysis problems, the plastic moment resistances / unit length γm_{pj}'s and the characteristic loads P_i's are known, while the yield-load factor λ is sought. On the other hand in design problems, the loads λP_i's and the characteristic plastic moment resistances / unit length m_{pj}'s are known, while the moment-resistance factor γ is sought. The program Yield Lines handles both analysis and design types of problems.

Since the yield line method leads to an upper bound solution, different mechanisms as well as different dimensions for each mechanism must be tried in order to find the lowest predicted load factor λ_{min}, or alternatively to find the maximum required moment resistance factor γ_{max}. In the conventional algebraic method, the optimum solution of simple problems can be found directly by differentiation. For complex problems, a trial and error technique is faster and usually satisfactory [2,3]. With the program Yield Lines, a simple searching procedure is used to find the optimum solution.

Numerical Method

A flow chart of the program Yield Lines is shown in Fig. 1, and its main features are discussed below.

The numerical method consists in computing the yield load or required bending resistance of a plate based on the geometry of an assumed mechanism defined by means of nodes, planes, and lines. Consider, as a simple example, an orthotropic square plate with fixed supports, subjected to a point load P at the center and assumed to form the yield line mechanism shown in Fig. 2. The yield lines are numbered from 1 to 8 with end nodes numbered from 1 to 5. The flat plate segments, or planes, are numbered from 1 to 5, including plane 1 which represents the plane containing the fixed supports. A right hand rectangular coordinate system is set with the origin located arbitrarily, say at the lower left corner, with the z axis pointing upward. The (x_i, y_i, z_i) coordinates of each node can hence be determined.

The energy dissipated by the yield lines is discussed first. This includes the calculation of the plastic moment, the rotation, and the length of the yield lines.

The length of each yield line is given by the distance between its end nodes p_1 (x_1, y_1, z_1) and p_2 (x_2, y_2, z_2), and is equal to

$$\overline{P_1 P_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (5)$$

where the z coordinates of the nodes, being very small, are not included.

The characteristic bending resistance per unit length, m_p, of a yield line making an angle α with the x axis is, in an orthotropic plate (Fig. 3a),

if the yield line is sagging:

$$m_p = m_{px} \cos^2 \alpha + m_{py} \sin^2 \alpha \qquad (6)$$

if the yield line is hogging:

$$m_p = m'_{px} \cos^2 \alpha + m'_{py} \sin^2 \alpha \qquad (7)$$

where the functions of α are found from

$$\cos^2 \alpha = \left(\frac{y_2 - y_1}{l}\right)^2, \qquad \sin^2 \alpha = \left(\frac{x_2 - x_1}{l}\right)^2 \qquad (8)$$

m_{px} and m_{py} are the characteristic sagging resistances in the x and y direction, respectively, and m'_{px} and m'_{py} are the characteristic hogging resistances. x_1, y_1 and x_2, y_2 are the x and y coordinates of the end nodes of the yield line and l is the length of the yield line.

In skew concrete slabs, the reinforcement may be placed parallel to the edges of the slab, and hence the plate is not orthotropic. Let the reinforcement be placed in the x direction and in the s direction, inclined at an angle β with the x axis ($0° < \beta < 180°$, positive when measured anticlockwise from the x axis to the s axis). The characteristic bending resistance, m_p, of a yield line making an angle α with the x axis is (Fig. 3b),

if the yield line is sagging:

$$m_p = m_{px} \cos^2 \alpha \bullet m_{ps} \cos^2 (\beta-\alpha) \tag{9}$$

if the yield line is hogging:

$$m_p = m'_{px} \cos^2 \alpha + m'_{ps} \cos^2 (\beta-\alpha) \tag{10}$$

where the functions of α are found from

$$\cos^2 \alpha = \left(\frac{y_2-y_1}{l}\right)^2 \tag{11}$$

if $(y_2-y_1)(x_2-x_1) > 0$:

$$\cos^2 (\beta-\alpha) = \left[\frac{(\cos \beta)(y_2-y_1)}{l} + \frac{(\sin \beta)(|x_2-x_1|)}{l} \right]^2 \tag{12}$$

if $(y_2-y_1)(x_2-x_1) <= 0$:

$$\cos^2 (\beta-\alpha) = \left[\frac{(\cos \beta)(y_1-y_2)}{l} + \frac{(\sin \beta)(|x_2-x_1|)}{l} \right]^2 \tag{13}$$

m_{px}, m'_{px}, m_{ps} and m'_{ps} are the characteristic sagging and hogging resistances in the x and s direction, respectively.

Before calculating the rotation of a yield line, planes must be defined, as follows, corresponding to the rigid plate segments of the assumed mechanism. For the plate shown in Fig. 2, plane 2 is defined by nodes 1, 2, and 3, plane 3 is defined by nodes 1, 3, and 4, etc. Given three points $p_0(x_0,y_0,z_0)$, $p_1(x_1,y_1,z_1)$, and $p_2(x_2,y_2,z_2)$, the algebraic equation of the plane through these points is

$$A\ x + B\ y + C\ z + D = 0 \tag{14}$$

where

$$A = (y_1-y_0)(z_2-z_0) - (z_1-z_0)(y_2-y_0)$$

$$B = (z_1-z_0)(x_2-x_0) - (x_1-x_0)(z_2-z_0)$$

$$C = (x_1-x_0)(y_2-y_0) - (y_1-y_0)(x_2-x_0)$$

$$D = - (A\ x_0 + B\ y_0 + C\ z_0)$$

In order to define a plane, the three points p_0, p_1, and p_2 must not be colinear. This can be checked by comparing the slope of a line from p_0 to p_1 and that of a line from p_1 to p_2 (Ref.[6]). For simplicity, the slopes in the x,y plane, $\frac{y_1-y_0}{x_1-x_0}$ and $\frac{y_2-y_1}{x_2-x_1}$ are compared. If the slopes are unequal the three points are not colinear and they can be used to calculate the algebraic equation of the plane.

Once the equation of a plane has been determined, it can be used to calculate the deflection of some nodes, which otherwise would have to be calculated by hand. Fig. 4 shows an example where the z coordinate of node 5 can be calculated using the equation of plane 3 or 4. The equation defining the plane of a plate segment can also be used to check whether the segment is indeed plane. This is done by comparing the z coordinates of each node to the value calculated based on the x and y coordinates of the node and using the equation of the plane, i.e.

$$z = -\frac{A\,x + B\,y + D}{C} \tag{15}$$

The mechanism shown in Fig. 5 is not kinematically admissible. This is detected by checking the z coordinates of nodes 3 and 4 on the contiguous rigid plate segments 2 and 5.

The rotation of each yield line is given by the angle θ between the two planes intersecting at that yield line (see Fig. 6). Given two planes m and n with the following algebraic equations

$$\begin{aligned} &\text{plane m:} \quad A_m\,x + B_m\,y + C_m\,z + D_m = 0 \\ &\text{plane n:} \quad A_n\,x + B_n\,y + C_n\,z + D_n = 0 \end{aligned} \tag{16}$$

the angle θ between these planes is equal to the acute angle between their normal vectors $\mathbf{n}_m$ and $\mathbf{n}_n$ and is given by

$$\theta \approx \tan\theta = \frac{|\mathbf{n}_m \times \mathbf{n}_n|}{|\mathbf{n}_m \cdot \mathbf{n}_n|} = \frac{\sqrt{(B_mC_n-C_mB_n)^2 + (C_mA_n-A_mC_n)^2 + (A_mB_n-B_mA_n)^2}}{|\,A_mA_n + B_mB_n + C_mC_n\,|} \tag{17}$$

where, since we consider virtual displacements, the angle can be considered small. Such small angles are obtained by assuming small deflections of the yield line mechanism. For example, choosing a maximum value of $1/10^{15}$ of the plate width, say, for the z coordinate of the nodes in the displaced plate leads to satisfactory results with less than $1/10^{10}$% error.

From a numerical description of a yield line mechanism, it is possible to determine the bending sign of the yield lines, i.e. whether they are sagging or hogging. Given a yield line with end nodes 1 and 2, bounded by planes m and n, and using the convention that plane m is on the left hand side of the yield line for an observer standing at node 1 and looking at node 2, then a point H with coordinates $[x_1+(y_2-y_1), y_1+(x_1-x_2)]$ is always on the right hand side of the yield line. (see Fig. 7). The difference between the z coordinate of point H on plane n and the corresponding coordinate using the equation of plane m indicates whether the yield line is sagging or hogging. When $z_{Hn}-z_{Hm} > 0$, the yield line is sagging. When $z_{Hn}-z_{Hm} < 0$, the yield line is hogging.

The plastic moment, the rotation, and the length of each yield line has been found using Eqs. (5) through (17), and the product of these values is then summed for all yield lines. The sum, $\Sigma\gamma m_p\theta l$, is equal to the total energy, D, dissipated by the yield lines.

The work done by the loads is now discussed. Point loads, line loads, and uniformly distributed loads (UDL) are treated.

Point loads can be defined by a characteristic value P_{PL}, a point (node) with coordinates x and y where the load is applied, and the plane on which it is applied. The deflection of the load is the z coordinate of the point where the load is applied. If the z coordinate is not specified at the load point, it can be calculated from Eq. (15). The work done by the point load is then

$$E_{PL} = \lambda\ P_{PL}\ z_{PL} \tag{18}$$

A uniform or linearly varying line load can be defined by two end nodes with coordinates x_1, y_1 and x_2, y_2, the characteristic value of the line load at each end, p_1 and p_2 (load/unit length), and the plane on which the line load is applied (see Fig. 8). Given this data, the work done by the line load is calculated as follows:

The length of the line load is

$$\ell = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \tag{19}$$

The characteristic resultant of the line load is

$$P_{LL} = \frac{p_1 + p_2}{2} \cdot \ell \tag{20}$$

The location of the resultant is at (x_c, y_c) where

$$x_c = x_1 + \frac{x_2-x_1}{\ell}\ c\ , \qquad y_c = y_1 + \frac{y_2-y_1}{\ell}\ c\ , \tag{21}$$

where

$$c = \frac{2\ p_2 + p_1}{3\ (p_2 + p_1)} \cdot \ell \tag{22}$$

The deflection at the point where the resultant load acts is

$$z_{LL} = -\ (A\ x_c + B\ y_c + D)\ /\ C \tag{23}$$

where A, B, C and D are the coefficients of the algebraic equation of the plane on which the load is applied.

Finally, the work done by the line load is

$$E_{LL} = \lambda\ P_{LL}\ z_{LL} \tag{24}$$

A uniformly distributed load (UDL) can be defined by the n vertices, with coordinates $x_1, y_1,\ x_2, y_2 \ldots x_n, y_n$, of the area covered by the UDL, the characteristic value of the UDL, p_{UDL} (load / unit area), and the plane on which the UDL is applied. Given this data, the work done by the load is calculated as follows:

The value and the location of the characteristic resultant are calculated in a manner similar to that by which the area of a traverse is calculated in surveying (see Fig. 9). The characteristic resultant load is

$$P_{UDL,x} = p_{UDL} \left[(y_2-y_1)(x_1+x_2)/2 + (y_3-y_2)(x_2+x_3)/2 + \dots + (y_1-y_n)(x_n+x_1)/2 \right] \quad (25)$$

alternatively,

$$P_{UDL,y} = p_{UDL} \left[(x_2-x_1)(y_1+y_2)/2 + (x_3-x_2)(y_2+y_3)/2 + \dots + (x_1-x_n)(y_n+y_1)/2 \right] \quad (26)$$

The location of the resultant is at the centroid of the area covered by the UDL, i.e. at (x_c, y_c) where

$$x_c = \frac{\Sigma\, \bar{x} p}{P_{UDL,y}}, \qquad y_c = \frac{\Sigma\, \bar{y} p}{P_{UDL,x}} \quad (27)$$

where

$$\Sigma\, \bar{x} p = p_{UDL} \left\{ \frac{y_2-y_1}{8} \left[(x_1+x_2)^2 + \frac{(x_2-x_1)^2}{3} \right] + \frac{y_3-y_2}{8} \left[(x_2+x_3)^2 + \frac{(x_3-x_2)^2}{3} \right] + \dots + \frac{y_1-y_n}{8} \left[(x_n+x_1)^2 + \frac{(x_1-x_n)^2}{3} \right] \right\} \quad (28)$$

and

$$\Sigma\, \bar{y} p = p_{UDL} \left\{ \frac{x_2-x_1}{8} \left[(y_1+y_2)^2 + \frac{(y_2-y_1)^2}{3} \right] + \frac{x_3-x_2}{8} \left[(y_2+y_3)^2 + \frac{(y_3-y_2)^2}{3} \right] + \dots + \frac{x_1-x_n}{8} \left[(y_n+y_1)^2 + \frac{(y_1-y_n)^2}{3} \right] \right\} \quad (29)$$

The deflection at the resultant is

$$z_{UDL} = -(A\, x_c + B\, y_c + D) / C \quad (30)$$

where A, B, C and D are the coefficients of the algebraic equation of the plane on which the load is applied.

Finally the work done by the UDL is

$$E_{UDL} = \lambda\, P_{UDL}\, z_{UDL} \quad (31)$$

Note that the abov~ procedure to determine the work done by a UDL is valid for UDL's covering areas of arbitrary polygonal shape, including areas with reentrant corners and with holes.

The work done by the various loads has been found using Eqs. (18) through (31), and the values are then summed for all the loads. This sum, $\Sigma\lambda P\delta$, is equal to the total work done by the loads, E.

Finally, the yield-load factor λ of the plate is found by using Eq. (3), i.e. by dividing E by the sum of $P\delta$. For the example in Fig. 2, with P=1 and all γm_p's=1 ($\gamma m_{px}=\gamma m_{py}=\gamma m'_{px}=\gamma m'_{py}=1$, i.e. an isotropic plate), the value obtained for λ is 16. Alternatively, the bending-resistance factor, γ, required to support a load of $\lambda P=1$ is found to be 0.0625 by using Eq. (4). These results agree exactly with the solution from a conventional algebraic yield line analysis of this mechanism.

Mechanisms involving fans (curved yield lines) can be treated by approximating the fan using a series of triangles placed one next to the other. Using 16 such triangles, a complete circular fan is approximated with a 1.3% error. Fans of elliptical, logarithmic shape, etc. can be approximated in a similar manner.

Slabs on beams and slabs with curtailed reinforcement can be treated by assigning the appropriate bending resistance to yield lines or segments of yield lines corresponding to the beams or to the actual placement of the reinforcement.

Optimization of a yield line mechanism is now discussed. To find the correct solution for a given mechanism, a series of patterns is defined by varying the dimensions of the mechanism. The yield-load factor is calculated for every pattern, and the least of these yield-load factors is retained as the solution.

Series of patterns are produced by specifying, for one or more nodes, initial and final positions in the x,y plane, and the number of steps between these positions. This data is used by an iteration procedure which creates a family of patterns. When a yield line mechanism involves several parameters, the iteration procedure is called as many times as there are parameters to generate all the families of patterns. The iteration procedures can be nested, so that all possible patterns are created in one solution. Recursive procedures are used advantageously for this purpose.

When generating a series of yield line patterns, it is often possible to relate the location of some of the moving nodes to the location of other nodes. This reduces the amount of data required for the optimisation of the yield line mechanism, and also it conveniently retricts the movements of the nodes within the limit of validity for the mechanism. One way of establishing the relationship is by locating a node at the intersection of two lines defined by two pairs of nodes. The node at the intersection is called a slave node, while the other four nodes guiding the slave node are called master nodes. Fig. 10 shows an example where node 3 is a slave node and nodes 1, 5 and 2, 4 are the master nodes; node 6 is also a slave node with master nodes 4, 8 and 5, 9. If a family of patterns is created by moving node 5, then the location of nodes 3 and 6 is adjusted automatically.

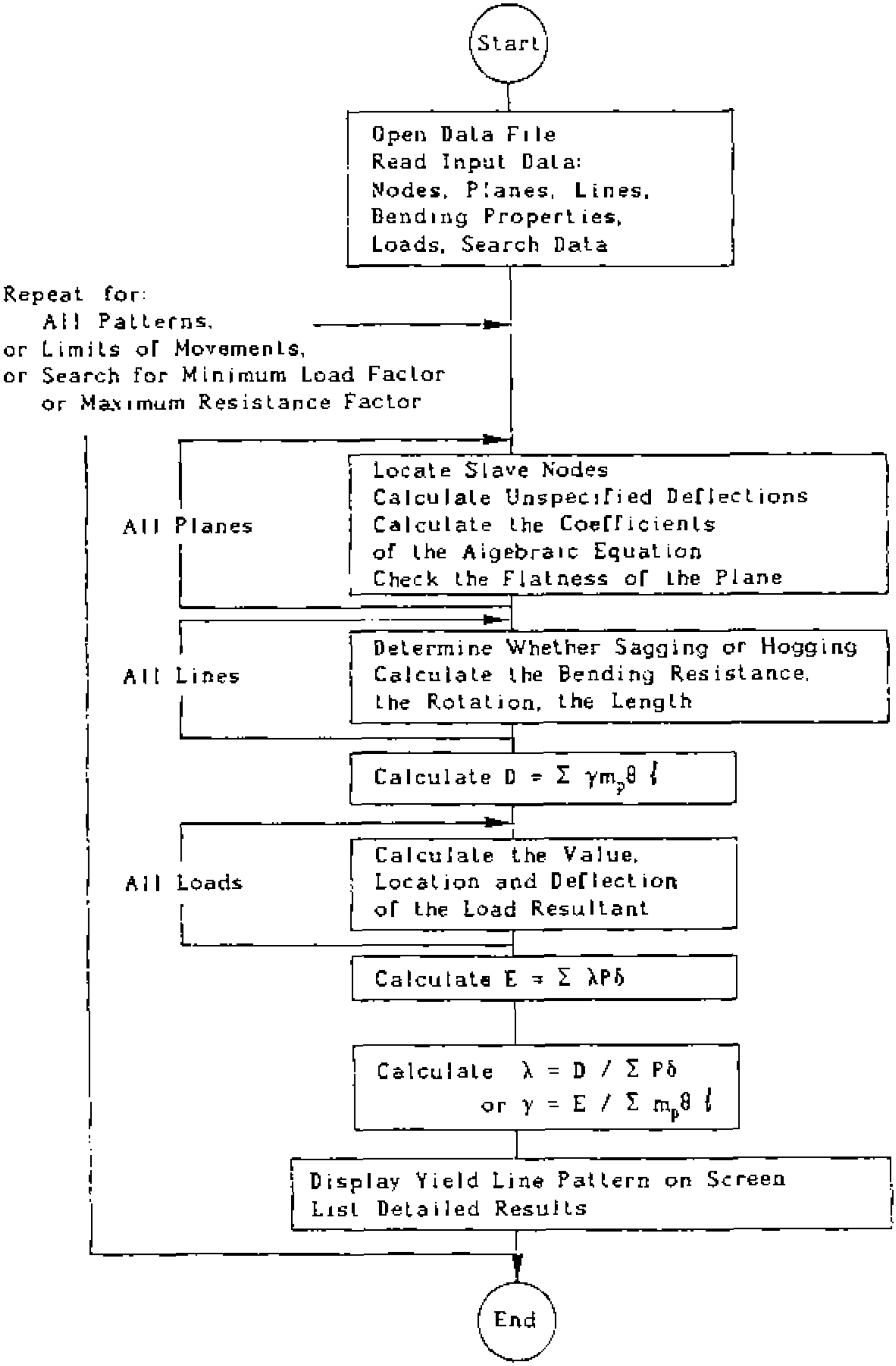


Figure 1 Flow chart of Yield Lines.

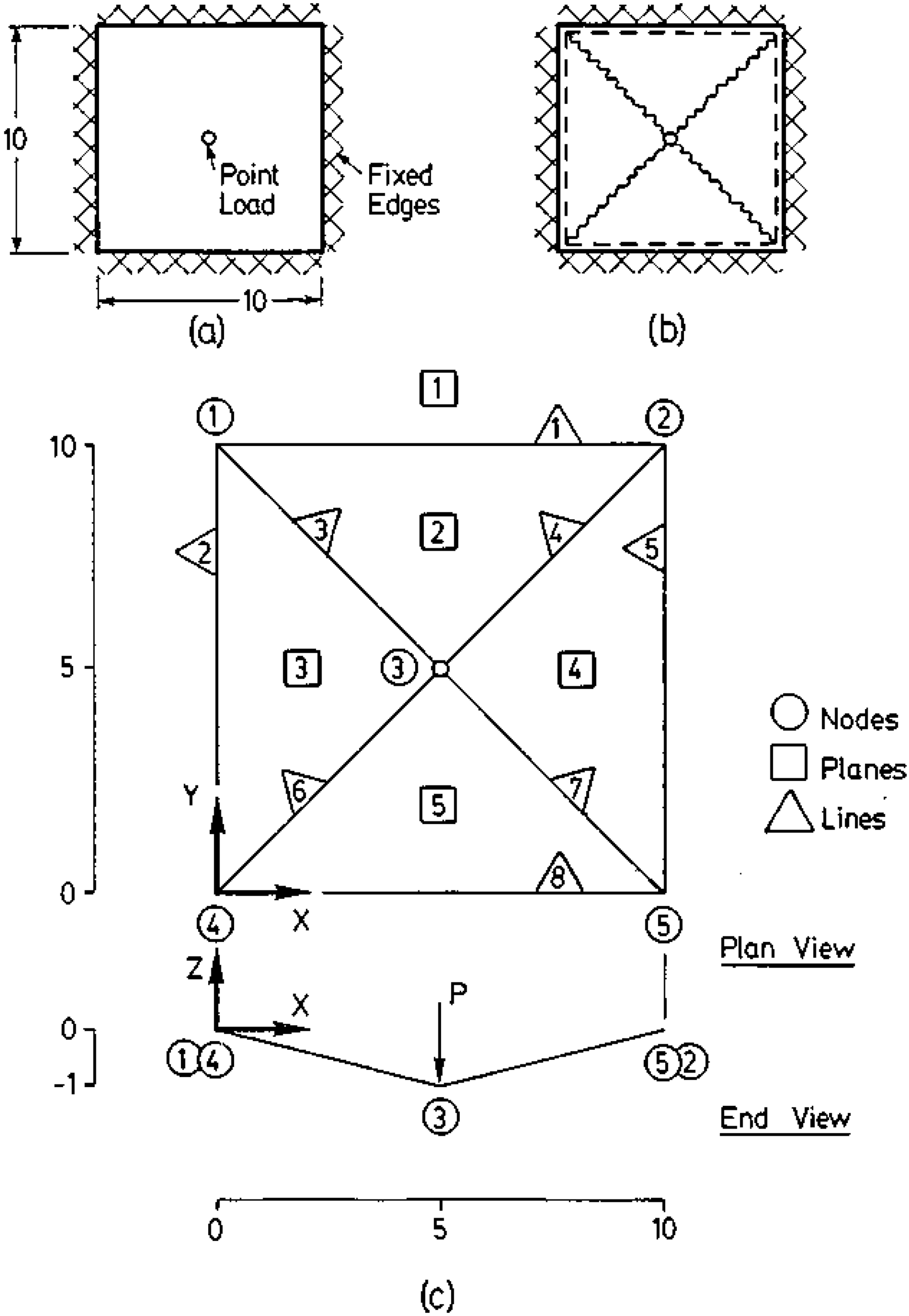


Figure 2 Fixed edges square plate with central concentrated load. (a) Plate. (b) Yield line pattern. (c) Model for numerical analysis.

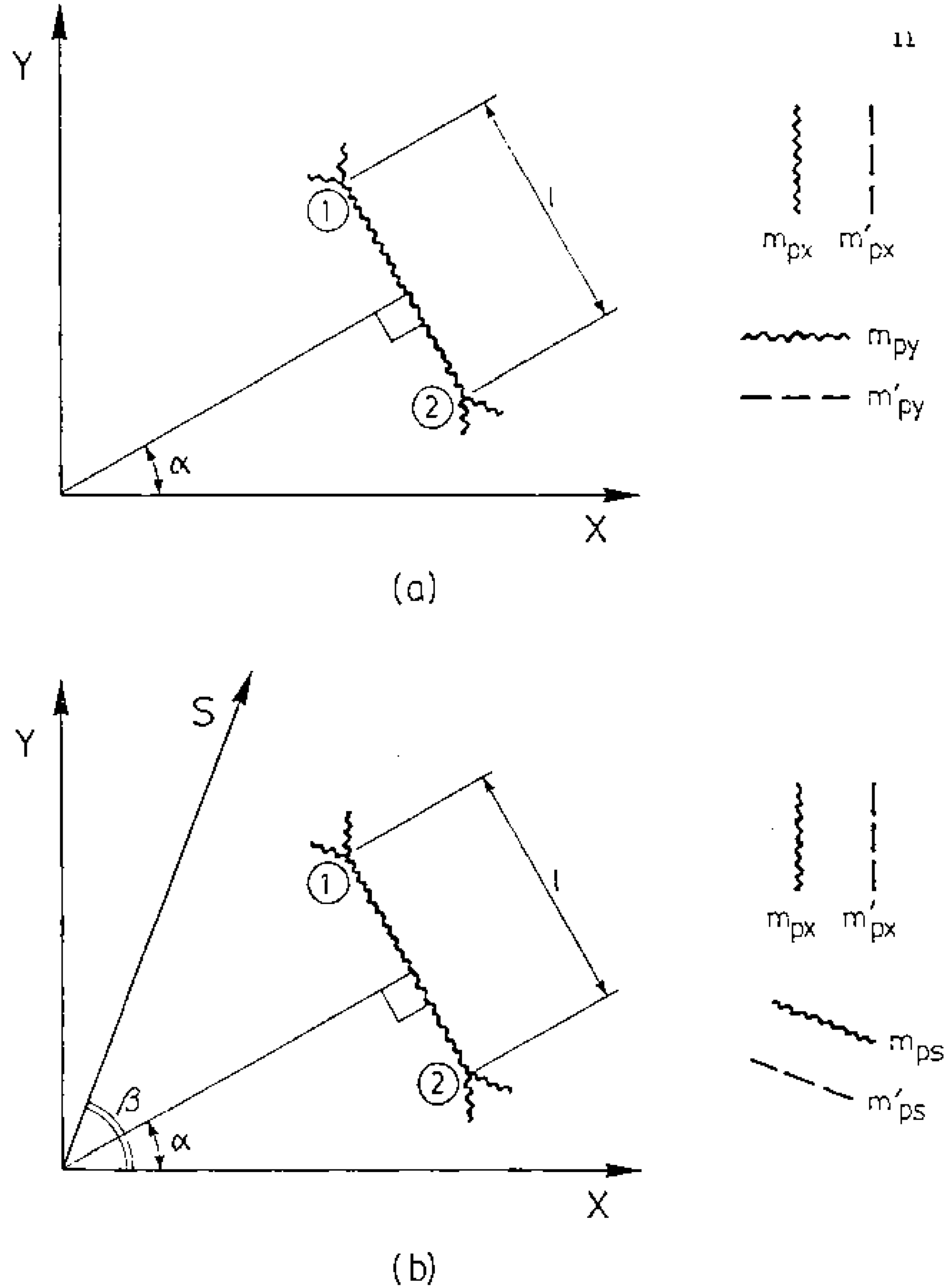
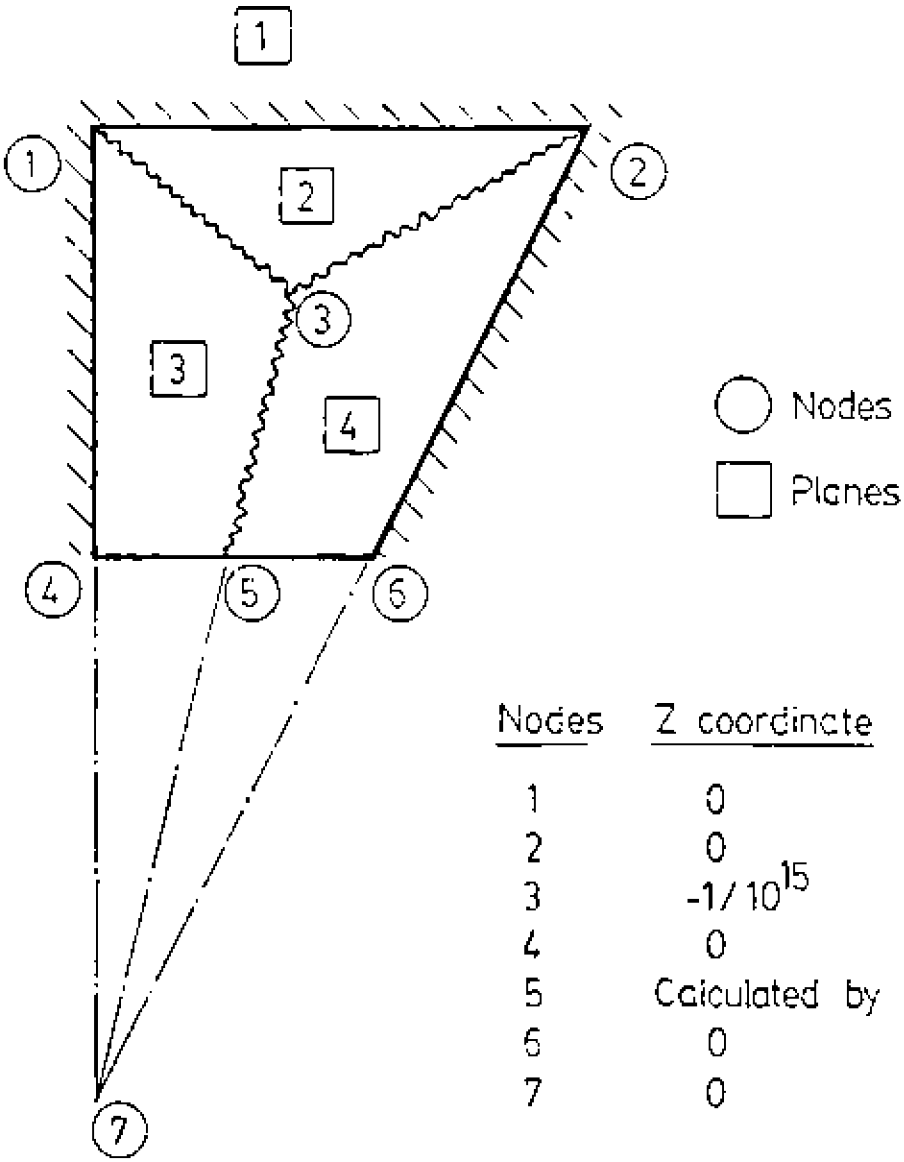


Figure 3 Yield line at general angle. (a) In orthotropic plate. (b) In skew concrete slab.



Nodes	Z coordinate
1	0
2	0
3	$-1/10^{15}$
4	0
5	Calculated by program
6	0
7	0

Figure 4 Example where the z coordinate of node 5 can be calculated using the equation of plane 3 (or 4).

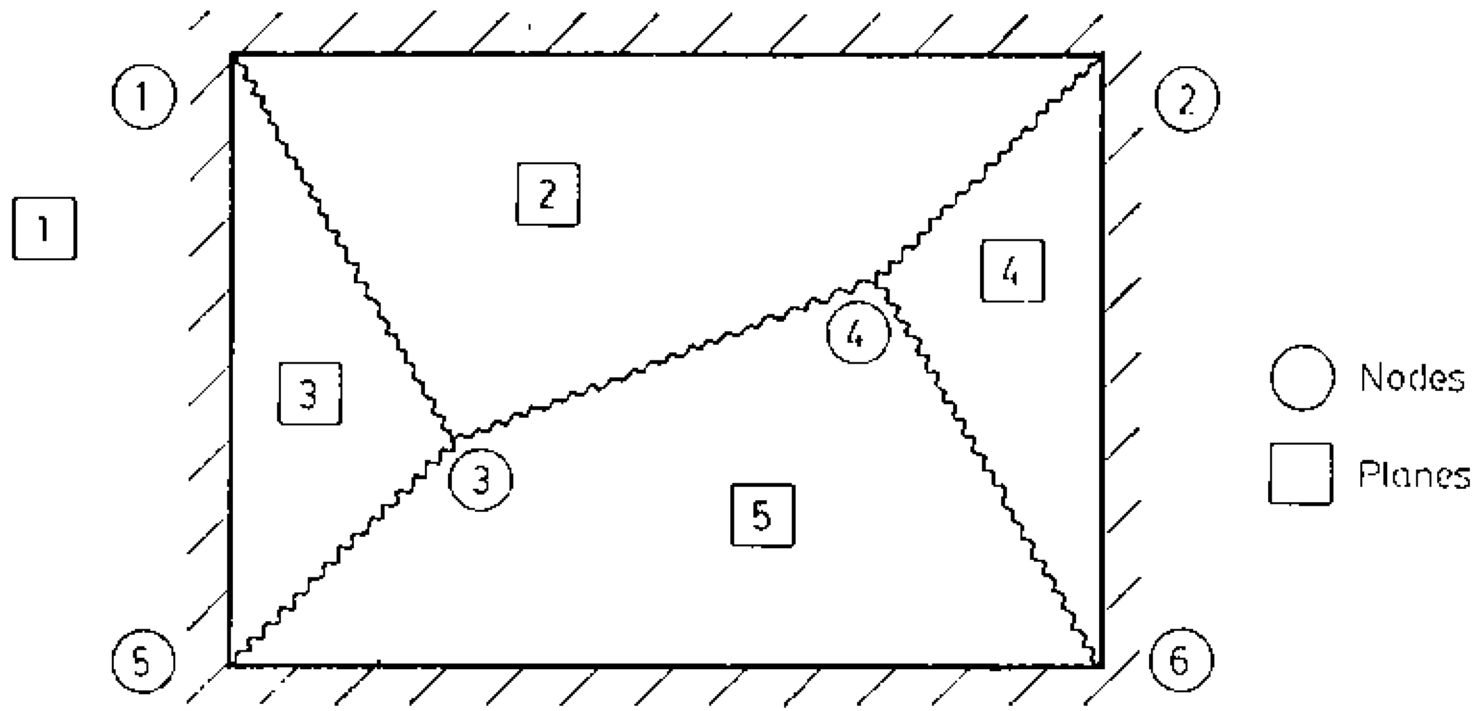


Figure 5   Example of a mechanism that is not kinematically admissible.

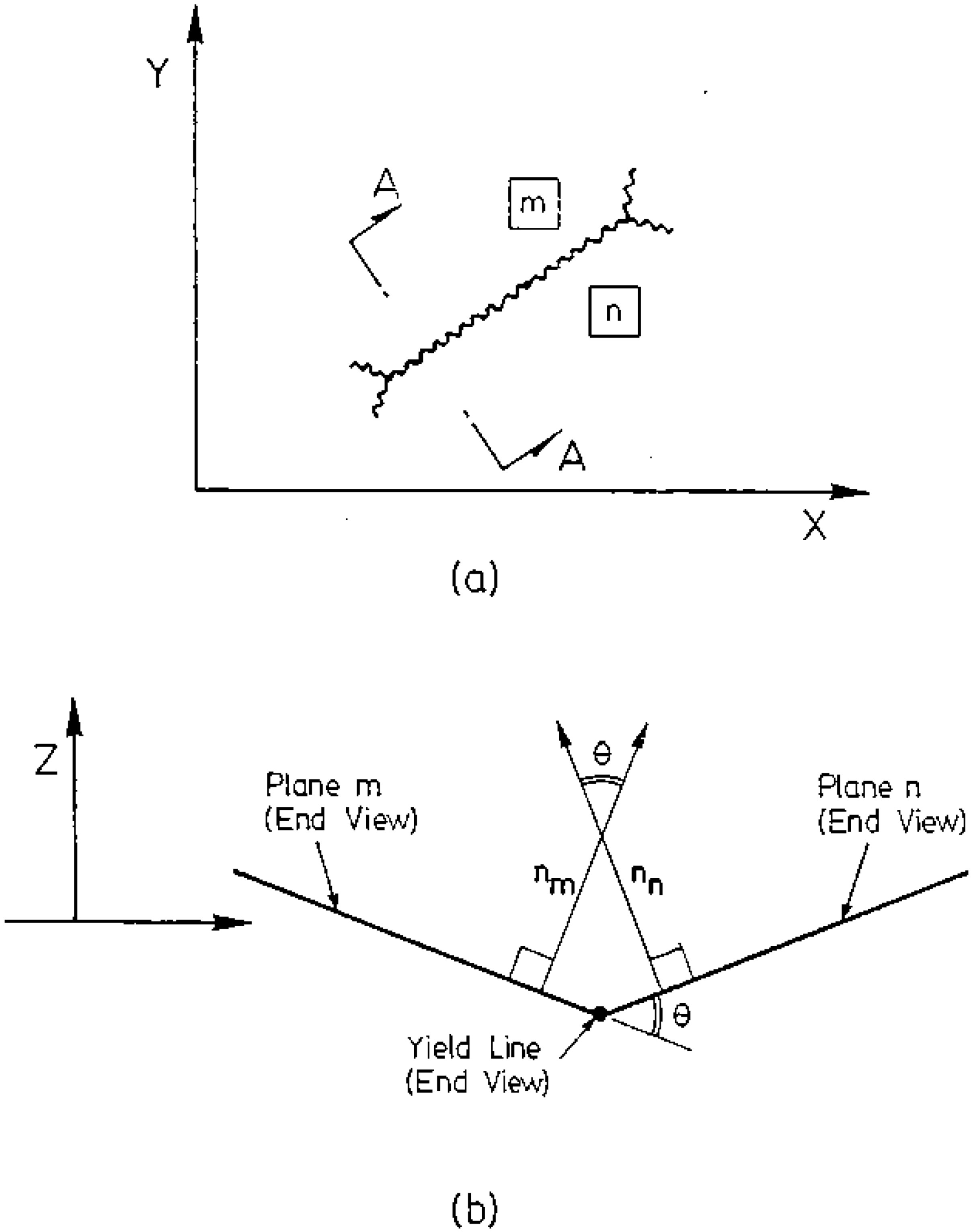


Figure 6 Rotation between the rigid plate segments. (a) Plan view. (b) Section A-A.

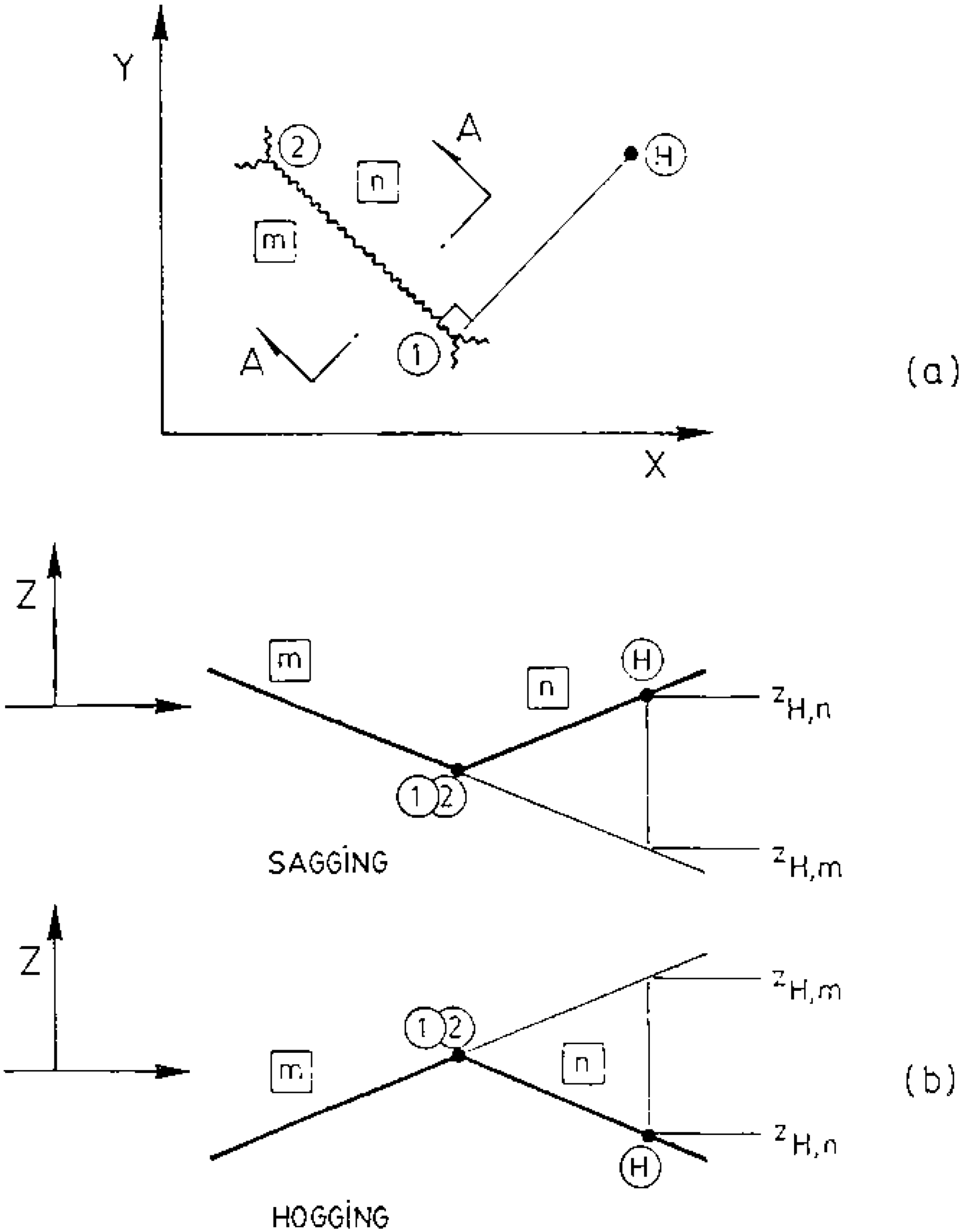


Figure 7 Bending sign (sagging or hogging) of yield line. (a) Plan view (b) Section A-A.

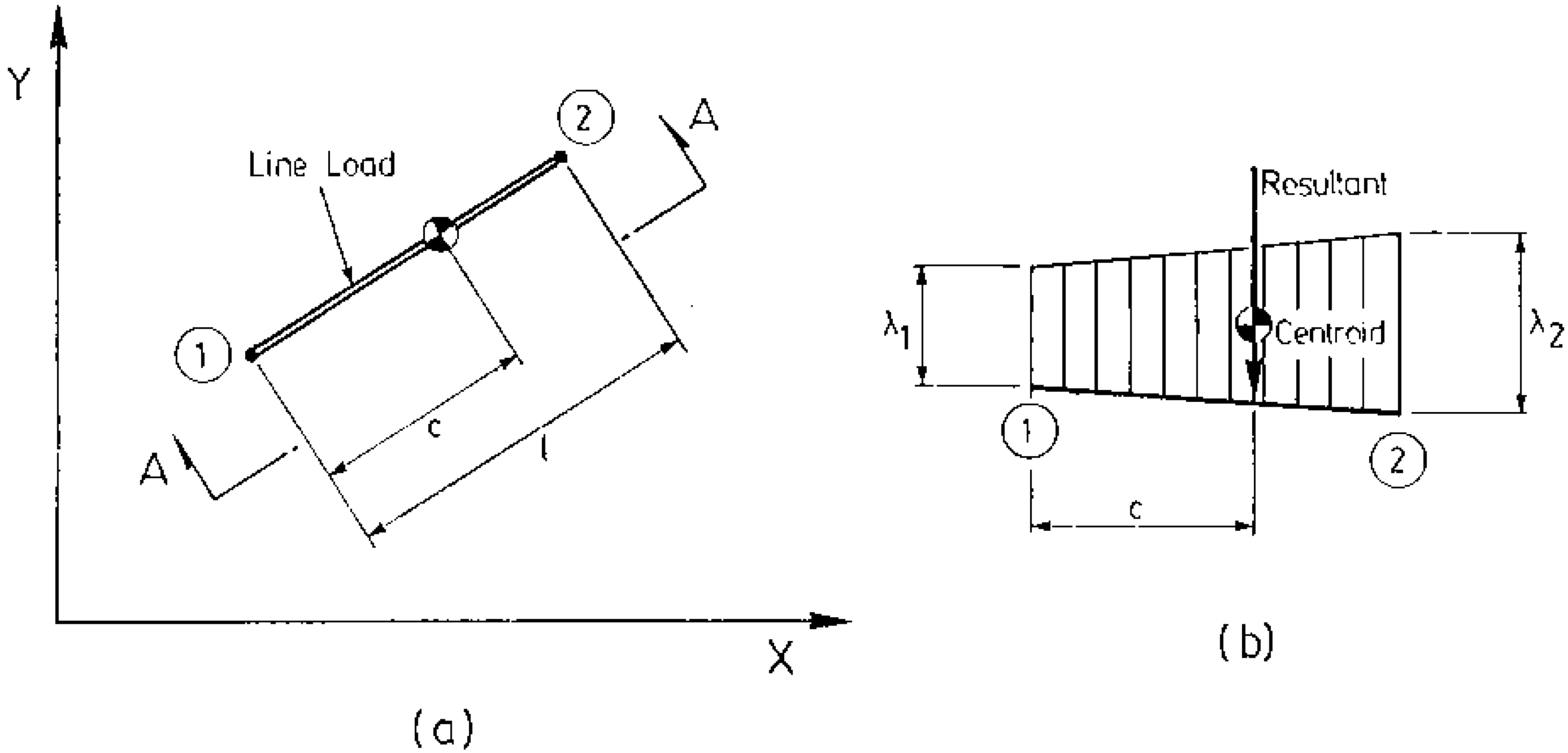


Figure 8   Line load.

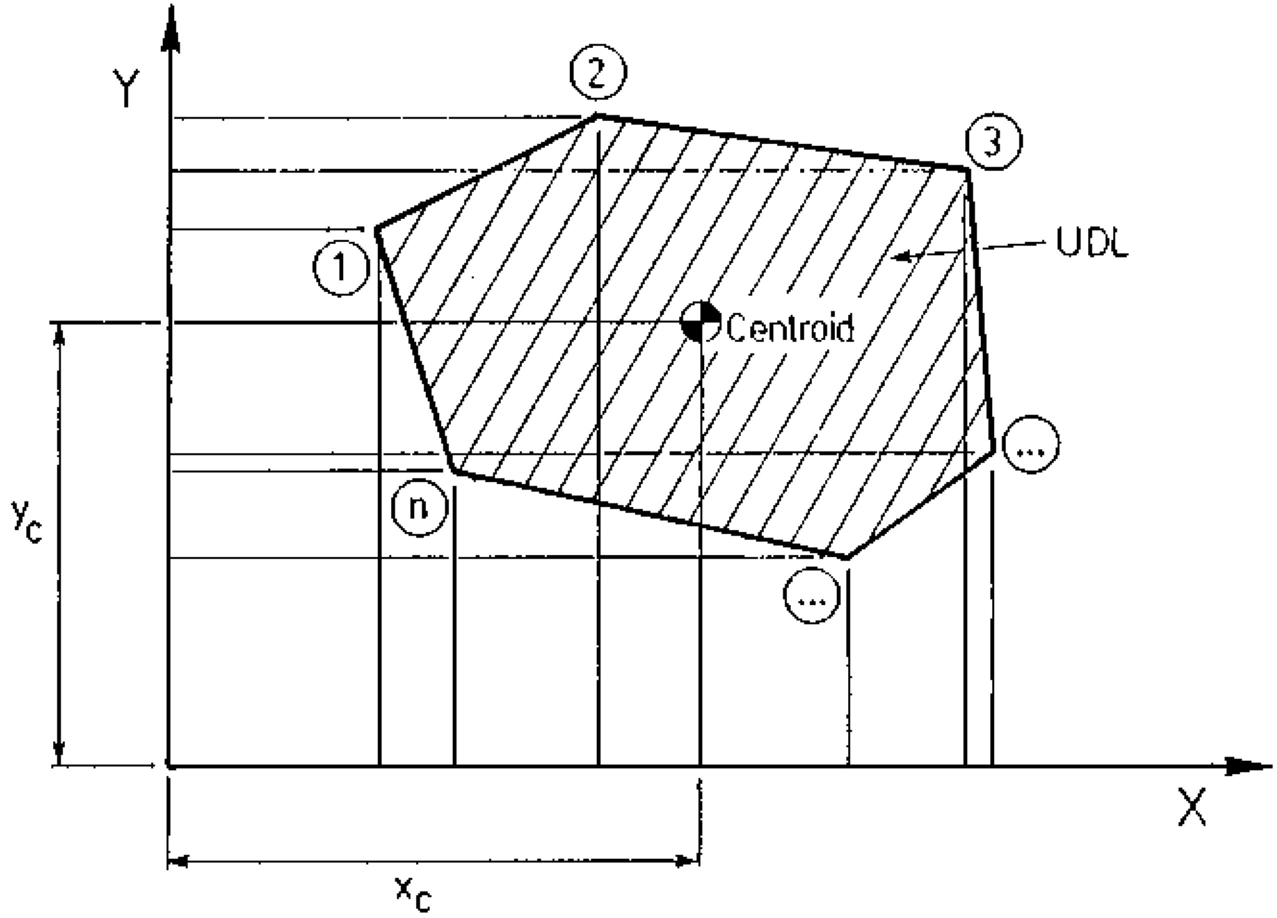


Figure 9 Uniformly distributed load (UDL).

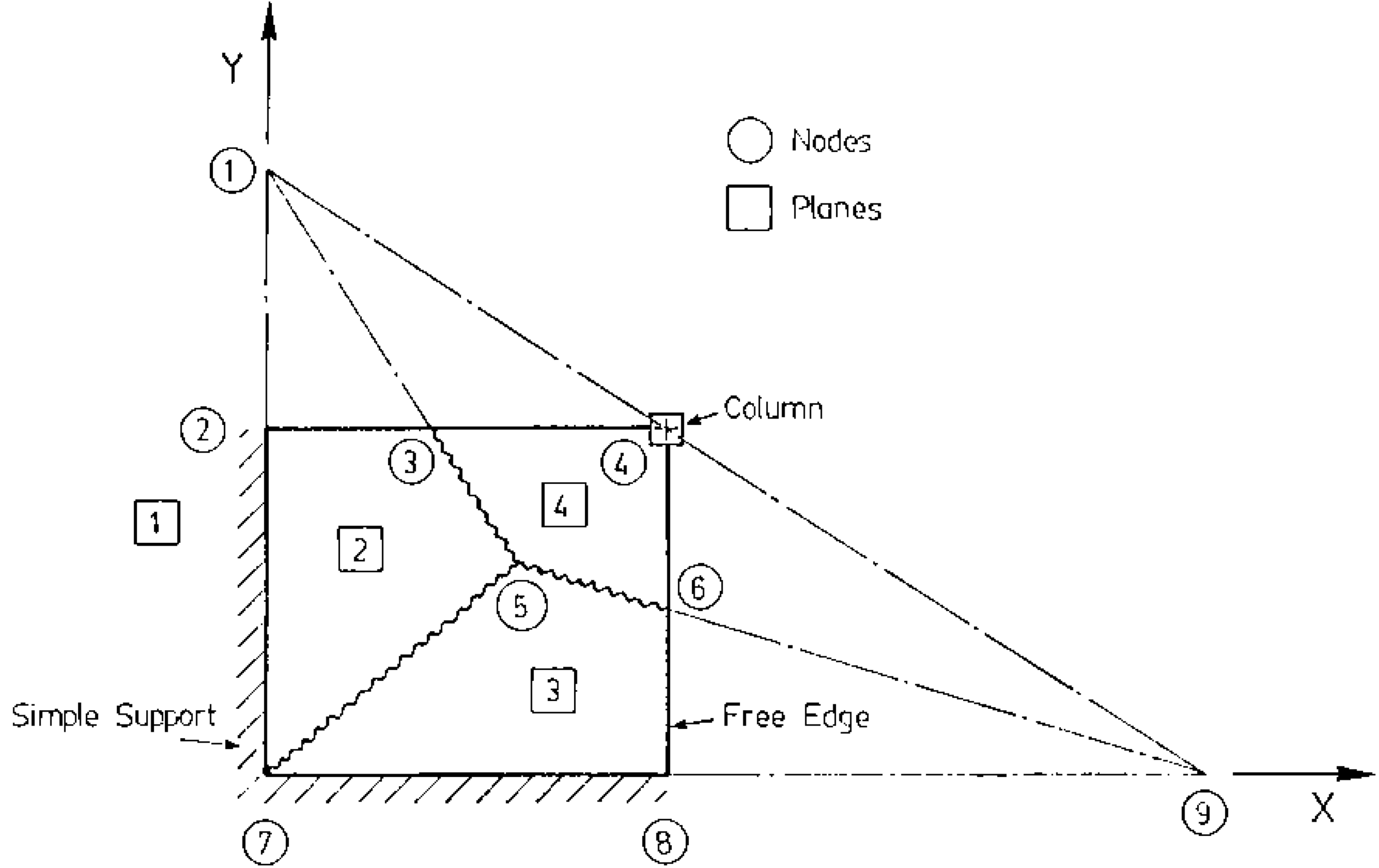


Figure 10 Example where nodes 3 and 6 are set as slave nodes.

Chapter 2 Input Data

In this chapter the input data, which must be carefully prepared by the user, is discussed in detail.

The input data must be created and written in a file by the user. Any text editor, such as Personal Editor, the editor of Turbo Pascal, Wordstar, etc., can be used to create the data file. The data may be written in free format, that is, numbers may be given in fixed or floating point notation. Data items must be separated by at least one blank space or one tab character (commas must not be used as separators). Each line of the input data must contain the specific information described below. The symbol / indicates where the data must continue on a new line. The data file must not contain blank lines.

Each item of the input data is described below. A summary of the input data is given in Table 1.

Maximum Deflection

/ zf : the z factor.

The z coordinate of the nodes with a non-zero deflection can be given a value between 1/5 to 1/100, say, of the plate width. However, this value is too large to satisfy the requirement that θ ≈ tan θ, and all the z coordinates are divided by the z factor for which a value of 10^{15} is usually satisfactory. zf is usually positive.

Nodes

/ ni : the number of nodes.

For each node

/ i : the node number,

x y : the x and y coordinates of node i,

or

* master1 master2 master3 master4 : the master nodes of slave node i (preceded by *),

z or * : the z coordinate of node i. An unspecified z coordinate must be indicated by *.

A right hand coordinate system must be used, with the z axis pointing upwards from the plate. Hence in most cases, deflecting nodes will have a negative z coordinate value. The origin of the coordinate system can be located anywhere convenient.

If the x and y coordinates of a node are not specified, the node is said to be a 'slave' node. Such a point is located by specifying 2 pairs of 'master' nodes. Each pair defines an infinite line, and the slave node is located at the intersection of the two lines. The master lines must not be parallel. Examples of valid slave nodes are shown in Fig. 11.

If a slave node has master nodes which themselves are slave of other nodes, the program will automatically sort the data to locate the nodes in order from the highest ranking master nodes to the lowest slave nodes. Hence the numbering of nodes can be done in any order.

Planes (Plate Segments)

/ nj : the number of planes.

For each plane

/ j : the plane number,

nnop : the number of nodes on the plane,

nop[n] (n=1...nnop) : the node number of all the nodes on the plane.

Although only 3 non-colinear nodes are enough to define a plane, the other nodes should be given since they can be used to:

- check the z coordinates of the nodes on the plane to verify that the plane is indeed flat.
- compute the z coordinate of the nodes with unspecified z coordinate.
- provide an alternative way of defining the boundaries of a UDL (uniformly distributed load).

In order to calculate the coefficients of the algebraic equation of the planes, the program attempts to locate the slave nodes, to calculate the unknown z coordinates, and to identify 3 non-colinear nodes with known x, y and z coordinates for each plane, and this is repeated until the entire geometry of the mechanism is defined. The data is sorted by the program so that the numbering of planes may be done in any order. Also, the nodes belonging to a plane are accepted in any order (random order, or in sequence when used to define a UDL). If the data is insufficient to define the mechanism, an error message is issued by the program.

Lines

/ nk : the number of yield lines.

For each yield line

/ k : the line number.

pi pj : the end nodes.

pm pn : the planes on each side of the line.

and

brn : the bending resistance number, which refers to bending properties (m_p's) of the plate through which the yield line passes.

or * : replaces pm, pn and brn for a construction line (free edge, simply supported edge, or axis of rotation).

Only the actual yield lines enter the calculations of the energy dissipated by the yield lines and hence are required as data. (By definition, the yield lines dissipate energy as they undergo rotations of the assumed mechanism). However, other lines may be specified in order to clarify the drawing of the yield line pattern displayed on the screen. Lines defining free and simply supported edges, axes of rotation, etc. are called construction lines. These lines do not dissipate energy and hence will be skipped during the calculations. The construction lines are specified by a *, replacing pm, pn and brn. Alternatively, the construction lines can be entered with the same plane specified for both pm and pn (any plane number can be used, e.g. pm= pn= 1). Yet an other way of specifying that a line does no work is by giving it a bending resistance number referring to all values of $m_p = 0$.

In connection with the planes on each side of a yield line, the plane on the left hand side of the line, pm, must be given first followed by the plane on the right hand side, pn. The left and right hand sides are those of an observer located at end node pi and looking towards node pj. Caution must be taken to correctly specify the left and right hand planes because this information is used to determine whether the yield lines are sagging or hogging and this affects the final results if the sagging and hogging plate bending resistances are different. Fig. 12 shows examples of valid data defining a line.

Plate Bending Resistances

/ nbr : the number of bending resistance sets. A 'set' includes four values of bending resistance (positive and negative, in two directions), and whether the reinforcement is isotropic, orthotropic, or skew (and the skew angle).

For each set of bending resistance:

/ br : the set number,

reinf : whether the reinforcement is orthotropic (including isotropic) or skew.

For concrete slabs with orthotropic reinforcement the axes must be oriented in the same way as the reinforcement and reinf must be specified as 0 (for Orthotropic). For isotropic steel plates reinf can be specified as 0 or I (for Isotropic). For concrete slabs with skew reinforcement, the X axis must be oriented along one set of reinforcing bars, the S axis must be oriented along the other set of reinforcing bars, and reinf must be specified as S (for Skew). The axes must be layed out as shown in Fig. 3 (see also discussion in Chapter 1).

If the reinforcement is orthotropic or isotropic (reinf = 0 or I):

/ mpxp mpyp : the sagging (positive) plate bending resistance per unit length in the X and Y direction,

mpxn mpyn : the hogging (negative) resistances.

If the reinforcement is skew (reinf = S)

/ mpxp mpsp : the sagging bending resistance per unit length due to the reinforcement in the X and S directions, respectively.

/ mpxn mpsn : the hogging resistances,

beta : the angle β, in degrees, measured anticlockwise from the X axis to the S axis. The value of β must be positive and $0^\circ < \beta < 180^\circ$.

Point Loads

/ nPL : the total number of point loads (=0 if there are no point loads).

For each point load (skip the following line if there are no point loads):

/ PL : the load number,

planePL : the plane on which the point load is applied.

loadPL : the value of the point load. loadPL is negative when the load is acting in the negative z direction, i.e. downwards,

nodePL : the node number where the point load is applied.

Line Loads (Uniform or Linearly Varying)

/ nLL	:	the total number of line loads (=0 if there are no line loads). A line load is acting on only one plate segment. For line loading continuous over several plate segments, several line loads must be defined.

For each line load (skip this line if there are no line loads):

/ LL	:	the load number,
planeLL	:	the plane on which the line load is applied.
loadLL1	:	the load/unit length at the end of the line load located at node nodeLL1. loadLL1 is negative when the load is acting downwards,
nodeLL1	:	the node number at one end of the line load.
loadLL2	:	the load/unit length at the other end of the line load located at node nodeLL2,
nodeLL2	:	the node number at the other end of the line load,

Uniformly Distributed Loads

/ nUDL	:	the total number of UDL's (=0 if there are no UDL's). A UDL is acting on only one plate segment. For loading over several plate segments, several UDL's must be defined.

For each UDL (skip this line if there are no UDL's):

/ UDL	:	the load number,
planeUDL	:	the plate segment on which the UDL is applied,
loadUDL	:	the load/unit area. loadUDL is negative when the load is acting downwards.
nnodesUDL	:	the number of nodes defining the UDL.
and nodeUDL[n] (n=1...nnodeUDL)	:	the node numbers defining the UDL.

or * : replaces nnodeUDL, nodeUDL[n] (n=1... nnodeUDL] when the nodes nop's, used to define a plate segment, are used instead of the nodes nodeUDL's.

The nodes defining a UDL are the vertices of the area covered by the UDL. nnodeUDL must be 3 or more, followed by the nodes nodeUDL[n] defining the UDL. The nodes must be given in sequence around the UDL, either clockwise or anticlockwise.

The nodes used to define a plate segment, nop's, may be used instead of the nodes nodeUDL's. To assign nodes nop's to the vertices of the UDL area, nnodeUDL and the nodeUDL's are replaced by a *. A common case for this is when the UDL covers an entire plate segment and there are no other nodes on the plate segment.

If there is a hole in a plate segment where a UDL is applied, the vertices of the UDL must be numbered in such a way as to create a full figure around the hole (see Fig. 13a) . Alternatively, two or more UDL's may be specified to act on the same plate segment (see Fig. 13b).

Search Data

The search data is used to move (i.e. relocate) nodes so that different yield line patterns and corresponding different yield-load factors and bending-resistance factors may be obtained. This is done in order to find the mechanism with the minimum yield-load factor (or alternatively, with the maximum bending-resistance factor).

/ nmoves : the number of movements (=0 for no search),

for each movement (skip the following lines if there is no search):

/ nnms : the number of nodes moving simultaneously in the movement,

steps : the total number of steps (i.e. positions) in the movement through which the simultaneously moving nodes are going, from their initial to their final position.

for each nodes moving simultaneously during a movement:

/ node : the node number,

xi : the initial x coordinate value,

yi : the initial y coordinate value,

xf : the final x coordinate value,

yf : the final y coordinate value.

The 'movements' correspond to a set of node positions ranging from an initial to a final x,y position. Several nodes can move simultaneously in a movement. Each step of every movement defines a yield line pattern for which the yield-load factor is calculated.

Since the loads are associated with nodes, the search data may be used to move loads on the plate structure. However, this is of limited interest since the concern here is to more the yield lines.

If there are more than one movement specified in a search, the steps of the second movement are repeated for each step of the first movement, and the steps of the third movement are repeated for each of these previous steps, and so on up to the last movement. Hence the movements are nested with the first movement being an outer loop of node positions and the last movement being an inner loop of node positions. To have a node cover a parallelogram area, the node and its initial and final x,y coordinates must be specified in two consecutive movements.

Precise solutions can be obtained by using a large number of steps in a search. Also, it may be efficient to 'close in' on the correct yield line pattern, that is, to adjust the limits of the movements to small ranges. Both ways require to modify the search data and run the program a few times.

Analysis versus Design Problems

In analysis problems, the bending resistances γm_p's and the characteristic loads P's are known, while the load factor λ is sought. When the input data is prepared for an analysis problem, the bending resistances mpxp, mpyp, etc. mean γm_p, and the loads loadPL, loadLL1, loadLL2, and loadUDL mean the characteristic loads P_{PL}, p_1, p_2, and p_{UDL}, respectively. The load factor λ is obtained in the output results.

On the other hand, in design problems the loads λP's and the characteristic bending resistances m_p's are known or at least assumed, while the bending resistance factor γ is sought. When the input data is prepared for a design problem, the bending resistances mpxp, mpyp, etc. mean the characteristic resistances m_p's, and the loads loadPL, loadLL1, loadLL2, and loadUDL mean λP_{PL}, λp_1, λp_2, and λp_{UDL}, respectively The resistance factor γ is obtained in the output results.

Numbering of Data

Nodes, planes, lines, bending properties, point loads, line loads and UDL's must be numbered consecutively from 1 to ni, 1 to nj, 1 to nk, 1 to nbr, 1 to nPL, 1 to nLL and 1 to nUDL, respectively. For example, if a structure is described using 5 nodes, 5 planes, 8 lines, etc., these must be numbered from 1 to 5, 1 to 5, 1 to 8, etc., respectively.

Units

Units are not specified in the data. However, the program assumes that the data is based on a consistent set of units. The consistency of units is assumed throughout the calculations and also for the output results. The following examples are consistent sets of units and either set could be implied in the input data and in the output results:

[length (x,y,z coordinates)]	m	m	ft
[m_p (bending resistance per unit length)]	(N.m)/m	(kN.m)/m	(lb.ft)/ft
[point load]	N	kN	lb
[line load (load per unit length)]	N/m	kN/m	lb/ft
[UDL (load per unit area)]	N/m^2	kN/m^2	lb/ft^2
[work done and energy dissipated]	N.m	kN.m	lb.ft

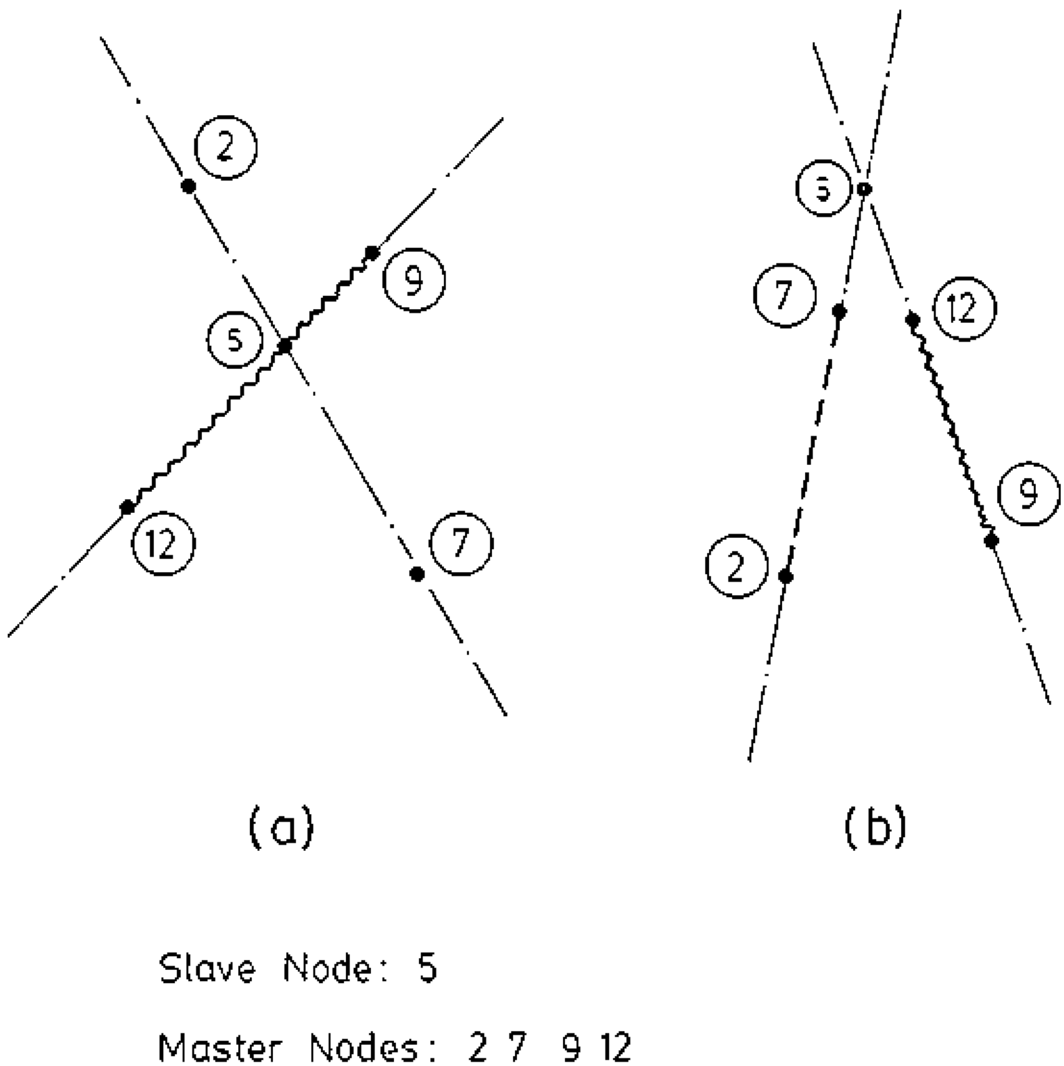
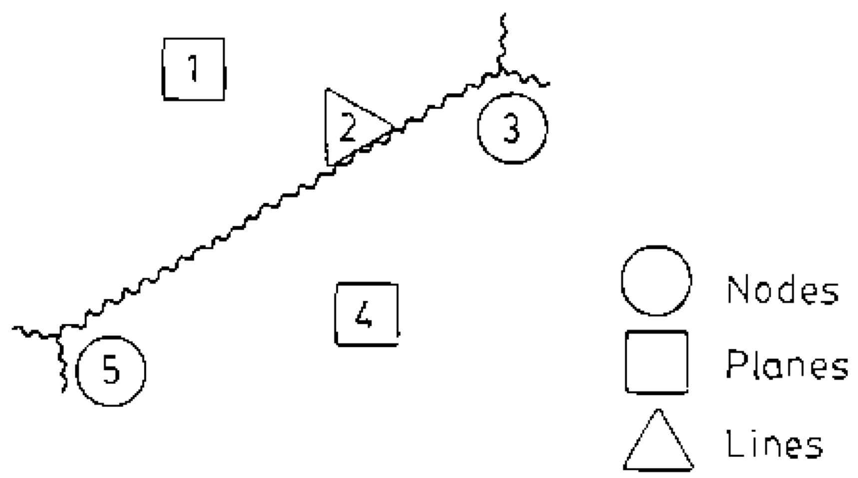


Figure 11 Examples of slave nodes. (a) and (b) are valid arrangements.



k	pi	pj	pm	pn	brn
2	5	3	1	4	1

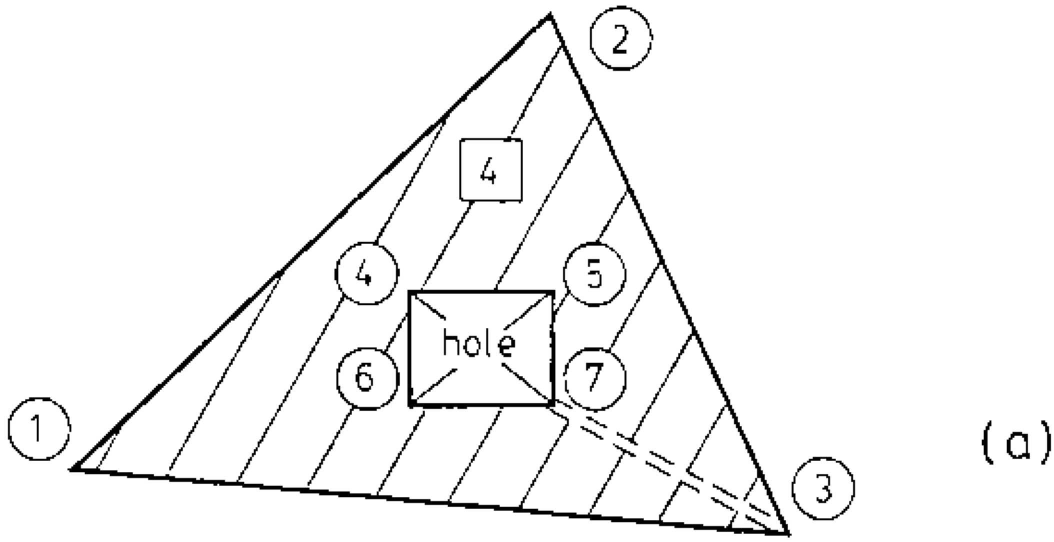
(a)

alternatively,

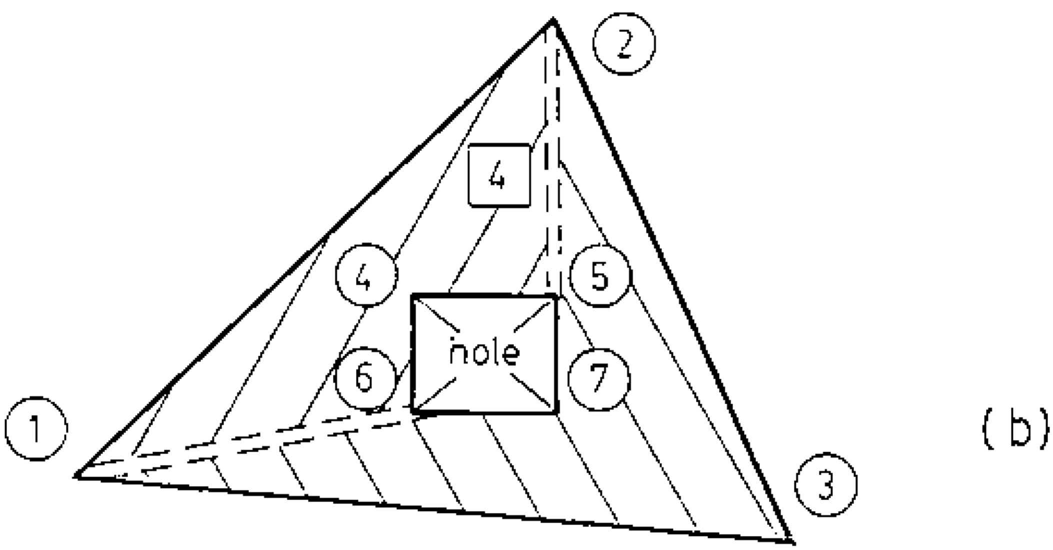
k	pi	pj	pm	pn	brn
2	3	5	4	1	1

(b)

Figure 12 Examples of data to define a line. Numbering in (a) and (b) is valid.



UDL	planeUDL	loadUDL	nnodesUDL	nodeUDL[n]'s
1	4	-1	9	1 2 3 7 5 4 6 7 3



UDL	planeUDL	loadUDL	nnodesUDL	nodeUDL[n]'s
1	4	-1	5	1 2 5 4 6
2	4	-1	6	2 5 7 6 1 3

Figure 13 Plate segment with hole and subjected to a UDL.

MAXIMUM DEFLECTION FACTOR:

```
                zf
                ________________________________________________


NODES:
                ni
                ________________________________________________

ni times    |   i    x   y    z
            | or
            |   i    x   y    *
            | or
            |   i    *   master1  master2  master3  master4   z
            | or
            |   i    *   master1  master2  master3  master4   *
            |   ________________________________________________


PLANES:
                nj
                ________________________________________________

nj times    |   j    nnop   nop[n] (n=1...nnop)
            |   ________________________________________________


LINES:
                nk
                ________________________________________________

nk times    |   k    pi   pj     pm   pn    brn
            | or
            |   k    pi   pj     *
            |   ________________________________________________


BENDING PROPERTIES:
                nbr
                ________________________________________________

nbr times   |   br    reinf
            |   ________________________________________________
            |
            |   mpxp  mpyp    mpxn  mpyn
            | or
            |   mpxp  mpsp    mpxn  mpsn     beta
            |   ________________________________________________
```

```
POINT LOADS:
                  nPL
                  ______________________________________________________

nPL times   :     PL    planePL    loadPL    nodePL
            :     ______________________________________________________


LINE LOADS:
                  nLL
                  ______________________________________________________

nLL times   :     LL    planeLL     loadLL1   nodeLL1     loadLL2  nodeLL2
            :     ______________________________________________________


UNIFORMLY DISTRIBUTED LOADS:
                  nUDL
                  ______________________________________________________

nUDL times  :     UDL    planeUDL    loadUDL    nnodesUDL
            :                                     nodeUDL[n] (n=1...nnodesUDL)
            :  or
            :     UDL    planeUDL    loadUDL    *
            :     ______________________________________________________


SEARCH DATA:
                  nmoves
                  ______________________________________________________

nmoves      :     nnms   steps
times       :     ______________________________________________________
            :
            :     nnms times      :    node    xi   yi    xf   yf
            :                     :    _________________________________
```

Note: The horizontal lines across the table indicate where a new line must be started. The vertical lines indicate repeated lines.

Table 1   Summary of input data.

Chapter 3 Features of the Program

Yield Lines is menu driven and relatively simple to use. An outline of the program, from the user's point of view, is given below.

Title Screen and Opening Menu

Yield Lines starts by reading the data file for a particular problem. The user has to simply specify the name of the data file. Drives and path names are supported.

Search Menu

If the data specifies a search for the optimum solution, Yield Lines lets the user choose between:

A: analysing All the patterns created by the search,

L: analysing only the patterns with the initial and final node positions (Limits) of each movement,

S: Searching directly for the minimum load factor (or maximum resistance factor).

If there are no search specified in the data, the search menu is not displayed. Analysis of the mechanism is performed and the screen-with-drawing is displayed immediately.

Screen with Drawing

Patterns analysed by Yield Lines are displayed on the screen. Nodes deflecting upwards, downwards, and not deflecting are each shown differently. Sagging, hogging, and construction lines are indicated by bright solid lines, bright dash lines, and light lines, respectively. Point loads, line loads, and UDL's are also shown. The calculated load factor or resistance factor are displayed at the bottom of the screen.

If A was chosen in the search menu, each pattern created by the search is displayed in turn by pressing <return>.

If L was chosen, the limits of each movement are displayed in turn by pressing <return>. The movement number and whether the position is initial or final are shown on the right hand side of the screen.

If S was chosen in the search menu, Yield Lines first analyses all the patterns created by the search. The patterns are not drawn to speed up execution, but a count of the patterns being analysed is indicated. YL then finds the optimum pattern (minimum load factor or maximum resistance factor). This pattern is drawn on the screen. The total number of patterns tried and the number of valid patterns are shown on the right hand side of the screen. (A pattern is not valid if not kinematically admissible or because of some other reasons. See the section on Error Messages).

A small menu on the right hand side of the screen-with-drawing lets the user ask Yield Lines to:

<return>	:	continue.
A or D	:	display either the load factor (Analysis) or the bending resistance factor (Design) at the bottom of the screen.
N	:	show or hide Nodes.
R	:	list detailed Results for the pattern drawn (see below).
Q	:	Quit to the previous menu.
C	:	Color. Allows the user to change the colors of the Screen with Drawing. For B/W graphic monitor, a black background (0) is preferable. The choice of colors can be saved for future session of Yield Lines. The file called YLINES-C.CNF contains the color information.

Detailed Results

The detailed results can be directed to the screen, to the printer or to a file. These results correspond to the pattern drawn and they include:

- an echo of the input data. Coordinates of slave nodes and nodes with unspecified z-coordinate are given as computed by YL.
- the coefficients of the algebraic equation of the planes.
- the bending sign (sagging or hogging), the plastic resistance m_p (from Eq. (6), (7), (9) or (10)), the rotation, the length, and the energy dissipated by each yield line.
- for the point loads, for the line loads, and for the UDL's: the value, the location, and the displacement of the resultant load, and the work done by each load.
- the total energy dissipated by the yield lines, the work done by all the loads and, finally, the load factor λ or the bending resistance factor γ for the pattern analysed.

Error and Warning Messages

Error 1: Insufficient master nodes.
Cannot locate slave node(s):
list of nodes

Meaning: Yield Lines could not find the x and y location of the master nodes for these slave nodes. Check the input data for the slave and master nodes.

Error 2: Invalid location
for slave node(s):
list of nodes

Meaning: The master nodes define parallel lines and the slave nodes cannot be located. Check the input data for the slave and master nodes and for the search.

Error 3: Insufficient nodes to
define plane(s):
list of planes

Meaning: For each of these planes, Yield Lines could not find at least 3 nodes for which the x, y, z coordinates could be determined. Check the input data for the planes.

Error 4: Mechanism not admissible.
Plane(s) not flat:
list of planes

Meaning: These planes are not flat. Check the input data for the planes and for the search. All the planes must be flat!

Warning 1: 0 resultant on UDL(s):
list of UDL's

Meaning: The resultant value of these UDL's is zero. This may be what you want. If not, the sequence of nodes around the UDL is probably incorrect. Check the input data for the UDL's and for the search.

Warning 2: MIN LF, MAX RF on moves(s):
list of movements

Meaning: The minimum Load Factor, or the maximum Resistance Factor was found at the initial or final limit of these movements during the search. This means either that:

- The optimum pattern is outside the limits of these movements. Change the the limits of the movements.
- The optimum pattern is inside the limits of these movements, but not enough steps were used. Increase the number of steps.
- The correct solution is indeed at the limit of these movements. The optimum pattern has been found. This situation arises in certain problems, usually when the limits of the movements correspond to some limits of validity of the mechanism studied.

Technical Aspects

Yield Lines was written in Pascal using the Turbo-Pascal package (Ref.[7]), and it was designed to run on an IBM PC or compatible.

Yield Lines version 1.0 can analyse problems with 100 nodes, 100 planes, and 100 lines. (These limits could be increased by modifying the program. The use of pointers would permit access to the available memory of the computer being used).

Computing time varies depending on the size of problem and the type of machine used. On an IBM AT running at 8 Mhz, with 640 K of memory and a 8087 math chip, 200 variations of a yield line mechanism with 20 nodes are analysed in approximately one minute.

System requirements:

- IBM PC, XT, AT, or compatible
- 140 Kbytes of free memory
- one $5\frac{1}{4}$" diskette drive
- PC- or MS-DOS 2.00 or later version
- 8087 math coprocessor
- B/W graphic or color graphic monitor
- IBM CGA (Color Graphics Adapter), EGA (Enhanced Graphics Adapter), or compatible. The Hercules Graphics Card adapter is not supported. (Modification of the program would permit support of the Hercules card).

Yield Lines is not copy protected and its source code is included with the program diskette.

Further information about the program can be obtained from:

McGill University
Department of Civil Engineering
and Applied Mechanics
817 Sherbrooke St. W.
Montreal, Que., CANADA H3A 2K6
c/o Professor Richard G. Redwood
Ref.: Yield Lines, a Numerical Yield Line Analysis Program

Yield Lines Diskette

The Yield Lines diskette contains all the files required to run the program Yield lines. These files are briefly described below.

Yield Lines Program Files

AUTOEXEC.BAT and YL.BAT are batch files that start Yield Lines.

YLINES.COM contains the program Yield Lines compiled and ready to be run.

Example Files

The eleven files EXAMPLE.1A, EXAMPLE.1B, EXAMPLE.2, ..., EXAMPLE.10 contain the input data for the examples of Appendix A.

Other Files

YLINES-C.CNF contains the selection of colors for the Screen with Drawing. This file is automatically read and written to by Yield Lines.

YL-READ.ME contains a message displayed by YL.BAT.

Source Code

Due to its large size (>64 K), the source code of Yield Lines is divided into the following files:

YLINES-1.PAS : main procedures (yield line calculations, menus, graphic displays, etc.).

YLINES-2.PAS : declaration of variables, constants, etc.

YLINES-3.PAS : procedures for reading the input data files.

YLINES-4.PAS : procedures for printing detailed results

YLINES-5.PAS : procedures for selecting and saving the colors of the Screen with Drawing.

The source code can be consulted, listed, or even edited and recompiled by the user.

Required DOS and Pascal Files

GRAPH.P and GRAPH.BIN contain graphic procedures required when compiling the source code. These files are supplied with the Turbo-Pascal package [7].

GRAPHICS.COM is required to allow the Shift-PrintScreen keys to produce a copy of the Screen with Drawing on the printer. This file is supplied with the MS-DOS or PC-DOS package.

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Appendix A Examples

This appendix presents several examples, with comments, of yield line problems illustratring the input data and use of the program **Yield Lines** for the numerical solution.

The words Z-FACTOR, EXAMPLE.xx, NODES, PLANES, LINES, etc. on some lines in the data files (Tables A.1 to A.10) are comments that were included for clarity. YL ignores comments on some lines if they are put at least one space to the right of the last character of the required input data. Lines on which comments can be put are: \zf, \ni, \nj, \nk, \nbr, \br reinf, \nPL, \nLL, \nUDL, \nmoves, \nnms steps. (By way of exception, comments can be put right next to reinf, e.g. RTHOTROPIC right next to 0). Comments must not appear on the other lines (\i x y z, \j nnop nop[n], etc.) as this would make the file unreadable by YL.

The hard copy of the screen displays (Fig. A1(c), A1(d), A2(c), etc.) were readily obtained on a printer using the Shift-PrintScreen keys on the IBM-PC. Note that the DOS Graphics program was run beforehand so that PrintScreen could work with such graphic displays.

Example 1A

Problem (analysis): A 12 ft (3.66m) by 18 ft (5.49 m) rectangular panel at the edge of a floor system is free (unsupported) along one long side and is continuous with adjacent panels at supporting beams along the other three sides. The slab is shown in Fig. A1-a. The slab has an ultimate moment resistance/unit length of 3690 lb-ft/ft of width (16.4 kN.m/m). Find the ultimate uniformly distributed load that can be carried by the panel.

Reference: Ref. 3, Section 7.9.3, p. 329, and Example 8.4., p 434.

Algebraic Solution:

Data: $\ell_x = 18$ ft
$\ell_y = 12$ ft
$i_1 = i_2 = 1, \quad \mu = 1$
$m_{uy} = 3690$ lb.ft/ft

Consider Mode 2:

$$K_3 = \frac{4}{\mu}\left(\frac{\ell_y}{\ell_x}\right)^2 \frac{1+i_1}{1+i_2} = \frac{4}{1}\left(\frac{12}{18}\right)^2 \frac{1+1}{1+1} = 1.7\dot{7}$$

$$\ell_1 = \frac{\sqrt{1+3K_3}-1}{k_3}\,\ell_y = \frac{\sqrt{1+3 \cdot 1.7\dot{7}}-1}{1.7\dot{7}}\,12 = 10.237 \text{ ft} \qquad (7.62) \text{ of Ref. 3}$$

$$w_u = \frac{6(1+i_2)m_{uy}}{\ell_1^2} = \frac{6(1+1)3690}{10.237^2} = 422.5 \text{ psf} \qquad (7.63) \text{ of Ref. 3}$$

Solution using Yield Lines:

Input Data: see Fig. A1-b and Table A1-a

Nodes: An assumed deflection of z=-1 (*zf) is specified at nodes 5 and 6. The other nodes do not deflect (z=0).

Planes: All the nodes belonging to every plane are given in the data for the planes. This allows Yield Lines to check that all the nodes belonging to a plane actually 'sit' on the plane, i.e. that the nodes define a 'flat' plane. If this check is not satisfied, an error message is issued.

The nodes defining planes 2, 3, and 4 are given in sequence around the planes. The purpose of this sequencing is to use the same nodes later when defining the UDL's applied on these planes.

Lines: Line 7 is a construction line, therefore a * replaces the planes on each side of the line. Construction lines are used only to clarify the drawing displayed on screen.

UDL's: The three UDL's correspond to the portions of the loading applied on the three plate segments. All P_{UDL}'s are equal, and they are set equal to unity for convenience (negative because acting downwards).

The number of nodes defining a UDL is given as *, therefore the UDL is defined by using the same sequence of nodes that was used to define the plane on which the UDL is applied. This reduces the amount of data in the input file, but it is optional. The nodes defining the UDL can always be specified in the section on UDL's.

Search Data: Node 5 is moved through 50 steps to find the pattern with the minimum ultimate UDL.

Results: (see Fig. A1-c and Table A1-b) The solution obtained is $\lambda = 422.526$.

Example 1B

Problem (design): Same as Example 1A above, but from a design point of view. The panel must carry a factored total uniformly distributed load of 425 psf (4.78 kN/m^2). Find the required ultimate moment resistance/unit length of the slab.

Algebraic Solution:

Data: $w_u = 425$ lb.ft/ft

Consider Mode 2:

$$m_{uy} = \frac{l_1^2 w_u}{6(1+i_2)} = \frac{10.237^2 * 425}{6(1+1)} = 3711.6 \text{ psf} \qquad \text{from (7.63) of Ref. 3}$$

Solution Using Yield Lines:

Input Data: see Fig. A1-b and Table A1-c

Bending Resistance: All m_p's are assumed equal, and they are set equal to unity for convenience.

UDL's: All λP_{UDL}'s are equal to 425 lb.ft/ft.

Results: (see Fig. A1-d) The solution obtained is $\gamma = 3711.603$.

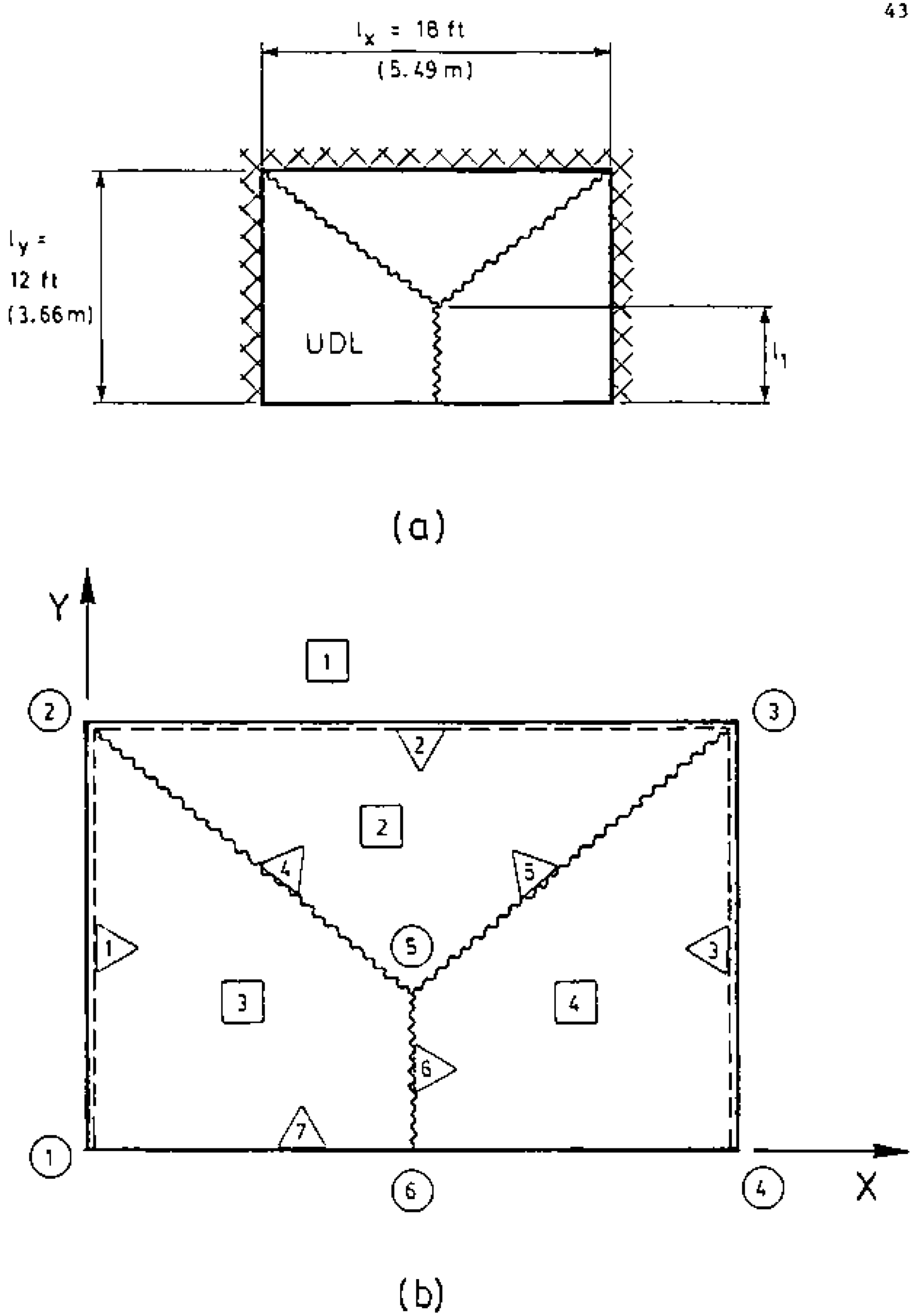


Figure A1 Example 1A. (a) Structure. (b) Model.

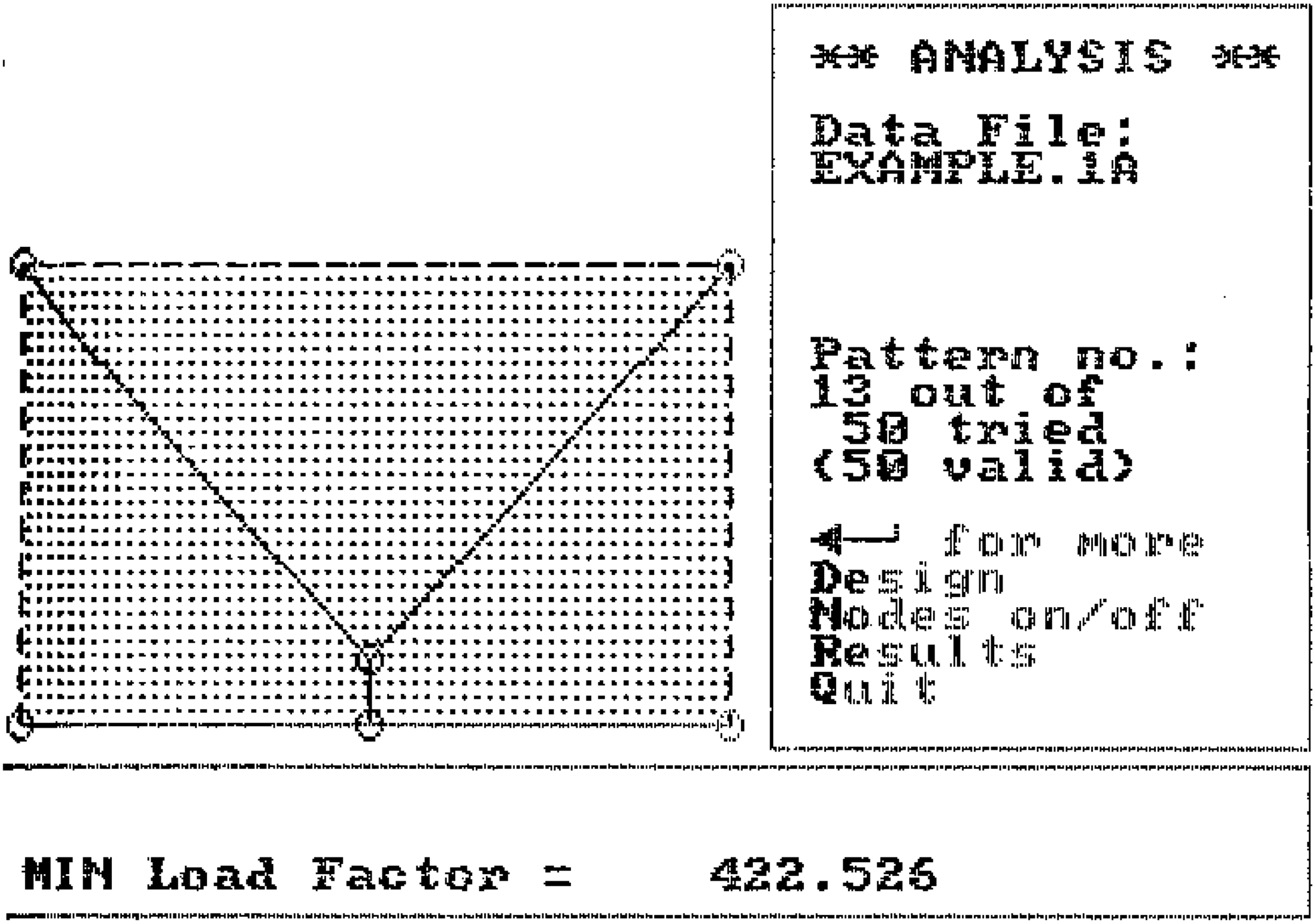


Figure A1 cont'd. (c) Screen display. Example 1A.

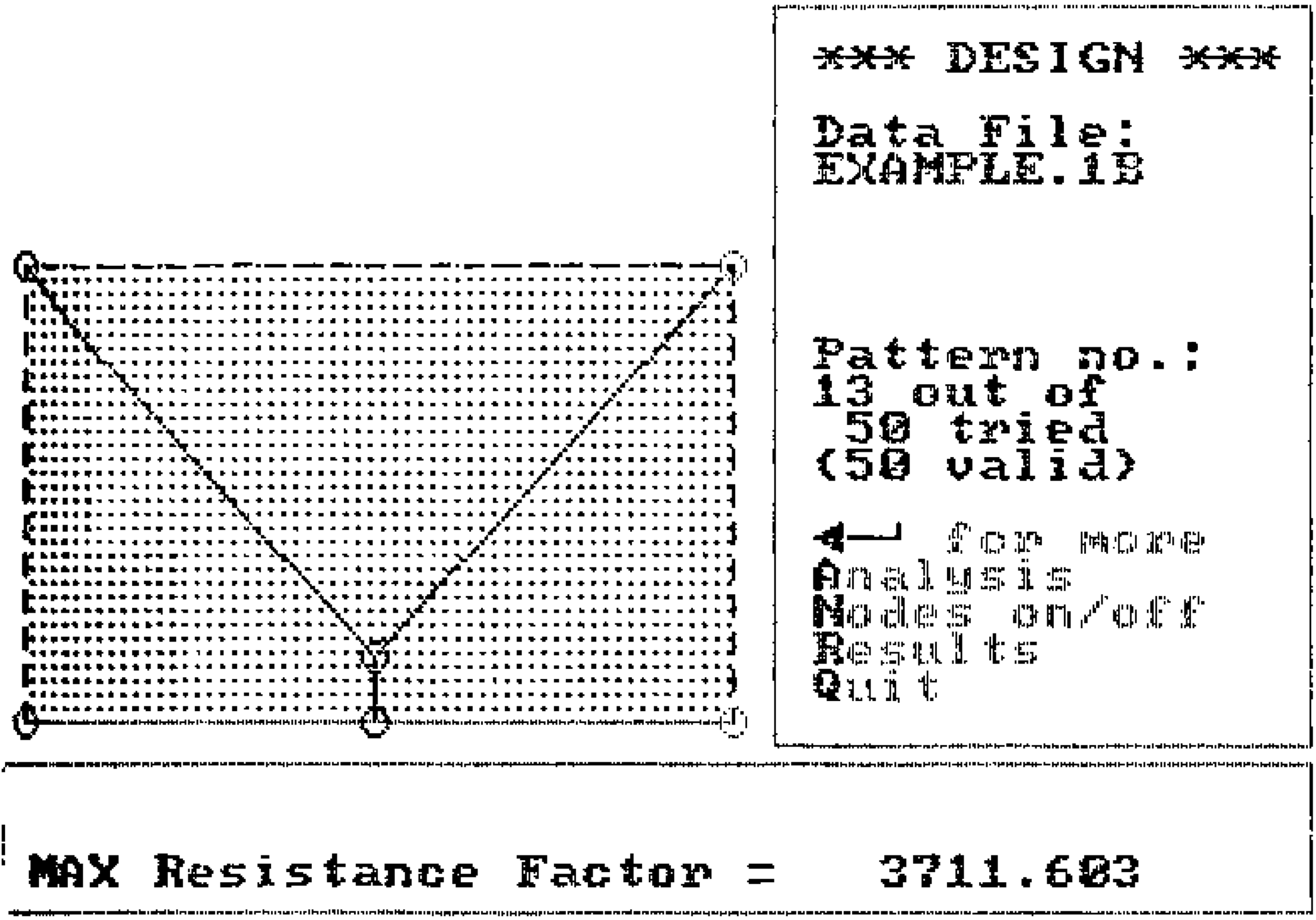


Figure A1 cont'd. (d) Screen Display. Example 1B.

```
1E-15  Z-FACTOR    EXAMPLE.1A
6  NODES
1   0  0  0
2   0 12  0
3  18 12  0
4  18  0  0
5   9  6 -1
6   9  0 -1
4  PLANES
1  4  1 2 3 4
2  3  2 3 5
3  4  1 2 5 6
4  4  3 4 6 5
7  LINES
1  1 2  1 3  1
2  2 3  1 2  1
3  4 3  4 1  1
4  2 5  2 3  1
5  5 3  2 4  1
6  6 5  3 4  1
7  1 4  *
1  BENDING RESISTANCE
1  ORTHOTROPIC
3690  3690  3690  3690
0  POINT LOAD
0  LINE LOAD
3  UDL'S
1  2  -1   *
2  3  -1   *
3  4  -1   *
1  MOVEMENT
1 50
5  9.0 0.5   9.0 5.5
```

Table A1 (a) Input data. Example 1A.

==========
DATA FILE : B:EXAMPLE.1A
==========

Z-COORDINATE FACTOR = 1.00000000000000E-015
=================

NODES
=====

NO.	COORDINATES X	Y	Z
1	0.000	0.000	0.000
2	0.000	12.000	0.000
3	18.000	12.000	0.000
4	18.000	0.000	0.000
5	9.000	1.724	-1.000
6	9.000	0.000	-1.000

PLANES
======

NO.	THROUGH THESE NODES P0	P1	P2	etc.
1	1	2	3	4
2	2	3	5	
3	1	2	5	6
4	3	4	6	5

PLANES
======

NO.	THROUGH THESE 3 NODES			ALGEBRAIC EQUATION A	B	C	D
1	1	2	3	0.000000	0.000000	-216.000000	0.000000
2	2	3	5	0.000000	1.80000E-014	-184.959184	-2.16000E-013
3	1	2	5	-1.20000E-014	0.000000	-108.000000	0.000000
4	3	4	6	1.20000E-014	0.000000	-108.000000	-2.16000E-013

LINES
=====

NO.	BETWEEN THESE 2 POINTS		BETWEEN THESE 2 PLANES		LENGTH	ROTATION (RAD)	BENDING SAN SIGN		MP	ENERGY DISSIPATED
1	1	2	1	3	12.000000	0.111111	Hogging	1	3690.000000	4920.000000
2	2	3	1	2	18.000000	0.097319	Hogging	1	3690.000000	6463.912612
3	4	3	4	1	12.000000	0.111111	Hogging	1	3690.000000	4920.000000
4	2	5	2	3	13.659653	0.147705	Sagging	1	3690.000000	7444.915490
5	5	3	2	4	13.659653	0.147705	Sagging	1	3690.000000	7444.915490
6	6	5	3	4	1.724490	0.222222	Sagging	1	3690.000000	1414.081633
7	1	4	*	*	18.000000	0.000000	no work	*	0.000000	0.000000

BENDING RESISTANCE PROPERTIES
==============================

NO.

1 ORTHOTROPIC
SAGGING: MX: 3690.000000 MY: 3690.000000
HOGGING: MX: 3690.000000 MY: 3690.000000

POINT LOADS
===========
WORK DONE BY ALL POINT LOADS = 0.000

NO POINT LOADS

LINE LOADS
==========
WORK DONE BY ALL LINE LOADS = 0.000

NO LINE LOADS

UNIFORMLY DISTRIBUTED LOADS
===========================
WORK DONE BY ALL UDLs = 77.173

UDL NO.	ON PLANE	LOAD VALUE	CENTROID COORDINATES X	Y	DISPLACEMENT (Z)	TOTAL LOAD	WORK DONE
1	2	-1.000000	9.000	8.575	-0.333333	-92.480	30.827
2	3	-1.000000	3.377	4.072	-0.375217	-61.760	23.173
3	4	-1.000000	14.623	4.072	-0.375217	-61.760	23.173

FINAL RESULTS
=============

ENERGY DISSIPATED BY THE YIELD LINES : 32607.8252234359

WORK DONE BY ALL THE LOADS : 77.1734693878

*** ANALYSIS *** LOAD FACTOR : 422.5263614831

*** DESIGN *** RESISTANCE FACTOR : 0.0023667162

Table A1 cont'd. (b) Detailed results. Example 1A.

```
1E-15  Z-FACTOR     EXAMPLE.1B
6  NODES
1   0  0  0
2   0 12  0
3  18 12  0
4  18  0  0
5   9  6 -1
6   9  0 -1
4  PLANES
1  4   1 2 3 4
2  3   2 3 5
3  4   1 2 5 6
4  4   3 4 6 5
7  LINES
1  1 2  1 3  1
2  2 3  1 2  1
3  4 3  4 1  1
4  2 5  2 3  1
5  5 3  2 4  1
6  6 5  3 4  1
7  1 4  *
1  BENDING RESISTANCE
1  ORTHOTROPIC
1  1  1  1
0  POINT LOAD
0  LINE LOAD
3  UDL'S
1  2  -425  *
2  3  -425  *
3  4  -425  *
1  MOVEMENT
1 50
5  9.0 0.5   9.0 5.5
```

Table A1 cont'd.   [d] Input Data.   Example 1B.

Example 2

Problem (analysis): Uniformly loaded orthotropic rectangular slab with all edges supported. Two simply supported edges, two fixed edges (see Fig. A2-a).

Reference: Ref. 3, section 7.9.2, p.327.

Algebraic Solution:

Data: $\ell_x = 10, \quad \ell_y = 6$
$m_{ux} = 1000, \quad m_{uy} = 3000$
$i_1 = 2, \quad i_2 = 2, \quad i_3 = 0, \quad i_4 = 0$

Solution

$\mu = m_{uy}/m_{ux} = 3000/1000 = 3$

$X = \sqrt{1+i_1}+\sqrt{1+i_3} = \sqrt{1+2}+\sqrt{1+0} = 2.732$
$Y = \sqrt{1+i_2}+\sqrt{1+i_4} = \sqrt{1+2}+\sqrt{1+0} = 2.732$

$$w_u = \frac{6 m_{uy} \mu Y^2}{\ell_y^2 (\ell_y/\ell_x)^2 \left\{\sqrt{(X/Y)^2+3\mu(\ell_x/\ell_y)^2}-(X/Y)\right\}^2}$$

$$= \frac{6*3000*3*(2.732)^2}{6^2(6/10)^2\left\{\sqrt{(2.732/2.732)^2+3*3*(10/6)^2}-(2.732/2.732)\right\}^2} = 1850.9$$

Solution Using Yield Lines:

Input Data: see Fig. A2-b and Table A2

Coordinates of Nodes: Any x and y coordinates can be specified for nodes 5 and 6 in the data for the nodes. The actual value of these coordinates is not important since the location of nodes 5 and 6 will be redefined during the search process which uses the search data.

Lines: Lines 3 and 4 represent simple supports The work done by these lines is made zero by specifying a * as for any construction line.

Bending Resistance: The first four sets of bending resistances correspond to the yield lines along the suppports. (Sets 3 and 4, corresponding to simple-support lines 3 and 4, will not be used). The fifth set correspond to the yield lines on the interior of the slab.

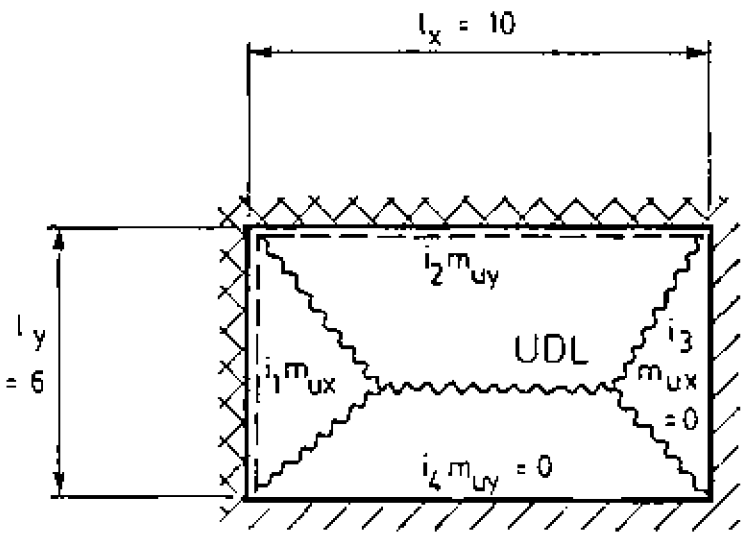
Search Data: The first movement (outer loop) moves node 5 horizontally. The second movement (intermediate loop) moves simultaneously both nodes 5 and 6 vertically. The third movement (inner loop) moves node 6 horizontally.

These movements will make nodes 5 and 6 cover rectangular areas on the slab. A node is moved over an area when two consecutive movements of the same node are specified. Note that the initial position of the nodes in the two consecutive movements are the same [position 2.5 2.0 for node 5, and 8.0 2.0 for node 6].

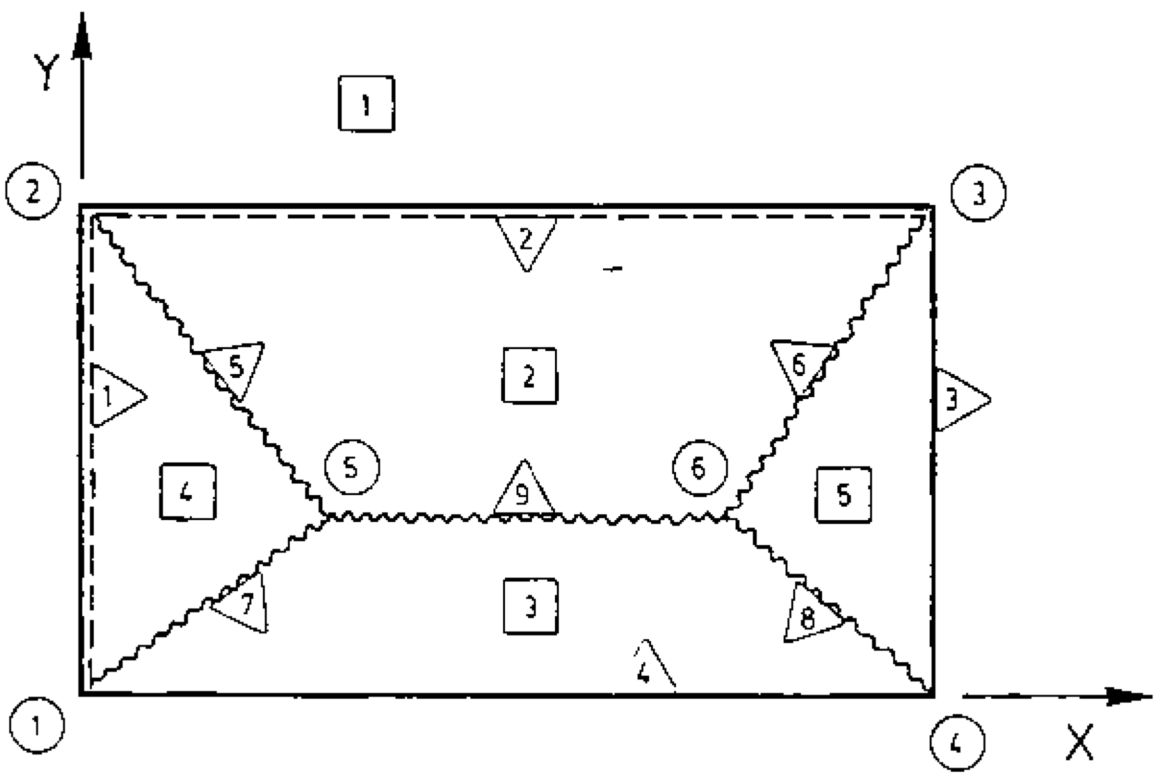
Note also that when the same node is specified in two consecutive movements, data for the node must be placed on the same line number of each movement in the data file. For example node 1 is placed as dummy data on the first line of the third movement in order to have node 6 on the second line, i.e. the same line number as in the second movement.

In the second movement, nodes 5 and 6 move down simultaneously in order to satisfy the kinematics of the problem.

Results: (see Fig. A2-c) The solution obtained is λ = 1851.957.



(a)



(b)

Figure A2 Example 2. (a) Structure. (b) Model.

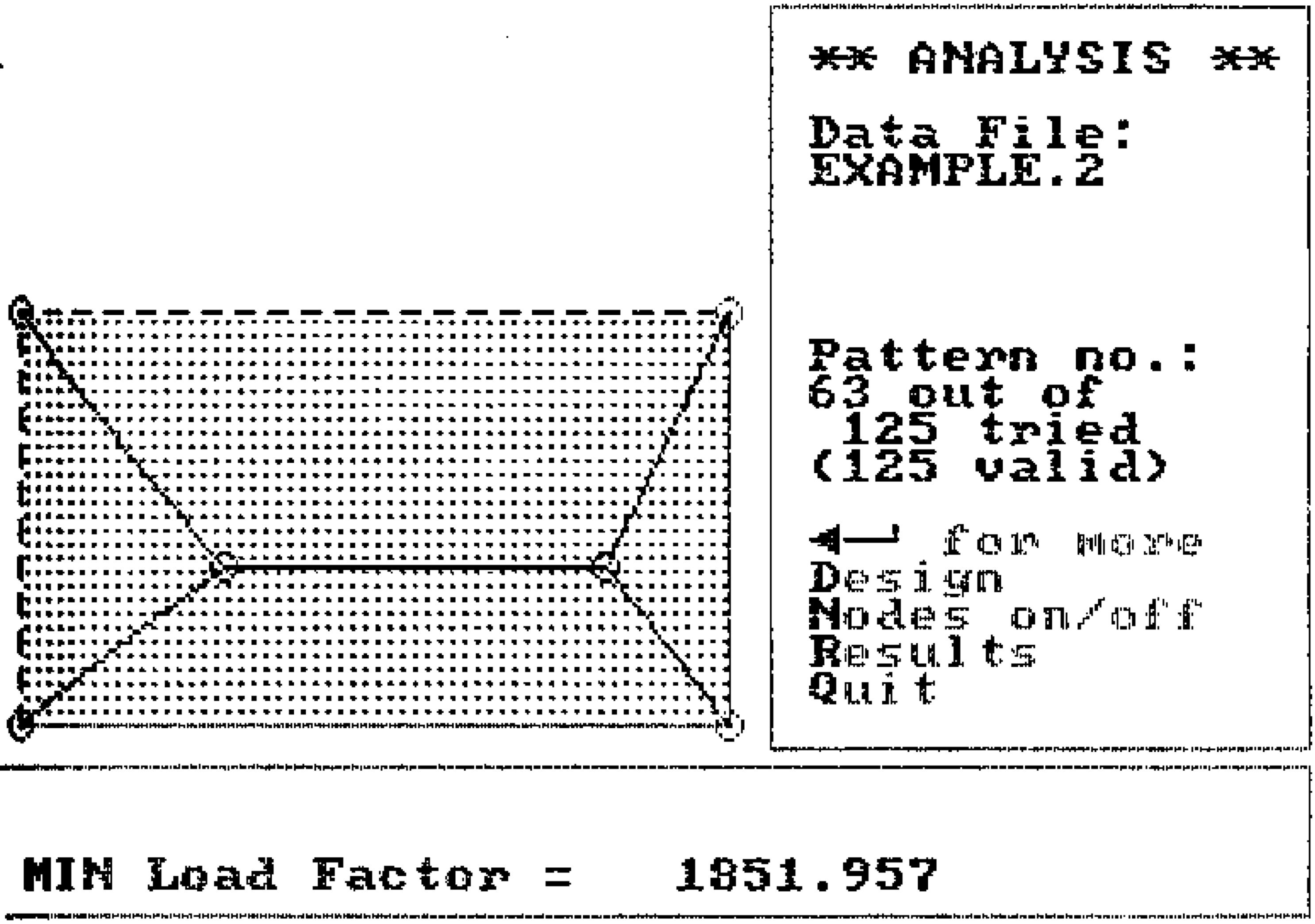


Figure A2 cont'd. (c) Screen Display.

```
1E-15  Z-FACTOR    EXAMPLE.2
6  NODES
1   0  0  0
2   0  6  0
3  10  6  0
4  10  0  0
5   2  3 -1
6   8  3 -1
5  PLANES
1  4  1 2 3 4
2  4  2 3 6 5
3  4  1 5 6 4
4  3  1 2 5
5  3  3 4 6
9  LINES
1  1 2  1 4  1
2  2 3  1 2  2
3  4 3  *
4  1 4  *
5  5 2  4 2  5
6  6 3  2 5  5
7  1 5  4 3  5
8  4 6  3 5  5
9  5 6  2 3  5
5  SETS OF BENDING RESISTANCE
1  ORTHOTROPIC
1000 3000 2000 6000
2  ORTHOTROPIC
1000 3000 2000 6000
3  ORTHOTROPIC
1000 3000 3000 7500
4  ORTHOTROPIC
1000 3000 3500 7500
5  ORTHOTROPIC
1000 3000 2000 6000
0  POINT LOAD
0  LINE LOAD
4  UDL'S
1   2  -1   *
2   3  -1   *
3   4  -1   *
4   5  -1   *
3  MOVEMENTS
1 5  1ST MOVEMENT
5  2.5 2.0   3.5 2.0
2 5  2ND MOVEMENT
5  2.5 2.0   2.5 2.5
6  8.0 2.0   8.0 2.5
2 5  3RD MOVEMENT
1 0 0 0 0
6  8.0 2.0   8.5 2.0
```

Table A2 Example 2. Input data.

Example 3

Problem (design): A corner panel of a floor system is continuous with adjacent panels at supporting beams along two edges and simply supported at the other edges except for a rectangular opening that is unsupported at its edges. Figure A3-a shows the panel. The factored loads are a uniformly distributed live load of 310 psf (14.84 kN/m^2) and a line load of 510 lb/ft (7.44 kN/m). Find the required ultimate moment resistance/unit length of the slab.

Reference: Ref. 3, Example 8.6, p.439.

Algebraic Solution:

Data: UDL: w_u = 310 psf, line load: p_u = 510 lb/ft (factored loads)

Consider the assumed yield line Pattern 2:

approximate solution: $m'_u = m_u$ = 3510 lb-ft/ft of width (Ref. 3)

Solution using Yield Lines:

Input Data: see Fig. A3-b and Table A3

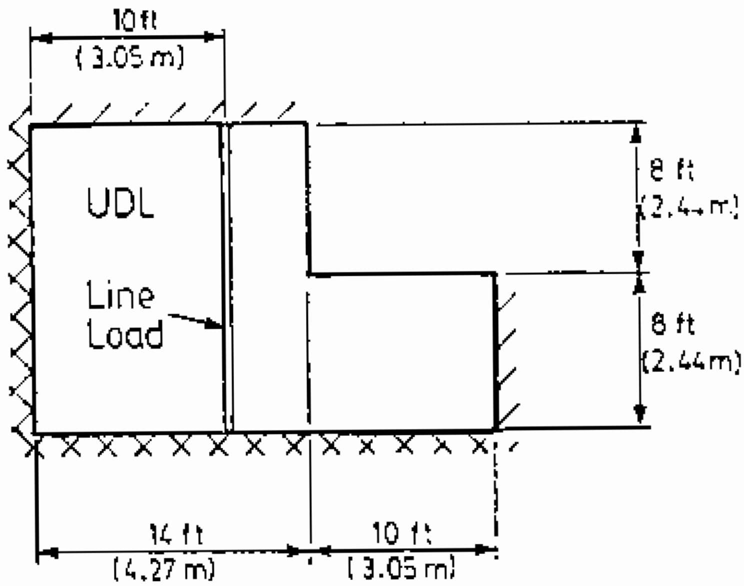
Nodes: Nodes 5, 7, 9 and 11 are slave nodes and the z coordinate of nodes 5, 7, 9, 10 and 11 is not specified. The location and deflection of these nodes are calculated by the program.

Lines Loads: The three line loads correspond to the portions of the line loading applied on three different plate segments.

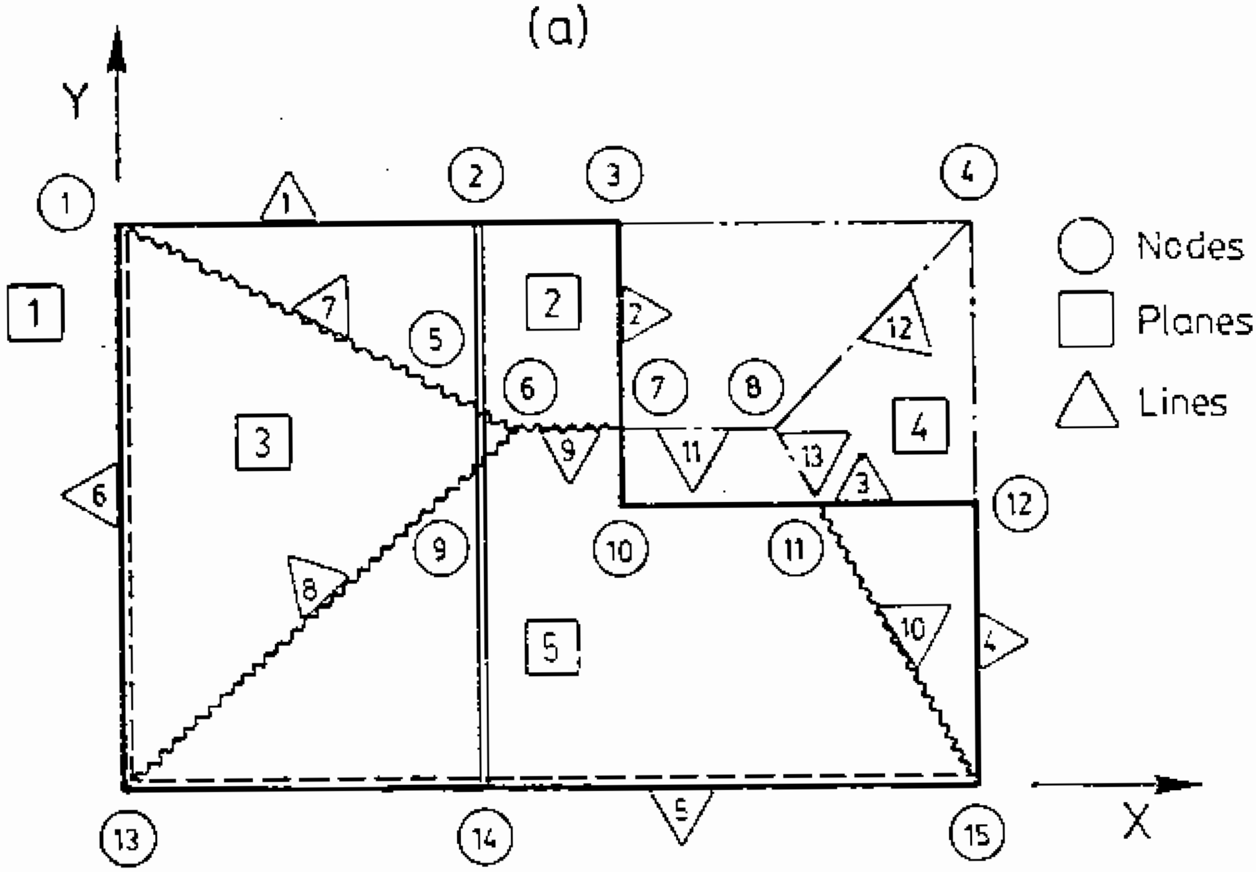
Search Data: Node 6 covers a rectangular area on the slab between the line load and the opening, while for each position of node 6, node 8 goes from left to right in the opening. In movement 3, node 13 serves as dummy data so that node 8 is on the second line in both the 2nd and 3rd movements.

Slave nodes 5, 7, 9 and 11 are carried along by master nodes 6 and 8 in all the patterns created by the search.

Results: (Fig. A3-c) The solution obtained is γ = 3514.959.



(a)



(b)

Figure A3   Example 3. (a) Structure. (b) Model.

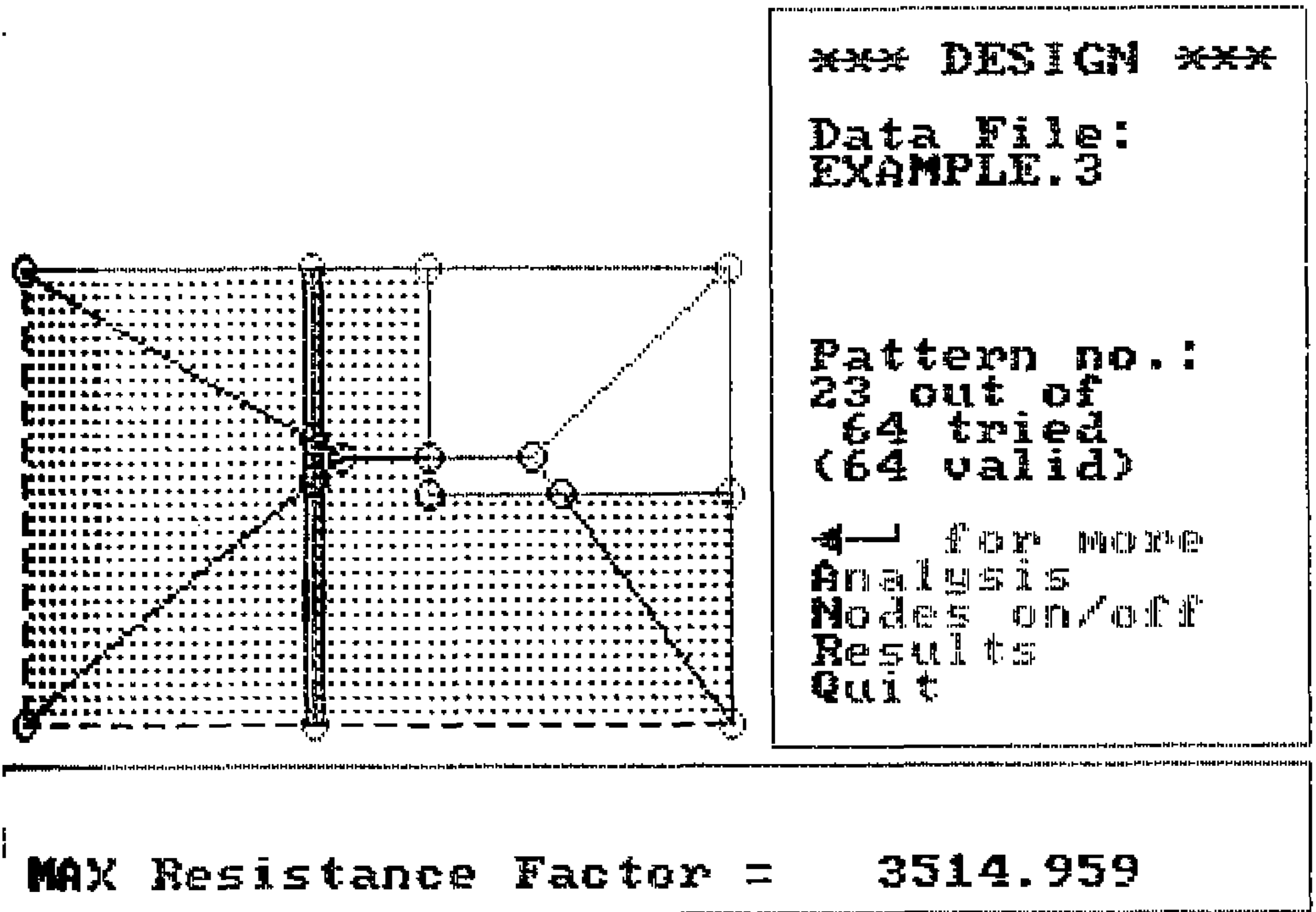


Figure A3 cont'd. (c) Screen Display.

```
 1E-15  Z-FACTOR    EXAMPLE.3
15  NODES
 1    0      16  0
 2   10      16  0
 3   14      16  0
 4   24      16  0
 5                    *  1  6  2 14 *
 6   11      10 -1
 7                    *  6  8  3 10 *
 8   17.125  10 -1
 9                    * 13  6  2 14 *
10   14       8  *
11                    * 10 12  8 15 *
12   24       8  0
13    0       0  0
14   10       0  0
15   24       0  0
 5  PLANES
 1   8   1  2  3  4 12 15 14 13
 2   8   1  2  3  4  5  6  7  8
 3   5   1  5  6  9 13
 4   5   4  8 11 12 15
 5   9   6  7  8  9 10 11 13 14 15
13  LINES
 1    1  4  *
 2    3 10  *
 3   10 12  *
 4    4 15  *
 5   13 15  5 1  1
 6   13  1  1 3  1
 7    1  6  2 3  1
 8   13  6  3 5  1
 9    6  7  2 5  1
10   15 11  5 4  1
11    7  8  *
12    8  4  *
13    8 11  *
 1  SET OF BENDING RESISTANCE
 1  ORTHOTROPIC
 1 1 1 1
 0  POINT LOAD
 3  LINE LOADS
 1   2   -510  2   -510  5
 2   3   -510  5   -510  9
 3   5   -510  9   -510 14
 4  UDL'S
 1   2   -310  4   1  3  7  6
 2   3   -310  3   1  6 13
 3   5   -310  6  13  6  7 10 11 15
 4   4   -310  3  11 12 15
 3  MOVEMENTS
 1 4  1ST MOVEMENT
 6 10.5 10.0  12.0 10.0
 2 4  2ND MOVEMENT
 6 10.5 10.0  10.5  8.2
 8 16.0 10.0  16.0  8.2
 2 4  3RD MOVEMENT
13  0 0  0 0
 8 16.0 10.0  18.0 10.0
```

Table A3 Example 3. Input data.

Example 4

Problem (analysis): The slab shown in Fig. A4-a is simply supported on two columns and along one edge. The slab has an ultimate moment resistance/unit length of 5898 lb.ft/ft of width (26.2 kN.m/m). Find the ultimate uniformly distributed load that can be carried by the slab.

Reference: Ref. 3, Example 7.16, p.375.

Algebraic Solution:

Data: slab width = 12 ft (3.66 m), slab length = 18 ft (5.49 m)
other dimensions of slab shown in Fig. A4-a
$m'_u = m_u$ = 5898 lb-ft/ft of width (26.2 kN.m/m)

Consider Mode 1: (see Ref. 3)
approximate solution: w_u = 687 psf (32.89 kN.m/m^2),
exact solution: w_u = 676 psf (32.37 kN.m/m^2)

Solution using Yield Lines:

Input Data: (Fig. A4-a and Table A4)

Nodes: Nodes 14 and 15 do not belong to the structure. They are included so that all the patterns are drawn at the same scale on the screen. The drawing of a pattern always include all the nodes, and it is scaled to fill the screen.

The z-coordinates of nodes 3, 4, 9, 10, and 12 are unspecified (*), and therefore these coordinates are calculated by Yield Lines when required, i.e. for each pattern tried during the search process. Note that nodes 3 and 4 will be deflecting upwards.

Slave Nodes: Nodes 7, 8, 9, and 10 are slave nodes. Nodes 9 and 10 have one of their master nodes (nodes 7 and 8, respectively) which itself is a slave node. This is accepted by Yield Lines.

The axes of rotation in the mechanism are used advantageously in establishing the arrangement of slave nodes.

Search Data: The use of slave and master nodes is directly related to the searching process. In this example, it is convenient to perform the search by moving only master nodes 11 and 13. The slave nodes are automatically carried along by their master nodes. Using slave nodes has the advantage of restricting the movements of the slave nodes within the limits of validity for the mechanism.

If slave nodes are not used, care must be taken that nodes 7, 8, 9 and 10 are not placed in a wrong position, i.e. nodes 7 and 8 must be outside the slab, and the x coordinate of nodes 9 and 10 must be smaller than that of node 11.

UDL's: The nodes defing the UDL's are given in the data for the UDL's. It was not possible to use the nodes defining the planes because the planes have nodes that do not belong to the UDL's.

Results: (Fig. A4-b) The solution obtained is λ = 682.411.

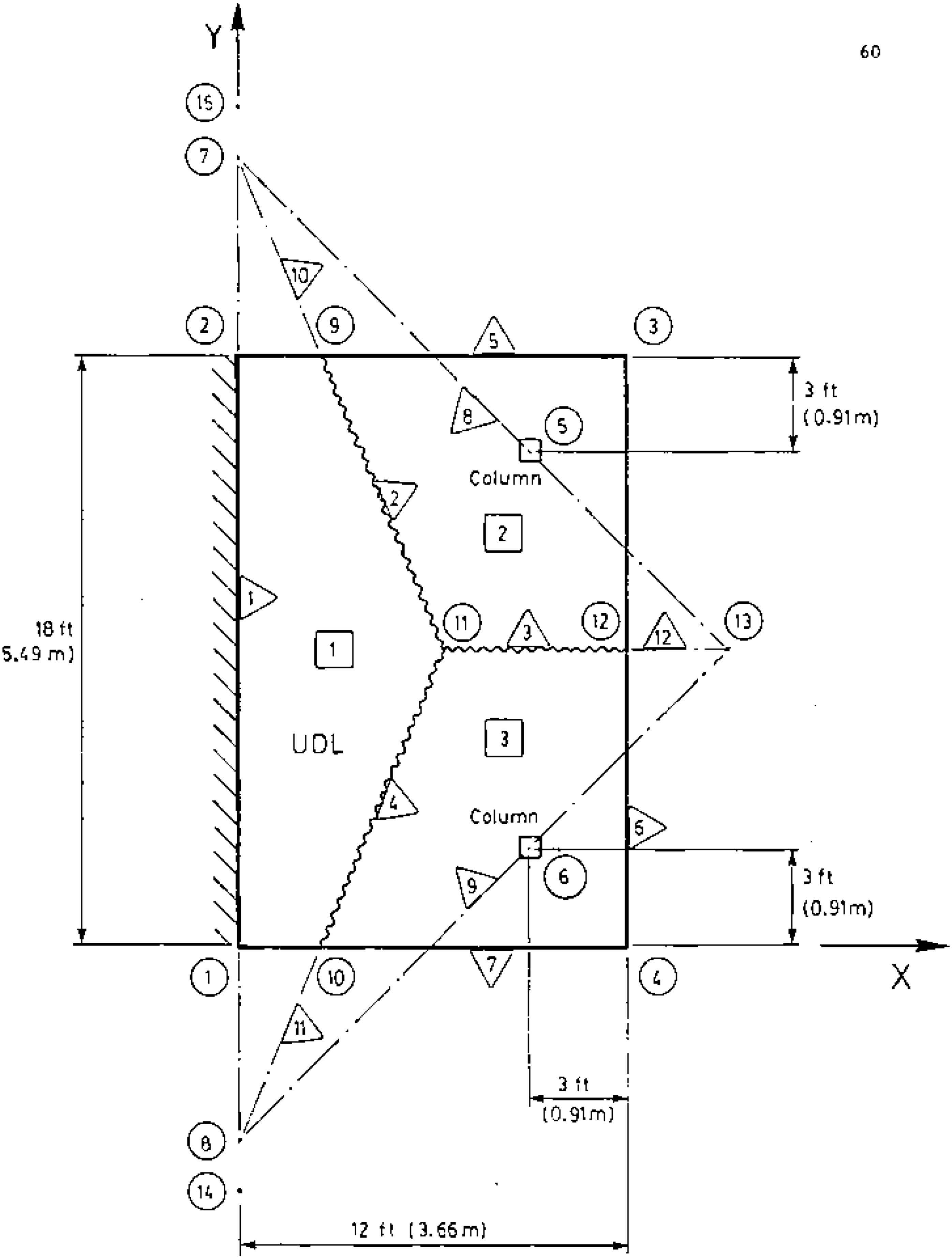


Figure A4 Example 4. (a) Structure and Model.

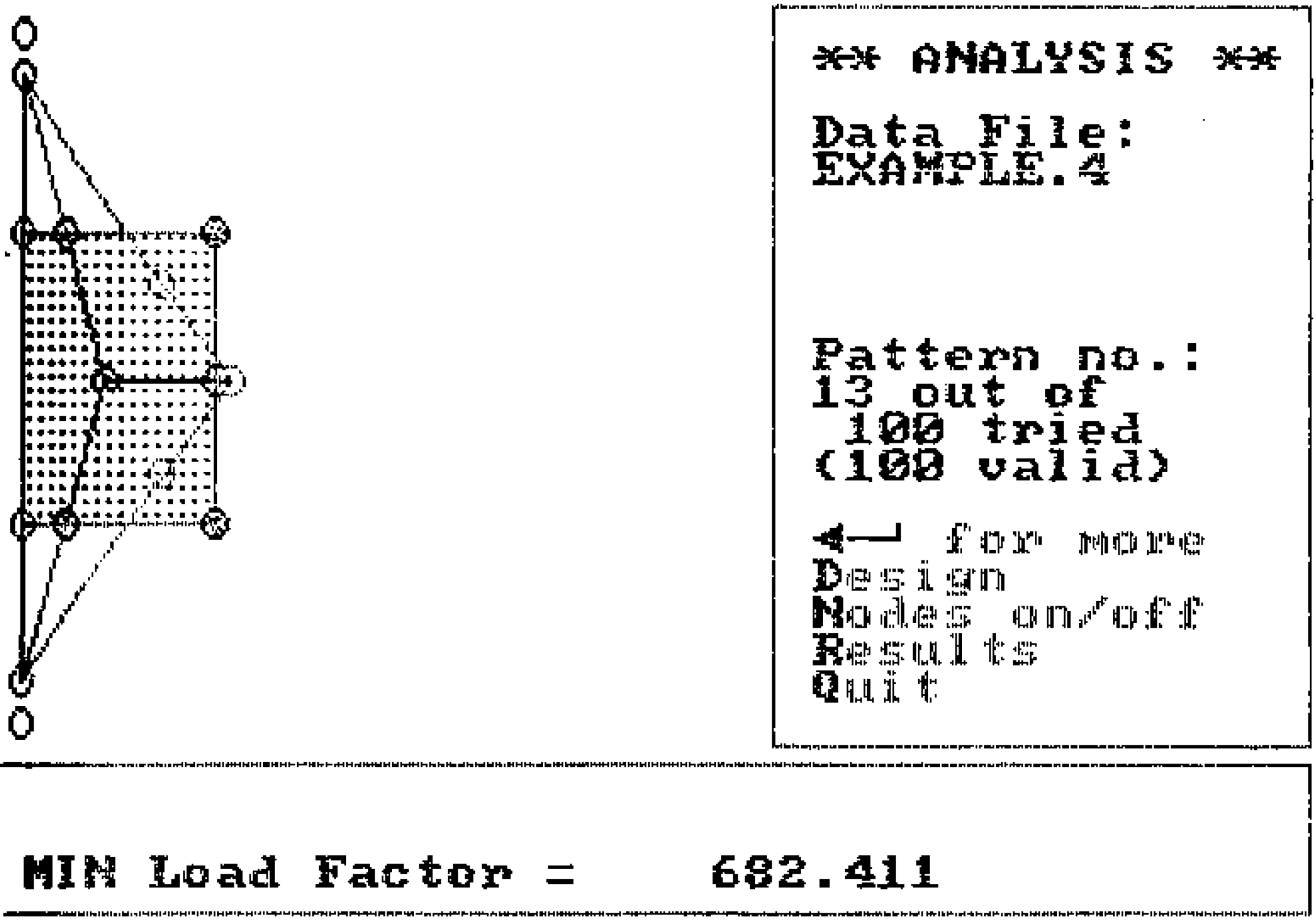


Figure A4 cont'd. (b) Screen Display.

```
 1E-15  Z-FACTOR     EXAMPLE,4
15  NODES
 1   0    0   0
 2   0   18   0
 3  12   18   *
 4  12    0   *
 5   9   15   0
 6   9    3   0
 7                  *  1  2  5 13  0
 8                  *  1  2  6 13  0
 9                  *  2  3  7 11  *
10                  *  1  4  8 11  *
11   6    9  -1
12  12    9   *
13  15    9   0
14   0 -12.5  0
15   0  30.5  0
 3  PLANES
 1   7   7  2  9 11  1 10  8
 2   7   7  9  3  5 11 12 13
 3   7  11 12 13  6 10  4  8
12  LINES
 1    8  7    *
 2   11  9    1 2   1
 3   11 12    2 3   1
 4   10 11    1 3   1
 5    2  3    *
 6    3  4    *
 7    1  4    *
 8    7 13    *
 9    8 13    *
10    7  9    *
11    8 10    *
12   12 13    *
 1  SET OF BENDING RESISTANCE
 1  ORTHOTROPIC
 5898  5898  5898  5898
 0  POINT LOAD
 0  LINE LOAD
 3  UDL'S
 1  1   -1  5   2  9 11 10 1
 2  2   -1  4   9  3 12 11
 3  3   -1  4  10 11 12  4
 2  MOVEMENTS
 1 10   1ST MOVEMENT
11    5.0 9.0   10.0 9.0
 1 10   2ND MOVEMENT
13  12.5 9.0   15.5 9.0
```

Table A4 Example 4. Input data.

Example 5

Problem (analysis): A uniformly loaded isotropically reinforced square slab is simply supported along two adjacent edges and on a column in the opposite corner as shown in Fig. A5-a. For a unit ultimate moment resistance/unit length of the slab find the ultimate uniform load per unit area.

Reference: Ref. 3, Example 7.17, p.380.

Algebraic Solution:

Let $m_u=1$ and $\ell=10$.

$$w_u = 10.67 \frac{m_u}{\ell^2} = 10.67 \frac{1}{10^2} = 0.106\dot{6}$$

Solution using Yield Lines:

Input Data: (Fig. A5-a and Table A5)

As in the previous example, slave nodes and unspecified z coordinates are used with advantage to facilitate the search.

Nodes: The column, at node 4, is simply a point that does not move nor deflect. It is not treated differently than the other nodes.

Slave Nodes: Node 6 is a slave of node 9, which itself is a slave of node 1.

Search Data: Node 1 moves along the y axis, while node 5 covers a square area on the slab.

Results: (Fig. A5-b) The solution obtained is λ=0.106667.

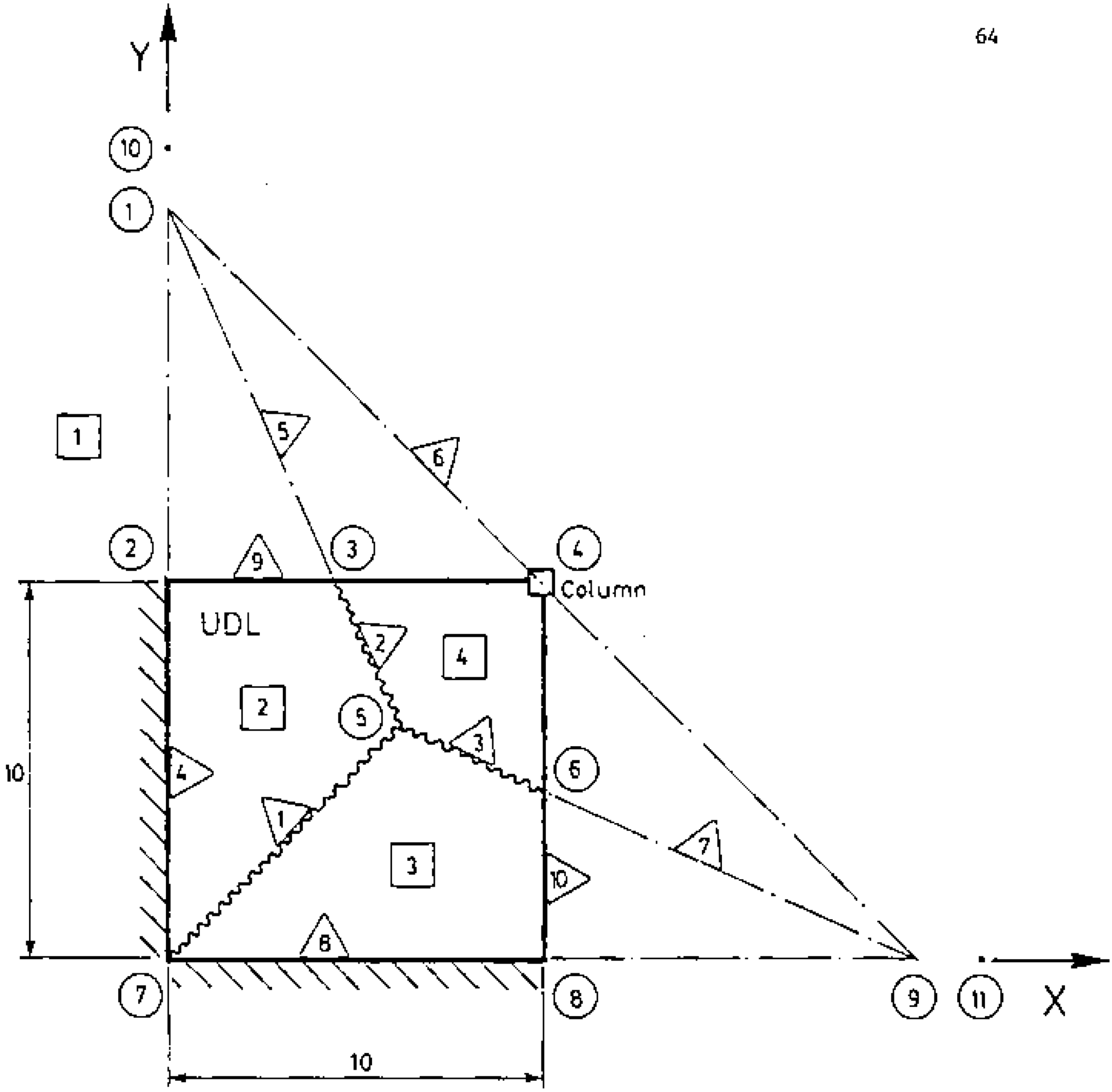


Figure A5 Example 5. (a) Structure and Model.

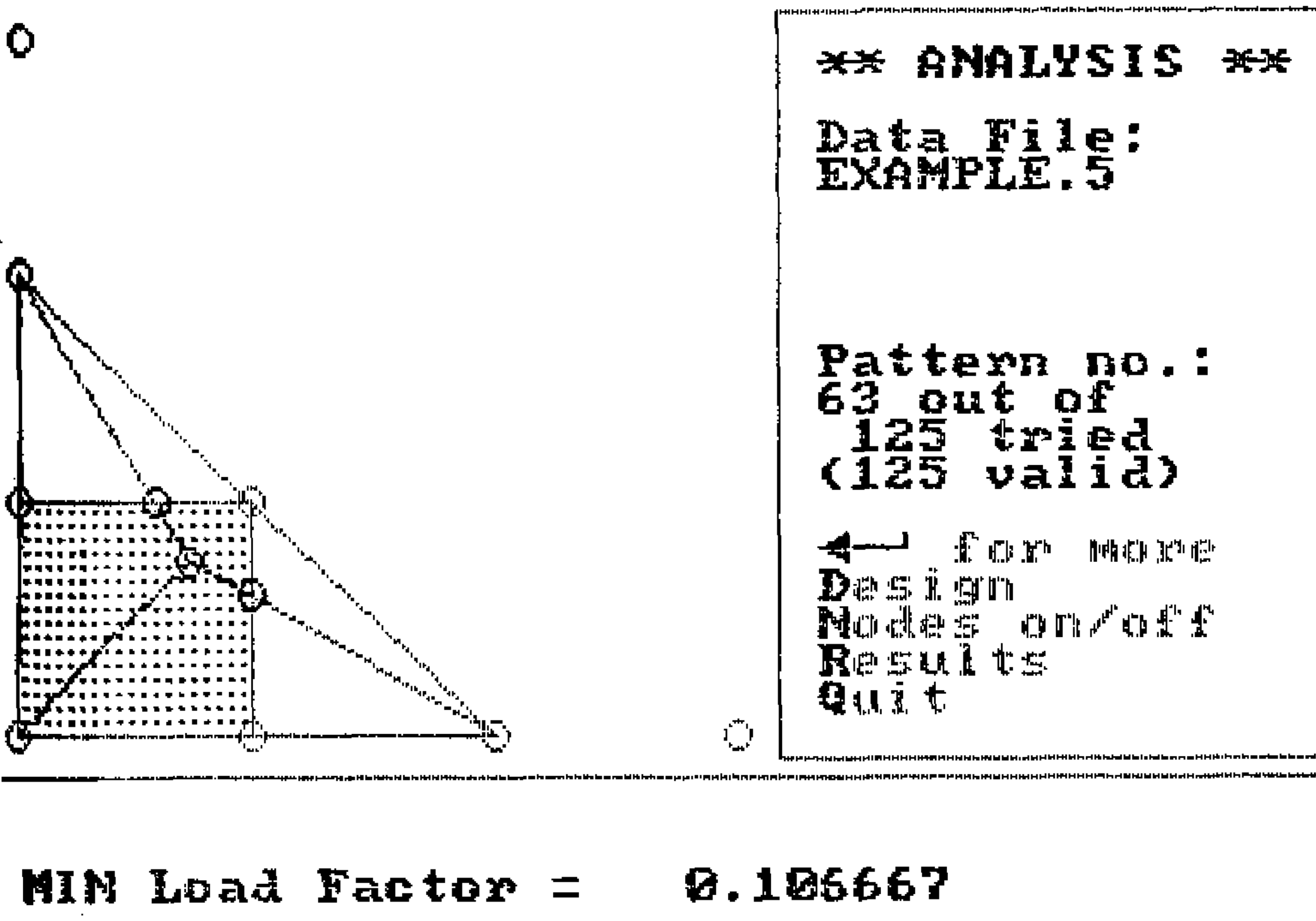


Figure A5 cont'd. (b) Screen Display.

```
 1E-15  Z-FACTOR     EXAMPLE.5
11   NODES
 1     0 20     0
 2     0 10     0
 3                    * 2 4 1 5  *
 4    10 10     0
 5   7.5  7.5 -1
 6                    * 4 8 5 9  *
 7     0  0     0
 8    10  0     0
 9                    * 1 4 7 8  0
10     0 30     0
11    30  0     0
 4   PLANES
 1   6    1 2 7 8 9 4
 2   5    1 2 3 5 7
 3   5    7 5 6 8 9
 4   6    1 3 4 5 6 9
10   LINES
 1   7  5   2  3   1
 2   3  5   4  2   1
 3   5  6   4  3   1
 4   1  7   *
 5   1  3   *
 6   1  9   *
 7   6  9   *
 8   7  9   *
 9   2  4   *
10   4  8   *
 1   SET OF BENDING PROPERTIES
 1   ORTHOTROPIC
 1   1  1  1
 0   POINT LOAD
 0   LINE LOAD
 3   UDL'S
 1   2   -1  *
 2   3   -1  *
 3   4   -1  4  3 4 6 5
 3   MOVEMENTS
 1   5  1ST MOVEMENT
 1   0 15   0 25
 1   5  2ND MOVEMENT
 5   5.5 9.5  9.5 9.5
 1   5  3RD MOVEMENT
 5   5.5 9.5  5.5 5.5
```

Table A5 Example 5. Input data.

Example 6

Problem (analysis): A skew slab is simply supported on opposite edges and subjected to a distributed load p/unit area and a point load P at the centre of the span. Analyse the slab for the assumed mechanism shown in Fig. A6-a. Note that the pattern analysed is not the only possible pattern nor necessarily the most critical pattern.

Reference: Ref. 2, Solution 3.6, p.78.

Algebraic Solution:

Data: slab width = 20, slab span = 10

$\psi = \tan^{-1}\frac{5}{10}$, skew angle $90^0-\psi = 63.434^0$

other dimensions of slab shown in Fig. A6-a

$m = 1$, $\mu = 0.8$, $i = 0.75$

$P = 1$, $p = 0.02$

$\beta = 0.5$

Solution:

$$m\left(\frac{2\mu(1+i)}{\beta} + 8\ \beta\ \cos^2\psi\right) = \frac{2}{3}\ \lambda p\ \beta\ L^2 + \lambda P \qquad (3.75)\text{ of Ref. 2}$$

$$1\left(\frac{2(0.8)(1+0.75)}{0.5} + 8(0.5)\cos^2\left(\tan^{-1}\frac{5}{10}\right)\right) = \frac{2}{3}\ \lambda(0.02)(0.5)(10^2) + \lambda(1)$$

solve and find $\lambda = 5.28$.

Solution using Yield Lines:

Input Data: (Fig. A6-b and Table A6)

Results: (Fig. A6-c) The solution obtained is $\lambda = 5.280$.

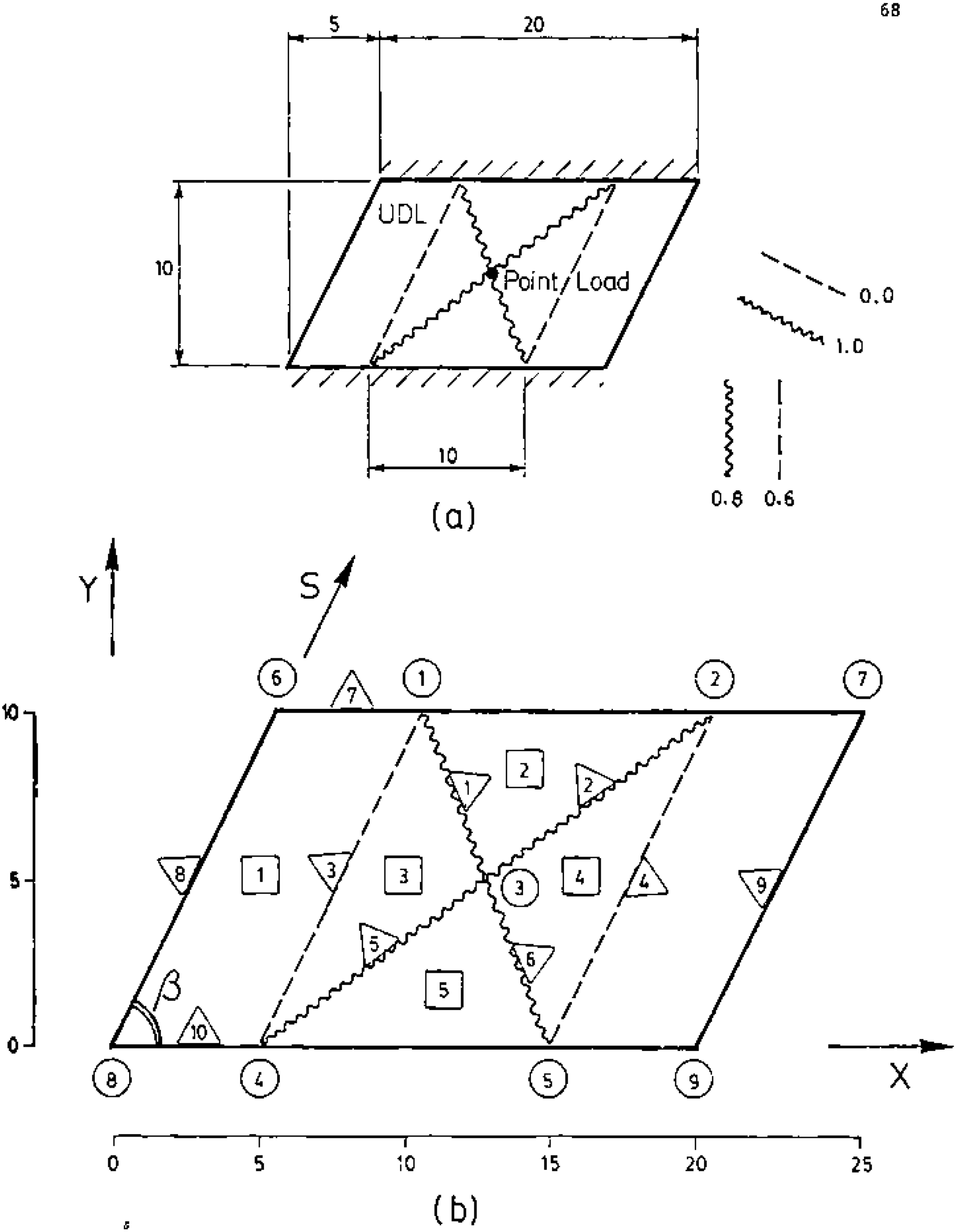


Figure A6 Example 6. (a) Structure. (b) Model.

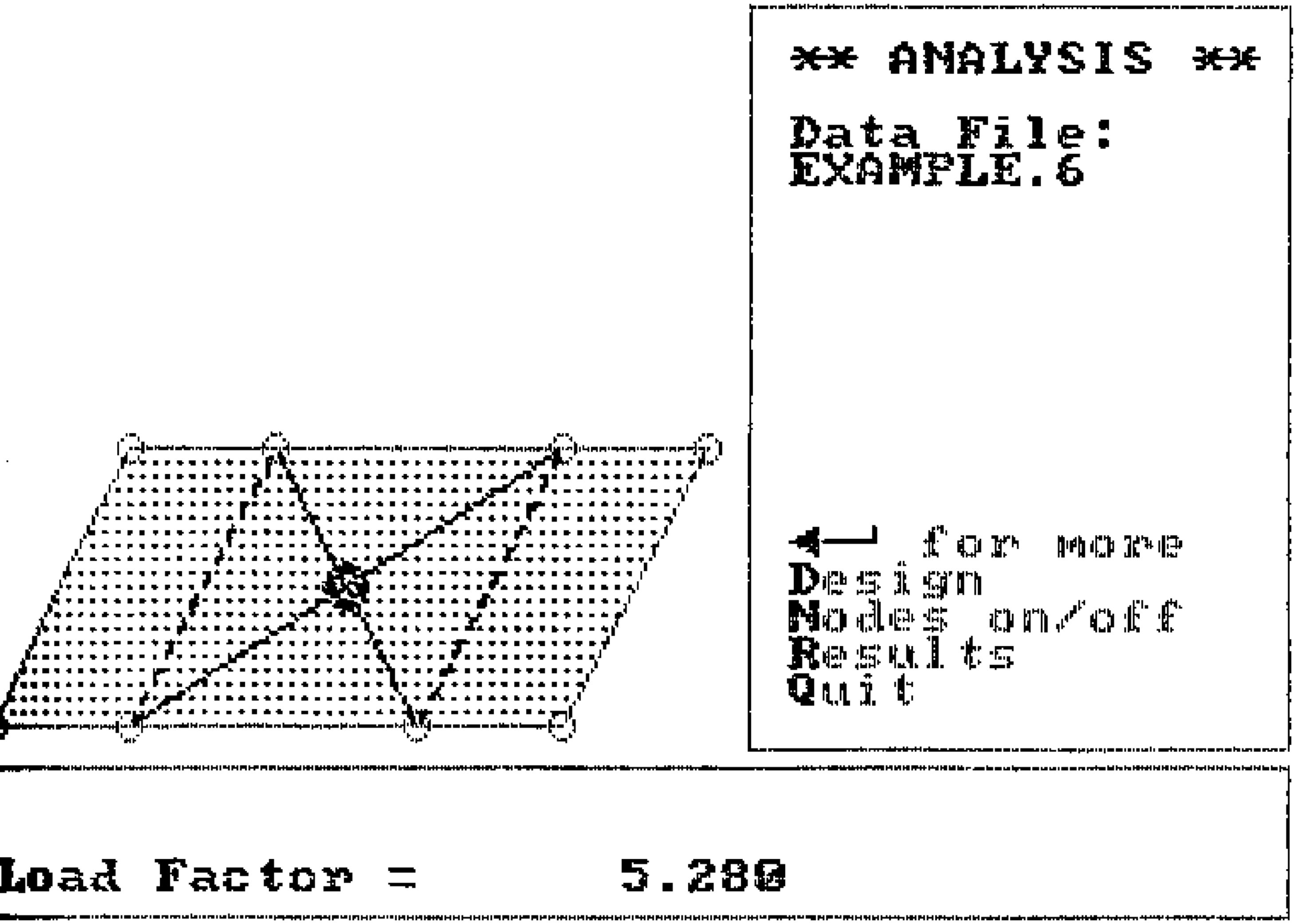


Figure A6 cont'd. (c) Screen Display.

```
 1E-15  Z-FACTOR    EXAMPLE.6
  9  NODES
  1    10 10  0
  2    20 10  0
  3 12.5  5 -1
  4     5  0  0
  5    15  0  0
  6     5 10  0
  7    25 10  0
  8     0  0  0
  9    20  0  0
  5  PLANES
  1  8  1 2 4 5 6 7 8 9
  2  3  1 2 3
  3  3  1 3 4
  4  3  2 3 5
  5  3  3 4 5
 10  LINES
  1   3 1  3 2  1
  2   3 2  2 4  1
  3   4 1  1 3  1
  4   5 2  4 1  1
  5   4 3  3 5  1
  6   5 3  5 4  1
  7   6 7  *
  8   6 8  *
  9   7 9  *
 10   8 9  *
  1  SET OF BENDING RESISTANCE
  1  SKEW
 0.8  1.0  0.6  0.0  63.43494882
  1  POINT LOAD
  1  2  -1  3
  0  LINE LOAD
  6  UDL'S
  1  1  -0.02   4  6 1 4 8
  2  1  -0.02   4  2 7 9 5
  3  2  -0.02   *
  4  3  -0.02   *
  5  4  -0.02   *
  6  5  -0.02   *
  0  MOVEMENT
```

Table A6 Example 6. Input data.

Example 7

Problem [analysis]: An isotropically reinforced square slab is supported along the four edges, and is subjected to a central concentrated load. The slab has a unit ultimate moment resistance/unit length. Find the ultimate concentrated load for the assumed circular fan mechanism shown in Fig. A7-a.

Reference: Ref. 3, Example 7.7, p.306.

Algebraic Solution:

Data: $m'_u = m_u = 1$

Solution:

$P_u = 2\pi\ (m'_u + m_u) = 2\pi(1+1) = 12.5664$ (7.34) of Ref. 3

Solution using Yield Lines:

Input Data: (Fig. A7-b and Table A7)

The input data appears somewhat lengthy for such a simple problem. The program could be modified in a later version to include the automatic generation of data for circular fans.

Complex problems involving elliptic fans, logarithmic fans, etc. can be solved using the same approach as for the circular fan.

Results: (Fig. A7-c) The solution obtained is $\lambda = 12.730$, conservative by 1.3% compared to the exact analytical solution.

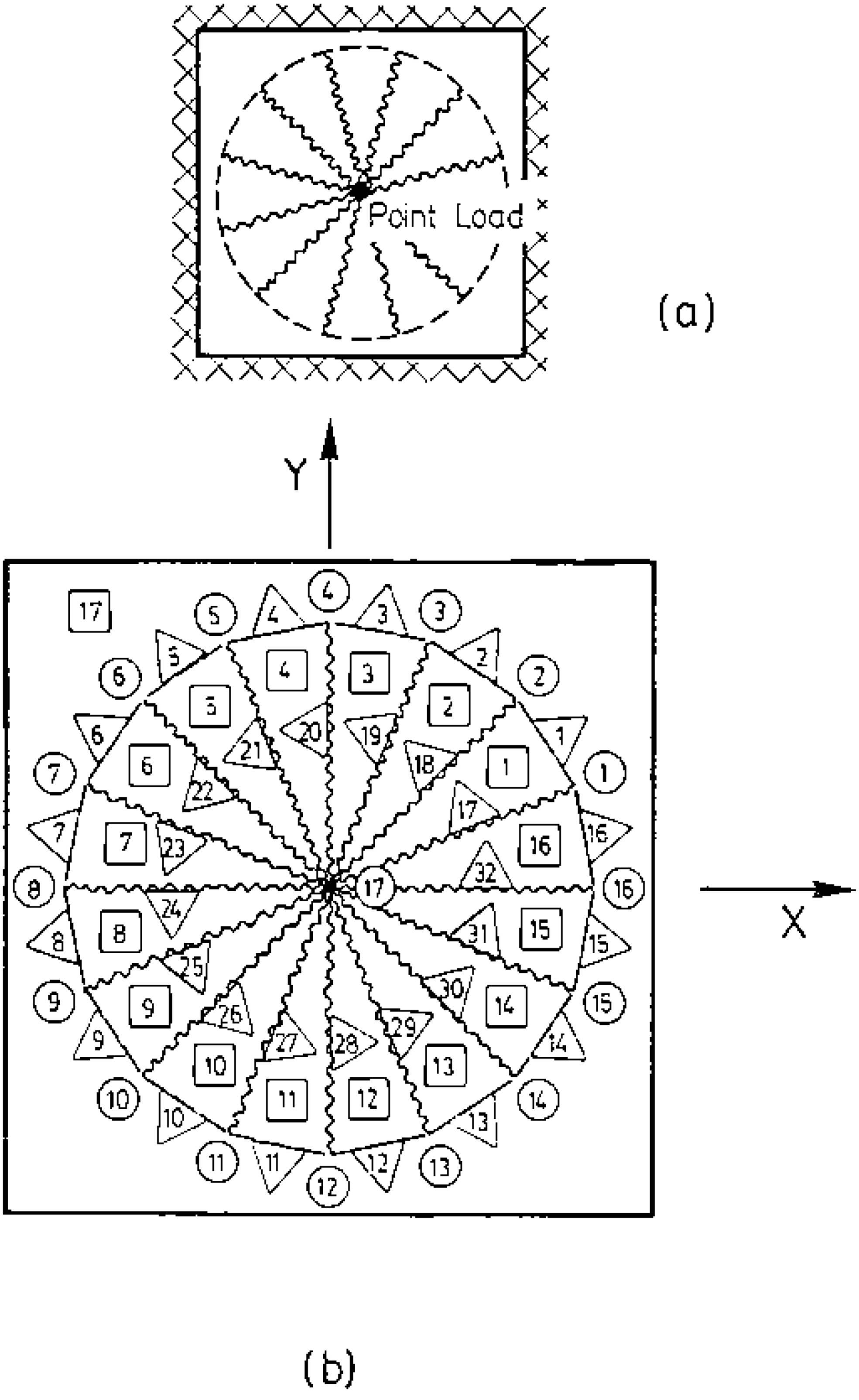


Figure A7   Example 7. (a) Structure. (b) Model.

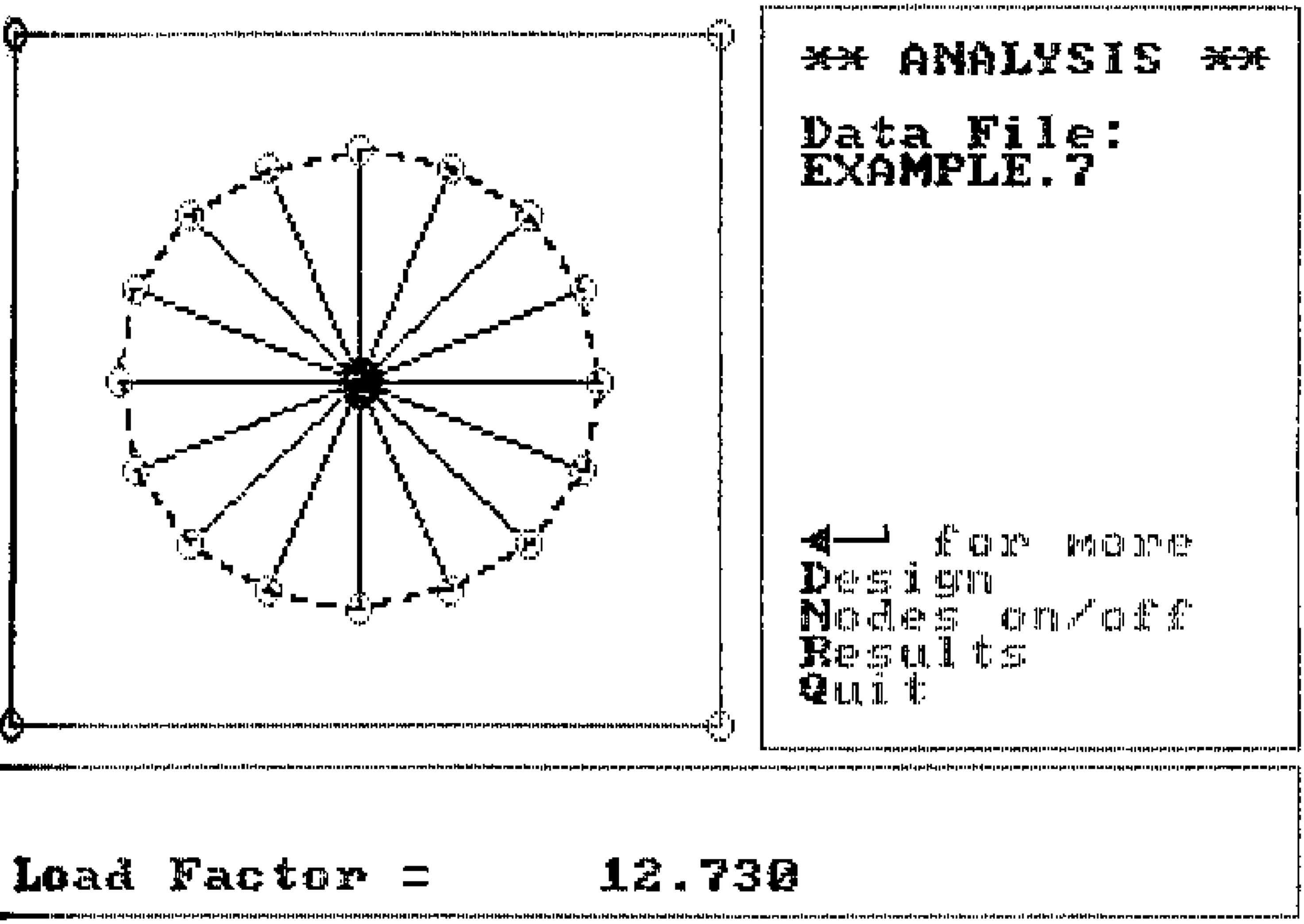


Figure A7 cont'd. (c) Screen Display.

```
 1E-15  Z-FACTOR    EXAMPLE.7
21   NODES
 1    0.923830    0.382683    0
 2    0.707107    0.707107    0
 3    0.382683    0.923580    0
 4    0.000000    1.000000    0
 5   -0.382693    0.923880    0
 6   -0.707107    0.707107    0
 7   -0.923880    0.382683    0
 8   -1.000000    0.000000    0
 9   -0.923880   -0.382683    0
10   -0.707107   -0.707107    0
11   -0.392683   -0.923580    0
12   -0.000000   -1.000000    0
13    0.382683   -0.923890    0
14    0.707107   -0.707107    0
15    0.923880   -0.382683    0
16    1.000000    0.000000    0
17    0           0          -1
18    1.5        -1.5         0
19    1.5         1.5         0
20   -1.5         1.5         0
21   -1.5        -1.5         0
17   PLANES
 1  3   1   2   17
 2  3   2   3   17
 3  3   3   4   17
 4  3   4   5   17
 5  3   5   6   17
 6  3   6   7   17
 7  3   7   8   17
 8  3   8   9   17
 9  3   9  10   17
10  3  10  11   17
11  3  11  12   17
12  3  12  13   17
13  3  13  14   17
14  3  14  15   17
15  3  15  16   17
16  3  16   1   17
17  3   1   5    9
36  LINES
 1   1   2    1  17    1
 2   2   3    2  17    1
 3   3   4    3  17    1
 4   4   5    4  17    1
 5   5   6    5  17    1
 6   6   7    6  17    1
 7   7   8    7  17    1
 8   8   9    8  17    1
 9   9  10    9  17    1
10  10  11   10  17    1
11  11  12   11  17    1
12  12  13   12  17    1
```

```
13  13  14  13  17   1
14  14  15  14  17   1
15  15  16  15  17   1
16  16   1  16  17   1
17   1  17  16   1   1
18   2  17   1   2   1
19   3  17   2   3   1
20   4  17   3   4   1
21   5  17   4   5   1
22   6  17   5   6   1
23   7  17   6   7   1
24   8  17   7   8   1
25   9  17   8   9   1
26  10  17   9  10   1
27  11  17  10  11   1
28  12  17  11  12   1
29  13  17  12  13   1
30  14  17  13  14   1
31  15  17  14  15   1
32  16  17  15  16   1
33  18  19  *
34  19  20  *
35  20  21  *
36  21  18  *
1  SET OF BENDING RESISTANCE
1  ORTHOTROPIC
1  1  1  1
1  POINT LOAD
1  1 -1 17
0  LINE LOAD
0  UDL
0  MOVEMENT
```

Table A7 Example 7. Input data.

Example 8

Problem (design): Consider the rectangular slab of Fig. A8-a supported by beams on the four edges. (The beams are supported by columns at each corner). The slab is subjected to a uniformly distributed load of 100/unit area. Find the required ultimate bending resistances of the slab and of the beam for the assumed mechanism shown in the figure.

Reference: Ref. 2, Solution 8.3, p.245.

Algebraic Solution:

Data: $L = 20, \ \beta = 0.3,$
$\lambda p = 100, \quad m = 1, \ M_{beam} = 30.4$

Solution:

$\bar{m} = M_b/(mL) = 30.4/(1*20) = 1.52$

$$\frac{\gamma m}{\lambda p L^2} = \frac{1-2\beta/3}{4\left(\frac{1+2\bar{m}}{\beta}+4\beta\right)} = \frac{1-2(0.3)/3}{4\left(\frac{1+2(1.52)}{0.3}+4(0.3)\right)} = 0.0137$$

(8.8) of Ref. 2

Solution using Yield Lines:

Input Data: (Fig. A8-b and Table A8)

Bending resistance: The lines representing hinges in the beam (lines 7 and 9) are of unit length, so that the plastic resistance of the beam is equal simply to the bending resistance of these lines.

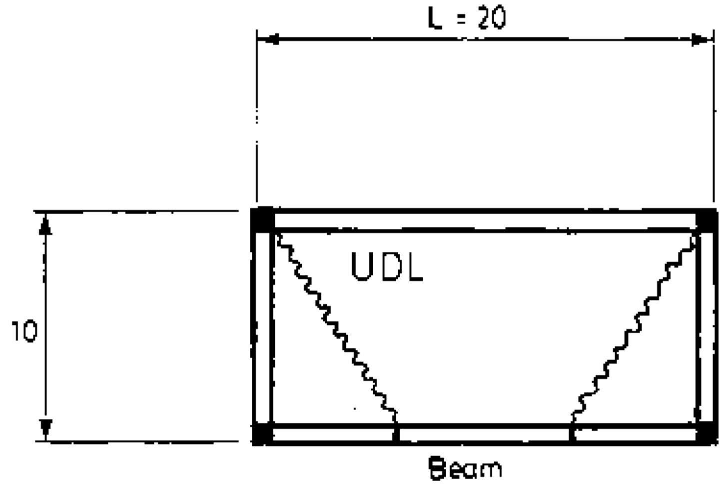
Planes: Due to the symmetry of the problem, the portion of the beam between the two plastic hinges deflects without rotating. Plane 4 represents the beam, and hence nodes 5, 6, 7 and 8 all deflect by the same unit amount.

Note that lines 7 and 9 (plastic hinges in the beam) are between planes 2 and 4, and 4 and 3, respectively.

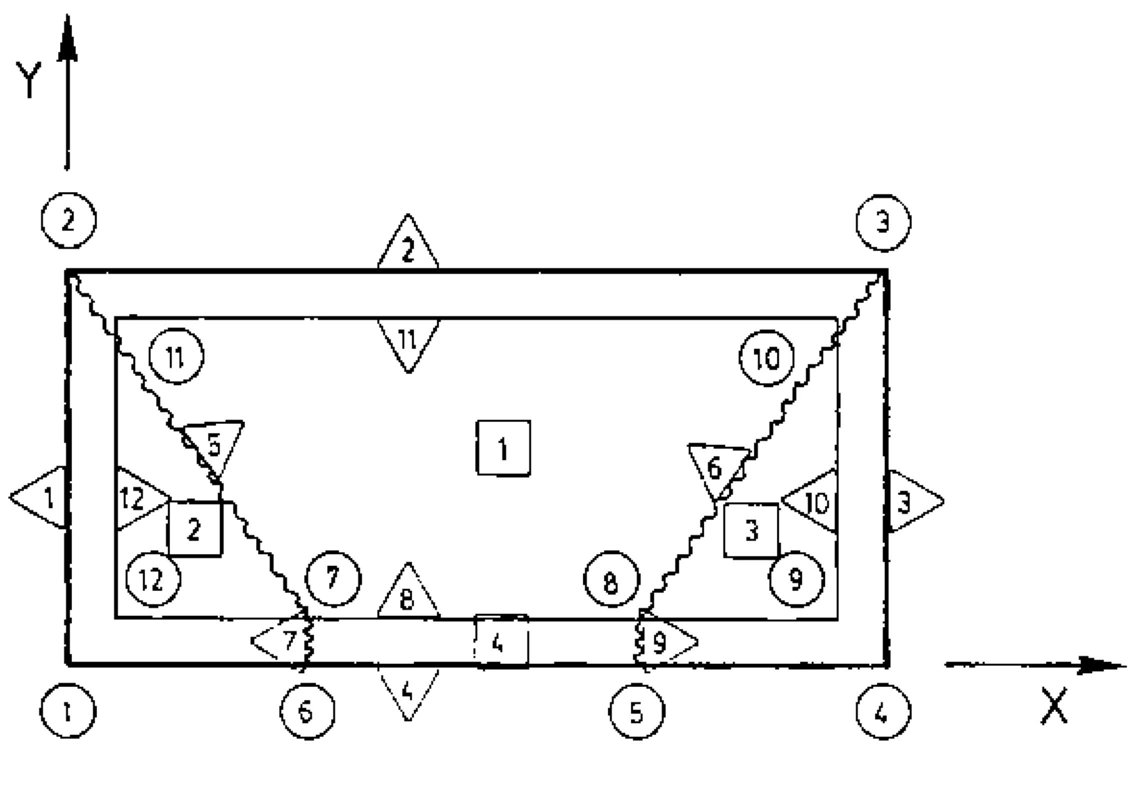
Yield Lines: The width of the beam is exagerated in the drawing of the mechanism (Fig. A8-b). The beam width is neglected in Ref. 2. In order to match the analytical solution, lines 5 and 6 go from nodes 2 to

6 and 3 to 5, instead of going from nodes 11 to 7 and 10 to 8.

Results: (Fig. A8-c) The solution obtained is y = 545.455, hence $\frac{ym}{\lambda p\, L^2} = \frac{(545.455)(1)}{(100)(20^2)} = 0.0137$.



(a)



(b)

Figure A8 Example 8. (a) Structure. (b) Model.

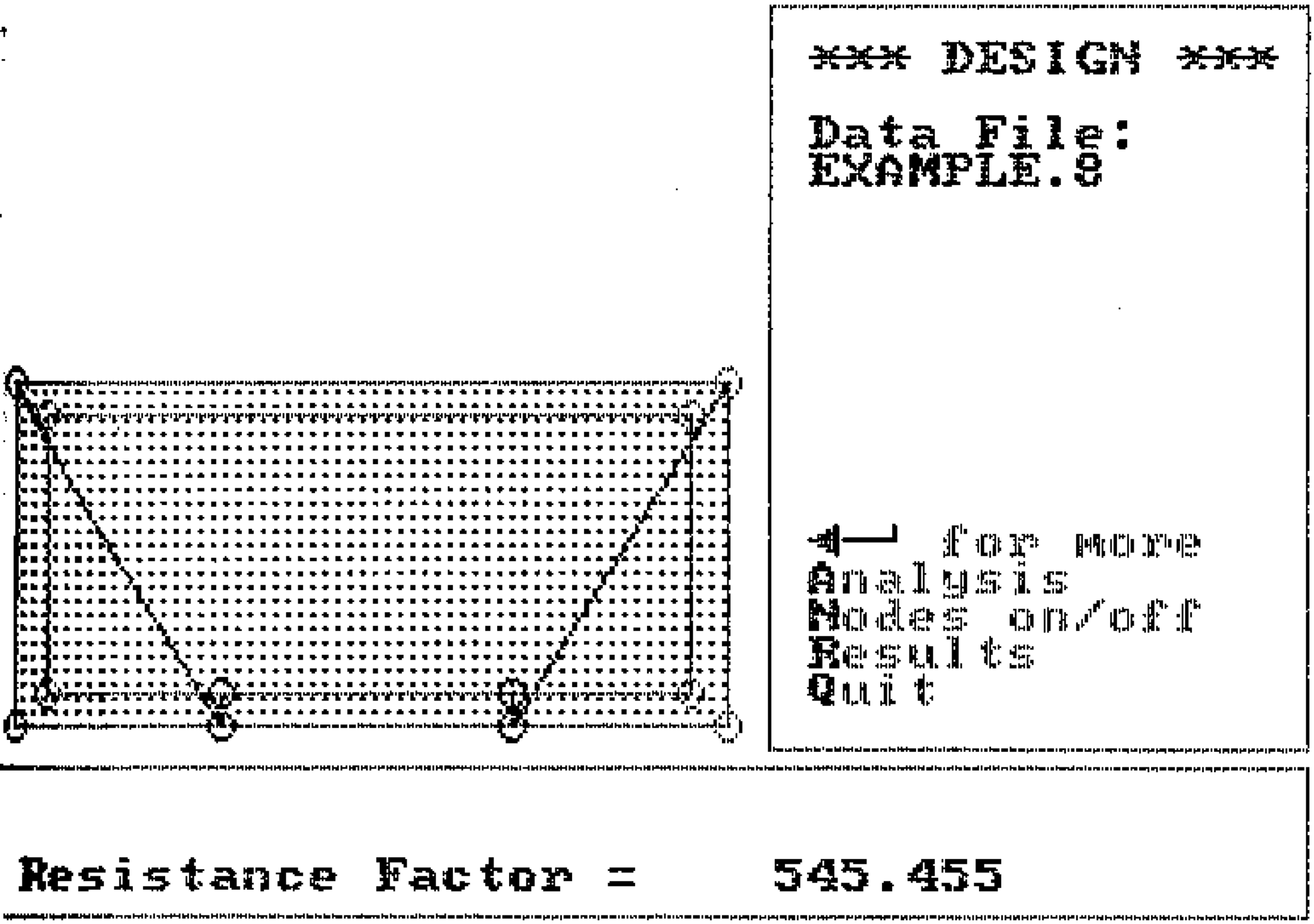


Figure A8 cont'd. (c) Screen Display.

```
 1E-15  Z-FACTOR    EXAMPLE.8
 12 NODES
 1    0  0  0
 2    0 10  0
 3   20 10  0
 4   20  0  0
 5   14  0 -1
 6    6  0 -1
 7    6  1 -1
 8   14  1 -1
 9   19  1  0
10   19  9  0
11    1  9  0
12    1  1  0
 4    PLANES
 1    4   2 3 5 6
 2    4   1 2 6 7
 3    4   3 4 5 8
 4    4   5 6 7 8
12   LINES
 1   1 2   *
 2   2 3   *
 3   3 4   *
 4   1 4   *
 5   2 6   1 2   1
 6   3 5   3 1   1
 7   6 7   2 4   2
 8  12 9   *
 9   5 8   4 3   2
10   9 10  *
11  10 11  *
12  11 12  *
 2   SETS OF BENDING RESISTANCE
 1   ORTHOTROPIC  PLATE
 1   1   1   1
 2   ORTHOTROPIC  BEAM
 30.4  30.4  30.4  30.4
 0   POINT LOAD
 0   LINE LOAD
 3   UDL'S
 1   1   -100    *
 2   2   -100    3   1 2 6
 3   3   -100    3   3 4 5
 0   MOVEMENTS
```

Table A8   Example 8. Input data.

Example 9

Problem (design): The slab shown in Fig. A9-a is subjected to a factored line load of 10 kN/m and a factored UDL of 20 kN/m^2. Find the required ultimate bending resistances of the slab for the assumed mechanism shown in Fig. A9-b.

Reference: Ref. 8, Assignment No.3.

Solution using Yield Lines:

Input Data: (Fig. A9-b and Table A9)

Results: (Fig. A9-c) The solution obtained is γ = 32.263.

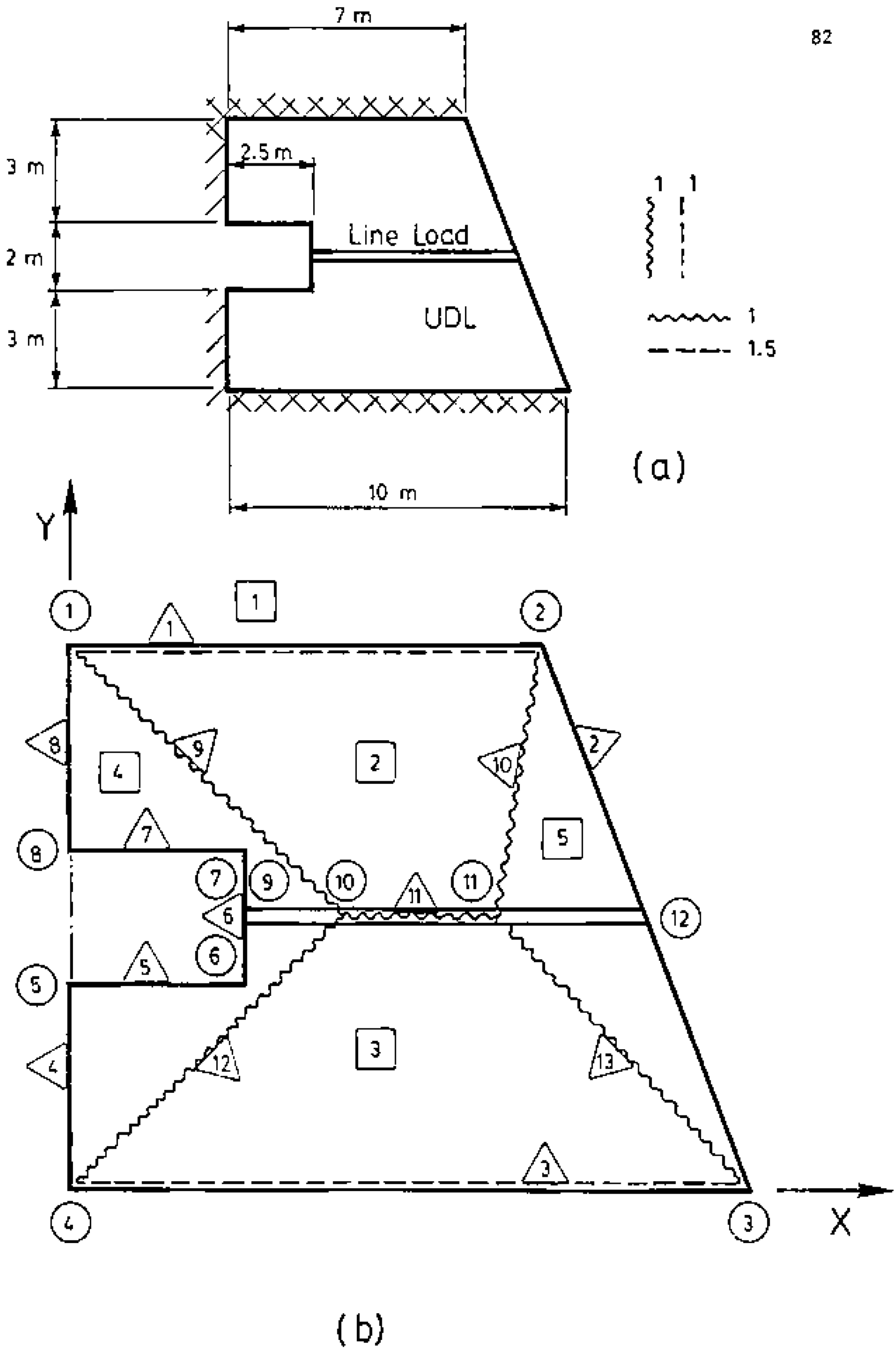


Figure A9 Example 9. (a) Structure. (b) Model.

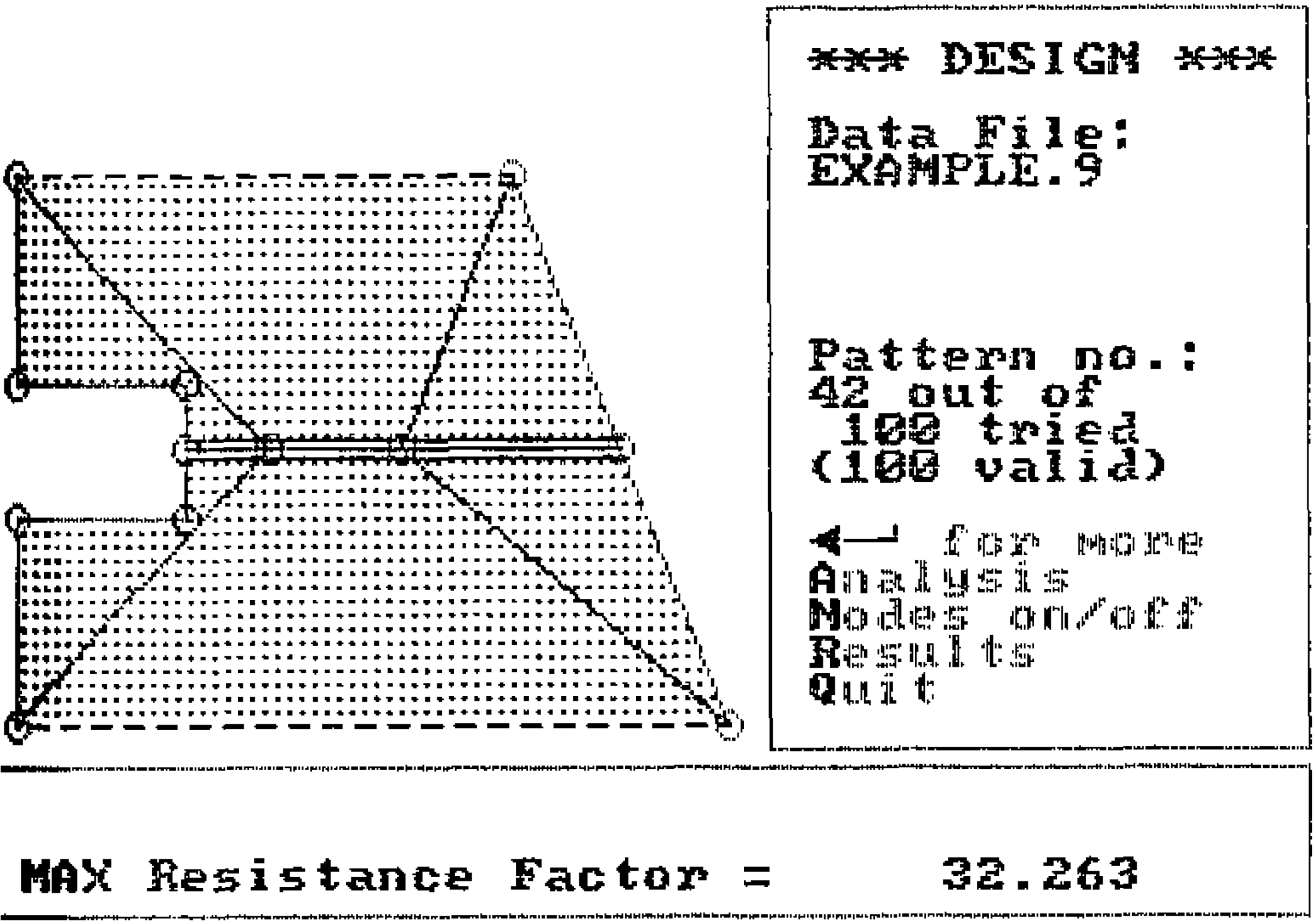


Figure A9 cont'd. (c) Screen Display.

```
 1E-15  Z-FACTOR    EXAMPLE.9
12  NODES
 1  0      8    0
 2  7      8    0
 3 10      0    0
 4  0      0    0
 5  0      3    0
 6  2.5    3    *
 7  2.5    5    *
 8  0      5    0
 9  2.5    4    *
10  3.5    4   -1
11  6.5    4   -1
12  8.5    4    0
 5  PLANES
 1  7    1   2 12   3   4   5   8
 2  4    1   2 11  10
 3  4   10  11  3   4
 4  8    1  10  4   5   6   9   7   8
 5  4    2  12  3  11
13  LINES
 1   1    2    1   2   2
 2   2    3    *
 3   3    4    1   3   2
 4   4    5    *
 5   5    6    *
 6   6    7    *
 7   7    8    *
 8   8    1    *
 9   1   10    2   4   1
10   2   11    5   2   1
11  10   11    2   3   1
12   4   10    4   3   1
13   3   11    3   5   1
 2  SETS OF BENDING RESISTANCE
 1  ORTHOTROPIC
 1  1  1  1
 2  ORTHOTROPIC
 1.5  1.5  1.5  1.5
 0  POINT LOAD
 3  LINE LOADS
 1   4   -10  9    -10 10
 2   2   -10 10    -10 11
 3   5   -10 11    -10 12
 4  UDL'S
 1   2   -20  *
 2   3   -20  *
 3   4   -20  *
 4   5   -20  *
 2  MOVEMENTS
 1  10  1ST MOVEMENT
11   4.6 4    6.5 4
 1  10  2ND MOVEMENT
10   3.5 4    4.4 4
```

Table A9 Example 9. Input data.

Example 10

Problem (analysis): Consider a joint on the tension chord of a triangular truss made of steel hollow structural sections (HSS), as shown in Fig. A10-a. The joint includes a 40 mm HSS compression web member and a 35 mm HSS tension web member welded on the face of a 100 mm square HSS chord with a 40 mm gap between the two web members.

Calculate the resistance of the joint for the assumed failure mechanism in which the compression web member is punching into the chord face and the tension web member is pulling out.

Reference: Ref. 9.

Algebraic Solution:

Data: Dimensions shown in Fig. A10-b.
Isotropic steel, all m_p's are equal.

Solution: P = 200.0 kN (from Ref. 9)

Solution using Yield Lines:

Input Data: (Fig. A10-c and Table A10)

Nodes: The deflection (z coordinate) of nodes 6, 7, 10, 11, 15 and 16 is unspecified, and therefore computed by the program.

Nodes 15 and 16 are slave nodes so that their location at the center of the web members is computed by the program. Point loads are assigned to these nodes.

Nodes 17 and 18 are included only to have all the patterns drawn at the same scale on the screen.

Search Data: Three movements are used to vary the position of nodes 2 and 13, nodes 1 and 12 (line 9), and nodes 3 and 14 (line 14).

Results: (Fig. A10-d) The solution obtained is λ = 200.86.

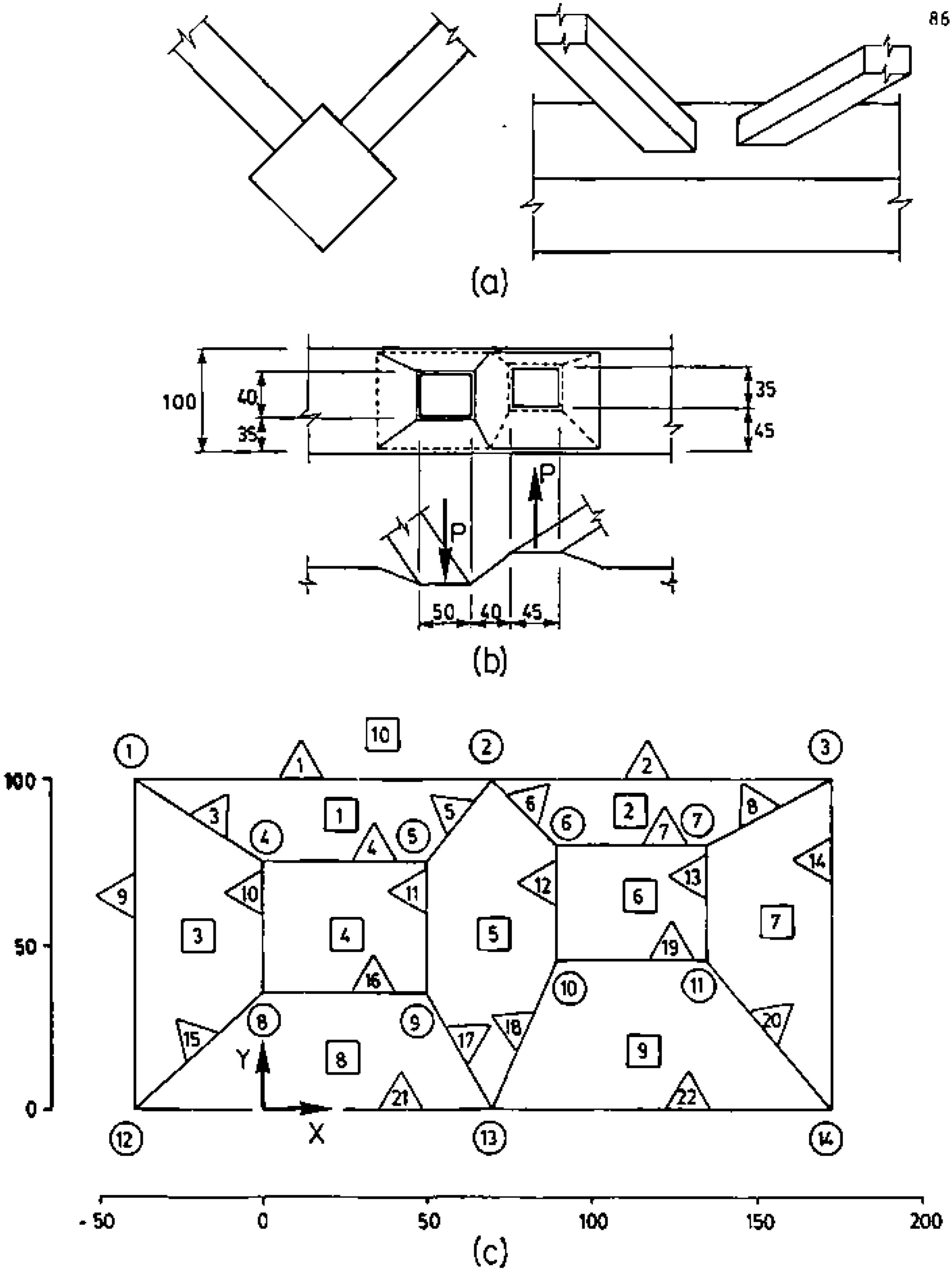


Figure A10 Example 10. (a) Structure. (b) Dimensions. (c) Model.

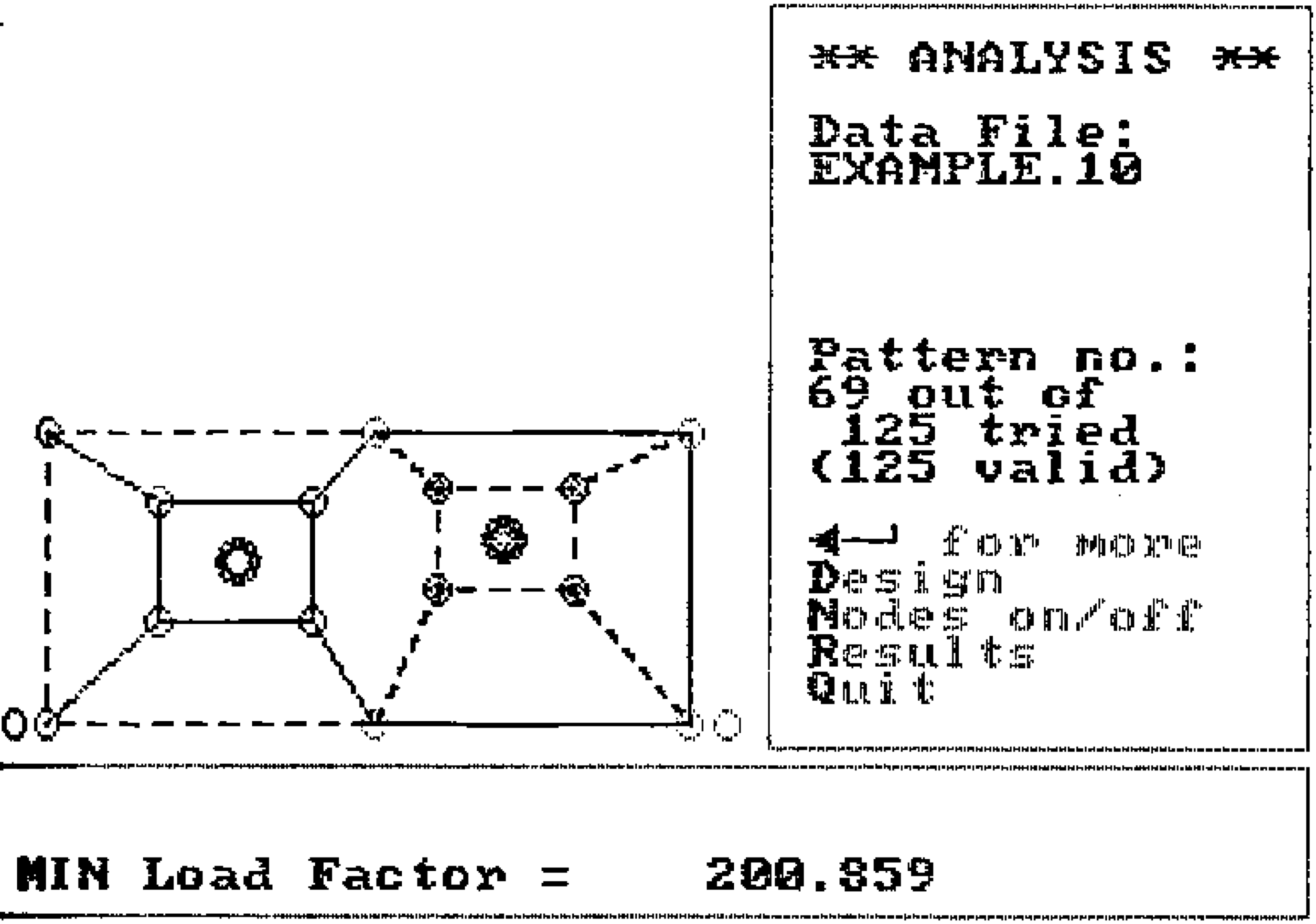


Figure A10 cont'd. (d) Screen Display.

```
 1E-15  Z-FACTOR     EXAMPLE.10
18  NODES
 1     -1.        100.      0.
 2     70.        100.      0.
 3    136.        100.      0.
 4      0.         75.     -1.
 5     50.         75.     -1.
 6     90.         80.      *
 7    135.         80.      *
 8      0.0        35.     -1.
 9     50.         35.     -1.
10     90.         45.      *
11    135.         45.      *
12     -1.          0.      0.
13     70.          0.      0.
14    136.          0.      0.
15    * 4   9   5   8       *
16    * 6  11   7  10       *
17    -50.          0.      0.
18    185.          0.      0.
10  PLANES
 1  4    1   2   4   5
 2  4    2   3   6   7
 3  4    1   4  12   8
 4  5    4   5   8   9  15
 5  6    2   5   6   9  10  13
 6  5    6   7  10  11  16
 7  4    7   3  11  14
 8  4    8   9  12  13
 9  4   10  11  13  14
10  6    1  12   2   3  13  14
22  LINES
 1     1  2   10  1     1
 2     2  3   10  2     1
 3     1  4    1  3     1
 4     4  5    1  4     1
 5     5  2    1  5     1
 6     2  6    2  5     1
 7     6  7    2  6     1
 8     7  3    2  7     1
 9    12  1   10  3     1
10     8  4    3  4     1
11     9  5    4  5     1
12    10  6    5  6     1
13    11  7    6  7     1
14    14  3    7 10     1
15    12  8    3  8     1
16     8  9    4  8     1
17     9 13    5  8     1
18    13 10    5  9     1
19    10 11    6  9     1
20    14 11    9  7     1
21    12 13    8 10     1
22    13 14    9 10     1
```

```
1   SET OF BENDING RESISTANCE
1   ISOTROPIC
8.   8.   8.   8.
2   POINT LOAD
1    4    -1   15
2    6     1   16
0   LINE LOAD
0   UDL
3   MOVEMENTS
2 5   1ST MOVEMENT
2     51. 100.   89. 100.
13   51.    0.   89.    0.
2 5   2ND MOVEMENT
1    -1. 100.   -50. 100.
12   -1.    0.   -50.    0.
2 5   3RD MOVEMENT
3    136. 100.  185. 100.
14   136.    0.  185.    0.
```

Table A10   Example 10. Input data.