# On Globally Optimizing a Mining Complex under Supply Uncertainty: Integrating Components from Deposits to Transportation Systems

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## CONTRIBUTION OF AUTHORS

The author of this thesis is the primary author in all papers included in the related chapters. Professor Roussos Dimitrakopoulos was the supervisor and is included as co-author in all four papers of this thesis. The four papers are:

- Stochastic mine production scheduling with multiple processes: Application at Escondida Norte, Chile. Montiel, L. and Dimitrakopoulos, R., 2013. Mining Science 49(4), 583-597.
- An extended stochastic optimization method for multi-process mining complexes. Montiel, L. and Dimitrakopoulos, R., 2014. To be submitted.
- Optimizing mining complexes with multiple processing and transportation alternatives: An uncertainty-based approach. Montiel, L. and Dimitrakopoulos, R., 2014. Submitted to European Journal of Operational Research.
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## ABSTRACT

Mining complexes are generally comprised of multiple deposits that contain several material types and grade elements, which are transformed in available processing destinations and transported to final stocks or ports as saleable products. These components, associated with a mining complex, encompass multiple sequential activities: *(i)* Mining the material from one or multiple sources; *(ii)* blending the material including stockpiling; *(iii)* transforming the material in different processing destinations considering operating modes; *(iv)* transporting the transformed material to final stocks or ports. Since these activities are strongly interrelated, their optimization must take place simultaneously. In the technical mining literature, this problem is known as global optimization of mining complexes. Conventional mining optimization methods suffer from at least one of the following drawbacks when optimizing mining complexes: some decisions are assumed when they should be dynamic; component based objectives are imposed, which might not coincide with global objectives; many parameters are assumed to be known when they are uncertain. Past research works have demonstrated that geological uncertainty is the main cause of the inability of meeting production targets in mining projects.

This thesis presents methods to optimize mining complexes that simultaneously consider different components and account for geological uncertainty. In this study, the term geological uncertainty refers to uncertainty related to grades and material types of the mineral deposits under consideration. This uncertainty is modelled through geostatistical orebody simulations of the different deposits.

A multistage methodology that uses simulated annealing algorithm to generate risk-based production schedules in mining complexes with multiple processing destinations is presented and implemented in Escondida Norte (Chile) copper dataset. The algorithm swaps periods of mining blocks seeking for minimizing the deviations from the capacities at the different processing destinations. Its implementation using Escondida Norte dataset generates expected average deviations of less than 5% regarding mill and waste targets, whereas a mine production schedule generated conventionally over a single estimated model generates expected average deviations of 20 and 12% for mill and waste targets respectively.

An iterative improvement algorithm that considers operating modes at different processing destinations is developed and applied to a copper complex. The objective function seeks for maximizing discounted profits along the different periods and scenarios (orebody simulations). The algorithm iteratively perturbs an initial solution by pushing profitable blocks to early periods and non-profitable ones to later periods while approaching mining and processing targets. The destinations of the mining blocks are also perturbed toward NPV improvements and attainment of production targets. The implementation of the method at a copper deposit allows reducing the expected average deviations from 9 to 0.2% regarding the capacity of the first process while increasing the expected NPV by 30% when compared with an initial solution generated conventionally.

A method that uses simulated annealing at different decision levels (mining, processing and transportation) is described and tested in a multipit copper operation. The method integrates three different types of perturbations: *(i)* Swapping periods and destinations of mining blocks; *(ii)* swapping operating modes at the different processing destinations; *(iii)* modifying the utilization of the transportation systems available in the mining complex. The implementation of the method in a multipit copper operation permits the reduction of the expected average deviations from the capacities at two mills from 18-22% to 1-3% and the expected average deviation from the targets regarding two blending elements from 7-1.8% to 0.3-0.6% when compared to an initial solution generated conventionally. The expected NPV also improves by 5%.

The previous method is extended to mining complexes that combine open pit and underground operations and it is tested in a gold complex in Nevada. The extended method also accounts for external blending material used for meeting the operational ranges of the metallurgical properties in some specific destinations. The implementation of the method at Twin Creeks gold complex in Nevada shows improvements in meeting the metallurgical blending requirements while increasing the expected NPV by 14%.

The formulations described in this thesis encompass a large number of integer variables given the discretization of the mineral deposits. To solve the problems, efficient optimization algorithms are implemented with significant improvements when compared with conventional deterministic approaches. These algorithms outperform conventional methods regarding expected NPV and meeting targets at the different components of the value chain.

# ABRÉGÉ

Les complexes miniers sont généralement composés de multiples gisements de minerai complexe qui contiennent plusieurs types de matériaux. Ces matériaux sont transformés dans différentes destinations de traitement et sont transportés au dépôt final où dans des ports comme produits finaux. Ces composantes associées au complexe minier englobent de multiples activités séquentielles : *(i)* l´abattage du minerai à partir d´un ou plusieurs gisements, *(ii)* l´homogénéisation des matériaux en incluant le stockage, *(iii)* la transformation des matériaux dans les différents destinations de traitement qui considèrent certains modes d´opération, *(iv)* le transport des matériaux transformés au dépôt final ou dans les ports. Puisque ces activités sont fortement inter-reliées, l´optimisation de ceux-ci doit être faite simultanément. Dans la littérature technique minière ce problème est connu sous le nom d´optimisation globale des complexes miniers. Les méthodes d´optimisation conventionnelles souffrent d´au moins une de ces inconvénients lorsqu´utilisées pour l´optimisation globale des complexes miniers : certaines décisions sont assumées fixes alors qu´elles devraient être dynamiques (les modes d´opération, destination des blocs d´exploitation, etc.), les objectifs sont basées sur certains composantes qui peuvent ne pas coïncider avec les objectifs globaux et plusieurs paramètres sont assumés connus alors qu´ils sont incertains. Les recherches passées ont démontré que l´incapacité à atteindre les objectifs de production dans les projets miniers est dûe à l´incertitude géologique du gisement.

Cette thèse présente des méthodes pour l´optimisation des complexes miniers

qui considèrent simultanément les différentes composantes ainsi que l´incertitude géologique. Dans cette étude, le terme incertitude géologique réfère au type de matériel et la qualité du matériel du gisement sous considération. Cette incertitude est modélisée par simulations géostatistiques des différents gisements. Une méthodologie en plusieurs étapes, qui utilise un algorithme de recuit simulé pour générer un calendrier de production basé sur le risque pour les complexes miniers avec plusieurs destinations de traitement de matériaux, est présentée et implémentée pour le complexe d´Escondida Norte, Chile. L´algorithme change la période d´abattage des blocs d´exploitation en cherchant à minimiser les écarts par rapport aux capacités des différentes destinations de traitement. Sa mise en œuvre à Escondida Norte génère des écarts moyens attendus de moins de 5% par rapport aux cibles du broyeur et des matériaux stériles produits alors qu´un calendrier de production générer à partir d´une méthode conventionnelle en utilisant seulement un modèle de gisement estimé génère des écarts moyens attendus de 20% et 12%. Un algorithme d´amélioration itérative qui considère les modes d´opérations aux différentes destinations de traitement est développé et implémenté à un complexe minier traitant du cuivre. La fonction objective cherche à maximiser le profit actualisé pour les différentes scénarios (simulations géostatistiques du gisement) et périodes. L´algorithme perturbe de manière itérative une solution initiale en devançant la période d´extraction de blocs d´exploitation rentables et en différant celle de blocs non-rentables tout en approchant les cibles des destinations de traitement et ceux d´extraction. La destination des blocs d´exploitation est aussi perturbée en favorisant une amélioration de la valeur nette actualisée et l´atteinte des objectifs de production. L´implémentation de cette méthode à un gisement de cuivre permet de réduire l´écart moyen attendu de 9% à 0.2% pour la capacité de la première destination tout en augmentant la valeur nette actualisée de 30% par rapport à celle de la solution initiale générée de manière conventionnelle. Une méthode qui utilise le recuit simulé pour les différentes étapes de décision (extraction, traitement, transport) est présentée et testée pour un complexe avec plusieurs mines de cuivre en opération. La méthode intègre trois différents types de perturbations : *(i)* le changement de périodes et de destinations des blocs d´exploitation, *(ii)* le changement du mode opérationnel aux différentes destinations de traitement et *(iii)* la modification de l´utilisation des modes de transport disponibles dans le complexe minier. L´implémentation de la méthode au complexe multi mines permet de réduire les écarts moyens attendus par rapport à la capacité de deux broyeurs de 18%-22% à 1%-3% et les écarts moyens attendus des éléments d´homogénéisation de 7%-1.8% à 0.3%-0.6% par rapport à la solution initiale générée conventionnellement. De plus, la valeur nette actualisée augmente de 5%. La méthode précédente est généralisée pour un complexe minier qui combine des opérations de mines à ciel ouvert et de mines sous-terraine et est testée dans un complexe traitant de l´or au Nevada. Cette méthode prend aussi en considération du matériel d´homogénéisation provenant d´une autre source qui est utilisée pour l´atteinte de contraintes opérationnelles de propriétés métallurgiques dans certaines destinations de traitement. L´implémentation de cette méthode au complexe Twin Creeks au Nevada montre une amélioration quant au respect des contraintes des propriétés métallurgiques tout en augmentant la valeur nette actualisée de 14%.

En considérant la discrétisation du gisement de minerai, les formulations décrites

dans cette thèse produisent un large nombre de variables en nombres entiers. Pour résoudre ces problèmes, des algorithmes d´optimisation efficients sont implémentés, produisant des améliorants significatifs lorsque comparés avec des méthodes déterministes conventionnelles. Ces algorithmes surpassent les méthodes conventionnelles aux points de vue de la valeur nette actualisée produite et d´atteinte des objectifs de production pour les différentes composantes de la chaîne de valeur.

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# CHAPTER 1
# General introduction

## 1.1 Overview

A mining complex is a value chain where raw material is extracted and transformed into saleable products. The value chain is comprised of different components: mineral deposits, stockpiles, processing paths, transportation systems. The mineral deposits are the sources of raw materials that can be classified into different material types based on their chemical and metallurgical properties (e.g. sulphide or oxide material in a copper operation). The stockpiles contribute to the blending operation, contain potential ore and can provide a backup supply of raw material. The material that comes directly from the deposits or from the stockpiles is transformed through multiple processing paths or destinations. In each destination, several operating modes can be used; for example, a mill can be operated at fine or coarse grinding. The operating mode defines the metallurgical recovery, processing cost, blending requirement and throughput of a given destination. Once the material is transformed, it is transported to final stocks or ports using transportation systems.

To optimize a mining complex, its components must be optimized simultaneously (deposits, stockpiles, processing destinations, transportations systems). The problem of simultaneously optimizing all the components of the value chain is known in the mining literature as global optimization of mining complexes [110, 111, 113]. Some efforts have been made to incorporate several components of the value chain during the optimization [12, 46, 50, 96, 110]. However, these methods have at least one of the following limitations: some decisions are fixed when they should be dynamic (operating modes, destination of mining blocks, etc.); component-based objectives are imposed, which might not coincide with global objectives; many parameters are assumed to be known when they are uncertain [110, 111].

Project risk may arise from three main sources: technical (geological and mining), financial and environmental [88]. Several authors [25, 106] have concluded that the major factor in the inability to meet production targets and to generate reliable project expectations in mining is geological uncertainty; that is, uncertainty in grades and material types. Mine optimization methods consider single estimated (average-type) orebody models in their calculation processes, ignoring the uncertainty associated with the spatial distribution of the attributes of interest. These models are generated by interpolation methods that provide smoothed representations of the mineral deposit attribute being estimated, typically metal grade. Smoothed models are inadequate for the assessment of variability in deposit characteristics. As opposite to interpolation methods, stochastic simulation is a technique that permits generating models that respect all that is known about the orebody in terms of the statistical distribution and spatial variability of the attributes as determined from available sampling information. Repeated simulations will produce different equally probable models of the orebody, which allows modelling the geological uncertainty associated with the deposit at a mining scale. The optimal design and sequence of an average-type model has poor performance over a set of orebody simulations derived from the smoothing effect associated with the estimated model. This misleading is originated from the fact that single average-type models do not generate solutions that perform well in average over a set of orebody simulations, given that the transfer function that relates grades and discounted economic values is non-linear.

Over the past decade, new stochastic optimization methods that take into account geological uncertainty by means of multiple orebody simulations have been developed for open pit designs and life-of-mine (LOM) production scheduling. These methods allow quantifying and minimizing the risks of deviating from production targets while increasing the expected NPV of the operation. Furthermore, some applications of stochastic methods reveal potential increases in recoverable metal, which contribute to the responsible utilisation of non-renewable resources [1, 2, 69].

## 1.2 Optimizing the components of the value chain

The global optimization of a mining complex demands the simultaneous optimization of all its components. Hoerger et al. [50] formulate the problem of optimizing the simultaneous mining of multiple pits and the delivery of ores to multiple plants as a mixed integer program. The model calculates the net present value of the mining complex by using variables that represent material sent from the mines to the stockpiles, material sent from the mines to the processes, and material sent from the stockpiles to the processes and their associated costs. The mining blocks are grouped into increments based on the metallurgical properties, which belong to sequences (or pushbacks). The integer variables are used to model mine sequencing constraints at a pushback level and plant startups and shutdowns. This formulation is based on the work done by Urbaez and Dagdelen [105]. Hoerger et al. [50] implement the formulation at Newmont's Nevada operations where 50 sources, 60 destinations and 8 stockpiles are considered. This has led to increased profitability in northern Nevada mine sites by taking advantage of the available synergies. However, due to the use of pushback sequencing constraints instead of block sequencing constraints to decrease the complexity of the problem, there is a loss of resolution in the solution generated from the method that may lead to the inability of meeting the blending and production requirements. Furthermore, the method does not consider multiple operating modes for each processing destination and ignores the geological uncertainty associated with the ore deposits.

Stone et al. [96] present the Blasor optimization software tool developed by the mine planning optimization group within BHP Billiton Technology. To determine the optimal extraction sequence in multiple pits, it formulates the problem as a mixed integer linear problem and solves it using ILOG CPLEX [53]. Material is assigned to bins on the basis of combination of grades and impurities. Blocks spatially connected and with similar properties are aggregated, which largely reduces the amount of integer variables in the formulation. The formulation accounts for slope angles, mining rates, capacities, and quality and grade constraints. The Blasor procedure starts with the aggregation of blocks, then it calculates the optimal extraction sequences and pit limits, later it generates mineable mining phases, and finally it provides the optimal panel extraction sequence, where a panel is the intersection of a mining phase and a bench. The software generates a solution within 0.5% bound of optimality in 6-10 hours when implemented in Yandi's operation comprised of 1000 aggregates, 11 pits and 20 periods over the LOM.

Rocchi et al. [91] implement Blasor at Illawarra Coal mining operations, New South Wales, Australia. BHP Billiton's Illawarra Coal operates several longwall coal extraction systems. Eight different domains are defined along two colliery complexes. Each domain is presented in Blasor as a distinct virtual open pit. The implementation of Blasor provides an optimized solution for coal hoisting and production within blend constraints that outperforms the mining sequences derived from the solution methodology using the XPAC scheduling software [93].

Zuckerberg et al. [119] present Blasor-InPitDumping or BlasorIPD that is a specialized version of Blasor to model waste handling. It uses the general approach of Blasor but incorporates variables representing when mined-out areas are filled with waste material. The refill is done in a way that maximum waste repose slope constraints are respected and no ore is overlaid with waste material. Despite of the additional complexity added in BlasorIPD, it provides fast solution times for full sized problems as in Blasor.

Bodor is an in-house BHP Billiton´s software developed to generate the resource extraction sequence at Boddington bauxite mine, south-western Australia. The operation is a bauxite mine comprised of bauxite pods clustered into several distinct mining envelopes. Pods are distinct bodies of ore lying close to the surface and of modest size. Zuckerberg et al.[120] outline the main features of Bodor and its implementation at the mine. Bodor focuses on the sequences in which pods are to be excavated to minimize capital and operational costs, meet blend targets at the refinery front gate, respect environmental and operational constraints, and allow for smooth utilization of trucking resources. The basic assumption is that the material in a given pod (or sub-pod) is homogeneous, which allows it to be extracted in any proportion. The formulation is a mixed-integer-linear model (MILP) that is solved using a standard CPLEX MILP optimization engine [53]. The objective function seeks for minimizing costs while ensuring the delivery of material to the refinery. The implementation of the software at Boddington mine generates a reduction of 5% of the costs when compared with a schedule generated using XPAC scheduling software under the policy of deferring the capital expenditure as long as possible. The software is deposit-specific as it works with bauxite pods instead of mining blocks as other formulations.

Chanda [16] formulates the delivery of material from different deposits to a metallurgical complex as a network linear programming optimization problem. The nodes of the network represent mines, concentrators, smelters, refineries and market regions. The arcs represent per-unit production and transportation costs. The objective is to minimize the costs throughout the network under a certain demand of metal. Although interesting to model the flow of material through a metallurgical complex, the model does not generate a mine production schedule by assuming a constant grade from the different sources.

Wooller [114] describes the COMET software that optimizes mill throughput / recovery and cut-off grade. COMET uses an iterative algorithm based on successive approximation dynamic programming [68]. Through successive iterations the algorithm searches for an operating policy and sequence that maximizes the value of a resource. In each iteration, it generates period operating policies until the depletion of the resource. The algorithm is able to define operating policies such as mill throughput/recovery or the choice of process routes such as heap leach versus concentration. However, one of the main limitations of the method is its ability to optimize deposits where blending is required to produce the final products, given that it does not consider minimum constraints required to respect operational ranges.

Whittle [110] introduces the global asset optimization tool incorporated in Whittle software. The tool is designed to optimize the sequence of extraction of multiple deposits considering complex processing and blending operations. The mining blocks are aggregated in parcels that are intersection of mining phases and benches and classified in grade bands originated from the different grade elements. This allows reducing orebody models of millions of blocks to grade banded databases of several thousand records. The method assumes the possibility of stockpiling each material type (grade band) and allows incorporating non-linear expressions, multi-stage paths and recycle loops in the processing database. Processing turns mined material into one or more blend feeds that can be stockpiled, discarded or combined to generate the final products. Prober is the algorithm used to solve the problem. It combines a mathematical search algorithm with a linear programming evaluation routine. The search algorithm samples the feasible domain of alternative LOM plans and the evaluation routine determines optimal COG, stockpiling, processing selection, blending and production plan, and determine the NPV. The prober works by repeatedly creating a random feasible solution and then finding the nearest local maximum. The algorithm stops when the top ten values lie within 0.1% of each other [113].

All the methods previously described ignore the uncertainty associated with key parameters. Groeneveld et al. [46] incorporate uncertainty in market price, costs, utilization of equipment, plant recovery and time for building options (infrastructure) while simultaneously optimizing mining, stockpiling, processing and port policies. The authors formulate the problem as a mixed integer program where the objective is maximizing NPV. To do that, the objective function accounts for the revenue from the sale of ore, the capital cost of building an option, the disposal cost of reducing capacity, the variable cost of processing ore and the fixed cost of maintaining an option. The formulation brings flexibility by considering options for: *(i)* mining, e.g., increase capacity by buying trucks, *(ii)* stockpiling, *(iii)* processing, which are characterized by their capacities, capital costs, fixed operating costs and grade limits, and *(iv)* port, e.g., different port capacities. The flexibility of the model allows an increment of 85% in the NPV of a hypothetical iron mine when compared to a design without flexibility. Although very flexible, the method has some limitations: it does not consider multi-product options and ignore geological uncertainty, which is the major factor in the inability to meet production targets and to generate reliable project expectations in mining [25, 106].

Bodon et. al [12] models the problem of supplying exports in a coal chain as a discrete event simulation model (DES). The model is able to asses various operating practices, including maintenance options and capital expenditure to determine the best infrastructure for a given system. The DES model allows determining the optimal capacity of the supply chain with its robustness under uncertainty. The optimization is a linear program with multiple objectives, in where some assumptions are made to linearize some non-linear constraints as the variation in stockpile quality over time. The implementation of the model on the export supply chain of PT

Kaltim Prima Coal in Indonesia shows the ability of DES to analyze multiple scenarios towards the increase of the value of the supply chain. However, the method is not able to generate LOM productions schedules or to account for non-linear expressions along the supply chain.

Interesting developments have being done in other industries to optimize value chains and incorporate uncertainty. Goel and Grossmann [39] model the construction of well and production platforms and pipelines in an offshore gas field as a multi-stage stochastic programming model. The model accounts for the uncertainty in the sizes and initial deliverabilities of the fields. The authors develop a decomposition based approximation algorithm that involves solving scenario sub-problems and a sequence of two-stage stochastic programming problems. Through different examples, the authors show improvements in expected net present value and good solutions in reasonable time. Tarhan et al. [103] develop a multistage stochastic programming approach for the planning of oil or gas infrastructure. The main uncertainties considered are the oil/gas flowrate, recoverable oil/gas and water breakthrough time of the reservoir. The probability distributions of the uncertain parameters are discrete, which allows representing the stochastic process by scenario trees. The model optimizes investment decisions such as number of wells to drill, facilities to build and operational decisions such as oil production rate from the reservoirs to maximize the expected net present value. To solve the problem, the authors propose a duality-based branch and bound algorithm that takes advantage of the problem structure. The solutions obtained with the algorithm are 10 and 22% better than the solutions obtained with the expected value approach in two different examples.

This result highlights the importance of stochastic optimizers in improving expected results when compared with deterministic implementations.

### 1.3 Stochastic mine planning

Geological risk is seen as the major contributor to not meeting project expectations. Vallee [106] notes that 60% of surveyed mines had an average rate of production less than 70% in the first year of production. Ravenscroft [87] proposes a methodology to measure the risk in mine production scheduling by using orebody models generated with conditional simulation. The methodology consists of testing a mine production schedule generated from an estimated model on a set of orebody simulations. The author performs a risk analysis in the schedule of a large open-pit mining operation and observes that in one of the periods evaluated, there was only 40% of chances of deviating less than 10% from the grade that was expected. Dimitrakopoulos et al. [25] point out that for any open-pit design, the uncertainty over grades, tonnages or geology can be readily modelled and integrated into the optimization and design process so as to provide accurate modelling and quantification of uncertainty and risk. The authors show the limits of conventional optimization in a test with a disseminated gold deposit where net present value determined from conventional optimization (estimated orebody model and pit optimization with industry conventional optimization standards) has only 2-4% probability of occurring with a 95% of probability of the project of returning a lower NPV than predicted. To summarize, traditional open pit optimization: *(i)* ignores uncertainty generating misleading NPV and thus suboptimal solutions and major deviations from production plans; *(ii)* is unable to assess thus manage risk regarding key project indicators.

Therefore, the implementation of risk-based approaches to value assets, operations or projects as well as quantifying and minimizing the associated risks is sought.

Dimitrakopoulos et al. [28] present one of the first efforts to include uncertain supply based on orebody simulations and conventional optimizers. To generate the pit designs and mining sequences using conventional methods, nested pit implementation of the Lerchs-Grossman algorithm and Milawa scheduler available in Whittle software are used [112]. The Lerchs-Grossman algorithm (LG) formulates the problem of generating the ultimate pit as a graph closure problem[72]. Seymour [94] proposes a pit limit parameterisation from the three-dimensional LG algorithm to discretize the pit space and account for discounting by iteratively modifying a given parameter, generating different pit shapes that are optimal under the specified parameter conditions. The nested pits (or pitshells) generated from parametric LG can be grouped into phases or pushbacks that respect operational constraints. Milawa scheduler from Whittle software is based on possible combinations of pushbacks and benches to generate the highest NPV [112]. Dimitrakopoulos et al. [28] propose a maximum upside / minimum downside approach to open pit optimization based on the quantification of geological uncertainty through the generation of a series of equally probable representations of the orebody. The approach consists of the following steps: *(i)* Stochastically simulate several orebody models using the available data; *(ii)* determine the final pit and generate the mining sequence design for each orebody simulation using the parametric implementation of Lerchs-Grossman algorithm and the Milawa algorithm incorporated in Whittle software; *(iii)* quantify the level of risk with each pit design for the key project indicators, such as net present value (NPV) of the project, ore production, metal production and cash flows; *(iv)* discard pit designs that do not meet the key project performance indicators deemed necessary; *(v)* calculate the upside potential and downside risk on the project indicators for the remaining designs using a point of reference and select the design that meet the stated decision making criteria. The method is applied at an epithermal gold deposit in where 13 orebody simulations are used to generate the set of possible designs. The key project indicators in the case study are discounting cash flows (DCF), periodical ore tonnage and metal content. Four mine designs are selected as they have more than 70% chance of producing at least one million tonnes of ore. Then, the upside potentials and downside risks of the 4 selected designs are calculated according to a minimum acceptable return (point of reference) of the DCF of the pushbacks. Two designs are selected as they have higher total upside potentials with less risk over their production life than the two others. To select the best schedule, the authors perform a sensitivity analysis of the two remaining designs to the gold price. Although upside potential values in both designs are comparable at the original price, the difference between upside potentials become significant when increasing the gold price, so that the selected final design provides the highest upside potential if there is an increment in gold prices. The approach permits selecting a single design preferable to the remaining in the group of designs being compared. However, the approach does not generate an optimal design that simultaneously accounts for all the possible scenarios (orebody simulations).

Dimitrakopoulos and Ramazan [26] develop a mathematical programming model to generate a mine production schedule that accounts for geological uncertainty and equipment mobility. The first term of the objective function of the model, that is linear, penalizes the deviation from having 100% probability of meeting the desired grade and ore quality and quantity. The last two terms of the objective function control the smoothness of the mining operation by accounting, for each block, the tonnages of some surrounding blocks belonging to a neighbourhood that are not mined simultaneously. These tonnages are used to calculate the deviations from pre-defined tonnage targets, which are also penalised. These two last terms are evaluated in the same manner but differ in the size of the neighbourhood and the penalty values, whereas penalties for deviations in the small neighbourhood should be greater than the penalties used on the bigger one. The model can be applied in multi-element deposits and can be easily extended to a mixed integer programming (MIP) model simply by defining the variables as binary instead of linear. The formulation contains constraints to calculate the probabilities, ensure the quality of the material, and respect the capacities for mining and processing. The model introduces the geological risk discounting; that is, the rate at which decrease the penalty values of not having 100% probability of getting the desired properties. It implies that blocks with highest probabilities will be scheduled in the early periods when the penalties are largest, and the uncertain blocks are scheduled later when more information will become available. The authors tested the method in a nickel-cobalt lateritic deposit in where attributes as Ni, Co, Mg, Al and % of rock types and ore were jointly simulated using minimum/maximum autocorrelation factors (Section 1.4). The realizations were used for calculating the probabilities of the blocks of having the grades within the desired intervals. The results of the simulation-based model exhibited a highest probability to achieve the desired properties in the first year (88.3%), a lower probability in the second year (84.3%) and the lowest in the last year (78.83%), which is what the formulation aims to do and is due to geological risk discounting. The traditional model for production scheduling does not control the risk, obtaining the highest probability to achieve the desired properties in the last period, when it is more likely to take risks as more information will be available at that time. The simulation-based model controls the risk of not having the desired properties while generating a practical schedule for mining due to the control of smoothness in the objective function. The schedule obtained with the traditional model has a pattern spread over the deposit and does not appear feasible in practice, which is a common concern with traditional mixed/linear integer programming scheduling models. Although the NPVs of both schedules appear to be similar, the one obtained with the traditional model is unreliable given the poor control of risk (year 1) and the lack of smoothness (infeasible schedule). The magnitude of the penalties that control the probability of having the desired properties and the smoothness of the schedule are defined as a trade-off between the quality of the solution in terms of meeting the processing plant requirements and the feasibility of the schedule based on the accessibility. The major drawback of the model is that the use of probabilities acts in a block-by-block basis while scenario-wise approaches make full use of joint local uncertainty. This limits the ability of probabilistic programming formulations in generating higher rewards with less risk. Similar results were obtained by Grieco and Dimitrakopoulos [45], who implement probabilistic programming in stope design optimization. The authors evaluate the probabilities of the different rings of being above specified cut-offs. The limitation of the probabilistic programming formulations come from the fact that probabilities are evaluated independent for each unit (blocks or rings), which discard the compound relationship between units.

Godoy [37] develops a multistage method for mine production scheduling that integrates the joint local uncertainty using simulated annealing (SA) algorithm [35, 64, 76] in the optimization stage. The method seeks for generating a risk-based production schedule that minimizes deviation from ore and waste production targets over a set of orebody simulations. The stages of the method are: *(i)* calculate the stable solution domain (all the feasible combinations of ore and waste); *(ii)* generate optimal mining rates using a linear programming formulation; *(iii)* generate mining sequences for the available simulations using a conventional scheduler; *(iv)* derive a single mining sequence using a combinatorial optimisation algorithm based on SA. The risk-based schedule minimizes expected deviations from annual ore and waste production targets. The algorithm iteratively perturbs an initial schedule by swapping the periods in where the blocks are mined and evaluating the deviations. The evaluation in every perturbation of the deviations from ore and waste production targets for each scenario incorporates the joint local uncertainty, preserving the spatial correlation in the simulations that are discarded in probabilistic formulations. The method accepts perturbations that reduce the deviations from mine production targets with 100% probability, while the perturbations that increase the deviations are accepted or rejected based on a probability function that accounts for the objective value of the new solution and the annealing temperature (T). T is reduced by a cooling factor which controls the annealing schedule that accepts unfavourable perturbations according to a negative exponential probability distribution [44]. At higher temperatures, the algorithm is more likely to accept unfavourable perturbations. The acceptance of unfavourable perturbations allows the algorithm to avoid local optimal while the cooling factor permits to converge to a final solution. Godoy and Dimitrakopoulos [38] test the method at Fimiston gold open pit (Superpit) in Western Australia. The expected deviations from production targets in the risk-based production schedule are shown to be less than 4% in all production periods while in a conventional schedule periods with expected deviations of the order of 13% are found. The conventional optimizer generates a solution that does not meet production targets leading to not meeting NPV forecast either. The risk-based schedule has an expected NPV 28% higher than the forecast of the conventional one (predicted by the conventional optimizer). This is originated from the fact that the forecast of the conventional optimizer evaluates the schedule assuming that the smoothed representation of the deposit denotes the reality, whereas the risk analysis on the stochastic schedule accounts for the different orebody simulations. Furthermore, the stochastic schedule allows producing more ore in the periods evaluated by minimizing the deviations. In addition, the swapping of the periods of the mining blocks is done considering the probabilities of the blocks of being mined in the different periods, which are calculated from the mining sequences that are generated so as to maximize NPV. Therefore, the stochastic formulation maximizes NPV in an indirect way.

Leite and Dimitrakopoulos [69] test the method in a low-grade disseminated copper deposit in where 20 orebody simulations were used. The authors obtain an increment of 26% in the NPV with the stochastic schedule when compared to the results obtained by a conventional approach. The average NPV of the stochastic schedule is 15% higher than the average from a risk profile of the conventional schedule; that is, testing the conventional schedule on the 20 orebody simulations. This 15% of increment may be seen as the value of the stochastic solution (VSS) which represents what is expected to be gained by implementing the stochastic schedule [9]. The authors also find that, for that particular case study, the stochastic schedule reduces the life of the mine one year caused by a lower tonnage of ore exhibited in the simulations when compared to the estimated model at the cut-off grade used (0.3% Cu). It is well known that interpolation methods tend to overestimate ore tonnages in low cut-off grades and underestimate ore tonnages in high cut-offs.

Even though there are substantial benefits of using risk-based schedules through simulated annealing algorithm, some additional aspects are further addressed. Albor and Dimitrakopoulos [1] use the same low grade disseminated copper deposit as in [69] to evaluate the best starting sequence; define the number of simulations (or mining sequences) required; and the ultimate pit limit. The authors perform a sensitivity analysis to select the best and the worst starting mining sequences regarding the production targets and the NPVs. They observed that using the same annealing parameters, the schedule obtained with the worst starting mining sequence did not meet the ore production targets and yielded a low cumulative net present value when compared to the production schedule that uses the best starting sequence. The authors implement the method using different number of starting sequences and notice that the production schedule obtained is not particularly sensitive after 10-15 mining sequences. Although a large number of orebody simulations may be generated, the study shows that for production scheduling 10-15 simulations or more lead to the same schedule. This is because the yearly schedule groups hundreds of mining blocks in one extraction period, which is a large volume and therefore it is affected by the volume-support effect; that is, the sensitivity of a schedule is not the same as observed at the scale of individual mining blocks. The method by construction uses predefined pit limits obtained conventionally. In the presence of uncertainty, a conventional (deterministic) optimization cannot provide an optimal solution, i.e., truly optimal pit limits. Albor and Dimitrakopoulos [1] extend the ultimate pit limit by adding pitshells (from Whittle´s nested pit implementation of LG) as a final pushback and allow the simulated annealing algorithm to decide whether or not to mine the blocks added. They find a stochastic ultimate pit limit 17% greater than the deterministic one while the net present value increases by an additional 9%. The method has several limits that can be overcome: it does not consider grade blending constraints; does not defer the risk for the latest periods (geological risk discounting); and does not optimize the other components of a given mining complex (multiple deposits, stockpiles, processing destinations, etc.).

Ramazan and Dimitrakopoulos [85] develop a stochastic integer programming formulation for mine production scheduling. The scheduling is formulated as a two-stage stochastic integer program with fixed recourse [9], where the binary variables that represent whether or not a block is mined in a given period are the first stage variables; and the deviations from production targets over the different scenarios (ore-body simulations) correspond to the second stage variables. The objective function seeks to maximize the net present value while simultaneously minimizing deviation from ore, waste and metal production targets. The deviations from production targets are controlled by penalties that decrease using the geological risk discounting introduced in the probabilistic formulation [26], which allows deferring risk to latest periods. The authors test the method on a hypothetical two-dimensional data set where the life of the mine is 3 periods. The problem is solved using the commercial software for optimization ILOG CPLEX [53]. Several schedules with different risk distributions are generated by considering different ore and grade penalty values. The model allows the selection of the best schedule based on the expected NPV and the risk profile defined.

Ramazan and Dimitrakopoulos [86] extend the formulation to include stockpiling, which allows processing material from the stockpile at any period. The amount of material processed from the stockpile depends on the simulated orebody models. The authors test the method on a gold deposit in where the implementation of the stochastic integer programming approach requires splitting the problem in two sub-problems because of its size. The first sub-problem considers periods 1-4 while the second sub-problem considers years 4-6. The authors state that the schedule obtained with the SIP formulation reduces the ore deviation in the first year of production to 500kt from 4Mt that is obtained with the conventional approach. Furthermore, the increment in net present value due to the stochastic implementation is 10%.

Leite and Dimitrakopoulos [71] apply the method in the same low-grade disseminated copper deposit where they implement the multi-stage approach with simulated annealing algorithm. Firstly, the authors define the final pit conventionally. They use 20 orebody simulations and a geological risk discounting of 20% with the magnitude of the penalties weighted accordingly with the first term of the objective function that accounts for NPV. The authors later apply a geological risk discounting of 30% with no substantial differences between both stochastic schedules. They compare the base stochastic schedule with a conventional schedule generated using Milawa NPV algorithm [112]. While the stochastic schedule control the deviation from production targets, the conventional one only meets the expected targets in two years (from 8 mining periods). Although the NPV forecasts of the conventional schedule are not reliable given the large deviations from targets, the stochastic schedule generates an expected NPV 29% greater than the value of the conventional one.

Bendorf and Dimitrakopoulos [11] expand the stochastic integer programming approach to multi-element deposits and include mineability constraints to facilitate accessibility and equipment size constraints [26]. The authors implement the approach at Yandi Central 1 iron deposit in Western Australia, in where $Fe$, $SiO_2$, $Al_2O_3$, $P$ and losses of ignition (LOI) are the key deposit attributes. The goal is to ship to customer material with specified geochemical characteristics by maintaining the various grades between target limits. The authors test the effect of the order of magnitude with three different penalty values to the elements to control ($1, $10 and $100 per unit of deviation in the first year) with a geological discount rate of 10%. They observe that higher the penalty value, more dispersed is the schedule generated. The authors find that with medium penalty values, the fluctuation of grades between periods decreases significantly and there are slight probabilities of deviating from target in few periods, while the higher penalties improve the results only marginally in comparison with the medium penalty values. Even though the schedule they obtain with the medium penalty values is more dispersed than the one obtained with low penalties, it is reworked to a feasible schedule using a hauling road design tool [22] with no significant impact on the results.

Typically, short-term production sequences deviate from the long-term plans and are adjusted to meet mill demand to avoid large deviations from forecasts. Jewbali [56] develops a multistage approach for production scheduling incorporating short-scale deposit information and related grade uncertainty into the scheduling process, thus allowing for the realistic integration of short- and long-term mine production schedules, as well as the generation of more reliable mine production forecasts. The first stage consists in simulating future grade control data from mined out parts of the deposit. The second stage calls for updating the current geological simulations with the grade control data by using conditional simulation with successive residuals [107, 57]. Once the orebody simulations are updated, generate a mine production schedule using a stochastic optimisation method handling multiple simulated orebody models, while accommodating both maximising net present value and minimising deviations from production targets [86]. The final stage consists in quantifying grade risk in the production schedules that have been generated. Dimitrakopoulos and Jewbali [31] implement the approach at a gold deposit in where 20 orebody simulations and 20 simulated grade control data are considered, which generates 400 simulations conditioned on both the exploration data and the simulated future grade control data. Twenty updated simulations are selected for scheduling given the volume-support effect [1]. The approach delivers 3.6 Mt of additional ore which matches better with the mine reconciliation and adds 7.7 million dollar to the expected NPV when compared to a stochastic schedule that does not consider grade control data.

Boland et al. [13] propose a multistage stochastic programming approach that considers the decision of processing block aggregates as posterior-stage variables. This approach provides a set of polices to follow according to the actual scenario (orebody simulation) obtained with the advance of the extraction. The set of aggregates with considerable differences in grade permits to differentiate two different scenarios. The mining and processing decisions can change when new information is revealed. However, mining decisions require a lag between the acquisition of new information and reacting to that information (they consider one year of production), whereas processing decisions react in real time. In multistage stochastic formulations, there exists large number of non-anticipativity constraints to ensure that if two scenarios cannot be distinguished until certain time, then the same decisions will be made under both scenarios. The authors propose two variations of the method: scenario dependent processing decisions, that consider mining decisions equal for all scenarios; and scenario dependent mining and processing decisions in which both decisions depend on the scenario considered. The authors use realistic mining data to compare the multistage stochastic approach with a base-case method that ignores multiple scenarios and find that the proposed approach increases the net present value around 3% (the expected value of perfect information is 5% higher). This formulation presents several drawbacks: it assumes that the scenarios used cover all possibilities, therefore no policy will be provided if reality happens to be different (as it might be); it assumes that the blocks inside an aggregate are mined in the same proportion, which can push cash flows in time where high grade blocks are at the bottom of the aggregate; it does not penalise the schedule in cases where the mill demand is not supplied in a scenario, which can lead to considerable shortfalls in ore processed.

Albor and Dimitrakopoulos [2] develop a methodology for pushback design that involves geological uncertainty by testing the stochastic integer programming formulation [85] for scheduling over different pushback designs. The available pushback designs are generated by grouping a set of nested pits. Given a set of N nested pits, the number of possible combination of pits for generating a pushback design is given by 2N-1, so that testing all the possible combinations with the SIP formulation is impractical. The authors suggest considering the best pushback design for different target number of pushbacks. A stochastic integer programing formulation for scheduling is implemented in the pushback designs selected previously, and the one that leads to the best results in terms of meeting production targets and net present value is selected. The approach was implemented in a porphyry copper deposit in where 20 orebody simulations were available. 17 nested pits were generated and 6 different pushback designs that have different number of pushbacks were considered. The SIP formulation is implemented on the 6 available pushback designs. The selected pushback is the one that is practical and leads to the highest NPV. Such design was obtained by combining the available pushback designs: the starting pushback that gives a higher NPV from the available designs was selected as starting pushback; the intermediate pushbacks were selected to avoid ore shortfalls and infeasible waste production rates; and, the bottom of the pit was discretized for being more selective and avoid negative cash flows at the end. The stochastic LOM production schedule originated with the stochastic pushback design method although capable of controlling deviation from targets, it is also able to generate a NPV around 30% higher than the value of the traditional pushback design and scheduling implementation.

Meagher et al. [73] develop a method for pushback design through a parametric implementation of the maximum flow/minimum cut algorithm [81] in where geological and market uncertainty where considered as well as discounting. Several stochastic models can be used to describe the evolution of metal prices and exchange rates with time. In particular, the authors suggest the use of the mean reversion method proposed by Schwartz [95] because of its availability to reproduce the cyclic nature of metal and currency markets. With multiple orebody realizations and price simulations, several block valuations are obtained. The mining blocks are treated as nodes that can be connected through arcs to a source node or a sink according to their economic values. Arcs connecting blocks with negative economic values to the sink are placed with capacities equal to the economic value of the block. The source node is connected to blocks with positive economic values by arcs with capacities equal to the economic value of the block. To deal with multiple realisations, negative and positive economic values are accumulated over all realisations and connected to the sink and the source node respectively. Slope constraints are respected by placing arcs with infinity capacities between corresponding blocks. A cut in a directed graph is a set of edges such that after the removal of these edges no direct path exists between the source node and the sink. A minimum cut is the set of edges where the sum of capacities is as small as possible over all cuts in the graph. Many efficient algorithms are known for computing the minimum cut of a graph in polynomial time with a well performance in practice. A parametric minimum cut algorithm replaces the capacities on the arcs leaving the source and arcs entering the sink with functions of a single parameter $p$. Different $p$ values generate different nested pits obtaining small pits at small $p$´s when they multiply the economic value directly. The discounting is involved in this modified parametric net flow algorithm by considering different values according to the different years of production and combining them as was done with the multiple realization values. The authors observe that in a copper deposit, conventional block valuation methods tend to under valuate mining blocks.

Chatterjee et al. [17] applied the method developed by Meagher et al. [73] at a copper-gold deposit by implementing the push-relabel algorithm [40] for solving the minimum cut problem. In this case study, only the geological uncertainty is considered (no metal price and exchange rate uncertainties) through 20 orebody simulations generated with direct block simulation algorithm [37]. The discount rate is not considered in the generations of pushbacks. The number of pushbacks generated with the method is 5, which minimizes the gap problem (large differences in pushback sizes). However, the authors observe a high increment in the stripping ratio in the 5th pushback that may indicate that is not profitable to mine after pushback 4. The authors generate bench-wise schedules in the stochastic pushback design obtained and the conventional pushback design of the average grade of the simulations (E-type). The uncertainty-based pushback design and schedule generated 11% higher NPV than schedule from the E-type orebody model. Asad and Dimitrakopoulos [6] introduce the subgradient method [101] to systematically update the parameters for addressing the gap problem. The authors apply the approach at a copper deposit involving copper price uncertainty as well as geological uncertainty. They generate two different pushback designs, one by scaling the parameters with the subgradient method and one that does not scale them. The authors observe that when scaling the factors they obtained approximately uniform size in phases as opposed to the inconsistent phases when the parameters are not scaled. Therefore, the scaling procedure played its role in addressing the gap problem. The stochastic approach leads to an optimal pit design 45% larger than the conventional one, providing a higher economic value and metal production. The extended version of the minimum cut algorithm is computationally very fast, thus integrating multiple uncertainties in the optimization process is feasible on a routine basis. The major drawbacks of the method are: *(i)* there is no geological risk discounting to defer the risk in time; *(i)* even though the method uses substantially more information than conventional network flow models, the fact that it accumulates economic values along different realizations averts the use of the joint local uncertainty thus not maximizing the upside potential of the deposit.

The cut-off grade is an economic-based criterion to discriminate between ore and waste in a mineral deposit, or to decide where to send the ore material among a set of processing streams. The methods that have been described so far use static cut-off grades that do not consider opportunity costs associated to grade-tonnage distributions of the deposit. A dynamic cut-off optimization policy would decide whether material should be stockpiled for future processing or processed immediately. Many publications can be found in the technical literature concerning estimation and optimization of cut-off grades [63, 67, 89]. An optimal cut-off policy depends on the mining extraction sequence, which is influenced by the choice of the cut-off; hence there is an interaction between mining sequence and cut-off policy.

Menabde et al. [75] develop and implement a method that accounts for geological uncertainty and simultaneously optimizes the extraction sequence and the cut-off grade. The approach developed is similar to the traditional mixed integer programming formulation for mine scheduling but extended to include multiple orebody simulations and multiple cut-off grade values from which one will be selected at each period. At first glance the formulation seems difficult to solve due to the large number of integer variables involved but the authors implement an algorithm for block aggregation that allows replacing the binary variables representing blocks for significantly less binary variables representing panels. The authors do not indicate the mechanism that uses the algorithm for aggregating blocks due to confidentiality reasons. Applying variable cut-off grades without considering grade uncertainty allows increasing the expected net present value 20% when compared to the schedule generated with marginal cut-off grades. Considering the orebody simulations and the variable cut-off grade simultaneously, the solution obtained increases the net present value 26% with respect to the base case, i.e., 6% with respect to the case of variable cut-offs without geological uncertainty.

### 1.4 Stochastic simulation

In the uncertainty-based methods described previously the geological uncertainty is modeled through spatial Monte Carlo stochastic simulations. Simulations are used to integrate joint local uncertainties in stochastic frameworks such as stochastic mine scheduling optimization. The basic idea of conditional spatial simulations is to generate multiple realisations (images) of a pertinent attribute reproducing all data/information available. For geological data, a conditional simulation must reproduce *(i)* data statistics, *(ii)* spatial correlation, and *(iii)* original data.

A random function in a set of $N$ locations is characterized by the $N$-variate or $N$-point cumulative distribution function: $F(u_1, \ldots, u_N : z_1, \ldots, z_N) = Prob\{Z(u_1) \leq z_1, \ldots, Z(u_N) \leq z_N\}$. In two-point geostatistics the analysis is limited to cdfs involving no more than two locations at a time and their corresponding moments, e.g., the (two-point) Z-covariance $C(u, u^{'}) = E\{Z(u).Z(u^{'})\} - E\{Z(u)\}.E\{Z(u^{'})\}$. The concept of second order stationarity implicates that the bivariate cdf depends on the distance $h$ of separation of locations $u - u^{'}$ instead of the locations itself.

Given a set of data and grid points, the covariance matrix $C$ can be generated including both conditioning and grid points. Davis [23] proposes a method for conditional simulation based on the Cholesky decomposition of the covariance matrix. Given that a covariance matrix $C$ is positive definite, it can be decomposed into the product of a lower triangular matrix $L$ and its conjugate transpose $T$ (named also $U$). A conditional simulation that reproduces the covariance matrix is obtained when multiplying the lower triangular matrix $L$ by a vector that contains weight values derived from conditioning data and independent random numbers. A major limitation of the method is the size of the covariance matrix $C$ (that considers both conditioning and grid points) which in practice cannot exceed few thousand of points.

Johnson [58] describes the conditional distribution approach that allows decomposing the problem of generating a $p$-dimensional random vector $X = (X_1, X_2, ..., X_p)$ into a series of $p$ univariate distribution problems. Isaaks [55] introduces the sequential conditional simulation method based on the decomposition of the multivariate *pdf* into a product series of conditional distributions. Sequential Gaussian simulation is the application of the decomposition of the multivariate *pdf* to the Gaussian random function model. Any stochastic simulation method is implemented considering neighbouring data around the grid points to simulate, which is known in the literature as screen effect approximation. The sequential Gaussian simulation with screen effect approximation reduces significantly the computational cost when compared to the LU conditional simulation algorithm. In a case study with $n$ conditioning points and $N$ grid points a realization of conditional LU algorithm [23] requires a number of floating point operations (flops) of the order of $O(n+N)^3$ while a SGS implementation with screen effect requires $O(NV_{max})^3$, in which $V_{max}$ is the neighbourhood size. Therefore the benefits in terms of number of flops depend on the neighbourhood size and the number of conditioning points. Furthermore, with SGS there is significant less requirement of memory as it is not necessary to consider the large covariance matrix with conditioning and grid points as in the LU.

Dimitrakopoulos and Luo [27] propose a generalized sequential Gaussian simulation algorithm (GSGS) to enhance the computational efficiency. The authors propose the decomposition of the multivariate probability density function Z(u) into groups of products of univariate posterior distributions, where each group is used to simultaneously generate realization at the corresponding grid nodes. A unique neighbourhood is considered in each group in where the grid points are simulated using LU decomposition method. The GSGS algorithm can be seen as a generalization of the sequential Gaussian simulation method, in which a decomposition with a single node in a group is identical to SGS, and a decomposition with all nodes in one group is identical to LU method. The authors also introduce a measure of the loss due to screen effect approximations (SEA loss) and find that they are function of the corresponding posterior variances. The number of flops in GSGS is given by $O(N/V(V_{max}^3 + V^3))$ where $N$ is the number of points to simulate, $V$ is the number of grid points in a group and $V_{max}$ is the size of the neighbourhood. The authors observe that the number of flops is minimal when the group size is approximately 80% of the neighbourhood size. They also observe that losses due to screen effect approximations were negligible in exponential and spherical covariance models while in Gaussian models they were high in most of the cases.

Godoy [37] propose a natural extension of the GSGS algorithm called direct block simulation (DBSIM). The author noticed that when simulating large grids, the new simulated values have to be retained as conditioning information. This generates increased memory requirements, issues of data management and, in general, leads in practice to performance decline [10]. DBSIM simulates directly at the block support scale based on GSGS, whereby the group of nodes discretizes a block. The implementation of the DBSIM proceeds as follows: *(i)* define a random path visiting each of the blocks to be simulated; *(ii)* normalise data; *(iii)* for each block, generate the simulated values in Gaussian space of the internal nodes discretizing the block; *(iv)* derive the simulated block value by averaging values of simulated nodes in one group in Gaussian space and calculate the block value in data space; *(v)* discard values of internal nodes and add the simulated block value in Gaussian space to the conditioning dataset; keep the block value in data space as the result; and, *(vi)* loop through steps three to five until all blocks are simulated. The author implements DBSIM at Fimiston Gold mine to generate 20 realizations. The Fimiston resource model consists of 321,937 ore blocks and a total of 20,843,814 nodes to be simulated (416,876,280 grade values for the 20 realizations). DBSIM generates the realizations and validations in practical time (1.8 times the processing time required to generate an estimated model). The results show a close reproduction of the block variograms and a consistent reproduction of the sample statistics. Bendorf and Dimitrakopoulos [10] observe through an application on a porphyry copper deposit that the application of DBSIM results in a substantial reduction of storage requirements and leads to improved data management when compared to GSGS. Boucher and Dimitrakopoulos [14] extend the method for multivariate simulations by using minimum/maximum autocorrelation factors.

Multiple point or MP statistics consider the joint neighbourhood of any number $n$ of points. Two-point statistics is a particular case where $n = 2$. MP statistics can be formulated using the multiple-point data event D with the central value A. The geometric configuration of D is called the template $\tau_n$ of size $n$. As MP statistics characterize spatial relations of closely spaced data, they may not be calculated directly from drilling data. Guardiano and Srivastava [47] propose the use of training images (TI) to infer MP statistics. The TI is regarded as a geological analogue, forms part of the geological input, and it should contain the relevant geometric features of the units being simulated. The geometries contained in the TI should be consistent with the geological concept and interpretation of the deposit. The authors proposed a direct algorithm for imposing MP statistics into stochastic simulation that is extremely CPU demanding: the full training image has to be scanned at each unsampled node to infer the node-specific conditional probability distribution.

Strebelle [98] proposes the single normal equation simulation algorithm (SNESIM) that can be seen as an extension of the method developed by Guardiano and Srivastava [47]. The TI image is scanned once and the events that occur are stored in a search tree in where probabilities of the data are calculated based on the replicates obtained in the scanning process. At each location the cpdf is calculated from the search tree based on the conditioning data event and a simulated value is drawn from the distribution. This process is repeated until all nodes have been simulated. The SNESIM algorithm represents a large improvement in computational costs as the TI is scanned once and only the events that occur in the TI are retained in the search tree. Strebelle and Zhang [99] present a modification of SNESIM to account for non-stationary information. The modified SNESIM is able to capture different directions of continuity of the training image and fluctuations in facies dimension. Strebelle and Cavelius [100] increase speed and decrease memory demand of SNESIM by introducing a new multiple-grid approach that includes intermediate grids, a new search neighborhood designing process to preferentially include previously simulated node locations and a method to optimize data template size. However, in large datasets memory is still an issue given the necessity of storing the data events in the search tree. Straubhaar et al. [97] present IMPALA, a revision of SNESIM where the search tree is replaced by a list to decrease RAM requirement. This also allows accounting for non-stationarity by having different lists at different simulation zones and parallelization by splitting the lists into as many sub-lists as the number of available processors.

Arpat and Caers [4] propose a pattern-based simulation algorithm to overcome the limitation of RAM usage. The algorithm uses the training image as a database of patterns, which are multi-pixel configurations identifying meaningful entities of the underlying spatial continuity. The algorithm randomly visits nodes along a random path and simulate/paste an entire pattern. The major random component of the method is the random path it uses to visit the nodes whereas the major tuning parameter is the template size. Although less memory demanding than SNESIM, its computational cost is much higher as it requires searching for most similar patterns at each node. Zhang et al. [117] propose FILTERSIM to reduce the dimensionality of the space of patterns. The method groups all the patterns from a TI into a set of classes using filter scores. At each location, the method identifies the training pattern class closes to the local conditioning data event, then samples a training pattern from the prototype class and pastes it onto the simulation grid. The selection of the closest pattern class is based on the wise-distance between the prototype of each training pattern class and the local conditioning data event. Wu et al. [115] propose a reduction of the computational cost of FILTERSIM by replacing that pixel-wise distance calculation with a filter score comparison, which allow speeding up FILTERSIM algorithm by a factor around 10 in 3D applications. Mustapha et al. [79] propose CDFSIM, a pattern-based simulation method that maps the patterns to one dimension. The method builds cumulative distribution functions (CDF) of the one-dimensional patterns and classifies them by decomposing the CDFs. The implementation of the method shows improvements in reproducing MP configurations and main characteristics of images when compared with FILTERSIM. Furthermore, CDFSIM appears to be less sensitive to the number of classes and the spatial templates. Chatterjee et al. [18] present a pattern-based simulation algorithm that uses wavelet analysis for dimensional reduction of the space of patterns. The method performs wavelet decomposition of the patterns generated using a given template and classify the approximate sub-band of the patterns using k-means clustering technique. The implementation of the method shows a better reproduction of images in 2D and 3D examples when compared with FILTERSIM. The method is sensitive to the number of clusters as the pattern based simulation methods and the orientation of the training image.

MP simulation methods although widely used, have a major drawback: they rely on the training image for probabilities or patterns, therefore when the statistics of the training image and the hard data are different, they reproduce the statistics of the training image, being the hard data information more certain (and costly) [41, 80, 90]. Robles and Dimitrakopoulos [90] apply both SNESIM and FILTERSIM at a kimberlitic diamond pipe in the Northwest Territories and observe that while the realizations tend to reproduce the high-order statistics of the TI, they do not reproduce those of the available data.

To overcome this limitation, a stochastic simulation algorithms that uses cumulants has being developed [77]. Cumulants are combinations of moment statistical parameters that allow complete characterization of non-Gaussian random variables [8]. A random variable is fully determined by its probability density function, its cumulative distribution function and its first or second characteristic function. Given a random variable Z, its first characteristic function is given by $\Phi(\omega) = E[e^{j\omega Z}] = \int_{-\infty}^{+\infty} e^{j\omega u} dF_Z(u)$ and its second characteristic function is $\Psi(\omega) = ln(\Phi(\omega))$. The moments of order $r$ at the origin are the coefficients of $((j\omega)^r)/r!$ of the MacLaurin series expansion of the first characteristic function: $Mom[Z^r] = E\{Z^r\} = (1/j^r)(d^r/(d\omega^r))[\Phi(\omega)]_{\omega=0}$. The cumulants of order $r$ are given by the derivative of order $r$ of the second characteristic function at $\omega = 0$: $Cum[Z, \ldots, Z] = (1/j^r)(d^r/(d\omega^r)[\Psi(\omega)]_{\omega=0}$.

The cumulants of the random variable $Z$ of order $r$ are related to the lower or equal order moments by:

$Cum[Z, Z \ldots, Z] = \sum((-1)^{p-1}(p-1)!E\{\prod_{iS_1} Z_i\}E\{\prod_{iS_2} Z_i\} \ldots E\{\prod_{iS_p} Z_i\}$, where the summation extends over all partitions $(s_1, s_2, \ldots, s_p)$, $p = 1, \ldots, r$. From the definition, it is possible to infer that the first order cumulant is the mean and the second order cumulant of a non-centered random function $Z(x)$ is the covariance. Mustapha and Dimitrakopoulos [77] develop an algorithm called HOSC (High Order Spatial Cumulants) to compute experimental high order spatial cumulants on regular and irregular grids. Mustapha and Dimitrakopoulos [77] develop HOSIM, a high-order simulation algorithm that implements a sequential simulation process, where local conditional distributions are generated using weighted orthonormal Legendre polynomials. These polynomials define the Legendre cumulants, which are high-order conditional spatial cumulants inferred from both the available data and training images. This approach is data-driven and reconstructs both high and lower-order spatial complexity. The authors validate the algorithm in a three-dimensional domain of complex channels.

## 1.5 Goal and objectives

The goal of this thesis is to develop and implement a new stochastic optimization framework for optimizing mining complexes while simultaneously accounting for the different components of the value chain (deposits, stockpiles, processing destinations, transportation systems) and geological uncertainty (grades and material types). To reach the goal, the following objectives are set:

(1) Critical review on recent developments in optimization of the different components of a mining complex, mine planning with uncertainty and stochastic simulations.

(2) Develop and implement a stochastic optimization model based on simulated annealing for mine production scheduling that accounts for geological uncertainty and integrates a single supply (pit) with multiple ore/waste destinations.

(3) Develop and implement an iterative improvement approach to optimize a multi-process mining complex accounting for geological uncertainty and operating modes at the different processing destinations.

(4) Develop and implement an uncertainty based heuristic that uses simulated annealing at different decision levels to optimize multipit mining complexes with operating and transportation alternatives.

(5) Extend the method developed in (4) to mining complexes comprised of open pit and underground operations and implement it at Twin Creeks Mining Complex, Nevada, USA.

(6) Discuss general conclusions and suggest future research avenues.

## 1.6 Thesis outline

The Chapter 2 presents the method to generate stochastic mine production schedules in mining complexes with multiple processing destinations and its implementation using Escondida Norte dataset.

The iterative improvement algorithm that accounts for operating modes at the different processing destinations is described and implemented in Chapter 3.

The Chapter 4 describes the method to optimize multipit mining complexes accounting for multiple operating modes at the different processing destinations and presents its implementation in a multipit copper complex.

The extension of the previous method to mining complexes comprised of open pit and underground operations is displayed in Chapter 5 as well as its implementation in Twin Creeks gold complex, Nevada, USA.

Conclusions and future research avenues are presented in Chapter 6.

## 1.7 Originality and contribution to knowledege

The major contributions of this thesis are highlighted in this section.

**I. Optimizing a mining complex by integrating flexibility at different**

**components of the value chain.**

Although standard practices in the mining industry are able to consider flexibilities at different components of the value chain such as material types, stockpiles, non-linear recovery curves, etc., this thesis considers, besides that, operating modes at the different processing destinations and transportation systems. Operating modes are result of different design and operating parameters at a given processing destination (processing times, temperature, pressure, rotation speed, etc.), differences in quality of the input material and different product specifications. Transportation systems are important given that they may limit the overall output of the system (bottlenecks).

**II. Extension of optimization to include both open pits and underground operations.**

Open pits and underground mines are usually optimized in isolation. This thesis integrates the optimization of both operations in the context of a global optimization of a mining complex while accounting for supply uncertainty.

**III. Development of new and efficient algorithms that can be implemented in a wide variety of mining operations.**

Given the flexibility associated with the different components of a mining complex and the discretization of the deposits in mining blocks (or activities in underground mines), the global optimization of a mining complex is a problem with millions or hundreds of thousands integer variables where standard optimization software packages are unable to solve or even find the linear relaxation. Because of this, heuristic methodologies and metaheuristics to solve the problems are presented in this thesis with results that outperform standard practices in terms of generating higher and more reliable NPVs. Simulated annealing is the metaheuristic used in the methods described in Chapters 2, 4 and 5.

**IV. Optimization moving beyond economic values of mining blocks to account for rock/material properties and blending requirements.**

Standard practices in mine optimization consider cut-off grade policies to define destinations of mining blocks which implies that the destination of a mining block is defined prior to optimization. The destination of a mining block not only depend on the properties of the block (grades, metallurgical properties) but also on the properties of the material sent to that particular destination given its blending and operating requirements. This thesis presents methods that swap destinations of mining blocks based on the properties of the compound material that go to those destinations.

**Full field testing and benchmarking.**

The methods described in this thesis are tested in full field case studies to evaluate their applicability in actual operations. For large size problems (millions of integer variables) the methods generate good quality solutions in practical times (less than 12 hours of running time using a common desktop computer).

# CHAPTER 2
# Stochastic mine production scheduling with multiple processes: Application at Escondida Norte, Chile

## 2.1 Introduction

A mine operation may be seen as a sequence of processes in which material is safely extracted from an ore deposit to feed one or several processing plants or waste dumps, given various requirements in quality and tonnage of the material extracted. Mine planners must guarantee the continuous operation of the plant(s) by generating mine production schedules that meet production targets and maximize discounted cashflows. Traditional methods for mine planning and open pit optimization consider single estimated (average type) orebody models in their calculation processes, ignoring the uncertainty associated with the spatial distribution of the attributes of interest. Several authors [25, 106] have concluded that geological uncertainty is the major factor in the inability to meet production targets and to generate reliable project expectations in mining. Over the past decade, new methods that take into account this uncertainty have been developed for open pit designs and life-of-mine (LOM) production scheduling [30]. These methods demonstrate effectiveness in controlling the risk of deviating from production targets while increasing the expected net present value of the operation as well as recoverable metal, thus contributing to the responsible utilisation of non-renewable resources [1, 2, 69]. Additional recent research work further extends the above methods to jointly address geological and market uncertainty [6, 73] and address issues of computational efficiency [65].

Stochastic optimization research in LOM production scheduling includes works by Godoy [37] and Godoy and Dimitrakopoulos [38] who develop a multistage method for LOM production scheduling that integrates geological uncertainty and is based on simulated annealing (SA) [35, 64, 76]. In [37] and [38], the authors test the method at Fimiston gold open pit mine, Western Australia. The expected deviations from production targets in the stochastic production schedule are shown to be less than 4% in all production periods, while in the schedule generated conventionally, periods with expected deviations of the order of 12% are observed. Furthermore, the net present value (NPV) increases by 28% when compared to conventional methods. Leite and Dimitrakopoulos [69] increase the expected NPV by 26% when applying the method in a low-grade disseminated copper deposit. Albor and Dimitrakopoulos [1] use the method in the same copper deposit with a pit 17% larger, obtaining an additional 9% to the expected NPV.

Even though the method integrates geological uncertainty, the solution generated is local in the sense that the method does not consider the entire mining complex [110]. A mining complex can contain several mine operations producing simultaneously with multiple processing streams, stockpiles and products (Figure 2–1). The optimal solution for a single processing plant differs from the global optimal solution for the mining complex, which highlights the necessity of generating global optimal solutions.

In the context of stochastic global optimization, Goodfellow and Dimitrakopoulos [42] implement a SA approach to design pushbacks in complex multi-process open-pit mines. The method attempts to minimize the variability around the target tonnage for each pushback while accounting for material types and grade uncertainty. The method proposed in this paper generates stochastic mine production schedules instead, and may be seen as an adaptation of the multi-stage method with SA [37, 38] to mining complexes with multiple material types and multiple processing streams. First, the method is described in detail, then it is applied to Escondida Norte copper deposit, and finally, some conclusions are drawn and future work is addressed.

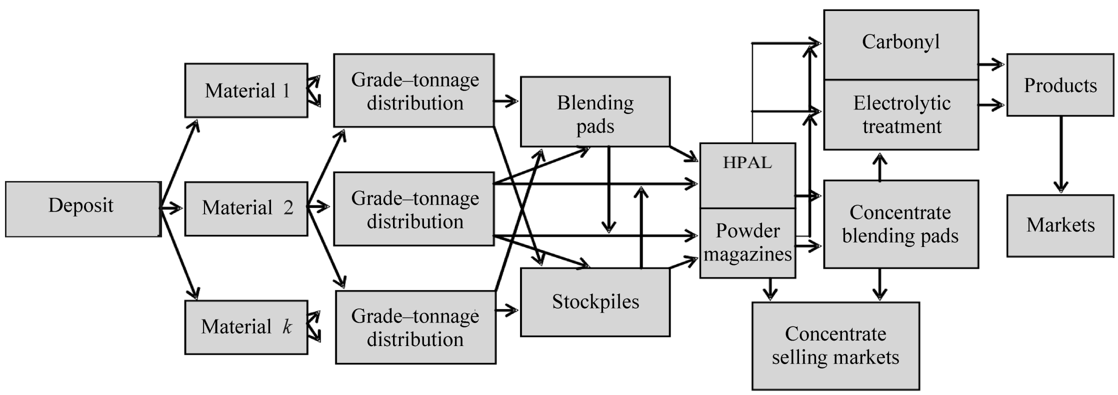


Figure 2–1: Example of a mining complex

## 2.2 LOM production scheduling with multiple ore/waste destinations via simulated annealing

The multi-stage method for generating stochastic long-term LOM production schedules (Figure 2–2) requires: *(i)* a set of stochastically generated orebody simulations , *(ii)* defined mining rates, *(iii)* a conventional scheduler to generate mine production schedules from the orebody simulations, and *(iv)* the implementation of a simulated annealing algorithm (SA). SA is a metaheuristic method for solving large combinatorial problems based on the principle of stochastic relaxation [35]. Stochastic relaxation is a class of optimisation algorithms that randomly perturb the current state of the system and determine the resulting change in performance, allowing temporary decreases in an objective function with nonzero probability. SA gradually perturbs an initial solution so as to match constraints, e.g., meeting production targets in a mine production scheduling problem. The algorithm requires an objective function that measures the deviation between the target and current values (productions) of the solution at each $i$-th perturbation. The steps of the algorithm are as follows:

(1) Define an initial solution.

(2) Compute the initial value of the objective function.

(3) Perturb the solution by some mechanism, such as swapping elements of the solution.

(4) Compute again the objective function, accounting for the modification of the previous solution.

(5) Accept or reject the new solution on the basis of a specified decision rule.

(6) If the new solution is accepted, update the solution to the new perturbed solution.

Repeat steps 3 to 6 until the target constraints are acceptably reached or the perturbations do not further reduce the objective function significantly.

Different criteria can be used to decide whether a given perturbation is accepted or rejected during the optimization process. The decision rule applied in the proposed

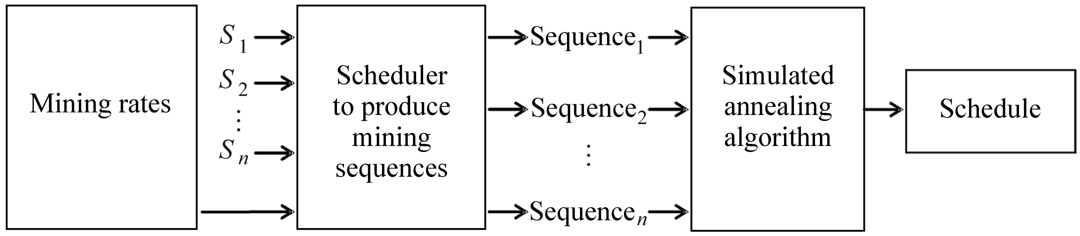


Figure 2–2: Multi-stage method to generate stochastic long-term production schedules [37]

method herein is the one commonly used, and amounts to accepting unfavourable perturbations according to a negative exponential probability distribution [35]:

$$P(\Delta O) = P(O_{new} - O_{current}) = \left\{ \begin{array}{rl} 1 & if(\Delta O \leq 0) \\ e^{\frac{-(\Delta O)}{T}} & if(\Delta O > 0) \end{array} \right\} \tag{2.1}$$

where $T$ is the annealing temperature. The probability of accepting an unfavourable perturbation is greater at higher temperatures. As the optimization proceeds, the temperature is gradually lowered by a reduction factor named the cooling factor. The objective is to find a balance between a too slow temperature reduction that unnecessarily increases the convergence time and a too fast one that may freeze the solution at some local minimum with values far from the targets.

The objective is to minimize the average deviations from production targets for $N$ mining periods, $S$ orebody simulations, and $P$ different processes:

$$minO = \sum_{n=1}^{N} \left( \sum_{s=1}^{S} \left( \sum_{p=1}^{P} \lambda_p |\theta^*_{np} - \theta_{np}(s)| \right) + \lambda_\omega |\omega^*_n - \omega_n(s)| \right) \tag{2.2}$$

where $\theta^*_{np}$ is the target for process $p$ at period $n$; $\omega^*_n$ is the waste target at period $n$; $\theta_{np}(s)$ is the actual production of process $p$ at period $n$ in the orebody simulation

$s$; $\omega_n(s)$ is the actual waste production at period $n$ in the orebody simulation $s$; $\lambda_p$ is the weight associated to process $p$ and $\lambda_\omega$ is the weight associated to waste. The $\lambda$ parameters are chosen to balance the capacities of the processes if they have different scales and to give more weight to the more critical ones.

In mining complexes with multiple material types and multiple processing streams, the mining blocks can be processed by some treating plants according to their material type; that is, a given processing plant may not accept all the available material types. Furthermore, if a given process accepts two or more different material types, their associated costs and metallurgical recoveries may differ. These considerations need to be taken into account in defining the perturbation strategy and the type of solution of the SA algorithm. When dealing with multiple processes, two possibilities can be distinguished: (a) there is a single material type, or, if there are multiple ones, for any block its material type remains constant over the orebody simulations; (b) there are multiple material types that can change for the same block in the different orebody simulations (material type uncertainty).

If the case evaluated corresponds to case (a), the solution obtained from the SA implementation corresponds to a long-term production schedule that determines the periods and destinations of the mining blocks. On the other hand, if the case is (b), the solution must be a mining sequence that determines the periods only without defining destinations to avoid infeasible material type-process combinations. In the last case, the algorithm must respect the destinations of the input mining sequences; that is, the algorithm provides a solution that states the mining period for each block, but evaluates the productions of the various scenarios (orebody simulations) based on the optimal destinations obtained from a conventional scheduler.

From the input mining sequences, the probabilities of each block to be mined in different periods are calculated. A certain block is a block that is mined in the same period in all the input mining sequences, e.g., it has 100% probability of being mined in a particular period. The set of candidate blocks is given by the uncertain´ blocks, in which the mining periods vary over the mining sequences. The perturbation mechanism depends on the two possible cases. If the case corresponds to case (a), the algorithm randomly selects a candidate block and swaps the current mining period to a different one based on the probabilities calculated from the input mining sequences and the respect of the slope angles. The available processing stream that leads to the best objective value is selected as the destination. If the case evaluated corresponds to case (b), the additional step of selecting the destination is not needed, as the destinations obtained from the mining sequences are respected to avoid infeasible material type-process combinations. Once a new solution is generated, it is accepted or rejected based on the decision rule described in Eq. (2.1).

The algorithm perturbs a given solution until a stopping criterion is reached. A stopping criterion may be that: the current solution yields to a satisfactory minimum; the total number of swaps equals a user-defined maximum; either the number of perturbations at any given temperature or the number of perturbations without a change in the objective function surpasses maximum acceptable values. These parameters must be established in the implementation of SA together with the initial temperature and the reduction or cooling factor.

The implementation of the method proposed herein has important concerns. The method considers the pit limits of the starting mining sequence. The optimal stochastic pit limits do not coincide with the ones obtained deterministically [1, 2, 69], so that the method can be iteratively implemented to various pit limit definitions in order to select the ones that generate the higher NPV. The cut-off grade policy in the method is implicitly considered when generating the mining sequences, in which the blocks are sent to the most profitable destinations based on the technical and economical parameters. The method assumes specified mining rates based on processing plant requirements.

## 2.3 Case study

The method is applied at Escondida Norte mine, Chile. The orebody is a porphyry copper deposit located 170 km south-east of Antofagasta, Chile. The deposit is formed by two major stages of sulphides and one stage of oxide mineralization. The material of the deposit is classified in 5 different types and can be processed through 3 available treating paths or sent to the waste dump. Figure 2–3 displays the possibilities for handling the material according to the material types.

A given block will be sent to the waste dump if it is waste rock or if the revenue obtained from the copper recovered does not compensate the operating costs. Both types of sulphides can be sent to the milling process or the bio-leaching plant. The mixed material can only be processed in the bio-leaching plant, while the oxides can only be processed in the acid-leaching plant. The metallurgical recoveries vary with the type of material and the process selected.

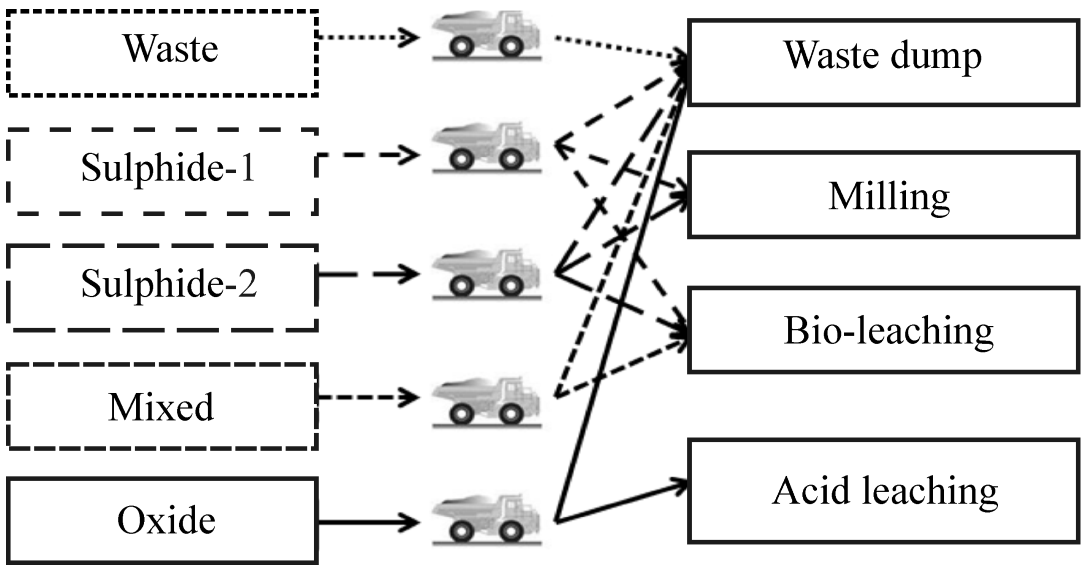


Figure 2–3: Possibilities for material handling according to the material type

Several stochastically simulated representations of the orebody of Escondida Norte are available [44, 92, 118]. The operating and financial parameters used to generate the input mining sequences are displayed in Table 2–1. The average metallurgical recoveries for the material types and processes are calculated from the simulations and are summarized in Table 2–2.

The orebody simulations consider uncertainty in both copper grades and material types (Figure 2–4); that is, for a given block, the material type can change among simulations. This implies that the final solution must state only the mining periods of the blocks to avoid infeasible material type-process combinations. The deposit has been divided in four geotechnical zones with their corresponding slope angle definitions (Figure 2–5), and these are 33, 35, 41 and 35, for zones 1, 2, 3 and 4, respectively.

Table 2–1: Operating and financial parameters

| Item | Value |
|---|---|
| Mine capacity | 500 ktpd |
| Mill capacity | 120 ktpd |
| Acid-leaching capacity | 60 ktpd |
| Bio-leaching capacity | unlimited |
| Copper price | \$2/lb |
| Mining cost | \$1.5/t |
| Milling cost | \$6.0/t |
| Bio-leaching cost | \$1.5/t |
| Acid-leaching cost | \$4/t |
| Discount rate | 8% |

Table 2–2: Metallurgical recoveries

| Material type | Milling (%) | Bio-leaching (%) | Acid-leaching (%) |
|---|---|---|---|
| Sulfide-1 | 81 | 31.8 | Infeasible |
| Sulfide-2 | 79.2 | 47.3 | Infeasible |
| Mixed | Infeasible | 37.5 | Infeasible |
| Oxide | Infeasible | Infeasible | 72.7 |

(a)

(b)

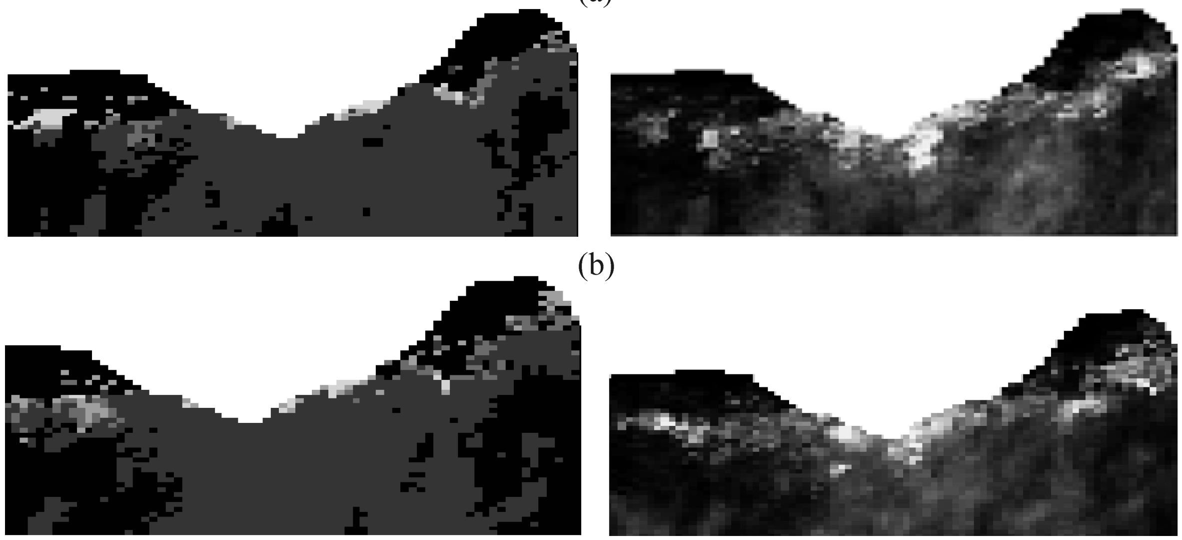

Figure 2–4: Material types (left) and copper grades (right) in section Y=114037.5 of (a) orebody simulation 5; (b) orebody simulation 10

### 2.3.1 Generation of the input mining sequences

Albor and Dimitrakopoulos [1] document that there is no significant improvement in the quality of the stochastic solution generated when more than $15 - 20$ simulations are used. Based on that result, 15 mining sequences are generated from the orebody simulations using the technical and financial information available. The robustness of the stochastic solution is further tested through a risk analysis [87] on a different set of simulations. The mining sequences are obtained using the Milawa algorithm from the software program Whittle [112]. This algorithm seeks to maximize the NPV in an approximate way based on the combination of benches and nested pits generated from the nested pit implementation of the LerchsGrossman algorithm in the Whittle software [112].

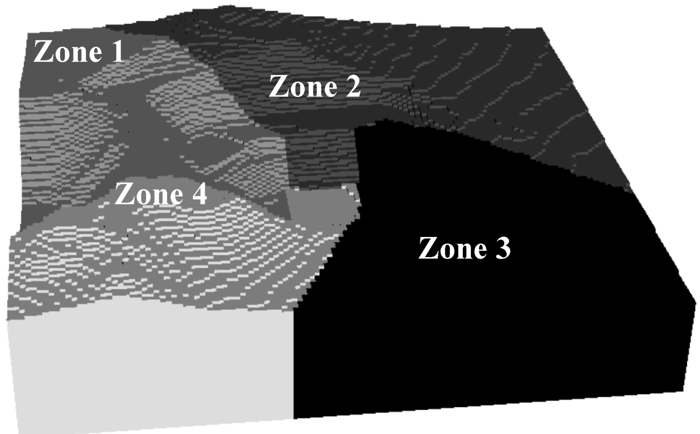

Figure 2–5: Geotechnical zones of the deposit

Figure 2–6 shows a particular section of four mining sequences generated from the orebody simulations. It is observed from the figure that different orebody simulations generate different mining sequences. Even though the number of years in the mining sequences varies between 45 and 50, only the first 15 years are considered in the comparison of results for production targets and cumulative NPV. The first 15 years represent more than 80% of the total NPV of the project.

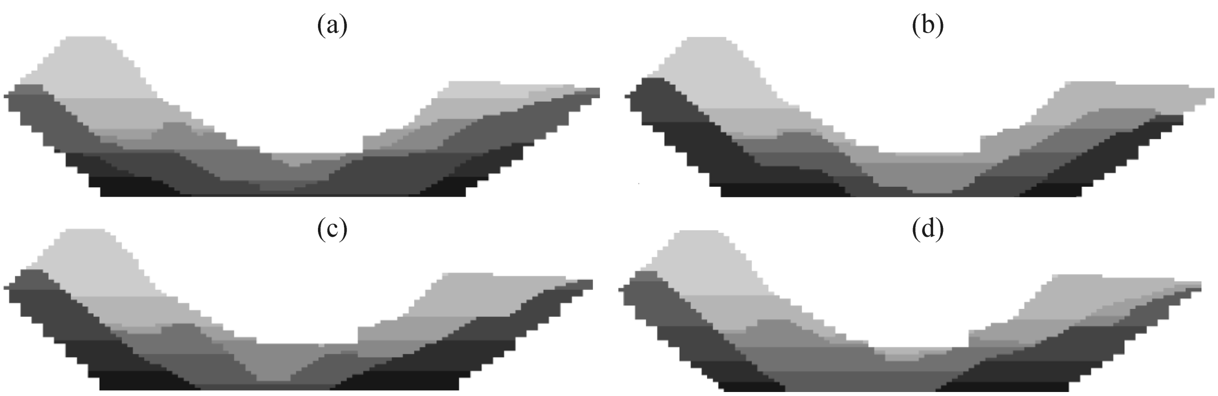

Figure 2–6: Cross-section of (a) mining sequence 1; (b) mining sequence 6; (c) mining sequence 10; (d) mining sequence 15

### 2.3.2 Selection of the starting sequence

The first step of implementing SA is selecting the starting mining sequence. A risk analysis on productions is performed on each of the input mining sequences. Figure 2–7 shows the expected mill production of the 15 mining sequences generated from the orebody simulations.

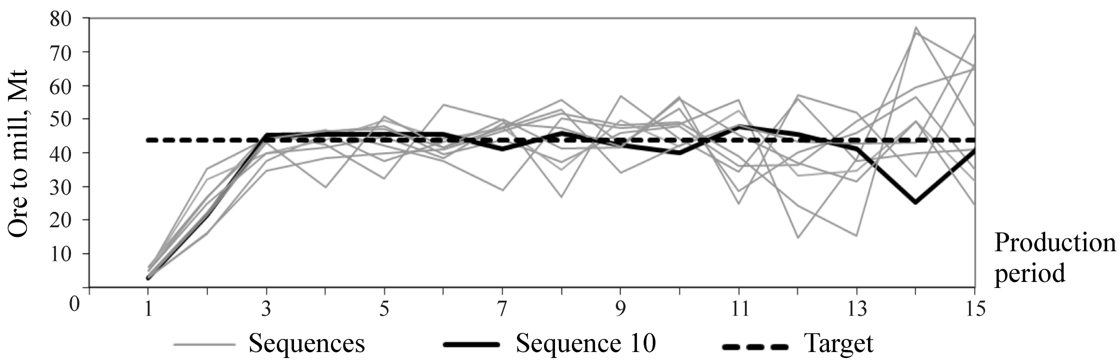


Figure 2–7: Expected ore sent to milling process of input mining sequences

The expected ore tonnages sent to the milling process have larger fluctuations in the latest periods due to the increment of fractioned blocks in these periods in the mining sequences generated. Between years 3 and 15, it is expected that the mining sequence 10 deviates on average 8.1% from the milling target, which is the smallest expected deviation from the set of input mining sequences. This mining sequence is highlighted in the figure. Figure 2–8 shows the expected material sent to the waste dump for the input mining sequences for the first 15 years of production. In the first two years, the waste removal is shown to be above the target, which compensates for the lower tonnage sent to mill at these periods. Between years 3 and 15, it is expected that mining sequence 10 deviates on average 7.0% regarding the waste removal target. These deviations are shown to be smaller than the expected deviations of the other input mining sequences.

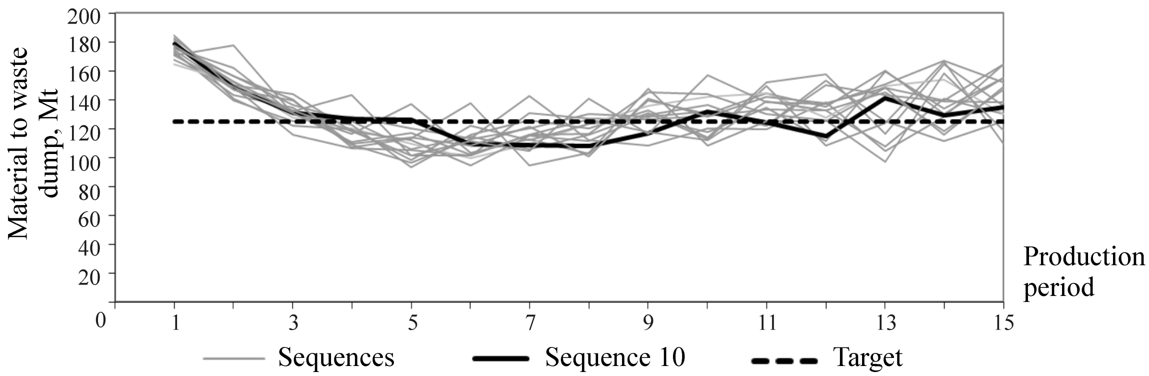


Figure 2–8: Expected material sent to the waste dump of input mining sequences

The criteria used to select the starting sequence is subjective in the sense that the best starting sequence may in some cases be the one that has productions nearest to the targets, the one that leads to the highest expected NPV, or can be selected arbitrarily. Due to the smaller expected deviations from mill and waste production targets, mining sequence 10 is selected as the starting mining sequence. This criterion is applied to facilitate the use of fewer mining sequences and less practical computing time to obtain a nearly optimal solution.

### 2.3.3 The stochastic mine production schedule

Once the starting mining sequence has been selected, it is possible to initiate the SA algorithm. The parameters that control the algorithm must be chosen to guarantee a considerable number of swaps that make possible a satisfactory improvement of the initial solution in a practical amount of time. After running several tests, the annealing parameters used are the ones displayed in Table 2–3.

Table 2–3: Annealing parameters

| Item | Value |
|---|---|
| Maximum number of swaps | 1.0e+11 |
| Low objective function value (convergence) | 1.0e+6 |
| Initial temperature (C) | 0.00001 |
| Reduction factor | 0.1 |
| Maximum number of perturbations at any given temperature | 85000 |
| Max. number of perturbations without a change | 1.0e+10 |

Figure 2–9 shows the mill production of the stochastic mine production schedule obtained through the implementation of SA. Between years 3 and 15, the expected deviation from mill production targets with the stochastic mine production schedule is 3.9%.The largest expected deviation is shown to be in year 14. The expected deviation from the mill target decreases from 8.1% in the starting mining sequence to 3.9% in the stochastic schedule due to the SA implementation. Figure 2–10 shows the bio-leaching production with the stochastic mine production schedule. The expected bio-leaching production is greatest in year 8, with 12.8 M tonnes.

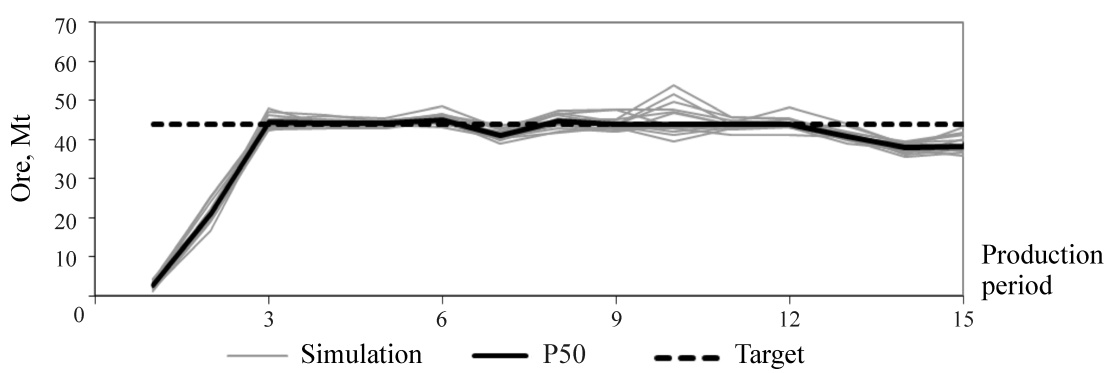


Figure 2–9: Ore sent to milling process with stochastic mine production schedule

Figure 2–11 shows the acid-leaching production for the oxide material with the stochastic mine production schedule. In the stochastic schedule, the acid-leaching

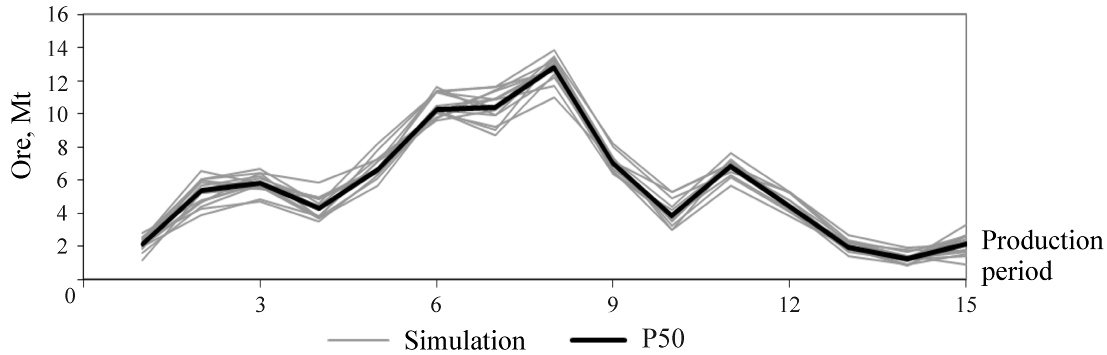


Figure 2–10: Ore sent to bio-leaching process with stochastic mine production schedule

production starts increasing from the second year until the sixth, when the capacity of the plant is reached for two consecutive periods, then starts decreasing from year 7 until year 12 (depletion of the oxide resources). Figure 2–12 shows the material sent to the waste dump using the stochastic mine production schedule. The expected deviations from waste production targets with the stochastic schedule are on average 4.5% between years 3 and 15. The waste targets are used to control the mining rates.

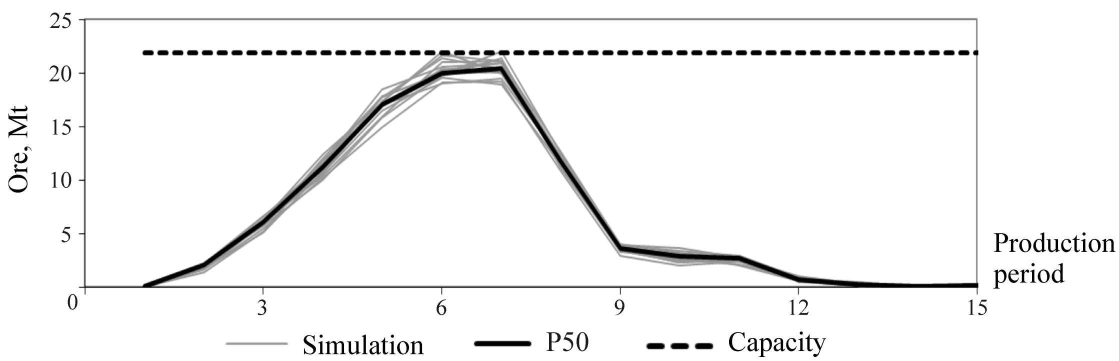


Figure 2–11: Ore sent to acid-leaching process with stochastic mine production schedule

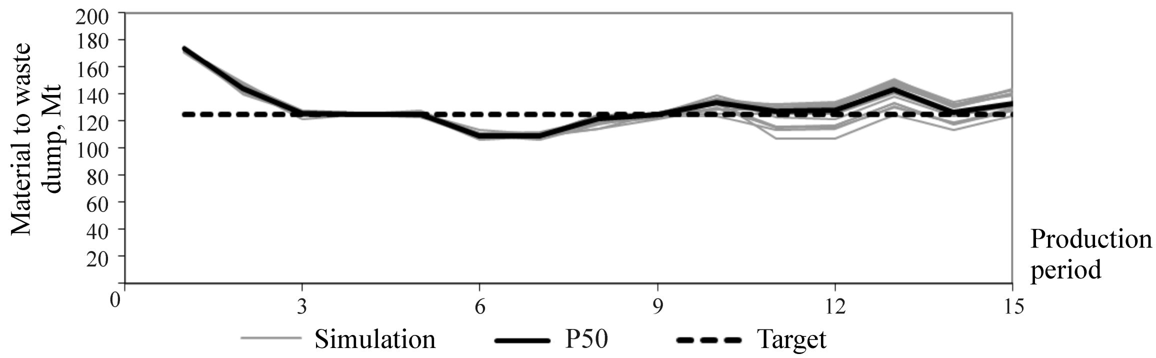


Figure 2–12: Material sent to the waste dump with stochastic mine production schedule

Figure 2–13 shows the copper production using the stochastic schedule. As expected, there are larger productions of copper in the early periods because of discounting (8%). The largest expected copper production is shown to be in year 5. Figure 2–14 shows the cumulative NPV of the stochastic schedule through the first 15 years of production.

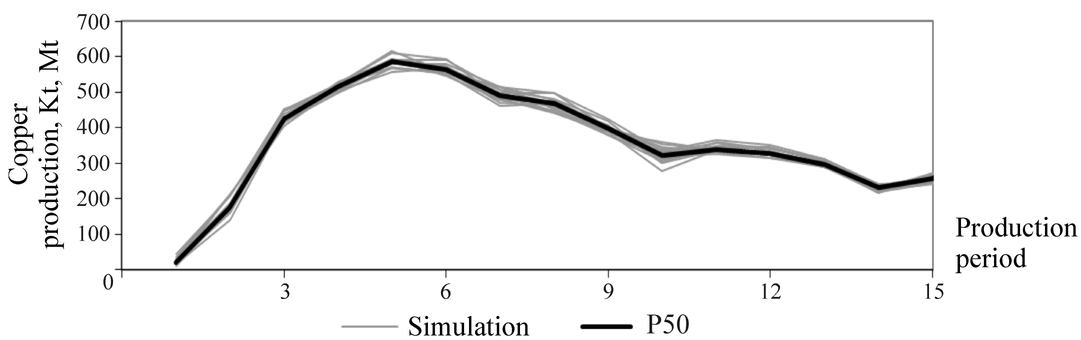


Figure 2–13: Copper production with stochastic mine production schedule

The cumulative NPV until year 15 has a negligible variation in comparison with any intermediate sequence when implementing the SA algorithm. However, the benefits of the SA implementation are evidenced in the expected reach of specified

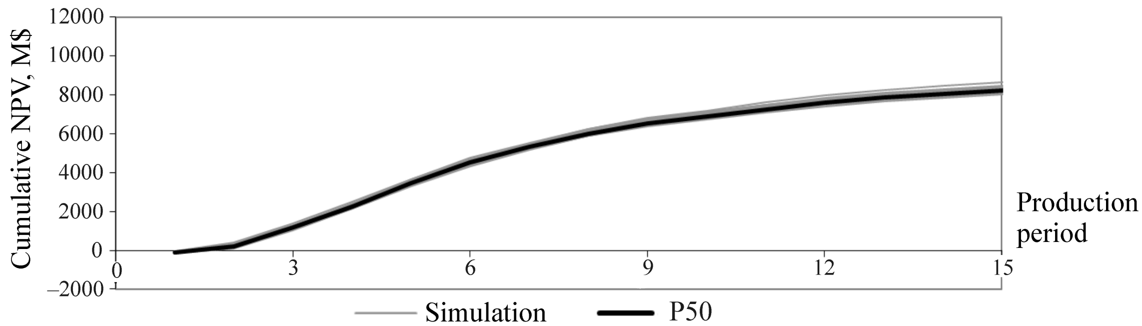


Figure 2–14: Cumulative NPV with stochastic mine production schedule

production targets, which avoid costs associated with idle capacities that are not taken into account in the calculation of the NPV.

Table 2 summarizes the results obtained from the risk analysis of the stochastic schedule. The stochastic mine production schedule has a 90% probability of deviating on average less than 8.5 and 8.4% from mill and waste production targets, respectively, and provides a cumulative NPV until year 15 greater than $8.11 billion.

Table 2–4: Results from the stochastic schedule

| | P10 | P50 | P90 |
|---|---|---|---|
| Average deviation from mill target (%) | 1.6 | 4.6 | 8.5 |
| Average deviation from waste target (%) | 2.8 | 5.7 | 8.4 |
| Average production in bio-leaching plant (Mt) | 5.1 | 5.7 | 6.4 |
| Average production in acid-leaching plant (Mt) | 8.3 | 8.9 | 9.5 |
| Cumulative NPV ($ billion) | 8.11 | 8.24 | 8.45 |

### 2.3.4 The robustness of the stochastic solution

The risk analysis performed so far on the stochastic mine production schedule considers the simulations from which the mining sequences were generated. To test the robustness of the schedule, a risk analysis over different orebody simulations is made. A set of 15 different orebody simulations is used. Figure 2–15 shows the ore sent to the mill, bio-leach, and acid-leach, and material sent to the waste dump with the stochastic schedule.

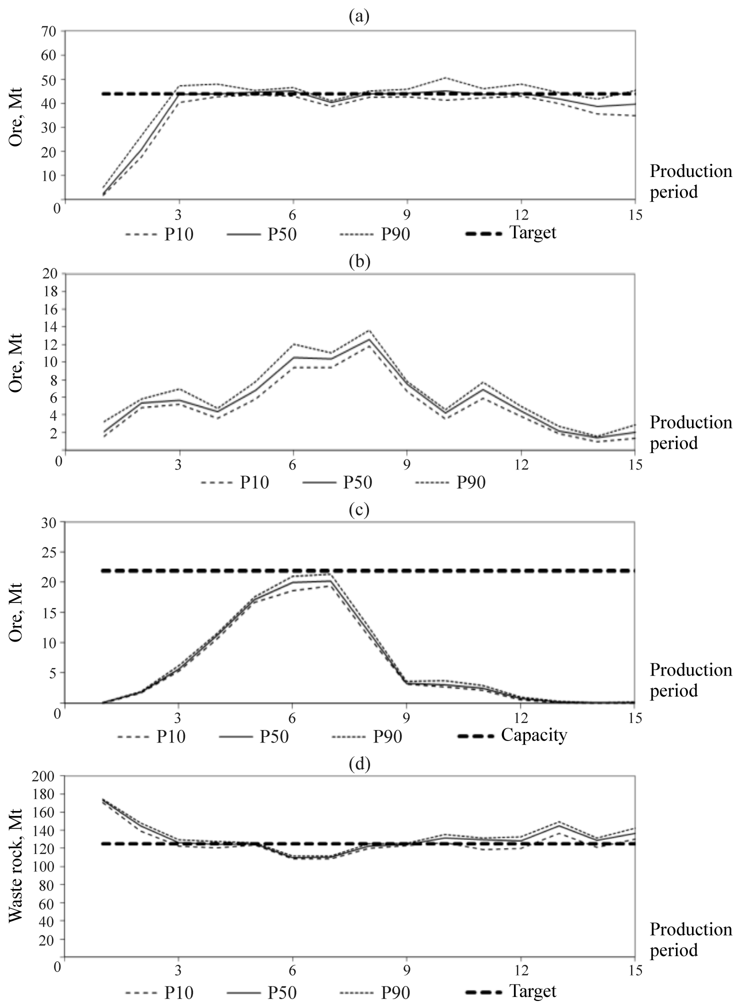


Figure 2–15: Material sent to processes and waste dump with the stochastic mine production schedule: (a) ore to mill; (b) ore to bio-leaching; (c) ore to acid-leaching; (d) material to waste dump

The expected average deviation from mill production targets according to this new risk analysis is 3.6%, which is on the same order as the 3.9% obtained with the previous risk analysis. The expected average deviation from waste production targets is 4.7%, which is very similar to the 4.5% obtained in the previous risk analysis. Figure 2–16 shows the cumulative NPV calculated with the new risk analysis. The difference between the expected cumulative NPVs in both risk analyses is negligible in practice (around 1.8%). Based on the forecasted productions and the cumulative NPV, it is possible to conclude that the schedule generated considering 15 orebody simulations is robust when compared to a different set of orebody simulations.

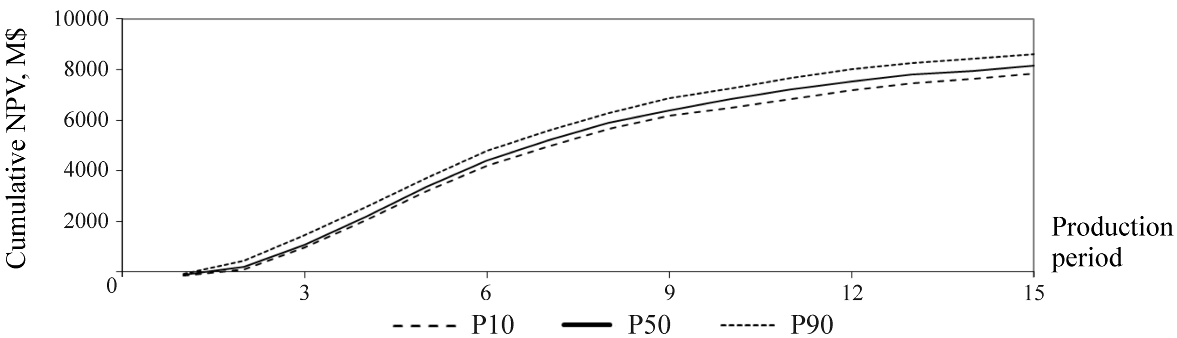


Figure 2–16: Cumulative NPV with the stochastic mine production schedule

### 2.3.5 Comparison with a conventional mine production schedule

In order to quantify the benefits of the stochastic mine production schedule, this schedule is compared to a schedule generated conventionally. A conventional mine production schedule that considers the 3 available processes is generated using the average grade of the 50 available orebody simulations (e-type) and using the Milawa scheduler of Whittle software [112]. Figure 2–17 shows the mill and waste productions with the e-type schedule. The conventionally generated schedule leads to large and impractical ore and waste productions. Figure 2–18 shows the NPV of the conventional schedule forecasted in Whittle and the expected NPV of the stochastic schedule.

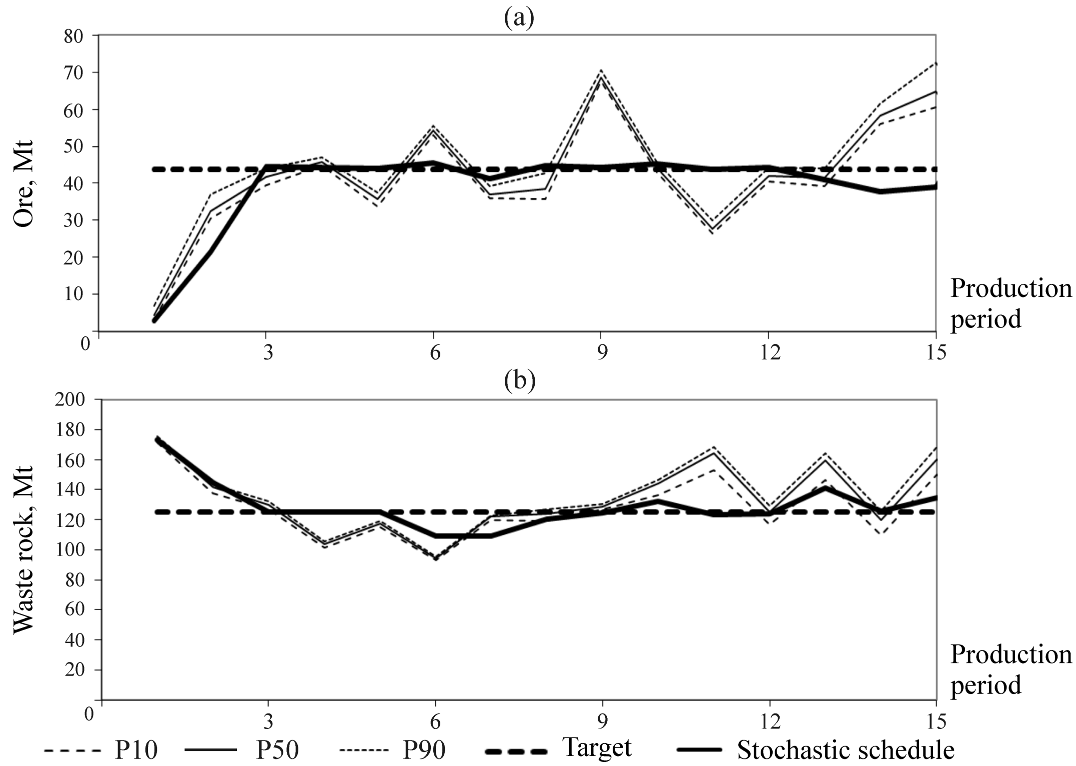


Figure 2–17: Material sent to mill and waste dump with the schedule of the e-type

The stochastic schedule provides an expected cumulative NPV 4% higher than that forecasted in Whittle software for the conventional schedule. However, the costs associated with idle capacities are not included in the calculation of the NPV. This indirect cost has a large influence in the conventional schedule, which reduces its NPV expectations.

## 2.4 Conclusions

The method proposed in this paper allows for the control of deviation from production targets in mining complexes with multiple ore types and multiple processing streams. The solution obtained from the implementation of the method depends on

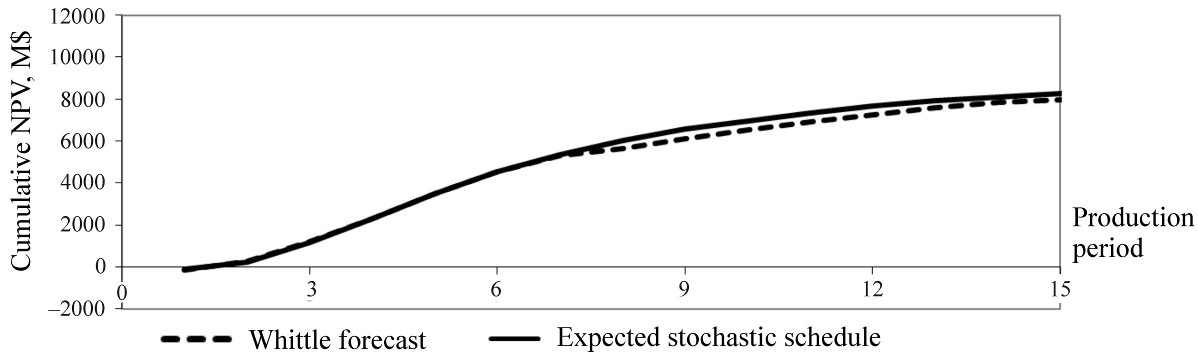

Figure 2–18: Cumulative NPV with conventional schedule

the uncertainty of material types of the related mining blocks: if the blocks present invariable material types over the orebody simulations, the stochastic mine production schedule states the periods and destinations of the blocks; if the material type of a given block can vary over the simulations, the solution obtained determines only the block mining periods to avoid sending blocks to improper processes.

The selection of the starting mining sequence may influence the generation of high quality solutions in a practical amount of time; it is recommended to perform a sensitivity analysis to select the best starting mining sequence. The application of the method in a copper deposit generates expected average deviations smaller than 5% from mill and waste production targets. The expected average deviations are around 20% from mill targets and 12% from waste targets when considering a schedule generated conventionally using the e-type of the orebody simulations.

During the definition of the annealing parameters, several tests were performed to determine the ones that generate good quality solutions with practical computing time; however, documenting the change of the quality of the solution with different annealing parameters can result in better assessment of the optimal ranges of the annealing parameters for solving mine production scheduling problems. Although the stochastic mine production schedule obtained is robust to a different set of simulations when considering 15 input mining sequences, a sensitivity analysis on the number of input mining sequences is also desirable. The method needs to be expanded to mining complexes with multiple sources (open pit mines) and stockpiles.

# CHAPTER 3
# An extended stochastic optimization method for multi-process mining complexes

## 3.1 Introduction

Mining complexes contain multiple sequential activities that are strongly interrelated: *(i)* mining the materials from one or multiple sources; *(ii)* blending the material considering stockpiling, *(iii)* transforming the material sent to different destinations or processing paths; *(iv)* transporting the products to final destinations, etc. The quality of the input material of a metallurgical process may determine its corresponding throughputs, costs and metallurgical recoveries. Mill throughput can be sensitive to rock hardness, work index, the ratio of clay materials, etc.; costs and reaction times in an autoclave depend on sulphur content; recoveries are affected by deleterious materials [108]. Multiple approaches have been developed to optimize the different parts of a mining complex in isolation: For example, Caccetta and Hill [15], Lerchs and Grossmann [72], Picard [81] for pit design and mine production scheduling. The process of optimizing all activities of a mining complex simultaneously is known in the mining literature as global optimization [113]. This is a problem with high complexity due to the link between time periods and discounting, the blending requirements, the flexibility generated from the stockpiles, the multiple destinations and operating alternatives, and the variability and uncertainty associated with grades and material types [110].

Over the last decade, several algorithms that seek for generating optimal solutions in mining and processing plans have been developed. Hoerger et al. [50] formulate the problem of optimizing the simultaneous mining of multiple sources (pits and underground mines) and the delivery of ores to multiple plants as a mixed integer program. The model calculates the net present value of the mining complex by using variables that represent material sent from the mines to the stockpiles, material sent from the mines to the processes, and material sent from the stockpiles to the processes and their corresponding associated costs. The blocks are grouped into increments based on the metallurgical properties, which belong to sequences (or mining phases). The integer variables are used to model mine sequencing constraints at a phase level and plant startups and shutdowns. Due to the use of phase sequencing constraints instead of block sequencing constraints to decrease the complexity of the problem, there is a loss of resolution in the solution generated from the method that may lead to the inability of meeting the blending and production requirements. Furthermore, the method does not consider multiple operating alternatives for each process and ignores the geological uncertainty associated with the ore deposits. Whittle [110] presents the Prober optimizer for global optimization that aggregates the mining blocks into parcels of mine material type. These material types are defined from different grade bin categories; that is, for each relevant grade or attribute, cut-offs are defined to allow flexibility for blending purposes. The method considers stockpiles for each material type that may be combined with the material obtained directly from the mines to satisfy the different process requirements. Prober uses a random sampling and an evaluation routine to generate the solution. The random sampling consists of a search algorithm that samples the feasible domain of alternative life-of-mine (LOM) mining plans; the evaluation routine determines the optimal cut-off grade, stockpiling, processing selection, blending and production plans and determines the net present value (NPV). The optimizer works by repeatedly creating a random feasible solution and then finding the nearest local maximum. The various NPVs that the algorithm finds are stored, and the run is usually stopped when the top ten values lie within 0.1 per cent of each other. Although very flexible and able to handle complex blending operations, the algorithm has some drawbacks: it groups the parcels into panels and assumes that the parcels are consumed in the same proportion within a panel; good solutions may be found but it does not guarantee optimality; and, geological uncertainty is discarded.

Regarding geological uncertainty; that is, uncertainty in grades and material types, some approaches have been developed in the last decade for pit design and mine production scheduling problems. Ramazan and Dimitrakopoulos [85] formulate the mine scheduling problem as a two-stage stochastic integer program (SIP) in which the first stage variables represent mining decision variables and the second stage variables represent deviation from grade and production targets evaluated on a set of orebody simulations. The formulation was later extended to include stochastically designed stockpiles, multiple processors and integrate short- term information [11, 31, 71, 86]. Menabde et al. [75] develop and implement a method that accounts for geological uncertainty and simultaneously optimizes the sequence of extraction of the mining blocks and the cut-off grade policy. The authors aggregate blocks into panels to reduce the number of binary variables and obtain an increase of 26% in expected NPV when compared to a solution that uses a deterministic static cut-off grade policy. Boland et al. [13] propose a multistage stochastic programming approach that considers the decision of processing block aggregates as posterior-stage variables. The approach provides a set of policies to follow according to the actual scenario (orebody) obtained with the advance of the extraction. The implementation of the approach using realistic mining data increases the expected NPV by 3% when compared to a conventional deterministic method. However, some drawback can be remarked in the approach: continuous variables with aggregates do not guarantee slope constraints; it assumes all scenarios can be covered with orebody simulations; and, it does not penalize deviation from production targets. Although, the SIP formulation generates substantial improvements in terms of NPV and meeting production targets, industry standard optimizers such as CPLEX are unable to solve big size problems due to the large amount of integer variables, thus alternative solution avenues are being sought [65]. Many different approaches are available to solve large combinatorial optimization problems. Some of them have been implemented for solving complex mine scheduling optimization problems. Godoy [37] develops a multi-stage method for mine production scheduling that integrates the joint local geological uncertainty and uses simulated annealing (SA) algorithm. The method seeks to generate a risk-based mine production schedule that minimizes deviation from ore and waste production targets. Leite and Dimitrakopoulos [69] apply the method at a copper deposit obtaining an expected NPV 20% greater than the ones obtained using conventional deterministic schedulers. Albor and Dimitrakopoulos [1] implement the method at the same copper deposit and observe that the schedule obtained was not sensitive after 15 orebody simulations. Furthermore, the authors point out that the stochastic final pit limit was 17% greater than the deterministic one, adding 9% to the expected NPV. Goodfellow and Dimitrakopoulos [42] develop a simulated annealing implementation for pushback design to control deviation from pushback size targets considering different material types and processing plants. Lamghari and Dimitrakopoulos [65] implement tabu search (TS) and variable neighbourhood search (VNS) for the mine scheduling problem obtaining near-optimal solutions while outperforming CPLEX in terms of computational time. Lamghari et al. [66] develop a hybrid approach that combines exact methods and metaheuristics for solving the LOM production scheduling problem.

In a mining complex, the different types of material extracted from a deposit are sent to stockpiles or the different processing streams (destinations). Processing a particular mining block can be profitable; however, the decision of mining and processing that particular block in a given period not only depends on the individual characteristics of the block (grades, tonnage, metallurgical properties) but in the compound characteristics of the total material sent to the destination, including both: the material sent directly from the deposit and the material sent from the stockpiles. At any destination, there may be multiple operating conditions depending on the quality of the input material, the design and operating parameters (processing time, temperature, pressure, rotation speed, etc.) and the desired properties of the output products; that is, a given destination (mill) may have several operating alternatives (operate the mill with high silica in the input material or with low silica content). Some additives may be considered at each destination, e. g., cyanide in a hydrometallurgical plant. In some cases additives are the bottleneck of the system. To acount for them, availability constraints are added to the formulation in the next section. Each operating alternative at a given destination may have its corresponding costs, metallurgical recoveries, blending requirements, and additive demand (Figure 3–1). Considering the example of the mill operated with higher silica content in the input material, given the hardness of the silica, the demand of energy (seen as an additive of the mill) to decrease the average particle size to 80m is higher when compared to a low silica content operation. This may affect the costs and recoveries there-after; that is, having different blending requirements in the input material for a particular process originates different consumption of additives, costs and metallurgical recoveries. However, these differences may also be originated from different conditions of operation in the process, or different product specifications; e.g., 120m instead of 80m as average particle size desired in the output material of the mill.

The production plan of a mining complex must state for any mining block when to mine it, but also where to send it; and, for each destination, which operating alternative to use.

### 3.2 Optimization model

Let $N$ be the set of mining blocks considered to discretize the deposit; $D$ the set of processing destinations; $X_{itd}$ a binary variable that denoted whether or not the block $i$ is mined in period $t$ and sent to destination $d$; and, $m_{is}$ the mass of block $i$ in simulation $s$. The amount of material mined at a given period and simulation, denoted as $tonne\_mn(s,t)$, is given by Eq. (3.1). For modelling purposes, destination

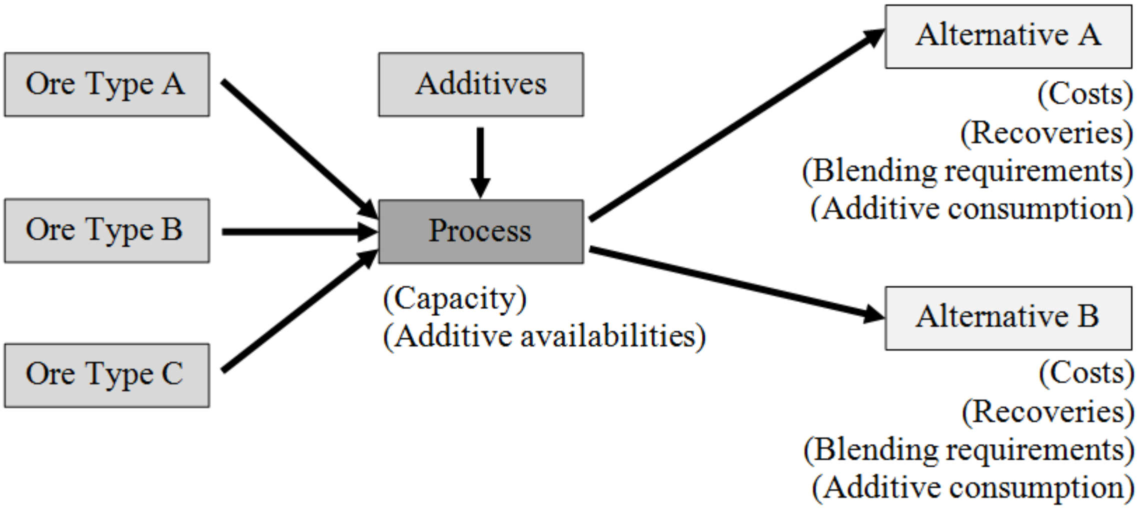


Figure 3–1: Process and operating alternatives

$d = 0$ represents sending the block to the waste dump, and $d = D + 1$ represents sending the block to the stockpiles.

$$tonne\_mn(s,t) = \sum_{i=1}^{N} \sum_{d=0}^{D+1} X_{itd} \cdot m_{is} \qquad \forall s,t \tag{3.1}$$

The amount of material that will be processed at a given destination accounts for the material that comes directly from the mine and the material that comes from the stockpiles as shown in Eq. (3.2). The material sent from the stockpiles to destination $d$ in period $t$ in simulation $s$ is represented as $tonne\_rehandle(s,t,d)$.

$$tonne\_destination(s,t,d) = \sum_{i=1}^{N} X_{itd} \cdot m_{is} + tonne\_rehandle(s,t,d) \quad \forall s,t,d \tag{3.2}$$

This amount of material rehandle depends on the tonnage available in the stockpiles and the idle capacity associated with a given destination, and follows the mass conservation expression showed in Eq. (3.3), where $tonne_stockpile(s, t)$ is the tonnage available in the stockpiles in period $t$ in simulation $s$.

$$\begin{aligned} tonne\_stockpile(s,t) = tonne\_stockpile(s,t-1) + \sum_{i=1}^{N} X_{itd} \cdot m_{is} \\ - \sum_{d=1}^{D} tonne\_rehandle(s,t,d) \qquad \forall s,t \end{aligned} \tag{3.3}$$

Eq. (3.4) is used to ensure that all the material sent to a particular destination $d$ is processed using any of the available operating alternatives $O(d)$, where $Y_{tdo}$ is a variable between 0 and 1 that represents which proportion of material sent to destination $d$ in period $t$ is processed using operating alternative $o$.

$$\sum_{o=1}^{O(d)} Y_{tdo} = 1 \qquad \forall t, d \tag{3.4}$$

The revenue in each period and destination ($revenue(s, t)$) is obtained by accounting for the different metals recovered from the available destinations. For each operating alternative at any given destination, there is a recovery expression associated to each material type and recoverable metal. The same level of flexibility is considered for processing costs and the requirement of additives; that is, they depend on the type of material input to a destination and the operating alternative implemented.

Eq. (3.5 - 3.8) represent the costs associated with the different activities of the mining complex where $mn\_cst(s, t)$ is the cost incurred by mining in period

$t$ in simulation $s$; $m_c$ is the per-unit mining cost; $pr\_cst(s,t)$ is the cost incurred by processing the material at the different destinations in period $t$ and simulation $s$; $p_c(d,o)$ is the per-unit processing cost in destination $d$ using alternative $o$; $stk\_cst(s,t)$ is the cost incurred by stockpiling the material in period $t$ and simulation $s$; $sp_c$ is the per-unit stockpiling cost; $rh\_cst(s,t)$ is the cost incurred by sending material from the stockpiles to the different destinations in period $t$ and simulation $s$; $rh_c$ is the per-unit rehandle cost.

$$mn\_cst(s,t) = tonne\_mn(s,t) \cdot m_c \qquad \forall s,t \tag{3.5}$$

$$pr\_cst(s,t) = \sum_{d=1}^{D} \sum_{o=1}^{O(d)} (tonne\_destination(s,t,d) \cdot Y_{tdo} \cdot p_c(d,o)) \qquad \forall s,t \tag{3.6}$$

$$stk\_cst(s,t) = \left( \sum_{i=1}^{N} X_{it(D+1)} \cdot m_{is} \right) \cdot sp_c \qquad \forall s,t \tag{3.7}$$

$$rh\_cst(s,t) = \left( \sum_{d=1}^{D} tonne\_rehandle(s,t,d) \right) \cdot rh_c \qquad \forall s,t \tag{3.8}$$

The objective function of the proposed formulation is given by the sum of the discounted revenues obtained by selling the different recoverable metals minus the discounted costs associated to the different parts of the operation throughout the different periods and simulations.

$$MaximizeO = \sum_{s=1}^{S} \sum_{t=1}^{T} \frac{\left( \begin{array}{c} revenue(s,t) - mn\_cst(s,t) - pr\_cst(s,t) \\ -stk\_cst(s,t) - rh\_cst(s,t) \end{array} \right)}{(1+d)^t} \tag{3.9}$$

where $S$ is the set of orebody simulations, $T$ is the number of years considered for the project and $d$ is the discount rate.

Given the time value of money and the geological uncertainty associated with the deposit, the blocks with higher and more certain profit must be mined in early periods and sent to their optimal destinations, whereas the blocks which are more certain to be non-profitable must be delayed for latest periods and sent to the waste dump.

At any given period and simulation it is possible to evaluate the amount of metal $v$ sent to a given destination using Eq. (3.10) where $g_{isv}$ is the grade of block $i$ in simulation $s$ considering property $v$; the amount of metal $v$ that will be processed using a particular operating alternative of a given destination using Eq. (3.11); and, the average grade of metal $v$ in that operating alternative using Eq. (3.12). The amount of metal $v$ sent from the stockpiles to a given destination $d$ in a period $t$ and simulation $s$ is given by $metal\_rehandle(s,t,d,v)$ and follows the mass conservation principle as $tonne\_rehandle(s,t,d)$. $Z_{tdov}$ is a variable between 0 and 1 that represents the proportion of metal $v$ sent to destination $d$ in period $t$ that will be recovered using the operating alternative $o$. To ensure that all the metal sent to a given destination will be processed, $\sum_{o=1}^{O(d)} Z_{tdo} = 1$ where $O(d)$ is the set of operating alternatives available in destination $d$.

$$metal\_destination(s,t,d,v) = \left(\sum_{i=1}^{N} (X_{itd} \cdot m_{is} \cdot g_{isv})\right) + metal\_rehandle(s,t,d,v) \qquad \forall s,t,d,v \tag{3.10}$$

$$metal\_alternative(s,t,d,o,v) = metal\_destination(s,t,d,v) \cdot Z_{tdov} \qquad \forall s,t,d,o,v \tag{3.11}$$

$$average\_grade(s,t,d,o,v) = \frac{metal\_alternative(s,t,d,o,v)}{tonne\_destination(s,t,d) \cdot Y_{tdo}} \qquad \forall s,t,d,o,v \tag{3.12}$$

The requirement of additive $a$ at each destination is obtained using expression (3.13). $k(d,o,a)$ represents the per-unit additive consumption coefficient of $a$ in destination $d$ using operating alternative $o$.

$$additive\_consumption(s,t,d,a) = \sum_{o=1}^{O(d)} (tonne\_destination(s,t,d) \cdot Y_{tdo} \cdot k(d,o,a)) \qquad \forall s,t,d,a \tag{3.13}$$

The feasible domain is constrained by Eq. (3.14 - 3.18). Eq. (3.14) represents capacity constraints, which imply that it is not possible to send to any destination more material than the amount that can be processed. Eq. (3.15) represents availability constraints, which avoid the usage of more additives than the amount available at a given destination. Eq. (3.16) represents blending constraints, which control that the grade of the different attributes falls in between some operational ranges for the different operating alternatives. Eq. (3.17) represents reserve constraints, which control that a block can be mined only once and sent to a unique destination. Eq. (3.18) represents block precedence constraints, which ensure that slope constraints at the different geotechnical zones are not violated.

$$tonne\_destination(s,t,d) \le capacity\_destination(d) \qquad \forall s,t,d \tag{3.14}$$

$$additive\_consumption(s,t,d,a) \le availability(d,a) \qquad \forall s,t,d,a \tag{3.15}$$

$$low\_range(d,o,v) \le average\_grade(s,t,d,o,v) \le high\_range(d,o,v) \qquad \forall s,t,d,o,v \tag{3.16}$$

$$\sum_{t=1}^{T}\sum_{d=0}^{D+1} X_{itd} = 1 \qquad \forall i \tag{3.17}$$

$$\sum_{d=0}^{D+1} X_{itd} - \sum_{k=1}^{t}\sum_{d=0}^{D+1} X_{jkd} \le 0 \qquad \forall i,t \quad and \quad \forall j \in P\{i\} \tag{3.18}$$

All variables must be greater or equal to zero. $Y_{tdo}, Z_{tdov} \le 1$ and $X_{itd} \in \{0,1\}$.

Given the complexity of the problem derived from the flexibility considered at the different stages of the mining complex, the use of an exact method incorporated in any conventional optimization software, such as CPLEX, will not be able to generate an optimal solution in a feasible amount of time. A heuristic methodology is proposed to generate risk-based solutions that outperform conventional deterministic ones in practical times. The methodology is presented in the next section. Capacity, availability and blending constraints are called target constraints in the method for simplicity.

### 3.2.1 Solution of the problem

The method proposed in this paper uses iterative improvement over an initial solution until it converges to a final one. The procedure that the method uses can be divided in three stages: *(i)* Assign periods and destinations to the mining blocks based on the initial solution; *(ii)* Calculate the overall profitability per block per destination based on the orebody simulations; and, *(iii)* perturb the solution until a stopping criteria is reached to generate the final solution.

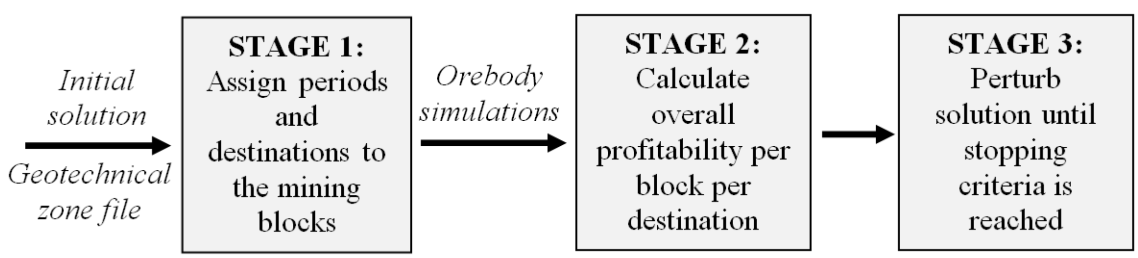


Figure 3–2: Stages of the method

#### Stage1

In this stage, the method assigns periods and destinations to the mining blocks from the initial solution. It also assigns a geotechnical zone for each block based on the geotechnical zone file. Different zones can have different slope angles and therefore different set of predecessor and successor blocks. If there are some slope constraint violations in the initial solution, block mining period corrections are performed based on the different slope angles. For doing so, when a slope constraint violation is found, the mining period of the block is moved to a feasible period based on the set of successor and predecessor blocks; that is, the range between the latest period of the predecessor blocks and the earliest period of the successor blocks (Figure 3–3).

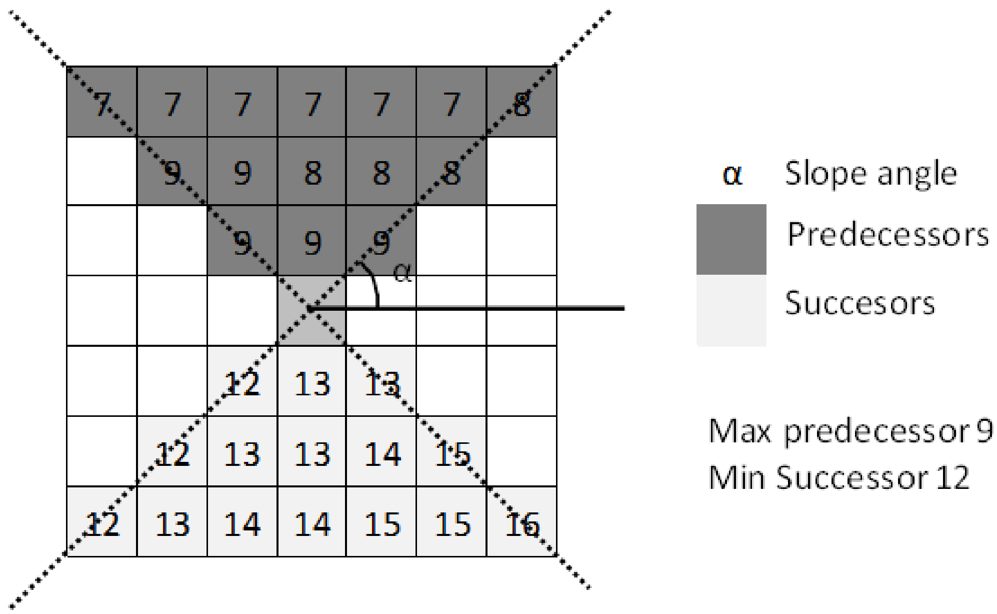


Figure 3–3: Two-dimensional example of predecessors and successors of a given block

**Stage2**

At this stage, the profits and costs for each simulation and period are evaluated. From the material types and grades at each simulation, the proposed method calculates, for each block, the overall profitability per available destination; that is, it evaluates the profit (or loss) obtained by sending a particular block to a given destination (considering the best operating alternative for that block at that destination) and accumulates it through the set of simulations (Figure 3–4). From there, the method evaluates the optimal destination for a particular block, but, this optimal destination may not be the final destination due to capacity, availability and blending constraints (target constraints).

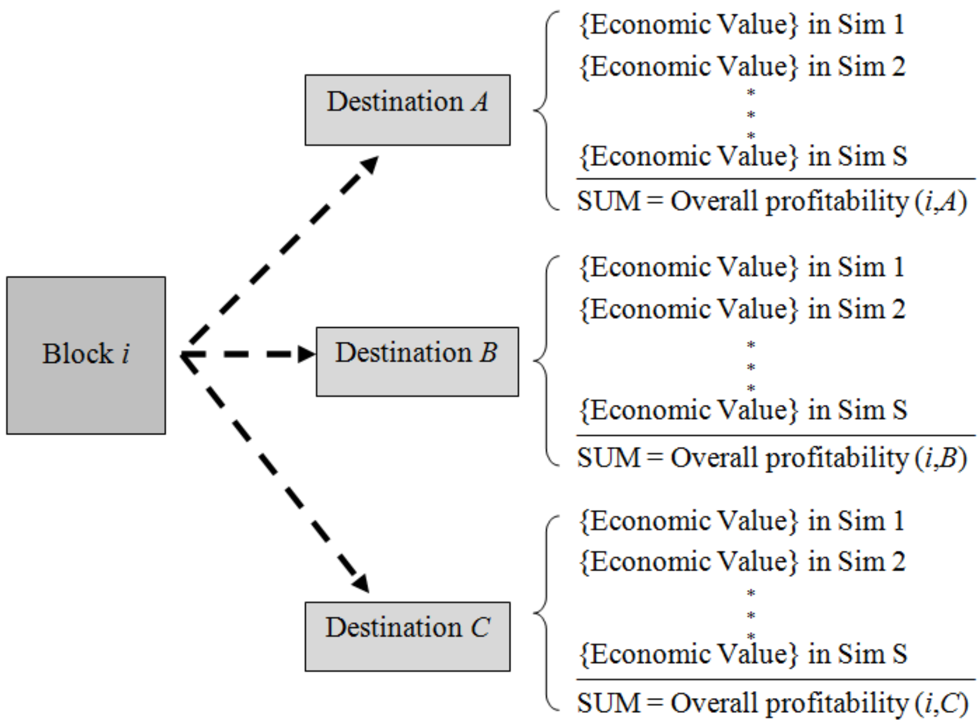


Figure 3–4: Overall profitability per block per destination

**Stage3**

This is the perturbation stage. A block is selected randomly and the available destinations for that particular block are sorted based on its overall profitability. If the best destination has a positive overall profitability, i.e., it increases the value of the objective function, the block is pushed to early periods, otherwise it may be pushed to later periods.

For positive overall profitable blocks, the method defines four possible options for periods and destinations (Figure 3–5). The *first option* is to send the block to its best destination in the previous period (current period  1). If there are no slope and target constraint violations this option is chosen. The *second option* is to mine the block in the previous period and send it to a profitable destination different from the optimal without violating slope and target constraints; that is, it considers the destinations with positive overall profit. The *third option* is to randomly select another block mined in the previous period from which a double swapping that increase the objective function can be performed without violating slope and target constraints. The double swapping consists of two different blocks switching mining periods. If the double swapping is non-feasible or non-profitable, the block is sent to the stockpile, which is the *last option*.

If the block has a negative overall profit for all the different destinations, it is sent to the waste dump. To decide the period when the block is going to be mined, the method evaluates the overall profitability of the set of closest successor blocks. If the sum of the overall profitability of the closest successor blocks is positive, the period of the block does not change to allow the successor blocks to move to early periods. This permits the schedule to access profitable areas early even when waste blocks are overlying them. If the sum is negative, the block and the predecessors belonging to the same period are sent to the next period without violating slope and mine

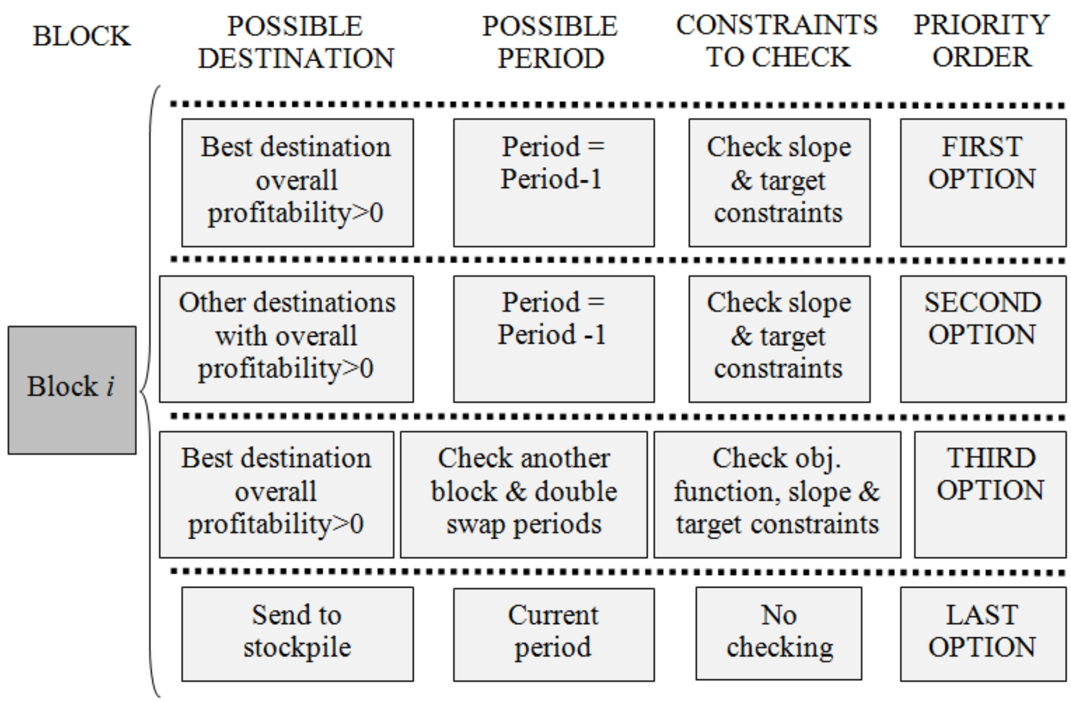


Figure 3–5: Possible destinations and mining periods of a block with positive overall profitability

capacity constraints. It should be noted that the method uses an overall revenue cut-off instead of a grade cut-off that conventional methods use to discriminate between ore and waste. The material sent to stockpiles is a profitable material that cannot be processed immediately due to capacity, availability and blending constraints.

After a certain number of iterations, the method re-evaluates the destination policy of the stockpiled material based on the tonnage available, its profitability and the respect of the target constraints at the different destinations.

The method stops after the maximum number of iterations, swaps, or iterations without substantial improvement of the objective value, are reached. The maximization of the objective value is driven by means of the swapping mechanism: sending the most profitable blocks to early possible periods and the best available destinations and sending to the waste dump the blocks with negative overall profit in later possible periods without violating slope and target constraints. The constraints are respected by means of the checking mechanism throughout the iterations of Stage 3. Figure 3–6 shows the flowchart of the method in the Stage 3.

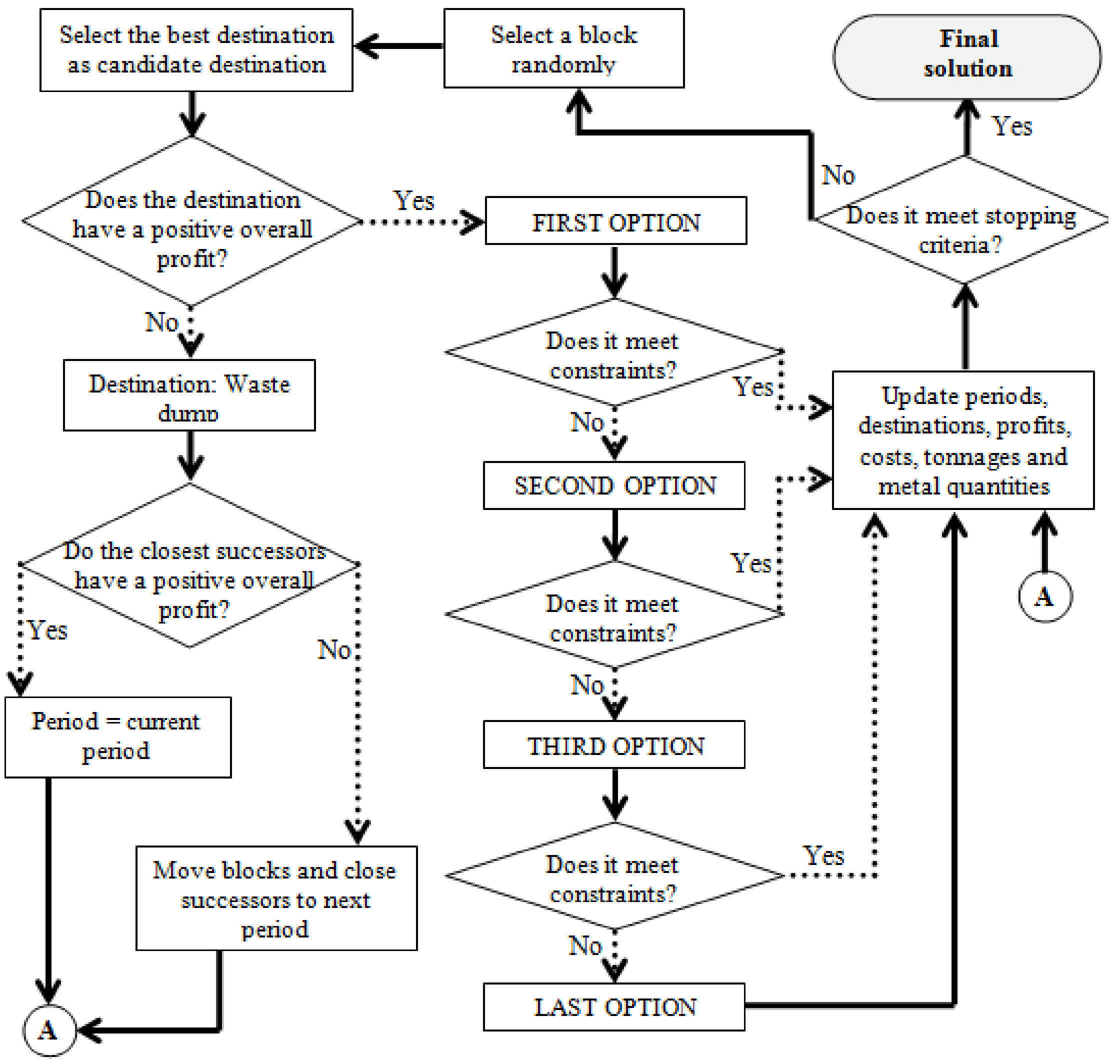


Figure 3–6: Stage 3 of the proposed method

## 3.3 Case study: a copper deposit

The method is implemented at a copper deposit, from which fifty orebody simulations are available for modelling geological uncertainty. Figure 3–7 shows the different material types of the deposit and the available processing destinations in the mining complex. Figure 3–8 shows different possible initial solutions generated using a conventional optimizer over different orebody simulations.

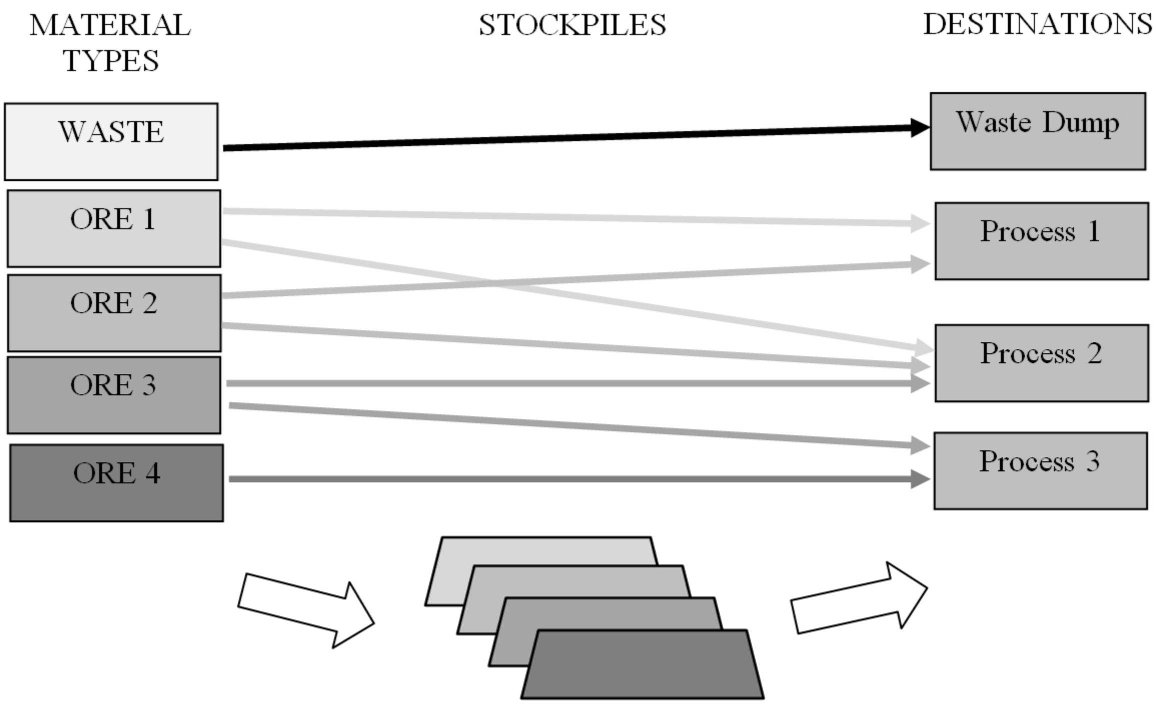


Figure 3–7: Available material types and destinations

The proposed method seeks to generate a mine and destination schedule that maximizes the NPV and respect capacity, availability and blending constraints. For doing so, an initial solution is iteratively perturbed to improve the objective value. To identify the number of perturbations required in the perturbation stage, different

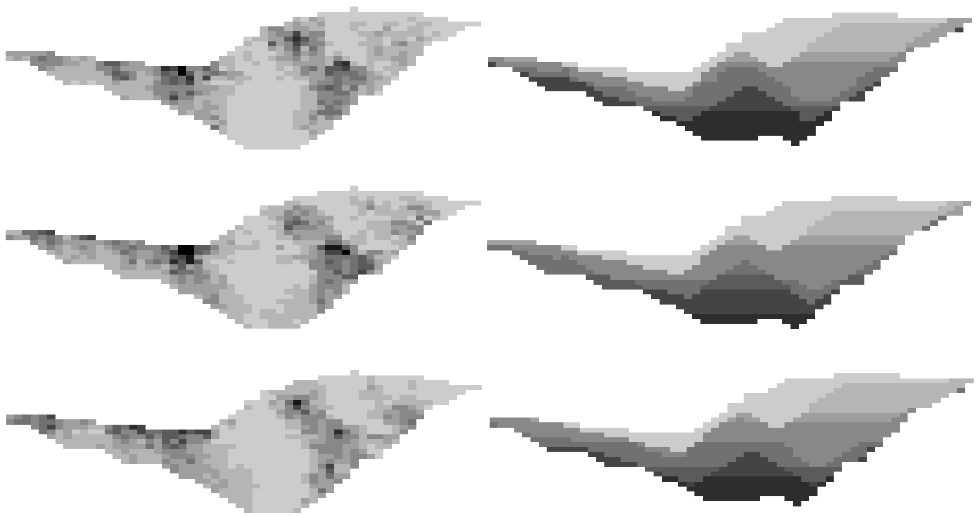

Figure 3–8: Orebody simulations (left), mining sequences (right)

numbers of perturbations are tested and the deviations from target constrains evaluated. Figure 3–9 shows the evolution of the 50th percentile (P50) of the tonnage sent to process 1 and the total tonnage mined with the number of perturbations. It is observed a large deviation from capacities at small number of perturbations and a substantial reduction in the deviation driven by the increment of the number of perturbations; i.e., the reduction in deviation from capacity of process 1 decreases from 9% in average to 0.2% when increasing the number of perturbations from 100 thousand to 1 million. Regarding the total mine production, the average deviation in the first 16 years remains in the same level (around 4% from the mine capacity).

An analysis based on the value of the expected NPV was performed. Figure 3–10 shows the evolution of the expected NPV with the number of perturbations and the number of simulations.

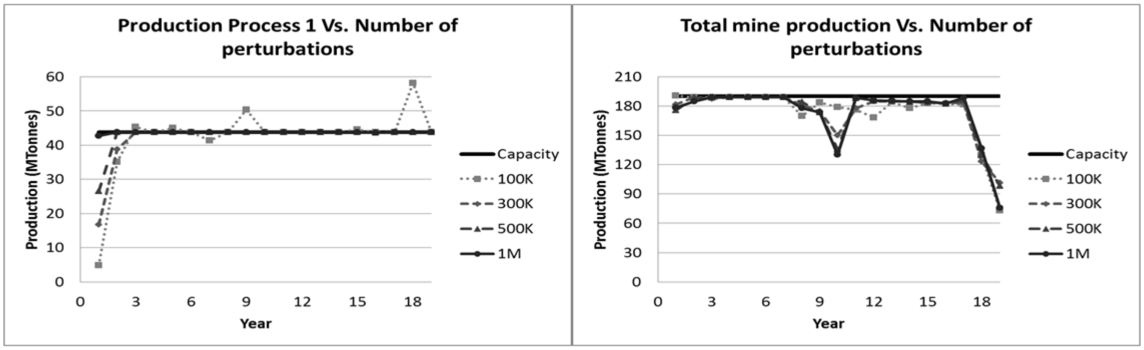


Figure 3–9: Productions vs. number of perturbations

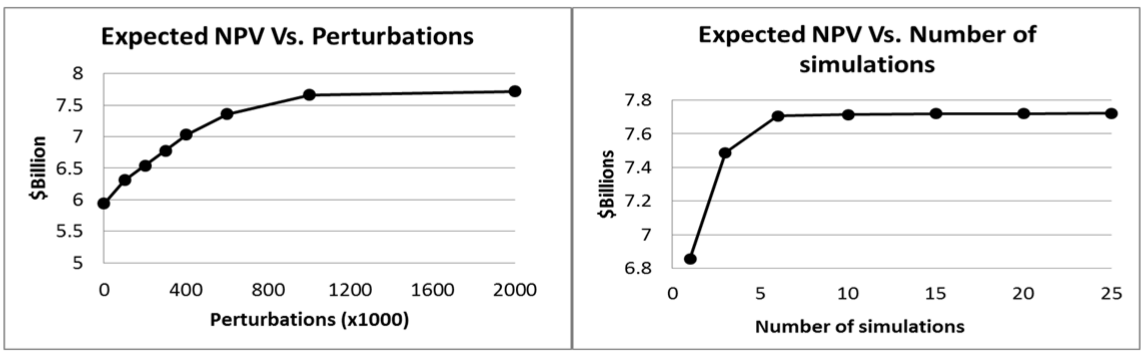


Figure 3–10: Evolution of expected NPV

It is observed that the increment of the expected NPV is marginal after 1 million perturbations. There is no substantial benefit in increasing the number of perturbations there-after. The objective value is increased by 30% when compared to the initial solution. The same analysis is done regarding the number of simulations required. It can be observed that after 15 simulations, no significant improvement in the expected NPV is presented.

Figure 3–11 shows the tonnage sent to Process 1, 2, 3 and the total tonnage mined. Given that the solution states the destination of the blocks, the differences in tonnage of the material sent to a given destination through the different simulations are negligible. These minor differences are generated from different tonnages of blocks among simulations derived from simulated densities. If the tonnage of the blocks were similar along the different simulations, no differences were presented in terms of tonnage among simulations. It can be observed that the Process 1 and the total tonnage mined are controlled by their corresponding capacities, whereas the Processes 2 and 3 are controlled by the amount of profitable reserves for those destinations.

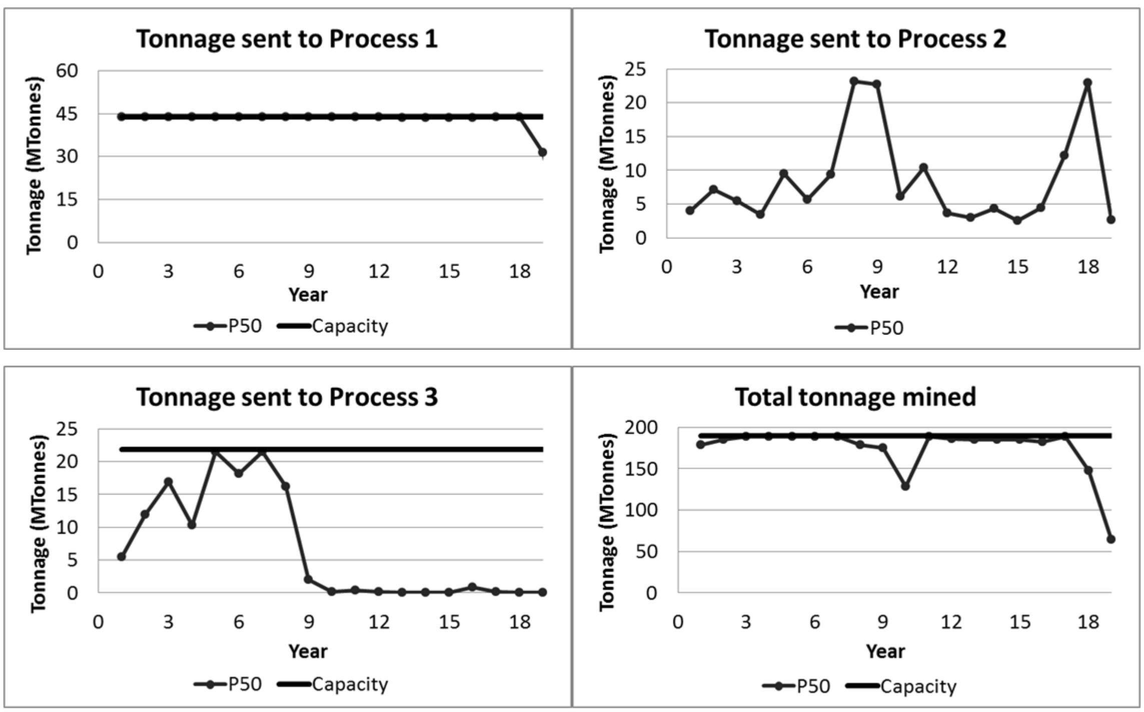


Figure 3–11: Tonnage sent to destinations

Although the material sent to the different destinations does not vary significantly between simulations, the amount of metal that can be recovered has significant fluctuations (Figure 3–12). This is originated from the grade and material type uncertainties; that is, the amount of metal sent to a process change in the simulations due to copper grade uncertainty, and the metallurgical recovery at a given destination vary in the simulations due to material type uncertainty.

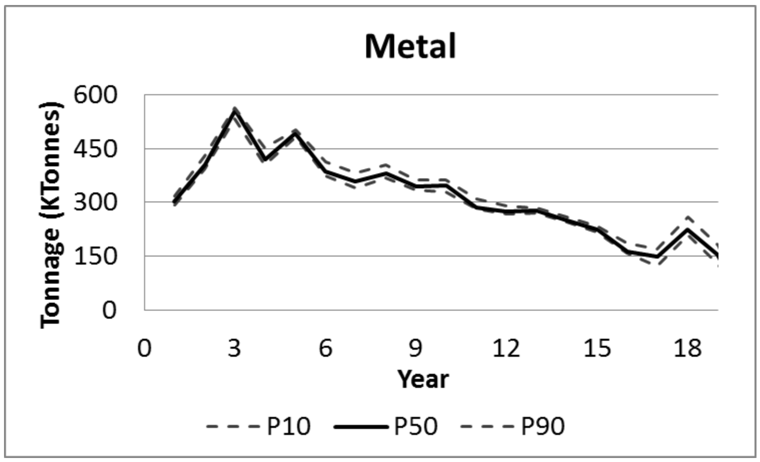


Figure 3–12: Metal

Figure 3–13 shows the amount of the different material types sent to the stockpiles. Sending waste material to the stockpile may be seen as a misclassification error. Although there are some risks of misclassifying material by following the stochastic solution generated, the algorithm seeks for minimizing the misclassification errors; that is, sending waste material to the stockpiles or sending a given material type to a non-profitable destination. A given material type sent to a wrong destination may produce a very high cost with low or negligible recovery. The way the algorithm controls misclassification errors is by maximizing the objective function, given that misclassification errors are very costly. The amounts of material type 3, 4 and 5 sent to stockpile are marginal and may be generated from misclassification; however, the algorithm will send that material to a particular destination if there is some profit associated. By having a look at the output material from the stockpiles, it was observed that only ore types 1 and 2 are rehandled and sent to process 1.

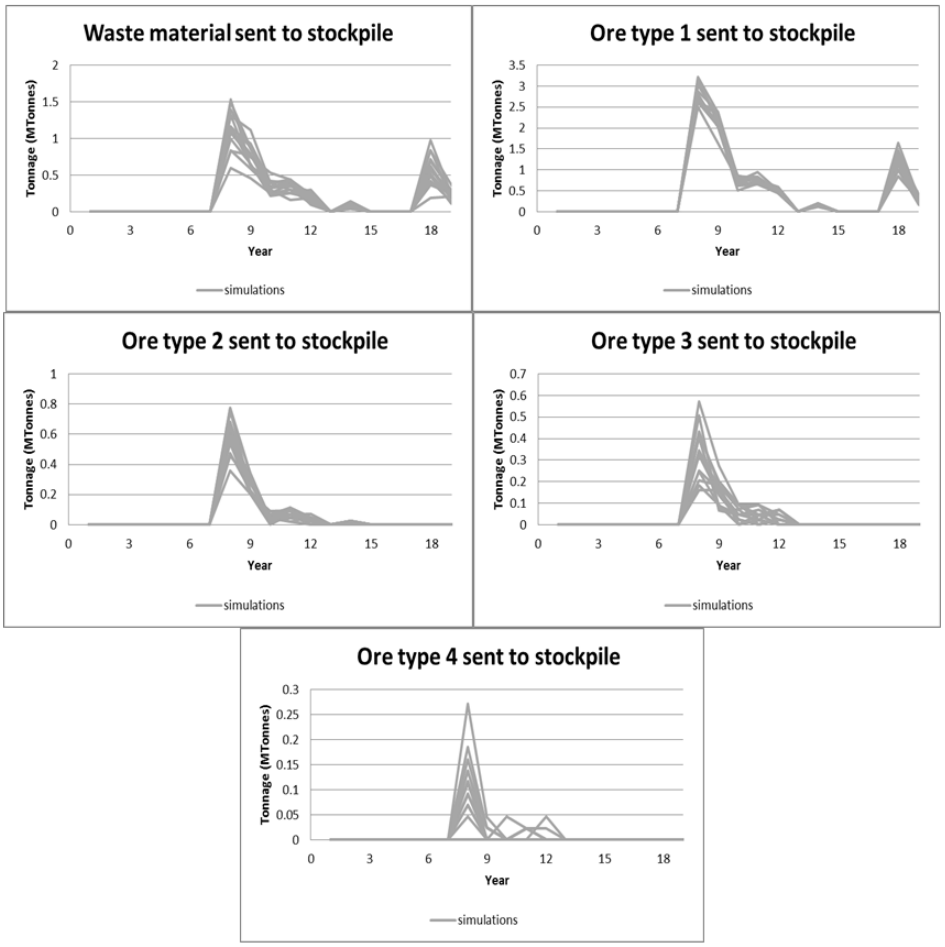


Figure 3–13: Material types sent to stockpile

Figure 3–14 shows the P10, P50 and P90 of the cumulative discounted cash flows. It is observed that during the first two decades of the project, the expected NPV is around $7.8 billion. Although, no blending constraints were considered in this case study, the method attempts to maximize net present value expectations while maintaining target constraints within acceptable tolerable limits. It discriminates blocks between ore and waste based on the overall profitability; that is, the profit (or loss) obtained by sending a block to a given destination accumulated through the set of simulations. When profitable material cannot be processed due to target constraints, it is sent to the stockpiles for being rehandled in future periods.

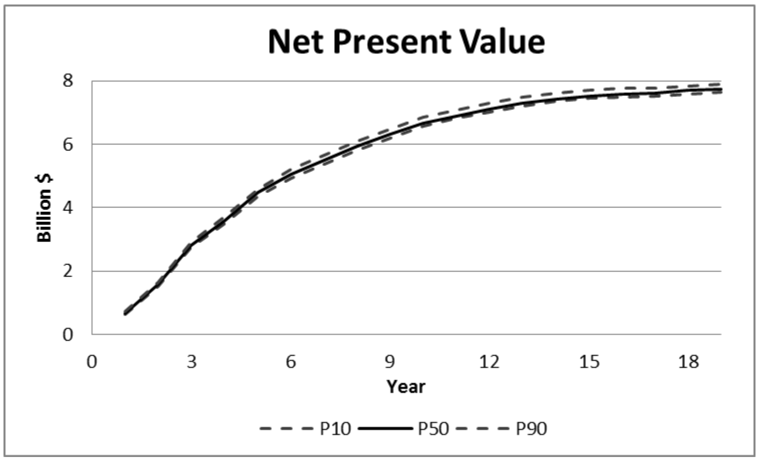


Figure 3–14: Net present value

To evaluate the benefits of the method, a comparison with the initial solution that was generated using conventional mining practices can be performed. Figure 3–15 shows the tonnage sent to process 1 when using a conventional scheduler. Large impractical deviations from the capacity of Process 1 can be observed.

Figure 3–16 shows the risk profile of the cumulative discounted cash flow of the conventional schedule. It is observed that during the first two decades of the project, the expected net present value of the risk-based schedule is 30% greater than the conventional initial schedule. This shows the ability of the method to handle two

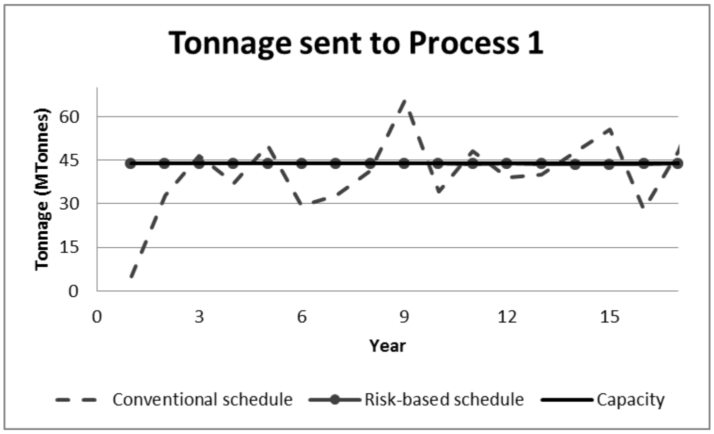


Figure 3–15: Process 1: Production forecast for the conventional initial solution

conflicting objectives: maximize expected net present value while approaching target constraints.

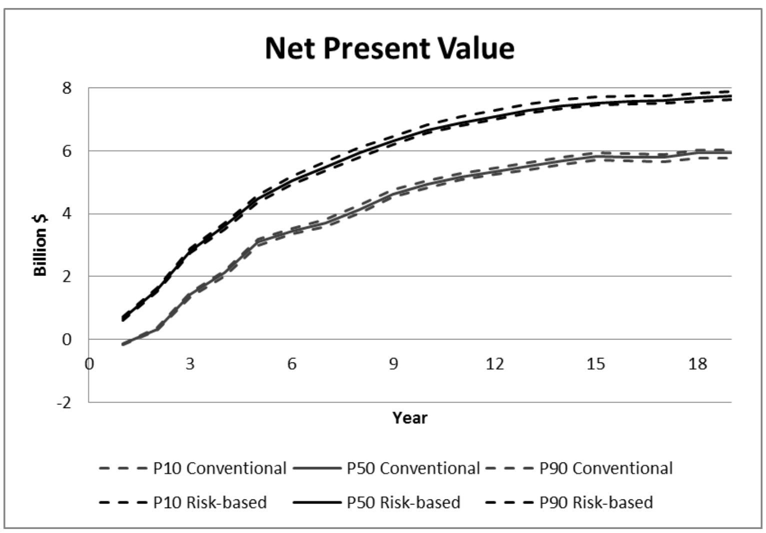


Figure 3–16: Net present value of the conventional initial solution

The method can handle processes with multiple operating alternatives, additives and blending constraints. Although it shows good results in this particular case-study, its ability to handle complex blending requirements needs to be tested in a future work. Regarding the heuristic process, even though it has some checking mechanisms to take advantage of the nature of the problem for having large improvements of the objective function, new heuristic mechanisms and diversification strategies should be evaluated to better explore the solution domain.

Figure 3–17 shows a cross section of the final schedule generated using the method. It can be observed that slope constraints are controlled by means of the correcting and checking mechanism utilized throughout the stages of the method.

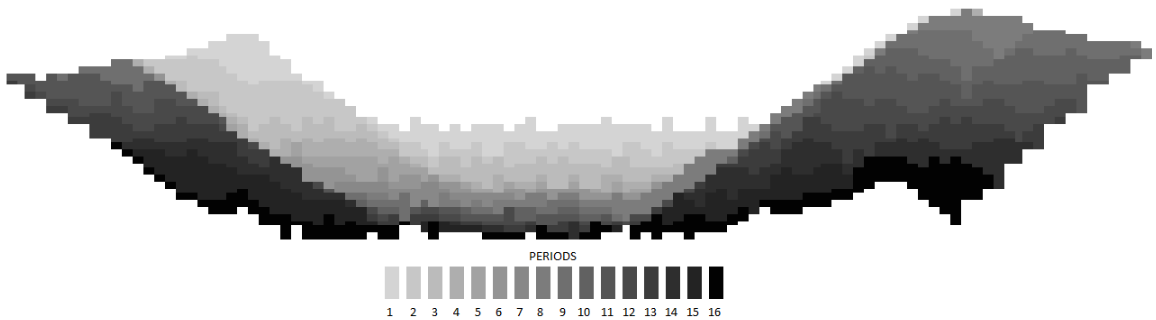


Figure 3–17: Cross-section of the risk-based schedule

## 3.4 Conclusions

An iterative improvement heuristic method is presented for generating mine production schedules in single-pit mining complexes that can contain multiple metals or attributes, multiple material types, stockpiles and processing options. The method considers relaxed capacity, availability and blending constraints. The implementation of the method in a copper deposit shows its ability to control target constraints by reducing the deviations from the capacity of Process 1 from 9% to 0.2% while increasing the expected net present value 30% when compared to an initial conventional solution.

An advantage of the method regarding previous developments is that it requires a single initial solution and the set of orebody simulations, whereas other implementations require multiple starting solutions, which increases substantially the labor of the mine planning engineer.

Regarding the expected NPV, there were no significant additional benefits from increasing the number of simulations after 15. However, the amount of simulations required to control complex blending operations needs to be addressed in future implementations.

Although the method allows for improving an initial solution in terms of meeting target constraints and net present value expectations, different heuristic strategies with diversification should be implemented to explore better the solution domain. Another possibility is to implement the method iteratively by considering several initial solutions simultaneously.

The possibility of adapting the method to multi-pit mining complexes is a future research avenue. Although the method requires practical amount of time for solving single-pit mining complexes (no more than 3 hours for dozens of millions of perturbations in a 1-million blocks deposit), its requirement in terms of computational time for multi-pit mining complexes needs to be addressed given the large size of the multi-pit problems.

# CHAPTER 4
# Optimizing mining complexes with multiple processing and transportation alternatives: An uncertainty-based approach

## 4.1 Introduction

A mining complex can be interpreted as a supply chain system where material is transformed from one activity to another [43]. The primary activities (or stages) consist of mining the materials from one or multiple sources (deposits); blending the material considering stockpiling; processing the material in different processing paths accounting for multiple operating alternatives; and transporting the products to port or final stocks using one or multiple transportation systems.

For a given processing path (e.g. mill-roaster in a refractory ore operation), it is possible to have multiple operating alternatives; for example, a mill may be operated using two different options: fine or coarse grinding (Figure 4–1). If the mill is operated using fine grinding, there is often a very high energy consumption, which is associated with a higher processing cost and also requires larger residence times for the material processed, thus limiting the mill throughput. A coarse grinding option requires less energy and residence time in the mill, which decreases the operating cost and increases the mill throughput, however, it results in a lower recovery in the roaster downstream. Furthermore, different processing alternatives often impose different blending requirements. For example, the tolerable amount of free silica of the input material may be lower when operating the mill at fine grinding given that the presence of this element increases the hardness of the material. Case studies have demonstrated [111] that for maximizing the net present value (NPV) when a mill is bottlenecking the system, it is better to use a coarse grind with higher throughput in the early periods of the life-of-mine (LOM), and, to use a finer grind to maximize recovery towards the end of the LOM. During the early periods, a mining complex incurs an opportunity cost for having material with large residence times in the mill, however, as the quantity of ore remaining in the mining complex diminishes, there is no opportunity cost.

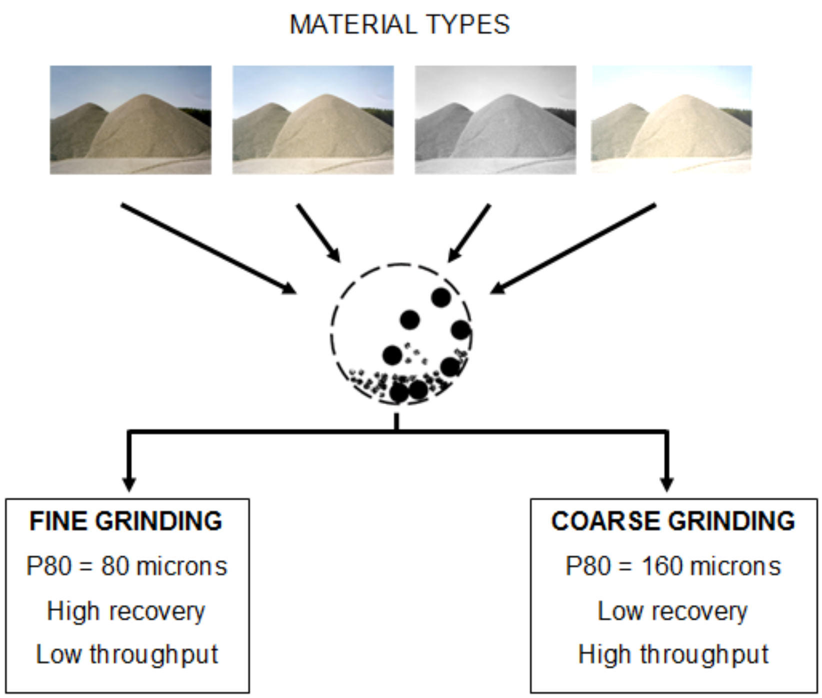


Figure 4–1: Operating alternatives for a mill

Once the material is processed through the different processing paths and using some available operating alternatives, existing transportation systems, either continuous (belt conveyors, pipeline transport) or batch (trucks, rail transportation), are used to transport the processed material to one or several ports or final stocks. Accounting for transportation systems in the optimization of mining complexes is important, given that they may limit the overall system output. In a mining complex, it is common to have multimodal transportation that involves the use of separate contractors or operators for each type of transport [116]. To account for the demand of transportation of material processed, it is necessary to establish the feasible relations between processing paths and transportation systems; specifically, a particular transportation system may be able to handle output material from some of the available processing paths: For example, in a pyro/hydrometallurgical complex, a hydraulic pipe may be able to transport the material output from the lixiviation plant whereas the material output from the pyrometallurgical plant is transported to the final stocks via trucks. Once the feasible transport relations are established, the demand for transportation is evaluated by considering the throughput relationships (output/input tonnages) for each processing path given the operating alternative implemented. For example, the output/input tonnage relation and the metallurgical recovery in a gold flotation plant change if the mass pull is 4 or 7% [48]. When the transportation of processed material is the bottleneck in the overall system, the operating conditions at the different processing paths must be evaluated. To overcome this limitation, it may be useful to re-evaluate throughput specifications of the processed material. Whittle [111] shows that by increasing the copper concentrate from 28 to 32% in some periods on a sulphide deposit, the metallurgical recovery decreases by 7%, but the NPV increases by 6% given the possibility of transporting more concentrated ore on the pipe, which is the bottleneck of the system.

Optimizing mining complexes by considering geological uncertainty and the different stages simultaneously is a large combinatorial optimization problem. Several efficient methodologies have been developed in stochastic environments for the mine production scheduling problem [37, 38, 42, 65, 66]. This paper presents a methodology for optimizing the mine plan and destination and transportation policies by considering all the stages of the mining complex simultaneously, and accounting for geological uncertainty by means of geostatistical orebody simulations of the deposit(s). The next section presents the formulation of the problem and the heuristic methodology proposed to solve it. Then, its implementation at a multi-pit operation is described; and, finally, some conclusions and future research avenues are addressed.

## 4.2 Method

### 4.2.1 Overview

In a mining complex, the material flows from the deposits as raw material to ports or final stocks as saleable products. To optimize the mining complex, the different stages that are involved must be considered simultaneously (Figure 4–3). First, the multiple material types coming from the mine(s) are sent to the available processes or to stockpiles where they are blended to meet the quality requirements. At each process the material is transformed into intermediate or final products, which are then transported to ports or final stocks. The goal when optimizing a mining complex is to maximize discounted cash flows while minimizing deviation

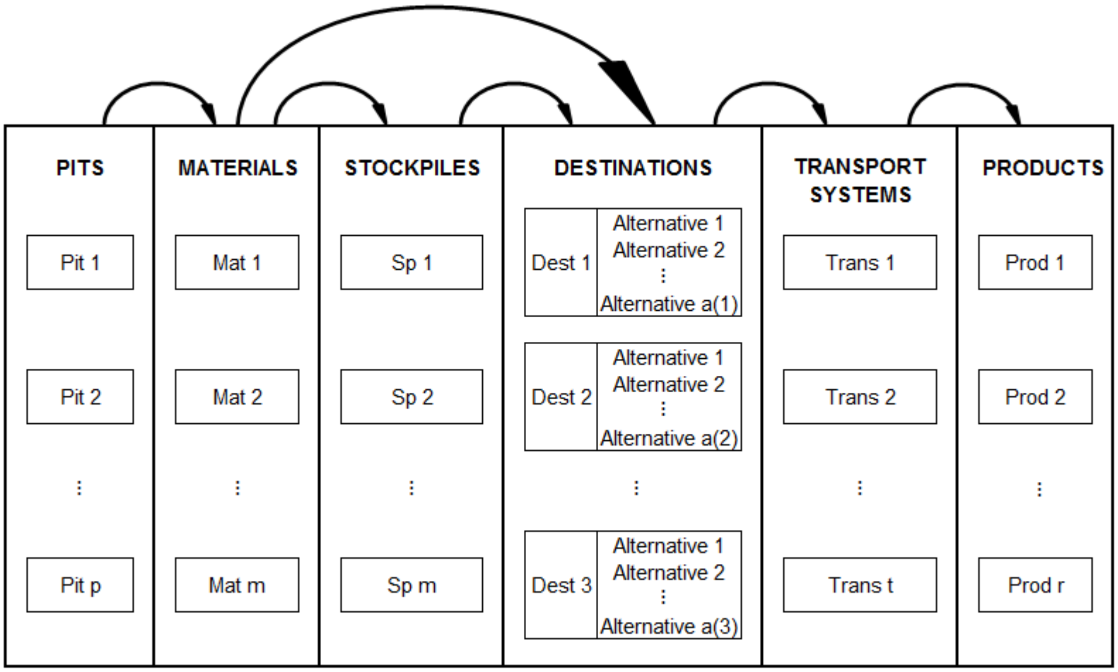


Figure 4–2: Flexibility of the mining complex

from mining and metallurgical processing targets, such as capacities associated to the different processing and transportation options and blending requirements regarding the different metallurgical properties. These metallurgical properties control the operation of the different processes and are calculated as mathematical expressions of the different grade elements, e.g., fuel value is a metallurgical property that controls the operation on a roaster.

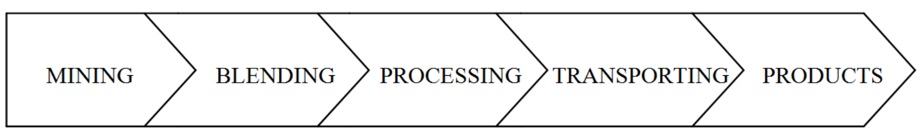


Figure 4–3: Activities of the mining complex

### 4.2.2 Optimization model

Table 4–1: Main variables

| | |
|---|---|
| $X_{itd}$ | binary variable denoting whether or not a block $i$ is mined in period $t$ and sent to destination $d$ |
| $Y_{tdo}$ | binary variable denoting whether or not a processing alternative $o$ is implemented in destination $d$ in period $t$ |
| $Z_{tdr}$ | continuous variable $\in [0,1]$ that represents the proportion of output tonnage from destination $d$ to be transported by transportation system $r$ in period t |
| $discprofit(s,t)$ | discounted profit obtained in period $t$ under scenario $s$ |
| $penalty(s,t)$ | penalty term of objective function in period $t$ under scenario $s$ |

Table 4–2: Deviation variables

| | |
|---|---|
| $D(s,t,p)_U$ | tonnage exceeding the capacity associated with pit $p$ in period $t$ under scenario $s$ |
| $D(s,t,p)_L$ | deficient amount of tonnage mined in pit $p$ during period $t$ under scenario $s$ regarding its associated capacity |
| $D(s,t,d,o)_U$ | tonnage exceeding the capacity associated with the operation alternative $o$ of destination $d$ in period $t$ considering the scenario $s$ |
| $D(s,t,d,o)_L$ | deficient amount of tonnage input to destination $d$ in period $t$ under scenario $s$ considering operating alternative $o$ and its associated capacity |
| $D(s,t,d,o,k)_U$ | over-deviation from the upper target regarding the metallurgical property $k$ in processing option $o$ of destination $d$ in period $t$ under scenario $s$ |
| $D(s,t,d,o,k)_L$ | under-deviation from the lower target regarding the metallurgical property $k$ in processing option $o$ of destination $d$ in period $t$ under scenario $s$ |
| $D(s,t,r)_U$ | tonnage exceeding the capacity associated with the transportation system $r$ in period $t$ considering the scenario $s$ |
| $D(s,t,r)_L$ | deficient amount of tonnage regarding the capacity associated with transportation system $r$ in period $t$ under scenario $s$ |

Table 4–3: Economic and penalty variables

| | |
|---|---|
| $revenue(s,t)$ | revenue in period $t$ under scenario $s$ |
| $mncost(s,t)$ | cost of mining the materials in period $t$ under scenario $s$ |
| $procost(s,t)$ | cost of processing the materials in period $t$ under scenario $s$ |
| $stkcost(s,t)$ | cost of stockpiling the materials in period $t$ under scenario $s$ |
| $rehandlecost(s,t)$ | cost of sending material from the stockpiles to the available destinations in period $t$ under scenario $s$ |
| $transcost(s,t)$ | cost of transporting the products to the ports or final stocks in period $t$ under scenario $s$ |
| $penalpit(s,t)$ | penalized deviations from pits capacities in period $t$ under scenario $s$ |
| $penalpro(s,t)$ | penalized deviations from operation alternatives capacities in period $t$ under scenario $s$ |
| $penalmetal(s,t)$ | penalized deviations from metallurgical operational ranges in period $t$ under scenario $s$ |
| $penaltrans(s,t)$ | penalized deviations from transportation systems capacities in period $t$ under scenario $s$ |

Table 4–4: Tonnage variables

| | |
|---|---|
| $mineproduction(s,t)$ | tonnage mined in period $t$ under scenario $s$ |
| $tonnesentmine(s,t,d)$ | tonnage sent from the pits to destination $d$ in period $t$ under scenario $s$ |
| $tonnestockpiles(s,t)$ | tonnage presented in the stockpiles in period $t$ under scenario $s$ |
| $tonnerehandle(s,t,d)$ | tonnage sent from the stockpiles to destination $d$ in period t under scenario $s$ |
| $tonneprocess(s,t,d)$ | tonnage of material processed in destination $d$ in period $t$ under scenario $s$ |
| $metalsentmn(s,t,d,m)$ | amount of metal $m$ sent from the pits to destination $d$ in period $t$ under scenario $s$ |
| $metalrehand(s,t,d,m)$ | amount of metal $m$ sent from the stockpiles to destination $d$ in period $t$ under scenario $s$ |
| $tonneoutprocess(s,t,d)$ | tonnage of material output from destination $d$ in period $t$ under scenario $s$ |
| $tonnetransport(s,t,r)$ | tonnage of material transported using transportation system $r$ in period $t$ under scenario $s$ |
| $metalrec(s,t,m)$ | amount of metal $m$ recovered in period $t$ under scenario $s$ |

Table 4–5: Parameters

| | |
|---|---|
| $m_{is}$ | mass of block $i$ under scenario $s$ |
| $P_{do}$ | proportion output/input tonnage in operating alternative $o$ of destination $d$ |
| $A_{dr}$ | 0-1 parameter indicating whether or not the output material from destination $d$ can be transported using transportation system $r$ |
| $rec_{dom}$ | metallurgical recovery of metal $m$ in destination $d$ using the operation alternative $o$ |
| $price_m$ | price of metal $m$ |
| $mc$ | per-unit mining cost |
| $pc_{do}$ | per-unit processing cost in destination $d$ using operation alternative $o$ |
| $kc$ | per-unit stockpiling cost |
| $hc$ | per-unit rehandle cost |
| $\tau_c$ | per-unit transportation cost using transportation system $r$ |
| $drate$ | discount rate |
| $C(t,p)_U$ | per-unit penalty cost associated with over-deviation of production in pit $p$ during period $t$ |
| $C(t,p)_L$ | per-unit penalty cost associated with under-deviation of production in pit $p$ during period $t$ |
| $C(t,d,o)_U$ | per-unit penalty cost associated with over-deviation of production in operation alternative $o$ of destination $d$ during period $t$ |
| $C(t,d,o)_L$ | per-unit penalty cost associated with under-deviation of production in operation alternative $o$ of destination $d$ during period $t$ |
| $C(t,d,o,k)_U$ | per-unit penalty cost associated with over-deviation from upper target of metallurgical property $k$ in period $t$ considering operation alternative $o$ of destination $d$ |
| $C(t,d,o,k)_L$ | per-unit penalty cost associated with under-deviation from lower target of metallurgical property $k$ in period $t$ considering operation alternative $o$ of destination $d$ |
| $C(t,r)_U$ | per-unit penalty cost associated with exceeding the capacity of transportation system $r$ during period $t$ |
| $C(t,r)_L$ | per-unit penalty cost associated with failing to meet the tonnage capacity of the transportation system $r$ during period $t$ |

Table 4–6: Sets

| | |
|---|---|
| $S$ | set of scenarios |
| $T$ | set of periods considered in the LOM |
| $P$ | set of mining pits |
| $I$ | set of mining blocks considering all available pits |
| $D$ | set of destinations (processing paths) available |
| $O(d)$ | set of operating alternatives at destination $d$ |
| $M$ | set of grade elements (including recoverable metals) |
| $K$ | set of metallurgical properties |
| $R$ | set of transportation systems |

### Objective function

The objective function is given by Eq. (4.1) and seeks for maximizing discounted profits and minimizing deviations from targets along all periods and scenarios (derived from orebody simulations). The first term of the objective function accounts for discounted profits by evaluating the revenues obtained by selling the different products and the costs associated to the different activities of the mining complex. The second term accounts for penalized deviations regarding mining, processing, transportation and blending targets and may be seen as a penalty cost it is incurred by not meeting the different targets. The value of *penalty(s,t)* depends on the deviations from the targets itself and the magnitude of the per-unit penalty costs associated. If the per-unit penalty costs are too high, the method will improve the reproduction of the targets ignoring the first term of the objective function generating poor improvement of expected NPV. Conversely, too small per-unit penalty costs will generate impractical solutions with large and non-realistic NPV forecasts given the large violations of the targets.

$$MaxO = \sum_{t=1}^{T} \left( \frac{1}{S} \left( \sum_{s=1}^{S} discprofit(s,t) - penalty(s,t) \right) \right) \tag{4.1}$$

To manage the risk along the different periods, the per-unit penalty costs can be discounted using the geological risk discounting rate (GRD) introduced by Dimitrakopoulos and Ramazan [26]. This allows deferring risks of not meeting targets for later periods when more information will be available. GRD can be applied to processing, blending and transportation targets.

**Model constraints**

When generating a strategic plan for a mining complex there are important questions that need to be answered: *(i)* at each mining block, when to mine it and where to send it; *(ii)* at each destination (processing path), which processing alternative to implement every period; *(iii)* which transportation systems should be used to transport the products.

The tonnage mined in a given period $t$ under a scenario $s$ can be evaluated as:

$$mineproduction(s,t) = \sum_{i=1}^{I} \sum_{d=0}^{D} X_{itd} \cdot m_{is} \tag{4.2}$$

Scenarios are obtained from orebody simulations and, due to grade and material type uncertainties, the tonnage of a block may differ from one scenario to another. Similarly, the tonnage sent from the pits to any particular destination $d$ can be evaluated as:

$$tonnesentmine(s,t,d) = \sum_{i=1}^{I} X_{itd} \cdot m_{is} \tag{4.3}$$

In a mining complex, different material types are storage in different stockpiles given that they may have different metallurgical properties. The model considers one stockpile for each material type that contributes to the blending operation. When a particular block is sent to the stockpiles, the assignment of any particular stockpile is a scenario-dependent decision derived from the material type uncertainty. In other words, for each scenario, a stockpiled block will be assigned to the corresponding pile related to its material type. For modeling purposes, stockpiling a block is represented as having destination *d=0*. Therefore, in a period $t$, the total tonnage presented in the stockpiles under a scenario $s$ is:

$$\begin{aligned} tonnestockpiles(s,t) = {} & tonnestockpiles(s,t-1) \\ & - \sum_{d=1}^{D} tonnerehandle(s,t,d) \\ & + tonnesentmine(s,t,0) \end{aligned} \tag{4.4}$$

The amount of material processed in a given destination $d$ during period $t$ under scenario $s$ is given by:

$$\begin{aligned} tonneprocess(s,t,d) = {} & tonnesentmine(s,t,d) \\ & + tonnerehandle(s,t,d) \end{aligned} \tag{4.5}$$

Similarly, the amount of metal $m$ input to a particular destination $d$ during period $t$ under scenario $s$ can be evaluated as:

$$\begin{aligned} metalprocess(s,t,d,m) = {} & metalsentmine(s,t,d,m) \\ & + metalrehandle(s,t,d,m) \end{aligned} \tag{4.6}$$

At any given period $t$, only one of the possible available operating alternatives $o(d)$ can be implemented in a particular destination $d$, which implies that:

$$\sum_{o=1}^{O(d)} Y_{tdo} = 1 \tag{4.7}$$

At each destination, every available operating alternative may have its corresponding associated capacity, operating cost, recoveries, operational ranges for metallurgical properties, and throughput specification (relation between output/input tonnages). The amount of material output from a given destination $d$ in period $t$ under scenario $s$ is given by:

$$tonneoutprocess(s,t,d) = \sum_{o=1}^{O(d)} (tonneprocess(s,t,d) \cdot Y_{tdo} \cdot P_{do}) \tag{4.8}$$

After evaluating the output material from the different destinations in a period $t$ under scenario $s$, the tonnage transported by a given transportation system $r$ can be calculated as:

$$tonnetransport(s,t,r) = \sum_{d=1}^{D} (tonneoutprocess(s,t,d) \cdot Z_{tdr}) \tag{4.9}$$

Eq. (4.10) and (4.11) are used to respect feasible process-transport configurations and to transport all output material obtained from a given destination.

$$Z_{tdr} \leq A_{dr} \tag{4.10}$$

$$\sum_{r=1}^{R} Z_{tdr} = 1 \tag{4.11}$$

The amount of metal $m$ recovered in period $t$ under scenario $s$ can be calculated as:

$$metalrec(s,t,m) = \sum_{d=1}^{D} \sum_{0=1}^{O(d)} (metalprocess(s,t,d,m) \cdot rec(d,o,m)) \tag{4.12}$$

Using the amount of metals recovered, it is possible to calculate the revenue obtained in period $t$ under scenarios $s$ as:

$$revenue(s,t) = \sum_{m=1}^{M} (metalrec(s,t,m) \cdot price(m)) \tag{4.13}$$

The costs associated with the different activities of the mining complex are given by Eq. (4.14 - 4.18):

$$minecost(s,t) = mineproduction(s,t) \cdot m_c \tag{4.14}$$

$$procost(s,t) = \sum_{d=1}^{D} \sum_{o=1}^{O(d)} (tonneprocess(s,t,d) \cdot P_c(d,o) \cdot Y_{tdo}) \tag{4.15}$$

$$stockcost(s,t) = tonnesentmine(s,t,0) \cdot k_c \tag{4.16}$$

$$rehandlecost(s,t) = \left( \sum_{d=1}^{D} tonnerehandle(s,t,d) \right) \cdot h_c \tag{4.17}$$

$$transcost(s,t) = \sum_{r=1}^{R} (tonnetransport(s,t,r) \cdot \tau_c(r)) \tag{4.18}$$

The discounted profit, which is the term that appears in the objective function, can be calculated by discounting the difference between the revenue and the costs associated with the different activities Eq. (4.19).

$$discprofit(s,t) = \frac{\begin{pmatrix} revenue(s,t) - minecost(s,t) - procost(s,t) \\ -stockcost(s,t) - rehandlecost(s,t) - transcost(s,t) \end{pmatrix}}{(1+drate)^t} \tag{4.19}$$

To control the operations at the different destinations (processing paths), various metallurgical properties must be considered. At any given operation alternative of a particular destination, these metallurgical properties must fall in between some operational ranges. The deviations from these operational ranges must be also minimized by means of penalty costs.

*penalty(s,t)* is the second term of the objective function and can be evaluated in each period and scenario using Eq. (4.20).

$$penalty(s,t) = \begin{matrix} penalpit(s,t) + penaltrans(s,t) \\ + penalpro(s,t) + penalmetal(s,t) \end{matrix} \tag{4.20}$$

The evaluation of the different penalized deviations that affect the penalty term of the objective function is displayed in Eq. (4.21 - 4.24):

$$penalpit(s,t) = \sum_{p=1}^{P} \left( C(t,p)_U \cdot D(s,t,p)_U + C(t,p)_L \cdot D(s,t,p)_L \right) \tag{4.21}$$

$$penaltrans(s,t) = \sum_{r=1}^{R} \left( C(t,r)_U \cdot D(s,t,r)_U + C(t,r)_L \cdot D(s,t,r)_L \right) \tag{4.22}$$

$$penalpro(s,t) = \sum_{d=1}^{D} \sum_{o=1}^{O(d)} \begin{pmatrix} C(t,d,o)_U \cdot D(s,t,d,o)_U \\ + C(t,d,o)_L \cdot D(s,t,d,o)_L \end{pmatrix} \tag{4.23}$$

$$penalmetal(s,t) = \sum_{d=1}^{D} \sum_{o=1}^{O(d)} \sum_{k=1}^{K} \begin{pmatrix} C(t,d,o,k)_U \cdot D(s,t,d,o,k)_U \\ + C(t,d,o,k)_L \cdot D(s,t,d,o,k)_L \end{pmatrix} \tag{4.24}$$

All variables must be greater or equal to zero. $X_{itd}, Y_{tdo} \in \{0,1\}$ and $Z_{tdr} \leq 1$.

The production and transportation capacities and the blending targets are controlled via penalties in the objective function. Other operational constraints, although not displayed in this paper, are considered in the formulation, such as precedence constraints, reserve constraints, etc. [83].

### 4.2.3 Solution approach

Given the complexity of the problem derived from the flexibility considered at the different activities of the mining complex, the use of an exact method incorporated in any conventional optimization software, such as CPLEX, will not be suitable as solution times will be impractical even for small instance problems (few thousands of blocks). To overcome this situation, a heuristic methodology is proposed to generate good-quality solutions. The proposed algorithm perturbs an initial solution iteratively to improve the objective function. In order to avoid local optimal solutions and to explore the solution domain (the set of all possible mine production schedules with operating policies for processing paths and transportation systems), the method allows deterioration based on a decision rule and uses diversification. The decision rule is the same implemented by the Metropolis algorithm [76] and allows the exploration of the solution domain while converging to a final good-quality solution. A diversification strategy over the solution domain is performed by means of perturbation at different decision levels of the mining complex (blocks, operating alternatives, transportation systems). The proposed algorithm can be implemented multiple times to improve the final solution by controlling the number of cycles.

**Decision rule**

Metropolis et al. [76] introduce an algorithm to provide a simulation of a collection of atoms in equilibrium at a given temperature. The Metropolis algorithm perturbs the initial state and, at each iteration, an atom is displaced and the resulting change in energy $\Delta E$ is computed. If $\Delta E \leq 0$, the displacement is accepted. The case $\Delta E > 0$ is accepted or rejected based on random sampling of a probability distribution $P(\Delta E) = exp(-\Delta E/k_B T)$ where $K_B$ is a constant and $T$ the temperature of the state. Kirkpatrick et al. [64] use a cost function in place of the energy and define configurations by a set of variables to generate a population of configurations of a given optimization problem at some temperature. This temperature acts as a control parameter of the same units as the cost function. Previous implementation of annealing schedule have demonstrated its ability to improve mine production scheduling and pit designs in terms of expected NPV and meeting production targets [1, 37, 42, 69].

Given the nature of the optimization problem considered in this paper, which is a maximization problem and not a cost or deviation minimization one, a perturbation that deteriorates the current solution is the one that decreases its objective value. Accounting for this, the probability distribution is given by Eq. (4.25) with $T$ being the annealing temperature.

$$P(\Delta O) = P(O_{new} - O_{current}) = \left\{ \begin{array}{rl} 1 & if(\Delta O \geq 0) \\ e^{\frac{-(\Delta O)}{T}} & if(\Delta O < 0) \end{array} \right\} \tag{4.25}$$

The probability of accepting an unfavourable perturbation is greater at higher temperatures. As the optimization proceeds, the temperature is gradually lowered by a reduction factor. When the temperature approaches zero, the probability of accepting an unfavourable swap tends to zero. This allows the algorithm to converge to a final solution.

The total number of swaps and the number of swaps at a given temperature control the end of the algorithm and the changes of temperature throughout the iteration process.

**Perturbation mechanism**

The proposed algorithm requires an initial mining sequence to assign periods and destinations to mining blocks and a set of orebody simulations for each deposit to evaluate profits, costs, productions and deviations at the different activities of the mining complex. While reading the orebody simulations, the algorithm evaluates the overall profitability of a block at a given destination by accumulating the economic value of the block in that destination through all scenarios. For simplicity, the overall profitability of a block at a given destination will be referred to as OPBD. Based on the OPBD, it is possible to determine the optimal destination of a given block. One or several waste dumps may be considered and they are treated as processing destinations with null recoveries.

The solution is improved by means of the perturbation mechanism. The algorithm performs perturbation at three different level or stages: blocks, operating alternatives and transportation systems. At any level of perturbation, a new solution will be accepted based on the decision rule explained in the previous section.

*Block Based Perturbations (BBP)*: The algorithm selects a block randomly and checks its OPBD in the different destinations. It perturbs the solution by modifying periods and destinations of mining blocks. Moving the extraction period of a mining block to a previous period will be referred to as pulling up the block, whereas moving the block to a following period is referred to as pushing down the block. If the block has a positive OPBD in the optimal destination, the algorithm iterates the candidate period from a previous to a following period, favouring first the chance of pulling up the block given the time value of money. In the case where a block has negative OPBD in all destinations, the algorithm iterates from the following period to the previous one, favouring first pushing down the block (Figure 4–4). Before accepting any candidate period, the algorithm checks that slope constraints are respected.

For a block with positive OPBD in its optimal destination, the set of candidate destinations are those with positive OPBD. The algorithm sorts candidate destinations based on the OPBD and iterates from the most profitable destination to the less profitable one (but still with positive OPBD). If the block has negative OPBD in all destinations, the only candidate destination is its optimal destination (the one with higher OPBD). This ensures that waste blocks are always sent to the waste dump(s) as they are treated as destinations with null recoveries. There may be cases where blocks have negative OPBD in all destinations but the optimal destination in not a waste dump; that is, although processing that block in a particular processing path generates a negative profit, the profit losses are less by processing the block than by sending it to a waste dump. In these cases, optimal destinations are also respected.

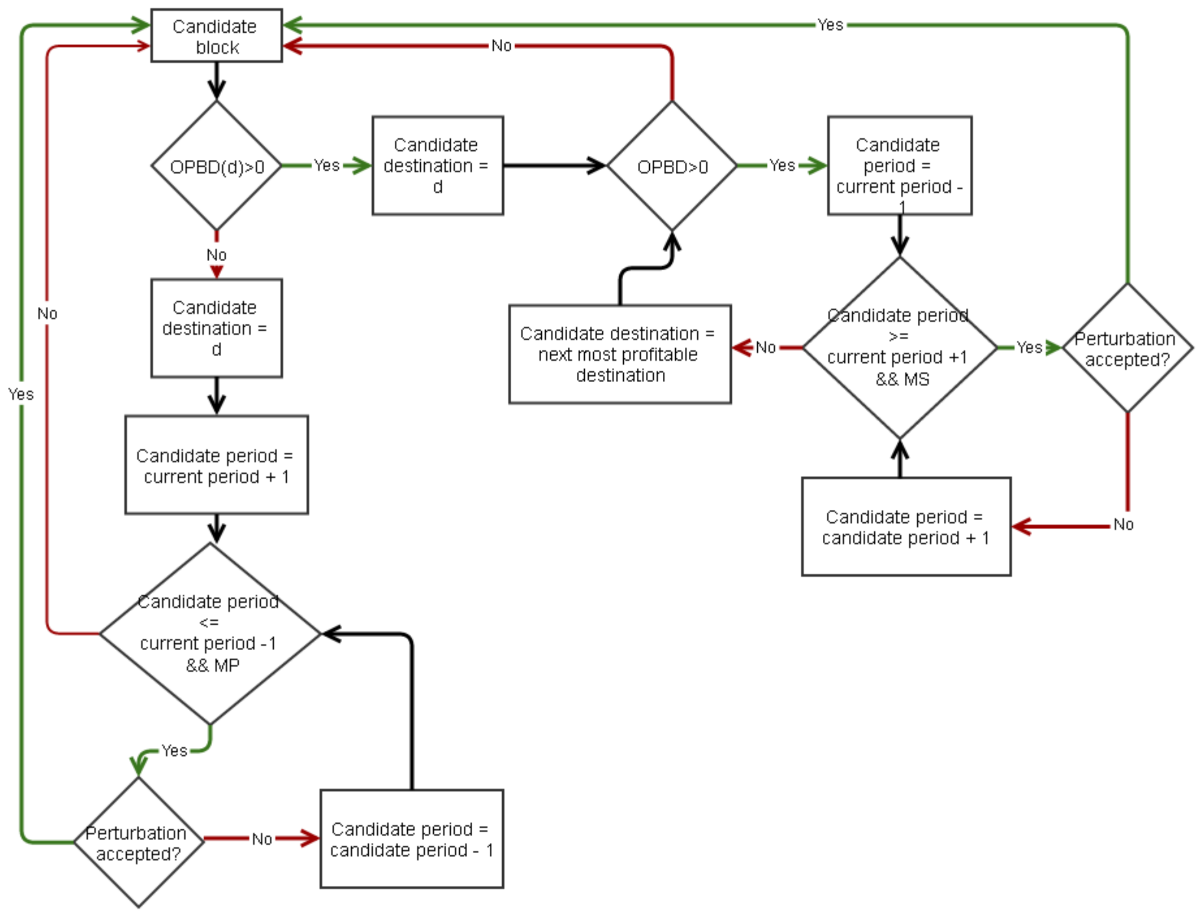


Figure 4–4: Block based perturbations

Although, pulling up a positive block and pushing down a negative one improve NPV expectations because of the time value of money, the objective value of the new solution generated does not necessarily increase as there is a penalty term also affecting the objective function. There may be cases where the NPV increases, but the new solution deteriorates the objective value due to the penalized deviations. A similar analysis can be made for the opposite case; a perturbation that decreases NPV expectations does not necessarily deteriorate the current solution. The objective value can be seen then as a trade-off between improving the NPV and decreasing deviation from operational targets.

*Operating Alternative Based Perturbations (OBP)*: In the previous section, block based perturbations were described; however, the operating alternatives implemented at each destination are not simultaneously modified. The set of perturbations at the OBP level refer to swapping operating alternatives at the different destinations (processing paths); e.g., swapping from fine to coarse grinding in the mill in some periods of the LOM. Modifying the operating alternative at a particular destination in a given period may vary the objective value as: *(i)* Processing cost and recovery may change, which may affect the expected NPV; *(ii)* Capacity and operational metallurgical ranges may change, which affect the penalty term in the objective function.

Given a particular period and destination, the algorithm selects randomly an available operating alternative as the candidate alternative, and evaluates the objective value when swapping the operating alternative to the candidate one (Figure 4–5). The new solution is accepted or rejected based on the decision rule explained previously.

*Transportation system based perturbations (TBP)*: As previously explained, the first level of perturbations modify period and destinations of mining blocks, whereas the second level of perturbations modify operating alternatives at the different periods and destinations. The third level of perturbations is referred to as the transportation system based perturbations. For a given destination and period, the algorithm attempts to perturb the proportion of output material transported using the available

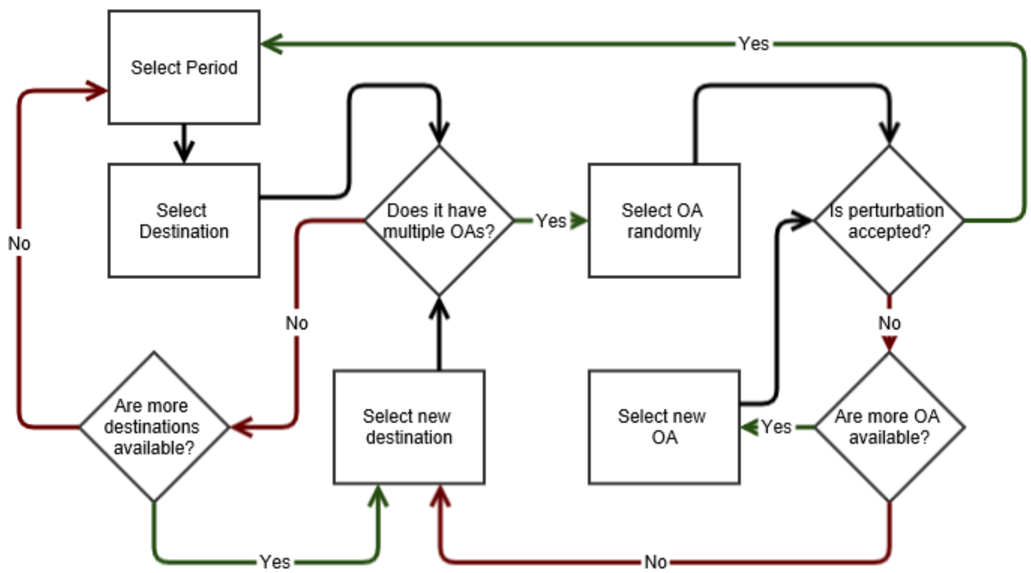


Figure 4–5: Operating alternative based perturbations

transportation systems; e.g., the mill-roaster processing path (destination) change its transportation arrangement for the output material from [70% trucks / 30% pipe] to [50% trucks / 50% pipe]. This set of perturbations seeks to minimize the transportation costs and penalized deviations in the objective function. The variations of the proportions of transportation systems utilized are generated using random numbers but ensuring that 100% of the output material from a given destination is transported using the feasible transportation systems (mass conservation). Perturbations are accepted or rejected based on the decision rule described previously.

*The heuristic approach*: The different activities of a mining complex are strongly interrelated. Any modification in a particular activity modifies the optimal operation at the other activities of the mining complex; e.g., modifying the mining sequence affects the optimal operating parameters at a given destination and the transportation system implemented. The same occurs when modifying operating parameters or transportation arrangements. Given the interrelation between the different activities, the algorithm integrates the multilevel perturbations in an iterative way (Figure 4–6).

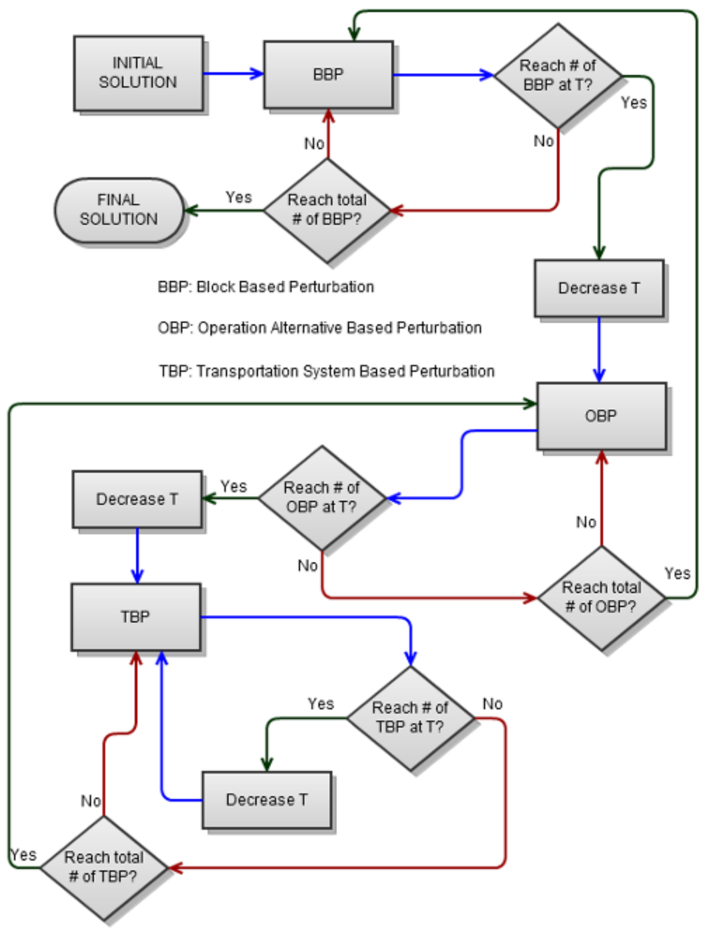


Figure 4–6: The heuristic approach

At any given temperature, a user-defined number of BBP is performed, when it reaches this predefined number, the temperature in this level of perturbation is lowered and the OBP starts. Similarly, when it reaches a user-defined number of OBP, the temperature in this level of perturbation is lowered and the TBP starts. When the three levels of perturbations are performed, the algorithm returns to the first level (BBP). It continues until the total number of BBP is reached.

The heuristic approach can be implemented iteratively by controlling the number of cycles. However, it must be important to establish a trade-off between the quality of the solution and computational time, given that it increases linearly with the number of cycles. Furthermore, there may be a number of cycles from where no significant improvement in the objective value of the solution is obtained.

## 4.3 Implementation of the method: A multipit operation

The method is implemented in a mining complex that produces copper and contains two different pits: Pit A and Pit B.

### 4.3.1 Overview of the operation

The material extracted from both pits has been classified in 5 different types that originate different metal recoveries at the different destinations. 5 destinations are available (Figure 4–7), including a waste dump.

Twenty orebody simulations of each deposit are used, which consider both uncertainty in grades and material types. Three variables of interest are considered in the orebody simulations: copper, which is the selling product of the mining complex; and two metallurgical properties that control the operation in the small and the big mill.

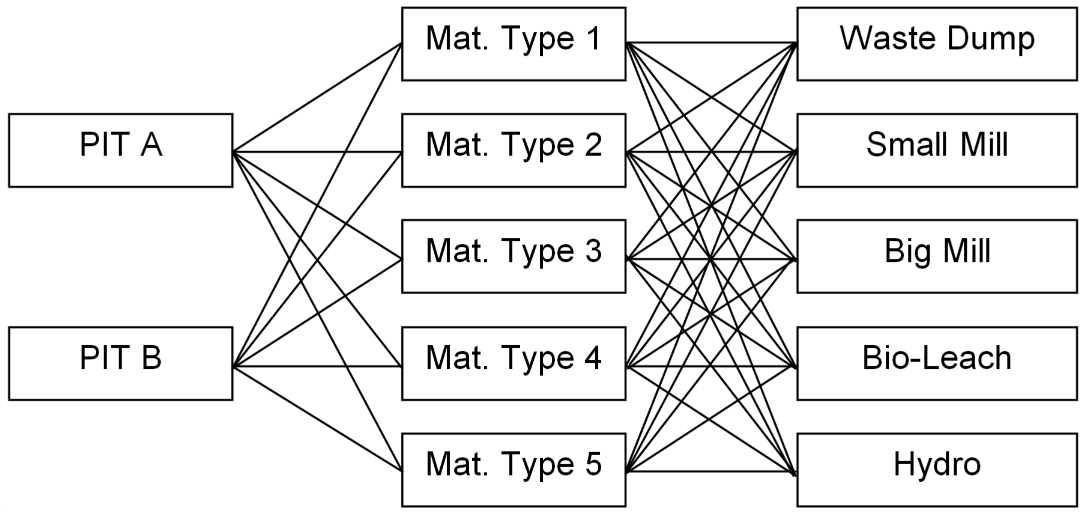


Figure 4–7: Multipit operation

### 4.3.2 Base case

An initial solution for the multipit multiprocess problem is generated using Whittle software. This initial solution contains the periods and destinations of mining blocks for both pits, and is generated considering the estimated geological models (E-types) of the two deposits; that is, the average grade of each block from the available simulations. This solution will be referred to as base case schedule and it is generated using a conventional optimizer widely used in the mining industry.

The results obtained by implementing the base case schedule are depicted in Figure 4–8. Large and impractical deviations in terms of capacities and blending targets are presented when implementing the base schedule over the different scenarios. After the pre-stripping years, deviations in the small mill are 18% in average and 22% in the big mill. Regarding the blending element 1 (BEL1) that controls the operation of the small mill, the deviations in the first 7 years of operation are in average 7%, whereas the blending element 2 (BEL2), that controls the operation of the big mill, deviates in average 1.8% in these periods.

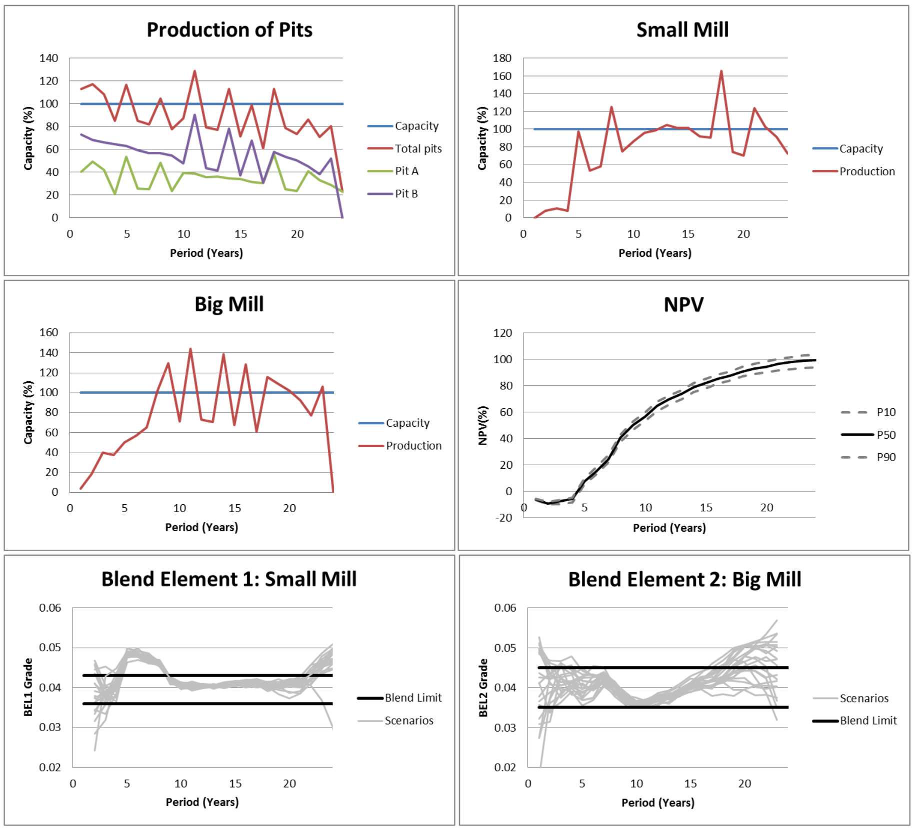


Figure 4–8: Base case schedule

Any conventional scheduler attempts to optimize the sequence of extraction of a given deposit using a single input model. Interpolation methods generate averaged-type models that smooth the grades and do not reproduce the spatial variability of the data. Furthermore, any mining sequence that performs well using an E-type model does not perform well in average with respect to a set of orebody simulations given that the transfer function that relate grades and economic values of blocks is non-linear. These two factors explain the poor performance of deterministic methods when performing risk analysis [32].

### 4.3.3 Optimization parameters

To implement the approach, optimization parameters such as initial temperature, reduction factors, penalties, cycles and number perturbation must be calibrated. The penalties must account for the order of magnitude of the different targets in order to balance the penalization in the objective function. Figure 4–9 displays the evolution of the terms of the objective function with the number of perturbations for 5 different temperatures. An initial temperature of 0 means that only perturbations that improve the objective value are accepted (pure iterative improvement) which limits the ability of the methodology of escaping from local optimal solutions. A very large initial temperature implies accepting both, favourable and unfavourable perturbations with high probability, which may not improve the initial solution as the solution will not necessarily converge to a final good solution.

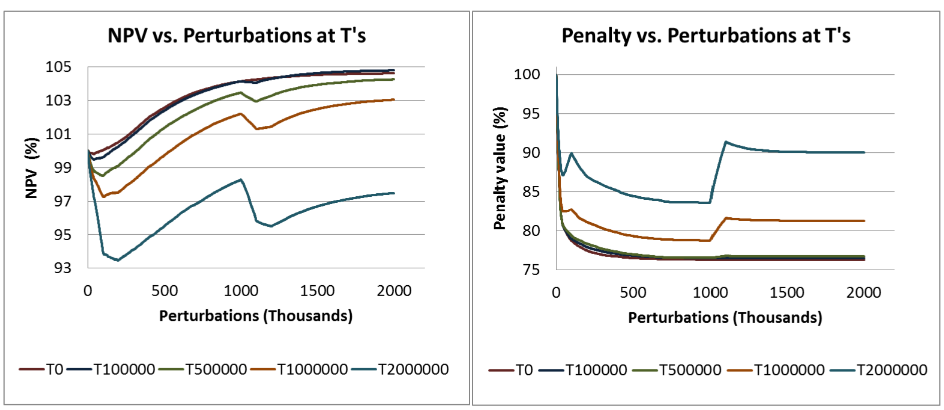


Figure 4–9: Objective function terms at different temperatures

The same analysis is performed to evaluate the number of cycles (Figure 4–10). It can be observed that after two cycles, the improvement in expected NPV is negligible, whereas no significant reduction in penalized deviations is attained after one cycle.

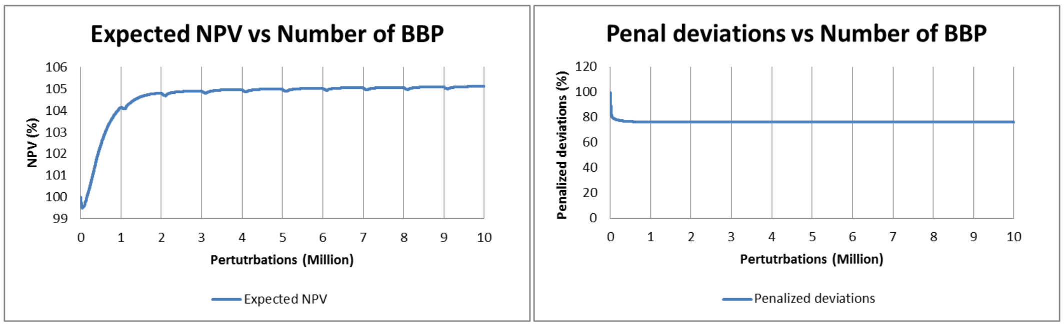


Figure 4–10: Objective function terms vs. number of cycles

### 4.3.4 Case 1: Multipit multiprocess

The proposed approach is implemented considering the economic and technical parameters used in the base case. The results obtained are displayed in Figure 4–11.

Low deviations from capacities and blending targets are expected. After the pre-stripping years, deviations from the capacity of both the small and big mills are 1% on average. The probability of deviating from the operational ranges of BEL1 are largely reduced, obtaining an average expected deviation of 0.4%. Larger probabilities of deviating are presented at the end, originated from the geological risk discounting applied to the penalties that allow deferring risk to later periods when more information becomes available. For BEL2, there are expected deviations of 1.3%. The expected NPV is 3% higher when compared to the base case; however,

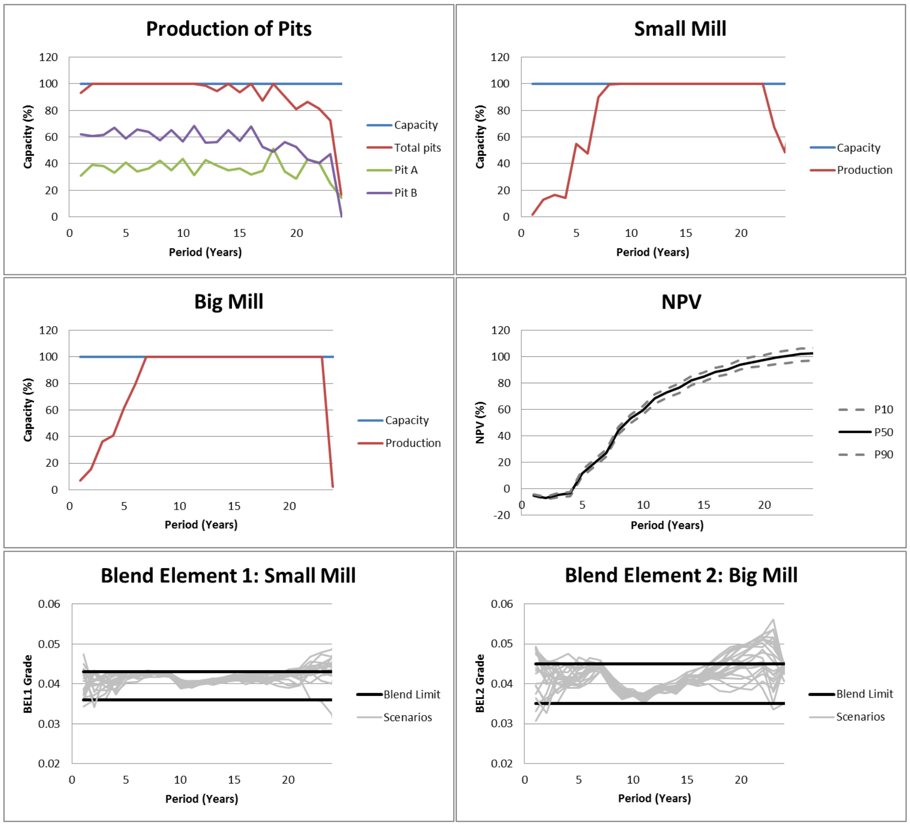


Figure 4–11: Case 1: Multipit multiprocess

given the large and impractical deviation from targets in the base case, its NPV forecast is not reliable.

### 4.3.5 Case 2: Multipit multiprocess with operating alternatives at the mills

The method is implemented considering the case where multiple operating alternatives are available in both mills. In case 1, fine grinding option is selected by default. The method is now able to choose which option to implement at each mill along the different periods. For both mills, when operating using a coarse grinding option, the capacities increase by 11.6% and the metallurgical recoveries decrease by

2%. The operational ranges for BEL1 and BEL2 also change with the two different operating alternatives. Given the flexibility in the operation of the mills, the method is able to perform second level perturbations (OAP). Figure 4–12 display the results obtained when implementing the method.

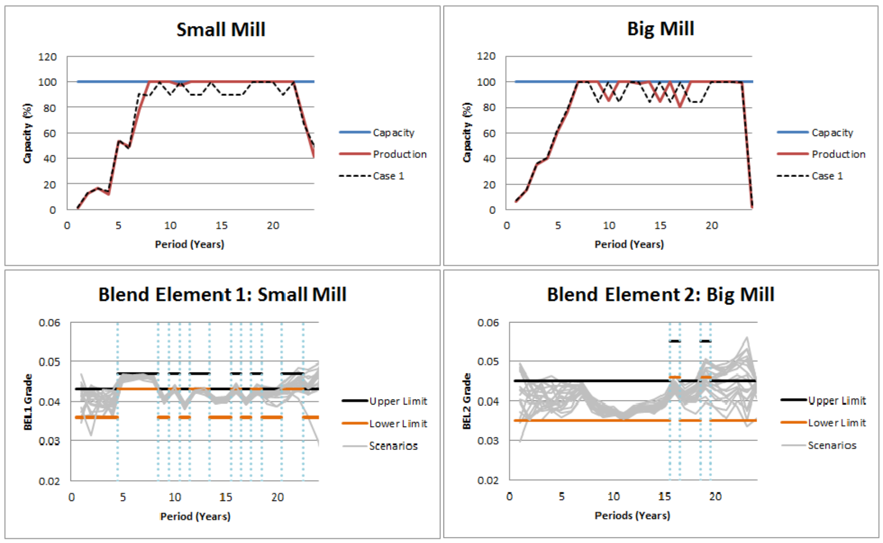


Figure 4–12: Case 1: Multipit multiprocess with operating alternatives at the mills

The coarse grinding option is selected in years 5-8, 10, 12, 13, 16, 18, 21 and 22 in the small mill, and in years 16, 19 in the big mill. Although capacities of the mill change when swapping from one alternative to another, the algorithm pushes material in a way that deviation from capacities of both mills are well controlled (in average 1% in small mill and 3% in the big mill). The same behaviour is observed with respect to the blend element targets; BEL1 jumps in periods when the small mill operates at coarse grinding to meet the new blending requirements, whereas BEL2 jumps in periods when the big mill swaps to coarse grinding. In average, BEL1 and BEL2 deviate 0.7 and 1.2% respectively.

The risk analysis of the NPV expected by implementing the solution generated in the case 2 is displayed in Figure 4–13. This solution generates an expected NPV 5% higher than the base case. As was previously mentioned, the NPV forecasts of the base case solution are not reliable given the large deviations from capacities and blending targets. The base case solution is improved by means of the two levels of perturbation implemented (BBP and OBP).

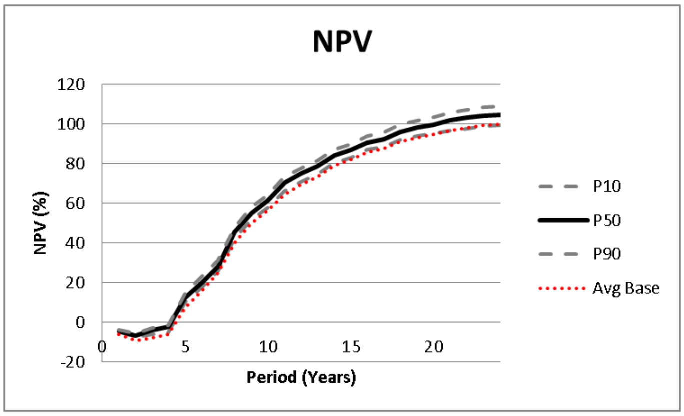


Figure 4–13: NPV of the case 2 solution

### 4.4 Conclusions

This paper proposes a method to generate solutions that outperform solutions obtained using conventional deterministic approaches in mining complexes with multiple pits, blending requirements, processing paths, operating alternatives and transportation systems. The solutions generated define the sequence of extraction of the mining blocks in the different pits, the operating alternative implemented at each processing path, and the transportation systems used to carry the processed material to the final stocks.

The implementation of the method in a multipit copper operation allows reducing the average deviations from capacities and blending targets considering an initial solution generated using a conventional optimizer over a single estimated model: from 18 to 1% regarding small mill capacities, from 22 to 3% regarding big mill capacities, from 7 to 0.3% regarding BEL1 in the first 7 periods, and from 1.8 to 0.6% regarding BEL2 in the first 7 periods.

Although NPV forecasts for the base case are not reliable given its large deviations from capacities and blending targets, the solution generated by implementing the proposed method generates an expected NPV 5% higher than the base case, which highlights the ability of the method to generate a higher NPV with less risk.

# CHAPTER 5
# Globally optimizing open-pit and underground mining operations under geological uncertainty, Twin Creeks Mining Complex, Nevada, USA

## 5.1 Introduction

A mining complex is a value chain with multiple components: deposits, stockpiles, processing destinations and transportation systems. Optimizing a mining complex demands the simultaneous optimization of all its components, a problem known in the mining literature as global optimization of mining [110, 113]. Several methods have been developed to incorporate multiple components of the value chain during the optimization. Hoerger et al.[50] formulate the problem of optimizing the simultaneous mining of multiple pits and the delivery of ore to multiple plants as a mixed integer program. The model groups blocks into increments and accounts for multiple stockpiles. The authors implement the model at Newmonts Nevada operations where 50 sources, 60 destinations and 8 stockpiles are present, and leads to an increase of profitability by taking advantage of the synergies. Stone et al. [96] present the Blasor optimization tool developed by the mine planning optimization group within BHP Billiton technology. Blasor formulates the problem of determining the optimal extraction sequence of multiple pits as a mixed integer linear program and solves it using ILOG CPLEX [53]. Blasor aggregates blocks spatially connected that have similar properties and generates nearly-optimal solutions in practical times in large-scale operations: Yandi (1000 aggregates, 11 pits, 20 periods) and Illawarra

Coal mine (8 domains) [91]. Zuckerberg et Al. [119] present Blasor-InPitDumping (Blasor IPD) that is an extension of Blasor that accounts for waste handling; that is, it incorporates refilling mined-out areas with waste. Zuckerberg et Al. [120] introduce Bodor to optimize the sequence of extraction of bauxite `pods´ at Boddington bauxite mine, south-western Australia. Pods are distinct bodies of modest-sized ore that are lying close to the surface. Chanda [16] formulates the delivery of material from different deposits to a metallurgical complex as a network linear programming optimization problem. The model attempts to minimize the costs through the network that encompasses mines, concentrators, smelters, refineries and market regions. Wooller [114] describes COMET, software used to optimize mill throughput/recovery and cut-off grade. COMET uses an iterative algorithm to define operating policies and process routes; e.g., heap leach versus concentration. Whittle [110] introduces the global asset optimization tool incorporated in Whittle software. The tool is designed to optimize the sequence of extraction of multiple deposits considering complex processing and blending operations.

Although efficient and able to incorporate several components of the value chain, the methods previously described have at least one of the following limitations when globally optimizing mining complexes: some decisions are fixed when they should be dynamic (operating modes, destinations of mining blocks, etc.); component-based objectives are imposed, which may not coincide with global objectives; many parameters are assumed to be known when they are uncertain [110].

All the methods previously described ignore the uncertainty associated with different parameters. Groeneveld et al. [46] incorporate uncertainty in market price, costs, utilization of equipment, plant recovery and time for building options (infrastructure) while simultaneously optimizing mining, stockpiling, processing and port policies. Bodon [12] models the problem of supplying exports in a coal chain as a discrete event simulation model (DES). The model is able to asses various operating practices, including maintenance options and capital expenditure to determine the best infrastructure for a given system. In both methods, geological uncertainty is discarded, which is the major contributor of not meeting production targets and NPV forecasts.

Several methods have been used to incorporate geological uncertainty for the open pit production scheduling problem [26, 37, 65, 66]; however, few work has been done regarding the production scheduling of underground mines. Grieco and Dimitrakopoulos [45] implement probabilistic programming in stope design optimization. The authors evaluate the probabilities of the different rings of being above specified cut-offs. However, the probabilistic programming formulation discards the compound relationship between rings as opposite of stochastic formulation that make full use of joint local uncertainty.

This paper describes a method for simultaneously optimizing different components of mining complexes comprised of open pits and underground operations under geological uncertainty. The method is described in the next section, later its implementation at Twin Creeks gold complex is displayed and finally conclusions are presented.

## 5.2 Optimizing the components of the value chain

### 5.2.1 Generalities

The components of a mining complex are strongly interrelated (Figure 5–1). Any decision taken in a particular component affects the decisions taken at the others. To optimize a mining complex the different components must be optimized simultaneously.

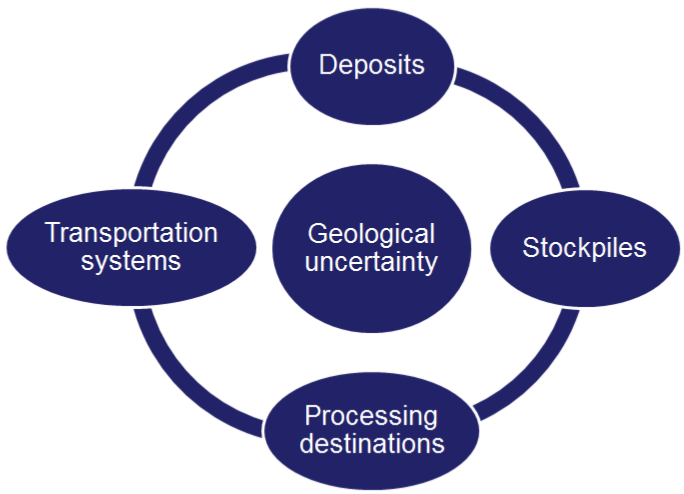


Figure 5–1: Components of a mining complex

Mineral deposits are the sources of material. The different ore types are extracted via open pits or underground mining methods. Open pits are discretized into mining blocks whereas underground mines are comprised of development, preparation and production activities. Different underground mining methods have different activities; however, regardless of the mining method, the mine design can be discretized in activities and dependencies; that is, each activity has its set of successor and predecessor activities, similar to slope constraints in an open pit.

Mining blocks in open pits and activities in an underground mine are named units in this paper. Each unit can be sent to a particular processing destination or to stockpiles. There may be as many stockpiles as metallurgical ore types available from the deposits. Stockpiles contain potential ore, contribute to the blending operation and serve as a backup supply of material.

Each processing destination may have operating modes that determine the operating costs, metallurgical recoveries, operational blending limits for the metallurgical properties and throughputs. For example, the capacity, operating cost and recovery of a milling plant change if it operates to generate fine material ($80\mu m$) or coarse material ($120\mu m$). The decision of which operating mode to select at a given processing destination must be taken by accounting for the decisions taken at the other components of the value chain. In some cases, the quality of the material extracted from the different deposits does not meet the specific blending requirements at a given processing destinations. To meet the quality targets, external blending materials are added to specific destinations (Figure 5–2). These materials come from external sources and have very specific quality. For example, in an autoclave, external material with high sulfide and low carbonate may be added if the ore extracted from the deposits have low sulfide, in order to meet the $SS/CO^3$ ratio.

The output material from the different processing destinations is transported to final stocks or ports using available transportation systems (Figure 5–3). It is important to account for transportation systems when optimizing a mining complex given that they can limit the overall throughput of the system (bottleneck). Each transportation system has its associated cost and capacity.

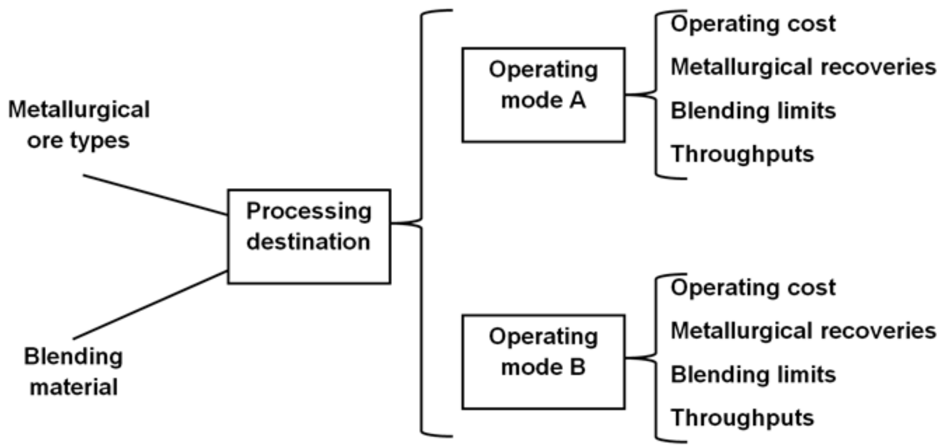


Figure 5–2: Processing destination

### 5.2.2 Mathematical model

The goal is to maximize expected NPV while minimizing deviations from targets associated with the different components of the value chain. The objective function (Eq. 5.1) has two terms: $discprofit(s,t)$ is the discount profit in period $t$ under scenario $s$; $penalty(s,t)$ is a term that accounts for the penalized deviations from the targets at the different components of the value chain in period $t$ under scenario $s$. Each scenario is a combination of orebody simulations of the different deposits.

$$maximize O = \sum_{t=1}^{T} \sum_{s=1}^{S} (discprofit(s,t) - penalty(s,t)) \tag{5.1}$$

The discounted profits at each period and scenario are calculated by accounting for the revenue obtained by selling the different products, the cost of mining the material at the different deposits, the cost of processing the material at the different destinations, the cost of stockpiling the material, the cost of sending material from the stockpiles to the available processing destinations and the transportation costs.

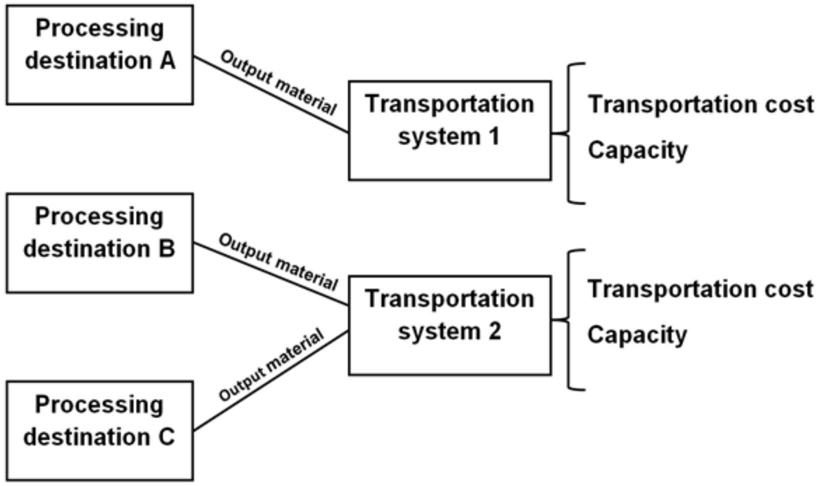


Figure 5–3: Transportation systems

$$discprofit(s,t) = \frac{\begin{pmatrix} revenue(s,t) - minecost(s,t) - procost(s,t) \\ -stockcost(s,t) - rehandlecost(s,t) - transcost(s,t) \end{pmatrix}}{(1+drate)^t} \tag{5.2}$$

The second term of the objective function that accounts for the penalized deviations is evaluated using Eq. 5.3.

$$penalty(s,t) = \begin{array}{l} penalmine(s,t) + penaltrans(s,t) \\ + penalpro(s,t) + penalmetal(s,t) \end{array} \tag{5.3}$$

where $penalmine(s,t)$ are the penalized deviations from the capacities of the different mines, $penaltrans(s,t)$ are the penalized deviations from the capacities of the different transportation systems, $penalpro(s,t)$ are the penalized deviations from the capacities at the different processing destinations and $penalmetal(s,t)$ are the penalized deviations from the operational ranges of the metallurgical properties.

Three main decision variables are used to evaluate revenues, costs, production and deviations at the different components of the value chain. $X_{i,t,d}$ is a binary variable that represent whether or not a particular unit $i$ is mined in period $t$ and sent to processing destination $d$. $Y_{t,d,o}$ is a binary variable that represent whether or not an operating mode $o$ is used in destination $d$ during period $t$. $Z_{t,d,r}$ represents the proportion of output material from destination $d$ transported using transportation system $r$ during period $t$.

For example, the amount of material extracted from the different deposits can be evaluated using Eq. 5.4, the output material from a given destination can be evaluated using Eq. 5.5 and the amount of material transported using a particular transportation system using Eq. 5.6.

$$mineproduction(s,t) = \sum_{i=1}^{I}\sum_{d=0}^{D} X_{itd} \cdot m_{is} \tag{5.4}$$

$$tonneoutprocess(s,t,d) = \sum_{o=1}^{O(d)} \left(tonneprocess(s,t,d) \cdot Y_{tdo} \cdot P_{do}\right) \tag{5.5}$$

$$tonnetransport(s,t,r) = \sum_{d=1}^{D} \left(tonneoutprocess(s,t,d) \cdot Z_{tdr}\right) \tag{5.6}$$

where $I$ is the set of all units from the deposits, $D$ is the set of processing destinations, $m_{i,s}$ is the tonnage of unit $i$ under scenario $s$, $O(d)$ is the set of operating modes available at destination $d$, $tonneprocess(s,t,d)$ is the tonnage sent to destination

$d$ from the deposits and the stockpiles in period $t$ under scenario $s$ and $P_{do}$ is the proportion output/input tonnage in destination $d$ using operating mode $o$.

A mining complex may contain millions or hundreds of thousands of units which implies a large number of integer variables in the optimization model. Because of this, finding an optimal solution using a standard optimizer is impractical.

### 5.2.3 Solution

Any solution of the optimization model must answer the questions associated with the three main decision variables: (*i*) Which units are going to be extracted in each period and where are they going to be sent? ($X_{itd}$ variables); (*ii*) Which operating modes are going to be used at the different processing destinations? ($Y_{tdo}$ variables); (*iii*) Which transportation systems are going to be used? ($Z_{tdr}$ variables).

Given a particular solution, it is possible to modify the objective value by generating perturbations at the three different decision levels. These perturbations should be done towards improvements in the objective value. Given the time value of money, profitable units should be pushed to be extracted in early periods and non-profitable ones should be pushed to later periods. Operating and transportation decisions should minimize processing and transportation costs and deviations.

#### The perturbation mechanism

For each unit $u$, it is possible to calculate the cumulative profit of $u$ in every destination by accumulating the economic value in each scenario (Figure 5–4). The cumulative profit provides a guidance of the most profitable destinations for a particular unit and controls the iterating process when swapping periods and destinations of a mining unit (Figure 5–5). If the greatest cumulative profit of a unit is positive, the iterating process for the candidate periods of extraction of the mining unit will favor extracting that unit in an earlier period; otherwise, the iterating process will favor extracting the unit in a later period. The set of candidate destinations is given by the destinations with positive cumulative profit if the unit is profitable, or the less unprofitable destination in the opposite case.

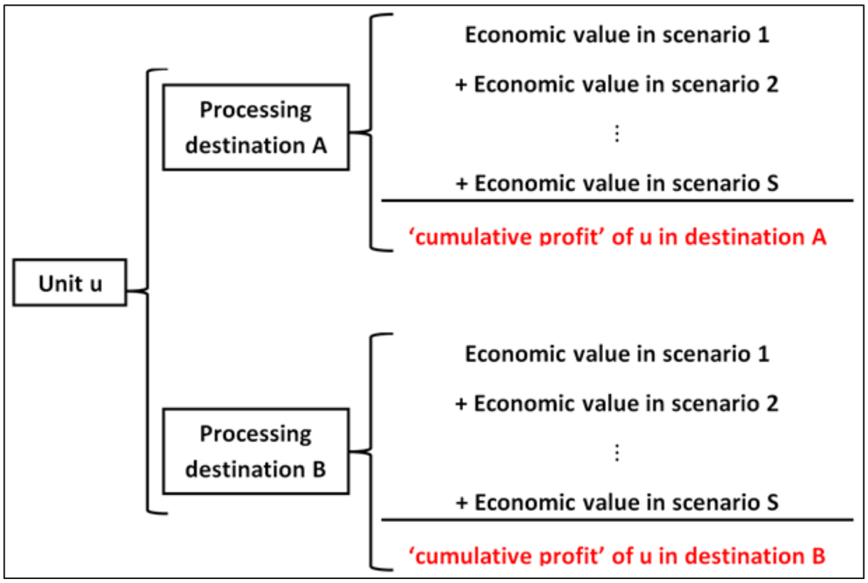


Figure 5–4: Cumulative profit of a unit

The iteration process over the candidate periods and destinations of a mining unit is designed to increase the expected NPV given the time value of money. However, the objective value of the perturbed solution is also affected by the penalized deviations, therefore, there might be cases when pushing a profitable unit to a later period or an unprofitable one to an early period increases the objective value given the lower deviations in the new solution. In these cases, the objective function can

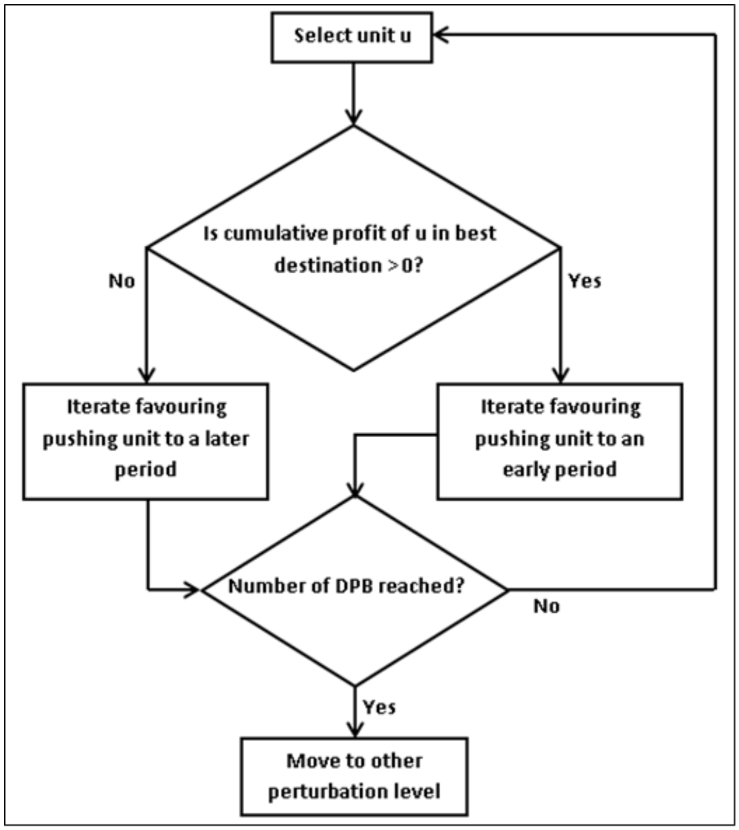


Figure 5–5: Perturbation of units

be seen as trade-off between maximizing the expected NPV and minimizing the penalized deviations.

The perturbations at an operating decision level consist in swapping operating modes at different processing destinations towards improvements in the objective value. The perturbations at the transportation decision level consist in modifying the transportation proportions of the output material from the different processing destinations; for example, changing the transportation of the output material from a mill from 50% trucks / 50% pipeline to 70% trucks / 30% pipeline. The transportation perturbation mechanism seeks for minimizing transportation costs and deviations.

At any level, perturbations are accepted or rejected using the Eq. 5.7 from Metropolis algorithm [64, 76].

$$P(\Delta O) = P(O_{new} - O_{current}) = \left\{ \begin{array}{rl} 1 & if(\Delta O \geq 0) \\ e^{\frac{-(\Delta O)}{T}} & if(\Delta O < 0) \end{array} \right\} \tag{5.7}$$

where $T$ is the annealing temperature. The probability of accepting an unfavourable perturbation is greater at higher temperatures. As the optimization proceeds, the temperature is gradually lowered by a reduction factor. When the temperature approaches zero, the probability of accepting an unfavourable swap tends to zero. This allows the algorithm to converge to a final solution.

**The method**

The method proposed to optimize a mining complex has three stages (Figure 5–6). The first stage consists in assigning periods and destinations to the mining units from an initial solution. In the second stage the method evaluates the profits, productions and deviations at the different scenarios. It also evaluates the cumulative profit of the mining units at the different destinations. The last stage is the perturbation mechanism at the three different decision levels. The algorithm stops when it reaches a user-specified number of iterations or poor improvement is presented in the objective value after a certain number of perturbations.

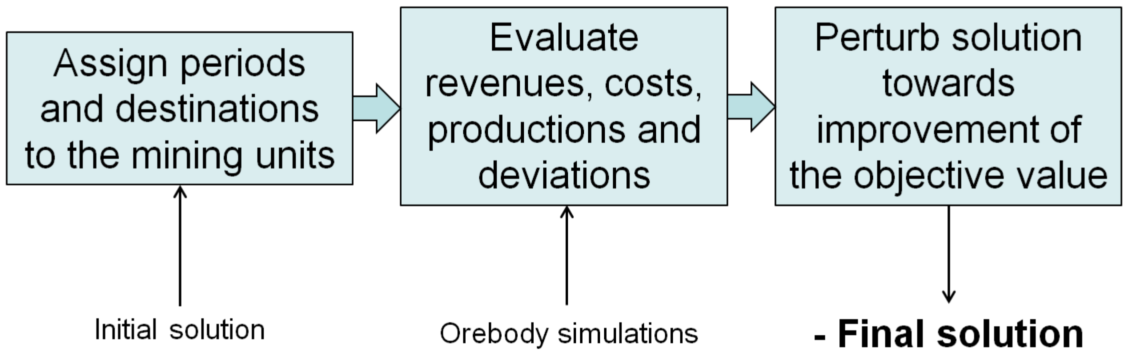


Figure 5–6: Method

## 5.3 Case study: Twin Creeks mining complex, Nevada

Twin Creeks is a gold mining complex located in northern Nevada, USA. Twin Creeks is one of the major ore producers in Newmont Nevada's operations [61]. The complex has two open pits named Mega Pit and Vista Pit and one underground mine named Turquoise Ridge that is a joint venture between Newmont Mining Corporation and Barrick Gold Corporation. Higher-grade oxide ore is processed at Juniper mill, lower-grade is treated on heap leach pads. Refractory ore is processed at the Sage autoclave. In this particular study, the focus is the interaction between the ore extracted from Mega pit and Turquoise Ridge (Figure 5–7).

The Mega pit provides oxide and refractory ore. Twenty orebody simulations for gold, sulfide sulfur, $CO_3$ and organic carbon are provided. The higher concentrations of gold are located in the north-east part of the deposit where the current mining phases are located (Figure 5–8). The gold and sulfide sulfur grades are controlled by the mineralized domains whereas the carbonate and organic carbon are spread in all the area of the deposit.

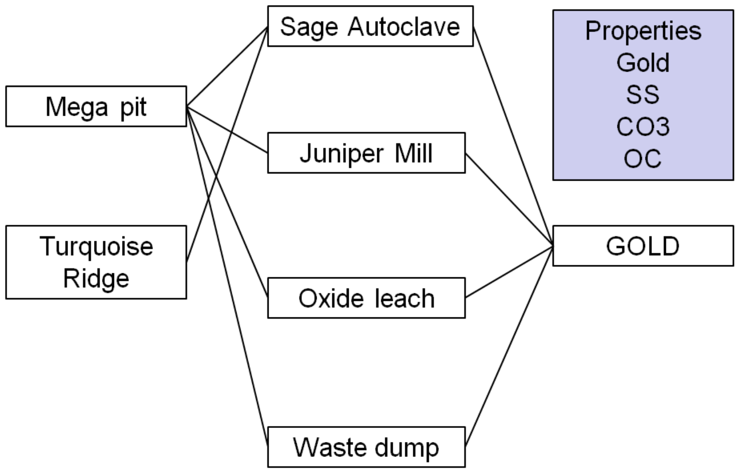


Figure 5–7: Twin Creeks gold complex

The Turquoise Ridge mine uses underhand-cut-and-fill due to the relatively low rock quality in the ore zones [54]. Intensity of gold mineralization is related to structural complexity and the location of rocks chemically receptive to mineralization. The production zones are located in the high grade areas (Figure 5–9).

Twenty orebody simulations are generated with direct block simulation using the drillhole data within the mineralized domain [37]. The Figure 5–10 shows the validation of the orebody simulations generated. It can be observed that the simulations respect the statistics of the drillhole data as they reproduce its histograms and variograms at the main directions of anisotropy. The simulated values at each unit are calculated by averaging the simulated points that fall inside; that is, given the different shapes and sizes of the underground mining units, there is no a single support size as in the open pits where the mining blocks have the same size. The Figure 5–11 shows three different orebody simulations and the production zones of the mine.

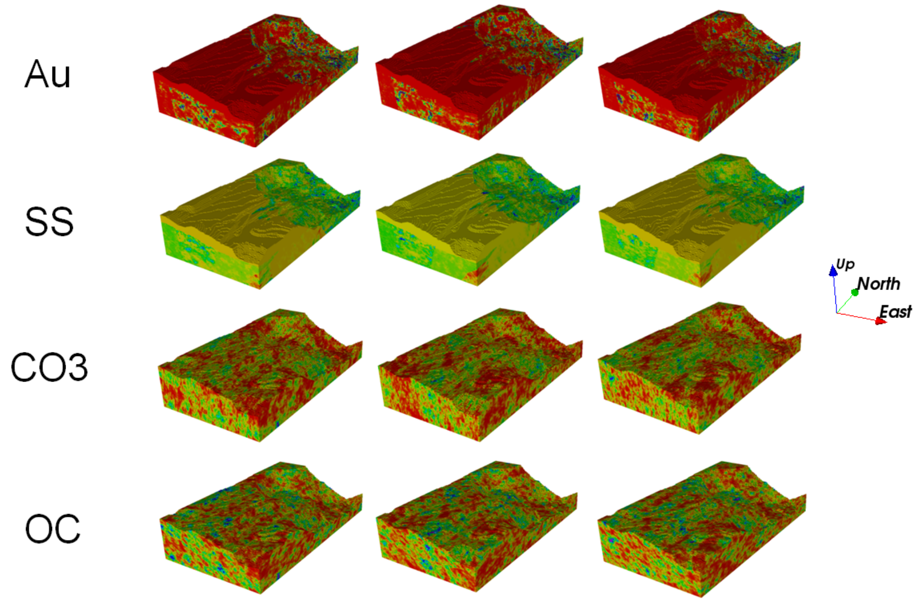


Figure 5–8: Three orebody simulations of Mega pit

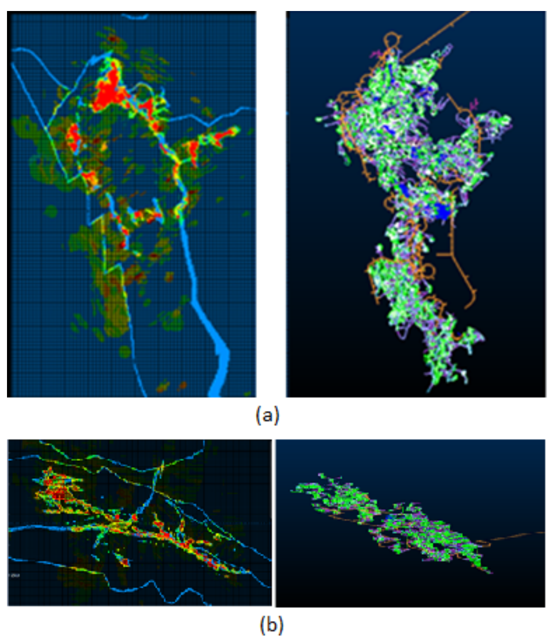


Figure 5–9: Gold grades (left) and production zones(right) in Turquoise Ridge: (a) Plan view; (b) Cross section

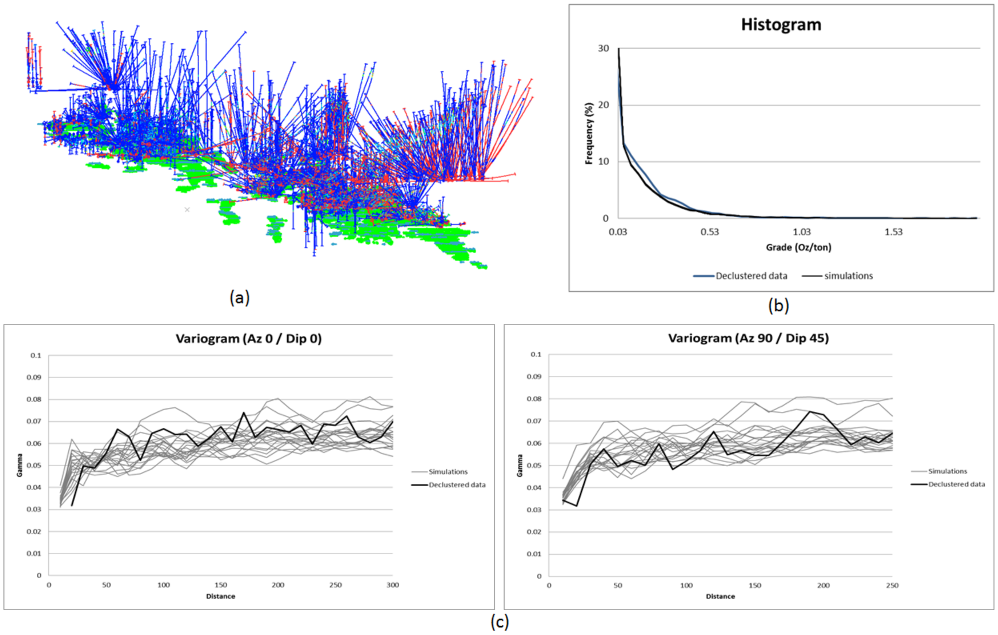


Figure 5–10: Validation of the simulations in Turquoise Ridge: (a) Drillhole data; (b) Histogram reproduction; (c) Variogram reproduction

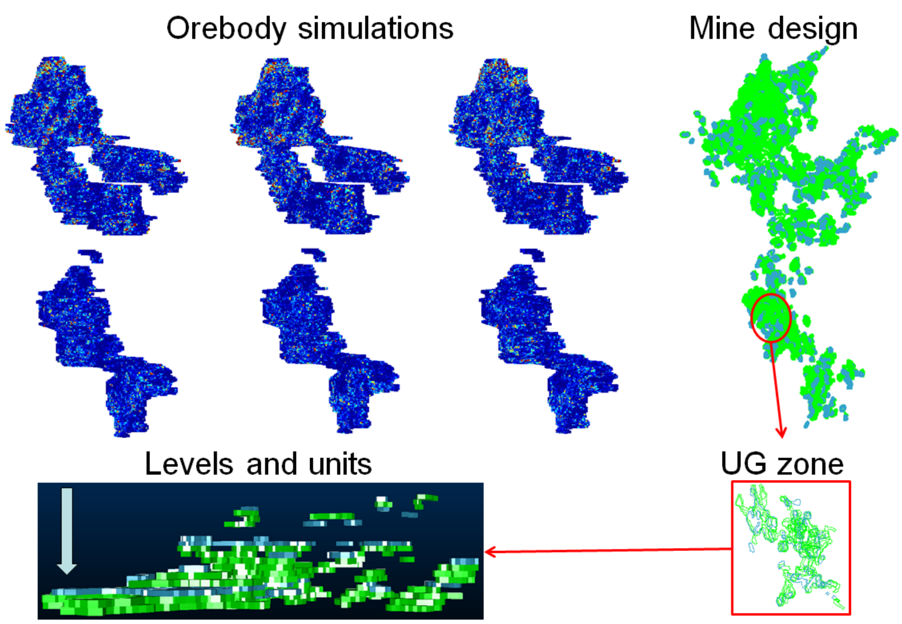


Figure 5–11: Orebody simulations and production zones of Turquoise Ridge

The Sage autoclave has tight operating ranges for $SS/CO_3$, $SS$, $CO_3$ and organic carbon [61]. To help metallurgical blending, concentrates from other plants are added to the process (Figure 5–12).

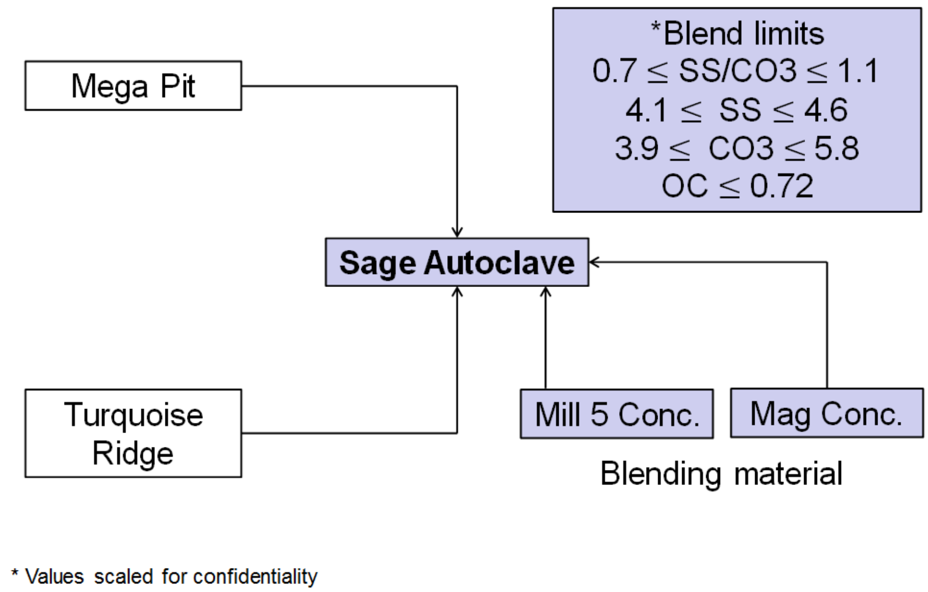


Figure 5–12: Sage autoclave

In the three processing destinations the metallurgical recovery of gold follows non-linear curves. In the sage autoclave the recovery curve is a function of the gold grades and the organic carbon whereas in the juniper mill and the heap leaching plant the recovery is a function of gold grades only.

### 5.3.1 Initial solution

An initial solution for the optimization of Twin Creeks gold complex was generated by: (*i*) Considering the current mining plan in Turquoise Ridge that was developed by the mine planners using Enhanced Production Scheduler (EPS) software; (*ii*) using Milawa scheduler in Whittle software for the Mega pit using the e-type of the orebody simulations; that is, the average grades of the mining blocks at the different realizations.

The amount of external blending material used in the Sage autoclave is considered when evaluating the results of the implementation of the initial solution over the different scenarios (combinations of orebody simulations of Mega pit and Turquoise Ridge). The productions of the two mines, the Sage autoclave and the Juniper mill are shown in Figure 5–13. It can be observed that the Turquoise Ridge mine operates below the target whereas Mega pit operates very close to its capacity until the depletion of the reserves. Although external blending material is added to the Sage autoclave, given the tight blending constraints imposed to this processing destination, the conventional scheduler only can find blended material to fill the capacity in three periods of the life of the mine (LOM). There is a big shortfall in production in the Sage autoclave in year 4, and after year 9 the tonnage sent to this processing destination is very marginal. Regarding the Juniper mill that processes oxide ore, the production is going to be close to the capacity in years 2-4 but deficient production is observed in the rest of the periods of the LOM. However, most of the oxide ore filled to this processing destination comes from Vista pit that is not considered in this study.

The low sulfide sulfur presented in the simulations generates that the initial solution is below the operational ranges for $SS$ in most of the years and for $SS/CO_3$ in the last years (Figure 5–14). The $CO_3$ increases with time and fall inside the operational ranges in most of the years. The organic carbon is well controlled in all the different scenarios.

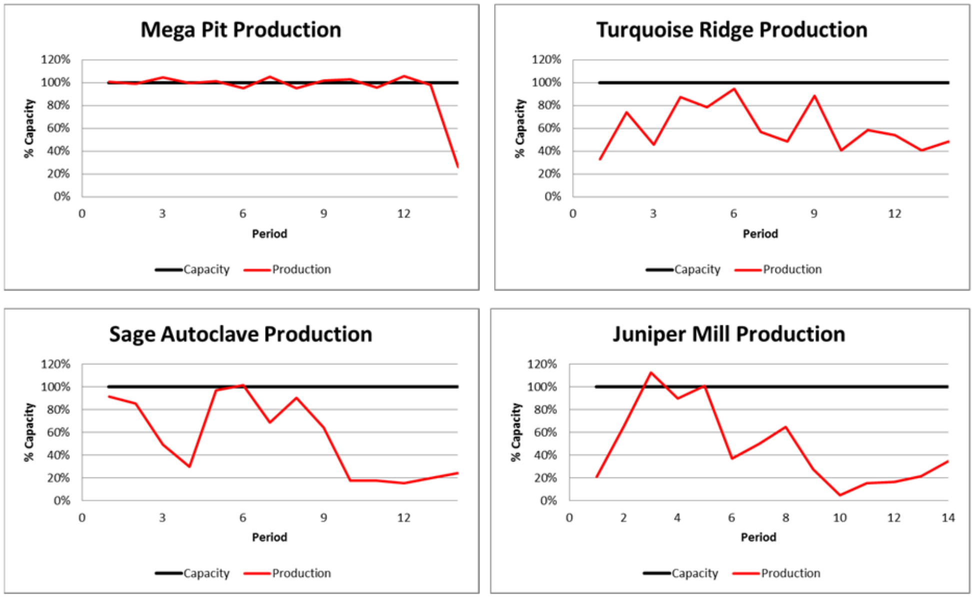


Figure 5–13: Productions with the initial solution

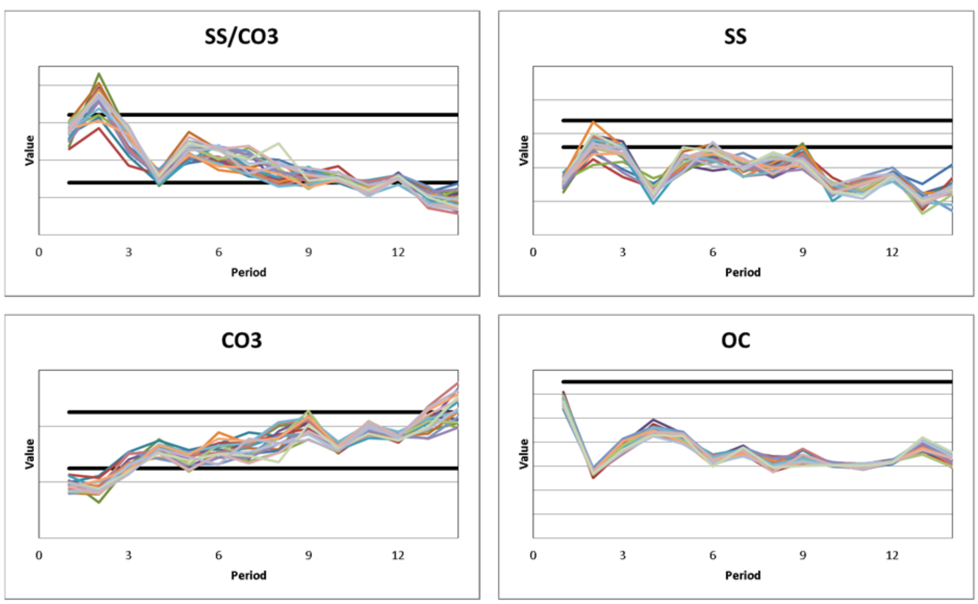


Figure 5–14: Metallurgical properties with the initial solution

The risk profile of the NPV is displayed in Figure 5–15. It is observed that after year 9 the cumulative NPV starts to decrease given the marginal tonnage sent to the Sage autoclave. It will be more profitable to stop the operation after year 9. For confidentiality, the values of the NPV are presented in percentages.

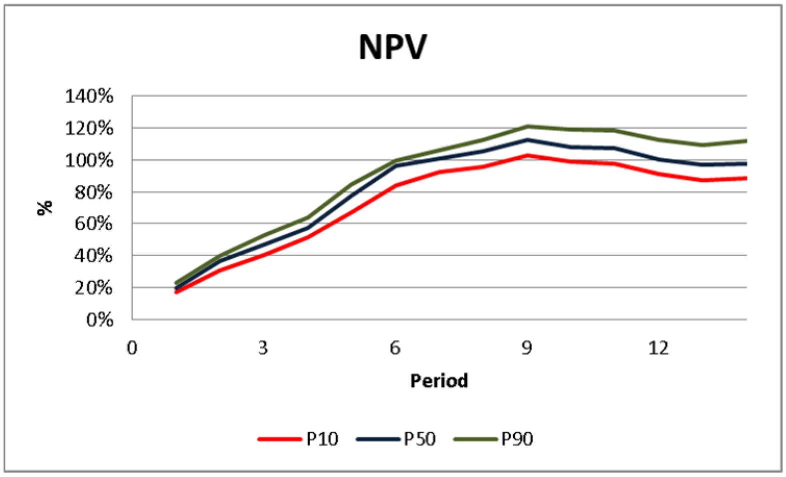


Figure 5–15: NPV with the initial solution

### 5.3.2 Optimization parameters

Different tests are performed to define the optimization parameters that lead to the largest improvement of the objective value. Different initial temperatures, reducing factors and number of perturbations were tested. The figure 5–16 shows the evolution of the objective value with the number of perturbations for six different initial temperatures. The largest improvement in the objective value is obtained when the initial temperature of the order of 1 million.

Other important parameters to define are the per-unit penalty values associated with the targets at the different components and the geological risk discounting. The magnitude of the penalties must be defined so as to balance the two terms of

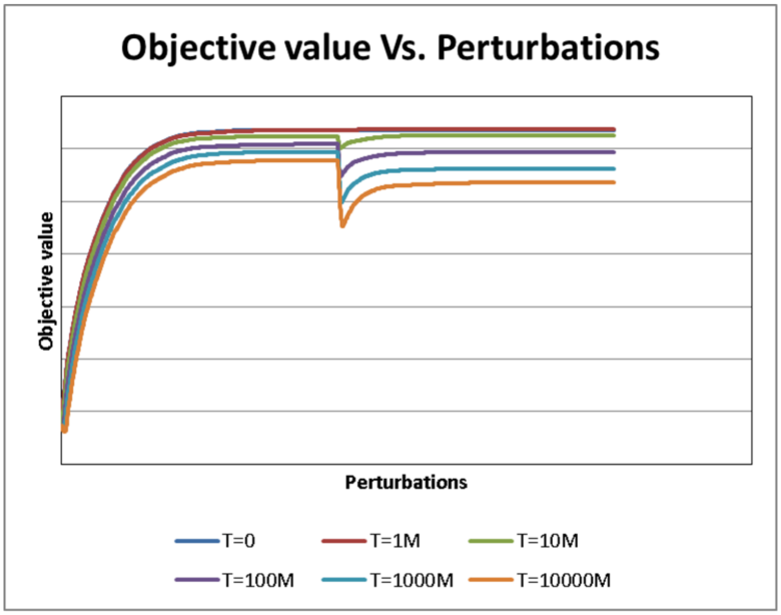


Figure 5–16: Evolution of objective value with different initial temperatures

the objective function. Too high penalty values will improve the reproduction of the targets ignoring the first term of the objective function generating poor improvement of expected NPV. Conversely, too small penalty values will generate impractical solutions with large and non-realistic NPV forecasts given the large violations of the targets.

### 5.3.3 Stochastic solution

The method is implemented after setting up the optimization parameters. It is possible to observe from Figure 5–17 that the solution operates the underground mine very close to its capacity except in year 12 where there is a big shortfall. However, the productions at the Sage autoclave and the Juniper mill are below their capacities in all the periods of the LOM. Regarding the blending properties, it is observed a similar behaviour when compared with the deterministic initial solution

5–18. Given the low sulfide sulfur presented in the simulations, the method is not able to accommodate the sulfide sulfur inside the operational ranges. The expected NPV is 14% greater than the one obtained with the deterministic initial solution 5–19.

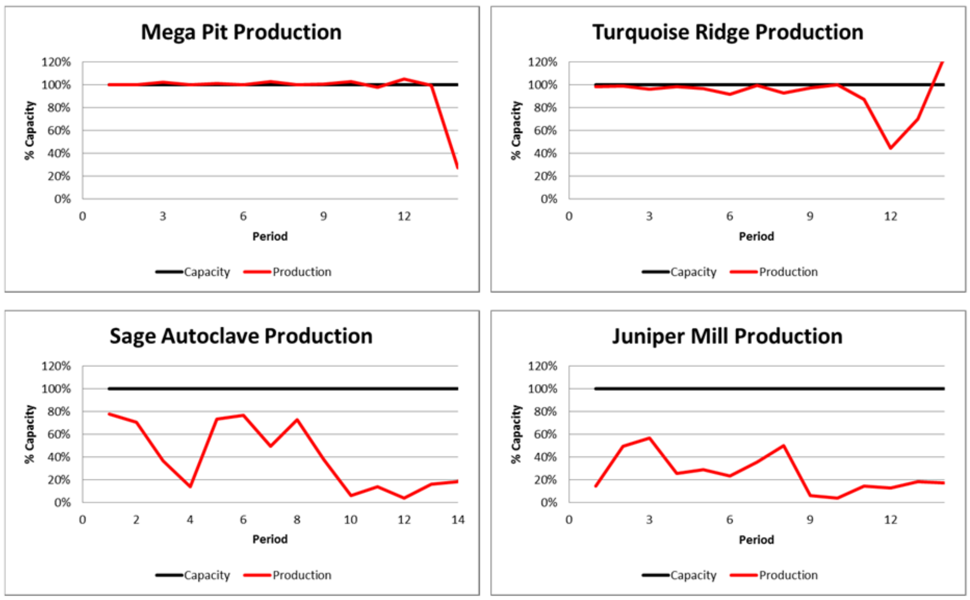


Figure 5–17: Productions with the stochastic solution

The method is implemented considering different amount of external blending material input to the Sage autoclave. In particular, the amount of Mill 5 concentrate fed to the Sage autoclave is increased five times given its high sulfide sulfur. The productions at the Turquoise Ridge mine and the Sage autoclave and the sulfide sulfur with the new stochastic solutions are displayed in Figure 5–20. It can be observed that by increasing the concentrate from mill 5 the method is able to find more material to blend to increase the production in the Sage autoclave. Furthermore, the sulfide sulfur approaches the operational ranges by increasing the external blending

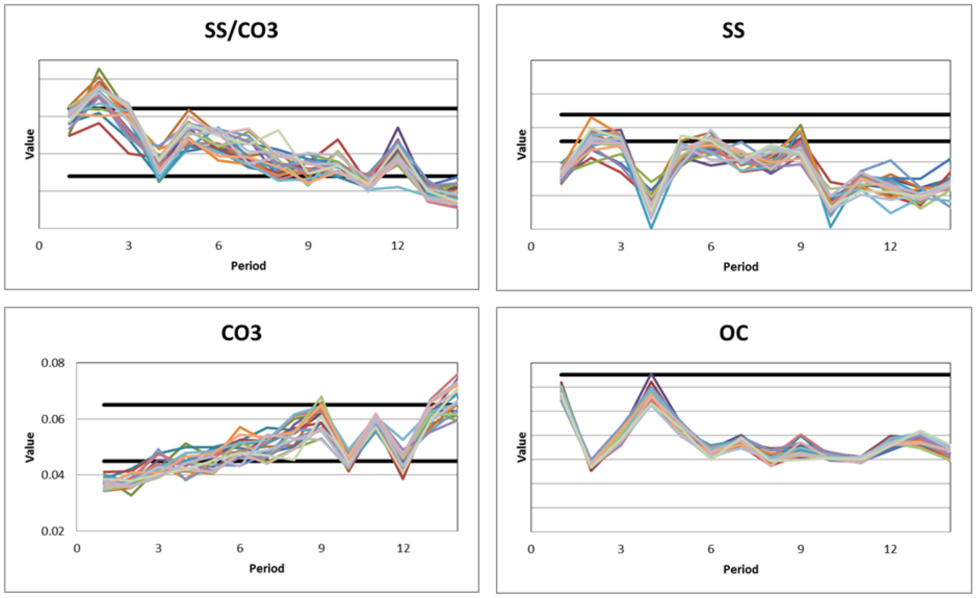


Figure 5–18: Metallurgical properties with the stochastic solution

material given its large sulfide sulfur content compared to the material extracted from Mega Pit and Turquoise Ridge. However, the content of sulfide sulfur is still below the operational ranges given the grade in the orebody simulations.

The expected NPVs of both stochastic solutions are very similar (Figure 5–21). However, the availability of mill 5 concentrate is a strong assumption. In the actual operation, the planning and production departments mitigate the negative effects of contaminants in the Sage autoclave by using acid. This is not considered in this case study. The shortfall in production in the Sage autoclave in the year 4 in both stochastic solutions suggests that the method got trapped in a local optima. To overcome this situation, a diversification strategy in the perturbations at the unit decision level is desired.

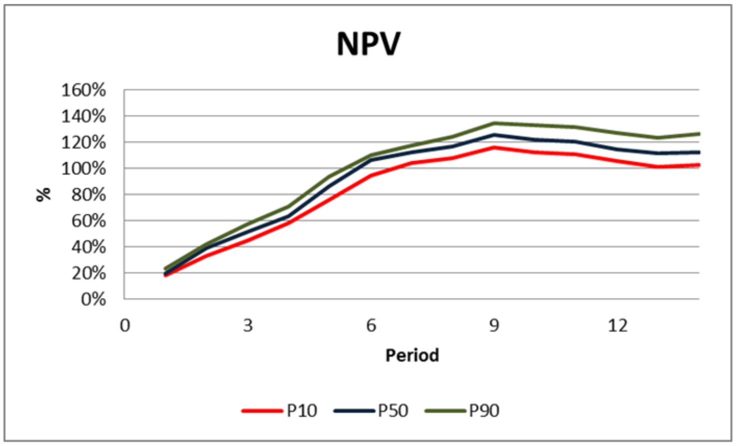


Figure 5–19: NPV with the stochastic solution

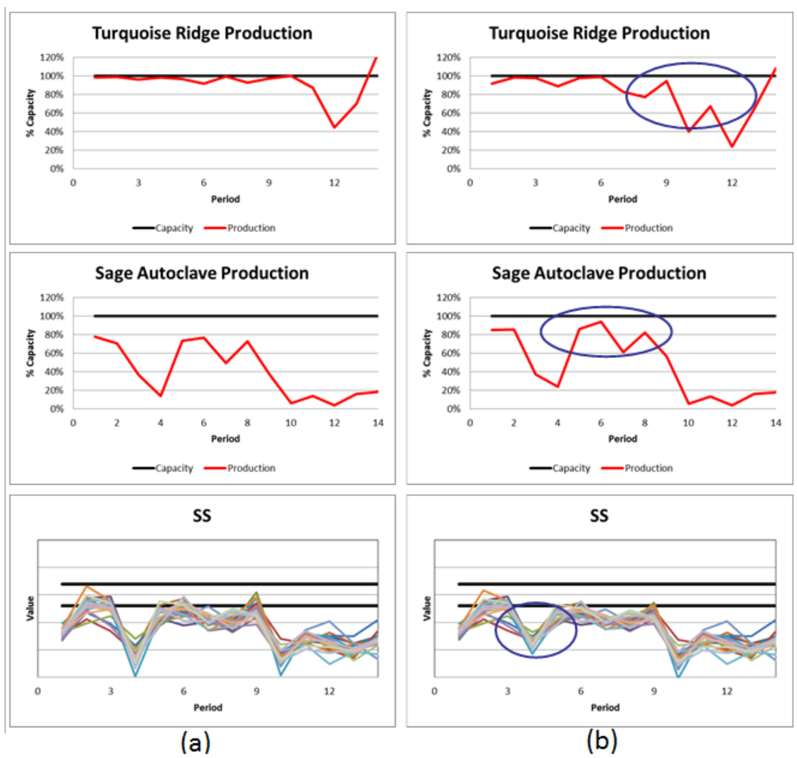


Figure 5–20: Productions with the new stochastic solution: (a) Mill 5 concentrate; (b) Mill 5 concentrate x 5

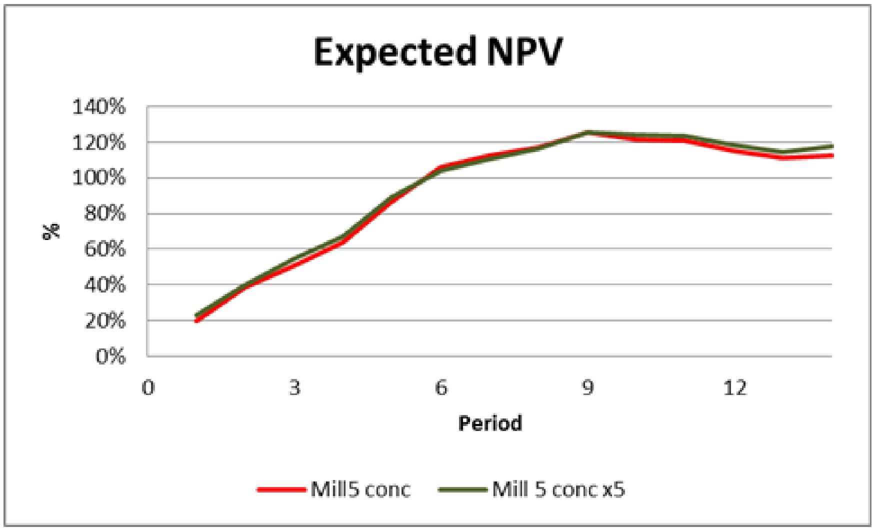


Figure 5–21: Expected NPVs of stochastic solutions

### 5.4 Conclusions

This paper presents a method to simultaneously optimize different components of mining complexes comprised of open pits and underground operations. The method is easily adapted to different underground mining methods.

At the different processing destinations, the method accounts for operating modes and external sources of material used for blending purposes. The implementation of the method at Twin Creeks shows substantial improvement in expected NPV (14% when compared with a deterministic initial solution); however, given the low sulfide sulfur in the orebody simulations, the stochastic solution is not able to meet the tight operational ranges for sulfide sulfur.

The perturbations at operating and transportation decision levels act as a diversification strategy for the unit-based perturbations. However, as in the case study no operating modes and transportation systems are considered, a stand-alone diversification strategy for the unit-based perturbations must be included to explore better the solution domain.

Future extensions of the method may consider geotechnical and environmental aspects of the underground activities and the optimal consumption rate of external blending material.

# CHAPTER 6
# General conclusions

Optimizing a mining complex demands the simultaneous optimization of all its components. This is a complex problem given the flexibility associated with the decisions taken at the different components of the value chain and the uncertainty associated with different parameters. Geological uncertainty is seen as the major contributor of not meeting project expectations in mining. Optimizing a mining complex while accounting for geological uncertainty is a large optimization problem given the amount of integer variables associated with the discretization of the mineral deposits.

This thesis presented several formulations for simulataneously optimizing different components of the value chain. To solve these large optimization problems, efficient algorithms are developed and implemented. These algorithms outperform conventional deterministic methods in generating higher expected NPV while minimizing deviations from the targets associated with the different components of the mining complex. This is originated from the fact that stochastic optimizers not only prioritize the mining of blocks with high economic values but also with high probabilities of being profitable, which lead to higher chances of meeting production targets and higher NPV; that is, lower risk and higher reward.

A multistage method that uses simulated annealing was presented in Chapter 2. The method shows its ability to minimize deviations from targets at different processing destinations. It also improves the expected NPV by swapping periods of mining blocks to most probable periods based on pre-optimized input mining sequences. The expected deviations of the stochastic solution are shown to be less than 5% for mill and waste targets, whereas a conventional solution generates average expected deviations of 20 and 12% from mill and waste targets respectively. It was observed that 15 orebody simulations were sufficient to generate a robust LOM production schedule. This is originated from the fact that mine planning works at a block scale thus it is affected by the support effect.

In Chapter 3 a heuristic approach that account for operating alternatives at the different processing destinations was presented. The method makes use of the structure of the problem and the time value of money by pushing profitable blocks to early periods and non-profitable blocks to later ones. The iteration process of the method controls the respect of the targets and the imposition of the discounted profits in the objective function leads to better improvements in expected NPV. The implementation of the method in a copper deposit reduces the deviations from capacity of processing destination 1 from 9 to 0.2% and increases the expected NPV by 30% regarding an initial solution generated with a conventional method.

A method that uses simulated annealing at different decision levels is described in Chapter 4. The method account for mining, blending, processing and transportation decision variables that incorporate multiple pits, material types, processing destinations, operating alternatives and transportation systems. Given the strong relationship between the different activities associated with the mining complex, the method performs perturbation at different decision levels in an iterative way. This heuristic implementation of simulated annealing allows exploring the solution domain; that is, the set of possible mining, blending, processing and transportation plans. The implementation of the method in a multipit copper operation allows controlling the deviations from capacities of the mills and the targets of the blending elements while increasing the expected NPV. The extension of the method to mining complexes comprised of open pits and underground operations is presented in Chapter 5. The method is able to incorporate underground activities while respecting their dependencies. The implementation at Twin Creeks mining complex, Nevada, USA shows its ability to increase the expected NPV while generating little improvement in meeting metallurgical blending targets. A diversification strategy in the unit-based perturbation mechanism must lead to larger improvements of the objective value.

Given the previous successful implementations of simulated annealing in stochastic LOM production scheduling, this metaheuristic was chosen to optimize the value chain. However, other metaheuristics must be tested seeking for a better exploration of the solution domain and a better integration with the nature of the problem. In particular, local search techniques with diversification strategies appear as a promising avenue of research.

Further improvements also include the incorporation of other sources of uncertainty (market, environmental, etc.), investment options at the different components, and operational constraints such as equipment mobility, drilling and blasting alternatives and location of infrastructure. Furthermore, the integration of underground operations can be extended to account for different mining methods, development activities and geotechnical constraints.

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