A CRITICAL STUDY OF THE TEACHING OF MATHEMATICS IN THE SCHOOLS OF MONTREAL AND THE SURROUNDING DISTRICT

A Critical Study of the Teaching of Mathematics in the Schools of Montreal and the Surrounding District.

A Thesis Submitted to the
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by

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C O N T E N T S.

CHAPTER I.

Introduction.

The title of this thesis was chosen within a few weeks of the student's arrival in Canada. As a result of lack of definite information concerning educational matters in the Province of Quebec, at that time he was unable to judge whether less vague delimitation of the topic was desirable. After observation of the teaching of mathematics ranging from the incidental number work of the Kindergarten to the mathematical studies of Grade XI in several elementary and high schools both in and near Montreal, concomitant with a rapid survey of some of the material that has been written during the last quarter of a century on the pedagogy of mathematical subjects at all stages and in practically every country, he decided to restrict the subject of the thesis more particularly to the study of the teaching of the higher mathematical branches, geometry, algebra and trigonometry for the most part, in the Protestant high schools of the district, though incidental reference will necessarily be made to the teaching of arithmetic in both elementary and high schools; to compare such instruction with the systems in use in Scotland and

England, where he has already made observation; and to cite some of the contributions that have been made by Britain and America, and, to a slighter extent, those of other European countries, of which he has acquaintance either directly or indirectly.

The teacher of mathematics to-day may well feel that his position is not one to be envied. But there is no doubt that he is not in as bad case now as he has already been during the present century. When the doctrine of formal discipline was discredited and it was first suggested that training was much more specific than had been formerly believed, many chose to think and to preach that the value of mathematics as a subject for study at school was lost entirely. In Britain, to be sure, the security of tenure of mathematics in the curriculum of the secondary school was not threatened to such extent as that of the classics. But the teacher of mathematics who gave heed to the question had certainly cause for alarm. The discussion of the value of the study of mathematics will show, however, that great benefit has accrued from the keen consideration of the whole question; that changes have resulted in a comparatively short time that are out of all proportion when viewed alongside the reforms of previous centuries.

The man in the street has a holy reverence for one who studies mathematics; for him the connotation of the term is more closely that of its Greek origin, 'mathema', than for either the expert or the dabbler. In consequence, he needs naught but his hazy notions of formal discipline to justify the inclusion of mathematics in the curriculum of the secondary school, and he is vigorous in his support of any scheme that embraces it and no less antagonistic to one that neglects it or seems to neglect it. It is for such that the traditional curriculum is justified merely on account of its being traditional. And if the subject is presented in uninteresting and difficult fashion, the training which he supposes derived by those compelled to pursue the course is merely an added incentive to his support.

The man who in his youth had a "sound general education" involving years of study of classics and mathematics, from which he broke loose into a successful career in the business world, closely approximates to this position in his regard for the educational value of mathematics. Even the teacher of mathematics has long relied on such unsatisfactory grounds.

It is to the expert in mathematics, and the expert who is also skilled in education and the kindred and subsidiary sciences, that one must look for guidance in the question of the place of mathematics in matters educational to-day. The turmoil in the minds of the teachers of mathematics is

perhaps the surest index that the question is claiming its due of attention. Who shall decide when doctors disagree? For even in the ranks of pedagogical specialists who have knowledge and training derived from the realms of mathematics there are opinions as diverse as the poles of heaven: though all agree that there is no doubt as to the necessity for teaching mathematics, there are those who advocate it on the grounds of disciplinary value and deprecate any utilitarian considerations and those who seek the justification of their profession in the practical value of the subject and discuss the issues of disciplinary value slightly and slightingly, if at all: some hold that the advantages of education in mathematics are sufficient to compel its study by all, others would restrict it for the few who can derive intellectual enjoyment from its pursuit.

It is necessary, therefore, to approach the question with an unbiassed mind: and unfortunately it is most difficult to eliminate prejudice in favour of, or against, this subject in discussing its educational value and topics deduced therefrom. From the very nature of his position the man who has lived in the clear intellectual atmosphere of pure mathematics finds it difficult to regard the matter as educationists demand. Hence the advice and commands laid on the secondary school by university and college entrance requirements are not always safe guides, though lack of consideration of such a point of view is as reprehensible as neglect of the

attitudes and requirements of the engineer, the business man and the tradesman.

CHAPTER II.

Brief Historical Survey of the Teaching of Mathematics in Scotland, England and the Province of Quebec.

There are several excellent works on the history of mathematics: some writers have dealt with the development of the subject from its very inception as a science to recent times, some with certain periods of the history of mathematics or with the history of particular topics in greater detail, while others have considered the contributions of notable mathematicians. For the most part, however, these pay comparatively slight attention to the teaching of the subject. Moreover, there are only occasional references in the standard works on the history of education, and even these are apt to be vague. Articles have appeared now and again dealing with this particular topic, but the only systematic contribution to the modern discussion of the question seems to be Dr. Alva W. Stamper's "A History of the Teaching of Geometry", published in 1909. No publication of similar investigation on the history of the teaching of algebra appears to have been made, though work on this subject would be no less profitable.

The modern teacher of mathematics has much to learn from the history of mathematics, but not more than the administrator of the secondary school may learn from the history of the pedagogy of the subject. With the former topic we

shall be concerned later, but one can evaluate most easily the study of mathematical subjects in school and the aim sought by their introduction through a careful examination of the place they obtained in the curricula of the schools from which our present-day secondary schools are descended.

At the end of the thirteenth century the Oxford student knew little more concerning mathematics than the definitions and first few propositions of Euclid. Two hundred years later only the first two books were being read, but an advance was made in the subject when Sir Henry Billingsley translated Euclid's "Elements" from Greek into English in 1570. At the same time Knox and educational reformers in Scotland were attempting to introduce mathematics in the second year at St. Andrews in a system based on the Trivium, the Quadrivium and the Three Philosophies, and Sir Henry Savile was lecturing on mathematics at Oxford, where fifty years later Briggs was Savilian professor. In 1663 the Lowndean professorship of mathematics was founded at Cambridge, and about the same time Isaac Barrow made a complete translation of Euclid there. In 1756 Robert Simson's edition of Euclid was published, and served more or less as a basis for subsequent editions, noteworthy among which were Playfair's text (1795) and Todhunter's (1862). This last was used in the High School of Montreal till 1893, when it was superseded by

Hall & Stevens' "Euclid".

The study of mathematics in the secondary schools of Scotland is comparatively modern. In 1660 James Gorss petitioned the council of Glasgow for permission to teach mathematics in his native burgh in the vernacular, and found sufficient encouragement to open a school for teaching mathematical subjects, he being "naturally addicted thereto from his infancy". Between 1700 and 1750 we hear of mathematics becoming increasingly common in secondary schools remote from the universities, which sought to make the secondary schools provide merely a thorough knowledge of the Latin language. In 1718 there is mention of a professor of mathematics in the grammar school of Perth; in 1721 of instruction in mathematics at Ayr where it was represented that "as the world now goes, the mathematical part of learning is a principal part of a gentleman's education"; in 1734 at Dunbar and in the next year at Dundee.

This was merely the stage preliminary to the reaction against the exclusive use of classics as the means of education. We come now to "happier times, when things begin to be valued according to their use and men of the greatest abilities have employed their skill in making the sciences contribute not only to the improvement of the physician, lawyer, and divine, but to the improvement of the merchant, mechanic, and farmer, in their respective arts. Must it not, then, be of importance to put it into the power of persons in these stations of life,

to reap that advantage science is capable to afford them?" The movement culminated in the foundation of Perth Academy, in 1761, where not only is there visible a complete break-away from the traditional curriculum, but even in the means of its communication, for all instruction is given in English. The curriculum included during the first year natural science, mathematics, navigation, astronomy, and English, and in the second, natural philosophy, practical geometry, civil history, logic and principles of religion. The course of study in mathematics embraced Euclid, Books I-VI; plane trigonometry; practical geometry, for example, mensuration, surveying and fortification in theory and practice; algebra; Euclid, Books XI and XII; spherical trigonometry; navigation; the practical part of conic sections and doctrine of projectiles; general principles and useful problems in astronomy in the first year, and practical methematics, illustrated by experiments in mechanics, and their applications and uses in life, in the second. In 1786 the curriculum at Dundee Academy comprised mathematics as far as fluxions, natural philosophy, astronomy, drawing, perspective, geography, French. In 1793 Inverness Academy was founded and offered as its course of study in mathematics during the fourth of the five years for which it retained its pupils Euclid's Elements with their application, plane and spherical trigonometry and mensuration of solids and surfaces, as well as geography, navigation, drawing and fortification. Ayr Academy followed closely in time of foundation and in course of study. At Edinburgh Academy in 1835

the fifth class studied the first book of Euclid, the sixth Books I-IV and began algebra, and the seventh and highest completed the six books of Euclid and studied algebra and trigonometry besides.

But despite these examples of the reactionary tendency towards mathematics in particular as the chief subject of study of the secondary school, the classical curriculum persisted till the middle of the nineteenth century in some grammar schools, especially in Aberdeen Grammar School, where even in 1867 algebra was scarcely studied at all.

In 1868 the burgh school commissioners reported that mathematics was best taught in Dumfries, Ayr and Perth Academies, Madras College of St. Andrews, Dundee High School and Dollar Institution. Geometry, which was regarded as mathematics in distinction to algebra and arithmetic, was taught with Euclid's Elements as the text book, though there were occasional protests for a more practical treatment of the subject at home. Edinburgh High School and Madras College supplemented the course in Euclid by instruction in the methods of modern geometry. Rote learning in trigonometry apart from the method of deduction of the rules was decreasing to such extent as to call for remark, but the teaching of algebra showed less change "as regards disciplinary character", though more of the subject was taught and more attention was paid to incidental parts of it. But more than the mere elements were taught and that efficiently: solid geometry, spherical trigonometry, pure and analytical

geometry of the conic sections, and differential calculus each found a place in some school.

The more recent developments in the study of mathematics in secondary schools came as a result of legislation, particularly the Education Acts of 1872, 1878 and 1882, which resulted in the establishment of the Leaving Certificate examinations, and the Universities Acts of 1859 and 1889 which established the entrance bursary competitions which have played a large part in the determination of the courses of study of secondary schools since that time. Their effects and those of the legislation of the present century will be considered more appropriately in the chapter in which recent change in the teaching of mathematics in Scottish schools is to be discussed.

The introduction of mathematics into the curriculum of the secondary schools of England probably dates from the invention of the differential calculus by Newton, when the universities concerned themselves more with this new mathematical subject. But it is difficult to discover the place that geometry was accorded among the studies of the secondary school pupil. The Public Schools were very unwilling to include it in their curricula,and it is safe to say that its place was not definitely recognised until the middle of the nineteenth century. At Harrow during the first three decades of last century Euclid was "lightly glanced at by the Sixth Form once a week" at Rugby about 1840 the Fourth Form studied the first fifteen propositions in Euclid,Book I, and began algebra: the Sixth Form completed the first six books of Euclid. The method of presenting

the subject was determined to a great degree by the editions of Euclid which were used as textbooks; as a result to this day the teaching of geometry in the secondary schools of England is more or less of the Euclidean type.

Even in the higher grades of the elementary schools the time spent on geometry was divided into two parts, one of which was devoted to Euclid and one to practical geometry, so that the two may be treated as separate subjects. The system of Local Examinations established about 1858 by Oxford and Cambridge has been the means of raising the standard of the mathematical instruction of both Grammar and Public Schools: but it has helped to strengthen the conservative tendency of the Public Schools and to keep Euclid as the textbook for geometry. As a result it is hopeless to look to English schools for reform in the teaching of mathematics, though the entrance scholarship competitions of Cambridge and Oxford colleges demand a very high standard of mathematical ability. The position of the teaching of mathematics in England in the years immediately preceding the war is discussed in several papers published in 1912 by the Board of Education in its "Special Reports on Educational Subjects, Vols. 26,27". But it will be more profitable to postpone its discussion till we come to consider the most recent changes in the teaching of mathematics.

The history of the teaching of geometry in other countries has been dealt with adequately by Dr. Stamper, but it is not so important for our uses as a comparative study of

the teaching of mathematics to-day in the various educational systems of the world.

It has been difficult to obtain connected sequence of the development of the teaching of mathematics in the Protestant Intermediate and High Schools of the Province of Quebec. The most useful sources have been the Calendars of McGill University from 1857, in which are reported the requirements for entrance to, and the courses of study in mathematics in, the different faculties and in McGill Normal School (till 1907 when it was transferred to Macdonald College), the examination papers set in the various faculties of McGill University and the curricula of the High School Department of McGill College from 1857 till 1870, when it was placed under the direction of the Protestant Board of School Commissioners; the Reports of the Superintendent of Education for the Province of Quebec, which give in some cases the courses of study of Protestant Model Schools and Academies and the rules and regulations for examination of candidates for Teachers' Certificates or Diplomas in the Province; and the Prospectus of the High School of Montreal from 1891 to the present day.

Some of the most useful information derived from a study of these sources is tabulated in the succeeding pages as it will be required in different connections throughout the thesis. But it must be remarked that no information has been obtained which deals with the history of the teaching of mathematics prior to 1850, though the High School of Montreal was

founded in 1843 by a Board of Directors, incorporated in 1845, substituted in 1846 by Order-in-Council with the Privileges and Duties of the Royal Grammar School (which was founded in 1816), and united with McGill College in 1853. Even the history since 1850 is very disconnected though attempt is made at the conclusion of the chapter to trace the general trend in the teaching of mathematics in the province.

LIST OF TABLES.

T A B L E I.

Hours Per Week Allotted to Mathematics in the High School of Montreal.

Year	III Cl., Sc. & Cm.	IV Cl., Sc., & Cm.	V Cl.	V Sc.	V Cm.	VI Cl.	VI Sc.	VI Cm.
1891 -92	4a	3a	1a 1½A 1½G	2½a 1A 1½G	2½a 1A 1½G	1½A 2½G & m	1½A 4G, m & T	1a 1½A 3G & m
1892 -93	3½a	3½a & A	2a 1½A 1½G	2½a 1½A 1½G	1½a 1½A 1½G	2½A 2G 1m & a	2½A 2G 1m & a 2T	2½A 2G 1m & a
1893 -94	3a	3½a & A	2a 1½A 1½G	2½a 1½A 1½G	1½a 1½A 1½G	2½A 2G 1m & a	2½A 2G 1m &a 2T	2½A 2G 1m & a

a Arithmetic
A Algebra
G Geometry

m Mensuration
g Graphical Algebra
T Trigonometry

E Extra Mathematics
EA Extra Algebra
EG Extra Geometry

Year	III Cl.,Sc.& Cm.	IV Cl.,Sc. & Cm.	V Cl.	V Sc.	V Cm.	VI Cl.	VI Sc.	VI Cm.
1895 -96	3½a	3½a & A	2a 1½A 1½G	2a 1½A 1½G	2a 1½A 1½G	2½A 2G 1m & a	2½A 2G 1m & a 2T	2½A 2G 1m & a
1896 -97		No Record.						
1897 -98	3a	2a 1A	2A 3G	2A 3G 2m	2A 3G 2m & a			
1898 -99	3a	3a 1A	2A 3G	2A 3G 2m & a	2A 3G	2½A 2½G	2½A 2½G 2m & T	
1899 -1900	3a	3a 1A	2A 3G	2A 3G 2m & a	2A 3G	2½A 2½G	2½A 2½G 2m & T	
1900 -01	3a	3a 1A	2A 3G	2A 3G 2m & a	2A 3G	2½A 2½G	2½A 2½G 2T	

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Year	III Cl., Sc. & Cm.	IV Cl., Sc., & Cm.	V Cl.	V Sc.	V Cm.	VI Cl.	VI Sc.	VI Cm.
1901 -02	3a	3a 1½A	2A 3G	2A 3G 2m & a	2A 3G 2m & a	2½A 2½G	2½A 2½G 2T	
1902 -03	3a	3a 1½A	2A 3G	2A 3G 2m & a	2A 3G 2m & a	2½A 2½G	2½A 2½G 2T	
1903 -04	3a	3a 1½A	2A 3G	2A 3G 1½m & a	2A 3G 1½m & a	2½A 2½G	2½A 2½G 2T	
1904 -05	3a	3a 1½A	2A 3G	2A 3G 1½m & a	2A 3G 1½m & a	2½A 2½G	2½A 2½G 2T	
1905 -06	3a	3a 2½A	2A 3G	2A 3G 1½m & a	2A 3G 1½m & a	2½A 2½G	2½A 2½G 2T	
1906 -07	3a	3a 2½A	2A 3G	2A 3G 1½m & a	2A 3G	2½A 2½G	2½A 2½G 2T	

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Year	III Cl., Sc., & Cm.	IV Cl., Sc., & Cm.	V Cl.	V Sc.	V Cm.	VI Cl.	VI Sc.	VI Cm.
1907 -08	3a	3a 2½A	2A 3G	2A 3G 1½m & a	2A 3G	2½A 2½G	2½A 2½G 2T	
1908 -09	3a	3a 2½A	2A 3G	2A 3G 1½m & a	2A 3G	2½A 2½G	2½A 2½G 2T	
1909 -10	3a	3a 2½A	2½A 3½G	2½A 3½G 1½m & a	2½A 3½G	2½A 2½G	2½A 2½G 2T	
1910 -11	3a	3a 2½A	2½A 3½G	2½A 3½G 1½m & a	2½A 3½G	Arts. 1½A 2½G	1½A 2½G 2T	
1911 -12	3a	3a 2½A	2½A 3½G	2½A 3½G 1½g	2½A 3½G	1½A 2½G	1½A 2½G 2T	
1912 -13	3a	3a 2½A	2½A 3½G	2½A 3½G 1½g	2½A 3½G	1½A 2½G	2½A 2½G 2T	

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Year	III Cl.,Sc.,& Eng.	IV Cl.,Sc., & Cm.,		V Cl.,	V Sc.	V Cm.	VI Arts	VI Sc.	
1914 -15	3a 1A	3a 2½A		2½A 3½G	2½A 3½G 1½g	2½A 3½G	1½A 2½G	1½A 2½G 2T	
		IV Cl.,& Sc.	IV Eng.						
1915 -16	3a 1A	3a 2½A 1E	3a 2½A	2½A 3½G	2½A 3½G 1½g	2½A 3½G	1½A 2½G	1½A 2½G 2T	
						V Eng.			
1916 -17	3a 1A	3a 2½A	3a 2½A 1E	2½A 3½G	2½A 3½G 1½g	2½A 3½G	1½A 2½G	1½A 2½G 2T	
1917 -18	3a 1A	2½a 2½A	2½a 2½A 1E	2½A 3½G	2½A 3½G 1½g 2½E	2½A 3½G	2A 2½G	2A 2½G 1½T 2E	
		Cl.,& Sc.	Eng.& Sc.						
1918 -19	3a 1A	2½a 2½A	2½a 2½A 1E	2½A 3½G	2½A 3½G 1½g 2½E	2½A 3½G	2A 2½G	2A 2½G 1½T 2E	

a Arithmetic
A Algebra
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EG Extra Geometry

Year	III Cl.,Sc.,& Eng.	IV Cl.,& Sc.	IV Eng.& Sc.	V Cl.	V Sc.	V.Eng.	VI Arts	VI Sc.
1919 -20	3a 1A	2½a 2½A	2½a 2½A 1E	2½A 3½G	2½A 3½G 1½g 2½E	2½A 3½G	2A 2½G	2A 2½G 1½T 2E
1920 -21	3a 1A	2½a 2½A	2½a 2½A 1E	2½A 3½G	2½A 3½G 1½g 2½E	2½A 3½G	2A 2½G	2A 2½G 1½T 2E
1921 -22	3a 1A	2½a 2½A	2½a 2½A 1E	2½A 3½G	2½A 3½G 1½g 2½E	2½A 3½G	2A 2½G	2A 2½G 1½T 2E
	VIII	IX Cl.,& Sc.	IX Eng.& Sc.	X Cl.	X Sc.	X Eng.	XI Arts	XI Sc.
1922 -23	3a 1A	2½a 2½A	2½a 2½A 1E	2½A 3½G	2½A 3½G 1½g 2½E	2½A 3½G	2½A 2G	2½A 2G 1½T 2E
1923 -24	3a 1A	3a 3A & G	3a 2½A 1G	2½A 2½G	2½A 3½G 1½g 2½E	2½A 3½G	2½A 2G	2½A 2G 1½T 2E

a Arithmetic
A Algebra
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Year	VIII	IX_1	IX_2	X	XI
1924 -25	2½a 1½A	3a 1½A 1½G	3a 2A 1G (1E)	2½A 3½G (1 EA) (1½EG)	2½A 2G 1½T
1925 -26	2½a 1½A	5a,A & G (1E)	5a, A & G (1E)	2½A 3½G (2½EA) (1½EG)	2½A 2½G (1½T) (2½M,Pt.II)
1926 -27	2½a 1½A	5a, A & G (1E)	5a, A & G (1E)	2½A 3½G (2½EA) (1½EG)	2½A 2½G (1½T) (2½M,Pt.II)
1927 -28	2½a 1½A	5a, A & G	5a, A & G	2½A 3½G (2½EA) (1½EG)	2½A 3½G (1½T) (2½M,Pt.II)
1928 -29	2½a 1½A	5a, A & G	5a, A & G	2½A 3½G (2½EA) (1½EG)	2½A 3½G 1½T (2½M,Pt.II)

a Arithmetic
A Algebra
G Geometry

m Mensuration
g Graphical Algebra
T Trigomometry

E Extra Mathematics
EA Extra Algebra
EG Extra Geometry

TABLE II.

Course of Study in Mathematics

in the High School Department of McGill College

1857-1870.

Years	Form IV	Form V	Form VI
1857-1860		Euclid Bks.I-IV,VI Alg. to end of simple equations	Logarithms Plane and spherical geom. Alg. from simple equations Plane trig. Problems in mensuration and mechanics
1860-1864	Euclid Book I	Do.	Logarithms Alg. from simple equations Plane trig. Problems in mensuration
1864-1870	Do.	Euclid Bks.I-IV Alg. to end of simple equations	Euclid, Bks. I-IV Defs. of Bks. V & VI Alg. from simple equations Plane trig.

Note:

Age of Pupils on Entrance to Course - 7 years.

Length of Course - 6 or 7 years.

TABLE III.

Course of Study in Mathematics

in the High School of Montreal from 1891 to the present day.

1891.	Grade V	Geometry	Euclid Bks. I,II & easy deductions.
		Algebra	to simultaneous equations.
	Grade VI	Geometry	Euclid Bks. I-IV, Defs. of Bks. V & VI and easy deductions.
		Algebra	Quadratic equations, involution, evolution, fractions, indices and surds.
		Trigonometry	Hamblin Smith's Trigonometry to page 100.
1896.	Grade IV	Algebra	C. Smith's Algebra to page 64 and easy problems.
	Grade V	Geometry	Euclid Bks. I-III.
		Algebra	to quadratic equations.
	Grade VI	as in 1891.	
1901.	Grade IV	Algebra	C. Smith's Algebra to page 82 and easy problems.
	Grade V	as in 1896.	
	Grade VI	Geometry	as in 1891.
		Algebra	C. Smith's Algebra
		Trigonometry	as in 1891.
1906.	Grade IV	Algebra	C. Smith's Algebra to H.C.F.
	Grade V	as in 1896.	
	Grade VI	Geometry	Euclid Bks. I-IV,VI and Defs. of Bk. V.
		Algebra & Trigonometry	as in 1901.

1911.	Grade IV	Algebra	Hall & Knight's Algebra to page 127 omitting cube root.
	Grade V	as in 1896.	
	Grade VI	Geometry	as in 1906 with easy deductions.
		Algebra	Hall & Knight's Algebra
		Trigonometry	as in A.A. Examination
1916.	Grade III	Algebra	Fundamental rules.
	Grade IV	as in 1911.	
	Grade V	Geometry	Hall & Steven's School Geometry to page 171.
		Algebra	as in 1896.
	Grade VI	as in University School Examinations.	
1921.	Grade III	as in 1916.	
	Grade IV	as in 1911.	
	Grade V	Geometry	as in 1916.
		Algebra	Hall & Knight's Algebra to page 221.
	Grade VI	as in 1916.	
1926.	Eighth Year	Algebra	Hall & Knight's Algebra to page 60.
	Ninth Year	Geometry	Hall & Steven's School Geom. to page 33 and pages 70-76.
		Algebra	as in 1911.
	Tenth Year	Geometry	Hall & Steven's School Geom. to page 138.
		Algebra	as in 1921.

Eleventh Year	Geometry	)	as for Matriculation
	Algebra	)	Examinations at McGill
	Trigonometry	)	University.

TABLE IV.

Mathematical Textbooks in Use in Montreal High School since 1891.

1891. Todhunter's Euclid.

Todhunter's Algebra.

Hamblin Smith's Elementary Trigonometry.

1892. Hall & Steven's Euclid.

C. Smith's Algebra.

1907. Hall & Knight's Algebra.

1910. Hall & Knight's Trigonometry.

1911. Hall & Steven's School Geometry.

TABLE V.

McGill Normal School (1860-1890).

Course of Study in Mathematics.

1860.	Junior class (elementary diploma)	Geometry	Euclid Bks. I-III.
		Algebra	to quadratic equations.
	Senior class (model diploma)	Geometry	Euclid Bks. IV & VI. Elements of solid geom.
		Algebra	to binomial and exponential theorems.
		Trigonometry	Elements.
1870.	Elementary school class	Geometry	Euclid Bks. I-III & deductions.
		Algebra	to quadratic equations.
	Model school class		Logarithmic, algebraic and geometric arithmetic, quadratic equations, ratios and progressions, theorem of undetermined coefficients, binomial & exponential theorems. Euclid Bks. IV & VI.
	Academy class		Trigonometry, solid geometry, theory of equations, mechanics and astronomy.
1880.	Elementary school class	Geometry	Euclid Bks. I,II and deductions.
			Algebra to simple equations in three unknown quantities.
	Model school class	Geometry	Euclid Bks. III, IV, VI; application to mensuration.
		Algebra	Quadratic equations, ratios and progressions, theorem of undetermined coefficients and binomial theorem.

	Academy Class	Trigonometry, solid geometry and mechanics.
1890.	Elementary school class	Geometry, Euclid Bk. I and deductions. Algebra to simple equations in one unknown and problems.
	Model school class	Geometry, Euclid Bks. I-IV and applications to mensuration. Algebra to quadratic equations Plane Trigonometry.
1900.	No record.	

TABLE VI.

REGULATIONS FOR ADMISSION TO MCGILL UNIVERSITY.

Faculty of Arts.

1860. Geometry Euclid Bks. 1-III.

Algebra to quadratic equations.

1870. First Year;

Geometry Euclid Bks. I-III.

Algebra to simple equations.

Second Year:

Geometry Euclid Bks. I-IV, VI, Defs. of Bk. V. omitting Bk. VI, 27, 28, 29.

Algebra to end of quadratic equations.

Trigonometry to solution of plane triangles.

1880. First Year: (passing only) as in 1870 first year.

(First Year (first class, second class, passing and exhibitions)

(Second Year: as in 1870 second year.

1890. First and Second Years: (passing only) as in 1880.

First Year Higher Examination Geometry Euclid Bks. I-IV.

Algebra to harmonic progressions.

1900. Mathematics Part I:

Geometry Euclid Bks. I-III and easy deductions or equivalent.

Algebra Hall and Knight's Algebra to end of surds or as in a similar textbook.

Mathematics Part II:

Geometry Euclid Bks. IV & VI, Defs. of Bk. V and easy deductions or equivalent.

Algebra Progressions, ratio, proportion, variation, permutations and combinations, binomial theorem, logarithms, interest and annuities.

Trigonometry Measurement of angles, trigonometrical ratios of one angle, two angles or multiple angle

Faculty of Applied Science.

1880 . First Year:

Geometry Euclid Bks. I-III.

Algebra to simple equations.

Second Year:

Geometry Euclid Bks. I-IV, VI and XI, Defs. of Bk.V.

Algebra to end of quadratics.

Trigonometry to solution of plane triangles.

1890. Junior Matriculation as in 1880 first year.

Senior Matriculation as in 1880 second year.

1900. Mathematics Parts I and II as in Faculty of Arts.

From these tables it can readily be seen that about 1855 mathematical subjects were taught in the two highest classes of the six years secondary school course merely as a preparation for more advanced mathematical study in the university. The course of study was ambitious, especially in geometry. In one year practically the whole of Euclid's Elements was studied: this was followed in the sixth form by a course in solid and spherical geometry, plane trigonometry and mensuration. In algebra the work proposed was less startling. In the fifth form the subject was studied to the end of simple equations, and the continuation in the next year had no specified limit. However, as the course in geometry during the fifth form must have proved overpowering, within five years the study of geometry was begun in the fourth form, and the solid and spherical geometry was omitted from the course of study in the last year. In 1864 the amount of geometry studied in the fifth form was still further curtailed,and only the definitions of the fifth and sixth books of Euclid were required, and that in the sixth form. The course in algebra meantime remained unchanged in the control of the fifth and sixth years. There was no alteration in this programme till 1870 when the school passed under the direction of the Protestant Board of School Commissioners, after which date for twenty years no record is available of the work attempted in mathematics.

In 1891 the school was reorganised with the last four of the six forms in the high school including secondary school subjects, especially Latin. But mathematics was again confined to Forms V and VI, though in the following year algebra was begun in the third form and during the next twenty years received increasing attention there: till in 1914 it was considered expedient to begin it one year earlier still. This may probably have been due to the demands of the Faculty of Applied Science. Geometry, however, remained a subject of study for only the last two forms till 1923, though the time per week allotted to it in Form V was more than doubled. In 1923 it was begun in the ninth grade (the former fourth form), and it receives even more attention now than it did then in that grade.

The study of geometry in Montreal schools has been strictly Euclidean in its nature as far as the records consulted show. The textbooks of geometry used in the Faculty of Arts in McGill University, as far back as the Calendars in the Redpath Library bear witness, have been various editions of Euclid till 1911, when Hall & Stevens' "School Geometry" was introduced; and these were also for the most part the books employed by the High School. In algebra the books used were Colenso's, Todhunter's, C. Smith's and Hall & Knight's in both High School and University. Hall & Knight's Algebra has been in use since 1911. In 1910 Hamblin Smith's "Elementary Trigonometry" was replaced by Hall & Knight's in the work of the

final year of the High School. Since 1911, therefore, the textbooks required for mathematical work in High School and in the first year of the Faculty of Arts at McGill University have remained unchanged.

CHAPTER III.

The Values of the Teaching of Mathematics.

The tripartite division of value into the practical or utilitarian, the disciplinary and the cultural is as convenient in the study of mathematics as in that of any other school subject. It must be borne in mind, however, that disciplinary value is in some measure utilitarian, while the cultural may also be included if utility is interpreted as fully as may be. The division is not then absolutely definite, though some of the argument that has been raised on this question would lead one to expect a natural boundary between each pair.

In the 1857-58 Prospectus of the University of McGill College, Henry Aspinwall Howe, Rector of the High School and Professor of Mathematics and Natural Philosophy, states in his summary of the activities of the pupils of the High School Department: "Mathematics have an hour daily assigned to them as soon as the faculties of the Pupil are sufficiently developed to cope with the difficulties of the subject. They are the grand means of strengthening and disciplining the reason; a knowledge of them is indispensable in the Engineering profession and in the Mechanical Arts; and the pursuit of them affords a very high and pure pleasure to the active and inquiring mind." In effect he claims for mathematics a unique standing among

other subjects for disciplinary value, utilitarian value in rather a restricted field and cultural value for most of its students.

More than forty years later Professor D. E. Smith quotes with approval from a paper "On the Teaching of Elementary Algebra", read by Professor W. H. H. Hudson before the Educational Society in 1886, "I maintain, therefore, that algebra is not to be taught on account of its utility, not to be learnt on account of any benefit which may be supposed to be got from it; but because it is a part of mathematical truth, and no one ought to be wholly alien from that important department of human knowledge." Smith adds: "Useful it (i.e. algebra) is, and that to a great degree in all subsequent mathematical work; but for the merchant, the lawyer, the mechanic, it is of slight practical value." The disciplinary value of algebra he discusses in the quotation "We need it also as an exercise in logic; to follow the logic of the process (i.e. finding the H.C.F. of three functions), to keep the mind intent upon the operation while performing it, herein lies much of the value of the subject—here is to be sought its chief raison d'être." On such grounds he introduces questions such as that mentioned above or the extraction of the fourth root of a polynomial, though he admits that it is highly improbable that necessity for such operations would occur even in subsequent mathematics. He cautions us, however, not to be extreme in pushing our

claims for the ethical value of algebra.

His reasons for the study of geometry are twofold, "the practical side of the subject in simple mensuration, and the culture side in the logic which enters into it to such a marked degree." The cultural value, "almost the only value which formal demonstrative geometry has," he subdivides into two phases. Geometry is necessary for general information, a defence which he states can be advanced as justification for the retention of any traditional subject in the curriculum, and for "its value as an exercise in logic, as a means of mental training, 'as a discipline in the habits of neatness, order, diligence, and above all, of honesty.'" His quotation is from G. B. Mathews in the School World, vol. 1 (1899).

Dr. J. W. A. Young devotes a long chapter to the exposition of the purpose and value of the study of mathematics. He summarises his discussion of the values of mathematics under the following heads:

(1) utilitarian, in which it is second only to the mother tongue;

(2) as a fundamental type of thought, characteristic of all civilisations;

(3) as a tool for the study of nature;

(4) as exemplifying especially well certain modes of thought, e.g., understanding statements, noting facts, and making inferences. In mathematics the "form of reasoning is

the ideal towards which all other reasoning strives; owing, however, to the simplicity and narrow range of mathematical inferences, mathematics can only give the beginnings of the requisite practice";

(5) as cultivating reverence for truth, the habit of self-scrutiny, the power of attention, and other minor miscellaneous values. From England Mr. G. St. L. Carson in an essay, "The Educational Value of Geometry", first published in 1912 in the 'Special Reports of the Board of Education on the Teaching of Mathematics, No. 15', warns against giving undue prominence to considerations of practical utility or to the plea that every educated man should have some idea of a subject of such wide utility, on the grounds that many other subjects have higher claims on such a basis of discrimination and that it would suggest that the aim of the teaching of geometry is to provide encyclopaedic knowledge. Such justification, he remarks, even if it is used as of secondary importance, is apt to defeat the aims of those proposing it and to harm educational ideals.

So he has to find reasons sufficient to explain how practically every university follows the lead of Plato and exhorts "Let no one enter without geometry." "Such recognition of the subject by educationalists who are not mathematicians implies an inherent value which must be expressible in non-technical terms," and which the mathematical teacher must be quick to appreciate and to realise in the school: such a statement, however, implies the infallibility of universities as

authorities on educational matters and neglects to take full account of their conservative tendency which makes for the preservation of obsolete and obsolescent content and methods. The basis on which he seeks to justify the universal teaching of geometry is threefold:

(1) "the presence of schemes of deduction based on statements which find universal acceptance as descriptions of our space impressions must make for good in the child development." He analyses the sequence of the geometrical processes as separation of essential from irrelevant considerations, erection on the data obtained of continuous chains of reasoning and discussion of the interdependence of the premisses.

(2) "The contemplation of unassailable mental structures such as are found in mathematics cannot but raise ideals of perfection different in nature from those found in the more emotional creations of literature and art."

(3) Geometry has peculiar value in relation to other branches of science.

Arthur Schultze in "The Teaching of Mathematics in Secondary Schools", published at the same time as Carson's essay, discusses the reasons for teaching mathematics under the same three heads: but, though his book is written with the stated purpose of making mathematical teaching less informational and more disciplinary, he does not fail to emphasise the great usefulness of mathematics to our civilisation. He shows, however, that this is not the same as usefulness to

every individual, and his final verdict is that though mathematics has some practical value for all students, even to an extent greater than many other high school subjects, its teaching in secondary schools can hardly be justified on utilitarian grounds. The chief disciplinary value of mathematics lies in the "fact that it exercises the reasoning power more and claims from the memory less than any other secondary school subject". The characteristics of the reasoning in mathematical work to which he pays particular attention are simplicity, accuracy, certainty of results, originality, similarity to the reasoning of life, and amount.

The claim for the similarity of mathematical reasoning and that used in daily life is probably the most astounding. He explains later that the mental qualities cultivated by mathematical study are a necessary but not a sufficient reason for success in the affairs of life. "More than one business man has testified that he owes his success in life to the habits of exact thinking which he formed while studying mathematics". We cannot doubt this statement or the good faith of the business men quoted; but Schultze does not, and, as far as we can see just now, cannot prove that such men would have had less success in life if they had studied no mathematics whatsoever. He is not unaware of the controversy that had started by this time when psychologists had denied the existence of general mental discipline: but he condemns pedagogy and psychology as not being exact sciences and their

findings as only approximately true, so thin and insecure as to be swept away by the verdict of any mathematical teacher on the strength of his own experience.

Some "minor functions" of mathematical study that he states are worth passing notice. He holds that mathematics can develop the power of concentration, the constructive imagination: that it promotes mental self-reliance and develops character by training in systematic and orderly habits and by stimulating the will power through mastery: that good teaching in mathematics increases the ability of the pupils to use the English language correctly, and finally that acquaintance with the fundamental facts and methods of mathematics ~~are~~ *is* necessary for general culture.

These are the opinions of four of the most eminent authorities on the teaching of mathematics of the present century, but one does not require to search far to find views that conflict more forcibly and fundamentally than the minor differences which they express.

Professor Franklin Bobbitt, for example, in his 'How to Make a Curriculum' takes a novel attitude to the question and treats it in a refreshing and rather unorthodox fashion. He emphasises from the outset the necessity of the quantitative aspect of things, so that from his point of view "in school life ... mathematics should be primarily not a matter of solving difficult problems, but rather a matter of continuously viewing for many years the quantitative aspect of things, and

of thinking in accurate terms": a position that we are unfortunately obliged to confess is seldom achieved even by those who have had what is now considered as a good mathematical education. His remedy is to decrease the abstract mathematics and to delegate to other departments quantitative treatment of their subject matter. The intellectual satisfaction, to which these others have drawn careful attention, he repudiates except for those of large native intellectual endowment, though I imagine he is unfortunate in his experience of the teaching of mathematics since he states that "even with the intellectualist, in the majority of cases, the intellectual hunger here is not particularly striking". He scoffs at the disciplinary value, basing his contempt on the waste of time and energy, which could be employed for activities, the worth of which has been substantiated.

In another recent book on "The Teaching of Elementary Algebra" Paul Ligda has concluded with a justification of the subject in school and in life. His aim is to teach the pupil to solve problems, rather than to teach him definite problems; to develop skill, habits and attitudes in preference to giving him knowledge. But he emphasises one point that most of his opponents have neglected to consider. The fact that few people use algebra in their daily life—and the argument is equally cogent for other branches of mathematics—is no sound reason for the conclusion that little of the subject should be

taught. From his own experience in industrial construction work he has found that many situations in which algebra would be of the highest value are not taken into account by attempts to make a survey of such cases, for the simple reason that these men who were placed in such circumstance were frequently unable to realise the need of the subject, through sheer inability to appreciate its potentialities. His discussion of the matter is extremely valuable: for though he was doubtless prejudiced, he was able to compare the efficiency of a worker with mathematical training of the kind he advocates with that of those who were denied it. He reaches a conclusion that is satisfactorily similar to that of Bobbitt's, which was attained on more theoretical considerations. The reiteration of the necessity of the ability to recognise the quantitative aspect and of subsequent manipulative skill and accuracy in its treatment shows that recent work is tending to a more general recognition of one of the major values of the subject that has been slighted by those reared in the mathematics of a more academic character.

Nearly thirty years ago Professor John Perry uttered his plea for the teaching of geometry on utilitarian grounds before an audience, some of whom must have been deeply grieved and shocked, to judge by the criticisms that have been published with the paper he read before the Mathematical Section of the British Association. He includes indeed "the develop-

ment of the mental power and soul" in his conception of usefulness, and even the passing of examinations, the latter being the only form which had not been almost entirely neglected. But the usefulness of the subject that he especially commends is in the aid to the study of science, in the criticism and mastery of matters usually accepted on authority and in the provision of mental tools under control.

D. E. Smith in collaboration with Professor W. D. Reeve has lately given in a few lines the purpose of the study of mathematics in the secondary school. The subject is presented to allow the pupils knowledge of its meaning and to make possible further advance according to their needs and inclinations: an aim that seems at first sight indefinite and of comparatively little importance to the teacher. It comprises, however, many of the arguments that other writers have advanced, and shows the modern tendency not to dogmatise on a topic which is of such universal moment. They elaborate it in a discussion of the aims of specific branches, intuitive geometry, algebra, numerical trigonometry and demonstrative geometry, and they reinforce it by an elaborate treatment of the objectives to be obtained in mathematics studied in its various forms and in different situations. These various views serve to show the divergence that has existed in the opinion of mathematicians and of teachers of mathematics as to the value of their subject in the curriculum of the secondary school. As a result of investigation along psychological and sociological lines much

that was held as sure has been discredited, much that failed to get recognition has been put on certain footing. But before we turn to consideration of the advances made by educational psychology and sociology in the determination of the value of mathematics it may be well to make several remarks on matters, neglect of which in the past has vitiated the whole discussion.

In the first place, there is lack of uniformity in the treatment of the value of mathematical education as to whether one should interpret it as mathematical instruction as it now is or as it might be. If we restrict ourselves to the present method of treatment of the subject, we face the further difficulty of meeting samples which vary considerably: some even yet teach not only the propositions as to be learned verbatim, but also a close approximation to this view with deductions as well. On the other hand, some advocate and successfully employ the practice of having the pupils develop the subject for themselves under careful and tactful guidance.

As a result, too, of this indefiniteness in the meaning of the teaching of mathematics, its value in practical matters often suffers considerably. Many realise that their mathematical education has not fitted them to meet in any sense adequately the situations of real life in which it is to be presumed that the greatest requisite is along mathematical lines.

The different needs of different pupils who are to have different careers is another consideration that must be faced before we can discuss the question of values authoritatively. At the present moment, it is openly admitted that the

curricula in mathematics in the Central Schools of England and the Advanced Divisions in Scotland is a mere makeshift--a truncation of the courses of study employed in the secondary schools, and there are few who pretend that this scheme approaches the ideal.

The main contributions of educational sociology can be classified under two headings. In the first place, it emphasises the need for recognition of the fact that various pupils have different needs as a result of the course which they intend to pursue at school: that mathematics as well as other subjects must be treated so as to give in each and every course its fullest value, and that it is no justification to claim that because it was first introduced in the secondary school which catered in the main for those who intended to proceed to the university, it is unnecessary to give equal consideration to the education of those others who have less ability and no desire for the higher reaches of intellectual attainment.

Secondly, the needs of various communities must be considered. The case in which this has been recognised effectively is in the university where the various faculties--arts, applied science, architecture--have mathematical courses which differ in great part from each other. But the municipal and the rural school, the pupil who intends to proceed to the university and the one whose aim is the engineering workshop or the office desk, have usually followed by compulsion the same course of

study in the classroom.

Psychology with quite a different point of view brings us to the same conclusion that greater attention must be paid to the differences between pupils. Euclid's "Elements" was an attempt at logical development of theorems that had been discovered in haphazard order during the preceding centuries. It is difficult for those exceptional cases who appreciated this logical order in the days of their youth to realise that it is quite unnatural for the boy of thirteen or fourteen. By the time he is reaching manhood the logical nature of the subject probably will appeal to him, but for earlier years the psychological order is much to be preferred, and here as in many other cases the psychological and the historical orders are very closely akin.

A study, then, of the value of mathematics in the field of education must take into account the various differences in communities, in schools, in pupils: it should be governed rather by consideration of the most effective teaching in each case rather than by a criticism of unsatisfactory existing systems. With this in view one finds it hard to criticise any of the objectives that have been mentioned in the previous chapter. Probably most enter in some degree, however slight, in different systems. It is rather in their relative importance that we seek their value. In the former supplementary course in Scotland, which has been replaced by

the advanced division, in the central schools of England, the junior high school in U.S.A. and the intermediate grades of Canadian schools, the aim of the mathematical instruction must approach more to the practical than the disciplinary and the cultural. As a general rule the useful, the disciplinary and the cultural follow in that order in historical development, and it is probable that they should appear in the same order in the education of the child. But the order is not invariable and often the cultural leads to the useful as is seen in recent applications of speculative science. So that to make a course of study for the first years of high school in which utilitarian value alone is recognised is as mistaken policy as to neglect the useful altogether. So, too, in the senior grades is it unwise to divorce mathematical study entirely from its application. Those who are to be engaged in applied science in after years will not hesitate to accept this view, but even the pure mathematician to-day finds that his work finds application so readily in allied sciences that few can afford to neglect their claims to attention.

Many of the works and much of the study that has been cited in addition to discussion of specific problems by leading authorities has been utilised and incorporated in ~~The~~ a Report of the National Committee on Mathematical Requirements which was organised in 1916 by the Mathematical Association of America. It attains comprehensiveness with brevity by discussing

the matter according to the parts played by the different aims--the practical, the disciplinary and the cultural and the various stages and topics in which one or other predominates. Under practical aims are mentioned the understanding of the language of algebra, the study of its fundamental laws, ability to interpret graphic representations: familiarity with common geometric forms, the development of space perception and the exercise.

The disciplinary value was made the subject of a special investigation by Miss Vevia Blair who reviewed the experiments and discussions of psychologists that were subsequent to those of Thorndike and Woodworth in 1901. The inferences derived from the consideration of these were submitted to some of the leading educational psychologists in America and conclusions based on their opinions were formulated. There is almost unanimous agreement that transfer does exist; that there is possibility of zero or negative transfer and that in very few experiments is the full amount of transfer shown. But the most notable contributions are that the amount of transfer is to large extent dependent on the methods of teaching and that it is most evident with respect to general elements--ideas, attitudes and ideals. The conclusion is, therefore, fairly definite that transfer of training is a valid aim in teaching. Thus as a result not of arguments based on introspection or on seemingly logical necessity, but in direct consequence of experimental work, the disciplinary aim of the teaching of mathematics is validated. It includes the acquisition of the con-

cepts which are required for quantitative thinking; the ability to think in such concepts, which necessitates training in the analysis of the situation, in the recognition and expression of relations between the essential factors and in generalisation; the acqqisition of attitudes and of the idea of relationship or dependence.

Some of the cultural aims must be delayed till the later stages of mathematical study, but some are valid right from the outset: such, for example, are the appreciation of beauty, ideals of perfection and recognition of the power of mathematics.

CHAPTER IV.

The Teaching of Mathematics in the Province of Quebec.

The Protestant schools in the Province of Quebec are divided into three types - Elementary, Intermediate and High Schools. The elementary school includes Grades I to VII, the intermediate school, Grades I to IX and the high school, Grades I to XI. In other words, the course of study is completed in eleven grades, which are divided on the 7-2-2 basis, though many schools have in addition a kindergarten or preparatory class. (This description of the grouping of the grades is not in accordance with that given in the "Memoranda of Instructions for Teachers of the Protestant Intermediate Schools and High Schools of the Province of Quebec", which states that the division is on the 6-3-2 plan, a classification that is not followed, however, in a publication from the Provincial Secretary's Department, "Educational Statistics of the Province of Quebec for the school year 1926-27", the most recent issue available. Here Grades I to VII are grouped as Elementary, Grades VIII and IX as Intermediate and Grades X and XI as High School. Grade VIII cannot be taught in an Elementary School, nor Grade X in an Intermediate School, unless special permission is obtained from the Director of Protestant Education. In only one elementary school in Montreal

is there teaching beyond that of Grade VII, while few elementary schools which have instruction as far as Grade VI discontinue it at that point. West Hill High School, however, begins its course in the seventh year, though a considerable number join Grade VIII from other schools of the neighbourhood. The division of the course of study into three periods, which last respectively for seven years, two years and two years, seems to have more justification from these facts than the other). At all events, the question, however important for purposes of administration, is a minor one here, as no advanced mathematics is taught prior to the eighth grade in any school.

The nature of the secondary grades - a term which is here understood to include Grades VIII to XI from both intermediate and high schools - has been changing rapidly during the present century in the Province of Quebec as elsewhere. The number of pupils in the intermediate and high school grades has increased greatly in the past few years, as even casual inspection of the figures in the tables on pages 51 and 52 will show. Unfortunately, it has been impossible to obtain exact figures or even reasonably reliable estimates of the Protestant population of secondary school age (roughly between thirteen and seventeen years) either in the Province of Quebec or in the City of Montreal. But a fairly satisfactory and sufficiently striking comparison can be drawn from the corresponding numbers enrolled in elementary and in secondary grades during the last few years.

TABLE VII.

NUMBER OF PUPILS ENROLLED IN PROTESTANT SCHOOLS OF THE PROVINCE OF QUEBEC.

Year	(a) Kindergarten-Grade VII	(b) Grade VIII-Grade XI	Ratio (a) : (b)
1914-15	50,975	4,127	12 : 1
1915-16	54,948	4,151	13 : 1
1916-17	54,271	4,205	13 : 1
1917-18	50,856	3,691	14 : 1
1918-19	54,032	4,134	13 : 1
1919-20	55,218	5,584	10 : 1
1920-21	56,536	6,307	9 : 1
1921-22	59,117	7,156	8 : 1
1922-23	62,326	7,729	8 : 1
1923-24	61,972	7,990	8 : 1
1924-25	62,420	8,086	8 : 1
1925-26	62,215	8,377	7 : 1

TABLE VIII.

NUMBER OF PUPILS ENROLLED IN THE SCHOOLS UNDER THE CONTROL OF THE PROTESTANT BOARD OF SCHOOL COMMISSIONERS OF THE CITY OF MONTREAL.

Year	(a) Kindergarten-Grade VII	(b) Grade VIII-Grade XI	Ratio (a):(b)
1907	9,918	471	21 : 1
1908	11,170	513	22 : 1
1909	11,329	509	22 : 1
1910	13,264	505	24 : 1
1911	14,560	533	27 : 1
1912	16,016	552	29 : 1
1913	18,102	614	29 : 1
1914	19,983	696	28 : 1
1915	20,888	790	26 : 1
1916	21,227	813	26 : 1
1917	21,601	855	25 : 1
1918	24,442	944	26 : 1
1919	25,565	1,094	23 : 1
1920	25,712	1,230	21 : 1
1921	28,153	1,861*	16 : 1
1922	28,786	2,866*	10 : 1
1923	28,431	2,936*	10 : 1
1924	28,288	2,900*	10 : 1
1925	27,915	2,951*	9 : 1
1926	27,915	3,206*	9 : 1
1927	27,969	3,578*	8 : 1
1928	27,590	3,809*	7 : 1

* These figures include numbers from Grade VIII in some elementary schools.

The High School of Montreal was founded in 1843 and transferred to the direction of the Protestant Board of School Commissioners of the City of Montreal in 1870. The High School for Girls, which was opened in private dwellings in 1875, was transferred to the High School Buildings in 1892. In 1906 the Commercial and Technical High School was opened, when pupils of the Senior School were transferred to it. The classes of the Commercial side were organised as classes of the Commercial High School in 1922, and the pupils of the Technical side were transferred to Baron Byng High School, which was opened in the same year. In 1921, West Hill High School was transferred to the Board.

As a result of the foundation and transference of these high schools about 1922, the figures which appear for the enrolment in the secondary grades of Montreal prior to 1922 and immediately succeeding cannot be interpreted as indicative of such a considerable increase as would at first sight appear: for the numbers are those in schools under the direction of the School Commissioners. But the statistics for the City of Montreal subsequent to that year show the trend of increase in scarcely less startling fashion. In 1923, the number of pupils in the elementary grades of the schools of Montreal was more than 28,000, and in the secondary grades less than 3,000, the proportion of the number of secondary to that of elementary pupils being approximately 1 : 10. In the registration of 1928-29 the number of pupils enrolled in the elementary grades

of all schools was less than 28,000, while in the secondary grades of the high schools the numbers had risen to over 3,800, an increment of about thirty per cent, the ratio of the figures for secondary and for elementary enrolment being now 1 : 7.

The educational statistics of the province, however, do not suffer from this defect of alteration of the number of schools, and of districts served thereby, under the direction of the Protestant Committee. Consequently, the figures for a longer period are valid, though these for the last two years have not been obtained. Since 1918-19 the ratio of the number of secondary pupils to that of the pupils attending elementary grades has increased from 1 in 13 to 1 in 7, which it attained in 1925-26. Though the change in the enrolment of the elementary school during this interval has not been negligible, it is overwhelmed by the numbers of the secondary grades which have more than doubled themselves in the space of seven years.

As is seen from the table on page 15, which shows the number of hours per week allotted to the study of geometry, algebra and trigonometry during the sessions from 1891-92 till 1928-29 in the High School of Montreal, mathematics has been gradually recognised as worthy of more attention in the curriculum of the secondary schools of the city. To-day the study of algebra is begun in Grade VIII, the first year of high school, and a beginning to geometry is made in the following year; arithmetic also holds a place in the course of study

for these two years, but it is discontinued in Grades X and XI. Geometry is studied by all for three and a half hours and algebra for two and a half hours per week in Grade X; but those pupils who do not study Latin have one and a half hours per week for extra geometry and two and a half of extra algebra. As a result the course of study in the case of these pupils is much more inclusive than that of the others. In the final year, two and a half hours per week are devoted to geometry and to algebra and one and a half to trigonometry. But only those pupils who do not take Latin study trigonometry, and they have two and a half hours weekly for extra mathematics.

The course of study is determined by certain pages or sections of the mathematical textbooks in use--Hall and Stevens' "School Geometry", Hall and Knight's "Algebra" and Hall and Knight's "Elementary Trigonometry". The "extra mathematics", which is studied by the pupils who do not take Latin in their course, is intended to be of assistance to the pupils mainly in their physical science, for the study of which the limited course that is possible in the regular period is insufficient. It consists for the most part of graphical algebra, and is taken with special sections of Hall and Knight's "Algebra" as the textbook.

This course of study may be taken as typical of that of most of the high schools of Montreal. In each case the study of algebra is commenced in Grade VIII, a year before geometry is studied. But only the very elements of geometry and algebra are treated of before Grade X, and for the two

last school years the subjects have quite an important place in the curriculum.

In the schools outwith the city mathematics is not begun so early in the school course and in most cases it is not treated so fully. In the "Memoranda of Instructions for Teachers in Protestant Intermediate Schools and High Schools of the Province of Quebec" it is stated that "Superior Schools that desire to do so may begin the study of Algebra and Geometry in Grade IX but departmental examinations in these subjects will not be provided for that grade. In Grade X either Algebra or Geometry is necessary and both may be taken: either has the value of one unit in a course for which the minimum is 9 units and the maximum 12. In Grade XI the same condition holds: one is necessary and both may be included in the course, each having a value of one unit in a total which ranges from 8 to 10: but advanced algebra is also optional as a single unit and it is possible to take advanced geometry and trigonometry which have two units for credit assigned to them.

The course of study in the subject is chosen in great part to correspond to the requirements of the examinations held at the end of the eleventh year: it is doubtful if it is more correct to say that the examinations are based on the course of study. These examinations are the High School Leaving Examinations and the Matriculation Examinations of McGill University. The limits in the subjects of the eleventh

year for the High School Leaving Examinations are given in the Prospectus of the High School of Montreal (1928-29) as follows:

Geometry: Hall & Stevens', pp.1-203, 219-265, 267-269, 274-276. Theorems 66, 70, 71, 72.

Algebra: Hall & Knight, pp. 1-281, omitting exercises marked with an asterisk.

That is, the course in geometry includes the substance of Euclid Books I,III,II and Vi with the omission of several theorems which are particularly difficult for the secondary school pupil, either in their intrinsic nature or through lack of ready application in deductions. That in algebra goes as far as the study of simultaneous quadratic equations, the theory of indices, elementary surds, ratio and proportion.

The course of study in advanced mathematics as given in the "Memoranda of Instructions for Teachers" includes

Algebra: Hall & Knight's Elementary Algebra (omitting chapters 40-42,inclusive);

Plane and Solid Geometry: Hall & Stevens' School Geometry to Theorem 98, inclusive;

Trigonometry: Hall & Knight's Elementary Trigonometry, Chapters I - XIX, inclusive, omitting pages 212 to 230.

In algebra this requires knowledge of the progressions, theory of quadratic equations, permutations and combinations, binomial theorem, logarithms, interest and annuities and graphical representation of functions. The further study of geometry requires a more systematic and extended knowledge of Euclid's Elements Books I - VI, of Book XI, and also of the elements of modern geometry, for example, the

theory of polars, inversion and the theorems of Menelaus and Ceva. Hall and Knight's Elementary Trigonometry as far as Chapter XIX deals with the elements of the subject up to and including the trigonometrical functions of compound angles, the solution of plane triangles and the use of inverse circular functions.

In the subjects of the Matriculation Examination for admission to the Faculty of Arts at McGill University Elementary Mathematics in which two papers, one on algebra and one on geometry, are set, is compulsory and allotted 200 marks out of a total of 1,000. Advanced Mathematics is optional and has value of 100 marks for the B.A. course, and 200 for the B. Sc. course in Arts, though in the Department of Architecture it is compulsory and carries 200 marks as its total. Candidates for admission to the Faculty of Applied Science for courses in the different branches of Engineering, must pass in Mathematics in the Senior Matriculation or First Year Arts Examination.

The matriculation requirements for McGill University are practically identical in wording as one would expect, since the same examination is set for both. But one important departure is noteworthy. In the scheme of work, though the same textbooks are mentioned, it is specifically stated that the subjects may be studied in similar textbooks. Any proof of a proposition is accepted, provided it forms part of a systematic treatment of the subject. In the course

prescribed for geometry in advanced mathematics certain parts of Godfrey and Siddons' Elementary Geometry are laid down as necessary supplements in modern geometry to that included in Hall and Stevens' textbook.

It is becoming increasingly rare, however, for pupils from the schools of the Province of Quebec to sit the Senior Matriculation Examination before proceeding to the University. The requirements in the School Leaving Examination or in Junior Matriculation are fulfilled by most entrants from the province, though some do have a more advanced knowledge approaching the level of Senior Matriculation standard.

Mathematics is compulsory for all students who enter the Faculty of Arts. The course offered embraces geometry, algebra and trigonometry: and the work done includes that proposed in the requirements for the Senior Matriculation Examination in mathematics. Though the mathematical knowledge of the students who enter their first year at McGill University is much slighter than that of the entrants to a Scottish or English university, there must be more homogeneity of attainments here than is possible in these other countries where examinations in different mathematical subjects may be taken in addition to those elementary examinations which must be passed by all. The teaching of the subject in school and in university can be articulated more closely in this province than in other places where the prerequisite attainment and course of study are less definite.

In many schools in the province mathematics is not taught by a specialist teacher, who has had interest and advanced study in mathematics at university and special training in the methods of teaching the subject. Usually it is taken by the grade teacher, though in certain cases mathematics and subjects more or less allied to it are put in the charge of one teacher who has particular ability in such subjects or in teaching them. But the differences, which are in most cases noticeable between the teaching of the mathematician and that of the teacher who has no aptitude or liking for the subject, but who is compelled to include it in his subjects of instruction, are sufficiently striking to counterpoise the advantages that accrue to a class in having the same teacher for most, if not for all, subjects.

Professor John Adams in his survey, "The Protestant School System in the Province of Quebec", made in 1902, remarks that the great defect in the teaching of mathematics is bookishness. The blackboard lesson in Euclid is usually a more or less free recitation of the book constructions and proofs, and he deplores the absence of deductions. His explanation is that it is expected of the teacher to cover too much ground in mathematics, that the time that can be spared for mathematical study is scarcely enough even for an understanding and thorough knowledge of the book work, which is necessary for examinations.

In those schools of the province, indeed, where it is expected to cover the course of study for the School Leaving

Certificate in mathematics in two years, it is almost impossible to free the teaching from an approximation to recitation of propositions: this ill tendency is emphasised by the nature of the examination questions for which the pupil has to prepare himself. A great proportion of the paper is taken up with propositions and even some of the deductions set are at times corollaries of the theorems in the textbook, more or less explicitly solved there for the pupil. Small wonder, then, that the teacher who is not particularly interested in his subject attempts to replace the teaching of mathematics by a compulsion to learn theorems and deductions by heart more or less perfectly. Schools, which spread the study of algebra over four years and that of geometry over three, as they do in Montreal, where, moreover, there are one or two mathematical specialists on the staff of each high school, show a pleasing lack of this defect; and if original deductions were given a more important place in the examination for school leaving and for matriculation, the teaching of geometry would show a great advance.

In algebra the tendency to rely on memory is not so prevalent, probably owing to the fact that the demand for ingenuity is more in evidence in algebraic exercises, which are shorter and more varied, than in geometrical deductions. This fact is noticeably true in the answers to questions set in matriculation examinations. But it doubtless owes something to the fact that in most schools in the Province of Quebec the study of algebra is begun before the study of geometry,

and consequently through longer acquaintance with the subject the pupil has a greater feeling of mastery than over geometrical proofs.

The main reason, however, is that even in Hall and Stevens' School Geometry, the approach to the subject is still too similar to the logical development in which Euclid proposed his textbook to appeal to the majority of high school boys. Algebra is sufficiently similar to arithmetic to carry the pupils over the initial difficulties, and its readiness of application to problems of everyday life, too often neglected in the presentation of the subject to secondary school pupils, shows its real scope and ensures that the pupils seek command of it.

The criticism that pupils are not sufficiently able to solve geometrical deductions or algebraic exercises is not by any means confined to the Province of Quebec. It is heard on every hand in Scotland and in England, from the very beginnings of the study of mathematics almost into the classes which deal with the higher branches of the subject. It is small wonder, then, that it is levelled at the mathematical education of pupils who have studied geometry and algebra for a period varying between a minimum of two years and a maximum of four.

The course of study offered for matriculation is crammed too much with Euclidean propositions that are seldom or never used in higher work. It would seem advisable, there-

fore, that the course should not demand logical proofs of geometrical facts that the pupil accepts unhesitatingly by intuition, that more emphasis be laid in class on the solution of deductions rigidly proved from these postulates, and that in the examination in geometry at the end of the eleventh grade of high school there should be more opportunity for pupils to show their ability in the solution of deductions with which they are unfamiliar.

In algebra the same recommendation can be made to omit much that is at present taught: not for the reason that the pupil accepts it intuitively, but on the grounds that it will be of no use to him in higher mathematical work. This would permit of progress in the school course of study to advanced sections of algebra, which can be presented in comparatively simple form and are of themselves much more interesting to the pupils.

And if this is a criticism levelled at the course of study from the point of view of the pupil proceeding to the university, what can we say of it for these others who leave Grade XI to take their places in the business world? For them its practical value is scant, its cultural value is almost non-existent, and if the disciplinary value is to depend mostly on the methods of teaching the subject, we find it hard to discover any justification. The improvements suggested above would help this difficulty probably to some extent: but it would not remove it entirely. Some suggestions can be obtained

from the recent improvements in the teaching of mathematics and work lately done in Great Britain and the United States of America--work that was undertaken to meet just such problems as the teacher of mathematics in the Province of Quebec is facing to-day.

CHAPTER V.

Recent Contributions to the Teaching of Mathematics in Great Britain and the United States of America.

As a result of new types of schools that have arisen lately in English-speaking educational system on both sides of the Atlantic Ocean, much interest has been stimulated in the curriculum and the course of study. In England the pupil in the elementary school when he reaches the age of approximately eleven and a half years, proceeds either to a secondary school for a six years' course or to a central school for three years. In Scotland the secondary school and the advanced division play corresponding parts in the education of the adolescent. In the United States of America the secondary school course of six years is now more frequently furnished by the Junior High School during the first three and the Senior High School during the second three years.

The mathematics taught in the advanced division includes the geometry of the first, third and fourth books of Euclid, algebra as far as simultaneous linear equations and easy quadratic equations, and arithmetic, that by use of mensuration on the one hand and algebraic notation on the other, helps to maintain a feeling of unity between the different branches of mathematical study. But the teacher is permitted to go further than the prescribed syllabus and advised to do so in cases where

continuation classes require further knowledge of any of the branches than is given in the course suggested.

In the secondary school, during the first three years, much the same course is followed, but Book II of Euclid and several theorems from Book VI are introduced; logarithms are studied in arithmetic and in algebra the theory of indices as far as it is necessary for intelligent use of logarithms. The elements of trigonometry as far as the solution of right angled triangles is required. It seems strange that this should be excluded from the instruction offered to those who are to leave school, as trigonometry has probably more practical application and is more readily understood than the elements of geometry.

The Leaving Certificate Course on the Higher Standard embraces the work for the Lower Standard; in addition, in geometry the content of Books VI and XI of Euclid's Elements is expected, in algebra the theory of the quadratic equation, indices and surds, and the progressions: and trigonometry to the general solution of plane triangles with logarithmic calculation. The Higher Leaving Certificate Examination is intended to be taken in the sixth year, but most schools find it possible and convenient to prepare their pupils for it by the end of the fifth year. This allows them to devote the time allotted to mathematics in the final year to additional mathematical subjects--dynamics, geometry, pure and analytical,

and elementary analysis. These subjects are not required for entrance to a Scottish university, but questions on them may be set in the Entrance Examination for Bursaries, and proficiency in these subjects may allow the pupil to go directly into the Special Class (normally for second year students) without spending a year in the General Class.

It is noteworthy that while the requirements for these examinations are given explicitly as certain theorems of Euclid, scarcely a school now-a-days uses an edition of Euclid as the textbook. In the advanced divisions books of practical mathematics are most common and these usually give a satisfactory three years' course in which the practical and the disciplinary values are both recognised.

In England in 1912 the position of mathematics in the secondary schools and in the public schools was regarded as highly unsatisfactory. The report prepared then for the Board of Education had perhaps ~~their~~ *its* chief value in the definite localisation of the difficulties and of the problems to be encountered. In the Public Schools the few who were prepared for the Scholarship Examinations at Oxford and Cambridge were given a thoroughly good course in mathematics, far beyond that given in any schools in Scotland, in Canada or in U.S.A. But the great majority of pupils had little mathematical instruction, and most of them had their courses cut short as a result of inability to keep up to the standard set by their more able companions. Several attempts were made to begin courses in

"practical" mathematics, and in several of the reports notable success in most of the experiments was the result. In the secondary schools, too, some of the pupils attained scholarship standard at Cambridge, while most were taken through a less ambitious course, which was intended as a preparation for entrance to provincial universities or technical colleges.

In the fourth Year-book of the National Council of Teachers of Mathematics, G. St. L. Carson discusses significant changes and trends in the teaching of mathematics in England since 1910. He discusses the changes in the nature of the schools and of the ability of the pupils who are now entering. But the changes in the syllabi and in the teaching of the subjects are of more interest to the present discussion. In algebra the scope of formal operations has been somewhat limited: in geometry the use of intuitional knowledge has resulted in substantial advance in the ability of the pupil. Numerical trigonometry is studied by practically all the pupils and has proved effective in increasing mathematical interest. For the first examination which is taken at the end of the normal secondary course these are the subjects most usually prescribed. But some of the more able pupils are now initiated in the "mysteries" of the calculus and a little analytical geometry is included in a paper on "Additional Mathematics". Through the calculus they find a real use for the formal algebra that is now taught and it is a tool of great promise for those who do not proceed to university work.

A two years' course subsequent to this First Examination or Matriculation standard is now proposed for the Second Examination, for which the pupils prepare by specialised study in two or three main subjects: if mathematics is one physics and chemistry are usually taken in addition. It results in a course in mathematical subjects and topics, which were formerly reserved for the universities, and the pupils who follow it are sent on to university work with a much finer and more careful preparation than was possible before.

In the Central Schools, which provide education for pupils between twelve years of age and fifteen, the advance in England has not been so striking. The matter still is in unfinished state and the temporary measures that have been adopted are fortunately regarded as such and as open to great improvement. What is required is"a close linkage of the leading ideas and process of mathematics with matters arising in science, industry and the like." Criticism, therefore, of the present condition is of little value, though it may confidently be expected that a scheme that will prove satisfactory will be evolved and put into operation within the next few years.

It would be unfair to conclude this discussion of the situation in Britain without brief mention of some of the men and works that are responsible for the successful measures that have been taken. The speech of Professor John Perry has already been alluded to: it was the spark which thirty

years ago set alight the enthusiasm of the progressive teachers of mathematics who recognised the needs of reform in schemes of work and methods of teaching. In a systematic treatise on method and textbooks to exemplify his ideas in algebra, Professor T. P. Nunn gave more definite and detailed suggestions: his is a book that is known to practically every mathematical teacher in Britain: for it must be remembered that almost without exception teachers there have not only university training in the special subjects which they profess to teach but also training subsequent in the methods of teaching the subject. This does not hold to such extent in England as it does in Scotland, which has absolute requirements on both heads, but the necessity for it is now recognised in England: in the Public Schools, of course, they still hold the view that sound knowledge of the subject is sufficient for a teacher and that professional training is of relatively slight importance. The work of the Mathematical Association cannot be over-emphasised. In a Report prepared on the teaching of geometry about seven years ago, they dealt with various points under dispute, such as the value of a definite sequence, the need for acceptance of the earlier theorems by intuition, the different stages of the course of geometry and the early introduction of the use of trigonometrical ratios. Its recommendations are seen indirectly in all subsequent discussions and are reflected in the course of study prescribed for mathematical examinations in schools throughout the whole country. It must not be imagined, however, that

Britain has been slow to profit by the studies made in the United States, where more attention has been directed to different aspects and problems in recent years than in any other country. To a summary of their findings, which are valuable to all mathematical teachers we now proceed.

A place of honour must, of course, be accorded to "The Psychology of Algebra" by E. L. Thorndike and his co-workers. It does not restrict itself to a mere discussion of the nature and constitution of algebraic abilities, but it takes into consideration allied matters--the uses of algebra, the interest of pupils in algebra in comparison with other subjects of instruction and in various features of algebraic learning and gives recommendations as to the arrangement of topics, the types of exercises and the methods of teaching on the strength of the psychological study. It is, however, in this psychological investigation that the book is unique and has its greatest value for mathematical teachers: and it gives grounds for a critical reply to the attitude of those authorities who do not require professional training for those who are to undertake instruction in the subject. The enthusiasm with which this work has been recognised, shows the value of the contribution of psychology to the subject: but as yet there has been no such study of the psychology of geometry, though occasional papers and certain sections in books dealing with

methods of teaching geometry show that the psychological point of view is now attracting the attention it deserves.

The present tendency across the border is to organise the schools on the 6-3-3 plan: the first three years of secondary school constituting the junior high school and the second three, the senior high school. The recent literature recognises that the needs of the pupils of these different types, and textbooks and papers have been written dealing with one or other in particular detail. Smith and Reeve in their book, "The Teaching of Mathematics in the Junior High School", emphasise their view that the introduction to mathematics for such pupils should not be of the formal, logical type that has hitherto been customary. The course is based on the actual or probable social needs of the pupils, takes account of psychological principles dealing with the learning of children and particularly of individual differences, suiting itself to the average or even to the under-average and lays stress on power rather than on technique. It aims at knowledge of, and accuracy in, modern business practice, an introduction to intuitive geometry as a prerequisite for the study of demonstrative geometry, the meaning and uses of algebra with the function concept as the central unifying principle, the nature and value of numerical trigonometry and perhaps an insight into the meaning of demonstration in geometry. The programme is certainly ambitious, but it should appeal to all who recognise the folly of presenting geometry with rigour, even approaching that of Euclid's treatment, to

pupils of twelve years of age. Of equal importance is the recommendation made in "The Reorganization of Mathematics in Secondary Education" that even in schools organised on the 8-4 plan, the mathematical work in Grades VII, VIII and IX should be altered to include the same material. But even where it is impossible to put an end to the wasteful process of teaching only arithmetic before high school, it is suggested that the mathematics studied in the first years there should be such as would be most valuable for information and training--as far as the pupils are capable of receiving it--with little reference to future courses which the pupil may not take. The figures showing the enrolment of the different high school grades in this province show this provision is undoubtedly wise.

In a paper on "The Mathematics of the Senior High School" in the "Teachers College Record" of 1927, W. D. Reeve recognises four groups of students: (1) those intending to enter college or technical school; (2) those undertaking commercial work requiring algebra; (3) intending specialists in science; and (4) those who like mathematics and desire further study. In view of their differences in outlook he suggests that there should be foundation courses in elementary algebra, plane and solid geometry, plane and spherical trigonometry and intermediate algebra: and probably alternative courses in college algebra, for future mathematicians, for vocations and in general mathematics. The elective nature of these courses

indicates the attitude that many of the pupils have not the mathematical ability necessary to profit by them. But it must be pointed out here that the proportion of pupils who enter the grades of the senior high schools in the United States is far greater than that which enters the final years of the schools of this province. Further it is recommended that "the high school should encourage many of its pupils to take" these elective courses that it offers in its last years. In connection with the question of the course of study of the senior high school, the nature of the requirements of college entrance naturally comes into question: and the changes suggested in this field are as novel as those offered in the school course, though they form an obvious corollary.

As regards methods of teaching, there are real contributions presented from the same quarter. That of Paul Ligda has already been mentioned in another connection. As we have seen he seeks for proper motivation by use of "practical" problems, on the ground that "a school or life task will be more expeditiously performed if the reason for its performance is plainly seen". He points out that the inductive method cannot lead by itself to a thorough knowledge of the fundamentals of the subject and that it is as liable to abuse as the deductive method, which has been generally rejected: a combination of the two is therefore the only satisfactory method of teaching algebra. But his main achievement has been to put in its proper place the "characteristic formula" as embodying a unifying idea in problem solving. This is his introduct-

ion of the function concept, which has played so large a part in recent discussions of the subject. It is clear to anyone with experience of mathematical teaching that it is lack of this unifying principle which lies at the root of many of the difficulties that pupils meet in elementary and advanced algebra. Most textbooks pay slight attention to it, and even though one has the impression that there is little new in the idea, its definite formulation and the examples given of its use are sufficient to demonstrate the importance of its recognition explicitly by the teacher.

That the functional concept is not to be restricted in its application to the teaching of algebra is shown by the discussion of relationships in geometry and trigonometry by the Mathematical Association. Apart from applications in mensuration, which immediately come to mind, the idea of relationship, though often lost sight of, is fundamental in the early theorems on congruence, in those concerned with inequalities, in the dynamic aspects of geometry, as in linkages or in imagined variations of a geometrical figure and in theorems of proportionality. Mathematics taught with this concept clearly in view is believed to have more than possibilities of transfer of training to the many problems of real life which involve the ability to think accurately about the relationships holding between quantities.

The question of the best method of approach to the study of mathematics is more valuable for the discussion it has aroused than for any conclusions that may have been reached.

To the proposal that algebra proves the best introduction since it bears closest relationship to the arithmetic that is already known, it is contended that it is in itself harder than the study of the advanced arithmetic or the intuitive geometry proper to the grade; there is good reason for beginning it early, however, since the final result is more satisfactory if the time devoted to its study is extended even if the course followed is not made longer. The pupils find it tedious to proceed with the study of arithmetic, which does not offer much novelty, even if this does not involve a review of work formerly done, while some teachers argue that a return to the subject later in the course is scarcely less disastrous to the interest of the class. For this reason, it is difficult to recommend that intuitive geometry be taken first, though it is part of the pupil's daily needs and has been proved to offer satisfaction to the pupils' desire for variety. One method of evading this question or at least the most serious disadvantages of any of the proposed recommendations is to accept such a compromise as is offered by Schorling and Reeve's textbook "General Mathematics", a sequel to which has been written by Reeve. This incorporates with the reorganisation of the material suggested for the junior high school course most of the valuable suggestions that have been made in methods of presentation of the subject.

Within the present century the testing of the efficiency of instruction has been revolutionised in mathematics as in most other school subjects. Mathematical tests

of prognostic value have been devised and used by Professor Agnes L. Rogers. These are useful to locate promise of mathematical ability, but they cannot safely be taken as the only basis for guidance or prediction, as factors other than those covered by the test may be of supreme importance. Several mathematicians of note have failed to show great promise till their interest was aroused by a problem occurring in the mathematical course which they had begun. Achievement tests can be classified as (1) the traditional "essay-type of examination", (2) the objective examination, and (3) the standardised test. With the disadvantages of the first everyone is more or less familiar: but in mathematics as elsewhere it is recognised as possessing a value which is not invalidated by the introduction of the objective type of examination. Professor C. B. Upton discusses standardised tests in mathematics for secondary schools very fully and illustrates his treatment with typical examples in an investigation, the results of which are embodied in the report of the National Committee. Though these tests are not applicable to the Province of Quebec, where the period and course of study are so different, the results obtained in the United States show that similar work here would go far to substantiate recommendations of change in the scheme of work. The value of such tests as establishing standards of achievement can be overrated, but they have proved their worth in the stimulation of the pupils to improve on their previous achievement.

Moreover, they have a certain value as diagnostic tests, though for this purpose those specifically prepared are to be preferred.

CHAPTER VI.

Conclusion.

Though only slight attention is given to mathematics in the curriculum of the schools of this province, and even that is kept for the most part for pupils in the last two grades of high school, it seems probable that such time as is devoted to this study will tend rather to decrease than to increase through lack of self-justification. It is imperative that those responsible for the organisation of the curriculum and of the course of study should take early opportunity to give mathematics its rightful place in the education of all high school pupils: at the same time the authorities in charge of the college entrance requirements must make corresponding changes. The teachers of the subject can make their contribution to the reform by acquainting themselves with the modern tendencies in other provinces and countries.

Unfortunately, the matter involves a vicious circle. University authorities and the School Leaving Examination Board require knowledge of certain parts of mathematics. School administrators only allow the teacher a very limited proportion of the school life and school year of the pupils in which to prepare them for this examination. Teachers are apt to employ methods which achieve this end,

even if the pupils fail to derive from their mathematical study the real value it should hold for them. Consequently, it is surprising that the pupils should ever show signs of mathematical ability sufficient to justify the view that other than the present state of affairs is possible. Any transition must necessarily be gradual, but where the need is so pressing, there can be no valid excuse for delay in action by anyone who recognises the need for reform.

In conclusion, the student desires to put on record his appreciation of the help he has received from many sources: in particular, that from Dr. H. J. Silver, Secretary and Superintendent of the Schools of the City of Montreal, from Mr. D. C. Logan, Assistant Secretary-Superintendent, from the Principals and Mathematical Teachers in the Schools of the City of Montreal and of the Province of Quebec which he has visited for purposes of observation. He takes special pleasure in thanking Professor S. Laird, Dean of the School for Teachers at Macdonald College, and Professor of Education in McGill University, Dr. W. D. Tait and Dr. C. E. Kellogg, Professors of Psychology in McGill University for introducing him to the educational system of the province and to much of the literature that bears on the subject of this thesis.

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