

# Fast Simulation of Cascading Outages with Islanding

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Electrical and Computer Engineering  
in partial fulfilment of the requirements for the degree of  
Master of Engineering

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## **ABSTRACT:**

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This thesis proposes an efficient power system simulator to estimate the automatic sequence of events that follow a fault contingency leading to islanding and cascading outages. The simulator is based on a quasi-steady state model that includes island identification, under-frequency load shedding, over-frequency generator tripping, and island load flow. Contingencies can include the outage of generators, loads, or transmission lines. Often times, a fault of one or two of these power system elements can lead to many cascaded outages and system islanding. The simulator utilizes an innovative method that analyzes the null space of the DC load flow susceptance matrix to identify system islands after each disturbance. Once system islands have been determined, each island power imbalance is calculated and the simulator determines based on the power imbalance in each island whether any load shedding, generator tripping, or primary frequency regulation is required. Once these corrective actions are completed each island will either have been found to balance power or will experience blackout. In the islands that have balanced power, a load flow is computed to see if all line flow constraints are satisfied. Any lines with flow constraint violations are faulted, and the iterative process is repeated under all line flow constraints are satisfied.

The results demonstrate the ability of the simulator to quickly and efficiently predict a system's response to contingencies leading to cascading outages and islanding. Simulations were conducted on a 10-bus 13-line network, a 24-bus 38-line network, and a 72-bus 119-line network.

This thesis also examined the highly complex mixed-integer linear problem of identifying the optimum initial outage in the sense that it would cause

the maximum amount of load shedding through islanding. The results on a three-line, three-bus test properly identified the line whose initial outage caused overflows leading to system separation and maximum loss of load.

## RÉSUMÉ:

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Cette thèse propose un simulateur efficace d'un réseau électrique pour estimer la séquence automatique d'événements suite à une faute menant à une séparation du réseau en îlots et à des coupures en cascade. Le simulateur est basé sur un modèle quasi-stationnaire qui inclut l'identification des îlots, le délestage de charge par relais sous-fréquence, le déclenchement de génération par relais sur-fréquence ainsi que l'écoulement de puissance. Une faute ou contingence peut inclure la perte de générateurs, de charges ou de lignes de transport. Des fois, une contingence comprennent un ou plusieurs de ces éléments peut mener à de pannes en cascade et à une séparation du réseau en îlots.

Le simulateur utilise une méthode innovatrice qui analyse l'espace nul de la matrice de susceptance d'écoulement de puissance CC pour identifier des îlots du réseau après chaque contingence. Une fois que les îlots ont été déterminés, le simulateur calcule le déséquilibre de puissance à chaque îlot et détermine s'il est nécessaire de délester de la charge, de déclencher de la génération ou de régler la fréquence. Une fois que ces actions correctives sont accomplies, chaque îlot sera dans un équilibre de puissance ou éprouvera l'arrêt total. Dans les îlots où un équilibre de puissance existe, un écoulement de puissance est calculé pour voir si toutes les lignes respectent les contraintes d'écoulement. Toutes les lignes avec des violations de contrainte d'écoulement sont déclenchées, et le processus itératif est répété jusqu'à que toutes les contraintes d'écoulement soient satisfaites.

Les résultats démontrent que le simulateur prévoit rapidement et efficacement la réaction du réseau aux contingences menant à des pannes en cascade et à une séparation du réseau en îlots. Des simulations ont été conduites sur des réseaux avec 13 lignes et 10

barres, 38 lignes et 24 barres, et 72 barres et 119 lignes.

Cette thèse a également examiné le problème très complexe de programmation linéaire-entière-mixte pour identifier l'événement initial optimal dans le sens qu'il causerait le maximum de délestage suite à une panne en cascade. Les résultats sur un réseau avec 3 lignes et 3 barres ont correctement identifié la ligne dont l'événement initial mène à la séparation du réseau et à la perte maximale de charge.

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Nader

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### **List of Abbreviations:**

|        |                                                   |
|--------|---------------------------------------------------|
| MILP   | Mixed Integer Linear Program                      |
| PFR    | Primary Frequency Regulation                      |
| NERC   | National Electric Reliability Council             |
| IEEE   | Institute of Electrical and Electronics Engineers |
| MATLAB | Matrix Laboratory                                 |
| GAMS   | General Algebraic Modeling System                 |

### **List of Symbols:**

#### **General Symbols:**

|                        |                                                                     |
|------------------------|---------------------------------------------------------------------|
| $n$ :                  | Node Index                                                          |
| $l$ :                  | Line index                                                          |
| $k$ :                  | Generator index                                                     |
| $j$ :                  | Load index                                                          |
| $s$ :                  | Load shedding Generator tripping iteration                          |
| $t$ :                  | Load flow iteration                                                 |
| $i$ :                  | Island index                                                        |
| $I^i$ :                | Set of nodes belonging to island $i$                                |
| $I_g^n$ :              | Set of generators belonging to node $n$                             |
| $I_d^n$ :              | Set of loads belonging to node $n$                                  |
| $\underline{A}$ :      | System Incidence Matrix                                             |
| $\underline{B}$ :      | DC load flow system susceptance matrix                              |
| $\underline{P}$ :      | Vector of Nodal Power Injections                                    |
| $\underline{\delta}$ : | Vector of nodal phase angles                                        |
| $\underline{X}$ :      | Matrix of null space vectors for system susceptance matrix          |
| $\underline{F}$ :      | Vector of power flows into the network from each node               |
| $\Delta f^{\max}$ :    | Maximum allowable system frequency deviation                        |
| $\Delta P^i$ :         | Power Imbalance at island $i$ prior to primary frequency regulation |

#### **Generation Symbols:**

|         |                                |
|---------|--------------------------------|
| $G^i$ : | Total generation at island $i$ |
| $g_n$ : | Total generation at node $n$   |

|                |                                      |
|----------------|--------------------------------------|
| $g_k :$        | Total generation of generator $k$    |
| $\Delta g_n :$ | Total generation tripped at node $n$ |
| $g_k^0 :$      | Initial setpoint for generator $k$   |
| $g_k^{\max} :$ | Maximum generation for generator $k$ |
| $g_k^{\min} :$ | Minimum generation for generator $k$ |
| $D_k :$        | Damping constant for generator $k$   |
| $H_k :$        | Inertia Constant for generator $k$   |

#### **Load Symbols:**

|                |                              |
|----------------|------------------------------|
| $D^i :$        | Total load at island $i$     |
| $d_n :$        | Total load at node $n$       |
| $d_j :$        | Total demand of $j$          |
| $\Delta d_n :$ | Total load shed at node $n$  |
| $d_j^0 :$      | Initial setpoint of load $j$ |

#### **Primary Frequency Regulation Symbols:**

|                |                                                    |
|----------------|----------------------------------------------------|
| $R_i :$        | Total primary frequency regulation at island $i$   |
| $R_i^{\max} :$ | Extreme limit on up regulation for island $i$      |
| $R_i^{\min} :$ | Extreme limit on down regulation for island $i$    |
| $r_k :$        | Primary frequency regulation for generator $k$     |
| $r_k^{\max} :$ | Extreme limit on up regulation for generator $k$   |
| $r_k^{\min} :$ | Extreme limit on down regulation for generator $k$ |
| $r_n :$        | Total primary frequency regulation at node $n$     |

## **CHAPTER 1: INTRODUCTION**

---

### **1.0 System Security in the 21<sup>st</sup> Century:**

As society progresses into the 21<sup>st</sup> century, power system security is becoming a growing concern. Costs on society for sustained periods of blackout are exceptionally high. Estimates for economic losses due to the August 14<sup>th</sup>, 2003 blackout in North America are in the \$7 billion to \$10 billion range [3]. Despite this, in North America we have let our utilities age to a point where in some cases over 50% of the components were installed prior to 1960 [6]. This is occurring at a time when demand levels are reaching record highs and we are pushing system components to their limits [4]. This aging trend, along with a push to integrate renewable forms of generation into the distribution grid, and the opening of electrical generation to market forces are combining to bring the reliability of the entire grid into question. Numerous recent large cascading outages serve as a further reminder that power systems are not indestructible [8]. This thesis focuses on large cascading outages and specifically on how we can efficiently simulate them.

### **1.1 Cascading Outages and Islanding**

A cascading outage is a power system failure that begins with a simple contingency, for example a single line or generator outage, and results in a widespread failure of the system. As an example of a recent cascading outage, we can look at the blackout in Eastern Denmark and Southern Sweden on September 23<sup>rd</sup>, 2003. Here, there were initially only two faults: the loss of a 1200MW nuclear power plant and a failure at a substation 300km away. The combination of these two faults led to the overloading and eventual failure of local components, and a domino effect of outages that led to the loss of power for 4 million people [8].

The Italian blackout on September 28<sup>th</sup>, 2003 is another example of a recent cascading outage where system islanding occurred. The initial outages occurred in Switzerland, from whom Italy imports power. Arcing between a tree and a transmission line in Switzerland caused the tripping of that transmission line. This failure led to the power that was being carried on this line being picked up by another nearby line that overflowed for 15 minutes. During this time the system operator attempted to make topological system changes that would relieve the overflow. However, the operator was unable to relieve the overflow before the line overheated and tripped, an event that, within seconds, led to the tripping of two more lines and to the separation of the Italian and Swiss systems. Once separated, the Italian system did not have enough generation to support the demand. Since load could not be shed quickly enough to prevent an under-frequency condition, this led to an entire system collapse [11].

#### *1.1.1 Example of a Cascading Outage and Islanding*

The example provided below shows how a cascading outage propagates through a network. First, as shown in Figure 1, the system is operated with all generators and loads operational and all line flow constraints satisfied. Then, the contingency where transmission line 2 is taken out of service is considered. The removal of line 2 leads to an over-current of line 3 as a result of power that was being carried on line 2 being transferred to line 3. The over-current of line 3 causes line 3 to overheat and eventually trip. This cascaded tripping of line 3 results in system splitting and the formation of 2 separate islands within the network; one with a generation shortage and the other with a generation surplus. The network at its various stages and its DC load flow solution is given below. This simple example demonstrates how a single line outage can lead to system islanding and collapse. The network data is given below:

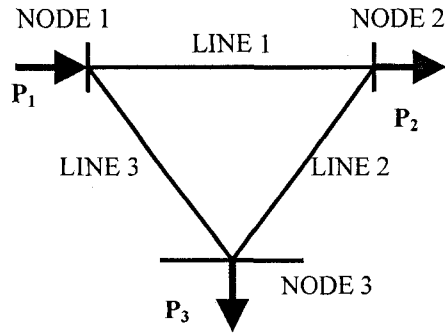


Figure 1: 3-Bus Network, all Flow Constraints Satisfied

$$P_1 = 90 \text{ MW}; P_2 = 30 \text{ MW}; P_3 = 60 \text{ MW} \quad \text{Power Injections}$$

$$F_1^{\max} = F_2^{\max} = F_3^{\max} = 50 \text{ MW} \quad \text{Line Flow Constraints}$$

Where  $P_n$  is the real power injection at node  $n$ , and  $F_l^{\max}$  is the maximum power flow on line  $l$ . The DC load flow solution with node 1 defined as the reference bus is,

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -40 \\ -50 \end{bmatrix} \text{ deg}, \quad \text{abs} \left( \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \right) = \begin{bmatrix} 40 \\ 10 \\ 50 \end{bmatrix} \text{ MW}$$

So with all lines in service the system is operating within limits. Now, if we take the contingency where line 2 is tripped, the resulting network is shown below,

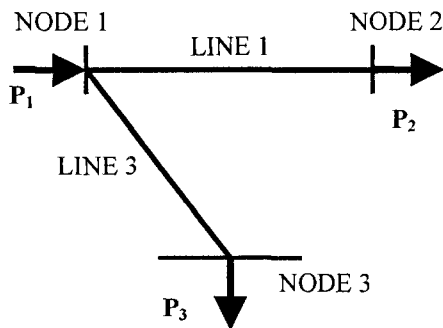


Figure 2: 3-Bus Network, Outage of Line 2

The new DC load flow solution is,

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -30 \\ -60 \end{bmatrix} \text{ deg,} \quad \text{abs} \left( \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \right) = \begin{bmatrix} 30 \\ 0 \\ 60 \end{bmatrix} \text{ MW}$$

We see that line 3, at 60MW, is carrying more power than its thermal limit of 50MW allows. Within a matter of minutes, line 3 will overheat and trip. This will result in the islanded system shown below.

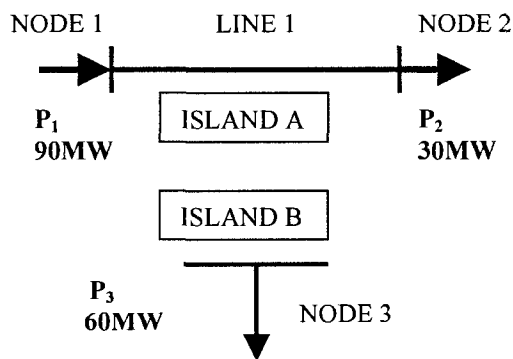


Figure 3: 3-Bus Network, Islanded

In this example island A has a generation surplus of 60MW while island B has a generation deficit of 60MW. This will result in the frequency at island A increasing and the frequency at island B decreasing. If primary frequency regulation within each island, can correct the respective power imbalances then no load shedding or generator tripping will be required. If there isn't enough regulation present within an island then that island will need to conduct generation tripping (Island A) or load shedding (Island B). In general, it is possible that the only balancing solution is for all generation to be tripped and all loads to be shed. This example demonstrates how a single line outage can lead to system islanding and a possible system wide collapse. The topics of frequency swing and primary frequency regulation, as well as load shedding and generator tripping will be covered extensively in chapter 2.

## 1.2 Historical Review

A real power system is far more complex than the network given in the example above and it is not always as simple to determine outages and islanding that may follow an initial contingency.

Many papers have been written on the topics of cascading outages, system islanding, and the restoration process. The literature review that follows summarizes some of the key papers in these areas.

### *1.2.1 Literature Review of Cascading Outages:*

In [10] Wang and Thorpe addressed the problems of simulating cascading outages and identifying the areas of greatest vulnerability within a network. Specifically, the authors addressed the propagation of line outages through a network due to the improper functioning of protection relays. The National Electric Reliability Council (NERC) has found that the improper functioning of protection relays has helped spread smaller system disturbances into much larger cascading system wide outages in a number of blackouts [10]. The aim of the paper [10] was to quantitatively identify the areas of greatest vulnerability within a network so that protection enhancements in the form of advanced protection relays could be installed at these locations. By enhancing the network in the areas of greatest vulnerability the aim is to prevent the spreading of smaller disturbances that would have otherwise not been caught by the standard relays already in place.

In order to find the areas of greatest vulnerability simulations of cascading outages needed to be conducted to identify the relay failures within a system that are most often involved in blackout scenarios. The two types of relays that were simulated in the model were line protection relays and generator tripping relays. Stochastic failure models were implemented for

both types of relays. A heuristic random search algorithm was applied along a blackout tree that took advantage of the fact that disturbances usually spread through a transmission network in only one direction. After all desired blackout scenarios were simulated, a quantitative vulnerability for each protection relay was computed based on the number and severity of blackout scenarios the relay was involved in. With this vulnerability calculated, a budget constrained optimization was then performed to indicate which relays were best suited for upgrading. The model was applied on the NYPP 3000-Bus System, and relay upgrading recommendations were made based on the results. While the paper made some simplifications on modeling to ease the computational burden, for example the inclusion of only two types of relays and a stochastic failure model, the results still provide a good basis from which a system planner can make restructuring decisions. Much of the ground work for this paper is contained in [5].

Another paper that utilizes a probabilistic approach to simulate cascading outages and deals with hidden failure modes of protective relays is [9]. In this paper a CASCADE model is developed to simulate cascading outages. The initial line flows are each based on a random variable. The model is based on the fundamental principle that if a line fails the power it was carrying is transferred to surrounding lines. A disturbance is characterized by an increase in flow on each line by an amount  $\Delta$ . The size of  $\Delta$  for a given disturbance and network is approximated by the average number of parallel paths in the network. For a well meshed network  $\Delta$  will be small. The simulation then determines the minimum increase in flow on each line that would cause an additional line to trip, thus helping propagate a cascade.

As noted in the paper one shortcoming in the model is that network structure has not been used to determine the actual amount that each line

flow should increase for each different disturbance. The variable  $\Delta$  is not disturbance or line dependent. There are very few outages if any that will cause a uniform increase in line flows. Here again we have a case where simplifications were made in order to make the model more applicable to very large systems.

In [4], an expert system approach is utilized in an attempt to eliminate erroneous protective relay operations. Here Tan et al propose employing an artificially intelligent area wide backup protection system. The goal is to precisely identify fault locations so that only circuit breakers that would act to isolate the fault are tripped. Once the fault location is identified, unnecessary trip signals generated by conventional relays would be blocked by the backup system to prevent a cascading outage. The BPES (Backup Protection Expert System) outlined here is comprised of four components: 1) Communication and Data Acquisition, 2) Data Monitoring, 3) Decision Making System, and 4) Tripping and Blocking System.

A BPES device is located in each substation control room and each device has an exact knowledge of the system architecture, operational status, and how each protection relay would respond to a given fault. It is important to note that a method for accurately and rapidly predicting each relay's response is needed. Each BPES device exchanges its data with the others via a substation communication network. If necessary, the BPES will transmit its data to the expert decision making entity. The decision operating system then activates the tripping and blocking system. The system would employ the same protection relays utilized in the current system but would alter the way a relay responds to a given fault by modifying the timing of the trip and blocking signals transmitted to a relay. This system provides an innovative solution for enhancing the security of a network, but it must be noted that a fair amount of time would be needed in order to completely test and put such an expert system into place.

### *1.2.2 System Islanding*

A number of papers have been written on system islanding, not only as the result of cascaded outages but also as an intentional last resort to avoid system collapse.

Vittal and You have worked on several papers that attempt to intentionally island a system in order to facilitate the post fault restoration process [14], [15], and [18]. In [15] a method for partitioning a collapsing network into islands which minimizes the total generation-demand imbalance within the network and facilitates the restoration process is provided. Slow coherency is used to help form groups of generators which appear to be the most strongly linked. The slow coherency model analyzes differential equations to see which groups of generators will experience similar phase angle swings when a network disturbance occurs. There are two classes of system swings, fast intra-area swings and slow inter-area swings. The method is termed slow coherency because it groups generators based on those that share the slower inter-area oscillations.

Once coherency groupings are made, the system can be partitioned into appropriate islands. Then, a load shedding scheme based on the rate of each island's frequency decline is employed. It is useful to note that while this paper [15] utilizes the rate of frequency decline to initiate load shedding, in most conventional systems it is the actual level of frequency that is used. Since the purpose of the model presented in this thesis is primarily simulation, the level of frequency is the method used to initiate load shedding as described in Chapter 2. However, as noted in [18], where an intelligent adaptive load shedding scheme based on the rate of frequency decline is proposed, the conventional method of load shedding based on level of frequency has a much slower time constant and can be overly conservative in certain cases.

In [16] Tiptipakorn proposes a spectral bisection partitioning method for forming islands within a power system. The work was conducted under the supervision of DeMarco who co-wrote [17] which contained much of the ground work for [16]. The work provides an alternative to the coherency based technique for intentionally forming islands. The author here uses a spectral method for identifying groups of strongly connected sub-networks within a given structure. One key advantage of this method over coherency based methods is that it provides partitions indicating both loads and generators within a strongly linked area. The coherency based method only indicated strongly connected generators.

The method proposed is based on the Fiedler value and Fiedler vector of the susceptance matrix of a given network. The Fiedler value and Fiedler vector are the second smallest eigenvalue and eigenvector for a given matrix, in this case a normalized susceptance matrix. For a lossless system, as assumed here, the smallest eigenvalue and eigenvector are zero and a column vector of ones respectively. The second smallest eigenvalue has the distinction that it defines for an arbitrary network the overall strength of connectivity of that network. Take for example the two networks shown below. The meshed network, system A, has a Fiedler value of 4. While the weakly connected chain network, system B, has a Fiedler value of only 0.5828. Many other examples can be given to further support this claim.

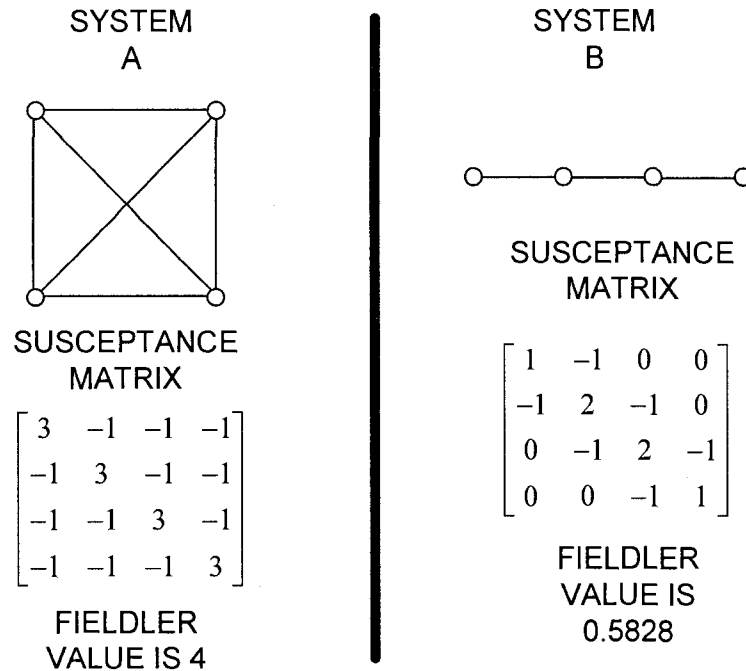


Figure 4: Fielder Values Indication of Network Connectivity

The process for determining system islands (or nodal groupings) proceeds as follows: once the normalized network susceptance matrix is found a recursive spectral graph bisection (RSB) is performed on the network, grouping the nodes into two groups, those whose corresponding entry in the fielder vector are below the mean of that vector, and those whose corresponding entry are above. In this manner the network is split in two. The bisection process continues until the number of islands which has been specified a priori is reached. One weak point with the RSB method versus coherency is that here a method isn't given to identify the optimal number of islands that should be formed. With coherency, grouping all nodes that oscillate together should theoretically form the appropriate number of islands.

### 1.2.3 Long Term Restoration Problem

The long term restoration process deals with the problem of maximizing the amount of load that can be served given that a network is operating in a degraded state. This optimization problem is a steady state problem and is entirely separate from the quasi-steady state problem of determining

system responses to faults as studied in this thesis. However, it is believed that by including this material in the literature review a more complete picture of a power system's response to a fault can be provided. Unlike the quasi-steady state problem where the actions are the automatic response of the system enacted by protection relays, here the actions and decisions are carried out by a system operator.

The main topics that were studied under restoration were bus-bar and line switching. A study was conducted to see how they could best be utilized by a system operator to maximize the amount of load served by a network operating at a reduced capacity. The goal was to see by allowing the system operator to perform line and bus-bar switching decisions more load could be served. One example of an instance where removing a line allows the network to serve more load is shown below.

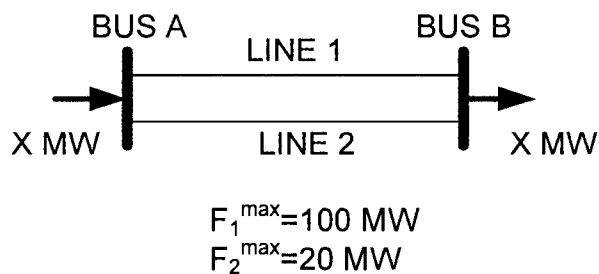


Figure 5: Line Switching Example

If lines 1 and 2 have the same reactance then it can be seen by applying a DC load flow that due to the flow constraint on line 2, the maximum power that can be transmitted from bus A to bus B is 40 MW. This follows from the fact that lines 1 and 2 must have the same phase angle differences across them since they are in parallel. Also, since they have identical reactances, their power flows must be equal. However, if we disconnect line 2 then the maximum power that can be transmitted is 100 MW. Clearly, with line 2 disconnected more load can be served. The goal in line

switching is to apply this notion to analogous but not as obvious situations in large networks.

In [1], Arroyo and Galiana study the terrorist threat problem, where the aim is to use line switching to minimize the amount of load shedding experienced by a network. The problem is posed as a bi-level problem where the outer problem is for the terrorist to maximize the amount of load shedding given a maximum number of lines they can remove, and the aim of the system operator is to minimize load shedding and balance power. The problem was formulated as a mixed integer linear program (MILP) so that powerful tools such as GAMS and CPLEX could be utilized. The transmission line operational status in this paper is characterized by the binary variable  $v_l$ , equal to 1 if line  $l$  is operational and 0 otherwise. The load flow formulated is based on the values of these  $v_l$ 's and a DC load flow approximation. The MILP models provided in this paper [1] were vital in the development of the optimization introduced later on in this thesis in Chapter 2.

In [7] Zaoui, Fliscounakis, and Gonzalez provided MILP models of bus-bar switching to maximize load served. Bus-bar switching was represented by a binary variable,  $f_{ij}$ . Here  $M$  is a large constant, and  $\theta_i$  and  $\theta_j$  are the phase angles at each end of the bus-bar  $ij$ . The bus-bar status,  $f_{ij}$ , is set equal to 1 if the bus bar linking nodes  $i$  and  $j$  merges  $i$  and  $j$  and 0 otherwise. Equations (1.1) and (1.2) force the angles of  $i$  and  $j$  to be equal when  $f_{ij}$  is set to 1.

$$M(f_{ij} - 1) \leq \theta_i - \theta_j \quad (1.1)$$

$$M(1 - f_{ij}) \geq \theta_i - \theta_j \quad (1.2)$$

Equation (1.3) defines  $T_{ij}$  as the equilibrium flow seen along the bus-bar  $ij$ . If  $f_{ij}$  is 0 then it is forced to 0 whereas if  $f_{ij}$  is 1 then it can range between  $-M$  and  $M$ .

$$-f_{ij}M \leq T_{ij} \leq f_{ij}M \quad (1.3)$$

In [2], Shao, and Vittal make several key points on bus-bar switching that help simplify the problem of substation bus-bar modeling. In this paper substation models are identified for the six most commonly used bus-bar layouts: single bus, double-bus-double breaker, main-and-transfer-bus, double-bus-single-breaker, ring bus, and breaker-and-a-half.

If we combine the conclusions made in [2] with the MILP's described in [1] and [7], a complete picture of bus bar modeling can be formulated. If we take the network from Figure 5 and assume that not only can lines 1 and 2 be switched but buses A and B can be split into buses A1, A2, and B1, and B2, then a revised model can be developed and a new maximum flow between buses A and B can be determined as follows:

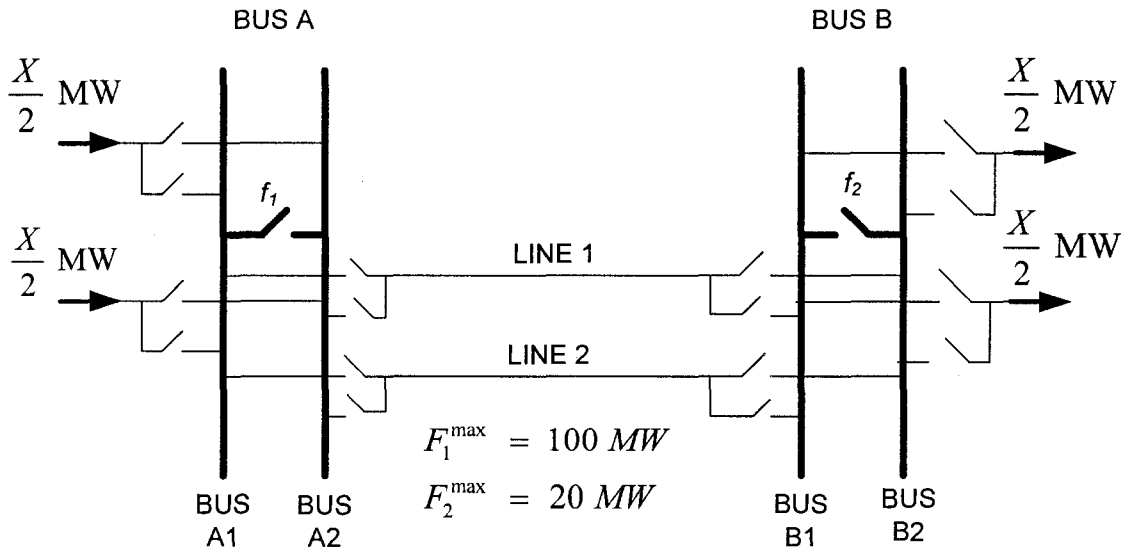


Figure 6: Bus Bar Switching Model

Here each injection is divided so that it can be split between sub-buses. If either switch  $f_1$ ,  $f_2$ , or both are opened, it would now be possible to transfer up to 120 MW of power from bus A to bus B. With both bus-bars  $f_1$  and  $f_2$  closed the maximum power flow is again 100 MW with line 2 switched off and 40 MW with line 2 switched on.

### 1.3 Motivation of Thesis

An inability to accurately predict a power system's corrective response, specifically corrective operations performed by protection relays, has led to area wide blackouts in many documented cases [5]. Protection relays have played a part in over 75% of major disturbances in recent history [4]. In a study of the August 2003 outage that blacked out much of the US Northeast and Southern Ontario, the US-Canada Power System Outage Task Force named inadequate understanding of the system corrective response as a primary cause [8]. Two other recent outages where a lack of such knowledge has been claimed to play a part in the system failure are the 1996 Western US blackout, and the 1999 Brazil blackout [8].

The motivation of the present thesis is to provide a quasi-steady state method to analyze power systems in the event of cascading outages that lead to system splitting. The proposed method should be both simple to implement and fast to calculate. Currently, simulators available are unable to account for islanding in a computationally efficient manner. The model presented in this thesis uses an efficient iterative approach to determine each island power imbalance and nominal settling frequency. Each relay action defines one iteration of the quasi-steady state model. This model is described in detail in Chapter 2.

### 1.4 Thesis Outline

Chapter 2 provides the theoretical development of this thesis. A simulator is presented that predicts a power system's response to fault

contingencies and specifically to those that lead to cascading outages and islanding. A novel approach is used to detect islanding, and the response of each individual island is simulated using a minimal amount of computational effort. The problem of automatic system response and islanding has been posed here in a manner that allows the entry of such a scenario into an optimization tool that would facilitate the identification of contingencies that would lead to the worst cases of islanding and large scale blackouts. The objective is to identify scenarios of line, generator, and substation outages that lead to the largest deterioration in system performance so that these areas of vulnerability can get more attention when protection enhancement is discussed.

In chapter 3 numerical results are provided from three different test systems. We also include a section here on the results obtained from an optimization model as applied to a 3-bus network. It should be noted that while the simulator can be applied to very large systems, the optimization model has so far only been applied to small systems.

In chapter 4 conclusions are made and recommendations for future topics of research are proposed.



## **CHAPTER 2: DETAILED CASCADING OUTAGE SIMULATION MODEL**

### **2.0 Quasi-Steady State Simulation Model**

The simulation model utilizes a quasi steady-state approach that provides a fast approximation of the reaction of a system to a contingency that leads to network islanding. The model behaves in a manner analogous to that of a DC load flow which provides a fast initial approximation of the power flows in a network. After line outages, load shedding, or generator tripping, the simulation model finds the settling frequencies of each island, the corresponding island power imbalances, and decides whether any additional corrective relay actions will take place. The method is termed quasi-steady state because it updates the settling frequency of each island after each corrective relay action resulting in the removal of a load or generator block.

### **2.1 Simulator Overview**

The simulator is described as a finite state machine in Figure 7. The various steps making up the simulator are:

Block (1): It first checks whether any islanding has occurred as a result of the initial contingency. This must be done first because only after determining which nodes belong to which islands can the simulator identify the islands' initial power imbalances and frequency deviations. In the event that there is only one island, all nodes are connected.

The identification of system islands is accomplished through the non-trivial vectors that span the null space of the system susceptance matrix,  $\underline{B}$ . A set of vectors spanning the null space of  $\underline{B}$  are stored in a matrix  $\underline{X}$ . The number of islands is equal to the dimension of the column space of  $\underline{X}$ . In addition, the set of nodes belonging to each island is defined by the set of non-zero entries in each column of  $\underline{X}$ . This is detailed in section 2.3.

Block (2): The initial power imbalance between generation and demand in each island can then be computed. If no imbalances exist, then the simulator proceeds to the first load flow iteration. If an imbalance exists, the simulator enters the primary frequency regulation block (3).

Block (3): Here, the first step is to compute for each island the primary frequency regulation from each generator that would bring the power imbalance as close to zero as possible. If the amount of available regulation in each island is enough to correct the imbalance, then no load shedding or generator tripping is required. The simulator then proceeds to the load flow iterations in block (5).

Block (4): If the available regulation is insufficient to balance power in some island, the simulator either sheds load or trips generation in a preordained block order to attempt to bring the island net power back into balance. At the end of this iterative process, each island will have either balanced generation and demand or will have shed its entire load and tripped all generation.

Block (5): The simulator then runs load flows in each surviving island. Lines whose flows exceed their limit trip out. The network structure must then be recalculated to identify any new islands. The process repeats until all line flows are within limits or have tripped.

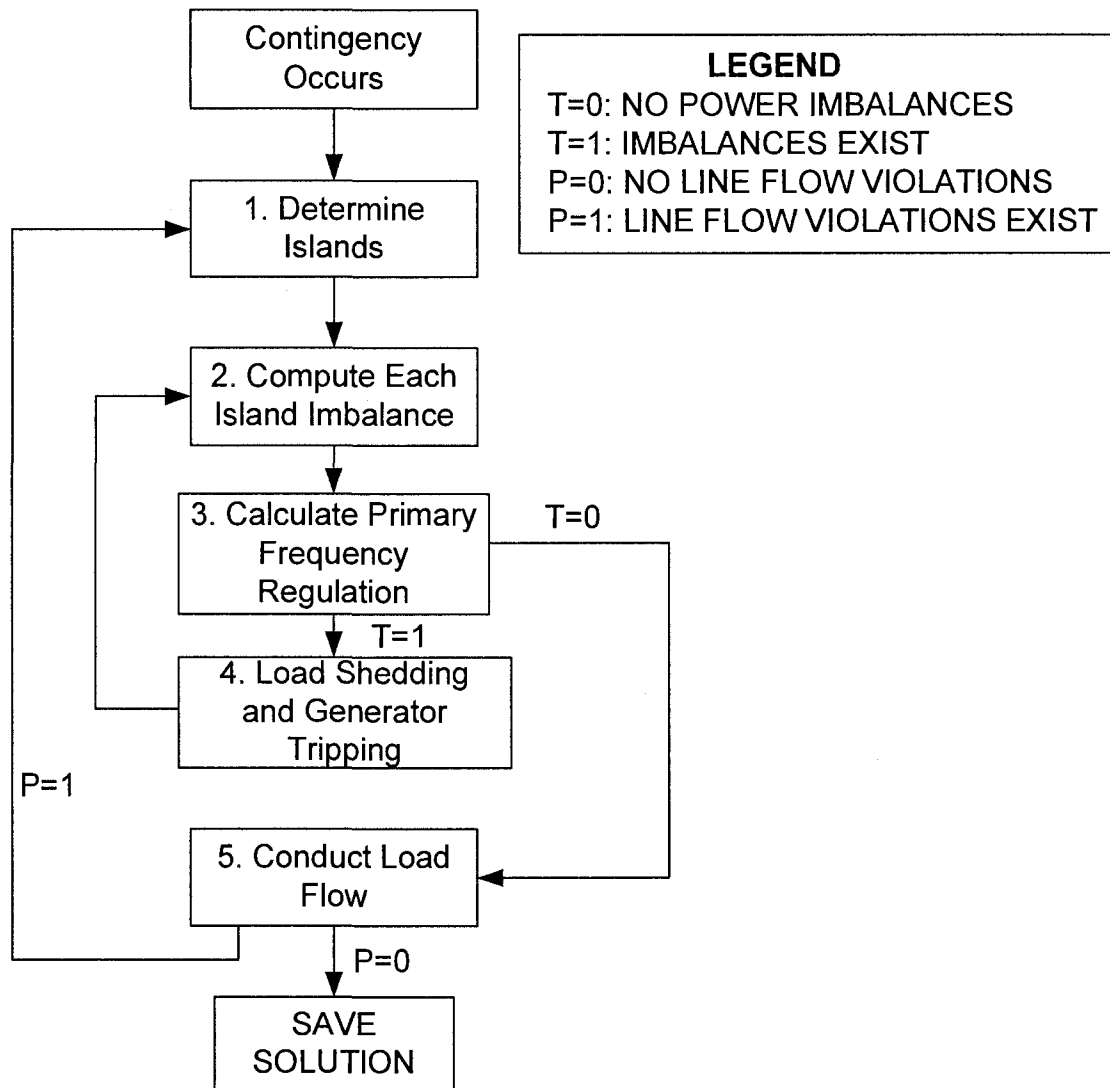


Figure 7: Finite State Machine Model of a Cascading Outage with Islanding

In the sections that follow, the individual components of the simulator are described in more detail. We however begin with the underlying simulator assumptions.

## 2.2 Underlying Assumptions

A basic assumption of the quasi-steady state model is that following the initial contingency and any subsequent relay trip, the frequency at each bus in an island settles to a common island frequency, typically within

seconds. The consequence of this basic assumption not being satisfied is discussed further on in this section.

In contrast to the quasi-steady-state model, in a transient model, after a contingency and any subsequent tripping action, each bus will experience a different frequency swing before possibly settling to a new frequency. The transient response of a power system can be modeled by a set of mixed differential and algebraic equations describing the dynamic power balance at each node. Denoting  $I^i$  as the set of buses belonging to island  $i$ , for every node  $n \in I^i$  where a generator is present, the power balance is given by,

$$\frac{d\Delta f_n(t)}{dt} = \frac{\omega^0}{2H_n} [g_n^0 - d_n^0 - F_n(\underline{\delta}) - D_n \Delta f_n(t)] \quad (2.1)$$

Where,

$$\Delta f_n(t) = f_n(t) - f^0 \quad (2.2)$$

$$\frac{d\delta_n(t)}{dt} = \Delta f_n(t) - \Delta f_r(t); r \in I^i \quad (2.3)$$

Here,  $\Delta f_n(t)$  is the instantaneous frequency deviation at bus  $n$  from nominal,  $f^0$ , while  $\Delta f_r(t)$  is the frequency deviation at the reference bus  $r$  of island  $i$ , where we recall that if the transmission network is split into islands, then each island must have its own separate reference bus  $r$ . The topic of island identification is fundamental to this thesis and will be covered in detail in the next section.

The term  $F_n(\underline{\delta})$  in (2.1) represents the net power flowing into the network

from bus  $n$ ,  $H_n$  is the machine inertia constant,  $D_n$  is the machine damping constant, while  $D_n \Delta f_n(t)$  is the corrective generation contribution from the primary frequency regulation feedback loop of generator  $n$  (see Appendix A.4). The quantity  $\delta_n(t)$  is the phase angle at bus  $n$  measured with respect to the reference phase angle at bus  $r$  whose value, for each island, is chosen arbitrarily to be zero without loss of generality.

Note also that since we allow for the possibility of more than one generator at each node, the variables and parameters in (2.1) must be interpreted as the combined contributions from all generators and loads at that bus. Thus,  $H_n$ ,  $D_n$ ,  $g_n^0$ , are respectively the summed inertias, damping factors, generation set points of all generators at bus  $n$ , while  $d_n^0$  is the sum of all initial demands at bus  $n$ .

For each node with no generation and only a load, the power balance becomes,

$$-F_n(\underline{\delta}) - d_n^0 = 0 \quad (2.4)$$

The power injections that flow into the network ( $\underline{F} = \{F_n, \forall n\}$ ) are related to the network bus phase angles,  $\underline{\delta} = \{\delta_n; \forall n\}$ 's, according to the well-known power flow equations, here represented by the DC load flow,

$$\underline{F} = \underline{F}(\underline{\delta}) \square \underline{B}\underline{\delta} \quad (2.5)$$

In steady-state, typically reached within seconds of a disturbance, the frequency at each bus  $n$  in an island  $i$  settles to a new constant

level,  $f_n(t) \rightarrow f^i = f^0 + \Delta f^i$ , characterized by setting the time derivatives in (2.1) and (2.3) to zero,

$$g_n^0 - d_n^0 - F_n(\underline{\delta}) - r_n(\Delta f^i) = 0; n \in I^i \quad (2.6)$$

Note that we have now adopted the notation  $r_n(\Delta f^i)$  for the primary frequency regulation at node  $n$  to account for any active regulation limits that may not permit linearity with respect to frequency deviation,  $D_n \Delta f^i$ . Equation (2.6) implies that it has been possible to balance power at each node  $n$ , in island  $i$  through primary regulation with a new constant island frequency deviation from nominal,  $\Delta f^i$ . If the frequency regulation capability in island  $i$  is insufficient to satisfy (2.6) then, depending on the direction of the imbalance, either an amount of generation,  $\Delta g_n$ , is tripped, or an amount of demand,  $\Delta d_n$ , is shed. This amount of shedding or tripping must bring the imbalance at each node within the range of primary frequency regulation to balance. Mathematically, this means that,

$$(g_n^0 - \Delta g_n) - (d_n^0 - \Delta d_n) - F_n(\underline{\delta}) - r_n(\Delta f^i) = 0; n \in I^i \quad (2.7)$$

Note that in practice the amount of load shed or generation tripped is not continuous but discrete. In the simulation model, shedding or tripping also occurs in discrete blocks, following a preset order of priority. The model decision whether to shed or trip is based on the sign of the frequency deviation computed in (2.6). If positive, then generation is tripped while if negative, then load is shed. Note that in some islands, power will balance only by shedding all loads and tripping all generation.

A point of contention against the quasi-steady-state model is that during the transient the frequency deviations may exceed the maximum allowed

by a protection relay, even though the settling frequency is below this maximum (e.g., the frequency deviation of generator 2 in Figure 8). This transient frequency limit violation could lead to the tripping of a generator by an over-frequency relay, an action that would have been missed by the quasi-steady state model. With this particular exception noted, it is therefore recognized that the steady-state model can in some cases be overly optimistic. Nonetheless, the computational efficiency gained by neglecting the transient response of each machine is well worth this compromise, since the simulation times are then of the order of one hundred to one thousand times faster. The difference being that in the quasi-steady-state model one solves only a set of coupled algebraic equations, while in the transient model one has to solve a set of coupled algebraic/differential equations.

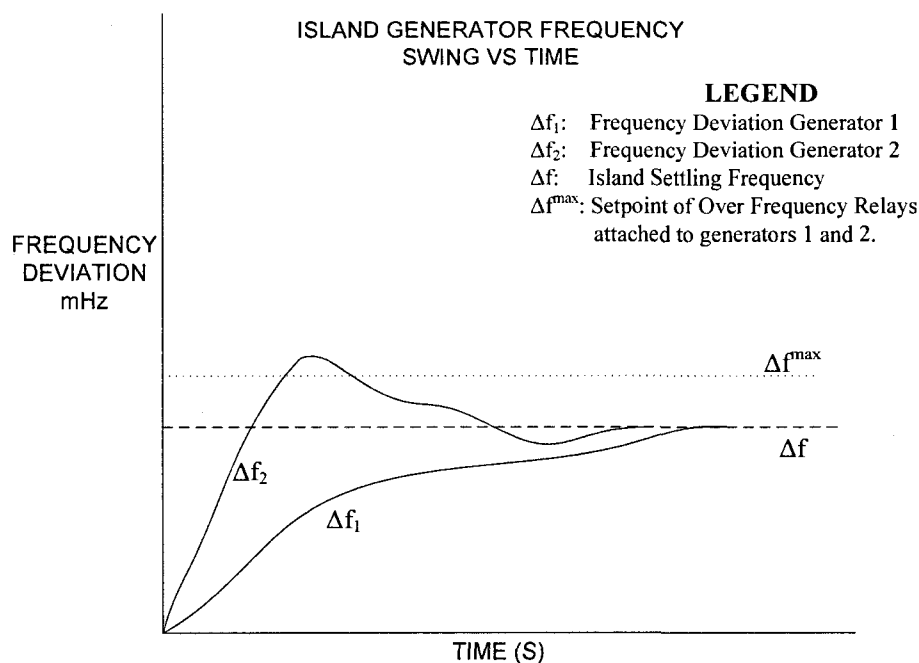


Figure 8: Post-Fault Generator Transient Frequency Response

Additional assumptions made by the model are that the transmission network is lossless, that the bus voltages are stable following a contingency, and that the system loads are known. The key point to stress

here is that the model is intended solely as a first approximation in the simulation of cascading events.

## 2.3 Identifying System Islands

The identification of system islands uses the notions of DC load flow network susceptance matrix (see Appendix A.1) and its null space.

### 2.3.1 Null Space of DC Load Flow Network Susceptance Matrix

The null space of a DC load flow network susceptance matrix  $\underline{B}$  is the space spanned by all non-zero vectors ( $\underline{x}$ ) satisfying,

$$\underline{B} \underline{x} = \underline{0} \quad (2.8)$$

It should be noted that the vector of ones,  $\underline{1}$ , belongs to the null space of any valid DC load flow susceptance matrix. To prove this, consider the following argument. Since under the DC load flow model, network branches have no shunt elements, the susceptance matrix diagonal elements are equal to the sum of all branch susceptances connected to the corresponding node. On the other hand, off-diagonal susceptance matrix terms are equal to the negative of the branch admittance connecting the two corresponding network nodes. Thus, the elements of the n-by-n  $\underline{B}$  matrix satisfy,

$$b_{nn} = -\sum_{m \neq n} b_{nm} ; \forall n \quad (2.9)$$

And as a result,

$$\underline{B} \underline{1} = \underline{0} \quad (2.10)$$

As an example, consider the 3-bus network used in the Introduction with,

$$\underline{B} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

For which,

$$\underline{B} \underline{1} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

With this noted, we proceed to the analysis of a general system that has experienced islanding.

Since the assignment of bus indices is arbitrary, if the system has split into  $ni$  islands, by reordering such indices, the  $\underline{B}$  matrix can be expressed without loss of generality as a block-diagonal matrix with  $ni$  diagonal blocks,

$$\underline{B} = \begin{bmatrix} \underline{B}^1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \underline{B}^{ni} \end{bmatrix} \quad (2.11)$$

Here, each diagonal block  $\underline{B}^i$  is the susceptance matrix of island  $i$ . Since each  $\underline{B}^i$  is a susceptance matrix, from (2.10) it follows that,

$$\underline{B}^i \underline{1}^i = \underline{0}^i \quad (2.12)$$

Where  $\underline{1}^i$  is a vector of ones of dimension equal to the number of nodes in island  $i$ . Similarly,  $\underline{0}^i$  is a vector of zeroes of dimension equal to the number of nodes in island  $i$ .

From (2.8), (2.11) and (2.12), it follows that the null space of the susceptance matrix of an islanded network, is spanned by the set of vectors,  $\underline{X} = \{\underline{x}^i; i = 1, \dots, ni\}$ , where,

$$x_n^i = \begin{cases} 0 & \text{if } n \notin I^i \\ 1 & \text{if } n \in I^i \end{cases} \quad (2.13)$$

Note that in the cascading outage simulator developed here, the Matlab command, null(B), directly finds the null space vectors spanning the susceptance matrix of an islanded network without the need to reorder the buses.

The null space matrix  $\underline{X}$  and the susceptance matrix  $\underline{B}$  then satisfy,

$$\underline{B} \underline{X} = \underline{0} \quad (2.14)$$

An alternative proof of this result is described next. Let the net power injection at bus  $n$  be denoted by  $p_n = g_n - d_n$ . Under the DC load flow assumption, for each island  $i$ , any feasible set of real power injections by energy conservation must satisfy,

$$\sum_{n \in I^i} p_n = 0 \quad (2.15)$$

Using the vector  $\underline{x}^i$  defined in (2.13) and defining  $\underline{P} = [p_1, \dots, p_N]^T$  as the vector of power injections for all network nodes, equation (2.15) can be expressed as,

$$(\underline{x}^i)^T \underline{P} = 0 \quad (2.16)$$

Now, since the DC load flow relating power injections  $\underline{P}$  and phase angles  $\underline{\delta}$  is of the form,  $\underline{P} = \underline{B}\underline{\delta}$ , then from (2.16),

$$(\underline{x}^i)^T \underline{B}\underline{\delta} = 0 \quad (2.17)$$

Since condition (2.17) must hold for any  $\underline{\delta}$ , it follows that for any island  $i$ ,

$$\underline{B}^T \underline{x}^i = 0 \quad (2.18)$$

Or since  $\underline{B} = \underline{B}^T$ ,

$$\underline{B} \underline{x}^i = 0 \quad (2.19)$$

## 2.4 Representation of Line Outages

Let the operational status of transmission line  $l$  be represented by the binary variable  $v_l$  equal to 1 if the line is in service and 0 if it is out of service. Let  $b_l$  be the corresponding line susceptance magnitude, and let  $\underline{b}$  denote the vector of all such line susceptance magnitudes. Also, let  $\underline{A}$  be the system incidence matrix (see Appendix A.1). Then, the system susceptance matrix can be represented as follows,

$$\underline{B} = \underline{A} \text{diag}(\underline{v}.*\underline{b})\underline{A}^T \quad (2.18)$$

Note that in (2.18) the operation  $.*$  represents the Matlab element by element multiplication, while the notation  $\text{diag}(\underline{x})$  represents a diagonal matrix whose diagonal is equal to the vector  $\underline{x}$ .

From (2.12) and (2.18), we can now establish an explicit relation between the null space binary vectors  $\underline{X}$  and the line operational status binary vector  $\underline{v}$ . This relation is important when seeking the worst possible initial outage in the sense that it will lead to maximum loss of load through islanding (see section 2.10),

$$\underline{B}\underline{X} = \underline{A} \text{diag}(\underline{v}.*\underline{b})\underline{A}^T \underline{X} = \underline{0} \quad (2.19)$$

#### 2.4.1 3-Bus Example

Consider the power system example from the Introduction in its islanded form (see Figure 3 repeated below),

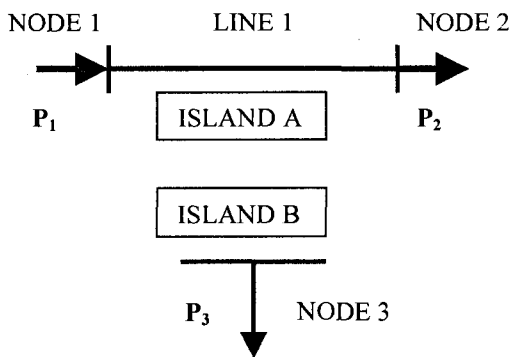


Figure 3: 3-Bus Network, Islanded

For this network the line status vector is:

$$\underline{v} = [1, 0, 0]^T$$

If we define all the line susceptance to be equal to 1 p.u then,

$$\underline{b} = [1, 1, 1]^T \text{ p.u.}$$

From (2.19), the system susceptance matrix is,

$$\begin{aligned} \underline{B} &= \underline{A} \text{diag}(\underline{v} * \underline{b}) \underline{A}^T \\ &= \left( \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} (1)(1) & 0 & 0 \\ 0 & (0)(1) & 0 \\ 0 & 0 & (0)(1) \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

From (2.12), the null space binary vectors are found to be,

$$\underline{X} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, two islands have been identified, one containing nodes 1 and 2, and the other containing node 3.

## 2.5 Power Imbalance with System Islanding

The reader is referred to the model block diagram of Figure 7. This section describes block number 2 in which the power imbalances in each island are computed prior to applying primary frequency regulation. In island  $i$ , such an imbalance is denoted by  $\Delta P^i$ , and is defined as the difference between the total generation and load in each island, including any previous contributions from load shedding and generation tripping. From the network null space vectors,  $\underline{X}$ , we can identify which nodes belong to which islands, and hence calculate the island imbalances as,

$$\Delta P^i = \sum_{n \in I^i} x_n^i \left[ (d_n^0 - \Delta d_n) - (g_n^0 - \Delta g_n) \right]; \forall i \quad (2.20)$$

Note that  $\Delta d_n$  and  $\Delta g_n$  are respectively the accumulated amounts of load shedding and generator tripping at bus  $n$  at the start of the current stage. Section (2.7) provides further details on how to calculate these quantities.

## 2.6 Primary Frequency Regulation

Block 3 of the simulation model in Figure 7 calculates the amount of primary frequency regulation that will bring the power imbalance in each island as close to zero as the available frequency regulation limits will allow.

In order to calculate the available regulation limits, we must first recognize that the frequency deviation in each island  $i$ ,  $\Delta f^i$ , must lie between strict limits (typically  $\pm 500$  mHz) so as to prevent the activation of frequency-dependent relays. Mathematically,

$$-\Delta f^{\max} \leq \Delta f^i \leq \Delta f^{\max} \quad (2.21)$$

Within these frequency deviation limits, the primary frequency regulation,  $r_k$ , provided by generating unit  $k$  located at node  $n$  in island  $i$  is of the form,

$$r_k = -D_k \Delta f^i \quad (2.22)$$

However, there exist two other limitations on  $r_k$ . The first is that the generator output including frequency regulation,  $g_k = g_k^0 - \Delta g_k + r_k$ , must lie between its allowable upper and lower limits,

$$g_k^{\min} \leq g_k \leq g_k^{\max} \quad (2.23)$$

The second is a hard limit on the amount of regulation a generator is physically able to provide within seconds from the energy stored in high pressure boilers,

$$|r_k| \leq ramp_k^{\max} \quad (2.24)$$

In summary, the limits on the amount of primary frequency regulation provided by each generator are,

$$\begin{aligned} r_k &\leq \min(D_k \Delta f^{\max}, ramp_k^{\max}, g_k^{\max} - g_k^0) = r_k^{\max} \\ r_k &\geq \max(-D_k \Delta f^{\max}, -ramp_k^{\max}, g_k^{\min} - g_k^0) = r_k^{\min} \end{aligned} \quad (2.25)$$

An example of the limits on regulation,  $r_k$ , as a function of the frequency deviation,  $\Delta f_k$ , acting on an arbitrary generator,  $k$ , is given in Figure 9.

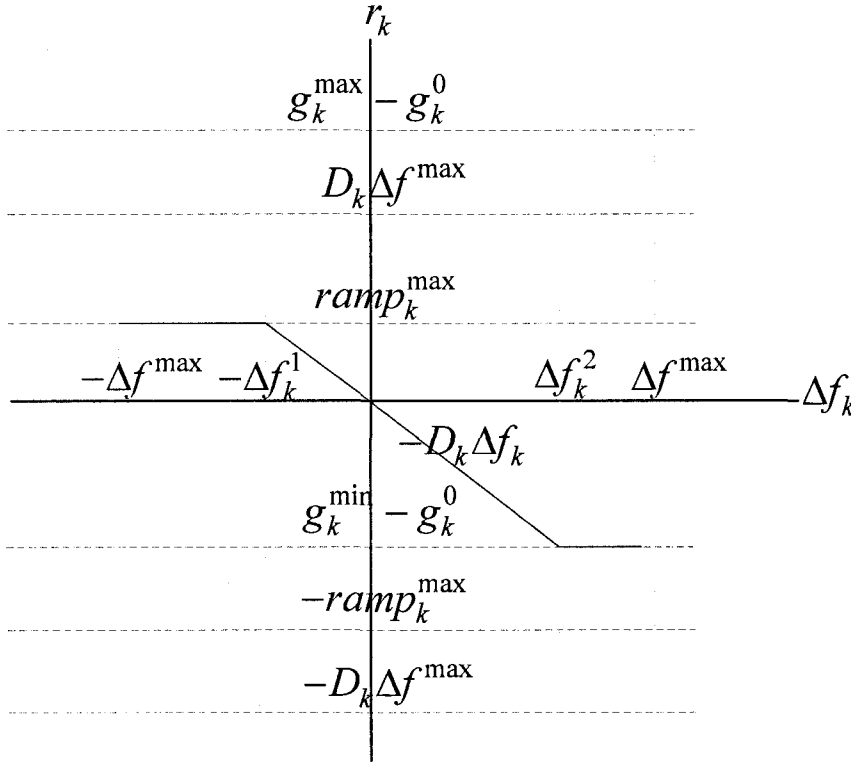


Figure 9: Primary Frequency Regulation Limits

For this example the following relationship holds for generator  $k$ :

$$r_k = \begin{cases} ramp_k^{\max} & \Delta f_k \leq -\Delta f_k^1 \\ -D_k \Delta f_k & -\Delta f_k^1 \leq \Delta f_k \leq \Delta f_k^2 \\ g_k^{\min} - g_k^0 & \Delta f_k \geq \Delta f_k^2 \end{cases}$$

So we see that below the frequency deviation,  $-\Delta f_k^1$ , the maximum regulation limit for generator  $k$  is reached, while above the frequency deviation,  $\Delta f_k^2$ , the hard limit on minimum generation is reached. In between these frequency deviations the amount of regulation provided by generator  $k$  is linear with respect to the frequency deviation. When  $\Delta f_k \leq 0$ , the binding limit on regulation is the maximum up regulation that can be given, determined by the minimum in (2.25). When  $\Delta f_k \geq 0$ , the binding limit is the minimum down regulation that can be given, determined by the maximum in (2.25).

In the simulation model we also allow for the possibility of more than one generator at each node and define the set of generating units belonging to bus  $n$  as  $I_g^n$ . The total primary frequency regulation at bus  $n$  is then,

$$r_n = \sum_{k \in I_g^n} r_k \quad (2.26)$$

This quantity will be needed in Section 2.8 in order to solve the load flow after all network islands have been identified and balanced.

The total frequency regulation in island  $i$  is given by,

$$R_i = \sum_{n \in I^i} \sum_{k \in I_g^n} r_k \quad (2.27)$$

From (2.25) and (2.27), the maximum up and minimum down regulation available in each island  $i$ , respectively,  $R_i^{\max}$  and  $R_i^{\min}$ , can be expressed as,

$$\begin{aligned} R_i^{\max} &= \sum_{n \in I^i} \sum_{k \in I_g^n} r_k^{\max} \\ R_i^{\min} &= \sum_{n \in I^i} \sum_{k \in I_g^n} r_k^{\min} \end{aligned} \quad (2.28)$$

If the island power imbalance before regulation,  $\Delta P^i$  (see equation (2.20)), is outside the range of regulation available, that is,  $\Delta P^i > R_i^{\max}$  or  $\Delta P^i < R_i^{\min}$ , then, in the first case, the amount of regulation provided by each generator in that island is set to its maximum,  $r_k = r_k^{\max}$ . Here, all possible up regulation is used. In the second case, the amount of

regulation provided by each generator in that island is set to its minimum,  $r_k = r_k^{\min}$ , thus using all possible down regulation. In both cases, the simulator then proceeds to block 4 in Figure 7. In block 4, in the first case when there is insufficient up regulation in an island, load shedding takes place within that island. In the second case, when there is insufficient down regulation, generator tripping is implemented.

However, if there is enough regulation within island  $i$  to correct the imbalance, then the equation  $R_i = \Delta P^i$  has a feasible solution  $\Delta f^i$  satisfying all constraints. This nonlinear equation in  $\Delta f^i$  can be readily solved through a binary search algorithm, which is implemented in block 3 of Figure 7.

## 2.7 Load Shedding and Generator Tripping

Load shedding and generator tripping actions are carried out by under and over-frequency relays respectively. These frequency-activated relays disconnect discrete blocks of generation and load, one at a time, in sequence. This ensures that not all load is shed or all generation is tripped simultaneously. If  $\Delta P^i > 0$  and  $\Delta P^i > R_i^{\max}$  then load shedding is required in island  $i$ . If  $\Delta P^i < 0$  and  $\Delta P^i < R_i^{\min}$  then generator tripping is required in island  $i$ .

### 2.7.1 Discrete Blocks

Load and generator blocks are disconnected from the system in a pre-defined order which is determined by coordinated relay settings. Each frequency-based relay can be programmed with a frequency deviation action level and a time delay. It is then possible to set a priority list for loads and generators to be disconnected by programming different time

delays in the relays. The relay of a low priority load may be given a shorter delay than that of a higher priority load to allow the latter to stay connected if by shedding the lower priority load the island's frequency deviation returns within the allowable range, (see figure 10).

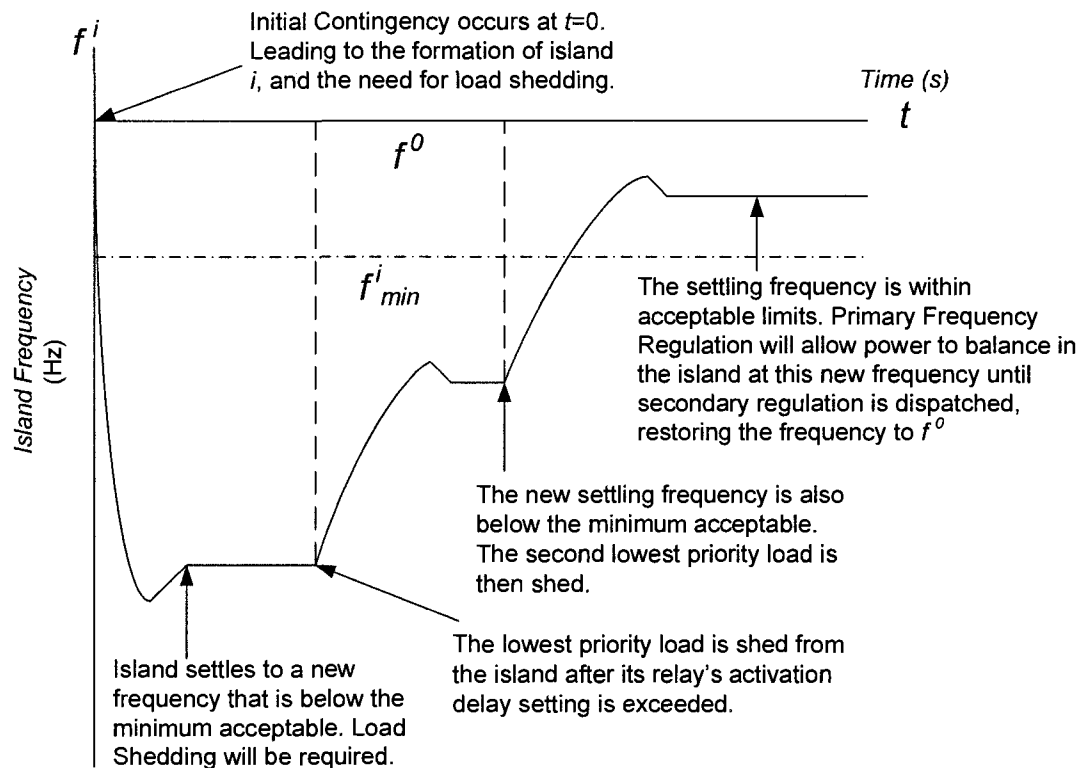


Figure 10: Under-Frequency Block Load Shedding Actions

From this figure it should be noted that the quasi steady-state approach can be overly optimistic. This is due to the fact that we assume that after each relay trip the network settles to a new frequency before the next relay action takes place. In practice, it may be the case that another relay times out before the island can settle to a new frequency. In the particular event where the tripping of a lower priority load would lead to an island settling frequency that is within the acceptable range, but a higher priority load's relay trips out due to a slow rise time in island frequency, that load in

practice would be shed whereas the simulator would miss this disconnection. This is one trade-off in using the quasi steady-state approach.

### 2.7.2 Simulator Block Priority Modelling

In the simulator, each load and generator are assigned a priority. In the first load shedding and generator tripping stage, the load and generator with the lowest priorities are shed or tripped if they belong to an island that requires their disconnection. If an island comes to balance before the completion of the load shedding and generator tripping stages then the remaining higher priority loads and generators within that island remain connected to that island.

## 2.8 Power Flow Model

Once the cumulative load shed,  $\Delta d_n$ , generation tripped,  $\Delta g_n$ , and primary frequency regulation,  $r_n$ , have been implemented at each bus  $n$ , a power flow is run (block 5). The net injection at each bus,  $p_n$ , must equal the power flowing into the network, that is,

$$p_n = (g_n^0 - \Delta g_n) - (d_n^0 - \Delta d_n) + r_n = F_n(\underline{\delta}) \quad (2.29)$$

This DC load flow is conducted for each island  $i$  by utilizing the  $x^i$  vector found in block 1 identifying which nodes belong to which island, as well as the  $\underline{B}^i$  susceptance matrix. In order to make each  $\underline{B}^i$  matrix invertible, a reference bus must be arbitrarily assigned to each island. Here, this is done by taking the minimum node number belonging to island  $i$  and making it the reference. The DC load flow then solves for  $\underline{\delta}^i$  from,

$$\underline{P}^i = \underline{B}^i \underline{\delta}^i \quad (2.30)$$

Where  $\underline{P}^i$  is the vector of  $p_n$ 's such that  $n \in I^i$ . Once  $\underline{\delta}^i$  has been determined for each island, the power flowing through each line,  $l$ , can be computed by identifying the sending ( $fr$ ) and receiving ( $to$ ) bus of that line,

$$F_l = (\delta^{fr} - \delta^{to}) b_l \quad (2.31)$$

If  $F_l$  is found to be outside the flow limits,  $-F_l^{\max} \leq F_l \leq F_l^{\max}$ , then the simulator must trip that line and return to block 1.

## 2.9 Mixed Integer Linear Programming Formulation – Optimization

The simulator described above, can now be used to examine worst-case scenarios leading to cascading outages. Such a scenario can be defined by a set of initial outages triggering cascading outages resulting in the maximum possible load shed. This can be formulated as an optimization problem in which the objective function is the total amount of load shed at the end of the cascading process, whose independent decision variables are the initial outaged lines. For each set of initial triggering variables, the simulator steps described above then define a unique set of continuous and binary dependent decision variables. The relations among these variables are also those described in the previous sections.

The difficulty with such an optimization problem is that in order to represent a cascading outage, one has to model many different binary decisions whose existence and value depend on the initial triggering outages and cannot be specified a priori. For example, binary decisions need to be made to determine: whether or not a node belongs an island,

whether each island has enough regulation to balance power, whether each island requires up or down regulation, whether a generator has reached a maximum or minimum generation level, whether each island requires load shedding or generator tripping, whether at each load shedding and generator tripping stage each particular load or generator needs to be shed or tripped, and finally, whether at each load flow stage, each line is under its flow limits.

In general, this leads to a potentially very large number of binary decision variables. For example, for a 10 bus network with a load and generator at each node and 10 lines, 20 load shedding generator tripping stages would be needed, and 10 load flow stages, in order to guarantee a solution for all possible initial triggering outages. Depending on the number of islands that are formed, the worst-case being 10, the number of binary variables needed will be of the order of 10000. In this example, a maximum of 1000 variables will be needed to fill the 10 by 10  $\underline{X}$  matrix for 10 load flow stages to determine at each stage which nodes belong to which islands.

As an example of how the simulator can be modeled in a mixed integer linear programming environment we can look at the problem of identifying whether an island has balanced power and if not, whether it requires up or down regulation. The power imbalance at island  $i$  before primary frequency regulation at load shedding generator tripping stage  $s$  at power flow stage  $t$  is described by,

$$\Delta P^{i,s,t} = \sum_n x_{ni}^t [d_n^0 - g_n^0] - \sum_n x_{ni}^t \Delta d_n^{s,t} + \sum_n x_{ni}^t \Delta g_n^{s,t} \quad (2.34)$$

Here  $x_{ni}^t$  is a binary variable that is equal to 1 if node  $n$  belongs to island  $i$  at power flow stage  $t$  and 0 otherwise,  $\Delta d_n^{s,t}$  is the cumulative amount of load shed at node  $n$  at load shedding generator tripping stage  $s$  at

power flow stage  $t$ , and  $\Delta g_n^{s,t}$  is the cumulative amount of generation tripped at node  $n$  at load shedding generator tripping stage  $s$  at power flow stage  $t$ .

Now, if  $\Delta P^{i,s,t} > 0$ , then there is a generation shortage and up regulation will be required. If  $\Delta P^{i,s,t} < 0$  then there is a generation surplus and down regulation will be required. A binary variable,  $w^{i,s,t}$ , must be set to 1 if up regulation is required and 0 if down regulation is required. A linear set of equations that can accordingly set  $w^{i,s,t}$  is,

$$\Delta P^{i,s,t} = PP^{i,s,t} - MP^{i,s,t} \quad (2.35)$$

$$PP^{i,s,t} \geq 0 \quad (2.36)$$

$$MP^{i,s,t} \geq 0 \quad (2.37)$$

$$PP^{i,s,t} \leq 1000w^{i,s,t} \quad (2.38)$$

$$MP^{i,s,t} \leq 1000(1-w^{i,s,t}) \quad (2.39)$$

Where,  $PP^{i,s,t}$  is non-zero only when  $\Delta P^{i,s,t} > 0$ , and  $MP^{i,s,t}$  is non-zero only when  $\Delta P^{i,s,t} < 0$ . The determination of whether or not there is sufficient regulation in island  $i$  would proceed from here. Equations dealing with up regulation would be multiplied by  $w^{i,s,t}$ , and those dealing with down regulation by  $(1-w^{i,s,t})$ . By doing this we ensure that only relevant constraints are imposed. This however, leads to the product of binary variables with other binary variables and binary variables with continuous variables. These equations have to be linearized to solved in a MILP, thus leading to the Introduction of linearization variables and additional equations which further add to the burden of the formulation. A prototype of this optimization is under development but its inclusion is deemed outside of the scope of this thesis.

## **2.10 Summary**

In this section a first order quasi-steady state simulator was developed to simulate a power system's automatic response to contingencies leading to cascading outages and islanding. The purpose in developing such a simulator was to develop a method that could quickly and accurately simulate the spreading of cascading outages and islanding within a network. The simulator requires a fraction of the computational time and resources that are required by a more complete model based upon a transient analysis of differential equations. The computational efficiency was numerically estimated through simulations to be of the order of 1000 to 10,000 times faster as a lower estimate (estimated using MATLAB differential equation solver). While we agree that in some cases the simulator could be optimistic, the level of detail in the results, as illustrated in Chapter 3, and the speed at which these can be obtained, demonstrate the merit of the approach. A mixed integer linear programming optimization based on the simulator is also under development.

## CHAPTER 3: NUMERICAL RESULTS

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### 3.0 Selection of Test Cases

Simulations were performed using the quasi-steady state simulator detailed in Chapter 2 to analyze the system response to contingencies leading to cascading outages and islanding. Simulations for a 10-bus, 13-line network, for the 24-bus, 38-line IEEE RTS network, and for the 3-area, 72-bus, 119-line IEEE RTS network are presented in this chapter. Complete network data, including branch, load, and generator data are given in Appendix B. Test cases were selected that led to cascading outages, islanding, frequency regulation, as well as generator tripping and load shedding actions. Simulations were performed on a Dual Core AMD Opteron™ Processor 280 running at 2.39Ghz with 4.00GB of RAM. Simulations for each network were performed in MATLAB 7.0.4.

In addition to performing simulations, the optimization program prototype introduced at the end of Chapter 2 was run on a 3-bus network whereby the worst contingencies were found by maximizing the amount of load shed given a limit on the number of lines that could be initially outaged.

### 3.1 10-Bus Example

In the 10-bus network depicted in Figure 11, 100MW generators are placed at nodes 1 through 5 and 100MW loads are placed at nodes 6 through 10. The 100kV lines have identical reactances of 50 ohms. A DC load flow analysis shows a flow of 100MW from top to bottom on lines  $l_5, l_6, l_7, l_8,$  and  $l_9$ . Each line has a flow limit of 150MW so the initial solution is feasible. All line flows and power injections in Figure 11 are in MW.

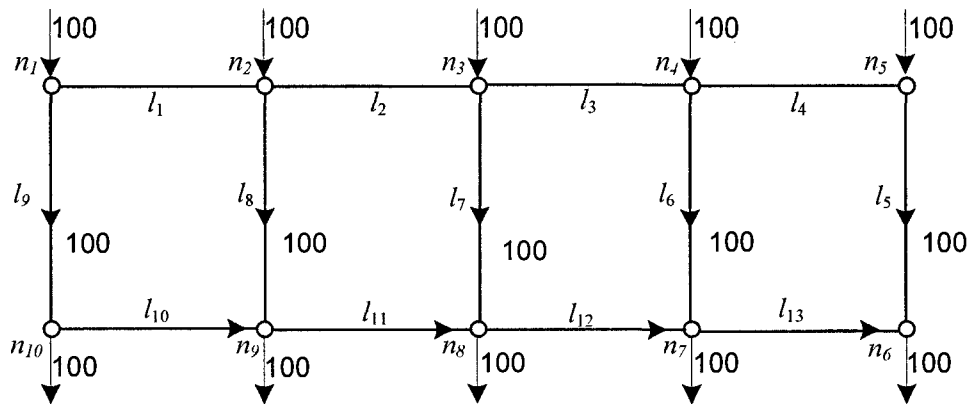


Figure 11: 10-Bus Network

### 3.1.1 Outage of Line 5

The initial contingency was the outage of line  $l_5$ , which led to the power flowing through that line to be transferred to the rest of the network via line  $l_4$ . This, in turn, led to the overflow and outage of line  $l_6$ , which caused the cascaded outage of lines  $l_1, l_3, l_7, l_8, l_9, l_{10}$ , and  $l_{12}$ . The sequence of cascaded line outages and corresponding islands is depicted in the figures below.

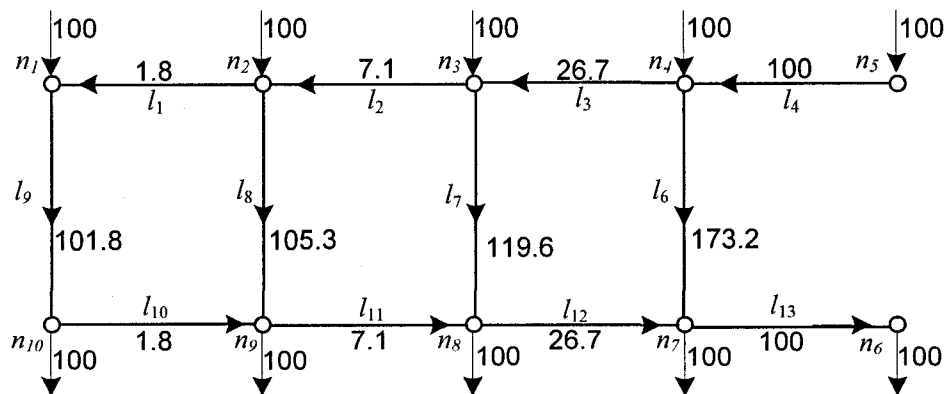


Figure 12: Network after outage of  $l_5$

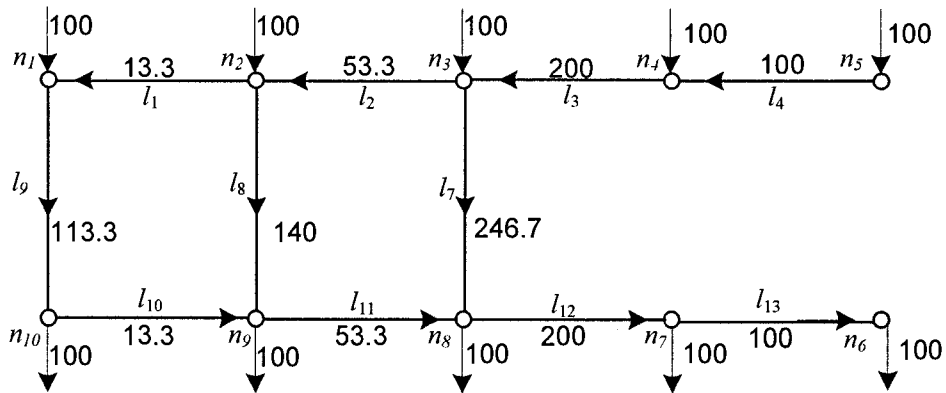


Figure 13: Network after outage of  $l_6$

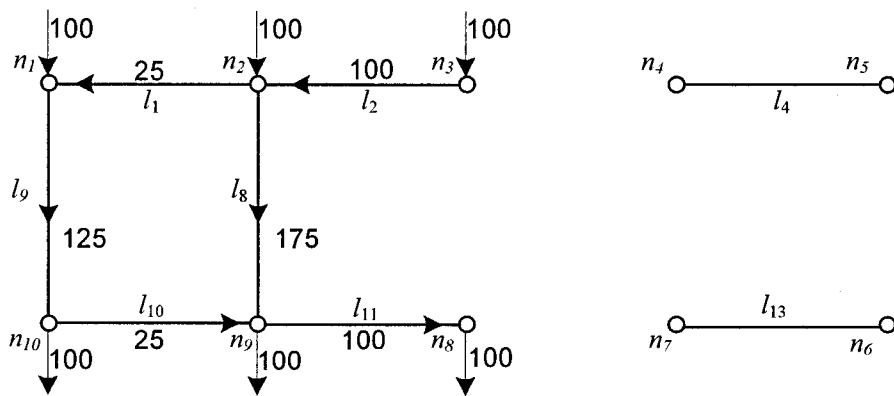


Figure 14: Network after outage of  $l_3$  and  $l_7$

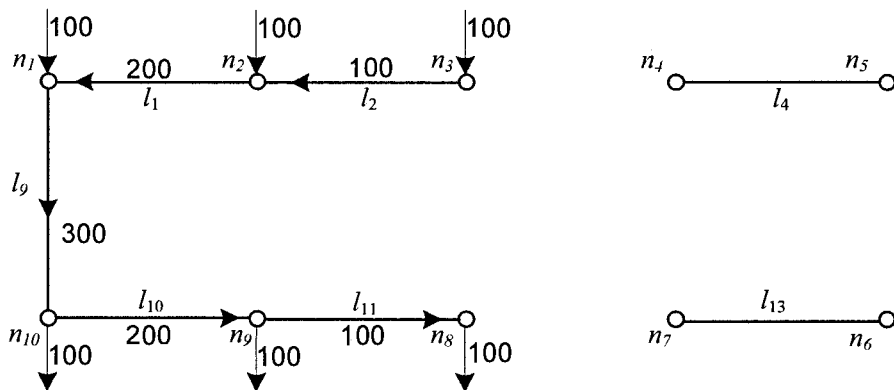


Figure 15: Network after outage of  $l_8$

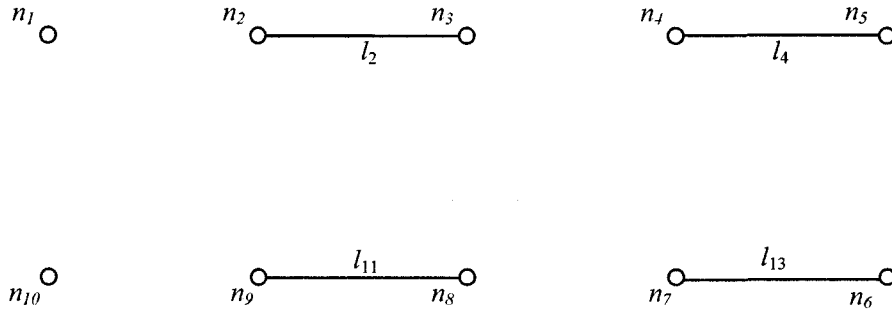


Figure 16: Network after outage of  $l_1, l_9$ , and  $l_{10}$

After all the cascading outages, only 4 lines remain operational,  $l_2, l_4, l_{11}$ , and  $l_{13}$ . These lines however do not interconnect generators to loads so at the final load flow stage, no power flows through the network, and full generation tripping and load shedding was necessary. The final grid has 6 islands defined by the null space vectors of the  $\underline{B}$  matrix:

$$\text{null}(\underline{B}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The Islands are:

- 1)  $n_1$
- 2)  $n_2, n_3$
- 3)  $n_4, n_5$
- 4)  $n_6, n_7$
- 5)  $n_8, n_9$
- 6)  $n_{10}$

The system is able to serve a diminishing amount of load as it becomes islanded as displayed in the figure below.

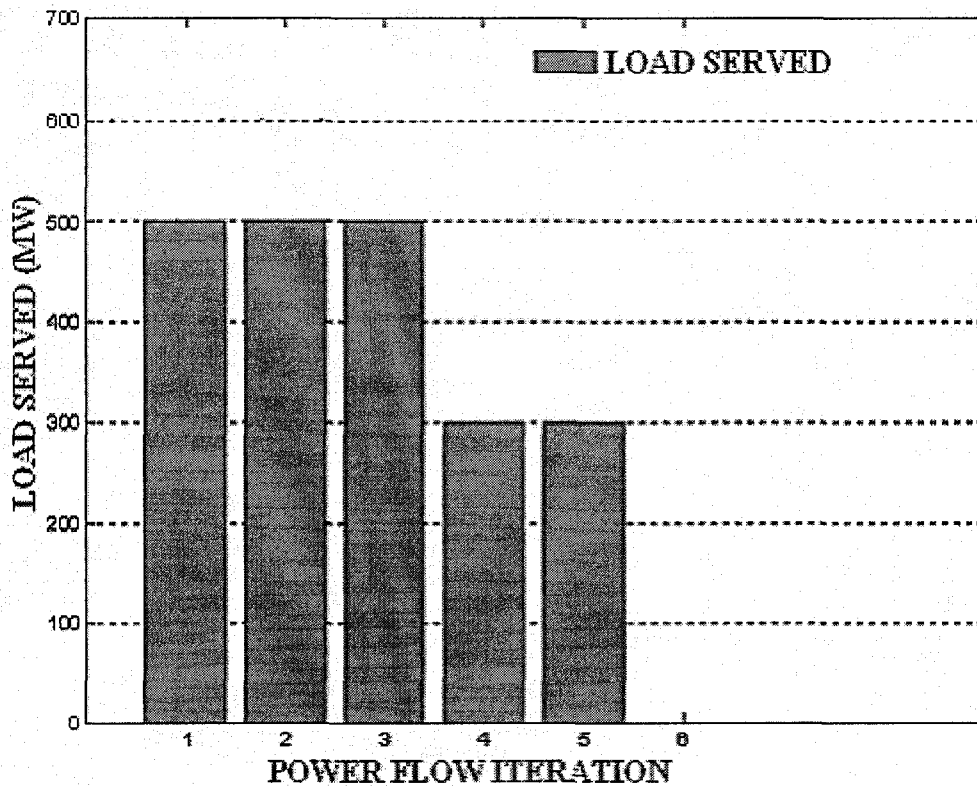


Figure 17: Diminishing Load Serving Capability, 10-Bus Network

It is evident from this graph that at the final islanding stage (6), all load has been shed and all generation has been tripped. This example demonstrated the simulator's ability to predict the propagation of a cascading outage leading to islanding and system collapse. In this example, we started off with a single line fault and ended up with a network split into six islands serving zero load. The entire simulation for all islanding stages took 0.06 seconds to complete.

### 3.1.2 Outage of Line 5 with Modification of System Generation and Loads

Next, we modify the generator and load data given in Appendix B so that all the loads at the bottom of the network are increased from 100 MW to 105 MW, 15MW generators are added at nodes 6, 7, 8, 9, and 10, and 10MW loads are added at nodes 6, 7, 8, 9, and 10. We see that although the network still experiences cascading outages, islanding, and massive load shedding, it is able to serve some load after the final islanding

iteration. The islands formed in this case were the same as in section 3.2.1. The line flows as they update from DC load flow stage to DC load flow stage are given below. Note that lines  $l_2$  and  $l_4$  still carry power in the final DC load flow stage.

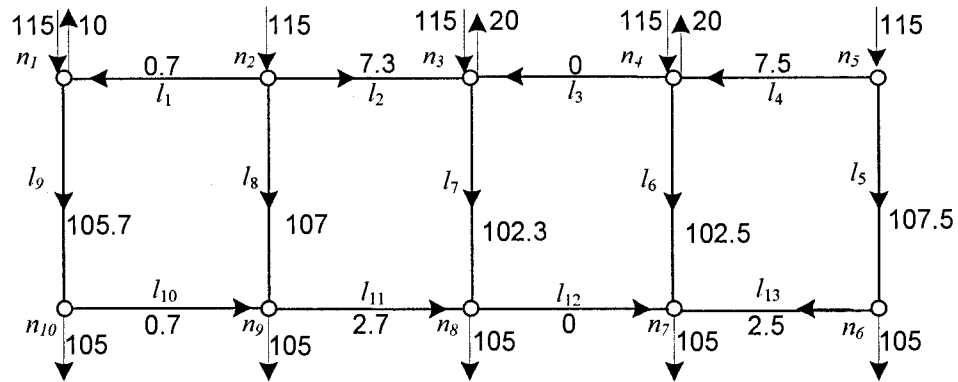


Figure 18: First Power Flow Stage

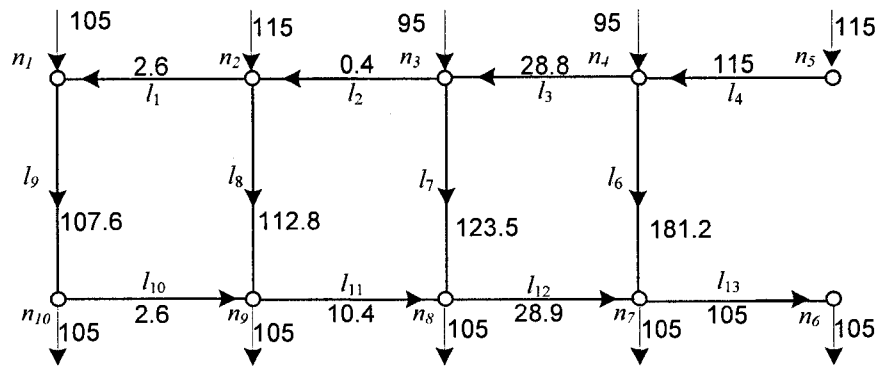


Figure 19: Outage of  $l_5$

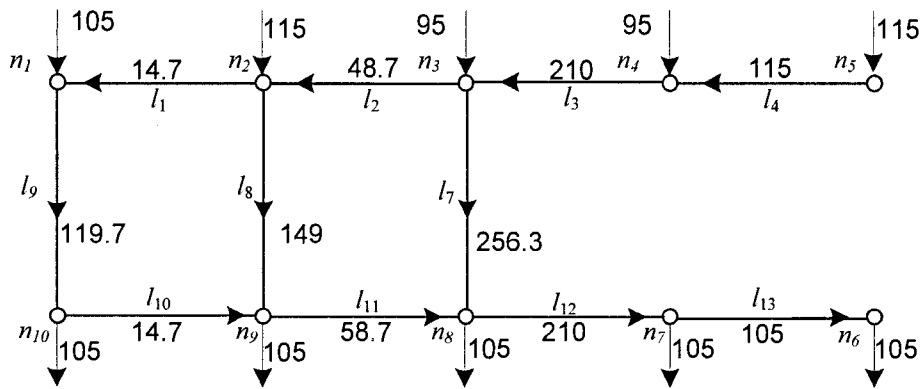


Figure 20: Outage of  $l_6$

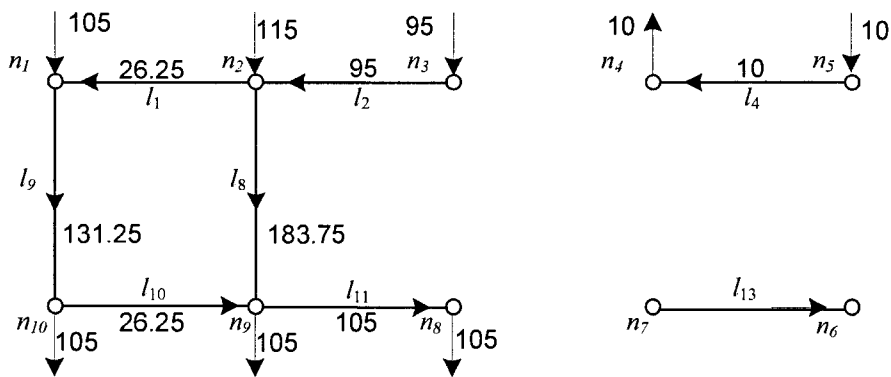


Figure 21: Outage of  $l_3, l_7$ , and  $l_{12}$

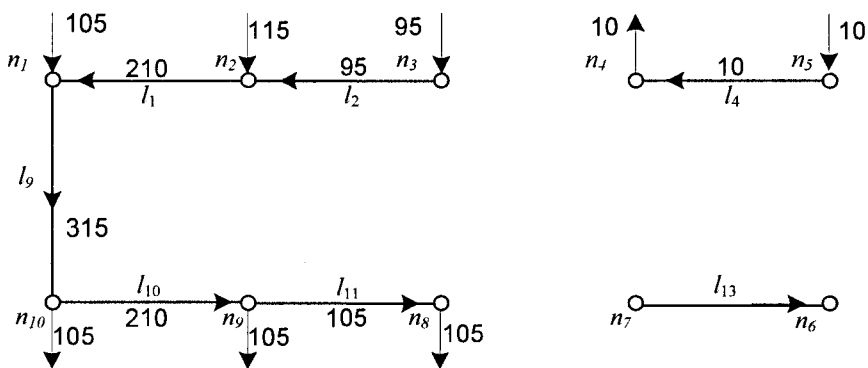


Figure 22: Outage of  $l_8$

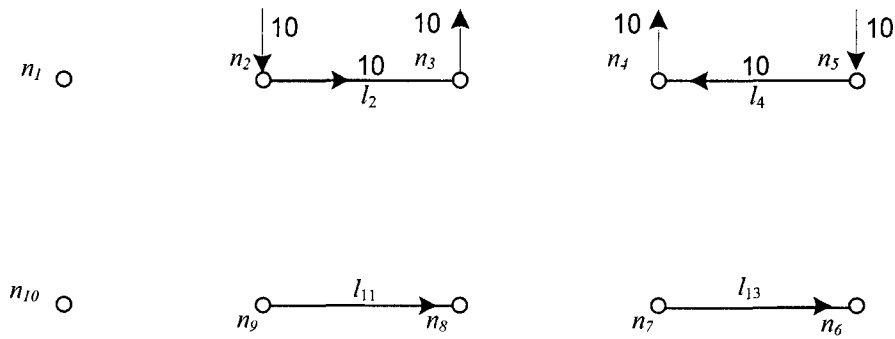


Figure 23: Outage of  $l_1$ ,  $l_9$ , and  $l_{10}$

In the last islanding stage, 10MW of load are being served at node 1 but this 10MW is being generated at node 1 so the net injection is zero. Also, at nodes 3 and 4, 20MW of load are being served but only 10MW are being supplied by the network because each of these nodes has 10MW of generation being supplied locally.

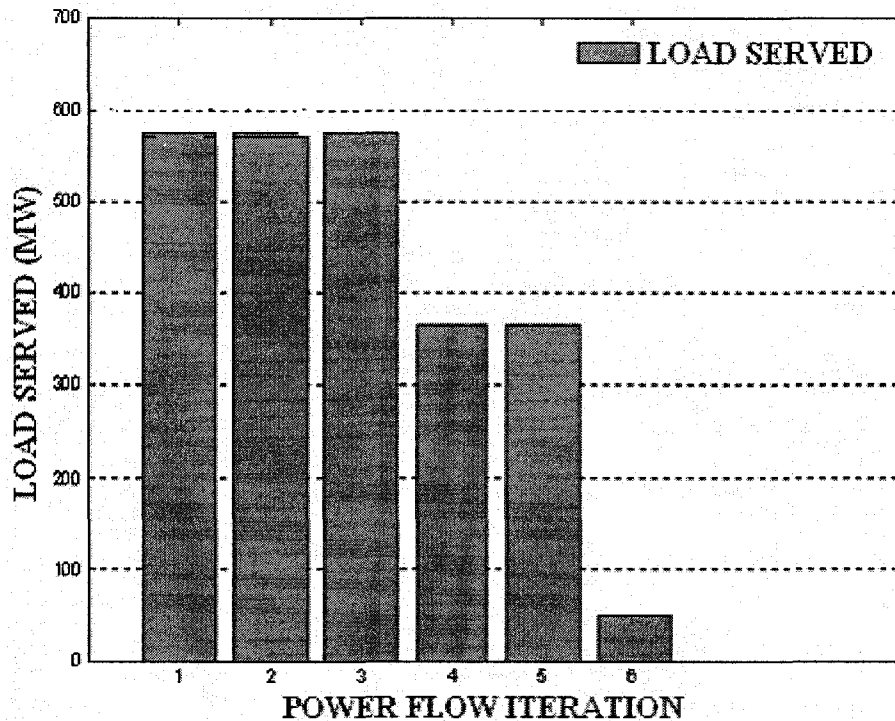


Figure 24: Diminishing of Load Serving Capabilities, 10-Bus Network

The data for the 3 surviving islands is given below:

Island 1:

| Nodes | Generation (MW)      | Generation Tripped (MW) | Regulation (MW) | Island Generation (MW) | Loads (MW)          | Load Shed (MW) | Island Load (MW) |
|-------|----------------------|-------------------------|-----------------|------------------------|---------------------|----------------|------------------|
| 1     | g <sub>1</sub> : 100 | 100                     | 0               | 10                     | d <sub>6</sub> : 10 | 0              | 10               |
|       | g <sub>6</sub> : 15  | 0                       | -5              |                        |                     |                |                  |

Table 1: 10 Bus Network, Island 1: Power Injections

Island 2:

| Nodes | Generation (MW)      | Generation Tripped (MW) | Regulation (MW) | Island Generation (MW) | Loads (MW)          | Load Shed (MW) | Island Load (MW) |
|-------|----------------------|-------------------------|-----------------|------------------------|---------------------|----------------|------------------|
| 2     | g <sub>2</sub> : 100 | 100                     | 0               | 20                     | NONE                | X              | 20               |
|       | g <sub>7</sub> : 15  | 0                       | -5              |                        |                     |                |                  |
| 3     | g <sub>3</sub> : 100 | 100                     | 0               |                        | d <sub>7</sub> : 10 | 0              |                  |
|       | g <sub>8</sub> : 15  | 0                       | -5              |                        | d <sub>8</sub> : 10 | 0              |                  |

Table 2: 10 Bus Network, Island 2: Power Injections

Island 3:

| Nodes | Generation (MW)      | Generation Tripped (MW) | Regulation (MW) | Island Generation (MW) | Loads (MW)           | Load Shed (MW) | Island Load (MW) |
|-------|----------------------|-------------------------|-----------------|------------------------|----------------------|----------------|------------------|
| 4     | g <sub>4</sub> : 100 | 100                     | 0               | 20                     | d <sub>9</sub> : 10  | 0              | 20               |
|       | g <sub>9</sub> : 15  | 0                       | -5              |                        | d <sub>10</sub> : 10 | 0              |                  |
| 5     | g <sub>5</sub> : 100 | 100                     | 0               |                        | NONE                 | X              |                  |
|       | g <sub>10</sub> : 15 | 0                       | -5              |                        |                      |                |                  |

Table 3: 10 Bus Network, Island 3: Power Injections

It can be seen here that islands 1, 2, and 3 were all generation rich so only generation tripping occurred. No load shedding occurred in these islands. It should also be noted that the higher priority generators, g<sub>6</sub> through g<sub>10</sub>, were also left running. If the priority of generator tripping is changed so that g<sub>6</sub> through g<sub>10</sub> are given tripping priority 1 through 5 instead of 6 through 10, then the network's entire generation is tripped and all loads must be shed.

Finally, we note that the total simulation time for the entire example was 0.06 seconds.

### 3.2 IEEE 24 Bus 38 Line Example

The example given in section 3.1 demonstrated how a cascading outage can propagate through a system and cause islanding. The objective now is to apply the simulator to a larger system.

The data for the IEEE 24-bus RTS system can be found in appendix B. The initial contingency studied is an outage of line 21. The line flow capacity of all lines has been reduced to 300 MW with the exception of lines 11 and 19 which have been reduced to 220 MW and 190 MW respectively. A prior unit commitment has selected Generators  $g_6, g_7, g_{16}$ , and  $g_{17}$  to be out of service. All loads are being served and the system is balanced.

It should be noted that in this system there is a large flow of power from the generation-rich north to the load-heavy south. The removal of line 21 from the system causes an overflow on other parallel lines running north to south which causes several cascading outages. These cascading outages lead to a separation between the north and south regions. Once the two regions have separated, the impact of the initial outage depends heavily on the operating state of the network.

An analysis of the post contingency susceptance matrix led to the following islands at the end of the cascading process,

$$\text{null}(\underline{B}) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

The Islands are:

- 1)  $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}, n_{12}, n_{13}$
- 2)  $n_{14}$
- 3)  $n_{15}$
- 4)  $n_{16}$
- 5)  $n_{17}, n_{18}, n_{21}, n_{22}$
- 6)  $n_{19}, n_{20}, n_{23}$
- 7)  $n_{24}$

The manner in which the outage spreads through the network is demonstrated in the plot below which shows how the number of line outages and islands grows.

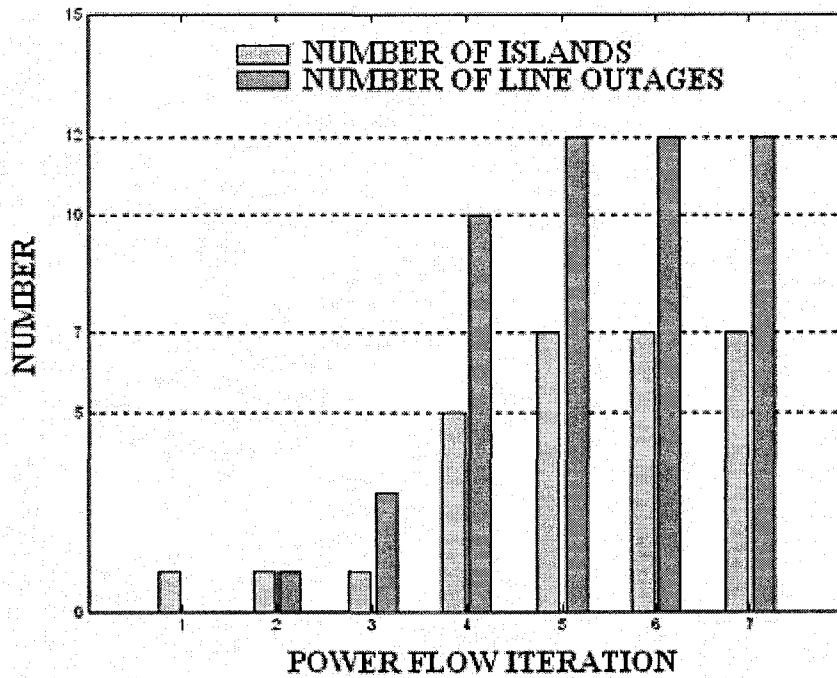


Figure 25: Cascading Outage, 24 Bus Network

As the cascading progresses, the amount of load the system serves continues to decrease due to load shedding actions. This is displayed in the following graph where the system goes from serving 2850 MW to 847 MW.

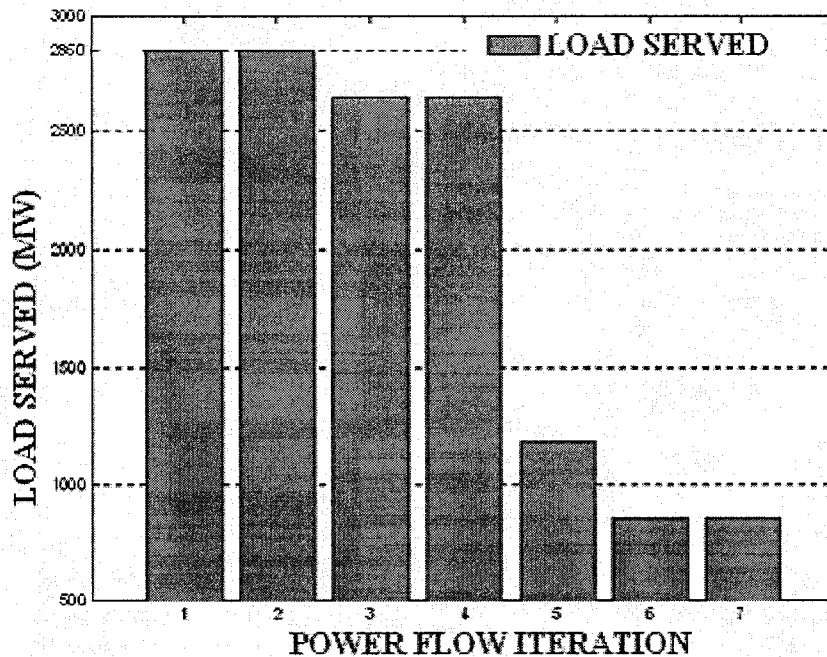


Figure 26: Diminishing Load Serving Capability, 24-Bus Network

In the last islanding stage, each island balances its power as illustrated in tables below. It is evident that islands 1 and 3 were able to balance power within their range of regulation whereas all other islands had to trip all initial generation and shed all initial load.

Island 1: Final frequency is 59.752Hz

| Nodes | Generation (MW)     | Generation Tripped (MW) | Regulation (MW) | Island Generation (MW) | Loads (MW)         | Load Shed (MW) | Island Load (MW) |
|-------|---------------------|-------------------------|-----------------|------------------------|--------------------|----------------|------------------|
| 1     | g <sub>6</sub> :0   | 0                       | 0               |                        | d <sub>1</sub> :54 | 54             |                  |
|       | g <sub>7</sub> :0   | 0                       | 0               |                        |                    |                |                  |
|       | g <sub>16</sub> :0  | 0                       | 0               |                        | d <sub>2</sub> :54 | 54             |                  |
|       | g <sub>17</sub> :0  | 0                       | 0               |                        |                    |                |                  |
| 2     | g <sub>8</sub> :43  | 0                       | -3.72           |                        | d <sub>3</sub> :97 | 97             |                  |
|       | g <sub>9</sub> :43  | 0                       | -3.72           |                        |                    |                |                  |
|       | g <sub>18</sub> :43 | 0                       | -3.72           |                        |                    |                |                  |
|       | g <sub>19</sub> :43 | 0                       | -3.72           |                        |                    |                |                  |
| 3     | NONE                | X                       | X               |                        | d <sub>4</sub> :90 | 90             |                  |
|       |                     |                         |                 |                        | d <sub>5</sub> :90 | 90             |                  |

|    |              |   |        |                |      |
|----|--------------|---|--------|----------------|------|
| 4  | NONE         | X | X      | $d_6$ :74      | 74   |
| 5  | NONE         | X | X      | $d_7$ :71      | 71   |
| 6  | NONE         | X | X      | $d_8$ :68      | 68   |
|    |              |   |        | $d_9$ :68      | 68   |
| 7  | $g_{20}$ :80 | 0 | -7.44  | $d_{10}$ :125  | 125  |
|    | $g_{21}$ :80 | 0 | -7.44  |                |      |
|    | $g_{22}$ :80 | 0 | -7.44  |                |      |
| 8  | NONE         | X | 0      | $d_{11}$ :85.5 | 85.5 |
|    |              |   |        | $d_{12}$ :85.5 | 85.5 |
| 9  | NONE         | X | 0      | $d_{13}$ :87.5 | 0    |
|    |              |   |        | $d_{14}$ :87.5 | 0    |
| 10 | NONE         | X | 0      | $d_{15}$ :97.5 | 0    |
|    |              |   |        | $d_{16}$ :97.5 | 0    |
| 11 | NONE         | X | 0      | NONE           | X    |
| 12 | NONE         | X | 0      | NONE           | X    |
| 13 | $g_{27}$ :95 | 0 | -8.432 | $d_{17}$ :88   | 0    |
|    | $g_{28}$ :95 | 0 | -8.184 | $d_{18}$ :88   | 0    |
|    | $g_{29}$ :95 | 0 | -8.184 | $d_{19}$ :89   | 0    |

Table 4: 24-Bus network, Island 1, Final Power Balance.

It must be remembered that in this island, generators  $g_6, g_7, g_{16}$ , and  $g_{17}$  started the simulation as being off. This island is a load heavy island. It should also be noted that the priority ordering is respected as loads are shed by the under frequency relays. Also note that the island frequency deviation will not be at its maximum, but In this case the island's frequency deviation is 248 mHz below nominal, which is within the maximum acceptable deviation of 500 mHz. In addition, since the block load-shedding scheme disconnects too much load, down regulation must be provided by the primary frequency regulation.

Island 2:

| Nodes | Generation (MW) | Generation Tripped (MW) | Regulation (MW) | Island Generation (MW) | Loads (MW)          | Load Shed (MW) | Island Load (MW) |
|-------|-----------------|-------------------------|-----------------|------------------------|---------------------|----------------|------------------|
| 14    | NONE            | X                       | X               | 0                      | d <sub>20</sub> :97 | 97             |                  |
|       |                 |                         |                 |                        | d <sub>21</sub> :97 | 97             |                  |

Table 5: 24-Bus network, Island 2, Final Power Balance

This island has blacked-out because it has no generation and all load had to be shed.

Island 3: Final frequency is 59.9625Hz

| Nodes | Generation (MW)     | Generation Tripped (MW) | Regulation (MW) | Island Generation (MW) | Loads (MW)           | Load Shed (MW) | Island Load (MW) |
|-------|---------------------|-------------------------|-----------------|------------------------|----------------------|----------------|------------------|
| 15    | g <sub>1</sub> :40  | 0                       | -0.45           |                        | d <sub>22</sub> :105 | 105            |                  |
|       | g <sub>2</sub> :40  | 0                       | -0.45           |                        |                      |                |                  |
|       | g <sub>3</sub> :40  | 0                       | -0.45           |                        | d <sub>23</sub> :105 | 0              |                  |
|       | g <sub>4</sub> :40  | 0                       | -0.45           |                        |                      |                |                  |
|       | g <sub>5</sub> :40  | 0                       | -0.45           |                        | d <sub>24</sub> :107 | 0              |                  |
|       | g <sub>23</sub> :15 | 0                       | -0.75           |                        |                      |                |                  |

Table 6: 24 Bus network, Island 3, Final Power Balance

This island was initially load heavy but after shedding load  $d_{22}$  it became slightly generation rich, which could be balanced within the range of primary frequency regulation. Also note that loads of the highest priority remained connected to the system. The final island frequency deviation was 37.5 mHz, below nominal, thus requiring down primary frequency regulation.

Island 4:

| Nodes | Generation (MW) | Generation Tripped (MW) | Regulation (MW) | Island Generation (MW) | Loads (MW)     | Load Shed (MW) | Island Load (MW) |
|-------|-----------------|-------------------------|-----------------|------------------------|----------------|----------------|------------------|
| 16    | $g_{24}$ : 155  | 155                     | 0               | 0                      | $d_{25}$ : 100 | 100            | 0                |

Table 7: 24 Bus network, Island 4, Final Power Balance

Although island 4 initially has both generation and load, the sizes of the generation tripping and load shedding blocks, 155MW and 100MW respectively, are too coarse to allow the system to balance within the range of frequency regulation. As a result, the island blacks out.

Island 5:

| Nodes | Generation (MW) | Generation Tripped (MW) | Regulation (MW) | Island Generation (MW) | Loads (MW)     | Load Shed (MW) | Island Load (MW) |
|-------|-----------------|-------------------------|-----------------|------------------------|----------------|----------------|------------------|
| 17    | NONE            | X                       | 0               | 0                      | NONE           | X              | 0                |
| 18    | $g_{31}$ : 400  | 400                     | 0               | 0                      | $d_{26}$ : 111 | 111            | 0                |
|       |                 |                         |                 |                        | $d_{27}$ : 111 | 111            | 0                |
|       |                 |                         |                 |                        | $d_{28}$ : 111 | 111            | 0                |
| 21    | $g_{32}$ : 400  | 400                     | 0               | 0                      | NONE           | X              | 0                |
| 22    | $g_{10}$ : 50   | 50                      | 0               | 0                      | NONE           | X              | 0                |
|       | $g_{11}$ : 50   | 50                      | 0               | 0                      |                |                | 0                |
|       | $g_{12}$ : 50   | 50                      | 0               | 0                      |                |                | 0                |
|       | $g_{13}$ : 50   | 50                      | 0               | 0                      |                |                | 0                |
|       | $g_{14}$ : 50   | 50                      | 0               | 0                      |                |                | 0                |
|       | $g_{15}$ : 50   | 50                      | 0               | 0                      |                |                | 0                |

Table 8: 24 Bus network, Island 5, Final Power Balance

Island 5 blacks out as a result of a disadvantageous list ordering. If generator's  $g_{10}$  through  $g_{15}$  were given a higher priority than  $g_{31}$  and  $g_{32}$  then the island would have been within the range of frequency regulation to balance after the tripping of generators  $g_{31}$  and  $g_{32}$  and all 333MW

could have been served. This highlights the interesting problem of optimizing the priority of generator tripping and load shedding over many different contingencies.

Island 6:

| Nodes | Generation (MW)     | Generation Tripped (MW) | Regulation (MW) | Island Generation (MW) | Loads (MW)           | Load Shed (MW) | Island Load (MW) |   |
|-------|---------------------|-------------------------|-----------------|------------------------|----------------------|----------------|------------------|---|
| 19    | NONE                | X                       | 0               |                        | d <sub>29</sub> 90.5 | 90.5           |                  |   |
|       |                     |                         |                 |                        | d <sub>30</sub> 90.5 | 90.5           |                  |   |
| 20    | NONE                | X                       | 0               |                        | d <sub>31</sub> 64   | 64             |                  |   |
|       |                     |                         |                 |                        | d <sub>32</sub> 64   | 64             |                  |   |
| 23    | g <sub>25</sub> 200 | 200                     | 0               |                        | NONE                 |                |                  | X |
|       | g <sub>26</sub> 200 | 200                     | 0               |                        |                      |                |                  |   |
|       | g <sub>30</sub> 111 | 111                     | 0               |                        |                      |                |                  |   |

Table 9: 24 Bus network, Island 6, Final Power Balance

Island 6 blacks out due to a lack of refinement in the load shedding and generator tripping blocks. If the 200MW blocks located at bus 23 were split into several smaller blocks, the island would have been able to operate at some reduced load.

Island 7:

| Nodes | Generation (MW) | Generation Tripped (MW) | Regulation (MW) | Island Generation (MW) | Loads (MW) | Load Shed (MW) | Island Load (MW) |
|-------|-----------------|-------------------------|-----------------|------------------------|------------|----------------|------------------|
| 24    | NONE            | X                       | 0               |                        | NONE       | X              |                  |

Table 10: 24 Bus network, Island 7: Power Injections

Node 24 never contained any generators or loads. It served as a feeder station from the 230kV system in the North to the 138kV system in the south. It was left on it's own due to the outages of lines 7 and 27.

The entire cascading outage simulation for this 24-bus network took 0.23 seconds to complete.

### 3.3 3-area 72-bus IEEE network

A 3-area network composed of 3 interconnected replicas of the 24 bus network utilized in section 3.2 was also simulated. As indicated in Table 11 below, five lines interconnect the 3 areas, resulting in a total of 119 lines.

| Line Number | Sending Bus, Area | Receiving Bus (Area) |
|-------------|-------------------|----------------------|
| $l_{115}$   | Bus 21, Area 1    | Bus 23, Area 3       |
| $l_{116}$   | Bus 23, Area 1    | Bus 17, Area 2       |
| $l_{117}$   | Bus 13, Area 1    | Bus 15, Area 2       |
| $l_{118}$   | Bus 7, Area 1     | Bus 3, Area 2        |
| $l_{119}$   | Bus 23, Area 2    | Bus 18, Area 3       |

Table 11: Inter-Area Connections

#### 3.3.1 3-Area Network Without Any Interconnections

When the same contingency from Section 3.2 is applied to each area of the 3-area network without any inter-area connections, we get three replicas of the solution found in 3.2. Instead of 7 islands we find 21, and instead of 12 line outages we get  $(3)(12)+5=41$  line outages.

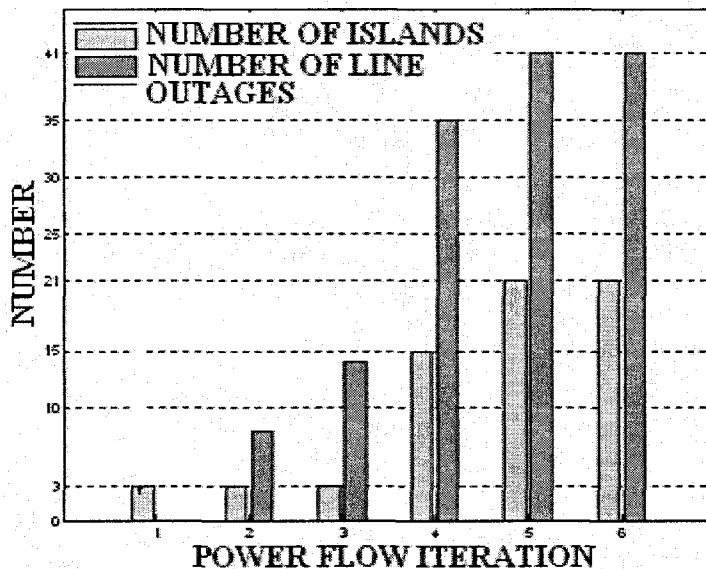


Figure 27: 72-Bus Network, 3 Areas Isolated

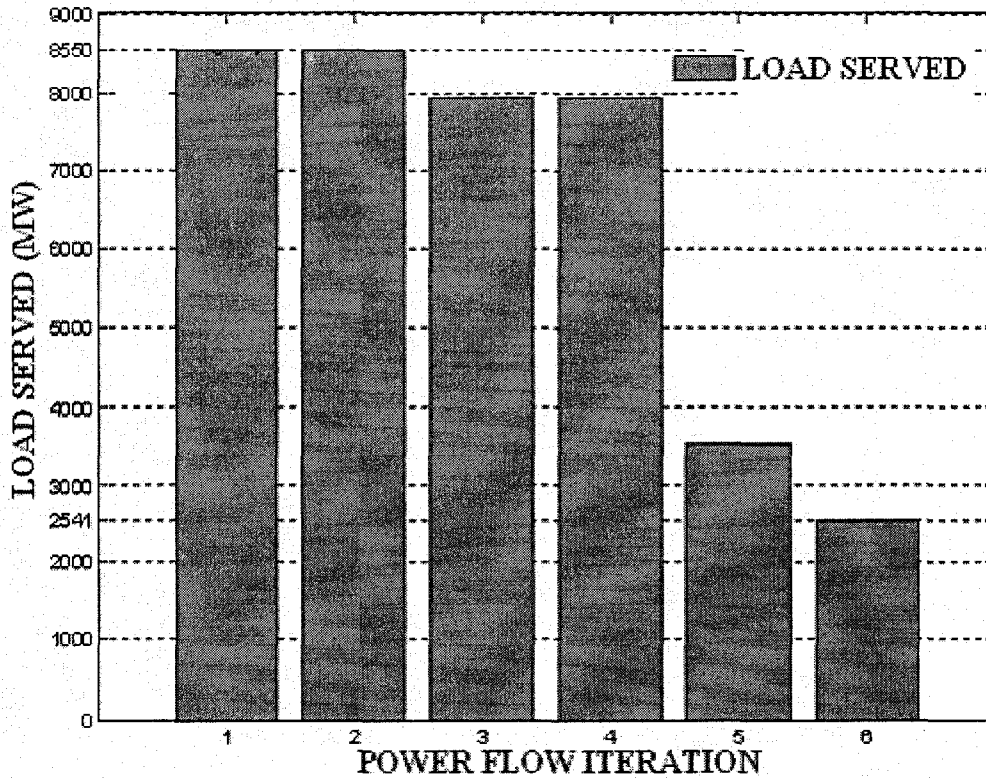


Figure 28: 72-Bus Network, Diminished Load Serving Capability

We can see from Figure 23 that the total load served is  $(3)(847) = 2541\text{MW}$ , as expected from the results in Section 3.2.

The entire simulation including output plots took under 4 seconds to complete.

### 3.3.2 Full 3-area network

With the interconnection of the 3-areas by the 5 additional transmission lines, we expect the network to be less sensitive to cascading outages. The simulator confirms this prospect when the interconnected network is subject to a contingency defined by the removal of line 21 in each area. The final interconnected network then serves 4807 MW (instead of 2541MW) and splits into 18 islands versus 21.

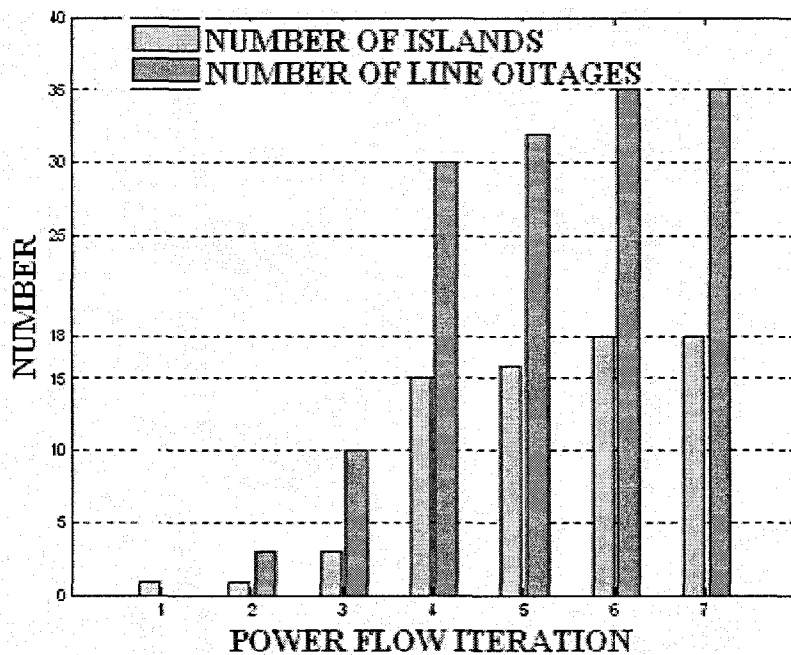


Figure 29: 72-Bus Network with Interconnected Areas

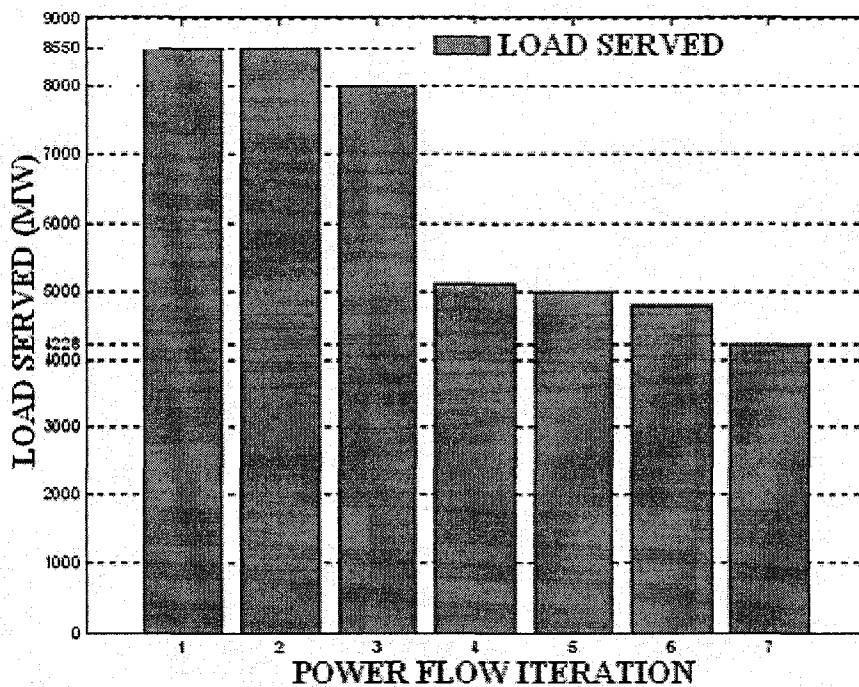


Figure 30: 3-Area, 72-Bus Network, Now Able to Serve 4807 MW

From Figures 24 and 25 we can see that the network, although still severely degraded, was able to serve almost twice as much load as in the previous case where the inter-area connections were not present. A complete summary of the results can be found in Tables 12, and 13

| ITER | Vector of Load Served<br>In Each Island (MW)              | Vector of Island Loads<br>Shed (MW)                                 | Vector of Island Generation<br>Setpoints (MW)                |
|------|-----------------------------------------------------------|---------------------------------------------------------------------|--------------------------------------------------------------|
| 0    | 8550                                                      | 0                                                                   | 8550                                                         |
| 1    | 8020                                                      | 530                                                                 | 8550                                                         |
| 2    | 654,6229,0                                                | 1137,530,0                                                          | 869,7681,0                                                   |
| 3    | 0,2195,0,635,0,0,0,750,<br>0,0,0,362.5,0,1059,0           | 125,530,180,782,194,181,128,<br>0,0,0,125,1109.5,194,0,0            | 240,2610,0,869,0,0,0,1470,511,<br>0,240,629,0,1981,0         |
| 4    | 0,0,2001,0,635,0,0,0,750,<br>0,0,0,362.5,0,1059,0         | 125,194,530,180,782,194,181,128,<br>0,0,0,125,1109.5,194,0,0        | 240,0,2610,0,869,0,0,0,1470,511,<br>0,240,629,0,1981,0       |
| 5    | 0,362.5,0,1059,0,0,635,0,0,<br>0,750,0,0,0,362.5,0,1059,0 | 125,1109.5,194,0,0,180,782,194,<br>181,128,0,0,0,125,1109.5,194,0,0 | 240,629,0,1981,0,0,869,0,0,0,1470,<br>511,0,240,629,0,1981,0 |

Table 12: Simulation Data, 72-Bus Network

| ITER | Vector of Total Island<br>Regulation (MW)               | Generation Tripped<br>(MW)                                | NUMBER OF<br>ISLANDS |
|------|---------------------------------------------------------|-----------------------------------------------------------|----------------------|
| 0    | 0                                                       | 0                                                         | 1                    |
| 1    | -14                                                     | 516                                                       | 1                    |
| 2    | -43,-1028,0                                             | 172,424,0                                                 | 3                    |
| 3    | 0,-163,0,-62,0,0,0,-50,0,<br>0,0,-51.5,0,-52,0          | 240,252,0,172,0,0,0,670,<br>511,0,240,215,0,870,0         | 15                   |
| 4    | 0,0,-317,0,-62,0,0,0,-50,<br>0,0,0,-51.5,0,-52,0        | 240,0,292,0,172,0,0,0,670,<br>511,0,240,215,0,870,0       | 16                   |
| 5    | 0,-51.5,0,-52,0,0,-62,0,0,<br>0,-50,0,0,0,-51.5,0,-52,0 | 240,215,0,870,0,0,172,0,0,<br>0,670,511,0,240,215,0,870,0 | 18                   |

Table 13: Simulation Data, 72-Bus Network Continued

Table 12 indicates the amount of loads served, loads shed, and initial generation in each island at each islanding iteration. Table 13 indicates the amount of regulation and generation tripped in each island at each iteration. It should be noted that the status of every load, every generator, and every transmission line, at every islanding iteration is determined by the simulator. At each islanding iteration within each island, the amount of load served is equal to the sum of generation setpoints in that island plus contributions from regulation, minus the generation tripped. The amount of load shed is equal to the amount of generation tripped minus contributions from regulation. The entire simulation took under 4 seconds to complete.

### 3.4 Optimization Problem Example

The optimization scheme to identify the initial outages leading to the maximum load-shedding scenario described in Section 2.9 is now illustrated. We apply the optimization to the 3-bus example from the Introduction. Here  $P_1$  is defined as 180 MW,  $P_2$  as 90 MW and  $P_3$  as 90MW. The line flow limits are set at 150 MW and the number of load flow iterations is set to three. The number of lines the destructive agent is allowed to remove is set to 1 and the goal is to inflict the maximum amount of load shedding. The results from the optimization are given in the table below:

|        | 1st Power Flow Iteration |        | 2nd Power Flow Iteration |        | 3rd Power Flow Iteration |        |
|--------|--------------------------|--------|--------------------------|--------|--------------------------|--------|
|        | LINE FLOW                | STATUS | LINE FLOW                | STATUS | LINE FLOW                | STATUS |
| LINE 1 | 90MW                     | ON     | 180MW                    | ON     | 0MW                      | OFF    |
| LINE 2 | 0MW                      | ON     | 90MW                     | ON     | 0MW                      | ON     |
| LINE 3 | -90MW                    | ON     | 0MW                      | OFF    | 0MW                      | OFF    |

Table 14: Optimization load flow stages

The solution that produced the maximum damage on the network was the removal of line 3, whose outage leads to an overload of line 1. This splits the system into two islands, one containing node 1 and the other containing nodes 2 and 3. Note that the same result could have been obtained by removing line 1 due to the symmetry in the network. At the final load flow iteration, all loads (180MW) have been shed and all generation (180MW) has been tripped. The progression of the solution from load flow to load flow is shown in the illustration below, all values are in MW.

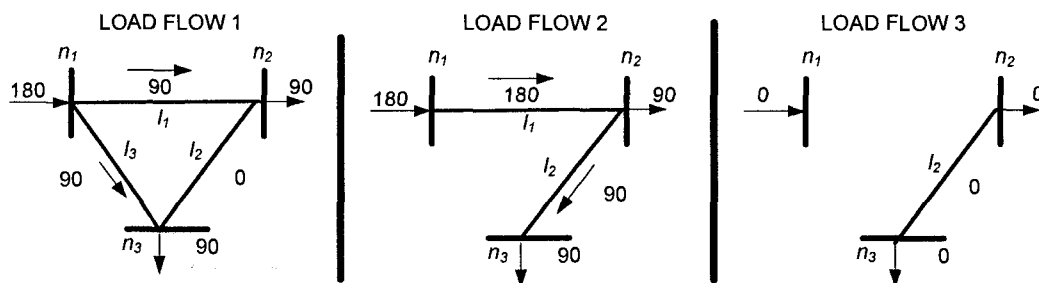


Figure 31: Optimization Solution by Load Flow Stage

As a sub-optimum example, we see that if line 2 had been selected as the line to be initially removed, no line overload would occur, and as a result no load shedding would take place.

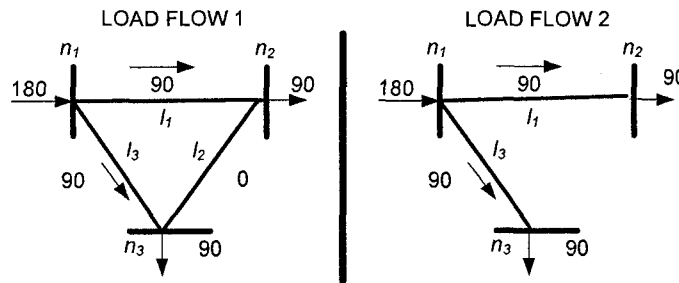


Figure 32: Sub-Optimal Initial Outage

### 3.5 Summary

In this section simulations were conducted using the quasi-steady state model developed in Chapter 2 on a 10-bus 13-line network, a 24-bus 38-line network, and a 72-bus 119-line network. The simulator is able to identify islands that result from a given contingency as well as the response of protection relays that enact load shedding and generation tripping actions, in addition to the primary frequency response of a network's generation. If an island is able to survive an outage, the simulator gives the island's new operating frequency before the dispatch of secondary regulation, as well as the state of all lines, loads, and generators in the island. If an island collapses the simulator shows that zero load is served at that island.

Since in the optimization problem the only contingencies under study were line outages, the only way to cause load shedding or system collapse was to cause islanding. The results corroborated this and the worst contingencies were those that led to line overflows, followed by cascaded outages, and finally followed by system separation.



## CHAPTER 4: CONCLUSIONS:

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### 4.0 Conclusions

The quasi-steady state simulator presented in this thesis provides a fast approximation of a power system's response to contingencies leading to cascading outages and islanding. The simulator utilizes power balance equations to determine island settling frequencies and necessary relay tripping events that will follow. An innovative null space analysis on the DC load flow susceptance matrix is utilized to identify which nodes belong to which islands. The simulator is termed quasi-steady state because after each relay tripping event a new island settling frequency is computed. No actual differential equation based transient analysis is conducted. This leads to a simulator that can be overly optimistic, but that requires only a small fraction of the computation time. When compared with the speed of a transient simulation that solves the governing differential equations through a MATLAB solver, the simulator presented in this thesis has been shown to be of the order of 1000 times faster in computation. The speed of the simulator can be attributed to the fact that it does not solve a set of differential equations from time step to time step. The simulator is intended to be utilized as a first approximation system analysis tool in the same manner that a DC load flow is applied to gain insight into the power flows through a network.

The results of Chapter 3 demonstrate the ability of the simulator to quickly and efficiently predict a system's response to contingencies leading to cascading outages and islanding. Simulations were conducted on a 10-bus 13-line network, a 24-bus 38-line network, and a 72-bus 119-line network. In all cases, the simulator was able to identify islands that result from a given contingency as well as the response of protection relays that enact load shedding and generation tripping actions, in addition to the primary frequency response of a network's generation. If an island

survived an outage, the simulator found the island's new operating frequency before the dispatch of secondary regulation, as well as the state of all lines, loads, and generators in the island. If an island was unable to balance load and generation, the simulator showed that zero load was served in that island.

This thesis also examined the very complex problem of identifying the optimum initial outage in the sense that it would cause the maximum amount of load shedding through islanding. This result identifies the most vulnerable initial outage (in terms of lines destroyed) that a terrorist could exploit in order to create the maximum amount of subsequent disruption. This information is useful to the system operator in order to identify and quantify network vulnerability to deliberate outages and, as a result, take preventive action, if possible. This problem was formulated as a mixed-integer linear program however because of its high dimensionality, only a three-bus example could be solved. The results properly identified the line whose initial outage caused overflows leading to system separation and maximum loss of load.

#### **4.1 Recommendations for Further Work**

A mixed integer linear programming optimization to find the most damaging initial contingency is under development and should be expanded to handle large systems. This result could be used by planners to enhance a network found to be particularly susceptible to deliberate outages.

In addition to this optimization problem, another that determines the optimal priority lists for generation tripping and load shedding over the set of all likely contingencies would be of great value. This could be seen from the analysis of islands in the 24-bus test case where a less than optimal priority list setting resulted in the unnecessary shedding of loads.

An analytical model, that serves as a middle ground between the simulator presented here and a full differential equation model seems like the most likely next step. By implementing an analytical model it is hoped that we can maintain many of the improvements made in computational speed without significantly sacrificing accuracy in modeling the dynamic swings in frequency between quasi-steady state steps.

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## Appendix A – Background Information

### **A.1 System Incidence and Susceptance Matrix**

The incidence matrix,  $\underline{A}$ , of a power network is a data structure that describes which buses of a network are connected by which transmission lines. For a network of  $l$  lines and  $n$  buses its dimension is  $n$  by  $l$ . If line  $l_1$  starts at bus 1 and terminates at bus 2 a 1 is placed in location  $A_{11}$  and -1 in  $A_{21}$ . As an example, the incidence matrix for the following network data would be:

| LINE | SENDING BUS | RECEIVING BUS | LINE SUSCEPTANCE |
|------|-------------|---------------|------------------|
| 1    | 1           | 2             | 1                |
| 2    | 2           | 3             | 1                |
| 3    | 3           | 1             | 1                |

Table A.1: Incidence Matrix Example

$$\underline{A} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$a_{nl} = \begin{cases} 1 & \text{if line } l \text{ starts at node } n \\ -1 & \text{if line } l \text{ ends at node } n \\ 0 & \text{if line } l \text{ is not connected to node } n \end{cases}$$

If we take the incidence matrix and multiply it by a diagonal matrix,  $diag(\underline{b})$ , of size  $l$  by  $l$  whose diagonal entries are the susceptance of each line and then multiply this product by  $\underline{A}^T$  we get an  $n$  by  $n$  matrix known as the susceptance matrix,  $\underline{B}$ .

$$\underline{B} = \underline{A} \, diag(\underline{b}) \, \underline{A}^T \tag{A.1}$$

In this example it would follow,

$$\underline{B} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

It should be noted that construction of a matrix in this manner will lead to a  $\underline{B}$  matrix that is not invertible. So it is necessary to define a reference bus whose corresponding row and column are removed from the  $\underline{B}$  matrix so that load flow calculations can be made. If node 1 is defined as the reference,

$$\underline{B}' = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

## A.2 DC Load Flow

The DC load flow is a first order approximation of the power flow that holds under the following network conditions:

- 1) Angle differences across lines are small,  $|\delta_n - \delta_m| \ll 1$  radian
- 2) Good voltage stability,  $V_n = 1 \quad \forall n$
- 3)  $\frac{R}{X}$  is small: true for well-designed systems

These conditions are in general reasonable for a network with short lines and good voltage support. The transmission line model used for the DC load flow neglects resistances so that the admittance matrix only contains line reactances. Any shunt elements are also neglected. The power

flowing through each line in a DC load flow is equal to the line susceptance multiplied by the angle difference in radians between the sending and receiving buses:

$$f_l = b_l (\delta^{fr} - \delta^{to}) \quad (A.2)$$

The relationship between power injections, phase angles, and the network susceptance matrix is then given by,

$$\underline{P} = \underline{B}\underline{\delta} \quad (A.3)$$

So if the power injections and network structure are known for a given system then all phase angles can be determined. From these phase angles all line flows can be found (equation A.2).

### A.3 Primary Frequency Regulation

Primary frequency regulation is a mechanism that allows each generator to counteract a power imbalance between mechanical power input and electrical power output. This is accomplished through a feedback signal proportional to the frequency deviation from nominal that is added to the generation set point. In order to explain this mechanism, let us revisit the electromechanical swing equation of a rotating machine without primary frequency regulation,

$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = P_m - P_e \quad (A.5)$$

We see here that in a system where the electrical load,  $P_e$ , is greater than the generator mechanical power input,  $P_m$ , the machine speed will start to decline as kinetic energy is drawn out of the rotating machine.

Alternatively, if the electrical load is less than the mechanical power input, the frequency will rise due to an increase in kinetic energy of rotation.

To counteract this effect, as shown in Figure A.1, each generator adds a frequency feedback term to either curtail or increase its input mechanical power when the system frequency starts to drift. This feedback term, proportional to the island frequency deviation  $\Delta f$ , is added to the centrally dispatched setpoint,  $g_k^0$ , which is recomputed every time an economic dispatch is performed (typically every 5 to 10 minutes).

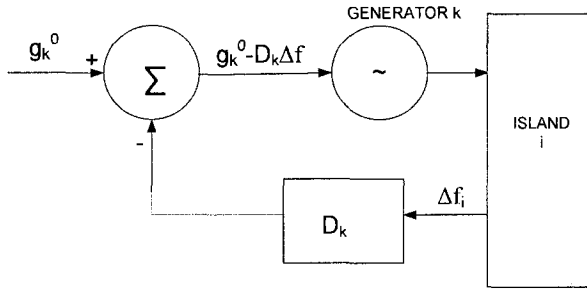


Figure A.1: Primary Frequency Regulation Feedback Model

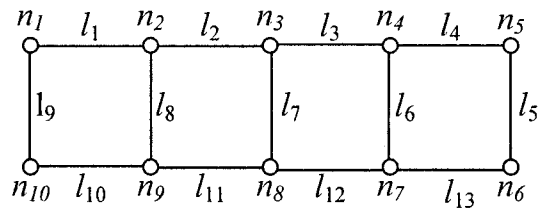
The combined mechanical power input to each island generator  $k$  is then,

$$g_k = g_k^0 - D_k \Delta f \quad (\text{A.6})$$

We note that each island has a different frequency deviation  $\Delta f$ . If  $\Delta f$  is positive then island  $i$  is generation rich and each generator receives a lower setpoint than demanded by central dispatch, thus reducing the surplus in generation in that island. If  $\Delta f$  is negative then island  $i$  is load rich and each generator receives a higher setpoint than centrally dispatched to reduce the demand surplus. In many cases, primary frequency regulation is adequate to balance a power imbalance. In these cases the island will stay at some acceptable deviated frequency until secondary regulation is dispatched, (every few minutes), and the  $g_k^0$  values are all updated.

## Appendix B – Test System Data

### B.1 10-Bus Network Data



Branch Data:

| LINE | SEND BUS | REC BUS | STATUS | MAX FLOW |
|------|----------|---------|--------|----------|
| 1    | 1        | 2       | 1      | 150      |
| 2    | 2        | 3       | 1      | 150      |
| 3    | 3        | 4       | 1      | 150      |
| 4    | 4        | 5       | 1      | 150      |
| 5    | 5        | 6       | 0      | 150      |
| 6    | 4        | 7       | 1      | 150      |
| 7    | 3        | 8       | 1      | 150      |
| 8    | 2        | 9       | 1      | 150      |
| 9    | 1        | 10      | 1      | 150      |
| 10   | 9        | 10      | 1      | 150      |
| 11   | 8        | 9       | 1      | 150      |
| 12   | 7        | 8       | 1      | 150      |
| 13   | 6        | 7       | 1      | 150      |

Generator Data (Section 3.2.1):

| UNIT # | PRIORITY | $G_0$ | LOC | $D_i$ | MAX | STATUS | PFR |
|--------|----------|-------|-----|-------|-----|--------|-----|
| 1      | 1        | 100   | 1   | 12    | 150 | 1      | 0   |
| 2      | 2        | 100   | 2   | 12    | 150 | 1      | 0   |
| 3      | 3        | 100   | 3   | 12    | 150 | 1      | 0   |
| 4      | 4        | 100   | 4   | 12    | 150 | 1      | 0   |
| 5      | 5        | 100   | 5   | 12    | 150 | 1      | 0   |

Load Data (Section 3.2.1):

| LOAD # | PRIORITY | $D_0$ | LOC | STATUS |
|--------|----------|-------|-----|--------|
| 1      | 1        | 100   | 6   | 1      |
| 2      | 2        | 100   | 7   | 1      |
| 3      | 3        | 100   | 8   | 1      |
| 4      | 4        | 100   | 9   | 1      |
| 5      | 5        | 100   | 10  | 1      |

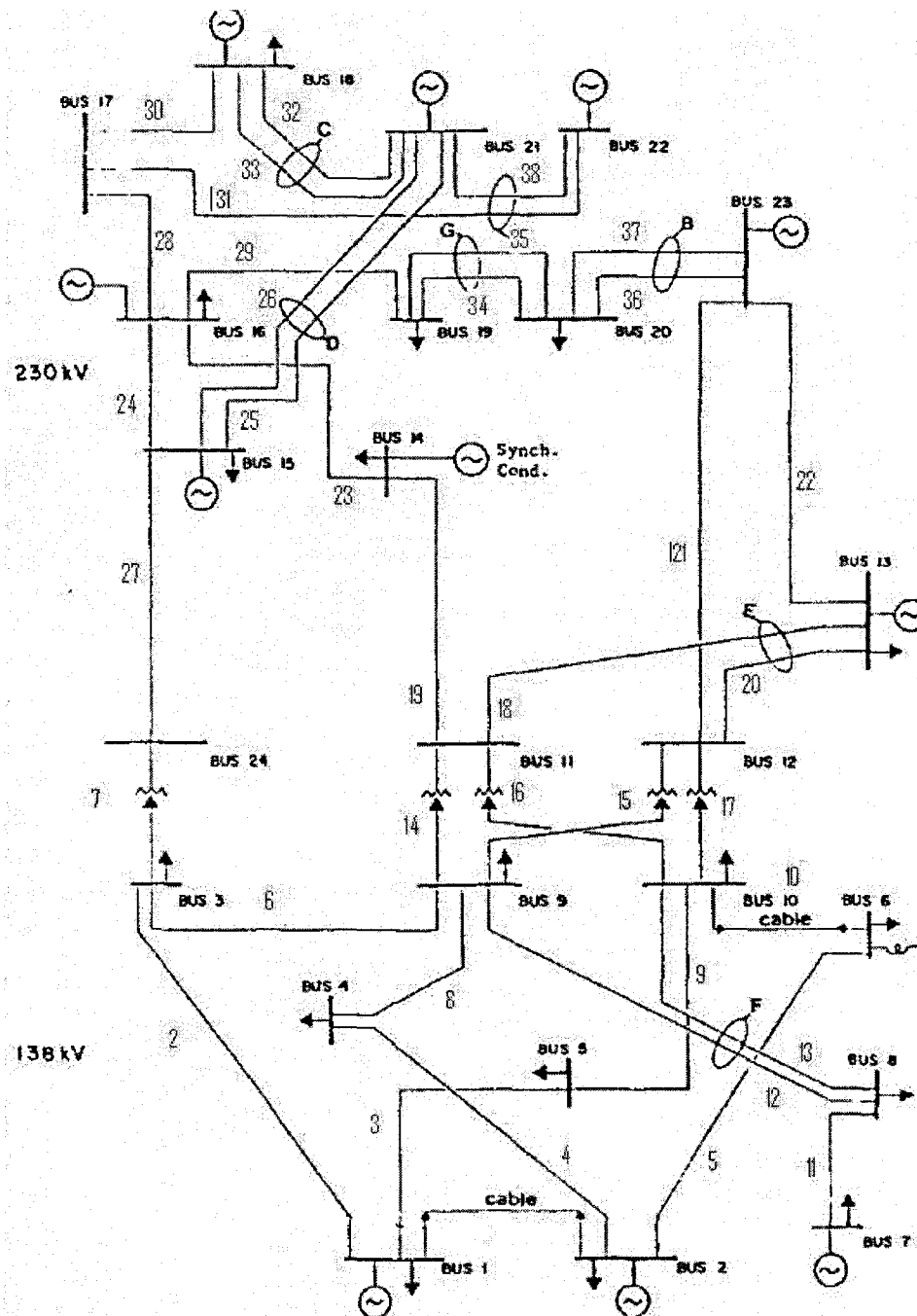
### Generator Data (Section 3.2.2)

| UNIT # | PRIORITY | $G_0$ | LOC | $D_i$ | MAX | STATUS | PFR |
|--------|----------|-------|-----|-------|-----|--------|-----|
| 1      | 1        | 100   | 1   | 12    | 150 | 1      | 0   |
| 2      | 2        | 100   | 2   | 12    | 150 | 1      | 0   |
| 3      | 3        | 100   | 3   | 12    | 150 | 1      | 0   |
| 4      | 4        | 100   | 4   | 12    | 150 | 1      | 0   |
| 5      | 5        | 100   | 5   | 12    | 150 | 1      | 0   |
| 6      | 6        | 15    | 1   | 12    | 50  | 1      | 0   |
| 7      | 7        | 15    | 2   | 12    | 50  | 1      | 0   |
| 8      | 8        | 15    | 3   | 12    | 50  | 1      | 0   |
| 9      | 9        | 15    | 4   | 12    | 50  | 1      | 0   |
| 10     | 10       | 15    | 5   | 12    | 50  | 1      | 0   |

### Load Data (Section 3.2.2)

| LOAD # | PRIORITY | $D_0$ | LOC | STATUS |
|--------|----------|-------|-----|--------|
| 1      | 1        | 105   | 6   | 1      |
| 2      | 2        | 105   | 7   | 1      |
| 3      | 3        | 105   | 8   | 1      |
| 4      | 4        | 105   | 9   | 1      |
| 5      | 5        | 105   | 10  | 1      |
| 6      | 6        | 10    | 1   | 1      |
| 7      | 7        | 10    | 3   | 1      |
| 8      | 8        | 10    | 3   | 1      |
| 9      | 9        | 10    | 4   | 1      |
| 10     | 10       | 10    | 4   | 1      |

## B.2 IEEE 24-Bus RTS System:



Branch Data:

| LINE # | SEND BUS | REC BUS | STATUS | MAX FLOW |
|--------|----------|---------|--------|----------|
| 1      | 1        | 2       | 1      | 300      |
| 2      | 1        | 3       | 1      | 300      |
| 3      | 1        | 5       | 1      | 300      |
| 4      | 2        | 4       | 1      | 300      |
| 5      | 2        | 6       | 1      | 300      |
| 6      | 3        | 9       | 1      | 300      |
| 7      | 3        | 24      | 1      | 300      |
| 8      | 4        | 9       | 1      | 300      |
| 9      | 5        | 10      | 1      | 300      |
| 10     | 6        | 10      | 1      | 300      |
| 11     | 7        | 8       | 1      | 220      |
| 12     | 8        | 9       | 1      | 300      |
| 13     | 8        | 10      | 1      | 300      |
| 14     | 9        | 11      | 1      | 300      |
| 15     | 9        | 12      | 1      | 300      |
| 16     | 10       | 11      | 1      | 300      |
| 17     | 10       | 12      | 1      | 300      |
| 18     | 11       | 13      | 1      | 300      |
| 19     | 11       | 14      | 1      | 190      |
| 20     | 12       | 13      | 1      | 300      |
| 21     | 12       | 23      | 0      | 300      |
| 22     | 13       | 23      | 1      | 300      |
| 23     | 14       | 16      | 1      | 300      |
| 24     | 15       | 16      | 1      | 300      |
| 25     | 15       | 21      | 1      | 300      |
| 26     | 15       | 21      | 1      | 300      |
| 27     | 15       | 24      | 1      | 300      |
| 28     | 16       | 17      | 1      | 300      |
| 29     | 16       | 19      | 1      | 300      |
| 30     | 17       | 18      | 1      | 300      |
| 31     | 17       | 22      | 1      | 300      |
| 32     | 18       | 21      | 1      | 300      |
| 33     | 18       | 21      | 1      | 300      |
| 34     | 19       | 20      | 1      | 300      |
| 35     | 19       | 20      | 1      | 300      |
| 36     | 20       | 23      | 1      | 300      |
| 37     | 20       | 23      | 1      | 300      |
| 38     | 21       | 22      | 1      | 300      |

Generator Data:

| UNIT # | PRIORITY | G <sub>0</sub> | LOC | D <sub>i</sub> | MAX | STATUS | PFR |
|--------|----------|----------------|-----|----------------|-----|--------|-----|
| 1      | 1        | 40             | 15  | 12             | 80  | 1      | 0   |
| 2      | 2        | 40             | 15  | 12             | 80  | 1      | 0   |
| 3      | 3        | 40             | 15  | 12             | 80  | 1      | 0   |
| 4      | 4        | 40             | 15  | 12             | 80  | 1      | 0   |
| 5      | 5        | 40             | 15  | 12             | 80  | 1      | 0   |
| 6      | 6        | 43             | 1   | 15             | 80  | 0      | 0   |
| 7      | 7        | 43             | 1   | 15             | 80  | 0      | 0   |
| 8      | 8        | 43             | 2   | 15             | 80  | 1      | 0   |
| 9      | 9        | 43             | 2   | 15             | 80  | 1      | 0   |
| 10     | 10       | 50             | 22  | 15             | 80  | 1      | 0   |
| 11     | 11       | 50             | 22  | 15             | 80  | 1      | 0   |
| 12     | 12       | 50             | 22  | 15             | 80  | 1      | 0   |
| 13     | 13       | 50             | 22  | 15             | 80  | 1      | 0   |
| 14     | 14       | 50             | 22  | 15             | 80  | 1      | 0   |
| 15     | 15       | 50             | 22  | 15             | 80  | 1      | 0   |
| 16     | 16       | 43             | 1   | 15             | 80  | 0      | 0   |
| 17     | 17       | 43             | 1   | 15             | 80  | 0      | 0   |
| 18     | 18       | 43             | 2   | 15             | 80  | 1      | 0   |
| 19     | 19       | 43             | 2   | 15             | 80  | 1      | 0   |
| 20     | 20       | 80             | 7   | 30             | 150 | 1      | 0   |
| 21     | 21       | 80             | 7   | 30             | 150 | 1      | 0   |
| 22     | 22       | 80             | 7   | 30             | 150 | 1      | 0   |
| 23     | 23       | 15             | 15  | 20             | 40  | 1      | 0   |
| 24     | 24       | 155            | 16  | 50             | 300 | 1      | 0   |
| 25     | 25       | 200            | 23  | 40             | 300 | 1      | 0   |
| 26     | 26       | 200            | 23  | 40             | 300 | 1      | 0   |
| 27     | 27       | 95             | 13  | 34             | 150 | 1      | 0   |
| 28     | 28       | 95             | 13  | 33             | 150 | 1      | 0   |
| 29     | 29       | 95             | 13  | 33             | 150 | 1      | 0   |
| 30     | 30       | 111            | 23  | 40             | 150 | 1      | 0   |
| 31     | 31       | 400            | 18  | 95             | 400 | 1      | 0   |
| 32     | 32       | 400            | 21  | 95             | 400 | 1      | 0   |

Load Data:

| LOAD # | PRIORITY | D <sub>0</sub> | LOC | STATUS |
|--------|----------|----------------|-----|--------|
| 1      | 1        | 54             | 1   | 1      |
| 2      | 2        | 54             | 1   | 1      |
| 3      | 3        | 97             | 2   | 1      |
| 4      | 4        | 90             | 3   | 1      |
| 5      | 5        | 90             | 3   | 1      |
| 6      | 6        | 74             | 4   | 1      |

|    |    |      |    |   |
|----|----|------|----|---|
| 7  | 7  | 71   | 5  | 1 |
| 8  | 8  | 68   | 6  | 1 |
| 9  | 9  | 68   | 6  | 1 |
| 10 | 10 | 125  | 7  | 1 |
| 11 | 11 | 85.5 | 8  | 1 |
| 12 | 12 | 85.5 | 8  | 1 |
| 13 | 13 | 87.5 | 9  | 1 |
| 14 | 14 | 87.5 | 9  | 1 |
| 15 | 15 | 97.5 | 10 | 1 |
| 16 | 16 | 97.5 | 10 | 1 |
| 17 | 17 | 88   | 13 | 1 |
| 18 | 18 | 88   | 13 | 1 |
| 19 | 19 | 89   | 13 | 1 |
| 20 | 20 | 97   | 14 | 1 |
| 21 | 21 | 97   | 14 | 1 |
| 22 | 22 | 105  | 15 | 1 |
| 23 | 23 | 205  | 15 | 1 |
| 24 | 24 | 107  | 15 | 1 |
| 25 | 25 | 100  | 16 | 1 |
| 26 | 26 | 111  | 18 | 1 |
| 27 | 27 | 111  | 18 | 1 |
| 28 | 28 | 111  | 18 | 1 |
| 29 | 29 | 90.5 | 19 | 1 |
| 30 | 30 | 90.5 | 19 | 1 |
| 31 | 31 | 64   | 20 | 1 |
| 32 | 32 | 64   | 20 | 1 |

### B.3 IEEE 72-Bus 3-Area RTS System:

