Risk Management under Information Asymmetry:

Applications in Homeland Security and Supply Disruption

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ABSTRACT

Risk management is the identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events or to maximize the realization of opportunities. The strategies to manage risks usually include transferring the risk to another party, avoiding the risk, or reducing the negative effect or probability of the risk. Using any of these strategies in real life situations, either transferring the risk to another party in the form of outsourcing or reducing the probability of the risk from a deliberate attack from an adversary, may require the interaction with another parties, necessitating the study of strategic decision making, namely game theory. In its very simple form, we have symmetric games in which information is the same for each player. Indeed, when players have an equal set of information then only their individual decisions will determine their success in the game. On the other hand, asymmetric games are those games where the players do not stand on equal ground. Specifically, a game under information asymmetry deals with the study of decisions where one player has more or better information than the other player(s). In real life, most of the games are played under information asymmetry, therefore, it is an important issue for players to analyze and to find the optimal decision in order to minimize the probability and/or impact of unfortunate events under such asymmetric environment. Motivated by the importance of strategic decision making under information asymmetry, this dissertation aims to develop normative recommendations in two different contexts; in government sector, in the context of homeland security, and in private sector, in the context of supply chain risk management. Our research in homeland security has resulted in one essay which explores the impact of terrorist's private information in government

defensive resource allocation decisions. Our research in supply chain risk management has resulted in two essays. In the first one, we explored how an audit program, which is employed by the manufacturer to manage its supplier's action, may bring value to the supply chain and its parties when supplier is risky and privileged with private information about supply disruptions. In the latter one, we analyzed a supply chain problem suitable for new products, where both buyer and supplier face ex-ante same uncertainty regarding the supply risk but the supplier can make a costly investment to acquire further information about the true risk and exert an effort to improve the reliability of his supply process based on the acquired information.

ABRÉGÉ

La gestion des risques est l'identification, l'évaluation et la hiérarchisation des risques suivie par l'application coordonnée et économique des ressources afin de minimiser, de surveiller et de contrôler la probabilité et/ou l'impact des événements malheureux ou de maximiser la réalisation de possibilités. Les stratégies de gestion des risques comprennent normalement le transfert des risques à une autre partie, d'éviter le risque, ou réduction de l'effet négatif ou de la probabilité du risque. L'utilisation de ces stratégies dans des situations de la vie réelle, soit le transfert du risque à une autre partie sous la forme de sous-traitance ou réduction de la probabilité du risque d'une attaque délibérée d'un adversaire, peut nécessiter l'interaction avec les autres partis, ce qui nécessite l'étude de stratégique la prise de décision, savoir la théorie des jeux. Dans sa forme la plus simple, nous avons des jeux symétriques dont l'information est la même pour chaque joueur. En effet, quand les joueurs ont un jeu égal de l'information alors que chaque leurs décisions détermineront le succès dans le match. D'autre part, asymétriques ces jeux sont des jeux où les joueurs se tiennent pas sur un pied d'égalité. Plus précisément, le jeu sous l'asymétrie d'information traite de l'étude de décisions où un joueur dispose d'informations plus ou mieux que l'autre joueur (s). Dans la vraie vie, la plupart des jeux sont joués dans l'asymétrie d'information, par conséquent, il est une question importante pour les joueurs d'analyser et de trouver la décision optimale afin de minimiser la probabilité et / ou l'impact des événements malheureux sous un tel environnement asymétrique. Motivée par l'importance de la prise de décision stratégique dans l'asymétrie d'information, cette thèse vise à élaborer des recommandations normatives dans deux contextes différents; dans le secteur de gouvernement, dans le contexte de la sécurité intérieure, et dans le secteur privé, dans le cadre de la

gestion des risques de la chaîne d'approvisionnement. Notre recherche dans la sécurité intérieure a donné lieu à un essai qui explore l'impact de l'information privée des terroristes dans les décisions de défense de l'allocation des ressources du gouvernement. Notre recherche dans la gestion des risques de la chaîne d'approvisionnement a donné lieu à deux essais. Dans la première, nous avons exploré la manière dont un programme de vérification, qui est utilisé par le fabricant pour gérer l'action de son fournisseur, peut apporter de la valeur à la chaîne d'approvisionnement et de ses partis est risqué et quand le fournisseur privilégié avec des informations privées sur les ruptures d'approvisionnement. Dans une derniers, nous avons analysé la chaîne d'approvisionnement le problème approprié pour de nouveaux produits, où l'acheteur et le fournisseur visage incertitude ex ante En ce qui concerne le même risque d'approvisionnement, mais le fournisseur peut faire un investissement coûteux pour acquérir plus d'informations sur le risque réel et exercer une efforts pour améliorer la fiabilité de son processus d'approvisionnement basé sur les informations acquises.

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DEDICATION

To my dear wife, Mahsa, who has been a constant source of practical and emotional support during the challenges of graduate school and life. I am truly thankful for having you in my life.

To my wonderful parents, Ahmad and Fatemeh, whose affection, encouragement, and prays of day and night make me able to get such success and honor.

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Chapter 1

Introduction

Risk management is the identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events or to maximize the realization of opportunities [Hubbard, 2009]. The strategies to manage risks (uncertainties with negative consequences) typically include transferring the risk to another party, avoiding the risk, reducing the negative effect or probability of the risk, or even accepting some or all of the potential or actual consequences of a particular risk. Using any of these strategies in real life situations, either transferring the risk to another party in the form of outsourcing or reducing the probability of the risk from a deliberate attack from an adversary, may require the interaction with another parties, necessitating the study of strategic decision making, namely *game theory*. Indeed, game theory attempts to determine mathematically and logically the actions that decision makers (players) should take to secure the best outcomes for themselves. According to Rasmusen and Blackwell [1994], a game must specify the following elements: the players of the game, the information and actions available to each player at each decision point, and the payoffs for each outcome. A game theorist typically uses these elements, along with a solution concept of their choosing, to deduce a set of equilibrium strategies for each player such that, when these strategies are employed, no player can profit by unilaterally deviating from their strategy.

In its very simple form, we have *symmetric games* in which information is the same for each player.

Indeed, when players have an equal set of information then only their individual decisions will determine their success in the game. On the other hand, *asymmetric games* are those games where the players do not stand on equal ground. Specifically, a game under *information asymmetry* deals with the study of decisions where one player has more or better information than the other player(s). In real life, most of the games are played under information asymmetry; the government suffers from lack of knowledge about terrorist attributes in its defensive budget allocation decision, an insurance company suffers from lack of knowledge about insurance buyers' health condition when designing the contract clauses, workers' potential productivity is unobservable by a hiring firm, and, supplier knows more about his production reliability than manufacturer when signing the contract. Therefore, it is an important issue for players, like government, insurance company, hiring firm, and manufacturer, to analyze and to find the optimal decision in order to minimize the probability and/or impact of unfortunate events under such asymmetric environment. Motivated by the importance of strategic decision making under information asymmetry, this dissertation aims to develop normative recommendations in two different contexts; in government sector, in the context of homeland security, and in private sector, in the context of supply chain risk management. Specifically, we provide three essays as follows.

Our study in the first essay, which is based on Nikoofal and Gümüş [2014a], helps the government to explore the impact of terrorist's private information in her defensive resource allocation decisions. In defense budget allocation problem, the government often needs to prioritize and distribute her limited resources among valuable targets to defend them against an unpredictable terrorist whose response strategy cannot be fully assessed a priori. Therefore, an important issue from the government's point of view is the development of a defense strategy that allows her to incorporate this unpredictability into her budget allocation decisions. In Chapter 2, i.e., the first essay, we explore the value of terrorist's private information on government's defense allocation decision. In particular, we consider two settings with different informational structures. In the first setting, the government knows the terrorist's target preference but does not know whether the terrorist is fully rational in his target selection decision. In the second setting, the government knows the degree of rationality of the terrorist, but does not know the terrorist's target preference. We analyze both settings in order to address three research questions raised in the first essay. First, we find that the government's equilibrium budget allocation strategy involves a set of thresholds for each target. Specifically, the government makes resource allocation decisions by comparing her valuation for each target with the thresholds. Second, our analysis shows that the set of thresholds that determines whether a target should be defended is more restrictive when the information asymmetry is about the terrorist's degree of rationality than when it concerns target preference. Finally, we derive the value of information (VOI) from the perspective of the government for each setting. As expected, terrorist's information has no value when government's a-priori belief matches with the true type of the terrorist. Otherwise, VOI mainly depends on the government's budget and the degree of heterogeneity among the targets. In general, VOI goes to zero when government's budget is high enough. But, the impact of heterogeneity among the targets on VOI further depends on whether the terrorist's target preference matches with government's or not. Specifically, if it matches then VOI decreases when the degree of mismatch among targets increases, otherwise, it increases. We also perform various extensions on the baseline model and show that the structural properties of budget allocation equilibrium still hold true.

Our second essay, which is based on Nikoofal and Gümüş [2014b], examines how information asymmetry between manufacturer and supplier would affect the contractual relations among supply chain members as well as total supply chain. Besides the many benefits of outsourcing, firms are still concerned about the lack of critical information regarding both the risk levels and actions of their suppliers that are just a few links away. Indeed, the information asymmetry between buyers and suppliers can be due to many reasons such as lack of process automation across supply chain [Industry Week, December 2009], lack of confidence among the channel partners [Cranfield University, 2002], and insufficient due diligence on the part of the supplier, etc. In the second essay, we focus on two sources of information asymmetry that naturally arise in the form of hidden information and action between a manufacturer and a supplier in a supply disruption setting. The former arises due to the mere fact that the supplier knows the extent of his true reliability better than the manufacturer because he is either closer to the source of risk factor than the manufacturer or affected by the exogenous factors and players to which the manufacturer may have no direct access. For example, Philips had better understanding than Nokia and Ericsson about the consequences of the catastrophic event that disrupted the production in its New Mexico plant, on March 2000, which, in turn, triggered in cellular phone industry the battle between Nokia and Ericsson [Sheffi, 2007]. The latter arises due to the lack of control that results from the suppliers' taking certain actions without informing the buyer. The main goal of Chapter 3, i.e., the second essay, therefore, is to shed light on the issues jointly caused by hidden information and hidden action and to explore the various means with which the resulting adverse effects can be mitigated. Specifically, we study the effectiveness of audit for the manufacturer in managing her supplier's process improvement effort when the supplier is privately informed about his disruption risk and actions. By comparing the agency costs associated with the optimal menu of contracts with and without audit, we completely characterize the value of audit for all the cases from the perspectives of both manufacturer, and supplier as well as total supply chain. First, the analysis of value of audit from the manufacturer's perspective shows that she can strictly benefit from auditing her supplier's actions. To the best of our knowledge, this result has not been documented before in the principal-agent literature under a standard setting where the agent is assumed to be risk-neutral and not protected by limited liability constraints. Second, we find that not only the manufacturer but also the supplier can strictly benefit from audit. Third, the audit enables the manufacturer to customize her contract offerings based on the reliability of the supplier. Finally, by analyzing the impact of problem parameters on the value of audit, we identify the conditions under which an audit would be beneficial for individual supply chain parties as well as total supply chain.

Finally, the last essay is based on a recent paper Nikoofal and Gümüş [2014c], in which we analyze a supply chain problem suitable for new products, where both buyer and supplier face ex-ante same uncertainty regarding the supply risk. When a new product project fails, the failure is usually blamed on culprits such as tough competition or weak market research. But, the practitioner findings reveal that the true causes of failure may lie in supply-end of the chain. Lack of experience in new product manufacturing not only leads to supply side problems, such as yield problems, in inflexibility in production capacity, lead-time variability and long set-up time, but also it may lead to huge loss on the demand side, such as lost sales, customer goodwill loss, and market share loss due to a fast follower. Therefore, in order to reduce the failure likelihood of a new production development project, the supplier may invest in a costly diagnostic test technology, e.g., running a test production, before commencing the final production. The third essay aims to spotlight the importance of such supply diagnostic test in a new product development project. We establish a dyadic supply chain with one buyer who procures for a new product from a supplier. Due to the lack of experience in manufacturing, the state of supply disruption is not known for both the buyer and supplier at the time of contract, hence both buyer and supplier face ex-ante same uncertainty regarding the supply risk. However, the supplier may invest in a diagnostic test to acquire information about his true reliability, and to use this information when deciding on an process improvement effort, which may reduce his exposure to disruption risk. Using this setting, we identify benefits and drawbacks of diagnostic test from buyer's perspective. Specifically, if the buyer offers a contract that avoids the supplier from investing in diagnostic test, then the supplier decides on process improvement based on his ex-ante belief about his true reliability. It brings two different inefficiencies for the buyer. The first one is related to inefficient improvement decision by uninformed supplier, comparing to the first-best scenario where buyer and supplier works together as an integrated firm. Specifically, due to lack of knowledge about its true reliability, an uninformed reliable supplier may overinvest in process improvement, while an uninformed unreliable supplier may underinvest in process improvement. The second inefficiency comes from the financial burden of trade with an uninformed supplier in the form of limited liability. On the other hand, by offering a contract that induces diagnostic test on the supplier the first-best level of improvement is implementable, hence the buyer can rectify channel loss. Furthermore, the buyer can get rid of limited liability payments. That being said, learning the true reliability by supplier creates information asymmetry between supply chain parties, hence the informed supplier armed with new information asks for information rent. In Chapter 4, we take all these pros and cons of diagnostic test into account and examine how the diagnostic test may brings value to the buyer and supplier as well as total supply chain. Our analysis shows that the buyer should be careful when inducing diagnostic test on its supplier. Specifically, when diagnostic test rectifies under-investment in process improvement, it would be a win-win strategy for both the buyer and supplier, however, it may hurt all the parties if it rectifies over-investment in process improvement.

Chapter 2

The Value of Terrorist's Private Information in Government's Defensive Resource Allocation

2.1 Introduction

In defense budget allocation problem, the government (defender, from now on) often needs to prioritize and distribute her [1] limited resources among valuable targets to defend them against an unpredictable terrorist (attacker, from now on) whose response strategy cannot be fully assessed a priori. Therefore, an important issue from the defender's point of view is the development of a defense strategy that allows her to incorporate this unpredictability into her budget allocation decisions. One of the main sources of unpredictability is related to the uncertainty in degree of rationality of the attacker in his target selection decisions. An extreme, though commonly assumed case, is one of a fully rational attacker who responds optimally to the defender's budget allocation decisions when he chooses which target to attack. However, this type of strategic behavior cannot fully explain some of the terrorist acts in the recent history. Indeed, the attack on the World Trade Center in New York

[1]Throughout the chapter, we use "he" and "she" to refer to "the attacker" and "the defender", respectively.

and the four hijackings of September 11, 2001, suggest that the attackers may indeed be choosing their targets regardless of the observed defense levels [2]. Furthermore, the target selection criteria of such nonstrategic attackers may be influenced by some other factors that are either not readily available to or difficult to observe for the defender. Therefore, the possibility that the attacker's degree of rationality and target selection criteria might come in many guises requires us to analyze various informational scenarios from the defender's point of view. In this study, we aim to accomplish this goal in two stages. First, we develop an incomplete (asymmetric) information model to capture the different degrees of unpredictability in the attacker's response. Then, we explore this model to address the following research questions:

Research Question 1: How should a defender prioritize multiple targets and allocate limited budget among them when faced with two types of asymmetric information (degree of rationality and target preference) about the attacker?

Research Question 2: What is the impact of partial information on the defender's equilibrium budget allocation strategy?

Research Question 3: From the defender's perspective, what is the value of information regarding the attacker's degree of rationality and target preference? How does the value of information depend on problem parameters such as defender's budget, targets' valuations, and effectiveness of the defender's budget?

We develop a game-theoretic model considering the following sequence of decisions. First, the defender distributes her limited defense budget across given targets. The attacker responds to that by first choosing a target and then deciding on his effort level, which ultimately determines the degree of damage inflicted upon the selected target. Target selection by the attacker depends on whether he is strategic or not. A strategic attacker, who shares a common target valuation with the defender, chooses a target by taking into account both the valuation of that target and the defender's budget allocation decision, and responds optimally to defender's decision, whereas a nonstrategic attacker,

[2]New York, where the attack happened, and Boston, where the hijackings happened, were among top urban areas to receive US Homeland Security Grant Program between 2000 and 2008, based on the US Federal Funding Assistant (http://www.fedspending.org/).

whose target valuation is unknown to the defender, decides on the target by considering only his own valuation. In both strategic and nonstrategic cases, after the target is selected, the attacker then decides on his effort level. Finally, the damage on a target depends stochastically on the amount of budget allocated by the defender and the attacker's effort level. To address the above questions, we focus on two types of information asymmetry: (i) the attacker's degree of rationality and (ii) his target preference. Considering two scenarios (symmetric and asymmetric information) along each dimension, we build four models, as shown in Table 2.1.1.

Table 2.1.1: Different Informational Models

		Rationality of attacker	
		Known	Unknown
Nonstrategic attacker's target preference	Known	Full symmetric information	Model A
	Unknown	Model B	Full asymmetric information

Addressing *first research question*, we characterize defender's equilibrium budget allocation under different informational scenarios depicted in Table 2.1.1. In particular, in each scenario above, we show that the equilibrium policy involves a set of thresholds for each target and that the defender makes her budget allocation decision by first ordering the targets with respect to a ranking rule, and comparing the valuations of targets with these thresholds. If the valuation of a target is sufficiently high, the defender invests non-zero budget to defend the target; otherwise, the target is left undefended.

To address our *second research question*, we examine the impact of asymmetric information on the ranking rule and thresholds. Our analysis shows that both the ranking rule and number and level of thresholds depend on the type of information available to the defender before she makes budget allocation decision. First, we show that in Model A (i.e., when the attacker's rationality is unknown to the defender), the defender employs valuation-based ranking rule and two thresholds for each target to decide whether the target should be defended or not, whereas in Model B (i.e., when the target preference is unknown to the defender) she uses belief-adjusted valuations to rank the targets and compare the valuation of each target with a single threshold to decide on her budget allocation. The reason behind this is that in Model A, the defender must trade off the threat from a strategic attacker with that of a nonstrategic one. Since a strategic attacker differs from its non-strategic version in terms of its attack strategy, the defender has to simultaneously examine two thresholds (one for strategic and one for non-strategic) to hedge herself against risks from both of the types. However, in Model B, the defender has perfect information about the type of attacker. Consequently, she uses a single threshold to decide on her defense strategy.

Finally, to address our *third research question*, we characterize the value of information along each dimension by analytically comparing the defender's payoffs under Model A and B with those under symmetric information. We identify two phenomena that significantly influence the value of attacker's rationality information. First, the defensive strategies against a strategic and a nonstrategic attacker differ fundamentally. Against a strategic attacker, on one hand, the defender implements a comprehensive defense strategy in which she allocates her budget among all the targets in a way that will bring down the expected damage across all the targets to the same level. On the other hand, against a nonstrategic attacker, the defender ranks the targets from the highest to lowest value, adjusts the ranking by her a-priori information, and defends only a few of them with a more concentrated and focused defensive effort. Employing a comprehensive strategy against a nonstrategic attacker or a concentrated one against a strategic attacker leads to significant losses on the defender's side. The second factor is related to the difference between strategic and nonstrategic attackers' behaviors. Namely, a defender can influence a strategic attacker's behavior indirectly via her defensive budget allocations, whereas a similar strategy has limited influencing power on a nonstrategic attacker. Therefore, knowing the degree of rationality of the attacker gives significant prediction and control capability to the defender, which she can use to improve the security of the overall system. To summarize, these two effects suggest that additional value that can be obtained by knowing the degree of rationality of an attacker is more than that by knowing his target preference. In addition to the above three questions, we also explore how the rationality and target information behave with respect to system parameters, and show that it is sensitive to the total defensive budget, the effectiveness ratio of an attack, and the degree of heterogeneity among the targets.

2.2 Related Literature

Our study in this chapter is related to the growing body of literature that considers the role of incomplete information in defender-attacker games. This literature can be further divided into two streams, depending on whether it is the attacker or the defender who holds asymmetric information regarding his/her opponent. We refer the reader to Sandler and Siqueira [2009] for an extensive review of game-theoretical models in this literature.

The papers in the first stream consider the cases where the attacker is uncertain about the target properties and/or the defensive allocation. Since our focus is to analyze the impact of asymmetric information from the defender's perspective, we discuss only a few representative papers in this stream. Powell [2007a], Zhuang and Bier [2011], Zhuang et al. [2010] develop models in which the defender holds private information and explore whether the defender should hide this information from the attacker (secrecy policy) or convey a noisy signal (deception policy) to the attacker [3]. Jenelius et al. [2010] analyze the impact of the degree of an attacker information asymmetry on the likelihood of his attack and show that a less informed attacker may cause more damage to a defender. Kaplan et al. [2010] also consider the uninformed attacker setting and characterize a simple rule for the attacker's optimal strategy.

The second stream of research in this literature, which is more relevant to our model, studies the cases in which the defender has incomplete information regarding the attacker's attributes [Bier et al., 2007, Powell, 2007b, Bier et al., 2008, Rios and Insua, 2009, Wang and Bier, 2011, Rothschild et al., 2012, Nikoofal and Zhuang, 2012, Zhang and Ramirez-Marquez, 2012]. Specifically, Bier et al.

[3] There is also a body of literature exploring the impact of secrecy and deception in defense strategy. It assumes that the defender may be the first mover [Brown et al., 2005, Zhuang and Bier, 2011, Zhuang et al., 2010, Jenelius et al., 2010], the second mover [Overgaard, 1994, Brown et al., 2005, Arce and Sandler, 2007], or that the defender and attacker may play simultaneously [Kjell and Hausken, 2010, Zhuang and Bier, 2007].

[2007] study the defender's first-mover advantage when she publicly announces the defensive allocation rather than keeps it secret. They compare the equilibrium in sequential and simultaneous games when attacker's preferences are unknown by the defender. Their results show that, in equilibrium, the defender is always better off in the sequential game. Powell [2007b] explore the existence of Nash equilibrium for the defender's resource-allocation problem in different settings, including the case where the defender is unsure of the terrorists' preferred targets. Bier et al. [2008] analyze a sequential defender-attacker game, where the defender acts first and considers that the attacker's valuation for each target follows a two-parameter Rayleigh distribution with mean value equal to the defender's valuation. Nikoofal and Zhuang [2012] also study a defender-attacker game with incomplete information in which the attacker has private information about the valuation of targets. They use robust optimization techniques to model the defender's uncertainty about the attacker's attributes. Brown and Cox [2011] show that traditional probabilistic risk assessment can lead to poor defensive decisions when the attacker holds private information about his attack probabilities. Recently, Wang and Bier [2011] develop a two-period model for the defensive resource allocation problem, in which the attacker decides on the targets to attack based on a multiattribute utility function. In their model, they assume that the defender has prior beliefs on the attacker's attributes, which she updates upon observing the actions of the attacker. Finally, Hausken and Zhuang [2011] analyze government's decision to allocate its resources between attacking to downgrade a terrorist's resources and defending against a terrorist attack. In their model, the terrorist also allocates its resources between attacking a government's asset and defending its own resources. Our model differs from this stream in two ways: first, while papers in this stream commonly assumed that the attacker is fully rational and responds optimally to the defender's strategy, this study models different degrees of rationality by considering two types of attacker behaviors: strategic and nonstrategic. Second, our objective of this study also differs from the above papers'. Most of the previous literature focuses only on one-dimensional information asymmetry (mostly target preference), and analyze its impact on the defender's budget allocation decision. In contrast, our work considers two types of information asymmetry (both target preference and degree of rationality of the attacker).

Our work is also related to a series of papers [Zhuang and Bier, 2007, Powell, 2007b, Levitin and Hausken, 2009, Golany et al., 2009, Hao et al., 2009, Hausken et al., 2009, Shan and Zhuang, 2013a] that consider both terrorist and non-terrorist attacks (natural disasters, for example). Do note however that our definition of a nonstrategic attacker is different from the definition of a non-terrorist attacker used in the above models. Indeed, a nonstrategic attacker can still adjust his effort endogenously in response to the defender's budget allocation decision, whereas a non-terrorist attacker is modeled in the above papers as a passive actor whose level of effort entirely depends on exogenous factors. The work that comes closest to ours in this context is Shan and Zhuang [2013a], in which, similar to our study, the terrorist might be strategic or non-strategic. To model the non-strategic attacker's behavior, Shan and Zhuang [2013a] assume that such an attacker attacks with an exogenously-determined probability. Our work differs from Shan and Zhuang [2013a] in different ways. First, we assume that the non-strategic attacker keeps different valuation of the targets than the government. We operationalize the valuation difference between non-strategic attacker and the government by creating different types for the non-strategic terrorist. Second, our definition of non strategic attacker differs from that in Shan and Zhuang [2013a], in the sense that the non-strategic attacker chooses its target irrespective of defensive budget allocation, but he can update his attack level based on the defense level observed on his preferred target, hence we develop a game-theoretical model to capture non-strategic attacker decision. Finally, our modeling approach for the non-strategic attacker helps us to answer one of the main research questions of this study, which is to explore the value of terrorist's private information in government defensive resource allocation.

2.3 Model Framework

To address the research questions raised in §2.1, we develop a two-stage non-zero-sum game between defender and attacker [4]. In the first stage, the defender distributes her limited budget D across

[4]The model of constant-sum simultaneous defender-attacker game is known as Colonel Blotto game [Shubik and Weber, 1981, Roberson, 2006, Adamo and Matros, 2009, Kovenock and Roberson, 2011], in which two players simultaneously

N different targets. Let v_i and D_i be the defender's valuation and budget allocation for target i, respectively. In the second stage, the attacker selects a target to attack and the level of effort to exert. Let A_i be the effort exerted by the attacker for target i. Once decisions over D_i and A_i are made, target i's damage is realized. We use a likelihood function $P(D_i, A_i)$ to capture the expected damage for target i as a joint function of defender's budget allocation decision D_i and attacker's effort A_i. We assume that $P(D_i, A_i)$ increases in A_i, and decreases in D_i. Also, it has the following regularity properties: (i) $P(D_i, A_i)$ is twice differentiable with respect to A_i and D_i and (ii) $\lim_{A_i \to 0} P(D_i, A_i) = 0$, and $\lim_{D_i \to \infty} P(D_i, A_i) = 0$. An appropriate candidate for the damage probability function that parsimoniously satisfies the above properties is the cumulative exponential function [Bier et al., 2008, Gerchak and Safayeni, 1996, Golalikhani and Zhuang, 2011, Shan and Zhuang, 2013b].

$$P(D_i, A_i) = 1 - \exp\left(\frac{-\lambda A_i}{D_i}\right) \tag{2.3.1}$$

where λ is effectiveness ratio of an attack and measures the sensitivity of the damage function with respect to attacker's effort level per unit of budget investment [Bier et al., 2008, Wang and Bier, 2011]. To avoid additional complexity, without loss of generality, we assume $\lambda \in [0, 1]$.

In order to model the defender's incomplete information on the attacker's degree of rationality, we use Harsanyi's transformation to create two types for the attacker: strategic (denoted by s-type) and nonstrategic (denoted by n-type). Let p_s and p_n be the defender's a-priori beliefs about the true type of attacker, where $p_s + p_n = 1$. The s-type (strategic) attacker has the same target valuations as the defender and responds to the defender's budget allocation decision optimally. Specifically, if the s-type attacker wants to make an attack on target i, he chooses attack effort A_i^s that maximizes his expected payoff:

$$\pi_i^s\left(A_i^s\right) = \max_{A_i^s \geq 0} v_i P(D_i, A_i^s) - A_i^s \tag{2.3.2}$$

distribute their fixed amount of resource across n battlefields. Within each battlefield, the player that allocates the higher level of the resource wins the battlefield, and each player's payoff is equal to the number of battlefields won. Our model differs from the literature in Colonel Blotto game in two ways. First, the players play sequentially. Second, the game is not a zero-sum game in which defender and attacker respectively consider the cost of defense and attack effort in their payoff functions.

The s-type attacker then chooses target i^* that provides him with the maximum payoff among the N targets,

$$i^* = \underset{i=1, \dots, N}{\arg\max}\ \pi_i^s\left(A_i^s\right) \tag{2.3.3}$$

The n-type (nonstrategic) attacker differs from the s-type attacker in two ways. First, he chooses his target without considering the defender's budget allocation decision. Second, his target preference is unknown to the defender. To model these two features in a tractable way, we use again Harsanyi's transformation to create N different n-type attacker for each target $i = 1, \dots, N$ and denote the type that attacks target i irrespective of defender's budget allocation decision by n_i-type. If the attack is successful, the attacker obtains a fixed value of w, where w is a positive number large enough so that n_i-type can never be deterred from attacking his preferred target i. Let q_i, $i = 1, \dots, N$ denote the defender's a-priori belief for the n_i-type attacker, where q_i is common knowledge and $\sum_{i=1}^{N} q_i = 1$. Even though n_i-type attacker's target selection decision does not depend on the defender's budget allocation decision, his effort level decision (denoted by A_{n_i}) does indirectly depend on it in the sense that his expected payoff is as follows,

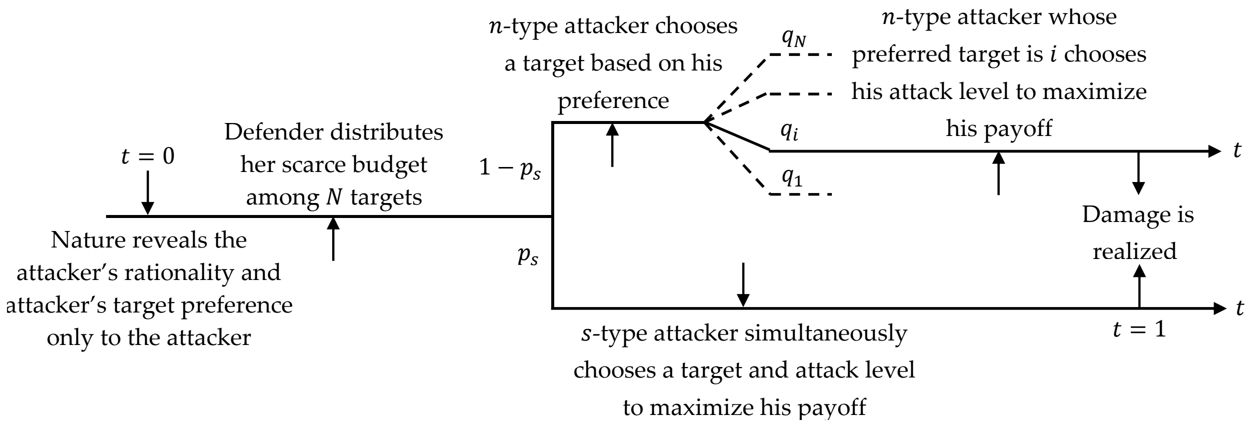
$$\pi_{n_i} = \max_{A_{n_i} \geq 0} wP(D_i,\, A_{n_i}) - A_{n_i} \tag{2.3.4}$$

The defender makes her budget allocation decision to minimize the total expected damage subject to budget and non-negativity constraints:

$$\begin{aligned}
\text{(Model I)} \quad \min \quad & p_s \sum_{i=1}^{N} v_i P(D_i,\, A_i^s) + (1 - p_s) \sum_{i=1}^{N} q_i v_i P(D_i,\, A_{n_i}) && (2.3.5)\\
\text{s.t.} \quad & \sum_{i=1}^{N} D_i \leq D && (2.3.6)\\
& D_i \leq v_i,\ i = 1,\ \dots,\ N && (2.3.7)\\
& D_i \geq 0,\ i = 1,\ \dots,\ N && (2.3.8)
\end{aligned}$$

where constraints (2.3.6) and (2.3.7) ensure that the total budget allocation to all targets is less than the defender's maximum budget, and that the defender never invests on a target more than its valuation. The timing of events is shown in Figure 2.3.1.

Figure 2.3.1: Timing of Events in Government-Terrorist Game



We use the *Bayesian Nash Equilibrium* (BNE) solution concept to analyze the above game. In our modeling framework, BNE consists of a collection of budget allocations for the defender and type-contingent efforts for the attacker that are required to yield a perfect Nash equilibrium for the two-stage game between the defender (leader) and s- and n_i-type attackers (followers). For a detailed definition of BNE, please refer to Fudenberg and Tirole [1991].

We begin our analysis of Model I by characterizing the attacker's type-contingent best-response function. The best-response function for s-type attacker, denoted by $R^s(D_{i^*})$, can be obtained by solving the first order condition for his payoff function (2.3.2) with respect to his effort as follows:

$$R^s(D_{i^*}) = \begin{cases} 0 & \text{if } D_{i^*} \geq \overline{D}_{i^*} \\ \frac{D_{i^*}}{\lambda} \ln\left(\frac{\lambda v_{i^*}}{D_{i^*}}\right) & \text{otherwise} \end{cases} \tag{2.3.9}$$

where i^* is the target that provides the s-type attacker with the maximum payoff among the N targets (Eq. 2.3.3). Note that $R^s(D_{i^*})$ initially increases with D_{i^*}, when $0 < D_{i^*} \leq \frac{\lambda v_{i^*}}{e}$ then decreases with D_{i^*} when $\frac{\lambda v_{i^*}}{e} \leq D_{i^*} < \lambda v_{i^*}$, and finally is equal to zero for $D_{i^*} \geq \lambda v_{i^*}$. Hence, $\overline{D}_{i^*} = \lambda v_{i^*}$ is the lowest possible level of defense required to deter s-type attacker from attacking target i. Next, we consider the best-response function of the n_i-type attacker. Similarly, obtaining the first order condition from

Equation (2.3.4) and solving it for A_{n_i} provide us with the n_i-type attacker's best-response function as follows:

$$R^{n_i}(D_i) = \frac{D_i}{\lambda} \ln\left(\frac{\lambda w}{D_i}\right) \tag{2.3.10}$$

Recall that we assume that n_i-type attacker's target selection decision cannot be deterred by the defender's budget allocation decision. This implies that $R^{n_i}(D_i)$ should be positive for all feasible values of budget allocation, which imposes the following condition on target valuations and λ:

Assumption 1. $w > \max_{i=1,\ldots,N} \{v_i\} / \lambda$.

Finally, substituting both n_i- and s-type attackers' best-response functions into the defender's optimization problem gives us the following optimization problem:

$$\min \quad p_s \sum_{i=1}^{N} v_i \left(1 - \frac{D_i}{\lambda v_i}\right) I_{i=i^*} + (1 - p_s) \sum_{i=1}^{N} q_i v_i \left(1 - \frac{D_i}{\lambda w}\right) \tag{2.3.11}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} D_i \leq D \tag{2.3.12}$$

$$D_i \leq v_i,\ i = 1, \ldots, N \tag{2.3.13}$$

$$D_i \geq 0,\ i = 1, \ldots, N \tag{2.3.14}$$

where $I_{i=i^*}$ is the binary indicator variable that takes 1 if $i^* = \arg\max_{i=1,\ldots,N}\{v_i P(D_i, A_i^s) - A_i^s\}$ and 0 otherwise. Note that in our game theoretical model, s- and n_i-type attackers maximize the expected damage on their selected targets (target i^* for s-type and target i for n_i-type), while the defender minimizes it. Hence, this leads to an equilibrium budget allocation for the defender that minimizes the attacker's maximum payoff. Using this observation, we can get rid of the binary indicator variables

Table 2.3.1: Notations and decision variables in Government-Terrorist Game

Notations	
N	Number of targets
p_s	The proportion of s-type attackers
q_i	The proportion of n-type attackers whose preferred target is i; $\sum_{i=1}^{N} q_i = 1$
D	Total budget of the defender
v_i	Defender's and s-type attacker's valuation of target i
w	The valuation of n-type attacker's preferred target
λ	Effectiveness ratio of an attack
$P(D_i, A_i)$	Likelihood of damage function given budget allocation D_i and level of attacker's effort A_i
Decision variables	
D_i, A_i^s, A_i^n	Defender's budget allocation, s- and n-type attackers' efforts on target i

$I_{i=i^*}$ by introducing a variable z that bounds the maximum payoff for the s-type attacker:

$$\text{(Model II)} \quad \min \quad p_s z + (1 - p_s) \sum_{i=1}^{N} q_i v_i \left(1 - \frac{D_i}{\lambda w}\right) \tag{2.3.15}$$

$$\text{s.t.} \quad v_i \left(1 - \frac{D_i}{\lambda v_i}\right) \leq z, \; i = 1, ..., N \tag{2.3.16}$$

$$\sum_{i=1}^{N} D_i \leq D \tag{2.3.17}$$

$$D_i \leq v_i, \; i = 1, ..., N \tag{2.3.18}$$

$$D_i \geq 0, \; i = 1, ..., N \tag{2.3.19}$$

Note that Model II is a linear programming problem with respect to D_i. Therefore, to characterize the defender equilibrium strategy, we can apply the optimality principle of linear programming, which effectively states that when the feasible set is nonempty and bounded, then at least one optimal solution is located at an extreme point [Dantzig, 1951]. Some of the constraints in Model II are therefore binding at the optimal extreme point, which enable us to determine the basic variables. Specifically, assuming that constraint (2.3.16) is binding for some of the targets, namely, $i \in I_D$, we have $D_i = \lambda(v_i - z)$, $i \in I_D$, and $D_i = 0$, $i \notin I_D$. Replacing these values in Model II gives a tractable model to characterize the defender's equilibrium strategy. Before analyzing the equilibrium strategy using this methodology, we summarize the list of notation used for the problem parameters and decision variables in Table 2.3.1.

2.4 Allocation Equilibrium under Symmetric Information

In this section, we characterize the defender's equilibrium strategy under symmetric information scenario, where the defender knows both the attacker's degree of rationality and target preference. The following proposition provides the symmetric information equilibrium:

Proposition 2.1. *In equilibrium,*

1. *[Prioritization of the targets] The defender prioritizes all the targets in decreasing order with respect to their valuations v_i.*

2. *[Distribution of budget] The defender distributes budget to the i^{th} most valuable target if and only if its valuation v_i exceeds a threshold t_i where $t_i = \frac{\lambda \sum_{j=1}^{i} v_j - D}{i\lambda}$.*

3. *[Budget allocation] The defender's optimal budget allocation decision is fully characterized by Algorithm 1 provided in the Proof.*

Note that from the threshold expression in Proposition 2.1, when the defender faces with an s-type attacker, the likelihood that she defends target i increases with both target value, v_i, and defender's budget, D, but decreases with λ. Let us describe the underlying rationale behind Proposition 2.1. First of all, it is obvious that the most valuable target is also the target that will be protected first by the defender. However, the more the defender allocates budget to protect this target, the less the attacker's payoff will be for it, until the attacker becomes better off by switching to the second most valuable target. The defender should now protect both targets, and allocate budget to them so that the attacker's payoff is the same across both targets. The defender continues in this way to make a subset of the most valuable targets less and less exposed while ensuring that no target suffers more damage than any other one. Therefore, the equilibrium thresholds characterized in Proposition 2.1 are determined just to equalize the expected damage across all the defended targets. In the next section, we use the same approach to characterize the defense equilibrium when the defender has only partial information about the attacker's attributes.

2.5 Allocation Equilibrium under Asymmetric Information

In the previous section, we showed how the defender optimally distributes her scarce budget among the targets when exact information about the attacker's attributes is available to her. In this section, we consider the cases where the defender is uncertain about one of these attributes (degree of rationality in §2.5.1 and target preference in §2.5.2 of the attacker. Specifically, we answer the following two questions: (i) How should defender prioritize targets for budget allocation under asymmetric information? (ii) How should she distribute budget among those targets that are to be defended?

2.5.1 Information Asymmetry about an Attacker's Degree of Rationality (Model A)

The aim of this section is to investigate the situation where the defender knows precisely the target preference of the non-strategic attacker, but has partial information about his true degree of rationality. More specifically, she has a-priori beliefs that p_s proportion of the attackers are s-type and $p_n = 1 - p_s$ of them are n_k-type, where k is the preferred target of non-strategic attackers that is known to the defender. Proposition 2.2 summarizes the defender's equilibrium in this case:

Proposition 2.2. *There exists a BNE in which*

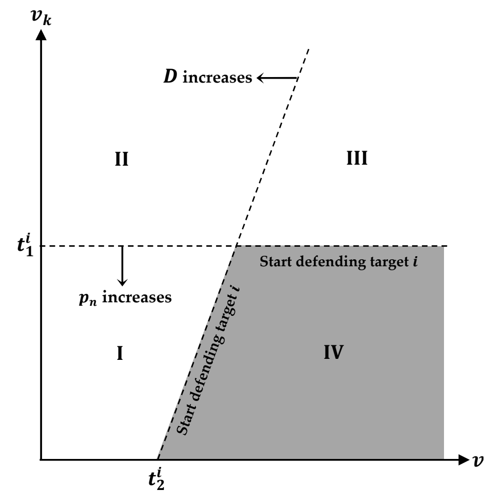
1. *[Prioritization of the targets] The defender prioritizes all the targets (except the target preferred by n_k-type attacker) in decreasing order with respect to their valuations v_i.*

2. *[Distribution of budget] The defender distributes budget to the i^{th} most valuable target if and only if the following two conditions are satisfied: $t_1^i \geq v_k$ and $v_i \geq t_2^i$, where $t_1^i = \frac{p_s w}{i(1-p_s)}$, and $t_2^i = \frac{\lambda \sum_{j=1}^{i} v_j + \lambda v_k - D}{\lambda(i+1)}$. Furthermore, let S be the subset of all defended targets except target k, i.e., $S = \{i \mid t_1^i \geq v_k,\ v_i \geq t_2^i\}$. Then, target k is defended if and only if $v_k > \frac{\lambda \sum_{i \in S} v_i - D}{\lambda |S|}$.*

3. *Finally, the optimal defense allocation is $D_i^* = \lambda(v_i - \mathscr{B})$, $i \in S$, $D_i^* = 0$, $i \notin S$, and $D_k^* = \min\{[D - \sum_{i \in S} [\lambda(v_i - \mathscr{B})]^+]^+,\ v_k\}$, where $\mathscr{B}$ is characterized in Appendix (see Algorithm 2).*

Note that even though prioritization scheme utilized in equilibrium under asymmetric information is same as that under symmetric one, the budget distribution policy is different. Specifically, there

Figure 2.5.1: Defender's optimal decision under uncertainty about attacker's rationality information



are two thresholds in asymmetric setting (as opposed to one threshold in symmetric information setting) that need to be checked by the defender in order to distribute non-zero budget to the i^{th} most valuable target. This implies that the conditions under which a target is defended become more restrictive under asymmetric information (as characterized in proposition 2.2) than under symmetric information (as characterized in Proposition 2.1). The rationale behind this difference relies on the fact that under asymmetric information, the defender needs to take into account the threats from not only an s-type attacker, (by comparing v_i's with t_2^i), but also an n_k-type attacker (by comparing v_k and t_1^i). In other words, the budget distribution for the i^{th} most valuable target depends on not only the characteristics of its own and targets of higher valuation (via t_2^i) but also that of a special target k (via t_1^i). Note also that the threshold t_1^i is the highest for the most valuable target and decreases as the target becomes less valuable. The joint effects of two thresholds on the budget allocation decision for the i^{th} most valuable target are shown in Figure 2.5.1. As highlighted by region IV, target i is defended only when the expected threat from n_k-type attacker is relatively low and its valuation is sufficiently high. Also note that the likelihood of target i being defended decreases as $p_n = (1 - p_s)$ increases. This is because when the threat from an n_k-type attacker increases, the defender shifts resources away from the targets that can potentially be attacked by an s-type to target k (i.e., n_k-type attacker's preferred target).

2.5.2 Information Asymmetry about Nonstrategic Attacker's Target Preference (Model B)

Now in this section, we assume that attacker's type is known by the defender. If true type is s-type, the budget allocation problem facing the defender becomes similar to the symmetric information game considered in Section 2.4. Note that, in Section 2.7.1, we extend the analysis when defender and s-type attacker share different target valuations. Hence, in this section, we only consider the case where the true type of attacker is n-type and the only information asymmetry is on the attacker's preferred target. To model this type of information asymmetry, we assume that the defender has a-priori beliefs that the q_i proportion of (non-strategic) attackers attacks target i. Next proposition characterizes the defender's equilibrium budget allocation under this scenario:

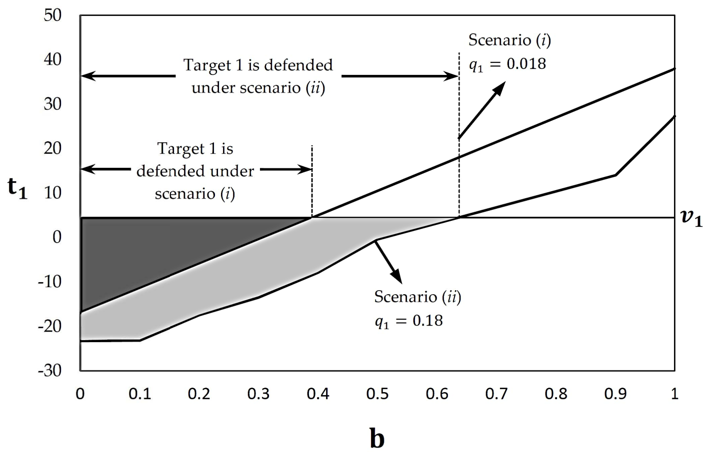
Proposition 2.3. *Assume that the attacker is n_i-type and his preferred target is unknown to the defender. Then, there exists a BNE in which*

1. *[Prioritization of the targets] The defender prioritizes targets in decreasing order with respect to $q_i v_i$, and*

2. *[Distribution of budget] The defender distributes budget to the i^{th} most valuable target (in the sense of $q_i v_i$) if and only if $v_i \geq t_i$, where $t_i = \sum_{j=0}^{i} v_j - D$, and $v_0 = 0$.*

Note that in Proposition 2.3, the defense strategy also consists of a single threshold (as in symmetric information case). However, the defender changes her prioritization scheme by ordering the targets based on not only their valuations but also her a-priori information. To illustrate how this new prioritization scheme affects the budget allocation decision, we consider two different scenarios depending on whether a-priori beliefs are correlated with target valuations or not. Specifically, in the first scenario (scenario (i)), we consider $v_i = \mathtt{a} + \mathtt{b}i$, and $q_i = \frac{i}{\sum_{j=1}^{N} j}$, while in the second one (scenario (ii)), we assume that $v_i = \mathtt{a} + \mathtt{b}i$, and $q_i = \frac{N-i+1}{\sum_{j=1}^{N} j}$. Note that in scenario (i) (resp., scenario (ii)), the valuation of target i is positively (resp., negatively) correlated with a-priori beliefs. Also, in both scenarios, $\mathtt{b}$ measures the degree of heterogeneity among target valuations. Arranging the targets in descending order with respect to $q_i v_i$ and comparing v_i with t_i as characterized in Proposition 2.3, we can determine whether target i is defended or not. For illustrative purposes, we consider only the optimal threshold for Target 1 (i.e., the target with smallest valuation) and numerically analyze how it behaves under each scenario with respect to b in Figure 2.5.2.

Figure 2.5.2: Optimal threshold to defend the target with the smallest valuation when $N = 10$, $a = 1$, and $0 \leq \mathtt{b} \leq 1$



We can make two observations from Figure 2.5.2. First, under both scenarios, t_1 increases in b, i.e., the likelihood of Target 1 being defended decreases as b increases. This is due to the fact that as the degree of target heterogeneity increases, the gap between the targets with smallest and highest valuations widens, which enforces the defender to shift more resources toward higher valuation targets. Second, the negative correlation between a-priori beliefs and target valuations makes the targets equally valuable from the defender's perspective according to $q_i v_i$-ranking hence it smoothens budget allocation among the targets. This is why the likelihood of Target 1 being defended under scenario (ii) is more than that under scenario (i).

2.6 Value of Attacker's Information

In §2.5, we characterized the defender's equilibrium budget allocation when she makes her decision without knowing either (i) true degree of rationality (Model A), or (ii) target preference (Model B) of an attacker. In this section, *addressing research question 3*, we explore how much the defender can potentially gain if she could make her decisions after she observes either one of these attacker's characteristics, and how this additional gain due to information depends on problem data such as targets valuations, v_i, effectiveness ratio of attack, λ, and maximum budget, D. In general, when the attackers and defenders interact repeatedly over time then the defender has opportunity to update her beliefs about the attacker's attributes. In reality, the attacker may arrange numerous trials to evaluate the defense level before making the final attack. For example, the attacker may make successive attacks (as the series of attacks by Al-Qaeda in the United States to World Trade Center in February 26, 1993 and September 11, 2001) or engage in successive attempts to probe a system before a successful attack (as in the case of computer security [Zhuang et al., 2010]). Under such scenarios, the defender can observe attacker's choice of target and level of effort, and thus update her beliefs about attacker's attributes for possible attack in the future. To define the value of information, we compare the defender's payoffs under symmetric and asymmetric settings. For analytical simplicity, we assume that there are only two targets k and k' where target k is more valuable than target k', i.e., $0 < v_{k'} \leq v_k \leq 1$. Recall that the maximum payoff the defender can generate is always under symmetric information setting, where the defender knows both characteristics of the attacker before allocating her budgets. On the other hand, under asymmetric information, the defender makes her budget allocation decision without knowing one of these characteristics. Therefore, the value of information in a defender-attacker game can be defined as the difference between the defender's expected payoffs under symmetric and asymmetric information scenarios. In our setting, it translates into the difference between the defender's payoffs under the symmetric (characterized in Proposition 2.1) and asymmetric (characterized in Proposition 2.2 for rationality and Proposition 2.3 for target preference) scenarios. Let $\mathbb{V}_R^i$ and $\mathbb{V}_T^i$ be the value of rationality and target preference information, respectively, when the attacker is i-type, where $i = s, n_k, n_{k'}$. Let us first establish the value of rationality information.

2.6.1 Value of Attacker's Rationality Information

Recall that the characterization of the value of rationality information (VOR) depends on the true type of the attacker (s- or n-type) as well as on the preferred target of the n-type attacker (target k or k'). We can therefore obtain four different cases, for which we characterize VOR as follows:

Proposition 2.4. *The value of rationality information (VOR) is fully characterized in Table 2.6.1 (when the true type is n) and Table 2.6.2 (when the true type is s).*

Note that in each case VOR depends on two main parameters: the defender's a-priori beliefs about the attacker's type and the defender's budget. Before discussing the impact of these and other parameters on VOR, we note that VOR is equal to zero if either the true type is s and $p_s > t_s$ in Table 2.6.2, or the true type is n and $p_s < t_n$ in Table 2.6.1. In other words, obtaining rationality information has no value to the defender if her prior beliefs correctly match with the true type of the attacker. Therefore, in what follows, we focus only on the cases where these two do not match. Those are the cases where the degree of information asymmetry between the defender and attacker is sufficiently high, i.e., $p_s \geq t_n$ in Table 2.6.1 and $p_s < t_s$ in Table 2.6.2.

- [The impact of defender's budget] In general, VOR initially increases then decreases in the defender's budget D. Recall from Proposition 2.1 that the defender's budget allocation decision under symmetric information depends on the true type of attacker. That is, if the true type is of n, she only defends the preferred target of the n-type attacker, whereas if the true type is of s, she distributes her budget among the most valuable targets proportionally so that these targets are protected equally well. However, the analysis of Model A shows that the defender uses both strategies in order to hedge herself against both s- and n-type attackers. As a result, due to the lack of rationality information, the defender ends up wasting money on the targets that are less likely to be attacked when her budget is limited. This implies that the VOR initially increases in D for small values of D. However, as D keeps increasing (see column "condition on defender's budget" in Table 2.6.1 from bottom to top), even though the defender wastes some money on

Table 2.6.1: Value of rationality information when the attacker is n-type

	Condition on defender's beliefs	Condition on defender's budget	VOR
Non-strategic attacker is n_k-type	$p_s < \bar{t}_n$	Any value of D	0
	$p_s \geq \bar{t}_n$	$v_k + \lambda v_{k'} < D$	0
		$v_k + \lambda v_{k'} \geq D$, $v_k < D$,$\lambda(v_k + v_{k'}) < D$	$v_k - D + \lambda v_{k'}$
		$v_k < D$,$\lambda(v_k + v_{k'}) \geq D$	$v_k - \frac{D+\lambda\Delta v}{2}$
		$v_k \geq D$,$\lambda(v_k + v_{k'}) < D$	$\lambda v_{k'}$
		$v_k \geq D$,$\lambda(v_k + v_{k'}) \geq D$, $\lambda\Delta v < D$	$\frac{D-\lambda\Delta v}{2}$
		$v_k \geq D$,$\lambda(v_k + v_{k'}) \geq D$, $\lambda\Delta v \geq D$	0
Non-strategic attacker is $n_{k'}$-type	$p_s < \underline{t}_n$	Any value of D	0
	$p_s \geq \underline{t}_n$	$v_{k'} + \lambda v_k < D$	0
		$v_{k'} + \lambda v_k \geq D$, $v_{k'} < D$,$\lambda(v_k + v_{k'}) < D$	$\kappa_v(v_{k'} - D + \lambda v_k)$
		$v_{k'} \geq D$,$\lambda(v_k + v_{k'}) < D$	$\lambda v_{k'}$
		$v_{k'} < D$,$\lambda(v_k + v_{k'}) \geq D$, $\lambda\Delta v < D$	$\kappa_v(v_{k'} - \frac{D}{2} + \lambda\Delta v)$
		$v_{k'} \geq D$,$\lambda(v_k + v_{k'}) \geq D$, $\lambda\Delta v < D$	$\kappa_v \frac{D+\lambda\Delta v}{2}$
		$v_{k'} \geq D$,$\lambda(v_k + v_{k'}) \geq D$, $\lambda\Delta v \geq D$	$\kappa_v D$
		$v_{k'} < D$,$\lambda(v_k + v_{k'}) \geq D$, $\lambda\Delta v \geq D$	$\kappa_v v_{k'}$

Notes. $\bar{t}_n = \frac{\lambda}{1+\lambda}$; $\underline{t}_n = \frac{\lambda v_{k'}}{v_{k'}+\lambda v_{k'}}$; $\Delta v = v_k - v_{k'}$; $\kappa_v = \frac{v_{k'}}{v_k}$

the less vulnerable targets, she still has sufficient funds to defend all the targets, which makes rationality information less of an issue for the defender.

- [The impact of λ] The impact of λ on VOR depends on whether defender's and attacker's target preferences match. In general, VOR decreases in λ if they match; otherwise, it increases. The rationale behind the above observation is that λ impacts the defender's budget differently depending on whether the true type is n or s. As shown in Proposition 2.1, under symmetric information, the defender adjusts her budget allocation using λ if the attacker is s-type, whereas her budget allocation does not depend on λ if the attacker is n-type. This means, ceteris paribus, that if the true type is s, VOR decreases with λ because under both symmetric and asymmetric information, the defender spends money only on the most valuable targets as λ increases. On the other hand, the impact of λ on VOR when the true type is n is further complicated by whether the n-type attacker's target preference matches with the defender's or not. If they

Table 2.6.2: Value of the attacker's rationality information when the attacker is s-type

	Condition on defender's beliefs	Condition on defender's budget	VOR
Nonstrategic attacker is n_k-type	$p_s < \bar{t}_s$	$\lambda(v_k + v_{k'}) < D$	$v_{k'}$
		$\lambda(v_k + v_{k'}) \geq D > \lambda\Delta v$	$\frac{D-\lambda\Delta v}{2\lambda}$
		$D \leq \lambda\Delta v$	0
	$p_s \geq \bar{t}_s$	Any value of D	0
Nonstrategic attacker is $n_{k'}$-type	$p_s < \underline{t}_s$	$\lambda(v_k + v_{k'}) < D$	v_k
		$\lambda(v_k + v_{k'}) \geq D$	$\frac{D+\lambda\Delta v}{2\lambda}$
	$p_s \geq \underline{t}_s$	Any value of D	0

Notes. $\bar{t}_s = \frac{\lambda}{1+\lambda}$; $\underline{t}_s = \frac{\lambda v_{k'}}{v_{k'}+\lambda v_{k'}}$; $\Delta v = v_k - v_{k'}$

match, then VOR decreases with λ, otherwise it increases with λ.

- Finally, we discuss the sensitivity of VOR with respect to the degree of heterogeneity in valuation between targets k' and k, i.e., $\Delta v = v_k - v_{k'}$. The result in this case also depends on the true type and the degree of mismatch between target preferences of the defender and attacker. In general, VOR increases with Δv when target preferences match otherwise it decreases in Δv.

2.6.2 Value of Target Information

We now consider the value of the target information (VOT) given that the defender knows the true type of the attacker. In contrast to the value of rationality information (VOR), VOT has only two subcases because VOT is zero when the type of the attacker is s. Therefore, in the following proposition, we characterize VOT only when the attacker is of n-type.

Proposition 2.5. *The value of the nonstrategic attacker's target information is fully characterized in Table 2.6.3.*

Similar to our discussion about VOR information, VOT is also non-zero only when the degree of information asymmetry between the defender and the n-type attacker is sufficiently high, i.e., when either $q_k \geq Q$ and the true target is k', or $q_k \leq Q$ and the true preferred target is k. Below, we briefly discuss the impact of D, λ, and Δv on VOT:

Table 2.6.3: Value of nonstrategic attacker's target information

	Condition on defender's beliefs	Condition on defender's budget	VOT
Nonstrategic attacker is n_k-type	$q_k < Q$	$v_k + v_{k'} < D$	0
		$v_{k'} < D; v_k < D; v_k + v_{k'} \geq D$	$v_k + v_{k'} - D$
		$v_{k'} < D; v_k \geq D$	$v_{k'}$
		$v_{k'} \geq D; v_k \geq D$	D
	$q_k \geq Q$	Any value of D	0
Nonstrategic attacker is $n_{k'}$-type	$q_k \leq Q$	Any value of D	0
	$q_k > Q$	$v_k + v_{k'} < D$	0
		$v_{k'} < D; v_k < D; v_k + v_{k'} \geq D$	$\kappa_v(v_k + v_{k'} - D)$
		$v_{k'} < D; v_k \geq D$	$\kappa_v v_{k'}$
		$v_{k'} \geq D; v_k \geq D$	$\kappa_v D$

Notes. $Q = \frac{v_{k'}}{v_{k'}+v_{k'}}$; $\kappa_v = \frac{v_{k'}}{v_k}$

- [The impact of defender's budget] In general, when the defender's budget D increases (see column "condition on defender's budget" in Table 2.6.3 from bottom to top), VOT initially increases then decreases (last column in Table 2.6.3). The rationale behind this is similar to the VOR case, hence, further discussion is omitted.

- [The impact of λ] As opposed to VOR, λ has no impact on VOT. This is because when the true type is n, the defender does not take λ into account in her budget allocation decision under either the symmetric or asymmetric information scenario.

- Finally, VOT decreases (resp., increases) with Δv if a higher (resp., lower) proportion of n-type attackers prefer the target that is also the most valuable from the defender's perspective. Put differently, the defender gains more under symmetric information if she orders the targets differently from the n-type attacker.

2.6.3 Numerical Study: VOR vs. VOT

Note that from characterizations in Propositions 2.4 and 2.5, the characterization of both VOR and VOT depend on the true type and target preference, hence it is analytically quite cumbersome to compare VOR and VOT and explore the impact of different parameters on the relative comparison. Therefore, in this section, we first take a holistic approach to define the expected VOR and VOT and then provide an illustrative numerical study to compare VOT and VOR. Recall from §§2.6.1 and 2.6.2 that VOR and VOT depend on the true type of the attacker as well as on the preferred target of the n-type attacker. To make a unified comparison, we first aggregate all the cases with their respective probabilities and obtain the expected VOR and VOT. Let $\overline{\mathbb{V}}_R$ and $\overline{\mathbb{V}}_T$ denote the expected VOR and VOT, respectively, where $\overline{\mathbb{V}}_R$ and $\overline{\mathbb{V}}_T$ are computed as follows:

$$\overline{\mathbb{V}}_R = p_s[q_k \mathbb{V}_R^{s,k} + (1-q_k)\mathbb{V}_R^{s,k'}] + (1-p_s)[q_k \mathbb{V}_R^{n_k,k} + (1-q_k)\mathbb{V}_R^{n_{k'},k'}] \tag{2.6.1}$$

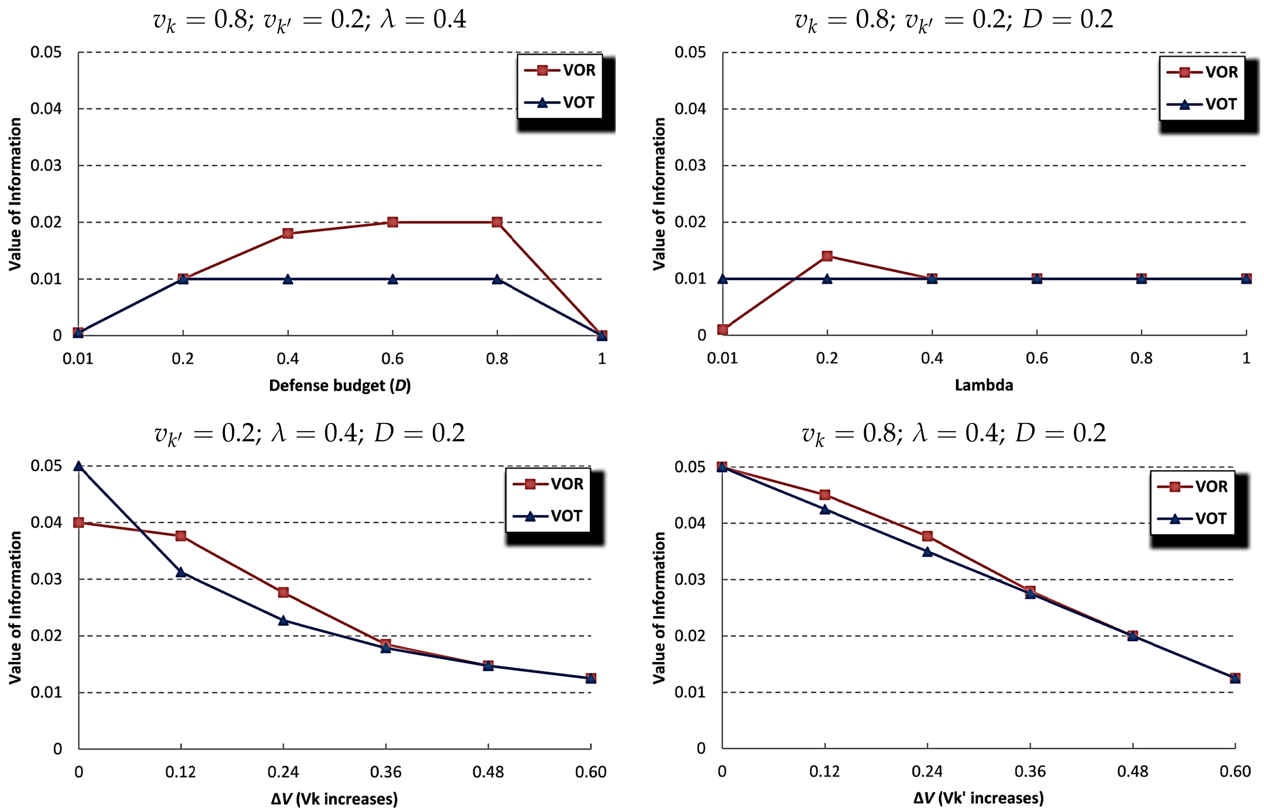
$$\overline{\mathbb{V}}_T = q_k[p_s \mathbb{V}_T^{s,k} + (1-p_s)\mathbb{V}_T^{n_k,k}] + (1-q_k)[p_s \mathbb{V}_T^{s,k'} + (1-p_s)\mathbb{V}_T^{n_{k'},k'}] \tag{2.6.2}$$

where $\mathbb{V}_T^{s,k}$ and $\mathbb{V}_T^{s,k'}$ are zero by definition. In Figure 2.6.1, keeping $p_s = q_k = 0.5$, we plot $\overline{\mathbb{V}}_R$ and $\overline{\mathbb{V}}_T$ with respect to D, λ and Δv. We make the following observations from the above comparisons:

- In general, from all of the above four figures, attacker's rationality information is more valuable than target information. In other words, lack of rationality information costs more to the defender than lack of target information does. The rationale behind this is as follows: as shown in the previous sections, the defender employs radically different defense strategies depending on whether the attacker is n- or s-type. Specifically, an n-type attacker requires for a more concentrated defensive effort, whereas an s-type attacker demands a more comprehensive defense strategy. This implies that using an s-type strategy against n-type attacker (or vice versa) imposes significant losses on the defender.

- The gap between VOR and VOT initially increases in D. To understand this, consider a unit-dollar increase in the total budget. Faced with an s-type attacker, the defender distributes this extra dollar among all the defended targets in such a way that the expected damage is evenly reduced for all of them. On the other hand, in the case of an n-type attacker, the extra dollar is fully spent on the preferred target, as long as there is room for budget allocation for this target.

Figure 2.6.1: The comparison between VOR and VOT



Therefore, as D increases, the degree of mismatch between equilibrium budget allocations for s- and n-type attackers increases, which in turn makes the defense against an attacker with unknown degree of rationality more costly. In addition, if the defender knows the true degree of rationality of the attacker, under certain cases, she can use this to manipulate the attacker in his target selection (this is especially true if she knows the attacker to be s-type). Therefore, rationality information gives extra value to the defender, which reduces the impact of target information asymmetry, $\overline{\mathbb{V}}_T$ on the defender's payoff. To summarize, these two factors increase VOR at a faster rate than VOT as the defender has access to more resources.

- In general, the difference between VOR and VOT diminishes as λ increases. The main reason for this comes from the fact that the equilibrium budget allocation rule employed for the s-type attacker becomes more similar to that for n-type attacker as λ increases. Because of this, for high values of λ, the defender is equally worse off with both rationality and target information asymmetries.

- In order to explore which information is more valuable when targets become heterogeneous in terms of their values, we can either increase the value of target k while keeping k' constant (see the third panel in Figure 2.6.1), or decrease the value of target k' while keeping k constant (see the fourth panel in Figure 2.6.1). Note that the difference between VOR and VOT vanishes for high values of Δv, ($\Delta v = 0.6$). When Δv is small and both targets are low-valued (third panel of Figure 2.6.1), the defender may easily deter an s-type attack on both targets, causing her to be more concerned about the n-type's preferred target information. On the other hand, when Δv is small and both targets are high-valued (fourth panel of Figure 2.6.1), it is quite costly for the defender to protect these targets from an s-type attacker and consequently the defender is interested to know whether the attacker is s- or n-type.

- Finally, the difference between VOR and VOT may be affected by other values of p_s and q_k. In general, when the n-type attacker preference on targets is more likely to be similar to the s-type attacker (i.e., when q_k increases) then it becomes less valuable for the defender to know whether the attacker is s- or n-type. That is to say, the relative advantage of VOR over VOT decreases when q_k increases. However, the impact of change in p_s on the comparison between VOR and VOT mainly depends on whether or not the n-type attacker shares the same preference with the s-type attacker. Specifically, for low values of p_s, knowing the n-type attacker's preferred target becomes important for the defender when the n-type attacker prefers the less valuable target (target k'), under which the VOR may lose its relative advantage over VOT.

2.7 Extensions

Throughout the analysis of our basic model, we assume that s-type attacker shares the same target valuations with the defender. In §2.7.1, we relax this assumption. In addition, in asymmetric information analysis, we consider the cases, where either only rationality or target information is unknown by the defender. In §2.7.2, we analyze full asymmetric information case where both rationality and target information are unknown to the defender.

2.7.1 Uncertainty in s-type Attacker's Preference

In this section, we consider the case where s-type attacker's target valuations are not necessarily same as the defender's. For analytical tractability, we focus on the same two-target setting analyzed in §2.6. Namely, there are two targets k and k' with valuations v_k and $v_{k'}$, where $0 < v_{k'} \leq v_k \leq 1$. Similar to our approach before, using Harsanyi's transformation, we can define two types of s-type attacker: type 1 who values targets in the same way as the defender does; and, type 2 who values targets in the opposite way. Furthermore, we assume that q_k proportion is of type 1. Specifically, if u_{i_j} show the target i's valuation for type $j = 1, 2$ of s-type attacker, then we have $u_{k_1} = v_k$, $u_{k'_1} = v_{k'}$, $u_{k_2} = v_{k'}$, and $u_{k'_2} = v_k$. The rest of model has the same specifications as in the basic model of §2.4. The following proposition characterizes the defense equilibrium for this case.

Proposition 2.6. *Assume that the attacker is s-type and his preferred target is unknown for the defender. In a two-target setting, the defender's more valuable target, i.e., target k, should be always defended, whereas the defender uses a threshold-type policy to decide whether to defend the less valuable target, k'. Specifically, target k' is defended if and only if either* $q_k < \frac{v_k v_{k'}}{v_k^2 + v_{k'}^2}$ *and* $D \geq \frac{\lambda v_{k'}(v_k - v_{k'})}{v_k}$*, or,* $q_k \geq \frac{v_k v_{k'}}{v_k^2 + v_{k'}^2}$ *and* $D > \lambda(v_k - v_{k'})$.

There are two main take-aways from Proposition 2.6. First, consistent with our earlier results, defender uses a threshold-type policy to decide whether to defend a target or not. Second, the defender always sorts the targets in terms of their valuations and allocates budget to the more valuable target regardless of the s-type attacker's preferred target. This is in contrast with our finding in Proposition 2.3, where the defender, facing with a n-type attacker with unknown target information, sorts the targets with respect to belief-adjusted valuations (i.e., $q_k v_k$), and consequently may or may not defend the target with the highest valuation. This comes from the difference between s- and n-type behaviors in the target selection. Specifically, the defender cannot influence n-type attacker's target selection decision with her budget allocation decision, whereas she can incentivize s-type attacker to choose his second most valuable target through budget allocation decision. Therefore, the defender can be always better off by defending her most valuable target when she faces with s-type attacker.

2.7.2 Allocation Equilibrium under Full Asymmetric Information

The following proposition provides the defender's equilibrium strategy under full asymmetric information.

Proposition 2.7. *Assume that the attacker is s-type with probability p_s and n-type with probability $1 - p_s$. If the attacker is n-type, he prefers target k with probability q_k. Then, there exists a BNE in which*

1. *[Partition of targets] All the targets are partitioned into two disjoint sets denoted by I_1 and I_2 (as characterized by Algorithm 3 in Appendix), where $I_1 \cap I_2 = \varnothing$ and $I_1 \cup I_2 = \{1, ..., N\}$.*

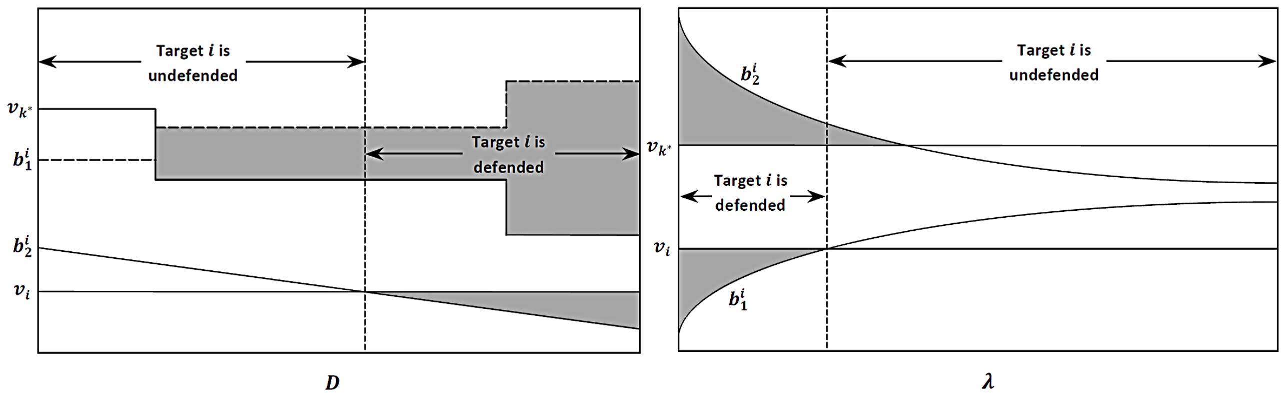
2. *[Prioritization of targets] The defender prioritizes all the targets in I_1 and I_2 with respect to v_i and $q_i v_i$, respectively.*

3. *[Distribution of budget in I_1 and I_2] The defender distributes budget to the i^{th} most valuable target (in the sense of v_i) in I_1 if and only if $b_1^i \geq v_{k^*}$ and $v_i \geq b_2^i$, where k^* denotes the least valuable target (in the sense of $q_i v_i$) in subset I_2. The defender distributes budget to the i^{th} most valuable target (with respect to $q_i v_i$) in I_2 if and only if $v_i \geq t_i$. The thresholds for each target in those subsets are fully characterized in Algorithm 3 provided in Appendix.*

Note that the above proposition can be viewed as an extension of Propositions 2.2 and 2.3. Specifically, the defender divides the targets in two categories (i.e., $I_1$and I_2) and adopts the same prioritization and budget distribution schemes of Propositions 2.2 and 2.3 to defend the targets in first and second categories against s- and n-type attackers, respectively. Namely, the defender allocates budgets to the targets in I_1 to make sure that all the defended targets in I_1 would face equal expected damage from s-type attacker. On the other hand, to defend targets in I_2 against n-type attacker, the defender allocates the maximum available budget to each target in I_2, bounded by either target valuation v_i or the remaining budget (whichever is minimum). However, there is a subtle difference

Figure 2.7.1: Optimal defensive thresholds as a function of D and λ under full asymmetric information

(a) Optimal threshold as a function of D (b) Optimal threshold as a function of λ



between the strategies used to defend in I_1 in the above Proposition and Proposition 2.2. Recall that in Proposition 2.2, when deciding whether to defend a specific target, say target i, the defender takes into account the valuation of not only that target but also another (critical) target, which is exogenously given by the preferred target of n-type attacker. In the above Proposition, the defender still employs two thresholds. However, both the critical target k^* and the threshold used for the critical target are endogenously determined by the partition scheme provided in Algorithm 3 of Appendix. To illustrate this, we provide a numerical example in Figure 2.7.1. Note that in this example, the critical target k^*, whose valuation needs to be checked in order to decide whether the i^{th} most valuable target in set I_1 is defended or not changes endogenously as defender's total budget D increases. This is because the size of I_2 increases with the inclusion of another target into the set I_2 as defender can allocate more budget, and the target included becomes the new critical target k^*. Using the algorithm in Proposition 2.7, we can compute two thresholds b_1^i and b_2^i and check whether each threshold is satisfied or not, i.e., $b_1^i \geq v_{k^*}$ or $v_i \geq b_2^i$. Thus, target i is defended whenever both shaded areas overlap on the horizontal axis. Figures 2.7.1a and 2.7.1b show the behavior of the thresholds b_1^i and b_2^i with respect to the total budget D and to λ, respectively. As shown in Figure 2.7.1a, target i is more likely to be defended in D and less likely to be defended in λ. The underlying rationale behind the latter observation is as follows. The defense investment becomes less cost-effective as λ increases. This causes the defender to concentrate her budget allocation on fewer targets in order to reduce the total expected damage from an s-type attacker. Hence, the total number of defended targets decreases.

2.8 Conclusion and Future Research Directions

In this chapter, we studied the impact of information asymmetry about the terrorist's various attributes on equilibrium defensive budget allocation decision. To address our research questions, we considered two critical information that affects the government's decision: (i) the degree of rationality of the terrorist, and (ii) the terrorist's target preference. To answer research question 1 (How should a defender prioritize multiple targets and allocate limited budget among them when faced with two types of asymmetric information about the attacker?), we fully characterized the equilibrium defense strategy under various information scenarios. We showed that, under both symmetric and asymmetric information scenarios, the defender first ranks the targets according to a scheme that depends on the type of information available to her. Next, she distributes the budget to the targets by using a set of thresholds starting from the most valuable targets according to ranking rule employed. Addressing research question 2 (What is the impact of partial information on the defender's equilibrium budget allocation strategy?), we compared both the target ranking and budget distribution schemes under symmetric and asymmetric information scenarios. Our analysis shows that the conditions under which a target is being defended involve more conditions under asymmetric information scenarios, especially when the government knows less about the terrorist's degree of rationality. Second, the targets are ranked according to their valuations if the attacker is strategic; otherwise, when the defender expects to face with non-strategic attacker with unknown target preference, she should adjust the ranking by using her a-priori beliefs. Finally, to address research question 3 (From the defender's perspective, what is the value of information regarding the attacker's degree of rationality and target preference? How does the value of information depend on problem parameters such as defender's budget, targets' valuations, and effectiveness of the defender's budget?), we compared how much additional value the defender would gain by using the rationality versus target preference information in her budgeting decision and explored how these comparisons are affected by the problem parameters. Our analysis shows that: (1) the value of information regarding either terrorist's rationality or target preference is nonzero only if the degree of information asymmetry is sufficiently high; (2) the value of information initially increases and then decreases in government's budget; (3) the value of information decreases (resp., increases) with the degree of heterogeneity between targets if the nonstrategic terrorist's preference highly (resp., weakly) matches with the government's preference; and, (4) the effectiveness ratio of attack has no impact on VOT information, but, the impact of that on VOR depends on the true type of the terrorist. We also provided two extensions. In the first one, the strategic terrorist's target valuations are not necessarily same as the government's, and, in the second extension, both rationality and target information are unknown to the government. Our findings showed that the structural properties of budget allocation equilibrium, specifically, the way to prioritize the targets and distribute the budget, still hold true. The model presented in this chapter can be extended in various ways. For example, our model is a non-zero-sum two-stage game in which players make decisions sequentially. An interesting extension would be to consider simultaneous-move games with asymmetric information, known as Colonel Blotto games in the literature. Another extension would be to explore dual-asymmetric information scenarios, where both government and terrorist have incomplete information about each other. Lastly, we believe that the analysis of information asymmetry in the defender-attacker problems presents fruitful research opportunities, and hope that our model will fuel future research in this field.

Chapter 3

The Value of Audit in Managing Supplier's Process Improvement

3.1 Introduction

Although outsourcing a certain function to an expert brings economic advantage and allows the outsourcing company to focus on what she does best, it comes at the cost of increasing risk, reducing visibility, and losing control over the way the function is performed by the outsourcee [Knowdell, 16 Apr 2010]. A recent survey by CFO Research Services [February 2009] reveals that nearly 40% of outsourcing companies experience a high correlation between their global sourcing strategies and greater risk exposure. The results of more recent papers and surveys in both academic and practitioner literatures consistently show that the underlying causes behind some of these supply chain failures not only come from the lack of information regarding the conditions under which the suppliers operate but also can be traced back to the lack of observability of the suppliers' actions. For example, Mattel's investigation regarding the causes of 2007 product recall unveiled that some of its contract manufacturers *intentionally* have avoided to perform the mandated test procedures on paint, which resulted in noncompliant levels of lead [Tang, 2008]. Similarly, dozens of deaths from blood thinner Heparin were traced back to a supplier in China who had employed an unapproved component in order to lower the production costs [Sheffi, 2007].

These supply chain failures have led firms that use contract manufacturers to directly involve in their vendors' process- and production-related decisions. For example, after the product recalls due to malfunctioning of fuel pumps [Harley, Decebmer 2009], Volvo has established a Supplier Evaluation Model under which all current suppliers are audited to verify whether their products meet the pre-specified quality and reliability standards or not and awarded with additional business if their combined scores pass certain level [Volvo, 2010]. Similarly, after its 2007 product recall, Mattel started to mandate from its vendors to purchase paint from a list of certified suppliers, who have been pre-audited to ensure compliance with lead level standards [Tang, 2008]. The examples are not only specific to the companies that experienced product recall. Indeed, according to Aberdeen's 2012 report [Limberakis, August 2012], the companies that closely monitor their suppliers and manage their actions enjoy best-in-class performance 1.36 times more likely than the others who don't. Similarly, in a more recent survey by Aberdeen Group [Group., May 2013], the top 20% best-in-class performers consist of companies who track end-to-end supply chain visibility at the item level. These results suggest that increasing visibility over the true risk factors faced by the suppliers as well as their actions becomes a critical strategy for the outsourcing companies who seek to reduce cost and improve operational performance in the presence of complex and multi-tiered global supply-demand networks. The following statements of an operations director at a medium-sized U.S. durable goods firm support these survey results [Group., May 2013]:

> "Having supply chain visibility translates into being able to meet customers' needs... We have good visibility after the product has left a foreign port, but would like to have more insight into our suppliers' subcontractors and what is happening in their incoming supply chains. We believe that higher visibility is partly contributing to lower lead time variability, reduced inventory, shorter lead times, increased fill rates and other supply chain operational improvements."

To summarize, the information asymmetry between buyers and suppliers can be due to many reasons such as lack of process automation across supply chain [Industry Week, December 2009], lack of confidence among the channel partners [Cranfield University, 2002], and insufficient due diligence on the part of the supplier, etc. In this chapter, we focus on two sources of information asymmetry that naturally arise in the form of hidden information and action between a manufacturer and a supplier in a supply disruption setting. The former arises due to the mere fact that the supplier knows the extent of his true reliability better than the manufacturer because he is either closer to the source of risk factor than the manufacturer or affected by the exogenous factors and players to which the manufacturer may have no direct access. For example, Philips had better understanding than Nokia and Ericsson about the consequences of the catastrophic event that disrupted the production in its New Mexico plant, on March 2000, which, in turn, triggered in cellular phone industry the battle between Nokia and Ericsson [Sheffi, 2007]. The latter arises due to the lack of control that results from the suppliers' taking certain actions without informing the buyer. As explained above, the buyer's lack of information over its suppliers' actions is one of the primary reasons behind recent problems in the toys, textiles and electronics industries [USA Today, October 2008]. The main goal of this study, therefore, is to shed light on the issues jointly caused by hidden information and hidden action and to explore the various means with which the resulting adverse effects can be mitigated.

As showcased by the above cases, a particularly important issue that faces a buyer is to find out how to incentivize its suppliers to take the costly actions to improve the reliability of their processes in the presence of information asymmetry. Numerous approaches with varying degrees of power have been proposed in both practitioner and academic circles. At one extreme is the so-called "arms-length relationship" approach, which advocates minimizing dependence on suppliers through standardized transactions and/or contracts. This has the benefit of minimizing the transaction costs and maximizing the bargaining power for the sake of buyers [Dyer et al., 1998]. At the other extreme is the "close relationship" approach, in which the manufacturers work closely with their suppliers and monitor their actions throughout every phase of the production process [Dyer et al., 1998]. As one moves to the latter end of spectrum, the buyers earn more transparency into the actions of their suppliers, but at the same time, it becomes more costly for them to establish and, more importantly, maintain such a close relationship with their suppliers. In reality, firms use hybrid approaches that consist of features reminiscent of these two extremes [Spekman et al., 1998]. For example, consider the case of Apple, which to a large extent, owns its success to its extensive contract-based outsourcing program [NY Times, 21 January 2012]. However, as indicated in its 2012 report [Apple Inc., 2012], Apple seems to increase its degree of control over its suppliers by implementing a very strong auditing program. In just 2011 alone, Apple conducted 229 audits — an 80 percent increase over the previous year. Its auditing program reaches all levels of its supply chain, including its final assembly and component suppliers. According to *the same report,* Apple continues to expand this program by adding more detailed and specialized audits to address safety and environmental concerns. Not only the users of contract-based manufacturers like Apple but also OEMs that run extensive supplier network at their back stages have increased their auditing efforts in order to preempt the publicity smear due to a possible noncompliance of their suppliers. For example, extensive audit program undertaken by Toyota after having to recall around 8 million vehicles due to malfunctioning gas pedals between 2009 and 2010 uncovered another covert action incidence for one of Toyota's component suppliers [see Bloomberg Business Week report, Kitamura et al., October 2010].

In this chapter, we study the value of audit by comparing following two mechanisms in the presence of hidden information and hidden action: (i) in the first one, the manufacturer incentivizes the supplier to take a particular action only by offering him a contract (hereafter referred to as "Induced-Effort (IE) contract"); and; (ii) in the second one, in addition to the contract, the manufacturer also incurs a cost to audit the supplier's action (hereafter referred to as "Audited-Effort (AE) contract"). The former is more cost-effective but leads to potentially higher agency costs for the manufacturer, whereas the latter provides more visibility at the expense of direct auditing costs for the manufacturer. By comparing these two settings, our aim is to study the following research questions:

Research Question 1: Which one of the above two mechanisms should be used by a manufacturer when she contracts with a supplier whose decisions as well as extent of the supply risk under which these decisions are taken are not observable by the manufacturer?

Research Question 2: What is the value of audit in this context for the manufacturer, the supplier and the total supply chain?

Research Question 3: How do the problem parameters affect the value of audit for each supply chain party and total supply chain?

By applying extended revelation principle [Myerson, 1982] to our setting, *without loss of generality*, we will implement both IE and AE via following three-term contracts: subsidy, contingent payment and penalty. The first term in the contract (i.e., *subsidy*) stands for the fixed payment made by the manufacturer to cover a fraction of supplier's investment in process improvement (see Tang et al. [2013] for a similar use of subsidies in a contractual setting between a manufacturer and an unreliable supplier). The contracts with only fixed terms, also called as "cost-plus" or "fixed-price" contracts, are quite common in regulation and procurement literatures (see Baron and Myerson [1982], Laffont and Tirole [1986] and the citations therein) and are known to be theoretically the best under symmetric information setting. However, in the presence of informational asymmetries, one needs to introduce incentive fees to the contract, which serve to increase the power of the contract at the expense of overall efficiency (see "cost-plus-incentive-fees" contracts in Laffont and Tirole, 1986). There are various ways in which incentive fees are introduced in the literature. For example, similar to the second term in our contract, Babich and Tang [2012] used also *contingent payment* in a quality uncertainty setting to reward the supplier for the items that are successfully delivered to the customer, whereas similar to our third term, Reyniers and Tapiero [1995], Baiman et al. [2000], Gurnani and Shi [2006] and Yang et al. [2009] embedded *penalty* terms into their contracts in order to recover damages for non-delivery or defective deliveries. As we show later in the model analysis, a judiciously designed "fixed-term-with-incentive-fees" contract will be quite instrumental for the manufacturer in maximizing the overall supply chain efficiency and minimizing the agency costs resulting from hidden information and action.

In order to answer the above research questions, in this chapter, we develop a dyadic supply chain model in which a manufacturer procures from a supplier whose production process is subject to disruption risk, the extent of which is *private information* for the supplier. Furthermore, to reduce the disruption risk, the supplier can exert a costly process improvement effort that is also *unobserved* by the manufacturer. Using this setting, we completely characterize the optimal IE and AE contracts. The first observation from this is on the power of optimal contracts under IE and AE settings. Relatively speaking, the optimal contracts under IE setting need more power (i.e., larger incentive fees) than those under AE setting. This is aligned with the established facts on principle-agent models that counteract reduced informational visibility with increased incentive power. The comparative analysis of agency costs and efficiency losses between IE and AE contracts lead to one of the main results of this study. Specifically, differently from the previous results [Laffont and Martimort, 2002], we show that observing the supplier's actions has an actual and positive effect on the manufacturer's payoff *even* under a standard setting (i.e., under risk-neutrality and in the absence of limited liability). The reason behind this comes from the interaction between supplier's actions and his private information, which in turn makes the audit as an additional screening device for the manufacturer to differentiate more reliable suppliers from less reliable ones. Through a series of robustness checks, we confirm that this result extends to the other settings (such as product quality uncertainty [Laffont and Martimort, 2002] and restricted contract space (please refer to §5.2 for a discussion on these extensions). Finally, we also analyze the value of the audit from the perspectives of the supplier and the total supply chain, and show when it can be a win-win strategy for all parties involved.

3.2 Literature Review

Our study in this chapter is related to two streams of research in operations management. The first one focuses on modeling improvement decisions of supply chain firms. The second stream relates to contract design under supply disruption. In what follows, we review each stream and relate them to our work.

The papers in the first stream vary in terms of whether the decision taken by the firm improves the quality of the product [Baiman et al., 2000, Balachandran and Radhakrishnan, 2005, Chao et al., 2009, Babich and Tang, 2012], reduces cost [Corbett et al., 2005, Bernstein and Kok, 2009, Li, 2012, Kim and Netessine, 2013], or increases the reliability of the process [Chopra et al., 2007, Wang et al., 2010, Tang et al., 2013]. In some of these models, the action taken by the firm cannot be observed by the other party in the supply chain; hence, the problem becomes a moral hazard type. The works that come closest to ours in this stream are Kim and Netessine [2013] and Babich and Tang [2012]. In the former paper, similar to our study, the manufacturer suffers not only from supplier's hidden action, but also from supplier's private information that could be acquired via a joint cost-reduction effort. Our work differs from Kim and Netessine [2013] mainly in two ways. First, in this study, we focus on the impact of auditing the supplier's action in reducing degree of the information asymmetry about the disruption risk, whereas Kim and Netessine [2013] focus on the impact of collaborative efforts in reducing cost uncertainty. Moreover, in Kim and Netessine [2013], both supply chain parties are, ex-ante, in equal footing in terms of information available to them with regards to the extent of [cost] uncertainty, and the action taken collaboratively by the parties creates ex-post information asymmetry among them. However, in our model, supply chain parties are endowed with asymmetric information with regards to the extent of [supply] uncertainty in both ex-ante and ex-post stages and the cost of action is incurred only by the supplier.

In the latter paper, Babich and Tang [2012] study the role of inspection in product adulteration setting, where, similar to our audit-based contract, the buyer uses inspection to detect whether or not the supplier has adulterated the product. Our work differs from Babich and Tang [2012] in two ways; first, in Babich and Tang [2012] the buyer only suffers from unobservability of supplier's action, hence they only study pure moral hazard problem, whereas in our model, there are both information asymmetry and moral hazard. Second, in our model, audit is exerted in advance of a potential disruption event hence it is used mainly as a risk-mitigation strategy, whereas in Babich and Tang [2012], inspection happens after the realization of disruption, therefore, it is used as a contingency tactic.

The papers in the second stream study hidden information problems in supply chain context (see Cachon 2003 for an excellent review of this literature). Similar to the first stream, the papers in this stream can also be categorized in terms of whether the hidden information is defined in terms of the supplier's cost (e.g., Corbett et al., 2004, Cachon and Zhang, 2006, Özer and Raz, 2011, Kim and Netessine, 2013) or reliability (e.g., Yang et al., 2009, Tomlin, 2009, Chaturvedi and Martinez-de Albeniz, 2011, Gümüş et al., 2012, Yang et al., 2012). Our study contributes to the latter case by developing a supply chain reliability model that explores how audit can be combined with the contracts to effectively reduce the agency costs associated with hidden information and hidden action. The most related paper to ours in this stream is Yang et al. [2009], which considers the value of the backup production option in a supply disruption setting, where the reliability of the supplier is private information for the supplier. Our work differs from Yang et al. [2009] mainly for two reasons. First, we consider supplier's hidden action on top of his private information. Second, the backup production option is a contingency tactic, which can be employed by the supplier in the case of disruption, however, process improvement is a mitigation tactic that stochastically reduces the exposure to disruption.

Lastly, our work is related to the mixed principal-agent models in information economics in the sense that under IE setting, the manufacturer faces an adverse selection problem followed by a moral hazard [Laffont and Tirole, 1993, Laffont and Martimort, 2002], whereas under AE setting, she faces a moral hazard with private information (refer to Chapters 3.6. and 4A.2. of Macho-Stadler and Perez-Castrillo [2001], Chapters 10.1 and 10.2 of Rasmusen [2006], and Milgrom [1987]). One of the main findings in this literature is that under a standard setting (i.e., when the principal is not subject to limited liability constraint, and the agent is risk-neutral), the moral hazard constraints imposed by the principal's unobservability of the actions of the agent have no impact on the principal's payoff (see Laffont and Tirole, 1993, Laffont and Martimort, 2002). Our analysis adds to this result a *caveat*. More specifically, when the hidden action taken by the agent affects the degree of information asymmetry between principal and agent (as it is the case in our model), then we show that moral hazard constraints have bite in the sense that the principal's payoff under pure adverse selection becomes strictly higher than that under mixed model. Mixed models find various applications in different branches of applied economics. For recent applications of them in health, environmental and resource economics, see Liu et al. [2011], Anthon et al. [2010], and Bellemare [2006], respectively. Finally, we refer the readers to Guesnerie et al. [1989] for a review of the mixed models in information economics.

3.3 Model Framework

In order to address the research questions raised in §3.1, we develop a stylized two-level supply chain model between a manufacturer (hereinafter referred to as "she") and a risky supplier (referred to as "he"). At the downstream level, the manufacturer faces a demand D and earns \$$r$ per unit sold in the market. To satisfy her demand, she needs to procure from a supplier whose production cost is \$$c$ per unit. In order to focus on the supply-side risks, we assume that the demand is known at the time of the contract and, without loss of generality, we normalize it to be one, i.e., $D = 1$.

At the upstream level, the supplier is unreliable in the sense that his ultimate production quantity $\tilde{q}$ is subject to a disruption risk[1]. The extent of disruption risk depends on exogenous and endogenous factors, both of which are unobservable by the manufacturer. The exogenous factor faced by the supplier is determined by his true reliability type denoted by θ. For the sake of analytical tractability, we assume that θ can take two values: h and l representing h- and l-type suppliers, respectively. Everything else remaining the same, the l-type supplier faces more disruption risk than the h-type does in the sense of first-order stochastic dominance (to be formally defined in Assumption 1). On the other hand, both l- and h-type suppliers can endogenously reduce their exposure to the disruption risk by exerting a costly and unobservable process improvement effort e_θ, where e_θ can be either 0 or 1, representing no-effort and effort cases, respectively. To develop analytical managerial insights, we assume that the supplier's production process is subject to all-or-nothing type disruption risk whose survival probability $p(\theta \mid e_\theta)$ is a function of his type $\theta \in \{l, h\}$ and effort $e_\theta \in \{0, 1\}$. Note that modeling disruption risk as all-or-nothing type is also common in the recent supply-risk literature (e.g., Babich et al., 2007, Yang et al., 2009, 2012, and references therein). Let $\tilde{q}_\theta$ be the realized production quantity for θ-type supplier. Then, it can be characterized by a Bernoulli random variable with the following probability mass function:

[1] We would like to point out that most of our analysis and results can be extended to a model, where the disruption risk can be defined in terms of the quality (rather than quantity) of the product.

$$\tilde{q}_\theta = \begin{cases} 1 & \text{with probability } p(\theta \mid e_\theta) \\ 0 & \text{with probability } 1 - p(\theta \mid e_\theta) \end{cases} \tag{3.3.1}$$

For the sake of notational simplicity, we let $p(\theta \mid e_\theta = 1) = \rho_\theta$ and $p(\theta \mid e_\theta = 0) = \varphi_\theta$. As mentioned above, the process-improvement effort helps the supplier to increase the likelihood of survival from disruption (i.e., $\tilde{q}_\theta = 1$), but it comes at a cost $C_\theta(e_\theta)$ which denotes the cost of process improvement for θ-type supplier. Without loss of generality, we equalize the increase in survival probability observed by both supplier types due to process-improvement effort, i.e., $\rho_h - \varphi_h = \rho_l - \varphi_l = \delta$, normalize the cost of no-effort at zero, i.e., $C_\theta(e_\theta = 0) = 0$ and let $C_\theta(e_\theta = 1) = \psi_\theta$ for both $\theta \in \{l, h\}$[2]. The below conditions characterize the impact of the supplier's reliability type and effort on the likelihood of disruption:

Assumption 1.

(A) For the same level of effort, $e_h = e_l = e \in \{0, 1\}$, the survival probability for the h-type is larger than that for the l-type: $\rho_h \geq \rho_l$, and $\varphi_h \geq \varphi_l$.

(B) The survival probability for both l- and h-type suppliers increases in e, i.e., $\rho_\theta \geq \varphi_\theta$.

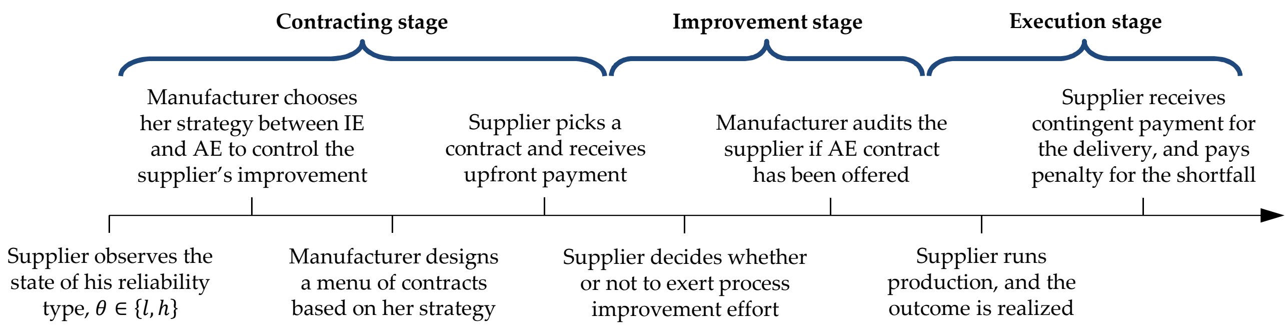
The above conditions are standard in the asymmetric information literature, where they are commonly referred to as "Spence-Mirrlees Conditions" [Laffont and Martimort, 2002]. Also, the standard mixed models in information economics (see Chapter 7 in Laffont and Martimort [2002]) assume differently from ours that the supplier's effort has the same effect on the likelihood of $\tilde{q}_\theta = 1$ and cost function for both types, i.e., (using our notation) $\rho_h = \rho_l$, $\varphi_h = \varphi_l$ and $\psi_h = \psi_l$. As we will show in §3.6, relaxing these equalities will play important role in the analysis of value of audit. Next, we discuss the distribution of information among the supply chain parties. As mentioned above, the true type of the supplier's reliability, as well as the process improvement effort exerted by the supplier, are only known to the supplier himself. The manufacturer has a-priori beliefs on the type of supplier, denoted by $\nu \in [0, 1]$ in the sense that the fractions of the h- and l-type suppliers are ν and $1 - \nu$, respectively. Finally, we assume that all the parameters are common knowledge among the players. Using the above modeling framework, we evaluate two different contractual settings for the manufacturer in order to analyze the value of audit in the presence of disruption risk with asymmetric information. In the first

[2]Note that process improvement cost and reliability of θ-type supplier are perfectly correlated with each other. In Appendix, we extend our model to the case where the process improvement cost and reliability are uncorrelated and show that the qualitative nature of all the results hold true.

setting, i.e., IE (*Induced-Effort*) contract, the manufacturer cannot observe the supplier's process improvement effort, hence, indirectly influences his decisions via contract terms. In the second setting, i.e., AE (*Audited-Effort*) contract, the manufacturer can monitor the supplier's effort in exchange for an auditing cost of $\mathscr{A}$. The contracts in both IE and AE consist of three terms: (i) an upfront transfer payment $\$\omega_\theta$, (ii) contingent payment $\$Y_\theta$, and (iii) penalty $\$\kappa_\theta$. The first term is a fixed payment irrespective of outcome $\tilde{q}_\theta$, whereas the second and third terms stand for the contingent payment (from the manufacturer to the supplier) and penalty (from the supplier to the manufacturer) that depend on the realizations of $\tilde{q}_\theta = 1$ and $\tilde{q}_\theta = 0$, respectively. Since under both contracts, the supplier's true type of reliability is private information, the manufacturer has to design a menu of contracts from which each supplier type self-selects the one that is designed for himself. Invoking the extended revelation principle for mixed adverse selection and moral hazard problems (see Myerson [1982]), without loss of generality, we can restrict our attention to direct-revelation mechanisms in which suppliers truthfully reveal their types and exert actions that are induced by the manufacturer. Thus, it suffices for the manufacturer to offer two contracts, one for each type, i.e., $(\omega_h, Y_h, \kappa_h)$ and $(\omega_l, Y_l, \kappa_l)$, where the θ-type supplier self-selects the one that is designed for him. We defer the detailed discussion about the optimal design of $(\omega_\theta, Y_\theta, \kappa_\theta)$ under each contractual setting to §3.5. Below, we provide the timing of events and actions (see Figure 3.3.1).

- At time zero, the supplier observes his type of reliability, i.e., $\theta \in \{l, h\}$.
- The manufacturer chooses her strategy between IE and AE contracts.
- Depending on the choice between IE and AE, the manufacturer offers a menu of contracts, $(\omega_\theta, Y_\theta, \kappa_\theta)$ for $\theta \in \{l, h\}$.
- The θ-type supplier then self-selects a contract designed for him from the menu.
- The θ-type supplier decides whether or not to exert process improvement effort, i.e., $e_\theta \in \{0, 1\}$.
- If AE is chosen in the second step, then the manufacturer incurs an auditing cost of $\mathscr{A}$ and observes the supplier's effort.
- Finally, the production quantity $\tilde{q}_\theta \in \{0, 1\}$ is realized, and the contract is executed according to its terms.

Figure 3.3.1: Timing of events in Audit Model



3.4 Optimal Contract under Symmetric Information

To study the value of audit in the presence of hidden information and action, we first analyze the manufacturer's problem under full information case. In other words, in this section, we assume that the supplier's true reliability type θ is known and his process improvement effort e_θ is observable and verifiable by manufacturer. Note that, because of this, supplier's effort can be included in a contract which can be enforced with appropriate out-of-equilibrium penalties if supplier deviates from the requested effort [Laffont and Martimort, 2002]. Given the contract $\left(\omega_\theta, Y_\theta, \kappa_\theta, e_\theta^{\mathrm{fb}}\right)$ offered by the manufacturer, the supplier takes action e_θ^{fb}, where superscript "fb" indicates the first-best level of effort. The manufacturer's optimization problem is then to find the optimal level of improvement effort e_θ^{fb} and its corresponding contract that satisfies the individual rationality constraint (Eq. 3.4.2) for the θ-type supplier:

$$\max_{\omega_\theta, Y_\theta, \kappa_\theta, e_\theta^{\mathrm{fb}}} \quad \pi_M^\theta\left(\omega_\theta, Y_\theta, \kappa_\theta, e_\theta^{\mathrm{fb}}\right) = p\left(\theta \mid e_\theta^{\mathrm{fb}}\right)(r - Y_\theta) + \left(1 - p\left(\theta \mid e_\theta^{\mathrm{fb}}\right)\right)\kappa_\theta - \omega_\theta \tag{3.4.1}$$

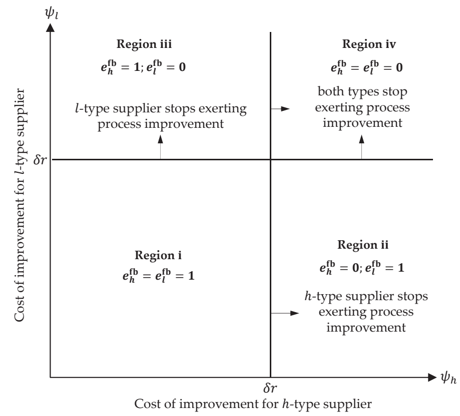
$$\text{s.t.} \quad \pi_S^\theta\left(\omega_\theta, Y_\theta, \kappa_\theta, e_\theta^{\mathrm{fb}}\right) = p\left(\theta \mid e_\theta^{\mathrm{fb}}\right)Y_\theta - \left(1 - p\left(\theta \mid e_\theta^{\mathrm{fb}}\right)\right)\kappa_\theta + \left(\omega_\theta - c - e_\theta^{\mathrm{fb}}\psi_\theta\right) \geq 0 \tag{3.4.2}$$

Note that by observing both supplier's reliability type and action, the manufacturer can enforce a contract that depends on both the supplier's type and effort. Since she only needs to satisfy θ-type supplier's participation constraint, she pays no rent to the supplier and extracts all of the supply chain profit under full information case. In Proposition 4.1, we characterize the optimal effort that the manufacturer wants to induce on the θ-type supplier, as well as the optimal contract parameters associated with the optimal effort (Note that the proofs for all propositions are delegated to Appendix).

Proposition 3.1. *Under full-information scenario, the supplier exerts process improvement effort iff $\delta r \geq \psi_\theta$. Furthermore, the first-best contract that implements this is fully characterized in Table 3.4.1.*

Table 3.4.1: Manufacturer's optimal contract and supplier's first-best effort under symmetric information

Region	Optimal contracts $(\omega_h, Y_h, \kappa_h), (\omega_l, Y_l, \kappa_l)$	Supplier's first-best level of effort
i. $\psi_h \leq \delta r; \psi_l \leq \delta r$	$(c + \psi_h, 0, 0), (c + \psi_l, 0, 0)$	$e_h^{\text{fb}} = 1, e_l^{\text{fb}} = 1$
ii. $\psi_l \leq \delta r < \psi_h$	$(c, 0, 0), (c + \psi_l, 0, 0)$	$e_h^{\text{fb}} = 0, e_l^{\text{fb}} = 1$
iii. $\psi_h \leq \delta r < \psi_l$	$(c + \psi_h, 0, 0), (c, 0, 0)$	$e_h^{\text{fb}} = 1, e_l^{\text{fb}} = 0$
iv. $\delta r < \psi_h; \delta r < \psi_l$	$(c, 0, 0), (c, 0, 0)$	$e_h^{\text{fb}} = 0, e_l^{\text{fb}} = 0$



There are three take-aways from Proposition 3.1. First, note that the optimal contract is a fixed-price contract. This is in alignment with the contract design literature, where it is shown that fixed-term contracts are theoretically the best in terms of achieving overall system efficiency [Laffont and Tirole, 1993]. Second, under fixed-term contract, the supplier's objective is aligned with the total supply chain profit, hence, he internalizes both the cost and benefit of the process improvement effort on the supply chain. The cost of process-improvement effort is simply ψ_θ. The benefit comes from the increase in survival probability due to the process improvement decision (which is, by assumption, equal to $\delta = p(\theta \mid e_\theta = 1) - p(\theta \mid e_\theta = 0)$) multiplied by the profit earned per unit sold in the market, r. To sum, as long as the cost ψ_θ is less than the (expected) benefit δr, the supplier would exert the process improvement effort. Finally, we note that since we do not impose limited liability constraints for the supplier and assume that all the parties are risk-neutral, the standard results of principal-agent models imply that Proposition 3.1 can also be extended to a pure moral hazard setting in which the manufacturer observes the true reliability type (but not the action) of the supplier (see Proposition 4.1 in Laffont and Martimort [2002]).

3.5 Optimal Contracts under Asymmetric Information

We now study the manufacturer's optimal contract design problem under asymmetric information. In this section, we assume that nature reveals whether the true state of supplier reliability is of type h or l only to the supplier, and the level of effort exerted by the supplier may or may not be observable to the manufacturer, depending on the manufacturer's contracting strategy. As opposed to the symmetric information case, the manufacturer can no longer enforce a contract that satisfies only participation constraints. Therefore, in the next two subsections, we develop optimal menu of contracts that are self-selected by the right supplier and induce desired actions on him at the same time.

3.5.1 Induced-Effort (IE) Contract

In this section, we characterize optimal menu of contracts for IE setting which induces a specific process improvement effort on each supplier type. Using backward induction, we first study the θ-type supplier's process improvement decision. Given a contract $(\omega_\theta, Y_\theta, \kappa_\theta)$ offered by the manufacturer, the θ-type supplier solves the following optimization problem to decide on whether or not to exert effort:

$$\pi_S^\theta(\omega_\theta, Y_\theta, \kappa_\theta \mid e_\theta) = \max_{e_\theta \in \{0,1\}} \{p(\theta \mid e_\theta) Y_\theta - (1 - p(\theta \mid e_\theta)) \kappa_\theta + \omega_\theta - (c + e_\theta \psi_\theta)\} \qquad (3.5.1)$$

where the first three terms denote the expected net payment transferred between manufacturer and θ-type supplier and the last term denotes the cost incurred by the supplier. Note that, by exerting effort, the supplier stochastically increases the likelihood of survival; however, he has to incur an additional cost, ψ_θ, that depends on his type. The following lemma characterizes θ-type supplier's best response function:

Lemma 3.1. *Given a contract $(\omega_\theta, Y_\theta, \kappa_\theta)$ offered by the manufacturer, the θ-type supplier exerts improvement effort, i.e., $e_\theta^*(\omega_\theta, Y_\theta, \kappa_\theta) = 1$ iff $Y_\theta + \kappa_\theta \geq \frac{\psi_\theta}{\delta}$.*

As opposed to the full information scenario, the fixed-price contracts are no longer sufficient to induce a process improvement effort on the supplier under asymmetric information. As shown in the above lemma, the manufacturer has to provide θ-type supplier with the right amount of incentive fees via contingent payment Y_θ and penalty κ_θ terms. Note that the net incentive seen by θ-type supplier (which also measures the incentive power of IE contract, see Laffont and Tirole, 1993) is equal to $Y_\theta + \kappa_\theta$, therefore, he exerts the process improvement effort as long as the power of contract is more than per-unit reliability improvement cost, i.e., $\frac{\psi_\theta}{\delta}$. Also, note that the manufacturer has to set $Y_\theta + \kappa_\theta = r$ if she wants to induce the first-best (i.e., supply-chain efficient) effort decision on the supplier. However, as we will see below, this is not always aligned with the manufacturer's incentives, which in turn, creates distortion in the total supply chain's efficiency.

With the help of Lemma 3.1, we can now formulate the manufacturer's optimal contract design problem. First of all, optimal menu of contracts needs to satisfy the following individual rationality (IR) constraints to ensure non-zero profit for θ-type supplier:

$$\pi_S^{\theta}\left(\omega_\theta, Y_\theta, \kappa_\theta \mid e_\theta^*\right) \quad = \quad p\left(\theta \mid e_\theta^*\right) Y_\theta - \left(1 - p\left(\theta \mid e_\theta^*\right)\right) \kappa_\theta + \left(\omega_\theta - c - e_\theta^* \psi_\theta\right) \geq 0,\ \theta \in \{h, l\} \qquad (3.5.2)$$

where e_θ^* denotes the θ-type supplier's effort provided that the manufacturer offers a menu of contracts $(\omega_\theta, Y_\theta, \kappa_\theta)$ i.e.,

$$e_\theta^* \quad = \quad \max_{e_\theta} \left\{ p\left(\theta \mid e_\theta\right) Y_\theta - \left(1 - p\left(\theta \mid e_\theta\right)\right) \kappa_\theta + \left(\omega_\theta - c - e_\theta \psi_\theta\right) \right\},\ \theta \in \{h, l\} \qquad (3.5.3)$$

Second, it has to satisfy the following incentive compatibility (IC) constraints, which ensure that the θ-type supplier self-selects the contract designed for him, i.e.,

$$\pi_S^{\theta}\left(\omega_\theta, Y_\theta, \kappa_\theta \mid e_\theta^*\right) \quad \geq \pi_S^{\theta}\left(\omega_{\check{\theta}}, Y_{\check{\theta}}, \kappa_{\check{\theta}} \mid \tilde{e}_\theta\right), \quad \theta, \check{\theta} \in \{h, l\}, \check{\theta} \neq \theta \qquad (3.5.4)$$

where $\tilde{e}_\theta$ is the optimal effort for θ-type supplier should he mimic $\check{\theta}$-type supplier (off-equilibrium decision), i.e.,

$$\tilde{e}_\theta \quad = \quad \max_{e_\theta} \left\{ p\left(\theta \mid e_\theta\right) Y_{\check{\theta}} - \left(1 - p\left(\theta \mid e_\theta\right)\right) \kappa_{\check{\theta}} + \left(\omega_{\check{\theta}} - c - e_\theta \psi_\theta\right) \right\},\ \theta, \check{\theta} \in \{h, l\} \qquad (3.5.5)$$

The manufacturer's problem is then to find the optimal menu of contracts $(\omega_\theta, Y_\theta, \kappa_\theta)$ that satisfies all the constraints (3.5.2-3.5.5):

$$\max_{(\omega_h, Y_h, \kappa_h), (\omega_l, Y_l, \kappa_l)} \quad \nu \pi_M^{h}\left(\omega_h, Y_h, \kappa_h \mid e_h^*\right) + (1 - \nu)\, \pi_M^{l}\left(\omega_l, Y_l, \kappa_l \mid e_l^*\right) \qquad (3.5.6)$$
$$\text{s.t.} \qquad \text{Constraints } (3.5.2 - 3.5.5)$$

where $\pi_M^{\theta}\left(\omega_\theta, Y_\theta, \kappa_\theta \mid e_\theta^*\right)$ denotes the manufacturer's expected profit provided that the θ-type supplier accepts the contract $(\omega_\theta, Y_\theta, \kappa_\theta)$ and subsequently exerts action e_θ^* as characterized in Lemma 3.1, i.e.,

$$\pi_M^{\theta}\left(\omega_\theta, Y_\theta, \kappa_\theta \mid e_\theta^*\right) \quad = \quad p\left(\theta \mid e_\theta^*\right) r - \left[p\left(\theta \mid e_\theta^*\right) Y_\theta - \left(1 - p\left(\theta \mid e_\theta^*\right)\right) \kappa_\theta + \omega_\theta\right] \qquad (3.5.7)$$

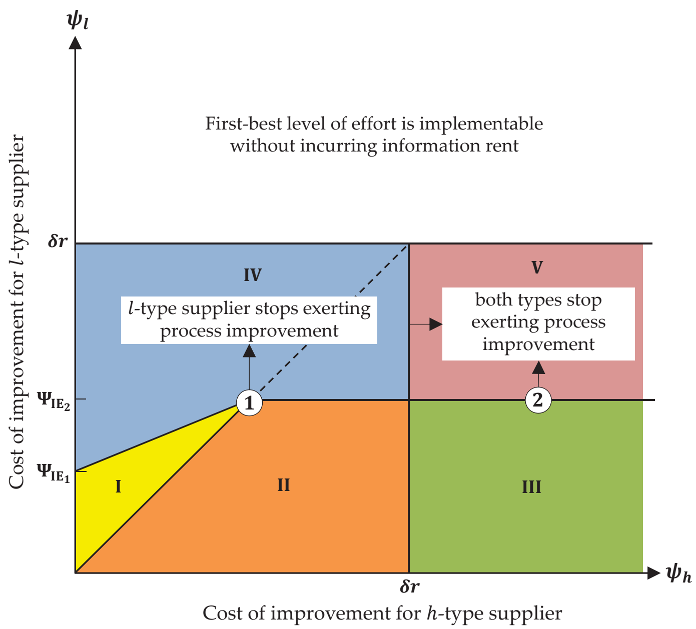
Note that the first term represents the manufacturer's expected revenue and the last three terms represent the net payment transferred between the manufacturer and θ-type supplier. Recall that, under symmetric information, θ-type supplier exerts the first-best effort level (denoted by e_θ^{fb}), and the manufacturer earns the entire channel profit. However, it can be easily verified that the same menu of contracts that induces e_θ^{fb} under symmetric information is not incentive-compatible for h-type (more reliable) supplier under asymmetric information. The manufacturer then has two options: either she needs to modify the contract terms to make it incentive-compatible for h-type supplier or induce the second-best level of effort (denoted by e_θ^*) on the suppliers. Each option comes at a cost for the manufacturer. The former leads to information rent, while the latter causes channel loss. Using the theory of mechanism design, one can re-formulate the manufacturer's optimization problem as the weighted average of these two costs with the weights represented by a-priori beliefs. We delegate the detailed analysis to the Appendix, and provide the complete equilibrium characterization of the optimal IE contract under asymmetric information in the following Proposition:

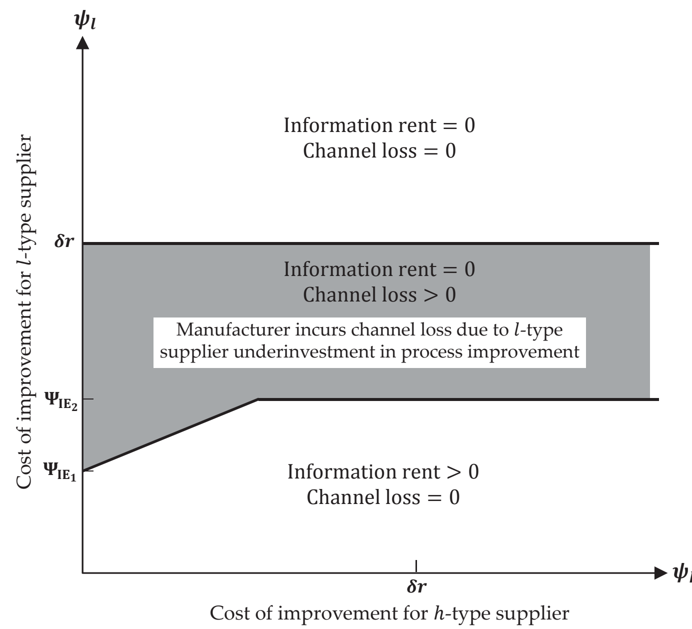
Proposition 3.2. *The second-best effort level e_θ^* and corresponding optimal IE contract $\left(\omega_\theta^*, \Upsilon_\theta^*, \kappa_\theta^*\right)$ are fully characterized in Table 3.5.1. Furthermore, Table 3.5.1 provides closed-form expressions for information rent and channel loss incurred by the manufacturer under each case.*

As shown in Lemma 3.1, the manufacturer has to provide right amount of incentives to each supplier type in order to induce process improvement under asymmetric information. The above proposition also shows that these effort-inducing incentives should be also self-selected by the right supplier types in order to prevent them from choosing each other's contract. To summarize, on top of the effort-inducing incentives, the manufacturer has to also provide additional incentives (called information rent). Luckily, it can be shown that information rent needs to be paid to only one of the supplier types. Due to the ranking (between supplier types) implied by Assumption 1, in our model, it is always h-type supplier who earns this information rent in equilibrium.

Table 3.5.1: Optimal contract, supplier's choice of action, information rent, and channel loss under IE contract

Region	Optimal menu of contracts $(\omega_h^*, Y_h^*, \kappa_h^*)\,;(\omega_l^*, Y_l^*, \kappa_l^*)$	Supplier's second-best effort	h-type supplier's off-equilibrium effort	Information rent	Channel loss
I	$\begin{cases}\omega_h^* &= c+\psi_h \\ Y_h^* &= \frac{1-\varphi_l}{\delta}\psi_l-\psi_h \\ \kappa_h^* &= \frac{\varphi_l}{\delta}\psi_l+\psi_h\end{cases};\begin{cases}\omega_l^* &= c+\psi_l \\ Y_l^* &= \frac{1-\rho_l}{\delta}\psi_l \\ \kappa_l^* &= \frac{\rho_l}{\delta}\psi_l\end{cases}$	$e_h^*=1; e_l^*=1$	$\tilde{e}_h=1$	$(\psi_l-\psi_h)+(\rho_h-\rho_l)\,\frac{\psi_l}{\delta}$	0
II	$\begin{cases}\omega_h^* &= c+\psi_h \\ Y_h^* &= \frac{1-\rho_h}{\delta}\psi_h+\frac{\rho_h-\rho_l}{\delta}\psi_l \\ \kappa_h^* &= \frac{\rho_h}{\delta}\psi_h-\frac{\rho_h-\rho_l}{\delta}\psi_l\end{cases};\begin{cases}\omega_l^* &= c+\psi_l \\ Y_l^* &= \frac{1-\rho_l}{\delta}\psi_l \\ \kappa_l^* &= \frac{\rho_l}{\delta}\psi_l\end{cases}$	$e_h^*=1; e_l^*=1$	$\tilde{e}_h=0$	$\psi_l+(\varphi_h-\rho_l)\,\frac{\psi_l}{\delta}$	0
III	$\omega_h^*=c; Y_h^*=\frac{1-\varphi_l}{\delta}\psi_l; \kappa_h^*=\frac{\varphi_l}{\delta}\psi_l;$ $\omega_l^*=c+\psi_l; Y_l^*=\frac{1-\rho_l}{\delta}\psi_l; \kappa_l^*=\frac{\rho_l}{\delta}\psi_l$	$e_h^*=0; e_l^*=1$	$\tilde{e}_h=0$	$\psi_l+(\varphi_h-\rho_l)\,\frac{\psi_l}{\delta}$	0
IV	$\omega_h^*=c+\psi_h; Y_h^*=\frac{1-\rho_h}{\delta}\psi_h; \kappa_h^*=\frac{\rho_h}{\delta}\psi_h; \omega_l^*=c; Y_l^*=0; \kappa_l^*=0$	$e_h^*=1; e_l^*=0$	$\tilde{e}_h=0$	0	$\delta r-\psi_l$
V	$\omega_h^*=\omega_l^*=c; Y_h^*=Y_l^*=0; \kappa_h^*=\kappa_l^*=0$	$e_h^*=0; e_l^*=0$	$\tilde{e}_h=0$	0	$\delta r-\psi_l$





Notes. $\Psi_{IE_1}=\left[\frac{\delta(1-\nu)}{\delta(1-\nu)+\nu(\rho_h-\varphi_l)}\right]\delta r$; $\Psi_{IE_2}=\left[\frac{\delta(1-\nu)}{\delta(1-\nu)+\nu(\rho_h-\rho_l)}\right]\delta r$; The manufacturer incurs the information rent and channel loss with probability ν and $1-\nu$, respectively. The above results hold when $p\,(h \mid e_h=0) \geq p\,(l \mid e_l=1)$ or equivalently, $\varphi_h \geq \rho_l$. The opposite case $\varphi_h < \rho_l$ leads to similar theoretical results and the results are available from the authors upon request.

We will now discuss information rent for h-type characterized in Proposition 3.2 (see Table 3.5.1), because it will play a key role in understanding the value of audit in §3.6. First of all, note that the manufacturer needs to pay information rent to h-type supplier only when l-type is induced to exert effort. This is because, in equilibrium, it is only h-type who has incentive to deviate, and if he deviates, he would do it in order to earn incentives provided by the manufacturer to induce l-type supplier to exert effort. Therefore, in order to satisfy IC constraint of h-type supplier, the manufacturer has to guarantee that h-type supplier would not earn more profit if he chose the contract designed for l-type. In other words, the information rent paid to h-type supplier must be exactly equal to the expected profit that he would earn if he chose l-type supplier's contract, $(\omega_l^*, Y_l^*, \kappa_l^*)$, and exerted the best effort under this contract, i.e., $\tilde{e}_h = e_h^*(w_l^*, Y_l^*, \kappa_l^*)$:

$$\text{Information rent paid to } h\text{-type } = \underbrace{\omega_l^* - (c + \tilde{e}_h \psi_h)}_{\text{fixed-price term}} + \underbrace{[p\,(h \mid \tilde{e}_h) - p\,(l \mid e_l^*)]\,(Y_l^* + \kappa_l^*)}_{\text{incentive-fee term}} \tag{3.5.8}$$

Note that information rent paid to h-type has two terms: (i) *fixed-price term*: This accounts for the upfront transfer payment that needs to be paid to the l-type supplier in order to satisfy his participation (IR) constraint; and (ii) *incentive-fee term*: This represents for the amount of incentives that needs to be paid to the l-type in order to induce him to exert process improvement effort. Furthermore, both terms are modified in order to account for h-type's effort decision $\tilde{e}_h$ when he deviates. Specifically, the fixed-price term is reduced by h-type's cost incurred due to his off-equilibrium decision, and incentive-fee term is multiplied by relative reliability of h-type supplier on the off-equilibrium path over l-type supplier (measured by the difference in survival probabilities between l- and h-type suppliers). To summarize, the higher are the fixed- and incentive-payments for l-type supplier, the more is the information rent. On the other hand, the lower is the relative reliability of h-type over l-type, and the higher is the cost of effort incurred by the h-type supplier on the off-equilibrium path, the lower is the information rent. We can further simplify Eq. (3.5.8) by using the fact that the manufacturer in equilibrium would always offer a break-even contract term to l-type that just satisfies his IR constraint (i.e., $\omega_l^* = c + \psi_l$) and induces him to exert process improvement effort (i.e., $Y_l^* + \kappa_l^* = \frac{\psi_l}{\delta}$ - see Lemma 3.1). Substituting these contract terms into above Eq. (3.5.8) would lead to the following simplified expression for the information rent:

$$\text{Information rent paid to } h\text{-type } = \underbrace{\psi_l - \tilde{e}_h \psi_h}_{\text{fixed-price term}} + \underbrace{[p\,(h \mid \tilde{e}_h) - p\,(l \mid e_l^*)]\,\frac{\psi_l}{\delta}}_{\text{incentive-fee term}} \tag{3.5.9}$$

By analyzing the impact of system parameters on the information rent, we can obtain the intuition behind why the optimal IE contract leads to channel inefficiency for (i) relatively higher values of ψ_l, and (ii) lower values of ψ_h (see Table 3.5.1):

- First of all, note that the information rent (Eq. 3.5.8) increases in ψ_l. This is because l-type supplier with high values of ψ_l needs larger incentives to exert process improvement effort and the manufacturer has to provide same incentives to h-type supplier in order to satisfy his IC constraint. This implies that when ψ_l becomes very high, the manufacturer would stop inducing l-type supplier to exert process improvement effort in order to reduce information rent for h-type supplier.

- Note that the region in which the optimal IE contract leads to channel-inefficiency depends on the parameters other than ψ_l. This is because the information rent depends on not only the contract terms offered to l-type supplier but also the extent of information asymmetry between l- and h-type suppliers. Recall that there are two ways in which the suppliers differ from each other. First one is caused by the difference between the costs of process improvement (measured by $\psi_l - \psi_h$) and the second is the degree of reliability $p(h|e) - p(l|e)$. Note that the cost asymmetry $\psi_l - \psi_h$ affects fixed-price term, whereas the reliability asymmetry $p(h|e) - p(l|e)$ affects incentive-fee term in information rent. The former implies that the more cost efficient is h-type (i.e., the lower is ψ_h), the more information rent would he benefit from the direct subsidy that l-type receives from the manufacturer in the form of upfront payment. On the other hand, the latter implies that the more reliable is h-type (i.e., the higher $p(h|e)$), the more information rent would he benefit from the indirect subsidy that l-type receives from the manufacturer in the form of contingent payments. These two observations imply that the optimal IE contract leads to channel-inefficiency more likely as ψ_h decreases and ρ_h increases because the manufacturer reduces the information rent for h-type supplier by not providing effort-inducing incentives to l-type supplier.

- Finally, an increase in a-priori beliefs for more reliable supplier (i.e., ν) also affects the likelihood of optimal IE contract creating channel-inefficiency. This is because the manufacturer pays information rent only to h-type supplier, the probability of which increases in ν. Therefore, *ceteris paribus,* the expected costs of channel loss and information rent, respectively, decrease and increase in ν.

3.5.2 Audited-Effort (AE) Contract

In the previous section, we show that the manufacturer's inability to observe the actions of the supplier has significant impacts on both information rent and channel inefficiency. In this section, we consider an auditing setting where the manufacturer receives a perfect signal on the action chosen by the supplier by incurring $\mathscr{A} > 0$, and uses this signal to enforce a particular action profile. It readily means that the manufacturer can include the supplier's effort in the contract. Let $\left(e_h^*, e_l^*\right)$ denote the optimal action profile that the manufacturer wants to induce on her supplier. Note that even though the manufacturer can verify the supplier's effort, she cannot observe his type and hence she does not verify whether the supper's effort decision is optimal or not. Therefore, the optimal contract design boils down to the standard moral hazard problem with private information (see Macho-Stadler and Perez-Castrillo [2001], Rasmusen [2006], and Milgrom [1987]) with the following IR and IC constraints to ensure participation, i.e.,

$$\pi_S^\theta\left(\omega_\theta, Y_\theta, \kappa_\theta, e_\theta^*\right) \quad = \quad p\left(\theta \mid e_\theta^*\right) Y_\theta - \left(1 - p\left(\theta \mid e_\theta^*\right)\right) \kappa_\theta + \left(\omega_\theta - c - e_\theta^* \psi_\theta\right) \geq 0,\ \theta \in \{h, l\} \tag{3.5.10}$$

and self-selection, respectively:

$$\pi_S^\theta\left(\omega_\theta, Y_\theta, \kappa_\theta, e_\theta^*\right) \geq \pi_S^\theta\left(\omega_{\check{\theta}}, Y_{\check{\theta}}, \kappa_{\check{\theta}}, e_{\check{\theta}}^*\right), \tag{3.5.11}$$

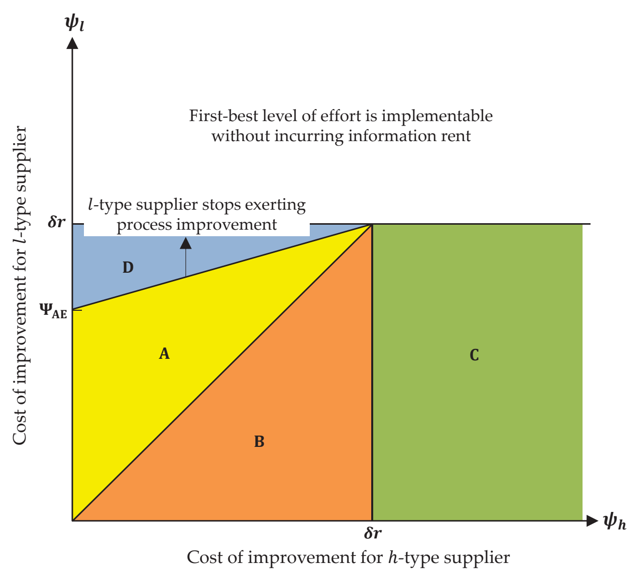
where $\check{\theta} \neq \theta$. Finally, the optimal AE contract has to maximize the manufacturer's payoff subject to the constraints 3.5.10-3.5.11:

$$\max_{\left(\omega_h, Y_h, \kappa_h, e_h^*\right), \left(\omega_l, Y_l, \kappa_l, e_l^*\right)} \quad \nu \pi_M^h\left(\omega_h, Y_h, \kappa_h, e_h^*\right) + (1-\nu)\, \pi_M^l\left(\omega_l, Y_l, \kappa_l, e_l^*\right) - \mathscr{A} \tag{3.5.12}$$

Note that observing the action profile modifies the optimal contract problem in two ways. First, the manufacturer does no longer need to enforce moral hazard constraints (i.e., Eq. (3.5.3) under IE contract) to induce the optimal action profile. Second, a deviating θ-type supplier has to choose not only the contract designed for $\check{\theta}$-type supplier but also the effort induced for him, i.e., $e_{\check{\theta}}^*$, where $\check{\theta} \neq \theta$. This contrasts with the IE setting, where a θ-type supplier can deviate by choosing the contract designed for $\check{\theta}$-type supplier and then keep exerting the best effort under the deviated contract according to Lemma 3.1. As it is shown in the following proposition, these two changes would make deviation under AE contract more costly for θ-type supplier, which, in turn, enables the manufacturer to reduce distortions caused by asymmetric information:

Table 3.5.2: Optimal contract, supplier's second-best level of effort, information rent, and channel loss under AE contract

Region	Optimal menu of contracts $\left(\omega_h^*, Y_h^*, \kappa_h^*\right);\left(\omega_l^*, Y_l^*, \kappa_l^*\right)$	Supplier's second-best effort	Information rent	Channel loss
A	$\omega_h^* = c + \psi_h, Y_h^* = \frac{\psi_l - \psi_h}{\rho_h}, \kappa_h^* = 0; \omega_l^* = c + \psi_l, Y_l^* = 0, \kappa_l^* = 0$	$e_h^* = 1; e_l^* = 1$	$\psi_l - \psi_h$	0
B	$\omega_h^* = c + \psi_h, Y_h^* = \frac{1-\rho_h}{\rho_h - \rho_l}\left(\psi_h - \psi_l\right), \kappa_h^* = \frac{\rho_h}{\rho_h - \rho_l}\left(\psi_h - \psi_l\right);$ $\omega_l^* = c + \psi_l, Y_l^* = 0, \kappa_l^* = 0$	$e_h^* = 1; e_l^* = 1$	0	0
C	$\omega_h^* = c, Y_h^* = 0, \kappa_h^* = 0; \omega_l^* = c + \psi_l, Y_l^* = 0, \kappa_l^* = 0$	$e_h^* = 0; e_l^* = 1$	0	0
D	$\omega_h^* = c + \psi_h, Y_h^* = 0, \kappa_h^* = 0; \omega_l^* = c, Y_l^* = 0, \kappa_l^* = 0$	$e_h^* = 1; e_l^* = 0$	0	$\delta r - \psi_l$



Notes. $\Psi_{AE} = (1-\nu)\delta r$; The manufacturer incurs the information rent and channel loss with probability ν and $1-\nu$, respectively.

Proposition 3.3. *The optimal AE contract* $\left(\omega_\theta^*, Y_\theta^*, \kappa_\theta^*\right)$ *and the effort levels* $\left(e_\theta^*\right)$ *induced in equilibrium, as well as the decomposition of the total agency costs into information rent and channel loss, are characterized in Table 3.5.2.*

First of all, note from Table 3.5.2 that under AE contract, the manufacturer still needs effort-inducing incentive fees (i.e., non-zero contingent payment and penalty terms) at least for one of the supplier types because even if she can verify the supplier's effort, she cannot observe his type. For example, consider Region A in Table 3.5.2. If the manufacturer does not provide effort-inducing incentives for h-type supplier, he can simply pretend to be l-type in order to get higher fixed-term payment that is offered to l-type because $\psi_l \geq \psi_h$ in Region A. Similarly, in Region B, the manufacturer uses effort-inducing incentives in order to prevent l-type from pretending to be h-type.

That being said, a closer examination of information rent and channel loss expressions characterized in Table 3.5.2 reveals that audit can significantly reduce the agency costs associated with hidden information. As a result, the manufacturer can induce the first-best level of effort on the supplier in a less costly fashion. In order to explain the intuition behind this, we need to revisit the information rent expression characterized in the previous section. Similar to IE contract, the information rent under optimal AE contract can be divided into two terms:

$$\text{Information rent paid to } h\text{-type } = \underbrace{\omega_l^* - (c + e_l^* \psi_h)}_{\text{fixed-price term}} + \underbrace{[p(h \mid e_l^*) - p(l \mid e_l^*)](Y_l^* + \kappa_l^*)}_{\text{incentive-fee term}} \tag{3.5.13}$$

We can further simplify the above expression for the information rent. Recall that under optimal IE contract, the manufacturer needs effort-inducing incentives for both supplier types, whereas audit eliminates the necessity of this for l-type supplier, i.e.,

$Y_l^* + \kappa_l^* = 0$ under AE, which removes the contribution of "incentive-fee" term to the information rent, i.e.,

$$\text{Information rent paid to } h\text{-type } = \underbrace{\psi_l - e_l^* \psi_h}_{\text{fixed-price term}} \tag{3.5.14}$$

In addition to this, audit provides a second benefit to the manufacturer. Namely, the manufacturer can employ audited action profile (e_h^*, e_l^*) as a screening device to distinguish h-type from l-type. For example, consider Regions B and C in Table 3.5.2. Note that as opposed to IE setting, h-type supplier is not free to exert any effort when he deviates. In fact, under AE contract, h-type supplier has to choose $\tilde{e}_h = e_l^* = 1$ if he deviates, which in turn makes his deviation non-profitable because $\psi_h \geq \psi_l$ in both Regions B and C.

Finally, our last observation from Table 3.5.2 is that even though AE helps to eliminate agency costs for Regions B and C, it does not completely eradicate them in Regions A and D. The reason for this comes from the trade-off between information rent and channel efficiency. On one hand, in Region A, inducing on both suppliers to exert effort helps the manufacturer to increase total supply chain surplus, but as explained above, it requires for her to pay information rent to h-type supplier. On the other hand, by inducing the suppliers to exert different actions, the manufacturer can completely eliminate information rent because she can use audit as a perfect screening device to separate h-type from l-type supplier, however, as shown in Region D in Table 3.5.2, this causes channel inefficiency.

In the next section, we compare optimal IE and AE contracts from the perspectives of the manufacturer, as well as the supplier and the total supply chain. It enables us to address the main research questions of this study, namely, when the manufacturer should audit, and how it affects the individual and total supply chain profits.

3.6 The Value of Audit

We analyze the value of audit (VOA) from the perspectives of the manufacturer, supplier and total supply chain. First, note that the comparison of channel loss expressions characterized in Propositions 3.2 and 3.3 for optimal IE and AE contracts, respectively, would directly imply that the audit is always valuable from total supply chain perspective. However, in order to determine the value of audit for both manufacturer and supplier, we need to take into account both information rent and channel loss under optimal IE and AE contracts:

Proposition 3.4. *The value of audit for the manufacturer, supplier and total supply chain are characterized in Table 3.6.1. Furthermore, the manufacturer is better off with IE contract in Region "I", otherwise, the AE contract is the superior strategy from the manufacturer's perspective.*

In what follows, we discuss how and when audit has an actual effect on individual and total supply chain profits. First, we start with the manufacturer.

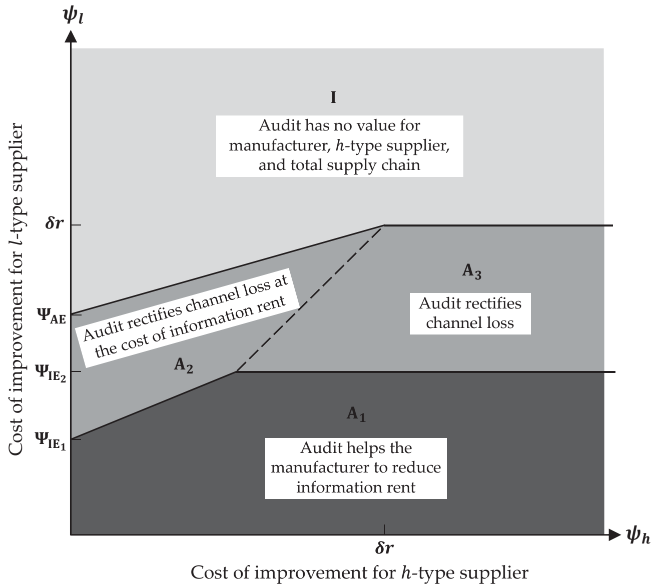
VOA for the manufacturer: As shown in Table 3.6.1, audit has strictly positive effect on the manufacturer's payoff by eliminating either information rent (in Region A_1) or channel inefficiency (in Regions A_2, and A_3) incurred under optimal IE contract. Even though the type of agency costs rectified by audit is different, the value of audit is ultimately driven by two factors:

- **[Efficiency-improving effect:]** On one hand, fixed-price contracts are theoretically the best in terms of achieving channel efficiency but they work only under symmetric information scenario (see Proposition 3.1). On the other hand, under hidden information and action scenario, the manufacturer needs to offer incentive-fees to both supplier types in order to induce them to exert process improvement efforts (see Proposition 3.2). Audit helps the manufacturer to restore channel efficiency by customizing the contractual form based on the type of supplier, in the sense that, with the help of audit, the manufacturer can offer fixed-price contract to less reliable suppliers, and use fixed-price-incentive-fees for more reliable suppliers (see Proposition 3.3).

- **[Screening effect:]** Due to the linkage between the supplier's type and his effort, observing the latter provides the manufacturer with valuable information about the former. This implies that audit can be

Table 3.6.1: Value of Auditing (VOA)

Region	Manufacturer	h-type supplier	Channel
A_1	$\nu\left(\frac{\rho_h-\rho_l}{\delta}\right)\psi_l$	$-\nu\left(\frac{\rho_h-\rho_l}{\delta}\right)\psi_l$	0
A_2	$(1-\nu)\left[\delta r-\psi_l\right]-\nu\left(\psi_l-\psi_h\right)$	$\nu\left(\psi_l-\psi_h\right)$	$(1-\nu)\left[\delta r-\psi_l\right]$
A_3	$(1-\nu)\left[\delta r-\psi_l\right]$	0	$(1-\nu)\left[\delta r-\psi_l\right]$
I	0	0	0



Notes. The name of the region indicates the superior contracting strategy from manufacturer's perspective, i.e., "I" indicates that IE contract is better than AE, and "A_i" indicates that AE contract is better than IE. The expressions for Ψ_{IE_1} and Ψ_{IE_2} are characterized in Proposition 3.2, and Ψ_{AE} in Proposition 3.3.

used as an additional screening device by the manufacturer to separate the more reliable suppliers from the less reliable ones. This in turn helps her to reduce the information rent incurred under optimal IE contract.

VOA for the supplier: Depending on whether audit has an efficiency-improving or screening effect, its value for the supplier can be positive or negative. First, consider Region A_1. As discussed above, audit is used in this region primarily as a screening device by the manufacturer and enables her to reduce information rent, which would then hurt the h-type supplier. On the other hand, in Regions A_2 and A_3, audit increases the channel efficiency. Consequently, depending on whether the manufacturer shares some of the channel surplus with h-type supplier through information rent (see Region A_2) or not (see Region A_3), the audit either increases the h-type supplier's profit or keep it unchanged.

VOA for the total supply chain: Finally, when audit has a screening effect, it does not have any actual value

Table 3.6.2: The Impact of system parameters on VOA

VOA (A_1, A_2, A_3)	ψ_l	ψ_h	r	δ	ρ_h and φ_h	ρ_l and φ_l	ν
Manufacturer	$(+,-,-)$	$(0,+,0)$	$(0,+,+)$	$(0,+,+)$	$(+,0,0)$	$(-,0,0)$	$(+,-,-)$
h-type supplier	$(-,+,0)$	$(0,-,0)$	$(0,0,0)$	$(0,0,0)$	$(-,0,0)$	$(+,0,0)$	$(-,+,0)$
Total supply chain	$(0,-,-)$	$(0,0,0)$	$(0,+,+)$	$(0,+,+)$	$(0,0,0)$	$(0,0,0)$	$(0,-,-)$

Notes. The profile $(\cdot,\cdot,\cdot)$ indicates the impact of system parameters on VOA in Regions A_1, A_2, and A_3, respectively. "$+$", "$-$", and "0" indicate that the increase in system parameter has increasing effect, decreasing effect and no-effect on the VOA, respectively.

on the total supply chain's surplus (see Region A_1). In other words, it simply transfers some of the surplus from the supplier to the manufacturer without changing the total supply chain value. On the other hand, when it has an efficiency-improving effect (i.e., in Regions A_2 and A_3), it indeed allows the total supply chain to achieve its first best efficiency (realized under symmetric information scenario). In the next section, we analyze the impact of system parameters on the value of audit.

Impact of System Parameters on the Value of Audit

Note that both the magnitude of VOA and the size of Regions A_1, A_2 and A_3 (see Table 3.6.1) are affected by changes in problem parameters. In this section, we focus on the sensitivity of VOA with respect to cost parameters ψ_l and ψ_h, unit revenue r, the impact of process improvement effort on the degree of reliability measured by δ, and distribution of risks faced by different types of supplier measured by ν, ρ_θ, and φ_θ, and use this analysis to identify the conditions under which the audit would be beneficial to the individual supply chain parties as well as total supply chain. The following Proposition shows the impact of system parameters on the VOA:

Proposition 3.5. *The impact of system parameters on the VOA for manufacturer, supplier as well as total supply chain is fully characterized in Table 3.6.2.*

Considering Table 3.6.2, we can analyze the impact of parameters on VOA for the manufacturer depending on whether they affect audit's role on reducing channel loss (i.e., *efficiency-improving effect*) or information rent (i.e., *screening effect*). Then, we consider the impact of parameters on VOA for the supplier and total supply chain.

Impact of ψ_l and ψ_h: First, note from Table 3.6.1 that VOA for the manufacturer is non-monotone in ψ_l. It first increases in ψ_l in Region A_1 and decreases in Regions A_2 and A_3. The intuition is as follows. Recall that higher values of ψ_l increase the information rent and reduce the channel loss incurred under optimal IE contract. Therefore, higher values of ψ_l amplify the screening effect (which increases VOA for the manufacturer in

Region A_1) and dampen the efficiency-improving effect of audit (which decreases VOA for the manufacturer in Regions A_2 and A_3). Also recall that higher values of ψ_h decrease the information rent transfer between manufacturer and supplier. Therefore, in Region A_2, where the manufacturer shares a portion of channel surplus with the supplier in the form of information rent transfer under optimal AE contract, she shares less as ψ_h increases, which in turn increases VOA for the manufacturer.

Impact of r **and** δ: Both r and δ increase the total supply chain profit. This amplifies the efficiency-improving effect of audit for the manufacturer. Therefore, VOA for the manufacturer increases in both r and δ in Regions A_2 and A_3. On the other hand, the trade-off between channel efficiency and information rent suggests that for higher values of r and δ, incurring channel loss would become more expensive for the manufacturer compared to incurring information rent. Therefore, relatively speaking, higher values of r and δ dampen the screening-effect of audit (and decrease VOA for the manufacturer) in Region A_1.

Impact of ν: Instead of total supply chain value, ν affects only the relative weight of channel loss and information rent for the manufacturer because ν and $1-\nu$ measure the likelihoods of the supplier being of h-type (when the manufacturer incurs the information rent) and l-type (when the manufacturer incurs channel loss), respectively. Since higher values of ν increases the relative importance of the information rent, an increase in ν amplifies the screening effect (which increases VOA for the manufacturer in Region A_1) and dampens the efficiency-improving effect (which decreases VOA for the manufacturer in Regions A_2 and A_3).

Impact of ρ_θ **and** φ_θ: Note that both $\rho_h - \rho_l$ and $\varphi_h - \varphi_l$ measure the degree of information asymmetry between manufacturer and supplier, which determines the amount of information rent transfer between them. Therefore, lower values of ρ_h and φ_h and higher values of ρ_l and φ_l dampen the screening-effect of audit, which decrease the VOA for the manufacturer in Region A_1. With regards to the impact on the efficiency-improving effect of audit, we need to consider how the regions A_2 and A_3 change in ρ_θ and φ_θ because in these regions, the VOA expressions for the manufacturer depend on neither of them (see Table 3.6.2). The analysis of Ψ_{IE_1} and Ψ_{IE_2} reveals that the former converges to Ψ_{AE} and the latter to δr (implying that both A_2 and A_3 vanish) as $\rho_h - \rho_l$ and $\varphi_h - \varphi_l$ go to zero. As shown in the following proposition, this in turn implies that not only the VOA for the manufacturer but also those for the supplier and total supply chain vanish as $\rho_h - \rho_l$ and $\varphi_h - \varphi_l$ go to zero:

Proposition 3.6. *The VOA goes to zero for the manufacturer, supplier as well as total supply chain as* $\rho_h - \rho_l \to 0$ *and* $\varphi_h - \varphi_l \to 0$.

As we discussed in §3.2, differently from our results, Laffont and Martimort [2002] show that auditing the agent's actions has no value for the principal under a standard mixed model (i.e., when the agent is risk-neutral and not protected by limited liability constraints) where it is assumed that the impact of the agent's action on the likelihood of $\tilde{q}_\theta = 1$ is same for both $\theta = h$ and $\theta = l$ (i.e., in terms of our notation, they assume $\rho_h = \rho_l$, and $\varphi_h = \varphi_l$). The above proposition shows that our results coincide with Laffont and Martimort [2002] in the limit when the difference between the survival probabilities for h- and l-type suppliers for the same level of effort goes to zero.

So far in this section, we only focus on the impact of parameters on VOA for the manufacturer (i.e., the first row in Table 3.6.2). However, by using the relationship between VOAs for manufacturer, supplier and total supply chain under efficiency-improving and screening effects of audit, we can easily extend the above discussion to the impact of parameters on VOAs for supplier and total supply chain (i.e., the second and third rows in Table 3.6.2, respectively). Note that when the audit's role is to reduce information rent, the VOAs for the manufacturer and the supplier move in opposite directions. This implies that all the parameters that increase the VOA for manufacturer in Region A_1 decrease the VOA for the supplier and do not affect the VOA for the total supply chain in the same region. This is because the audit redistributes the surplus from the supplier to the manufacturer without changing the total supply chain value. On the other hand, when the audit's role is to increase the channel efficiency, a change in the value of a parameter that causes an increase in VOA for the manufacturer always causes an increase in VOA for the total supply chain. However, depending on whether the resulting surplus is shared with the supplier (in Region A_2) or not (in Region A_3), an increase in VOA for the manufacturer either causes an increase in the VOA for the supplier or keeps it unchanged.

3.7 Conclusions

Besides the many benefits of outsourcing, the increase in information asymmetry between supply chain parties due to the lack of control becomes the main concern for the users of outsourcing. In this chapter, we explore the value of audit for a supply chain where a manufacturer has to contract with a supplier whose true state of delivery reliability and actions are not observable. We analyzed two contractual mechanisms for the manufacturer to interact with such suppliers. In the first mechanism, the manufacturer offers to the supplier a menu of contracts both to screen his reliability and to induce him to exert a process improvement effort (henceforth called Induced-Effort (IE) contract). In the second one, in addition to offering a menu of contracts, the manufacturer also audits the supplier's effort (henceforth called Audited-Effort (AE) contract).

The characterization and comparison of optimal contracts under two mechanisms yield insights regarding the impact of audit on equilibrium decisions and payoffs of individual supply chain parties as well as total supply chain. In terms of the impact on equilibrium decisions, we find that the audit enables the manufacturer to customize her contract offering based on the reliability type of the supplier. Namely, the optimal menu of contracts under AE consists of a fixed-price contract for a low-reliable supplier and a fixed-price-incentive-fees for a high-reliable one. In comparison with the optimal menu of contracts under IE, which consists of fixed-price-incentive-fees for both supplier types, AE has two effects: (i) efficiency-improving effect, and (ii) screening effect. First one is related to the restoration of channel efficiency caused by removal of effort-inducing incentives from the low-reliable-type's contract offering and replacing them with audited effort. Second one is related to the exploitation of linkage between supplier's reliability and his effort via audit. In other words, by observing supplier's effort, the manufacturer can screen more reliable supplier from less reliable one.

The analysis of the value of audit from each supply chain party's perspective reveals that the first effect of the audit may benefit both the manufacturer and supplier due to an increase in supply-chain efficiency, whereas the second effect benefits the manufacturer and hurts the supplier. Finally, the sensitivity analysis with respect to system parameters help us to understand when audit is beneficial (resp., when it is not) for the manufacturer, the supplier and total supply chain.

The above-mentioned results shed some managerial insights into the role of auditing for different supply chains. Consider an original equipment manufacturer, like Toyota, whose supply base mostly consists of local suppliers. In such cases, the degree of information asymmetry between the manufacturer and its suppliers would be relatively mild. Hence, the manufacturer mainly enjoys the efficiency-improving effect of audit to rectify channel loss. Note that in this case, audit may be of interest to the suppliers as well especially when the resulting increase in the efficiency of channel is shared between the channel parties. On the other hand, in the cases of companies that extensively use global sourcing (such as Apple), they would benefit from screening-effect of the audit to reduce the information rent transfers. Finally, the companies that do not have prior experience with their suppliers around the globe can also enjoy the screening effect of audits and use it to increase the overall end-to-end visibility of their supply chains.

The model presented in this chapter can be extended in multiple directions. In our study, we focus only on the quantity uncertainty. However, we can show that both the analysis and the results of our study can be extended to a similar setting where the supplier's actions affect the uncertainty about the quality of the product (as analyzed in Laffont and Martimort [2002]). Also, both IE and AE contracts used in our study consist of three terms. We can show that qualitative nature of all the results of our work can be extended to a (restricted) two-term contractual setting where the supplier's incentive fees consist of only positive contingent payments, i.e., $Y_\theta \geq 0$ and $\kappa_\theta = 0$ (The details of additional analysis for the quality uncertainty and two-term contractual settings are available from the authors upon request).

We also discuss the implications of relaxing some of the assumptions made in our model. We assume that supply chain firms obtain a perfect signal by auditing the actions of their partners. One possibility is to introduce noise into the audit. We expect that this extension would dilute the screening effect of the auditing, which may increase the information rent paid under the AE contract. Hence, under the noisy-audit scenario, AE may lose some of its appeal against IE. Another possibility is to consider a probabilistic audit scenario, where the manufacturer randomizes between audit and no-audit scenarios in order to save some of the auditing cost and at the same time enjoy the two benefits explained above. On the contrary, our detailed analysis (please refer to Appendix B) show that a randomized auditing strategy would never be sustained in equilibrium. In other words, depending on the cost-benefit tradeoff, the manufacturer would always opt for either audit or no-audit option. Another extension is to consider the impact of multiple risk-types, and multiple effort-levels for the supplier. Even though the analysis would considerably become more complicated, we expect that these extensions would make audit even more valuable for the manufacturer by amplifying either its efficiency-improving effect (under the multiple-effort case) or screening effect (under the multiple risk-type scenario). Last but not least, we believe that audit has become an increasingly important issue for many companies as they expand their supply bases locally and globally. We hope that our model will contribute to understanding the key factors of this issue.

Chapter 4

The Value of Diagnostic Test in Contract Manufacturing under Supply Risk

4.1 Introduction

When a new product project fails, the failure is usually blamed on culprits such as tough competition, insufficient promotion and advertising strategies or weak market research. But, the results from a survey of 252 new product launches at 123 firms reveal that the true causes of failure may lie elsewhere [Cooper, 1988]. The survey indicates that among the commonly prescribed activities in new product introduction process, trial sell and trial production were undertaken only in less than half of the projects studied. Motivated by such findings, a large body of academic and practitioner literature addresses the importance of customer involvement into new product development (NPD) efforts, but, it is relatively recently that supplier involvement has received significant attention in practitioner (Handfield et al., 1999, Primo and Amundson, 2002, Petersen et al., 2005) and academic (Kim and Netessine, 2013).

Lack of experience in new product manufacturing not only leads to supply side problems, such as yield problems, inflexibility in production capacity, lead-time variability and long set-up time, but also it may lead to huge loss on the demand side, such as lost sales, customer goodwill loss, and market share loss due to a fast follower. On the other hand, the empirical evidences indicate that the use of contract manufacturing in the form of outsourcing to a supplier in engineering of new product leads to performance improvement in the form of reduced cycle time, improved manufacturability, greater technical improvements, and reduced costs

(Clark, 1989, Helper, 1991). That being said, due to the reasons provided above, the actual implementation of outsourcing in new product development requires utmost care. For example, the IBM's $150 million first-quarter loss in 2004 is attributed mostly to the yield problems faced with a new process technology introduced to manufacture a new semiconductor-based product at its microelectronics plant in East Fishkill, New York [Tang and Tomlin, 2008]. According to an independent industry analyst, yield problems are not uncommon in semiconductor industry when a chip maker shifts to a new process technology. Although integrating a new process technology is a difficult undertaking, companies such as IBM hedge their operational risks by employing a costly trial production in which they learn the true yields of new process and fine-tune them over a *testing period*. In a closely related PC industry, Original Equipment Manufacturers (OEMs) face similar challenges when they try to launch new products. For example, IBM had to suffer a huge loss in its earnings due to back order lasted over a month for its newly launched introduced ThinkPad series (T20 and A40 models). The backorder is caused mainly because of its key component suppliers' (particularly the suppliers of DVD and CD-RW parts) inability to cope with the problems surfaced during initial production stages, which is exacerbated due to parts suppliers' lack of flexibility in ramping up their production rates to match up with the growth in demand [Sheffi, 2007]. Similarly, Apple Computer Inc. did not completely materialize the growth in consumer demand because of the several delays they faced in increasing production rate of new products [Hendricks and Singhal, 2003]. These examples spotlight how disruption on supply end of the chain can directly result in opportunity costs on demand end of the chain, which may in some cases lead to erosion in competitive edge. The importance of managing end-to-end supply chain process in NPD is not restricted to only technologically intensive firms. Indeed, being capable enough to appreciate the importance of matching supply and demand sides, Nine West, a fashion wholesale and retail company, decides to run a *test production* [Sheffi, 2007] with its suppliers in order to not only minimize the supply-side risks but also to improve its forecasts and adjust its production decisions on the realized sale data. As suggested, this competency brings two benefits for Nine West. On the supply-side, Nine West and its suppliers gather early information about the potential technical production issues, which further enables them to fine-tune the process for the final production by investing on appropriate disruption mitigation strategies. On the demand-side, the test production helps Nine West to build a critical sales index that can be used to adjust the production rate.

The main focus of this study is therefore to explore the benefits and potential consequences of such a test production on the supply chains that face unique challenges of new product development. Particularly, in this study, we restrict our attention to diagnostic tests that are taken on the supply side and define them as "any costly effort, either conducted directly by the supplier or subsidized by the buyer, to identify the nature and source of a random disruption in supplier's production process". Due to novelty in both the product and its production process, the supplier may face manufacturability problems in the form of disruption risk. In response to this, the supplier can take a costly diagnostic test on its production system, learn the extent of its disruption risk, and use the results to fine-tune the process for the final production period.

Similarly to the previous chapter, we assume that the supplier's production process is subject to a random catastrophic disruption (i.e., an all-or-nothing yield) where the probability of disruption depends on both exogenous and endogenous factors [1]. The exogenous factor faced by the supplier is determined by his state of reliability θ that can be of two types: h-type or l-type. The h-type supplier is fully reliable, therefore, the probability of disruption is zero for him. The l-type supplier, on the other hand, is partially unreliable supplier and subject to disruption. Furthermore, he can reduce his exposure to the disruption risk by exerting a costly process improvement effort (endogenous factor). Since the supplier is uninformed about his true reliability, his decision on process improvement could potentially lead to inefficiency. This is true especially if he exerts process improvement without taking a diagnostic test because exerting a process improvement by a fully reliable supplier brings negative value to the channel. However, with the help of investing in diagnostic test, the supplier can identify his true reliability before deciding on process improvement, and proceed with the improvement decision if and only if the test reveals lack of reliability. Besides its benefits, supply diagnostic test can potentially create two issues in a decentralized supply chain setting. On one hand, employing a diagnostic test leads to a natural information asymmetry between the buyer and the supplier, where the latter armed with the new and private information would demand its rent from the former. On the other hand, if the diagnostic test is not used, the buyer would have to financially subsidize process improvement costs not only for the unreliable- but also for the reliable-type supplier. To summarize, in the presence of such information- and incentive-related frictions, it is not clear whether a diagnostic test is beneficial in a decentralized setting, and if yes, how its benefit would be divided between the parties of the chain. Motivated by the above issues, our aim in this chapter is to address the following research questions:

Research Question 1. Should the buyer leave it to the supplier to decide whether or not to take a diagnostic test or provide extra incentives to him?

Research Question 2. If the buyer decides to provide incentives, what would be the optimal incentive contract?

Research Question 3. Can information acquisition hurt the supply chain? The buyer? The Supplier?

[1] Note that random disruption model can be seen as a special case of the random yield model, where the realized yield is either 100% or 0% [Wang et al., 2010].

To answer the above research questions, we develop a dyadic supply chain model where a buyer delegates the production of a new product to a supplier whose production may be subject to disruption risk. Furthermore, due to lack of experience in production, the extent of disruption risk is unknown by both the buyer and supplier. However, the supplier can invest in a costly diagnostic test (such as running a test production) to learn the true state of his reliability, which further enables him to make optimal process improvement decision to reduce the disruption risk.

To manage the supplier's diagnostic and improvement decisions, the buyer may consider various incentive mechanisms. In particular, in this study, we examine a *deferred* payment strategy that consists of two parts: the *subsidy* payment and *contingent* payment. In practice, a deferred payment strategy can be implemented in the form of trade credit (see Smith [1987], Long et al. [1993]), which is the largest source of external short-term financing for the firms both in the United States [Petersen and Rajan, 1994] and internationally [Rajan and Zingales, 1995] [2]. The first payment term, subsidy, enables a buyer to directly contribute in supplier's diagnostic investment [3]. Subsidies are also commonly used in regulation, where a regulator faces a firm with a privately known productivity parameter (see Baron and Myerson, 1982, Laffont and Tirole, 1986 and the citations therein). In OM literature, subsidy is also considered as a direct incentive mechanism by which the buyer can directly involve in supplier's reliability improvement program [Tang et al., 2013] [4]. We examine *contingent payment* for delivery items by the supplier. In a recent paper, Babich and Tang [2012] used contingent payment scheme to allow the buyer to learn about supplier's product quality and to withhold contingent payments in case the supplier produced defective products. As opposed to the first term, the second one in the contract lets the buyer to penalize the supplier for non-delivery items. Note that penalty clauses in contracts are a common means for the manufacturers to recover damages for non-delivery. Reyniers and Tapiero [1995] embedded the penalty cost in contract components such that the supplier has to pay the penalty for producing defective products. Baiman et al. [2000] considered various penalties to be paid by the supplier to the manufacturer in case certain events occur. Gurnani and Shi [2006] proposed nondelivery penalty contract to address the manufacturer's lack of confidence in the supplier's ability to deliver the order. Recently, Yang et al. [2009] considered the penalty to provide an incentive to the supplier to use backup production option to satisfy the order quantity.

In our model, combination of subsidy and penalty terms would make the incentive contract an effective risk

[2] For more on deferred payment, we refer to Babich and Tang [2012] and references therein.

[3] The reason for why subsidy should be deferred is discussed later in Section 4.3.

[4] See Krause et al. [2007] to find empirical evidences in the US where direct involvement is an effective mechanism to improve supplier reliability

management strategy for the buyer, in the sense that it enables the buyer to provide an incentive to the supplier to look for some disruption mitigation strategy to satisfy his obligations in delivery. We call such a mitigation strategy *process improvement*. In practitioner literature on new product development, process improvement is viewed as an investment such as automation and flexible manufacturing technology that increases the reliability in manufacturability. For example, in an semiconductor manufacturing industry, Bohn and Terwiesch [1999] discussed the value of investment in automation technology in improving new product process yields. Flexible manufacturing technology is also strongly recommended in NPD process, which allows a company to respond quickly to disruption [Sheffi, 2007]. In particular, the corrective design changes ("redesign") due to mismatches between product solution and customer needs are inevitable in product development. Having operational flexibility enables the supplier to be quick and cost-efficient in redesign process.

4.2 Related Literature

This study contributes to the supply disruption literature with a unique feature of exploring the value of supply diagnostic test (e.g., test production) and process improvement in NPD. In our model we assume that the buyer can incentivize the supplier to gather reliability information by investing in diagnostic test, therefore it is related to the contracting literature under information asymmetry in the presence of supply disruptions. Our work also contributes to the stream of NPD that explores the impact of testing on NPD project. Finally, we explore some of related papers in economics literature.

The models developed in the supply chain contracting literature under information asymmetry generally assumes that one of the supply chain parties has superior information than the other parties at the ex-ante stage. For example, Yang et al. [2009] considers a case where the supplier is informed about his reliability at the contracting stage and studies how an uninformed buyer provides an incentive to the supplier to elicit his true information. A Bayesian model of supply learning developed by Tomlin [2009] explores how supply learning influences both sourcing and inventory strategies in dual-sourcing and single-sourcing models. Chaturvedi and Martinez-de Albeniz [2011] analyze optimal procurement strategies when there exists two dimensional information asymmetry on supplier's production cost and reliability. Gurnani and Shi [2006] consider the case of a first-time interaction between a buyer and its supplier when they have different estimates of the supplier's reliability. In Gümüş et al. [2012], an unreliable supplier offers price and quantity contract to compete with a reliable supplier. They study the underlying motivation for the guarantee offer and its effects on the competitive intensity and the performance of the chain partners. These and the majority of other papers (see Aydin et al. [2010], Tomlin and Wang [2010], Tang [2006] for recent reviews of supply-risk literature) in the supply-risk literature assume that the distribution (likelihood) of supply-side uncertainties is asymmetrically known only by the supplier. In contrast, motivated by NPD, we assume that the distribution of supply uncertainty may be also unknown by the supplier himself. However, we let the supplier to invest in a diagnostic test (e.g., test production) to acquire information about his reliability, and use this information when deciding on process improvement effort, which reduces his exposure to disruption risk.

In this aspect, our study is also related to the NPD literature. In general, this specific literature explores the importance of testing activities that are carried out to evaluate novel product concepts and designs in new product introduction. The information gathered from testing can reduce uncertainty for the involved parties; for the supplier in the form of technical production problems, and for the retailer by revealing mismatches between product solution and customer needs (see Thomke and Bell [2001], Dahan and Mendelson [2001], Erat and Kavadias [2008]). Upon finding such problems, either on supply or demand side, various corrective actions (such as changes in the design of both product and production process) can be implemented. Therefore, the timing of information gathering activity can significantly affect the economics of a NPD (Krishnan et al. [1997], Thomke [1998], Terwiesch et al. [2002]). Note that the main goal of above papers is to develop mathematical models to treat testing as an activity that generates information about technical or demand-side uncertainties in a centralized system. However, in our study, we develop a game-theoretical model in which the outcome of a testing activity is learned only by the supplier. Using this decentralized setting, we examine not only the learning effect of the test on supplier's improvement decisions, but also the incentive-related problems that may arise due to privately acquired information by the supplier. The most related paper to ours in NPD literature is the recent paper by Kim and Netessine [2013], which compares the efficiency of a screening contract (an ex-post strategy) to a commitment contract (an ex-ante strategy) in a decentralized supply chain setting where the buyer and supplier jointly collaborate in a cost-reduction effort, which lowers the expected production cost and its related uncertainty. They assume that the new information on the production cost is realized only for the supplier, which creates information asymmetry between channel parties. They show that ex-post contracting leads to a hold-up problem for the supplier, which in turn hinders the benefits of collaboration. Similar to Kim and Netessine [2013], we assume that the reliability information acquired from diagnostic test is only available for the supplier. That being said, our model differs from Kim and Netessine [2013] in different ways. First, we assume that both information-gathering and process-improvement efforts are exerted only by the supplier, which effectively leads to one-side moral hazard problem. Therefore, the hold-up problem does not arise in our setting. Secondly, the information acquired during information-gathering stage is an input for the supplier when deciding on unobservable process improvement, hence the buyer faces with an inter-temporal two-stage moral hazard problem. Third, Kim and Netessine [2013] assume that the cost-reduction effort reduces both average and variance of marginal production cost, whereas in our study, we divide the supplier's decision to two steps. In the first step, he decides whether or not to invest in a diagnostic test, which effectively reduces the variance of his true reliability, and in the second step, based on the information acquired in the first step, he decides whether or not to invest in process improvement, which effectively reduces the mean of disruption risk. Considering a sequential decision leads to two contrasting effects on the different sides of the supply chain. On the upstream side, it enables the supplier to make more efficient mean-reduction effort in the second step. Specifically, if the outcome of the variance-reduction effort indicates that the supplier is reliable, then the supplier can avoid wasting money by investing in a mean-reduction effort. On the downstream side, however, it restricts the power of the buyer in influencing the supplier to exert a certain action profile because of the inter-temporal relation between first- and second-stage moral hazard problems. Last but not least, we believe that there is a crucial difference between analyses of the diagnostic test in a supply disruption context (studied here) and procurement context (studied by Kim and Netessine, 2013).

In our model the supplier (who plays the role of an agent) does not have superior information on his reliability than does the buyer (who is the principal) at the time of contract. Hence, our model is related to the principal-agent models of adverse selection where the agent is also uninformed. In Crémer and Khalil [1992], Lewis and Sappington [1993], Kessler [1998], Cremer et al. [1998], the agent chooses whether or not to gather private information on a relation-specific parameter before contracting takes place. It is shown that remaining uninformed has a positive strategic value for the agent. Contrary to our model, the above papers assume that the agent may acquire information before the principal offers the contract. In a more related paper to our model, Lewis and Sappington [1991] develop a model in which the principal chooses the probability p with which the agent receives perfect private state information. They show that the agent information rent increases when principal increases p. Similarly, in our model, the principal offers the contract before agent decides on information-gathering effort. Moreover, the agent may use the acquired information to increase the social welfare, which benefits both principal in terms of increasing channel efficiency and agent himself in terms of information rent. Finally, our comparative analysis reveals that the principal can benefit from uninformed agent if increasing agent's observability over his type leads to high information asymmetry.

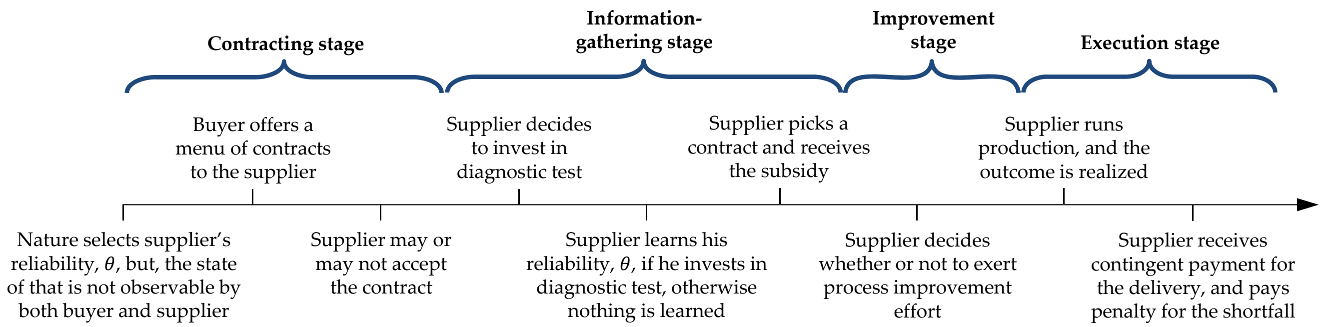


Figure 4.3.1: Timing of events in Supply Diagnostic Model

4.3 Model Framework

We develop a dyadic supply chain in which a buyer ("she") outsources the production of a new product to a supplier ("he"). The buyer faces a fixed demand [5] $D = 1$ and the expected profit per unit sold in the market is \$$r$. To satisfy her demand, the buyer procures the product from a supplier whose production cost is \$$c_s$/unit. Since the product is new, the extent of the supplier's true reliability, denoted by θ is unknown by both the buyer and supplier at the time of contract. Further, we assume that supplier's true reliability can be of two types, i.e., $\theta \in \{l, h\}$, and both the buyer and supplier share a common a priori beliefs about θ, i.e., $\theta = h$ with probability α and $\theta = l$ with probability $1 - \alpha$, where $0 \leq \alpha \leq 1$. The h-type supplier is fully reliable in the sense that the probability of disruption is zero for him, whereas the l-type supplier is unreliable and is subject to the disruption, the extent of which can be reduced by exerting a process improvement effort at cost c_p. As we discuss latter, the uncertainty of θ may lead the supplier to make inefficient improvement decision. For example, a process improvement effort has a negative net present value for an uninformed h-type supplier because he incurs a cost to exert a process improvement, and in return, gets no return. We let the supplier choose to invest in a diagnostic test at cost c_d, which enables him to learn his true reliability, before deciding on process improvement. The sequence of events is presented in Figure 4.3.1 and also provided below.

- *Contracting stage:* At time zero, nature selects the state of supplier's reliability, i.e., $\theta \in \{h, l\}$, which is not observable by both the buyer and supplier. Our model setting starts with the buyer by offering a menu of contracts that consists of three terms. The first term is a deferred subsidy payment, ω, which may cover for different costs (production, diagnostic, and process improvement) incurred by the sup-

[5] To avoid more complexity, and to focus on supply-side risk in new product launch and the impact of diagnostic test on that, we assume that the market demand is known at the time of the contract and, without loss of generality, we normalize demand to be one unit, i.e., $D = 1$.

plier and is payable to the supplier once he chooses the contract from the menu, which occurs at the end of information-gathering stage. The second term of the contract is a contingent payment Y, which is payable to the supplier only for the delivered items. Finally, the supplier pays penalty κ per unit of shortfall. Considering a deferred subsidy is an important feature of our model because of the fundamental difference between our problem and the regular adverse selection problem. Note that, in the regular adverse selection the principal designs a menu of incentive compatible contracts so that the agent, who knows his true type at the time of contract, self-selects the contract designed for him and therefore truthfully reveals his private information to the principal. However, in our model, the supplier (i.e., agent) does not know his true reliability at the time of contract. Thus, the contract terms should satisfy the participation constraints of the supplier irrespective of his decision on diagnostic test. Moreover, if the contract incentivizes the supplier to invest in a diagnostic test, which consequently enables the supplier to learn his true reliability, the menu should be also incentive compatible so that the supplier self-selects the contract designed for him. As we will show latter, this fundamental difference, which is particularly important in different situations such as new product development project, causes different source of inefficiencies in buyer's optimal contract design problem [6].

- *Information-gathering stage:* The supplier can learn his true reliability by investing in a diagnostic test, e.g., running a test production, at cost c_d. Let $d \in \{0,1\}$ show the supplier's decision on diagnostic test, where $d=1$ and $d=0$ respectively correspond to supplier's investing and not investing in diagnostic test. We assume that by investing in diagnostic test, the supplier receives a perfect signal about his true reliability. Let $\tilde{\alpha}$ denote the supplier's a-posterior beliefs on his true reliability being of h-type. Then, we have

$$\tilde{\alpha} = \begin{cases} 1 & \text{if } d = 1 \text{ and } \theta = h \\ 0 & \text{if } d = 1 \text{ and } \theta = l \\ \alpha & \text{if } d = 0 \end{cases} \tag{4.3.1}$$

Using updated beliefs, we can define $\tilde{\theta} = \tilde{\alpha}h + (1-\tilde{\alpha})l$ that represents the supplier's expected reliability updated after his diagnostic decision. Clearly, when the supplier invests in diagnostic test then $\tilde{\theta} = h$ when he is of h-type and $\tilde{\theta} = l$ when he is of l-type, otherwise he uses his a priori beliefs and his expected reliability would be $\tilde{\theta} = \bar{\theta} = \alpha h + (1-\alpha)l$. Based on the decision on diagnostic test the supplier then

[6] This type of problem is known as adverse selection with *"endogenous information structures"*. See Laffont and Martimort [2002] for a detailed discussion.

picks a contract and receives the subsidy.

- *Improvement stage:* Based on the results from information-gathering stage the $\tilde{\theta}$-supplier decides on process improvement. Let $e_{\tilde{\theta}} \in \{0,1\}$ denote the supplier's improvement decision when his expected reliability is $\tilde{\theta}$, where $e_{\tilde{\theta}} = 0$ and $e_{\tilde{\theta}} = 1$ represent no-effort and effort, respectively. We assume that the supplier's production outcome $q_{\tilde{\theta}}$ is subject to an all-or-nothing type uncertainty whose survival probability p is a function of the supplier's expected reliability $\tilde{\theta}$ and improvement effort $e_{\tilde{\theta}}$:

$$q_{\tilde{\theta}} = \begin{cases} 1 & \text{with probability } p\left(\tilde{\theta}, e_{\tilde{\theta}}\right) \\ 0 & \text{with probability } 1 - p\left(\tilde{\theta}, e_{\tilde{\theta}}\right) \end{cases} \tag{4.3.2}$$

where $p\left(\tilde{\theta}, e_{\tilde{\theta}}\right)$ is provided in Table 4.3.1:

Table 4.3.1: Supplier's survival probability as a function of expected reliability and improvement effort

		Supplier's expected reliability		
		$d = 1$		$d = 0$
		$\tilde{\theta} = l$	$\tilde{\theta} = h$	$\tilde{\theta} = \bar{\theta}$
Supplier's	$e_{\tilde{\theta}} = 0$	φ	1	$\alpha + (1-\alpha)\varphi$
improvement effort	$e_{\tilde{\theta}} = 1$	ρ	1	$\alpha + (1-\alpha)\rho$

- *Execution stage:* Finally, in the execution stage the supplier runs the production, realizes the supply uncertainty, receives the contingent payment per unit delivered to the buyer and pays the penalty per unit of shortfall. Let $\pi^B_{\theta}(\omega_\theta, Y_\theta, \kappa_\theta \mid e_{\tilde{\theta}})$ and $\pi^S_{\theta}(\omega_\theta, Y_\theta, \kappa_\theta \mid e_{\tilde{\theta}})$ respectively show the profits of buyer and supplier when supplier's true reliability is $\theta \in \{h, l\}$, and he takes improvement effort $e_{\tilde{\theta}}$ based on his expected reliability $\tilde{\theta}$.

4.4 The First-Best Outcome

In order to establish a benchmark, in this section, we consider a case in which the buyer and supplier work together as an "*integrated firm*", i.e., both diagnostic and improvement decisions are taken by the integrated firm. If diagnostic test is not employed, the firm uses its a priori beliefs $\bar{\theta} = \alpha h + (1-\alpha)l$ and chooses the optimal process improvement $e^*_{\bar{\theta}}$ accordingly. However, by investing in a diagnostic test, the firm learns his true reliability type, and would invest in a process improvement if its true reliability is of l-type. The integrated firm's optimization problem can be written as follows:

$$\max_{d} \quad d\left[\alpha\left(r-c_{s}\right)+(1-\alpha)\left(p\left(l, e_{l}^{*}\right) r-c_{s}-e_{l}^{*} c_{p}\right)-c_{d}\right]+(1-d)\left[p\left(\bar{\theta}, e_{\bar{\theta}}^{*}\right) r-c_{s}-e_{\bar{\theta}}^{*} c_{p}\right] \tag{4.4.1}$$

$$\text{s.t.} \quad e_{l}^{*}=\arg\max_{e_{l}}\left\{e_{l}\left[p(l, 1) r-c_{s}-c_{p}\right]+\left(1-e_{l}\right)\left[p(l, 0) r-c_{s}\right]\right\} \tag{4.4.2}$$

$$e_{\bar{\theta}}^{*}=\arg\max_{e_{\bar{\theta}}}\left\{e_{\bar{\theta}}\left[p(\bar{\theta}, 1) r-c_{s}-c_{p}\right]+\left(1-e_{\bar{\theta}}\right)\left[p(\bar{\theta}, 0) r-c_{s}\right]\right\} \tag{4.4.3}$$

Clearly, if the cost of process improvement effort is sufficiently high, then, neither l- nor h-type firms would exert improvement action, therefore investing in a diagnostic test brings no value. Hence, the optimal solution would be $d = 0; e_{\bar{\theta}} = 0$. However, when the cost of process improvement is relatively low, then it would be optimal to invest in diagnostic test for the integrated firm. Especially, if the cost of diagnostic test is less than the expected savings obtained due to avoiding costly process improvement effort, then, diagnostic test would be optimal. Mathematically, we can express the optimality condition for diagnostic test by comparing the cost of diagnostic test c_d to its benefit αc_p, or equivalently, $\frac{c_d}{c_p}$ to α. Note that $\frac{c_d}{c_p}$ corresponds to the relative cost of diagnostic test with respect to the cost of process improvement, and α represents the likelihood of being h-type, who would save from process improvement cost. The following Proposition 4.1 summarizes the above discussion (note that the proofs for all propositions are presented in Appendix).

Proposition 4.1. *Under first best scenario, the optimal diagnostic test and process improvement decisions are characterized in Figure 4.4.1, where:*

- *If the cost of process improvement is sufficiently low, then*
  - *If $\frac{c_d}{c_p} \leq \alpha$: it is optimal to invest in diagnostic test and improves its process if and only if he is of l-type - see Region (i);*
  - *If $\alpha < \frac{c_d}{c_p}$: it is optimal to exert process improvement effort without investing in diagnostic test - see Region (ii).*
- *If the process improvement cost is high (i.e., Region (iii)), then neither diagnostic test nor process improvement effort is exerted in equilibrium.*

The main takeaway from Proposition 4.1 is that the optimal diagnostic and improvement decisions in a centralized channel mainly depends on the relative cost of learning to expected saving ($\frac{c_d}{c_p}$) and the likelihood of

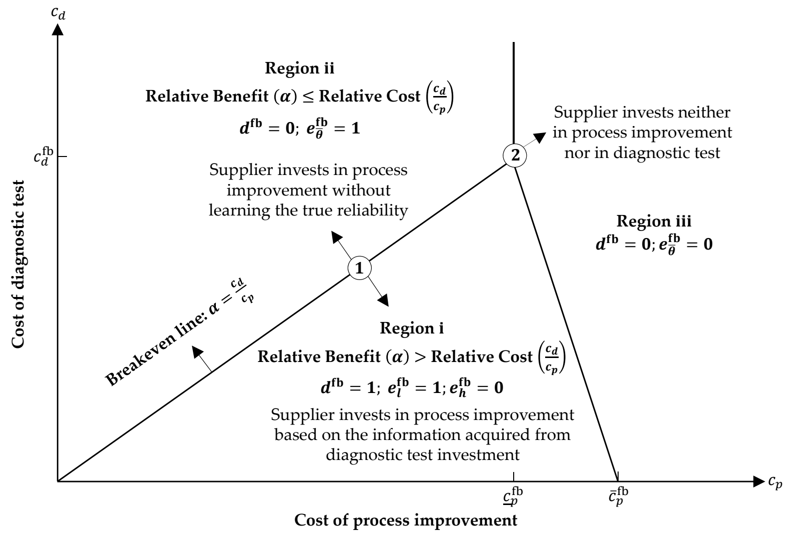


Figure 4.4.1: The First-Best Outcome

Notes. $c_d^{fb} = \alpha(1-\alpha)\delta r$; $\underline{c}_p^{fb} = (1-\alpha)\delta r$; $\bar{c}_p^{fb} = \delta r$.

possessing a fully reliable (i.e., h-type) production system (denoted by α). Below, we analyze the impact of diagnostic test in a decentralized supply chain, where the buyer delegates both the diagnostic test and process improvement decisions on the supplier.

4.5 Optimal Contract in Decentralized Supply Chain

In this section, we analyze the decentralized channel where channel parties maximize their own profits. Furthermore, the supplier's diagnostic test and process improvement decisions are unobservable by the buyer, hence they are not contractible. In other words, if the supplier invests in a diagnostic test, then the acquired information is no longer available to the buyer, which, as we discuss below, creates an information asymmetry between supply chain parties. Note that, to understand the value of diagnostic test in a decentralized supply chain, in §4.5.1, we first consider a case in which diagnostic test is not available. It helps us to identify different agency problems that may arise in the absence of diagnostic test in a decentralized channel. Later, in §4.5.2, we analyze the model in the presence of diagnostic test where the supplier has the opportunity to learn his true reliability. By comparing the results in §§4.5.1 and 4.5.2, we characterize the value of diagnostic test for the supply chain parties as well as total supply chain.

4.5.1 Optimal Contract in the Absence of Diagnostic Test

Suppose that diagnostic test is not available, and therefore, the supplier cannot learn his true reliability. Under this condition, both the supplier, when deciding on process improvement, and the buyer, when designing the optimal contract, share the same a priori beliefs $\bar{\theta} = \alpha h + (1-\alpha)l$ on the supplier's true reliability. This in turn implies that employing a type-dependent improvement action is not feasible. Consequently, under certain cases, the uninformed supplier's improvement decision may inevitably lead to either over- or under-investment in process improvement compared to the first-best outcome. On the other hand, because the supplier cannot learn his true reliability, the buyer would offer a pooling contract $(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}})$, and given the contract, the supplier would make the process improvement decision $e^*_{\bar{\theta}}$ based on the a-priori beliefs.

In the next paragraph, we derive the list of constraints that need to be satisfied by a feasible and incentive-compatible contract. First of all, the buyer must offer a contract that satisfies supplier's *ex-ante* participation constraint, where supplier's reservation profit is normalized to zero:

$$\pi^S_{\bar{\theta}}\left(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e^*_{\bar{\theta}}\right) = \alpha\pi^S_h\left(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e^*_{\bar{\theta}}\right) + (1-\alpha)\pi^S_l\left(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e^*_{\bar{\theta}}\right) \geq 0 \tag{4.5.1}$$

where $\pi^S_{\theta}(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e^*_{\bar{\theta}})$ indicates the profit of an uninformed θ-type supplier who decides on $e^*_{\bar{\theta}}$. In addition to the above ex-ante participation constraint, the contract should also satisfy the reservation profit of uninformed supplier for any realization of $\theta \in \{h, l\}$. Note that the uninformed supplier exerts the improvement effort $e^*_{\bar{\theta}}$ induced by the buyer only if he is financially protected for the cost of improvement for any realization of $\theta \in \{h, l\}$. We capture these financial limitations by considering *limited liability* constraints in buyer's contract design problem (see Laffont and Martimort, 2002 for the detailed analysis of principal-agent models with limited liability constraints). The limited liability constraints can be written as

$$\pi^S_h\left(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e^*_{\bar{\theta}}\right) = \omega_{\bar{\theta}} - c_s + Y_{\bar{\theta}} - e^*_{\bar{\theta}} c_p \geq 0 \tag{4.5.2}$$

$$\begin{aligned}\pi^S_l\left(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e^*_{\bar{\theta}}\right) = \omega_{\bar{\theta}} - c_s + e^*_{\bar{\theta}}\Big[p(l,1)Y_{\bar{\theta}} - \big(1-p(l,1)\big)\kappa_{\bar{\theta}} - c_p\Big] \\ + \big(1-e^*_{\bar{\theta}}\big)\Big[p(l,0)Y_{\bar{\theta}} - \big(1-p(l,0)\big)\kappa_{\bar{\theta}}\Big] \geq 0\end{aligned} \tag{4.5.3}$$

Now, the problem for the buyer is to decide whether or not to induce improvement effort on the uninformed supplier whose expected reliability is $\bar{\theta}$. Since the supplier's improvement effort is not observable, the buyer can induce the optimal level of effort $e^*_{\bar{\theta}}$ by satisfying the following moral hazard constraint:

$$\pi^S_{\bar{\theta}}\left(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e^*_{\bar{\theta}}\right) \geq \pi^S_{\bar{\theta}}\left(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e_{\bar{\theta}} \neq e^*_{\bar{\theta}}\right) \tag{4.5.4}$$

where

$$\begin{aligned} e^*_{\bar{\theta}} = \arg\max_{e_{\bar{\theta}}} \Big\{ & e_{\bar{\theta}}\Big(\omega_{\bar{\theta}} - c_s + p(\bar{\theta},1)Y_{\bar{\theta}} - (1-p(\bar{\theta},1))\kappa_{\bar{\theta}} - c_p\Big) + \\ & (1-e_{\bar{\theta}})\Big(\omega_{\bar{\theta}} - c_s + p(\bar{\theta},0)Y_{\bar{\theta}} - (1-p(\bar{\theta},0))\kappa_{\bar{\theta}}\Big)\Big\} \end{aligned} \tag{4.5.5}$$

The buyer's optimization problem is then to find the optimal level of improvement effort $e^*_{\bar{\theta}}$ and the contract terms:

$$\max_{e^*_{\bar{\theta}}} \max_{\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}}} e_{\bar{\theta}}\Big[p\left(\bar{\theta},1\right)\left(r - Y_{\bar{\theta}}\right) + \left(1 - p\left(\bar{\theta},1\right)\right)\kappa_{\bar{\theta}}\Big] + \left(1 - e^*_{\bar{\theta}}\right)\Big[p\left(\bar{\theta},0\right)\left(r - Y_{\bar{\theta}}\right) + \left(1 - p\left(\bar{\theta},0\right)\right)\kappa_{\bar{\theta}}\Big] - \omega_{\bar{\theta}} \tag{4.5.6}$$

s.t. Constraints (4.5.1-4.5.5)

In order to solve the buyer's problem, we first need to work backward and solve the supplier's process improvement decision given a contract offered by the buyer. The expected benefit of exerting a process improvement comes from the supplier's marginal profit $(Y_{\bar{\theta}} + \kappa_{\bar{\theta}})$ multiplied by the increase in uninformed supplier's expected reliability due to process improvement. Note that since the reliability improves only for l-type, the increase in expected reliability can be expressed as $(1-\alpha)\delta$, where $\delta = p\left(l,1\right) - p\left(l,0\right)$. The expected benefit of exerting process improvement is therefore $(1-\alpha)\delta\left(Y_{\bar{\theta}} + \kappa_{\bar{\theta}}\right)$. The supplier then exerts process improvement if the cost of improvement effort c_p is less than its expected benefit, or:

Lemma 4.1. *Given the contract $(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}})$ offered by the buyer, the uninformed supplier exerts process improvement (i.e., $e^*_{\bar{\theta}} = 1$) iff $Y_{\bar{\theta}} + \kappa_{\bar{\theta}} \geq \frac{c_p}{(1-\alpha)\delta}$.*

The above Lemma 4.1 shows that the fixed-price contract (i.e., subsidy) is not enough to induce process improvement on an uninformed supplier in a decentralized supply chain, and that the buyer has to provide incentive to the supplier via contingent payment and/or penalty terms. As we discuss latter, both contingent payment and penalty terms should be part of the contract whenever the buyer induces process improvement on the supplier.

Let us look at different incentive-related problems that may arise for the buyer in managing an uninformed supplier's improvement decision. The first one is related to inefficiency due to supplier's process improvement decision without knowing his own type. To evaluate the amount of inefficiency, consider first Proposition 4.1 where, thanks to diagnostic test, the supplier takes informed improvement decision in Region (i); $e_l = 1$ and $e_h = 0$. However, in the absence of diagnostic test, the buyer uses her a priori belief $\bar{\theta}$ to decide whether or not to induce process improvement effort on the supplier. On one hand, by inducing $e_{\bar{\theta}} = 0$ the supply chain is not as reliable as the first-best outcome, therefore buyer may incur channel loss due to underinvestment in process improvement by l-type supplier. On the other hand, by inducing $e_{\bar{\theta}} = 1$ although the supply chain is as reliable as the first-best outcome, but it is not aligned with h-type supplier's best interest, therefore buyer may incur channel loss due to overinvestment in process improvement.

The second inefficiency comes from the limited liability constraints. Note that the incentive contract should satisfy limited liability constraints for both l- and h-type. Unfortunately, the incentives that satisfy the l-type supplier's limited liability constraint is more than enough to satisfy that of the h-type supplier, which means that the h-type supplier receives limited liability rent whenever the buyer induces process improvement on the supplier. To summarize, the buyer should take both above inefficiencies into account when deciding on the optimal level of process improvement to be induced on the uninformed supplier. In the following proposition, we present the equilibrium characterization when diagnostic test is not available:

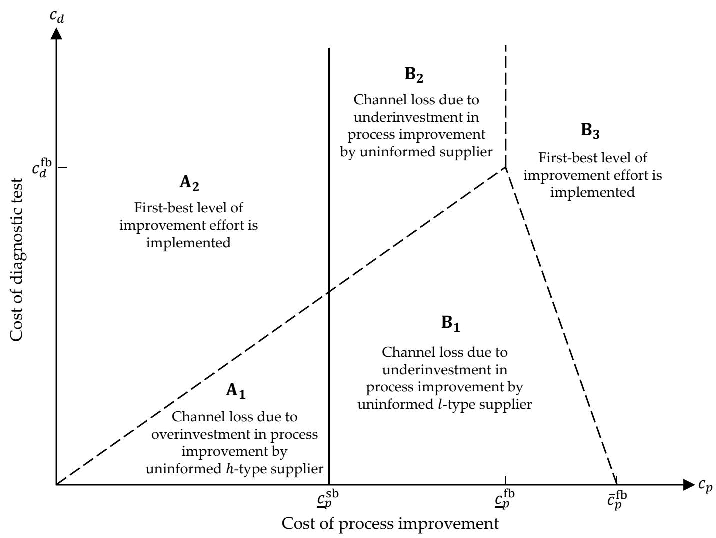
Proposition 4.2. *When diagnostic test is not available, the supplier is induced to exert process improvement effort if and only if $c_p \leq \underline{c}_p^{sb}$. The optimal contract, supplier's induced effort, and agency costs are fully characterized in Table 4.5.1.*

Below, we summarize the main observations from Proposition 4.2:

- Note that both contingent payment and penalty terms are part of the contract whenever buyer induces process improvement on the uninformed supplier (Regions A_1 and A_2). Indeed, each term plays different role for the buyer. Note that penalty term is enough, and the costless way, for the buyer in order to incentivize the uninformed supplier to invest in process improvement (see Lemma 4.1). Unfortunately, the l-type supplier's limited liability constraint (4.5.3) cannot be satisfied if the buyer only considers penalty term in the contract, which necessitates for embedding contingent payment. On the other hand, the penalty term has no impact on h-type supplier's profit, which means that the incentive that goes through contingent payment in order to satisfy l-type supplier's limited liability constraint brings positive profit for the h-type supplier in the form of limited liability rent. Now, because the limited liability rent is increasing in contingent payment, therefore, the buyer would like to consider a break-even contract that satisfies l-type limited liability constraint (i.e., $\rho Y_{\bar{\theta}} - (1-\rho)\kappa_{\bar{\theta}} = 0$), and, at the same time, it

Table 4.5.1: Optimal contracts and supplier's second-best level of effort when diagnostic test is not available

Region	Optimal contract	Supplier's second-best effort	Limited liability rent	Channel loss
A_1	$\omega^*_{\bar{\theta}} = c_s + c_p$; $Y^*_{\bar{\theta}} = \frac{1-\rho}{\delta(1-\alpha)} c_p$; $\kappa^*_{\bar{\theta}} = \frac{\rho}{\delta(1-\alpha)} c_p$	$e^*_{\bar{\theta}} = 1$	$\alpha \frac{(1-\rho)}{(1-\alpha)\delta} c_p$	$\alpha c_p - c_d$
A_2				0
B_1	$\omega^*_{\bar{\theta}} = c_s$; $Y^*_{\bar{\theta}} = 0$; $\kappa^*_{\bar{\theta}} = 0$	$e^*_{\bar{\theta}} = 0$	0	$(1-\alpha)(\delta r - c_p) - c_d$
B_2				$(1-\alpha)\delta r - c_p$
B_3				0



Notes. $\underline{c}_p^{\text{sb}} = \frac{\delta(1-\alpha)^2}{\alpha(1-\rho)+(1-\alpha)\delta}\delta r$; $\underline{c}_p^{\text{fb}} = (1-\alpha)\delta r$; $\bar{c}_p^{\text{fb}} = \delta r$.

induces the uninformed supplier to exert process improvement effort (i.e., $Y_{\bar{\theta}} + \kappa_{\bar{\theta}} = \frac{c_p}{(1-\alpha)\delta}$ - see Lemma 4.1). By satisfying these binding conditions, it is easy to verify that the limited liability rent payable to the h-type supplier, which occurs with probability α, is as follows:

$$\text{Limited liability rent payable to } h\text{-type supplier} = \alpha \times Y_{\bar{\theta}} = \alpha \frac{(1-\rho)}{\delta(1-\alpha)} c_p \quad (4.5.7)$$

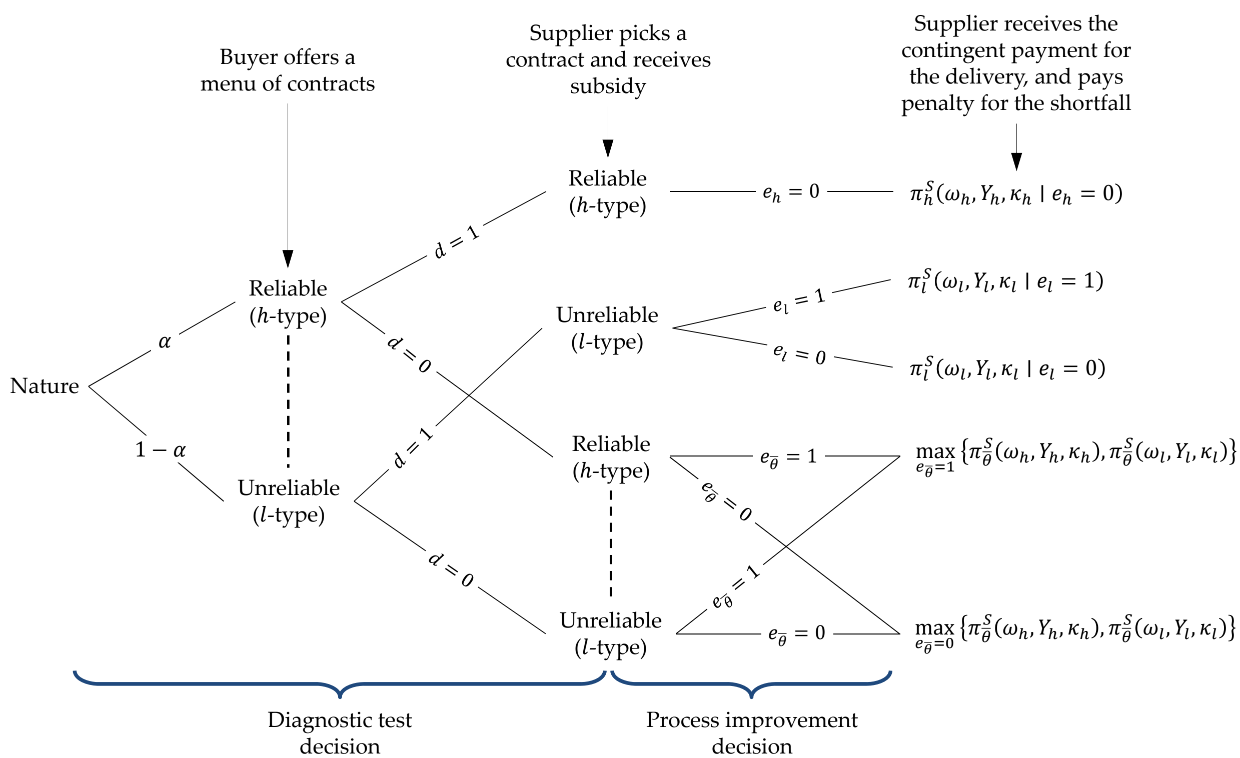
- From Eq. (4.5.7), note that the limited liability rent increases in cost of process improvement. Therefore, the buyer may prefer to induce second-best level of improvement effort on the supplier rather than paying high amount of limited liability rent, which leads to channel inefficiency. Specifically, by comparing Proposition 4.2 to the first-best outcome, we can verify that the buyer suffers from channel inefficiency for different reasons; in Region A_1 due to overinvestment in process improvement by an uninformed h-type supplier; in Region B_1 due to underinvestment by an uninformed l-type supplier; and finally in Region B_2 due to underinvestment by either h- or l-type supplier. The former (i.e., overinvestment) makes the channel as reliable as the first-best, however, the last two lead to a less reliable supply chain than the first-best. Indeed, the buyer trades off the benefits of inducing process improvement on an uninformed supplier to its costs. By inducing improvement effort on the uninformed supplier, she enjoys from a channel as reliable as the first-best at the cost of channel loss due to overinvestment by the h-type supplier (αc_p) and limited liability rent payable to the h-type supplier. By inducing no-effort, on the other hand, the buyer can rectify the limited liability rent, but in turn she suffers from less reliable channel due to l-type supplier not exerting improvement effort. By comparing the benefits and costs, we observe that the buyer is better off by not inducing process improvement when the cost of improvement is greater than $\underline{c}_p^{\text{sb}}$ (Regions B_1, B_2 and B_3).

4.5.2 Optimal Contract in the Presence of Diagnostic Test

In this section, we study the case where diagnostic test is available, and therefore the supplier has the opportunity to learn his true reliability. As we discuss below, inducing diagnostic test on the supplier brings its costs besides its benefits. On one hand, it enables the buyer to induce type-dependent level of process improvement on the supplier, hence the buyer can induce the first-best level of improvement on the supplier. In addition, because the supplier can decide on process improvement based on his true reliability, the buyer can get rid of limited liability rent. On the other hand, it creates information asymmetry between supply chain parties, hence the informed supplier armed with new information asks for information rent. Furthermore, as we will show later, considering sequential decisions for the supplier (i.e., diagnostic test and process improvement decisions) restricts the power of the buyer in influencing the supplier to exert a certain action profile because of the inter-temporal relation between first- and second-stage moral hazard problems. Below, we start our analysis by working backward and analyze supplier's two-stage decision problem.

Note that, in the presence of diagnostic test, the supplier may invest in diagnostic test and learns his true reliability. Therefore, the buyer may design a menu of contracts $(\omega_\theta, Y_\theta, \kappa_\theta), \theta \in \{h, l\}$ in order to extract supplier's true reliability information. Given the menu of contracts $(\omega_\theta, Y_\theta, \kappa_\theta), \theta \in \{h, l\}$ offered by the buyer, the supplier has two decisions (see Figure 4.3.1). In the first stage, he decides whether or not to invest in a (costly) diagnostic test $d \in \{0, 1\}$ that would provide him with perfect information regarding his true reliability. In the second stage, based on the information acquired at the diagnostic test stage, the supplier decides whether or not to exert a (costly) improvement effort, which increases his reliability only if he is of l-type. Figure 4.5.1 shows the supplier's decision tree. From Figure 4.5.1, we note that the buyer can

Figure 4.5.1: Supplier's Decision Tree



induce four different action profiles on its supplier: (i) $d = 0, e_{\bar{\theta}} = 0$; (ii) $d = 0, e_{\bar{\theta}} = 1$; (iii) $d = 1, e_h = 0, e_l = 1$; and (iv) $d = 1, e_h = 0, e_l = 0$. First of all, profile (i) is always implementable simply because the buyer can always offer zero subsidy and contingent payments to the supplier under which he has no incentive to exert diagnostic and process improvement efforts. Secondly, profit (iv) is always dominated by profile (i), because under the case when the supplier does not exert a process improvement effort, investing in a costly diagnostic test also does not create any value to him. Now, we discuss the conditions under which it is feasible to implement profiles (ii) and (iii). If the buyer wants to prevent the supplier from investing in diagnostic test $(d = 0)$, she can offer a contract based on his expected reliability $\bar{\theta}$. Note that the analysis of this scenario is different than the case discussed in Section 4.5.1, where diagnostic test was not available. Indeed, when diagnostic test is available, then the supplier would consider its benefit and cost to himself before making a diagnostic investment decision. Therefore, in order to incentivize the supplier to exert process improvement effort without investing in diagnostic test, the contract $(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}})$ should satisfy the following incentive-compatibility constraint for the $\bar{\theta}$-type (i.e., uninformed) supplier:

$$\pi^S_{\bar{\theta}}\left(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e^*_{\bar{\theta}}\right) \geq \alpha\pi^S_h\left(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e^*_h = 0\right) + (1-\alpha)\pi^S_l\left(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e^*_l\right) - c_d \tag{4.5.8}$$

Now, we can show that the buyer's contract design problem in order to induce profile (ii) is similar to model presented in §4.5.1 with the additional constraint (4.5.8). Upon receiving the contract from buyer, similar to our earlier discussion under centralized channel, we can show that the supplier decides on diagnostic test by comparing its benefits with the costs. On the benefit side, he can avoid incurring process improvement cost c_p with probability α and receive information rent from the buyer. On the cost side, he has to incur a cost c_d for the diagnostic test. First of all, since the information rent is always greater than or equal to zero, we can infer that the supplier will always exert a process improvement decision after he larks his type. In other words, if $\frac{c_d}{c_p} \leq \alpha$, a supplier cannot be induced to exert a process improvement effort without learning his type. This implies that incentive-compatibility constraint (4.5.8) for profile (ii) is not implementable when $\frac{c_d}{c_p} \leq \alpha$:

Lemma 4.2. *When diagnostic test is available, the supplier cannot be induced to exert process improvement effort without learning his true reliability (i.e., exerting diagnostic test) if* $\frac{c_d}{c_p} \leq \alpha$.

Now suppose that the buyer wants to induce diagnostic test on the supplier (i.e., profile (iii)). In order to do that, the buyer has to customize the contract terms based on supplier's true reliability type, i.e., offer $(\omega_\theta, Y_\theta, \kappa_\theta)$, where $\theta \in \{h, l\}$. Furthermore, the supplier invests in diagnostic test if his expected profit when he learns his true reliability and exerts process improvement decision is greater than his profit if he remains uninformed. To summarize, the supplier would invest in diagnostic test if and only if the following *ex ante* incentive-compatibility constraint is satisfied for the supplier:

$$\begin{aligned} &\alpha\pi^S_h\left(\omega_h, Y_h, \kappa_h \mid e_h = 0\right) + (1-\alpha)\pi^S_l\left(\omega_l, Y_l, \kappa_l \mid e_l = 1\right) - c_d \geq \\ &\qquad\qquad \max\left\{\pi^S_{\bar{\theta}}\left(\omega_h, Y_h, \kappa_h \mid e^*_{\bar{\theta}}\right), \pi^S_{\bar{\theta}}\left(\omega_l, Y_l, \kappa_l \mid e^*_{\bar{\theta}}\right)\right\} \end{aligned} \tag{4.5.9}$$

Note that maximization function in right-hand side of Eq. (4.5.9) comes from the fact that the uninformed supplier would choose the contract from the menu that maximizes his profit if he decides to deviate. Further analysis of the constraints (4.5.8) and (4.5.9) would provide the conditions under which we can solve the buyer's contract design problem. Specifically, from Lemma 4.2 it is straightforward to conclude that the buyer's contract design problem boils down to a two-wise comparison between the expected profit from Profiles (i) and (iii) when $\frac{c_d}{c_p} \leq \alpha$. However, when $\frac{c_d}{c_p} > \alpha$, then the buyer needs to compare the expected profit from all Profiles (i), (ii), and (iii). Below, we discuss the other conditions that should be satisfied in buyer's contract design problem in order to induce Profile (iii). Given a menu of contracts $(\omega_\theta, Y_\theta, \kappa_\theta), \theta \in \{h, l\}$ offered by the buyer, the supplier invests in diagnostic test if the moral hazard constraint (4.5.9) is satisfied. Furthermore, from mechanism design theory, the menu of contracts needs to satisfy two sets of constraints: (i) the *ex post* participation constraints (4.5.10-4.5.11), which assure that the reservation profit of θ-type supplier is met when he invests in diagnostic test and takes the induced improvement effort; and (ii) incentive compatibility constraints (4.5.12-4.5.13), which ensure that h- and l-type suppliers self-select the contracts designed for them, and that they cannot be better off by mimicking the other type:

$$\pi_h^S(\omega_h, Y_h, \kappa_h \mid e_h^* = 0) - c_d \geq 0 \tag{4.5.10}$$

$$\pi_l^S(\omega_l, Y_l, \kappa_l \mid e_l^* = 1) - c_d \geq 0 \tag{4.5.11}$$

$$\pi_h^S(\omega_h, Y_h, \kappa_h \mid e_h^* = 0) \geq \pi_h^S(\omega_l, Y_l, \kappa_l \mid e_h^* = 0) \tag{4.5.12}$$

$$\pi_l^S(\omega_l, Y_l, \kappa_l \mid e_l^* = 1) \geq \max_{e_l \in \{0,1\}} \pi_l^S(\omega_h, Y_h, \kappa_h \mid e_l) \tag{4.5.13}$$

Finally, with the help of following moral hazard constraint (4.5.14), the buyer makes sure that the l-type supplier, who has already learned his true reliability, exerts process improvement effort:

$$\pi_l^S(\omega_l, Y_l, \kappa_l \mid e_l^* = 1) \geq \pi_l^S(\omega_l, Y_l, \kappa_l \mid e_l = 0) \tag{4.5.14}$$

The buyer's optimization problem in order to induce profile (iii) can be written as follows

$$\max_{(\omega_h, Y_h, \kappa_h), (\omega_l, Y_l, \kappa_l)} \quad \alpha \pi_h^B(\omega_h, Y_h, \kappa_h \mid e_h = 0) + (1-\alpha)\pi_l^B(\omega_l, Y_l, \kappa_l \mid e_l = 1) \tag{4.5.15}$$
$$\text{s.t.} \quad \text{Constraints (4.5.9-4.5.14)}$$

where the objective function (4.5.15) is the sum of buyer's expected profits from the h- and l-type suppliers, each weighted by the probability of drawing that type of supplier. The above optimization problem contains many constraints while some of them are redundant. The approach to solve the above problem is provided in Appendix. We characterize the buyer's optimal contract, supplier's decisions, and all agency costs in the following Proposition 4.3:

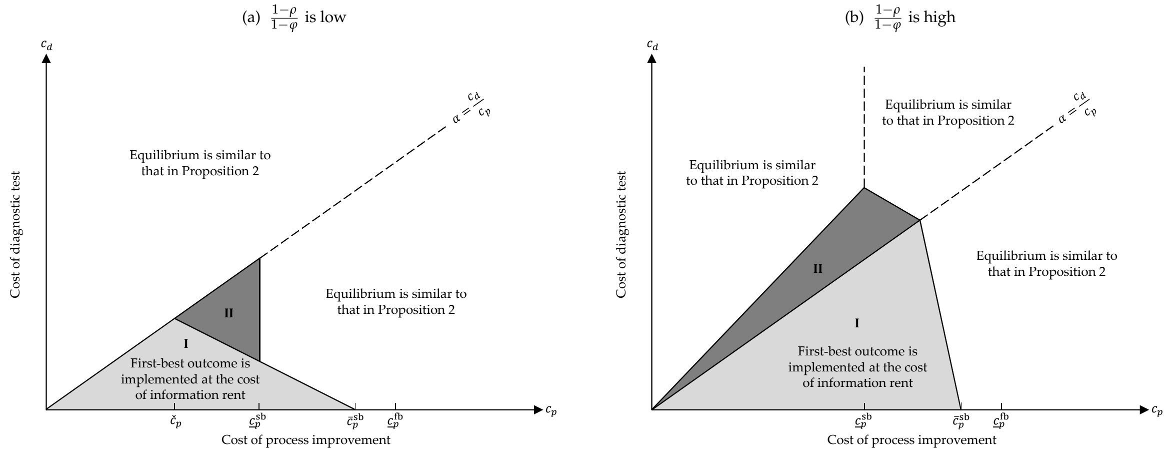
Proposition 4.3. *When diagnostic test is available, the buyer's optimal contract, supplier's optimal decisions, and dif-*

ferent agency costs are characterized in Table 4.5.2.

Table 4.5.2: Optimal Contract and Supplier's Optimal Diagnostic and Improvement Decisions in the Presence of Diagnostic Test

Region		Optimal menu of Contracts	Diagnostic decision	Improvement decision	Information rent	Channel loss
$\frac{1-\rho}{1-\varphi}$ is low	I	$\begin{cases}\omega_h &= c_s + c_d \\ Y_h &= \frac{1-\varphi}{\delta}c_p + \frac{1-\rho}{\delta(1-\alpha)}c_d \\ \kappa_h &= \frac{c_p+c_d}{\delta(1-\alpha)}\end{cases}$; $\begin{cases}\omega_l &= c_s + c_d + c_p \\ Y_l &= \frac{1-\rho}{\delta}c_p + \frac{1-\rho}{\delta(1-\alpha)}c_d \\ \kappa_l &= \frac{\rho}{\delta}c_p + \frac{\rho}{\delta(1-\alpha)}c_d\end{cases}$	$d^* = 1$	$\begin{cases}e_h^* = 0 \\ e_l^* = 1\end{cases}$	$\frac{\alpha(1-\varphi)}{\delta}c_p + \frac{\alpha(1-\rho)}{\delta(1-\alpha)}c_d$	0
	II	$\begin{cases}\omega_h &= c_s \\ Y_h &= 0 \\ \kappa_h &= 0\end{cases}$; $\begin{cases}\omega_l &= c_s \\ Y_l &= 0 \\ \kappa_l &= 0\end{cases}$	$d^* = 0$	$\begin{cases}e_h^* = 0 \\ e_l^* = 0\end{cases}$	0	$(1-\alpha)(\delta r - c_p) - c_d$
	Rest	Similar to the equilibrium in Proposition 2				
$\frac{1-\rho}{1-\varphi}$ is high	I	$\begin{cases}\omega_h &= c_s + c_d \\ Y_h &= \frac{1-\varphi}{\delta}c_p + \frac{c_d}{\alpha} \\ \kappa_h &= \frac{(c_p/\delta)+(c_d/\alpha)}{(1-\alpha)(1-\rho)}\end{cases}$; $\begin{cases}\omega_l &= c_s + c_d + c_p \\ Y_l &= \frac{1-\rho}{\delta}c_p \\ \kappa_l &= \frac{\rho}{\delta}c_p\end{cases}$	$d^* = 1$	$\begin{cases}e_h^* = 0 \\ e_l^* = 1\end{cases}$	$\frac{\alpha(1-\varphi)}{\delta}c_p + c_d$	0
	II					$c_d - \alpha c_p$
	Rest	Similar to the equilibrium in Proposition 2				

(a) $\frac{1-\rho}{1-\varphi}$ is low (b) $\frac{1-\rho}{1-\varphi}$ is high



Notes. $\check{c}_p = \frac{\delta(1-\alpha)^2}{\alpha(1-\varphi)+\delta(1-\alpha)-\alpha^2\delta}\delta r$; $\underline{c}_p^{sb} = \frac{\delta(1-\alpha)^2}{\alpha(1-\rho)+\delta(1-\alpha)}\delta r$; $\bar{c}_p^{sb} = \frac{\delta(1-\alpha)}{\alpha(1-\varphi)+\delta(1-\alpha)}\delta r$; $\underline{c}_p^{fb} = (1-\alpha)\delta r$.

The following can be achieved from Proposition 4.3:

- The first observation from Proposition 4.3 is that the equilibrium characterization not only depends on important factors discussed before, i.e., probability of being h-type supplier, α, and relative cost of learning to expected saving, $\frac{c_d}{c_p}$, but also it depends to $\frac{1-\rho}{1-\varphi}$, which indicates the amount of information that can be obtained from diagnostic test. At one extreme is that the supplier invests in diagnostic test and takes informed improvement decisions, hence the reliability difference between h- and l-type would be $1-\rho$. At the other extreme is that the supplier remains uninformed and does not invest in process improvement due to lack of knowledge about his true reliability, hence the reliability difference would be $1-\varphi$. Therefore, diagnostic test is informative when it leads to higher reliability difference than that when the supplier remains uninformed and takes no improvement effort. This explains why diagnostic test is more applicable from buyer's perspective when it is informative, i.e., $\frac{1-\rho}{1-\varphi}$ is high (Figure *b* in Table 4.5.2), than that when diagnostic test is uninformative i.e., $\frac{1-\rho}{1-\varphi}$ is low (Figure *a* in Table 4.5.2).

- The above Proposition 4.3 also shows different agency costs in a decentralized supply chain when diagnostic test is available. If the buyer prevents the supplier from learning his true reliability, the agency costs are similar to those that have been discussed in Proposition 4.2, where diagnostic test is not available, hence further explanation would be omitted for the sake of redundancy. However, by inducing diagnostic test on the supplier, we observe that the buyer can get rid of financial burden of trade with an uninformed supplier in the form of limited liability rent. Furthermore, the supplier can take informed improvement decisions, hence the first-best level of effort is achievable (Region I in both Figures *a* and *b*). Besides these benefits, increasing supplier's visibility over his true reliability brings its own problem. Specifically, it creates information asymmetry between supply chain parties, hence, the supplier armed with this new information would demand for its rent from the buyer. When solving the buyer's contract design problem, we find that constraint (4.5.9) is binding at optimality. It means that the supplier demands for information rent at the ex-ante stage because of his ability to receive the subsidy, but to deviate and not invest in diagnostic test. The amount of ex-ante information rent depends on (i) how much the supplier can obtain if he deviates and remains uninformed, therefore his average reliability is $\bar{\theta}$; (ii) the contract he picks from the menu that maximizes his profit; and finally, (iii) the optimal improvement effort under deviation contract, $e^*_{\bar{\theta}}$. As discussed in the proof of Proposition 4.3 in Appendix, it is easy to show that if the supplier deviates and remains uninformed, then he would be better off by choosing the contract designed for l-type supplier $(\omega_l, Y_l, \kappa_l)$ and then not exerting improvement effort, i.e., $e^*_{\bar{\theta}} = 0$. That means, the buyer has to provide the following amount to the supplier as information rent in order to induce him to learn his type:

$$\text{Ex-ante information rent payable to the supplier} = (\omega_l - c_s) + p\big(\bar{\theta}, 0\big)Y_l - \big(1 - p\big(\bar{\theta}, 0\big)\big)\kappa_l \qquad (4.5.16)$$

Note that by plugging the optimal contract terms from Proposition 4.3 into above formula, we can verify that the information rent is exactly equal to the contingent payment payable to the h-type supplier multiplied by the probability of drawing an h-type supplier. It means that it is only the h-type supplier who receives information rent at the *ex-post* stage. Let us describe the rationale behind this observation. Note that the ex-ante information rent should be offered to the supplier, regardless of his true reliability, in order to avoid him from deviation and not investing in diagnostic test. Therefore, we can assure that the supplier would invest in diagnostic test and would learn his true reliability. However, at the ex-post stage, when the supplier learns his true reliability, the contracts are incentive-compatible and therefore each type self-selects the contract that designed for himself. Whereas the h-type supplier is fully reliable, the penalty term κ_h has no impact on h-type supplier's profit, hence the incentive that goes through contingent payment Y_h, that satisfies constraint (4.5.9) at the ex-ante stage, brings positive profit for the h-type supplier in the form of information rent at the ex-post stage. Therefore, the ex-post information rent payable to h-type supplier is

$$\text{Ex-post information rent payable to } h\text{-type supplier} = \alpha \times Y_h \tag{4.5.17}$$

The above discussion provides pros and cons of diagnostic test in a decentralized supply chain. In the next section, we compare the optimal contract in the presence of diagnostic test (Proposition 4.3) to that in the absence of diagnostic test (Proposition 4.2) from the perspectives of the buyer, as well as the supplier and the total supply chain. It enables us to address the main research question of this chapter, namely, how information acquired by the supplier may affect the buyer's and supplier's, as well as total supply chain profits.

4.6 Value of Diagnostic Test

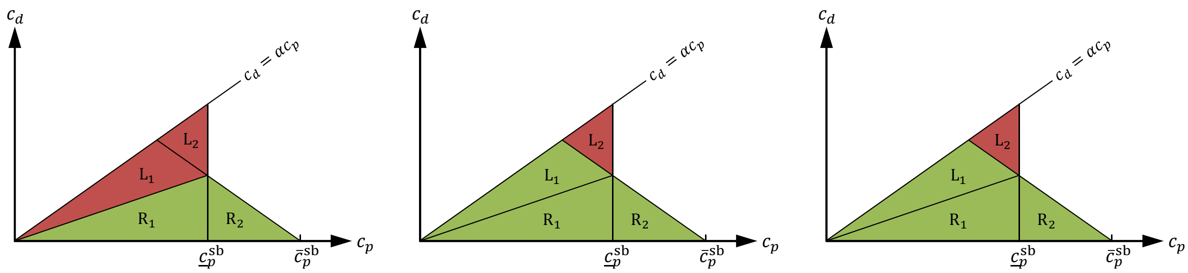
In §§4.5.1 and 4.5.2, we designed two contracts between a buyer and a supplier whose relationships might be hindered by hidden information, as well as hidden action. In this section, we first compare these contracts by taking into account all sources of inefficiencies that may arise in such a relationship; namely, channel loss due to a conflict of interest, limited liability rent due to the unobservability of the underlying reliability type, and information rent due to unobservability over supplier's acquired information and actions.

First of all, diagnostic test has no value for supply chain when it is not employed under the first-best scenario (i.e., Region iii in Proposition 4.1). However, in all the other regions, the availability of diagnostic test has a significant effect on the agency costs, hence, it may affect the buyer's or supplier's profits. Therefore, in order to characterize the value of diagnostic test for total supply chain and its parties, we need to compare only Regions I and II in Proposition 4.3 to the corresponding regions in Proposition 4.2 (Regions A_1, A_2 and B_1). We provide the full comparison in the following proposition:

Proposition 4.4. *The value of diagnostic test (VOT) for buyer, supplier and total supply chain are characterized in Table 4.6.1, where Π^T and Π^{NT} show supply chain partner's profit with and without diagnostic test, respectively.*

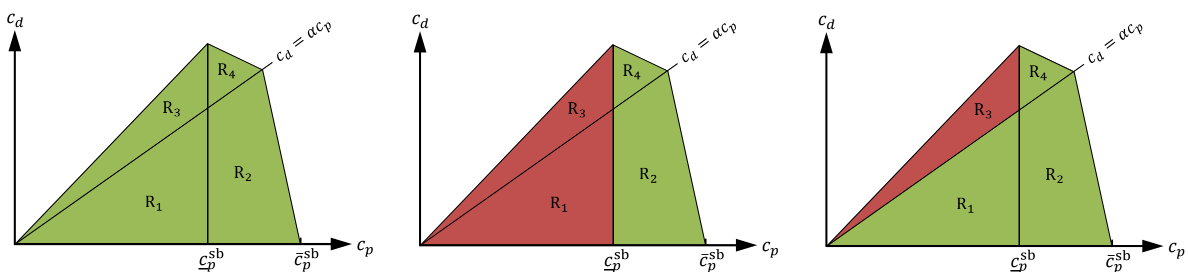
Table 4.6.1: Value of Diagnostic Test (VOT)

$\frac{1-\rho}{1-\varphi}$ is low



Region	Buyer	h-type supplier	Channel
R_1	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$
L_1	$\Pi^T \leq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$
R_2	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$
L_2	$\Pi^T \leq \Pi^{NT}$	$\Pi^T \leq \Pi^{NT}$	$\Pi^T \leq \Pi^{NT}$

$\frac{1-\rho}{1-\varphi}$ is high



Region	Buyer	h-type supplier	Channel
R_1	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \leq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$
R_2	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$
R_3	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \leq \Pi^{NT}$	$\Pi^T \leq \Pi^{NT}$
R_4	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$	$\Pi^T \geq \Pi^{NT}$

Notes. The different colored regions in the above figures denote whether or not the diagnostic test brings value for each supply chain partner: green regions - value of diagnostic test is positive; red regions - value of diagnostic test is negative; and, white regions - diagnostic test has no value.

From Proposition 4.4, it is clear that the value of diagnostic test from buyer's perspective mainly depends on whether diagnostic test is informative or uninformative. Therefore, we discuss each scenario below.

- When diagnostic test is uninformative ($\frac{1-\rho}{1-\varphi}$ is low): Note that the buyer can rectify channel loss due to either overinvestment (Regions R_1 and L_1) or underinvestment (Region R_2) at the cost of information rent. Therefore, diagnostic test is valuable in Regions R_1, R_2 and L_1 from total supply chain perspective. In addition, rectifying underinvestment in Region R_2 brings value to both buyer and h-type supplier, hence it is a win-win strategy; for the buyer in terms of inducing first-best action on its supplier and enjoying from more reliable supply chain, and for the supplier in terms of receiving information rent. However, when diagnostic test helps the buyer to rectify channel loss due to overinvestment then inducing diagnostic test may hurts the buyer (Region L_1). This occurs due to two reasons: (i) buyer's inability to induce a less costly action profile on the supplier, which is to incentivize the supplier to exert process improvement effort without learning his true reliability (as discussed in Lemma 4.2); and (ii) the high burden of information rent. Let us describe the rationale behind this observation. Note that, in Regions R_1 and L_1, the buyer rectifies limited liability rent and channel loss due to overinvestment at the cost of information rent. But information rent is increasing in the cost of diagnostic test, therefore, the buyer is better off by inducing test only when the resulted amount of information rent is less than total inefficiency if diagnostic test is not induced on the supplier (Region R_1), otherwise, she would worse off (Region L_1). It also means that the h-type supplier is better off due to receiving more information rent, comparing to limited liability rent if he remained uninformed, when c_d is sufficiently high (Region L_1). Note that the better strategy for the buyer could be to incentivize the supplier to exert improvement effort without learning his true reliability, but this is not feasible due to the restricted contract space that discussed in Lemma 4.2. This phenomenon also can explain why all parties, as well as total supply chain, are worse off in Region L_2. Due to restriction from Lemma 4.2, the buyer can induce only two profiles; either $d = 1, e_h = 0, e_l = 1$, or $d = 0, e_{\bar{\theta}} = 0$. The former leads to information rent, and the latter brings channel loss due to underinvestment by the l-type supplier. From buyer's best interest, it is optimal to incur channel loss due to underinvestment in improvement effort by the supplier rather than giving information rent in Region L_2, hence the buyer induces $d = 0, e_{\bar{\theta}} = 0$. Unfortunately, this is not in channel's best interest, since the channel loss due to underinvestment is greater than that where channel incurs loss due to overinvestment (which occurs in the absence of diagnostic test), hence both the channel and buyer are worse off comparing to the case when diagnostic test is not available. Finally, the h-type supplier is also worse off in Region L_2 because he earns zero profit, while he could receive limited liability rent if diagnostic test is not available.

- When diagnostic test is informative ($\frac{1-\rho}{1-\varphi}$ is high): Under this condition, the availability of diagnostic test always brings value to the buyer in all Regions R_1, R_2, R_3, and R_4, however, the reason for each case is different. In general, if diagnostic test rectifies overinvestment by h-type supplier (see Regions R_1 and R_3), then it always hurts the supplier, but may or may not benefit the total supply chain. Note

Table 4.6.2: Impact of System Parameters on the Value of Diagnostic Test

Relative cost $\left(\frac{c_d}{c_p}\right)$	Acquired information from test $\left(\frac{1-\rho}{1-\varphi}\right)$	Probability of being h-type (α)	Buyer	Supplier	Total supply chain
Low	Low/High	Low/High	Win	Win	Win
Medium	High	High	Win	Lose	Win
Medium	Low	Low	Lose	Win	Win
High	Low	Low	Lose	Lose	Lose

that diagnostic test helps the buyer to get rid off limited liability rent in Regions R_1 and R_3 at the cost of information rent, and because the amount of information rent is lower than that of limited liability rent, the h-type supplier is worse off by investing in diagnostic test. It also benefits the channel when it is consistent with the first-best outcome (Region R_1). But, in Region R_3, the buyer's interest conflicts with the channel's. Specifically, because the information rent due to inducing diagnostic test on the supplier is less than limited liability rent payable to the h-type supplier, the buyer is better off by deviating from first-best outcome and inducing diagnostic test on the supplier, which is not consistent to the best of channel's interest. Therefore, the value of diagnostic test is negative from total supply chain perspective. However, when diagnostic test rectifies underinvestment by the supplier (either by only l-type in Region R_2 or by both types in Region R_4) then it brings value not only for the buyer, but also for the supplier in the form of information rent.

Finally, we can analyze the impact of diagnostic test on supply chain and its parties' profits based on the important factors that discussed so far, i.e., the probability of being h-type supplier, α, the relative cost of diagnostic test to process improvement, $\frac{c_d}{c_p}$, and the relative information acquired from diagnostic test, $\frac{1-\rho}{1-\varphi}$. Table 4.6.2 summarizes the impact of these factors on the value of diagnostic test.

4.7 Conclusion

When a new product project fails, the failure is usually blamed on culprits such as tough competition or weak market research. But, the practitioner findings reveal that the true causes of failure may lie in supply-end of the chain. Lack of experience in new product manufacturing not only leads to supply side problems, such as yield problems, lack of flexibility in production capacity, lead-time variability and long set-up time, but also it may lead to huge loss on the demand side, such as lost sales, customer goodwill loss, and market share loss due to a fast follower. Therefore, in order to reduce the likelihood of failure of a new production development project, the supplier may invest in a costly diagnostic test technology, e.g., running a test production, before commencing the final production. This chapter aims to spotlight the importance of such supply diagnostic test in a new product development project.

We establish a dyadic supply chain with one buyer who procures for a new product from a supplier. Due to the lack of experience in manufacturing, the state of supply disruption is not known for both the buyer and supplier at the time of contract, hence both buyer and supplier face ex-ante same uncertainty regarding the supply risk. However, the supplier may invest in a diagnostic test to acquire information about his true reliability, and to use this information when deciding on process improvement effort, which may reduce his exposure to disruption risk. Using this setting, we identify benefits and drawbacks of diagnostic test. Specifically, if the buyer offers a contract that avoids the supplier from investing in diagnostic test, then the supplier decides on process improvement based on his ex-ante belief about his true reliability. It brings two different inefficiencies for the buyer. The first one is related to inefficient improvement decision by uninformed supplier, compared to the first-best scenario where buyer and supplier works together as an integrated firm. Specifically, due to lack of knowledge about its true reliability, an uninformed h-type supplier may overinvest in process improvement, while an uninformed l-type supplier may underinvest in process improvement. The second inefficiency comes from the financial burden of trade with an uninformed supplier in the form of limited liability. Indeed, the incentives that go through contract should satisfy the participation constraint for an uninformed supplier, whether he is h- or l-type supplier. But, such incentives are more than enough to satisfy h-type supplier's participation, and therefore, the buyer incurs limited liability rent. On the other hand, by inducing diagnostic test on the supplier the supplier takes type-dependent level of process improvement, hence the first-best level of improvement is implementable. Furthermore, the buyer can get rid of limited liability rent. That being said, it creates information asymmetry between supply chain parties, hence the informed supplier armed with new information asks for information rent. Therefore, the buyer should take a holistic approach and consider all pros and cons of inducing diagnostic test when she decide which contract to offer to the supplier.

Our results suggest that the value of diagnostic test mainly depends on the level of information that could be acquired from diagnostic test. In general, when diagnostic test is sufficiently informative, i.e., when test is induced and the realized reliability difference between types is high, then diagnostic test brings value to the buyer as well as supply chain because of rectifying channel loss. However, its value to the supplier further depends whether diagnostic test rectifies underinvestment or overinvestment. By rectifying underinvestment, the supplier is better off by receiving information rent, hence it also brings value to the supplier, however, when it rectifies overinvestment, then the amount of information rent payable to the supplier is less than limited liability rent if he remained uninformed, hence the supplier is worse off by learning his true reliability. Interestingly, when diagnostic test is uninformative, our results show that diagnostic test can hurt all the parties as well as supply chain. This is mainly related to the restriction on contract space that diagnostic test brings in buyer's side. Namely, if the relative cost of diagnostic test is less than the expected saving due to not investing in process improvement when the true reliability of the supplier is high, then the buyer cannot incentivize the supplier to exert process improvement effort without learning his true reliability. Under this condition, the buyer prefers to incur channel loss due to underinvestment in improvement effort by supplier rather than giving information rent. Unfortunately, this is not in channel's best interest, since the channel loss due to underinvestment is greater than that where channel incurs loss due to overinvestment, therefore, both the channel and buyer are worse off comparing to the case when diagnostic test is not available. The h-type supplier is also worse off because he earns zero profit, while he could receive limited liability rent if diagnostic test is not available.

Chapter 5

Appendix: Proofs for Propositions

5.1 Proofs of Propositions and Lemmas for Chapter 2

Proof. of Proposition 2.1. First, assume that the attacker is of s-type. So, let $p_s = 1$ in Model II. Clearly, Model II is a linear optimization problem in terms of D_i and z. Therefore, the optimal solution is an extreme point of feasible region, at which constraint 2.3.16 is binding for some of the targets. Let I_D and I_{ND} indicate the set of targets for which constraint 2.3.16 is binding and non-binding, respectively. This implies that $D_i = \lambda(v_i - z)$, $\forall i \in I_D$, and $D_i = 0$, $\forall i \in I_{ND}$. By plugging these values into the objective function, the defender's optimization problem can be rewritten as follows:

$$\min \quad z \tag{5.1.1}$$

$$\text{s.t} \quad v_i \leq z,\ i \in I_{ND} \tag{5.1.2}$$

$$\sum_{i \in I_D} \lambda(v_i - z) \leq D \tag{5.1.3}$$

Note that the above model is a linear optimization model with respect to z. Since the s-type attacker's valuation of targets is the same as defender's, the defender should first protect against the most valuable target. The more the defender spends on the most valuable target, the smaller the attacker's payoff function will be until the strategic attacker maximizes his payoff by attacking to the second most valuable target. The defender should now protect both targets to make the success probability of the attack for both targets as small as possible such that the strategic attacker maximizes his payoff by hitting the third most valuable target. The defender keeps distributing the budget in this way to make a set of most valuable targets less and less damageable until she fully allocates her limited budget, or all targets being fully defended. The above discussion yields the following solution approach: Let us first order the targets with respect to their valuations such that $i = 1$ indicates the most valuable target and $i = N$ shows the least one. We can then reduce constraint set 5.1.2 to only one constraint; $\max_{i \in I_{ND}} \{v_i\} \leq z$. That means, given a partition of target into I_D and I_{ND}, the objective function will be limited by the maximum valuation in undefended subset of the targets, i.e., $\max_{i \in I_{ND}} \{v_i\}$. In order to reduce the objective function further, the defender has to pick the most valuable target from I_{ND} and allocate optimal budget to defend it as long as her total budget constraint 5.1.3 is satisfied. The optimal solution therefore is of a threshold-type policy, where targets are added to set I_D in order of their values until either the constraint 5.1.3 becomes binding, or all targets are being defended. We summarize our proposed approach in the following Algorithm 1 in order to characterize the defender's equilibrium.

Algorithm 1 (Defense equilibrium under symmetric information):

1. Order the targets in decreasing order of their values such that $i = 1$ indicates the most valuable target and $i = N$ shows the least valuable target. Initialize $k = 1$.
2. Calculate $b_k = \frac{\lambda \sum_{i=1}^{k} v_i - D}{\lambda k}$ for each $k \in \{1, \ldots, N-1\}$, and stop first time when $b_k \geq v_{k+1}$. Then, it is not optimal to defend targets whose valuations are less than or equal to v_{k+1}. The optimal defense allocation is $D_i = \lambda(v_i - b_k)$, $i \leq k$, and $D_i = 0$, $i \geq k+1$.
3. If $b_k < v_{k+1}$ for all $k \in \{1, \ldots, N-1\}$, then it is optimal to defend all the targets. The optimal defense allocation is then $D_i = \lambda v_i$, $\forall i$.

Note that by using the above Algorithm, we can define a threshold $t_i = \frac{\lambda \sum_{j=1}^{i} v_j - D}{i\lambda}$ for each target i such that target i would be defended in equilibrium as long as $v_i \geq t_i$. Now, assume that the attacker is of n-type, i.e., $p_s = 0$ in Model II. The problem then boils down to a simple knapsack problem. Since the defender knows the n-type attacker's preferred target, namely target i, then the defender would minimize her objective function by allocating the maximum budget to target i subject to feasibility constraints 2.3.17, 2.3.18 and 2.3.19, i.e., $D_i = \min\{v_i, D\}$. □

Proof. of Proposition 2.2. Recall that this Proposition considers the case where the defender has asymmetric information regarding the true type of attacker. In other words, the attacker is of s-type with probability p_s and n_k-type with probability $1 - p_s$. Note that this case can be modeled by letting $q_k = 1$ and $q_i = 0$, $i \neq k$ in

Model II. The defender optimization problem can then be written as follows:

$$\min \quad p_s z + (1-p_s) v_k \left(1 - \frac{D_k}{\lambda w}\right) \tag{5.1.4}$$

$$\text{s.t.} \quad v_i \left(1 - \frac{D_i}{\lambda v_i}\right) \le z,\ i = 1, ..., N \tag{5.1.5}$$

$$\textstyle\sum_{i=1}^{N} D_i \le D \tag{5.1.6}$$

$$D_i \le v_i,\ i = 1, ..., N \tag{5.1.7}$$

$$D_i \ge 0,\ i = 1, ..., N \tag{5.1.8}$$

Note that the above 5.1.4 is a linear programming model with respect to D_i and z. That means, the constraint 5.1.5 is either binding or non-binding for each target $i = 1, ..., N$. We denote the subset of targets i (excluding target k) for which the constraint 5.1.5 is binding and non-binding by I_D and I_{ND}, respectively. Note that solving the binding and non-binding constraints for D_i would lead to $D_i = \lambda(v_i - z)$, $\forall i \in I_D$, and $D_i = 0$, $i \in I_{ND}$. Substituting these values for D_i into the above linear program would yield the following optimization problem for the defender:

$$\min \quad p_s z + (1-p_s) v_k \left(1 - \frac{D_k}{\lambda w}\right) \tag{5.1.9}$$

$$\text{s.t.} \quad v_i \le z,\ i \in I_{ND},\ i \ne k \tag{5.1.10}$$

$$v_k \left(1 - \frac{D_k}{\lambda v_k}\right) \le z \tag{5.1.11}$$

$$D_k \le v_k \tag{5.1.12}$$

$$\textstyle\sum_{i \in I_D}^{N} \lambda(v_i - z) + D_k \le D \tag{5.1.13}$$

Since D_k appears in the objective function with a negative sign, we are interested to find the maximum value of D_k subject to constraints 5.1.11, 5.1.12 and 5.1.13. Because the upper bound on D_k implied by the constraint 5.1.12 is greater than the lower bound on D_k implied by the constraint 5.1.11, we can conclude that $D_k = \min\{v_k,\ D - \sum_{i \in I_D} \lambda(v_i - z)\}$. Plugging this value in 5.1.9 and removing the fixed values from objective function, the defender's optimization problem can be reduced to either 5.1.14 or 5.1.17 depending on whether $D_k = v_k$ or $D_k = D - \sum_{i \in I_D} \lambda(v_i - z)$. Below, we analyze each case separately:

- If $D_k = v_k$ or, equivalently, $v_k \le D - \sum_{i \in I_D} \lambda(v_i - z)$, then, the defender's optimization problem is expressed as follows:

$$\min \quad p_s z + (1 - p_s) v_k \left(1 - \frac{D_k}{\lambda w}\right) \tag{5.1.14}$$

$$\text{s.t.} \quad v_i \le z,\ i \in I_{ND},\ i \neq k \tag{5.1.15}$$

$$\textstyle\sum_{i \in I_D}^{N} \lambda(v_i - z) + v_k \le D \tag{5.1.16}$$

Clearly, all the constraints in 5.1.15 can be reduced to a single constraint; $\max\limits_{i \in I_{ND}} \{v_i\} \le z$. Also note that the constraint 5.1.16 models the availability of defensive budget, D. At the optimality, we can show that at least one of the two constraints, i.e., $\max\limits_{i \in I_{ND}} \{v_i\} \le z$ or $\sum_{i \in I_D}^{N} \lambda(v_i - z) + v_k \le D$ would be binding. That means, the defender in equilibrium would defend as many targets as possible subject to the budget constraint in 5.1.16. As a result of this, the optimal budget allocation will be of a threshold-type policy; i.e., starting with the most valuable target, the defender would keep adding targets with lower valuations until either the constraint 5.1.16 becomes binding or there is no target left undefended.

- If $D_k = D - \sum_{i \in I_D} \lambda(v_i - z)$ or, equivalently $v_k > D - \sum_{i \in I_D} \lambda(v_i - z)$, then the defender's optimization problem is expressed as follows:

$$\min \quad \left(p_s - \frac{(1-p_s) v_k |I_D|}{w}\right) z + \left(\frac{(1-p_s) v_k}{w}\right) \textstyle\sum_{i \in I_D} v_i \tag{5.1.17}$$

$$\text{s.t.} \quad v_i \le z,\ i \in I_{ND},\ i \neq k \tag{5.1.18}$$

$$\textstyle\sum_{i \in I_D}^{N} \lambda(v_i - z) + \lambda(v_k - z) \le D \tag{5.1.19}$$

Again, constraints 5.1.18 can be reduced to only one constraint; $\max\limits_{i \in I_{ND}} \{v_i\} \le z$. Similar to the above case, we can show that at least one of the constraints 5.1.18 and 5.1.19 would be binding in the optimal solution. So, in equilibrium, the defender would defend as many targets as possible subject to constraint 5.1.19. But, note that the first and second terms in the objective function in 5.1.17 are decreasing and increasing in $| I_D |$, respectively. Combining increasing and decreasing cases, we can characterize the optimal policy as follows: Keep adding the targets with respect to their valuations to I_D and stop as soon as either no improvement occurs in the objective value or constraint 5.1.19 becomes binding. Note that if D is sufficiently high, then all targets would be defended in equilibrium. We summarize above discussion in Algorithm 2.

Algorithm 2 (Defense equilibrium under asymmetric information about attacker's rationality):

1. Suppose that target k is the nonstrategic attacker's preferred target. Order the targets, excluding target k, in decreasing order of their values such that 1 and $N-1$ indicate the most and the least valuable targets, respectively. Initialize $j = 1$.
2. Let $b_1^j = p_s - (j)\frac{(1-p_s)v_k}{w}$, and $b_2^j = \frac{\lambda(\sum_{i=1}^{j} v_i + v_k) - D}{\lambda(j+1)}$.
3. If $b_1^j \geq 0$ and $b_2^j < v_{j+1}$ then check whether j is the least valuable target (i.e., $j = N-1$) or not. If $j = N-1$, then set $\mathscr{B}$ =0 and go to 6, otherwise, set $j = j+1$ and return to 2.
4. If $b_1^j < 0$, then set $\mathscr{B}$ =v_j and go to 6.
5. If $b_2^j \geq v_{j+1}$, then set $\mathscr{B} = \frac{\lambda(\sum_{i=1}^{j} v_i + v_k) - D}{\lambda(j+1)}$ and go to 6.
6. Optimal defense allocation is as follows: set $D_i = \lambda(v_i - \mathscr{B})$, $i \leq j$, $i \neq k$, $D_i = 0$, $i > j$, $i \neq k$, and $D_k = \min\{[D - \sum_{i=1}^{j}[\lambda(v_i - \mathscr{B})]^+]^+, v_k\}$.

By using the above algorithm iteratively from $j = 1$ to $j = N-1$, one can characterize the conditions under which target $j \neq k$ is defended as well as the optimal defense budget allocated to that target. Note that the optimal defense allocation mainly depends on the value of $\mathscr{B}$ in Step 6, which comes from Step 4 or 5, depending on whether $b_1^j < 0$ or $b_2^j \geq v_{j+1}$ (whichever case is satisfied first). Below, we discuss each scenario separately:

- If $b_1^j < 0$ is satisfied first: from Step 4, assume that k^* is the smallest index for which $b_1^{k^*} < 0$ while $b_2^{k^*} < v_{k^*+1}$. It implies that $\mathscr{B}$ =v_{k^*} in Step 6. Furthermore, we have $b_1^i \geq 0$ and $b_2^i < v_{i+1}$, $\forall i < k^*$. But, $v_{i+1} \leq v_i$ suggests that $b_2^i < v_i$, $\forall i < k^*$. So, from Step 6, the optimal defense allocation is $D_i = \lambda(v_i - v_{k^*})$, $i < k^*$, $i \neq k$, and $D_i = 0$, $i > k^*$, $i \neq k$, specifically, $D_{k^*} = 0$. Finally, because b_1^j is decreasing in j, we have $b_1^i < 0$, $\forall i > k^*$.
- If $b_2^j \geq v_{j+1}$ satisfied first: from Step 5, assume that k^* is the smallest index for which $b_2^{k^*} \geq v_{k^*+1}$ while $b_1^{k^*} \geq 0$. It implies that $\mathscr{B}$ =$\frac{\lambda\left(\sum_{i=1}^{k^*} v_i + v_k\right) - D}{\lambda(k^*+1)}$ in Step 6. Furthermore, we have $b_1^i \geq 0$ and $b_2^i < v_{i+1}$, $\forall i < k^*$. But, $v_{i+1} \leq v_i$. Therefore, $b_2^i < v_i$, $\forall i < k^*$. Now, we show that $b_2^{k^*} \leq v_{k^*}$. Suppose that $b_2^{k^*} > v_{k^*}$. It concludes $b_2^{k^*-1} > v_{k^*}$, which contradicts with the latter result that $b_2^i < v_{i+1}$, $\forall i < k$. So, from step 6, it is clear that $D_i = \lambda(v_i - \mathscr{B})$, $i \leq k^*$, $i \neq k$, and $D_i = 0$, $i > k^*$, $i \neq k$. It is also easy to check that $v_i \leq b_2^i$, $\forall i > k^*$. Suppose that $v_i > b_2^i$, $\forall i > k^*$. It concludes $v_{k^*+1} > b_2^{k^*}$, which contradicts with the immediate assumption that $b_2^{k^*} \geq v_{k^*+1}$.

From the above algorithm, we can show that target i is defended if and only if $b_1^i \geq 0$ and $v_i > b_2^i$. Note that $b_1^i \geq 0$ is equivalent to $\frac{p_s w}{i(1-p_s)} \geq v_k$. If $t_1^i = \frac{p_s w}{i(1-p_s)}$ then $b_1^i \geq 0$ is equivalent to $t_1^i \geq v_k$. For notational consistency, we define $t_2^i = \frac{\lambda \sum_{j=1}^{i} v_j + \lambda v_k - D}{\lambda(i+1)}$. So, $v_i \geq b_2^i$ is equivalent to $v_i \geq t_2^i$. Now, let $S = \{i \mid t_1^i \geq v_k,\, v_i \geq t_2^i\}$. From step 6, since $v_k > 0$, target k is defended if and only if $D - \sum_{i=1}^{j} \lambda(v_i - \mathscr{B}) > 0$. By substituting optimal value of $\mathscr{B}$, it is straightforward to conclude that target k is defended if and only if $v_k > \frac{\lambda \sum_{i \in S} v_i - D}{\lambda |S|}$. □

Proof. of Proposition 2.3. This Proposition considers the case where attacker is of n-type, however, the defender does not know the true preferred target of the attacker. Note that this case can be modeled by letting $p_s = 0$ in Model II. The reduced problem is then a knapsack problem with a minimizing objective function. Let $k_1 = \arg\max_i \{q_i v_i\}$. We assign defensive budget to target k until either the constraint 2.3.17 or 2.3.18 becomes binding. If the constraint 2.3.18 becomes binding first, we can then set $k_2 = \arg\max_{i \neq k_1} \{q_i v_i\}$ and use the following iterative approach: In each step, if all the targets have unique $q_i v_i$ values, then the equilibrium budget allocation is $D_i = \min\{v_i,\, [D - \sum_{j=0}^{i-1} v_j]^+\}$, $\forall i$, or, equivalently, target i is defended iff $D - \sum_{j=0}^{i-1} v_j \geq 0$ or $v_i \geq t_i$ where $t_i = \sum_{j=0}^{i} v_j - D$. Now, assume that there are some targets with the same $q_i v_i$. Let k, $(k+1)$, ..., $(k+n)$ indicate these targets. If we still have leftover budget after allocating the budget to the targets $i < k$, then we can show that any distribution of remaining budget, i.e., $D - \sum_{i=0}^{k-1} v_i$, among targets k, $(k+1)$, ..., $(k+n)$ can be an equilibrium. That means, we will have multiple defense equilibria if $D - \sum_{i=0}^{k-1} v_i > 0$, whereas we have unique equilibrium if $D - \sum_{i=0}^{k-1} v_i \leq 0$. Specifically, if $D - \sum_{i=0}^{k-1} v_i > 0$, then, any combination of α_k, $\alpha_{(k+1)}$, ..., $\alpha_{(k+n)}$, $0 \leq \alpha_i \leq 1$, such that $\alpha_k + \alpha_{(k+1)} + ... + \alpha_{(k+n)} = 1$ would yield the same equilibrium, where the defender would allocate the following budgets to defend the targets: $D_i = v_i$, $i < k$ and $D_i = \min\{v_i,\, \alpha_i[D - \sum_{j=0}^{i-1} v_j]^+\}$, $k \leq i \leq k+n$, and $D_i = \min\{v_i,\, [D - \sum_{j=0}^{i-1} v_j]^+\}$, $i > k+n$. On the other hand, if $D - \sum_{i=0}^{k-1} v_i \leq 0$, then the unique defender's equilibrium is $D_i = \min\{v_i,\, [D - \sum_{j=0}^{i-1} v_j]^+\}$, $i < k$ and $D_i = 0$, $i \geq k$. □

Proof. of Proposition 2.4. We need to compare the defender's objective in equilibrium under symmetric and asymmetric information. Table 5.1.1 shows the optimal defense allocation under symmetric information. From assumption 1, $w > \max_{i=1,...,N}\{v_i\}/\lambda$. Since $v_k > v_{k'}$ by assumption, in order to simplify the proof, without loss of generality, we can reduce w so that $w = \frac{v_k}{\lambda} + \epsilon$, where ϵ is infinitesimal number. Table 5.1.2 shows the defensive budget allocation when the defender has only partial information about the attacker's rationality. We can calculate the defender's expected loss due to asymmetric information by comparing the defender's objective in different regions in Table 5.1.1 with corresponding regions in Table 5.1.2. Regarding Equation

Table 5.1.1: Optimal defense allocation under different symmetric information scenarios

Defensive budget	Attacker is n-type whose preferred target is k (Figure a)	Attacker is n-type whose preferred target is k' (Figure b)	Attacker is s-type (Figure c)		
			$a. \Delta v \geq \frac{D}{\lambda}$	$b. v_k + v_{k'} \geq \frac{D}{\lambda}$	$c. v_k + v_{k'} < \frac{D}{\lambda}$
D_k	$\min\{D, v_k\}$	0	$\min\{D, v_k\}$	$\frac{D+\lambda\Delta v}{2}$	λv_k
$D_{k'}$	0	$\min\{D, v_{k'}\}$	0	$\frac{D-\lambda\Delta v}{2}$	$\lambda v_{k'}$

Figure a

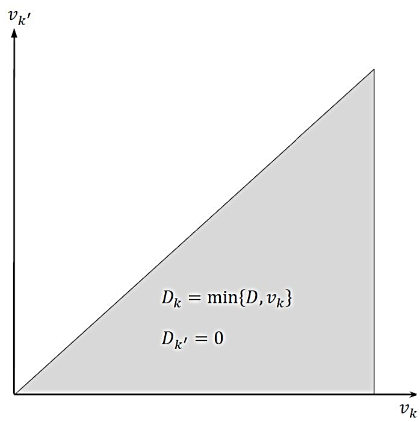


Figure b

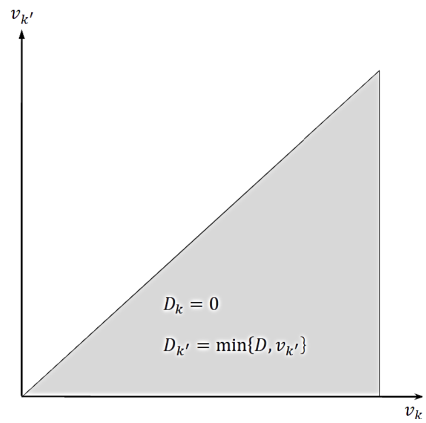
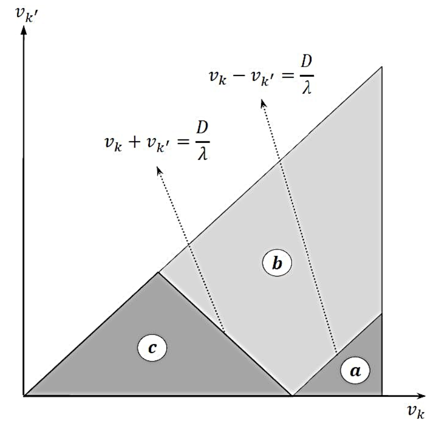


Figure c



2.3.11, the defender's loss function in our two-target problem is:

$$p_s \sum_{i \in k, k'} v_i \left(1 - \frac{D_i}{\lambda v_i}\right) I_{i=i^*} + (1 - p_s) \sum_{i \in k, k'} q_i v_i \left(1 - \frac{D_i}{v_k}\right) \tag{5.1.20}$$

In what follows, we denote the value of information by γ. Below, we categorize all possibilities when there is uncertainty about p_s.

- When the true type of the attacker is n-type and his preferred target is k (i.e., the most valuable target): We need to compare Figure a in Table 5.1.1 with different regions of Figure a in Table 5.1.2. From Equation 5.1.20, under symmetric information, the defender's loss is equal to $v_k - \min\{D, v_k\}$. Since the defensive budget allocated to target k in regions 1, 2 and 5 (see Figure a in Table 5.1.2) is equal to that of Figure a in Table 5.1.1, we have $\gamma_1 = \gamma_2 = \gamma_5 = 0$. Let $p_s = 0$ and $q_k = 1$ in Equation 5.1.20 which would yield $v_k - D_k$ as the defender's loss. Therefore, in regions 3 and 4, the value of information for the defender is $\min\{D, v_k\} - D_k$, which gives $\gamma_3 = \min\{D, v_k\} - \min\{D - \lambda v_{k'}, v_k\}$, and $\gamma_4 = \min\{D, v_k\} - \frac{D+\lambda\Delta v}{2}$.
- When the true type of the attacker is n-type and his preferred target is k' (i.e., the least valuable target): We need to compare Figure b in Table 5.1.1 with different regions of Figure b in Table 5.1.2. From Equation 5.1.20, under symmetric information, the defender's loss is equal to $v_{k'} - \kappa_v \min\{D, v_{k'}\}$. There-

Table 5.1.2: Defender's budget equilibrium under partial information about p_s

n-type attacker's preferred target is k (Figure a)			
Region	Condition	D_k	$D_{k'}$
1	$p_s < \bar{t}, v_k + v_{k'} < \frac{D}{\lambda}, \Delta v < \frac{D}{\lambda}$	$\min\{D, v_k\}$	0
2	$p_s < \bar{t}, v_k + v_{k'} \geq \frac{D}{\lambda}, \Delta v < \frac{D}{\lambda}$	$\min\{D, v_k\}$	0
3	$p_s \geq \bar{t}, v_k + v_{k'} < \frac{D}{\lambda}$	$\min\{D - \lambda v_{k'}, v_k\}$	$\lambda v_{k'}$
4	$p_s \geq \bar{t}, v_k + v_{k'} \geq \frac{D}{\lambda}, \Delta v < \frac{D}{\lambda}$	$\frac{D+\lambda\Delta v}{2}$	$\frac{D-\lambda\Delta v}{2}$
5	$p_s \geq \bar{t}, \Delta v \geq \frac{D}{\lambda}$	$\min\{D, v_k\}$	0
n-type attacker's preferred target is k' (Figure b)			
Region	Condition	D_k	$D_{k'}$
1	$p_s < \underline{t}, v_k + v_{k'} < \frac{D}{\lambda}$	0	$\min\{D, v_{k'}\}$
2	$p_s \geq \underline{t}, v_k + v_{k'} < \frac{D}{\lambda}$	λv_k	$\min\{D - \lambda v_k, v_{k'}\}$
3	$p_s \geq \underline{t}, v_k + v_{k'} \geq \frac{D}{\lambda}, \Delta v < \frac{D}{\lambda}$	$\frac{D+\lambda\Delta v}{2}$	$\frac{D-\lambda\Delta v}{2}$
4	$p_s \geq \underline{t}, v_k + v_{k'} \geq \frac{D}{\lambda}, \Delta v \geq \frac{D}{\lambda}$	$\frac{D+\lambda\Delta v}{2}$	0
5	$p_s < \underline{t}, v_k + v_{k'} \geq \frac{D}{\lambda}$	0	$\min\{D, v_{k'}\}$

Figure a

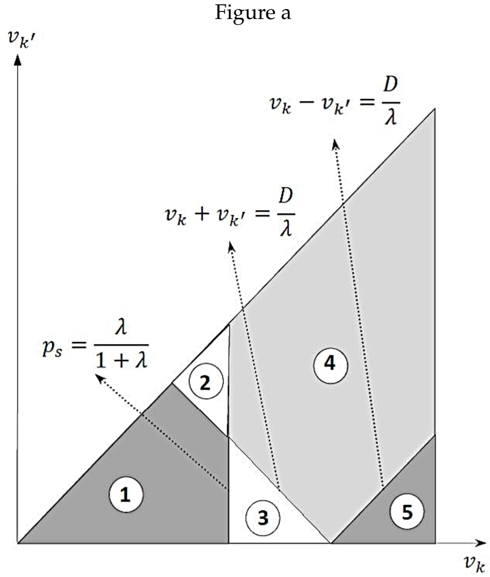
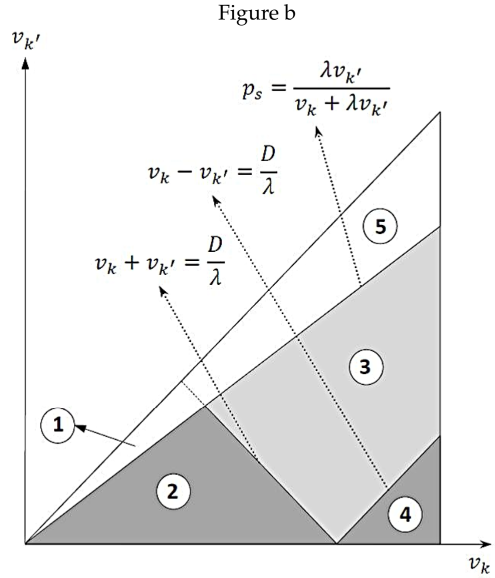


Figure b



Notes. $\bar{t} = \frac{\lambda}{1+\lambda}$; $\underline{t} = \frac{\lambda v_{k'}}{v_k + \lambda v_{k'}}$

fore, the information has no value for the defender in regions 1 and 5 of Figure b in Table 5.1.2, i.e., $\gamma_1 = \gamma_5 = 0$, since the budget allocated to target k' equals to that under symmetric case. Let $p_s = 0$ and $q_{k'} = 1$ in Equation 5.1.20 which gives $v_{k'} - \kappa_v D_{k'}$ as defender's loss. The value of informational in regions 2, 3, and 4 is $\kappa_v[\min\{D, v_{k'}\} - D_{k'}]$, which gives $\gamma_2 = \kappa_v[\min\{D, v_{k'}\} - \min\{D - \lambda v_k, v_{k'}\}]$, $\gamma_3 = \kappa_v[\min\{D, v_{k'}\} - \frac{D-\lambda\Delta v}{2}]$, and $\gamma_4 = \kappa_v \min\{D, v_{k'}\}$.

- Attacker is s-type: The defender only has partial information about attacker's rationality, but she exactly knows the n-type attacker's preferred target. Let $p_s = 1$ and $I_{i=k} = 1$ in Equation 5.1.20 whenever the strategic attacker attacks on target k, and $I_{i=k'} = 1$ whenever he strikes target k'. We have to consider two scenarios:

1. The n-type attacker's preferred target is k: We need to compare different regions of Figure c in Table 5.1.1 with corresponding regions of Figure a in Table 5.1.2. It is clear that $\gamma_4 = \gamma_5 = 0$. Region 3 of Figure a in Table 5.1.2 corresponds to region c of Figure c in Table 5.1.1. In region 3 (Figure a in Table 5.1.2), the defense on target k' is the same as what we have in region c (Figure c in Table 5.1.1), which is $\lambda v_{k'}$, and also the budget allocated to target k under asymmetric information is strictly greater than that under symmetric information. It readily means that $\gamma_3 = 0$. Note that, region 1 of Figure a in Table 5.1.2 corresponds to region c of Figure c in Table 5.1.1. In region c, the defensive budget allocated to both targets is big enough to deter any attack from both targets which means defender's loss is zero in that region. However, in Figure a (Table 5.1.2), the budget allocated to target k is big enough to deter any attack by a strategic attacker. The strategic attacker therefore attacks on target k' and benefits the entire value of target k' which is $v_{k'}$. That is to say $\gamma_1 = v_{k'}$. Finally, region 2 of Figure a in Table 5.1.2 corresponds to region b of Figure c in Table 5.1.1. In region b, it is easy to verify that the defensive budget allocated to each target is not sufficient to deter an attack on that target. Therefore, the strategic attacker may strike target k or k' depending on problem parameters. We can show that, under symmetric information, the defender's loss is $v_k - \frac{D+\lambda\Delta v}{2\lambda}$ if the attacker strikes target k, and it is $v_{k'} - \frac{D-\lambda\Delta v}{2\lambda}$ if he strikes target k'. On the other hand, considering region 2 of Figure a in Table 5.1.2, the attacker can benefit the whole value of target k', i.e., $v_{k'}$ if he attacks on k'. While $D_k = \min\{D, v_k\}$ in region 2 (Figure a in Table 5.1.2), the s-type attacker is deterred from k and strikes target k' when $D \geq \lambda v_k$. Consequently, the value of information is $\gamma_2 = \frac{D-\lambda\Delta v}{2\lambda}$. If $\lambda\Delta v \leq D < \lambda v_k$, the s-type attacker strikes target k' and $\gamma_2 = \frac{D-\lambda\Delta v}{2\lambda}$.
2. The n-type attacker's preferred target is k': We need to compare different regions of Figure c in Table 5.1.1 to their corresponding regions of Figure b in Table 5.1.2. Since our approach is similar to part (1), we only summarize the results in Table 5.1.3.

It is trivial to derive the results in Proposition 2.4 from Table 5.1.2 and 5.1.3. □

Proof. of Proposition 2.5. We need to compare the defender's loss under symmetric information when the attacker is n-type (Figures a and b in Table 5.1.1) and defender's loss when she has only partial information about q (Table 5.1.4). It leads us to consider two cases: attacker is n-type who prefers (i) the most valuable target (target k); and, (ii) the least valuable target (target k'). Note that, to obtain the defender's loss, we set

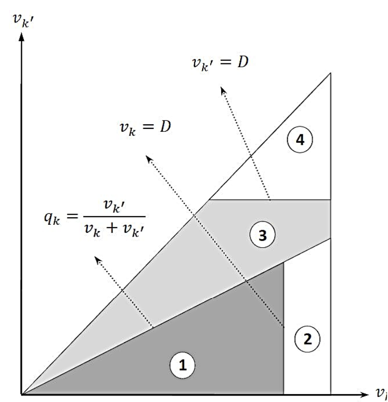
Table 5.1.3: Value of attacker's rationality information

	Attacker's type			
Region	n-type whose preference is k	n-type whose preference is k	s- type attacker	
			n-type prefers k	n-type prefers k'
1	0	0	$v_{k'}$	v_k
2	0	$\kappa_v[\min\{D, v_{k'}\} - \min\{D - \lambda v_k, v_{k'}\}]$	$\frac{D-\lambda\Delta v}{2\lambda}$	0
3	$\min\{D, v_k\} - \min\{D - \lambda v_{k'}, v_k\}$	$\kappa_v \left[\min\{D, v_{k'}\} - \frac{D-\lambda\Delta v}{2}\right]$	0	0
4	$\min\{D, v_k\} - \frac{D+\lambda\Delta v}{2}$	$\kappa_v \min\{D, v_{k'}\}$	0	0
5	0	0	0	$\frac{D+\lambda\Delta v}{2\lambda}$

Table 5.1.4: Defender's defense equilibrium under partial information about q

Region	Condition	D_k	$D_{k'}$
1	$q_k > Q, D \geq v_k$	v_k	$\min\{v_{k'}, D - v_k\}$
2	$q_k > Q, D < v_k$	D	0
3	$q_k \leq Q, D \geq v_{k'}$	$\min\{v_k, D - v_{k'}\}$	$v_{k'}$
4	$q_k \leq Q, D < v_{k'}$	0	D

Note. $Q = \frac{v_{k'}}{v_k + v_{k'}}$



$p_s = 0$ in Equation 5.1.20.

- When the true type of the attacker is n-type and his preferred target is k (i.e., the most valuable target): Let $q_k = 1$ in Equation 5.1.20. We need to compare different regions of Table 5.1.4 to Figure a in Table 5.1.1. Note that, under symmetric information, only target k is defended (see Figure a in Table 5.1.1), and the optimal defense budget that should be allocated to target k is $\min\{D, v_k\}$. Verify that this budget is equal to the budget that should be allocated to target k in regions 1 and 2 in Table 5.1.4. This observation concludes $\gamma_1 = \gamma_2 = 0$. In region 3 in Table 5.1.4, the defender allocates $D - v_{k'}$ to target k which is different to what she allocated to target k under symmetric information scenario. The defender's loss under symmetric information is $v_k - \min\{D, v_k\}$, while it is $v_k - \min\{v_k, D - v_{k'}\}$ under asymmetric information. That means $\gamma_3 = \min\{D, v_k\} - \min\{v_k, D - v_{k'}\}$. Finally, in region 4, the defender leaves target k undefended and the attacker benefits the entire value of target k under asymmetric information scenario. Since in region 4 we have $\min\{D, v_k\} = D$, the defender's expected loss under symmetric scenario is $v_k - D$. Therefore, we have $\gamma_4 = D$.

- When the true type of the attacker is n-type and his preferred target is k' (i.e., the least valuable target):

Table 5.1.5: Value of nonstrategic attacker's preference information

Region	n-type whose preference is k	n-type whose preference is k
1	0	$\kappa_v[\min\{D, v_{k'}\} - \min\{v_{k'}, D - v_k\}]$
2	0	$\kappa_v \min\{D, v_{k'}\}$
3	$\min\{D, v_k\} - \min\{v_k, D - v_{k'}\}$	0
4	D	0

Let $q_{k'} = 1$ in Equation 5.1.20. We need to compare different regions of Table 5.1.4 to Figure b in Table 5.1.1 where only target k' is defended in equilibrium, and the optimal level of defense is $\min\{D, v_{k'}\}$. Note that, this budget is equal to the budget that should be allocated to target k' in regions 3 and 4 in Table 5.1.4. That is to say, $\gamma_3 = \gamma_4 = 0$. In region 1, the defender allocated $D - v_k$ to target k' which is different to what she allocated to target k under symmetric information. The defender's loss under symmetric information is $v_{k'}(1 - \kappa_v)$, however it is $v_{k'} - \kappa_v(D - v_k)$ under asymmetric information. That means $\gamma_1 = \kappa_v(v_k + v_{k'} - D)$. Finally, in region 2, the defender leaves target k' undefended and the attacker obtains the whole value of target k' under asymmetric information. On the other hand, the defender's loss under symmetric scenario is $v_{k'} - \kappa_v \min\{D, v_{k'}\}$. Therefore, we have $\gamma_2 = \kappa_v \min\{D, v_{k'}\}$.

We summarize our results in Table 5.1.5. It is straightforward to derive the results in Proposition 2.5 from Table 5.1.5.

□

Proof. of Proposition 2.6. The defender's optimization problem is:

$$\min \quad q_k \sum_{i=k}^{k'} v_i \left(1 - \frac{D_i}{\lambda u_{i_1}}\right) I_{i=i^*} + (1 - q_k) \sum_{i=k}^{k'} v_i \left(1 - \frac{D_i}{\lambda u_{i_2}}\right) J_{i=i^*} \tag{5.1.21}$$

$$\text{s.t.} \quad D_k + D_{k'} \leq D \tag{5.1.22}$$

$$D_i \leq v_i,\ i = k, k' \tag{5.1.23}$$

$$D_i \geq 0,\ i = k, k' \tag{5.1.24}$$

where $I_{i=i^*}$ and $J_{i=i^*}$ are the binary indicators for the s-type attacker whose preferred target is k and k', respectively, where $i^* = \underset{k,k'}{\arg\max} \left\{ u_{i_j} P(D_i, A_i^{s_j}) - A_i^{s_j} \right\}, j = 1, 2$. Similar to our earlier approach, we can remove binary indicators by introducing variables z_1 and z_2 that bound the maximum payoff for the s-type attacker:

$$\begin{aligned}
\min \quad & q_k z_1 + (1-q_k) z_2 && (5.1.25)\\
\text{s.t.} \quad & v_i\left(1 - \frac{D_i}{\lambda u_{i_1}}\right) \le z_1,\ i = k, k' && (5.1.26)\\
& v_i\left(1 - \frac{D_i}{\lambda u_{i_2}}\right) \le z_2,\ i = k, k' && (5.1.27)\\
& D_k + D_{k'} \le D && (5.1.28)\\
& D_i \le v_i,\ i = k, k' && (5.1.29)\\
& D_i \ge 0,\ i = k, k' && (5.1.30)
\end{aligned}$$

Note that the objective function of the above problem is a knapsack problem in terms of z_1 and z_2. If $q_k \ge 0.5$ then the defender would minimize z_1 as much as possible. Since z_1 corresponds to the s-type attacker whose preferred target is k, decreasing z_1 corresponds to the defender's defending target k as per constraint 5.1.26 for $i = k$. By assigning defense to target k, i.e., increasing D_k, both z_1 and z_2 decrease according to constraints 5.1.26 and 5.1.27. However, the rate of decrease in z_2 is greater than that of z_1 because $\mid \frac{\partial z_2}{\partial D_k} \mid \ge \mid \frac{\partial z_1}{\partial D_k} \mid$. While $D_{k'} = 0$, the defender can increase D_k until one of the constraints 5.1.27 (for $i = k'$) or 5.1.28 becomes binding. Specifically, if 5.1.28 becomes binding first, the optimal solution is $D_k = D$ and $D_{k'} = 0$, otherwise if constraint 5.1.27 becomes binding, then the solution would yield $z_2 = v_{k'}$. In the latter case, we can obtain the value of D_k by solving the constraint 5.1.27 for D_k when $i = k$: $D_k = \frac{\lambda v_{k'}(v_k - v_{k'})}{v_k}$. Since constraint 5.1.28 is not binding, we still have some leftover budget. Then, we can continue optimizing objective function by decreasing z_1 until either constraint 5.1.28 or 5.1.26 (for $i = k'$) becomes binding. In the first case, the optimal solution is $D_k = D$ and $D_{k'} = 0$. However, in the latter case, the optimal solution would yield $D_k = \lambda(v_k - v_{k'})$. By comparing these two cases, we can show that the constraint 5.1.28 cannot be binding when $D \ge \lambda(v_k - v_{k'})$. This in turn implies that the defender can be better off by decreasing z_2, i.e., defending target k'. To summarize, we can conclude that target k' is defended when D is sufficiently large, i.e., $D \ge \lambda(v_k - v_{k'})$.

Now assume that $q_k < 0.5$. The defender would then minimize z_2 as much as possible. Note that, from constraints 5.1.27, the rate of decrease in z_2 is higher when the defender starts defending target k than when she defends target k', i.e., $\mid \frac{\partial z_2}{\partial D_k} \mid \ge \mid \frac{\partial z_2}{\partial D_{k'}} \mid$. By allocating budget to target k, both z_1 and z_2 decrease according to constraints 5.1.26 and 5.1.27 (for $i = k$). Because the rate of decrease in z_2 is greater than that of z_1, the defender can increase D_k until one of the following three conditions happens: (i) constraint 5.1.28 becomes binding; (ii) the objective function can be improved by decreasing z_1, which happens when $z_2 = \frac{q_k}{1-q_k} z_1$ or $D_k =$

$\frac{\lambda(1-2q_k)v_k v_{k'}}{(1-q_k)v_k - q_k v_{k'}}$; (iii) the objective function can be improved by decreasing z_2 by defending target k'. Assuming that constraint 5.1.28 is not binding, condition (iii) is satisfied before the condition (ii) when $q_k \leq \frac{v_k v_{k'}}{v_k^2 + v_{k'}^2}$, which is satisfied when constraint 5.1.26 (for $i = k'$) becomes binding before $z_2 = \frac{q_k}{1-q_k} z_1$ holds. So, (i) is not satisfied when $D \geq \frac{\lambda v_{k'}(v_k - v_{k'})}{v_k}$. If $\frac{v_k v_{k'}}{v_k^2 + v_{k'}^2} < q_k \leq 0.5$, then condition (ii) holds before condition (iii). Constraint 5.1.28 is then not the binding constraint when $D \geq \frac{\lambda(1-2q_k)v_k v_{k'}}{(1-q_k)v_k - q_k v_{k'}}$. In this case, the defender keeps decreasing z_1 by increasing defense on target k until one of constraints either 5.1.27 (for $i = k'$) or 5.1.28 becomes binding. Similar to the above discussion where $q_k \geq 0.5$, we can show that target k' is defended when $D \geq \lambda(v_k - v_{k'})$. □

Proof. of Proposition 2.7. From Model II, when defender is fully uninformed about both the attacker's type and the n-type attacker's preferred target, after removing the fixed terms from objective function, the defender's optimization problem can be written as follows:

$$\min \quad p_s z - (1-p_s)\sum_{i=1}^{N} \frac{q_i v_i D_i}{\lambda w} \tag{5.1.31}$$

$$\text{s.t.} \quad v_i\left(1 - \frac{D_i}{\lambda v_i}\right) \leq z,\ i = 1, ..., N \tag{5.1.32}$$

$$\sum_{i=1}^{N} D_i \leq D \tag{5.1.33}$$

$$D_i \leq v_i,\ i = 1, ..., N \tag{5.1.34}$$

$$D_i \geq 0,\ i = 1, ..., N \tag{5.1.35}$$

If I_D shows the set of targets for which constraint 5.1.32 is binding, then $D_i = \lambda(v_i - z),\ \forall i \in I_D$. Let I_{ND} shows the remaining targets. The defender's optimization problem can be then reduced to:

$$\min \quad \left(p_s + (1-p_s)\sum_{i\in I_D} \frac{q_i v_i}{w}\right) z - (1-p_s)\left(\sum_{i\in I_D} \frac{q_i v_i^2}{w} + \sum_{i\in I_{ND}} \frac{q_i v_i D_i}{\lambda w}\right) \tag{5.1.36}$$

$$\text{s.t.} \quad v_i\left(1 - \frac{D_i}{\lambda v_i}\right) \leq z,\ i \in I_{ND} \tag{5.1.37}$$

$$\sum_{i\in I_D} \lambda(v_i - z) + \sum_{i\in I_{ND}} D_i \leq D \tag{5.1.38}$$

$$D_i \leq v_i,\ i \in I_{ND} \tag{5.1.39}$$

$$D_i \geq 0,\ i \in I_{ND} \tag{5.1.40}$$

It is clear that 5.1.36 is a knapsack problem with respect to D_i, $i \in I_{ND}$ with minimization objective function. Since the coefficients in constraint 5.1.38 are the same for all $i \in I_{ND}$, the optimal solution is to allocate the remaining defensive resources, i.e., $D - \sum_{i \in I_D} \lambda(v_i - z)$, to target j_1 as much as possible, where $j_1 = \arg\max_{i \in I_{ND}} \{q_i v_i\}$, until one of constraints 5.1.38 or 5.1.39 becomes binding. If 5.1.39 first turns into a binding constraint, then, regarding the optimal solution of knapsack problem, the remaining defensive budget, i.e., $D - \sum_{i \in I_D} \lambda(v_i - z) - v_{j_1}$, should be allocated to target j_2, where $j_1 = \arg\max_{i \in \{I_{ND} - j_1\}} \{q_i v_i\}$. We continue this approach until either constraint 5.1.38 becomes binding, i.e., D is entirely allocated, or all targets $i \in I_{ND}$ are defended. Let us define I_{ND_1} as the targets for which the relevant constraint 5.1.39 becomes binding, i.e., $D_i = v_i$, $i \in I_{ND_1}$. After some simplifications, it is easy to verify that the defender's optimization problem can be reduced to:

$$\min \quad \left(p_s - (1 - p_s)\frac{q_k v_k |I_D|}{w}\right) z + (1 - p_s) \sum_{i \in I_D} \frac{q_i v_i (z - v_i)}{w} + (1 - p_s) \frac{q_k v_k \sum_{i \in I_D} v_i}{w} + \mathscr{F}_{I_{ND_1}} \tag{5.1.41}$$

$$\text{s.t.} \quad v_i \leq z, \; i \in I_{ND} \tag{5.1.42}$$

$$\lambda \sum_{i \in I_D} v_i + \sum_{i \in I_{ND_1}} v_i - \lambda \mid I_D \mid z \leq D \tag{5.1.43}$$

where $\mathscr{F}_{I_{ND_1}}$ is a function of $\sum_{i \in I_{ND_1}} v_i$. Since the targets are ordered with respect to their valuations, we can reduce all constraints in 5.1.42 to only one constraint; $\max_{i \in I_{ND}} \{v_i\} \leq z$. Note that the first part of the objective function is decreasing function in I_D, the second part is always negative (because $z < v_i$, $i \in I_D$), and the third part is increasing function in I_D. Together, this implies that, we have to keep adding the targets with respect to their valuations to I_D and stop as soon as either no improvement occurs in the objective value or constraint 5.1.43 becomes binding. We summarize our proposed approach in Algorithm 3 to characterize the defender's equilibrium.

Algorithm 3 (Defense equilibrium under full asymmetric information):

1. Order the targets with respect to $q_i v_i$, such that j_1 indicates the target with the highest $q_i v_i$ and j_N shows the target with the lowest $q_i v_i$. Set $v_{j_0} = 0$ and $k = 1$. Moreover, partition all targets to two disjoint subsets $I_1 = \{1, 2, ..., N\}$ and $I_2 = \emptyset$, where targets in I_1 are prioritized in decreasing order with respect to their valuations.
2. Pick target j_k from set I_1 and put it into set I_2. Reorder all the remaining targets in I_1 with respect to their valuations, such that 1 indicates the target with the highest valuation and $N - k$ shows the target with the lowest valuation. Set $j = 1$.
3. Let $b_1^j = p_s + (1 - p_s)\left(\sum_{i=1}^{j} \frac{q_i v_i}{w} - (j)\frac{q_{j_k} v_{j_k}}{w}\right)$, $b_2^j = \frac{\lambda\left(\sum_{i=1}^{j} v_i + v_{j_k}\right) + \sum_{t=0}^{k-1} v_{j_t} - D}{\lambda(j+1)}$.
4. If $b_1^j \geq 0$, and $b_2^j < v_{j+1}$, then check whether j is the least valuable target in set I_1 or not. If $j < N - k$, i.e., j is not the least valuable target, then let $j = j + 1$ and return to 3, however, if j is the least valuable target, i.e., $j = N - k$, then $\mathscr{B} = 0$ and go to 7.
5. If $b_1^j < 0$, then $\mathscr{B} = v_j$ and go to 7.
6. If $b_2^j \geq v_{j+1}$, then $\mathscr{B} = \frac{\lambda\left(\sum_{i=1}^{j} v_i + v_{j_k}\right) + \sum_{t=0}^{k-1} v_{j_t} - D}{\lambda(j+1)}$ and go to 7.
7. If $D - \sum_{i=1}^{j} \lambda(v_i - \mathscr{B}) - \sum_{t=0}^{k-1} v_{j_t} > v_{j_k}$, then let $k = k + 1$ and return to 2. Otherwise, go to 8.
8. Optimal defense allocation is as follows: set $D_i = \lambda(v_i - \mathscr{B})$, $i \leq j$, $D_i = 0$, $i > j$, $D_i = v_i$, $i \in I_2$, and $D_{j_k} = D - \sum_{i=1}^{j} \lambda(v_i - \mathscr{B}) - \sum_{t=0}^{k-1} v_{j_t}$.

By iterating the above algorithm for all targets, we can show that under fully incomplete information, only a combination of the most valuable targets and the nonstrategic attacker's most preferred targets will receive the defensive resources. In particular, the nonstrategic attacker's most preferred targets are fully defended, i.e., $D_i = v_i$, $i \in I_2$, and among the remainders, only a set of most valuable targets are defended. Specifically, if k^* indicates the smallest k for which one goes from step 7 to step 8, it is easy to show that target $i \notin I_2$ is defended if and only if $t_1^i \geq v_{k^*}$, and $v_i > t_2^i$, where: $t_1^i = \frac{p_s w}{i(1-p_s)q_{k^*}} + \sum_{j=1}^{i} \frac{q_j v_j}{i q_{k^*}}$, and $t_2^i = \frac{\lambda(\sum_{j=1, j \neq k^*}^{i} v_j + v_{k^*}) + \sum_{j \in F} v_j - D}{\lambda(i+1)}$. □

5.2 Proofs of Propositions and Lemmas for Chapter 3

Proof. of Proposition 3.1. From the manufacturer's perspective, exerting process improvement effort by θ-type supplier is profitable when expected profit if he exerts effort, i.e., $e_\theta^{fb} = 1$, is more than that if he does not, i.e., $e_\theta^{fb} = 0$. Therefore, we need to solve the manufacturer's problem for two possible actions, i.e., $e_\theta^{fb} = 0, 1$, and then compare the manufacturer's profit under each scenario to find the optimal supplier's action and contract terms. Assume that the manufacturer wants to induce $e_\theta^{fb} = 1$ on the θ-type supplier. Note that, under symmetric information, manufacturer pays no rent to the supplier, which means that the supplier's participation constraint should be binding. Therefore, any combination of ω_θ, Y_θ and κ_θ that satisfies $\rho_\theta Y_\theta - (1-\rho_\theta)\kappa_\theta + (\omega_\theta - c - \psi_\theta) = 0$ is an optimal contract. For simplicity, we consider $Y_\theta = 0$, $\kappa_\theta = 0$ and $\omega_\theta = c + \psi_\theta$. The manufacturer's profit is then $\rho_\theta r - c - \psi_\theta$. Similarly, if the manufacturer induces zero effort on the θ-type supplier, the optimal contract would be $Y_\theta = 0$, $\kappa_\theta = 0$ and $\omega_\theta = c$, and manufacturer's profit is $\varphi_\theta r - c$. Therefore, exerting process improvement effort by the supplier is profitable from manufacturer's perspective iff $\rho_\theta r - \psi_\theta \geq \varphi_\theta r$ or equivalently $\delta r \geq \psi_\theta$. Finally, because the cost of exerting effort is type-contingent, there are four different regions according to Table 3.4.1. □

Proof. of Lemma 3.1. Given the contract $(\omega_\theta, Y_\theta, \kappa_\theta)$ offered by the manufacturer, the θ-type supplier exerts improvement effort iff $\pi_S^\theta(\omega_\theta, Y_\theta, \kappa_\theta \mid e_\theta = 1) \geq \pi_S^\theta(\omega_\theta, Y_\theta, \kappa_\theta \mid e_\theta = 0)$, or equivalently $Y_\theta + \kappa_\theta \geq \frac{\psi_\theta}{\delta}$. □

Proof. of Proposition 3.2. From manufacturer's perspective, there are four possible action profiles to induce on the supplier: inducing improvement effort on both types (i.e., $e_h = e_l = 1$); inducing improvement effort only on the more reliable supplier (i.e., $e_h = 1, e_l = 0$); inducing improvement effort only on the less reliable supplier (i.e., $e_h = 0, e_l = 1$); and, inducing no effort on both types (i.e., $e_l = e_h = 0$). Below, we first find the manufacturer's profit under all action profiles and then compare them to find the optimal one, as well as the optimal menu of contracts. Note that there are two ways to compensate the cost of process improvement to the supplier; either directly through subsidy payment ω, or indirectly through contingent payment and penalty clause. It is easy to verify that there are multiple solutions at optimality and the optimal contract can be obtained through either way. However, to use the whole contract space, which further enables us to simplify the exposition, we let the manufacturer compensate the cost of process improvement through subsidy whenever she wants to induce $e_\theta = 1$. Finally, let $T_{e_\theta}^\theta$ show the total channel profit when the θ-type supplier chooses action e_θ. From mechanism design, the manufacturer can design the optimal contract so that only the h-type supplier receives information rent at optimality, i.e., the l-type supplier earns zero; $\pi_S^l(\omega_l, Y_l, \kappa_l \mid e_l^*) = 0$. Moreover, the information rent payable to the h-type supplier is what he receives if he mimics the l-type supplier and chooses whatever action $\tilde{e}_h$ that maximizes his profit. Therefore, we can rewrite the manufacturer's profit function as $\nu T_{e_h}^h + (1-\nu) T_{e_l}^l - \nu \pi_S^h(\omega_l, Y_l, \kappa_l \mid \tilde{e}_h)$, where $\pi_S^h(\omega_l, Y_l, \kappa_l \mid \tilde{e}_h)$ indicates the information rent which comes from the h-type supplier's ability to mimic the l-type and to choose action $\tilde{e}_h$ under the deviated contract. Note also that writing h-type supplier's profit function in terms of the contract terms for l-type supplier helps us to suppress the dependence of manufacturer's objective function on the contract terms for h-type supplier, which further gives a tractable way to solve the manufacturer's contract design problem.

- Inducing improvement effort on both types ($e_h = e_l = 1$): The manufacturer subsidizes the production and improvement costs through subsidy; $\omega_\theta = c + \psi_\theta, \theta \in \{h, l\}$. To incentivize the θ-type supplier to exert effort, from Lemma 3.1, we should satisfy $Y_\theta + \kappa_\theta \geq \frac{\psi_\theta}{\delta}$. Moreover, the right-hand side of incentive compatibility constraint (3.5.4) depends on the level of effort the θ-type supplier chooses under deviated contract in constraint (3.5.5). Specifically, if the θ-type supplier picks the contract for the $\check{\theta}$-type, where $\theta \neq \check{\theta}$, then $\tilde{e}_\theta = 1$ iff $Y_{\check{\theta}} + \kappa_{\check{\theta}} \geq \frac{\psi_\theta}{\delta}$, otherwise, $\tilde{e}_\theta = 0$. It readily means that when $\psi_h > \psi_l$, then satisfying constraint (3.5.3) for the h-type supplier leads to $\tilde{e}_l = 1$, and, if $\psi_h \leq \psi_l$, then satisfying constraint (3.5.3) for the l-type supplier leads to $\tilde{e}_h = 1$. First, assume that $\psi_h > \psi_l$. The manufacturer can induce $\tilde{e}_h = 1$ by satisfying $Y_l + \kappa_l \geq \frac{\psi_h}{\delta}$, or $\tilde{e}_h = 0$ by satisfying $Y_l + \kappa_l < \frac{\psi_h}{\delta}$; each leads to different amount of information rent. As we show below, $Y_l + \kappa_l < \frac{\psi_h}{\delta}$, corresponding to $\tilde{e}_h = 0$, leads to lower information rent. The manufacturer's optimization problem can be then rewritten as follows:

$$\max_{\omega_l, Y_l, \kappa_l} \nu T_1^h + (1-\nu) T_1^l - \nu \left[\varphi_h Y_l - (1-\varphi_h)\kappa_l + \psi_l\right] \tag{5.2.1}$$

$$\text{s.t. } \pi_S^l \left(\omega_l, Y_l, \kappa_l \mid e_l = 1\right) = 0 \tag{5.2.2}$$

$$Y_l + \kappa_l \geq \frac{\psi_l}{\delta} \tag{5.2.3}$$

$$Y_l + \kappa_l \leq \frac{\psi_h}{\delta} \tag{5.2.4}$$

Note that, from mechanism design theory, the l-type supplier's participation constraint (5.2.2) is binding at optimality. It is easy to verify that the optimal contract for the l-type supplier depends whether $\varphi_h \geq \rho_l$ or not. Let us consider $\varphi_h \geq \rho_l$. The other case leads to the same theoretical results and are available from the authors upon request. The optimal contract is then the intersection of constraints (5.2.2) and (5.2.3), which gives $\left(Y_l^*, \kappa_l^*\right) = \left(\frac{1-\rho_l}{\delta}\psi_l, \frac{\rho_l}{\delta}\psi_l\right)$. By plugging the optimal contract in the manufacturer's optimization problem, we can extract the optimal contract for the h-type supplier by solving the following problem:

$$\max_{\omega_h, Y_h, \kappa_h} \nu T_1^h + (1-\nu) T_1^l - \nu \pi_S^h \left(\omega_h, Y_h, \kappa_h \mid e_h = 1\right) \tag{5.2.5}$$

$$\text{s.t. } \pi_S^h \left(\omega_h, Y_h, \kappa_h \mid e_h = 1\right) \geq 0 \tag{5.2.6}$$

$$\pi_S^h \left(\omega_h, Y_h, \kappa_h \mid e_h = 1\right) \geq \frac{\varphi_h - \rho_l}{\delta}\psi_l + \psi_l \tag{5.2.7}$$

$$\pi_S^l \left(\omega_h, Y_h, \kappa_h \mid \tilde{e}_l = 1\right) \leq 0 \tag{5.2.8}$$

$$Y_h + \kappa_h \geq \frac{\psi_h}{\delta} \tag{5.2.9}$$

Using graphical approach, it is easy to verify that the optimal contract for the h-type supplier is $Y_h = \frac{1-\rho_h}{\delta}\psi_h + \frac{\rho_h-\rho_l}{\delta}\psi_l$; $\kappa_h = \frac{\rho_h}{\delta}\psi_h - \frac{\rho_h-\rho_l}{\delta}\psi_l$, and the h-type supplier earns the information rent $\psi_l + (\varphi_h - \rho_l)\frac{\psi_l}{\delta}$. Note that if the manufacturer induces $\tilde{e}_h = 1$ on the h-type supplier under deviated contract, the information rent will be $\psi_l + (\varphi_h - \rho_l)\frac{\psi_h}{\delta}$, which is greater than that when $\tilde{e}_h = 0$ under the assumption of $\psi_h > \psi_l$. Now, assume that $\psi_h \leq \psi_l$. Under this condition, satisfying $Y_l + \kappa_l \geq \frac{\psi_l}{\delta}$ for the l-type supplier leads to $\tilde{e}_h = 1$. Therefore, we have:

$$\max_{\omega_l, Y_l, \kappa_l} \nu T_1^h + (1-\nu)T_1^l - \nu\left[\rho_h Y_l - (1-\rho_h)\kappa_l + \psi_l - \psi_h\right] \tag{5.2.10}$$

$$\text{s.t. } \pi_S^l\left(\omega_l, Y_l, \kappa_l \mid e_l = 1\right) = 0 \tag{5.2.11}$$

$$Y_l + \kappa_l \geq \frac{\psi_l}{\delta} \tag{5.2.12}$$

The optimal solution for the above optimization problem is similar to the previous case, which is $\left(Y_l^*, \kappa_l^*\right) = \left(\frac{1-\rho_l}{\delta}\psi_l, \frac{\rho_l}{\delta}\psi_l\right)$. By plugging the optimal contract in the manufacturer's optimization problem, we can find the optimal contract for the h-type supplier by solving the following problem:

$$\max_{\omega_h, Y_h, \kappa_h} \nu T_1^h + (1-\nu)T_1^l - \nu\pi_S^h\left(\omega_h, Y_h, \kappa_h \mid e_h = 1\right) \tag{5.2.13}$$

$$\text{s.t. } \pi_S^h\left(\omega_h, Y_h, \kappa_h \mid e_h = 1\right) \geq 0 \tag{5.2.14}$$

$$\pi_S^h\left(\omega_h, Y_h, \kappa_h \mid e_h = 1\right) \geq \psi_l - \psi_h + \frac{\rho_h - \rho_l}{\delta}\psi_l \tag{5.2.15}$$

$$\pi_S^l\left(\omega_h, Y_h, \kappa_h \mid \tilde{e}_l = 1\right) \leq 0 \tag{5.2.16}$$

$$Y_h + \kappa_h \geq \frac{\psi_l}{\delta} \tag{5.2.17}$$

It is easy to verify that the optimal contract for the h-type supplier is the intersections of constraints (5.2.15) and (5.2.17), which is $Y_h^* = \frac{1-\varphi_l}{\delta}\psi_l - \psi_h$; $\kappa_h^* = \frac{\varphi_l}{\delta}\psi_l + \psi_h$, and the h-type supplier earns the information rent $\psi_l - \psi_h + \frac{\rho_h-\rho_l}{\delta}\psi_l$.

- Inducing improvement effort only on the more reliable supplier ($e_h = 1, e_l = 0$): For the sake of brevity, and due to similarity, we omit the proof and only provide the results. To satisfy the l-type supplier's participation constraint, the manufacturer only compensates the production cost through upfront payment, hence $\omega_l^* = c$; $Y_l^* = 0$; and $\kappa_l^* = 0$. However, for the h-type supplier, the manufacturer subsidizes the production and process improvement via $\omega_h^* = c + \psi_h$. Note that the h-type supplier has no incentives to mimic the l-type, otherwise he cannot receive the subsidy on process improvement. It readily means that information rent is zero. Now, the manufacturer should design Y_h and κ_h to avoid the l-type supplier to not mimic the h-type. By plugging the l-type optimal contract and solving the manufacturer's optimization problem, we have $Y_h^* = \frac{1-\rho_h}{\delta}\psi_h$; and $\kappa_h^* = \frac{\rho_h}{\delta}\psi_h$.

- Inducing improvement effort only on the less reliable supplier ($e_h = 0, e_l = 1$): The manufacturer subsidizes the production cost for the h-type (i.e., $\omega_h = c$), but both production and improvement cost for the l-type (i.e., $\omega_l = c + \psi_l$). Now, to incentivize the l-type supplier to exert improvement effort, contract should satisfy $Y_l + \kappa_l \geq \frac{\psi_l}{\delta}$ according to Lemma 3.1. Furthermore, if the h-type supplier mimics the l-type he can benefit from subsidy and not exert process improvement, i.e., $\tilde{e}_h = 0$. To incentivize him to exert improvement effort under deviated contract ($\tilde{e}_h = 1$), the contract for the l-type should also satisfy $Y_l + \kappa_l \geq \frac{\psi_h}{\delta}$. But, it is easy to verify that it leads to greater information rent. Therefore, the manufacturer is better off by letting the h-type not to exert effort under deviated contract. By satisfying the binding condition for participation constraint and considering the moral hazard constraint for the l-type supplier, we can find the optimal contract for the l-type, which is $Y_l^* = \frac{1-\rho_l}{\delta}\psi_l$ and $\kappa_l^* = \frac{\rho_l}{\delta}\psi_l$. Finally, by solving the manufacturer's optimization problem, we can extract the optimal contract terms for the h-type supplier, which is $Y_h^* = \frac{1-\varphi_l}{\delta}\psi_l$; and $\kappa_h^* = \frac{\varphi_l}{\delta}\psi_l$.
- Inducing no effort on both types ($e_l = e_h = 0$): The manufacturer needs to compensate only the production cost to both types, hence $\omega_\theta = c$; $Y_\theta = 0$; $\kappa_\theta = 0$, where $\theta \in \{h, l\}$.

So far, we obtained the optimal contract for different action profiles as well as the corresponding information rent. Now, by comparing the manufacturer's profit for different action profiles we can characterize the equilibrium. This comparison results in the full characterization presented in the bottom-left panel in Table 3.5.1. Finally, by comparing induced action profile in different regions of Table 3.5.1 to that in Proposition 3.1, we can find channel loss wherever they are different. □

Proof. of Proposition 3.3. Similar to our approach in Proposition 3.2, we need to consider four possible action profiles to induce on the supplier. We then compare the manufacturer's profit under each of action profiles to find the optimal one.

- $e_h = e_l = 1$: The manufacturer compensates the cost of process improvement through direct subsidy, hence $\omega_\theta = c + \psi_\theta$. Furthermore, it is enough to set $Y_l^* = 0$ and $\kappa_l^* = 0$ to satisfy the binding condition of the l-type supplier's participation constraint. Furthermore, when the manufacturer induces effort on both types the supplier cannot deviate by not exerting effort, otherwise the manufacturer simply can detect the deviation. It readily means that $\tilde{e}_h = 1$, which leads us to rewrite the h-type supplier's incentive compatibility constraint as follows:

$$\pi_S^h(\omega_h, Y_h, \kappa_h \mid e_h = 1) \geq \psi_l - \psi_h \tag{5.2.18}$$

Therefore, the h-type supplier has incentive to mimic the l-type only if $\psi_h \leq \psi_l$. The optimal contract for the h-type is then any combination of Y_h and κ_h that satisfy constraint (5.2.18). Specifically, we consider $Y_h^* = \frac{\psi_l - \psi_h}{\rho_h}$ and $\kappa_h^* = 0$. However, when $\psi_h > \psi_l$ then the h-type supplier is worse off by mimicking the l-type. Therefore, constraint (5.2.18) is not an effective constraint, however, the contract should be designed such that it avoids the l-type supplier from mimicking the h-type. The optimal contract is then $Y_h^* = \frac{1-\rho_h}{\rho_h - \rho_l}(\psi_h - \psi_l)$ and $\kappa_h^* = \frac{\rho_h}{\rho_h - \rho_l}(\psi_h - \psi_l)$. Therefore, the h-type earns the information rent only when $\psi_h \leq \psi_l$; the amount of that is the difference between the cost of improvement between types, which is $\psi_l - \psi_h$.

- $e_h = 1, e_l = 0$: Note that inducing process improvement effort only on the h-type supplier is optimal if $\psi_l > \psi_h$. The manufacturer subsidizes the cost of process improvement through subsidy only to the h-type supplier, hence $\omega_h = c + \psi_h$ and $\omega_l = c$. We can simply satisfy the binding condition of the l-type supplier's participation constraint the by setting $Y_l^* = 0$ and $\kappa_l^* = 0$. Therefore, the h-type supplier gets nothing but the compensation of the production cost if he mimics the l-type. It readily means that the h-type supplier's participation and incentive compatibility constraints are the same at optimality, hence the information rent is zero. Furthermore, if the l-type supplier mimics the h-type, then he has to exert effort, i.e., $\tilde{e}_l = 1$, otherwise the manufacturer catches him. But, the subsidy payment under deviated contract, which is ψ_h, is less than the cost of effort for the l-type; meaning that the l-type never mimics the h-type. The manufacturer can simply sets $Y_h^* = 0$ and $\kappa_h^* = 0$.

- $e_h = 0, e_l = 1$: To induce the first-best level of effort, the manufacturer may induce effort only on the l-type supplier if $\psi_h > \psi_l$. The manufacturer subsidizes the cost of process improvement through subsidy only to the h-type supplier, hence $\omega_h = c$ and $\omega_l = c + \psi_l$. With the same rational similar to the case when ($e_h = 1, e_l = 0$), if the h-type mimics the l-type, he has to no way but exerting effort (i.e., $\tilde{e}_h = 1$), which costs ψ_h. But the subsidy under deviated contract, which is ψ_l, is less than the cost of effort; meaning that the h-type never mimics the l-type, hence the information rent is zero. The manufacturer simply sets $Y_h^* = Y_l^* = 0$ and $\kappa_h^* = \kappa_l^* = 0$.

- $e_l = e_h = 0$: The manufacturer only compensates the production cost to both types, hence $\omega_\theta = c$;

Table 5.2.1: The comparison of information rent and channel loss: IE vs. AE

Region	IE		AE	
	Information rent	Channel loss	Information rent	Channel loss
1	$(\psi_l - \psi_h) + \frac{\rho_h - \rho_l}{\delta} \psi_l$	0	$\psi_l - \psi_h$	0
2	$\psi_l + \frac{\varphi_h - \rho_l}{\delta} \psi_l$	0	0	0
3	$\psi_l + \frac{\varphi_h - \rho_l}{\delta} \psi_l$	0	0	0
4	0	$(1-\nu)\left[\delta r - \psi_l\right]$	$\psi_l - \psi_h$	0
5	0	$\delta r - \left[\nu \psi_h + (1-\nu)\psi_l\right]$	0	0
6	0	$(1-\nu)\left[\delta r - \psi_l\right]$	0	0
7	0	$(1-\nu)\left[\delta r - \psi_l\right]$	0	$(1-\nu)\left[\delta r - \psi_l\right]$
8	0	0	0	0

$\Upsilon_\theta = 0$; $\kappa_\theta = 0$, where $\theta \in \{h, l\}$.

The comparison between the manufacturer's profit for different action profiles results in the full characterization presented in the bottom-left panel in Table 3.5.2. Finally, by comparing induced action profile in different regions of Table 3.5.2 to that in Proposition 3.1, we can find channel loss wherever they are different. □

Proof. of Proposition 3.4. We need to compare the total inefficiency in corresponding regions under IE and AE contracts. From the h-type supplier's perspective, the VOA is the difference between information rent, whereas, from overall supply chain perspective, the VOA can be obtained by comparing channel loss in two contracts. By comparing corresponding regions, one can distinguish nine different sub-regions (see Table 5.2.1). It is straightforward to conclude the results in Proposition 3.4 from Table 5.2.1. Specifically, Region "A_1" in Proposition 3.4 corresponds to Regions 1, 2, and 3 where audit helps the manufacturer to completely remove (Regions 2 and 3) or at least decrease the information rent (Region 1); Region "A_2" corresponds to Region 4 where audit removes channel loss at the expense of information rent; Region "A_3" corresponds to Regions 5 and 6 where audit totally removes channel inefficiency; and finally, Region "I" corresponds to Regions 7 and 8 where audit brings no value for the manufacturer and total supply chain.

□

Proof. of Proposition 3.5. The results in Table 3.6.2 can be extracted from the closed-form statements in Table 3.6.1 in Proposition 3.4. □

Proof. of Proposition 3.6. First, let $\rho_h = \rho_l$. From the characterization of VOA in Proposition 3.4, the VOA becomes zero in Region A_1. Moreover, if one plugs $\rho_h = \rho_l$ into the characterizations of Ψ_{IE_1} and Ψ_{IE_2} in Proposition 3.2, it is then straightforward to verify that Ψ_{IE_1} under IE matches with Ψ_{AE} under AE, and $\Psi_{IE_2} =$

δr, which means that under the assumption of $\rho_h = \rho_l$ Regions A_2 and A_3 will be vanished. $\square$

Probabilistic Audit Scenario

In the main body of the chapter, we assume that if the manufacturer takes AE contract then audit takes place with probability 1. In this section, we study the case where the manufacturer randomizes between audit and no-audit options. We show that considering probabilistic audit policy in our setting always results in one of the extreme scenarios; IE or AE contract.

Assume that the manufacturer incurs an auditing cost $\mathscr{A}$ with probability $z \in [0,1]$. Similar to the original setting, we assume that the audit provides the manufacturer with the perfect information of the supplier's action e_θ, i.e., if she decides to audit, she can verify the actual action decided by the supplier with probability 1. Since there is a probability that the manufacturer may skip auditing her supplier (with probability $1-z$), the supplier may get caught by the manufacturer if he exerts an effort that does not comply with the effort e^*_θ that comes with the contract $(\omega^*_\theta, Y^*_\theta, \kappa^*_\theta)$. If such thing happens, we assume that the supplier has to pay a penalty A^p_θ to the manufacturer. Under such a setting, the manufacturer's objective is as follows:

$$\max_{(\omega_h, Y_h, \kappa_h),(\omega_l, Y_l, \kappa_l)} \nu \pi^h_M(\omega_h, Y_h, \kappa_h \mid e^*_h) + (1-\nu)\pi^l_M(\omega_l, Y_l, \kappa_l \mid e^*_l) - z\mathscr{A} \tag{5.2.19}$$

Next, we need to modify the incentive compatibility (IC) constraints as follows. Recall that when the supplier of type θ decides to deviate and choose a contract designed for $\check{\theta} \neq \theta$, then he has two options: (i) if he chooses the effort $e^*_{\check{\theta}}$ that is associated with the contract designed for $\check{\theta}$-type supplier, then he does not need to worry about getting caught. In this case, his payoff will be $\pi^\theta_S(\omega_{\check{\theta}}, Y_{\check{\theta}}, \kappa_{\check{\theta}} \mid e^*_{\check{\theta}})$; (ii) if he chooses the effort $e_\theta \neq e^*_{\check{\theta}}$, then he gets away with his decision with probability $1-z$ and gets caught and pays the penalty with probability of z. In this case, his payoff will be equal to $\pi^\theta_S(\omega_{\check{\theta}}, Y_{\check{\theta}}, \kappa_{\check{\theta}} \mid e_\theta) - zA^p_{\check{\theta}}$. To summarize, if he chooses the contract $(\omega_\theta, Y_\theta, \kappa_\theta)$ and exerts e_θ, his incentive compatibility constraint should be satisfied:

$$\pi^\theta_S(\omega_\theta, Y_\theta, \kappa_\theta \mid e_\theta) \geq \max\left(\pi^\theta_S(\omega_{\check{\theta}}, Y_{\check{\theta}}, \kappa_{\check{\theta}} \mid e^*_{\check{\theta}}), \pi^\theta_S(\omega_{\check{\theta}}, Y_{\check{\theta}}, \kappa_{\check{\theta}} \mid e_\theta \neq e^*_{\check{\theta}}) - zA^p_{\check{\theta}}\right) \tag{5.2.20}$$

Furthermore, in order to incentivize the θ-type supplier to take the optimal action in equilibrium, the following moral hazard constraint should be satisfied:

$$\pi_S^\theta(\omega_\theta, Y_\theta, \kappa_\theta \mid e_\theta^*) \geq (\omega_\theta, Y_\theta, \kappa_\theta \mid e_\theta \neq e_\theta^*) - zA_\theta^p \tag{5.2.21}$$

Finally, the IR constraints stay the same as in Equation (3.5.10). From moral hazard constraint (5.2.21), note that the θ-type supplier exerts process improvement effort if $Y_\theta + \kappa_\theta \geq \frac{\psi_\theta}{\delta} - z\frac{A_\theta^p}{\delta}$. That is to say, increasing the frequency of audit helps the manufacturer to reduce the incentive-fees. Let us obtain the information rent formula under frequent-audit scenario. Recall that information rent is payable to h-type supplier only when the manufacturer induces improvement effort on the l-type supplier. It is easy to verify that the information rent term in Eq. (3.5.9) can be rewritten as follows:

$$\text{Information rent paid to } h\text{-type} = \underbrace{\psi_l - \tilde{e}_h \psi_h}_{\text{fixed-price term}} + \underbrace{[p(h \mid \tilde{e}_h) - p(l \mid e_l^*)]\left[\frac{\psi_\theta}{\delta} - z\frac{A_\theta^p}{\delta}\right]}_{\text{incentive-fee term}} \tag{5.2.22}$$

Note that, the fixed-price term can also be affected by z in the sense that if $\tilde{e}_h = 1$ then the h-type supplier has no worry about getting caught, therefore, information rent is $(\psi_l - \psi_h) + (\rho_h - \rho_l)[\frac{\psi_l}{\delta} - z\frac{A_h^p}{\delta}]$. However, if $\tilde{e}_h = 0$, then the h-type supplier is caught with probability z and penalized by A_l^p, hence, information rent is $(\psi_l - zA_l^p) + (\varphi_h - \rho_l)[\frac{\psi_l}{\delta} - z\frac{A_h^p}{\delta}]$. Now, in order to find the optimal frequency of audit z^*, the manufacturer trades off the cost of audit $z\mathscr{A}$ with the reduction in information rent due to audit, which is $z \times [(\rho_h - \rho_l)\frac{A_h^p}{\delta}]$ when $\tilde{e}_h = 1$, and $z \times [A_l^p + (\varphi_h - \rho_l)\frac{A_h^p}{\delta}]$ when $\tilde{e}_h = 0$. Therefore, the optimal frequency of audit is always a binary solution, i.e., if $\mathscr{A}$ is less than the reduction in information rent then $z^* = 1$, otherwise, $z^* = 0$.

Product Quality Uncertainty

As discussed in Laffont and Martimort [2002], there are mixed models in the literature where similar to our model, the effects of type and effort on the contract variable are stochastic, and yet in contrast to our setting, the moral hazard does not have any actual effect on the principal's payoff. The reason behind this seemingly conflicting outcome lies on the crucial property of such models, in which the hidden action exerted by the agent does not change the information asymmetry between different agent types. To be specific, let's consider the classical mixed model of quality improvement analyzed in Chapter 7 of Laffont and Martimort [2002]. Let

q denote the quality (as opposed to quantity in our model), where $q = 1$ represents the high-quality outcome. In this model, the likelihood of high quality, i.e., $p(q = 1 \mid \theta, e)$ depends on both the agent's type and effort, however, it is assumed that $p(q = 1 \mid \theta = h, e) = p(q = 1 \mid \theta = l, e)$ for all $e \in \{0, 1\}$, i.e., in words, exerting a process improvement action changes the likelihood of quality of the product in the same fashion for all agent types. In addition, it is also assumed that the cost of process improvement effort is the same for all types, i.e., $c(e, \theta = h) = c(e, \theta = l)$ for all $e \in \{0, 1\}$. On the other hand, in our model, both the impact of process improvement on the likelihood of $q = 1$ and the cost of process improvement effort are type-dependent, i.e., $p(q = 1 \mid \theta = h, e) \neq p(q = 1 \mid \theta = l, e)$ and $c(e, \theta = h) \neq c(e, \theta = l)$ for all $e \in \{0, 1\}$. This crucial difference implies that the principal's information rent expression under IE is more than that under AE, which essentially implies that observing the agent's action has an actual effect on the principal's payoff. In order to check the robustness of this observation, we also modified the mixed model of quality improvement analyzed in Chapter 7 of Laffont and Martimort [2002] in a similar fashion (i.e., changed the likelihood function so that $p(q = 1 \mid \theta = h, e) \neq p(q = 1 \mid \theta = l, e)$ for all $e \in \{0, 1\}$ and found that moral hazard has also an actual effect on the principal's payoff function under in this modified model. Below, we provide the detailed analysis.

Let us first briefly describe the model and results in Laffont and Martimort [2002]. In their model, the agent is risk neutral and is in two types: efficient (h-type) and inefficient (l-type) whose marginal costs are $\underline{\theta}$ and $\bar{\theta}$, respectively. The agent produces the quantity q ordered by the principal. Both types can invest in an effort $e \in \{0, 1\}$ that improve the quality of the product sold by the agent. They assume that exerting effort will cost the agent a non-monetary disutility $\psi(e)$ with the normalizations $\psi(e = 0) = 0$ and $\psi(e = 1) = \psi$. Finally, they assume that, with probability π_e (resp, $1 - \pi_e$) the quality of outcome is high (resp. low) and the principal benefits $S_h(q)$ (resp. $S_l(q)$) with $S_h(q) > S_l(q)$. The principal offers a three-term contract: the payment t_θ^h if the quality of product is high; t_θ^l if the quality of product is low; and, order quantity q_θ. The utility function for the θ-type agent if he exerts effort (i.e., $e_\theta = 1$) is:

$$U_\theta = \pi_1 t_\theta^h + (1 - \pi_1)\, t_\theta^l - \theta q_\theta - \psi \tag{5.2.23}$$

Under this setting, when inducing effort to both types, they show that moral hazard is not an issue and the information rent payable to the efficient agent is $\left(\bar{\theta} - \underline{\theta}\right) q_l^{\text{sb}}$, which is the same as rent in a pure adverse selection problem.

Let us now apply our model setting to the above procurement model. Note that our model is different to

Laffont and Martimort [2002] mainly in two ways. First, we assume that probability of high quality (high outcome in our paper) is not only a function of agent's level of effort, but also a function of agent's type. In particular, in our model, with probability π_e^θ (resp, $1 - \pi_e^\theta$) the quality of outcome is high (resp. low). Second, the cost of effort is type contingent in our model; $\psi_\theta(e = 0) = 0$ and $\psi_\theta(e = 1) = \psi_\theta$. Specifically, in the revised manuscript, we consider all realizations of cost of effort: $\psi_l \geq \psi_h$ and $\psi_l < \psi_h$. The utility function for the θ-type agent if he exerts effort (i.e., $e_\theta = 1$) can be written as follows:

$$U_\theta = \pi_1^\theta t_\theta^h + \left(1 - \pi_1^\theta\right) t_\theta^l - \theta q_\theta - \psi_\theta \tag{5.2.24}$$

Assume that the manufacturer wants to induce effort on both types; i.e., $e_h = e_l = 1$. If one apply IE contract (endogenous-control mode) presented in our paper, the information rent is as follows:

$$\text{Information rent under IE} = \underbrace{(\psi_l - \psi_h) + (\bar{\theta} - \underline{\theta})\, q_l^{\text{sb}}}_{\text{IR term}} + \underbrace{\left(\pi_1^h - \pi_1^l\right) \frac{\psi_l}{\Delta\pi}}_{\text{IC term}} \tag{5.2.25}$$

In consistent to our definitions in the manuscript, "IR term" is the amount that the h-type agent benefits from the payments that satisfy the l-type agent's participation constraint, whereas "IC term" represents for the amount of incentives that needs to be offered to the l-type in order to induce him to exert improvement action. Verify that if one sticks to the original assumptions of Laffont and Martimort [2002], i.e., $\pi_1^h = \pi_1^l = \pi_1$ and $\psi_h = \psi_l = \psi$, then the information rent is exactly the same as that in pure adverse selection problem. Now, let us study the AE contract (close-control mode). Thanks to audit, enforcing the action to both types can be done in a more costless fashion, hence the "IC term" is totally removed. Moreover, the principal can satisfy the l-type agent's participation constraint in more cost-efficient fashion, which reduces the "IR term" as follow

$$\text{Information rent under AE} = \underbrace{(\psi_l - \psi_h) + (\bar{\theta} - \underline{\theta})\, q_l^{\text{sb}} - \left(\pi_1^h - \pi_1^l\right) \frac{\bar{\theta} q_l^{\text{sb}} + \psi_l}{1 - \pi_1^l}}_{\text{IR term}} \tag{5.2.26}$$

Verify that the value of audit is then $\left(\pi_1^h - \pi_1^l\right) \frac{\psi_l}{\Delta\pi} + \left(\pi_1^h - \pi_1^l\right) \frac{\bar{\theta} q_l^{\text{sb}} + \psi_l}{1 - \pi_1^l}$, which is zero under the assumption of $\pi_1^h = \pi_1^l = \pi_1$ in classical principal-agent model Laffont and Martimort [2002].

IE vs. AE in Restricted Contractual Setting

Under restricted contractual setting the manufacturer pays subsidy ω_θ for the cost of process improvement and pays contingent payment Y_θ per unit of delivery. Importantly, there is no penalty clause in the contract, which means that the distressed supplier who has faced with disruption never pays penalty to the manufacturer. We solve for the optimal contract terms $\left(\omega_\theta^*, Y_\theta^*\right)$ when the manufacturer wants to induce process improvement effort on both types, i.e., $e_h = e_l = 1$. The below Table compares the inefficiencies (in the form of information rent) incurred by the manufacturer with (AE contract) and without (IE contract) auditing the supplier's action. The expressions under "Information Rent" column clearly shows that the manufacturer can reduce the cost of information asymmetry by observing the supplier's action.

Table 5.2.2: Optimal contract and information rent with two-term contract

Contract		Optimal menu of contracts $\left(\omega_h^*, Y_h^*\right); \left(\omega_l^*, Y_l^*\right)$	Information rent
IE		$\omega_h^* = 0, Y_h^* = \frac{\psi_h}{\delta}$ $\omega_l^* = \psi_l, Y_l^* = \frac{\rho_l\psi_h - \delta\psi_l}{\delta\rho_l}$	$\underbrace{\nu\left[\frac{\varphi_h}{\delta}\psi_h\right]}_{h\text{-type rent}} + \underbrace{(1-\nu)\left[\frac{\rho_l\psi_h - \delta\psi_l}{\delta}\right]}_{l\text{-type rent}}$
AE	$\psi_h \leq \frac{\rho_h}{\rho_l}\psi_l$	$\omega_h^* = 0, Y_h^* = \frac{\psi_h}{\rho_h}; \omega_l^* = \psi_l, Y_l^* = 0$	0
	$\psi_h > \frac{\rho_h}{\rho_l}\psi_l$	$\omega_h^* = 0, Y_h^* = \frac{\psi_h}{\rho_h}; \omega_l^* = \psi_l, Y_l^* = \frac{\rho_l\psi_h - \rho_h\psi_l}{\rho_h\rho_l}$	$\underbrace{(1-\nu)\left[\frac{\rho_l\psi_h - \rho_h\psi_l}{\rho_h}\right]}_{l\text{-type rent}}$

Uncorrelated Reliability-Cost Scenario

In the main body of Chapter 3, we assume that the type of supplier is defined by supply reliability (high or low). Moreover, each type is characterized by two attributes; supply reliability and cost of effort in improving reliability. Note that in the setup of the baseline model, once the type is known, the cost of effort can also be perfectly determined, indicating that the two attributes are perfectly correlated. In this section, we study the case where the supplier's reliability and cost of effort are uncorrelated. Under this condition, supplier's type has two components; (i) reliability component which can be high (h) or low (l), and (ii) cost (of effort) component which can be also high (h) or low (l). Different to the baseline model, the cost of process improvement for a h-type cost component is greater than that of l-type cost component, i.e., $\psi_h \geq \psi_l$. From buyer's perspective, the supplier's reliability is high with probability ν and low with probability $1-\nu$, and also supplier's cost component might be high with probability γ and low with probability $1-\gamma$. That is to say, the supplier comes in four different types: hh with probability $\nu\gamma$, hl with probability $\nu(1-\gamma)$, lh with probability $(1-\nu)\gamma$, and ll

with probability $(1-\nu)(1-\gamma)$. In what follows, we consider a specific case, and show that audit brings value to the buyer by reducing information rent similar to the results we proved in the main body of 3.

Suppose it is optimal that the buyer induces improvement effort on all types; i.e., $e_{hh} = e_{ll} = e_{hl} = e_{lh} = 1$. Under IE contract, because the supplier's effort is not observable, the buyer needs to use incentive-fees $(Y_\theta, \kappa_\theta)$ to satisfy supplier's moral hazard constraint. Note that inducing process improvement on the supplier only depends on the cost component of the supplier's type. Therefore, the buyer needs to satisfy $Y_{ll} + \kappa_{ll} \geq \frac{\psi_l}{\delta}$ and $Y_{hl} + \kappa_{hl} \geq \frac{\psi_l}{\delta}$ to induce $e_{ll} = e_{hl} = 1$, and $Y_{hh} + \kappa_{hh} \geq \frac{\psi_h}{\delta}$ and $Y_{lh} + \kappa_{lh} \geq \frac{\psi_h}{\delta}$ to induce $e_{hh} = e_{lh} = 1$. Similar to the results in our baseline model, the buyer can design break-even contract that just satisfies θ-type IR constraint ($\omega_\theta = c + \psi_\theta$) and induces him to exert process improvement effort ($Y_\theta + \kappa_\theta = \frac{\psi_\theta}{\delta}$). It is easy to show that all other types can make profit (information rent) by mimicking the lh-type supplier whose reliability is low and his cost of process improvement is high (the worst type supplier from buyer's perspective). Specifically, if the θ-type supplier, $\theta \in \{hh, hl, ll\}$, mimics the lh-type supplier, the information rent can be written as follows

$$\text{Information rent paid to } \theta\text{-type} = \underbrace{\omega^*_{lh} - (c + \tilde{e}_\theta \psi_\theta)}_{\text{fixed-price term}} + \underbrace{[p(\theta \mid \tilde{e}_\theta) - p(lh \mid e^*_{lh})](Y^*_{lh} + \kappa^*_{lh})}_{\text{incentive-fee term}} \tag{5.2.27}$$

Note that because of the moral hazard constraint $Y_{lh} + \kappa_{lh} \geq \frac{\psi_h}{\delta}$, if either one of hh-,hl-, or ll-type supplier mimics the lh-type, he will invest in process improvement effort, therefore, the level of improvement effort under deviated contract is $\tilde{e}_{hh} = \tilde{e}_{hl} = \tilde{e}_{ll} = 1$. By plugging the break-even contract for the lh-type supplier and the level of effort under deviated contract for other types, the information rent payable to hh-,hl-, and ll-type suppliers are $\frac{(\rho_h - \rho l)\psi_h}{\delta}$, $(\psi_h - \psi_l) + \frac{(\rho_h - \rho l)\psi_h}{\delta}$, and $(\psi_h - \psi_l)$, respectively. The total information rent payable to different types is as follows

$$\text{Information rent under IE } = \nu\gamma\left[\frac{(\rho_h - \rho l)\psi_h}{\delta}\right] + \nu(1-\gamma)\left[(\psi_h - \psi_l) + \frac{(\rho_h - \rho l)\psi_h}{\delta}\right] + (1-\nu)(1-\gamma)\left[(\psi_h - \psi_l)\right] \tag{5.2.28}$$

Let us explore the impact of audit in buyer's contract design problem for action profile $e_{hh} = e_{ll} = e_{hl} = e_{lh} = 1$. Under AE contract because the supplier's effort is observable and verifiable, the buyer can include the supplier's action in the contract. The buyer can induce improvement effort on the lh-type supplier through subsidy and there is no need to incentive-fees, i.e., $\omega_{lh} = c + \psi_h, Y_{lh} = \kappa_{lh} = 0$. Now, assume that the hh-type supplier wants to mimic the lh-type supplier. Because his action is observable by the buyer, the hh-type supplier has to exert improvement effort under deviated contract, and because the cost of effort for him is similar to that of lh-type, he never receive information rent under AE contract and incentive-fees is zero for him. Now assume that one of ll- or hl-type wants to mimic the lh-type supplier. Although they have to exert improvement effort under deviated contract, but, because the cost of effort for them is low, they can make a positive information rent which is $\psi_h - \psi_l$. To avoid mimicking phenomena, the buyer needs to design incentive-fees for ll- and hl-type suppliers such that they can receive the same amount ($\psi_h - \psi_l$) if they self-select the contract designed for them, e.g., $Y_{ll} = \frac{\psi_h - \psi_l}{\rho_l}, Y_{hl} = \frac{\psi_h - \psi_l}{\rho_h}$ and $\kappa_{ll} = \kappa_{hl} = 0$. The total information rent under AE contract is as follows

$$\text{Information rent under AE} = \nu(1-\gamma)\left[(\psi_h - \psi_l)\right] + (1-\nu)(1-\gamma)\left[(\psi_h - \psi_l)\right] = (1-\gamma)\left[(\psi_h - \psi_l)\right] \tag{5.2.29}$$

In other words, under AE contract, the buyer pays information rent only based on the cost (of effort) component of supplier's type. Specifically, the supplier receives information rent only if his cost of effort is low, which occurs with probability $1-\gamma$. By comparing the amount of information rent under IE and AE contracts, one can verify that considering uncorrelated scenario leads to the similar theoretical results in the main body of 3.

5.3 Proofs of Propositions and Lemmas for Chapter 4

Proof. of Proposition 4.1. To find the optimal diagnostic decision, we first need to characterize e_l^* and $e_{\bar{\theta}}^*$ in constraints (4.4.2) and (4.4.3). Specifically, $e_l^* = 1$ iff $c_p \leq \delta r$, otherwise $e_l^* = 0$. Similarly, $e_{\bar{\theta}}^* = 1$ iff $c_p \leq (1-\alpha)\delta r$, otherwise $e_{\bar{\theta}}^* = 0$. By considering the above optimality conditions, we can plug e_l^* and $e_{\bar{\theta}}^*$, in order to find the optimal value for d in the objective function. In particular, when $c_p \leq (1-\alpha)\delta r$, then $e_l^* = 1$ and $e_{\bar{\theta}}^* = 1$. Under this condition, investing in diagnostic test ($d = 1$) is optimal only if $\alpha\,(r - c_s) + (1-\alpha)\left(\rho r - c_s - c_p\right) - c_d \geq (\alpha + (1-\alpha)\rho)r - c_s - c_p$, or equivalently, $c_d \leq \alpha c_p$. However, when $(1-\alpha)\delta r < c_p \leq \delta r$, then $e_l^* = 1$ and $e_{\bar{\theta}}^* = 0$. Investing in diagnostic test is then optimal only if $c_p + \frac{c_d}{1-\alpha} \leq \delta r$. Finally, when $c_p > \delta r$, then $e_l^* = 0$ and $e_{\bar{\theta}}^* = 0$ and $d^* = 0$. Figure 4.4.1 in Proposition 4.1 summarizes the above discussion. □

Proof. of Lemma 4.1. Since the supplier does not know his true reliability, he decides on process improvement based on his expected reliability $\bar{\theta}$. Given a contract offered by the buyer, and according to Table 4.3.1, if the supplier exerts improvement effort, his expected profit is $\omega_{\bar{\theta}} + (\alpha + (1-\alpha)\rho)\,Y_{\bar{\theta}} - (1 - (\alpha + (1-\alpha)\rho))\,\kappa_{\bar{\theta}} - c_p$. However, if he does not exert improvement effort, his expected profit is $\omega_{\bar{\theta}} + (\alpha + (1-\alpha)\varphi)\,Y_{\bar{\theta}} - (1 - (\alpha + (1-\alpha)\varphi))\,\kappa_{\bar{\theta}}$. Thus, exerting process improvement is profitable as long as the cost of improvement effort c_p is less than the increase in expected profit $(1-\alpha) \times \delta \times (Y_{\bar{\theta}} + \kappa_{\bar{\theta}})$ where $\delta = \rho - \varphi$. □

Proof. of Proposition 4.2. First, we need to find and then compare buyer's profit for $e_{\bar{\theta}} = 1$ and $e_{\bar{\theta}} = 0$. Assume that the buyer wants to induce $e_{\bar{\theta}} = 1$. The buyer can simply subsidize the costs of production c_s and process improvement c_p, therefore $\omega_{\bar{\theta}} = c_s + c_p$. By plugging $e_{\bar{\theta}} = 1$ and $\omega_{\bar{\theta}} = c_s + c_p$ into constraints 4.5.1-4.5.4, it is straightforward to verify that the only effective constraints are l-type limited liability constraint (4.5.3) and moral hazard constraint (4.5.4). The optimal solution is then the intersection of these two constraints which gives $\omega_{\bar{\theta}} = c_s + c_p$; $Y_{\bar{\theta}} = \frac{1-\rho}{(1-\alpha)\delta} c_p$; and $\kappa_{\bar{\theta}} = \frac{\rho}{(1-\alpha)\delta} c_p$. Furthermore, when inducing $e_{\bar{\theta}} = 0$, the buyer only need to subsidize the production cost, therefore $\omega_{\bar{\theta}} = c_s$; $Y_{\bar{\theta}} = \kappa_{\bar{\theta}} = 0$. By comparing buyer's profit, it is easy to verify that $e^*_{\bar{\theta}} = 1$ is optimal when $c_p \leq \underline{c}^{\text{sb}} = \frac{\delta(1-\alpha)^2}{\alpha(1-\rho)+(1-\alpha)\delta} \delta r < \delta r$. Now, we can find the channel loss wherever the second-best level of effort $e^*_{\bar{\theta}}$ is different to the first-best effort in Proposition 4.1. Different regions presented in Table 4.5.1 come from this comparison. □

Proof. of Lemma 4.2. In order to induce $d = 0$; $e_{\bar{\theta}} = 1$, the supplier's profit if he does not learn his true reliability should be greater than that if he learns his type and take informed improvement decisions, i.e., $\pi^S_{\bar{\theta}}(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e_{\bar{\theta}} = 1) \geq \alpha \pi^S_h(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e_h = 1) + (1-\alpha)\alpha \pi^S_l(\omega_{\bar{\theta}}, Y_{\bar{\theta}}, \kappa_{\bar{\theta}} \mid e_l)$. It is straightforward to verify that this constraint can be simplified and rewritten as $c_d \geq \alpha c_p$ for $e_l = 1$. □

Proof. of Proposition 4.3. Based on the results of Lemma 4.2, when $c_d \leq \alpha c_p$, the buyer's problem boils down to a two-wise comparison between the expected profit from Profiles (i) and (iii). However, when $c_d > \alpha c_p$, the buyer needs to compare the expected profit from all profiles (i), (ii), and (iii). Note that the optimal contract for profiles (i) and (ii) are similar to those provided in Proposition 4.2. Therefore, below we find the optimal menu of contracts for profile (iii), which is to induce $d = 1; e_l = 1; e_h = 0$.

First of all, by learning the true reliability we know that the h-type supplier does not invest in process improvement, but the l-type supplier invests in that, therefore we can set subsidy payments as $\omega_h = c_s + c_d$ and $\omega_l = c_s + c_p + c_d$. It helps us to alleviate difficulty in contract design problem, and we can show that the solution from such subsidy scheme is always an optimal contract. Second, the number of constraints in buyer's optimization problem (4.5.9-4.5.15) is already huge, and our first goal should be to identify the binding constraints. Note that, if one ignores constraint (4.5.9), the resulting problem, i.e, objective function (4.5.15) with constraints (4.5.10-4.5.14), is the standard model of adverse selection followed by moral hazard for the l-type supplier. From mechanism design theory, it is straightforward to show that the following constraints are binding at optimality: the participation constraint (4.5.11) and moral hazard constraint (4.5.14) for the l-type supplier, and the incentive compatibility constraints for the h-type supplier (4.5.12). In other words, we can design the contract such that the l-type supplier earns zero and the h-type supplier earns information rent due to his ability to mimic the l-type. The optimal contract is then $\omega_h = c_s + c_d; Y_h = \frac{1-\varphi}{\delta} c_p; \kappa_h = \frac{\varphi}{\delta} c_p$ and $\omega_l = c_s + c_d + c_p; Y_l = \frac{1-\rho}{\delta} c_p; \kappa_l = \frac{\rho}{\delta} c_p$. Now, let us verify whether this menu of contracts satisfy constraint (4.5.9) or not. Note that constraint (4.5.9) should be satisfied for four different scenarios depending on which contract would be chosen by the uninformed supplier and whether or not he invests in process improvement. Verify that the menu of contracts obtained above cannot satisfy constraint (4.5.9) when the uninformed supplier picks any contract from the menu and decides not to invest in process improvement (i.e., $e_{\bar{\theta}} = 0$). In other words, under current scheme, the supplier is better off by not investing in diagnostic test, picking any contract, and not investing in process improvement. It readily means that constraint (4.5.9) is a binding constraint at optimality. Let us discuss how to rectify this problem.

Note that the h-type supplier is fully reliable and penalty term has no impact on his profit. Therefore, we can take advantage of κ_h and set it big enough to deter not only the informed l-type supplier, but also the uninformed supplier from mimicking the h-type. Note that it's the only costless way to deter the uninformed supplier from mimicking the h-type. In other words, we can always chooses κ_h such that if the supplier decides to remain uninformed, he is better off by picking the contract designed for the l-type supplier. That is to say, the right-hand side of constraint (4.5.9) can be rewritten as $\max_{e_{\bar{\theta}} \in \{0,1\}} \pi^S_{\bar{\theta}} (\omega_l, Y_l, \kappa_l \mid e_{\bar{\theta}})$. With the same logic, we can set κ_h big enough to deter the l-type supplier from mimicking the h-type. That is to say, constraint (4.5.13) is also redundant at optimality. Therefore, the buyer's contract design problem boils down to the following problem:

$$\max_{(Y_h, \kappa_h), (Y_l, \kappa_l)} \quad \alpha [r - Y_h] + (1-\alpha) \left[\rho (r - Y_l) + (1-\rho) \kappa_l - c_p\right] - c_s - c_d \tag{5.3.1}$$

subject to the following constraints:

$$Y_h \geq Y_l + \frac{c_p}{\alpha} + \frac{c_d}{\alpha} - \frac{(1-\alpha)\delta}{\alpha}(Y_l + \kappa_l) \quad \left[\text{when } Y_l + \kappa_l < \frac{c_p}{\delta(1-\alpha)}\right] \tag{5.3.2}$$

$$Y_h \geq Y_l + \frac{c_d}{\alpha} \quad \left[\text{when } Y_l + \kappa_l \geq \frac{c_p}{\delta(1-\alpha)}\right] \tag{5.3.3}$$

$$\rho Y_l - (1-\rho)\kappa_l \geq 0 \tag{5.3.4}$$

$$Y_h \geq 0 \tag{5.3.5}$$

$$Y_h \geq Y_l + c_p \tag{5.3.6}$$

$$Y_l + \kappa_l \geq \frac{c_p}{\delta} \tag{5.3.7}$$

where constraints (5.3.2) and (5.3.3) are the extensions of constraint (4.5.9) when inducing $e_{\bar{\theta}} = 0$ and $e_{\bar{\theta}} = 1$, respectively. Constraints (5.3.4) and (5.3.5) are participation constraints for l- and h-type suppliers, respectively. Constraint (5.3.6) is h-type supplier's incentive compatibility constraint, and finally, constraint (5.3.7) is moral hazard constraint to induce $e_l = 1$ on the informed l-type supplier.

Now, we can design the optimal menu of contracts in two different ways depending on whether it induces $e_{\bar{\theta}} = 1$, or $e_{\bar{\theta}} = 0$. Below, we find the menu of contracts under each scenario and then characterize the optimal one.

- Inducing $e_{\bar{\theta}} = 1$: First, we need to satisfy $Y_l + \kappa_l \geq \frac{c_p}{\delta(1-\alpha)}$. Under this condition, the only effective constraints are (5.3.3), (5.3.4), and (5.3.6). Furthermore, by comparing the right-hand sides of constraints (5.3.3) and (5.3.6), verify that when $c_d > \alpha c_p$ then constraint (5.3.6) is redundant, and when $c_d \leq \alpha c_p$ then constraint (5.3.3) is redundant. Using graphical approach of linear programming, it is easy to find the optimal contract:
  - $c_d > \alpha c_p$: $Y_l = \frac{1-\rho}{\delta(1-\alpha)} c_p$; $\kappa_l = \frac{\rho}{\delta(1-\alpha)} c_p$, and $Y_h = \frac{1-\rho}{\delta(1-\alpha)} c_p + \frac{c_d}{\alpha}$,
  - $c_d \leq \alpha c_p$: $Y_l = \frac{1-\rho}{\delta(1-\alpha)} c_p$; $\kappa_l = \frac{\rho}{\delta(1-\alpha)} c_p$, and $Y_h = \frac{1-\rho}{\delta(1-\alpha)} c_p + c_p$.
- Inducing $e_{\bar{\theta}} = 0$: We need to satisfy $Y_l + \kappa_l < \frac{c_p}{\delta(1-\alpha)}$. Note that the only effective constraints are (5.3.2), (5.3.4), (5.3.6), and (5.3.7). However, the right-hand side of constraint (5.3.2) is greater than that of (5.3.6) when $Y_l + \kappa_l < \frac{c_p}{\delta} + \frac{c_d}{\delta(1-\alpha)}$. But since we have $Y_l + \kappa_l < \frac{c_p}{\delta(1-\alpha)}$ (in order ro induce $e_{\bar{\theta}} = 0$), one can readily conclude that constraint (5.3.6) is redundant when $c_d > \alpha c_p$. Under this condition $(c_d > \alpha c_p)$ we can find the optimal value of Y_h from constraint (5.3.2); $Y_h = Y_l + \frac{c_p}{\alpha} + \frac{c_d}{\alpha} - \frac{(1-\alpha)\delta}{\alpha}(Y_l + \kappa_l)$. By plugging this value in the objective function (5.3.1), we can characterize the optimal contract. Specifically, from

Table 5.3.1: Optimal contract to induce profile (iii): $d = 1; e_l = 1; e_h = 0$

$\alpha \leq \frac{\delta}{1-\varphi}$ ($\frac{1-\varphi}{1-\rho}$ is high)	
$c_d \leq \alpha c_p$	$\alpha c_p < c_d$
$\begin{cases} \omega_h^* = c_s + c_d \\ Y_h^* = \frac{1-\varphi}{\delta} c_p + \frac{1-\rho}{\delta(1-\alpha)} c_d \\ \kappa_h^* = \frac{c_p + c_d}{\delta(1-\alpha)} \end{cases}$ $\begin{cases} \omega_l^* = c_s + c_d + c_p \\ Y_l^* = \frac{1-\rho}{\delta} c_p + \frac{1-\rho}{\delta(1-\alpha)} c_d \\ \kappa_l^* = \frac{\rho}{\delta} c_p + \frac{\rho}{\delta(1-\alpha)} c_d \end{cases}$	$\begin{cases} \omega_h^* = c_s + c_d \\ Y_h^* = \frac{1-\rho}{\delta(1-\alpha)} c_p + \frac{c_d}{\alpha} \\ \kappa_h^* = \frac{(c_p/\delta)+(c_d/\alpha)}{(1-\alpha)(1-\rho)} \end{cases}$ $\begin{cases} \omega_l^* = c_s + c_d + c_p \\ Y_l^* = \frac{1-\rho}{\delta(1-\alpha)} c_p \\ \kappa_l^* = \frac{\rho}{\delta(1-\alpha)} c_p \end{cases}$
$\alpha > \frac{\delta}{1-\varphi}$ ($\frac{1-\varphi}{1-\rho}$ is low)	
$\begin{cases} \omega_h^* = c_s + c_d \\ Y_h^* = \frac{1-\varphi}{\delta} c_p + \frac{c_d}{\alpha} \\ \kappa_h^* = \frac{(c_p/\delta)+(c_d/\alpha)}{(1-\alpha)(1-\rho)} \end{cases}$ $\begin{cases} \omega_l^* = c_s + c_d + c_p \\ Y_l^* = \frac{1-\rho}{\delta} c_p \\ \kappa_l^* = \frac{\rho}{\delta} c_p \end{cases}$	

graphical presentation of linear programming we can show that

- If $\alpha \leq \frac{\delta}{1-\varphi}$ (or $\frac{1-\varphi}{1-\rho}$ is high): $Y_l = \frac{1-\rho}{\delta(1-\alpha)} c_p; \kappa_l = \frac{\rho}{\delta(1-\alpha)} c_p$ and $Y_h = \frac{1-\rho}{\delta(1-\alpha)} c_p + \frac{c_d}{\alpha}$,
- If $\alpha > \frac{\delta}{1-\varphi}$ (or $\frac{1-\varphi}{1-\rho}$ is low): $Y_l = \frac{1-\rho}{\delta} c_p; \kappa_l = \frac{\rho}{\delta} c_p$ and $Y_h = \frac{1-\varphi}{\delta} c_p + \frac{c_d}{\alpha}$

Now, if $c_d \leq \alpha c_p$, then there are two possibilities; setting $Y_l + \kappa_l < \frac{c_p}{\delta} + \frac{c_d}{\delta(1-\alpha)}$ under which constraint (5.3.6) is redundant, or $Y_l + \kappa_l \geq \frac{c_p}{\delta} + \frac{c_d}{\delta(1-\alpha)}$ under which constraint (5.3.2) is redundant. By solving the buyer's problem for both scenarios, it is easy to verify that the optimal solution can be obtained by setting Y_l, κ_l such that constraint (5.3.6) is redundant. The optimal contract is as follows:

- If $\alpha \leq \frac{\delta}{1-\varphi}$ (or $\frac{1-\varphi}{1-\rho}$ is high): $Y_l = \frac{1-\rho}{\delta} c_p + \frac{1-\rho}{\delta(1-\alpha)} c_d; \kappa_l = \frac{\rho}{\delta} c_p + \frac{\rho}{\delta(1-\alpha)} c_d$ and $Y_h = \frac{1-\varphi}{\delta} c_p + \frac{1-\rho}{\delta(1-\alpha)} c_d$,
- If $\alpha > \frac{\delta}{1-\varphi}$ (or $\frac{1-\varphi}{1-\rho}$ is low): $Y_l = \frac{1-\rho}{\delta} c_p; \kappa_l = \frac{\rho}{\delta} c_p$ and $Y_h = \frac{1-\varphi}{\delta} c_p + \frac{c_d}{\alpha}$

Let us now obtain the optimal menu of contracts corresponding to profile (iii). Because the h-type supplier is fully reliable, we can give the right incentives through contingent payment Y_h to avoid him mimicking the l-type supplier, however, the l-type supplier earns zero at optimality. Therefore, characterizing the optimal contract boils down to comparison of contingent payment to h-type supplier. It is straightforward to verify that the contract that induces $e_{\bar{\theta}} = 0$ brings always lower Y_h than that induces $e_{\bar{\theta}} = 1$ (see Table 5.3.1). Finally, We can find the optimal penalty term for the h-type supplier such that both l- and $\bar{\theta}$-type suppliers deter from mimicking the h-type.

Full equilibrium characterization

The optimal characterization depends on whether $c_d \leq \alpha c_p$ or $c_d > \alpha c_p$:

- $c_d \leq \alpha c_p$: The buyer's problem is to compare the expected profit from profiles (i) and (iii). The optimal contract for profile (iii) comes from Table 5.3.1, and the optimal contract for profile (i) has been already characterized in Proposition 4.2 associated with Region B_1.

- $c_d > \alpha c_p$: The buyer needs to compare the expected profit from all profiles (i), (ii), and (iii). The results of comparison between profiles (i) and (ii) has been already appeared in Proposition 4.2 associated with Regions A_2 and B_2, respectively. Therefore, we only need to compare the optimal contract characterized in Table 5.3.1 to those in Regions A_2 and B_2 in Proposition 4.2.

The results provided in Table 4.5.2 comes from the above comparison. □

Proof. of Proposition 4.4. We need to compare the profits in Propositions 4.2 and 4.3. When $\frac{1-\rho}{1-\varphi}$ is low, then the value of diagnostic test boils down to the comparison of Regions I and II in Proposition 4.3 to Regions A_1 and B_1 in Proposition 4.2. However, when $\frac{1-\rho}{1-\varphi}$ is high, then the value of diagnostic test boils down to the comparison of Region I in Proposition 4.3 to Regions A_1 and B_1 in Proposition 4.2, as well as the comparison of Region II in Proposition 4.3 to Regions A_2 and B_2 in Proposition 4.2. The results in Table 4.6.1 comes from the above comparisons. □

Bibliography

Physical risks to the supply chain: the view from finance. Technical report, CFO Research Services, February 2009.

T. Adamo and A. Matros. A blotto game with incomplete information. *Economics Letters*, 105(1):100–102, 2009.

S. Anthon, S. Garcia, and A. Stenger. Incentive contracts for natura 2000 implementation in forest areas. *Environmental and Resource Economics*, 46(3):281–302, 2010.

D. G. Arce and T. Sandler. Terrorist signalling and the value of intelligence. *British Journal of Political Science*, 37(04):573–586, 2007.

G. Aydin, V. Babich, D. Beil, and Z. Yang. *Decentralized supply risk management*. John Wiley & Sons: Handbook of Integrated Risk Management in Global Supply Chains, 2010.

V. Babich and C. S. Tang. Managing opportunistic supplier product adulteration: Deferred payments, inspection, and combined mechanisms. *Manufacturing & Service Operations Management*, 14(2):301–314, 2012.

V. Babich, A.N. Burnetas, and P.H. Ritchken. Competition and diversification effects in supply chains with supplier default risk. *Manufacturing & Service Operations Management*, 9(2):123–146, 2007.

S. Baiman, P.E. Fischer, and M.V. Rajan. Information, contracting, and quality costs. *Management Science*, pages 776–789, 2000.

K.R. Balachandran and S. Radhakrishnan. Quality implications of warranties in a supply chain. *Management Science*, 51:1266–1277, 2005.

D. P. Baron and R. B. Myerson. Regulating a monopolist with unknown costs. *Econometrica: Journal of the Econometric Society*, pages 911–930, 1982.

M. F. Bellemare. Testing the effect of monitoring in production contracts: evidence from madagascar. Technical report, Mimeo, Terry Sanford Institute of Public policy, Duke University, 2006.

F. Bernstein and A.G. Kok. Dynamic cost reduction through process improvement in assembly networks. *Management Science*, 55(4):552–567, 2009.

V. M. Bier, S. Oliveros, and L. Samuelson. Choosing what to protect: Strategic defensive allocation against an unknown attacker. *Journal of Public Economic Theory*, 9(4):563–587, 2007.

V. M. Bier, N. Haphuriwat, J. Menoyo, R. Zimmerman, and A. M. Culpen. Optimal resource allocation for defense of targets based on differing measures of attractiveness. *Risk Analysis*, 28(3):763–770, 2008.

R. E. Bohn and C. Terwiesch. The economics of yield-driven processes. *Journal of Operations Management*, 18(1): 41–59, 1999.

G. Brown, M. Carlyle, D. Diehl, J. Kline, and K. Wood. A two-sided optimization for theater ballistic missile defense. *Operations Research*, 53:745–763, 2005.

Gerald G. Brown and Louis Anthony (Tony) Cox, Jr. How probabilistic risk assessment can mislead terrorism risk analysts. *Risk Analysis*, 31(2):196–204, 2011.

G. P. Cachon. Supply chain coordination with contracts. *Handbooks in operations research and management science*, 11:229–340, 2003.

G. P. Cachon and F. Zhang. Procuring fast delivery: Sole sourcing with information asymmetry. *Management Science*, 52(6):881, 2006.

G. H. Chao, S. M. R. Iravani, and R. C. Savaskan. Quality improvement incentives and product recall cost sharing contracts. *Management Science*, 55(7):1122–1138, 2009.

A. Chaturvedi and V. Martinez-de Albeniz. Optimal procurement design in the presence of supply risk. *Manufacturing & Service Operations Management*, 13(2):227–243, 2011.

S. Chopra, G. Reinhardt, and U. Mohan. The importance of decoupling recurrent and disruption risks in a supply chain. *Naval Research Logistics*, 54(5):544–555, 2007.

K. B. Clark. Project scope and project performance: the effect of parts strategy and supplier involvement on product development. *Management science*, 35(10):1247–1263, 1989.

R. G. Cooper. Predevelopment activities determine new product success. *Industrial Marketing Management*, 17(3):237–247, 1988.

C. J. Corbett, D. Zhou, and C. S. Tang. Designing supply contracts: Contract type and information asymmetry. *Management Science*, 50(4):550–559, 2004.

C. J. Corbett, G. A. DeCroix, and A. Y. Ha. Optimal shared-savings contracts in supply chains: Linear contracts and double moral hazard. *European Journal of Operational Research*, 163(3):653 – 667, 2005.

J. Cremer, F. Khalil, and J. C. Rochet. Strategic information gathering before a contract is offered. *Journal of Economic Theory*, 81(1):163–200, 1998.

J. Crémer and F. Khalil. Gathering information before signing a contract. *The American Economic Review*, pages 566–578, 1992.

E. Dahan and H. Mendelson. An extreme-value model of concept testing. *Management Science*, 47(1):102–116, 2001.

G.B. Dantzig. Application of the simplex method to a transportation problem. *Activity Analysis of Production and Allocation*, 13:359–373, 1951.

J. H. Dyer, D. S. Cho, and W. Chu. Strategic supplier segmentation: The next "best practice" in supply chain management. *California Management Review*, 40:57–77, 1998.

S. Erat and S. Kavadias. Sequential testing of product designs: Implications for learning. *Management Science*, 54(5):956–968, 2008.

D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, 1991.

Y. Gerchak and F. Safayeni. Perfect information with potentially negative value: An intriguing war story and a possible explanation. *Journal of the Operational Research Society*, 47:710–714, 1996.

M. Golalikhani and J. Zhuang. Modeling arbitrary layers of continuous-level defenses in facing with strategic attackers. *Risk Analysis*, 31(4):533–547, 2011.

B. Golany, E. H. Kaplan, A. Marmur, and U. G. Rothblum. Nature plays with dice-terrorists do not: Allocating resources to counter strategic versus probabilistic risks. *European Journal of Operational Research*, 192(1):198 – 208, 2009.

Aberdeen Group. Supply chain visibility: A critical strategy to optimize cost and service. *Aberdeen Group*, May 2013.

R. Guesnerie, P. Picard, and P. Rey. Adverse selection and moral hazard with risk neutral agents. *European Economic Review*, 33(4):807–823, 1989.

M. Gümüş, S. Ray, and H. Gurnani. Supply side story: Risks, guarantees, competition and information asymmetry. *Management Science*, 58 (9):1694–1714, 2012.

H. Gurnani and M. Shi. A bargaining model for a first-time interaction under asymmetric beliefs of supply reliability. *Management science*, 52(6):865, 2006.

R. B. Handfield, G. L. Ragatz, K. Peterson, and R. M. Monczka. Involving suppliers in new product development? *California management review*, 42:59–82, 1999.

M. Hao, S. Jin, and J. Zhuang. Robustness of optimal defensive resource allocations in the face of less fully rational attacker. *Proceedings of the 2009 Industrial Engineering Research Conference*, pages 886–891, 2009.

M. Harley. Volvo recalls over 140k 2001-2005 and 2010 my vehicles over fuel system issues. *Autoblog*, http://www.autoblog.com/volvo/recalls/, Decebmer 2009.

K. Hausken and J. Zhuang. Governments' and terrorists' defense and attack in a t-period game. *Decision Analysis*, 8(1):46–70, 2011.

K. Hausken, V. M Bier, and J. Zhuang. Defending against terrorism, natural disaster, and all hazards. *Game Theoretic Risk Analysis of Security Threats*, pages 65–97, 2009.

S. Helper. Strategy and irreversibility in supplier relations: the case of the us automobile industry. *Business history review*, 65(04):781–824, 1991.

K. B. Hendricks and V. R. Singhal. The effect of supply chain glitches on shareholder wealth. *Journal of Operations Management*, 21(5):501–522, 2003.

Douglas W Hubbard. *The failure of risk management: why it's broken and how to fix it*. John Wiley and Sons, 2009.

Apple Inc. Apple supplier responsibility. *Progress Report*, www.apple.com/supplierresponsibility, 2012.

E. Jenelius, J. Westin, and J. Holmgren. Critical infrastructure protection under imperfect attacker perception. *International Journal of Critical Infrastructure Protection*, 3(1):16–26, 2010.

E. H. Kaplan, M. Kress, and R. Szechtman. Confronting entrenched insurgents. *Operations Research*, 58:329–341, 2010.

A. S. Kessler. The value of ignorance. *The Rand Journal of Economics*, pages 339–354, 1998.

S. H. Kim and S. Netessine. Collaborative cost reduction and component procurement under information asymmetry. *Management Science*, 59(1):189–206, 2013.

M. Kitamura, A. Ohnsman, and Y. Hagiwara. Toyota safety audit finds misunderstanding with suppliers. *Bloomberg Business Week*, http://www.bloomberg.com/news/2010-10-13/toyota-safety-audit-finds-misunderstanding-with-suppliers.html, October 2010.

Kjell and Hausken. Defense and attack of complex and dependent systems. *Reliability Engineering & System Safety*, 95(1):29 – 42, 2010.

J. Knowdell. The benefits and disadvantages of contract manufacturing. Technical report, Industrial Quick Search, Inc, 16 Apr 2010.

D. Kovenock and B. Roberson. A blotto game with multi-dimensional incomplete information. *Economics Letters*, 113(3):273 – 275, 2011.

D. R. Krause, R. B. Handfield, and B. B. Tyler. The relationships between supplier development, commitment, social capital accumulation and performance improvement. *Journal of Operations Management*, 25(2):528–545, 2007.

V. Krishnan, S. D. Eppinger, and D. E. Whitney. A model-based framework to overlap product development activities. *Management science*, 43(4):437–451, 1997.

J. J. Laffont and D. Martimort. *The theory of incentives: the principal-agent model*. Princeton Univ Pr, 2002.

J. J. Laffont and J. Tirole. Using cost observation to regulate firms. *The Journal of Political Economy*, pages 614–641, 1986.

J. &J. Laffont and J. Tirole. *A Theory of Incentives in Procurement and Regulation*. Cambridge: MIT Press, 1993.

G. Levitin and K. Hausken. Redundancy vs. protection in defending parallel systems against unintentional and intentional impacts. *IEEE Transactions On Reliability*, 58(4):679–690, 2009.

T. R. Lewis and D. E. Sappington. All-or-nothing information control. *Economics Letters*, 37(2):111–113, 1991.

T. R. Lewis and D. E. Sappington. Ignorance in agency problems. *Journal of Economic Theory*, 61(1):169–183, 1993.

C. Li. Sourcing for supplier effort and competition: Design of the supply base and pricing mechanism. *Working paper*, 2012.

C. G. Limberakis. Supplier lifecycle management. Technical report, Aberdeen Group, August 2012.

X. Liu, D. Nestic, and T. Vukina. Estimating adverse selection and moral hazard effects with hospital invoices data in a government-controlled healthcare system. *Health Economics*, 21(8):883–901, 2011.

M. S. Long, I. B. Malitz, and S. A. Ravid. Trade credit, quality guarantees, and product marketability. *Financial Management*, pages 117–127, 1993.

I. Macho-Stadler and D. Perez-Castrillo. *An Introduction to the Economics of Information: Incentives and Contracts*. Oxford university Press, 2001.

P. Milgrom. Adverse selection without hidden information. *Working paper-UC Berkeley*, 1987.

R. B. Myerson. Optimal coordination mechanisms in generalized principal-agent problems. *Journal of mathematical economics*, 10(1):67–81, 1982.

M. E. Nikoofal and M. Gümüş. On the value of terrorist's private information in government's defensive resource allocation problem. *IIE Transactions*, forthcoming, 2014a.

M. E. Nikoofal and M. Gümüş. The value of audit in managing supplier's process improvement. *Working paper*, McGill University, 2014b.

M. E. Nikoofal and M. Gümüş. The value of diagnostic test in contract manufacturing under supply risk. *Working paper*, McGill University, 2014c.

M. E. Nikoofal and J. Zhuang. Robust allocation of a defensive budget considering an attacker's private information. *Risk Analysis*, 32:930–943, 2012.

P. B. Overgaard. The scale of terrorist attacks as a signal of resources. *Journal of Conflict Resolution*, 38(3): 452–478, 1994.

O. Özer and G. Raz. Supply chain sourcing under asymmetric information. *Production and Operations Management*, 20(1):92–115, 2011.

K. J. Petersen, R. B. Handfield, and G. L. Ragatz. Supplier integration into new product development: coordinating product, process and supply chain design. *Journal of operations management*, 23(3):371–388, 2005.

M. A. Petersen and R. G. Rajan. The benefits of lending relationships: Evidence from small business data. *The Journal of Finance*, 49:3–37, 1994.

R. Powell. Allocating defensive resources with private information about vulnerability. *American Political Science Review*, 101(04):799–809, 2007a.

R. Powell. Defending against terrorist attacks with limited resources. *American Political Science Review*, 101(03): 527–541, 2007b.

M. A. Primo and S. D. Amundson. An exploratory study of the effects of supplier relationships on new product development outcomes. *Journal of Operations Management*, 20(1):33–52, 2002.

R. G. Rajan and L. Zingales. What do we know about capital structure? some evidence from international data. *The Journal of Finance*, 50:1421–1460, 1995.

E. Rasmusen. *Games and Information: An Introduction to Game Theory*. Wiley-Blackwell; 4th edition, 2006.

Eric Rasmusen and Basil Blackwell. *Games and information*, volume 2. Cambridge, 1994.

D. J. Reyniers and C. S. Tapiero. Contract design and the control of quality in a conflictual environment. *European journal of operational research*, 82(2):373–382, 1995.

J. Rios and D. R. Insua. Adversarial risk analysis: Applications to basic counterterrorism models. In *Algorithmic Decision Theory*, volume 5783 of *Lecture Notes in Computer Science*, pages 306–315. 2009.

B. Roberson. The colonel blotto game. *Economic Theory*, 29(1):1–24, 2006.

C. Rothschild, L. McLay, and S. Guikema. Adversarial risk analysis with incomplete information: A level k approach. *Risk Analysis*, 32(7):1219–1231, 2012.

T. Sandler and K. Siqueira. Games and terrorism. *Simulation & Gaming*, 40(2):164–192, 2009.

X. Shan and J. Zhuang. Hybrid defensive resource allocations in the face of partially strategic attackers in a sequential defender-attacker game. *European Journal of Operational Research*, 228:262–272, 2013a.

X. Shan and J. Zhuang. Cost of equity in homeland security resource allocation in the face of a strategic attacker. *Risk Analysis*, 33(6):1083–1099, 2013b.

Y. Sheffi. *The resilient enterprise: overcoming vulnerability for competitive advantage*. The MIT Press, paperback edition, 2007.

M. Shubik and R.J. Weber. Systems defense games: Colonel blotto, command and control. *Naval Research Logistics Quarterly*, 28(2):281–287, 1981.

J. K. Smith. Trade credit and informational asymmetry. *The Journal of Finance*, 42.4:863–872, 1987.

R. E. Spekman, J. W. Kamauff, and N. Myhr. An empirical investigation into supply chain management: a perspective on partnerships. *Supply Chain Management: An International Journal*, 3(2):53–67, 1998.

C. Tang and B. Tomlin. The power of flexibility for mitigating supply chain risks. *International Journal of Production Economics*, 116(1):12–27, 2008.

C. S. Tang. Robust strategies for mitigating supply chain disruptions. *International Journal of Logistics: Research and Applications*, 9(1):33–45, 2006.

C. S. Tang. Making products safe: process and challenges. *International Commerce Review*, 8(1):48–55, 2008.

Y. Tang, H. Gurnani, and D. Gupta. Managing disruptions in decentralized supply chains with endogenous supply process reliability. *Production and Operations Management*, 2013.

C. Terwiesch, C. H. Loch, and A. De Meyer. Exchanging preliminary information in concurrent engineering: Alternative coordination strategies. *Organization Science*, 13(4):402–419, 2002.

S. Thomke and D. E. Bell. Sequential testing in product development. *Management Science*, 47(2):308–323, 2001.

S. H. Thomke. Managing experimentation in the design of new products. *Management Science*, 44(6):743–762, 1998.

NY Times. How the u.s. lost out on iphone work. *Report*, http://www.nytimes.com/2012/01/22/business/apple-america-and-a-squeezed-middle-class.html, 21 January 2012.

USA Today. Economy rocks china factories. http://usatoday30.usatoday.com/money/world/2008-10-21-red-dragon-china-factories-economy-N.htm, October 2008.

B. Tomlin. Impact of supply learning when suppliers are unreliable. *Manufacturing & Service Operations Management*, 11(2):192–209, 2009.

B. Tomlin and Y. Wang. *Operational strategies for managing supply chain disruption risk*. John Wiley & Sons: Handbook of Integrated Risk Management in Global Supply Chains, 2010.

Cranfield University. Supply chain vulnerability. *Executive report*, 2002.

Volvo. Supplier quality assurance manual. Technical report, 2010.

C. Wang and V. M. Bier. Target-hardening decisions based on uncertain multiattribute terrorist utility. *Decision Analysis*, 8(4):286–302, 2011.

Y. Wang, W. Gilland, and B. Tomlin. Mitigating supply risk: Dual sourcing or process improvement? *Manufacturing & Service Operations Management*, 12(3):489–510, 2010.

Industry Week. Lack of visibility causing elevated supply chain risk. http://www.industryweek.com/planning-amp-forecasting/lack-visibility-causing-elevated-supply-chain-risk, December 2009.

Z. B. Yang, G. Aydin, V. Babich, and D. R. Beil. Supply disruptions, asymmetric information, and a backup production option. *Management Science*, 55(2):192–209, 2009.

Z. B. Yang, G. Aydin, V. Babich, and D. R. Beil. Using a dual-sourcing option in the presence of asymmetric information about supplier reliability: Competition vs. diversification. *Manufacturing & Service Operations Management*, 14(2):202–217, 2012.

C. Zhang and J. E. Ramirez-Marquez. Protecting critical infrastructures against intentional attacks- a two-stage game with incomplete information. *IIE Transactions*, (ja), 2012.

J. Zhuang and V. M. Bier. Balancing terrorism and natural disasters defensive strategy with endogenous attacker effort. *Operations Research*, 55(5):976–991, 2007.

J. Zhuang and V. M. Bier. Secrecy and deception at equilibrium, with applications to anti-terrorism resource allocation. *Defence and Peace Economics*, 22(1):43–61, 2011.

J. Zhuang, V. M. Bier, and O. Alagoz. Modeling secrecy and deception in a multiple-period attacker-defender signaling game. *European Journal of Operational Research*, 203(2):409 – 418, 2010.