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ECONOMETRICS OF MONEY DEMAND: WITH APPLICATIONS TO THE CANADIAN ECONOMY

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ABSTRACT

This thesis seeks to contribute to the theoretical and empirical debate surrounding five key issues in the demand for money. These issues are identified as: stability, functional form, causality, dynamics and competing theories. Each is examined through the application of current econometric methods to Canadian data. In addition to providing information about Canadian money demand, efforts are made to assess the practical nature of the econometric techniques employed.

Contributions include: an assessment of relative sensitivity of various stability tests; a discussion of stability of monetary aggregates wherein a demand shift in the current account component of narrow money in the mid-1970s is identified; empirical and theoretical analyses of the appropriateness of a semilogarithmic functional form; technical improvements in the study of causality for Canada; discussion and assessment of variable dynamics in the equilibrating adjustment process; and construction of a statistically-optimum and economically-rational price expectations series.

RÉSUMÉ

Le but de cette thèse est de contribuer au débat théorique et empirique concernant cinq questions majeures sur la demande de monnaie. Ces questions sont identifiées comme: la stabilité, la forme fonctionelle, la causalité, la dynamique et les théories concurrentes. Chacune est examinée en appliquant des méthodes économétriques aux données canadiennes. En plus de fournir des renseignements sur la demande de monnaie au Canada, des efforts sont déployés afin d'évaluer les techniques économétriques utilisées.

Les contributions se résument à: une évaluation de la sensibilité relative de divers tests de stabilité; une discussion de la stabilité des agrégats monétaires dans lesquels un déplacement de la demande des comptes courants, au milieu des années 1970, est identifié; des analyses empirique et théorique sur l'application de la forme semi-logarithmique; des améliorations techniques dans une étude de causalité canadienne; la discussion et l'évaluation de la dynamique variable dans le processus d'adjustement à l'équilibre; et la construction d'une série des attentes optimales et rationelles sur les prix.

FOREWORD

Econometrics, the interface of statistical methods and economic theory, has done much in recent decades to sharpen our focus of the economic environment. Rapid advances in computer technology and improved data collection have contributed to a world in which econometrics, as a discipline, has truly blossomed. Econometric technique, however, does not exist as an end in itself. Despite the fact that the bulk of textbook material on econometrics, and one author estimates at least 80 per cent, is devoted exclusively to econometric theory, it is the merging of theory and data which is the crucial step in what may be termed the 'econometric approach'. It is my view that an over-emphasis of 'theory-only' and insufficient written attention to the problems which actually confront the practicing econometrician has prevailed in the literature. This thesis seeks to survey, develop and apply econometric methods in a balanced presentation of theory and practice.

The principal thrust of this study will be to address the structural relation which exists between real money balances and other predetermined variables of the economy. In so doing, attention will be focused not only on the way in which our perception of the macroeconomy has evolved but also on how this perception has been misdirected through faulty or incomplete econometric analysis. Though this work cannot hope to resolve the myriad of problems which econometricians must face in estimating structural relationships, it can point to directions in which quality of estimation can be improved. The Lucas critique of econometric models, for example, applies to structural equations in general. We are concerned with the estimation of parameters which cannot be 'structural' if they change, as they must, with changes in policy. Neither can we hope to resolve problems of data quality. Apart from using the 'best' data that are currently available, little can be done about measurement errors of which we are unaware. In spite of these and other pervasive problems facing econometric research the objective must be to learn as much, but not more, about the economy as current data can tell us. This maximization can be approached by means of prudent application of econometric theory.

By necessity, the analysis of the present thesis is partial equilibrium in nature. Attempts will be made to isolate demand for money functions for Canada and to examine them as to questions judged to be of current interest. Although a serious attempt has been made to cover as broad a range of related topics as possible, a certain subjective narrowing of focus was necessary for adequate treatment of the chosen issues. Unfortunately, and not for their lack of relevance in a broader economic setting, topics such as: the importance of money demand to the formation and strategy of monetary policy, linkages between the money market and other sectors of the economy, substitutability of monetary assets, and the proper economic definition of money; have been given only summary attention. The issues which are considered to be of immediate concern involve stability of the demand relation, proper functional and dynamic specifications, the relevant economic variables and their causal interaction, and appropriate estimation techniques. It is judged that adequate resolution of the basic questions is necessary, though not sufficient, for consideration of the more complex ones. The latter are relegated to a future volume.

CHAPTER 1

AN OVERVIEW

Most excursions into the theory of money demand begin at a familiar starting point. Early theoretical formulations provide the anchor and, in some cases, the vehicle for many narratives on this topic. This thesis will not attempt to break with that tradition. My intention, though ultimately to gain a better understanding of the larger economic environment, is specifically aimed at the resolution of currently-debated issues relating to the demand for money. A well-defined view of the evolution of monetary theory is therefore judged essential to the realization of my goals.

The present introductory chapter traces the development of key issues in contemporary monetary theory. Since the ancestry of much of what is currently debated can be found in early quantity and liquidity-preference theories, Section 1.1 is devoted to theories of money up to, and including, that of Keynes (1936). This first stage of development saw the emergence of two fairly distinct views of the demand for money. The second, or 'synthesis', stage brought these views closer together in what is termed, in Section 1.2, the neoclassical view. Section 1.3 provides the background for current debates based upon transactions and asset views of money demand. One interesting dimension of the present debate is that the two competing views share a common heritage. Relevant cross-currents are identified in Section 1.3.

Common to all three phases of theoretical development is the recurring dispute as to the proper definition of money. Section 1.4 describes the background of this dispute. In the next section, the empirical studies on the demand for money are surveyed. Since this literature is vast, it has been necessary to group the important studies into five major areas of concern. These are the five areas which are judged to be of primary interest to demand for money in Canada. Each of these areas will be examined in turn in successive chapters of this thesis. They are: stability, functional form, causality, dynamics and competing theories.

1.1 <u>The Forerunners</u>

Early formulations of the quantity theory (as seen in the works of Bodin, Cantillon, Hume, Ricardo and Mill) established a relation between the quantity of money and the demand for commodities. The 20th century saw refinements to this classical version in two specific directions. The first, the transactions version, is associated with Newcomb and Fisher. The second, the cash-balance version, is associated with Marshall and Pigou.

Fisher began his analysis with the familiar exchange identity:

$$MV \equiv PT, \tag{1.1}$$

where M is the quantity of money, V is its velocity of circulation, P the general price level and T the volume of transactions. Since the volume of transactions was expected to persist in a fixed relation to the volume of full employment output, T was taken to be given. He viewed V as a variable dependent upon such institutional factors as the payment habits of the public, the extent of the use of credit and the speed of transportation and communication -- all of which could be treated as fixed in the short run. The exchange identity was thus transformed to the quantity equation:

$$M\bar{V} = P\bar{T}, \tag{1.2}$$

and Fisher (1911, p. 164) concluded,

> "The quantity theory of money thus rests, ultimately upon the fundamental peculiarity which money alone of all goods possesses -- the fact that it has no definite relation to the satisfaction of human wants, but only the power to purchase things which do have such satisfying power".

Though the Fisherian demand for money can still be found in contemporary work (see, for example, Pesek, 1970) a more fruitful line of development came through the work of Marshall and Pigou and their realization that money is capable of yielding utility through the provision of convenience and security. The Cambridge economists were able to formulate a demand theory in microeconomic terms from the choice-making behavior of individuals. The randomness in the timing of receipts and expenditures and the possibility of unforeseen contingencies led to the demand of a specific quantity of money. This quantity was postulated to vary proportionately with the volume of final transactions or the level of money income, that is,

$$M_d = KY = KPQ, \tag{1.3}$$

where M_d is the quantity of money demanded, Y the level of money income, Q an index of the real level of output in the economy (as opposed to T in the Fisherian version which was a measure of total transactions including transfers of used goods, intermediate goods, financial assets, etc.), and K the factor of proportionality.

In short, the Cambridge theory under the same institutional constraints imposed by Fisher, and equilibrium in the money market, yields the same quantity theory conclusion that price varies in direct proportion with the size of the money stock. Its advantage over the Fisherian version derives partly from its inception in microeconomic terms and partly from its greater flexibility. The most important contribution of the Cambridge economists was undoubtedly the suggestion that money demand was no different in principle from the demand for any other good.

It was, however, no more than a mere suggestion. Hicks (1935, p. 2) wrote that Marshall and his followers

> ".....were aware that money ought to be subjected to marginal utility analysis; but they were so dominated by the classical conception of money as a 'veil' (which is valid enough at a certain level of approximation) that they persisted in regarding the demand for money as a demand for the things which money can buy -- 'real balances'. As a result of this, their invocation of marginal utility remained little more than a pious hope".

The real break with classical tradition came in Keynes' Treatise which contained three separate theories of money. Hicks referred to the first as a glorified quantity theory, the second a Wicksellian natural rate theory, and the third, judged by Hicks to be the most important, one of relative preference between bank deposits and securities. History was to prove Hicks right for, in the following year, the General Theory was published and Keynesian 'liquidity preference' was formally launched.

In the General Theory, Keynes outlined three separate motives for the holding of cash. The first was a "transactions motive" which described the necessity of holding cash to bridge the gap between receipts and regular planned payments. For the payment of unexpected bills or for the meeting of sudden emergencies, there was to have existed a second motive which Keynes termed "precautionary". The third, or "speculative motive", arose from consideration of the choice, open to all individuals, of holding money or bonds. He assumed that the individual's financial wealth consisted of either all bonds, which he described as consols yielding a fixed nominal sum in perpetuity, or all money. The relevant rate of interest was therefore a long-run rate of interest. The individual was assumed to alter his holdings between money and bonds depending upon the relation between the market rate of interest and his own expected rate. Since this expected rate was subjectively formed and allowed to differ among individuals, a downward-sloping aggregate demand curve was possible.

It is at this juncture that the views of much of what is currently debated in monetary theory seem to diverge. It is clear, for example, that Keynes felt that the asset demand for money had theoretical substance and that he sought to deal with it by means of the precautionary motive (see the General Theory, p. 170).

This was glossed over in the neoclassical restatement of Keynes but reborn in a new and dominant role in subsequent portfolio approaches. Important extensions of the liquidity preference theory came about through the relaxation of certain simplifying assumptions. The consideration of portfolio diversification as behavior towards risk is one such example. The importance of relevant holding periods for money and term structure arguments are others. The Keynesian extension of Cambridge monetary theory might well be viewed a cornerstone in the development of monetary thought.

1.2 The Neoclassical Synthesis

In 1936, Hicks presented a paper at the Oxford meeting of the Econometric Society which was to have an impact, at least pedagogically, rivalling that of the General Theory itself. The contribution was a vehicle of expression, to become known as ISLM analysis, which incorporated the teachings of Keynes within a classical equilibrium model. In this model a liquidity-preference schedule is merged with a 'Cambridge-style' quantity equation to produce the LM or liquidity-money relation between the market rate of interest and real income. The IS curve is defined as the relationship between interest and real income consistent with equilibrium in the goods market. According to Hicks and his followers, notable among whom were Hansen, Klein, Modigliani and Samuelson, the essence of the General Theory could be reduced to a comment on the interest elasticities of money demand and investment.

The realization that this was not an adequate representation of Keynes' own view developed slowly. Davidson (1977, P. 277) wrote

> "By the 1950's, this mutant 'Keynesian' analysis was sufficiently entrenched in the orthodox macroeconomic literature that the few economists who were still faithful to developing Keynes' monetary analysis (as opposed to the Keynesian analysis) began to warn that what had been propagated as the Keynesian theory of output, employment, inflation and money was actually a perversion of Keynes' own views about the real world".

Weintraub (1957) was one of the first among many in the United States to make this point. He drew attention to the inflation slip of "classical Keynesianism" -- a model totally inadequate for the discussion of inflation. Robinson labelled it "bastard Keynesianism" and berated its timeless equilibrium nature. Leijonhufvud (1968) devoted an entire volume to the distinction between Keynesian economics and the economics of Keynes. Shackle emphasized the importance of uncertainty, historical time and money as the essence of Keynes' analysis and noted their conspicuous absence in the ISLM treatment. The realization, though slow to develop, nevertheless did and Hicks, himself, felt compelled to write nearly forty years after publication of his 1936 article

> "I begin....with the old ISLM (or SILL) diagram ....I must say that that diagram is now much less popular with me than I think it still is with many other people. It reduces the General Theory to (general) equilibrium economics; it is not really in time".*

*From "Some Questions of Time in Economics" mimeographed 1975 and referred to in Paul Davidson "Post-Keynes Monetary Theory and Inflation" in S. Weintraub ed. (1977).

Patinkin (1949) developed, at the microeconomic level, a monetary theory which was not far removed from the classical view of the world. He incorporated money as the nth good in a Walrasian general equilibrium system of n-1 commodities. He argued that the demand for real balances, in his simple model, was invariant under a change in the price level. He viewed the real balance effect as a mechanism through which the economy reacted to changes in the quantity of money in both the short- and long-runs. The essential difference between the short- and the long-run was that the long-run outcome of a change in the price level, exactly proportional to the change in money, was arrived at through a series of less-than proportionate steps. Although, at the end of any short run, the position attained was 'stable' in the sense that total supply equalled total demand, excess supply and demand for individuals in the market were not eliminated until long-run equilibrium was attained.

As with the Hicksian model, Patinkin's analysis met with considerable opposition. Archibald and Lipsey (1958) attacked it on the grounds that the real balance effect is a short-run disequilibrium phenomenon and, since the neoclassical theory is everywhere comparative statics, this type of dynamic mechanism did not belong. Other criticisms attacked the rigidity and narrowness of Patinkin's model. Meltzer, for example, demonstrated that with the inclusion of 'outside money' the neutrality of money was disturbed. Through open-market operations, the relative amounts of interest-bearing and non-interest-bearing assets change -- thus changing the net wealth of the system. Gurley and Shaw reinforced this point. Nobay and Johnson (1977) labelled Patinkin's analysis "pre-Wicksellian" monetary theory.

Patinkin's (1965) demand-for-money model is interesting for its explicit account of a stochastic payments process. He assumed that cash flows occur within time periods and that cash management decisions are made in the intervals between periods. All transactions were carried out in lump sums of m real dollars. The costs associated with holding cash were assumed directly proportional to the amount held. The cost associated with running short of cash was a fixed charge assessed only once per period. Given the possibility of random occurrence of N receipts and N payments during any period, the transactor was faced with the problem of optimizing his initial cash position. If, for example, he began the period with m dollars but encountered two payment transactions in succession then he had to pay a penalty. If, on the other hand, his first transaction was a receipt and was followed by an alternate succession of payments and receipts he had to forego rm dollars in income, where r is the cost per period of holding a dollar of cash. Dvoretzky proved in an appendix to chapter V of Patinkin (1965) that, at any stage of a randomly ordered sequence of N receipts and N payments, the probability that the payments will outnumber the receipts by M or more is approximately equal to exp $(-M^2/N)$. If initial holdings were mM dollars then the problem choice becomes one of minimizing the expected cost of cash management:

$$E = rmM + b\exp(-M^2/N), \tag{1.4}$$

where b is the penalty incurred because of a shortage of cash.

The first-order condition for a minimum on E is:

$$m = 2b\exp(-M^2/N)\,/(rN). \tag{1.5}$$

Transactions and interest elasticities may be computed from equation (1.5). Though Patinkin's model has passed relatively unnoticed in the literature, its contribution to the modern-era 'cash-flow' variant of the transactions view of money demand has been considerable.

1.3 Transactions and Asset Views

The importance of brokerage costs and the payments process had already been well-established prior to Patinkin's (1965) account. Hicks (1935) recognized the impact of "frictions" and paper-work costs on the equilibrating process of the money market. Both Baumol (1952) and Tobin (1956) built upon this theme in developing transactions models of the demand for cash.

The so-called "inventory model" begins at the level of the individual transactor. He receives an income payment once per time period and spends it all during the period at a uniform rate. At every moment, except at the very last instant in the period between the expenditure of his last cent and the receipt of his next pay, he will be holding financial assets. The cost of maintaining his portfolio will be the sum of two components: a brokerage cost assumed to be inversely proportional to the amount of cash held, and an opportunity cost assumed to be directly proportional to cash holdings. Minimizing total costs with respect to cash holdings yields the familiar 'square-root rule' whereby the demand for real balances is directly proportional to the square-root of brokerage costs and real income and inversely proportional to the square-root of the rate of interest.

The Baumol model abstracts from the question of choosing a representative interest rate by considering only one alternative interest-bearing asset. Later transactions theorists would claim that the representative rate is the rate on that asset which is most closely substitutable with money. Its maturity is sometimes referred to as the relevant holding-period of money. While most transactions theorists would accept that there exist many possible interest-bearing substitutes for money and that the holding-period relevant for one period may not be that which is relevant for the next, explicit account of the term structure as an explanatory variable is usually associated with the portfolio view of the demand for money.

The Baumol model does not guarantee homogeneity of degree one in prices. It will be so only as long as brokerage costs increase in step with the implicit price index for transactions. To the extent that structural changes cause relative price movements, however, homogeneity of degree one will not hold. Technological changes such as the increased use of computers or institutional changes such as increased competitiveness of the banking system are examples of suitable structural changes.

Despite the richness of the inventory model in emphasizing the importance of cash flow and costs associated with investing idle funds, its rigidity in transactions and interest elasticities is often cited as a disturbing feature. Miller and Orr (1966), for example, suggest that the problem with the Baumol model lies specifically in its assumptions regarding cash flow. While the familiar 'saw tooth' representation of the operating cash balance may be accurate for the household sector where income earners frequently receive payments at regular intervals, such is not the case for the business sector. The Miller-Orr model assumes a random-walk cash flow pattern in which cash flow is constrained within upper and lower bounds. In this formulation, it is shown that an important determinant of the demand for cash is the variance of cash flow. Depending upon whether an increase in cash flow variance is brought about by an increase in the frequency of transactions or by an increase in the real value of transactions, the transactions elasticity can be shown to lie in the range from 1/3 to 2/3. This model consolidates the lumpy portfolio adjustment cost feature of Baumol with the stochastic flow concept of Patinkin to produce a model more amenable to the explanation of business cash management than the early inventory models.

Responding, perhaps, partially to a desire to explain portfolio diversification and partially to the claim of transactions theorists that the speculative motive was inadequate as an explanation of money demand in the presence of an asset which dominates money, Tobin (1968) developed a more sophisticated version of liquidity preference. He viewed the liquidity preference of an individual as being dominated by behavior towards risk. Through indifference curve analysis he showed how portfolio diversification could be viewed as utility maximizing behavior for risk-averse individuals. The portfolio-selection theory initiated by Tobin was found to fit comfortably within the Keynesian framework. It relied essentially upon a price adjustment mechanism. An injection of cash into the banking system by the monetary authority, for example, would be expected to be felt immediately by short-term rates and, eventually, through a chain of portfolio substitutions, all along the term structure. The increase in bank reserves would be expected to reinforce, through the making of loans at more favourable terms and through a greater demand for securities, the decline in yields on all financial assets. With the supply price of, and the expected yield from, new real capital remaining virtually unchanged the portfolio positions of households and firms would be out of equilibrium. The direction of adjustment would naturally be toward those assets which have become relatively more valuable; namely, real capital.

Friedman and the 'Chicago school' adopt a portfolio approach in developing quite a different view of the economy. Here the emphasis is placed upon a quantity rather than a price adjustment. At any given point in time, an individual's wealth is in many forms, including financial assets, consumer durables and non-durables, and human and non-human capital. There is some preferred position in which these various quantities stand in relation to one another. With an injection of cash which causes the level of real balances to be in excess of desired real balances, agents will be working simultaneously to re-establish preferred positions. The combined effect is increased demand for consumer goods and physical capital. Subsequent price inflation will restore real balances to their desired level.

Friedman (1956) restated the quantity theory of money as a theory of the demand for money. Each wealth-owning unit was assumed to divide his wealth so as to maximize utility. Friedman grouped the numerous forms of wealth into five categories: 1) money, M, recognized as the generally acceptable means of payment at a fixed nominal value; 2) bonds, B, or claims to time streams of nominal income; 3) equities, S, or claims to stated pro-rata shares of returns of enterprises; 4) physical goods, G; and 5) human capital, H. The demand for money was written:

$$M_d = M_d \left(P, r_B, r_S, \frac{I}{P}\frac{dP}{dt}, W, Y, u\right), \tag{1.6}$$

with r_B the rate on bonds, W the ratio of non-human to human wealth, Y nominal income, and u standing for all other variables which might affect tastes and preferences. Friedman assumed homogeneity of degree one in prices and nominal income and rewrote (1.6) in terms of velocity, or Y/M. The velocity of money became a function of other variables. Its stability over time is often taken to be fundamental to the monetarist position.

1.4 What is Money?

To address the definition of money only now, after a survey of seventy years of demand for money literature, may strike the reader as a peculiar ordering of priorities. It is, in fact, consistent with the manner in which monetary thought has evolved. For much of the 'classical' era preceding Keynes, the concept of money as a lubricant to trade was one which dominated theory. Though neither Fisher nor Marshall held constant-velocity views of the monetary system, the general thrust of both theories upheld the classical neutrality of money. The Keynesian era added a new dimension to the economic perception of money. It became recognized that money was useful not only as a medium of exchange, but also as a store of value.

In addition to the new dimensionality of money as yielding utility, the Keynesian era saw the introduction of a new mechanism linking the monetary and real sectors. Though it is usually associated with Pigou as a reaction against Keynesian theory, the wealth effect is also found in the General Theory. In the simple Keynesian model with a horizontal LM curve (or a vertical IS curve] any shift to the right in LM brought about by a fall in prices is not effective in raising demand. The fall in prices, or interest rates as the case may be, will, however, have a positive impact upon wealth and consequently upon demand for goods and services. Both the price-induced and the interest-induced wealth effects can be found in the General Theory (pp. 92-94).

The suggested relevance of money to the real economy thrust the issue of its definition to the forefront of the monetary debate. The conventional view at the time was that money must act simultaneously as an asset to its holder and as a liability to its issuer. This suggested that money had no 'net' existence and hence was of no 'real' importance. An increase in fiat or 'outside' money, for example, is an increase in the indebtedness of the Government. Gurley and Shaw contend that a non-zero net wealth effect is possible from an issue of 'inside' money. Inside money is defined as being government debt (fiat money) issued in payment for government purchases of private securities. The increase in government debt is backed by increased indebtedness of the private sector to Government. The value of this construction might be questioned on the grounds that households must ultimately perceive that they own the businesses whose indebtedness has been increased. This, however, may be carrying an abstract argument to its extreme. Pesek and Saving (1967) defend the opposite extreme in defining wealth as being any commodity that yields a positive income to the owner and no negative income to the non-owner. They stand apart from the mainstream in their belief that all forms of money, whether fiat money, demand deposits, or travellers' cheques, represent monetary wealth.

The definition of money which would be relevant for transactions demand is a narrow one. Money is thought of as being primarily a medium of exchange and hence necessarily highly liquid. For the asset demand for money the definitional lines are less clearly drawn and the choice has become one of empirical preference. Since demand functions for the broader aggregates normally exhibit greater stability, these are generally favoured by the monetarist school. Laidler (1980), for example, argues that since currency plus demand deposits (M1) is more susceptible to shifts arising from newly-invented instruments than any of the broader aggregates and since financial innovation is difficult to forecast, then a broader aggregate is preferable to M1 for policy-making as well as empirical analysis.

With respect to the institutional environment of the United States, evolving definitions of money are especially relevant. The Federal Reserve Board, for example, has recently (February 7, 1980) implemented new definitions for the monetary aggregates. The redefinitions were necessary to account for the emergence of new deposit instruments. They had threatened to make existing aggregates obsolete as intermediate targets for monetary policy.

Broaddus (1978) gives a detailed account of the historical development of the automatic transfer service (ATS) afforded by the commercial banks. In June 1972, state-chartered mutual savings banks in Massachusetts began offering negotiable orders of withdrawal (NOW) accounts following a favourable ruling of the Massachusetts Supreme Court. NOW accounts are effectively interest-bearing chequing accounts. By January 1974, this ability was extended by federal legislation to all depository institutions. In April 1975, commercial banks were authorized to transfer funds from savings to chequing accounts upon receipt of a depositor's telephone call. As of November 1, 1978, member banks of the Federal Reserve System were allowed to transfer funds from a depositor's savings account to his chequing account automatically subject to certain prearranged conditions. The ATS accounts enable the depositor to earn interest on funds which otherwise would be held in chequing accounts. The argument is made that since cheques may be written against balances held in ATS accounts then they should be included in M1, the aggregate which is generally taken to represent the economy's medium of exchange. Any transfer of funds from demand deposits to ATS deposits will reduce M1 without any consequent reduction in money. Also, on an institutional level, since required reserves for ATS accounts are less than that for demand deposits such a transfer will increase excess reserves in the system and may operate against the authority's wishes for restraint.

Repurchase agreements (RPs), although not new to the financial system, have grown rapidly in the 1970s. Securities are sold with the accompanying agreement that the seller will repurchase them at a later date. A typical RP transaction may take place when a corporate customer of a commercial bank has funds, say $1 million, which will not be needed for a day or more. He may earn interest at or near the federal funds rate in the interim by purchasing a government security from his bank. The bank will simultaneously agree to repurchase the security at a specified future date. Since the maturities of the RPs are frequently so short that cheques may be written on them, they too could be included as a component of the medium of exchange. This instrument, in fact, allows demand deposits to be much larger during the day when business is being conducted than at the close of the day when the demand deposits are recorded for purposes of reserve accounting.

The Canadian experience of the 1970s, though similar to that of the U.S., is not as well documented. This is perhaps due to the fact that the Canadian banks operate under less formal regulatory control. The emergence of new instruments in Canada often appear in response to competitive pressures and are not necessarily accompanied by federal legislation. Such a phenomenon occurred in early 1972 when the major chartered banks began offering their corporate customers cash management services. These services, though different across banks, in general allowed the automatic transfer of funds and consolidation of balances in different accounts. They gave to corporate clients the options of:

1) earning interest on their current accounts (CAs),

2) the automatic transfer of funds in excess of a pre-specified balance into certificates of deposit (CDs), or

3) the automatic paydown of loans in multiples of $10,000 to $25,000 for funds in excess of some maximum desired balance.

The result of increased cash management efficiency has been a sharp reduction in the demand for CAs, and consequently for M1. Financial innovation in the 1970s has been a cause for concern of monetary authorities in both Canada and the United States.

1.5 Empirical Studies

Early empirical work on the demand for money was undertaken in an effort to validate certain precepts of Keynesian theory. The notion of a speculative motive for holding money encouraged the movement of research activity in the 1930s and 1940s toward empirical investigation. Two pioneering studies by Brown (1939) and Tobin (1947) launched a body of literature on the relevance of interest rates to the demand for money. The evidence supported the theory well as to the importance of an interest rate variable. Since then a voluminous empirical literature on the many facets of money demand has been generated. The present section attempts to summarize that literature by grouping the important studies under five subject headings. These headings are considered to be the issues most relevant to the demand for money in Canada for the time period under study. Each will be examined in turn in a subsequent chapter of this thesis.

The question of stability of money demand has been one of surviving interest in the monetary debate. Laidler (1977) frames the familiar monetarist-Keynesian dispute in terms of the relative stability of money demand and expenditure functions. Boorman (1972) has called the stability of demand, together with the capacity of the monetary authority to influence the relevant money aggregate, "necessary conditions for the successful implementation of monetary policy". Though both Teigen (1964) and Meltzer (1963) were able to isolate stable demand for money relations, the first serious attempt at determining relative stability of narrow and broad money appeared in Laidler (1966a). He favoured the broad aggregate. This has been the consensus of most U.S. studies to date [Goldfeld (1976), Enzler, Johnson and Paulus (1976), and Porter, Maukopf and Simpson (1979)]. Studies which found stability for both broad and narrow aggregates include Weintraub and Hosek (1970) and Khan (1974). Recent success in isolating stable functions for both broad and narrow money has been reported in Heller and Khan (1979) and Cargill and Meyer (1979). The Canadian data seem to support the relative stability of narrow money. Clinton (1973), Foot (1977), Poloz (1979) and Cameron (1979) all report that M1 is more stable while only Rausser and Laumas (1976) hold the opposite view. For France, Melitz (1976) reports that broad money has a relatively stable demand function. For the United Kingdom, Hacche (1974) supports a narrow aggregate.

The second question for empirical investigation concerns the choice of appropriate functional form. The early empirical work of Brown (1939) specified the demand for idle money as a linear function of the rate of interest, the change in the rate of interest and the change in the level of prices. Bronfenbrenner and Mayer were early proponents of a logarithmic functional form. Indecision as to whether linear or logarithmic is the more appropriate form has been registered in Eisner (1963) and Chow (1966). Mixed functional forms have broadened the field considerably. Latané (1954), for example, tried a demand equation linear in income and reciprocal in interest rates. Konstas and Khouja (1969) used a similar form with an interest rate floor to explore the existence of a liquidity trap. Cagan (1956) hypothesized that the demand for money is semilogarithmic with the level of anticipated inflation explaining the logarithm of real balances. A more recent variant of the semilogarithmic form appeared in Hacche (1974). This equation is linear in the interest rate and logarithmic in all other variables. It has gained contemporary acceptance in Klein (1974), White (1976) and Cameron (1979).

Box and Cox (1964) developed a statistical technique for estimating the power transformation which best suits the data. Such a procedure has been applied to the demand for money in several studies. Zarembka (1968) pioneered this work and found that the logarithmic formulation is more appropriate than the linear for the demand for money. Furthermore, he found that this result is not sensitive to model specification. White (1972) used the Box and Cox transformation in a re-examination of the liquidity trap. Spitzer (1976) tried a generalized Box-Cox function and Spitzer (1977) constructed a simultaneous equation model of demand and supply using the generalized forms. Mills (1978) gave evidence from the U.K. economy that the demand for narrow money (as modelled by conventional specifications) is not sensitive to choice of functional form.

The third topic of interest concerns causality. Sims (1972a), using post-war U.S. data, found that he could reject the exogeneity of GNP affecting money but that he could not reject the exogeneity of money affecting GNP. He concluded that one should not estimate money demand relations which treat GNP as an exogenous explanatory variable. Pierce (1974) performed cross-correlation analyses on four components of the money supply, two interest rates, bank reserves and retail sales and concluded that predictions of money supply can be only marginally improved, if at all, by including as explanatory variables past, present and future values of any series other than money supply itself. Barth and Bennett (1974) could not support Sims' finding of unidirectional causality from money to GNP for Canada. They, in fact, reported unidirectional causality in the opposite direction when an index of industrial production was used as a scale variable. Williams, Goodhart and Gowland (1976) emphasized the complexity of the causal pattern when prices and exchange rate policy are taken into consideration. Sargent and Wallace (1973) investigated the direction of causality between money and prices and could not reject unidirectional causality from money to prices. In a multivariate analysis, Mehra (1978) could not reject the Sims result as long as nominal variables were used. When real balances were regressed upon real income and nominal interest rates, however, he could not reject that proper causality flowed from right to left. In an important refutation of Barth and Bennett, Auerbach and Rutner (1978) illustrated the damaging effects of improper filtering techniques. Putnam and Wilford (1978) rationalized the different causal patterns of the U.S. and U.K. economies on the basis of different exchange rate regimes. Mills and Wood (1978) supported this hypothesis. Hsiao (1979) performed a bivariate causality test for Canadian money and income. His technique employed the Akaike final prediction error criterion for choosing the appropriate lag length in the Sims framework. He reported bidirectional causality between M1 and GNP and unidirectional causality from GNP to M2.

The fourth area of concern deals with the issue of dynamics. The discussion begins with the partial adjustment model since it is the dynamic specification which has predominated the early empirical work. Chow (1966) appears to have been the first with a theoretical description of the geometrically declining lag structure for money demand. He felt that the distinction between short-run and long-run demands for money was too sharply drawn. Short-run demand for money was believed to be dominated by a 'transactions' motive with emphasis on current income and the long-run demand for money was believed to be governed by an 'assets' motive, with the emphasis on permanent income. Chow designed a partial adjustment model which captured both the short- and long-runs and used it to empirically determine the relative importance of permanent versus current income. While Chow's approach served as a good description of the model, a more theory-oriented derivation appeared in Feige (1967). He isolated two different theoretical justifications for the use of a stock-adjustment model. The first, due to Cagan, dealt with 'adaptive' expectations and the second involved costs of adjustment. A thorough examination of this and other partial adjustment models was given by Griliches (1967).

Applications of the partial adjustment-adaptive expectations demand for money model have appeared in many studies. Their estimates of the speeds at which various economies adjust toward their long-run demand curves have covered a wide range of possibilities. De Leeuw (1967), for example, estimated unrealistically long lags of adjustment for the U.S. economy. This conflicted with results from Feige (1967) and Starleaf (1970) which suggested that of the two effects -- expectational and partial adjustment -- the expectional lag is the dominant one. Furthermore, when permanent income was used as the income constraint, there was found to be no lag of adjustment for the U.S. economy. A similar diversity has appeared in Canadian studies. Clinton (1973) estimated that the mean lag from M1 between the first quarter of 1955 and the fourth quarter of 1970 (1955 I - 1970 IV), varied from 2.7, when the rate on 90-day finance company paper (R90) was used as the interest variable, to 7.3 quarters when the over-10-year Government of Canada bond rate was used. The mean lag for broader money (M1 plus personal savings plus non-personal term and notice deposits] was found to vary from 7.5 to 22.8 quarters. Villaneuva and Arya (1975), using permanent income, found complete adjustment within one quarter for both broad and narrow aggregates. The period of the study was 1958 I to 1971 I. Al-Khuri and Nsouli (1975) estimated mean lags which varied from 2.3 to 2.5 quarters for M1 and from 6.7 to 10.1 quarters for M2 using Canadian data for the period 1960 I to 1970 IV.

Much of the work considered thus far has been based upon the adjustment in the reallocation of the existing portfolio. A second possibility for adjustment is through the reallocation of the flow of savings. When the desired level of an asset differs from its actual level this divergence can be dissipated either by changing the levels of all existing assets or by redirecting the flow of savings toward or away from this asset. Brainard and Tobin (1968), by explicitly considering cross-adjustment effects between assets in the adjustment process, were among the first to model this type of behavior. Modigliani (1972) and Friedman (1977) also emphasized the importance of the reallocation of savings. White (1977) considered this effect under moderate inflation. Santomero and Seater (1978) appealed to search behavior as an explanation of the presence of adjustment. Brillemburg (1979) reformulated the partial adjustment under conditions of uncertainty.

Although the partial adjustment model has prevailed in the money demand literature other dynamic specifications have been used. White (1976) and Cameron (1979) used Almon distributed lags on income and interest variables in Canadian demand for money equations. Lieberman (1978) used the Shiller technique in estimating money demand for the U.S. economy. Cargill and Meyer (1979) consider, within the context of variable parameter regression, the possibility that the parameters of money demand have evolved through time.

The fifth area of research centres upon the debate between transactions and asset theorists. While this debate has not easily lent itself to empirical investigation, it is possible to isolate at least three salient points on which the two views fail to converge. The first involves choice of the scale variable, the second regards the relevance of the term structure, and the third concerns the role of expectations.

As has already been noted, the transactions approach to money demand focuses on income as the scale variable while the asset approach uses long-run variables such as permanent income or wealth. As an either/or proposition the empirical results are inconclusive. Brunner and Meltzer (1963) using annual U.S. data for 1910-40 and 1951-58 found that the demand for M1 is more stable when constrained by wealth rather than either measured or permanent income. On the basis of coefficients of determination, R^2s, for regressions performed over various subsamples from 1892-1960 using annual U.S. data, Laidler (1966) concluded that permanent income is a better explanatory variable than either income or non-human wealth. His evidence came, however, primarily from a broader definition of money and was less decisive with regard to M1. Meltzer

(1963) noted that, with both real income and real wealth as explanatory variables for M1, the wealth variable entered with an elasticity of 0.97 and a t-statistic of 9.5 and the income elasticity positive but not significantly different from zero. Due to the possible collinearity of income and wealth, however, this does not constitute a convincing rejection of the transactions argument. Studies using this same approach have, in fact, come to opposite conclusions. Bronfenbrenner and Mayer (1960) using annual data from 1919 to 1956 show significance for real GNP and insignificance for real wealth. Heller (1965) using quarterly data from 1947 to 1958 also reported a significant income elasticity and an insignificant wealth elasticity. Modigliani, Rasche and Cooper (1970) tried the value of stock transactions in the demand for money and found it to have the expected positive sign but not significantly different from zero. Alchian and Klein (1973) included the Standard and Poor's 500 Common Stock Price Index divided by the GNP deflator. The corresponding coefficient was significantly different from zero and equal to .0369.

The problem of near-collinearity between income and wealth variables has effectively been avoided in several cross-sectional studies. The bulk of these studies show joint determination of the demand for money by both income and wealth. Lee (1964) using the 1957-58 U.S. Survey of Consumer Finances showed significant coefficients of 1.27 and 0.39 on income and net worth respectively. Nieuwenburg (1969) using the 1960 Dutch Savings Survey reported a current account income elasticity of .42 and a current account net worth elasticity of .04. Both estimates were significantly different from zero. Peterson (1975) using the 1960-62 U.S. Survey of Consumer Finances supported the findings of Lee. In an equation with current income, the estimated income and wealth elasticities were 0.98 and 0.11, respectively. With permanent income, they were 1.47 and .09. As a check on the credibility of these cross-sectional estimates we may compare them with the wealth elasticities as computed in Thomson, Pierce and Parry (1975). They defined wealth as the sum of currency plus privately-held deposits plus Treasury bill holdings plus other asset holdings less loans to the banking system. They then estimated a monthly money market model for the U.S. for the period 1960:1 through 1968:6. A dollar increase in wealth was estimated to result in a \$.03 increase in desired currency holdings and \$.16 increase in demand deposit holdings. Although Lee's estimates are high in comparison, both Nieuwenburg and Peterson compare favourably with these time series results. The results of these studies are summarized in Table 1.

In reference to the second point of differentiation, I believe that the transaction approach leans more towards the use of a single representative rate, as determined by the appropriate holding period, than the asset approach. Goldfeld (1973, 1976), for example, considers many single interest rate demand models. The asset view, as theoretically demonstrated in Friedman (1977), is quite explicit about the relevance of the term structure.

TABLE 1.1: INCOME AND WEALTH ELASTICITIES IN THE DEMAND FOR NARROW MONEY

Study	Data	Period	Dependent Variable	Wealth Variable	Income Elasticity	Wealth Elasticity
Meltzer (1963, p. 232)	U.S. Annual	1900-58	M1	Real non-human Wealth	0.13 (1.4)	0.97 (9.5)
Bronfenbrenner and Mayer, (1960, p. 817)	U.S. Annual	1919-56	M1	Goldsmith Total*** Wealth (1956)	0.34 (4.0)	0.12 (1.3)
Heller (1965, p. 301)	U.S. Quarterly	1947-58	M1	Goldsmith Total Wealth (1962) minus gov't assets	0.82 (3.3)	-0.21 (-0.9)
Chow (1966, p. 119)	U.S. Annual	1897-1958 (ex. war yrs.)	M1	Total private assets	0.39 (3.6)	0.64 (5.6)
Goldfeld (1973, p. 614)	U.S. Quarterly	1961-72	M1	Net Worth (MPS model)	0.60*	0.11*
Lee (1964, p. 754)	U.S. Survey (cross-section)	1957-58	Chequing deposits	Net Worth (ex. real estate)	1.27**	0.39**
Nieuwenburg (1969, p. 262)	Dutch Survey (cross-section)	1960	Current accounts	Net Worth (incl. real estate)	0.42 (11.0)	0.04 (19.6)
Peterson (1975, pp. 84-5)	U.S. Survey (cross-section)	1960-62	Chequing deposits	Total non-monetary assets (incl. govt. and corp. bonds)	0.98 (8.4)	0.11 (5.7)

* These are steady-state coefficients. The short-run income coefficient had a t-statistic of 3.8 and the short-run wealth coefficient a t-statistic of 1.4 .

** t-values are not available. Both are significant at the 1 per cent level.

*** The principal difference between Goldsmith Total Wealth and Goldsmith Net Worth is that Net Worth contains equity assets and Total Wealth does not (see Meltzer, 1963, p. 228).

In Heller and Khan (1979), the slope, intercept and curvature of a quadratic term structure are estimated over time and entered as explanatory variables in the demand for money. They reported a stable function.

On the issue of expectations, the dividing line between transactions and asset views is less clearly drawn. Clower and Howitt (1978), for example, would claim the relevance of expectations to the transactions demand for money. Goldfeld (1976), on the other hand, stated "On a strict transactions view of the demand for money, a variable measuring anticipated inflation seems to have no place". A unanimous view in favour of the relevance of expectations is held by the Chicago school. The failure of early empirical evidence to support the relevance of expectations, however, caused them a degree of concern. In 1963, Friedman and Schwartz were moved to comment

> "Failure has marked every attempt we know of to find a systematic relation between the quantity of money demanded in the United States and either the current rate of change in commodity prices or a weighted average of past rates of change in prices, taken as an estimate of the rate of change expected to prevail in the future".

Cagan (1956) established price expectations as an important variable in the demand for money for seven hyperinflations. This led to the hypothesis that agents respond to price expectations in setting their demand for money only if inflation is very high. This 'threshold effect' argument is formally considered in Barro (1970). What made this hypothesis questionable, however, was the observation that expectations were also inoperative at very high rates of inflation. All studies of the German hyperinflation including Cagan (1956), Barro (1970), Evans (1978), Frenkel (1978), Garber (1976), and Sargent (1977) were forced to ignore data in the final months of 1923. During these months, real money balances moved (contrary to accepted theory) positively with inflation. Cagan's explanation that rumours of impending monetary reform had a dampening effect on price expectations during these months is tested and supported in Flood and Garber (1980).

Recent exceptions to the empirical failure observed by Friedman and Schwartz have been Shapiro (1973) for the U.S., Smith and Winder (1971) for Canada, and Valentine (1977) for Australia. The inability of any one study to make a definite statement is due to the nature of expectations -- they are not directly observable. Empirical results are highly dependent upon the process by which expectations are assumed to be generated.

* * * * *

Chapter 2 of this thesis considers the empirical question of stability of demand for money in Canada between 1955 and 1977. The chapter begins with a description of the econometric tests to be used and a Monte Carlo examination of their reliability. Having established the properties of the tests, the chapter proceeds with a stability analysis of demand for two different aggregates. The results of the analysis are then discussed within the context of the changing institutional framework of the Canadian financial system.

Chapter 3 considers the choice of appropriate functional form. It begins with a theoretical description of the Box and Cox procedure. The procedure in its simplest form is then applied to a demand for narrow money formulation. Subsequent relaxation of constraints within the simple model allows for the testing of mixed functional forms and of mixed functional forms with errors autoregressive of order one. The final section of this chapter discusses the theoretical implications of the semilogarithmic form. Alternative forms are compared on both theoretical and empirical levels.

Chapter 4 extends and improves upon our knowledge of causality in the Canadian economy. The emphasis of this chapter is upon technical improvement. It begins with a review of the published literature. Six basic flaws in the methodology of previous causality studies are isolated. Through their systematic elimination from the methodology described in Chapter 4, it is hoped that the empirical results produced will give an accurate assessment of causality for Canadian money demand.

Chapter 5 examines the dynamics of money demand in Canada. It consists of four main sections. The first discusses the Koyck distributed lag and considers associated theoretical problems. The second examines the Almon distributed lag. The third deals with error structures in the partial adjustment-adaptive expectations model. The fourth examines the realism of the Koyck constraint to equality of the response pattern of money to all arguments of the system, and tests empirically the hypothesis that the adjustment of demand for real balances in Canada has changed over time.

Chapter 6 focuses on empirical issues which appear to differentiate transactions and asset views of money demand. In particular, two rather unrelated issues are chosen. The first deals with the observation of instability in the demand for M1 in the mid-1970s and its apparent causes. Transaction-style models for the components of narrow money are designed and estimated in an effort to isolate the apparent demand shift. The second issue concerns the formation of expectations and their relevance to money demand. The expectations literature is surveyed and a series of 'economically rational' expectations is constructed. In addition, a partial adjustment model of the demand for money is developed in which alternative expectations hypotheses may be tested. The results are reported with the intention of improving upon the existing knowledge of money demand in Canada.

CHAPTER 2

STABILITY

Relative constancy of regression parameters over time has been a key issue in the evaluation of economic theory. The demand for money is one topic which has received wide attention. Unfortunately, there remains an unsettling lack of consistency in the studies that have been conducted in this area. Not only has there failed to emerge a general consensus as to the relative stability of the various money aggregates, techniques employed in assessing stability have had methodological weaknesses. The conventional approach has been to propose a plausible single-equation model, to choose a reference time period which encloses a potential breakpoint, and to test for constancy of the regression coefficients over the two sub-periods before and after the assumed breakpoint. Fortunately, techniques have been developed which do not require advance knowledge of the position in time of the structural shift. Three such techniques will be described in this chapter and examined for sensitivity to different forms of change. Two of the three techniques, the cumulative sum of recursive residuals (CUSUM) and the cumulative sum of squares of recursive residuals (CSQ) tests, are developed in Brown, Durbin and Evans (1975).* The third, developed by Riddell (1978), is a computational simplification of the Chow test for structural change applied at each observation point from k+1 to T-k-1, where k is the number of explanatory variables and T is the number of observations for each variable.

In Section 2.1, I describe each test in turn. Then in 2.2, I apply each in Monte Carlo simulations of various forms of structural change and assess the relative merits of the tests. In the following two sections respectively, I describe and analyse the stability of conventional money demand models for Canada. In 2.5, I reexamine the stability of both broad and narrow money relationships using cubic splines. My conclusions are: 1) the CSQ test is the most powerful of the three tests in the detection of coefficient, error variance shift; 2) the Stepwise Chow test is most powerful in the detection of pure coefficient shift; 3) instability is observed at various points in time for both broad and narrow aggregates but the narrow aggregate exhibits greater long-run stability; and 4) local instabilities emerge at points of known legal or institutional change.

2.1 Description of Tests

Consider the linear model:

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*These tests are embodied in TIMVAR, a computer program obtained from the Central Statistical Office, London, England.

$$y_t = x_t'\beta_t + u_t, \quad t = 1, \ldots, T;$$

where x_t' is a 1 by k vector of non-stochastic explanatory variables, β_t is a k by 1 coefficient vector and u_t is a random error term normally and independently distributed with zero mean and variance σ_t^2 [henceforth $NID(0,\sigma_t^2)$]. The hypothesis of constancy over time, call it H_0, is that β_t and σ_t are invariant with respect to t and equal to β and σ. Letting $X_r' = [x_1, x_2, \ldots, x_r]$ and $Y_r' = [y_1, y_2, \ldots, y_r]$, the least squares estimate of β based upon the first r observations is given by:

$$b_r = (X_r' X_r)^{-1} X_r' Y_r.$$

The T-k 'recursive residuals' are defined:

$$w_r = (Y_r - X_r' b_{r-1})/\delta \quad \text{for } r = k+1, k+2, \ldots, T;$$

where $\delta = [1 + X_r' (x_{r-1}' x_{r-1})^{-1} X_r]^{1/2}$. (2.1)

Under H_0, the w_r are $NID(0,\sigma^2)$ and their cumulative sums normalized by the estimated standard deviation, $\hat{\sigma}$, are written:

$$W_r = (1/\hat{\sigma}) \sum_{k+1}^{r} w_j .$$

W_r are approximately normal variables with means, variances and covariances given by:

$$E(W_r) = 0, \quad V(W_r) = r-k, \text{ and } COV(W_r,W_s) = MIN(r,s)-k .$$

If, however, the β_t are constant up to a point t^{+} but differ from then on:

$$E(W_r) = 0 \quad \text{for} \quad r = k+1, \ldots, t^{+}; \text{ but}$$

$$E(W_r) \neq 0 \quad \text{for} \quad r = t^{+}+1, \ldots, T.$$

The cumulative sum of recursive residuals (CUSUM) test is designed to detect significant departures of W_r from its mean-value line.

The cumulative sums of squares of recursive residuals normalized by the estimated error variance are written:

$$S_r = \left(\sum_{k+1}^{r} W_j^2\right)/\left(\sum_{k+1}^{T} W_j^2\right) \quad \text{for} \quad r = k+1, \ldots, T.$$

If T-k is even there are ½(T-k)-1 independent observations in the set $\{S\} = \{S_{k+2}, S_{k+4}, \ldots, S_{T-2}\}$; whereas, if T-k is odd $\{S\}$ has ½(T-k)-(3/2) elements. $\{S\}$ can be shown to be an ordered sample drawn from the uniform (0,1) distribution. The statistics:

$$C^{+} = \mathrm{MAX}(S_{k+2j} - j/m), \text{ and}$$

$$C^{-} = \mathrm{MAX}(j/m - S_{k+2j}), \quad \text{for } j = 1,2, \ldots, m-1 \text{ and } m = \tfrac{1}{2}(T-k),$$

are distributed as Pyke's (1959) modified Kolmogorov-Smirnov statistic, the significance values of which have been tabulated. Alternatively, C^{+} and C^{-} are the maximum positive and negative deviations of the elements of $\{S\}$ from their hypothetical mean-value line. Given that the maximum positive and negative deviations of the <u>whole</u> set of S_r from their mean-value line are close enough approximations to C^+ and C^-, they can be compared with the tabulated distribution to test for significant departure from constancy. This is known as the cusum of squares or CSQ test.

If it is known that a linear regression model obeys two distinct regimes and that its change point occurs at point t^+, the sample can be split into two subsamples of t^+ pre-shift and $T-t^+$ post-shift observations. The conventional Chow test consists of comparing the statistic:

$$F_2(t^+) = (T-2k)\left[S-(S_1+S_2)\right]/k(S_1+S_2),$$

with an F distribution having k and T-2k degrees of freedom, where S is the residual sum of squares (RSS) taken from the linear regression on the full sample, S_1 is the RSS taken from the linear regression on the first t^+ observations and S_2 the RSS taken from the last $T-t^+$ observations. Riddell (1978) developed a computationally simple technique for performing this test at each of the potential breakpoints from k+1 to T-k-1. S_1 can be shown to be equal to the cumulative sum of squares of the <u>forward</u> recursive residuals from k+1 to t^+, and S_2 to the cumulative sum of squares of the <u>backward</u> recursive residuals from T-k-1 to t^++1. The series of $F_2(t^+)$ for t^+ = k+1, ... , T-k-1 can be computed from one set of forward and one set of backward recursive regressions instead of the 3+2(T-2k) OLS regressions previously required. This will be referred to as the Stepwise Chow test.

Unfortunately, doubt as to the merits of all three tests has been registered in the literature. Johnson and Bagshaw (1974), for example, found that the CUSUM tests are not robust to departure from independence in the error term. Garbade (1977) applied CUSUM, CSQ, and variable parameter (VPR) tests in Monte Carlo simulations of parametric instability and found that both the CUSUM and CSQ tests were less powerful than VPR. The CUSUM test was judged "quite weak". Brown, Durbin and Evans (1975) stress that the CSQ statistics should be thought of as "yardsticks against which to assess the observed sample path rather than providing formal tests of significance." The inherent problem with the Stepwise Chow test is that for it to be exact at each point in time change must take place nowhere else in the sample. $F_2(t^+)$ is only distributed as F when t^+ coincides with the true breakpoint. Riddell (1978) illustrates two ways in which inappropriate inferences as to the timing of structural shocks can be made from the Stepwise Chow test. He shows how a point at which structural change actually occurs can go undetected and how points at which no-change occurs can be identified as significant breakpoints.

2.2 Shock Simulations and Comparative Reliability

Stochastic simulations of different forms of structural change were introduced to the model:

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t ,$$

where $\beta_0 = -1.4181$, $\beta_1 = -0.0681$, $\beta_2 = 0.6349$, $\sigma^2 = 0.25$ and u_t

is a randomly generated $N(0,\sigma^2)$ error term.* The data consisted of 87 observations and each experiment was replicated 10 times. Eight different shocks were tried: 1) discrete change in β_0 at midsample (trials 1 - 6 of Table 2.1); 2) discrete change in β_0 early in sample (trials 7 - 11); 3) discrete change in β_0 late in sample (trials 12 - 16); 4) gradual change in β_0 at midsample (trials 17 - 21); 5) discrete change in σ^2 at midsample (trials 22 - 26); 6) discrete change in σ^2 early in sample (trials 27 - 31); 7) discrete change in both β_0 and σ^2 (trial 32); discrete change in β_0 with autoregressive errors (trials 33 - 44). The incidence of rejection of the null hypothesis is recorded in Table 2.1.

The results from the CUSUM tests are shown in the first and second columns of Table 2.1. When structural shift occurs in the coefficients of the underlying model with no change in the error variance, the CUSUM test is fairly sensitive to it. Its sensitivity increases with the magnitude of the shift. There is no appreciable difference in the power of the test according to the time location of the shift, although the test performed on data arranged chronologically does better for late sample changes

*The model is based upon a logarithmic demand for money model. The variables X_1 and X_2 are interest and income series, respectively. The data are described in Section 2.3.

TABLE 2.1: INCIDENCE OF REJECTION OF NULL HYPOTHESIS OF STABILITY AT THE 90 PER CENT LEVEL FOR THREE STABILITY TESTS UNDER VARIOUS FORMS OF STRUCTURAL SHIFT

Model: $y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t$

TRIAL	DESCRIPTION OF STRUCTURAL SHIFT	Rejection Incidence for 10 Replications: CUSUM Back.	CUSUM For.	CSQ Back.	CSQ For.	Step. Chow
1	No change	0	1	2	0	0
2	20 per cent discrete change in β_0 at midsample*	2	3	2	2	1
3	40 per cent	5	5	5	3	8
4	60 per cent	2	6	10	2	10
5	80 per cent	8	7	10	9	10
6	100 per cent	10	8	10	10	10
7	20 per cent discrete change in β_0 early in sample	3	1	3	0	2
8	40 per cent	4	8	5	0	7
9	60 per cent	9	10	10	4	10
10	80 per cent	10	10	10	3	10
11	100 per cent	10	10	10	5	10
12	20 per cent discrete change in β_0 late in sample	1	2	1	1	0
13	40 per cent	3	4	1	5	0
14	60 per cent	6	6	5	9	3
15	80 per cent	5	10	8	10	9
16	100 per cent	6	10	10	10	9

*This represents a change of roughly 0.6 standard errors of estimate.

TRIAL	DESCRIPTION OF STRUCTURAL SHIFT	Rejection Incidence for 10 Replications: CUSUM Back.	CUSUM For.	CSQ Back.	CSQ For.	Step. Chow
17	20 per cent gradual change in β_0 at mid-sample	1	2	1	0	0
18	40 per cent	2	4	2	1	3
19	60 per cent	9	8	4	8	10
20	80 per cent	8	9	7	6	10
21	100 per cent	10	10	10	10	10
22	10 per cent change in σ^2 at midsample	1	1	4	3	0
23	20 per cent	0	1	2	3	0
24	30 per cent	0	2	5	4	0
25	40 per cent	0	1	10	10	0
26	50 per cent	0	3	10	10	0
27	10 per cent change in σ^2 early in sample	1	0	1	1	0
28	20 per cent	0	1	5	5	0
29	30 per cent	0	1	7	6	0
30	40 per cent	0	0	4	6	0
31	50 per cent	0	3	9	8	0
32	10 per cent change in β_0 and 40 percent change in σ^2 at midsample	1	3	10	8	1
33	No change with AR(1) in error (RHO = .2)	2	1	2	0	0

TRIAL	DESCRIPTION OF STRUCTURAL SHIFT	Rejection Incidence for 10 Replications: CUSUM Back.	CUSUM For.	CSQ Back.	CSQ For.	Step. Chow
34	20 per cent change in β_0 with AR(1) in error (RHO = .2)	3	3	5	1	1
35	40 per cent	4	4	5	2	7
36	60 per cent	3	6	9	4	10
37	80 per cent	9	7	9	9	10
38	100 per cent	10	8	10	10	10
39	No change with AR(1) in error (RHO = .8)	7	7	7	7	8
40	20 per cent change in β_0 with AR(1) in error (RHO = .8)	9	6	7	6	2
41	40 per cent	6	6	8	6	5
42	60 per cent	9	8	7	6	5
43	80 per cent	7	6	8	9	9
44	100 per cent	9	7	8	9	8

than that performed on data arranged reverse-chronologically (trials 12 - 16). Furthermore, it seems not to matter whether the change is discrete or gradual; i.e., whether abrupt at one point in time or spread out evenly over five observations on either side of that point.

The poor performance of the CUSUM test in detecting changes in error variance (trials 22 - 32) is entirely expected. Since the presence of heteroscedasticity affects not the unbiasedness but rather the minimum variance property of least-squares estimators, it is not surprising that such a phenomenon is not reflected in the mean of the cumulative sum of recursive residuals. When the independence assumption is relaxed; i.e. with an error autoregressive of order one whose autoregressive coefficient (RHO) is equal to 0.8, the CUSUM test becomes highly unreliable (trials 39 - 44). For moderately autoregressive schemes, however, with RHO equal to 0.2 (trials 33 - 38), the power of the test is not adversely affected.

Results of Monte Carlo trials of the CSQ tests appear in the third and fourth columns of Table 2.1. These tests respond to coefficient changes, either discrete or gradual, and to changes in the error variance. The sensitivity to heteroscedastic error is easily explained since $\sum_{j=k+1}^{r} W_j^2$ can be shown to be equivalent to the residual sum of squares taken from a linear regression based upon the first r observations. Trial 32 illustrates the greater reliability of the CSQ test for one combination of coefficient and variance change.

The general insensitivity of the forward test to early sample change and the backward test to late sample change is demonstrated in trials 7 - 16. Like the CUSUM test, the CSQ test becomes highly unreliable in the presence of autoregressive error. For weak AR(1) processes of the sort that would go undetected by conventional means (trials 33 - 38), the tests are unaffected, however.

Column 5 records the results of the Stepwise Chow test. It performs well in detecting coefficient change. It appears more powerful than the CUSUM test for discrete change and most powerful of the three for gradual change -- no matter where in the sample period the change occurs. Complete insensitivity to change in the error variance is demonstrated by trials 22 - 26. This is to be expected. The Chow test is designed to pick up changes in the coefficient vector and nothing else. Its robustness to heteroscedastic error, is, in fact, a positive attribute. Just as with the other two tests, its power suffers badly in the presence of autocorrelation.

In ranking the three tests for the purposes of this study, the CSQ test is judged more versatile than the other two because of its ability to pick up changes in σ^2 as well as in β. Of the two tests for coefficient change, the Stepwise Chow test comes out ahead in being more powerful and computationally simpler. These results also illustrate how a combination of tests can provide a clearer picture of the underlying structure than any one of the tests taken alone. Such is the technique that will be applied to conventional demand for money models for Canada.

2.3 The Data and Models

The data are quarterly and seasonally adjusted. They consist of: currency plus demand deposits (M1); currency plus privately-held deposits (M2C); the rate on 90-day finance company paper (R90); the rate on 90-day swapped deposits (RSWAP); the McLeod, Young, Weir average of ten provincial bond yields (RPROV); gross domestic product (GDP); gross national expenditure (GNE); and the GNE implicit price deflator (PGNE).

A conventional demand for money model which has received wide acceptance is the partial adjustment model of Clinton (1973). The demand for money balances, M^d, depends upon a constant; real income, Y; a representative rate of interest, R; the price level, P; and a random error term u:

$$M_t^d = e^{\beta_0} Y_t^{\beta_1} R_t^{\beta_2} P_t e^{u_t} . \tag{2.2}$$

As full adjustment of actual money stock, M_t, to the desired money stock is assumed to take more than one quarter, an adjustment mechanism is required. It is specified as:

$$(M_t/P_t)/(M_{t-1}/P_{t-1}) = \left[(M_t^d/P_t)/(M_{t-1}/P_{t-1})\right]^g , \tag{2.3}$$

where g is an adjustment coefficient. Substituting (2.2) into (2.3) and taking natural logarithms yields demand equation A:

$$(m_t-p_t) = g\beta_0 + g\beta_1 y_t + g\beta_2 r_t + (1-g)(m_{t-1}-p_{t-1}) + u_t \, , \quad \text{(A)}$$

where lower-case symbols denote the natural logarithms of the initial variables defined above.

Two alternative models which allow for a different dynamic structure have also been considered. The first, model B, explains the demand for real M1 balances by Almon distributed lags on income and two interest rate variables:

$$(m1_t-p_t) = a_0 + \sum_{i=0}^{1} b_i y_{t-i} + \sum_{i=0}^{2} c_i \, rswap_{t-i} + \sum_{i=0}^{2} d_i \, rprov_{t-1}$$
$$+v_t \, . \qquad \text{(B)}$$

The scale variable, y_t, is assumed to impose a faster constraint upon the demand for real money balances than the interest rate variables. The delayed response of market participants to changing interest rates is allowed to be non-linear through choice of a second order Almon polynomial. A linear polynomial is chosen for the income variable.

The second, model C, explains the demand for real M2C by current income and a distributed lag on R90:

$$(m2c_t-p_t) = A_0 + A_1 y_t + \sum_{i=0}^{7} B_i \, r90_{t-i} + w_t \, . \qquad \text{(C)}$$

The error terms u_t, v_t and w_t are assumed throughout to be normally distributed white noise random variates.

Problems affecting the choice of specification and proper variables for the demand for money are numerous and have been discussed at length elsewhere in the literature.* Several problems are particularly relevant to the present analysis. The issue of simultaneous equation bias affecting estimation in single-equation models is one example. Recent studies generally concur, however, that simultaneity is not of sufficient magnitude to warrant the use of systems estimation methods.** Data deficiencies should also be noted. The disrupting effects of mail strikes in the second quarter of 1974 and the fourth quarter of 1975 are examples. The actual money supply figures during these periods are inflated by unusually long delays in the clearing of cheques. Suitable corrections have been applied to the money aggregates for mail-strike float.*** A second data problem involves the measurement of

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*See, for example, Laidler (1977).

**The CSQ tests can be used in conjunction with systems estimation methods (Poloz, 1979). Goldfeld (1976, p.702) and Laidler (1977, p.117) suggest, however, that simultaneity has not been, at least historically, of over-riding concern.

***The procedure involves estimating models A, B and C with strike dummy variables, D_1 and D_2. In model A, to prevent the impact of D_1 and D_2 from having a gradual run-off over time, the terms $g\beta_3 D_{1t} - (1-g)\beta_3 D_{1t-1} + g\beta_4 D_{2t} - (1-g)\beta_4 D_{2t-1}$ are included. [See Gregory and MacKinnon, (1980)]. The estimates are then subtracted from the relevant aggregates to preserve the models in a form compatible with the stability tests described in Section 2.2.

opportunity cost. For the broader aggregate, an 'own rate' adjustment is applied to the interest rate. The rate is adjusted by a weighted average of the rates on the components of M2C. While it is true that M1, too, has an implicit own rate (below-cost services on demand deposits, for example) it is assumed constant and ignored here for lack of data.*

Estimates for all three models for the full sample and various subsamples have been recorded in Table 2.2. The R90-GNE combination was found to perform better than other combinations in model A for both money aggregates. The long rate, RPROV, performs adequately for the full sample period with M1 and M2C but fails to explain the variation in either aggregate over the shorter subperiods. GDP performs as well as GNE for the full period with the narrow aggregate for the shorter period but does not explain M2C for 1968 II to 1977 IV. Both R90 and RPROV enter significantly in model B. Theoretical justification for such a specification involves the argument that term structure plays a role in the determination of money demand [see Friedman (1977)]. Model C is similar to the broad money equation analysed in Cameron (1979) except that the real income term enters contemporaneously instead of with a distributed lag of four to six quarters. Unfortunately, this equation shares with that of Cameron the undesirable feature

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*Startz (1979) offers several empirical measures of the implicit rate on demand deposits.

of high residual autocorrelation. A first-order autoregressive correction with RHO equal to 0.9 was necessary to raise the D.W. to a level of 1.72.

TABLE 2.2: MONEY DEMAND ESTIMATES

			Estimated Long-Run Elasticities*						
Model	Period	Dep. Variable	GNE	GDP	R90	RSWAP	RPROV	Adj. Coeff.	SEE
A	56II-77IV	M1	.79		-.29			.2037	.0114
A	"	M1	.81				-.30	.3334	.0133
A	"	M1		.81	-.30			.1711	.0119
A	"	M1		.82			-.35	.2358	.0140
A	"	M2C	1.29		-.20			.1019	.0113
A	"	M2C		1.29	-.23			.0855	.0117
A	68II-77IV	M1	.81		-.43			.1016	.0106
A	"	M1		.74	-.55			.1067	.0106
A	"	M2C	1.38		-.03			.4097	.0119
B	"	M1	.68			-.015	-.044		.0124
B	62II-77IV	M1	.68			-.016	-.042		.0113
C	68II-77IV	M2C	.49		-.13				.0129

* All short run elasticities for equations reported here are significant at the 95 per cent level. The long run elasticities for models B and C are sums of lag weights.

2.4 Stability Analysis

For the purposes of stability analysis, model A requires a further transformation. It is seen from equation (2.1) that if X contains a stochastic variable then the W_r will not be normally distributed. The validity of the CUSUM and CSQ tests, however, requires normality of the recursive residuals. Dufour (1979) suggested a method of dealing with this problem. He noted that if the parameter g of model A were known then the term $(1-g)(m_{t-1}-p_{t-1})$ could be subtracted from both sides of the equation. In practice, g is not known but a consistent estimate of it, say $\hat{g}$, may be obtained through application of ordinary least squares (OLS). The version of A which will be analysed here is of the form:

$$(m_t-p_t)-(1-\hat{g})(m_{t-1}-p_{t-1}) = g\beta_0 + g\beta_1 \tilde{y}_t + g\beta_2 r_t + u_t \,. \qquad \text{(A')}$$

Table 2.3 records the incidence of rejection of H_0 for the three models for various time periods. The general picture seems to be that stability can be rejected for both broad and narrow money. With M1, significant instability is observed only by the Stepwise Chow test. All three tests detect instability in M2C. As to the point in time of structural change these tests are inconclusive.

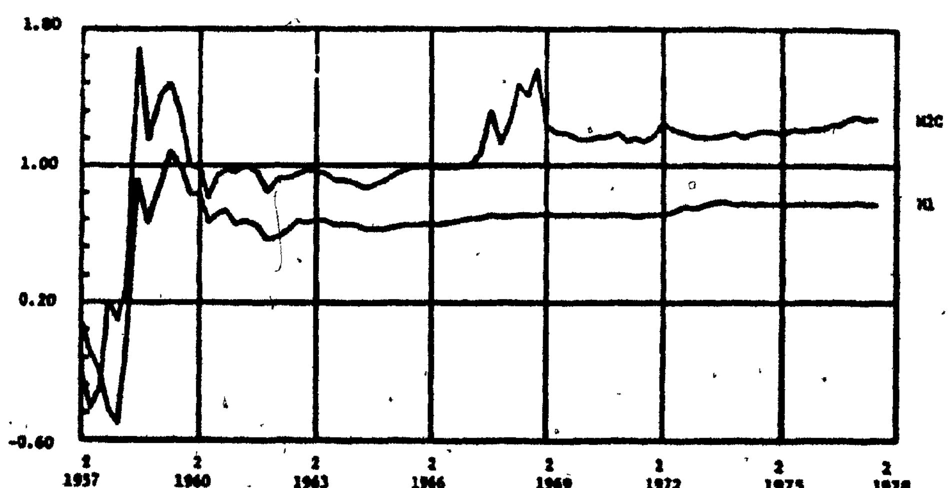
For the 1956 - 70 subsample, a period comparable to that of Clinton (1973), the hypothesis of stability is rejected by the Stepwise Chow test for M1 and by the CUSUM and CSQ tests for M2C.

TABLE 2.3: INCIDENCE OF REJECTION OF NULL HYPOTHESIS OF STABILITY

Model	Sample Period	Variable	CUSUM Forward	CUSUM Backward	CSQ Forward	CSQ Backward	Stepwise Chow	D.W.
A'	M1	(56II-77IV)					***	2.01
A'	M1	(68II-77IV)					***	1.96
A'	M1	(56II-70IV)					***	1.99
B	M1	(68IV-77IV)						2.00
B	M1	(62IV-77IV)						1.82
A'	M2C	(56II-77IV)					***	1.62
A'	M2C	(56II-70IV)	**		**	**		2.17
C	M2C	(68II-77IV)						1.75

Legend: ** - significance at the 90 per cent level
*** - significance at the 95 per cent level

CHART 2.1: LONG-RUN INCOME ELASTICITIES FOR M2C AND M1



One implication of this finding is that the instability in M1 is different in nature from that in M2C. The M2C shift seems to arise in the error variance as well as in the coefficients. The stronger parametric instability of M2C is further supported by Chart 2.1. This figure plots long run elasticities for M2C and M1. These elasticities are calculated over time from 1957 II to 1977 IV; with each successive point arrived at by application of OLS to a sample beginning in 1956 II and augmented by one observation. Strong instability in the income elasticity of M2C coincides with the 1967 Bank Act revisions. Expansion of new savings instruments as a result of removal of interest rate ceilings is one possible cause. The movement into term deposits which resulted appears not to have affected the M1 income elasticity. The behavior of the M2C elasticity indicates an initial over-reaction followed by a decline and stabilization at a higher level. These results are in accordance with Clinton's finding that demand for the broader aggregates is unstable relative to that for M1, and that this instability arises, at least in part, from a changed legal structure. There is, however, no confirmation of Clinton's finding of abrupt change corresponding to the movement from floating to fixed rates in 1962.

A similar situation is observed in the full sample, 1965 to 1977. The Stepwise Chow test rejects stability for both narrow and broad aggregates. The CUSUM and CSQ tests, however, do not. The apparent inconsistency of this result with that of the 1956 - 70 subsample might be explained by the high degree of autocorrelation evident in the full sample, M2C equation.

The desirability of M2C as an asset was further enhanced in the first two quarters of 1972 by a sudden rise in term deposit rates. Bank loans were expanding rapidly at this time and, due to the low liquidity in the banking system, the banks bid strongly for term deposits. Chart 2.1 shows a peak in the M2C income elasticity at precisely this time. The Winnipeg Agreement of June 1972, which limited rates of interest offered on deposits of $100,000 or more for less-than-a-year maturities to a maximum 5½ per cent, removed instability in the market and appears to have reestablished the M2C income elasticity at its former level.

For the period 1962 to 1977 in model B and for 1968 to 1977 in models B and C, no instability is detected for either aggregate. The post-1968 finding of stability in M1 by the CUSUM and CSQ tests, however, is called into doubt by a systematic string of negative residuals from model B following 1976 I. To investigate this finding, a shift term (which was constrained to zero prior to 1976 I, increased transitionally through to 1977 III and held constant thereafter) was introduced to model B. This shift was found to be significantly different from zero and to represent a decline of roughly 6.7 per cent in M1.

Coincident with this downward shift in M1 was a period of rapid improvements in the techniques available to corporations for the management of transactions balances. Active competition on the part of chartered banks in offering cash management plans to their corporate customers resulted in a reduced demand for current accounts. When real current accounts (CA) are regressed on distributed lags of GNE and RSWAP the same shift variable that was introduced to model B shows significance with a coefficient roughly twice as large. Since current accounts make up approximately one-half of M1, this supports the hypothesis that the M1 shift predominated in the CA component. This hypothesis will be examined further in Chapter 6.

The failure of the Stepwise Chow test to detect the 1976 shift in M1 is explained by its late occurrence in the sample. As defined above, the Stepwise Chow test checks for instability at every point from k+1 to T-k-1. In the present sample, T-k-1 corresponds to 1976 II. The failure of the CSQ test, however, is less easily explained. If the answer is that the CSQ test checks for a different and broader hypothesis of constancy and so is less sensitive to pure coefficient change than the Chow test, then it would be instructive to determine empirically its relative sensitivity. Very often, in a forecasting context, large prediction errors are encountered due to late-sample structural change. It is useful, therefore, to be aware of the degree of resolution which each test affords.

TABLE 2.4: INCIDENCE OF REJECTION OF NULL HYPOTHESIS OF STABILITY FOR MODEL (B) SUBJECTED TO SHOCKS OF VARYING DEGREES

% decrease in constant term	% of increase in error variance	CUSUM Forward	CUSUM Backward	CSQ Forward	CSQ Backward	D.W.
6.7	0					2.08
20.0	0					2.05
25.0	0					2.11
30.0	0		**	***		1.86
35.0	0		**	***		1.74
6.7	6.7					2.09
20.0	10.0			*		2.08
25.0	25.0			*		2.14
30.0	30.0		**	***		1.89
35.0	35.0		**	***		1.78

LEGEND: * significance at the 80 per cent level
** significance at the 90 per cent level
*** significance at the 95 per cent level

The results of subjecting the constant term of model B to gradual shifts of varying degrees and applying the three tests are shown in Table 2.4. It is seen that for a pure coefficient shock alone, a decrease in the constant term of 30 per cent is required before a rejection of the null hypothesis is indicated by either the CUSUM or the CSQ test. When this constant term shock is combined with an equal shock to the error variance, the CSQ test is more sensitive and picks up the change at 20 per cent. The CUSUM test is, as expected, unaffected. In other words, for the 1976 M1 shift to have been detected by 1977 IV with either the CUSUM or CSQ test, it would have had to have been from three to four times greater than 6.7 per cent. This causes one to question the applicability, for forecasting purposes, of the CUSUM and CSQ tests to the analysis of stability of money demand.

2.5 Cubic Splines and Money Demand

One of the results of the stability analysis of Section 2.4 was the suggestion that a discontinuity, or abrupt change, occurred in the demand for M2C at a point in time coinciding with a change in the legal structure of the Canadian banking system. One alternative explanation, aside from structural change or misspecification, might be that the dependent variable responds in some nonlinear fashion to its explanatory variables. Although the issue of functional form will be examined in depth in Chapter 3, consideration will be given here as to its relevance in the context of stability. It is known that through the application of cubic spline regression a 'free-form' curve, continuous in the second derivative may be fitted to the data. If instability in the M2C demand function can be removed through the use of a non-linear functional form, the discontinuity hypothesis referred to above is left severely in doubt. If, on the other hand, no significant improvement can be made the hypothesis remains unrejected.

The terminology commonly used in the description of splines will be used here. The 'knots' are the set of k join points denoted by: $\underline{x}_j$ for j = 1, ... , k. If we parameterize the variable x according to:

$$(x - \underline{x}_j)_+ = \begin{bmatrix} 0 & , x < \underline{x}_j \\ x - \underline{x}_j & , x \geq \underline{x}_j \end{bmatrix} ;$$

then the linear spline, $S^1(x)$, is given by:

$$S^1(x) = \sum_{j=1}^{k} \beta_j (x-\underline{x}_j)_+ + \beta_k + \beta_{k+1}\, x ,$$

and the spline polynomial of degree n, $S^n(x)$, is given by:

$$S^n(x) = \sum_{j=1}^{k} \beta_j (x-\underline{x}_j)_+^n + \beta_k + \beta_{k+1}\, x + \ldots + \beta_{k+n}\, x^n .$$

The terms in $(x-\underline{x}_j)^n$ are the terms which provide the discontinuities in the nth derivative of $S^n(x)$.

Applying a cubic polynomial with k internal knots to the income variable of model A yields:

$$m_t-p_t = d_0 + d_1r_t + d_2(m_{t-1}-p_{t-1}) + d_5(y_t-\underline{y}_0) + d_6(y_t-\underline{y}_0)^2$$
$$+ d_7(y-\underline{y}_0)^3 + \sum_{j=1}^{k} c_j(y_t-\underline{y}_j)^3 D_j^* + u_t ,$$

where $D_j^* = \begin{bmatrix} 0, & y<\underline{y}_j \\ 1, & y>\underline{y}_j \end{bmatrix}$. (2.4)

The ability to accurately fit a segmented polynomial model is constrained not by estimation technique, for standard regression methods may be applied to (2.4), but rather by number and location of the internal knots. On the one hand, too many knots will ensure a flexible model with a high risk of imprecise estimates due to collinearity between spline variables. On the other hand, too few knots may falsely constrain the estimates. For the present study, it is judged preferable to err on the side of constrained estimates than on the side of ill-conditioning. Cubic splines with one and two internal knots will be used here.

Regarding the location of knots, three different criteria will be used. Poirier (1976) suggested that if a point of structural change were known, a priori, then it might be chosen as a knot in the spline polynomial. Columns 2 and 3 of Table 2.5 employ this knot selection criterion. Ahlberg, Nilson and Walsh (1967) suggest the use of equal intervals between knot points. Columns 4 and 5 employ this criterion. McCulloch (1978)

TABLE 2.5: ESTIMATES FROM DEMAND FOR M2C MODEL WITH AND WITHOUT CUBIC SPLINES

	1	2	3	4	5	6
	Original Model	Cubic Spline on M-1 (One) Knot at 1967 III)	Cubic Spline on Y (One Knot at 1967 III)	Cubic Spline on Y (Two Knots Equal INTVLS)	Cubic Spline on R (Two Knots Equal INTVLS)	Cubic Spline on R (Two Knots Equal # of Observations)
Coeff:						
C	-1.3910 (-23.04)	-2.9132 (-4.35)	.5323 (0.95)	1.8814 (3.62)	-1.3921 (-23.32)	-1.3884 (-23.01)
Y	.1209 (3.74)	.1711 (2.74)			.1306 (4.01)	.1267 (3.87)
R	-.0206 (-5.72)	-.0221 (-5.80)	-.0237 (-6.22)	-.0239 (-6.41)		
M_{-1}	-.0915 (-3.20)		-.1482 (-2.84)	-.1985 (-3.59)	-.0997 (-3.46)	-.0960 (-3.31)
D_5		-.0168 (-2.06)	-.0664 (-0.45)	.7807 (1.72)	-.0796 (-0.93)	-.1236 (-1.44)
D_6		.0054 (0.66)	.0563 (1.70)	-.2217 (-1.49)	.0324 (0.16)	.1544 (0.76)
D_7		-.0039 (-0.51)	-.0391 (-1.63)	.2573 (1.61)	.0204 (0.15)	-.0688 (-0.50)
C_1		.0022 (0.61)	.0845 (1.73)	-.3296 (-1.78)	-.0102 (-0.54)	.0030 (0.13)
C_2				.2139 (2.61)	.0018 (1.01)	.0004 (0.20)
Resid:						
RSS	.01095	.01062	.01027	.00981	.01010	.01019
$\bar{R}^2$	.326	.321	.344	.365	.347	.341
SEE	.01149	.01152	.01133	.01114	.01131	.01136
DW	1.64	1.60	1.68	1.67	1.90	1.77

suggests the use of equal numbers of observations between knot points. Column 6 of Table 2.5 employs the McCulloch criterion.

In Table 2.5, the cubic spline on income, Y, with two knots (column 4) outperforms that on either the lagged dependent term, M_{-1}, or interest, R, on the basis of t-statistics of cofficients of the spline variables.* Four of five spline coefficients are judged significantly different from zero. Three of four of the spline coefficients on Y with one knot (column 3) are significant. One of four of the spline coefficients on M_{-1} (column 2) is significant. One of five spline coefficients on R with equal numbers of observations per interval (column 6) is significant. None of the five spline coefficients on R with equal intervals is significant.

On the basis of fit, the income spline with two knots, (column 4), again outperforms all others. This model shows the lowest standard error of estimate (SEE) of the six regressions. It is also noted that the SEE is lower for all but one of the spline models than for the original model. Column 2 shows the only SEE higher than that of column 1.

In Table 2.6, forecasting performance, as measured by mean square prediction error over the last 25 observations (MSPE(25)),

*The t-test is used as a test for significance from zero of individual coefficients, although it is recognized that to do so in a lagged dependent model involves an approximation affecting the validity of the test.

and over the last 5 observations (MSPE(5)), are compared for the original model and three alternative cubic spline models. The original model outperforms all three spline models in terms of long-run performance, MSPE(25). In terms of late-sample performance, MSPE(5), however, the spline models fare better than the original model.

TABLE 2.6: FORECASTING THE DEMAND FOR M2C WITH AND WITHOUT CUBIC SPLINES

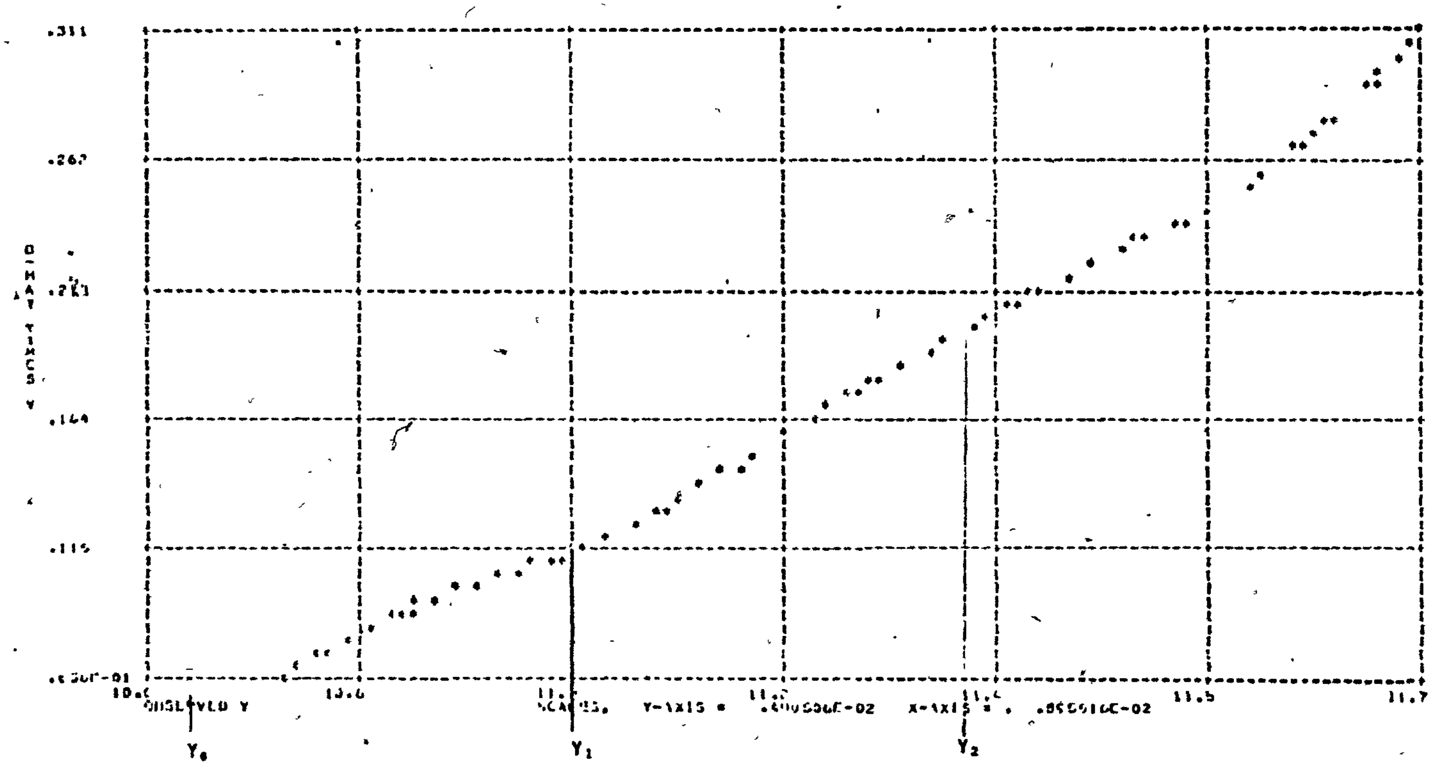
	1	4	5	6
	Original Model	Cubic Spline on Y (Two Knots Equal INTVLS	Cubic Spline on R (Two Knots Equal INTVLS	Cubic Spline on R (Two Knots Equal # Observations
MSPE (25) X 10^{-4}	1.185	2.859	1.484	1.617
MSPE (5) X 10^{-4}	0.610	0.590	0.467	0.482

Chart 2.2 plots the weighted sum of the income spline variables times their regression coefficients against the dependent variable. It shows the extent of the estimated nonlinearity in income response which this model allows. The first derivative of the contour, which is the short-run income elasticity of demand for M2C, is everywhere positive as expected. The slope increases with Y. The average slope of the first interval (approximately corresponding to years 1956 to 1962) is 0.204

TABLE 2.7: ESTIMATES FROM DEMAND FOR M1 MODEL WITH AND WITHOUT A CUBIC SPLINE ON THE INCOME VARIABLE

	Original Model	Cubic Spline on Y (2 Knots - Equal Intervals)
Coefficients:		
C	-.8042 (-.474)	.9746 (1.94)
Y	.1513 (5.59)	
R	-.0506 (-8.96)	-.0509 (-8.75)
M_{-1}	-.1865 (-4.21)	-.2052 (-4.05)
D_5		.3781 (0.92)
D_6		-.0852 (-0.61)
D_7		.0994 (0.66)
C_1		-.1259 (-0.72)
C_2		.0121 (0.13)
Residuals:		
SEE	.01153	.01137
DW	1.75	1.84
Predictions:		
MSPE (25) $\times 10^{-4}$	1.56	3.85
MSPE (5) $\times 10^{-4}$	1.60	1.00

CHART 2.2: SPLINE VARIABLES WEIGHTED BY REGRESSION COEFFICIENTS FOR Y-SPLINE 2-KNOT MODEL (M2C)



for a long-run elasticity of 1.03. The average slope in the second interval (1963 to 1969) is 0.237, for a long-run elasticity of 1.19. The average slope in the third interval (1970 to 1977) is 0.297, for a long-run elasticity of 1.48. This evidence concurs with that of Chart 2.1.

Table 2.7 records the results of application of a cubic spline to the income variable for M1 (model A). It is seen that on the basis of fit and significance of coefficients, the cubic spline represents little improvement over the original model. The cubic spline does, however, improve the late-sample predictive performance of the equation. It is notable that the MSPE(5) is higher than MSPE(25) for M1, while for M2C, as shown in Table 2.6, the reverse is true. This reflects the worsening performance of the M1 model since 1976.

2.6 Concluding Comments

Numerous techniques for the detection of instability in regression relations have been developed in the few short years since Brown, Durbin and Evans popularized the use of recursive residuals. These include tests for heteroscedasticity, serial correlation, functional misspecification, independence, and combinations thereof. Three of these tests, which have received attention in recent literature, have been applied here in an effort to; first, gain an insight into their comparative reliabilities and; second, to answer some long-standing questions about the stability of money demand in Canada. From simulation exercises it is concluded that the Stepwise Chow is superior in power and efficiency to both CUSUM and CSQ when the instability is manifested entirely in the coefficients. To the extent that such a phenomenon is unnatural and that one would expect instability to be manifested in all parameters of a model simultaneously, then the CSQ test, because of its sensitivity to variance as well as coefficient change is preferable. The choice of either CSQ or Stepwise Chow will depend upon the relative weight one attaches to coefficient and error variance shifts. On the one hand, the error variance is an important parameter of the system we wish to estimate and demands the same attention as the coefficients. On the other hand, for forecasting purposes one may wish a more finely tuned instrument such as the Chow test for detection of coefficient shift. Evidence has been presented here that, not only is the Chow test more sensitive than the CSQ to coefficient shift, this result is relatively robust to departure from homoscedasticity.

On a strictly empirical level, certain characteristics of the log-linear money demand model for Canada can be improved upon by allowing the logarithmic response of money demand to its explanatory variables, expressed in logs, to be nonlinear. Application of cubic splines to both income and interest variables in the conventional log-linear partial adjustment money demand model have shown significant improvements in overall fit and in late-sample predictive performance. An improved mean squared prediction error based on predictions of the five last observations was shown to be the case when cubic splines were applied to the income variable in both M1 and M2C equations. The cubic spline must, however, be appreciated for what it is -- a flexible curve-fitting tool and nothing more. While ex post forecasts can always be improved with this method, such is not necessarily the case on an ex ante basis.

On the issue of stability of money demand the general conclusion emerging from this study is that although instability has been observed in both M1 and M2C in Canada between 1956 and 1977, it has been relatively more pronounced in the broader aggregate. The data suggest two instances of abrupt change arising in the term deposit component of M2C. The 1967 change coincides with the expansion of new term deposit instruments by the chartered banks and after an adjustment period of roughly six quarters, results in a permanently higher income elasticity of demand. The 1972 instability stems from unusually competitive bidding by the chartered banks for term deposits, which stabilized with the introduction of the Winnipeg Agreement.

As for the narrower aggregate, there is evidence that there has been a downward shift in the demand for M1 in the mid-1970s. Institutional data on financial innovation during this period and a coincident downturn in current accounts suggest that this shift may have been the result of increased cash management efficiency.

CHAPTER 3

FUNCTIONAL FORM

The inability of economic theory to differentiate between alternate functional forms in the demand for money has traditionally left the choice of form to the discretion of the researcher. A variety of different forms has surfaced in the literature. Brown (1939), for example, wrote the demand for money as a linear function of the rate of interest, the change in the rate of interest and the change in the level of prices. Bronfenbrenner and Mayer (1960) used a logarithmic functional form. The convenient economic interpretation afforded by the coefficients of the logarithmic formulation enhanced its popularity for empirical work. The semilogarithmic form, as first appeared in Cagan (1956), became increasingly popular in the 1970s. A recent variant of this form, which is linear in the interest rate variable and logarithmic in all other variables, (henceforth called the 'semi-log' model) has appeared in Haache (1974), Klein (1974), White (1976) and Cameron (1979).

The field of possible alternatives is broad. Fortunately, the Box and Cox procedure provides a technique for estimating the power transformation which best suits the data. Studies pioneered by Zarembka (1968) and extended by White (1972), Spitzer (1976, 1977) and Mills (1978) applied this procedure to the demand for money. The objective of the present chapter will be to examine Canadian demand for money using a generalized functional form. Efforts will be made to relax as much as possible the constraints of functional form which are typically imposed.

Section 3.1 considers a simple Box-Cox model in which the power coefficients of all independent variables are constrained to equality but are allowed to differ from the power coefficient of the dependent variable. Searching over a grid of different combinations of these coefficients shows, for both money aggregates, that the logarithmic form performs the best and that all mixed forms can be statistically rejected. Section 3.2 extends the basic model to allow a third power dimension. In this extended model, the power coefficient of the interest variable is allowed to differ from that of all other independent variables. No significant improvement over the more constrained model of Section 3.1 is noted here. Section 3.3 considers the extended Box-Cox model with errors assumed to be autoregressive of order 1. These results strengthen those of Section 3.2. They suggest that the semi-log transformation induces a moving average in the residual of the M2C equation. Section 3.4 considers theoretical implications of the semi-log model and concludes, both on theoretical and empirical grounds, that the logarithmic model is preferable.

3.1 The Model and Box-Cox Estimation

Desired real balances in the economy are assumed to behave according to the rule:

$$(M_t^d - P_t)^{(\mu)} = B_0 + B_1\, y_t^{(\mu)} + B_2\, r_t^{(\mu)} + u_t, \tag{3.1}$$

where u_t are independent, normally distributed random variables with zero mean and constant variance. The symbol $Z^{(\theta)}$ denotes the Box-Cox power transformation, written as:

$$Z^{(\theta)} = \begin{cases} (Z^{\theta}-1)/\theta, & \theta \neq 0,\ Z > 0 \\ \ln Z & , \theta = 0. \end{cases} \tag{3.2}$$

If actual real balances adjust to desired real balances according to the rule:

$$(M_t-P_t)^{(\mu)} - (M_{t-1}-P_{t-1})^{(\mu)} = g\left[(M_t^d-P_t)^{(\mu)} - (M_{t-1}-P_{t-1})^{(\mu)}\right], \tag{3.3}$$

where g as before is the adjustment coefficient, the demand for real balances can be written as:

$$(M_t-P_t)^{(\mu)} = gB_0 + gB_1\, y_t^{(\mu)} + gB_2\, r_t^{(\mu)} + (1-g)(M_{t-1}-P_{t-1})^{(\mu)} + u_t. \tag{3.4}$$

A more general version of (3.4) might allow the power coefficient of the dependent variable, say λ, to vary independently of μ. The log-likelihood function of this more general model is given by:

$$L\{\lambda,\mu,\beta,\sigma^2;M,X\}$$
$$= -\frac{T}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}(M^{(\lambda)} - X^{(\mu)}\beta)'(M^{(\lambda)} - X^{(\mu)}\beta)$$
$$+ \ln J, \tag{3.5}$$

where M, for convenience, denotes the T by 1 dependent variable vector, X is the T by 4 matrix of explanatory variables, and

$$J = \det\left(\frac{\partial M^{(\lambda)}}{\partial M}\right) = \prod_{t=1}^{T} (M_t - P_t)^{\lambda-1}$$

is the Jacobian of the transformation of $M^{(\lambda)}$ to M. Maximizing L with respect to β and σ^2 yields:

$$\hat{\beta}(\lambda,\mu) = (X^{(\mu)\prime}X^{(\mu)})^{-1}X^{(\mu)\prime}M^{(\lambda)}$$

$$\hat{\sigma}^2(\lambda,\mu) = \frac{1}{T}(M^{(\lambda)} - X^{(\mu)}\hat{\beta}(\lambda,\mu))'(M^{(\lambda)} - X^{(\mu)}\hat{\beta}(\lambda,\mu)).$$

Substituting for β and σ^2 in (3.5) gives the concentrated likelihood function:

$$L(\lambda,\mu;M,X) = -\frac{T}{2}\{\ell n(2\pi)+1\} - \frac{T}{2}\ell n(\hat{\sigma}^2(\lambda,\mu)) + (\lambda-1)\sum_{t=1}^{T} \ell n(M_t - P_t) \quad (3.6)$$

Maximum likelihood (ML) estimators of λ and μ are found by searching over a (λ,μ) grid and the ML estimators of β and σ^2 are $\hat{\beta}(\hat{\lambda},\hat{\mu})$ and $\hat{\sigma}^2(\hat{\lambda},\hat{\mu})$.*

The long-run income and interest elasticities are computed as:

$$\eta_{y_t} = \beta_1 \frac{y_t^{(\mu)}}{(M_t - P_t)^{(\lambda)}}, \text{ and } \eta_{R_t} = \beta_2 \frac{r_t^{(\mu)}}{(M_t - P_t)^{(\lambda)}} \quad (3.7)$$

---

* The estimates of β and σ^2 are computed as OLS estimates from given values of λ and μ in this procedure.

respectively. It is noted from (3.3) that for $g > 1$ the adjustment becomes explosive. The size of $\hat{g}$ can thus be considered grounds for possible acceptance or rejection of a particular functional form. The primary criterion of choice will be a likelihood ratio test where an aproximate 95% confidence region for λ and μ can be obtained from

$$L\max(\hat{\lambda},\hat{\mu}) - L\max(\lambda,\mu) < \frac{1}{2}\chi_2^2(.05) = 3.00.$$

Alternative forms will be ranked according to the size of their log likelihood maximum. Forms within the stated confidence region will be judged superior to those outside, given that their estimated adjustment coefficients are less than unity.

The results of the estimation when applied to Canadian data from 1956 II to 1977 IV are recorded in Tables 3.1 and 3.2.* The data are quarterly and seasonally adjusted. Two monetary aggregates: M1, or currency plus demand deposits; and M2C, or M1 plus privately-held deposits, are examined. The income measure used was real GNE in 1971 dollars. The interest rate was the rate on 90-day finance company paper. Price deflation of the money aggregates was performed by the GNE implicit price deflator, PGNE.

Fifteen different combinations of λ and μ were tested. The power coefficient on the dependent variable, λ, was set equal to 0, .5 and 1.0. The power coefficient on the explanatory variables, μ, was set equal to 1.0, 0.5, 0.0, -0.5 and -1.0. These models are ranked in order of decreasing log-likelihood maxima in Tables 3.1 and 3.2. For both money aggregates the logarithmic form ($\lambda = 0$, $\mu = 0$) appears to be superior followed closely by the square root ($\lambda = .5$, $\mu = .5$)

*The program used was obtained from Huang, Moon and Chang (1978).

TABLE 3.1: DEMAND FOR M1 WITH BOX-COX (BC) TRANSFORMATION

λ	μ	L-MAX	LIKELIHOOD RATIO	COEFF. OF INCOME	COEFF. OF INTEREST	COEFF. OF LAGGED M1	η_y	η_r
0.00	0.00	197.221		0.151	-0.051	0.814	0.75	-0.21
				(5.59)	(-8.96)	(18.36)		
0.50	0.50	196.558	0.66	0.138	-0.065	0.858	0.86	-0.32
				(4.98)	(-9.10)	(20.42)		
1.00	1.00	194.864	2.36	0.118	-0.081	0.905	1.06	-0.11
				(4.32)	(-9.31)	(23.36)		
1.00	0.50	187.339	9.88*	0.381	-0.189	2.645	-0.10	0.11
				(4.12)	(-7.92)	(18.88)		
0.50	0.00	185.287	11.93*	0.366	-0.139	2.581	-0.12	0.18
				(3.93)	(-7.17)	(16.94)		
0.00	-0.50	182.152	15.07*	0.324	-0.100	2.560	-0.12	0.27
				(3.64)	(-6.51)	(16.05)		
0.00	0.50	180.043	17.18*	0.049	-0.023	0.281	0.07	1.77
				(4.41)	(-7.85)	(16.56)		
0.50	1.00	177.577	19.64*	0.037	-0.027	0.301	0.09	1.60
				(3.37)	(-7.65)	(19.11)		
1.00	0.00	160.576	36.65*	0.820	-0.386	8.234	-0.02	0.04
				(2.21)	(-4.99)	(13.57)		
0.50	-0.50	158.242	38.98*	0.564	-0.259	8.374	-0.02	0.07
				(1.61)	(-4.27)	(13.30)		
0.00	-1.00	155.921	41.30*	0.347	-0.170	8.517	-0.02	0.13
				(1.11)	(-3.75)	(13.70)		
0.00	1.00	152.578	44.64*	0.012	-0.009	0.101	0.04	0.58
				(2.37)	(-5.81)	(14.42)		
1.00	-0.50	135.479	61.74*	0.431	-0.665	27.423	0.0	0.02
				(0.32)	(-2.82)	(11.18)		
0.50	-1.00	134.754	62.47*	-0.369	-0.401	28.188	0.0	0.04
				(-0.31)	(-2.31)	(11.86)		
1.00	-1.00	116.326	80.90*	-5.445	-0.901	93.122	0.0	0.01
				(-1.23)	(-1.40)	(10.57)		

* Indicates significance at the 95% level.

TABLE 3.2: DEMAND FOR M2C WITH BOX-COX (BC) TRANSFORMATION

λ	μ	L-MAX	LIKELIHOOD RATIO	COEFF. OF INCOME	COEFF. OF INTEREST	COEFF. OF LAGGED M2C	μ_y	η_r
0.00	0.00	303.970		0.121	-0.021	0.908	2.74	-0.24
				(3,74)	(-5.72)	(31.79)		
0.50	0.50	302.776	1.19	0.064	-0.020	0.293	0.25	-0.03
				(3.50)	(-5.44)	(34.69)		
1.00	1.00	297.906	6.06*	0.034	-0.019	0.095	0.15	-0.02
				(3.20)	(-5.22)	(37.37)		
0.50	1.00	280.731	23.24*	0.128	-0.017	0.028	0.70	-0.03
				(16.37)	(-5.97)	(15.07)		
0.00	0.50	276.993	26.98*	0.229	-0.017	0.085	0.91	-0.03
				(15.17)	(-5.73)	(12.24)		
0.50	0.00	254.148	49.82*	-0.701	-0.021	2.32	0.82	0.01
				(-7.52)	(-1.99)	(28.16)		
1.00	0.50	254.104	49.87*	-0.440	-0.020	0.757	3.56	-0.06
				(-8.47)	(-1.90)	(31.48)		
0.00	-0.50	252.143	51.83*	-1.058	-0.022	7.009	0.23	0.00
				(-6.33)	(-2.17)	(25.04)		
0.00	1.00	238.473	65.50*	0.140	-0.013	0.001	1.00	0.03
				(17.91)	(-4.78)	(0.67)		
1.00	0.00	205.205	98.77*	-2.729	-0.014	5.312	0.71	0.00
				(10.25)	(-0.47)	(22.58)		
0.50	-0.50	203.045	100.93*	-4.174	-0.017	15.725	0.27	0.00
				(-8.74)	(-0.58)	(19.65)		
0.00	-1.00	200.180	103.79*	-6.050	-0.020	45.453	0.12	0.00
				(-7.22)	(-0.75)	(17.14)		
1.00	-0.50	166.320	137.65*	-11.148	0.003	33.495	0.24	0.00
				(-9.41)	(0.04)	(16.87)		
0.50	-1.00	164.585	139.39*	-16.126	-0.006	94.657	0.11	0.00
				(-7.86)	(-0.09)	(14.57)		
1.00	-1.00	135.684	168.29*	-37.444	0.034	192.107	0.09	0.00
				(-8.05)	(0.22)	(13.05)		

* Indicates significance at the 95% level.

and the linear ($\lambda = 1$, $\mu = 1$) forms. According to the likelihood ratio test the logarithmic, the square root and the linear forms cannot be rejected for M1; and the logarithmic and square root forms for M2C. For both M1 and M2C all mixed functional forms, (i.e. $\lambda \neq \mu$), can be statistically rejected. No change in these findings came about as a result of extending the range of λ to include negative values or of tightening the μ dimension of the (λ,μ) grid. None of the forms accepted by the likelihood ratio test has a negative adjustment coefficient. All forms for both M1 and M2C for which $\mu < 0$, however, have strongly negative adjustment coefficients.

3.2 Extended Box-Cox (EBC) Estimation

The model thus far developed, while being more flexible than conventional models, still does impose one constraint which might be considered unrealistic, namely; the power coefficient on the interest variable is constrained to equal that on all other explanatory variables. Not only are the units of measurement of r different from all other variables but also there exists ample theoretical support for the separate treatment of r. The rationale, for example, which allows the interest elasticity to rise as the interest rate rises need not apply to the income elasticity. There is sufficient justification for extension of the model to include a third power dimension, γ which transforms the interest variable.

The model now becomes:

$$(M_t-P_t)^{(\lambda)} = g\beta_0 + g\beta_1\, y_t^{(\mu)} + g\beta_2\, r_t^{(\gamma)} + (1-g)(M_{t-1}-P_{t-1})^{(\mu)} + u_t.$$

Estimators of λ, μ and γ are found by searching over a (λ,μ,γ) volume for the combination which maximizes the concentrated likelihood function:

$$L(\lambda,\mu,\gamma\ ;\ M,X) = -\frac{T}{2}\{\ln(2\pi) + 1\} - \frac{T}{2}\ln(\hat{\sigma}^2(\lambda,\mu,\gamma))$$

$$+ (\lambda-1)\sum_{t=1}^{T}\ln(M_t - P_t). \qquad (3.6a)$$

As before, an approximate 95% confidence region for λ, μ and γ is obtained from

$$L\max(\hat{\lambda},\hat{\mu},\hat{\gamma}) - L\max(\lambda,\mu,\gamma) \leq \frac{1}{2}\chi^2_3(.05) = 3.91.$$

The search for the highest L-max was performed over λ, μ and γ with λ varied from 0.0 to 1.0 in increments of .5, and μ and γ varied from 0.0 to 2.0 in increments of 0.25. Since space would not allow the tabulation of these 243 models, the estimates from certain selected models appear in Tables 3.3 and 3.4. These include all unrejected models together with the semi-log model for M1 and the same for M2C plus the full linear model. Within the logarithmic family (λ=0, μ=0), which was found to have had the highest likelihood under BC estimation, no improvement appeared as a result of allowing $\lambda \neq \mu$ for M1 (Table 3.3). Within the square root (λ=.5, μ=.5) and linear (λ=1, μ=1) families, however, strong improvement was noticed. The new L-max of 199.231 for the form λ=.5, μ=.5, and γ=.25 was, in fact, sufficiently high for rejection of the full linear form which previously had been accepted in Table 3.1.

TABLE 3.3 DEMAND FOR M1 WITH EXTENDED BOX-COX (EBC) TRANSFORMATION

λ	μ	γ	L-MAX	LIKELIHOOD RATIO	COEFF. OF INCOME	COEFF. OF INTEREST	COEFF. OF LAGGED M1
0.50	0.50	0.25	199.231		0.236	-0.026	0.852
					(5.24)	(-9.54)	(20.72)
1.00	0.25	0.25	198.935	0.3	0.807	-0.085	0.477
					(6.77)	(-10.33)	(22.85)
1.00	1.00	0.25	198.776	0.46	0.666	-0.081	0.870
					(5.32)	(-9.83)	(22.81)
1.00	1.00	0.50	198.286	0.95	0.403	-0.082	0.869
					(5.20)	(-9.83)	(22.16)
0.50	0.50	0.50	196.558	2.67	0.138	-0.065	0.858
					(4.98)	(-9.10)	(20.42)
0.50	0.75	0.25	197.908	1.33	0.270	-0.28	0.474
					(6.14)	(-9.91)	(20.35)
0.00	0.00	0.00	197.221	2.01	0.151	-0.051	0.814
					(5.59)	(-8.96)	(18.36)
0.00	0.00	0.25	196.929	2.3	0.080	-0.009	0.845
					(5.02)	(-9.03)	(19.43)
1.00	1.00	0.75	196.849	2.38	0.226	-0.082	0.882
					(4.89)	(-9.63)	(22.35)
0.50	0.25	0.50	196.261	2.97	0.134	-0.025	1.494
					(4.82)	(-8.84)	(20.47)
1.00	1.25	0.50	196.219	3.01	0.473	-0.086	0.48
					(6.17)	(-10.02)	(21.55)
0.50	0.25	0.75	196.202	3.03	0.075	-0.025	1.517
					(4.81)	(-8.85)	(22.10)
0.50	0.50	0.75	196.139	3.09	0.071	-0.026	0.888
					(4.46)	(-9.11)	(22.08)
1.00	0.75	0.75	195.804	3.43	0.218	-0.080	1.542
					(4.56)	(-9.19)	(22.04)
1.00	1.50	0.25	195.684	3.55	0.968	-0.088	0.258
					(8.25)	(-10.37)	(21.88)
1.00	0.75	1.00	195.574	3.66	0.121	-0.080	1.565
					(4.50)	(-9.17)	(23.58)
0.00	0.00	1.00	191.348	7.89*	0.010	-0.008	0.933
					(3.49)	(-8.10)	(25.02)

* Indicates significance at the 95% level.

TABLE 3.4 DEMAND FOR M2C WITH EXTENDED BOX-COX (EBC) TRANSFORMATION

λ	μ	γ	L-MAX	LIKELIHOOD RATIO	COEFF. OF INCOME	COEFF. OF INTEREST	COEFF. OF LAGGED M2C
0.00	0.00	0.50	305.198		0.075 (4.22)	-0.008 (-6.04)	0.829 (18.90)
0.00	0.00	0.25	304.620	0.58	0.098 (4.06)	-0.008 (-5.86)	0.871 (24.62)
0.50	0.50	1.00	304.615	0.59	0.039 (4.03)	-0.013 (-5.89)	0.270 (21.30)
0.50	0.50	1.25	304.062	1.14	0.027 (3.87)	-0.013 (-5.84)	0.263 (17.14)
0.00	0.00	0.00	303.970	1.23	0.121 (3.74)	-0.021 (-5.72)	0.908 (31.79)
0.50	0.50	0.75	303.931	1.27	0.051 (3.83)	-0.012 (-5.71)	0.283 (27.47)
0.00	0.00	0.75	303.846	1.35	0.049 (3.84)	-0.008 (-5.86)	0.814 (15.62)
0.50	0.50	0.50	302.776	2.42	0.064 (3.50)	-0.002 (-5.44)	0.293 (34.62)
0.50	0.75	0.25	302.964	2.24	0.267 (12.58)	-0.013 (-6.11)	0.107 (42.54)
0.50	0.50	0.25	302.384	2.82	-0.086 (-3.36)	-0.012 (-5.36)	0.299 (42.24)
0.50	0.50	0.00	302.271	2.93	0.062 (3.38)	-0.032 (-5.32)	0.294 (34.75)
0.50	0.75	0.50	302.159	3.04	0.186 (12.40)	-0.014 (-6.29)	0.102 (34.23)
0.50	0.50	1.50	301.341	3.86	0.013 (3.00)	-0.012 (-5.32)	0.273 (16.41)
0.00	0.00	1.00	300.334	4.87*	0.021 (2.68)	-0.007 (-5.21)	0.871 (16.22)
1.00	1.00	1.00	297.906	7.29*	0.034 (3.20)	-0.019 (-5.22)	0.095 (37.37)

* Indicates significance at 95% level.

Considering, for simplicity, those forms in Table 3.3 for which $\lambda=\mu$ we are left with 8 possible functional transformations. In order of likelihood-ratio acceptability they are (.5, .5, .25), (1, 1, .25), (1, 1, .5), (0, 0, 0), (0, 0, .25), (.5, .5, .5), (1, 1, .75) and (.5, .5, .75). This last reduction in numbers is not as ad hoc as it might at first appear. Recalling that μ is the power coefficient of both income and lagged money and that adjustment of real balances is assumed to behave according to equation (3.3), the imposition of $\lambda=\mu$ is not wholly unrealistic.

For M2C (Table 3.4) a slightly different picture emerges. In both the logarithmic and square root families improvement in L-max is obtained by relaxation of the $\mu=\gamma$ constraint. Setting, as before, $\lambda=\mu$ we have 11 possible models which in order of decreasing L-max are: (0, 0, .5), (0, 0, .25), (.5, .5, 1), (.5, .5, 1.25), (0, 0, 0), (.5, .5, .75), (0, 0, .75), (.5, .5, .5), (.5, .5, .25), (.5, .5, 0) and (.5, .5, 1.5). For both M1 and M2C the full logarithmic transformation fares well. The semi-log and full linear models are rejected for both aggregates.

Another perspective of the effect of relaxing the $\mu=\gamma$ constraint is gained from Charts 3.1a and 3.1b. These charts plot the means of estimated income elasticities (M1 and M2C), as computed from (3.7), for the square root and logarithmic families for various values of γ. It is noted that the income elasticity exhibits higher variability for M2C in the logarithmic form than in the square root form as γ is varied. It is also exhibits greater variability for M1 in the square root form as γ is varied. In this case it might be appropriate to choose that functional form

CHART 3.1a: MEAN INCOME ELASTICITIES IN THE SQUARE ROOT FAMILY OF THE EBC MODEL FOR A RANGE OF POWER COEFFICIENTS ON THE INTEREST VARIABLE.

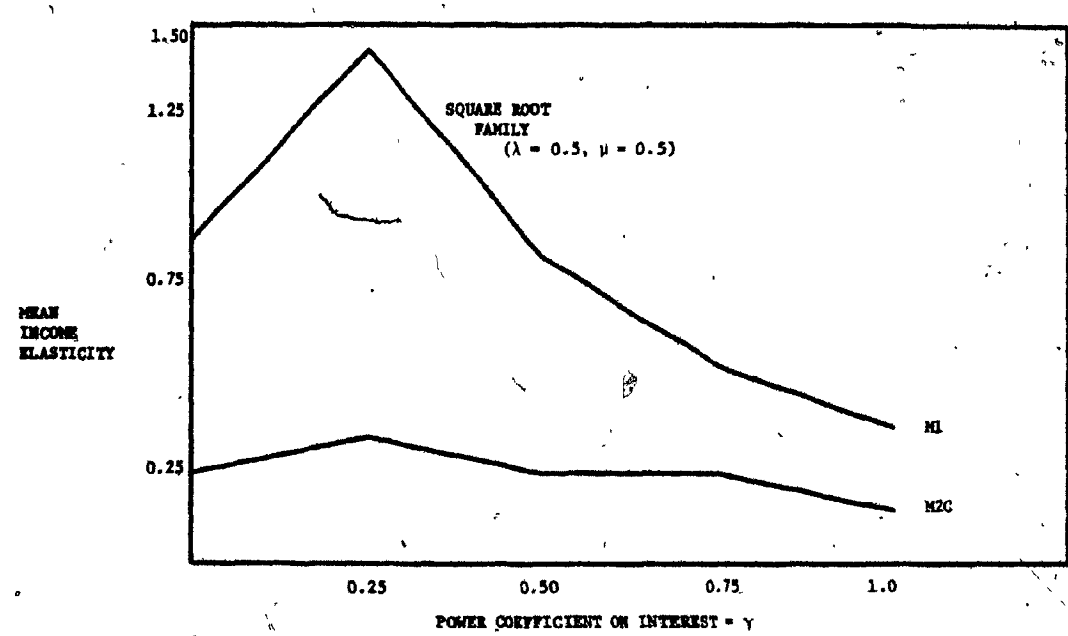


CHART 3.1b: MEAN INCOME ELASTICITIES IN THE LOGARITHMIC FAMILY OF THE EBC MODEL FOR A RANGE OF POWER COEFFICIENTS ON THE INTEREST VARIABLE.

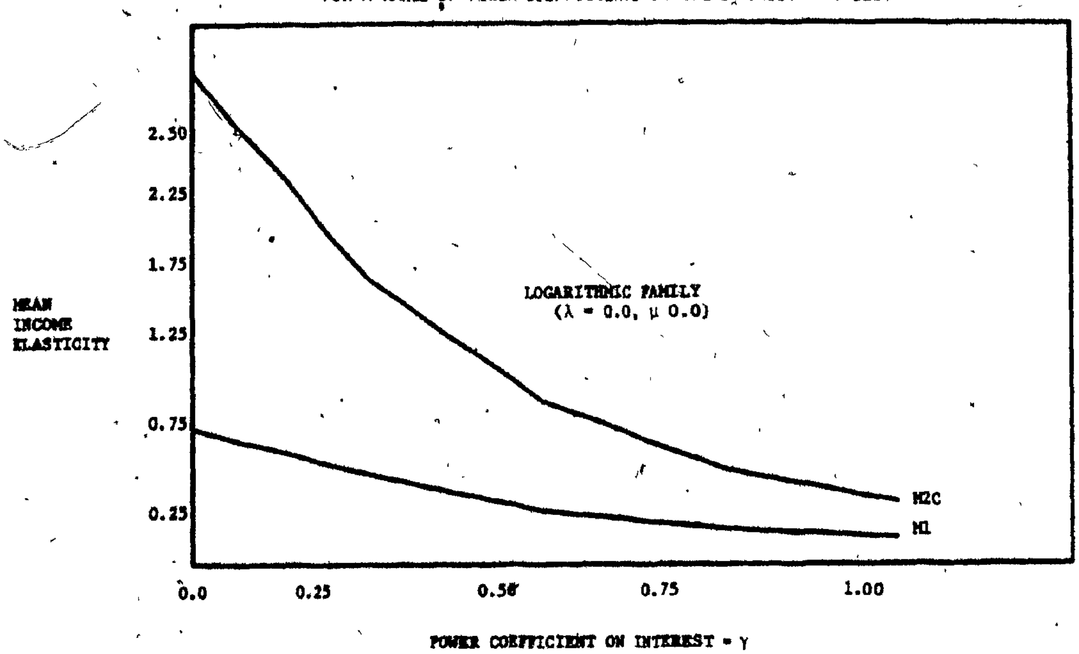


TABLE 3.5: BOX-PIERCE TESTS FOR AUTOCORRELATION IN REGRESSIONS EMPLOYING THE EXTENDED BOX-COX (EBC) TRANSFORMATION

MONEY	λ	μ	γ	FIRST ORDER AUTOCORRELATION	AUTOCORRELATION OF ORDERS ONE TO FOUR
M1	0.50	0.50	0.25	0.44	7.15
	1.00	1.00	0.50	0.44	7.86*
	0.50	0.50	0.50	0.72	7.38
	0.00	0.00	0.00	1.29	8.58*
	1.00	1.00	1.00	1.09	8.23*
	0.00	0.00	1.00	1.76	7.50
M2C	0.00	0.00	0.50	4.76**	9.90**
	0.00	0.00	0.25	3.60*	9.02*
	0.50	0.50	1.00	4.36**	9.02*
	0.00	0.00	0.00	2.92*	9.14*
	0.50	0.50	0.50	3.25*	8.21*
	0.00	0.00	1.00	9.22**	19.16**
	1.00	1.00	1.00	3.25*	7.53

* Indicates significant autocorrelation at the 90% level.
** Indicates significant autocorrelation at the 95% level.

for which the estimates are least sensitive to an improperly transformed interest variable; namely, the logarithmic form for M1 and the square root form for M2C.

For selected forms of the M1 and M2C models Box-Pierce tests for autocorrelation* of order 1, and of orders 1 through 4, were applied to the residuals. The results appear in Table 3.5. The models chosen for testing were four accepted models from Table 3.3, five accepted models from Table 3.4, the semi-log model and the full linear model. It is seen that the M1 models are relatively free of autocorrelation. This supports the widely-held belief that the demand equation for M1 is properly specified and that its performance is generally insensitive to choice of functional form [(see Mills, (1978)]. M2C, however, shows significant autocorrelation for all of the seven forms tested. This evidence suggests that, at least in the case of M2C, our modelling is not complete. The next step in the development of our model will be to design a method for removing the AR(1) component of this autocorrelation within the Box and Cox framework.**

3.3. Extended Box-Cox Autoregressive of Order One (EBCAR1) Estimation

Consider the model:

$$(M_t-P_t)^{(\lambda)} = g\beta_0 + g\beta_1\, y_t^{(\mu)} + g\beta_2\, r_t^{(\gamma)} + (1-g)(M_{t-1}-P_{t-1})^{(\mu)} + u_t, \tag{3.4b}$$

where the disturbances, u_t, follow the stationary AR(1) process defined by:

* See Box and Pierce (1971), Ljung and Box (1978)

** In addition to the Box-Pierce tests for autocorrelation, the Shapiro-Wilk test for normality was applied to all thirteen sets of residuals. The S-W statistic was uniformly less than .13 while the critical value needed for rejection of normality was upwards of .95.

$$u_t = \rho u_{t-1} + e_t, \ |\rho| < 1$$

where the e_t's are independent $N(0,\sigma^2)$ random variables. For this model $E(u) = 0$ and $E(uu') = V$ where

$$V^{-1} = \begin{bmatrix} 1 & -\rho & 0 & \cdots & & 0 \\ -\rho & (1+\rho^2) & -\rho & & & \vdots \\ 0 & -\rho & (1+\rho^2) & & & \vdots \\ \vdots & & & & (1+\rho^2) & -\rho \\ 0 & \cdots & \cdots & & & 1 \end{bmatrix}.$$

The concentrated log-likelihood function is given by

$$L(\lambda,\mu,\gamma,\rho;M,X)$$
$$= -\frac{T}{2}\{\ell n(2\pi) + 1\} - \frac{T}{2}\ \ell n\{\hat{\sigma}^2(\gamma,\mu,\gamma,\rho)\}$$
$$+ \ \frac{1}{2}\ \ell n(1-\rho^2) + (\lambda-1) \sum_{t=1}^{T} \ell n\,(M_t-P_t)\,. \tag{3.6b}$$

The estimation of ρ then simply involves adding a fourth dimension to the maximization procedure described above with the new likelihood function (3.6b). The results are summarized in Tables 3.6 and 3.7.

Since the new dimensionality allows ρ to vary from -.9 to .9 in increments of .1 the number of potential models increases by a factor of 19 to a new total of 1425. Of these 122 were not rejected by the likelihood ratio test. Table 3.6 shows the incidence of these 122 models in the various families (λ,μ) and sub-families (λ,μ,γ). It is seen, for example, that in the square root family (λ=.5, γ=.5) with γ=0 there are five forms which meet the likelihood ratio requirements. Of these (which correspond to 5 different values of ρ between -.9 and .9) the form (λ=.5, μ=.5, γ=0, ρ=.2) reaches the highest L-max. Again we can reduce our field by considering only those models for which $\lambda=\mu$ and we are left with the ten models appearing in Table 3.7.

Table 3.8 registers the results of Box-Pierce tests applied to the residuals of the EBCAR1 transformed model. Comparing Tables 3.8 and 3.5 then shows that the EBCAR1 extinguishes the first order autocorrelation from six of the seven models tested. The one model which is not improved is the semi-log model. In order to get a clearer understanding, the autocorrelation functions (AGF) and partial autocorrelation functions (PACF) of the seven sets of residuals are plotted in charts 3.4 - 3.10. In time series analysis it can be shown that the ACF of an AR(p) process decays while its PACF cuts off after the pth autocorrelation. Similarly the ACF of an MA(q) process cuts off after the qth autocorrelation while its PACF decays. Charts 3.4 - 3.8 and Chart 3.10 show the complete removal of the first order component. Chart 3.9 shows no improvement. The semi-log transformation apparently, intoduces a moving average form of autocorrelation in the residual of the M2C equation which the AR(1) correction cannot remove.

TABLE 3.6: THE INCIDENCE OF ACCEPTABLE MODELS FOR DEMAND FOR M2C EMPLOYING THE EBCARI TRANSFORMATION

λ	μ	γ	Number in Family*	Family Max Occurs at ρ	L-Max
0.5	0.5	0.0	5	0.2	300.203
		0.25	5	0.2	300.479
		0.5	5	0.2	300.716
		0.75	5	0.2	300.911
		1.0	6	0.2	301.060
			26		
0.5	0.75	0.0	5	0.4	301.497
		0.25	5	0.4	301.924
		0.50	7	0.4	302.294
		0.75	7	0.4	302.600
		1.0	7	0.4	302.834
			31		
0.0	0.0	0.0	7	0.2	302.082
		0.25	7	0.2	302.148
		0.5	7	0.2	302.165
		0.75	7	0.2	302.134
		1.0	7	0.2	302.057
			35		
0.0	0.25	0.0	6	0.4	301.943
		0.25	6	0.4	302.170
		0.5	6	0.4	302.325
		0.75	6	0.4	302.406
		1.0	6	0.4	302.414
			30		

* This column contains the number of models for each λ,μ,γ combination which are not rejected by the likelihood ratio test. There are five such models in the (λ=.5, μ=.5, γ=o.o) family, each of which has a different value of ρ.

TABLE 3.7: DEMAND FOR M2C WITH EBCART TRANSFORMATION: THE TOP-10

λ	μ	γ	ρ	COEFF. OF INCOME	COEFF. OF INTEREST	COEFF. OF LAGGED M2C	L-MAX
0.0	0.0	0.0	0.2	0.137 (3.47)	-0.020 (-4.81)	0.894 (25.84)	302.082
		0.25	0.2	0.138 (3.50)	-0.016 (-4.83)	.661 (25.79)	302.148
		0.5	0.2	0.139 (3.52)	-0.012 (4.83)	.489 (25.75)	302.165
		0.75	0.2	0.139 (3.53)	-0.009 (-4.83)	.362 (25.70)	302.134
		1.0	0.2	0.140 (3.53)	-0.007 (-4.81)	.267 (25.65)	302.057
0.5	0.5	0.0	0.2	0.071 (3.20)	0.031 (-4.48)	0.529 (28.38)	300.203
		0.25	0.2	0.072 (3.25)	-0.025 (-4.55)	.391 (28.36)	300.479
		0.5	0.2	0.073 (3.30)	-0.019 (-4.62)	.289 (28.34)	300.716
		0.75	0.2	0.074 (3.34)	-0.015 (-4.67)	.214 (28.33)	300.911
		1.0	0.2	0.075 (3.38)	-0.012 (-4.70)	.158 (28.31)	301.060

TABLE 3.8: BOX-PIERCE TESTS FOR AUTOCORRELATION EMPLOYING THE EXTENDED BOX-COX AUTOREGRESSIVE OF ORDER ONE (EBCAR1) TRANSFORMATION (M2C)

λ	μ	γ	ρ	FIRST ORDER AUTOCORR.	AUTOCORR. OF ORDERS ONE TO FOUR
0.00	0.00	0.50	0.2	0.00	7.79*
0.00	0.00	0.25	0.2	0.00	7.80*
0.50	0.50	1.00	0.2	0.01	7.87*
0.00	0.00	0.00	0.2	0.01	8.20*
0.50	0.50	0.50	0.2	0.00	7.50
0.00	0.00	1.00	0.2	9.70**	26.56**
1.00	1.00	1.00	0.2	0.01	6.21

* Indicates significant autocorrelation at the 90% level.
** Indicates significant autocorrelation at the 99.5% level.

3.4 The Semi-Logarithmic Model and Interest Rate Response

A recent, and increasingly popular, variant of the logarithmic demand for money function is the semi-log specification where the interest rate is entered as a level and all other dependent and independent variables are entered logarithmically. This model dates at least to Cagan (1956) who regressed the logarithm of real balances upon the level of anticipated inflation. Hacche (1974) resurrected the idea in his usage of interest rate levels as a variable in a logarithmic demand for money function. The economic thinking behind such a specification is revealed in the statement by Hacche:

> "The log-linear form ..... constrains the elasticities (short-term and long-term) of the demand for money with respect to each explanatory variable to be constant, and in particular to be independent of the level of the variable. This implicit assumption is convenient and, generally speaking, not implausible. In the case of the interest rate variable, however, it is perhaps less likely that, for example, a doubling in the rate from 1 per cent to 2 per cent will have the same proportionate effect on the demand for money as will a doubling from 10 per cent to 20 per cent."

He goes on to show that by using the variable (1+R) instead of R, a rise from 10 per cent to about 11.1 per cent would have the same proportionate effect upon money demand as would a rise from 1 per cent to 2 per cent. Furthermore, since $\ln(1+R) \simeq R$ for small R, then the semi-log specification is intuitively justified. This seemingly innocuous modification has become firmly rooted in contemporary empirical work.

Statistical tests for stability as developed in Chapter 2 do not reject stability in the logarithmic specification for M1 but do reject stability for the semi-log model. Furthermore, the CSQ statistic from the backward recursive regression when plotted shows strong vertical movement for the semi-log model between 1958:3 and 1959:2. Since the recursive residuals are themselves a series of one-period-ahead prediction errors a pronounced movement in their normalised cumulative sum of squares indicates the position in time of structural shift. Since the backward CSQ test is more sensitive to early sample instability than the forward test (see Chapter 2), it is not inconsistent that the forward test does not indicate rejection.

While it is known that the late 1950s were a rather turbulent period of Canadian financial history,* the task at hand is to explain why one model appears to be stable while the other does not. One possible explanation is summarized in Charts 3.2 and 3.3. In Chart 3.2 it is seen that there exist two intervals between 1956 and 1962 -- 1958:3 and 1959:4 -- in which RPROV (the McLeod, Young and Weir average of ten provicial bond yields) and R90 move in opposite directions. It is seen also that the logarithmic R90 series exhibits much less variability than its level R90 counterpart. If money demand responds to changes in both short and long rates,** and if factors causing an imbalance in the bond market force long rates upward but not short

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* It is a period, for example, of accelerating growth in near-banks, of vigorous growth in short-term money market instruments and the time of an unprecedentedly large sales campaign of government securities.

** Friedman (1977) develops the theoretical basis for this argument.

CHART 3.2: R90, LOG(R90) AND RPROV 1956 II to 1962 II

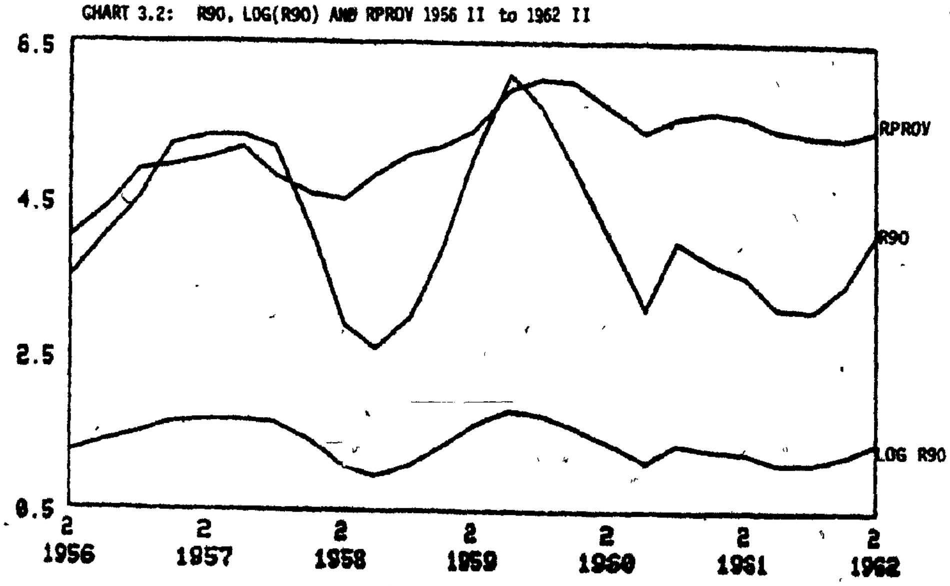
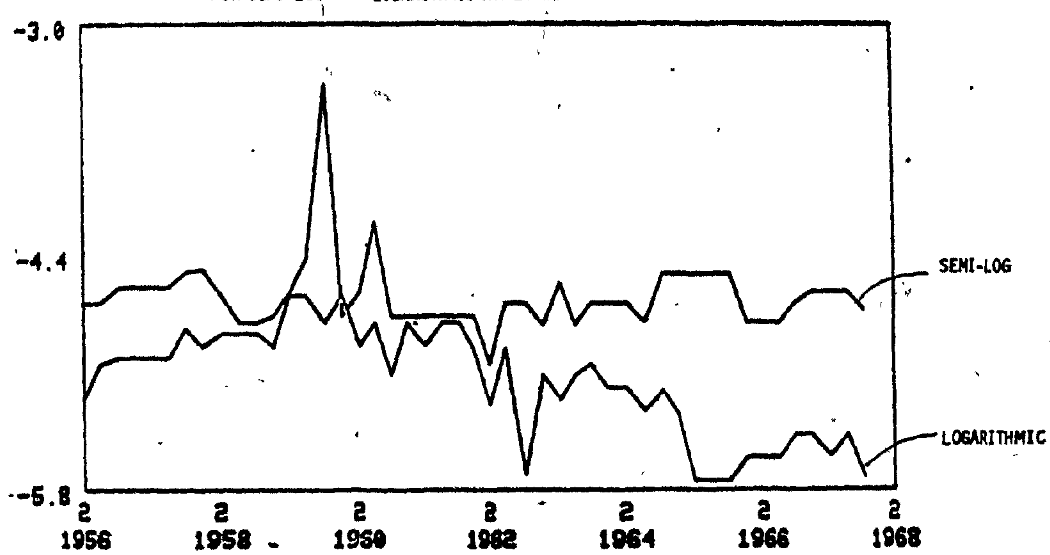


CHART 3.3: ESTIMATED SHORT-RUN INTEREST ELASTICITIES ($X10^{-2}$) OVER TIME FOR SEMI-LOG AND LOGARITHMIC MONEY DEMAND 1956 II TO 1966 IV



rates, then a regression relating money demand only to R90 might appear unstable. The effect of RPROV rising would likely be to reduce money demand. The absence of a simultaneous rise, or presence of a fall, in R90 would then be reflected in a less negative interest elasticity of money demand. In Chart 3.3 the estimated short-run interest elasticity is plotted over time for both models. The extreme left values of this plot are the regression coefficients based upon the full sample of 87 observations. In moving from left to right each new position corresponds to the coefficient produced by a successively smaller sample with the earliest observation removed. Both models show a smaller absolute value (less negative) for the interest elasticity in the late 1950s. In the case of the semi-log model the effect is more pronounced.* The relative instability of the level R90 regression might then be attributed to the greater variability exhibited by R90 than by its logarithmic transformation.

Box-Cox estimation of the appropriate functional from is summarized in Table 3.3. These results show that the log-linear model ($\lambda = 0$, $\mu = 0$, $\gamma = 0$) is clearly statistically superior to the semi-log model ($\lambda = 0$, $\mu = 0$, $\gamma = 1$) for these data.

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* The long-run elasticity as measured by $a_2\overline{R}$, where $\overline{R}$ is the mean of the interest rate series, shows the same peak in 1959:4.

3.5. Concluding Comments

This study has been designed to demonstrate some of the rigidities inherent in conventional demand for money models and to illustrate the consequence of their removal upon demand for money estimates in Canada. It is found that the logarithmic transformation works well for both narrow and broad money aggregates. The square root transformation works equally well and, on the basis of the likelihood ratio test, no statistical preference is indicated for one or the other. When the power transformation of the interest variable is allowed to vary from that of other variables, the logarithmic form for M1 shows less variability in income elasticity than the square root form. For M2C the square root form shows a more stable income elasticity. The specification of the M1 model does not appear sensitive to functional form -- the residuals of the M1 equation being almost universally free of autocorrelation. The type of autocorrelation present in the M2C model does, however, appear to be sensitive to the choice of functional form. The semi-log model, for example, seems to inject a form of autocorrelation into the error which conventional corrections do not remove.

While it may be intuitively appealing to suggest that the effect upon the demand for money of an increase in rates from 1 per cent to 2 per cent should be proportionately the same as the effect of an increase from 10 per cent to 11.1 per cent, the theoretical implications bear closer scrutiny. The aggregate demand curve for real money balances in such a world would be nearly linear with respect to interest rates. Given the usual independence assumptions between individuals, market participants would display this nearly linear response to the full spectrum of interest rates. This linearity assumption imposes upon the world a homogeneous response to interest rates whether they are high or low. That is to say, individuals would respond to a changing income in precisely the same way vis-a-vis their cash balances whether the economy was in recession or expansion. Such a view negates the possibility that uncertainty plays a role in the formation of portfolio decisions.

A more realistic view of the world, and one which dates back to the foundation of the Keynesian theory of liquidity preference, is that all market participants entertain a notion as to what is the normal rate of interest at any point in time. There is a point for each consumer where market rates are sufficiently below this normal rate that demand becomes inelastic. When rates fall to a critical level, suspicion that bond prices can do nothing but fall prevents any movement out of money and into bonds in spite of variations in the opportunity cost of holding money. Any new increments of income are fed directly into the cash component of the individual's portfolio. It is the existence of different ideas as to what is the normal rate of interest at any point in time which bends the aggregate demand curve convex to the origin. The logarithmic demand curve is of this form.

The early Keynesian view, it has been argued, is no longer relevant in a modern world of high inflation and high nominal interest rates. It is precisely this argument which suggests an asymmetric effect of the omitted expectations variable upon the interest rate response of money demand. It is possible that, in buoyant periods, when inflation and inflationary expectations are high money demand responds linearly with respect to the rate of interest. In recessionary periods with low inflation and interest rates, however, expectational effects are likely to be overshadowed by general feelings of uncertainty and the simple Keynesian model is likely to be the relevant one.

This theoretical exposition includes, of course, the possibility that interest rates could fall to a level at or below the critical level for all market participants. Such a situation typifies the liquidity trap hypothesis where aggregate demand becomes perfectly inelastic to interest rates. Many studies, including Kostas and Khouja (1969), White (1972), Spitzer (1976), Barth, Kraft and Kraft (1976) and McCulloch (1978), have examined the possibility of existence of a liquidity trap. The general concensus has been that there is no empirical evidence to support its existence. This does not, however, imply linearity of the money demand function. The true aggregate demand curve may lie somewhere between the hyperbolic function of constant elasticities whose horizontal asymptote represents an interest rate floor and the linear function of the semilogarithmic specification. For quarterly data from 1956 II to 1977 IV, the lifting of the the constraint of constant elasticity and imposition of that of linear response appears neither theoretically nor empirically justified.

CHART 3.4:
ACF AND PACF OF RESIDUALS FROM EBC MODEL: $\lambda=0,\ \mu=0,\ \gamma=.5$ (M2C)

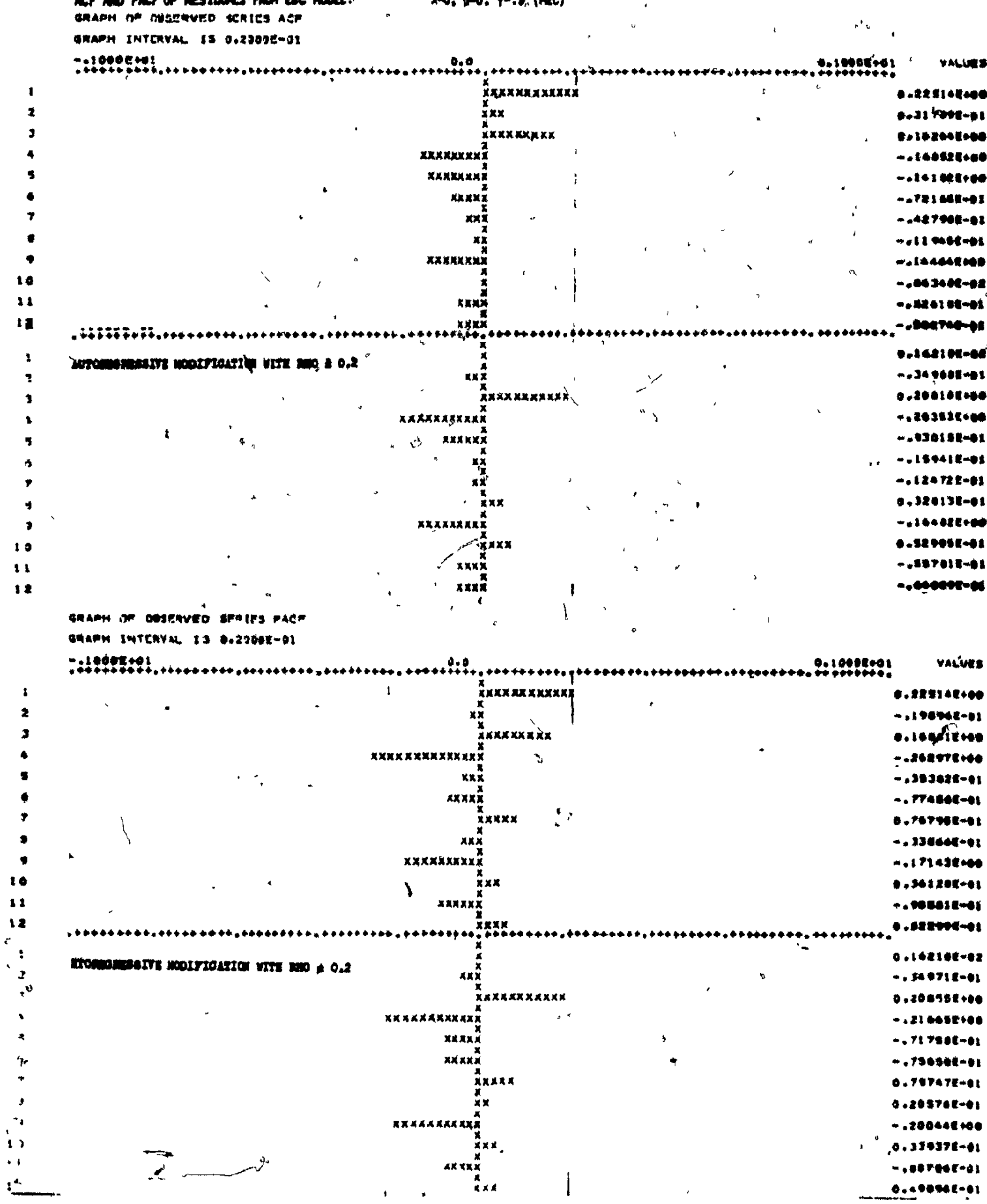
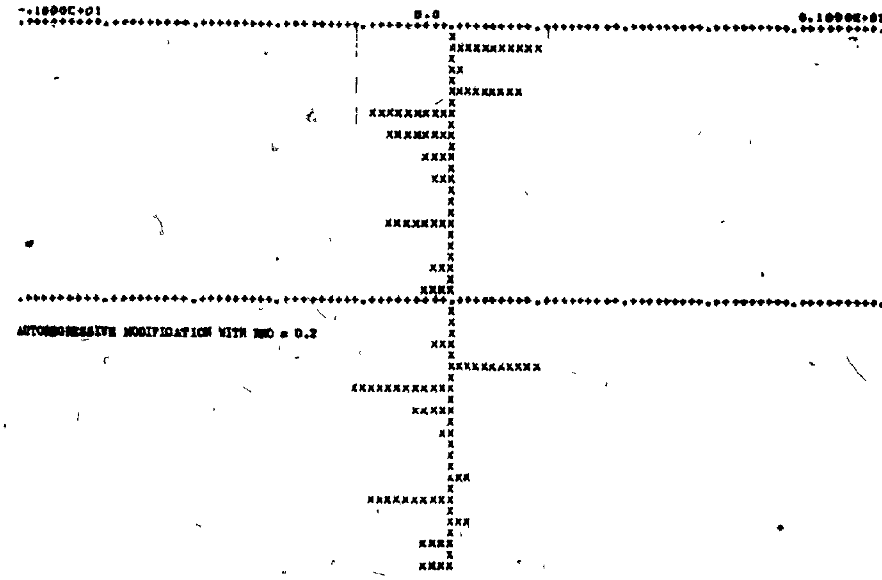


CHART 3.5:
ACF AND PACF OF RESIDUALS FROM EDC MODEL: λ=0, ν=0, γ=.25 (MEC)

GRAPH OF OBSERVED SERIES ACF

GRAPH INTERVAL IS 0.2000E-01



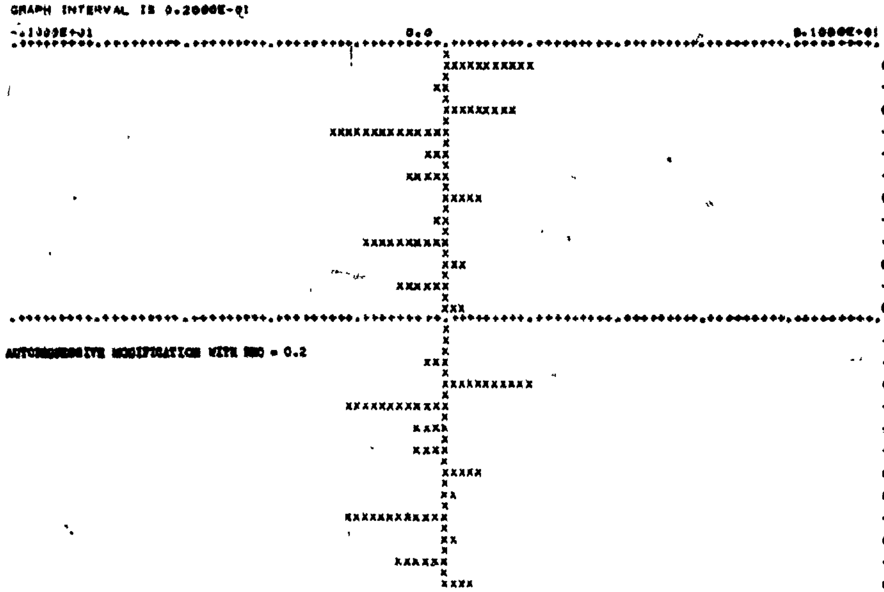
	VALUES
1	0.20441E+00
2	0.22360E-01
3	0.16183E+00
4	-.17629E+00
5	-.14543E+00
6	-.60283E-01
7	-.37708E-01
8	-.43037E-02
9	-.14085E+00
10	0.51564E-01
11	-.66250E-01
12	-.63196E-01

AUTOREGRESSIVE MODIFICATION WITH RHO = 0.2

	VALUES
1	-.26603E-02
2	-.32326E-01
3	0.20353E+00
4	-.21179E+00
5	-.87114E-01
6	-.16716E-01
7	-.82709E-02
8	0.32818E-01
9	-.17463E+00
10	0.46412E-01
11	-.66829E-01
12	-.60036E-01

GRAPH OF OBSERVED SERIES PACF

GRAPH INTERVAL IS 0.2000E-01



	VALUES
1	0.20441E+00
2	-.20272E-01
3	0.16841E+00
4	-.26194E+00
5	-.46052E-01
6	-.73760E-01
7	0.76745E-01
8	-.24431E-01
9	-.17564E+00
10	0.46926E-01
11	-.90068E-01
12	[illegible]

AUTOREGRESSIVE MODIFICATION WITH RHO = 0.2

	VALUES
1	-.20603E-02
2	-.32331E-01
3	0.20361E+00
4	-.22277E+00
5	-.68531E-01
6	-.47707E-01
7	0.85235E-01
8	0.14724E-01
9	-.21295E+00
10	0.20576E-01
11	-.91107E-01
12	0.55440E-01

CHART 1.6:
ACF AND PACF OF RESIDUALS FROM ESC MODEL: λ=.5, μ=.5, γ=1 (REC)

GRAPH OF OBSERVED SERIES ACF
GRAPH INTERVAL IS 0.2000E-01

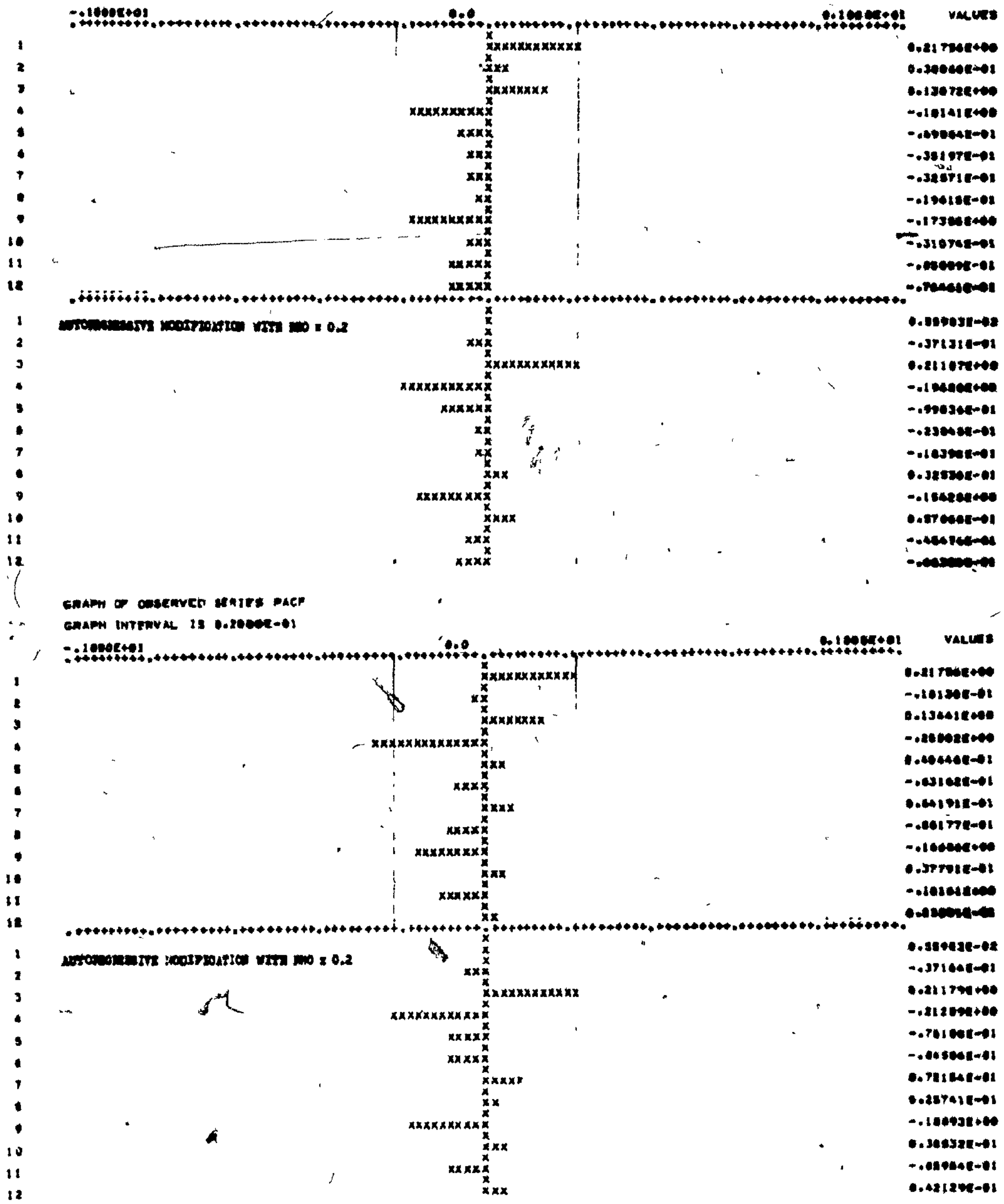


CHART 3.7:
ACF AND PACF OF RESIDUALS FROM EBC MODEL: δ=0, μ=0, γ=0 (NEC)

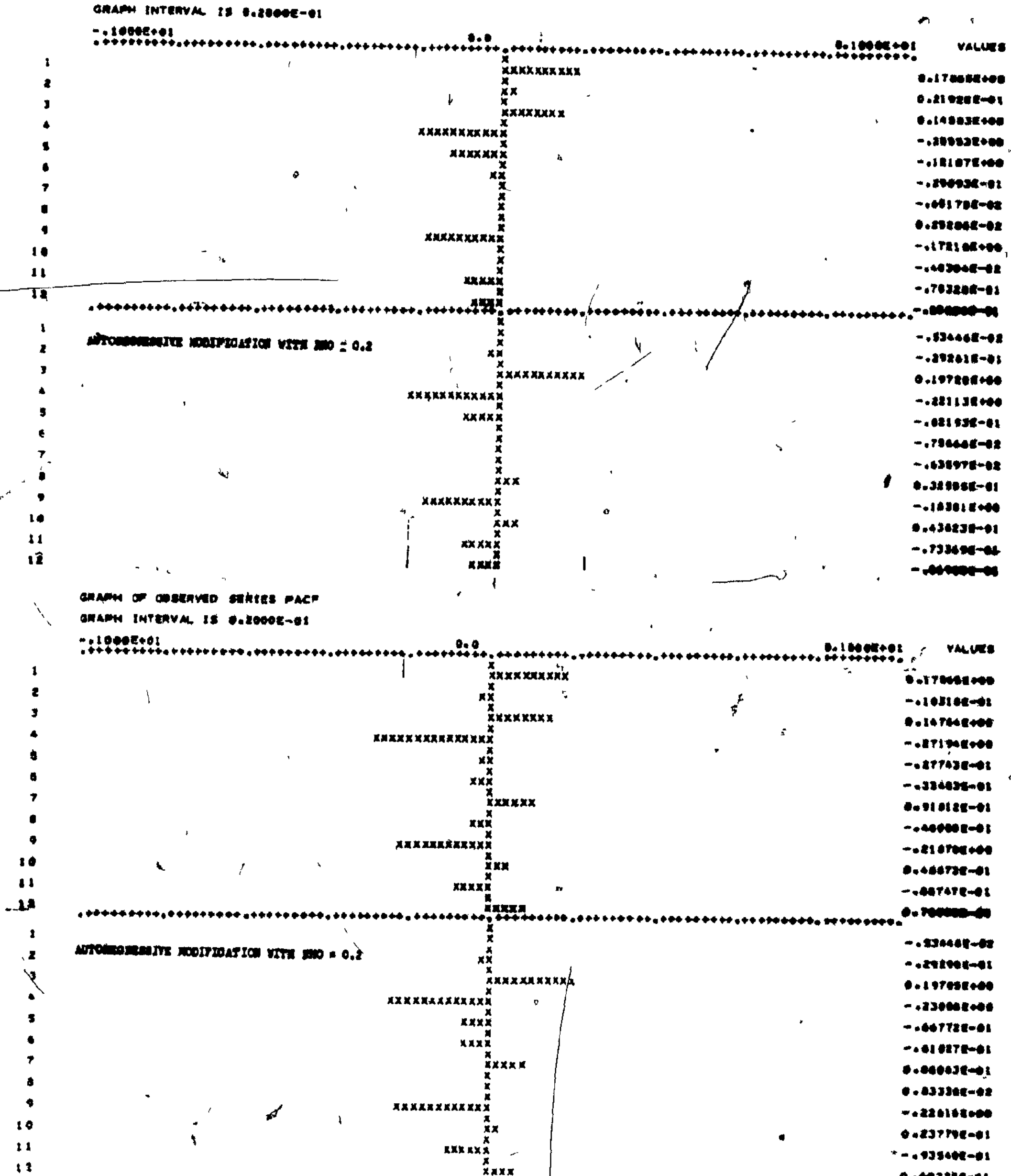


CHART 3.8:
ACF AND PACF OF RESIDUALS FROM EBC MODEL: λ=.5, μ=.5, γ=.5 (MRC)

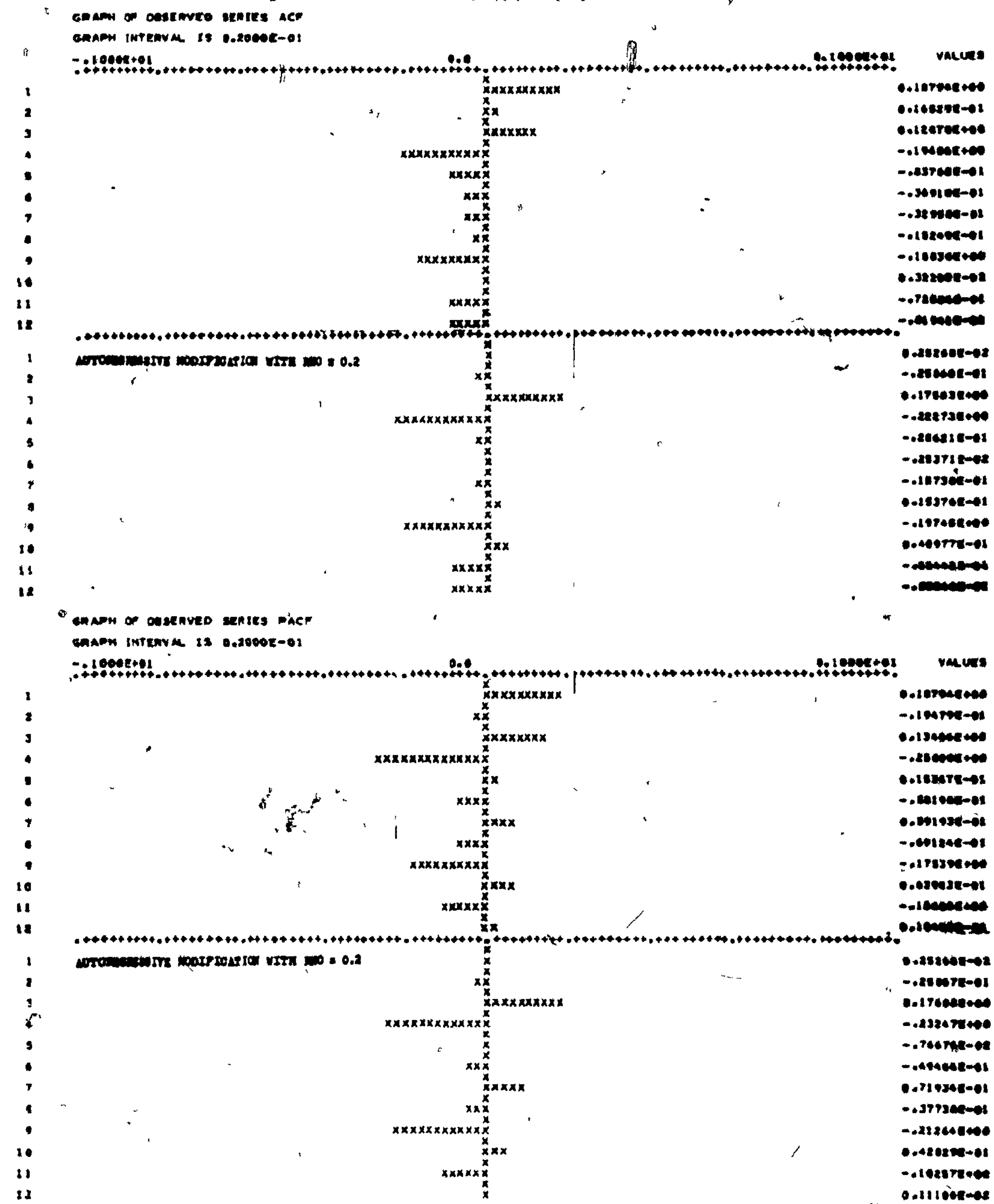


CHART 3.9:
ACF AND PACF OF RESIDUALS FROM EBC MODEL: $\lambda=0$, $\mu=0$, $\gamma=1$ (NEC)

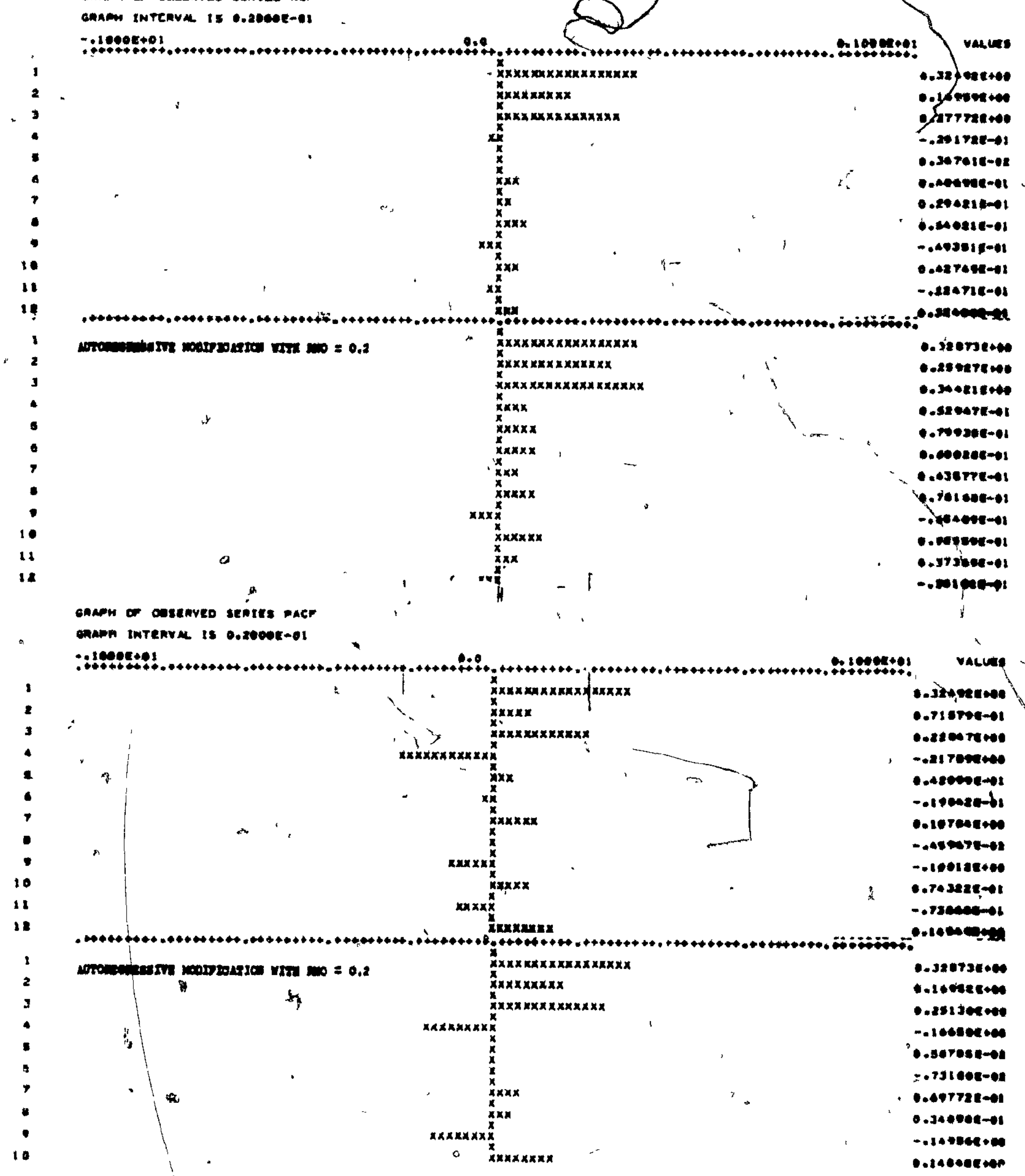
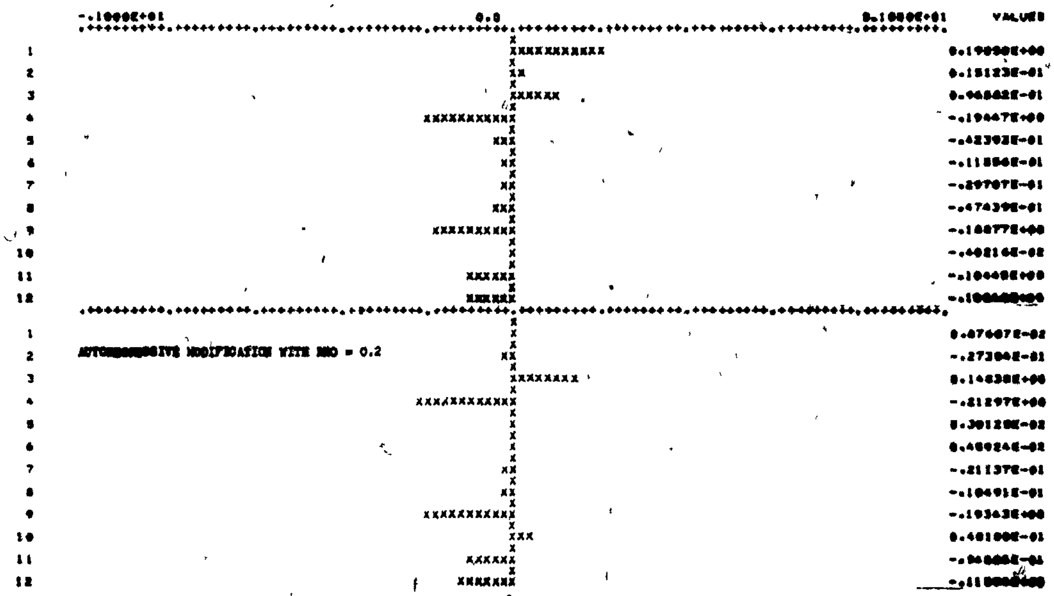


CHART 3.10:
ACF AND PACF OF RESIDUALS FROM EDC MODEL: $\lambda=1$, $\mu=1$, $\gamma=1$ (MEC)

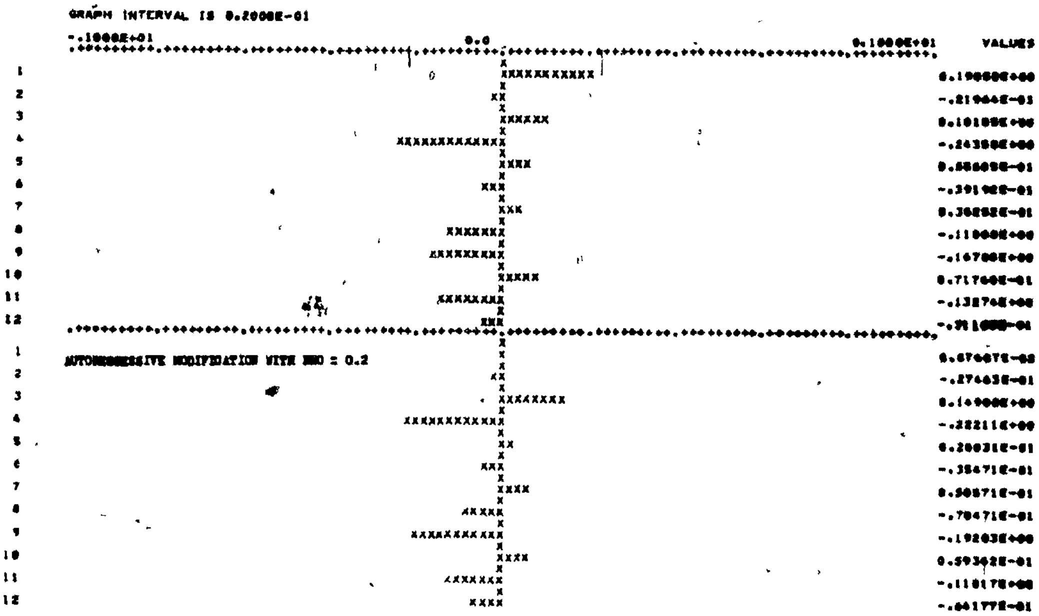
GRAPH OF OBSERVED SERIES ACF

GRAPH INTERVAL IS 0.2000E-01



GRAPH OF OBSERVED SERIES PACF

GRAPH INTERVAL IS 0.2000E-01



CHAPTER 4

CAUSALITY

Econometrics is intimately tied to questions of causality. Implicit in many single-equation econometric models where an endogenous variable is regressed upon predetermined variables is the assumption of causality running from the right-hand 'independent' variables to the left-hand 'dependent' variable. Unbiased and consistent estimates of the parameters of these models can be found through the application of ordinary least squares (OLS). If, as suggested by Wold (1952), unidirectional causal chains can in general be mapped out between economic variables, then the world can be modelled recursively. Requirements for a recursive system are that 1) the matrix of coefficients be triangular and that 2) the contemporaneous error dispersion matrix be diagonal. The relations of these systems may be given unidirectional causal interpretation.

An alternative causal structure was advanced by Haavelmo (1944), and later by Koopmans (1949) as being more relevant to economics. Their model, known as the simultaneous equation model, allowed for the existence of feedback, or bidirectional causality. Given identifiability of a structural equation, consistent and asymptotically unbiased estimates of the structural parameters can be obtained via two stage least squares and alternative methods. Strotz and Wold (1960) considered causal interpretation within a simultaneous equation model. They suggested that, for practical purposes, it might be desirable to impose recursivity upon such models through the use of control variables.

At least three extensions appeared in the literature around this time. Simon (1964) and Blalock (1962) built upon the foundations of path anaylsis, as originated by Wright (1934), in the study of causality in recursive models. Zellner (1962) considered recursive models with cross-equation correlation in the error terms. Feedback does not enter explicitly in such models but is manifested in the error terms of 'seemingly-unrelated' regressions. Fisher (1965) suggested a compromise between the simple but naive recursive models of Wold and the more complex simultaneous equation models of the Cowles Commission. In Fisher's 'block-recursive' models, causation is simultaneous within blocks of equations but is uni-directional between blocks.

Granger (1969), drawing upon work by Wiener (1956), explicitly laid out an econometric definition of causality. This definition has been subsequently used in many empirical studies. Pierce and Haugh (1977) give a thorough review of the definitions and methodology involved. Section 4.1 of the present chapter defines various terms with which causality studies are typically concerned. Zellner (1978) points to several problems encountered in transforming the Wiener-Granger definition of causality to an empirically testable definition. These problems, and others, are described in Section 4.2. In Section 4.3 the empirical results of multivariate causality tests applied to Canadian money, income and interest series are presented. Conclusions of this analysis are summarized in Section 4.4.

4.1 Definitions

As an illustration of the basic methodology, consider X and Y as covariance stationary processes. X is said to be causing Y, in the Granger sense, if we are better able to predict Y_t using all information than all information excluding X.* We may write

$$X_t = au_t + bv_t$$
$$Y_t = cu_t + dv_t, \qquad (4.1)$$

where u and v are mutually uncorrelated, white noise processes. Suppose that u_t and v_t may be represented by distributed lags on X_t and Y_t together with white noise errors η_t and γ_t respectively:

$$u_t = \mu(L)X_t + \eta_t$$
$$v_t = \xi(L)Y_t + \gamma_t. \qquad (4.2)$$

*Zellner (1978) presents a survey of philosophical definitions of causality and their relation to econometric definitions.

Sims (1972a) proved that Y does not cause X (or Granger non-causality of X by Y) if and only if b in (4.1) is identically zero. Substituting (4.2) into (4.1) and setting b = 0 yields:

$$\begin{aligned} X_t &= a\mu(L)X_t + a\eta_t \\ Y_t &= c\mu(L)X_t + d\xi(L)Y_t + c\eta_t + d\gamma_t. \end{aligned} \tag{4.3}$$

In the second equation of (4.3) we solve for Y_t to give the expression:

$$Y_t = c\mu(L)\left[1-d\xi(L)\right]^{-1} X_t + \left[1-d\xi(L)\right]^{-1}\psi_t \tag{4.4}$$

where $\psi_t = c\eta_t + d\gamma_t$. Unidirectional causality from X to Y is consistent only with the situation that the prediction of Y is not improved with the inclusion of future X as explanatory variables. Sims suggested that this can be tested by regressing Y_t on future, current and past X and checking for the significance of coefficients on future X as a group.

It is seen that equation (4.4) is not in a form suitable for testing since its error is a moving average of white noise. Only under the strict assumption that $\xi(L)$ is of zero order will the errors be free of autocorrelation. This describes the need for filtering. Empirical methods for determining the appropriate filter for 'whitening' the residual will be discussed in the analysis of Section 4.3. A second important condition for applicability of the conventional significance tests to model (4.4) is that the errors be normally distributed. This assumption is rarely emphasized and seldom tested for in the empirical literature.

The finding of Granger non-causality of the RHS of a regression relation by the LHS is often referred to as exogeneity of the independent variables. Hansen and Sargent (1979) correctly point out that Granger non-causality of X by Y is a necessary but not sufficient condition for strict exogeneity of X in (4.4). An additional requirement is that $E\left[X_t \psi_{t-j}\right] = 0$, for all j. The conditions for exogeneity are therefore more stringent than those under which Y fails to Granger-cause X. This distinction is recognized by Wu (1973) who proposed a test for exogeneity within the simultaneous equation model. No attempt at establishing exogeneity will be attempted in the present study.

Feedback is said to occur, according to Granger, if X is causing Y and also Y is causing X. Instantaneous causality occurs if Y_t is better predicted with X_t included in the prediction than with X_t excluded. There exists a causality lag of length m if knowing the values of X_{t-j}, $j \geq m$ improves the prediction of Y_t. Spurious causality is said to exist between two variables when no causal-link exists between them but both are caused by a third variable.

The definitive work of Granger and Sims precipitated an explosion of empirical work on causality in economics. I limit my field of attention to studies relevant to money demand. My review cites the major studies together with their empirical results and lists the main econometric problems associated with them.

4.2 Survey and Critique

Sims (1972a) made the first authoritative statement on causality within a bivariate money-income relationship. He argued that a central tenet in the disagreement "between the monetarists and the skeptics" was regarding the direction of influence between business conditions and the quantity of money. The major thrust of the monetarist position was that nominal money had a direct impact upon nominal income, while non-monetarists argued that business conditions in general influenced the supply of money. Sims felt that empirical evidence as to the direction of this causality might help in resolving this dispute. He found, using post-war U.S. data, evidence of unidirectional causality from money to GNP.

Barth and Bennett (1974) using Canadian data from 1957 to 1972 found no evidence of unidirectional causality from money to GNP nor from GNP to money. Furthermore, when the index of industrial production (IIP) was used as the income measure they found unidirectional causality running from IIP to money. This evidence, which conflicts with the monetarist view and with the result of Sims, was attributed to the openness of the Canadian economy and fixed exchange rates which severely limited the scope of monetary policy over most of the period. Auerbach and Rutner (1978) cast doubt on the Barth and Bennett finding of bidirectional causality between GNP and money on the grounds that they filtered their data inappropriately.

Williams, Goodhart and Gowland (1976) examined the money-income relationship using data for the United Kingdom. They found that evidence as to causality was less clear cut for the U.K. than Sims reported for the U.S. There was some evidence of unidirectional causality running from nominal income to money but also some evidence of unidirectional causality running from money to prices. Their conclusion favoured a more complicated relationship between money and income in which both are determined simultaneously. Mills and Wood (1978) examined U.K. data for the period 1870-1914 when Britain was under the gold standard and fixed exchange rates. Their findings supported those of Barth and Bennett whose data also spanned a period predominated by fixed exchange rates. Their contention was that exchange rate policy exerts a powerful influence on the perceived relation between income and money.

Mehra (1978), using U.S. data in a multivariate model, could reject non-causality of the RHS variables; namely, nominal GNP, the rate on commercial paper (RCP) and the rate on time deposits (RTD); by nominal money. With real money and real GNP, however, he could not reject non-causality of the RHS by the LHS.

Most economists would agree that results of these causality tests should be interpreted cautiously. Seldom, however, are the reasons for such skepticism precisely spelled out. I have enumerated six complaints that have, at various times, been levelled at one or more of the above studies.

1. One pervasive conclusion emanating from these studies involves the specification of money demand relations. The trouble is, most causality studies have been set in bivariate money-income models. As Mehra noted:

> Sims' conclusion ... clearly implies that nominal money stock which is exogenous in a bivariate distributed-lag framework of income on money will still be exogenous in a multivariate distributed-lag framework of money-demand relations."*

Such is not necessarily the case. As noted earlier, two variables may appear to be causally linked when, in fact, they are not -- the phenomenon known as 'spurious causality'. The equivalent situation might exist when correlation is spuriously low instead of spuriously high. Pierce (1974), for example, referred

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*Mehra consistently used the term 'exogeneity' when in fact he meant 'Granger non-causality'. As noted above, evidence of a one-sided relation between money and contemporaneous and lagged values of explanatory variables implies only that money does not cause the explanatory variables in the Granger sense. Strict exogeneity is only indicated by additional evidence of lack of covariance between the error term and the explanatory variables themselves.

to the situation where two variables appear to be independent because of a common but opposite association with a third variable as 'spurious independence'. If we are to make conclusions about causality within a money demand relation, then, at least three variables are important -- money, income and a measure of the opportunity cost of holding money.

2. A related problem also considers the ultimate use to which the studies are directed. If we are to make inferences about money demand after having established the direction of causality between nominal income and money, then the demand function to which we should address ourselves is the demand for nominal balances. It is the demand for real balances, however, which is judged to be the economically relevant one. Sims (1972), Barth and Bennett (1974), and Mills and Wood (1978) all consider only nominal variables. Among the studies looking at both the real and nominal variables are Williams, Goodhart and Gowland (1976) and Mehra (1978). The present study looks at nominal, real and per capita money and income.

3. Most of the studies thus far mentioned have used seasonally adjusted data. The reason is, of course, the necessity of having 'whitened', covariance stationary data for applicability of the tests. As Sims freely admits, however, "It can be shown that in distributed-lag regressions relating two variables which have

been deseasonalized by procedures with different assumed rates of shift in the seasonal pattern, spurious 'seasonal' variation is likely to appear in the estimated lag distribution." On the other hand, the deseasonalizing procedure could be taking too much out of the data, thereby weakening the true causal pattern. Sims used deseasonalized data, included seasonal dummies to remove 'spurious seasonality' and then filtered throughout to remove autocorrelation. Williams, Goodhart and Gowland felt they could dispense with the seasonal dummies since they used the same seasonal adjustment on both sides of the equation. The present study uses unadjusted data and relies upon seasonal dummies for removal of seasonality and a pre-whitening filter to ensure residual white noise.

4. An equally harmful preoccupation is with 'ad hoc' filtering techniques. Sims, for example, uses the filter $(1 - .75L)^2$ on the grounds that "this filter approximately flattens the spectral density of most economic time series, and the hope was that regression residuals would be very nearly white noise with this prefiltering". Williams, Goodhart and Gowland used a combined filter of the form $(1 - L)(1 - a_1L - a_2L^2)$ where a_1 and a_2 were estimated from the fitted residuals. Mehra used the filter $(1 - kL)^2$ where k was estimated from the data. Barth and Bennett used the same filtering device as Sims. Auerback and Rutner used a more sophisticated

estimation technique for deriving their filter. It is based on a technique developed by Durbin and outlined in Anderson (1971, p. 214) which uses an infinite autoregressive procedure arbitrarily truncated. Each technique contains an element of arbitrariness either in the choice of the lag weights, in the length of the lag, or in both. The consequences of inadequate removal of autocorrelation upon the tests are studied in Granger and Newbold (1974) and in Pierce and Haugh (1977). If autocorrelation remains in the filtered residuals then bias will occur in the estimates of the variances of the least square coefficients. Very often the bias is downward [see Granger and Newbold (1974)] which produces inflated F statistics. As a result, non-causality of X by Y may be rejected when it really does exist. Feige and Pearce (1974) suggest the possibility of such an occurrence in the study of Sims (1972).

Hsiao (1979) developed a technique for reducing the arbitrariness in choice of number of leads and lags in the Sims framework. His method is based upon the Akaike (1969) technique which optimizes the tradeoff between loss in efficiency of too long a lag and bias resulting from too short a lag. Although Hsiao does remove some of the arbitrariness of the Sims methodology, he too chooses an <u>ad hoc</u> filter. In the present study I fix the number of leads and lags arbitrarily at four and use the Akaike criterion for choice of an optimum filter.

5. The methodology for testing for autocorrelation is another area which demands attention. Sims used Durbin's periodogram bounds test which failed to give conclusive results in some instances. He stated: "The conclusion from this list of approximate or inconclusive tests can only be that there is room for doubt about the accuracy of the F-tests on regression coefficients". Furthermore, even in instances where the tests were conclusive, Pierce and Haugh (1977) express doubt as to the choice of test: "Possibly the periodogram test used by Sims is not always a reliable indicator of the type of serial correlation patterns likely to be of importance." Sims (1972, page 549) may have been justified in his choice of test because, as he notes, at the time his paper was written there existed no appropriate tests for whiteness of residuals. Box and Pierce (1971) developed a test which detects autocorrelation of any order and of either moving average or autoregressive type in univariate time series. A subsequent modification of this test developed by Ljung and Box (1978) will be referred to in the present study as the 'Q-test'. See Davies, Triggs and Newbold (1977) and Davies and Newbold (1979) for discussions of finite sample properties and power studies for this particular test.

6. Even Mehra's expansion of the analysis to a multivariate framework may not have been sufficient to account for external influences on the causal pattern. There is evidence, beginning with

Barth and Bennett's cursory observation on the openness of the Canadian economy, achieving analytical basis in the work of Putnam and Wilford (1978) and acquiring empirical support in Mills and Wood (1978), that exchange rate policy has had an important influence upon apparent money-income patterns. The present study compares the results of causality tests applied to data from the full period 1956 to 1977 with those from tests applied to data with the fixed exchange rate period (1962-1970) excluded.

4.3 Empirical Results

It is known that the dynamic response pattern of the partial adjustment model follows a geometrically decaying infinite lag. Moreover, the partial adjustment model A of Chapter 2 imposes the same response pattern of the dependent variable for all arguments in the system. Lifting this constraint and allowing current values of the dependent variable to be influenced by future values of the independent variables allows us to write the more general money demand function:

$$(m_t - p_t) = \alpha_0 + \sum_{i=-\infty}^{\infty} \alpha_{1i}\, y_{t-1} + \sum_{i=-\infty}^{\infty} \alpha_{2i}\, r_{t-i} + e_t\,, \qquad (4.5)$$

specified, as before, in natural logarithms.*

*This functional form makes easier the required assumption of e_t being a normal and independently distributed random variable. For other functional forms it is likely that the positive and increasing money and income series would impose bounds upon the distribution of the error, whether the estimation was performed on stock or flow variables. It is noted also that the choice of a logarithmic form is empirically justified in Chapter 2.

The data are quarterly and seasonally unadjusted. Series on M1, R90, GNE and PGNE from 1956:II to 1977:IV were obtained from the CANSIM data base. For removal of trend, first differencing was applied to the logarithmically transformed data. The insertion of dummy variables (D_i) to remove seasonal variation and the constraint to four leads and four lags on the explanatory variables of equation (4.5) yields:*

$$(m_t - p_t) = a + \sum_{i=1}^{3} b_i D_i + \sum_{i=-4}^{4} c_i y_{t-i}$$
$$\sum_{i=-4}^{4} d_i F_{t-i} + n_{1t} . \qquad (4.6)$$

Demand for money equations have been inverted in various studies [for example, Poole (1970) and Goldfeld (1973)], to give formulations with income or interest as the dependent variable. Corresponding formulations for model (4.6) would look like:

$$y_t = e + \sum_{i=1}^{3} f_i D_i + \sum_{i=-4}^{4} j_i r_{t-i} + \sum_{i=-4}^{4} h_i (m_{t-i} - p_{t-i}) + n_{2t} , \qquad (4.7)$$

and

$$r_t = k + \sum_{i=1}^{3} n_i D_i + \sum_{i=-4}^{4} S_i (m_{t-i} - p_{t-i}) + \sum_{i=-4}^{4} q_i y_{t-i} + n_{3t} . \qquad (4.8)$$

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*This constraint is imposed for reasons of parsimony. Even the present version has 22 explanatory variables. Any more leads or lags would dangerously reduce our degrees of freedom, especially for the smaller samples. Mehra tested the appropriateness of this constraint and found that "the accumulated lag weights on income and interest rates do not change significantly after the inclusion of current and four lagged terms."

The error terms η_{1t}, η_{2t}, and η_{3t} are assumed to be normal, independently distributed random variables with zero mean and constant variances of σ_1^2, σ_2^2 and σ_3^2, respectively. To test for freedom from autocorrelation a Q-test was applied to the residuals of all regressions. In those cases where the Q-statistic rejected the hypothesis of freedom of autocorrelation, it was necessary to prefilter the data. The method of estimation of the appropriate filter, together with the algorithm used, are outlined in Appendices 4.A and 4.B. Briefly, it is an application of the Akaike final prediction error (FPE) criterion for choice of the appropriate order of an autoregressive process using Yule-Walker estimates of the filter coefficients.* In all cases of autocorrelated residuals (7 of the 15 models estimated) one pass of the respective filter reduced the Q-statistic to an acceptable level -- that is, lower than 40.26 at the 90 per cent level. When the Shapiro-Wilk test was applied to the resulting sets of residuals the hypothesis of normality could be rejected, at the 90 per cent level, for none.

Tables 4.1 to 4.5 summarize the estimation. In Table 4.1 the methodology is applied to nominal variables. M, Y and R denote

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*Geweke and Meese (1979) show that model-fitting criteria such as that of Akaike are, in general, asymptotically inefficient. They suggest that the Schwarz (1978) approach, which incorporates nested alternatives formally in a Bayesian model, has better asymptotic properties. For sample sizes used here, however, such potential gains are judged to be unimportant.

nominal M1, nominal GNE and R90, respectively. In Table 4.2, real variables are used; that is, nominal M1 and nominal GNE deflated by PGNE. In Table 4.3 the methodology is applied to real per capita money and income. Total Canadian population is used as the deflator. In Table 4.4 nominal variables are examined with the fixed exchange rate period data omitted. Table 4.5 presents the results for real variables, again with fixed exchange rate omitted.

In Table 4.1 it is seen that the filter $1 + .34L + .09L^2 + .18L^3 - .24L^4$ was required in the regression of Y on M and R [equation (4.7)] The notation $L^n(X_t) = X_{t-n}$, signifies the lag operator. Neither of the other two regressions of Table 4.1 required, from the point of view of the Q-statistic, an autoregressive filter. The resulting residuals of equation (4.5) gave a Q-statistic of 23.65 which is much below the critical value at the 90 per cent level. It is noteworthy, however, that the D.W. statistic still indicates first order autoregression. Throughout the exercise it will be noticed that the filtering procedure used here is more effective in eliminating orders of autocorrelation higher than first order.

Table 4.2 shows filters applied to real variable models for equations (4.6) and (4.8) Equation (4.6) had a Q-statistic of 33.60 but indicated significant first order autoregression

TABLE 4.1: ESTIMATION SUMMARY FOR NOMINAL VARIABLES

	EQUATION 4.6 M=f(Y,R)		EQUATION 4.7 Y=f(M,R)		EQUATION 4.8 R=f(Y,M)	
FILTER	NONE		$1+.34L+.09L^2+.18L^3-.24L^4$		NONE	
	Y	R	M	R	Y	M
Coefficients on						
Lag of -4	.136	.002	-.045	-.018	.547	-.153
-3	.044	.009	.000	-.016	.667	1.095
-2	.151	.012	.044	.003	.775	-2.502
-1	.279	.004	.333	.007	1.236	-1.554
0	.064	-.039	.079	.015	1.032	-3.423
1	.270	-.016	.245	-.001	.828	.916
2	.479	-.026	.164	.033	.240	.932
3	-.131	.004	.002	.005	-.443	1.197
4	-.034	.003	.087	.011	-.777	.412
NOBS	83		79		83	
Q	22.20		23.65		26.64	
RSS	.0118		.0239		.7560	
D.W.	2.06		1.50*		1.78	
	Y	R	M	R	Y	M
Coefficients on						
Lag of -4	0.0	0.0	0.0	0.0	0.0	0.0
-3	0.0	0.0	0.0	0.0	0.0	0.0
-2	0.0	0.0	0.0	0.0	0.0	0.0
-1	0.0	0.0	0.0	0.0	0.0	0.0
0	.110	-.444	.309	.012	.782	-3.273
1	.323	-.278	.163	-.002	1.015	.870
2	.172	-.389	.283	.044	.581	1.219
3	.504	.838	-.197	.002	-.482	2.232
4	.128	-.115	.236	.001	-1.315	.392
RSS	.0156		.0267		.9321	
D.W.	1.79		1.52*		1.74	

* Indicates significant first order autoregression at the 99 per cent level.

TABLE 4.2: ESTIMATION SUMMARY FOR REAL VARIABLES

	EQUATION 4.6 $M=f(Y,R)$		EQUATION 4.7 $Y=f(M,R)$		EQUATION 4.8 $R=f(Y,M)$	
FILTER	$1+.37L+.18L^2$ $-.03L^3-.13L^4$		NONE		$1+.02L+.03L^2$ $+.28L^3$	
	Y	R	M	R	Y	M
Coefficients on						
Lag of -4	-.008	.025	.273	-.748	.751	-.609
-3	.119	.002	-.182	-.207	-.180	-.078
-2	.075	.053	.084	.056	.709	-1.401
-1	.153	-.020	.171	.012	-.013	-1.925
0	.181	-.053	.192	.002	.997	-1.934
1	.281	-.017	-.062	-.021	1.147	-.079
2	.107	.001	-.073	.047	1.404	1.852
3	-.141	-.034	-.015	-.018	.602	1.054
4	.274	-.009	.191	-.011	-.276	1.170
NOBS	79		83		79	
Q	33.60		24.21		23.37	
RSS	.0205		.0332		.5323	
D.W.	1.25*		2.87*		2.29	
	Y	R	M	R	Y	M
Coefficients on						
Lag of -4	0.0	0.0	0.0	0.0	0.0	0.0
-3	0.0	0.0	0.0	0.0	0.0	0.0
-2	0.0	0.0	0.0	0.0	0.0	0.0
-1	0.0	0.0	0.0	0.0	0.0	0.0
0	.324	-.056	.267	.000	1.002	-2.054
1	.335	-.037	-.127	-.048	1.667	-.727
2	.855	-.009	.039	.059	1.662	1.122
3	-.011	-.044	.076	-.032	.077	1.496
4	.204	.001	.232	-.009	-.546	1.767
RSS	.0286		.0422		.7672	
D.W.	1.11*		2.77*		2.07	

* Indicates significant first order autoregression at the 99 per cent level.

TABLE 4.3: ESTIMATION SUMMARY FOR REAL PER CAPITA VARIABLES

	EQUATION 4.6 M=f(Y,R)		EQUATION 4.7 Y=f(M,R)		EQUATION 4.8 R=f(Y,M)	
FILTER	$1+.38L+.18L^2$ $-.03L^3-.13L^4$		$1+.56L+.25L^2$ $+.27L^3$		NONE	
	Y	R	M	R	Y	M
Coefficients on						
Lag of -4	-.012	.026	.211	.978	.590	-.020
-3	.111	.003	-.076	-.030	.256	-.107
-2	.068	.054	.007	.033	.725	-.594
-1	.150	-.021	.264	.003	-.663	-2.233
0	.177	-.053	.256	.016	.504	-1.956
1	.277	-.017	-.028	-.021	.579	-.116
2	.107	.001	.034	.047	.640	1.776
3	-.145	-.035	-.048	-.013	1.066	1.644
4	.268	-.009	.188	-.002	-.336	0.824
NOBS	79		79		83	
Q	34.20		39.77		23.71	
RSS	.0207		.0262		.6191	
D.W.	1.23*		1.11*		2.00	
	Y	R	M	R	Y	M
Coefficients on						
Lag of -4	0.0	0.0	0.0	0.0	0.0	0.0
-3	0.0	0.0	0.0	0.0	0.0	0.0
-2	0.0	0.0	0.0	0.0	0.0	0.0
-1	0.0	0.0	0.0	0.0	0.0	0.0
0	.333	-.056	.443	.011	.776	-2.164
1	.336	-.037	-.166	-.016	1.146	-.185
2	.826	-.009	.187	.053	1.492	1.809
3	-.007	-.045	-.134	-.025	.126	1.502
4	.197	.003	.216	-.005	-.024	1.302
RSS	.0291		.0304		.8132	
D.W.	1.08*		1.17*		1.96	

* Indicates significant first order autoregression at the 99 per cent level.

TABLE 4.4: ESTIMATION SUMMARY FOR NOMINAL VARIABLES WITH FIXED EXCHANGE RATE PERIOD DATA (1962II - 1970II) EXCLUDED

	EQUATION 4.6 M=f(Y,R)		EQUATION 4.7 Y=f(M,R)		EQUATION 4.8 R=f(Y,M)	
FILTER	NONE		$1+.42L-.01L^2+.36L^3$		NONE	
	Y	R	M	R	Y	M
Coefficients on						
Lag of -4	.141	.011	.303	-.051	-.372	.018
-3	-.088	.006	-.668	-.014	1.654	1.775
-2	.323	-.001	.775	-.002	3.334	-2.152
-1	.437	.015	-.393	-.007	1.029	-5.602
0	-.009	-.017	.887	.007	.120	-1.556
1	.279	-.037	-.525	-.022	-.249	1.433
2	-.132	-.023	.907	.056	-2.643	2.873
3	-.259	.007	-.285	.027	-.405	-1.216
4	.078	-.000	.254	-.025	1.060	2.121
NOBS	51		47		51	
Q	22.47		33.40		20.39	
RSS	.0046		.0077		.3909	
D.W.	2.26		1.70		2.14	
	Y	R	M	R	Y	M
Coefficients on						
Lag of -4	0.0	0.0	0.0	0.0	0.0	0.0
-3	0.0	0.0	0.0	0.0	0.0	0.0
-2	0.0	0.0	0.0	0.0	0.0	0.0
-1	0.0	0.0	0.0	0.0	0.0	0.0
0	.124	-.026	.140	-.299	-.499	-2.646
1	.300	-.045	.155	.019	.740	.994
2	.135	-.033	.594	.036	-.066	2.289
3	.035	-.007	-.094	.047	-.930	1.797
4	.022	-.015	.207	-.042	-.665	1.359
RSS	.0102		.0158		.6943	
D.W.	1.65		1.74		1.83	

TABLE 4.5: ESTIMATION SUMMARY FOR REAL VARIABLES WITH FIXED EXCHANGE RATE PERIOD DATA (1962II - 1970II) EXCLUDED

	EQUATION 4.6 M=f(Y,R)		EQUATION 4.7 Y=f(M,R)		EQUATION 4.8 R=f(Y,M)	
FILTER	NONE		$1+.4L+.05L^2$ $+34L^3-.02L^4$		NONE	
	Y	R	M	R	Y	M
Coefficients on						
Lag of -4	-.276	.003	.196	-.028	1.268	.024
-3	.057	-.013	-.359	-.022	2.492	-1.014
-2	.479	.043	.133	-.006	2.903	-1.088
-1	.438	-.005	.050	.010	-.600	-3.467
0	.033	-.024	.565	-.024	-.655	-1.471
1	.205	-.081	-.241	-.014	-.559	.107
2	-.152	.003	.385	.026	-1.173	1.913
3	-.240	-.004	-.127	.040	2.181	1.399
4	.737	-.003	.368	-.044	.293	.771
NOBS	51		47		51	
Q	21.88		15.94		33.44	
RSS	.0125		.0085		.3180	
D.W.	2.48*		1.62		2.11	
	Y	R	M	R	Y	M
Coefficients on						
Lag of -4	0.0	0.0	0.0	0.0	0.0	0.0
-3	0.0	0.0	0.0	0.0	0.0	0.0
-2	0.0	0.0	0.0	0.0	0.0	0.0
-1	0.0	0.0	0.0	0.0	0.0	0.0
0	-.040	-.038	.361	-.039	-.270	-1.627
1	.182	-.084	-.233	.013	1.644	-.230
2	.102	.005	.404	.036	1.319	2.148
3	-.005	-.042	.161	.020	.164	1.368
4	.452	-.031	.380	-.067	.742	1.390
RSS	.0171		.0140		.5993	
D.W.	2.37		1.91		1.95	

* Indicates significant first order autoregression at the 99 per cent level.

at the 99 per cent level with a D.W. of 1.25. Equation (4.7) was not seen to require an autoregressive filter but did show a significant D.W. of 2.87. Table 4.3 reports significant autoregression of order 1 for Equations 4.6 and 4.7, with D.W.s of 1.23 and 1.11 respectively. In Table 4.4 all three equations passed the Q and D.W. tests. In Table 4.5 only equation (4.6) indicated significant first order autoregression with a D.W. of 2.48.

Keeping in mind the apparent limitations of these filtering procedures, I examined the results of the causality tests as reproduced in Table 4.6. The elements of this table are the F-statistics computed from the residual sum of squares (RSS) of the constrained and unconstrained estimations reported in Tables 4.1 through 4.5. When marked by asterisks they indicate significant non-zero coefficients on the future variables as a group. This indicates significant rejection of non-causality of the RHS variables by the dependent variable at the 95 per cent level. As shown by Granger and Newbold (1974), however, the effect of inadequate removal of autocrrelation is to inflate the F-statistics. The significant F-statistics which are produced from regressions whose D.W. statistics are unable to reject first order autoregression are therefore inconclusive. Table 4.7 replaces with the symbol '?', those elements of Table 4.6 for which first order autoregression was not properly removed and which showed significant F-statistics.

TABLE 4.6: CAUSALITY IN M, Y AND R RELATIONS (PRELIMINARY RESULTS)

EQUATION	(1) NOMINAL Y AND M	(2) REAL Y AND M	(3) REAL PER-CAPITA M AND Y	(4) NOMINAL Y AND M EXCLUDING 1962II - 1970IV	(5) REAL Y AND M EXCLUDING 1962II - 1970IV
Equation 4.6 $M=f_1(Y,R)$	2.45*	2.83*	2.90*	4.46*	1.32
Equation 4.7 $Y=f_2(M,R)$	0.82	2.08	1.12	3.29*	2.01
Equation 4.8 $R=f_3(M,Y)$	1.78	3.14*	2.23*	2.81*	3.21*

TABLE 4.7: CAUSALITY IN M, Y AND R RELATIONS (FINAL RESULTS)

EQUATION	(1) NOMINAL Y AND M	(2) REAL Y AND M	(3) REAL PER-CAPITA Y AND M	(4) NOMINAL Y AND M EXCLUDING 1962II - 1970IV	(5) REAL Y AND M EXCLUDING 1962II - 1970IV
Equation 4.6 $M=f_1(Y,R)$	2.45*	?	?	4.46*	1.32
Equation 4.7 $Y=f_2(M,R)$	0.82	2.08	1.12	3.29*	2.01
Equation 4.8 $R=f_3(M,Y)$	1.78	3.14*	2.23*	2.81*	3.21*

*Indicates significantly non-zero coefficients on the future variables as a group (i.e. significant rejection of non-causality of the RHS variables by the dependent variable) at the 95 per cent level.

Column 1 of Table 4.7 supports the finding of Sims and of Mehra that there exists unidirectional causality running from nominal money to the RHS variables. Granger non-causality is rejected for Equation (4.6) but not for either of the other two. When the same experiment is performed with fixed exchange rate data removed (column 4), Granger non-causality is rejected for all three equations. This would indicate simultaneous determination with no clear causation pattern. The non-causality of the RHS variables by the dependent variable of equation (4.6) is more emphatically rejected in column 4 than in column 1.

Columns 2 and 3 of Table 4.7 repeat the exercise with real and real per capita variables respectively. In neither case can the non-causality of the RHS by the LHS be rejected (a desirable property). For column 2 the statistic of equation (4.7) is very nearly significant at the 95 per cent level (compared to a critical value of 2.10). Were it so, one could state statistically that unidirectional causation flowed from right to left in equation (4.6). The closeness of the result speaks favourably, in an econometric sense, of our models of demand for real balances.

Column 5 repeats the test of column 2 with fixed exchange rate data excluded. Again, one cannot reject the non-causality of the RHS of equation (4.6) by the LHS. Again, one must stop just short of saying that unidirectional causation runs from right to left in equation (4.6) The presence or absence of fixed exchange rate data seems not to affect the causal pattern between variables in the real demand for money model.

4.4. Concluding Comments

Much has been done on money-income causality since the study of Sims (1972a). Fortunately, the sheer volume of studies and their sometimes conflicting views have injected a greater caution in the interpretation of results. The use of causality testing has undergone, and is undergoing, a process of maturation. Recent work on the consequences of inadequate removal of residual autocorrelation has led to greater emphasis on the choice of the appropriate filter and to the use of empirically determined filters. The effects of measurement error upon filter specification and the consequent effects upon causality tests is a topic of growing concern. Improper model specification, aggregation bias and the inevitable problem of defining 'causality' econometrically must all be reckoned with before final conclusions are drawn.

In concrete terms this study makes six basic improvements to the methodology of Sims. Some of these six improvements have appeared in recent articles but all of them have appeared in none. They are: (1) the use of a multivariate framework as done by Mehra (1978); (2) the consideration of real versus nominal variables, also done by Mehra (1978), and subsequent consideration of real per capita variables; (3) the use of undeseasonalized data in order to avoid unnecessary smoothing and the spurious seasonality described by Sims; (4) the use of the Akaike final prediction error criterion for estimation of the whitening filter; (5) the use of the Box-Pierce Q-test in conjunction with the Durbin-Watson test for verification of the applicability of the F-tests; and (6) consideration of the effects of different exchange rate regimes upon causal patterns as done by Mills and Wood (1978).

Mehra advanced a multivariate framework for the testing of causality. Unfortunately, the identification of causal patterns in such a model is much more complicated than in the bivariate case. For example, if two variables can be related by three different causal links then three variables can be related in twenty-seven different causal patterns. Perhaps Mehra seeks to avoid the complexity of multivariate causality by speaking in terms of exogeneity. Exogeneity is, however, stronger in an econometric sense than the form of Granger 'non-causality' he observed.

It is believed that the results of this study confirm the information provided by the bulk of the published literature. For nominal variables the non-causality of the RHS by money is rejected. When income or interest is the dependent variable non-causality of the RHS is not rejected. This is in agreement with Sims and Mehra. For nominal variables, with the fixed exchange rate data excluded, the non-causality of equation

(4.6) is even more emphatically rejected. This agrees with Mills and Wood (1978) for the United Kingdom and with Montmarquette and Forest (1979) for Canada.

The economic interpretation of this observation is that there is little scope for monetary policy in an open economy with fixed exchange rates. The causal link from money to income should be strengthened, therefore, by leaving out the fixed rate data. This is observed with the less emphatic rejection of non-causality in column 1 than in column 4 for equation 4.6 in Table 4.7.

Recent studies [for example, Poloz (1979)] have examined the question of simultaneous equation bias in the demand for money. The argument is that if the Bank of Canada sets interest rates in response to the behaviour of money aggregates then one requires an interest rate reaction function in addition to a money demand function. Unfortunately, the present study has not answered this question. What it has done, at least in the case of demand for real balances, is to verify a necessary condition for exogeneity of the RHS. Further support for the belief that simultaneity was not a problem for the time period of this study comes from the knowledge that prior to mid-1975 the Bank of Canada followed an interest rate rule and paid little attention to monetary aggregates. Except for the last ten observations of the sample period of this study, when the Bank of Canada was actively manipulating interest rates with a view towards controlling the growth of M1, it is probably adequate to treat interest rates as exogenous.

Appendix 4.A: The Akaike FPE Criterion for Choice of the Appropriate Autoregressive Filter

Wold's theorem tells us that any zero mean, covariance stationary process excluding deterministic components can be represented as a moving average of a sequence of uncorrelated random variables with zero mean and constant variance. If X_t, $(t = 1, \ldots T)$, is a sample drawn from such a process, then

$$X_t = e_t + \Psi_1 e_{t-1} + \Psi_2 e_{t-2} + \ldots, \tag{4.9}$$

where e_t' are NID$(0,\sigma^2)$ random variables. Equation (4.9) may be rewritten:

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + e_t. \tag{4.10}$$

Consider the finite representation of (4.10):

$$X_t = \sum_{j=1}^{N} \alpha_j X_{t-j} + e_t.$$

Covariance stationarity of X assures invertibility, i.e. that the roots of the polynomial $(1-\alpha_1 Z- \ldots -\alpha_N Z^N)$ lie outside the unit circle. An estimate of this polynomial, to be termed the autoregressive filter of X_t, is what we desire. The problem before us is that asymptotically unbiased estimation of the α_i requires knowledge of the true N and that choice of an appropriate N requires estimates of the α_i.

Akaike (1969) suggests:

(1) Choose an upper limit for N, say L, which is sufficiently large as not to exclude the efficient model.

(2) Calculate the sample autocovariances

$$\gamma_S = \frac{1}{T} \sum_{t=1}^{T-S} X_{t+S} X_t \quad \text{for } S = 0, 1, \ldots L.$$

(3) Minimum mean square prediction estimates of $\alpha_i^{(N)}$ for N = 1, ... L, are found by solving the Yule-Walker equations:

$$\begin{bmatrix} \gamma_0 & \gamma_1 & \cdots & \gamma_{N-1} \\ \gamma_1 & \gamma_0 & & \\ \vdots & & & \\ \gamma_{N-1} & & \cdots & \gamma_0 \end{bmatrix} \begin{bmatrix} \hat{\alpha}_1(N) \\ \hat{\alpha}_2(N) \\ \vdots \\ \hat{\alpha}_N(N) \end{bmatrix} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_N \end{bmatrix} .$$

(4) Define FPE as the mean square prediction error:

$$\text{FPE} = E\left[(X_t - \hat{X}_t)^2\right]$$

where $\hat{X}_t = \sum_{i=1}^{N} \hat{\alpha}_i X_{t-i}$. Consider another realization of this same

AR process and denote it as Y_t. The prediction of Y_t would be given by:

$$Y_t = \sum_{i=1}^{N} \hat{\alpha}_{x,i} Y_{t-i}.$$

The FPE for Y_t would be

$$\text{FPE} = E\left[\left(Y_t - \sum_{i=1}^{N} \hat{\alpha}_{x,i} Y_{t-i}\right)^2\right], \tag{4.11}$$

which is shown in Ulrych and Bishop (1975) to reduce to the sum of two components. The first one corresponds to the minimum residual sum of squares of the Nth order AR fit to Y_t, S_N^2. When N is less than the true order of the process S_N^2 includes not only contribution from the variance of the innovation but an additional contribution from the bias resulting from misspecification. The estimate S_N^2 is artificially high and will decrease as N increases to its true value. The second component of FPE, however, measures the statistical deviation of $\hat{\alpha}_{x,i}$ from $\hat{\alpha}_{y,i}$ which increases with N.

The Akaike criterion is to adopt as the optimum of N that value that minimizes the FPE of (4.11). Akaike (1969) has developed an efficient estimate of this minimum criterion. It is written

$$\text{FPE}(N) = \frac{T + (N+1)}{T - (N+1)} S_N^2$$

A FORTRAN program for the computation of the $\hat{\alpha}_i(N)$ and FPE(N) is included in Appendix 4.B.

Appendix 4.B: A FORTRAN Program for the Computation of the Akaike Final Prediction Error

```
C          THIS PROGRAM COMPUTES THE YULE-WALKER ESTIMATES OF THE
C     PREDICTION ERROR FILTER COEFFICIENTS AND THE AKAIKE FINAL
C     PREDICTION ERROR.  THE ALGORITHM IS DESCRIBED IN AKAIKE (1969)
C     ANN. INST. STATIST. MATH.  PROGRAMMED J.P.COCKERLINE (1979).
C
      DIMENSION X(200),C(50,50),CINV(50,50),WK(200),D(50,1),
     1 A(50,1),AT(1,50),XL(50,200),W(1,200),R(50),FPE(50)
      REAL * 8 RHO,SUM,SUM1
      DOUBLE PRECISION C,CINV,D,A,AT,XL,W,R,FPE
      READ(5,1) N,M
      DO 10 I=1,N
      READ(5,2) X(I)
10    CONTINUE
1     FORMAT (2I4)
2     FORMAT (F15.8)
3     FORMAT (12X,'P = ',I2,11X,'A( 1) ',F8.4,5X,'FPE(AR',I2,') ',F12.9
4     FORMAT (1H0,10X,'YULE-WALKER ESTIMATES OF PREDICTION ERROR
     1FILTER COEFFS. AND THE AKAIKE FINAL PREDICTION ERROR (FPE).'//)
5     FORMAT (29X,'A(',I2,') ',F8.4)
7     FORMAT (/)
8     FORMAT (10X,'-----------',9X,'------------',8X,'-------------
     1----',/)
9     FORMAT (10X,'ORDER AR(P)',9X,'COEFFICIENTS'8X,'FINAL PRED ERRORS')
      WRITE (6,4)
      WRITE (6,9)
      WRITE (6,8)
      SUM=0.0
      DO 15 I=1,N
      SUM=SUM + X(I)*X(I)
15    CONTINUE
      RHO = SUM/FLOAT(N)
      DO 99 MM=1,M
      D(MM,1)=0.0
      N20 = N-MM
      DO 20 J=1,N20
      D(MM,1)=D(MM,1)+(X(J+MM)*X(J))
20    CONTINUE
      D(MM,1)=D(MM,1)/FLOAT(N)
      DO 30 I=1,MM
      C(I,I) = RHO
      IF (I.EQ.MM) GO TO 30
      DO 40 J=2,MM
      IF (J.LE.I) GO TO 40
```

```
      C(I,J) = D(J-1,1)
      C(J,I) = C(I,J)
40    CONTINUE
30    CONTINUE
      CALL LINV1F (C,MM,50,CINV,10WK,IER)
      CALL VMULFF (CINV,D,MM,MM,1,50,50,A,50,IER)
      DO 50 I=1,MM
      AT(1,I) = A(I,1)
50    CONTINUE
      N60 = N-MM+1
      DO 60 J=1,N60
      DO 60 I=1,MM
      M2 = MM+J-I
      XL(I,J) = X(M2)
60    CONTINUE
      KK = N-MM
      CALL VMULFF (AT,XL,1,MM,KK, 1,50,W,  1,IER)
      SUM1 = 0.0
      DO 70 I=1,KK
      LL=MM+1
      SUM1 = SUM1 + (X(LL)-W(1,I))**2
70    CONTINUE
      R(MM)=SUM1/FLOAT(N)
      FPE(MM)=R(MM)*(FLOAT(N+MM+1)/FLOAT(N-1-MM))
      WRITE (6,3) MM, A(1,1),MM,FPE(MM)
      IF (MM.EQ.1) GO TO 81
      DO 80 J=2,MM
      WRITE (6,5) J,A(J,1)
80    CONTINUE
81    WRITE (6,7)
99    CONTINUE
      STOP
      END
```

CHAPTER 5

DYNAMICS

The distributed lag framework

$$y_t = \theta_k(L)X_t + u_t,$$

where L is the lag operator, may be seen to represent a variety of specific forms. The lag polynominal

$$\theta_1(L) = \alpha \sum_{i=0}^{n} (n + 1 - i)L^i$$

for example, denotes the Fisher lag. It is seen to be linearly-declining and parsimonious in the sense that only α and n need be estimated. The Koyck lag is written

$$\theta_2(L) = \alpha \sum_{i=0}^{\infty} (1 - \lambda)\lambda^i L^i, \; 0 < \lambda < 1.$$

The shape is geometrically declining with the estimated λ providing a measure of the 'speed of adjustment'. The Almon lag, written

$$\theta_3(L) = \sum_{i=0}^{n} \sum_{j=0}^{r} \alpha_{ij}\, i^j L^i$$

is of variable shape depending upon pre-specified values of order, r, and lag length, n. The Pascal lag (Solow, 1960),

$$\theta_4(L) = \alpha \sum_{i=0}^{\infty} \frac{(i + r - 1)!}{i!\,(r - 1)!} (1 - \lambda)^r \lambda^i L^i,$$

takes on the polynominal 'inverted-V' shape or the immediately declining Koyck shape depending upon pre-specified values of r

/see Kmenta (1971, p. 488)/. The rational lag,

$$\Theta_5(L) = \frac{\sum_{i=0}^{s} \alpha_i L^i}{(1 - \sum_{i=1}^{n} \delta_i L^i)},$$

generalizes for any possible function of adjustment speed, λ, and order of polynominal, r, and is unconstrained as to shape. (Jorgenson, 1966). Rational lags may be identified and estimated by ARIMA methods /Box and Jenkins (1970)/.

The two most prevalent lag forms appearing in the money demand literature have been the Koyck and Almon specifications. Each will be examined more closely in successive sections of this chapter. Section 5.3 will describe and apply a statistical test for alternative error structures within the partial adjustment-adaptive expectations model. Section 5.4 will consider the possibility that the dynamic structure in the demand for real balances has evolved through time. Three methods of variable parameter regression will be employed in partial adjustment models for two money aggregates.

5.1 The Koyck Lag

Usage of a geometrically declining lag form in the demand for money dates at least to Cagan (1956)* who modelled expectations adaptively in proportion to the previous period's forecast error.

---

*Maurice Allais uses the Koyck lag in "Explication des Cycles Economiques par un Modèle Non-linéaire a Régulation Retardée", presented at the European Meeting of the Econometric Society, Uppsala, 1954 and published in Metroeconomica, vol. 8, 1956, pp. 4-83.

Chow was the first to publish a theoretical description of the geometrically declining lag structure for money demand. He felt that the traditional distinction between short- and long-run demands for money was too sharply drawn. Short-run demand for money was believed to be strongly dominated by a 'transactions' motive with emphasis on current income. Long-run demand for money was believed to be governed by an 'assets' motive, with the emphasis on permanent income. Chow designed a partial adjustment model which captured both demands.

While Chow's approach served as a good description of the model a more theory-oriented derivation appeared in Feige (1967). He offered two different theoretical justifications for the lagged dependent variable model. The first dealt with expectations. In the familiar log-linear model, equilibrium demand for real balances, m_t^d, was written:

$$m_t^d = a_0 + a_1 y_t^e + a_2 r_t + u_t. \tag{5.1}$$

The determinants were expected real income, y_t^e, and a representative rate of interest, r_t. With the expectation-generating function written as

$$y_t^e = y_{t-1}^e + \lambda(y_t - y_{t-1}^e), \tag{5.2}$$

then equilibrium in the money market, $m_t^d = m_t$, allowed

$$m_t = b_0 + b_1 y_t + b_2 r_t + b_3 r_{t-1} + b_4 m_{t-1} + b_5 u_{t-1} + u_t \tag{5.3}$$

where

$$b_0 = \lambda a_0,\ b_1 = \lambda a_1,\ b_2 = a_2,\ b_3 = -(1 - \lambda)a_2,\ b_4 = (1 - \lambda),$$

and $b_5 = -(1 - \lambda)$.

Feige's second theoretical derivation of the lagged dependent variable model involved costs of adjustment. He identified two costs associated with any particular cash balance position as 1) the cost of being out of long-run equilibrium; and 2) the direct cost of getting there in terms of brokerage charges. The first results from utility foregone in the case of a shortage of cash or income foregone in the case of a surplus. It is typically represented as a quadratic loss function of the deviation of desired from actual balances. The second cost is likewise assumed to be a quadratic function of the change in actual balances in any one period. The total cost associated with cash balance m_t is therefore represented as

$$C = \alpha(m_t - m_t^d)^2 + \gamma(m_t - m_{t-1})^2. \qquad (5.4)$$

The problem is to choose the level of actual balances which minimizes C, for a given level of desired balances, m_t^d. Differentiating (5.4) with respect to m_t and setting the derivative equal to zero yields the optimal adjustment path:

$$m_t - m_{t-1} = \delta(m_t^d - m_{t-1}), \qquad (5.5)$$

where

$$\delta = \alpha/(\alpha + \gamma).$$

In a situation where both costs of adjustment and adaptive expectations play a role in the formation of adjustment lags the model becomes

$$m_t = c_0 + c_1 y_t + c_2 r_t + c_3 r_{t-1} + c_4 m_{t-1} + c_5 m_{t-2} + c_6 u_{t-1} + c_7 u_t, \qquad (5.6)$$

where

$$c_0 = \lambda\delta a_0,\ c_1 = \lambda\delta a_1,\ c_2 = \delta a_2,\ c_3 = -(1 - \lambda)a_2,\ c_4 = (2 - \lambda - \delta),$$
$$c_5 = -(1 - \delta)(1 - \lambda),\ c_6 = -(1 - \lambda) \text{ and } c_7 = \delta.$$

This model will be referred to as the partial adjustment-adaptive expectations (PAAE) model.

Several variations of the PAAE model have appeared in the literature. Friedman [Chow (1966, note 3, p. 114)], Genberg (1975) and Villaneuva and Arya (1974) have suggested modifications to the model which have altered the adjustment path described by (5.5). Villanueva and Arya applied non-linear two-stage least squares to models for Japan and Canada and concurred with Starleaf (1970) and Feige (1967) that the expectational effect dominates the partial adjustment effect. Furthermore, when permanent income was used as the income constraint, no lag in adjustment could be found.

Much of the theoretical discussion considered thus far has centred around adjustment in the re-allocation of the existing portfolio. A second possibility for adjustment, which was originally advanced by Chow (1966) and has recently gained support, is through the re-allocation of the flow of savings. When the desired level of an asset differs from its actual level this divergence can be dissipated either by changing the levels of all existing assets or by redirecting the flow of savings toward or away from this asset. Brainard and Tobin (1968) explicitly considered cross-adjustment effects between assets in a portfolio model. Modigliani (1972) and Friedman (1977) also emphasized the importance of savings. White (1978) considered this effect under moderate inflation. Santomero and Seater (1978) appealed to search behaviour as an explanation of the presence of adjustment lags. Brillemburg (1979) reformulated the partial adjustment model under conditions of uncertainty.

One powerful objection to the use of the Koyck lag, at least for quarterly or annual data, concerns the problem of time aggregation. Mundlak (1961) showed that if complete adjustment takes less than one data period then the presence of the lagged endogenous variable as a regressor biases upward the estimate of the mean adjustment lag. The common finding of relatively slow speeds of adjustment, for example in de Leeuw (1967), when the reaction time at the micro level might be only a few weeks, suggests such a problem. Bryan (1967) tested the appropriateness of the Koyck model for bank adjustments to monetary policy. An excess reserve model containing a lagged dependent model was estimated for each of 19 separate commercial banks using weekly data. This relation was then reestimated pooling the data over the banks. For the micro data the average lag was 3.2 weeks and from the pooled data was 2.6 weeks. When the aggregate weekly data were averaged into months the regression yielded an estimate of 28.7 months for adjustment.

Griliches (1967) demonstrated a second potentially serious problem. Even if adjustment is instantaneous the lagged dependent variable may seem to contribute very significantly to the explanation of money stock. This could happen if the unexplained portion of money stock, the residual, is autocorrelated. In this case, the estimate of the coefficient on lagged money reflects the serial correlation as well as the speed of adjustment and is upward biased.

Breen (1971) suggested a third source of confusion emerging from the stock-adjustment hypothesis. While it is clear that partial adjustment on an aggregate level implies some sort of habit persistence of the population as a whole, it is not clear what this implies about the individual micro unit. It is possible that agents act individually in the same way that they act aggregatively -- adjusting their demand only partially to a given change in supply. It is also possible, and more likely says Breen, that individuals are either complete adjusters or non-adjusters in any one period.

In such a world the coefficient upon the lagged dependent variable would measure more closely the proportion of non-adjusters in the sample than the speed of adjustment of the micro units. The presence of long lags of adjustment would then become more of an institutional factor than a behavioural characteristic of decision makers. In Breen's words: "Thus from the policy maker's viewpoint, the cumulative effects of a policy decision cannot be counted on for the gradual achievement of a particular policy goal. While other lags may exist within the monetary transmission mechanism, the long lag in adjustment to changes in policy variables may be much less important than indicated in recent econometric studies".

The permanent income approach to money demand disputes partial stock adjustment on two points [White (1978)]. The first is that the relevant income constraint in the demand for money is permanent income and because of high substitutability between money and other financial assets the impact of a change in permanent income upon the cash component of an individual's portfolio is relatively immediate. This, says Friedman (1959), is supported in the aggregate by the behaviour of permenent velocity. Given that actual balances are everywhere equal to desired balances then permanent velocity, unlike measured velocity, behaves in an intuitively-expected, counter-cyclical manner. The second argument is that changes in the desired share of money in a given portfolio due to changes in interest rates take place very rapidly. White (1978) explains this phenomenon with the observation that the desired addition to narrow money accompanying a fall in rates coincides with a desired reduction in other assets. Through explicit account of interactions among financial assets, the portfolio models entertain the possibility of more rapid adjustment.

Finally, it has been noted that the reduced form equation derived from the adaptive expectations hypothesis (5.3) differs from that derived from the partial adjustment-adaptive expectations hypothesis (5.6). Waud (1966,1968) considered the extent of small sample bias and departure from normality of the distribution of the estimated regression coefficients due to choice of one model when the other was more appropriate. He concluded that the effects of such misspecification are considerable both in estimation of the mean lag and, for very small samples, in testing for significance of the regression coefficients.

5.2 The Almon Lag

Almon (1965) developed a technique for estimating the weights of a polynominal distributed lag. This technique has been usefully employed in several demand for money studies. White (1976) and Cameron (1979) are examples using Canadian data. It is to be thought of as alternate, and not necessarily superior, to the Koyck specification. Theoretical problems with the technique are well documented, as for example in Schmidt and Waud (1973).

We begin with the distributed lag model:

$$y_t = w_o X_t + \ldots\ldots + w_n X_{t-n} + u_t, \ t = 1, \ldots, T. \qquad (5.7)$$

As long as n is finite and known, the X_t are non-stochastic, and u_t are independently distributed random errors with zero mean and constant variance, then unbiased and efficient estimates of w_o, ..., w_n can be obtained through ordinary least squares. Problems arise, however, in that large values of n may lead to imprecise estimates of the w_i. High collinearity among explanatory variables may make positive inference difficult or impossible. The desire, then, is for a parsimonious specification of (5.7). The Almon approach consists of mapping W, a vector of dimension n+1, onto a vector δ of dimension r+1 whose elements are the ordinates of an rth degree polynominal, where $r < n$. Algebraically this may be written

$$W = A.\delta, \qquad (5.8)$$

where A is an n+1 by r+1 transformation matrix. Rewriting (5.7) in matrix form and substituting for W yields

$$y = XA\delta + u. \qquad (5.9)$$

Given that (5.8) is true, or that the n+1 elements of W can be fitted exactly by an rth order polynominal, and that the error term u has the above-mentioned properties then unbiased and efficient estimates of the w_i can be derived through application of OLS to (5.9).

Unfortunately, the fit of the rth degree polynominal to the n+1 lag weights is rarely ever exact. Because any given continuous function within a closed interval can only be approximated by a polynominal of suitable degree and because, in practice, r is usually chosen small, (5.8) should be rewritten

$$W = A.\delta + e \tag{5.8a}$$

where e is an n+1 by 1 vector of errors. This yields

$$y = XA\delta + Xe + u. \tag{5.9a}$$

Application of OLS to (5.9a) yields the estimate

$$\hat{\delta} = (A'X'XA)^{-1} A'X'y'. \tag{5.10}$$

The derived estimate of W is

$$\hat{W} = A\hat{\delta} = A(A'X'XA)^{-1} A'X'y \tag{5.11}$$

substituting (5.9a) into (5.11) gives

$$\hat{W} = A\delta + A(A'X'XA)^{-1} A'X'Xe + A(A'X'XA)^{-1} A'X'u \tag{5.12}$$

The bias of $\hat{W}$ is written

$$E(\hat{W}) - W = \left[A(A'X'XA)^{-1} A'X'X - I_{n+1}\right](W-A\delta) \tag{5.13}$$

Since the bias of $\hat{W}$ depends upon the true parameter vector W then we know nothing of its size or direction. Furthermore, the dependence of the bias upon X may demand the complete re-estimation of a structural model for minor data revisions.

The residual sum of squares from (5.9a) is

$$\hat{u}'\hat{u} = e'X'MXe + u'Mu + 2e'X'Mu \qquad (5.14)$$

where $M = I_T - XA(A'X'XA)^{-1}A'X'$ and I_T is the T by T identity matrix. It can be seen that $\hat{u}'\hat{u}$ will not have the usual χ^2 distribution due to the first and third terms on the RHS of (5.14). Conventional F- and t- tests are therefore invalid. The practice of using significance tests to choose appropriate values for n and r is incorrect.

Having observed the properties of the Almon estimates, then, it is not surprising that they fall within the general class of 'biased estimators'. Shiller modified the Almon methods by imposing a degree of smoothness to the lag distribution. Instead of allowing (5.8) to be stochastic, as in (5.8a), he set

$$\Delta^3 w_i = \xi_i, \qquad (5.15)$$

where $\Delta w_i = w_i - w_{i-1}$ and the $\xi_i \sim NID(o, \sigma_\xi^2)$. Merging these priors with the sample information via Bayesian techniques then yields a posterior distribution function for w_i. Maddala (1977, p. 385) shows that the mean of this posterior distribution is a ridge estimator of w. When $\sigma_\xi^2 = 0$, the Shiller estimate reduces to the Almon estimate. Lieberman (1978) uses the Shiller methods in estimating money demand for the U.S. economy.

5.3 Testing for Alternative Error Structures

It is seen from equation (5.6) that the error structure of the partial adjustment-adaptive expectation (PAAE) model is complex. As the adaptive expectation assumption is extended to other variables in the multivariate model this complexity increases. This obviously has implications for estimation. Specifications involving autoregressive error structures, for example, would not be appropriate for either equation (5.3) or (5.6) since their error terms are generated, at the very least, by moving average processes. Neither would an MA(1) correction be appropriate if the errors were of higher order moving average or of ARMA processes.* Pertinent, therefore, to a discussion of dynamics in the demand for money is a description of how one would discriminate between alternative error structures within such a model.

Consider, for example, a general version of (5.1):

$$m_t{}^d = \alpha_0 + \alpha_1 y_t^e + \alpha_2 r_t{}^e + u_t, \tag{5.16}$$

where $m_t{}^d$ is generated by partial adjustment mechanism:

$$m_t{}^d = \frac{(1 - \theta_1 L)m_t}{\lambda_1} \tag{5.17}$$

and y_t^e and r_t^e are generated by adaptive expectation mechanisms:

---

*In Monte Carlo work by Hendry and Trivedi (1972) it is suggested that specifying the correct order of the error process is more important in terms of bias and mean square error of the estimators than specifying its correct form, i.e., whether MA or AR.

$$y_t^e = \frac{\lambda_2}{(1 - \theta_2 L)} y_t; \tag{5.18}$$

$$r_t^e = \frac{\lambda_3}{(1 - \theta_3 L)} r_t; \tag{5.19}$$

with L the lag operator; $\theta_i = 1 - \lambda_i$; $0 < \lambda_i < 1$ for $i = 1, 2, 3$; and $u_t \sim \text{NID}\,(0, \alpha_u^2)$. Substitution of (5.17) , (5.18) and (5.19) into structural model (5.16) yields the equation of observables:

$$\begin{aligned}(1 - \theta_1 L)(1 - \theta_2 L)(1 - \theta_3 L)m_t &= \lambda_1 \alpha_0 \,(1 - \theta_2 L)(1 - \theta_3 L) \\ &+ \lambda_1 \lambda_2 \alpha_1 \,(1 - \theta_3 L)y_t + \lambda_1 \lambda_3 \alpha_2 (1 - \theta_2 L) r_t \\ &+ \lambda_1 \,(1 - \theta_2 L)(1 - \theta_3 L)u_t\,,\end{aligned} \tag{5.20}$$

whose error term can be seen to be generated by MA(2). If we further generalize the model to allow u_t to be generated by an ARMA (p,q) process then the generating process of the error of (5.20) can be shown to be ARMA (p, q + 2).

Equation (5.20) reduces to

$$\begin{aligned}m_t = A_0 &+ A_1 m_{t-1} + A_2 m_{t-2} + A_3 m_{t-3} + A_4 y_t + A_5 y_{t-1} + A_6 r_t + \\ &A_7 r_{t-1} + V_t;\end{aligned} \tag{5.21}$$

where $A_0 = \alpha_0 \lambda_1 \lambda_2 \lambda_3$, $A_1 = 3 - \lambda_1 - \lambda_2 - \lambda_3$,

$A_2 = -[(1 - \lambda_1)(1 - \lambda_2) + (1 - \lambda_1)(1 - \lambda_3) + (1 - \lambda_2)\,(1 - \lambda_3)]$,

$$A_3 = -(1 - \lambda_1)(1 - \lambda_2)(1 - \lambda_3),\ A_4 = \alpha_1\lambda_1\lambda_2,\ A_5 = \alpha_1\lambda_1\lambda_2(1 - \lambda_3),$$

$$A_6 = \alpha_2\lambda_1\lambda_3,\ A_7 = \alpha_2\lambda_1\lambda_3(1 - \lambda_2) \text{ and}$$

$$V_t = \lambda_1 u_t - \lambda_1(2 - \lambda_2 - \lambda_3)u_{t-1} + \lambda_1(1 - \lambda_2)(1 - \lambda_3)u_{t-2}.$$

Given that V_t is generated by an MA(2) process, a suitable non-linear algorithm may be applied to (5.21) to produce estimates of model parameters and error terms. Godfrey (1978b) proposes tests of the null hypothesis that the errors, V_t, are generated by ARMA (p, q) against logical alternatives. Two alternatives are considered particularly relevant to the present case.

The first tests the null hypothesis, H_0; that the V_t are generated by the MA(r) process; against the alternative, H_A, that the V_t are generated by ARMA (p, r). For the present case with r = 2 this is equivalent to testing the null hypothesis that the u_t are serially independent against H_A: the u_t are generated by AR(p).

The second tests the null hypothesis H_0 against the alternative hypothesis, H_B; that the V_t are generated by MA(q + r). Again, with r = 2, this is equivalent to testing the null hypothesis that the u_t are generated by MA(q). The more general alternative hypothesis H_c, that V_t are generated by ARMA (p, q + r), cannot be entertained within the Godfrey framework (Godfrey, 1978a).

Derivation of these tests has been fully developed in Godfrey (1978 a,b) and need not be attempted here. What is considered useful, however, is a simple description of the mechanics involved for the two specific applications mentioned above. Some notation is required. Let V represent the T by 1 vector of errors from model (5.21). Let U represent the T by 1 vector of errors from the structural model (5.16). Let

$$V = N(\alpha)U \;\ldots\ldots.\mathrm{MA}(q)$$

denote the property that V is generated by an MA(q) of U where $N(\alpha)$ is the T by T matrix:

$$N(\alpha) = \begin{bmatrix} 1 & & & & 0 \\ \alpha_1 & \ddots & & & \\ \vdots & \ddots & \ddots & & \\ \alpha_q & & \ddots & \ddots & \\ & \ddots & & \ddots & \\ & & \ddots & & \ddots \\ 0 & & \alpha_q & \cdots\alpha_1 & 1 \end{bmatrix}$$

Let $N(\gamma)V = U$ .... AR(p) denote the property that U is generated by an AR(p) of V. Let $Z_j(U)$ represent the T by j matrix:

$$Z_j(U) = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ u_1 & 0 & & \vdots \\ \vdots & & & \vdots \\ \vdots & u_1 & & \vdots \\ \vdots & \vdots & & u_1 \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ u_{t-1} & u_{t-2} & & u_{t-j} \end{bmatrix}$$

Then it can be shown that to test

H_0: $V = N(\alpha)U$ .... MA(r)

against

H_A: $N(\gamma)V = N(\alpha)U$.... ARMA (p, r)

one regresses the vector $\hat{U}$ against the regressor set

$N(\hat{\alpha})^{-1}$ [X: $Z_r(\hat{U})$: $Z\ p(\hat{V})$]

where $\hat{\alpha}_i$ is an r by 1 vector of the estimates of α, and $\hat{U}$ and $\hat{V}$ are T by 1 vectors of residuals obtained through non-linear estimation subject to parameter constraints applied to (5.21). The R^2 of that regression times T, call it ϕ, is then distributed as χ^2_p.

To test

H_0: $V = N(\alpha)U$ .... MA(r) against

H_B: $N(\gamma)V = N(\alpha)U$ .... ARMA (0, q + r)

one regresses $\hat{U}$ against the regressor set

$N(\hat{\alpha})^{-1}$ [X: Z_{q+r} (U)].

ϕ is then distributed as χ^2_q .

The test that the errors V_t of model (5.21) are generated by an MA(2) process against the alternative that they are generated by an ARMA (1, 2) process has been performed using quarterly Canadian data. As shown above, this is equivalent to testing that the structural errors u_t, of model (5.16) are independently distributed against the alternative that the u_t are AR(1). The test statistic is computed to be 47.95. As compared with the critical value for the χ_1^2 distribution at the 95 per cent level, this indicates strong rejection of the null hypothesis. The test of the same null hypothesis against the alternative that the V_t are generated by an MA(3) yields a test statistic of 49.22. Although neither test is sufficient for identification of the true error structure of model (5.16) both strongly suggest the presence of autocorrelation in u_t. This could be an indication of misspecification in model (5.16).

5.4. Variable Dynamics

Several recent articles have considered the possibility that dynamic processes underlying money supply and demand have not been constant over time. White (1978) argued that adjustment to desired levels of money balances involves rebuilding or depleting stocks of financial assets. Since this may require changes in the rate of saving, it is a gradual process achieved over a period of time. Factors such as the size and direction of the necessary adjustment and the inflationary climate all have a bearing on the speed with which the adjustment is carried out. Carr and Darby (1979) suggested that money supply shocks influence the demand for money. Since unanticipated changes in the money supply, and hence in the level of prices, affect the synchronization of purchases and sales of assets then they may result in temporary changes in the desired level of money balances.

A simple test of the hypothesis that adjustment in the demand for money has not been constant through time is possible within model (2.2). It is rewritten

$$m_t^d - p_t = \beta_0 + \beta_1 y_t + \beta_2 r_t + u_t \qquad (5.22)$$

where all variables are expressed as logarithms. The 'nominal' adjustment mechanism, appearing in Clinton (1973),

$$(m_t - m_{t-1}) = g(m_t^d - m_{t-1}) \qquad (5.23)$$

assumes that the adjustment of real balances occurs at the same rate with respect to prices as with respect to any of the other variables in the model. Substitution of (5.23) into (5.22) yields

$$(m_t - p_t) = g\beta_0 + g\beta_1 y_t + g\beta_2 r_t + (1 - g)(m_{t-1} - p_t) + gu_t. \qquad (5.24)$$

An alternative adjustment mechanism, specified in real terms, is:

$$(m_t - p_t) - (m_{t-1} - p_{t-1}) = g[(m_t^d - p_t) - (m_{t-1} - p_{t-1})]. \qquad (5.25)$$

Substituting (5.25) into (5.22) yields

$$(m_t - p_t) = g\beta_0 + g\beta_1 y_t + g\beta_2 r_t + (1 - g)(m_{t-1} - p_{t-1}) + gu_t, \qquad (5.26)$$

which differs from (5.24) in that the true lagged dependent variable appears in the RHS. Model (5.26) states that the response of real balances is more rapid with respect to prices than with respect to any of the other variables. Both models (5.24) and (5.26) may be seen to be constrained versions of the more general model

$$m_t = b_0 + b_1 y_t + b_2 r_t + b_3 \bar{m}_{t-1} + b_4 p_t + b_5 p_{t-1} + u_t.$$

The three models were estimated for the sample periods 1956 II to 1977 IV. Using conventional statistical criteria neither model (5.26) nor model (5.24) could be rejected. With quarterly Canadian data there appears no strong support either for Goldfeld's (1976) preference for nominal adjustment or for Laidler's (1980) preference for real adjustment. The real adjustment model (5.26) is used in the present analysis.

Variable adjustment may be viewed as a form of parametric instability. Three specific forms are considered in this thesis. The first involves a finite number of abrupt parameter changes over the sample. This discrete parameter variation has been examined by Quandt (1958, 1972), Chow (1960), Hinkley (1970), Goldfeld and Quandt (1976) and Riddell (1978 a, b). Chapter 2 of this thesis addresses this particular form of instability.

The second form is random movement of parameters. This variation has been studied by Hildreth and Houck (1968), Swamy (1970), Rosenberg (1972), Cooley and Prescott (1973) and surveyed by Goldfeld and Quandt (1976). Of the random coefficient models, two will be developed and used in this study. These are the (Hildreth and Houck) stochastic parameter, and the (Cooley and Prescott) adaptive regression models.

The third form of instability is systematic parameter variation. Meyer (1972) and Cargill and Meyer (1978, 1979) consider variation which is 'state-of-the-economy' dependent. Their model allows for systematic variation in the coefficient vector according to some exogenous factors in the system. A similar model will be developed here. Both a time trend and the rate of inflation will be used as state variables.

5.4.A. Stochastic Parameter Regression

I applied stochastic parameter regression, as developed in Hildreth and Houck (1968), to the partial adjustment model (5.26). The first difference of logarithms of real money was regressed upon a constant, the logarithm of GNE, the logarithm of R90 and the logarithm of lagged real money for both M1 and M2C for the period 1956 II to 1977 IV. Tables 5.1 and 5.2 record the results of OLS and Hildreth-Houck estimation of M1 and M2C respectively. Column 1 contains the ordinary least squares estimates. Column 3 contains the GLS estimates of the means of the four stochastic parameters. Column 5 contains the Hildreth-Houck estimates of the variance components. Associated with each estimate is an approximate t- value. The inclusion of the lagged dependent

TABLE 5.1: HILDRETH AND HOUCK ESTIMATION IN THE DEMAND FOR M1

	Ordinary Least Squares		GLS Estimation		Variance Components	
	Estimate	t-value	Estimate	t-value	Estimate	t-value
CONSTANT	0.0896	0.71	0.0952	0.76	0.0008	0.81
GNE	0.1513	5.59	0.1530	5.63	-0.0000	-0.08
R90	-0.0506	-8.96	-0.0505	-8.95	0.0000	0.97
LAGGED M1	0.8135	4.21	0.8108	4.25	0.0000	0.18
R^2	0.514		0.514			
DW	1.75		1.75			
RSS	0.0110		0.0110			

TABLE 5.2: HILDRETH AND HOUCK ESTIMATION IN THE DEMAND FOR M2C

	Ordinary Least Squares		GLS Estimation		Variance Components	
	Estimate	t-value	Estimate	t-value	Estimate	t-value
CONSTANT	-0.3929	-4.74	-0.3927	-4.81	-0.0002	-0.28
GNE	0.1209	3.74	0.1213	3.75	0.0000	0.58
R90	-0.0206	-5.72	-0.0208	-5.80	0.0000	0.49
LAGGED M2C	0.9085	3.20	0.9080	3.20	0.0000	0.60
R^2	0.350		0.350			
DW	1.64		1.65			
RSS	0.0110		0.0110			

variable, of course, allows for only asymptotic validity of tests based on such statistics.

It is seen from both Tables 5.1 and 5.2 that estimates from the classical linear model and those from the stochastic parameter model are very nearly identical. In both cases the estimates of the parameter variances are insignificantly different from zero. The stochastic parameter model seems particularly insensitive to the M2C parameter variation witnessed in Chapter 2 and suggests that allowance for random variability in the speed of adjustment of actual to desired real balances does not significantly alter or improve our ability to model the demand for money.

5.4.B Adaptive Regression

The original Cooley and Prescott article (1973a) allowed for stochasticity of only the constant term. In a subsequent paper (1973b) they generalized the model to allow for all parameters to be stochastic. They have termed it the "varying-parameter regression model". The simpler model, called "adaptive regression" because of the autoregressive-type adaptation of the constant term, will be applied here.

Tables 5.3 and 5.4 record the results of application of adaptive regression to our money demand model. As in the case of stochastic parameter regression, the results are not noticeably different from those of ordinary least squares. There is one statistic, however, which suggests that the adaptive model might

TABLE 5.3: COOLEY AND PRESCOTT ESTIMATION IN THE DEMAND FOR M1

	Ordinary Least Squares		Cooley and Prescott	
	Estimate	t-value	Estimate	t-value
CONSTANT	0.0896	0.71	0.0779	0.64
GNE	0.1513	5.59	0.1487	5.67
R90	-0.0506	-8.96	-0.0504	-9.19
LAGGED M1	0.8135	4.21	0.8179	4.24
R^2	0.514		0.581	
DW	1.75		1.87	
RSS	0.0110		0.0107	

TABLE 5.4: COOLEY AND PRESCOTT ESTIMATION IN THE DEMAND FOR M2C

	Ordinary Least Squares		Cooley and Prescott	
	Estimate	t-value	Estimate	t-value
CONSTANT	-0.3929	-4.74	-0.3907	-4.83
GNE	0.1209	3.74	0.1193	3.78
R90	-0.0206	-5.72	-0.0205	-5.83
LAGGED M2C	0.9085	3.20	0.9099	3.23
R^2	0.350		0.660	
DW	1.64		2.04	
RSS	0.0110		0.0108	

represent an improvement over the classical version. The DW statistic in Table 5.4 indicates rejection of first-order autoregression for the adaptive model but is inconclusive for the classical model. Two factors, however, make this a very tentative conclusion. The DW is, of course, not the proper test statistic to use in the presence of a lagged dependent variable (see Durbin, 1970). Secondly, even if the DW statistic is accurately indicating a lack of AR(1) in the error, it says nothing about moving average and higher orders of autoregressive autocorrelation. In summary, very little information is derived from the application of adaptive regression to money demand model (5.26). If the coefficients of (5.26) are indeed variable we have no evidence to suggest that this variability is random.

5.4.C State-Varying Parameter Regression

Meyer (1972) suggests an approach to parameter variability whereby the variation is systematic and determined by other pre-determined variables in the system. He assumes the model;

$$y_t = \sum_{i=1}^{k} X_{it}\, \beta_{it} + w_t, \tag{5.27}$$

where y_t is the t-th observation on the dependent variable, X_{it} is the t-th observation on the i-th explanatory variable and w_t is the value of the disturbance at time t. The subscript, t, on β_i indicates that the coefficient is allowed to vary over the sample. More specifically,

$$\beta_{it} = z_{it}\gamma_i + \xi_{it}, \tag{5.28}$$

where β_{it} is the t-th element of the T by 1 coefficient vector β_i.

z_{it} is the t-th element of the state variable associated with coefficient i, γ_i is the scale coefficient to be estimated, and ξ_{it} is a disturbance term. Substitution of (5.28) into (5.27) yields

$$y_t = \sum_{i=1}^{k} X_{it} z_{it} \gamma_i + \sum_{i=1}^{k} X_{it} \xi_{it} + w_t. \tag{5.29}$$

Unbiased, consistent and efficient estimates of γ_i can be obtained through application of OLS to (5.29) given the assumptions of the classical linear model and that $\xi_{it} \equiv 0$. This is equivalent to saying that the movement over time of the β coefficients is purely systematic with no random component. If, however, $\xi_{it} \not\equiv 0$ and Z is a vector of constants then the movement of β can be said to be purely random. With the additional assumptions that $E(\xi_{it}) = 0$, $E(\xi_t \xi_t') = \sigma_t^2 I_T$ and $E(\xi_t w_t') = 0$ then model (5.29) can be seen to reduce to a form similar to that of the stochastic parameter model of Hildreth and Houck.

Incorporating this approach into the partial adjustment model yields:

$$\begin{aligned}(m_t - p_t) - (m_{t-1} - p_{t-1}) = {} & g\alpha o + g\alpha_1 y_t + g\alpha_2 r_t - g(m_{t-1} - p_{t-1}) + \\ & g\gamma o Z_t + g\gamma_1 Z_t y_t + g\gamma_2 Z_t r_t - \\ & g\gamma_3 Z_t (m_{t-1} - p_{t-1}) + v_t.\end{aligned} \tag{5.30}$$

The results of estimating equation (5.30) for M1 and M2C, when Z represents a linear time trend and when it represents the rate of inflation, are shown in Tables 5.5 and 5.6. T- values which, again, are only approximate due to the lagged dependent variable

TABLE 5.5: STATE-VARYING PARAMETERS IN THE DEMAND FOR M1: 1956-1977

Coeff.	Time-Dependent Parameters Estimate	t- value	Inflation-Dependent Parameters Estimate	t- value
g	0.4543	3.92	0.3032	3.44
α_0	1.2408	0.81	1.4221	2.85
α_1	0.7196	4.90	0.7083	13.86
α_2	-0.1142	-3.21	-0.1455	-3.24
γ_0	-0.0223	-2.60	-1.1918	-3.69
γ_1	-0.0060	-4.77	-0.1409	-2.18
γ_2	-0.0001	-0.12	-0.0181	-0.98
γ_3	-0.0099	-4.56	-0.3066	-3.27
RHO	0.0396	0.32	0.1785	1.50
$\bar{R}^2$	0.525		0.558	
D.W.	1.96		1.97	
RSS	0.0097654		0.0090875	

TABLE 5.6: STATE-VARYING PARAMETERS IN THE DEMAND FOR M2C: 1956-1977

Coeff.	Time-Dependent Parameters Estimate	t- value	Inflation-Dependent Parameters Estimate	t- value
g	0.1897	1.52	0.0957	1.71
α_0	7.7632	0.93	-4.6848	-2.56
α_1	0.1914	0.25	1.3622	7.39
α_2	-0.0502	-1.39	-0.0809	-1.63
γ_0	-0.0939	-1.18	-1.4862	-0.94
γ_1	0.0178	0.81	0.2459	0.48
γ_2	0.0002	0.62	0.0127	1.03
γ_3	0.0096	0.60	0.1377	0.32
RHO	0.2941	2.01	0.3320	2.79
$\bar{R}^2$	0.369		0.509	
D.W.	2.01		2.04	
RSS	0.0096327		0.0074995	

are included with the estimates. The hypothesis that the coefficients are independent of time and independent of the rate of inflation are then tested by checking for significance of the group of state-varying coefficients of (5.30) in all four regressions. F-statistics are recorded in Table 5.7.

Table 5.7 suggests that the coefficients in the stock adjustment demand for money model are not constant over the period 1956 II to 1977 IV, and that it is more likely that this variability is more closely linked to the rate of inflation than to a linear time trend. This of course, is not conclusive evidence that adjustment from actual to desired real balances is variable, since instability in the regression parameters could come from diverse directions and is not necessarily related to the adjustment parameter. It is, however, consistent with such an hypothesis.

TABLE 5.7: TESTS OF THE NULL HYPOTHESES OF TIME AND INFLATION INDEPENDENT COEFFICIENTS IN THE DEMAND FOR MONEY

Aggregate	H_0: Parameters Independent of a Linear Time Trend	H_0: Parameters Independent of the Rate of Inflation
M1	2.177	3.813*
M2	1.983	8.165*

* Significance at 99 per cent.

5.5 Concluding Comments

A simple approach to problems of dynamic adjustment in demand analyses is to assume them away. That is, to assume that full adjustment occurs within any one period and that every observation coincides with the intersection of aggregate demand and supply relations. While such an assumption may be justifiable in demand for money studies with data periodicities of one year or longer, it is not obviously so for quarterly or monthly models. This has led to the development of demand for money models where the actual level is allowed to lag behind the desired level.

The partial adjustment model is one such example. In this model a shock to one of the exogenous arguments which determines the desired dependent variable will affect, only partially, the actual dependent variable. The divergence between actual and desired levels is then postulated to diminish geometrically with the passage of time. As long as the period of shock to the explanatory varables is shorter than the length of time required for full adjustment, such a system could persist in a state of disequilibrium. Many theoretical arguments exist as to the inappropriateness of such a model. Griliches (1967) discusses problems associated with residual autocorrelation. Mundlak (1961) illustrates how temporal aggregation may present a problem if adjustment of the micro units is actually faster than the period of the data. Breen (1971) extends this argument by claiming that micro units are either full-adjusters or non-adjusters in any one period and that the adjustment coefficient is closer to a measure of the proportion of non-adjusters in the sample than it is to a measure of the speed of adjustment of individuals. Permanent income proponents would favour the temporal aggregation argument on the grounds of high substitutability between money and other financial assets and of the double-edged effects of interest rates upon relative portfolio positions. Finally, Waud (1966) considers the effects of misspecification error upon the properties of unbiasedness and normality of the coefficients and finds them to be substantial. One popular rationale for the presence of lagged dependent variables in demand for money models is the joint partial adjustment-adaptive expectations argument as developed by Feige (1967). Casual inspection of such a model reveals that the error term must follow, at the very least, a moving average process. A test procedure developed by Godfrey allows us to make inferences about the structural error of the PAAE model. The hypothesis of autoregressive errors is tested against specific alternative hypotheses. These tests have implications as to the validity of the original adaptive expectations assumption. Such a procedure has indicated possible misspecification within a Canadian PAAE demand for money model.

A second approach to adjustment is to assume polynominal distributed lag weights. This approach, though apparently less restrictive and hence enjoying wider acceptance among practicing econometricians, is no less subject to criticism than the Koyck model.

The criticism is essentially based upon the bias which results from incorrectly specifying the appropriate order and lag length. Furthermore, procedures based upon significance tests which are often employed in choosing the order and lag length for an Almon polynominal, are statistically invalid.

As regards the adjustment process itself in the demand for money, it can be rationalized from two points of view. The traditional way is to proceed from the idea that the agent, upon recognizing his shortfall (or surplus) in cash balances, will react by transferring wealth from (to) other existing assets. The transaction costs of such a decision are then weighed against the risk and inconvenience of being out of equilibrium. An alternate adjustment procedure is through the reallocation of the flow of savings. This mechanism of adjustment does not entail transaction costs and allows for the possibility of variability over time in the speed of adjustment.

From the present study, when a state-varying parameter model was employed, the hypothesis that the coefficients were independent of the rate of inflation was rejected for M1 and M2C. The hypothesis that the coefficients were independent of a linear time trend was not rejected for M1 or for M2C. My principal conclusion is that there is substance to the hypothesis of variability in the speed of adjustment of money demand and that this variability coincides with periods of rapidly rising prices and is due, perhaps, to prevailing uncertainty and to the presence of unanticipated price shocks.

CHAPTER 6

COMPETING THEORIES

In seeking explanations of monetary phenomena, theorists frequently find themselves in support of one or the other of two competing theories of money demand. The asset view emphasizes utility gained through the holding of money as one of many assets in a given portfolio. Formalized in the work of Friedman (1956) is the proposition that the demand for money is determined in the main by permanent income and the rates on competing assets. The transactions view of the demand for money emerges more from the classical stream. Here, money is thought of as being in demand solely to facilitate transactions. It is derived, independently of a speculative motive, within the inventory models of Baumol and Tobin. Determinants are the level of transactions, the cost of transactions, and the opportunity cost of holding money.

In Friedman's formulation, the expected rate of inflation is viewed as the implicit rate of return on inventories of physical goods. It should, therefore, be recognized by the money holder as the rate of return of an alternate form of wealth. Despite theoretical relevance of inflationary expectations to the demand for money, however, empirical studies have generally failed to give supporting evidence.

The transactions school is less decisive with respect to the theoretical relevance of expectations. For example, Clower and

Howitt (1978) seek to demonstrate the importance of expectations to the transactions demand for money, while Goldfeld (1976) summarily dismisses expectations as having no place in the transactions model.

Explanations of recent instability in conventional M1 equations typically fall into either the transactions or asset school of thought. One 'transaction-style' argument is that changes in the efficiency of cash management have had an impact on desired money holdings. Porter, Simpson and Mauskopf (1979), for example, emphasize the importance of these changes for the U.S. economy. In the Miller-Orr model, the variance of the firm's cash flow is a determinant, along with brokerage costs and interest rates, of demand deposit balances. One contribution of Porter et al. was the recognition that the link between the firm's scale of operations and the variance of its cash flow may have altered significantly since 1974.

Most 'asset-style' explanations of the M1 shift stem from the Friedmanian proposition that all interest rates are relevant to the determination of money demand. If demand for M1 depends upon a spectrum of interest rates then a function employing only a short rate, say, will remain stable only as long as the yield curve does not move about. An unstable yield curve under such conditions will coincide with an unstable M1 equation. Heller and Khan (1978) estimate quadratic yield curves based upon seven interest rates from 1960 III to 1976 IV and enter the parameters of these curves into a money demand equation. They report a stable equation. The estimated equation, however, when dynamically simulated to 1977 IV, does not produce smaller errors than conventional models over the same period /see Porter and Mauskopf (1978)/.

The present chapter consists of two distinct sections. The first deals with a shifting demand for M1 in the mid-1970s, and its apparent causes. The second deals with inflationary expectations. Though both topics can be loosely joined under the general heading of 'competing theories', they are treated separately here. The results of section 6.1 bear in no way upon the assumptions or conclusions of section 6.2.

In section 6.1, transactions models of the demand for components of narrow money are developed. When shift variables are introduced to each equation it is concluded that the mid-1970s shift in M1 originated in the current account component. Furthermore, when cross-equation constraints on the shift coefficients are applied within a systems framework, M1 appears to have shifted downward by an amount ranging from 4 to 8 per cent.

In section 6.2 various theories of expectation generation are examined. One contribution of subsection 6.2.A is in the derivation of an optimum 'error-learning' series of autoregressive expectations. It is seen that under the assumed conditions for optimality, both the order and coefficients of the autoregressive generating function change over time. In subsection 6.2.B, a partial adjustment model is developed in which alternative expectations hypotheses may be tested. Although empirical testing of static versus rational expectations proves not to be informative due to high residual autocorrelation, a general framework for such a test is examined.

6.1 Components of Narrow Money

In Chapter 2, it was observed that M1 models which fit quarterly data well prior to 1976 produce large negative residuals for the subsequent two years. When a shift term (zero prior to 1976 I, increased linearly to 1977 III and held constant at 7 in 1977 IV) was included in the regression its coefficient was found to be significantly different from zero and to represent a decrease in M1 of roughly 6.7 per cent. This procedure, as shown in Wilton (1975), may be interpreted as a test for structural change in the constant term. It was concluded from knowledge of institutional behavior that the source of this instability was in corporate cash management policy. A more rigorous econometric examination of this phenomenon is undertaken in the present chapter.

A common transactions explanation of the mid-1970s shift in money demand arises from the Miller-Orr model. As noted in Chapter 1, an important determinant of the demand for cash may be the variance of cash flow. With improved cash management techniques a firm's cash flow variance may be reduced relative to its scale of operation. Akerlof (1978) shows that movement from loosely-monitored accounts to tightly-monitored accounts will have a downward impact on the demand for money. While Canadian bank deposit data are not directly separable as to holder of deposit, it is probably safe to assume that all personal chequing accounts are held by individuals and that most current accounts are held by firms. If the transactions hypothesis is true, then the observed M1 shift should be more closely associated with the current account component than with either currency or personal chequing accounts.

An asset-style explanation of the M1 shift claims that the source of instability is misspecification of the fitted equation. Friedman (1977) has shown that the opportunity cost of holding cash balances is some weighted average of rates on assets across the spectrum of maturities. To the extent that rates on assets of different maturities are not included in the specification of demand for money, pronounced movements in the term structure will appear as instabilities in the regression equation. Heller and Khan (1979) tested this hypothesis by entering a quadratic yield curve directly into a regression equation for the demand for money. While they reported a stable equation, Porter and Mauskopf (1978) subsequently questioned their findings on the basis of dynamic simulations. Subsection 6.1.A reports on single equation estimates of the demand for components of narrow money. This is done both for models containing interest rates explicitly, and for models containing estimated parameters of a quadratic yield curve, as measures of the opportunity cost of holding money.

6.1.A Single Equation Estimates

Aggregate data in the following analysis include: currency outside banks (CURR), current accounts (CA), personal chequing accounts (PCA), personal chequable savings deposits at chartered banks (PCSDS), and currency plus demand deposits (M1). They are seasonally adjusted and cover monthly intervals. End-of-month (EOM) data are collected from 1968: 1 to 1978: 9. Average-of-Wednesdays (AOW) data are available from 1974: 1 to 1978: 9. Both series are used below. Interest rate data include: the rate on 90-day swapped deposits (RSWAP), the rate on non-chequable savings deposits at chartered banks (RSDB), the McLeod, Young and Weir average of provincial bond yields (RPROV). Again, both EOM and AOW series are used. Two measures of income are real gross national expenditure (YGNE) and real personal disposable income (YDP). POP denotes the size of the total Canadian population, CPI denotes the consumer price index and PGNE denotes the GNE deflator. Three different shift variables are used. SHIFTEX is a linear time trend beginning in 1974: 1 and held constant after 1977: 9. SHIFT1 is a linear time trend beginning in 1974: 1 and held constant after 1975:12. SHIFT2 is a linear time trend beginning in 1976: 1 and held constant after 1977: 9.

Two different dynamic specifications are used in modelling M1 and the narrow money components. The first involves Almon distributed lags. These models, apart from dummy variables for periods affected by mail strikes and error terms, are summarized as follows:

$$\log\left(\frac{CURR}{POP*PGNE}\right)_t = a1_0 + \sum_{i=0}^{7} b1_i \, RSWAP_{t-i} + \sum_{i=0}^{3} c1_i \log\left(\frac{YGNE}{POP}\right)_{t-i} , \quad (6.1)$$

$$\log\left(\frac{CA}{PGNE}\right)_t = a2_0 + \sum_{i=0}^{7} b2_i \, RSWAP_{t-i} + \sum_{i=0}^{3} c2_i \log(YGNE)_{t-i}$$

$$+ d2_0 \, SHIFTEX , \quad (6.2)$$

$$\log\left(\frac{PCA}{CPI}\right)_t = a3_0 + \sum_{i=0}^{7} b3_i \, RSDB_{t-i} + \sum_{i=0}^{3} c3_i \log(YDP)_{t-i} , \quad (6.3)$$

$$\log\left(\frac{PCSDS}{PGNE}\right)_t = a4_0 + \sum_{i=0}^{7} b4_i \, RSWAP_{t-i} + \sum_{i=0}^{3} c4_i \log(YGNE)_{t-i}$$

$$+ d4_0 T + d4_1 T^2 , \quad (6.4)$$

$$\log\left(\frac{M1}{PGNE}\right)_t = a5_0 + \sum_{i=0}^{7} b5_i \, RSWAP_{t-i} \quad \sum_{i=0}^{3} c5_i \log(YGNE)_{t-i}$$

$$+ d5_0 \, SHIFT2 + \sum_{i=0}^{7} g5_i \, RPROV_{t-i} . \quad (6.5)$$

Here T is a linear time trend. Almon coefficients on income follow a first degree polynomial and those on interest follow a second degree polynomial, as in models B and C of Chapter 2.

Estimates for these five models using AOW data from 1974: 1 to 1978: 9 are reported in Table 6.1. In all cases, corrections for first order autoregression were applied. The autocorrelation coefficient is labelled AUTO1. In the four component equations the Durbin-Watson (DW) statistics are acceptable. In the aggregate M1 equation, however, the transformed residuals still indicate the presence of first order autoregression. With assumptions

TABLE 6.1: SINGLE EQUATION ESTIMATES FOR SELECTED MONEY MEASURES
(AOW, 1974:01 to 1978:09)

Independent Variables/ Summary Stats.	Dependent Variable CURR	CA	PCA	PCSDS	M1
Constant	-7.1005 (-22.3)*	-3.4681 (-1.2)	-8.1033 (-10.4)	-8.1457 (-3.29)	-2.5282 (-0.84)
SHIFTEX		-0.0044 (-5.6)			
SHIFT2					-0.0023 (-1.9)
T				-0.0139 (-19.42)	
T^2				0.0001 (12.36)	
Short Rate (Σ)	-0.0064 (-3.6)	-0.0211 (-7.4)	-0.0129 (-3.3)	-0.0208 (-11.05)	-0.010 (-2.63)
Long Rate (Σ)					-0.021 (-2.39)
Income	0.5214 (2.7)	0.6797 (2.7)	0.9720 (14.5)	1.0605 (4.95)	0.6490 (2.47)
AUTO1	0.969	0.213	0.338	0.630	0.789
DW	1.70	2.03	2.00	2.01	1.47
$\bar{R}^2$	0.951	0.872	0.950	0.996	0.660
SER	0.00353	0.01379	0.01391	0.00501	0.0076

CORRELATION OF OLS RESIDUALS

CURR	1.0000			
CA	0.3089	1.0000		
PCA	0.1925	0.7864	1.0000	
PCSDS	0.1447	0.5900	0.6699	1.0000

of normality, zero mean, homoscedasticity, and lack of autocorrelation in the error of each of these models, the t-values reported in parentheses below the estimates of Table 6.1 may be interpreted for tests of significance. All regressions report significantly negative interest elasticities and significantly positive (between 0.5 and 1.0) income elasticities. The correlation matrix was computed for the OLS residuals from the CURR, CA, PCA and PCSDS equations. This information, when taken in conjunction with the lack of similarity of explanatory variables of these equations, suggests that improvements in asymptotic efficiency over OLS estimation can be achieved.

The second dynamic specification involves the Koyck distributed lag. The Heller-Khan (1979) methodology has been applied to Koyck-style models for CURR, CA, PCA and M1. The term structure variables are the intercept, slope and curvature of a quadratic yield curve based on monthly observations for seven interest rates. Estimates from these four models are recorded in Table 6.2.

One remarkable feature of Table 6.2 is that even when the term structure is entered explicitly in the M1 equation, the coefficient of SHIFT2 indicates a downward shift of approximately 9.3 per cent.* This would seem to reject the asset view that

*Since SHIFT2 is entered in a form which prevents its run-off over time, the percentage shift is computed by multiplying the coefficient by the value of SHIFT2 at its endpoint.

TABLE 6.2: SINGLE EQUATION ESTIMATES OF TERM STRUCTURE IMPACT IN KOYCK-STYLE MODELS FOR SELECTED MONEY MEASURES.

(AOW, 1974: 1 to 1978: 9)

Independent Variables/ Summary Stats.	Dependent Variables			
	CURR	CA	PCA	M1
Constant	-0.05225 (-0.13)*	-0.4265 (-0.31)	0.1573 (0.37)	-1.3688 (-1.79)
SHIFTEX		-0.0079 (-3.82)		
SHIFT2				-0.0044 (-3.05)
Income	0.0131 (2.04)	0.2880 (2.34)	0.1184 (0.98)	0.2003 (2.59)
TS-Intercept**	0.0029 (0.23)	-0.1274 (-2.29)	-0.0624 (-1.24)	-0.0754 (-2.54)
TS-Slope**	0.2542 (1.37)	-1.2276 (-1.55)	-0.6124 (-0.87)	-0.5484 (-1.25)
TS-Curvature**	3.1386 (1.48)	-16.8584 (-1.84)	-7.9989 (-1.06)	-5.8972 (-1.16)
Lagged Dependent	0.9975 (15.69)	0.3668 (2.71)	0.7916 (7.25)	0.8318 (14.01)
DW	1.77	1.99	2.11	1.64
$\bar{R}^2$	0.954	0.846	0.958	0.926
SER	0.00359	0.0148	0.0126	0.0070

*t-values are reported in parentheses. They are only asymptotically valid due to presence of a lagged dependent variable.

**These variables are the estimated intercept, slope and curvature of a quadratic yield curve based on rates on 30-, 60-, and 90-day financial paper, and 1-3, 3-5, 5-10 and 10 years and over Government of Canada bonds. I am indebted to Ron Parker for guidance in this area.

instability in the demand for M1 is due largely to omission of relevant interest rate data. Furthermore, in both the Almon specifications of Table 6.1 and the Koyck specifications of Table 6.2, the only component for which a shift variable apparently contributes explanatory power is CA. This would seem to support the transactions view that the downward shift in M1 resulted from increased cash management efficiency of corporations.

Since the shift variables used in Tables 6.1 and 6.2 were chosen on purely empirical grounds, it is useful to summarize our prior views as to possible shifts in CA, PCA and PCSDS. Examination of institutional data reveals that banks began offering cash management plans to their corporate customers in 1972. This suggests that the downward shift in CA began before 1976:1.

As for PCA, the picture is less sharply defined. In the early 1970s, the chequing costs to customers for CAs relative to PCAs increased dramatically. One would normally associate this with an upward shift in the demand for PCA. However, at the end of 1974, a legislative change allowed tax exemption on $1000 of interest income. If the relevant opportunity cost for PCA is some tax-adjusted interest rate, then this legal change should have had a downward impact upon PCA. The combined effect on PCA is not, therefore, unambiguously defined. It is notable from the equations of Tables 6.1 and 6.2 that the extended shift variable (SHIFTEX) is relevant to CA and that no shift is relevant to PCA. When the extended shift is included in the M1 equation, however, it contributes explanatory power only for the period 1976: 1 to 1977: 9.*

The final component considered here is chequable personal savings deposits at chartered banks (PCSDS). While PCSDS is not a component of M1, it may be considered a transactions balance and may be relevant because of its potential substitutability with the components of M1. With the advent of non-chequable savings in 1967, PCSDS fell off from a level more than $2 billion higher than demand deposits to a level in 1970 roughly $0.5 billion below demand deposits. After a two-year interval of positive growth, PCSDS from 1973 to the present stayed relatively flat and declined steadily as a fraction of demand deposits. Throughout the period explored in the present study, chequable savings deposits have been rather uncompetitive.

The PCSDS equation (6.4) is also a simple transactions model. An important feature of this model is the presence of a quadratic trend. Although this trend has been chosen simply for the improvement it gives to the fit of the equation, an <u>ex post</u> justification for its presence may be that these deposits are held by a particular segment of the population. The holders are assumed to be averse to complications associated with maintaining separate savings and chequing accounts. The estimated impact of

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*SHIFT1 is the difference between SHIFTEX and SHIFT2. When SHIFT1 and SHIFT2 are entered in the M1 equation only SHIFT2 is significantly different from zero. When they are entered in the CA equation both are significant.

the quadratic time trend, as reported in Table 6.1, might then be the result of two factors: 1) the demographic effect of a holding population which is declining over time, and 2) a delayed learning response to the diminished competitiveness of these instruments which occurred in the late 1960s and early 1970s. The rate of change of this impact is seen to be strong initially and to moderate gradually through the sample period. The estimated polynomial, $-.0139\,T + .0001\,T^2$, has zero slope at T = 70, corresponding to 1979:10.

6.1.B <u>Systems Estimates</u>

As shown in the correlation matrix of Table 6.1 there exists considerable cross-equation correlation in the least squares residuals, particularly among CA, PCA and PCSDS. In view of this correlation, improved asymptotic efficiency is possible with the application of Zellner-efficient (ZEF) estimation. Results for this alternative approach are shown in Table 6.3 for the four equation model.

Comparing the estimates and t-values of Table 6.3 with their corresponding entries in Table 6.1 confirms what one would expect; namely, only slight changes in the estimates and higher t-values on all coefficients in the equations for CA, PCA and PCSDS. These were the equations least similar in explanatory variables and demonstrating the highest cross-equation residual correlation. The t-values reported, of course, are appropriate for statistical inference only asymptotically. Even in the present sample of 57 observations, however, the asymptotic result appears to hold.

TABLE 6.3: ZELLNER EFFICIENT ESTIMATION OF COMPONENTS OF NARROW MONEY

(AOW 1974: 1 TO 1978: 9)

Independent Variables/ Summary Statistics	Dependent Variable			
	CURR	CA	PCA	PCSDS
Constant	-6.9447 (-22.56)*	-3.7836 (-1.75)	-9.6541 (-9.59)	-8.8794 (-4.53)
SHIFTEX		-0.0046 (-7.42)		
T				-0.0143 (-23.13)
T^2				0.0001 (16.33)
Interest (Σ)	-0.0067 (-3.76)	-0.0207 (-8.12)	-0.0090 (-2.60)	-0.0209 (-11.49)
Income (Σ)	0.4300 (2.33)	0.7069 (3.77)	1.1022 (12.82)	1.1240 (6.63)
AUTO1	0.9693	0.2258	0.3506	0.6302
DW	1.68	1.97	1.97	2.01
$\overline{R}^2$	0.949	0.864	0.951	0.996
SER	0.0057	0.0223	0.02174	0.00804

*t-values, valid asymptotically, are in parentheses.

Aside from the potential gain in asymptotic efficiency, a second potential benefit of ZEF estimation stems from the feasibility of cross-equation constraints. In the present context it is considered useful to constrain the magnitudes of shifts in the various components as a means of incorporating prior knowledge. For example, if equal and opposite shifts occur simultaneously in two components of M1 then the aggregate should be unaffected. Conversely, any shift in one component which is not matched by an equal and opposite shift in another component should be reflected in the aggregate numbers. Suitable modifications to the CA, PCA, and PCSDS equations appearing in Table 6.3 allow for the interrelation of shift coefficients. The modifications are: 1) SHIFTEX of the CA equation is split into SHIFT1 and SHIFT2, 2) SHIFT1 and SHIFT2 are included in the PCA equation, 3) YGNE replaces YDP in the PCA equation to avoid the collinearity which was apparent between YDP and the shift, and 4) SHIFT1 and SHIFT2 replace the quadratic trend of the PCSDS equation.

Results from the unconstrained model estimated with AOW data from 1974:1 to 1978:9 appear in Table 6.4. It is seen that for CA both shift coefficients are negative and significantly different from zero. For PCA both are positive but not significantly different from zero while for PCSDS neither shift coefficient is significantly different from zero. This supports the inference drawn from the OLS estimates that the M1 shift can be largely identified as a shift in CA. It seems to conflict, however, with the inference drawn from the M1 regression that the shift began in the first quarter of 1976. The CA equation of Table 6.4 suggests that funds began shifting out in early 1974.

TABLE 6.4: ZELLNER EFFICIENT ESTIMATION OF UNCONSTRAINED THREE EQUATION MODEL (AOW 1974: 1 TO 1978: 9)

Independent Variables/ Summary Statistics	Dependent Variable		
	CA	PCA	PCSDS
Constant	-6.1239 (-1.90)*	-5.9444 (-0.86)	-5.1747 (-1.36)
SHIFT1	-0.0040 (-5.11)	0.0033 (1.52)	0.0018 (0.46)
SHIFT2	-0.0058 (-5.11)	0.0025 (0.10)	-0.0006 (-0.30)
Interest (Σ)	-0.0188 (-2.74)	-0.0141 (-2.00)	-0.0172 (-5.36)
Income (Σ)	0.9057 (3.24)	0.7459 (1.23)	0.7561 (2.35)
AUTO1	0.3239 (2.88)	0.6380 (6.57)	0.9651 (107.82)
DW	2.31	2.43	2.39
$\bar{R}^2$	0.864	0.945	0.995
SER	0.02068	0.02124	0.00825

*t-values, valid asymptotically, are in parentheses

The first row of Table 6.5 summarizes the relevant regression results of Table 6.4 in terms of estimated dollar amounts shifted to each of the components. An estimated exodus of $876 million came

TABLE 6.5: CROSS-EQUATION CONSTRAINTS ON SHIFT COEFFICIENTS IN CA, PCA AND PCSDS EQUATIONS

(AOW 1974: 1 TO 1978: 9)

Constraint	Test Stat.	Estimated SHIFT1 (1974:1-1975:12) $ Million			Estimated SHIFT2 (1976:1-1977: 9) $ Million			Estimated Total M1 Shift (1974:1-1977:9)	
		CA	PCA	PCSDS	CA	PCA	PCSDS	$ Million	% Shift
1		-876	0	0	-1249	0	0	-2125	-11.7
2	1.72	-793	+177*	0	-1229	0*	0	-1845	-10.1
3	2.27	-673	+226*	+912	-1198	0*	0	-1645	-9.0
4	2.94	-581	+259*	+1054	-1167	0*	0	-1489	-8.2
5	2.00	-940	+93*	-291	-1229	0*	0	-2076	-11.4
6	1.74	-912	+113*	-116	-1239	0*	0	-2038	-11.2
7	4.33	0*	+208	0	-795	0*	0	-587	-3.2

* Indicates constrained estimate.

from CA between 1974: 1 and 1975:12. Another $1,249 million shifted from CA between 1976: 1 and 1977: 9 with no shifts appearing in either of the other two components. The combined shift in M1 is then computed to be $2,125 million or 11.7 per cent of its average level over the period.

In the subsequent rows of Table 6.5, a series of arbitrarily chosen cross-equation constraints were applied to the coefficients of the shift variables as a means of assessing the robustness of the unconstrained shift estimates reported in the previous paragraph. The coefficients of the shift variables in the assumed models imply specific percentage changes in the dependent variables. It is possible, therefore, to constrain the coefficients across equations in such a way as to impose a given relation on the shift magnitudes of the various components. The method for testing the appropriateness of such constraints is to compute the systems analogue of the F-ratio. This ratio is distributed asymptotically as chi-square with q degrees of freedom, where q is the number of constraints. If the ratio exceeds the tabulated critical value at an appropriate level of significance the validity of the null hypothesis, i.e., the constrained model, is statistically rejected.

For constraint 2 of Table 6.5, the null hypothesis is that the influx of funds to PCAs between 1974: 1 and 1975:12 is approximately equal to 1/5 of the exodus of funds from CA, with no shift in PCA between 1976: 1 and 1977: 9. The constraint to zero of SHIFT2 in the PCA equation is repeated in all six constrained models of Table 6.5.

The test statistic shows that these constraints cannot be rejected. Constraint 3 imposes a SHIFT1 in PCA approximately equal to -1/3 times the shift in CA. Constraint 4 imposes a SHIFT1 in PCA approximately equal to -2/5 times the shift in CA. Constraint 5 imposes a SHIFT1 in PCA approximately equal to -1/3 the shift in PCSDS. Constraint 6 imposes a SHIFT1 in PCA approximately equal and opposite to the shift in PCSDS. Constraint 7 constrains SHIFT1 in CA to zero. In none of the six constrained models was the test statistic (compared with χ^2_2 at the 90 per cent level of 4.61) high enough for rejection of the null hypothesis. It is notable that even constraint 7, which constrains the early shift in CA to zero is not rejected. Also, a wide variation in possible movement of PCSDS from +$1,054 million to -$291 million is observed. The range of M1 shifts implied by these seven runs is from 3.2 per cent to 11.7 per cent. It is therefore possible that some of the long shift in CAs could be masked by an offsetting shift in PCAs; however, with this relatively short sample period it is not possible to impute a very high degree of precision to the shift estimates.

To investigate the possibility that the early CA shift is a spurious result peculiar to the short sample period, I tried four different shift variables in the M1 equation for two sample periods. Since average-of-Wednesdays (AOW) data are not available prior to 1974: 1, the longer sample period consists of end-of-month (EOM) data. The results, summarized in Table 6.6, would appear to support the claim that the early shift in CAs is spurious. While the extended shift appears significant to M1 for both sample periods, its significance, at least for the

TABLE 6.6: M1 EQUATION, LEAST SQUARES ESTIMATION OF THE SHIFT

Independent Variables/ Summary Stats.	Dependent Variables (AOW 1974: 1-1978: 9) Double Shift	(AOW 1974: 1-1978: 9) Extended Shift	(EOM 1968:12-1978: 9) Double Shift	(EOM 1968:12-1978: 9) Extended Shift
SHIFT1	-0.0027 (-1.79)*		0.0001 (0.05)	
SHIFT2	-0.0032 (-2.20)		-0.0032 (-4.19)	
SHIFTEX		-0.0029 (-3.77)		-0.0016 (-2.88)
RSWAP (Σ)	-0.021 (-4.67)	-0.021 (-4.77)	-0.016 (-5.93)	-0.016 (-5.00)
RPROV (Σ)	0.004 (0.25)	0.007 (1.08)	-0.023 (-2.88)	-0.012 (-1.54)
Income (Σ)	1.042 (3.90)	1.018 (4.42)	0.846 (16.92)	0.825 (11.79)
AUTO1	0.6196	0.6175	0.8208	0.8844
DW	1.42	1.42	1.69	1.74
$\bar{R}^2$	0.715	0.724	0.829	0.728
SER	0.00783	0.00755	0.00751	0.00611
% Shift	-13.2	-13.2	-6.6	-7.2

*t-values are in parentheses

EOM data, seems to derive from the 1976 and 1977 segment. The early shift is significant using AOW data, but is not using EOM data.

The CA equation, though not reported here, was estimated using EOM data for the period 1968:12 - 1978: 9. This regression gave supporting evidence that the early CA shift is peculiar to the AOW data and the shorter estimation period. While the extended shift was significant for the longer sample period, most of its explanatory power came from the 1976: 1 - 1977: 9 segment. When the extended period was split into SHIFT1 and SHIFT2, only the latter remained significant. SHIFT1 indicated a negative shift although it was not significantly different from zero. When Zellner-efficient estimation was applied this result did not change.

As a further test for possible spuriousness in the results from the short sample period, the cross-equation constraints of Table 6.5 were applied to the same model using end-of-month (EOM) data for the period 1968:12 - 1978: 9. The results appear in Table 6.7. Several points are worth mentioning. The first is that the early shift in CA, though unconstrained in rows 1 through 6, is never significantly different from zero. The second is that the positive early shift in PCA appears unimportant as a fraction of the negative early shift in PCSDS. This is not surprising since most of the shift between PCSDS and PCA is thought to have occurred between 1968 and 1971. Third, estimated M1 shifts range from 4.2 per cent to 5.6 per cent -- smaller than the aggregate measures and much less variable than the corresponding measures for AOW, 1974 - 1978 data.

TABLE 6.7: CROSS-EQUATION CONSTRAINTS ON SHIFT COEFFICIENTS IN CA, PCA AND PCSDS EQUATIONS

(EQM 1968:12-1978: 9)

Constraint	Test Stat	Estimated SHIFT1 (1974:1-1975:12) $ Million			Estimated SHIFT2 (1976:1-1977: 9) $ Million			Estimated Total M1 Shift (1974:1-1977:9)	
		CA	PCA	PCSDS	CA	PCA	PCSDS	$Million	% of avg.
1		0	0	-1526	-763	0	-901	-763	-4.2
2	4.44	0	+12*	-1514	-878	0*	-910	-866	-4.8
3	4.55	0	+9*	-1514	-1022	0*	-910	-1013	-5.6
4	4.63**	0	+3*	-1514	-898	0*	-910	-895	-4.9
5	7.86**	0	+419*	-1313	-1084	0*	-944	-666	-3.7
6	7.31**	0	+375*	-375	-1001	0*	-716	-626	-3.4
7	2.93	0*	0	-1520	-1022	0*	-904	-1022	-5.6

* Indicates constrained estimates.

** Indicates significant rejection of the null hypothesis at the 90 per cent level.

Such wide swings in the point estimates coming from different data illustrate the need for caution when attempting to attach a dollar amount to the shift out of M1. These results indicate, however, that such a shift did occur and that it predominated in the current account component of M1.

6.2 Expectations

6.2.A. Formation of Expectations

One popular hypothesis regarding the formation of expectations is that current and past levels of a given variable are sufficient for the modelling of expected future levels. One begins with the assumption that the expectation of prices, for example, can be represented as:

$$E_{t-1}(P_t) = A(L)P_{t-1} + e_t , \tag{6.6}$$

where A(L) is a polynomial of the distributed lag operator L. The symbol e is used to represent a random error term. For convenience, the same symbol is used for the error of subsequent equations though they need not represent the same error process. $E_{t-1}(P_t)$ is used to represent the expectation formed in period t-1 of prices in period t. Referral to the expectations of 6.6 as 'autoregressive' involves another important assumption -- one which is rarely mentioned. If the process represented by 6.6 is autoregressive in the traditional sense of the word, then

$$E_{t-1}(P_t) = P_t \quad . \tag{6.7}$$

This implies that expectations are always realized. This condition may or may not be desirable for any given application.

One example of autoregressive expectations, given condition (6.7), is written:

$$E_{t-1}(P_t) = b_1 P_{t-1} + b_2(P_{t-1} - P_{t-2}) + e_t \quad . \tag{6.8}$$

This may be compared to (6.1) with $\alpha_0 = b_1 + b_2$, $\alpha_1 = -b_2$ and $n = 1$. With $b_2 > 0$ then past trends are expected to continue and expectations are said to be extrapolative. With $b_2 < 0$ then past trends are expected to reverse and expectations are said to be regressive. With $b_2 = 0$ expectations are static.

An alternate form of expectation generation is provided by the adaptive expectation model. Such a model first appeared in the demand for money literature in Cagan (1956). Here, price expectations are assumed to adapt to last period's expectations in proportion to last period's forecast error as follows:

$$E_{t-1}(P_t) = E_{t-2}(P_{t-1}) + \lambda\left[P_{t-1} - E_{t-2}(P_{t-1})\right] + e_t \quad .$$

With $0<\lambda<1$, this is equivalent to expressing expected prices as a geometrically declining distributed lag on actual prices:

$$E_{t-1}(P_t) = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i P_{t-i-1} + e_t \quad . \tag{6.9}$$

The idea that expectations are formed 'rationally' is formally credited to Muth who wrote in 1961 that "expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory ...". More precisely, the rational expectations hypothesis states that expectations are the true mathematical expectations of future variables conditional upon all information available at time t. Within the context of a simple macro model, rational expectations on prices might be represented as a reduced form equation containing lagged values of prices and all other relevant information on the exogenous variables. The assumption that prices behave according to a stable, known reduced form endogenizes price expectations to the model. That agents utilize full information in the place of the limited information contained in past inflation rates to form inflation expectations underlies, for example, the work of Modigliani and Shiller (1973) and Sargent and Wallace (1973).

Midway between the concepts of autoregressive expectations and rational expectations lies the recognition that there is an economic cost associated with the acquisition and use of information. Feige and Pearce (1976) introduce the notion of 'economically rational' expectations to represent those expectations which are formed on the reduced information for which the marginal improvement in forecast is just offset by the marginal cost of acquiring information. Feige and Pearce assume that information on inflation history is of negligible cost and that agents go to autoregressive expectations models first. Using the cross-correlation procedure of Haugh (1972) they concluded that neither

TABLE 6.8: AKAIKE - OPTIMUM AUTOREGRESSIVE FILTERS FOR PGNE

FILTER COEFFICIENTS OF X_{T-I}

Forecast Observation (T)	I = 1	I = 2	I = 3	I = 4	I = 5	I = 6	I = 7	I = 8	I = 9	I = 10
1967 :3	.0305	.1120	.2056	.4394						
4	.1304	.2444	.3288							
1968 :1	.2676	.2993								
2	.0807	.0654	.2588	.4009						
3	.2213	.0846	.5085							
4	.3125	.2580								
1969 :1	.4810									
2	.1179	-.0503	.3527	.4153						
3	.2663	.0859	.4977							
4	.3689	.3107								
1970 :1	.5582									
2	.3447	.2044	.5480	.4823	-.6693					
3	.0873	.0879	.3298	.3732						
4	.2599	.2291	.3820							
1971 :1	.3917	.4040								
2	.6423									
3	.0907	.1724	.3508	.3008	-.1291	.0971	.0487			
4	.0707	.2654	.1915	.3976	-.0686	.0615				
1972 :1	.1371	.1911	.2470	.2764	.0573					
2	.1459	.2443	.2020	.3215						
3	.1990	.3687	.3251							
4	.3862	.4520								
1973 :1	.2067	.2002	.1560	.2156	.0234	-.0858	.1124	.0611	.0590	.0090
2	.2685	.2459	.1177	.1109	.0972	.0273	-.0220	.1762	-.0771	
3	.3690	.2295	.1062	.0532	.0248	.0609	.0246	.0350	.0238	.0172
4	.3690	.2976	.0763	.0750	.0030	.0107	.0898	.1107	-.1068	
1974 :1	.4819	.1871	.1190	.0285	.0077	-.0262	.0204	.1172		
2	.5121	.2511	.0135	.0743	-.0164	-.0121	-.0005	.0543	.0141	.0395
3	.6220	.2078	.032	.0038	.0162	-.0198	.0077	.0442	.0040	
4	.6648	.2794	-.0278	.0082	-.0242					
1975 :1	.5604	.3942	.0115	-.0498						
2	.5815	.3519	.1052	-.0276	-.0572	-.0545	-.0213	.0373	-.0301	.0331
3	.4735	.4366	.0448	.9871	-.0138	-.0944	-.0103	.0013	-.0123	
4	.5119	.2817	.1456	.0399	.0609	-.0619	-.0495	-.0038		
1976 :1	.5418	.2987	.0557	.0773	.0312	-.0275	-.0413			
2	.4358	.3915	.0815	-.0116	.0994	-.0531				
3	.4722	.3341	.1154	-.0303	.0581					
4	.3954	.5204	-.0247	.0562						
1977 :1	.4122	.4729	.0617							
2	.3945	.5530								
3	.8798									
4	.4221	.5217								

monetary policy variables (three different money aggregates) nor a fiscal policy variable (high-employment budget surplus) could significantly improve upon the forecasts of the autoregressive models.

One drawback of the rational expectation approach is that the structure of the expectation generating mechanism is assumed constant and known. There appears no scope for 'error-learning' or for relevant changes as new and better information becomes available. An attempt to deal with this problem is made in the present study by applying the Akaike technique (described in Chapter 4) to a subsample of 50 observations on PGNE and rolling this subsample one observation at a time through the full sample from 1956 II to 1977 IV. This allows not only the autoregressive coefficients, but also the order of the autoregressive process, to change over time. In addition, the expectation generating function is optimum, in a statistical sense, at every point in time. The coefficients of these 'Akaike-optimum' autoregressive filters are recorded in Table 6.8. It is seen that as the sample extends into the price-volatile mid-1970s the optimum order of autoregression lengthens. This is consistent with the view that, as uncertainty increases, agents will rely less upon the immediate past and will rely more upon a longer historical record in forming expectations of the future.

6.2.B. Static Vs. Rational Expectations

In this section, the partial adjustment model of Kennan (1979) is adapted for testing between static and rational expectations in the demand for money. We denote, as before, short-run desired real balances,

M_t^d, as:

$$M_t^d = \alpha_0 + \alpha_1 y_t + \alpha_2 r_t + u_t \quad . \tag{6.10}$$

We define long-run, desired real balances, $\overline{M}_t$, as a rational distributed lag of current and future short-run, desired real balances. It is assumed that individuals seek to minimize the expected present value of the quadratic loss function:

$$\sum_{t=1}^{\infty} R \left[a_1(M_t-M_t^d)^2 + a_2(M_t-M_{t-1})^2\right] \quad , \tag{6.11}$$

with respect to actual real balances, M_t. R is a known and constant discount factor. Setting the first derivative of this expected present value with respect to M_t to zero yields Kennan's first-order optimality condition. With suitable modification and the assumption that:

$$E_t(\Delta M_{t+1}) = \Delta M_{t+1} , \tag{6.12}$$

the optimality condition reduces to

$$\left[1 - \frac{(1+a+R)L}{R} + \frac{L^2}{R}\right] M_{t+1} = \frac{-a}{R} M_t^d , \tag{6.13}$$

where L is the backward operator, and a is a_1/a_2.

If λ is one root of the quadratic operator for L in (6.13), then it can be shown [Kennan (1979, p.1443)]

$$(1-\lambda L)M_t = (1-\lambda)\frac{(1-\lambda R)}{(1-\lambda RL^{-1})} M_t^d , \qquad t \geq 2. \tag{6.14}$$

Assuming that the long-run desired level of real balances, $\overline{M}_t$, is given by

$$\overline{M}_t = \frac{(1-\lambda R)}{(1-\lambda R L^{-1})} \; M_t^d \; ,$$

allows us to write

$$M_t = M_{t-1} + (1-\lambda)(\overline{M}_t - M_{t-1}). \tag{6.15}$$

This is recognized as the partial adjustment mechanism described in section 5.1. In short, the optimal adjustment path (6.13) is shown here to satisfy the partial adjustment rule.

In general, the long-run desired level of real balances, $\overline{M}_t$, will differ from the short-run target, M_t^d. This is accomodated by combining the optimal adjustment path (6.13) with structural model (6.10). Setting the discount rate R at unity, for convenience, yields the general reduced form:

$$M_t = \beta_0 + \beta_1 \, y_t + \beta_2 \, r_t + \beta_3 \, M_{t-1} + \beta_4 \, M_{t+1} + v_t \; , \tag{6.16}$$

where $\beta_0 = a\alpha_0/(2+a)$, $\beta_1 = a\alpha_1/(2+a)$, $\beta_2 = a\alpha_2/(2+a)$, $\beta_3 = 1/(2+a)$, $\beta_4 = 1/(2+a)$, and $v_t = a \, u_t/(2+a)$.

With static expectations, $E_t(M_{t+s})=E_t(M_t)=M_t$ for all s. In this case, the long-run target, $\overline{M}_t$, is indistinguishable from the short-run target, M_t^d. Substituting M_t^d for $\overline{M}_t$ in (6.15) then yields the reduced form model:

$$M_t = \gamma_0 + \gamma_1 y_t + \gamma_2 r_t + \gamma_3 M_{t-1} + w_t \quad , \tag{6.17}$$

where $\gamma_0 = (1-\lambda)\alpha_0$, $\gamma_1 = (1-\lambda)\alpha_1$, $\gamma_2 = (1-\lambda)\alpha_2$, $\gamma_3 = \lambda$ and $w_t = (1-\lambda)u_t$.

While, in principle, models (6.16) and (6.17) can be seen as special cases of a more general model, it is found that the traditional significance tests cannot be applied here because of strong autocorrelation in the residuals from (6.16) and the general model. The unconstrained model, when estimated for data extending from 1956 II to 1977 III, has a Durbin h statistic of -3.86 and a DW statistic of 2.76. Model (6.16), when estimated over the same period, shows a Durbin h of -4.73 and a DW of 2.73. Model (6.17), however, rejects first-order autoregression with a Durbin h of 1.37. A similar result is obtained for five subsamples.

While the theoretical derivation of the test is sound, in application it appears less than satisfactory. This is due, in part, to our failure to observe expectations directly. For example, to make the above test operational, Kennan's adjustment path is transformed to (6.13) with the assumption that $E_t(\Delta M_{t+1}) = \Delta M_{t+1}$. Within Muth's rational world such may be the case, but it is not at all clear that this should be so when expectations are static. Direct tests of expectation formation when the expectations variable itself must be proxied must always be subject to doubt.

6.3. Concluding Comments

As in most areas of economic research, several theories abound as to the nature and origin of the demand for money. In the foregoing chapters of the present work I have sought to identify the prevalent issues which have been open to disagreement in the literature. These have been stability, functional form, causality and dynamics. The present chapter deals with two issues which may be viewed as distinct from one another, except that both are examined here under the general theme of competing theories.

Different theories as to the motives for holding money yield different views as to the source of recent instability in demand for money equations. The transactions approach points to the possibility that instabilty has arisen in the demand for cash by firms, while the portfolio approach suggests misspecification in the opportunity cost variables of conventional equations. The foregoing empirical analysis both supports the transactions view that the demand for CAs has shifted, and rejects the portfolio view that functional instability is related ~~to the~~ absence of a spectrum of rates in the demand equation.

The unconstrained estimate of the M1 shift implied from component equations using AOW data from 1974:1 to 1978:9 was 11.7 per cent. This may be too high an estimate for reasons peculiar to the shortness of the sample period. As an illustration of possible imprecision in this estimate, constraints applied to this model which yielded an estimate of 3.2 per cent could not be statistically rejected. The lowest estimate of the M1 shift from Table 6.5 with an unconstrained early CA shift was 8.2 per cent. Similar equations when estimated with EOM data from 1968:12 to 1978: 9 produce estimates of the shift ranging from 4.2 per cent to 5.6 per cent. These numbers suggest a realistic confidence region for the shift in M1 to be from 4.2 to 8.2 per cent.

The second section of Chapter 6 deals with expectations. Competing theories may or may not be in conflict as to the relevance of expectations to the demand for money. While asset theorists agree among themselves that price expectations should be a determinant of money demand, transactions theorists are less in agreement. Whether the dispute is about relevance to the demand for money or about the formation of expectations, empirical resolution has been hampered by lack of observed expectations data. In spite of this, simplifying assumptions have been made in dealing with two expectations-related topics.

In subsection 6.2.A. an optimum autoregressive filter for prices was computed at each point in time for the period 1967 III to 1977 IV. The criterion for optimality was minimization of Akaike's final prediction error. It was seen that not only do the autoregressive coefficients change over time, so does the optimum order of autoregression. With expectations being formed in an 'economically rational' manner, then expectations formed in mid- to late-1970s were based upon a longer historical record than those formed a decade earlier.

In subsection 6.2.B. a procedure developed by Kennan (1979) to test for static versus rational expectations was adapted to the demand for money. While the theoretical derivation may be sound, its application proved unsuccessful. The lagging and leading dependent variables on the RHS of the more general models seemed to contribute to strong residual autocorrelation. It was concluded that, common to most empirical tests designed to choose between alternative expectations hypotheses, this procedure suffers from lack of observed data on price expectations.

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