Simultaneous stochastic optimization: Integrating waste management and feasible capital investments

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Abstract

A mining complex is a fully integrated logistics network that represents the transportation and transformation of material from the source, open-pit and underground mines, to the customers and/or the spot market. Mining enterprises around the world aim to create a strategic mine plan for each of their assets that maximizes the value generated for a company and its stakeholders. Simultaneous stochastic optimization is used to generate a production schedule that defines the extraction sequence, stockpiling, processing, blending, capital investment and waste management decisions under supply uncertainty. The optimization approach exploits synergies within the mining complex by considering the contribution of each interconnected component in a single mathematical formulation. These components may include multiple mines, processors, stockpiles, waste facilities, and methods of transportation. In this thesis, a study of simultaneous stochastic optimization is completed in two operating gold mining complexes focusing primarily on the integration of waste management and capital investment decisions under supply uncertainty. The appropriate timing of each capital investment can highly influence the net present value of an operation due to the large up-front costs, making it a necessity to properly time these investments. In addition, there are several benefits of integrating waste management into the production scheduling process including a conceptual understanding of material uncertainty, smaller environmental footprints, lower operating costs, and lower capital costs.

The first application presents the simultaneous stochastic optimization of a gold mining complex focusing on waste management, particularly the uncertain aspects of acid generating waste. Typically, when optimizing the production schedule, the primary focus is to deliver valuable products to the market. However, this tends to ignore the environmental and economic impact of simplifying waste management requirements, including the storage and disposal of waste material. Stricter regulations and engineering requirements are transforming past mining practices to develop more sustainable operations. These transformations increase the financial cost of waste management and identify the requirement to integrate waste management into the production schedule. Additionally, misrepresenting the material uncertainty and variability associated with the amount of waste produced can impact, both, the stakeholders and the profitability of a mining complex. In this case study, a simultaneous stochastic optimization approach is applied to generate a long-term production schedule that considers waste management. The resulting schedule leads to a 6% increase in the net present value when compared to a conventional approach, while

minimizing the likelihood of deviating from production targets and ensuring permit constraints are satisfied.

Second, an innovative strategic mine planning approach is applied to a multi-mine and multi-process gold mining complex that simultaneously considers feasible capital investment alternatives and capacity management decisions that a mining enterprise may undertake. The simultaneous stochastic optimization framework determines the extraction sequence, stockpiling, processing stream, blending, waste management and capital investment decisions in a single mathematical model. A production schedule branches and adapts to uncertainty based on the likelihood of purchasing a feasible investment alternative that may increase mill throughput, acid consumption, and tailings capacity. Additionally, the mining rate is determined simultaneously by selecting the number of trucks and shovels required to maximize the value of the operation. The mining complex contains several sources – two open-pit gold mines and externally sourced ore material – stockpiles, waste dumps, tailings and three different processing streams. The simultaneous optimization framework integrates the blending of sulphates, carbonates, and organic carbon at the autoclave for refractory ore while managing acid consumption. The resulting production schedule indicates an increase in net present value as the optimization model adapts to uncertainty and manages the technical risk of capital investment decisions.

Résumé

Un complexe minier est un réseau logistique intégré représentant le transport et la transformation de matériel partant d'une source (mine à ciel ouvert ou mine souterraine) allant jusqu'aux clients et/ou au marché au comptant. Les entreprises minières du monde entier créent un plan à long terme visant à maximiser valeur de leurs actifs pour leurs parties-prenantes. L'optimisation stochastique simultanée génère un horaire de production définissant la séquence d'extraction, le stockage du minerai, le traitement, le mélange de minerai, les investissements en capital, et la gestion des résidus miniers tout en prenant en compte l'incertitude géologique du gisement. Cette approche prend avantage des synergies qui existent dans un complexe minier en combinant la contribution de chaque composante interconnectée en une seule formulation mathématique. Les composantes peuvent inclure plusieurs mines, installations de traitement de minerai, stockages de minerai, haldes à stériles, et modes de transport. Dans cette thèse, une étude d'optimisation stochastique simultanée est appliquée à deux complexes miniers d'or avec un accent sur la gestion de résidus miniers et les investissements en capital sous incertitude. L'échéancier des investissements en capital peuvent influencer la valeur nette actuelle d'une exploitation minière due aux coûts initiaux élevés. De plus, l'intégration de la gestion des résidus miniers au processus de planification peut améliorer la compréhension conceptuelle de l'incertitude présente dans les matériaux miniers, réduire l'empreinte écologique de l'entreprise, réduire les coûts opérationnels, et réduire les coûts en capitaux.

Dans la première application, l'optimisation stochastique simultanée d'un complexe minier d'or, avec un accent particulier sur la gestion de résidus miniers et son potentiel de génération d'acide, est examinée. Généralement, l'optimisation de la séquence d'extraction vise à livrer des produits valables aux clients et au marché au comptant. Toutefois, ceci ignore les impacts environnementaux et économiques associés à la simplification des exigences de la gestion des résidus miniers, telles que l'entreposage et l'élimination du matériel. La réglementation et les exigences techniques de plus en plus strictes requièrent un changement des pratiques minières traditionnelles, poussant l'industrie à développer des exploitations durables. Ces changements augmentent le coût associé à la gestion des résidus miniers et souligne le besoin d'incorporer cet aspect au processus de planification minière. De plus, mal représenter l'incertitude et la variabilité associée au contenu et au montant de résidus produits peut avoir un impact sur les parties-prenantes ainsi que sur la marge de profits d'un complexe minier. Cette étude de cas applique l'optimisation

stochastique simultanée afin de produire une séquence d'extraction à long terme qui prend en compte la gestion des résidus miniers. La séquence qui en résulte démontre qu'un gain de 6% en terme de valeur nette actuelle est possible lorsque nous comparons cette méthode à la méthode traditionnelle, tout en réduisant la probabilité de manquer les objectifs de production de l'exploitation et s'assurant que les contraintes misent en place par les permis d'opération soient satisfaites.

Dans la deuxième application, une approche innovatrice à la planification minière à long terme, qui prend en compte les alternatives viables d'investissements en capital ainsi que les décisions de gestions de capacité qui devront se faire au cours de l'exploitation, est appliquée à un complexe minier d'or composé de plusieurs mines et installations de traitements de minerai. Cette approche définit les décisions à prendre en termes de séquence d'extraction, stockage de minerai, installations de minerai à utiliser, gestion de résidus miniers, et investissements en capital. Une séquence d'extraction peut se bifurquer et s'adapter à l'incertitude du gisement selon la probabilité d'achat d'une alternative viable d'investissement en capital qui pourrait augmenter la capacité d'une installation de traitement de minerai, la consomption d'acide, et/ou la capacité de la digue à résidus. De plus, le taux d'extraction est déterminé de manière simultanée puisque le model permet de choisir le nombre de camions et de pelles requises afin de maximiser la valeur de l'exploitation. Le complexe minier inclut plusieurs sources – deux mines d'or à ciel ouvert ainsi que du matériel provenant d'une source externe – aires de stockages, haldes à stériles, digues à résidus, et trois différentes installations de traitement de minerai. Le modèle d'optimisation simultanée incorpore le mélange de sulfates, carbonates, et de carbone organique du minerai réfractaire à l'autoclave tout en contrôlant l'utilisation d'acide. La séquence d'extraction qui en résulte démontre qu'une augmentation de la valeur nette actuelle est possible puisque le model s'adapte à l'incertitude et gère la risque technique associé aux investissements en capital.

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1. Introduction and literature review

A mining complex is an integrated logistics network designed to extract materials from in-situ mineral reserves and transform them into valuable products that generate a profit for a mining enterprise and its stakeholders (Pimentel et al. 2010; Montiel and Dimitrakopoulos 2015; Goodfellow and Dimitrakopoulos 2016). The projects undertaken by a mining enterprise require large capital investments and careful planning to successfully operate a mining complex. Uncertainty in terms of the supply of material from the mines, commodity pricing, technical and environmental issues cause challenges in forecasting the projected value of a mining complex, inherently making mining operations risky ventures to undertake. In particular, supply uncertainty and local variability are the primary reasons for not meeting production targets and can also lead to environmental consequences. Several studies including those undertaken by Baker and Giacomo (1998) show that the major reason for not meeting production targets is misunderstanding the reserves/resources due to uncertain material supply, out of 48 Australasian mining projects 13 of them had 20% more and 9 had 20% less then the projected reserves. Similarly, in a World Bank survey undertaken in Canada and the US, Vallee (2000) shows evidence of significant deviations between the published ore reserve estimate and the realized outcome for the first year of operations further stressing the need to account for supply uncertainty. Nearly 73% of the mining projects in this study were closed prematurely due to problems in their ore reserve estimates. Since these publications were released, several studies have discussed the topic (Dimitrakopoulos 2011). These failures highlight the importance of understanding the geological representation of the deposit and the expected material supply demonstrating most projects fail for this reason, not market price, late delivery, equipment uncertainty, etc. These are some of the critical reasons that recent advancements have taken place in the simultaneous stochastic optimization of mining complexes, which will be discussed in depth throughout this thesis.

Simultaneous stochastic optimization generates a production schedule for an industrial mining complex, which includes all the components that are required to generate valuable products from mineral reserves. These components may include multiple mines, external sources, stockpiles, autoclaves, oxide mills, refractory ore mills, waste facilities, ports, and various transportation mechanisms (Hoerger et al 1999; Whittle 2007; Pimentel et al. 2010). Mathematically, a mining complex is formulated to model both the linear and non-linear transformation of materials that occur between the source, underground and open-pit mines, and the final products that are delivered to the commodity market or disposed of as waste (Montiel and Dimitrakopoulos 2015; 2017; 2018;Goodfellow and Dimitrakopoulos 2016; 2017). The primary objective of modelling a mining complex in this manner is to simultaneously determine the annual long-term production schedule including the extraction sequence, stockpiling, processing, waste management, blending and capital investment decisions that will maximize the net present value (NPV) of a mining project, while satisfying a set of operational and environmental constraints and managing technical risk. Stochastic simulations of the raw material supply are integrated into the optimization allowing the optimizer to directly manage supply uncertainty and variability. Optimizing all components of the mining complex simultaneously improves the operations ability to capture synergies between the different components that help manage risk and increase project value. This coined the term, 'strategic mine planning' and since the early 1990s it has become a clear advantage to holistically consider several component of the mining complex during the optimization process (Hoerger et al. 1999).

The remainder of this chapter will cover fundamental literature in strategic mine planning including early advancements in global optimization, geostatistical simulation methods for quantifying supply uncertainty, and the simultaneous stochastic optimization framework.

1.1. Deterministic global optimization approaches in a mining complex

The conventional approach for determining a single mine's long-term production schedule assumes an exact understanding of geological boundaries and their grade distribution as well as deterministic or fixed mining costs and recoveries that are used for evaluating the NPV of an operation. Conventional approaches in open-pit mine planning locally optimize various components of a mine using a sequential framework that first determines the optimal cut-off grade (Lane 1964). Then, Lerchs-Grossman (1965) algorithm is employed to determine the ultimate pit limit by maximizing the cumulative discounted cashflow of each mine individually. Pushbacks or phases are defined within the ultimate pit limit using a similar process that accounts for feasible mining widths (Hustrulid and Kutcha 2006). Finally, within each pushback an annual production schedule is defined. The main drawback of this approach remains in the use of deterministic inputs and parameters which vary significantly in reality, further discussed in Section 1.2, and the downfall of independently optimizing the various components of the mining complex, which will be addressed in this section. Each critical business decision is optimized locally, or independently, disregarding the behaviour of downstream (or upstream) decisions and supply uncertainty. For example, a production schedule for a mining complex often involves several mines that are each optimized independently of other mines, however, there may be opportunities to blend materials and evaluate the available synergies by optimizing them together. Then the metallurgical group optimizes several mineral processing aspects on its own accord, tuning the processors demand and quality requirements but, disregarding the ability to change the production schedule to gain further value. For example, blending materials of different qualities can improve the recovery at certain processing facilities and potentially lead to higher profitability. Afterwards the transportation of products to the market is optimized based on the expected supply of products from each processing facility and the market demand. Capital expenditure planning is also a critical strategic mine planning decision and can lead to expansions of the mining and processing capacities through the acquisition of equipment and the construction of new facilities. Capital investment decisions are optimized locally through various testing strategies and sensitivity analysis. The schedule must be repetitively reoptimized to consider each capital investment, making for an exhaustive evaluation process. A waste management plan is also assembled to sequence waste placement, ensuring future reclamation and environmental constraints are not violated. The conventional optimization process is completed in a step-wise framework where the interconnectivity between these components are not actively considered, ensuing a sub-optimal decision-making process for the mining complex. These are several drawbacks of conventional mine planning which, highlight the advantage of optimizing all the components in a mining complex simultaneously.

Newmont's Nevada Operations recognized that independently optimizing each mine and their components limited their ability to maximize the value of their assets (Hoerger et al. 1999). Several operations, in the Carlin Trend, could share processing capacities, mining capacities, and blend materials of various qualities providing opportunities to capture major synergies between these operations. Although, there was a desire to model each of their assets using a simultaneous approach they could not fully solve the extremely large multifaceted problem, even to this day. Hoerger et al. (1999) describes the development of a mixed integer linear program that simultaneously optimizes the mining of multiple pits with several material types and the processing of ores through multiple plants in the Carlin Trend area. The model aims to maximize the NPV of the mining complex by determining the optimal flow of materials from the mine sources, i.e. pushbacks and underground mine subdivisions, to stockpiles and then from stockpiles to processing plants. Costs for each of the allowable material movements are directly incorporated into the optimization model, detailed in the work of Urbaez and Dagdelen (1999). However, there are several limitations of this model: i) mining blocks are grouped into pushbacks that are predefined based on their metallurgical properties, reducing the granularity of the problem and potentially leading to infeasible mine plans; ii) Hoerger et al. (1999) discusses the need to incorporate the timing of capital expenditures, although, it has not been directly integrated into the optimization process itself; iii) it assumes there is a fixed annual production schedule describing the precedence relationships; iv) can not deal with non-linearities in stockpile destinations and operational modes; and finally v) the method ignores supply uncertainty and variability in material sources. However, the approach described is completed in a single optimization formulation unlike the step-wise approach described subsequently.

Similarly, Whittle (2007) identifies the need to jointly optimize several components of a mining complex including multiple pits and underground mines, stockpiles, blending elements and processing routes substantially increasing the challenge of solving the long-term production schedule. The global asset optimisation framework commences by using a pre-determined grouping of nested pit-shell, for each asset, that satisfy the stripping ratio and are prioritized based on economic value (Whittle 2014). Then, mining blocks are aggregated into panels that are grouped on similar grade attributes, a priori, and segregated into grade bands. All becoming a fixed production schedule. Aggregating and pre-determining the pushbacks and ultimate pit limits reduces the number of decision variables that must be considered throughout the optimization process, simplifying the approach. Panels and underground sequences are scheduled together to find an overarching mine plan that will satisfy processing and blending constraints. In addition, the method can model more advanced material transformations including the throughput and recovery relationship for each type of ore at each processor and jointly determines the optimal cut-off grade during the optimization process. The solution approach begins by repeatedly creating random feasible extraction sequences followed by a linear solver that determines the downstream components, i.e. processing, blending and transportation decisions (Whittle 2010; 2014). Although global considerations are made, the method is locally or independently optimizing certain parts of the mine production schedule and eliminating the ability to capture synergies between different components. For example, locally optimizing the cut-off grade decisions to maximize the utilization of the processing plants may lead to more metal production in a gold mine but, frequently a higher NPV can be generated by not maximizing throughput at one or more destinations and instead focusing on maximizing the cashflows and integrating the cut-off into the optimization process. Additionally, several shortcuts are taken including independently determining the ultimate pit limits and pushbacks, manually creating panels, and allowing for fractional extraction of mining blocks. The limitations of panels include allowing for fractional extraction which, assumes the average contribution of all the blocks within a panel will be obtained even if a fraction of the panel is extracted in an operating period. In addition, panels represent benches in a phase and must be mined completely before progressing to the next one (top to bottom. Therefore, a bench-by-bench extraction of each phase is commenced leading to worse case mining. Each of these shortcuts are taken because the computational requirements to formulate a single mathematical program is extremely large and challenging in a reasonable amount of time. Granted, the advancements addressed by Whittle (2007) and Whittle (2010; 2014) provide a commercialized framework that considers substantially more interconnectivities than a conventional optimization approach.

Stone et al. (2007) present BHP's mine planning tool Blasor. Developed by the BHP Billiton Technology division, the MIP formulation maximizes NPV over the life of operation by sequencing multiple pits and determining the ultimate pit limits using a commercial solver. Spatially connected blocks with similar material properties are aggregated to reduce the number of integer decision variables in the mathematical programming model. In addition, the materials are not classified as ore or waste a priori, as the optimizer makes decisions on how to blend the material extracted from the pits to produce marketable ore. Material in the pits are assigned to bins based on chemistry. After blocks are aggregated, the optimization tool generates the optimal extraction sequence and ultimate pit limits. Then, mining phase design is performed on each pit independently and panels are generated based on the intersection of a mining phase and bench. Panels are represented as the total tonnage of each attribute in each bin. The optimal extraction sequence of the panels is calculated in the same way as the aggregates. The process is used to minimize the compromise made by constructing mineable phases and to prevent "rat-holing". Blasor was specifically designed to jointly optimize eleven pits and blending requirements at the Yandi Joint Venture in the Pilbara region, which is more challenging to solve than the sequential approach used to generate the ultimate pit limit for a single mine due the interdependency of blended materials. Zuckerberg et al. (2007) extend the use of Blasor to consider waste handling, in particular, in-pit dumping, for operations where space available outside the pit limits may be limited. This may also be required at some operations due to other environmental requirements. The Blasor-InPitDumping (BlasorIPD) model simultaneously decides the extraction sequence for each mine and the location where waste material can will be placed, while satisfying blending and capacity constraints. BlasorIPD works similar to the standard version of Blasor but, considers the movement of waste from the mine to the haulage network and from the road to either a final waste dump or in-pit waste location by tracking the previously extracted blocks and available locations for dumping waste material. The optimization formulation ensures that material will not be dumped in-pit if there is ore beneath the dumping location and the in-pit dumping will not violate the waste repose slope constraints.

Integrating waste management into the production scheduling process is a critical measure for successful open-pit mining operations. Ben-Awuah and Askari-Nasab (2013) attempt to integrate waste management into the long-term production schedule for oil sand mining operations. In oil sand operations, inpit and expit dykes are created to store the tailings produced during the extraction of bitumen products. The aim of the model is to determine the optimal extraction sequence of ore, dyke and waste material that maximizes the NPV and minimizes the cost of constructing dyke material storage areas. Major limitation of the model is it uses the pre-determined ultimate pit limits and groups block into aggregates; however, it does allow for the sequencing of in-pit tailings while considering the required amount of dyke construction for tailings disposal and the costs incurred. Fu et al. (2019) develop a more in depth approach of waste management, where not only the quantities are described but the exact placement of waste material is optimized directly in the optimization framework. This overcomes past frameworks that given a production schedule, a waste dump schedule is then optimized subsequently to determine the optimal destination for waste dump disposal leading to a sub-optimal mine plan. However, the optimization framework assumes a block-based economic value and allows for the fractional extraction of mining blocks. Fractional extraction is allowed based on the continuous variables $x_{b,i,m,t}$, $x_{b,i,s,n_s,t,}$, and $x_{b,i,e,d,t}$ which are used in the formulation and represent the amount of material extracted from the mine and sent to the mill m, dump d, stockpile s from block b in period t. Different proportions of a block can be sent to several destinations as long as the whole block is mined; this is a long-recognized limit for strategic mine planning optimization, as it misrepresents mining selectivity, thus generating overoptimistic results. Lastly, the approach is used on a very small instance that considers only 4 periods and 1688 mining blocks in 24 minutes and generally long-term production scheduling for large operation is completed with greater than 1 million block-based decisions, making it a slow decision-making process.

These global optimization approaches attempt to jointly optimize many components of the mining complex, but it is clear there are still a variety of performance inhibiting short-cuts that are either taken to speed up the optimization process or that have not been successfully integrated into the optimization framework. In addition, the integration of supply uncertainty into the optimization framework is infeasible and there is no way to hedge the risk of highly variable and uncertain material within each mine. There also has been minimal effort to incorporate capital expenditures directly into the optimization model.

1.2. Modelling supply uncertainty and spatial variability

The uncertainty and spatial variability of the mineral deposit, or the material supply, is the largest contributor of risk in a mining complex (Baker and Giacomo 1998; Vallee 2000). Therefore, the uncertainty and spatial variability must be quantified and managed when determining the production schedule, as it directly influences the cashflows and profitability of a mining enterprise (Ravenscroft 1992; Dowd 1994; 1997;Dimitrakopoulos et al. 2002).

Orebody models are used to represent the available supply of material when optimizing a mining complex. These models represent the spatial distribution of the quality and quantity of material available for extraction at each mine. Stochastic simulations can be generated that reproduce the critical statistics, variogram and histogram models, including the spatial variability, mean and variance of the mineral deposit by generating a number of equally-probable stochastic orebody models. These simulations can be used to measure the impact of supply uncertainty by using transfer functions to simulate the flow of material through the mining complex and, then, observe the resulting impact on the key performance indicators (Dimitrakopoulos et al. 2002). Traditionally, estimated orebody models are used to represent the supply of material to the mining complex. Estimation approaches find the average of the possible grade values in a certain location, but they tend to smooth the local variability of the material properties (Journel and Huijbregts 1978; David 1988). For example, if the characteristics of kriging are considered (Isaaks and Srivastava 1989; Goovaerts 1997; Rossi and Deutsch 2014), naturally low-grades are overestimated and high grades are underestimated misrepresenting the proportions of material that exist within a mineral deposit. Consequently, the average input of the material supply may not provide an average assessment of the long-term production schedule in the presence of supply uncertainty. The risk of meeting the expected production targets using a production schedule generated from an estimated orebody model has been demonstrated in the work of several researchers (Ravenscroft 1992; Dowd 1994; Dimitrakopoulos et al. 2002).

Supply uncertainty arises from imperfect knowledge of mineral deposits largely due to the sparse amount of geological information available and the modelling techniques used to quantify the materials available for extraction and subsequently processing (Goovaerts 1997; Godoy 2002). Geological information is gathered through exploration campaigns (i.e. diamond and reverse circulation drilling, trenching, and geophysical techniques) and this data is used to characterize a model of the orebodies. Simulations of the pertinent material properties are generated and spatially represented in an orebody model as a block. The orebody models and all the corresponding blocks are used as input into the subsequent simultaneous stochastic optimization frameworks to manage supply uncertainty directly during the optimization. The remainder of this section explains the foundations of simulation methods and explores several common simulation approaches.

1.2.1. *Sequentially simulating univariate and multivariate mineral deposits*

There are a number of simulation methods that can be used for different purposes including simulating continuous and discrete variables (Journel and Huijbregts 1978; David 1988; Journel and Alabert 1989; Goovaerts 1997; Boucher and Dimitrakopoulos 2009; Remy et al. 2009; Rossi and Deutsch 2014). Each simulation method must be able to reproduce the model statistics this means the: (i) data values; (ii) histogram; and (iii) spatial correlation of the original data (Goovaerts 1997). The most common class of simulations algorithms are known as sequential simulation algorithms. These models use a conditional cumulative distribution function (ccdf) which is modeled and sampled at each node ($n \in N$) visited along a random sequence and conditional to, both, the original data points and the simulated values in the distribution. Isaaks (1991) uses different random paths for visiting each node to reduce the likelihood of creating artificial continuity and retrieving similar simulations by taking the same path during each simulation.

Sequential Gaussian simulation (SGS) is based on the multiGaussian random field model. The Gaussian framework is used because all conditional distributions are Gaussian and kriging provides the estimated Gaussian mean and variance (Rossi and Deutsch 2014). This simulation approach is most common in the mining industry and is integrated into a range of commercial mining packages for geologists. A summarized description of the SGS algorithm follows:

1) Apply a normal score transform to the sample data to obtain the Gaussian distribution
2) Compute and model the variogram, covariance, or correlogram for the normalized model
3) Define a random path that visits each node of the grid representing the deposit
4) Estimate using simple or ordinary kriging of the normalized value at the selected node using the sample data and previously simulated grid nodes to estimate the normal local conditional distribution
5) Simulate the value by sampling the estimated normal local conditional distribution
6) Add the simulated value to the conditioning data for nodes to be simulated later and move to the next node
7) Repeat the process until all nodes are simulated
8) Back-transform the Gaussian values to the original data space
9) Validate the results by checking the reproduction of the data

A limitation of using SGS is the values show less connectivity due to the maximum entropy property associated with a Gaussian distribution, meaning it provides a highly 'disorganized' spatial arrangement. In addition, SGS can be computationally expensive requiring a significant amount of time to create a set of realizations for a large deposit (Luo 1998).

Davis (1987) introduces a conditional simulation technique performed using lower-upper (LU) triangular decomposition which uses vector processing capabilities. Drawbacks still exist in terms of the amount of memory required (Luo 1998) but, the simulation and conditioning are performed together and can be applied to an arbitrary covariance structure speeding up the process. Luo (1998) proves the decomposition algorithm is equivalent to SGS and then develops a generalized sequential gaussian simulation (GSGS) method. GSGS overcomes many of the limitations of SGS as it takes advantage of the large and dense set of nodes that are required to be simulated and considers sharing neighborhood searches and kriging operations at adjacent nodes, overcoming the node-by-node sequential process in SGS. Equivalent to SGS, GSGS is a sequential process but instead of going node by node it is performed group by group. For definitions sake, N is the total number of nodes to be simulated and v is the number of the nodes in the neighbourhood. Dimitrakopoulos and Luo (2004) explore how to size the neighbourhood of the GSGS method noting that if the group size of the neighbourhood v=1 then the approach is equivalent to SGS where as if v=N then it is identical to the LU decomposition method. In order to optimize, a balance must be found between computational efficiency and the precision of the results. This is accomplished by using the screen-effect approximation loss; the mean square difference between the simulated value using the information.

Godoy (2002) demonstrates a sequential conditional simulation method direct block simulation (DBSIM). The method overcomes many of the computational limitations of SGS for simulating continuous attributes by directly simulating at block support instead of point support, discarding the internal points of simulated blocks reducing memory allocation and simulating using groups as in GSGS. Stochastic optimizers require multiple realizations of the supply uncertainty and large deposits can have millions of nodes that require simulation. By simulating at block support the DBSIM algorithm eliminates the need for reblocking to a selective mining unit which is required as input for stochastic optimization (Dimitrakopoulos and Ramazan 2004; Leite and Dimitrakopoulos 2014; Montiel and Dimitrakopoulos 2015; 2018;Goodfellow and Dimitrakopoulos 2016; 2017).

Geological phenomena frequently have multiple attributes of interest. The additional information obtained by sampling and simulating multiple attributes is vital for maximizing the value of a mining complex. For example, the impact of integrating secondary elements, managing deleterious elements or minimizing the impact of hazardous waste. In order to integrate these attributes into the optimization model, the simulation process must be able to jointly consider all the attributes and model their correlations. The more information that is available regarding the supply of material the closer the mining complex can perform to reality. A class of simulation methods, known as multivariate, are used to simulate multiple variables of interest while preserving any spatial cross-correlations (Journel and Huijbregts 1978; David 1988). David (1988) first used PCA (primary component analysis) to co-simulate correlated variables, however, the number of variogram and cross-variograms models that were required was large. Desbarats and Dimitrakopoulos (2000) apply the minimum/maximum autocorrelation factors (MAF) to simulate multi-variate orebodies by: (i) decorrelating the attributes of interest; (ii) independently simulating these attributes; (iii) and then back transforming simulated values to the correlated space.

Eliminating the need to model cross-variograms or simulate each attribute using a series of univariate simulations. Furthermore, Boucher and Dimitrakopoulos (2009; 2012) average point support scale values, similar to the DBSIM algorithm, to jointly simulate multivariate datasets directly at block support using a multistage process. The method termed DBMAFSIM performs a linear transformation to the original multivariate vector into a new set of independent MAF to reproduce the statistics of multiple spatially correlated attributes. For example, consider a stationary and ergodic non gaussian vector of k random functions $Z(u) = \{Z^1(u), \ldots, Z^k(u)\}$ at each location u and measured at point support scale. A normal score transformation is performed on the vector to move to Gaussian space, $Y(u) = \left\{ \phi_1\left(Z^1(u)\right), \ldots, \phi_k\left(Z^k(u)\right)\right\}$. MAF are defined as a random field, $M(u) = \{M^1(u), \ldots, M^k(u)\}$ where the k RFs are independent and obtained from vector $Y(u)$ using the coefficient matrix A:

$$M(u) = A^T Y(u) \qquad [1]$$

The MAF, $M(u)$ are linear combinations of the multiGaussian vector $Y(u)$. The derivation of A is equivalent of performing two successive PCA decompositions. This is all completed at block support, however, similar to DBSIM these can be upscaled directly to block support. After, the variables to be jointly simulated are transformed and the MAF are simulated independently. Then, direct simulation at the block support scale (V) with MAF is based on the up-scaled vector

$$Z_v(x) = \frac{1}{N} \sum_N \phi^{-1}\left(A^{-T} M(u)\right) \qquad [2]$$

Jointly simulating multi-variate mineral deposits improves the ability to properly understand the behaviour of secondary elements in a multi-element deposit, such as, a copper-gold porphyry deposit (Goodfellow et al. 2012; Kumar and Dimitrakopoulos 2019). In addition, the geochemical properties of hazardous contaminants that may lead to acid mine drainage are often caused by the presence of attributes that can be simulated. Simulating these attributes, specifically carbon and sulphur grades associated with the mineralization of carbonates and sulphides, is useful for modelling the quantities of acid generators that could violate the environmental constraints in a mining complex (Dold 2008; Kumral and Dimitrakopoulos 2008). There have also been various case studies where material uncertainty plays a critical role in blending operations, for example, iron ore quality requirements and gold autoclaving operations (Spleit and Dimitrakopoulos 2017; Montiel and Dimitrakopoulos 2018; Vallejo and Dimitrakopoulos 2018).

1.2.2. *Multi-point and high-order simulation methods for simulating spatially complex non-linear and non-Gaussian geological phenomena*

The previous geostatistical simulation techniques are based on the reproduction of second-order statistics, which satisfy the requirements for modelling multiGaussian random functions. However, there are limitations when simulating non-linear and non-Gaussian geological phenomena that have complex non-linear spatial patterns (Guardiano and Srivastava 1993; Journel 1997; 2002). These geological formations are often found in mineral deposits, as they are shaped based on natural means of deposition or other geological processes, and there are opportunities to use the data retrieved through production and exploration sampling to infer high order statistics and continuities. Therefore, multi-point simulation frameworks have been presented to simulate both continuous and categorical random functions that can spatially reproduce these structures, unlike traditional two-point statistical methods (Guardiano and Srivastava 1993; Strebelle 2002; Journel 2003; 2005;Zhang et al. 2006; Remy et al. 2009).

Multi-point statistical frameworks have been used by the oil and gas industry for decades to reproduce curvilinear structures, i.e. sand channels in clastic reservoirs (Strebelle 2002). Moving away from the traditional random field simulation frameworks, multi-point methods extract patterns from a training image (TI) a geological representation of the attribute of interest and its spatial data distribution to simulate categorical (Strebelle 2002; Zhang et al. 2006; Strebelle and Cavelius 2014; Mariethoz and Caers 2015) and continuous variables (Guardiano and Srivastava 1993; Zhang et al. 2006; de Iaco and Maggio 2011; Mariethoz and Caers 2015). Each TI does not necessarily carry any locally accurate information on the geological phenomena but instead reflects the critical spatial structures and geological behaviours of the simulated attributes. Training images can be generated using various object based algorithms, simulated realizations of an analogous field, or a geologists inference of the specific shapes (Strebelle 2002). In mining, examples include the use of dense grade control data to generate training images which act as supplemental data that honours the geological behaviour and works in tandem with the exploration data while simulating the deposit (Osterholt and Dimitrakopoulos 2007). There are several algorithms that use TIs including ENESIM (Guardiano and Srivastava 1993), SNESIM (Strebelle

2002; Strebelle and Cavelius 2014), FILTERSIM (Zhang et al. 2006), Direct Sampling (Mariethoz and Renard 2010), WAVESIM (Chatterjee and Dimitrakopoulos 2012) and IMPALA (Mariethoz and Caers 2015).

A major limitation of using a TI is they are computationally expensive to produce, particularly for jointly simulating multivariate deposits, and the data required to build accurate models is frequently unavailable. Moreover, multi-point methods derived from training images are occasionally unable to reproduce the statistics of the exploration data due to the heavy focus on TI patterns, resulting in simulations that reproduce the training image but, not necessarily the statistics of the hard exploration drill hole data. These shortcomings become more interpretable when observing the behaviour in the dense data sets of mining operations where there are certain challenges including reproducing the statistics of particular areas due to widely spaced drilling and reproducing the variability of hard data due to overfitting on spatial behaviour of the TI (Osterholt and Dimitrakopoulos 2007; Goodfellow et al. 2012). The previous approaches replace the two-point variogram model with a TI to account for spatial connectivity or high-order dependencies in the geological models.

Dimitrakopoulos et al. (2010) introduce a high-order simulation (HOSIM) method to reproduce non-linear and non-Gaussian high-order geostatistics using spatial cumulants, a spatial connectivity measure, as an alternative approach to reproduce the complex non-linear and non-Gaussian geological features. The mathematical definitions of non-Gaussian spatial random functions and their high-order statistics are presented including the use of anisotropic experimental cumulant calculations using spatial templates. Cumulants by definition are a combination of statistical moments and are derived from the logarithm of the moment generating function. The work presented goes over the definition of spatial cumulants and an understanding of the relationship between the cumulant characteristics and in-situ behaviour of the geological process. Then, Mustapha and Dimitrakopoulos (2010) present a stochastic HOSIM method that uses high-order Legendre polynomials to approximate the conditional distribution. The coefficients of the Legendre polynomials are calculated from the cumulants population. This provides a framework that reproduces the spatial statistics of multiple points as an ensemble and generates more accurate realizations of complex geological patterns and the distribution of categorical variables. The training image is only used to complement the simulation when there is insufficient data available.

Minniakhmetov and Dimitrakopoulos (2017) extend this framework to jointly simulate multivariate deposits and categorical data. Similarly, a data driven high-order simulation that approximates high-order spatial indicator moments was developed (Minniakhmetov and Dimitrakopoulos 2017). The high-order spatial moments proved that higher-order statistics are connected with lower-orders based on boundary conditions. Therefore, a recursive B-spline approximation algorithm could be used to reproduce the high-order statistics from the hard data to improve computational issues with Legendre polynomials. The data-driven approach can simulate without a training image and still reproduce the high-order statistics of hard data. Additionally, there is the option to simulate using a training image for sparse datasets. de Carvahlo et al. (2017) extend the work of Minniakhmetov and Dimitrakopoulos (2017) by directly simulating at block-support, which minimizes the memory requirements and improves the efficiency of simulating at point support by considering neighbourhoods as explained in Godoy (2002) and Boucher and Dimitrakopoulos (2009; 2012).

Applications of high order and multi-point statistic methods are observed in several mining case studies. Minniakhmetov et al. (2018) test a HOSIM approach using Legendre-like orthogonal splines in a gold deposit. Three different systems of functions for HOSIM are compared to the sequential gaussian simulation, Legendre like splines and Legendre polynomials using order 10 and 20. The HOSIM using Legendre like splines shows stable reproduction of spatially connected structures, whereas the polynomial approach is less connected. The HOSIM with Legendre like splines demonstrates the advantages of HOSIM by improving the reproduction of the spatial distribution of grades and the continuity of high grades. Vallejo and Dimitrakopoulos (2018) simulate the boundaries of geological domains using WAVESIM, an algorithm developed by Chatterjee et al. (2012), overcoming the limitations of SNESIM and FILTERSIM. WAVESIM begins by classifying a pattern database and then the prototypes of the classes are calculated. These prototypes are compared with the conditional data event and the class with the highest similarity is assigned. A Monte-Carlo type sampling approach is commenced on the cumulative probability density function to determine the class of the central node. Furthermore, de Freitas Silva (2016) perform a similar application of simulating the boundaries of the geological zones of a nickel-laterite deposit using an unwrinkling process and then jointly simulating the attributes of interest using DBMAFSIM within the simulated domains, providing a framework that is capable of quantifying geological uncertainty and spatial variability. Finally, de Carvahlo et al. (2017)

generates high-order simulations using Legendre-like splines at block support for a gold deposit. A case study is completed which reports that the simulation approach increases the connectivity of high grades and reproduces the complex behaviour of the mineral deposit. The final outcome results in a noticeably different optimal mine extraction sequence.

Multi-point and high order simulation frameworks provide methods to reproduce non-Gaussian and non-linear spatial patterns, such as, the connectivity of high-grades in a mineral deposit. This can aid the simultaneous stochastic optimization process by creating more realistic realizations of the deposit that better guide the optimizer and lead to more informed decisions. Multi-point and high-order frameworks overcome the limitations of the entropy spatial disorder of more traditional multiGaussain approaches that use two-point statistics. In addition, the simulation of categorical attributes, i.e. boundaries and volumes, may be able to assist in waste management. These categorical simulations could be used as a tool for mapping the uncertainty of waste material and improve the ability to integrate waste management into the simultaneous optimization framework. For example, the lithologies of overburden, till, and the host rock of acid generating waste materials could be simulated to assist with reclamation and satisfy permitting constraints.

1.3. Strategic mine planning with uncertainty

The previous sections address the limitations of using an estimated orebody model and the risk of misguiding the optimization process due to the smooth representation of a mineral deposit. In addition, several examples identified the importance of jointly optimizing a number of components in the mining complex to take advantage of available synergies. This section showcases the importance of directly managing the risk of uncertain material supply when generating the optimal mine production schedule, which can be accomplished by generating a set of equally probable stochastic simulations of the mineral deposit and using them as input into the optimization.

1.3.1. *Managing risk in strategic mine planning*

An early approach for managing supply uncertainty is tackled by generating a conventional production schedule for a group of simulated orebody scenarios. The production schedule is evaluated to determine the maximum-upside and minimum-downside of each production schedule by testing the performance of the scenarios that were not used to generate the initial production schedule (Dimitrakopoulos et al. 2007). This is repeated for all scenarios and a final design is selected based on satisfying the minimum acceptable return and maximizing the upside potential.

The evaluation approach quantifies the risk of meeting production targets and the selection of the optimal design aims to minimize the amount of risk and variability of the production schedule. This approach does not generate a mine plan that jointly manages the uncertainty of all the scenarios, likely leading to sub-optimal designs. Another limitation is the exhaustive approach requires a large amount of time to generate a production schedule for each scenario. Therefore, considerations for optimizing the mine production schedule while jointly considering each of the simulated scenarios could lead to improved performance.

Godoy (2002) develops an alternative method for assessing grade uncertainty by deriving an optimal schedule for each simulated scenario. In the first stage the stable solution domain is defined by determining all feasible extraction rates for mining two products, ore and waste. The stable solution domain is generated by considering the worst case (bench-by-bench) and best case (pit-shell-by-pit-shell) mining. The worst and best case scenarios are defined for each simulations and the stable solution domain is the area all the simulations share. Using the stable solution domain, a mathematical programming formulation determines the optimum mining rate over the life of an asset. The value must be within the stable solution domain. These rates are then used to create an extraction sequence for several orebody simulation, generating each schedule separately. A simulated annealing optimization approach is then used to minimize deviations from the optimal ore and waste production targets using the previously defined extraction sequences. The result is a single production schedule that aims to minimize deviations from the optimized processing capacity, while respecting the mining rate. This approach is applied at the Fimiston open-pit gold mine and compared to the base case schedule generated with a single estimated orebody model. The comparison with the base case production schedule shows the approach increases the projects NPV by approximately 28% and substantially reduces the risk of deviating from production targets (Godoy and Dimitrakopoulos 2004). In addition, a risk analysis was performed on the base case production schedule to test the performance based on the simulated outcomes, which resulted in 11% less NPV then the initial economic analysis (Godoy 2002). This demonstrates the overestimation of metal quantities due to the effects of smoothing in an estimated orebody model. Although the method integrates uncertainty into the optimization process it remains a stepwise procedure that can not consider the available synergies between components.

Del Castillo et al. (2015) uses the notion of the stable solution domain to provide an optimal annual extraction rate, jointly optimizing the utilization of the mining fleet and creating a purchasing schedule that helps eliminate unnecessary capital expenditures. The optimal solution in this study refers to the production schedule that maximizes the NPV within the stable solution domain. In the objective function, the costs of adding new equipment to increase production capacity and the ownership cost for parking equipment are considered. The method leads to a 40% reduction in equipment purchases over the first seven years of the mine life. A limitation of the method is no extraction sequence is created, therefore, the mining rate obtained must be used subsequently to generate a new mine production schedule.

Dimitrakopoulos and Ramazan (2004) use a set of stochastic orebody simulations to calculate the probability of a block containing grades above relevant cut-offs or within a given range. The mixed-integer program (MIP) aims to maximize the probability of meeting ore tonnage and grade targets, while deferring blocks of lower probability to later periods. This concept is termed an orebody risk discount rate or geological discount rate in future work. The probabilistic approach also generates feasible mining patterns by adding equipment accessibility and mobility constraints smoothing the mineability of the production schedule. These scheduling constraints prevent the optimizer from hand picking high probability blocks leading to an erratic and infeasible production schedule. The MIP is applied at a Nickel-Laterite deposit where there is a feasible extraction sequence in terms of mineability and decreased risk of meeting the forecasted ore production by 6%. The main limitation of these probabilistic approaches is the probability does not directly account for the joint uncertainty of groups of blocks in the optimization problem and remains a block-based approach.

Stochastic integer programming (SIP) can be used to optimize problems under uncertainty and find a solution that is ideal under all circumstances, ensuring feasibility while managing or hedging the associated risk (Birge and Louveaux 2011). Ramazan and Dimitrakopoulos (2005; 2013) and Dimitrakopoulos and Ramazan (2008) implement a two-stage SIP with recourse generating a production schedule that maximizes the expected NPV of the mineral deposit and minimizes the deviations from production targets including ore tonnage, grade and quality. The first stage decisions are the mining decision variables and the second stage recourse variables measure the resulting deviations from production targets due to uncertainty, which are a result of the first stage decisions. The optimization framework forces the optimizer to balance the trade off between managing the uncertainty of meeting production targets and maximizing the expected NPV (Albor and Dimitrakopoulos 2009; Dimitrakopoulos 2011; Ramazan and Dimitrakopoulos 2013; Leite and Dimitrakopoulos 2014). This overcomes substantial limitations of the maximum-upside and minimum-downside approach. In addition, the NPV increases in the various studies.

Leite and Dimitrakopoulos (2014) test the two-stage SIP approach addressing the impact of adjusting the geological discount rate and probability cut-offs, introduced in the work described previously (Dimitrakopoulos and Ramazan 2004; 2008;Ramazan 2006; Ramazan and Dimitrakopoulos 2013). The production schedule appears to be insensitive to changes in the geological discount rate, which is applied to the penalty cost for deviating from production targets. A probability cut-off is used to classify blocks as ore or waste. This is done by determining the number of simulated grade values that are greater than the cut-off and dividing it by the number of scenarios. Then a probability cut-off is used to define whether a block is considered ore or waste during the optimization. Although, all blocks should ideally be described as binary decision variables the time required to solve the solution can be quite large and using linear decision variables can improve the solution time while, maintaining the optimality of the solution (Ramazan and Dimitrakopoulos 2005; 2013). Therefore, ore blocks are forced to be binary decisions and waste blocks are allowed to be linear leading to a feasible production schedule. The probability cut-off reduces the number of binary decisions as a pre-processing step and speeds up the optimization process by using less binary variables. The authors test the SIP formulation in a copper deposit where the resulting outcome increases the expected NPV by 29% while managing the uncertainty of ore and total mine production over the operating life. This improvement overcame the results of the conventional mine production schedule that had less than a 5% chance of reaching the desired ore production by accounting for uncertainty directly in the optimization framework. This case study showcases the value of the stochastic solution that outperforms any conventionally produced production schedule by accounting for uncertainty and managing the risk of meeting production targets.

Managing supply uncertainty in the quantity and quality of ore is critical for achieving desirable material to deliver to the market. Benndorf and Dimitrakopoulos (2013) demonstrate the use of a two-stage SIP to manage blending constraints in a multi-element iron ore deposit. The method integrates the use of a geological discount rate, while generating practical mining shapes using the approach described in the work of Dimitrakopoulos and Ramazan (2004). The optimization approach manages the blend of iron, silica, alumina, LOI, and phosphorous limits at the Yandi Central 1 deposit in Western Australia. The SIP uses a set of equally probable stochastic orebody models as input. The production schedule generated with the SIP is compared with the output from Blasor, BHP's mine planning tool, and an estimated orebody model. The stochastic approach directly considers the uncertain ore blending properties and minimizes the risk of deviating from blending production targets throughout the optimization process. The SIP results in a production schedule that can increase the project value and better manage the risk of deviating from blending targets.

Menabde et al (2007) present a mixed integer programming formulation that is generalized to include a group of conditional orebody simulations and variable cut-off grade decisions in the optimization approach. The MIP does not contain recourse variables that directly manage uncertainty and instead the framework ensures that production targets are achieved using the average outcome of the simulations. The method optimizes cut-off grade decisions by dividing the range of grades into a number of pre-defined bins and the cut-off grade is optimized in each period to maximize the NPV. The percentage of ore in each aggregate at each cut-off grade and in every scenario is a pre-processed result along with its relative value. Therefore, the cut-off grade is mainly effective for discriminating ore and waste for a single destination and attribute. Adding several different processors would drastically increase the dimensionality of the problem. In addition, blocks are averaged into panel and fractional extraction is permitted. The averaging of grades over panels and allowance of fractional extraction over different periods causes problems due to the effects of averaging. For instance, the lower grades in the pre-defined panel may be heavily weighted on one side and the fraction extracted may be taking this lower grade portion due to the mining sequence. Therefore, the average expected grade from that panel in the earlier period will be lower than expected and is not properly informing the optimizer of its true characteristics. Another limitation is the MIP formulation constrains the average outcome of the production targets, which is misleading if there is variability in the material being mined.

Riméle et al. (2018) develops a two-stage SIP formulation that can incorporate uncertainty and consider in-pit dumping for both tailings and waste materials, while simultaneously determining the extraction sequence and destination policy. This approach overcomes the deterministic limitations of BlasorIPD that was discussed previously. The goal of this approach is to reduce the size of waste dumps and stockpiles by considering placement of waste directly within the pit limits. Transportation and rehabilitation costs decrease along with environmental damages contributing to a higher NPV. In each period, a top and bottom strip are considered to delineate the available in-pit storage during the optimization. The delineated area represents the space available for in-pit storage. In-pit storage variables are first stage decisions that show whether a strip is available for storage, the bottom strip, or the top strip. A strip can only be filled with material if it is available for storage and the quantity is capacitated. There is also a limitation on the amount of tailings and stockpiled material that can be stored ex-pit. The framework is applied on a low-dipping iron ore deposit where tailings and waste are successfully stored in-pit. A large reduction in rehandling costs are obtained by minimizing the rehandle for the reclamation process.

Multistage stochastic programming models provide an approach that can adapt to uncertainty, particularly geological uncertainty (Boland et al. 2008) and can evolve to include the randomness of equipment longevity or uncertain demand in different industries (Ahmed et al. 2003). Boland et al. (2008) use a multistage stochastic programming approach without recourse to yield a number of different production schedules in response to geological uncertainty. A set of stochastic geological simulations are used as input into a mixed integer multistage stochastic programming approach, where future decisions are dependent on the geological properties of the material previously mined. As the production schedule is optimized and if there are significant differences between the grades of simulated orebody scenarios the solution is allowed to change the processing and mining decision variables between scenarios. However, if there is no large difference in the scenarios before some time period non-anticipativity constraints are enforced. Non-anticipativity constraints ensure that if it is not possible to distinguish between two groups of scenarios then the same decisions must be undertaken in those scenarios. The problem here is these decisions directly relate to the geological uncertainty, which is highly variable. Therefore, decisions will vary drastically and the optimization process will end up overfitting the mine production schedule to a single production schedule for each scenario. A predetermined differentiator α checks the difference in grade between scenarios and determines whether non-anticipativity constraints are applied. The method is capable of handling multiple geological simulation of the supply uncertainty and shows the ability to allow processing and mining decisions to adapt to these uncertain attributes. However, the major limitation of this approach is it ends up generating a production schedule for each scenario and by doing so it is able to precisely control the production rates. This leads to a production schedule that is best fit for a scenario and if any different outcome occurs it is not robust enough to handle these types of changes leading to an over optimistic vision of the potential project value. The reason the overfitting leads to a problem in generating a strategic mine production schedule is addressed in the following descriptive example: the physical mine design in the second year of production is conditional to the extraction sequence in the first year and once a plan has commenced it becomes infeasible to mine the second year of extraction in any of the scenarios that differ in action from the first year of the initial plan. As a result, once the first decision is made the sequence is guided by this decision and the multistage framework that is suggested no longer becomes feasible due to the previous extraction sequence decision. This limitation is overcome in the subsequent section in the work of Del Castillo and Dimitrakopoulos (2019).

Groeneveld and Topal (2011) attempt to evaluate the flexibility of mine designs under uncertainty by using a mixed integer programming model and Monte Carlo type simulations. Stochastic parameters (price, capital expenditures, equipment utilization, etc.) are simulated using a Monte Carlo approach providing a given state of the world and than are optimized using the mixed integer program. Each scenario is being optimized individually and using the group of optimized scenarios they expect to be able to determine the flexibility that provides the best risk profile based on the options most frequently visited. Several limitations of this model are quickly observed including the inability to incorporate geological uncertainty, the oversimplification of the Monte Carlo simulation techniques used, no production schedule is produced, and many more. In essence, this method simply provides a risk analysis of potential outcomes and assumes the design chosen strictly based on the frequency of occurrence is the 'optimal' option. Groeneveld et al. (2012) expand on this model by creating a hybrid model that fixes the initial periods for choosing different design options and reopens them up. Providing a more realistic approach that only allows the design to actively adjust after a number of periods allowing for appropriate lead times. Additionally, the model described as robust considers numerous states at once and generates one design that best satisfy a range of simulated price and cost conditions.

There are suggestions that SIP formulations are not computationally feasible when there are a large number of blocks (hundreds of thousands to millions) that must be scheduled. This has been overcome in the subsequent contributions to this thesis by applying a SIP in two large real-world applications where the number of blocks used for scheduling is over 2.3 million. Mai, Topal and Erten (2018) suggest an alternative aggregation approach to reduce the problem size using a TopCone algorithm (TCA) instead of panel aggregation as a preprocessing step. The blocks are clustered into a number of TopCones, predefining the quantity and shape of each TopCone and eliminating the need for pushback design due to their shape. The linear programming approach for clustering TopCones aims to group blocks with positive and negative economic values into clusters that are positive in value. The minimization problem reduces the number of connections between nodes, so the size of clusters is minimal, and the previous conditions are satisfied. A major limitation of TCA is that if the TopCones become too large the model may become infeasible to solve towards the bottom of the pit due to incremental value that could be achieved by smaller cones, which is not feasible due to the size of the cone. An advantage here is they do not assume fractional extraction and their shape is advantageous as the approach can lead to an ultimate pit limit by determining the optimal extraction of TopCones.

Mai et al. (2018) generate an open pit production schedule by combining the TCA with a SIP framework. The combined approach requires a three step process (i) the geological simulations are averaged to generate an E-type orebody model (ii) an E-type model is used to solve the TCA algorithm and the simulated values are reinput into the TopCones (iii) the stochastic values of the TopCones are used as input into the SIP production scheduling algorithm that is solved using CPLEX, which is now possible due to the significant reduction in the number of integer variables. Similar to previous work (Dimitrakopoulos and Ramazan 2008; Ramazan and Dimitrakopoulos 2013), the SIP formulation aims to maximize the NPV and minimize deviations from production targets. This resulting production schedule leads to a 2.28% increase in NPV due to the management of technical risk in the optimization formulation. However, limitations of this method still exist similar to panels where the flexibility to change the extraction sequence is limited and the top cones are generated based on E-type model that is not representative of the simulations as a group but, rather their averaged value. Aggregation approaches continue to limit the ability to generate additional value as they limit the flexibility of the optimization algorithm being applied. This highlights the need to continue building faster heuristic solving methods, described subsequently, to overcome the requirement of reducing the size of the optimization problem.

1.3.2. *Simultaneous stochastic optimization*

Simultaneous stochastic optimization delivers an approach that can optimize all the components of a mining complex in a single optimization model (Montiel and Dimitrakopoulos 2015; 2017; 2018;Goodfellow and Dimitrakopoulos 2016; 2017). These components may include open-pit and underground mines, stockpiles, leach pads, autoclaves, oxide mills, waste dumps, tailings facilities, and various methods of transportation (Pimentel et al. 2010). The two-stage SIP formulation with fixed recourse provides a framework that maximizes the NPV, while managing technical risks and is a major extension of the work previously described by Ramazan and Dimitrakopoulos. Moving away from the conventional sequential mine-to-mill optimization approach, the simultaneous stochastic optimization framework generates a long-term production schedule that considers the influence of all the components in the mining complex from the source, open-pit and underground mines, to the customers and spot market. The holistic approach provides opportunities to identify and capitalize on synergies between the different components and determine the optimal decisions that account for the interconnectivity of a mining complex. These opportunities arise due to the change in notion from earlier conventional approaches that consider the economic value of a block determined a priori to the new approach that directly maximizes the profit generated from selling valuable products. The change in the optimization formulation allows for stockpiling, blending, destination policy and capital expenditure decisions to be directly integrated into the optimization resulting in additional value from the available synergies in the mining complex. The production schedule generated using a simultaneous stochastic optimizer increases the NPV and improves the ability to satisfy production targets by searching for combinations of decisions that extract valuable minerals from in-situ mineral reserves and transform them into saleable products. A production schedule defines the resulting extraction sequence, destination policy, blending, stockpiling, processing and capital expenditure decisions.

Montiel and Dimitrakopoulos (2013) take a mining complex in Chile, Escondida Norte, and perform a stochastic optimization using a simulated annealing approach. The simulated annealing approach does not simultaneously optimize the entire complex but integrates a number of critical decisions into the production scheduling approach. The optimization changes the extraction sequence of several ore types to minimize deviations from production targets and ore quality requirements then, models the flow of material that can be transferred to multiple processing streams. A destination policy checks each block to see if it has an invariable material type in all the simulations. If it does then a destination is assigned, however if the material type varies over simulations the scheduler assigns only a period to the block and sends it to the appropriate destinations. The solution performs best if it starts from an initial mining sequence and decreases deviations from production targets. A case study is commenced at Escondida Norte which reduces deviations from 20% to 5% for the processing and grade targets. Although, the objective function does not explicitly maximize NPV it still increases the value of the mining complex by 4% when compared to a conventional production schedule.

Montiel and Dimitrakopoulos (2015; 2017; 2018) continue to extend this framework into a two-stage SIP that maximizes NPV and minimizes deviations from mining and metallurgical processing targets by simultaneously optimizing the extraction sequence, destination policy, operating modes and transportation system alternatives. The NPV is driven by maximizing the value of the products sold and minimizing the costs incurred at each component in the mining complex. This overcomes the limitations of focusing on the economic value of the block. The solving approach considers three different types of perturbation mechanisms within a mining complex, which will be accepted based on the simulated annealing framework (Metropolis et al. 1953; Kirkpatrick et al. 1983). A simulated annealing framework for optimizing a mine production schedule was first introduced by Godoy (2002). Here, the first level block-based perturbations modify the period of extraction, destination of each mining block and the reclamation from stockpiles. The destination is chosen based on the overall profitability of a block at each destination in all the simulated scenarios. Operating alternatives are randomly selected to see if they lead to an increase in objective function by reducing the deviations from production targets, increasing the NPV or a combination of both. For example, the operating alternatives at the processing facility may consider different grinding sizes, such as, coarse or fine by changing the operating mode of the comminution process. The alternative results in different costs, throughputs and recoveries. Last the transportation alternative perturbations are explored by randomly changing the proportion of material sent between different transportation mechanisms, such as, trucks or pipe transportation. Integrating several operating and transporting alternatives into the optimization framework demonstrates the ability to consider the interactions of several components in the mining complex, while determining the extraction sequence. Operating and transportation alternatives can be incorporated in the optimization model by focusing on the value of the products sold instead of the conventional approach that considers the economic value of the block. These decisions optimized together ultimately improve the NPV of the mining complex and lead to a higher probability of meeting production targets when compared to the locally optimized conventional approaches. This is demonstrated at Newmont's Twin Creek operations where the simultaneous stochastic optimization framework increases the NPV by 6% and improves the management of autoclave blending constraints (Montiel and Dimitrakopoulos 2018). The improved blending strategy generated using the simultaneous stochastic optimization approach ensures the autoclave throughput can be utilized without violating the constraint on acid consumption, therefore, a larger difference in NPV is likely to be achieved as the material variability and uncertainty is not accounted for in the deterministic production schedule created for Twin Creeks.

Goodfellow and Dimitrakopoulos (2016; 2017) also propose a two-stage SIP for simultaneously optimizing the production schedule of a mining complex with uncertainty. Large real-life mining complexes can be solved where the number of blocks in the geological model can be in the order of millions with a mine life of 25 years or more. The first-stage decision variables are based on the optimization of each mines extraction sequence and destination policy and the second-stage recourse decisions change the downstream processing decisions. A metaheuristic solution approach is used to solve the large non-linear problem, to ensure stockpiling and blending considerations can be integrated into the simultaneous stochastic optimization of a mining complex. The optimization model defines three types of decision variables; i) scenario independent binary extraction sequence variables that define the period each block is mined; ii) scenario independent binary destination decision variables which define the destination of each cluster membership, defined subsequently; and iii) continuous processing stream variables that define the proportion of material sent from one location to another. The optimization approach is generalized to consider the extraction of mineral reserves and the transformation of this material into different products through blending and processing, by considering two attribute types. Primary attributes are additive variables that originate at the source and flow through the mining complex to generate valuable products. Examples include tonnage, metal quantity and volumes. Hereditary attributes are variables that represent important aspects of the optimization model expressed as function of primary attributes, however, they are not necessarily passed between the different components. Some examples include costs, energy consumption, feed material chemistry and recovery. The flexibility of the simultaneous optimization framework allows for the integration of non-linear relationships through the use of hereditary attributes, which have generally been ignored in existing models because of the challenges associated with non-linear optimization. This limitation is overcome by the use of smart metaheuristics discussed subsequently.

Goodfellow and Dimitrakopoulos (2016) also develop a robust destination policy framework that can be used in multi-variate mineral deposits. Similar to the binning methodology proposed by

Menabde et al. (2007), clusters are generated by assigning blocks, using k-means++ clustering algorithm (Arthur and Vassilvitskii 2007), to a membership based on the properties of interest. The membership for each block is scenario dependent due the local variability of the simulated grades and is determined in a pre-processing step. The number of clusters is a user defined parameter, where more clusters leads to a higher degree of flexibility for the optimizer and can potentially lead to overfitting as the number of clusters approaches the number of blocks in the optimization model. The overarching destination policy is scenario independent leading to one policy that can be used for operational guidance. This works well with multivariate deposits as the dimensionality of the clustering algorithm permits a higher degree of freedom in terms of managing deleterious elements or contaminants that are jointly simulated in the orebody model. A limitation of the approach is the interpretability of the clustering decisions becomes challenging to visualize at higher dimensions as the boundaries can become highly complex. This means the method requires a mathematical formulation to determine the cluster each block is a member of at the operational level and it can not simply be described as high, medium, and low grade based on linear cut-offs. These cut-offs are not desirable due to the impact of secondary products and deleterious elements that are not based on a single grade. However, the clustering approach allows the destination policy decisions to be determined during the optimization process, which overcomes major limitations of conventional methods that pre-determine the cut-off using Lane's cut-off grade optimization approach (Lane 1964; Lane et al. 1984; Rendu 2014).

Furthermore, three metaheuristics are tested and combined to assess the speed and quality of the simultaneous stochastic optimization framework (Goodfellow and Dimitrakopoulos 2016). It was found that particle swarm optimization (PSO) and differential evolution (DE) metaheuristics when applied in the optimization of a mining complex were not suitable for determining an optimal extraction sequence as these approaches were sensitive to initial sequences and destination policies determined by the population. The PSO and DE algorithms required a large amount of computational time to normalize the extraction sequence and enforce slope constraints, however, they appeared to work well for optimizing the downstream destination policies and processing stream decisions because they could change both sets of the interconnected decisions simultaneously. Goodfellow and Dimitrakopoulos (2016) compare the traditional simulated annealing framework with two alternatives: (i) simulating annealing with downstream PSO and (ii) simulating annealing with downstream DE (Metropolis et al. 1953; Kirkpatrick et al. 1983;

Geman and Geman 1984; Kennedy and Eberhart 1995; Price et al. 2005). The two optimization techniques are tested in a copper-gold mining complex to demonstrate the advantage of the new optimization approaches when compared to the traditional simulated annealing framework. Based on the performance comparisons it can be seen that both methods are capable of improving the average NPV of the mining complex. The production schedule obtained in the copper-gold mining complex using simulating annealing with PSO and DE is compared to the basic simulating annealing algorithm and results in a 1.91% and a 2.57% increase in NPV with the requirement of 2.4 and 2.9 times longer computing time, respectively.

Goodfellow and Dimitrakopoulos (2017) further demonstrate the criticality of focusing on the value of the products sold and integrating supply uncertainty into the simultaneous optimization framework in two mining complexes. A nickel-laterite mining complex with multiple stockpiles and strict blending constraints is optimized and the results highlight that ignoring uncertainty can lead to a sub-optimal destination policy that can severely impact material quality requirements. The approach is also tested on a copper-gold mining complex, where the non-linear grade-recovery relationship for copper and gold grades are integrated into the simultaneous optimization process to highlight the importance of maximizing recovery through blending. This advancement is possible as the optimization model now focuses on the value of products sold and the homogenization of materials can now be calculated at the different processes. In the past, non-linear recoveries could only be calculated at the block level, which assumes that each block is processed independently. However, there are often several material sources that supply material to the processing and stockpiling destinations, therefore, a block-based approach is not ideal for understanding the mixing of materials throughout the mining complex. The recovery should instead be examined at the location where the material is being blended or mixed in the processing stream.

Another challenge in the mining industry is determining the optimal time to invest in large capital investments and even more so determining the optimal mining rate to deliver a life-of-mine production schedule. Quite commonly, a fixed deterministic value is tested and evaluated until a mining and processing rate performs well using conventional methods. Predetermining the mining rate prior to the optimization is likely to lead to a locally optimal solution, limited by a schedule that does not consider increasing or decreasing capacity. Hence, the strong relationship between capacities their operating costs and the overall mining complex performance must be considered together in a simultaneous framework to obtain favourable results.

Goodfellow (2014) and Goodfellow and Dimitrakopoulos (2015) integrate capital investment decisions that can either increase or decrease capacities into the optimization. The method considers the cost of investments in the objective function by including capital expenditures in the cashflow model and linking the unitary increase or decrease in capacity to the accompanying capacity constraints. Using the generalized two-stage stochastic integer formulation described previously (Goodfellow and Dimitrakopoulos 2016), the first stage decisions are modified to include each mines extraction sequence, destination policy, and now capital expenditures. The first-stage decisions are decided prior to revealing uncertainty; then, the second-stage recourse variables adapt to uncertainty by adjusting the processing stream decisions and managing deviations from production targets. The capacity decisions are able to be increased or decreased by a unitary amount depending on the number of capital investments undertaken. A new investment decision variable defines how many capital expenditure options are exercised in each period. In addition, realistic lead times and equipment life are considered so feasible purchase plans can be developed. The method is then tested on a copper mining complex that considers changes to the production schedule by increasing or decreasing the mining rate through the acquisition of shovels and trucks leading to a 5.7% higher NPV than the deterministic equivalent design. An advantage of this method is that the target production rates are being decided during the production scheduling process capturing opportunities to contract and extract capacities to gain further value. This overcomes limitations of the sequential approach that predetermines the rates prior to determining the production schedule.

Farmer (2016) simultaneously determines the processing and mining capacities during the life-of-mine production schedule using the method previously described (Goodfellow 2014; Goodfellow and Dimitrakopoulos 2015). The optimization simultaneously considers the sizing of a processing facility, which highly influences the optimal mining rate. This provides a method that can balance the amount of load and haul equipment with the corresponding processing capacity, while determining the extraction sequence, destination policy, stockpiling, and processing decisions. Additionally, there are considerations of project financing within the revenue calculation, in this case a streaming contract was considered under price uncertainty. The resulting production schedule attempted to produce higher grade metal during high price periods and lower cost low-grade material during lower price periods leading to a 12% increase in NPV.

Kumar and Dimitrakopoulos (2019) integrate uncertainty in geo-metallurgical variables alongside grade and material type uncertainty. Two non-additive geo-metallurgical properties are considered in the optimization approach; the simulated semi-autogenous power index and bond work index. The material hardness is then calculated using the geo-metallurgical properties to define the 12 different material types used in the optimization process. Specific geo-metallurgical targets are introduced to maximize the utilization of the processing facilities. The integrative approach is tested at a large copper-gold mining complex and indicates a higher chance of satisfying production targets particularly the ratio of hard/soft material entering the three processing destinations. This increases the NPV by 19.3% when compared to a conventionally produced mine production schedule, which is primarily due to higher metal production achieved through the simultaneous optimization.

Kizikale and Dimitrakopoulos (2014) present a distributed dynamic programming framework that determines the production rates of multiple mines under financial uncertainty. The iterative method assumes that each mine's extraction rate is a function of the simulated prices and the other mines' extraction rates. The individual mines are solved independently and then an iterative approach is used until the iterations converge satisfying the global optimization of multiple mines. Unfortunately, the method is not capable of producing a production schedule and does not account for supply uncertainty, but it is able to understand the interactions between the different mines under financial uncertainty and confirm they do not react proportionally.

Zhang and Dimitrakopoulos (2017) develop a decomposition method to optimize a multi-mine mining complex. The decomposition approach optimizes the upstream mine production schedule and the downstream transportation of materials simultaneously to maximize the NPV, under both supply and market uncertainty. Typically, when optimizing the downstream components of a mining complex the production schedule is treated as fixed and the goal is to maximize the utilization of each processor and transportation method, while simultaneously maximizing the NPV. Depending on the complexity of the mining complex, there may be many material transformations and non-linear relationships between the input and output of each component in the mining complex, making it difficult to simultaneously schedule the extraction sequence and downstream components. In their work, a dynamic-material-value-based decomposition approach is developed to synchronize the upstream and downstream optimization of a mining complex through iterations. This leads to a solution that first solves the extraction of material from the mine (mine production scheduling (MPS)) and then the tonnage of each material type produced are sent to the downstream optimization approach that transports and transform material into a valuable material (material flow planning (MFP)) which, then sends the updated value of the material back to the MPS model that solves the extraction sequence. When the amounts of purchasing and selling of each material type in each period equals zero then the optimization approach has converged and an optimal solution for the upstream and downstream components has been found. Zhang and Dimitrakopoulos (2018) also formulate a two-stage non-linear SIP to optimize a mining complex under both geological and market uncertainty, while simultaneously integrating forward sales contracts. An effective and efficient heuristic is designed to manage the throughput and head-grade dependent recovery in a processing facility. The approach provides a method for evaluating whether a forward contract should be undertaken by observing one of the following: (i) contract should be taken if the best-and worst case NPV increases; (ii) a contract should not be taken if the best and worst case NPV decreases; (iii) a contract may be taken if it is reduces the worst case but decreases the expected NPV.

Contributions leading to the simultaneous stochastic optimization of mining complexes have drastically improved the strategic mine planning framework over the past two decades. The two-stage SIP approaches explained allows robust production schedules to be created that manage uncertainty, while providing mining enterprises the ability to quantify and manage technical risk associated with their assets and achieve a higher NPV. However, a limitation with these approaches is the policies and production schedules produced are fixed or static and they do not consider opportunities to consider feasible alternatives.

Del Castillo and Dimitrakopoulos (2019) use the simultaneous stochastic optimization framework to develop an adaptive optimization approach that considers feasible investment alternatives over the long-term production schedule of a mining complex. These investments change the production capacity of different components within the mining complex resulting in large changes to the production schedule. The approach expands on the previous work proposed by Goodfellow and Dimitrakopoulos (2016; 2017) by integrating a branching mechanism that chooses the opportune time to undertake an investment and whether the production schedule should branch, resulting in a number of feasible alternatives. This is accomplished by allowing the optimizer to consider the probability of purchasing an investment alternative in different groups of simulated orebody scenarios, which represent the supply uncertainty of the mines. When the decision is counterbalancing, where one large representative group of scenarios invest and another large representative group does not, the production schedule branches into alternative mine plans. The innovative approach overcomes the limitations of the previously described multistage SIP (Boland et al. (2008)) as each of these alternatives are fully optimized based on the investment undertaken, however, the decisions made prior to the investment can not be changed once branching occurs. This prevents the optimization framework from anticipating investment decisions and changing the decisions made prior to the investment choice, as the investment decision remains uncertain until it is executed. Each alternative mine plan is optimized fully leading to a production schedule that can be actively followed and depending on whether the investment is undertaken a continuous sequence is available. Overfitting the production schedule to a scenario is eliminated by ensuring there are a minimum number of scenarios within each branch and only allowing the production schedule to branch when a certain percentage of scenarios choose to invest. This is based on the representativity parameter R. Three different outcomes are possible based on probability of investing in each feasible investment alternative:

1. If the probability of investing in a capital expenditure is less than the representative parameter, then the investment is not taken.
2. If the probability of investing in a capital expenditure is greater or equal to the representative parameter and less than $1 - R$ then branching of the production schedule occurs.
3. Lastly, if the probability of investing in a capital expenditure is greater than or equal to $1 - R$ than the investment are taken over all scenarios and there is no branching.

Non-anticipativity constraints are also used to ensure that in each branch of the production schedule all the decision variables are equal over the scenarios until branching occurs. The method is applied on a multi-mine copper mining complex and considers the purchase of trucks and shovels and a secondary crusher for increasing the mining and processing capacity, respectively. When comparing the solution with the results from the initial two-stage stochastic integer programming approach a $170 million increase in NPV is observed. Del Castillo (2018) further expands this model to integrate operational alternatives including drill and blast pattern designs and grind size factoring the trade-off between throughput and recovery at a copper-gold deposit. The end result entails a 10.5% increase in NPV of the mining complex.

1.3.3. *Smarter solving algorithms*

The simultaneous stochastic optimization approaches discussed previously are computationally expensive to solve as the number of integer variables are in the order of billions when integrating uncertainty into the suggested framework. Montiel and Dimitrakopoulos (2017) use a heuristic method to generate life-of-mine production schedules that can consider operational alternatives and geological uncertainty. The heuristic iteratively improves the solution by swapping the periods of blocks within the extraction sequence and switching the destination of those blocks.

Lamghari and Dimitrakopoulos (2012) develop a metaheuristic based on the Tabu search algorithm and apply it to an open-pit mine production optimisation process. They suggest two diversification strategies to explore the large feasible solution domain. First, a long-term memory approach is used to remember where different blocks have been scheduled in the search history and moving them to the least frequented periods. The second approach uses a variable neighbourhood search algorithm. Based on the test results of 10 practical case studies the Tabu search combined with variable neighbourhood modification produces similar results as the long-term memory strategy. However, as the applications grow the variable neighbourhood applications is not nearly as effective as the long-term memory strategy.

Two variants of the variable neighbourhood descent algorithm have been applied to the two-stage stochastic integer formulation and later tested on large-scale applications to improve the efficiency of achieving good solutions (Lamghari et al. 2014). Both methods decompose the solution to a set of smaller sub-problems. The first variant solves the problem exactly, whereas, the second method sub-problems are solved using a greedy heuristic. The exact method slightly outperforms the greedy heuristic in solution quality; however, the greedy heuristic is nearly twice as fast. This method still requires an initial mining sequence, but it can substantially improve the initial solution in a reasonable amount of time. These methods substantially improve the solving time of stochastic optimization framework and have allowed the development of more complex models like described in Goodfellow and Dimitrakopoulos (2016) and Montiel and Dimitrakopoulos (2015). Additionally, there have been various applications of heuristic optimization approaches that aim to optimize larger problems in faster times achieving near optimal results these methods need to continue to be developed for stochastic frameworks and enhance the ability to simultaneously optimized detailed models under uncertainty (Asad and Dimitrakopoulos 2013b; Gilani and Sattarvand 2016; Sari and Kumral 2016) Lastly, a newer hyper heuristic approach is described by Lamghari and Dimitrakopoulos (2018) that provides a learning mechanism that can select or generate heuristics to solve computationally challenging problems, including the optimization of a mining complex, using reinforcement learning and Tabu search. This method helps determine the heuristic method when given a particular problem as some search strategies perform far better on one case compared to another. Therefore, it is desirable to be able to switch between and determine the heuristic approach that best suits the problem at hand. This is being addressed in future work by using smart artificial intelligence and machine learning frameworks that are able to choose the best heuristic approach to apply to the problem at hand.

1.4. Goal and objectives

The goal of this thesis is to investigate simultaneous stochastic optimization through major applications and further develop the method by integrating waste management and capital investment decisions to generate a feasible long-term production schedule in a mining complex. The following set of objectives are to be addressed:

1. Review the technical literature related to strategic mine planning including conventional and stochastic approaches that maximize the value of a mining complex and the methods required to generate conditional orebody simulations that are used to evaluate the response of uncertainty in a mine production schedule and manage supply uncertainty.
2. Apply a simultaneous stochastic optimization approach to a gold mining complex that manages the uncertain production of acid generating waste. Furthermore, address the requirement to quantify material uncertainty and variability of waste as a mining product.
3. Apply an adaptive simultaneous stochastic optimization to a large multi-mine and multi-process gold mining complex that adapts to uncertainty by branching the solution based on a set of feasible investment alternatives.
4. Summarize the main contributions and conclusions of the research completed and provide suggestions for future research.

1.5. Thesis outline

Chapter 1 presents a technical literature review on strategic mine planning focusing on the significance of an integrative optimization approach that accounts for the global impact of each decision within the mining complex. In addition, methods for simulating supply uncertainty using stochastic orebody simulations are described. These aspects are discussed including the integration of the current state-of-the-art simultaneous stochastic optimizers. Lastly, the goals and objectives are provided.

Chapter 2 presents an innovative application of a simultaneous stochastic optimization framework that demonstrates the ability to manage the production of acid generating waste directly in the optimization formulation. The gold mining complex considered includes a large deposit with 2.3 million blocks and a 25-year mine life. The production schedule generated with the simultaneous stochastic optimization framework is compared with a conventional mine production schedule and identifies the underlying uncertainty of waste production.

Chapter 3 presents a major case study in a multi mine and multi process gold mining complex, where an adaptive simultaneous stochastic optimization approach strategically considers investment alternatives in mining equipment, process plant upgrades and the tailings management area. The production schedule branches on feasible investments providing alternative mine plans and the mining rate is determined directly in the optimization formulation.

Chapter 4 reviews the conclusions obtained from these applications and explains the value of simultaneous stochastic optimization frameworks, while considering suggestions for future research.

2. Simultaneous stochastic optimization of an open-pit gold mining complex with waste management

2.1. Introduction

A mining complex is an integrative logistic network that represents the extraction, transportation and transformation of materials between their sources, open-pit and underground mines, and final products delivered to the commodity market (Pimentel et al. 2010; Montiel and Dimitrakopoulos 2015; Goodfellow and Dimitrakopoulos 2016; 2017). New technological advancements in strategic mine planning and optimization simultaneously consider all the components of a mining complex, including multiple mines, processors, stockpiles, and waste facilities, in a single optimization framework with uncertain raw material supply (Montiel and Dimitrakopoulos 2015; Goodfellow and Dimitrakopoulos 2017). These advancements provide an integrative approach that can be adapted to accurately consider waste as a mining product and manage the uncertain quantity and quality of waste produced, thus minimize potential environmental impacts and assist rehabilitation. The simultaneous stochastic optimization approach is used to generate the extraction sequence, cut-off grade, stockpiling, processing, and waste management decisions that maximize the value of the mining complex, while managing risk. Waste management decisions are a fundamental aspect of strategic mine planning; however, they are frequently simplified when optimizing the long-term production schedule and are managed subsequently. In this work, a simultaneous stochastic optimization approach is applied to a gold mining complex that actively manages the uncertain production of acid generating waste when generating the production schedule.

There have been several applications of simultaneous stochastic optimization that focus on delivering the appropriate quality and quantity of valuable products to the market, but do not consider the gross impact of generating waste (Saliba and Dimitrakopoulos 2018; Kumar and Dimitrakopoulos 2019; Del Castillo and Dimitrakopoulos 2019). In strategic mine planning, waste management is typically simplified by considering a single unconstrained waste dump destination (Albor and Dimitrakopoulos 2009; Leite and Dimitrakopoulos 2014). Additionally, the uncertainty and local variability of waste's chemical composition is ignored. These simplifications can lead to deviations from production targets, such as, exceeding the permitting constraints at the waste dump destinations and misrepresenting the amount of each waste product produced due to its uncertain properties. Violating permitting constraints and mismanaging waste production can lead to detrimental effects on the environment (Adibee et al. 2013; Jain and Das 2017), emphasizing the importance of integrating waste management into the long-term production schedule.

Waste management is integrated into the simultaneous stochastic optimization approach by first, simulating equally probable stochastic orebody models to identify the underlying uncertainty and variability of the different material types. Second, each component of the mining complex is optimized simultaneously to capture any synergies that may exist (Hoerger, Hoffman, et al. 1999; Whittle 2007; 2010). The uncertainty and variability of the geochemical properties in the mined material can lead to challenges in managing waste production. For example, the oxidation of sulphide materials in a gold deposit can lead to acid mine drainage when exposed to air and water (Johnson and Hallberg 2005; Akcil and Koldas 2006; Simate and Ndlovu 2014). Acid mine drainage may also contain dissolved heavy metals, which can have adverse effects on the surrounding environment. The composition of sulphides and carbonates is highly uncertain and deterministic methods for estimating these properties may result in substantial material misclassification. Estimated models smooth the representation of the grades and all the pertinent properties within the mineral deposit misrepresenting the proportions of material by under and overrepresenting the extreme and average grades of the deposit, respectively (Rossi and Deutsch 2014). The smoothing effect directly impacts the ability to properly predict the geochemical behaviour of waste material extracted. This includes the classification of acid generating material, for example, by evaluating the ratio of neutralization (NP) and acid potential (AP) in a gold deposit (Pedretti et al. 2017). Therefore, strategic plans must be developed that integrate waste management and uncertainty into the mine production schedule to alleviate the cost of remediation, long-term site monitoring, and further environmental impacts (Gray 1997; Johnson and Hallberg 2005). Costs and environmental impacts can be minimized by decreasing the production of acid generating waste rock, reducing surface disturbance and satisfying permit constraints. Opportunities to minimize the cost of rehabilitation and long-term monitoring can be achieved by decreasing the size of waste dumps and ensuring there is an appropriate amount of material to encapsulate acid generating waste. For instance, modelling the haulage costs of transporting waste material to the appropriate destination and considering the impact of rehandling material to confine waste dumps during the reclamation process can help reduce costs (Fu et al. 2019).

Several applications of waste management in long-term production scheduling have been developed to address the overproduction and misallocation of waste. Rimélé and Dimitrakopoulos (Rimélé et al. 2018) and Zuckerberg et al. (2007) each propose a model that opens in-pit waste dump and tailings disposal areas to minimize the footprint of expit waste. Fu et al. (Fu et al. 2019) and Badiozamani and Askari-Nasab (Badiozamani and Askari-Nasab 2014) define mixed integer programs that simultaneously optimize the open-pit production schedule while considering waste disposal, however, these models consider substantially smaller optimization problems and do not account for uncertainty. Each of these methods expand on past research (Ben-Awuah and Askari-Nasab 2013; Li et al. 2016) that independently optimize the waste dump schedule after pre-determining the extraction sequence, leading to sub-optimal solutions. Several case studies successfully apply blending quality constraints on processing material at iron ore and autoclave operations using stockpiles (Benndorf and Dimitrakopoulos 2013; Montiel 2014). However, the blending and layering of waste rock material to mitigate or prevent the migration of contaminants into the surrounding environment should also be considered in the production schedule optimization (Mehling Environmental Management 1998).

The work presented is an innovative application of the simultaneous stochastic optimization formulation proposed by Goodfellow and Dimitrakopoulos (2016), demonstrating the ability to manage the production of acid generating waste directly in the optimization framework. The gold mining complex considered includes a large deposit with 2.3 million blocks, a 25-year mine life and acid generating waste. In the following sections, the simultaneous stochastic optimization approach is first outlined. Subsequently, a case study at the above-mentioned gold deposit demonstrates the integration of waste management under uncertainty into the simultaneous stochastic optimization framework and includes comparisons to a conventional mine production schedule and related waste management. Lastly, the conclusion and recommendations for future work are presented.

2.2. Simultaneous stochastic optimization of a gold mining complex

2.2.1. *Two-stage stochastic optimization model*

An application of the simultaneous stochastic optimization method detailed in Goodfellow and Dimitrakopoulos (2016) is applied to a case study that focuses on waste management and cut-off grade optimization. Stochastic mathematical programming techniques provide the means to incorporate various sources of uncertainty into the optimization of the mine production schedule. The model aims to define the extraction sequence, destination policy, and processing stream decisions while simultaneously managing the targets and capacities at waste, processing, and stockpile facilities. This is successfully implemented by focusing on the value of the products sold. The desired output from the simultaneous stochastic optimization is an optimized and feasible production schedule.

A binary decision variable $x_{b,t}$ is equal to one if block b is mined in period t and zero otherwise. Predecessors and successors for each block are calculated in a preprocessing step. Before mining any block b the set of predecessors $\mathbb{O}_b$ must be extracted in the same period or earlier. Additionally, a robust destination policy determines the operational cut-off grades required to maximize the value of the mine. Discretized grade bins are created, similar to the approach proposed by Menabde et al. (2007), and blocks are categorized into groups based on their grade in each simulation. A binary decision variable $z_{g,j,t}$ controls the destination policy for the bin or group ($g \in \mathbb{G}$) of material that is sent to a destination $j \in \mathcal{O}(g)$ in each period t. Groups are generated using k-means++ clustering algorithm to define the boundaries of the bins. Blocks may be sent to different destinations across various scenarios but are optimized using a scenario independent destination policy that assigns membership to the group g for each block b in scenario s, determined a priori.

Materials in the model are described as any product that is generated from the mine or created by blending, separation or processing. Each of these materials contain multiple attributes that pertain to a specific property of the material. These attributes are categorically separated into primary ($p \in \mathbb{P}$) and hereditary ($h \in \mathbb{H}$) attributes. Primary attributes are additive and can be sent from one location to another in the mining complex. Hereditary attributes are parts of the model that may be of interest at specific locations and help facilitate the inclusion of non-linearities into the mathematical programming model. The mining complex ($\mathcal{C}$) may be composed of several mines or external sources ($m \in \mathcal{M}$), stockpiles ($\mathcal{S}$), waste facilities ($\mathcal{W}$), processors ($\mathcal{P}$) and other destinations ($\mathcal{D}$). Geological uncertainty is considered by simulating attributes for each block, $b \in \mathbb{B}_m$, each scenario $s \in \mathbb{S}$ has equal probability of occurring. For each location $i \in \mathcal{S} \cup \mathcal{W} \cup \mathcal{P} \cup \mathcal{D}$ in the mining complex, materials and their attributes are sent to and received from a distinct set of locations. $\mathcal{I}(i)$ and $\mathcal{O}(i)$ represent the set of locations that i can receive and send material to, respectively. The stockpiling decision variables $y_{i,j,t,s} \in [0,1]$ define the proportion of material sent from stockpile $i \in \mathcal{S}$ to location $j \in \mathcal{P}$ in period t and scenario s. Lastly, the value of a primary or hereditary attribute is denoted as $v_{p,i,t,s}$ and $v_{h,i,t,s}$, respectively, for each location i in period $t \in \mathbb{T}$, and scenario s.

Revenues and expenditures are incurred throughout the mining complex and these attributes are directly incorporated into the objective function by calculating a discounted value $p_{h,i,t} = \frac{p_{h,i,1}}{(1+d)^t}$, where d is the discount rate utilized. Typically, the discounted expenses are more specifically broken down into mining ($MC_{h,i,t}$), processing ($PC_{h,i,t}$) and rehandling ($RH_{h,i,t}$) categories for each hereditary attribute h, location i, and period t. To formulate the stochastic integer program deviation variables $d^{+}_{h,i,t,s}$ and $d^{-}_{h,i,t,s}$ are introduced and their respective penalty $c^{+}_{h,i,t}$ and $c^{-}_{h,i,t}$ for managing deviations from production targets. Waste management applications require a penalty for overproducing waste ($WP_{h,i,t}$) and for exceeding the stockpile capacity $\left(SP_{h,i,t}\right)$. The waste management considerations are an extension to the original model, which inform the optimizer of the impact waste has on the mining complex. A geological discount rate (d_{geo}) is applied to the cost of deviation to defer the riskiest material to later periods, $c^{+}_{h,i,t} = \frac{c^{+}_{h,i,1}}{\left(1+d_{geo}\right)^{t}}$.

2.2.2. *Objective function*

$$Max \frac{1}{|S|}\sum_{s\in S}\sum_{t\in \mathbb{T}}\left\{\underbrace{\sum_{i\in \mathcal{D}}\sum_{h\in \mathbb{H}} p_{h,i,t}v_{h,i,t,s}}_{Part\ I} - \underbrace{\sum_{m\in \mathcal{M}}\sum_{h\in \mathbb{H}} MC_{h,i,t}v_{h,i,t,s}}_{Part\ II} \right.$$
$$- \underbrace{\sum_{i\in \mathcal{P}}\sum_{h\in \mathbb{H}} PC_{h,i,t}v_{h,i,t,s}}_{Part\ III} - \underbrace{\sum_{i\in \mathcal{S}}\sum_{h\in \mathbb{H}} RH_{h,i,t}v_{h,i,t,s}}_{Part\ IV} - \underbrace{\sum_{i\in \mathcal{W}}\sum_{h\in \mathbb{H}} WP_{h,i,t}d^{+}_{h,i,t,s}}_{Part\ V} \qquad [3]$$
$$\left. - \underbrace{\sum_{i\in \mathcal{S}}\sum_{h\in \mathbb{H}} SP_{h,i,t}d^{+}_{h,i,t,s}}_{Part\ VI} - \underbrace{\sum_{i\in \mathcal{P}}\sum_{h\in \mathbb{H}} \left(c^{+}_{h,i,t}d^{+}_{h,i,t,s} + c^{-}_{h,i,t}d^{-}_{h,i,t,s}\right)}_{Part\ VII}\right\}$$

The objective function (Eq. 3) of the mathematical model maximizes the net present value (NPV) that is generated by producing a set of marketable materials (Stage I) and minimizes the risk of failing to meet production targets and environmental requirements (Stage II). Part I maximizes the profits generated from the different products produced. Part II, III, and IV aim to minimize the cost of mining, processing, and rehandling, respectively. Part IV and V minimize the deviations from waste and stockpile facility capacities, while Part VII minimizes deviations from additional production targets over the life-of-mine.

2.2.3. *Constraints*

Mining complexes can have multiple waste facilities. These facilities can only accept certain material types based on their geochemical properties. For instance, the quantity of potentially acid-generating waste material must be reduced to minimize the size of collection dykes and long-term monitoring wells; these are expensive to install, maintain, and monitor. Furthermore, there is an annual waste production limit due to environmental permitting constraints. The capacity constraints are managed to satisfy the operational capabilities of the mine by calculating the deviations from the upper ($U_{h,i,t}$) and lower $\left(L_{h,i,t}\right)$ limits and penalizing the deviations in the objective function. Similarly, stockpiles are bound by the available space in the run-of-mine stockpiling facility. Stockpiles are designed to store valuable material preventing ore gaps at the process plant or when excess ore material is available the lower grade material is stockpiled for use in a later period. This considers the time value of money. Capacity constraints are demonstrated as follows:

$$v_{h,i,t,s} - d^{+}_{h,i,t,s} \leq U_{h,i,t} \quad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{D} \cup \mathcal{M}, t \in \mathbb{T}, s \in \mathbb{S} \qquad [4]$$

$$v_{h,i,t,s} + d^{-}_{h,i,t,s} \geq L_{h,i,t} \quad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{D} \cup \mathcal{M}, t \in \mathbb{T}, s \in \mathbb{S} \qquad [5]$$

Additionally, a mining and processing target are managed by measuring the deviations from production targets (Eq. 4 and Eq. 5) to properly utilize the available equipment.

Operationally, multiple stockpiles for different material types may be desired to prevent mixing of high and low-grade material. The ability to increase selectivity of the material sent from the stockpile to the mill is improved substantially and minimizes the impact of homogenizing different grade ore material. Higher-grade stockpiles are likely to be built-up and depleted faster than low-grade stockpiles which may accumulate material until the mine reaches an ore gap or end of life. When considering multiple stockpiles, an additional constraint (Eq. 6) is used in conjunction with the stockpile capacity to force the optimizer to split the material between all destinations based on the grade received in each period. A global attribute calculates the grade ($G_{i,t,s}$) of the low-grade ($\mathcal{S}_{LG}$) and high-grade stockpiles ($\mathcal{S}_{HG}$) and penalizes the objective function when the grade within the low-grade stockpile exceeds that of the higher-grade stockpile:

$$G_{i,t,s} - G_{j,t,s} \geq 0 \quad \forall\ i \in \mathcal{S}_{HG}, j \in \mathcal{S}_{LG}\ , t \in \mathbb{T}, s \in \mathbb{S} \qquad [6]$$

Lastly, a destination policy constraint ensures that a bin or group can only be sent to one destination within the mining complex:

$$\sum_{j \in \mathcal{O}(g)} z_{g,j,t} = 1 \quad \forall g \in \mathbb{G}, t \in \mathbb{T} \qquad [7]$$

2.2.4. *Applying a metaheuristic solution method*

Mine production scheduling is inherently challenging to solve for a mining complex due to the large number of decision variables that must be considered during the simultaneous optimization. Commercially available solvers for conventional optimization that consider a single estimated model substantially reduce the complexity as they require a single input to the optimizer. Comparatively, the stochastic methods used in this research substantially increase the challenge of solving the optimization problem and require a heuristic solving method. Goodfellow and Dimitrakopoulos (2016; 2017) and Lamghari et al. (2014) explain various heuristic techniques. In this work, a combination of multi-neighbourhood simulated annealing with adaptive neighbourhood search is used, and it is described in Appendix A1.

2.3. Application at a gold mining complex

2.3.1. *Overview of the gold mining complex*

The proposed simultaneous stochastic optimization method is tested in an open-pit gold mining complex comprised of a large open pit mine that supplies material to multiple stockpiles, waste dumps, and an ore processing facility. The objective of the simultaneous stochastic optimization approach is to maximize the NPV of the mining complex, while managing the uncertain production of acid generating waste. Figure 1 describes the configuration of the mining complex that the mathematical model considers and restricts the allowable flow of materials to either the waste dumps, stockpiles, or processing facilities based on the material characteristics, while managing the related risk. The orebody is directly simulated at block support with an efficient method for quantifying the uncertainty and variability of a large deposit (Godoy 2002). In addition, material uncertainty is considered by simulating carbon and sulphur grades to determine the neutralization potential (NP) and acid potential (AP), respectively. The AP is based on the content of sulphates derived from pyrite and, similarly the NP, is related to the quantity of carbonates in the gold deposit. The carbonates act as buffer and reduce the likelihood of producing acid mine drainage (Lawrence and Scheske 1997). The net producing ratio (NPR) is a classifier that defines the material type. The mine contains three main material types; overburden, non-acid generating rock (NAG), and potentially acid-generating rock (PAG). These three materials must be sent to stockpiles and waste facilities that accept them. Industry standard practice assumes a $NPR = \frac{NP}{AP}$

greater than two is required to prevent acid mine drainage and, if less than one the material is likely to produce acid mine drainage (Table 1).

The NPR classification between 1 and 2 is uncertain and a cut-off is defined based on the operation's risk tolerance. In this case, a NPR of 1.5 is used as the defining boundary between NAG and PAG material based on the mine's current policy. Existing stockpile grades are simulated to quantify the geological risk of material that is used for blending and future processing (2014). Monte Carlo simulation methods are used to simulate existing stockpiles based on historical data from dump locations, which are provided by grade control and the fleet management system. However, infill drilling is a more common approach for simulating the stockpile before reclamation (Dirkx and Dimitrakopoulos 2017). Typically, the expected grade of material to be mined from an existing stockpile is assumed to be the average grade of the stockpile (Groeneveld and Topal 2011). This is unrealistic as stockpiles are heterogeneous in nature and are constantly being mixed, making simulations more practical.

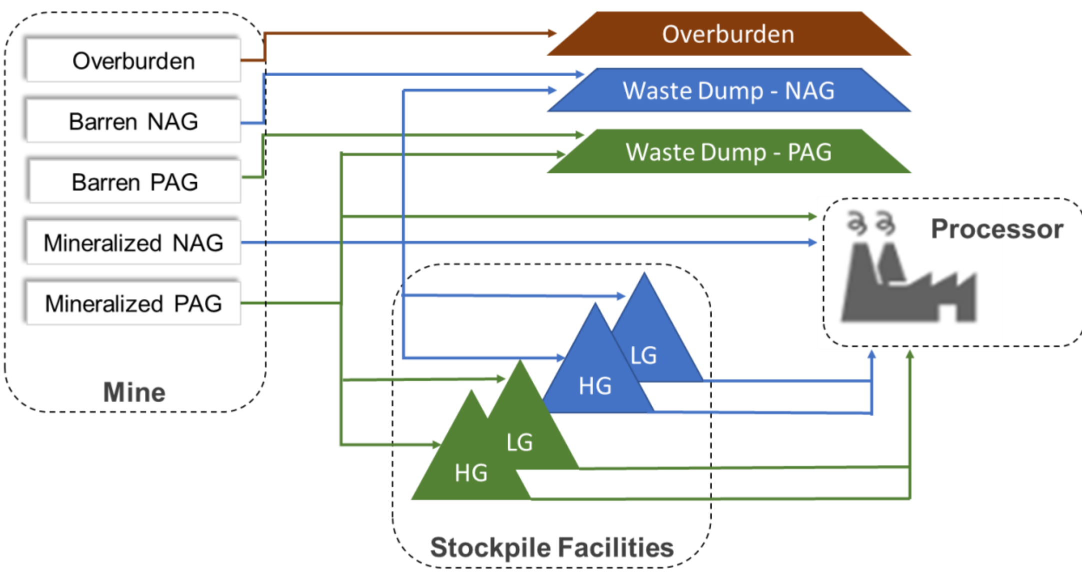

Figure 1. Material flow diagram at a gold mining complex

Table 1. Material classification

NPR	Classification
> 2	NAG
≤ 1	PAG
$> 1\ AND \leq 2$	Uncertain

PAG material must be segregated from NAG when stored at stockpiles and waste dumps. Separating the two material types allows the operation to efficiently control the production of contaminants and their migration into the surrounding environment, reducing the extensive requirement for long-term monitoring and reclamation. NAG and PAG are further separated into two sub-materials, barren and mineralized, depending if the grade is below or above 0.1 g/t, respectively. This reduces the number of decisions required during the optimization process as barren materials are sent to the appropriate waste facility.

The base mining cost accounts for sending material from the pit to the NAG waste dump facility and additional costs are incurred for a longer hauling distance to the PAG waste dump, stockpile, and processing facility. Furthermore, an incremental mining cost is included to account for the increasing cost of mining deeper into the pit. The costs parameters in Table 2 were scaled for confidentiality purposes. Table 3 summarizes the targets for each component of the mining complex including the stockpiles, processors, and waste facilities. Table 4 denotes two scheduling constraints used to create smooth mineable schedules. The objective function incurs a penalty if the blocks within a distance of 60 m are not mined in the same period. Furthermore, penalties are imposed if the mine is operating on two benches that are separated by more than 120 m. These constraints lead to a feasible mining schedule. An overview of smoothing and sink rate penalty strategies can be found in past literature (Caccetta and Hill 2003; Dimitrakopoulos and Ramazan 2004). In addition, grade differential constraint measures the difference in grade being sent to the high-grade and low-grade stockpile further explanation can be found in Section 2.2.3.

Table 2. Economic parameters

Parameter	**Units**	**Value**
Economic Discount Rate	%	10
Geological Discount Rate	%	15
Gold Price	USD $	1200
Exchange Rate	USD/CAD	1.25
Base Mining Cost	$/t	2.15
Incremental Mining Cost	$/bench	0.03
Additional PAG Waste Mining Cost	$/t	0.02
Additional NAG Waste Mining Cost	$/t	0.00
Additional Processor Mining Cost	$/t	0.04
Additional Stockpile Mining Cost	$/t	0.04
Processing Cost	$/t	7.59
Rehandle Cost	$/t	0.45

Table 3. Capacity constraints

Description of constraint	**Lower, upper bound (Mt)**	**Penalty ($/t)**
Mine capacity[a,b]	125, 156	10
High-Grade Stockpile capacity[a]	-, 3	5
Processing capacity[a,b]	26, 28	18
PAG waste dump capacity[a]	-, 27	10

[a] scaled capacities for confidentiality purposes

[b] lower bound is not enforced in period 15-25

Table 4. Scheduling constraints

Description of constraint	Distance (m)	Penalty ($/deviation)
Smoothness	60	12000
Max sink rate	120	12000

Lastly, the mine consists of 2.3 million blocks with dimensions of 20x10x12 m^3. Geological uncertainty is considered by constructing twenty-five equally probable simulations that consider gold, carbon, and sulphur grades. Albor and Dimitrakopoulos (2009), and Montiel and Dimitrakopoulos (2017) demonstrate that 10-15 simulations generate stable production schedules, due to the related support-scale effects.

2.3.2. *Risk analysis*

A base case mine production schedule was generated at the mine site using Hexagon's MSSO and a single estimated orebody model to create a conventional mine plan. The parameters and constraints used for generating this schedule are described in Table 3 and 4. Conventional approaches aim to strictly maximize NPV in the objective function and use hard constraints during optimization. The base case production schedule is used alongside the mine's cut-off grade policy to perform a risk analysis and assess the limitations of using an estimated orebody model with a conventional optimization approach. For the risk analysis, scheduled blocks in each period are sent to the destination based on their cut-off grade, material type and scheduled period determined in the conventional production schedule. The risk analysis tests the robustness of the current life-of-mine production schedule and cut-off grade policy. The high and medium-grade ore are sent directly to the processing facility; low-grade NAG and low-grade PAG are stockpiled or sent to the mill if the processing capacity is not satisfied; and the remaining waste material is sent to either the NAG, PAG, or overburden waste facility. The risk analysis, then, shows the performance of a conventional mine production schedule given the quantification of grade uncertainty associated with each material type, thus assessing the probability of meeting the forecasted production targets.

Material uncertainty and variability is evident when assessing the NP and AP of the blocks within the deposit. In Figure 2, the simulations show variance in the number of blocks that report to each classification when comparing them to the base case model. Whereas the stochastic model shows a higher likelihood of PAG material that must be appropriately handled to reduce the risk of acid-mine drainage.

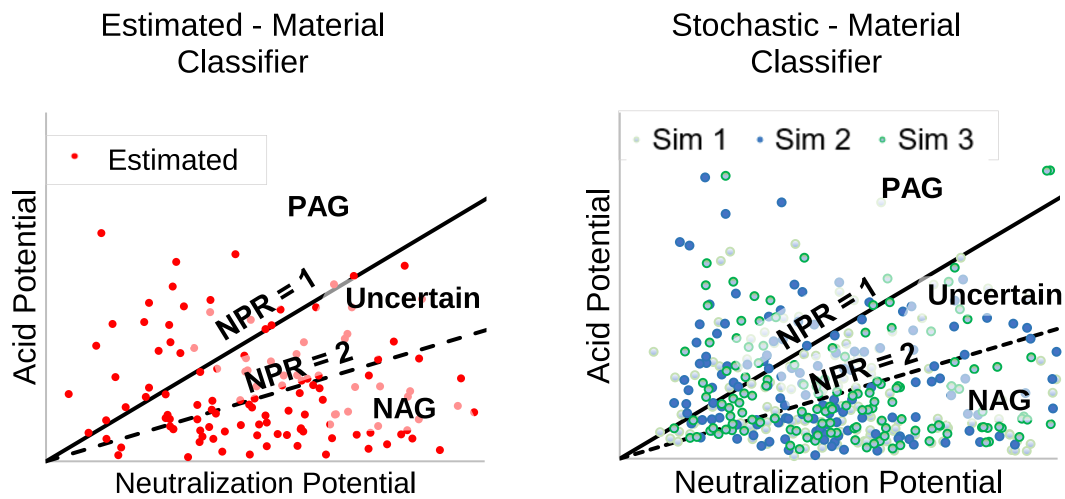

Figure 2. Material classification of acid generators

The flat topography and numerous bodies of water in the region make potentially acid generating waste rock placement critical as space is limited and run-off must either return to the pit or collection ditches. The environmental footprint must be minimized due to the sensitivity of the surrounding watershed. In addition, federal environmental regulations protect migratory birds during nesting season, which runs from mid-March to August and prevents the operation from extending the waste dump during this time (Canada 2017). This forces the operation to increase the elevation of the waste dump and limits the annual waste dump expansions due to the geotechnical constraints associated with stability.

The P10, P50, and P90 in the following figures represent a 10%, 50%, and 90% probability of having values below this amount. In Figure 3c, it is obvious that the base case tends to underestimate the amount of PAG waste material by approximately 12% on an annual basis. This leads to over a 90% chance of violations to the permissible waste dump expansion in periods 3 to 8. In addition, Figure 3a demonstrates that when using the mines current fixed cut-off grade policy there is an extremely high probability of not satisfying the processor in more than 15 periods. Geological simulations reproduce the local variability of the deposit and are used as a tool to show the limitations of estimated orebody models. The effects of smoothing misrepresent the proportion of material that exist in the deposit severely affecting the expected grade-tonnage relationship of material attributes at different grades. Since the cut-off grade decisions are determined prior to the optimization process this can result in highly unrealistic forecasts, similar to those observed in Figure 3. Reinforcing the requirement to include geological uncertainty and determine the cut-off grade policy within the simultaneous optimization to follow. As discussed earlier, an understanding of this common phenomenon is frequently considered for ore material types but neglected for those elements that pertain to waste production. The base case underestimates the quantities of PAG rock due to the smoothing of sulphur and carbon grades in the estimated model that directly result in the misclassification of material. Additionally, after the first three periods of production, the mine is no longer achieving the forecasted gold production (Figure 3d) in the estimated model leading to an 18% decrease of gold produced and a 20% decrease in the NPV (Figure 3e). The large reduction in NPV is mainly due to large ore deficit at the processor. This shortage directly influences the expected ounce profile over the life of mine.

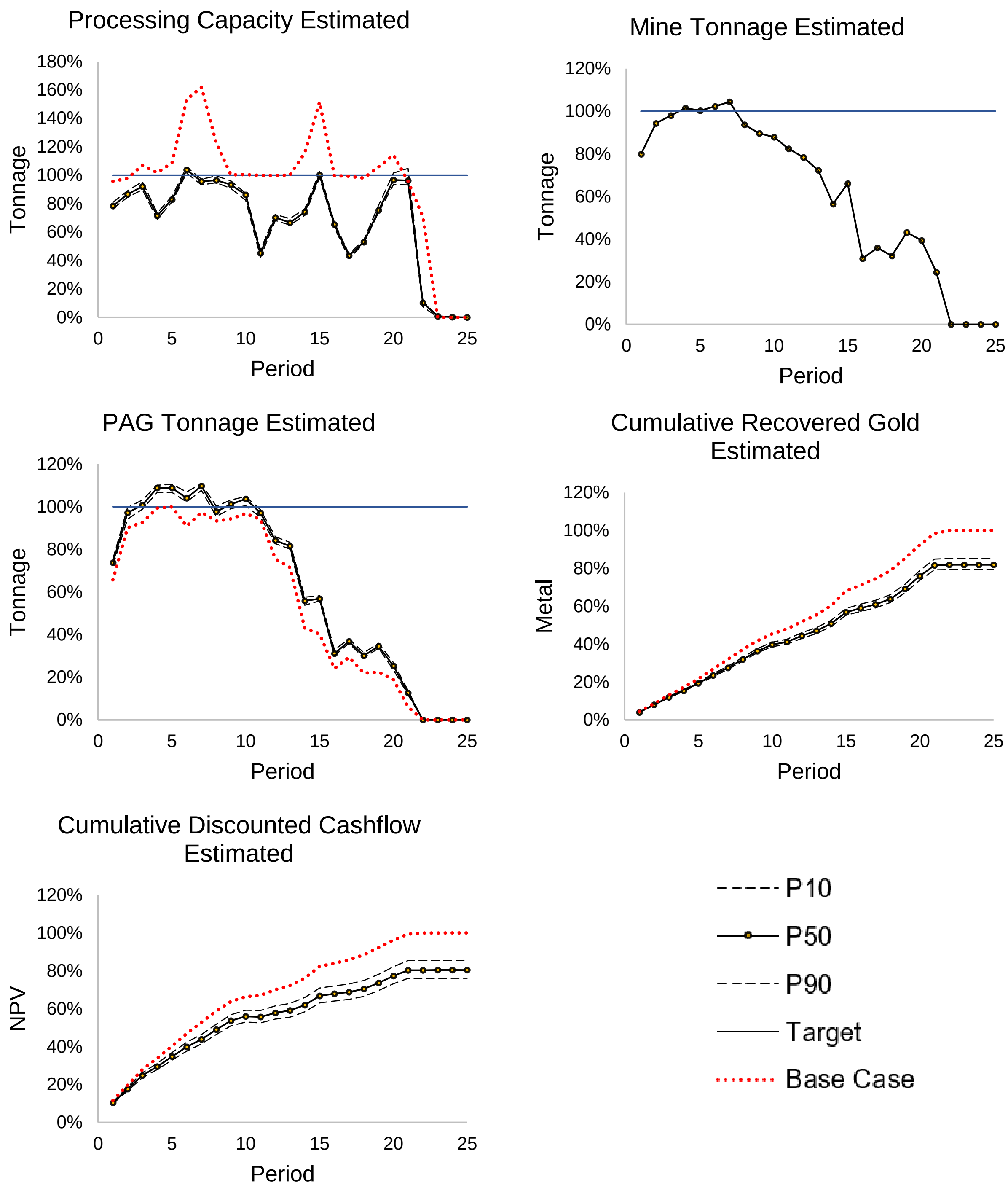

Figure 3. Risk analysis of base case: (a) processing capacity; (b) mine tonnage; (c) PAG tonnage; (d) recovered gold; and (e) cumulative discounted cashflow

2.3.3. *Simultaneous stochastic optimization*

A simultaneous stochastic optimization approach is applied to overcome many of the limitations of the conventional production schedule assessed in Section 2.3.2. The optimization process considers the impact of uncertain attributes and simultaneously optimizes waste management, cut-off grade, stockpiling and processing decisions while determining the production schedule. Figure 4 shows major visual differences between the production schedule produced in the base case and those produced in the simultaneous optimization process. In period 10, the base case schedule shows mining occurring primarily in the west compared to the stochastic schedule which mines deeper into the pit and more towards the east. Then, looking at the north-south cross-section on the lower portion of the figure it is visible that the base case schedule mines bench-by-bench compared to the more aggressive stochastic schedule. These significant differences are a result of the simultaneous optimizer attempting to minimize the deviations from production targets while bringing the most valuable products forward in the mine life.

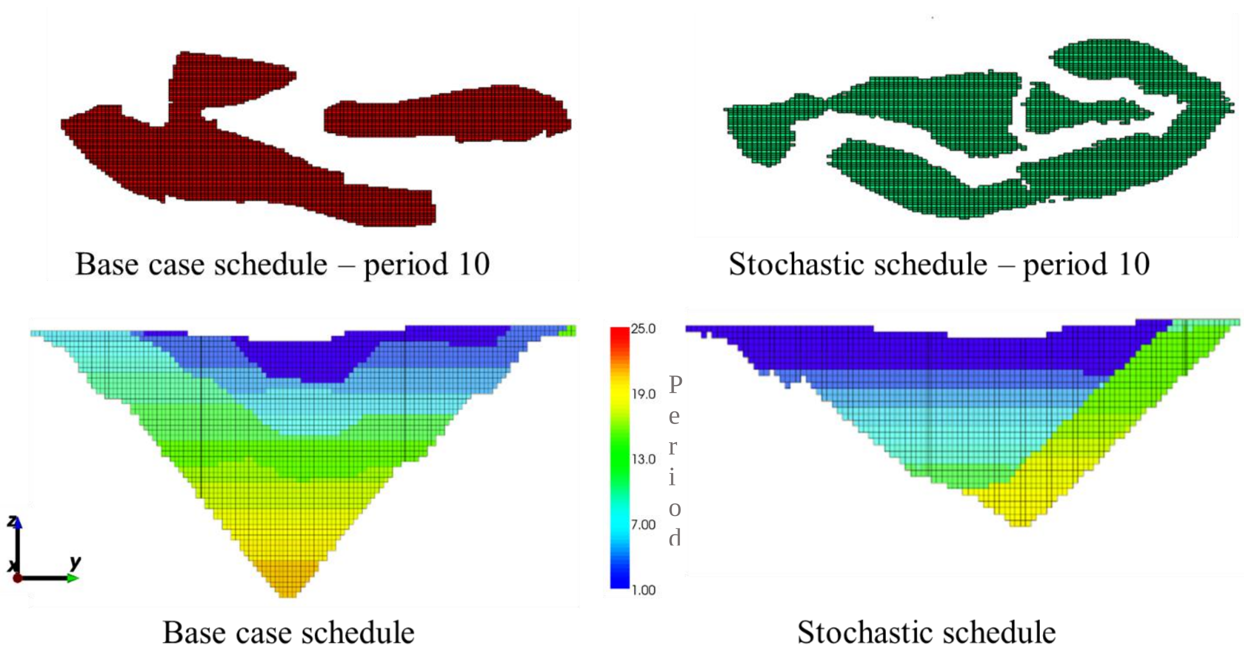

Figure 4. Comparison of the large differences between base case (conventional) and simultaneously optimized stochastic production schedule: (a) base case schedule – plan view period 10; (b) stochastic schedule – plan view period 10; (c) base case schedule – N-S cross section; and (d) stochastic schedule – N-S cross section

The base case in Section 2.3.2 is the P-50 performance of the conventional mine production schedule using the geological simulations. Earlier, it was determined that the base case schedule was unable to satisfy the processing stream due to the use of a predetermined cut-off grade policy. The predetermined cut-off grades fail to consider the effects of uncertainty. Therefore, leading to a sub-optimal production schedule that is poorly informed.

Figure 5 demonstrates that the new production schedule has a high likelihood of satisfying the processor in all periods until the end of the mine life. This is obtained by using a dynamic cut-off grade policy that is determined during the simultaneous optimization. Cut-off grades are reduced to ensure the processing stream is satisfied, yielding a higher throughput. Figure 6 shows the variable cut-off grade throughout the life-of-mine compared to mine’s current fixed cut-off grade decisions. Long-term stockpiling is strategically planned and is only required during periods 5, 7, 11, 13, 18, and 19 instead of all periods in the base case schedule. The working life of the processor is also decreased by 2 periods. Considering waste management during the optimization process reduces the size of the pit significantly, resulting in more than a 30% decrease in the annual material movement (Figure 5b). The mining rates decrease due to a lower stripping ratio shrinking the environmental footprint and the total amount of waste to be managed by each facility.

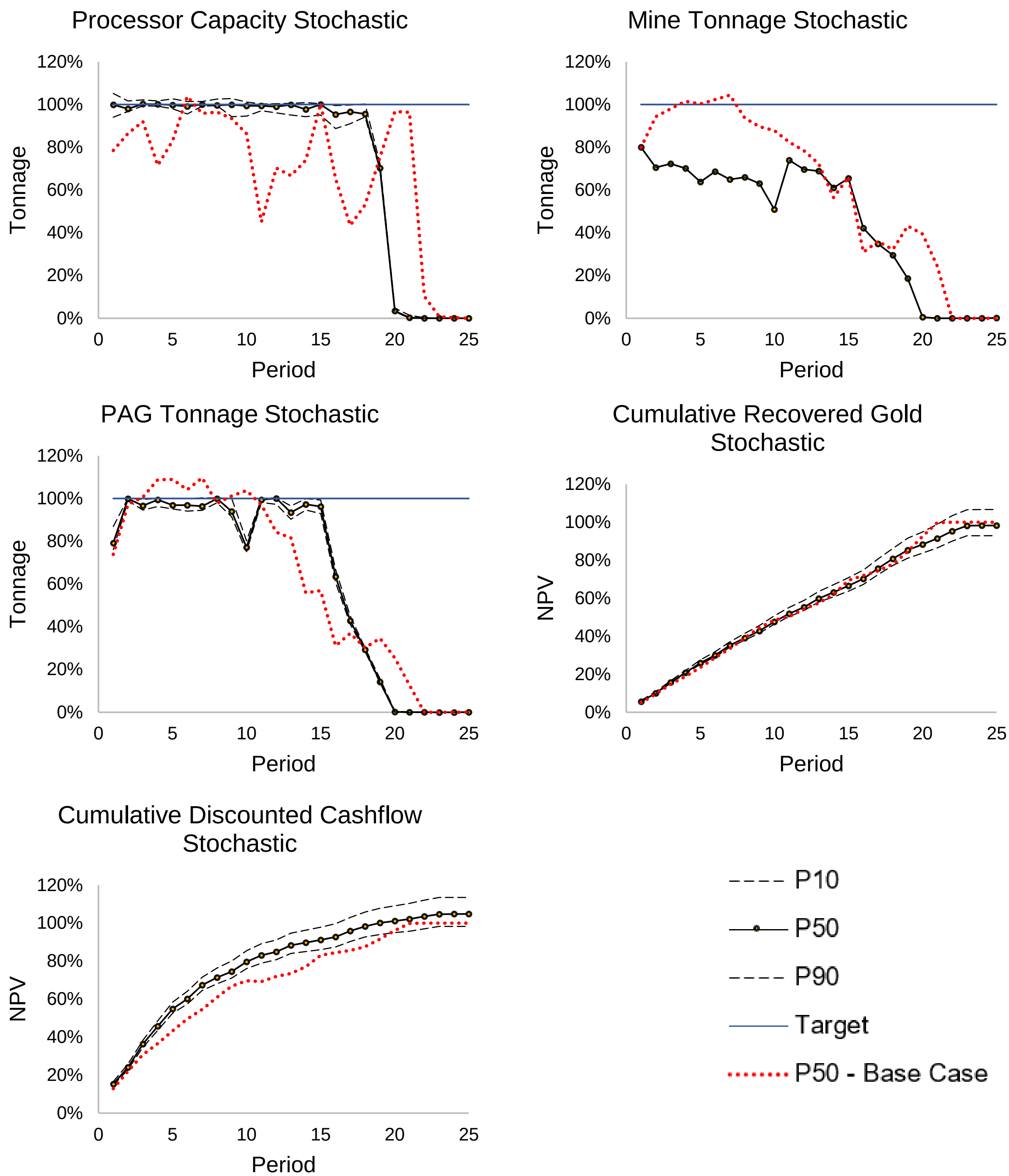

Figure 5. Results from the simultaneous stochastic optimization: (a) processor capacity; (b) mine tonnage; (c) PAG tonnage; (d) recovered gold; and (e) cumulative discounted cashflow

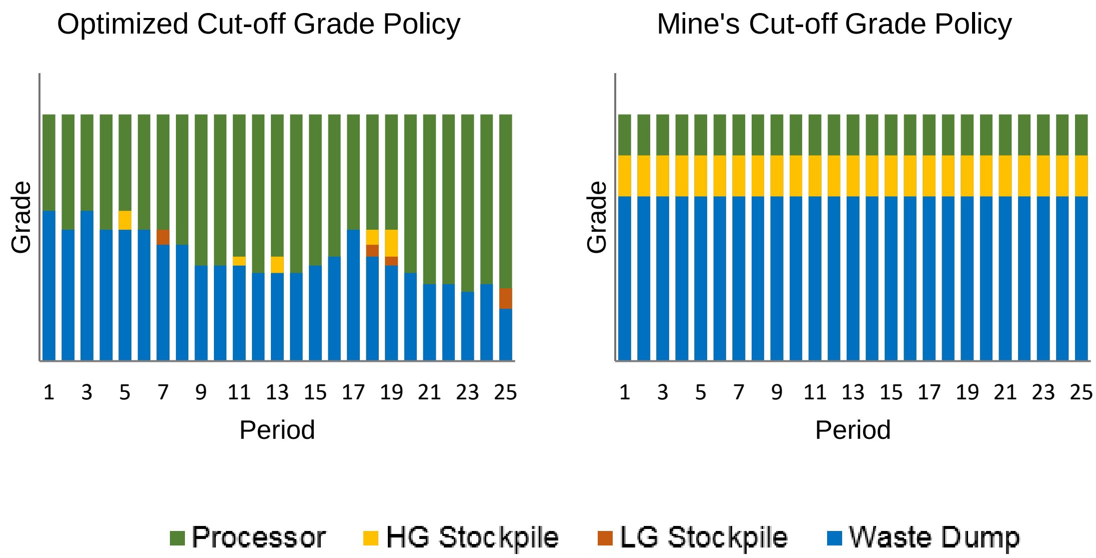

Figure 6. Cut-off grade optimization

Additionally, the problems associated with overproducing PAG waste and breaching the permit guidelines are now controlled and satisfied over all periods (Figure 5c). This is achieved by integrating waste management into the optimization and considering the potential effect of adjusting the cut-off grades in terms of waste production. A lower cut-off grade indicates some material that would have been considered waste is now sent to the processor, further reducing the quantity of waste produced, and ensuring that the processor capacity is fully satisfied. The higher throughput in combination with lower cut-off grade leads to a similar ounce profile observed in Figure 5d. Where Figure 5e indicates a 6% increase in the NPV of the project. This outcome is partially due to the consequences of discounting and producing more gold in earlier periods. However, the main influence is the significant reduction in mining costs during periods 2-10. A lower bound is still enforced on the mining rate for the first fifteen periods to increase the cut-off grade and ensure that the profit generated in earlier periods is high enough to overcome the general and administrative expenses and capital cost for the investments required, i.e. infrastructure and equipment. Although, the objective is to manage the amount of PAG waste produced there is a correlated effect on the total amount of waste that must be stored and reclaimed. Ignoring the uncertain behaviour of waste material will likely lead to unachievable mine plans as miners aim to minimize environmental consequences and satisfy permitting constraints. In Figure 7, the total waste produced is decreased by 16% reducing future monitoring requirements as the waste dump extents are decreased and less infrastructure is required to contain and monitor contaminants. In addition, the probability of contamination is reduced with less reactive material exposed while also increasing the NPV.

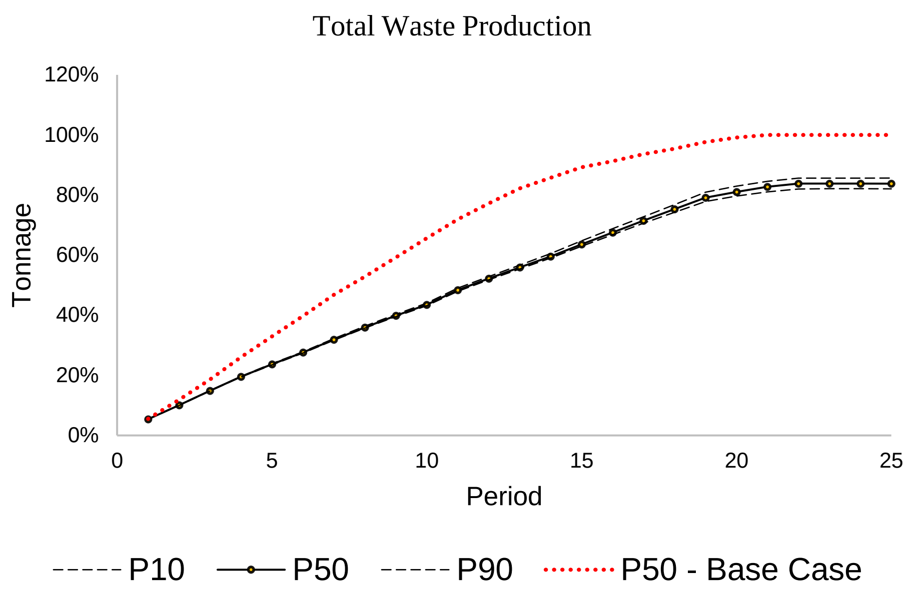

Figure 7. Cumulative waste production over life-of-mine

It should be noted that the mining complex requires 57.5 million block-based extraction sequence decisions, 4,500 destination policy decisions and 1000 stockpiling decisions to be determined in a single optimization run.

2.4. Conclusions

An innovative application of the simultaneous stochastic optimization of a mining complex aims to integrate waste management at a gold mining complex. The contribution considers waste as a mining product and the importance of quantifying material uncertainty and variability. PAG waste rock must be managed to satisfy permitting constraints, minimize surface disturbance, and prevent the production of harmful contaminants. The forecasts generated from the simultaneous stochastic optimization approach balance the requirements of the processing facility and waste management by simultaneously optimizing the cut-off grade policy and considering uncertainty. Comparing the results with the base case production schedule it is evident there are large improvements in the ability to satisfy environmental, permitting, and processing targets. A controlled extraction rate of PAG waste material is successfully obtained by holistically considering waste management, cut-off grade, processing stream, and stockpiling decisions during the production scheduling process.

The new production schedule reduces the total amount of waste produced. Therefore, a lower mining rate is required to satisfy these targets. Although less material is mined and lower grades are sent to the processing facility, a similar amount of metal is produced over the life of mine. The change in mining rate lowers mining costs and equipment expenditures, resulting in a 6% increase in the NPV. Future work should consider complex and integrative waste management approaches. For example, controlling water infiltration by layering and co-disposal opportunities (Aubertin et al. 2016; Antonaki et al. 2018). Material uncertainty is a critical component in understanding waste management and requires further data such as simulated boundaries of overburden material to manage the risk associated with material supply to ongoing projects within the mining complex.

3. Adaptive simultaneous stochastic optimization of a mining complex: A case study

3.1. Introduction

Mining operation are capital-intensive ventures that require smart decisions to strategically time each investment and sustainably produce valuable products. The simultaneous stochastic optimization approach generates an optimal production schedule for a mining complex, using a single mathematical formulation (Montiel and Dimitrakopoulos 2015; 2017; 2018;Goodfellow and Dimitrakopoulos 2016; 2017;Del Castillo and Dimitrakopoulos 2019). The optimized production schedule defines the extraction sequence, stockpiling, processing stream, blending, waste management and capital investment decisions that maximize the net present value (NPV). These decisions are obtained by considering the interactions throughout the entire mining complex that may consist of open pit and underground mines, several processing facilities, crushers, stockpiles, and waste destinations (Pimentel et al. 2010). The stochastic approach also manages technical risk during the optimization by integrating a set of stochastic geostatistical simulations of the in-situ material supply, which reproduce the uncertainty and local variability of the material sourced from the mines. Selecting the appropriate time to undertake a capital investment during the life of mine is challenging due to a combination of supply uncertainty, high upfront costs and prolonged payback periods for each investment. Nevertheless, investments in shovels, trucks, crushers, process plant upgrades, and waste facilities are critical for maximizing the NPV of the long-term production schedule.

The uncertain aspects of mine planning and forecasting, which arise from supply uncertainty, indicate there is large risk of undertaking capital investments (Ravenscroft 1992; Dowd 1994; Groeneveld and Topal 2011; Groeneveld et al. 2012; Asad and Dimitrakopoulos 2013a; Del Castillo and Dimitrakopoulos 2014). In particular, supply uncertainty makes it challenging to produce an optimized production schedule with an investment plan that will satisfy the various futures that may unfold. The optimal investment decision for one future outcome may be very different from another scenario. This generates an interest in developing strategic mine plans that can adapt to uncertainty, by considering feasible investment alternatives that directly impact the production rate of certain components in the mining complex and manage technical risk.

Del Castillo and Dimitrakopoulos (2019) present an adaptive simultaneous stochastic optimization approach that considers a number of feasible investment alternatives and determines the optimal time to branch the production schedule to manage the potential risk of supply uncertainty. A set of orebody simulations are generated for each mine to quantify supply uncertainty. Then, an adaptive approach considers the probability of undertaking an investment in different groups of scenarios. If the decision is counterbalancing, where a representative group of simulations takes on an investment and another representative group does not, the production schedule splits or branches into alternative mine plans based on these investments. Each of these branching alternatives are fully optimized based on the investment that is undertaken, however, decisions made prior to the investment can not be changed once branching occurs. This prevents the optimization model from anticipating the investment decision and changing the previous decisions that were made prior to choosing to invest, as the future investment choice remains uncertain until it is executed. The adaptive optimization approach integrates non-anticipativity constraints into the optimization formulation, similar to the long-term multistage stochastic optimization approach proposed by Boland et al. (2008). Non-anticipativity constraints ensure that the same decisions are taken unless there is an investment alternative that branches the mine production schedule. If branching occurs, the resulting mine plan of each branch should be distinguishably different based on the investment choice. Otherwise, the non-anticipativity constraints should be enforced and the same decision is taken over all the simulated scenarios of the mine. The single production schedule generated with feasible investment alternatives provides an advanced method for determining the optimal time to invest and identifies the risk of investing in new equipment, plant improvements, and other infrastructure purchases (Dixit and Pindyck 1994). Evaluating feasible alternatives and the resulting mine plan creates opportunities to delay, pre-plan or undertake sizeable capital investments (De Neufville and Scholtes 2011).

Boland et al. (2008) use non-anticipativity constraints to ensure when scenarios are similar equal actions are taken across these scenarios. These simulated orebody scenarios are differentiated based on metal grades, which ends up overfitting the production schedule to generate one mine plan for each simulated scenario. In strategic mine planning, the operational flexibility this method allows does not exist. For example, the physical mine design in the second year of production is conditional to the extraction sequence in the first year and once a plan has commenced it becomes infeasible to mine the second year of extraction in any of the scenarios that differ in action from the first year of the initial plan. As a result, a decision must be made on how to execute the mine plan ahead of time. Similarly multistage frameworks have been applied to strategically time the purchase of capital investments and expand the production capacity in other industries (Ahmed et al. 2003; Li et al. 2008; Singh et al. 2009; Gupta and Grossmann 2017), which remain impractical for mine planning and design purposes. Multistage frameworks lead to production schedules with one plan per a scenario, which is optimistic in the ability to change capacities and is the major limitation of multistage approaches. The adaptive approach described herein branches based on investment alternatives that change the operational capabilities of the mining complex and all of the associated mine planning decisions. In addition, there must be a representative number of scenarios in each group to produce stable results that are replicable using another group of simulations overcoming the limitation of overfitting. Furthermore, when considering the execution of the long-term production schedule, operations can not proceed without fixed guidance for the current year of production. Groeneveld et al. (2012) suggest fixing the initial periods of the mine production schedule, to address this limitation, ensuring that operations have the appropriate production guidance and lead time to consider different mining and plant options for the future.

The adaptive simultaneous stochastic optimization approach manages technical risk and delivers a mine production schedule that can identify synergies between different components of the mining complex. For example, in a Nevada type gold mining complex, the metal recovery of refractory ore is influenced by the composition of sulfates and carbonates in the material that is delivered to an autoclave processing facility (Thomas and Pearson 2016; Montiel and Dimitrakopoulos 2018). Blending the material from several sources in the mining complex to maximize recovery may lead to a higher NPV over the operating life and captures value that is unidentifiable using traditional sequential optimization methods (Gershon 1983; Whittle 1999; Hustrulid and Kutcha 2006). Additionally, waste management considerations such as the production of acid generating waste and tailings can be integrated into the optimization to minimize environmental detriments and ensure permitting constraints are satisfied (Saliba and Dimitrakopoulos 2018; Levinson and Dimitrakopoulos 2019). These advancements are achieved by maximizing the value of the products sold (Montiel and Dimitrakopoulos 2015; Goodfellow and Dimitrakopoulos 2017), instead of the traditional approach that considers the economic value of a block determined a priori and sequentially optimizes the extraction sequence, cut-off grade and transportation of materials downstream (Hustrulid and Kutcha 2006).

Furthermore, the proceeding case study strategically determines the optimal production rate during the mine production scheduling process using an adaptive simultaneous stochastic optimization. Several frameworks directly integrate investments into the optimization to achieve a certain level of production and increase the value of the operation (Groeneveld and Topal 2011; Groeneveld et al. 2012; Goodfellow 2014). These integrative frameworks allows the optimizer to decide the most suitable time to invest in capital investment overcoming limitations of defining the optimal mining and processing rates prior to optimizing the production schedule (Godoy and Dimitrakopoulos 2004; Del Castillo and Dimitrakopoulos 2014).

This work presents a major case study in a multi-mine and multi-process gold mining complex, where an adaptive simultaneous stochastic optimization approach strategically considers investment alternatives in mining equipment, process plant upgrades and the tailings management area. In the following sections, the adaptive simultaneous stochastic optimization approach is outlined, followed by a comprehensive case study at a gold mining complex containing two open-pit mines, twelve material types, twelve stockpiles, three external sources (including an underground mine) and three processing stream alternatives. Subsequently, the conclusions and future work are presented.

3.2. Method

This section summarizes the method used for the adaptive simultaneous stochastic optimization approach proposed by Del Castillo and Dimitrakopoulos (2019), which allows the production schedule to branch on a set of feasible investment alternatives.

3.2.1. *Definitions and notation*

A mining complex is designed to include a set of open-pit and underground mines ($\mathcal{M}$), stockpiles ($\mathcal{S}$), processors ($\mathcal{P}$), and waste facilities ($\mathcal{W}$) (Montiel and Dimitrakopoulos 2015; 2017; 2018;Goodfellow and Dimitrakopoulos 2016; 2017). There can be many material types that are either extracted from the mine or generated through blending and processing. Each material has a set of attributes which can be transferred through the mining complex (i.e. mass, metal content, etc.). Attributes are further divided into two sub-categories; primary attributes that define the composition of the material passed between various locations in the mining complex; and hereditary attributes which are a derived through linear and non-linear expressions. Hereditary attributes track important information in the model including the costs incurred at different locations, revenues from the various processing streams, and metal grade. Two variables $v_{p,i,t,s}$ and $v_{h,i,t,s}$ quantify the value of primary ($p \in \mathbb{P}$) and hereditary ($h \in \mathbb{H}$) attributes at each location $i \in \mathcal{M} \cup \mathcal{S} \cup \mathcal{P} \cup \mathcal{W}$ in period $t \in \mathbb{T}$ under scenario $s \in \mathbb{S}$, respectively. Hereditary attributes allow both non-linear and linear functions to be incorporated into the model and are a function of the primary attributes, $f_h(p, i, k)$ for each primary attribute $p \in \mathbb{P}$ at location $i \in \mathcal{M} \cup \mathcal{S} \cup \mathcal{P} \cup \mathcal{W}$ and considering each available capital investment $k \in \mathbb{K}$. The primary source of material for the mining complex is obtained by extracting a set of mining blocks $b \in B_m$ from mine $m \in \mathcal{M}$. Every block b has a set of simulated primary attributes $\beta_{p,b,s}$, which are inputs into the optimization model (Boucher and Dimitrakopoulos 2012). The recovery of each attribute p at location $i \in \mathcal{P}$ in each scenario s is defined as $r_{p,i,t,s}$ and are calculated using a non-linear recovery function (Goodfellow 2014; Farmer 2016; Del Castillo 2018).

3.2.2. *Decision variables*

Considering a life-of-mine of $\mathbb{T}$ time periods, the adaptive simultaneous stochastic optimization approach aims to maximize the NPV of the mining complex and minimize deviations from the annual production targets. This is accomplished by simultaneously determining the optimal decisions for four decision variables: (i) the mining block extraction sequence; (ii) destination policy; (iii) processing stream decisions; and (iv) capital investment plan. The method uses a set of binary decision variables $x_{b,t,s}$ that denote whether a block b is extracted in period t, in simulation s. The destination policy is then defined by discretizing the range of metal grades into a set of bins to determine the cut-off grade policy during the optimization process (Menabde et al.

2007). Bins or groups $g \in \mathcal{G}$ are defined using k-means++ clustering algorithm for the primary block attributes $\beta_{p',b,s}\ \forall p' \subseteq \mathbb{P},\ b \in \mathbb{B}_m,\ m \in \mathcal{M},\ s \in \mathbb{S}$ of each material type (Goodfellow and Dimitrakopoulos 2016). The destination policy decision variable $z_{g,j,t,s} \in \{0,1\}$ determines if the blocks in group g are sent to destination $j \in O(g)$ in period t, where $O(g)$ is the set of locations where the group of materials can be delivered in scenario s. After the material reaches the first set of destinations, based on the extraction sequence decisions, the downstream material flow is controlled by the processing stream decision variables $y_{i,j,t,s} \in [0,1]$. The processing stream variable defines the portion of product that is sent from destination $i \in \mathcal{S} \cup \mathcal{P}$ to destination $j \in O(i) \subseteq \mathcal{S} \cup \mathcal{P}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$. Lastly, the capital investment decision variable $\omega_{k,s,t}$ defines if a capital investment $k \epsilon K$ is executed in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$. Subsequently explained in Section 2.3.

3.2.3. *Branching the production schedule*

Two different sets are used to describe the different types of investments branching (K^*) and non-branching ($K^=$), where $K^* \cup K^= = K$. Branching alternatives are large capital investments decisions that are only purchased once during the life of the mining complex. For example, purchasing large crushers or constructing a new tailings facility. The non-branching investments may occur multiple times over the planning horizon, for instance truck and shovel purchases. The decision tree outlines the optimal timing of the branching investments and a new node n is created for each branching decision; this is defined as a stage. An optimized mine plan is produced for each branch that is created. The representativity measure $R \in (0\ , 0.5)$ is a user defined parameter, which is used to describe the confidence interval for branching. The representativity measure outlines the probability required to invest over all scenarios, branch the production schedule, or not invest in each capital investment (Eq. 8).

$$\begin{cases} if\ the\ probability\ of\ investing\ in\ k^* < R \ \rightarrow\ do\ not\ invest\ in\ k^*\ during\ t^\omega \\ if\ probability\ of\ investing\ in\ k^* \in [R, 1-R] \ \rightarrow\ branch\ during\ t^\omega \\ if\ the\ probability\ of\ investing\ in\ k^* > 1-R \ \rightarrow\ invest\ in\ k^*\ during\ t^\omega \end{cases} \quad [8]$$

The branching mechanism is described in the subsequent steps:

1. Calculate the probability of investing in all alternatives $k^* \in K^*$ in each time period t.

2. If there are a representative number of scenarios that choose to purchase the investment alternative, within an allotted time window, the solution branches and a new stage is created. Although, if the probability of investing is less than the threshold then the optimization will not branch, and the investment is not purchased. On the contrary, if the probability is greater than $(1 - R)$ there is no branching and the investment is made over all scenarios. This is mathematically described in Eq. 8.

3. Given there are $\mathbb{S}_n \subseteq \mathbb{S}$ scenarios that belong to the root, these scenarios are partitioned into $\mathbb{S}_{n1}$ and $\mathbb{S}_{n2}$ when branching occurs. Therefore, when combined all the simulations from each branch are at the root ($\mathbb{S}_{n1} \cup \mathbb{S}_{n2} = \mathbb{S}_n$) and when the simulations are partitioned each simulation can only report to one of the two partitions ($\mathbb{S}_{n1} \cap \mathbb{S}_{n2} = \emptyset$).

A time window, $t_\omega = \{t - \omega,\ t + \omega\}$, is used to stabilize the solution as often there may be a representative number of scenarios between one or two consecutive periods making it more desirable to invest in one of those two years rather than completely ignoring the investment opportunity. ω is set as an integer value that allows the model to expand the time window of the branching mechanism. The branching or new stage will begin during the floor of the expected time period of investment k^* and is denoted as t^*. Lastly, N defines the minimum number of scenarios in a branch required to allow for further branching in periods $t + 1\ \in \mathbb{T}$.

3.2.4. *Capital investments*

Capital investments are critical decisions that require a lead time (τ_k) to assemble or construct. For each investment alternative $k \in K$ there is a life expectancy (λ_k) and a unitary increase in capacity $\left(\kappa_{k,h}\right)$ that comes at a discounted purchase cost $\left(p_{k,t}^K\right)$ for each period $t \in \mathbb{T}$. The periodicity (ψ_k) of the investment decisions is also incorporated into the optimization model to simplify the optimization process and ensure a practical plan. The number of investments undertaken is denoted by $\sigma_{k,t,s}$ for each investment $k \in K$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.

3.2.5. *Objective function and constraints*

$$\max \frac{1}{\|\mathbb{S}\|} \sum_{s\in\mathbb{S}} \sum_{t\in\mathbb{T}} \left\{ \underbrace{\sum_{i\in\mathcal{M}\cup\mathcal{S}\cup\mathcal{P}\cup\mathcal{W}} \sum_{h\epsilon H} p_{h,t} v_{h,i,t,s}}_{\substack{Profit\ of\ the\ mining\ complex \\ \boldsymbol{Part\ I}}} - \underbrace{\sum_{k\epsilon K} p_{truck,t} \sigma_{truck,t,s}}_{\substack{Cost\ of\ Truck\ Investments \\ \boldsymbol{Part\ II}}} \right.$$

$$- \underbrace{\sum_{k\epsilon K} p_{shovel,t} \sigma_{shovel,t,s}}_{\substack{Cost\ of\ Shovel\ Investments \\ \boldsymbol{Part\ III}}} - \underbrace{\sum_{k\epsilon K} p_{k^*,t}^{K^*} \sigma_{k^*,t,s}}_{\substack{Cost\ of\ One\ Time\ Capital\ Investments \\ \boldsymbol{Part\ IV}}} \qquad [9]$$

$$\left. - \underbrace{\sum_{i\in\mathcal{M}\cup\mathcal{S}\cup\mathcal{P}\cup\mathcal{W}} \sum_{h\epsilon H} (c_{h,t}^{+} d_{h,i,t,s}^{+} + c_{h,t}^{-} d_{h,i,t,s}^{-})}_{\substack{Penalties\ for\ Deviations \\ \boldsymbol{Part\ V}}} \right\}$$

The objective function (Eq 2) maximizes the expected profit obtained by summing the revenues generated from the metal produced and subtracting the various costs, for example, transportation, mining, processing and refining costs (Part I). In addition, the objective aims to minimize the costs of investing in trucks and shovels (Part II & III), and one-time capital investments (Part IV). Part V minimizes the deviation from production targets, actively managing uncertainty. The adaptive optimization approach will only purchase investments when they lead to an increase in overall profitability and/or improve the capability to meet production targets in the mining complex.

Integrating the feasible investment alternatives into the optimization model changes the standard formulation of capacity constraints, from static lower ($L_{h,i,t}$) and upper ($U_{h,i,t}$) bounds, to dynamically changing capacities that are determined during the optimization. The capacities reflect changes in the corresponding investment decisions $\omega_{k,s,t}$. $\kappa_{k,h}$ represents the unitary increase in production capacity:

$$v_{h,i,t,s} - d_{h,i,t,s}^{+} \leq U_{h,i,t} + \sum_{k\ \epsilon\ K;\ t>\tau_k} \sum_{t'=t-\lambda_k-\tau_k}^{t-\tau_k} \kappa_{k,h} \cdot \omega_{k,s,t'} \qquad [10]$$

$$v_{h,i,t,s} + d^-_{h,i,t,s} \geq L_{h,i,t} + \sum_{k \,\epsilon\, K;\ t > \tau_k} \sum_{t'=t-\lambda_k-\tau_k}^{t-\tau_k} \kappa_{k,h} \cdot \omega_{k,s,t'} \qquad [11]$$

$$\forall\, h \,\epsilon\, H, i \in \mathcal{M} \cup \mathcal{S} \cup \mathcal{P} \cup \mathcal{W}, t \,\epsilon\, \mathbb{T}, s \,\epsilon\, \mathbb{S}$$

$$d^+_{h,i,t,s}, d^-_{h,i,t,s} \geq\ 0 \qquad [12]$$

When investments are activated the capacity expansions and contractions can be explored allowing for changes to the extraction rate, processing capacity, and storage of waste materials.

In addition, non-anticipativity constraints ensure that all scenarios within the same branch must undertake the same decisions. The problem is initialized with the solution from a two-stage stochastic integer program and then non-anticaptivity constraints are enforced for the first period. Subsequently, the mechanism for branching iteratively solves a series of sub-problems to determine the optimal period to invest. The non-anticipativity constraints are then dynamically enforced over an iteratively increasing time frame T^{α} when a branching investment is undertaken. For example, once the first branching period is established non-anticipativity constraints become active for all periods up to t^*, the period a branching investment is undertaken. This ensures that the optimization framework will not change earlier decisions in anticipation of the investments made in future periods. A binary variable $u^n_{k^*,t}$ equals one when the design branches over option $k^* \,\epsilon\, K^*$ in node n in period $t \,\epsilon\, \mathbb{T}$ and otherwise zero. Therefore, the variable A determines whether the non-anticipativity constraints are activated (0) or not (1) for a given partition of scenarios in a single branch:

$$A = \left\lceil \frac{\sum_{k^* \epsilon K^*} u^n_{k^*,t}}{|K^*|} \right\rceil = \{0,1\} \qquad [13]$$

hen there is no branching all decision variables must be the same for all scenarios. However, when branching occurs the scenarios partition $\mathbb{S}_{n1} = \{s; w_{k^*,t^*,s} = 1, \forall s \in \mathbb{S}_n\}$, $\mathbb{S}_{n2} = \mathbb{S}_n \backslash \mathbb{S}_{n1}$. Examples of the non-anticipativity constraints are below:

$$(1 - A)\left(x_{b,(t+1),s} - x_{b,(t+1),s'}\right) = 0, \quad \forall t \in T^{\propto}; b \in M \qquad [14]$$

$$(1 - A)\left(z_{g,j,(t+1),s} - z_{g,j,(t+1),s'}\right) = 0, \quad \forall t \in T^{\propto}; g \in G;\ j \in \mathcal{M} \cup \mathcal{S} \cup \mathcal{P} \cup \mathcal{W} \qquad [15]$$

$$(1 - A)\left(w_{k,(t+1),s} - w_{k,(t+1),s'}\right) = 0, \quad \forall t \in T^{\propto}; k \in K \qquad [16]$$

The destination policy, extraction sequence, and capital investment decisions are the same for all scenarios within each branch of the decision tree. Lastly, in order to ensure stochastic solution stability there must be a minimum number of simulated scenarios in each partition.

3.2.6. *Solution method*

A multi-neighbourhood simulated annealing metaheuristic is used to solve the optimization model. Metaheuristics are required as the number of decision variables are in the order of hundreds of millions when considering multi-mine long term production schedule. The metaheuristic used in this work explores a neighbourhood or class of perturbations that are used to change decision variables and achieve near optimal solutions in a short period of time (Montiel and Dimitrakopoulos 2015; 2017;Goodfellow and Dimitrakopoulos 2016; 2017). Del Castillo (2018) introduces perturbations that change capital investment decisions including adding or removing multiple investments in a period and swapping two investments between periods. The simulated annealing algorithm then uses an acceptance probability to determine whether the new solution is accepted or rejected to further explore the solution space (Kirkpatrick et al. 1983). The modified simulated annealing approach, used in the subsequent case study, updates the probability of choosing a neighbourhood depending on its ability to improve the objective function (Goodfellow and Dimitrakopoulos 2016).

3.3. Case study at a gold mining complex

The adaptive simultaneous stochastic optimization approach is applied to a gold mining complex that consists of two large open-pit mines with twelve different material types. These materials can be transported to a number of destinations; an autoclave processing facility, oxide mill, oxide leach, twelve stockpiles (one for each material type), waste facility, and a tailings management area. Each mine contains a mixture of sulphide ores, which must be pretreated at the autoclave before processing, and oxide ores that can be sent to the oxide processor or oxide leach. The mining complex, including each of its component, and the allowable material routing are presented in Figure 8. Sulphide materials, a refractory ore type, can be extracted from either of the open-pit mines and sent to the autoclave, stockpile or waste dump facility. Stockpiles are separated for each material type to provide accessibility to materials of certain chemistry compositions, shown in

Table 5. Externally sourced material is used to supplement the ore feed that is produced from the two open-pit mines and sent to the autoclave to help meet blending requirements. The optimizer seeks opportunities to increase value and more effectively blend materials to obtain a satisfactory product quality for effectively running the autoclave. Sulphide or refractory ores must be blended to achieve the permissible operating criterion for the autoclave, by controlling the grades of sulphide sulphur (SS), carbonates (CO_3), organic carbon (OC), and the SS/CO_3 ratio. Therefore, these deleterious attributes must be managed within the optimization framework to ensure blending requirements will be meet. A constraint is added to the model to maintain the grade of SS and CO_3 from 3.8-4.2% and 4.5-6.5%, respectively. The deviations from these targets are penalized in the objective function to manage the risk similar to all the other production targets. Acid is used to pre-treat the ore by neutralizing CO_3 and ensuring the appropriate SS/CO_3 ratio (0.8-1.2) is entering the autoclave circuit. This becomes critical as there is variability in the material received from the different sources and often there are not enough materials with the desired qualities readily available. There is a maximum amount of acid (38,400 t) that can be used on an annual basis which introduces a constraint in the optimization process. The autoclave's target production is 2.5 Mt/y. Oxide materials can either report to the oxide mill, leach, or stockpile destinations and there are no constraints on the blending requirements for the oxide ore material. The oxide mill has a production target of 1.4 Mt/y and the leach pad is not constrained. After processing, the volume of mine tailings that are generated from the processing facilities are continuously examined to ensure there is a large enough containment area to continue mining, which then introduces a constraint on the available tailings capacity. Stockpiling facilities are used as intermediate locations to assist with blending and can be extracted from throughout the mine life. Lastly, any material that does not positively contribute to the NPV of the mining complex is sent to the corresponding waste dump facility.

Table 5. Material classification for blending and material routing

Material	Chemistry			
Type	**CO3**	**SS**	**OC**	**Oxide**
1	Med-Low	Low	-	-
2	Med-Low	High	-	-
3	Low	Med	-	-
4	Low	Low	-	-
5	Low	Med-High	-	-
6	High	-	-	-
7	Med-High	Low	-	-
8	Low	High	-	-
9	Very High	-	-	-
10	High	-	Med-High	-
11	-	-	High	-
12	-	-	-	High

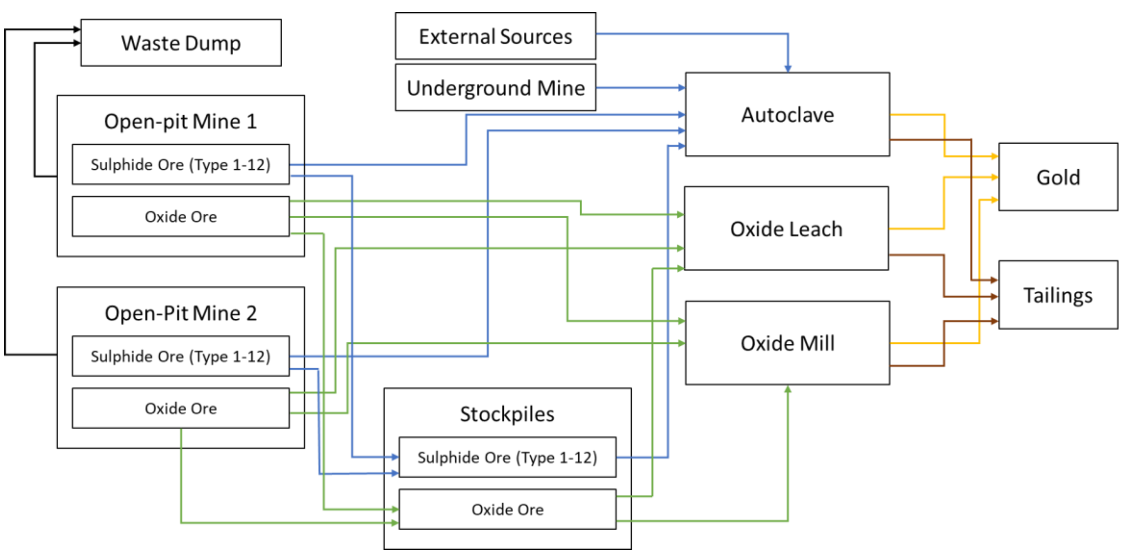

Figure 8. Mining complex and allowable material routing

In this case study, there are three one-time feasible investment alternatives considered throughout the optimization process to test the adaptive optimization approach. First, the annual autoclave processing throughput may be expanded by investing in two additional positive displacement piston-diaphragm pumps (Eichhorn et al. 2014). Second, an investment in the process plant autoclave circuit is evaluated to increase the allowable acid consumption and manage blending. Third, an investment alternative that considers the construction of a new tailings storage area increases the life-of-mine by allowing for the processors to continue operating. The pump installation increases throughput at the autoclave by 25% which allows for more refractory ore to be processed. The capital cost of this expansion is minimal, however, the cost of implementation and loss of production during the pump installation is also considered in the capital investment decision, resulting in a $1M investment. Acid is ordered annually to satisfy production requirements, but storage areas and adaptations to the autoclave pre-treatment circuit are required to safely utilize the additional acid. The expected investment is $0.2M. The most significant investment decision is related to the addition of a new tailings containment area which is expected to cost $200M to construct completely. The new tailings area results in a 33% increase in tailings storage for the mining complex. Once any of the three investments are purchased, they can be continuously used for the remainder of the mine life. Additionally, these three capital investment decisions can potentially allow the production schedule to branch. In this case study, a representativity measure $R = 0.3$ is used based on the acceptable risk of investing in capital at this mining operation. Therefore, the production schedule branches when a representative number of scenarios, between 30 and 70 %, invest in one of these three feasible alternatives. The scenarios are then split, and further branching considerations are assessed in future periods. Further details on the parameters considered for each of the capital investments are described in Table 6.

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Table 6. Parameters and cost of capital investments

Parameters	Non-branching		Branching Expansions		
	Shovel	Truck	Tailings	Autoclave	Acid
Lead Time (years)	2	2	3	2	3
Capital Cost (M$)	6.7	1.1	200	1	0.2
Life of Equipment (years)	7	7	13	13	13
Periodicity of Decision (years)	3	3	13	13	13
Increase in Capacity	Feed for 5 trucks/unit	0.93 Mt/unit	5.75 MCM	925 kt	9.6 kt

The mine initially begins with 45 haul trucks and 9 shovels that have two years remaining in their productive life before salvaging. The model dynamically considers the purchase of trucks and shovels throughout the thirteen-year production schedule. Truck and shovel purchases define the annual mine production rate. The cost per truck and shovel is $1.1M and $6.7M, respectively, which is accounted for in the annual cashflows. Allowing for the optimizer to decide on the appropriate time to invest in trucks and shovels throughout the mine life. The mining operation has an aging fleet and it is planning to replace the originally purchased haul trucks with a new fleet. The ability to consider the purchase of new equipment during the optimization provides an opportunity to re-establish the optimal mining rate to satisfy the processor requirements and maximize the value of the operation. The trucks and shovels have a corresponding lead time of two years to provide a suitable amount of time for purchasing equipment from the manufacturer, shipping, and on-site assembly. In addition, they have an expected equipment life of seven years and a purchase can be made every three years stabilizing the production rate.

3.3.1. *Base case mine production schedule*

A base case mine production schedule is defined herein using a simultaneous stochastic optimization approach that considers capital investment decisions within the optimization framework while managing uncertainty, however, branching is not considered. The base case mine production schedule can choose to invest in trucks, shovels, and the available expansions, but it can not branch and adapt to uncertainty by considering alternatives; it must either choose to invest or not invest. This is different then the adaptive simultaneous stochastic optimization that can be used to evaluate different alternatives and their corresponding value, as there is a fixed production schedule that must be executed in one way, which does not consider the value of having alternative options to manage uncertainty to a greater extent. The results from the base case mine production schedule are compared with the adaptive branching approach that considers feasible capital investment alternatives. Each method uses a set of multi-variate stochastic simulation of the orebody for each open-pit as input into the optimization model (Boucher and Dimitrakopoulos 2012; Rossi and Deutsch 2014). The external sources are simulated based on historical data associated with variability in the supply and quality of material received from other mines in the region. The variability and uncertainty of the material sources are accounted for directly in the optimization framework, unlike conventional frameworks that use a single estimated orebody model as input (Hustrulid and Kutcha 2006). Lastly, the open-pit mines have a block size of 30 m x 30 m x 20 m, representing the selective mining unit and contain 296k and 172k blocks in Mine 1 and Mine 2, respectively. The results from the base case production schedule including the extraction sequence, capital investments, stockpiling, blending, mining rate and processing decisions follow.

Figure 9 defines the base case mining rate alongside the truck and shovel investment decisions. Noticeably the amount of equipment that is required is decreasing as the mine life proceeds and as the older equipment is approaching the end of its operational life. An opportunity arises to operate the two mines at a lower mining rate. Although a lower mining rate is utilized, the ability to satisfy the autoclave processor (Figure 10a) and oxide mill (Appendix A1) is fulfilled and a resulting NPV of \$3.65B is achieved in the 50th percentile (P-50). The base case mine production schedule invests in both the expansion of the tailings management area and the additional acid storage facility. The investment in additional pumps do not contribute an increase in the mining complex’s NPV when accounting for all scenarios, consequently the pumps are not purchased. The blending constraints are satisfied, between the upper (UB) and lower bounds (LB), in most periods through the utilization of stockpiles and other available material (Figure 10b, Figure 10c). However, during the first period, the blending constraints are unachievable as the material that can be extracted during that period does not have the appropriate properties to meet the blending requirements. As the production schedule proceeds, stockpiles are established to help with blending in future periods. The operational costs of stockpiling these materials are integrated into the optimization to ensure that the stockpiling decisions contribute to the profitability of the mining complex and help manage the supply uncertainty.

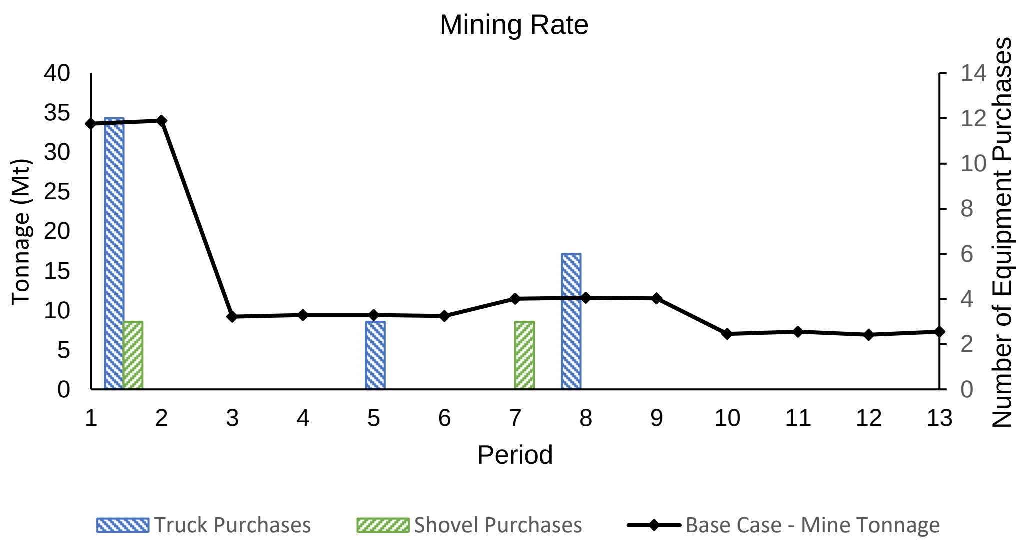

Figure 9. The mining rate and shovel/truck purchase plan for the base case production schedule with no branching

Lastly, the base case production schedule invests in a tailings expansion in period 7. This investment increases the storage capacity and becomes available in period 10 (Figure 11). The increased tailings storage prolongs the mine life by three periods and allows for 1-2 more years of gold production if the duration of this schedule was increased. This results in an additional $0.7B in discounted cashflows generated. Waste management considerations, such as tailings disposal, are important to optimize directly in the mine production scheduling process in order to generate feasible life of mine designs. Additionally, the processor upgrade that allows for additional acid consumption was purchased in period 3 allowing for 20% increase in additional acid consumption in subsequent periods (Figure 12) This controls the blending requirements at the autoclave processing stream.

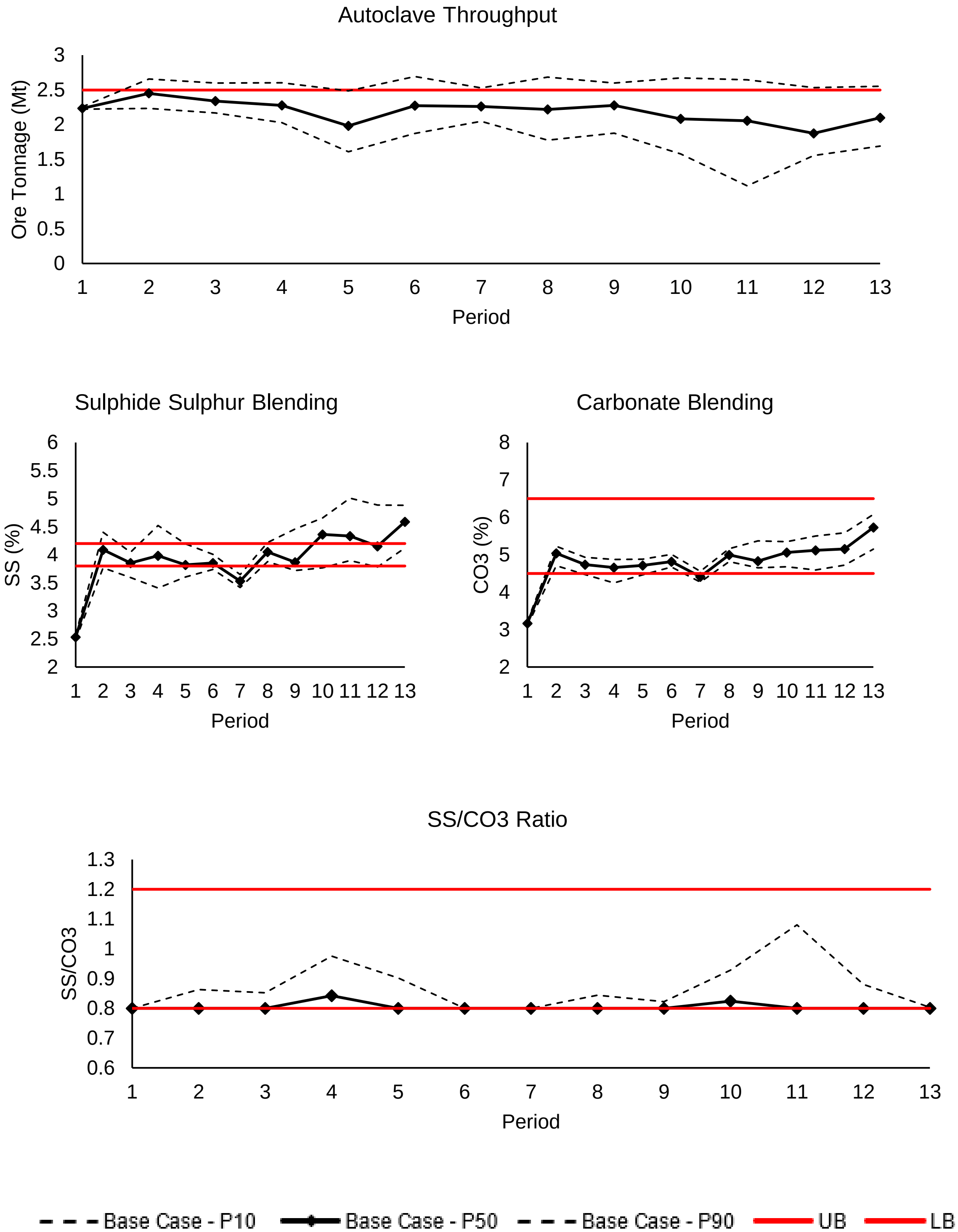

Figure 10. Base case autoclave throughput and blending: (a) no expansion taken in the optimization for additional throughput; (b) blending of SS; (c) blending of CO_3; and (d) maintaining the SS/CO_3 ratio for ideal operating conditions

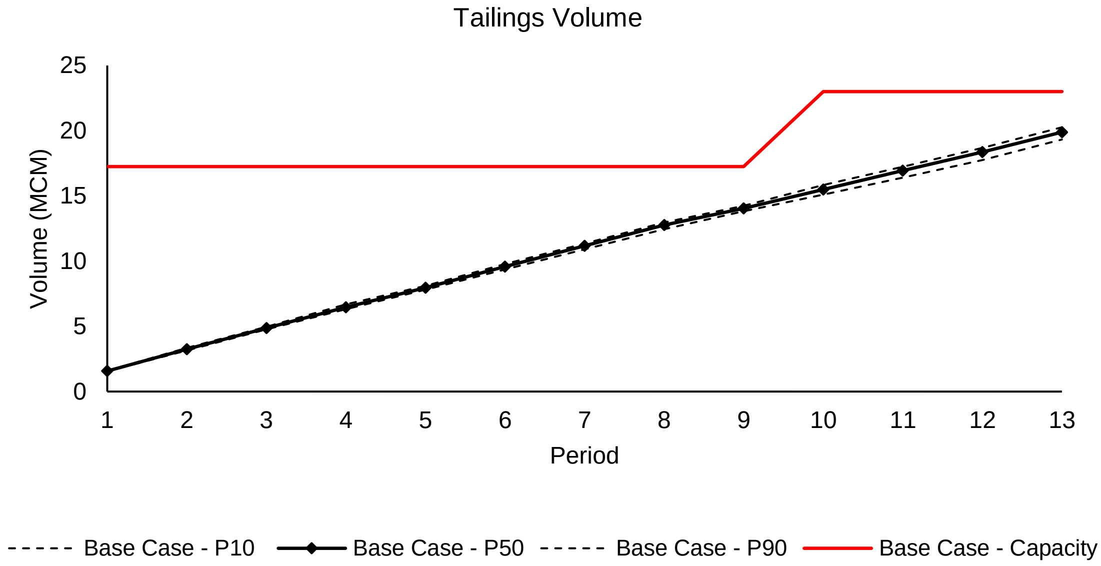

Figure 11. Tailings production over the long-term production schedule and the available capacity expanded in period 10

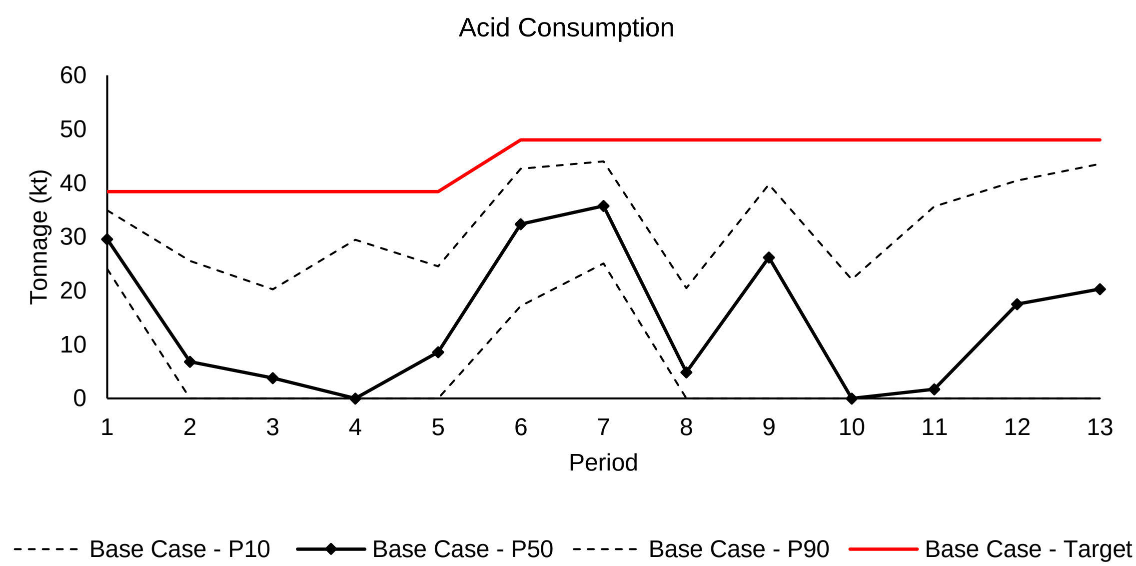

Figure 12. Annual acid consumption with additional capacity obtained in period 6

3.3.2. *Adapting to supply uncertainty in a gold mining complex*

The previously mentioned results will be compared with the adaptive stochastic optimization that considers branching on feasible investment alternatives. During the adaptive simultaneous stochastic optimization groups of scenarios are optimized to determine if there is a beneficial time to invest in any of the one-time capital investments alternatives described previously. The scenarios that lead to a branching decision are separated based on those that invest and those that choose not to invest in the time window. The scenarios that choose not to invest maintain the ability to invest in the capital investment in future periods, while the scenarios that invest lock-in that decision for that period activating the non-anticipativity constraints. The scenarios are grouped into separate branches and optimized to produce a feasible alternative for both investing and not investing in the solution. A representative number (over 30%) of scenarios must undertake the same decision for the solution to consider branching or investing in these alternatives, which reduces the number of branches and prevents overfitting the decision tree to each scenario. It is important to note that the scenarios in each branch all undertake the same decisions until a new branching decision is made.

Based on the available capital investments, it was first determined that the additional acid capacity was a suitable investment for greater than 70% of the scenarios leading to a non-branching investment decision. The first investment helped improve the ability to meet the quality requirements of the autoclave. After considering all the simulated scenarios (geostatistical simulations of each open-pit mine and an uncertain external source) and the branching mechanisms criterion, the first branching decision is undertaken allowing for the expansion of the autoclave throughput by installing two additional positive displacement pumps. This separates the number of scenarios into a group of 115 scenarios in branch 1 (B1) that invest and 205 scenarios in branch 2 (B2) that do not invest. After the branching occurs, the optimizer also decides to invest in the additional tailings capacity in more than 70% of the scenarios, for both branches, preventing further growth of the scenario tree. The resulting feasible alternatives both produce a higher NPV then the base case production schedule achieving a value of $3.89B and $4.66B in B1 and B2, respectively (Figure 13). This accounts for a 6.4% and 27.5% increase in NPV when comparing the P-50 of each alternative to the base case production schedule. Each of the branches or feasible alternatives perform better than the base case production schedule, however, this may not always be the case as there could be a group of scenarios that underperforms the base case production schedule. The method prevents overfitting by ensuring a number of scenarios do not become too few within each branch and that there is a significant difference in the number of scenarios that either invest or maintain the same operating conditions, hence the representativity parameter which ensures between 30-70% of the scenarios will be split and not a small group of outliers. This substantially reduces the number of branches and ensures feasible stable solutions. The changes in the investment decisions result in a very different response in the production scheduling process, as shown in Figure 14, when comparing the N-S cross sections. First observing, the solution is the exact same until branching occurs and then noticing the schedules change dramatically to take advantage of the new capital investments. There are a number of similarities between the base case and B2 in terms of depth and extents of the mine. However, in B1 there is a large portion of the mine that is no longer extracted in the north, when compared to the other two mine plans. This entails there is some high material variability and uncertainty in this section of the mine that leads to large changes in the resulting mine plan.

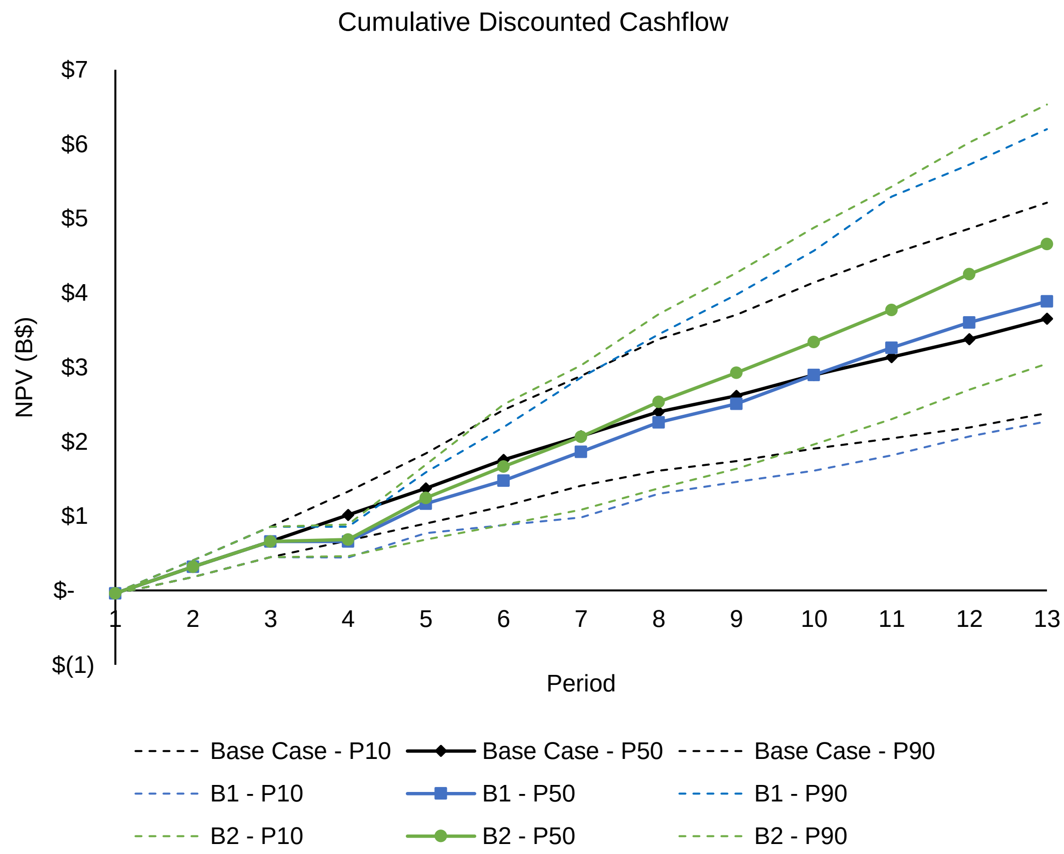

Figure 13. A comparison of the resulting NPVs from the adaptive branching and non-branching base case production schedule

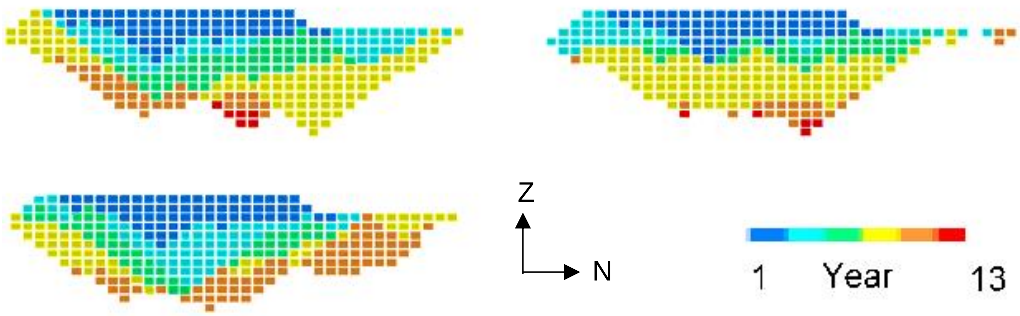

Figure 14. N-S cross section of mine production schedule Mine 1: a) base case (top left); b) branch 1 (top right); and c) branch 2 (bottom left)

B1 invests in the autoclave expansion (Figure 15), which can be fully utilized in period six, and has the lowest mining rate over the long-term production schedule. A comparison of the mining rates are reviewed in Figure 16, where the resulting production rates directly correlate to the amounts of trucks and shovels purchased. The autoclave expansion results in lower grade refractory ore material being processed and a higher throughput being used at the autoclave. Over the long-term production schedule, there is a 9% reduction in the number of gold ounces produced over the life of mine when compared with the P-50 of the base case scenario (Figure 17). However, the reduction in mining costs due to the lower mining rate overcomes the loss in revenue and results in a higher NPV. The lower mining rate is feasible as the throughput outweighs the grade of material through the autoclave changing the selectivity between ore and waste material. Lower utilization of the oxide processing facilities also decreased the operating costs. In B1, the optimizer has a challenging time meeting the blending constraints and is unable to provide the appropriate material to attain the blending targets, making the acid investment a critical decision for ensuring there is a suitable SS/CO_3 ratio. These indicators are shown in Appendix A2.

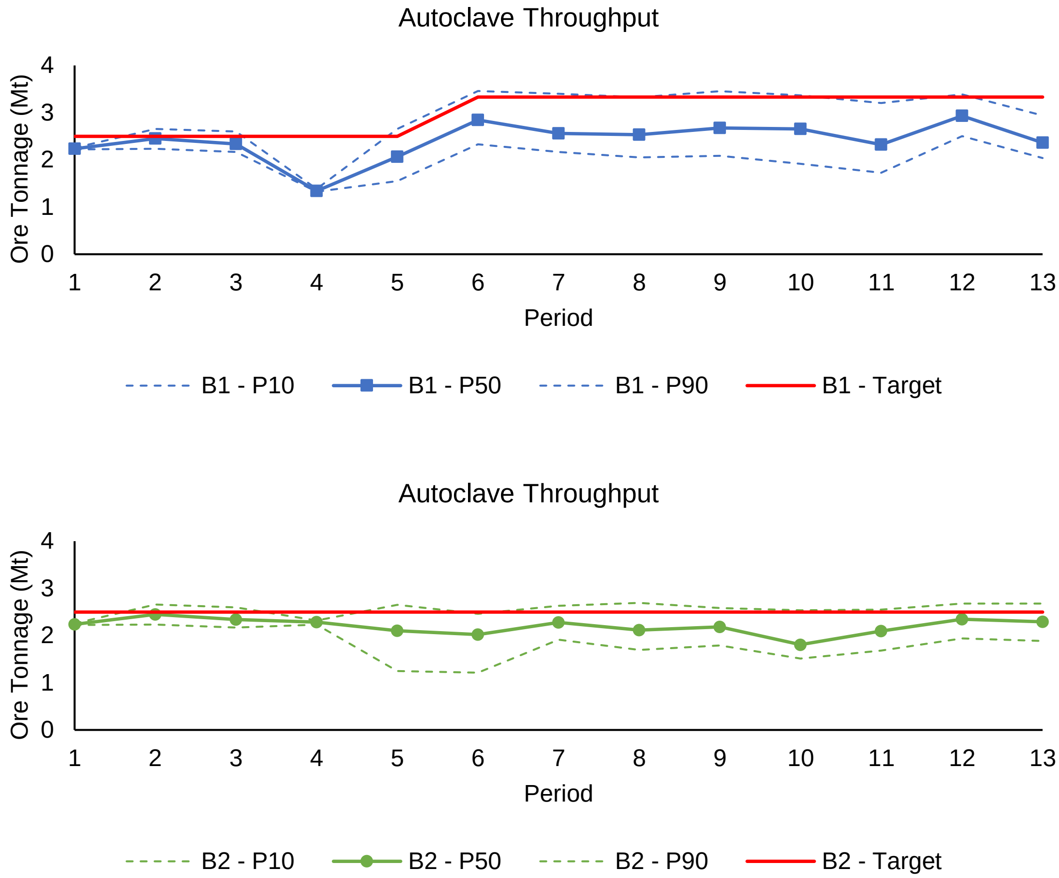

Figure 15. Autoclave throughput and targets: a) B1; and b) B2 with investments

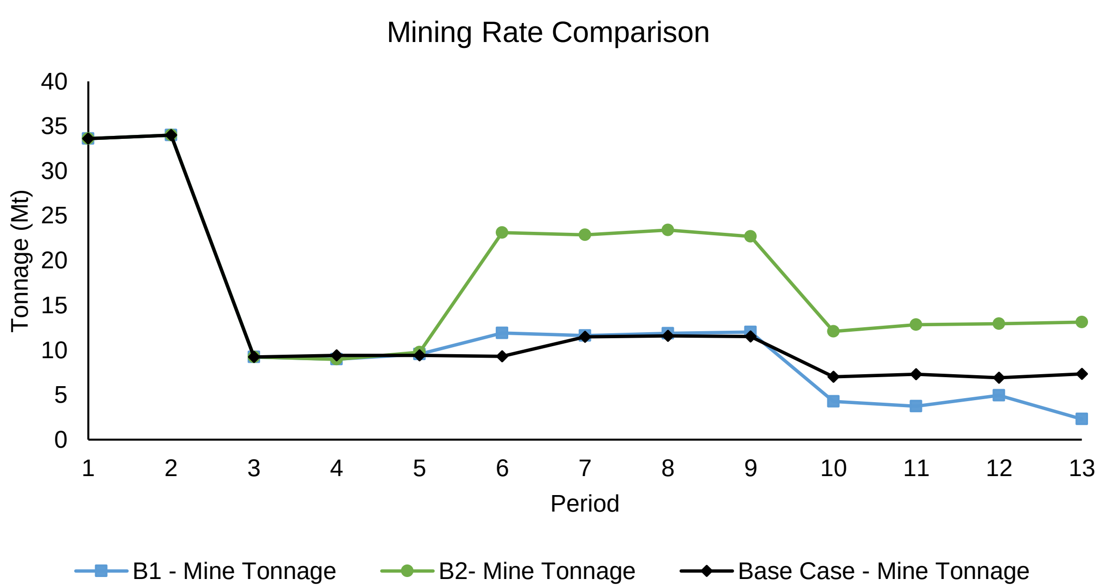

Figure 16: A comparison of the mining rates required to satisfy each production schedule

B2 performs quite differently and instead increases the size of the truck and shovel fleet, which results in a higher extraction rate and ensures that higher-grade refractory ore is being sent to the processor. The oxide processing streams are utilized far more in B2 than in B1 and their target production is maintained during most periods. A higher stripping ratio is required to move the additional waste between periods five and nine (Figure 18), which is the reason for the additional truck and shovel requirements. Increasing the selectivity, between ore and waste, results in a substantially higher NPV, which B1 was unable to achieve even with the autoclave capacity expansion. The larger contribution in NPV is primarily due to the accessibility to oxide materials in the different groups of simulations and the uncertainty and variability in the gold, SS, CO_3, and OC grades. Here the adaptive approach is able to take advantage of understanding the inherent variability of the mineral deposits and identifies there is an important investigation to commence. This includes more information with regards to the mineralization of oxide materials and stricter guidelines in terms of the quality of material received from external sources before deciding on the autoclave expansion. B2 produces 10% more gold by fully utilizing all the processing stream capacities and better satisfying the blending constraints. The increased utilization of the oxide leach and mill contribute significantly more gold ounces.

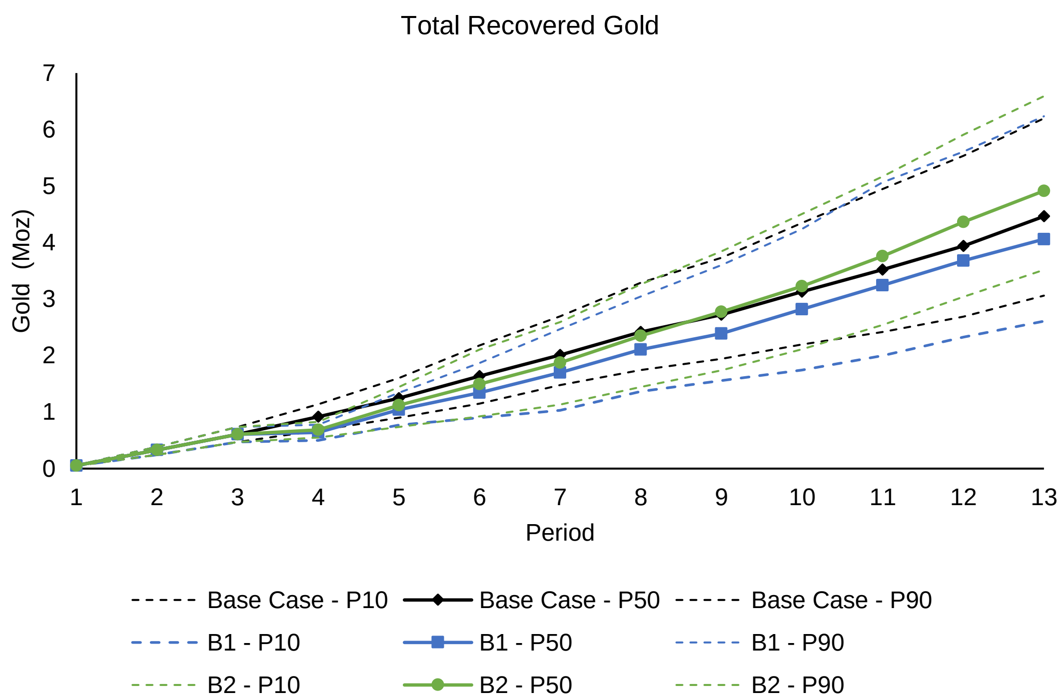

Figure 17. Comparison of the total recovered gold over the long-term production schedule

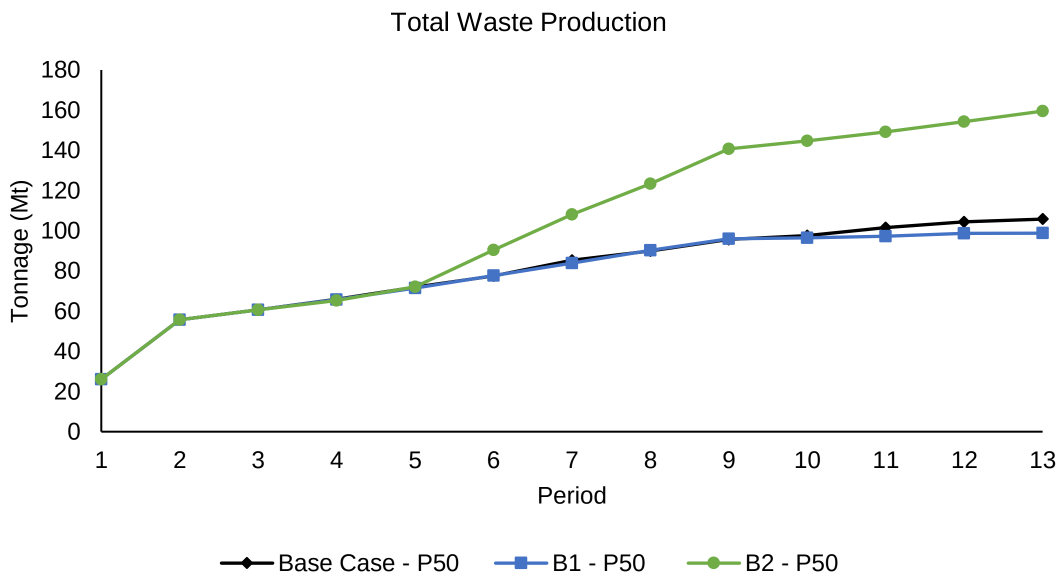

Figure 18. Total waste production over long-term production schedule

Both of the feasible alternatives B1 and B2 invest in the additional tailings containment area in period seven and receive the additional capacity in period ten, similar to the base case. Although, B1 produces less gold it still produces a similar volume of tailings due to the additional throughput at the autoclave processor (Figure 19). Had the tailings expansion not been considered during the optimization process, processing would have been required to stop in period ten and a loss of $1B and $1.3B of additional cashflow would be lost in B1 and B2, respectively. This would be a larger loss than the resulting $0.7B in the base case production schedule. The potential loss highlights the importance of simultaneously optimizing the entire mining complex to further understand the intrinsic value of each investment decision.

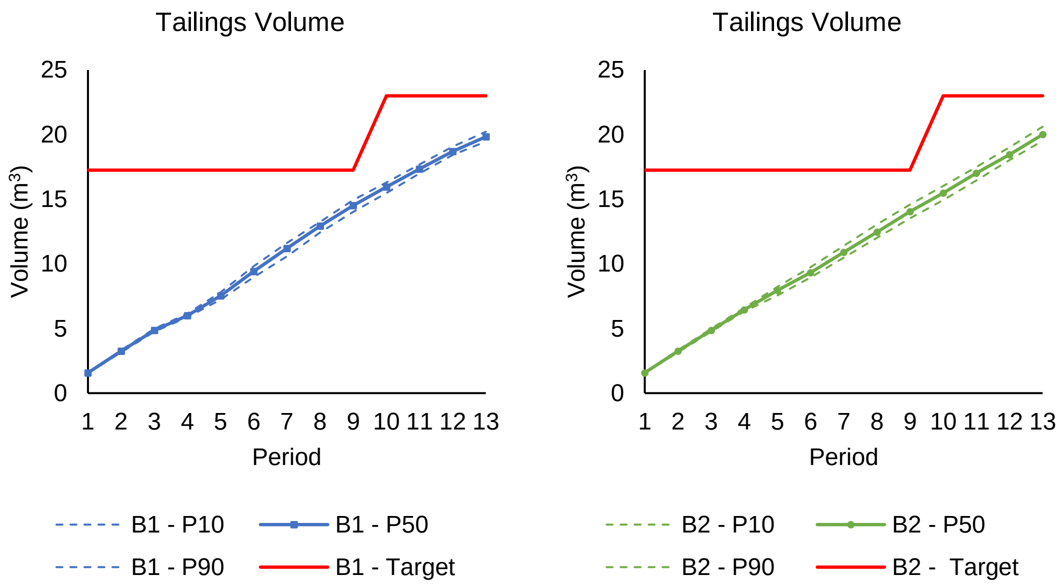

Figure 19. Total tailings production with investment decisions

3.4. Conclusions

The simultaneous stochastic optimization of a gold mining complex is presented using an adaptive method that integrates feasible capital investment alternatives. The framework capitalizes on synergies and adapts to uncertainty resulting in a 6.4% and 27.5% increase in NPV in B1 and B2, respectively, while satisfying a wide array of production targets and managing supply uncertainty. Investments in trucks and shovel define a new mining rate that minimizes capital expenditures and satisfies each processors capacity. Additionally, an investment in a tailings facility expansion and additional acid consumption increase the life of the mining complex and manage variable material quality at the autoclave processor. Integrating tailings management into the optimization process increases the NPV by 0.7B in the base case production schedule and leads to an additional \$1B and \$1.3B in B1 and B2, respectively. This emphasizes the importance of considering waste and tailings management in the optimization process to capitalize on the available synergies. The optimizer chooses to branch the production schedule when the autoclave expansion is considered and identifies uncertainty and local variability associated with the supply of oxide and refractory ores sent to each processor. This leads to different mine plans and operating requirements for the processing streams and mining equipment, which is dependent on whether the investment alternative is purchased. The feasible investment alternatives provide a high-level insight on the appropriate attributes to investigate including highly variable areas of the deposit and large differences in the quantity of oxide materials being mined. The optimized production schedule does not branch for the first three years and provides the appropriate lead time to evaluate each alternative decision and gather the required information to make an informed final production schedule.

If either of the feasible alternatives are executed, the expected NPV increases substantially. The base case and adaptive approaches capitalize on the synergies that exist between the different components of the mining complex helping to manage the challenging blending constraints and determine the appropriate size of the mining fleet directly in the optimization. The results from this case study emphasize the importance of modelling the entire mining complex in a single optimization process. In addition, the branching mechanism and adaptive ability of the optimizer provides a method to easily evaluate several feasible alternatives and further understand the variability and uncertainty associated with the mining complex.

4. Conclusions

The simultaneous stochastic optimization framework is presented through innovative applications that strategically generate a production schedule in two gold mining complexes. These applications showcase the ability to integrate waste management and capital investment decisions directly into a single optimization formulation that maximizes the NPV based on the value of the products sold and manages technical risk. The resulting production schedule simultaneously determines the extraction sequence of multiple mines, stockpiling, processing, destination policy, waste management, and capital investment decisions, while managing uncertain raw material supply.

The first work, explained in Chapter 2, demonstrates the importance of quantifying uncertainty of both ore and waste using a set of stochastic orebody simulations to reproduce the uncertainty and variability of the raw material supply. Overcoming the limitations of previous methods, which primarily focus on satisfying the processing stream requirements, this approach considers the implications of acid generating waste on the long-term production schedule at an operating mining complex. The uncertain production of acid generating waste requires management to satisfy permitting constraints and minimize disruptions to the environment through contamination and surface disturbance due to regulations issued by the government that help protect important natural resources. The results generated using the simultaneous stochastic optimization framework are compared with a base case production schedule that uses as an estimated orebody model as input and a pre-determined cut-off grade policy. The base case production schedule is tested using the set of stochastic orebody simulations and it is determined that the schedule generated has 50% probability of producing 18% less gold then expected. In addition, the schedule is highly likely to produce 12% more acid generating waste than originally expected. This identifies that the deterministic production schedule is highly likely to cause substantial challenges throughout the remaining mine life. The production schedule generated using the simultaneous stochastic optimization approach balances the requirements of the processing facility and waste management by simultaneously optimizing the cut-off grade policy and managing technical risk during the optimization. The production schedule reduces the total amount of waste produced and utilizes lower mining rate, while still satisfying the other production targets. The large change in mining rate lowers mining costs and equipment expenditures resulting in a 6% increase in the NPV.

The second application, discussed in Chapter 3, presents an adaptive approach to the simultaneous stochastic optimization framework that considers branching the production schedule on a number of feasible investment alternatives. Similar, to the first application the two-stage SIP aims to maximize the value of products sold and minimize risk of deviating from production targets as a result of supply uncertainty. The adaptive nature of the method applied allows the production schedule to branch when an investment decision is undertaken and there is polar behaviour between groups of scenarios, where one large group of scenarios chooses to invest and another large group of scenarios does invest in the same time period. The branching mechanism splits the production schedule based on these decisions. This approach provides a number of feasible investment alternatives that can be switched between depending on how new information unfolds and provides a strategy evaluating different investments and the underlying uncertainty of undertaking different capital investments. In addition, the branching mechanism identifies areas of the production schedule that should be investigated due to supply uncertainty, providing an understanding of the most uncertain components in the mine plan. The framework capitalizes on synergies and adapts to uncertainty resulting in a 6.4% and 27.5% increase in NPV in B1 and B2 when compared to a two-stage SIP with capital investments, respectively. Investments in trucks and shovels define a new mining rate that minimizes capital expenditures and satisfies each processors capacity. Additionally, an investment in a tailings facility expansion and additional acid consumption increase the life of the mining complex and manage variable material quality at the autoclave processor. The integration of tailings management into the optimization increases the

NPV by $0.7B in the base case production schedule and leads to an additional $1B and $1.3B in B1 and B2, respectively. The adaptive simultaneous stochastic optimization framework considers aspects of waste management including the expansion of the tailing management area and stresses the need to integrate waste management into the long-term production schedule for determining the appropriate size and location for tailings placement. The optimizer chooses to branch the production schedule when the autoclave expansion is considered and identifies uncertainty and local variability associated with the supply of oxide and refractory ores sent to each processor. This leads to a number of feasible alternatives with different operating requirements for the processing streams and mining equipment that are dependent on whether the investment alternative is purchased. The feasible alternatives provide a high-level insight on the appropriate attributes to investigate including highly variable areas of the deposit and large differences in the quantity of oxide materials being mined. The optimized production schedule does not branch for the first three years and provides the appropriate lead time to evaluate each alternative decision and gather the required information to make an informed final production schedule.

Both of these studies indicate the importance of considering the entire mining complex during the optimization process, while managing and quantifying supply uncertainty. The ability to capitalize on synergies and manage technical risk leads to schedules that are highly probable to outperform the production schedules generated using conventional mine planning approaches. In addition, the adaptive simultaneous stochastic optimization approach provides an innovative approach to test several feasible alternatives and further understand the variability and uncertainty associated with the mining complex.

Future work in simultaneous stochastic optimization should consider complex and integrative waste management approaches, for example, controlling water infiltration by layering and co-disposal opportunities (Aubertin et al. 2016; Antonaki et al. 2018). Through the work discussed herein, there are relevant observations that suggest material uncertainty is a critical component in understanding waste management and requires further understanding of the material boundaries and zones in a mining complex. The simulation of domain boundaries related to overburden, till, and acid generating material in a mineral deposit should help generate reclamation strategies that minimize long-term costs by reducing further material handling in future periods. In addition, research in choosing the fastest algorithms for solving the optimization problem using artificial intelligence and machine learning techniques will help lead to faster solution times and improve the ability to rapidly test a number of capital investment alternatives, while rapidly solving the production schedule for a multi-mine and process mining complex.

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Appendix A: Multi-neighbourhood simulated annealing with adaptive neighbourhood search

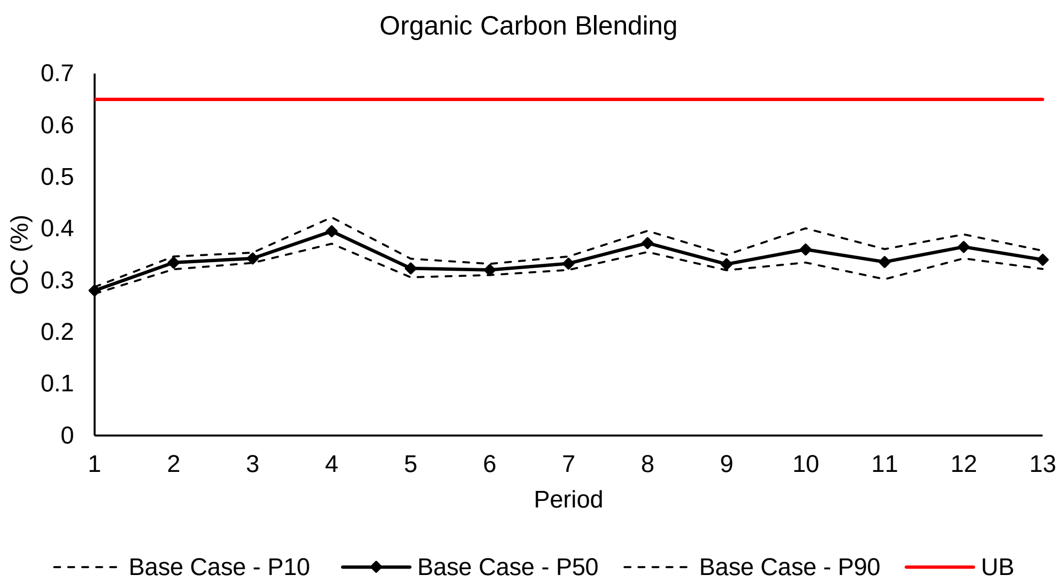
The multi-neighbourhood simulated annealing with adaptive neighbourhood search is the metaheuristic technique used to solve the simultaneous stochastic optimization of a mining complex and is detailed by Goodfellow and Dimitrakopoulos (2016). The solution approach requires an initial feasible solution and is generated based on a greedy algorithm that attempts to maximize the objective function and satisfy slope constraints. The initial solution is then optimized by a set of perturbations to generate a new solution. Based on the simulated annealing framework the probability of acceptance (P) is given by following formula:

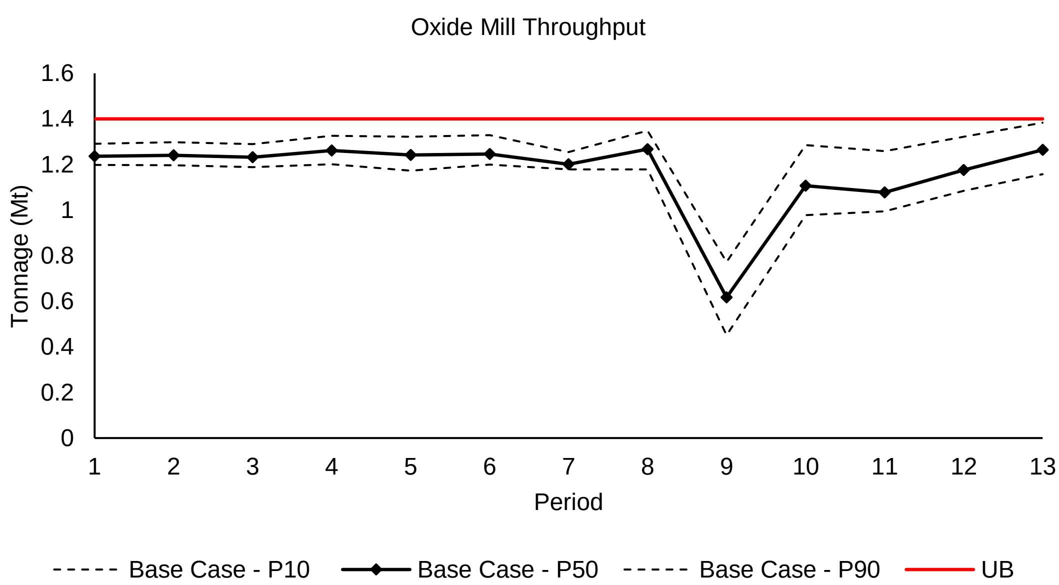
$$P(g(\Phi), g(\Phi'), T) = \begin{cases} 1, & if\ g(\Phi) \leq g(\Phi') \\ \exp\left(-\frac{|g(\Phi') - g(\Phi)|}{T}\right), & otherwise \end{cases} \qquad [17]$$

Suppose the solution vector Φ has an objective value of $g(\Phi)$. Then the multi-neighbourhood simulated annealing algorithm randomly selects a neighbourhood that corresponds to a specific change in the long-term production schedule by modifying: (i) the extraction sequence ($x \in \Phi$), (ii) destination policy ($z \in \Phi$) or (iii) processing stream and stockpiling decisions ($y \in \Phi$). The new solution Φ' is either accepted or rejected based on Eq. 17 and an initial temperature T. After a number of iterations, the temperature is decreased by a cooling factor $k \in [0,1]$. The difference between this approach and the standard simulated annealing framework is that instead of using a single temperature T the optimizer uses a parameter $\delta \in [0,1]$ that represents the probability of accepting a non-improving solution. Based on the parameter δ a temperature is retrieved from a cumulative probability distribution function that is updated based on the feedback from non-improving perturbations and instead of applying the cooling factor on k it is applied to δ. Some examples of the perturbations that modify the initial solution include (i) swapping the periods of different blocks, (ii) changing the destination of a group of materials, or (iii) changing the proportion of materials sent from the stockpiles. These mechanisms allow the solution space to be explored and the adaptive neighbourhood search helps guide the optimizer to choose neighbourhoods that have a higher probability of improving the solution.

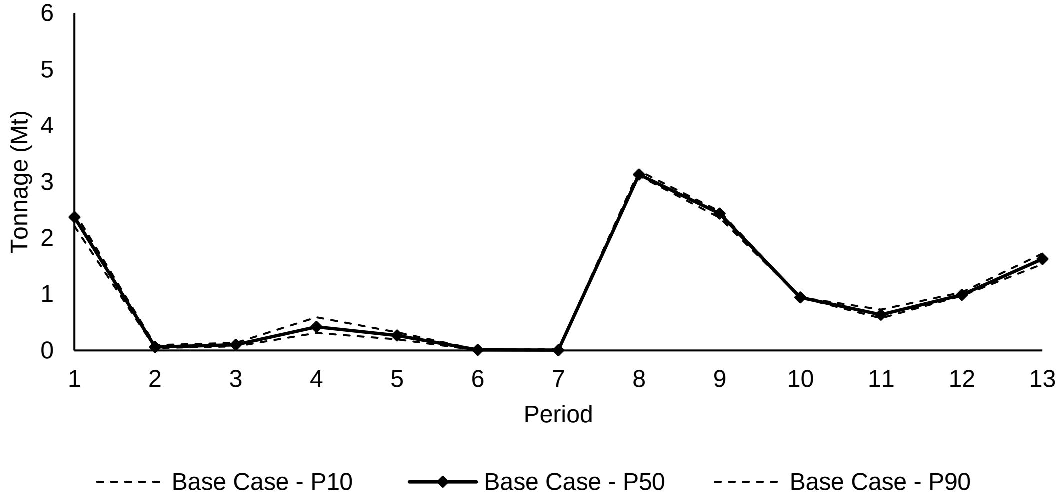
Appendix B: Additional results

B1 – Base case results



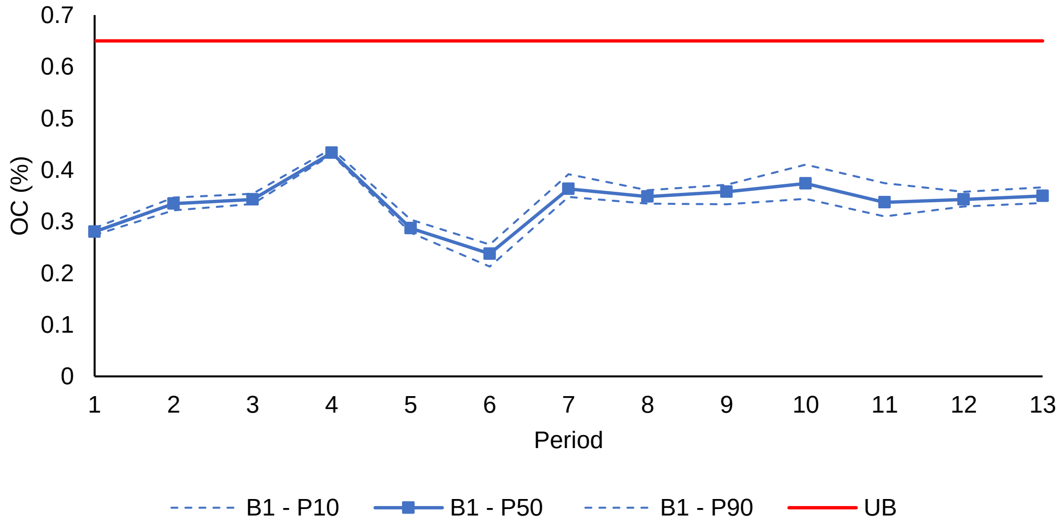


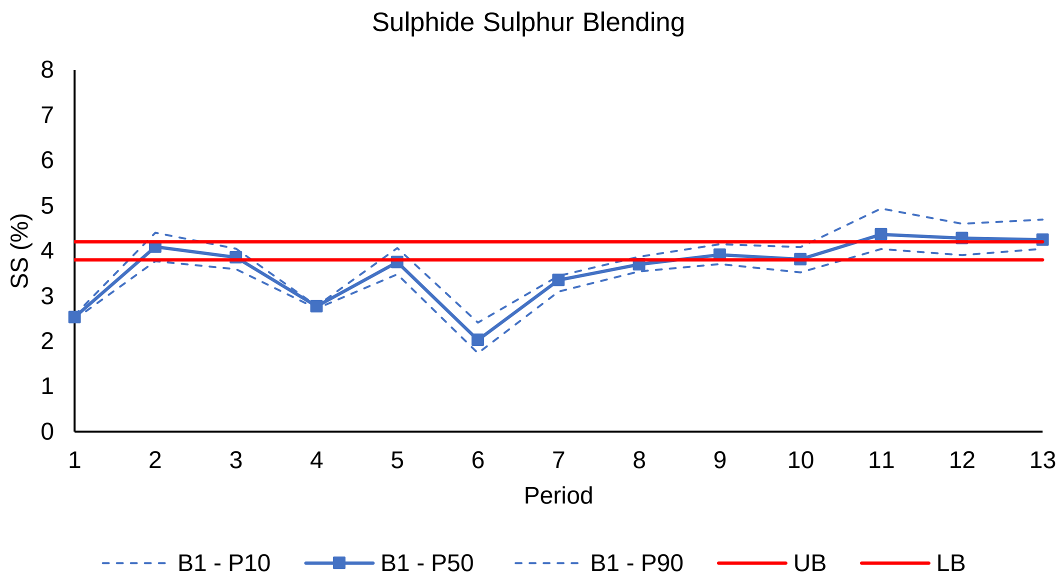
Oxide Leach Tonnage

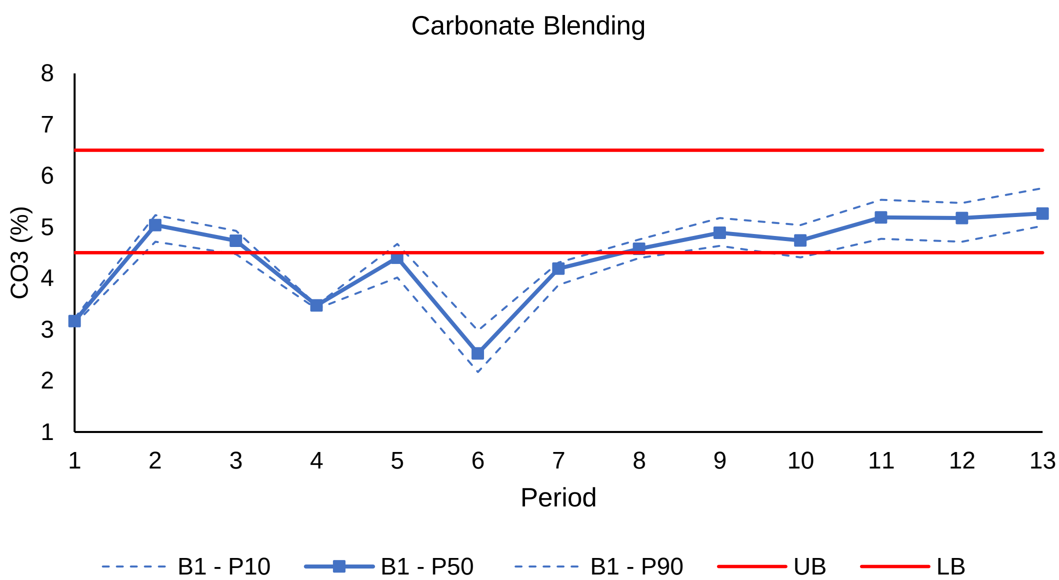


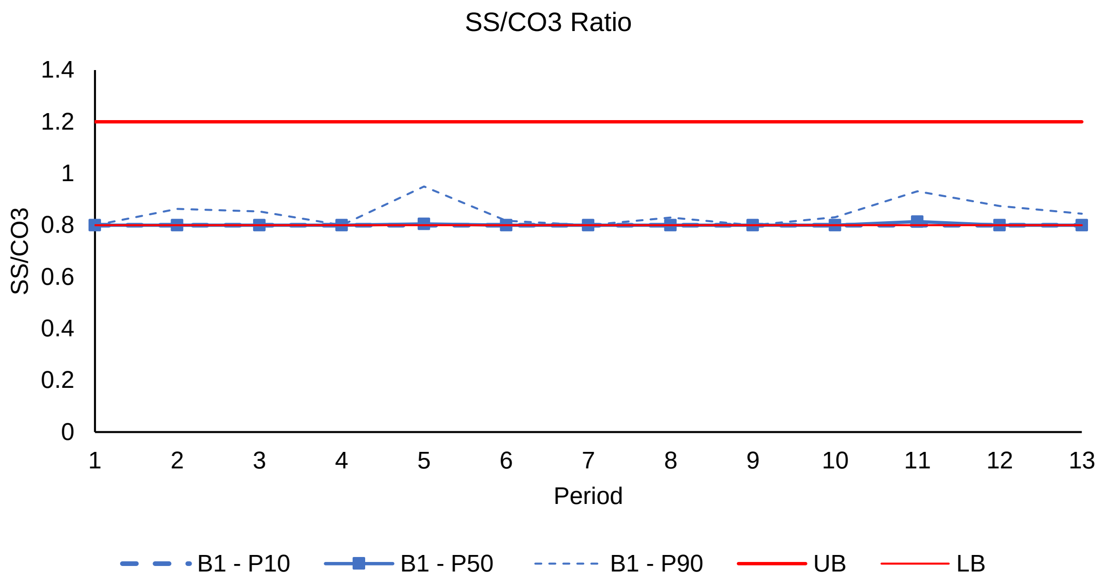
B2 – Simultaneous stochastic optimization results with branching B1

Organic Carbon Blending

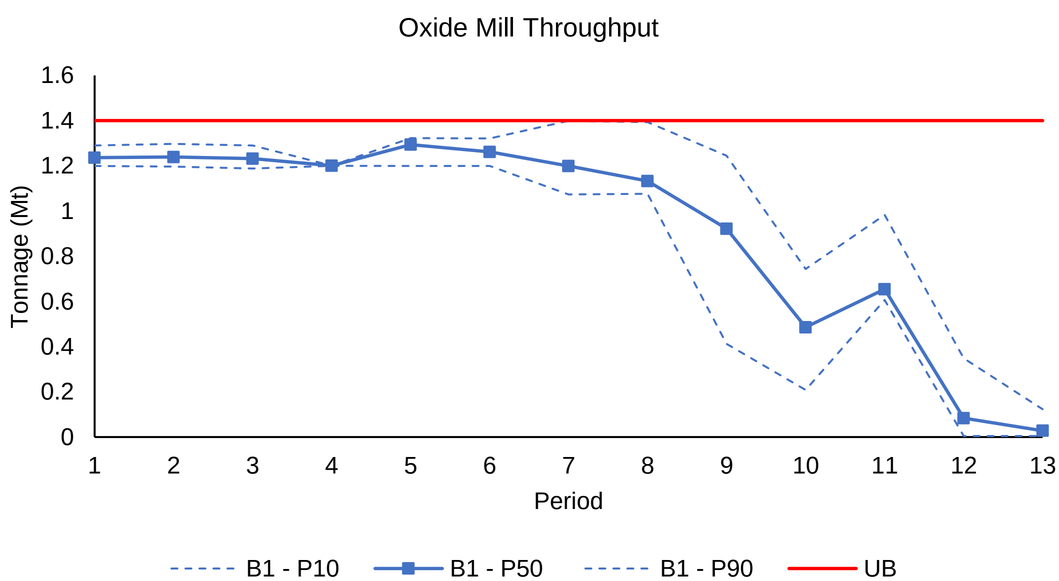



Sulphide Sulphur Blending
SS (%)
0
1
2
3
4
5
6
7
8
1
2
3
4
5
6
7
8
9
10
11
12
13
Period
B1 - P10
B1 - P50
B1 - P90
UB
LB

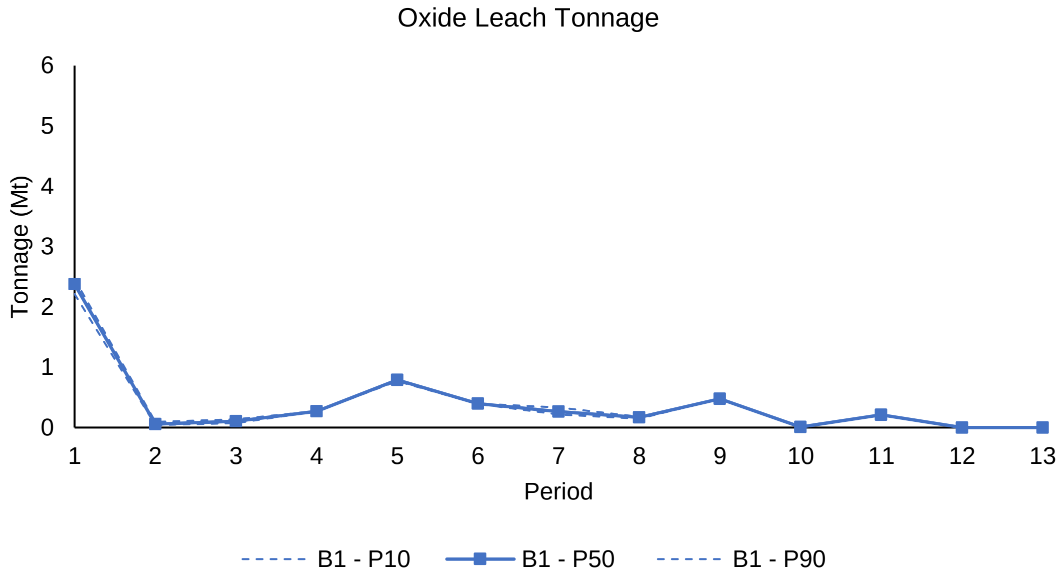

Carbonate Blending
CO3 (%)
1
2
3
4
5
6
7
8
1
2
3
4
5
6
7
8
9
10
11
12
13
Period
B1 - P10
B1 - P50
B1 - P90
UB
LB



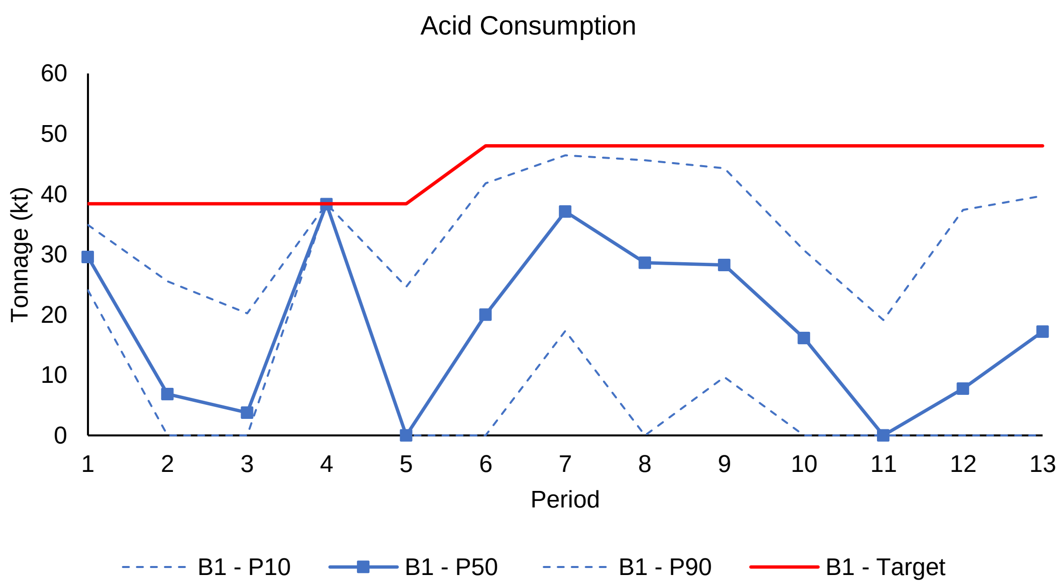
SS/CO3 Ratio
1.4
1.2
1
0.8
0.6
0.4
0.2
0
SS/CO3
1
2
3
4
5
6
7
8
9
10
11
12
13
Period
B1 - P10
B1 - P50
B1 - P90
UB
LB



Oxide Mill Throughput
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Tonnage (Mt)
1
2
3
4
5
6
7
8
9
10
11
12
13
Period
B1 - P10
B1 - P50
B1 - P90
UB

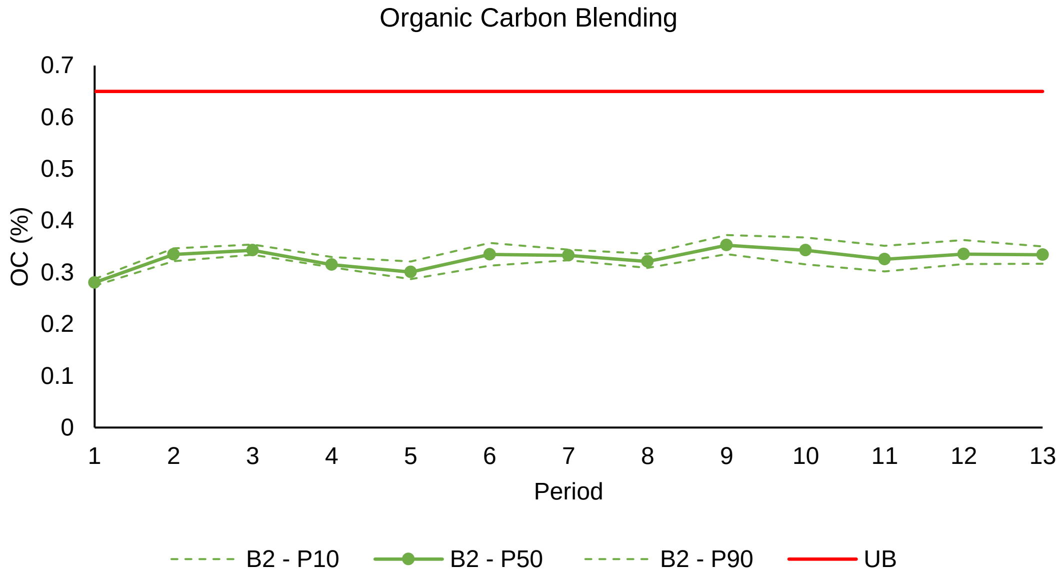


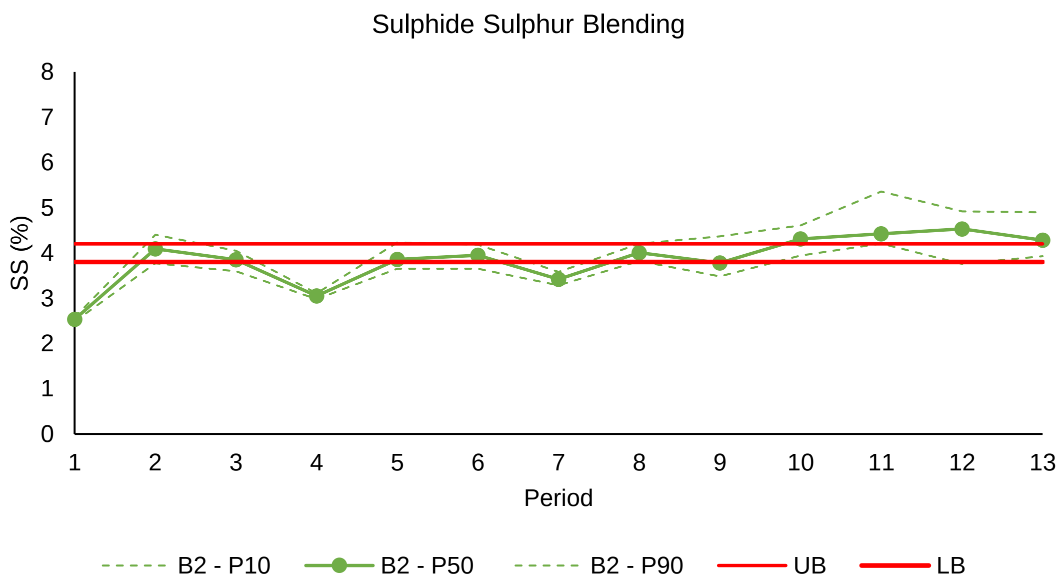
Oxide Leach Tonnage
Tonnage (Mt)
0
1
2
3
4
5
6
1
2
3
4
5
6
7
8
9
10
11
12
13
Period
B1 - P10
B1 - P50
B1 - P90

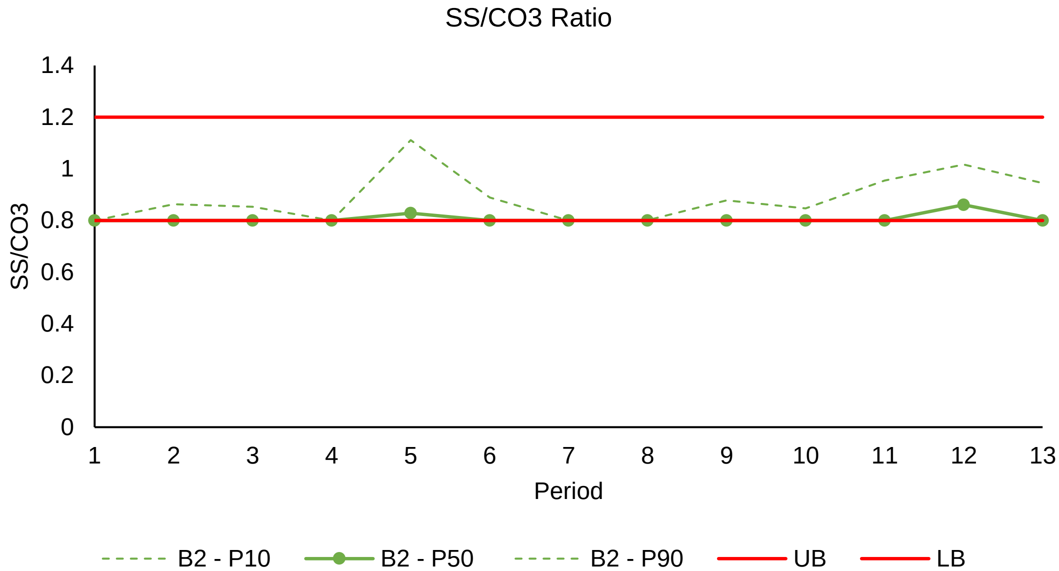


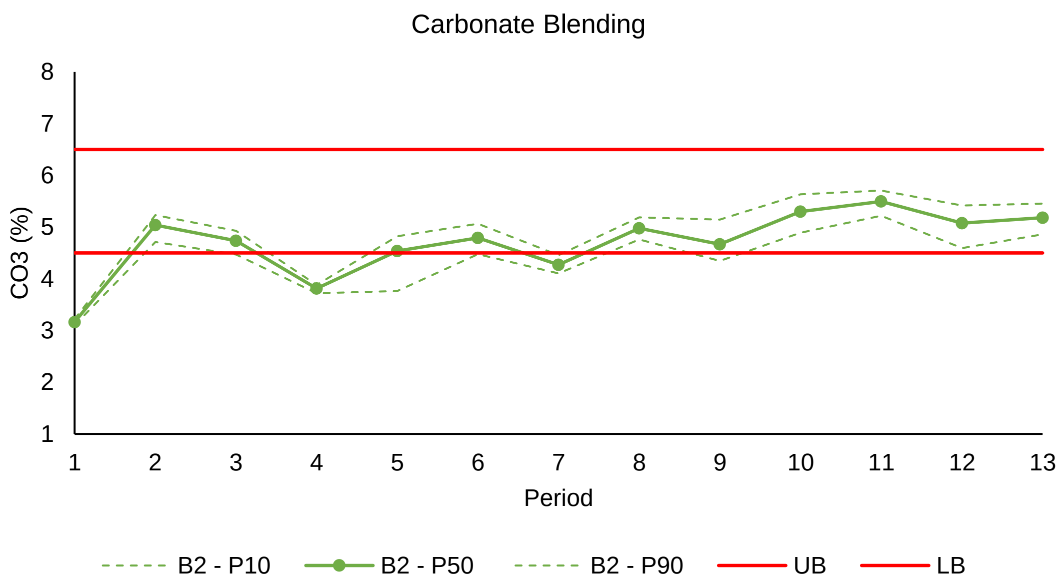
Acid Consumption
Tonnage (kt)
0
10
20
30
40
50
60
1
2
3
4
5
6
7
8
9
10
11
12
13
Period
B1 - P10
B1 - P50
B1 - P90
B1 - Target

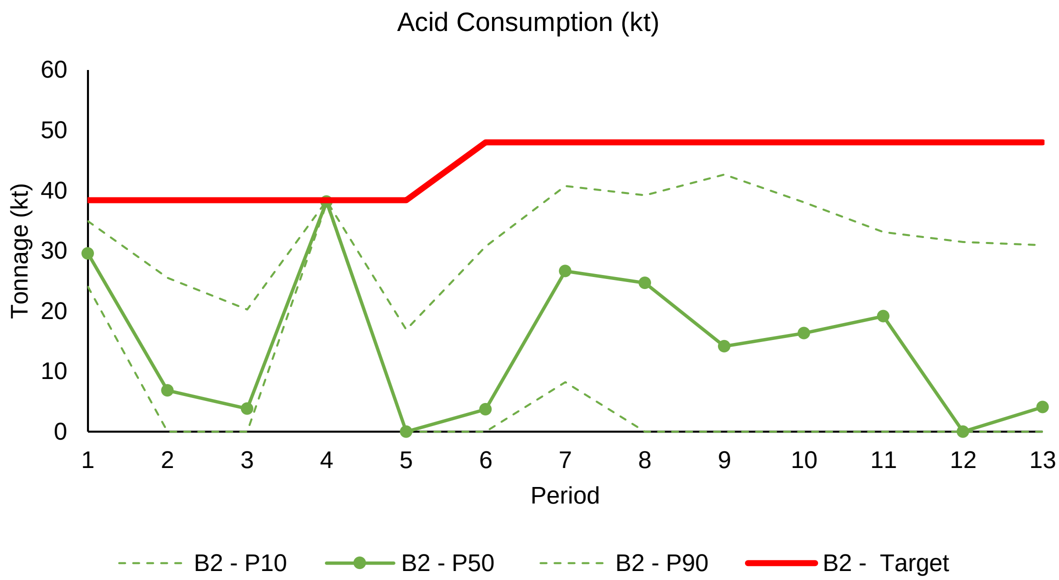
B3 – Simultaneous stochastic optimization results with branching B2

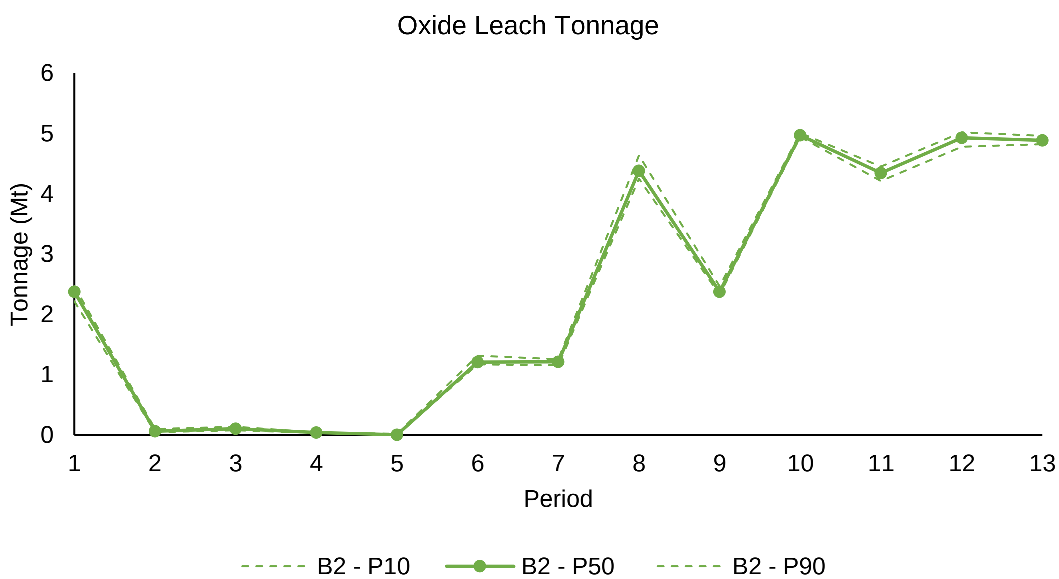


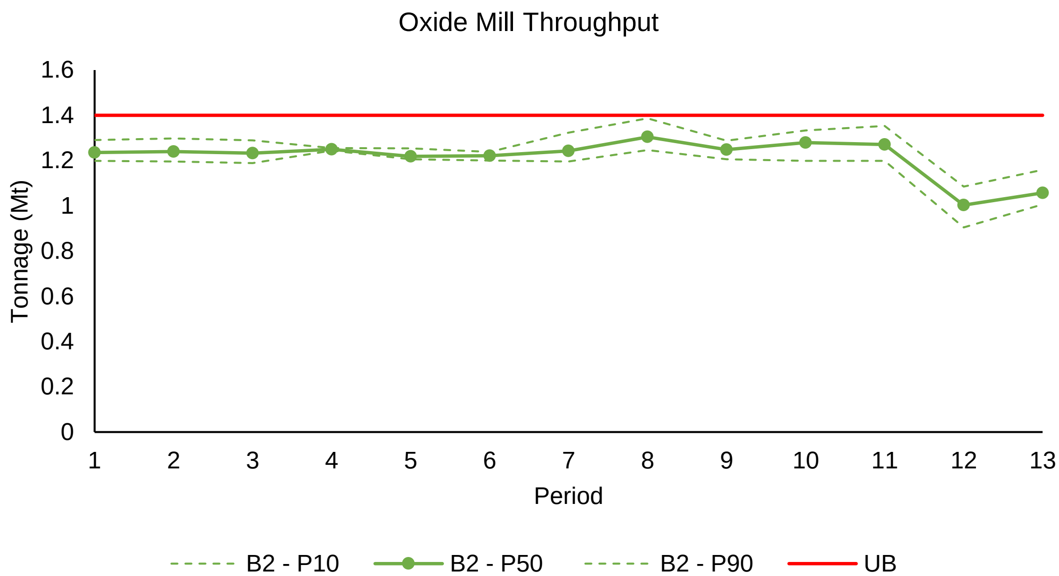



SS/CO3 Ratio
SS/CO3
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10 11 12 13
Period
B2 - P10
B2 - P50
B2 - P90
UB
LB


Carbonate Blending
CO3 (%)
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13
Period
B2 - P10
B2 - P50
B2 - P90
UB
LB


Acid Consumption (kt)
60
50
40
30
20
10
0
Tonnage (kt)
1
2
3
4
5
6
7
8
9
10
11
12
13
Period
B2 - P10
B2 - P50
B2 - P90
B2 - Target


Oxide Leach Tonnage
6
5
4
3
2
1
0
Tonnage (Mt)
1
2
3
4
5
6
7
8
9
10
11
12
13
Period
B2 - P10
B2 - P50
B2 - P90


Oxide Mill Throughput
Tonnage (Mt)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
Period
B2 - P10
B2 - P50
B2 - P90
UB