A DIRECT READING PHASE ANGLE METER

A thesis submitted to the Faculty of Graduate Studies and

Research of McGill University in part fulfilment

of the requirements for the degree of

Master of Science.

by

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April 28th, 1948.

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SUMMARY

A meter is constructed for the measurement of phase differences of voltages in the lower audio range. This meter possesses many distinct advantages over the prevailing types in range, accuracy, and ease of operation. To test the meter, calculations are made of the phase change in a transmission line at various frequencies, and the results checked with the readings on the phase meter. A very good agreement was found between experimental and predicted values.

In the construction of the meter two voltages 90° out of phase are required. These are obtained by the use of a three phase oscillator and Scott transformer connection. This method is selected as it will not spoil the adjustment of the phase meter with frequency change.

INTRODUCTION

The prevailing types of meters for the measurement of phase differences in voltages at audio frequencies are inadequate in that they are not direct reading; do not differentiate between the positive and negative angle, or angles greater or less than 90°; have a small range; have low reading accuracy; or are suitable for only one frequency. Most meters have one or more of these faults so it would be desirable to design one that would eliminate them all.

Before discussing this meter it would be convenient to give a brief review of the types now in use[1]. Most of the methods have one characteristic in common in that they use a cathode ray tube as an indicator.

The first method consists of putting the two voltages whose phase difference is required on the horizontal and vertical plates of an oscilloscope. An ellipse is formed and the phase difference is given by the formula

$$\sin \theta = B/A \quad (2)$$

where θ is the phase difference, B the vertical distance of the ellipse from its centre, and A the maximum projection of the ellipse on the vertical axis. This method is illustrated in Figure 1.

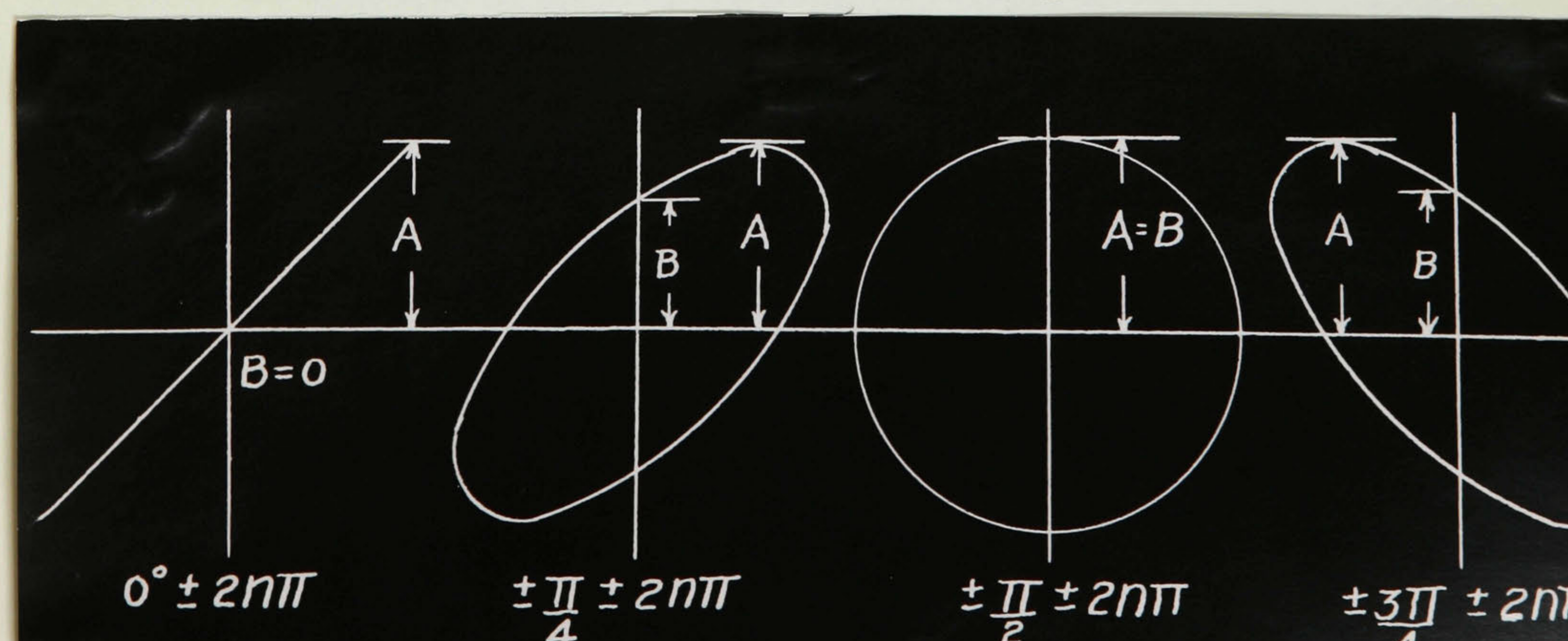


Fig 1 - Equal amplitudes and varying phase differences

This method has a number of disadvantages.

(1) It is not direct reading.

(2) In the majority of cases the measurement of A or B will be in error by around 5%.

(3) It is difficult to tell whether the phase angle is positive or negative without the use of special refinements[3].

(4) The ellipse is in the same position for $90^\circ + \theta^\circ$ as for $90^\circ - \theta^\circ$

A refinement of the above method consists of changing the ellipse to a straight line with a calibrated phase shift network[4].

Another method of measuring the phase difference between two voltages is to measure their amplitudes separately and then measure the amplitude of their sum. Then a phase diagram can be constructed and their phase difference measured from this. This method suffers in that it is quite slow and not direct reading.

An additional method consists of triggering the sweep with an external reference voltage, and then separately placing the two voltages on the vertical plates. The phase angle can be directly calculated from the amount of horizontal travel of the wave form. Low reading accuracy is the chief difficulty with this method.

The phase meter in this discussion also makes use of an oscilloscope as a detector. Two voltages 90° out of phase and equal amplitudes are put on the horizontal and vertical plates to form a circle. A blanking pulse is then introduced which causes a spot on the circle. This spot is moved around the circle by external voltages applied to the scope. The angle through which the spot moves gives the phase difference between the external voltages. This meter is direct reading from 0 to $\pm$ 360° over a frequency range of 200-3000 c.p.s. and with a reading accuracy of $1/2^\circ$.

PROCEDURE USED

To obtain the circle on the scope the circuit in Figure 2 was used

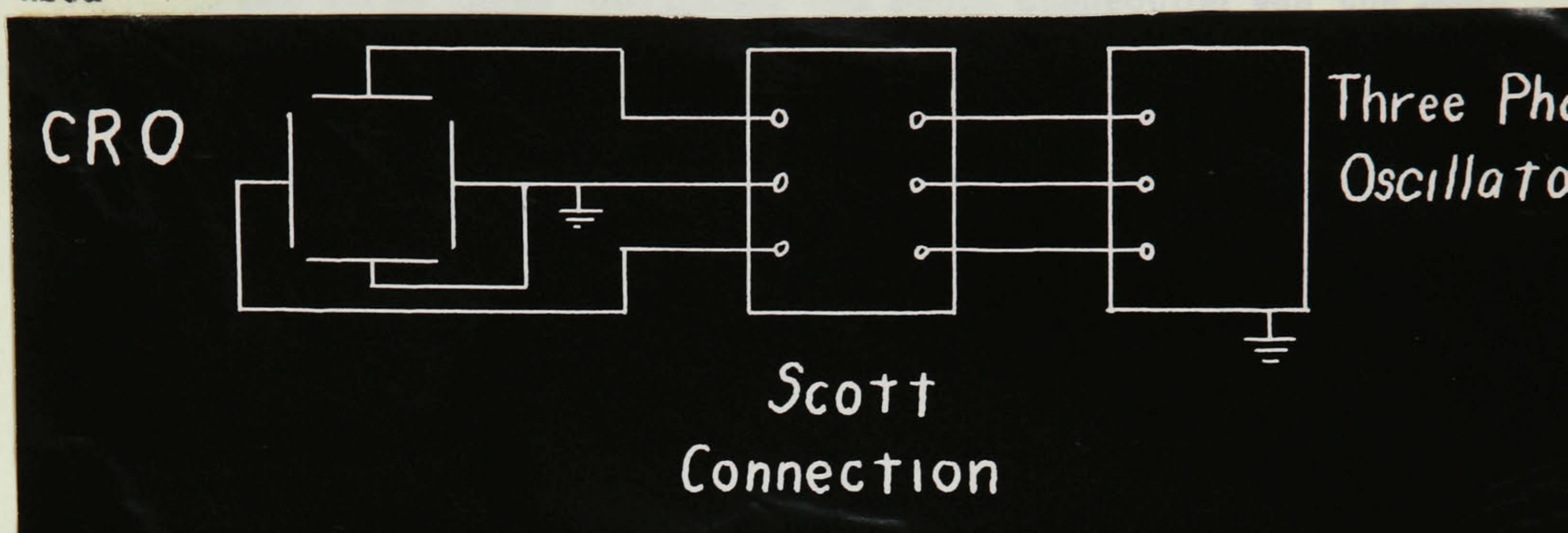


Fig 2

A three phase oscillator (voltages varying by 120°) feeds into the three contacts of a Scott Transformer Connection, and two voltages 90° out of phase are obtained at the output of the transformer. When put on the plates of the C.R.O., these voltages produce a circle. A circle of diameter five cm. was engraved on a celluloid screen and mounted in front of the display screen of the cathode ray tube to correspond to the circle on the screen. This circle was marked every three degrees to give a reading accuracy of one half degree.

The blanking pulse was obtained directly from the slope. In the sweep circuit of the scope there is a blanking pulse to remove the return trace in applications using the standard linear sweep. The combination of the blanking pulses for the horizontal and vertical plates gives a spot on the circle. As the frequency goes up the spot becomes wider but one side retains a sharp edge which can be used.to indicate the angle. The timing of the blanking pulse around the circle can be controlled through a terminal on the scope.

APPARATUS USED IN THE CONSTRUCTION OF A PHASE METER.

(1) The Three Phase Oscillator.

The three phase oscillator and Scott Connection was the most convenient way available to obtain two voltages 90° out of phase over a range of frequencies. The oscillator selected was a three phase resistance capacitance oscillator(5).

The oscillator (Figure 3) consists of three identical stages, each one feeding into the next. Each stage of the oscillator has a vacuum tube amplifier and a phase shift network so that the output has the proper magnitude and phase as is required to produce the output. When a signal is fed back to the grid circuit so as to aid oscillations it is called positive feedback, if the signal tends to reduce the oscillations it is called negative feedback. In this oscillator the resistance capacitance phase shift coupling network produces positive feedback at the selected frequency and negative feedback for the harmonics. This fact tends to guarantee a distortion free wave form.

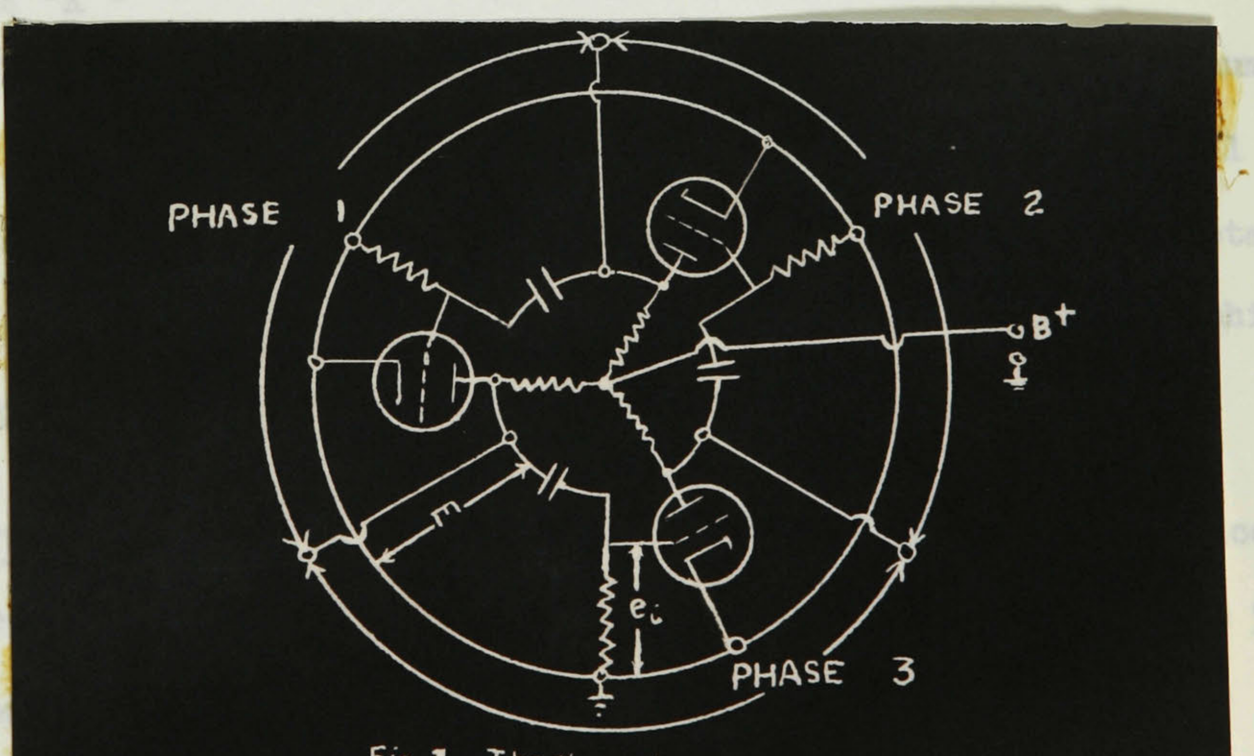


Fig 3 The three phase oscillator

Three conditions must be satisfied to produce self sustaining three phase oscillations.

(1) Each stage must have identical characteristics.

(2) The output of the amplifier must be in phase with the input.

(3) The overall gain of the network must be equal to or greater than unity.

Let the amplifications parameter of the Kth stage be A_K, and B_K be the fraction of the voltage introduced into the input of the next stage.

$$A_1B_1 \times A_2B_2 \times A_3B_3 \geq 1$$

If $A_K = |A_K|\angle\theta$ and $B_K = |B_K|\angle\phi$.

As the gain must be greater than one.

$$|A_1||B_1| \times |A_2B_2| \times |A_3||B_3| \geq 1$$

As the output must be in phase with the input,

$$(\theta_1 + \phi_1) + (\theta_2 + \phi_2) + (\theta_3 + \phi_3) = 2n\pi \quad (n = 0,1,2 \ldots)$$

The requirements for the three individual stages are as follows

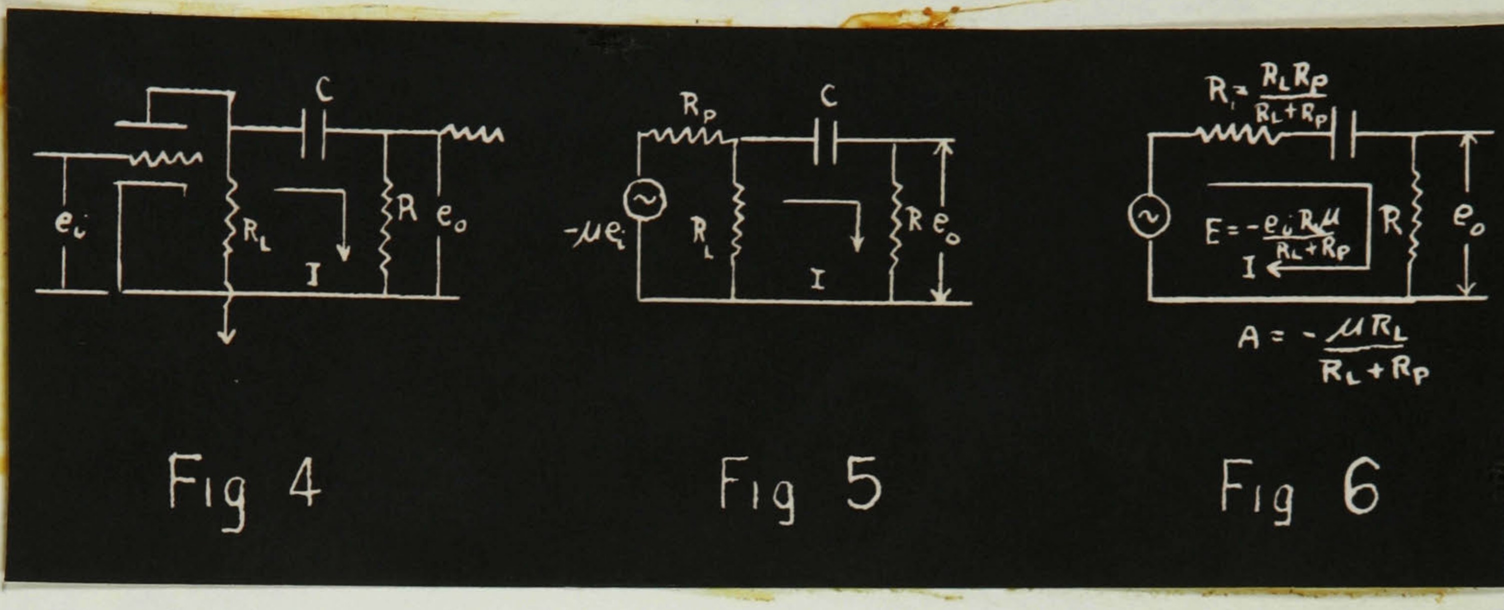
(1) The amplification of the stage must be equal or greater than one

$$A_K B_K \geqslant 1$$

(2) The stage output will have a phase shift depending on the number of stages. The phase shift for the amplifiers equals 180°, or a total of 540° for the three of them. In order to get positive feedback the total phase shift must equal 720°. Therefore, each coupling network must shift the phase by 60°.

(a) <u>Theoretical Analysis</u> - Calculation of frequency and amplificati

Figure 4 represents a single stage of the amplifier, Figure 5 i an equivalent representation, and Figure 6 is Figure 5 transformed by Thévènin's Theorem.



From Figure 6.

(1) $$E = I\,(R_1 + R - jX_C)$$

(2) $$\frac{e_o}{E} = \alpha + j\beta = \sqrt{\alpha^2 + \beta^2}\ \underline{/\tan^{-1} \beta/\alpha}$$

(3) $$e_o = IR$$

Substituting (1) in (3)

(4) $$\frac{e_o}{E} = \frac{R}{(R_1 + R - JX_C)} = \frac{R(R + R_1)}{(R_1 + R)^2 + X_C^2} + \frac{jX_C R}{(R_1 + R)^2 + X_C^2} = \alpha + j\beta$$

The phase shift $= \tan^{-1} \beta/\alpha = 60^\circ$

$$\therefore\ \beta/\alpha = \sqrt{3} = \frac{RX_C}{R(R + R_1)} = \frac{1}{2\pi fC(R_1 + R)} \quad \text{from (4)}$$

(5) $$\therefore\ f = \frac{1}{2\pi C\sqrt{3}\,(R + R_1)}$$

The minimum gain for oscillations can be calculated from (2)

$$E = Ae_i$$

For oscillations $e_i = e_o$

$$\therefore\ \frac{E}{e_o} = A = \frac{(R_1 + R) - jX_C}{R} = \frac{R_1 + R}{R}\quad (1 - j\sqrt{3})$$

(6) $$\therefore\ |A| = 2(1 + R_1/R)$$

(b) <u>Frequency Stability.</u>

The frequency of oscillation depends on R_1 which includes R_p. R_p varies due to plate voltage variations. It is therefore useful to calculate frequency stability with reference to plate resistance variation.

Frequency stability is defined as

$$\frac{\text{the incremental change in oscillator frequency}}{\text{frequency of the oscillations}}$$

$$= -K_1' \frac{\text{Incremental change in } (R_1 + R)}{(R_1 + R)}$$

$$\frac{\Delta f}{f} = -K_1' \frac{\Delta (R_1 + R)}{(R_1 + R)}$$

$$\frac{df}{f} = -K_1' \frac{dR_1}{R_1 + R} = -K_1 \frac{dR_1}{R_1}$$

By a change of variable this equation becomes

$$\frac{df}{f} = -K_2 \frac{dR_P}{R_P} \tag{7}$$

From equation 5 and Figure 6

$$f = \frac{1}{2\pi C\sqrt{3}\,(R + R_1)} \qquad \text{and } R_1 = \frac{R_L R_P}{R_L + R_P}$$

Differentiate with respect to R_P.

$$df = -\frac{1}{2\pi C\sqrt{3}} [R_1 + R]^{-2}\, dR_1$$

$$dR_1 = \left[-\frac{R_L R_P}{[R_L + R_P]^2} + \frac{R_L(R_L + R_P)}{(R_L + R_P)^2}\right] dR_P = \frac{R_L^2\, dR_P}{[R_L + R_P]^2}$$

$$df = -\frac{1}{2\pi C\sqrt{3}} \frac{R_L^2}{(R_1 + R)^2} \frac{dR_P}{R_P^2(1 + R_L/R_P)^2}$$

$$\frac{df}{f} = \frac{-R_L^2}{(R_L + R_P)\,[R_L + R(1 + R_L/R_P)} \frac{dR_P}{R_P}$$

Comparing with (7) gives

$$K_2 = \frac{R_L^2}{(R_L + R_P)\left[R_L + R(1 + R_L/R_P)\right]}$$

Taking a typical example

$R_L = 10,000\,\Omega$ $\qquad$ $R = 1,000,000\,\Omega$

$R_P = 1,000,000\,\Omega$ $\qquad$ $f = 1,000$ c.p.s.

$\therefore\ K_2 = 9.7 \times 10^{-5}$

A change in plate resistance of 10% would make a change in frequency of $9.7 \times 10^{-4}\% = 0.0097$ (cycles/sec)/(1000 cycles/sec.). This makes any variations in frequency due to change in R_p negligible.

(c) Amplitude Control.

In the circuit of Figure 3 a need was found for amplitude control. The method selected consisted of a thermistor controlled negative feedback bridge. The circuit in Figure 3 is further modified by the addition of a cathode follower to each stage. The low output impedance of the cathode follower allows R_1 to be ignored in the formula for frequency The complete oscillator circuit is shown in Figure 7[5].

When the amplitude of oscillations increases the thermistor resistance goes up, this increases the negative feedback and the oscillations die down; as the oscillations die down the thermistor resistance decreases which causes the negative feedback to decrease thus allowing the oscillations to build up. This causes the amplitude to settle at a constant intermediate value.

To eliminate spurious oscillations at high frequencies in the feedback bridge, .001 μf condensors are put in to bypass the 700 Ω resistors.

(2) The Scott Transformer Connection.

This connection is used for converting two phase circuits to three and vice versa. The circuit is shown in Figure 8.

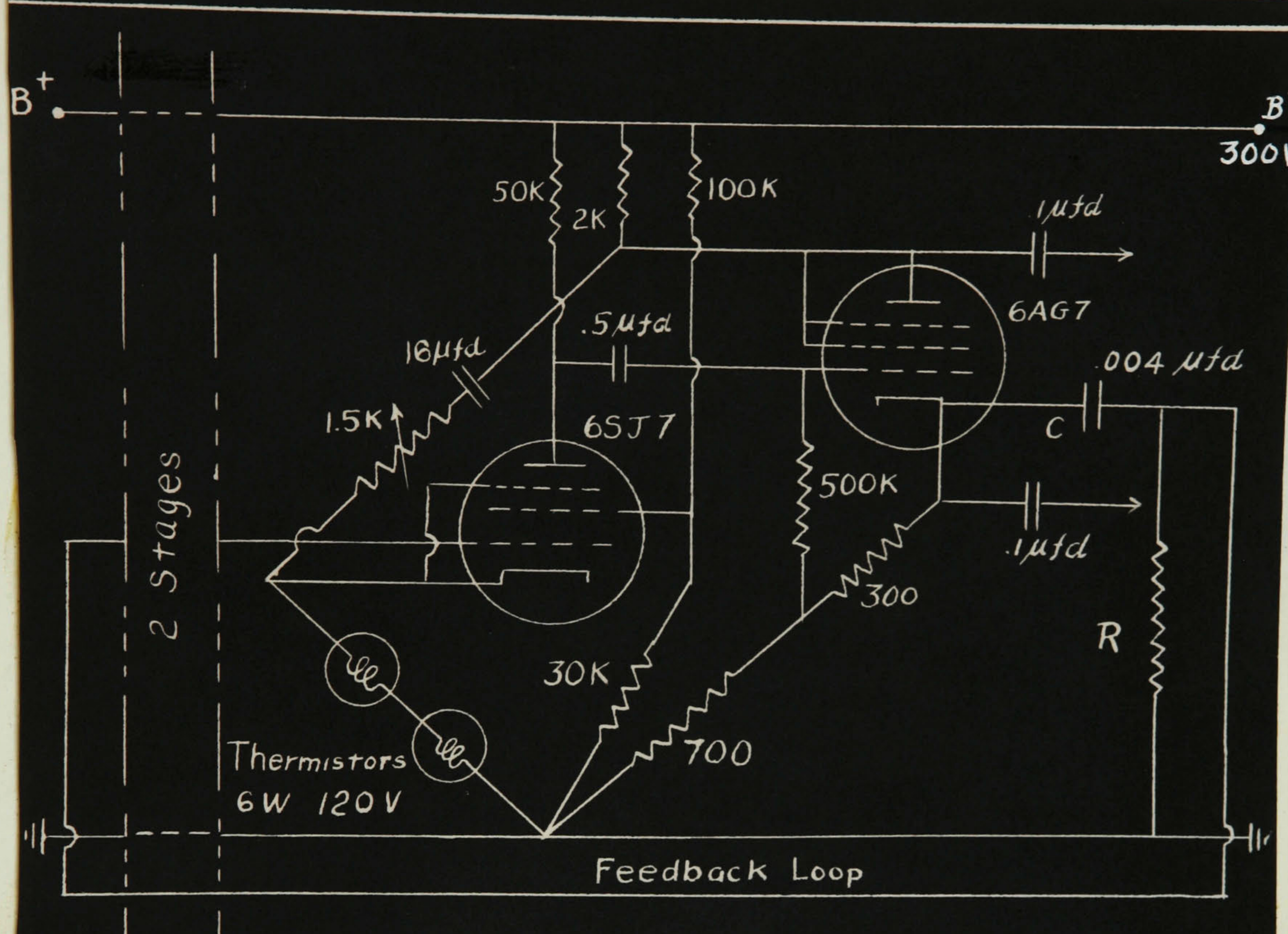


Fig 7 - Three Phase Oscillator

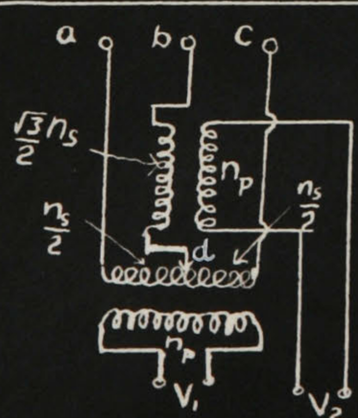


Fig 8

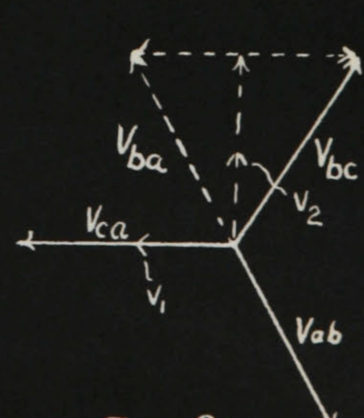


Fig 9

The voltage relations can be found directly from the figure.

$V_{ca} = \frac{n_s}{n_p} V_1$

$V_{ba} = V_{bd} + V_{da} = \sqrt{3}/2 \quad \frac{n_s}{n_p} V_2 + \frac{1}{2} \frac{n_s}{n_p} V_1$

$V_{cb} = V_{cd} - V_{db} = - \sqrt{3}/2 \quad \frac{n_s}{n_p} V_2 + \frac{1}{2} \frac{n_s}{n_p} V_1$

In our case $|V_{ca}| = |V_{bc}| = |V_{ab}|$ and they differ in phase from each other by 120°. (Figure 9).

Let $\frac{n_s}{n_p} = 1/K$

$V_1 = KV_{ca}$

$$V_2 = \frac{K \begin{vmatrix} V_{ba} & 1/2 \\ V_{cb} & 1/2 \end{vmatrix}}{\begin{vmatrix} \sqrt{3}/2 & 1/2 \\ \sqrt{3}/2 & 1/2 \end{vmatrix}} = \frac{K}{\sqrt{3}} (V_{ba} - V_{cb})$$

V_1 is in the same line as V_{ca} and K times as long.

$(V_{ba} - V_{cb})$ is a vector making an angle of 90° with V_1. Its magnitude is $\sqrt{3}\ |V_{ca}|$. Therefore $|V_2| = |V_1|$ and they are at right angles to each other.

As no transformer was available with the number of turns $\sqrt{3}/2\ N_s$ one with n_s turns was used. This does not alter the phase of the output but it does change its magnitude. As the output must be amplified in any case this does not cause any difficulties.

In this case

$V_1 = K V_{ca}$

$$V_2 = \frac{K \begin{vmatrix} V_{ba} & 1/2 \\ V_{cb} & 1/2 \end{vmatrix}}{\begin{vmatrix} 1 & 1/2 \\ -1 & 1/2 \end{vmatrix}} = \frac{1}{2} K(V_{ba} - V_{cb}) = \frac{\sqrt{3}}{2} K\,|V_{ba}|$$

$\therefore\ |V_2| = \frac{\sqrt{3}}{2} |V_1|$ and the phase differs by ninety degrees.

The transformers used to construct the Scott connection were two Hammond shielded transformers (23033). When the connection was tried out it was found that the transformers were not properly matched as the C.R.O. has a high impedance and the oscillator a low impedance output. This caused the windings of the connection to act as inductances producing the phase shift which distorted the circle on the C.R.O. This effect could be eliminated by placing small resistors across V_1 and V_2 or the same effect could be accomplished by placing large resistors (60K) across the primaries. In practice the latter was found to be preferable. The circle then retains its shape over the range 200-3000 c.p.s.

TRANSMISSION LINE

To check the meter as a means of measuring phase a number of measurements were made on an artificial line. A diagram of the line and the derivation of the equations for its characteristic impedance and phase shift are included in the Appendix.

The phase shift β is given by

$$(8) \qquad \beta = \frac{(R^2+\omega^2L^2)^{\frac{1}{4}}\,(\omega C)^{\frac{1}{2}}}{2}\left[\sqrt{1+\frac{R}{\sqrt{R^2+\omega^2L^2}}}+\sqrt{1-\frac{R}{\sqrt{R^2+\omega^2L^2}}}\right]$$

The characteristic impedance Z_o is

$$(9) \qquad Z_o = \frac{1}{2}\sqrt{\frac{L}{C}}\;\frac{(R^2+\omega^2L^2)^{\frac{1}{4}}}{(\omega L)^{\frac{1}{2}}}\left\{\left[\sqrt{1+\frac{R}{\sqrt{R^2+\omega^2L^2}}}+\sqrt{1-\frac{R}{\sqrt{R^2+\omega^2L^2}}}\right]\right.$$

$$\left. + j\left[\sqrt{1-\frac{R}{\sqrt{R^2+\omega^2L^2}}}-\sqrt{1+\frac{R}{\sqrt{R^2+\omega^2L^2}}}\right]\right\}$$

The circuit used is drawn up in Figure 10. The input for the line was taken from the oscillator. A single stage amplifier was connected between the oscillator and the line to boost the output and isolate the transmission line from the oscillator. The line was terminated by a shunt resistance and capacitance, the value of which is given by equation (9). A movable lead connected the line to the external triggering circuit of the scope.

The need for an amplifier preceding the triggering circuit will be seen from a consideration of Figure 11. V_T is the minimum voltage level that triggers the sweep. Consider two voltages V_1 and V_E used to trigger the sweep. If $|V_1| >> |V_E|$ the sweep will be triggered later for V_E than V_1 in terms of phase. The amplifier is put into the circuit to overload V_E. This overloaded voltage is represented in the figure by V_{EO}. It will be seen that V_{EO} and V_1 now trigger the sweep in the same phase.

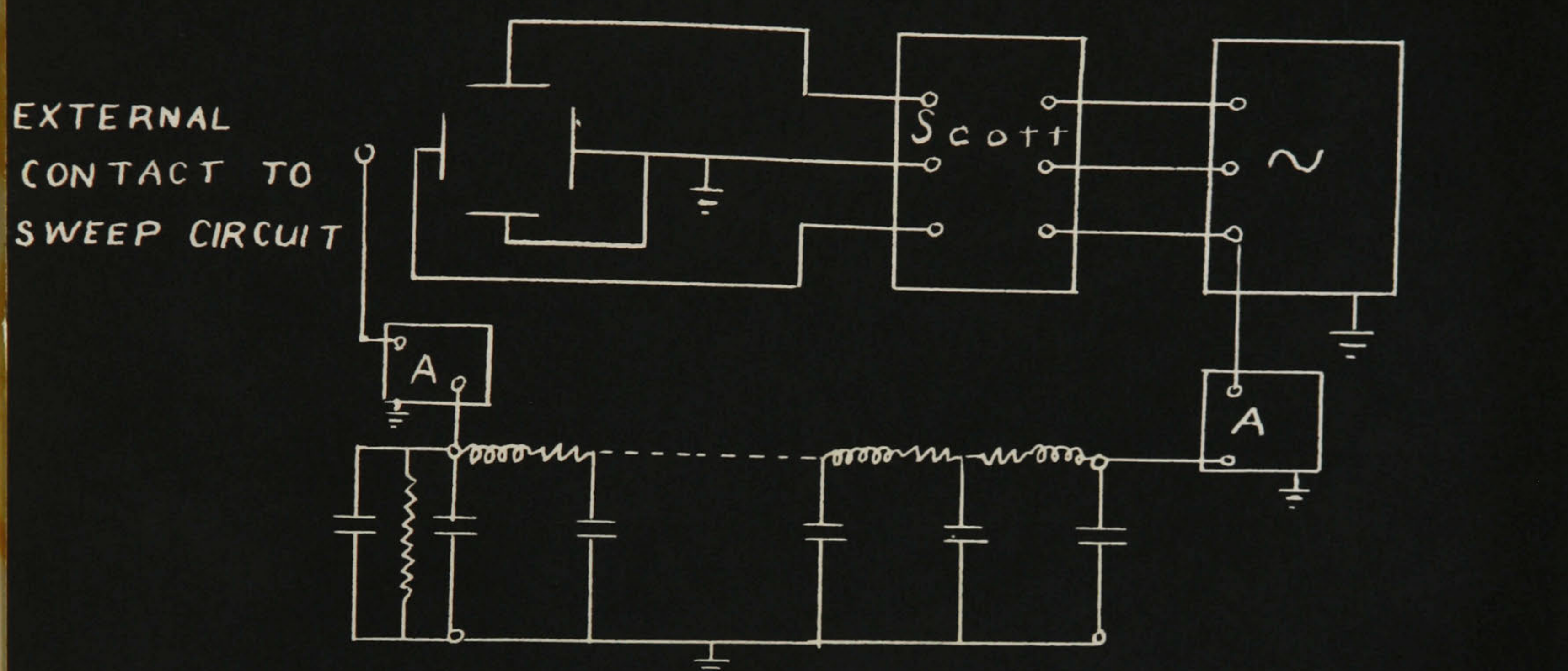


FIG 10 - PHASE METER AND TRANSMISSION LINE

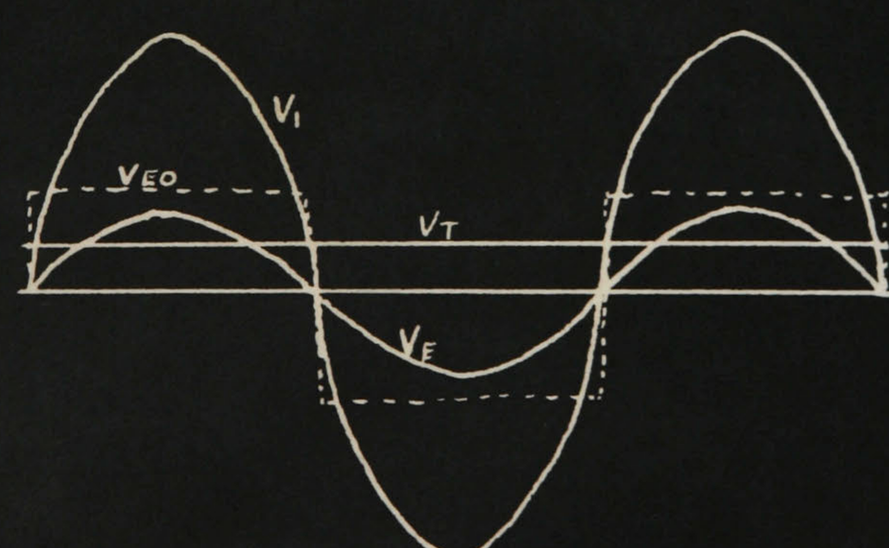


FIG 11

DATA

Readings were made on the phase shift of the transmission line at frequencies of from 200 to 3000 c.p.s. The phase shift was calculated by formula 8 and the characteristic impedance from 9.

TABLE I

f	C_o	R_o	Formula 1	Meter Readings	
cycles/sec	μf.	Ω	Degrees	Degrees	
	For Z_o			Trial 1	Trial 2
200	.146	6880	52.3	54	54
400	.0870	4530	83.2	84	84
700	.0520	2340	124.	123	124
1000	.0326	2020	168.	167	167
1300	.0219	1860	214.	215	212
1600	.0160	1780	254.	256	252
2000	.0108	1710	317.	317	315
2400	.00773	1680	378.	377	378
2800	.00586	1640	437.	438	435
3200	.00456	1630	498.	495	495

The transmission line was composed of 27 sections, with an unattached section for the measurement of components. As it was not feasible to measure every component in the line the following values were taken

$C = 0.010_0 \pm 0.0001\ \mu f$

$L = 250 \pm 2$ mh.

$R = 153 \pm 1\,\Omega$

From a consideration of Table I and equation 8 it will be seen that the choice of R was slightly low. When f becomes equal to 400, $\omega^2 L^2$ becomes large enough so that $(\omega^2 L^2 + R^2)^{\frac{1}{2}}$ and hence Z_o and β are not affected by this low value of R. For all frequencies from 400 and up the probable errors assumed for the components would more than account for the deviations between theory and calculation.

The frequency is known to within one percent. At high frequencies over 1000 c.p.s. β varies directly as ω. Therefore this would account for the deviations between the experimental values of β.

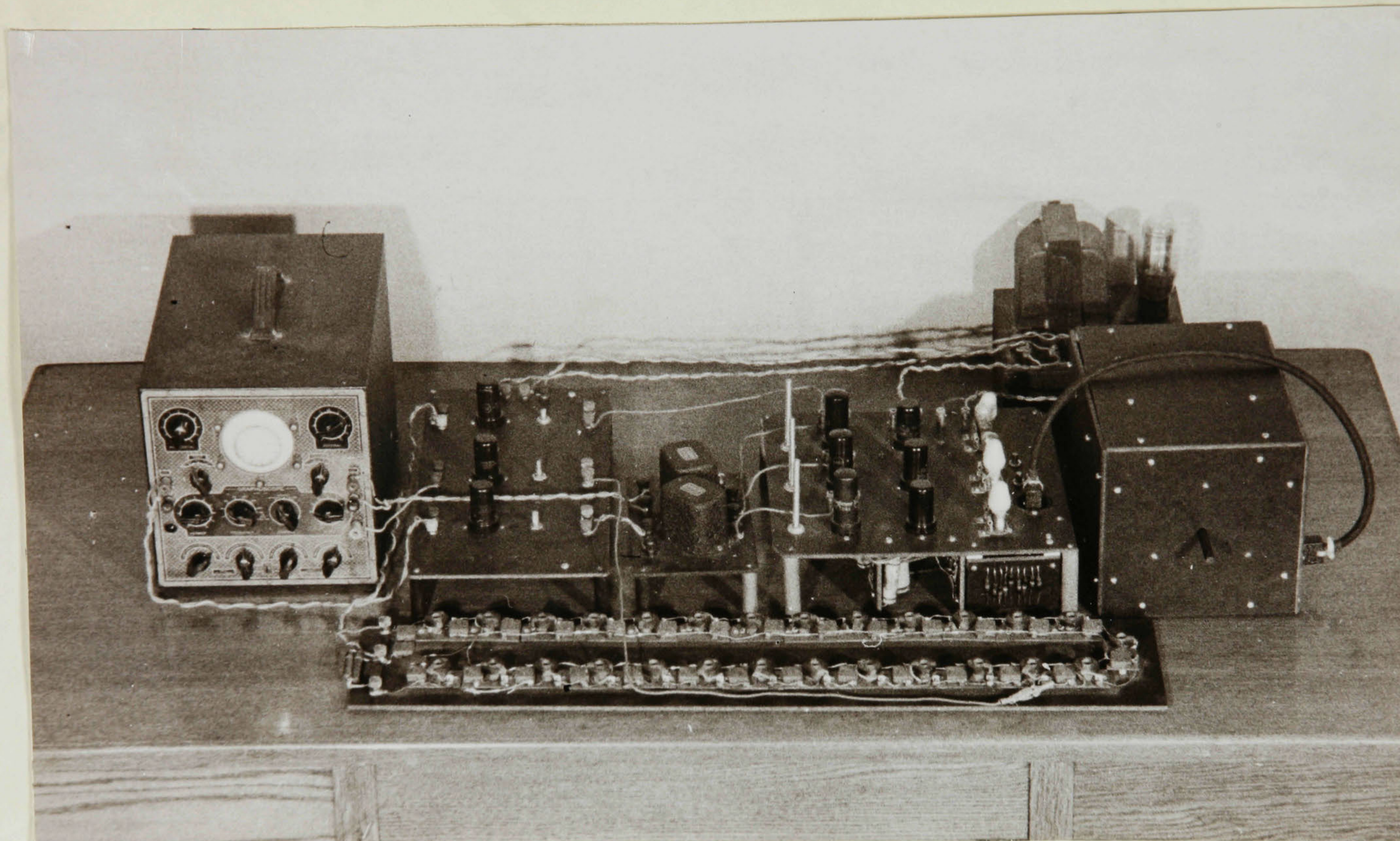
Using the probable errors mentioned in this section β is calculated at 400 c.p.s.

$$\beta = (12.9 \pm \tfrac{1}{4}\%)(5.02 \pm 1\%)(1.44 \pm 3/8\%) = 84^\circ \pm 2\%$$

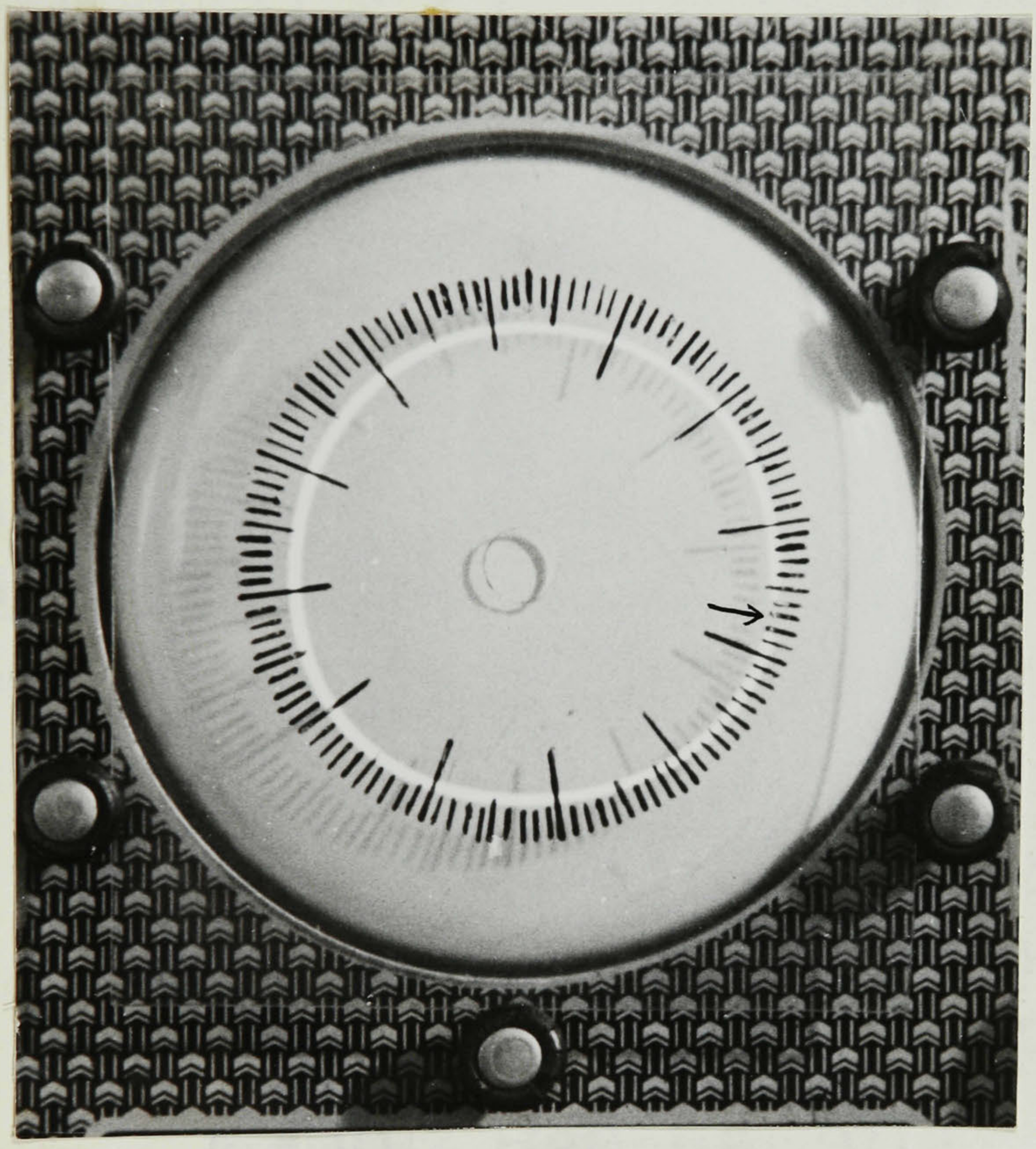
CONCLUSIONS

The phase meter operates satisfactorily in that the circle does not change shape over the range 200-3000 c.p.s. It has a reading error less than 1° caused by a $1/2^{\circ}$ uncertainty in the two positions of the spot. In the case of the transmission line, to this error must be added the errors caused by frequency uncertainty. One disadvantage is that the diameter of the circle changes with frequency. This difficulty is not serious as the value is relatively small, 4% for each frequency change of 500 c.p.s.

This meter is superior to most of the present types in accuracy, and phase range. The spot travels 360° before it starts to repeat its path. The direction of phase change is clearly indicated. The frequency range of the meter could be enlarged or shifted by altering the components in the three phase oscillator.



Phase Meter and Transmission Line.



C.R.O. Circle with Blanking Pulse Indicated by Arrow.

APPENDIX

Transmission Line[6]

The equation for the attenuations and phase shift is

(10) $$\alpha + j\beta = \sqrt{(R_1 + j\omega L_1)(G_1 + j\omega C_1)}$$

Equation (10) cannot be used for calculating phase shift in the above form. In the case of the transmission line used in the experiment no errors were introduced by assuming G = 0. Therefore (10) becomes

$$\alpha + j\beta = \left[\sqrt{R^2 + \omega^2 L^2} \quad \epsilon^{j\theta} \omega C \ \epsilon^{j\pi/2}\right]^{\frac{1}{2}}$$

where $\theta = \tan^{-1} \frac{\omega L}{R}$

$$\therefore \alpha + j\beta = (R^2 + \omega^2 L^2)^{\frac{1}{4}}(\omega C)^{\frac{1}{2}} \epsilon^{j(\theta/2 + \pi/4)}$$

Equating in-phase and quadrature components

$$\beta = (R^2 + \omega^2 L^2)^{\frac{1}{4}}(\omega C)^{\frac{1}{2}} \sin(\theta/2 + \pi/4)$$

define $K = (R^2 + \omega^2 L^2)^{\frac{1}{4}}(\omega C)^{\frac{1}{2}}$

$$\therefore \beta = K(\sin\theta/2 \cos\pi/4 + \cos\theta/2 \sin\pi/4)$$

$$= \frac{K}{\sqrt{2}} (\sin\theta/2 + \cos\theta/2)$$

$$= \frac{K}{2}\left[\sqrt{1 + \cos\theta} + \sqrt{1 - \cos\theta}\right]$$

$$= \frac{K}{2}\left[\sqrt{1 + \frac{R}{\sqrt{R^2 + \omega^2 L^2}}} + \sqrt{1 - \frac{R}{\sqrt{R^2 + \omega^2 L^2}}}\right]$$

The characteristic impedance Z_o is defined as follows:

(11) $$Z_o = \frac{\sqrt{R_1 + j\omega L_1}}{\sqrt{G_1 + j\omega C_1}} = \frac{\sqrt{R_1 + j\omega L_1}}{\sqrt{j\omega C_1}}$$

When the line is terminated by its characteristic impedance the phase shift for each section is the same.

$$Z_o = \sqrt{\frac{L}{C}} \sqrt{1 + \frac{R}{j\omega L}} = \sqrt{\frac{L}{C}} \sqrt{\frac{R + j\omega L}{j\omega L}}$$

$$= \sqrt{\frac{L}{C}} \quad \frac{(R^2 + \omega^2 L^2)^{\frac{1}{4}}}{(\omega L)^{\frac{1}{2}}} \times \frac{\epsilon^{j\theta/2}}{\epsilon^{j\pi/4}} = K' \ \epsilon^{j(\theta/2 - \pi/4)}$$

where $K' = \dfrac{(R^2 + \omega^2 L^2)^{\frac{1}{4}}}{\sqrt{\omega C}}$ and $\theta = \tan^{-1} \dfrac{\omega L}{R}$

$$\therefore \; Z_o = K' \left(\cos\left(\theta/2 - \pi/4\right) + j \sin\left(\theta/2 - \pi/4\right)\right)$$

$$= \frac{K'}{\sqrt{2}} \left[(\cos \theta/2 + \sin \theta/2) + j(\sin \theta/2 - \cos \theta/2)\right]$$

$$= \frac{K'}{2} \left\{ \left[\sqrt{1 + \frac{R}{\sqrt{R^2 + \omega^2 L^2}}} + \sqrt{1 - \frac{R}{\sqrt{R^2 + \omega^2 L^2}}} \right] + j \left[\sqrt{1 - \frac{R}{\sqrt{R^2 + \omega^2 L^2}}} - \sqrt{1 + \frac{R}{\sqrt{R^2 + \omega^2 L^2}}} \right] \right\}$$

This gives Z_o as a series resistor and condenser.

They are transformed into parallel components by the formula

$$G = \frac{R}{Z^2}, \quad B = -\frac{X}{Z^2}$$

The fact that R is large makes it impossible to use the simplifying assumption $\dfrac{R}{\omega L} = 0$.

BIBLIOGRAPHY

(1) T.E. Terman, Radio Engineer's Handbook, McGraw-Hill, New York, 1943.

(2) R.B. Lindsay, Physical Mechanics, D.Van Nostrand Company, New York, 1933.

(3) E.R. Mann, A Device for Showing the Direction of Motion of the Oscillograph Spot, Rev.Sci. Instruments, Vol. 5, p. 214, June 1934.

(4) J.P. Taylor, Cathode-Ray Antenna Phasemeter, Electronics, Vol. 12, p. 62, April 1939.

(5) R.M. Barrett, "N"- Phase Resistance Capacitance Oscillators, Proc. I.R.E., Vol. 33, p.541, August 1945.