Simultaneous Short-term Decision-Making in Mining Complexes Integrating Geometallurgy Assisted by Production Data

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Abstract

A mining complex is an integrated business of mines and downstream facilities that extracts raw materials, converts extracted materials into sellable products, and transports products to markets and customers. Conventionally, individual components of a mining complex are optimized locally and independently of each other, which causes underperformance of the mineral value chain. Simultaneous stochastic optimization of mining complexes has shown to create strategic mine plans that increase the net present value while reducing risk of meeting production targets by incorporating geological and price uncertainty. While these developments jointly optimize strategic decisions of a mining complex, short-term planning makes daily/weekly/monthly decisions to best meet long-term production targets and maximize value. These decisions include, but are not limited to, short-term extraction sequence, destination of materials, and downstream material flow in mining complexes. Furthermore, the optimal allocation of the mining fleet is an important aspect of short-term planning; however, the joint stochastic optimization of short-term production schedules and fleet management in mining complexes has not yet been developed. Additionally, geometallurgical properties that drive revenues, costs, and the ability to meet production targets, are not integrated in the optimization of short-term production schedules in mining complexes and main issues related to the upscaling and blending of non-additive geometallurgical properties need to be addressed. Digital technologies enable the central storage of a large amount of production data in mines and processing plants, however, new ways must be found to integrate these new sources of information into short-term decision-making.

This thesis expands the simultaneous stochastic optimization of mining complexes for long-term planning into a decision-making framework for short-term mine planning through the incorporation of fleet management and geometallurgical prediction models of plant performances into the short-term optimization of mining complexes, which is assisted by the utilization of collected datasets from production processes in mines and processing plants. The planning horizon of interest for the short-term planning framework developed in this thesis spans several weeks to months of future scheduled materials.

First, a new stochastic integer programming model for short-term planning is developed that extends the simultaneous stochastic optimization of mining complexes to allow the scheduling of a heterogeneous truck fleet and shovel allocations while considering the costs and loss of production caused by shovel relocation. Next to geological uncertainty, equipment performance uncertainties related to production rates, availabilities, and truck cycle times are integrated. The new method is applied at a gold mining complex and compared to a conventional two-step approach, where the short-term production schedule is optimized first before optimizing the allocation of the mining fleet. The developed method reduces costs generated by shovel movements by 56% and decreases total operating costs of trucks by 3%.

Next, a geometallurgical model for the prediction of ball mill throughput in mining complexes is developed which utilizes penetration rates from measurement while drilling and recorded throughput rates of the operating plant. The creation of hardness proportions avoids biases typically introduced by the change of support and blending of non-additive geometallurgical hardness properties. A case study at a gold mining complex shows that throughput can be predicted with a Pearson correlation coefficient of up to 0.8. By integrating the throughput prediction model into the simultaneous stochastic optimization formulation, weekly planned production can be achieved reliably because scheduled materials match with the predicted mill performance. Comparisons to optimization using conventional mill tonnage constraints reveal that expected production shortfalls of up to 7% can be mitigated by integrating the novel throughput prediction model.

The throughput prediction model is extended thereafter by including recorded measurements of ball mill power draw and feed/product particle size distributions. Since the addition of new features increases the possibilities of non-linear interactions between predictive variables and ball mill throughput, a neural network is used, replacing a multiple linear regression model. Comparisons show that adding ball mill power and product particle size decreases the throughput prediction error by 10.6%. Furthermore, hardness proportions created from penetration rates can decrease the prediction error by 6.5% compared to the use of average penetration rates per mining block. This quantifies the effect of ignoring non-additivity for hardness-related geometallurgical variables, which has rarely been considered.

The prediction of metallurgical responses of the operating plant and their incorporation into short-term stochastic production scheduling in mining complexes is finally extended beyond ball mill throughput. This is achieved by creating prediction models of consumption rates of reagents and consumables in a gold mining complex, considering caustic soda, and hydrochloric acid, and grinding media consumptions. Because all sources of information stem from centrally collected datasets, the prediction models developed in this thesis can be constructed instantly and give a potential financial advantage compared to laboratory tests for steel ball consumptions, rock hardness and strength, etc. Lastly, the developed simultaneous stochastic optimization model optimizes the short-term extraction sequence and destination of materials in a gold mining complex and integrates the prediction models of reagents and consumables into the optimization resulting in an added profit of 3.2% compared to a conventionally created short-term production schedule.

With the new developments presented in this thesis the simultaneous stochastic optimization of mining complexes can now be applied for short-term planning, modelling the operational aspects and uncertainties of the mining fleet and metallurgical behaviour of processing plants based on geometallurgical properties in greater detail. The integration of these pertinent short-term aspects leads to short-term mine plans that are more likely to align with long-term production targets while benefitting from synergistic effects that maximize the profit of the mineral value chain.

Résumé

Un complexe minier est une entreprise intégrée de mines et d'installations en aval qui extrait des matières premières, transforme les matières extraites en produits vendables et transporte les produits vers les marchés et les clients. Conventionnellement, les composants individuels d'un complexe minier sont optimisés localement et indépendamment les uns des autres, ce qui entraîne une sous-performance de la chaîne de valeur minérale. Il a été démontré que l'optimisation stochastique simultanée des complexes miniers permet de créer des plans stratégiques de mines qui augmentent la valeur actuelle nette tout en réduisant le risque d'atteindre les objectifs de production en intégrant l'incertitude géologique et des prix. Alors que ces développements optimisent conjointement les décisions stratégiques d'un complexe minier, la planification à court terme prend des décisions quotidien/hebdomadaire/mensuel pour atteindre au mieux les objectifs de production à long terme et maximiser la valeur. Ces décisions comprennent, sans s'y limiter, la séquence d'extraction à court terme, la destination des matériaux et le flux de matériaux en aval dans les complexes miniers. De plus, l'allocation optimale de la flotte minière est un aspect important de la planification à court terme ; cependant, l'optimisation stochastique conjointe des planifications de production à court terme et de la gestion de la flotte dans les complexes miniers n'a pas encore été développée. De plus, les propriétés géométallurgiques qui déterminent les revenus, les coûts et la capacité à atteindre les objectifs de production ne sont pas intégrées dans l'optimisation des planifications de production à court terme dans les complexes miniers et les principaux problèmes liés à l'augmentation de l'échelle et au mélange des propriétés géométallurgiques non additives doivent être adressés. Les technologies numériques permettent le stockage centralisé d'une grande quantité de données de production dans les mines et les usines de traitement, cependant, de nouveaux moyens doivent être trouvés pour intégrer ces nouvelles sources d'information dans la prise de décision à court terme.

Cette thèse étend l'optimisation stochastique simultanée des complexes miniers pour la planification à long terme à un cadre décisionnel pour la planification minière à court terme par l'incorporation de modèles de gestion de flotte et de prédiction géométallurgique des performances des usines dans l'optimisation à court terme des complexes miniers, qui est assistée par l'utilisation

d'ensembles de données collectées à partir des processus de production dans les mines et les usines de traitement. L'horizon de planification d'intérêt pour le cadre de planification à court terme développé dans cette thèse s'étend sur plusieurs semaines à plusieurs mois de matériaux futurs programmés.

On développe d'abord un nouveau modèle de programmation stochastique en nombres entiers pour la planification à court terme qui étend l'optimisation stochastique simultanée des complexes miniers pour permettre la planification d'une flotte de camions hétérogènes et l'allocation de pelles tout en considérant les coûts et la réduction de la production causés par le déplacement des pelles. En plus de l'incertitude géologique, les incertitudes de performance des équipements liées aux taux de production, aux disponibilités et aux temps de cycle des camions sont intégrées. La nouvelle méthode est appliquée à un complexe minier aurifère et comparée à une approche conventionnelle en deux étapes, où la planification de la production à court terme est d'abord optimisée avant d'optimiser l'allocation de la flotte minière. La méthode développée réduit les coûts générés par les mouvements de pelle de 56% et diminue les coûts totaux d'exploitation des camions de 3%.

Ensuite, un modèle géométallurgique pour la prédiction du débit des broyeurs à boulets dans les complexes miniers est développé. Il utilise les taux de pénétration mesurés lors du forage et les débits enregistrés de l'usine en opération. La création de proportions de dureté évite les biais typiquement introduits par le changement de support et le mélange de propriétés de dureté géométallurgiques non additives. Une étude de cas dans un complexe minier aurifère montre que le débit peut être prédit avec un coefficient de corrélation de Pearson allant jusqu'à 0,8. En intégrant le modèle de prédiction de débit dans la formulation d'optimisation stochastique simultanée, la production hebdomadaire planifiée peut être atteinte de manière fiable car les matériaux planifiés correspondent à la performance prédite du broyeur. Des comparaisons avec l'optimisation utilisant des contraintes de tonnage conventionnelles révèlent que les déficits de production prévus jusqu'à 7% peuvent être atténués par l'intégration du nouveau modèle de prédiction du débit.

Le modèle de prédiction du débit est ensuite amélioré en incluant des mesures enregistrées de la consommation d'énergie du broyeur à boulets et des distributions granulométriques de la matière première. Puisque l'ajout de nouvelles caractéristiques augmente les possibilités d'interactions non linéaires entre les variables prédictives et le débit du broyeur à boulets, un réseau neuronal est utilisé, remplaçant un modèle de régression linéaire multiple. Les comparaisons montrent que l'ajout de la puissance du broyeur à boulets et de la granulométrie du produit réduit l'erreur de prédiction du débit de 10,6%. En plus, les proportions de dureté créées à base des taux de pénétration peuvent diminuer l'erreur de prédiction de 6,5% par rapport à l'utilisation des taux de pénétration moyens par bloc minier. Cela quantifie l'effet de l'ignorance de la non additivité des variables géométallurgiques liées à la dureté, qui a rarement été prise en compte.

La prédiction des réponses métallurgiques des installations en fonctionnement et leur incorporation dans la planification stochastique à court terme de la production dans les complexes miniers est enfin étendue au-delà du débit des broyeurs à boulets. Ceci est réalisé en créant des modèles prédictifs des taux de consommation de réactifs et de consommables dans un complexe minier aurifère, en considérant les consommations de soude caustique, d'acide chlorhydrique et de médias de broyage. Puisque toutes les sources d'information proviennent de données collectées de manière centralisée, les modèles prédictifs développés dans cette thèse peuvent être construits instantanément et offrent un avantage financier potentiel par comparaison aux tests de laboratoire pour la consommation de boulets en acier, la dureté des roches, et autres. Enfin, le modèle d'optimisation stochastique simultanée développé optimise la séquence minière à court terme et la destination des matériaux dans un complexe minier aurifère et intègre les modèles prédictifs des réactifs et des consommables dans l'optimisation, ce qui se traduit par un avantage supplémentaire de 3,2% par rapport à une planification de production à court terme créée de manière conventionnelle.

Avec les nouveaux développements présentés dans cette thèse, l'optimisation stochastique simultanée des complexes miniers peut désormais être appliquée pour la planification à court terme, en modélisant plus en détail les aspects opérationnels et les incertitudes de la flotte minière et le comportement métallurgique des usines de traitement basé sur les propriétés géométallurgiques. L'intégration de ces aspects significatifs à court terme permet d'élaborer des plans miniers à court terme plus susceptibles de s'aligner sur les objectifs de production à long terme, tout en bénéficiant d'effets synergiques qui maximisent le profit de la chaîne de valeur minérale.

Contribution of Authors

The author of this thesis is the primary author for all manuscripts included in the dissertation. Professor Roussos Dimitrakopoulos is the supervisor and is included as co-author in all of the following articles:

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Chapter 4 – Both, C., and Dimitrakopoulos, R. (2021). Applied Machine Learning for Geometallurgical Throughput Prediction – A Case Study using Production Data at the Tropicana Mining Complex. *Minerals*. https://doi.org/10.3390/min11111257

Chapter 5 – Both, C., and Dimitrakopoulos, R. (Under Review). Utilization of Geometallurgical Predictions of Processing Plant Reagents and Consumables for Production Scheduling under Uncertainty. *International Journal of Mining Reclamation and Environment.*

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1 Introduction

1.1 Overview

A mining complex is an integrated business that comprises open pit or underground mines and a group of downstream facilities connected by various material handling methods which transform mined raw materials into sellable products (Figure 1-1). This integrated business involves numerous tasks which can be summarized into: (i) extraction of raw materials using heavy equipment such as shovels and trucks; (ii) beneficiation of valuable materials into products that can be sold; (iii) safe disposal of non-valuable materials such as waste rock and tailings; (iv) connecting the different downstream facilities by material handling systems to deliver products to customers and/or the spot market.

The goal of the mathematical optimization of mining complexes is to maximize the net present value (NPV) of the complete integrated business while managing uncertainties, obeying operational constraints, and minimizing the environmental impact of the operation both during and after the depletion of the mineral reserves. This is a complex task given all the components, variables and decisions to be taken for a mining complex over the life-of-mine planning horizon (Whittle and Whittle, 2007). Furthermore, decisions must be taken at different time scales: strategic (over the complete life of mine), tactical (shorter time scale, typically within a year) and operational (daily basis) (Hartmann and Mutmansky, 2002). Conventionally, single mines in the mining complex are optimized individually using several optimization algorithms (Dagdelen, 2005; Hustrulid et al., 2013). These algorithms optimize decisions such as which portions of material will be extracted or left in the ground (pit limit optimization), the sequence of extraction (production scheduling) and distinguishing between ore and waste material (cut-off grade optimization). Downstream facilities of the mining complex, such as stockpiles, processing plants, and material handling infrastructure (mining fleet, conveyors, railway, and port), are typically regarded as separate entities and are optimized locally and independently.

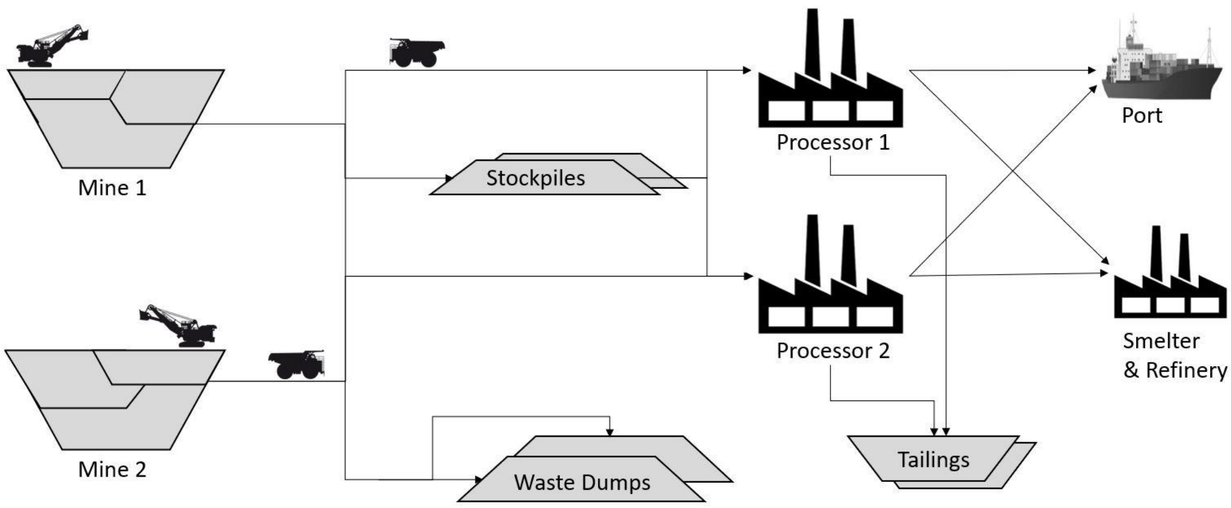


Figure 1-1 A hypothetical mining complex

The simultaneous or joint optimization of mining complexes aims to move away from the local and independent optimization of several parts of the mining complex, which is suboptimal, towards a globally optimal solution that can unlock synergies between its components. Simultaneous optimization of mining complexes is currently performed on the strategic planning level and includes the optimization of long-term multi-mine production schedules, the destination policy of the mined materials, strategic stockpiling, material flows to downstream locations and operational policies of processing facilities (Hoerger et al., 1999; Stone et al., 2007; Whittle, 2007, 2010, 2014; Whittle and Whittle, 2007). Although the simultaneous optimization methods show an increase in NPV compared to locally optimized mining complexes, they ignore the inherent uncertainties of a mining complex, most prominently geological uncertainty stemming from the imperfect knowledge of the materials in the ground (Dowd, 1994, 1997; Vallée, 2000).

Stochastic optimization methods for long-term mine planning utilize a set of orebody simulations to account for grade and material type uncertainty, leading to production schedules that not only maximize NPV but also reduce the risk of not meeting production targets and grade blending targets over the life-of-mine (Ramazan and Dimitrakopoulos, 2005, 2013; Benndorf and Dimitrakopoulos, 2013). The stochastic optimization models have recently been extended to consider the whole mining complex, resulting in the simultaneous stochastic optimization of mining complexes for long-term planning (Montiel and Dimitrakopoulos, 2015, 2018; Goodfellow and Dimitrakopoulos, 2016, 2017; Montiel et al., 2016). These models do not only provide a risk-based approach to optimize life-of-mine production schedules, destination policy, and downstream material flows simultaneously, but also shift the focus from economic block values to the value of products sold. However, a comprehensive framework that simultaneously optimizes the pertinent decisions in a mining complex on short-term scale has not yet been developed and additional sources of uncertainty in the short-term must be considered.

Short-term mine planning generally aims to make daily, weekly or monthly to decisions on a horizon of less than one to two years in order to meet production targets received by strategic, long-term plans (Wilke and Reimer, 1977; Fytas and Calder, 1986; Fytas et al., 1987; Hartmann and Mutmansky, 2002; Hustrulid et al., 2013; Blom et al., 2018). Within the broader spectrum and definition of short-term planning, this thesis focuses on decision-making that optimizes weekly to monthly periods up to a planning horizon of one year. Conventionally, short-term production scheduling ensures that the different processing facilities of the mining complex are effectively supplied with material that fulfills tonnage and grade blending constraints while following the outline of the long-term production schedule. The focus generally lies on minimizing operational costs instead of maximizing the profit of a mining complex as a whole. Moreover, existing short-term optimization models cannot deal with the non-linear transformation of extracted and blended materials in processing facilities and are limited in the type of decisions optimized simultaneously.

Fleet management is an important aspect of short-term planning and efforts to include optimal allocation decisions of mining equipment, primarily shovel allocation, into short-term mine planning optimization can be found in varying extent in the literature (Wilke and Reimer, 1977; Fytas et al., 1987; L'Heureux et al., 2013; Mousavi et al., 2016c). Stochastic optimization of short-term mine plans considering geological and equipment uncertainty has recently been shown (Villalba Matamoros and Dimitrakopoulos, 2016; Quigley and Dimitrakopoulos, 2019). However, the joint stochastic optimization of fleet management and short-term production scheduling must be extended to mining complexes, considering its inherent uncertainties such as production rates, availabilities, cycle times, and geological uncertainty.

Furthermore, geometallurgical properties, i.e., rock attributes that drive revenues, costs, and the ability to meet production targets, as well their inherent uncertainty, are relevant and should be considered and integrated into the simultaneous stochastic short-term optimization of mining complexes. However, challenges related to the non-additivity of geometallurgical properties, such as hardness and grindability of the rock, need to be overcome. Main issues lie in the upscaling and blending of non-additive properties, and the corresponding evaluation of their metallurgical response (e.g., ball mill throughput, reagent consumptions, metal recoveries) within the optimization of mine production schedules in mining complexes, which are addressed in this thesis. This is especially important in the short-term, where the degree of blended materials in a single period is typically high.

Another critical aspect that will be explored in this thesis is the utilization of collected datasets stemming from production processes in the mining complex to build links between the rock characteristics of the mineral resource and their metallurgical responses in the processing plant. Some datasets provide spatially dense information, such as measurement while drilling (MWD) data and can be used to massively increase geometallurgical information which is typically sparse. Other datasets collected at the processing plants provide sensor information and measured responses in the correct operating scale. These collected data sources have been routinely overlooked for mine planning optimization so far and new methods have to be found to capitalize on their information for better short-term decision-making in mining complexes.

1.2 Literature Review

This section contains a review of the literature pertinent to the topics discussed in this thesis. Section 1.2.1 provides an overview of strategic mine planning, leading towards the development of simultaneous optimization of mining complexes for long-term production scheduling. As will be discussed thereafter, these developments serve as a foundation for the short-term decision-making framework for mining complexes developed in this thesis. The relevant technical literature for short-term mine planning is critically reviewed in Section 1.2.2, and their current limits are pointed out. Section 1.2.3 addresses geometallurgical properties and concepts, whose integration into mine planning optimization can have a considerable effect on the business' value both for short and long-term planning. Finally, the utilization of production data generated in operating mining complexes is reviewed in Section 1.2.4 to identify new ways of how production data can be used to improve geometallurgical modelling linked to short-term mine planning in mining complexes.

1.2.1 Strategic mine planning

Strategic, or life-of-mine planning of a mining complex aims to make long-term, annual decisions that maximize the net present value (NPV) of the integrated business over its complete life cycle. Important strategic decisions in a mining complex include the optimization of the yearly extraction sequence of the mines, the destination policy of the materials extracted, long-term stockpile management, procurement of capital investments (e.g., purchase of mining equipment, processing plant expansion, etc.), and the optimization of the downstream facility network of the mineral value chain. Conventionally, the optimization of these strategic decisions is divided into individual computationally manageable steps because of their large complexity (Whittle, 1988, 1999; Dagdelen, 2005; Hustrulid et al., 2013; Caccetta, 2016). One example of individual strategic decision-making is the cut-off grade optimization, which aims to define the destination of the extracted material over the mine life, given a set of available processing facilities and waste dumps. In the simplest case, cut-off grade optimization distinguishes between valuable (ore) and waste material. Several methods and algorithms have been proposed that provide optimized cut-off grades over the life-of-mine for single elements (Henning, 1963; Lane, 1964, 1988; Taylor, 1972; Dagdelen, 1992; Whittle and Wharton, 1995; Menabde et al., 2007; Rendu, 2014), multiple elements (Lane, 1984; King, 2001; Osanloo and Ataei, 2003), accounting for supply uncertainty (Asad and Dimitrakopoulos, 2013), commodity price uncertainty (Dowd, 1976; Githiria and Musingwini, 2019), and stockpiles (Asad, 2005). A review of cut-off grade optimization can be found in Asad et al. (2016). Recently, clustering techniques have been used for the optimization of material destinations as well (Del Castillo and Dimitrakopoulos, 2016; Sepúlveda et al., 2018; Li et al., 2020).

The yearly extraction sequence of a mine from its current state until its depletion (life-of-mine production schedule) can be optimized using mathematical programming techniques such as linear programming (LP), integer programming (IP), mixed-integer programming (MIP), and dynamic programming (DP) (Johnson, 1968; Gershon, 1982, 1983; Tolwinski and Underwood, 1996; Hustrulid et al., 2013). The mathematical optimization models take as input an array of three-dimensional blocks dividing the mineral reserve into equally sized mining blocks, or selective mining units (SMU). Each block contains information of its mineral content and other pertinent geological and geometallurgical information. The goal is to find the optimal grouping of blocks, forming yearly periods of material extraction, such that the NPV is maximized while obeying mining and processing capacities, mining precedence, and grade quality constraints (Johnson, 1968). One important limitation is the typically large size of mineral resource models (up to 10s of millions of blocks) which can make it impossible to obtain a solution even with today's available commercial solution techniques. Thus, many heuristic and metaheuristic algorithms have been proposed to obtain a life-of-mine production schedule at the cost of losing the guarantee of obtaining the mathematically optimal solution (Gershon, 1987; Tolwinski and Underwood, 1996; Kumral and Dowd, 2005; Ferland et al., 2007; Bienstock and Zuckerberg, 2010; Sattarvand and Niemann-Delius, 2011). Other researchers propose block aggregation techniques to reduce the problem size for single-mine life-of-mine production scheduling (Tolwinski, 1998; Ramazan, 2006; Tabesh and Askari-Nasab, 2011; Mai et al., 2018). Many of the proposed methods create life-of-mine production schedules within pre-defined ultimate pit limits and pre-defined pushbacks which are optimized beforehand using separate methods (Lerchs and Grossmann, 1965; Alford and Whittle, 1986; Zhao and Kim, 1992; Hochbaum, 2001). This stepwise approach is not mathematically required and leads to suboptimal solutions. However, it is often used to reduce the problem size so that production schedules can be obtained in reasonable time spans.

Other individual efforts in strategic mine planning include the optimization of strategic mine designs and layouts. Godoy (2002) and Del Castillo et al. (2015) provide a mathematical model to optimize annual mining rates, which includes the procurement and ownership costs of the mining fleet, i.e. shovels and trucks. Bradley et al. (1985) optimize the number of required storage capacities and train loading facilities in open pit coal mines and show an application at the Wyoming's Powder River Basin. Optimization of downstream facilities such as mineral processing plants in mining complexes are proposed by other researchers. Examples are the optimization of grinding circuits (Huband et al., 2006; Farzanegan and Vahidipour, 2009), optimization of leaching operations (Fleming et al., 2011), and Grade Engineering (Carrasco et al., 2016). These performance improvements are conventionally considered separately from other strategic decisions discussed above.

Next to the described individual optimization approaches, researchers have also proposed methods to simultaneously optimize important strategic decisions in a mining complex. The so-called simultaneous, or global optimization of mining complexes has demonstrated to outperform approaches that separately optimize parts of the mining complex by providing higher NPV and meeting production targets better. Various proposed simultaneous optimization approaches are reviewed in Section 1.2.1.1. Another aspect that is commonly not addressed by strategic mine planning is the uncertainty of input parameters, most prominently geological and grade uncertainty stemming from the imperfect knowledge of the geological reserve. Efforts in strategic mine planning under uncertainty are reviewed in Section 1.2.1.2. Recently, simultaneous stochastic optimization has emerged, which aims to capitalize on joint decision-making in mining complexes while accounting for various sources of uncertainty, which is reviewed in Section 1.2.1.3.

1.2.1.1 Simultaneous optimization of mining complexes

Hoerger et al. (1999) simultaneously optimize strategic allocation, stockpiling and blending decisions for material streams in a mining complex in a single model, based on a mixed integer programming formulation developed by Urbaez and Dagdelen (1999). The model considers more than 90 defined metallurgical ore types that are provided from a set of 50 sources within a gold mining complex. Parcels of material can be sent to either one of 60 processors or eight stockpiles available in the mining complex. More specifically, three continuous variables are used to (i) define the amount of material extracted from a mine (source) and sent to a processor (ii) sent from a mine to a stockpile and (iii) sent from a stockpile to a processor. This approach maximizes NPV and satisfies blending constraints well on a strategic level, but it has drawbacks. Each mine (source) is divided beforehand into a fixed set of five or more sequences. The first sequence has to be mined out completely before mining of the second sequence can start and so on. This is equivalent to a division of material to be mined into a sequence of pushbacks before optimization, which is not optimal. Furthermore, by only considering the partial extraction of a certain metallurgical ore type (increment) within one sequence, there is no guarantee that the scheduled amount of material can be provided from the mine in a certain period in the short-term since other increments (e.g., waste rock) may have to be removed before accessing the desired material.

Another framework that aims to simultaneously optimize various components of a mining complex is proposed by Whittle (2007) and further described by Whittle and Whittle (2007). Instead of using one single optimization formulation, the authors aim to connect several optimization procedures along the mineral value chain by utilizing several algorithms. Whittle (2010) provides a general overview of one of their optimization algorithms ProberC. Given a sequence of pushbacks in an optimized pit limit, the algorithm searches global solutions in a mining complex by creating production schedules that approximate optimally blended products further downstream. A heuristic algorithm iteratively creates feasible solutions, until a local optimum is reached. Whittle (2014) states that although recognized in theory that all decisions should be optimized simultaneously, ProberC is only able to optimize some parts simultaneously given the large number of decision variables in typical mining complexes. Specifically, the described approach suffers from excluding final pit limit definition and computation of nested pit shells prior to sequencing from its simultaneous optimization framework. Further, Whittle (2014) suggests that optimizers should include the natural flexibility of processing facilities by adapting their performance (e.g., throughput) to optimize metal production.

Wooller (2007) provides an overview of the COMET mine planning software that optimizes a life-of-mine production schedule and operational policies of processing facilities. Possible operational policies include optimized cut-off grades and other policies such as mill throughput/recovery rules (Wooller, 1999). The algorithm uses a heuristic successive approximation dynamic programming approach proposed by King (2001). Successive iterations are performed that heuristically search for an operational policy and mining sequence that maximizes the NPV of the mining complex. However, several drawbacks are identified. The scheduling algorithm requires a previously optimized sequence of pushbacks as input and uses an aggregation of blocks called ‘increments’ of variable size for scheduling. Furthermore, certain blending requirements cannot be controlled because minimum grade blending constraints cannot be modelled.

Stone et al. (2007) outline basic mechanisms of BLAZOR, which has been designed to provide a production schedule for material extracted from several mines to provide an optimally blended product. The smallest units considered by BLAZOR for scheduling are aggregates (AGG’s) that consist of a larger number of spatially connected mining blocks. No AGG shall be mined before all its predecessor AGGs are completely mined out. Any proportion of a certain material contained in an AGG may be extracted in a certain period if all constraints related to mining, crushing, screening and market requirements are obeyed. No material portion is defined as waste a priori but evaluated by the optimization procedure which respects the capacity of the downstream value chain infrastructure for realistic product/value evaluation. Rather than defining the pit limits of each mine individually and dividing the pit limit into pushbacks afterward, BLAZOR creates production schedules that result in optimal pit limits. Aggregation of material is deemed necessary to reduce the size of the optimization problem which creates drawbacks. Blocks in the orebody model are spatially aggregated, whereas one aggregate can be scheduled in several periods (years) and partially be sent to several processing streams and available waste dumps.

Topal and Ramazan (2012b) propose a network flow model for strategic mine planning using linear programming. The model determines annual tonnages of materials sent though the network from one node to another. The network consists of mines, stockpiles, processors, ports, and delivery points to customers. While the model optimizes material flows in mineral value chains, the model cannot provide detailed production schedules of the mines because the granularity of mining blocks and the required predecessor constraints are not implemented.

1.2.1.2 Strategic mine planning under uncertainty

Conventionally, mine planning optimization models use a single geological orebody model as input for optimization. These orebody models are generated using geostatistical estimation methods (David, 1977; Journel and Huijbregts, 1978; Isaaks and Srivastava, 1989; Goovaerts, 1997) which provide average-type estimates of metal grades for each mining block in the deposit. However, estimation methods are limited by the smoothing effect, leading to a misrepresentation of the true grade distribution in the deposit by pushing the grade distribution towards its mean. Thus, proportions of very low grade and very high grade are underrepresented in the estimated block model while the proportion of the medium-grade material is overrepresented. Furthermore, the natural variability of grades within the deposit is underrepresented in the estimated block model. Geostatistical simulation methods (Journel and Huijbregts, 1978; Goovaerts, 1997; Remy et al., 2009; Rossi and Deutsch, 2014) aim to generate a set of equally probable representations of the orebody that better represent the spatial distribution of grades and correlations within the orebody. As a group, geostatistical simulations may be used to quantify the geological uncertainty associated with mine production planning to evaluate the financial and production forecasts of the short-term and long-term production schedules (Ravenscroft, 1992; Dowd, 1994, 1997; Dimitrakopoulos et al., 2002). Rather than solely assessing the risk of a mine production schedule, stochastic mine planning and production scheduling methods aim to utilize stochastic orebody simulations as inputs to optimization, which are important keystones for the stochastic optimization methods developed in this thesis.

Stochastic mine production scheduling models are typically formulated as a two-stage stochastic mixed integer programming model with fixed recourse (Birge and Louveaux, 2011). Ramazan and Dimitrakopoulos (Ramazan and Dimitrakopoulos, 2005, 2013) develop a stochastic integer programming formulation for long-term production scheduling which takes a set of geostatistical orebody simulations as input for optimization instead of a single, deterministic orebody model. In this way, the optimization process is supplied with a more accurate representation of grade distribution and grade variability, which results in a higher expected NPV of the life-of-mine production schedule. Several recourse variables are introduced that minimize deviations of stochastic targets such as ore tonnage, metal production and minimum metal grades. Ramazan and Dimitrakopoulos (2013) extend their formulation to integrate the option to stockpile marginal-grade ore while approximating the average grade of the stockpile.

Benndorf and Dimitrakopoulos (2013) apply the model developed by Ramazan and Dimitrakopoulos (Ramazan and Dimitrakopoulos, 2005, 2013) to control grade targets of multiple elements under grade uncertainty in an iron ore deposit. Leite and Dimitrakopoulos (2014) report a 29 % increase in expected NPV when applying the model to a copper deposit compared to deterministically optimized life-of-mine production schedules. This confirms the superiority of stochastic solutions compared to their deterministic counterparts, which has also been observed in earlier studies (Godoy and Dimitrakopoulos, 2004; Leite and Dimitrakopoulos, 2007; Albor Consuegra and Dimitrakopoulos, 2009). Morales et al. (2019) use a similar stochastic integer programming formulation to include uncertainty of metal recovery and mill throughput next to metal uncertainty into life-of-mine production scheduling. Mai et al. (2019) utilize a block aggregation technique to reduce the size of their stochastic integer program for mine production scheduling. Vallejo and Dimitrakopoulos (2018) and Maleki et al. (2020) create stochastic production schedules that consider grade uncertainty and the volumetric uncertainty of rock types and lithologies of the mineral deposit (Goovaerts, 1997; Osterholt and Dimitrakopoulos, 2005; Mariethoz and Caers, 2015). Rimélé et al. (Rimélé et al., 2018) add the option of in-pit waste and tailings disposal to stochastic life-of-mine production scheduling and demonstrate their stochastic model at an iron ore mine with a moderately dipping orebody. Several metaheuristic and hyperheuristic algorithms have been proposed to solve large-scale stochastic integer mine production scheduling models in reasonable time spans (Lamghari and Dimitrakopoulos, 2012, 2016a, 2016b, 2020; Lamghari et al., 2014).

Menabde et al. (2007) optimize the extraction sequence of block aggregates (benches) and simultaneously optimize single-element cut-off grades per period, while accounting for metal uncertainty. The proposed mathematical formulation divides the continuous space of cut-off grades into ‘grade bins.’ An additional decision variable is introduced, which defines the optimal grade bin that discriminates extracted material into ore and waste for every period, next to optimizing the life-of-mine production schedule of a single mine.

1.2.1.3 Simultaneous stochastic optimization of mining complexes

Montiel and Dimitrakopoulos (2015) propose a non-linear optimization model for the simultaneous stochastic optimization of mining complexes, which optimizes multiple pits, multiple processors, and stockpiles while accounting for geological uncertainty. In addition, new decision variables are introduced that optimize operational modes in processing plants and transportation alternatives. As a result, a life-of-mine production schedule is obtained that maximizes NPV and evaluates the value of products sold accounting for non-linear behaviour of blended materials instead of relying on economic values of blocks. The model is extended by Montiel et al. (2016) for optimizing mining complexes that comprise both open pit and underground mines. Montiel and Dimitrakopoulos (2018) showcase their developed model by optimizing the life-of-mine production schedule of Newmont’s Nevada operations. The mining complex consists of two open pit mines, three external sources of concentrate, multiple rehandling stockpiles for sulfide and oxide ore, an autoclave for refractory material, a heap leaching facility, and a processing facility for oxide ore.

Goodfellow and Dimitrakopoulos (2016, 2017) develop a non-linear stochastic mixed integer optimization formulation that simultaneously optimizes the life-of-mine production schedule, destination of extracted materials, and downstream material flow decisions for multi-pit, multi-process mining complexes. The authors model a mining complex as a directed graph of material streams and introduce primary and hereditary attributes in their formulation. The purpose of these attributes is to calculate potentially non-linear hereditary attributes as a function of additive, primary material flows at a set of locations consisting of mines, stockpiles, and processors. Examples of frequently utilized hereditary attributes are recovered metal, cost of mining, cost of processing, cost of stockpiling and consumption of additives. In this way, non-linear transformations in the beneficiation processes of ore are realistically accounted for and it is possible to maximize the value of products that are sold to customers. Thus, the simplified assumption of economic block values, implying that blocks are processed independently of each other in a pre-defined destination can be overcome. The number of destination decisions to be optimized is considerably reduced by grouping mining blocks bearing similar attributes together utilizing a clustering algorithm (Arthur and Vassilvitskii, 2007). A modified Simulated Annealing algorithm, paired with particle-swarm optimization (PSO) and differential evolution (DE) is used to obtain solutions to the typically large-scale optimization in reasonable times spans.

Several case studies in real-world mining complexes have demonstrated the advantage of simultaneous stochastic optimization over conventionally optimized mine plans (de Carvalho and Dimitrakopoulos, 2019; Del Castillo and Dimitrakopoulos, 2019; Kumar and Dimitrakopoulos, 2019; Levinson and Dimitrakopoulos, 2019, 2020; Saliba and Dimitrakopoulos, 2019a, 2019b; LaRoche-Boisvert and Dimitrakopoulos, 2021). The case studies also underline the generality of the underlying optimization model (Goodfellow and Dimitrakopoulos, 2016) and its high adaptability to mining complexes of any kind. Levinson and Dimitrakopoulos (2019) apply simultaneous stochastic optimization to optimize the life-of-mine extraction sequence, cut-off policy, stockpile policy and waste management focusing on the save disposal of acid generating waste in a gold mining complex. Saliba and Dimitrakopoulos (2019a) apply simultaneous stochastic optimization in a gold mining complex while considering a tailings expansion by integrating a capital expenditure option through simultaneously optimized capital expenditure decisions (Goodfellow and Dimitrakopoulos, 2015). Capital investment decisions are extended to dynamic simultaneous stochastic optimization in mining complexes by Del Castillo and Dimitrakopoulos (2019) using multistage stochastic programming. The multistage approach has been applied at a gold mining complex considering an investment option of an autoclave expansion (Levinson and Dimitrakopoulos, 2020). Saliba and Dimitrakopoulos (2019b) incorporate joint commodity price and grade uncertainty into the stochastic optimization of a multi-pit multi-processor gold mining complex. De Carvalho and Dimitrakopoulos (2019) study the influence of geostatistical high-order stochastic simulations of metal grades (Mustapha and Dimitrakopoulos, 2010; Minniakhmetov et al., 2018; de Carvalho et al., 2019) on the simultaneous stochastic optimization of mining complexes. The authors show that high-order simulations reproduce the spatial connectivity of high and low grades better, which results in a long-term production schedule that generates 5% higher NPV compared to production schedules optimized on simulations using two-point statistics such as Sequential Gaussian Simulation. Kumar and Dimitrakopoulos (2019) introduce constraints for non-additive geometallurgical hardness properties for the simultaneous optimization of mining complexes. Paithankar et al. (2020) propose simultaneous stochastic optimization of a single-mine production schedule, long-term stockpiling and cut-off grades using a maximum flow algorithm paired with a genetic algorithm. LaRoche-Boisvert and Dimitrakopoulos (2021) apply simultaneous stochastic optimization at the Rosebel Gold mining complex. They simultaneously optimize the production schedule of three mines and implement an SAG mill availability constraint utilizing deterministic mill throughput rates for each of the three available ore types.

While the reviewed literature shows the benefits of simultaneous stochastic optimization of mining complexes, it has so far only been applied for strategic decision-making producing life-of mine production schedules on yearly discretization. The literature on mine planning and optimization methods for shorter time horizons is reviewed next to identify key decisions and uncertainties pertinent to shorter time scales that must be integrated to apply simultaneous stochastic optimization of mining complexes in the short-term.

1.2.2 Short-term mine planning

Short-term mine planning generally aims to make optimal decisions over a timeframe of days to months, bound by current operational conditions and constraints, to meet production targets imposed by the optimized strategic mine plan (Hustrulid et al., 2013). Consequently, several operational decisions need to be addressed by short-term planning such as what to mine, i.e. the monthly, weekly or daily extraction sequence, where to send mined material, where to allocate loading and hauling equipment optimally in the mine(s), how to utilize stockpiles, how to operate processing facilities optimally considering geometallurgical properties and how to implement other necessary actions in the mine such as blasting, ramp building, grading, fleet maintenance, and more. To account for all these complex tasks, short-term mine planning typically proceeds in several stages (Alarie and Gamache, 2002; Hustrulid et al., 2013; Moradi Afrapoli and Askari-Nasab, 2017; Blom et al., 2018) Given a long-term strategic plan, the short(er)-term extraction sequence and material destinations are defined first. Blom et al. (2018) review the literature of short-term production scheduling for production horizons ranging from one week to two years. Successively, mining equipment is allocated to the optimized sequence and finally, operational, intra-day decisions such as truck dispatching are made (Alarie and Gamache, 2002; Moradi Afrapoli and Askari-Nasab, 2017). In the following, methods for short-term production scheduling are reviewed first in Section 1.2.2.1. The optimization of fleet allocation decisions is reviewed thereafter in Section 1.2.2.2. Combined efforts of short-term production scheduling and fleet management are reviewed in Section 1.2.2.3. As it will be seen in the literature reviewed herein, the developed short-term planning methods in the first three sections routinely ignore relevant sources of uncertainty for short-term decision-making, such as geological uncertainty and equipment performance uncertainty. Henceforth, these methods will be called conventional methods. Section 1.2.2.4 then discusses developments in stochastic short-term mine planning that integrate some of the relevant sources of uncertainty to create short-term mine plans that are more likely to meet production targets.

1.2.2.1 Short-term mine production scheduling

Wilke and Reimer (1977) develop a linear program (LP) for short-term production scheduling of a single weekly period by considering upper and lower processing capacities as well as upper and lower blending requirements for multiple elements. The first goal in this model is to define the extraction of blocks for the next week of production that meet ore blending requirements. The authors also aim to create a production schedule that is in conformity with the strategic mine plan by assigning priority factors to each mining block in such a way that blocks which are to be mined sooner receive higher priority. Although satisfying needs for short-term mine planning in some respects, the linear program cannot ensure block accessibility by shovels. Furthermore, the destination of materials is fixed beforehand, and only a single period is considered for planning, lacking the foresight of a multi-period planning approach.

Fytas and Calder (1986) and Fytas et al. (1987, 1993) present optimization models for long-range and short-range planning in open pit mines. Among other algorithms, the authors present an LP model for monthly short-term production scheduling which divides the volume to be scheduled on the short-term horizon into multiple shovel regions. A priority factor is assigned to each region depending on its accessibility and compatibility of blending requirements for sellable products. An iterative process is proposed where the user reduces the priorities of non-practical regions to receive a practical solution. While it cannot be guaranteed that the short-term extraction sequence follows a given long-term extraction plan, the linear programming model has other limitations such as allowing block aggregation and mining of partial blocks.

Chanda and Wilke (1992) propose a short-term production scheduling approach for stratiform orebodies by combining linear goal programming and post-LP simulation of ore and waste production. Their approach aims to select the optimal shovel regions such that metal tonnage is maximized and deviations in grade and ore tonnage are minimized. However, the optimization only considers a single period and mining precedence of shovel regions is not enforced. Mann and Wilke (1992) develop a combination of computer-aided design (CAD) techniques and linear programming for short-term mine planning. The linear program minimizes deviations from quality targets of the processed ore for multiple periods. The minimization of shovel moves is attempted indirectly by choosing the solution with the lowest number of active extraction points. Chanda and Dagdelen (1995) apply linear goal programming for single-period short-term planning considering fractional removal of blocks to satisfy blending requirements of the processed ore.

As linear programming can lead to infeasible production sequences for mining problems due to partially scheduled blocks, mixed integer linear programming has since been deemed more appropriate for optimizing mine production schedules (Gershon, 1983). Youdi et al. (1992) propose a two-step procedure for short-term mine planning consisting of a mixed integer goal program for quarterly mine plans and a graphic design system to arrange production and stripping ratio in the short-term. Smith (1998) utilizes mixed integer programming for short-term mine production scheduling using the programming modeling language AMPL. Gholamnejad (2008) adds horizonal precedence constraints to previously developed mixed integer linear programs to account for short-term accessibility of blocks located on the same bench extracted by truck-and shovel operations. Huang et al. (2009) present details of the MineSight Schedule Optimizer (MSSO) which provides short-term production schedules using mixed integer linear programming. The software considers multiple mining areas, mining fleet capacities, and multiple processing destinations including blending and quality requirements.

Eivazy and Askari-Nasab (2012) propose a mixed integer programming formulation for short-term production scheduling of monthly periods, which incorporates decision variables to send material to multiple processors and stockpiles while considering one mine. The authors include the ability to reclaim material from a fixed-grade stockpile, which is a necessary simplification to keep the model linear. Furthermore, decisions related to optimal access-ramp placements are included. One drawback of the solution approach is the spatial aggregation of neighboring blocks bearing similar grade and rock type using a clustering technique (Tabesh and Askari-Nasab, 2011) which is conducted to achieve a solution utilizing a commercial solver and to receive a mineable schedule. Aggregation, however, ignores the ability to blend favorable ore types together on a resolution of selective mining units and thus leads to suboptimal solutions. Furthermore, the discussed conventional short-term production scheduling models utilize a single, deterministic orebody model which misrepresents proportions of high and low-grade material and grade variability. This can result in the failure of meeting production targets in the short-term and thus also missing production targets in the long-term.

1.2.2.2 Fleet management and equipment allocation optimization

In the following, mathematical models and optimization approaches that aim to schedule and allocate trucks, shovels and other mining equipment in open pit mines are reviewed. These developments are relevant because a model for joint stochastic short-term production scheduling and fleet allocation in mining complexes is developed in Chapter 2 in this thesis. However, literature dealing with operational truck dispatching, e.g. Temeng (1997), is not discussed because the short-term extraction schedules of weekly to monthly periods do not intersect with the intra-day operational activities carried out in minute-wise intervals.

Many models have been proposed to optimize truck-to-shovel allocation decisions in open pit mines using mathematical programming techniques. Li (1990) presents a linear program that optimizes the number of trucks operating on a travel path from a fixed loading point in a mine (shovel) to a destination by minimizing total transportation work. Transportation work is defined as the transported distance of material multiplied by the material tonnage. This approach assumes homogeneous truck fleets. Furthermore, grade-quality requirements of transported materials are not considered. Temeng et al. (1998) propose a goal programming approach that aims to achieve competing goals, such as maintaining production levels of shovel production and meeting ore quality targets. The approach also assumes a homogeneous truck fleet. Chang et al (2015) develop a mixed integer programming model that maximizes transport revenue of all truck loads hauled in a finite planning horizon and propose a heuristic to solve this large-scale problem. Problems arise because the necessary removal of waste rock is not enforced, and a homogeneous truck fleet is assumed. Moreover, all listed approaches above consider deterministic inputs in their optimization. There are many sources of uncertainty to consider, however, such as production rates, and equipment availability, and geological uncertainty which are all not accounted for.

Ta et al. (2005) optimize truck allocation in open pit mines using chance-constrained stochastic optimization. The model incorporates certain uncertainties by modelling truck loads and truck cycle times as stochastic parameters. However, uncertain shovel production is not considered, and ore quality targets are omitted. Ta et al. (2013) propose a stochastic integer program that minimizes the number of required trucks by allocating trucks to shovels based on the idle time probability of a shovel. They also integrate constraints keeping grade requirements; however, they assume a constant ore grade delivered by every shovel. Bakhtavar and Mahmoudi (2018) utilize robust programming to optimize multi-period truck allocation to fixed shovel positions considering uncertain shovel production and deterministic grade constraints. None of the described models optimizes shovel allocations in the mines, thus assume their positions at fixed loading locations, mostly divided in ore and waste loading points. Also, geological uncertainty of the mineral reserve is ignored.

Zhang and Xia (2015) propose a mixed integer program to determine the number of required single-route truck trips in one shift such that the fuel consumption of the truck fleet is minimized. Bajani et al. (2017) extend this approach to also account for and minimize fuel consumption of shovels. Similar efforts of minimizing fuel consumption are reported by Patterson et al. (2017). The authors use a Tabu Search variant to solve their large mixed integer programming model. All described cost-driven approaches can lead to sub-optimal solutions because shorter routes will always be preferred if overall grade and tonnage constraints are met. Higher-yielding solutions are ignored this way and ore quality can be impacted throughout the shift leading to loss in revenue that surpasses the cost savings in fuel, especially if geological uncertainty is ignored.

Topal and Ramazan (2010) optimize the truck fleet for the total life of a open pit mine considering maintenance cost due to equipment aging and achieve cumulative cost savings of 19.5 % compared to a conventional purchasing plan. Topal and Ramazan (2012a) extend their model considering maintenance cost as a stochastic parameter and solving a stochastic integer program. While the consideration of maintenance costs is important, the models take a separately optimized life-of-mine production schedule as input. However, it is preferred to optimize the production schedule simultaneously with truck purchases and utilization, as seen in Goodfellow and Dimitrakopoulos (2015) and Del Castillo and Dimitrakopoulos (2019).

Liu and Kozan (2012), Liu et al. (2013), and Kozan and Liu (2016, 2018) optimize the completion of equipment jobs in open pit mines. The mixed integer program considers drilling, blasting and excavation activities which have to be applied to so-called mining jobs. One mining job consists of an aggregation of same-grade blocks which are located on the same bench in the same pit. A schedule for completing mining jobs is created which maximizes the throughput and utilization of mining equipment by minimizing overall completion time and idle times of mining equipment units at each stage.

Besides mathematical programming models, other optimization techniques are proposed as well for making equipment allocation decisions in open pit mines. Ercelebi and Bascetin (2009) apply a closed queuing network model to obtain an optimal number of truck assignments to shovels. However, a homogeneous fleet is assumed, and the queuing model relies on the assumption that all stochastic operations are Markovian. Burt and Caccetta (2007) and Chaowasakoo (2017) develop match factors for heterogeneous truck and shovel fleet sizes to avoid under-trucked and over-trucked mines. Discrete event simulation models are utilized to evaluate truck-shovel systems in open pit mines (Awuah-Offei et al., 2003, 2012; Askari-Nasab et al., 2007; Ben-Awuah et al., 2010; Torkamani and Askari-Nasab, 2015; Ozdemir and Kumral, 2017) and continuous mining systems (Shishvan and Benndorf, 2016) considering their stochastic nature. In an attempt to combine the strengths of mathematical optimization and Monte Carlo type simulation, so-called simulation-optimization approaches are developed for multi-objective short-term production scheduling (Fioroni et al., 2008; Upadhyay et al., 2015; Upadhyay and Askari-Nasab, 2018), haul fleet sizing (Moradi Afrapoli et al., 2018), truck and shovel allocation in multiple pits (Ozdemir and Kumral, 2019) and job-shop scheduling in continuous mining systems (Shishvan and Benndorf, 2019).

1.2.2.3 Multi-objective short-term planning

Recently, some developments in short-term mine planning have combined aspects of fleet management and short-term extraction sequencing for open pit mines (Fioroni et al., 2008; L’Heureux et al., 2013; Torkamani and Askari-Nasab, 2015; Mousavi et al., 2016c; Villalba Matamoros and Dimitrakopoulos, 2016; Blom et al., 2017; Kozan and Liu, 2018; Upadhyay and

Askari-Nasab, 2018). The general motivation is to create synergies between the steps that are conventionally optimized separately. The synergistic developments are reviewed in the following, which play an important role for proposed developments in Chapter 2 of this thesis.

Fioroni et al. (2008) propose a mixed integer program to create a short-term production schedule for a single period, consisting of optimized tonnages of extracted materials from mining areas, assignments of mining shovels to areas, required number of trucks trips to those areas, and assuring grade quality constraints. Souza et al. (2010) develop a similar model defining the mining rate of each mining area, shovel allocation to mining areas, and necessary truck trips. They develop a hybrid algorithm consisting of greedy randomized search procedures and variable neighborhood search. Drawbacks of both models are that metal grades are assumed constant over a mining area, which can lead to considerable quality violations due to ore variability. An a priori classification into ore and waste areas aggravates the problem, defining the destination of aggregated materials beforehand. Also, the optimization of a single period is myopic and can lead to difficulties meeting production targets later in mine life.

L'Heureux et al. (2013) create short-term production schedules that consider up to 90 daily periods of extraction, while incorporating equipment-related activities related to material extraction, such as optimized drilling and blasting and minimized shovel movements between mining faces. While jointly accounting for many decisions in short-term mine planning, computational limitations regarding the size of the optimization model are reported by the authors, and the development of heuristic solution approaches is suggested. Furthermore, all sources of uncertainty are ignored, most prominently metal uncertainty and equipment production uncertainty.

Upadhyay and Askari-Nasab (2016, 2018, 2019) propose a multi-objective goal programming approach which aims to (i) minimize the negative deviation in production by shovels compared to their capacities, (ii) minimize the deviation of tonnage and grade targets in processors and (iii) minimize movement times of shovels. Their model uses binary decision variables to assign shovels to a mining face in a period. Furthermore, the number of trips made by a truck from a mining face to a destination is optimized. A mining face is considered as the smallest planning unit, which is an aggregation of spatially connected blocks within the same bench, bearing similar grade. The model optimizes proportions of material from a face, extracted by a certain shovel and sent to a destination per period. Upadhyay and Askari-Nasab (2016) only consider the next period of extraction, wheras the later publications extend the approach to a multi-period schedule. Unfortunately, all presented models use pre-optimized aggregation of faces which has drawbacks. Firstly, geometries of faces are fixed beforehand, which limits the possible extraction sequences to a subset of suboptimal solutions. Secondly, the model assumes a constant grade for material types within a mining face, which is unrealistic even for a few clustered blocks. This assumption leads to unwanted deviations from ore tonnage and ore quality targets of the mine plan.

Mousavi et al. (2016a) develop a MIP model for short-term production scheduling considering feasible geometrical mining shapes and shovel production capacities. The model optimizes the destination per selective mining unit (block) by minimizing misclassification losses. Stockpiling is seen as an important factor in the presented short-term model, as they are used to feed the processors in some periods. However, material coming from stockpiles is assumed to have an average-feed grade to avoid non-linear grade-blending constraints. Furthermore, the model cannot account for uncertainties relevant to short-term mine planning. Mousavi et al. (2016b) test their previously developed optimization model with three different metaheuristic algorithms, which are Tabu Search (TS), Simulated Annealing (SA) and a combined version of TS and SA. Swapping mechanisms exist to change either periods or destinations of selected blocks. The combined TS/SA approach applies TS to maintain a search history of preventing revisiting found solutions and uses SA to allow moves to inferior solutions. A maximum problem size of 1200 blocks, 6 excavators and 6 time periods is tested, yielding solutions within 4% optimality gap.

Blom et al. (2017) develop a multi-objective short-term optimization model which produces multiple short-term schedules using a split-and-branch approach. A user-defined sequence of objectives, in descending priority, is given prior to optimization. The compliance with the long-term extraction sequence is modelled as a possible objective to be maximized. A rolling horizon-based algorithm solves a MIP representation of the short-term planning problem consecutively for each period in the planning horizon. This algorithm takes the next period to be solved and unifies the rest of the planning horizon to a second period. In this way, the large optimization problem is heuristically solved. Optimality gaps are not reported in the case study.

Liu and Kozan (2017) integrate a series of three different planning horizons into one mathematical model. The mine design, which seeks the determination of the ultimate pit, the production scheduling, which seeks the most profitable block extraction sequence, and a job-shop scheduling for creating a detailed schedule of drilling, blasting and excavating activities are unified in one single modelling approach. The developed mathematical model schedules mining activities over a series of ‘operational stages’. Working benches and available equipment can change from stage to stage. A set of same-grade block units on the same bench are aggregated to so-called ‘mining jobs’, which are mined at the same production rate. The resulting schedule of drilling, blasting and excavating mining jobs maximizes throughput and reduces the idle time of equipment units. However, grade blending requirements in the processors are disregarded during this scheduling approach. The case study maximizes total mined throughput on an 18-week planning horizon, given capacities and drilling rates of available mining equipment.

1.2.2.4 Stochastic short-term production scheduling

Geological uncertainty, as already discussed for strategic mine planning, is also a crucial aspect to consider for short-term production scheduling. Additionally, since fleet management is an important part of short-term production scheduling, uncertain availability and productivity of the mobile equipment should be accounted for. Truck queuing and shovel wait times inevitably occur due to unforeseen breakdowns and random fluctuations of operating conditions which can change due to weather, haul road conditions, and maintenance levels. If scheduled material cannot be extracted, the resulting shortfalls of production in the short-term can lead to under-achieving long-term mine plans that may cause rippling effects throughput the life-of-mine. Ravenscroft (1992) shows that optimization methods that use deterministic, average-type orebody models fail to provide a robust short-term production schedule resulting in an ore feed that lies outside the minimum and maximum grade blending constraints. Smith and Dimitrakopoulos (1999) assess the influence of deposit uncertainty using stochastic orebody models for short-term production scheduling and the mixed integer programming language AMPL. The stochastic integer programming formulation developed for life-of-mine production scheduling by Ramazan and Dimitrakopoulos (2005) has been utilized by Dimitrakopoulos and Jewbali (2013) taking orebody information in the form of simulated grade control data into account. The simulated grade control data is based on exploration data and grade control data in previously mined out parts of the deposit. Production schedules are created on a time scale of trimesters.

A combination of stochastic single-mine production scheduling and equipment allocation is presented by Matamoros and Dimitrakopoulos (2016). The stochastic integer goal programming formulation considers supply uncertainty together with uncertainty of truck and shovel availability and uncertain truck cycle times. Uncertainty of availability is represented through equipment performance scenarios by randomly sampling Gaussian distributions derived from historical data. All uncertainty scenarios combined serve as an input to stochastic optimization that gives a more informed decision of realistic mining sequences and equipment allocation decisions. A case study in an iron ore mine shows planned production targets can be met more likely than approaches that ignore the uncertainties included. Quigley and Dimitrakopoulos (2019) extend the stochastic formulation of Matamoros and Dimitrakopoulos (2016) to optimize the production schedule and fleet assignment in multiple mines. The objective function aims to reduce shovel movement costs and minimizes deviations from production targets subject to grade and equipment uncertainty. Several processing options are included; however, stockpiling is not considered, and material destinations are not optimized. Thus, by minimizing deviations alone, the model cannot optimize the value chain as a whole, and trade-offs resulting from the cost of equipment in operation versus the profit made by blended products cannot be evaluated. Notably, the equipment performance scenarios of shovels and trucks are co-simulated (Desbarats and Dimitrakopoulos, 2000), honoring correlations of availability and utilization among similar shovel types and truck types. Truck cycle times are sampled from Gaussian cycle time distributions derived from historical data. All described stochastic approaches use the CPLEX solver to obtain a solution, which restricts their approach to instances of a relatively small size. Kumar and Dimitrakopoulos (2021) adapt short-term extraction sequence, material destinations, and downstream material flows in a mining complex using a reinforcement learning algorithm that combines a Monte-Carlo tree search with a deep neural network agent. Geological uncertainty is provided by a set of stochastic orebody simulations. Although initial fleet assignments are not adapted, equipment performance uncertainty is considered by the algorithm. Empirical distributions represent production rates of shovels, trucks, and crushers which are built with historical production data These empirical distributions are sampled to create a set of equipment performance scenarios. If new production data is available, the distributions of production rates are adjusted, and new equipment scenarios are sampled.

Although many decisions and uncertainties have recently been included in the described multi-objective short-term optimization models, their ability of conventional and stochastic short-term optimization methods to meet production targets such as mill throughput and metal production are limited because complex, non-linear processes of blended materials in mineral value chains cannot be assessed. This is partly due to the linear nature of the described optimization models and their focus on cost minimization, instead of maximizing value of final products sold shown in the simultaneous stochastic optimization of mining complexes for strategic planning. Particularly geometallurgical aspects have not been considered in short-term production scheduling although they can have profound influence on the ability to meet (short-term) production targets, which will be reviewed in the following section.

1.2.3 Modelling and integration of geometallurgy in mine planning

Geometallurgy aims to describe, model and harness the relationships between spatially distributed rock characteristics to be extracted from mineral deposits and its metallurgical and processing behaviour further downstream in a mineral value chain. Richmond and Shaw (2009) state that conventional mine planning optimization is largely based upon modeling of ore grade without considering other geometallurgical properties. However, many authors see a large potential in the technical literature to positively impact the value of mine planning decisions by identifying and integrating key geometallurgical properties which drive project costs and revenues in a fundamental way (Dunham and Vann, 2007; Coward et al., 2009; Dunham et al., 2011; Deutsch et al., 2016; Dowd et al., 2016). Section 1.2.3.1 discusses critical geometallurgical properties of the rock, including how to quantify them, problems related to non-additivity, and consequences for geostatistical modelling of these properties. The subsequent sections review the literature on the integration of geometallurgy into various aspects of mine planning on different planning horizons, such as geometallurgy for processing plant optimization (Section 1.2.3.2), geometallurgy for evaluation of uncertainty in mineral value chains (Section 1.2.3.3), and geometallurgy for mine production scheduling (Section 1.2.3.4).

1.2.3.1 Modelling geometallurgical properties

One of the most studied rock characteristics that influences the processing plant performance is the rock's comminution behaviour, which can also be described as the ore's resistance to crushing and grinding. Bond (1952, 1961) develops the Work index (Wi) in combination with the third theory of comminution, which calculates the specific energy (kWh/t) that is required to grind an ore sample from a known feed size to a required product size using ball mills and rod mills. With the advent of semi-autogenous grinding (SAG), new indices have been developed that quantify the energy requirements of ore grinding, such as the SAG Power Index (SPI) (Starkey and Dobby, 1996), the resistance to impact breakage (A×b) and resistance to abrasion (t_a) (Napier-Munn et al., 1996), and the Drop Weight index (DWi) (Morrell, 2004b). A review of ore comminution indices and their measurement techniques is given by Lynch et al. (2015). For the quantification of rock comminution before grinding, a crushing index (Flores, 2005) and a blast index (Segui and Higgins, 2002) have also been developed.

Traditionally, the ore comminution indices are used by processing engineers for optimal circuit design, using averages or ranges of ore hardness and grindability of the mineral deposits to be extracted. In contrast, geometallurgical programs account for the heterogeneity of geometallurgical variables within the mineral reserves and their effect on downstream processes over time. A spatial orebody model of hardness and other pertinent geometallurgical properties is typically built using geostatistical techniques, commonly referred to as the geometallurgical model. One major challenge to build reliable geometallurgical models is that information of geometallurgical properties is typically derived from very sparse geometallurgical sampling. Exemplary, laboratory tests on the samples' comminution behaviour are typically costly and time-consuming (Hunt et al., 2013; Deutsch et al., 2016). Mwanga et al (2015) review the required test procedures for comminution indices and evaluate their potential for geometallurgical use. The authors conclude that cost effective and rapid test procedures of the mineral resource are required for the efficient quantification of geometallurgical variability of hardness within the orebody. The limitation of sparse sampling is addressed in Chapter 3 of this thesis where the use of measurement while drilling (MWD) data is proposed to inform about spatial hardness, strength, and comminution behaviour of the mineral reserve.

With the widespread use of comminution indices, studies followed that assessed their additivity through laboratory grinding tests by mixing two distinct ore types together in various ratios. Yan and Eaton (1994) test the additivity of Wi, whereas Amelunxen (2003) and Amelunxen et al. (2014) investigate the additivity of SPI. The studies conclude that Wi and SPI are non-additive, i.e., ore blends behave differently than the calculated average of the ores comminuted individually. The non-additivity of other geometallurgical properties has also been shown (Carrasco et al., 2008; Coward et al., 2009; Richmond and Shaw, 2009; Van Tonder et al., 2010; Deutsch, 2013). When modelling non-additive geometallurgical properties, caution is required for their spatial interpolation using geostatistical estimation and simulation techniques. Next to the smoothing issues discussed previously, estimation methods (e.g., kriging) may introduce biases when modelling non-additive properties through the calculation of linear weighted averages (Coward et al., 2009; Deutsch, 2013). Instead, Gaussian simulation methods have been used for the simulation of non-additive properties to reproduce variability and account for spatial uncertainty (Newton and Graham, 2011; Boisvert et al., 2013; Deutsch et al., 2016). Other proposed methods that aim to avoid the direct simulation of non-additive properties are discussed later. Barnett and Deutsch (2012) introduce conditional standardization and discuss the systematic application of non-linear transformations such as log ratios, minimum/maximum autocorrelation factors (MAF), normal scores, and stepwise conditionals to create multivariate simulations of potentially non-linear and heteroscedastic geometallurgical properties. Deutsch et al (2016) create multivariate simulations of grades, metallurgical, and geotechnical variables using an intrinsic supersecondary model (Babak and Deutsch, 2009). Ortiz et al. (2020) simulate copper grades and SPI using sequential Gaussian co-simulation to account for the joint spatial uncertainty of the attributes. However, the change of support, or scaling, of the non additive SPI values from simulated grid nodes to mineable volumes, or selective mining units (SMU), is not addressed. Generally, the challenge for any change of support (volume) of non-additive properties is to avoid biases caused by the calculation of arithmetic or weighted averages. Typical applied changes of support in geostatistics include drillhole compositing and upscaling from measured sample volumes or simulated grid nodes to SMU scales. Deutsch (2015) discuss power-law re-expressions of non-additive geometallurgical properties to linearize them for further use. However, experimental data is required of the non-additive behaviour from individual mineral deposits, which is mostly not available. Garrido et al. (2019) address the problem of drillhole compositing of non-additive recovery samples through additive auxiliary variables. These auxiliary variables are simulated using a Gibbs sampler for inferring the recovery samples on smaller support. However, upscaling of non-additive simulated values in point support (grid nodes) to mineable units is not addressed. Note that conventional geostatistical estimation or simulation algorithms that include the change of support, such as Block Kriging (Journel and Huijbregts, 1978; Goovaerts, 1997) and Direct Block Simulation (Godoy, 2002), assume additivity through the calculation of averages. The change of support of non-additive hardness-related geometallurgical properties from the measurement scale to mineable volumes (upscaling) is addressed in Chapter 3 of this thesis. Comparisons of the new proposed approach to conventional, average-type upscaling methods are subsequently presented in Chapter 4. After obtaining a spatial geometallurgical model in the scale of mineable units, non-additive properties can lead to more biases if mine planning and production scheduling methods assume so-called machine additivity for blended materials (Bye, 2011; Newton and Graham, 2011). Machine additivity implies that the metallurgical behaviour of heterogeneous ore blends is equal to the calculated average of the individual components of the blend. As seen above in the examples of geometallurgical properties related to hardness, averages of blended materials may not reflect the true response of the processing plant. A new approach that can account for machine-related non-additive geometallurgical properties in short-term production scheduling for mining complexes is proposed in Chapter 3. Other approaches in the technical literature dealing with non-additive geometallurgical properties are reviewed next.

Coward et al. (2009) introduce a geometallurgical primary-response framework, which separates all geometallurgical properties into ‘primary variables’ and ‘response variables’. Primary variables are intrinsic rock attributes that can be directly measured such as mass, metal grade, mineral and chemical compositions, degree of alteration, and rock texture. On the other hand, response variables are a result of applying energy, mechanical or chemical processes on excavated material. Response variables can also be described as the resulting metallurgical performance of the processed material in the downstream facilities of the mineral value chain. These variables are often non-additive and typically non-linearly related to primary rock attributes. Examples of economically important response variables are the metal recovery, energy consumption, throughput, consumption of chemical reagents, additives and grinding media, product particle size distribution, grindability, leachability and more. Primary, additive rock attributes can be well spatially interpolated for the complete mineral deposit from conditioning data with geostatistical simulation methods (Journel and Huijbregts, 1978; Goovaerts, 1997; Chilès and Delfiner, 2012). Response variables can then be deduced as an empirical function or with the use of general prediction models. A model that calculates response variables from primary variables is often called a *proxy model*. Richmond and Shaw (2009) warn that proxy models are typically constructed using samples in drill-core support, thus do not guarantee correct mapping for larger volumes (SMU). Also, the calibration of these proxy models is typically performed on an extremely limited number of sample pairs of input variables and measured metallurgical responses using laboratory tests. Several proxy models proposed in the technical literature are reviewed in the following.

Everett and Howard (2011) use multiple linear regression (MLR) models to predict the grade of crushed material (post-crusher properties) using primary rock attributes such as rock density, material type and metal grades collected from grade control data. The authors demonstrate their method by generating a daily crusher plan with the aim of maintaining the desired product grade after crushing. Advantages over the conventionally utilized run-of-mine (ROM) crusher trials are discussed. Hunt et al. (2013) use MLR to estimate the comminution indices SPI, Wi, and A×b, based on sample data that is widely available at mine site, including mineralogy and chemistry of drill core samples, as well as lithology and alteration. Boisvert et al. (2013) create several multiple linear regression models to populate a geometallurgical model of response variables such as Cu and U_3O_8 recovery, acid consumption, net recovery, DWi, and Wi at the Olympic dam mine. After removing redundant and poorly correlated variables the authors merge 112 remaining input variables in their case study into four super secondary variables in a two-step process before creating the linear regression models for the six response variables. The input variables consist of drill core samples of metal grades, the P20, P50, and P80 grain size of minerals, and detailed mineralogy of ore. Keeney and Walters (2011) propose a PCA analysis on primary rock attributes, before using MLR to predict the comminution indices Wi and A×b. Keeney et al. (2011) apply the developed method for geometallurgical domaining and throughput prediction at the Cadia East Porphyry Deposit. All studies discussed above utilize multiple linear regression models, which leads to the limitation that linear dependencies between input and response variables need to be assumed.

Sepúlveda et al. (2017) use projection pursuit regression (PPR) for the prediction of metallurgical response variables. Quantitative rock attributes such as metal grades as well as qualitative attributes such as rock type are used as input properties to predict Wi, as well as Cu and Au recovery. Projection pursuit aims to project the data space onto a set of directions that optimize the fit of the model with the aim of revealing underlying non-linear relationships. Improvements over MLR models are demonstrated. Furthermore, the authors claim that uncertainty associated with response properties can be quantified if a whole set of validated PPR models is used. Lishchuk et al. (2019) compare ten different machine-learning methods for populating a spatial drillhole database of an iron ore deposit with metallurgical response variables such as mass pull and recovery using wet low intensity magnetic separation (WLIMS) and Davis tube magnetic separation (DT). Input variables comprise chemical assays of 13 elements and minerals and the density of the samples. Results show that the different machine learning models have similar prediction error (relative standard deviation), whereas some metallurgical variables to be predicted have lower accuracy than others. Using the discussed approaches, metallurgical response variables are typically incorporated in the spatial geometallurgical model. One general drawback in this workflow is that that blended responses of non-additive metallurgical variables from different parts of the resource cannot be assessed accurately if they are expected to be processed together. This applies especially for short-term production scheduling.

Another method for the spatial modelling of geometallurgical properties is the generation of geometallurgical domains or clusters of materials, grouping them together based on their similarity of expected plant behaviour. Sepúlveda et al. (2018) use fuzzy clustering resulting in clusters that contain similar geometallurgical characteristics and are grouped in as few contiguous and compact spatial locations as possible. An iterative method of maximizing separation and choosing the optimal weighting of attributes is applied first. Spatial dependencies between samples are added next which converts the weighted fuzzy clustering (WFC) method into a spatially informed, weighted fuzzy clustering approach (SWFC). The authors conclude that the fuzzy clustering approach is superior to the conventional domaining based on cut-off grades since the ore selection process by grades does not consider the energy consumption due to different hardness and grindability of processed material. Rajabinasab and Asghari (2019) use hierarchical clustering, k-means clustering and self-organizing maps to create geometallurgical domains in an iron ore deposit. The authors describe geometallurgical domains as homogeneous regions characterized by similar processing behaviour. Del Castillo and Dimitrakopoulos (2016) use coalition-formation clustering to create groups of mining blocks that shall be processed together in mineral processors based on a spatial geometallurgical model. However, the authors do not consider that certain coalitions may not be available to be processed at the same time, because their spatial location in the deposit is not considered.

Mineral texture is known to play a major influence on metallurgical behaviour such as the ore's fracture pattern during breakage and its influence on metal recovery (Richmond and Dimitrakopoulos, 1997; Lund et al., 2013; Jardine et al., 2018; Voigt et al., 2019). However, some challenges remain for integrating textures in geometallurgical models, mainly due to the lack of methods for texture quantification and change of scale (Richmond and Shaw, 2009). Richmond and Dimitrakopoulos (1997) simulate mesotextures using Sequential Indicator Simulation (SIS) and upscale the simulations by building distributions of mesotextures in larger mineable volumes. Nguyen (2013) develop a texture analysis technique using grey level radial shell association (GLRSA) and spectral angle mapping (SAM) to differentiate texture images. Jardine et al. (2018) perform texture analysis in 3D using x-ray computed tomography grey scale volumes of drill cores. Voigt et al. (2019) build upon the method presented by Jardine et al. (2018) and test the spatial mineral texture quantification method for drill cores. Their case study investigates the classification of a polymetallic sulfide deposit into three distinct mineral textural types, based on the grain structure of the ore. However, more research is required to accurately quantify and differentiate textures and to model their variability in orebodies.

According to van den Boogaart et al. (2013), another challenge is that many relevant geometallurgical variables are measured in non-Euclidean scales, such as compositional mineralogy and particle size distributions. Compositional or modal mineralogy measures the mineral composition of samples as relative contributions, summing up to a constant, which is conveniently chosen to be 1 or 100 % (Tolosana-Delgado et al., 2019). Particle size distributions are often reduced to single-valued passing diameters of the 50th percentile (P50) or 80th percentile (P80) instead of modelling the distribution itself. Tolosana-Delgado et al. (2019) discuss shortcomings of current multi-variate geostatistical practices for spatial interpolation of compositional data. Van den Boogaart et al. (2018) propose an approach for the joint simulation of discrete and continuous geometallurgical variables, using multi-point geostatistics (Mariethoz and Caers, 2015), combined with a generalization of logistic regression to estimate conditional distributions of arbitrary scales from different sources of information. In an earlier approach Desbarats and Dimitrakopoulos (2000) demonstrate the joint geostatistical simulation of discretized pore-size distributions using min/max autocorrelation factors (MAF).

1.2.3.2 Geometallurgy for processing plant optimization

Geometallurgical models, i.e., orebody models that include pertinent geometallurgical variables and their variability within the deposit, have been utilized for the general technical design of processing plants. Bulled and McInnes (2005) design a flotation plant using a geometallurgical model that is built with the Flotation Economic Evaluation Tool (FLEET) (Dobby et al., 2002). Bulled (2007) designs a grinding circuit using the Comminution Economic Evaluation Tool

(CEET) which is an empirical process model developed by Dobby et al. (2001) to obtain SAG throughput rates. Powell (2013) proposes flexible circuit designs that can respond to a wide variability in ore properties and throughput requirements. Bueno et al. (2015) minimize risk in a comminution circuit design by estimating throughput variability based on a fixed mine production schedule and the associated extraction of ten different ore types. A Monte Carlo approach then simulates energy consumption and throughput of the grinding circuit which results in a risk profile. The drawback of the presented process models and studies above is that they take mine production schedules as input instead of simultaneously optimizing the extraction sequence with the predicted performance of the processing plant. Except of Bueno et al. (2015), none of the models accounts for uncertainty of the geometallurgical variables due to imperfect knowledge of the orebody which is part of the inherent supply uncertainty discussed in previous chapters.

Other studies have used geometallurgical models for temporal performance prediction of processing plants over the depletion of the mineral deposits over shorter time intervals of weeks to months. These approaches are generally described as tactical, or operational geometallurgy (McKay et al., 2016), compared to strategic decision-making frameworks. Flores (2005) utilizes kriged models of SPI and Wi to create a throughput prediction model of the grinding circuit of the Escondida copper mining complex. Bond’s comminution law (Bond, 1952, 1961) and CEET is used for throughput prediction of the grinding circuit consisting of crushers, SAG mills and ball mills. The performed reconciliation of monthly plans reveals weaknesses to correctly forecast throughput on shorter time scales, with larger over-estimations of up to 17 %, or 800 t/h, for a given month. Several authors have pointed out systematic biases of kriging for geometallurgical variables (Coward et al., 2009; Deutsch, 2013; Deutsch et al., 2016). Further, Flores (2005) ignores non-additivity of blended materials, and downtime events in the plant are not considered. Alruiz et al. (2009) present a geometallurgical throughput prediction model of a grinding circuit of the Collahuasi copper mine consisting of SAG mills and ball mills. Instead of populating a block model, hardness and grindability indices are collected for six distinct geometallurgical orebody domains. The authors use the work index (Wi) for ball mill throughput prediction, whereas the hardness and abrasion indices A×b and t_a are used for SAG throughput prediction. Reconciliation with weekly observed throughputs over a period of one year resulted in high prediction accuracy, with an R^2 value of 0.95. Suazo et al. (2010) present a geometallurgical model for the prediction of flotation performance at the same mine studied by Alruiz et al. (2009). The authors utilize mineralogical compositions of the same six geometallurgical domains and conduct flotation test work on samples for each domain. A floatability index is created which can be scaled up to estimate copper recoveries at industrial scale with high accuracy, showing an average relative error of less than 2 %. A serious limitation of all studies above is that they require fixed extraction schedules to provide forecasts of future plant responses, such as throughput and metal recovery. A simultaneous approach should be strived for, optimizing the short-term mine production schedule together with integrated geometallurgical models of the processing plant(s), which is addressed in Chapter 3 of this thesis.

1.2.3.3 Geometallurgy for evaluation of uncertainty in mineral value chains

Coward et al. (2013), Jackson et al. (2014), and Coward and Dowd (2015) use simulated geometallurgical models to quantify the effects of multiple uncertainties on the NPV generated in a mineral value chain. The most comprehensive approach is presented by Coward and Dowd (2015), who assess the uncertainty of the net smelter return (NSR) using a set of simulated orebody models, create multiple possible grade-recovery curves using process simulations, and use a set of future scenarios reflecting uncertain metal prices. A risk analysis of the compound uncertainties shows that the NPV spread (P10 - P90) is as large as 70 % of the expected project NPV. MacFarlane and Williams (2014) present a geometallurgical value chain approach for a copper mine which results in revised cut-off grades, improved annual ore and waste extraction tonnages and modified stockpile management over the mine life.

In contrast to the presented life-of-mine approaches, Shaw and Khosrowshahi (2013) assess day-to-day variability of the process plant feed to quantify short-term uncertainty of metallurgical response variables such as reagent consumption and plant recovery. Lechuti-Tlhalerwa et al. (2019) propose an integrated geometallurgical value chain model (IGVCM), which links spatial models of primary rock attributes to observed process responses. The process simulation model simulates material flows from mines and stockpiles and attempts to forecast metallurgical plant performance. A review of integrated approaches of geometallurgy for mineral value chains is presented by Dominy et al. (2018).

1.2.3.4 Geometallurgy for mine production scheduling

Integration of geometallurgy into mine production scheduling has been investigated for long-term planning in single mines. Kumral (2011) creates a long-term production schedule considering variable mining costs, processing costs and recoveries, which are proposed to be modelled as functions of geometallurgical properties. Mining costs, processing costs and recoveries are calculated individually for each mining block using regression models. Grade uncertainty is accounted for using sequential indicator simulation. A two-stage stochastic integer program with fixed recourse is solved to determine the yearly extraction sequence and destination of material. Navarra et al. (2018) calculate economic block values using a geometallurgical factor while considering two distinct plant operating modes for long-term production scheduling. The geometallurgical factor aggregates several attributes including mineral composition, texture, liberation, and mineral chemistry. However, no details are shown how to spatially interpolate these attributes for each mining block in the mineral reserve, which is not trivial, and part of ongoing research as discussed earlier. Morales et al. (2019) optimize a long-term production schedule using simulated orebody models that include simulated metal grades, mill throughput rates (processed tons per hour) and metal recoveries. Each variable is simulated independently, ignoring multi-variate dependence. While simulated ore grade and metal recovery are used to calculate economic values of blocks per orebody scenario, the simulated mill throughput is converted into the required mill processing time per mining block. A linear constraint in a stochastic integer program ensures that processed blocks do not exceed the available mill hours per year. All production scheduling models discussed above quantify geometallurgical variables and their metallurgical and economical responses block by block, which implies that bocks are processed independently from each other. This is unrealistic in a typical mining environment where blending and stockpiling of materials is ubiquitous and where the notion of a block disappears altogether when material is mined. Exemplary, non-linear metallurgical processes applied on blended materials in processing facilities are responsible for total metal recovered in a mineral value chain. Further, the hardness-related geometallurgical variables and their derived variables mill throughput and mill processing time are added up linearly in the discussed production scheduling models, which assumes additivity. In the case of non-additive comminution behaviour of blended materials (Yan and Eaton, 1994; Amelunxen, 2003), these approaches lead to a biased assessment of the ore tonnage that can be processed, as well as the energy required for grinding. Finally, to derive mill throughput rates per mining block, a constant energy input as well as constant feed and product particle sizes need to be assumed (Keeney et al., 2011), which is unrealistic.

Kumar and Dimitrakopoulos (2019) introduce new constraints for rock hardness and grindability in the long-term stochastic simultaneous optimization of mining complexes. The constraints aim to achieve a consistent throughput by sending pre-defined ratios of hard and soft rock to the processing plant(s). The mineral reserve is categorized into hard and soft rock through spatially simulated Wi and SPI values. While the novel constrains do not need to assume additivity, the method requires simulated Wi and SPI values in a geometallurgical block model, which suffers from non-additive upscaling and sparse information, as discussed earlier. Furthermore, the hardness ratios are chosen arbitrarily and do not guarantee optimal material selection for throughput maximization.

Summarizing, the integration of geometallurgical properties in production scheduling has been exclusively considered for long-term planning. With the exception of Kumar and Dimitrakopoulos (2019), mill throughput rates, metal recoveries and other non-additive properties are calculated per mining block, ignoring that extracted materials are continuously blended in stockpiles and in processing facilities. These simplifications may still be accepted in long-term plans which tend to have lower degrees of mixing relative to the length of a single period (yearly) among the materials extracted during this period. However, modelling the non-additive attributes block by block can be highly impeding in short-term production scheduling where the degree of blended materials mixed in a single short-term period (days/weeks/months) can be very high. The assumed plant responses of shorter planning periods are biased this way and short-term targets may not be met, leading to readjustments and profit loss. One way to inform the plant responses of blended materials more accurately in the short-term is to utilize observed production data generated at the operating processing plant, which is reviewed in the next section.

1.2.4 Use of production data for geometallurgy

Recently, the mining industry has witnessed a massive increase of collected and centrally stored production data along the mineral value chain which is fueled by the use of digital technologies such as automation, cloud computing, Internet of Things (IoT) and other recent technological advances (Bartnitzki, 2017; Barnewold and Lottermoser, 2020; Faz-Mendoza et al., 2020; Sishi and Telukdarie, 2020). Some examples of centrally stored datasets in operating mines are records of fleet management systems (Moradi Afrapoli and Askari-Nasab, 2017), measurement while drilling (MWD) (Rai et al., 2015), measurements of material characteristics using sensor techniques (Lessard et al., 2014), and other key performance indicators at the processing plants. Some studies use parts of this growing data pool to create geometallurgical prediction models for short-term and tactical planning horizons spanning several weeks to months. One favourable aspect of using production data to create geometallurgical models is that responses of the operating processing plant (e.g., throughput, metal recovery, reagent consumptions) are measured in true operating conditions and in the correct support scale. This stands in contrast to conventional laboratory tests, which use batch testing conducted at much smaller scale (Suazo et al., 2010; Truter, 2010; Mwanga et al., 2015). The need for upscaling complicates the modelling for the correct prediction of geometallurgical variables in an operating mining complex.

Carpenter and Saunders (2017) use observed throughput rates of the SAG mill at the Phu Kham copper-gold mine to build an empirical throughput prediction model based on the monthly mill feed consisting of varying proportions of lithology types. After solving a set of linear equations to obtain weight factors for each lithology, linear programming is used to identify the optimized mill feed blend to achieve a maximum SAG mill throughput. The authors make throughput predictions for two consecutive months, based on eight observed months of the blended material feed. Although the use of production data at the operating ball mill is promising, the approach is limited because it only considers proportions of lithologies of the mill feed for throughput prediction, ignoring other relevant geometallurgical properties of the rock. Carpenter et al. (2018) use observed residual gold grades in the tailings fraction of Ban Houayxai gold-silver mine to build a machine learning model that predicts gold recovery. The monthly feed grades of copper, lead, zinc, and silver, and acid neutralizing capacity of the rock serve as inputs to the model. The input-output data pairs are created by linking the mined portions and associated feed grades to the observed gold tailings grades in monthly intervals. Some drawbacks of the described case studies are that they require a fixed production schedule to make predictions instead of integrating the prediction models into production scheduling to make more informed decisions. Also, geological/geometallurgical uncertainty is ignored. Furthermore, the case studies do not consider temporary stockpiling of ore which can lead to incorrect assumptions of the material compositions sent to the processing plant when building prediction models. Thus, ignoring stockpiling is only recommended if stockpiled tonnages are insignificant thus having high percentages of direct tipping ore hauled from the pits to the crusher.

There are more studies that use machine learning models such as neural networks to predict (metallurgical) performance parameters using recorded production data of the processing plant. Some examples are the prediction of SAG energy consumption (Avalos et al., 2020), froth flotation processes (Kalyani et al., 2007), hydro cyclone performance (Karimi et al., 2010), and jig performance (Panda et al., 2012). This development is interesting due to many possible non-linear interactions between the measured performance variables, such as observed throughput, power draw, particle size distributions, and more. Since these studies do not link or consider the spatial information of rock characteristics being processed, they do not follow a geometallurgical approach. It is possible, however, to extend these prediction models based on machine learning by considering spatially informed geometallurgical features of the ore, which is demonstrated in Chapter 4 of this thesis.

Wambeke et al. (2018) use an Ensemble Kalman filter approach (Evensen, 2009; Benndorf, 2015; Wambeke and Benndorf, 2016) to assimilate performance measurements from an operating ball mill back into a spatial geometallurgical model. The observed throughput, power, feed particle size and product particle size measurements are utilized to update a set of geostatistical orebody simulations of the Ball Mill Work Index (Wi) in four-hourly intervals. The case study shows that the expected error of Wi estimates of the orebody decreases by 72 % for already extracted parts (historical) and decreases by 26 % the future scheduled material. The case study also describes a detailed material tracking approach including run-of-mine (ROM) stockpiles using truck cycle data from fleet management software. The consideration of material tracking is deemed important for high percentages of rehandled material.

Whereas the literature above uses observed production data from the processing plants to build or update geometallurgical models, other collected data originates from the operating equipment in the mines. One of these equipment-based datasets is measurement while drilling (MWD) data which is a collection of mechanical performance indicators obtained from drilling machinery. The monitored performance indicators of the drilling process include, among other entries, rate of penetration, downhole pressure, rotational pressure, rotational speed of the drill bit, air pressure, and vibration. MWD is collected in operating mines during drilling activities such as exploration, grade control and blasthole drilling. The potential of these spatially dense datasets has already been exploited in mining for enhanced blast design (Segui and Higgins, 2002), detection of rock fractures (Schunnesson, 1996; Babaei Khorzoughi et al., 2018), coal seam detection (Leung and Scheding, 2015), rock characterization and classification in various commodities (Schunnesson, 1998; Zhou et al., 2012; Vezhapparambu et al., 2018), and more. Several authors have highlighted the ability of drill penetration rates to indicate rock strength and to differentiate rock types of the mineral reserve (Horner and Sherrell, 1977; Sugawara et al., 2003; Babaei Khorzoughi and Hall, 2016; Vezhapparambu et al., 2018; Park and Kim, 2020). Babaei Khorzoughi and Hall (2016) characterize ROP as the most effective drill variable in ground characterization derived from MWD. Another motivation to use ROP rather than established but typically sparsely informed hardness and grindability indicators, such as SPI and Wi, is the spatially dense information of penetration rates provided by MWD data. Although repeatedly suggested in literature (Segui and Higgins, 2002; Mwanga et al., 2015; McKay et al., 2016), MWD has not yet been utilized to create a direct link between the mineral reserve and its comminution performance in milling and grinding circuits, which is presented in Chapter 3 of this thesis. Together with addressing the fleet management aspects discussed in Section 1.2.2 and geometallurgical aspects pointed out in Section 1.2.3, the integration of the production data reviewed in this section will help to overcome shortcomings of current methods for short-term production scheduling in complex multi-mine multi-process operations.

1.3 Research Goal and Objectives

The goal of this thesis is to expand the simultaneous stochastic optimization of mining complexes into a decision-making framework for short-term mine planning focusing on operational aspects of the mining fleet and processing plants; More specifically, equipment allocation decisions and geometallurgical prediction models of plant performances are integrated into the optimization while accounting for the additional uncertainties of equipment and geometallurgy and capitalizing on new datasets to build links between the mineral reserve and processing plants.

To achieve this goal, the following objectives are outlined:

(1) Review past work related to the simultaneous stochastic optimization of mining complexes, existing methods and shortcomings of conventional and stochastic short-term mine planning, the current practices of geometallurgical modelling for mine planning, and the potential use of production data generated in mines and processing plants.

(2) Extend the simultaneous stochastic optimization model for long-term planning in mining complexes to incorporate detailed fleet management decisions of shovels and trucks in the short-term, considering the uncertainties related to equipment production rates, availabilities, and truck cycle times.

(3) Develop a novel geometallurgical model for the prediction of ball mill throughput in mining complexes by: (a) exploring the utilization of new datasets in the form of measurement while drilling data to inform rock hardness/strength and measured throughput rates at the operating plant; (b) addressing issues of upscaling and blending of non-additive geometallurgical properties; and (c) enabling throughput prediction of blended rock properties within the optimization of a short-term production schedule by integrating the throughput prediction model into the simultaneous stochastic optimization for short-term planning developed in objective (2).

(4) Extend the performance of the geometallurgical throughput prediction model developed in objective (3) by adding sensor measurements in the comminution circuit affecting mill throughput rates and building a supervised machine learning model for throughput prediction that captures non-linear relationships between prediction and response variables.

(5) Develop geometallurgical prediction models of pertinent consumption rates of reagents and consumables in a gold mining complex utilizing blended rock properties and observed metallurgical responses of the operating plant and integrating these prediction models in the simultaneous stochastic optimization of mining complexes for short-term planning.

(6) Outline the contributions and limitations of the methods developed and suggest opportunities for future research.

1.4 Thesis Outline

This manuscript-based thesis is organized into the following six chapters:

Chapter 1 provides a literature review of all pertinent topics addressed herein. This includes related developments in strategic mine planning leading to the simultaneous stochastic optimization of mining complexes, conventional and stochastic short-term mine planning, integration of geometallurgical properties into mine planning, and the utilization of production data in mining complexes.

Chapter 2 expands the simultaneous stochastic optimization model for long-term planning in mining complexes by incorporating short-term equipment allocation decisions and their associated uncertainties. The newly integrated components include the scheduling of a heterogeneous truck fleet and individual shovel allocation decisions which are simultaneously optimized with the short-erm extraction sequence, material destinations, and downstream material flows in mining complexes. The benefits of jointly optimizing a short-term production schedule and fleet management are demonstrated at a gold mining complex.

Chapter 3 develops a novel geometallurgical throughput prediction model using production data in a gold mining complex and integrates the prediction model of the metallurgical response into the simultaneous stochastic optimization for short-term planning. This enables the matching of the scheduled materials with the predicted throughput of the ball mill. Recorded penetration rates from blasthole drilling are used to inform the hardness and strength of the rock, and the subsequent creation of hardness proportions avoids biases typically introduced by the change of support and blending of non-additive geometallurgical properties.

Chapter 4 improves the geometallurgical throughput prediction model developed in Chapter 3 by: (i) Including real-world measurements in the comminution circuit that likely affect ball mill throughput rates in a non-linear way. (ii) Utilizing a supervised learning model in the form of a neural network to approximate non-linear relationships between predictor and response variables. And lastly, (iii) testing if compositional approaches can account for non-additive geometallurgical variables better than average-type information. The additions are tested in a gold mining complex, and their performance is compared to the developed model in Chapter 3.

Chapter 5 extends the empirical prediction of metallurgical responses at the operating scale of the plant and their incorporation into short-term stochastic production scheduling in mining complexes. This is achieved by creating geometallurgical prediction models of consumption rates of reagents and consumables from production data in a gold mining complex and integrating these prediction models in the simultaneous stochastic optimization model.

Chapter 6 outlines the conclusions of this thesis and suggests future research directions.

1.5 Original Contributions

I. Development of a non-linear stochastic optimization model that simultaneously optimizes short-term extraction sequence, shovel allocation including cost and lost tons of relocation, scheduling of a heterogeneous hauling fleet, material destination, and downstream material allocation in open-pit mining complexes

Existing optimization models for short-term mine planning are unable to jointly create short-term production schedules and fleet allocation decisions in mining complexes that maximize the profit of the mineral value chain while accounting for geological and equipment performance uncertainty. The newly developed stochastic integer programming model is applied in a real-world gold mining complex which reveals that considerable synergies exist between short-term production scheduling and fleet management, thus adding value to the mineral value chain when optimized simultaneously.

II. Successful use of a dense spatial measurement while drilling dataset for geometallurgical prediction of ball mill throughput in mining complexes

Measurement while drilling data has not yet been utilized for the performance prediction of milling and grinding circuits. In this thesis, a spatial dataset of penetration rates collected through measurement while drilling is used for the prediction of ball mill throughput. The new approach offers a fast, accessible, and cost-effective method for throughput prediction compared to conventional geometallurgical test work of hardness and grindability.

III. Quantification of the effect of ignoring non-additive geometallurgical properties related to hardness for throughput prediction

Non-additivity for hardness-related geometallurgical variables is mostly ignored and current methods do not quantify this effect. This thesis shows that hardness proportions created from penetration rates can decrease the error within a ball mill throughput prediction model compared to the use of average penetration rates, which ignores non-additivity for upscaling and blending.

IV. Integration of a ball mill throughput prediction model into short-term production scheduling that is based on blended geometallurgical properties

Existing geometallurgical throughput prediction models typically ignore the non-additive nature of hardness and are not designed to interact with mine production scheduling. In this thesis, throughput rates are predicted within the optimization as a function of blended rock properties, overcoming the shortcomings of previous attempts of integrating throughput into production scheduling.

V. Integrating geostatistically informed prediction models of consumption rates of reagents and consumables into short-term production scheduling

Consumption rates of reagents and consumables are typically accounted for in mine production scheduling by cost adjustment factors per rock type or mining block, ignoring geological uncertainty, blending, and geometallurgical information. This thesis overcomes these limitations by creating empirical geometallurgical prediction models of reagents and consumables by tracking blended rock properties that are matched with observed consumption rates of the operating processing plant and integrating them in a simultaneous stochastic optimization model for short-term production scheduling.

2 Joint Stochastic Short-term Production Scheduling and Fleet Management Optimization for Mining Complexes

A new stochastic integer programming model for short-term planning in mining complexes is developed in this chapter. By extending a non-linear simultaneous stochastic optimization formulation for long-term planning, the resulting model simultaneously optimizes pertinent decisions in a mining complex on short-term scale, namely short-term extraction sequence, shovel allocation including the costs and loss of production by shovel relocation, scheduling of a heterogeneous truck fleet, destination of extracted materials, and downstream material flow in mining complexes. Next to geological uncertainty, uncertain shovel production rates, truck availabilities and truck cycle times are added to account for the additional inherent uncertainties of the mobile fleet in short-term planning. The benefits of jointly optimizing a short-term production schedule and fleet management decisions are demonstrated in a gold mining complex.

2.1 Introduction

Short-term mine planning generally aims to make optimal decisions over a timeframe of days to months to best meet annual production targets given by the long-term mine production plan (Wilke and Reimer, 1977; Fytas et al., 1987; Hustrulid et al., 2013). This task is typically accomplished in two separate steps. In the first step, the physical short-term extraction sequence is optimized, which is guided by the long-term mine plan and other pertinent short-term objectives (Blom et al., 2018). The second step optimizes the assignment of mining equipment (trucks and shovels) in open pit mines and is referred to as fleet management (Moradi Afrapoli and Askari-Nasab, 2017). Fleet management optimization includes two parts. The first part optimizes the shovel positions in the mine as well as the allocation of a certain number of trucks to the related shovels. The second part optimizes truck dispatching to allocate single trucks to their next destination (Alarie and Gamache, 2002; Moradi Afrapoli and Askari-Nasab, 2017). Note that shovels are typically large in size to facilitate the cost-efficient extraction of materials, which leads to their difficult and costly relocation over long distances within mining operations.

Recent developments combine aspects of fleet management and short-term extraction sequencing for open pit mines with the motivation of creating synergies between the steps that are conventionally optimized separately; this, in turn, generates more efficient short-term plans (Fioroni et al., 2008; L'Heureux et al., 2013; Torkamani and Askari-Nasab, 2015; Mousavi et al., 2016c; Villalba Matamoros and Dimitrakopoulos, 2016; Blom et al., 2017; Kozan and Liu, 2018; Upadhyay and Askari-Nasab, 2018). Most of these recent approaches integrate shovel allocation decisions, shovel capacities and the cost of shovel movements in the production scheduling model. Current research, however, has not incorporated the physical loss of shovel production due to time-consuming shovel movements within an industrial mining complex. Some recent models focus on extending equipment plans towards drilling and blasting activities (L'Heureux et al., 2013; Kozan and Liu, 2018), while others include additional decision variables related to optimal truck allocation (Fioroni et al., 2008; Torkamani and Askari-Nasab, 2015; Villalba Matamoros and Dimitrakopoulos, 2016; Blom et al., 2017; Upadhyay and Askari-Nasab, 2018).

In addition to integrating mining equipment allocation decisions, important developments in short-term mine planning include modelling and optimizing components of an industrial mining complex simultaneously. Simultaneous optimization may include multiple mines, multiple processing streams, multiple waste dumps, the option to stockpile material and the transportation of products to the port or the customer (Howard and Everett, 2008; Blom et al., 2016). Recent developments in simultaneous long-term mine planning shift the focus away from modelling the economic value of blocks towards the value of products sold, which can account for non-linear transformations of blended material in stockpiles and processing streams (Montiel and Dimitrakopoulos, 2015, 2018; Goodfellow and Dimitrakopoulos, 2016, 2017; Montiel et al., 2016). To date, this development has not yet been accounted for in the short-term optimization of mining complexes, which provides one motivation for the work presented herein. Note that different simultaneous or integrated optimization developments are available in the technical literature with regards to integrated logistics in the context of general supply chains (Darvish and Coelho, 2018). However, these do not address issues pertaining to typically unstructured mining operational environments or the management and extraction of ore from mineral deposits, where the location and supply of material quantities and qualities are uncertain, and are thus part of the short-term mine production scheduling optimization.

In addition, despite the listed manifold advances in modelling the short-term mining environment more accurately, most developed optimization models ignore pertinent uncertainties related to short-term mine planning, such as: (i) geological uncertainty stemming from imperfect knowledge of the material mined; (ii) equipment performance uncertainty due to equipment downtime, queuing times or external factors (e.g., weather), and (iii) plant performance uncertainty.

The impact of ignoring geological uncertainty has been widely discussed for long-term and short-term mine planning (Ravenscroft, 1992; Dowd, 1994, 1997; Dimitrakopoulos et al., 2002). Villalba Matamoros and Dimitrakopoulos (2016) simultaneously optimize a short-term extraction sequence and truck-shovel allocation decisions of a single mine while integrating metal uncertainty and equipment performance uncertainty into their optimization formulation. Several sets of equally likely uncertainty scenarios, including stochastic orebody simulations (Goovaerts, 1997; Rossi and Deutsch, 2014) and equipment availability scenarios, serve as an input to a stochastic integer programming (SIP) model, which is based on previous developments in stochastic mine planning (Ramazan and Dimitrakopoulos, 2005, 2013; Mai et al., 2019). Quigley and Dimitrakopoulos (2019) extend the formulation from Villalba Matamoros and Dimitrakopoulos (2016) to suit the optimization of multiple pits, processing streams, and material types, as well as access constraints for the location of hauling ramps. Villalba Matamoros and Dimitrakopoulos (2016) and Quigley and Dimitrakopoulos (2019) use the CPLEX solver (IBM, 2013) to obtain a solution, which restricts their approach to instances of a relatively small size, and thus limits their application.

Although short-term mine planning models naturally consider fewer mining blocks than their counterparts in long-term mine planning, their optimization formulations can have a high number of variables and constraints due to the optimization of many periods, additional decision-making, and stochasticity of various input variables. Some formulations for short-term mine planning use the aggregation of blocks as a pre-processing step to reduce problem size (Eivazy and Askari-Nasab, 2012; Kozan and Liu, 2018). This grouping is undesirable because the aggregation of mining blocks misrepresents mining selectivity and ore dilution. In addition, it ignores the ability to blend favorable ore types together given the mine’s selectivity, as well as fixes the geometries of mining fronts beforehand and the varying distances of truck hauling and shovel movement that correspond to the aggregated mining blocks.

Metaheuristics provide a useful platform for the optimization of large, potentially non-linear optimization formulations. Kumral and Dowd (Kumral and Dowd, 2005) apply a simulated annealing (SA) algorithm (Kirkpatrick et al., 1983; Geman and Geman, 1984a) to improve a suboptimal short-term extraction sequence. Mousavi et al. (2016c) solve their mixed integer programming model for short-term mine planning by a hybridized combination of large neighborhood search and branch-and-bound. Mousavi et al. (2016b) test three different variants of local search algorithms to optimize a short-term extraction sequence. However, mechanisms that consider the perturbation of extraction decisions, equipment assignment decisions and downstream allocation of material altogether remain to be developed.

In this article, a new simultaneous stochastic optimization approach to jointly optimize short-term mine production schedules and fleet management for industrial mining complexes is presented. The proposed approach extends previous developments for long-term mine production planning (Goodfellow and Dimitrakopoulos, 2016) by adding major components linked to short-term planning, notably fleet management and its associated uncertainties. The new components that are integrated into the mine production scheduling optimization model for short-term planning include the scheduling of a heterogeneous truck fleet and individual shovel allocations that consider the costs and loss of production caused by their relocation. These decisions are jointly optimized with the short-term extraction sequence as well as with the downstream allocation of extracted materials to multiple processing facilities and stockpiles in a mining complex. The stochastic optimization formulation considers uncertainty related to equipment performances and truck cycle times in addition to geological or material supply uncertainty. A simulated annealing metaheuristic is adapted to account for all newly added decision variables related to short-term production planning. The metaheuristic is designed to optimize large linear and non-linear problem instances that typically occur in open-pit mine planning.

In the next sections, the mathematical formulation of the developed stochastic programming model is presented first, along with details of the metaheuristic solution approach. Subsequently, a case study is performed at a gold mining complex to demonstrate the applied aspects of the proposed optimization model. Conclusions follow.

2.2 Mathematical Formulation

The mathematical model for joint stochastic short-term production scheduling and fleet management optimization for mining complexes is formulated as a stochastic integer programming model with fixed recourse (Birge and Louveaux, 2011), and builds upon the simultaneous stochastic optimization of the components of a mining complex that are pertinent to long-term mine planning (Goodfellow, 2014; Goodfellow and Dimitrakopoulos, 2016, 2017). In this section, a general notation and objective function of the mathematical model are presented first. Subsequently, the decision variables for optimal shovel positioning and optimal truck scheduling in a mining complex are introduced, along with new stochastic components of the mining fleet, such as equipment uncertainty and uncertain truck cycle times.

2.2.1 Modelling a mining complex

The mathematical formulation models a mining complex as a set of locations, $i \in \mathbb{A} \cup \mathbb{D}$, whereas material flows from a set of mining areas, $\mathbb{A}$, towards a set of destinations, $\mathbb{D}$. Extracted material is then processed, stockpiled, sent to secondary destinations, or is disposed of in waste dumps or tailing facilities. A mining area is a location in one of the pits belonging to a mining complex, where material is excavated. Hence, mining areas account for the fact that extraction normally takes place in several locations (benches) within the pits in parallel, leading to different excavation and hauling requirements. All mining blocks, *b*, that belong to one area of the mining complex are summarised in the set $\mathbb{B}_a$. When material flows through the mining complex, a set of primary, additive attributes of the extracted materials, $p \in \mathbb{P}$ (e.g., ore tonnage, metal tonnage, etc.), is defined. A set of hereditary attributes, $\mathrm{h} \in \mathbb{H}$, is then derived from the linear or non-linear conversion of primary attributes (e.g., metal recoveries, profits, costs). For more information about modelling primary and hereditary attributes, the reader is referred to Goodfellow and Dimitrakopoulos (2016).

2.2.2 Notation

This section defines the indices and sets, parameters, and decision variables that are used in the mathematical model.

2.2.2.1 Indices and sets

$t \in \mathbb{T}$ Index of a time period for the discretized planning horizon $\mathbb{T}$

$s \in \mathbb{S}$	Index of an orebody scenario in the set of orebody scenarios $\mathbb{S}$
$s_e \in \mathbb{S}_E$	Index of an equipment performance scenario in the set of equipment scenarios $\mathbb{S}_E$
$\varepsilon \in \mathbb{P} \cup \mathbb{H}$	Index of an attribute in the unified set of primary attributes $\mathbb{P}$ and hereditary attributes $\mathbb{H}$ in the mining complex
$d \in \mathbb{D}$	Index of a destination in the set of destinations $\mathbb{D}$ in the mining complex
$a \in \mathbb{A}$	Index of a mining area in the set of all mining areas $\mathbb{A}$
$i \in \mathbb{A} \cup \mathbb{D}$	Index of a location *i* in the mining complex, whereas $\mathbb{A} \cup \mathbb{D}$ contains all locations in the mining complex
$b \in \mathbb{B}$	Index of a mining block in the set of all blocks $\mathbb{B}$
$\mathbb{B}_a$	Set of blocks belonging to area *a*
$g \in \mathbb{G}$	Index of a group *g* of material in the set of all material groups $\mathbb{G}$
$l \in \mathbb{L}$	Index of a shovel operating in the mining complex in the set of all shovels $\mathbb{L}$
$m \in \mathbb{M}$	Index of a truck-type operating in the mining complex in the set of all truck-types $\mathbb{M}$

2.2.2.2 Parameters for mining complex

$p_{h,i,t}$	The unit price of hereditary attribute *h* in location *i* in period *t*, which can be positive (revenue) or negative (cost)
$c^-_{\varepsilon,t}$, $c^+_{\varepsilon,t}$	Associated costs for shortage (-) or excess (+) of a primary or heredetary attribute $\varepsilon \in \mathbb{P} \cup \mathbb{H}$ in period *t*
Ton_b	The tonnage of block b
H^t	Scheduled working hours per period
$CostSmooth$	Penalty cost for unconnected blocks, enforcing the extraction of blocks in a connected (smooth) pattern

2.2.2.3 Equipment-related parameters

L_a^{Max} The maximum number of shovels that can work simultaneously in the same area *a*

T_m^{Max}, T_m^{Min} The maximum and minimum number of trucks of truck-type *m* that are to be scheduled for any time period

$Prod_{l,s_e}^t$ Stochastic production rate of shovel *l* (tons per period) for equipment scenario s_e in period *t*

Cap_m Nominal truck capacity (tons) of truck-type *m*

A_{m,s_e}^t Stochastic availability factor of truck-type *m* for equipment scenario s_e in period *t*

$CT_{b,s}$ Truck cycle time required to haul material of block *b* to its destination for orebody scenario *s*

$CostOpTruck_m$ Cost of having one truck of truck-type *m* in operation ($ per period)

$CostMove_{l,a,a'}$ Cost of moving a shovel *l* from area *a* to area *a'* between period *t-1* and *t* ($ per move)

$LostProd_{l,a',a}$ Lost shovel production (tonnes per period) that occurs when moving a shovel *l* from area *a'* to area *a* between period *t-1* and *t*

$CostShovelShortage$ Penalty cost per tonne of not extracted material ($ per tonne) due to missing shovel capacity

$CostTruckShortage$ Penalty cost for not transported material ($ per tonne and truck hour) due to missing truck capacity

2.2.2.4 Decision variables

x_b^t Binary variable which equals 1, if block *b* is mined in period *t*, 0 otherwise

$z_{g,d}^t$ Binary variable which equals 1 if a group of material *g* is sent to destination *d* in period *t*, 0 otherwise

$\lambda_{l,a}^t$ Binary variable which equals 1 if shovel *l* is located in area *a* at period *t*, 0 otherwise

$\omega^t_{l,a,a'}$ Binary variable which equals 1 if a shovel *l* has moved from area *a* to area *a'* between period *t-1* and *t*, 0 otherwise

τ^t_m Integer variable which reflects the number of trucks of truck-type *m* scheduled in period *t*

y^t_b Integer smoothing variable which reflects the number of adjacent blocks of block *b* that are not extracted in the same period *t*

$\delta^t_{d,r,s}$ Continuous variable [0,1] which represents the proportion of material sent out from destination d (e.g., a stockpile) to a receiving destination r (e.g., a concentrator) for an orebody scenario *s* in period *t*

$dshovel^t_{a,s_e}$ Continuous variable which represents shovel production deviations in area *a* for equipment scenario s_e in period *t*

$dtruck^t_{s,s_e}$ Continuous variable which represents haulage capacity deviations for each orebody scenario *s* and equipment scenario s_e in period *t*

$v_{p,i,t,s}$ Continuous variable which represents the value of primary attribute *p* in location *i* for orebody scenario *s* in period *t*

$v_{h,i,t,s}$ Continuous variable representing the value of hereditary attribute *h* in location *i* for orebody scenario s in period *t*, which is obtained by applying linear or non-linear functions $f_{h,i}(v_{p,i,t,s})$ on the amount of primary attributes

$d^-_{\varepsilon,i,t,s}, d^+_{\varepsilon,i,t,s}$ Continuous variables modelling either shortage (-) or surplus (+) of primary or heredetary attribute $\varepsilon \in \mathbb{P} \cup \mathbb{H}$ at location *i* in period *t* in orebody scenario *s*

2.2.3 Objective function

Unlike most optimization formulations for short-term mine planning, the objective is to maximize metal production and profit of the mining complex as a whole, instead of minimizing operational costs. The minimization of costs alone neglects the fact that material can be blended and possibly sent to different available processing streams, which affects generated revenues from metal products that should be maximized. In the following, the objective function, (2.1), of the proposed mathematical model is presented.

$$\text{Maximize } \underbrace{\frac{1}{\|\mathbb{S}\|}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{h\in\mathbb{H}}\sum_{i\in\mathbb{A}\cup\mathbb{D}} p_{h,i,t}\cdot v_{h,i,t,s}}_{\textit{Revenues and costs in mining complex}\ \text{(I)}}$$

$$-\underbrace{\frac{1}{\|\mathbb{S}\|}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{i\in\mathbb{A}\cup\mathbb{D}}\sum_{\varepsilon\in\mathbb{P}\cup\mathbb{H}}\left(c^{+}_{\varepsilon,t}\cdot d^{+}_{\varepsilon,i,t,s}+c^{-}_{\varepsilon,t}\cdot d^{-}_{\varepsilon,i,t,s}\right)}_{\textit{Penalties for deviations from production targets}\ \text{(II)}}$$

$$-\underbrace{\frac{1}{\|\mathbb{S}_E\|}\sum_{t\in\mathbb{T}}\sum_{s_e\in\mathbb{S}_E}\sum_{a\in\mathbb{A}} CostShovelShortage\cdot dshovel^{t}_{a,s_e}}_{\textit{Reduce risk of not achieving shovel production target per mining area}\ \text{(III)}}$$

$$-\underbrace{\frac{1}{\|\mathbb{S}\cdot\mathbb{S}_E\|}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{s_e\in\mathbb{S}_E} CostTruckShortage\cdot dtruck^{t}_{s,s_e}}_{\textit{Reduce risk of falling short of truck haulage capacity}\ \text{(IV)}}$$

$$-\underbrace{\sum_{t\in\mathbb{T}\setminus\{1\}}\sum_{l\in\mathbb{L}}\sum_{a\in\mathbb{A}}\sum_{a'\in\mathbb{A}\setminus\{a\}} CostMove_{l,a,a'}\cdot\omega^{t}_{l,a,a'}}_{\textit{Shovel movement cost}\ (V)}$$

$$-\underbrace{\sum_{t\in\mathbb{T}}\sum_{m\in\mathbb{M}} CostOpTruck\cdot\tau^{t}_{m}}_{\substack{\textit{Account for costs of trucks}\\ \textit{in operation}\ \text{(VI)}}}-\underbrace{\sum_{t\in\mathbb{T}}\sum_{b\in\mathbb{B}} CostSmooth\cdot y^{t}_{b}}_{\textit{Enforce smooth mining schedule}\ \text{(VII)}} \tag{2.1}$$

Part (I) of the objective function, (2.1), summarizes revenues from products and operational costs that are generated in all modelled locations in the mining complex. Penalties for positive or negative deviations from production targets are accounted for in part (II). As in Goodfellow and Dimitrakopoulos (2016), these two parts form the basis of the simultaneous stochastic optimization of mining complexes. The remaining parts of the objective function are new, reflecting the aspects related to mining fleet management. The new parts are explained in conjunction with all newly added constraints in the remainder of this section.

Parts (III) and (IV) of the objective function are equipment-related, aiming to reduce the risk of not meeting short-term production targets associated with the mining fleet. Stochastic shovel production targets are controlled in part (III), whereas stochastic truck haulage capacity is controlled in part (IV). Additional cost factors related to equipment use are accounted for in parts (V) and (VI). Part (V) sums up the costs related to moving a shovel from one area to another. Part (VI) includes the operational costs of the trucks that need to be in operation, so as to be able to haul the scheduled material per period. Part (VII) enforces the grouping of mining blocks in connected (smooth) patterns to generate physically mineable shapes (Dimitrakopoulos and Ramazan, 2004). Typically, a practical extraction sequence is created this way.

2.2.4 Constraints

Reserve constraints, precedence constraints, constraints related to ore targets, metal targets and deleterious elements, as well as the downstream allocation of materials in the mining complex, considering stockpiles and multiple processing facilities, have been defined and are available in previous publications (Goodfellow and Dimitrakopoulos, 2016). However, all equipment-related constraints are new and defined next.

2.2.4.1 Constraints related to shovel allocation

While most of the literature that includes fleet management in optimization formulations assumes constant shovel production rates, some articles have considered their stochastic nature by employing (i) discrete-event simulation models (Awuah-Offei et al., 2003; Upadhyay and Askari-Nasab, 2018), (ii) robust optimization techniques for mathematical programming (Ta et al., 2005; Bakhtavar and Mahmoudi, 2018), and (iii) stochastic integer programming with recourse (Villalba Matamoros and Dimitrakopoulos, 2016; Quigley and Dimitrakopoulos, 2019).

In this article, stochastic shovel production scenarios (in tons per period) are created by sampling a distribution of historical production rates for each shovel and quantify the available shovel production and its associated risk of underperformance in the mining complex. These shovel production rates can be optimally allocated to each area using the binary shovel-allocation variables, $\lambda_{l,a}^{t}$, which together form the first term of constraint (2.2). Note that production losses associated with shovel movements are included, whereas their absence in modelling is seen as one of the limitations of current algorithms (Moradi Afrapoli and Askari-Nasab, 2017). Here, the shovel production in area a will be reduced if a shovel l was moved from another area a' to a

between period *t-1* and *t*. By summing the scheduled extraction per area (note extraction variables x_b^t), the shortfalls of shovel production are offset in constraint (2.2) by the deviation variable $dshovel_{a,s_e}^t$ for each scenario, area, and time period, which will be penalized in part (III) of the objective function.

$$\underbrace{\sum_{l\in\mathbb{L}}\left(Prod_{l,s_e}^t\cdot\lambda_{l,a}^t-\sum_{a'\in\mathbb{A}\backslash\{\boldsymbol{a}\}}LostProd_{l,a',a}\cdot\omega_{l,a',a}^t\right)}_{\substack{Scenario-dependent\,shovel\\production\,per\,area}}-\underbrace{\sum_{b\in\mathbb{B}_a}x_b^t\cdot Ton_b}_{\substack{Scheduled\\extraction\\per\,area}}+dshovel_{a,s_e}^t\geq 0$$

$$\forall\, t\in\mathbb{T},\ a\in\mathbb{A},\ s_e\in\ \mathbb{S}_E \qquad (2.2)$$

Other shovel-related constraints ensure that a shovel is only allocated to one area per period in constraint (2.3), and limit the maximum number of shovels, L_a^{Max}, that are allowed to work in one area in constraint (2.4). The usefulness of the latter constraint will be emphasized later in the case study.

$$\sum_{a\in\mathbb{A}}\lambda_{l,a}^t=1\ \forall\ \ t\in\mathbb{T},l\in\mathbb{L} \qquad (2.3)$$

$$\sum_{l\in\mathbb{L}}\lambda_{l,a}^t\leq L_a^{Max}\ \ \forall\ \ t\in\mathbb{T},a\in\mathbb{A} \qquad (2.4)$$

2.2.4.2 Constraints related to truck haulage

Truck allocation has been considered in recent publications for combined production scheduling and fleet management optimization (Torkamani and Askari-Nasab, 2015; Villalba Matamoros and Dimitrakopoulos, 2016; Blom et al., 2017; Upadhyay and Askari-Nasab, 2018; Quigley and Dimitrakopoulos, 2019). The number of required trips of individual trucks or truck types to individual shovels is typically chosen as a decision variable, which ignores the fact that single trips of trucks to shovels are often decided by truck dispatching algorithms in real-time. Instead, the proposed short-term optimization model optimizes the decisions of how many trucks of truck-type *m* should be optimally utilized per period. Here, the mine planner has the possibility of setting an upper and lower bound on the number of trucks per truck-type *m*, which is seen in constraint (2.5).

$$T_m^{Min} \leq \tau_m^t \leq T_m^{Max} \; \forall \; t \in \mathbb{T}, m \in \mathbb{M} \tag{2.5}$$

The transportation of mined materials by the hauling fleet is defined by the tonnage, as well as the required time for a truck to haul materials from the excavation location to the destination and the return to the excavation point (commonly referred to as truck cycle time). Ignoring cycle time requirements in short-term production scheduling can lead to unexpected haulage requirements to be accomplished by the truck fleet, which is clarified in Figure 2-1. In this example, a constraint (red line) that keeps mined tonnage per period under a threshold is obeyed consistently in every period by the resulting mine plan (blue line). However, when measuring the required haulage capacity (black line), defined as the product of tonnage and truck cycle time, haulage requirements differ substantially over the time horizon than previously indicated by extracted tonnage. The requirement of truck haulage is overestimated in the first period where cycle times are typically shorter due to shallower depths in the pit but are underestimated in periods 8 to 10, as noted through the steep rise above previous periods. Furthermore, it is possible that there is an increase of required truck haulage (black line) despite a decrease of total tonnage (blue line), as seen in periods 7 and 12. All these differences can be explained by different cycle times of the materials to be removed, depending on their respective position in the mining complex.

Constraint (6) accounts for the abovementioned cycle time requirements by defining required haulage capacity as a product of cycle time of individual blocks, $CT_{b,s}$, and the respective tonnage of a block to be hauled, Ton_b. Since the material type and metal quantity of a mining block can change from one orebody simulation to another, the destination of the material in a mining block can change due to two underlying reasons. First, a different material type (e.g., oxide vs. sulfide material) might require a different mineral processing method. Second, the grade of a block can change so that the most profitable destination might now be a different processor or the waste dump[1]. Hence, the required cycle time to transport the material to a certain destination depends on orebody scenario s. The effect of geological uncertainty on the cycle time of material is also the

[1] This footnote provides additional information to the published and unaltered version of the journal article presented in Chapter 2. In the presented case study in this chapter, pre-defined cut-off grades define the material destination, which renders the stochastic cycle time, $CT_{b,s}$, a parameter. In general, the presented simultaneous stochastic optimization formulation enables the simultaneous optimization of material destinations and extraction sequences, as exemplary shown in Chapter 5. This way, $CT_{b,s}$ will be a variable, because cycle times per block and orebody scenario can vary based on optimized cut-off grades.

reason why the median (P50) of the required haulage capacity is reported in Figure 1. By modelling cycle times utilizing exact block positions in the mining complex and their possible destination, limitations due to mining block aggregation, as noted earlier, are removed.

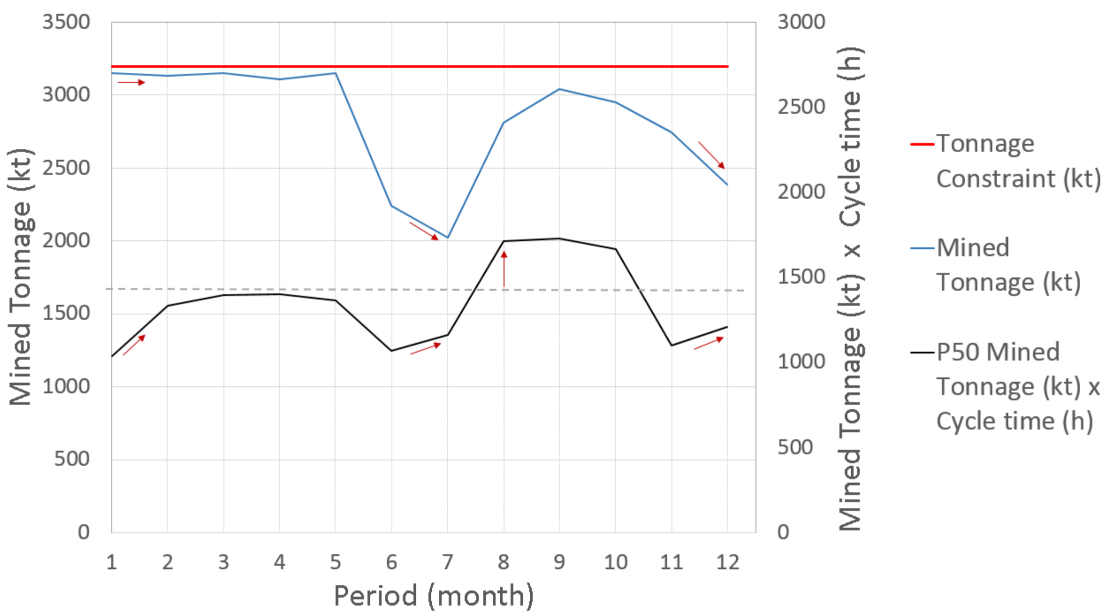


Figure 2-1 Analysis of truck haulage requirement comparing mined tonnage (upper line, blue) with the product of mined tonnage and required haulage cycle time (lower line, black)

In constraint (2.6), the required haulage capacity is opposed with the scenario-dependent haulage capacity of the heterogeneous truck fleet, which is modelled as a product of truck payload, Cap_m, stochastic availability, A^t_{m,s_e} per truck-type m, accounting for the uncertainty that the truck fleet has a natural risk of underperforming, planned working hours of the planning horizon, H^t, and the integer truck decision variable, τ^t_m. Deviations are, similar to shovel production, offset by the deviation variable $dtruck^t_{s,s_e}$ for each equipment scenario, orebody scenario, and time period.

$$\underbrace{\sum_{m\epsilon\mathbb{M}} Cap_m \cdot A^t_{m,s_e} \cdot H^t \cdot \tau^t_m}_{\substack{Scenario-dependent\ haulage\\ capacity\ of\ truck\ fleet}} - \underbrace{\sum_{b\in\mathbb{B}} CT_{b,s} \cdot Ton_b \cdot x^t_b}_{Required\ haulage\ capacity} + dtruck^t_{s,s_e} \geq 0$$

$$\forall\, t \in \mathbb{T}, s_e \in \mathbb{S}_E, s \in \mathbb{S} \tag{2.6}$$

2.3 Metaheuristic Solution Method

For the optimization of large and potentially non-linear instances of the mathematical model for joint short-term optimization of mining complexes (note the possibility of blending materials including stockpiled material and applying non-linear beneficiation processes on blended material), a simulated annealing (SA) metaheuristic is adopted herein, extending that in Goodfellow and Dimitrakopoulos (2016, 2017). The use of SA stems from its past successful use for single open-pit mine planning and production scheduling (Godoy, 2002; Kumral and Dowd, 2005; Kumral, 2013; Mousavi et al., 2016a), as well as its excellent performance in major case studies for long-term simultaneous stochastic optimization of production planning in mining complexes (Montiel et al. 2016; Montiel and Dimitrakopoulos 2018; and others).

Let Φ be the current solution of the optimization problem, which is partitioned into solution vectors of individual decision variables $\Phi = (x, z, \delta, \lambda, \tau)$. Here, the solution Φ consists of the vector of the extraction sequence, *x*, which stores the period of extraction for each block, the vector of the destination decisions, *z*, which stores the current destination of each material group for each period, the vector of downstream allocation decisions, δ (all similar to Goodfellow and Dimitrakopoulos 2016), and the newly added fleet management components, consisting of the vector of shovel allocation decisions, λ, and the vector of hauling fleet decisions, τ. Note that all other variables are calculated as a result of the described decision vectors above.

Let $v(\Phi)$ be the objective function value of the current solution and $v(\Phi')$ the objective function value of a feasible modified solution, Φ'. For a maximization problem, the Metropolis (1953) acceptance probability criterion for SA is given in Eq. (2.7).

$$P(v(\boldsymbol{\Phi}'), v(\boldsymbol{\Phi}), temp) = \begin{cases} 1, & if\ v(\boldsymbol{\Phi}') \geq v(\boldsymbol{\Phi}) \\ exp\left(-\dfrac{|v(\Phi') - v(\boldsymbol{\Phi})|}{temp}\right), & otherwise \end{cases} \tag{2.7}$$

As the algorithm progresses, the annealing temperature, *temp*, gradually decreases using a cooling schedule. This cooling schedule is defined by the initial temperature, $temp_0$, a reduction factor, *k*, and the number of iterations, n^{iter}, before the reduction factor is applied. Goodfellow and Dimitrakopoulos (2016) introduce a modification, which uses multiple annealing temperatures depending on the type of decision variable that is perturbed for obtaining a feasible modified solution, Φ'. Furthermore, a diversification strategy reinitiates the cooling schedule after a defined set of iterations, n^{divers}, to broaden the search of the solution space.

A modified solution, Φ', is obtained by applying a perturbation rule to the solution vector, chosen from a set of available perturbation rules, commonly referred to as a neighbourhood N_k ($k = 1 \dots k_{max}$) in a finite set of neighbourhood structures (Mladenovic and Hansen, 1997). For example, the extraction sequence, x, can be modified by (i) changing the extraction period of a single block, (ii) swapping the periods of extraction of two blocks, or (iii) changing the extraction period of a block and its entire predecessor set, and so on. Many perturbation rules for the mining block extraction sequence, destination decisions, and the utilization of processing streams are detailed in previous publications (Goodfellow, 2014; Goodfellow and Dimitrakopoulos, 2016).

The model presented herein introduces new fleet management decisions to the simultaneous optimization of mining complexes, thus new perturbation rules are introduced for alternating shovel and truck allocation decisions. All newly defined perturbation structures are detailed in Appendix 1. Perturbation rules are chosen with equally likely probability at the start, and as the algorithm progresses, the probability of selecting each perturbation rule is adapted depending on their performance in terms of improving the objective function (Burke et al., 2013; Lamghari and Dimitrakopoulos, 2020). An initial solution to the metaheuristic is obtained by randomly assigning values to variables within the feasible solution domain while obeying slope constraints. All utilized parameters for the metaheuristic are given in Table 2-1.

Table 2-1 Parameters for simulated annealing metaheuristic

Parameter	**Value**
Initial temperature[2] $temp_0$	0.15
Reduction factor k	0.95
Iterations before cooling n^{iter}	300
Iterations before diversification n^{divers}	80,000
Number of diversifications	4

[2] This footnote provides additional information to the published and unaltered version of the journal article presented in Chapter 2. The initial temperature used in the described Simulated Annealing algorithm can be described as global temperature that is used to determine annealing temperatures for individual neighborhoods, such as block extraction neighborhood or shovel allocation neighborhood. Several temperatures are used since a single temperature may prevent some neighborhoods from being altered prematurely because of their strong influence on the objective function value. As the algorithm progresses, cumulative distribution functions of previous non-improving objective function changes are used as look-up tables to determine the appropriate temperature per neighborhood. More details can be retrieved in Goodfellow and Dimitrakopoulos (2016).

2.4 Application at a Gold Mining Complex

The gold mining complex considered includes two open pits that are sub-divided into six spatially distinct mining areas[3] (A1 - A6) for short-term production scheduling, depicted in Figure 2-2. This mining complex operates with a shared mining fleet, whereas material is excavated and hauled to a milling and grinding circuit (first processing stream), a stockpile connected to the mill, a heap leach facility (second processing stream), or the waste dump that is located closest to each pit, as seen in Figure 2-3.

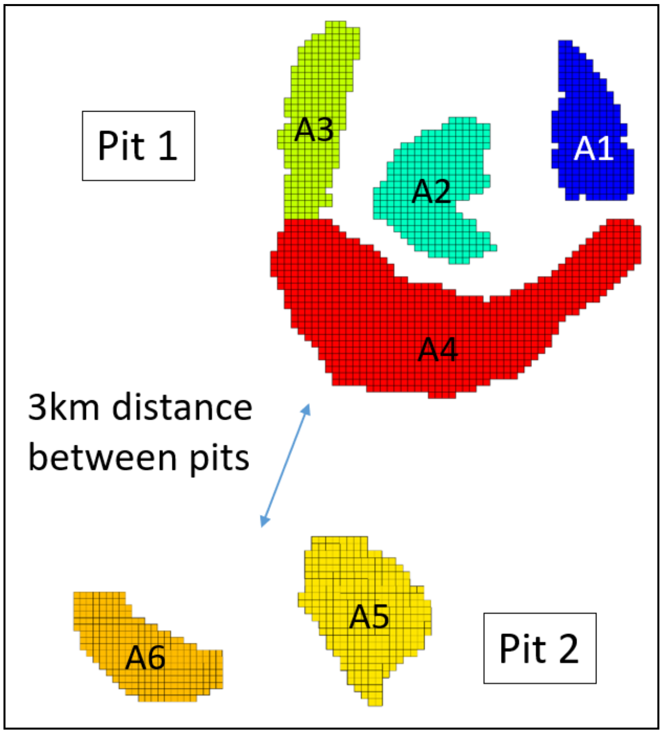


Figure 2-2 Mining areas in the gold mining complex

[3] This footnote provides additional information to the published and unaltered version of the journal article presented in Chapter 2. The mining areas shown in Figure 2-2 are manually defined by grouping the materials together that form a mining face, are spatially connected, and can be accessed by the same haul route.

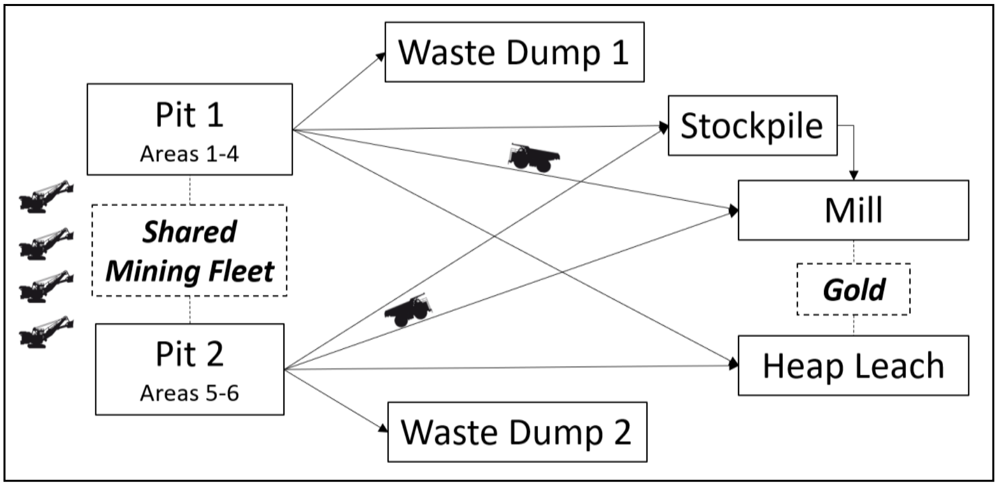


Figure 2-3 Components of the gold mining complex

2.4.1 Optimization parameters

Economic parameters for the optimization are given in Table 2-2. The scheduled material comprises the year of extraction defined by a long-term production schedule. The annual production horizon, to be scheduled on a monthly discretization, comprises 6,382 blocks (15x15x12 m^3). Uncertainty of metal grade (Au) is accounted for by utilizing fifteen equally probable orebody scenarios using geostatistical simulation techniques (Goovaerts, 1997). Other optimization parameters, given in Table 3, define the targets of the mining complex to be met by optimization. The milling and grinding circuit has a maximum throughput capacity of 360 kt of ore per month. Penalty costs are applied for exceeding this tonnage and consider metal and material type uncertainty stemming from the geological reserve, as seen in Table 2-3. Furthermore, shovel allocation is controlled by applying penalty costs if uncertain shovel production falls short of the material to be extracted per mining area and period using constraint (2.2). The schedule of a heterogeneous truck fleet is optimized by applying penalty costs if required truck haulage capacity, measured as a product of tonnage and required truck cycle time, is not met by the scheduled truck fleet, using constraint (2.6). Generally, higher penalty costs reflect more risk-averse behaviour for meeting the respective production target.

Table 2-2 Economic parameters in mining complex

Parameter	Value
Periods	12 months
Gold price	$1250/oz
Refinery cost	$13/oz
Cut-off grade[4] (Waste/Leaching)	0.0041 oz/t
Cut-off grade[5] (Leaching/Mill)	0.0105 oz/t
Leach cost	$2.30/t
Recovery leach	45%
Mill cost	$7.80/t
Recovery mill	88%
Stockpile rehandling cost	$0.15/t
Operational costs for moving a shovel	$950/h

Table 2-3 Targets and penalty costs for optimization

Description of constraint	Target	Uncertainties involved	Penalty costs
Ore target mill	360,000 t/month	Metal uncertainty	$20 per tonne that cannot be processed
Shovel production per area	Varies (depending on allocated shovels in each area and in each period)	Equipment performance uncertainty	$10 per tonne that cannot be extracted
Truck haulage capacity	Varies (depending on the utilized number of trucks in each period)	Equipment performance and metal uncertainty	$10 per not transported tonne and truck hour
Mining pattern (smoothness)	Mine out blocks in a connected pattern	-	$20,000 per unconnected block

Total truck cycle time is measured as a combination of two components. The first component consists of the time required to haul material from its position in the pit, measured per mining block, to the pit exit of the nearest connected ramp, which is visualized in Figure 2-4 and Figure 2-5. The second component defines the time required to haul material from the pit exit to its

[4] Optimal economic cut-off grade that distinguishes waste and leaching material, which has been provided by the strategic mine plan for the year of extraction
[5] Optimal economic cut-off grade that distinguishes leaching and milling material, which has been provided by the strategic mine plan for the year of extraction

destination, shown in Table 2-4. Both cycle time calculations also include the time for the truck to return to the shovel.

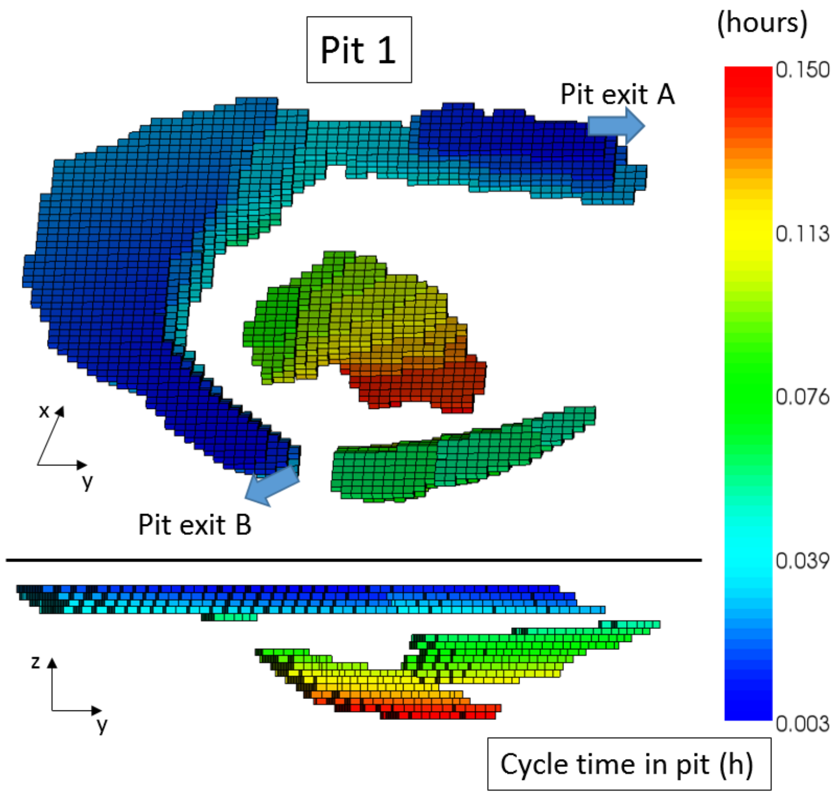


Figure 2-4 Horizontal (top) and vertical (bottom) sections of required truck cycle time in Pit 1 based on block positions

Table 2-4 Truck cycle times from pit exit to destinations and return

Location	Waste Dump 1	Waste Dump 2	Heap Leach	Stockpile	Crusher (Mill)
Pit 1	0.21 h	-	0.35 h	0.39 h	0.43 h
Pit 2	-	0.19 h	0.31 h	0.35 h	0.39 h

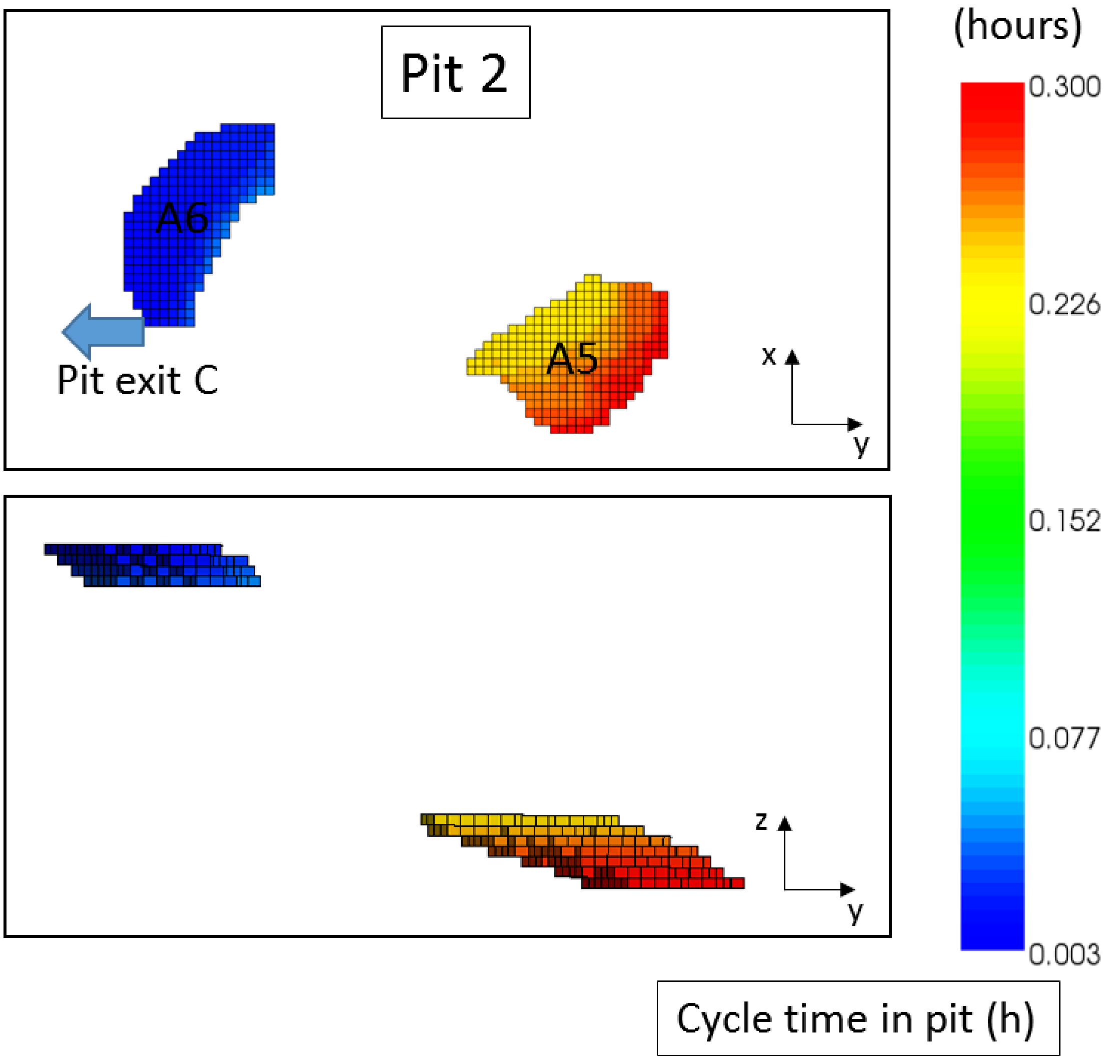


Figure 2-5 Horizontal (top) and vertical (bottom) sections of required truck cycle time in Pit 2 based on block positions

The available heterogeneous truck fleet in the mining complex incudes two truck-types, whose respective payloads, operational costs, and stochastic availability factors are presented in Table 2-5. The expected production rates and standard deviation of the four individual shovels in the mining complex are presented in Table 2-6. Costs of shovel movements are provided in Table 2-7., which are calculated by multiplying the expected travel time (h), approximated by the length of connecting ramp segments and shovel travelling speeds, and the mine-specific operational costs ($/h) of moving a shovel to a different mining area. This hourly rate includes a premium for the higher consumption of resources and the increased maintenance costs, along with the shovel's standard operational costs. Lost production due to shovel movements are presented in Table 2-8, which are the product of expected travel time (h) and production rate (t/h) per shovel.

Table 2-5 Payload, costs and stochastic availabilities of heterogeneous hauling fleet

Hauling Equipment	**Payload (t)**	**OPEX ($/h)**	**Availability (%)**		**Number of Trucks in place**	**Parameters used for joint optimization**	
			Mean	**Standard Deviation**[6]		**Maximum Number (T_m^{Max})**	**Minimum Number (T_m^{Min})**
Truck Type 1	100	124	0.81	0.0405	10	8	5
Truck Type 2	140	176	0.83	0.0415	14	11	7

Table 2-6 Expected production rate and standard deviation of individual shovels based on historical data

Loading Equipment	**Production (t/h)**	
	Mean	**Standard deviation**[4]
Large Shovel 1	2,597	129.8
Large Shovel 2	2,538	126.9
Small Shovel 1	1,310	65.5
Small Shovel 2	1,283	64.1

[6] This footnote provides additional information to the published and unaltered version of the journal article presented in Chapter 2. For uncertainty quantification of mining equipment in this case study, the underlying distribution for shovel production rates and truck availabilities is assumed to be Gaussian, with a mean equal to historical averages and a standard deviation equal to 5 % of the respective mean values. It should be noted, however, that the variability of these factors, i.e., the standard deviation, changes based on the length of the defined short-term interval. For example, a larger standard deviation is typically observed in daily production intervals compared to monthly production intervals (variance of sample mean decreases with n=number of days). If daily production data is available, it is possible to draw values at random from the histogram of daily production rates and sum them until the desired short-term interval is reached. The process can be repeated to create a set of uncertainty scenarios for multiple periods. Related topics are discussed in Kumar (2020).

Table 2-7 Costs caused by shovel movement from one area to another, calculated by multiplying expected travelling time (h) with shovel moving cost ($/h), exemplary for Large Shovel 1

$ in ‘000s	To A1	To A2	To A3	To A4	To A5	To A6
From A1	0.0	5.7	8.6	6.7	39.0	36.1
From A2	6.7	0.0	10.5	10.5	39.9	37.1
From A3	9.5	8.6	0.0	3.8	39.9	37.1
From A4	7.6	9.5	2.9	0.0	39.0	37.1
From A5	40.9	40.9	43.7	39.9	0.0	8.6
From A6	38.0	39.0	40.9	37.1	7.6	0.0

Table 2-8 Lost production caused by shovel movement from one area to another, calculated by multiplication of expected travel time (h) and shovel production rate (t/h), exemplary for Large Shovel 1

in kt	To A1	To A2	To A3	To A4	To A5	To A6
From A1	0.0	15.6	23.4	18.2	106.6	98.8
From A2	18.2	0.0	28.6	28.6	109.2	101.4
From A3	26	23.4	0.0	10.4	109.2	101.4
From A4	20.8	26	7.8	0.0	106.6	101.4
From A5	111.8	111.8	119.6	109.2	0.0	23.4
From A6	104	106.6	111.8	101.4	20.8	0.0

2.4.2 Optimization results

The following results compare the proposed joint short-term optimization of a mining complex to a two-step approach where a production schedule is generated first using a stochastic model that optimizes all components of a mining complex. The latter model (Goodfellow and Dimitrakopoulos, 2016) excludes all the equipment-related components presented earlier in this manuscript. In a separate second step, equipment allocation is performed, which minimizes the number of shovel moves and the truck hauling effort, based on the fixed production schedule given by the previous step. In this second step, variable equipment performance is also accounted for by using a set of stochastic availability scenarios per equipment type, as discussed earlier in the manuscript. Generally, both optimization methods schedule mining equipment with high variability in performance rather conservatively. It is shown next that the proposed joint optimization of production schedule and equipment allocation in a mining complex is superior to the described two-step approach.

Figure 2-6 visualizes the optimized shovel allocation to mining areas, as well as the resulting shovel movements both for the two-step approach and the joint optimization. By observing the case where shovels are assigned separately to a previously optimized production schedule (upper part of Figure 6), there is an inevitably high number of shovel movements from area to area. This is because the previously optimized production schedule is only concerned with providing ore to the processors taken from any location of the mining complex. By performing a joint optimization, however, the production schedule adapts to the available equipment (lower part of Figure 6). As a result, the lower number of shovel moves leads to a 56% decrease in shovel movement costs and a 54% decrease in shovel production losses caused by relocation, as summarized in Table 2-9. Most notably, the number of necessary shovel moves between pits is reduced from five to two moves. Furthermore, the joint optimization of the production schedule and mining fleet can also limit the number of shovels working in the same area in the same period, which is visualized in Figure 2-7. By re-arranging the quantities of material to be handled by one shovel alone, as seen in areas A1 and A2 in this case study, space and safety requirements are facilitated and, at the same time, truck traffic on the supply ramp to these areas is also reduced, especially when they are located at the bottom of the pit where only one ramp is available for hauling.

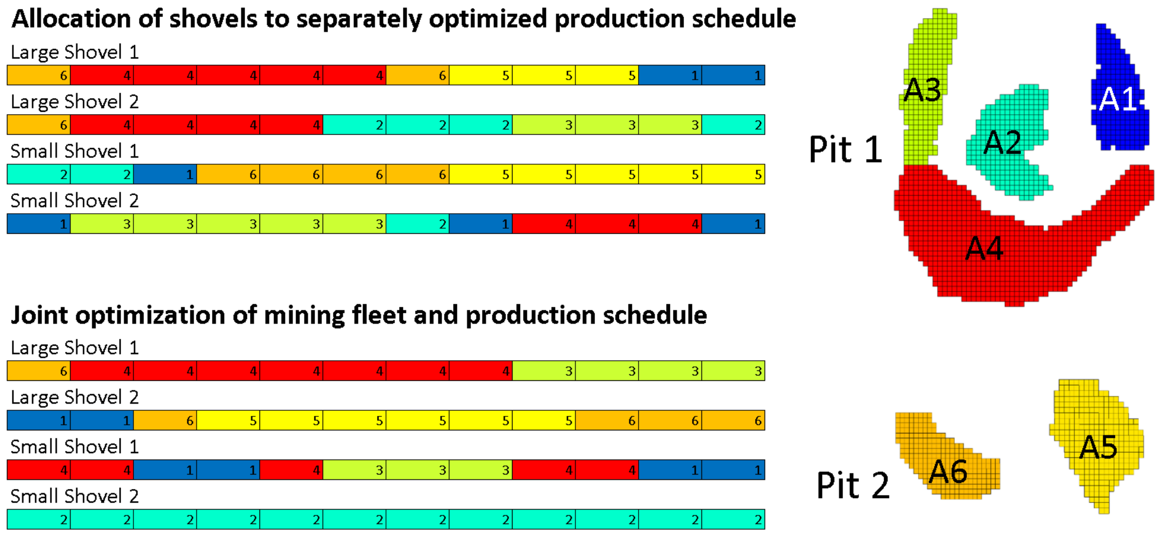


Figure 2-6 Comparison of shovel allocation per period (12 months) to mining areas (colour-coded from A1 to A6) for separately optimized production schedule (top) and joint optimization (bottom)

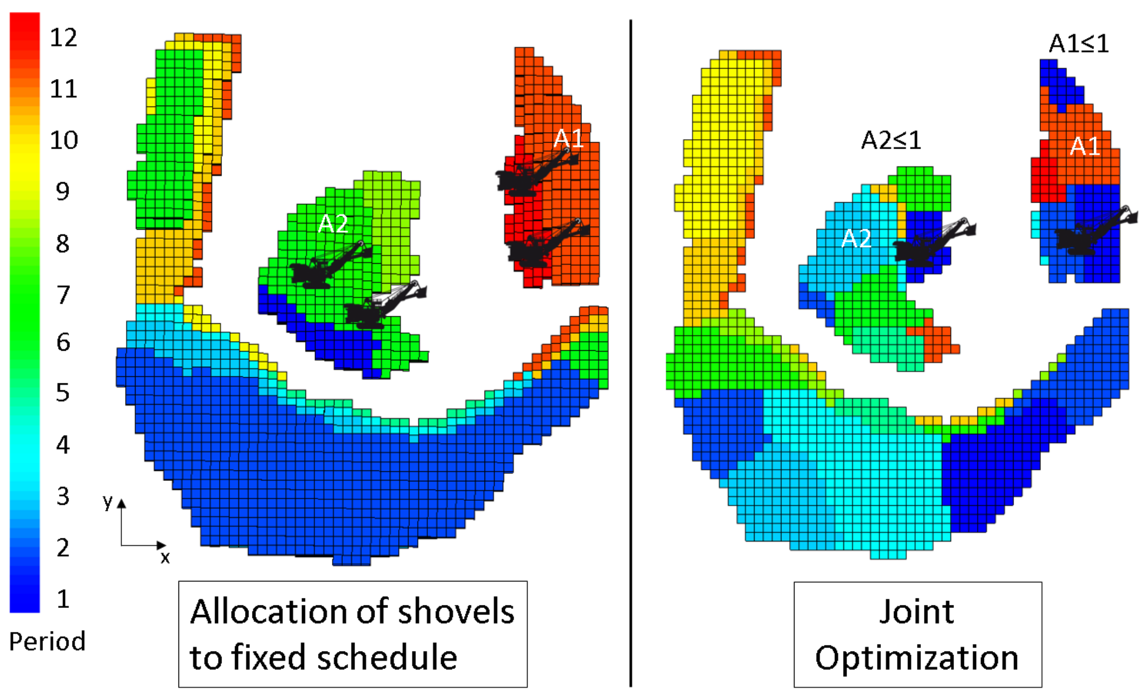


Figure 2-7 Comparisons of the fixed production schedule of Pit 1 (left) and jointly optimized fleet allocation and production schedule (right), whereas joint optimization includes constraints of limiting the number of shovels working in the same area for A1 and A2

The advantages of simultaneously optimizing the production schedule and truck fleet are discussed herein. As seen in Figure 2-8, the assignment of trucks on a fixed production schedule can cause high fluctuations that lead to, on the one hand, periods where the complete fleet of 24 trucks needs to be assigned to haul extracted material (periods 8 and 9) and, on the other hand, periods where less than half of the truck fleet is in use (periods 6 and 12). Problems can arise when (i) the maximum required number of trucks over the planning horizon is unavailable, (ii) the preventive maintenance cannot be accomplished adequately if a maximum number of required trucks is in operation, or (iii) if the workforce cannot be flexibly assigned over a short-term planning horizon.

By analyzing optimized truck assignments in Table 2-10, one notes that the joint optimization of the mining fleet and production schedule succeeds in consistently having up to 80% of trucks and over 50% of trucks of each truck-type in use. As a result, trucks of both truck-types are more efficiently matched to the production schedule of the mining complex, so that total truck operational costs are reduced by 3.1% over the annual planning horizon compared to the two-step optimization approach. Furthermore, the more balanced utilization of the truck fleet reduces the hauling effort of haulage-intensive periods effectively (compare periods 8 - 10), which allows for preventive truck maintenance on the remaining part of the fleet in every period and minimizes disruptions caused by events of unexpected breakdowns. A summary of all key improvements of the jointly optimized mine plan is given in Table 2-9.

Table 2-9 Key improvements of the joint optimization of short-term production schedule and fleet management in mining complexes

Optimization Parameter	**Two-step optimization**	**Joint optimization**	**Difference**
Shovel movement costs ($ in ‘000s)	276.5	120.7	-56%
Lost production through shovel movement (kt)	635.7	291.2	-54%
Total shovel moves (Number of Moves)	16	10	-38%
Total truck operational costs (M $)	23.46	22.73	-3.1%
Most haulage-intensive period (Number of Trucks)	24	18	-25%

Table 2-10 Comparison of the number of trucks per period and associated truck operational costs obtained for truck optimization on a fixed schedule and joint optimization

	Period	1	2	3	4	5	6	7	8	9	10	11	12	Total
Fixed schedule	Truck Type 1 (# Trucks)	3	3	6	6	5	2	7	10	10	9	3	0	64
	Truck Type 2 (# Trucks)	11	14	13	13	13	10	10	14	14	14	14	12	152
	Operational costs trucks (M$)	1.55	1.90	2.05	2.05	1.96	1.35	1.79	2.53	2.53	2.44	1.90	1.40	23.46
Joint optimization	Truck Type 1 (# Trucks)	7	7	7	7	7	8	8	7	8	7	7	6	86
	Truck Type 2 (# Trucks)	11	11	11	11	11	10	10	11	10	11	11	11	129
	Operational costs trucks (M$)	1.91	1.91	1.91	1.91	1.91	1.88	1.88	1.91	1.88	1.91	1.91	1.82	22.73

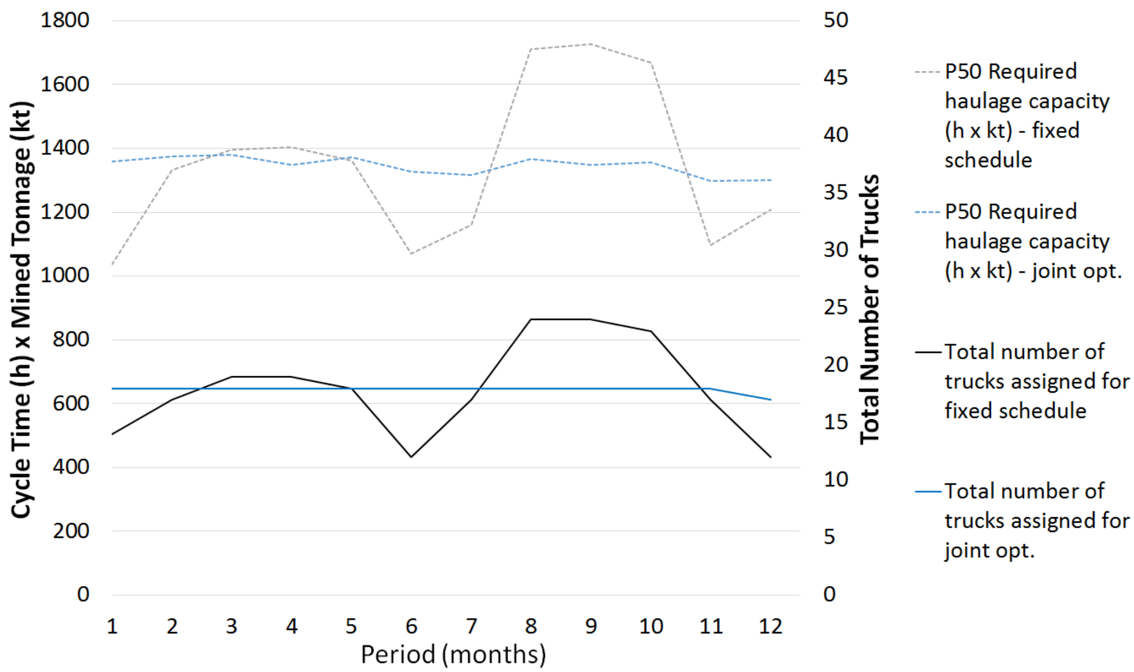


Figure 2-8 Comparison of required haulage capacity per period (top lines) and the total assigned number of trucks per period (bottom lines) for fixed schedule and joint optimization of production schedule and mining fleet

2.5 Conclusions

In this article, a novel stochastic mathematical model for the joint optimization of a short-term production schedule and fleet management in a mining complex is presented. The model simultaneously optimizes the short-term extraction sequence, the shovel allocation while considering costs and lost production from relocation, the scheduling of a heterogeneous hauling fleet, and the downstream allocation of extracted materials to multiple processing facilities and stockpiles along the mineral value chain. Several sources of uncertainty are integrated into the mathematical optimization formulation, including metal and material type uncertainty stemming from the geological reserve, uncertainty in shovel production, uncertainty in the availability of haul trucks, and the uncertainty of truck cycle time.

To model required truck hauling capacity more accurately, the product of tonnage and required truck cycle time per block is newly defined. Furthermore, truck cycle times are precisely quantified block by block, considering the uncertain destination of the material due to metal and material type uncertainty. The case study presented shows how the joint optimization of a production schedule and fleet management in a mining complex results in synergies that cannot be achieved through a conventional two-step approach, where fleet management optimization is applied on a previously optimized, fixed production schedule.

Future work could integrate additional sources of uncertainty into the proposed framework of joint short-term optimization of mining complexes, including uncertainties related to plant throughput, the density of mined material, and metal recovery in various processing facilities. In addition, spatial clustering techniques may be used for sub-dividing the material to be mined into distinct mining areas. While the SA-based solution approach utilized herein provides good results in reasonable time, further research can explore the performance of other state-of-the-art metaheuristics (e.g. Hansen and Mladenović 2018; Laguna 2018). Lastly, with the recent advancements of sensor information collection during operations in mining complexes, such as real-time measurements of rock properties, equipment availability and utilization, as well as the performance of processing plants, the related data collected may be integrated into the proposed framework to facilitate real-time updates of short-term production plans.

2.6 Appendix: Newly defined perturbation rules to obtain a modified solution for truck- and shovel decisions using a simulated annealing metaheuristic

The basic aspects of the metaheuristic solution method have been described in the main part of the manuscript. However, all newly developed perturbation rules for truck and shovel decisions, which are commonly referred to as neighbourhood structures, are listed here. For optimization, the truck and shovel allocation variables are encoded into separate solution vectors, which are perturbed during the simulated annealing process to obtain modified solutions. The shovel allocation vector, λ, stores an encoded version of shovel assignment variables $\lambda_{l,a}^{t}$ whereas each element $\lambda_{l}^{t} \in \lambda$ stores the current area *a* that has been assigned to shovel *l* in period *t*. Note that shovel movements $\omega_{l,a,a'}^{t}$ are obtained as a direct result of the current state of shovel allocation vector λ, which are newly evaluated after an accepted perturbation of λ. Further, it is assumed that the position of each shovel is fixed in the first period without requiring shovel movements in advance. The following perturbation rules are developed to modify λ:

Randomly-swap-shovels perturbation: Randomly select a period *t* and two shovels *l* and *l'*. If the area assignment *a* of the first shovel differs from the area assignment *a'* of the second shovel, swap their area assignment, otherwise repeat the random selection.

Targeted-swap-shovels perturbation: Create a list of preferred swaps[7] of shovels by scanning solution vector λ first. If the list is non-empty, pick and apply one preferred swap randomly. Otherwise, perform perturbation (1).

Randomly-change-one-area perturbation: Randomly select a period *t* and a shovel *l* allocated to area *a*. Randomly select a new area $a' \neq a$. After ensuring that a maximum number of shovels in *a'* is not exceeded, assign *a'* to shovel *l*.

Targeted-change-one-area perturbation: Create a list of preferred shovel moves[8] by scanning the entire solution vector λ. If the list is non-empty, pick and apply one preferred move randomly after checking if a maximum number of shovels is not exceeded in *a'*. Otherwise, perform perturbation (3).

Balance-capacity perturbation: Scan current extraction sequence vector *x* and enlist total extracted tonnage from each area. Scan current solution vector λ and enlist total expected shovel production allocated to each area. Randomly select a shovel *l* that has been assigned to the area of the largest

[7] Preferred swaps occur when shovel *l* is allocated to area *a* in two consecutive periods but allocated to a different area in the previous or following period, whereas another shovel *l'* is allocated to *a;* thus, swapping these two shovel-to-area assignments possibly reduces the necessary shovel moves of shovel *l* by one.

[8] Preferred shovel moves are closing 'gaps' in a sequence of shovel-to-area assignments. Exemplary, shovel *1* may be allocated to areas 1,1,2,1,1 in periods 1-5. Thus, a gap is identified in the third period. Closing this gap can lower the necessary shovel moves by two.

surplus of shovel production a^{surpl} in period t and attempt to reallocate this shovel to the area of largest shortfall of shovel production a^{short} after checking if a maximum number of shovels is not exceeded in a^{short}.

New developed perturbation rules for the integer hauling fleet variables $\tau_m^t \in \tau$ are as follows:

Gradually increase truck haulage capacity: Randomly select a period t and truck-type m. If $\tau_m^t < T_m^{Max}$, increase the number of trucks in this period by one.

Gradually decrease truck haulage capacity: Randomly select a period t and truck-type m. If $\tau_m^t > T_m^{Min}$, decrease the number of trucks by one.

2.7 Chapter Discussion and Next Steps

Chapter 2 provides a new stochastic integer programming model for short-term planning which demonstrates benefits of jointly optimizing a short-term production schedule and fleet management decisions in mining complexes. Main decisions in short-term planning, i.e., extraction sequence, downstream material allocation, shovel allocations, truck fleet assignments, and their related uncertainties (geological, equipment-related) are included in the new optimization model. It should be noted that the uncertain mechanical availability of trucks is used as an input parameter, whereas the utilization of trucks is an outcome of the optimization. This is because utilization depends, among other factors, on the number of trucks in operation which is a variable to be optimized. Similarly, the modelled uncertain shovel production rates should include downtime due to planned and unplanned maintenance (availability) but exclude waiting times for trucks (utilization). Up to this point, the ability of the presented optimization model to meet short-term production targets such as mill throughput and metal production is limited, since metallurgical processes in processing plants are severely simplified in short-term production scheduling. New ways must be found to integrate non-additive geometallurgical properties into the short-term optimization of mining complexes, which can be assisted by the available information of collected production data in the short-term. This topic is addressed in the following chapters of this thesis.

3 Integrating Geometallurgical Ball Mill Throughput Predictions into Short-term Stochastic Production Scheduling in Mining Complexes

This chapter develops a novel geometallurgical throughput prediction model using production data in the form of measurement while drilling and observed throughput rates from an operating mining complex. The throughput prediction model is then integrated into the simultaneous stochastic optimization for short-term planning in mining complexes developed in Chapter 2. This way, the scheduled materials in the short-term can be matched with the predicted throughput of the ball mill, while accounting for supply and geometallurgical uncertainty. Recorded penetration rates are used to inform rock hardness and the subsequent creation of hardness proportions avoids biases typically introduced by the change of support and blending of non-additive geometallurgical properties. A case study in a gold mining complex shows substantial improvements of meeting mill production targets when compared to the use of conventional mill tonnage constraints.

3.1 Introduction

Short-term mine production planning aims to make daily, weekly, or monthly operational decisions that best meet strategic production targets under existing operating conditions and constraints (Hustrulid et al., 2013). Blom et al. (2018) review past advancements in short-term planning for open pit mines, while recent developments consider the simultaneous stochastic optimization of a short-term mine production schedule and fleet management decisions in mining complexes under supply and equipment performance uncertainty (Quigley and Dimitrakopoulos, 2019; Both and Dimitrakopoulos, 2020). However, the incorporation of pertinent geometallurgical properties into short-term mine production scheduling has not been addressed sufficiently.

Geometallurgy describes the relationships between rock characteristics extracted from mineral deposits and its metallurgical/processing behaviour further downstream in a mineral value chain. Key geometallurgical properties should thus be integrated into short-term mine production scheduling, which would lead to better-informed mine plans (Dunham et al., 2011; Dowd et al., 2016). Important properties to consider are the rock hardness and grindability, which largely control the throughput in the comminution circuit of the processing plant(s). The integration of a throughput prediction model is crucial for achieving short-term production targets since plant throughput directly influences metal production. Having a detailed modelling of the comminution circuit in production scheduling is also beneficial from a cost and environmental perspective since a substantial portion of a mine’s energy consumption can be attributed to the comminution of the ore in hard rock open pit mines (Norgate and Jahanshahi, 2011). Although many geometallurgical models that predict throughput/comminution performance of the mineral reserve based on ore hardness and grindability have been developed (Flores, 2005; Alruiz et al., 2009; Bueno et al., 2015; Ortiz et al., 2020), they are typically not designed to interact with mine production scheduling. These geometallurgical throughput models typically rely on very sparse measurements of hardness and typically ignore the non-additive nature of hardness when changing the support scale from points (measurement scale) to blocks (mining scale), as well as when materials are blended from various sources within a mining complex. It is important to note that blends of additive rock properties (e.g., metal grades) can be correctly calculated by their (simple or weighted) averages, whereas blends of non-additive properties behave differently than their calculated average. Prediction models that rely on non-additive hardness properties can thus only be accurate in mine settings where no or very limited blending occurs.

Comminution performance of the mineral reserve is conventionally assessed by different hardness and grindability indices. Lynch et al. (2015) review the most used indices to date, including Bond’s Ball Mill Work Index (Wi), the SAG Power Index (SPI), the Drop Weight Index (DWi) and the resistance to impact breakage (A×b). To date, the limited integration of throughput/comminution performance into mine production scheduling involves a spatial model of the mentioned hardness and grindability indices (Coward and Dowd, 2015; Deutsch et al., 2016; Dowd et al., 2016; Morales et al., 2019). The spatial model of geometallurgical properties is built using geostatistical techniques and is commonly referred to as the geometallurgical model. This endeavor is challenging due to the non-additive nature of most hardness and grindability indices (Yan and Eaton, 1994; Amelunxen, 2003; Amelunxen et al., 2014). Specific care has to be taken for spatial interpolation (Deutsch, 2013; van den Boogaart et al., 2013), change of support, including drill-hole compositing and upscaling to mineable units (Garrido et al., 2019; Ortiz et al., 2020), and blending of materials in downstream processes (Newton and Graham, 2011). The latter directly contradicts the common assumptions of mine production scheduling models that use mixed integer linear programming. These models need to assume the additive behaviour of variables to enforce blending constraints and processing capacities. The other major challenge is that information about hardness properties is typically derived from very sparse geometallurgical sampling (Hunt et al., 2013; Deutsch et al., 2016). Part of the reason for sparse sampling is that drilling campaigns and laboratory tests on the samples' comminution behaviour are typically costly and time-consuming (Mwanga et al., 2015).

Several proxy models have been tested for predicting hardness variables as a function of primary rock attributes, which is also termed a primary-response framework (Coward et al., 2009). Applications include multiple linear regression (MLR) (Keeney and Walters, 2011; Boisvert et al., 2013), projection pursuit regression (Sepúlveda et al., 2017), and a variety of supervised learning techniques (Lishchuk et al., 2019). However, these proxy models are typically constructed using samples in drill-core support and thus do not guarantee the correct mapping of larger volumes (Selective Mining Unit, SMU) (Richmond and Shaw, 2009). Furthermore, the calibration of these proxy models is typically performed on an extremely limited number of hardness samples.

In contrast to the above, the use of penetration rates stemming from measurement while drilling (MWD) data is investigated in this article to inform the mineral reserve about hardness and comminution performance. The potential of these spatially dense datasets has already been exploited in mining for enhanced blast design (Segui and Higgins, 2002), the detection of rock fractures (Schunnesson, 1996; Babaei Khorzoughi et al., 2018), rock characterization in various commodities (Schunnesson, 1998; Zhou et al., 2012; Vezhapparambu et al., 2018), and other applications. Although suggested in the technical literature (Segui and Higgins, 2002; Mwanga et al., 2015), MWD has not yet been utilized to create a direct link between the mineral reserve and its comminution performance in milling and grinding circuits. In the approach presented herein, the rate of penetration (ROP) serves as one of the main spatial indicators for ball mill throughput. The utilization of ROP is motivated by the ability to indicate rock-type, strength, and alteration (Vezhapparambu et al., 2018; Park and Kim, 2020).

The next step in a conventional workflow of integrating throughput into mine production scheduling involves empirically derived process models, which are used to calculate the expected mill throughput rates using as input the geometallurgical block model of hardness and grindability indices. Examples of frequently used process models to date are the Bond equation (Bond, 1952), the SMC model (Morrell, 2004a), and the CEET model (Dobby et al., 2001). The mill throughput rates are either calculated per resource block or per geometallurgical domain (Alruiz et al., 2009; Coward and Dowd, 2015). Morales et al. (2019) present a stochastic formulation for long-term single mine production scheduling accounting for uncertain throughput rates through geostatistical simulations. A linear constraint ensures that processed blocks do not exceed the available mill hours per production year. The above efforts towards integrating throughput into mine production scheduling are limited in that the non-additive upscaling behaviour of hardness is ignored, and the resulting single-mine production schedule is informed by assessing the mill throughput and economic value per mining block independently. In this way, the schedule ignores that materials are blended and processed together with other materials, which is typical in multi-pit, multi-processor mining complexes. As a result, the non-additive comminution behaviour of blended materials in the short-term cannot be captured, and the components of the mining complex are not optimized simultaneously.

As an advantage over the conventional mixed integer linear programming models for production scheduling, the simultaneous optimization of mining complexes successfully takes non-linear blending processes in stockpiles and processing facilities into account to maximize NPV (Montiel and Dimitrakopoulos, 2018; Saliba and Dimitrakopoulos, 2019a). Initially developed for long-term planning (Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016), simultaneous optimization of mining complexes capitalizes on synergies between decisions that are conventionally optimized separately. Kumar and Dimitrakopoulos (2019) introduce geometallurgical constraints related to hardness in the simultaneous optimization of mining complexes for long-term planning. The constraints aim to achieve a consistent throughput by sending pre-defined ratios of hard and soft rock to the processing plant(s). The mineral reserve is categorized into hard and soft rock by stochastically simulated Wi and SPI values, where the hardness ratios are chosen arbitrarily and do not guarantee optimal material selection for throughput maximization. Another recent publication (Fathollahzadeh et al., 2021) accounts for Grade Engineering in long-term production scheduling in another attempt to integrate geometallurgical variables in mine planning optimization.

In the present article, an alternative approach is proposed which integrates geometallurgical hardness properties into short-term mine production scheduling in mining complexes. More specifically, a ball mill throughput prediction model informed by blended rock attributes related to hardness is constructed, which is then integrated into the simultaneous stochastic optimization of mining complexes. The utilized datasets for throughput prediction include penetration rates from blast hole drilling and measured throughput rates of the operating ball mill, supplemented by geological domains, material types, and rock density. Furthermore, truck cycle data is used for material tracking. Given that the penetration rates informing rock hardness and the measured throughput rates of the ball mill stem from production data, the novel approach gives a potential financial advantage compared to laboratory grinding tests for throughput prediction. Additionally, the metallurgical response of the processed rock, i.e., throughput, is measured directly in the appropriate support scale, and non-additivity issues of hardness are alleviated using a compositional approach. Section 3.2 presents the newly proposed approach, including the stochastic integer programming formulation in detail. In Section 3.3, the proposed approach is applied at the Tropicana Gold Mining Complex in Western Australia. Conclusions follow.

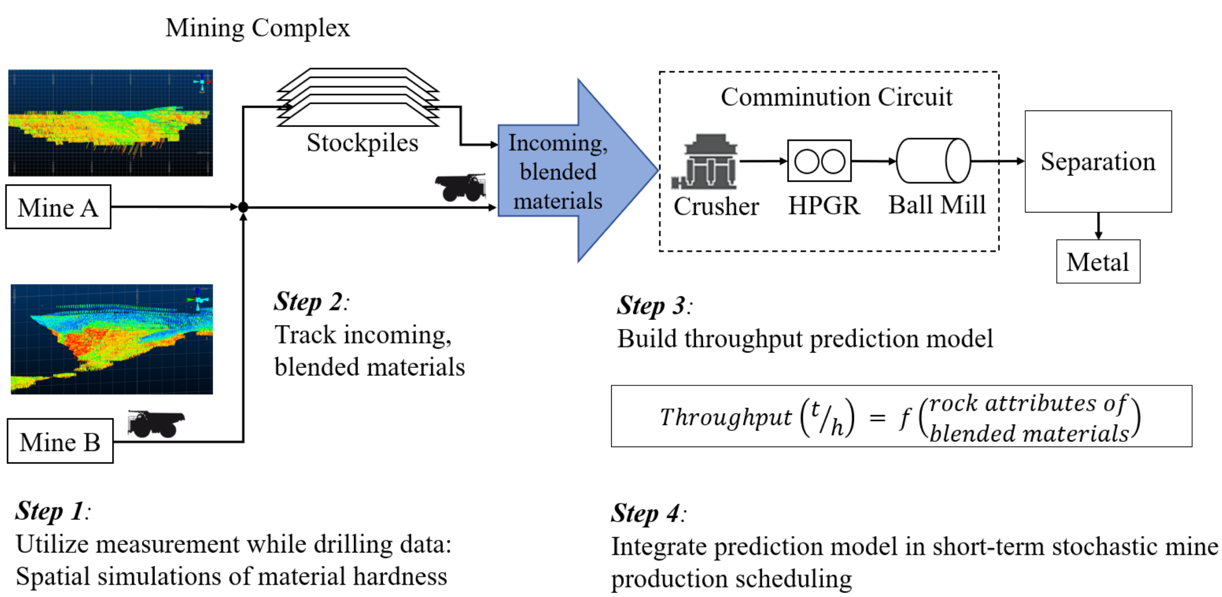


Figure 3-1 An example of a mining complex and steps for the proposed approach for integrating a geometallurgical throughput prediction model into short-term stochastic mine production scheduling

3.2 Proposed Approach

The proposed approach for integrating a geometallurgical throughput prediction model into short-term stochastic mine production scheduling consists of four consecutive steps, which are discussed in detail in this section. Figure 3-1 shows an overview of the required steps for the proposed approach. In the first step, the comminution behaviour of the mineral reserve is geostatistically simulated by building additive hardness proportions using penetration rates from blasthole drilling, which is complemented by additional orebody information such as geological domains, material types, and density. In the second step, a material tracking approach considers all material movements from the various pits to the processing plant, including short-term, run-of-mine stockpiles, to inform the throughput prediction model about properties of blended materials that are sent to the processor. In the third step, measured throughput rates of the operating ball mill are utilized and linked with the previously obtained blended rock characteristics of the processed ore. The link is created through a multiple regression model, which predicts ball mill throughput as a function of rock attributes of blended material. Finally, the proposed throughput prediction model is integrated into short-term production scheduling.

3.2.1 Spatial simulation of material hardness using penetration rates

Measurement while drilling (MWD) data is a centralized collection of mechanical performance indicators obtained from drilling machinery. The monitored performance indicators of the drilling process include, among other entries, rate of penetration, downhole pressure, rotational pressure, rotational speed, air pressure, and vibration. These measurements are routinely collected in operating mines during drilling activities such as exploration, grade control and blast hole drilling. The latter typically comprises dense drilling patterns (up to 6m × 6m). A compositional approach of geostatistically simulating the hardness of the mineral reserve is presented next, consisting of creating point simulations of penetration rates first, and creating hardness proportions thereafter.

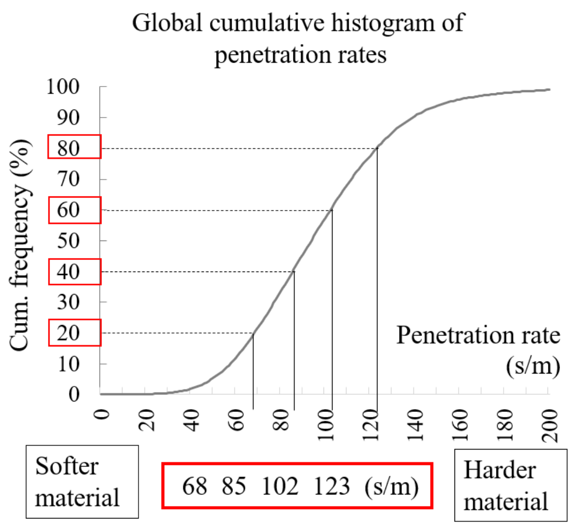
3.2.1.1 Spatial simulation of drilling rate of penetration

The rate of penetration (ROP) is used in this article for the spatial simulation of material hardness. Note that details regarding data cleaning and pre-processing of ROP entries are discussed in the case study presented in Section 3.4. The ROP measurements are converted to the unit $(^{s}/_{m})$ for down-the-hole compositing. This unit is preferred since compositing (averaging) is performed by down-hole length. The unit $(^{s}/_{m})$ is kept for the remainder of the article. The ROP composites form a set of n conditioning data points distributed in space $\{z_{ROP}(u_\alpha), \alpha = 1, \ldots, n\}$. Conventional geostatistical simulation techniques (Goovaerts, 1997) can be used to create a set of equiprobable spatial simulations of ROP, which is performed on a regularly spaced grid (point support).

A challenge arises for the change of support of hardness-related variables from simulated grid nodes (point support) to mineable volumes (SMU). Yan and Eaton (1994) indicate that comminution behaviour is disproportionally affected by harder fractions of material. Thus, a simple averaging of ROP will create biases (Carrasco et al., 2008; Richmond and Shaw, 2009; Deutsch, 2013; Ortiz et al., 2020). An alternative approach is proposed herein, which generates additive geometallurgical variables related to rock hardness and strength. Additivity of variables becomes especially important for material tracking and assessing the properties of blended materials, which is discussed in a subsequent section.

3.2.1.2 Construction of additive hardness proportions

Instead of changing the support scale from grid nodes to mineable volumes, a compositional approach is proposed, which involves the creation of K proportions of softer (easier-to-penetrate) and harder (harder-to-penetrate) material within a larger volume (SMU). Note that ROP for the prediction of throughput and comminution behaviour does not only relate to rock hardness but also to rock strength, as these properties have a clear dependence (Meulenkamp and Grima, 1999; Zhang et al., 2011). Considering all simulated values at the point support scale within a mining block, each proportion represents a fraction of point penetration rates that fall within a specific interval of the simulated ROP. All K proportions naturally sum up to 1, or 100 %, within each mining block. An illustration of hardness proportions is shown in the table on the right in Figure 3-2. The required intervals are delimited by a set of global ROP thresholds derived from a set of percentiles of the global cumulative histogram of ROP, as seen on the left in Figure 3-2. The number of required intervals (percentiles) is discussed in the case study.

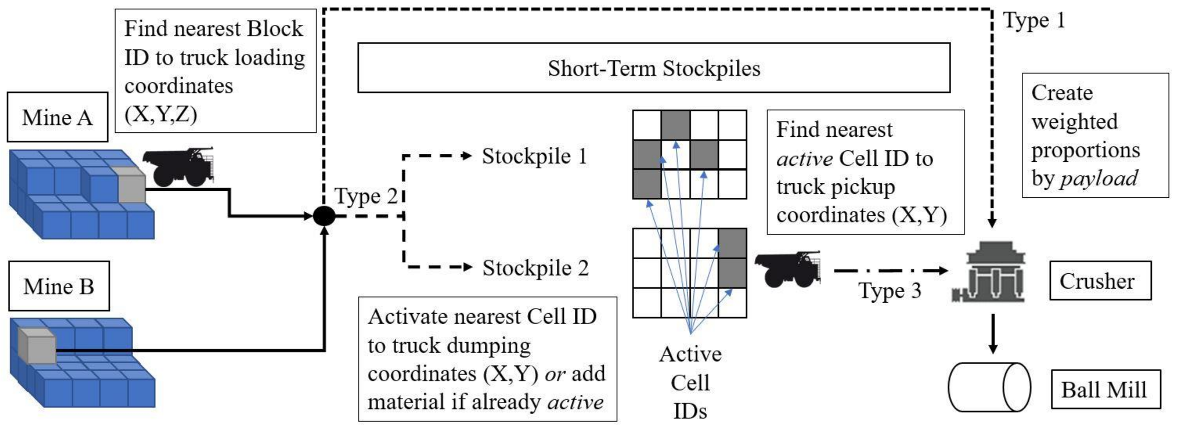


Block ID	< 68	[68, 85)	[85,102)	[102,123)	≥ 123
1	0.20	0.22	0.16	0.22	0.20
2	0.64	0.16	0.14	0.06	0.00
3	0.05	0.05	0.22	0.38	0.31
4	0.44	0.19	0.22	0.11	0.05
5	0.41	0.39	0.13	0.08	0.00
6	0.13	0.13	0.19	0.36	0.20
7	0.59	0.20	0.17	0.03	0.00
8	0.42	0.27	0.17	0.13	0.02
9	0.05	0.14	0.27	0.33	0.22
10	0.28	0.14	0.19	0.25	0.14

Figure 3-2 Construction of penetration rate thresholds (left), used to create hardness proportions per mining block (right)

3.2.2 Tracking blended materials through the mining complex

The main challenge for linking the simulated rock characteristics of the mineral reserve to the observed production data at a processing plant is to keep track of the materials extracted from the pits and sent to the processing facilities, as indicated in the second step of the proposed framework (Figure 3-1). The aim of this step is to characterize the pertinent rock attributes of blended materials entering the comminution circuit so that they can be matched later with observed responses (i.e., throughput) of the ball mill. In addition to the created hardness proportions, other orebody attributes, such as density, geological domains, and material types of the extracted blocks, are tracked as well. The material tracking approach is schematically shown in Figure 3-3. The design accounts for multiple mines and includes all short-term stockpiling that occurs before materials enter the processing facilities.



Block ID	< 68	[68, 85)	[85,102)
1	0.20	0.22	0.16
2	0.64	0.16	0.14
3	0.05	0.05	0.22

Hardness proportions are *additive*

< 68	[68, 85)	[85,102)	Year Day
0.54	0.22	0.14	58
0.51	0.22	0.16	59
0.45	0.22	0.19	60

Figure 3-3 Schematic representation of material tracking of hardness proportions through the mining complex, including short-term stockpiles

The material movements in the various pits and stockpiles are replicated in detail using truck cycle data. The necessary information includes the start and end times of truck cycles, as well as spatial coordinates of truck loading and dumping locations. Three types of truck cycles are relevant for material tracking from the mines to the processing facilities: (1) ore is hauled directly from one of the pits to the crusher; (2) ore is hauled from one of the pits to one of the short-term stockpiles; (3) ore is reclaimed from a short-term stockpile and transported to the crusher. In the presented material tracking scheme (Figure 3-3), all truck cycles starting in the mine (Type 1 and 2) are first linked to their nearest Block ID of the simulated resource model (hardness proportions, rock type, density) using a nearest-neighbor search. Cycles that end at the crusher location (Type 1 and 3) are utilized for calculating the properties of incoming, blended materials.

While Type 1 cycles are easy to trace back to the resource model, a more elaborate tracing scheme is constructed for stockpiled material (Figure 3-3). Other than blending beds or homogenization facilities (Kumral, 2006), short-term stockpiles (run-of-mine pads or fingers) may not have a fixed build/remove scheme, and their position and dimensions change over time. Thus, an initial grid of cells (10m × 10m) is constructed, one for every stockpile, covering the area of potential material placement. When a truck dumps material on one stockpile (Type 2), the closest cell is determined by a nearest neighbor search using its dump coordinates. If this cell has been active before (previously dumped material), the associated Block ID of the truck cycle is added to a list. Otherwise, the cell is activated, and the associated Block ID of the truck cycle becomes its first entry. When material is hauled from a stockpile to the crusher (Type 3), the closest active Cell ID is found using truck loading coordinates. The respective list of Block IDs informs about all material that has been deposited there for calculation of blends (one-to-many relationship, detailed in Wambeke et al. 2018). An action of deleting all active cells in an individual stockpile is triggered after a stockpile is depleted.

Weighted proportions of all tracked properties are constructed by truck payload (weighting factor) from all Type 1 and Type 3 cycles that are recorded in a specific interval arriving at the crusher. The time interval for material tracking can be as small as a few minutes or can comprise several days, constrained by the minimum truck dumping frequency recorded in the fleet database.

3.2.3 Building a throughput prediction model

A multiple linear regression model is constructed to predict ball mill throughput (response variable) as a function of rock properties of blended materials (predictor variables or features), which have been obtained previously in the material tracking step. In the regression model, the i^{th} observed throughput, y_i, in a sample of N observations, is expressed as a linear combination of M predictor variables of the tracked rock properties, $f_{i,j}$, as shown in Eq. (3.1).

$$Throughput_i \left({}^{t}/_{h}\right) = y_i = w_0 + \sum_{j=1}^{M} w_j \cdot f_{i,j} + e_i \tag{3.1}$$

Note that w_j, $j = 0,1, \dots, M$, are the regression coefficients, and e_i is a random variable representing the prediction error. The observed throughput rates, y_i, are average observed ball mill throughput rates (milled ore tonnage per operating hour), collected for the same time intervals as the tracked rock properties, $f_{i,j}$. The goal of regression is to find the vector of weights, $w = [w_0, w_1, \dots, w_M]$, that minimize the sum of squared errors (SSE),

$$SSE(w) = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} (w^T f_i - y_i)^2 \tag{3.2}$$

where $f_i = [1, f_{i1}, \dots, f_{iM}]$ is a vector of predictor variables for the i^{th} observation. The closed-form solution of the minimization of SSE with respect to the vector of weights, written in matrix form, is

$$w = (X^T X)^{-1} X^T y \tag{3.3}$$

(Everett and Howard, 2011; Rencher and Christensen, 2012), where $X \in \mathbb{R}^N \times \mathbb{R}^{M+1}$ is the feature matrix and $y \in \mathbb{R}^N$ is the vector of all observations. The resulting weights, $w^* = [w_0^*, \dots, w_M^*]$, are used as input for production scheduling. A discussion of feature selection and regression results, including an analysis of correlation to ball mill throughput, are presented in Section 3.

3.2.4 Integration of throughput prediction model into stochastic short-term mine production scheduling

The mathematical model which integrates the throughput prediction model into the simultaneous stochastic short-term optimization of mining complexes is presented next. The stochastic optimization model is formulated as a non-linear two-stage stochastic mixed integer program with recourse (Birge and Louveaux, 2011), and builds upon the long-term simultaneous stochastic optimization of mining complexes (Goodfellow and Dimitrakopoulos, 2016, 2017). A simulated annealing metaheuristic is used to solve the non-linear optimization model. Simulated annealing is chosen due to its successful utilization for open pit mine production scheduling for single open pit mines (Kumral and Dowd, 2005; Kumral, 2013; Mousavi et al., 2016b) and mining complexes (Goodfellow and Dimitrakopoulos, 2017; Montiel and Dimitrakopoulos, 2018). The typical runtimes documented for the simulated annealing algorithm for long-term production scheduling in mining complexes with more than 100,000 blocks to be scheduled (Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016) are reduced significantly for short-term production scheduling (typically 1000s of blocks), staying below two hours runtime to reach a solution. Details of the adapted simulated annealing algorithm and parameter selection for short-term planning can be seen in Both and Dimitrakopoulos (2020). All new parts related to the integration of the developed ball mill throughput prediction model for short-term planning are discussed in the following. The utilized indices and sets of the stochastic optimization model are listed in Table 3-1, whereas the used parameters and variables are presented in

Table 3-2 and Table 3-3, respectively.

Table 3-1 Indices and sets used in the stochastic optimization model

Index	Description
$t \in \mathbb{T}$	Index of a time period for the short-term planning horizon $\mathbb{T}$
$s \in \mathbb{S}$	Index of an orebody scenario in the set of orebody scenarios $\mathbb{S}$
$l \in \mathbb{L}$	Index of a location l in the set of all locations $\mathbb{L}$ in a mining complex, which includes the subsets of Mining Areas ($\mathcal{A} \subset \mathbb{L}$), Processing facilities ($\mathcal{P} \subset \mathbb{L}$), Stockpiles ($\mathcal{S} \subset \mathbb{L}$) and Waste Dumps ($\mathcal{W} \subset \mathbb{L}$)
$j \in \mathbb{J}$	Index of an attribute j in the set of all attributes $\mathbb{J}$ in a mining complex, which includes the subsets of throughput-related attributes ($\mathbb{H}_{TPH} \subset \mathbb{J}$), metal-related attributes ($\mathbb{H}_M \subset \mathbb{J}$) and tonnage-related attributes ($\mathbb{H}_T \subset \mathbb{J}$)
$b \in \mathbb{B}$	Index of a mining block in the set of all blocks $\mathbb{B}$
$g \in \mathbb{G}$	Index of a group g of material in the set of all material groups $\mathbb{G}$

Table 3-2 Parameters used in the stochastic optimization model

Parameter	Description
$p_{j,l}$	The unit price for attribute j in location l, which can be positive (revenue) or negative (cost)
${c_{j,l}}^{+}, {c_{j,l}}^{-}$	Penalty cost of positively (+) or negatively (-) deviating from the attributes $j \in \mathbb{H}_M \cup \mathbb{H}_T$ at location l
${c_l}^{TPH+}, {c_l}^{TPH-}$	Penalty cost of positively (+) or negatively (-) deviating from the throughput that can be realized in the location l
c_{smooth}	Penalty cost for unconnected blocks, enforcing the extraction of blocks in a connected pattern
$\beta_{b,j,s}$	Value of attribute j for block b in orebody scenario s, including metal content, density, hardness proportions (fractional), geological domain (binary), and weathering type (binary)
$\theta_{g,b,s}$	Parameter indicating that block b belongs to material group g in orebody scenario s
Ton_b	Tonnage of block b
$Ton_{l',t-1}$	Tonnage of stockpile $l' \in \mathcal{S}$ in the previous period $(t-1)$

Table 3-3 Variables used in the stochastic optimization model

Variable	Description
$d^{+}_{j,l,s,t}$, $d^{-}_{j,l,s,t}$	Continuous variables modelling either surplus (+) or shortage (-) of attribute $j \in \mathbb{H}_M \cup \mathbb{H}_T$ at location *l* in period *t* for orebody scenario *s*
$d^{TPH+}_{l,s,t}$, $d^{TPH-}_{l,s,t}$	Continuous variables modelling positive (+) and negative (-) deviations from the throughput that can be realized in the location *l* in period *t* for orebody scenario *s*
$v_{j,l,s,t}$	Continuous variable representing the quantity of an attribute *j* in location *l* for scenario *s* in period *t*
$x_{b,t}$	Binary variable which equals 1, if block *b* is mined in period *t*, 0 otherwise
$y_{b,t}$	Integer smoothing variable which reflects the number of adjacent blocks of block *b* that are not extracted in the same period *t*
$z_{g,l,t}$	Binary variable which equals 1 if a group of material *g* is sent to location *l* in period *t*, 0 otherwise
$\delta_{l,l',t,s}$	Continuous variable [0,1] which represents the proportion of material sent out from location *l* to a receiving destination *l'* in period *t* for orebody scenario *s*

The various parts of the objective function shown in Eq. (3.4) aim to maximize the profit generated in the mining complex within the short-term planning horizon while simultaneously minimizing the risk of not meeting short-term production targets.

$$\text{Maximize} \quad \underbrace{\frac{1}{\mathbb{S}}\sum_{j\in\mathbb{J}}\sum_{l\in\mathbb{L}}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}} p_{j,l}\cdot v_{j,l,s,t}}_{Part\ 1}$$

$$-\underbrace{\frac{1}{\mathbb{S}}\sum_{j\in\mathbb{J}}\sum_{l\in\mathbb{L}}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\left(c_{j,l}{}^{+}\cdot d^{+}_{j,l,s,t} + c_{j,l}{}^{-}\cdot d^{-}_{j,l,s,t}\right)}_{Part\ 2}$$

$$-\underbrace{\frac{1}{\mathbb{S}}\sum_{l\in\mathbb{L}}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\left(c_{l}{}^{TPH+}\cdot d^{TPH+}_{l,s,t} + c_{l}{}^{TPH-}\cdot d^{TPH-}_{l,s,t}\right)}_{Part\ 3}$$

$$-\underbrace{\sum_{t\in\mathbb{T}}\sum_{b\in\mathbb{B}} c_{smooth}\cdot y_{b,t}}_{Part\ 4} \tag{3.4}$$

Part 1 of the objective function sums all revenues and costs generated within the mining complex. Part 2 is a stochastic component that penalizes deviations from production targets given orebody uncertainty. Both parts are well-established building blocks of simultaneous stochastic optimization of mining complexes, both in long-term (Goodfellow and Dimitrakopoulos, 2016) and short-term planning (Villalba Matamoros and Dimitrakopoulos, 2016; Both and Dimitrakopoulos, 2020). Part 3 is a new component introduced in this article and penalizes deviations from the mill tonnage that can be processed based on the throughput that is calculated using the proposed prediction model. Part 4 ensures that the resulting production schedule is mined in a connected (smooth) pattern. Details of the new constraints related to ball mill throughput are given below. Other included constraints are mine capacity constraints, blending constraints, reserve constraints, slope constraints, destination constraints, and material flow constraints, which are detailed in Goodfellow and Dimitrakopoulos (2016, 2017). Note that the reserve constraints in this formulation enforce all blocks to be extracted by the end of the short-term planning horizon.

Complementing Part 3 of the objective function, new components are added to integrate the developed throughput prediction model into the simultaneous optimization of mining complexes. Consider the periods t for a short-term horizon $\mathbb{T}$. The resulting short-term production schedule is informed by a set of orebody simulations, $\mathbb{S}$, containing attributes that are used for throughput evaluation, $j \in \mathbb{H}_{TPH}$, including hardness proportions and other attributes important for scheduling, such as metal grades. All throughput-related attributes of blended materials sent to the processing location, $l \in \mathcal{P}$, in period t, and orebody scenario s, are represented by $v_{j,l,s,t}$, $j \in \mathbb{H}_{TPH}$. This variable is calculated as a weighted average of block attributes, $\beta_{b,j,s}$, extracted in *t* by their respective tonnage, Ton_b, and sent to the processor *l*, shown in Eq. (3.5). The formulation also considers material attributes sent from locations other than mines, $l' \in \mathcal{S}$, i.e., stockpiles and external sources, to the processor *l*.

$$v_{j,l,s,t} = \frac{\sum_{g \in \mathbb{G}} \sum_{b \in B} x_{b,t} \cdot \beta_{b,j,s} \cdot Ton_b \cdot \theta_{g,b,s} \cdot z_{g,l,t} + \sum_{l' \in \mathcal{S}} v_{j,l',s,t-1} \cdot Ton_{l',t-1} \cdot \delta_{l',l,t,s}}{\sum_{b \in B} x_{b,t} \cdot Ton_b + \sum_{l' \in \mathcal{S}} Ton_{l',t-1} \cdot \delta_{l',l,t,s}}$$

$$\forall\, t \in \mathbb{T}, s \in \mathbb{S}, l \in \mathcal{P}, j \in \mathbb{H}_{TPH} \tag{3.5}$$

Note that the calculation of $v_{j,l,s,t}$ in Eq. (5) is non-linear, since it represents the attributes *j* of the blended material stream sent to the processor *l* in a short-term period *t* under orebody scenario *s*, such as the grade or density of the blend, the proportions of harder and softer material sent to the processing plant, the fraction of weathered materials etc. The throughput calculation for a processing location for any given period t and orebody scenario s at location l is shown in Eq. (3.6).

$$Throughput_{l,s,t}\left({}^{t}/_{h}\right) = w_0^* + \sum_{j \in \mathbb{H}_{TPH}} w_j^* \cdot v_{j,l,s,t} \;\; \forall\, t \in \mathbb{T}, s \in \mathbb{S}, l \in \mathcal{P} \tag{3.6}$$

Throughput rates are calculated utilizing the optimized weights, w^*, obtained in Eq. (3.3). Specifically, the prediction model is queried at every iteration of the metaheuristic solution method, using the current production schedule. The throughput prediction obtained in Eq. (3.6) is used to assess the tonnage that can be processed by the ball mill, as shown in Eq. (3.7).

$$Mill\ Tonnage_{l,s,t}^{processed} = Throughput_{l,s,t} * h_l^{avail} \;\; \forall\, t \in T, s \in S, l \in \mathcal{P} \tag{3.7}$$

Available hours of the ball mill, h_l^{avail}, are obtained by the use of an empirical availability factor, representing scheduled and unscheduled downtime of the comminution circuit. As the last component, a stochastic constraint is added to the mathematical model, as shown in Eq. (3.8).

$$Mill\ Tonnage_{l,s,t}^{processed} - Mill\ Tonnage_{l,s,t}^{scheduled} - d_{l,s,t}^{TPH+} + d_{l,s,t}^{TPH-} = 0$$
$$\forall\, t \in T, s \in S, l \in \mathcal{P} \tag{3.8}$$

This constraint penalizes a deviation between the scheduled tonnage sent to the processing unit and the calculated tonnage in Eq. (3.7). The latter tonnage represents the ore tonnage that can actually be processed by processing location l, which is now based on pertinent rock attributes related to hardness.

The above approach of modelling mill constraints stands in contrast to conventional tonnage constraints which do not take the hardness properties of blended materials into account. The deviation variables $d_{l,s,t}^{TPH+}$ and $d_{l,s,t}^{TPH-}$ are penalized in Part 3 the objective function (Eq. (3.4)), similar to previously developed stochastic constraints for stochastic mine planning (Ramazan and Dimitrakopoulos, 2005, 2013).

3.3 A Case Study at the Tropicana Gold Mining Complex

In this section, the proposed approach of integrating a ball mill throughput prediction model into simultaneous stochastic short-term production scheduling is applied at the open pit Tropicana Gold Mining Complex, which is located 330 km East-North-East of Kalgoorlie in Western Australia. The gold mining complex comprises four contiguous pits extending six kilometers in strike length from North to South. Ore is sent to a single processing facility, consisting of a comminution circuit and a carbon in leach (CIL) facility. The comminution circuit comprises a primary crusher, a high-pressure grinding roll (HPGR), and a ball mill. In this case study, the material to be extracted on the short-term horizon of one year is mined from two of the four pits located in distinct mining areas within each pit. The first six months of production data (blast hole drilling, truck cycle data, and measured ball mill throughput rates) are used to inform the throughput prediction model, whereas three months out of the latter half will be used for production scheduling. The gold mining complex is operated as a typical truck and shovel open pit operation. After fragmentation through conventional drilling and blasting, ore is hauled by truck directly to the crusher or to one of eight short-term, run-of-mine (ROM) stockpiles located in the vicinity of the primary crusher. Temporarily stockpiled material amounts to 80-90 % of processed ore; thus, it is crucial to include material of ROM stockpiles in material tracking, as shown in the case study (Section 3.2).

3.3.1 Hardness proportions

The raw, recorded penetration rates used to create hardness proportions stem from MWD data collected by all drilling activities (blast hole, grade control, pre-split drilling) routinely performed in the mining complex. Figure 3-4 presents cross-sections of the recorded rate of penetration (ROP), shown in seconds per drilled meter (inverse penetration rate, Figure 3-4a), and drilled meters per hour (Figure 3-4b), which can be visually compared to the geological domains of the orebody (Figure 3-4c) and the weathering profile/regolith/material types of the deposit (Figure 3-4d). Large-scale geological structures, such as dipping geological units displayed in Figure 3-4c, can be clearly distinguished by consistent differences in ROP. Furthermore, the five stratigraphic regolith units or material types of the orebody (weathering profile, Figure 3-4d) are strongly reflected in ROP, indicating weathered, easier-to-penetrate rock towards the surface (blue colors in Figure 3-4a, red colors in Figure 3-4b). Also, the harder-to-penetrate, fresh rock, situated in greater depth, is easily detectable by penetration rates (red colors in Figure 3-4a, blue-green colors in Figure 3-4b).

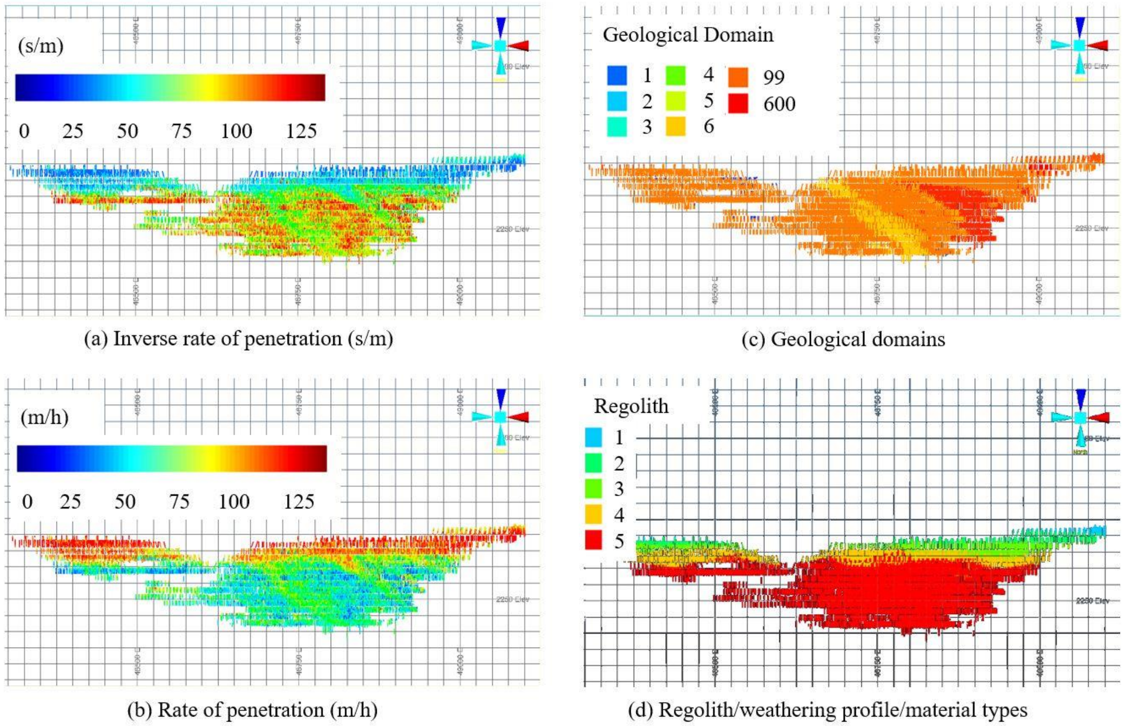


Figure 3-4 Cross sections (E-W) of penetration rates (a, b) retrieved from measurement while drilling data, compared to cross sections (E-W) of geological domains (c) and weathering profile (d) of the orebody

It is important to note that biases in this database may exist due to several factors, including the utilization of different drill rig types, multiple rig operators, and varying drilling tasks. Also, the drill-bit wear should to be accounted for, which gradually affects penetration rates during the drill-bit life (Rafezi and Hassani, 2021). These sources of noise contribute to data entries that are less correlated to properties of the penetrated rock but rather relate to operational procedures. Many normalization procedures have been proposed in the technical literature (Teale, 1965; Schunnesson, 1998; Zhou et al., 2012). However, these normalization procedures need to be applied carefully. Internally created variables of the MWD system in the present dataset, such as the Blastability Index (BI) (Segui and Higgins, 2002), were found to be highly biased towards individual drilling machines. Furthermore, the application of the adjusted penetration rate (APR) (Zhou et al. 2012) resulted in new, machine-related biases and reduced geological correlation compared to ROP. This shows that site-specific data handling is required. The following targeted outlier removals of ROP data entries were applied on the present dataset, proving to be effective in this case study:

- Production drilling was utilized exclusively (blast holes), removing pre-split and grade control drilling from the dataset. The reason for this is that blast hole drilling is conducted solely with drill rigs of the same type in this mining complex.
- Due to pre-fragmented rock from previous blasting activities (easier to penetrate, resulting in higher ROP), the first 2m of every hole were discarded.
- Data outliers due to some irregular events (enlarging the drill rod, flushing the hole, etc.) were identified and removed.

3.3.2 Material tracking in the mining complex

A fleet management system installed in the mining complex records each individual truck cycle in a central database and can therefore be used to replicate material movements in the whole mining complex, as described in Section 2.2. Figure 3-5 visualizes how material tracking in short-term stockpiles (Run-of-Mine Fingers) is performed at the mining complex.

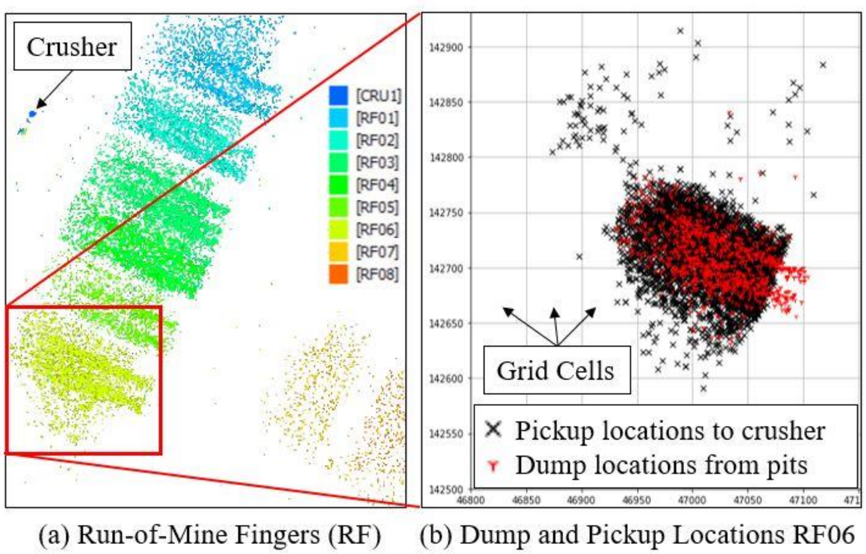


Figure 3-5 Top view of (a) material tracking in short-term stockpiles (Run-of-Mine Fingers), and (b) a close-up of a single stockpile (RF06) (right)

The left-hand side in Figure 3-5 shows a top view of all ROM Fingers (stockpiles) located in the vicinity of the primary crusher. A close-up of the sixth stockpile (RF06) shows all dumping events of stockpiled material coming from pits (red markers) and reclaimed material sent from the stockpile to the crusher (black markers). Furthermore, the created grid cells are indicated, which are used to spatially inform and distinguish material within the stockpiles depending on where it has been deposited.

3.3.3 Multiple regression implementation and parameter testing

The material tracking is conducted in daily time intervals for a period of one year. As part of data preparation for multiple regression, a moving average is calculated for both blended rock properties (predictor variables) and observed ball mill throughput (response variable). In this way, similar time intervals are preserved for inputs and outputs of the regression. Figure 3-6 shows the application of a seven-day moving average on daily average ball mill throughput observed at the mining complex.

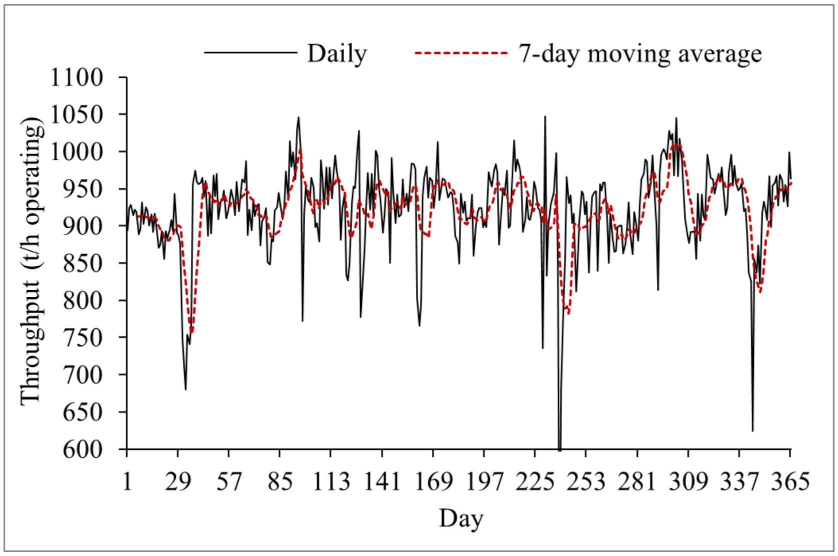


Figure 3-6 Observed ball mill throughput showing daily average(blue) and 7-day moving average (red)

The moving average is applied because the daily throughput rates observed at the mine show very noisy behaviour (blue line). It can be seen in Figure 3-6 that a seven-day moving average helps emphasize trends of higher and lower throughput rates over longer periods, which are more likely connected to rock properties of the processed material. At the same time, the influence of technical disruptions and short-term delays of ore feed is reduced. Additionally, an important reason for applying a seven-day moving average is that the prediction model is aimed to predict a ball mill throughput for production scheduling comprising weekly periods. By applying the seven-day moving average, data fits the time scale on which ball mill throughput predictions will be used for in production scheduling. From the available 365 days, a 180-day window is retained for regression analysis, discarding data entries at the beginning and end of the observed interval. This ensures that (i) material deposited in the short-term stockpiles is known, and (ii) material in later time intervals is retained so that it can be used for production scheduling. Correlations between rock attributes of blended materials and observed ball mill throughput are shown in Table 4-2 using Pearson's correlation coefficient shown in Eq. (3.9),

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2} * \sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{3.9}$$

with x_i and y_i individual sample points and $\bar{x}, \bar{y}$ indicating the respective sample means.

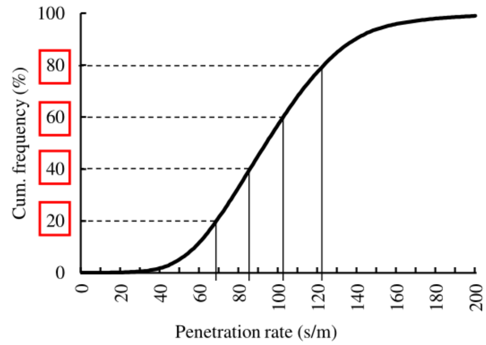
An important observation of Pearson's correlation coefficients shown in Table 4-2 is that the strongest linear relationships exist between the constructed hardness proportions (A1 - A10) and ball mill throughput. The softer hardness proportions (easier-to-penetrate, A8 - A10) correlate positively with ball mill throughput, indicating that the throughput increases when more of this softer material is processed in the ball mill. On the other hand, increased proportions of harder-to-penetrate materials lead to a decreased ball mill throughput, indicated by a negative correlation coefficient (A1 - A6). The consistency of positive and negative correlation among hardness proportions supports the hypothesis that penetration rates recorded by drilling machines have the capacity to predict comminution performance/grindability of the rock to a certain degree. Other correlation coefficients in Table 4-2 indicate that the originating geological orebody domains (B1 - B4) and average material density (D1) have less influence on ball mill throughput in this case study. However, the different degrees of weathering/regolith/material types (C1 - C2) show a stronger correlation to throughput, whereas higher proportions of weathered material are positively correlated with ball mill throughput. This result is not surprising since higher degrees of weathering can cause degradation in rock competency, leading to better grindability in the comminution circuit (Bhuiyan et al., 2019). However, there is still a large part of the variability in the throughput rate that is not explained by the attributes considered. Other factors such as operating conditions of the processing plant should be considered as well to improve throughput prediction, which is addressed in Chapter 4.

Table 3-4 Pearson's correlation coefficient between rock attributes of blended materials in the comminution circuit and observed ball mill throughput

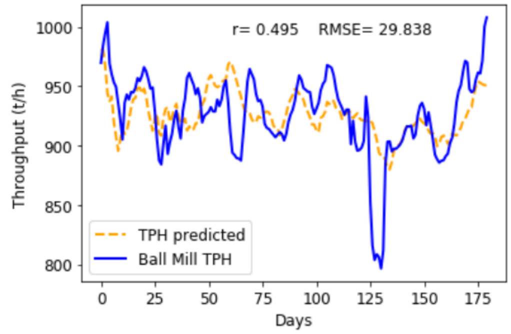
No.	Category	Feature	Unit	Pearson's correlation coefficient to ball mill throughput
A1	(Harder material)	> 139 s/m	(%)	-0.274
A2		123 - 139 s/m	(%)	-0.370
A3		112 - 123 s/m	(%)	-0.525
A4	Material hardness using proportions of penetration rate categories	102 - 112 s/m	(%)	-0.361
A5		94 - 102 s/m	(%)	-0.416
A6		85 - 94 s/m	(%)	-0.318
A7		78 - 85 s/m	(%)	0.028
A8		68 - 78 s/m	(%)	0.356
A9		58 - 68 s/m	(%)	0.386
A10	(Softer material)	< 58 s/m	(%)	0.498
B1	Proportions of geological domains	Fraction of domain 2	(%)	0.113
B2		Fraction of domain 6	(%)	-0.074
B3		Fraction of domain 600	(%)	0.161
B4		Fraction of domain 99	(%)	0.046
C1	Weathering proportions	Fraction of weathered rock	(%)	0.341
C2		Fraction of fresh rock	(%)	-0.341
D1	Density	Average material density	(t/m^3)	-0.086

3.3.3.1 Number of hardness proportions

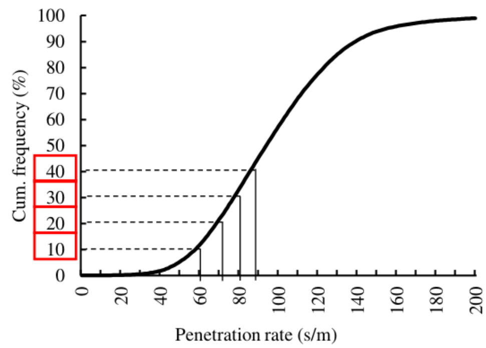
The construction of hardness proportions requires a choice of the number of thresholds and their distribution of percentiles, which can be chosen to be evenly or unevenly distributed. Figure 3-7 shows variations of splitting penetration rates into different hardness proportions and their effect on the multiple regression using only the constructed proportions as predictor variables (features). To avoid multi-collinearity issues, redundant features were removed from the dataset including one hardness, domain, and weathering proportion. Leave-one-out cross-validation is applied (Hastie et al., 2009), which consecutively builds a regression model using all data points but one, then predicts the value of the left-out data point. This process is repeated for every data point.



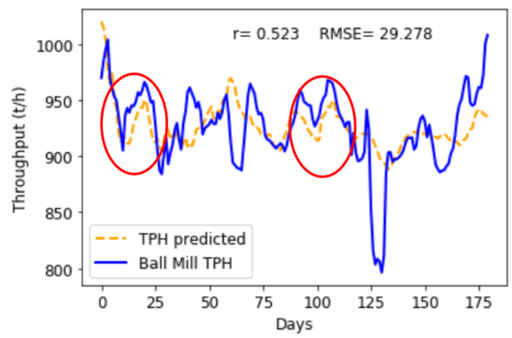
(a) Cumulative histogram of 5 hardness proportions (even split)



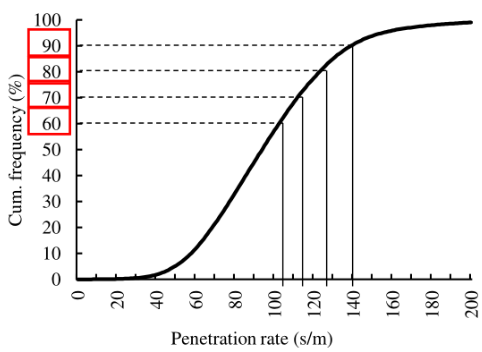
(b) Multiple regression of 5 hardness proportions (even split)



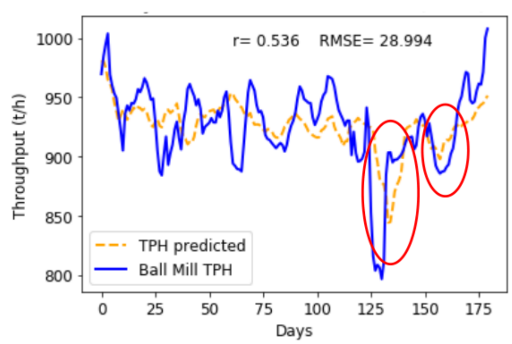
(c) Cumulative histogram of 5 hardness proportions (soft tail)



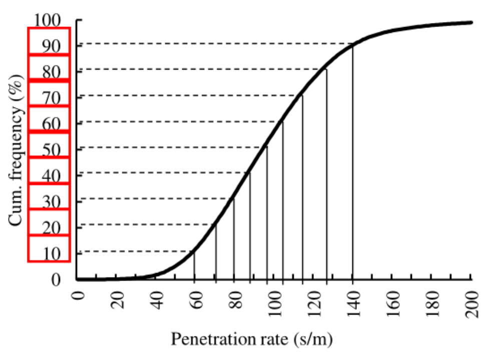
(d) Multiple regression of 5 hardness proportions (soft tail)



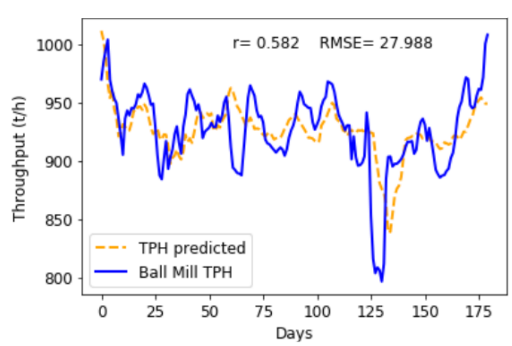
(e) Cumulative histogram of 5 hardness proportions (hard tail)



(f) Multiple regression of 5 hardness proportions (hard tail)



(g) Cumulative histogram of 10 hardness proportions (even split)



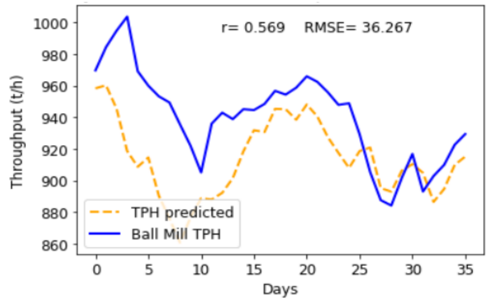
(h) Multiple regression of 10 hardness proportions (even split)

Figure 3-7 Regression analysis testing multiple splits of hardness proportions

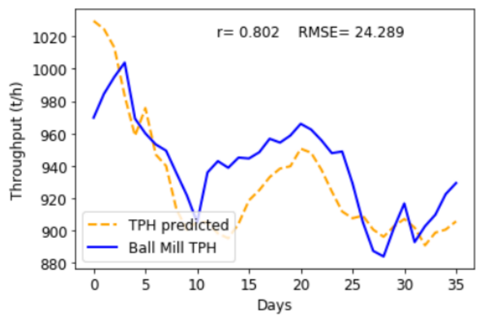
A first result of regression analysis in Figure 3-7 is that hardness proportions enable a prediction of ball mill throughput (orange line) with a root mean squared error (RMSE) of less than 30 t/h. Additionally, it can be observed that splitting the cumulative histogram of penetration rates differently (even split, soft tail, hard tail) leads to different prediction behaviour. The split of softer material appears to predict medium/high observed throughput rates better, whereas the split of harder material clearly predicts lower throughput rates better. Note that average-type approaches of hardness relying on single-valued hardness indicators such as Wi and SPI cannot capture these observed differences, which are driven by varying proportions of harder and softer rock. When comparing the five hardness and ten hardness proportions, splitting into more categories appears to have lower RMSE.

3.3.3.2 Generalization potential of various features

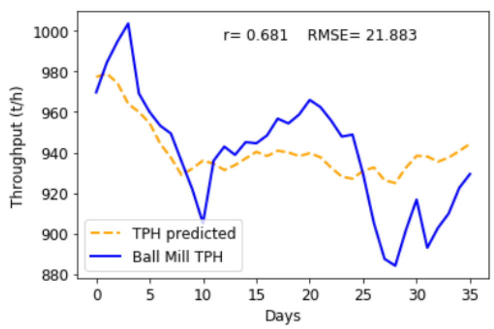
The material density, geological orebody domains, and degree of weathering are also tracked next to hardness proportions, as listed in Table 4-2. A comparison of their potential to predict ball mill throughput in the present case study is shown in Figure 3-8. Here, the various regression models are built on a data set comprising 80 % of data points. The models are then tested on the remaining 20 % of data points, revealing the generalization potential to unseen data of the tested feature sets. Note that testing on a consecutive time series of throughput data is more informative than a randomly picked test set since random test samples can be very similar to neighboring training sample points. Also, note that the y-axis changes the scale for some graphs in Figure 3-8 depending on the spread of throughput predictions.



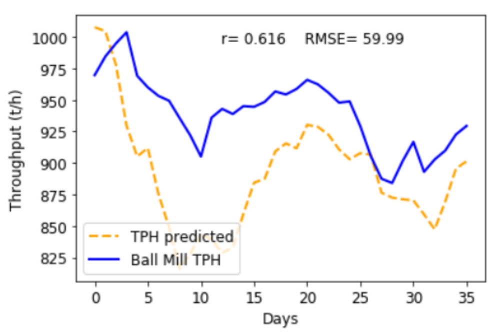
(a) 5 hardness proportions (even split)



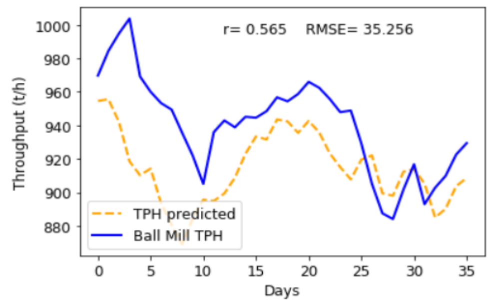
(b) 5 hardness proportions (soft tail)



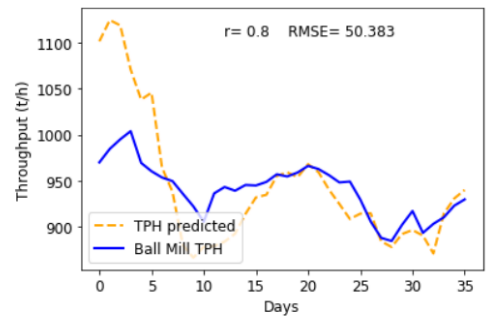
(c) 5 hardness proportions (hard tail)



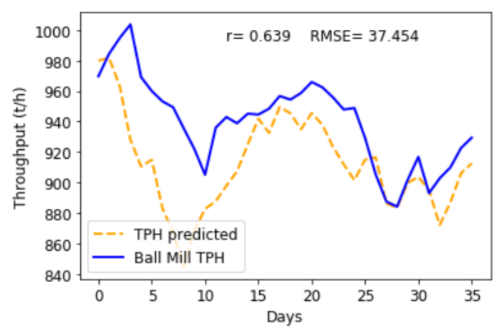
(d) 10 hardness proportions



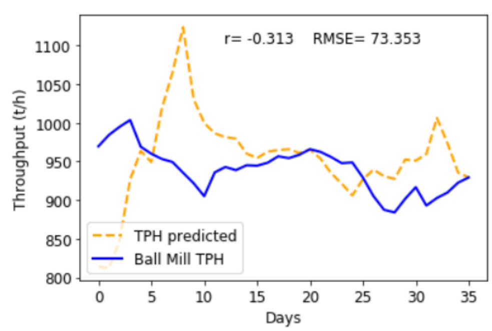
(e) 5 hardness proportions (even split) - density



(f) 5 hardness proportions (even split) - regolith



(g) 5 hardness proportions (even split) - domains



(h) 5 hardness proportions (even split) - 2nd order pol.

Figure 3-8 Generalization potential various of rock attributes testing on 20% unseen test data

The approach presented in this article is general and allows the utilization of any rock attributes contributing to more accurate throughput prediction. This can also potentially include head grades of metals and other (deleterious) elements fed to the ball mill next to the geological attributes used herein. In the present case study, the most robust predictions are obtained from the utilization of five hardness proportions, as seen in Figure 3-8. This result forms the basis for the attributes used for short-term production scheduling, presented next.

3.4 Short-term Production Scheduling

For weekly short-term production scheduling of the described mining complex, three months of planned production are used as input to optimization (3874 mining blocks, 12m × 12m × 12m). Various sets of geostatistical orebody simulations serve as input for production scheduling, displayed for Mining Area A in Figure 3-9. Uncertainty of metal grade (Au) is accounted for by twenty equally probable orebody scenarios. The required number of orebody scenarios for mine production scheduling has been examined by previous authors, showing that about 15 simulations provide proper stable schedules and forecasts (Albor Consuegra and Dimitrakopoulos, 2009; Montiel and Dimitrakopoulos, 2017). Note that each of the five hardness proportions forms an individual attribute for production scheduling and is represented by twenty orebody simulations each, as indicated in Figure 3-9.

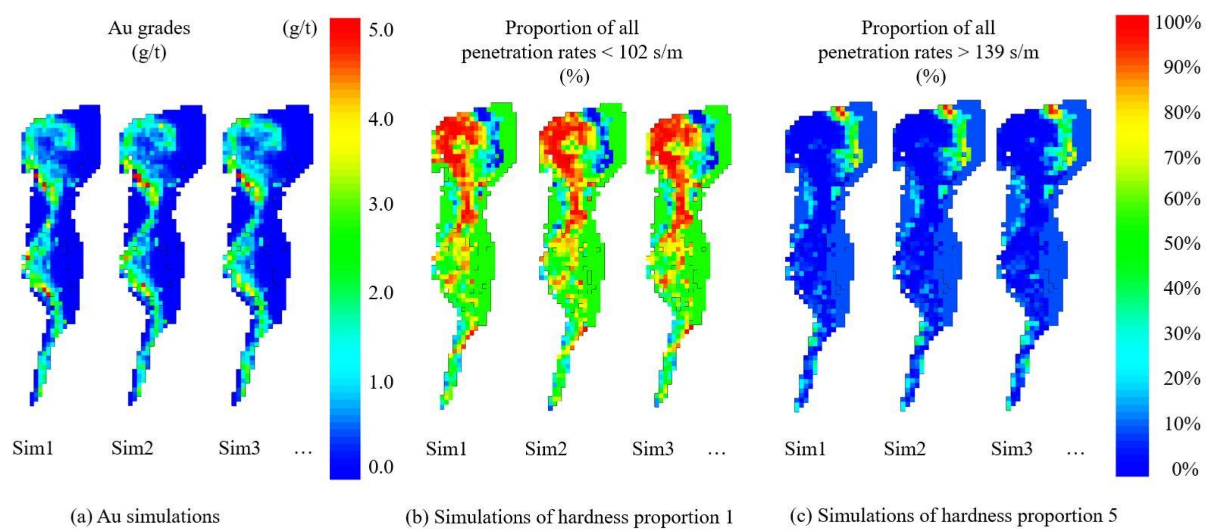
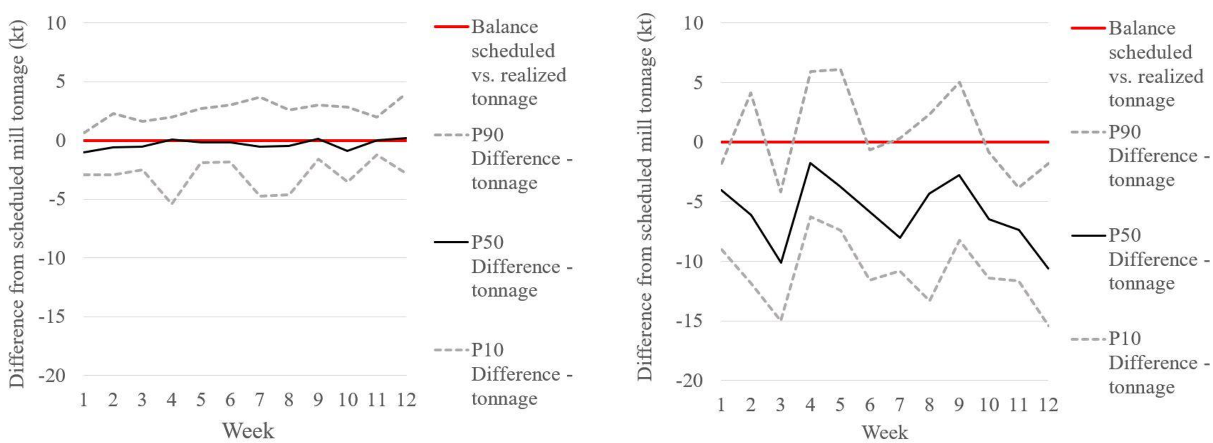


Figure 3-9 Orebody simulations of Mining Area A serving as input for short-term production scheduling using an integrated throughput prediction model

A 12-week short-term production schedule is generated for the mining complex with the stochastic optimization model described in Section 3.2.4. The simulated annealing algorithm to solve the model is written in C++ programming language and is run on a computer with an Intel Core i5-7200U CPU @ 2.50 GHz processor and 8 GB RAM, using Windows 10 Home as operating system. The algorithm is run for 200,000 iterations with a running time of 1.7 hours. Figure 3-10a shows a risk analysis of the tonnage that is expected to be processed in the ball mill. More specifically, Figure 3-10a shows the difference between the scheduled ore tonnage (blocks scheduled for production) and the tonnage that can be processed by the ball mill using the relevant geometallurgical attributes and the integrated throughput prediction model. The new stochastic constraints used in this optimization model (Eq. (3.8)) are directly linked to the objective function, as explained earlier.



(a) Risk analysis of mill tonnage (all new components included)

(b) Risk analysis of mill tonnage (conventional tonnage constraints)

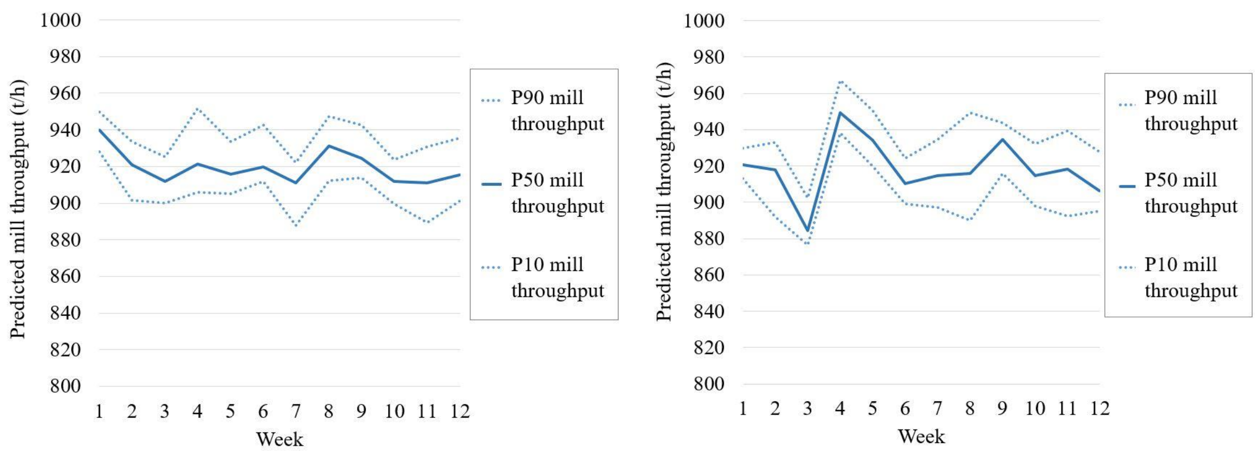
Figure 3-10 Risk analysis showing the difference between scheduled mill tonnage and realized tonnage of a production schedule (a) using all newly developed components and (b) using conventional mill tonnage constraints

It can be seen in Figure 3-10a that the expected difference between scheduled and processed ore (Figure 3-10a, solid black line) is small in all scheduled periods, with a maximum expected deviation of 1 kt (0.6 %) per period and a total deviation from production targets of less than 5 kt (0.3 %) for the complete scheduling horizon (three months). Note that the use of a set of geological orebody scenarios as input to optimization enables the evaluation of the risk of falling short of or surpassing the required ore tonnage per period. There is a remaining risk involved of over and underfeeding the ball mill indicated by P10 and P90 risk profiles (Figure 3-10a dashed gray lines).

However, the spread of the profile is kept well below 5 kt per period. The result indicates that the risk of not meeting scheduled ball mill production using the proposed method is low and can be controlled.

To evaluate the benefits of the new components for short-term production scheduling, the case study is repeated without the use of the throughput prediction model. A conventional mill capacity constraint is used instead, which limits the processed material per period based on a constant tonnage capacity. Figure 3-10b shows that the differences between scheduled tonnage and processed tonnage can be significant by using conventional capacity constraints. The largest deviations can be recognized in periods three and twelve. In both periods, an expected value of over10 kt of ore cannot be processed by the ball mill, which amounts to a deficit of 7% of the planned tonnage in each of these periods. This is indicated by the P50 value of the risk profile in Figure 3-10b (black line, solid). The risk evaluation also shows that there is a small probability of underfeeding the ball mill in some periods, indicated by the P90 risk profile exceeding the balance line (red). However, the expected cumulative shortfall of ore over the complete scheduling horizon accumulates to an expected value of 61 kt, which is equivalent to 3 % of total production that cannot be achieved.

Mill capacity constraints that limit the amount of ore tonnage sent to the processing facility per period are standard practice, both in stochastic and deterministic production scheduling. However, these tonnage constraints assume a constant, average throughput of the milling and grinding circuit for every period and thus ignore the natural heterogeneity of rock attributes of processed ore, leading to highly variable throughput in the short-term, as it can be observed from daily and weekly production data displayed in Figure 3-6. The newly presented and integrated prediction model, however, is capable of assessing the throughput and the associated risk profile to be expected from scheduled material sent to the processing plant. Figure 3-11 shows the ball mill throughput prediction for both schedules generated.



(a) Risk analysis of mill throughput (all new components included) (b) Risk analysis of mill throughput (conventional tonnage constraints)

Figure 3-11 Risk analysis of expected ball mill throughput of a production schedule (a) using all newly developed components and (b) using conventional mill capacity tonnage constraints

While variability in throughput can be observed for both schedules, the variations in expected throughput from period to period are particularly large when conventional tonnage constraints are used (Figure 3-11b). There is a decrease of expected throughput from 920 t/h in period two to 884 t/h in period three (-4%). Considerable risks in production shortfall are the consequence, as seen in Figure 3-10b. These results emphasize the need for an integrated geometallurgical approach in mine production planning that creates short-term production schedules while integrating key metallurgical processes further downstream in mining complexes.

3.5 Conclusions

This article presents a novel four-step approach to integrate a geometallurgical throughput prediction model of the ball mill into short-term stochastic production scheduling. The usefulness of the approach is shown in a large-scale open pit mining complex in Western Australia using real-world production data, whereas the key achievements and conclusions are as follows:

(1) Penetration rates stemming from measurement while drilling data are used for the first time for the prediction of ball mill throughput, informing the hardness, strength and comminution behaviour of the mineral reserve.

(2) The creation of hardness proportions with the use of penetration rates avoids biases typically introduced by the change of support and blending of non-additive geometallurgical properties related to hardness, whereas the hardness proportions

created show the highest correlation to ball mill throughput among all tracked rock attributes.

(3) The use of five hardness proportions enables throughput predictions with an RMSE of less than 30 t/h and a correlation coefficient of up to 0.8.

(4) By integrating the throughput prediction model and the newly developed stochastic constraints into the stochastic optimization of mining complexes, the weekly production schedule can achieve planned production with high certainty (expected shortfall from total planned production < 0.3 %) by enabling the matching of the scheduled materials with the predicted performance of the ball mill.

(5) Future ball mill throughput rates are predicted within the optimization model as a function of blended rock properties, overcoming the shortcomings of previous geometallurgical models integrated into production scheduling that assess geometallurgical properties block by block.

(6) Risk analysis of a weekly stochastic production schedule using conventional tonnage capacity constraints reveals that an expected value of more than 7 % of planned tonnage cannot be processed by the ball mill in certain periods, which amounts to an expected deficit of 3 % of planned tonnage over the three-month planning horizon.

(7) Given that the penetration rates informing rock hardness and the measured throughput rates of the ball mill stem from production data, the novel approach can give a financial advantage compared to laboratory grinding tests for throughput prediction, which are typically costly and time-consuming to obtain.

Future work aims to further improve the prediction of non-additive metallurgical responses of processing plants by utilizing supervised learning models beyond multiple linear regression, which can account for non-linear relationships between predictor and response variables. For future prediction models, measured processing parameters, such as ball mill power and particle size distributions, should be considered given their direct influence on throughput rates. In addition, production data generated from other parts of the processing plant, such as mineral separation processes, should be considered to better integrate value-driving geometallurgical properties into short-term mine production scheduling.

3.6 Chapter Discussion and Next Steps

Chapter 3 presents the integration of a novel geometallurgical prediction model of ball mill throughput into the short-term optimization of mining complexes. The throughput prediction model uses an innovative dataset in the form of recorded penetration rates from drilling activities in the mines which provides information about the hardness and strength of the processed materials in the ball mill. One of the advantages of converting geostatistically simulated penetration rates into hardness proportions in Chapter 3 is that potentially non-additive variables do not need to be averaged in upscaling and blending processes, and thus improve throughput prediction, which will be further validated in Chapter 4. Issues due to the different support of samples for hardness and grindability tests compared to other geological samples (Deutsch et al., 2016; Garrido et al., 2019) can also be alleviated that way. However, the geostatistical simulation of penetration rates using Sequential Gaussian Simulation involves weighted averaging of conditioning data for the determination of means and variances of conditional distributions, assuming additivity. Non-parametric simulation techniques such as Sequential Indicator Simulation or Successive Co-indicator Simulation (Vargas-Guzmán and Dimitrakopoulos, 2002) could be used instead to avoid the additivity assumption for geostatistical simulation. Generally, there is a large part of the variability in the throughput rate that is not explainable by penetration rates. Thus, other factors need to be considered as well to improve throughput prediction. Up to this point, the effect of changing operational conditions of the comminution circuit and their effect on the observed throughput have not been accounted for, which will be addressed in Chapter 4. The use of neural networks will allow to unveil potential non-linear relationships in the data, which can overcome assumptions of linearity that had to be made by the multiple linear regression models utilized in Chapter 3.

4 Applied Machine Learning for Geometallurgical Throughput Prediction – A Case Study using Production Data at the Tropicana Gold Mining Complex

This chapter extends the geometallurgical throughput prediction model developed in the previous chapter by including recorded measurements of ball mill power draw, as well as measurements of particle size distributions before and after entering the ball mill. Instead of the multiple linear regression model employed in Chapter 3, a supervised learning model in the form of a feed-forward neural network is used to approximate non-linear relationships between the extended set of predictor variables and throughput response. The case study at the Tropicana Gold mining complex also evaluates the usefulness of hardness proportions for throughput prediciton, created from penetration rates in the previous chapter, by comparing the compositional approach to the use of average-type penetration rates.

4.1 Introduction

In recent years, the amount of collected and centrally stored production data in the mining industry has increased massively with the implementation of digital technologies. Some examples of centrally stored datasets in operating mines are records of fleet management systems (Moradi Afrapoli and Askari-Nasab, 2017), measurement while drilling (MWD) (Rai et al., 2015), measurements of material characteristics using sensor techniques (Lessard et al., 2014), and other key performance indicators at the processing plants. While potentially all mine planning activities can benefit from the analysis of production data (data analytics), interdisciplinary fields such as geometallurgy can particularly gain from this growing data. Geometallurgy aims to capture the relationships between spatially distributed rock characteristics and its metallurgical behaviour when the mined materials are processed and transformed into sellable products. One pertinent part of geometallurgy is the optimization of comminution circuits and the prediction of comminution performance indicators such as throughput in the mineral processing facilities (Dobby et al., 2001; Williams, 2013; Bueno et al., 2015). However, value is only added to the operation when the gained geometallurgical knowledge is integrated into decision-making processes, whereas appropriate methods are still mostly lacking for the tactical or short-term production planning horizon (McKay et al., 2016). Another current limitation is the cost-intensive sampling and laboratory testing of rock hardness and grindability (Mwanga et al., 2015). The present article shows a case study at the Tropicana Gold mining complex that demonstrates how production data combined with machine learning can be used to construct a data-driven geometallurgical throughput prediction model and how such a model can subsequently be utilized for short-term mine production scheduling.

The optimization of comminution circuits has traditionally relied on well-accepted comminution laws and ore hardness and grindability indices for ball/rod mills (Bond, 1952, 1961) and SAG mills (Starkey and Dobby, 1996; Morrell, 2004b; Amelunxen et al., 2014). These comminution models are routinely used for optimized grinding circuit design, using averages or ranges of ore hardness tests of the mineral deposits to be extracted. Instead of using constant values representing whole deposits, geometallurgical programs account for the heterogeneity of geometallurgical variables within the mineral reserve and their effect on downstream processes over time (Dominy et al., 2018). A typical geometallurgical workflow includes a spatial model, which comprises geostatistically simulated or estimated variables (e.g., grindability). Several case studies have demonstrated how throughput rates of a comminution circuit can be predicted using spatial geometallurgical models of hardness and grindability indices in combination with comminution theory (Flores, 2005; Alruiz et al., 2009; Bulled et al., 2009; Keeney et al., 2011). Although some of these throughput models have demonstrated high accuracy in reconciliation studies, there are notable challenges to use and integrate them into decision-making processes such as short-term production scheduling. First, the geometallurgical sampling program requires cost-intensive laboratory testing to obtain the abovementioned hardness and grindability indices (Amelunxen et al., 2014; Mwanga et al., 2015). The high associated costs spent in early project stages can be prohibitively large and typically result in very sparse sampling, although research is being conducted to increase the number of samples by using alternate data measurement tools and small scale processing tests (Keeney and Walters, 2011). Second, the throughput prediction models are built to evaluate the weekly or monthly performance of mine production schedules *a posteriori* instead of integrating them into short-term production scheduling. Third, none of the models accounts for the inherent uncertainty of the geometallurgical variables stemming from the imperfect knowledge of the orebody.

There have been efforts to incorporate geometallurgical hardness properties and their associated geological uncertainty into mine production scheduling in single open pit mines (Morales et al., 2019) and in mining complexes (Kumar and Dimitrakopoulos, 2019). The stochastic optimization models are developed for long-term production scheduling and require that hardness and grindability indices are geostatistically simulated for volumes of selective mining units (mining block). However, most of the frequently utilized hardness and grindability indices are non-additive (Yan and Eaton, 1994; Amelunxen, 2003; Amelunxen et al., 2014). Geometallurgical samples are also collected on large support scales (Deutsch et al., 2016; Garrido et al., 2019) and are typically very sparse, as mentioned earlier. These complicating factors make the joint spatial interpolation of geometallurgical variables and their change of support from point measurements to mining blocks challenging (Deutsch, 2013; van den Boogaart et al., 2013; Deutsch et al., 2016; Ortiz et al., 2020). Morales et al. (2019) optimize the mine production schedule using pre-calculated mill throughputs and economic values for each block independently. The method thus ignores that extracted materials are blended in stockpiles and in processing facilities; consequently, the non-additive comminution behaviour of blended materials and resulting metal production cannot be correctly assessed. Kumar and Dimitrakopoulos (2021) optimize a mining complex while including pre-defined ratios of hard and soft rock to achieve a consistent throughput in processing streams. However, these ratios are defined arbitrarily, and details of short-term planning are not addressed.

Both and Dimitrakopoulos (2021b) present a new approach that integrates a geometallurgical throughput prediction model into short-term stochastic production scheduling for mining complexes. The stochastic production scheduling formulation builds upon simultaneous stochastic optimization of mining complexes (Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016) which optimizes pertinent components of a mining complex in a single mathematical model and incorporates geological uncertainty to minimize technical risk. Instead of using block throughput rates, the production scheduling formulation calculates the throughput of blended materials using an empirically created throughput prediction model learning from previously observed throughput rates at the ball mill (Both and Dimitrakopoulos, 2021b). One limitation of this work is that the integrated throughput prediction model so far has only considered rock hardness, density, lithology, and weathering degree of the mineral reserve. This ignores that mill throughput rates also depend on operating factors of the processing plant, such as power draw, utilization rates, and particle size distributions. Second, a multiple linear regression (MLR) has been used for throughput predictions, which is unable to capture potential non-linear relationships among input variables and geometallurgical response.

The case study at the Tropicana Gold mining complex shown in this article expands the method presented in Both and Dimitrakopoulos (2021b) in multiple ways. First, the recorded plant measurements power draw, feed particle size, and product particle size of the ball mill are newly considered to improve the prediction of ball mill throughput rates. Second, a more powerful supervised learning method in the form of an (artificial) neural network is tested and compared to MLR, since the addition of the new comminution-related features increases the possibilities of non-linear interactions between predictive and response variables. The plant measurements, including the observed ball mill throughput, are retrieved from the comminution circuit at the Tropicana Gold mining complex. The other dataset used in this case study to predict ball mill throughput comprises penetration rates from measurement while drilling (MWD). The use of this dataset is motivated by its ability to indicate the strength and hardness of the intact rock (Rai et al., 2015; Vezhapparambu et al., 2018; Park and Kim, 2020). The penetration rates are converted into a set of hardness proportions per selective mining unit (SMU) which has recently been proposed to build a link between intact rock hardness and comminution performance of the rock in milling and grinding circuits (Both and Dimitrakopoulos, 2021b). The present article also compares the prediction capabilities of hardness proportions to averages of penetration rates. In this way, the effect of ignoring non-additivity of hardness-related geometallurgical variables can be quantified, an issue that has had little attention in the literature thus far.

In the following sections, the components of the Tropicana Gold mining complex are introduced first, together with all utilized production data that is used for the prediction of ball mill throughput. The supervised machine learning model is discussed next, including a statistical analysis of the present dataset and a hyperparameter calibration. Analysis of results, discussion, and conclusions follow.

4.2 The Tropicana Gold Mining Complex and Utilized Production Data for Ball Mill Throughput Prediction

The Tropicana Gold mining complex is located in Western Australia in the west of the Great Victoria Desert. The gold deposit is mined from four pits, Boston Shaker, Tropicana, Havana, and Havana South (from North to South), as can be seen in the aerial view in Figure 1. In addition, the mining complex contains a processing plant, stockpiles, a tailings facility, and multiple waste dumps. Gold is produced on-site in a single processing stream, consisting of a comminution circuit and a carbon-in-leach (CIL) plant.

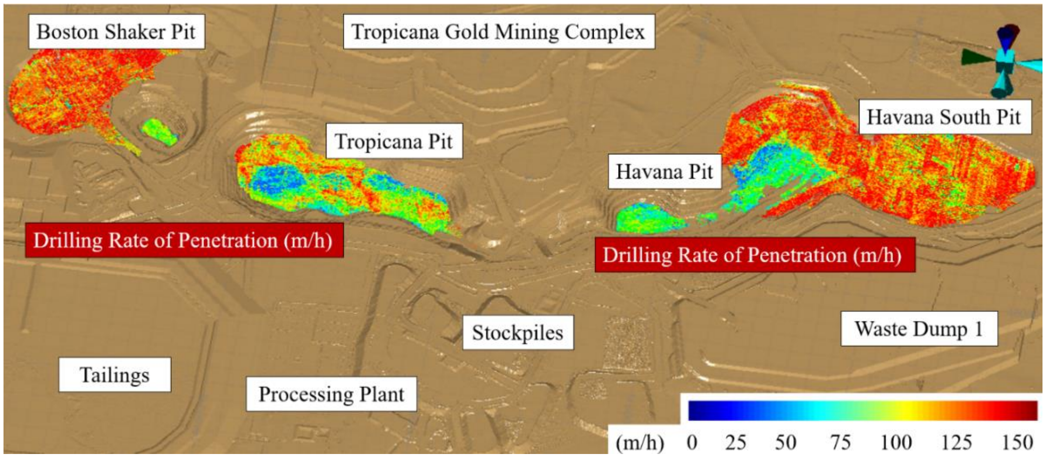


Figure 4-1 Components of the Tropicana Gold Mining Complex and a heat map of drilling rate of penetration (ROP) retrieved from measurement while drilling (MWD)

The displayed dataset in the four pits in Figure 1 shows the drilling rate of penetration (ROP) from production drilling (blastholes), which is part of the measurement while drilling (MWD) dataset collected at the Tropicana Gold mining complex. It is clearly visible how ROP reflects the heterogeneity of the rock and decreases with depth. Exemplary, easy-to-drill (softer) rock is found towards the surface (red colors at Havana South Pit and Boston Shaker Pit), whereas difficult-to-drill (harder) rock is located deeper in the pits (green-blue colors in Havana Pit, Tropicana Pit, and deeper cutback of Boston Shaker Pit). Both and Dimitrakopoulos (2021b) demonstrate strong correlations between the rate of penetration (ROP) of drilled rock and ball mill throughput when these rock parcels are sent to the processing plant. They subsequently present a method that predicts ball mill throughput using ROP. This article extends this work by utilizing additional measurements in the processing plant related to ball mill throughput.

The relevant material flow in the mining complex is shown together with all utilized production data in

Figure 4-2. Detailed material tracking in daily intervals is performed using truck cycle data, starting from the material extraction in the pits and ending at the crusher. Crucially, material tracking includes all dumping and rehandling activities at run-of-mine (ROM) stockpiles since rehandled material accounts for 80-90% of processed ore in the Tropicana Gold mining complex. In this way, ROP entries recorded in the pits can be successfully linked to observed measurements in the processing plant, including the observed throughput of the ball mill. Details of successful implementations of material tracking that include stockpiles can be found in Wambeke et al. (2018) and Both and Dimitrakopoulos (2021b).

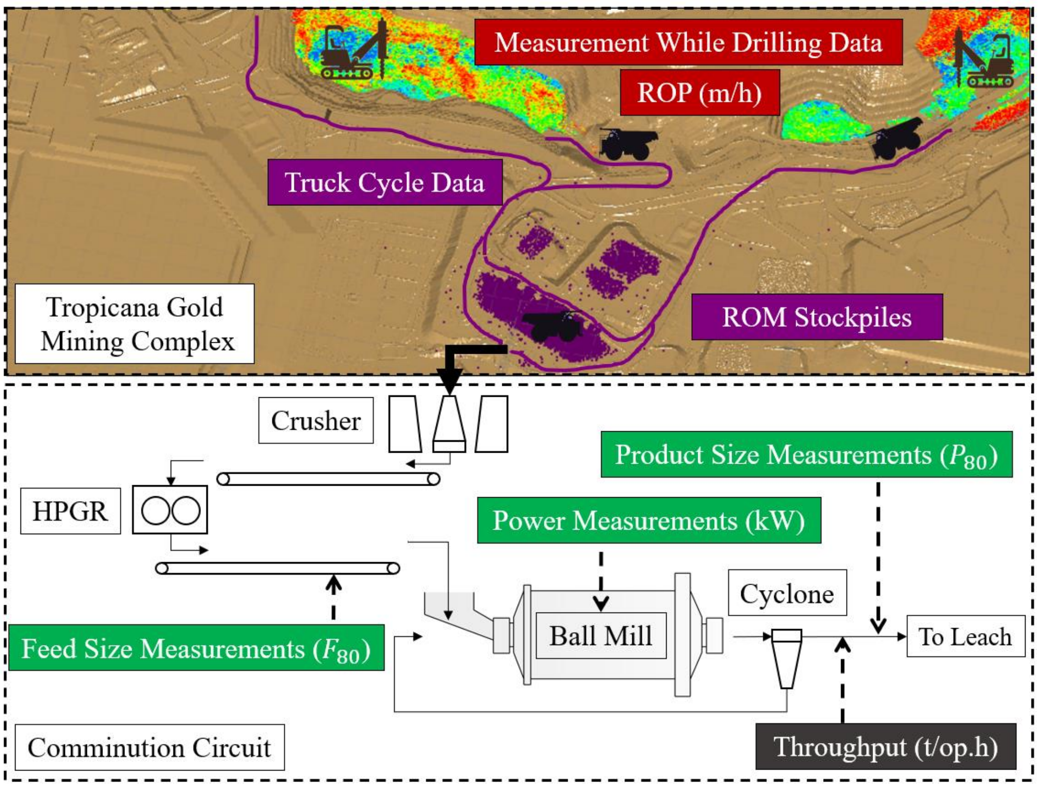


Figure 4-2 Material flow and utilized production data for ball mill throughput prediction in the Tropicana Gold mining complex

The comminution circuit at Tropicana Gold mining complex comprises three stages: crushing (primary and secondary crusher), grinding (high pressure grinding roll, HPGR), and milling (ball mill). The cyclone overflow is sent to the CIL plant to extract the gold. The recorded average power draw of the ball mill and the particle size distributions entering and leaving the ball mill are of particular interest for throughput prediction. Note that the feed and product particle size distributions are subsequently defined by their 80 % passing diameters in μm. The feed particle size measurements (F_{80}) are performed using image analyzers on the conveyor belt of the HPGR product. Shift composites of cyclone overflow samples are used for product particle size measurements (P_{80}).

The relevance of all presented measurements above can be derived from comminution theory, such as Bond's law of comminution (Bond, 1952, 1961). The Bond equation (Eq. 4.1) calculates the specific energy of the ball mill (W in kWh/t) required to grind the ore from a known feed size (F_{80}) to a required product size (P_{80}).

$$W = Wi * \left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right) \tag{4.1}$$

The Work index (Wi in kWh/t) is a measure of the ore's resistance to crushing and grinding (Bond, 1952). In this article, it is useful to substitute the specific energy of the ball mill (energy delivered per ton of ore in kWh/t) by the quotient of mill power draw (kW) and mill throughput (processed tonnes per operating hour), as shown in Eq. 4.2.

$$\frac{Power}{TPH} = Wi * \left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right) \tag{4.2}$$

Eq. 4.3 is obtained by re-arranging Eq. 4.2 for ball mill throughput (TPH).

$$TPH = \frac{Power}{Wi * \left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right)} \tag{4.3}$$

Next to the measured power draw and particle size distributions, it is clear that throughput predictions of the ball mill must include some kind of information about ore hardness. Generally, the harder the material, the higher its resistance against comminution, thus needing to reside longer in the ball mill to reach the desired product size, given constant power draw and particle feed size. In Bond's equation, TPH is inversely proportional to Wi, as shown in Eq. 4.4.

$$TPH \propto \frac{1}{Wi} \tag{4.4}$$

As introduced above, the role of informing ore hardness is taken over by ROP measurements in this article. By utilizing cost-effective and easily accessible production data (MWD information generated by drilling machines), costly and time-consuming laboratory tests spent for Wi estimates of the geological reserve can be replaced. Mwanga et al. (2015) report that the typical sample volume required for Bond tests is relatively large (2-10 kg, depending on test modification), and requires crushed ore smaller 3.35 mm (passing a 6 mesh sieve). Furthermore, several grinding cycles are necessary to reach the steady state of the simulated closed circuit. The alternative utilization of ROP is especially promising as a substitute for Wi because of its demonstrated ability to indicate rock-type, strength, and alteration (Horner and Sherrell, 1977; Sugawara et al., 2003; Yue et al., 2004; Vezhapparambu et al., 2018). In general, high ROP (in m/h) indicates less competent rock, bearing lower Wi. In turn, TPH is expected to increase, as shown in Eq. 4.5.

$$ROP\left({}^{m}/_{h}\right) \nearrow \implies Wi\left({}^{kWh}/_{t}\right) \searrow \implies TPH \nearrow \tag{4.5}$$

Note that the dependencies in Eq. 4.5 may be non-linear. Rather, potentially non-linear dependencies call for more sophisticated prediction models for TPH prediction, which are subsequently discussed in Section 4.3.

4.3 Application of Supervised Machine Learning for Throughput Prediction

This section discusses the use of supervised machine learning to create a throughput prediction model at the Tropicana Gold mining complex. Supervised machine learning models require labelled datasets for training, consisting of data pairs $\{\boldsymbol{x}_i, y_i\}$, $i = 1, \dots, N$, whereas $\boldsymbol{x}_i$ is a vector of predictor variables, and y_i is the known response. In this article, the known response (label) is the observed ball mill throughput, and the M predictor variables (features) are comprised of the geological attributes of the ore and measured variables in the comminution circuit. Throughput responses are recorded on a continuous scale, rendering the supervised learning problem a regression task ($y_i \in \mathbb{R}$).

4.3.1 Neural networks

A feed-forward neural network is chosen as a supervised learning model for the potentially non-linear task of ball mill throughput prediction. In its essence, feed-forward neural networks are fully connected, layered combinations of neurons that find their origins in the perceptron model (Rosenblatt, 1958). A single neuron (perceptron) calculates the inner product between its internal weight vector, $\boldsymbol{w}^{\boldsymbol{T}}$, and the input vector, $\mathbf{x}$. After adding a bias term, $b \in \mathbb{R}$, the resulting value is passed through a non-linear activation function, $g(\ \cdot\ )$, creating a scalar output $z = g(\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x} + b)$. Several connected neurons to $\mathbf{x}$ form the so-called first hidden layer of the neural network. If the outputs of the first hidden layer are passed through another layer of neurons, a multilayer neural network is built (Hornik et al., 1989). The output layer comprises a single neuron that receives as input the vector of hidden outputs, $\boldsymbol{z}$, and provides an estimate, $\hat{y} \in \mathbb{R}$. Neural networks are the method of choice in this article because they have the proven advantage of being capable of approximating every arbitrary function using either one hidden layer of exponentially many neurons or multiple consecutive neural layers consisting of fewer neurons (Hornik, 1991). This gives neural networks theoretical advantages over linear prediction models, such as multiple linear regression, which has been tested in previous work for throughput prediction (Both and Dimitrakopoulos, 2021b). Univariate statistics and correlations in the present dataset, including potential non-linearities, are discussed next, followed by the discussion of the utilized neural network architecture, and tuning of its hyperparameters.

4.3.2 Dataset and statistical analysis

The dataset for throughput prediction contains the hardness-related rate of penetration (ROP) of the ore, which has been tracked in the Tropicana Gold mining complex, as presented in Figure 4-2. The power draw, F_{80}, and P_{80} measurements, as well as a ball mill utilization factor reflecting ball mill up- and downtime, are also included. A 7-day moving average of the data is calculated for an observed time horizon of six months (Feb 2018 – Aug. 2018), which reduces noise in the dataset and helps recognize trends of higher and lower throughput rates that are more likely connected to rock properties of the material processed. In the six-month interval, extraction mainly occurs in two pits, the Tropicana and Havana Pit, and material is continuously stockpiled at the ROM stockpiles. Univariate statistics of the predictive variables and the response variable (throughput) are shown in Table 4-1.

Table 4-1 Univariate statistics of predictive variables (features) and ball mill throughput (response)

	Average ROP (m/h)	**Ball Mill Power (kW)**	**Ball Mill Utilization (%)**	$\boldsymbol{P_{80}}$ **(μm)**	$\boldsymbol{F_{80}}$ **(mm)**	**Ball Mill Throughput (t/op.h)**
Minimum	35.0	9996	0.7	76.5	10.3	796.4
Mean	41.4	13002	1.0	83.3	13.1	926.5
Maximum	53.6	13435	1.0	93.2	15.0	1007.9
Std. Dev.	3.45	685.9	0.052	3.12	1.00	34.8
Coeff. of Var. (CV)	0.083	0.053	0.053	0.037	0.077	0.038
Skewness	0.88	-3.01	-3.03	0.57	-0.36	-1.07
Kurtosis	1.20	9.26	9.30	0.94	-0.13	3.05
Count	181	181	181	176	153	181

Table 4-2 shows linear correlations between pertinent features and observed TPH using Pearson's correlation coefficient, in Eq. 4.6 below, with x_i and y_i representing individual sample points and $\bar{x}, \bar{y}$ indicating sample means. Note that correlations in Table 4-2 can be inflated because they are calculated after applying the moving average.

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2} * \sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{4.6}$$

The tracked ROP entries are henceforth used in two different ways to inform material hardness. The feature 'Average ROP' is comprised of weighted averages of continuous ROP values linked to the materials that are transported to the crusher in the same observed time interval. In contrast, Both and Dimitrakopoulos (2021b) propose a compositional approach, which partitions ROP into easier-to-drill (softer rock) and difficult-to-drill (harder rock) categories, using a set of ROP intervals. The split in multiple intervals results in proportions of harder or softer materials sent to the comminution circuit in a given time interval. A detailed explanation of how to calculate these hardness proportions is given in Both and Dimitrakopoulos (2021b). The listed features in Table 2 can broadly be distinguished into three categories, whereas the first two categories are related to ore hardness. Average ROP comprises the first category (A1), and hardness proportions built by intervals of penetration rates comprise the second category (B1 – B10). The third feature category reflects measurements at the comminution circuit (C1 – C4).

Table 4-2 Pearson's correlation coefficient between predictor variables (geological, comminution-related) and ball mill throughput

No.	Category	Feature	Unit	Pearson's correlation coefficient to ball mill throughput
A1	Ore hardness using average pen-rate values	Average ROP	(m/h)	0.440
B1	(Harder material)	< 26 m/h	(%)	-0.274
B2		26-29 m/h	(%)	-0.370
B3		29-32 m/h	(%)	-0.525
B4	Ore hardness expressed by proportions of penetration rate intervals	32-35 m/h	(%)	-0.361
B5		35-38 m/h	(%)	-0.416
B6		38-42 m/h	(%)	-0.318
B7		42-46 m/h	(%)	0.028
B8		46-53 m/h	(%)	0.356
B9		53-62 m/h	(%)	0.386
B10	(Softer Material)	> 62 m/h	(%)	0.498
C1	Measurements in the comminution circuit	Feed size F80	(mm)	0.046
C2		Product size P80	(µm)	0.063
C3		Power	(kW)	0.382
C4		Mill Utilization	(%)	0.374

By comparing the Pearson correlation coefficients in Table 4-2, it can be seen that some variables correlate stronger with TPH, whereas other variables do not. A stronger positive correlation of TPH for 'Average ROP' (in m/h) gives first evidence of the usefulness of this feature (A1). The compositional approach effectively partitions the distribution of penetration rates into multiple hardness categories. Here, a higher percentage of difficult-to-penetrate material in the processed ore blend (B1 - B6) indicates harder material, thus lowering TPH, which is confirmed by the negative correlation in Table 4-2. Conversely, a higher fraction of easier-to-penetrate material in the blend is expected to increase TPH, which is equally confirmed in Table 4-2 through positive correlations of categories B8 - B10. Interestingly, some hardness categories show a stronger correlation (positive and negative) than the average ROP feature (A1). This indicates that additional information may be conveyed through the creation of hardness categories. The prediction potential of average penetration rates and hardness proportions is compared in detail in Section 4.4.1.

According to Eq. 4.5, the relationship between ball mill power and TPH is directly proportional. This theoretical relationship is empirically well reflected in Table 4-2, showing a stronger positive correlation between ball mill power (C3) and TPH. The power measurements thus comprise an important part of throughput prediction, subsequently performed in Section 4 of this article. Although the ball mill utilization (C4) is not part of Bond's equation, it is not surprising to see a stronger correlation to TPH. At events of planned and unplanned ball mill downtime, i.e., utilization < 100 percent, ramp-up and ramp-down processes are among the effects that also lower the effective throughput per operating hour. A redundancy between ball mill utilization and ball mill power is observed, confirmed by similar statistics of power and utilization in Table 4-1, which explains similar correlation in Table 4-2. Relationships between TPH and particle sizes of the ore that result from Bond's law (Eq. 4.1) are shown in Eqs. 4.7 and 4.8.

$$P_{80} \nearrow \;\Longrightarrow\; TPH \nearrow \tag{4.7}$$

$$F_{80} \searrow \;\Longrightarrow\; TPH \nearrow \tag{4.8}$$

On the one hand, a coarser product particle size (larger P_{80} value) results in higher TPH (Eq. 4.7), given that ore characteristics, energy input and feed particle size stay the same. On the other hand, a finer-grained feed size (smaller F_{80} value) can also lead to an increased TPH because less grinding work needs to be applied to reach the desired product size (Eq. 4.8). In the present dataset, the particle size measurements (C1 - C2) show very little correlation in Table 4-2. This can have several reasons. Contrary to power draw, the relationships in Eqs. 4.7 and 4.8 are non-linear, and the particle-size measurements are incomplete for some periods, as indicated in Table 4.1. Also, one must consider that particle size measurements over running belts are error-prone, especially when using image analyzers for F_{80}. It is analyzed in Section 4.4 if particle size measurements can enhance throughput prediction in practice. Note that all comminution variables are scaled before usage by dividing by their maximum value. Compositional data naturally comprises fractional values in [0,1] and thus does not have to be scaled.

4.3.3 Network architecture and hyperparameter search

In its implementation, the architecture of a feed-forward neural network requires the calibration of several hyperparameters. The hyperparameter setting is relevant to the evaluation process and robustness of the approach. Therefore, it becomes obvious to explore the hyperparameter space in order to find a stable region of this space (Bengio and Grandvalet, 2004). However, due to the small size of the dataset (181 data points) and the need to test on the entire horizon (181 days) to extrapolate the overall performance of the proposed approach, the dataset cannot be split. Instead, k-fold cross-validation is used to measure the configuration quality, thus minimizing the information loss (Hastie et al., 2009). Different periods are used for different folds (20 folds) to simulate the more realistic scenario where a prediction is made over a new period. The network architecture is implemented in Python using the scikit-learn package (Pedregosa et al., 2011). The squared error between the observed throughput, $y_i,\ i = 1, \dots, N$, and predicted throughput, $\hat{y}_i,\ i = 1, \dots, N$, is chosen as the loss function to be minimized during training and the rectified linear unit is chosen as activation function. The quasi-Newtonian L-BFGS algorithm (Liu and Nocedal, 1989) is used to minimize the loss function, which proved to converge faster on the small dataset compared to stochastic gradient methods. Finally, the root mean squared error (RMSE) is used for comparisons.

Early stopping of training is important to prevent overfitting in neural networks, and therefore, the number of training iterations is a hyperparameter that needs to be calibrated (Goodfellow et al., 2016). It was found that the validation error was minimal after five iterations. L2 regularization was tested but did not significantly increase generalization potential in this application.

4.3.4 Number of layers and neurons

Figure 4-3 shows a sensitivity analysis of the number of neurons for two selected feature sets. In Figure 4-3a, only hardness-related features are used, whereas Figure 4-3b includes more features. Given the stochastic processes involved during training, each network configuration is repeated 20 times using random initializations of weights. This procedure results in a sample of errors that are shown by boxplots.

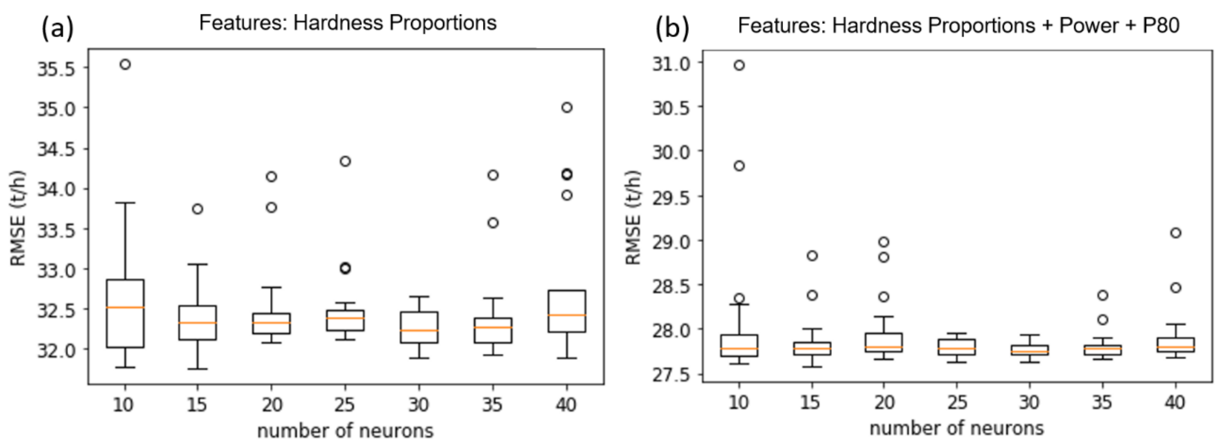


Figure 4-3 Comparison of the number of neurons for two selected feature sets: (a) hardness proportions and (b) hardness proportions, ball mill power and product particle size (P80)

Figure 4-3 shows that the average error and error variance reduce for both feature sets as the number of neurons increases. A plateau is reached at twenty-five to thirty neurons. This is expected since a too small number of neurons is not able to adequately map the underlying function. Note that this behaviour can be observed independently of the number of layers. Two fully connected hidden layers are used in Figure 4-3a, whereas a single connected hidden layer was used for the sensitivity analysis in Figure 4-3b. For the best choice of layers, another sensitivity analysis is performed by varying the number of hidden layers from one to four. Figure 4-4 shows the results performed on the same selected feature sets.

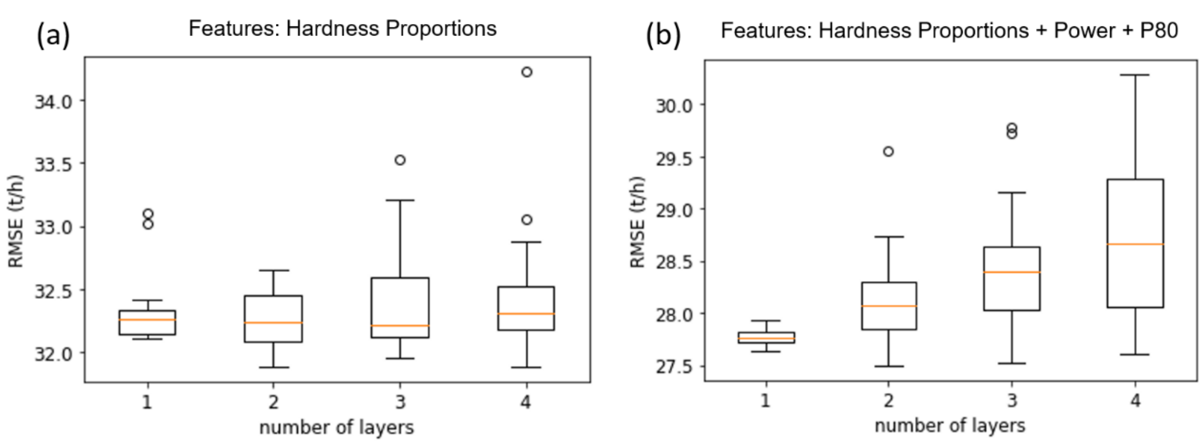


Figure 4-4 Comparison of the number of hidden layers for two selected feature sets: (a) hardness proportions and (b) hardness proportions, ball mill power and product particle size (P80)

Figure 4-4 indicates that one hidden layer delivers the most stable results on all tested feature sets. Although the addition of more layers can reduce the error in individual runs, as seen in Figure 4-4a, the network appears more prone to overfitting and the error variance increases. For larger feature sets (Figure 4-4b), overfitting appears to be exacerbated the more layers are used. The obtained results demonstrate the strength of parsimony of parameters (POP), as the model with the smallest size (i.e., one hidden layer) is performing best.

4.4 Results and Analysis

Section 4 is subdivided into two separate parts that aim to analyze the effects of different feature sets on throughput prediction and then benchmark the presented neural network against a multiple regression model. Subsection 4.1 addresses the prediction of ball mill throughput using hardness-related variables only. In Subsection 4.2, pertinent comminution variables are added individually, and their effect on throughput prediction is evaluated.

4.4.1 Hardness-related variables (effect of non-additivity)

This subsection aims to answer how different ways of informing the hardness and grindability of the geological reserve using penetration rates from blasthole drilling perform for throughput prediction. Specifically, the prediction potential of the average rate of penetration (ROP) is compared to the prediction behaviour of hardness proportions created using penetration rate intervals. Figure 4-5a shows a graphical comparison of ball mill throughput (left axis) and average ROP of the processed ore (right axis). Figure 4-5b and Figure 4-5c illustrate the evolution in time of two distinct hardness proportions compared to throughput and are discussed subsequently.

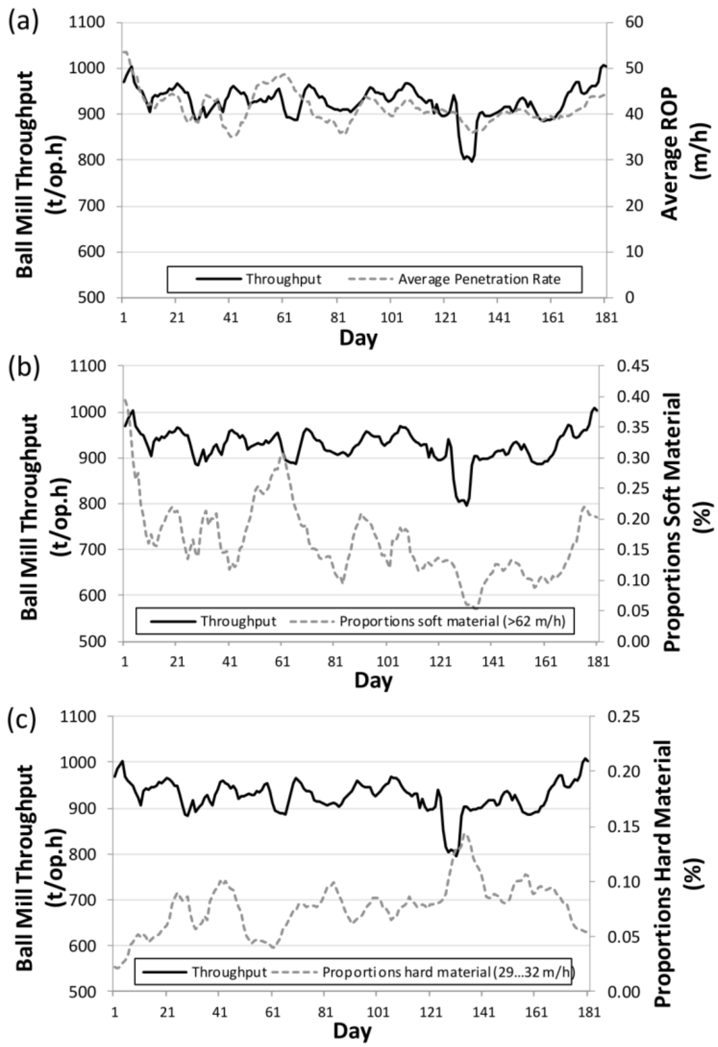


Figure 4-5 Moving average of ball mill throughput compared to moving average of (a) average rate of penetration (ROP), (b) proportions of softer material (high penetration rates in the interval > 62 m/h), and (c) proportions of harder material (low penetration rates falling in the interval of 29-32 m/h)

It can be seen in Figure 4-5a that average ROP follows ball mill throughput well in many periods of the observed time horizon. Together with the strong positive correlation reported in Table 4-2, the similar behaviour of both variables in Figure 4-5a confirms the hypothesis that penetration rates recorded by drilling machines can contribute to inform the comminution performance and grindability of the processed ore. Next, this feature is tested using 20-fold cross-validation. The performance of average ROP as a single feature for throughput prediction is shown in Figure 4-6a (neural network) and Figure 4-6b (multiple regression).

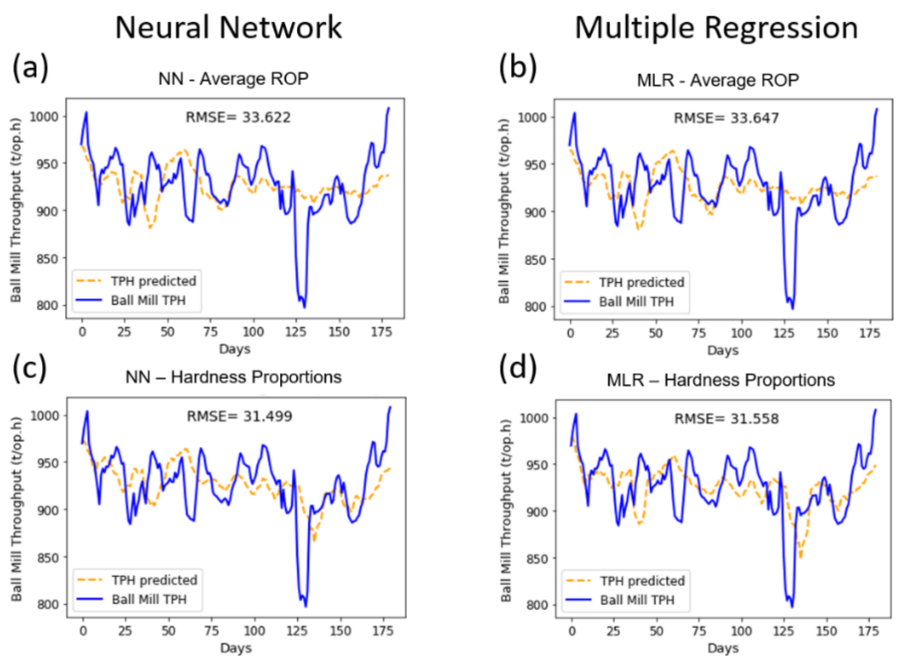


Figure 4-6 Ball mill throughput prediction (20-fold cross-validation) using (a) average ROP (NN), (b) average ROP (MLR), (c) hardness proportions (NN), and (d) penetration rate categories (MLR)

When comparing Figure 4-6a and b, there appear to be no obvious advantages of the neural network compared to multiple regression, which can be explained by the fact that only one single feature is used. Although following the general trends of throughput in most of the observed time intervals, the results reveal weaknesses in predicting the right magnitudes of low and high throughputs. A possible explanation for this weakness can be found when considering penetration rates as a non-additive variable. Non-additivity is present if linear averages of a variable, for instance, penetration rates of two separate rock entities, are different from the expected value of the combined (blended) sample. Thus, taking mathematical averages can be detrimental to such variables. Other well-known examples are metal recovery (Carrasco et al., 2008) and other variables representing product quality (Coward et al., 2009).

In fact, the feature 'average ROP' has gone through an averaging process twice. First, penetration rates are averaged within a mining block when changing the support from simulated grid nodes (point support) to mineable volumes (SMU) to reflect mine selectivity. This standard process is only innocuous for additive variables such as metal grades (at constant density). Second, a weighted average by tonnage of each truckload is calculated per day, accounting for all sources of material that are blended. For the alternative feature set of hardness proportions, penetration rates in point support are split into several categories using penetration rate intervals. This procedure avoids the averaging of harder and softer parts within the geological reserve. Instead, proportions of softer and harder material are preserved in the ore blends that are processed in the mill (compositional approach). A discussion of how to build hardness proportions and how many hardness categories are needed can be found in Both and Dimitrakopoulos (Both and Dimitrakopoulos, 2021b).

Figure 4-5b and Figure 4-5c illustrate the evolution of two distinct hardness proportions compared to TPH. Figure 4-5b shows the proportions of soft material arriving at the mill, informed by the percentage of high penetration rates (greater than 62 m/h) in the ore blend. Here, higher throughputs are expected to occur when more of this soft material arrives at the mill. Indeed, large proportions of softer material in Figure 4-5b coincide with high mill throughput, which is most visible for days 1 - 10 as well as for days 170 - 181 of the observed period. Figure 4-5c shows the proportions of harder material, which is reflected by penetration rates that fall in the interval of 29 to 32 m/h. Larger proportions of this material category should have a negative effect on throughput. Interestingly, Figure 4-5c shows that the lowest mill throughput (days 128-133) coincides with the peaking of the fractions of harder material. Conversely, the highest throughput is achieved when the proportions of this harder-to-penetrate material are the smallest.

The performance of hardness proportions for throughput prediction is shown in Figure 4-6c (neural network) and Figure 4-6d (multiple regression). The highs and lows of throughput are more closely predicted, leading to a reduction of the prediction error by 6.3% for both prediction models. This indicates that classification into hardness proportions is advantageous over using a single, continuous hardness variable. The difference between the neural network and the multiple regression model is relatively small.

4.4.2 Effect of comminution variables on prediction

Several comminution variables were identified as potential candidates to improve throughput prediction in Sections 4.2 and 4.3. In this subsection, the hardness feature set comprising hardness categories is enhanced by one additional comminution variable at a time. To analyze the effects of the neural network, a comparison to a multiple linear regression model is provided for each experiment.

4.4.3 Ball mill power

The ball mill power measurements showed the potential to improve the prediction of ball mill throughput due to its proportional relationship to TPH in Bond's law (Eq. 4.1) and its strong correlation in the present dataset shown in Table 4-2. Figure 4-7a shows a graphical comparison between the daily average power draw of the ball mill and TPH. Power draw stays mostly constant for the observed time horizon, including some distinctive drops in power in the second half of the observed time horizon. These power drops tend to occur at times when the mill throughput decreases as well. It is thus not surprising that adding ball mill power as a feature for throughput prediction especially enhances the periods of sharp throughput decrease, as shown in Figure 4-8a.

By comparing the predictive performance of the neural network (NN) with the performance of the multiple linear regression model (MLR) in Figure 4-8b, the superiority of the neural network becomes apparent. MLR overestimates the influence of ball mill power, seen in the sharp decrease in days 120-125. The neural network predicts closer to the true throughput, which can be noticed visually and statistically. Compared to the sole utilization of hardness proportions (Section 4.1), the RMSE decreases by 5.3 % when using the neural network, whereas the error for MLR rises by 1.5 %.

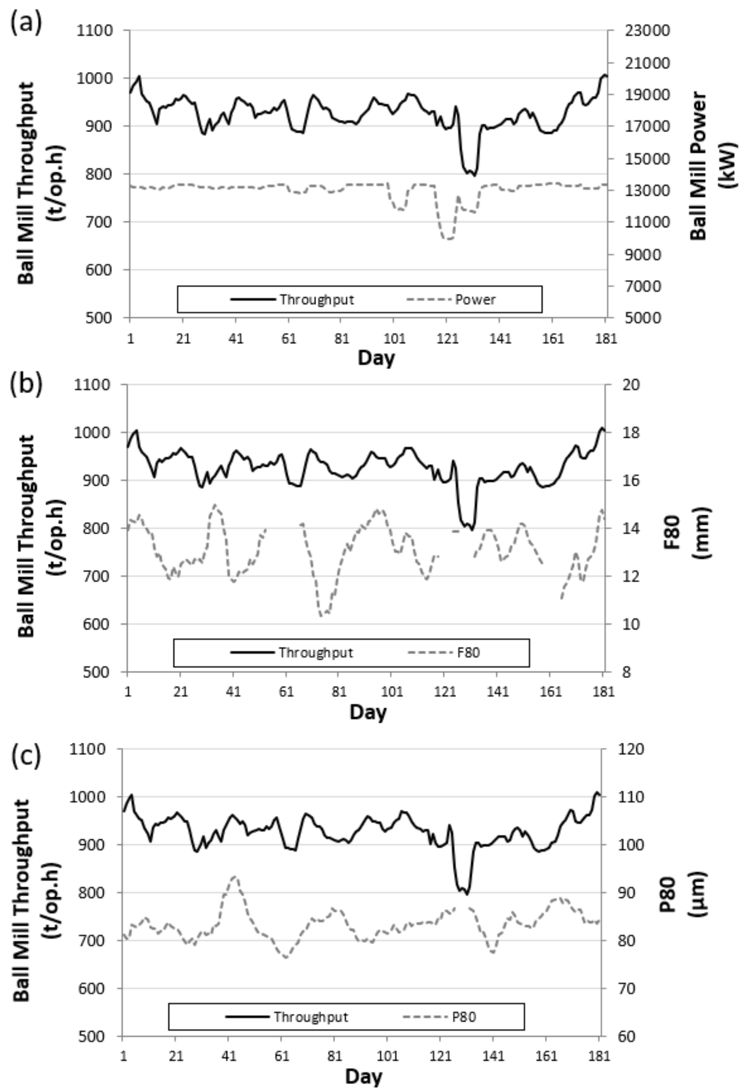


Figure 4-7 Moving average (7-days) of ball mill throughput compared to moving average of (a) ball mill power (b) feed particle size: (F80), and (c) product particle size (P80)

4.4.4 Particle sizes

Compared to ball mill power measurements, particle size measurements indicate a low empirical correlation in the present dataset between particle sizes and TPH (Table 4-2). The theoretical relations to throughput (Eqs. 4.5, 4.8, 4.9) cannot be confirmed by visual analysis in Figure 4-7b and Figure 4-7b alone. The graphs also show a large amount of missing data, especially for feed particle size (F80) measurements. No visible trends are recognizable.

By comparing the prediction behaviour adding particle sizes in Figure 4-8c-f, the following conclusions may be drawn. Adding F80 measurements seems to not significantly enhance throughput prediction in this case study since the RMSE decreases only marginally when using the NN (-0.6 %, Figure 8c) and increases when using MLR (+1.4 %, Figure 4-8d). The addition of product size measurements (P80) seems to have a positive effect on throughput prediction in this case study, which is noticeable for both prediction models. However, the NN prediction error (-6.5 %) in Figure 4-8e reduces notably more than the MLR prediction error (-3.0 %) in Figure 4-8f, showing the superiority of the NN when dealing with non-linear features. The biggest gain in prediction accuracy can be obtained when using both well-performing features, power draw and P80, together. Here, the strengths of the neural network become most apparent, showing the lowest error in Figure 4-8g and a 10.6 % error reduction compared to ore hardness only. The MLR also shows the lowest recorded error (-4.2 %, Figure 4-8h), but the error decreases much less than the NN. To summarize, as more features are added, their interdependencies can be better interpreted by NN.

4.5 Discussion

Next to the superior performance of hardness proportions combined with power draw and product size measurements, the results obtained above show that the use of neural networks can decrease the ball mill throughput prediction error compared to using multiple regression. Short-term decision-making, such as short-term mine production scheduling, can benefit from the demonstrated improvements in throughput prediction presented in this article. A conventional short-term production schedule for the Tropicana Gold mining complex is shown in Figure 4-9.

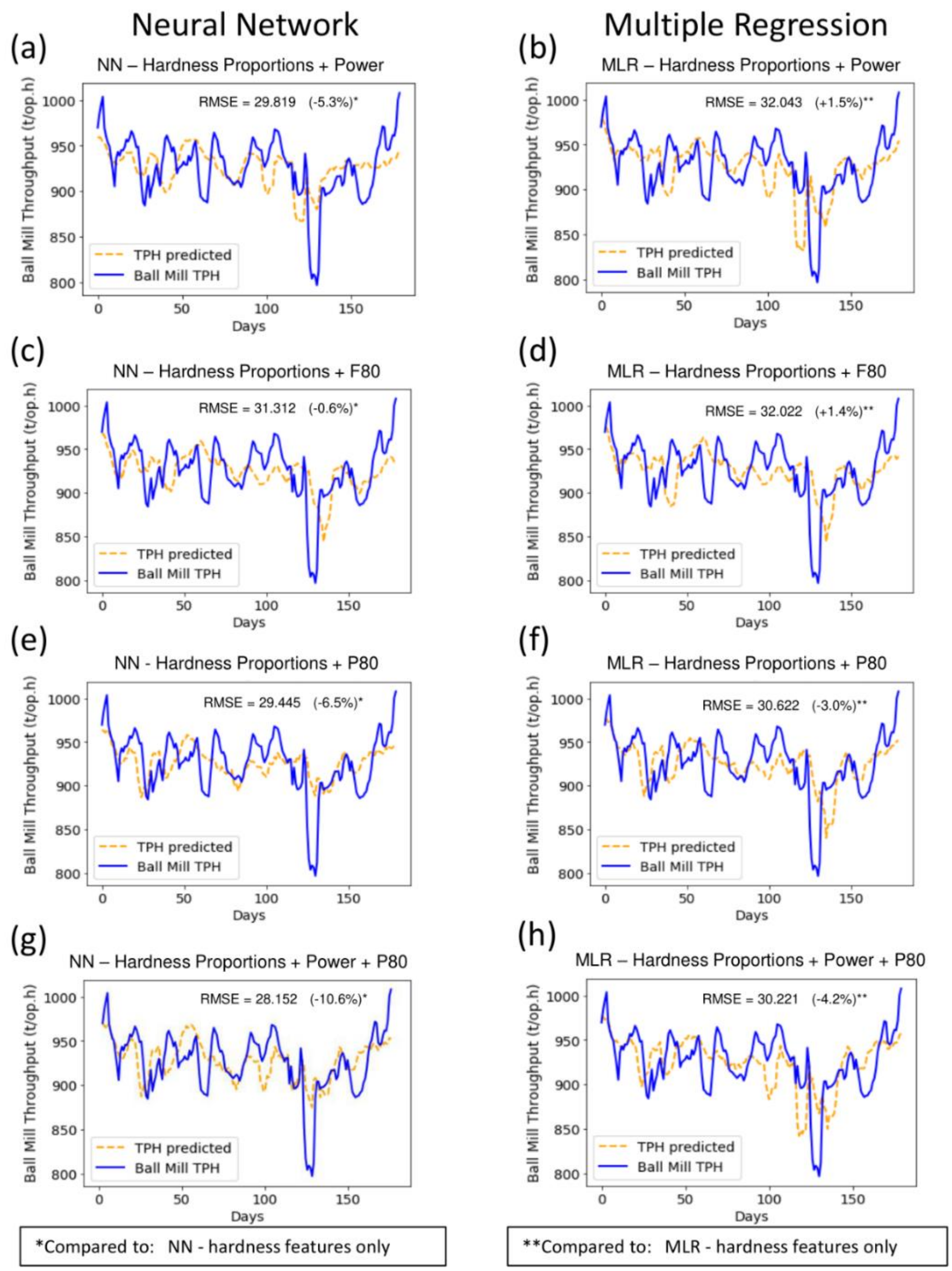


Figure 4-8 Ball mill throughput prediction (20-fold cross-validation) using as additional features: (a) ball mill power (NN), (b) ball mill power (MLR), (c) feed particle sizes (NN), (d) feed particle sizes (MLR) (e) product particle sizes (NN), (f) product particle sizes (MLR) (g) power and P80 (NN), (h) power and P80 (MLR) – RSME is compared in brackets to respective model predictions (neural network/ multiple regression) using hardness features only

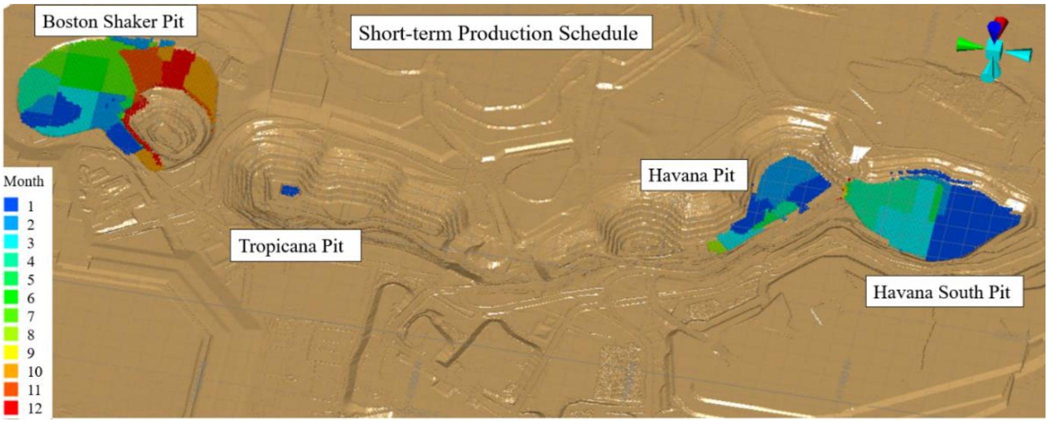


Figure 4-9 Example of a monthly short-term production schedule in the Tropicana Gold mining complex

As it can be seen in Figure 4-9, short-term extraction can take place in multiple pits and different mining areas within the pits in the same period of extraction, leading to blended material streams at the processing plant(s). As a recent development in short-term mine planning, the incorporation of a geometallurgical throughput prediction model into short-term production scheduling has been demonstrated in Both and Dimitrakopoulos (2021b). Instead of building pre-defined throughput estimates per mining block, the authors predict the ball mill throughput of blended materials using a multiple regression model and use these predictions for short-term production scheduling in a stochastic optimization model. Figure 4-10 illustrates how the trained neural network in this article, together with comminution variables at the ball mill, can replace the multiple regression model for production scheduling optimization.

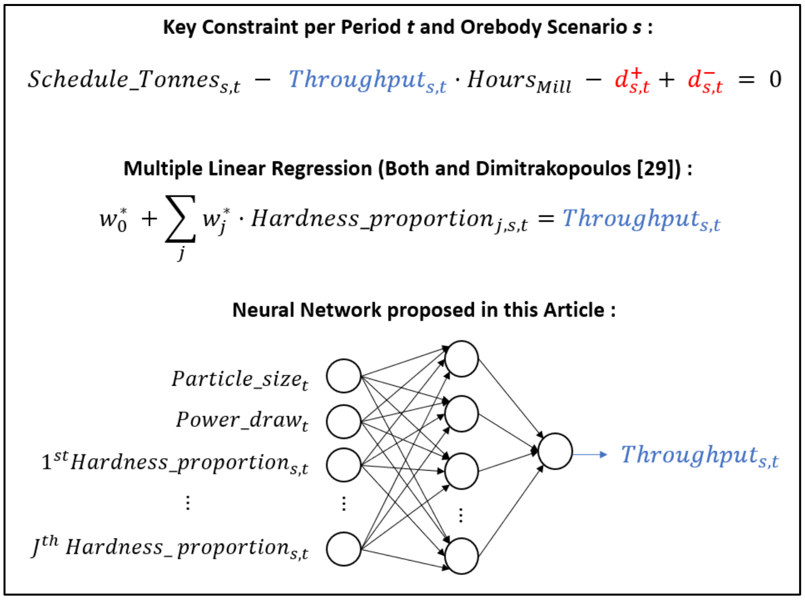


Figure 4-10 Comparison of models for ball mill throughput prediction and integration into short-term production scheduling

The stochastic constraint shown in Figure 4-10 ensures that for every period and simulated orebody scenario, the scheduled ore tonnage equals the tonnage resulting from the predicted hourly throughput and available mill hours. The deviation variables, $d^{+}_{,t,s}$ and $d^{-}_{,t,s}$, penalize deviations between the scheduled tonnage and realizable mill tonnage in the objective function of the mathematical program, which is discussed in detail in Both and Dimitrakopoulos (2021b). The hardness proportions serving as input to the neural network represent the weighted hardness proportions of the materials to be scheduled together in a single short-term period. Furthermore, the planned power draw, as well as the planned feed and product particle sizes for the future scheduled materials, can now serve as input to the production scheduling optimization, since the neural network has been trained on these attributes. Note that non-linear production scheduling formulations combined with a metaheuristic solution method, such as simulated annealing, can handle these internal non-linear computations in the optimization process, which have been developed for long-term and short-term planning (Goodfellow and Dimitrakopoulos, 2016; Both and Dimitrakopoulos, 2020).

4.6 Conclusions

This article shows a case study at the Tropicana Gold mining complex that demonstrates improvements of a geometallurgical throughput prediction model using collected production data in mines and processing plant combined with supervised machine learning. The key improvements over a previous publication are: (i) including and testing the influence of measurements in the comminution circuit that likely affect ball mill throughput rates in a non-linear way, (ii) utilizing a supervised learning model in the form of a neural network to approximate non-linear relationships between predictor and response variables, and (iii) testing if compositional approaches can account for non-additive geometallurgical variables better than average-type information. Finally, recommendations are given on how to integrate the prediction model into short-term production scheduling.

Results show that adding ball mill power draw and product particle size measurements can decrease the prediction error of throughput by 10.6 % compared to throughput prediction using geological hardness variables only. This result can only be achieved with the trained neural network, whereas the linear regression model shows improvements of up to 4.2 %. Available feed size measurements in the presented case study appear too imprecise to positively affect the throughput prediction. A neural network structure of one hidden layer comprising thirty neurons delivers the most stable predictions and shows the lowest error variance. However, the advantages of the neural network are partly offset by the more time-intensive hyperparameter search compared to the linear model, which is easy to apply and shows comparative performance in some cases.

Finally, hardness proportions decrease the prediction error compared to the use of averages of penetration rates. This underlines the importance of compositional approaches for non-additive geometallurgical variables. A key takeaway is that the shown compositional approach is not limited to ore hardness variables. Instead, it is conceivable to utilize compositional approaches for other non-additive (geometallurgical) variables as well.

Future work aims to create more data-driven prediction models of metallurgical responses in mining complexes using production data generated in the mines and processing plants. Next to the demonstrated prediction of comminution performance, the data-driven prediction of metal recovery, consumption of reagents, and other revenue and cost factors should be considered. The integration of these prediction models into decision-making processes, such as short-term production scheduling, is pertinent for meeting key production targets in mineral value chains.

4.7 Chapter Discussion and Next Steps

Chapter 4 shows how prediction accuracy of the developed throughput prediction model in Chapter 3, relying on geometallurgical rock attributes, can be improved by including power and product particle size measurements of the processing plant and the use of a neural network to unveil non-linear relationships within the enlarged feature set. Feed particle size measurements are also tested but appear too noisy in the underlying dataset to significantly reduce the prediction error and ball mill utilization rates appear to be a redundant feature compared to ball mill power in this case study. It appears that also after adding processing plant measurements to the prediction model, variability in the observed throughput rate is not yet fully explained. From this point, several research avenues are possible. More parameters of the closed-loop grinding cycle, consisting of ball mill and hydrocyclone, may be examined and included in the prediction model, such as circulation load, slurry viscosity, solid concentration, mill load, and more. Operational changes in the fragmentation stages of blasting and crushing that influence ore characteristics beyond particle size distributions may be considered as well. Further reduction of throughput prediction errors can potentially be achieved also from utilizing machine learning techniques that can unveil existing temporal dependencies in the data (Hochreiter and Schmidhuber, 1997; Cho et al., 2014). Improvements are also expected if the accuracy of material tracking in pits and stockpiles can be enhanced, as it is outlined in the section of future research. Chapter 5 continues with the expansion of geometallurgical prediction models to other pertinent metallurgical responses observed in processing plants, which can further improve the short-term decision-making in mining complexes.

5 Utilization of Geometallurgical Predictions of Processing Plant Reagents and Consumables for Production Scheduling under Uncertainty

The previous two chapters created prediction models of ball mill throughput using production data collected in mines and processing plants from an operating mining complex. This chapter extends the empirical prediction of metallurgical responses at the operating scale of the processing plant and their incorporation into short-term stochastic production scheduling in mining complexes to consumption rates of reagents and consumables. Specifically, empirical prediction models of caustic soda, and hydrochloric acid, and grinding media consumption rates are created at the Tropicana Gold mining complex by tracking blended rock properties that are matched with observed consumption rates at the operating processing plant. A risk analysis quantifies the uncertain reagent consumptions of a conventional short-term production schedule and improvements of simultaneously optimizing short-term extraction sequence and destination of materials in the mining complex while accounting for better informed reagent consumptions are demonstrated.

5.1 Introduction

Geometallurgy aims to describe, model, and exploit the relationships between spatially distributed rock characteristics to be extracted from mineral deposits and their impact on processes further downstream in a mineral value chain. Geometallurgical relationships have long aided the design and risk assessment of processing plants (Dobby et al., 2004; Bulled and McInnes, 2005; Powell, 2013; Bueno et al., 2015). However, the impacted processes are not restricted to metallurgical processing plants; rather, they can comprise activities in mines, waste dumps, tailings, material transport, smelting, and others. Today, geometallurgy is seen as interdisciplinary approach to maximize the economic value and reduce the technical risk of the entire value chain (McKay et al., 2016; Dominy et al., 2018). One important aspect of the modern geometallurgical approach is the incorporation of geometallurgical components, as well as their geological uncertainty, into mine production scheduling (Dunham et al., 2011; Coward and Dowd, 2015; Dowd et al., 2016).

Some stochastic optimization models incorporate geometallurgical information into long-term production scheduling by including geostatistically simulated metal grades, mill throughput rates, and metal recoveries that lead to adjusted revenues, mining costs, processing costs, and processing durations per mining block (Kumral, 2011; Morales et al., 2019). However, by expressing metallurgical responses block by block, the models ignore that materials are blended and converted into products by non-linear metallurgical processes. Blending problems especially occur if non-additive properties of the rock are involved (Bye, 2011; Newton and Graham, 2011; Kumar and Dimitrakopoulos, 2019). Recently developed simultaneous stochastic optimization methods shift the focus away from economic block values towards maximizing the value of products generated in mineral value chains (Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016, 2017; Montiel et al., 2016). These non-linear optimization methods are well suited for integrating geometallurgical aspects that link uncertain orebody attributes of the rock to non-linear metallurgical responses in processing plants and other downstream facilities. To date, only geometallurgical properties related to hardness and grindability, and their effect on mill throughput, have been considered for the simultaneous stochastic optimization of mining complexes (Kumar and Dimitrakopoulos, 2019; Both and Dimitrakopoulos, 2021b).

While case studies for the incorporation of mill throughput and metal recovery into production scheduling exist in the technical literature, mine production scheduling models have yet to incorporate geometallurgically informed consumption rates of reagents and consumables that go beyond simple cost estimates per ton of ore or consumption per rock type. Reagents and consumables have been identified in the technical literature as important aspects of geometallurgical modelling due to their essential role in converting mineral reserves into products, their environmental impacts, and their direct contribution to processing costs (Richmond and Shaw, 2009; Bye, 2011; Boisvert et al., 2013; Dominy et al., 2018). Richmond and Shaw (2009) mention the consumption of cyanide and hydrochloric acid used in gold leaching processes as part of such metallurgical responses that should be considered in geometallurgical models. Bye (2011) uses a processing cost adjustment factor per mining block that increases processing cost due to increased grinding media usage and energy consumption as a function of ore hardness. However, cost adjustment factors oversimplify responses in value chains by ignoring blending, non-additivity of hardness properties, and effects on other related processes.

Some geometallurgical prediction models aim to forecast metallurgical responses in processing plants directly instead of estimating a geometallurgical response for each individual block within a spatial geometallurgical model. Suazo et al. (2010) develop a monthly prediction model of copper recovery for the flotation circuit at the Collahuasi copper mine in Chile. Geometallurgical prediction models have also been developed to forecast the monthly throughput of comminution circuits based on spatially distributed hardness and grindability indices (Flores, 2005; Alruiz et al., 2009; Amini et al., 2021). Other authors have developed geometallurgical proxy models using several rock attributes to predict metallurgical plant responses, which can be described as primary-response frameworks (Coward et al., 2009). Boisvert et al. (2013) use mineral compositions including metal grades, grain size distributions, and rock density of drill core samples to predict metallurgical copper and uranium recoveries as well as acid consumption used in a leaching process and two grindability indices. The prediction models consist of several multiple linear regression (MLR) models, which are applied on a reduced set of supersecondary variables. Lishchuk et al. (2019) use various machine learning models for predicting the metallurgical response variables mass pull and iron recovery using metal grades and rock density from drill cores in an iron ore deposit.

All described models above rely on laboratory tests performed on samples obtained from drilling activities, which provide the metallurgical response to be predicted. One common challenge for these models is scaling up the laboratory results of metallurgical responses to processing plants on an industrial level (Richmond and Shaw, 2009; Suazo et al., 2010; Garrido et al., 2019; Ortiz et al., 2020). In addition, the number of samples containing all relevant geometallurgical data is typically very sparse, although research has been conducted to facilitate geometallurgical sampling through the development of small-scale tests and new data measurement tools (Keeney and Walters, 2011). As a result, the support (volume) of the samples required for geometallurgical testing can vary between the geometallurgical variables, which makes the spatial estimation and simulation of these attributes challenging (Deutsch et al., 2016).

Recently, there have been efforts to utilize recorded production data at the processing plants to build empirical prediction models of metallurgical responses directly in the operational support scale. Carpenter and Saunders (2017) use observed throughput rates of the grinding circuit at the Phu Kham mine, Laos, to build a throughput prediction model based on monthly mill feed proportions of lithology types. Similarly, Carpenter et al. (2018) use production data at the Ban Houayxai mine, Laos, to build a prediction model for gold recovery. Monthly residual gold grades in the tailings fraction of the operating processing plant serve as observations that are matched with monthly feed grades of copper, lead, zinc, silver, and the acid neutralizing capacity of the rock to predict future recovery rates. Both and Dimitrakopoulos (2021a) utilise a spatial dataset of penetration rates from blasthole drilling to inform ore hardness and plant measurements, such as power draw and particle size distributions, as inputs to a neural network to predict ball mill throughput rates in a mining complex. Material tracking of extracted materials from their origin in the ground towards the processing facilities plays an important role for the empirical prediction models mentioned. This way, the spatial rock characteristics of the mineral reserve can be linked to the observed production data at the processing plant in any desired interval spanning from hours to weeks (Wambeke et al., 2018; Both and Dimitrakopoulos, 2021b).

Observed consumption rates of reagents and consumables in operating processing plants are suitable candidates to build data-driven prediction models that are similar to the case studies discussed in the previous paragraph. These models can allow for the improved assessment of future utilization, as well as the cost of reagents and consumables based on pertinent geometallurgical attributes. Note that the predictions are not performed per mining block but at the operational scale of blended and processed materials. However, the mineral value chain can only be optimized if the gained geometallurgical knowledge of consumption rates is integrated into decision-making processes, such as in short-term production scheduling, which has had little attention in the literature thus far. Both and Dimitrakopoulos (2021b) integrate an empirically created geometallurgical throughput prediction model into simultaneous stochastic short-term production scheduling in mining complexes. The simultaneous prediction of throughput rates and production scheduling ensures that future scheduled materials match the rock-dependent performance of the ball mill, while accounting for geological uncertainty and non-additive hardness properties. The integration of predicted consumption rates can similarly lead to more informed production schedules moving away from fixed cost and profit assumptions per ton of rock towards a risk-based integration of geometallurgical attributes.

This article creates geometallurgical prediction models of consumption rates for reagents and consumables from production data in a gold mining complex and utilizes these prediction models in a simultaneous stochastic optimization model for short-term production scheduling. In the method section, the creation of the empirical geometallurgical prediction models is shown first: the models utilize blended rock attributes as predictive variables and observed consumption rates of reagents and consumables from an operating gold leaching plant as response variables. Subsequently, it is shown how the prediction models can be integrated into a non-linear, stochastic integer program for short-term production scheduling. A case study at the Tropicana Gold mining complex in Western Australia shows the benefits of the proposed approach and compares the newly created stochastic production schedule to a short-term production schedule that is generated conventionally.

5.2 Method

The approach of integrating geometallurgical predictions of consumables and reagents into short-term stochastic production scheduling in mining complexes comprises two main steps, shown in Figure 5-1. First, the prediction models for processing plant reagents and consumables are created utilizing rock attributes of blended materials and the observed consumption rates of reagents and consumables at the plant, which is described in detail in the following section. The stochastic optimization formulation integrating the created prediction models into short-term production scheduling is presented subsequently.

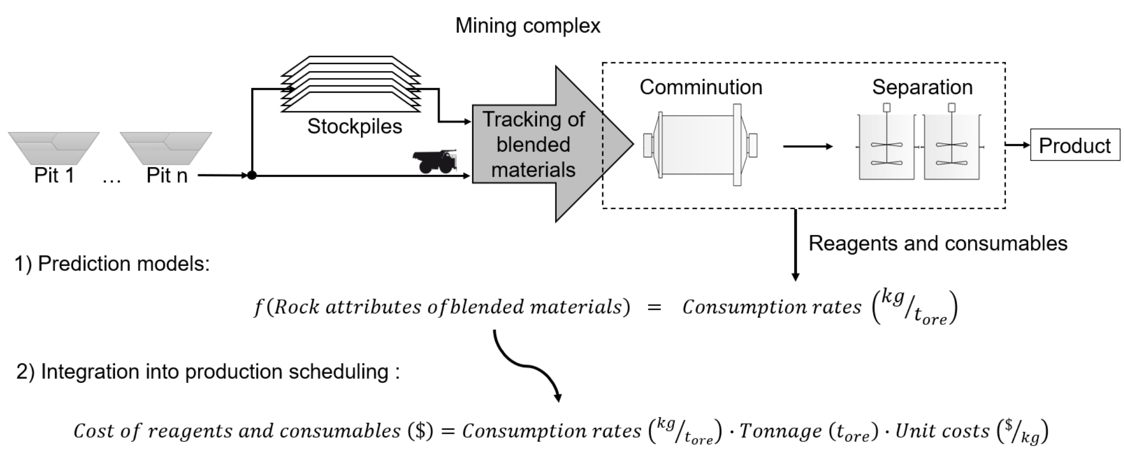


Figure 5-1 Concept of creating prediction models of consumables and reagents in a mining complex using production data and integration of prediction models into production scheduling

5.2.1 Prediction models for processing plant reagents and consumables

Several multiple linear regression models are constructed to predict the consumption rates of individual reagents and consumables as a function of rock properties of blended materials processed in the plant. The consumption rates of reagents and consumables (response variables) are taken directly from observed production data of the processing plant, whereas the rock properties of blended materials (predictor variables) are obtained using a material tracking approach in the mining complex.

Material tracking uses individual recorded truck cycles from a fleet management system to replicate the material movements of extracted materials from the mining faces to the processing plant in detail. This includes the tracking of materials sent to, and reclaimed from (short-term) run-of-mine stockpiles (Wambeke et al., 2018; Both and Dimitrakopoulos, 2021b). Spatial rock characteristics are provided by geological orebody models informing the spatial distribution of material types, rock density, and metal grades. Note that the presented approach can account for geological uncertainty by allowing sets of geostatistical orebody simulations (Goovaerts, 1997; Rossi and Deutsch, 2014) as inputs for material tracking (e.g., simulated metal grades), resulting in multiple equiprobable scenarios of blended rock attributes at the processing plant.

5.2.1.1 Regression models

In the set of all reagents and consumables, $\mathbb{C}$, let c be the index of a reagent or consumable, whose consumption rates will be predicted using multiple linear regression. In the c^{th} regression model, the i^{th} observed consumption rate, $y_{c,i}$, $i = 1, \ldots, N$, is expressed as a linear combination of M_c predictor variables, $h = 1, \ldots, M_c$, related to different rock attributes of the blend, as shown in Eq. (5.1).

$$Consumption\ rate_{c,i}\ \left({}^{kg}/_{t_{ore}}\right) = y_{c,i} = w_{0,c,s} + \sum_{h=1}^{M_c} w_{c,h,s} \cdot f_{c,i,h,s} + e_{c,i,s} \tag{5.1}$$

Some rock attributes, such as metal grades, are provided in the form of a set of orebody simulations. Thus, the parameter $f_{c,i,h,s}$ denotes the tracked rock attribute, h, of the blended material stream in an orebody scenario, $s \in \mathbb{S}$, belonging to the observed consumption rate, $y_{c,i}$. Note that $w_{0,c,s}$ and $w_{c,h,s}$ are the regression weights to be optimized and $e_{c,i,s}$ represents the prediction error. For each reagent or consumable, c, and orebody scenario, s, the sum of squared errors, $SSE(\boldsymbol{w}_{c,s})$, shown in Eq. (5.2), is minimized with respect to the weight vector $\boldsymbol{w}_{c,s} = [w_0, w_1, \dots, w_{M_c}]$.

$$SSE(\boldsymbol{w}_{c,s}) = \sum_{i=1}^{N} (e_{c,i,s})^2 = \sum_{i=1}^{N} (\boldsymbol{w}_{c,s}{}^T \boldsymbol{f}_{c,i,s} - y_{c,i})^2 \tag{5.2}$$

Here, $\boldsymbol{f}_{c,i,s} = [1, f_{c,i,1,s}, \dots, f_{c,i,M_c,s}]$ represents a vector of blended rock attributes within a single time interval to predict reagent or consumable, c. The closed-form solution to obtain the optimal weight vector, $\widehat{\boldsymbol{w}}_{c,s}$, is shown in Eq. (5.3) (Rencher and Christensen, 2012).

$$\widehat{\boldsymbol{w}}_{c,s} = (\boldsymbol{X}_{c,s}{}^T \boldsymbol{X}_{c,s})^{-1} \boldsymbol{X}_{c,s}{}^T \boldsymbol{y}_c \tag{5.3}$$

Eq. (5.3) uses the feature matrix, $\boldsymbol{X}_{c,s} \in \mathbb{R}^N \times \mathbb{R}^{M_c+1}$, and the vector of all observations for a reagent or consumable, $\boldsymbol{y}_c \in \mathbb{R}^N$. It should be noted that a different number of rock attributes, $h = 1, \dots, M_c$, can be used for each reagent or consumable. For example, consumables of the comminution circuit, such as grinding media, may be more sensitive to certain rock types, or elemental compositions, whereas other rock types or elements in the ore feed affect reagent consumptions in the leaching process. A preliminary correlation analysis is required to obtain the feature sets for each reagent or consumable.

5.2.1.2 Uncertainty quantification

Given that one or more simulated attributes are used for the prediction of the consumption rate of a reagent or consumable, c, a set of optimal weight vectors, $\{\widehat{\boldsymbol{w}}_{c,1}, .., \widehat{\boldsymbol{w}}_{c,S}\}$, one for each orebody scenario, can be obtained. The modeller has two choices in terms of how to utilize this set of weights in successive short-term production scheduling. One option is to use the complete set of weight vectors, which can be interpreted as an assessment of uncertainty of the prediction model itself. This modelling approach leads to a large risk envelope when predicting reagent consumption rates for future scheduled materials and increases computational complexity by adding another source of uncertainty to the optimization. Alternatively, the weight vector with the best-performing statistics can be used. For example, this can be the weight vector with the largest Pearson correlation coefficient, or the smallest root mean squared error (RMSE) obtained by predictions on test data. In this case, the weights that fit best with real plant observations are carried forward, whereas poorly fitting weights are discarded and prevented from artificially enlarging the risk spread of consumption rates for future scheduled materials. Note that by choosing the best-performing weight vector, the ability of quantifying geological risk is not lost, since geological orebody scenarios of future materials can be utilized on either one or multiple weight vectors. In this article, the best-performing weight vector is used, reducing computational complexity for the mine production scheduling, which is presented next.

5.2.2 Integration of prediction models into simultaneous stochastic short-term production scheduling

The mathematical optimization model that integrates the prediction models of reagents and consumables into short-term production scheduling in mining complexes is formulated as a non-linear stochastic mixed integer program. The formulation builds upon recent developments in long-term production scheduling, which simultaneously optimize the extraction sequence, the destination of extracted materials, and downstream material flow decisions in mining complexes (Goodfellow and Dimitrakopoulos, 2016, 2017).

5.2.2.1 Modelling a mining complex

The mining complex is modelled as a set of locations, $\mathbb{L}$, where material is extracted from a set of mining areas, $\mathcal{A} \subset \mathbb{L}$, and sent to processing facilities, $\mathcal{P} \subset \mathbb{L}$, stockpiles, $\mathcal{S} \subset \mathbb{L}$, and waste dumps, $\mathcal{W} \subset \mathbb{L}$. The extracted materials have associated primary attributes, $p \in \mathbb{P}$, such as mass or metal quantity, which flow through the mining complex and are quantified by the variable $v_{p,l,s,t}$ at each location, *l*, orebody scenario, *s*, and period, *t.* The non-linear optimization formulation allows the calculation of derived attributes at each location in the mining complex, so-called hereditary attributes, $h \in \mathbb{H}$. Linear and non-linear functions can be used to compute the quantity of a hereditary attribute, $v_{h,l,s,t} = f_{h,l}(v_{p,l,s,t})$. The quantity of recovered metal at a processing location, $l \in \mathcal{P}$, is a hereditary attribute, which can be calculated as a function of total metal received per period, accounting for a non-linear metal recovery function.

The consumption rate of reagents and consumables, $CR_{c,l,s,t}$, is a novel hereditary attribute that is developed and added to the optimization formulation in this article. The consumption rate is computed in kg per tonne of ore for each consumable *c* in location *l* in orebody scenario *s* for period *t* as shown in Eq. (5.4).

$$CR_{c,l,s,t} = \widehat{w}_{0,c} + \sum_{h \in \mathbb{H}_c} \widehat{w}_{h,c} \cdot v_{h,l,s,t} \; \forall \; t \in \mathbb{T}, s \in \mathbb{S}, l \in \mathcal{P}, c \in \mathbb{C} \tag{5.4}$$

The set of hereditary attributes relevant to predict a specific consumption rate is denoted $\mathbb{H}_c \; \forall \; c \in \mathbb{C}$. The optimized weights, $\widehat{w}_{0,c}$ and $\widehat{w}_{h,c}, h \in \mathbb{H}_c$, are received from the prediction models built in the previous step in this article. Note that the developed prediction models are based on the rock attributes of blended materials, such as the blended gold grade (head grade), or the fraction of weathered materials. These are hereditary attributes in the optimization model since the primary, additive attributes, such as the total gold processed per period, are divided by the total ore tonnage per period. Note that the ability to calculate consumption rates of blended materials within the production scheduling optimization is a distinct feature of the non-linear simultaneous optimization formulation. In contrast, linear production scheduling optimization formulations need to make simplifications by calculating the economic value of each mining block individually to maintain linearity in the model.

5.2.2.2 Decision variables

The simultaneous stochastic optimization of mining complexes (Goodfellow and Dimitrakopoulos, 2016, 2017) includes multiple jointly optimized decision variables. Block extraction variables, $x_{b,t} \in \{0,1\}$, define whether (1) or not (0) a block $b \in \mathbb{B}$ is mined in period $t \in \mathbb{T}$. The destination of the extracted materials is defined by the variable $z_{g,l,t} \in \{0,1\}$, which equals one if a group of material, $g \in \mathbb{G}$, is sent to location l in period t, and zero otherwise. The group membership parameter, $\theta_{g,b,s} \in \{0,1\}$, indicates whether (1) or not (0) block b is a member of material group g in scenario s. A pre-processing step defines the scenario-dependent group membership of each block, depending on the primary attributes of the block, $\beta_{b,p,s}$. This way, a destination policy based on any desired material attribute can be optimized simultaneously. For example, the materials can be grouped into grade bins using metal grades, enabling the simultaneous optimization of a production schedule and cut-off grades per period (Menabde et al., 2007), which is discussed in more detail in the case study. Alternatively, a clustering approach can be used to assign group memberships based on multiple elements (Goodfellow and Dimitrakopoulos, 2016). The third type of decision variables are the downstream flow variables, $\delta_{l,l',s,t} \in [0,1]$, which represent the proportion of material sent out from location l to a receiving destination l' in scenario, s, and period, t. These variables are typically used for defining the amount of stockpiled materials sent to a processing facility per period. Contrary to extraction and destination decisions, downstream flow variables are scenario-dependent, assuming that the uncertainty of materials is revealed once arriving at its first destionation. Additionally, the continuous variables, $d^{+}_{h,l,s,t} \in [0,1]$ and $d^{-}_{h,l,s,t} \in [0,1]$, are the scenario-dependent deviation variables that model either a surplus (+) or a shortage (-) of hereditary attributes, h, at location l in period t for orebody scenario s. The associated penalty costs for a shortage (-) or an excess (+) of the attribute, h, are denoted with $c^{-}_{h,l}$, $c^{+}_{h,l}$. Finally, the integer smoothing variable, $y_{b,t}$, indicates the number of adjacent blocks of block b that are not extracted in the same period.

5.2.2.3 Objective function

The objective function of the optimization model integrating the predicted consumption rates of reagents and consumables into short-term production scheduling in mining complexes is shown in Eq. (5.5).

$$Max \ \frac{1}{|\mathbb{S}|}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\sum_{l\in P}\left\{\underbrace{\sum_{h\in\mathbb{H}_p} p_{h,l}\cdot v_{h,l,s,t}}_{Part\ 1} \underbrace{-\sum_{c\in\mathbb{C}} rc_c\cdot CR_{c,l,s,t}\cdot v^{ore}_{l,s,t}}_{Part\ 2} \underbrace{-\sum_{h\in\mathbb{H}_{pc}} pc_{h,l}\cdot v_{h,l,s,t}}_{Part\ 3}\right\}$$

$$-\frac{1}{|\mathbb{S}|}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\left\{\underbrace{\sum_{l\in\mathcal{S}}\sum_{h\in\mathbb{H}_s} rh_{h,l}\cdot v_{h,l,s,t}}_{Part\ 4} + \underbrace{\sum_{l\in\mathcal{A}}\sum_{h\in\mathbb{H}_m} mc_{h,l}\cdot v_{h,l,s,t}}_{Part\ 5}\right\}$$

$$\underbrace{-\frac{1}{|\mathbb{S}|}\sum_{h\in\mathbb{H}}\sum_{l\in\mathbb{L}}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\left(c^{+}_{h,l}\cdot d^{+}_{h,l,s,t} + c^{-}_{h,l}\cdot d^{-}_{h,l,s,t}\right)}_{Part\ 6}$$

$$\underbrace{-\sum_{t\in\mathbb{T}}\sum_{b\in\mathbb{B}} c^{smooth}\cdot y_{b,t}}_{Part\ 7} \tag{5.5}$$

Part 1 of the objective function sums the revenues from products produced in the processing facilities $l \in \mathcal{P}$. All hereditary attributes that generate a profit, $h \in \mathbb{H}_p$, have an associated unit selling price, $p_{h,l}$.The costs generated in the processing facilities of the mining complex are split into two parts in this article. Part 2 of the objective function reflects the newly added component of accounting for variable consumption rates of reagents and consumables in the optimization based on rock attributes. This stands in contrast to the conventional and simplified approach of modelling processing costs solely by the tonnage of ore processed. In this work, the costs of reagents and consumables are determined by their predicted consumption rates in kg per tonne, $CR_{c,l,s,t}$, which are based on the rock attributes of the blended materials processed, as seen in Eq. (5.3). Geological uncertainty is accounted for by predicting a consumption rate per orebody scenario, *s*. For each reagent and consumable, their individual consumption rates are multiplied with their unit costs, rc_c, and the ore tonnage processed per period, $v_{l,s,t}^{ore}$. This way, the ore processing costs related to reagents and consumables are more realistically modelled, because they now directly depend on influencing rock attributes (metal grades, deleterious elements, rock type proportions, and more), as well as their geological uncertainty. Part 3 of the objective function considers all other remaining costs at the processing plant. Those costs, with unit cost $pc_{h,l}$, can be either directly linked to the processed ore tonnage, or to some other processing-related hereditary attributes, $\mathbb{H}_{pc}$. Part 4 models rehandling costs, $rh_{h,l}$, generated at stockpiling facilities, $l \in \mathcal{S}$, considering ore tonnage and other hereditary attributes, $\mathbb{H}_s$. Part 5 sums the mining costs generated in mining areas, $l \in \mathcal{A}$. Different unit costs, $mc_{h,l}$, can be applied, considering that drilling, blasting, loading, and hauling costs vary depending on different rock attributes, pit depth, and other mining-related attributes, $\mathbb{H}_m$. Part 6 of the objective function penalizes all positive or negative deviations from production targets in the mining complex. Part 7 enforces physically mineable shapes for short-term production schedules using the smoothing variable, $y_{b,t}$, and a penalty cost, c^{smooth}, to group mining blocks in connected (smooth) patterns (Dimitrakopoulos and Ramazan, 2004).

5.2.2.4 Constraints

The simultaneous stochastic optimization model is subject to the following constraints:

a) Mining constraints

$$\sum_{t \in \mathbb{T}} x_{b,t} = 1 \ \forall \ b \in \mathbb{B} \tag{5.6}$$

$$x_{b,t} \leq \sum_{t'=1}^{t} x_{u,t'} \ \forall \ b \in \mathbb{B}, u \in \mathbb{O}_b, t \in \mathbb{T} \tag{5.7}$$

$$\|K_b\| \cdot x_{b,t} - \sum_{b' \in K_b} x_{b',t} \leq y_{b,t} \;\; \forall \, b \in \mathbb{B} \, , t \in \mathbb{T} \tag{5.8}$$

Constraints (5.6) ensures that every block in the short-term horizon is exactly mined once. Note that the blocks to be extracted in the short-term come from the long-term plan and must be mined out completely to follow the long-term objectives. Constraints (5.7) are predecessor constraints that only allow the extraction of a block if the set of overlaying blocks,$\mathbb{O}_b$, has been extracted. Smoothing constraints (5.8) count the number of surrounding blocks, K_b, not extracted in the same period as block b, which is penalized in Part 7 of the objective function.

b) Capacity constraints

$$v_{h,l,s,t} - d^{+}_{h,l,s,t} \leq U_{h,l,t} \; \forall \; h \in \mathbb{H}, l \in \mathbb{L} \, , s \in \mathbb{S} \, , t \in \mathbb{T} \tag{5.9}$$

$$v_{h,l,s,t} + d^{-}_{h,l,s,t} \geq L_{h,l,t} \; \forall \; h \in \mathbb{H}, l \in \mathbb{L} \, , s \in \mathbb{S}, t \in \mathbb{T} \tag{5.10}$$

$$v_{h,l,s,t} = f_{h,l}\big(v_{p,l,s,t}\,\big) \; \forall \; h \in \mathbb{H}, l \in \mathbb{L} \, , s \in \mathbb{S} \, , t \in \mathbb{T} \tag{5.11}$$

Capacity constraints (5.9) and (5.10) impose upper limits, $U_{h,l,t}$, and lower limits, $L_{h,l,t}$, on attributes at the various locations in the mining complex, such as limiting the tonnage mined per period, the ore tonnage sent to processing plants per period, and grade blending constraints. Whereas some geometallurgical capacity constraints have been added in case studies for long-term planning (Kumar and Dimitrakopoulos, 2019; LaRoche-Boisvert and Dimitrakopoulos, 2021), other short-term related capacity constraints can be included as well, modelling the capacities and requirements of the mining fleet and the throughput in processing facilities in more detail (Both and Dimitrakopoulos, 2020, 2021b). The hereditary attributes, $v_{h,l,s,t}$,, which can be constrained by the capacity constraints (5.9) and (5.10) at any location in the mining complex, can be derived linerarly or non-linearly from any primary attribute, as shown in constraints (11).

c) Destination constraints

$$\sum_{l \in \mathcal{O}(g)} z_{g,l,t} = 1 \; \forall \; g \in \mathbb{G} \;\, , t \in \mathbb{T} \tag{5.12}$$

Constraints (5.12) ensure that the materials belonging to the grade bin, or group, g, can only be sent to one location per period, which are chosen in the optimization from the set of available destinations, $\mathcal{O}(g)$. There are several additional constraints that calculate the quantity of primary attributes received at each location in the mining complex and ensure mass balancing at all destinations such as stockpiles and processing plants. These constraints are listed in detail in previous publications and can be retrieved from there (Goodfellow and Dimitrakopoulos, 2016, 2017).

5.2.2.5 Solution method

Although the short-term production schedules typically comprise less mining blocks than long-term production scheduling, the number of variables and constraints for the short-term optimization formulation can be large due to the optimization of many periods, stochasticity of multiple inputs, and simultaneous decision-making. Furthermore, non-linear constraints are an integral part of the simultaneous stochastic optimization of mining complexes. This is why a simulated annealing metaheuristic with multiple, adaptive neighborhoods (Goodfellow and Dimitrakopoulos, 2016, 2017; Both and Dimitrakopoulos, 2020) is used to solve the non-linear optimization formulation presented above. During the optimization, the solution vector containing extraction, destination, and downstream allocation decisions is modified by applying perturbation rules, which are commonly referred to as neighborhoods. Multiple neighborhoods are available to perturb the solution vector, such as changing the period of extraction of a block, changing the destination of a material group to another processor or stockpile, and changing the proportion of materials sent from stockpiles, among others (Goodfellow, 2014). The modified solutions are then accepted or rejected based on simulated annealing (SA) (Kirkpatrick et al., 1983; Geman and Geman, 1984b) until a certain number of iterations is reached. Modifications to the standard SA include multiple annealing temperatures depending on the type of decision variable that is perturbed (Goodfellow and Dimitrakopoulos, 2016) and adapting the probability of selecting perturbation rules depending on their performance of improving the objective function (Lamghari and Dimitrakopoulos, 2020).

5.3 A Case Study at the Tropicana Gold Mining Complex

The Tropicana Gold mining complex is located 330 km East-North-East of Kalgoorlie, Western Australia and consists of four open pits extending six kilometers in strike length from North to South. The mining complex is run as a typical drill and blast, truck and shovel operation and the ore is either hauled directly to the primary crusher or stockpiled at the run of mine stockpiles in the vicinity of the crusher. The gold is extracted in a single processing stream consisting of several comminution stages and a carbon-in-leach process. The case study demonstrates the prediction of reagents and consumables using collected production data at Tropicana and the optimization of a 12-month short-term production schedule of the mining complex using the mathematical model presented above. A process flow diagram of the processing plant at the Tropicana Gold mining complex is presented in Figure 5-2. The figure also indicates where individual reagents and consumables are added at different stages of the gold extraction process.

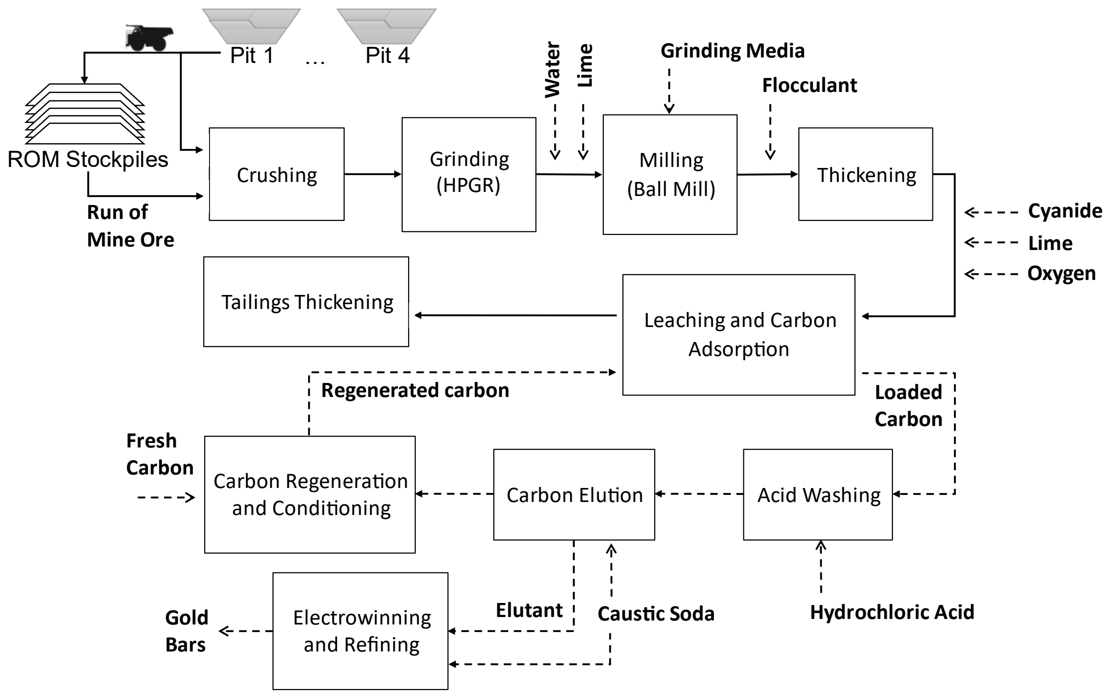
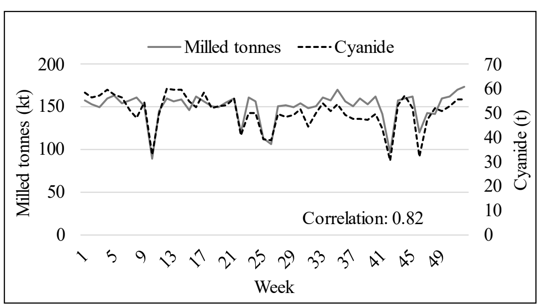
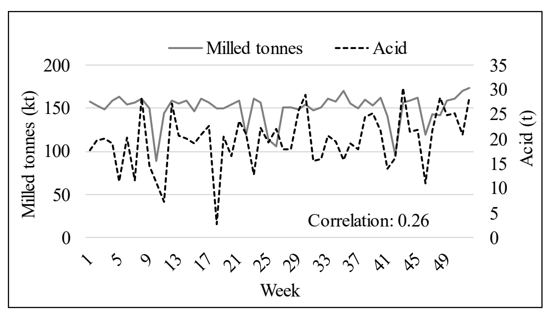


Figure 5-2 Process flow diagram of the comminution and carbon-in-leach (CIL) circuit at the Tropicana Gold mining complex, modified after Stange (1999)

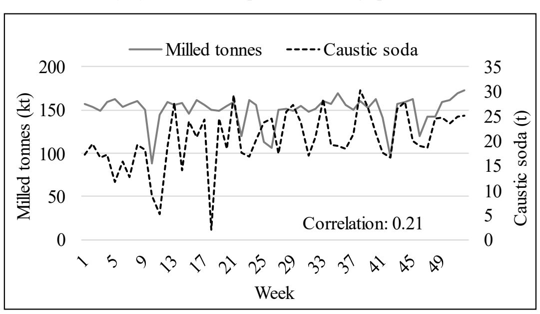
After extraction, the run of mine ore is sent to primary and secondary crushing using gyratory and cone crushers. A grinding stage follows using a high-pressure grinding roll (HPGR). Water and lime are added afterwards, creating a pulp that is sent to the ball mill. One of the most cost-intensive consumables in the processing plant are the grinding media inside of the ball mill consisting of steel balls that are cascading together with the rock inside the trommel. This way, the required particle size is achieved at which the gold is liberated sufficiently to be leached. The high-cost factor of grinding media consumption serves as incentive in this case study to monitor and predict its real-world consumption using the methodology introduced in this article. After milling, flocculant is added to the pulp for thickening. The carbon in the leach process requires the addition of cyanide, lime, and oxygen to the ore feed. Whereas the addition of lime is required to elevate the pH level of the solution to prevent the formation of deadly hydrogen cyanide gas, the cyanide dissolves the gold and forms an aurocyanide complex in the presence of oxygen (Elsner, 1846). The aurocyanide molecules are adsorbed onto activated carbon in the same set of agitators using the carbon-in-leach method. The loaded carbon is then sent to acid washing using hydrochloric acid, which helps remove any built up calcium carbonate, silica, sulfates, base metals and other undesired deposits on the carbon (Davidson and Veronese, 1979). At the carbon elution stage, the loaded carbon is sent through a stripping vessel at an elevated temperature where a strip solution consisting of cyanide and caustic soda is used to elude the aurocyanide from the carbon. Gold bars are obtained from the strip solution using electrowinning and refining, whereas the carbon is conditioned and regenerated, and sent back to the leaching and adsorption stage. The observed weekly total consumptions of several selected reagents (cyanide, hydrochloric acid, caustic soda) and consumables (grinding media) used at the processing plant of the Tropicana Gold Mine are displayed in Figure 5-3.



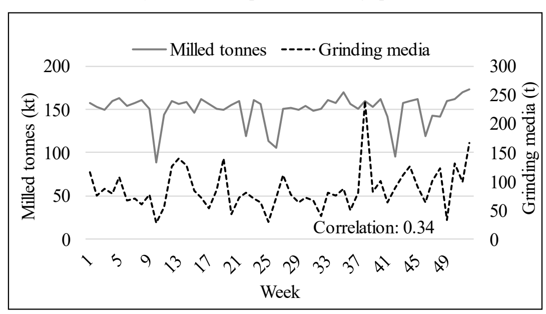
(a) Weekly cyanide consumption vs. tonnage processed



(b) Weekly acid consumption vs. tonnage processed



(c) Weekly caustic soda consumption vs. tonnage processed



(d) Weekly grinding media consumption vs. tonnage processed

Figure 5-3 Weekly comparisons of (a) cyanide consumption (b) acid consumption (c) caustic soda consumption and (d) grinding media consumption compared to weekly processed tonnage at the Tropicana Gold mining complex

It can be seen in Figure 5-3 that the weekly cyanide consumption highly correlates with the ore tonnage processed in the present dataset, showing a Pearson correlation coefficient (Eq. 13) of 0.82.

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2} * \sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{13}$$

However, the weekly consumptions of hydrochloric acid (r = 0.26), caustic soda (r = 0.21), and grinding media (r = 0.34), show a weak correlation with the weekly processed ore tonnage. Thus, these three consumption rates are chosen as candidates for prediction in this case study to unveil relationships with other pertinent contributing attributes of the rock in the ore feed. Next, the total weekly consumptions of acid, caustic soda, and grinding media are divided by the weekly ore tonnage resulting in consumption rates of kg per ton ore. This removes the remaining bias of larger consumptions when more ore is processed. Thereafter, a four-week moving average of consumption rates is built for two reasons. First, the observed total consumptions obtained by weekly silo filling levels show noisy behaviour, especially when larger shipments of reagents are received. A moving average emphasizes trends of higher and lower consumption rates that are more likely connected to rock attributes of the material processed. Second, the optimization of the mine production schedule comprises monthly periods. Thus, after applying the moving average, the data for training the prediction models fits the time scale and variability of consumption rates required for production scheduling. The rock attributes of the ore reserve used for the prediction of the chosen consumption rates are presented in Table 5-1. A feature selection step determines the utilized rock attributes for each regression model to predict the three consumption rates. The individual feature selections per consumption rate take into account the (linear) dependencies between predictor variables and response variables, expert knowledge, and a grid search on a smaller subset of features. As described in the method section and shown in Figure 5-1, the feature sets consist of blended materials that are processed at the same time intervals as the observed consumption rates.

Figure 5-4 shows the regression results for all three consumption rates using k-fold cross validation (five folds) (Hastie et al., 2009). K-fold cross validation allows the analysis for the entire dataset and avoids splitting the small dataset further (52 data points). Pearson's correlation coefficient is calculated, comparing observed consumption rates, y_i (blue color), to the predicted consumption rates, $\hat{y}_i$ (orange color) , $i = 1, \ldots, N$.

Table 5-1 Rock attributes available for prediction of consumption rates

Rock attributes	Simulated?	Unit	Choice for prediction of consumption rates:		
			Caustic soda	**Acid**	**Grinding media**
Gold grades	Yes	(g/t)	✓	✓	✓
Sulfur content	Yes	(%)		✓	
Density	No	(t/m^3)			✓
Regolith 4 (transitional rock)	No	Fraction (%)			✓

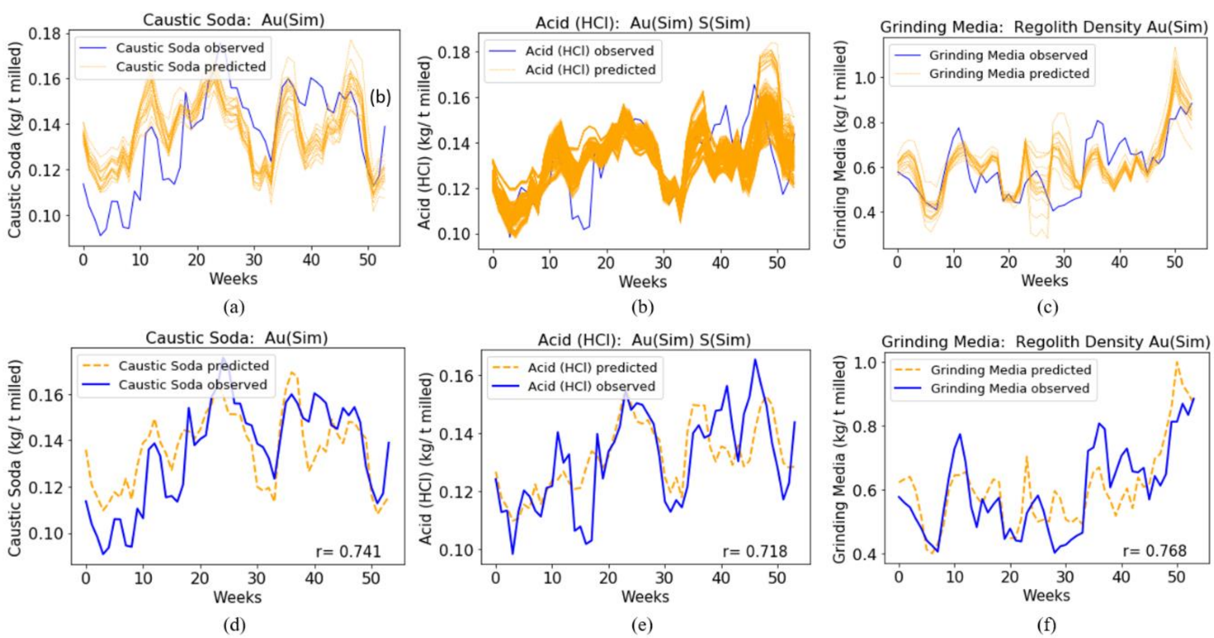


Figure 5-4 Geometallurgical predictions of caustic soda consumption rates (a, d), acid consumption rates (b, e) and grinding media consumption rates (c, f) using a group of orebody simulations to quantify uncertainty (upper row) and using best performing orebody simulations (lower row)

As noted in Table 5-1, gold grades and sulfur content are provided by a set of 20 geostatistical orebody simulations each (Remy et al., 2009). The complete set of orebody simulations can be used to create multiple fitted regression models, which are displayed for each consumption rate in Figure 5-4a, 5-4b, and 5-4c. Figure 5-4b includes all 400 combinations of simulated gold grades and sulfur content to predict acid consumption leading to a large number of weight vectors. Using these weight vectors in stochastic optimization largely increases computational complexity and includes weight vectors with poor fitting. Instead, a single orebody simulation of gold grades providing the highest correlation respectively for each consumption rate can be utilized together with the non-simulated rock attributes, which is shown in Figure 5-4d, 5-4e, and 5-4f. The best performing orebody simulation of sulfur content is added for the prediction of acid consumption in Figure 5-4e. Table 5-2 shows the regression weights obtained for the best performing predictions shown in Figure 5-4d, 5-4e, and 5-4f.

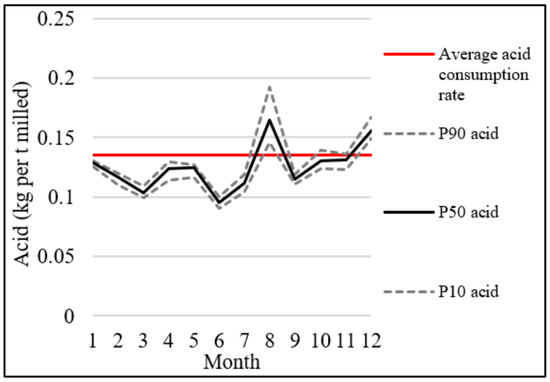
Table 5-2 Regression weights for prediction of consumption rates

Rock attributes	**Weights for prediction of consumption rates:**		
	Caustic soda	**Acid**	**Grinding media**
Intercept	-0.022	-0.061	-0.250
Gold (simulated)	0.099	0.092	0.382
Sulphur (simulated)		-0.030	
Density			0.081
Regolith 4			1.096

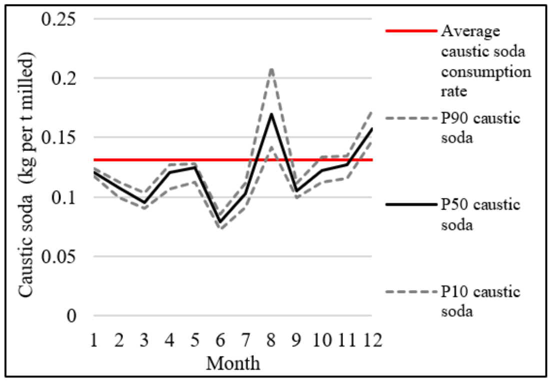
In this case study, the gold content of the processed ore appears to be the main driver to predict caustic soda and acid consumption rates. Soda and acid consumptions correlate strongly with each other (correlation coefficient of r=0.74, calculated using weekly consumption rates in 2018). Both reagents are added after carbon adsorption, which may be influenced by the amount of extracted gold adsorbed onto carbon. Whereas caustic soda consumption is more sensitive to gold grades, the prediction of acid improves when sulfur grades in the ore feed are accounted for. The fraction of transitional rock (Regolith 4) in the ore feed is identified as a critical driver for the forecast of grinding media consumption. The positive weight indicates abrasive behaviour leading to more grinding media consumption. The other rock types are omitted in the prediction because their fractions are either too low or lead to poor generalizations when added to the regression model. Higher gold grades and higher material density also appear to increase grinding media consumption in this case study, according to the obtained regression weights. Note that these empirical prediction models are site-specific and should only be used for future production planning of materials that are expected to behave similarly in processing plants. Therefore, the obtained prediction models are used next for short-term production scheduling within a yearly horizon. Also, major changes in the processing plant configuration can lead to large inaccuracies in forecasts when applying the empirically obtained prediction models proposed in this article. A new dataset stemming from the operation of the newly configured processing plant would need to be collected.

5.3.1 Risk analysis of conventional monthly production schedule

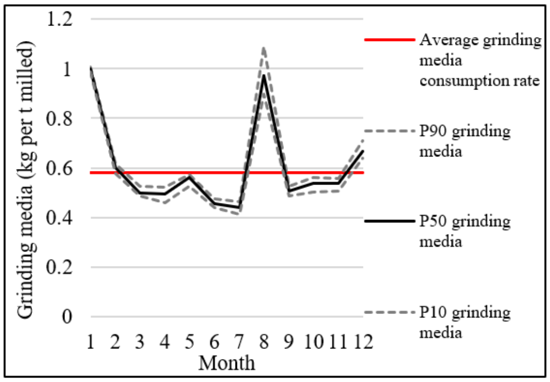
This section presents a risk analysis of reagent consumptions using the created prediction models to assess a conventional monthly production schedule at the Tropicana Gold Mining Complex for the year 2019. While the reagent consumptions are conventionally assumed constant per ton of processed rock or rock type, the created prediction models for reagents and consumables can provide a more nuanced forecast of reagent consumptions based on pertinent blended rock attributes. Figure 5-5 compares the predicted consumption rates in kg per tonne for caustic soda, acid, and grinding media, with their average consumption rates of the year 2018. The average consumption rates are shown in red and are calculated as the ratio of total yearly consumption of the reagent in the previous year divided by the total ore tonnage processed in the same year.



(a) Risk analysis of acid consumption rate



(b) Risk analysis of caustic soda consumption rate



(c) Risk analysis of grinding media consumption rate

Figure 5-5 Risk analysis of (a) acid consumption rate (b) caustic soda consumption rate and (c) grinding media consumption rate showing average consumption rate of 2018 (red lines) compared with monthly risk profile of 2019 production schedule using prediction models and blended rock attibutes of monthly extracted materials (P10, P50, P90)

As seen in Figure 5-5a, the predicted acid consumption rates for the 2019 production schedule fall below the average 2018 acid consumption rate in most of the periods, as shown by the risk profiles (P10, P50, P90). The risk profiles account for the different materials and their geological uncertainty of the blended rock attributes processed in the plant (simulated gold grades and sulfur content). Note that by using a single set of regression weights, shown in Table 5-2, uncertainty of reagent consumptions can still be quantified because every orebody simulation provides a different realization of blended materials at the processing plant. Table 5-3 shows the differences of total consumptions accumulated over the entire year of 2019.

Table 5-3 Risk analysis of acid, caustic soda and grinding media consumption for 2019 schedule

Reagent/ Consumable	**Average consumption rate 2018**	**Median predicted consumption rate (P50) 2019**	**Diff.**	**Max. diff. per period (P50)**
Acid (HCl)	0.131 kg/t ore	0.123 kg/t ore	-8.7 %	-29.3 %
Caustic soda	0.135 kg/t ore	0.117 kg/t ore	-10.6 %	- 39.5 %
Grinding media	0.579 kg/t ore	0.597 kg/t ore	+3.2 %	+ 72.4 %

The median predicted acid consumption rate for the entire year 2019 is 8.7 % smaller than the average 2018 consumption rate, while the difference per period (P50) can be as large as 29.3 %. This new predicted consumption rate is obtained by multiplying the median consumption rates per month (P50) with the monthly scheduled ore tonnage. The predicted consumption rates for caustic soda in Figure 5-5b show similar behaviour compared to the monthly predicted acid consumption rate. This is expected since both reagent consumptions are strongly correlated, which has already been discussed above. Note that both reagents are utilized for the treatment of loaded carbon after the carbon adsorption phase (compare Figure 5-2). Predicted caustic soda consumption rates show more variability than acid consumption rates, deviating by up to 39.5% per period, which may be explained by the even stronger dependency on the gold content in the ore feed compared to acid consumption. The median predicted consumption rate of caustic soda in 2019 is 10.6 % lower than the 2018 consumption rate. Figure 5-5c shows the monthly predicted grinding media consumption, whereas Table 5-3 reveals that the total median predicted consumption of grinding media exceeds the average 2018 consumption by 3.2 %. Periods one and eight largely exceed the average 2018 consumption rate, while other periods remain closer to the average grinding media consumption. Figure 5-6 shows pertinent blended rock attributes sent to the processing plant per period.

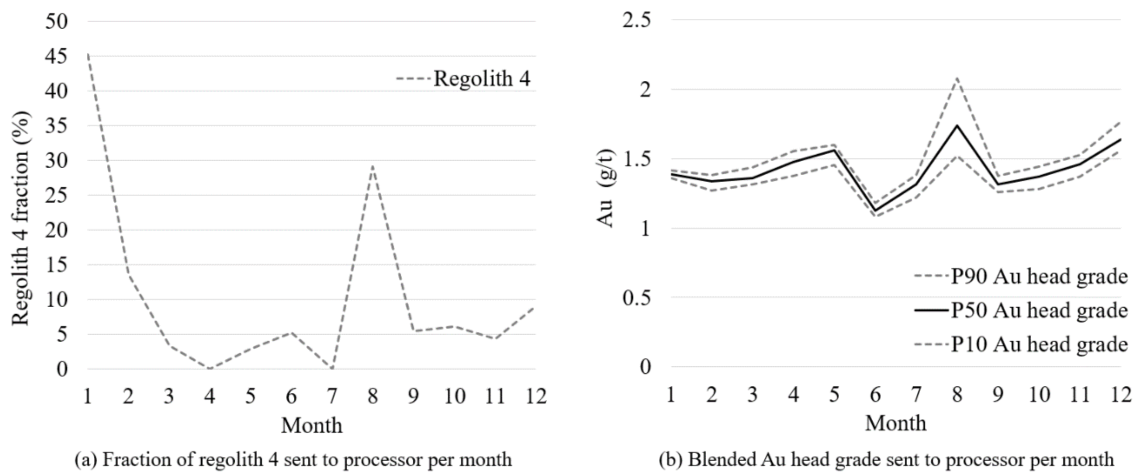
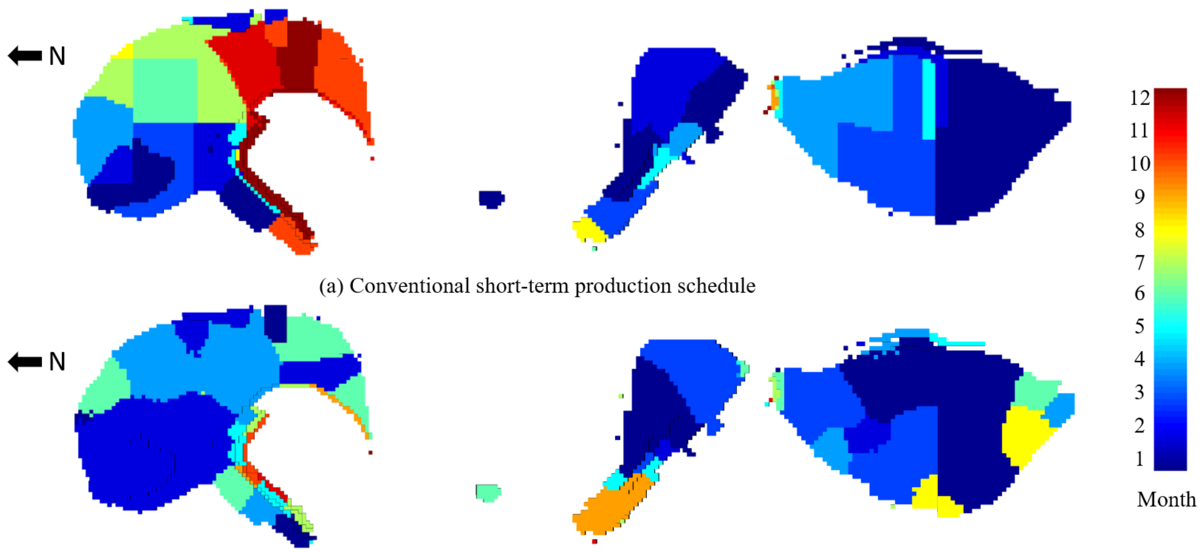


Figure 5-6 Risk of blended material attributes sent to the processor

Figure 5-6a demonstrates that the fraction of transitional material (Regolith 4) sent to the mill is much larger in periods one and eight than in other periods. This material is associated with high grinding media consumption, as the weights of the prediction model show. Furthermore, the gold content in period eight is high, as shown in Figure 5-6b. However, the set of geostatistical orebody simulations reveals a larger risk in gold content in period eight due to geological uncertainty. Clearly, period eight comprises distinct materials leading to different consumption rates which can be noticed for acid and caustic soda consumptions as well. The monthly and yearly predictions presented in this section account for these heterogeneous and uncertain material blends and can thus create awareness of the effects in processing plants due to changed ore feed characteristics and more accurate budget planning. The benefits of integrating the prediction models for stochastic production scheduling are presented in the next section.

5.3.2 Simultaneous stochastic short-term production scheduling using prediction models

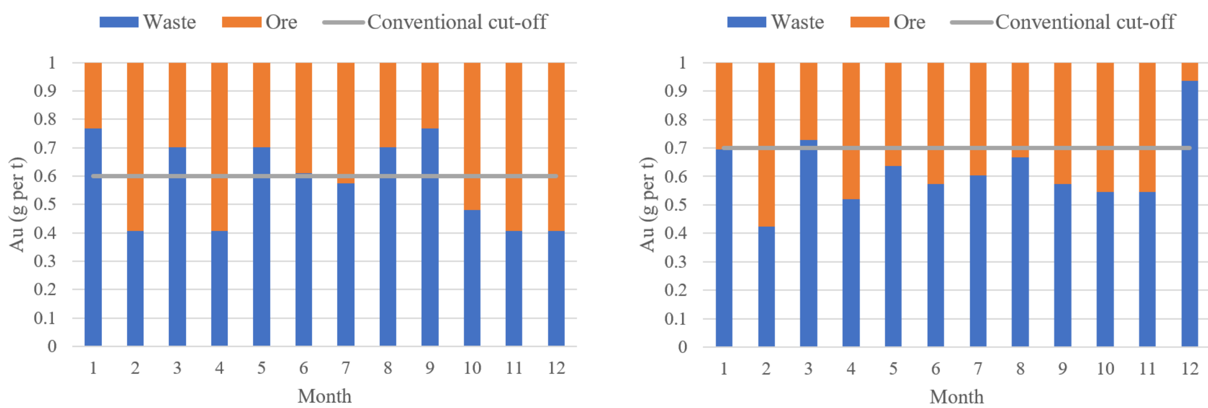
The stochastic optimization model integrating the prediction models of reagents and consumables presented above produces a monthly production schedule showing a different extraction sequence compared to the conventional production schedule of the Tropicana Gold mining complex. The schedules are displayed in a plan view in Figure 5-7. Both schedules take as input the same blocks, given by the long-term production schedule. The stochastic optimization model presented in the methodology section is capable of simultaneously optimizing the block extraction sequence and material destinations. The material destination strategy of the conventional production schedule is compared to the optimized material destinations from the simultaneous stochastic optimization in Figure 5-8. The conventional plan defines the material destination by a fixed cut-off grade of gold per material type for the complete year (grey lines). Conversely, the presented optimization model has the flexibility to change the cut-off grades per period, allowing different materials to be sent to the mill. These blended materials are chosen by the profit and cost that they generate in the processing facilities, whereas the costs of reagent consumptions are evaluated within the optimization by the newly added prediction models.



(b) Simultaneous stochastic short-term production schedule integrating predictions of reagents and consumables

Figure 5-7 Plan views of (a) conventional short-term production schedule and (b) simultaneous stochastic short-term production schedule integrating predictions of reagents and consumables

Table 5-4 shows the differences of key performance indicators between the conventional production schedule and the simultaneous stochastic production schedule. The mined tonnage of both schedules is identical, since all blocks to be scheduled in the yearly horizon are extracted. However, the flexibility of creating different extraction sequences, paired with optimizing cut-off grades, allows the simultaneous stochastic optimization model to send different blended materials to the processing plant.



(a) Monthly optimized cut-offs and conventional cut-off for weathered materials

(b) Monthly optimized cut-offs and conventional cut-off for transitional/fresh materials

Figure 5-8 Monthly optimized cut-off grades for two material types in simultaneous production scheduling compared to conventional cut-off grades

Table 5-4 End-of-year difference of key performance indicators between conventional short-term production schedule and simultaneous stochastic short-term production schedule integrating predictions of reagents and consumables

Category	Performance indicator (P50)	Risk analysis of conventional production schedule	Risk analysis of simultaneous stochastic production schedule	Difference
Tonnage/ Production	Tonnes mined (t)	90,685,404	90,685,404	+/- 0.00 %
	Au grams recovered (g)	18,087,380	18,302,797	+ 1.19 %
	Tonnes processed (t)	12,907,133	12,912,205	+ 0.04 %
Reagents and Consumables	Acid consumption (t)	1,594,221	1,609,475	+ 0.96 %
	Caustic soda consumption (t)	1,515,424	1,535,555	+ 1.33 %
	Grinding media consumption (t)	7,702,495	8,026,676	+ 4.21 %
Financials	Proc. costs of reag. and cons. (AUD)	14,968,098	15,557,870	+ 3.94%
	Revenue (AUD)	845,640,690	855,711,694	+1.19 %
	Profit (AUD)	301,109,237	310,803,366	+3.22 %

Although the cut-off grades are raised in some periods and lowered in others compared to the fixed cut-off grades, the simultaneous stochastic optimization appears to send higher-grade materials overall, as more gold is recovered (+1.19 %) by processing a similar total amount of ore (+0.04 %). The resulting higher head grades of gold explain the increased overall consumptions of acid and caustic soda, as their consumption rates positively correlate with the gold grades. The grinding media consumption increases as well, given the different proportions of regolith and increasing gold grades, which also correlate positively with the grinding media consumption in this case study (not to be generalized). Notably, the processing costs related to the reagents and consumables increase due to higher consumptions, while the processed tonnage does not. This exemplifies the usefulness of integrating reagent prediction models into optimization. Processing costs on a per-ton basis would not account for the blended materials causing different reagent consumptions. Finally, added revenue from the additional recovered gold outweighs the additional processing costs and results in an added profit of 3.22 %.

5.4 Conclusions

This article presents the construction of geometallurgical prediction models of consumption rates of reagents and consumables from production data in a gold mining complex and the subsequent integration of these prediction models into a simultaneous stochastic optimization model for short-term production scheduling. Instead of relying on laboratory tests on rock samples, prediction models for processing plant reagents and consumables use the rock attributes of blended materials, which are processed over the horizon of one year, and link them to the observed consumption rates of the operating plant. In addition to capturing the responses of blended materials at the operating scale, these data-driven prediction models are cost-effective because of their use of recorded production data.

Furthermore, simplifications of adjusted consumption rates and processing costs per mining block in a spatial geometallurgical model can be overcome by integrating the prediction models of consumption rates into a non-linear stochastic optimization model for production scheduling. The model simultaneously optimizes the extraction sequence and destinations of future scheduled blocks and evaluates the metallurgical responses of blended materials while accounting for the geological uncertainty of rock attributes. The approach is tested at the Tropicana Gold mining complex using real-world production data. In the case study, the consumption rates of grinding media, caustic soda, and hydrochloric acid can be predicted with a correlation coefficient > 0.7 using blended rock attributes. A risk analysis quantifies the uncertain reagent consumptions for a given production schedule and reveals that consumption rates of future scheduled materials can deviate up to 70% in a single month compared to average consumptions based on the previous year. Total yearly consumptions of hydrochloric acid and caustic soda are overestimated by 8.6 % and 10.6 %, respectively, and grinding media consumption is underestimated by 3.2 %. The results create better awareness of the effects in processing plants due to changed ore feed characteristics and enable more accurate budget plans. The simultaneous stochastic optimization model finally integrates the prediction models of reagents and consumables and creates an improved 12-month production schedule with an added profit of 3.22 %.

One of the limitations of the proposed approach is that the created prediction models of consumption rates are site-specific; thus, they may only be valid for the future production planning of materials that are expected to behave similarly in processing plants, thereby limiting the proposed approach to short-term production scheduling. Furthermore, the generated prediction models are strongly dependent on the site-specific processing configurations within the monitoring time span, e.g., product particle size distributions from grinding and residence time of the pulp in the leach tanks. Since the aggregated time intervals for material tracking span at least one to several days, small intra-day changes do not affect prediction results in a mayor way. However, the models may be adjusted or retrained when larger modifications are made, such as a mill expansion. It should also be further investigated how certain reagents can affect the consumption rates of other reagents.

Future research may investigate other geometallurgical prediction models based on the production data and integrate them into simultaneous stochastic production scheduling. This can improve the estimates of parameters that are pertinent to production scheduling, such as mining and processing costs, as well as mining and processing capacities. Especially, new geometallurgical prediction models of metal recovery beyond conventional grade-recovery relationships should be integrated into short-term decision-making. The metal recovery is a key performance indicator in a mining operation and plays an important part in the integration of geometallurgical information (Suazo et al., 2010; Kumral, 2011; Carpenter et al., 2018).

6 Conclusions and Future Work

6.1 Conclusions and Summary

Simultaneous stochastic optimization of mining complexes for long-term planning has recently shown to create strategic mine plans that increase the net present value and metal production by jointly optimizing the components of a mining complex while reducing the risk of meeting strategic production targets by incorporating geological uncertainty and price uncertainty into the optimization. This planning stage produces life-of-mine plans comprising yearly periods, which must be broken down into smaller time periods to make operational decisions. The work presented in this thesis expands the simultaneous stochastic optimization of mining complexes to short-term planning, optimizing weekly to monthly periods up to a planning horizon of one year, by integrating fleet management decisions and geometallurgical prediction models of plant performances into the optimization. Furthermore, additional sources of uncertainty stemming from the scheduled mining equipment and the metallurgical processes of the plant are integrated into the optimization to reduce the risk of meeting short-term production targets leading to better alignments of production targets imposed by strategic mine plans. The geometallurgical prediction models for ball mill throughput and consumption rates of reagents and consumables constructed in this thesis utilize datasets from production processes such as measurement while drilling data, records of fleet management systems, and measured metallurgical responses of the processing plant. The prediction modes can thus be easily constructed and updated from centrally stored databases and give a potential financial advantage over geometallurgical laboratory tests. Moreover, non-additivity of rock hardness is addressed by a compositional approach for throughput prediction and non-linear relationships between predictor and response variables are addressed by a supervised machine learning model. By modelling the operational aspects and uncertainties of the mining fleet and metallurgical behaviour of processing plants based on geometallurgical properties in greater detail, the resulting short-term mine plans that are not only more likely to align with long-term production targets, but also benefit from synergistic effects that maximize the profit of the mineral value chain. The individual developments and results can be summarized as follows:

In Chapter 2, new decision variables and constraints that allow the scheduling of shovels and trucks are integrated into a non-linear simultaneous stochastic optimization model for long-term planning (Goodfellow and Dimitrakopoulos, 2016). The resulting model simultaneously optimizes pertinent decisions in a mining complex on short-term scale, namely short-term extraction sequence, shovel allocation including the costs and loss of production caused by shovel relocation, scheduling of a heterogeneous truck fleet, destination of extracted materials, and downstream material flow in mining complexes. The new method is applied at gold mining complex consisting of two open pits, stockpiles, a mill, and a leach pad. Compared to a conventional two-step approach, where the short-term production schedule is optimized first before optimizing the allocation of the mining fleet, the developed method reduces costs generated by shovel movements by 56%, and lost production due to shovel relocation is cut by 54%. Furthermore, the required number of trucks shows a more balanced profile, reducing total truck operational costs by 3.1% over an annual planning horizon, as well as the required haulage capacity in the most haulage-intense periods by 25%. The results show that significant synergies exist between short-term production scheduling and fleet management, thus adding value to the mineral value chain when optimized simultaneously.

Chapter 3 integrates a novel geometallurgical throughput prediction model for the ball mill into the simultaneous stochastic optimization model for short-term planning developed in Chapter 2. The datasets for the throughput prediction model include penetration rates from blast hole drilling (measurement while drilling), geological domains, material types, rock density, and throughput rates of the operating mill, offering an accessible and cost-effective method compared to other geometallurgical models for throughput prediction. First, the orebody’s comminution behaviour is reflected by hardness proportions built from geostatistically simulated penetration rates. A multiple linear regression model is constructed that predicts throughput rates as a function of blended rock properties. A case study at the Tropicana Gold Mining Complex shows that throughput can be predicted with an error of less than 30 t/h (RMSE) and a correlation coefficient of up to 0.8. By integrating the prediction model and new stochastic components into optimization, the production schedule achieves weekly planned production with high certainty, showing a total expected deviation from the mill production target of 0.3% over a 3-month horizon. This is because the new method successfully matches scheduled materials with the predicted performance of the mill. Comparisons to optimization using conventional mill tonnage constraints show expected production shortfalls of up to 7%, demonstrating superiority of the proposed method to meet production targets.

Chapter 4 extends the geometallurgical throughput prediction model developed in Chapter 3 by including recorded measurements of ball mill power draw, feed particle size distributions and product particle size distributions. Instead of the multiple linear regression model employed in Chapter 3, a supervised learning model in the form of a feed-forward neural network is used to approximate non-linear relationships between predictor and response variables. Comparisons in the Tropicana Gold mining complex show that when adding ball mill power and product particle size, the throughput prediction error decreases by 10.6%. This result can only be achieved with the neural network, whereas the multiple linear regression shows improvements of 4.2%. Furthermore, the case study shows that hardness proportions created from penetration rates using the proposed approach in Chapter 3 can decrease the throughput pediction error by 6.5% compared to the use of average penetration rates per mining block. This result underlines the importance of compositional approaches for non-additive geometallurgical variables, which has rarely been considered.

In Chapter 5, the empirical prediction of metallurgical responses at the operating scale of the plant and their incorporation into short-term stochastic production scheduling in mining complexes is extended to consumption rates of reagents and consumables. Specifically, empirical prediction models of caustic soda, and hydrochloric acid, and grinding media consumption rates are created at the Tropicana Gold mining complex, tracking blended rock properties that are matched with observed consumption rates at the operating processing plant. The created simultaneous stochastic optimization model in this thesis optimizes the short-term extraction sequence and destination of materials in a gold mining complex and integrates the prediction models of reagents and consumables into the optimization resulting in an added profit of 3.2 % compared to a conventionally created short-term production schedule.

6.2 Discussion and Future Work

There are several possible directions for future research based on the work presented in this thesis. Next to the integrated fleet management decisions and uncertainties into the simultaneous stochastic optimization of mining complexes in Chapter 2, other short-term related activities can be integrated into the optimization. Particularly, the scheduling of drilling and blasting activities is relevant (L'Heureux et al., 2013; Kozan and Liu, 2018), as well as auxiliary processes such as ramp building (Eivazy and Askari-Nasab, 2012) and truck maintenance scheduling (Topal and Ramazan, 2010, 2012a). Whereas the number of trucks in operation can be actively managed with the presented formulation in Chapter 2, exemplary to perform scheduled maintenance, the effects of truck maintenance on operating costs could be included as well. Also, the operation of processing plants in different settings and configurations (operating modes) can be relevant in short-term planning. This has so far only been considered for simultaneous optimization in long-term planning (Montiel and Dimitrakopoulos, 2015; Del Castillo and Dimitrakopoulos, 2019). Other improvements can consider the automation of the definition of mining areas for shovel allocation presented in Chapter 2 of this thesis. The mining areas shown in Chapter 2 are manually defined by grouping the materials together that form a mining face, are spatially connected, and can be accessed by the same haul route. Instead of the manual division, this process could be automated in the future to improve the shovel allocation process, possibly with the use of clustering algorithms. It is also imperative for future research to develop methods that lead to an integrated planning approach of short-term and long-term planning. While short-term plans typically comply with the outline of long-term production schedules, it is not yet sufficiently addressed how short-term planning results can influence the long-term plan. Some initial research of combining different planning horizons in single optimization models (multi-horizon planning) has been provided (Kozan and Liu, 2017). Other methods use discrete event simulation techniques to simulate the behaviour of the long-term plan in the context of uncertain operational and equipment-related constraints in the short-term (Ben-Awuah et al., 2010).

The integration of geometallurgical aspects into short-term production scheduling presented in this thesis can be expanded in several ways and the developed geometallurgical prediction models of ball mill throughput, and consumption rates of reagents and consumables in Chapters 3, 4 and 5 can be improved. Because of the unique operating conditions of each mine, the generalization potential of the constructed prediction models is expected to be site-specific and only valid in short-term planning horizons for future production planning of materials that behave similarly in the operating processing plants. Furthermore, it should be noted that the reliability of the constructed prediction models is strongly dependent on the accuracy of the material tracking systems in place in pits and stockpiles. While this thesis proposes to use recorded truck cycle data, there are other possible data sources that can improve tracking accuracy, including the monitoring of blast movements (Dowd and Dare-Bryan, 2005; Hmoud and Kumral, 2022). Generally, the uncertainty introduced by inaccurate material tracking should not be ignored and deserves more studies to be quantified and managed in future work.

Other future research may attach to how this thesis treats the input-output pairs consisting of prediction variables and metallurgical responses of the plant, which are seen as independent samples. However, the utilized measurements originate from time series that can include temporal correlations. Other supervised machine learning techniques that explore temporal feature dependencies are available, such as recurrent neural networks including long short-term memory (Hochreiter and Schmidhuber, 1997) and gated recurrent units (Cho et al., 2014). Finally, the developed prediction models in this thesis provide single estimates of plant performances which may underestimate the variability of future responses. Conditional simulations of time series can be considered to create outputs that better reproduce the observed variability of the metallurgical response.

Measurement while drilling is an example of ‘soft data’ containing indirect and noisy measurements of rock attributes related to strength, hardness, and comminution behaviour. Noise and biases in measurement while drilling data can be caused by different drill rig types, rig operators, drilling tasks, planned and unplanned disruptions in the drilling process, and drill-bit wear. Thus, the prediction models constructed in this thesis need to be developed further to better quantify the ‘softness’ of the used data for an improved understanding of how noisy measurements link to reality. So far, the proposed methods have been applied at a gold mining complex and should also be tested in other commodities. This can provide insight if measurement while drilling can be consistently applied for throughput prediction. A requirement for similar studies at other mines will be a centralized data management system that collects production data from drilling, hauling, grinding, and separation processes. Other datasets containing ‘soft’ information also can be considered to improve the prediction of metallurgical responses of processing plants. Of particular interest are spatial datasets such as X-ray fluorescence and hyperspectral scanning of grade control samples (Dominy et al., 2018; Wambeke et al., 2018). Other potential datasets are measurements of extracted material characteristics using sensor techniques (Lessard et al., 2014). It should be noted, however, that information on extracted materials will not be readily available for future scheduled materials in the ground, which needs to be considered when prediction models are integrated into production scheduling.

As digital technologies increasingly provide production data in mines and processing plants in real-time, future research should also focus on assimilating this new information into short-term decision-making in (near) real-time. In the context of the research presented in this thesis, this can include updating the proposed geometallurgical prediction models of plant performances, as well as other models used as input to short-term optimization, such as updating the geostatistically simulated orebody models (Benndorf, 2015; Wambeke and Benndorf, 2016; Wambeke et al., 2018; Prior et al., 2020; Kumar and Dimitrakopoulos, 2022), and updating equipment performance scenarios (Kumar and Dimitrakopoulos, 2021). Finally, the short-term decisions made in this thesis, such as extraction sequence, material destinations, downstream material flows, shovel positions, and truck allocation can be adapted in light of new information (Paduraru and Dimitrakopoulos, 2018; Kumar and Dimitrakopoulos, 2020, 2021; de Carvalho and Dimitrakopoulos, 2021).

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