SHORT TITLE

JOINT EFFECTS OF UNEQUAL SLOPES AND VARIANCES IN ANCOVA

THE JOINT EFFECTS OF SIMULTANEOUSLY VIOLATING THE HOMOGENEITY OF REGRESSION AND HOMOGENEITY OF VARIANCE ASSUMPTIONS ON THE F-TEST IN THE ANALYSIS OF COVARIANCE - A MONTE CARLO SIMULATION

by

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ABSTRACT

The present study was designed as an attempt to evaluate the effects on the Analysis of Covariance F-test of varying combinations of degrees of violation of the homogeneity of variance and the homogeneity of regression assumptions. Measures of violation, invariant under stated constraints on the covariate, were derived for each of the two assumptions. Sampling distributions of simulated ANCOVA experiments embodying combinations of values of these measures were generated. The effect of each combination on the F-test was determined by a comparison of obtained and theoretical percentage points. Results suggested that the two violations tend to neutralize each other in their effects, leaving the F-test remarkably robust with respect to their joint presence. An attempt was made to establish a predictive relationship between levels of violation of the two assumptions on the one hand, and their effect on probability of Type 1 error on the other.

PREFACE

This investigation was supported by a grant from the National Research Council of Canada (Grant No. 246-07) to Dr. James O. Ramsay.

The contribution of the study is to show how the violation of the homogeneity of regression and the homogeneity of variance assumptions in the analysis of covariance, tend to cancel each other out in their effects on the F-test, leaving the test remarkably robust with respect to their joint presence.

I wish to thank my supervisor Dr. James O. Ramsay for his continued encouragement and helpful direction throughout the study.

I am also indebted to Mr. Nick Paul and Dr. Kassi Ananthanasayanan both of the School of Computer Science, McGill University, for their very willing advice on matters related to the random number generator.

Finally I wish to express my thanks to my wife Susan for help in preparing the text.

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APPENDIX A

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CHAPTER 1

The application of statistical tests to the analysis and interpretation of data implies that the assumed conditions of the test are met in the experimental situation. In practice, however, these assumptions are never fully justified. Thus, an important area of investigation has necessarily arisen to answer questions concerning the extent to which actual experimental conditions can depart from those assumed in any test before its application is rendered invalid. In other words, this field of research attempts to determine how 'robust' any given test is. lf a statistical test is not robust, and if, in actual experimental practice, its underlying assumptions are not upheld, then it becomes extremely difficult for the researcher to isolate that component of his results which is attributable to the violation of the assumptions, from that part which purports to answer his experimental question.

Two ways of approaching questions concerning the robustness of statistical tests have been developed. The first employs traditional mathematical analysis and attempts to determine the behaviour of the test under less than ideal conditions. A difficulty with this analytic approach is that once the assumptions of the test are relaxed the statistics frequently become difficult to handle, and in many cases the statistician has to resort to distributional approximations (often asymptotic) or other simplifying procedures, which yield results that are more indicative than definite.

The second approach is that of Monte Carlo simulation in which the robustness of a statistical test is evaluated by generating an empirical sampling distribution of the statistic, derived from the simulated sets of data embodying some specified degree of violation of one or more of the test's underlying assumptions. This distribution can then be compared with its corresponding theoretical, violationfree distribution and the effect of that degree of violation is determined from the comparison.

This kind of work is much more costly than the first approach, and harbours some intrinsic limitations; whereas there is only one way in which the assumptions can be fully satisfied, there is no practical limit to the number of ways in which, singly or in combination, they can be violated. A Monte Carlo investigator must consequently choose a more or less arbitrarily circumscribed set of possibilities to study, without any guarantee that his findings are generalizable to other contingencies of violation.

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The present study was designed as an attempt to ascertain, through the use of the Monte Carlo procedure, the effects of the simultaneous violation of two assumptions underlying a statistical test commonly applied to experimental data in Educational and Psychological research - the Analysis of Covariance. The two subjects of this investigation were the homogeneity of regression and the homogeneity of variance assumptions.

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THE ANALYSIS OF COVARIANCE:

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The Model and Its Application

Covariance analysis (ANCOVA) incorporates two systems of data analysis, Analysis of Variance and Regression Analysis, which have come to be thought of by many psychologists as distinct, but which in fact are both subsumed under the general linear model.

The mathematical model in the Analysis of Covariance (one-way, linear, fixed effects) is as follows:

 $Y_{ij} = \mu + \tau_j + \beta (X_{ij} - \overline{X} ...) + e_{ij}(1.1)$

where Y_{ij} is the measure of the dependent variable and X_{ij} is a concomitant variable or covariatee with grand mean $\overline{X}_{...,}$ on which Y_{ij} has a linear regression with regression coefficient β . The constants μ and τ_j are the grand mean of treatment populations and the effect of the jth treatment level, respectively, with $\sum_{j} \tau_j = 0$. The variable e_{ij} is the random error term assumed to be normally and independently distributed with zero mean and constant variance.

From the point of view of Analysis of Variance, the model can be expressed as:

$$Y_{ij} - \beta (X_{ij} - \overline{X}..) = \mu + \tau_j + e_{ij}, \dots (1.2)$$

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where a modification (due to the estimation of β) of the usual F-test is applied to the Y-scores now adjusted for X.

On the other hand, considered from the point of view of Regression Analysis, the treatment effects can be conceptualized as dummy variables, so that the design takes on the form of an F-test for multiple regression. These two F-tests are equivalent.

The Covariance model is designed for the following.experimental situation: different levels of a treatment, or independent variable, are being applied to randomly selected groups of experimental units. The purpose of the experiment is to determine whether or not these treatment levels differentially affect some dependent response Y. Before the treatments are applied, however, a measure of another variable X is taken on the experimental units. The treatment levels are then effected, and in each instance the measure of variable Y is recorded. In the usual Analysis of Variance situation, these data are then subjected to an F-test for differences in treatment means. In the Analysis of Covariance situation, however, before the Y data are subjected to the Ftest, they are first adjusted to remove the influence of the variable X.

This procedure has the advantage of increasing the precision of the treatment comparisons by reducing extraneous variability in the experiment. Cochran (1957) reports that

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the gain in precision resulting from covariance adjustment is a function of the size of the correlation coefficient ρ between Y and X on experimental units that receive the same treatment. He states that "... if σ_y^2 is the experimental error variance when no covariance is employed, the adjustments reduce this variance to a value which is effectively about

$$\sigma_{y}^{2} (1 - \rho^{2})(1 + \frac{1}{f_{e} - 2}), \qquad (1.3)$$

where f_e is the error number of degrees of freedom." (Cochran, 1957, p. 262).

Assumptions Underlying the Analysis of Covariance

The F-test for the Covariance Analysis, like all parametric tests, assumes for its valid application that the data to which the model is applied behave in a certain way. The following are the basic assumptions of Covariance Analysis.

- The experimental units are randomly assigned to treatment groups.
- The dependent or criterion scores have a linear regression on the covariate and the regression coefficient is constant across treatment levels.
- 3. The covariate is measured without error.
- 4. The dependent or criterion scores are a linear combination of independent components - an overall mean, a treatment effect, a linear regression on X, and an error term.

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5. For each treatment/covariate combination, the error term e_{ij} is independently and normally distributed with a mean of zero and constant variance.

Finally, it is usual in applying the test to a set of data to make the assumption that $\tau_j = 0$ for all treatment groups (the null hypothesis).

If any of these six conditions is not satisfied, the sampling distribution of the F-ratio may differ from the central F-distribution. This means that a significant, or for that matter, a non-significant F-ratio could result from a failure to fulfill any one of these assumptions. Consequently, before concluding from a significant or nonsignificant F that the sixth assumption is or is not sustained, one must be able to satisfy oneself that failure to meet the other five assumptions is not seriously affecting the behaviour of the F-ratio distribution.

Effects of Departures from the Underlying Assumptions in ANCOVA - A Selected Review

Some of the studies to be cited in this section pertain directly to the ANCOVA design; others focus primarily on ANOVA, but their results are in most cases directly generalizable to the ANCOVA situation. This review consists of a brief summary of the effects of departures from each of several

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underlying assumptions not directly connected to the subject of the present study, and of a more elaborate documentation of the results of studies dealing with those directly related assumptions - homogeneity of regression and homogeneity of variance.

The consequences of violating the assumption of random assignment of experimental units to treatment groups have been clearly stated by Lord. He points out that "If the individuals are not assigned to treatments at random, then it is not helpful to demonstrate statistically that the groups after treatment show more difference than would be expected by random assignmental" (Lord, 1967, p. 305). The ANCOVA test is particularly open to violation of this assumption since in many empirical situations in psychology to which the test is applied, randomization is not feasible.

Evans and Anastasio (1968) distinguish in this context, three different uses of ANCOVA:

- Random assignment of experimental units to groups and random assignment of treatments to groups;
- Already-existing groups used as treatment groups, but treatments randomly assigned to them;
- 3. Already-existing groups used as treatment groups, with some intrinsic attribute considered as "treatment".

They conclude that only usage 1 leaves interpretation of results unequivocal, and that interpretation becomes less and less meaningful as one goes from usage 2 to usage 3.

As to the assumptions related to properties of the conconcomitant variable, Lord, (1960) examined the situation where the covariate X contains errors of measurement, and concluded that under these circumstances, "... the usual ANCOVA fails to adjust adequately for initial differences between groups." (Lord, 1960, p. 307). He constructed a langemsample significance test in ancattempt to deal withathesproblem) but imitedontst usefulnessginwmasytempirieal.situationsebyover quiring two sets of measures for the covariate.

Cochran (1968) shows how the situation of X measured with error decreases the precision of the experiment by increasing the error variance by a factor determined by the reliability of X. Porter (1967) developed a covariance design for the situation where X is measured with error and incorporates the reliability of X into his model.

Atiqullah (1964) has examined the effect of nonnormality on the ANCOVA F-test. He demonstrates how in the balanced lay-out the sensitivity of the test to nonnormality depends on the behaviour of the covariate. X. He concludes that the test is robust to non-normality when the distribution of the concomitant variable is normal.

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Atiqullah measures non-normality by kurtosis. He does not deal with skewness, which as Peckham (1968) remarks, is also relevant to non-normality in the behavioural sciences.

Scheffé (1959) has examined the effect of serial correlation (violating the assumption of independence of errors) for large samples in ANOVA. He derives probabilities corresponding to a 95% confidence interval for various values of ρ and concludes that "... the effect of serial correlation on inferences about means can be serious" (Scheffé, 1959, p. 338).

Atiqullah (1964) has considered the situation where the usual ANCOVA (linear) test is applied to data which contain a quadratic component of regression. In the case of two treatments, he finds that the expected value of the adjusted difference between the two groups is unbiased only if the covariates in both groups are members of the same normal population. Even this does not hold, however, for the case of more than two groups. He concludes that the presence of a quadratic component, if large, may have serious effects on the ANCOVA F-test (Atiqullah, 1964, p. 372).

Homogeneity of Regression

Until quite recently, very little information was available on the effects of violating the assumption of homogeneity of regression in ANCOVA. The assumption implies that the re-

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lationship between Y and X is a linear one, and that the regression coefficient of Y on X is constant across treatment groups.

A preliminary check on the assumption of homogeneity of regression is given by Winer (1971). He provides a sampling F-distribution based on the assumption that $\beta_1 = \beta_2 \dots = \beta_k$. This is a useful though indirect precaution, but as Evans and Anastasio point out, "... the decision that assumptions have been met rests on the acceptance of the null hypothesis. Thus the user has only a roundabout procedure (using a large value for α) to guard against the relevant class of error, Type II. He cannot even determine, much less control, the probability of detecting violations which are serious enough to affect his conclusions" (Evans and Anastasio, 1968, p. 226).

The two sources of information on the robustness of the test with respect to this assumption are those by Atiqullah (1964) and Peckham (1968). The former is a theoretical paper while the latter consists of a Monte Carlo empirical investigation.

Atiqullah sets up the following two models:

$$M_{1} Y_{ij} = \mu + \tau_{j} + \beta(X_{ij} - \overline{X}..) + e_{ij}$$
(1.4a)

$$M_{2} Y_{ij} = \mu + \tau_{j} + \beta_{j}(X_{ij} - \overline{X}..) + e_{ij}$$
(1.4b)

and examines the effect of employing the F-test based upon model M_1 , when in fact model M_2 is the appropriate one. He makes use of the following notation:

and a

$$W_{jj} = \Sigma (X_{ij} - \overline{X}_{j})^2; \quad W_2 = \Sigma W_{jj}$$

His contribution may be divided into three parts.

- 1. In the case of a comparison of two treatments in a twogroup experiment, he found that when $\beta_1 \neq \beta_2$, that is when model M_2 is the appropriate one, but model M_1 is applied, the expected value of the difference in adjusted treatment means is biased unless either $\overline{X}_1 = \overline{X}_2$, or $W_{11} = W_{22}$. He suggests that "... in the absence of a prior presumption that B_1 and B_2 are nearly equal, the model M_2 should be used, separate regressions fitted, and the treatment effects estimated as a function of X.
- 2. Atiquilah next considers a comparison of two treatments in an experiment involving more than two groups. Again, in the case of model M₂ being the appropriate one, but model M₁ being applied, he finds that the expected value of the adjusted differences between pairs of treatments is biased unless both $\overline{X}_1 = \overline{X}_2$ and $W_{11} = W_{22}$.
- 3. In the case of experiments involving more than two treatment groups, Atiqullah derived an asymptotic approximation to the ANCOVA F-distribution using Fisher's Ztransformation, and then examined the consequences of applying model M₁ when model M₂ obtains. He concluded that the application of the standard covariance F-test, under these circumstances, may yield misleading results

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unless the X's are normally distributed, $\sigma_x^2/\sigma_e^2 > 1$, and the variance of the inhomogeneous regression coefficients $\sigma_{\beta j}^2$ is in the order of $\frac{1}{\kappa^2}$ where κ denotes number of treatment groups.

Peckham (1968) conducted a Monte Carlo investigation into the effects of violating the homogeneity of regression assumption in ANCOVA. His method consisted of generating a sampling distribution of the ANCOVA F-statistic, where each sample consisted of an ANCOVA experiment embodying a specified degree of heterogeneity of regression, all other assumptions of the test being satisfied. (To avoid bias due to nonnormality in the concomitant variable, the values of X were chosen to approximate a normal distribution). He then compared the empirical sampling distribution with its corresponding central F-distribution.

The study proceeded in two phases. In phase one, Peckham combined varying degrees of heterogeneity of regression with different sample sizes and numbers of treatment groups. In this phase, he fixed $\overline{X}_j = 0$ and $MS_{x,j} = 1$ for all groups. In phase two, he used only the two-group case and arranged the data so that the relationships among the Y, X and β were such that the expectation of the adjusted mean for each group was equal to the same constant.

In both phases, Peckham found for the degree of violation studied that the analysis was not seriously affected by departures from the assumption, and that as the degree of heterogeneity increased, the test became more conservative with respect to Type I error (see Table 1.1).

<u>TABLE 1.1</u>

Effects of Violating the Assumption of Homogeneity of Regression on the ANCOVA (1-way, fixed-effects) F-test (Abridged from Peckham, 1968, Phase 1).

No. of <u>Groups</u>	<u>Group Size</u>	Regression Coeofficsiont Values	Probability ceeding <u>5% pt.</u>	of ex- <u>1% pt.</u>
2	10	.4 .6	.050	.010
	10	.1 .9	.029	.004
3	10	.4 .5 .6	.050	.009
3	10	.1 .5 .9	.035	.006
5	10	.4 .4 .5 .6 .6	.048	.007
5	10	.1 .3 .5 .7 .9	.041	.010

Homogeneity of Variance

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ANCOVA also relies for its validity on the assumption that the population variance σ^2 of the error term e_{ij} for each treatment-covariate combination is a constant. As Elashoff (1969) points out, there are two main ways in which this assumption is likely to be violated:

- 1. The variance σ^2 is dependent on the covariate X, but for a given X is constant across treatments,
- 2. σ^2 is constant within each treatment group, but differs across treatments.

For the purposes of the present study, attention will be focussed exclusively on the second contingency, since it better represents the empirical situation to which ANCOVA is usually applied.

To date, no theoretical or empirical work has been specifically devoted to studying the effects on the ANCOVA F-test of departures from the assumption of equal variances. However, a good deal of both kinds of work has been carried out on the ANOVA design, and since the effect of unequal variances on the two models is almost identical under certain conditions of the covariate (see below page 26), these ANOVA results will be reviewed as relevant in the context of the present study.

Since the 1930's, statisticians have considered the effects of departures from the assumption of homogeneous variances in ANOVA (Welch, 1937; Daniels, 1938; Horsnell, 1953). The results of these studies suggested that the test is robust with respect to the violation of this assumption. Several tests for homogeneity of variance have been developed (Bartlett, 1937; Cochran 1941; Hartley, 1950), but these tests suffer from the same drawbacks as those outlined above in reference to the test for homogeneity of regression (see section immediately preceding).

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The first important Monte Carlo study on the issue was carried out by Norton (1952). As reported in Lindquist (1953, p. 78), Norton obtained samples of ANOVA F's by randomly sampling from 3 card populations, which were normally distributed with constant mean, but whose variances were 25, 100 and 225 respectively. He obtained the percentage of sample F8 ss that exceeded the theoretical 5% and 1% values (expected under the null hypothesis), and used the discrepancy between the expected and obtained percentages as a measure of the effects of the violation. His results indicated that the ANOVA F-test is remarkably insensitive to violation of the assumption of equal variances, when equal numbers are assigned to treatment groups (see Table 1.2).

TABLE 1.2

Norton Study - Percentage Counts of Mean-Square Ratios in Empirical Distributions Exceeding Theoretical Percentage Levels in Normal-Theory F-distribution. (Abridged from Lindquist 1953, p. 84).

No. of Groups	<u>Group Size</u>	Percen <u>Expected</u>	itages <u>Obtained</u>
3	3	5%	7.26%
3	10	5%	6.56%
3	3	1%	2.13%
3	10	1%	2.00%

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Box (1954), using theorems on quadratic forms in multinormally distributed variables, developed a very close approximation to the distribution of ANOVA F (one-way), when the assumption of equal variances is relaxed. Utilizing this approximation he calculated the probability of Type 1 error corresponding to the .05 level of normal theory, for various levels of violation. His results, in the case of equal groups, support those of Norton. In the case of unequal groups however, he showed that the violation of the homogeneity of variance assumption is drastically disturbing to the distribution of F.(see Table 1.3). Both the results of Norton and Box were later empirically supported by those of Boneau (1960).

TABLE 1.3

Effects of Violating the Assumption of Homogeneity of Variance on the ANOVA (1-way) F-test (Abridged from Box 1954, p.299).

No. of <u>Groups</u>	Variances	<u>Group Sizes</u>	Prob. (%) of exceeding 5% point
3	1:2:3	5: 5: 5	5.78
3	1:2:3	7: 5: 3	9.57
3	1:1:5	5: 5: 5	5.82
3	1:1:3	7:5:3	9.78
5	1:1:1:1:3	5: 5: 5: 5: 5	6.86
5	1:1:1:1:3	9: 5: 5: 5: 1	15.56

ORIGIN OF THE PRESENT STUDY

The preceding review has considered only the effects on ANCOVA and ANOVA of violating each single underlying assumption in turn. This limitation reflects the paucity of existing work evaluating the joint effects of violating two or more of the basic assumptions in ANCOVA. The use of these results as a guide to data analysis would seem to depend upon the validity of the assumption that, if in any set of data to which ANCOVA is being applied more than one assumption is being violated (as is most likely to be the case), the effects of the violations are independent. That is, the violation of one assumption does not ameliorate or exacerbate the effect of another assumption's being violated.

The present study arose out of indications that such a state of affairs does not hold for at least two of the assumptions underlying the ANCOVA F-test. Peckham (1968) found that, as the violation of the homogeneity of regression assumption increased in severity, fewer and fewer simulated ANCOVA F-values exceeded the theoretical percentage levels; that is, the test became more and more conservative with regard to Type 1 error (see Table 1.1). On the other hand, generalizing from the results of Norton and Box (see Tables' 19.2 and 1.3), it seems likely that in ANCOVA, the number of Fvalues exceeding nominal probability levels increases as the degree of violation of the homogeneity of variance assumption increases.

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The present study was therefore an attempt to evaluate the effects on the ANCOVA F-test (1-way, linear, fixed effects, equal group sizes) of simultaneously violating the homogeneity of variance and the homogeneity of regression:assumptions: to ascertain in general, the extent to which these violations in their varying degrees of severity, tend to cancel each other out, and to determine the general relationship between cumulative percentage counts and violation levels of the two assumptions.

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CHAPTER 2

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DESIGN OF THE STUDY

The procedure was first to choose the combinations of degrees of violation of the two assumptions to be investigated; then for each combination, to construct a pseudo-random sampling distribution of F-ratios, derived from a series of simulated ANCOVA experiments embodying the degrees of violation of that combination. For each sampling distribution a count was then taken of the proportion of those F-values exceeding the critical tabled F-values for the corresponding central F-distribution $(df_1 = k-1, df_2 = N-k-1)$ at the 0.05 and 0.01 levels of significance. An estimate of the joint effects of each combination of violation-levels of the two assumptions on the probability of Type 1 error was thus determined.

MEASURES OF VIOLATION OF THE TWO ASSUMPTIONS

In order to proceed systematically, it was necessary at the outset to define an index of violation for each of the two assumptions; that is, to determine a quantitative relationship among the k parametric values (beta's, variances) such that as it increased in magnitude, so also did the effect on the probability of Type 1 error.

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I - For the purpose of deriving a measure of violation for the homogeneity of regression assumption, the following models of ANCOVA were compared. (The models are simplified so as to ensure full rank in matrices that are to be inverted).

$$L_{1}: Y_{ij} = \tau_{j} + \beta X_{ij} + e_{ij} \qquad j = 1, 2..., k$$

$$L_{2}: Y_{ij} = \tau_{j} + \overline{\beta} X_{ij} + (\beta_{j} - \overline{\beta}) X_{ij} + e_{ij}, \qquad (2.1a)$$

$$(2.1b)$$

where L_1 is the usual model for ANCOVA and L_2 incorporates a set of parameters $(\beta_j - \overline{\beta})$ to accommodate the presence of inhomogeneous regression coefficients.

Under L_2 , the design matrix X, the coefficient vector β , and the error vector E are represented as follows:

X =	$\begin{bmatrix} 1 & 0 & \dots & 0 & X_{11}X_{11}0 \\ 1 & 0 & \dots & 0 & X_{21}X_{21}0 \\ 1 & 0 & \dots & 0 & X_{n1}X_{n1}0 \\ 0 & 1 & \dots & 0 & X_{12}0X_{12} \\ 0 & 1 & \dots & 0 & X_{22}0X_{22} \\ \dots & \dots & \dots & \dots \\ 0 & 1 & \dots & 0 & X_{n2}0X_{n2} \\ \dots & \dots & \dots & \dots \\ 0 & 1 & \dots & 0 & X_{n2}0X_{n2} \end{bmatrix}$	··· 0 ··· 0	$\beta = \frac{\tau_1}{\beta_1 - \beta_1}$ $\beta_1 - \beta_2 - \beta_1$	$\begin{bmatrix} e_{11} \\ e_{21} \end{bmatrix}$ $\stackrel{e_{n1}}{\stackrel{e_{12}}{\stackrel{e_{22}}{\stackrel{e_{22}}{\stackrel{e_{22}}{\stackrel{e_{22}}{\stackrel{e_{22}}{\frac{e_{n2}}{$
	$\begin{array}{c} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 &$	X _{1K} X _{1K} X _{2K} X _{nk}	βκ-β	elk e2k enk

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where

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 $Y = X\beta + E$ and $E(Y) = X\beta$ (2.2)

Now, let Model L be (incorrectly) applied and the following quantities calculated:

1. error sume of squares

2. sum of squares due to treatments

Let S denote the matrix containing the first (K + 1) columns of X

c denote the vector containing the first (K + 1) elements of $\boldsymbol{\beta}$

T denote the wactor containing the $(\kappa + 1)$ th. column of X d denote the seater containing the $(\kappa + 1)$ th. element of β

Let
$$U = Y - S\hat{c}$$
 be a vector of residuals.

$$= Y - S \left[(S'S)^{-1} S'Y \right]$$

$$= \left[I - S(S'S)^{-1}S' \right] Y$$
Let $Q = \left[I - S(S'S)^{-1}S' \right]$
(2.3)

Q is a symmetric idempotent matrix since

$$Q^{\phi} = I - S(S'S)^{-1}S'$$
 (2.4a)

$$Q^{2} = \left[1 - S(S'S)^{-1}S' \right] \left[1 - S(S'S)^{-1}S' \right]$$
(2.4b)
= 1 - 2S(S'S)^{-1}S' + S(S'S)^{-1}S'S(S'S)^{-1}S'

$$= 1 - S(S'S)^{-1}S'$$

= 0

Thus the error sum of squares is:

$$U'U = (QY)'QY = Y'QY$$
 (2.5)

Let W = Y - Td be a vector of residuals

$$= \left[I - T(T'T)^{-1} T' \right] Y$$

where R = $I - T(T'T)^{-1}T'$ is a symmetric idempotent matrix (using 2.4a, and 2.4b). (2.6) Thus treatment sum of squares = Y'RY - Y'QY (2.7a) = Y'(R - Q)Y (2.7b)

where (R - Q) is also a symmetric idempotent matrix, (2.8) (for proof see Appendix A). Further $\left[(R - Q)Y \right]^{1}QY = Y^{1}(R - Q)^{4}QY = 0$ (2.9)

(forpproof see Appendix A).

Thus the two vectors are orthogonal and hence any two quantities based on them respectively will be independently distributed.

A necessary and sufficient condition that Y'AY is distributed as a chi-square is that A is idempotent; the degrees of freedom of such a chi-square are equal to the rank of A, and its non-centrality parameter, λ , has the value of the quadratic form Y'AY when the variables have been substituted by their expected values. (Rao, 1965, p. 150).

Thus, using (2.4a; 2.4b; 2.2), Y'QY the error sum of squares, is distributed as a non-central chi-square with degrees of freedom equal to the rank of Q, and non-centrality parameter = $\beta'X'QX\beta$. So, too, Y'(R-Q)Y the treatment sum of squares, is also distributed as a chi-square with degrees of freedom equal to the rank of (R-Q), and non-centrality parameter = $\beta'X'(R-Q)X\beta$, (using 2.8; 2.2). Furthermore, the two chi-square distributions are independent (using 2.9).

The ratio of these two independent chi-square distributions $\frac{Y'(R-Q)Y}{Y'QY}$ yields a doubly non-central F-distribution whose probability density function is given by:

$$dH(\mu) = e^{-\frac{1}{2}\lambda} \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \left(\frac{\frac{1}{2}\lambda_{1}}{r!}\right)^{r\left(\frac{1}{2}\lambda_{2}\right)^{s}} \mu^{s\left(\frac{1}{2}V_{1}+r-1\right)} \left(\frac{1}{1+\mu}\right)^{\frac{1}{2}V+r+s}$$

$$\times \frac{du}{B(\frac{1}{2}V_{1}+r,\frac{1}{2}V_{2}+s)}, \quad (2.10)$$
where
$$\mu = \frac{\gamma}{(R+Q)} \sqrt[3]{\chi} \frac{\beta}{Q} \gamma$$

$$\lambda_{1} = \beta^{1} \chi^{1} (R-Q) \chi\beta$$

$$\lambda_{2} = \beta^{1} \chi^{1} Q \chi\beta$$

$$\lambda = \lambda_{1} + \lambda_{2}$$

$$V_{1} = Rank \text{ of } (R-Q)$$

$$V_{2} = Rank \text{ of } Q$$

$$V = V_{1} + V_{2}$$

(Kendall and Stuart, 1961, Vol. II, pp. 252)

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From (2.10) it can be seen that the shape of the F-distribution is uniquely determined by the values of the two non-centrality parameters. Since the particular values of the covariate X_{ij} as well as those of the β_j are involved in the determination of the values of λ_1 and λ_2 , an invariant measure of the **viola**tion of the homogeneity of regression assumption may be expressed in terms of the β_j alone only if the values of X_{ij} are known. For the purposes of convenience in this investigation therefore, \overline{X}_j was set to zero and ΣX_{ij}^2 was given a constant value n - 1 for all j, thereby reducing the value of $\lambda_1 = 0$ and $\lambda_2 = n - 1 \begin{bmatrix} \Sigma & \beta_j^2 & - & \overline{\beta}^2 \end{bmatrix}$ (for proofs see Appendix A).

Thus under the present constraints on the covariate X_{ij} , $\sigma_{\beta_j}^2$, the variance of the inhomogeneous β_j becomes an invariant measure of the violation of the homogeneity of regression assumption.

II - The measure of violation of the homogeneity of variance assumption used in this study is the squared coefficient of variation of the k population variances:

$$c_{\sigma_{i}}^{2} = \sigma_{\sigma_{i}}^{2} / \mu_{\sigma_{i}}^{2}$$
 (2.11)

where the numerator is the variance of the k variances and the denominator is the squared mean of the k variances.

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Although this measure has not been directly derived for the ANCOVA situation, Box (1954) found in the case of ANOVA, with equal group sizes, under the condition of unequal variances, the ratio of mean squares does not follow the distribution $F(\kappa-1, N-\kappa)$ but is distributed approximately as:

$$F\left[(K-1)\varepsilon', (N-K)\varepsilon\right], \qquad (2.12)$$

where $\epsilon' = (1 + \frac{\kappa-2}{\kappa-1} c^2)^{-1}$; $\epsilon = (1+\epsilon^2)^{-1}$ and c^2 is the squared coefficient of variation of the κ variances. Thus when the variances are unequal ϵ' and ϵ are less than unity and the "significance of effects is somewhat overestimated" (p. 300); the larger the value of c^2 , the larger the overestimation.

Box's approximate measure of violation of the homogeneity of variance assumption in ANOVA extends itself to the ANCOVA situation provided that the behaviour of the covariate does not execceebrable the situation. Potthoff (1965) as reported by Elashoff (1969) suggests that the effect of inequality of variances in the Y scores is minimized in the ANCOVA situation when σ_{xj}^2 and n_j are constant across groups. For the purposes of the present study, therefore these restrictions on the X_{ij} obtained.

Thus the measures of violation of the homogeneity of regression and the homogeneity of variance assumptions used in this study are $\sigma_{\beta_i}^2$ and $c_{\delta_i}^2$ respectively.

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DETERMINATION OF INPUT VALUES OF β , AND σ_{j}^{2} CORRESPONDING TO VALUES OF σ_{j}^{2} and σ_{j}^{2}

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For any given value of $\sigma_{\beta_j}^2$, the variance of the regression slopes, or of $c_{\delta_j}^2$, the squared coefficient of variation of the variances, a correspondingly unique set of β_j or σ_j^2 does not exist. Consequently the following non-arbitrary methods of determining values of β_j and σ_j^2 corresponding to given values of $\delta_{\beta_j}^2$ and $c_{\sigma_j}^2$ respectively were derived and applied uniformly throughout the present investigation.

The set of $\kappa \beta_j$'s corresponding to a given value of $\sigma_{\beta_j}^2$ were selected so that they were centred on unity and were separated from one another by equal intervals.

Since variances are proportional to a chi-square distribution, an attempt was made to select k variances corresponding to k points on the abscissa of such a distribution so that the areas bounded by their vertical projections were equal. To do this, k points on the abscissa of the normal distribution were chosen so that the areas bounded by their vertical projections were equal and centred on the mean. The corresponding χ^2 values were derived by the following approximation due to Wilson and Hilferty (1931):

$$(\chi^2/\mathbf{v})^{1/3} = 1 - 2/9\mathbf{v} + Z(2/9\mathbf{k})^{1/2}$$
(2.13)

where the value of \mathbf{v} was chosen to yield a desired value of $c_{\sigma_1}^2$ between the k values. The values were then transformed so

as to have a mean of unity. The pair-wise sets of β_j and σ_j^2 were then combined in the same ascending order of magnitude. (The actual values of β_j and σ_j^2 derived in the above manner corresponding to given values of $\sigma_{\beta_j}^2$ and $c_{\delta_j}^2$ for different values of k are recorded in Appendix B).

THE RANDOM NUMBER GENERATOR

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Necessary to the success of any Monte Carlo study is an adequate procedure for generating random numbers described by specific probability density functions. Computer simulation of stochastic processes usually depends on the internal generation of 'pseudo-random' sequences of numbers. The most common procedure is to generate sequences of numbers which are randomly 'sampled' from a uniform distribution; samples from other distributions are then produced by some transformation of these uniform variates.

The McGill Random Number Package 'Super-Duper' developed by Marsaglia, Ananthanarayanan and Paul (1972) was used to generate random normal variates in this study. 'Super-Duper' capitalizes on the idiosyncrasies of the I.B.M./360 hardware on which the present simulations were run. The Package contains a fast uniform random number generator which combines a multiplicative congruential generator and a 'shift-register' generator; both generators were chosen so as to have the greatest possible period and to minimize any regularity patterns. The random number generator, which is very fast relative to other available generators (15,000 normal variates per second) uses the uniform generator to generate the normal variate 95% of the time from a rectangular distribution which is close to the normal curve; for the remaining 5% of the time, the variate is generated so as to compensate for the discrepancy. The result is a sequence of variates whose distribution is described by the normal curve. (A listing of 'Super-Duper' appears in Appendix C).

CONTROL OF CONCOMITANT VARIABLES

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In order to evaluate the effects on the size of probability of Type 1 error of any given violation combination, each sampling distribution of simulated ANCOVA experiments was constructed so that any deviation from theoretical percentage levels could be attributed as exclusively as possible to the violation of the two assumptions. This implied that:

- I. in each simulated ANCOVA experiment, all assumptions of the test except those under study were upheld; and
- 2. a large enough series of ANCOVA experiments was simulated; that is, the sampling distribution was sufficiently large to keep the standard error of the probability of Type l error acceptably low.

Upholding the Remaining Assumptions of the Test

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Throughout the entire study the following specifications for each ANCOVA experiment obtained:

- a) the K error term populations, from which the K treatment groups were randomly drawn, were independently and normally distributed with zero mean.
- b) the fixed (for a given sampling distribution) values of the covariate **X** were transformed such that they were approximately normally distributed with $\overline{X}_j = 0$ and $S_{x_i} = 1$ for all treatment groups.
- c) Tj was set to zero for all treatment groups (the null hypothesis situation obtained).
- d) individual values on the dependent variable were determined by 'predicting' from the X values and then adding a random error term (the additivity assumption).

Size of the Sampling Distribution

3000 ANCOVA F-values constituted each sampling distribution. This ensured that under normal Central-F theory conditions, the standard error of the probability of Type 1 error ≈ 0.004 for $\alpha = .05$ and ≈ 0.002 for $\alpha = .01$.
THE MONTE CARLO SIMULATION PROCESS

In addition to the above specifications, values were also given to the following parameters in the construction of each sampling distribution:

- a) k, the number of treatment groups per ANCOVA experiment.
- b) n, the number of experimental units per treatment group.
- c) the κ population variances as determined by the value of $c_{\sigma^2}^2$ and
- d) the κ population regression slopes as determined by the value of $\sigma_{\beta_1}^2$.

With these specifications given as input, each sampling distribution is constructed by the following steps:

- 1. The simulation program causes K groups of random normal variates each of size n to be generated. These values are transformed in such a way as to conform to the specifications of the covariate (see above). The values are then used as the covariate values for each ANCOVA experiment in the sampling distribution.
- 2. Next, k groups of random normal variates each of size n are generated. The elements of the k groups are then transformed so that the k groups constitute k random samples from k error populations with respective variances as specified.

- 3. The values for the dependent variable are determined by $Y_{ij} = X_{ij} + \beta_j + \epsilon_{ij}$. The data are now ready for ANCOVA computation, and the calculation of F is carried out using a specially written ANCOVA program.
- 4. Steps 2 and 3 are repeated 3000 times, yielding an empirical sampling distribution of ANCOVA F-values for a particular combination of degrees of violation of the two assumptions. (A listing of the simulation program appears in Appendix C).

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CHAPTER 3

THE EXPERIMENTS

THE SCOPE OF THE STUDY

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Considerations of time and money kept the number of combinations of degrees of violations of the two assumptions investigated necessarily low, and since the primary purpose of the study was to establish a functional relationship between combinations of degrees of violation on the one hand, their effects on the probability of Type 1 error (α) on the other, it seemed desirable to sample from over a fairly wide range of violation of each assumption. A preliminary study was carried out to ascertain roughly the range of effects on α level of varying degrees of violation of each separate assumption. The results of this pilot suggested in the case of the homogeneity of regression assumption, a maximum level of σ_{β}^2 = 0.36 which yielded empirical percentages of the order of 2% and 0.02% corresponding to 5% and 1% nominal levels respectively; and in the case of the homogeneity of variance assumption a maximum level of $c_{2}^{2} = 1.80$ which yielded empirical percentages of the order of 10% and 5% corresponding to the nominal 5% and 1% levels respectively.

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PARAMETRIC SPECIFICATIONS

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The effects of simultaneously violating the two assumptions under study were examined using the following levels of violation:

 $\sigma_{\beta j}^{2}$ **0.00**, 0.04, 0.08, 0.12, 0.16, 0.20, 0.24, 0.28, 0.32, 0.36 $c_{\sigma j}^{2}$ 0.00, 0.18, 0.36, 0.54, 0.72, 0.90, 1.08, 1.26, 1.44, 1.62, 1.80.

The investigation proceeded by constructing six blocks of sampling distributions according to the procedure described in Chapter 2; each of the first four blocks contained 110 sampling distributions each of which embodied one of the 110 combinations of $\sigma_{\beta_j}^2$ and $c_{\delta_j}^2$ above. Each of the last two blocks contained 48 sampling distributions each of which embodied one of the 48 combinations of the first eight levels of $\sigma_{\beta_j}^2$ and the first six levels of $c_{\sigma_j}^2$. (The reason for the reduced number of sampling distributions in the last two blocks was due to inherent limitations on the procedure for deriving values of σ_j^2 corresponding to higher levels of $c_{\sigma_j}^2$ in the $\kappa = 2$ case).

The size of the simulated ANCOVA experiment in each block was as follows (k is the number of treatment groups, and n is group size)

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BLOCK	1	K = [5,	n	=	15	
BLOCK	2	k = !	5,	n	=	5	
BLOCK	3	κ = 3	3,	n	=	15	
BLOCK	4	к = 3	3,	n	=	5	
BLOCK	5	k = 2	2,	n	=	15	
BI OCK	6	к = 3	2,	n	=	5	

A violation-free sampling distribution $(\sigma_{\beta_j}^2 = 0.00, c_{\sigma_j}^2 = 0.00)$ was included in each block as a check on the accuracy of the simulation procedure. For each sampling distribution in each block, the number of ANCOVA F-values (referred to as -Ecounts) exceeding the F-values corresponding to nominal 5% and 1% levels of significance respectively, were recorded. An attempt was then made to establish a predictive relationship between Fcounts and levels of $\sigma_{\beta_j}^2$ and $c_{\sigma_j}^2$ and to examine any changes in this relationship which occurred with changes in

a) the ANCOVA design size

b) the $_{lpha}$ level of which Fcounts were taken.

RESULTS

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Tables 3.2 and 3.3 contain the matrices of Fcounts (expressed in proportions) of the six blocks of sampling distributions corresponding to the 0.05 and 0.01 levels of signifiance respectively. Table 3.1 below shows the 95% confidence regions for P = 0.5 and P = 0.1; they were calculated using the following angular transformation to normality for binomial variates:

$$A = \frac{1}{2} \left[\sin^{-1} \left(\frac{\chi}{N+1} \right)^{\frac{1}{2}} + \sin^{-1} \left(\frac{\chi+1}{N+1} \right)^{\frac{1}{2}} \right], \quad (3.1)$$

where X/N = P and A is asymptotically normally distributed with variance within $\pm 6\%$ of $821/(N+\frac{1}{2})$. (Keeping, 1962, p. 72).

TABLE 3.1

95% Confidence Limits for Estimating P = 0.05 and P = 0.01 for Sample Size N = 3000

Lower Limit	<u>Upper Limit</u>	<u> </u>
0.043	0.058	0.05
0.005	0.014	0.01

The first entry of each matrix in Tables 3.2 and 3.3 is an Fcount on a violation-free sampling distribution and as can be seen from Table 3.1 each one falls inside the 95% confidence intervals for P = 0.05 and P = 0.01 as the case may be, thereby validating the accuracy of the simulation process; these intervals are also useful in interpreting the **sexious**ness of violation effects.

Table 3.4 contains the results of multiple regression analysis applied to each block with Fcounts as the dependent variable and levels of violation of the two assumptions as predictorss. A subsequent examination of plotted residuals suggested the inclusion of the product of the two measures of violation as a third predictor. The regression equations for the three-predictor case are displayed in Table 3.5; with

the exceptions of $\hat{\beta}_3$ for Block 1 and for Block 5, in Table 3.2, all parameter estimates proved to be significantly different from zero at least at the 0.05 level of significance (For details of statistical tests see Appendix D).

As can be seen from inspection of Table 3.5, the values of the regression weights in the six equations corresponding to $\alpha = 0.05$ differ substantially from those of the six equations corresponding to $\alpha = 0.01$. In order to ascertain whether the regression equations depended on the size of k (number of treatment groups), tests for differences between the three sets of regression equations, (collapsed across n = 5 and n = 15) at each level of α were carried out, and yielded significant results in each case. Further, pairwise tests between regression equations with the same value of k and the same α level, but differing in the size of n were carried out, and yielded significant differences in all twelve cases. (For details of statistical tests, see Appendix D).

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 α (a): Matrix of Fcounts for Block l. k = 5, n = 15, $\alpha = .05$

	*				the second s	a subsection of the second					
- Cho	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.051	.066	.0 68	Q072	.073	• 085	.084	.088	.098	.104	.095
0.04	.045	.052	.059	.068	.067	.077	.075	.077	.082	.093	.091
0.08	.038	.046	.050	.054	.062	.069	.077	.068	.075	.082	.095
0.12	.0 29	.032	.045	.055	.055	.064	.069	.064	.076	.076	.079
0.16	.033	.036	.035	.052	.053	.048	.052	•068	.066	.083	.070
0.20	.026	.027	.039	.042	.048	.051	.056	.069	.065	.064	.070
0.24	.022	.027	.033	.033	.039	.049	.050	.051	.051	.064	.067
0.28	.016	.020	.027	.039	.030	.037	.045	.052	.048	.060	.061
0.32	.014	.023	[.] .026	.032	.032	.033	.042	051	.054	.055	.048
0.36	.013	.010	0022	.029	.031	.034	.038	.038	.041	.048	.053

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TABLE $3.2(b)$:	Matrix of	Fcounts	for Blo	оск 2.	k =	5. n =	25.α	05
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α α β j	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.047	.061	.075	.073	.100	.091	.105	.099	.107	.113	.119
0.04	.052	.055	.057	.066	.082	.078	.081	.091	.094	.099	.091
0.08	.037	.045	.055	.052	.062	.065	075	.082	.090	.097	.097
0.12	.034	.042	.052	.054	.060	.055	.063	.079	.081	.089	.089
0.16	.026	.032	.046	.051	.055	.056	.069	.0 6 6	.075	.075	.080
0.20	.027	.027	.038	.046	.054	.055	.057	•059	.061	.074	.068
0.24	.020	.028	.034	.041	.045	.056	.049	.050	.064	.065	.068
0.28	.026	.025	.030	.039	.035	.048	.049	.050	.056	.064	.065
0.32	.019	.023	.031	.027	.039	.040	.054	.050	.053	.049	.063
0.36	.018	.018	.024	.027	.034	.039	.041	.044	.044	0.50	.050

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	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.056	.057	.061	.066	.070	.076	.072	.081	.084	.077	.090
0.04	.045	.047	.047	.055	.059	.060	.063	.078	.081	.078	.089
0.08	.040	.044	.053	.049	.060	.065	.067	.067	.076	.,083	.088
0.12	.037	.041	.043	.044	.053	.059	. 052	•064	.063	.069	.065
0.16	.030	.034	.046	.039	.048	.047	.052	.051	•058	. 056	.061
0.20	.024	.029	.030	.041	.042	.044	.049	.051	.055	.067	•054
0.24	.025	.021	.030	.034	.032	.046	.047	.044	.056	.060	.056
0.28	.023	.025	.025	.030	.037	.037	.042	.048	.050	.051	.050
0.32	.019	.020	.026	.031	.033	.032	.045	.045	.048	.045	.051
0.36	.019	.021	.031	.028	.031	.037	.036	.036	.040	.047	.046

TABLE 3.2(c): Matrix of Fcounts for Block 3. $\kappa = 3$, n = 15, $\alpha = .05$

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α α α β β β β β	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.053	.061	.066	.070	.073	.073 .	.090	•095	.105	.110	.105
0.04	.039	.049	.057	.057	.070	.067	.084	.080	.094	.086	.110
0.08	.042	.046	.055	.063	.061	.070	.080	.084	.089	.097	.097
0.12	.034	.045	.049	.049	.052	.061	.073	.072	.075	.093	.086
0.16	.034	.038	.039	.044	.049	.054	.065	.067	.074	.076	.086
0.20	.033	.032	.048	.048	.041	.045	.059	.065	.066	.076	.071
0.24	.031	.027	.040	.033	.050	.051	.047	.050	.058	.064	.067
0.28	.028	.032	.037	.040	.042	.045	.044	.055	.059	.058	.056
0.32	.026	.027	.029	.040	.044	•035	.048	.050	.056	.054	.063
0.36	.020	.024	.025	.038	.034	.038	.043	•037	.051	.055	.054

TABLE 3.2(d): Matrix of Fcounts for Block 4. $\kappa = 3$, n = 5, $\alpha = .05$

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TABLE 3.2(e): Matrix of Fcounts for Block 5. $\kappa = 2$, n = 15, $\alpha = .05$

Bron Ngw	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.048	.054	.052	.053	.051	. 068 .					
0.04	.041	.050	.046	.046	.059	.049					
0.08	.045	.042	.042	.046	.052	.052					
0.12	.033	.043	.040	.043	.045	.047					
				· ·		in a sa s	-				
0.16	.038	.041	.035	.043	.036	.040					
0.16	.038 .030	.041 .037	.035 .042	.043 .035	.036 .036	.040 .039					
0.16 0.20 0.24	.038 .030 .025	.041 .037 .022	.035 .042 .031	.043 .035 .028	.036 .036 .039	.040 .039 .037					
0.16 0.20 0.24 0.28	.038 .030 .025 .030	.041 .037 .022 .025	.035 .042 .031 .030	.043 .035 .028 .034	.036 .036 .039 .032	.040 .039 .037 .027					
0.16 0.20 0.24 0.28 0.32	.038 .030 .025 .030	.041 .037 .022 .025	.035 .042 .031 .030	.043 .035 .028 .034	.036 .036 .039 .032	.040 .039 .037 .027					

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TABLE 3.2(f): Matrix of Fcounts for Block 6. $\kappa = 2$, n = 5, $\alpha = .05$

a by ch w ch ch ch ch ch ch ch ch ch ch ch ch ch ch c	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.048	.055	.061	.063	.073	.071					
0.04	.056	.043	.057	.059	.060	.070					
0.08	.036	.048	.047	.058	.056	.064					
0.12	.037	.044	.044	.048	.049	.058					
0.16	.033	.038	.044	.048	.047	.053					
0.20	.037	.029	.038	.040	.043	.051					
0.24	.032	.032	.033	.037	.041	.049					
0.28	.029	.026	.033	.034	.036	.041					
0.32			•								
0.36											

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TABLE 3.3(a): Matrix of Fcounts for Block 1. $\kappa = 5$, n = 15, $\alpha = .01$

				T							
N N N N N N N N N N N N N N N N N N N	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.011	.017	.016	.024	.025	.028	.032	.039	.042	.050	.043
0.04	.006	.011	.019	.020	.025	.025	.028	.032	.034	.044	.037
. 0.08	.006	.011	.017	.019	.020	.022	.028	.030	.034	.033	.042
0.12	.003	.009	.010	.020	.020	.021	.029	.025	.028	.031	.031
0.16	.005	.008	.010	.018	. 0 06	.013	.015	.023	.026	.031	.030
0.20	.004	.094	.009	.009	.014	.016	.017	.025	.024	.027	.023
0.24	.002	.004	.007	.010	.010	.014	.015	.019	.019	.024	.027
0.28	.001	.003	.004	.010	.006	.012	.014	.015	.017	.019	.024
0.32	.002	.003	.003	.007	.011	.008	.013	.016	.019	.022	.017
0.36	.001	.001	.004	.006	.009	.011	.011	.011	.014	.016	.020

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NA SUNCE	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.007	.012	.023	.023	.032	.030	.035	.040	.048	.048	.059
0.04	.011	.011	.016	.020	.031	.024	.031	.036	.038	.042	.043
0.08	.007	.013	.017	.012	.020	.024	.029	.032	.039	.039	.046
0.12	.004	.009	.013	.011	.017	.020	.018	.032	.034	.034	.041
0.16	.004	.006	.010	.014	.016	.018	.026	.020	.027	.028	.033
0.20	.007	.006	.010	.010	.015	.014	.022	.020	.024	.028	.02 6
0.24	.003	.006	.008	.010	.012	.016	.016	.017	.024	.025	.023
0.28	.004	.003	.005	.000	.010	.014	.016	.019	.018	.019	.023
0.32	.002	.004	.005	.008	.008	.012	.017	.014	.019	.018	.022
0.36	.002	.003	.005	.005	.011	.011	.013	.014	.015	.015	.020

TABLE 3.3(b): Matrix of Fcounts for Block 2. $\kappa = 5$, n = 5, $\alpha = .01$

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TABLE $3.3(c)$:	Matrix of	Fcounts	for Block 3.	k = 3, n =	15, $\alpha = .0$	21
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	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.011	.011	.015	.016	.021	.026	.023	.030	.034	.029	.034
0.04	.010	.010	.011	.013	.017	.018	.022	.026	.034	.028	.033
0.08	.008	.008	.013	.014	.021	.022	.026	.025	.031	.026	.033
0.12	.007	.011	.013	.010	.016	.017	.015	.017	.025	.024	.023
0.16	.004	.004	.007	.007	.013	.015	.015	.016	.014	.022	.022
0.20	.005	.004	.005	.013	.009	.015	.013	.015	.017	.023	.017
0.24	.005		.004	.009	.007	.013	.014	.018	.018	.018	.019
0.28	.003	.006	.005	.007	.009	.008	.011	.013	.016	.015	.012
0.32	.002	. 0 0 3	.006	.005	.008	.008	.012	.012	.015	.013	.017
0.36	.003	.002	.007	.007	.008	.010	.012	.010	.012	.014	.014

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NAN	0.00	0.18	0.36	0.54	0.72	0.90	80.1	1.26	1.44	1.62	1.80
0.00	.009	.015	.016	.020	.020	.025	.031	.037	.039	.046	.043
0.04	.007	.012	.014	.014	.019	.019	.027	.030	.037	.034	.049
0.08	.006	.010	.010	.015	.018	.025	.025	.027	.034	.036	.041
0.12	007	.007	.011	.014	.012	.017	.022	.026	.034	.038	.034
0.16	.004	.007	.008	.010	.014	.017	.021	.022	.025	.027	.032
0.20	.006	.006	.011	.014	.012	.0}0	.019	.021	.0244	.029	.022
0.24	.005	.005	.009	.009	.010	.013	.012	.018	.019	.024	.026
0.28	.005	.004	.010	.011	.008	.013	.012	.015	.018	.020	.018
0.20	.005	.003	.006	.011	.009	.006	.012	.012	.019	.015	.019
0.36	.004	.004	.004	.009	.007	.011	.013	.009	.014	.017	.020
	1	1	i	i		astssss till					•

TABLE 3.3(d): Matrix of Fcounts for Block 4. $\kappa = 3$, n = 5, $\alpha = .01$

-- TABLE 3.3(e): Matrix of Fcounts for Block 5. $\kappa = 2$, n = 15, $\alpha = .01$

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							A	<u> </u>			· · · · · · · · · · · · · · · · · · ·
NAC CONCERNENCE	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.009	.008	.011	.014	.014	.015 .					
0.04	.008	.014	.009	.008	.015	.014					
0.08	.009	.009	.007	.009	.010	.013					
0.12	.008	.006	.010	.012	.008	.011					
0.16	.005	.008	.007	.007	.008	.011					
0.20	.004	.007	.007	.007	.005	.007					
0.24	.003	.002	.006	.006	.004	.008					
0.28	.005	.005	.007	.005	.005	.003		•			
0.32			•								
0.36			•								

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NOC CUB	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
0.00	.008	.013	.012	.013	.020	.022					
0.04	.011	.006	.017	.013	.011	.021					
0.08	.006	.010	.011	.014	.013	.017					
0.12	.004	.009	.010	.013	.012	.015					
0.16	.007	.006	.009	.009	.011	.013					
0.20	.006	.005	.007	.007	.009	.014					
0 . 24	.006	.005	.009	.008	.011	.008					
0.28	.006	.004	.006	.005	.008	.009		•			
0.32			•			•					
0.36											

TABLE 3.3(f): Matrix of Fcounts for Block 6. $\kappa = 2$, n = 5, $\alpha = .01$

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TABLE 3.4: Multiple Regression Weights for 6 Blocks of Sampling Distributions. Note: $\hat{\beta}_0$ = constant; $\hat{\beta}_1$ = estimated regression weight for $c^2_{\sigma_j^2}$; estimated regression weight for $\delta^2_{\beta_j}$

BLOCK NO.	<u> </u>	K	n	Â.	βì	β ₂	R
1	.05	5	15	161.93	 74 84	-383 87	0.076
2	.05	5	5	174.65	81.89	-421.07	0.965
3	.05	3	15	152.42	58.56	-318.50	0.964
4	.05	3	5	165.20	77.79	-355.58	0.963
5	.05	2	15	147.09	27.86	-266.37	0.914
6	.05	2	5	153.65	62.56	-302.33	0.959
ו	.01	5	15	44.68	43.37	-167.48	0.960
2	.01	5	5	46.67	47.56	-178.44	0.946
3	.01	3	15	38.02	30.59	-123.40	0.934
4	.01	3	5	39.69	42.55	-145.22	0.937
5	.01	2	15	30.33	10.60	- 77.18	0.831
6	.01	2	5	J 31.30	25.79	- 88.00	0.877

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TABLE 3.5: Multiple Regression Weights for 6 Blocks of Sampling Distributions Note: $\hat{\beta}_0$ = constant; $\hat{\beta}_1$ =estimated regression weight for c_{02}^2 ; $\hat{\beta}_2$ estimated regression weight for $\delta_{\beta_j}^2$; $\hat{\beta}_3$ = estimated regression weight for $\sigma_{\beta_j}^2 \cdot c_{0j}^2$

BLOCK NO.	<u> </u>	ĸ	<u> </u>	β _o	β ₁	β ₂	β <u>3</u>	<u>R</u>
1	.05	5	15	156.14	81.28	-351.68	- 35.80	0.976
2	.05	5	5	155.74	102.91	-316.00	-116.80	0.972
3	.05	3	15	145.16	66.62	-278.17	- 44.80	0.965
4	.05	3	5	141.33	104.31	-223.00	-147.30	0.975
5	.05	2	15	156.14	38.06	-233.59	- 72.80	0.917
6	.05	2	5	155.74	77.20	-255.29	-104.50	0.963
1	.01	5	15	31.53	57.99	- 94.41	- 81.20	0.974
2	.01	5	5	25.79	70.54	- 63.51	-127.70	0.975
3	.01	3	15	27.16	42.66	- 63.08	- 67.00	0.951
4	.01	3	5	19.43	65.06	- 32.66	-125.10	0.973
5	.01	2	15	26.00	20.23	- 46.23	-6 68880 0	0.856
6	.01	2	5	25.22	39.30	- 44.59	- 96.50	0.902

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CHAPTER 4

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INTERPRETATION OF RESULTS

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The results of this study belong to that body of findings which attempts to apprise the scientist of the consequences of applying statistical tests to his data when the requirements of the underlying assumptions of the test are not fulfilled. For the most part, previous findings have related to situations where only one assumption is being violated with all other demands of the test satisfied. The present study arose out of the suggestion implicid in several separate findings that the simultaneous presence of violation of the homogeneity of regression and homogeneity of variance assumptions in ANCOVA would tend to cancel each other out in their effects.

The overall findings contained in Chapter 3 show that this tendency is in fact the case; for all 6 blocks of sampling distributions the effect on probability of Type 1 error of a given degree of violation of one assumption is strongly dependent on the degree of violation present in the other. Furthermore, the two violations are seen to have a neutralizing effect on one another. Inspection of Tables

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3.2 and 3.3 reveals that many combinations of σ_{β}^2 and $c_{\sigma^2}^2$ have sampling distributions with Fcounts falling within the 95% confidence interval for P = 0.05 or P = 0.01 as the case may be (see Fcounts within bold lines). It is only those combinations involving a low level of violation of one assumption in combination with a high level of violation of the other that yield Fcounts extending beyond the confidence limits. Finally, for any given violation level of one assumption, the most serious departure from nominal probability levels occurs when the other assumption is fully satisfied (see Column 1 and Row 1 of each block).

From inspection of the multiple regression data of Tables 3.4 and 3.5, it is suggested that the inclusion of the product of the two violation measures as a third predictor is important not only because most of its regression estimates proved statistically significant but also because the resultant regression equations yield values of the constant term $\hat{\beta}_0$ which more readily approximate the expected numbers of Fcounts when $\sigma_{\beta_j}^2 = 0.0$ and $c_{\sigma_j}^2 = 0.0$ for both α levels (150 and 30 respectively) than those emerging from the twopredictor equations.

The differences between the 6 regression equations corresponding to α = .05 and those corresponding to α = 0.1 in Table 3.5 appear to be accounted for by changes in the

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relative weightings of $\sigma_{\beta j}^2$ and $c_{\delta j}^2$. It is clear from inspection that the role of $c_{\delta j}^2$, the measure of violation of the homogeneity of variance assumption, is more important in the equations corresponding to $\alpha = .01$ than in those corresponding to $\alpha = .05$. This is also evident in the different locations of the areas circumscribed by bold lines in Tables 3.2 and 3.3 respectively. The confounding presence of the interaction term β_3 notwithstanding, it also appears that within each α level, significant differences between

regression equations corresponding to different values
 of k (collapsed over n), and

2. pairs of regression equations corresponding to different values of n for a given value of K,

are also attributable to changes in the relative weightings of $\sigma_{\beta j}^2$ and $c_{\beta j}^2$. While it is difficult to interpret a trend in the former case, it is suggested that the latter differences are due to the increasing importance of $c_{\beta 2}^2$ when n is relatively small (compare n = 5 with n = 15).

Referring again to Table 3.5, it can be seen from the column of multiple correlation coefficients (R), that the three predictors account for a very large proportion of the dependent variable (Fcounts); these figures testify to the high degree of linearity in the parameters.

QUAL IF ICAT IONS TO RESULTS

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Any Monte Carlo researcher who attempts to examine the effects of violating some specified assumption or set of assumptions underlying a statistical test is invariably faced with a methodological paradox: in order to evaluate the specific effects of violating the particular assumption or assumptions of his study, he must ensure that all other assumptions of the test are upheld. This precaution, however, reduces the generalizability of his results to situations where some of the other assumptions of the test are also not upheld. Consequently he can only hope that the general trend of his findings obtained in these latter situations. Often it is a hope that is not realized as the present study has attempted to show for one specific situation.

Apart from these general considerations, perhaps the greatest impediments to generalizability in this investigation are the restrictions on the behaviour of the covariate, X_{ii}. These were:

1. ΣX_{ij} and hence \overline{X}_{j} were set to a constant for all j. 2. ΣX_{ij}^{2} was set equal to a constant for all j. These two constraints on the covariate permitted a derivation of an invariant measure of violation of the homogeneity of reregression assumption, while the second constraint permitted the extension of Box's approximate measure of violation of the

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homogeneity of variance assumption (derived for ANOVA) to the present ANCOVA context. Another qualification to the general aplicability of the present results is due to the arbitrary if systematic way in which,

1. sets of β_j and σ_j^2 were derived to correspond to given levels of $\sigma_{\beta_j}^2$ and $c_{\beta_j}^2$ respectively, and 2. these two sets, so derived, were matched in the simulated ANCOVA experiments. More generally, even though it was possible to derive invariant (or nearly so) measures of violation of the two assumptions separately, there is no guarantee that the combination of these two measures constitutes an invariant measure of their joint violation.

APPLICATION OF RESULTS TO PRACTICAL SITUATIONS

Despite the qualifications outlined in the preceding section, the general results of the present study strongly points to the neutralizing effect on the F-test in ANCOVA of the simultaneous violation of the homogeneity of regression and homogeneity of variance assumptions. It is very unlikely that in any practical ANCOVA situation either of these two assumptions will be fully met, and consequently the ANCOVA F-test appears from the present findings to be even more robust in practice, than results from previous studies taking these violations one at a time have indicated. It is difficult to estimate the upper limits on the violations of these two assumptions

as they occur in practice, but they are practically certain to fall within the range of violations encompassed by the present study, and unless the departure from one assumption is very big and is accompanied by only a slight departure from the other, it is safe to conclude that the ANCOVA Ftest is only inconsequently affected by the presence of joint violations of the two assumptions. Finally, the findings of the present study suggest the limited usefulness of any test of homogeneity of regression slopes carried out on ANCOVA data without due regard being given to possible differences between treatment group variances.

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APPENDIX A

1. Proof that (R-Q) is symmetric and idempotent:

R-Q is symmetric, since

Let L be a matrix with 1 as its last element and zero's everywhere else, such that SL = T.

Then RQ =
$$\begin{bmatrix} I - SL(L'S'SL)^{-1}L'S' \end{bmatrix} \begin{bmatrix} I - S(S'S)^{-1}S' \end{bmatrix}$$

= $I - S(S'S)^{-1}S' - SL(L'S'SL)^{-1}L'S' + SL(L'S'SL)^{-1}$
 $L'S'S(S'S)^{-1}S'$
= $I - S(S'S)^{-1}S'$
= Q (A.1)

(R-Q) is also idempotent, for $(R-Q)^2 = R^2 - RQ - QR + Q^2$ = R - Q - Q + Q, since $R^2=R$, $Q^2=Q$ and QR=Q'R'=(RQ)'=Q'=Q

= R - Q

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APPENDIX A (cont'd)

2.

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Proof that
$$(R - Q)'Q = 0$$
:
 $(R - Q)'Q = (R - Q)Q$
 $= (RQ - QQ)$
 $= Q - Q$ using (A1)
 $= 0$

3. Proof that under the constraints placed on the covariate X_{ii} as stated in Chapter 2,

$$\lambda_1 = 0$$
 and $\lambda_2 = n - 1 \left[\sum_{j} \beta_j^2 - \kappa \overline{\beta}^2 \right]$:

The constraints on X_{ij} are:

$$\Sigma X_{ij} = 0 \quad \text{for all } j \qquad (A3)$$

$$\Sigma X_{ii}^2 = n - l \quad \text{for all } j \qquad (A4a)$$

$$\sum_{i j} X_{ij}^2 = (n - 1)\kappa$$
 (A4b)

Let the design matrix X be partitioned into the following submatrices:

 $X = \begin{bmatrix} A & T & B \end{bmatrix},$

where A is the nK x K submatrix whose columns correspond to treatment assignment, T is the already defined nK x l vector of covariate values and B is the nK x K submatrix whose columns correspond to $(\beta_j - \overline{\beta}), j = 1, \dots, K$. APPENDIX A (cont'd)

then
$$A'T = T'A = 0$$
 (using A3) (A5)

$$A'B = B'A = 0 \quad (using A3) \quad (A6)$$

$$\lambda_{1} = \beta^{*}X^{*}(R - Q) X\beta$$

= $\beta^{*}X^{*}\left[1 - T(T^{*}T)^{-1}T^{*} = 1 + S(S^{*}S)^{-1}S^{*}\right]X\beta$
= $\beta^{*}X^{*}\left[S(S^{*}S)^{-1}S^{*} - T(T^{*}T)^{-1}T^{*}\right] X\beta$

Now

$$S = \begin{bmatrix} A & | T \end{bmatrix} \text{ and } S'S = \begin{bmatrix} A'A & | A'T \\ T'A & | T'T \end{bmatrix}$$
$$\begin{bmatrix} A'A & | O \\ --- & | T'T \end{bmatrix}, \text{ (using A5)}$$

$$(S'S)^{-1} = \begin{bmatrix} -1 \\ (A'A)^{-1} & 0 \\ 0 & (T'T)^{-1} \end{bmatrix}$$
, using a theorem on the inverse of partitioned matrices (Graybill,

1969, p. 165).

$$S(S'S)^{-1}S' = \left[A \mid T\right] \left[\frac{(A'A)^{-1}}{0} \mid \frac{0}{(T'T)^{-1}}\right] \left[\frac{A'}{T'}\right]$$
$$= A(A'A)^{-1}A' + T(T'T)^{-1}T'$$
(A7)

Thus

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$$\begin{aligned} \mathbf{x}_{1} &= \boldsymbol{\beta}' \mathbf{X}' \begin{bmatrix} \mathbf{A} (\mathbf{A}'\mathbf{A})^{-1} \mathbf{A}' + \mathbf{T} (\mathbf{T}'\mathbf{T})^{-1} \mathbf{T}' - \mathbf{T} (\mathbf{T}'\mathbf{T})^{-1} \mathbf{T}' \end{bmatrix} \mathbf{X} \mathbf{\beta} \\ &= \boldsymbol{\beta}' \mathbf{X}' \begin{bmatrix} \mathbf{A} (\mathbf{A}'\mathbf{A})^{-1} \mathbf{A}' \end{bmatrix} \mathbf{X} \mathbf{\beta} \end{aligned}$$

APPENDIX A (cont'd)

$$A'X = \begin{bmatrix} A'A & A'T & A'B \end{bmatrix}$$
$$= \begin{bmatrix} A'A & O & O \end{bmatrix} \quad (using A5 and A6)$$

A'A is $K \times K$. Under the null hypothesis, $\tau_1 = \tau_2 \cdots \tau_K = 0$, the leading K elements of β are zero.

Hence $A'X\beta = 0$ (A8) Hence $\lambda_1 = 0$ (A9)

$$\lambda_{2} = \beta' X' \left[1 - S(S'S)^{-1}S' \right] X\beta$$

$$= \beta' X' X\beta - \beta' X' \left[A(A'A)^{-1}A' \right] X\beta - \beta' X' \left[T(T'T)^{-1}T' \right] X\beta,$$
(using A7)
$$= \sum_{j} \beta_{j}^{2} \sum_{i} X_{ij}^{2} - 0 - \beta' X' T(T'T)^{-1}T' X\beta, \text{ (using A9)}$$

$$(T'T) = \sum_{i} \sum_{j} X_{ij}^{2} \text{ and } (T'T)^{-1} = 1/\sum_{i} \sum_{j} X_{ij}^{2}$$
Thus $\beta' X' T(T'T)^{-1}T' X\beta = 1/\sum_{i} \sum_{j} X_{ij}^{2} \left[\beta' X' TT' X\beta \right]$

$$\beta' X' T = T' X \beta = \sum_{j=1}^{\infty} \beta_{j} \sum_{i=1}^{\infty} X_{ij}^{2}$$

Thus

$$\lambda_{2} = \sum_{j} \beta_{j}^{2} \sum X_{ij}^{2} - 1/\Sigma \sum X_{ij}^{2} \left(\sum_{j} \beta_{j} \sum X_{ij}^{2}\right)^{2}$$
$$= n-1 \left(\sum_{j} \beta_{j}^{2}\right) - \frac{(n-1)^{2}}{(n-1)\kappa} \left(\sum_{j} \beta_{j}\right)^{2}$$
$$= n-1 \left[\sum_{j} \beta_{j}^{2} - \kappa \overline{\beta}^{2}\right]$$

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APPENDIX B

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Values of β_j and σ_j^2 corresponding to levels of $\sigma_{\beta_j}^2$ and $c_{\sigma_j^2}^2$.

к = 5

σ ² βj	β	β2	^β 3	β <u>4</u>	β <u>5</u>
0.00 0.04 0.08 0.12 0.16 0.20 0.24 0.28 0.32 0.36	1.000000 0.717157 0.600000 0.510102 0.434315 0.367544 0.307180 0.251669 0.200000 0.151472	1.000000 0.858579 0.800000 0.755051 0.717157 0.683772 0.653590 0.625834 0.600000 0.575736	1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000	1.000000 1.141421 1.200000 1.244949 1.282843 1.316228 1.346410 1.374166 1.400000 1.424264	1.000000 1.282843 1.400000 1.489898 1.565685 1.632456 1.692820 1.748331 1.800000 1.848528
cos:	σ_{l}^{2}	0 ² 2	6 <mark>2</mark> 3	<u>64</u>	6 ² 5
0.00 0.18 0.36 0.54 0.72 0.90 1.08 1.26 1.44 1.62 1.80	1.000000 0.690374 0.558518 0.457049 0.372529 0.302851 0.244267 0.188417 0.143651 0.104365 0.071038	1.000000 0.838605 0.752084 0.677277 0.608515 0.546432 0.489463 0.429794 0.376756 0.324585 0.273825	1.000000 0.962725 0.923455 0.882294 0.839373 0.769713 0.769713 0.76622 0.661098 0.613175 0.563104	1.000000 1.098524 1.119061 1.124999 1.122283 1.113687 1.100861 1.082341 1.082341 1.036069 1.006574	1.000000 1.298665 1.420338 1.512725 1.589612 1.653676 1.708745 1.763235 1.809415 1.853195 1.894650

o ² J	β	β ₂	<u>β</u> 3
0.00	1.000000	1.000000	1.000000
0.04	0.755051	1.000000	1.244948
0.08	0.653590	1.000000	1.346410
0.12	0.575736	1.000000	1.424263
0.16	0.510102	1.000000	1.489898
0.20	0.452278	1.000000	1.547722
0.24	0.400000	1.000000	1.599999
0.28	0.351926	1.000000	1.648073
0.32	0.307180	1.000000	1.692820
0.36	0.265153	1.000000	1.734846

	σ_1^2	0 ²	0 ² 3
0.00 0.18 0.36 0.54 0.72 0.90 1.08 1.26 1.44 1.61	1.000000 0.725208 0.594683 0.493581 0.400668 0.315367 0.236368 0.163549 0.98270 0.041036 0.005997	1.000000 0.961686 0.918516 0.873828 0.823106 0.766848 0.704075 0.633189 0.551836 0.449345 0.325273	1.000000 1.244632 1.342640 1.411666 1.470361 1.520687 1.564739 1.603846 1.638847 1.672246 1.701224

к = 3

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٥ ² ا	β ₁	β <u>2</u>
0.00	1.000000	1.000000
0.04	0.800000	1.200000
0.08	0.717157	1.282843
0.12	0.653590	1.346410
0.16	0.600000	1.400000
0.20	0.552786	1.447214
0.24	0.510102	1.489898
0.28	0.470850	1.529150
0.32	0.434315	1.565685
0.36	0.400000	1.600000

к = 2

c ² 2 ² ;	σ_1^2	02 02
0.00	1.000 000	1.000000
0.18	0.755864	1.195270
0.36	0.635216	1.263527
0.54	0.512068	1.318251
0.72	0.390689	1.359177
0.90	0.223631	1.396420

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APPENDIX C

LISTING OF SIMULATION PROGRAM AND SUPER-DUPER

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PROGRAM SIMULATION C . . 1 2 C., 3 C., REAL *8 SUMD, SUMXD, SMIJX2, SMIJY2, SMIJXY, TJXI, TJYI, BIGTX, BIGTY, 4 1SMTJX2, SMTJY2, SMTJXY, BIGTX2, BIGTY2, BIGTXY, ADJTOT, WITHIN, F 5 DIMENSION TUTAL(10000), X(100), VARX(5), Y(100), 6 1STANDE(5), BETAXY(5), FVALUE(3000) 7 VARX(1), VARX(2), VARX(3), VARX(4), VARX(5)/5*1.0 / DATA 8 READ (5,100, END=200) ND, N, K, INTEG1, INTEG2, CRIT5, CRIT1, MUCH, NEED 9 1 FORMAT (1X, I3, 4X, I2, 4X, I1, 4X, 2I3, 5X, F6, 2, 4X, F6, 2, 4X, I4, 4X, I2) 100 10 READ (5,40) (STANDE(1), I=1,K) 11 (BETAXY(L),L=1,K) READ (5,40) 12 FORMAT (5F9.6) 13 40 , 14 C., 15 C . . C., THE FOLLOWING BLOCK - PART 1 - USES SUPERDUPER TO GENERATE 16 C. RANDOM NORMAL VARIATES, AND THEN ARRANGES THE XIS AND Y'S 17 C., ACCORDING TO PARAMETRIC SPECIFICATIONS. 18 19 С., 20 C . . SN=N 21 SK=K 22 23 NK=N*K SNK=NK 24 CALL START (INTEG1, INTEG2) 25 DO 18 I=1,NK 26 X(I) = RNOR(0)27 18 CONTINUE 28 LL=1 29 INDEX=1 30 31 L=1 NN = N32 5 SUMD=0.0 33 SUMXD=0.0 34 SUMD=SUMD+X(L) 35 6 SUMXD=SUMXD+X(L)*X(L) 36 L=L+1 37 GO TO 6 IF(L.LE.NN) 38 AMEAN=SUMD/SN 39 VARIAN=(SUMXD*SN-(SUMD*SUMD))/(SN*(SN-1.0)) 40 FACTOR=VARX(INDEX)/SQRT(VARIAN) 41 X(LL)=(X(LL)-AMEAN)*FACTOR 7 42 LL = LL + 143 GO TO 7 IF(LL,LE,NN) 44 INDEX=INDEX+1 45 45 NN=NN+N GO TO 5 IF(NN,LE,NK) 47 LOT=MUCH*NK 48 MORE=MUCH 49 LEAP=150 I=1,NEED DD 20 51 J=1,LDT DO 19 52 TOTAL(J) = RNOR(0)53 CONTINUE 19 54 JUMP=0 55 56 24 INDEX=1 NN = N57 58 L=1

	59) 10	Jal + HIMp
	60)	
\$ 9	- A1	-	I - I - I - I - I - I - I - I - I - I -
Z.	۰. ۲	• •	TE / L Frenkins and me -
	62	-	AF (L+LE+NN) GU TO 10
	66		
	0 T 4 E	, :	
	02		IF(NN.LE.NK) GD TO 10
	00		CUNTINUE
	67	C	
	68	C	
	69	C	THE FOLLOWING BLOCK - PART 2 - COMPUTES ANOUVA IN DOUBLE PRECISION
	70	C	THE TELEVISION AND THE DOUDLE PRECISION
	71	C	
	. 72		SMIJX2=0,0
	73		SMIJY2=0.0
	74		SMIJXY=0.0
	75		L=1
	76	13	SMIJX2=SMIJX2+X(1)+X(1)
	77		SMIJXY=SMIJXY+X/I)+X/I)
	78	•	SMTJY2=SMTJY2+V/LY+V/LY
	79		
	03		IF(L. F. NK) CO TO 12
	81		SMTJX2=0.0
	82		SMTJY2=0.0
	83		SMTJXY=0.0
	84		BIGTX=0.0
	85		BIGTY=0.0
	86		
	87		
	88	15	TJYI=0.0
	89		TJXI=0.0
	90	14	TJXI=TJXI+X(I)
	91		TJYI = TJYI + Y(I)
	92		L=L+1
	93		IF(L.LE.NN) GD TD 14
	94		SMTJX2=SMTJX2+((TJX5#TJX7)/SM)
	95		SMTJXY=SMTJXY+((TJYT+TJYT)/SN)
	96		SMTJY2=SMTJY2+((TIVI*TIVIX/GM)
-	97		BIGTX=BIGTX+TIXT
	98		BIGTY=BIGTY+TIVI
	99		NN=NN+N
	100		IF(NN.LF.NK) GD TD 15
	101		BIGTX2=(BIGTX#BIGTX)/SNK
,	102		BIGTXY=(BIGTX#BIGTY)/CHI
•	103		BIGTY2=(BIGTY*BIGTY)/SNK
	104		ADJTOT=(SMIJY2-BIGTY2)-///SMI INV DICTWYAT/CHI INV DICTWYA
	105	1	(SMIJX2-BIGTX2)
	106		WITHIN= (SMIJY2-SMTJY2)-(((SMT)YY-CHT)YY)+(CHT)YY)
	107	1	(SMIJX2=SMTJX2))
	108		F=((ADJTOT=WITHIN)/(SK-1 O))//WITHIN//CNW CK 1 ON
	109		FVALUE(LEAP) = F
•	110		LEAP=LEAP+1
	111		JUMP=JUMP+NK
	112		IF(LEAP.LE.MORE) GD TD 24
	113		MORE=MORE+MUCH
	114	20	CONTINUE
	115	C	
	116	С.,	

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117	C., T	HE FOLLOWING BLOCK - PART 3 - COMPARES THE 3000 F-VALUES
118	C. W	ITH NOMINAL F-VALUES AT THE 0.05 AND 0.01 LEVELS RESPECTIVELY
119	C	
120	C.	
121		INDEX5=0
122		INDEX1=0
123	17	DD 16 K=1,3000
124		FVAL=FVALUE(K)
125		IF(FVAL.LT.CRIT5) GD TO 16
126		INDEX5=INDEX5+1
127		IF(FVAL.LT.CRIT1) GO TO 16
128		INDEX1=INDEX1+1
129	16	CONTINUE
130		WRITE (6,102) NO,LEAP, INDEX5, INDEX1
131	102	FORMAT (////>2X;I4>4X>I9>4X;I4>4X>I4)
132		GD TO 1
133	200	STOP
134		END

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1 - 粋 2 * MCGILL UNIVERSITY SCHOOL OF COMPUTER SCIENCE 3 * * 4 RANDOM NUMBER GENERATOR PACKAGE - 'SUPER-DUPER' 5 * × 6 7 * UNIFORM, NORMAL AND EXPONENTIAL RANDOM NUMBER GENERATOR 8 * 9 * G. MARSAGLIA, K.ANANTHANARAYANAN, N.PAUL. × 10 11 * RANDOM NUMBER GENERATOR PACKAGE-REGISTER USAGE 12 * GPR O - STORES RESULT OF IUNI, IVNI 13 * ¥ GPR 1 - (REGB) CALCULATION OF RESULTS 14 GPR 2 - (REGC) CALCULATION OF RESULTS 15 * * GPR 3 - (REGD) CALCULATION OF RESULTS 16 17 GPR13 - ADDRESS OF SAVE AREA OF CALLING PROGRAM, OR OF THIS * PROGRAMSIS SAVE AREA ON CALL TO RNORTH OR REXPTH 18 * GPR14 - CUNTAINS RETURN ADDRESS. 19 ¥ GPR15 - USED AS BASE REGISTER. 20 * FPR O - RESULT OF UNI, VNI, REXP, RNOR. 21 × 22 * DEFINE ENTRY POINTS RANDOM START O 23 24 ENTRY START CALL START(11,12) 25 ENTRY UNI U=UNI(0)ENTRY VNI 26 V = V N I (0)ENTRY RNDR 27 X = RNOR(0)28 ENTRY REXP Y = REXP(0)ENTRY IUNI 29 K = IUNI(0)ENTRY IVNI 30 J = IVNI(0)31 EXTRN RNORTH FORTRAN FUNCTIONS REQUIRED-RNORTH(I) EXTRN REXPTH REXPTH(I) 32 33 REGB EQU 1 REGC EQU REGISTER EQUATES 34 2 REGD 35 EQU 3 36 뇨 CALL START(I1, I2) 11,12 ARE USED FOR STARTING THE TWO 37 ¥ * SEQUENCES INCONI AND ISRGNI. 38 39 USING START, 15 40 START STM REGB, REGD, 24(13) SAVE REGISTERS 1,2,3 LM REGC, REGD, O(1) LOAD ADDRESSES OF 11,12 INTO REGC,REGD 41 REGC > O(REGC) LOAD VALUE DF I1 INTO REGC 42 L LTR REGC, REGC 43 IF ZERD, STORE AT IMCGNIJELSE 44 BC 8,ST1 ENSURE ODD, TO KEEP PERIOD OF 'MCGN' LARGE 45 0 REGC X1 REGC, MCGN STORE AT IMCGNI ST1 ST 46 LOAD I2 INTO REGD REGD, O(REGD) 47 L REGD, REGD LTR 48 IF ZERD, STORE AT 'SRGN', ELSE 49 BC 855T2 REGD:X7FF TAKE RESIDUE MODULO 2048 50 N 0 AND ENSURE NON-ZERO 51 REGD X1 AND STORE AT 'SRGN', ST 52 ST2 REGD, SRGN RETRNO LM REGB, REGD, 24(13) RESTORE REGISTERS 1,2,3 53 BCR 15,14 AND RETURN 54 55 * U=UNI(0)RESULT IS NORMALIZED FLOATING POINT VALUE * 56 UNIFORMLY DISTRIBUTED ON (0.0,1.0). 57 * USING UNI:15 58

· · · · · · · · · · · · · · · · · · ·	590123456789012345678	UNI RDIGT1	ST RL RRRLSXSLMSXSASLAL BRRLLSXSLMSXSASLAL	REGB; REGD; 24(13) REGB; SRGN REGC; REGB REGC; 15 REGB; REGC REGC; REGB REGC; 17 REGB; REGC REGB; REGC REGD; REGB REGD; MCGN REGD; REGB REGD; B REGD; CHAR REGD; FWD O; Z REGB; REGD; 24(13)	SAVE REGISTERS 1,2,3 LOAD SRGN INTO REGB AND INTO REGC SHIFT REGC RIGHT 15 BITS AND XOR INTO REGB COPY REGB INTO REGC SHIFT IT LEFT 17 BITS, AND XOR INTO REGB SAVE THE NEW 'SRGN' LOAD MCGN INTO REGD AND MULTIPLY BY 69069 STORE RESULT, MODULO 2**32, AS NEW 'MCGN' XOR NEW 'MCGN' AND 'SRGN' IN REGD SHIFT REGD RIGHT 8 BITS FOR F.P. FRACTION ADD CHARACTERISTIC X'40' INTO FIRST BYTE STORE AT FWD, LOAD INTO FPR 0, AND ADD NORMALIZED TO ZERD LEAVING RESULT 'UNI' IN FPR 0.
	78	*	BCR	15,14	RETURN
	80	* /	V=VNI(0)	RESULT IS NORMALIZED FLOATING POINT VALUE
	81	*		VNT • 1 5	UNIFORM ON (-1.0/1.0)
	83	VNI	STM	REGB, REGD, 24(13)	SAVE REGISTERS 1,2,3
2 	84	RDIGT2		REGB SRGN	LOAD SRGN INTO REGB
	85		SRL	REGC/15	SHIFT REGC RIGHT 15 BITS
	87		XR	REGB, REGC	AND XOR INTO REGB
	88			REGCAREGB	COPY REGB INTO REGC
	90		XR	REGB; REGC	AND XOR INTO REGB
i. L	91		ST	REGB, SRGN	SAVE THE NEW ISRGN
•	92		L	REGDIMCGN	LDAD MCGN INTO REGD
	93		ST	REGDIMOLI	STORE RESULT, MODULO 2**32, AS NEW MCGN
	95		XR	REGDAREGB	XDR NEW IMCGNI AND ISRGNI IN REGD
•	96		SRA	REGD 7	SHIFT RIGHT 7 BITS DE FIRST BYTE
	97		AL	REGD, CHAR	ADD CHARACTERISTIC X'40' TO FIRST BYTE
	99		ST	REGD, FWD	STORE AT FWD, LOAD INTO FPR O
	100		LE	O.FWD	AND ADD NORMALIZED TU ZERU Turantiko pesult ivnit in FPR 0.
	101	RETRN2	AE LM	U#4 REGB,REGD,24(13)	PERATIAD VERABL LANTI TH IIN AN
•	103		BCR	15,14	RETURN
	104	**		101	DECHLT IS STANDARD NORMAL VARIATE.
2 1	105	*	X=KNUR	(0)	VEDAPI TO DIVINUALA MANINE ANTALES
	107	*	METHOD		
1	108	*	0a		HTHE & BANDTH HEYADECIMAL DIGITS.
•	109	* l• *	GENE	KATE MIHZH3H4H5H6	LIUOTO KUMUDA LEVADECTARE DIGILIT
	111	÷ 2.	IF H	1H2 .LT. 68, SET	IRNOR! TO
(.	112	* 3.	те н	(NTBL 11H2 .LT. DOJ SET	(H1H2)+.H3H4H5H6H7H8)/16, AND QUIT, !RNOR! TO
	114	*		(-NTB	L(H1H2-68)H3H4H5H6H7H8)/16, AND QUIT.
• •	115	* 4.	IF H	1H2H3 ,LT, E2F, S	ET IRNOR! TO

	1.7	т. Е	។ខ មា	10202 JT. 555. 55	
	118	* 2*	TL U1	(-NTRL	(H1H2H3-F17)- H4H5H6H7H8)/16, AND QUIT.
\$ }	119	* 6.	FLSE	GENERATE IRNORI F	ROM THE NORMAL TOOTH-TAIL SUBPROGRAM.
	120	*			
•	121	*			
	122		USING	RNDR+15	
	123	RNDR	STM	REGB, REGD, 24(13)	SAVE REGISTERS 1,2,3
	124	RDIGT3	L	REGB, SRGN	LUAD SRGN INTO REGB
	125		LR	REGC = REGB	AND INTO REGO
	126		SRL	REGC/15	SHIFT REGC RIGHT 15 BITS
	127		XR	REGBIREGU	AND XUR INTO REGO
	128		LR	REGUREGE	CUPY REGBINID REGG
	129		SLL	REGUII (AND YOP INTO REGR
	130		AK CT	RECORNECC	SAVE THE NEW ISRGNI
	132		31 1	REGD MCGN	LIAD MCGN INTO REGD
	122		M	REGCIMULT	AND MULTIPLY BY 69069
	124		ST	REGD, MCGN	STORE RESULT, MODULO 2**32, AS NEW IMCGNI
	135		XR	REGD, REGB	XOR NEW IMCGNI AND ISRGNI IN REGD
	136	NRCT	SLR	REGC, REGC	ZERD DUT REGC
	137		CL	REGD,X68	IF REGD GE 68000000,BRANCH TD 'ND2'
	138		BC	11,ND2	
	139	ND1	SLDL	REGC > 8	SHIFT FIRST 2 HEX DIGIIS INTO REGU
	140		IC	REGCINTBL (REGC)	FEICH CURRESPONDING DITE FRUM NIDE
,	141		STC	REGC>PSTWRD+1	STURE AS 2ND BITE OF FSTWED TAKE DEMAINING 24 BITS DE BEGD
	142		SRL		TARE REMAINING 24 DITS OF RECOU
	143		AL		AND STORE AT LERACI
	144		21		ADD IPSTWRDI AND IFRACI
•	142		ΔE	O • FRAC	LEAVING RESULT IN FPR O
	140		I M	REGB, REGD, 24(13)	
	148		BCR	15,14	RETURN
	149	ND2	CL	REGD, XDO	IF REGD GE DODOOOOO,BRANCH TO IND31
	150		BC	11,ND3	
	151		SLDL	REGC 8	SHIFT FIRST 2 HEX DIGITS INTO REGC
	152		SL	REGCX68R	AND SUBTRACT 00000068
	153		IC	REGC, NTBL (REGC)	FEICH CURRESPUNDING BYTE PE NSTWPD
	154		STC	REGUINSIWRD+1	TAVE DEMAINING 24 BITS DE PEGD
	155		.SRL		EDRA ELANTING POINT FRACTION CHAR XI3FL
	150		АL С Т		AND STORE AT IERAC!
	151		16	O.NSTWRD	SUBTRACT IFRAC' FROM INSTWRD!
	150		SE	OF FRAC	LEAVING RESULT IN FPR O
	160		LM	REGB, REGD, 24(13)	
•	161		BCR	15,14	RETURN
	162	ND3	CL	REGD, XE2F	IF REGD GE E2F00000,BRANCH TO 'ND4'
	163		BC	11 º ND4	
	164		SLDL	REGC 12	SHIFT FIRST 3 HEX DIGITS INTO REGL
	165		SL	REGC XCE8	AND SUBTRACT DODUCLES
	166		IC	REGCANTEL (REGC)	CTODE AS AND RATE OF DETARD
	167		STC	KEOUSKSIMKD+T	TAKE REMAINING 20 BITS OF REGD
	168	•	5KL	KEUUJO NECD-DCHAD	FORM FLOATING POINT FRACTION, CHAR X13FT
(169		АЦ 5 Т	DECOFERAC	AND STORE AT IFRAC!
٠	170		1 E 2 I	O.PSTWRD	ADD IPSTWRDI AND IFRACI
	172		ΔE	O FRAC	LEAVING RESULT IN FPR O
	172		L M	REGB, REGD, 24(13)	
	174		BCR	15,14	RETURN

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175 ND4 CL REGD, XF5E IF REGD GE XF5E00000, BRANCH TO INTTHTL! 176 BC 11,NTTHTL 177 SLDL REGC 12 SHIFT FIRST 3 HEX DIGITS INTO REGC 178 SL REGC XE17 AND SUBTRACT 00000E17 179 IC REGC NTBL (REGC) FETCH CORRESPONDING BYTE FROM NTBL 180 STC REGC NSTWRD+1 STORE AS 2ND BYTE OF NSTWRD 181 SRL REGD 8 TAKE REMAINING 20 BITS OF REGD 182 AL REGD, PCHAR FORM FLOATING POINT FRACTION, CHAR XI3F! 183 ST REGD, FRAC AND STORE AT IFRACI 184 LE O, NSTWRD SUBTRACT IFRACI FROM INSTWRDI 185 SE O, FRAC LEAVING RESULT IN FPR O 186 LΜ REGB, REGD, 24(13) 187 BCR 15,14 RETURN 188 NTTHTL ST REGD, ARG STORE REGD AS ARGUMENT FOR RNORTH ROUTINE .189 STM 14,0,12(13)SAVE ALL REGISTERS FROM 14 TO 3. 190 LR 3113 COPY PREVIOUS SAVE AREA ADDRESS TO GPR3 191 LA 13, SVAREA LUAD ADDRESS OF SVAREA INTO GPR13 192 ST 13,8(0,3)STORE ADDRESS OF SVAREA IN SAVE AREA 193 ST 3,4(0,13) STORE ADDRESS OF PREVIOUS SAVE AREA 194 1. ARGLST LA PLACE ADDRESS OF ARGUMENT LIST IN GPR 1 195 L 15, ADNTH 196 BALR 14,15 BRANCH TO SUBPROGRAM 197 LR 13,3 RESTORE ADDRESS OF SAVE AREA IN GPR13 198 MVI 12(13),XIFFI SET RETURN INDICATOR 199 RETRN3 LM 14, REGD, 12(13) RESTORE ALL REGISTERS 200 BCR 15,14 RETURN 201 × 202 * Y=REXP(0) RESULT IS STANDARD EXPONENTIAL VARIATE. 203 * 204 * METHOD 205 * -----GENERATE H1H2H3H4H5H6H7H8, 8 RANDOM HEXADECIMAL DIGITS 206 * 1. 207 * 208 * 2. IF H1H2 .LT. D5, SET IREXP! TO 209 ¥ (ETBL(H1H2)+.H3H4H5H6H7H8)/16, AND QUIT. 210 × IF H1H2H3 .LT. F17, SET 'REXP! TO з. 211 * (ETBL(H1H2H3-CFF)+ H4H5H6H7H8)/16, AND QUIT 212 ¥ ELSE GENERATE IREXPI FROM THE EXPONENTIAL TOOTH-TAIL SUBPROGRAM. 4. 213 * 214 USING REXP,15 215 REXP REGB, REGD, 24(13) SAVE REGISTERS 1,2,3 STM 216 RDIGT4 L REGB, SRGN LOAD SRGN INTO REGB 217 LR REGC, REGB AND INTO REGC 218 SRL REGC, 15 SHIFT REGC RIGHT 15 BITS 219 XR REGB, REGC AND XOR INTO REGB 220 LR REGC, REGB COPY REGB INTO REGC 221 SLL REGC, 17 SHIFT IT LEFT 17 BITS, 222 XR REGB, REGC AND XOR INTO REGB 223 ST REGB SRGN SAVE THE NEW ISRGNI 224 L REGD, MCGN LOAD MCGN INTO REGD 225 M REGC + MULT AND MULTIPLY BY 69069 226 ST REGD, MCGN STORE RESULT, MODULO 2**32, AS NEW MCGNI 227 XR REGD, REGB XOR NEW IMCGNI AND ISRGNI IN REGD 228 ERCT SLR REGC, REGC ZERD DUT REGC 229 CL REGD XD5 IF REGD GE D5000000, BRANCH TO 'ED2' 230 BC 11,ED2 231 ED1 SLDL REGC.8 SHIFT FIRST 2 HEX DIGITS INTO REGC 232 IC REGC, ETBL(REGC) FETCH CORRESPONDING BYTE FROM ETBL

233 234 235 236 237 238 239		STC SRL AL ST LE AE LM	REGC, PSTWRD+1 REGD, 8 REGD, PCHAR REGD, FRAC O, PSTWRD O, FRAC REGB, REGD, 24(13)	STORE AS 2ND BYTE OF PSTWRD TAKE REMAINING 24 BITS OF REGD FORM FLOATING POINT FRACTION, CHAR X'3F' AND STORE AT 'FRAC' ADD 'PSTWRD' AND 'FRAC' LEAVING RESULT IN FPR O RETURN
240 241 242 243 244 245	ED2	CL BC SLDL SL IC	REGD, XF17 11, ETTHTL REGC, 12 REGC, XCFF REGC, ETBL(REGC) PEGC, PSTWRD+1	IF REGD GE F1700000, BRANCH TO TETHING SHIFT FIRST 3 HEX DIGITS INTO REGC AND SUBTRACT 00000CFF FETCH CORRESPONDING BYTE FROM ETBL STORE AS 2ND BYTE OF PSTWRD
246 247 248 249 250 251		STC SRL AL ST LE AE	REGD, B REGD, PCHAR REGD, FRAC O, PSTWRD O, FRAC PEGB, REGD, 24(13)	TAKE REMAINING 20 BITS OF REGU FORM FLOATING POINT FRACTION, CHAR X'3F' AND STORE AT 'FRAC' ADD 'PSTWRD' AND 'FRAC' LEAVING RESULT IN FPR O
252 253 254 255 256 257 258	ETTHT	BCR ST L ST LR LA ST	15,14 REGD,ARG 14,0,12(13) 3,13 13,SVAREA 13,8(0,3) 2,4(0,13)	RETURN STORE REGD AS ARGUMENT FOR REXPTH ROUTINE SAVE ALL REGISTERS FROM 14 TO 3. COPY PREVIDUS SAVE AREA ADDRESS TO GPR 3 LOAD ADDRESS OF SVAREA INTO GPR13 STORE ADDRESS OF SVAREA IN SAVE AREA STORE ADDRESS OF PREVIOUS SAVE AREA
259 260 261 262 263 264 265	RETRI	LA L BALR LR MVI V4 LM	1, ARGLST 15, ADETH 14, 15 13, 3 12(13), X!FF! 14, REGD, 12(13)	PLACE ADDRESS OF ARGOMENT LIGT ON AN BRANCH TO SUBPROGRAM RESTORE ADDRESS OF SAVE AREA IN GPR13 SET RETURN INDICATOR RESTORE ALL REGISTERS RETURN
266 267 268	* *	K=IUN	10)	UNIFORMLY DISTRIBUTED POSITIVE INTEGER.
269 270 271 272 273 273 273	IUNI RDIG	USIN STM T5 L LR SRL XR	IG IUNI,15 REGB,REGD,24(13 REGB,SRGN REGC,REGB REGC,15 REGB,REGC REGC,REGB	B) SAVE REGISTERS 1,2,3 LOAD SRGN INTO REGB AND INTO REGC SHIFT REGC RIGHT 15 BITS AND XOR INTO REGB COPY REGB INTO REGC
27 27 27 28 28 28 28 28	5 7 8 9 0 1 2 3	SLL XR ST L M ST XR	REGC, 17 REGB, REGC REGB, SRGN REGD, MCGN REGC, MULT REGD, MCGN REGD, REGB REGD, 1	SHIFT IT LEFT 17 BITS AND XOR INTO REGB SAVE THE NEW 'SRGN' LOAD MCGN INTO REGD AND MULTIPLY BY 69069 STORE RESULT; MODULO 2**32; AS NEW 'MCGN' XOR NEW 'MCGN' AND 'SRGN' IN REGD XOR NEW 'MCGN' AND 'SRGN' IN REGD SHIFT LEFT 1 BIT; LEAVING SIGN BIT ZERD
28	4 5 6 RET	LR RN5 LM BCR	0, REGD REGB, REGD, 24() 15, 14	AND MOVE RESULT FIGHT TO STATE
28	8 * 39 * 30 *	J = I \	(0)	UNIFORMLY DISTRIBUTED INTEGER.

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		291 292	* *	METHOD		THE BASIC RANDOM NUMBER IS A COMBINATION OF TWO SEPARATELY GENERATED NUMBERS,
		293	光 光	1. TE	MP=XOR(SRGN, SRGN	SHIFTED RIGHT 15 BITS)
:		295	*	2. SR	GN=XOR (TEMP, TEMP	SHIFTED LEFT 17 BITS)
		296	*	3, MC	GN=MCGN*69069,MDD Sin t-XDC/MCGN,SRC	ULD 2**32 N1
		297	** **	4 NC		
		299		USING	IVNI)15	CAVE DECTERS 1.2.3
	·	300	IVNI RDIGT6	S1M L	REGBIREGDJ24(13) REGBISRGN	LOAD SRGN INTO REGB
		302	(01010	LR	REGCAREGB	AND INTO REGC
÷		303		SRL	REGC/15 PEGB+REGC	AND XOR INTO REGB
		305		LR	REGCIREGB	CDPY REGB INTO REGC
		306		SLL	REGC:17	SHIFT IT LEFT 17 BITS, AND YOP INTO REGR
		307		XK ST	REGBISRGN	SAVE THE NEW ISRGN!
•		309		Ĺ	REGD, MCGN	LOAD MCGN INTO REGD
		310		M	REGORMULI	STORE RESULT, MODULO 2**32, AS NEW MCGN
-1		312		XR	REGD, REGB	XOR NEW IMCGNI AND ISRGNI IN REGD
ļ		313		LR	DJREGD DECB.REGD.24(13)	LEAVE RESULT TIVNI' IN GPRO
		314 315	KEIKNC	BCR	15,14	RETURN
1 1 2		316	**	CONSTA	NTS SECTION	
		317 318	SRGN	DC	F1010731	
		319	X7FF	DC	X1000007FF1	
1		320	MCGN	DC DC	F123451 X1000000011	
ţ		322	FWD	DC	FIOI	
	•	323		DC	E10,01	
		324 325	SIGN	DC	X180FFFFFF	
		326	XD5	DC	X1D50000001	
:		327	XF17 XCFF		X100000CFF1	•
-		329	X68	DC	X168000001	
		330	XDO	· DC	X1000000001 X1000000681	• ,
		332	XE2F	DC	X1E2F000001	
		333	XCE8	DC	X100000CE81	
		334 325	XF5E XE17		X100000E171	
		336	PSTWR	D DC	X141AA00001	
		337		D DC	X121AA00001 X13F0000001	
		339	FRAC	DC	F101	
		340	ARG	DS	F A (RNDRTH)	
÷		341 342	ADNIN	DC	A(REXPTH)	
;		343	ARGLS	TDC	X1801	
:	(344	SVARE		ацэ(ркб) 18F	
	١	346	NTBL	DC	1X1001	TABLE USED FOR NORMAL LOOK-UP
:		347		DC	1X1011	FIRST PART HAS 104 ELEMENTS
•		348		μι	24.461	

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	349	DC	4X1031
	350	DC	5X1041
	351	DC	1X1091
	352	DC	5X1041
	353 354 255		1X 121
	356 357		5X1001
	358		4X1021 2X1031
	360	DC	1X'04'
	361	DC	5X'05'
	362	DC	5X1061
	363	DC	5X1071
	364	DC	5X1081
	365	DC	4X1091
	366	DC	4X10B1
	367	DC	4X10C1
	368	DC	4X10D1
	369	DC	1X10E1
·	370	DC	3X10F1
	371	DC	3X1101
	372 373	DC DC	2X1121
	374 375 376		2X1141
	377	DC	2X'16'
	378	DC	1X'17'
	379	DC	1X1181
	380	DC	1X1191
	381	DC	1X 1A
	382	DC	1X 1B
	383	DC	1X'1C'
	384	DC	1X'1D'
	385	DC	10X1051
	386	DC	7X1061
	387 388		2X1081
	387 390 301		5X1001
,	392	DC	10X'0F! 7X'10!
	394	DC	3X'11'
	395	DC	12X'13'
	396	DC	9X1141
	397	DC	5X1151
	398	DC	2X1161
	399	DC	13X1181
•	400	DC	10X'19'
	401	DC	7X'1A'
	402 403	DC DC	2X1101
	404 405	DC DC	13X11F1 13X11F1
	400		101.001

START OF SECOND PART OF NORMAL TABLE 223 ELEMENTS

	444444444444444444444444444444444444444	ETBL	10X 21 9X 22 8X 23 7X 24 6X 25 5X 26 4X 25 5X 26 4X 27 3X 28 3X 29 2X 28 15X 00 13X 02 5X 00 10 5X 00 10 10 10 10 10 10 10 10 10 10 10 10 1
Ċ	4449012345678901234 5555789012345678901234		3X'11' 4X'12' 4X'13' 4X'14' 3X'16' 3X'16' 3X'17' 3X'18' 2X'10' 2X'16' 2X'110' 2X'110' 2X'110' 2X'110' 2X'110' 2X'110' 1X'22' 1X'23'

START OF TABLE FOR EXPONENTIALS FIRST PART HAS 213 ELEMENTS

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SECOND PART OF EXPONENTIAL TABLE 455 ELEMENTS

	4 4 I ^m	DC	1 1 2 5 1						
	462	0C	1X1201						
·	460	DC	1X1271						
	468	DC	1X'28'						
•••	469	DC	1X1291			•			
	470	DC	1X12A1						
	471	DC	5X1051				· ·		
	472	DC	2X1071						
	473	DC	1X1091						
	474	DC	IXIOBI						
•	475	DC	4X10D1						
	476	DC	9XIOFI						
	477	DC	3X1101						
	478	DC	10X1121						
	479	DC	57,122,						
	480		981101						
	481		271181						
	484		1321101						
	482		10X/18/						
	404		7X1101						
	402	00	5X1101						
	487	DC	2X11E1		•				
	488	DC	13X'21'			• •			
	489	DC	11X 221	7					
	490	DC	981231	ī					
	491	DC	8X1241						
	492	DC	6X1251						
	493	DC	521261						
•	494	DC	4X1271						
	495	DC	221201						
	496		1571201						
	497		1411201						
	490	50	13X12E1				•		
	499 500	00	12X12F1						
	501	. DC	1121301	•	-				
	502	DC	11X1311						
	503	. DC	10X1321			·			
	504	DC	981331						
	505	DC	921341						
,	506	DC	8X1351						
	507	DC	821301						
	508		721291						
	509		141301						
	510		6X1341						
	513		6X13B1						
	513	DC	5%1301		,				
	514	DC	5X13D1						
	515	DC	4X13E1						
	516	DC	4X13F1						
\sim	517	END	RANDUM						
(518	C RNOR TOUT	TH FUNCTION	4					
_ د	519	FUNC	TION RNURTH	4(K)					
	520	DIME	ISIUN C(45)) ====================================	2DEF 7/4		740551648-	74053249	6,
	521	DATA	C/Z4OFU2B	フドメム 40FD	2001364(UPAAYAN! 5.740041	2905340903	E40 240C	 887BE-
	522	\$ Z401	EZ131+440	20901A#Z	40F198B;	512400AI	J767640060		

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<pre>\$23</pre>	•				
<pre>523 \$ Z40C102A6,Z4086F8DD,Z40ACF513,Z40A2EE4A,Z4098E780,Z40916269; 524 \$ Z4027BE,Z40A54072,Z40734E00,Z4056C8F6,Z405E022,Z40403D) 525 \$ Z402A9CB0,Z40259973,Z4020960E,Z401E145E,Z401910F7,Z40168F45; 526 \$ Z402A9CB0,Z40259973,Z4020960E,Z401E145E,Z401910F7,Z40168F45; 527 \$ Z471093,Z401108E0,Z3FF024624,Z3F50364C/ 528 \$ Z402A9CB0,Z40259973,Z4020960E,Z01E145E,Z401910F7,Z40168F45; 528 \$ Z401093,Z401108E0,Z3FF02462,Z3F50364C/ 529 \$ DATA 11/ZFR635400/12/ZFF79702E/ 520 \$ DATA 11/ZFR635400/12/ZFF79702E/ 521 TUN1(0) 523 TUN1(0) 524 TUN1(0) 525 \$ TUN1(0) 526 \$ TUN1(0) 527 TUN1(0) 528 \$ RNRTH=, 0625*(X*SIGN(B,X1) 529 \$ RNRTH=, 0625*(X*SIGN(B,X1) 529 \$ RNRTH=2,739VNI(0) 529 \$ 1F(X,G7,12)GD TD 5 538 \$ RNRTH=2,69817E-3 540 \$ 0 F1(J-1)18,1497466E-2 541 \$ 0 F1(J-1)18,1497466E-2 542 \$ GD TD 8 543 \$ 7 P1(B9-J_J)*,409766E7 544 \$ 1F(UNI(0),66,79,788468(EXP(-5*RNDTH*RNDTH)) 545 \$ - C(J)-P*(J-6*A85(RNDTH))) \$ GDTU4 740 \$ RETURN 545 \$ C(J)-P*(J-6*A85(RNDTH))) \$ GDTU4 740 \$ RETURN 545 \$ V=VNI(0) 546 \$ RETURN 547 \$ V=VNI(0) 547 \$ RETURN 548 \$ RIVEN' 549 \$ C(J)-P*(J-6*A85(RNDTH))) \$ GDTU4 740 \$ RETURN 759 \$ RETURN 750 \$ IF(UNI(0),67,79,788468(EXP(-5*RNDTH*RNDTH))) 751 \$ IF(UNI(0),67,79,788468(EXP(-5*RNDTH*RNDTH))) 752 \$ RETURN 753 \$ RNRTH=5ER(X,Y) 754 \$ RETURN 755 \$ RNRTH=5ER(X,Y) 755 \$ RNRTH=5ER(X,Y) 755 \$ RNRTH=5ER(X,Y) 755 \$ RAVENT+5ER(X,Y) 755 \$ RETURN 756 \$ C REXP TDTH FUNCTION 757 \$ DATA C/240F0000,2405E10000,24060000,2405E0000,240580000, 756 \$ Z40530000,2405E10000,240620000,240580</pre>				•	
<pre>523 \$ Z40C102A6,Z4086FBDD,Z40ACF513,Z40A2EE4A,Z4098E780,Z40916269, 524 \$ Z403781A0,Z40705406,Z40734EDD,Z40ACE84,Z40372554,Z402FA33D15, 525 \$ Z402A0CDHZ,Z4025973,Z402006CE,Z401E45C,Z401910F7,Z40168FA3, 527 \$ Z40140093,Z401188E0,Z3FF0A2E4,Z3FC387NE,Z3FA06C98,Z3F785172, 528 \$ Z40140093,Z401188E0,Z3FF0A2E4,Z3FC387NE,Z3FA06C98,Z3F785172, 529 DATA 11/ZF8C35A00/J2/ZFF03702E/ 520 DATA 11/ZF8C35A00/J2/ZFF03702E/ 520 DATA 11/ZF8C35A00/J2/ZFF03702E/ 521 FE(K,CT,11)00 T0 3 523 BeAINT(7, CS+T)+37,*A85(S-T)) 524 ANNRTH=0625*(X*SIGN(B,X)) 755 RNNRTH=0625*(X*SIGN(B,X)) 756 RNNRTH=0625*(X*SIGN(B,X)) 757 J IF(K,CT,12)00 T0 5 538 A RNNRTH=2,75*VN1(0) 539 J IF(K,CT,12)00 T0 5 539 J IF(K,CT,12)00 T0 5 536 J IF(K,CT,12)00 T0 5 536 J IF(K,CT,12)00 T0 5 537 J IF(K,CT,12)00 T0 5 538 A CCL,12,57,77 541 C Pe(J-J-1)*,69817E-3 543 7 Pe(80-J-J)*,69817E-3 545 J IF(K,CT,12)00 T0 5 556 R CCL,12,75160 T0 5 557 S IF(K,CT,12)00 T0 5 558 A CCL,12,75160 T0 5 558 A CCL,12,75160 T0 5 558 A CCL,12,75160 T0 5 559 R CL,1100 R CCL,27,5160 T0 5 550 R CL,27,75625-2,*4L0C(A85(V))) 550 R CLUN 551 R CLUN 555 R NORTH=5160(X,V) 555 R NORTH=5160(X,V) 555 R CACCO00,Z40710000,Z40800000,Z40070000,Z40880000, 556 C 4040000,Z40710000,Z40800000,Z40070000,Z40880000, 557 DATA C/Z407000,Z40710000,Z40800000,Z40800000,Z40880000, 558 C 4040600,Z40710000,Z40800000,Z40800000,Z40880000, 559 Z40780000,Z40710000,Z4070000,Z40800000,Z40880000, 550 DATA C/Z407000,Z40710000,Z4080000,Z40800000,Z40880000, 551 Z40780000,Z40710000,Z4070000,Z40800000,Z40800000,Z40880000, 553 Z40780000,Z40710000,Z4070000,Z4070000,Z40800000,Z40800000,Z40800000,Z40200000,Z4020000,Z24020000,Z4020000,Z24020000,Z4020000,Z24020</pre>			•	•	
<pre>\$23 \$ Z40C102A6,Z40B6FBDD,Z40ACF513,Z40AZEE4A,Z4098E780,Z40916269, \$24 \$ Z4087BK0,Z407D54007,Z40734E0D,Z405BCF6,Z405BC22C,Z403D15 \$25 \$ Z402A9CD0,Z40729973,Z402906DE,Z401E145C,Z401910F7,Z40108F45, \$26 \$ Z402A9CD0,Z40259973,Z402906DE,Z401E145C,Z401910F7,Z40108F45, \$27 \$ Z401A0093,Z401108ED7,Z1F7032E4,Z1F2037E,Z2F70502E4, \$28 \$ DATA 11/2FR635400/12/ZFE79702E4 \$29 DATA 11/2FR635400/12/ZFE79702E4 \$20 DT 8 \$21 FUNI(0) -UNI(0) \$23 BEA1WI(T,*(5+T)+37,*A85(S-T)) \$24 ANURTH-2059*(X+SIGN(B,X)) \$25 RNURTH-2059*(X+SIGN(B,X)) \$26 RETURN \$27 D 16, *A85(RNURTH)+1. \$26 DT 8 \$28 ANURTH-2,759VNI(0) \$29 J=16,*A85(RNURTH)+1. \$20 DT 8 \$24 DT 8 \$24 DT 8 \$24 DT 8 \$24 DT 8 \$25 C(1)-P*(1)-16,*A85(RNURTH))) GOTU4 \$25 RETURN \$26 RETURN \$27 SV=VNI(0) \$28 LF(V,C,0) GD TD 5 \$29 X=SQRT(T,5625-2,*ALD0(AB5(V))) \$29 X=SQRT(T,5625-2,*ALD0(AB5(V))) \$29 X=SQRT(T,5625-2,*ALD0(AB5(V))) \$29 X=SQRT(T,5625-2,*ALD0(AB5(V))) \$20 RETURN \$20 RETURN \$20 RETURN \$21 REXPTUG1H FUNCTIDN \$25 RNURTH-5IGN(X,V) \$25 RETURN \$25 END \$24 CR2000,2/40710000,2/40780000,2/40780000,2/40880000, \$24 CA780000,2/40780000,2/40780000,2/40780000,2/40880000, \$24 CA780000,2/40780000,2/40780000,2/40780000,2/40780000,2/40880000, \$24 CA780000,2/40780000,2/40780000,2/408</pre>			.•		•
<pre>523 \$ Z40C10246.74086FBD.7400CF513.74002E44.74098F780.7401620%, 524 \$ Z405207EF.Z406832F7.7400430D0.7403C28K9.Z40372354.7402FA03D, 525 \$ Z40240D8.740138E0.734602080E.7401E45C.7401910F7.240186F4. 526 \$ Z40240D8.740138E0.73FF042E4.73EC837RE.Z3FA06C98.Z3F785172. 527 \$ Z40140093.7401188E0.73FF042E4.73EC837RE.Z3FA06C98.Z3F785172. 528 \$ Z40140093.7401188E0.73FF042E4.73EC837RE.Z3FA06C98.Z3F785172. 528 \$ Z40140093.7401188E0.73FF042E4.73EC837RE.Z3FA06C98.Z3F785172. 528 \$ Z40140093.7401188E0.73FF042E4.73EC837RE.Z3FA06C98.Z3F785172. 528 \$ Z40140093.7401188E0.73FF042E4.73EC837RE.Z3FA06C98.Z3F785172. 528 \$ Z4017.74*(5+T)+37.*A85(S-T)) 538 \$ S4007.102 530 \$ Ba1N17.7*(5+T)+37.*A85(S-T)) 534 \$ S+UNI(0) 535 \$ RAURTH-2, 73FVN1(0) 535 \$ RAURTH-2, 73FVN1(0) 536 \$ RETURN 537 \$ IF(K, G1, I2)60 TD \$ 538 \$ RAURTH-2, 73FVN1(0) 539 \$ JF(K, G1, I2)60 TD \$ 539 \$ RETURN 540 \$ IF(J-14) \$ 6,67 541 \$ 6 \$ P4(J+1)*.14497466E-2 542 \$ GD T0 \$ 543 \$ 7 \$ P4(B-1.J)*.4407466E-2 544 \$ IF(UNI(0).5.77,7884664(EXP(5*RNDRTH*RNDRTH)) 545 \$ \$ -C(J)-F*(J-16*A85(RNDRTH))] \$ GDTD4 546 \$ RETURN 547 \$ V=VN1(0) 547 \$ V=VN1(0) 548 \$ IF(V, E0, 0) & TD \$ 549 \$ X=SQRT(7, 5625-2,*ALDG(AB5(V))) 550 \$ IF(UNI(0)*X.CT.2,75) \$ GD TD \$ 551 \$ RNDRTH-5164(X,V) 552 \$ RETURN 553 \$ END 554 \$ C REXP TDDTH FUNCTION 555 \$ END 555 \$ RETURN 555 \$ END 555 \$ RETURN 556 \$ C REXP TDDTH FUNCTION 557 \$ Z4078000.740620000,740910000,740620000,740880000,7 558 \$ Z4078000,740620000,74090000,740620000,740280000,7 559 \$ Z4078000,740240000,740920000,740420000,740280000,7 550 \$ Z4078000,740240000,740920000,740280000,740280000,7 551 \$ Z4078000,740240000,740920000,740420000,740280000,740280000,7 552 \$ Z4078000,740240000,740490000,740420000,740280000,740280000,7 553 \$ Z4078000,7402420000,740270000,740280000,740280000,740280000,7 554 \$ Z4078000,7402420000,74010000,74040000,740420000,740280000,740280000,7 555 \$ Z4078000,7402420000,74010000,74040000,74040000,740280000,7402800000,7402800000,7402800000,7402800000,7402800000,7402800000,7402800000,7402800000,7402800000,7402800000,7402800000,7402800000</pre>					
<pre>524 5 2,0575EAD,2407D54D5240734ED0.2405H26F0.240012227,4405A2013, 525 5 2,020FF,24049327.54043AD0.2405422897.240372554,24027403D, 526 5 240249CDb.240259973.24020906F.2401E145C,2401910F7,240160F45, 527 5 2,010093.740118BE0.23FF0.2264.23F50364C/ 528 5 23F785172.23F50364C.23FF03264.23F50364C/ 529 DATA 11/2FR635400/12/2FF79702E/ 530 IF(K,0T,11100 TD 3 531 T=UNI(0) 532 T=UNI(0) 533 S=UNI(0) 534 X=UNI(0) 535 RETURN 535 RETURN 537 3 IF(K,0T,1210C TD 5 538 RETURN 539 J=16,#A85(RNORTH)+1. 540 J=1,440546(X+SIGN(8,X)) 544 RETURN 545 J=1,4685(RNORTH)+1. 540 J=1,4685(RNORTH)))) G0TD4 544 B IF(UNI(0),6T.79.788464(EXP(-,5*RNDRTH*RNORTH) 545 S - C(1)-F*(,16,458(RNORTH)))) G0TD4 546 RETURN 547 5 V=VNI(0) 548 IF(UNI(0),6T.79.788464(EXP(-,5*RNDRTH*RNORTH) 549 S = C(1)-F*(,16,458(RNORTH)))) G0TD4 546 RETURN 547 5 V=VNI(0) 548 IF(UNI(0),6T.79.788464(EXP(-,5*RNDRTH*RNORTH) 549 S = C(1)-F*(,16,455(RNORTH)))) G0TD4 549 S = C(1)-F*(,16,25,22,44LDC(ABS(V))) 540 IF(V,60,0) GD TD 5 540 X=SQRT(7,5625-2,44LDC(ABS(V))) 551 RNORTH=51CM(X,V) 552 RETURN 553 END 554 C REXP T0DTH FUNCTION 554 C REXP T0DTH FUNCTION 555 FUNCTION REXPTH(K) 556 J Z 40780000.240710000.24050000.24050000.24050000.24050000. 557 DATA C/240F0000.240450000.24050000.24050000.24050000.24050000. 558 Z 40780000.240450000.240540000.24050000.24050000.24050000. 559 Z 40780000.240450000.24050000.24050000.24050000.24050000.24050000. 550 DATA C/240F0000.240170000.24052000.24050000.257500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000.23F600000</pre>		523	•		\$ Z40C102A6, Z40B6FBDD, Z40ACF513, Z40A2EE4A, Z4098E780, Z40916269,
<pre>-</pre>		524			\$ Z40875BA0, Z407D54D6, Z40734E0D, Z406BC8F6, Z4061C22C, Z405A3D15,
<pre>4. 52 52 52 52 52 52 52 52 52 52 52 52 52 5</pre>		525			\$ Z405287FE, Z404B32E7, Z4043ADD0, Z403C2889, Z40372554, Z402FA03D
<pre>\$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$</pre>		526			\$ Z402A9CD8, Z40259973, Z4020960E, Z401E145C, Z401910F7, Z40108F45,
<pre>\$ 23F785172.23F50364C/23F50364C/23F50364C/ DATA 11/ZF643A900/12/ZF5702E/ \$ DATA 11/ZF643A900/12/ZF5702E/ \$ DATA 11/ZF643A900/12/ZF5702E/ \$ DATA 11/ZF64A91 \$ S=UNI(0) \$ DEALTHOR \$ DEA</pre>		527			\$ Z40140D93,Z40118BE0,Z3FF0A2E4,Z3FC887BE,Z3FA06C98,Z3F785172,
<pre>55 0ATA 11/2F8C35400/,12/2FE79702E/ 1F(K,GT,11)G0 T0 3 331 532 T=UNI(0) 533 0 = ALNT(7,*(S+T)+37.*ABS(S-T)) 534 X=UNI(0)-UNI(0) 535 7 RNGRTH=.0625*(X+SIGN(B,X)) 536 RETURN 537 3 IF(K,GT,12)GD T0 5 538 4 RNGRTH=.0625*(X+SIGN(B,X)) 536 7 F2(4)-14) 6/6/7 541 6 P2(1+j-1)*.1497466E=2 542 60 T0 8 544 8 IF(UNI(0),6T.79,78846*(EXP(-,5*RNGRTH*RNORTH)) 545 1F(U)-14) 6/6/7 544 8 IF(UNI(0),6T.79,78846*(EXP(-,5*RNGRTH*RNORTH)) 545 1F(U)-14) 6/6/7 544 8 IF(UNI(0),6T.79,78846*(EXP(-,5*RNGRTH*RNORTH)) 545 1F(U)-14) 6/6/7 547 5 V=VNI(0) 548 1F(UNI(0),6T.79,78846*(EXP(-,5*RNGRTH*RNORTH)) 549 1F(V,EQ,0) 6D T0 5 549 X=SQRT(7,5625-2,*ALD6(ABS(V))) 550 1F(UNI(0)*X,6T.2,75)GD T0 5 551 RNGRTH=516H(X,V) 553 END 554 CREXP T0DTH FUNCTION 555 FUNCTION REXPTH(K) 555 END 556 0 X=SQRT(7,5625-2,*ALD6(ABS(V))) 557 DATA (/40F00000,240E10000,240E10000,240E70000,240B0000, 558 240APD000/A0A50000,24090000,2404910000/240890000,24050000, 559 2404P0000/240500000,24090000,24050000,24050000,24050000, 550 5 2404P000/240250000,240320000,240400000,240420000,240500000, 551 240290000/2403500000,240320000,24040000,24040000,24040000,24025000,24025000,560 5 2404P000/2403500000/240320000/240420000,240420000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,240250000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,24025000,240250000,24025000,240250000,240250000,240250000,23F600</pre>		528			\$ Z3F785172,Z3F50364C,Z3F50364C,Z3F50364C/
<pre>50 FF(K,GT,1)1GD TD 3 531 S=UNI(0) 532 T=UNI(0) 533 B=AINT(7, #(S+T)+37.*ABS(S=T)) 534 X=UNI(0)-UNI(0) 535 RNDRTH=.0625*(X+SIGN(B,X)) 536 RTURN 537 3 FF(K,GT,1):0D TD 5 538 4 RNDRTH=2,75*VNI(0) 539 J=16,*ABS(RNDRTH)*1. 540 FF(J=14) 6:07 541 6 P=(J+J=1)*.1497466E=2 542 GD TD 8 543 7 P=(89-J=J)*.69831TE=3 543 7 P=(89-J=J)*.69831TE=3 544 B FF(UNI(0).GT.79.78846*(EXP(5*RNDRTH*RNDRTH)) 545 \$ -((J)-P*(J=16.*ABS(RNDRTH)))) GDTD4 546 RETURN 547 5 V=VNI(0) 548 IF(V,E0,0) GD TD 5 549 X=SQR(7,:5625-2,*ALDG(ABS(V))) 550 IF(UNI(0)*X:GT.2,75):GD TD 5 549 X=SQR(7,:5625-2,*ALDG(ABS(V))) 551 RNDRTH=SIGH(X,V) 552 RETURN 553 END 554 C REXP TDDTH FUNCTIDN 555 FUNCTION REXPTH(K) 556 DATA C/240F0000,240E10000,240910000,240E0000,24080000, 558 \$ 240780000,240F10000,240980000,240910000,240800000, 559 \$ 240780000,240510000,240980000,240980000,240580000, 559 \$ 240780000,240510000,240980000,240580000, 559 \$ 240780000,240510000,240980000,240580000, 550 DATA C/240F00000,240510000,24050000,240580000, 551 \$ 240780000,240510000,240980000,240580000,240580000, 552 \$ 240780000,240510000,240510000,240580000,240580000, 553 \$ 240780000,240510000,240510000,240580000,240580000, 554 \$ 240780000,240510000,240580000,240580000,240580000, 555 \$ 240780000,240510000,240580000,240580000,240580000, 555 \$ 240780000,240540000,240580000,240580000,240580000, 556 \$ 240780000,240540000,240580000,240580000,240580000, 557 DATA C/240F0000,240580000,240580000,240580000,240580000, 558 \$ 240780000,240580000,240580000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,235500000,23540000</pre>		529			DATA 11/2FBC35400/+12/2FE79702E/
<pre>551 S=UNI(0) 532 T=UNI(0) 533 B=AINT(7,*(S+T)+37.*ABS(S=T)) 534 X=UNI(0)-UNI(0) 535 RNGTH=.0625*(X*SIGN(B,X)) 536 RETURN 537 3 IF(K,GT,I2)GD TD 5 538 4 RNDRTH=.0625*(X*SIGN(B,X)) 536 RETURN 537 3 IF(K,GT,I2)GD TD 5 538 4 RNDRTH=.0625*(X*SIGN(B,X)) 536 RETURN 537 3 IF(K,GT,I2)GD TD 5 538 4 RNDRTH=.0625*(X*SIGN(B,X)) 536 RETURN 540 F=(UNI(0),GT.79,78846*(EXP(5*RNDRTH*RNDRTH) 541 6 P=(J+-1)*.497466E=2 542 GD TD 8 543 7 P=(0?-J-J)*.698817E=3 544 8 IF(UNI(0),GT.79,78846*(EXP(5*RNDRTH*RNDRTH) 545 \$ -C(J)+P*(J-16.*ABS(RNDRTH))) GDTD4 546 RETURN 547 5 V=VNI(0) 548 X=SQRT(7,5625-2,*ALDG(ABS(V))) 559 IF(UNI(0),GT.79,76846*(EXP(5*RNDRTH*RNDRTH) 549 IF(UNI(0),GT.79,78846*(EXP(5*RNDRTH*SID000,Z4085000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F400000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F00000,Z3F400000,Z3</pre>		530			IF(K.GT.I))GD TD 3
<pre>522 T=UNI(0) 533 B=AINT(7; k(S+T)+37.*ABS(S-T)) 534 X=UNI(0)-UNI(0) 535 RNORTH=.0625*(X+SIGN(B,X)) 536 RETURN 537 3 IF(K,GT,I2)GD TO 5 538 4 RNORTH=2,75*VNI(0) 539 J=16,*ABS(RNORTH)+1, 540 F(J=14) 5-677 541 6 P=(J+J=1)*.1497466E=2 542 GD TO 8 543 7 P=(89-J=J)*.6986ITE=3 544 8 IF(UNI(0).6(7.79,76846*(EXP(=.5*RNORTH*RNORTH)) 545 5 -C(J)=P*(J=16.*ABS(RNORTH))) GDTD4 546 RETURN 547 5 V=VNI(0) 548 IF(V,EQ,0) GD TO 5 549 X=SCR(T,5525-2,*ALDG(ABS(V))) 550 IF(UNI(0)(X,CT.2,75)GD TO 5 551 RNORTH=5IGH(X,V) 552 RETURN 553 ETURN 554 C REXP TODTH FUNCTION 555 FUNCTION REXPTH(K) 556 JACCOUNC.Z40F0000,Z40E10000,Z40F10000,Z40F0000,Z4080000,J 557 DATA C/Z40F0000,Z40E10000,Z40910000,Z405E0000,Z40580000,J 558 Z 40780000,Z40450000,Z4098000,Z40910000,Z40580000,Z40580000,J 559 S Z40780000,Z40450000,Z4098000,Z40590000,Z40580000,Z40580000,J 550 DATA C/Z40F0000,Z40250000,Z40250000,Z40580000,Z40580000,J 551 DATA C/Z40F0000,Z40250000,Z40250000,Z40580000,Z40580000,J 552 S Z40780000,Z40510000,Z40590000,Z40540000,Z40580000,Z40580000,J 553 S Z40720000,Z40540000,Z40590000,Z40540000,Z40580000,Z40580000,J 554 S Z40270000,Z40540000,Z40540000,Z40540000,Z40520000,Z40580000,J 555 S Z401A0000,Z40190000,Z40240000,Z40540000,Z40580000,Z40580000,J 564 S Z40120007,Z40100000,Z40120000,Z3FE00</pre>	•	531			S = UNI(0)
<pre>533 B=AINT(7;*(5+T)+7;*AB5(S-T)) 534 X=UNI(0)-UNI(0) 535 RNDRTH=.0625*(X+SIGN(B,X)) 536 RETURN 537 3 IF(K,GT,I2)GD TD 5 538 4 RNDRTH=2;75*VNI(0) 539 4 RNDRTH=2;75*VNI(0) 539 4 RNDRTH=2;75*VNI(0) 539 4 F(J-14) 6;6;7 54 6 F(J-14) 6;6;7 54 7 P=(B9-J-J]**698B17E-3 54 8 IF(UNI(0),GT.79;78846*(EXP(5*RNDRTH*RNDRTH) 545 \$ -((J)-P*(J-16,*AB5(RNDTH)))) GDTD4 546 RETURN 547 5 V=VNI(0) 548 IF(UNI(0),CT.75;GG TD 5 551 RNDRTH=SIGN(X,V) 552 RETURN 553 E ND 554 C REXP TODTH FUNCTION 555 FUNCTION REXPTH(K) 556 DIMENSION Z4050000;Z40540000;Z40540000;Z40580000;A605000; 557 DATA (Z40F000;Z4054000;Z4054000;Z4058000;Z4058000;Z40580000;Z46020000;Z4602000;Z4602000;Z4602000;Z4602000;Z4602000;Z4602000;Z4602000;Z4602000;Z4602000;Z4602000</pre>		532			T=UNI(O)
<pre>534 X=UNI(0)-UNI(0) 535 RNDRH=,0625*(X*SIGN(B,X)) 536 RETURN 537 3 IF(K,GT,I2)GD TD 5 538 4 RNDRTH=2,75×VNI(0) 539 J=16,*ABS(RNDRTH)*1. 540 IF(J=14) 6.677 541 6 P=(J+J=1)*.1497466E=2 542 GD TD 8 543 7 P=(89-J+J)*.69817E=3 544 8 IF(UNI(0).GT.79.78846*(EXP(=.5*RNDRTH*RNDRTH) 545 5 -C(J)-P*(J=16,*ABS(RNDRTH)))) GDTD4 546 RETURN 547 5 V=VNI(0) 548 IF(V,E0,0) GD TD 5 549 X=SQR1(7,562=2,*ALDG(ABS(V))) 550 IF(UNI(0)*X.6T.2.75)GD TD 5 551 RNDRTH=5IGN(X,V) 552 END 554 C REXP TDDTH FUNCTION 555 FUNCTION REXPTH(K) 555 DIMENSION C4651 557 DATA C/240F0000,240E10000,240E40000,240E80000,240880000, 558 5 Z40AF000,240750000,240980000,24056000,240580000,240580000, 559 5 Z40780000,240350000,24040000,240540000,240580000,240580000, 560 \$ Z4058000,240350000,240440000,2405E0000,240580000,240580000, 561 \$ Z4027000,240350000,24044000,24040000,24050000,240580000, 562 \$ Z4027000,240350000,24044000,24040000,2405E0000,240520000,240580000, 563 \$ Z4027000,240350000,24042000,240250000,240250000,240520000,25F00000,23F00000,24050000,25F00000,23F00000,25F00000,23F000000,25F00000,23F00000,25F00000,23F000000,25F00000,23F400000,25F00000,23F400000,25F0000</pre>		533			B=AINT(7,*(S+T)+37.*ABS(S-T))
<pre>535 RNDRTH=.0623*(X*SIGN(B,X)) 537 3 IF(X,GT,I2)GD TD 5 538 4 RNDRTH=2,75*WNI(D) 539 J=16,*ABS(RNDRTH)+1, 540 IF(J=14) 6/6/7 541 6 P=(J=1)*.698817E-3 542 GD TD 8 543 7 P=(89-J=J)*.698817E-3 544 8 IF(UNI(0).61.79,78846*(EXP(5*RNDRTH*RNDRTH) 545 \$ -C(J)-P*(J=16.*ABS(RNDRTH)))) GDTD4 546 RETURN 547 5 V=VNI(D) 548 IF(UNI(0).61.79,78846*(EXP(5*RNDRTH*RNDRTH) 549 X=SQRT(7,5625-2,*ALDG(ABS(V))) 540 IF(V,E0,0) GD TD 5 540 X=SQRT(7,5625-2,*ALDG(ABS(V))) 551 RNDRTH=SIGN(X,V) 552 RETURN 553 EN 554 C REXP TDDTH FUNCTIDN 555 DIMENSION C(65) 557 DATA (7240F0000,240E10000,240E70000,240E80000,4008000,240E8000,240E80000,240E80000,23FE00000,24F00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FF00</pre>		534			X=UNI(O)-UNI(O)
<pre>536 RETURN 537 3 IF(K,GT,I2)CD TD 5 538 4 RNDRTH=2,75*VNI(0) 539 J=16,*ABS(RNDRTH)+1. 540 IF(J=14) 6/6,7 541 6 P=(J+J=1)*,1497466E=2 542 GD TD 8 543 7 P=(80-J_J)*,698617E=3 544 8 IF(UNI(0).617.97.78846w(EXP(-,5*RNDRTH*RNDRTH) 545 s -C(J)=P*(J=16.*ABS(RNDRTH)))) GDTD4 546 RETURN 547 V=VNI(0) 548 IF(V,E0,0) GD TD 5 549 X=SGRT(7,5625=2.*ALDG(ABS(V))) 550 IF(UNI(0)*X.GT.2.75)GD TD 5 551 RNDRTH=SIGH(X,V) 552 RETURN 553 EN0 554 C REXP TDDTH FUNCTION 555 FUNCTION REXPTH(K) 556 DINENSION C(65) 557 DATA C/Z40F0000,Z40E10000,Z40E10000,Z40E70000,Z408B0000, 558 \$ Z402F0000,Z405X0000,Z405X0000,Z405E0000,Z40580000,Z40580000, 560 \$ Z4033000,Z405X0000,Z40450000,Z402F0000,Z405E0000,Z40250000, 561 \$ Z4030000,Z405X0000,Z402F0000,Z402E0000,Z402E0000,Z40250000, 562 \$ Z4027000,Z40250000,Z40250000,Z402F0000,Z402E0000,Z40250000, 563 \$ Z402F0000,Z405X0000,Z40250000,Z402F0000,Z402E0000,Z40250000, 564 \$ Z4027000,Z40350000,Z40250000,Z402F0000,Z402E0000,Z40250000, 565 \$ Z402F0000,Z40350000,Z40250000,Z402F0000,Z402E0000,Z40250000,Z3FE000000,Z3FE00000,Z3FE000000,Z3FE00000,Z3FE00000,Z3</pre>		535			RNORTH=,0625*(X+SIGN(B,X))
<pre>537 3 IF(K,GT,I2)GD TD 5 538 4 RNDRTH=2,75*VNI(0) 539 J=16,*ABS(RNDRTH)+1. 540 IF(J=14) 6/6/7 541 6 P=(J+J=1)*,1497466E=2 542 GD TD 8 543 7 P=(89-J=J)*,698B17E=3 544 8 IF(UNI(0).GT.79.78846*(EXP(5*RNDRTH*RNDRTH)) 545 \$ -C(J)=P*(J=16.*ABS(RNDRTH)))) GDTD4 546 RETURN 547 5 V=VNI(0) 548 IF(UNI(0)*X.GT.2.75)GD TD 5 549 X=SQRT(7.5625-2.*ALDG(ABS(V))) 550 IF(UNI(0)*X.GT.2.75)GD TD 5 551 RNDRTH=51GN(X,V) 552 RETURN 553 END 554 C REXPTODTH FUNCTION 555 FUNCTION REXPTH(K) 556 DIMENSION C(65) 557 DATA C/240F00007,240E10000,740E700007,240B0000,74030000, 559 \$ Z40780007,240A50007,2404500007,2404500007,240320007,240320007, 560 \$ Z40450007,240450007,2404500007,2404200007,240320007, 561 \$ Z40270007,240450007,2404500007,2404200007,240320007, 562 \$ Z40270007,240450007,2404500007,240220007,240220007,240220007,240220007,240150007,240130007, 563 \$ Z40120007,240350007,240120007,241260007,240150007,240130007, 564 \$ Z40120007,247800007,247700007,23F600007,23F600007,23F600007,23F600007,23F600007,23F600007,23F600007,23F6000007,23F600007,23F400007,2</pre>		536			RETURN
<pre>538 4 RNDRTH=2,75*VH1(0) 539 J=16.*AB5(RNDRTH)+1. 540 IF(J=14) 6/67 541 6 P=(J+J]*.1497466E=2 542 CD TD 8 543 7 P=(89-JJ)*.698817E=3 544 8 IF(UNI(0).CT.79.78846*(EXP(=.5*RNDRTH*RNDRTH) 545 \$ -C(J)-P*(J=16.*AB5(RNDRTH)))) GDTD4 546 RETURN 547 5 V=VN1(0) 548 IF(V,E0,0) GD TD 5 549 X=SQRT(7,5625=2.*ALDG(AB5(V))) 550 IF(UNI(0)*X.CT.2.75)GD TD 5 551 RNDRTH=SIGH(X,V) 552 RETURN 553 END 554 C REXP TDDTH FUNCTIDN 555 FUNCTIDN REXPTH(K) 556 J DATA (/240F0000,2405H000,2405E000,2405B0000,2405B0000,2405B0000,2405B0000,2405B0000,2405B0000,2405B0000,2405B0000,2405B0000,2405B0000,2405B0000,2405B0000,2405B0000,23F60000,23F60000,23F60000,23F60000,23F60000,23F60000,23F60000,23F60000,23F400000,23F40000,23F40000,23F40000,23F40000,23F40000,23F400000,23F400</pre>		537	3		IF(K.GT.I2)GD TD 5
<pre>539 J=16,*ABS(RNDRTH)+1. 540 IF(J=14) 6,677 541 6 P=(J+J=1)*.1497466E=2 542 G0 TD 8 543 7 P=(89-J=J)*.698817E=3 544 8 IF(UNI(0).GT.79.78846*(EXP(=.5*RNDRTH*RNDRTH)) 545 s -C(J)-P*(J=16.*ABS(RNDRTH))) G0TD4 546 RETURN 547 5 V=VNI(0) 548 IF(V,E0.0) GD TD 5 549 X=SQRT(7.5625=2.*ALDG(ABS(V))) 550 IF(UNI(0)*X.CT.2.75)G0 TD 5 551 RNDRTH=SIGH(X,V) 552 RETURN 553 END 554 C REXP T0TH FUNCTIDN 555 FUNCTION REXPTH(K) 556 DIMENSION C(65) 557 SAAG0000.Z40E10000,Z40D40000,Z40E70000,Z40B80000, 558 \$ Z40AF0000.Z40E10000,Z40P80000,Z40B90000,Z40B80000, 559 \$ Z40F0000.Z40A50000,Z40E10000,Z40E0000,Z40B0000,Z40B50000,Z40B0000, 560 \$ Z40AF0000.Z40E000,Z40E10000,Z40E0000,Z40E0000,Z40B0000,Z40B0000, 561 \$ Z4039000.Z40A50000,Z40E10000,Z40E0000,Z40E0000,Z40E0000,Z40E0000,Z40E000,Z40E000,Z40E000,Z40E0000,Z40E000,Z40E0000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F400000,</pre>	•	538	4		RNDRTH=2,75*VNI(0)
<pre>540 IF (J-14) 6,677 541 6 P=(J+-1)*.1497466E=2 542 G0 T0 8 543 7 P=(89-J_J)*.698817E=3 544 8 IF (UNI(0).61.79.78846*(EXP(5*RN0RTH*RN0RTH) 545 \$ -C(J)=P*(J-16.*ABS(RN0RTH))) G0T04 546 RETURN 547 5 V=VNI(0) 548 IF (V,E0.0) G0 T0 5 549 X=SQRT(7,5625=2.*AL0G(ABS(V))) 550 IF (UNI(0)*X.67.2.75)G0 T0 5 551 RN0RTH=SIGH(X,V) 552 RETURN 553 END 554 C REXP T0DTH FUNCTION 555 FUNCTION REXPTH(K) 556 DIMENSION C(65) 557 DATA C/240F0000.24050000.240910000.240580000.240580000.5 558 \$ 240A7000.24070000.240490000.240580000.240580000.240580000.5 559 \$ Z4053000.24046000.240490000.240580000.240580000.5 561 \$ 24053000.24046000.240490000.240420000.240580000.240580000.5 561 \$ 24053000.24042000.24020000.24020000.240120000.240120000.5 563 \$ Z4012000.24019000.240120000.24020000.240120000.240120000.5 563 \$ Z4012000.24019000.240170000.240580000.240120000.240120000.5 564 \$ Z40270000.240190000.23F800000.23F800000.23F500000.23F400000.23F400000.23F400000.23F400000.23F400000.23F400000.23F400000.23F500000.23F500000.23F500000.23F400000.23F400000.23F400000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F500000.23F</pre>		539			J=16, *ABS(RNORTH)+1
<pre>541 6 P=(J+J)*,149/460E+Z GU TD 8 542 GU TD 8 543 7 P=(89-J+J)*,698617E+3 544 8 IF(UNI(0).6[.79,76846*(EXP(-,5*RNDRTH*RNDRTH) 545 \$ -C(J)+P*(J-16.*ABS(RNDRTH))) GDTD4 546 RETURN 547 5 V=VNI(0) 548 IF(V,E0.0) GD TD 5 549 X=SQRT(7,5625-2,*ALDG(ABS(V))) 550 IF(UNI(0)*X.CT.2,75)GO TD 5 551 RNDRTH=SIGH(X,V) 552 RETURN 553 END 554 C REXP TDDTH FUNCTIDN 555 JUNCTION REXPTH(K) 556 DIATA (/240F0000,240E10000,2400F000,2408F0000,2408B0000, 557 DATA (/240F0000,2409B0000,240910000,2408F0000,24080000, 558 \$ Z40AF0000.240710000,2406A0000,2405F0000,240580000, 560 \$ Z40530000.240710000,240490000,240580000,240580000, 561 \$ Z4078000.240710000,24020000,240250000,24015000,24015000,24015000,24015000,24015000,24015000,24015000,24015000,24015000,24015000,24015000,23F00000,23F00000,23F00000,23F00000,23F60</pre>		540			IF(J-14) 6, 6, 7
<pre>542</pre>		541		6	P = (J + J = 1) * * 149/400 = *2
<pre>543 7 P=(89-J-1)**(9901/t=2) 544 8 IF(UNI(0):61.79.78846*(EXP(-,5*RNURTH*RNURTH) 545 \$ -((J)-P*(J-16.*ABS(RNURTH)))) GDTU4 546 RETURN 547 5 V=VNI(0) 548 IF(V,E0.0) GD TD 5 549 X=SQRT(7,5625-2,*ALDG(ABS(V))) 550 IF(UNI(0)*X.GT.2.75)GD TD 5 551 RNURTH=SIGH(X,V) 552 RETURN 553 END 554 C REXP TDDTH FUNCTION 555 FUNCTION REVENTH(K) 556 DIMENSION C(65) 557 DATA (/240F0000,240E10000,240E10000,240E0000,240B0000,240B0000,2405E0000,2405E0000,2405E0000,2405E0000,2405E0000,2405E0000,2405E0000,2405E0000,2405E0000,2402C000,2402C000,2402C000,2402C000,2402E0000,2402E0000,2402E0000,2402E0000,2402E0000,240E0000,23FE000000,23FE00000,23FE00000,23FE00000,23FE00000,23FE00000,23FE0</pre>		542		_	
<pre>544 8 IP(UNITO).01.79.1034041CATCT)TANDALAMAN 545</pre>		543	-	7	$P = (89 = J_{HJ})^{\#} \cdot 69001(272)$
<pre>545 5</pre>		544	8		$\frac{1}{1} = \frac{1}{1} = \frac{1}$
<pre>546 RELINN 547 5 V=VNI(0) 548 IF(V,EQ.0) GD TD 5 549 X=SQR1(7,5625-2,*ALDG(ABS(V))) 550 IF(UNI(0)*X.GT.2.75)GD TD 5 551 RNDRTH=SIGH(X,V) 552 RETURN 553 END 554 C REXP TDDTH FUNCTIDN 555 FUNCTION REXPTH(K) 556 DIMENSION C(65) 557 DATA C/240F0000,Z40E10000,Z409H0000,Z40E0000,Z408B0000, 558 \$ Z40AF0000,Z40A50000,Z409H0000,Z405E0000,Z40580000, 559 \$ Z40780000,Z40450000,Z40540000,Z405E0000,Z40580000, 560 \$ Z40530000,Z404E0000,Z40490000,Z405E0000,Z40250000,Z40250000, 561 \$ Z40390000,Z40450000,Z404920000,Z402E0000,Z40220000, 562 \$ Z4050000,Z40350000,Z404220000,Z402E0000,Z402E0000,Z402E0000, 563 \$ Z401A0000,Z40190000,Z40170000,Z401E0000,Z401E0000,Z40120000, 564 \$ Z40120000,Z40190000,Z45F00000,Z3FF000000,Z3FE00000,Z3F500000, 565 \$ Z3F600000,Z3F500000,Z3F500000,Z3F600000,Z3F600000,Z3F600000, 566 \$ Z3F600000,Z3F500000,Z3F700000,Z3F600000,Z3F600000, 567 \$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F400000, 568 \$ Z4050000,Z3F500000,Z3F500000,Z3F400000,Z3F400000, 569 IF(K,GT,I1)GD TD 5 570 1 U1=UNI(0) 571 IF(U1,GT,.77917049) GD TD 3 572 T=1,-1,239962#U1 573 REXPTH==ALDG(T) 574 J=16,*REXPTH+1, 575 IF(UNI(0)*(,0604*T+,0039),GT,T-C(J))GDTD1 576 RETURN 577 3 REXPTH=19,20352*U1-15,20352 578 J=16,*REXPTH+1, 579 EX=EXP(-REXPTH) 590 EX=EXP(-REXPTH) 590 EX=EXP(-REXPTH)</pre>		545			2
 547 5 V=VN10; 548 IF(V,E0,0) GD TD 5 549 X=SQRT(7,5625-2,*ALDC(ABS(V))) 550 IF(UN1(0)*X,GT,2,75)GD TD 5 551 RNDRTH=SIGN(X,V) 552 RETURN 553 END 554 C REXP TDDTH FUNCTIDN 555 FUNCTIDN REXPTH(K) 556 DIMENSION C(65) 557 DATA C/240F00000, 240E10000, 240910000, 240890000, 24080000, 558 \$ Z40AF0000, Z40A50000, 240980000, 240910000, Z40890000, 24080000, 559 \$ Z40780000, 240710000, 240940000, 240440000, 2405E0000, 240360000, 560 \$ Z40530000, 240450000, 240490000, 240440000, 2405E0000, 240260000, 561 \$ Z40390000, 240350000, 240220000, 240260000, 240260000, 240260000, 562 \$ Z40270000, 240350000, 240220000, 240160000, 240150000, 240150000, 240160000, 240160000, 240130000, 563 \$ Z401A0000, 240190000, 240170000, 23F000000, 23F5000000, 23F5000000, 23F5000000, 23F5000000, 23F5000000, 23F500000, 23F500000, 23F600000, 23F600000, 23F600000, 23F600000, 23F600000, 23F600000, 23F600000, 23F400000, 23F500000, 23F500000, 23F400000, 23F500000, 23F400000, 23F400000, 23F400000, 23F400000, 23F500000, 23F400000, 23F400000, 23F400000, 23F400000, 23F5000000, 23F500000, 23F500000,		546	~		
<pre>540 IF (V10.0, 0.0, 0.0, 2.4 AL DG (ABS (V))) 550 IF (UN1(0)*X.GT.2.75)GD TD 5 551 RNURTH=SIGH(X,V) 552 RETURN 553 END 554 C REXP TODTH FUNCTION 555 FUNCTION REXPTH(K) 556 DIMENSION C(65) 557 DATA C/240F00000, Z40E10000, Z40910000, Z40890000, Z40800000, 558 \$ Z40450000, Z40450000, Z40910000, Z4050000, Z40800000, 559 \$ Z40780000, Z40450000, Z4090000, Z4050000, Z4050000, Z4050000, 560 \$ Z40530000, Z40450000, Z40490000, Z4020000, Z40250000, Z4029000, 561 \$ Z4039000, Z40250000, Z4022000, Z4022000, Z4026000, Z4029000, 562 \$ Z4027000, Z40240000, Z4022000, Z4020000, Z4016000, Z4016000, Z4016000, 563 \$ Z4014000, Z40190000, Z40120000, Z3F500000, Z3F500000, Z3F500000, 564 \$ Z3F60000, Z3F800000, Z3F500000, Z3F600000, Z3F600000, Z3F600000, 566 \$ Z3F60000, Z3F500000, Z3F500000, Z3F400000, Z3F400000, Z3F400000, 567 \$ Z3F60000, Z3F500000, Z3F500000, Z3F400000, Z3F400000, Z3F400000, 568 DATA 11/ZF84FA91/ 569 IF (K,GT,11)GD TD 5 570 1 U1=UNI(0) 571 IF (UI.GT.7917049) GD TD 3 572 T=1,-1.239962×U1 573 REXPTH=10,20352×U1-15,20352 578 J=16,*REXPTH+1, 579 EX=EXP(=REXPTH) 500 If (UN1(0)*(,0604*EX+,0039),GT,EX=C(J))GDTD1 570 IF (UNI(0)*(,0604*EX+,0039),GT,EX=C(J))GDTD1 571 IF (UN1(0)*(,0604*EX+,0039),GT,EX=C(J))GDTD1 572 F30000,F30000,F30000,F30000,F3000000,F300000,F300000,F3000000,F300000,F3000000,F300000,F30000000,F3000000,F3000000,F30000000,F3000000,F3000000,F3000000</pre>		547	2		V = V N I (0) T = V = 0 (0) (0) TO 5
<pre>547 ALSERIG() #X.CT.2.75)GD TD 5 550 IF (UNIC) #X.CT.2.75)GD TD 5 551 RNDRTH=SIGH(X,V) 552 RETURN 553 END 554 C REXP TDDTH FUNCTIDN 555 FUNCTIDN REXPTH(K) 556 DIMENSION C(65) 557 DATA C/Z40F0000,Z40E10000,Z40D40000,Z40E70000,Z40BB0000, 558 \$ Z40AF0000,Z40A50000,Z409B0000,Z405E0000,Z40580000, 560 \$ Z40530000,Z404E0000,Z40490000,Z4040000,Z405E0000,Z40580000, 561 \$ Z40390000,Z404E0000,Z40490000,Z40440000,Z405E0000,Z40290000, 561 \$ Z40390000,Z404E0000,Z40490000,Z402E0000,Z402E0000,Z402E0000, 562 \$ Z40270000,Z4040000,Z40220000,Z402E0000,Z401E0000,Z40100000, 563 \$ Z40120000,Z40190000,Z40120000,Z40150000,Z40130000, 564 \$ Z40120000,Z40110000,Z40100000,Z3FE00000</pre>		540			$Y = SCRT(7, 5625=2, *A) \Pi G(ABS(V)))$
<pre>500 17 (007(0) 707</pre>		547			$TE/INT(0)*X_GT_2,75)GT_TD_5$
<pre>551 RETURN 553 END 554 C RETY TODTH FUNCTION 555 FUNCTION REXPTH(K) 556 DIMENSION C(65) 557 DATA C/Z40F00000,Z40E10000,Z4009000,Z40890000,Z40800000, 559 \$ Z40AF0000,Z40710000,Z406A0000,Z40560000,Z40580000,Z40580000, 560 \$ Z40530000,Z40710000,Z40490000,Z40440000,Z40580000,Z40290000, 561 \$ Z40390000,Z40350000,Z40420000,Z40240000,Z40260000,Z40220000, 562 \$ Z40270000,Z40240000,Z40220000,Z402E0000,Z401E0000,Z40120000, 563 \$ Z401A0000,Z40190000,Z40170000,Z40160000,Z40150000,Z40120000, 564 \$ Z40120000,Z40190000,Z40100000,Z3FF00000,Z3FF00000,Z3FF00000, 565 \$ Z3F600000,Z3F500000,Z3F500000,Z3FF00000,Z3F900000,Z3F900000, 566 \$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F900000,Z3F400000, 567 \$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F400000,Z3F400000, 568 DATA 11/ZFB4FAA91/ 569 IF(K,GT,11)GD TD 5 570 1 U1=UNI(0) 571 IF(U1.GT,.7917049) GD TD 3 572 T=11.239962*U1 573 REXPTH=ALDG(T) 574 J=16,*REXPTH+1. 575 IF(UNI(0)*(.0604*T+.0039).GT,T-C(J))GDTD1 576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16,*REXPTH+1 579 EX=EXP(=REXPTH) 500 IF(U1(0)*(.0604*EX+.0039).GT,FX=C(J))GDTD1</pre>		55V			
<pre>552 REVENT 553 END 554 C REXP TODTH FUNCTION 555 FUNCTION REXPTHI(K) 555 DIMENSION C(65) 557 DATA C/Z40F00000,Z40E10000,Z40910000,Z40890000,Z408800000, 558 \$ Z40AF0000,Z40A50000,Z409B0000,Z405E0000,Z40580000, 559 \$ Z40780000,Z40710000,Z4090000,Z40540000,Z405E0000,Z40580000, 560 \$ Z40530000,Z404E0000,Z40490000,Z402F0000,Z4026000,Z40290000, 561 \$ Z40390000,Z40250000,Z40220000,Z402E0000,Z401E0000,Z4012000, 562 \$ Z40270000,Z40240000,Z40220000,Z402E0000,Z401E0000,Z40120000, 563 \$ Z40120000,Z40190000,Z40170000,Z40160000,Z40150000,Z40130000, 564 \$ Z40120000,Z401100000,Z40100000,Z3FF00000,Z3FF00000,Z3F900000, 565 \$ Z3FE00000,Z3F800000,Z3F700000,Z3F400000,Z3F900000,Z3F900000, 566 \$ Z3F800000,Z3F500000,Z3F700000,Z3F400000,Z3F400000,Z3F400000, 567 \$ Z3F600000,Z4F500000,Z3F500000,Z3F400000,Z3F400000,Z3F400000, 568 DATA I1/ZF8FAA91/ 569 IF(K,GT.I1)GD TD 5 570 1 U1=UNI(0) 571 IF(U1.GT7917049) GD TD 3 572 T=1,-1:239962*U1 573 REXPTH=14. 575 IF(UNI(0)*(,0604*T+,0039).GT,T-C(J))GDTD1 574 J=16,*REXPTH+1. 575 IF(UNI(0)*(,0604*T+,0039).GT,T-C(J))GDTD1 576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16,*REXPTH+1 579 EXEEXP(-REXPTH) 500 IF(U1(0)*(,0604*EX+,0039).GT,FX=C(J))GDTD1 571 IF(U1.0)*(,0604*EX+,0039).GT,FX=C(J))GDTD1 572 IF(U1.0)*(,0604*EX+,0039).GT,FX=C(J))GDTD1 573 REXPTH=19.20352*U1-15.20352 574 J=16,*REXPTH+1 579 EXEEXP(-REXPTH)</pre>		221			RETURN
<pre>553 C REXP TOOTH FUNCTION 555 FUNCTION REXPTH(K) 556 DIMENSION C(65) 557 DATA (7240F0000,740E10000,7409H0000,740890000,740880000, 558 \$ 740AF0000,740A50000,7409B0000,740910000,7405E0000,740580000, 559 \$ 740780000,7404E0000,740640000,740460000,7405E0000,740580000, 560 \$ 740530000,7404E0000,740490000,740460000,740450000,740580000, 561 \$ 740390000,7404350000,740420000,740420000,740420000,74020000, 562 \$ 74027000,740420000,740220000,74026000,74012000, 563 \$ 74012000,740190000,740170000,740160000,73FE00000,740150000,740150000, 564 \$ 73FE00000,73FE00000,73FE00000,73FE00000,73FE00000,73FE00000, 565 \$ 73FE00000,73FE00000,73FF00000,73FE000000,73FE00000,73FE00000,73FE00000,73FE00000,73FE00000</pre>		226			END
<pre>555 FUNCTION REXPTH(K) 556 DIMENSION C(65) 557 DATA C/Z40F00000,Z40E10000,Z400J000,Z40C70000,Z408B0000, 558 \$ Z40AF0000,Z40A50000,Z409B0000,Z409J0000,Z40890000,Z40580000, 559 \$ Z40780000,Z40710000,Z4040000,Z40540000,Z405E0000,Z40580000, 560 \$ Z40530000,Z40710000,Z40490000,Z40240000,Z405E0000,Z403C0000, 561 \$ Z40390000,Z40350000,Z40320000,Z402F0000,Z402C000,Z40290000, 562 \$ Z40270000,Z40240000,Z40220000,Z40220000,Z40150000,Z40120000, 563 \$ Z401A0000,Z40190000,Z40120000,Z40160000,Z40150000,Z40130000, 564 \$ Z40120000,Z40190000,Z40120000,Z3FE00000,Z3FE00000,Z3FE00000,Z3FE00000,Z3F000000, 566 \$ Z3F800000,Z3F500000,Z3F700000,Z3F600000,Z3F600000,Z3F600000, 566 \$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F400000,Z3F400000, 567 \$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F400000,Z3F400000, 568 DATA 11/ZFB4FAA91/ 569 IF(K,GT,I1)GD TD 5 570 1 U1=UNI(0) 571 IF(U1,GT,.7917049) GD TD 3 572 T=1,=1.239962*U1 573 REXPTH=ALDG(T) 574 J=16,*REXPTH+1. 575 IF(UNI(0)*(,0604*T+,0039).GT,T-C(J))GDTD1 576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16,*REXPTH+1. 579 EX=EXP(-REXPTH) 590 IF(UNI(0)*(,0604*EX+,0039).GT,EX-C(J))GDTD1</pre>		554	c		REXP TONTH FUNCTION
<pre>556 DIMENSION C(65) DATA C/Z40F0000,Z40E10000,Z40D40000,Z40C70000,Z408B0000, 557 DATA C/Z40F0000,Z40A50000,Z409B0000,Z40910000,Z40890000,Z40800000, 558 \$ Z40AF0000,Z40A50000,Z409B0000,Z40910000,Z405E0000,Z40580000, 560 \$ Z40530000,Z40710000,Z40490000,Z40440000,Z405E0000,Z403C0000, 561 \$ Z40390000,Z40350000,Z40320000,Z402F0000,Z402C0000,Z40290000, 562 \$ Z40270000,Z40240000,Z40220000,Z402F0000,Z401E0000,Z401C0000, 563 \$ Z401A0000,Z40190000,Z40170000,Z40160000,Z40150000,Z40130000, 564 \$ Z40120000,Z40190000,Z40170000,Z3FF00000,Z3FF00000,Z3FF00000,Z3F900000, 565 \$ Z3FC00000,Z3F800000,Z3F700000,Z3F700000,Z3F900000,Z3F900000, 566 \$ Z3F800000,Z3F800000,Z3F700000,Z3F400000,Z3F900000,Z3F900000, 567 \$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F400000,Z3F400000,Z3F400000,Z3F500000,Z3F500000,Z3F500000,Z3F500000,Z3F500000,Z3F500000,Z3F400000,Z3F400000,Z3F5000000,Z3F500000,Z3F500000,Z3F500000,Z3F5000000,Z3F500000,Z3F500000</pre>		555	v		FUNCTION REXPTH(K)
557 DATA C/Z40F00000,Z40E10000,Z40D40000,Z40C70000,Z40BB0000, 558 \$ Z40AF0000,Z40A50000,Z409B0000,Z40910000,Z40890000,Z40880000, 559 \$ Z40780000,Z40710000,Z409B0000,Z40560000,Z40580000,Z40580000, 560 \$ Z40530000,Z404E0000,Z40490000,Z404640000,Z40400000,Z40320000, 561 \$ Z40390000,Z40240000,Z40220000,Z40220000,Z40220000,Z40220000,Z40220000,Z40220000,Z40220000,Z40120000,Z40120000,Z40120000,Z40150000,Z40150000,Z40130000,S63 564 \$ Z40120000,Z40110000,Z40120000,Z3FF000000,Z3FF000000,Z3FD00000,Z3FD00000,Z3FD00000,Z3FD00000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F000000,Z3F4000000,Z3F4000000,Z3F4000000,Z3F4000000,Z3F400000,Z3F4000000,Z3F400000,Z3F40000		556			DIMENSION C(65)
<pre>558 \$ Z40AF0000,Z40A50000,Z409B0000,Z40910000,Z40890000,Z40800000, 559 \$ Z40780000,Z40710000,Z4040000,Z405E0000,Z40580000, 560 \$ Z40530000,Z404E0000,Z40490000,Z405E0000,Z40580000, 561 \$ Z40390000,Z40350000,Z40220000,Z402C0000,Z402290000, 562 \$ Z40270000,Z40240000,Z40220000,Z401E0000,Z401E0000,Z40120000, 563 \$ Z401A0000,Z40190000,Z40170000,Z40160000,Z40150000,Z40130000, 564 \$ Z40120000,Z40190000,Z40100000,Z3FF00000,Z3FE00000,Z3FD00000, 565 \$ Z3FE00000,Z3FB00000,Z3FF00000,Z3FF00000,Z3F900000,Z3F900000, 566 \$ Z3FE00000,Z3FB00000,Z3F700000,Z3F400000,Z3F900000,Z3F900000, 567 \$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F400000,Z3F400000, 568 DATA I1/ZFB4FAA91/ 569 IF(K,GT,I1)GD TD 5 570 1 U1=UNI(0) 571 IF(U1,GT,.7917049) GD TD 3 572 T=1,=1,239962*U1 573 REXPTH=ALDG(T) 574 J=16,*REXPTH+1. 575 IF(UNI(0)*(,0604*T+,0039).GT,T-C(J))GDTD1 (576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16,*REXPTH+1 579 EX=EXP(-REXPTH) 579 IF(=CUNI(0)*(,0604*EX+,0039).GT,EX=C(J))GDTD1</pre>		557			DATA C/Z40F00000, Z40E10000, Z40D40000, Z40C70000, Z40BB0000,
<pre>559 \$ Z40780000,Z40710000,Z406A0000,Z40540000,Z40580000,Z40380000, 560 \$ Z40530000,Z404E0000,Z40490000,Z4040000,Z4040000,Z40320000, 561 \$ Z40390000,Z40350000,Z40220000,Z402E0000,Z40120000,Z40120000, 562 \$ Z40270000,Z40240000,Z40220000,Z40220000,Z40150000,Z40130000, 563 \$ Z401A0000,Z40190000,Z40170000,Z40160000,Z40150000,Z40130000, 564 \$ Z40120000,Z40110000,Z40100000,Z3FF000000,Z3FE000000,Z3FF000000, 565 \$ Z3FE00000,Z3FB00000,Z3FF00000,Z3FF000000,Z3F900000, 566 \$ Z3F800000,Z3F800000,Z3F700000,Z3F4000000,Z3F600000, 567 \$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F600000, 568 DATA I1/ZFB4FAA91/ 569 IF(K,GT_11)GD TD 5 570 1 U1=UNI(0) 571 IF(U1.GT7917049) GD TD 3 572 T=11.239962*U1 573 REXPTH=ALDG(T) 574 J=16,*REXPTH+1. 575 IF(UNI(0)*(,0604*T+,0039).GT.T-C(J))GDTD1 (576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16,*REXPTH) 579 EX=EXP(-REXPTH) 579 IF(UNI(0)*(,0604*EX+,0039).GT.EX=C(J))GDTD1</pre>		558			\$ Z40AF0000, Z40A50000, Z409B0000, Z40910000, Z40890000, Z40800000
<pre>560 \$ Z40530000,Z404E0000,Z40490000,Z40440000,Z4040000,Z4029000, 561 \$ Z4039000,Z4035000,Z4022000,Z402F000,Z402C000,Z4029000, 562 \$ Z4027000,Z4024000,Z4022000,Z4020000,Z401E000,Z401C000, 563 \$ Z4012000,Z4019000,Z4017000,Z40160000,Z40150000,Z40130000, 564 \$ Z4012000,Z4019000,Z4010000,Z3FF00000,Z3FF00000,Z3FF00000, 565 \$ Z3FC0000,Z3FB00000,Z3FF00000,Z3FF00000,Z3F900000, 566 \$ Z3F600000,Z3F800000,Z3F700000,Z3F400000,Z3F600000,Z3F600000, 567 \$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F400000,Z3F400000, 568 DATA 11/ZFB4FAA91/ 569 IF(K,GT,I1)GD TD 5 570 1 U1=UNI(0) 571 IF(U1.GT7917049) GD TD 3 572 T=11.239962*U1 573 REXPTH=ALDG(T) 574 J=16,*REXPTH+1. 575 IF(UNI(0)*(,0604*T+,0039).GT,T-C(J))GDTD1 576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16,*REXPTH) 579 EX=EXP(-REXPTH) 590 IF(UNI(0)*(,0604*EX+,0039).GT,EX=C(J))GDTD1</pre>		559			\$ Z40780000, Z40710000, Z406A0000, Z40640000, Z405E0000, Z405E0000, Z403C0000,
<pre>561 \$ Z40390000,Z40350000,Z40320000,Z402C0000,Z402C0000,Z4020000, 562 \$ Z40270000,Z40240000,Z40220000,Z401E0000,Z401C0000, 563 \$ Z401A0000,Z40190000,Z40160000,Z40150000,Z40130000, 564 \$ Z40120000,Z40110000,Z40100000,Z3FF000000,Z3FF000000,Z3FF000000, 565 \$ Z3F800000,Z3F800000,Z3FF00000,Z3F600000,Z3F900000, 566 \$ Z3F800000,Z3F800000,Z3F700000,Z3F400000,Z3F600000, 567 \$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F400000,Z3F400000, 568 DATA 11/ZF84FAA91/ 569 IF(K,GT.11)GD TD 5 570 1 U1=UNI(0) 571 IF(U1.GT7917049) GD TD 3 572 T=11.239962*U1 573 REXPTH=ALDG(T) 574 J=16,*REXPTH+1. 575 IF(UNI(0)*(.0604*T+.0039).GT.T-C(J))GDTD1 576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16,*REXPTH+1. 579 EX=EXP(-REXPTH) 590 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GDTD1</pre>		560			\$ Z40530000, Z404E0000, Z40490000, Z40440000, Z4040000, Z40400000, Z4030000
<pre>562 \$ Z40270000,Z40240000,Z40220000,Z401200000,Z40120000,Z40120000, 563 \$ Z401A0000,Z40190000,Z40170000,Z40150000,Z401300000, 564 \$ Z40120000,Z40110000,Z4010000,Z3FF00000,Z3FE00000,Z3F900000, 565 \$ Z3FE00000,Z3FB00000,Z3FA00000,Z3F900000,Z3F900000, 566 \$ Z3F800000,Z3F500000,Z3F700000,Z3F400000,Z3F600000, 567 \$ Z3F600000,Z3F500000,Z3F400000,Z3F400000,Z3F400000, 568 DATA 11/ZFB4FAA91/ 569 IF(K,GT,11)GD TD 5 570 1 U1=UNI(0) 571 IF(U1.GT,.7917049) GD TD 3 572 T=11.239962*U1 573 REXPTH=ALDG(T) 574 J=16,*REXPTH+1. 575 IF(UNI(0)*(.0604*T+.0039).GT.T-C(J))GDTD1 (576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16,*REXPTH+1. 579 EX=EXP(-REXPTH) 580 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GDTD1</pre>		561			\$ Z40390000, Z40350000, Z40320000, Z402F0000, Z402C0000, Z402F0000, Z402F00000, Z402F0000, Z402F000, Z402F0000, Z402F000, Z40000, Z4000, Z4000, Z4000, Z4000, Z4000, Z4000
<pre>563 \$ Z401A0000,Z40190000,Z40170000,Z3F100000,Z3F100000,Z3F100000, 564 \$ Z4012000,Z4011000,Z4010000,Z3F100000,Z3F100000,Z3F100000, 565 \$ Z3F00000,Z3F800000,Z3F100000,Z3F00000,Z3F900000, 566 \$ Z3F800000,Z3F500000,Z3F700000,Z3F600000,Z3F600000, 567 \$ Z3F600000,Z3F500000,Z3F400000,Z3F400000,Z3F400000/ 568 DATA I1/ZF84FAA91/ 569 IF(K,GT,I1)GD TD 5 570 1 U1=UNI(0) 571 IF(U1,GT,.7917049) GD TD 3 572 T=11.239962*U1 573 REXPTH=ALDG(T) 574 J=16.*REXPTH+1. 575 IF(UNI(0)*(,0604*T+,0039).GT,T-C(J))GDTD1 576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16.*REXPTH+1. 579 EX=EXP(-REXPTH) 580 IF(UNI(0)*(,0604*EX+,0039).GT,EX=C(J))GDTD1</pre>		562			\$ Z40270000, Z40240000, Z40220000, Z40200000, Z40120000, Z40100000,
<pre>564 \$ Z40120000,Z40110000,Z40100000,Z3FF00000,Z3F00000,Z3F900000, 565 \$ Z3F00000,Z3F800000,Z3F700000,Z3F700000,Z3F600000, 566 \$ Z3F800000,Z3F500000,Z3F700000,Z3F600000,Z3F600000, 567 \$ Z3F600000,Z3F500000,Z3F400000,Z3F400000,Z3F400000/ 568 DATA 11/ZFB4FAA91/ 569 IF(K,GT.I1)GD TD 5 570 1 U1=UNI(0) 571 IF(U1.GT7917049) GD TD 3 572 T=1.=1.239962*U1 573 REXPTH=ALDG(T) 574 J=16.*REXPTH+1. 575 IF(UNI(0)*(.0604*T+.0039).GT.T-C(J))GDTD1 (576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16.*REXPTH+1. 579 EX=EXP(-REXPTH) 580 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GDTD1</pre>		563			\$ Z401A0000, Z40190000, Z40170000, Z40100000, Z40150000, Z401500000, Z40150000, Z40150000, Z40150000, Z40150000, Z401500000, Z401500000, Z401500000, Z401500000, Z401500000, Z4001500000, Z400150000, Z4000000, Z40000000, Z40000000, Z40000000, Z40000000, Z40000000, Z40000000, Z40000000, Z400000000, Z40000000000
<pre>565 \$ Z3FC00000,Z3F800000,Z3F800000,Z3F400000,Z3F600000,Z3F600000, 566 \$ Z3F800000,Z3F500000,Z3F700000,Z3F600000,Z3F600000, 567 \$ Z3F600000,Z3F500000,Z3F400000,Z3F400000,Z3F400000/ 568 DATA 11/ZF84FAA91/ 569 IF(K,GT.11)GD TD 5 570 1 U1=UNI(0) 571 IF(U1.GT7917049) GD TD 3 572 T=11.239962*U1 573 REXPTH=ALDG(T) 574 J=16.*REXPTH+1. 575 IF(UNI(0)*(.0604*T+.0039).GT.T-C(J))GDTD1 576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16.*REXPTH+1. 579 EX=EXP(-REXPTH) 590 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GDTD1</pre>		564			\$ Z40120000, Z40110000, Z40100000, Z3FF00000, Z3F900000, Z3F900000, Z3F900000,
<pre>566 \$ Z3F800000,Z3F800000,Z3F700000,Z3F700000,Z3F00000,Z3F400</pre>		565			\$ Z3FC00000, Z3FB00000, Z3FB00000, Z3FA00000, Z3F600000, Z3F600000,
<pre>567 \$ Z3F600000,Z3F5000000,Z3F50000000,Z3F500000000,Z3F500000000,Z3F5000000000000000000000000000000000000</pre>		566			\$ Z3F800000, Z3F800000, Z3F700000, Z3F700000, Z3F400000, Z3F400000,
568 DATA 11/2FB4FAA91/ 569 IF(K.GT.I1)GD TD 5 570 1 01=UNI(0) 571 IF(U1.GT7917049) GD TD 3 572 T=11.239962*U1 573 REXPTH=ALDG(T) 574 J=16.*REXPTH+1. 575 IF(UNI(0)*(.0604*T+.0039).GT.T-C(J))GDTD1 576 RETURN 577 3 578 J=16.*REXPTH+1. 579 EX=EXP(-REXPTH) 579 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GDTD1		567			\$ Z3F600000723F500000723F500000725F400000725F400000725F
569 IF(K.GT.II)0B TO 5 570 1 U1=UNI(0) 571 IF(U1.GT7917049) GD TO 3 572 T=11.239962*U1 573 REXPTH==ALOG(T) 574 J=16.*REXPTH+1. 575 IF(UNI(0)*(.0604*T+.0039).GT.T-C(J))GDTD1 576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16.*REXPTH+1. 579 EX=EXP(=REXPTH) 580 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GDTD1		568			
570 1 01=0N1(0) 571 IF(U1.GT.7917049) GD TD 3 572 T=11.239962*U1 573 REXPTH==ALDG(T) 574 J=16.*REXPTH+1. 575 IF(UNI(0)*(.0604*T+.0039).GT.T-C(J))GDTD1 576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16.*REXPTH+1. 579 EX=EXP(=REXPTH) 580 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GDTD1		569			$= \frac{1}{16} \left(\frac{1}{6} \right) \left(\frac{1}{16} \right) \left(\frac{1}{16} \right)$
571 IF(UI.GT7917049) 00 TO 5 572 T=11.239962*U1 573 REXPTH=ALOG(T) 574 J=16.*REXPTH+1. 575 IF(UNI(0)*(.0604*T+.0039).GT.T-C(J))GOTO1 576 RETURN 577 3 REXPTH=19.20352*U1-15.20352 578 J=16.*REXPTH+1. 579 EX=EXP(-REXPTH) 580 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GOTO1		570	1	-	U = U V (U)
572 573 REXPTH==ALOG(T) 574 J=16,*REXPTH+1. 575 IF(UNI(0)*(.0604*T+.0039).GT.T-C(J))GOTO1 576 RETURN 576 S77 3 REXPTH=19.20352*U1-15.20352 578 J=16,*REXPTH+1. 579 EX=EXP(=REXPTH) 580 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GOTO1		571			
573 574 574 J=16,*REXPTH+1. 575 IF(UNI(0)*(,0604*T+,0039).GT,T-C(J))GDTD1 576 RETURN 577 S77 S77 J=16,*REXPTH+1. 579 EX=EXP(-REXPTH) 580 IF(UNI(0)*(,0604*EX+,0039).GT.EX=C(J))GDTD1		572			1=10/77000000000000000000000000000000000
574 J=10,4KEXFILL 575 IF(UNI(0)*(,0604*T+,0039).GT,T-C(J))GDTD1 576 RETURN 577 3 577 3 578 J=16,*REXPTH+1. 579 EX=EXP(-REXPTH) 580 IF(UNI(0)*(,0604*EX+,0039).GT,EX=C(J))GDTD1	•	573			NEAF18-88697111 1416 486971111
575 RETURN 576 REXPTH=19.20352*U1-15.20352 577 3 578 J=16.*REXPTH+1. 579 EX=EXP(-REXPTH) 580 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GOTO1		574			V-10;***CFAF 117 1 * TFC(J))GDTD1
577 3 REXPTH=19.20352*U1-15.20352 578 J=16.**REXPTH+1. 579 EX=EXP(=REXPTH) 580 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GOTO1	(5/2			RETIRN
578 J=16,*REXPTH+1. 579 EX=EXP(~REXPTH) 580 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GOTO1	Γ.	.)(0		2	REXDTH=19.20352*U1-15.20352
579 EX=EXP(#REXPTH) 580 IF(UNI(0)*(*0604*EX+*0039)*GT*EX=C(J))GOTO1		5/1	-	. נ	J=16.*RFXPTH+1.
580 IF(UNI(0)*(.0604*EX+.0039).GT.EX=C(J))GOTO1		210			FX=FXP(_RFXP7H)
		2 (7 5 Q N			IF(UNI(0)*(,0604*EX+,0039).GT.EX=C(J))GOTO1

(:	581 582 583 584	5	RETURN REXPTH=4.HALAG(UNI(O)) RETURN END			
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APPENDIX D

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Denotations ***: p << .01 **: p < .01 *: p < .05

TABLE D.1: Tests of Signifiance on the Parameter Estimates of the 12 Multiple Regression Equations of Table 3.5

<u>BLOCK 1</u> $\alpha = 0.05$, $\kappa = 5$, n = 15

Regression Coefficients	D.F.	T-value
β _o	106	33•99***
Â	106	18.84***
β ₂	106	-16.35***
β ₃	106	- 1.77

Multiple Comprehension Coefficient (R)

 $F = 1455.74 ** DF_1 = 3 DF_2 = 106$

<u>BLOCK 2</u> $\alpha = 0.05, k = 5, n = 5$

Regression Coefficients	D.F.	T-value
Â	106	28.06***
Â	106	19.74***
β ₂	106	-12.16***
ρ_ β ₃	106	- 4.78**
Multiple Commentation Coeff	icient (R)	
F = 1213.63*** DF	= 3 DF ₂ =	106

APPENDIX D (contid) <u>BLOCK 3</u> α = 0.05, k = 3, n = 15 T-value D.F. Regression Coefficients 106 32.05*** β_o βı 15.67*** 106 ^β2 -18.12** 106 β̂3 - 2.25* 106 Multiple Coorgree Santicon Coefficient (R) F = 983.54*** $DF_1 = 3$ $DF_2 = 106$ <u>BLOCK 4</u> α = 0.05, k = 3, n = 5 Regression Coefficients D.F. T-value 30.20*** β_o 106 βı 23.74*** 106 <mark>،</mark> β -10.18*** 106 - 7.16*** β₃ 106 Multiple & egrelation Coefficient (R) $F = 1387.57 * * DF_1 = 3 DF_2 = 106$ <u>BLOCK 5</u> α = 0.05, k = 2, n = 15 Regression Coefficients T-value D.F. 25.80*** 44 β̂ο βı 3.76** 44 - 7.08*** β₂ 44 - 1.20 44 β₃ Multiple ConrelationCoefficient (R) $F = 161.26 \times DF_1 = 3$ $DF_2 = 44$

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APPENDIX D (cont'd) <u>BLOCK 6</u> α = 0.05, k = 2, n = 5 D.F. T-value **Regression Coefficients** 31.74*** β_o 44 β₁ 9.08*** 44 Â2 - 9.22*** 44 - 2.06* 44 Multiple Roegnebation Coefficient (R) $F = 380.05 * * PF_1 = 3 PF_2 = 44$ <u>BLOCK 1</u> α = 0.01, k = 5, n = 15 **Regression Coefficients** D.F. T-value 12.68*** β_o 106 βĵ 24.8 *** 106 β₂ - 8.1 *** 106 - 7.4 *** β̂3 106 Multiple Correlation Coefficient (R) $F = 1330.00 \times T$ $DF_1 = 3$ $DF_2 = 106$ <u>BLOCK 2</u> α = 0.01, k = 5, n = 5 T-value **Regression Coefficients** D.F. 106 9.50*** β_o 27.69*** βı 106 ^β2 - 5.00** 106 -10.70*** β₃ 106 Multiple ConrelationCoefficient (R) F = 1356.42*** $DF_1 = 3$ $DF_2 = 106$

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APPENDIX D (cont'd) <u>BLOCK 3</u> α = 0.01, k = 3, n = 15 Regression Coefficients D.F. T-value β_ο 10.85*** 106 β_β 106 18.15*** Â2 - 5**.3**8*** 106 β̂ 106 - 6.09*** Multiple Corgredisation Coefficient (R) $F = 690.66*** DF_1 = 3 DF_2 = 106$ <u>BLOCK 4</u> α = 0.01, k = 3, n = 5 Regression Coefficients D.F. T-value β_β 7.91*** 106 β₁ 106 28.18*** ^β2 - 2.84** 106 β_β 106 -11.57*** Multiple Correlation Coefficient (R) $F = 1260.22*** DF_1 = 3 DF_2 = 106$ <u>BLOCK 5</u> α = 0.01, k = 2, n = 15 T-value Regression Coefficients D.F. β_βο 44 11.01*** βı 44 4.67** ^β2 - 3.28** 44 β̂3 - 2.66** 44 Multiple Coegresation Coefficient (R) $F = 87.41 * * * DF_1 = 3 DF_2 = 44$

APPENDIX D (contid)

<u>BLOCK 6</u> α = 0.01, k = 2, n	= 5	
Regression Coefficients	5 D.F.	T-value
β	44	9.29***
β ₁	44	7.89***
β _ρ	44	- 2.75**
β ₃	44	- 3.24**
Multiple Cogresation Coe	efficient (R)	
F = 135.23***	DF, = 3	$DF_{2} = 44$

TABLE D.2: Analysis of Variance for Testing the Equality of Six Regression Equations at Each Level of α

 α = .05

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Due to	DF	SS	, F
Dev. from hypothesis	20	99017.96	480.32***
Separate regressions (residual)	512	101434.00	
Common regression (residual)	532	200452.00	
$\alpha = .01$			
Dev. from hypothesis	20	43913.85	80.57***
Separate regressions (residual)	512	20452.55	
Common regression (residual)	532	64366 .4 0	

APPENDIX D (cont'd)

TABLE D.3: Analysis of Variance for Testing the Equality of Three Regression Equations Corresponding to the Three Values of κ , at each Level of α

 α = .05

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Due to	DF	SS	F
Dev. from hypothesis	8	36870.00	14.76***
Separate regressions (residual)	524	163582.00	
Common regression (residual)	536	200452.00	

 $\alpha = .01$

Dev. from hypothesis	8	26030.5	44.47***
Separate regressions (residual)	524	38335.87	
Common regression (residual)	536	64366.4	

APPENDIX D (cont'd)

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TABLE D.4: Analysis of Variance for Testing the Equality of Pairs of Regression Equations Corresponding to the Two Values of n at Each Level of κ , at Each Level of α

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α = .05, k = 2

Due to	DF	SS	F
Dev. from hypothesis	4	10066.66	21.14***
Separate regressions (residual)	88	10477.24	
Common regression (residual)	92	20543.90	
α = .05, κ = 3			
Dev. from hypothesis	4	40227.00	52.20***
Separate regressions (residual)	212	40899.50	
Common regression (residual)	216	81126.50	
α = .05, κ = 5			
- · · ·	1.	11954 20	10 50***

Dev. from hypothesis	4	11854.30	12.00^^
Separate regressions	212	50057.30	
(residual)		<u> </u>	
Common regression (residual)	216	61911.60	

APPENDIX D (cont'd)			
α = .01, K = 2			
Due to	DF	SS	F
Dev. from hypothesis	4	1512.11	12.75***
Separate regressions (residual)	88	2609.66	
Common regression (residual)	92	4121.77	
α = .01, κ = 3		· ·	
Dev. from hypothesis	4	7666.97	34.23***
Separate regressions (residual)	212	11869.63	
Common regression (residual)	216	19536.60	
α = .01, κ = 5			

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Dev. from hypothesis	4	1614.42	6.55***
Separate regressions	212	13063.08	
(residuar)	~~~~~	**************************************	
Common regressions (residual)	216	14677.50	

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