Opportunistic Multicasting Scheduling using Erasure-Correction Coding over Wireless Channels

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This thesis is dedicated to my parents and my teachers.

ABSTRACT

In wireless communications, the broadcast nature can be explored to efficiently support multicast services while the difference in channel gains among the wireless links of users promotes multiuser diversity which can be used to improve multicast performance. This thesis proposes an opportunistic multicast scheduling scheme using erasure-correction coding to jointly exploit multicast gain, multiuser diversity, and time/frequency diversity in wireless communications. The proposed scheme sends only one copy to all users in the multicast group at a selected transmission threshold on time/frequency slots and using erasure-correction coding to recover erased packets when the instantaneous signal-to-noise ratio (SNR) of the link between the Base Station and a user is insufficient.

On flat fading channel, an analytical framework is developed to establish the optimum selection of transmission threshold and erasure-correction code rate to achieve the best multicast throughput on different fading conditions. Numerical results show that the proposed scheme outperforms both the Worst-User and Best-User schemes for a wide range of SNR. The results also show that multiuser diversity is superior in the low SNR region while multicast gain is most significant at high SNR region. Moreover, to study the role of channel knowledge, the proposed scheme is considered in two cases: (i) with full channel gain knowledge and (ii) with only partial knowledge of fading type and average SNR. Our study indicates that full channel knowledge is beneficial for small multicast groups but at large group size it is sufficient to have partial channel knowledge as the difference in achievable throughput between the two cases is just marginal.

The proposed scheme is further extended for applications to Orthogonal Frequency Division Multiplexing systems to take advantage of frequency diversity in a frequency-selective fading environment. Our study on the effect of frequency correlation on multicast throughput shows that by making use

of frequency diversity, significant delay reduction can be achieved with minimal penalty on multicast throughput.

ABRÉGÉ

Dans les communications sans-fil, la diffusion se prête naturellement aux services efficaces de multidiffusion tandis que les variations des gains de canal, parmi les liens avec les différents utilisateurs, permettent de promouvoir la diversité multiutilisateur qui peut être utilisée pour améliorer la performance de multidiffusion. Ce mémoire propose une méthode d'ordonnancement multidiffusion opportuniste en utilisant un code correcteur d'effacement pour exploiter conjointement le gain de multidiffusion ainsi que les diversités temporelle, spectrale et multiutilisateurs disponibles dans les communications sans fil. La méthode proposée n'envoie qu'une seule copie, sur les créneaux temporels et spectraux, à un seuil de transmission sélectionné pour tous les utilisateurs appartenant au groupe de multidiffusion et en utilisant un code correcteur d'effacement pour récupérer les paquets effacés lorsque le rapport signal sur bruit (SNR) instantané du lien entre Station de Base et un utilisateur est insuffisant.

Sur un canal à évanouissement uniforme, un cadre analytique est développé pour établir la sélection optimale du seuil de transmission et le taux de codage à effacement pour atteindre le meilleur débit de multidiffusion sous l'effet de différentes conditions d'évanouissement. Les résultats numériques montrent que la méthode proposée surpasse en performance à la fois les méthodes Pire-Utilisateur et Meilleur-Utilisateur pour une ample plage de SNR. Les résultats montrent aussi que la diversité multiutilisateur est supérieure à bas SNR tandis que le gain multidiffusion est plus significatif à haut SNR. De plus, pour étudier le rôle de la connaissance du canal, la méthode proposée est considérée pour deux cas: (i) avec connaissance complète du canal et (ii) avec seulement une connaissance partielle du type d'évanouissement et SNR moyen. Notre étude indique que la connaissance complète du canal est avantageuse pour les petits groupes multiutilisateurs, mais que dans un large groupe il est suffisant de n'avoir

que la connaissance partielle puisque la différence du débit réalisable entre les deux cas est marginale.

La méthode proposée est généralisée pour être appliquée aux systèmes de multiplexage par répartition orthogonale de la fréquence (OFDM) qui permettent de profiter de la diversité spectrale dans un milieu à évanouissement progressif de fréquences. Notre étude sur l'effet de la corrélation spectrale sur le débit de multidiffusion montre qu'en utilisant la diversité spectrale, une réduction significative du délai est atteignable tout en minimisant l'effet sur le débit de multidiffusion.

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LIST OF ACRONYMS

AMC	Adaptive Modulation and Coding.
BS	Base Station.
BU	Best User.
cdf	Cumulative distribution function.
ECOM	Erasure-Coding based Opportunistic Multicast.
ECOMP	Erasure-Coding based Opportunistic Multicast with Partial channel information.
ECOMF	Erasure-Coding based Opportunistic Multicast with Full channel information.
$GF(2^8)$	Galois field with 2^8 elements.
i.i.d.	independent and identically distributed.
LOS	Line-of-sight.
OFDM	Orthogonal Frequency Division Multiplexing.
pdf	Probability density function.
RS	Reed-Solomon.
SNR	Signal to Noise Ratio.
WU	Worst User.

LIST OF SYMBOLS

arg	argument.
B	Bandwith of OFDM system.
$CN(\mu,\sigma)$	Circularly complex Gaussian distribution with mean μ and variance σ.
$\mathrm{E}[\cdot]$	Expected value operator.
$f_x(x)$	Probability density function of random variable x.
$F_x(x)$	Cumulative distribution function of random variable x.
h	Scalar h.
$\underline{h}$	Vector $\underline{h}$.
$h_i(t)$	Channel gain of user i at time slot t.
H_{m_x}	Channel gain on subcarrier m_x.
i	User index.
j	$\sqrt{-1}$.
k	Number of information packets within Reed-Solomon code.
l	Resolvable path index.
L	Number of resolvable paths.
lim	limit.
max	maximum.
min	minimum.
m	Nakagami-m fading parameter.
M	Number of subcarriers of considered OFDM system.
m_x	subcarrier x.
n	Size of Reed-Solomon code.
N	Number of multicast users.
$\mathcal{N}_0$	Additive white Gaussian noise variance.
N'	Subgroup selection of ECOM.
$n_i(t)$	Additive white Gaussian thermal noise at user i.
p	Probability that $\rho_i(t) \geq \rho_*$.
q	Number of bits for each Reed-Solomon code symbol.
$r(t)$	Instantaneous multicast transmission rate at timeslot t.
$\bar{r}$	Average multicast transmission rate.
$s(t)$	Transmitted signal.
S	Number of transmission time-slots in EECOM.
t	Timeslot index.
T	Subgroup selection of Threshold-T.
x	Counting index.

$y_i(t)$	Received signal of user i at timeslot t.
Δf	Bandwidth of one subcarrier in OFDM system.
Δt	Sampling time of OFDM system.
$\overline{\gamma}$	Average Signal to Noise Ratio.
$\Gamma(m)$	Gamma function.
$\rho_i(t)$	Magnitude squared of channel gain of user i.
ρ_*	Channel gain threshold for ECOM.
$\upsilon(m, x)$	Lower complete Gamma function.
$\nabla_x f(x, y)$	Partial derivative of $f(x, y)$ with respect to x.

Chapter 1:
Introduction

1.1 Motivation

Multicast is the method of delivering the same information to a subset of network nodes [18]. It has been widely used in many standards such as IP multicast for wired networks [19], Multimedia Broadcast Multicast Services (MBMS) for cellular networks [20], to support a wide range of applications including broadcasting applications (e.g., IPTV [19]), pushing media (e.g., delivering news to subscribers [22]), voice/video conference [23] and shared editing and collaboration applications (e.g., white board [24]). Multicast has also been developed as a necessary part of the next evolution of Internet: multicast for Internet 2 [25].

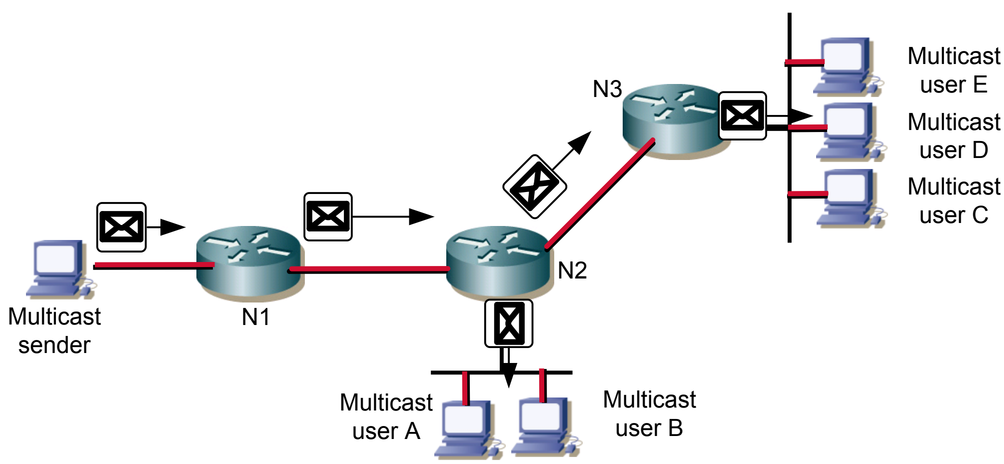

Fig. 1-1: A multicast transaction example.

In multicasting, a set of intended receivers (users) is called multicast group and is represented as a single multicast address. Users need to join a multicast group to receive multicast data for that multicast group. The

Multicast sender considers the whole multicast group as a single destination and sends only one copy to the multicast group address. The network infrastructure then intelligently replicates the copy only when needed to direct data to the intended users. As an example, Fig. 1-1 shows a multicast transaction for a group of 5 users via 3 network nodes in a network. Node N1 just forwards *one* copy to node N2 while node N2 needs to send *two* copies: *one* for users A and B over a broadcast channel and the other to node N3 that finally forwards *one* copy to users C, D and E over another broadcast channel. The example indicates that whenever the connection to multicast users has broadcast nature, (i.e., one signal transmitted through the network can be received by many users, such as Ethernet), multicast information can be transferred by just only one copy to *all* connected multicast users instead of sending one copy for each user. In this way, in supporting multicast services, broadcast nature of the transmission medium can be exploited to provide resource (bandwidth) saving, which can be effectively represented by the *multicast gain* – a gain in bandwidth efficiency as compared to a unicast (host-to-host) service of the same amount of information. It can be seen that the maximum achievable *multicast gain* is N if all N users of the multicast group are connected to a broadcast medium with the same transmission quality such as Ethernet.

As point-to-point wireless channel is inherently broadcast in nature, it is quite natural to think that the source (e.g., base-station) can exploit *full* multicast gain by sending each packet once in order to reach all the intended users of the multicast group. This would substantially reduce bandwidth and power consumptions in wireless multicast. The transmission rate is selected in such a way that *all* the intended users can reliably receive the information. In other words, the selected transmission rate is dictated by the worst-user (WU), i.e., by the wireless user link of the lowest transmission quality. If all the wireless user links have about the same transmission quality, this worst-user link rate is also similar to the rates of the other user links, which can only occur in early fixed communications systems using line-of-sight (LOS) transmission. However, since current wireless communications systems cannot afford LOS transmission, small-scale fading due to multipath propagation introduces random and large variation in signal attenuation at different time instants and over different user links. As a consequence, the selected transmission rate dictated by the worst-user can be very low, especially for a large multicast group. In other words, *full* multicast gain can be achieved but at the cost of inferior transmission rate. On the other hand, the random and large variation in signal attenuation at different time instants and over different user links due to small-scale fading promotes *multiuser diversity*, which has been exploited in opportunistic scheduling of unicast transmission to maximize both bandwidth and power efficiency as follows. At a given time, the source schedule is to send information to the best user (BU), i.e., user who has the best link quality. Of course, this BU approach can also be used to support multicast services but clearly it omits the multicast gain. In other words, the BU approach can make use of the best wireless channel capacity but the Base-Station (BS) needs to repeatedly send each packet many times since each transmission serves only one user.

At this point, two questions arise. First, between WU and BU, which one can best support multicast over a wireless fading channel? Second, whether or not and how we can combine multicast gain and multiuser diversity to improve multicast throughput. These questions inspired many studies (e.g., [2]-[9]) on opportunistic multicast to find schemes to enhance the multicast throughput, which will be discussed in Chapter 2. Various schemes proposed in these studies and corresponding performance evaluation results indicate the consensus that by sending multicast information to N' of N multicast users where $1 \le N' \le N$, both multicast gain and multiuser diversity can be partially but jointly exploited, and N' can be selected to achieve higher multicast throughput than the WU and BU approaches. Since N' is often smaller than N and can be variable in many proposed schemes, $\left(N - N'\right)$ users do not receive information in the first transmission. Efficient techniques and strategies to reduce and/or to compensate for this loss have been investigated but are still an open issue. Furthermore, in all the above mentioned studies, full knowledge of all instantaneous user channel responses is assumed at the transmitter. This assumption could be prohibitively expensive, especially for large multicast groups.

1.2 Contributions of this thesis

Inspired by the above observations, the main focus of this thesis is on opportunistic multicast scheduling that can jointly explore multicast gain, multiuser diversity and time/frequency diversity in a wireless fading environment. In particular, we propose an opportunistic multicast scheduling scheme using erasure-correction coding, in which each packet is sent only once to all users in the multicast group at a transmission rate determined by a selected channel gain threshold. When the instantaneous channel gain of a given user happens to be inadequate, i.e., below the selected threshold, the packet is considered to be erased. Reed-Solomon (n, k) erasure-correction code is applied to a block of transmitted packets such that erased packets can be recovered as long as the number of erased packets in a block does not exceed the erasure correction capability, which is $(n - k)$. As each packet can be transmitted in a time or a frequency slot, erasure-correction coding to a block of transmitted packets effectively exploits the time/frequency diversity in a wireless fading environment. Since both the selected channel gain threshold and erasure-correction code parameters contribute to the multicast throughput, they are jointly optimized to achieve the best multicast throughput. Furthermore, to study the role of channel knowledge, the proposed scheme is considered in two cases: (i) with full channel gain knowledge and (ii) with only partial knowledge of fading type and average SNR. An analytical framework has been developed to evaluate the multicast throughput of the proposed erasure-correction coding opportunistic multicast scheduling (ECOM) scheme as well as the BU and WU approaches. We prove that the effective multicast throughput (i.e., the multicast rate that each user can receive) of WU and BU asymptotically converges to zero as the group size increases while that of our proposed scheme is bounded from zero depending on the SNR. Besides, using optimization methods, we developed a mechanism to find the optimal threshold and code rate in the case the BS possesses partial channel knowledge of BS-user links.

Taking into account the change of the optimal transmission threshold in different fading conditions, the trade-off between multicast gain and multiuser diversity is studied. Our numerical results illustrate that multicast gain is most pronounced at high SNR while multiuser diversity is superior at low SNR region. Throughput comparison shows that with the ability of combining both gains, the proposed scheme outperforms both BU and WU for a wide range of SNRs. Regarding the role of channel knowledge, our study shows that for small multicast group size, full channel gain knowledge can offer better multicast throughput than partial channel knowledge; however, for large group size, the difference in multicast rates of these two cases is just negligible. This makes the proposed ECOM using partial knowledge more attractive for its low complexity.

Furthermore we consider extending ECOM for applications to Orthogonal Frequency Division Multiplexing (OFDM) systems. In particular, we explore frequency diversity in a frequency-selective fading environment by sending coded packets over subcarriers. However, since there is a correlation in channel gains among the subcarriers, deep fade on one subcarrier may result in insufficient instantaneous SNR on neighbouring subcarriers. Hence, we have investigated the effects of correlation in subcarrier channel gains on the achievable multicast throughput of the proposed scheme. Numerical results indicate that by exploring frequency diversity, we can significantly reduce the delay with negligible degradation in multicast throughput.

1.3 Thesis outline

The rest of this thesis is organized as follows. Chapter 2 provides the background and the literature review on opportunistic multicast. Afterwards, in Chapter 3, the proposed Erasure-Coding based Opportunistic Multicast scheduling (ECOM) schemes are described and the analytical framework on multicast throughput of the proposed schemes and BU, WU is provided. Then performance evaluation and comparisons are discussed to illustrate the trade-off between multicast gain and multiuser diversity, and the significance of full and partial channel knowledge. In Chapter 4, ECOM scheme is extended for applications to OFDM systems. The effects of correlation in subcarrier channel responses in a frequency-selective fading environment on the multicast throughput and the trade-off between throughput and delay are discussed. Finally, Chapter 5 concludes the thesis.

Chapter 2:

Background and Literature Review

In this chapter, background information on multicast transmission over wireless communications systems is covered. In particular, at first, a brief overview of the characteristics and model of radio propagation is presented to introduce and explain the effects of path loss, shadowing and multipath fading on transmitted signal when travelling through a wireless channel. Then, the system model is described and the wireless propagation in the context of multicast is discussed to present the benefit of multicast gain and multiuser diversity. Trying to explore both gains, many approaches for opportunistic multicast in the literature are reviewed. Motivated by these works, the comments on open and promising questions to be explored are given, and the issues to be studied in this thesis are proposed.

2.1 Radio propagation characteristics and model: a brief overview

Consider a wireless communication link between a transmitter and a receiver. While travelling through this wireless link, depending on the distance between the transmitter and receiver and due to obstacles, reflections and diffractions along the traveling path, the transmitted signal is attenuated. Generally speaking, this signal attenuation is considered to be a product of three main components: path loss, shadowing and multipath fading.

Path loss is the attenuation in power of the transmitted signal, which can be expressed as $L_0 d^{-\alpha}$ where L_0 depends on the operating frequency, atmospheric and terrain conditions, and d is the distance between the transmitter and receiver, and $2 \leq \alpha \leq 8$ is the path loss exponent. Shadowing is the random variation in the signal power due to blockage from objects in the signal paths. Since path loss and shadowing occur over relatively large distances and change slowly with time, they can be estimated when the distance between the transmitter and receiver is known and are usually referred as large-scale-fading. Beside path loss and shadowing, the transmitted signal also experiences random fluctuations of multipath fading due to the constructive and destructive multipath signal components resulting from reflections, scattering and diffractions. Different from path loss and shadowing, multipath fading occurs over a very short distances (in the order of the signal wavelength) and is changing randomly and rapidly with time and hence is usually referred as small-scale fading.

In the early days, traditional wireless communication systems, such as point-to-point microwave systems, try to mitigate the effect of fading by employing very high antennas at the transmitter and receiver. In this way, line-of-sight (LOS) communication link between the communication peers can be achieved and reflected signals, if they exist mainly due to slow atmospheric condition changes, are very weak compared to the line-of-sight component. However, in today's communication systems, e.g., personal wireless or cellular networks, due to the limited size and portability of user communications devices, building high antennas is not possible. Besides, reflected signals from buildings and moving objects are unavoidable and very significant. As a result, small-scale fading and the effects of its randomness cannot be excluded in consideration of wireless link.

2.2. System model and wireless propagation in the context of multicast.

Consider a wireless point-to-multipoint downlink system supporting multicast service for a group of N users. For simplicity, without loss of generality, downlink transmission from the BS to users is assumed to consist of non-overlapping time-slots; each slot can accommodate one equal-length packet. Let $s(t)$ be the transmitted signal in the time-slot t, which already comprises the effects of path loss and shadowing; $n_i(t) \sim CN(0, \mathcal{N}_0)$ be additive white Gaussian thermal noise with $\mathcal{N}_0$ noise power at user i. The average SNR which is denoted as $\overline{\gamma}$, represents the average link quality of the channel including the path loss and shadowing and is assumed to be the same for all BS-user links[1]. The received signal $y_i(t)$ at user i, is then given by

$$y_i(t) = h_i(t) * s(t) + n_i(t) \tag{2.1}$$

where $h_i(t)$ is the instantaneous channel gain in the time-slot t on the link from the BS to user i. $h_i(t)$ represents the small-scale fading[2] on wireless link from the BS to user i, with a normalized power of $E\{|h_i(t)|^2\} = 1$. $h_i(t)$'s are further assumed to be independent and identically distributed (i.i.d.) and quasi-static, i.e., any BS-user link fade remains unchanged during a given time slot, but varies independently from one time slot to another.

In the case of perfect channel knowledge at the transmitter, i.e., the BS knows exactly the instantaneous channel gains, $h_i(t)$'s, of all BS-user links, adaptive modulation and coding (AMC) can be applied to achieve the maximum transmission rate. For a frequency-flat fading channel, this transmission rate for user i in the time-slot t in terms of bandwidth efficiency, b/s/Hz can be given as

$$r_i(t) = \log_2[1 + \overline{\gamma}\rho_i(t)], \qquad \rho_i(t) \triangleq |h_i(t)|^2 \tag{2.2}$$

Since wireless environment is broadcast in its nature, the BS can transmit each multicast packet to the whole multicast group using only one transmission by sending at the supportable rate of the user with lowest channel response, i.e.,

$$r_{\mathrm{WU}}(t) = \log_2\left(1 + \overline{\gamma} \min_{i=1,2,\ldots,N}\{\rho_i(t)\}\right). \tag{2.3}$$

This is known as the worst-user (WU) approach. In the case of LOS, the wireless links only suffered from path loss and shadowing which results in small difference among the channel gains, i.e., $\rho_i(t) \approx 1$.Hence the achievable rates of the multicast users are similar and close to the selected $r_{\mathrm{WU}}(t)$. In

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[1] As our main focus in this thesis is to study opportunistic multicast schemes for wireless communications in presence

[2] In this thesis, for short, we use the word fading when referring to small-scale fading.

this scenario, it can be seen that by using WU approach[3], the full multicast gain can be achieved.

However, when taking into account small-scale fading, instantaneous channel gains of various user links at a given time can be largely different. Hence, $\min_{i=1,2,\ldots,N}\{\rho_i(t)\}$ and accordingly, $r_{\mathrm{WU}}(t)$ is likely to be very low when N is large[4], which may lead to inefficient use of available resource (bandwidth) although multicast gain is exploited.

In fact, this difference in instantaneous channel responses among the users promotes multiuser diversity that has been explored in unicast services by sending information to the best-user (BU), i.e., the user with the best instantaneous channel gain. This opportunistic approach can be also used to support multicast services with the transmission rate of

$$r_{\mathrm{BU}}(t) = \log_2(1 + \bar{\gamma} \max_{i=1,2,\ldots,N}\{\rho_i(t)\}), \tag{2.4}$$

In this way, the resource utilization can be maximized in each time slot at the cost of sending each packet N times. Since each packet requires at least N transmissions to cover the whole multicast group, the *effective multicast rate* that each user receives can be expressed as[5]

$$r_{\mathrm{BU_{eff}}}(t) = \frac{1}{N}\log_2(1 + \bar{\gamma} \max_{i=1,2,\ldots,N}\{\rho_i(t)\}). \tag{2.5}$$

As shown in equation (2.5), this effective multicast rate of the BU opportunistic approach is likely to be reduced when N increases[6].

From the previous discussion, it can be seen that if we try to take advantage of multicast gain by using WU approach, the BS needs to send multicast packets only once but the consequence is that the transmission rate must be chosen as the lowest rate of all the users. On the other hand, if we try to make use of multiuser diversity by using BU approach, the BS can

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[3] The WU approach for multicasting has been proposed for IEEE 802.16, e.g., see page 189 of [1].
[4] In our study in Chapter 3, $r_{WU}(t)$ is proved to asymptotically converge to zero as N increases.
[5] It is noted that the effective multicast rate for WU is $r_{WU}(t)$.
[6] In our study in Chapter 3, $r_{BU_{eff}}(t)$ is proved to asymptotically converge to zero as N increases.

maximize its transmission rate at each time slot; however, each packet needs to be sent many times.

Which one of the two approaches (i.e., WU and BU) gives better effective multicast rate? While the detailed throughput analysis to address this question will be given in Section 3.2 (of Chapter 3), here we can make the following observations. First, both equation (2.3) for WU and (2.5) for BU indicate that their achievable effective multicast rates reduce as the multicast group size N increases. This implies that both WU and BU approaches may not be efficient to support multicast services for large group size. Second, the rate function, as shown in (2.3) or (2.5), is logarithmically increasing with $\overline{\gamma}$: Its increasing rate is large at low values of $\overline{\gamma}$, and greatly compressed at sufficiently high $\overline{\gamma}$. Therefore, it can be conjectured that at sufficiently high $\overline{\gamma}$, the variation in the instantaneous user channel gain, $\overline{\gamma}\rho_i(t)$ does not make a large difference in the corresponding rates, and, accordingly, the worst-user (WU) approach can offer better effective multicast rate than the best-user (BU) approach, while the BU can outperform the WU at low $\overline{\gamma}$.

The above observations raise a legitimate question: can we explore both multicast gain and multiuser diversity in order to achieve a better effective multicast rate than both WU and BU approaches? One possible strategy is to select a transmission rate $r_T(t)$ which can support T of N multicast users in each transmission where $1 \leq T \leq N$, in order to enhance the effective multicast rate

$$r_{\mathrm{mul}_{\mathrm{eff}}} = \frac{T}{N} r_T(t). \tag{2.6}$$

Following this general strategy, there have been several proposed schemes that will be reviewed in the next section.

2.3 Approaches for opportunistic multicast over wireless fading channels

In [2], the authors proposed a threshold-T multicast scheduling to maximize multicast performance subject to the stability condition. In this work, the authors considered the multicast scheduling problem at MAC layer by using a queue with a fixed service rate, which represents the selected transmission rate to the multicast users. The channel conditions and receiver of each multicast user are represented by a two-state model. A user in its *ready* state is considered to have sufficiently good channel that can accommodate the transmission rate and hence can receive the packet. Otherwise, its *not-ready* state indicates that the user cannot receive the packet. This two-state model is defined with predetermined transition probabilities and is assumed to be identical for all the users but independent from each other. In each time slot, the BS sends multicast packet if there are at least T users ready to receive it; otherwise, the BS will back off by a random duration, and then resend the packet. The authors then derive the optimum value of T to maximize the effective multicast rate while maintaining the system stable. The stability region is defined as the maximum value of the arrival rate at which the mean queue length is still bounded. The performance of the optimum scheme is compared with that of two reference schemes: threshold-0 (i.e., in which the BS sends multicast packets regardless of the readiness of users) and unicast-based multicast (i.e., the BS transmits each packet separately to each receiver in a round-robin manner). Their numerical results show that since threshold-0 scheme allows transmission without caring about the readiness of multicast users, there are lost packets, which result in very low effective multicast rate, but has achieved the largest stability region. On the other hand, in the unicast-based multicast scheme, each packet needs to transmit N times, which yields the smallest stability region but since there is no packet loss, the scheme offers the highest effective multicast rate in its stability region. With the ability of adjusting T according to the packet arrival rate, the achievable multicast rate of the optimal threshold-T scheme outperforms both threshold-0 and unicast-based multicast. As the two-state model cannot fully describe fading channels, the work in [2] could not give relevant details on the behaviour of the scheme such as the selection of T or at different fading conditions (e.g., SNRs, different fading types).

Another approach is to select a threshold for multicast transmission in each time slot following a predetermined criterion. This approach is different from threshold-T in the sense that the BS always transmits packets at the beginning of each time slot but the transmission rate is adaptive according to some criterion. One of the criteria in the literature is to enhance the effective multicast rate. Specifically, in this criterion, T of N multicast users with the best link quality are selected and the transmission rate is determined as the lowest transmission rate corresponding to the worst-user in these T best users in each time slot. In this way, in each time slot, only T users can reliably receive the packet while the other $(N - T)$ users with insufficient channel gains cannot. Taking into account this loss, in [3], the optimization problem for finding optimal T to achieve the best effective multicast rate as in equation (2.6) is formulated. The throughput performance of the proposed scheme is analyzed over Rayleigh fading channel. Using the effective multicast rate of the WU scheme (r_{WU}) as the reference, the authors considered the throughput gain which is represented by $\frac{r_{\mathrm{mu\,l_{eff}}}}{r_{\mathrm{WU}}}$. Numerical results on three different group sizes of 10, 50 and 100 users at 10dB show that there is a linear increase in the peak of this throughput gain with respect to the multicast group size. The authors further formalized this observation by a theorem stating that over Rayleigh fading channels, this throughput gain increases linearly with N. The authors also investigated the proposed scheme over three different SNR levels and it is shown that as the SNR increase, the improvement of the proposed scheme over WU reduces and the optimal selection T tends to increase. Following this trend it can be expected that at a very high SNR, the proposed scheme becomes WU; however, since its gain in throughput as compared to WU is reduced from above, the achievable throughput of the proposed scheme is always better than WU. Also in this work, the author suggests that reliability of the scheme could be improved by using erasure-correction code, and the optimal code rate is conjectured as $\frac{T^*}{N}$ where T^* is the optimal value of T. However, the authors did not propose the structure of the scheme using erasure-correction code, nor analyze its throughput performance.

Following the same approach in [3], various schemes have been proposed to make sure all N users can reliably receive the multicast packet by retransmission. In the scheme proposed in [4] the BS also transmits to the group of T best users using the lowest supportable rate of this group, and keeps transmitting each multicast packet in this manner until the entire group receives the packet. Let K be the number of required transmissions, the effective multicast rate for this scheme can be written as $r_{\text{mu l}_{\text{eff}}} = \frac{E[r_T(t)]}{E[K]}$. The scheme is then analyzed in a Rayleigh fading environment at different SNRs. The study on the changes of user selection according to SNR levels from -30dB to 40dB illustrates that the optimal $\frac{T}{N}$ increases from around 0.5 to 1 as the SNR increases and hence the scheme tends to converge to WU at high SNR. In this work the authors have also compared the throughput performance of the proposed scheme and the BU and WU approaches at different SNR levels to show the superiority of the proposed scheme. Numerical results illustrate that the proposed scheme can provide significant improvement at low SNR region but as the SNR increases, this improvement reduces and at 25dB, its performance converges to that of WU. It can be seen that the scheme proposed in [4] is very simple in implementation as the BS needs to maintain only one queue for the multicast service. However, since the BS always transmits to T users with best channel conditions regardless of their receipt of the packet in the previous time slots, there may be some users that receive the same packet many times. This duplication is a waste of resources (bandwidth and power) and makes the scheme inefficient. To avoid this, in [5] and [6], a different approach to retransmission has been taken to provide reliability while keeping multicast efficiency at the same time. In these works, the BS maintains a queuing system consisting of $\binom{N}{T}$ queues, each for a combination of T users. These queues are further divided into $\frac{N}{T}$ sets such that the combinations of users served by queues in each set are mutually exclusive and collectively exhaustive. An example of such a queuing system for the case of $N = 4, T = 2$ is illustrated in Fig. 2-1. In this example, each queue is a combination of 2 out of 4 users and each set, e.g. $\{(1,2);(3,4)\}$, is a mutually exclusive and collectively exhaustive combination of all 4 users. For each multicast packet, the BS replicates the packet into $\left\lceil\frac{N}{T}\right\rceil$ copies for each of $\left\lceil\frac{N}{T}\right\rceil$ queues in one set. In each time slot, the BS transmits the packet to T users with best channel gain by using the transmission rate of the user with the worst channel response among these T users and a packet in the appropriate queue of this group of users is chosen to be served. It can be seen that this queuing system ensures that each user receives each packet only once while the BS can still maintain a transmission rate for the T best users. The authors then propose an intuitive selection of $T = \frac{N}{2}$ to cover half of the multicast group in each transmission (median user scheme) and study the multicast throughput of all the users $Nr_{\mathrm{mul}_{\mathrm{eff}}}(t)$ of this scheme when the multicast group size increases in comparison to the worst-user ($T = N$) and best-user ($T = 1$) schemes. Their study shows that as N increases, the multicast throughput of median scheme increases linearly with N while it slowly increases as $\log(\log N)$ for BU scheme and is saturated at a certain level for the case of WU. Although their simulation results on Rayleigh fading channel at 0dB confirms that such median scheme can improve multicast rate as compared to WU and BU, these works have not solved the question of the optimal selection T for the best multicast throughput. Besides, since the BS has to maintain $\binom{N}{T}$ queues, it introduces complexity into the design; in addition, when N varies this queuing structure has to change accordingly. Moreover, for this particular queuing system to have best efficiency without duplication, it requires $\left(\frac{N}{T}\right)$ to be an integer which may not be satisfied in general.

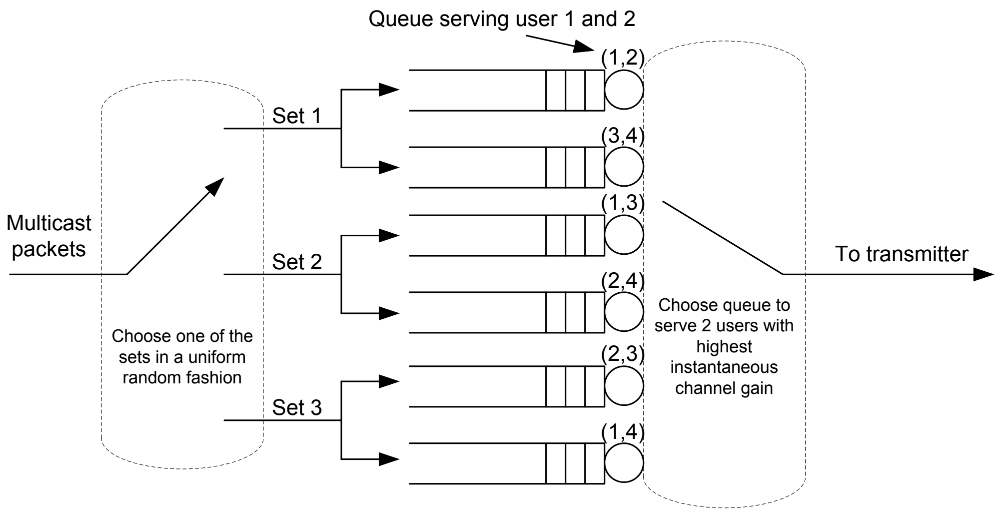


Fig. 2-1: A queuing model for a system with N=4, T=2.

Another criterion for choosing the multicast rate is based on the fairness among the users as in [7]-[9]. In these works proportional fair scheduling schemes have been proposed for multicast to not only take advantage of temporal variations of users' channel responses but also to guarantee fairness among the users by looking at the long-term average rate of users rather than the instantaneous rate in selection of transmission rate.

Let us denote the transmission rate supported by user i at time slot k as $r_i(t_k)$. Assume that channel responses of users are available at BS before each transmission. At time slot k, the BS schedules the packet to user j with

$$j = \arg\max_i \left(\frac{r_i(t_k)}{\overline{r}_i(t_k)}\right), \tag{2.7}$$

where $\overline{r}_i(t_k)$ is the average data rate of user i observed over a predefined sliding window of length T time slots until time slot k. At time slot $(k+1)$, $\overline{r}_i(t_k)$ is updated through[7]

[7] In [9], many ways of updating $\overline{r}_i(t_k)$ are proposed in which the author integrates not only average throughput but also delay and in this case fairness can be understood as the combination of throughput and delay.

$$\bar{r}_i(t_{k+1}) = \begin{cases} \left(1-\frac{1}{T}\right)\bar{r}_i(t_k) + \frac{1}{T} r_i(t_k) & \text{if user } i \text{ is served in time slot } k \\ \left(1-\frac{1}{T}\right)\bar{r}_i(t_k) & \text{if user } i \text{ is not served in time slot } k \end{cases} \quad (2.8)$$

From (2.7), it can be seen that users are not ranked according to their instantaneous channel gains but rather to their instantaneous channel gains relative to the average of their own channel conditions. Therefore, users with high instantaneous channel gain do not necessarily have advantage over the others and in this way fairness is preserved while channel variations of users are also considered. Since the transmission rate is chosen based on fairness, it is expected that the average rate of proportional fairness cannot compete with the schemes using the criteria of maximizing multicast rate at the optimal selection T^*; however, in [7], the authors have shown that proportional fairness can provide improvement over the median scheme which can support half of the users in each transmission. It can be seen that the selection of transmission rate as in (2.7) provides an indication of fairness; however, it is difficult to say that the scheme can guarantee fairness for all the users, as the user with lower priority in (2.7) may have higher supportable rate than the chosen multicast rate and can still receive the packet.

2.4 Proposed studies

Inspired by the works that have been done in the area, first we reckon that the approach of transmitting to T best users using a transmission rate threshold in each time slot is a good approach for getting better multicast throughput by exploring both the multicast gain and multiuser diversity. We also notice that the idea of using erasure-correction code for better reliability is very promising and can be improved. It can be seen that channel responses do not only vary among the users, but also change from one time slot to another. Hence, if we apply erasure-correction coding a block of transmitted packets, an erased packet in one time slot can be successfully recovered as long as the number of erased packets in a block does not exceed the correction capacity of the erasure-correction code. In this way, time diversity can be exploited to improve multicast performance. Although erasure-correction coding introduces overhead, it can help to increase the throughput performance due to its erasure correction capability. If we increase the transmission threshold then there is more chance that users experience an erased packet, and as a consequence, erasure code rate must be reduced accordingly to increase its erasure-correction capability and vice versa. Hence, the selection of transmission threshold and code rate must be jointly considered for optimal multicast throughput. Another important remark is that all of the reviewed papers assume perfect knowledge of all multicast user instantaneous channel responses at the transmitter. Although channel feedback and estimation may be quite advanced nowadays, this assumption might still be very hard to achieve and multicast performance can be greatly degraded if only some of the channel estimations are incorrect. Hence, if we can remove the assumption of full channel knowledge and assume only partial knowledge of channel responses (for example, distribution type and average SNR of channel response) we can significantly reduce the resources for feedback and signalling. However, how to do so, whether it can provide some improvement as compared to BU, WU and how much is the gain/loss in the two cases are the open questions to be addressed in this thesis.

Chapter 3:

Opportunistic Multicast Scheduling using Erasure-Correction Coding over Frequency-flat Fading Channels[8]

In this chapter, erasure-correction coding based opportunistic multicast scheduling (ECOM) schemes for single-carrier wireless systems in a block frequency-flat fading environment are presented. At first, the operation of ECOM using Reed-Solomon erasure-correcting codes is described. Then, the analytical framework for multicast throughput of BU, WU and ECOM in both cases of full channel knowledge and partial channel knowledge is developed. The chapter concludes with providing numerical results and discussions on the trade-off between multicast gain and multiuser diversity, the throughput comparison and the significance of full and partial channel knowledge.

3.1 Erasure-coding based opportunistic multicast (ECOM)

In this chapter, without loss of generality, downlink transmission from the BS to users is assumed to consist of non-overlapping time-slots, each can accommodate one equal-length packet. Further, fades over BS-user links in each time-slot are assumed to be block frequency-flat fading channels, i.e. the channel impulse response can be expressed as $h(t) = h_t \delta(t)$, where h_t is assumed to be unchanged during a given time-slot and varies independently from one time-slot to another. An example of block frequency-flat fading channel is illustrated in Fig. 3-1 with $\rho(t) \triangleq |h_t|^2$.

[8] This chapter is partially presented in [16].

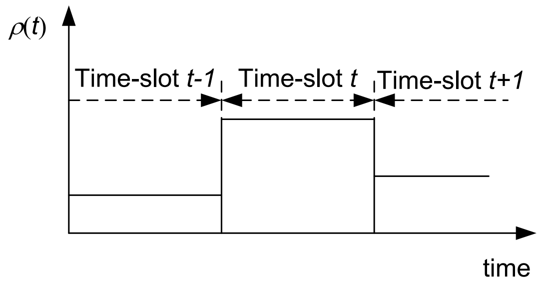


Fig. 3-1 Illustrative example of block fading channel.

Taking into account both multi-user diversity and multicast gain, the proposed erasure coding based opportunistic multicast (ECOM) schemes try to maximize the achievable multicast throughput. ECOM schemes make use of an erasure-correcting code, e.g., Reed-Solomon (RS) code, to encode the transmitted packets[9] as shown in Fig. 3-2.

Each *information* packet is partitioned into E symbols, each symbol has q bits. Organizing the k *information* equal-length packets (to be sent) in a row-wise manner, they are encoded in a column-wise manner by using a Reed-Solomon code $\mathrm{RS}(n,k)$ defined over the Galois field $\mathrm{GF}(2^q)$, as follows. Each RS codeword contains k information q-bit symbols and $(n-k)$ *parity* q-bit symbols. The k information symbols of the RS codeword e, $e=1,2,\dots,E$, are the e-th symbols of the k *information* packets, and are used to generate the $(n-k)$ *parity* symbols of the RS codeword e. Each of these $(n-k)$ parity symbols forms the e-th symbol of one of $(n-k)$ parity packets. In other words, for k *information* packets, the proposed ECOM scheme send n packets, in which $(n-k)$ additional packets contain parity symbols as overhead.

[9] A similar packet-level coding structure used for a different purpose has been proposed for DVB-S2, e.g., see [11].

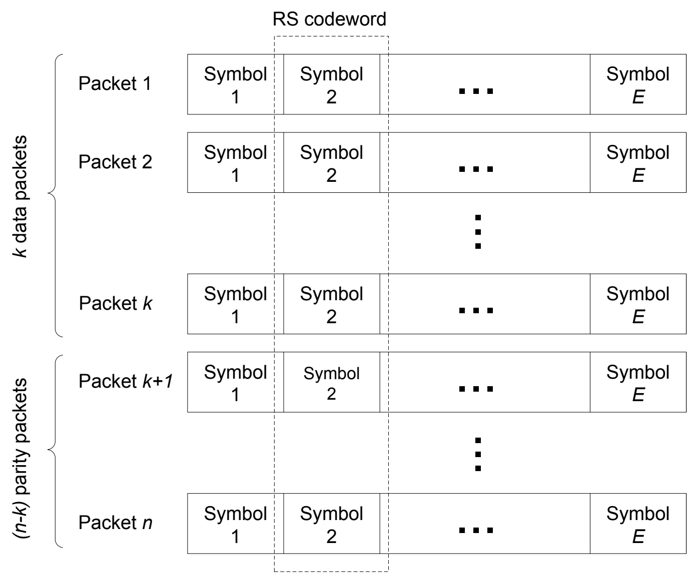


Fig. 3-2: Packet-level coding structure using a RS(n,k) code.

The transmission rate (in b/s/Hz) to send n packets is selected as

$$r_{\text{ECOM}} = \log_2(1 + \overline{\gamma}\rho_*) \tag{3.1}$$

where ρ_* is the predetermined channel gain threshold. The choice of ρ_* for certain criterion will be discussed later. Taking into account the overhead of the *parity* packets, the *effective* transmission rate in the proposed ECOM scheme is $\left(\frac{k}{n}\right) r_{\text{ECOM}}$.

It can be seen that in the time-slot t, users with $\rho_i(t) \geq \rho_*$ can correctly receive the packet. For other users with $\rho_i(t) < \rho_*$, the packet is likely in error due to insufficient instantaneous SNR. In this case, the erroneous packets can be assumed to be erased and this event can be denoted at the receiver.

It is well known that a RS(n, k) code can correct up to $(n - k)$ erased symbols, e.g., [10]. Therefore, in the proposed ECOM scheme, user i can correctly decode all k packets when the number of events that $\rho_i(t) < \rho_*$, is not exceeding $(n - k)$ within the n time-slots. It can be seen that the proposed ECOM schemes exploit multicasting gain by sending only one copy to all N users while making use of both multi-user diversity (by selecting ρ_*)

and time diversity (with erasure-correcting codes). Although RS code is used as an illustrative example in this paper, other erasure-correcting codes can be applied in the proposed ECOM schemes.

Regarding the choice of ρ_*, interesting questions are raised: whether possessing exact channel gain knowledge of all users can help to increase multicast throughput? And if it can, in which case channel gain knowledge is most pronounced and in which case the gain provided by this side information is negligible? Motivated by these questions, the selection of ρ_* is considered for 2 following scenarios.

ECOM with full channel knowledge (ECOMF): Inspired by WU and BU as extreme cases of multicast gain and multi-user diversity and threshold-T scheme, if the base-station transmitter has full knowledge of the instantaneous channel gains, $\rho_i(t)$'s, of all users in every timeslot, the BS can sort users in the descending order of their instantaneous channel gains, i.e., $\rho_1 \geq \rho_2 \geq \cdots \geq \rho_{N'} \geq \cdots \geq \rho_N$, and selects a subgroup of N' users ($N' \leq N$) that have the highest channel gains and ρ_* as $\rho_* = \rho_{N'}$.

Interestingly, WU and BU can be considered as two specific cases of ECOMF, i.e., WU is ECOMF with $N' = N$ (all users), $k = n$ (no coding) while BU is ECOMF with $N' = 1$ (best user), $k = 1$ (repetition code).

The choice of the subgroup size N', and code rate k/n is crucial in optimizing the required transmission rate and will be discussed in the next sections.

ECOM with partial channel knowledge (ECOMP): As the full knowledge of the instantaneous channel gains, $\rho_i(t)$, of all users at any time-slot t comes at the costs of required fast and accurate channel measurements and signalling between the BS and users, it is interesting to consider the case without perfect channel information at transmitter. In particular, we investigate an approach called ECOMP to select $\rho_* = \rho_{\text{th}}$ that maximizes the average multicast rate based on the partial knowledge of the channel stochastic properties of the BS-user links. The throughput analysis of ECOMP is to be discussed in the next sections.

3.2 Throughput analysis

We consider a quasi-static i.i.d. fading environment so that the channel gain $\rho_i(t)$ can be represented by a random variable ρ with the probability density function (pdf) $f_\rho(\rho)$ and the instantaneous SNR is denoted by the random variable $\gamma \triangleq \bar{\gamma}\rho$.

3.2.1 Worst-user (WU) scheme: In this scheme, only one copy is sent to all N users using the transmission rate corresponding to the channel gain of the worst user. Under the assumption of a quasi-static i.i.d. fading environment, the cumulative distribution (cdf) of the channel gain of the worst user is given by

$$F_{\rho_{\mathrm{WU}}}(\rho) = 1 - \left(1 - F_\rho(\rho)\right)^N \tag{3.2}$$

where $F_\rho(\rho)$ is the cdf of ρ.

As only one copy is sent to all N users, effectively, the average achievable *multicast* rate of the WU scheme is N times the average transmission rate, i.e.,

$$\bar{r}_{\mathrm{WU}} = N\int_0^\infty \log_2(1+\bar{\gamma}\rho) f_{\rho_{\mathrm{WU}}}(\rho)d\rho, \tag{3.3}$$

where the pdf $f_{\rho_{\mathrm{WU}}}(\rho) = N\left(1 - F_\rho(\rho)\right)^{N-1} f_\rho(\rho)$

According to (3.3), the effective average throughput of WU for each user is given by

$$\bar{r}_{\mathrm{WU_{eff}}} = \frac{\bar{r}_{WU}}{N} = \int_0^\infty \log_2(1+\bar{\gamma}\rho) f_{\rho_{WU}}(\rho)d\rho, \tag{3.4}$$

For Rayleigh fading channel, $f_{\rho_{\mathrm{WU}}}(\rho) = Ne^{-N\rho}$ and therefore, according to Jensen's inequality

$$\begin{aligned}\bar{r}_{\mathrm{WU_{eff}}} &= E_{\rho_{\mathrm{WU}}}[\log_2(1+\bar{\gamma}\rho)] \\ &\leq \log_2\left(1+\bar{\gamma}E_{\rho_{\mathrm{WU}}}[\rho]\right) = \log_2\left(1+\frac{\bar{\gamma}}{N}\right) \leq \frac{\bar{\gamma}}{N}.\end{aligned} \tag{3.5}$$

Since $\frac{\overline{\gamma}}{N} \xrightarrow[N\to\infty]{} 0$, the effective throughput of WU approaches zero as the multicast group size N grows large; therefore, for large multicast group, exploiting only multicast gain is not an efficient way to do multicast.

3.2.2 Best-user (BU) scheme: In this scheme, each packet is sent N times at the rate of the user with best channel condition. Under the assumption of a quasi-static i.i.d. fading environment, the cdf of the instantaneous SNR of the best user is given by

$$F_{\rho_{\mathrm{BU}}}(\rho) = \prod_{i=1}^{N} F_{\rho_i}(\rho) = \left(F_{\rho}(\rho)\right)^{N}. \tag{3.6}$$

The expected transmission rate for the best user in any given time-slot is given by

$$E_{\rho_{\mathrm{BU}}}[r_{\mathrm{BU}}] = \int_{0}^{\infty} \log_2(1+\overline{\gamma}\rho) f_{\rho_{\mathrm{BU}}}(\rho) d\rho, \tag{3.7}$$

where the pdf $f_{\rho_{\mathrm{BU}}}(\rho) = N\left(F_{\rho}(\rho)\right)^{N-1} f_{\rho}(\rho)$.

As one copy is sent to each user, effectively, the average achievable *multicast* rate of BU scheme over n time-slots can be expressed as

$$\begin{aligned} \overline{r}_{\mathrm{BU}} &= \frac{N}{n}\sum_{x=1}^{n}\binom{n}{x} E_{\rho_{\mathrm{BU}}}[r_{\mathrm{BU}}] x p^{x}(1-p)^{n-x} \\ &= \frac{N}{n} E_{\rho_{\mathrm{BU}}}[r_{\mathrm{BU}}] np. \end{aligned} \tag{3.8}$$

Since the channel distributions of users are i.i.d., $p = \frac{1}{N}$ is the probability that a given user can receive the packet, (3.8) then becomes

$$\overline{r}_{\mathrm{BU}} = N E_{\rho_{\mathrm{BU}}}[r_{\mathrm{BU}}]\frac{1}{N} = E_{\rho_{\mathrm{BU}}}[r_{\mathrm{BU}}]. \tag{3.9}$$

According to (3.9), the effective average throughput of BU for each user is given by

$$\overline{r}_{\mathrm{BU_{eff}}} = \frac{1}{N} E_{\rho_{\mathrm{BU}}} [r_{\mathrm{BU}}]. \tag{3.10}$$

It is noted that since $p = \frac{1}{N}$, the probability that a given user can receive the packet after N consecutive transmissions according to binary probability law is not 1. Hence, further implementation is needed for BU to achieve (3.10). One of such implementations is illustrated in [6] with a separated queue for each user.

For Rayleigh fading channel, $f_{\rho_{\mathrm{BU}}}(\rho) = N(1 - e^{-\rho})^{N-1} e^{-\rho}$ and therefore, according to Jensen's inequality,

$$\begin{aligned} \overline{r}_{\mathrm{BU_{eff}}} &= \frac{1}{N} E_{\rho_{\mathrm{BU}}} [\log_2(1 + \overline{\gamma}\rho)] \\ &\leq \frac{1}{N} \log_2\left(1 + \overline{\gamma} E_{\rho_{\mathrm{BU}}} [\rho]\right) \\ &= \frac{1}{N} \log_2\left(1 + \overline{\gamma} \sum_{i=1}^{N} \frac{1}{i}\right) \\ &\leq \frac{1}{N} \log_2\left(1 + \overline{\gamma} + \overline{\gamma}\frac{N-1}{2}\right). \end{aligned} \tag{3.11}$$

Using L'Hospital rule for (3.11) at the limit $N \to \infty$, we have

$$\lim_{N\to\infty} \frac{1}{N} \log_2\left(1 + \overline{\gamma} + \overline{\gamma}\frac{N-1}{2}\right) = \lim_{N\to\infty} \frac{\overline{\gamma}/2}{1 + \overline{\gamma} + \overline{\gamma}(N-1)/2} = 0. \tag{3.12}$$

Equations (3.11)-(3.12) prove that the effective throughput of BU approaches zero as the multicast group size N grows large; therefore, for large multicast group, exploiting only multiuser diversity is also not an efficient way for multicasting.

3.2.3 Proposed ECOM schemes: In this scheme, a user can correctly decode its information if it can receive k or more non-erased packets within n transmitted packets. Under the assumption of a quasi-static i.i.d. fading environment, the probability p that channel gain of a certain user is greater than channel gain threshold ρ_* is the same for all users i in all time-slots, and the probability that each user can receive at least k non-erased packets can be expressed as

$$\Pr\{x \geq k\} = \sum_{x=k}^{n} \binom{n}{x} p^x (1-p)^{n-x}. \tag{3.13}$$

3.2.3.1 ECOMF: As discussed the ECOMF selects a subgroup of N' users ($N' \leq N$) that have the highest channel gains and ρ_* as $\rho_{N'}$ the lowest instantaneous channel gain of the N'-th user. Under the assumption of a quasi-static i.i.d. fading environment, according to order statistics, the cdf of ρ is given by

$$F_{\rho_{N'}}(\rho) = \sum_{i=N-N'+1}^{N} \binom{N}{i} F_\rho(\rho)^i \left(1 - F_\rho(\rho)\right)^{N-i}, \tag{3.14}$$

and the corresponding pdf is

$$f_{\rho_{N'}}(\rho) = \frac{N}{(N'-1)!\,(N-N')!} F_\rho(\rho)^{N-N'} \left(1 - F_\rho(\rho)\right)^{N'-1} f_\rho(\rho). \tag{3.15}$$

It is obvious that, in a given time-slot, the channel gain of a certain user is greater than channel gain threshold $\rho_{N'}$ if this user belongs to the selected sub-group of N' users. Since the distributions of the users are i.i.d., the probability that a user is in this selected sub-group is $\frac{N'}{N}$. In other words, the probability p_{ECOMF} that the channel gain of a certain user is greater than channel gain threshold $\rho_{N'}$

$$p_{\text{ECOMF}} = \frac{N'}{N}. \tag{3.16}$$

As a result, the average achievable *multicast* rate of the ECOMF scheme with $\text{RS}(n,k)$ is given by

$$\begin{aligned} \bar{r}_{\text{ECOMF}} &= \frac{k}{n} N \left(\int_0^\infty \log_2(1+\bar{\gamma}\rho)\, f_{\rho_{N'}}(\rho)\, d\rho \right) \Pr\{x \geq k\} \\ &= \frac{k}{n} N \left(\int_0^\infty \log_2(1+\bar{\gamma}\rho)\, f_{\rho_{N'}}(\rho)\, d\rho \right) \sum_{x=k}^{n} \binom{n}{x} p_{\text{ECOMF}}^x \,(1 - p_{\text{ECOMF}})^{n-x}. \end{aligned} \tag{3.17}$$

When $N' = N$, $k = n$, (3.17) becomes

$$\begin{aligned}\bar{r}_{\mathrm{ECOMF}} &= \frac{n}{n} N\left(\int_0^\infty \log_2(1+\bar{\gamma}\rho)\, f_{\rho_N}(\rho) d\rho\right)\binom{n}{n}\left(\frac{N}{N}\right)^n \\ &= N\int_0^\infty \log_2(1+\bar{\gamma}\rho)\, N\left(1-F_\rho(\rho)\right)^{N-1} f_\rho(\rho) d\rho.\end{aligned} \tag{3.18}$$

and ECOMF becomes WU.

When $N' = 1$ and $k = 1$, (3.17) becomes

$$\begin{aligned}\bar{r}_{\mathrm{ECOMF}} &= \frac{1}{n} N\left(\int_0^\infty \log_2(1+\bar{\gamma}\rho)\, f_{\rho_1}(\rho) d\rho\right)\sum_{i=1}^{n}\binom{n}{i}\left(\frac{1}{N}\right)^i\left(1-\frac{1}{N}\right)^{n-i} \\ &= \frac{1}{n} N\left(\int_0^\infty \log_2(1+\bar{\gamma}\rho)\, N\left(F_\rho(\rho)\right)^{N-1} d\rho\right)\left(1-\left(1-\frac{1}{N}\right)^n\right) \\ &\xrightarrow[N\to\infty]{} \frac{1}{n} N\left(\int_0^\infty \log_2(1+\bar{\gamma}\rho)\, N\left(F_\rho(\rho)\right)^{N-1} d\rho\right)\left(1-\left(1-\frac{n}{N}\right)\right) \\ &= \int_0^\infty \log_2(1+\bar{\gamma}\rho)\, N\left(F_\rho(\rho)\right)^{N-1} d\rho.\end{aligned} \tag{3.19}$$

As can be shown in (3.19), when the number of users is very large, ECOMF can approach BU.

For a given channel fading type denoted by $f_\rho(\rho)$, the average achievable multicast rate of ECOMF $\bar{r}_{ECOMF}$ can be optimized by selecting N' and k/n.

3.2.3.2 ECOMP: In this approach, for a selected channel gain threshold ρ_{th}, the probability p_{ECOMP} that channel gain of a certain user is greater than channel gain threshold ρ_* is

$$p_{\mathrm{ECOMP}} = Pr\{\rho > \rho_{\mathrm{th}}\} = \int_{\rho_{\mathrm{th}}}^{\infty} f_\rho(\rho) d\rho = 1 - F_\rho(\rho_{\mathrm{th}}). \tag{3.20}$$

For example, $p_{\mathrm{ECOMP}} = e^{-\rho_{\mathrm{th}}}$ for a Rayleigh fading channel. In average, there are only $NPr\{x \geq k\}$ users that can successfully receive the multicast packets at an effective transmission rate of $\left(\frac{k}{n}\right) r_{\mathrm{ECOM}}$. Therefore, effectively, the average achievable *multicast* rate of the ECOM scheme with RS(n, k) code is given by

$$\begin{aligned}\bar{r}_{\text{ECOMP}} &= \frac{k}{n} N r_{\text{ECOMP}}\ \Pr\{x \geq k\} \\ &= \frac{k}{n} N \log_2(1+\bar{\gamma}\rho_{\text{th}}) \sum_{x=k}^{n} \binom{n}{x} p_{\text{ECOMP}}^{x}\,(1-p_{\text{ECOMP}})^{n-x}.\end{aligned} \tag{3.21}$$

For a given channel fading type denoted by $f_\rho(\rho)$, ρ_* and k/n can be selected to maximize the above average achievable *multicast* rate of the ECOMP scheme.

It is straight forward to see that $\frac{\bar{r}_{\text{ECOMP}}}{N}$ does not depend on the multicast group size N; there always exist k and ρ_{th} so that $\bar{r}_{\text{ECOMP}}$ is bounded from zero regardless of N. However, the previous statement may be misleading without taking into account the effect of the average SNR. Equation (3.21) shows that at a given SNR, $\bar{r}_{\text{ECOMP}}$ is always bounded from zero but as the average SNR reduces, the multicast rate of ECOMP also reduces. In other words, if the SNR is sufficient high, the achievable multicast rate for each user offered by ECOMP is unchanged and bounded from zero regardless of the multicast group size.

3.2.3.3 Comparison between ECOMF and ECOMP: In this part, an analytical derivation is given to show the performance comparison between ECOMF and ECOMP. It is shown in the previous part that at a given SNR, the performance of ECOMP is always bounded from zero and the gap between ECOMP's performance and zero or how far it is bounded from zero depends only on the average SNR of the BS-user links. The following derivations will show that the performance of ECOMF, although it depends on multicast group size N as shown in equations (3.15) and (3.17), is bounded from zero and more importantly, is lower bounded by ECOMP's performance.

Recall that the rate equation for ECOMF in (3.17) is given by

$$\bar{r}_{\text{ECOMF}} = \frac{k}{n} N . E_{\rho_{N'}}\,[\log_2(1+\bar{\gamma}\rho)] . \Pr\{x \geq k\}, \tag{3.22}$$

where $E_{\rho_{N'}}\,[\log_2(1+\bar{\gamma}\rho)] = \int_0^\infty \log_2(1+\bar{\gamma}\rho)\, f_{\rho_{N'}}(\rho) d\rho$,

and $\Pr\{x \geq k\} = \sum_{x=k}^{n} \binom{n}{x} p_{\text{ECOMF}}^{x}\,(1-p_{\text{ECOMF}})^{n-x}$ with $p_{\text{ECOMF}} = N'/N$.

Using the Jensen inequality, $E_{\rho_{N'}}\,[\log_2(1+\bar{\gamma}\rho)]$ can be approximated as

$$E_{\rho_{N'}}\,[\log_2(1+\bar{\gamma}\rho)] \approx \log_2\left(1+\bar{\gamma}E_{\rho_{N'}}\,[\rho]\right). \tag{3.23}$$

For a Rayleigh fading channel, we have

$$\begin{aligned} E_{\rho_{N'}}\,[\rho] &= \int_0^\infty \Pr(\rho_{N'} > \rho)\, d\rho \\ &= \int_0^\infty \sum_{i=0}^{N-N'} \binom{N}{i} (1-e^{-\rho})^i e^{-\rho(N-i)} d\rho \\ &= \sum_{i=0}^{N-N'} \binom{N}{i} X(N,i) \end{aligned} \tag{3.24}$$

where

$$\begin{aligned} X(N,i) &\triangleq \int_0^\infty (1-e^{-\rho})^i e^{-\rho(N-i)} d\rho \\ &= \sum_{j=0}^{i} (-1)^{i-j} \binom{i}{j} \int_0^\infty e^{-\rho(N-j)} d\rho \\ &= \sum_{j=0}^{i} (-1)^{i-j} \binom{i}{j} \left[\frac{-1}{N-j} e^{-\rho(N-j)}\right]_{\rho=0}^{\rho\to\infty} \\ &= -\sum_{j=0}^{i} \frac{(-1)^{i-j}\binom{i}{j}}{N-j}. \end{aligned} \tag{3.25}$$

It follows that

$$\begin{aligned} X(N,i+1) &= -\sum_{j=0}^{i+1} \frac{(-1)^{1+i-j}\binom{i+1}{j}}{N-j} \\ &= -\frac{(-1)^{1+i}\binom{i+1}{0}}{N} - \sum_{j=1}^{i} \frac{(-1)^{1+i-j}\binom{i+1}{j}}{N-j} - \frac{\binom{i+1}{i+1}}{N-i-1}. \end{aligned} \tag{3.26}$$

Using the relation $\binom{i+1}{j} = \binom{i}{j} + \binom{i}{j-1}$, we can write

$$X(N,i+1) = -\frac{(-1)^{1+i}\binom{i+1}{0}}{N} - \sum_{j=1}^{i}\frac{(-1)^{1+i-j}\binom{i}{j}}{N-j} - \sum_{j=1}^{i}\frac{(-1)^{1+i-j}\binom{i}{j-1}}{N-j} - \frac{\binom{i+1}{i+1}}{N-i-1}$$

$$= \sum_{j=0}^{i}\frac{(-1)^{i-j}\binom{i}{j}}{N-j} - \sum_{j=0}^{i}\frac{(-1)^{i-j}\binom{i}{j}}{N-1-j} = -X(N,i) + X(N-1,i), \quad (3.27)$$

with $X(N,0) = 1/N$. Using the above recursive relation, we obtain

$$X(N,1) = X(N-1,0) - X(N,0) = \frac{1}{N-1} - \frac{1}{N} = \frac{1}{\binom{N}{1}(N-1)},$$

$$X(N,2) = X(N-1,1) - X(N,1) = \frac{1}{\binom{N-1}{1}(N-2)} - \frac{1}{\binom{N}{1}(N-1)} = \frac{1}{\binom{N}{2}(N-2)}.$$

For i=3,...,N it can be verified that $\frac{1}{\binom{N-1}{i-1}(N-i)} - \frac{1}{\binom{N}{i-1}(N-i+1)} = \frac{1}{\binom{N}{i}(N-i)}$, and, hence $X(N,i) = X(N-1,i-1) - X(N,i-1) = \frac{1}{\binom{N}{i}(N-i)}$.

From the above results, for a Rayleigh fading channel, we have

$$E_{\rho_{N'}}[\rho] = \sum_{i=0}^{N-N'}\binom{N}{i}X(N,i) = \sum_{i=0}^{N-N'}\frac{1}{N-i}$$

$$= \sum_{i=N'}^{N}\frac{1}{i} > \int_{N'}^{N}\frac{1}{x}dx = \ln\left(\frac{N}{N'}\right) \triangleq \rho'. \quad (3.28)$$

From (3.22), (3.23), and (3.28), the lower bound of ECOMF multicast rate can be expressed as

$$\bar{r}_{\mathrm{ECOMF}} > \frac{k}{n}N\log_2(1+\bar{\gamma}\rho')\sum_{x=k}^{n}\binom{n}{x}\left(e^{-\rho'}\right)^{x}\left(1-e^{-\rho'}\right)^{n-x} \quad (3.29)$$

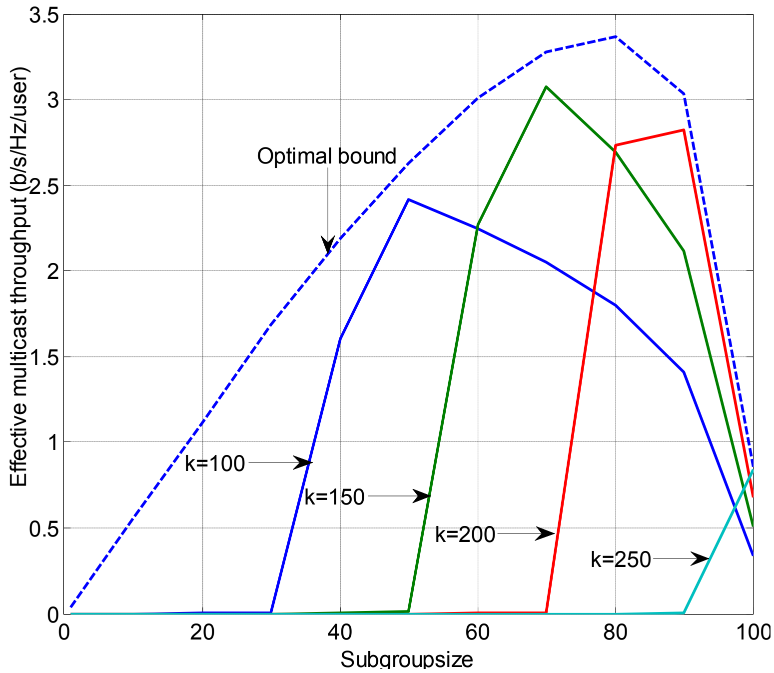
It is interesting to see that the right-hand side of inequality (3.29) is equivalent to the multicast rate of ECOMP as in (3.21) with $\rho' \equiv \rho_{\mathrm{th}}$. In other words, the multicast rate of ECOMF is lower-bounded by that of ECOMP and therefore is also bounded away from zero. The relationship $e^{-\rho'} = \frac{N'}{N}$ further shows that when the multicast group size N is sufficient large, ECOMP can converge to ECOMF by setting $\rho_{\mathrm{th}} = \ln\left(\frac{N}{N'}\right)$.

3.3 Illustrative results

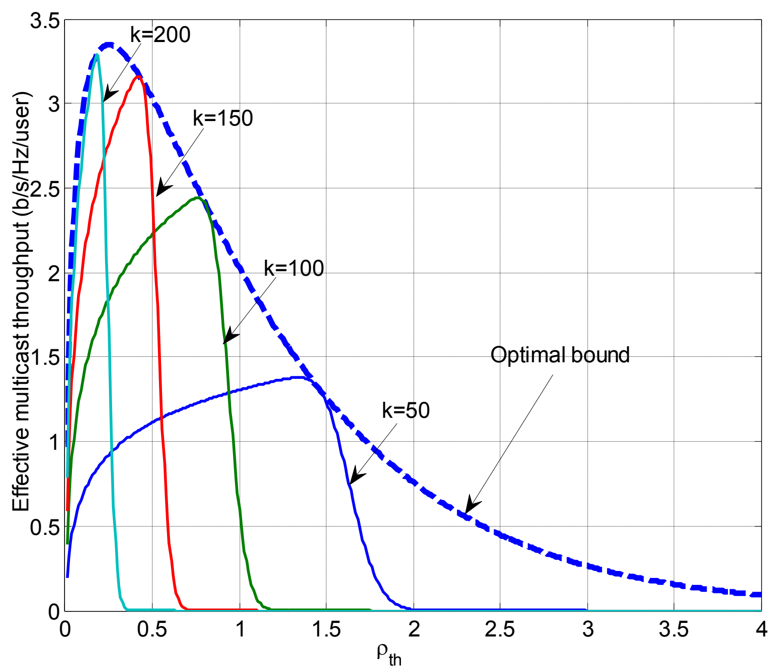
In this section, the numerical results and discussions on the behavior and performance of ECOM schemes are presented. As a figure of merit to evaluate and compare the performance of different schemes, we define the *effective multicast throughput* in units of b/s/Hz/user as the ratio of the average achievable *multicast* rate (as shown in Equations (3.3), (3.9), (3.17) and (3.21)) to the multicast group population, N. Our numerical results are based on (3.3), (3.9), (3.17) and (3.21) and are confirmed by simulation at a very good agreement with difference of less than 1%.

3.3.1 Effect of $\boldsymbol{\rho_*}$ on throughput

We first analyze the effect of the selected ρ_* on the achievable throughput of ECOM schemes over different Rayleigh fading conditions and code rates. In this case, as an illustrative example, a multicast group size of $N = 100$ and RS $(255, k)$ defined over GF(2^8) is considered.



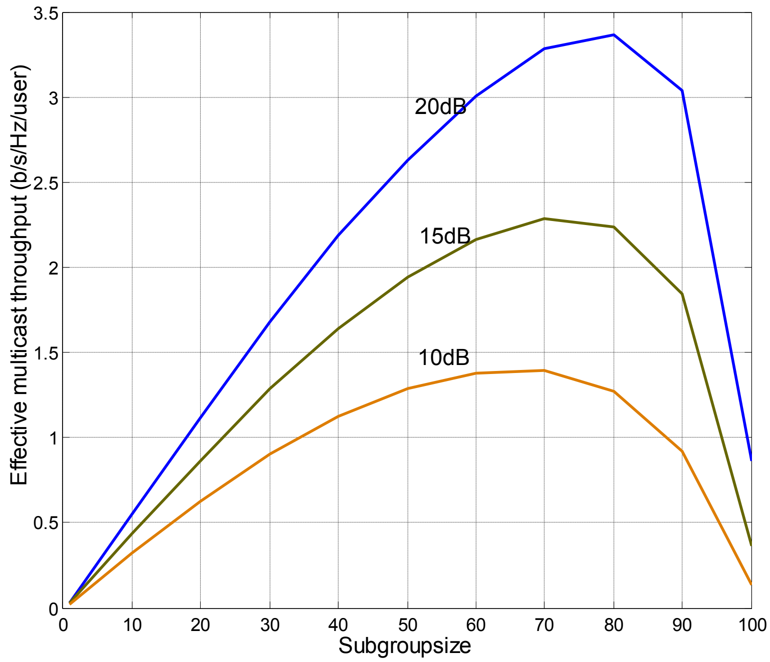
a. ECOMF



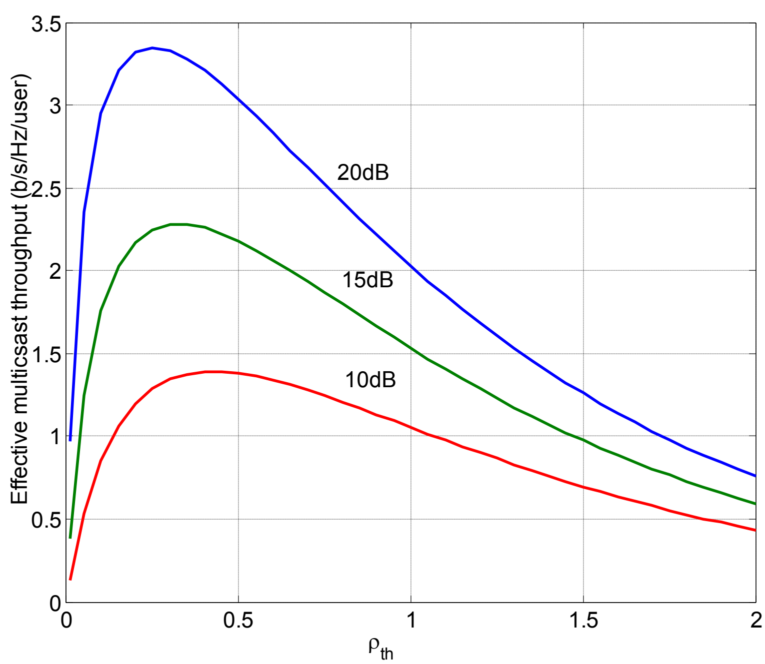
b. ECOMP

Fig. 3-3: Effective multicast throughput versus $\boldsymbol{\rho}_*$ for ECOM schemes using RS (256,k)on a Rayleigh fading channel with an average SNR of 20dB.

Consider a Rayleigh fading environment with average SNR of 20dB, the effect of subgroup size and cut-off threshold selection is depicted in Fig. 3-3. It is shown that for a given value of k, there is an optimum value of ρ_* that maximizes the multicast rate. It is noted that from the derived relationship $\rho_* = \rho_{\text{th}} = \ln\left(\frac{N}{N'}\right)$ in the last section, increasing the subgroup size N' in ECOMF is equivalent to decreasing the cut-off threshold ρ_{th} in ECOMP. Keeping this inverse relationship in mind, the selection of ρ_* has similar effect on both ECOM schemes. It is shown that for a given value of k, there is an optimum ρ_* that maximizes the effective multicast throughput of ECOM. It is observed that these optimum ρ_*'s decrease as k increases, which indicates that multicast gain is preferred over multiuser diversity as more users can receive the packet. It is noted that the normalized throughput of ECOM's schemes drops sharply after this optimal point when ρ_* increases (accordingly with the increase of ρ_{th} and decrease of N'). In this case, a lower k with its corresponding ρ_* is a better choice since it provides better erasure correction capability at the expense of more coding overhead. The optimal bound (dashed line) presents the maximum achievable multicast throughput over all possible values of k for each scheme. The results in Fig. 3-3 show that the optimum throughput increases with ρ_* until reaching its peak and decreases afterwards, which implies that if we try to increase a short term rate in each timeslot, the payoff will be the long-term average throughput as the erasure correction capability has to be high to compensate for packet loss, which makes multicast transmission inefficient after its optimal point. It is shown in Fig. 3-3 that over Rayleigh fading channel at an average SNR of 20dB, the optimal k for best multicast throughput is 190 for ECOMF and 184 for ECOMP with an appropriate optimum threshold value at $N' \approx 80$ for ECOMF and $\rho_{\text{th}} = 0.25$ for ECOMP. The optimal values of ρ_{th} and k for ECOMP for each channel condition can be found through optimization method as illustrated in the Appendix 3A. For ECOMF, the optimization problem is the integer programming problem on three integer variables N, N' and k; the optimum selection of these parameters requires an exhaustive search through all possible values. However, by using the approximation in (3.23) and the result in (3.28), infinite integration in (3.22) can be transferred into finite summation, and hence can significantly reduce the complexity.



a. ECOMF



b. ECOMP

Fig. 3-4: Optimal effective multicast throughput versus $\boldsymbol{\rho}_{*}$ for ECOM schemes in a Rayleigh fading channel at different average SNR's.

We are now extending our observation of the *optimal* throughput versus ρ_* for different SNR's as shown in Fig. 3-4. It is observed that the peak throughput decreases with SNR as expected. As the average SNR decreases, the optimum channel gain threshold ρ_* increases which illustrates that erasures occur more often at lower average SNR and k has to be reduced to increase the erasure-correction capability of $\text{RS}(255, k)$ at the expense of lower coding rate (and hence lower achievable throughput). The results also show that as the average SNR increases the proposed ECOM schemes select a lower transmission rate, as shown in (3.1), implying that the multicast gain becomes more dominant at higher SNR as more users can receive multicast packet in each timeslot.

The above results and discussions confirm that the proposed ECOM schemes can flexibly combine the multicast gain with the multiuser diversity and time diversity via the use of erasure correction coding to achieve optimum achievable throughput in various fading conditions.

3.3.2 Effect of group size on multicast throughput

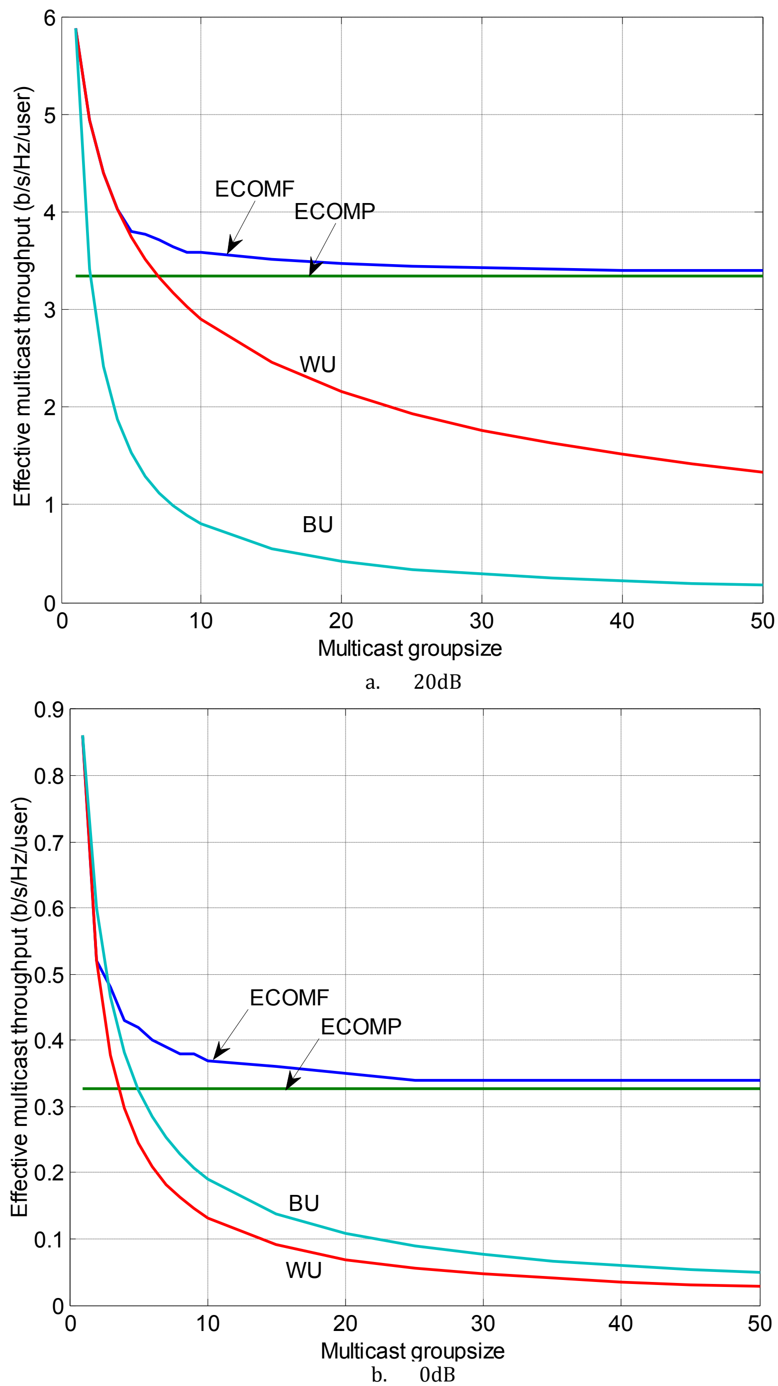


Fig. 3-5: Effective multicast throughputs of different schemes vs. number of users (Rayleigh fading channel, with average SNR of (a.) 20dB and (b.) 0dB).

The effect of multicast group size on multicast throughput on Rayleigh fading channel at 20dB is shown in Fig. 3-5a for WU, BU and ECOM schemes. As defined at the beginning of Section 3.3, the effective multicast throughput in terms of b/s/Hz/user represents the effective rate *each* user of the multicast group can expect. When the number of users increases, the achievable multicast rate of the WU and BU schemes is quickly reduced to zero, as indicated by equations (3.5) and (3.12), while the proposed ECOM schemes achieve a high multicast rate with the effective multicast throughput of ECOMP unchanged with the multicast group size, shown by (3.21). This can be explained by the fact that, in the proposed ECOMP scheme, the probability of successful decoding/reception of the multicast copy does not depend on the multicast group size and is the same for every user in the group in an i.i.d. fading environment while the decision for transmission in WU, BU and ECOMF cases requires the consideration of the whole multicast group for determining transmission rate at each timeslot. Further comparisons on the performance of the four schemes at a comparable low SNR (0dB) are shown in Fig. 3-5b. Fig. 3-5a and b confirm the previous observation that WU and accordingly, multicast gain is more favorable at high SNR while BU or multi-user diversity is superior at low SNR. However, regardless of the SNR, the performance of BU and WU quickly decreases as the multicast group size increases, as shown in (3.5), (3.12) and confirmed by this result, which illustrates that at large group size, neither multicast gain nor multi-user diversity alone can fully exploit multicast capacity and a hybrid treatment is more suitable. A close observation at small multicast group size reveals that while the ECOMF can easily converge to WU which yields the best throughput at high SNR and small multicast group as shown in (3.18), it cannot converge to BU since the approximation in (3.19) is only correct at very large multicast group size. This weakness emerges from the use of erasure code as at least k successful transmissions are required to reconstruct the original packets while in case of BU, no coding is used and therefore no restriction on the number of successful transmission is needed. However, for the case of BU, as discussed in Section 3.2, a complicated queuing system is needed to guarantee loss free transmission to achieve (3.10). Some additional results on the maximum multicast rate of ECOMF at different SNR are provided in Appendix 3B.

Fig. 3-5 also answers the interesting question on the importance of full channel gain knowledge in opportunistic multicast. As shown in this figure, when the multicast group size is small, without channel gain knowledge, ECOMP is the worst performer with significantly worse performance than of ECOMF at group size less than 20 and even worse than WU and BU at smaller group size. However, when the multicast group size increases, this advantage becomes negligible as the effective multicast throughput of WU and BU are much inferior to ECOMP and the performance of ECOMF asymptotically converges to that of ECOMP with minimal difference as shown in Section 3.2.3.3. This can be explained as follows: when N is sufficient large the transmission rate for N'-th user is nearly unchanged from one time slot to another and hence the knowledge of channel gain information loses its significance and fixing multicast transmission rate can eliminate the requirement of channel gain knowledge with marginal difference in performance. In spite of not being the best among the four compared schemes, it is noted that the group size independence characteristic of ECOMP along with its partial knowledge requirement are very beneficial and quite desirable in practice where the multicast group size may be quite large, changing with time and even unpredictable.

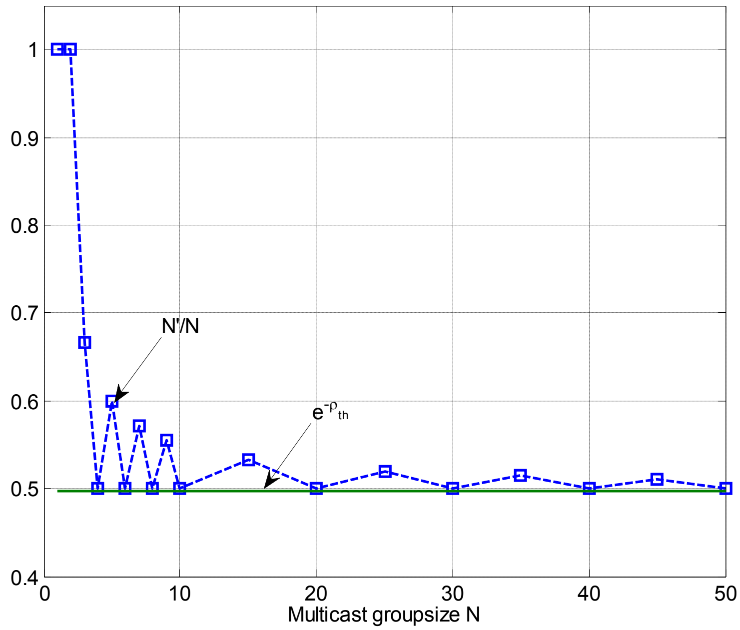


Fig. 3-6: The convergence of $\frac{\boldsymbol{N}'}{\boldsymbol{N}}$ to $\mathbf{e}^{-\boldsymbol{\rho}_{\mathbf{th}}}$ (0dB).

The convergence of ECOMF to ECOMP in Fig. 3-5 confirms the discussion in 3.2.3.3 that the two ECOM variations are indeed having the same root. The relationship $e^{-\rho_{\mathrm{th}}} = \frac{N'}{N}$ which is derived in the last section is confirmed in Fig. 3-6. It is shown in Fig. 3-6 at an average SNR of 0dB that the ratio $\frac{N'}{N}$ converges to $e^{-\rho_{\mathrm{th}}}$ from above. This indicates that the average of selected transmission rate of ECOMF is always larger than that of ECOMP which also illustrates the advantage of possessing channel gain information. The convergence of $\frac{N'}{N}$ to $e^{-\rho_{\mathrm{th}}}$ can be explained as when N is sufficient large, the transmission rate for the user with N' best channel is nearly unchanged in each time slot hence probabilistic characteristic of user's channel gain is more pronounced and letting $p_{\mathrm{ECOMF}} = p_{\mathrm{ECOMP}}$ results in $e^{-\rho_{\mathrm{th}}} = \frac{N'}{N}$.

3.4.3 Performance comparison and the trade-off between multicast gain and multiuser diversity

In this part, the performance of the two proposed ECOM schemes will be evaluated and compared with the WU and BU schemes.

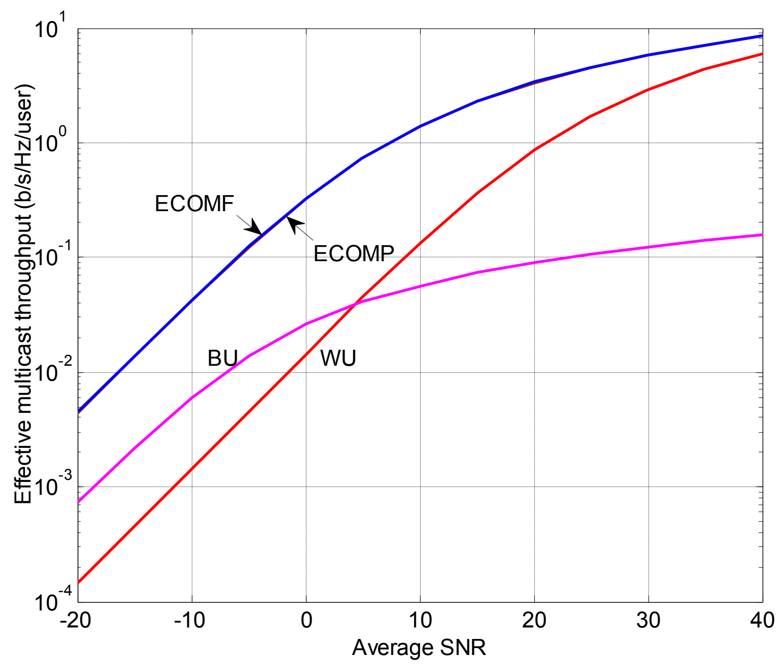


Fig. 3-7: Throughputs given by different schemes versus average SNR's (Rayleigh fading channel, 100 users).

Fig. 3-7 compares the effective multicast throughput of the WU, BU and proposed ECOM schemes in a Rayleigh fading environment for a wide range of average SNR from -20dB to 40dB with a multicast group size of $N = 100$ users. It is observed that the BU has higher throughput than the WU in the low SNR region, but as the average SNR increases above the crossover point of 5dB, the BU scheme has inferior performance with an almost saturating throughput. The results indicate that when the average SNR is sufficiently high, the various BS-user links are sufficiently good, and, as a consequence, it is more likely that all N users in the multicast group are able to successfully receive the transmitted packets. Hence, it is better to explore multicast gain (i.e., transmission only one copy for all N users) to achieve higher normalized throughput in the case of high SNR. However, at a low *average* SNR (e.g., below 5dB in Fig. 3-7), the *instantaneous* SNR's in various BS-user links are likely more different, i.e., some users may be in deep fades while the others have adequate SNR's. This suggests a more pronounced role of multiuser diversity, and hence the BU scheme outperforms the WU scheme as confirmed in Fig. 3-7. It is interesting to note that, by optimizing the subgroup size N' or the threshold value, ρ_{th}, and code rate according to the *average* SNR, as well as fading type (e.g., Rayleigh) of the channel, the proposed ECOM schemes can jointly adjust the use of multicast gain and the multiuser diversity (and time diversity) to obtain a much larger achievable throughput over a wide SNR range, e.g., 18 times better than that of the BU and WU schemes at an average SNR of 5dB. At a very high average SNR, the performance of the WU scheme asymptotically approaches that of the proposed ECOM schemes. This implies that at high average SNR, the proposed ECOM schemes will select a very high coding rate (i.e., k approaches n, or without coding), and essentially explore only the multicast gain. Fig. 3-7, also confirms that for large multicast group size the gain provided by ECOMF is just marginally larger than that provided by ECOMP and hence, it is enough to have only the knowledge of the channel distribution which varies much more slowly than the channel itself and is much easier to estimate than the instantaneous channel. Without the required knowledge of the instantaneous user channel responses $h_i(t)$'s, the proposed ECOMP scheme can significantly reduce the system complexity and resources for channel estimation and feedback signaling. Furthermore, it can cope with fast time-varying fading channels, especially in mobile wireless communications systems.

3.4.4 Effect of different Nakagami-*m* fading environments on ECOM.

Consider a quasi-static i.i.d. Nakagami-m fading environment with pdf

$$f_\rho(\rho) = (m)^m \frac{\rho^{m-1}}{\Gamma(m)} \exp(-m\rho) \quad with \;\; E\{\rho\} = 1, \tag{3.30}$$

and cdf

$$F_\rho(\rho) = \frac{\nu(m, m\rho)}{\Gamma(m)}, \tag{3.31}$$

where $\Gamma(m)$ is the Gamma function, $\Gamma(m) = \int_0^\infty t^{m-1}e^{-t}dt$ and $\nu(m, m\rho)$ is the lower incomplete Gamma function, $\nu(m, x) = \int_0^x t^{m-1}e^{-t}dt$.

In this part, the effect of different Nakagami-m fading environments on ECOM is investigated. Since both ECOMP and ECOMF have the same characteristics as shown in the last parts, for simplicity, only the results of ECOMP are illustrated.

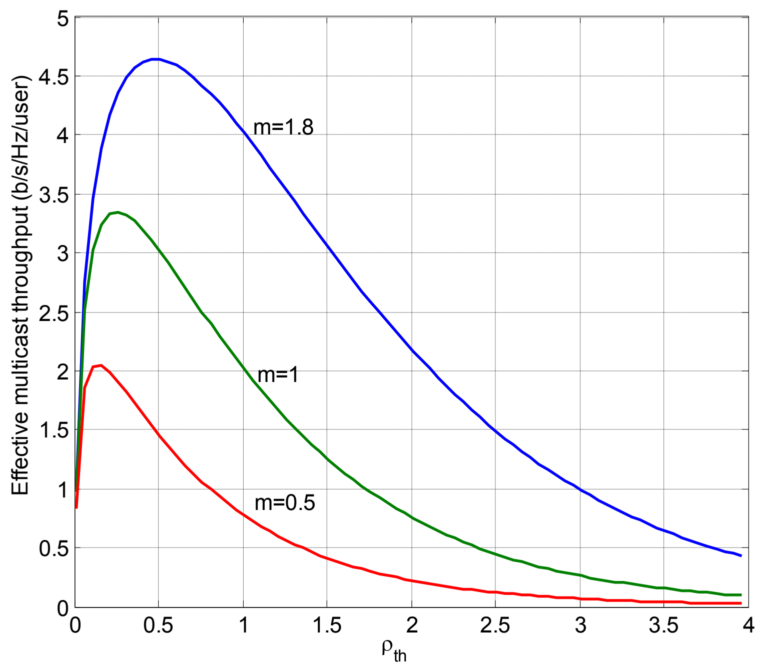


Fig. 3-8: Optimal normalized throughput versus ρ_{th} for ECOMP scheme in different Nakagami-m channels with average SNR of 20dB.

In Fig. 3-8, performance comparison of ECOMP on different fading type conditions is investigated. Consider Nakagami-m channels at the same average SNR of 20dB for different values of m: $m = 1$ for a Rayleigh channel, $m = 1.8$ for a milder situation, equivalent to a Ricean channel, and $m = 0.5$ for a considerably severe fading channel. The results in Fig. 3-8 illustrate that as the fading becomes less severe (i.e., with larger value of m), the optimum achievable throughput is increased as we can expect. Correspondingly, the optimum value of threshold ρ_{th} is increased in a milder fading environment. This can be explained as follows. When m increases, the peak of the

Nakagami-m probability density function occurs at a higher value and its variance decreases, in other words, more users have good channels and therefore are less likely to receive erased packets. Hence the proposed ECOMP scheme can select higher transmission rate, $\overline{r}_{ECOMP}$, and a higher code rate k/n as shown in Equation (3.21) for multicast transmission.

3.5 Chapter summary

In this chapter, an opportunistic multicast scheduler with erasure-correction was proposed aiming at exploiting both multicast gain and multiuser diversity on a block flat fading environment. Considering two cases of possessing full channel gain knowledge and only partial channel gain knowledge of average SNR and fading type, the proposed scheme along with the usage of Reed-Solomon coding scheme were described and explained. An analysis framework for throughput evaluation was developed and the effective multicast throughput for the proposed scheme and BU, WU were derived. Numerical results based on analysis and simulations are provided in various cases to confirm the ability of exploiting both multicast gain and multiuser-diversity of the proposed scheme. Performance comparisons were provided to show the superiority of the proposed scheme at different SNR level and the trade-off between multicast gain and multiuser diversity. Furthermore, throughput evaluation over different multicast group sizes illustrated that the full knowledge of channel responds is only pronounced at small group size; at large multicast group size, this advantage becomes just marginally better than only partial channel gain knowledge of fading type and average SNR of multicast users.

Appendix 3A: Rate Optimization for ECOMP

Recall that the rate equation for ECOMP in (3.21) is given by

$$\bar{r}_{\text{ECOMP}} = \frac{k}{n} N \log_2(1+\bar{\gamma}\rho_{\text{th}}) \sum_{j=k}^{n} \binom{n}{k} p^j (1-p)^{n-j}. \tag{3A.1}$$

Using Normal approximation to Binomial, equation (3.21) can be rewritten as

$$\bar{r}_{\text{ECOMP}} = \frac{k}{n} N \log_2(1+\bar{\gamma}\rho_{\text{th}}) \left(1 - \frac{1}{2}\left(1 + \text{erf}\left(\frac{k-np}{\sqrt{2npq}}\right)\right)\right). \tag{3A.2}$$

The optimization problem for ECOMP can be expressed as the following

$$\min_{k,\rho_{\text{th}}}\{-\bar{r}_{\text{ECOMP}}\} = \min_{k,\rho_{\text{th}}}\left\{-\frac{k}{n} N \log_2(1+\bar{\gamma}\rho_{\text{th}}) \left(1 - \frac{1}{2}\left(1 + \text{erf}\left(\frac{k-np}{\sqrt{2npq}}\right)\right)\right)\right\}. \tag{3A.3}$$

Subject to

$$\rho_{\text{th}} > 0, \tag{3A.4}$$

$$k > 0, \tag{3A.5}$$

$$k < n+1. \tag{3A.6}$$

Larangian function can be defined as

$$L(\rho, k) = -\bar{r}_{\text{ECOMP}} - \lambda_1\rho - \lambda_2 k - \lambda_3(n-k+1). \tag{3A.7}$$

Since all the constrains (3A.4-3A.6) are strictly inequality, they are inactive constrains and according to the Karush-Kuhn-Tucker conditions, $\lambda_1 = \lambda_2 = \lambda_3 = 0$ [17][17]. Therefore, the derivative of the Larangian is given by

$$\nabla_{\rho_{\text{th}}} L(\rho_{\text{th}}, k) = -\nabla_{\rho_{\text{th}}} \bar{r}_{\text{ECOMP}}, \tag{3A.8}$$

$$\nabla_k L(\rho_{\text{th}}, k) = -\nabla_k \bar{r}_{\text{ECOMP}}. \tag{3A.9}$$

Consider Rayleigh fading channel, $p = e^{-\rho_{\text{th}}}$, we then solve $\nabla L(\rho_{\text{th}}, k) = 0$ for the peak rate.

$$\nabla_k L(\rho_{\text{th}}, k) = -\frac{1}{2}\frac{\ln(1+\overline{\gamma}\rho_{\text{th}})}{n\ln(2)}\left[1-\operatorname{erf}\left(\frac{\sqrt{2}}{2}\frac{(k-ne^{-\rho_{\text{th}}})}{\left(ne^{-\rho_{\text{th}}}(1-e^{-\rho_{\text{th}}})\right)^{1/2}}\right)\right]$$
$$+\frac{\sqrt{2}}{2}\frac{k\ln(1+\overline{\gamma}\rho_{\text{th}})}{n\ln(2)\sqrt{\pi}\left(ne^{-\rho_{\text{th}}}(1-e^{-\rho_{\text{th}}})\right)^{1/2}}\exp\left(-\frac{1}{2}\frac{(k-ne^{-\rho_{\text{th}}})^2}{\left(ne^{-\rho_{\text{th}}}(1-e^{-\rho_{\text{th}}})\right)}\right)$$
$$=0, \tag{3A.10}$$

$$\nabla_{\rho_{\text{th}}} L(\rho_{\text{th}}, k) = -\frac{1}{2}\frac{k\overline{\gamma}}{n\ln(2)(1+\overline{\gamma}\rho_{\text{th}})}\left[1-\operatorname{erf}\left(\frac{\sqrt{2}}{2}\frac{(k-ne^{-\rho_{\text{th}}})}{\left(ne^{-\rho_{\text{th}}}(1-e^{-\rho_{\text{th}}})\right)^{1/2}}\right)\right]$$
$$+\frac{\sqrt{2}}{2}\frac{k\ln(1+\overline{\gamma}\rho_{\text{th}})}{n\ln(2)\sqrt{\pi}\left(ne^{-\rho_{\text{th}}}(1-e^{-\rho_{\text{th}}})\right)^{1/2}}\exp\left(-\frac{1}{2}\frac{(k-ne^{-\rho_{\text{th}}})^2}{\left(ne^{-\rho_{\text{th}}}(1-e^{-\rho_{\text{th}}})\right)}\right)\Bigg[ne^{-\rho_{\text{th}}}$$
$$-\frac{1}{2}\frac{(k-ne^{-\rho_{\text{th}}})(-ne^{-\rho_{\text{th}}}(1-ne^{-\rho_{\text{th}}})+ne^{-2\rho_{\text{th}}})}{\left(ne^{-\rho_{\text{th}}}(1-e^{-\rho_{\text{th}}})\right)}\Bigg]=0. \tag{3A.11}$$

Solve $\nabla_k L(\rho_{\text{th}}, k) = 0$ gives us

$$1-\operatorname{erf}\left(\frac{\sqrt{2}}{2}\frac{(k-ne^{-\rho_{\text{th}}})}{\left(ne^{-\rho_{\text{th}}}(1-e^{-\rho_{\text{th}}})\right)^{1/2}}\right)$$
$$=\frac{\mathrm{k}\sqrt{2}}{\sqrt{\pi}\left(ne^{-\rho_{\text{th}}}(1-e^{-\rho_{\text{th}}})\right)^{1/2}}\exp\left(-\frac{1}{2}\frac{(k-ne^{-\rho_{\text{th}}})^2}{\left(ne^{-\rho_{\text{th}}}(1-e^{-\rho_{\text{th}}})\right)}\right). \tag{3A.12}$$

Plug this relationship into $\nabla_{\rho_{\text{th}}} L(\rho_{\text{th}}, k) = 0$ gives us

$$k = \frac{(1+\overline{\gamma}\rho_{\text{th}})\ln(1+\overline{\gamma}\rho_{\text{th}})ne^{-\rho_{\text{th}}}}{2\overline{\gamma}(1-e^{-\rho_{\text{th}}})+(1+\overline{\gamma}\rho_{\text{th}})\ln(1+\overline{\gamma}\rho_{\text{th}})(-1+2e^{-\rho_{\text{th}}})}. \tag{3A.13}$$

The above equation gives the relationship between k and ρ_{th} at the peak rate of $\overline{r}_{\text{ECOMP}}$. Plugging (3A.13) back to (3A.10), subject to (3A.4)-(3A.6), the optimal pairs of k and ρ_{th} can be found numerically, since in (3.21) the code rate is integer number, the nearest integer of k is the result code rate[10]. Another way of finding this optimal pair of k and ρ_{th} is using the relationship in (3A.13), do the search on ρ_{th} to find the peak multicast rate and use the constraints on (3A.4)-(3A.6) to limit the search.

[10] Results computed using this approximation approach are found to be in a very good agreement with those in Fig. 3.7 obtained by exhaustive search using the exact binomial distribution, i.e., (3A.1). As a result, the simplified method in this Appendix can be used to find the optimal operational pairs of (ρ_{th}, k) for ECOMP.

Appendix 3B: ECOMF's Optimized Parameter Sets at Different SNR and Multicast Group Size

Table 1: Optimized parameter sets at different SNRs and Multicast group sizes of ECOMF.

	ECOMF's parameters	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**
SNR= 40dB	k	255	255	255	255	255	255	255	255	255	255	228	218	224	218	222	218	221	218
	N'	1	2	3	4	5	6	7	8	9	10	14	18	23	27	32	36	41	45
	r_{ECOMF}	12.46	11.46	10.87	10.46	10.14	9.88	9.66	9.46	9.29	9.14	8.92	8.79	8.75	8.71	8.68	8.66	8.64	8.63
SNR= 30dB	k	255	255	255	255	255	255	255	211	215	218	209	204	212	209	206	204	209	207
	N'	1	2	3	4	5	6	7	7	8	9	13	17	22	26	30	34	39	43
	r_{ECOMF}	9.14	8.15	7.58	7.17	6.85	6.60	6.38	6.29	6.26	6.23	6.07	5.99	5.97	5.94	5.92	5.90	5.89	5.89
SNR= 20dB	k	255	255	255	255	190	199	206	211	184	190	190	190	190	190	190	190	184	185
	N'	1	2	3	4	4	5	6	7	7	8	12	16	20	24	28	32	35	39
	r_{ECOMF}	2.65	2.94	2.71	2.80	2.73	2.75	2.74	2.72	2.75	2.70	2.74	2.71	2.73	2.72	2.71	2.72	2.71	2.71
SNR= 10dB	k	255	255	255	177	190	155	167	177	155	164	155	151	158	155	152	157	155	153
	N'	1	2	3	3	4	4	5	6	6	7	10	13	17	20	23	27	30	33
	r_{ECOMF}	2.91	2.15	1.76	1.72	1.64	1.59	1.57	1.54	1.53	1.52	1.47	1.45	1.44	1.43	1.43	1.42	1.42	1.42
SNR= 0dB	k	255	255	155	113	138	113	131	113	127	113	121	113	118	113	117	113	116	113
	N'	1	2	2	2	3	3	4	4	5	5	8	10	13	15	18	20	23	25
	r_{ECOMF}	0.86	0.52	0.48	0.43	0.42	0.40	0.39	0.38	0.38	0.37	0.36	0.35	0.34	0.34	0.34	0.34	0.34	0.34
SNR= - 10dB	k	255	113	155	113	89	113	96	113	99	113	105	101	98	97	96	95	94	98
	N'	1	1	2	2	2	3	3	4	4	5	7	9	11	13	15	17	19	22
	r_{ECOMF}	0.13	0.08	0.07	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.04

Chapter 4:

Opportunistic Multicast Scheduling with Erasure-Correction Coding over Frequency-selective Multipath Fading Wireless Channels

In this chapter, considering frequency-selective fading environment, an expansion of ECOMP scheme for applications to OFDM-based system is studied. At first, motivations for developing such a scheme and OFDM system model for multipath fading environment and the considered OFDM system model are presented. Subsequently, the extended ECOMP scheme for OFDM is described for two cases: (i) using only frequency domain by sending all erasure coded packets on all subcarriers and (ii) using both frequency and time domains by sending erasure coded packets in many time-slots on equally spaced subcarriers. Then simulation results and discussions on the effects of multipath fading on normalized multicast throughput and the trade-off between throughput and delay are provided.

4.1 Motivation

In this chapter we want study the performance of ECOM scheme on environment with correlated channels to see the effect of correlation on the achievable multicast rate. In particular, we are interested in extending ECOM scheme onto OFDM system to explore frequency diversity in a frequency-selective fading environment by sending coded packets over subcarriers. It can be seen in Chapter 3 that the proposed ECOM scheme can provide a much better multicast rate than BU and WU. However, since packets are coded in blocks, each user needs to receive at least a certain amount of number of packets in each block before it can decode the original information and since the coded packets are sent in time, there is a delay due to this decoding mechanism. Hence, if we can make use of frequency diversity by sending many coded packets at the same time on many subcarriers it is possible that we can reduce this delay.

Nevertheless, in multicarrier OFDM systems, fading on each subcarrier is correlated with that of the neighboring subcarriers. Hence, deep fade in one subcarrier can result in deep fade in other subcarriers as well. As a consequence, if this correlation is large, it may greatly reduce the multicast rate as packets sent on these subcarriers are likely to be erased. At this point, two questions arise: how this correlation factor affects the multicast throughput and how we can take advantage of frequency diversity with as little as possible degradation in the achievable multicast rate.

These two questions motivate us to apply ECOM scheme to OFDM system and are also the key issues to be addressed in this chapter. As discussed in Chapter 3, for large group size, the throughput performance of ECOMP is as good as ECOMF. Furthermore, ECOMP requires only the knowledge of average SNR and fading type of BS-user links. Therefore, when applied to OFDM the ECOMP scheme is more suitable than ECOMF as it can significantly reduce feedback signaling. Hence, in this chapter we focus only on ECOMP scheme. First, using only frequency diversity to study the effect of correlation on multicast rate, all the coded packets are sent in only one time slot, each on one subcarrier. ECOMP is then expanded to make use of both time and frequency diversity to investigate the trade-off between throughput and delay.

4.2 OFDM system model and correlation factor among the OFDM subcarriers.

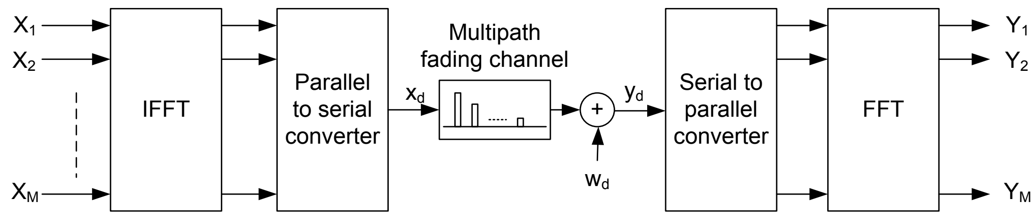


Fig. 4-1: OFDM system model.

Consider a wireless downlink OFDM system with M subcarriers over a bandwidth $B = M\Delta f$, where Δf is the subcarrier bandwidth, as shown in Fig. 4-1. Cyclic prefix is not shown in the diagram for simplicity, as it does not affect the following study. In a given time slot t, $\{X_m\}_{m=1,\ldots,M}$ is a block of complex signals in frequency domain where X_m is the transmitted signal on frequency m. X_m is assumed to have zero mean and power $E[X^2] = P$. These transmitted signals are transformed into samples $x_{d,t}$ in time domain through the discrete inverse Fourier transform with sampling time $\Delta t = 1/(M\Delta f)$. Theses time samples are serially transmitted through a multipath fading channel and are contaminated by additive white Gaussian noise $w_{m,t} \sim CN(0, \mathcal{N}_0)$ with $\mathcal{N}_0$ noise power. The received signal $y_{n,t}$ can be expressed as

$$y_{d,t} = h_{d,t} * x_{d,t} + w_{d,t}. \tag{4.1}$$

Let L be the number of resolvable paths in the multipath fading channel, with h_l be the tap gain of the l^{th} resolvable path. Similar to Chapter 3, h_l's are assumed to remain unchanged in a given time-slot and vary independently from one time-slot to another. The impulse response of the multipath channel in time domain is illustrated in Fig. 4-2 and can be written as

$$h_{d,t} = \sum_{l=0}^{L-1} h_{l,t}\delta[d-l]. \tag{4.2}$$

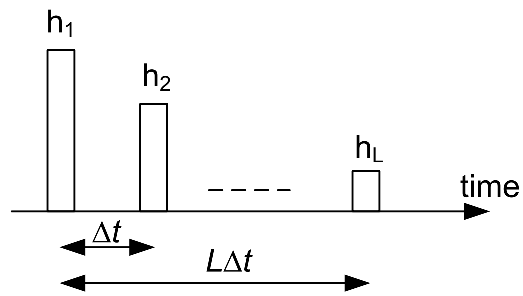


Fig. 4-2: Impulse response of multipath channel.

In this chapter, h_l's are assumed to follow power delay profile in COST 259 [12], i.e., $h_{l,t} \sim CN(0, \sigma_l^2)$ with $\sigma_l^2 = \frac{e^{-l/4}}{\sum_{l=0}^{L-1} e^{-l/4}}$.

From equation (4.1), the received signal $Y_{m,t}$ in frequency domain (after FFT block in Fig. 4-1) can be given by

$$Y_{m,t} = H_{m,t} X_{m,t} + W_{m,t} \, , \tag{4.3}$$

where $W_{m,t}$ is the fast Fourier transform (FFT) of $w_{m,t}$ and $H_{m,t}$ denotes the channel response in frequency domain, which is the FFT of (4.1), i.e.,

$$H_{m,t} = \sum_{l=0}^{L-1} h_{l,t} e^{-j2\pi(l\Delta t)(m\Delta f)} = \sum_{l=0}^{L-1} h_{l,t} e^{-\frac{j2\pi lm}{M}}, \tag{4.4}$$

$H_{m,t}$ indicates the fade level of the signal on subcarrier m and hence determine the channel quality at that frequency. Since $h_{l,t} \sim CN(0, \sigma_l^2)$ with $\sigma_l^2 = \frac{e^{-l/4}}{\sum_{l=0}^{L-1} e^{-l/4}}$, $H_{m,t} \sim CN(0,1)$. Based on the assumption that $h_{l,t}$'s remain unchanged in a given time-slot and vary independently from one time-slot to another, $H_{m,t}$'s also remain unchanged in a given time-slot and vary independently from one time-slot to another.

From equation (4.3), the correlation between subcarriers m_1 and m_2 can be calculated as follows

$$E[H_{m_1}H_{m_2}^*] = E\left[\sum_{l=0}^{L-1}\sum_{l'=0}^{L-1} h_l e^{-\frac{j2\pi l m_1}{M}} h_{l'}^* e^{\frac{j2\pi l' m_2}{M}}\right]$$

$$= E\left[\sum_{l=0}^{L-1} |h_l|^2 e^{-\frac{j2\pi l(m_1-m_2)}{M}}\right] \text{ since } E[h_l h_{l'}^*] = 0$$

$$= E\left[\sum_{l=0}^{L-1} \alpha_l^2 e^{-\frac{j2\pi l(m_1-m_2)}{M}}\right]. \quad (4.5)$$

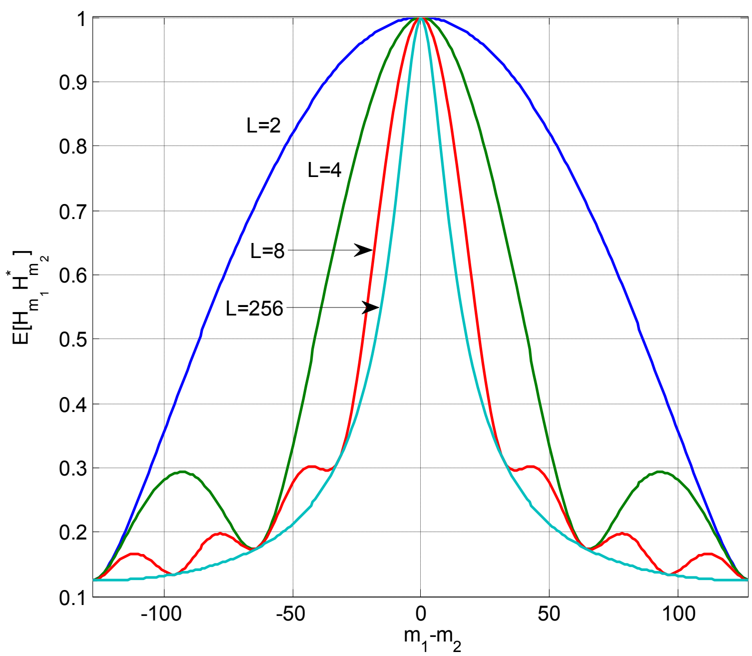


Fig. 4-3: Correlation between OFDM subcarriers.

From equation (4.4), this correlation factor depends on the distance between two subcarriers and the number of resolvable paths L. Fig. 4-3 further illustrates this correlation factor for an OFDM system of 256 subcarriers over a multipath Rayleigh fading channel with 2, 4, 8 and 256 tap gains. It can be seen in Fig. 4-3 that this correlation is large when the frequency separation between m_1 and m_2 is small, hence if one subcarrier is in deep fade, the other is also likely to be in deep fade which may result in packet loss on both subcarriers if we use these two for transmission. On the other hand, when this correlation factor is small, deep fade on one subcarrier is less likely to affect the other subcarrier. Moreover, it is observed from Fig.

4-3 that, for a given level of correlation, when the number of resolvable paths increases the frequency separation decreases. For example, when the number of resolvable paths increases from 2 to 4, the minimum frequency separation to achieve a correlation of 0.3 decreases from 105 to 52 subcarriers, which is approximately two times. The same observation applies when L increases from 4 to 8 taps. However, when L is larger than 8 this observation is no longer valid as shown in Fig. 4-3. Hence, multipath fading channel introduces frequency diversity that can be used, especially for L from 2 to 8.

4.3 ECOMP for OFDM

Using the system model as described in Section 4.1 to support multicast scenario for N users, we first derive the relationship between average SNR and the instantaneous SNR on each subcarrier and then describe the operation of ECOMP in the case of OFDM.

Applying (4.3), the instantaneous channel gain for user i on subcarrier m is given by,

$$H_{i,m} = \sum_{l=0}^{L-1} h_{i,l} e^{-\frac{2\pi ml}{M}}. \tag{4.6}$$

Let $X_{m,t}$ be the transmitted signal on subcarrier m at timeslot t with normalized power of 1, the average SNR on subcarrier m of user i can be expressed as

$$\begin{aligned} \overline{\gamma}_{i,m} &= \frac{E\left[\left|X_{m,t}H_{i,m,t}\right|^2\right]}{N_0} = \frac{E\left[\left|X_{m,t}\right|^2\right]E\left[\left|H_{i,m,t}\right|^2\right]}{N_0} = \frac{P}{N_0} \\ &= \overline{\gamma}, \end{aligned} \tag{4.7}$$

and the instantaneous SNR on subcarrier m of user i at timeslot t is given by

$$\overline{\gamma}_{i,m,t} = |H_{i,m,t}|^2 \frac{E\left[\left|X_{m,t}\right|^2\right]}{N_0} = \rho_{i,m,t}\overline{\gamma}, \tag{4.8}$$

where $\rho_{i,m,t} \triangleq \left|H_{i,m,t}\right|^2$.

Following the discussions in the previous chapter, the transmission rate for

WU and BU in OFDM system in a given time slot can be given by

$$r_{\mathrm{WU}\,m,t} = \log_2(1 + \overline{\gamma} \min_{i=1,2,\dots,N}\{\rho_{i,m,t}\}), \tag{4.9}$$

$$r_{\mathrm{BU}\,m,t} = \log_2\left(1 + \overline{\gamma} \max_{i=1,2,\dots,N}\{\rho_{i,m,t}\}\right). \tag{4.10}$$

4.3.1 ECOMP for OFDM using only frequency diversity

Expanding ECOMP to OFDM system, the transmission rate to send multicast packet on each subcarrier can be selected base on the channel gain threshold ρ_{th} as follows

$$r_{\mathrm{ECOM}\,m} = \log_2(1 + \overline{\gamma}\rho_{\mathrm{th}}). \tag{4.11}$$

To recover the erased packets for users with inadequate instantaneous SNR, ECOMP makes use of RS code with the same encoding scheme as described in Chapter 3. For simplicity, we assume that the number of RS coded packets is equal to the number of subcarriers, i.e., $n = M$ and hence, the whole RS-coded packet block is sent in one time-slot with each coded packet transmitted on one subcarrier as illustrated in Fig. 4-4 for time-slot t.

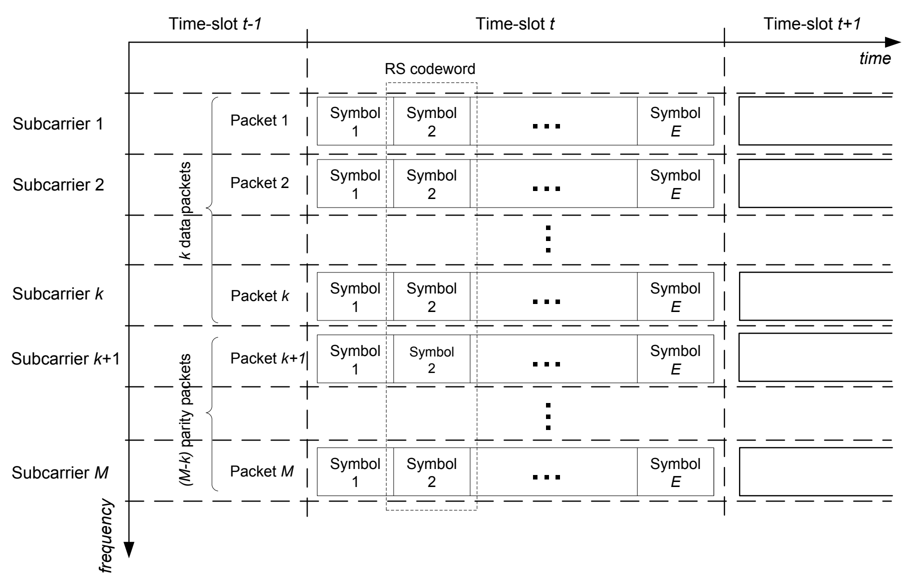


Fig. 4-4: Transmission of RS coded packets of ECOMP over OFDM subcarriers.

The optimization problem for multicast rate when applying ECOMP to

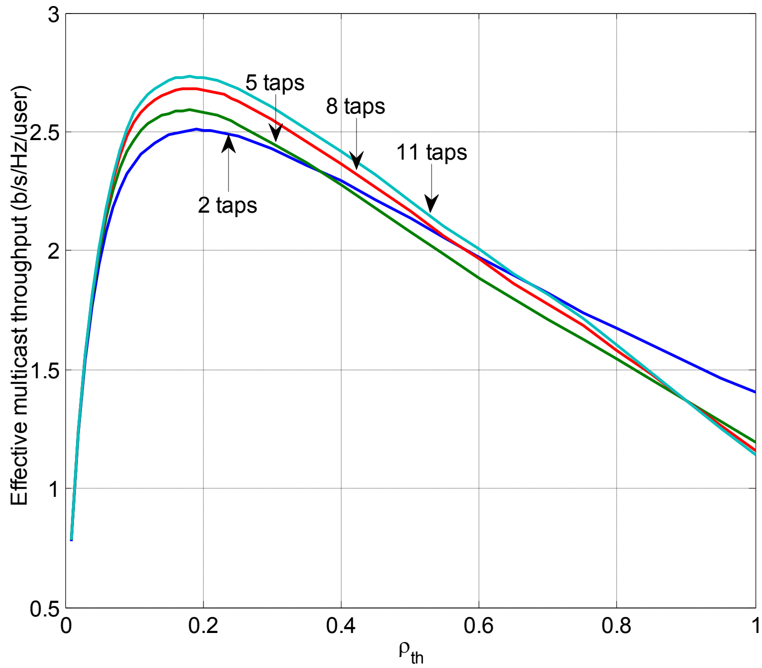
OFDM then becomes

$$\arg\max_{\rho_{\mathrm{th}},k} \frac{k}{n}\log_2(1+\bar{\gamma}\rho_{\mathrm{th}})\Pr\{x \geq k\}, \tag{4.12}$$

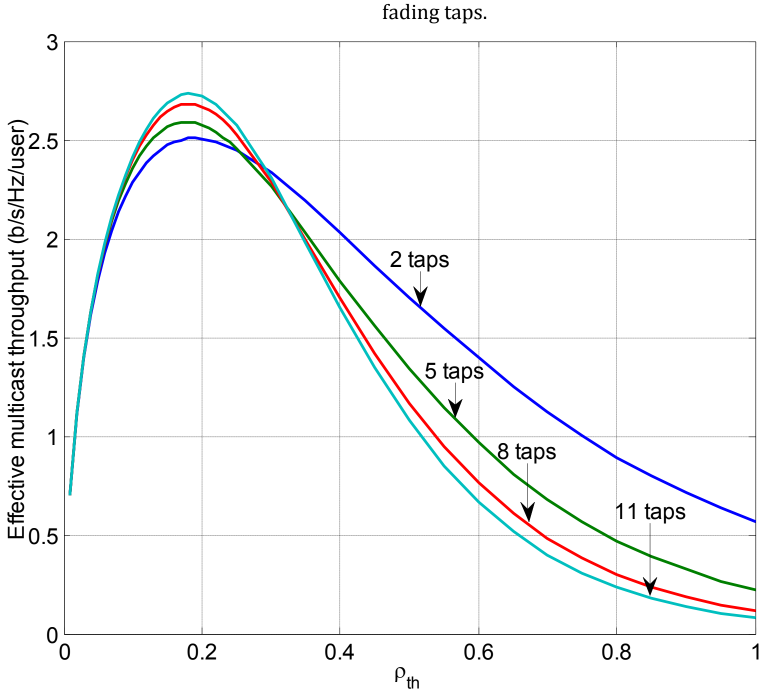
where $\Pr\{x \geq k\}$ is the probability that a given user can receive at least k non-erased packets on all the subcarriers. At the first look, this probability is similar to that in the last chapter; however, it is noted that, in the scenario of OFDM, channel gains on subcarriers are correlated and a simple expression using Binomial distribution as in the last chapter is not applicable. Also, to the best of our knowledge, there is even no closed-form expression for $\Pr\{x \geq k\}$ in general.

To study the throughput performance of ECOMP for OFDM at first the effects of multipath on multicast throughput is illustrated and then the throughput comparison of the proposed scheme with BU and WU is presented. In our simulations, multicasting is done on a multicast group of $N = 100$ users. For simplicity, an OFDM system with $M = 256$ subcarriers and $\mathrm{RS}(256, k)$ is considered. Similar to the previous chapter, in our simulation results and discussions, the *effective multicast throughput* in units of *b/s/Hz/user* is used as a figure of merit, to evaluate and compare the performance of different schemes.

4.3.1.1 Effects of the number of tap gains on multicast throughput



a. Effective multicast throughput comparison of optimal bound of environments with different numbers of fading taps.



b. Effective multicast throughput comparison when using RS (256,180).

Fig. 4-5: Effective multicast throughput versus ρ_{th} for ECOMP scheme in different numbers of taps on multipath fading channel with average SNR of 20dB.

In Fig. 4-5 the effect of different fading conditions on the achievable normalized throughput over multipath fading channels is examined at the same average SNR of 20dB with the number of taps $L = 2, 5, 8$ and 11. The results in Fig. 4-5a show that the effective multicast throughput depends on the number of resolvable paths. When L increases, the correlation among the subcarriers reduces, i.e., less chance that packets are erased at the same time, and hence the achievable multicast rate increase as L increases. This is not the case for unicast where the ergodic capacity is independent of the number of resolvable paths as shown in Appendix 4A and [13]-[15]. However, as shown in Fig. 4-5b, this gain comes with a cost: at the same code rate, as the number of taps increases, the throughput curve becomes more sensitive to the channel gain threshold ρ_{th}. This effect can be explained as follows. At lower multipath tap gains, the channel responses of OFDM subcarriers are highly correlated; hence, they follow similar trend and changes in ρ_{th}, which results in smaller change in the multicast throughput as compared to the weakly correlated case.

4.3.1.2 Performance comparison:

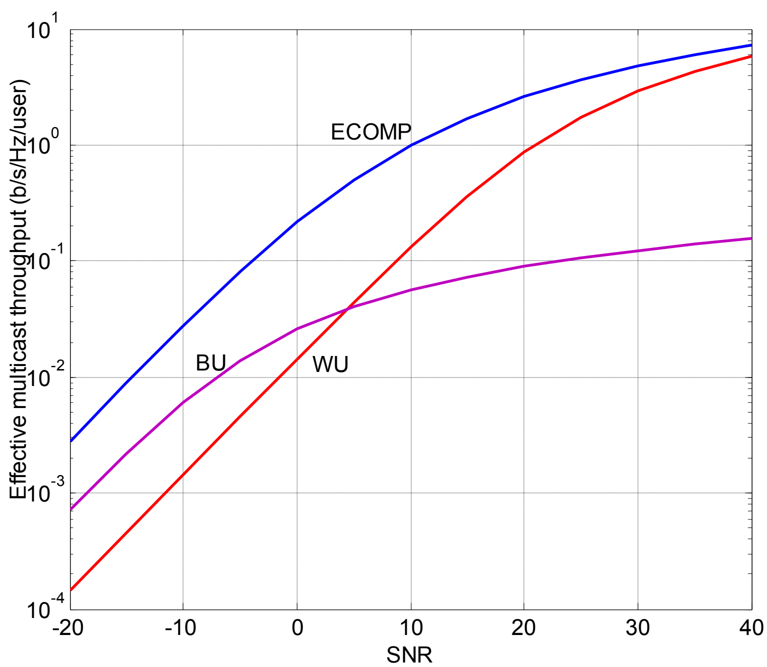


Fig. 4-6: Throughput given by different schemes versus average SNR's (5 taps, 100 users).

Fig. 4-6 compares the effective multicast rate of the WU, BU and ECOMP scheme in a 5-tap multipath fading environment for a wide range of SNR. Similar to the performance comparison in the Chapter 3, BU is better than WU in the low SNR region while WU outperforms BU after the crossover point of around 5dB. Combining both multicast gain and multiuser diversity, the throughput performance of ECOMP is superior to both BU and WU in the considered SNR range and asymptotically converges to WU at high SNR. However, it is noted that the improvement in throughput of ECOMP on OFDM system compared with BU and WU is smaller than in the case of single carrier. For instance, at 5dB, ECOMP offers an improvement in multicast rate of 10 times higher than BU and WU in the case of multicarrier while, as shown in Chapter 3 (Fig. 3-7), its improvement in multicast rate in the case of single carrier is 18 times higher. This can be explained by the fact that, due to the high correlation in frequency responses, channel gains of adjacent subcarriers are similar and consequently, changes in the SNR threshold ρ_{th} to adjust multicast gain and multiuser diversity give less effect on the multicast throughput than in the case of independent time slots. In other words, correlation in channel responses will decrease the benefits of combining multicast gain and multiuser diversity.

4.3.2 ECOMP for OFDM using both time and frequency diversity

When applying ECOMP to OFDM by sending all coded packets on all subcarriers, it can be seen that we gain n times reduction in the delay as each RS block can be sent in only one time-slot. However, as the channel gains of OFDM subcarriers are correlated, if one subcarrier of a given user is in deep fade (i.e., $\rho_{i,m}(t)$ is very low) it is likely that the subcarriers close to it are also in deep fade and the packets that are sent on these subcarriers will likely be erased. To compensate for the erased packets ECOMP has to select a lower transmission rate and lower RS code rate k to gain more erasure correction capability and hence this reduces the multicast rate. To enhance this throughput performance, it is necessary that the correlation among the subcarriers be as low as possible. As shown in Fig. 4-3, when the frequency separation between the two subcarriers increases, the corresponding correlation in channel gains decreases. This frequency separation depends on how many resolvable paths the environment has. Let us make the same assumption as in the previous part that $n = M$, hence by transmitting coded packets on subcarriers far from each other we can achieve lower correlation, i.e., higher multicast throughput. However, fewer packets can be transmitted on one time-slot and as a consequence more time-slots are needed for transmitting each RS block. Hence, in this case, a trade-off between throughput and delay is introduced to ECOMP.

The extended erasure-correction coding based opportunistic multicast (EECOM) is an extension of ECOM scheme, in which instead of sending all coded packets on all subcarriers in one timeslot, these coded packets are sent over many timeslots on only a subset of all OFDM subcarriers. This subset of subcarriers is chosen to have the largest equal frequency separation among the subcarriers for smallest correlation.

Let S be the number of time-slots for transmission, Fig. 4-7 illustrates the transmission mechanism of EECOMP for the case $S = 2$. First, the BS encodes k data packets using RS erasure code to form a RS block of M packets in the same way as in Chapter 3. Since $S = 2$, these coded packets are sent in 2 time-slots on equally separated subcarriers with the frequency separation of one subcarrier. In Fig. 4-7, as an illustrative example, RS coded packets are sent on odd subcarriers. After all the packets in the block are sent, the next k packets will be encoded and sent in the same manner.

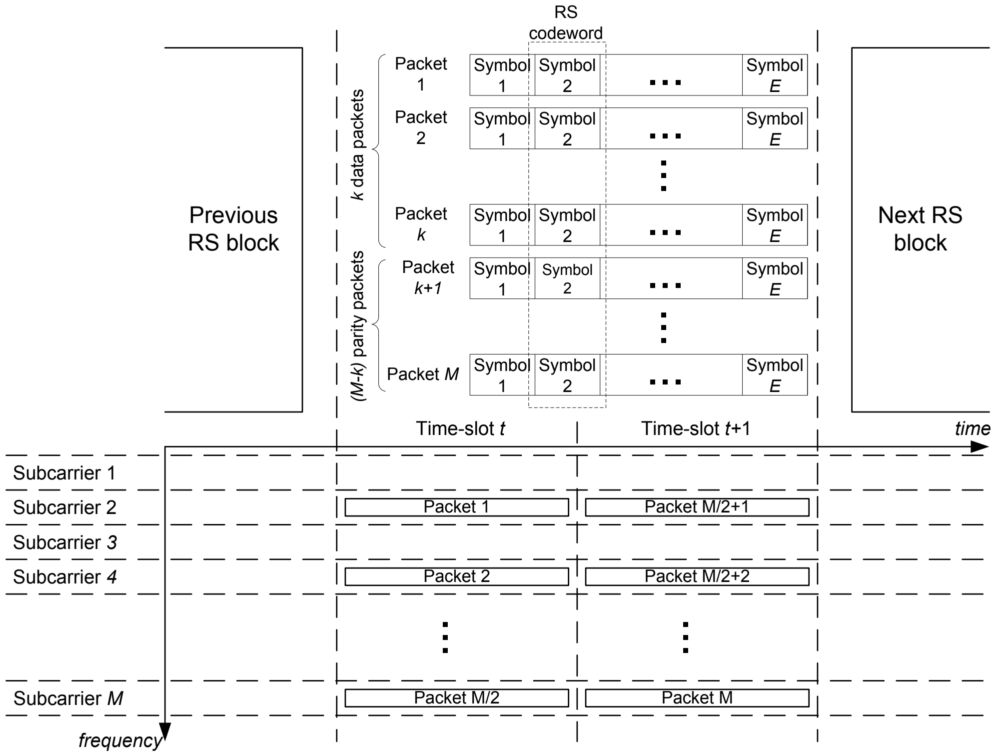


Fig. 4-7: Transmission of RS coded packets for EECOMP.

The optimization problem for EECOM can be modified from (4.11) as the following

$$\arg\max_{\rho_{\text{th}},k} \frac{k}{n}\log_2(1+\overline{\gamma}\rho_{\text{th}})\Pr\{x \geq k\}_S, \tag{4.13}$$

where $\Pr\{x \geq k\}_S$ is the probability that a given user can receive at least k non-erased packets over S time-slots. For the same reason as in the last part, the probability $\Pr\{x \geq k\}_S$ does not have close form mathematical expression and throughput of EECOM is analyzed by means of simulations. Similar to Section 4.3.1, our simulation results are based on a group of 100 users on an OFDM system with M=256 subcarriers.

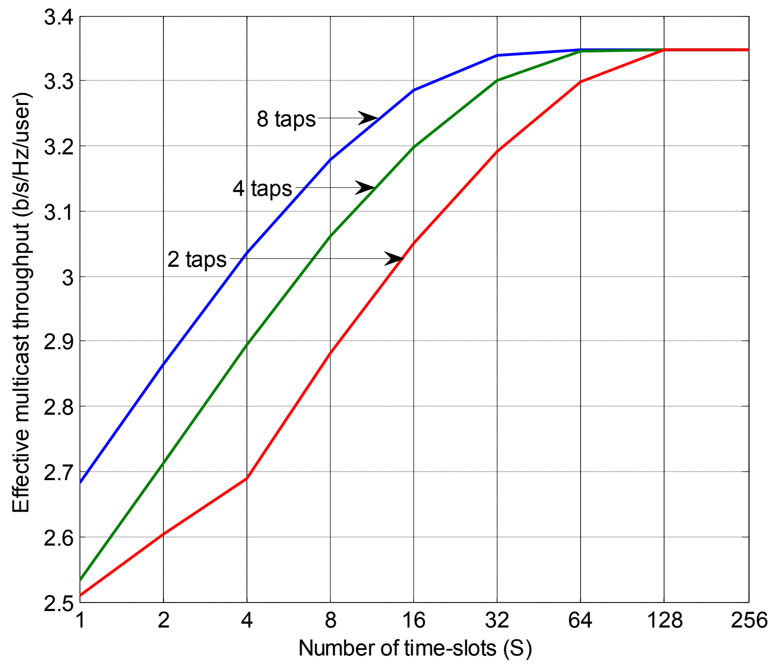


Fig. 4-8: Performance of EECOMP for different numbers of timeslots on multipath fading channel at average SNR of 20dB.

Fig. 4-8 depicts the performance of EECOMP for different numbers of timeslots. In this graph, the X axis represents the number of time-slot S in $\log_2$ scale. It is observed that as the number of time-slot S increases, the multicast throughput monotonically increases, which illustrates the trade-off between throughput and delay. When S=1, EECOMP exploits only frequency diversity by sending all coded packets of one RS block in one time-slot on all subcarriers. The effective multicast throughput for EECOMP is the same as in Fig. 4-5 a. When S=256, EECOMP exploits only time diversity by sending only one coded packet in each time-slot on one subcarrier and the multicast rate in this case is the same as in Fig. 3-3 b, i.e., around 3.34 b/s/Hz which is about 25% higher than in the case of S=1. Moreover, when S is large enough, the effective throughput for EECOMP is approximately equal to that of S=256. For example, when S is larger than 32 for 8-tap channels, 64 for 4-tap channels or 128 for 2-tap channels, the achievable multicast rates for EECOMP are roughly the same as the case of S=256. This indicates that there is a significant delay reduction with virtually no penalty in multicast rate at these points and S reduces when the number of resolvable paths increases, e.g., S reduces 2, 4, 8 times when L=2, 4, 8 taps accordingly. However, when L is very large, as shown in Fig. 4-3, the correlation between two subcarriers is similar. Hence, for larger L, the delay cannot be reduced further. By calculating the frequency separations in these cases and referring to Fig. 4-3 for the correlation values, it is shown that the correlation levels in these cases are less than 0.3.

In addition, it is observed that for points in Fig. 4-8 with the same multicast rate, they yield the same correlation level as shown in Fig. 4-3. For instance, at S=32 for L=4 and S=64 for L=2, the multicast rate is about 3.3 b/s/Hz/user (as shown in Fig. 4-8). At these points, the subcarrier separations are 32 and 64 subcarriers when L=4 and L=2 respectively, for the same correlation factor of about 0.7 (as shown in Fig. 4-3). The same observation applies for other points with approximately the same effective multicast throughput in Fig. 4-8.

4.4 Chapter summary

In this chapter, we consider to extend ECOMP to OFDM system, aiming at exploring frequency diversity in the frequency-selective fading environment. System throughput of the extended scheme is investigated by simulations to study the effect of correlation on multicast rate. The results show that the proposed scheme can make use of multipath fading channel to enhance multicast rate.

Moreover, the proposed scheme is extended to take advantage of both time and frequency diversity and the trade-off between delay and throughput is studied. Simulation results illustrate that when the correlation between two consecutive subcarriers is less than 0.3, the extended scheme can achieve the same multicast rate with significant reduction in delay as compared to the case of purely time diversity.

Appendix 4A: Ergodic Capacity of OFDM System for One User

Proposition 1: *The marginal distributions of* $|H_k|^2$*'s are the same regardless of* L *and* m.

Proof:

Recall that $H_m = \sum_{l=0}^{L-1} h_l e^{-\frac{2\pi ml}{M}}$ is the channel response on m^{th} subcarrier. The channel gain is then given by

$$|H_m|^2 = H_{mr}^2 + H_{mi}^2. \tag{A4.1}$$

where $\begin{cases} H_{mr} = Re\left\{\sum_{l=0}^{L-1} h_l e^{-\frac{2\pi ml}{M}}\right\} = \sum_{l=0}^{L-1}\left(h_{lr}\cos\left(\frac{2\pi ml}{M}\right) + h_{li}\sin\left(\frac{2\pi ml}{M}\right)\right) \\ H_{mi} = Im\left\{\sum_{l=0}^{L-1} h_l e^{-\frac{2\pi ml}{M}}\right\} = \sum_{l=0}^{L-1}\left(h_{li}\cos\left(\frac{2\pi ml}{M}\right) - h_{lr}\sin\left(\frac{2\pi ml}{M}\right)\right) \end{cases}$;

h_{lr}, h_{li} denote the real and imaginary part of the l^{th} resolvable path accordingly.

Since all h_l's are independent complex Gaussian random variables, H_{mr} is a linear combination of L Gaussian random variables. Therefore, H_{mr} is a Gaussian random variable with zero mean since all h_l's have 0 mean and the variance of

$$Var(H_{mr}) = \sum_{l=0}^{L-1}\left[Var(h_{lr})\cos^2\left(\frac{2\pi ml}{M}\right) + Var(h_{li})\sin^2\left(\frac{2\pi ml}{M}\right)\right]$$

$$= \sum_{l=0}^{L-1}\frac{{\sigma_l}^2}{2}\left[\cos^2\left(\frac{2\pi ml}{M}\right) + \sin^2\left(\frac{2\pi ml}{M}\right)\right] = \frac{1}{2}. \tag{A4.2}$$

Similarly, we then have $H_{mi} \sim N(0, \frac{1}{2})$

Since if we have $X \sim N(0, \sigma^2)$ and $Y \sim N(0, \sigma^2)$, then $Z^2 = X^2 + Y^2$ has the distribution given by

pfd $f_Z(z) = \frac{1}{2\sigma^2} e^{-\frac{z}{2\sigma^2}}$ for $z \geq 0$

and the CDF of $F_Z(z) = 1 - e^{-\frac{z}{2\sigma^2}}$ for $z \geq 0$.

Therefore, regardless of L and m, $|H_m|^2$'s follow the same distribution with the distribution functions as the following

Pfd $f_{|H_m|^2}(|H_m|^2) = e^{-|H_k|^2}$

and the CDF of $F_{|H_m|^2}(|H_m|^2) = 1 - e^{-|H_m|^2}$.■

Let C_m be the ergodic capacity of the channel on subcarrier m. The total average capacity of SISO-OFDM system for single user is given by

$$\begin{aligned} E[C] &= E\left[\sum_{m=1}^{M} C_m\right] = \left[\sum_{m=1}^{M} E[C_m]\right] \\ &= \sum_{m=1}^{M} \int_0^{\infty} \log_2(1+\bar{\gamma}|H_m|^2) f_{|H_m|^2}(|H_m|^2) d\,|H_m|^2. \end{aligned} \tag{A4.3}$$

From proposition 1, we know that all $|H_k|^2$ follow the same distribution; therefore the capacity in (A4.3) is given by

$$\begin{aligned} E[C] &= E\left[\sum_{m=1}^{M} C_m\right] = ME[C_m] \\ &= M \int_0^{\infty} \log_2(1+\bar{\gamma}|H_m|^2) f_{|H_m|^2}(|H_m|^2) d\,|H_m|^2. \end{aligned} \tag{A4.4}$$

As shown in (A4.4) and Proposition 1, capacity for SISO-OFDM system with single user does not rely on number of resolvable taps as well as the distribution of power over the taps; therefore, multipath diversity cannot be made use in SISO-OFDM system with single user.

Chapter 5:

Conclusion

In this work, we have proposed and studied an erasure-correction coding based opportunistic multicast scheduling scheme aiming at exploiting multicast gain, multiuser diversity and time/frequency diversity to enhance the throughput performance over wireless fading channels. In the proposed scheme the BS sends each packet only once at a transmission rate determined by a channel gain threshold and using erasure correction capability of RS (n, k) to recover erased packets due to insufficient instantaneous SNR on BS-user links. RS coding scheme is applied to a block of packets and coded packets are sent in time or frequency slots to effectively explore time/frequency diversity. The channel gain threshold and erasure code rate are jointly optimized for best multicast throughput.

On frequency-flat fading channels, the selection of channel gain threshold is considered in two cases of full channel knowledge and partial knowledge of average SNR and fading type of wireless channel. An analytical framework has been developed to analyze the effective multicast throughput of BU, WU and of the proposed scheme. In this framework, we prove that while the effective multicast rates of both BU and WU asymptotically converge to zero as the multicast group size increases, this effective multicast rate of the proposed scheme is bounded from zero depending on the average SNR. We further prove that, for the proposed ECOM scheme, the benefit of full channel knowledge is only pronounced at small multicast group sizes. As the group size increases, partial knowledge of channel response is sufficient in providing approximately the same throughput performance but significantly reducing resources (bandwidth, power) for feedback signalling.

In addition, numerical results illustrate that multiuser diversity is most pronounced at low SNR region since the difference in supportable rates of various users is large while multicast gain is superior at high SNR region where the difference in channel gain is compressed by the log-function that results in small difference in supportable rates among the users. The throughput comparison illustrates that with the ability of combining multicast gain and multiuser diversity, the proposed scheme outperforms both BU and WU for a wide range of SNR.

Furthermore, in this thesis, we have extended ECOM for applications to OFDM system aiming at exploiting both time and frequency diversity in a frequency-selective fading environment. The effects of frequency correlation on multicast rate is investigated and our study shows that by exploiting both time and frequency diversity, we can significantly reduce transmission delay with negligible degradation in multicast throughput.

Suggested future work

In this thesis the throughput performance of the proposed scheme is studied in i.i.d. and correlated channels. However, in the case of correlated channel, the study is based on simulation results only. Developing a mathematical framework to analyze throughput performance would give better understanding about the proposed scheme on correlated channels.

Furthermore, the proposed technique assumes the use of Reed-Solomon erasure-correction codes. One can go further by analyzing the multicast throughput with other codes and come up with the optimal bound that opportunistic multicast can achieve.

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