The Effect of Digital Educational Games on Students' Attitudes towards Mathematics

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Abstract

Children's math attitudes start to decrease at an early age. Previous literature suggests that digital educational games have the potential to promote positive math attitudes. However, the research on math attitudes has taken divergent paths with ambiguous definitions and unified measurements. Moreover, few studies have used the value-added approach to systematically investigate the impact of specific game features on math attitudes. The purpose of this dissertation is to address fundamental questions on the definitions and components of math attitudes, and how digital educational games can be leveraged to enhance math attitudes. In the first manuscript, a systematic review was conducted to explore the definitions, components of math attitude, as well as their relations with math performance. Based on this review, a tripartite math attitude scale was proposed and validated in the second manuscript. The third manuscript presents a meta-analysis of effects of different game features that support positive math attitudes and provides guidelines for choosing digital games to promote better learning experience and outcomes. Building on the meta-analysis, the fourth manuscript presents an empirical study to examine the impact of a particular game feature (i.e., an incentive system) on math attitudes. Two versions of digital math games were created and tested with children, a base version and a similar version with one added feature. The findings from this empirical study showed that games with incentive systems were more effective in fostering positive math attitudes and improving in-game performance compared to those without. Based on systematic reviews, meta-analysis, and empirical research, this dissertation provides insights into the factors that shape math attitudes and suggest potential avenues for improving math education.

Résumé

Les attitudes mathématiques des enfants commencent à diminuer dès leur jeune âge. Les travaux antérieurs suggèrent que les jeux éducatifs numériques ont le potentiel de promouvoir des attitudes mathématiques positives. Cependant, la recherche sur les attitudes mathématiques a suivi des chemins divergents avec des définitions ambiguës et des mesures unifiées. De plus, peu d'études ont utilisé l'approche de valeur ajoutée pour enquêter systématiquement sur l'impact des caractéristiques spécifiques des jeux sur les attitudes mathématiques. Le but de cette thèse est d'aborder des questions fondamentales sur les définitions et les composantes des attitudes mathématiques, ainsi que sur la manière dont les jeux éducatifs numériques peuvent être exploités pour améliorer ces attitudes. Dans le premier manuscrit, une revue systématique a été menée pour explorer les définitions, les composantes des attitudes mathématiques et leurs relations avec les performances mathématiques. Sur la base de cette revue, une échelle tripartite des attitudes mathématiques a été proposée et validée dans le deuxième manuscrit. Le troisième manuscrit présente une méta-analyse des effets des différentes caractéristiques de jeu qui favorisent des attitudes mathématiques positives et fournit des lignes directrices pour choisir des jeux numériques afin de promouvoir une meilleure expérience d'apprentissage et de meilleurs résultats. S'appuyant sur la méta-analyse, le quatrième manuscrit présente une étude empirique visant à examiner l'impact d'une caractéristique particulière du jeu (c'est-à-dire un système d'incitation) sur les attitudes mathématiques. Deux versions de jeux mathématiques numériques ont été créées et testées auprès des enfants, une version de base et une version similaire avec une fonctionnalité supplémentaire. Les résultats de cette étude empirique ont montré que les jeux avec des systèmes d'incitation étaient plus efficaces pour favoriser des attitudes mathématiques positives et améliorer les performances en jeu par rapport à ceux sans cette fonctionnalité. En se

basant sur des revues systématiques, des méta-analyses et des recherches empiriques, cette thèse offre des aperçus sur les facteurs qui façonnent les attitudes mathématiques et suggère des pistes potentielles pour améliorer l'éducation mathématique.

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Contributions of Authors

I am the primary author of all chapters included in this dissertation and I am solely responsible for the content. I independently wrote Chapters 1 to 5, and Dr. Adam K. Dubé provided feedback on them. As part of my comprehensive exam, I wrote initial versions of Chapter 2 independently, and received constructive feedback from Dr. Adam K. Dubé and Dr. Nathan C. Hall.

Chapter 2

Citation

Wen, R., & Dubé, A. K. (2022). A systematic review of secondary students' attitudes towards mathematics and its relationships with math achievement. *Journal of Numerical Cognition, 8*(2), 295-325. https://doi.org/10.5964/jnc.7937

Contributions

I conducted the systematic review and wrote this chapter in its entirety. An initial version of this paper was written in fulfilment of my comprehensive exam, for which Dr. Adam K. Dubé and Dr. Nathan C. Hall provided constructive feedback. The current version was modified based on their feedback.

Chapter 3

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Contributions

I was primarily responsible for the development of research question, ethic submission, data collection and analysis, as well as writing up the manuscript. At every stage of development, I received a thorough review and detailed feedback from Dr. Adam K. Dubé.

Chapter 4

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Contributions

I proposed the research question, conducted the literature review, calculated the effect size, and wrote this chapter in its entirety. Dr. Adam K. Dubé provided invaluable feedback on each stage of the development.

Chapter 5

Citation

Wen, R., & Dubé, A. K. (2023). *Effects of digital educational games on students' in-game performance, math achievement, and math attitudes: A value-added approach.* Manuscript in preparation.

Contributions

I was chiefly responsible for developing the research question, submitting ethical applications, collecting and analyzing data, and writing the manuscript. The Ululab team contributed to the design of the intervention games and assisted in logfile data collection. Dr. Adam K. Dubé provided invaluable feedback throughout each stage of the project, including the final manuscript structure.

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Chapter 1. Introduction

Math attitudes have been studied for decades due to their potential influence on students' conceptual understanding of math content and their overall achievement in math (Ashcraft & Kirk, 2001; Hembree, 1990; Ma & Kishor, 1997; Markovits & Forgasz, 2017; Neale, 1969; Passolunghi et al., 2016; Smetackova, 2015; Woodard, 2004). Research has reported a decline in students' math attitudes at an early age (Aiken, 1970), and the phenomenon appears to be prominent among secondary students (Ashcraft & Faust, 1994; Hembree 1990; Wigfield & Meece, 1988). The decline in math attitudes requires urgent attention and researchers need to identify an effective strategy to promote student's positive math attitudes from an early stage. However, there continues to be confusion in the literature regarding what math attitudes are and how they influence math performance (Di Martino & Zan, 2010). Some researchers define math attitudes mainly as individual's feelings and disposition towards math as a subject, such as anxiety, or enjoyment (Hemmings et al., 2011; Mavridis et al., 2017; Mohr-Schroeder et al., 2017), some researchers believe math attitudes are the combination of affective feelings and cognitive beliefs (Ifamuyiwa & Akinsola, 2008; Kiwanuka et al., 2017), where others argue that behavioral intentions (i.e., the tendency to engage or avoid math learning) should also be considered (Etuk et al., 2013; Koyuncu & Dönmez, 2018; Mutohir et al., 2018). Due to these disjointed definitions, the measures of math attitudes vary considerably (Hannula, 2002; Zan & Di Martino, 2007). The lack of consensus on the definitions and measures of math attitudes has affected researchers' understanding of math attitudes and their relation to math achievement.

Given decreasing math attitudes, one critical question for educators and researchers is how to improve them? Digital educational games have been shown to be an effective solution in terms of promoting positive attitudes (Dempsey et al., 1996; Randel et al., 1992; Wouters et al., 2013). For decades, meta-analyses have provided empirical evidence of the impact of educational games on learners' interests, attitudes, cognitive skills, and knowledge gains in some arears (Randel et al., 1992; Sitzmann, 2011; Vogel et al., 2006; Wouters et al., 2013). Studies have yet to provide substantial evidence of the impact of educational games on math (Byun & Joung, 2018; Dubé et al., 2019). Moreover, it has been argued that not all digital educational games are equally effective in enhancing positive attitudes and learning outcomes (Habgood & Ainsworth, 2011; Ke, 2009). To optimize the impact of educational games on math attitudes and learning outcomes, it is essential to identify which game feature(s) are most effective and to understand why and how they contribute to success.

Mayer (2019) proposes the value-added approach as a way of investigating a given game feature. This approach involves creating two versions of game: a base version and a value-added version which is identical to the base version but includes one additional feature (Mayer, 2019). By comparing the outcomes of the two versions of a game, researchers can draw more rigorous conclusions. While the value-added approach provides a systematic and controlled means of examining the effects of a particular game feature, few studies have used this approach to explore the effects of digital educational games on math outcomes. This is partially due to the approach requiring the creation of two different versions of the same game, increasing the cost and duration of development.

The present thesis aims to provide a comprehensive investigation into the components of math attitudes, their relationship between math achievement, and how games can promote positive math attitudes and math achievement. To achieve this goal, the thesis aims to clarify the definitions and measures of math attitudes, examine their relationship with math achievement, identify game features that contribute to positive math attitudes and learning outcomes, and investigate the effectiveness of a particular feature through the value-added approach.

The thesis comprises several chapters, each addressing a specific research question. Chapter 2 presents a literature review on definitions and measures of math attitudes, and their relationship with math achievement. Chapter 3 describes an empirical study which proposes and validates a tripartite measure of math attitudes. Chapter 4 reviews game features that promote positive math attitudes and better math performance. Finally, Chapter 5 presents a study on the effects of a particular game feature (i.e., incentive systems) on math performance and math attitudes.

Overall, this thesis seeks to contribute to the literature on math education by clarifying the essential components of math attitudes, shedding light on the complex relations between math attitudes and math achievement, while providing educators with effective tools and guidance to foster positive math attitudes and enhance learning outcomes.

Overview of the Chapters

Chapter 2 presents a literature review on math attitudes. First, it classifies the definitions of math attitudes and adopts a tripartite model as a theoretical framework to guide the comprehensive understanding of the term. Moreover, this chapter suggests a tripartite construct of math attitudes and clarifies the subdimensions under the three proposed components of math attitudes (i.e., cognitions, affects, and behaviours) through synthesizing the most commonly measured variables in research practice. Finally, this chapter unpacks the relations between each component of math attitudes and math achievement based on previous research findings.

Chapter 3 presents the validation of a comprehensive tripartite scale that measures math attitudes through cognitive, affective, and behavioural components, incorporating gender beliefs, confidence, value, anxiety, enjoyment, boredom, anger, pride, shame, and behavioural intentions. This scale not only captures the well-established dimensions of math attitudes, but also encompasses previously overlooked aspects of math attitudes. Additionally, this chapter explores the interconnections between dimensions of math attitudes under the guidance of control-value theory (Pekrun, 2006). The findings showed significant associations between cognitive, affective, and behavioural math attitudes. The results not only simultaneously reveal overall relations within the construct, but also provide a basis for further investigation into more complex relations using control-value theory.

Chapter 4 presents a meta-analysis on the impact of game features on learners' emotional experience and learning outcomes. The results suggested that game features that promoted control and value-related appraisals tend to lead to positive emotions and greater learning gains. The findings provide valuable insights for designing or selecting games for better learning outcomes.

Chapter 5 builds on the findings of Chapter 4 by further testing a specific game feature (i.e., incentive systems) that showed the most significant effects. To rigorously investigate incentive systems' impact on math learning, a value-added approach was used, where two versions of game were tested with children: a base version and a value-added version. The findings showed that games with incentive system were more likely to foster better in-game performance and positive math attitudes. Finally, Chapter 6 provides a summary of the contributions and limitations of the thesis and offers suggestions for future research directions to advance the field of study.

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Chapter 2. Manuscript 1

A Systematic Review of Secondary Students' Attitudes Towards Mathematics and Its Relations with Mathematics Achievement

Wen, R., & Dubé, A. K. (2022). A systematic review of secondary students' attitudes towards mathematics and its relationships with math achievement. *Journal of Numerical Cognition, 8*(2), 295-325. https://doi.org/10.5964/jnc.7937

Abstract

For a significant number of students, attitudes towards mathematics decrease notably during secondary education. Thus, there is an urgent need to improve students' mathematics attitudes because attitudes may negatively affect conceptual understanding of mathematics or mathematics performance. However, without a clear unified construct of mathematics attitudes, the ambiguity surrounding this construct prevents researchers from drawing broad conclusions about how to improve students' overall mathematics attitudes. Therefore, we conducted a systematic review of 95 studies focused on mathematics attitudes to clarify the construct and measurement of mathematics attitudes, and to provide a holistic picture of the relations between mathematics attitudes and math achievement. The review suggested the adoption of a multidimensional definition that regards mathematics attitudes as a combination of specific mathematical cognitions (value, gender roles/beliefs, confidence, self-concept), affects (enjoyment, anxiety), and behavioural intentions (i.e., willingness and tendency to spend more time learning mathematics subjects). The review then explored the relations between each subdimension of attitudes and mathematics performance. In general, anxiety and gender roles were negatively correlated with mathematics performance (r = -.27 to -.48; -.21) whereas enjoyment, self-concept, confidence, perceived value, and behavioural intentions were positively related to achievement (r = .27 to .68; .21 to .76; .34 to .42; .11 to .30; .21 to .34, respectively). Thus, mathematics attitudes appear to comprise three components with several subdimensions that each uniquely contribute to mathematics achievement. Going forward, researchers of mathematics attitudes should a) specify the components of mathematics attitudes used to guide their investigation b) adopt measures in line with their chosen components, and c) investigate how each subdimension of mathematics attitudes uniquely and cumulatively contribute to mathematics ability.

Keywords: mathematics attitudes, attitudes towards mathematics, mathematics achievement, control-value theory

Mathematics attitudes have long been studied in mathematics education, as ‘attitude’ is considered important for mathematics achievement (Neale, 1969). Critically, numerous studies report a significant decrease in students’ mathematics attitudes, and the phenomenon is particularly visible among secondary students (secondary students are defined based on US grading system and refer to students from Grades 7 to 12, ages 12 to 18 in this paper; Aiken, 1985; Fredricks & Eccles, 2002; Jacobs et al., 2002). For example, in an international mathematics test of 15-year-old students, less than one-third of students across all countries expressed positive attitudes towards mathematics (The Organisation for Economic Co-operation and Development [OECD], 2003). In fact, students in countries like Canada, Australia, and United Kingdom score below average on tests of interest and enjoyment in mathematics. Surprisingly, students with high mathematics achievement in countries like Finland and Japan also show negative mathematics attitudes (OECD, 2003). This is aligned with Fennema and Sherman (1976)’s claim that even students who are successful at mathematics may choose to avoid learning it.

Poor mathematics attitudes matter because they may reflect students’ prior experiences with mathematics (Ma & Kishor, 1997), may affect students’ conceptual understanding of mathematics (Markovits & Forgasz, 2017), and may even negatively impact mathematics performance (Ashcraft & Kirk, 2001; Hembree, 1990; Ma & Kishor, 1997; Passolunghi et al., 2016; Smetackova, 2015; Woodard, 2004). Low mathematics performance may further lead to avoidance of mathematics-related learning, higher drop-out rate from mathematics courses, and may eventually lead to job placement with lower incomes (Joensen & Nielsen, 2009; Rose & Betts, 2004). As such, mathematics attitudes are vital in achieving high performance and maintaining continued interests (Eccles et al., 1985; Haladyna et al., 1983; Lester et al., 1989).

Clearly mathematics attitudes are important, so there is an urgent need to find an effective way to improve secondary students' mathematics attitudes before they decrease irrevocably. However, researchers define mathematics attitudes inconsistently, making it difficult to draw wide-reaching conclusions.

Mathematics Attitudes: A Disjointed and Unclear Construct

Despite an existing body of research on the connections between mathematics attitudes and mathematics ability, dimensions of 'mathematics attitudes' may need to be better organized under a theoretical framework. Currently, the dimensions of mathematics attitudes being explored include confidence (Ganley & Vasilyeva, 2011), gender beliefs (also known as gender roles or gender stereotypes), which include beliefs that male students are more capable to learn mathematics compared to female students (Spencer et al., 1999), anxiety (Hauge, 1991), enjoyment (Ma, 1997), and actions associated with either seeking or avoiding learning mathematics (Simsek, 2016). However, solely focusing on an individual dimension of mathematics attitudes may prevent researchers from drawing overarching conclusions about mathematics attitudes and their connections to mathematics performance.

Researchers may benefit from a strong theoretical framework that more coherently defines mathematics attitudes as a construct and identifies its unique dimensions. The current lack of a theoretical framework is demonstrated by the explicit but idiosyncratic definitions used across studies, none of which are widely adopted (Di Martino & Zan, 2010), the many researchers who avoid clearly defining the term in their research (Hart, 1989), and others that implicitly define the construct via their measurements (Leder, 1985; Daskalogianni & Simpson, 2000). For example, some researchers define mathematics attitudes as mainly affective feelings, such as "individual's like or dislike toward mathematics" (Mohr-Schroeder et al., 2017, p. 215).

Some researchers argue attitude is the combination of affective feelings and cognitive beliefs, such as “a person’s disposition towards a subject, beliefs a person held about that subject” (Mirza & Hussain, 2018, p. 12). Besides affective feelings and cognitive beliefs, some researchers include behavioural intentions (i.e., the willingness to spend more time in mathematics learning) when defining mathematics attitude and describe it as “a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless” (Sengül & Dereli, 2013, p. 2527). These definitions each consider mathematics attitudes as comprised of different dimensions. As a result, the specific dimension and construct researchers adopt remains disjointed and unclear. Therefore, a single theoretical framework may facilitate more consistent categorization of the many dimensions of mathematics attitudes, which will in turn support researchers’ efforts to study relations among those dimensions.

Mathematics Attitudes and Mathematics Achievement

Positive mathematics attitudes may improve mathematics achievement (Ma & Kishor, 1997; Ma & Xu, 2004). However, due to the lack of a theoretical framework and construct for mathematics attitudes, the exact relations between mathematics attitudes and mathematics achievement are ambiguous (Ma & Kishor, 1997). In Ma and Kishor’s (1997) meta-analysis of 113 studies, evidence of a causal relation between mathematics attitudes and math achievement seem too weak to reach statistical and clinical significance, which the authors attribute to the divergent definitions and measurement practices in the field (Ma & Kishor, 1997). Since Ma and Kishor’s work, no study has systematically synthesized the definitions in use, categorized the various measurement practices, and analyzed the specific relations between each dimension of mathematics attitudes and mathematics achievement. Consequently, the definitions and measurements of mathematics attitudes in empirical work have continued down multiple divergent paths, and the relations between mathematics attitudes and mathematics achievement remain unclear.

Theoretical Frameworks for the Construct of Mathematics Attitudes

Zan and Di Martino (2007) suggest that the explicit definitions of mathematics attitudes used in research can be grouped into three categories: simple unidimensional definitions (affect focus), bi-dimensional definitions (affective and cognitive), and multidimensional definitions (affective, cognitive, behavioural). The use of multidimensional definitions by researchers suggests that a tripartite model could serve as the theoretical framework for mathematics attitudes. The tripartite model was originally used in the field of social psychology (Eagly & Chaiken, 1998), gradually gained a foothold in attitude theory (Breckler, 1984), and was later adopted in mathematics education (Leder, 1992; Ruffell et al., 1998). The tripartite model regards attitude as a complex construct with affective, cognitive, and behavioural components. It reflects not only the feelings associated with the object, but also the knowledge, ideas and beliefs about the object, and behavioural intentions towards the object. These three components are distinguishable as they each have their own antecedent: affects are the products of emotional stimuli, cognitions are products of previous exposure, and behavioural intentions are products of past reinforcement (Triandis, 1971). Though the three components of attitudes are unique, they may each influence actions (Breckler, 1984). Therefore, the tripartite model suggests that all three components (affective, cognitive, and behavioural) of mathematics attitudes may affect learning outcomes. The question at hand is whether the subdimensions of mathematics attitudes studied in the extant research align with the three components, as suggested by the tripartite model.

Control-Value Theory: A Theoretical Framework for the Relations Among Mathematics Attitudes and Achievement

One of the goals of improving mathematics attitudes is to promote academic achievement, thus the relations between the two phenomena has long been proposed and studied (Ethington & Wolfle, 1984, 1986; Lester et al., 1989; Ma & Xu, 2004; Nicolaidou & Philippou, 2003; Sherman, 1982; Suydam & Weaver, 1975). However, the existing research has not consistently found strong relations among mathematics attitudes and mathematics achievement: some researchers have claimed there is no causal relation between the two constructs (Quinn & Jadav, 1987), some suggest the correlation is weak (Aiken, 1970, 1976; Ma & Kishor, 1997), while others indicate a relatively strong correlation (Eldersveld, 1983; Kloosterman, 1991; Ma & Xu, 2004; Minato, 1983; Nicolaidou & Philippou, 2003; Yenilmez & Duman, 2008). Given the divergent definitions and measurement practices identified in this review, the inconsistent evidence could be due to researchers' use of disparate instruments (Ma & Kishor, 1997). Further, if mathematics attitude is a multidimensional construct then the relations among mathematics attitudes and mathematics achievement might not be a simple linear causal connection. Pekrun's control-value theory is well-suited for understanding how the three components of mathematics attitudes influence each other, and further affect learning outcomes.

Appraisals to Achievement Emotions

According to control-value theory, there are two types of appraisals that may affect achievement emotions (i.e., emotions closely related to achievement) and they are organized temporally, as proximal or distal (Pekrun & Perry, 2014). For proximal, two sub-types of appraisals are critical for the arousal of achievement emotions: control-related appraisals and value-related appraisals (Pekrun & Perry, 2014). Control-related appraisals are appraisals of one's controllability over achievement activities and outcomes. They consist of expectancies (confidence in one's ability to perform the action and achieve the goal successfully), causal attributions, and self-concepts of ability in the subject (e.g., "I'm good at mathematics"). Value-related appraisals describe the importance one places on mathematics activities and outcomes, either intrinsically or extrinsically (Pekrun & Perry, 2014). For example, an intrinsic value for mathematics can arise from the interests and fun during the learning process while an extrinsic value can arise from the benefits that being good at mathematics will produce (e.g., a better career and life). Achievement emotions are products of perceived value and control, with high value and control may lead to enjoyment while high value but low control may result in anxiety (Pekrun, 2006). Distal appraisals, such as achievement goals and gender roles, can also impact achievement emotions by influencing control-value appraisals (Pekrun, 2006). For example, girls with stereotypical gender beliefs/roles may have lower confidence and may evaluate themselves as possessing a low ability in learning mathematics compared to boys; this may generate achievement emotions such as anxiety.

Achievement Emotions to Behaviours

Control-value theory argues that the effects of achievement emotions on students' performance may be mediated by factors such as motivational behaviours (Pekrun, 2006). Positive achievement emotions (e.g., enjoyment) may reinforce the learning behaviour's, leading to reengagement in the learning activity, and result in better performance. In contrast, negative achievement emotions (e.g., anxiety) may undermine motivation and lead to avoidance of the learning task, which may further affect achievement (Pekrun, 2006).

To sum up, control-value theory can be used as a theoretical framework to predict the relations among the components of mathematics attitudes and mathematics achievement. From a control-value lens, mathematics attitudes are likely to affect achievement via the cognitive components influencing the affective components, which in turn may trigger different behaviours that lead to changes in mathematics achievement (Pekrun, 2006; Pekrun & Perry, 2014; Pekrun et al., 2007). Within the parlance of the theory, the cognitive and affective components of mathematics attitudes are referred to as appraisals and achievement emotions, respectively. Given this lens, this paper aims to explore the complex relations among mathematics attitudes and mathematics performance revealed in the current literature, and to evaluate whether they align with control-value theory.

Research Questions

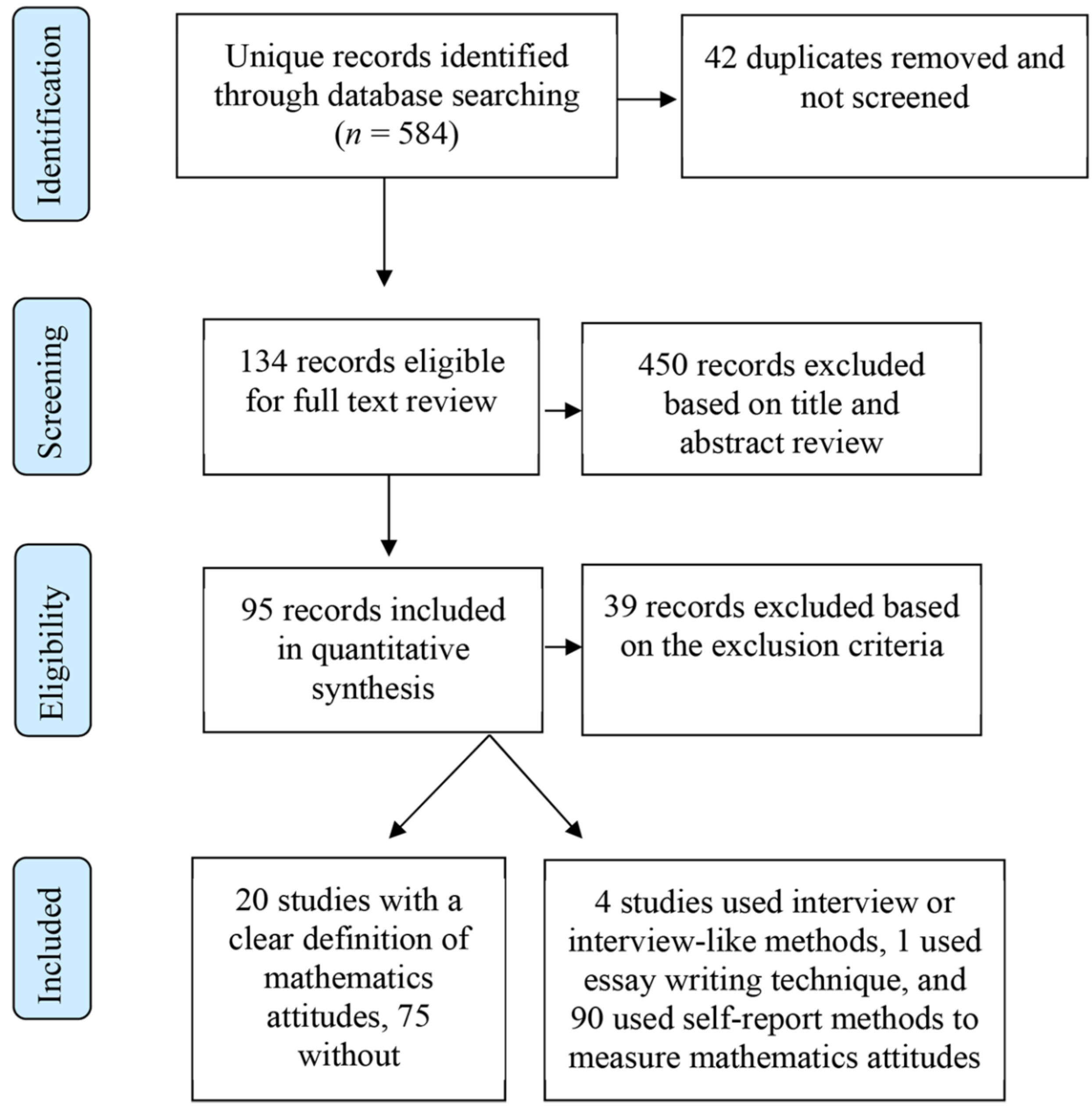
Two questions guided a two-phased systematic review: for secondary students, (a) what are the components of mathematics attitudes that predominate the research literature and (b) what is the relation between each component and mathematics achievement? The present review focuses on secondary education because many students start to develop negative mathematics attitudes during the last two years of elementary education (Haciomeroglu, 2017) and these negative attitudes became particularly visible during secondary education (Aiken, 1985; Pajares & Miller, 1994). Further, mathematics attitudes in this article refer to feelings, beliefs, and intentions that are closely related to mathematics achievement (e.g., classroom and test performance); as such, studies of adults were excluded as their attitudes are not test-specific (e.g., attitudes towards daily usage of numbers, such as figuring out sales tax). Studies focusing on college and university students were also excluded because attitudes towards college or university level mathematics subjects, such as statistics and calculus, differ from attitudes about secondary mathematics (Hedges & Hardness, 2017). Secondary students who hold positive mathematics attitudes may not transfer those attitudes towards statistics, possibly due to statistics involving different reasoning and relying heavily on language (Carmona et al., 2005, as cited in Hedges & Hardness, 2017). Therefore, it is critical to identify the components of secondary students' mathematics attitudes so that researchers and educators may purposefully investigate which subdimensions of mathematics attitudes undergo an important negative shift at this stage.

Phase 1. Components of Mathematics Attitudes

To answer question (a), 'math*' AND 'attitude* OR view* OR disposition* OR perspective* OR perception* OR perceive*OR think* OR opinion*' were searched in three databases (PsycINFO, ERIC EBSCO, Scopus), limited to English peer-reviewed articles. Based on these preliminary criteria, the initial search returned 626 articles. After removing duplicates, 584 articles were left for further review. Subsequently, a screening process was conducted. After reviewing titles and abstracts, 134 records were eligible for further review. Articles that did not just mention attitudes, but specifically examined them as part of the empirical work in secondary education (grades 7 to 12), were included for further analysis. If grades were not reported, ages (12 to 18) were used as inclusion criteria. Studies that overlapped with targeted grades or ages (e.g., grade 6-8 or ages 10-15) were also included. Several exclusion criteria were applied: dissertation/conference proceedings; annotated bibliographies; special education; teachers'/parents' attitudes towards mathematics; a focus solely on non-targeted students (such as elementary students, university students, and adults). After a full-text analysis of these 134 studies, 95 studies meeting the criteria were included in the review (see Figure 1). The 95 studies included were then organized according to the following codes: 1) author(s), 2) year of publication, 3) definition of mathematics attitudes, 4) techniques used to measure mathematics attitudes, 5) aspects of mathematics attitudes being measured, 6) reported validity and reliability of the instruments.

Figure 1

Systematic Review Flow Diagram



Definitions

Among 95 articles, only 20 clearly stated their definition of mathematics attitudes, while the remaining articles did not define the term at all. Di Martino and Zan (2010) suggested that definitions of mathematics attitudes can be categorized into one of three types: unidimensional definitions reflecting emotional dispositions (affective components), bi-dimensional definitions (affective and cognitive components, e.g., emotions and beliefs), or multidimensional definitions (affective, cognitive, and behavioural components, e.g., emotions, beliefs, and behaviours). However, there has been no review to examine if these three types of definitions are adopted within and across studies of mathematics attitudes. Thus, a three-type definition framework was adopted to guide the organization of studies. Findings revealed that most of the studies could be classified into Zan and Di Martino's three categories, except one study from the bidimensional category that defined attitudes from affective and behavioural perspective (see Table 1).

Table 1

Mathematics Attitudes Definitions

n	Types of definition	Aspects of attitudes	Examples of definitions	Citation
7	Unidimensional definition	Affective components	"the emotional tendencies developed by individuals"	(Eskici et al., 2017, p. 64)
			"a general emotional disposition toward the school subject"	(Haladyna et al., 1983, p. 20)
			"a positive or negative response towards mathematics that is relatively stable, similar to what some might call dispositions"	(Hemmings et al., 2011, p. 692)
			"one's general feeling of favor or otherwise toward some stimulus objects"	(Mallam, 1993, p. 223)
			"someone's basic liking or disliking of a familiar target"	(Mavridis et al., 2017, p. 1452)
			"individual's like or dislike toward mathematics"	(Mohr-Schroeder et al., 2017, p. 215)
			"predisposition to respond favourably or unfavourably to mathematics"	(Murimo, 2013, p. 75)

6	Bi-dimensional definition	Affective & cognitive components	"the sum total of a man's inclinations, feelings, prejudice or bias, preconceived notions, ideas, fears, threats and conviction about any topic"	(Ifamuyiwa & Akinsola, 2008, p. 570)
			"an aggregated measure of mathematics self-confidence, perceived usefulness, and enjoyment of mathematics"	(Kiwanuka et al., 2017, p. 3)
			"either positive or negative responses, in terms of importance, difficulty, and enjoyment"	(Ma, 1997, p. 222)
			"a person's disposition towards a subject, beliefs a person held about that subject"	(Mirza & Hussain, 2018, p. 12)
			"the result of highly interdependent aspects of beliefs and emotions"	(Moyer et al., 2018, p. 117)
		Affective & behavioural components	"emotional disposition towards mathematics, such as a positive or negative response towards mathematics, or a liking or disliking of mathematics, or a tendency to engage or avoid mathematical activities"	(Yang, 2015, p. 252
7	Multidimensional definition	Affective, cognitive & behavioural components	"A persons' attitude to an idea or object determines what the person thinks, feels and how the person would like to behave towards that idea or objects"	(Etuk et al., 2013)
			"emotional reaction to an object, behavior tendency towards an object and beliefs about the object"	(Idil et al., 2016, p. 210)
			"a mental, emotional and behavioural reactionary predisposition a person develops toward mathematics"	(Koyuncu & Dönmez, 2018, p. 1631)
			"an aggregated measure of a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless"	(Mutohir et al., 2018)
			"attitudes consist of cognitive, affective and behavioural reactions that individuals display towards an	(Sanchal &Sharma, 2017)

	object or the surrounding based on their feelings or interest”	
	“a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless”	(Sengül & Dereli, 2013, p. 2527)
	“a tendency attributed to the individual and regularly constitutes his/her thoughts, feelings and behaviours related to the psychological incident”	(Zakariya, 2017)

Unidimensional Definition

Seven articles used unidimensional definitions, all of which referred to emotional dispositions towards mathematics. Two of these studies defined mathematics attitudes as “general emotional disposition toward the school subject” (Eskici et al., 2017, p. 64; Haladyna et al., 1983, p. 20). Five made more specific reference to either liking or disliking mathematics. For example, Mallam (1993, p. 223) defined mathematics attitudes as “one's general feeling of favor or otherwise toward some stimulus objects.” Two studies from this category used measures that seemed to contradict the definitions they adopted. Specifically, Mavridis et al. (2017, p. 1452), defined mathematics attitudes as “someone’s basic liking or disliking of a familiar target” which these researchers’ measured with survey items not only tapping affective components (e.g., “I have really enjoyed studying mathematics in school”), but also involving value (e.g., “Mathematics is important in everyday life”) and behavioural intensions (e.g., “I am willing to take more than the required amount of mathematics”). Likewise, Murimo (2013) measured students’ perceived value and usefulness of mathematics together with affective components. The disconnect between measures and definitions of mathematics attitudes implicitly suggests there may be a better way to define the term rather than focusing solely on one dimension.

Bi-Dimensional Definition

Six articles used bi-dimensional definitions, all acknowledging the importance of affects. Five articles suggested that mathematics attitudes are the combination of affects and beliefs towards the subject (Ifamuyiwa & Akinsola, 2008; Kiwanuka et al., 2017; Ma, 1997; Mirza & Hussain, 2018; Moyer et al., 2018). For example, Moyer et al. (2018, p. 117) defined it as “the result of highly interdependent aspects of beliefs and emotions.” Similarly, Mirza and Hussain, (2018, p. 12) defined attitudes as “a person’s disposition towards a subject, beliefs a person held about that subject.” Two other studies identified components of mathematics attitudes via their measures. Kiwanuka et al. (2017, p. 3) suggested measuring “mathematics self-confidence, perceived usefulness, and enjoyment of mathematics” while Ma (1997, p. 222) proposed “importance, difficulty, and enjoyment.” Both suggested measuring enjoyment in terms of the affective components but hold different opinions on cognitive components. Another study defined the term as including affective and behavioural aspects (Yang, 2015, p. 252), suggesting that attitude is an “emotional disposition towards mathematics, such as a positive or negative response towards mathematics, or a liking or disliking of mathematics, or a tendency to engage or avoid mathematical activities.” The same study suggested to mainly measure attitudes via “self-confidence, usefulness of mathematics, and motivation.” Studies that regard mathematics attitudes as two dimensions held divergent views on definitions and components of mathematics attitudes. Therefore, studies adopting bi-dimensional definition seemed to not agree with one another.

Multidimensional Definition

Seven articles used multidimensional definitions, which treated mathematics attitudes as consisting of three dimensions: cognitive (i.e., the knowledge of mathematics, ideas and beliefs towards mathematics), affective (i.e., feelings associate with mathematics) and behavioural (i.e., actions towards mathematics; Etuk et al., 2013; Idil et al., 2016; Koyuncu & Dönmez, 2018; Mutohir et al., 2018; Sanchal & Sharma, 2017; Sengül & Dereli, 2013; Zakariya, 2017). For example, Idil et al. (2016, p. 210) proposed that a mathematics attitude is an "emotional reaction to an object, behavior tendency towards an object and beliefs about the object." Koyuncu and Dönmez (2018, p. 1631) suggested mathematics attitude is "a mental, emotional and behavioural reactionary predisposition a person develops toward mathematics."

Given that there are three approaches to define mathematics attitudes, one may wonder which approach is the most suitable for studying mathematics attitudes. Unlike the other two definitions, the multidimensional definition is supported by a theoretical framework—the tripartite model of attitudes (Di Martino & Zan, 2003). Moreover, with the support of the tripartite model, the multidimensional definition may help resolve the confusion caused by different measures of mathematics attitudes being used in the field. For example, McLeod (1992) argued that the confusion/inconsistency is caused by a lack of guiding theoretical framework. Finally, the multi-dimensional definition contains all aspects of attitudes used in research and theory. Therefore, this paper will adopt a multidimensional definition of mathematics attitudes to reflect both theoretical viewpoints and practical uses.

Measurements

Having adopted the multidimensional definition, the next step is to identify how the specific components of affect, cognition, and behaviour are most commonly measured. Doing so will further clarify what each dimension of mathematics attitudes is actually being assessed. This was achieved through a systematic analysis of the mathematics attitudes measurement tools used in the identified 95 studies.

There are three commonly used techniques to measure mathematics attitudes (Corcoran & Gibb, 1961): observations, interviews, and self-report methods (e.g., questionnaires/scales). In this review, none of the studies used observations, four used interview or interview-like methods, one used essay writing techniques, and ninety used self-report methods (see Table 2). Observations made by teachers are suggested to be a good indicator of students' attitudes (Bialangi et al., 2016), but previous literature showed mixed results. Ellingson's study (1962) showed a positive correlation between teacher's observation and students' self-reported attitudes while Brown and Abell's study (1965) suggested that observation was inadequate. This could be explained by individual differences, where teachers in different studies interpret behaviour differently. Interviews may overcome some of the limitations of observation by asking students' feelings directly. Four studies used interview or interview-like methods (open ended questions) to get students talking about their mathematics attitudes (Hannula, 2002; Joffe & Foxman, 1984; Kaiser-Messmer, 1993; Moyer et al., 2018). These interviews covered the value of learning math, perceived difficulty, confidence, emotions, and time spent on mathematics. One potential limitation to this approach is that students may not feel comfortable expressing their negative feelings face-to-face. One study used an essay writing technique, where students were asked to write an essay on their attitudes towards mathematics (Di Martino & Zan, 2010). Common topics covered in the essays were emotional dispositions, vision of mathematics, and perceived competence of learning mathematics.

For the 90 studies using self-reports, the specific instruments, latent variables, reliability, and citation were documented (see Table 3 and Appendix A). Though most of the studies in this review used self-reports, different instruments were applied. Interestingly, only 25 studies used well-known scales, including Fennema-Sherman Mathematics Attitudes Scale (Fennema & Sherman, 1978), Aiken's Math Attitude Scale (Aiken, 1970), Attitudes Toward Mathematics Inventory (Tapia & Marsh, 2004), The Mathematics Attitude Scale (Aşkar, 1986), and Sandman's Mathematics Attitude Inventory (Sandman, 1980), among which, Fennema-Sherman Mathematics Attitudes Scale and Aiken's Math Attitude Scale being the most cited (see Table 2). In contrast, 34 studies used scales modified from literature (including ones adapted from PISA and TIMSS's), 24 studies developed their own measures, and seven studies did not clearly state which scales they used.

Table 2

Techniques Used for Measuring Mathematics Attitudes in 95 Reviewed Articles

Techniques	**Number of studies**	
Observations	0	
Essay writing	1	
Interviews	4	
Self-reported methods (scales)	90	
	Cited in 95 studies reviewed	Used in 95 studies reviewed
Fennema-Sherman Mathematics Attitudes Scales (FSMAS)	39	10
Aiken's Math Attitude Scale	31	4
Attitudes Toward Mathematics Inventory (ATMI)	10	6
The Mathematics Attitude Scale	5	3
Sandman's Mathematics Attitude Inventory (MAI)	5	2
Modified scales	1	34
Self-developed scales	NA	24
No specific measure described or identified	NA	7

Table 3

Summary of the Instruments Used

Name	**Cognitive components**	**Affective components**	**Behavioural components**	**Type of scale**	**Used in studies reviewed**
FSMAS	Confidence (9)[b], Value (7), Gender roles (7), Attitude towards success (6), Perception of parents/teacher's attitudes (5)	Anxiety (3), Affect (3),		Bi-dimensional	10
ATMI	Confidence (5), Value (5)	Enjoyment (6), Motivation (5)		Bi-dimensional	6
Aiken's Math Attitude Scale	Value (4)	Enjoyment (4)		Bi-dimensional	4
The Mathematics Attitude Scale		Enjoyment (3)		Unidimensional	3
MAI	Perception of math teacher (2), Value (2), Self-concept in math (2)	Anxiety (2), Enjoyment (2), Motivation (2)		Bi-dimensional	2
Modified Scales	Value (18), Confidence (12), Gender roles (5), self-concept (5), attention (1), Perceived difficulty (1), Attitudes towards school (1), Learning (1), math teaching (1), Teacher's attitudes (1), Nature of math (1), Attitudes towards school (1)	Enjoyment (16), Anxiety (8), Motivation (2), Feelings (1)	Behavioural Intentions (4)	Multidimensional	34

Self-developed Scales	Value (13), Gender roles (8), Self-concept (8), Confidence (4), Perception of difficulty (3), Parents' perception (3), Teachers' impact (3), Perceived control (1), Rules (1), Intention (1), Home-support (1), Home-process (1), Society (1) Fatalism (1)	Enjoyment (13), Boredom (4), Anxiety (5), Feelings (1)	Behavioural Intentions (7), More attention (1)	Multidimensional	24
No specific measure described or identified					7

[a]The number of studies in the review that used a given scale as a measurement tool.

[b]The number of studies in the review that adopted each component.

Fennema-Sherman Mathematics Attitudes Scales

Fennema-Sherman Mathematics Attitudes Scale (FSMAS) was one of the most popular scales in this review, with 39 articles out of 95 citing it and 10 studies applying it. FSMAS was initially designed to explore gender differences in mathematics learning and other factors that influence the selection of mathematics courses (Fennema & Sherman, 1978). FSMAS is a bi-dimensional scale. It contains nine subcategories, among which, seven are cognitive components, including students' confidence, gender roles (see math as a male domain subject), perceived value, attitudes toward success, and parents' and teacher's mathematics attitudes. The other two are affective components, such as anxiety and motivation. However, only two studies adopted the full FSMAS scale, while other studies only adopted part of the scale without providing details as to how and why certain components were chosen. As shown in Table 3, the most commonly used components in FSMAS were confidence, followed by value, gender roles, attitudes towards success, and perception of parents'/teachers' attitudes, with the percentage of 90%, 70%, 70%, 60%, and 50% respectively.

Attitudes Toward Mathematics Inventory

The Attitudes Toward Mathematics Inventory (ATMI) developed by Tapia and Marsh (2004) was another commonly adopted scale, with 10 articles citing it and six applying it (Tapia & Marsh, 2004). ATMI is a bi-dimensional scale, with confidence and value as cognitive components, and enjoyment and motivation as affective components. As shown in Table 3, six studies adopted ATMI or short versions of it. Confidence was adopted in all six studies, while the other three components in the scale (value, enjoyment, and motivation) were equally applied in 83% of the studies.

Aiken’s Math Attitude Scale

Aiken’s Math Attitude Scale was cited by 31 studies in this review but only four used it to measure mathematics attitudes. Aiken’s Math Attitude Scale is a bi-dimensional scale, which only contains enjoyment as the affective component and value of mathematics as the cognitive component. In this review, all four studies that adopted Aiken’s Math Attitude Scale applied both enjoyment and value components.

The Mathematics Attitude Scale

The Mathematics Attitude Scale developed by Aşkar (1986) was cited by five but applied by three studies. Compared to others, this scale was not widely accepted, perhaps because it is unidimensional and solely assesses the affective aspects of attitudes. The affective aspect is measured through 20 items focused on students’ enjoyment of learning mathematics. As such, this scale was limited to measuring enjoyment and this may explain its infrequent use.

Sandman’s Mathematics Attitude Inventory

Sandman’s mathematics attitude inventory (MAI) was cited by five studies and applied by two. It is designed to measure students’ mathematics attitudes from Grade 7 to Grade 12 (Sandman, 1980). MAI is a bi-dimensional scale, with perception of mathematics teachers, value of mathematics, and self-concept as cognitive components, and anxiety, enjoyment, and motivation as affective components. The two studies using the MAI did not adopt the full scale, with each study only using half of the six components covered by the measure. This selective use of mathematics attitude measures further reinforced the disconnect between researchers’ theoretical approach and the tools used to study mathematics attitudes.

No Clear Statements Regarding Scales

Surprisingly, 7% of the studies in this review did not provide any information about the measures they used, but still reported an increase or decrease in student's mathematics attitudes. While it may be possible to contact the authors for more details and infer the theoretical approach by applying an item-by-item analyses of the measures, it is critical for researchers to specify the measures used in the study so that future work can meaningfully compare their results against other studies.

Modified Scales

Approximately 36% of studies adapted existing measures (n = 34), with the majority of the adapted measures only being used in a single study. The modified measures covered all three dimensions of mathematics attitudes (affective, cognitive, behavioural). The reliability varied from study to study as different measures were applied. After removing unreliable measures (α less than .7), the commonly used components of mathematics attitudes being indexed by these modified measures were identified (see Table 3). Among all the components, value, enjoyment, and confidence were most common, with each being cited in 53%, 47%, and 35% of the studies, respectively. The second most common components were anxiety, behavioural intentions, gender roles, and self-concept, applied in 24%, 24%, 15%, and 15% of the studies, respectively. The high number of modified scales and their diverse composition suggests that researchers may need a more comprehensive tool to reflect many dimensions of mathematics attitudes.

Self-Developed Scales

Approximately 25% of the studies used self-developed measures (n = 24). With the exception of three that did not identify the components indexed by their instruments, the other self-developed measures covered all three dimensions of mathematics attitudes (affective, cognitive and behavioural components). The Cronbach alpha for these measures ranged substantially, from .34 to .98. Given this, five studies' measures with a reliability coefficient less than .7 were not considered further. Among all the components covered by self-developed measures, value, gender roles, self-concept, and confidence were commonly used to represent cognitive aspects of attitudes, each were cited in 54%, 33%, 33%, and 17% of studies. Enjoyment, anxiety and boredom were commonly used to assess affective components of mathematics attitudes, each were cited 54%, 21%, and 17% of studies respectively. Finally, behavioural intentions were often used to represent behavioural components (29% of studies). Similar with modified scales, the high number of self-developed scales further suggests that the available measures of mathematics attitudes may not align with researchers' approach to the subject.

A Praxis Construct of Mathematics Attitudes—Tripartite Construct

In general, 58 measures identified in this review were either self-developed or adapted from literature. This lack of agreement may represent the numerous facets of "attitudes" that researchers deem worthy of study. Even for scales commonly cited in the literature (e.g., Fennema-Sherman Mathematics Attitudes Scale and Aiken's Mathematics Attitude Scale), even fewer studies actually used them; suggesting these scales may fail to represent researchers' view of mathematics attitudes. Thus, there is a need to clarify the facets of mathematics attitudes and identify the dominant components of attitudes found in the literature. Based on the frequency of components used in this systematic review (see Table 3 and Appendix A), a praxis derived construct of mathematics attitudes was generated to represent the understanding of mathematics attitudes as indicated by research practice (see Table 4).

Table 4

Praxis Construct of Mathematics Attitudes

Cognitive components	Affective components	Behavioural components
Value (49)[a]	Enjoyment (44)	Behavioural intentions (11)
Confidence (30)	Anxiety (18)	
Gender roles (20)		
Self-concept (15)		

[a]The number in parentheses represent the number of citations in this review.

There are three aspects to mathematics attitudes in the tripartite construct of mathematics attitudes: cognitive, affective, and behavioural. For the cognitive aspect, value of mathematics, confidence in solving mathematical problems, gender role beliefs in learning mathematics, and self-concept of mathematics were the most cited components. Among the 90 studies that used self-reports in this systematic review, each cognitive component was reported in 54%, 33%, 22%, and 17% of the studies respectively. Importantly, confidence (or self-efficacy) and self-concept (sometimes called self-beliefs, self-evaluation) are very similar terms, with one measuring the degree to which students believe they can handle mathematics' difficulties and get good outcomes while the other measures one's perception of themselves with mathematics. One example item for confidence is "I can get good grades in mathematics" (Fennema & Sherman, 1978). An example item for self-concept is "I have always believed that mathematics is one of my best subjects" (Stankov et al., 2012). For the affective aspect, enjoyment and anxiety were the common components being cited, each affective component was reported in 49% and 20% of studies respectively. For the behavioural aspect, behavioural intentions was the only commonly cited component. It was reported in 12% of studies. As such, these specific components were adopted into our tripartite construct of mathematics attitudes and will be used in the subsequent analysis of the relations between mathematics attitudes and mathematics achievement.

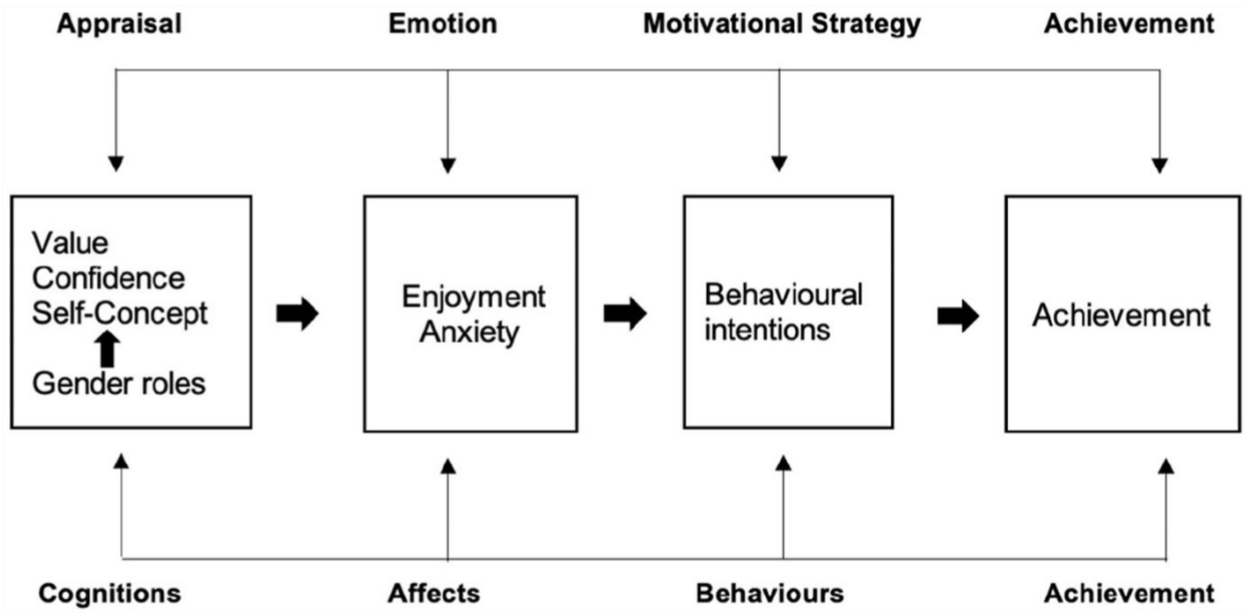
Other components such as perception of parents'/teachers' attitudes and motivation were also frequently used in different studies but were excluded due to the overlap with other components. Perception of parents' and teachers' attitudes was used in 14 studies out of 95. We agree with the importance of both parents' and teachers' attitudes in affecting students' mathematics attitudes and corresponding performance (Aiken, 1970). However, students' perception of parents' and teachers' attitudes are not appropriate in representing their own mathematics attitudes for two reasons. One, parents influence children's subject-specific attitudes in three ways: through support and encouragement, through expectation of children's performance, and through their own value beliefs of mathematics (Poffenberger & Norton, 1959). These three influences are reflected in students' self-concept, perception of gender roles in the subject, and value of mathematics, which are already present in the praxis construct of mathematics attitudes. Moreover, there may be differences between student's perception of parents'/teachers' attitudes and the actual attitudes held by parents/teachers. Therefore, students' perception of parents'/teachers' attitudes are not suitable to represent students' own attitudes due to overlap and lacking accuracy. Another commonly used component, motivation, was also excluded. Though 10% of the studies in this review considered motivation, others think that motivation can be expressed through other subdimensions of attitude (Chamberlin, 2010). For example, one may feel joy when they are highly motivated to learn math. Also, items used to measure motivation were mainly measuring student's desire to spend more time on math (e.g., "I would like to spend more time doing mathematics after class"), which overlap with the behavioural component. Therefore, motivation was removed due to overlapping with other included constructs.

Phase 2. Relations Between Mathematics Attitudes and Mathematics Achievement

Having clarified the subdimensions and constructs of mathematics attitudes in phase 1, we will unpack the relations between mathematics achievement and each component of mathematics attitudes. Theoretically, the relations among components in mathematics attitudes and mathematics achievement can be organized based on control-value theory. For instance, cognitive components of mathematics attitudes such as value, confidence, self-concept, and gender roles line up with control-value theory' appraisal factors, while the behavioural component fits the motivational strategy (either seeking or avoiding the task). Affective components such as enjoyment and anxiety align with achievement emotions, and link cognitive and behavioural components as a whole (see Figure 2). This assumption was tested in the following literature review of the relations between mathematics performance and each component in mathematics attitudes.

Figure 2

Control-Value Theory Framework of How Mathematics Attitudes Relate to Mathematics Achievement

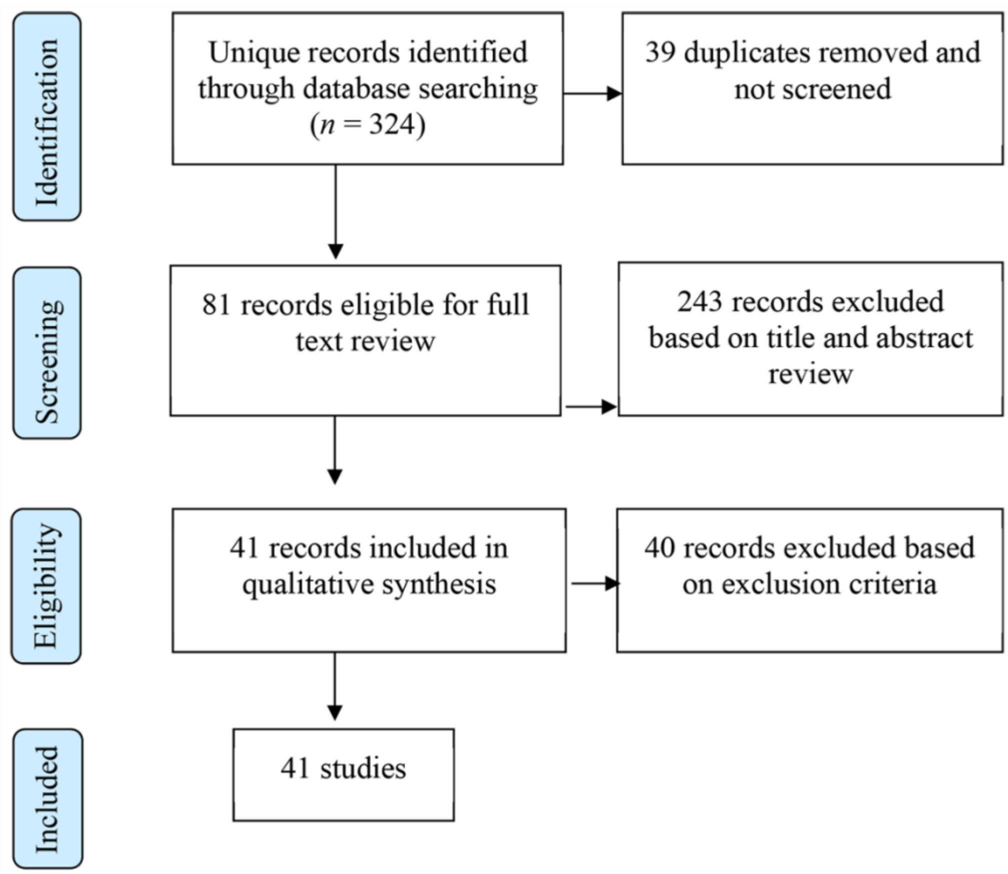


Note. Control-value theory serves as a framework to understand the relationship between mathematics attitudes and mathematics achievement. Adapted from "The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice," by R. Pekrun, 2006, *Educational Psychology Review*, *18*(4), 315-341. Copyright 2006 by Springer.

For the relations between each component in mathematics attitudes and mathematics performance, keywords were searched together (e.g., value*, gender* OR stereotype*, confidence*, self-concept*, enjoy*, anxi*, engag* AND "math* AND performance* OR achievement* OR grade* OR score* OR success"). Snowball sampling and forward citing techniques were also applied to add articles. The same exclusion criteria used in Phase 1 of the study was applied when reading abstracts. Finally, 41 research studies were identified and reviewed in total (see Figure 3). Among reviewed studies, 95% did not report socioeconomic status. In terms of country, except the review studies or the one that applied PISA data from different countries, 86% of them were conducted in North America (e.g., United States, Canada) and Europe (e.g., Germany, Greece, Portuguese, Netherlands), with 14% focused on Asian students (e.g., India, China, Singapore).

Figure 3

Systematic Review Flow Diagram

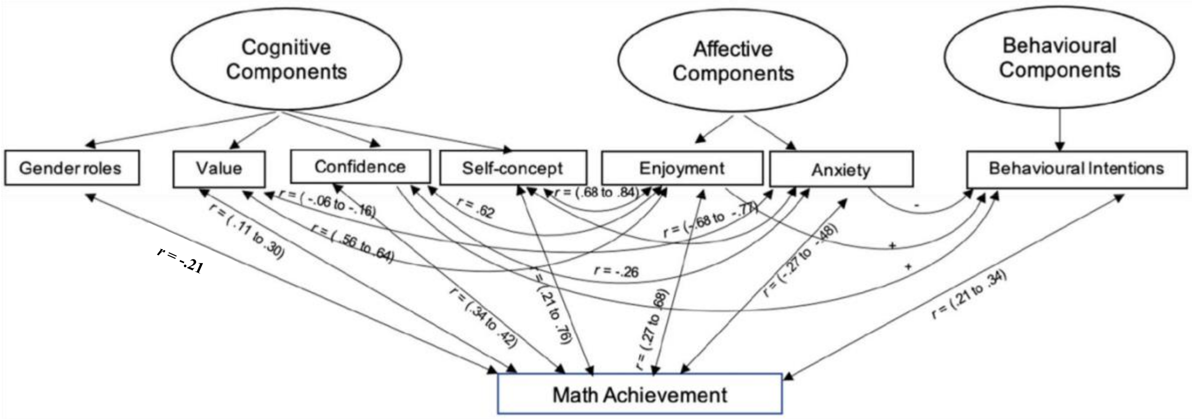


Relations Between Mathematics Attitudes and Mathematics Achievement

Based on the result of the review, Figure 4 illustrates the unique relations among each component of mathematics attitudes and mathematics achievement. In what follows, we briefly explain these results.

Figure 4

Relations Among Mathematics Attitudes and Mathematics Achievement



Note. This figure demonstrates the relations in the reviewed empirical studies. The ranges represent correlations from multiple studies while a single *r* is one estimation from one study.

Gender Roles/Beliefs/Stereotypes

Generally, having strong gender roles/beliefs/stereotypes makes students regard mathematics as a male dominated subject (Hyde et al., 1990). Gender role/belief is a product of cultural reinforcement and results in the view of boys being better skilled at mathematics than girls (Fennema et al., 1990). Historically, Aiken (1970) argued that this gender stereotype does not develop until the secondary school level. More recently, a study of 1300 adolescents asked students whether they see mathematics as a male domain, female domain, or gender-neutral domain subject. Most students viewed it as male domain subject, with this belief being particularly stronger amongst boys (Brandell & Staberg, 2008). This result is in line with Cai et al.'s (2016) finding, who argue that a lot of adolescents possess gender stereotypes in mathematics and science subjects.

Both control-value theory and empirical studies indicated that internalized gender roles may affect students' mathematics achievement (Li, 1999; Pekrun, 2006). For example, Fennema and Sherman (1978) examined 1,233 students' gender stereotypes and the results showed that girls' gender roles/beliefs negatively correlated with their mathematics achievement (r = -.21). Control-value theory also implies that gender roles may affect students' achievements by influencing control-value appraisals (e.g., value, self-concept, confidence) and achievement emotions (Pekrun, 2006). For instance, female students who have stronger gender roles/beliefs tend to evaluate themselves as possessing a low capacity in learning mathematics compared to boys. This may generate more anxiety while doing mathematics and result in lower mathematics achievement. However, none of the studies reviewed examine the mediating effect of control-value appraisals and achievement emotions in secondary education. So, whether value, self-concept, confidence, and achievement emotions such as anxiety play a role in mediating the effects of gender roles/ beliefs on mathematics achievement remains unclear.

Value

Perceived value measures how much importance students place on mathematics. Control-value theory suggests that student's value appraisals affect their achievement emotions and further affect motivational behaviour and academic achievement (Pekrun, 2006). In line with this assumption, studies showed that value positively predicted enjoyment (r = .56 to .64) and negatively predicted anxiety (r = -.06 to -.16; Luo et al., 2016; Peixoto et al., 2017). As for studies examining the direct relation between value and achievement, the results indicated a weak correlation. For example, Chouinard et al.'s (2007) analysis of 759 Grade 7 to Grade 11 students showed that perceived value had a significant but small relationship with mathematics performance (r = .29). In Hammouri's (2004) analysis of TIMSS data, students' perceived value of mathematics also related to mathematics performance (r = .11). Moreover, female students seemed to produce a stronger correlation between value and achievement than male students (r = .30/.26 respectively; Lim & Chapman, 2015).

Confidence

Confidence in mathematics is the degree to which students believe they can handle mathematics' difficulties and get good outcomes. Based on control-value theory, confidence is supposed to affect achievement through influencing achievement emotions and motivation (Pekrun, 2006). In line with this assumption, research indicates a strong positive correlation between confidence and enjoyment (r = .62) and a negative correlation between confidence and math anxiety (r = -.26; Luo et al., 2016). Moreover, confidence was also a good predictor of students' continuing participation in mathematics (Barkatsas et al., 2009). In fact, students with high levels of confidence spent more time learning mathematics (Barkatsas et al., 2009). The empirical studies that explore the direct relations between confidence and achievement suggested that confidence was an important predictor of mathematics performance (Lee, 2009; Marsh & Yeung, 1997). An analysis of PISA 2003 data indicated that confidence was a significant predictor of mathematics achievement for fifteen-year-old students (Liu & Wilson, 2009). Hammouri's (2004) analysis of TIMSS 2003 data also indicated that students' confidence was correlated with mathematics performance (r = .34). Similarly, Perry et al. (2016) tested 84 high school students and showed a moderately sized positive relation between confidence and algebra achievement (r = .42). In sum, both theory and empirical evidence suggest that confidence is a good predictor of emotions, behavioural intentions, and mathematics achievement. The high level of confidence may lead to positive achievement emotions, which may further result in more engagement in learning mathematics and better learning outcomes.

Self-Concept

Self-concept is one's perception of themselves in a certain environment (Shavelson et al., 1976). Mathematics self-concept is students' perception of themselves in mathematics. Based on control-value theory, self-concept's impact on mathematics achievement is mediated by achievement emotions and motivational behaviours (Pekrun, 2006). Control-value theory implies that appraisal has a reciprocal relation with emotions such that self-concept and emotions predict each other (Pekrun, 2006). In line with this assumption, empirical research has reported strong positive correlations between self-concept and enjoyment and negative correlations between self-concept and anxiety (Goetz et al., 2010; Jain & Dowson, 2009). Hembree's (1990) meta-analysis examined the relation between self-concept and anxiety and revealed a strong correlation (r = -.71). Ahmed and colleague's (2013) longitudinal design assessed self-concept and anxiety in three different time points across one academic year and revealed a consistent reciprocal relation in which self-concept's effect on anxiety was twice larger than anxiety's effect on self-concept. Goetz et al. (2010) systematically tested the relation between self-concept and five emotions (enjoyment, pride, anxiety, anger, and boredom), and the results also suggested strong correlations between self-concept and enjoyment/anxiety in mathematics (r = .68 to .84/ r = −.68 to −.77). Moreover, the relationship was stronger for higher grade levels in both cases (Goetz et al., 2010). Meanwhile, empirical studies that explore the direct relations between self-concept and mathematics achievement reported a consistent positive correlation (Lee, 2009) but strength differed from study to study. In a meta-analysis conducted by Hansford and Hattie (1982), self-concept was positively correlated with mathematics performance, but the relation was weak (r = .21 to .26). In contrast, a study of 1,710 Grade 8 and 11 students found an increasing, strong correlation (r = .68/.76; Goetz et al., 2010). This discordant strength could be explained by age and Grade level differences, as Rech (1994) suggested that the interaction of self-concept with achievement significantly changes across grades. In sum, both theory and empirical evidence suggest that self-concept in mathematics is significantly associate with enjoyment, anxiety, and achievement.

Enjoyment

Control-value theory implies that achievement emotions like enjoyment and anxiety are affected by students' gender and appraisals of their value, self-concept, and confidence (Pekrun, 2006). This assumption is supported with empirical studies discussed above, where value, confidence, and self-concept had a moderate relation with enjoyment (Goetz et al., 2008; Luo et al., 2016; Peixoto et al., 2017). Meanwhile, many studies exploring the direct relation between enjoyment and achievement indicate a moderate to strong positive interrelation (Van der Beek et al., 2017). For example, in a cross-cultural study of 891 Grade 8 students from Germany and China, enjoyment was positively correlated with mathematics achievement (r = .39 to .66; Frenzel et al., 2007). Similarly, Ahmed et al. (2013) assessed 495 Grade 7 students' achievement and enjoyment three times in a year in a short-term longitudinal study and found that students' achievement systematically changed with their enjoyment. This same relationship was reported across many grade levels by Jerusalem and Mittag (1999) and Pekrun et al. (2017), with enjoyment in mathematics positively predicting mathematics achievement (r = .27/.43/.68 in Grade 7/10/12, respectively). Therefore, both theory and empirical results reveal a clear positive correlation between enjoyment and value, self-concept, confidence, as well as mathematics achievement.

Anxiety

Math anxiety is a negative feeling combined with fear and tension when dealing with mathematical problems (Richardson & Suinn, 1972). As with enjoyment, anxiety is influenced by students' gender and appraisal of their value, self-concept, and confidence (Pekrun, 2006). Empirical studies showed that girls who hold gender stereotype tend to be more anxious about mathematics (Casad et al., 2015). Also, a negative correlation between value, self-concept, confidence, and anxiety were reported as discussed above (Luo et al., 2016; Peixoto et al., 2017). Moreover, both theory and empirical evidence suggested that anxiety affects students' performance and achievement in mathematics (Ashcraft & Faust, 1994; Pekrun, 2006). For example, students with mathematics anxiety showed more difficulty in computing and were less likely to use new strategies compared to their peers (Ashcraft & Faust, 1994). Negative correlation between anxiety and mathematics achievement were reported in most of the reviewed studies (r = -.27 to -.48), with high math-anxious students tending to perform worse than those with low anxiety (Brassell et al., 1980; Hembree, 1990; Lee, 2009; Ma, 1999; Reese, 1961; Zakaria & Nordin, 2008), and the phenomenon has been observed in all grades (Ma, 1999).

Behavioural Intentions

Behavioural intentions measure students' action or potential behaviours towards mathematics. An example item is "I think about mathematics problems outside school and like to work them out" (Tocco, 1971). Control-value theory holds that students' emotions affect their motivational strategy (i.e., time spent on mathematic tasks), which further affects academic performance (Pekrun, 2006). Studies indicated that anxiety was negatively influencing students' behavioural intentions where students with mathematics anxiety tried to avoid math-related activates (Beilock & Maloney, 2015; Hembree, 1990). In contrast, students who reported higher enjoyment spent more time on mathematics (Tulis & Fulmer, 2013). Interestingly, control-value theory implies that confidence affects behavioural intentions via emotions whereas the reviewed empirical studies suggested confidence is a significant predictor of choosing more elective mathematics courses (Hackett & Betz, 1989). The direct contribution of confidence to behavioural intentions seems to overlook the mediated effect of emotions. However, control-value theory argues that emotions can become nonreflective and routinized over time such that appraisals and emotions are directly linked (Pekrun, 2006). Therefore, confidence can be directly linked to a certain level of enjoyment and anxiety, which may further influence behavioural intentions in a more direct way. Finally, a reciprocal relation was also reported between behavioural intentions and mathematics achievement (r = .21 to .34), where students with high performance in mathematics tended to spend more time on mathematics while lower performing students demonstrated the tendency to avoid mathematics-related situations (Barkatsas et al., 2009; Sciarra & Seirup, 2008).

Summary

The theoretical framework suggests that cognitive factors have an impact on affective factors, which further influence learning behaviours and academic achievement. However, empirical evidence only supported some of these relationships. Overall, this may be due to researchers only examining the direct effect of each attitude's component on mathematics achievement, while ignoring many mediating effects. The direct relationships identified in this review showed that, in general, anxiety and gender role beliefs were negatively correlated with mathematics performance (r = -.27 to -.48/-. 21) while enjoyment, self-concept, confidence, perceived value, and behavioural intentions were positively related to achievement (*r* = .27 to .68/.21 to .76/.34 to .42/.11-.30/.21 to .34 respectively). As for the mediating effects, more needs to be done. For example, how appraisal of value, confidence, and self-concept mediate gender roles' effect on enjoyment and anxiety remains unstudied.

Discussion and Recommendations

Construct of Mathematics Attitudes

Research has explored many dimensions of mathematics attitudes but has not generated wide-reaching conclusions. We argue this is due to the lack of a theoretical framework for the construct of mathematics attitudes (Ma & Kishor, 1997; Zan & Di Martino, 2007). This paper applied the tripartite model (Triandis, 1971) as a theoretical framework to the concept of mathematics attitudes and therefore adopted a multidimensional definition of mathematics attitudes, which regards them as a construct with cognitive, affective, and behavioural components. Adopting the multidimensional definition of mathematics attitudes not only enables researchers to describe the term from a theoretical perspective, but also guides future research to investigate the many dimensions of the topic. Based on this definition, a tripartite construct of mathematics attitudes was proposed, where affective components contained enjoyment and anxiety; cognitive components contained value of mathematics, gender roles, confidence, and self-concept in mathematics, and the behavioural component was behavioural intentions. This tripartite construct of mathematics attitudes was rooted in a theoretical definition while also representing the most commonly measured components of mathematics attitudes, as found in research with secondary students. Thus, it helps researchers organize the many aspects of mathematics attitudes currently studied and provides researchers with a holistic picture of what mathematics attitudes are and how the sub-components relate to each other in secondary education.

With the guidance of a tripartite construct of mathematics attitudes, educators and researchers may more purposefully study which components of attitudes decrease in secondary education and which ones need to be improved. Improving mathematics attitudes not only entails increasing students' confidence in mathematics, but also involves improving students' self-perceptions with mathematics, finding utility in mathematics, improving learning experience with mathematics, and increasing the likelihood of engaging in more mathematics-related activities.

Though our tripartite construct of mathematics attitudes represents the most commonly measured components of mathematics attitudes in research with secondary students, some of the components largely overlap with what has been studied in other age levels. Articles on children's mathematics attitudes, which were excluded from this review, suggest that value, confidence, gender roles, enjoyment, and anxiety are also commonly investigated among children (Ayuso et al., 2020; Cheeseman & Mornane, 2014; Dove & Dove, 2017; Haciomeroglu, 2017; Mazzocco et al., 2012; Tossavainen & Juvonen, 2015; see Appendix B). The term 'self-concept' and 'confidence' is often used interchangeably with children to measure their perceived efficacy for solving math problems. Merging two components into one could occur for two reasons: 1) younger children may have difficulty distinguish higher-order conceptual conclusions (Harter, 1990), and 2) when reporting on self-concept, children may have difficulty telling the real and false selves apart and their "false selves" are more reflective of parental values rather than their own perception of themselves (Harter, 1990). Similarly, articles on college/university students and adults suggest that value, confidence, self-concept, gender roles, enjoyment, anxiety, and behavioural intentions are also the most frequently tested components (Afari et al., 2013; Code et al., 2016; Eldersveld &Baughman,1986; Hedges & Harkness, 2017; Royster et al., 1999; Serin & Incikabi, 2007; Watson, 1983; see Appendix C). The overlapping components illustrate that mathematics attitudes share commonality across ages.

Despite the common aspects of mathematics attitudes across different age levels, some components may be more age specific. Articles with children explore a broader range of emotions rather than focusing on enjoyment and anxiety. For instance, some studies measure how much children worry about their performance in mathematics (Lauermann et al., 2017) and some explore bored, surprised, and unhappy feelings associated with mathematics (White & McCoy, 2019). All these achievements related emotions are worthy of study, as different emotions may occur during the learning process and they could mediate cognitions' impact on mathematics achievement (control-value theory, Pekrun, 2006). Therefore, different types of emotions should be considered under the affective components of mathematics attitudes, as these emotions expand the construct of mathematics attitudes, may capture learners' attitudes more precisely, and illuminate the relations between mathematics attitudes and mathematics achievement in a more detailed manner. Articles containing college/university students and adults explore perceived difficulty of mathematics (Royster et al., 1999; Serin & Incikabi, 2017). Though perceived difficulty is sometimes measured in research with primary and secondary education, it became more visible in higher education. One possible reason could be that the subject difficulty increased in higher education (Hedges & Hardness, 2017), which may cause more frustration, thus has increased the need to understand students' perceptions.

Studies on mathematics attitudes should not only focus on components in the tripartite construct, but also need to take other influencing factors into consideration. For example, factors excluded from this review (e.g., parents'/teachers' values and attitudes) may not be appropriate and accurate to represent students' own attitudes, but these factors play an important role in affecting children's attitudes (Gunderson et al., 2012). Meanwhile, genetic and environmental factors play an important role in the development of mathematics anxiety (Wang et al., 2014). Studies need to further investigate both biological pathways and individual-specific learning experiences' impact on mathematics attitudes. In addition, the perceptions of lecturers/ teachers can influence college students' growing interest in the subject (Serin & Incikabi, 2017). Thus, future research on attitudes may measure the aforementioned factors in addition to students' attitudes, thereby having a comprehensive understanding of the development of mathematics attitudes.

Relations of Mathematics Attitudes and Mathematics Achievement

Research on the relations among mathematics attitudes and mathematics achievement were overly linear and unclear, due to lacking a theoretical framework and an inappropriate use of instruments (Ma & Kishor, 1997). This paper applied control-value theory as a framework for understanding the relation among mathematics achievement and each component of mathematics attitudes. It treated mathematics attitudes as a multifaceted concept and explored the distinctive connection between each individual component and mathematics achievement. According to control-value theory, mathematics achievement is mediated through control-value appraisal (e.g., value, self-concept, confidence) and achievement emotions (such as enjoyment and anxiety). Most of the studies in this review only assumed linear causal relationships between individual components and math achievement. The assumption of a linear relation may lead the field to overlook some critical mediating effects. As current findings suggested, how control-value appraisals mediate the effect of gender roles/beliefs on achievement emotions remains unclear.

Going forward, research on mathematics attitudes can continue to look for direct effects but also needs to look at how mathematics achievement is mediated through other components.

The relations between mathematics anxiety and mathematics achievement in this review showed a negative linear correlation (Lee, 2009; Zakaria & Nordin, 2008). However, this could be more complex for two reasons: 1) the degree of anxiety was not considered, and 2) the type of anxiety was not specified. Previous studies suggest that, among young adolescents and adults who are highly motivated to learn mathematics, extreme low or high anxiety leads to poor mathematics performance, but moderate amounts of anxiety could result in optimal learning outcomes by improving attention during cognitive processing (Wang et al., 2015). Meanwhile, different types of anxiety could lead to differential impacts on achievement. Lukowski and colleagues (2019) have classified mathematics anxiety into calculation performance related anxiety, classroom related anxiety, and test related anxiety. Their study with early adolescents revealed that only calculation related anxiety significantly predicts math achievement while classroom and test related anxiety are not significant predictors (Lukowski et al., 2019). Future studies on secondary students' mathematics anxiety need to further examine levels of stress and types of anxiety to see if these forementioned patterns still hold.

Conclusion

For future work on mathematics attitudes, researchers should clearly specify the components of attitudes being explored, as this will allow others to interpret the relation between attitudes and achievement or the relations amongst each component of mathematics attitudes. Further, studies on the relations between mathematics attitudes and mathematics achievement should not only explore how each component of mathematics attitudes contributes to mathematics ability but also explore the mediating role of each component and their combinatorial contributions to mathematics achievement. As our review focuses solely on secondary education, future work is needed to systematically test whether the components in our construct of mathematics attitudes for secondary students are suitable for the study of children and adults.

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Note. Asterisk (*) marks references included in the literature review. The rest of the references are the articles included in the review but not cited in the article.

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Appendices

Appendix A

Summary of the Instruments Used in Literature

Name of scale	**Cognitive components**	**Affective components**	**Behavioural components**	**Reliability** (coefficient α)	**Number of studies**	**Citation**
Fennema-Sherman Mathematics Attitudes Scales (FSMAS)	Confidence (9)[a], Value (7), Gender roles (7), Attitude towards success (6), Perception of parents/teacher's attitudes (5)	Anxiety (3), Affect (3)		NA .84 .79-.96 .92 .86 - .93 .86 - .98 .84 - .94 .87 - .89 .87 - .93 .66 - .68	10	Broadbooks et al. (1981) Casem (2016) Fennema and Sherman (1976) Ganley and Vasilyeva (2011) Hyde et al. (1990) Iben (1991) Norton and Rennie (1998) Pedro et al. (1981) Sherman (1982) Yang (2015)
Attitudes Toward Mathematics Inventory (ATMI; by Tapia & Marsh, 2004)	Confidence (5), Value (5)	Enjoyment (6), Motivation (5)		.88 - .95 .93 .71 - .90 NA .86 .88 - .95	6	Awofala et al. (2013) Lim and Chapman (2013) Mavridis et al. (2017) Mirza and Hussain (2018) Mohr-Schroeder et al. (2017) Tapia and Marsh (2004)
Aiken's Math Attitude Scale	Value (4)	Enjoyment (4)		.87 - .93 .85 - .95 .77 - .82 .68 - .88	4	Adwere-Boamah (1986) Sarouphim and Chartouny (2017) Tran (2012) Watson (1983)
The Mathematics Attitude Scale		Enjoyment (3)		.93 .96 .93	3	Dursun (2015) Erdik (2018) Idil et al (2016)

developed by Aşkar (1986)						
Sandman's Mathematics Attitude Inventory (MAI)	Perception of math teacher (2), Value (2), Self-concept in math (2)	Anxiety (2), Enjoyment (2), Motivation (2)		.68 - .89	2	Rech (1994) Sandman (1980)
Mixed	Value (3), Self-concept (2), Confidence (1), Gender roles (1)	Enjoyment (2)	Behavioural intentions (1)	.58 - .60 .60 - .73 NA	3	Gwizdala and Steinback (1990) Kiwanuka et al. (2017) Yáñez-Marquina and Villardón-Gallego (2016)
Not clear					7	Bilican et al. (2011) Kamoru and Ramon (2017) Martinez (2017) Mukherjee and Umar (1978) Muthulakshmi and Veliappan (2016) Oyedeji (2017) Yasar (2016)
Modified from previous literature	Value (9) Confidence (5) Gender roles (4) self-concept (1), attention (1), Perceived difficulty (1), Attitudes towards school (1), Learning (1), math teaching (1), Teacher's attitudes (1), Nature of math (1),	Enjoyment (8) Anxiety (8) Motivation (2), Feelings (1)	Behavioural intentions (3)	.64 .96 .83 NA NA NA NA .94 .89 NA .85 NA .85 - .90 .54	23	Arhin (2015) Arslan et al. (2012) Awofala et al. (2013) Aydın (2015) Elçi (2017) Geiser (1999) Kontas (2016) Koyuncu and Dönmez (2018) Mallam (1993) Mandina et al. (2013) Murimo (2013) O'Reilly (1980) Rice et al. (2013) Fisher and Rickards (1998)

	Attitudes towards			.79		Ritzhaupt et al. (2011)
	school (1)			.52 - .57		Sanchal and Sharma (2017)
				NA		Sengül and Dereli (2013)
				.73		Simsek (2016)
				NA		Soliman and Hilal (2016)
				.96		Swetz et al. (1983)
				NA		Van Eck (2006)
				.44 - .91		Mumcu and Aktas (2015)
				.96		Yenılmez (2007)
PISA 2012/2005	Self-concept (2)			NA	2	Cheung et al. (2018)
				.86		Perry et al. (2016)
TIMSS	Value (6),	Enjoyment		NA	6	Bilican et al. (2011)
2007/2011/1990-	Confidence (6)	(6)		86		Choi and Chang (2011)
2007/2009/2003/				.84 - .93		Hwang et al. (2017)
				NA		Ruthven (2011)
				.73 - .86		Smith et al. (2014)
				.83 - .93		Vandecandelaere et al. (2012)
Self-developed	Value (13),	Enjoyment	Behavioural	NA	24	Brandell and Staberg (2008)
	Gender roles (8),	(13),	intentions (7),	.91		Butty (2001)
	Confidence (4),	Boredom (4),	More	NA		Charles et al. (2014)
	Self-concept (8),	Anxiety (5),	attention (1)	.65 - .84		Cheung (1988)
	Perception of	Feelings (1),		NA		Elliott et al. (2001)
	difficulty (3),			.93		Eskici et al. 2017
	Parents’ perception			.94		Etuk et al. (2013)
	(3),			NA		Funkhouser (1993)
	Teachers’ impact (3),			NA		Haladyna et al. (1983)
	Perceived control (1),			.84 - .89		Hemmings et al. (2011)
	Rules (1),			NA		Higgins (1997)
	Intention (1),			.83		Ifamuyiwa and Akinsola (2008)
	Home-support (1),			.58 - .83		Jacobs (1991)
	Home-process (1),			.65		Jurdak and Abu Zein (1998)
	Society (1),			.89		Lipnevich et al. (2011)
	Fatalism (1)			NA		Ma (1997)

	NA	Minato and Yanase (1984)
	.60 -. 65	Mutohir et al. (2018)
	NA	Steinback and Gwizdala (1995)
	NA	Thomas (2000)
	.34 - .87	Tocci and Engelhard (1991)
	.91	Yenılmez (2007)
	.84	Zakariya (2017)

[a]The number in the brackets following each component represents the frequency of that component being applied.

Appendix B

Summary of Mathematics Attitude Components Used in Primary Education

Citations	**Cognitive components**	**Affective components**	**Behavioural components**	**Type of scale**
Ayuso et al. (2020)	Confidence Value	Anxiety		Bi-dimensional
Cheeseman and Mornane (2014)	Confidence Value			Uni-dimensional
Dove and Dove (2017)		Anxiety		Uni-dimensional
Lauermann et al. (2017)	Value Self-concept	Worry		Bi-dimensional
Rech (1994)	Value Self-concept	Enjoyment Anxiety	Behavioural intentions	Multidimensional
Tossavainen and Juvonen (2015)	Value	Enjoyment		Bi-dimensional
White and McCoy (2019)		Worry Boredom Surprise Happiness		Uni-dimensional
Haciomeroglu (2017)	Value Confidence/ Self-concept	enjoyment	Behavioural intentions	Multidimensional
Mazzocco et al. (2012)	Perceived difficulty Value	Enjoyment (likability)		Bi-dimensional

Appendix C

Summary of Mathematics Attitudes Components Used in Higher Education

Citations	Cognitive components	Affective components	Behavioural components	Type of scale
Afari et al. (2013)	Confidence	Enjoyment		Bi-dimensional
Code et al. (2016)	Confidence Value Math beliefs		Behavioural intentions	Bi-dimensional
Eldersveld and Baughman (1986)	Confidence	Anxiety		Bi-dimensional
Hedges and Harkness (2017)	Nature of statistics Previous experience		Behavioural intentions	Bi-dimensional
Royster et al. (1999)	Value Self-concept Confidence Perceived difficulty	Enjoyment Scariness	Behavioural intentions	Multidimensional
Serin and Incikabi (2017)	Perceived difficulty Interest			Uni-dimensional
Watson (1983)	Value	Enjoyment		Bi-dimensional

Bridging Text

Chapter 2 presents a literature review on math attitudes and addresses fundamental yet unanswered questions, specifically: what are math attitudes? what components are in math attitudes? and what is the relation between math attitudes and math achievement? It adopts a tripartite model as a theoretical framework to guide the understanding of the term, under which, math attitudes are defined as ones' beliefs, feelings, and intentional behaviours towards math. Building on this definition, Chapter 2 further clarifies the subdimensions of three components of math attitudes. The results suggest that gender beliefs, value, confidence, and self-concept were the most commonly measured subdimensions under cognitive attitudes, while anxiety, and enjoyment were often studied under affective attitudes, and behavioural intentions was the variable often tested under behavioural attitudes. Finally, Chapter 2 reveals the unique relations between each subdimension and math achievement, drawing on previous research findings. Overall, Chapter 2 provides a solid foundation for understanding math attitudes and their relations to math achievement. One key issue the chapter leaves undressed is the lack of a measure to reflect the many dimensions of math attitudes. Furthermore, it points out that the relation between math attitudes and math achievement has been studied linearly in the current literature, and far more complex relations among the components of math attitudes need to be further investigated.

Building on the foundation laid in Chapter 2, Chapter 3 provides a comprehensive measure of math attitudes while addressing the complex interconnections among components of math attitudes. It begins by proposing and validating a thorough math attitudes scale which includes the well-known subdimensions identified in Chapter 2, as well as other affective subdimensions previously overlooked in literature, such as anger, boredom, pride, and shame.

The measurement model of this newly created Tripartite Math Attitude Scale (TMAS) is validated via multiple psychometric assessments using SmartPLS 3.0 software. Following this, Chapter 3 uses control-value theory as a framework to explore the structural relationship between the dimensions of math attitudes. The Structural Equation Modeling (SEM) analysis suggested some significant associations within the construct, which are consistent with the theory. This chapter contributes to the field of math attitudes by providing researchers with an exhaustive measure that reflects the overall picture on the topic. It also advances the field by drawing attention to the interactions among different components of math attitude, providing insights into the mechanics of math attitudes.

Chapter 3. Manuscript 2

Validating A Tripartite Math Attitudes Scale and Exploring Relations in the Construct

Wen, R., & Dubé, A. K. (2023). *Validating a tripartite math attitudes scale and exploring relations in the construct.* Manuscript in preparation.

Abstract

Research on math attitudes has used different definitions and measures of the construct. Further, relationships among components of math attitudes have only been studied individually. There is an urgent need for an updated measure of math attitudes that reflects the many facets of the concept and reveals the complex relations among each component. Therefore, the aim of this online study with 222 adults was to 1) propose and validate a tripartite math attitude scale (TMAS) consisting of cognitive, affective and behavioural components, including gender beliefs, confidence, value, anxiety, enjoyment, boredom, anger, pride, shame, and behavioural intentions; and 2) explore the relationships among components guided by the control-value theory. The analysis supported TMAS as a reliable and valid tool to measure math attitudes. The findings suggested a significant association between cognitive math attitudes and affective math attitudes, as well as between affective math attitudes and behavioural math attitudes. In addition, gender beliefs were found to be a factor in determining confidence in cognitive math attitudes. These findings are consistent with the relationships predicted by control-value theory. Overall, the findings of this study contribute to the development of a more robust and comprehensive understanding of math attitudes, which has important implications for improving math education.

Keywords: math attitudes, measure, control-value theory

Students start to develop attitudes towards math in early elementary school (Aiken, 1970). These attitudes are shaped and become stable due to the influence of different factors, such as environmental influences (i.e., teachers' and parents 'attitudes) and learning experience (Beilock et al., 2010; Eccles & Jacobs, 1986; Gunderson et al., 202; Jacobs et al., 2005; Keller, 2001; Ma & Xu, 2004; Mueller & Dweck, 1998). Parents' and teachers' math gender roles/beliefs (also known as math stereotypes) affect their expectancy of children's math abilities and their attributions of children's math success, in turn, influencing children's perceptions of their abilities (Eccles & Jacobs, 1986; Jacobs et al., 2005; Keller, 2001). Also, parents' and teachers' math anxiety impact children's math attitudes (Beilock et al., 2010; Mueller & Dweck, 1998). As students gain more experience learning math in a formal setting, their prior achievement also plays an important role in shaping their math attitudes (Ma & Xu, 2010). Once these attitudes have been formed, they can last long into adulthood (Morrisett & Vinsonhaler, 1965, as cited in Aiken, 1970).

Math attitudes play a critical role in children's lives via their effect on math achievement, math involvement (i.e., course-taking), and even career orientation (Joensen & Nielson, 2009; Ma & Kishor, 1997; Neale, 1969; Nugent et al., 2015; Rose & Betts, 2004; Trusty, 2002). Students with positive math attitudes tend to have better math performance than those with negative math attitudes (Ma & Kishor, 1997; Neale, 1969). Relatedly, they are likely to spend more time learning math and involved in math-related courses (Nugent et al., 2015; Trusty, 2002). In contrast, individuals with negative math attitudes tend to avoid math-related activities and are less likely to pursue a math-related career (Joensen & Nielson, 2009; Rose & Betts, 2004). Clearly, math attitudes have a considerable effect on how well and how often individuals learn math.

Different Definitions and Measures of Math Attitudes

Though math attitudes have been studied for decades, the definition of the term varies significantly (Hannula, 2002; Zan & Di Martino, 2003). A systematic review conducted by Wen and Dubé (2022) on math attitudes reveals that the current existing definitions can be grouped into three types: 1) unidimensional definitions describing math attitudes via affective dimensions (e.g., "individual's like or dislike toward mathematics," Murimo, 2013, p. 75); 2) bi-dimensional definition depicting math attitudes as containing either affective and cognitive dimensions (e.g., "a person's disposition towards a subject, beliefs a person held about that subject," Mirza & Hussain, 2018, p. 12) or affective and behavioural dimensions (e.g., "a liking or disliking of mathematics, or a tendency to engage or avoid mathematical activities," Yang, 2015, p. 252); and 3) multidimensional definition including all three dimensions ("a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless," Sengül & Dereli, 2013, p. 2527). This finding is supported by Di Martino and Zan's (2001, 2007) category of 'simple', bi-demensional, and three-component definitions. Besides the various existing definitions, in the systematic review of 95 studies, a large number of studies (*n* = 75) do not provide a clear definition at all (Wen & Dubé, 2022). The fact that the term under investigation is not explicitly defined indicates a serious ambiguity in studies about attitude towards math

This unclear definition is paired with the use of similarly ambiguous constructs and measures in the field (Hannula, 2002; Zan & Di Martino, 2003). The Fennema-Sherman Mathematics Attitudes Scale (FSMAS; Fennema & Sherman, 1978) is the most widely cited and used measure in math attitudes research (Pepin, 2011; Wen & Dubé, 2022). FSMAS contains eight scales that measure math attitudes from cognitive and affective dimensions, namely attitudes toward success, beliefs towards gender (math as male domain beliefs), parents' and teachers' math attitudes, confidence, motivation, perceived usefulness, and anxiety (Fennema & Sherman, 1978). The Attitudes Toward Mathematics Inventory (ATMI; Tapia & Marsh, 2004) is another well recognized measure in mathematics research. ATMI also evaluates math attitudes via cognitive and affective dimensions, but its subdimensions are different from FSMAS in that it only measures confidence, value, enjoyment, and motivation (Tapia & Marsh, 2004). Aiken's Math Attitude Scale (MAS, 1974) and Sandman's Mathematics Attitude Inventory (MAI) are also popular math scales, which contains cognitive and affective dimensions. The former measures value and enjoyment while the later measures perception of mathematics teacher, value of mathematics, self-concept, anxiety, enjoyment, and motivation (Aiken, 1974; Sandman, 1980).

The Mathematics Attitude Scale developed by Aşkar (1986) is another math scale, but it only measures affective dimension of math attitudes, namely students' enjoyment of learning math. Although these well-known scales all measure similar dimensions of math attitudes, the subcomponents vary substantially. In addition to these well-established measures, a large body of research tends to develop new measures to update and reflect different dimensions of math attitudes (Wen & Dubé, 2022; Zan & Di Martino, 2003). All this suggests that many facets of math attitudes have been studied but none of the currently existing measures contain all subdimensions of math attitudes.

The Tripartite Math Attitudes Model and Scale

To reflect the multidimensions of math attitudes that have been studied in the current literature, the present study adopted the tripartite model as a framework to define math attitudes (Leder, 1992; Ruffell et al., 1998; Wen & Dubé, 2022; Zan & Di Martino, 2007). The tripartite model suggests that math attitudes consist of beliefs and ideas towards math (e.g., a belief that math is useful), feelings and emotions towards math (e.g., enjoyment), and a tendency to engage in or avoid learning math (Wen & Dubé, 2022). This tripartite model of math attitudes reflects both theory of attitudes and practical components (i.e., cognitive, affective and behavioural components) that have been studied in the field (Triandis, 1971).

With the guidance of tripartite model, the subdimensions of each component of math attitudes can be further clarified, providing researchers with a holistic view on the topic and encouraging them to investigate the inner relationships among subdimensions. In a systematic review of 95 studies on math attitudes in secondary education, Wen and Dubé (2022) identified the most frequently studied subdimensions. The findings showed that gender beliefs, confidence, and value are the central subdimensions under cognitive components, anxiety and enjoyment are the main subdimensions under affective components, and behavioural intentions are the core focus under behavioural component of math attitudes (Wen & Dubé, 2022). Since these components are largely overlap with those in other age levels (Wen & Dubé, 2022), they were adopted into the current study. Despite the commonality, Wen and Dubé (2022) suggest that there are age-specific subdimensions that may be overlooked, such as a broader range of emotions children may experience in addition to anxiety and enjoyment. To account for this, the present study also included a set of achievement emotions in affective math attitudes based on Pekrun's (2006) work. The following outlines each subdimension included in the final math attitudes construct.

Cognitive Components

Gender Beliefs

Gender beliefs towards math (also known as gender roles) reflect a math-related stereotype that male students are in a better position to learn math and can master math skills faster and get better results in math than girls (Fennema et al., 1990). This stereotype is influenced by adults, with children adopting the implicit gender beliefs of their parents or teachers with whom they identify (Lane, 2012). Research on the topic shows that negative gender beliefs may undermine students' performance and may prevent them from taking advanced math courses and pursuing math-related careers (Fennema & Sherman, 1978).

Value

Value refers to how much importance students place on math as a subject. Existing literature has examined the relationship between value, emotions, and learning achievement and indicates that value is a significant predictor of emotions; where high perceived value most likely correlates with high level of enjoyment while low perceived value often correlates with higher anxiety (Luo et al., 2016; Peixoto et al., 2017). In addition, students' perceived value of math relates positively with math performance, and this connection is stronger for female students (Chouinard et al., 2007; Lim & Chapman, 2015).

Confidence

Confidence or self-efficacy is how assured students are in their ability to deal with math problems (e.g., "I can get high score in math"). Researchers sometimes use the term 'self-concept' or 'self-evaluation' to refer to confidence, but these two terms are slightly different, in that they refer to how one perceives themselves to be in relation with math as a subject (e.g., "Math is my favourite subject"). Besides the similarity in the definition, both confidence and self-concept are reported to have a negative correlation with anxiety and positive correlation with enjoyment and learning outcomes (Wen & Dubé, 2022). Since these terms are similar and can be easily confused, this paper only adopted the widely used term confidence into our math attitudes construct.

Affective Components

Anxiety

Anxiety towards math refers to anxious feelings experienced when involved in math-related scenarios (e.g., solving math problems, taking math tests; Richardson & Suinn, 1972). Research on math anxiety shows that anxiety is closely related to gender beliefs, value, confidence, and math performance (Ashcraft & Faust, 1994; Casad et al., 2015; Luo et al., 2016; Peixoto et al., 2017; Zakaria & Nordin, 2008). Generally, students with higher levels of gender beliefs (stronger stereotype that male students have advantages when learning math), low confidence, and high value for math are more likely to develop math anxiety when dealing with math problems (Casad et al., 2015; Luo et al., 2016). Their worries and tensions disrupt working memory capacity and influence their strategy choices when computing, thus affecting their math performance (Ashcraft & Faust, 1994; Beilock, 2008).

Enjoyment

Enjoyment refers to how much students take delight or pleasure in learning math. Students with higher levels of enjoyment are believed to have better learning outcomes (Van der Beek et al., 2017). Research indicates a moderate to strong positive correlation between enjoyment and confidence (r = .62) and enjoyment and performance (r = .39 to .66; Frenzel et al., 2007; Luo et al., 2016).

Anger, Boredom, Pride, and Shame

Although anxiety and enjoyment are the two commonly studied emotions in math attitudes, it is important to note that other emotions of equal importance to learning have been overlooked. To capture a broader range of emotions in math attitudes, this study adopted Pekrun's (2006) concept of "achievement emotions," which describe emotions one may experience during learning or after receiving learning outcomes. These achievement emotions are critical as they have the possibility to change one's motivation towards learning, and hence affect learning outcomes (Pekrun, 2006). Pekrun has extensively researched these emotions in relation to math and has identified the most frequently experienced math-related emotions such as anger, boredom, pride, and shame (Pekrun & Goetz, 2007). These emotions were adopted in the current study as is essential for research on math attitudes to consider the entire spectrum of achievement emotions that could contribute to math attitude development and learning outcomes.

Behavioural Components

Behavioural intentions refer to how much students would like to engage in math-related scenarios. Factors influencing students' behavioural intention are confidence, enjoyment, and anxiety, with high level of confidence, enjoyment and low level of anxiety are more likely to lead students to spend more time doing math (Barkatsas et al., 2009; Beilock & Maloney, 2015; Tulis & Fulmer, 2013). Moreover, a reciprocal relation has been reported between behavioural intentions and math performance (Barkatsas et al., 2009).

The proposed tripartite construct of math attitudes and its subdimensions offer a new lens to understand and examine math attitudes. However, in order to fully understand the construct, it is essential to explore the interrelationships among each component. Surprisingly, no previous study has simultaneously and comprehensively investigated the relationships among the subdimensions of math attitudes. This knowledge gap presents a critical challenge to educators and researchers seeking to improve math attitudes through targeted teaching practices and interventions. To fill this gap in our understanding, it is necessary to examine the internal workings of the construct and elucidate the connections between its subdimensions. By doing so, we will be better equipped to design effective interventions that target specific subdimensions, rather than just overall math attitudes, thereby promoting more positive attitudes towards math among students.

Theoretical Background—Control-Value Theory of Achievement Emotions

As previously described, “achievement emotions” are emotions students experience that are closely related to achievement activities and learning outcomes (e.g., enjoyment anxiety, anger, shame, pride; Pekrun, 2006; Pekrun et al., 2007). Control-value theory (CVT) puts achievement emotions at the centre and provides an integrated framework to understand the antecedents and effects of achievement emotions (Pekrun, 2006). Control-value theory considers control appraisals and value appraisals to be the proximal antecedents of achievement emotions (Pekrun, 2006). Control appraisals refer to perceived control over achievement activities and outcomes, which include causal expectancies and evaluation of one’s ability. Value appraisals refer to both intrinsic and extrinsic value over the achievement activities and outcomes. One could value math due to passion (intrinsic value) or because of the benefits it brings, such as high grades or a better career (extrinsic value).

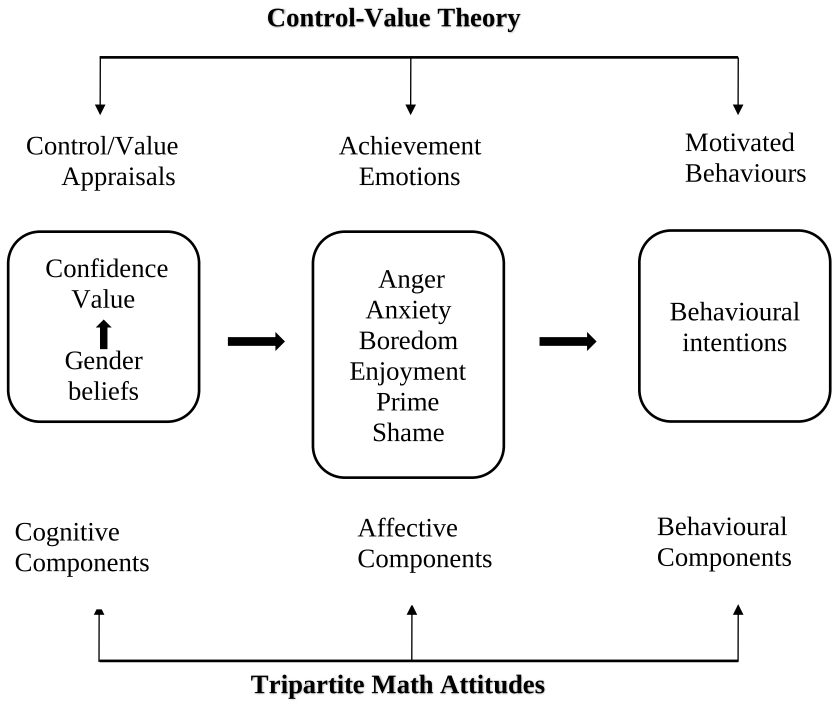
Together, different control appraisals and value appraisals are assumed to activate different achievement emotions. For example, a student who has high appraisal of ability and high value for math may get excited for a math exam. In contrast, a student who is uncertain of one’s ability but wants to get good grades may get anxious for an exam. Besides proximal antecedents, distal antecedents such as achievement goals and beliefs also can have an impact on achievement emotions. However, the effect is first placed on control and value appraisals (Pekrun, 2006). Control-value theory also explores the effects of achievement emotions. It holds that achievement emotions may affect students' learning outcomes through their academic engagement such as cognitive resources and motivation (Pekrun, 2006).

Connections Between TMAS and CVT

The proposed tripartite construct of math attitudes aligns with control-value theory (see Figure 1). Figure 5 outlines the relationship among cognitive, affective and behaviour components of math attitudes suggested by control-value theory (Pekrun & Perry, 2014). In this figure, cognitive components (gender beliefs, confidence, value) of math attitudes are in line with control and value appraisals in control-value theory. Affective components of math attitudes come from achievement emotions. Finally, behavioural intentions can be considered as similar to motivated behaviours (i.e., academic engagement). Therefore, from a control-value lens, we could say that affective math attitudes may closely relate to cognitive and behavioural math attitudes, with the former being antecedents and latter being effects.

Figure 5

Connection between Control-Value Theory and Tripartite Math Attitudes



Note. Adapted from "The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice," by R. Pekrun, 2006, *Educational psychology review*, *18*(4), 315-341. Copyright 2006 by Springer. Adapted from "A systematic review of secondary students' attitudes towards mathematics and its relations with mathematics achievement," by Wen & Dubé, 2022, *Journal of Numerical Cognition, 8*(2), 295-325. Original figure shows subdimensions of math attitudes based on a systematic review. The figure has been modified and updated to reflect more subdimensions.

Goals of The Present Study

Math attitudes are important and many facets of it have been explored. However, a comprehensive measure that reflects all components of math attitudes is lacking. Moreover, relations within the construct have been studied individually, but no study to date has simultaneously examined the interrelations among all subdimensions. To address this gap, the main purposes of the current study is to validate a Tripartite Math Attitudes Scale and to further examine the relationship between subdimensions of the TMAS as suggested by control-value theory. The following research questions/goals guided the investigation:

RG1: To validate a tripartite math attitude scale

RG2: To investigate if there are any significant relationships among cognitive math attitudes (gender role beliefs, value, confidence), affective math attitudes (enjoyment, anxiety, boredom, pride, anger, shame) and behavioural math attitudes (intentional behaviour). Based on this research goal, the following hypotheses were proposed:

Hypothesis 1 (H1): Distal cognitive math attitudes (gender beliefs) impact one or more proximal cognitive math attitudes (value, confidence).

Hypothesis 2 (H2): Proximal cognitive math attitudes (value, confidence) impact affective math attitudes (enjoyment, anxiety, boredom, pride, anger, shame).

Hypothesis 3 (H3): Affective math attitudes (enjoyment, anxiety, boredom, pride, anger, shame) impact behavioural math attitudes (behavioural intentions).

Methodology

Participants

English-speaking adults were recruited via an online recruitment tool “Prolific.” These participants are chosen as previous research has found that poor math attitudes last well into adulthood (Aiken, 1985; Fredricks & Eccles, 2002; Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002). A total of 311 adults participated in the study. All participants completed a questionnaire containing six demographic questions, five attentional check questions, and 29 math attitudes related questions. Attentional check questions used explicit instructions to guide respondents to select a specific answer from the scale. These attention check items were designed to identify participants who do not read the instructions or questions properly (Tourangeau et al., 2000), also known as carless respondents (Meade & Craig, 2012) or insufficient effort respondents (Huang et al., 2012, as cited in Shamon & Berning, 2019). After identifying careless respondents (those who failed two or more attentional check questions out of five), 222 participants' data were retained. Participants ages ranged from 18 to 76 (*M* = 33.26, *SD* = 11.76). The demographic profile of the sample can be found in Appendix D.

Sample Size Calculation

There are various of rules for minimum sample size requirements for factor analysis. Some authors argue the minimum sample size should be at least 100 or 200 (Boomsma, 1982; Kline, 1994). Some suggest using the ratio of subjects to the number of variables, which ranges from two subjects per variable to 15 subjects per variable (Hatcher, 1994; Kline, 1994; Nunnally, 1978). All these rules have the limitation of not being model-specific (Wolf et al., 2013). Westland's (2010) formula takes number of observed variables and number of latent variables into consideration, which is more model-specific, thus was adopted for calculating sample size. The minimum required sample size of 216 is sufficient to achieve statistical power (0.8) with 29 items and 10 latent variables. Therefore, the current sample size meets the requirement for sampling adequacy.

Instruments

This study's main measurement for students' cognitive, affective, and behavioural components of math attitudes are items adopted and adapted from those in the literature (see Appendix E; Aiken & Dreger, 1961; Dutton, 1951; Fennema & Sherman, 1976; Frenzel et al., 2007; Mulhern & Rae, 1988; Tapia & Marsh, 2004). The adopted items have previously established good face validity and convergent validity with loadings higher than 0.7 and reliability with highest Cronbach's alpha. As a result, 29 items assessing 10 subdimensions of math attitudes were included: value (three items), gender role beliefs (three items), anxiety (three items), enjoyment (three items), boredom (three items), pride (three items), anger (three items), shame (three items), and behavioural intentions (two items). A five-point Likert-type scale was used for each item, ranging from "strongly disagree" to "strongly agree."

Data Analysis

To analyze the data, SPSS version 27 and SmartPLS version 3.0 were used. Normality analysis, multicollinearity analysis, and descriptive statistics were done in SPSS. Kolmogorov-Smirnov Test (K-S Test) and Shapiro-Wilk Test (S-W Test) were performed to check the normality of data. The result indicated a violation of normality according to Kline's (1994) criteria. Variable Inflation Factors (VIF) were used to detect potential multicollinearity. VIF above ten (VIF > 10) is considered as a sign of severe multicollinearity (Naser & Hassan, 2013). The result showed VIF values were less than three, indicating no multicollinearity issue (Hair et al. 2014).

Partial least squares based structural equation modeling (PLS-SEM) analysis was used as 1) it is well established to deal with nonnormality in data (Afthanorhan, 2013, as cited in Mâță et al., 2020), and 2) its advantages in predicting relationships explained by R^2 (proportion of total variance explained; Hair et al., 2017). PLS-SEM has two stages of analysis—measurement model analysis and structural model analysis (Hair et al., 2014). Measurement model analysis uses Confirmatory Factor Analysis (CFA) to assess the measurement theory (the relationship between observed variables and latent variables). Structural model analysis uses Structural Equation Modeling (SEM) to test the structural relationship between latent variables.

Results

RG1. Measurement Model Assessment

CFA was used to assess the measurement model. Indicators of reliability, convergent validity, and discriminant validity are reported in Table 5 and evaluated below.

Table 5

Reliability and Validity Assessment of Measurement Model

Constructs	Items	Convergent validity		Reliability	
		Factor loadings	AVE	Cronbach α	Composite reliability
Value (Val)	Val1	.872	.706	.793	.878
	Val2	.807			
	Val3	.839			
Confidence (Con)	Con1	.872	.788	.866	.918
	Con2	.890			
	Con3	.901			
Gender beliefs (GB)	GB1	.825	.544	.645	.779
	GB2	.751			
	GB3	.621			
Enjoyment (Enj)	Enj1	.948	.875	.928	.954
	Enj2	.952			
	Enj3	.906			
Anxiety (Anx)	Anx1	.916	.794	.870	.920
	Anx2	.911			
	Anx3	.845			
Boredom (Bor)	Bor1	.865	.786	.864	.917
	Bor2	.897			
	Bor3	.898			
Pride (Pri)	Pri1	.903	.762	.845	.905
	Pri2	.895			
	Pri3	.817			
Anger (Ang)	Ang1	.907	.729	.816	.890
	Ang2	.849			
	Ang3	.804			
Shame (Sha)	Sha1	.822	.777	.721	.874
	Sha2	.886			
	Sha3	.681			
Behavioural intentions (BI)	BI1	.899	.760	.686	.864
	BI2	.843			

Reliability

Internal consistency reliability was tested through Cronbach Alpha coefficient (α) and Composite Reliability (CR). Cronbach alpha of .70 and above is a good indicator of internal consistency (Cronbach, 1951; Lord & Novick, 1968). Table 5 shows most variables' Cronbach's alpha ranged from .7 to .93, indicating an acceptable internal consistency, but gender beliefs and behavioural intentions' Cronbach's alpha were less than .7. As Cronbach's alpha holds the assumption of tau-equivalence (equal factor loading for all items), it may underestimate true reliability when the assumption is violated (Peterson & Kim, 2013). Alternatively, composite reliability was calculated to correct for the underestimation bias. Composite reliability describes how well observed variables indicate the latent variables (McDonald, 1970). The recommended value is .7 (Hair et al., 2014). Table 5 shows all composite reliability exceeded the satisfactory value of .7 (Hair et al., 2014), indicating good reliability.

Convergent Validity

Convergent validity describes to what extent observed variables under the same latent variable are correlated with each other or produce consistent results (Hair et al., 2012). It was assessed through factor loading and Average Variance Extracted (AVE). The factor loading should exceed .5 for newly developed items and .6 for established items (Chin et al., 2008). Table 5 shows that all item loadings exceeded .6, with most of the loadings ranged from .7 to .9. AVE value depicts the overall amount of variance in observed variables explained by a latent variable. In the rule of thumb, the recommended AVE value is .5 (Hair et al., 2014). Table 5 shows that AVE values exceeded the recommended value. Therefore, the construct for each latent variable is well established.

Discriminant Validity

Discriminant validity describes to what extent are latent variables unique (Henseler et al., 2015). It was assessed through Forell-Larcker criterion and cross-loadings. The Forell-Larcker criterion compares the squared root of AVE for a latent variable and correlations between other latent variables (Fornell & Larcker, 1981). Table 6 shows that the squared root of AVE for each latent variable were higher than the correlations with any other latent variables, indicating adequate discriminant validity (Fornell & Larcker, 1981). Cross-loading assesses discriminant validity at indicator/item level and evaluates whether each item loading is higher than other cross-loadings (Henseler et al., 2015). Table 7 shows that the loading of specific items was higher on the intended construct than on other constructs, which has established discriminant validity.

Table 6

Discriminant Validity Assessment (Forell-Larcker Criterion)

Latent variables	1	2	3	4	5	6	7	8	9	10
Anger	**.85**									
Anxiety	-.69	**.89**								
Boredom	.71	-.61	**.89**							
Confidence	-.61	.75	-.58	**.89**						
Enjoyment	-.63	.79	-.71	.78	**.94**					
Gender-B	-.02	.12	.06	.15	.03	**.74**				
Behav-I	-.42	.57	-.51	.59	.77	-.01	**.87**			
Pride	-.53	.69	-.54	.78	.78	.11	.62	**.87**		
Shame	.72	-.65	.57	-.56	-.53	-.08	-.27	-.45	**.80**	
Value	-.36	.49	-.37	.63	.57	.04	.59	.61	-.23	**.84**

*The bolded values are square root of the AVE; other values are correlations

Table 7

Discriminant Validity Assessment (Cross-loadings)

Constructs & items	1	2	3	4	5	6	7	8	9	10
1. Anger										
Ang1	**.907**	.675	.479	.702	.598	.647	.011	.564	.645	.392
Ang2	**.849**	.588	.33	.583	.494	.522	.038	.413	.624	.283
Ang3	**.803**	.469	.233	.52	.464	.418	.078	.345	.645	.207
2. Anxiety										
Anx1	.661	**.916**	.512	.558	.706	.733	.099	.652	.621	.478
Anx2	.592	**.911**	.505	.554	.643	.7	.045	.631	.556	.445
Anx3	.581	**.845**	.506	.52	.652	.687	.151	.56	.6	.378
3. Behavioural intentions										
BI1	.391	.508	**.894**	.546	.52	.732	.028	.536	.246	.504
BI2	.345	.485	**.849**	.329	.502	.602	.024	.549	.251	.538
4. Boredom										
Bor1	.61	.473	.423	**.864**	.474	.552	.051	.419	.502	.329
Bor2	.682	.589	.461	**.897**	.555	.666	.034	.521	.493	.346
Bor3	.605	.556	.473	**.898**	.507	.651	.088	.477	.521	.312
5. Confidence										
Con1	.502	.607	.458	.462	**.871**	.634	.178	.642	.514	.489
Con2	.586	.69	.479	.526	**.889**	.682	.114	.711	.535	.577
Con3	.544	.693	.615	.546	**.903**	.752	.103	.72	.479	.596

6. Enjoyment										
Enj1	.608	.735	.775	.636	.749	**.948**	.051	.739	.493	.544
Enj2	.614	.767	.722	.707	.775	**.952**	.06	.755	.527	.537
Enj3	.55	.725	.655	.634	.654	**.906**	.02	.679	.498	.512
7. Gender beliefs										
GB1	.039	.098	.002	.058	.129	.073	**.853**	129	.033	.068
GB2	.095	.109	.01	.024	.105	.024	**.69**	.044	.114	.007
GB3	.075	.001	.004	.083	.087	.013	**.718**	.025	.02	.026
8. Pride										
Pri1	.498	.638	.576	.484	.694	.742	.09	**.903**	.437	.504
Pri2	.583	.713	.599	.542	.795	.772	.057	**.895**	.05	.607
Pri3	.247	.405	.422	.343	.509	.465	.143	**.817**	.141	.472
9. Shame										
Sha1	.527	.527	.214	.41	.405	.388	.045	.278	**.84**	.166
Sha2	.735	.637	.28	.574	.583	.546	.073	.467	**.922**	.258
Sha3	.506	.498	.234	.476	.424	.439	.052	.321	**.843**	.145
10. Value										
Val1	.328	.447	.565	.382	.585	.557	.054	.588	.26	**.87**
Val2	.265	.324	.408	.268	.453	.37	.088	.425	.165	**.809**
Val3	.298	.442	.505	.271	.526	.48	0.27	.505	.186	**.84**

In summary, the measurement model indicates adequate reliability, convergent validity, and discriminant validity. Followed by structural model is examined.

RG2. Structural Model Assessment

SEM with bootstrapping of a sample of 5,000 was used to assess the structural model. The typical criteria were examined, such as variance inflation factor (VIF), estimates for path coefficients, effect size f^2, and R^2 of endogenous latent variables (Hair et al., 2017; Henseler et al., 2015).

Variance Inflation Factor (VIF)

To examine multicollinearity, VIF was calculated for each independent variable in the model. *VIF* values closer to 3 or lower than 3 indicates the absence of multicollinearity (Choy et al., 2019; Mason & Perreault, 1991). The results showed that most VIF values were closer to or below 3 (M_{VIF} = 2.31), aligning with the ideal conditions where low correlation among variables is typical ideal. It is noteworthy that while the overall *VIF* values are within the recommended range, there may be some potential correlations in specific cases; for instance, Anxiety (*VIF* = 3.05) and Anger (*VIF* = 3.15). Such correlations, however, do not undermine the overall finding of minimal multicollinearity as *VIF* values under 5 is deemed to be reasonable (see Table 8).

Table 8

Multicollinearity Assessment with VIF

Independent variable	*VIF*
Anger	3.15
Anxiety	3.05
Boredom	2.66
Confidence	1.69
Enjoyment	2.54
Gender beliefs	1.03
Pride	2.63
Shame	2.35
Value	1.66
Mean *VIF*	2.31

Effect Size (f^2)

Effect sizes f^2 were computed for each path in the model. Cohen's (1988) guidelines on effect size was followed, which suggests 0.02 is a small effect, 0.15 is a medium effect, and 0.35 or above is a large effect.

R^2 of Endogenous Latent Variables

R^2 of endogenous latent variable measures the amount of variance explained by exogenous latent variable(s). Chin's (1998) guideline on R^2 was followed, with 0.19, 0.33,0.67 being weak, moderate, and substantial, respectively.

Assessment of Direct Effects

The path model was examined with subdimensions from cognitive, affective and behavioural math attitudes. Table 9 shows the direct effects being examined.

Table 9

Assessing Direct Effects

Hypotheses	Path	Beta	P value	T value	Decision supported	f^2
H1	GB → Con	-.120	.042	2.030	Yes	0.024
	GB → Val	-.045	.944	0.070	No	0
	Val → Con	.622	.000	13.944	Yes	0.651
H2	Val → Ang	-.364	< .001	5.684	Yes	0.153
	Val → Anx	-.488	< .001	9.416	Yes	0.313
	Val → Bor	-.371	< .001	5.981	Yes	0.16
	Val → Enj	.568	< .001	12.372	Yes	0.477
	Val → Pri	.610	< .001	13.333	Yes	0.593
	Val → Sha	-.248	< .001	3.514	Yes	0.066
	Con → Ang	-.614	< .001	12.111	Yes	0.605
	Con → Anx	-.749	< .001	26.041	Yes	1.279
	Con → Bor	-.578	< .001	11.12	Yes	0.502
	Con → Enj	.779	< .001	28.021	Yes	1.542
	Con → Pri	.781	< .001	27.888	Yes	1.563
	Con → Sha	-.573	< .001	10.789	Yes	0.489
H3	Ang → BI	-.028	.717	0.363	No	0
	Anx → BI	.007	.928	0.091	No	0
	Bor → BI	.002	.983	0.022	No	0
	Enj → BI	.815	< .001	9.293	Yes	0.388
	Pri → BI	.067	.36	0.916	No	0.004
	Sha → BI	.197	.01	2.584	Yes	0.042

H1. Distal cognitive math attitudes (gender beliefs) impact one or more proximal cognitive math attitudes (value, confidence)

Gender beliefs significantly impact confidence with a small effect size (β = -.120, p = .042, $f^2 = 0.024$) but was not a significant predictor of value (β = -.045, p =.944; see Table 9). Gender beliefs explained 2% of variance in confidence (R^2 = .020). However, value was a significant predictor of confidence with a large effect size (β = .622, p < .001, $f^2 = 0.651$). Value and gender beliefs together explained 40.7% of variance in confidence (i.e., R^2 = .407). This leads to acceptance of H1 that gender beliefs have one or more significant impacts on proximal cognitive math attitudes.

H2. Proximal cognitive math attitudes (value, confidence) impact affective math attitudes (anger, anxiety, boredom, enjoyment, pride, shame)

Value significantly affected anger (β = -.364, p < .001), anxiety (β = -.488, p < .001), boredom (β = -.371, p < .001), enjoyment (β = .568, p < .001), pride (β = .610, p < .001), and shame (β = -.248, p < .001). Moreover, value explained 37.2% of variance in pride (i.e., R^2 = .372), 32.3 % of variance in enjoyment, 23.9% of variance in anxiety, 13.8% of variance in boredom,13.2% of variance in anger, and 6.2% of variance in shame. Except for the small effect on shame ($f^2 = 0.066$), value had a medium to large effect on other emotions (f^2 = 0.153 to 0.477; see Table 9).

Confidence significantly affected anger (β = -.614, p < .001), anxiety (β = -.749, p < .001), boredom (β = -.578, p < .001), enjoyment (β = .779, p < .001), pride (β = .781, p < .001), and shame (β = -.573, p < .001). Moreover, confidence explained 61% of variance in pride (i.e., R^2 = .610), 60.7 % of variance in enjoyment, 56.1% of variance in anxiety, 37.7% of variance in anger, 33.4% of variance in boredom, and 32.9% of variance in shame. Confidence had a large effect on all emotions (f^2 = 0.489 to 1.563; see Table 9). Taken together, this leads to acceptance of H2 that value and confidence significantly predict affective math attitudes.

H3. Affective math attitudes (anger, anxiety, boredom, enjoyment, pride, shame) impact behavioural math attitudes (behavioural intentions).

Behavioural intentions were significantly and positively affected by two emotions, namely enjoyment (β = .815, p < .001) and shame (β = .197, p = .01). Shame and enjoyment had a small ($f^2 = 0.042$) and large effect ($f^2 = 0.388$) on behavioural intentions, respectively. Enjoyment and shame together explained 62.1% of variance in behavioural intentions (i.e., R^2 = .621). Other emotions were not significant predictors of behavioural intentions, which includes anger (β = -.028; p = .717), anxiety (β = .007; p = .928), boredom (β = .002; p = .983), and pride (β = .067; p = .36; see Table 9). This result leads to partial acceptance of H3, that affective math attitudes were significant predictors of behavioural math attitudes.

Discussion

This study aims to validate a tripartite math attitudes scale and use it to further examine the relationships among components in math attitudes. The results revealed that 1) the TMAS is a valid measure of attitudes towards math; 2) distal cognitive math attitudes (i.e., gender beliefs) have a significant impact on proximal cognitive math attitudes; 3) proximal cognitive math attitudes (both confidence and value) are significant predictors of affective math attitudes; 4) some affective math attitudes significantly influenced behavioural math attitudes.

Validate a Tripartite Math Attitude Scale

This study proposed and validated a tripartite math attitude scale, which measures math attitudes from three components (cognitive, affective, and behavioural) with 10 subdimensions. The measurement model confirmed that this multidimensional scale is valid by examining reliability, convergent validity, and discriminant validity. Though previous studies on math attitudes have used existing scales, such as Aiken's Math Attitude Scale, Fennema-Sherman Mathematics Attitudes Scale, Sandman's Mathematics Attitude Inventory, The Mathematics Attitude Scale, and The Attitudes Toward Mathematics Inventory (Aiken 1980; Fennema & Sherman, 1978; Sandman, 1980; Tapia & Marsh, 2004), these scales are out of date and each one misses at least one important dimension captured by other scales. Also, recent studies on the topic started to develop new scales (Wen & Dubé, 2022), indicating that the existing ones need to be updated. The tripartite math attitude scale is the most up-to-date and thorough scale that not only includes all the important concepts that are currently being studied in the field (such as confidence, anxiety), but also embraces additional critical achievement emotions that students may experience during math (such as anger and shame; Pekrun et al., 2007). Therefore, researchers should adopt this valid scale to guide their investigation of math attitudes.

Relationships Among Cognitive, Affective, and Behavioural Math Attitudes

Distal and Proximal Cognitive Math Attitudes

Control-value theory implies that distal antecedents (such as individual goals and control-related beliefs) have an impact on proximal antecedents (i.e., control/value appraisals) of achievement emotions (Pekrun, 2006; Pekrun & Perry, 2014). In line with this assumption, this study demonstrated that gender beliefs, which are considered as a distal cognitive math attitude, had a significant impact on confidence. This means that one who holds high gender beliefs (i.e., low evaluation of female students' competences) may lead to lower levels of confidence in learning math. The finding is consistent with previous literature that women who believe they are less capable of mastering mathematical skills are more likely to have low level of confidence in their competence, and thus less likely to continue studying math (Schmader et al., 2004). To improve one's confidence, parents and educators should start by addressing gender stereotypes and encouraging students to believe that they have the ability to manage math skills. However, the current study did not explore gender differences concerning the effects of gender beliefs. As such, the extent to which females and males experience similar connections between gender beliefs and confidence remains unclear. Future studies could employ a combination of approaches to explore gender disparities in the relationship between gender beliefs and confidence. This might involve longitudinally tracking the connections between gender beliefs and confidence for both males and females to observe changes over time, as well as utilizing a mixed-methods approach to capture both quantitative trends and qualitative nuances in how gender beliefs shape individuals' confidence levels.

Cognitive and Affective Math Attitudes

Control-value theory argues that control and value appraisals influence achievement emotions (Pekrun, 2006). The results confirmed this connection, with both confidence and value being significant predictors of all achievement emotions, including anger, anxiety, boredom, enjoyment, pride, and shame. To be specific, control and value appraisals explained more variance in positive emotions such as pride and enjoyment than the variance in negative emotions such as anxiety, anger, boredom, and shame. The findings are consistent with previous studies that appraisals of control and value are closely related to positive emotions (Frenzel et al., 2007; Goetz et al., 2010). Thus, the results have provided further empirical support for the relationship between appraisals and achievement emotions and have demonstrated this for achievement emotions in math. Going forward, educators should act to increase students' perceptions of control and value to generate more positive achievement emotions. The current study did not explore the interaction effects of control and value appraisals in triggering different achievement emotions. Future research may further examine these complex interactions.

Affective and Behavioural Math Attitudes

According to control-value theory, different achievement emotions may trigger different levels of motivation and behavioural intentions (Pekrun, 2006). The findings showed that enjoyment from affective math attitudes had a significant influence on behavioural math attitudes. When one feels joy, he or she is more likely to spend more time and effort learning math. This finding is supported by literature where perceived enjoyment tends to lead more behavioural reengagement in a given task (Bashir et al., 2015; Dickinger et al., 2008). It is interesting to note that shame was also a significant predictor of behavioural actions. Goldberg (1991) considers shame to be “the master emotion” as it plays a vital role in influencing behavioural intentions that other emotions cannot achieve. Consistent with some motivational theories, shame is the most destructive emotions that impacts motivational behaviours (Atkinson, 1957; Weiner, 1985). Negative activating emotions like shame may result in increased extrinsic motivation, which lead to more behavioural intentions (Pekrun 2006).

For example, fear of feeling shame may generate greater extrinsic motivation to get higher grades in math, thus spending more time learning it. Also, experiencing shame may force one to re-evaluate expectancy and value in the subject, and the combined effects of both can lead to certain motivational behaviours (Turner & Schallert, 2001). For example, after experiencing shame, if one still values the task and believes that they have the capacities to perform better, one will adjust actions to spend more time mastering the task. In contrast, if one does not value the task or does not believe in their competences, shame feelings may prevent further investment of time in the task. The current study did not explore how expectancy and value interact to affect behavioural intentions, but future research may continue down this direction. As for other emotions (e.g., anxiety, boredom, pride) that did not show significant effects on behavioural intentions, the reason could be that their effects are topic-specific or age-specific.

Limitations and Future Studies

Although the present study provides insights into the relationships among components of math attitudes, there are several limitations that should be acknowledged. Firstly, due to the nature of the cross-sectional design, the findings cannot establish direct causal relationships (Solem, 2015). For example, while we found a significant association between confidence and all types of emotions, we cannot determine whether confidence caused these emotions. Secondly, the cross-sectional data does not provide information on how the relations of subdimensions in the construct change over time, which may limit the assessment of the temporal stability of the relations (Solem, 2015). Therefore, future studies using longitudinal data are needed to establish the causal effects and stability of the relations more conclusively.

Secondly, the study relied solely on self-report measure to collect data. While self-report measures are a widely used and valid approach for measuring attitudes, they can lead to single-source bias. Single-source bias could influence the relations between variables, resulting in inflated or inaccurate association between variables (Podsakoff et al., 2003). To address this issue, future studies should consider including control variables that help reduce the impact of single-source bias. For example, social desirability could be a control variable used to identify and correct for participants' tendencies to provide socially acceptable responses that do not reflect their true attitudes. Including control variables can help reduce single-source bias and improve the accuracy and generalizability of the findings. Therefore, it is recommended for future studies.

The current study focused on adult participants, which offers valuable insights into the construct of math attitudes among this age group. However, it is important to note that developmental differences could potentially impact the construct of math attitudes and inner connections among its subdimensions. Although the construct itself may be similar across ages (Wen & Dubé, 2022), the way in which individuals weigh each subdimension of math attitudes may vary depending on their age, which could in turn affect the internal relationships between the subdimensions. For example, younger individuals may place more emphasis on the "value" subdimension of math attitudes, which refers to the perceived usefulness of math skills in real-world settings, whereas older individuals may place more emphasis on the "confidence” or “enjoyment" subdimensions, which reflect students’ affective experience derived from engaging with math. Thus, in order to gain a more comprehensive understanding of the relationships between the subdimensions of math attitudes, it will be important for future studies to test these relationships among other age groups to determine whether the current connections still hold across different developmental stages.

Conclusion

This study aimed to validate a multidimensional measure of math attitudes and to investigate the relationship among each subdimension. The findings not only confirmed that tripartite math attitude scale is a valid measure for math attitudes but also indicated significant associations between cognitive, affective, and behavioural math attitudes. Practically, this study contributes to the body of knowledge in the math attitudes field by proposing and validating a comprehensive scale that reflects a variety of math attitudes. The proposed tripartite math attitude scale is the result of a comprehensive literature review (Wen & Dubé, 2022). This updated and exhaustive scale not only reveals the overall picture of math attitudes that have been studied, but also provides future researchers with a readily validated scale that can be either adopted entirely or selectively within the domain of math attitudes. Theoretically, the current study not only proposes and supports a tripartite model of math attitudes, it simultaneously examines the various relations among subdimensions of math attitudes under the guidance of control-value theory. Hence, the findings confirm the idea that affective math attitudes are closely related to cognitive and behavioural math attitudes. It brings awareness of the interconnections among each subdimensions of math attitudes and provides a comprehensive picture of the relations within the construct. The findings may serve as a starting point for exploring more complex interactions within the tripartite construct of math attitudes.

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Appendices

Appendix D

Descriptive Statistics of Participants' Demographic Profile

Demographic variables	*Mean*	*SD*	*n* (%)
Age	33.26	11.76	
18-20			16 (7.2)
21-30			95 (42.8)
31-40			66 (29.7)
41-50			23 (10.4)
51-60			16 (7.2)
61-70			3 (1.4)
71-80			3 (1.4)
Gender			
Male			127 (57.2)
Female			91 (41.0)
Other			4 (1.8)
Education level			
Less than high school degree			7 (3.2)
High school graduate			31 (14.0)
Some college but no degree			43 (19.4)
Associate degree in college (2-year)			17 (7.7)
Bachelor's degree in college (4-year)			90 (40.5)
Master's degree			28 (12.6)
Doctoral degree			3 (1.4)
Professional degree (JD, MD)			3 (1.4)
Ethnicity			
White			186 (83.8)
Black or African American			8 (3.6)
American Indian or Alaska Native			1 (0.5)
Asian			26 (11.7)
Other			5 (2.3)

Appendix E

Tripartite Math Attitudes Scale

Type	Items
	Value
Val1	I study math because I know it is useful.
Val2	Math is important in everyday life.
Val3	Math is important to me in my work.
	Confidence
Con1	I'm confident I can learn new math skills.
Con2	I can get good grades in math.
Con3	I can solve difficult math problems.
	Gender Beliefs
GB1	Females can do just as well as males in math.
GB2	Males are not naturally better than females in math.
GB3	Females are good enough to do well in math.
	Enjoyment
Enj1	Math is fun to me.
Enj2	Math is very interesting, and I enjoy math class.
Enj3	I like math just as much as other subjects.
	Anxiety
Anx1	I am usually relaxed during math classes.
Anx2	I am usually relaxed during math tests.
Anx3	Math doesn't scare me at all.
	Boredom
Bor1	I start yawning in math class because I'm so bored
Bor2	Math material is so boring that I find myself daydreaming.
Bor3	Mathematics lecture bores me.
	Anger
Ang1	I am annoyed during my math classes.
Ang2	I get angry because the material in math is so difficult.
Ang3	I am so angry that I would like to tear the math exam paper into pieces.
	Shame
Sha1	My face is getting hot because I am embarrassed that I cannot answer math teacher's questions.
Sha2	After taking a test in math, I feel ashamed.
Sha3	When I don't understand something in my math homework, I don't want to tell anybody.
	Pride
Pri1	After a math test, I am proud of myself.
Pri2	I'm proud of my knowledge in math.
Pri3	After having done my math homework, I am proud of myself.
	Behavioural intentions
BI1	I would like to spend more time working on math.
BI2	I think about mathematics problems outside of work.

Bridging Text

Chapters 2 and 3 provide the foundation for this thesis by offering a comprehensive literature review on math attitudes, introducing a validated measure that reflects the comprehensive dimensions of math attitudes, and investigating the complex relations within the construct. The findings not only establish a better understanding on math attitudes, but also shed light on the important role of affective math attitudes. The question at hand is: How do we effectively improve math attitudes?

Chapter 4 introduces a novel approach to improving math attitudes, through digital educational games. Specifically, it focuses on affective math attitudes (i.e., different achievement emotions) given their importance identified in Chapter 3. Chapter 4 argues that emotional design principles in a game context may have positive effects on students' achievement emotions and math performance. To support this, it presents a meta-analysis examining the effects of five emotional design principles, namely visual aesthetic design, musical score, game mechanics, narrative, and incentive system. The findings suggest that the inclusion of emotional design principles in a game have a moderate impact on both achievement emotions and learning outcomes. Notably, game features that influence learners' controllability over the game and the value of the game tend to have stronger effects. This chapter provides practical guidance for how to design educational games that enhance math attitudes and learning outcomes.

Chapter 4. Manuscript 3

Design Principles for Digital Mathematical Games that Promote Positive Achievement Emotions and Achievement

Wen, R., & Dubé, A. K. (2023). Design principles for digital mathematical games that promote positive achievement emotions and achievement. In K. M. Robinson, D. Kotsopoulos & A. K. Dubé (Eds.), *Mathematical cognition and understanding: Perspectives on mathematical minds in elementary and middle school years.* Springer, Cham. https://doi.org/10.1007/978-3-031-29195-1_8

Abstract

Digital educational games can be an enjoyable way to improve students' mathematics achievement. However, players may experience other emotions besides enjoyment when learning about mathematics, such as anxiety and boredom. These emotions are also important as they affect learning outcomes via multiple pathways. Loderer and colleagues' (2020) propose five foundational emotional design principles for use in digital game-based learning. However, empirical evidence to support these principles is lacking. This paper conducted a meta-analysis on the effects of emotional design principles. The results showed that among the studies reviewed (n = 17), most of games applied multiple principles. In general, emotional design principles had a medium effect size on both achievement emotions (g = .50) and learning outcomes (g = .66). Principles that influence control and value appraisals had stronger effects (g = .60/.63 respectively) compared to those that did not contribute to control and value appraisals. These principles should be adopted for a better emotional experience and learning outcome.

Keywords: achievement emotions, mathematical games, digital games, game design.

A great deal of research has been conducted on the effectiveness of digital mathematical games and their ability to improve learning outcomes. In Byun and Joung's (2018) meta-analysis of 17 studies on K-12 mathematics education, digital games are shown to have a positive effect on mathematics learning outcomes ($d = .37$). In a narrative review of 25 studies, Dubé et al. (2019) similarly conclude that digital mathematical games can be effective learning tools for engaging students with learning content, increasing performance, and improving attitudes. However, digital games are not equally effective in all domains of learning and some learning outcomes are more studied than others (Ke, 2009). This chapter discusses why digital mathematical games are effective tools for students in the elementary and middle school years by investigating which emotional design features contribute to better mathematics learning experience and outcomes.

Why Are Digital Mathematical Games Effective?

Dubé et al. (2019) argue that digital educational games are often designed to be "interactive" and "fun," and these characteristics of games may have the potential to overcome the "boring" mindsets and anxious feelings students hold towards mathematics. Students start to develop mathematics anxiety (fear and tension towards mathematics) at an early age, and it grows over time (Aiken, 1970). The level of mathematics anxiety students experience does not differ as a factor of mathematical ability as high performing students also report feeling stressed and anxious towards mathematics (OECD, 2018). Mathematics anxiety matters because it can affect students' performance during assessment as well as their willingness to pursue mathematics as a field of study (Ashcraft & Faust, 1994). In fact, a negative relationship between anxiety and mathematics performance has been reported in different studies spanning all grades (Brassell et al., 1980; Hembree, 1990; Lee, 2009; Ma, 1999; Zakaria & Nordin, 2008).

In contrast to the anxious feelings experienced with mathematics, students report having fun and enjoyment during digital educational games regardless of academic subject (Mekler et al., 2014). Mekler et al.'s (2014) systematic review of 87 digital game studies find that descriptors such as "enjoyable," "enjoying," "fun," and "interest" have appeared in 89% of studies. For mathematics, enjoyment is often reported by students themselves when interacting with mathematical games (Chen et al., 2012). Putwain et al. (2018) further report a reciprocal relation between enjoyment and mathematics achievement among 5th to 6th grade students. Their finding suggests that higher levels of enjoyment experienced during mathematical game learning leads to better learning outcomes (Putwain et al., 2018). Therefore, it seems that the fun part of digital mathematical games not only offsets the boring and anxious feelings students experience with mathematics but can also generate positive emotions and lead to better mathematics performance.

Emotions

Besides enjoyment, there are a range of emotions students may experience during game-based learning and how they affect the learning process is complex. Emotions refer to affective states, either positive or negative (Lazarus, 1993). Ekman and Friesen (1987) propose six basic emotions: fear, anger, joy, sadness, disgust, and surprise. However, these general emotions are insufficient to capture the complex roles of emotions in learning (Kapoor et al., 2001; Kort et al., 2001). Since this earlier work, there are a broad range of emotions that researchers consider when studying learning. Craig et al. (2004) includes confusion, frustration, boredom, flow/engagement, interest, and being stuck as affective sates that learners may encounter. Similarly, D'Mello et al. (2007) argue that confusion, frustration, and boredom are inevitable emotions during learning. Woolf et al. (2009) adapt Ekman's (1999) basic emotions, while focusing on learning-related subsets of emotions such as pleasure, frustration, boredom, anxiety, novelty, and confidence.

Achievement Emotions

Pekrun's (2017) three-dimensional taxonomy of achievement emotions is a systematic way to organize and define learning related emotions. Pekrun defined learning related emotions as achievement emotions, which are "emotions that are tied to achievement activities (e.g., studying) or achievement outcomes (success and failure)" (Pekrun, 2017, p.143). There are 17 types of achievement emotions that can be categorized according to three dimensions: object focus, valence, and activation (Pekrun & Perry, 2014). The object dimension distinguishes achievement emotions by activity and outcome (both prospective and retrospective). For example, enjoyment is an activity achievement emotion that occurs during learning, hope is an outcome prospective achievement emotion that occurs before learning, and pride is an outcome retrospective achievement emotion. The valence dimension groups achievement emotions into positive achievement emotions (joy) and negative achievement emotions (sadness). The activation dimension distinguishes the activating achievement emotions (anger) from deactivating achievement emotions (boredom).

Therefore, we adopted Pekrun and Perry's (2014) definition of achievement emotions and refer to learning related emotions as achievement emotions. Achievement emotions can manifest themselves overtly or be internalized. Moreover, they have the potential to impact learning and the pleasure of learning, which is described next.

Importance of Achievement Emotions

Mathematics attitude is closely related to mathematics achievement (Ma & Kishor, 1997; Ma & Xu, 2004). It contains cognitive (beliefs towards mathematics), affective (emotions associate with mathematics), and behavioural (intentional behaviours towards mathematics) components (Wen & Dubé, 2023). Achievement emotions are considered as affective components of mathematics attitudes in this Chapter and believed to play an important role in learning as suggested by Pekrun and Perry's (2014) control-value theory. Control-value theory argues that achievement emotions are products of cognitive appraisals of learning events and predictors of students' performance and achievement (Pekrun, 2006; Pekrun et al., 2007).

There are two types of appraisals that play vital roles in arousing achievement emotions: (1) control-related and (2) value-related appraisals. Control-related appraisal refers to the evaluation of one's controllability over achievement activities and achievement outcomes. Value-related appraisal refers to one's value of both achievement activities and achievement outcomes, which can be either intrinsically or extrinsically valued. Achievement emotions are products of control-/value-related appraisals, with high control and value resulting in positive achievement emotions (e.g., enjoyment) while high value but low control leading to negative achievement emotions (e.g., anxiety; Pekrun, 2006).

Achievement emotions are not only the results of control and value appraisals, but also indicators of students' mathematics achievement (Pekrun, 2006). In fact, achievement emotions have an impact on academic performance through motivational factors (Pekrun & Perry, 2014). For example, students with positive achievement emotions (e.g., enjoyment) are more likely to reengagement in the activity and are more likely to have a better performance. Students with negative achievement emotions (e.g., anxiety), on another hand, are more likely to avoid the task and underperform in their learning (Pekrun, 2006). This relationship between achievement emotions and learning achievement proposed by control-value theory is supported by empirical evidence. A study of 3,425 5th to 9th grade students find strong evidence for a reciprocal relationship between the two (Pekrun et al., 2017). From a control-value lens, achievement emotions are critical in learning as they reflect student's appraisals and predict their achievement.

So, how do digital mathematical games affect students' achievement emotions, and hence, influence their learning? Given the aforementioned theoretical lens, there must be features/design elements of digital mathematical games that affect students' control and value appraisals, which result in different achievement emotions, and further influence students' mathematics ability. To be specific, game features that contribute to students' control/value appraisals are assumed to lead to positive achievement emotions and better learning outcomes. The next step is to identify what these potential game features could be, and to test if these features lead to better mathematics learning experience and outcomes.

Emotional Foundations of Digital Game Design

Though achievement emotions have been widely studied in the field of psychology, few researchers have explored achievement emotions in the context of digital mathematical games. Loderer et al.'s (2020) recent study explored the emotional design of digital learning environments resulting in a model that explores the design of emotional foundations for game-based learning. Loderer et al. (2020) argues that emotional support can improve learning for all individuals and identifies five general principles for game design from an emotional design perspective: visual aesthetic design, musical score, game mechanics, narrative, and incentive system.

Visual Aesthetic Design

Loderer et al. (2020) propose that visual aesthetic design elements contain bright colours, round shapes, and the presence of learner-resembled avatars. Bright colours are saturated warm colours (e.g., orange, pink; Loderer et al., 2020). Round shapes refer to graphic images and user interfaces that adhere to a rounded form (cf., square; Loderer et al., 2020). Avatars are graphical representations of certain characters, whose faces and expressions evoke learners' emotions (Loderer et al., 2020). Generally, bright colours, round shapes, and avatars are connected with positive achievement emotions (Arroyo et al., 2013; Boyatzis & Varghese, 1994; Kao & Harrell, 2015a, b; Mayer & Estrella, 2013; Plass et al., 2014; Um et al., 2012). Particularly, avatars are better at inducing learners' positive achievement emotions compared to bright colours, which likely occurs because the control over avatars provides players a sense of power, and increased control-related appraisals further arouses positive achievement emotions.

Musical Score

Music has a direct impact on a learner's emotions via tones and rhythms (Loderer et al., 2020). Musical score includes emotional tones, vocal sound, and sound feedback in the game (Loderer et al., 2020). Studies on tones indicate that higher brightness of tones are associated with positive emotions such as happy and joyful (Wu et al., 2013). Human generated sounds are also more likely to evoke positive emptions compared to computer-generated sounds, due to a sense of social presence or connection they provide (Baylor, 2011). Sound feedback occurs when sounds are provided based on learners' performance (e.g., recognize mistakes or celebrate success; Loderer et al., 2020). It is a particularly important complementary source to visual feedback, and both are essential for learning (Fiorella et al., 2012).

Game Mechanics

Game mechanics are “methods invoked by agents for interacting with the game world” (Sicart, 2008, p. 338). A well-designed game mechanic should 1) match with learning goals, 2) have a clear task, 3) have learner-appropriate difficulty, 4) provide social interaction, and 5) provide scaffolding (Loderer et al., 2020). Loderer et al. (2018) suggest that good game mechanics provide students with a sense of control over the challenges, and thus are more likely to produce positive achievement emotions like enjoyment.

Narrative

Narrative means that a game has a storyline that contextualizes the gameplay situation and provides a sense of belonging to a world (Dickey, 2007). Narrative can be either relevant or irrelevant to learning mechanics. Relative storylines make learning part of the story, with the goal of increasing motivation, engagement, and learning gains (Cordova & Lepper., 1996). Though a meta-analysis found that most games adopting irrelevant storylines had better learning outcomes (Clark et al., 2016). Clark et al. suggests that games with overly developed, or ‘thick’ narratives distract players from learning. Therefore, games with relevant narratives to learning should avoid complex storylines as they may make it hard for learners to follow and understand.

Incentive Systems

Incentive systems contain rewards, unlocking mechanisms, and learner choice that keeps them motivated (Loderer et al., 2020). The basic incentives are rewards, which could be in the form of points, scores, stars, or badges. More advanced incentives include unlocking mechanisms, which allow learners to get access to game levels (new levels or new mini games). The opportunity to unlock an unknown game level acts as an intrinsically motivating factor, that captures learners’ curiosity (Malone, 1981). Another incentive is learner choice, which gives learners the ability to choose rewards. For example, learners can choose which gifts are earned or change the avatar used in the game. Learner choice provides learners a sense of control over the game (Loderer et al., 2020). Thus, incentive systems link directly to learners' control and value appraisals of the learning activity (McNamara et al., 2010). However, the number of incentives has to be considered carefully as overly frequent rewards can undermine learner's intrinsic value of learning (Abramovich et al., 2013).

Do Emotional Design Principles Promote Positive Achievement Emotions and Learning Outcomes?

There is limited evidence to support the effectiveness of a particular emotional design principle and whether these five principles facilitate positive achievement emotions during mathematical game-based learning needs to be systematically investigated. Further, control-value theory holds that emotional design principles that contribute to control/value appraisals are more likely to result in positive achievement emotions, but this assumption also needs empirical support. To address this gap, we developed a coding scheme based on Loderer and colleagues' (2020) five emotional design principles and conducted a meta-analysis of digital mathematical game research to explore 1) which emotional design principles are used in digital mathematical game research and 2) how effective each emotional design principle is at improving achievement emotions and learning outcomes.

Methodology

Review Process

First, a systematic review was conducted using keywords combinations of mathematics, game, and emotions searched in three databases: PsycINFO, ERIC EBSCO and Scopus (see Table 10). The initial search returned 171 articles. After removing duplicates, 144 articles were entered into a two-stage screening process. Second, selected articles were then coded for the following: study (authors and publication year), grade level, sample size, intervention duration, study design, and emotion measured (see Table 11). Third, a content analysis was conducted to categorize games in the selected studies based on Loderer et al.'s (2020) emotional design principles. Table 12 shows the coding framework. Fourth, effect sizes were calculated to compare the effect of the different principles on mathematics achievement emotions and outcomes across studies. This was done due to the advantages effect sizes provide in representing true effects and comparing across studies (Cohen, 1988; Ellis, 2010). Effect size for each study was calculated by using the data provided in the study.

Table 10

Keywords for Information Retrieval

Mathematics	Game	Emotions
Math*	Game*	Emotion*
	Educational game*	Enjoy*
	Digital game*	Hop*
	Serious game*	Anxi*
	Video game*	Boredom*
	Mobile game*	Frustrat*
	Mobile app*	Sad*
	Tablet game*	Anger*
	Tablet app*	Relief*
		Relax*
		Shame*

Note. The asterisk at the end of each keyword indicates truncation searching, allowing researchers to search for a term and its various spellings.

Table 11

Summary of Reviewed Studies

Studies	Grade level	Sample size	Durations	Design	Achievement emotions	*g* on emo	*g* on achi
(Adamo-Villani & Dib, 2013)	1-5	13	No data	qualitative	enjoyment	NA	NA
(Beserra et al., 2019)	2	110	14-week	mixed	enjoyment	NA	1.7
(Chen et al., 2012)	4	53	4-week	quantitative	enjoyment	.45	.43
(Chiang & Qin 2018)	7	89	4-week	mixed	enjoyment	.25	.52
(Conati & Gutica, 2016)	6-7	15	1-2h	mixed	enjoyment	NA	.64
(Gilliam et al., 2017)	9-12	133	5-week	mixed	enjoyment boredom	NA NA	NA NA
(Godfrey & Mtebe, 2018)	1-3	111	6-week	mixed	enjoyment frustration	NA NA	NA NA
(Gresalfi et al., 2018)	3	95	3-day	mixed	enjoyment	.69	.43
(Hensberry et al., 2015)	4	46	4-day	mixed	enjoyment	NA	1.2
(Hill et al., 2016)	8-11	322	3-week	quantitative	enjoyment	.09	.34
(Howard-Jones & Demetriou, 2009)	5-6	50	1 session	mixed	frustration	NA	NA
(Huang et al., 2014)	2	56	8-week	quantitative	reduced anxiety	.02	.32
(McLaren et al., 2017)	6	153	No data	quantitative	enjoyment	.95	.37
(Pareto et al., 2012)	3	47	9-week	mixed	enjoyment	NA	NA
(Plass et al., 2013)	6-8	58	1 session	quantitative	enjoyment	1.27	.86
(Sedig, 2008)	6	58	1 session	quantitative	enjoyment	NA	.72
(Van Eck,2006)	7-8	123	1 session	quantitative	reduced anxiety	.36	NA

Table 12

Operationalization of Emotional Design Principles

Emotional design principles	Operationalization	Operationalized keywords and instructions
Visual aesthetic design	Basic emotional related visual designs, such as warm colors and round shapes. Avatars or agents that resemble players in the game.	Shape, round, face-liked shapes, oval. warm, bright, red, yellow, pink, orange. agents, avatars, peers, experts, virtual selves.
Musical score	Musical score refers to auditory stimulus in the game. Higher musical tempo, vocal sound, and sound feedback promote positive emotions.	Rhythms, tones, higher musical tempo, vocal sound, human voice, volume, pitch, prosody, rate of speech, sound feedback.
Game mechanics	Game mechanics that align with learning goals, task clarity, task demands, scaffolding, and social interaction.	Learning activity matches with learning goals/skills. No extraneous content in the game. The task is simple and clear. The difficulty level matches with players' competencies. Scaffolding is provided such as providing examples, adjusting difficulty, providing hints, offering explanations, repeating contents. Social interactions are allowed between players in the game.
Narrative	Narrative refers to a story in the game. Well-structured narratives have compelling story lines.	Storyline contextualizes the skill to be learned.
Incentive system	Incentives such as rewards and other systems that provide extra control over game progression.	Reward, punishment, progress bars, scores, badges, access to game levels, unlock game levels, trade for gifts.

Which Emotional Design Principles Are Used in Mathematical Game Research?

Among the 17 reviewed studies, many (71%) used more than one emotional design principle, while only one study adopted all five (see Table 13). Among the five emotional design principles, visual aesthetic design was the most applied (71%). These studies were more likely to use bright colours and avatars in their games. Three types of avatars controlled by players were found in the reviewed studies, expert avatars (e.g., a scientist agent or a teacher agent), peer avatars (e.g., a student or boy agent), and animal avatars (e.g., a monkey agent). The second most common principle was game mechanics (65%). These included having a clear task, relevant difficulty, social interactions, and scaffolding. For social interaction, two of the five studies with this principle used collaboration, one study used competition, and the remaining two used both. Incentive systems were found in 53% of the reviewed studies. The most common incentive system was rewards, while unlocking mechanism and learner choice were seldom provided.

Musical score (29%) and narrative (24%) were the least applied principles in the reviewed studies. Studies that applied musical score mostly did so by providing sound feedback while fewer used vocal sounds. Notably, all studies that implemented musical score also adopted visual aesthetic design. This is likely because the combination of sound and visuals are more effective at engaging students than sound alone (Wolfson & Case, 2000). Finally, studies applied narrative all used simple narratives that were relevant to the learning content (cf. complex ones that distract). For example, in Beserra et al.'s (2019) work, the story involves an avatar progressing forward by building a bridge with stones, each stone has a number on it, and the avatar needs to choose the correct number to progress. No other distracting elements were presented and there was a central story that was linked to the game mechanic.

Table 13

Summary of Emotional Design Principles and Achievement Emotions in the Reviewed Studies

Citations	Visual			Musical score			Game mechanics					Narrative	Incentive system			Emotions
	V1	V2	V3	MS1	MS2	MS3	GM1	GM2	GM3	GM4	GM5		IS1	IS2	IS3	
(Adamo-Villani & Dib, 2013)			√												√	enjoyment
(Beserra et al., 2019)												√				enjoyment
(Chen et al., 2012)	√	√	√					√		√			√			enjoyment
(Chiang & Qin, 2018)	√	√				√	√	√					√			enjoyment
(Conati & Gutica, 2016)			√						√		√	√				enjoyment
(Gilliam et al., 2017)			√							√		√				enjoyment boredom
(Godfrey & Mtebe, 2018)			√	√	√	√		x	√			√	√	√		enjoyment frustration
(Gresalfi et al., 2018)		√				√							√	√		enjoyment
(Hensberry et al., 2015)		√					√	√			√				√	enjoyment
(Hill er al., 2016)		√			√	√			√							enjoyment
(Howard-Jones & Demetriou, 2009)													x			frustration

(Huang, Huang & Wu, 2014)								√		reduced anxiety
(McLaren et al., 2017)		√	√							enjoyment
(Pareto et al., 2012)							√	√		enjoyment
(Plass, et al, 2013)					√	√	√		√	enjoyment
(Sedig, 2008)	√	√		√		√			√	enjoyment
(Van Eck, 2006)			√				√			reduced anxiety

Note. A check means the game in the reviewed study has covered the given feature while a cross means the feature was included but was deemed poorly implemented. V1 = round shape, V2 = bright colour, V3 = avatars, MS1 = tones, MS2 = vocal sound, MS3 = sound feedback, GM1 = match with learning goals, GM2 = clear task, GM3 = relative difficulties, GM4 = social interactions, GM5 = scaffolding, IS1 = rewards, IS2 = unlocking mechanism, IS3 = learner choice.

Results

How Effective Are Emotional Design Principles at Improving Achievement Emotions and Learning Outcomes?

Both qualitative and quantitative studies were considered when exploring game's effectiveness, as Ke (2009) suggests. Among the reviewed studies, one was qualitative, nine were mixed methods, and seven were quantitative. For the studies that did not have statistical data to calculate effect size, a meta-thematic analysis was conducted based on the themes that students used to describe game-based learning in the interview (Mekler et al., 2014; Talan et al., 2020; see Table 14). Meta-thematic analysis indicated that students enjoyed learning with mathematical games, with "fun" and "interesting" being reported in multiple studies (Adamo-Villani & Dib, 2013; Beserra et al., 2019; Chiang & Qin, 2018; Conati & Gutica, 2016; Gresalfi et al., 2018). Not only did games generate positive achievement emotions, but students believed the game also helped their learning, fostered collaboration, and increased interests in the subject (Chen et al., 2012; Gilliam et al., 2017; Gresalfi et al., 2018).

Table 14

The Most Frequently Used Terms Regarding Games

Emotion-related themes	Examples	Frequency
Fun	"It was fun to play the game every session."	3
Interesting	"Scratch is interesting."	2
Enjoyable/enjoying	"I enjoyed participating in the activities using the game."	1
Awesome	"This is awesome because you feel like you are really in a bakery…"	1
Future play intention	"I would want to play this app again"	1

Learning-related terms	Examples	*N*
Helpful	"I think it's helpful" "I like this game because it helps you learn, for strugglers."	1
Collaboration	"…it was all about collaboration and helping with the group…" "All that collaboration between eight of us, that really helped"	1
Increased interest in learning	"It makes me feel like I want to know more about things"	1

For quantitative studies, effect sizes were calculated. Below, we provide forest plots that visualize the results as well as report key results (overall average effect size, heterogeneity, and predicted interval range). Forest plots can be interpreted as follows: boxes depict the effects of individual studies while the diamond at the bottom of each plot depicts the average effect for all studies combined. The horizontal lines for each study depict 95% confidence intervals, with narrow ones represent more precision effect sizes. A narrative explanation and interpretation for these results is presented in the subsequent discussion. First, we report the overall effect of mathematical games on achievement emotions and achievement.

Overall, emotional design principles in digital mathematical games had a medium effect on achievement emotions (n = 9, g = .50, 95% CI [.19, .80], p = .006; see Figure 6). The between-study heterogeneity variance was estimated at τ^2 = .11 (95% CI [.03, .59]), with an I^2 value of 81.6% (95% CI [.66, .90]). The prediction interval ranged from g = -.34 to 1.33, indicating that negative intervention effects cannot be ruled out for future studies. Similarly, a medium effect on mathematics achievement was found (see Figure 7; n = 12, g = .66, 95% CI [.38, .94], p < .001). The between-study heterogeneity variance was estimated at τ^2 = .16 (95% CI [.06, .48]), with an I^2 value of 88% (95% CI [.80, .92]). The prediction interval ranged from g = -.28 to 1.6, indicating that negative intervention effects may occur for future studies.

Figure 6

Forest Plot for Overall Digital Mathematical Game Effect on Achievement Emotions

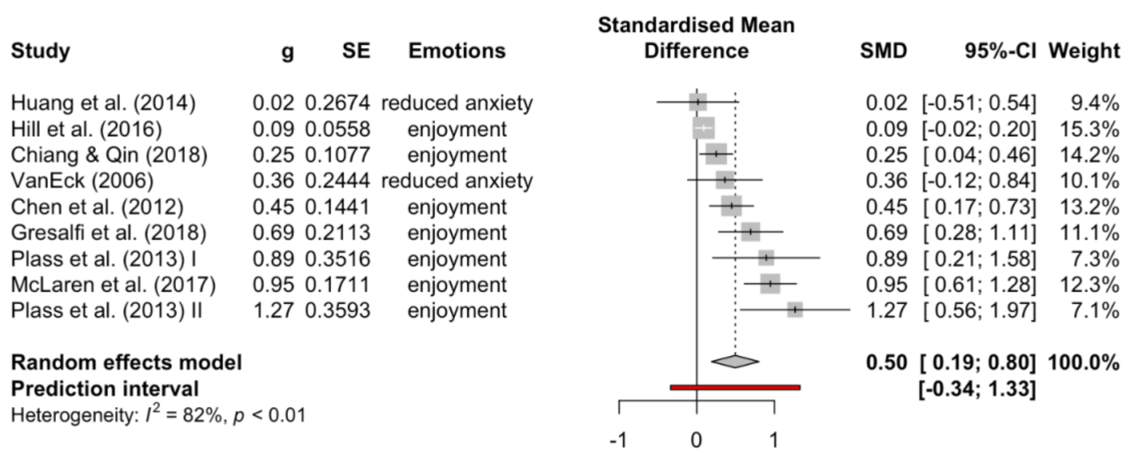
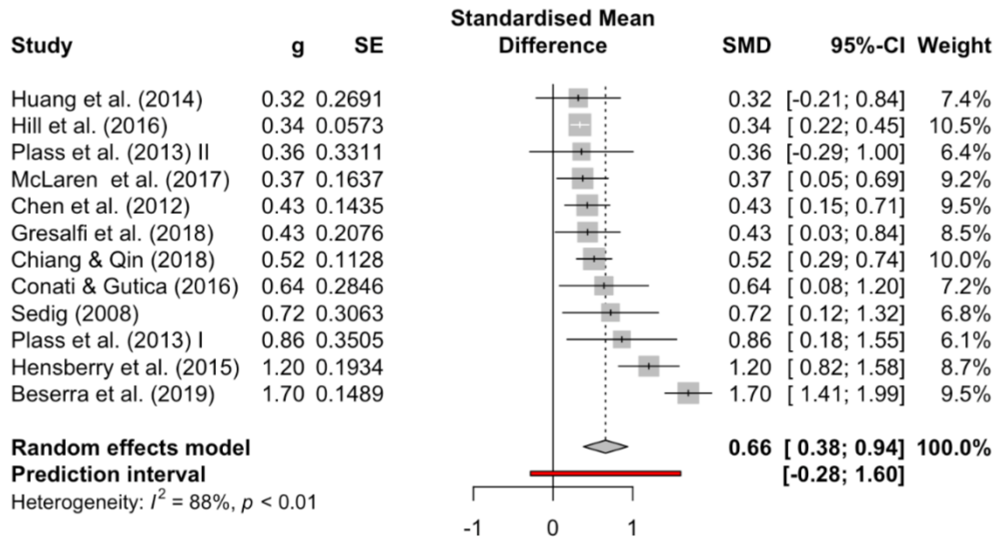


Figure 7

Forest Plot for Overall Digital Mathematical Game Effect on Mathematics Achievement



Second, we explore each design principles' effect on achievement emotions and learning outcomes. This was achieved by grouping studies based on their design principles and calculating effect sizes for each grouping.

Visual Aesthetic Design

Visual aesthetic design had a small to medium effect on enjoyment and reduced anxiety (see Figure 8.3; n = 6, g = .44, 95% CI [.11, .77], p = .02). The between-study heterogeneity variance was estimated at τ^2 = .08 (95% CI [.02, .56]), with an I^2 value of 84% (95%CI [.67, .92]). The prediction interval ranged from g = -.43 to 1.31, indicating that negative intervention effects cannot be ruled out for future studies. Similarly, a slightly higher medium effect on mathematics achievement was found (see Figure 8; n = 8, g = .54, 95% CI [.31, .78], p < .001). The between-study heterogeneity variance was estimated at τ^2 = .05 (95%CI [0.01, 0.30]), with an I^2 value of 66% (95%CI [.28, .84]). The prediction interval ranged from g = -.05 to 1.13, indicating that negative intervention effects may occur for future studies.

Musical Score

Musical score had a small but non-significant effect on enjoyment (see Figure 8; $n = 3$, $g = .29$, 95% CI [-.43, 1.02], $p = .22$). The between-study heterogeneity variance was estimated at $\tau^2 = .06$ (95%CI [.01, 3.85]), with an I^2 value of 77% (95%CI [.25, .93]). The prediction interval ranged from $g = -3.46$ to 4.05, indicating that negative intervention effects cannot be ruled out for future studies. Meanwhile, a significant small to medium effect on mathematics achievement was found (see Figure 8; $n = 6$, $g = .43$, 95% CI [.28, .59], $p < .001$). The between-study heterogeneity variance was estimated at $\tau^2 = .01$ (95%CI [.00, 0.18]), with an I^2 value of 3% (95%CI [.00, .75]). The prediction interval ranged from $g = .15$ to .72, indicating that negative intervention effects can be ruled out for future studies.

Game Mechanics

Game mechanics had a small to medium effect on enjoyment and reduced anxiety (see Figure 8; $n = 8$, $g = .42$, 95% CI [.11, .73], $p = .02$). The between-study heterogeneity variance was estimated at $\tau^2 = .07$ (95%CI [.01, .66]), with an I^2 value of 73% (95%CI [.45, .87]). The prediction interval ranged from $g = -.32$ to 1.15, indicating that negative intervention effects cannot be ruled out for future studies. Meanwhile, a medium effect on mathematics achievement was found (see Figure 8; $n = 10$, $g = .55$, 95% CI [.35, .75], $p < .001$). The between-study heterogeneity variance was estimated at $\tau^2 = .04$ (95%CI [.00, .22]), with an I^2 value of 60% (95%CI [.19, .80]). The prediction interval ranged from $g = .02$ to 1.08, indicating that negative intervention effects can be ruled out for future studies.

Narrative

There was not enough statistical data to estimate narrative's effectiveness on achievement emotions. A large non-significant effect size on mathematics achievement was found (see Figure 8; $n = 2$, $g = 1.2$, 95% CI [-5.53, 7.92], $p = .27$). The between-study heterogeneity variance was estimated at $\tau^2 = 0.51$, with an I^2 value of 91% (95%CI [.67, .97]).

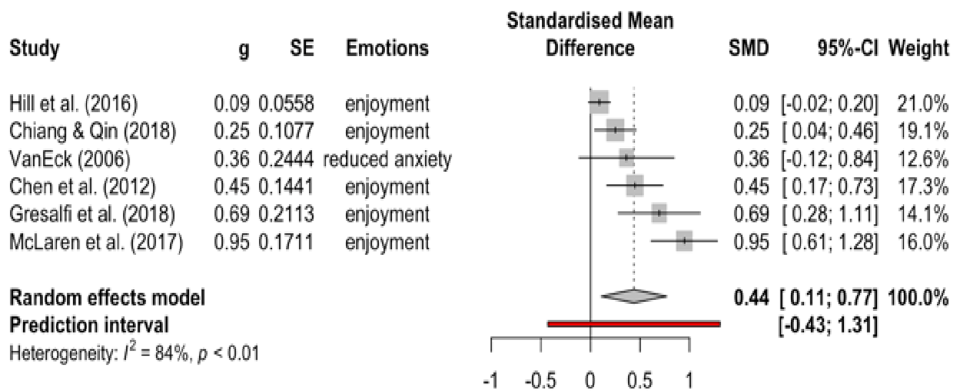
Incentive System

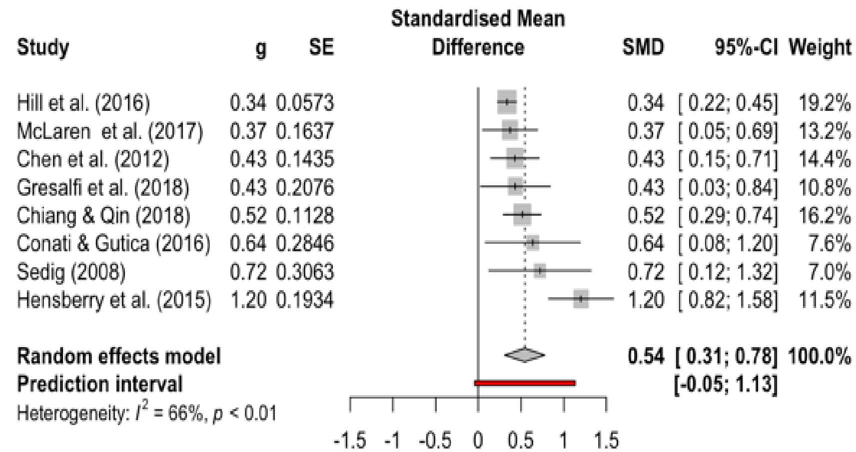
Incentive system had a medium effect size on enjoyment (see Figure 8; $n = 5$, $g = .60$, 95% CI [.14, 1.06], $p = .02$). The between-study heterogeneity variance was estimated at $\tau^2 = .08$ (95%CI [.00, 1.21]), with an I^2 value of 65% (95%CI [.09, .87]). The prediction interval ranged from $g = -.43$ to 1.64, indicating that negative intervention effects cannot be ruled out for future studies. Meanwhile, a medium effect size on mathematics achievement was found (see Figure 8; $n = 7$, $g = .63$, 95% CI [.35, .92], $p = .002$). The between-study heterogeneity variance was estimated at $\tau^2 = .06$ (95%CI [.00, .40]), with an I^2 value of 56% (95%CI [.00, .81]). The prediction interval ranged from $g = -.04$ to 1.31, indicating that negative intervention effects may occur for future studies.

Figure 8

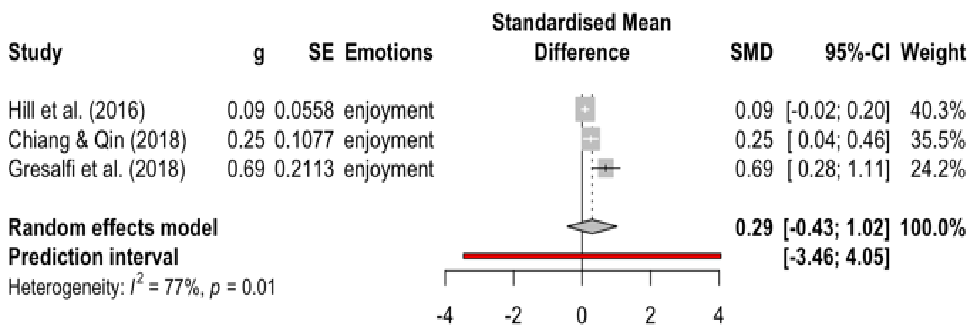
Forest Plots for Effect Size on Achievement Emotions and Mathematics Achievement

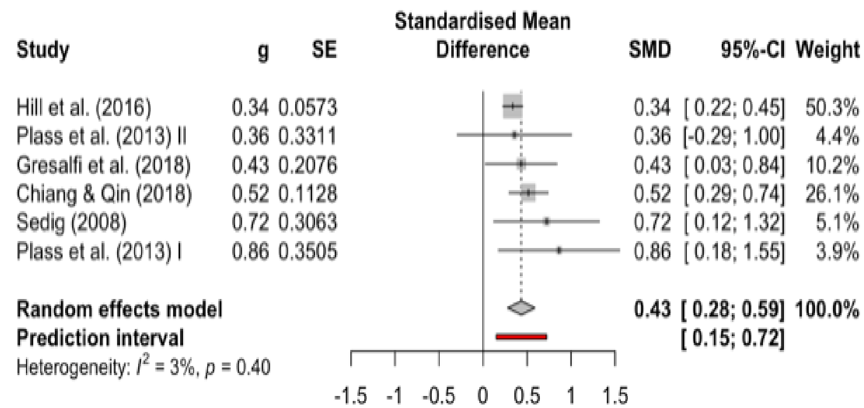
Visual Aesthetic Design



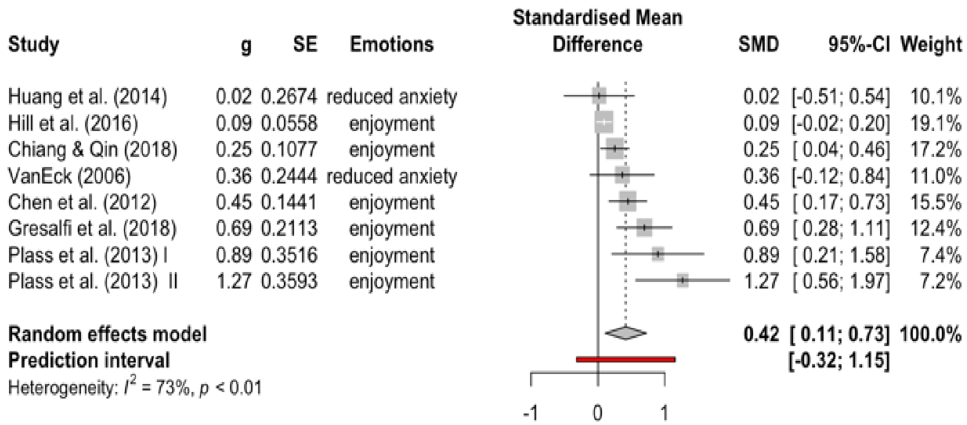


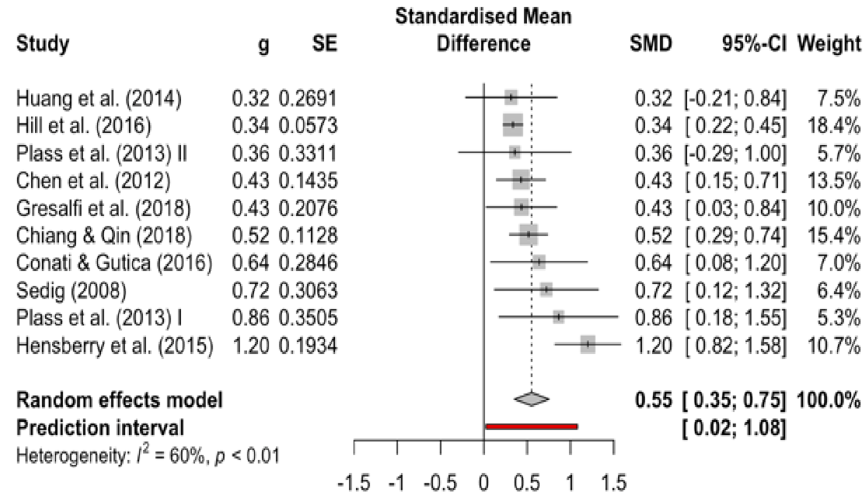
Musical Score



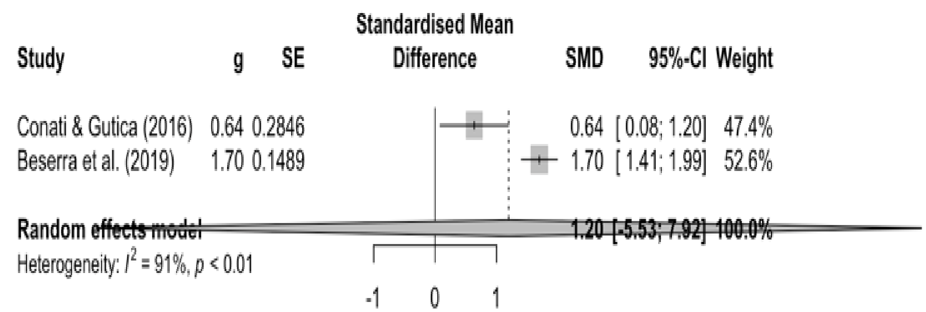


Game Mechanics

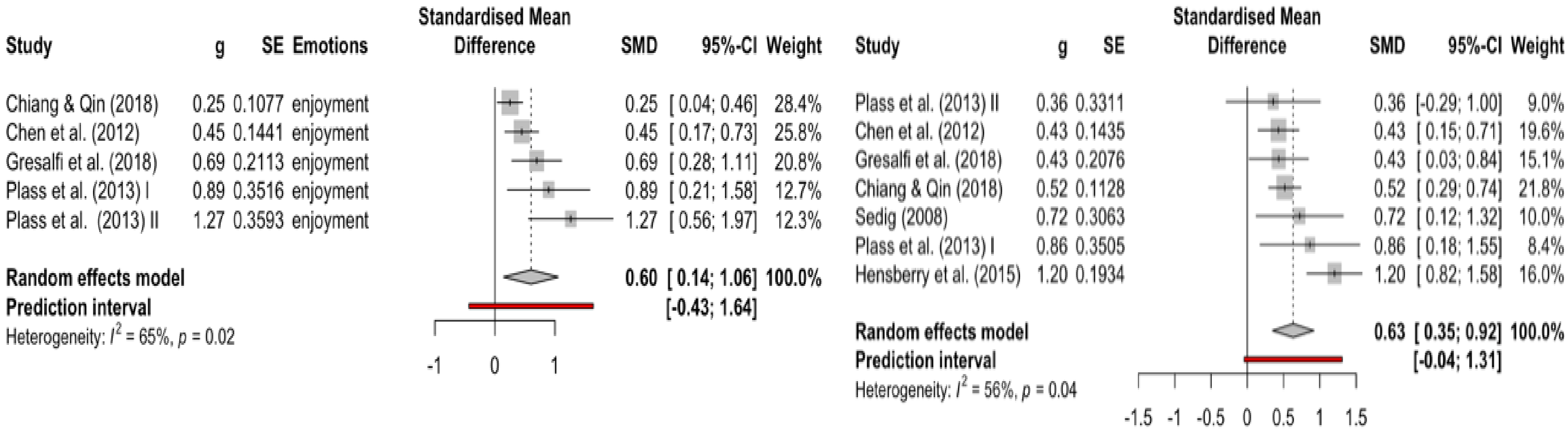




Narrative



Incentive System



Note. The diamond for the overall effect size of narrative is so large because too few studies use this principle. This is not a graphical error, but rather a sign that the effect is highly suspect.

Summary

Most games from reviewed studies applied more than one emotional design principle, indicating that students' emotional experience during digital game-based learning is being considered by mathematics researchers. The most used principles were visual aesthetic design, game mechanics, and incentive system, while musical scores and narrative were the least used. Though multiple emotional design principles were employed to some extent, most games adopted features that are easier to integrate (e.g., bright colours, avatars) while neglecting ones that should be just as important to learning but are difficult to integrate. For example, features such as narratives, unlocking mechanisms, and learner choice were largely neglected in the reviewed studies. The possible reasons could be that writing a simple story that aligns with learning goals is more challenging than presenting the learning content alone (Clark et al., 2016); having levels that unlock requires building a larger game world and system in addition to the core game; and providing learner choice requires developing and designing content that is not absolutely necessary. These design principles should be included in future mathematical game studies as they have the potential to intrinsically motivate learners and give them a sense of control over games (Habgood & Ainsworth, 2011).

Both the qualitative and quantitative studies indicate that digital mathematical games have a positive impact on achievement emotions and mathematics achievement. Incentive systems had the highest effect on both achievement emotions and learning outcomes, followed by visual aesthetic design and game mechanics. Narrative had a large but non-significant effect on achievement. However, due to small sample size, more studies are needed to draw further conclusions. Musical score showed the least effect on achievement emotions and mathematics achievement.

From a control-value theory lens, design principles that contribute to players' control and value appraisals are more likely to generate better achievement emotions and learning outcomes (Pekrun, 2006). Emotional design principles that had stronger effects (incentive system, visual aesthetic design, and game mechanics) likely provide learners with a sense of control over the task (control of avatars, adjusted difficulty level) or add extra value (rewards) to the task (Chen et al., 2012; Gresalfi et al., 2018). Similarly, narrative sets up challenges or goals in the game (Lindley, 2005), which offers extrinsic value to players. In contrast, emotional design principles that showed small effect size (i.e., musical scores) may not impact control appraisals (Beserra et al., 2019; Conati & Gutica, 2016), as sound and music are either pre-set or reflect in-game actions and cannot be predicted or determined by players.

All five emotional design principles had larger effects on learning outcomes than achievement emotions. One possible reason is that fewer studies measured achievement emotions than performance. Therefore, future works not only need to explore the effect of mathematical games on learning, but also need to examine their effect on achievement emotions, and how achievement emotions mediate the effects on achievement (Pekrun, 2006). The results provide guidance on choosing mathematical games for educational purposes. Teachers and parents should pick games that embed more than one forementioned emotional design principle, particularly those that provide a sense of control to players and add value to the activities. For instance, games that have avatars for learners to control or adjust difficulty level to different learners can give learners a sense of control whereas games with rewards add extra value for learners. This increased control and value may potentially lead to better learning experience and outcomes. A good digital educational game not only points out the mistakes made by players (e.g., sound feedback), but also scaffolds them with proper hints and adjusted difficulty levels, while motivating them with appealing visuals, an integrated narrative, and rewarding systems.

Conclusion

Research on achievement emotions and the effects of emotional design principles on game-based learning is lacking. This meta-analysis systematically examines each emotional design principles' impact on both achievement emotions and mathematics achievement. The findings show good evidence that adopting emotional design principles which promote control and value of the achievement activity can lead to positive achievement emotions and better learning outcomes. One limitation of the current work is that games in the reviewed studies covered more than one emotional design principles, thus the reported effect could be the result of an interaction. To better distinguish individual principle's effects, future work needs to adopt a value-added approach that systematically integrates and evaluates one principle at a time (Mayer, 2019). The current results not only provide insights to developers, educators, and parents on how to design and identify digital mathematical games that better facilitate students' learning but also provide a direction for future researchers to investigate the effect of different types of achievement emotions on game-based learning.

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Bridging Text

Chapter 4 presents a meta-analysis of the impact of game features on affective math attitudes and math achievement. The results indicate that certain game features, such as those that promote control/value appraisals, can lead to positive emotions and increase learning gains. Among the game features, incentive system had the strongest effects. These findings offer insights into the design and selection of digital educational games that have the potential to enhance students' math learning experience and outcomes.

Building on Chapter 4's findings, Chapter 5 further investigates the impact of incentive systems on math learning using the rigorous value-added approach. The chapter presents an experiment comparing two versions of a game, a base version and a value-added version, to rigorously examine the effect of incentive systems on children's math attitudes and achievement. The results suggest that incentive systems have the potential to foster better math performance and positive math attitudes. This empirical evidence provides valuable information for the development of effective digital educational games.

Chapter 5. Manuscript 4

Effects of Digital Educational Games on Students' In-game Performance, Math Achievement, and Math Attitudes: A Value-added Approach

Wen, R., & Dubé, A. K. (2023). *Effects of digital educational games on students' in-game performance, math achievement, and math attitudes: A value-added approach.* Manuscript in preparation.

Abstract

Children's math attitudes start to decrees at an early age. Literature and theories have suggested that intrinsic integration games have the potential to promote positive math attitudes and better performance, especially ones with incentive systems. However, no study to date has used the value-added approach to systematically investigate incentive systems' effects. The current study addressed this gap and used the value-added approach to examine incentive systems' impact on math outcomes. The findings revealed that games with incentive systems seem to generate higher math attitudes and better in-game performance than games without. However, there were no significant differences on math achievement across groups. Thus, games with incentive systems are more likely to promote positive math attitudes (e.g., higher value and less boredom) than games without and this yields practical implications for identifying and designing quality digital educational games for mathematics.

Keywords: digital games, educational games, math attitudes, incentive system

Math attitudes have received increasing attention in math education over previous decades (Carey et al., 2016; Neale, 1969). The term has been defined and investigated divergently across studies (Wen & Dubé, 2022). To provide a comprehensive picture of research on the topic, Wen and Dubé (2022) conducted a systematic review of math attitudes across 96 studies. Their review proposed a multidimensional definition, that math attitudes are cognitive beliefs about math, emotions associated with math, and behavioural tendency to learn math (Wen & Dubé, 2022). Correspondingly, a tripartite construct of math attitudes was proposed to reflect the cognitive, affective, and behavioural components of math attitudes (Wen & Dubé, 2022). The updated construct includes 10 subdimensions, with gender beliefs, confidence, and value representing cognitive attitudes; anger, anxiety, enjoyment, boredom, pride, and shame reflecting affective attitudes; and behavioural intentions representing behavioural attitudes (Wen & Dubé, under review). The present study leverages this updated construct to investigate the effectiveness of a popular method of addressing students' poor math attitudes, namely digital mathematics games (Dubé et al., 2019).

Importance of Math Attitudes

A number of researchers have investigated the relationships between math attitudes and achievement, and findings show that the former is a significant predictor of the later (Ganley & Vasilyeva, 2011; Ma & Kishor, 1997; Wen & Dubé, 2022). In a meta-analysis, Ma and Kishor (1997) summarized the results from 143 empirical studies, and the weighted effect size indicated that the correlation between math attitudes and math achievement is statistical reliable. Similarly, a systematic review by Wen and Dubé (2022) showed that each subdimension of math attitudes makes a unique and cumulative contribute to math achievement. The correlation between math attitudes and math achievement has been observed across different age levels, from young children (Vukovic, et al., 2013; Wu et al., 2012) to middle school students (Ganley & Vasilyeva, 2011; Wen & Dubé, 2022), and even undergraduate students (Buelow & Frakey, 2013; Pourmoslemi et al., 2013).

There is a clear value in investigating math attitudes due to their vital role in affecting math achievement (Neale, 1969; Wen & Dubé, 2022). Critically, research has revealed a significant decrease in students' attitudes towards math (Aiken, 1985; Fredricks & Eccles, 2002; Jacobs et al., 2002). These negative attitudes start to develop at an early age, and without intervention are likely to remain into adolescence and even adulthood (Aiken, 1985; Barroso et al, 2021; Morrisett & Vinsonhaler, 1965). Taking math anxiety for example, a subdimension of math attitudes that has received the most attention in research, children as early as Grade 1 and Grade 2 start to report anxious feelings towards math learning (Jameson, 2014; Ramirez et al., 2013). This math anxiety continuously develops into secondary education, where girls report higher levels of anxiety and worries than boys (Ashcraft & Faust, 1994; Hembree 1990; Wigfield & Meece, 1988). Once the math anxiety has been formed, it could last into adulthood (Barroso et al, 2021). In sum, math attitudes are important, and research has identified a continuously developing pattern of negative math attitudes over time. Researchers need to find an effective way to increase students' attitudes, especially from an early age, as early treatment and intervention may prevent math attitudes from decreasing irreversibly (Wigfield & Meece, 1988).

Digital Educational Games' Potential Effects on Math Attitudes

Digital educational games have the potential to generate positive attitudes towards learning content (Kirriemuir & McFarlane, 2004; Malouf, 1988; Oblinger, 2006; Pascale, 1974). Numerous studies have demonstrated the positive effects of educational games on attitudes towards various subjects: VanSickle (1986) reviewed 42 studies on educational games and found that almost half of the studies demonstrated better attitudes towards the subject in experimental groups than control groups. Similarly, Divjak and Tomić (2011) conducted a meta-analysis of 32 empirical studies on digital games, among which, 14 showed positive attitudes resulting from gaming. Researchers argued that the positive effectiveness of games should and can be applied specifically to mathematics, as students' attitudes towards mathematics need to be improved (Sedighian & Sedighian, 1966; Wen & Dubé, in press).

However, there is a dearth of empirical study on digital educational games' effects on students' math attitudes. For example, in a review of more than 200 digital educational math game studies published from 2010 to 2017, there was little research on the effects of educational games on students' math attitudes (Dubé et al., 2019). This is in line with eight major reviews of research on the effectiveness of educational games over the past twenty years (Mayer, 2011). Similarly, in Bai and colleagues' meta-analysis of 12 empirical studies on effectiveness of digital educational games, 10 tested impacts on learning achievement while only one study examined the effects on students' attitudes (2012). Clearly, previous studies on digital educational games in math education mainly focus on effectiveness in terms of providing engaging learning environments and improving academic achievement (Afari et al., 2013; Kiili et al, 2014; Outhwaite et al., 2017). But most research on engagement and achievement largely ignore the fact that students are engaged by the game content more than the math content (Dubé & Keenan, 2016; Falloon, 2013; McEwen & Dubé, 2016) and that improved math performance does not necessarily lead to better attitudes towards math learning. In fact, higher performing students also hold negative attitudes towards mathematics (Chen et al., 2012). Therefore, it is essential to explore digital educational game's impact on math attitudes.

What Makes a Good Digital Educational Game?

Not all digital educational games will be effective in supporting learning while sustaining students' interest and building positive attitudes (Habgood &Ainsworth, 2011; Ke, 2008). To start with, it is critical to understand what makes a good digital educational game. Digital educational games can be categorized into intrinsic integration games and extrinsic integration games (Habgood &Ainsworth, 2011). Intrinsic integration games are the ones in which learning contents are closely integrated into core game mechanics. According to Habgood et al. (2005), there are two main components in intrinsic integration games: a) the learning content is delivered through the most fun part of the game, without interrupting the flow of the game, and b) learning content is part of the core mechanics of the game, that players could interact with, and build representations from. As players interact with the most fun part of the game, they are self-directed to learn the content, complete challenges, or achieve desired goals while experiencing enjoyment from the process. In contrast, extrinsic integration games can be seen as "chocolate-covered broccoli" (Bruckman, 1999), in which core game mechanics provide the fun parts of the game that cover the learning contents. Learning contents are not delivered through the fun part of the game and are not tightly connected to core game mechanics. For example, a game named Toon Math Endless Run requires players to continually run to become a math ninja by solving a serious of arithmetic problems (e.g., 2 x 3 = ?) presented along the runner's path. The constantly appearing math problems block the way and interrupt the flow of the game. Here, there is no strong connection between learning content (arithmetic) and fantasy context (run to rescue friends). One could replace math task with a phonic task (e.g., cat, hat, b?t) without having any impact on game mechanics. As a result, fun parts of the games only serve as rewards or chocolate coating for learning content, while learning content seems to be the demotivating part that constantly interrupt the fun of the game.

Compared to extrinsic integration games, intrinsic integration games have the potential to promote intrinsic motivation and generate more autonomy and better learning outcome as implied by self-determination theory and related literature (Habgood &Ainsworth, 2011; Ryan & Deci, 2000). According to self-determination theory (Ryan & Deci, 2000), when three types of psychological needs (i.e., competence, autonomy, and relatedness) are met, people feel intrinsically self-motivated. In intrinsic integration games, players may gain high level of competence through accomplishing learning tasks and achieving goals in a fantasy context. They may feel high levels of autonomy by controlling the flow of the games, and taking actions based on individual choices. From self-determination theory's perspective, intrinsic integration games motivate players to self-directed learning in games. Previous research suggests that intrinsic integration games are more interesting and produce better learning outcomes than extrinsic integration games (Habgood &Ainsworth, 2011; Malone, 1980). Therefore, when selecting digital educational games, one should consider intrinsic integration games for better learning experience and educational benefits.

Game Features that Promote Positive Emotions—Incentive System

Choosing the intrinsic integration games over extrinsic integration games is just the first step. It is vital to understand what game features in the intrinsic integration games are most effective in maximizing learning and enhancing attitudes (Boyle et al., 2016). Loderer et al. (2020) argue that game features that support players' positive emotions are more likely to generate better learning outcomes. Based on this, five emotional design features were proposed for better learning experience in digital educational games, including visual aesthetic design, musical score, game mechanics, narrative, and incentive systems (Loderer et al., 2020). Visual aesthetic design refers to visual elements such as bright colors, round shapes, and presence of avatars. Musical score involves music, human generated sounds, and sound feedback. Game mechanics mean that the game has a clear task, which is related to learning goals, while it also provides scaffolding and adaptive difficulty levels. Narrative means that the game has a compelling storyline that contextualizes learning. Incentive systems refer to the features of rewards (e.g., starts, points, badges, avatars), the freedom to customize avatars or choose gifts, and unlocking mechanisms that provide opportunities to unlock new game levels (Loderer et al., 2020). Empirical evidence has revealed the effectiveness of these five design features in supporting better attitudes and learning (Arroyo et al., 2013; Baylor, 2011; Clark et al., 2016; Loderer et al., 2018; McNamara et al., 2010; Wen & Dubé, in press).

Of the five emotional design features mentioned above, incentive systems are probably the one that is the easiest to incorporate into games while contributing to better learning outcomes, if designed well. Wen and Dubé (in press) conducted a meta-analysis to examine the effects of these five design features, and the results showed that games with incentive systems had better learning outcomes comparing to games with other features. Specifically, incentive systems had a medium effect size on students' enjoyment (g = .60) and math performance (g = .63; Wen & Dubé, in press). In line with this finding, previous studies show that incentive systems in digital educational games can increase students' motivation to play and enhance students' performance (Rapp, 2017). Therefore, incentive systems seem to be a feature that can be added into games with less effort while promoting player's motivation, game play experience, and even learning outcomes.

Control-value Theory

The effects of an incentive system can be understood using control-value theory (Pekrun, 2006; Pekrun & Perry, 2014). Control-value theory suggests that emotions that relate to learning, such as enjoyment and anxiety, are products of control- and value-related appraisals. Control-related appraisals refer to one's evaluation of their controllability over learning activities and outcomes (Pekrun & Perry, 2014). Students with higher level of control-related appraisal tend to have positive emotions, such as enjoyment and pride, whereas students with low level of control are more likely to experience negative emotions such as frustration (Pekrun, 2006). Value-related appraisals refer to one's evaluation of the importance of the learning activity and outcomes (Pekrun & Perry, 2014). Both intrinsic value and extrinsic value account for value-related appraisals. When combining effects of two types of appraisals, high value and high controllability may arouse enjoyment while high control but low value may result in anger (Pekrun, 2006). An incentive system can provide students with a high degree of control over the game by allowing players to choose which levels to play or avatars to represent themselves. At the same time, they can increase players' extrinsic value by giving rewards. As incentive systems have the potential to generate positive emotions by influencing players' control and value appraisals, it could be used in digital educational games for promoting better math attitudes.

Value-added Approach for Examining Specific Game Feature

Though research has found some positive effects of incentive systems on learning (Rahimi et al., 2021), the inherently complex design of digital educational games, with their multiple features, makes it difficult to separate the effects of incentive systems from those of other features. To better investigate a specific feature's effects, a rigorous scientific approach is needed. Mayer (2019) suggests the value-added approach to investigate a given game feature. In the value-added approach, two versions of a game are needed; a base version and a value-added version, which is the same game as the base version but with one additional feature added (Mayer, 2019). By comparing outcomes of two versions of the same game, researchers can be more confident in the conclusions they draw on the effectiveness of one particular game feature.

Research Goals

Overall, students' attitudes towards math need to be improved at an early age, and intrinsic integration games have the potential to do so. Specifically, games with incentive systems are believed to promote learner's motivation, positive emotions, and performance. However, no study to date has used the value-added approach to rigorously test incentive systems' effects on learning. Therefore, the goal of the present study was to investigate the effects of digital educational games on math attitudes and math performance. Particularly, the study adopted a value-added approach, and rigorously examined incentive systems' effects on learning with younger kids. For this purpose, two high-quality versions of a game called "Math Maker" were specially designed for use in the study. The base version is an intrinsic integration math game containing four of the five emotional design features (Loderer et al., 2020) and the value-added version is identical but adds an incentive system. The games were designed to practice basic arithmetic skills for primary school students. A three-group (base game, value-added game, control) between-subject experimental design is used to test the effects of incentive systems on students in game-behaviours, learning gains, and math attitudes. The following research questions guided the investigation: 1) How do students' in-game behaviours differ between games with incentive systems and games without? 2) To what extent do digital educational games with incentive systems affect students' math performance? 3) To what extent do digital educational games with incentive systems affect students' math attitudes?

Methodology

Participants

72 first graders were recruited from suburban public schools in China. This particular age of children was chosen because research shows that negative shifts in math attitudes start at this stage (Jameson, 2014; Ramirez et al., 2013). Some participants dropped out due to internet access. Finally, 45 students (53% female) with a mean age of 7.58 years (*SD* = 0.75) were included in this study. The study was evaluated and approved by institution's ethics board and all students and their parents gave consent prior to participation.

Procedure

All participants were administered a pre-test on math attitudes and math performance online and completed the questionnaire and math test with the assistance of their parents. Then participants were randomly assigned to two modes of supplementary learning during the summer holiday: a math game with incentive system (value-added game group, VGG), and a math game without incentive system (base game group, BGG). As the two assigned games are browser-based, students who failed to load the game due to technical reasons but were still interested in participating were then organized into a control group (CG). Students in the experiment groups were suggested to play the assigned math game online for up to 15 minutes a day, five days a week, for four weeks. Students in the control group received no treatment. After the four-week intervention, all participants completed the math attitudes survey and math performance test online as post-tests. In addition to intervention, all participants were assigned the same math homework for summer holiday by their math teachers. The summer homework was designed to practice their math arithmetic skills learned in the previous semesters.

Materials

An intrinsic integrated digital math game called ‘Math Maker’ was used in this study. In ‘Math Maker’, math contents are intrinsically integrated into the core game mechanic by having players clear a path in each level to help the game character progress. Clearing the path involves matching the number of ice cubes in the level to the number of blocking lava cubes (see Figure 9 for the tutorial level). After the tutorial, ice cubes are generated by the player applying their understanding of a fundamental math concepts including operations, missing terms, tens and ones, symbols, and place value (see Figure 10 for operations on small numbers). Players make progresses by practicing these concepts. Therefore, the learning content is integrated into the most fun part of the game, the central game mechanic.

Two storylines are included in ‘Math Maker’, with one focusing on operations with small numbers (numbers less than 20) and the other with big numbers (numbers greater than 20). For each storyline, the aforementioned big concepts were integrated into the game, with each concept being practiced with several game levels (one math problem per game level), resulting in 50 game levels in total (28 levels for small numbers, and 22 levels for big numbers).

Two versions of ‘Math Maker’ were created for this study, a base version and a value-added version. In the base version, the game has implemented the four emotional design principles (i.e., visual aesthetic design, musical score, game mechanics, narrative) where students use arithmetic skills to progress from level-to-level as just described. In the value-added version, the core mechanic of the base version and four emotional design principles remain with one particular function added - an incentive system. When students completed each level by successfully solving one math problem, they were rewarded with different collections of avatars or hats. Players have the extra autonomy to customize avatars or change hats (see Figure 11).

Figure 9

Screen Shot of Tutorial in Math Maker

Figure 10

Screen Shot of Basic Level in Math Maker

Figure 11

Screen Shot of Incentive System in Math Maker

Measures

In-game Performance Measure

Log file data from games were collected to track participants in-game behaviours. These included participant's total time spent in game, total number of levels completed, and average time spent per level. Total time spent in game was calculated in minutes by adding up the time spent in each game level. It represents the total amount of time a player spent actively playing the game during the study. Total number of levels completed was calculated by adding up all the game levels player successfully played through, and it is a measure of the level of practice and exposure to math problems in the game experienced by the player. Average time spent per level is calculated by dividing total time spent in game by total number of levels completed. It represents the average amount of time a player spent for each level in the game, and it is an indicator of how fast a player is progressing, with shorter time indicating more effective solutions and better understanding of the math content involved.

Math Performance Measure

Participants were given an online math ability test as pre- and post-test of arithmetic learning. The test was designed by the lead researcher to assess the specific math skills taught in Math Maker, including operations (four items), missing terms (four items), tens and ones (four items), symbols (two items), and place value (four items; see Appendix F). Each math concept was measured through two-to-four math problems, with half the problems being small numbers (numbers less than 20, e.g., 2 + 4 = ?) and the other half containing big numbers (numbers greater than 20, e.g., 20 + 7 = ?). All 18 problems were in the format of fill in the blanks, and each problem is weighted for one point towards a total score of 18. The post-test was identical to pre-test but presented in a different randomized order.

Math Attitudes Measure

Math attitudes were assessed using items adapted from the Tripartite Math Attitudes Scale (TMAS; α ranges from .69 to .93) and Academic Emotions Questionnaire — Mathematics (α ranges from .91 to .96; Bieleke et al., 2022; Pekrun et al., 2011; Wen & Dubé, 2023; see Appendix G). The measure contains ten latent variables, namely gender beliefs (also known as stereotype, that boys are more capable to learn math and get good scores than girls), confidence, value of math subject, anger, anxiety, boredom, enjoyment, pride, shame, and behavioural intentions toward math. Each latent variable was measured through four items, resulting in 40 items in total. Students responded to all items by indicating their level of agreement to the provided statements ranging from ‘strongly disagree’, ‘disagree’, ‘neutral’, ‘agree’ to ‘strongly agree’. The scale was translated from English to Chinese with bilingual experts involved. To assist the comprehension of participants, the language of the scale was simplified, and visual face expressions with 5 emotions varying from sad to happy were used for each level.

Results

RQ 1. How Do Students' In-Game Behaviours Differ Between Games with Incentive Systems and Games Without?

To investigate whether students' in-game behaviours differ between games with incentive systems and that without, three paired *t*-tests were performed on total time spent, total number of levels completed, and average time spent per level (see Table 15). Students in BGG spent significantly more time in game than students in VGG, $t\ (28) = 11.52$, $p = .002$, $d = .64$. However, there was no significant difference in total game levels completed between BGG and VGG, $t\ (28) = 1.502$, $p = .231$, $d = .042$. Importantly, completing each level requires students to successfully solve the math problems. The lack of difference in game levels completed between groups still means that students had successfully solved a similar amount of math problems in both groups. However, students in VGG spent significant less time in solving math problems per game level than students in BGG, $t\ (28) = 10.53$, $p = .003$, $d = .72$. These findings indicate that students in both groups completed a similar number of game levels, by correctly solving math problems in each level, but students in the VGG (with incentive systems) progressed faster than students in the BGG (without incentive systems).

Table 15

In-game Behaviours by Game Conditions

	Base game group *M (SD)*	Value-added game group *M (SD)*
Total time spent (minutes)	399.73 (379.18)	215.27 (155.39)
Total levels completed	221.80 (145.53)	216.40 (112.14)
Average time spent per level (minutes)	1.45 (0.99)	0.93 (0.28)

RQ 2. To What Extent Do Digital Educational Games with Incentive System Affect Students' Math Performance?

Students' score on the math ability test were summed to represent their overall math performance with a total score of 18. Due to the fact that the math ability test aims to measure math skills covered in the game, and participants had already learnt the concepts covered in the game by the time of the intervention, censored data (or ceiling data) occurred for both pretest and post-test across groups (see Table 16). Treating censored data as if they were true scores are problematic, as it leads to weakened variance estimates (Liu & Wang, 2021). Therefore, raw data were first adjusted to estimate the true scores with ceiling thresholds while adjusting the ceiling effects and controlling for Type I error rates (Liu & Wang, 2021). The adjusted means and variances were generated for each group (see Table 17).

To investigate the effects of learning conditions on math performance, the Brown-Forsythe's F^* Test[1] was chosen as it deals with skewed data and unequal variance while keeping type I error close to .05 (Glantz & Slinker, 2001; Liu & Wang, 2021; Tomarken & Serlin, 1986). The Brown-Forsythe's F^* Test was performed with adjusted values in gain scores (the difference between post-test and pre-test scores) on five types of math concepts. The findings showed that there were no significant differences of students' gain scores on concepts of operations, missing terms, tens and ones, symbols, place value, and overall math performance cross groups, $F_{operations}$ (1, 41) = -0.28, p = 1, partial f^2 = -0.01; $F_{missing\ terms}$ (1, 41) = 5.15, p = 0.42, partial f^2 = 0.23; $F_{tens\ and\ ones}$ (1, 41) = -11.21, p = 1, partial f^2 = -0.50; $F_{symbols}$ (1, 41) = -0.28, p = 1, partial f^2 = -0.01;

[1] Welch's F test and Brown-Forsythe's F test are more consistent across all levels of variance heterogeneity and unequal sample size in terms of controlling for type 1 error while ANOVA and Kruskal-Wallis Test break down when larger variance with small sample sizes occur (Tomarken & Serlin, 1986). Moreover, Brown-Forsythe's F test is suggested to use when the data is censored (Glantz & Slinker, 2001; Liu & Wang, 2021).

$F_{place\ value}$ (1, 41) = 1.71, p = .19, partial f^2 = 0.76; $Fo_{verall\ performance}$ (1, 41) = 1.43, p = .28, partial f^2 = .06. Overall, the results indicated that students' performance in CG were not significantly different from students' performance in BGG and VGG.

Table 16

Mean Math Performance in Percentage

	Pretest			Post-test		
	CG (%)	BGG (%)	VGG (%)	CG (%)	BGG (%)	VGG (%)
Operations	100	100	98.25	100	100	100
Missing terms	98.25	100	98.25	98.25	96.75	100
Tens and ones	96.75	91.75	91.75	86.75	96.75	93.25
Symbols	96.50	96.50	96.50	96.50	96.50	100
Place value	80.00	81.75	90.00	91.75	88.25	88.25
Total	94.06	93.72	94.83	94.44	95.56	95.94

Note. CG = control group BGG = base game group, VGG = value-added game group

Table 17

Adjusted Means and Variance by Concepts and Conditions

	Pretest			Post-test		
	CG	BGG	VGG	CG	BGG	VGG
	M (V)	*M (V)*	*M (V)*	*M (V)*	*M (V)*	*M (V)*
Operations	4.00 (0)	4.00 (0)	3.93 (0.26)	4.00 (0)	4.00 (0)	4.00 (0)
Missing terms	3.93 (0.26)	4.00 (0)	3.93 (0.26)	3.93 (0.26)	3.87 (0.35)	4.00 (0)
Tens and ones	3.87 (0.35)	3.69 (0.53)	3.82 (0.59)	3.12(0.42)	3.62 (0.23)	3.52 (0.34)
Symbols	1.93 (0.26)	1.93 (0.26)	1.93 (0.26)	1.93 (0.26)	1.93 (0.26)	2 (0)
Place value	3.16 (0.59)	3.22 (0.46)	3.41 (0.41)	3.98 (1.00)	3.73 (1.07)	3.73 (1.07)
Total	16.89 (0.42)	16.84 (0.35)	17.02 (0.42)	16.96 (0.21)	17.15 (0.75)	17.25 (0.70)

Note. V stands for variance, CG = control group, BGG = base game group, VGG = value-added game group

RQ 3. To What Extent Do Digital Educational Games with Incentive System Affect Students' Math Attitudes?

Effects of Learning Conditions on Overall Math Attitudes

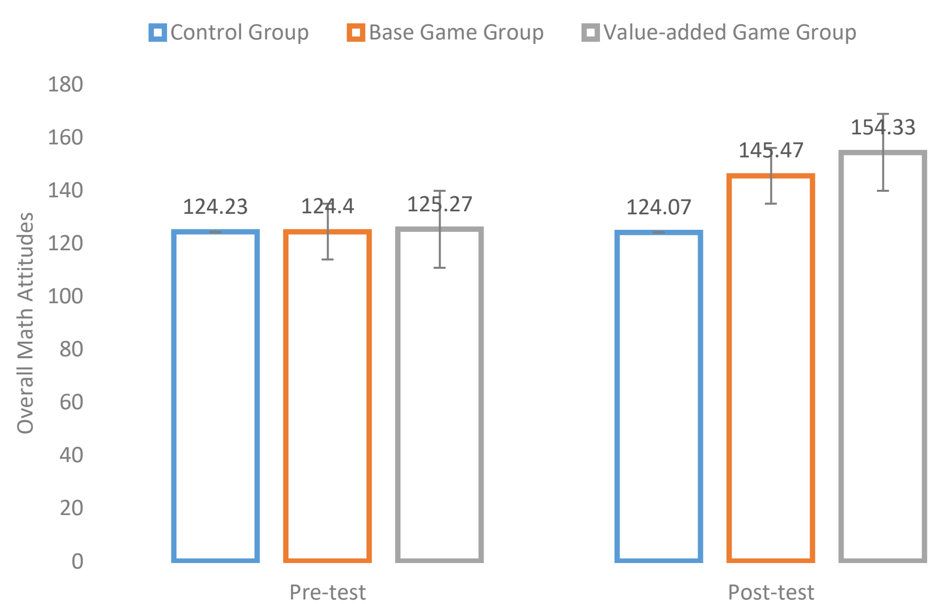
Students' scores on each subdimension of math attitudes were totaled to represent their overall attitudes (see Figure 12). The higher the score, the better attitudes towards math. One-way ANOVAs were conducted on the gain scores (pre-test/post-test score differences) of overall math attitudes[2]. Due to the violation of homogeneity of variance, Welch's ANOVA at an alpha

[2] To deal with pre-test and post-test design, ANOVA with gain scores and ANCOVA outperform ANOVA with post-test scores only and repeated-measures ANOVAs in terms of keeping Type-I error rate and statistical power (Jan Vanhove, 2014). Meanwhile, ANOVA with gain scores outperforms ANCOVA in very small samples (Jan Vanhove, 2014).

level of .05 and Games-Howell post hoc test were used (Tomarken & Serlin, 1986). The analysis yielded a significant difference in students' gain scores of overall math attitudes among learning conditions, F (2, 24.52) = 34.91, $p < .001$, $\eta^2 = .63$. The Games-Howell post hoc test for multiple comparisons found that students' gains scores in three learning conditions were significantly different from each other. To be specific, students in VGG (M = 29.07, SD = 5.41) had significant higher average gain scores in overall math attitudes than students in the other two groups. Moreover, students in BGG (M = 21.07, SD = 10.53) had significant higher average gain scores in overall math attitudes than students in CG (M = -0.20, SD = 12.37). This indicates that students who played the value-added game had the most gains in their positive math attitudes, followed by students who played the base game, while those in control group did not see any improvement in math attitudes.

Figure 12

Overall Math Attitudes by Time and Learning Conditions



Effects of Learning Conditions on Subdimensions of Math Attitudes

To further investigate which subdimension contributed to the difference in overall math attitudes among conditions, Welch's ANOVAs were conducted on gain scores (see Table 18) for the ten subdimensions under cognitive, affective, and behavioural math attitudes.

In cognitive components of math attitudes, the analysis did not yield a significant difference in students' gender beliefs among conditions, F (2, 24.06) = 1.49, p = .246, η^2 = .03. There was a significant difference in students' gain scores for confidence, F (2, 27.93) = 3.64, p = .039, η^2 = .15. Despite the means indicated that the BGG (M = 0.35, SD = 0.41) and VGG (M = 0.33, SD = 0.36) had higher confidence than the CG (M = 0.02, SD = 0.37), the conservative Games-Howell post hoc test did not yield a significant difference. There was a significant difference in students' gain scores for value across groups, F (2, 26.04) = 7.92, p = .002, η^2 = .31. Two game groups had significantly higher average gain scores than students in CG (M = -0.27, SD = 0.56). Moreover, students in VGG (M = 0.33, SD = 0.26) had significantly higher gain scores than students in BGG (M = 0.10, SD = 0.23). The results suggested that students in the math game groups gained more value towards math learning than those in the control group, and those who played the game with an incentive system gained more value for math than those played who played the base game.

In affective components of math attitudes, gain scores for different achievement emotions were compared across learning conditions. There was not a significant difference in students' gain scores for anger, F (2, 24.68) = 0.46, p = .638, η^2 = .02. Similarly, no significant difference was found in students' gain scores for anxiety, F (2, 23.63) = 1.34, p = .281, η^2 = .08. There was a significant different in gain scores for boredom, F (2, 23.88) = 48.86, p < .001, η^2 = .76.

Students in both BGG (*M* = 2.12, *SD* = 0.52) and VGG (*M* = 2.55, *SD* = 0.29) had significant higher average gain scores for boredom than students in CG (*M* = 0.02, *SD* = 0.95). Moreover, the gain score for VGG was significantly higher than that for BGG. The results indicated that students who played the value-added game had significant greater decrease in their boredom level than students who played the base game, followed by students in the control group. For the effect of learning conditions on students' enjoyment, there was a significant difference on gain scores, F (2, 26.95) = 14.98, $p < .001$, $\eta^2 = .38$. Students in both BGG (*M* = 0.92, *SD* = 0.50) and VGG (*M* = 1.17, *SD* = 0.86) had significant higher average gain scores than students in CG (*M* = 0.02, *SD* = 0.53). This indicated that students in both game conditions experienced higher levels of enjoyment than students in the control group. The analysis did not yield a significant difference in gain scores for pride F (2, 20.39) = 3.45, $p = .051$, $\eta^2 = .11$ or shame F (2, 26.93) = .01, $p = .987$, $\eta^2 = .00$.

In behavioural components of math attitudes, there was a significant different in gain scores for behavioural intentions, F (2, 27.36) = 12.09, $p < .001$, $\eta^2 = .32$. Both BGG (*M* = 0.98, *SD* = 1.09) and VGG (*M* = 1.52, *SD* = 0.74) had significant higher average gain scores than students in CG (*M* = 0.07, *SD* = 0.86). The result suggests that students in the game conditions gained more willingness to learn math than students in the control group.

Table 18

Gain Scores on Math Attitudes by Subdimensions and Learning Conditions

Math attitudes	Subdimensions	CG *n* = 15		BGG *n* = 15		VGG *n* = 15	
		M	*SD*	*M*	*SD*	*M*	*SD*
Cognitive components	Gender beliefs	-0.08	0.35	0.05	0.73	0.10	0.21
	Confidence	0.02	0.37	0.35	0.41	0.33	0.36
	Value	-0.27	0.56	0.10	0.23	0.33	0.26
Affective components	Anger	0.07	0.78	0.12	0.74	0.20	0.37
	Anxiety	0.22	0.59	0.45	0.40	0.48	0.20
	Boredom	0.02	0.95	2.12	0.52	2.55	0.29
	Enjoyment	0.02	0.53	0.92	0.50	1.17	0.86
	Pride	-0.17	0.98	0.18	0.89	0.47	0.21
	Shame	0.07	1.03	0.10	0.66	0.12	0.59
Behavioural components	Behavioural intentions	0.07	0.86	0.98	1.09	1.52	0.74

Note. CG = control group, BGG = base game group, VGG = value-added game group

Discussion

This study aims to investigate digital educational games' effects on students' attitudes towards math and their math performance. Further, it used the value-added approach to test the effects of a particular game feature--incentive systems. The findings revealed that students played games with incentive systems showed greater math ability and motivation as indicated by their faster in-game progression. Although there were no significant differences in students' math performance across the three learning conditions, students who played the value-added game were more proficient in solving in-game math problems than students who played the base game. In addition, digital educational games were better at increasing students' overall math attitudes. To be specific, games with incentive systems seemed to be more effective at improving math attitudes than games without.

Effects on In-game Behaviours

There was a significant difference in students' behaviours across the two versions of the games. Specifically, while students in the two conditions made similar amounts of progress (i.e., successfully solving a similar number of math problems), students in VGG took less time than those from BGG. This finding can be explained by the self-determination theory; with higher levels of autonomy satisfied, learners may be more motivated (Ryan & Deci, 2000). The value-added game provided the autonomy to customize avatars, which may promote higher level of motivation to engage in game learning. Similarly, control-value theory (Pekrun & Perry, 2014) also suggests that value-related appraisal alters achievement emotions, which can induce and modulate motivation. Thus, incentive systems may result in higher value-related appraisal, better learning experience, and higher motivation to progress as it adds extrinsic value to play the game. Taken together, games with incentive systems seem to promote higher levels of motivation than games without. Therefore, game designers, educators and parents should consider this particular game feature when designing or selecting games for motivational purposes.

Effects on Math Achievement

Due to Covid-19, the intervention was postponed until the end of the semester when participants had already learnt the concepts covered in the game and math ability test. As a result, the findings did not show a significant difference on math learning among groups. Both the control group and the treatment groups started with high scores at pre-test and ended with similar scores at post-test. Also, there was no time limit to answer the math ability test. Therefore, the extent to which the math ability test reflects students' math proficiency remains unclear. Yet, the data from in-game behaviours may indirectly reflect participants' proficiency in solving math problems. Calculation proficiency refers to the fluency in retrieving the solutions from long-term memory or the application of efficient strategies (Cowan et al., 2011). Research shows that processing speed is correlated with calculation proficiency (Bull & Johnston, 1997; Hitch et al., 2001). Students in the VGG spent significantly less time solving similar number of math problems than students in the BGG, indicating that their processing speed is faster, and they may be more effective at retrieving knowledge or applying strategies than peers. This finding is supported by control-value theory (Pekrun & Perry, 2014), which implies that value-related appraisals induce positive achievement emotions, higher level of motivation, and better performance. Incentive systems may lead to higher value-related appraisals, thus result in better performance in solving math problems in games. In line with control-value theory, empirical research also shows that incentive systems have enhanced in-game performance (Rapp, 2017). However, more research is needed to directly and clearly show the games' effect on students' math performance.

Effects on Math Attitudes

The findings revealed that students who played the games had significantly higher gains in their positive attitudes towards math than students in the control group. Specifically, students who played the games gained a higher value for math, experienced higher levels of enjoyment, lower levels of boredom, and more willingness to learn math than those in the control group.

Further, students in the VGG had significantly higher value and lower levels of boredom than students in the BGG. These results are supported by control-value theory in that control-related and value-related appraisals influence students' achievement emotions, which further affect their motivation and learning outcomes (Pekrun & Perry, 2014). Compared to the control group, students in the game conditions had more autonomy to explore the games and, thus, a higher level of control appraisals. Moreover, students in VGG experienced higher levels of value appraisals than the BGG, due to the incentive systems. With higher control and value appraisals, students in VGG are more likely to generate positive emotions than students in BGG and CG (Pekrun & Perry, 2014). The results are also in line with previous research that games supporting control/value appraisals enhance students' positive emotions (Wen & Dubé, in press). While the generalizability of the results from this study is limited by a small sample size, they are bolstered by the robustness of the study design and the researched-informed design of the custom digital math game. Further research with larger sample size is needed to replicate these results.

Limitations

The lack of randomization in the control group is a limitation of the current study. While it was practical to use the convenience sampling method (i.e., assign those students who had difficulty accessing the games to control group) given the constraints of the study, it may result in selection bias and confounding variables that could have affect the internal validity. While the design used pre/post-test to control for pre-existing differences and gain scores were used to represent the effects during the intervention, the results need to be interpreted with caution, as the control group may not be representative of the broader population. Future research with randomization methods could help address this limitation and improve the validity and generalizability of the results.

Due to COVID-19, the original plan of data collection during the school year had to be moved to the summer months. This unforeseen change resulted in a reduction in the number of available participants. Consequently, another limitation of this study is the small sample sizes, which may affect the statistical power and generalizability of the results. While the findings provide valuable insights into the effectiveness of the incentive systems and serves as an excellent basis for future studies using the value-added approach to test specific game feature's impact, caution is needed when interpreting the results. Moreover, the small sample may not be representative of the larger population, thus limit the ability to generalize the findings to broader populations. To address this limitation, future research with large sample size is necessary. Additionally, the replication with diverse populations and settings can help establish the robustness and reliability of the findings and provide more empirical evidence on the effectiveness of incentive systems.

Another potential limitation of the current study is the lack of control for executive functions as potential confounding variables. Executive functions, which encompass cognitive processes such as attention, working memory, and inhibitory control, are known to influence math achievements (Best et al., 2011). These functions might vary among individuals in the experimental groups, potentially impacting the observed outcomes. While the study focused on the effects of the intervention, the omission of executive functions as a controlled variable could introduce unaccounted variability that might have contributed to the observed differences in math attitudes and performance among the groups. To address this limitation, future research could benefit from incorporating measures of executive functions. This would enable us to explore whether variations in executive functions might have influenced the observed effects of the math game interventions.

Conclusion

This study aimed to investigate the effect of digital educational games on students' in-game behaviours, math performance, and math attitudes. Particularly, it is the first study to implement the five emotional design principles in a math game intervention while adopting a value-added approach to specifically test the unique contribution of incentive systems. The findings suggested that educational games containing four emotional design principles (i.e., visual aesthetic design, musical score, game mechanics, narrative) result in higher value for learning math and better attitudes towards math (e.g., more enjoyment, less boredom, more behavioural intention) than regular math homework alone. The addition of one additional feature, incentive systems, leads to even better in-game performance and math attitudes than games with only four principles. Thus, the findings from this study not only address the gap in the literature on digital game's effects on math attitudes, but also emphasize the importance of emotional design principles, particularly how incentive system can have a critical impact on in-game performance and math attitudes. Future work should expand this study to larger sample size to test if the current findings still hold. Once additional research is done supporting this finding, parents and educators should be choosing games with incentive systems for better learning experience and outcomes.

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Appendices

Appendix F

Math Ability Test

Operations on Numbers
2+4=___
5+3=___
20+7=__
5+60=___

Missing Terms
4 -1- __=0
7 -3- __=0
56 -16- __=0
28 -12- __=0

Tens and Ones
14 = one ten+ __ones.
15 = one ten + __ones.
34 = __tens+ __ones.
67+10 = __tens + 10+ __ones.

Symbols
Rank the following numbers from the smallest to biggest: 16， 9， 6，
__<__<__
Rank the following numbers from the smallest to biggest: 54， 14， 51,
_<__<__

Place Value
10 +1 = __tens + __ones
4 +9 +6 = __tens + __ones
38 = __tens+ __ones
3+40 +4 = __tens+ __ones

Appendix G

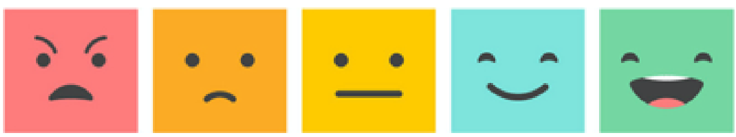
Math Attitudes Measure

Subdimensions	Items
Value	
	I study math because I know it is useful.
	Math is important in everyday life.
	Math is important to me.
	Math is important like other subjects.
Confidence	
	I'm confident I can learn new math skills.
	I can get good grades in math.
	I can solve difficult math problems.
	Math is difficult for me, even though I study hard.
Gender beliefs	
	Females can do just as well as males in math.
	Males are not naturally better than females in math.
	Females are good enough to do well in math.
	It's hard to believe that girls can become math geniuses.
Enjoyment	
	Math is fun to me.
	Math is very interesting, and I enjoy math class.
	I like math just as much as other subjects.
	I like the challenge of math problems.
Anxiety	
	I am usually relaxed during math classes.
	I am usually relaxed during math tests.
	Math doesn't scare me at all.
	Math makes me feel uncomfortable and impatient.
Boredom	
	I start yawning in math class because I'm so bored
	Math material is so boring that I find myself daydreaming.
	Mathematics lecture bores me.
	I never get bored at math class.
Anger	
	I am annoyed during my math classes.
	I get angry because the material in math is so difficult.
	I am so angry that I would like to tear the math exam paper into pieces.
	I am annoyed with my math homework.
Shame	
	My face is getting hot because I am embarrassed that I cannot answer math teacher's questions.
	After taking a test in math, I feel ashamed.

	When I don't understand something in my math homework, I don't want to tell anybody.
	After receiving math test score, I try to avoid eye contacts with others.
Pride	
	After a math test, I am proud of myself.
	I’m proud of my knowledge in math.
	After having done my math homework, I am proud of myself.
	I am proud of being able to answer my math teacher's questions.
Behavioural intentions	
	I would like to spend more time working on math.
	I think about mathematics problems outside of work.
	Even if math is boring, I make sure to spend some time studying it after class.
	I am willing to discuss math problems with my peers after class.

Visual Aids

Strongly disagree	**Disagree**	**Neutral**	**Agree**	**Strongly Agree**



Chapter 6. Final Discussion

The literature on math education highlights a decline in math attitudes, starting from early primary school and becoming more visible in secondary education (Aiken, 1985; Jameson, 2014; Morrisett & Vinsonhaler, 1965). While there is an urgent need to address this phenomenon, the lack of a theoretical framework on the topic has led to a vague understanding of what math attitudes are and how they affect math performance (Di Martino & Zan, 2001; Ma & Kishor, 1997). Furthermore, although digital educational games have been reported to be effective tools for improving learning outcomes for some subjects (Divjak & Tomić, 2011; VanSickle, 1986), empirical evidence of their impact on math attitudes is lacking (Dubé et al., 2019). Specifically, no study to date has systematically investigated the following questions: “Are games effective in improving math attitudes? If so, which game features contribute to the improvement?” To address these gaps, this thesis aims to clarify the definitions and components of math attitudes, investigate their relations with math achievement, identify effective game features, and examine their impact on math attitudes and math performance through the value-added approach.

General Study Findings

First and foremost, it is critical to clarify the meaning and components of math attitudes and understand their role in affecting math achievement. The literature review in Chapter 2 aims to provide a comprehensive understanding of the components and subdimensions of math attitudes and how they affect math achievement. By adopting a tripartite model as the theoretical framework and identifying the commonly studied subdimensions in research practice, the review suggests that math attitudes consist of cognitive, affective, and behavioural components with seven subdimensions being most commonly studied in secondary education. The review also shows that while anxiety is negatively correlated with math performance, other subdimensions such as gender beliefs, value, confidence, self-concept, enjoyment, and behavioural intentions have a positive impact on math achievement. This chapter provides a holistic picture on math attitudes and a solid foundation for further investigation of the complex relationship between math attitudes and math achievement.

While the definition and components of math attitudes become clear after the systematic review in Chapter 2, there lacks a comprehensive measure to reflect various dimensions of math attitudes. Chapter 3 proposes and validates a tripartite measure for math attitudes, consisting of both the well-studied subdimensions identified from Chapter 2 and overlooked subdimensions under affective math attitudes. This comprehensive measure not only presents a holistic view on the topic but also provides researchers with a valid tool that can be readily adopted. Chapter 3 further investigates the interconnections between the components of math attitudes. In addition to the partial linear relations that have been previously studied, the results reveal a more comprehensive picture of interconnections among the components of math attitudes. For example, distal cognitive math attitudes are closely related to proximal cognitive math attitudes, and proximal cognitive math attitudes are significantly associated with affective math attitudes. Similar patterns exist in affective and behavioural math attitudes.

Chapter 4 proposes digital educational games as a means for improving math attitudes, especially affective math attitudes, given their critical roles identified in Chapter 3. The chapter aims to explore game features that promote positive affective math attitudes and math achievement. The effects of five game features (visual aesthetic design, musical score, game mechanics, narrative, and incentive system) were investigated through a meta-analysis, and the findings showed that these game features have a positive impact on both affective math attitudes and math achievement. Moreover, the results suggest that the incentive systems might be the most effective game feature in promoting positive math attitudes and better math performance.

Chapter 5 presents an experimental study on the effects of incentive systems on math attitudes and math performance. The study employed Mayer's (2019) value-added approach, which is used to tease out the effects of incentive system from those of other game features. Two versions of game were created and tested with children, a base version consisting of four emotional design principles (i.e., visual aesthetic design, musical score, game mechanics, and narrative), and a value-added version with the fifth principle added—incentive system. The results showed that children's math attitudes in both game conditions outperformed those in control group, but the value-added version of the game resulted in higher level of value and less boredom. Moreover, although the comparison of math test scores did not yield a significant difference, games with incentive systems led to better in-game performance that that without.

In conclusion, the results of these four manuscripts suggest that math attitudes are a multidimensional construct that consist of cognitive, affective, and behavioural components. While each component contributes uniquely to math achievement, affective math attitudes play a central role within the construct. Digital educational games can be effective tools for promoting positive math attitudes and math performance. Specifically, games with incentive system appear to generate higher level of positive math attitudes and better in-game performance.

Contributions

Theoretical Contributions

This thesis makes important contributions to the theoretical understanding of math attitudes, and how they can be improved through digital educational games. Math attitudes have been studied for decades due to their vital role in affecting students' math performance and their choices of math-related courses and careers (Ashcraft & Kirk, 2001; Joensen & Nielsen, 2009; Markovits & Forgasz, 2017; Rose & Betts, 2004). However, the term is defined implicitly and divergently through the measures of it (Daskalogianni & Simpson, 2000; Di Martino & Zan, 2015; Wen & Dubé, 2022). The disjointed definitions in existing literature are due to the lack of a consistent theoretical framework to guide the understanding of math attitudes (Di Martino & Zan, 2001, 2015). Chapter 2 introduces the tripartite model (Triandis, 1971) as a theoretical framework to guide the definition of math attitudes. The adoption of the tripartite model in math education not only fills the gap created by the lack of a guiding theoretical framework (Di Martino & Zan, 2001, 2015), but also provides an inclusive way of defining math attitudes. Moreover, components of math attitudes have been studied individually and sometimes divergently, resulting in unclear relations between math attitudes and math achievement (Di Martino & Zan, 2001, 2015). Chapter 2 is the first attempt to identify the commonly studied components and subdimensions of math attitudes, and conclusively reveal their unique relations with math achievement based on existing literature. The findings from Chapter 2 provide researchers with a holistic view of what have been studied on the topic and how they relate to math achievement.

To better understand math attitudes, researchers have developed various measures assessing individuals' math attitudes, ranging from confidence, gender beliefs, enjoyment, anxiety, to willingness to learn math (Arhin, 2015; Dursun, 2015; Ganley & Vasilyeva, 2011; Mirza & Hussain, 2018; Simsek, 2016). While these dimensions reflect math attitudes to some extent, a measure that capture all dimensions of math attitudes is needed to provide a comprehensive understanding on the topic. However, existing measures are insufficient to capture the multifaceted nature of math attitudes (Wen & Dubé, 2022). For instance, the Fennema-Sherman Mathematics Attitudes Scale (FSMAS) is one of the most popular measures over decades. While it measures nine subdimensions of math attitudes, there are other crucial subdimensions that are missing, such as enjoyment and behavioural intentions (Wen & Dubé, 2022). To address this gap, Chapter 3 proposes and validates a tripartite math attitudes scale that consists of well-studied subdimensions as well as important but previously overlooked subdimensions. This comprehensive measure provides researchers with an overall picture of the components and subdimensions of math attitudes, and a readily available tool for measuring math attitudes as a whole. Furthermore, the construct of math attitudes has been suggested as an interpretive tool to depict the interactions between components (Di Martino & Zan, 2015; Ruffel et al. 1998). Chapter 3 is the first study that adopts control-value theory to guide the investigation of interrelations amongst constructs within math attitudes. The findings not only provide support for previously identified relations, but also shed light on the intricate connections between the components of math attitudes for the first time. Specifically, the results highlight the important role of affective math attitudes in shaping overall math attitudes. These findings suggest a need for further investigation into more complex relations in future research.

Although digital games have been reported to be effective in enhancing attitudes for some subjects (Kirriemuir & McFarlane, 2004; Malouf, 1988; Oblinger, 2006), few studies have explored their effectiveness in promoting math attitudes (Dubé et al., 2019). Chapter 4 addresses this gap while further looking into the specific game features that contribute to positive math attitudes and math performance. The investigation was based on Loderer and colleagues' (2020) emotional design principles, which are claimed to be effective in enhancing emotions (affective math attitudes) in game-based learning contexts. The impact of each emotional design principle was revealed through a meta-analysis. The results revealed that games with visual aesthetic design, musical score, game mechanics, and incentive system are associated with small to moderate effects on emotions and math achievement. The findings offer empirical evidence on effects of digital games on math attitudes and provide valuable insights on the development of effective math educational games. Research shows that educational games that promote positive learning outcomes may not necessarily lead to good experience (Dubé & Keenan, 2016). In fact, some educational games could demotivate students and negatively impact their learning (Ke, 2008; Shin et al., 2012). Designing effective educational games that motivate learners while optimizing learning outcomes has become a challenge (Hamari et al., 2016; Shute et al., 2019), prompting researchers to investigate game features that support both learners' emotions and learning outcomes (Boyle et al., 2016). Chapter 4 addresses this gap by proposing aforementioned emotional design principles to enhance both gaming experience and learning outcomes. The findings inform both game developers and educators that game features which promote controllability over the game and value of the game are more likely to promote positive math attitudes and better learning outcomes.

Incentive systems, often taking the form of points, scores, badges, and customized avatars, are populator feature in commercial games but not frequently incorporated in educational games (Rahimi et al., 2021). Leading scholars have suggested that this feature should be considered for educational games, as they have the potential to motivate players to improve their in-game performance and hence, strengthen their content knowledge (Garris et al., 2002; Kelle et al., 2013; Pierce et al., 2012). While incentive systems are recognized as an important feature, few studies empirically and scientifically examine their impacts on learning (Huang et al., 2010, Wang & Sun, 2011). This highlights the need for more rigorous investigation to better understand the effects of incentive systems in educational games. Mayer's (2019) value-added approach can be utilized as a means to scientifically evaluate the effects of incentive systems, by comparing a base version of a game with a value-added version that includes an incentive system.

Chapter 5 presents a first attempt to use the value-added approach to assess the impact of incentive systems on math learning. The findings indicate that the value-added version of math game with inclusion of incentive systems outperform the base version in terms of enhancing math attitude and math in-game performance. This improvement occurred even though the base version incorporated four of the emotional design principles (i.e., aesthetic design, musical score, game mechanics, and narratives) known to result in better learning outcomes (see Chapter 4). The results align with the assumption of control-value theory, that the increase in control and value-related appraisals may reduce negative emotions (Pekrun & Perry, 2014). The value-added version of game with incentive systems provided students with extrinsic value through rewards. Meanwhile, they increased students' sense of control over the game by allowing them to customize their avatars. Together, the improved control and value-related appraisal are associated with a reduction in students' boredom levels. This chapter provides empirical and scientific evidence on the isolated effects of incentive systems, and sheds light on their crucial roles in game-based learning contexts. The inclusion of incentive systems can positively influence students' control and value appraisals, which may further impact their achievement emotions and learning outcomes. Therefore, they should be included into educational games to improve learning. Additionally, Chapter 5 serves as a valuable foundation for future studies using the value-added approach to investigate the impact of a particular game feature.

Methodological Contributions

As previously discussed, existing measures of math attitudes failed to capture the various dimensions (Wen & Dubé, 2022). In addressing this, Chapter 3 proposes and validates a tripartite math attitudes scale to reflect the multifaceted dimensions of math attitudes. The scale was developed through a systematic review of the literature on math attitudes and was updated to capture both well-studied and overlooked submissions of math attitudes. The inclusion of previously overlooked subdimensions of math attitudes allows for a more nuanced understanding of the construct. Further, this tripartite scale provides researchers with a new tool to either comprehensively measure math attitudes as a whole or selectively measure their subdimensions. In addition, the proposed tripartite scale is the first to use control-value theory to guide the investigation of the interconnections within the construct. This integration can help advance research in the field by providing a more comprehensive understanding of math attitudes. Overall, the proposed tripartite math attitudes scale and the integration of control-value theory make important methodological contributions to the field of math education.

Limitations and Future Directions

There are some limitations in this thesis. Although some of the limitations have been discussed in previous individual manuscripts or addressed in subsequent ones, there are other critical issues worthy of further discussion when considering the thesis as a whole piece. First, the tripartite math attitudes scale presented in Chapter 3 consists of both well-studied components and overlooked subdimensions of math attitudes. It is important to note that the established subdimensions were originally derived from secondary education in Chapter 2. While these subdimensions largely overlap with those studied in other age groups (Wen & Dubé, 2022), more boarder range of emotions were included into the updated scale to reflects more comprehensive view on the topic. However, it is still possible that some other subdimensions specific to age groups might be missing. For this reason, future research is necessary to identify the components of math attitudes across different age groups, including children, college/university students and adults, as doing so will enable researchers to establish the commonalities across ages and reveal age-specific subdimensions.

Moreover, although the tripartite math attitudes scale has been validated in its English version with adults in Chapter 3, there are limitations to using this scale directly with child participants in a Chinese version in the study from Chapter 5. First, the cultural context in which the scale was validated in English may be different from the cultural context used in Chinese. The potential difference may affect how participants interpret and respond to the items. Secondly, children may have difficulty understanding the language used in the scale, especially if they are not fluent in reading. In Chapter 5, several efforts were made to address these issues: 1) scale items were carefully translated and simplified into Chinese with the support of bilingual experts; 2) parents were asked to assist their children's participation by reading the items to them and explaining the meaning when necessary; and 3) visual images were added to each response option to facilitate children's understanding. Notwithstanding these efforts, future research should validate the scale with the target population before its use in large-scale studies.

While Chapter 5 provides empirical evidence supporting the effectiveness of incentive systems in game-based learning contexts, further research is necessary to address other important questions. For instance, researchers need to determine the types of incentives, understand the optimal timing for delivering them, and balance the frequency of using them. Knowing the types of intrinsic and extrinsic incentives allows researchers to leverage their roles (Deci et al., 1999; Wang & Sun, 2011), while understanding the timing of delivery can facilitate game flow (Sweetser & Wyeth, 2005). Most importantly, it is vital to deliver the right number of incentives in order for players to maintain interests, foster intrinsic motivation while preventing boredom

(Richter et al., 2015; Sweetser & Wyeth, 2005). Answering these questions is crucial for implementing effective incentive systems that enhance learning outcomes.

Factors such as culture, age, and gender were not explored in detail in this thesis. For example, while Chapter 3 used data from 222 English-speaking adults to demonstrate the associations between cognitive, affective, and behavioural math attitudes, it is unclear whether the same relations hold for individuals from different age groups or cultural backgrounds. Therefore, future studies are needed to test if the identified direct relations are consistent across different populations. Similarly, while Chapter 5 tested the effects of incentive systems on children from a suburban area in China, the data source is not fully representative of the broader population. Future studies could explore how different types of learners (e.g., age, gender, cultural background) response to incentive systems, and how these systems could be personalized to meet individual needs for better learning outcomes. Addressing these limitations can provide more comprehensive insights on the complex relations within math attitudes and the effectiveness of incentive systems in promoting learning.

Chapter 5 investigated the effects of incentive systems on children's math attitudes in a short-term period. While the findings suggest that games with incentive systems promote positive math attitudes, the short period of the study limits the generalizability to long-term effects. To start with, the findings show significant changes of math attitudes in value, enjoyment, and boredom. It is important to note that enjoyment and boredom are short-term experiences that are often the result of achievement related activities (Graesser et al., 2022). Thus, it is unclear whether the changes in affective math attitudes would fade over time or transfer into long-term changes. On the other hand, components such as confidence, gender beliefs and behavioural intentions are long-term trait that may be less affected by short-term interventions (Graesser et al., 2022). Whether these components can be improved through long-term treatment remains unknown. To address these limitations, longitudinal studies that allow changes over longer periods are needed. Such studies would allow researchers to assess the long-term impact of incentive systems on math attitudes and identify subdimensions that contribute to the changes. By doing so, researchers can gain a more comprehensive understanding on the changing of math attitudes, thus develop effective interventions to enhance math attitudes.

Finally, multimodal measurement should be considered for future research. The current study relies solely on self-report measures to assess participants' math attitudes, which may be subject to response biases. To address this issue, future studies could incorporate multichannel methods to triangulate the findings. For example, researchers could use think-aloud protocols, diaries, observations, and interviews to supplement self-report measure (Di Martino & Zan, 2015; Ke, 2008; Hannula 2002). Additionally, automated measures of affective math attitudes such as facial expression recognition system could be used to capture participants' emotional changes during game play (Lyu et al., 2022). This is critical as it allows real-time assessment without interrupting the flow of game and learning (Loderer et al., 2020). By incorporating multimodal measurement, researcher can gain a more in-depth understanding of participants' changes in affective math attitudes and develop more effective digital games to improve overall math attitudes.

Conclusions

Research has identified a worrisome decrease in math attitudes across childhood and adolescence, and there is an urgent need to enhance them before they decline irreversibly (Aiken, 1985; Fredricks & Eccles, 2002; Jacobs et al., 2002; Wigfield & Meece, 1988). This thesis makes important contributions to our understanding of math attitudes and how they can be enhanced through the use of digital educational games. It establishes a robust foundation for a comprehensive understanding of what math attitudes entail and how they relate to math achievement. In addition to providing a holistic overview on what has been studied on math attitudes, this thesis also makes methodological contributions by validating an inclusive measure of math attitudes and introducing control-value theory to guide the investigation of interrelations within the construct.

Furthermore, this thesis proposes educational games as a means for improving math attitudes and examines the impact of specific game features on math attitudes and math performance using the value-added approach. The findings provide empirical evidence of incentive systems' impact, as well as practical guidance for designing effective educational games to enhance learning outcomes. Specifically, the findings suggest that educational games that incorporate incentive systems appropriately can positively influence math attitudes and learning outcomes. These results contribute to the ongoing debate about the role of fun in educational games (Dubé & Keenan, 2016), demonstrating that fun is not only to improve the game experience but also to serve learning when incentive systems are included.

As such, this thesis has significant implications for game developers, educators, and parents who are looking for an effective way to improve math learning. Educational game developers should consider incorporating incentive systems not only because they contribute to an enjoyable learning experience, but also due to their potential to improve attitudes towards academic content in the game. Educators and parents can choose math games with incentive systems as a tool to supplement traditional learning, as they may increase learners' value for learning math and reduce the boredom that is often associated with the subject (Dubé et al., 2019). This in turn, may enhance learners' math attitudes and math performance. In sum, this thesis has made significant contributions to the field of math education by advancing our understanding of math attitudes and providing insights on effective ways of improving them.

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