Double-Porosity Modelling of Groundwater Flow Through Fractured Rock Masses

by

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ABSTRACT

One of the key factors in ensuring safe slopes in open pit mines is the control of the groundwater flow within the slope. When analyzing the flow regime and its characteristics, traditional numerical methods designed for soil-like materials may not always apply. In the present thesis, the two-dimensional finite element code FlowD was developed to analyze steady-state seepage through fractured rock masses, under saturated conditions. The program provides the user with two modelling options: porous (soil) materials, or fractured rock masses. The double-porosity model was incorporated in the code, in order to better model flow through anisotropic, heterogeneous fractured rock masses, where Dirichlet and Neumann boundary conditions can apply. FlowD program calculates total pressure head, flow gradients and velocities within the specified domain. Drains, drainage galleries and wells, as well as aquifer recharge can be simulated. A real large-scale case study, with complex geological features has been simulated in order to demonstrate the application of the double porosity model. Three different scenarios have been modeled for the same slope , which are natural groundwater regime, vertical well simulation, and drainage galleries simulation. The results show good agreement with the predictions of both the consultant and the mine's engineering division.

RÉSUMÉ

La percolation de l'eau souteraine dans les talus rocheux des mines à ciel ouvert est l'un des facteurs les plus importants à considérer vis-à-vis de la stabilité des pentes. Dans les analyses du régime d'écoulement des eaux souterraines et de ses caractéristiques dans les talus rocheux, les méthodes numériques traditionnelles, conçues pour des matériaux granulaires, ne sont pas toujours applicables. Dans cette thèse, un programme à éléments finis à deux dimensions, FlowD, a été conçu afin de permettre l'analyse de l'écoulement à travers les milieux rocheux fracturés ou les milieux granulaires poreux. Un modèle à double porosité a été incorporé dans le code-source, afin de permettre une meilleure modélisation de l'écoulement à travers un milieu rocheux fracturé, anisotrope et hétérogène, où les conditions des frontière de type Dirichlet et Neumann sont applicables. Le logiciel FlowD calcule les pressions totales, les gradients et les vitesses de l'écoulement de la région étudiée. Les forages, les galeries de drainage, ainsi que les recharges de l'aquifère peuvent aussi être modélisées. Une étude d'un cas réel à grande échelle, ayant une géologie complexe, a été effectuée pour valider et démontrer l'applicabilité du modèle à double porosité. Trois différents scénarios ont été ainsi simulés: le régime naturel, l'existence d'un puits, puis celle de deux galeries de drainage dans le même talus rocheux. Les résultats obtenus concordent avec les prévisions du consultant et du département d'ingénierie de la companie minière.

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Chapter 1

Introduction

1.1 General

Surface mining is one of the oldest forms of mining around the world. In the early days of mineral extraction, it was the easiest, safest and most affordable mining method being widely used in small scale mining operations. In recent years, surface mining has attained great depths, which implies new problems related to this activity.

Costs of ore recovery have increased considerably, but new technology was developed in order to help the mining industry. New sciences have been developed in order to help and better understand different aspects related to this form of mining. Geological formations have a better description and the geophysical concepts can give interpretations of discontinuities' pattern. New rock mechanics concepts have been established, that help explain rock mass behavior. Hydrogeology as an engineering science, in particular, has taken new steps in explaining water influence on the earth crust behavior.

Open pit operations have become, in the last few years, challenging engineering projects, mainly due to the large pit sizes and ore recovery methods. These issues have led to the development of a variety of engineering tools that have become necessary to meet the needs of the mining industry.

Hydrogeology and rock mechanics sciences attempt to better understand phenomena that occur in rock masses. The water interaction with different types of geological formations changes the behavior of rock itself. Rock mass characterization and discontinuities' description is needed to apply safe and economic mining methods. Particularly, engineering design of high slopes in open pit mines has to take into consideration all of the above mentioned factors.

In pit slope stability problems, monitoring of hydrogeological regime plays a major role in evaluating safety. In most cases, groundwater regime should be well understood in combination with rock mechanics characterization of geological formations. These aspects have to be addressed in order to solve engineering problems related to water table, water pressure, water inflows and poor drainage, and structural anomalies in rock slope that can affect their stability. In general, continuous monitoring and depressurization reduce failure risks. These tasks are not always easy to achieve, and in some cases they become mandatory for efficient and safe mining operations.

Seepage laws in fractured media were formulated from the original groundwater laws in soil materials. They were completely reviewed to take into account the flow phenomenon in fractured media. Rock mass and discontinuity characterization were needed in order to look into hydrogeological models. Various models have been proposed for both rock mass and flow phenomenon. These models are briefly reviewed with respect to their applicability to surface mining related problems.

1.2 Objectives and methodology

The objective of this research is to develop an easy tool, in the form of a computer code, that will simulate confined groundwater flow in fractured rock slopes. Different assumptions had to be made related to the rock itself, structural discontinuities, and the flow regime.

The use of numerical modeling in combination with slope monitoring systems can give good feedback in designing and operating large open pit slopes. In this idea, a simple tool to estimate groundwater flow through a fractured rock mass is proposed in this thesis, that can be easily used in a mining operation environment. Fractured rock slopes geometry, discontinuities' pattern and characteristics, as well as rock mass behavior were the main aspects to include with the tool.

FlowD code is a finite element computer program for two-dimensional seepage analysis. It employs 4-node isoparametric elements, uses 4×4 integration scheme. It performs seepage analysis in soil or rock slopes. Double-porosity model was used to simulate flow in the rock media. Discontinuities' orientation (dip/dip direction), aperture, average number of fractures and their permeability describe their pattern in the slope. The rock blocks were considered with very low permeability that can be nil compared to the discontinuities' permeability.

Verifications of this code were made on literature examples. An open pit slope was modeled with the data from Black Lake Mine, LAB Chrysotile Inc., Québec.

Chapter 2

Basic concepts of groundwater flow

2.1 General groundwater classification

Water is often present in geologic media of the earth crust. Groundwater may be defined as the whole amount of water which is stationary or flows through the ground surface of the earth. In addition to its physico-chemical effects, its influence on the mechanical behavior of the soil or rock masses is of utmost importance. The presence of water flowing in the underground implies the existence of water pressure and seepage forces which have to be taken into consideration when various geotechnical problems are considered.

Subsurface waters are classified in different ways, considering their various properties as the classification basis, i.e. temperature, chemical composition, movement character, origin, etc. A general classification of groundwater is illustrated in Figure 2.1 flowchart and is based on the origin of groundwater, the occurrence depth and different types of rock formations. This classification shows only water which is stationary in the ground, but at the same time, water can move through the porous media and rock formations. From an engineering point of view, both forms of water in the ground are important because there is an interaction between water and media, with repercussions on the media's properties. This study will consider only the groundwater that moves freely in the underground.

When studying the groundwater movement, the place of movement, the time dependency and the type of flow must be taken into consideration. The water flow problem is actually divided into two groups: saturated flow when different media are completely saturated with water meaning all pores are filled with water, and unsaturated flow when voids are partially filled with water and partially with air. A third group is actually a combination of these two, when flow is saturated in part of the domain and unsaturated in another.

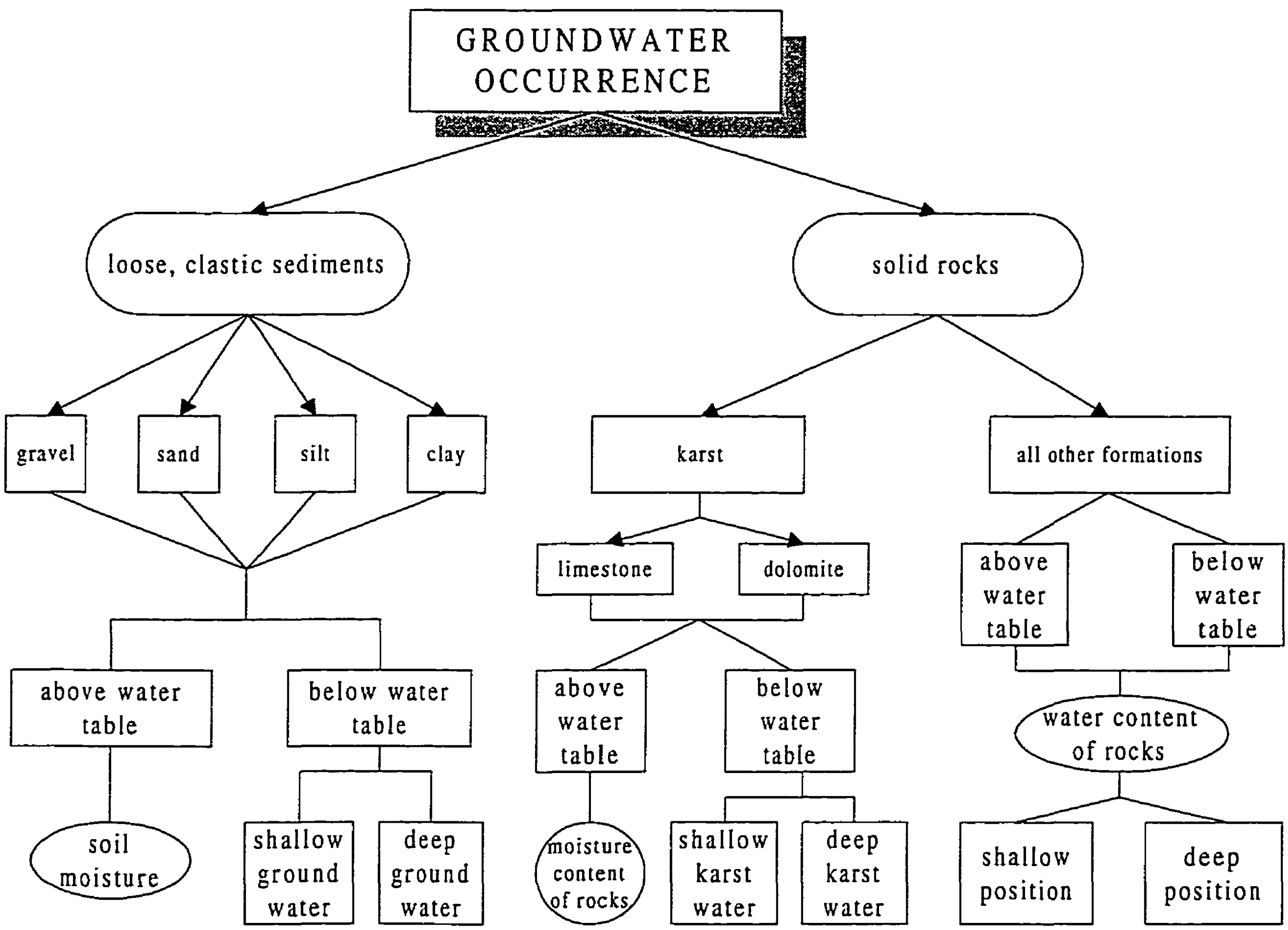


Figure 2.1 General groundwater classification

Other well-known classification methods used for groundwater flow are different types of problems like: dimensional character of flow, time dependency of flow phenomena, boundaries of flow domain, or medium and fluid properties.

The flow regime can be determined by using the Reynold's number (**Re**) which will be discussed later in this chapter. It can be steady or unsteady depending on time factor: steady when the flow parameters remain constant during a period of time and unsteady when the flow parameters change with time. Flow can be laminar, non-laminar, or turbulent. Fluid characteristics are important in solving flow equations and its properties should, if possible, be determined a priori in any further study.

The flow of groundwater is produced by different forces; the most important are gravity and pressure forces. Meanwhile, some other forces can produce water movement like thermo-osmoses forces, being the result of differences in temperature in the unsaturated zone. The increase of surface tension which accompanies the decrease in temperature can cause water movement. At the same time, the increase in surface tension causes a greater capillary attraction in the heat flow direction. The actual flow of water under temperature gradient is probably a combination of capillary and vapor transport. If two types of water have different dissolved solids concentrations, and if they are separately by a semipermeable membrane, water will move from the low concentration side to the high concentration side. The movement will continue until concentrations are the same or until some other counteracting forces, such as hydrostatic pressure, balance the chemical forces. The tendency of water to move in the direction of increasing chemical concentration is called osmosis.

In recent years, the new groundwater contamination studies have addressed other types of problems, such as site remediation, with many applications in waste disposal. As long as the groundwater is contaminated with reactive or non-reactive contaminants, it can transfer the latter to the medium through contaminated groundwater flows.

2.2 Overview of flow phenomena

Linear flow laws - Darcy's Law

The basic relationship of the subterrain hydrodynamics, the seepage law, establishes the relationship between the seepage velocity and the pressure field that generates the flow. This relationship was discovered experimentally in 1856 by a French engineer, Henry Darcy, who formulated it for one dimensional flow in an isotropic and homogenous porous medium,

$$\frac{Q}{A} = K\frac{\Delta h}{L} = v = K\,i \tag{2.1}$$

where: Q = flux of water discharge through porous medium $[L^3T^{-1}]$;

A = area of the cross section closed in the porous medium $[L^2]$;

Δh = difference in hydraulic head between the measurements points [L];

L = distance between the measurements points [L];

K = hydraulic conductivity or coefficient of permeability which depends on the fluid properties and porous medium $[LT^{-1}]$;

i = hydraulic gradient [L/L].

Darcy's Law was later derived from the Navier-Stokes equations by means of their statistical integration (Slattery, 1972, and Greenkorn, 1982). This study does not contain details about the demonstrations of Darcy's Law but, it will cover its different formulations and limitations.

Fluid flow in a porous medium differs from the fluid motion considered in ordinary hydrodynamics, because, in any open macro volume, there is immovable solid phase (the solid matrix, or skeleton) at the boundary of which the fluid is also immovable. In the latest demonstration of Darcy's Law, the porous medium is regarded as a system of pore channels of an elementary macro volume which is hydrodynamically equivalent to a system of interconnected tubes. Flow velocity characterizes discharge through this system. On the other hand, the discharge is determined by pressures at the channel entries and exits. Since the bulk discharge is the sum over many channels, it is governed by the pressure drop, i.e. by the gradient of the fluid mean pressure.

When considering the small velocity of an arbitrary point of the porous medium, the flow velocities field can be assumed to be continuous and all porous medium and fluid parameters can be assumed to be constant. Only pressure variation that is very small, can not be nucleated, because if the pressure does not exist or is constant over the space, the flow is absent. The basic assumption leading to the seepage law statement is as follows: the pressure gradient at a given point of porous medium is governed by the seepage flow velocity vector as well as by the fluid and porous medium properties.

Before extending Darcy's Law to a three-dimensions analysis a few basic definitions regarding porous media and flow phenomena must be given. Porous media can be characterized as homogenous or heterogeneous, as well as isotropic or anisotropic.

Homogenous: medium properties do not change from one place to another i.e. they are independent of location.

Heterogeneous: medium properties vary spatially and are dependent on location.

Isotropic: medium properties are independent of the measurement direction.

Anisotropic: medium properties vary with measurement direction within the formation or geologic units.

Here are the four possible combinations of anisotropy and heterogeneity:

- Homogeneous - isotropic;
- Homogeneous - anisotropic;
- Heterogeneous - with different isotropic layers;
- Heterogeneous - anisotropic.

The extension of one-dimensional Darcy's Law to three-dimensions law is:

$$\mathbf{v} = -\mathbf{K}(\nabla p - \rho g) \tag{2.2}$$

where: $\mathbf{v}$ = velocity tensor;

$\mathbf{K}$ = hydraulic conductivity tensor;

p = hydrostatic pressure [M/TL^2];

ρ= density of the fluid [M/L^3];

g = gravitational acceleration [L/T^2].

In the formula above porous medium is homogenous and isotropic; flow takes place in a saturated zone, and the fluid motion is inertialess. If the medium is anisotropic and inertia effects are considered, the formulation of Darcy's Law will differ.

In most of Darcy's Law formulations hydraulic conductivity is a second rank tensor. Its components depend on the characteristics of porous medium as well as on fluid properties. Science domain and material properties determine the formulation of this law. As an example in 1931, Richards formulated the expression of Darcy's Law for unsaturated zone used in geotechnical engineering:

$$\frac{\partial \theta}{\partial t} = \nabla [\, K \Delta (\psi + z)] \tag{2.3}$$

where: $\theta = \theta(\psi)$ moisture content [dimensionless];

$K = K(\theta)$ or $K = (\psi)$ unsaturated hydraulic conductivity [L/T];

ψ = pressure head [L];

t = time [T];

z = elevation from the arbitrary datum [L].

Variables are moisture content, pressure head and hydraulic conductivity. In recent years, there has been considerable work involving flow in unsaturated media with applications, particularly in waste disposal.

Darcy's Law holds well in most practical situations like:

1. saturated flow;
2. steady-state flow;
3. transient flow;
4. flow in heterogeneous and anisotropic porous media;
5. flow in granular media.

In some cases, the validity of this law is questionable. A plot of the groundwater velocity versus hydraulic conductivity gradient would reveal a straight line for all gradients between 0 and ∞, if linearity is maintained. For granular material flow, there are at least two situations where the validity of this linear relationship is questionable: flow through low permeability sediments under very low gradients and high flow rates through highly permeable media.

The first major concern of Darcy's Law is the macroscopic behavior. This assumption refers to the lower limit of this law. In a porous media considered, one has to be able to sample meaningful values for physical properties that are averaged over a volume sample and the Darcy's Law is valid. If not, the applicability of this law is under question. It is shown in Figure 2.2 that this volume sample is referred to as a Representative Elemental Volume (or REV) (Freeze et al.,1979). To apply Darcy's Law, three conditions has to be met:

1. One must be able to define REV which is homogenous;
2. One must be able to characterize the ensemble of grains as a continuum;
3. The continuum must be macroscopic.

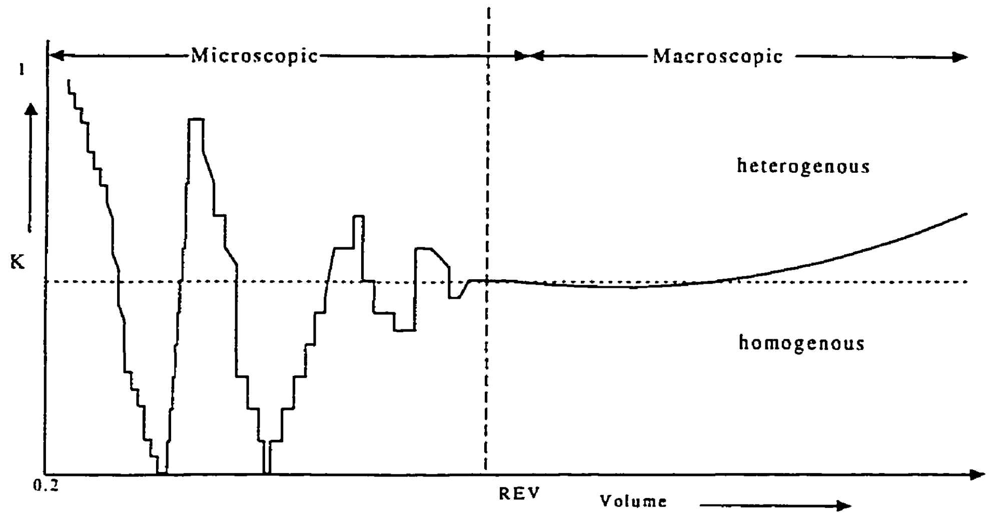


Figure 2.2 Physical properties as it might be measured for increasing sample volume (Therrien,1994)

The second major concern of Darcy's Law is the flow rate. If this law is universal, flow would occur for all infinitesimal gradients in low permeable materials, and laminar flow would occur for high flow rates under high gradients in permeable materials. Figure 2.3 illustrates Darcy's Law and its validity range.

For low permeability fine-grained materials, it has been suggested, based on laboratory evidence, that there may be a threshold hydraulic gradient below which there is no flow. Swartzendruber (1962) and Bolt et al. (1969) reviewed the evidence and summarized the phenomena.

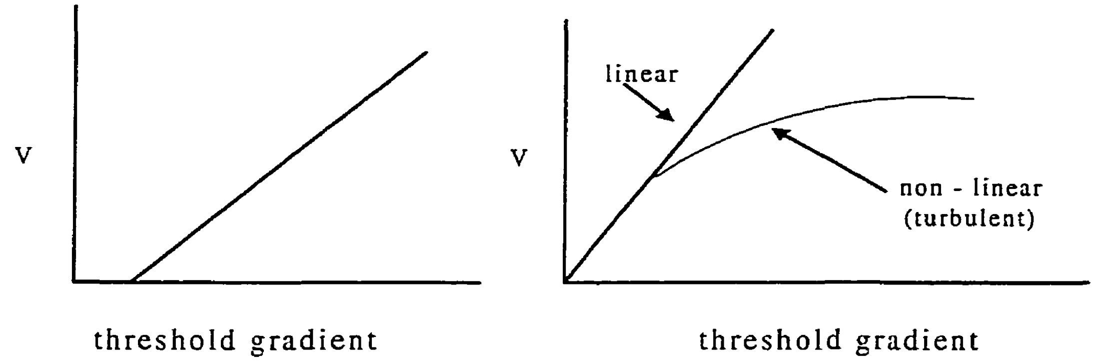


Figure 2.3 Range of validity of Darcy's Law (Therrien,1994)

Of greater practical importance is the upper limit on the range of validity of Darcy's Law. At very high flow rates, the linear law breaks down. This evidence is reviewed in detail by Todd (1959) and Bear (1972). The upper limit is identified with Reynold's number (**Re**), a dimensionless number that expresses the ratio of inertial to viscous forces during flow; it is defined for the flow through porous media as:

$$Re = \frac{\rho v d}{\mu} \tag{2.4}$$

where: ρ = density of the fluid [M/L^3];

v = specific discharge or velocity [L/T];

μ = dynamic viscosity of the fluid [M/LT];

d = the representative length of a sample of porous medium [L].

This number is widely used in fluid mechanics to distinguish between laminar flow at low velocities and turbulent flow at high velocities. In 1972, Bear summarized the experimental evidence with the following statement: " Darcy's Law is valid as long as Reynold's number based on average grain diameter is within the 1 to 10 range". For this range of Reynold's number flow through porous media is laminar and obeys the linear law.

Darcy's Law does not cover all solutions for groundwater flow. Deviations from the linear law are due to either aquifer material composition or flow velocities leading to turbulent flow regime. This situation can be explained by the variation of hydraulic gradients, which increase proportionally with the specific discharge in a nonlinear manner. It is also the case when velocities are increased, even in porous media. In coarse grained porous media and fractured media due to relatively high hydraulic conductivity, groundwater flow may be turbulent.

Nonlinear flow laws

In fractured media, a nonlinear relationship was observed by Louis (1969), Snow (1968) and Maini (1971). The nonlinearity was attributed to different factors such as kinetic energy, nonlinear pressure flow laws, leakage packers, and increase in fracture aperture. Both laws can be summarized as follows:

$$v = -K\left(\frac{dh}{dl}\right)^m \qquad (2.5)$$

where: v = seepage velocity [L/T];

K = hydraulic conductivity [L/T];

dh/dl = hydraulic gradient [L/L];

m= power of the hydraulic gradient;

m = 1 flow is linear and laminar, (Darcy's Law);

$m \neq 1$ flow is nonlinear and turbulent (Louis, 1974).

Flow rates exceeding the upper limit of Darcy's Law are common in rock formations such as karstic limestone's, dolomites and cavernous volcanic. Darcian flow rates are almost never exceeded in nonindurated rocks and granular materials. Fractured rocks which are more permeable through joints, fissures and cracks will be discussed later.

Other more or less empirical nonlinear expressions have been proposed; generally expressed as:

$$i = av + bv^2 \qquad (2.6)$$

where: v = specific discharge [L/T];

i = hydraulic gradient [L/L];

a = parameter dependent on medium and fluid properties [T/L];

b = parameter dependent on medium and fluid properties [T^2/L^2].

Fractured medium flow is quite different from the porous medium flow. Flow in an individual fracture is rather similar to pipe flow. Different from porous media, the moving water particle is not subjected to resistance everywhere, fractures' intersection are the major point were resistance forces are developed. In highly fractured media flow can be considered similar to that of porous media, if the rock-fractures properties can be viewed as a common unit.

Flow laws in individual fractures were investigated by Louis (1974) for laminar and turbulent flow by assuming a uniform flow velocity over the total aperture of the fracture. In any single fracture, flow may be considered one-dimensional or two-dimensional. For the one-dimensional flow, it can be assumed to be parallel or nonparallel. When stream lines are not parallel a two-dimensional flow appears in the fracture.

The roughness of the fracture walls plays a dominant role in a single fracture flow. This parameter influences the flow regime and the validity of the Reynold's number. Louis (1974) summarized flow laws in a single fracture in five different combinations of flow regimes and roughness of the fracture, as mentioned below:

1. Smooth-laminar flow regime (a linear low similar to Darcy's Law);
2. Rough-laminar flow regime (the relative roughness plays an important role);
3. Smooth-turbulent flow regime (fracture aperture plays on important role);
4. Rough-turbulent flow regime (nonlinear flow law);
5. Very rough-turbulent flow regime (similar to the rough-turbulent flow regime).

All above mentioned flow laws follow the general law (2.5) with the flow gradient at a certain power which is always less than one. All these laws and their empirical formulations were obtained under steady-state flow conditions similar to Darcy's Law.

In rock masses, the rock blocks are separated by fractures which may or may not be interconnected (Figure 2.4). These fractures have different directions and different apertures (with no superiority one over other), and the rock mass is made of blocks of irregular size and shape. Based on this idea, Barenbaltt et al. (1960) assumed in formulating a new approach growndwater flow, that any small volume of rock consists of a large number of porous blocks as well as fissures.

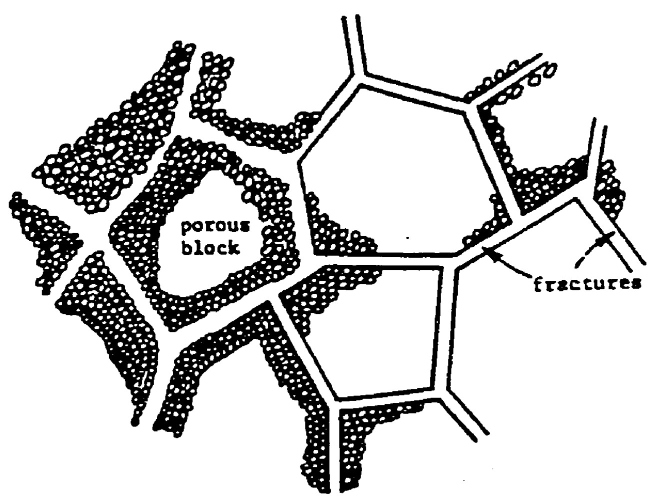


Figure 2.4 Schematic representation of fractured medium (Bear et al.,1993)

Therefore, within a small area of the same rock mass, there are two different media: fractures and blocks. Their hydraulic behaviors are different, depending on the type of flow considered. Under steady-state conditions, fractures and blocks act together as one unit. In unsteady flow, because fractures' hydraulic conductivity is greater than blocks' hydraulic conductivity and fractures' storativity less than blocks' storativity, the whole rock mass must be considered as consisting of two different but coexisting porous media with different hydraulic heads. Under such conditions, three different types of flow will appear. These are: flow in the fractures, flow in porous blocks, and flow from blocks to fractures. The flow law in the two mentioned media can be linear or nonlinear, as described above. However, the same laws cannot be used for block-to-fracture flow.

For the block-to-fracture flow, the law was formulated by Barenblatt et al. (1960). In formulating this law, they assumed that the flow is pseudo-steady state, and water exchange between blocks and fractures depends only on the difference between the head in the fractures, h_f, and the average head in the block, h_b . No consideration is given to the blocks geometry. This law is formulated as follows:

$$q = \alpha \ (h_b - h_f) \tag{2.7}$$

where: q = specific discharge [L/T];

α= parameter depending on the geometry of the fractured rock [1/T];

h_f = head in the fracture [L];

h_b = head in the porous block [L].

Other laws for block-to-fracture are based on the geometry of the porous blocks. Different restrictions could be imposed for block shapes, which can either be cubes or parallelepipeds, depending on the model used.

Karstic media differ from the above mentioned cases, because the medium is very heterogeneous. Water flows in a system of interconnected cracks, caverns, and channels. The flow could be similar to flow in conduits (large pipes) but, in most cases, the water may not fill the whole cross section, and the water may not be under pressure. In this medium, the presence of large cavities and caves could suggest that the flow can be either laminar or turbulent, and Darcy's Law in granular medium is no longer valid. It is also possible that in time, the karstic features might become larger and the flow regime might change. Such cases should be each studied individually.

2.3 Groundwater flow equations in porous media

The basic flow law in underground hydrology is Darcy's Law, which was analyzed in previous sections (with its different forms). This law, combined with the law of conservation of mass, results in the equation of continuity. The latter practically describes the conservation of mass during the flow through porous medium. The continuity equation for saturated groundwater flow is as follows after introducing the Darcy's Law in the conservation of mass law:

$$\nabla \rho g + \frac{\partial}{\partial t}(\eta \rho) = 0 \tag{2.8}$$

where: ρ = density of the water flowing [M/L^3];

g = gravitation acceleration [L/T^2];

η = porosity [%];

t = time [T].

In this section, some forms of the continuity equation are reviewed for granular materials under different conditions. The steps followed in their derivation will not be presented herein. The solution of the relevant differential equations should take into account the boundary-value conditions, and analytical or numerical methods should be used for solving the system of equations.

Steady State Saturated Flow

For steady state conditions, (flow is not time dependent and medium is saturated), groundwater flow in a homogenous and isotropic medium can be described by the following equation :

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{2.9}$$

where: $h = h(x, y, z)$ = hydraulic head [L],

x, y, z = coordinates of a point in a three-dimensional Cartesian system.

$$\nabla^2 \mathbf{h} = \mathbf{0} \tag{2.10}$$

This equation is one of the fundamental partial differential equations known in mathematics- Laplace's equation. Its solution in this case is a function h (x, y, z) that describes the hydraulic head h at any point in a three dimensional flow field and depends on boundary conditions imposed.

Transient Saturated Flow

If the groundwater flow occurs in a heterogeneous anisotropic medium, the continuity equation is expressed in a different way, in accordance with medium conditions, as follows:

$$\nabla (K \Delta h) = S_s \frac{\partial h}{\partial t} \tag{2.11}$$

where : $h = h(x, y, z)$ = hydraulic head [L];

x, y, z = coordinates of a point in the flow field;

S_s = specific storativity of the porous medium [dimensionless];

K = hydraulic conductivity (tensor).

The solution of this equation is a function h (x, y, z, t) which describes the hydraulic head at a point in the field, at any time. If the medium is homogenous and isotropic this equation becomes:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K}\frac{\partial h}{\partial t} \qquad (2.12)$$

and is known in mathematics as the diffusion equation. Any solution requires knowledge of the hydrogeological parameters, K = hydraulic conductivity [L/T], n = porosity [%], α = coefficient of vertical compressibility and fluid properties [dimensionless], ρ = density of the fluid [M/L^3], μ = dynamic viscosity[M/LT], and β_w = isothermal volumetric compressibility of the fluid [LT^2/M].

When developing this equation, it is assumed that changes in stresses within geological media occur only vertically. These changes are included in the vertical compressibility parameter. Such approach couples three-dimensional groundwater flow to one dimensional stress field. The more general approach which couples three-dimensional groundwater flow system to a three-dimensional stress field was examined in detail by Biot (1941).

Transient Unsaturated Flow

The partial differential equation which describes flow in a partial saturated medium must take into account changes in moisture content. These changes will occur in time that will produce complex changes in voids' space that are practically pores' expansion.

Like in the formulation of the Darcy's Law for unsaturated zone, the moisture content θ is a function of the pressure head ψ, and the hydraulic conductivity $K = K(\theta) = K(\psi)$ is a function of moisture content or pressure head. The equation has the following form:

$$\frac{\partial}{\partial x}[K(\psi)\frac{\partial \psi}{\partial x}] + \frac{\partial}{\partial y}[K(\psi)\frac{\partial \psi}{\partial y}] + \frac{\partial}{\partial z}[K(\psi)\frac{\partial \psi}{\partial Z}] = C(\psi)\frac{\partial \psi}{\partial t} \tag{2.13}$$

where: $C(\psi)$ = specific moisture content [dimensionless];

ψ = pressure head [L];

$K = K(\psi) =$ hydraulic conductivity [L/T];

x, y, z = coordinates of a point in the flow field.

The solution of Equation (2.13) is the pressure head a function $\psi = \psi(x, y, z, t)$. The equation is known as Richard's equation and is only valid for the unsaturated zone when studying groundwater flow. In a coupled saturated-unsaturated model the Richard's equation should be combined with the diffusion equation.

Chapter 3

Rock mass and discontinuity models in hydrogeology

3.1 Introduction

Discontinuities present in rock formations change the qualitative aspect of the rock mass and affect its behavior. Dealing separately with the discontinuities' characterization is the only way to reach a point where the interaction between rock mass and discontinuity can be analyzed. Understanding the behavior of the rock-joints ensemble is extremely important when studying flow in this type of medium.

This chapter presents the engineering properties of rock masses needed in groundwater flow problems, followed by a description of discontinuities' characterization. An overview of flow models used in hydrogeology to describe flow phenomena in rock masses, discontinuities, as well as the interaction between the two is presented.

3.2 Rock mass characterization

A general rock mass characterization starts with a general rock mass classification dividing rocks into two major classes, intact (or continuous) rock and fractured/jointed (or discontinuous) rock. The properties used to classify rocks will vary according to the purpose of the study and may include various criteria: shear strength, flexural strength, tensile strength, elasticity, permanent deferability, creep rate, water flow, water storage properties, in-situ stress, drillability, formations characteristics, and sometimes density, thermal expansion, mineralogy and color.

The aims of a rock mass characterization, as described by Bieniawski (1986), are therefore:

1. identify the most significant parameters influencing the behavior of a rock mass;
2. divide a particular rock mass formation into groups of similar behavior, i.e. rock mass classes of various qualities;
3. provide a basis for understanding the characteristics of each rock mass class;
4. compare the experience of rock condition at one site to the conditions and experience encountered at other sites;
5. derive quantitative data and guidelines for the engineering design;
6. provide a common basis between engineers and geologists.

Engineering characterization of rock masses may be examined from different viewpoints and structured in different groups like: **general characterization** and **direct classification.** The first one is based on the geological structure of the rock mass and on its basic characteristics and properties which are independent of the conditions that will occur to them later after investigation. The second one is adjusted to the engineering problems in tunneling, mining, civil engineering, etc.

General characterizations are usually made when building physical and mechanical models of rock masses, whereas engineering properties are generally defined after the constitutive models have been chosen.

At the beginning of this century, when the study of rock masses was starting, scientists tried to establish procedures and guidelines in this new science. During the last decades, rock engineering developed many tools to create a new image of rock and rock masses. Their effort is now used in rock mass classification and characterization. Many classification systems have been developed: "Rock Load" (Terzaghi,1946), RQD (Deere et al.,1967), RMR (Bieniawski,1973, modified 1979), etc.

When studying flow in fractured rocks, the characterization of the medium has a strong influence over the description of flow phenomenon. Depending on the flow model chosen, the discontinuities' characterization and their oncoming parameters which affect seepage are very important. Both play a major role in choosing a combine model for further studies.

Engineering properties are presented with respect to flow phenomena in fractured rocks. The geological aspect of the problem has an importance when establishing the physical and mechanical behavior of the rock mass. According to the International Society of Engineering Geology, different guidelines are based on the following features which affect the physical and mechanical properties of the rock:

1. the mineral composition related directly to the weight of solids;
2. the structure and the texture, which determine the unit weight of the solids and the rock porosity;
3. the water content and the degree of weathering, which describe the physical condition of the rock and affect its strength, deformability and permeability.

In the categories above, it is assumed that the physical and mechanical properties of a rock, in its present state, result from different processes, such as: genesis, metamorphism, tectonics and surface weathering. Thanks to these processes, one can explain not only the lithological and physical features of the rock, but also their locations on Earth. A proposed classification distinguishes the following rock mass units according to the degree of homogeneity:

- geotechnical unit;
- lithological unit;
- lithological complex.

In 1981, the International Society of Rock Mechanics (Bieniawski,1979) proposed that a basic geotechnical description of the rock masses should include the following characteristics:

1. rock name with a simplified geological description;
2. the layer thickness and fractures (discontinuities) that intercept the rock mass;
3. the unconfined compressive strength of the rock material and the angle of friction of the fractures.

The classification proposed by the International Society of Rock Mechanics (ISRM, 1973) is “less geological” in its character than the one presented above. It is based on compressive strength, on the weathering degree and on joint spacing; the range values can be determined relatively easily (compressive strength and joint spacing). As for the weathering degree, it can only have a descriptive character and cannot assign any precise value. According to variations in compressive strength, weathering degree and joint spacing, rock masses are divided into three categories and five corresponding classes, which are illustrated in Table 3.1 below.

Table 3.1 Rock mass classification with respect to basic characteristics (Goodman,1976)

[redacted]					
Class	Compressive strength [MPa]	Class	Weathering degree	Class	Joint spacing [m]
S_1	<5	A_5	slight or no weathering	F_1	<0.1
S_2	5-10	A_4	weathering along joints wall	F_2	0.1-0.3
S_3	10-15	A_3	total slight	F_3	0.3-1.0
S_4	50-100	A_2	total moderate	F_4	1.0-3.0
S_5	>100	A_1	total heavy	F_5	>3.0

Based on the three features above, and using an additional general geological classification, it is relatively easy to estimate the quality of the rock mass. Many other classifications are available, based on engineering criteria referring to specific works.

The velocity of seismic waves propagation throughout a rock mass is another criterion that can be used to split rock masses in different classes. Its value is a function of the type of rock within the rock mass and its mineral composition, density, elasticity, weathering degree and compactness. According to this criterion, rock masses are divided into four groups: limestones, shiest and adesites, granites and gneiss, and finally sandstone.

Flow in fractured rocks could take place either in the fracture or in the porous blocks. The porosity of the solid rock depends on the size of the pore opening and is called "primary porosity". The dry density and the primary porosity of the rock are the parameters controlling flow phenomenon in the intact rock or through blocks. According to the International Society of Engineering Geology in 1978, rocks have been classified in five classes as shown in Table 3.2.

Table 3.2 Rock classification according to dry density and porosity (ISRM,1976)

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	less than 1.8	very low	over 30	very high
2	1.8-2.2	low	30-15	high
3	2.2-2.55	moderate	15-5	medium
4	2.55-2.75	high	5-1	low
5	over 2.75	very high	less than 1	very low

Generally, the rock structure is composed of primary and secondary pores. Some rock formations have considerable primary porosity, which represents the expansion of pores formed between grains and microcracks. In fragmented rocks, porosity depends on whether the grain-size distribution, shapes, arrangement, cementation degree of the grains or water pressure. As for sedimentary rocks, the primary porosity is relatively high and could significantly influence the hydraulic behavior of the rock..

The rate of inactive primary pores is higher than the rate of fractures (secondary pores), but, in most of the cases secondary pores control the flow through rock masses. The primary pores of igneous and metamorphic rocks are generally negligible. Discontinuities found in rocks have the greatest influence when defining the hydraulic properties of rock mass for further studies. Hydraulic properties of rock masses will be discussed later in this chapter, after the discontinuities characterization.

3.3 Characterization of discontinuities

The presence of discontinuities in rock formations distinguishes rock from rock mass. Any type of rock formation contains many discontinuities. The origin of this is found in the orogenic and/or tectonic movements, weathering processes, etc. Conventionally, discontinuities are considered as either faults or joints. An example of discontinuities classification based on descriptive-structural criteria is presented in Figure 3.1.

The properties of discontinuity control the hydraulic behavior of the rock mass. These are orientation, spacing, frequency, intensity, shape, roughness and aperture. Each of these parameters can have a statistical interpretation because, in nature, their variation is wildly spread. There are no rules when considering their influence on the rock behavior. It is hard to measure these parameters and to generalize them when dealing with large scale studies.

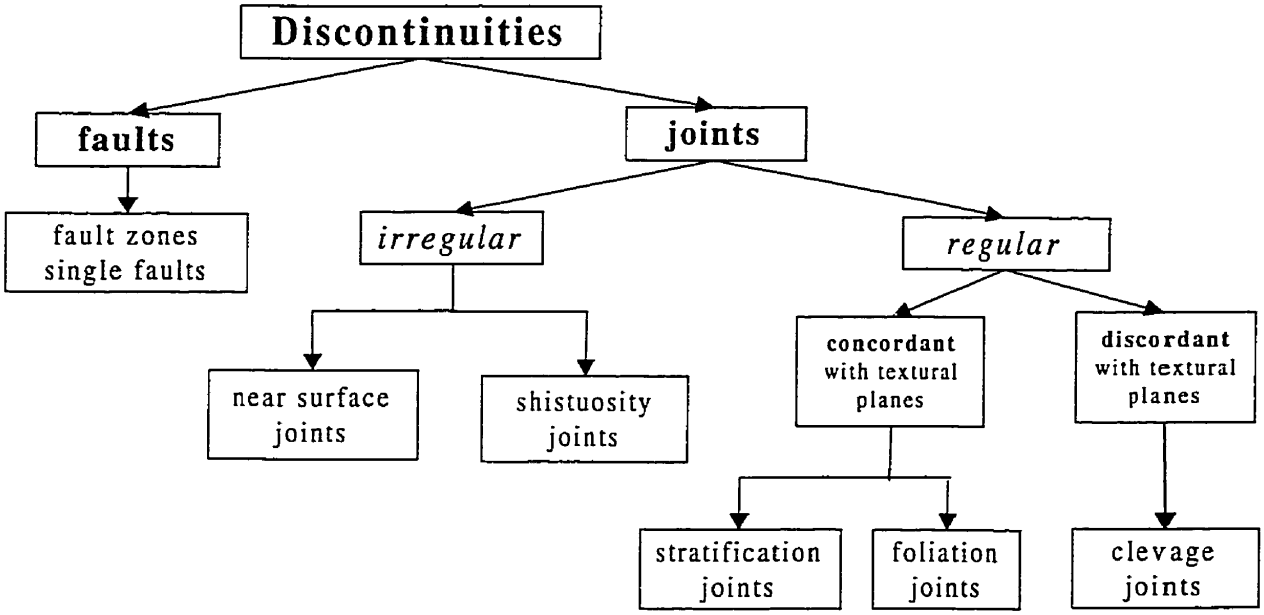


Figure 3.1 Classification of discontinuities using descriptive- structural criteria (Thiel, 1989)

Fractures' orientation can be measured from cores or exposures. It is quantified by discontinuities dip and dip direction. Methods for discontinuity orientation measurements, presentation and analysis have been described by Priest (1985), Goodman (1976), Einstein and Baecher (1983). Discontinuities' orientation could give the mean direction of the flow in a rock slope, and it can influence the directional permeability of the rock-discontinuity ensemble, if studied as such.

Discontinuity spacing, frequency and intensity have been defined by many authors, such as Robertsom (1970), Hudson & Priest (1979,1983). Deere (1964) developed an empirical assessment of rock quality to describe the weathering of the rock cores - Rock Quality Designation (**RQD**) index. In 1981, the Association Française des Travaux Souterrains (AFTES,1981) released a classification of the discontinuity pattern according to the joint spacing and to RQD index designation. This classification is presented in Table 3.3 below.

Table 3.3 Rock mass classification according to joint spacing and RQD (Thiel,1989)

[illegible]	[illegible]	[illegible]
S1	>200	90
S2	60-200	75-90
S3	20-60	50-75
S4	6-20	25-50
S5	<6	<25

Discontinuity shape refers to the relation between the trace length and its orientation. Simplified discontinuity shapes are usually assumed to be circular, square, elliptical or rectangular. The number of joint sets and the inter-connection control the hydraulic conductivity of the rock masses.

Roughness and aperture of the discontinuity raise questions related to the actual space through water flows. In the literature, Barton (1973) introduced the Joint Roughness Coefficient (JRC) as an empirical approach. The commission of International Society of Rock Mechanics, published in 1978 a classification of discontinuity roughness based on description of rock joints and their aspect. There are some other studies which simulates fracture roughness, e.g. Maini (1971), or fracture aperture, e.g. Amadei et al., (1995). These become more important in hydro-environmental problems of fractured rocks rather than in groundwater flow in mine slopes.

Estimation of the permeability of rock masses is very important in large scale studies. In 1976, Goodman proposed to evaluate the secondary permeability from discontinuity frequency, presented in Table 3.4.

Table 3.4 Estimation of secondary permeability from discontinuity frequency (Goodman,1976)

Very closely to extremely closely spaced discontinuities	highly permeable	10^{-2}-1
Closely to moderately closely discontinuities	moderately permeable	10^{-5}-10^{-2}
Widely to very widely spaced discontinuities	slightly permeable	10^{-9}-10^{-5}
No discontinuities	effectively impermeable	less than 10^{-9}

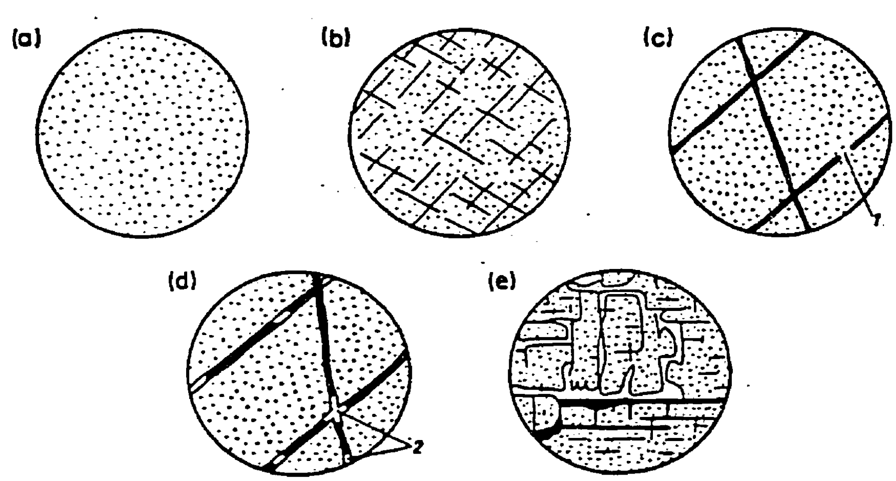
These are a few guidelines for the estimation of discontinuity permeability; tests should be conducted to determine the real permeability for each case to be analyzed. The International Association of Engineering Geologists suggested the following grades of permeability together with a classification used in a further rock mass characterization (Table 3.5).

Table 3.5 Permeability limits (Goodman,1976)

1	greater than 10^{-2}	very high
2	10^{-2}-10^{-4}	high
3	10^{-4}-10^{-5}	moderate
4	10^{-5}-10^{-7}	slight
5	10^{-7}-10^{-9}	very slight
6	less than 10^{-9}	practically impermeable

Some researchers (Louis, 1976) proposed a rock mass classification taking into account the discontinuities' appearance in the geologic formations. Five distinct groups have been proposed for the rock masses with respect to their hydraulic properties (Thiel, 1989):

1. **porous media** (Figure 3.2a), generally homogenous, containing only small pores; this group comprises jointed rock masses which lie at great depth and in which joints have been scaled by the action of high stresses;
2. **porous jointed media** (Figure 3.2b), in which discontinuities determine the hydraulic properties of the rock mass; two types have been distinguished: impervious rock and pervious rock;
3. **porous media containing impermeable barriers** (Figure 3.2c), in which joints are filled with a fine impervious material;
4. **porous media with small channels** (Figure 3.2d), in which large joints filled with an impervious material contain channels through which water can flow;
5. **karstic media** (Figure 3.2e) containing wide passages and caverns of various geometrical forms , created by chemical reaction between water and rock or by removal of the rock in underground.



1= rock bridge
2= channel

Figure 3.2 Types of media occurring in rock masses (Louis,1976)

All parameters regarding the rock masses and the discontinuities are taken into account in establishing the model used for solving groundwater flow. Depending on the desired level of accuracy, they are going to be either estimated or measured in field and/or lab test. The estimation of the hydraulic conductivity with a degree of accuracy could be crucial in further studies.

3.4 Models in hydrogeology

Hydrogeology is a relatively new field at the border between rock mechanics and geology. It deals with problems related to groundwater flow, coupled groundwater flow and stress analysis, determination of the hydraulic properties of the medium, test and data prediction, well recovery, dewatering systems and groundwater contamination problems. In solving these problems, different types of models were adopted for such studies. It is practically impossible to solve all these complex problems taking in account all the parameters existing in nature.

When choosing one model over another, one takes the scale of the analysis of each study into account. There are no strict rules in doing so, and the analyst should decide what is best for the study.

When considering the flow of water in jointed rock masses, two major points have to be considered regarding this phenomenon; models for the medium and models for the flow. Later, the method to resolve the system of equation could be chosen. Models regarding fractured rock medium could be divided in continuous and discontinuous models, each of them with some other subdivisions. Each model could be applied depending on the degree of fracturing, on available data and on the accuracy of the study. Groundwater flow equations could be solved in different ways, using analytical methods, numerical methods or analog methods.

In the literature, there are two classes of hydrogeological models: one for the network of fractures and the other for the flow description. The network of fractures could be regarded as continuum or discontinuum models. The main difference between them is the method used to simulate the fractured media and whether or not an equivalent continuum can be defined.

When relating the real network of fractures with the model used for flow, there are two major concerns: understanding the fracture network with correct representation, and deciding whether the fractures are suitable for that type of analysis. These two have to work together in order to obtain good results.

In the early stages of hydrogeology, most of the work was focused on generating an equivalent medium having the equivalent properties of the rock and the fractures together. As a first definition, this assumption does not take in account the location of the fractures or their real geometry. The main goal is to define an equivalent porous medium.

If it is possible to do so, then it should be possible to determine the representative elemental volume (REV in the fractured medium), as mentioned herein. In fractured media, there is much less certainty in the assumption of a representative elemental volume being valid (Schwartz and Smith, 1987). In fractured rock masses, REV could only be sampled when the fracture density is above the critical density. The latter is defined as the density of fractures that provides the network connectivity. Below the critical density, the network is not connected and the mean hydraulic conductivity will be zero, no matter how large REV is (Sahimi,1995).

Fractured rock can behave as a porous medium when the fractured density increases; apertures are constant rather than distributed over a large range of values; orientations are more distributed than constant, and a large scale analysis is performed. These conditions satisfied it may be possible to find a mean value for the equivalent medium.

Long et al. (1982) summarized the conditions that have to be met for a fractured rock to behave as a continuum. These are:

1. An insignificant change in the value of the equivalent permeability with a small addition or substraction of the test volume;
2. The existence of an equivalent permeability tensor predicting the correct flux when the direction of a constant gradient is changed.

The equivalent medium under these conditions behaves like a porous medium. Groundwater problems could be solved using the classical equations for different types of flow in soil material.

In addition , another model for the fractured rock which was formulated in 1960 by Barenbalatt et al. (1960): the double-porosity model. In their model, the fractured rock is considered as consisting of two porous systems: the rock matrix with high porosity and low permeability, and the fractures, with low porosity and high permeability, allowing an exchange of fluids between the systems at their interfaces. In the conceptual model for the double-porosity aquifer, the porous matrix blocks are assumed to act as sources which feed the fractures.

By using the conceptual framework of a double-porosity medium, three alternative flow models have been developed. The most significant difference among the various models is the treatment of rock matrix versus fracture leakage.

Later, Warren and Root (1963) proposed an idealization of the original double-porosity model, where the fractured rock is represented as regular, fully connected network, embedded in a porous matrix represented by parallelepiped blocks. Bringing this limitation to the initial model, some parameters could be estimated from matrix properties, like the size and shape of the blocks. Limiting the network of fractures at a uniform distribution through the system, Kazemi et al. (1970, 1976) proposed methods for estimating various parameters.

The naturally fractured rocks are very difficult to model with the above mentioned approach, because they have complex morphological properties such as incomplete fracture connectivity, fracture surface roughness, and fractal characteristics over certain length scales. There are many unanswered questions in hydrogeology and without a considerable amount of assumptions, none of the real rock mass behavior could be studied.

There are techniques to investigate the geometry of the fracture network: statistical analysis of the field data, geophysical techniques for seeing into the rock, and prediction of the fracture patterns (Witherspoon and Long, 1987). Simulating the fracture network could lead to modeling the flow in a discrete manner. In a two-dimension fracture network, rock discontinuity is represented by one dimensional finite line segment (Long, 1987) and in three dimensional fracture networks, fractures are represented either by discs of finite radii (Dershowitz,1987) or by flat planes of finite dimensions.

New techniques like simulated annealing and synthetic model (Bolton et al., 1987) are used to generate the network of fractures. These two techniques are applied where classical methods could not be applied; they involve a large amount of data generated and used .

The methods used for simulating the network of fractures should be combined with the different types of flow for solving groundwater problems. In the case of equivalent medium, classical flow equations for flow in porous media should be used. Discrete models have the choice over the flow in a single fracture or between parallel plates. Continuity equation is different for each type of flow; it combines parameters describing the flow and the media. Resolving the system of equations raises another kind of problem.

Over the years, methods for solving groundwater flow problems have been developed. Depending on type of study and its complexity, one can choose one of the following methods in resolving the system of equations:

1. Analytical methods;
2. Methods based on the use of models and analogs;
3. Numerical methods.

Each of these could be applied depending on the problem, on the human resource availability, on the time and cost required for reaching a solution. One should take into account the objectives of the investigation and how the results are going to be used for further studies

The conclusions of this chapter are summarized in the following flowchart, which reviews different types of models used in hydrogeology.

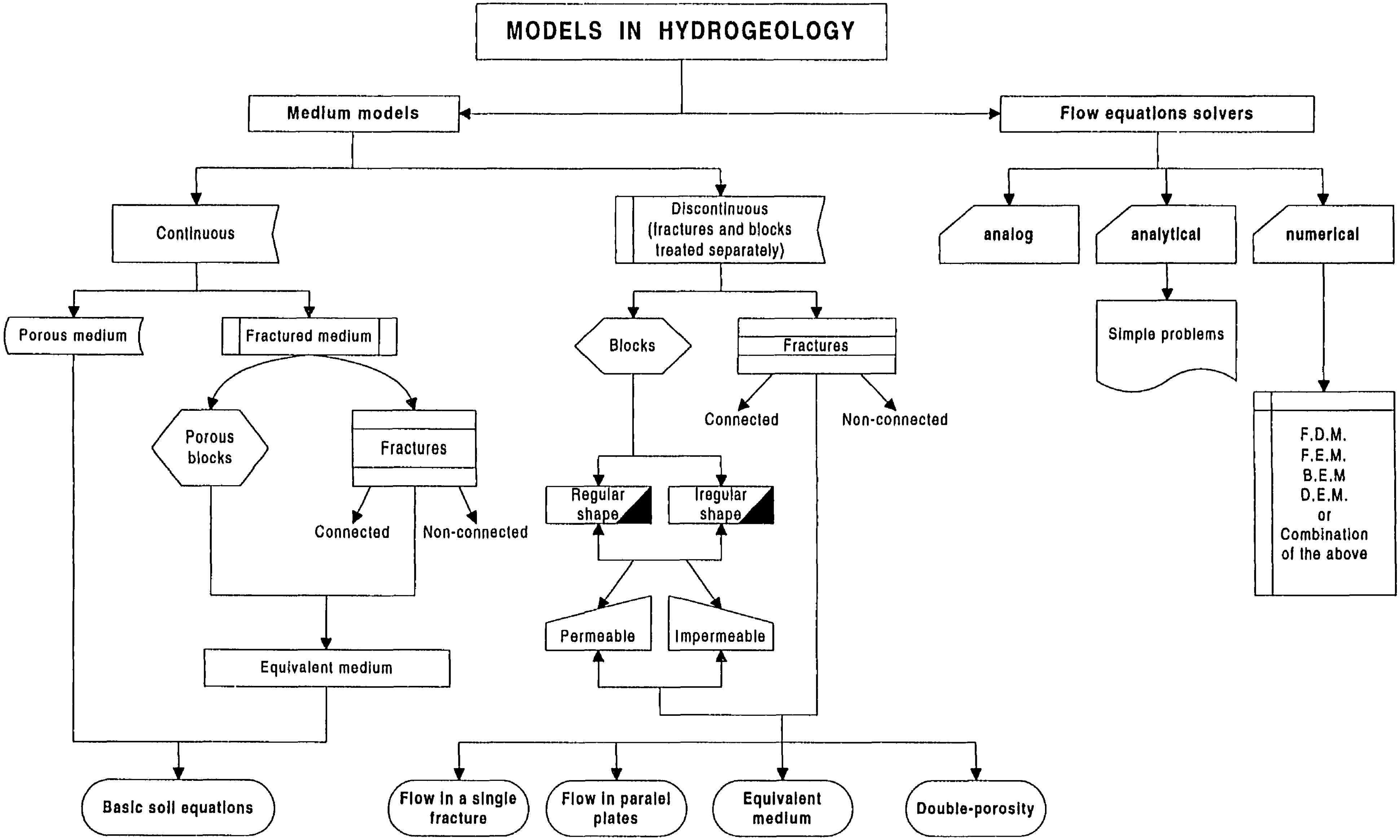


Figure 3.3 Models in hydrogeology

Chapter 4

Double-porosity model for flow in fractured rocks

4.1 Introduction

Flow in fractured rocks is a complex phenomenon, and several continuum, and discontinuum models have been developed for studying it. Different studies, that depend on study scale and data available, showed their applicability to engineering problems.

In the mining industry, water flow is important in slope stability studies of open pit mines. Building high water pressure could favor and induce slope failure. On the other hand, tailings dams which are made out of mine waste, allow water to flow freely, affecting their stability. The two processes are totally different, because of the media through which water flows; one is a naturally fractured slope and the other is a man-made structure out of granular material.

Seepage in fractured slope walls of open pit mines has become of more and more concern. The lack of detailed data describing the network of fractures, as well as the large scale of the study, could favor the application of certain models, like the double-porosity model, which is presented in this chapter.

4.2 Hydraulic conductivity

In general, fractured media may be regarded as two coexisting systems of voids: the aperture of the fractures and the porosity of the blocks of rock separating the fractures. In 1974, Louis, established some formulations for the equivalent permeability of fractured media in which the existence of the double system was pointed out. The hydraulic conductivity of the blocks is added to that of the fractures. That way, different formulations for the ensemble block-fracture hydraulic conductivity are developed (Louis,1974). Fractures network could be continuous or discontinuous depending on the connectivity of the discontinuities.

For continuous fractures: $$K = \frac{e}{b} K_f + K_m; \tag{4.1}$$

where: e = average aperture of the set of fractures [L];

b = mean distance between fractures in the set [L];

K_m = hydraulic conductivity of the rock matrix [L/T];

K_f = average hydraulic conductivity of the fractures [L/T].

For discontinuous fractures: $$K = K_m + [1 + \frac{1}{2}(\frac{l}{(L-l)} - \frac{l}{L})]; \tag{4.2}$$

where: K = hydraulic conductivity in the direction of discontinuous fractures [L/T];

K_m = hydraulic conductivity of rock matrix [L/T];

L = average distance between (center) fractures [L];

l = mean extension of the fractures (order of 10^{-1}) [L].

For large scale problems (in terms of special dimensions) the two-media approach of the "far field model" may be used. The conceptualization of the two-media approach, originally proposed by Barenblatt et al. (1960), is also known as the 'overlapping continua', 'double continuum', or 'double-porosity' approach. In this conceptual model, the fractured porous medium is represented by two distinct, but interacting systems, one consisting of fracture network and the other, of porous blocks. Each system is visualized as a continuum occupying the entire investigated domain. Interaction phenomena between the two continua are included to account for the exchange of fluid between fractures and porous blocks. Since the definition of two continua is required, one for porous blocks and one for fracture network, it follows that the application of the model assumes the existence of a Representative Element of Volume (REV) for the fractures and also for the porous medium (as defined in the previous chapters). But, in the definition of REV for both systems a common plateau should be identified, meaning the 'overlap' of the 'plateaus' defining the REV, and the behavior of the two can be studied together. The representation of the ensemble is shown in Figure 4.1.

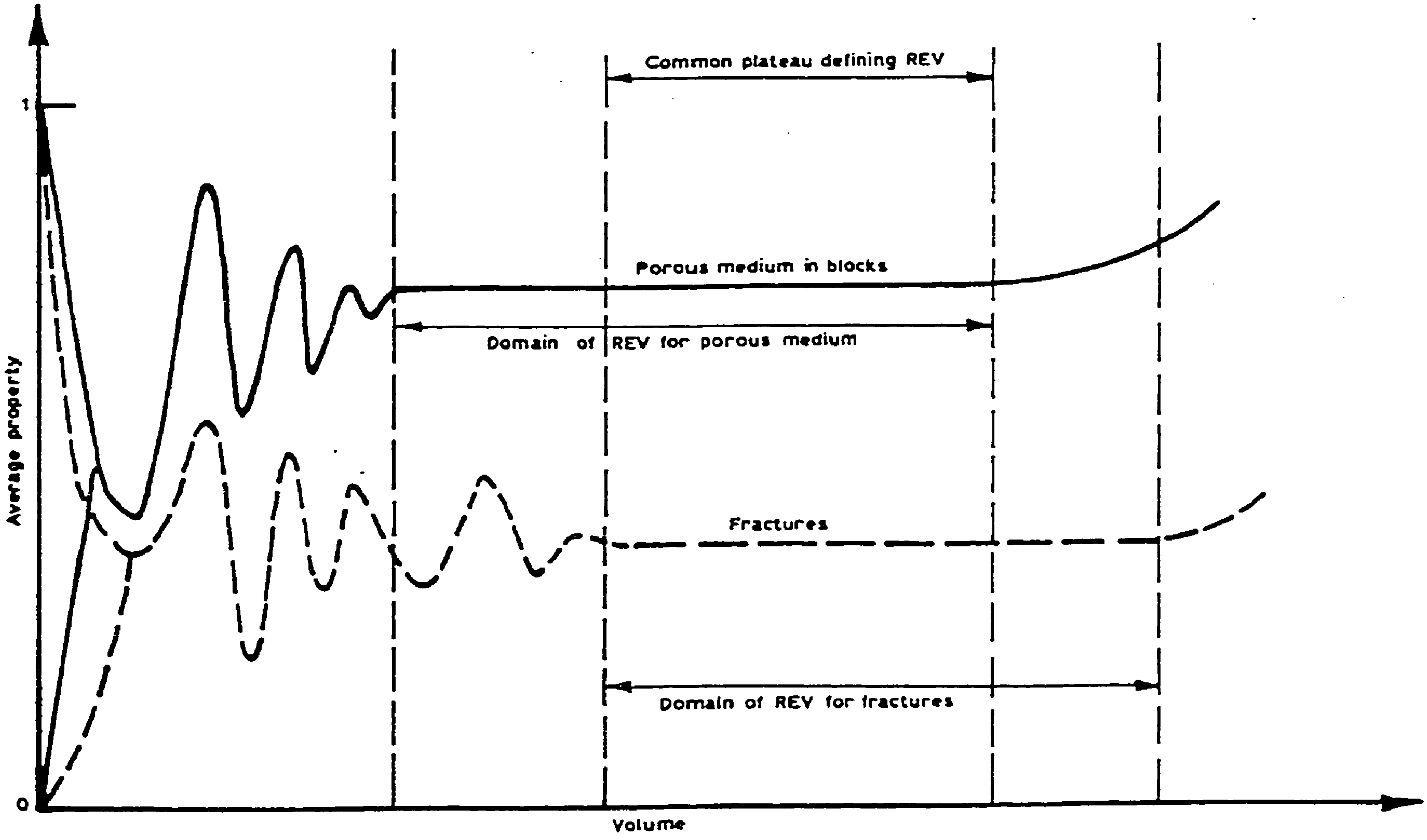


Figure 4.1 Definition of representative elementary volumes for the fractures and porous block domain (Bear J. and B. Berkowitz, 1987)

4.3 Flow velocity

Considering that fractured porous medium consists of a system of porous rock blocks separated one from another by fissures with irregular shapes. The elemental macrovolume, that is a large volume comparable to the size of an individual block, has to be define. This means that the study domain is much larger than the block size. Furthermore, the size of the block is much larger than the size of the pores.

The overlapping continua (Barenblatt et al.,1960) simplifies the flow domain by visualizing the fracture network (often characterized by high permeability and low porosity) and the porous blocks (often characterized by low permeability and high porosity) as separate, but interacting continua. It is assumed that any small representative volume element includes numerous random fractures and porous blocks, as shown in Figure 4.2. The movement of fluid within such system is then described by two balance equations, one for each continuum (or medium), coupled by a fluid exchange term. The later depends on the differences in piezometric head between the two continua at each point.

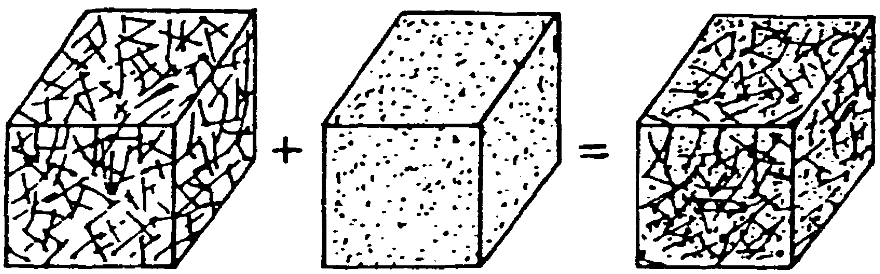


Figure 4.2 Coexistence of two porosities as random double-porosity (Şen,1995)

The permeability of a porous block is so low that it can be neglected when describing the macroscopic fluid motion. Assuming that flow through fissures is slow and inertialess, it may be possible to formulate Darcy's Law for a fractured domain on the basis of a dimensional analysis. In this way, formulations of the seepage law could be written taking into account the possible anisotropy of fissured system, and fractures' geometrical characteristics. This law is defined as (Barenblatt et al.,1990):

$$u_i = -\frac{k_{i\alpha}}{\mu}\partial_\alpha p = \frac{h^3}{\mu l}k^0_{i\alpha}\partial_\alpha p \tag{4.3}$$

where: u_i = components of flow velocity vector [L/T];

k = fissure permeability tensor [tensor];

h = mean fissure opening [L];

l = characteristic length size of a block [L];

μ = viscosity of the fluid [M/LT];

p = pressure [M/TL^2];

α = index for each fracture of the set.

The specific form of dimensionless permeability tensor may be determined by the geometry of the fissure system for a medium consisting of impermeable blocks and several systems of flat and regularly arranged fissures. It can be evaluated using Boussinesq's formula for the laminar flow of a viscous fluid through a narrow gap between parallel walls.

In some cases, it is difficult to determine the components of fracture permeability tensor through calculations. In such situations, experimental data should be used in order to accurately determine the components of the hydraulic conductivity tensor. By choosing properly the system of coordinates or by using tensor transformation operations, the hydraulic conductivity tensor can be transformed into its principal axes. However, if the pressure gradient is directed along a principal axis, flow velocity should have the same direction.

4.4 Flow in fractured media

To establish the basic equations for flow in fractured media, and later, the double-porosity equations, a series of conditions must be met:

- porous blocks are considered impermeable, and their permeability is neglected;
- the boundary of a fractured porous reservoir has the initial fluid pressure P_0 and the pressure drops to a lower value P_1;
- fluid motion through the fissures can be described with classical relations of flow theory through porous media;
- after transient process has occurred, a new steady-state distribution of pressure is established in fissures and, at any place close to reservoir boundary, the pressure will be considerably lower than the initial pressure;
- as a result of blocks impermeability and of the fact that their pressure could not be changed, a significant pressure difference will be set up between the fluid in blocks and the fluid in fissures; (in the order of $(P_0\text{-}P_1)$);
- local pressure gradients $(P_0\text{-}P_1)/l$ will be created in blocks, and these will be considerably higher than the pressure gradient in reservoir's fissures which is in order of $(P_0\text{-}P_1)/L$.

Under these conditions, local flows are produced in the reservoir, even if the blocks have very low permeability. The fluid flows from blocks to fissures and equalizes the local pressures (blocks and fissures). A schematic representation considered by Barenblatt in its demonstrations is presented below (Figure 4.3).

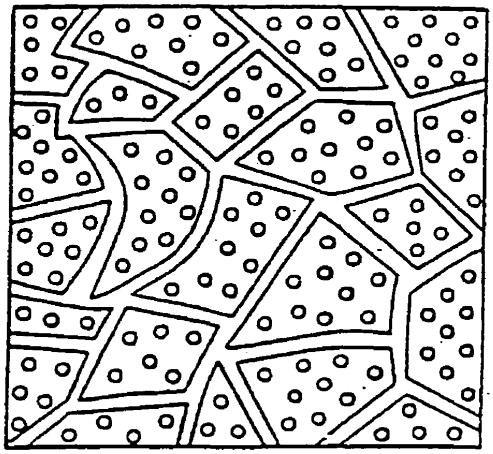


Figure 4.3 A fractured porous medium. The low-permeable porous matrix is dissected by a system of high-conductive fractures which have a small storage capacity (Barenblatt,1990)

Instead considering of having only one fluid pressure at a given point in the medium, it is consider the existence of two pressures, one in the fissures (p_1), and the other, in the pores of the block (p_2). Both pressures are mean pressure values averaged over scales, sufficiently large compared to the scale of the blocks, but small compared to the size of the flow region. Assuming that the permeability of the porous matrix is very low, it is possible to use Equation (4.3) in order to determine the flux through the studied area, by substituting into it pressure in the fissures p_1 .

The fluid balance equation (conservation of masses) in the fissures as follows (Barenblatt et al., 1990):

$$\partial_t(m_1\rho) + \nabla(\rho \mathbf{u}) - q = 0 \qquad (4.4)$$

where: m_1 = fissure porosity (the ratio of the fissure volume to the bulk volume of the medium)[%];

ρ = fluid density [M/L^3];

q = amount of fluid flow per unit time, from porous blocks to the fissures, per unit volume of the medium [L/TL^3];

$\mathbf{u}$ = flow velocity tensor [tensor].

It is possible to ignore the seepage flux from the blocks and write the continuity equation for the blocks as follows (Barenblatt et al., 1990):

$$\partial_t(m_2\rho) + q = 0 \tag{4.5}$$

where: m_2 = porosity of blocks (relative to the bulk volume of the medium) [%];

q = amount of fluid flow per unit time, from porous blocks to the fissures, per unit volume of the medium [L/TL^3];

ρ = fluid density [M/L^3].

Thus, the quantity of liquid flowing into the fissures equals the quantity of liquid flowing out of blocks. However, the volume of fissures in the blocks is considerably smaller than the volume of pores. Therefore, the influence of fluid pressure in the fissures (p_1) on the porosity of blocks can be disregarded compared to the influence of the fluid pressure in pores (p_2), and it can be assumed that (Barenblatt et al.,1990):

$$dm_2 = \beta_{c2} p_2 \tag{4.6}$$

where: m_2= porosity of blocks (relative to the bulk volume of the medium) [%];

β_{c2}= coefficient of blocks compressibility.

The fluid flow from the pores into the fissures (per unit time), per unit volume of rock has the following expression:

$$q = \frac{\rho\alpha}{\mu}(p_2 - p_1) \tag{4.7}$$

where: q= fluid flow from pores to fissures [L/TL^3];

p_2-p_1 = pressure drop between the pores and fissures [M/TL];

α= dimensionless characteristic of the fissured rock;

μ= viscosity of the fluid [M/LT].

The fluid is slightly compressible, and:

$$\rho=\rho_0+\beta\ \delta p \tag{4.8}$$

where: ρ = density of fluid [M/L^3];

ρ_0 = density of fluid at the standard pressure [M/L^3];

β = coefficient of compressibility of the fluid;

δp = change in pressure relative to the standard pressure.

Combining the last two equations and neglecting the terms of higher order, the following equation is obtained:

$$(b_{c2}+m_0\beta)\frac{\partial p_2}{\partial t}+\frac{\alpha}{\mu}(p_2-p_1)=0 \tag{4.9}$$

Assuming that the medium is homogenous and neglecting the smaller higher order terms:

$$k_1\ \Delta p_1+\alpha(p_2-p_1)=0 \tag{4.10}$$

Furthermore, eliminating p_2 from these equations, the following equation is obtained for the pressure of the fluid in the fissure p_1:

$$\frac{\partial p_1}{\partial t} - \eta \frac{\partial(\Delta p_1)}{\partial t} = \chi \Delta p_{11}$$

$$\text{where: } \chi = \frac{k_1}{\mu(\beta_{c2} + m_0\beta)} \quad \text{and} \quad \eta = \frac{k_1}{\alpha} \tag{4.11}$$

where: χ = coefficient of piezo-conductivity of the fissured rock (it corresponds to the porosity and compressibility of the blocks);

η = coefficient which tends to 0 (corresponds to a reduction in block dimension and an increase in degree of fissuring)

This equation will tend to coincide with the ordinary equation of seepage under deformable conditions. The above equations describe the motion of a uniform fluid in a fractured rock in the general case. Models like double-porosity, start from these equations and go further in modeling the parameters needed for solving the equations.

4.5 The double-porosity model

For the motion of homogenous fluids in double-porosity medium, two types of media are considered (Figure 4.3):

1. porous medium, consisting of relatively wide pores of first order, fissures and blocks (first order porosity is equal to m_1);
2. porous blocks themselves separated by fine pores of second order (second order porosity is equal to m_2).

The equations for fluid mass conservation for both media are as (4.4) and (4.5). Assuming that the flow in both media is inertialess, and Darcy's Law can be written as:

$$u_1 = -\frac{k_1}{\mu} \text{grad } p_1 \quad \text{and} \quad u_2 = -\frac{k_2}{\mu} \text{grad } p_2 \tag{4.12}$$

where: u_1 and u_2= components of the velocity vector for the two media considered [L/T];

k_1 and k_2= porosities of the system of pores of first order and second order [%];

μ= fluid viscosity [M/LT].

Assuming that both increments in porosity are linear functions and that they depend on fracture pressures, it would be possible to write their expressions as follows:

$$dm_1 = \beta_{c1} dp_1 - \beta_* dp_2 \qquad \text{and} \qquad dm_2 = \beta_{c2} dp_2 - \beta_{**} dp_1 \tag{4.13}$$

Inserting the relation (4.12) into Equations (4.4) and (4.5) will give a system of equations similar to that of heat transfer in a homogenous medium:

$$\begin{aligned} \frac{k_1}{\mu}\Delta p_1 &= (\beta_{c1} + m_{10}\beta)\frac{\partial p_1}{\partial t} - \beta_* \frac{\partial p_2}{\partial t} - \frac{\alpha}{\mu}(p_2 - p_1) \\ \frac{k_2}{\mu}\Delta p_1 &= (\beta_{c2} + m_{20}\beta)\frac{\partial p_2}{\partial t} - \beta_{**} \frac{\partial p_1}{\partial t} + \frac{\alpha}{\mu}(p_2 - p_1) \end{aligned} \tag{4.14}$$

where: m_{10} and m_{20} = values of the first and the second order porosity at standard pressure (Barenblatt et al.,1960).

It is advisable to consider the double-porosity model for a medium in which the porosity of each order discontinuities depends only on the appropriate pressure. Therefore, β_* and β_{**} coefficients in Equation (4.13) can be considered small, and the appropriate terms in Equation (4.14) can be disregarded. With these assumptions, Equation (4.14) will take a form similar to heat transfer:

$$\frac{k_1}{\mu}\Delta p_1 = (\beta_{c1} + m_{10}\beta)\frac{\partial p_1}{\partial t} - \frac{\alpha}{\mu}(p_2 - p_1)$$

$$\frac{k_2}{\mu}\Delta p_1 = (\beta_{c2} + m_{20}\beta)\frac{\partial p_2}{\partial t} + \frac{\alpha}{\mu}(p_2 - p_1) \tag{4.15}$$

Disregarding the terms in Equation (4.14) representing a change in liquid mass due to compressibility of the first medium and the compression of the liquid in the first order pores, as well as the change in liquid mass as a result of seepage inflow along the pores of the second order, the same Equation (4.15) of fluid motion in a fissured medium is obtained.

To resolve the last equation, some boundary-values must be set up for each case. Methods to solve this system of equations vary but are available. Numerical methods are widely used in this type of problems. In some cases, the convergence of the solution is not obvious, but it can be achieved after considering series of iterations or more assumptions should be considered.

Chapter 5

FlowD finite element code

5.1 Flow equations - analytical formulations

Based of the formulations of the double-porosity model, the author tried to develop a simple and practical finite element model to determine flow nets in rock media such as open pit rock slopes.

Flow in fractured rock slopes becomes very complicated when dealing with large scale studies. Different orientations of sets of fractures, as well as their hydraulic properties must be carefully determined. On the other hand, due to the lack of in-situ testing, hydraulic parameters have to be estimated from published data.

In this model, presented herein, as a first step in the development of finite element code, we have considered the following:

- steady-state flow conditions;
- no fluid exchange between fractures and blocks;
- heavily jointed rock with sets of fractures identified as controlling groundwater flow;
- fractures orientation and their hydraulic properties are considered;
- rock blocks are considered to have very low permeability, and their hydraulic conductivity is nil;
- double-porosity or double-conductivity are accounted for by the motion of equivalent hydraulic conductivity.

The governing differential equation used in formulation of the FlowD code is:

$$\frac{\partial}{\partial x}(k_x \frac{\partial H}{\partial x}) + \frac{\partial}{\partial z}(k_z \frac{\partial H}{\partial z}) + Q = 0 \qquad (5.1)$$

where: H = total head [L];

k_x = hydraulic conductivity in the x - direction [L/T];

k_z = hydraulic conductivity in the z - direction [L/T];

Q = applied boundary conditions [L^3/T].

Under steady-state conditions, the flux entering and leaving an element volume is the same all the time. In this particular case, the global system of coordinates is important not only in defining the mesh but also in the formulation of the hydraulic conductivity tensor which differentiates between the soil material and the rock mass domain.

Hydraulic conductivity tensor is calculated differently for soil and rock materials. For soil it takes into account directional hydraulic conductivity in x and z direction for each material, and the angle between the directional hydraulic conductivity and the system of coordinates (Figure 5.1).

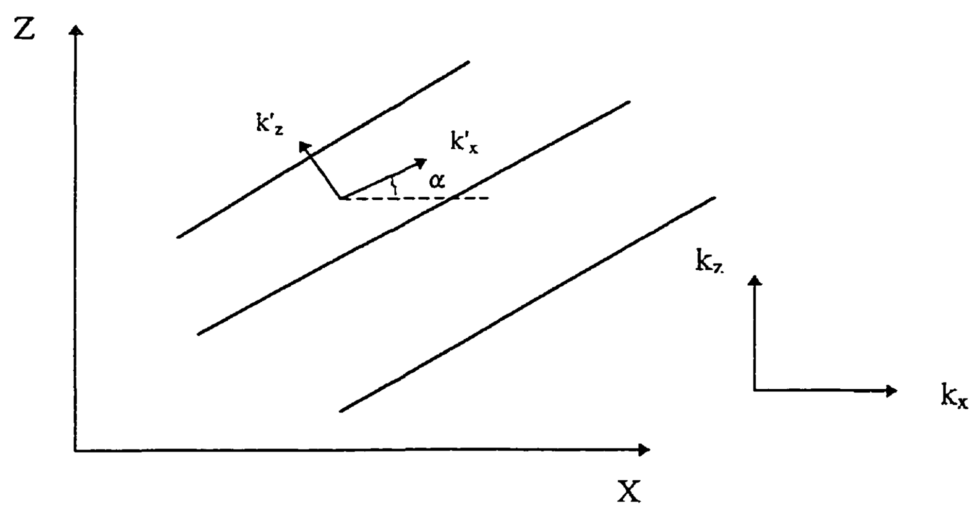


Figure 5.1 Hydraulic conductivity direction for soil material

For the rock version of the computer code, hydraulic conductivity tensor for fractured rock has been calculated as a summation of tensors representing hydraulic conductivity of each set of fractures taking in account their hydraulic properties. This concept was for the first time formulated by Feuga (1981) and is as follows (de Marsily, 1986):

$$\mathbf{K} = \frac{1}{l}\sum_{i=1}^{N} e_i \, k_i \mathbf{R}_i \tag{5.2}$$

where: $\mathbf{K}$ = hydraulic conductivity tensor;

l = arbitrary dimension of the side of a square block of the fractured medium, large enough to statistically sample all the families of fractures [L];

N = number of fractures in the block of side l;

e_i = the aperture of each individual fracture [L];

k = hydraulic conductivity of each individual fracture [L/T];

$\mathbf{R}_i$ = matrix depending on the direction d and the dip of each fracture, its form will be presented in the next chapter.

The matrix $\mathbf{R}_i$ has the following form for each set of fractures:

$$R_i = \begin{bmatrix} 1-\cos^2 d_i \sin^2 p_i & \frac{1}{2}\sin 2d_i \sin^2 p_i & -\frac{1}{2}\sin 2p_i \cos d_i \\ \frac{1}{2}\sin 2d_i \sin^2 p_i & 1-\sin^2 d_i \sin^2 p_i & \frac{1}{2}\sin 2p_i \sin d_i \\ -\frac{1}{2}\sin 2p_i \cos d_i & \frac{1}{2}\sin 2p_i \sin d_i & \sin^2 p_i \end{bmatrix} \tag{5.3}$$

where: d_i = direction of the set i of fractures [degree];

p_i = dip of the set i of fractures [degree].

The total hydraulic conductivity vector **K** is rotated in the plane of the studied cross section, after which is extrapolated for a 2D analysis. For the FlowD code, the global system of coordinates is represented in Figure 5.2. The Oy axis represents also the North direction and the analysis is performed in the xOz plane, which is practically the plane of a vertical cross section in a slope.

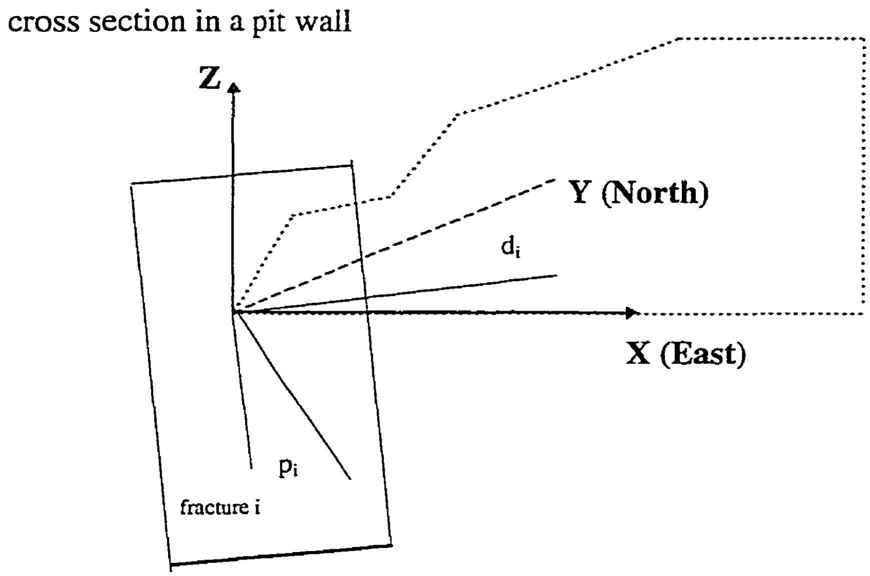


Figure 5.2 Global system of coordinates for the hydraulic conductivity tensor(de Marsily,1986)

Blocks are considered practically impermeable and flow takes place mostly in fractures. Their orientation becomes very important in studying flow. It is difficult to approximate the hydraulic properties of fractures when field tests do not exist. But, dealing with large scale studies, where fractures' orientations are available and the rock properties are known, assumptions regarding their hydraulic properties can be estimated from literature.

5.2 The quadrilateral isoparametric element

Finite element method has often been applied in solving groundwater flow problems. The finite element mesh in this case, employs 4-node isoparametric elements represented in Figure 5.3 with their local system of coordinates (s, t). The global coordinates (x, z) of any point in the element are related to the local coordinates (s, t) through the following equations:

$$x = \langle N \rangle \{ X \}$$
$$z = \langle N \rangle \{ Z \} \tag{5.4}$$

where $\langle N \rangle$ is an array of interpolating shape functions and $\{ X \}$ as well as $\{ Z \}$ are the global x, z coordinates of the element nodes. The shape functions are expressed in terms of local coordinates and have the following form for the 4 - nodes isoparametric element:

$$\begin{aligned} N_1 &= 1/4\,(1\text{-}s)(1\text{-}t) \\ N_2 &= 1/4\,(1+s)(1\text{-}t) \\ N_3 &= 1/4\,(1+s)(1+t) \\ N_4 &= 1/4\,(1\text{-}s)(1+t) \end{aligned} \tag{5.5}$$

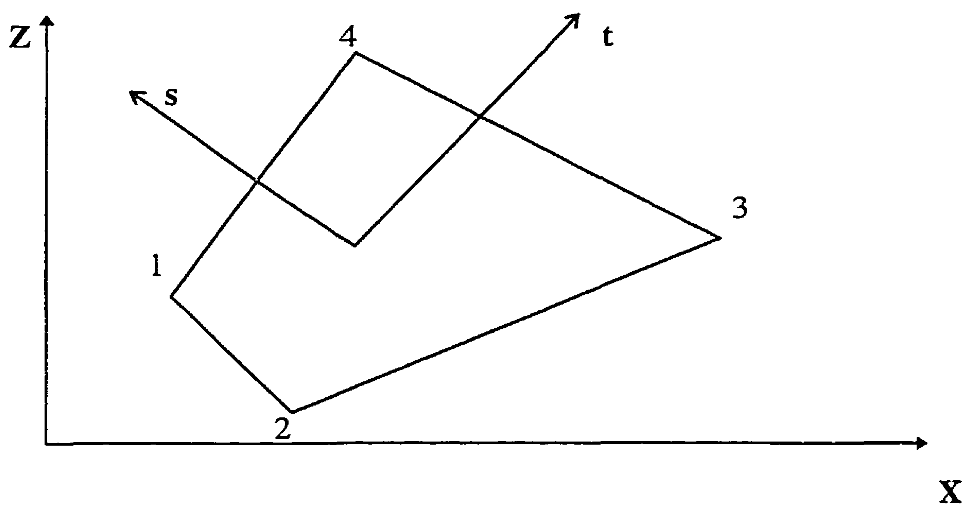


Figure 5.3 Isoparametric 4-node element in the global and local system of coordinates

In order to formulate the finite element equations, it is necessary to adopt a model for the field variable within the element. In the seepage analysis, the field variable is the total head (**H**) and a model has to be adopted for its variation within the element. FlowD code considers that the total head distribution within the element follows the interpolating functions presented above, which means that the head distribution is linear. The distribution of the total head in the element could be summarized by the following equation:

$$h = < N > \{H\}^e \qquad (5.6)$$

where: h = head at any point;

$< N >$ = array shape functions;

$\{H\}$ = vector of heads at the nodes.

The constitutive law for the seepage analysis is Darcy's Law previously presented. For FlowD code this law is expressed by the conventional equation:

$$q = k\, i \qquad (5.7)$$

where: q = groundwater flux [L/T];

k = hydraulic conductivity [L/T];

h = gradient [L/L].

In the finite element formulation, the hydraulic gradient 'i' is a key parameter. The procedure used in the FlowD code to compute the gradient is presented below.

The gradients in x and z direction are:

$$
\begin{aligned}
i_x &= -\frac{\partial h}{\partial x} = -\frac{\partial \langle N \rangle}{\partial x}\{H\}^e \\
i_z &= -\frac{\partial h}{\partial z} = -\frac{\partial \langle N \rangle}{\partial z}\{H\}^e
\end{aligned}
\tag{5.8}
$$

The shape functions are given in terms of local coordinates s and t, and must be computed in terms of the global coordinates. In this case, the derivatives must be determined by the chain rule of differentiation as follows:

$$
\begin{aligned}
\frac{\partial \langle N \rangle}{\partial s} &= \frac{\partial \langle N \rangle}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial \langle N \rangle}{\partial z}\frac{\partial z}{\partial t} \\
\frac{\partial \langle N \rangle}{\partial t} &= \frac{\partial \langle N \rangle}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \langle N \rangle}{\partial z}\frac{\partial z}{\partial t}
\end{aligned}
\tag{5.9}
$$

Which can be written as:

$$
\begin{bmatrix} \dfrac{\partial \langle N \rangle}{\partial s} \\ \dfrac{\partial \langle N \rangle}{\partial t} \end{bmatrix} = [J] \begin{bmatrix} \dfrac{\partial \langle N \rangle}{\partial x} \\ \dfrac{\partial \langle N \rangle}{\partial z} \end{bmatrix}
\tag{5.10}
$$

where: [J] is the Jacobian matrix. Performing matrix operations, the derivative of the interpolating function with respect to x and z can be determined, and the equation is rewritten as :

$$
\begin{bmatrix} \dfrac{\partial \langle N \rangle}{\partial x} \\ \dfrac{\partial \langle N \rangle}{\partial z} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \dfrac{\partial \langle N \rangle}{\partial s} \\ \dfrac{\partial \langle N \rangle}{\partial t} \end{bmatrix}
\tag{5.11}
$$

All the above vectors will be used in the finite element formulations of the seepage equations. The hydraulic gradient in matrix form can be written as:

$$[I] = - [B]\{H\}^e \tag{5.12}$$

where: [B]= characteristic gradient matrix;

[I]= gradient vector;

{H}= hydraulic head vector = < H_1, H_2, H_3, H_4>.

5.3 Finite element formulation

The finite element equation chosen for the seepage analysis performed by the FlowD code results from the application of the Galerkin method of weighed residuals to the governing differential equation (5.1), and is given by:

$$\int_v ([B]^T [C][B]) \, dv \, \{H\}^e = q \int_A (\langle N \rangle^T) \, dA \tag{5.13}$$

where: [B] = characteristic gradient matrix,

[C] = element hydraulic conductivity matrix;

{H} = vector of nodal heads;

q = unit flux across the side of an element;

< N > = vector of the shape functions.

For the two-dimensional analysis, the thickness of the element is considered to be constant over the entire domain. The finite element equation could be written as :

$$t \int_A [B]^T [C][B]) \, dA \, \{H\}^e = qt \int_A (\langle N\rangle^T) \, dL \tag{5.14}$$

where t is the element thickness. The abbreviated form of the element equation is:

$$[K]^e \{H\}^e = \{Q\}^e \tag{5.15}$$

where : $[K]^e$ = element characteristic matrix;

$\{H\}^e$= vector of nodal head;

$\{Q\}^e$= applied flux vector.

Special attention is given to the element characteristic matrix. For soil materials the general form of hydraulic conductivity matrix is:

$$[C] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{5.15}$$

where: $C_{11} = k_{x*} \cos^2 \alpha + k_{z*} \sin^2 \alpha$

$C_{22} = k_{x*} \sin^2 \alpha + k_{z*} \cos^2 \alpha$

$C_{11} = k_{x*} \cos \alpha \sin \alpha - k_{z*} \sin \alpha \cos \alpha$

$C_{21} = C_{12}$

Parameters k_{x*} and k_{z*} correspond to hydraulic conductivity in x* and z* directions; α is the angle between the system of coordinates (x, z) and the hydraulic conductivity directions (x*, z*) (Figure 5.1).

In the case of double-porosity model the hydraulic conductivity matrix is determined by taking into account the sets of fractures with their hydraulic conductivity properties, the average number of fractures in a set, the average aperture and their spatial orientation as in Equation (5.2). Extrapolation of the two dimensional characteristics matrix is needed in order to formulate the fractured flow. The projection of the dip and dip direction of the sets of fractures and their hydraulic properties are needed. The hydraulic conductivity of each set of fracture has to be positive defined, the angle between the fractures orientation and the system of coordinates gives the flow direction.

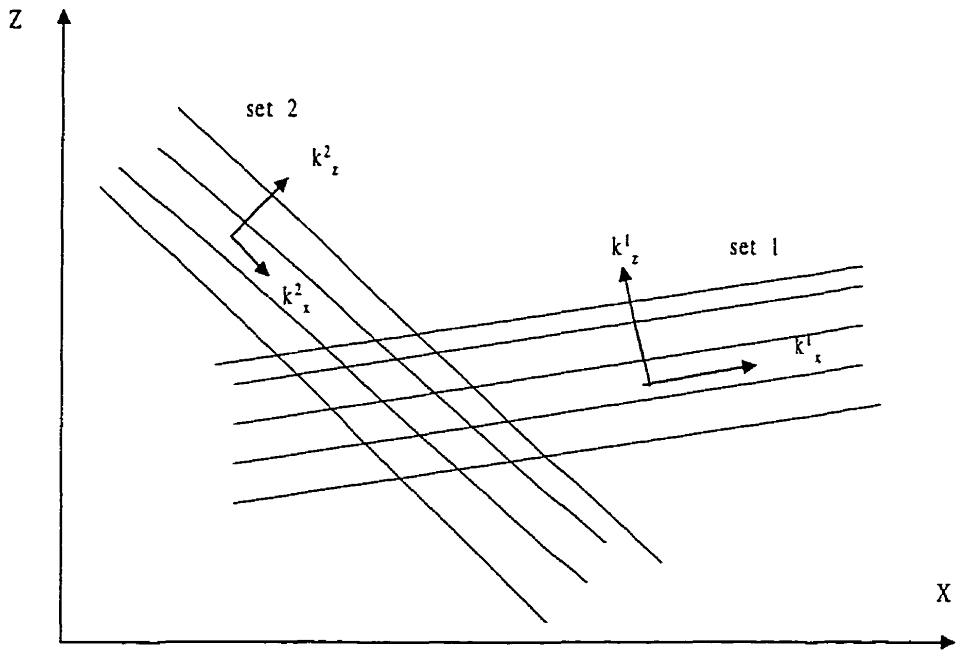


Figure 5.4 Hydraulic conductivity of sets of fractures

In order to resolve the system of equations, the boundary conditions must be imposed. FlowD code takes two types of boundary conditions. The first one is Dirichlet boundary condition of specified pressure head at a certain node of the mesh; the second one is the Neumann boundary condition which specifies groundwater flow rate over the side of an element or a punctual flux in a node of the mesh. Practically, Neumann boundary condition is counted as a nodal flux boundary in the program as follows:

$$\left\{F_i^{(e)}\right\} = \int_{\Gamma} N_i^{(e)} q^{(e)} \, d\Gamma \qquad (5.16)$$

where: F_i = nodal boundary flux;

Ni = shape function for that element;

q = unit flux over the side element;

Γ= studied domain.

The flux over the boundary could have constant variation along the side of the element. In this case, an equivalent force are calculated in the nodes of the side. The equivalent value depends on the variation of the flux over the side. FlowD code could handle only the linear variation as presented in Figure 5.5 below. Using this boundary conditions pumping or injecting wells could be simulated, they are represented as point source of sink in the domain and are considered as a punctual or distributed flux. Drain holes can be simulated as a well; along the drain hole, a uniform distributed flux which lives the element with pressure head nil.

Neumann boundary condition specifies piezometric head at the nodes of the mesh. At least one head should be imposed to have a unique solution of the system of equations. Consistency of units should be maintained for valid result. More details will be given in Appendix 1 which contains the user's manual of the FlowD program.

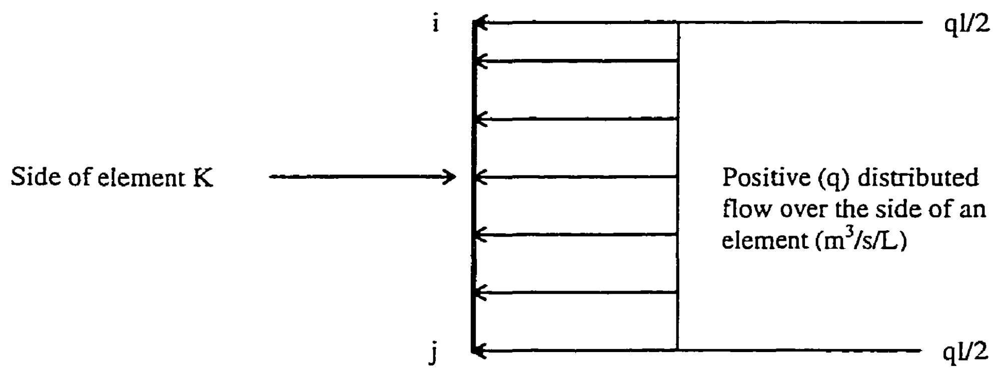


Figure 5.5 Distributed flux over the side of an element

5.4 Equation solver

FlowD uses Choleski decomposition technique to solve the finite element system of equations. The objective of solving the finite element equations is to compute total head at each node. The equations are linear, as being the assumption of steady state flow is made. Figure 5.6 presents the numerical integration scheme used and the definition of the element parameters.

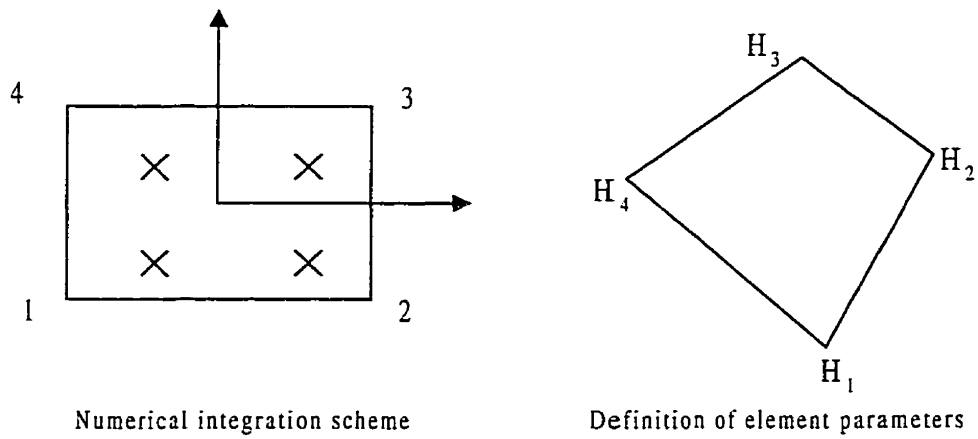


Figure 5.6 Numerical integration scheme

Once the equations are solved, the nodal heads are known, FlowD computes the hydraulic gradients and the Darcian velocities at the center of each element, using Equation (5.11). The Darcian velocities are calculated as follows:

$$\begin{Bmatrix} v_x \\ v_z \end{Bmatrix} = [C][B]\{H\}^e \qquad (5.15)$$

In the output file, the hydraulic head is specified at each node of the mesh and hydraulic gradients and hydraulic velocities, at the center of each element.

5.5 FlowD computer code

FlowD computer code is a finite element program that performs two-dimensional confined steady-state seepage analysis, homogenous or nonhomogenous soil and/or rock, and isotropic or anisotropic media. Boundary conditions are: (1) specified total head at the mesh boundary nodes, (2) specified flow at the mesh nodes. Nonhomogenous media are handled by allowing zones or elements to have different material properties, and the anisotropy is modeled by using the general 2×2 hydraulic conductivity tensor in the expression of Darcy's Law. A special treatment is given to the hydraulic conductivity tensor when the double porosity model is applied, sets of fractures are considered in the modeling process and their orientation and spacing modify the formulation of this tensor.

The code employs 4-node isoparametric elements with 2×2 Gauss integration scheme and is written in FORTRAN 77. To solve the equation system, a banded technique is used. The mesh is generated automatically, using an in-house mesh generator developed at the Numerical Modeling Laboratory of McGill University.

The computer program calculates velocities, hydraulic gradients and total heads at the nodes. Figure 5.7 presents the flowchart of the program .

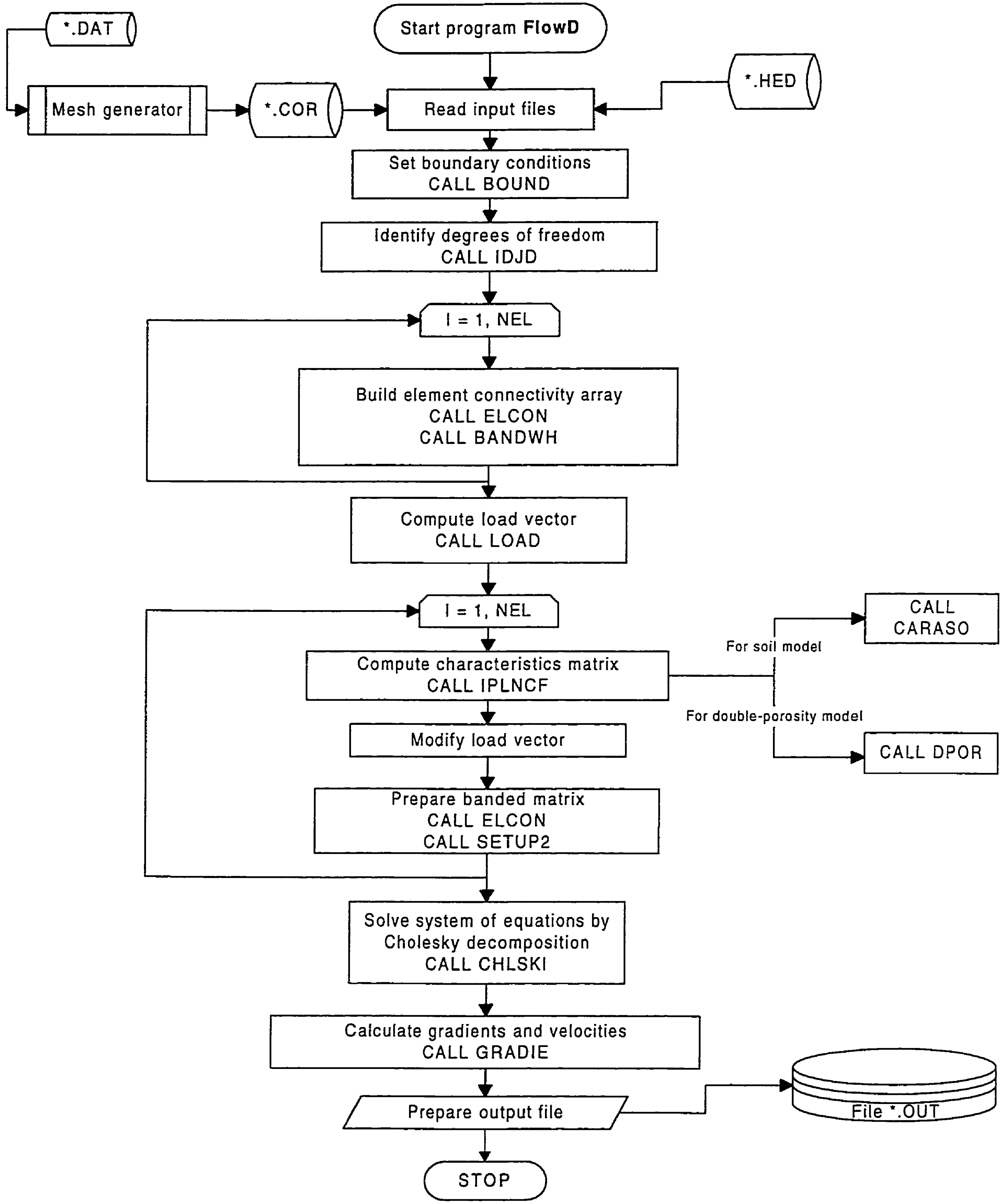


Figure 5.7 Flowchart of the FlowD code

The computer code could perform seepage analysis in the horizontal and vertical plane depending on the section studied. Figure 5.8 shows two types of analysis that FlowD can annualize. The soil version of the program has two features for horizontal and vertical cross section. The double-porosity version of the program has only the vertical cross section option.

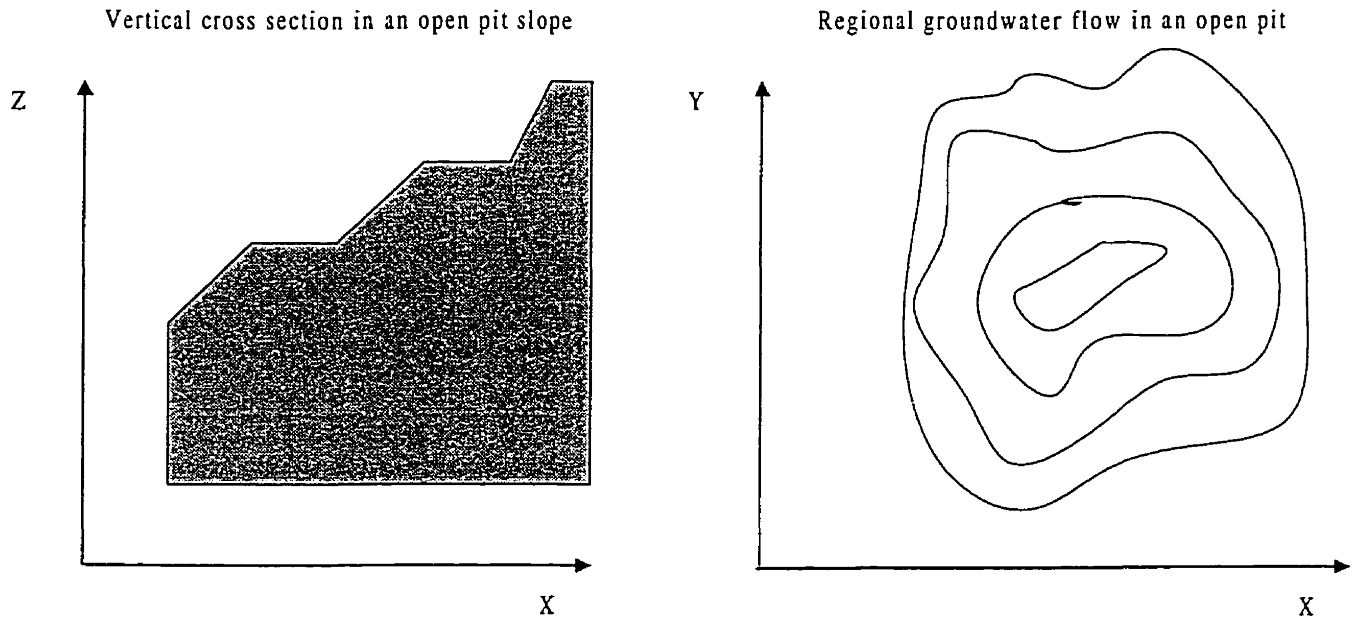


Figure 5.8 Types of analysis performed by FlowD

Chapter 6

Model verification and application

6.1 FlowD code verification

The verification of FlowD code and validation were performed on simple examples which were taken from different reference books. The double-porosity version of the code could not give the same results as other codes, having a different approach but in most of the cases the results could be pertinent. Two simple examples were run in order to verify the soil version of the code.

The first one consists of two-dimensional confined aquifer with two types of materials (silty sand and sandy gravel) considered isotropic with the same hydraulic conductivity in both directions; 12 elements mesh was design, with two types of boundary conditions: constant head along one edge of the mesh and a discharge well situated in one of the mesh nodes. This example was taken from the book by Istok J. (1989, page 259). The results of the FlowD code were very close to the ones specified in the reference. The margin of error was lower than 0.01%.

The second test of the code was seepage analysis under a dam. The dam itself was considered impermeable and its foundation was assumed to be layered. Just the constant head boundary condition was imposed. The two materials were considered anisotropic, with different horizontal and vertical hydraulic conductivity (1/30 and 1/25). The solutions were verified manually, giving a difference of 0.01% from the code calculation.

Some other verifications were performed like mesh density analysis, but they are not described here. From the verification process it can be concluded that the computer code gives accurate results for the total head, hydraulic gradients and flow velocities, in such type of analysis.

6.2 Case study

A large scale application was modeled in order to better understand the seepage phenomenon in a open pit slope and to validate the double-porosity model. With the help of mining staff from LAB Chrysotile Inc., we were able to test our model on a real case using the data provided from Black Lake Mine, Québec, where asbestos is the main mineral to be extracted.

The rock slope considered in this application is the East wall of the B-Pit of the Black Lake Mine property of the LAB Chrysotile Inc. in Eastern Townships, Québec. The crest of the pit which will have eventually an ultimate height of 1450 ft., is immediately adjacent to Provincial Highway 112, as shown in Figure 6.1 below. Behind the slope are located the BC Waste Dumps.

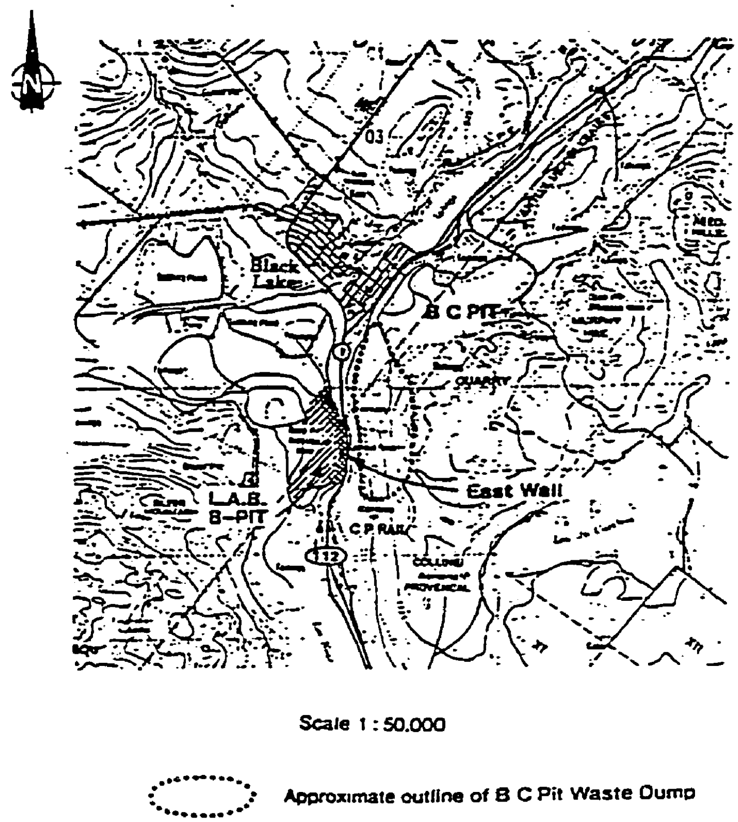


Figure 6.1 B Pit Mine location (Mine reports)

During mining operation, the design of the B Pit east wall was modified according to different engineering design studies, behavior of the slope, slope stability, water pressure in the wall and slope response to mining activity. Since the beginning of the mine the Highway 112 was relocated in order to expand the mining activity in a safety way and to better monitor slope stability.

6.2.1 East wall of Black Lake Mine

The original design for the slope was based on an overall angle of 45° for the initial cut, followed by a 600 ft. pushback to the ultimate, which will have a multiple angle slope, gradually flattening with depth. This design was changed during mining operations.

The general geology of the B Pit reflects the existence of three major rock types: massive, unaltered or weakly serpentinized peridotite; semi-schistose or schistose serpentinite (totally altered peridotite), and talc-carbonate shear zone. Narrow granitic veins occur through the wall; these are typically up to 10 ft. thick and strike down in the wall parallel to the regional structure (Figure 6.2).

From the hydrogeological point of view the high water table in the slope, the poor drainage performance and high groundwater recharge from the east through the BC Mine dumps rise more concerns about the real seepage phenomenon in the slope. During the years of operation, intensive studies were conducted for a better understanding of water flow phenomenon in the slope. Two pumping tests, one in the massive peridotite and the other in the schistos serpentinite were performed. Monitoring piezometers were installed in the crest of the ultimate pit. Inclined drain holes were drilled for depresurization. Different other methods of mine dewatering were studied, as pumping wells system or the design of an infiltration gallery in the slope.

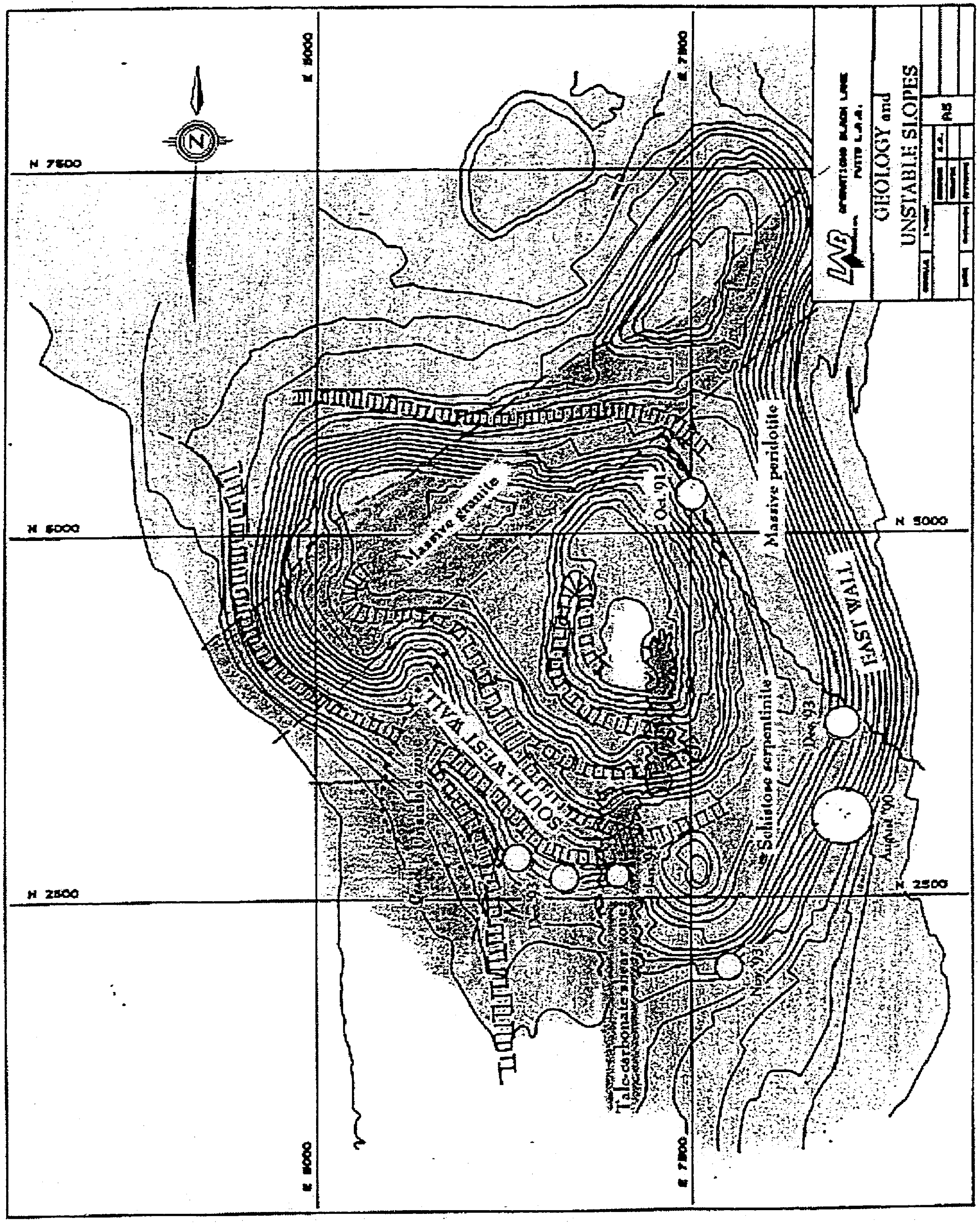


Figure 6.2 B Pit Mine geological plan view (Mine reports).

Based on the piezometric response to mining activity and drainage, the following hydrogeological model can be developed for the East Slope of the B Pit:

- In the upper section of the slope (peridotite), where there is a strong anisotropy in permeability, with vertical hydraulic conductivity being appreciably grater than the horizontal conductivity. This probably reflects the steeply dipping joints and shear in peridotite. Joints tend to be continuous, but irregular with steep dip. According with the stereographic projection of the discontinuities in this type of material, two sets of joints were identified with dip and dip direction 79°/337° and respectively 62/342. Major shears (faults) have been observed with similar trends and steep east-oriented dip; they were not monitored and valid information about their location is missing.

- Serpentinite sets of rock joints are typically continuous and frequently contain talc or fibrous minerals, although the surface is often curved. About four families of joints were identified in this type of rock, with the following dip and dip directions: 35°/194°; 56°/324°; 85°/127° and 86°/273°.

- The ore zone presents a strongly developed schistosity that dips to the east-northeast at 35° to 45°. Asbestos veining frequently parallels this foliation that also reflects the regional structural trend. This rock type presents many families of discontinuities five of which seems to be more important. They have the following dip and dip directions:62°/177°, 48°/194°, 65°/199°, 54°/207° and 80°/212°.

The connection between the two regimes is poor, possibly reflecting fault gouge / shearing along the peridotite contact with the ore zone and associated alteration zone. This concept is supported by the particularly high flows from the holes that pass through the contact. The horizontal drain holes, which were drilled in the slope, may intersect the orientations of maximum hydraulic conductivity.

Depresurization of the slope was needed in order to lower the water table and to drop the water pressure. Holes were drilled along the entire length of the wall, although there should be concentration in the areas of highest flows. A general spacing of 150 ft. along the wall on every second bench is recommended, with the spacing reduced to 50 ft. at the holes ends where the peridotite, schistose serpentinite contact is intersected. Installation of piezometers for monitoring the water pressure level was performed. Their location is along the highway at 1000 ft. intervals along the crest of the wall.

For a better depressurization of the pit walls pumping wells installation solution was studied. In the area of schistuosity associated with the ore zones, it should be possible to achieve deeper depresurization with vertical wells, which would be most suitable drilled from the proposed ramp on the ultimate wall. An other method of depresurization considered by the consultant was the design of a drainage gallery in the slope which will collect the same amount of water as the pumping wells system.

6.2.2 Slope geometry and hydrogeological regime

A typical cross section in this wall was chosen for the analysis. The height of the slope was 458m. with an overall angle of 27°. In the slope, three types of materials, were identified, and their characteristics follow the tests performed during the operation years.

The geometry of the slope is shown in Figure 6.4. Because of the recharge from the waste dumps, water table elevation is very high and was considered to be at the face of the slope. The two piezometers installed from the upper bench confirmed this fact and also indicate high water pressure in the wall. The geology consists in massif peridotite rock in the upper part in where two families of fractures are predominant. The same type of rock was found in the lower part closed to the toe of the slope. The ore body dips down east in the slope with an angle of 27° from the vertical. This part is highly fractured, having the discontinuities filled with fine material. Five families of fractures were identified to be predominant in this part of the slope. Parallel with the ore body there are some intercalation of massive serpentinite highly fractured and altered which are dipping down into the slope at the same inclination.

For each type of geological structure families of discontinuities were identified. Their dip and dip direction, determined from the stereonets, were used in running the model. Spacing of the family of discontinuities as well as their number were estimated, data being not available. They were approximated from the literature.

The 'Schematic Flow Diagram for East wall of the Pit' was used in order to determine boundary conditions for studied cross section. This diagram is presented in Figure 6.3 below.

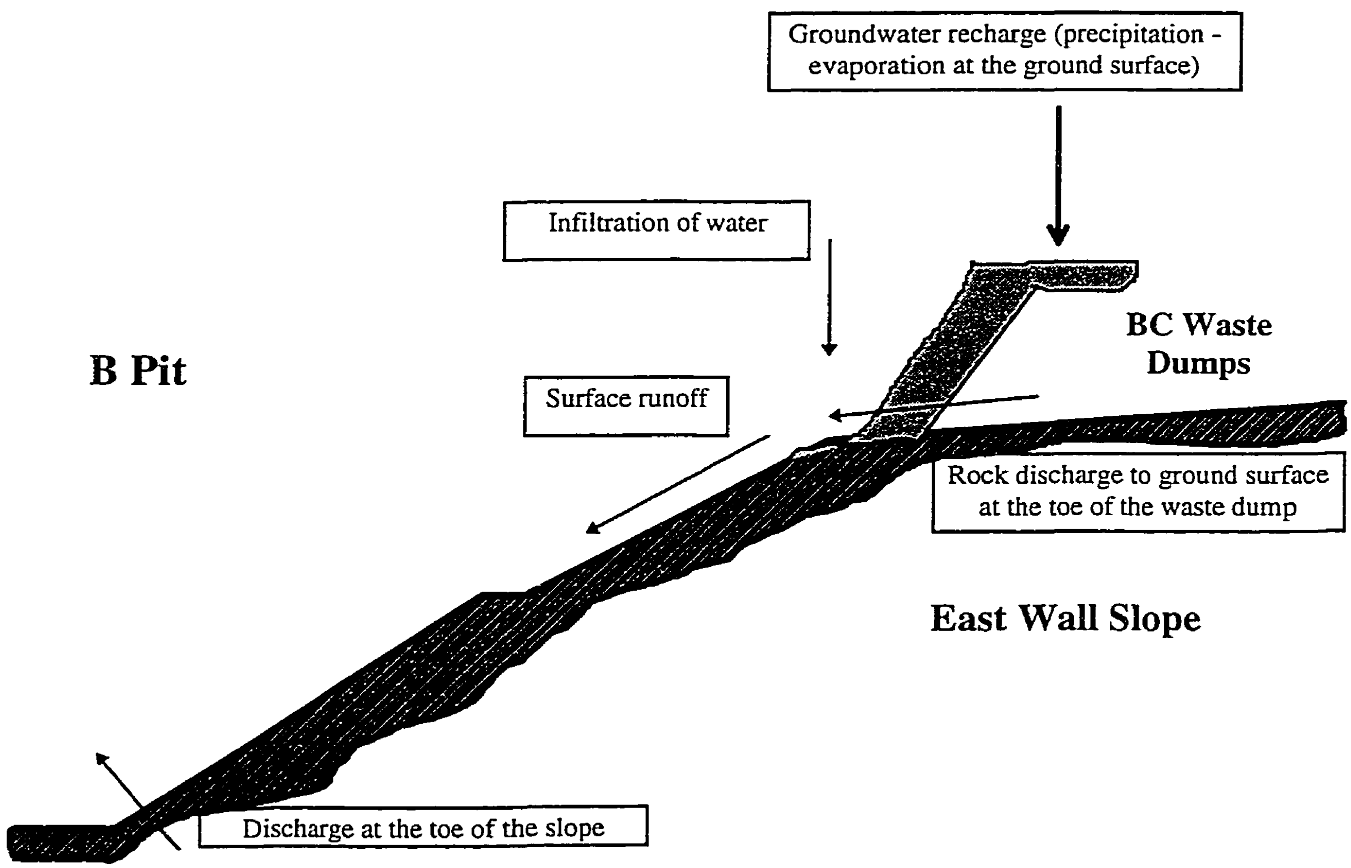


Figure 6.3 Schematic flow diagram (Mine reports)

The generated mesh for this example had 265 nodes and 225 isoparametric elements. Five types of materials were modeled and their properties were taken from the Mine reports or where missing, from literature, an example is the fracture aperture. Different boundary conditions were considered. Figure 6.4 shows the mesh designed for this example. Figure 6.5 shows the five materials identified in the slope as having different properties. In each material different number of families of fractures were identified, and modeled to run the double-porosity analysis.

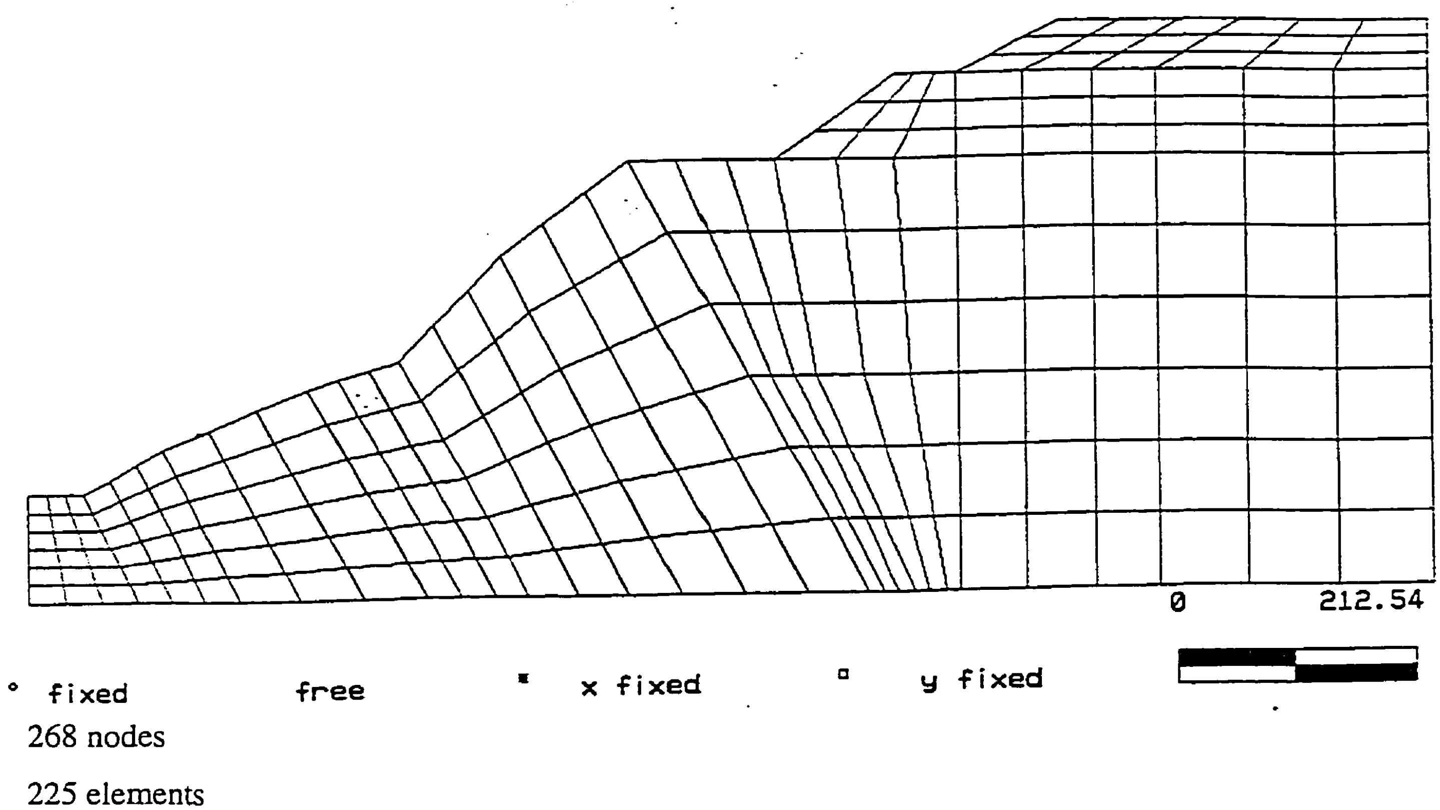


Figure 6.4 Generated mesh for the FE analysis

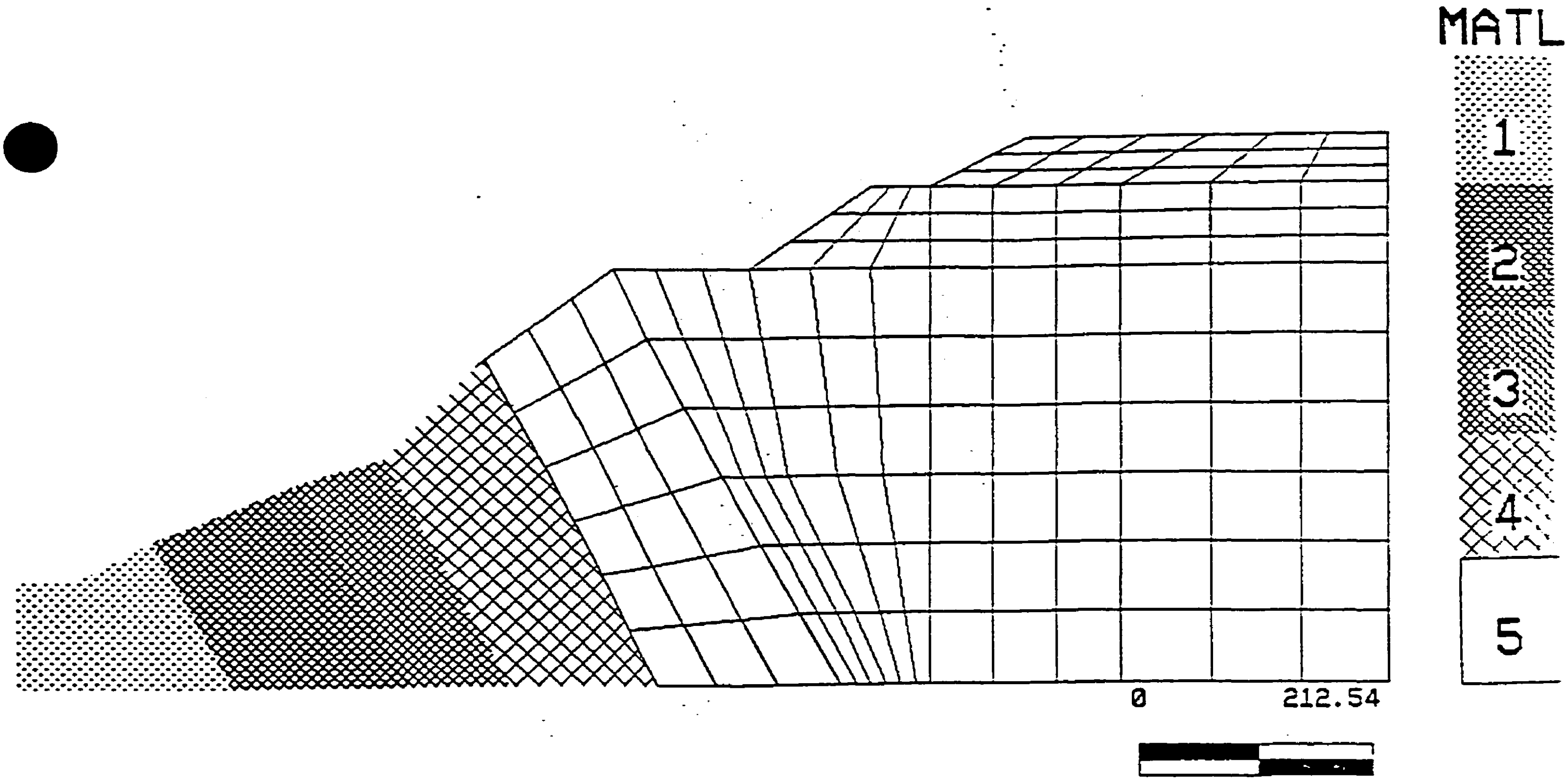


Figure 6.5 Materials identified in the 5000 N Section

In order to determine the water regime in the slope, three models were designed:

- **Model I**: water regime in the slope without any measure of dewatering, representing the flow diagram (Figure 6.3).
- **Model II**: well simulation, as a dewatering option.
- **Model III**: drainage gallery simulation, as a dewatering option.

Running the program for the **Model I** showed that the water regime in the slope present a high vertical anisotropy with high water pressure that can be explained by very low hydraulic conductivity of the materials. The slope zone that is highly fractured, and the fractures orientation dipping down into the slope practically control the water flow. The permanent recharge of the slope from the waste dumps and the poor drainage, caused by low hydraulic conductivity of the materials explain the high water pressure in the wall. Figure 6.6 shows the water velocity vector in the slope. As it can be seen, the third material modeled drains the entire slope and do not allow water to be drain properly from the surface. The equipotential lines are presented in Figure 6.7.

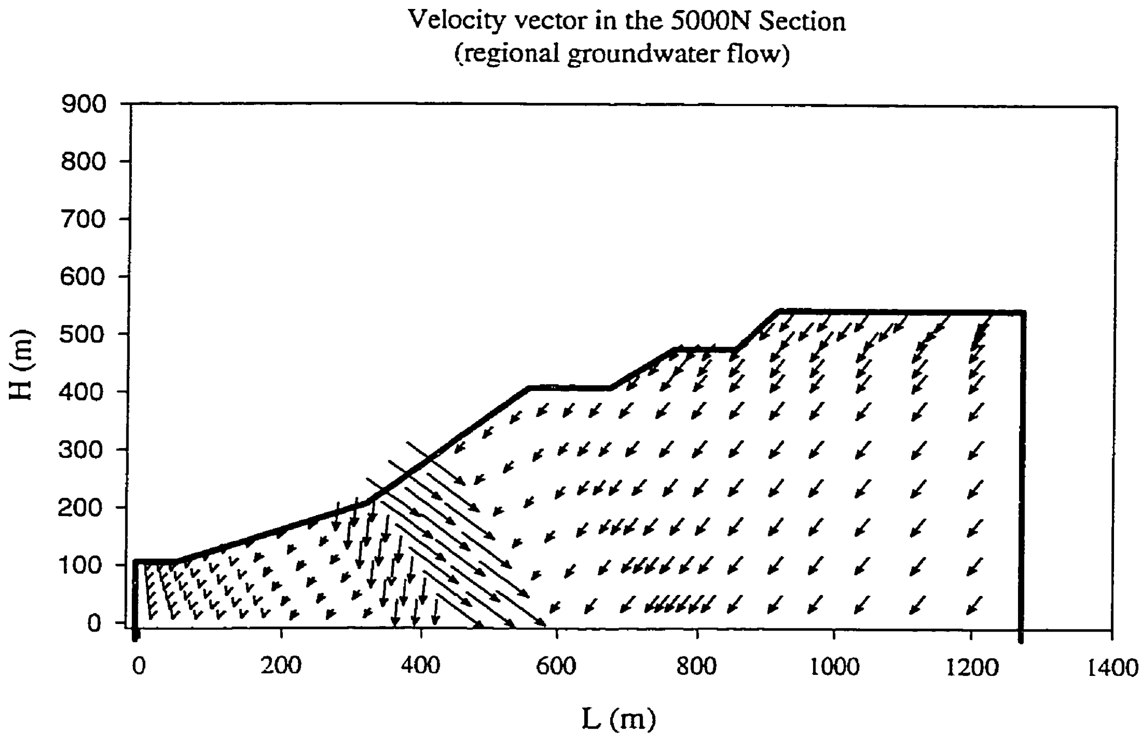


Figure 6.6 Groundwater flow velocity

Model II and **Model III** simulate dewatering methods considered by the Mine. Figure 6.8 and Figure 6.9 show the total head in the slope and the equipotential lines surrounding the well and the drainage galleries The low hydraulic conductivity of different materials and the middle highly fractured formation dipping down in the slope make very difficult the process of slope dewatering. The water pressure decreases, but not in a way that can be satisfactory enough.

Probably, a more complex method of dewatering or a combination of different methods should be used in order to considerably lower the water pressure in the slope. Free drainage of the slope could not be possible in such materials because of their properties and irregular internal geological features.

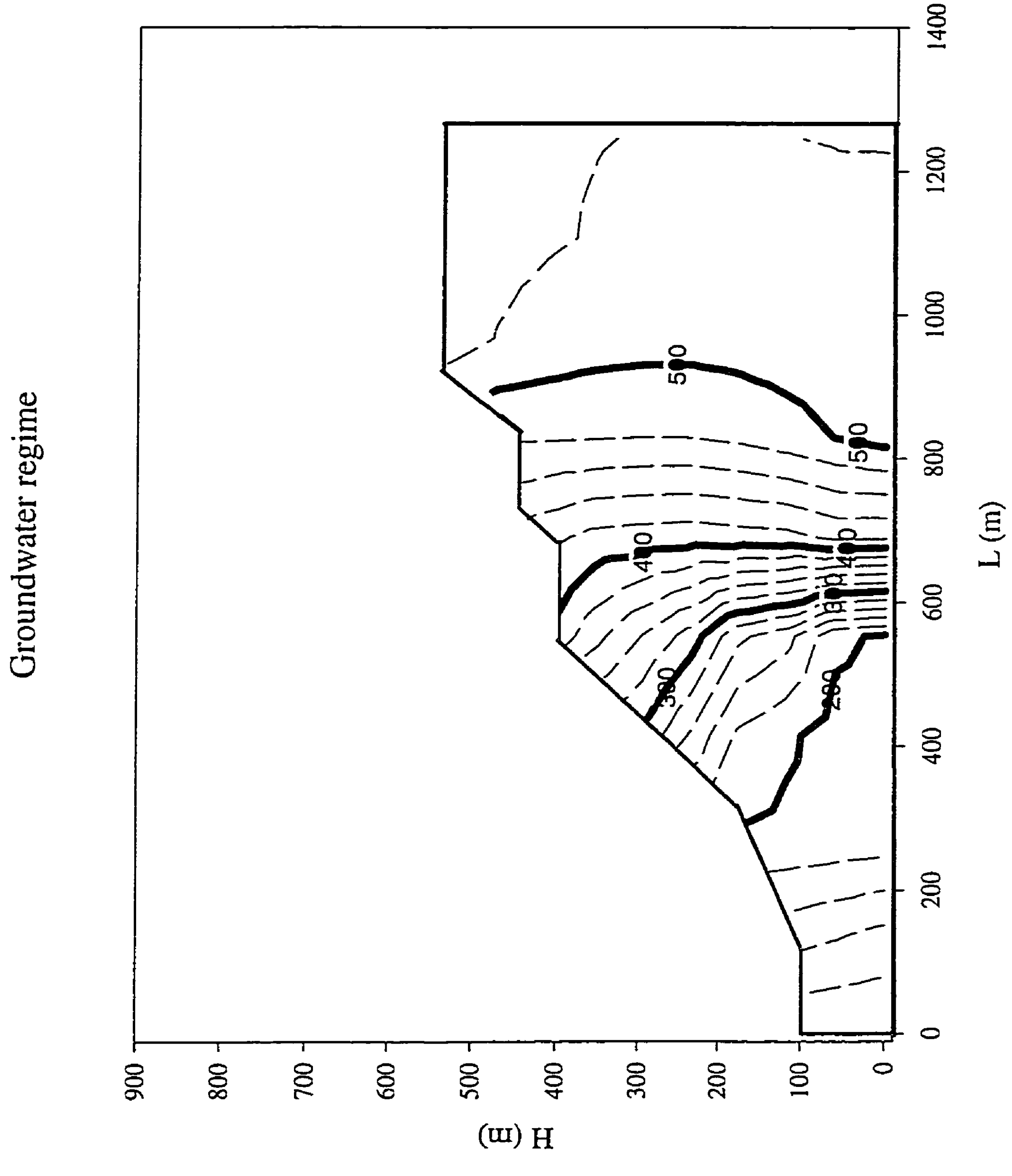


Figure 6.7 Total head - 5000N Section - Regional growndwater regime

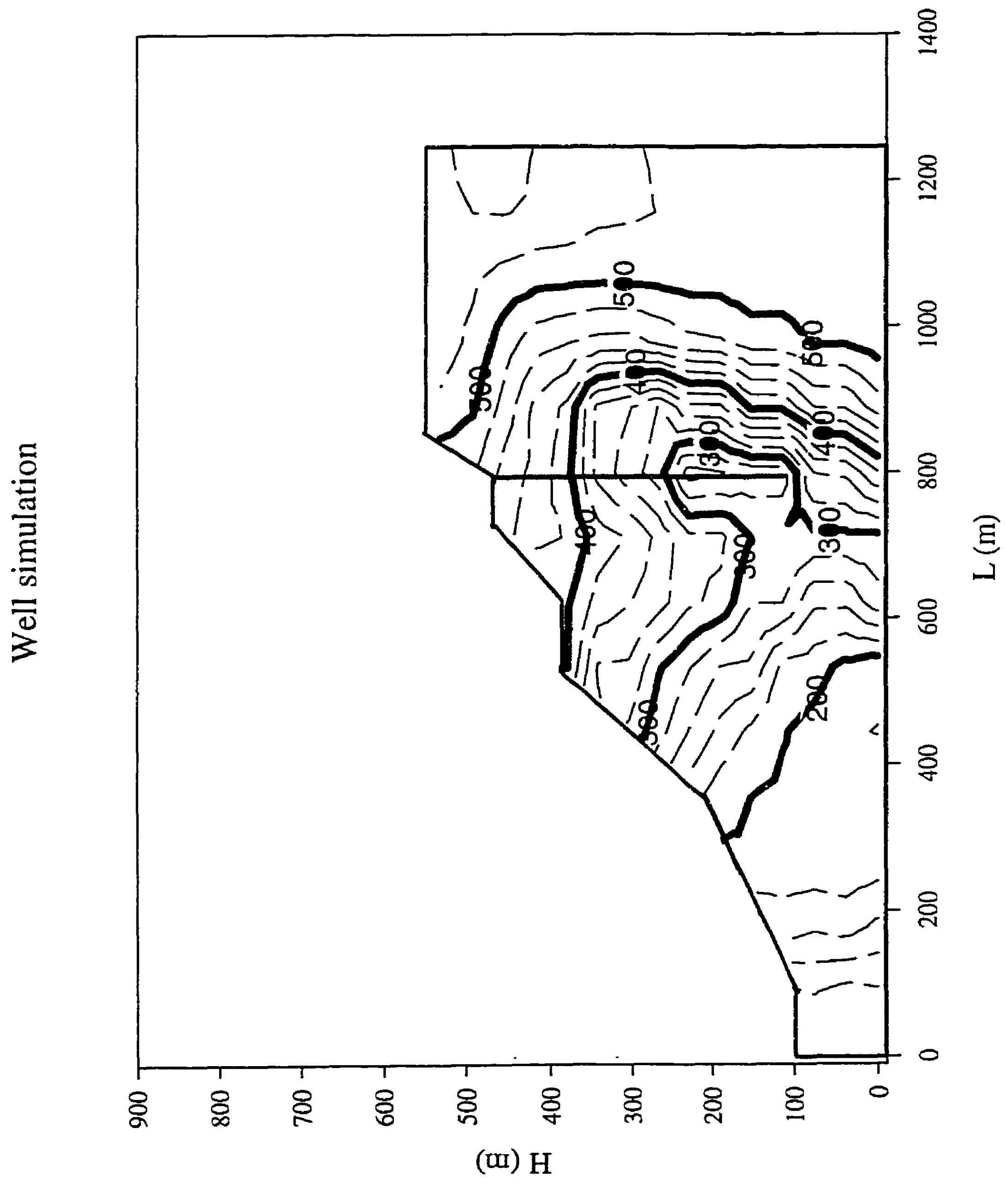


Figure 6.8 Total head - 5000N Section - Well simulation

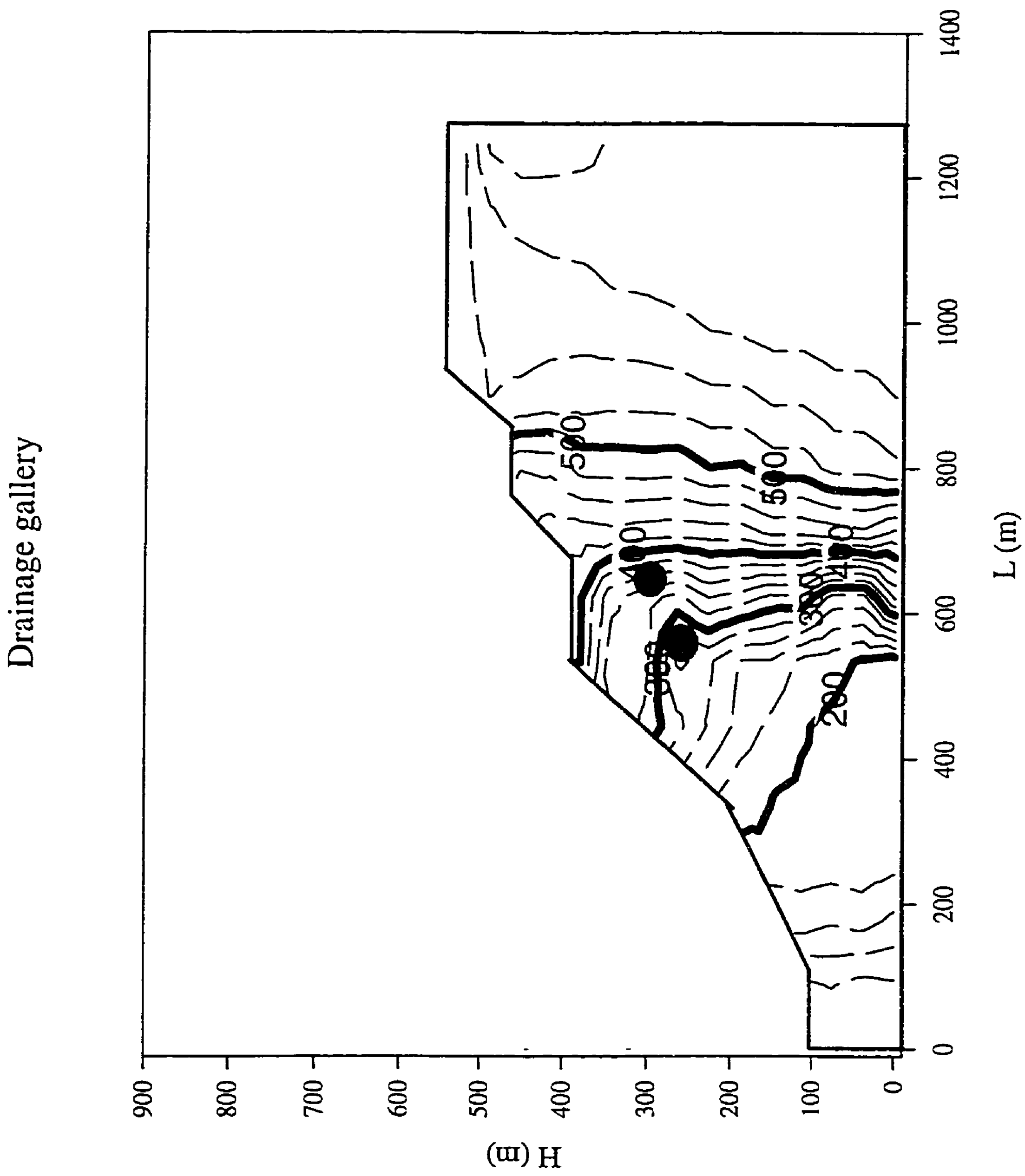


Figure 6.9 Total head - 5000N Section - Drainage gallery simulation

Chapter 7

Conclusions and recommendations for further research

Open pit slopes have become of more and more interest in geotechnical engineering for safe and economic surface mining operations. Stability of these slopes could be influenced by different geological, hydrogeological and structural factors. Groundwater regime in high slopes could badly damage their stability. Rock mass properties changes in contact with water and structural behavior of any slope changes when groundwater outflows appear at the face.

These were just a few reasons why the groundwater flow in fractured open pit slopes should be determine. Flow in fractured rocks differs from flow in porous media and, in some cases, traditional flow equation could not describe the real state of media. Double-porosity model for flow in fractured rocks could reflect, with a higher grade a confidence, the groundwater regime.

As a first step in application of the double-porosity flow to large scale studies in mining industry, the FlowD finite element code was developed. It is the basis for further applicability of this model to mining related problems. It performs steady-state flow analysis, and calculates total pressure head, gradients and velocities in the specific section of a slope.

FlowD code was initially verified on soil examples where no double-porosity models is applied to this type of analysis. Later, a study case was conducted on a real mining slope using double-porosity model. The results obtained validate this model showing consistency with actual flow state in that particular slope, as determined by monitoring and various other analytical methods used by the mine consultant. In the same time, the introduction of fracture orientations and hydraulic properties could model better the flow regime in the studied slope.

FlowD code performs steady-state seepage analysis in blocky rock slopes in which families of discontinuities could be sampled and their properties determined. It models Dirichlet and Neumann boundary conditions. Drains, wells and drainage galleries could be modeled.

FlowD computer code could be integrated in a stress-deformation analysis. It can be improved by adding the determination of the phreatic surface and in this way to model double-porosity unsteady-state flow in blocky rocks. For an easier way to use it, a pre and post-processors could be added, which can give the user a better filling and ease in analyzing large scale problems.

Chapter 8

References

1. Amadei, B. et al., 1995,'Effect of turbulence on fracture flow and advective transport of solutes', Technical Note, Int. J. of Rock Mech. & Min. Sci. & Geotechnical Abstracts, Vol. 32, No 4, p. 342-356.

2. Bai M. et al.,'Fluid flow and heat flow in deformable fractured porous media',1994, Int. J. Engng Sci. Vol. 32,No. 10,pg 1615-1633.

3. Barenblatt, G.I. et al.,' Basic concept in the theory of seepage of homogenous liquids in fissured strata',1960, Journal of Applied Mathematics and Mechanics, Vol. 24, 1286-1303.

4. Barenblatt, G.I. et al.,'Theory of fluid flow through natural rocks',1990, Kluwer Academic Publishers.

5. Bear, J. et al.,'Groundwater fow and pollution in fractures rocks aquifers',1987, Developments in Hydraulic Engineering - No. 4, Elsvier.

6. Bear, J. et al., 1972,'On the flow of two imiscibile fluids in fractured porous media', in Fundamentals of Transport Phenomenon in Pouous Media, Elsevier.

7. Bear, J. et al., 1993, 'Flow and Contaminant Transport in Fractured Rock', Academic Press Inc.

8. Bear, J.,'Modelling flow and contaminant transport in fractured rocks',1993, Flow and Contaminant Transport in Fractured Rocks, Ed. Bear, J. et al., Pb. Academic Press.

9. Bear, J., 1979,' Hydraulics of Groundwater', Pb. Mc Graw - Hill.

10. Bieniawski, Z.T.,1989, 'Engineering Rock Mass Classifications', Pb. John Wiley & Sons.

11. Boca, R.G et al.,'Modelling fluid flow in fractured-porous rock masses by finite element techniques',1984, International Journal for Numerical Methods in Fluids, Vol. 4, pg. 337-348.

12. Brady,B.H.G. et al.,1993,'Rock mechanics foe underground mining',Chapman & Hall.

13. Chan, T. et al.,'Finite-element simulation of groundwater flow and heat and radionuclide transport in plutonic rock mass',1987, 6th International Congress in Rock Mechanics, Proceedings, Ed. Herget, G. et al., Pb. Balkena.

14. Chisholm, J.S.R.,'Vectors in three-dimensional space',1978, Cambridge University Press.

15. de Marsily, G.,'Qualitative hydrogeology for engineers', 1986, Academic Press.

16. Dershowitz, W.S. et al.,'Three dimensional flow modeling in jointed rock masses',1987, 6th International Congress in Rock Mechanics, Proceedings, Ed. Herget, G. et al., Pb. Balkena.

17. Deskowitz, W.S. et al.,' Fracture modelling with JINX pakage',1987, 28th US Symposium on Rock Mechanics, Proceedings, Tucson, June 29th - July 1st, 1987.

18. Fan, Y. et al.,'Generation of flow netsusing f.e.m. nodal potential and bilinear shape functions',1992,International Journal for Numerical and Analytical Methods in Geomechanics.

19. Feuga, B, 'Modelling of flow through fractured rocks. Geostatistical generation of fractured networks. Stress - flow relationship in fractures.', 1987, Groundwater Flow and Quality Modelling, Ed. Custodio, E. et al., NATO/ ASI Series.

20. Freeze, A. et al.,'Groundwater', 1979, Pb. Prentice - Hall Inc.

21. Goodmann,'Methods in geological engineering in discontinuous rocks',1976, West Publishing Company.

22. 'Groundwater Models, Scientific and Regulatory Applications',1990, National Research Concil, National Academic Press.

23. Hoek, E. et al.,'Rock slope engineering',1981,The Institution of Mining and Metallurgy, London.

24. Huyakon, P.S. et al., ' Finite Element Technique for Modelling Groundwater Flow in Fractured Aquifers',1983, Water Resources Research, Vol. 19, No 4, p. 1019-1035

25. Istok, J.,' Groundwater modeling by the Finite Element Method',1989, Water Resources Monograph, No. 13, American Gephysical Union

26. Kavács & Associates,1981,' Subteran Hydrogeology' , Water Resources Publications.

27. Khaleel, R.,'Scale dependence of continuum models for fractured basalts',1987,Water Resources Research, Vol.23,N0.8,pg 1847-1855.

28. Lee, C.-H. et al., 'Fluid Flow in Discontinous Rocks',1993, Pb. Chapman & Hall.

29. Long,J.C.S. et al.,'Porous media equivalents for networks of fractures',1982, Water Resources Research, Vol. 18, No.3,pg 645-658.

30. Lopez,D. et al.,'Fluid flowin falt zones: Analysis of the interplay of convective circulation and topographically driven groundwater flow',1995, Water Resources Researche, Vol.31,No. 6,pg 1489-1503.

31. Louis, C. et al.,'Interaction between water flow phenomena and the mechanical gehaviour of soil or rock masses',1975,"Finite elements in geomechanics", Ed. Gudehms G.

32. Louis, C.,'Introduction a l'hydraulic des roche', 1974, Bulletin du Bureau de Recherches Geologiques et Miners, Section III, No. 4.

33. MacQuairrie, K.T.B. et al.,'On the interpretation of drains into three-dimentional variably saturated grounwater flow models', 1996, Water Resources Researche, Vol. 32,No.2, pg 477-482.

34. Matheson, C.H., Engineering geology',1981, Charles E. Merrill Publishing Company.

35. Mining Research Laboratoy,'Pit slope manual', Chapter 4, Groundwater, 1977,CANMET.

36. Mining Research Laboratoy,'Pit slope manual', Computer manual for seepage analysis, supplement 4-1, 1977,CANMET.

37. Novak, P., 'Developments in Hydraulic Engineering - 4', 1987, Pb. Elsvier Applied Science.

38. Parkin, G.W. et al.,'Analytical solution for one-dimensional drainage: Water stored in a fixed depth', 1995, Water Resources Research, Vol. 31, No. 5, pg. 1267-1271.

39. Pinder, G.P. et al., 'Simulation of flow and transport in fractured porous meddia', 1993, Flow and Contaminant Transport in Fractured Rocks, Ed. Bear, J. et al., Pb. Academic Press.

40. Pyrak-Nolte,L.J. et al.,'Hydraulic and mechanical properties of natural fractures in low permeability rock' 1987, 6th International Congress in Rock Mechanics, Proceedings, Ed. Horget G. et al., International Society of Rock Mechanics.

41. Rushton,K.P., 'Groundwater models',1987,in Developments in Hydraulic engineering-4',1987, Ed. Novak, Elsevier.

42. Ruton,J et al.,'Multiple seepage faces on layerd slopes and their inplication for slope stability analysis',1985, Canadian Geotechnical Journal, Vol. 22,pg. 347-356.

43. Sahimi, M.,'Flow and transport in porous media and fractured rocks',1995, Pb. Verhagesellschaft.

44. Sharp, J.C. et al.,'Fundamental considerations on the hydraulic characteristics of joints in rock',1972, Symposium Percolation Throgh Fissured Rock, Proceedings, Ed. Gessllschaft D. and W. Wittke.

45. Stock, Q.,'Groundwater Mechanics',1989,Prentice Hall Inc.

46. Şen,Z.,'Applied hydrogeology for scientists and engineers',1995,Lowis Publisher.

47.Therrien, R., 1994,'Hydrogogeology', Course notes, McGill University.

48. Thiel, K, 'Rock Mechanics in Hydroengineering', 1989, Developments in Geotechnical Engineering, Vol. 51.

49. Walton, W.,' Principles of grundwater engineering', 1991, Lewis Publshers.

50. Wang, H.F.,'Introduction to groundwater modelling, Finite difference and finite elenent methods',1982.

51. Wei,Z.Q. et al.,'Permeability prediction for jointedrock masses',1995, Int. J. Rock Mech. Min. & Geomech. Abstr., Vol 32,No. 3, pg 251-261.

52. Witherspoon,P.A. et al.,'Some recent developments in understanding the hydrology of fractured rocks',1987, 28th US Symposium on Rock Mechanics, Proceedings, Tucson, June 29th - July 1st, 1987.

53. Wittke,W.,'Rock mechanics, Theory and Applications',1990, Springer-Verlag Berlin Heidelberg.

54. Bouchamp,Y.,' Blake Lake Operations', Presentation, November 1994.

55. 'Report to LAB CHRISOTILE INC. on a Riview of slope designs for the B Pit East Wall', 1990, Golder Asociates Ltd.

56. 'Interim raport to Lake Asbestos of Quebec Ltd. on A stability review of B Pit East Wall',1978, Golder Asociates Ltd.

57. 'Draft report to Lake Asbestos of Quebec Ltd. on the 178 Stability Pogram for the B Pit East Wall', 1979, Golder Asociates Ltd.

Appendix A: User's manual

The present version of the FlowD code runs on IBM compatible computer machine equipped with a mathematical processor and a hard disk (minimum 100MB free memory). This first version of the code does not have a mesh generator. The mesh can be generated with any 2-dimentional mesh generator for 4-nodes isoparametric elements. The code accepts a maximum of 1000 nodes.

To run the program, the user has to create two separate files: one reflecting the nodal coordinates and the connectivity of the mesh (*.COR), and the other describing specified where material properties and the boundary conditions are specified (*.HED). The file name has to be the same for *.COR and *.HED. The program automatically search for the same name for both files. Also the program will automatically create the output file with the same name as the *.COR and *.HED. For example: create file PROGRAM.COR, with boundary conditions in file PROGRAM.HED. Output file will be PROGRAM.OUT.

A. Input file (*.COR and *.HED)

A.1 Data file *.COR (nodes, coordinates and elements)

1. First card:

Title of the problem to be solved = no more than 80 characters, including spaces

2. Number of nodes in the mesh,

NNODES= total number of nodes in the mesh (maximum 1000)

3. Nodal coordinates cards*:

NNODE= node number

CONT= 0

CONT= 0

X(NNODE)= X coordinate of the node NNODE (in meters)

Z(NNODE)= Z coordinate of the node NNODE (in meters)

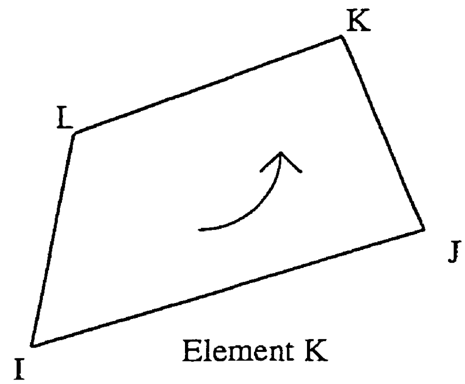
*Card no. 3 should repeat for as many nodes are in the mesh.

4. Elements control card:

NEL= no of elements in the mesh

NMAT= total number of materials

4-node isoparametric element



Element K

5. Element connectivity*,

IELEM= element identification number

I(IELEM)= I node number

J(IELEM)= J node number

K(IELEM)= K node number

L(IELEM)= L node number

IMAT(IELEM)= material number for the element IELEM

*Card no. 5 should repeat for as many elements are in the mesh.

<u>**A.2 Data file *.HED (material properties and boundary conditions)**</u>

1. Control card:

NMAT= number of materials (maximum 10)

NAL= number of analysis

1= soil version

2= rock version (double porosity model)

2. Material properties card:

Soil Version: (one card for each material, see figures in Chapter 5)

IMAT= material identification number

PERMX= permeability in X direction (in m/s)

PERMZ= permeability in Z direction (in m/s)

ALFA= angle between the principal axes of permeability and the global system of coordinates (in degrees)

Rock Version: (one card for each material, see figures Chapter 5)

a. Material identification card

IMAT= material identification number

NSET= number of sets of fractures in material (maximum 10) (in m/s)

b. Fractures Hydraulic properties (one card for each set, maximum 10)

PERM(X)= permeability along the sets of fractures(in m/s)

PERM(Z)= permeability transversal to the set of fracture(in m/s)

PLE= arbritary dimension of the rock block large enough to statistically sample the fractures (m)

EI= average aperture of the fractures in the set (in meters)

XF= average number of fractures sampled in the set

BETA= angle between the direction of fractures set , taken from sterenets and projected on the studied section, measured clockwise from the Ox ax of the global system of coordinates (in degrees)

3. Boundary conditions:

a. Specified total head at the nodes (Dirichlet Boundary Condition)

a.1 Number of nodes with specified total head; initially all the nodes are free, at least one node has to have a specified total head that has to be different from 0; as an example; if a slope is saturated, the nodes at the slope face have total heads equal to the Z coordinate of each particular node ($p/\gamma = 0$)

NHEAD= number of nodes with specified head

a.2 Specified head (for each node one card)

IHEAD= node identification where the total head is specified

HEAD= total head in node IHEAD (m)

Total head = Z(IHEAD)+pressure head

b. Point source or point sink (Neumann Boundary condition)

b.1 Number of nodes with specified flow rate

NQNODE= number of nodes with specified flow rate

b.2 Specified flow rate

IQNODE= node number point sink or source

QNODE= positive or negative flux magnitude at the node (in m^3/s)

- positive when flow exits the domain
- negative when flow recharges the domain

c. Distributed flow rate over the side of an element

c.1 Number of elements that have edges with specified distributed head

NQED= number of elements with edges that have specified head

c.2 Specified distributed flow over the edge

NQELEM= element number

MNODEq= edge node number (as example: node i)

NNODEq= edge node number (as an example: node j)

QDISTR= distributed flow rate over the edge (in m^3/s/meter)

- positive = enters into the element
- negative = exits from the element

d. Drain, well, and drainage gallery simulation

Drain: - drain position (node number)

- total head along the drain = elevation head (in meters)

Well: - well position (node number)

- total head at the botton of the well = elevation head (in meters)
- distributed or point source at the bottom of the well

Drainage gallery: - location of the gallery (node number)

- total head = elevation head (in meters)
- point source

B. Output file (*.OUT)

1. Header
2. Node number and the total head at that node (one line for each node)
3. Element number and the directional gradients, total gradient, directional flow velocity, total flow velocity vector, and its orientation angle at the center of the element (one card for each element).

Appendix B : FlowD computer code listing

```
C*************************************************************************
C*************************************************************************
C                                                                        *
C                                                                        *
C                        PROGRAM FlowD                                   *
C                                                                        *
C                                                                        *
C                                                                        *
C                   developed by Doina Priscu  1997                      *
C                                                                        *
C               Dept. of Mining and Metallurgical Engineering            *
C                          McGill University                             *
C                                                                        *
C                                                                        *
C                                                                        *
C*************************************************************************
C*************************************************************************
C
C
C   THIS PROGRAM PERFORMS FLOW ANALYSIS OF 2-DIMENSIONAL PROBLEMS USING
C   4-NODE ISOPARAMETRIC ELEMENTS & STEADY STATE CONDITIONS OF FLOW
C   IN HOMOGENEOUS, ISOTROPIC OR ANISOTROPIC DOMAINS,USING SINGLE
C   OR DOUBLE POROSITY MODEL FOR FLOW THROUGH FRACTURED ROCKS.
C
C
C   NOTATIONS USED IN THIS PROGRAM ARE LISTED BELOW:
C
C   X(1000),Y(1000)= COORDINATES OF THE NODES
C   XCG(1000),YCG(1000) = COORDINATES OF THE ELEMENT CENTER
C   ID(1000)= ID ARRAY OF THE DOF FOR ALL THE NODES
C   JD(1000)= JD ARRAY OF THE DOF FOR ALL THE NODES
C   NNODES=NUMBER OF NODES
c  NEQ= NUMBER OF EQUATION
C   HD= HYDRAULIC HEAD VECTOR
C   TH= THICKNESS OF THE DOMAIN (=UNITY)
C   NMAT= NUMBER OF MATERIALS
C   IMAT= IDENTIFICATION OF MATERIAL
C
C   GRAD(1000)= GRADIENT MATRIX
C   VELO(1000)= VELOCITY MATRIX
C   FL(1000)= LOAD VECTOR
C   PERMX(10,1)= PERMEABILITY IN THE X DIRECTION
C   PERMY(10,1)= PERMEABILITY IN THE Y DIRECTION
C   ALFA(10,1)= ANGLE BETWEEN THE SYSTEM OF COORDINATES AND THE PRINCIPAL
C       DIRECTIONS OF PERMEABILITY  (SOIL VERSION )
C   PERX(10,10)=MAJOR PERMEABILITY OF A SET OF FRACTURE
C   PERY(10,10)=MINOR PERMEABILITY OF A SET OF FRACTURE
C   PLE(10,10)= ARBITRARY DIMENSION OF THE SIDE OF THE BLOCK OF FRACTURED
C           MEDIUM, LARGE ENOUGH TO STATISTICALLY SAMPLE THE FAMILIES
C           OF FRACTURES
C   XF(10,10)= AVERAGE NUMBER OF FRACTURES IN THE SET
C   EI(10,10)= AVERAGE APERTURE OF THE FRACTURES IN THE SET
C   BETA(10,10)= ANGLE BETWEEN FRACTURES' PREDOMINANT PERMEABILITY &
C           GOBLAL SYSTEM OF COORDINATES
```

```
C   NAL= NUMBER OF ANALYSIS = 1 SOIL MEDIUM
C                  = 2 FRACTURED ROCK MEDIUM
C
C*******************************************************************
C*******************************************************************
C
    IMPLICIT REAL*4(A-H,O-Z)
    DIMENSION X(1000),Y(1000),ICO(4,1000),CON(2,2),LJ(4),HD(1000)
    DIMENSION BF(2,4),TH(1),HEADF(40000),IMAT(1000),ID(1000)
    DIMENSION CF(4,4),CFG(40000),GRAD(1000),VELO(1000),JD(1000)
    DIMENSION FL(40000),VELOS(1),DIRVELO(1),FLE(4),PERMX(10,1)
    DIMENSION PLE(10,10),EI(10,10),NSET(10),PERMY(10,1),ALFA(10,1)
    DIMENSION PERX(10,10),PERY(10,10),MAT(10),XF(10,10),BETA(10,10)
    DIMENSION XCG(1000),YCG(1000)
C
    CHARACTER TITLE*72
    CHARACTER*20 FILENAME
    CHARACTER*24 CORFILE
    CHARACTER*24 HEADFILE
    CHARACTER*24 OUTFILE
C
    WRITE(*,*) 'INPUT FILE NAME WITHOUT EXTENSION (MAX 8 CHARACTERS):'
    READ(*,*) FILENAME
C
    DO 1 ITA=LEN(FILENAME),1,-1
      IF(FILENAME(ITA:ITA).NE.' ') GOTO 2
  1 CONTINUE
  2 CORFILE=FILENAME(1:ITA)//'.COR'
    HEADFILE=FILENAME(1:ITA)//'.HED'
    OUTFILE=FILENAME(1:ITA)//'.OUT'
C
    WRITE(*,*) 'OPEN FILES'
    OPEN (UNIT=20, FILE=CORFILE, STATUS='UNKNOWN',
   $     ACCESS='SEQUENTIAL', FORM='FORMATTED')
C
    OPEN (UNIT=30, FILE=HEADFILE, STATUS='UNKNOWN',
   $     ACCESS='SEQUENTIAL', FORM='FORMATTED')
C
    OPEN (UNIT=50, FILE=OUTFILE, STATUS='UNKNOWN',
   $     ACCESS='SEQUENTIAL', FORM='FORMATTED')
C
    WRITE(*,*) 'READING THE DATA FILES *.COR & *.HED'
    READ(20,*) TITLE
    WRITE(50,*) TITLE
C
    READ(20,*) NNODES
C
    WRITE(*,*) 'READ THE NODES'
    DO 10 I=1,NNODES
      READ(20,*) INODE,DUMY1,DUMY2,X(I),Y(I)
 10 CONTINUE
C
    WRITE(*,*) 'READ THE ELEMENTS'
    READ(20,*) NEL,NMAT
    DO 21 K=1,NEL
```

```
      READ(20,*) IELEM,(ICO(L,K),L=1,4),IMAT(K),DUMY3,DUMY4,DUMY5,
     $DUMY6,DUMY8
C
C-------- CALCULATE COORDINATES OF THE ELEMENT CENTER---------------
C
      XCG(K)=(X(ICO(1,K))+X(ICO(2,K))+X(ICO(3,K))+X(ICO(4,K)))/4
      YCG(K)=(Y(ICO(1,K))+Y(ICO(2,K))+Y(ICO(3,K))+Y(ICO(4,K)))/4
   21 CONTINUE
C
      DO 22 I=1,10
   22 NSET(I)=0
C
      CALL PRESET(PERX,10,10)
      CALL PRESET(PERY,10,10)
      CALL PRESET(PLE,10,10)
      CALL PRESET(EI,10,10)
      CALL PRESET(BETA,10,10)
      CALL PRESET(XF,10,10)
C
      CALL PRESET(PERMX,10,1)
      CALL PRESET(PERMY,10,1)
      CALL PRESET(ALFA,10,1)
C
      WRITE(*,*) 'READ MATERIAL PROPERTIES'
      READ(30,*) NMAT, NAL
      DO 40 LL=1,NMAT
C
        IF (NAL.EQ.1) THEN
          READ(30,*) IMA, PERMX(LL,1), PERMY(LL,1), ALFA(LL,1)
        ELSE
          READ(30,*) MAT(LL),NSET(MAT(LL))
          NSETM=NSET(MAT(LL))
          DO 25 M25=1,NSETM
            READ(30,*) PERX(LL,M25),PERY(LL,M25),PLE(LL,M25),
     $        EI(LL,M25),XF(LL,M25),BETA(LL,M25)
   25     CONTINUE
        ENDIF
   40 CONTINUE
C
      NNODEL=4
      NICO=4
      NVR=1
      NLJ=NVR*NICO
      NMAX=NVR*NNODES
      NVEL=4
C
C-------------COMPUTE NUMBER OF NODES WITH SPECIFIED HEAD-----------
C              & BUILD ID ARRAY
C
      CALL BOUND(ID,HD)
C
      CALL IDJD(ID,NMAX,JD,NEQ)
C
      DO 60 I=1,NEL
        CALL ELCON(I,ICO,NICO,NEL,NNODEL,LJ,NLJ,JD,NMAX)
```

```
      CALL BANDWH(LJ,NNODEL,NVR,NLJ,LBAND)
   60 CONTINUE
C
      WRITE(*,*)'NUMBER OF NODES,    NNODES = ',NNODES
      WRITE(*,*)'NUMBER OF ELEMENTS,    NEL = ',NEL
      WRITE(*,*)'NUMBER OF EQUATIONS,   NEQ = ',NEQ
      WRITE(*,*)'BANDWIDTH,           LBAND = ',LBAND
C
      LSIZE = NEQ * LBAND
C
      CALL PSET(CFG,LSIZE)
C
C
C-----------------INITIALIZE AND COMPUTE LOAD VECTOR---------------
C               FROM BOUNDARY CONDITIONS
C
      WRITE(*,*) 'COMPUTE THE LOAD VECTOR'
      CALL PSET(FL,1000)
C
      CALL LOAD(X,Y,JD,FL)
C
C----------------DO LOOP TO ASSEMBLY THE GLOBAL CHARACTERISTICS
C                     MATRIX AND LOAD VECTOR
C
      WRITE(*,*) 'ASSEMBLY THE LOAD VECTOR'
      TH=1.0
C
      DO 200 INEL=1,NEL
C
       CALL IPLNCF(INEL,IMAT,X,Y,ICO,BF,TH,CON,CF,NAL,NSET,PLE,BETA,
     $EI,XF,PERX,PERY,PERMX,PERMY,ALFA)
C
C------------------MODIFY LOAD VECTOR - KNOWN HEAD -----------------
C              -NEUMANN BOUNDARY CONDITIONS-
C
      CALL PSET(FLE,4)
        DO 13 J=1,NNODEL
          KI=ICO(J,INEL)
          IF (ID(KI).EQ.0) THEN
          DO 14 K=1,NNODEL
            KJ=ICO(K,INEL)
            IF (ID(KJ).NE.0) THEN
              FLE(J)=FLE(J)-CF(J,K)*HD(KJ)
            END IF
   14       CONTINUE
          END IF
   13     CONTINUE
C
        CALL ELCON(INEL,ICO,NICO,NEL,NNODEL,LJ,NLJ,JD,NMAX)
C
        CALL SETUP2(CFG,FL,CF,FLE,NLJ,LJ,LBAND)
C
  200 CONTINUE
C
C---------------SOLVE SYSTEM WITH CHOLESKY TECHNIQUE---------------
```

```
C
   WRITE(*,*) 'SOLVE SYSTEM OF EQUATIONS'
   LLT=1
   CALL CHLSKI(CFG,FL,NEQ,LBAND,LLT,WDETJ)
C
C------------------PRINT HEAD AT NODES----------------------------
C
   DO 300 IB=1,NNODES
         IF(ID(IB).EQ.1) THEN
        HEADF(IB)=HD(IB)
      ELSE
        K=JD(IB)
        HEADF(IB)=FL(K)
      END IF
C
C
      WRITE(50,2030) IB,HEADF(IB)
 2030 FORMAT(/,'AT NODE',I4,11X,'HEAD=',F10.4)
  300 CONTINUE
C
C------------CALCULATE GRADIENTS & SEEPAGE VELOCITIES--------------
C
   WRITE(*,*) 'CALCULATE GRADIENTS & SEEPAGE VELOCITIES'
   WRITE(50,3000)
 3000 FORMAT(/,1X,'ELEMENT',3X,'XCG',5X,'YCG',7X,'GRADX',7X,'GRADY',
  $5X,'GRADIENT',6X,'VELOX',7X,'VELOY',4X,'VELOCITY',5X,'ANGLE')
C
C----------------------PREPARE OUTPUT---------------------------
C
   WRITE(*,*) 'PREPARE OUTPUT'
   DO 400 K=1,NEL
      CALL GRADIE(K,IMAT,PERMX,PERMY,ALFA,ICO,PERX,PERY,
  $PLE,EI,BETA,NSET,XF,CON,GRAD,VELO,VELOS,GRADOS,DIRVELO,HEADF,NAL)
   WRITE(50,4000)K,XCG(K),YCG(K),GRAD(1),GRAD(2),GRADOS,VELO(1),
  $VELO(2),VELOS,DIRVELO
 4000 FORMAT(/,2X,I4,1X,F7.2,1X,F7.2,6(3X,F9.5),3X,F7.2)
  400 CONTINUE
C
   CLOSE (UNIT=20, STATUS='KEEP')
   CLOSE (UNIT=30, STATUS='KEEP')
   CLOSE (UNIT=50, STATUS='KEEP')
C
   STOP
   END
C
C
C--------------------- SUBROUTINES --------------------------------
C
C
   SUBROUTINE BOUND (ID,HD)
   IMPLICIT REAL*4(A-H,O-Z)
C*******************************************************************
C   ROUTINE TO COMPUTE THE SPECIFIED VALUES OF THE FIELD VARIABLES
C   (HYDRAULIC HEAD)
```

```
C
C   OUTPUT: SPECIFIED VALUES OF THE FIELD VARIABLES (HEAD/NODES)
C
C   DEFINITION OF VARIAABLES:
C
C   NSH= NUMBER OF NODES WITH SPECIFIED HEAD
C   INODE= NODE NUMBER
C
C******************************************************************
      DIMENSION ID(1000),HD(1000)
C
      READ(30,*) NSH
      DO 100 IK=1,NSH
        READ(30,*) INODE,HD(INODE)
        ID(INODE)=1
  100 CONTINUE
      RETURN
      END
C
      SUBROUTINE PRESET(A,M,N)
C********************************************************************
C   A ROUTINE TO ZERO A TWO DIMENSIONAL ARRAY
C********************************************************************
      IMPLICIT REAL*4 (A-H,O-Z)
      DIMENSION A(M,N)
C
      DO 1 I=1,M
      DO 2 J=1,N
    2 A(I,J)=0.D0
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE PSET(A,M)
C********************************************************************
C   A ROUTINE TO ZERO A ONE DIMENSIONAL ARRAY
C********************************************************************
      IMPLICIT REAL*4 (A-H,O-Z)
      DIMENSION A(M)
C
      DO 1 I=1,M
    1 A(I)=0.D0
      RETURN
      END
C
      SUBROUTINE IDJD(ID,NMAX,JD,NEQ)
C**********************************************************************
C   A ROUTINE TO COMPUTE THE GLOBAL JD (OR JX) FROM THE ID ARRAY
C   INPUT
C   ID(NMAX) = I.D. ARRAY OF DOF FOR ALL NODES
C   NMAX     = MAX NO. OF DOF = NODE*NVR
C   OUTPUT
C   JD(NMAX) = J.D. ARRAY OF DOR FOR ALL NODES
C   NEQ= NO. OF THE GLOBAL EQNS OF EQUILIBRIUM
C**********************************************************************
```

```
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION ID(1000),JD(1000)
C
      NEQ=0
      DO 1 I=1,NMAX
        JD(I)=0
        IF(ID(I).EQ.1)GO TO 1
        NEQ=NEQ+1
        JD(I)=NEQ
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE ELCON(INEL,ICO,NICO,NEL,NNODEL,LJ,NLJ,JX,NMAX)
C*********************************************************************
C
C   A ROUTINE TO BUILD ELEMENT CONNECTIVITY ARRAY LJ. THE ROUTINE IS
C   CALLED IN A LOOP OVER ALL ELEMENTS.
C  INPUT:
C  IE            =ELEMENT NO. IN THE DO LOOP
C  ICO(NICO,NEL)  =GLOBAL CONNECTIVITY MATRIX
C  NICO          =NO. OF ROWS OF CONNECTIVITY MATRIX
C  NEL           =TOTAL NO. OF ELEMENTS
C  NVR           =NO. OF VARIABLES (DOF) PER NODE
C  NNODEL        =NO. OF NODES PER ELEMENT
C  JX(NMAX)      =GLOBAL ID ARRAY
C  NMAX          =MAX NO. OF DOF= NODES*NVR
C  OUTPUT:
C  LJ(NLJ)       =ELEMENT CONNECTIVITY ARRAY
C
C*********************************************************************
      IMPLICIT REAL*4 (A-H,O-Z)
      DIMENSION LJ(NLJ),JX(NMAX),ICO(NICO,NEL)
C
      DO 2 J=1,NNODEL
        J2=(ICO(J,INEL))
        lj(j)=jx(j2)
    2 CONTINUE
      RETURN
      END
C
      SUBROUTINE LOAD (X,Y,JD,FLUX)
      IMPLICIT REAL*4 (A-H,O-Z)
C*********************************************************************
C
C   A ROUTINE TO COMPUTE LOAD VECTOR OVER THE BOUNDARY OF THE
C    DOMAIN--NEUMANN BAUNDARY CONDITIONS.
C
C   INPUT:
C   FLUX(INODE)= SPECIFIED FLOW RATE IN NODES
C   QFLUX= VALUES OF THE FLUX OVER THE BOUNDARY (DISTRIBUTED LOAD)
C   OUTPUT:
C     NODE AND FLUX IN THE NODE
C     LOAD VECTOR
C
```

```
C*********************************************************************
   DIMENSION FLUX(1000),JD(1000),QFLUX(100),QNODE(100)
   DIMENSION X(1),Y(1)
C
C--------------COMPUTE NODE WITH SPECIFIED FLOW RATE --------------
C           ---NEUMANN BOUNDARY CONDITION---
C
C
   READ(30,*) NQNODE
C
   DO 175 N=1,NQNODE
     READ(30,*) INODE,QNODE(INODE)
     K=JD(INODE)
C
     FLUX(K)=FLUX(K)+QNODE(INODE)
C
 175 CONTINUE
C
C-------------COMPUTE UNIT FLUX OVER THE SIDE OF AN ELEMENT---------
C           ---NEUMANN BOUNDARY CONDITION---
C
   READ(30,*) NQSIDE
C
   DO 185 N=1,NQSIDE
     READ(30,*) IELE,NODEI,NODEJ,QFLUX(IELE)
     LIJ=((X(NODEI)-X(NODEJ))**2+(Y(NODEI)-Y(NODEJ))**2)**(1/2)
     QSIDEI=(QFLUX(IELE)*LIJ)/2
     QSIDEJ=(QFLUX(IELE)*LIJ)/2
     KI=JD(NODEI)
     KJ=JD(NODEJ)
     FLUX(KI)=FLUX(KI)+QSIDEI
     FLUX(KJ)=FLUX(KJ)+QSIDEJ
 185 CONTINUE
   RETURN
   END
C
   SUBROUTINE IPLNNN(S,T,SF,DSFS,DSFT)
C*********************************************************************
C   A ROUTINE TO COMPUTE THE SHAPE FUNCTIONS AND THEIR DERIVATES
C   AT A POINT (S,T) WITHIN THE 4-NODE ISOPARAMETRIC ELEMENT.
C   INPUT DATA:
C   S  = NONDIMENSIONAL S-COORDINATE OF THE POINT
C   T  = NONDIMENSIONAL T-COORDINARE OF THE POINT
C   IN  = INDEX FOR CALCULATE/SKIP THE SHAPE FUNCTION SF(4)
C   OUTPUT
C   SF  = SHAPE FUNCTIONS (4)
C   DSFS= (DN/DS)
C   DSFT= (DN/DT)
C*********************************************************************
   IMPLICIT REAL*4 (A-H,O-Z)
   DIMENSION SF(4),DSFS(4),DSFT(4)
C
   SF(1)=1./4.*(1.+S)*(1.+T)
   SF(2)=1./4.*(1.-S)*(1.+T)
   SF(3)=1./4.*(1.-S)*(1.-T)
```

```
      SF(4)=1./4.*(1.+S)*(1.-T)
C
      DSFS(1)=1./4.*(1.+T)
      DSFS(2)=-1./4.*(1.+T)
      DSFS(3)=-1./4.*(1.-T)
      DSFS(4)=1./4.*(1.-T)
      DSFT(1)=1./4.*(1.+S)
      DSFT(2)=1./4.*(1.-S)
      DSFT(3)=-1./4.*(1.-S)
      DSFT(4)=-1./4.*(1.+S)
      RETURN
      END
C
      SUBROUTINE IPLNBJ (SF,DSFS,DSFT,X,Y,AJ,AI,WDETJ,BF,S,T)
      IMPLICIT REAL*4 (A-H,O-Z)
C*******************************************************************
C
C   A ROUTINE TO COMPUTE THE STRAIND-DESPLACEMENTS MATRIX BF(2,4) AT
C   A POINT (X,Y) WITHIN THE 4-NODE ISOPARAMETRIC ELEMENT
C
C   SF  = SHAPE FUNCTION
C   DSFS & DSFT DERIVATES OF THE SHAPE FUNCTIONS
C   AJ  = JACOBIAN MATRIX
C   WDETJ  = DETJERMINANT OF THE JACOBIAN MATRIX
C   AI = INVERSE OF JACOBIAN MATRIX
C*******************************************************************
      DIMENSION SF(4),DSFS(4),DSFT(4),X(4),Y(4),BF(2,4),AJ(2,2),AI(2,2)
      DIMENSION S(4),T(4)
C
      CALL PRESET(AI,2,2)
C
      CALL IPLNNN(S,T,SF,DSFS,DSFT)
      CALL PRESET(AJ,2,2)
C
      DO 1 K=1,4
        AJ(1,1)=AJ(1,1)+DSFS(K)*X(K)
        AJ(1,2)=AJ(1,2)+DSFS(K)*Y(K)
        AJ(2,1)=AJ(2,1)+DSFT(K)*X(K)
        AJ(2,2)=AJ(2,2)+DSFT(K)*Y(K)
    1 CONTINUE
C
      WDETJ=AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)
      CDUM1=AJ(1,1)/WDETJ
      AI(1,1)=AJ(2,2)/WDETJ
      AI(1,2)=-AJ(1,2)/WDETJ
      AI(2,1)=-AJ(2,1)/WDETJ
      AI(2,2)=CDUM1
C
      DO 5 J=1,4
        BF(1,J)=AI(1,1)*DSFS(J)+AI(1,2)*DSFT(J)
        BF(2,J)=AI(2,1)*DSFS(J)+AI(2,2)*DSFT(J)
    5 CONTINUE
      RETURN
      END
C
```

```
C
   SUBROUTINE IPLNCF(I,IMAT,X,Y,ICO,BF,TH,CON,CF,NAL,NSET,PLE,BETA,
  $EI,XF,PERX,PERY,PERMX,PERMY,ALFA)
   IMPLICIT REAL*4 (A-H,O-Z)
C******************************************************************
C  A ROUTINE TO COMPUTE THE ELEMENT CHARACTERISTICS MATRIX
C
C  INPUT:
C   XX,YY=ELEMENT NODAL X,Y COORDINATES (EACH 4)
C   X,Y= NODAL X,Y COORDINATES
C   TH= ELEMENT THICKNESS
C   CON(2,2)= ELEMENT CONDUCTIVITY MATRIX
C   BF(2,8)= B MATRIX
C
C  OUTPUT:
C   CF(4,4)= ELEMENT CHARACTERISTICS MATRIX
C
C*******************************************************************
   DIMENSION XX(4),YY(4),X(1000),Y(1000),CF(4,4),BF(2,4),SF(4)
   DIMENSION DSFS(4),DSFT(4),W(2),ICO(4,1000),CON(2,2),PLE(10,10)
   DIMENSION VSI(4),VTI(4),WE(4,4),AK(4,4),IMAT(1),NSET(10)
   DIMENSION ALFA(10,1),PERMX(10,1),PERMY(10,1),PERX(10,10)
   DIMENSION PERY(10,10)
   DIMENSION XF(10,10),BETA(10,10),EI(10,10)
C
   W(1)=1.0
   W(2)=W(1)
C
   CALL PRESET(CF,4,4)
C
C--------- COMPUTE HYDRAULIC CONDUCTIVITY MATRIX-------------------
C          FOR DIFFERENT TYPES OF ANALYSIS
C      NAL=1 SOIL VERSION
C      NAL=2 DOUBLE POROSITY VERSION
C
   IF (NAL.EQ.1) THEN
     CALL CARASO(I,IMAT,PERMX,PERMY,ALFA,CON)
   ELSE
C
     CALL DPOR(I,IMAT,PLE,EI,CON,BETA,NSET,XF,PERX,PERY)
   ENDIF
C
   DO 25 LL=1,4
     LNODE=ICO(LL,I)
     XX(LL)=X(LNODE)
     YY(LL)=Y(LNODE)
 25 CONTINUE
C
C------------------COORDINATES OF THE GAUSS INTEGRATION POINTS
C
   VSI(1)=-1/SQRT(3.0)
   VTI(1)=VSI(1)
   VSI(2)=VSI(1)
   VTI(2)=-VSI(1)
   VSI(3)=-VSI(1)
```

```
      VTI(3)=VSI(1)
      VSI(4)=-VSI(1)
      VTI(4)=-VSI(1)
C
C-------------------------MAIN DO LOOP--------------------------
C
      DO 110 GG=1,4
        SI=VSI(GG)
        TI=VTI(GG)
c
          CALL IPLNBJ(SF,DSFS,DSFT,XX,YY,AJ,AI,WDETJ,BF,SI,TI)
          TH=1.0
          CAST=WDETJ*TH
          CALL TRANS(CON,BF,AK,WE,2,4,CAST)
          DO 35 NL=1,4
            DO 15 ML=1,4
              CF(NL,ML)=CF(NL,ML)+AK(NL,ML)
 15         CONTINUE
 35       CONTINUE
 110  CONTINUE
      do 1111 j=1,4
        do 1121 m=1,4
1121  continue
1111  continue
      RETURN
      END
C
      SUBROUTINE GRADIE(I,IMAT,PERMX,PERMY,ALFA,ICO,PERX,PERY,
     $PLE,EI,BETA,NSET,XF,CON,GRAD,VELO,VELOS,GRADOS,DIRVELO,HEADF,NAL)
C
      IMPLICIT REAL*4 (A-H,O-Z)
C*****************************************************************
C
C   A ROUTINE TO COMPUTE GRADIENTS & VELOCITIES AT CENTRE OF THE
C   ELEMENT IN X & Y DIRECTION.
C
C   INPUT:
C     HD(INODE)=VALUES OF THE HYDRAULIC HEAD IN NODE INODE
C     BF(2,4)= MATRIX
C     SC & TC= LOCAL COORDONATES OF THE CENTER OF THE ELEMENT
C
C   OUTPUT:
C     GRADX & GRADY= GRADIENTS IN X & Y DIRECTION.
C
C*****************************************************************
      DIMENSION GRAD(2),VELO(2),ICO(4,1),HEADF(1000),BF(2,4),CON(2,2)
      DIMENSION PERMX(10,1),PERMY(10,1),ALFA(10,1),VELOS(1),GRADOS(1)
      DIMENSION DIRVELO(1),PLE(10,10),PERX(10,10),PERY(10,10)
      DIMENSION EI(10,10),XF(10,10),BETA(10,1)
C
C----------COMPUTE GRADIENTS AT THE CENTER OF THE ELEMENT----------
C
      CALL PSET(GRAD,2)
      SC=0.0
      TC=0.0
```

```
      CALL IPLNBJ(SF,DSFS,DSFT,X,Y,AJ,AI,WDETJ,BF,SC,TC)
      DO 55 K=1,2
        DO 65 L=1,4
          GRAD(K)=GRAD(K)+(BF(K,L)*HEADF(ICO(L,I)))
C
 65     CONTINUE
 55   CONTINUE
C
      IF (NAL.EQ.1) THEN
        CALL CARASO(I,IMAT,PERMX,PERMY,ALFA,CON)
      ELSE
        CALL DPOR(I,IMAT,PLE,EI,CON,BETA,NSET,XF,PERX,PERY)
      ENDIF
      CALL PSET(VELO,2)
C
      DO 70 L=1,2
        DO 75 K=1,2
          VELO(L)=VELO(L)-(CON(L,K)*GRAD(K))
 75     CONTINUE
 70   CONTINUE
C
      GRADOS=((GRAD(1))**2+(GRAD(2))**2)**(0.5)
      VELOS=((VELO(1))**2+(VELO(2))**2)**(0.5)
      DIRVELO=(ATAN2(VELO(2),VELO(1)))*180/3.141593
      RETURN
      END
C
      SUBROUTINE TRANS(S,T,R,W,M1,M2,CONST)
      IMPLICIT REAL*4 (A-H,O-Z)
C*******************************************************************
C   THIS ROUTINE MULTIPLIES 3 MATRICES IN THE FORM:
C   (R) = CONSTANT * (T)TRANSPOSE * (S) * (T)
C*******************************************************************
      DIMENSION S(M1,M1),T(M1,M2),W(M2,M1),R(M2,M2)
C
      CALL PRESET(R,M2,M2)
      CALL PRESET(W,M2,M1)
      DO 10 I=1,M2
      DO 10 J=1,M1
      DO 10 K=1,M1
 10   W(I,J)=W(I,J)+CONST*T(K,I)*S(K,J)
      DO 20 I=1,M2
      DO 20 J=1,M2
      DO 20 K=1,M1
 20   R(I,J)=R(I,J)+W(I,K)*T(K,J)
      RETURN
      END
C
      SUBROUTINE BANDWH(LJ,NNODEL,NVR,NLJ,LBAND)
      IMPLICIT REAL*4 (A-H,O-Z)
C*******************************************************************
C   THIS ROUTINE CALCULATES  HALF BAND WIDTH OF THE GLOBAL STIFFNESS
C   MATRIX. THE ROUTINE SHOULD BE CALLED IN A LOOP FOR ALL ELEMENTS.
C   INPUT
C   LJ(NLJ)= ELEMENT DOF OR CONNECTIVITY ARRAY
```

```
C   NNODEL= NO. OF NODES PER ELEMENT
C   NVR= NO. OF VARIABLES(DOF) PER NODE
C   NLJ=NVR*NNODEL
C   OUTPUT
C   LBAND= MATRIX HALF BANDWIDTH INCLUDING THE DIAGONAL
C*******************************************************************
      DIMENSION LJ(NLJ)
C
      MAX=0
      MIN=10000
      NV=NVR*NNODEL
      DO 10 J=1,NV
        IF(LJ(J).EQ.0) GO TO 10
        IF(LJ(J).GT.MAX)MAX=LJ(J)
        IF(LJ(J).LT.MIN)MIN=LJ(J)
 10   CONTINUE
      NB=MAX-MIN+1
      IF(NB.GT.LBAND) LBAND=NB
      RETURN
      END
C
C
      SUBROUTINE CHLSKI(A,B,NEQ,LBAND,LLT,WDETJ)
      IMPLICIT REAL*4 (A-H,O-Z)
C*******************************************************************
C   THIS ROUTINE SOLVES THE EQUATION : P= (K).(DELTA) , FOR THE DIS-
C   PLACEMENT VECTOR (DELTA), BY MEANS OF CHOLESKI DECOMPOSITION
C  INPUT
C  A     =GLOBAL MATRIX IN BANDED FORM
C  B     =GLOBAL LOAD VECTOR
C  NEQ   =NO. OF EQUATIONS
C  LBAND =HALF BANDWIDTH OF GLABAL STIFFNESS MATRIX
C  LT    =INDEX, =1 FOR FORWARD ELIMINTATION AND BACKSUBSTITUTION
C             =ANY OTHER VALUE FOR SUBSTITUTION ONLY
C  NMAT  =STORAGE SIZE OF GLOBAL STIFFNESS MATRIX = NEQ*LBAND
C  OUTPUT
C  B   =DISPLACEMENT VECTOR
C*******************************************************************
      DIMENSION A(1),B(neq)
      LLT=1
      WDETJ=0.0
      MM=LBAND-1
      NM=NEQ*LBAND
      NM1=NM-MM
      IF (LLT.NE.1) GO TO 55
      MP=LBAND+1
      KK=2
      FAC=WDETJ
      A(1)=1./SQRT(A(1))
      BIGL=A(1)
      SML=A(1)
      A(2)=A(2)*A(1)
      A(MP)=1./SQRT(A(MP)-A(2)*A(2))
      IF(A(MP).GT.BIGL)BIGL=A(MP)
      IF(A(MP).LT.SML)SML=A(MP)
```

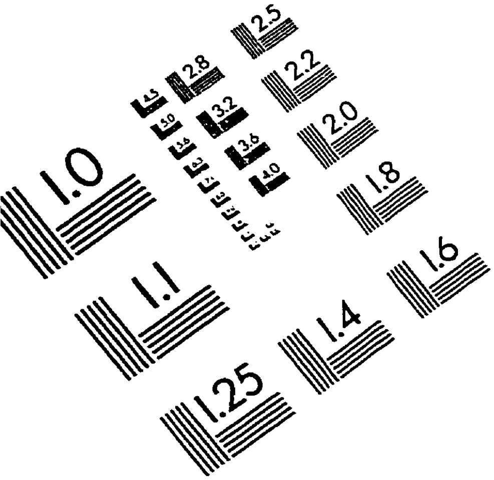
```
      MP=MP+LBAND
      DO 62 J=MP,NM1,LBAND
      JP=J-MM
      MZC=0
      IF(KK.GE.LBAND) GO TO 1
      KK=KK+1
      II=1
      JC=1
      GO TO 2
1     KK=KK+LBAND
      II=KK-MM
      JC=KK-MM
2     DO 65 I=KK,JP,MM
      IF(A(I).EQ.0.)GO TO 64
      GO TO 66
64    JC=JC+LBAND
65    MZC=MZC+1
      ASUM1=0.
      GO TO 61
66    MMZC=MM*MZC
      II=II+MZC
      KM=KK+MMZC
      A(KM)=A(KM)*A(JC)
      IF(KM.GE.JP)GO TO 6
      KJ=KM+MM
      DO 5 I=KJ,JP,MM
      ASUM2=0.
      IM=I-MM
      II=II+1
      KI=II+MMZC
      DO 7 K=KM,IM,MM
      ASUM2=ASUM2+A(KI)*A(K)
7     KI=KI+MM
5     A(I)=(A(I)-ASUM2)*A(KI)
6     CONTINUE
      ASUM1=0.
      DO 4 K=KM,JP,MM
4     ASUM1=ASUM1+A(K)*A(K)
61    S=A(J)-ASUM1
      IF(S.LT.0.)WDETJ=S
      IF(S.EQ.0.)WDETJ=0.
      IF(S.GT.0.)GO TO 63
      NROW=(J+MM)/LBAND
      WRITE(6,99) NROW
99    FORMAT(35H ERROR CONDITION ENCOUNTERED IN ROW,I6)
      RETURN
63    A(J)=1./SQRT(S)
      IF(A(J).GT.BIGL)BIGL=A(J)
      IF(A(J).LT.SML)SML=A(J)
62    CONTINUE
      IF(SML.LE.FAC*BIGL)GO TO 54
      GO TO 53
54    WDETJ=0.
      RETURN
53    WDETJ=SML/BIGL
```

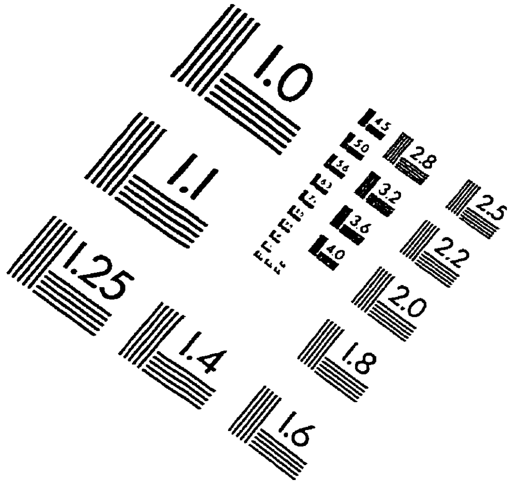
```
55  B(1)=B(1)*A(1)
    KK=1
    K1=1
    J=1
    DO 8 L=2,NEQ
    BSUM1=0.
    LM=L-1
    J=J+LBAND
    IF(KK.GE.LBAND)GO TO 12
    KK=KK+1
    GO TO 13
12  KK=KK+LBAND
    K1=K1+1
13  JK=KK
    DO 9 K=K1,LM
    BSUM1=BSUM1+A(JK)*B(K)
    JK=JK+MM
9   CONTINUE
8   B(L)=(B(L)-BSUM1)*A(J)
    B(NEQ)=B(NEQ)*A(NM1)
    NMM=NM1
    NN=NEQ-1
    ND=NEQ
    DO 10 L=1,NN
    BSUM2=0.
    NL=NEQ-L
    NL1=NEQ-L+1
    NMM=NMM-LBAND
    NJ1=NMM
    IF(L.GE.LBAND)ND=ND-1
    DO 11 K=NL1,ND
    NJ1=NJ1+1
    BSUM2=BSUM2+A(NJ1)*B(K)
11  CONTINUE
10  B(NL)=(B(NL)-BSUM2)*A(NMM)
    RETURN
    END
C
    SUBROUTINE SETUP2(A,BB,S,FL,NLJ,LJ,LBAND)
    IMPLICIT REAL*4 (A-H,O-Z)
C***********************************************************************
C   A ROUTINE TO ASSEMBLE THE GLOBALAL STIFFNESS MATRIX USING A UNIFORM
C   BAND WIDTH LBAND. IT ALSO ASSEMBLES THE GLOBAL LOAD VECTOR.  THIS
C   ROUTINE SHOULD BE CALLED IN A LOOP OVER ALL ELEMENTS.
C INPUT
C INEL       =ELEMENT NUMBER IN DO-LOOP
C S(NLJ,NLJ) =ELEMENT STIFFNESS MATRIX
C FL(NLJ)    =ELEMENT LOAD VECTOR
C NLJ        =NO. OF DOF PER ELEMENT
C LJ(NLJ)    =ELEMENT CONNECTIVITY ARRAY
C LBAND      =HALF BANDWIDTH
C OUTPUT
C A          =GLOBAL STIFFNESS MATRIX IN BANDED FORM
C B          =GLOBAL LOAD VECTOR
C***********************************************************************
```

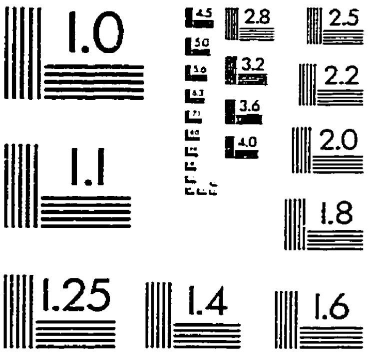
```
      DIMENSION A(1),BB(1),S(NLJ,1),FL(NLJ),LJ(NLJ)
C
      LB=LBAND - 1
      DO 12 I=1,NLJ
        LJR=LJ(I)
        IF(LJR.EQ.0) GO TO 12
        BB(LJR)=BB(LJR)+FL(I)
        DO 11 J=I,NLJ
          LJC=LJ(J)
          IF(LJC.EQ.0) GO TO 11
          IF(LJR-LJC) 9,10,10
 10          K=(LJC-1)*LB + LJR
      GO TO 13
 9          K=(LJR-1)*LB + LJC
 13          A(K)=A(K)+S(I,J)
 11     CONTINUE
 12  CONTINUE
      RETURN
      END
C
      SUBROUTINE CARASO(I,IMAT,PERMX,PERMY,ALFA,CON)
      IMPLICIT REAL*4(A-H,O-Z)
C******************************************************************
C   A ROUTINE TO COMPUTE THE CHARATERISTICS MATRIX FOR THE SINGLE
C    POROSITY CASE
C
C   INPUT:
C    I= ELEMENT NUMBER
C    PERMX= PERMEABILITY X DIRECTION
C    PERMY= PERMEABILY IN Y DIRECTION
C    ALFA= ANGLE BETWEEN THE GLOBAL SYSTEM OF COORDINATES &
C          DIRECTIONAL PERMEABILITIES
C    CON(2,2)= CHARACTERISTICS MATRIX
C
C******************************************************************
      DIMENSION PERMX(10,1),PERMY(10,1),ALFA(10,1),CON(2,2),IMAT(1)
      MID=IMAT(I)
      ALFAR=(ALFA(MID,1)*3.141593)/180
      CALL PRESET(CON,2,2)
        CON(1,1) = PERMX(MID,1)*(COS(ALFAR))**2 +
     $              PERMY(MID,1)*(SIN(ALFAR))**2
        CON(2,2) = PERMX(MID,1)*(SIN(ALFAR))**2 +
     $              PERMY(MID,1)*(COS(ALFAR))**2
        CON(1,2) = PERMX(MID,1)*(SIN(ALFAR))*(COS(ALFAR))-
     $              PERMY(MID,1)*(SIN(ALFAR))*(COS(ALFAR))
        CON(2,1) = CON(1,2)
      RETURN
      END
C
      SUBROUTINE DPOR(I,IMAT,PLE,EI,COND,BETA,NSET,XF,PERX,PERY)
      IMPLICIT REAL*4 (A-H,O-Z)
C******************************************************************
C   A ROUTINE TO CALCULATE THE ELEMENT CONDUCTIVITY MATRIX
C    IN THE DOUBLE POROSITY APPROACH, TAKING IN ACCOUNT
C    ORIENTATION OF THE FRACTURES AND THEIR HYDRAULIC PROPERTIES
```

```
C
C   INPUT:
C
C
C*******************************************************************
      DIMENSION COND(2,2),PLE(10,10),EI(10,10),PERX(10,10),PERY(10,10)
      DIMENSION CONI(2,2)
      DIMENSION NSET(10),IMAT(1),XF(10,10),BETA(10,10)
C
C------------------COMPUTE DIRECTIONAL MATRIX---------------------
C
      CALL PRESET(CONI,2,2)
      CALL PRESET(COND,2,2)
      KMID=IMAT(I)
      DO 1000 II=1,NSET(KMID)
      BETAR=(BETA(KMID,II)*3.141593)/180
      CONI(1,1)= PERX(KMID,II) * (COS(BETAR))**2 +
     $        PERY(KMID,II) * (SIN(BETAR))**2
      CONI(2,2)= PERX(KMID,II) * (SIN(BETAR))**2 +
     $        PERY(KMID,II) * (COS(BETAR))**2
      CONI(1,2)=(PERX(KMID,II)-PERY(KMID,II))*
     $                (SIN(BETAR))*(COS(BETAR))
      CONI(2,1)=CONI(1,2)
C
      DO 1200 KK=1,2
        DO 1300 JJ=1,2
        COND(KK,JJ)=COND(KK,JJ)+(1/PLE(KMID,II))*(EI(KMID,II))*
     $       (XF(KMID,II))*(CONI(KK,JJ))
 1300       CONTINUE
 1200      CONTINUE
 1000 CONTINUE
C
      RETURN
      END
```

IMAGE EVALUATION TEST TARGET (QA-3)

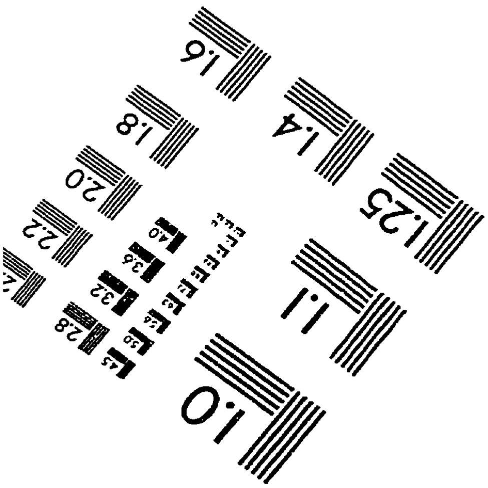


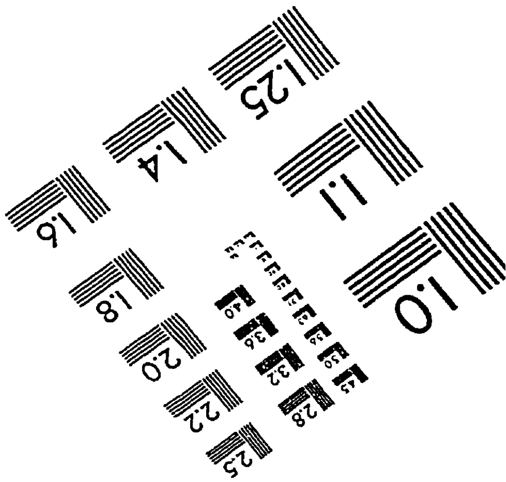




150mm

6"





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