A Study of the Factors influencing Laser Grade Control System Accuracy on Trenchless Plow-Type Drainage Machines.

by

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ABSTRACT

A study of the principal factors influencing laser grade control accuracy is described.

An investigation of the effect of the travel speed of trenchless plow-type drainage machines on the drain grade deviations revealed a positive relationship for the range of speeds observed under different soil conditions.

A dynamic study on a plow-model with double trapezoidal linkage permitted the establishment of a relation between the plow point response, the machine speed and the hydraulic cylinder velocity. The relation is shown in a generalized formula applicable to any similar plow-model.

ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks to Professor P.J. Jutras for his guidance throughout the duration of this study.

Appreciation is extended to some drainage Contractors who willingly cooperated with the author, and especially Messrs. M. Poirier, G. Edes, P. Laporte.

Thanks also to Mrs. G. Langlois for typing this project.

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INTRODUCTION

The Project

Today, more than one half of the drain pipe installed in Canada is installed with trenchless plow-type drainage machines. These plows are controlled by an automatic laser beam grading system which is becoming a common method of grade control for all types of drainage machines.

The new technology allows designers and engineers to design faster and more efficient installation equipment. But at higher machine speeds, the accuracy of the grade control system becomes more critical.

To insure good quality workmanship, drain inspection is carried out on a regular basis. There are many methods of inspection depending whether the installation is completed or not. If the plow has completed the installation, the tile can be probed or excavated, but if the plow is installing drain the consistency of the grade control systems of drainage machines or drain grade deviations can be readily checked through a transit instrument with the telescopic lens installed parallel to the drain gradient.

In this study the transit method was used with field observations to check the relationship between the ground speed of trenchless plow-type drainage machines and the drain grade deviations under different soil conditions. Also, other factors which cause grade deviations were studied in order to improve quality of drain installation by trenchless plow-type drainage machines.

Objectives and Scope

The following objectives were established:

1. To study the principal factors influencing the accuracy of the laser grade control systems of trenchless plow-type drainage machines.
2. To investigate the relationship between the travel speed of trenchless plow drainage machines and the drain grade deviations under different soil conditions.
3. To establish a relationship between the plow point response, the travel speed and the hydraulic cylinder velocity, by a dynamic study of a plow-model with double trapezoidal linkage.

REVIEW OF LITERATURE

Laser Adaptation to Drainage Machines

"Plowing-in" corrugated plastic tubing with plow-type equipment, automatically controlled by a laser grade control system, is a modern subsurface drainage method which is intensively used in several areas of Europe, the United States and Canada. The successful operation of high speed trenchless plow drainage machines is dependent upon the ability to control and maintain the gradient of the draintube installed.

Fouss and Reeve (1), (2) in 1968 studied by analog computer simulation the adaptation of laser control system to draintube plow machines. An attempt was made to simulate the entire control system (receiver, controller, hydraulic valve and cylinder). The system performance was evaluated on the basis of the accuracy of maintaining the plow hitch point near the zero elevation. For the test, the plowing depth was assumed to follow the hitch elevation with a time lag behind it. Comparison of results obtained from the simulation for a hydraulic cylinder velocity of 6 cm/sec (2½ in/sec) and 7.5 cm/sec (3 in/sec) indicated that the higher piston speed improved the control system accuracy. The computer simulation offered an excellent method to analyze and adjust the system for optimum performance on a simulated ground surface consisting of step, ramp, sawtooth, sinusoidal function and other combinations. To accomplish this using field tests would have been difficult, expensive and time-consuming because of the random variation of ground surface encountered in the field.

Fouss (4) in 1972 studied the same laser adaptation but in modelling the dynamics of the plow in the computer simulation. The prototype used for this study was an ARS draintube plow equipped with a laser automatic grade control system produced by the Laserplane Corporation of Daytona, Ohio, The draintube plow and the laser beam feedback control system were represented in block diagram to study their stability and sensitivity. However, as concluded in the study, the two previously mentioned parameters did not guarantee accurate grade control. It was found that the more critical factors for grade control accuracy were the laser receiver position and the machine ground speed. Field tests conducted on the ARS plow-model at an average ground speed of 46 m/min (150 ft/min) showed that the optimum position to mount the laser receiver was just over and forward of the plow point. Another field test on the same prototype showed that the grade accuracy was noticeably improved by decreasing the machine ground speed.

Plow Design

Fouss (10) in 1971 studied the basic response characteristics of a long floating beam and he concluded that for a wide range of topography and changing soil types, accurate grade control on a long floating beam plow could not be achieved by controlling the hitch point such that it travels on a line parallel to the desired gradient.

In 1970, designers developed new plow-models utilizing the floating beam principle. One of these was manufactured in England and was called the "Badger Minor". It was patterned after a unique design developed by Ede (1961-1965) in England. Later a similar drainage implement called the "Zor Plow" was developed in Canada. It featured a double trapezoidal linkage which has become the standard used by many manufacturers. The quality in the design of any trenchless plow is based on its ability to "grade" and "float" independently of ground surface irregularities, within certain speed limits.

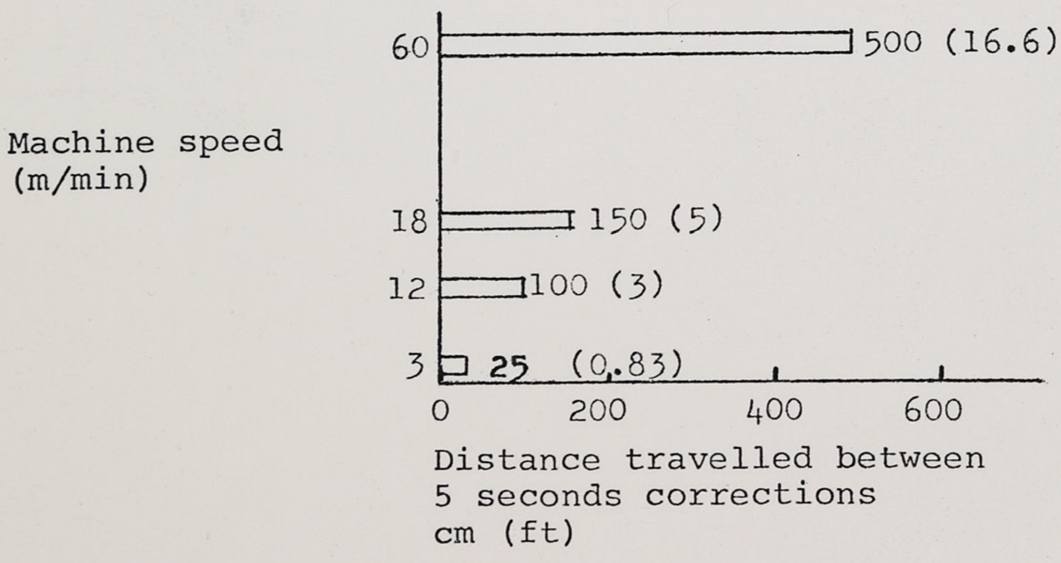
INVESTIGATION

Need for Automatic Grade Control

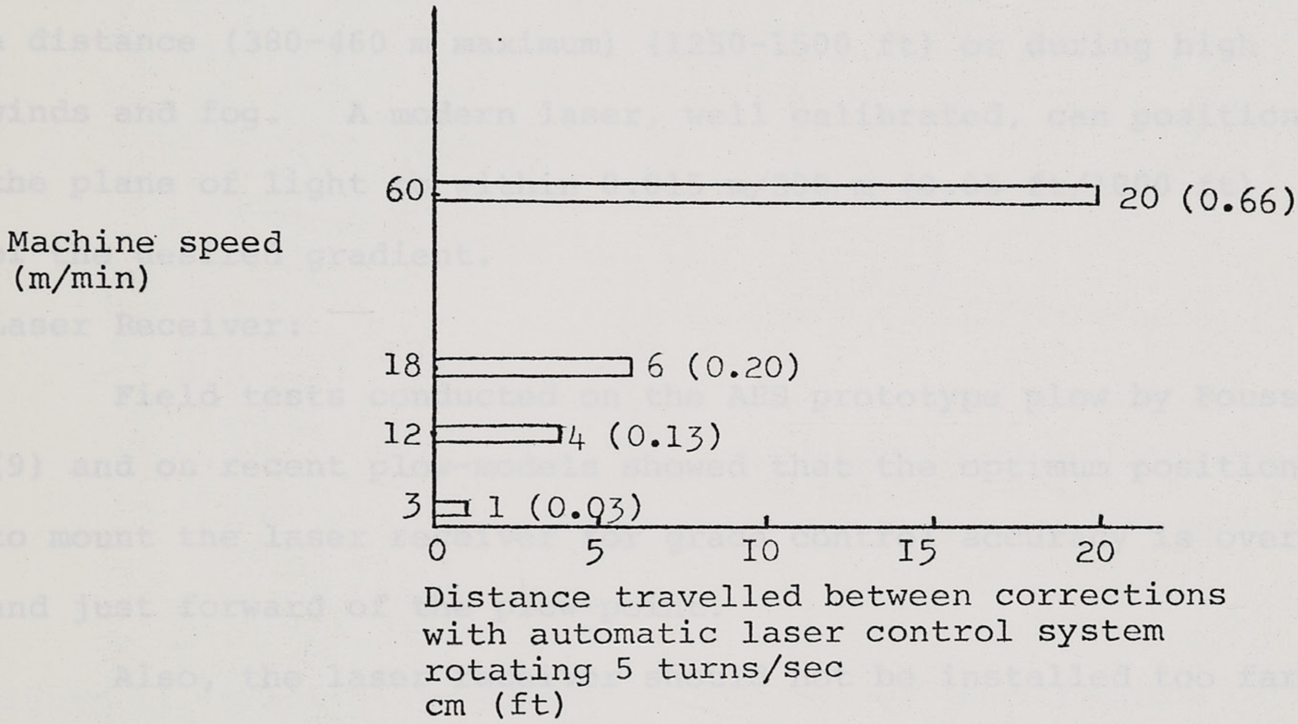
New technology has increased the potential speed of installation to as much as 50-60 m/min (165-195 ft/min). Around 1947, the wheel trenching machine's top speed was approximately 3 m/min (10 ft/min), under ideal conditions.

The need for automatic grade control on drainage machines can be considered from the standpoint of distance travelled between corrections. A good operator on an old wheel trencher machine would check grade and make corrections, if necessary, perhaps once every 5 seconds. Thus, at a speed of 3 m/min (10ft/min) grade was corrected as often as every 25 cm (10 in) which produced acceptable results.

Today, modern machines travel much faster. A typical wheel machine may dig at 12 m/min (40 ft/min), a drain machine 18 m/min (60 ft/min) and a plow may potentially operate at more than 60 m/min (195 ft/min). If these were corrected manually every 5 seconds the distance travelled between corrections would be as shown in the figure below:



The laser emitter, rotating at some 5 turns/sec or more solved the correction problems at higher machine speeds. At a rate of 5 corrections/sec, the distance travelled between corrections is reduced considerably, as shown in the figure below:



It can be concluded that the need for automatic grade control becomes very important even on a modern wheel-type drainage machine for better accuracy of installation.

Factors Influencing the Laser Grade Control System Accuracy

Laser Transmitter:

Self-levelling laser transmitters are important for accurate work and they must be calibrated regularly because they can drift off grade. They should not be used at too great a distance (380-460 m maximum) (1250-1500 ft) or during high winds and fog. A modern laser, well calibrated, can position the plane of light to within 0.015 m/300 m (0.05 ft/1000 ft) of the desired gradient.

Laser Receiver:

Field tests conducted on the ARS prototype plow by Fouss (9) and on recent plow-models showed that the optimum position to mount the laser receiver for grade control accuracy is over and just forward of the plow-point.

Also, the laser receiver should not be installed too far from the center line of the plow. If installed too far, the drain depth will be affected when the machine is installing drain across a slope, as shown in Figure 1. The more the tractor is tilted, the more the drain depth deviation is increased.

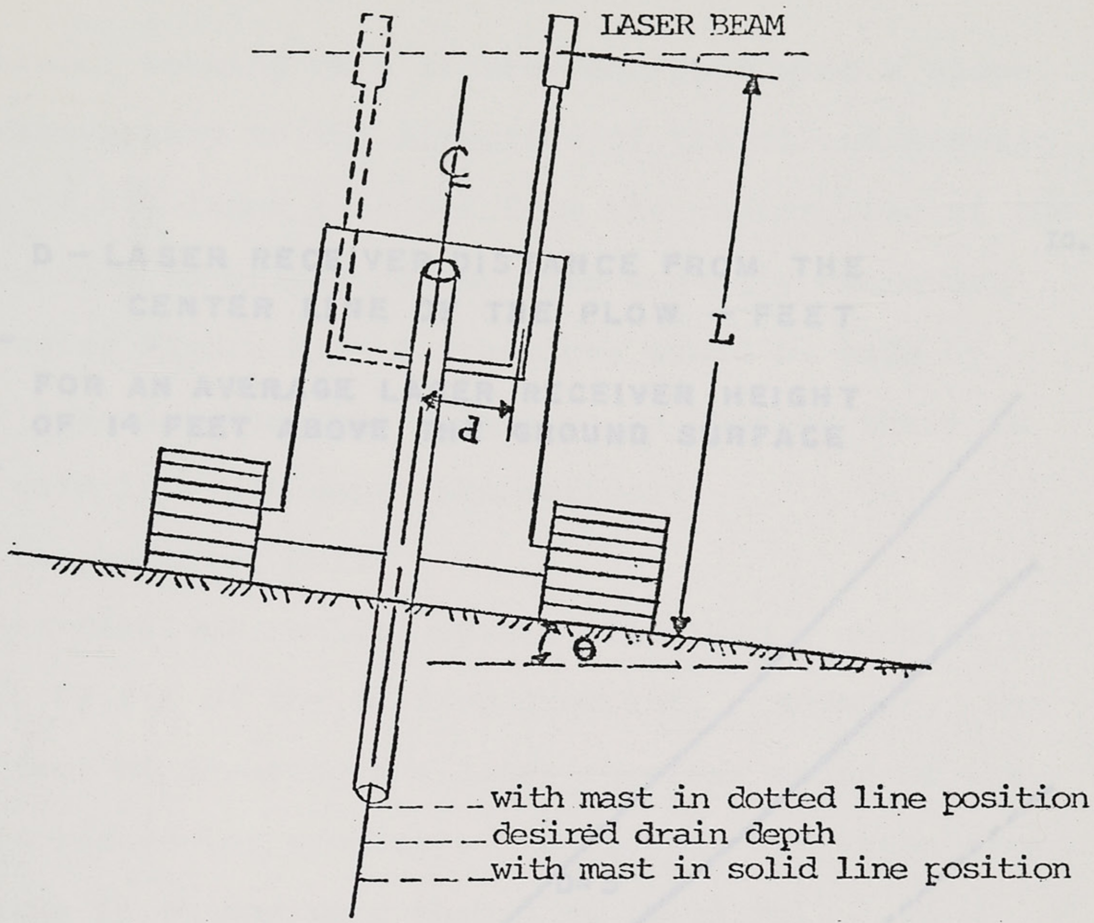


Figure 1: Drain depth deviation encountered when tractor travel across a slope.

A general formula involving all the factors causing the depth deviation would be as follows:

$$D = L/\cos\theta - L + d \times \tan\theta$$

where

D = Drain depth deviation - (m)

L = Average laser receiver height from the ground surface - (m)

d = Laser receiver distance from the center line (℄) of the plow - (m)

θ = Tractor tilt angle - (degree)

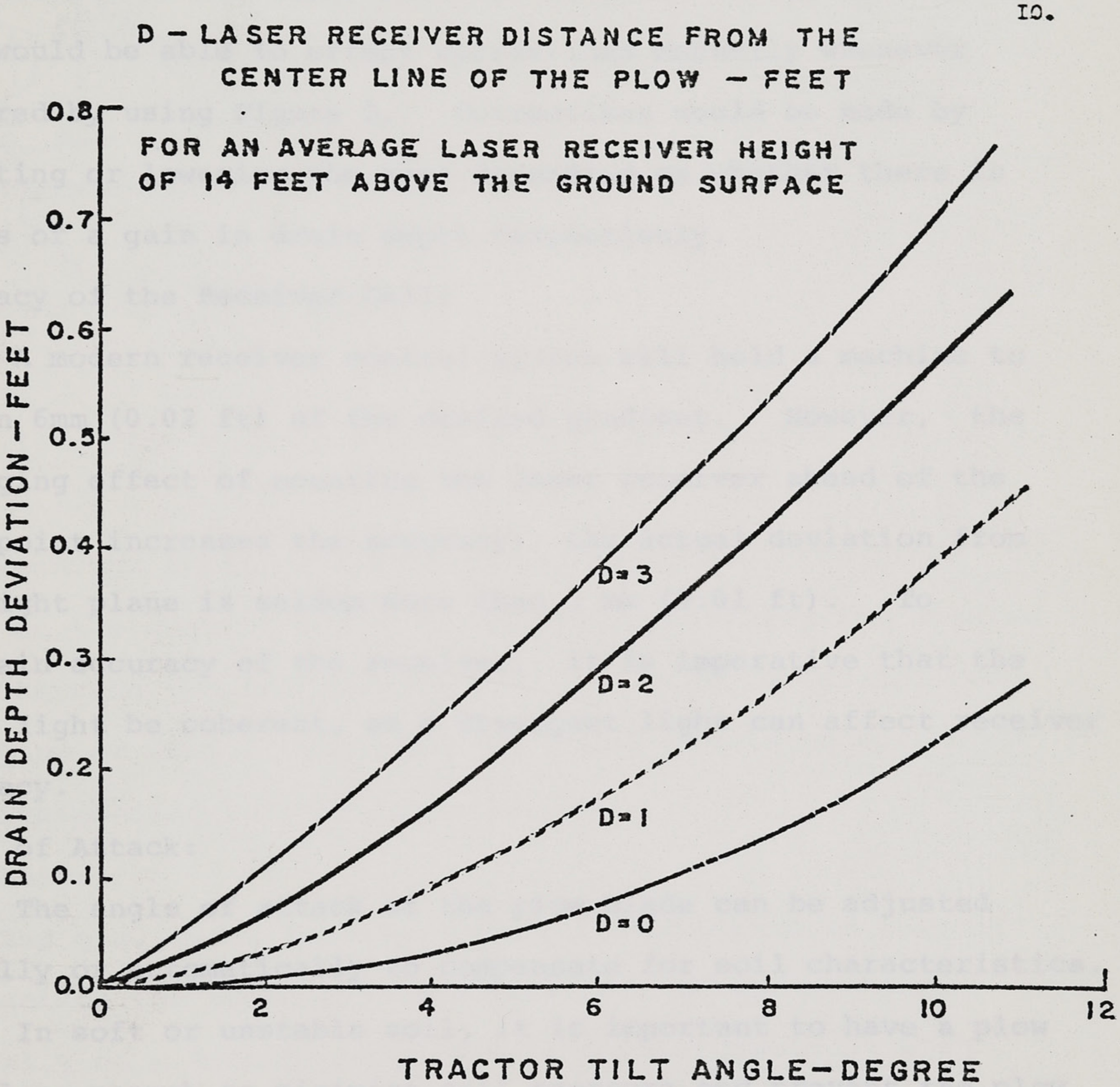


Figure 2. Estimated drain depth deviation at various tractor tilt angle and laser receiver distance from the center line of the plow.

An operator working on a tractor equipped with a slope meter set transversely to the direction of travel and knowing the distance of his laser receiver from the center line of the plow would be able to effect corrections manually whenever required by using Figure 2. Corrections would be made by elevating or lowering the mast depending on whether there is a loss or a gain in drain depth respectively.

Accuracy of the Receiver Cell:

A modern receiver control system will hold a machine to within 6mm (0.02 ft) of the desired gradient. However, the averaging effect of mounting the laser receiver ahead of the plow point increases the accuracy; the actual deviation from the light plane is seldom more than 3 mm (0.01 ft). To maintain accuracy of the receiver, it is imperative that the laser light be coherent, as a divergent light can affect receiver accuracy.

Angle of Attack:

The angle of attack of the plow blade can be adjusted manually or automatically to compensate for soil characteristics.

In soft or unstable soil, it is important to have a plow sole long enough to minimize soil pressure and prevent the plow from running with the point up and the heel down affecting the drain depth as shown in Figure 3. Shoe level correcting devices are now available that will put a partial pressure on the rear cylinder to correct the angle of attack in such cases.

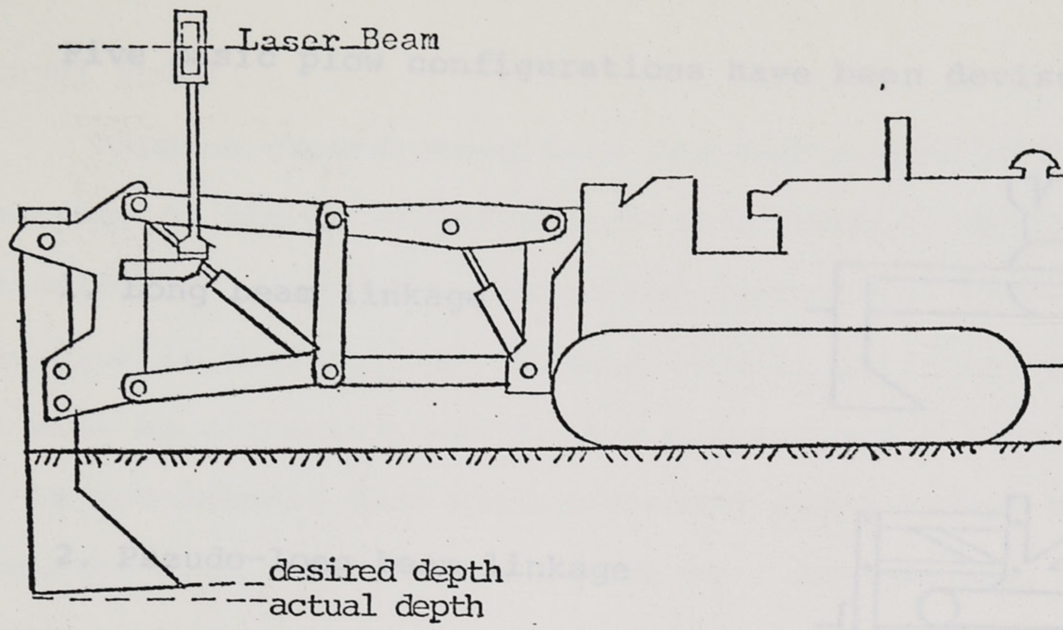


Figure 3: Plow blade position in soft or unstable soil.

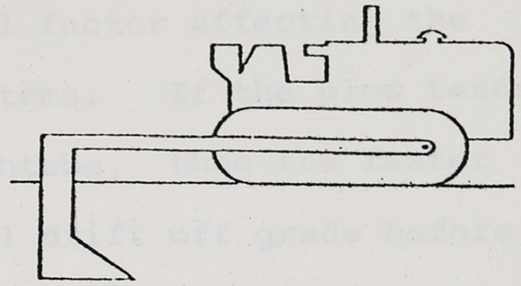
When the angle of attack is well adjusted, the grade lights on the control box will oscillate regularly from high to low, otherwise the hydraulic system will tend to correct constantly up or down. The angle of attack should be corrected for corresponding soil characteristics.

Plow Design:

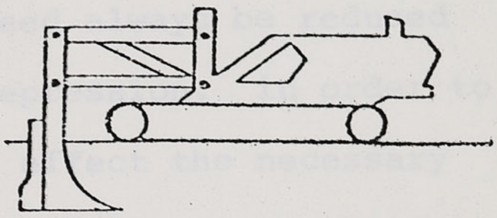
The quality of drainage tubing installation is largely dependent on the accuracy characteristics of the machine, especially on a trenchless plow-type which travels at high speed.

Five basic plow configurations have been devised:

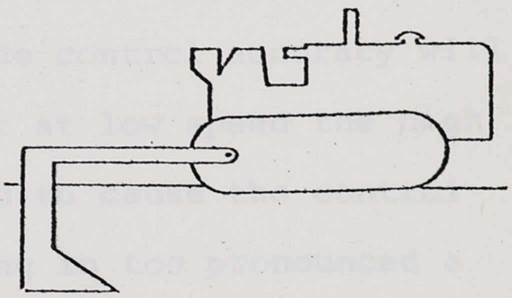
1. Long beam linkage



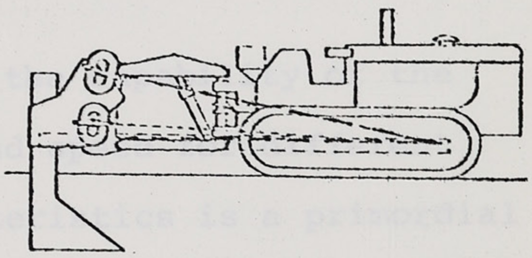
2. Pseudo-long beam linkage



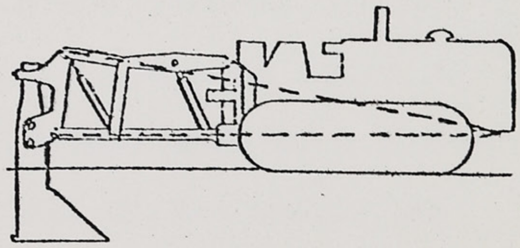
3. Short beam linkage



4. Simple trapezoidal linkage with free roller; "virtual" hitch point near the front of the tracks



5. Double trapezoidal linkage with frictionless bearing; "virtual" hitch point in front of the tracks



The last two configurations are the most popular in the U.S. and Canada, because of the geometry of their linkages which produces even ground pressure along the track area, thus resulting in maximum traction efficiency, less soil slippage and compaction.

Travel Speed:

Machine ground speed is a critical factor affecting the accuracy of the laser grade control systems. If the plow tends to drift off grade when installing draintube, then the faster the plow is moving, the further it will drift off grade before it will be corrected back to the desired grade line. Therefore, it is recommended that machine ground speed always be reduced when the plow travels over a rise or a depression, in order to allow time for the hydraulic cylinder to effect the necessary corrections. If the rate of hydraulic cylinder movement to make corrections is increased, the grade control accuracy will be improved at higher machine speed, but at low speed the high rate of the hydraulic cylinder will tend to cause the control system to "hunt" rapidly, thus resulting in too pronounced a correction.

On present field installations, the capability of the operator to determine an adequate ground speed for different soil conditions and topographic characteristics is a primordial factor to insure the quality of work.

Field Study Procedure

Tests were performed to investigate the relationship between the travel speed and the drain grade deviations.

Because of limitations such as time and machine availability, it was decided to check three trenchless plow-type drainage machines which were working in different soil conditions. The grades were verified during drain installation by the transit reading method. For each machine, many drain lines of 100 mm (4 in) diameter, one percent slope and not more than 180 m (600 ft) for reading accuracy were considered in this study.

Preliminary field tests were made to check the consistency of laser grade control systems of machines at very low travel speeds. This was achieved by running the plow at shallow depths (no drain was laid during this test) along a predetermined path. Then the plow travelled 12 to 16 m (40 to 55 ft) and stopped to insure grade. A transit (T 1 WILD) was set up directly behind the plow blade with the line-of-sight of the telescopic lens pointing in the line of travel and also aligned parallel to the projected plane of light of the laser transmitter. A reference mark was made on a surveyor's rod attached to the back of the blade holder which lay on the line-of-sight (i.e. cross-hairs on the transit). As the plow moved away along the path any vertical displacements of the blade were indicated through the transit and could be noted. The three machines were verified for travelled distances varying from 25 to 30 m (82 to 98 ft). No noticeable deviations could be registered. It was concluded that the laser grade control system of each machine showed very good consistency at travel speeds of 3.5 to 7 m/min (12 to 24 ft/min).

For each drain line the same method of verification was used. At faster machine speeds the drain grade deviations were more apparent and they were registered at intervals of 5 to 10 seconds along the lateral line depending on the travel speed.

The machine speed may be influenced by many factors such as plowing depth, uneven soil conditions, soil slippage, etc. However, for these tests the operator attempted to keep as constant a speed as possible. A person travelling on the tractor utilized a stop-watch to check installation time for every drain line under test and the initial and final reading on the distance counter of the command box to determine drain line length. Thus, an average travel speed for each drain line was obtained.

Results and Discussion

A total of fifteen drain lines corresponding to 1960 m (6425 ft) of drain installed were inspected. Data and drain line characteristics are presented in Figures 4(a-b), 5 (a-b), 6 (a-b) for machines A, B, C, respectively. Frequency polygons of the average grade deviations of the three machines working at common travel speeds are presented in Figure 7. Pertinent machine characteristics are tabulated in Table 1.

A simple linear regression analysis was used to investigate the relationship between the travel speed and the drain grade deviations for each group of drain lines installed by each machine. The form of the regression used was:

$$Y = b_o + b_1 x$$

Where

Y = Average grade deviation - mm.

x = Average travel speed - m/minute

b_o = Sample estimate of the population parameter

b_1 = Sample estimate of the population parameter

The results in Table 2 show a positive relationship between x and Y in all the three groups, as expressed by the positive values of b_1. Further analysis to test the hypothesis $H_o : \beta_1 = 0$ revealed that this hypothesis could be rejected with 99 percent confidence, as indicated by the value of F for all the three groups. In other words, the statistical analysis (5) suggests that a positive relationship does exist between the travel speed and the drain grade deviations over the range of speeds observed.

The extent to which the variation in Y could be attributed to X was investigated by the correlation analysis. The resulting coefficients of determination r^2 as listed in Table 2 show that from 35 to 49 percent of the observed variations in grade deviations were due to travel speed. The remaining "unexplained" variations must have been due primarily to soil stoniness.

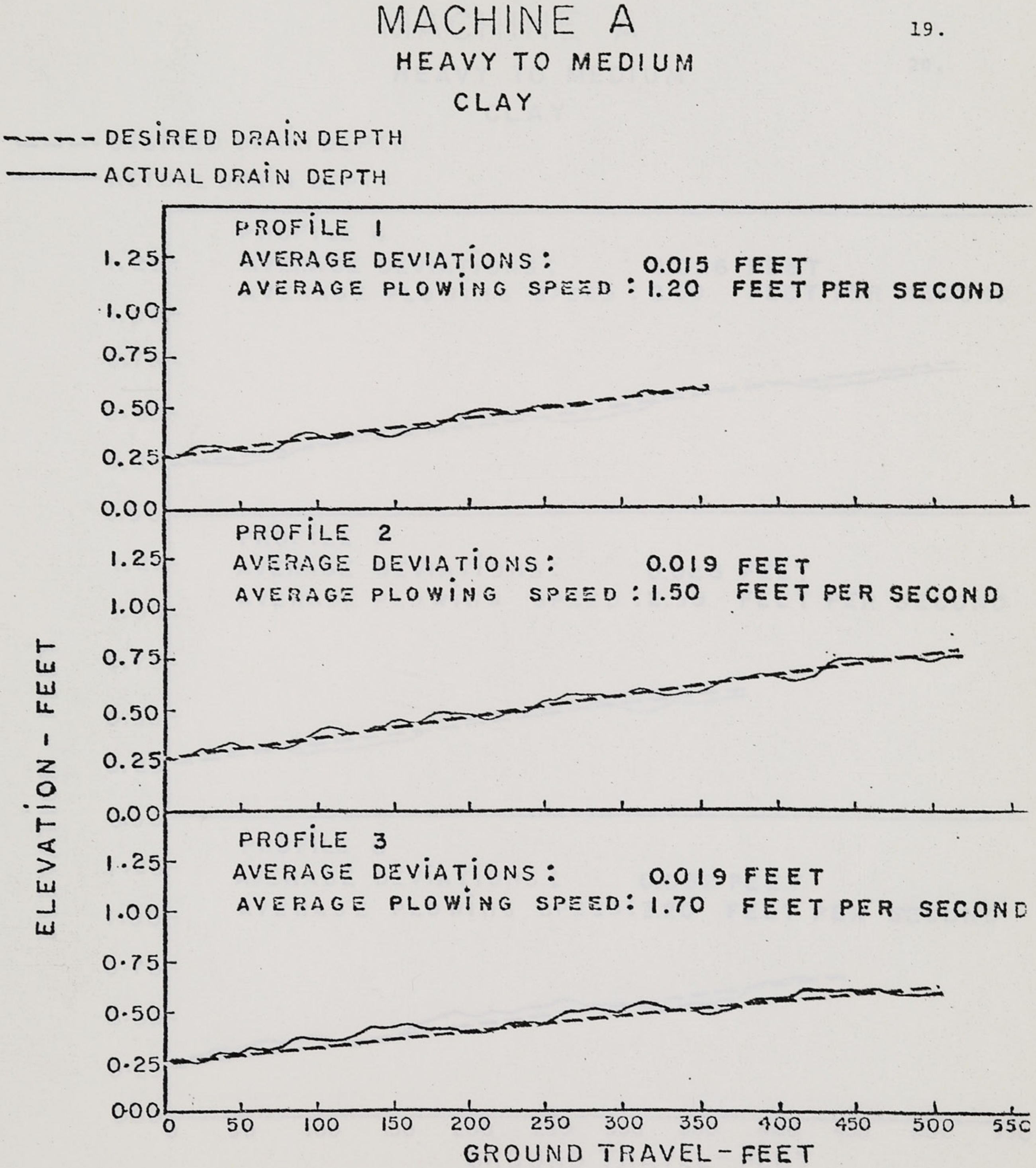


Figure 4(a). Observated drain grade deviations at various travel speeds of machine A in heavy to medium clay.

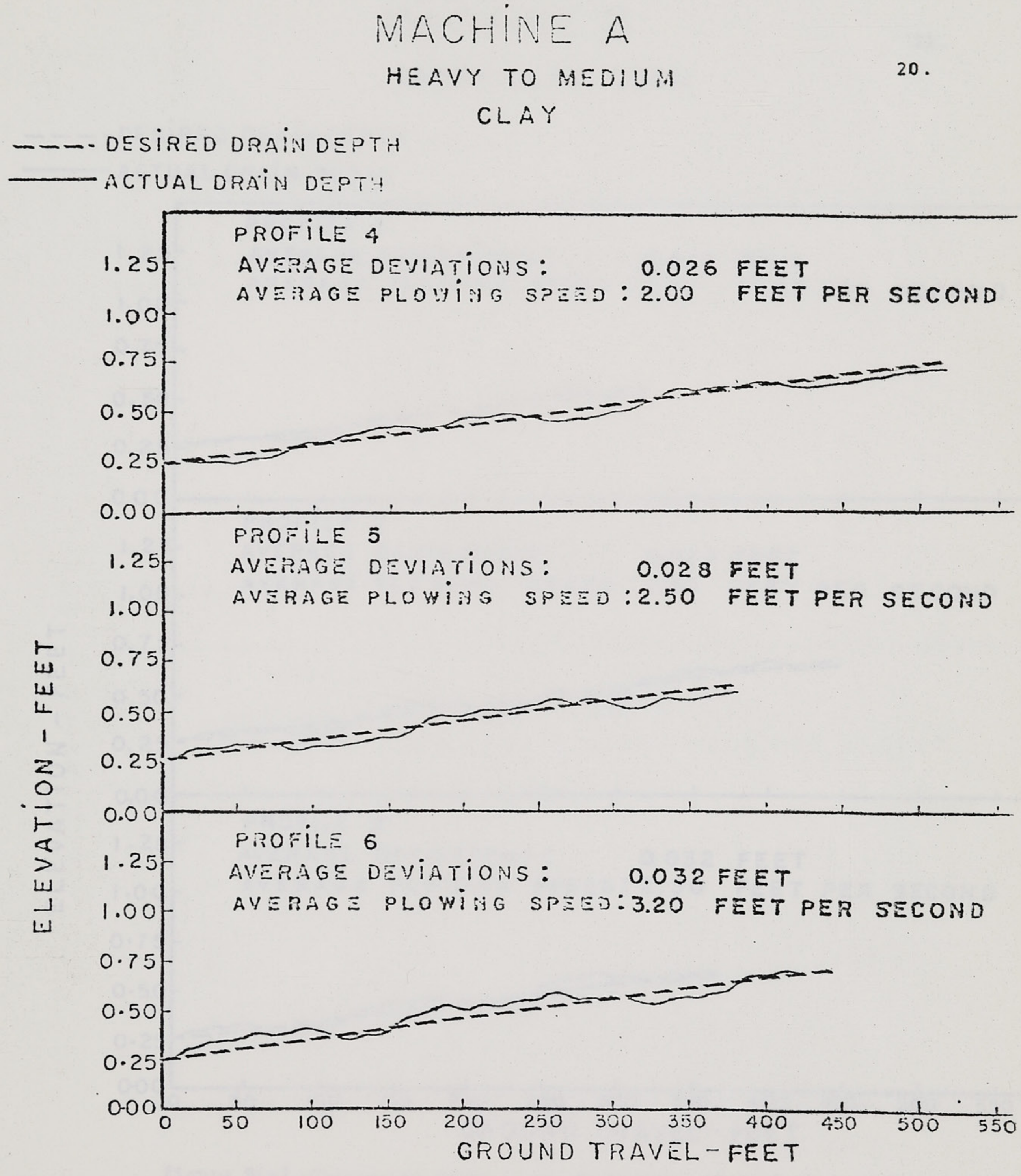


Figure 4(b). Observated drain grade deviations at various travel speeds of machine A in heavy to medium clay.

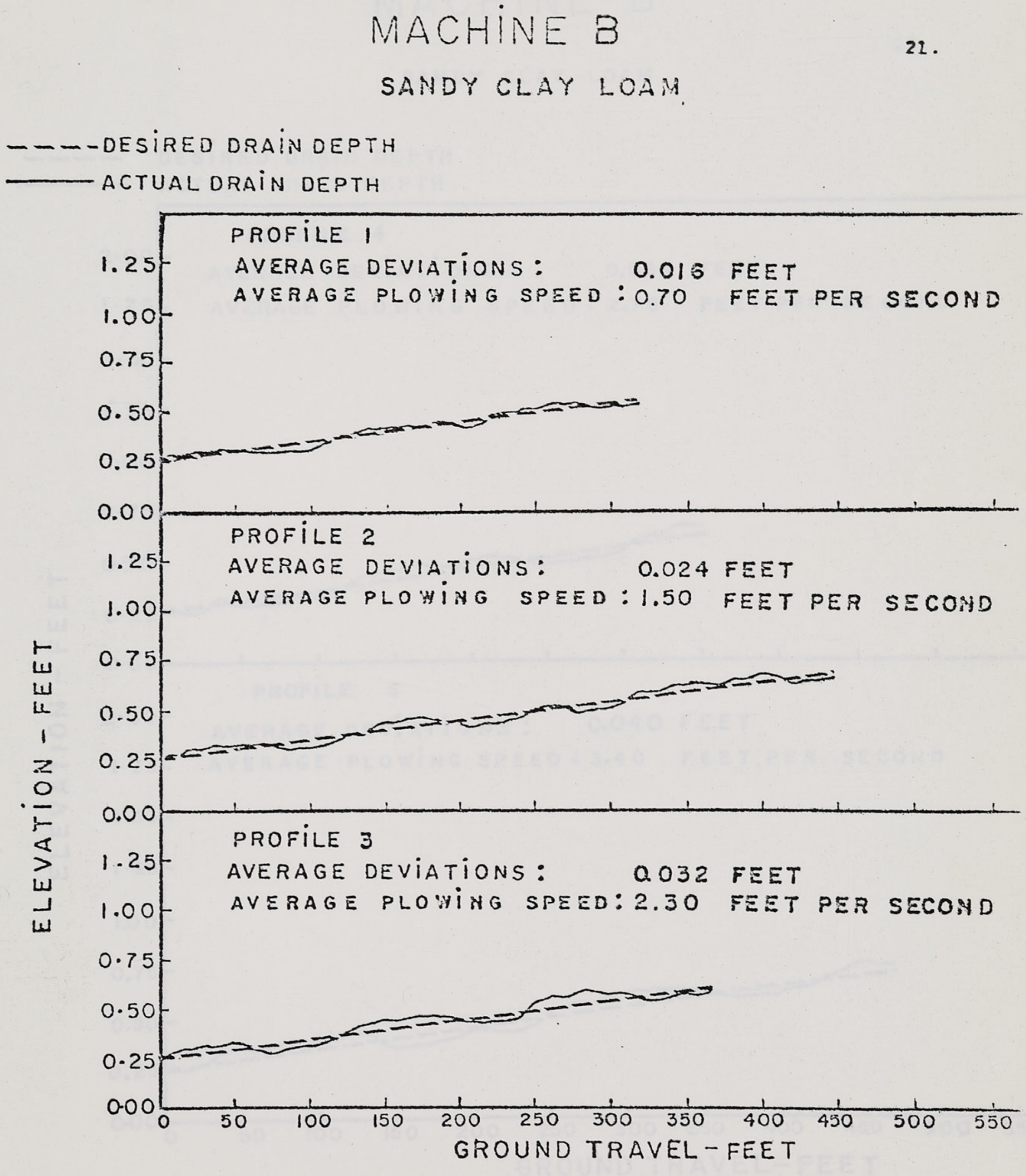


Figure 5(a). Observated drain grade deviations at various travel speeds of machine B in sandy clay loam.

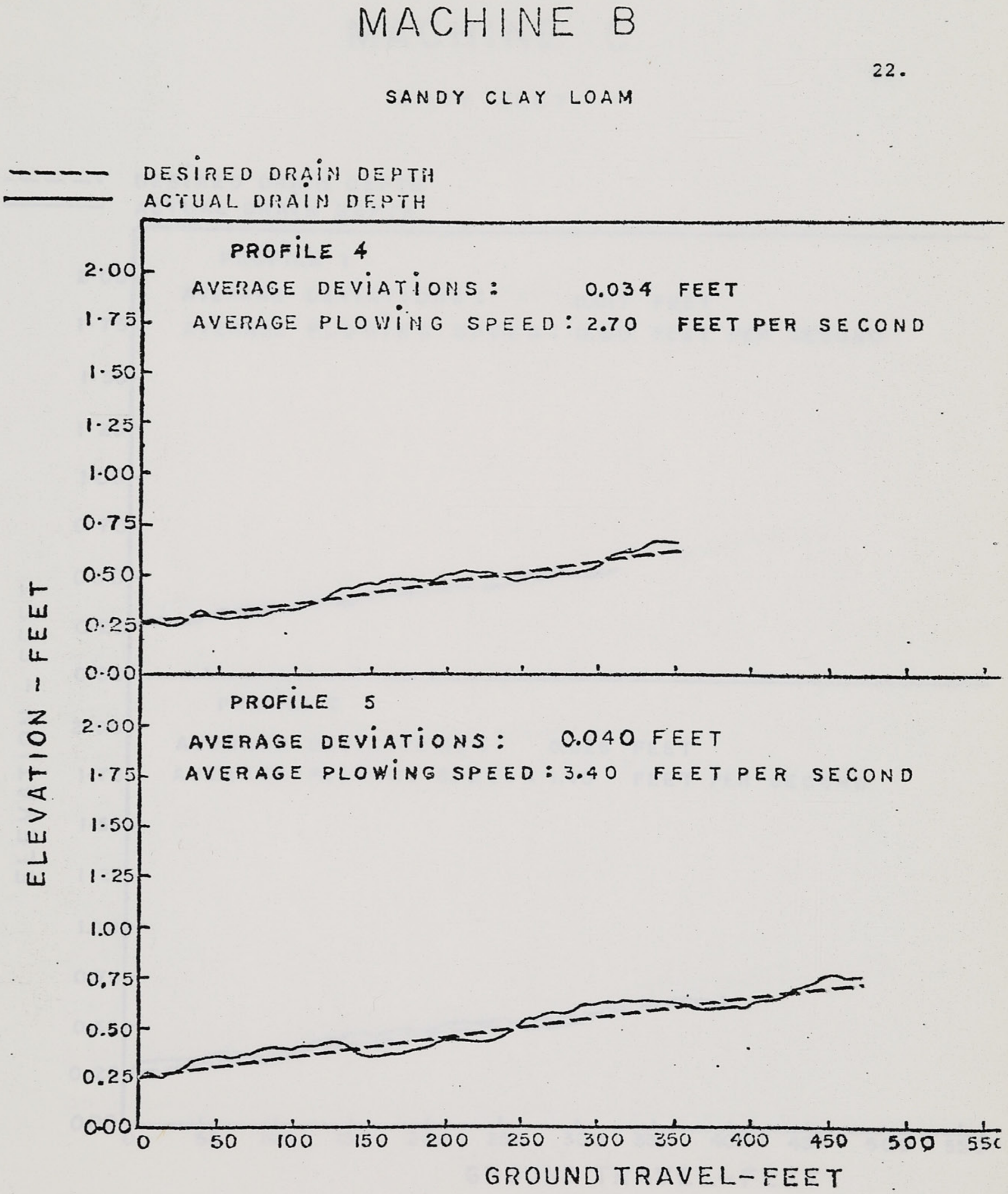


Figure 5(b). Observated drain grade deviations at various travel speeds of machine B in sandy clay loam.

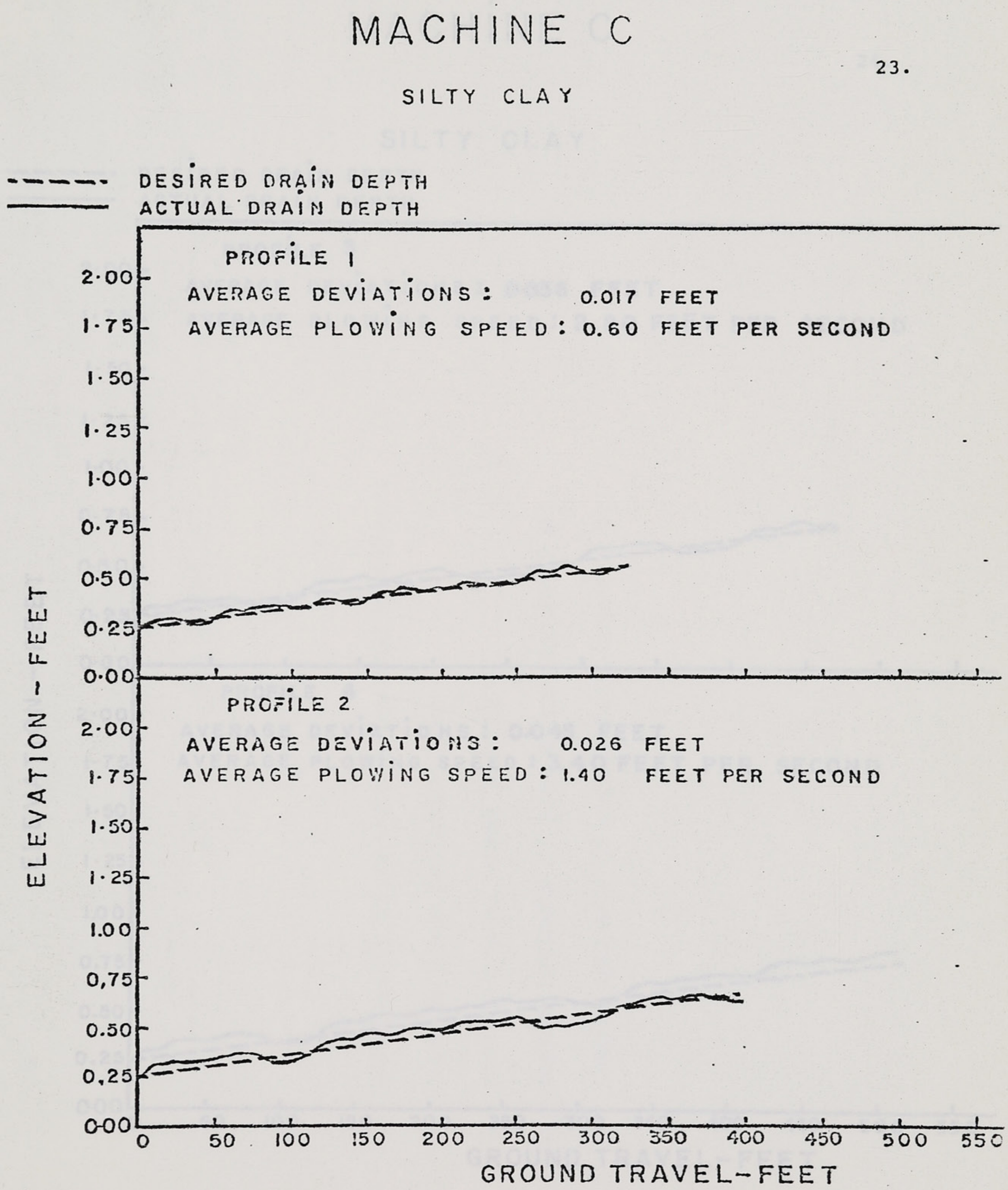


Figure 6(a). Observated drain grade deviations at various travel speeds of machine C in silty clay.

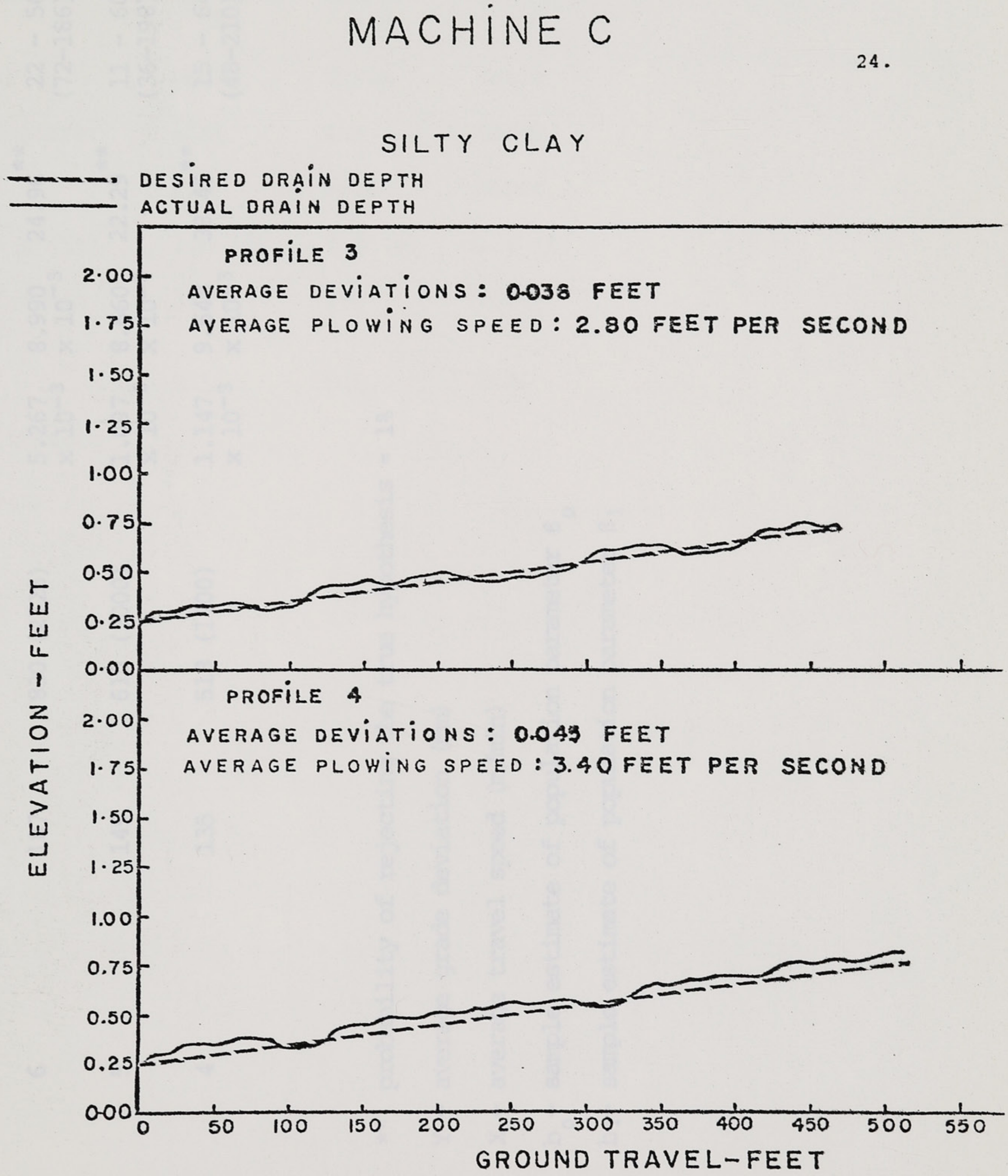


Figure 6(b). Observated drain grade deviations at various travel speeds of machine C in silty clay.

Table 1. Results of Regression on Travel Speed - Grade Deviations Relationship of the form $Y= b_0+ b_1x$

Machine	Soil texture	Number of drain profiles	Number of observations	Drain line length observed m (ft)	b_0	b_1	F $H_0:\beta_1$ 0	Range of X observed m/min (ft/min)	r^2
A	Heavy to medium clay	6	187	830 (2725)	5.267×10^{-3}	8.990×10^{-3}	24.96**	22 - 56 (72-186)	0.35
B	Sandy clay loam	5	149	610 (2000)	1.097×10^{-3}	8.660×10^{-3}	22.29**	11 - 60 (36-198)	0.42
C	Silty clay	4	135	518 (1700)	1.147×10^{-3}	9.648×10^{-3}	38.45**	15 - 64 (48-210)	0.49

** probability of rejecting the true hypothesis = 1%

Y - average grade deviation (mm)

X - average travel speed (m/min)

b_0- sample estimate of population parameter β_0

b_1- sample estimate of population parameter β_1

Table 2: Machines Characteristics of the Three Machines Observed.

Characteristics	Machine A	Machine B	Machine C
Laser model	Genie	Genie	Genie
Plow manufacturer	Link	Zor	Badger
Plow model	250	DP200	Minor
Max. depth m (ft)	1.8 (6)	1.8 (6)	1.8 (6)
Required drawbar (H.P.)	180	200	200
Grading*	D.L.	D.L.	D.L.

* Double linkages

Discussion of Figures

Figure 7 shows that for a travel speed equal to greater than 38 m/min (125 ft/min) the maximum grade deviations would be greater than the permissible grade deviation or ± 15 mm (0.050 ft).

The linear regression equations obtained from the drain observations and their confidence belts are presented graphically in Figures 8, 9 and 10. These figures show that the predicted maximum grade deviation within the confidence interval for travel speeds equal to or slightly greater than 54 m/min (180 ft/min) would be close to the permissible drain grade deviation.

It can be seen on Figures 8, 9 and 10 that, although the grade deviation varies positively with the travel speed, there are apparent differences between each group of drain profiles. However, it was not within the scope of this project to determine how the soil texture or machine characteristics could affect the drain grade deviations.

This study does point out that the plowing speed is a critical factor for good quality of drain installation. If normal ground surface irregularities are encountered, these results can serve as general guidelines to operators for the establishment of an optimum machine ground speed.

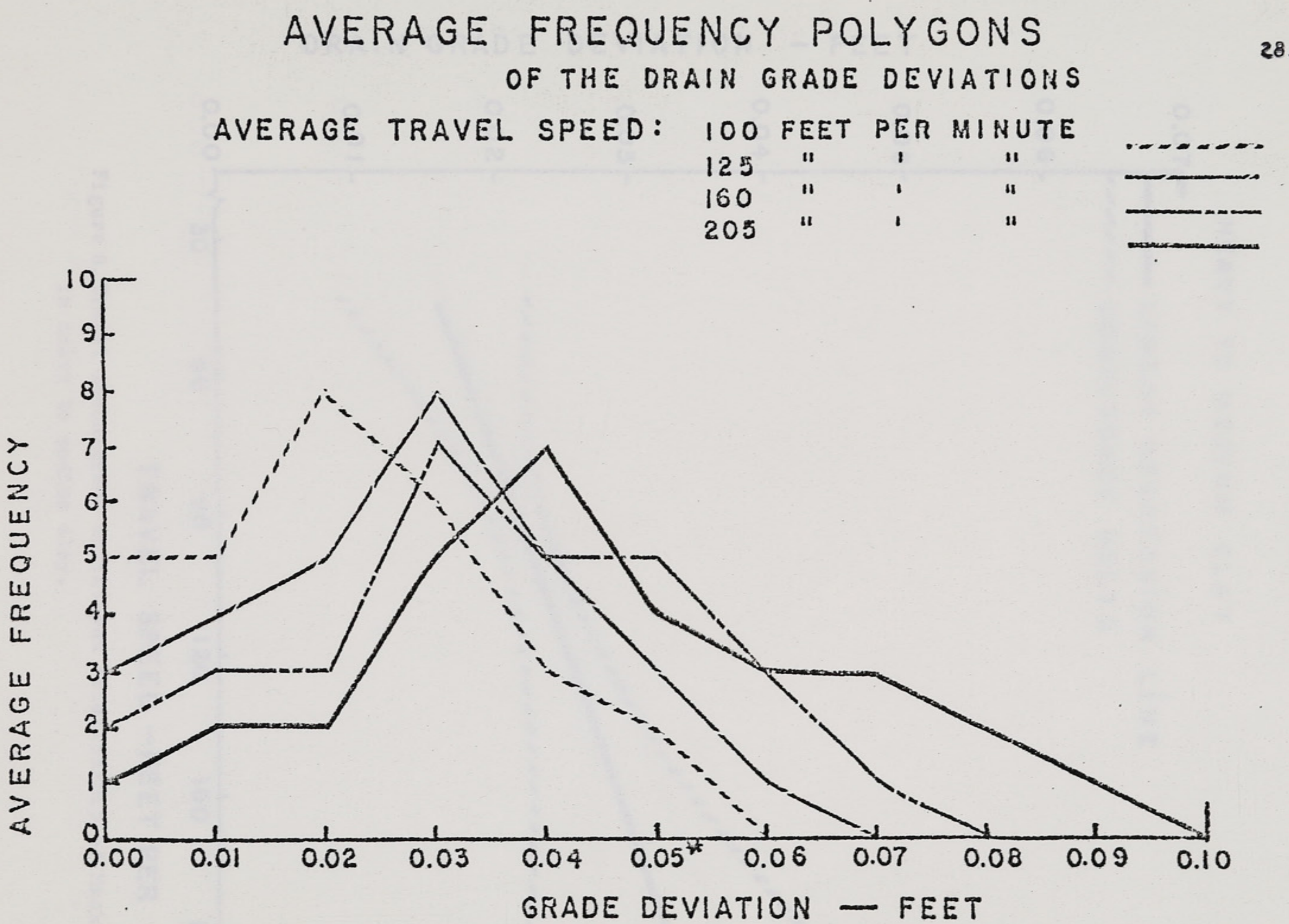


Figure 7. Frequency polygons of the drain grade deviations observed for the three machines.

MACHINE A

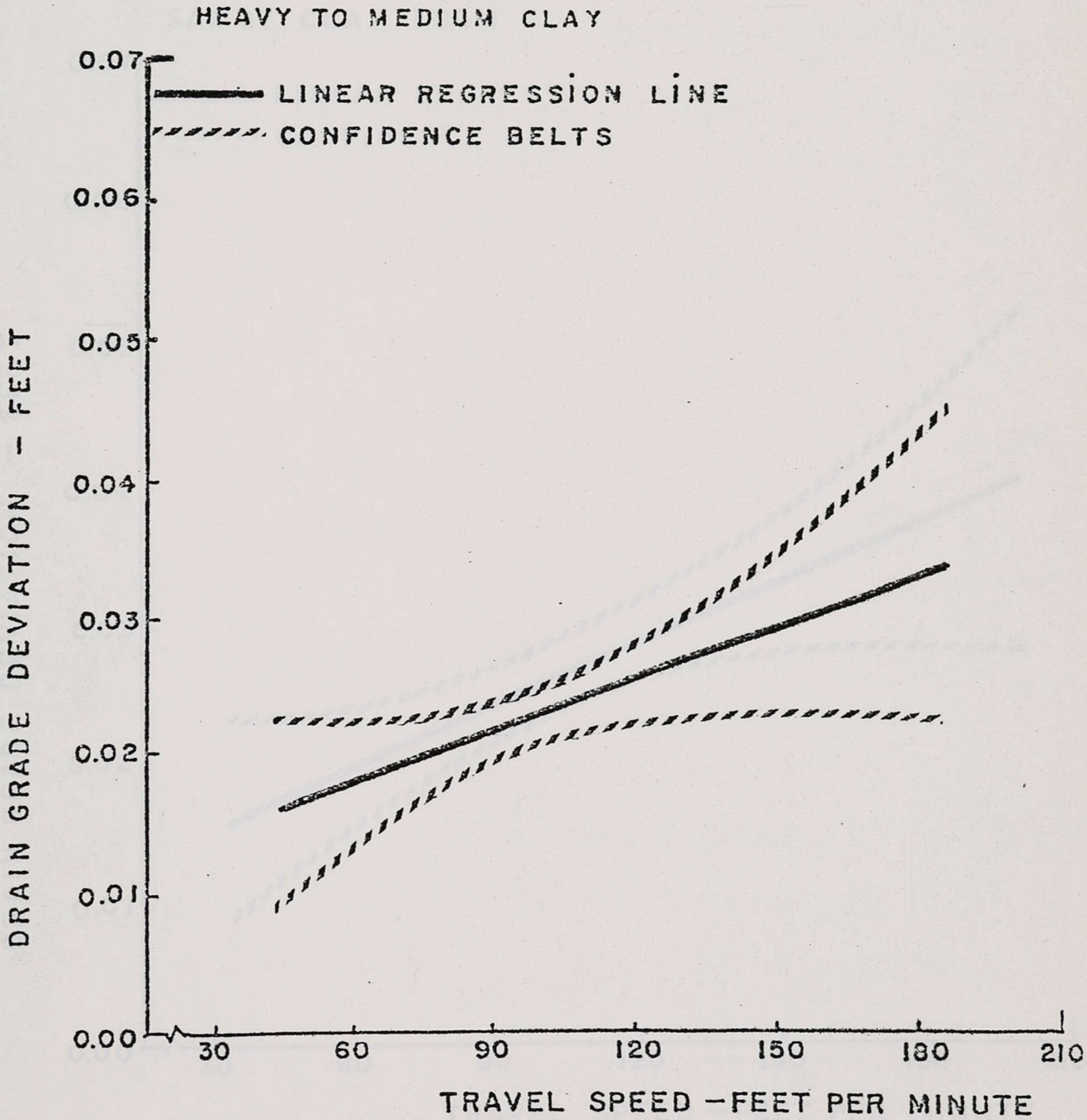


Figure 8. Travel speed-grade deviations relationship of Machine A in heavy to medium clay.

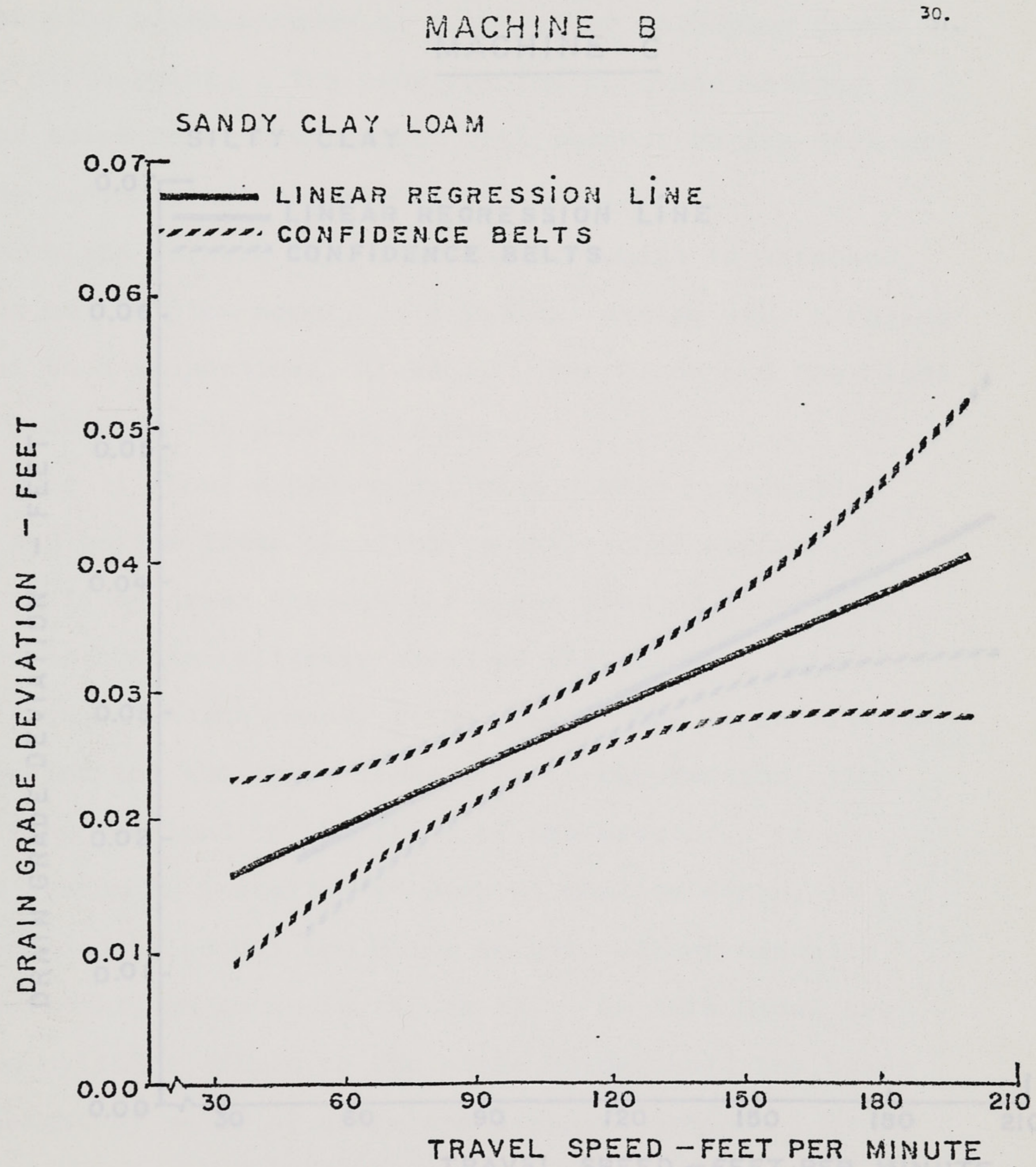


Figure 9. Travel speed-grade deviations relationship of Machine B in sandy clay loam.

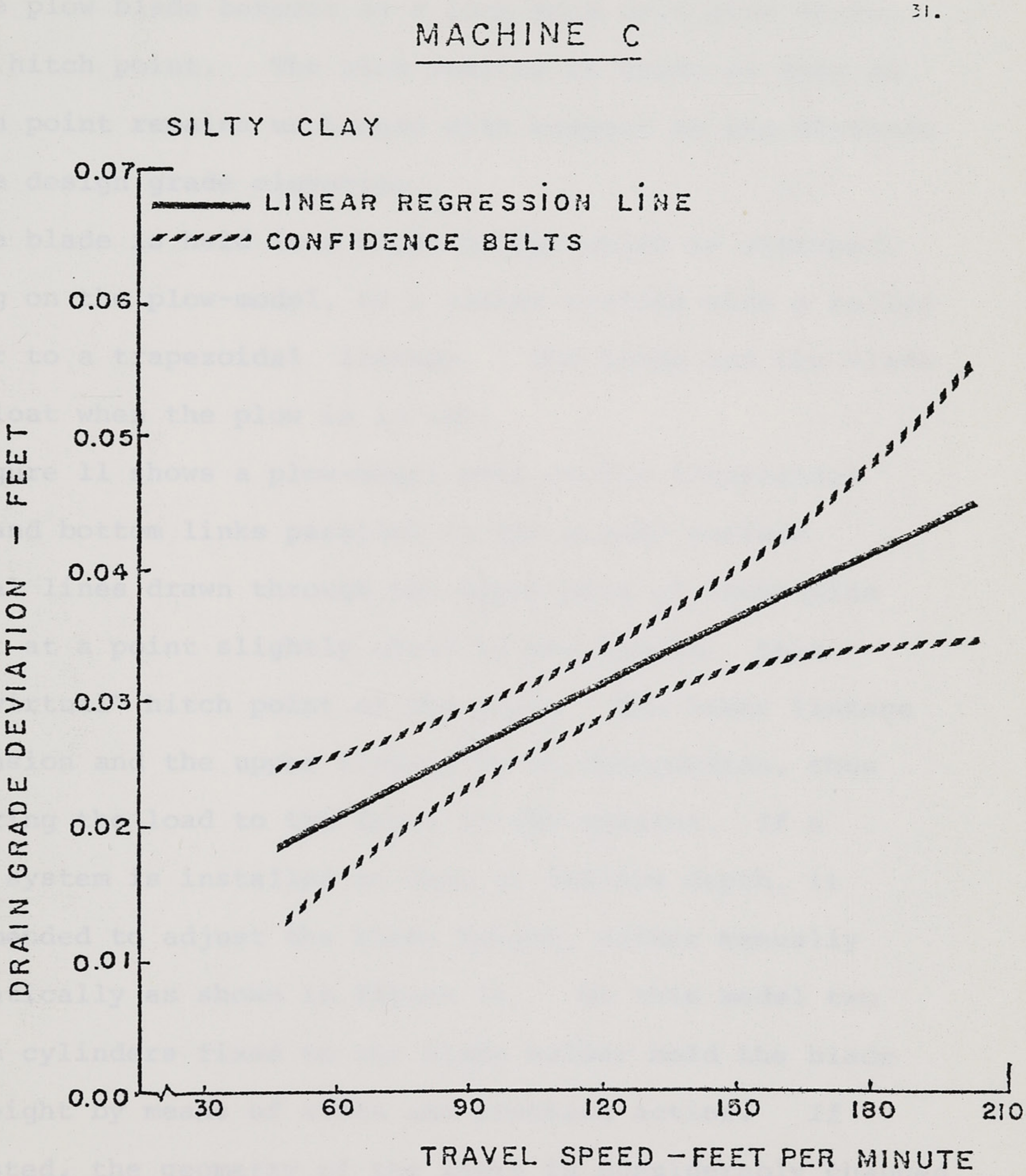


Figure 10. Travel speed-grade deviations relationship of Machine C in silty clay.

DYNAMIC STUDY OF A PLOW

Plow Action:

The plow blade behaves as a long beam male plow drawn from the hitch point. The plow remains on grade as long as the hitch point remains unchanged with respect to its distance above the design grade elevation.

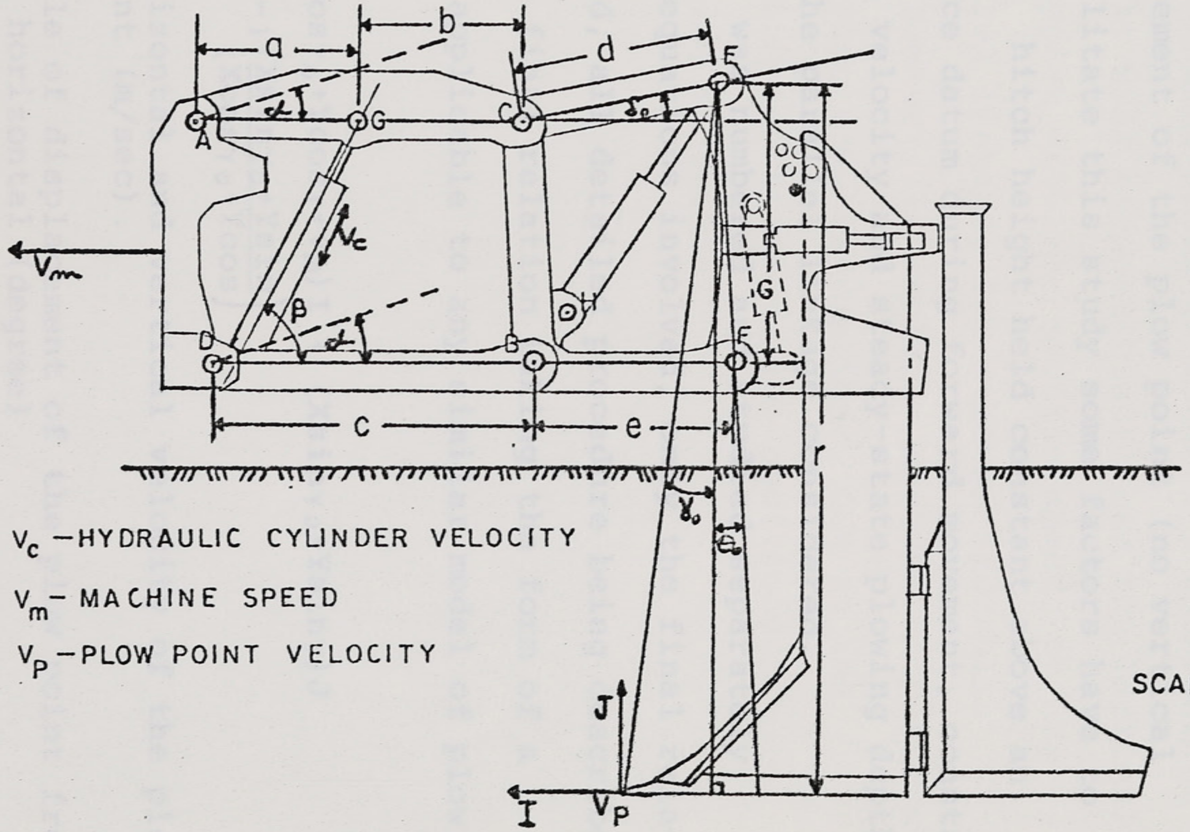
The blade is held in a blade holder which is attached, depending on the plow-model, to a radius casting with a roller system or to a trapezoidal linkage. The blade and the blade holder float when the plow is in use.

Figure 11 shows a plow-model with double trapezoidal linkage and bottom links parallel to the ground surface. Horizontal lines drawn through the upper pins or lower pins intersect at a point slightly ahead of the tracks; this is the "virtual" hitch point of the plow. The lower linkage is in tension and the upper linkage is in compression, thus transferring the load to the front of the tractor. If a drainage system is installed at deep or shallow depth, it is recommended to adjust the blade height, either manually or automatically as shown in Figure 11. On this model two hydraulic cylinders fixed to the blade holder hold the blade at any height by means of slots and pressing action. If not adjusted, the geometry of the links is considerably changed and the "virtual" hitch point may no longer be along the track area.

The front parallel linkage is attached to the grading ramp which corrects for the vertical movement of the tractor relative to the grade line. The hydraulic cylinder in the parallel linkage changes the relative position of the hitch point, and the one in the trapezoidal linkage assists in raising the plow behind the tractor for transport. When the plow is in use, it is in floating position.

PLOW - MODEL

WITH DOUBLE TRAPEZOIDAL LINKAGE



SCALE : 1. MM = 0.106 Feet

Figure 11.

Study Procedure:

A dynamic study (6) was performed to establish a relation between the plow point response, the machine speed and the hydraulic cylinder velocity.

Figure 11 shows a geometrical description of the plow-model studied at an initial position ($\alpha = 0^{o}$) corresponding to a horizontal displacement of the plow point (no vertical movement). To facilitate this study some factors have to be assumed, such as: hitch height held constant above an average ground surface datum during forward movement, constant machine and cylinder velocity and steady-state plowing depth at any position of the parallel linkage considered.

Each plow link was numbered and studied separately, but due to the many equations involved, only the final relationship is presented, all detailed procedure being described in Appendix A. The final relation taking the form of a generalized formula applicable to any similar model of plow, as follows:

$$V_P = (X\cos\gamma_o + Y\cos\gamma + v_m)I + (X\sin\gamma_o + Y\sin\gamma)J$$

$$\Theta_P = \tan^{-1}\left(\frac{X\sin\gamma_o + Y\sin}{X\cos\gamma_o + Y\cos}\right)$$

where V_P = Horizontal and vertical velocity of the plow point (m/sec).

Θ_P = Angle of displacement of the plow point from the horizontal (degree)

X, Y, γ - described in Appendix A

v_m = machine speed

γ_o - shown in Figure 11

Results and Discussion

Figure 12 shows the vertical displacement of the plow point per foot of ground travel at various machine speeds, at a parallel linkage position $\alpha = 7^{\circ}$. This figure shows that the vertical response of the plow point decreases exponentially as the machine speed increases. Considering initial assumptions, it will be possible to see the vertical response of the plow point with the machine speed, at any position of the parallel linkage, for a known hydraulic cylinder velocity. However, it would be arduous to perform this study without making some assumptions, due to the numerous soil conditions and tractor-plow positions encountered when working in fields.

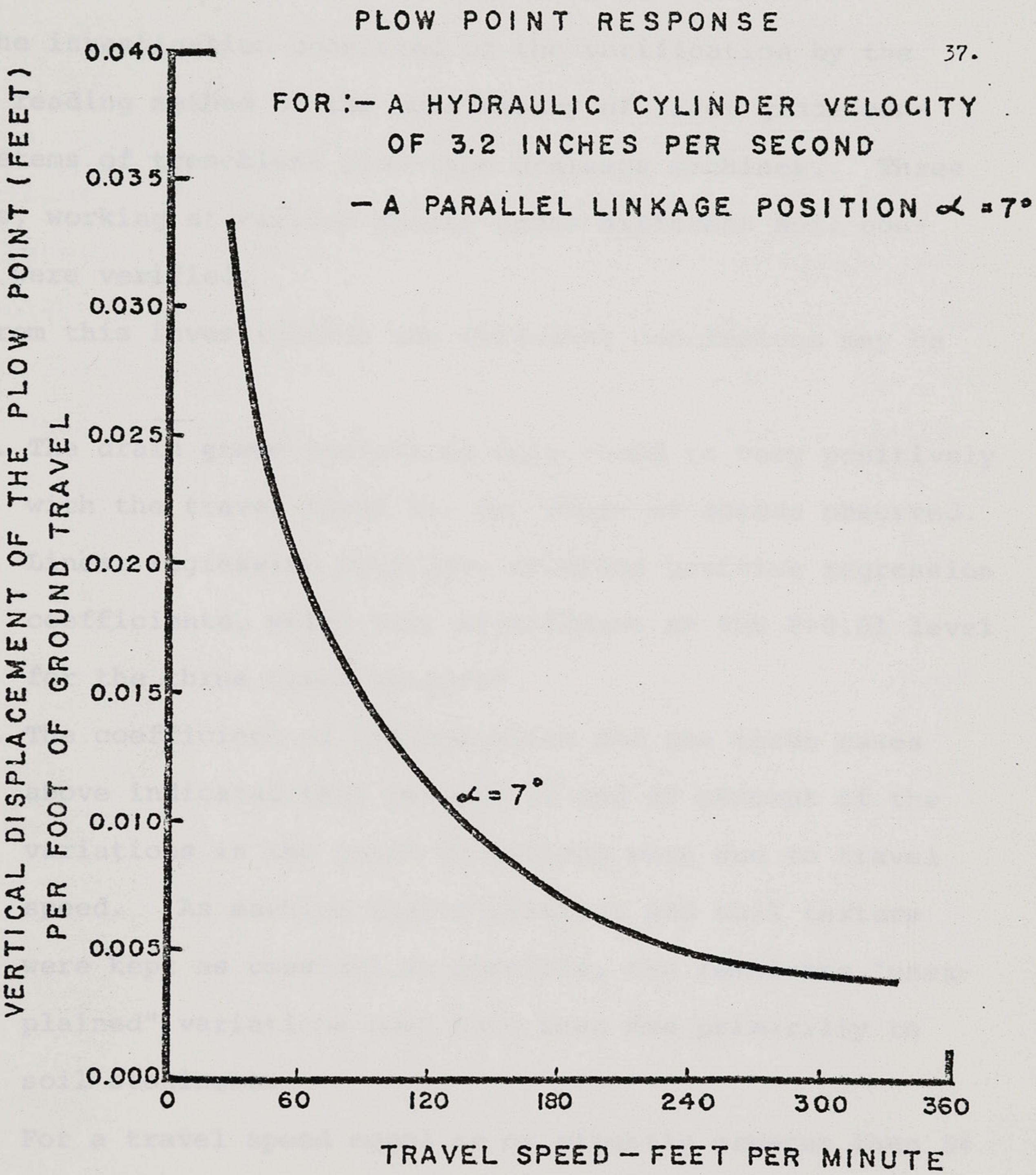


Figure I2. Plow point response with the machine ground speed.

SUMMARY AND CONCLUSIONS

The investigation consisted of the verification by the transit reading method of the consistency of laser grade control systems of trenchless plow-type drainage machines. Three machines, working at various speeds under different soil conditions were verified.

From this investigation the following conclusions may be drawn:

1. The drain grade variations were found to vary positively with the travel speed for the range of speeds observed. Linear regression equations provided positive regression coefficients, which were significant at the P=0.01 level for the three cases observed.
2. The coefficient of determination for the three cases above indicated that between 35 and 49 percent of the variations in the grade deviations were due to travel speed. As machine characteristics and soil texture were kept as constant as possible, the remaining "unexplained" variations must have been due primariliy to soil stoniness.
3. For a travel speed equal to or slightly greater than 54 m/min (180ft/min), linear regression equations obtained from the three cases predict that the maximum grade deviations within the confidence interval would be close to the maximum permissible grade deviation or 15mm (0.050 ft).

4. For a travel speed equal to or greater than 38 m/min (125ft/min) the maximum grade deviations would be greater than the permissible grade deviation or 15mm (0.050ft).

A dynamic study performed on a plow-model with double trapezoidal linkage, permitted the establishment of a relationship between the plow point response, the machine speed and the hydraulic cylinder velocity for any similar plow-model. Sample calculations showed that the vertical displacement of the plow point decreases exponentially as the machine speed increases for a constant hydraulic cylinder velocity.

RECOMMENDATIONS FOR FURTHER RESEARCH

Development of an electronic instrument, which when installed on the tractor unit will record all the grade deviations. This instrument should be adapted to the laser grade control system with the function to detect and record how far within certain limits the plow is above or below grade when the red lights on the command post are displayed. Both distance from main drain and grade deviations should be registered to facilitate drain pipe inspection, when grade deviations registered by the instrument would be greater than the permissible ones.

This instrument would be very useful in helping the operators to determine the optimum machine speed for different field conditions. Also, it would be of great interest to the farmers in insuring the quality of their drainage system installation.

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APPENDIX A

Detailed Procedure

All the calculations are refered to Figure 11:

1. Velocity and direction of the points E and F

$$V_E = (Y\cos\gamma + v_m)I + (Y\sin\gamma)J$$

$$\gamma_1 = \tan^{-1}\left(\frac{Y\sin\gamma}{Y\cos\gamma+v_m}\right)$$ - direction of the velocity of point E with the horizontal

$$V_F = (Y\cos\psi + v_m)I + (Y\sin\psi)J$$

$$\psi_1 = \tan^{-1}\left(\frac{Y\sin\psi}{Y\cos\psi+v_m}\right)$$ - direction of the velocity of point F with the horizontal

where:

$$\gamma = \tan^{-1}\left[\frac{(a+b)\sin\alpha - d\sin\delta_o}{L+d}\right]$$

$$L = \sqrt{\{(a+b)(\alpha-\alpha_o)\}^2-\{(a+b)\sin\alpha-d\sin\delta_o\}^2}$$

$$\Psi = \tan^{-1}\left[\frac{c\sin\alpha}{\sqrt{(\alpha c)^2-(c^x\sin\alpha)^2}+e}\right]$$

$$\alpha_o = \sin^{-1}\left(\left(\frac{d}{a+b}\right)\sin\delta_o\right)$$

$$Y = \frac{(a+b)\sin\beta \times v_c \times \sin(\alpha+\gamma)}{a \times \cos\alpha}$$

For $\alpha<\alpha_o \Rightarrow \gamma=0,\ V_E=v_m$ (machine speed)

2. Direction and velocity of the plow point P.

$$V_P = (X\cos\gamma_o+Y\cos\gamma+v_m)I + (X\sin\gamma_o+Y\sin\gamma)J$$

$$\theta_P = \arctan\left(\frac{X\sin\gamma_o+Y\sin\gamma}{X\cos\gamma_o+(Y\cos\gamma+v_m)}\right)$$ - direction of the velocity of point P with the horizontal;

Where:

$$X = \frac{r \times \cos\gamma_o \times |(V_F-V_E)|}{\cos(\theta+\theta_o) \times G}$$

$M = (Y\sin\psi) - (Y\sin\gamma)$

$N = (Y\cos\psi) - (Y\cos\gamma)$

$|(V_F - V_E)| = \sqrt{M^2 + N^2}$

$\Theta = \tan^{-1}(M/N)$

$\Theta_o, G, r, \gamma_o, \delta_o$ - (Shown in Figure 11)