A MODEL TO SUPPORT
RADIOGRAPHIC EQUIPMENT ALLOCATION DECISIONS
BY GOVERNMENT

A Model to Support Radiographic Equipment Allocation Decisions by Government

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A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Master of Engineering

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Montréal, Québec, Canada
August 1975

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ABSTRACT

In this work a linear programming production model of a diagnostic radiology department is presented. The model is intended to support the radiographic equipment allocation decisions pursued by government planners in Quebec.

This construct was developed from a set of model building guidelines which were derived from an examination of both the health care system of interest and the decision making environment in which this model is to be used.

It is shown that this diagnostic radiology department model is applicable to any such department in an acute hospital regardless of departmental size, estimates a feasible departmental capacity output, allows investigation of the effect of various resource changes on this capacity output, and can be used to calculate the technical efficiency of a department relative to the most efficient departments in the province. Moreover, the identical modeling procedure is applicable to medical departments in hospitals, other than diagnostic radiology, for which similar allocation decisions are required of government.

Information theoretic indices are proposed to quantify the resource mixes of diagnostic radiology departments. It is demonstrated by classifying radiology departments in terms of these information measures, applied to the radiographic equipment mix, radiology manpower mix and hospital diagnostic case mix, that the technical efficiency of a department is correlated with, if not conditioned by, a balance in these mixes.

In addition, an entropy index is presented which measures the inequality in the inter-departmental distribution of a resource, such as a particular type of radiographic equipment, and is shown to be capable of monitoring the allocation of such resources with respect to the government's policy of centralizing specialized health care resources.

RESUME

Ce travail présente un modèle de programmation linéaire représentant la production d'un département de radiologie diagnostique. Ce travail est destiné à faciliter aux planificateurs québécois l'allocation d'équipement radiologique.

Ce modèle a été établi à partir d'une série de critères basés sur un examen du système de santé québécois et du processus décisionnel qui y a cours.

Il est demontré que ce modèle peut être appliqué à tout département radiologique hospitalier à court terme quelqu'en soit la taille, qu'il estime une capacité de production dans un domaine réalisable, qu'il permet l'étude de l'impact sur cette capacité des variations du niveau de différentes ressources, et, qu'il peut être utilisé pour comparer l'efficacité technique d'un département particulier de radiologie à celle des plus efficaces au niveau provincial. De plus, une mise en modèle identique peut être appliquée aux départements hospitaliers médicaux, autres que la radiologie diagnostique, pour lesquels seraient requises des décisions similaires portant sur l'allocation des ressources.

Des indices théoriques de l'information sont proposés afin de quantifier l'intéraction des différents niveaux de ressources. En classifiant les départements de radiologie, en termes de ces indices appliqués aux ressources en équipement et personnel de même qu'au type de cas traités, il est démontré que l'efficacité technique d'un département est corrélée à un état d'équilibre de ces indices.

De plus, un indice d'entrpoie est présenté. Ce dernier mesure l'inégalité de la distribution des ressources départementales; tel un type particulier d'équipement radiologique. Il est démontré que cet indice permet la surveillance de l'allocation de telles ressources en rapport direct avec la politique gouvernementale dans le domain de la santé visant à centraliser les ressources spécialisées.

ACKNOWLEDGEMENTS

I wish to express my appreciation to Drs. C. A. Laszlo and M. D. Levine for the encouragement and critical appraisal each unstintingly gave throughout the course of this project and writing of the thesis. I am indebted for both the expertise they provided as teachers and the counsel they gave as friends.

I would like to thank the staff of Scidem Inc. (Montréal), and in particular Dr. J. M. Rouseau, whose collaboration and support made this work possible.

I am especially grateful to J. Cooper and E. Wong for having typed this manuscript, and to S. Schecter for proof reading the thesis.

Finally, I must thank the members of the Biomedical Engineering Unit at McGill University, my family, and friends for their generosity, patience, and encouragement.

A. J. Hosios

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NOTATION

Particular symbols will be defined where appropriate throughout the thesis. In general, bold-face majuscules will be used to denote matrices, and bold-face minuscules will denote column vectors. Light-face italics will indicate scalars. Transposition of matrices and vectors will be denoted by t's ($\mathbf{A}^t$, $\mathbf{a}^t$) and a matrix inversion by -1 ($\mathbf{A}^{-1}$).

CHAPTER 1

INTRODUCTION

All hospital operating costs, including those for medical and hotel services, research, maintenance and capital expenditures, are directly supported by the provincial government in Quebec. Of these costs, the federal government reimburses the provincial government for 50 percent of those supporting acute hospitals. This work concerns the decision making process in the provincial government for allocating funds for medical equipment acquisition within the hospital sector. In particular, we present mathematical tools directed towards expediting this decision process.

The project originated from a series of informal discussions held in 1973 with planners in the Ministry of Social Affairs (Government of Quebec). At that time they expressed a strong desire for some formal scheme to help them evaluate either the operation of the hospitals and hospital departments requiring new equipment, the merits of particular allocation decisions, or both. In addition, they indicated a growing concern by the Ministry for the costs, utilization, and effectiveness of different technologies in the health care system, and in particular, the equipment used in diagnostic radiology, therapeutic radiology, laboratories and surgery. This concern for medical technology underscores the need for evaluation techniques.

Specifically, this thesis contains an examination of the mechanism whereby the provincial government allocates funds to acute hospitals for radiographic equipment. In particular, we focus attention on those allocations which result from applications made directly to the government by diagnostic radiology departments. These requests for equipment funding are independent of the departmental operating budget.

Resource allocation decisions comprise the means for governmental health care policy implementation. These decisions require procedures for evaluation which must be consistent with established policy, yet attuned to a set of specific decision making situations. As evidenced by our discussions with the medical equipment planners, such procedures are all but absent from their decision making process. Yet the basic problem confronted by planners is the necessity of evaluation; that is, defining the merits and faults of equipment applications made by different diagnostic radiology departments as seen in the context of broad qualitative allocation guidelines elucidated in the government's health care policy.

The work in this thesis is directed towards some part of this problem by providing government planners with:

i) a mathematical model of a diagnostic radiology department which estimates the level of resource utilization in a department, can gauge the effect of a resource change (allocation) on a department's capacity output, and is applicable to any radiology department in an acute hospital, regardless of departmental size, and;

ii) an index which indicates whether the allocation of a particular type of radiographic equipment to a particular department is consistent with the government's policy of (de)centralizing (non)specialized resources in the health sector.

The principal part of this thesis is devoted to the derivation and estimation of the diagnostic radiology department model. The success of a model, or any quantitative tool aimed at improving a decision making process, is measured by the extent to which it is actually used by decision makers. This will reflect upon both the representational credibility of the model, and equally important, whether the model responds to the demands of the decision making

environment in which it is to be implemented. These demands encompass the need to provide information directly relevant to the decisions that must be made, and the requirements for integrating an analytic construct into the intellectual process of making a decision.

The decision making environment in government for allocating radiographic equipment is examined in Chapter 2 with the express purpose of establishing model building guidelines that will facilitate the eventual implementation of a model in this setting. To this end, we discuss the existing mechanism for allocating equipment to radiology departments and the specific decision making criteria that are used. Subsequently, decision making is described as a multi-phase process, and a particular facet of this process is delineated for which a decision aid (model) can successfully be developed, as well as the concomitant model building guidelines requisite to introducing a construct to this particular part of the decision process.

The operation of diagnostic radiology departments, radiology facilities' planning methodologies, and the data base that was provided for this project are described in Chapter 3. From this discussion, we propose a set of features which a diagnostic radiology department model must exhibit in terms of: i) the properties of the system under investigation; ii) the type of information it should supply to planners, and; iii) a general model format which is required by the limitations of the available data.

Chapter 4 presents the theoretical derivation and estimation technique for a mathematical model of a diagnostic radiology department model which satisfies the implementation criteria described in Chapter 2, and demonstrates the desired system characteristics elaborated upon in Chapter 3.

The quantitative estimation of the departmental model from

a sample of observations is pursued in Chapter 5. Prior to the identification of this model, we present a detailed investigation of the data upon which it is to be based, with particular emphasis on the validity of using parts of this data in terms of the basic assumptions underlying the model. Next, a method is described for discriminating between models derived from different data sets, and for choosing the most suitable representation (data set) of a radiology department. This method is unique with respect to the proposed model estimation technique. And finally, twenty-two feasible models are presented, each of which has the same mathematical form, are estimated by the same technique, yet are based on slightly different data sets. From this set of models, two are chosen as the more credible representations of a diagnostic radiology department.

Two output measures per radiology department can be derived from each of the two models cited above; that is, a department's capacity output, and a measure of departmental resource utilization. An investigation of these measures is provided in Chapter 6, and consists of: i) a sensitivity analysis of the capacity output, and; ii) the establishment of a scheme to quantitatively determine the sources of departmental resource utilization. As part of this latter effort, we construct indices to measure the heterogeneity of the variables used in the models, and then classify or group departments in terms of these measures. In addition, it is demonstrated that one of these indices can be used to monitor the allocation of technological resources in the health sector with respect to the government's policy of (de)centralizing (non)specialized resources.

Finally, in Chapter 7 we summarize the major results of the

thesis, discuss a number of directions in which the work should be extended, and provide suggestions for related studies.

Our goal throughout this project has been to apply the methods most appropriate to the realities of a problem, rather than subvert the nature of a problem to a set of predetermined techniques. As a result, we have used concepts from a variety of disciplines, such as management science, medical economics and electrical engineering. Since the background of our readership will be equally diverse, the fundamentals of each of the approaches that have been adopted are presented. As some readers will find parts of this material familiar, we indicate where a section may be omitted by the reader with the appropriate background.

CHAPTER 2

MODELS AND THE DECISION MAKING PROCESS

2.1 Introduction

All too often, the process of building models for decision making is portrayed as evolving through two successive and independent stages: development and implementation. In recent years we have witnessed an accelerating growth in the number of technical journals, books, conferences and seminars dedicated to espousing the state-of-the-art of model building and computer-based decision environments. Hence, one can see that the bulk of discussion amongst analysts has revolved about the developmental stage. For example, from January 1971 to June 1973 *Management Science: Application* contained over 150 articles of which less than 3 percent represented implementation in an organization (Urban, 1974).

The neglect of implementation criteria and constraints in the development process can be one of the greatest stumbling blocks to the success of a model. As Little (1970) has candidly stated, "The big problem with management science models is that managers practically never use them." Much of the difficulty lies in the alienation of the decision maker from the decision construct. What is seen as a "good" model from the analyst's point of view may not be appreciated as such by the decision maker.

> "Convincing models that include the manager's control variables and so contain direct implications for action are relatively difficult to build, particularly in the areas that are of greatest concern. ... People tend to reject what they do not understand. The manager carries the responsibility for outcomes. We should not be surprised if he prefers a simple analysis that he can grasp, even though it may have a qualitative structure, broad assumptions, and only a little relevant data, to a complex model whose assumptions may be

> partially hidden or couched in jargon and whose parameters may be the result of obscure statistical manipulation. .... The manager feels that to get deep into the model and find out what is really going on is totally out of the question because he lacks the time or background. The solution to this predicament is often to pick on some seeming flaw in the model, usually a consideration left out, and make that the basis for postponing use into the indefinite future." (Little, 1970)

This situation can be remedied by tempering the development of a model to the needs of the organization in which it is to be used. These needs encompass both the information required to make specific decisions and the prerequisites to integrating an analytical construct into the decision making process of individual planners.

In this chapter we examine the decision making environment for the allocation of new equipment to diagnostic radiology departments in Quebec. Our goal is to pinpoint the major decision areas, and those problems most attuned to the introduction of information derived from a mathematical construct.

It should be noted that many of the problems in allocating radiographic equipment to hospitals are common to those of the other hospital departments for which the same government agency is responsible. Hence the following description will remain of a general nature unless the specifics of diagnostic radiology are uniquely appropriate.

2.2 Allocation of Equipment to Hospitals

Major sites of medical instrumentation in a hospital include the departments of diagnostic and therapeutic radiology, laboratories and surgery. Corresponding to each, there is a group of individuals in the government that is responsible for the allocation of new equipment funds to all institutions in the province. Their decisions, rather than being performed in a vacum, are conditioned, at least explicitly, by two

outstanding factors: i) a series of independent decisions outside of government in the hospital sector, and; ii) the decision environment within government.

2.2.1 Initiation of the Decision Process. There are three sources of funds available to hospitals in Quebec for the purchase of equipment: i) philanthropy; ii) the global hospital budget which is approved and supplied by the government (Anon., 1971), and; iii) direct equipment grants from the government (Anon., 1972f). We shall only consider the latter two sources, as they are directly sponsored by the provincial government.

If the supply contract for the desired equipment is less than 1 percent of the operating costs of the department, it may be included in the fixed capital expansion section of the departmental budget (Anon., 1972k, 1974a). This budget must be approved as a total entity, first by the hospital manager and then by the Ministry of Social Affairs (Anon., 1972i). If the equipment cost is greater than the 1% criterion, direct authorization from the Minister must be requested. This is only for equipment to be used in the following fields: diagnostic or therapeutic radiology; nuclear medicine and isotopes; data processing; automation of laboratories; hemedialysis; electronic monitoring equipment for patients receiving intensive or coronary care; and anaesthesia and resuscitation (Anon., 1972g). In addition, above $50,000, tenders and other special procedures are required.

Although our concern is with the Ministry's problem of operating on these direct requests, i.e., outside departmental budgets, their origin is of sufficient relevance to merit examination.

2.2.1.1 Local Decision Criteria. The decision to acquire new equipment originates with the perception of a need for change. The basic forces converging in this process are common to all managerial decisions, as articulated by Easton (1973):

> "There may be an intense dissatisfaction with the existing state of affairs or merely an urge for the improvement of a good condition. The motivation for change may arise

within a single individual or it may be imposed on the decision maker by situational or organizational imperatives."

Thus, a technological environment which was deemed acceptable in the past, is now found wanting. The source for this transformation may be considered as one of two phenomena: functional degradation of equipment or super-structure change.

Terborgh (1949) examines two types of functional degradation, quantitative and qualitative. While his concern is for an industrial environment, the approach is equally applicable to medical technology in a non-profit institutional setting. Both types of degradation are described in an absolute as well as a relative sense. In the first case, there may either be a noticeable decrease in the amount of services per appropriate unit of time, or a deterioration in the quality of services provided by a particular machine. In the second instance, either one of these variables may appear to decline relative to the superior service available from competitive modern substitutes; that is, those available on the medical equipment market.

Discrimination between real or imagined functional decline of equipment is substantially more difficult in a medical environment. Complementing those problems faced by business concerns in making investment decisions, there are the added problems of: i) a high degree of equipment specialization and low standardization (Brown and Lowell, 1972), and; ii) efforts on the part of manufacturers to enhance product differentiation, though sometimes only superficially (Peterson and MacPhee, 1973). This situation is further aggrevated by the large number of suppliers and dealers of medical equipment. Yet this, in turn, reflects the historical trend towards proliferation of specialized equipment in diagnostic services by the medical community (Anon., 1970b).

The services of the diagnostic radiology department, as with most ancillary departments in a hospital, are not sought directly by patients. Traditionally these specialized services are utilized on behalf of the patients by physicians. Thus, these services are demanded as a part of a

larger process through which the citizen seeks the maintenance of his health or the treatment of disease. Hence we define the super-structure of a radiology department to be the enveloping hospital, i.e., inpatient facilities, outpatient clinics, all physicians outside the hospital referring their patients for diagnostic procedures, and the community that utilizes this hospital system.

Either one of two modifications in this structure can ferment a desire for a technological change. First, a shift in the demographic or epidemiological profile of the community may alter either the volume of patients, diagnostic mix of patients, or both, presented to the hospital system. This, in turn, may result in a somewhat different volume and/or mix of services demanded of the radiology department. Second, a change in the hospital itself, such as in the admissions policy, size of the emergency department or clinics, or the number of beds, might also impose a new demand situation on the department. Hence, the decision maker in radiology is faced with the problem of, at best, anticipating future needs, a situation wrought with uncertainties and random fluctuation or, at worst, attempting to rectify a "crisis" development.

When the decision has been made that a particular type of new equipment is required, the problem of selecting from the many different brands arises. The actual application for equipment to the government does not require an indication of specific suppliers, unless the particular type of machine is unique (Anon., 1972*l*; see section 2.2.2). In all likelihood though, the specifications of the request are written with a particular brand in mind. In evaluating medical equipment manufacturers, the decision maker in diagnostic radiology must confront: i) the problems of product differentiation, as described above; ii) the unique properties of major medical equipment which often preclude real competitive bidding (Muller, 1971), and; iii) the need to consider eventual parts replacement, where most probably only the products of the original vendor will be compatible (Muller, 1971).

The completed application, which essentially is a rationalization of the need for new equipment, is then sent to the government. In this

manner, the decision making problem has been shifted from a local to the global environment, and a new decision process initiated.

2.2.2 The Decision Environment. The following description of the allocation problem and problem solving environment is not intended to be a detailed representation of the actual situation, but is meant to provide a clear picture of the task facing the decision makers. As discussed in the previous section, direct requests for new equipment are deposited with a department in the government dedicated to handling this type of allocation problem. In order to appreciate the constraints imposed upon their "decision space", we will examine this group within the context of the Ministry of Social Affairs. Fig. 2.1 depicts an extraction from the Ministry's organizational flow chart, indicating the relevant departments and sub-departments.

Applications for medical equipment are processed through two successive stages (Dahman, 1975). First, they are delivered to the group responsible for major medical equipment in Resource Development (see Fig. 2.1). This group's function is to evaluate each request; that is, to define the merits and faults of an application for a particular type of machinery in terms of the needs of the applying hospital department, and subsequently to recommend its acceptance or refusal.

If an application is refused, the process stops there, but if accepted, the second stage is initiated and is pursued by a group in Medical Apparatus. They must decide, in collaboration with the applying hospital department, on the specifications of the equipment. This would include the brand, dealer, supply contract, size and power requirements, and all other details relevant to the purchase of the equipment and its integration into the present hospital technological environment. In the case of an accepted application, the complete two-stage process will take approximately 8 to 9 months (Dahman, 1975).

We will focus our attention on the evaluation process pursued in Resource Development because it is felt to have the greater decision problems, and in addition, is a prerequisite for the second stage.

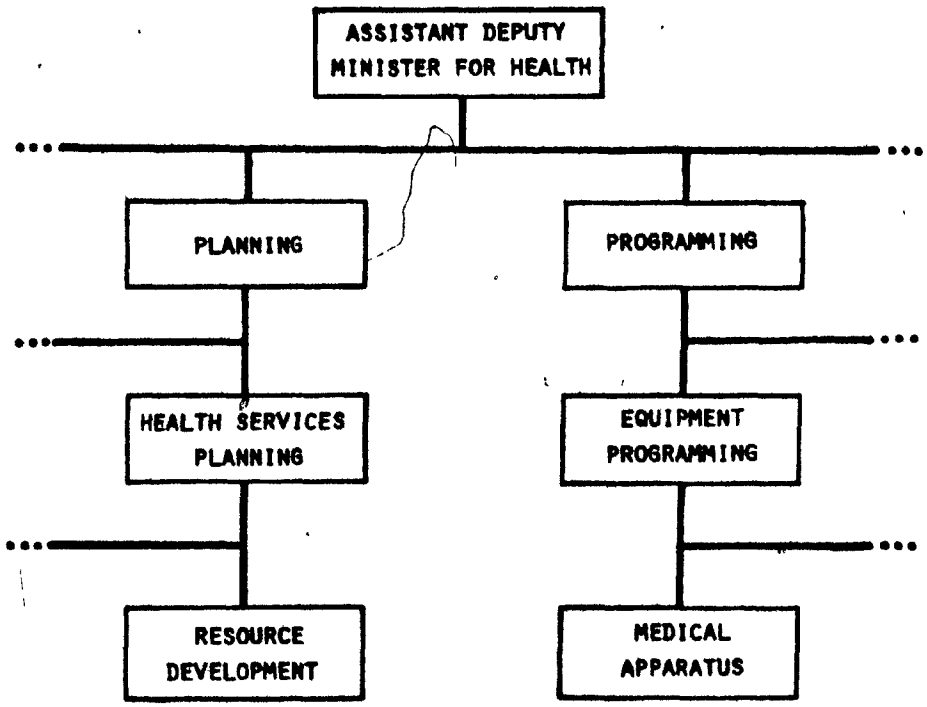


Fig. 2.1. Extraction from the Ministry of Social Affairs (Government of Quebec) organizational flow chart showing those departments involved with medical equipment planning (Anon., 1974b).

2.2.1.1 Global Decision Criteria and Decision Making. The policy vis-a-vis resource allocation in the health sector adopted by Planning, and hence Resource Development, is essentially the same as that proposed by the Quebec Commission of Inquiry on Health and Social Welfare (Anon., 1970a). The main thrust of their report was towards efficiency in allocation, as seen in the context of disparities in the distribution of facilities, spiraling costs, and limited availability of funds. The means chosen to achieve this goal were, and are: centralization of specialized services; decentralization of non-specialized services (in particular primary care facilities); and matching community needs with resources. In sum and substance this forms the essential philosophy towards equipment allocation.

The adoption and implementation of policy may be seen to occur at various levels within government. On descending an organizational hierarchy we see the scope, length of time for, and time horizon of decisions decreasing (Mesarovic *et al.*, 1970). On might argue that Cabinet policy, with respect to long term objectives for society, is implemented over a period of years by the assignment of Ministerial budgets. Ultimately, levels are reached within government where resources are distributed directly into the society. In addition to the scope and time factors cited above, the decision variables can differ significantly at various decision making strata. This is particularly true of Resource Development in the context of the overall management of the health care system.

For example, in the allocation of hospital budgets, the locations are known but not the dollar amounts. For the most part though, and excluding wage settlements which usually affect more than one hospital, the new budget for a hospital is usually not significantly different from the previous year's budget. In Resource Development, the locations are unknown *a priori* they differ significantly from year to year, and are financially conditioned solely in terms of the total government budget for all new equipment.

The allocation of budgets for new medical equipment over several years, relative to the other government health care budgets, may reflect a long term goal for the health care system. Yet each year, a series of

decisions must be made within the confines of budget and consistent with the overall planning policy, having an immediate effect on the institutions involved, and possible long-term consequences for at least the regional delivery of health care.

2.2.2.2 Decision Making. In a broad sense, almost any decision making or evaluation process can be characterized as an interpretation of present structure in the context of what is known of the past and desired of the future (Gore and Dyson, 1964). Such decisions may be considered to be either programmed (routine) or non-programmed (innovative). Simon (1960) provides a succinct definition of these terms:

> "Decisions are programmed to the extent that they are repetitive and routine, to the extent that a definite procedure has been worked out for handling them so that they don't have to be treated *de nova* each time they occur. ... Decisions are non-programmed to the extent that they are novel, unstructured and consequential. There is no cut and dried method for handling the problem either because it hasn't arisen before or because its precise nature and structure are elusive or complex, or because it is so important that it deserves a custom tailored treatment."

Above all, the distinguishing feature is that routine decisions yield predictable results, whereas non-routine decisions involve a large degree of uncertainty (Gore and Dyson, 1964).

While most decisions fall somewhere between these two extremes (Edstrom, 1973), equipment allocation and, in fact, many health planning decisions lie towards the non-programmed end of the spectrum. These uncertainties reflect the planners' incomplete knowledge of the operations of the health care system, in addition to the non-deterministic behaviour of many of the underlying processes composing this system. The non-programmed nature of the decision making process in Resource Development is indicated by the complete absence of any mechanism for converting

planning guidelines and policy into an operable format. The planner's need for a yardstick to compare and evaluate equipment applications is acute.

Two features of non-programmed decision making are most relevant to the present discussion. First, in this type of problem solving a large proportion of the total time is involved with information gathering (Edstrom, 1973). Numerous examples of this exist in the planning for diagnostic equipment allocation.

If an application is felt to be incomplete, or if more specific information is required of the radiology department, then a dialogue must be established between the hospital and government. Data may be required on either the present equipment inventory, departmental superstructure, or both, and possibly similar data on other hospitals for comparative purposes. Most of this information can only be acquired from sources outside Resource Development in the Provincial Government (Depts. of Planning, Programming or Finance in the Ministry of Social Affairs) or Federal Government (Statistics Canada and the Dept. of National Health and Welfare). Sometimes the required data may take from several months to a year to accumulate. For example, the high priority project on a province-wide inventory of radiographic equipment (Anon. 1972a) started in 1972, and took over a year to compile and another year to computerize.

A second type of information is introduced in the form of outside expertise. Although Medical Apparatus is not officially involved in the evaluation process, their opinion may be sought on some facet of the present technological setup of an applicant. In addition, the Quebec College of Radiologists may be consulted by either Resource Development or Medical Apparatus, if some aspect of a request needs further clarification.

The second major feature of non-programmed decision making is the difficulty in handling non-comparable criteria (Edstrom, 1975). The tendency is to emphasize those variables that can be quantified in some

common unit of measure, such as the dollar. This often results in the implicit assignment of lesser importance to either qualitative considerations or those variables that can be measured, but in dissimilar units. This problem becomes particularly acute when comparing the costs of a device, its diagnostic capabilities, and the "needs" of the radiology department super-structure. The problem also arises when comparing these same criteria between applicants. In addition, there is difficulty in translating the centralization/decentralization policy into a means of measuring the degree of equipment specialization, and judging the suitability of adding a particular type of machine to a particular hospital.

After Medical Apparatus has established the specifications for each of the accepted applicants, a budget must be established to fund these requests.

Budgeting for new equipment involves a one year lag. Each fiscal year, the previous years budget is used as a guideline for accepting applications. After all the accepted requests have been examined by Medical Apparatus during that year, the Equipment section in Programming (see Fig. 2.1) proposes a budget for the following year to fund the accepted applications.

The proportion of new equipment funds to be given for diagnostic radiology (vs surgery, laboratories, etc.), is dependent both on the relative number of applications and the previous years sum. For 1972-73, 24 percent of the new equipment money was allocated for diagnostic and therapeutic radiology (Anon., 1972c). Yet if Health Services Planning as part of its grand scheme, feels that a change should be made in either the total amount, or the amount for a particular class of new equipment, it can institute this policy through a directive to Resource Development and by subsequently putting its weight behind the resulting budget proposed by Equipment.

To summarize then, the problem-solving environment in Resource Development can be characterized as the application of a weakly-defined strategy (exemplified by the difficulty in quantifying goals and objectives), on

an uncontrolled input (equipment applications arising from the hospital sector as a function of a decision process, independent of government) under fuzzy constraints (demonstrated by the variability in budget).

2.3 Integration of a Model in the Decision Process

In discussing the allocation problem solving environment we delineated programmed from non-programmed decision making, having recognized that equipment allocation generally falls within the latter category. In the context of this framework there are two outstanding features: i) the large amount of information gathering and processing requisite to decision making, and; ii) the difficulty in handling multiple (and often non-comparable) decision criteria. In this section, we examine the decision making process in somewhat greater detail. This will allow the identification of the most effective means of introducing a formalized quantitative construct to this process.

2.3.1 The Decision Process. Decisions have classically been viewed as the choice among alternative courses of action in which the decision maker can generate all possible alternatives, their consequences, and has a mechanism available for choosing an optimal course based on a single criterion (Edstrom, 1973). Recognizing the inadequacy of this "rational" scheme, Simon (1958, 1960) divides the decision making process into three phases:

1. *the intelligence-phase:* searching the environment for conditions calling for decisions,
2. *the design-phase:* inventing, developing and analyzing possible courses of action,
3. *the choice-phase:* selecting a particular course of action from those available.

Simon's view differs from the classical definition in two fundamental ways. First, neither the problem and the alternative actions nor their consequences may be given, but instead must be determined by analysis. Second, a choice based on a single criterion is often unrealistic, especially where intangible consequences may arise. In this situation, the search is for a satisfactory rather than an optimal solution.

In the previous section we observed that the problems which Resource Development must confront are (in the form of applications) generated in the hospital sector. Although this sector is outside the direct operational domain of government, the process of applying for medical equipment is at least partially conditioned by legislation (see section 2.2.1). Hence, the intelligence-phase, or at least the process of examining the raw data for potential problems, is absent in the medical equipment funding decision process in the Ministry.

For each problem or application, the set of alternative actions is defined as a binary choice: acceptance or refusal. We have previously suggested that these decisions, both individually and collectively, are subject to a process of evaluation. In particular, a request may be refused by its failure to meet certain criteria, either in an absolute sense or in a relative sense, when competing for part of the overall new equipment budget with applications in the same or possibly different equipment class.

If we now contract our description of the evaluation process in Resource Development to be just that, an evaluation of the merits and faults of applications, we can readily associate this process with Simon's design-phase. The process of weighing the merits and faults of an applicant, or comparing these same factors between applicants, in order to resolve a decision (course of action), is encompassed by Simon's choice-phase.

2.3.2 <u>The Choice Phase</u>. The inputs to the choice-phase are some selected outputs from the design/evaluation phase per application. At

present, some indicators do exist to monitor and evaluate the performance of diagnostic radiology departments. Unfortunately, they are primarily geared to in-hospital planning and, as such, they either require a detailed data set unavailable to government or are so crude as to provide little new information. In addition, they are independent of the Quebec scene. These measures will be discussed in greater detail as part of the review of quantitative planning approaches for diagnostic radiology departments in Chapter 3. For the most part, though, these choice phase inputs are of a qualitative nature, indicating more a sense of approval or disapproval than anything else.

Assuming that some meaningful quantitative measures could be attached to each application, then the choice-phase can be seen as a maximization of some multiple governmental utility function, through an allocation of application acceptances subject to resource (budget) constraints. This could, in theory, be programmed as part of the decision process. However, it became apparent during the early stages of this project, that pursuing such a line would be a fruitless venture.

A global decision making program such as described above was abandoned for two reasons; one inherent to the construction of this type of mechanism, and the other relating to its probable utilization in a decision environment.

Shuman *et al.*(1974) have tried to explain why these operational research methodologies have failed to result in useful tools for regional health planning. Their basic critique, after reviewing numerous techniques and philosophies, is the tendency of investigators to dissociate the analysis from the realities, including the goals and operational properties, of the system under investigation.

> "....the problems that have been attempted have not been well formulated; the objective functions have been naively constructed and the measures of effectiveness yield suboptimizations; quantitative variables are considered of prime importance, but behavioral variables are neglected."
>
> "Typical constraints, e.g., plant capacity or available capital, which can be represented by well known relations do not necessarily result in solutions that are politically or socially feasible."

Hoos (1972) provides a damning critique of the "track record" of these approaches in the public sector, of which health planning is of course a subset:

> "....form takes precedance over and even determines content ...This leads to a chain reaction of poor conceptualization, gathering data more because it is available than indicative, and dependence on factors only because they can be counted in the ongoing analysis and not because they are known to be important in the final analysis."
>
> "....a technological conception of a problem limits the focus to those aspects which can be expressed quantitatively and which fit certain models. The technological solution which results may be satisfactory from the engineering point of view but because it has encompassed only a few selected facets, vital dimensions may have been neglected. Such violation of the essence of the problems may, in the long run, exacerbate rather than ameliorate the troublesome condition."

We have already alluded to some of these problems in our previous discussion on the difficulty in quantifying the allocation goals of the Department of Planning. A machine that may be specialized in one

radiology department may not be in another hospital. If centralization of specialized services is considered a good thing, then to what degree? Is the situational demand for a facility defined strictly in terms of the spatial distribution of the population or are the socio-economic characteristics equi-determinant?

Unlike the bulk of cost-benefit studies for health program investment, e.g., a syphilis control program or a dialysis unit (Klarmen, 1974), it is not always possible to define a specific diagnostic class of patients that would utilize a particular piece of radiographic equipment. In fact, radiography is one of the most generally used diagnostic tools in almost all medical specialties.

In addition, the global perspective of a regional planning scheme is attuned to policy decisions, rather than operational considerations (Shuman *et al.*, 1974). As we have previously indicated, policy decisions are not explicitly the concern of Resource Development. Furthermore, such a regional planning system could not accommodate implementation of policy beyond a one-year period, as allocations are confined to equipment applications which represent an uncontrolled input from the hospital sector per annum.

Even if a viable construct were developed, implementation would still be open to pitfalls. It is obvious that such an algorithm would be tremendously complex, at least in terms of any one decision maker's own perception of the health care system. This can lead to overconfidence in the capabilities of an allocation scheme, which cannot possibly account for all the qualitative goals of planning. Hence, an element of danger can be introduced to the decision making process "simply because the clarity and apparent objectivity of quantitatively measureable subordinate goals can easily lead to failure to bear in mind that they are in fact subordinate" (Boulding, 1966).

An even greater possibility would be the complete neglect of the decision system by planners. The reasons for this can vary from an incomplete understanding of the construct, to the observation of a missing variable in the construct, to the feeling that by using a

decision making program their personal integrity and viability in the organization will suffer.

It is apparent, then, that a decision making system as embodied in a medical equipment allocation scheme would pose tremendous difficulties for development and implementation. As yet, we have only discussed a scheme which operates on many, if not all, of the applications made in a particular year. By operate, we of course mean the same process pursued in the choice-phase; that is, the assignment of a utility measure to each application and the subsequent choosing of a subset of applications to maximize some global utility function. The next question which would naturally arise is: if a global scheme is inappropriate, could a decision making scheme that would deal with only one application be more readily devised?

Having posed the question, I criticize it as missing the point. That is, the development and implementation problems we have discussed are quite independent of the mandate of a decision making construct; whether for one or many applications. Therefore the choice-phase, which essentially amounts to the assignment of acceptance or refusal labels to an application, or group of applications, is an inappropriate process to quantize in a formalized construct.

The first phase of the decision process, i.e. the intelligence-phase, has been shown to proceed independent of the government and thus is not available for quantification. In the following section, we examine the design-phase in order to identify a suitable facet of this component of the decision process in which to quantitatively intervene.

2.3.3. The Design Phase. Morton (1971) has further divided Simon's design-phase into three subphases, in order to reflect the emphasis on information gathering and scanning which are endemic to an evaluation process. These subphases are:

(a) *generation* of input data for the manipulation subphase,

(b) *manipulation* of the data to arrive at some appropriate input for the selection subphase, and

(c) *selection* of some output to go to the next (Choice) phase.

In section 2.2.2, we discussed the different types of information (raw data, reports, consultations), and their sources, which Resource Development acquires in the course of evaluating equipment applications. This defines the generation-subphase. Of particular importance is the diversity of agencies, departments, and groups who must be approached, and the resulting variations in data volume, accuracy and time required to aggregate.

The lack of a unified data base is a source of inefficiency in the process. This can be appreciated both in terms of the time factor and the direct relationship between effort and detail. While we accept that computerization, in the form of some information storage and retrieval system, would ultimately result in a more efficient generation-subphase, it does not necessarily follow that it would be more effective in terms of the overall decision process. There is a significant difference between increasing the amount and accessibility of raw data, and increasing the amount and quality of information relevant to solving a problem.

Our discussion of the manipulation and selection subphases has been rather scant, owing in large part to the present state of the evaluation process or design-phase. I would describe this "state" as the general absence of a formalized scheme to investigate and 'manipulate' the data in order to derive information upon which a decision can be based. This weakness in the design-phase mirrors the lack of quantitative organization in the choice-phase.

In addition to the indices previously mentioned (and to be examined in Chapter 3), simple input/output ratios are often constructed in order to evaluate the performance of a department. A typical example would be the average number of daily diagnostic radiology examinations performed in each x-ray room. A particularly interesting problem arises when deriving measures based on a single variable, and which can only reflect one facet of an applicant's performance. This is the natural tendency to 'select' the particular measure that seemingly provides the

maximum amount of information; that is, the index which is most deviant from the norm or expected value. This can lead to inappropriate appraisals.

It was decided that broadening the scope of the present manipulation-subphase would add greater depth to the selection-subphase, and hopefully strengthen the subsequent choice-phase. We shall introduce quantitative instruments that encompass more than one variable and can be applied in an unbiased fashion to all applicants. The advantage of focusing on the manipulation-subphase is that we can both provide "new" information and yet minimize the difficulty in implementing a scheme; that is, this additional information will not attempt to encompass the qualitative facets, and will not infringe on the authority of the decision makers, which is really exercized in the selection-subphase and choice-phase.

The goal of the design-phase is to provide information to support the acceptance/refusal of an application. Currently missing is a means to evaluate the performance and estimate the effect an expansion of facilities would have on the department under investigation. To fill this void, a mathematical model of a diagnostic radiology department will be constructed that can provide both types of information.

Complementing the criteria relevant to model purpose and scope, explicit development criteria should be set. Little (1970) has proposed such a list. He suggests a model be i) simple, ii) robust, iii) easy to control, iv) adaptive, v) as complete as possible, and vi) easy to communicate with.

> "By simple is meant easy to understand; by robust, hard to get absurd answers from; by easy to control, that the user knows what input would be required to produce a desired output; adaptive means that the model can be adjusted as new information is acquired; completeness means that the important phenomena will be included ...; and easy to communicate with means that the manager can quickly and easily change inputs and obtain and understand outputs."

The last element in this set relates primarily to the medium of implementation and will not be explicitly considered in this thesis. The five remaining criteria will be investigated in greater detail in Chapters 4 and 5 when describing the derivation and estimation of the model.

In accepting this list of guidelines as basic to effective model building, we also recognize that they are quite independent of the particular situation at hand. Since this project arose from the problems of a real decision environment, the model must: i) deal solely with data that is presently available, and; ii) be sufficiently versatile to be applicable to all hospital radiology departments in the province. We have noted that the planners in Resource Development are responsible for new equipment allocation beyond radiology. Thus, it is desirable that the chosen methodology be applicable to some of their other decision problems.

2.4 Summary

In this chapter we have described the decision making environment for the allocation of new equipment to diagnostic radiology departments, determined the point at which the integration of a quantitative approach can be most successful, and established a set of guidelines for the development of an appropriate methodology.

Throughout our discussion, we have conspicuously omitted that sometimes determinant, yet innocently underrated facet of decision making: the exercise of power and influence. We may safely assume its existence, or at least the potential for it in all organizations. But what of it? In terms of our analysis, it will suffice to acknowledge the political element and recognize the validity of our actions independent, and possibly in spite of, its presence.

Looking beyond the particulars of this discussion, it is apparent that we have touched on an area of growing importance in our society; that is, the problems of direct government intervention (investment) in

a process operating independent of the central authority. These processes may vary in scope from encompassing only one sector, to directly or indirectly affecting the welfare of the population at large. Their common thread is the legislative umbilical cord. Health and social services immediately come to mind, as they are, for the most part, supported by government. Although in recent years we have seen the other end of the spectrum (labour, industry and commerce, natural resources) come into focus.

We have witnessed a two-fold effort at improving the decision making required in the health sector. Many of the techniques of management science and industrial engineering, which were originally developed for the profitable operation of business concerns, are now used for more effective planning and coordination of activities in individual institutions, such as clinics and hospitals. Concurrently, economists and others are increasingly contributing quantitative analyses for policy making in the health sector. Caught in between are the departments and agencies of government who provide input (allocations) directly into a system which often appears as a "black box", and for whom the techniques of neither group have as yet to be fruitfully applied.

This project is directed towards the decision making problems of such a group, wherein the ultimate success or failure of our goals and policies lie.

CHAPTER 3

THE DIAGNOSTIC RADIOLOGY DEPARTMENT

3.1 Introduction

The previous chapter centered about the decision making process for allocating equipment to hospitals. It sufficed therein to consider diagnostic radiology as one of many departments in a hospital which uses specialized equipment for the provision of services necessary to patient care. This chapter will provide a more general description of a diagnostic radiology department.

Model building often appears to involve a trade-off between the inclusion of all system features deemed important, and those properties which are identifiable from the data. In fact, a model must include all the important system features. Data may provide information on the functional relationships among variables, but usually not all of the features can be described fully. Thus assumptions must be made. The trade-off that will be introduced is as follows: the more one assumes, the more inaccurate is the resulting model, and hence, the less trusted are the model's predictions. We fully recognize that this may be the price for not having enough data.

In the context of the present examination of diagnostic radiology, i.e. the system and its properties, we include both a review of radiology facilities' planning approaches and an introduction to the available data. With this discussion and the model building criteria from the previous chapter, a complete framework will have been established in which to propose a model in chapter 4.

3.2 The Diagnostic Radiology System

3.2.1 A New Kind of Rays. On December 28, 1895, Wilhelm Rontgen announced his discovery of an unknown type of rays (which he labelled x-rays) in a paper entitled: "About a New Kind of Rays" (Van Der Plaats, 1969). From rather humble and hazardous beginnings, clinical applications of these "unknown rays" have proliferated into the medical specialty called radiology (Knowles, 1969). As a result of scientific and technological advances, this field has divided into three subspecialities: diagnostic radiology; therapeutic radiology; and nuclear medicine.

> "The medical utilization of radiology has today progresses to the point where there is scarcely a human disease in which the science of radiology is not employed in diagnosis, in the evaluation of the efficiency of its treatment by medical or surgical methods, or in its treatment per se." (Dowdy, 1964)

The remainder of this discussion is concerned specifically with diagnostic radiology.

Radiography is the making of a record or photograph by means of the action of x-rays on a sensitized surface (Jacobi & Paris, 1968). A radiogram (or roentgenogram) is a record of the differences in density in the part of the body through which these rays have passed (Robbins, 1972). Hence we define diagnostic radiology as the nondestructive search for, or investigation, of human pathology through the use of radiological techniques.

3.2.2 The Radiographic Subsystem. Diagnostic radiology is but one of a group of departments in a hospital whose function is to generate highly specific information for patient care. In particular, the activities of this department are primarily geared

to the interpretation of the radiograms for the physician in charge of the patient. This service requires the cooperative effort of radiologists, a team of x-ray technicians, and a supportive staff of file clerks working in a highly technological environment. While the radiologist is ultimately responsible for the analysis of a radiogram, the preparation of the patient, the taking of the x-ray, the development of the film and all associated clerical tasks are contributed primarily by other people as shown in Fig. 3.1.

The radiology departments of general hospitals provide only diagnostic services. The spectrum of activities pursued by radiology departments in teaching hospitals includes, in addition, teaching, training and research (Drui, 1970). Teaching is generally conducted in either large acute or specialized hospitals, and therefore the volume and mix of diagnostic radiology services will normally be greater or more complex than those of non-teaching institutions.

Figure 3.2 shows the interrelationships of the radiology department in the Massachusetts General Hospital (Boston) with the rest of the hospital. Radiology departments in all hospitals may be mapped in a similar fashion using some subset or modification of these links depending on the specific administrative and medical structures. This description emphasizes the information generating nature of this department, and its dependency on the hospital superstructure for definition of the range of services it must provide.

Lindheim (1971) categorizes the numerous activities that take place in the provision of diagnostic services into six groups;

1. those that get the patient and x-ray machine together,
2. those required for the x-ray examination,
3. those involved in retaining the x-ray,
4. those required for the diagnosis,
5. those involved in the transmission of the diagnosis to the proper medical authority,
6. those relating to the storage and retrieval of the information obtained.

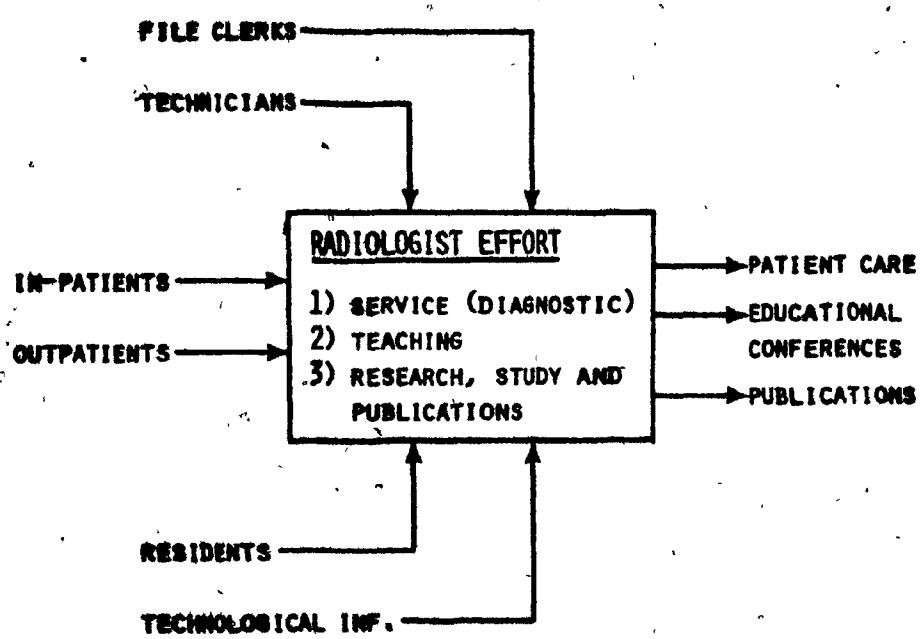


Fig. 3.1. Radiologist input and output in a teaching hospital (Drui, 1970, p. 1126).

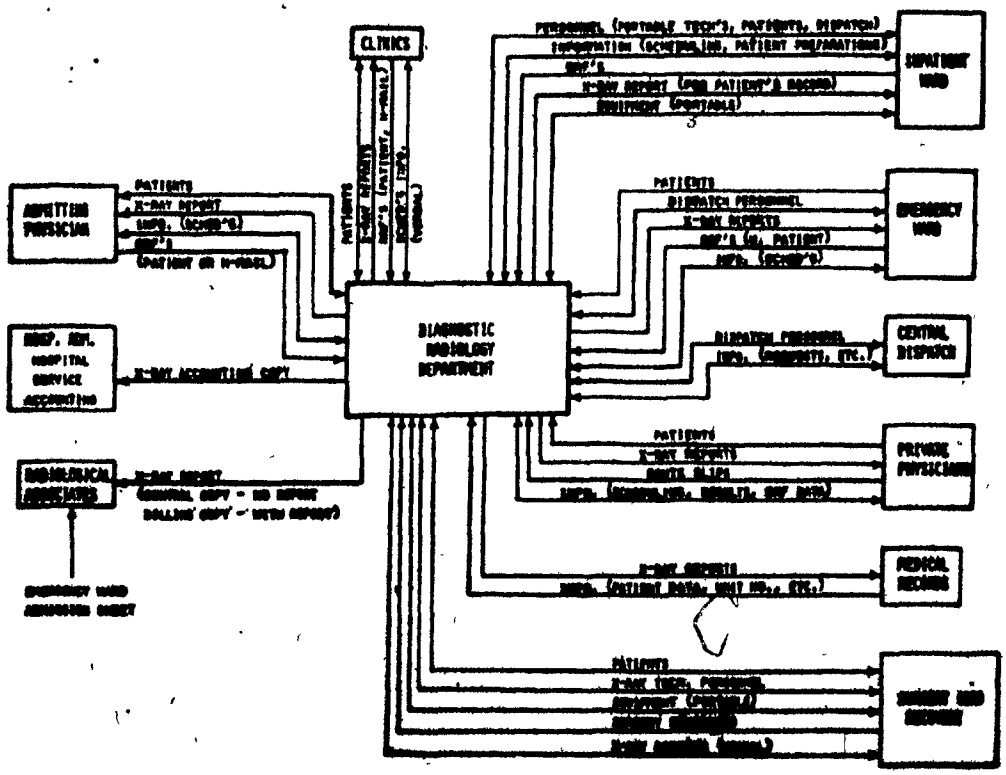


Fig. 3.2. Inputs and outputs for the radiology system. The complexity of the interrelationships of the radiology department to the hospital environment is illustrated. Note: RRF=Radiology Request Form for consultation (Ferrucci *et al.*, 1970, p. 19)

Fig. 3.3 illustrates these six activities diagramatically.

Several important features can now be observed. First, within this set of six activities, two distinct processes emerge; one limited to the actual taking of x-rays (activities 1 to 3), and the second involved with the processing of information (activities 4 to 6). Sullivan (1971) characterizes this by labelling the former process as the radiographic subsystem within diagnostic radiology. He then defines diagnostic radiology as a larger system: radiography plus interpretation of x-rays plus provision of consultations to referring physicians. In light of our concern for equipment allocation, we will focus our attention on the first of these processes as it includes the site for equipment usage and the environment for man/machine interaction.

We should note that depending on the size of the department, there may be one or more different rooms in which activity 2 is pursued. These "x-ray rooms", by virtue of the equipment set up they contain, may be dedicated to handling general or specialized procedures, and often may be used for both. In addition, only one patient is examined in a room at a time.

In Fig. 3.3 the radiologist is explicitly depicted as performing two functions. First, the radiologist or resident is usually present at the x-ray table either to supervise or actively participate in the more complicated procedures. Secondly, he is solely responsible for the interpretation of x-rays. Recognizing that the radiographic subsystem involves a complex interaction of labour, material and technological resources, we see that in addition to the functions cited above, the management role of the radiologist is critical to the smooth operation of the department.

Excluding physicians, the three major input resources to the department are the technical staff, materials, and equipment. Both the radiological examination and the processing of film require

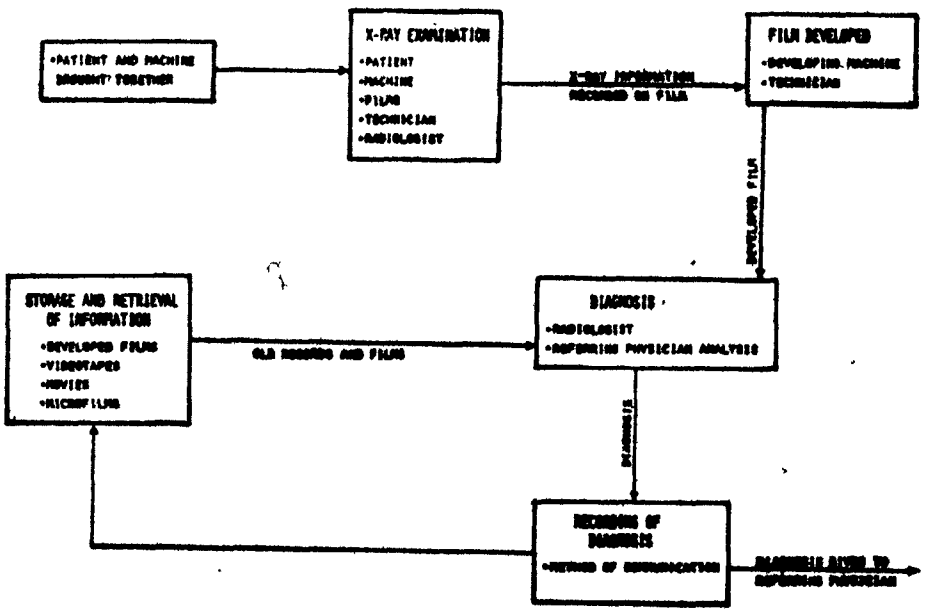


Fig. 3.3. Flow of activities in a diagnostic radiology department, indicating the six basic groups of activities (Lindbeim, 1971, p. 56).

specialized equipment, which in turn utilize different materials. It is obvious that to produce an x-ray both of these resources must simultaneously be consumed; material in terms of a quantity and equipment in terms of time. For each examination, the amounts of each are dependent on the nature of the procedure and the capabilities of the machinery. Film is processed in a separate room, with one such room usually servicing many x-ray rooms. Film may be replaced by continuous video recording during an examination for special procedures.

The technician is responsible for most of the film-taking (Lindheim, 1971). He must study the request for what is to be done, get the patient on the table, see that the correct films are available, adjust the machine, read the films for adequacy, consult with the radiologist, and clean up the room after each examination. Because of their training with respect to equipment maintenance and operation, technicians form a critical component of the system. This was evident from the data on hospital radiology departments in Quebec, where all departments are seen to have at least one technician on the payroll.

As the four inputs to the radiographic subsystem of diagnostic radiology, radiologists, non-MD staff, materials and equipment, form a necessary and integral part of the whole, each should be present in some explicit form in a mathematical representation of a department.

3.3 Radiology Facilities Planning

3.3.1 Source of Concern. In recent years there has been a growing interest in the planning of radiology facilities. Lindheim (1971) attributes this interest to both the medical role and operational requirements of diagnostic radiology: i) it renders services essential to the operation of other departments, such as surgery, emergency, medicine, and orthopedics; ii) it requires relatively large numbers

of personnel whose activities are interdependent; iii) it is utilized by patients in all degrees of sickness and health, and; iv) it requires a major outlay in terms of plant and equipment. In addition, two related trends have catalyzed concern: the accelerating demand for radiological services and a growing awareness of the inappropriate use of some of these services.

The burgeoning number and the increased complexity of consultations requested has resulted in the need for major facility expansion and/or modernization of some departments every 7-10 years (Robbins, 1964; Copestake, 1967). A study in France showed that from 1959 to 1972 the number of examinations provided expanded at more than 10 percent per year, and that from 1967 to 1972 all of Europe saw a total increase of well over 50 percent (Couder & Sandier, 1974). In 1966, the American College of Radiologists estimated an annual rate of growth in excess of 7.2 percent (Anon,, 1966). One author (Sullivan, 1971; Sullivan & Thuesen, 1971) estimates a doubling of radiographic procedures every eight to ten years, with work loads doubling every two to four years. These figures are not radically different from the Canadian scene. At the Winnipeg General Hospital, MacEwan (1973) documents a 60% increase in radiology examinations from 1962 to 1970. From 1971 to 1972 alone, the number of insured examinations performed by private radiologists in Quebec increased by 44 percent (Anon., 1973).

Some part of these increases may reflect medical trends, such as the introduction of screening programs, or the development of new technologies (Rourke, 1968; Lindheim, 1971). Despite these allowances, concern has been voiced both within and outside the radiology profession about the inappropriate use of services (Knowles, 1969; McClenahan, 1970; Gellman, 1971; Middlemass, 1972; Rousseau, 1974). MacEwan (1973) estimates utilization at about 30 percent greater than necessary. Campbell (1973) claims the number of radiologic studies in the last decade to be out of proportion to either population increase or patient visits by 4 to 1. He

cites failure to discriminate high from low yield exams, unnecessary repetitive and follow up studies, failure to prevent duplicate exams, substituting radiologic for physical exams, lack of cost deterent and abuse of routine examinations to rule out unlikely possibilities as major causes of facility overutilization. A good part of these problems are reflected in a study by Childs (1970) which concluded that "patterns of chest examinations may vary with physician characteristics, independent of diagnosis and other patient characteristics."

3.3.2 Government Intervention The sources of facility overutilization cited by Campbell are a function of the present day practice of medicine, and as such, are most amenable to changes within the medical community itself. Typical examples would be increased peer review and educational improvements. A financial incentive may utlimately prove to be the most productive tool in discouraging waste. Yet this mechanism strikes to the very core of health care planning, and hence would require a major policy initiative and thus is not easily implemented.

Discouraging overconsumption of services or inappropriate utilization of facilities is not within the mandate of Resource Development. As described in chapter 2, Resource Development is charged with the evaluation of equipment applications and hence cannot directly intervene in, or influence the medical practice of radiology. The question then arises as to whether the expansion of facilities in a radiology department, as provided by the acceptance of equipment applications, is related to the overconsumption of services.

Overconsumption implies that radiographic facilities are being utilized to a certain extent merely because they are available. This phenomenon is implicitly encouraged by the third party payment scheme we have in Quebec which effectively eliminates a market mechanism for controlling utilization. It is apparent then, that

increasing the availability of facilities through an expansion of the equipment environment of a radiology department may explicitly sponsor and implicitly encourage inappropriate usage. Obviously this will not be the case for all departments, and so, how can one determine when it might?

For some interval of time, such as a month or a year, the input resources to a radiology department (see section 3.2.2) may be considered fixed. The volume of services which the department can supply during this period will depend on the extent to which these resources are actively utilized. Yet each of the inputs can only be consumed to a finite extent. For example, the operation of equipment is limited by either the number of hours in a day or the manpower work schedule. Therefore some finite maximum service capacity must exist per department over the constant input time interval.

An application by a radiology department for new equipment is effectively an application to increase the service capacity of that department. This in turn is a reflection of either degradation in the present technological environment or an expected increase in the demand for services (see section 2.2.1.1). If a department which is utilizing most or all of its present service capacity expands its facilities, we would expect that the bulk of its additional capabilities would be consumed by the present or expected level of demand.

Let us assume that some departments are operating at well below their capacity level. If one of these departments is allowed to increase its equipment input, then the availability of facilities in that department will increase as well. Because it originally had excess availability, as observed by the underutilization of its capacity, then the expanded capabilities of the department will not be consumed by the expected needs of the department. As well, it may eventually be inappropriately consumed by virtue of its availability. This would be observed as an overconsumption.

If the utilization of the service capacity of a radiology department can be measured, then this information can be used by Resource Development, through its ability to accept or refuse equipment applications, to check eventual overconsumption of services in a particular department. Therefore the primary thrust of the diagnostic radiology department model will be towards the perception and quantification of non-capacity behaviour in a department, and thus will provide this previously unavailable information to planners.

Very few departments can be expected to fully utilize their service capacities. A study by the Nuffield Provincial Hospitals Trust (Anon., 1962) assumed "that a usage of 70 percent.... can be achieved with good organization." Hence the model must estimate a realizable capacity, which in fact is some percentage of the actual capacity, rather than a theoretical and unattainable level.

3.3.3 Radiology Facilities Planning. Traditionally, radiology department planning, both within the hospital sector and government, has relied heavily on prototype designs prepared by government or radiographic equipment manufacturers (Taylor *et al.*, 1959; Langlois, 1973). The size of a department is usually specified in terms of a desired ratio of x-ray rooms to hospital beds for hospitals of different size and orientation (Anon., 1950). This approach has been criticized because it ignores the emergency and outpatient departments which may have special radiology service needs (Lindheim, 1971; Altman, 1973).

A large number of studies have addressed themselves to the problem of radiology department size, design and location in relation to the total needs of a hospital (Reeves, 1959; Anon., 1962; Cusdin, 1964; Shaw, 1964; Wheeler, 1964; Scott, 1966; Anon., 1968; Lindheim, 1971; Sullivan, 1971; Sullivan & Thuesen, 1971). The common theme throughout the literature is the interest in planning

at the institutional level. This may readily be appreciated both historically in terms of hospitals' past autonomy from each other and from the government, and because of the very real need for detailed systematic design procedures for such a complex man-machine environment.

Typically analysis is confined to one or two institutions wherein a large data collection effort must be pursued. Lindheim (1971) provides a list of the type of information that is requisite to calculating the number of diagnostic radiology rooms required by a given hospital:

i) the nature of the population to be served;
ii) the number of inpatients, outpatients, and emergency patients who will use the facility immediately as well as in the future;
iii) the types of examinations each group is likely to require;
iv) the length of time required for these examinations;
v) the expected type of utilization for each room, and;
vi) the hours the radiology department will be open.

Other data of relevance would include: the peak loads; the time required to develop various films and analyze different procedures; and the physical and locational setup of the radiology department with respect to patient flow and proximity to other hospital departments. The spectrum of time intervals for which this type of data would be collected may vary from a per diem to a yearly basis.

A detailed review of the techniques used in the studies cited above is not necessary here. Typically, the analytic tools presented in these studies are geared to the utilization of a unique data collection effort. Hence the viability of the resulting schemes is directly dependent on the availability of the appropriate information. The type and detail of information we have discussed, per department over time, is presently not available to government (see section 3.4).

We decided against pursuing data beyond that available directly from the government for two reasons: i) it would require a tremendous amount of time to investigate and possibly visit each diagnostic radiology department in an acute hospital in Quebec, and; ii) our experience with the government's data gathering efforts indicates that a large number of omissions and errors would accompany new data (see chapters 5 and 6).

Adopting a scheme that is not attuned to our own data base would, as discussed in section 3.1, require the introduction of assumptions, which in turn would reflect on the credibility of the model. In addition, most analyses result in some prescription, such as the number of x-ray rooms per institution, and as such are really decision making schemes. In chapter 2 we ruled out the introduction of a decision making program to the allocation process in Resource Development as unrealistic.

Hence the unique characteristics of radiology facilities planning for hospitals pursued by government, i.e. the global decision perspective and the data availability, require the development of a new analytical approach. We present such a technique in the next chapter.

3.3.4 The Role of Government in Facilities Planning. Detailed design and planning of departments, radiology or otherwise, should not be the responsibility of a central authority. Two factors would promote failure. Regardless of the amount of data that could be collected, only those individuals working in a department on a day to day basis can fully appreciate their own problems and needs. To impose upon this group a design which does not coincide with their perceptions of need, would be foolish and possibly catastrophic. Equally important is the emphasis placed on individual decision making within the medical profession. To infringe upon the sovereignty of the radiologist could not only be politically dangerous, but would in sum, be the same as telling a manager how

to manage - not a good idea.

The Quebec Government fully recognized the faults inherent in detailed centralized planning. It therefore established a legal framework in which the decision of how, when, and where to expand a department's facilities lies solely with the department itself (see section 2.2.1). In order to guide the health care system, the government has reserved the right through its control of the purse to pass final judgement on such plans. It should be recalled that applications are a function of a cost criterion (see section 2.2.1), which presumably reflects the magnitude of the effect a technological expansion will have on an institution.

In essence, this approach by government reflects the difference between a planned and a planning society (Bergwall *et al.*, 1974). Rather than dictate how the health care system must evolve and operate, the government has chosen to pursue an interactive approach or dialogue in which both planning and society are allowed to evolve simultaneously.

The present application form for new diagnostic radiology equipment basically asks for an inventory of labour and equipment, a brief review of the procedures performed in the last year, what specialized departments there are in the hospital, and a run down of the proposed equipment purpose and cost (see Appendix A). This data is presented as a list of facts and figures with no unifying theme or semblance of cause and effect.

A major lightening of the governments decision burden could be achieved by establishing a radiology facilities planning procedure that could be independently executed by each hospital. This would allow presentation of a more detailed and organized equipment application to government, and would as a side effect, bolster the governments data base. In addition, this type of program

would encourage a greater and more uniform "rationality" in planning at the local level.

3.4 The Data Base

Three data sets were merged to form the data base for this project. The years for which they were collected, their sources, and a general description of their content is shown in Table 3.1. A detailed description is provided in Appendix B.

On combining the three data sets it was found that hospitals present in some were absent from others. A total of 95 acute hospitals were common to all three sets, with an additional 44 contained in only D.1 and D.2 for a total of 139 hospitals. All hospital data was recoded to maintain confidentiality.

The two most important features of the data base resulting from the merger of the three data sets are:

i) it consists of observations on a cross section of radiology departments (only those in acute hospitals), most of which is at a single point in time, and;

ii) the data is essentially of an input/output nature, that is, we are given a detailed description of the input resources to the system and what is produced, but rarely how the resources are utilized in providing services and for whom.

The data sets D.1 and D.2 were derived from annual surveys taken by the federal and provincial governments respectively, whereas D.3 resulted from a survey conducted outside the normal data collection mechanisms of the provincial government. Both D.1 and D.2 report information for 1971. The period 1971-1973 for D.3 indicates when the data was collected, yet as shown in Appendix B

DATA SET	YEAR	SOURCE	DESCRIPTION
D.1	1971	Statistics Canada & Department of National Health and Welfare, Government of Canada, Ottawa, Ontario.	Diagnostic Radiology Department Finances, Facilities & Services
D.2	1971	Department of Planning, Ministry of Social Affairs, Government of Québec, Québec, Québec.	Hospital In-Patient Data and Radiology Department Finances and Services
D.3	1971-73	Programming Department, Ministry of Social Affairs, Government of Québec, Québec, Québec.	Diagnostic Radiology Department Equipment Inventory

Table 3.1. The three data sets that were merged to form the data base for this project.

much of the information is dated, such as when a particular x-ray room was established. The only information of a time series nature is the number of examinations and exposed films per year from 1968 to 1972.

Even the most cursory examination of the data reveals a "black box" appearance for both radiology departments and hospitals. As to the radiology departments, we are given the financial, technological and labour inputs, and a set of outputs depicting what was produced or the services provided. For the rest of the hospital we have the volume of inpatients in each of 98 diagnostic classes, their average length of stay diagnosis, and the financial inputs and services provided by 22 different departments in the hospital.

The basic problem in utilizing this data base is its inconsistency; that is, the merging of micro- and macro-data, neither of which is complete. For example, we are given the number of inpatients but not the number of outpatients, and yet are given the number of radiological examinations provided to each group. The rest of the data, such as the number of man-hours, machine-hours and films, are not broken down as to a recipient group. We know the number of each of a variety of different machines and of different x-ray rooms, but not which machines are in which rooms.

It is apparent that all the data cannot be used. The mixture of gross and detailed information, of time related and stationary (for one year) data, necessitates choosing some consistent subset upon which to base a model. This can be accomplished by using data on the departmental inputs and outputs over a one year period. The resulting diagnostic radiology model must then represent some mapping from an input set to an output set, and hence, will require neither quantitative nor qualitative assumptions as to the internal operation of a department.

3.5 Summary

Within this chapter and Appendix B all the requisite information has been provided for building a model of a diagnostic radiology department.

We have investigated the general operation of the diagnostic radiology department: discussed the recent concern for radiologic facilities planning; indicated how the government could improve the application forms for new equipment in order to more readily achieve its goals, and; introduced the data base available to this project. From this discussion we proposed three features which a model should demonstrate:

i) representation in some form of the four basic departmental input resources (radiologists, technicians, materials and equipment);

ii) the ability to estimate an attainable service capacity as well as the utilization of that capacity for a department, and;

iii) represent a mapping of the input resource set into an output service set per annum.

The next chapter is devoted to the derivation of a diagnostic radiology department model using the above criteria and the guidelines of the previous chapter.

CHAPTER 4

A MODEL FOR DIAGNOSTIC RADIOLOGY DEPARTMENTS

4.1 Introduction

In this chapter we present a mathematical model which can be applied to the diagnostic radiology departments represented in the data. Before doing so, we review exactly what information the model is to be capable of providing.

In Chapter 2 we examined the decision making environment in government for the allocation of new equipment to hospitals. It was decided that the most profitable input to this decision process would be a model to evaluate the performance of a department and estimate the effect of an expansion of facilities or a change in the input resources. We elaborated on the first of these criteria as an underutilization of facilities in Chapter 3, and described the importance of its perception in terms of the accelerating consumption of radiological services.

Incrementing the input labour or technological resources of a department will not in itself change the amount of services provided by the department. Rather, it will affect the capacity of the department for an increased volume or mix of services. This service capacity can also be used as the standard to which observed department output can be compared to indicate an underutilization (non-capacity) performance. It would appear, then, that both the underutilization of facilities and the effect of resource changes can be monitored in a model that represents a mapping of inputs into capacity outputs.

In section 2.3.3 we stated that this function or mapping should be simple and easy to understand, and include all important system phenomena. This can imply a trade-off, for both these goals may be difficult to achieve simultaneously. A compromise has been resolved by using the concept of a production function, which at once is basic to understanding any process for transforming inputs into outputs, and can be made to

include most, if not all, the important variables and their relationships.

Given the nature of our data base (see section 3.4), and the prohibitive time required to search out more detailed information on all departments, one production function will be estimated from the entire sample of departments. Individual production functions per department cannot be derived with the present level of data. Hence we decided that more information on the operation of diagnostic radiology departments can be secured by examining the complete sample within a common framework, that is, a single production function.

The resulting construct will naturally emphasize the common properties or behaviour of the departments in the sample. As such, it will be applicable to all radiology departments. This, in fact, is consistent with our original goal of providing the decision makers in government with a scheme in which any and all departments can be examined and compared.

4.2 Production Theory

The purpose of this section is to provide an introduction to production theory. No claim is made to a detailed investigation of the subject matter, but rather to the establishment of a framework in which to specify a model. The reader familiar with the concept of a production function and its analysis may proceed, without loss, to the next section.

4.2.1 Production, Inputs and Outputs. Almost all discussions of production classify the producers' decision variables into only two categories, inputs and outputs.

> "An input is simply anything which the firm (*producer*) buys for use in its production or other processes. An output is any commodity which the firm produces or processes for sale." (Baumol, 1965)

As Baumol indicates, the terms production and processing are used in the general sense of a transformation, rather than specifically implying

a literal physical operation on raw materials to yield physical final products. Frish (1965) provides an elaboration of this concept,

> "The term transformation indicates that there are certain things (goods or services) which enter into the process, and lose their identity in it, i.e., ceasing to exist in their original form, while other things (goods or services) come into being in that they emerge from the process. The first category may be referred to as production factors (input elements) while the last named category are referred to as products (the output of the resultant elements)."

Typical production decisions involve: the total amount to be spent on all inputs and how to divide this sum among the different inputs; and the allocation of certain quantities of each input to the production of each type of output and how much of each final product or output to produce (Baumol, 1965).

4.2.2 The Production Function. Decisions on how much of each input to consume and the amount of output to be produced cannot be pursued independently, but are dependent on specific engineering or technical information; that is, the state of technology available to the firm. It is apparent that if the techniques of production should change, as by the discovery of a new process or the introduction of an invention, then the output obtainable from given inputs will increase. Yet at any particular time "there will be a maximum obtainable amount of output for any given amounts of factor inputs" (Samuelson, 1970).

This technological relationship between inputs and outputs is called a production function, and has been defined by Samuelson (1970) as follows,

> "The production function is the technical relationship telling the maximum amount of output capable of being produced by each and every set of specified inputs (or factors of production). It is defined for a given state of technical knowledge."

Mathematically a production function is indicated:

$$Y = f(X_1, X_2, \ldots) \qquad (4.1)$$

where Y is the output, the X_i are the inputs, and f represents the relationship between inputs and outputs determined by the technology. Thus Eqn. 4.1 states that Y is the maximum amount of output y which can be produced with X_i units of input x_i, $i = 1, 2, \ldots etc.$

Equation 4.1 implicitly includes a time variable, i.e., both the dependent and independent variables represent a rate of flow. The exact time period encompassed by the production function will depend on the data from which it estimated. Typically, X_i and Y may represent a weekly, monthly, or yearly input and output to a production process.

It is sometimes stated that the production function is a fictitous concept, unused by either engineers or managers. Brown (1966) counters this by indicating first, that it is possible to derive such a relationship from purely engineering data and second, that even if this were not the case "the employment of production functions can be justified simply on the grounds that it produces highly useful and verifiable hypotheses". He then points to the physical sciences for examples of a "theory which postulates relations and quantities that cannot be immediately measureable, but which, nonetheless, produces valuable hypotheses".

4.2.3 The Production Surface. In this, and the next two sections we elaborate some general properties of production functions by following the discussion and examples of Baumol (1965, pp. 252-259). The specific production example he develops is the simple case of two inputs (man-hours and yards of cloth), and one output (aprons).

If for any quantitative combination of the two inputs, such as OM_R of labour and OC_R of cloth, as represented by the point R in Fig. 4.1, we know the maximum quantity of output or aprons to be RR', then we can map the production surface or function by examining all such combinations. In Fig. 4.2 a portion of this surface is shown.

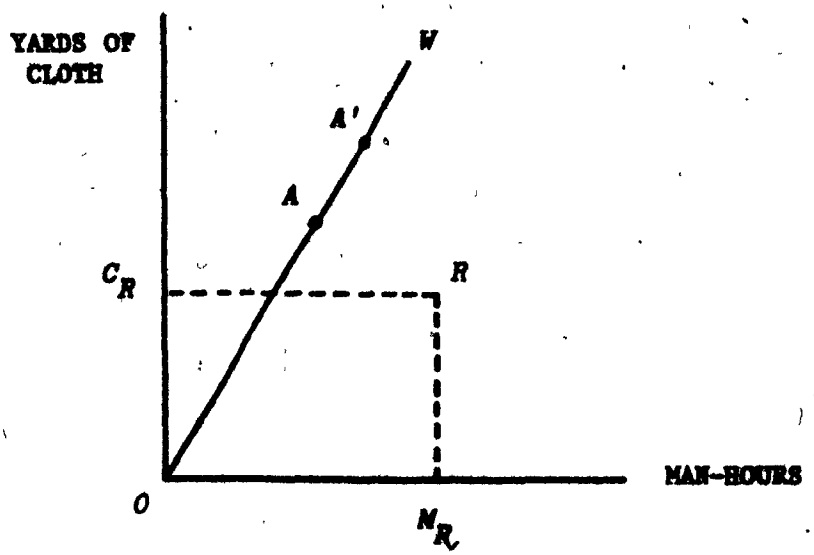


Fig. 4.1. Quantitative combinations of two production variables, yards of cloth and man-hours (Baumol, 1965, p. 252).

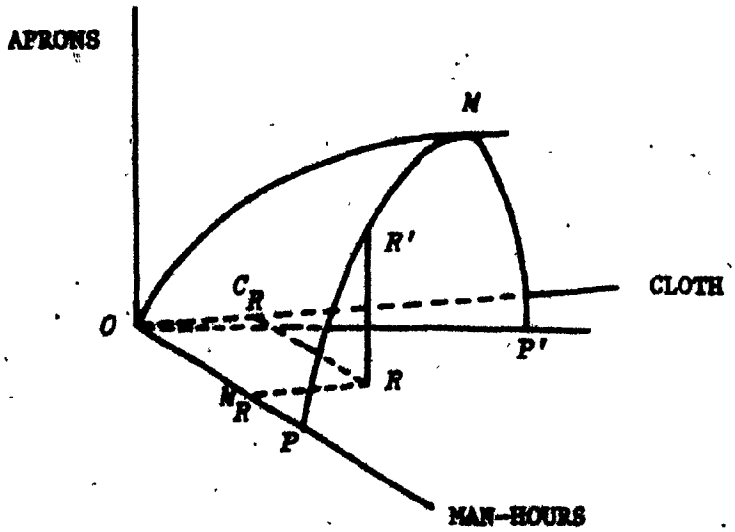


Fig. 4.2. A portion of the production function describing the transformation of yards of cloth and man-hours into aprons (Baumol, 1965, p. 252).

From Fig. 4.2 we note that to produce a positive amount of output, both inputs must be simultaneously consumed in positive amounts. In addition, there are regions in this input-output space where the output is constrained to be zero. That is, to produce aprons both cloth and labour must be used. If either one, or both of these inputs are zero, it follows that nothing can be produced. These properties are what cause the production function to be roughly concave to the input subspace.

The shape of the production surface, as seen in the cross section PMP' in Fig. 4.2, may neither be symmetrical nor smooth. In the first instance, a technology favoring the consumption of one input factor over the other would result in a skewed surface. Secondly, dents or kinks may be introduced if for example, the labour/cloth ratio becomes so low as to require the introduction of, or switching to, a labour saving device, which in turn may produce a sharp rise in output (Baumol, 1965).

Although point M is the highest on the cross section PMP', it represents only a technical optimum and excludes input cost considerations. These may be conducive to the same or a decreased level of output with a different cost per unit output combination of inputs. More will be said of this in section 4.3.

4.2.4 Marginal Productivity and Diminishing Returns. The marginal productivity is the change in output caused by a unit change in one factor input, holding all other independent variables constant. Mathematically, the marginal productivity of factor i is the first order partial derivative of the production function with respect to the quantity of factor i (Frish, 1965). From Eqn. 4.1 we have,

$$MP_i = \frac{\partial Y}{\partial X_i} = \text{marginal productivity of factor } i. \qquad (4.2)$$

The "law" of diminishing returns or diminishing marginal productivity may be stated as follows,

> "As more and more of some input, i, is employed, all other input quantities being held constant, eventually a point will be reached where additional quantities of input i

will yield diminishing marginal contributions to total product." (Baumol, 1965)

The existence of diminishing marginal productivity is highly plausible for any production environment where different inputs must be simultaneously consummed if any output is to be produced. For the particular example at hand, we would expect that a sufficiently high cloth to labour ratio could be reached, whence the addition of more cloth will grow less helpful. Two courses of action are available to rectify this situation: first, the labour input could be increased, thus shifting production to a new constant labour plane; second, one could replace an older machine by one that is more efficient in its utilization of labour, thus changing the available technology and hence the production function itself. Similar analysis could be pursued for a high labour to cloth ratio.

Diminishing returns may be observed by intersecting the production surface with a constant cloth plane at *LSTB* as shown in Fig. 4.3. Examining this curve, the total product versus labour with cloth = constant in Fig. 4.4, we see that as the labour input is increased, the slope of *LST* will decrease as dictated by the "law".

In effect, the marginal productivity allows one to gain an insight into the shape of the production surface when moving along a plane parallel to an input axis, and hence an insight into the functional role of each input in the production process. Another production index, called the substitution ratio, can be used to further provide information on the specific shape of the production surface, and thus on the nature of the production technology itself.

The substitution ratio expresses the relationship between the marginal productivities of two factor inputs, that is,

$$\frac{MP_i}{MP_j} = \frac{\frac{\partial Y}{\partial X_i}}{\frac{\partial Y}{\partial X_j}} = \text{substitution ratio for factor } i \text{ versus factor } j \qquad (4.3)$$

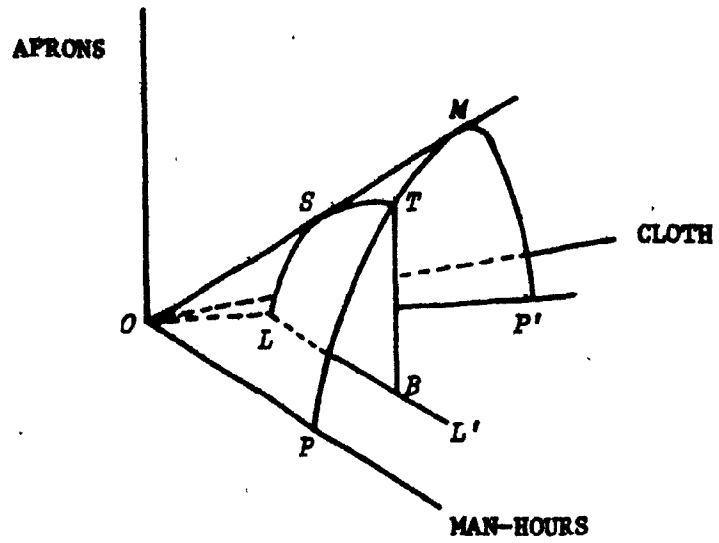


Fig. 4.3. Intersection of the aprons versus cloth and man-hours production function with a constant cloth plane (Baumol, 1965, p. 254).

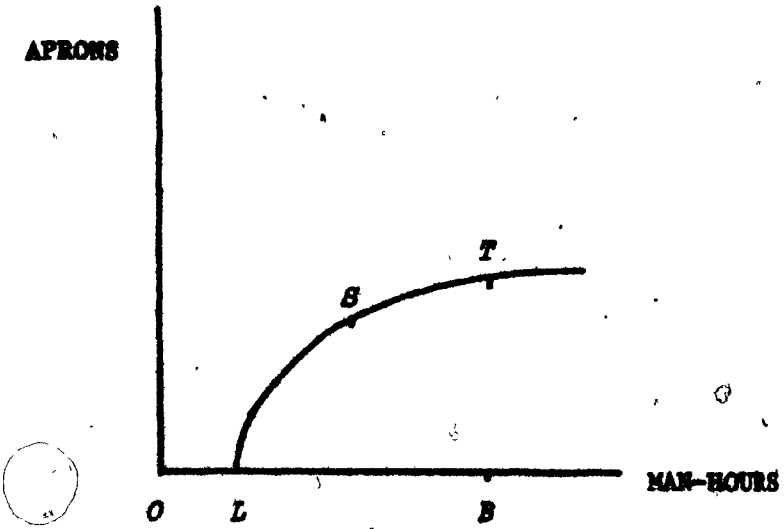


Fig. 4.4. Illustration of diminishing marginal productivity of man-hours with cloth-constant (Baumol, 1965, p. 254).

> "The substitution ratio tells us in which proportion the two factors are capable of replacing (substituting) one another if we insist that the product quantity be unchanged." (Frish, 1965)

We can best expediate relating this entity to function shape by defining the product indifference curve or isoquant. "A product indifference curve is defined as the locus of input combinations all of which are capable of producing the same output level" (Baumol, 1965). An isoquant for any level of output may be derived by intersecting the production surface with a constant output plane. A set of four such isoquants for a two input-one input process is shown in Fig. 4.5, with output levels Y_1, Y_2, Y_3, & Y_4 where $Y_1 < Y_2 < Y_3 < Y_4$.

The four basic properties of product indifference curves described by Baumol (1965) are:

i) they have a negative slope;

ii) if one indifference curve lies above and to the right of a second indifference curve, then the first will normally correspond to a higher output than the second;

iii) no two indifference curves intersect; and

iv) the curves are convex to the origin.

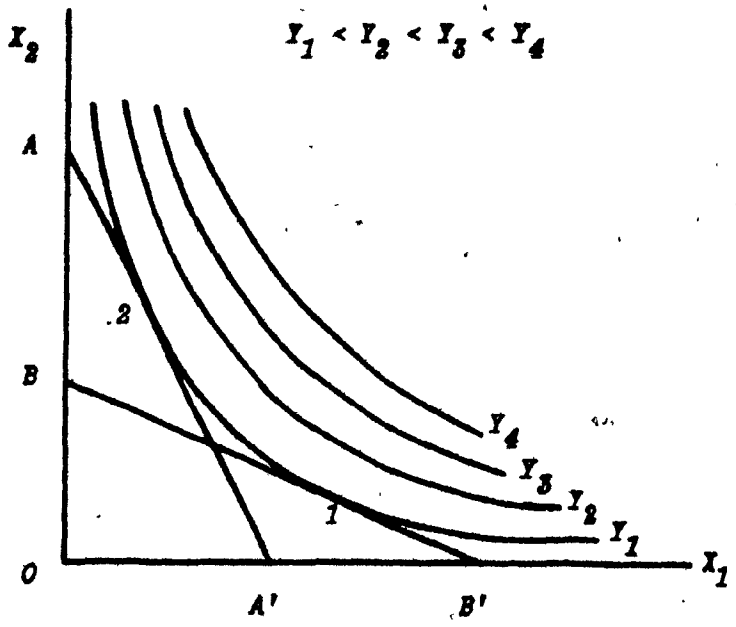
Each of these properties follows from one description of the production surface and the manner of intersection.

The slope of an isoquant is simply the negative of the substitution ratio. We may write the differential of f in Eqn. 4.1 as

$$dY = \frac{\partial f}{\partial X_1} \cdot dX_1 + \frac{\partial f}{\partial X_2} \cdot dX_2$$

Being on an indifference curve requires the $dY = 0$, so that by rearranging terms we have,

$$\frac{dX_2}{dX_1} = -\frac{\frac{\partial f}{\partial X_1}}{\frac{\partial f}{\partial X_2}} = -(\text{substitution ratio})$$



$$\frac{OA}{OA'} = \left.\frac{dX_2}{dX_1}\right|_2 \qquad \frac{OB}{OB'} = \left.\frac{dX_2}{dX_1}\right|_1$$

Fig. 4.5. A set of four product indifference curves for a two input–one output production process.

where the term on the left is the slope of the isoquant. By examining the substitution ratios at several points along any isoquant, such as points *1* & *2* on isoquant Y_1 in Fig. 4.5, information is provided on the degree of convexity of the isoquant. Yet of greater importance is the knowledge it provides on the relationship between inputs at a fixed level of output under the technology described by the production function.

4.2.5 Returns to Scale. In the preceding section we examined the output change corresponding to a change in only one of the inputs. Returns to scale deals with the phenomena of an output change when all inputs are simultaneously increased in the same proportion. This type of input change occurs when moving along a ray from the origin in the input space. An example is shown in Fig. 4.1 in moving from *A* to *A'* on *OW*. Returns to scale are defined as follows:

> "for a given proportional increase in all inputs, if output is increased by a larger proportion, the firm enjoys increasing returns (or economies of scale); if output is increased in the same proportion, there are constant returns to scale; and if output is increased by a smaller proportion, decreasing returns result (or diseconomies of scale)." (Brown, 1966)

These three production states are depicted in both the input-output plot of Fig. 4.6 and the indifference curves of Fig. 4.7a,b and c.

The type of returns to scale is determined by the relative positions of a production function's isoquants, regardless of their shape, whereas diminishing marginal productivity arises from isoquant convexity and hence is shape dependent. Thus a production function can exhibit both constant returns to scale and diminishing marginal productivity as depicted in Fig. 4.7b.

Brown (1966) dichotomizes the source of variation in returns to scale into: i) those resulting from the general development of an industry, as from the state of technology available to all producers within a common sector, and; ii) those dependent on the resources of the individual firms,

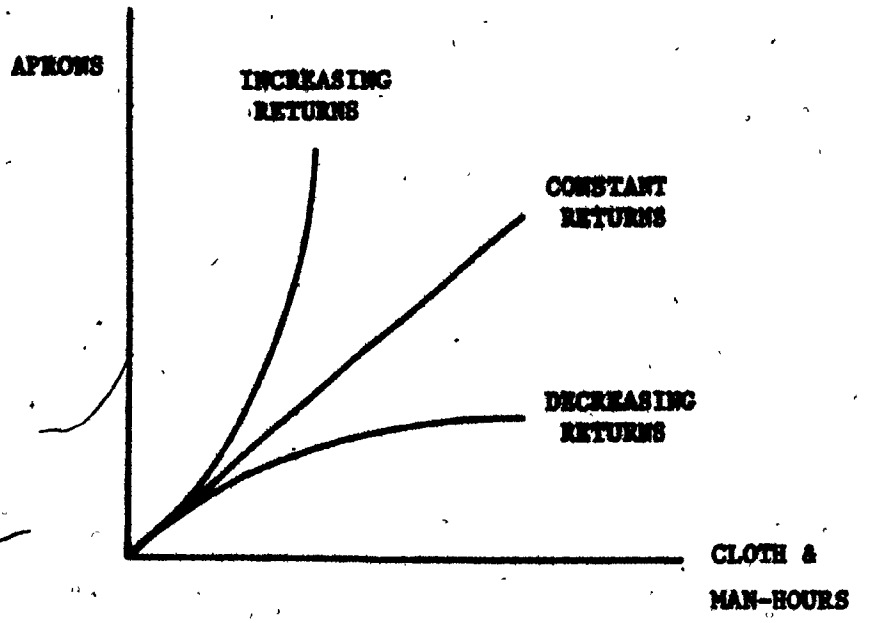
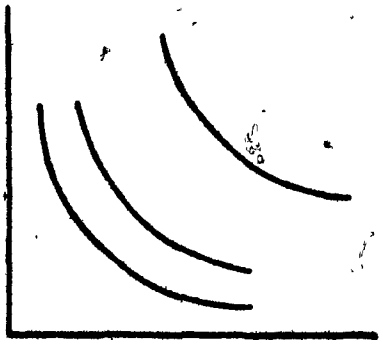
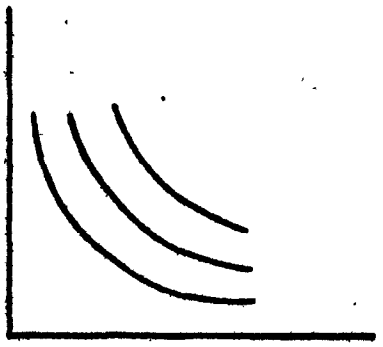


Fig. 4.6. A plot of output versus the production inputs showing the three possible modes of production (Baumol, 1965, p. 245).



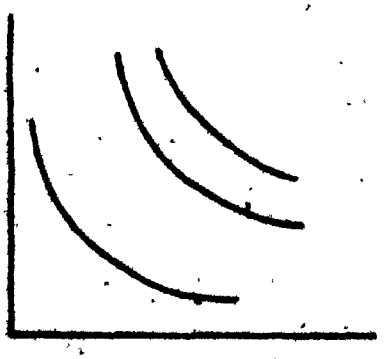
INCREASING RETURNS

(a)



CONSTANT RETURNS

(b)



DECREASING RETURNS

(c)

Fig. 4.7. Sets of isoquants showing the three possible modes of production (Brown, 1966, p. 14).

i.e. their organizational and managerial efficiency.

Various arguments can be postulated for the case of either constant, increasing or decreasing returns in a firm. In the first case, one may reason that if all inputs are identically doubled, then the output will double as well. If, for example, when starting out at lower levels of output the division of labour is such as to carry out production more efficiently as output rises, then the second situation of increasing returns is expected. In some instances, as the inputs to production are increased the problems of management and organization may become increasingly more difficult, resulting in proportionately less efficient processes, and hence decreasing returns to scale will be introduced.

The type of returns to scale is ultimately dependent on the specific properties of the system under investigation, whereas the ability to estimate the extent of these returns is dependent on the data available to estimate the production function.

4.3 Measurement of Productive Efficiency

The concept of a production function was initially introduced in response to the need for quantifying the effect of input resource changes and measuring the extent of underutilization of facilities in diagnostic radiology departments. It is shown below how the properties of a production function developed in the previous section can be used to achieve these ends.

4.3.1 Input Changes. A production function describes the maximum output attainable with different input combinations for a particular technology. Given the availability of such a function for a producing system and knowledge of its inputs, the corresponding output capacity can then be calculated. Unless some inovation is introduced so as to change the overall shape of the production surface, output capacity is fixed per input combination and can only be modified by incrementing one or more of the inputs.

It may be desirable to investigate two types of input changes.

Firstly, one could pursue a sensitivity analysis using the marginal productivities and substitution ratios described in section 4.2.4. By comparing the magnitudes of the marginal productivities for different inputs, one effectively gains an insight into the nature of the production process and the relative importance of the inputs at their present levels.

If the marginal productivity of one input is far greater than the others, this may indicate an instability in the production process. That is, the viability of the system could be questioned if the supply of this critical input were suddenly changed. Factor supply is usually outside the control of an individual producer. An imbalance in the marginal productivities would also indicate the most fruitful direction for input investment in terms of a capacity expansion.

The substitution ratio indicates the ease with which inputs may be interchanged at the same level of output. All things being equal, the motivation for substituting inputs is an economic one.

In Fig. 4.8 we have depicted an isoquant and a series of isocost lines. An isocost line shows the different input combinations that may be purchased for the same total cost (TC). A series of such lines are parallel, each having a slope equal to the negative of the input prices (P) ratio. The total cost increases as one moves away from the origin.

Inputs are substituted on an isoquant to drive a production process to a least cost per unit output position. In Fig. 4.8 this is accomplished by moving from point *1* to point *2*, where the price ratio and substitution ratio are equal, i.e., the isoquant and isocost curves are tangent. Hence knowledge of the input marginal productivities and prices would indicate in which direction to economically change inputs while retaining the same output level.

While the marginal analysis described may be highly instructive, it does not usually allow for testing of implementable input changes. For example, if the production unit of cloth is square yards but must be purchased in lots of 100 square yards, then if cloth corresponds to input *1*,

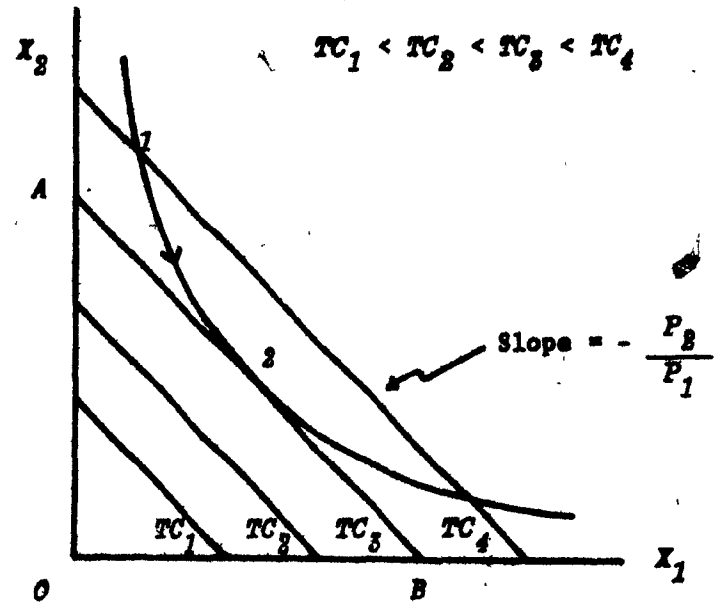


Fig. 4.8. An isoquant and a series of isocost curves showing how to economically change inputs at a constant level of output.

we have that,

$$MP_i = \frac{\partial Y}{\partial X_i} \neq \frac{f(X_1+100, X_2 \ldots X_n) - f(X_1, X_2 \ldots X_n)}{100}$$

Thus, one would like to estimate the effect of a realistic, rather than an incremental, input change.

In Fig. 4.9, the capacity changes are shown for different input changes ΔX_1, ΔX_2, and both, from A resulting on isoquants Y_1, Y_2, and Y_3 respectively, where $Y_1 < Y_2 < Y_3$. These input changes can be used to test the existence and extent of returns to scale at different input levels. In addition we see that if expansion to Y_3 is desired and the isocost line TC is available, then the economically optimal change in inputs is from A to B.

The locus of tangent points formed between a family of isoquants and a family of isocost lines defines the optimal expansion path for a firm. An example of such an expansion path is shown in Fig. 4.10.

Incremental and absolute changes in the inputs to a production function provide information on production behaviour unavailable from a qualitative or raw data description of the process and, if price data is available, allows for minimum cost planning.

4.3.2 Technical Efficiency. We have agreed that the production function establishes a capacitive upper bound for output, yet there is no reason why a firm must be observed to operate at all times on this production surface. In fact, one would expect that there should be some firms, whose observed output will be less than the possible maximum, over the time period encompassed by the production function. This could result from either inefficient operational organization, insufficient demand for output, without resource layoff, or some combination of the two.

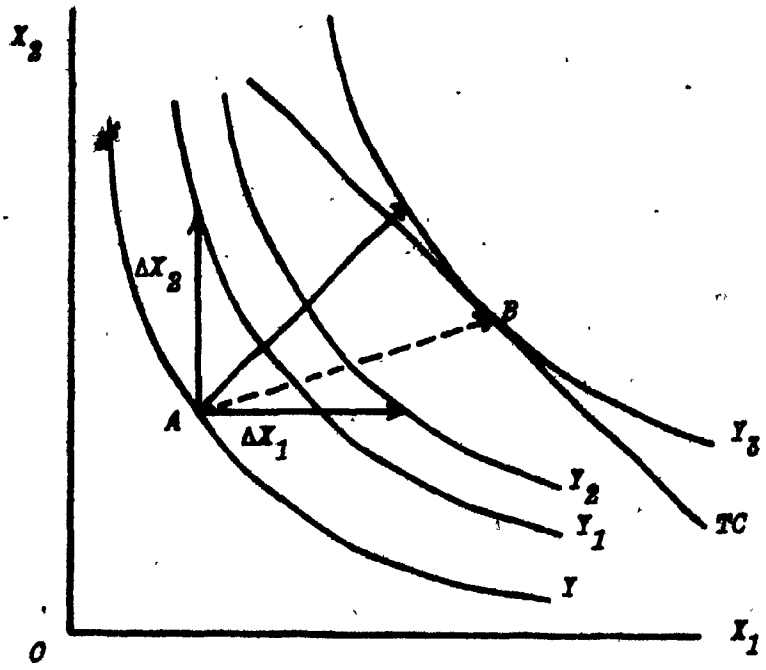


Fig. 4.9. The effect of realistic input changes on the capacity output of a production process.

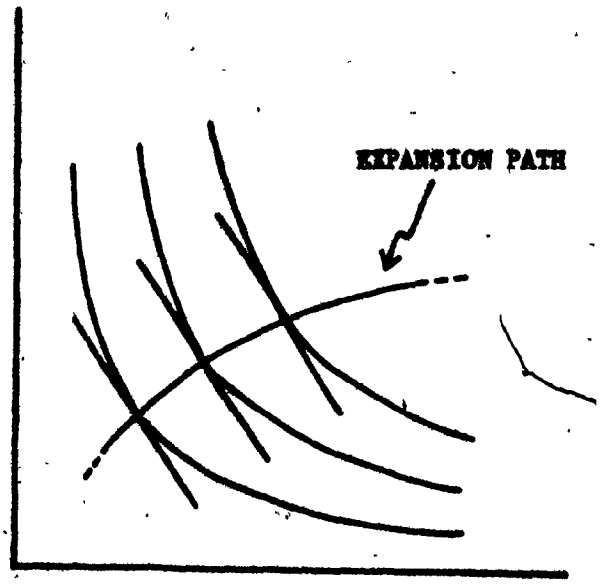


Fig. 4.10. The economically optimal expansion path of a firm.

Consider the example of section 4.2 with a firm having inputs OM_R and OC_R and producing RR'' aprons, a quantity less than its capacity output RR', as shown in Fig. 4.11. The ratio of RR'' to RR' indicates the extent to which the observed firm utilizes its capacitive ability. Since the productive capacity is defined by the input levels, a value of less than 1 for this ratio indicates an underutilization of facilities. We shall call this ratio, multiplied by 100, the technical efficiency of the firm as it is defined strictly in terms of a technological standard - the production function - and will use it as a proxy measure of the underutilization of facilities.

The name "technical efficiency" is appropriate. As Farrell (1957) states, "when one talks about the efficiency of a firm one usually means its success in producing as large as possible an output from a given set of inputs".

If for a given firm we know its production function, and can observe its inputs and output, we may then indicate the presence and extent of an underutilization of facilities by measuring the technical efficiency in terms of the observed and capacity output.

4.3.3 Linear Homogeneous Production Functions. The class of production functions having constant returns to scale are known as linear homogeneous production functions. Production systems which comply to this condition have a particularly unique and attractive property for planning purposes. Baumol (1965) describes it as a theorem,

> "The expansion path of a linear homogenous production function is always a straight line through the origin. This means that given the prices of its inputs, the optimal proportion of the inputs of the firm will not change with the size of the firm's input budget."

This result is shown diagramatically in Fig. 4.12 when expanding the output from Y_1 to Y_2 by moving on OL_1 from A to $k \cdot A$. Yet of greater significance to the present discussion is the fact that the cost

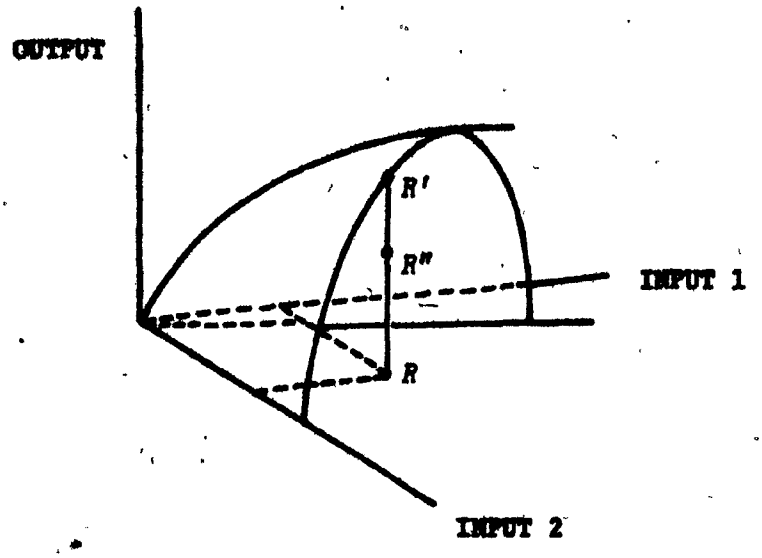
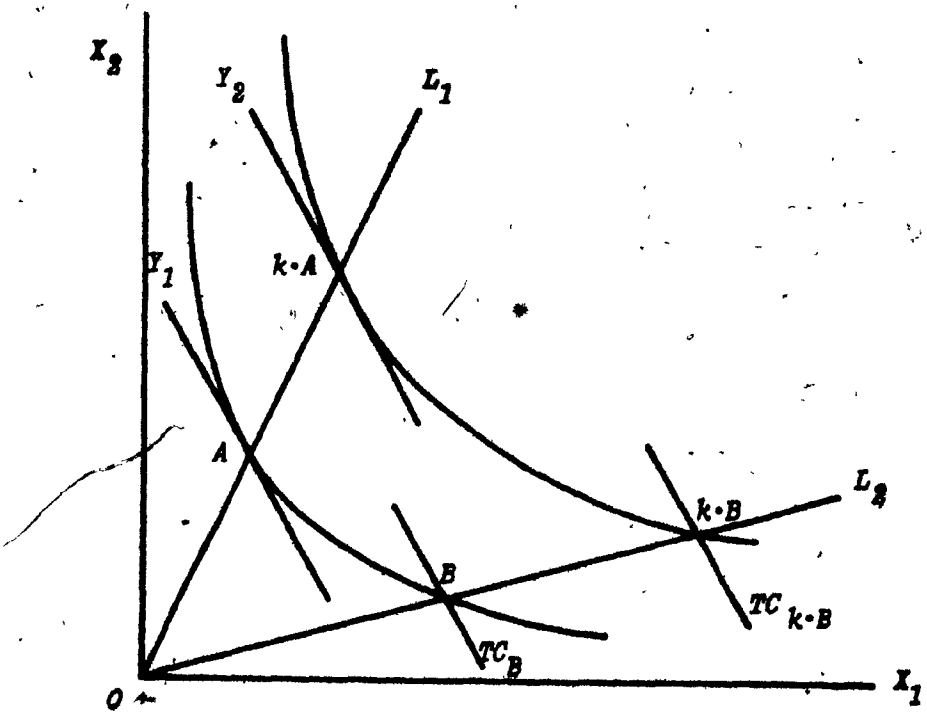


Fig. 4.11. A production function showing a firm with specified inputs *(R)* and an observed output *(R")* which is less than its capacity output *(R')*.



$$B = (b_1, b_2)$$

$$k \cdot B = (k \cdot b_1, k \cdot b_2)$$

$$\therefore \quad Y_2 = k \cdot Y_1$$

Fig. 4.12. Expanding the output with different production processes (OL_1) under constant returns to scale.

per unit output remains fixed on any ray from the origin. This is seen in Fig. 4.12 in going from B to $k \cdot B$;

$$Y_2 = k \cdot Y_1$$

$$\text{total cost}(TC) = \text{a linear function of } X_1 + X_2$$

$$TC_{k \cdot B} = k \cdot TC_B$$

$$\text{cost per unit output}\Big|_{k \cdot B} = \frac{TC_{k \cdot B}}{Y_2} = \frac{k \cdot TC_B}{k \cdot Y_1} = \frac{TC_B}{Y_1} = \text{cost per unit output}\Big|_B$$

Therefore the relative costs at points on the production surface are dependent only on the quantitative ratio between inputs, and not on their absolute values. In addition, if a firm is observed to be inefficient, i.e., below the production function, and its inputs and output are multiplied by the same positive constant k, then under constant returns to scale both the technical efficiency and the cost per unit output remain the same - independent of the magnitude of k.

This result is clarified by examining a linear homogeneous production function in a space where the coordinates represent the amount of input per unit output. In Fig. 4.13 Z_1 and Z_2 represent the input per unit output variables for a two factor production system. The curve SS' represents the overlay of all isoquants of a production function and P represents an observed input-output rate for a particular firm.

The point Q uses inputs in the same proportion as P but is observed to be on the production surface. This means that either: i) Q has a greater output for the same input values as P; ii) Q has the same output but proportionately less of each input than P, or; iii) some combination of the two. In any event, P is less efficient than Q. Farrell (1957) defines the technical efficiency of the firm P as OQ/OP. This is consistent with our previous definition of efficiency when applied to the constant returns to scale case.

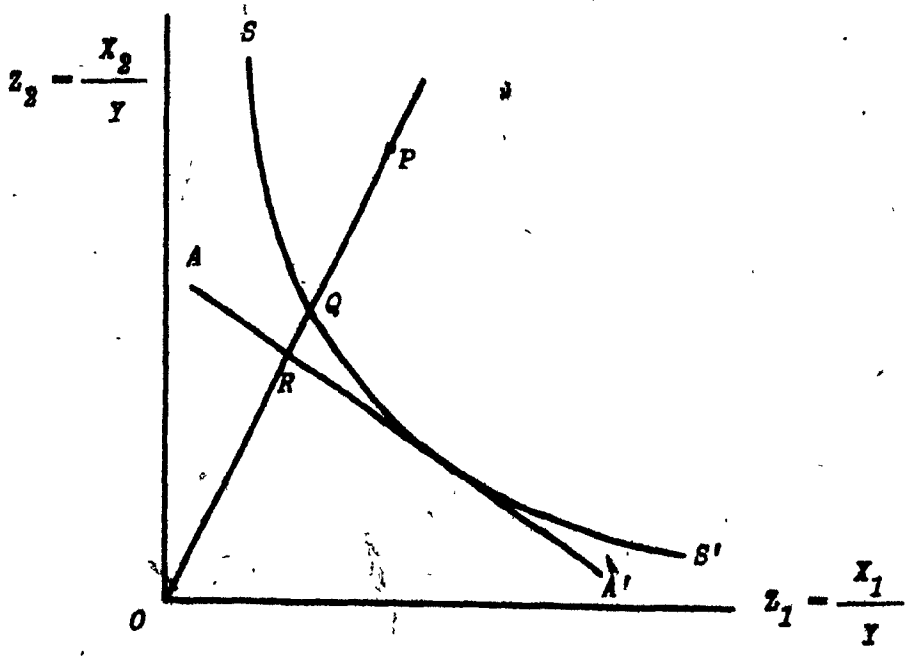


Fig. 4.13. The measurement of the technical and price efficiencies of a firm *(P)* relative to the isoquant *(SS')* and isocost curve *(AA')* under constant returns to scale in an input per unit output space (Farrell, 1957, p. 254).

Let X_1 = input of factor 1

X_2 = input of factor 2

Y = output

ϵ = constant, $\epsilon>0$

then from Fig. 4.13 we have,

$$Q = \left(\frac{X_1}{Y}, \frac{X_2}{Y}\right) \qquad P = \left(\frac{X_1}{Y-\epsilon}, \frac{X_2}{Y-\epsilon}\right)$$

$$OQ = \frac{(X_1^2+X_2^2)^{\frac{1}{2}}}{Y} \qquad OP = \frac{(X_1^2+X_2^2)^{\frac{1}{2}}}{Y-\epsilon}$$

$$\left.\begin{array}{l} \text{Let } Y = RR' \\ Y-\epsilon = RR'' \end{array}\right\} \text{ see Fig. 4.9}$$

$$\text{then } \frac{OQ}{OP} = \frac{Y-\epsilon}{Y} = \frac{RR''}{RR'}$$

The line AA' has a slope equal to the negative of the factor price ratio and thus represents the overlay of all isocost lines. Hence Q' and not Q is the overall optimal means of production. Farrell (1957) defines the price efficiency of point P as OR/OP.

What is important to remember at this point is that the measurement of a price efficiency is valid only under the assumption that the firm is a profit maximizer. While this assumption may be quite natural for a business environment, it is not necessarily the case for a medical "firm".

Discussion as to the existence of a profit maximizing motive for diagnostic radiology departments will be postponed to section 4.5.

From a planning perspective, the existence of non-constant returns to scale can be dangerous. Increasing (decreasing) returns can be seen to encourage both economically and from an "efficiency" - output per unit input - viewpoint, the proliferation of relatively large (small) units, independent of the environment in which the firm must operate. In some instances, such as ancillary departments in a hospital, the bond between

the firm and its environment can be critical.

If constant returns to scale is the rule of the day, then both large and small operations may be meaningfully compared in terms of their technical efficiencies without ascribing an inherent advantage to one group over the other.

4.4 Estimation of Continuous Production Functions

The estimation of continuous production functions usually proceeds by fitting some predetermined algebraic form to observed input-output rates.

Nerlove (1965) groups these observations into four basic types: a cross section of firms at a single point in time; observations for a single firm over time; observations for an industry over time; and observations on a cross section of firms over time. The last type of data represents the most complete set in terms of observing the effects of both time and interfirm differences. Yet as Nerlove points out, both the first and last types of data are suitable for considering interfirm differences alone.

Our goal at this point, is to derive a construct in which to examine and compare the production behaviour of radiology departments. This scheme will be used solely for an analysis of interfirm differences, and hence excludes consideration of trends over time and forecasting. The predominately cross sectional nature of our data base (see section 3.4), is thus conducive to quantifying the principle measures of interfirm difference of interest; that is, the technical efficiency, the marginal productivities, and the effect of implementable resource changes.

4.4.1 Different Albegraic Forms. The choice of functional form is dependent on the observed properties of the system under study, and, to a certain extent, on the methodological bias of the model builder (Urban, 1972). Reinhardt (1973) summarizes the problem of *a priori* selection of

a particular model structure,

> "In principle it should be possible to derive the precise algebraic form of the production function from knowledge of the underlying technology. In practice, however, the best that can be done is to specify the functional form on the basis of an educated hunch. In selecting the *a priori* specification it is tempting to trade off realism for the sake of mathematical convenience.... Actually, the *a priori* specification must be made with great care, lest the purely mathematical properties of the specification preordain the economic properties inferred from the estimate."

Three basic types of continuous production functions have been dominant in health care studies: the linear, with terms to different powers; the multiplicative; and; the mixed multiplicative-exponential.

Kovner (1968) in his analysis of outpatient departments, used the following specification:

$$Y = \sum_{j=1}^{m} a_j X_j^{n_j}$$

where Y denotes a rate of output, vector $\mathbf{X}$ is a set of input rates, n_j are either 1 or 2 depending on the corresponding X_j element, and the elements of vector α are the parameters of the function to be estimated. This equation implies: i) that output may be generated in the absence of all medical inputs other than administrative staff (one of the inputs); ii) the marginal product of any input is independent of all other inputs; and iii) diminishing marginal productivity does not exist for the system. These assumptions conflict with prior knowledge clinic behaviour (Reinhardt, 1973).

A second form, the Cobbs-Douglas production function or multiplicative function has the following general structure:

$$Y = C \prod_{j=1}^{m} X_j^{a_j}$$

where all parameters and variables are the same as described above and C is a constant to be estimated. This function has been used by Kimbell & Lorant (1973a) and Yett *et al.*,(1970) to describe physician services, by Boaz (1972) for a family planning clinic, and by Feldstein (1968) for a hospital production function. It has the advantage of forcing output to zero if any one input is zero, displays diminishing marginal productivity and, depending on the values of elements of vector $\mathbf{a}$, can exhibit either increasing, decreasing or constant returns to scale. Yet as Boaz (1972) points out, "if all inputs except X_k are held constant, output is still expected to increase by a constant a_k percent whenever X_k is increased by 1 percent from its initial level, whatever this may be, $X_k = 10$ or $X_k = 1000$." For this reason she investigates a modified exponential form. Reinhardt (1972) uses a similar approach in defining a production function for medical practices, which he describes as having the following general form (Reinhardt, 1973):

$$Y = A \prod_{j=1}^{m} \left[X_j^{a_j} \exp\left(-b_j X_j\right) \right] \exp\left[g(\mathbf{X}, \mathbf{c}) \right]$$

where X_j, $j = 1..,m$ denotes rates of essential inputs, i.e. those that must be used at positive rates such as physician time, X_j, $j = m+1,..,n$ denotes rates of inputs that are not essential such as some paramedical personnel, $\mathbf{a}$, $\mathbf{b}$, & $\mathbf{c}$ are vectors of parameters to be estimated and $g(\mathbf{X}, \mathbf{c})$ may be a nonlinear function in nonessential inputs.

The advantage of this form lies in its retention of the redeeming properties of the Cobbs-Douglas function while admitting greater variability in the marginal productivity by introducing the $\mathbf{b}^t\mathbf{X}$ and $g(\mathbf{X}, \mathbf{c})$ terms in exponential form. As the most general form the multiplicative exponential would seem to have the greatest promise.

4.4.2 Empirical Estimates. The parameters of the production functions mentioned above are normally estimated by converting the equation to a tractable form, and then applying a technique such as linear least squares. Typically, a production function is converted to a linear form (if not already in one) by taking logarithms of the output and the function. A major advantage of using conventional statistical methods is that the resulting estimate is accompanied by indices of the statistical significance of the results (Reinhardt, 1973). Yet because of the very techniques used in estimation, the resulting functions will yield expected or average output rates. Depending on the goal of the analysis this may be acceptable.

If, on the other hand, a function reflecting the theoretical definition of maximum technical efficiency is desired, these statistical procedures are no longer applicable.

The concept of the production function for diagnostic radiology departments was introduced as a tool to estimate the technical efficiency of the individual departments; in effect, their underutilization of facilities. Yet, as shown in the previous section, a function satisfying the theoretical definition is required to pursue this end. Rather than abandon this approach, we shall use the non-statistical technique for identifying a capacity output, or maximally efficient production function, developed in the next section.

An additional problem in using a continuous production function becomes apparent in trying to integrate the radiologist input as represented in the data into a functional form. Since radiologists aren't paid by the hour, it is not surprising that in both data set D.1 and D.3 (see section 3.4), this input is provided as a number of full-time and part-time man years. The majority of radiology departments have only full-time or part-time radiologists, but not both.

We cannot differentiate between the two types of radiologist input as they both perform identical functions in the radiographic subsystem of diagnostic radiology. Even if one did consider the two groups as distinct, and equi-necessary to the operation of a department, this would result in

the multiplicative production function being zero for any department not having both full and part-time input.

If one merely examined the total number of radiologists present this would be equivalent to ascribing an advantage to departments with full-time radiologists. For example, consider the case of three part-time radiologists in one department producing 1000 examinations over a certain time period and another department with three full-time radiologists producing 2000 examinations over the same period. There is no valid reason why the second department, producing more examinations per "radiologist", should be observed as more efficient - but it would by aggregating radiologists. In addition, any effort to equate *X* full-time radiologists with *Y* part-time radiologists can only be viewed as somewhat hopeless, ultimately arbitrary, and finally, a homogenization of the only distinctive feature of this input.

The only reasonable approach then, would be to divide the sample into three subsets as to radiologist input and independently estimate a production function for each. Not only would the estimation of a function for the subset of departments having both full and part-time radiologists be untenable in terms of the sample being too small, but this approach would defeat the original idea of developing a common framework in which all departments could be compared.

The technique used in this work to identify the production function integrates full and part-time labour inputs in a unified framework and approximates the theoretical definition of a production function.

4.5 Identification of a Production Function

The non-statistical identification of a production function or production model will be accomplished by using a linear programming (L.P.) formulation. The origin of this type of model can be traced to military, economic-industrial, and mathematical sources, which after World War II, merged for the development of the linear programming model as it is known today (Dantzig, 1963).

T.C. Koopmans (1951) originally proposed a scheme for recognizing production as an efficient combination of activities. The idea behind almost all L.P. production models and programming allocation schemes to date can be traced to this classic paper. Somewhat later, Farrell (1957), using a simplified version of Koopmans work, first described a technique for approximating a maximally efficient production function. He applied this technique to measure the productive efficiency of the agricultural sectors of various nations. Recently, Reinhardt (1973) suggested using what he labels the "little known method" of Farrell, for the identification of a production function for medical practices.

We shall first investigate Reinhardt's formulation and then convert it to a form comparable to Koopmans and more amenable to our analysis. In discussing this technique it will become apparent that its relative obscurity is related to standard problem definition, rather than a weakness in the method itself.

4.5.1 A Linear Programming Production Function for Diagnostic Radiology. "The word linear applied to the model means that if all inputs to a process are multiplied by a non-negative constant then the outputs are multiplied by the same constant. In other terms, the production function is homogeneous of degree one" (Gale, 1965). Thus the linear programming model of production is one type of constant returns to scale production function.

The rational for adopting the linear programming approach has been described by Reinhardt (1973):

> "The characterization of a production process by a single continuous function proceeds on the assumption that any given rate of output can be produced with a literally infinite number of distinct input combinations. From an analytic standpoint, it is often just as plausible to pretend that given rates of output can be produced only by a finite number of 'production processes', each characterized by a distinct combination of inputs per unit output."

The critical point in this work is not whether a continuous production function, having constant returns to scale, can be approximated by a piecewise continuous function (the linear program). We initially accept, as proposed by Reinhardt's statement, that such an estimation will suffice for the needs of our analysis. Rather, what is crucial is whether diagnostic radiology departments can be expected to obey constant returns to scale in the first place.

Returns to scale is a hotly debated issue in health services research, with contradictory conclusions having been reached (Bailey, 1970; Hefty, 1970; Migue and Belanger, 1974). The inputs to health care are easily quantified in either physical or monetary units and have thus escaped controversy. It then follows, from Fig. 4.6, that the main difficulty in estimating returns to scale for "producers" in the health system lies with identification of their product or output. This is true whether examining solo practices or large university hospitals.

Even when focusing attention to the provision of medical care, either by or under the direction of a licensed physician (i.e. excluding hotel services, teaching, research, etc.), it is apparent that there is a great deal of product heterogeneity. "As we move along the spectrum toward larger firms with more physicians, it is natural to find a more diversified product line" (Bailey, 1970). As the resources of a health care system (both manpower and equipment) increase, there will be a greater capacity for patient treatment. This is seen both in terms of an increased volume of patients and a greater mix of health care problems.

The question then arises of how to define treatment. Some analysts, in rejecting such measures as patient visits or discharged cases, have suggested something more abstract, such as patient health or quality of medical care (Reder, 1969), neither of which readily lend themselves for quantification.

The problem of defining one homogeneous output for diagnostic radiology departments is greatly reduced. Admittedly, the number of different types of examinations a department can provide is large. Yet almost without exception, every examination requires utilization of the same resources;

i.e., a machine, materials, technician and radiologist input, although to varying degrees in each case. This is contrasted with the patient or case-oriented outputs of other parts of the health system, where the spectrum of resources consumed is much greater and the specific set is dependent on the diagnostic and therapeutic needs of the patient.

In addition, the services provided by radiology compose but one part of the total treatment program of a patient. As such we need not be concerned with the diagnostic value of these services to patient care, but rather with the capabilities of a department for their provision. This point of view is reinforced by our interest in the radiographic subsystem of diagnostic radiology which excludes consultations - the direct departmental input to the treatment process.

If we consider the output of this system to be diagnostic examinations two contradictory trends might be expected. First, as departments increase in size, more automated equipment will be introduced, such as for film processing, causing a proportionately increasing change in capacity and hence increasing returns to scale.

Second, as there is a high correlation between department size and the size and specialization of the hospital in which it is situated, there would of necessity be some x-ray rooms in the larger hospitals dedicated to providing highly specialized procedures for particular types of patients which are only cared for in these more specialized institutions. Typically these specialized rooms are used for only a few hours a day or sometimes only on certain days of the week because the number of patients needing their services, even in these large hospitals, is relatively small. The result is an idle capacity. It is unlikely that this capacity, however necessary, will ever be fully utilized. Thus the departmental superstructure effectively imposes a decreasing returns to scale situation which it would be unfair to attribute to inefficient operation.

The adoption of the constant returns to scale assumption for diagnostic radiology departments was promoted by: i) the fact that only one examination can be provided at a time per machine (regardless of department size),

which lends plausibility to the proportionate input increase - proportionate output increase arguement, and ii) the weighing of the two opposing trends cited above.

In constructing the payment schedule for radiologists in hospitals the Quebec Government assigned to each type of x-ray examination a certain number of equivalent units, the actual number being based on the complexity of the procedure and time involved. The total number of diagnostic radiology units produced by a department represents a weighted sum of the number of examinations, and hence is a more standardized measure of output. As such, it represents another measure of department output which should satisfy our assumption.

The existence of a "distinct combination of inputs per unit output", that is, a fixed quantitative ratio between inputs and between inputs and outputs in a department, is highly unlikely on a daily basis. As the period of consideration increases, the validity of the assumption becomes more acceptable. Since the time period the model of production will encompass is one year (see section 3.4), the assumption becomes all the more palatable.

We proceed on this assumption for the present, and postpone to a later chapter the empirical verification of this hypothesis.

The model to be examined in the next few sections embodies both the relationship between inputs, and the process of transforming inputs into outputs utilizing the technology available to the economy. As stated previously, the term technology is taken here to mean the apparatus which defines the purely technical possibilities of production.

Before proceeding, it is well to point out that linear programming or activity analysis formulations of production are well established in the literature (Baumol, 1965; Boulding and Spivey, 1964, Dorfman *et al.*, 1958; Henderson and Quanot, 1971), and therefore the following discussion will be kept relatively non-rigorous. As Reinhardt (1973) provides a much simpler derivation of the technique than either Koopmans (1951) or Farrell (1957), we will follow at least part of his discussion explicitly (Reinhardt, 1973, pp. 212-214 and 221-223).

4.5.2 The Linear Activity Model. Consider the situation of k different and unique production configurations, each consuming a maximum of m distinct factor inputs, and all geared to the production of the same output or final product. The set of inputs available to every production unit is the same. Each production process may be represented by a unique production function.

Let Y_r = rate of output generated by operating the r-th production process

X_j = rate at which the j-th input is consumed in the above process

From Eqn. 4.1, the r-th production function is written as

$$Y_r = f_r(X_1, X_2, \ldots, X_m) \quad r = 1,2,\ldots, k \tag{4.4}$$

We assume that the k processes are each characterized by a fixed quantitative ratio between inputs and a distinct combination of inputs per unit output (see section 4.5.1). This implies that for the individual processes: i) no substitution amongst inputs is possible, and ii) returns to scale are constant. The first condition requires that the function f_r, Eqn. 4.4, be a Leontief-fixed proportions production function (Feldstein, 1968), that is

$$Y_r = \min\ (g_{1r}(X_1), g_{2r}(X_2), \ldots, g_{mr}(X_m)) \tag{4.5}$$

The second condition requires that,

$$g_{ir}(X_i) = \frac{X_i}{a_{ir}} \qquad r = 1,2,\ldots,k \qquad i = 1,2,\ldots,m$$

where a_{ir} = minimum amount of the i-th input required to produce one unit of output by the r-th process,

and therefore,

$$Y_r = \min\ \left(\frac{X_1}{a_{1r}}, \frac{X_2}{a_{2r}}, \ldots, \frac{X_m}{a_{mr}}\right) \tag{4.6}$$

A series of product indifference curves for such a production function is shown in Fig. 4.14. Note how both conditions are depicted diagramatically.

The input coefficients a_{ir} define the technically attainable output per given set of inputs for each production process. Therefore the entire set of $m \times k$ coefficients defines the complete technology encompassed by the k production processes and these k different activities constitute the production function for this technology.

Although substitutability between inputs is defined to be zero per production function, it can be observed in the overall production process by simultaneously operating two or more processes at various levels.

Let Q = rate of output from simultaneous operation of k processes,

$\alpha_r = \frac{Y_r}{Q}$ = proportion of the output produced by the r-th process,

then the overall production function for the technology is,

$$Q = \min \left(\frac{X_1}{\beta_1}, \frac{X_2}{\beta_2} \ldots, \frac{X_m}{\beta_m} \right) \qquad (4.7)$$

where $\beta_i = \sum_{r=1}^{k} [\alpha_r a_{ir}] \quad i = 1,2,\ldots,m$

The proportions α_r are known as activity levels and denote the extent to which the r-th process is contributing to the overall production process. Corresponding to any particular input combination, X_i, $i = 1,2,\ldots m$, we should like to choose values for the activity levels α_r, $r = 1,2,\ldots k$, so as to maximize output Q. "If such solutions and the corresponding maximum output rates can be found for a sufficient variety of fixed input combinations, the approximate shape of the overall production function can be traced without *a priori* specification of the algebraic form" (Reinhardt, 1973).

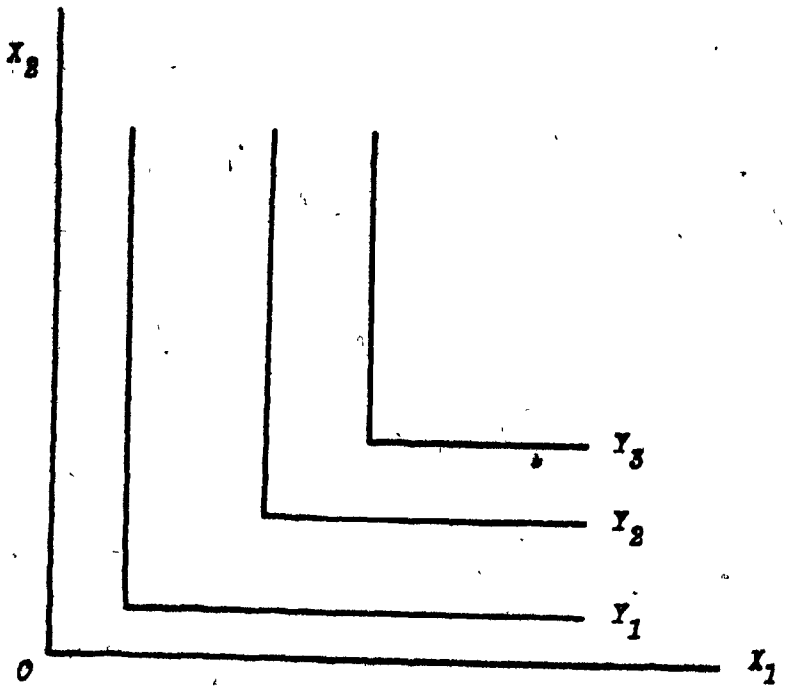


Fig. 4.14. Product indifference curves (isoquants) for the production function described by Eqn. 4.6.

4.5.3 The Efficient Production Function. The difficulty in making a linear activity model operable lies in the estimation of the input coefficients, a_{ir}. The choice of estimation technique is closely related to model purpose.

Reinhardt (1973) suggests that the estimated coefficients either be i) some average or model value for a representative sample of producers, or ii) values that correspond to the theoretical concept of a production function. "For estimating a base line on which to assess aggregate loss due to inefficiency in the health sector, the 'ideal' definition would clearly be favoured; for estimating the effect that might be expected on average", for example, "from more extensive task delegation in the health care sector at large, the first definition would probably be preferred. And for deriving optimal staffing patterns for a particular facility, coefficient estimates would have to be developed from that facility" (Reinhardt, 1973).

The measures of efficiency defined in sections 4.3.2 and 4.3.3 are based on the assumption that the theoretical production function is available, and therefore the second approach will be pursued.

While many techniques for deriving the efficient production function coefficients exist, Farrell (1957) suggests that two dominate - "a theoretical function specified by engineers and an empirical estimate based on the best results observed in practice ". Rather than engineers, a panel of radiologists and radiology technicians would be approached to identify alternative processes and to specify the corresponding coefficients. Yet the more complex a process, the less accurate is the theoretical function likely to be and, as a result of human frailty, the more optimistic (Farrell, 1957). As well, the perceptions of health care producers often fail to coincide with reality (Reinhardt, 1973).

In light of the above observations, and the fact that "it is far better to compare performances with the best actually achieved than with some unattainable ideal," (Farrell, 1957), we reject the use of experts in favour of the method to estimate an observed standard of efficiency described by Reinhardt (1973) and outlined below. We postpone the choice

of production variables for diagnostic radiology, and the application of this technique to these variables, to the next chapter.

We initially focus our attention to an observed set of production processes in the same two inputs and one output. Given that the i-th producer uses X_{1i} units of input 1 and X_{2i} units of input 2 to produce Q_i units of output, then the observed input coefficients, denoted as a vector $\mathbf{a}_i$, are written as follows:

$$\mathbf{a}_i = \left(\frac{X_{1i}}{Q_i}, \frac{X_{2i}}{Q_i} \right)^t = (a_{1i}, a_{2i})^t$$

A sample of observed input coefficients is shown diagramatically in Fig. 4.15.

Each sample point is thought to belong to a distinct production process described by the ray from the origin on which it is located. Consider the observation on the k-th producer at $\mathbf{a}_k$ on ray OL. The process OL represents the production of output with a certain fixed ratio between inputs, a_{2k}/a_{1k}. Movement outward along the ray indicates ever decreasing degrees of technical efficiency, and conversely, movement towards the origin represents increasingly efficient production. A lower bound need not exist to define how far one can move away from the origin; that is, we admit observations with finite inputs and infinitely small output. For a realistic technology we must define an upper bound dictating the maximum technically feasible output, or how close one can move towards the origin.

The sample of all observations in Fig. 4.15 represents the technology available for the overall production process. From the relative positions of $\mathbf{a}_k$, and observations $\mathbf{a}_i$ and $\mathbf{a}_j$, it is clear that the k-th producer was technically inefficient relative to the entire sample, and hence to the technology. This is because the k-th producer could have operated the two processes identified by $\mathbf{a}_i$ and $\mathbf{a}_j$ as a composite, resulting in the process $\mathbf{r}_k$, i.e.,

$$\mathbf{r}_k = \alpha_k \mathbf{a}_i + (1-\alpha_k)\mathbf{a}_j \qquad 0 < \alpha_k < 1$$

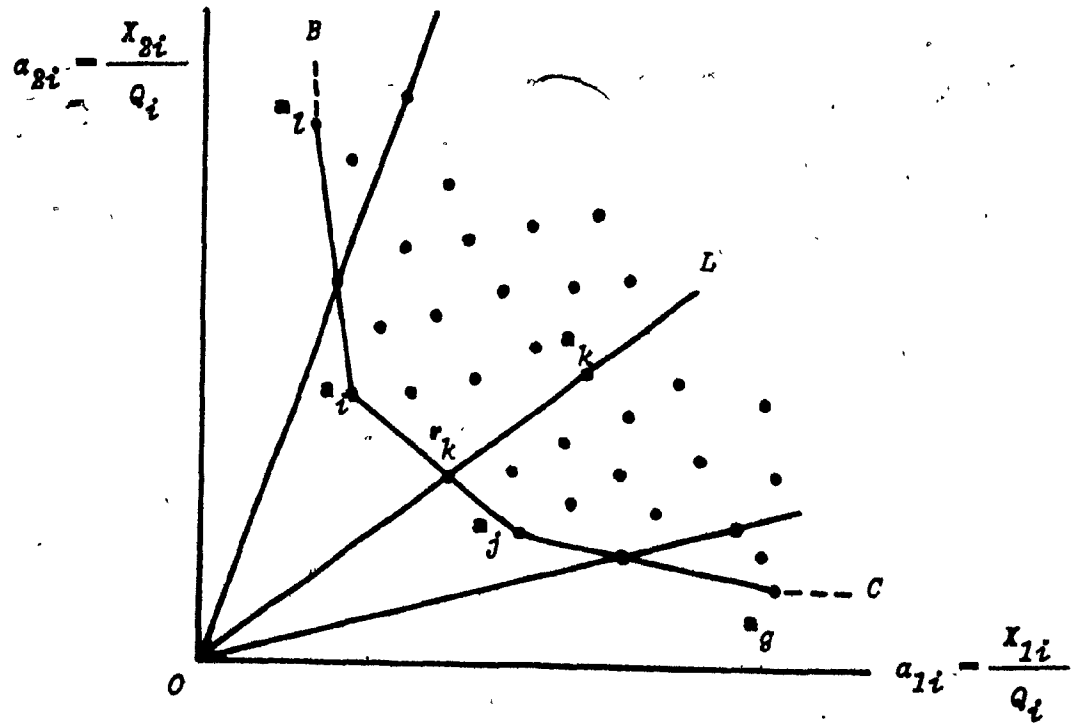


Fig. 4.15. A sample of observed input coefficients showing those processes which are identified as technically efficient (Reinhardt, 1973, p. 222).

r_k consumes the same amount of each input but produces more output, or equivalently, uses proportionately less input and produces the same amount of output. Since a_k and r_k are both on OL, then

$$a_k = \gamma r_k \qquad \gamma > 1$$

$$= \gamma \alpha_k a_i + \gamma(1-\alpha_k) a_j$$

$$a_k = \lambda_{ki} a_i + \lambda_{kj} a_j \tag{4.9}$$

where $\lambda_{ki}, \lambda_{kj} > 0$ and $\lambda_{ki} + \lambda_{kj} > 1$

There will, in general, be many sets of processes for which the process a_k may be written as a composite. The most efficient composite corresponds to the maximum possible value of γ. With this basic idea and the formulation of Eqn. 4.9, we can solve for the most efficient - maximum value of γ - composite by using a linear programming formulation (Reinhardt, 1973),

P1: $$\max \; e^t \lambda$$

such that $$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1s} & L_1 & 0 \\ a_{21} & a_{22} & \cdots & a_{2s} & 0 & L_2 \end{bmatrix} [\lambda] = a_k \qquad \lambda \geq 0$$

where e — unit vector

$\lambda = (\lambda_{k1}, \lambda_{k2}, \cdots, \lambda_{ks}, \lambda_{L_1}, \lambda_{L_2})^t$

s — total number of observations

L_i — large numbers introduced to permit a convex closure about the observations to approach the axes.

Problem P1 can be solved for every point in the sample. The non-zero elements of the vector λ for each solution will correspond to the two most efficient processes, of which the point process under examination is thought to be an inefficient composite. By examining every process in a sample, a_k, through the above formulation, the efficient frontier for the sample may be identified. In Fig. 4.15 the maximally efficient frontier for the given sample is identified by the line segment $B a_t a_i a_j a_g C$.

By comparing Figs. 4.13 and 4.15, it then follows that this efficient frontier is merely a piecewise linear approximation to the continuous theoretical linear homogeneous production function for this sample of producers. In addition, the technical efficiencies defined in terms of these two diagrams are the same, that is,

$$\frac{OQ}{OP} = \frac{1}{\gamma} = \frac{1}{e^t \lambda} \tag{4.10}$$

It is important to remember that because the efficient frontier is defined by the relatively most efficient points in the sample, the measures of inefficiency become all the more relevant. Efficiency is defined in terms of production levels we observe to be possible, rather than some theoretical and unattainable upper bound. Thus encouraging a producer, observed as inefficient, towards a 100% efficiency, may be viewed as a realistic goal.

Problem P1 is easily generalized to m inputs as shown below in problem P2. In this case the m non-zero elements of the optimal solution vector λ corresponds to the points that map one facet of the efficient frontier.

P2: $\max \; e^t \lambda$

such that $\left[\begin{array}{ccc|ccc} a_{11} & a_{21} \cdots & a_{1s} & L_1 & & 0 \\ \vdots & & \vdots & & \ddots & \\ a_{m1} & & a_{ms} & 0 & & L_m \end{array}\right] [\lambda] = a_k \qquad \lambda \geq 0$

Solving P2 for $k = 1,2,\ldots,s$ will identify the entire production function, or efficient frontier, and provide an estimate of the technical efficiency of each point by Eqn. 4.10.

4.5.4 Reformulation of the Production Model. In the section 4.5.5. we will discuss the interpretation of the measure of technical efficiency derived from the solution of problem P2. Rather than deal with problem P2 explicitly we investigate an equivalent formulation which is then applied to the data in the next chapter.

Consider the standard linear programming problem in which a performance measure P is specified to be maximized, i.e.,

$$P = \mathbf{o}^t \boldsymbol{\lambda}$$

subject to $\quad \mathbf{A}\boldsymbol{\lambda} = \mathbf{b} \qquad \boldsymbol{\lambda} \geq 0$

where

$\mathbf{A}$ = $m \times n$ matrix, with elements a_{ij}, $m < n$

$\mathbf{b}$ = $m \times 1$ " " " b_i

$\mathbf{o}$ = $n \times 1$ " " " o_i

$\boldsymbol{\lambda}$ = $n \times 1$ " " " λ_i

The above problem can be set up as indicated in Table 4.1. If the columns of this table are scaled, the solution of the resulting linear program will differ from the original problem only in the value of the λ_i's but not in which of the λ_i's correspond to non-zero values (Pierre, 1969). Table 4.2 shows such a scaling operation. Solving the linear programming problem indicated in Table 4.2 yields the following result (Pierre, 1969),

$$\left.\begin{aligned} P &= \frac{(\text{scaled } P)}{a_p} \\ \lambda_i &= (\text{scaled } \lambda_i)\ \frac{a_i}{a_p} \end{aligned}\right\} \qquad (4.11)$$

P	c_1	c_2	. . .	c_n
b_1	a_{11}	a_{12}	. . .	a_{1n}
b_2	a_{21}	.		.
.	.			.
.	.			.
.	.			.
b_m	a_{m1}	.	. . .	a_{mn}

Table 4.1. Initial scaling table for the standard linear program (Pierre, 1969, p. 219).

α_P	α_1	α_2		α_n
$\alpha_P P$	$\alpha_1 c_1$	$\alpha_2 c_2$	. . .	$\alpha_n c_n$
$\alpha_P b_1$	$\alpha_1 a_{11}$	$\alpha_2 a_{12}$		$\alpha_n a_{1n}$
$\alpha_P b_2$	$\alpha_1 a_{21}$	.		.
.	.			.
.	.			.
.	.			.
$\alpha_P b_m$	$\alpha_1 a_{m1}$	.	. . .	$\alpha_n a_{mn}$

Table 4.2. Scaling completed by columns for the standard linear program as indicated in Table 4.1 (Pierre, 1969, p. 220).

We now apply this scaling operation to problem P2, but first:

Let d_{ij} = amount of input i used in process j

o_j = " " output produced from process j

X_{ik} = " " input i used in department k

Q_k = " " output produced in department k

$\varepsilon_i = \frac{1}{L_i}$

Using the above notation problem P2 may be rewritten as:

$$\max \; e^t \lambda$$

such that

$$\begin{bmatrix} \frac{d_{11}}{o_1} & \frac{d_{12}}{o_2} & \cdots & \frac{d_{1s}}{o_s} & \vdots & L_1 & & 0 \\ \frac{d_{21}}{o_1} & & & \vdots & & & L_2 & \\ & & & & & & \ddots & \\ \frac{d_{m1}}{o_1} & & & \frac{d_{ms}}{o_s} & \vdots & 0 & & L_m \end{bmatrix} [\lambda] = \begin{bmatrix} \frac{X_{1k}}{Q_k} \\ \vdots \\ \frac{X_{mk}}{Q_k} \end{bmatrix}$$

$$\lambda = (m+s) \times 1 \text{ and } \lambda \geq 0$$

Multiplying column i by o_i for $i = 1,2,\ldots,s$, column j by ε_{j-s} for $j = s+1,\ldots,m+s$ and the right hand side of the matrix equation by Q_k, we have the following linear programming problem:

P3:

$$\max \sum_{i=1}^{s} c_i \lambda_i + \sum_{j=s+1}^{m+s} \lambda_j \varepsilon_{j-s}$$

such that

$$\left[\begin{array}{ccc|ccc} d_{11} & d_{12} \cdots & d_{1s} & 1 & & 0 \\ d_{21} & & \vdots & & 1 & \\ \vdots & & \vdots & & & \ddots \\ d_{m1} & \cdots\cdots & d_{ms} & 0 & & 1 \end{array}\right] [\Lambda] = \left[\begin{array}{c} x_{1k} \\ \vdots \\ x_{mk} \end{array}\right]$$

$$\Lambda = (m+s) \times 1 \qquad \Lambda \geq 0$$

Problem P3 is a scaled version of problem P2. This relationship between these linear programs is independent of the values of the L_i, $i = 1,2,\ldots,m$, and remains so as the L_i become infinitely large. In addition, the derivation of problems P1 and P2 as production models only require that the L_i be very large numbers. As $L_i \to \infty$ in problem P2, $\varepsilon_i \to 0$ in problem P3. Applying this limiting procedure to both of these problems results in the following new, but equivalent formulation:

P4:

$$\max \sum_{i=1}^{s} c_i \lambda_i$$

such that

$$\left[\begin{array}{ccc|ccc} d_{11} & d_{12} \cdots & d_{1s} & 1 & & 0 \\ d_{21} & & \vdots & & 1 & \\ & & & & & \ddots \\ d_{m1} & & d_{ms} & 0 & & 1 \end{array}\right] [\Lambda] = \left[\begin{array}{c} x_{1k} \\ \vdots \\ x_{mk} \end{array}\right]$$

$$\Lambda = (m+s) \times 1 \qquad \Lambda \geq 0$$

Problem P4 is equivalent to the statement: maximize the output attainable by using the same amount of factor inputs consumed in department k with the technology represented in all departmental processes. As the non-zero elements of the Λ vector in the optimal solution to problem P4 correspond to the same indices of the non-zero elements of the vector in the optimal solution to P2, it follows that output is maximized by simultaneously operating the same efficient processes in a scaled form. This is shown diagramatically in Fig. 4.16 (note the similarity to Fig. 4.11).

By solving problem P4 for each department we can explicitly map out the production function for the technology represented by all departments in the sample. This function describes the capacity output for different input combinations; that is, the output attainable from a given input combination with maximally efficient production. In addition, the efficiencies derivedfrom both problems P2 and P4 are identical for each and every department in the sample as shown below.

Let Λ^{P4} — optimal solution to problem P4

TE_k^{P4} — technical efficiency of department k calculated by solving problem P4

then,

$\sum_{i=1}^{s} o_i \lambda_i^{P4}$ — capacity output for department k

$$TE_k^{P4} = \frac{\text{observed output}}{\text{capacity output}} = \frac{Q_k}{\sum_{i=1}^{s} o_i \lambda_i^{P4}}$$

Using similar notation for the solution of problem P2, we have from Eqn. 4.11

$$P = e^t \Lambda^{P2}$$

$$\text{scaled } P = \sum_{i=1}^{s} o_i \lambda_i^{P4}$$

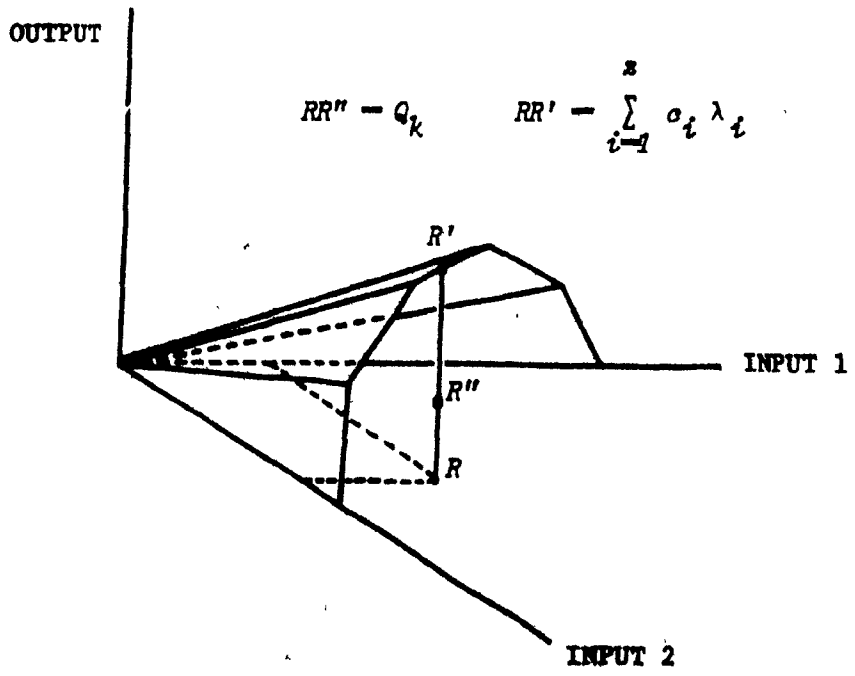


Fig. 4.16. A piecewise linear approximation to a continuous production function that is identified by solving a linear program (P4) for each of the observed production processes in a sample.

$$\alpha_P = Q_k \quad (\ \alpha_P \text{ from Eqn. 4.11 }\ )$$

$$\text{therefore } {}^{t}\lambda^{P2} = \frac{\sum_{i=1}^{s} c_i \lambda_i^{P4}}{Q_k}$$

But Eqn. 4.10 states,

$$TE_k^{P2} = \frac{1}{{}^{t}\lambda^{P2}} = \frac{Q_k}{\sum_{i=1}^{s} c_i \lambda_i^{P4}}$$

and therefore,

$$TE_k^{P2} = \frac{Q_k}{\sum_{i=1}^{s} c_i \lambda_i^{P4}} = TE_k^{P4}$$

Problem P4 is in the explicit form derived by Koopmans (1951) for analyzing production as an efficient combination of activities. Although the efficiency measure is the same for both P2 and P4, the latter formulation is preferred because of its ease in dealing with the input variables. Either incremental or absolute changes in the inputs are more readily implemented and observed in a function that operates explicitly on these inputs.

In order to appreciate the relationship of Koopmans formulation to production planning in the firm, we consider the situation where each production process has been estimated from many observations on each process available in the plant. Let us assume that the output measure is in dollars of profit for each process. The solution to P4 then implies that output or total profit is maximized for a given set of inputs, by operating some subset of the relatively most efficient processes in the plant. This result is not surprising. The linear

programming solution not only identifies which processes are relatively most efficient, but also dictates the level at which to operate them to maximize profit.

From this simple example and the general discussion on estimating input coefficients at the beginning of section 4.5.3, it is apparent that the more exacting and specific the technique for estimating the elements of the $(m \times s)$ matrix **D** and the $(n \times 1)$ vector **c** of problem P4, the more literal an interpretation of the solution is allowed.

We have identified three approaches to estimating the coefficients of problem P4: i) a detailed investigation of a single unit as just described; ii) estimation of average coefficient values from a sample of each type of process, and iii) the method we have elaborated throughout this section. These estimation techniques differ significantly with respect to the questions their resulting programs are capable of answering.

It is apparent that the technique that we are pursuing is the weakest of these three in terms of specifically monitoring the exact or even average behaviour of each producing unit. By ordering all observations in an input-output space, such that the efficiency of each point relative to the sample can be approximated, our methodology investigates only relative behaviour.

Thus little or no meaning can be attached to the actual values of the marginal productivities and other output sensitivity measures. Yet the order of magnitude comparisons of these indices can be interpreted as meaningful. They will indicate the relative roles of various resources in production for particular departments relative to the sample and hence to the observed technology. This is not disadvantageous. The decision process in government for the allocation of new equipment does not require absolute numbers, but merely general indicators of the overall process (see Chap. 2).

The difficulties with Farrell's methodology can now be appreciated as a question of problem definition. Almost all linear programming

production formulations are geared to solving a planning problem and making a decision rather than an analysis of past behaviour. In most instances, the estimation of a capacity, or a maximum possible output, as opposed to an average or expected maximum, is not really important, unless there is an accompanying rigorous presentation of how this output measure is affected by input changes.

Part of the rationale for adopting our linear programming production formulation lies with the difficulty in integrating both full and part-time radiologist man-years into a continuous function. At this point we can investigate how to pursue this in our model.

Just as we recognised the impossibility of introducing both full and part-time inputs into a continuous form, it follows that their effectively indistinguishable roles in production deny simultaneous entry into the same process. Any input combination in the optimal solution to P4 is represented by a composite of efficient processes; i.e., the columns of the **D** matrix corresponding to the efficient processes. As these efficient processes map the production surface it is only this subset of processes which cannot contain both full and part-time inputs.

Let $d_i = \begin{bmatrix} d_{1i} \\ d_{2i} \\ d_{3i} \end{bmatrix}$

where d_i – a column of the **D** matrix corresponding to an efficient process

d_{1i} – number of full-time radiologist man-years used in process i

d_{2i} – number of part-time radiologist man-years used in process i

d_{3i} – $(m-2) \times 1$ vector of all other inputs

Consider the efficient processes d_1 and d_2 where $d_{12} = d_{21} = 0$. Using these two processes we can represent an observed department with both full

and part-time radiologists as indicated below,

$$\begin{bmatrix} d_{11} \\ 0 \\ d_{31} \end{bmatrix} \lambda_1 + \begin{bmatrix} 0 \\ d_{22} \\ d_{32} \end{bmatrix} \lambda_2 - \begin{bmatrix} X_{1k} \\ X_{2k} \\ X_{3k} \end{bmatrix}$$

$$\lambda_1 , \lambda_2 > 0 \quad X_{1k} \neq 0 \quad X_{2k} \neq 0$$

Admittedly, the above example is a gross simplification, yet it demonstrates the kind of approach that is allowed with the linear programming model. We shall elaborate on this somewhat in the next chapter when constructing specific models.

Finally, problem P4 has an interesting interpretation in terms of a profit maximizing view of diagnostic radiology departments. The government ultimately pays for the materials, non-MD labour, and equipment in radiology departments through the hospital budget. For the most part, radiologists are paid according to the services they provide, which as described in section 4.5.1 are measured in diagnostic radiology professional units; each type of examination being equivalent to a fixed number of professional units, each unit having a fixed dollar value.

Thus by maximizing output, either in terms of professional units or diagnostic examinations, the volume of services provided by radiologists is also maximized and so, specifically in the case of professional units and approximately in the case of examinations, is their incomes. The idea of price efficiency introduced in section 4.3.3, calls for a minimization of the cost per unit output. This is not relevant to diagnostic radiology as the individuals accumulating an income from production do not bear the cost of paying for inputs. Hence there is not a preferred input combination along any isoquant with respect to profit maximization.

This should not be interpreted as implying that radiologists are concerned solely with their incomes, oblivious to the effective cost management of their departments, but rather, that the model can be interpreted as estimating a total income capacity as well as a service capacity.

4.5.5 . The Technical Efficiency of a Radiology Department. The technical efficiency is a measure of a firm's, or in our case, a radiology department's, success in producing maximum output from a given set of inputs. A number of qualifications should be made added to this seemingly simple concept.

As has been stated previously, the construction of the production function is such that all measures of efficiency are relative to the sample for which the function is identified, and therefore the introduction of additional departments may reduce but cannot increase the efficiencies of any departments initially in the sample (Farrell, 1957). This is desirable as a department may, for example, be highly efficient in a local or regional context, but not by a province-wide standard.

Throughout the discussion and development of the production model a basic underlying assumption was never stated explicitly, and so we now rectify the situation: all production variables, inputs and outputs, are qualitatively homogeneous within their defined subclasses (Koopmans, 1951). This assumption allows for comparison between the resources and products of all departments in a sample. Specifically it means that we can add inputs from various departments which are defined to be in the same qualitatively uniform input class. The same is also true of the output measure. For example, consider the i-th row of the matrix equation in problem P4, that is,

$$\sum_{j=1}^{s} d_{ij} \lambda_j + \lambda_{s+i} = X_{ik}$$

where d_{ij} = amount of input i used in process j

X_{ik} = amount of input i consumed in department k.

The validity of this equation is based on the premise that, as stated above, the i-th input to each of the s processes and to department k are qualitatively identical. If the quality of input d_{il} were different from that of X_{ik}, then $d_{il} \lambda_l$ could not meaningfully compose any fraction

of X_{ik}, and thus the i-th process would have to be omitted from the linear program examining department k. The inputs to every department are in some sense unique. Hence, without the homogeneity assumption, a linear programming production model consisting of only one observed process would have to be used for each department; that is, a common framework would be missing in which to compare the production behaviour of different departments.

The establishment of an input or output class attaches an average quality to that input or output in all departments and thus a department with relatively "high grade" factors should be favoured with a higher technical efficiency.

If the heterogeneity of factors is spread evenly over all departments it will not be of consequence, "it is when there are differences between firms in the average quality (or, more strictly, in the distribution of qualities) of a factor, that a firm's technical efficiency will reflect the quality of its inputs as well as the efficiency of its management." (Farrell, 1957) Inputs can be made more qualitatively uniform by defining a greater number of subclasses.

Managerial efficiency in a radiology department may be seen as having two facets. The first, and most obvious, deals with the productive organization of a department. This can include such factors as the physical set-up of the department and the location of the department in the hospital, allocation of tasks amongst individuals in the department, the utilization of materials, the qualitative and quantitative functional degradation of equipment (see section 2.2), and the balancing of the labour and machine mixes.

The second, and equally important, criterion for managerial efficiency involves estimation of the demand for radiographic services. As diagnostic radiology is but one part of a larger hospital structure, the department must gear its growth and development to the needs of the hospital. Thus some part of the technical efficiency of a department will reflect the demand for its services. Inefficient behaviour can be observed when demand is not sufficient to fully utilize the capabilities

of the department, which in turn suggests poor planning, at least for the short term, on the part of the department's management.

After constructing measures of technical efficiency for the radiology departments in the next chapter, we will quantitatively investigate the two factors mentioned above which can affect the efficiency of a department as observed through our construct; that is, the homogeneity assumption and the effect of departmental management.

4.6 Summary

In this chapter we have introduced the basics of production theory, investigated the problems inherent to the empirical estimation of a continuous production function, and described a technique for identifying a production function as a capacity output or maximally efficient concept.

In section 2.3.3 we listed the six model building criteria proposed by Little (1970). Admittedly our production function, as embodied in a linear program, is not "simple", yet we maintain that its essentials are conceptually easy to understand. If we consider a sample of producers with only one input and one output, find the particular member with the lowest input per unit output ratio, and then define the relative efficiency of all other producers as a fraction of this best producer, then in effect we would be pursuing the identical process involved in solving the linear programming production model. We increased the number of inputs (to m), and as a result made this process somewhat more complicated in order to attain the equally important goal of "completeness".

As discussed at the beginning of section 4.5.5, this model readily allows for the introduction of new information (departmental observations) in a manner that is intuitively appealing, and hence the model is "adaptive". Whether this type of model is "robust" and "easy to control" can only truly be measured on implementation. Yet even at this stage, it would seem that in a suitable man-machine computer environment a linear program will be "easy to communicate with". With the discussion in this, and the previous two chapters, we have established a complete framework in which to pursue an analysis of the data, and we proceed to do so in the next two chapters.

CHAPTER 5

TWO PRODUCTION FUNCTIONS FOR DIAGNOSTIC RADIOLOGY DEPARTMENTS

5.1 Introduction

A production function for diagnostic radiology facilities describes the technological relationship between departmental inputs and outputs. This functional relationship dictates the maximum amount of output capable of being produced by each and every set of specified inputs. In this chapter we employ the linear programming formulation of the production model developed in the previous chapter to identify a piece wise continuous approximation of the production function for radiology departments.

Two different models will be investigated. The distinction between these models lies in the specification of their production variables. Before proceeding to the elaboration of these models, a review of the data and the possible quantification of inputs and outputs will be provided.

Recall that the solution of problem P4 (see section 4.5.4) for each department in a sample results in an approximation to the corresponding capacity output. These model outputs are then compared to the respective observed departmental outputs to define a technical efficiency for each department. As indicated below, the frequency distribution of these technical efficiencies for the sample of departments from which a model is constructed can be used to test the plausibility of the model itself.

5.2 Production Variables - Definition & Quantification

A radiology department requires the concurrent utilization of manpower, materials, and equipment in the provision of diagnostic services. In this section we investigate the manner in which to introduce these variables in a production model.

Nerlove (1965) defines eight different methods for quantifying the production variables for a two input - one output production function:

> "I. All variables measured in physical terms. II. All variables measured in value terms. III. Output measured in physical terms and factors measured in value terms. IV. Output measured in value terms, factors measured in physical terms. V. & VI. Output measured in value terms and one factor measured in physical terms. VII. & VIII. Output measured in physical terms and one factor measured in value terms."

The choice of a unit of measurement for any variable is ultimately dependent on the intersection of two equally important factors.

> "Generally, the methods used to quantify the variables will depend on the type of data available and,..., the aims of the analysis" (Kovner, 1968).

This, in turn, reflects a crucial element in socio-economic research. For the most part, the investigator is confronted with a non-experimental environment. The data to be used has been collected by an independent agent whose criteria are usually very different from those of the investigator, and for whom the "experiment" cannot be repeated. In addition, if any errors or omissions occur in the collected data, it is rarely possible to rectify them at a later point in time. This is the case of the present study.

A data base (D.1, D.2 & D.3, see section 3.4) was established by aggregating the data collected by different departments in the federal and provincial governments. This information was collated at least one year before this study was initiated.

Hence, the definition and quantification of production variables requires a compromise between what we would ideally choose, and that which the realities of the data require.

5.2.1 Production Inputs. In this section we discuss the data representation of the four basic inputs to diagnostic radiology departments described in section 3.2.2. To reiterate then, we shall be considering: radiologists; x-ray technicians; materials; and equipment. While each of these four inputs must simultaneously be present in a department in order that diagnostic services be provided, it is possible to define models which may not include variables representative of all four input types. Yet, if for any model, or equivalently any production variable set, a department has a zero entry for any one of the input types included in that model, then this department must be deleted from the sample upon which that model is to be based.

5.2.1.1 Radiologists. The presence of a radiologist input to departments is indicated on data sets D.1 and D.3. As radiologists are not hourly wage earners, it is hardly surprising that on both data sets we are merely presented with the total number of full-time and part-time radiologists in each department. While it would obviously be preferable to have an aggregate hourly input per annum per department for radiologists, the available representation may still be profitably used.

Consider the case of having only full-time radiologists. This may be represented diagrammatically as a radiologist man-year per unit output axis in Fig. 4.15. Moving along any ray from the origin towards the efficient production function would represent increasing output per full-time radiologist man-year, which surely describes increasingly efficient production.

As a result of our homogeneity assumption (see section 4.5.5) we ascribe an average quality to each man-year. In terms of education and training we can safely assume a large degree of homogeneity in this measure as a result of the relatively small number of training centers and the requirement that all radiologists sit the same fellowship examinations.

Yet, it is also apparent, that the same number of man-years in two departments can represent vastly different hourly inputs for the radiologists. In this case one would expect a corresponding difference in the actual output produced in these departments. Hence the department with the larger output per radiologist man-year ratio would be recognized as the more efficient one, at least insofar as the radiologist input, although the output per hour ratios may be comparable. This is not necessarily undesirable. It would reflect relatively poor managerial efficiency on the part of the department with the smaller output per man-year ratio in estimating the hospital demand for services and establishing its radiologist staff. For example, consider two departments that both have only one full-time radiologist. In the first department the radiologist puts in 1000 hours to produce 10,000 diagnostic radiology professional units, whereas in the second department, the radiologist puts in 2000 hours to produce 20,000 professional units. Although their output per radiologist man-hour ratios are the same, it is apparent that the radiologist in the second department is more highly utilized, as an input production resource, than the radiologist in the first department. We have previously agreed that managerial efficiency, as reflected in the staffing pattern, should be a component of the technical efficiency (see section 4.5.5).

The production period encompassed by the production function is one year and will be estimated from 1971 data. Data set D.3 (the inventory of radiographic equipment) was collected throughout 1971-73, while data set D.1 (hospital statistics collected by the federal government) is strictly for 1971. The discrepancy which often exists in the number of full-time and part-time radiologists in the same department between these two data sets reflects a staff change during 1971-73. To keep the data consistent in time, we use the radiologist input data from data set D.1.

Two types of medical manpower input, both divided into full and part-time categories, are indicated on D.1; radiologists and "other medical staff". Six of the 139 departments in acute hospitals on D.1

provide a zero value for both types of medical manpower input, either full or part-time. Hence the maximum sample size, considering any medical manpower, will be 133 departments or 96% of the initial sample. Of these remaining 133 departments only 6, or 4.5%, indicate a medical manpower input other than radiologists. Thus the "other medical staff" is included as part of the radiologist input.

The sample of 133 departments may be further broken down: 57 departments, or 43%, have only full-time radiologists; 61 departments, or 46%, have only part-time radiologists; and 15 departments, or 11%, have both full and part-time radiologists. Hence the majority of departments in the possible radiologist input sample, i.e. 89%, have only full or part-time radiologists, but not both. This factor alone, allows simultaneous introduction of both full and part-time radiologists, as separate inputs, to the same linear programming production model. This could not be accomplished with a continuous production function (see section 4.3.2).

5.2.1.2 Technicians. The second major category of manpower input is that of x-ray technicians, and is represented on all three data sets, though in different forms. Data set D.3 is immediately discounted as a source for this input, both because of the time consideration discussed above, and the fact that only the total number of full and part-time technicians is provided. Although data set D.2 provides the total number of non-MD hours worked in 1971 for each department, we once again defer to data set D.1. This data set provides the number of full-time and part-time technicians, the number of technician separations, and the total number of man-hours, all for each of seven non-MD staff classes.

Table 5.1 lists the 7 mutually exclusive and collectively exhaustive technician classes which comprise the non-MD staff on D.1. In 8 of the 139 departments on D.1 all seven classes were indicated to have zero entries, thus resulting in a technician sample of 131 departments or 95% of the possible maximum. Beside each labour class in Table 5.1 is entered the number of departments where the volume of man-hours for the respective classes is indicated to be non-zero.

Non-Medical Manpower Categories	Number of Diagnostic Radiology Departments on Data Set D.1 with a Non-Zero Volume of Man-Hours
1 Non-medical specialists	11
2 Technicians with diplomas	128
3 Technicians without diplomas	11
4 Combined laboratory and radiology technicians	1
5 Other technical staff	62
6 Nurses in Radiology	27
7 Others - incl. file clerks and secretaries	117

Table 5.1. The seven mutually exclusive and collectively exhaustive technician categories which comprise the non-MD staff of a diagnostic radiology department as provided on data set D.1.

For purposes of introducing a technician input to a production model we have aggregated these seven classes into three major groups. The reason for doing so is two-fold. First, it is hardly obvious that the members of each of the original classes provide a unique and distinct contribution to the production process in radiology departments. This is easily appreciated in light of the fact that for a large proportion of the departments at least four of the seven inputs are zero, as shown in Table 5.1.

Second, by defining a large number of input classes, relative to the number of departments in a model sample, the methodology we have adopted (i.e. the linear programming model described in Chapter 4) will recognize some departments as maximally efficient, only because they cannot be represented as a linear combination of the production processes defined by the other departments in the sample. Efficiency and uniqueness are not considered to be equivalent terms. This idea is elaborated in section 5.3.

We thus define the following new technician groups:

A	Technicians (Diplomas) & Non-MD Specialist	= Class 1 + Class 2
B	Technicians (No Diplomas)	= Class 3 + Class 4 + Class 5 + Class 6
C	"Others"	= Class 7

The unit of measure for each of these groups is man-hours.

It is not necessary that all three groups be introduced to a model together. We may, for example, consider group A as the most important and exclude the others. In terms of the linear programming production model, this would call for maximizing the output per "Tech (Dipl) & Non-MD Spec" man-hour. Conversely, we may consider two, or possibly all three groups as equally critical to the production process, and would want to introduce them. Yet by virtue of their definitions, the introduction of group B requires the presence of group A, and the introduction of group C requires the presence of both

groups A and B. This does not preclude the possibility of combining groups, such as group A + B defining a total technician input, and group A + B + C defining a total non-MD staff input.

The seven technician classes represent the most homogeneous breakdown of the non-MD labour available in the data. As indicated above, they may neither be suitable for introduction to the linear programming model, nor truly representative of distinct roles in the production process of diagnostic radiology. By aggregating these seven inputs (groups A, B, & C or groups A + B, & C or group A + B + C) we introduce, to a varying extent, a degree of heterogeneity to the resulting production variables. While there may be some highly trained non-MDs who perform narrowly defined tasks, we assume that radiology departments function in a roughly similar fashion to medical practices. There is evidence that physicians in a practice environment tend to use medical technicians, nurses, and even clerical manpower without formal training, in essentially the same manner (Reinhardt, 1973). This would imply that the heterogeneity of aggregation, in at least the three level case, will not be a dominant factor as a source of technical inefficiency in the model.

5.2.1.3 Materials. A material input is indicated on all three data sets. On D.1 and D.2 we are provided with the total non-labour expenses in dollars, whereas on D.3, we have the total number of exposed films. All three of these measures are for 1971. In this instance, we reject the physical measure in favour of the value term. Several factors indicate such a decision.

First, exposed films are in reality an output measure of what was produced, whereas non-labour expenses is a budgetary input for each fiscal year. Second, the value term includes the cost of radioactive materials and other supplies in addition to the cost of films. And third, only 86 departments on D.3 of a possible 139, or 62% of the maximum sample indicate a non-zero value for the number

of exposed films. This should not be interpreted as a statement that some diagnostic radiology departments can provide services without films, but rather, that on the survey that preceded the construction of D.3, the question relating to films was either improperly answered or omitted by 53 departments.

Although D.1 and D.2 both provide the value term, only 110 departments on D.2 have non-zero values, while 137 departments on D.1 have non-zero values. Thus we chose the latter data set as the material input data source.

Fig. 5.1 shows a plot of the number of films, from D.3, versus the non-labour expenses, from D.1; each point represents a department. The approximately linear relationship between these two variables further supports the decision to adopt non-labour expenses as the measure of material input, if only on the grounds of maximizing the sample size.

In effect, the value term represents a weighted sum of the physical material inputs to a department, but measured in dollars. We assume that 1971 dollars are a truly homogeneous unit of measure between all departments.

5.2.1.4 Fixed Capital. The quantification of fixed capital presents a much more difficult problem than either manpower or materials. The bulk of fixed capital in diagnostic radiology departments is in the form of equipment, and floor space or x-ray rooms. The major stumbling block to integrating capital into a production model is that "differences in vintage result in different capabilities for capital (Kovner, 1968)". That is, capital is subject to technological change.

Our present concern is for real technological change, which may be seen as having two facets. First, we can observe a functional degradation of capital over time; that is, a noticeable decrease in the amount (per appropriate unit) or deterioration in the quality of services provided. And second, there may be a major change in the

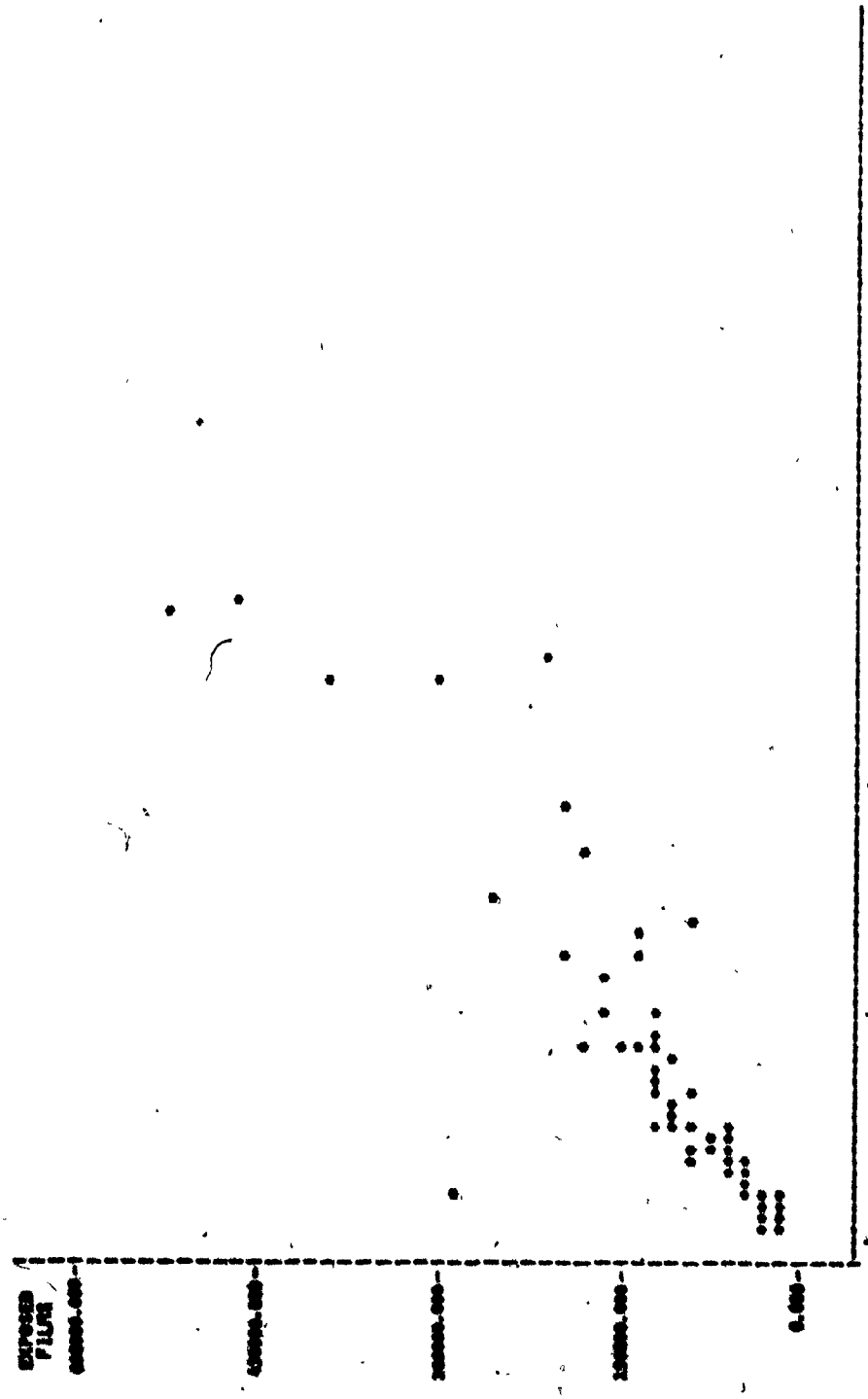


Fig. 5.1. The number of exposed films versus the non-labour budget in diagnostic radiology departments for 1971.

technology available from suppliers to an industry, in this case, from the manufacturers of radiographic equipment.

The most common technique for aggregating capital is to take the gross investment minus depreciation (at a fixed rate), and arrive at a cumulative net investment which is used as the amount of capital stock (Klein, 1962; Kovner, 1968). This approach, or in fact any method for dealing with the dollar value of capital, requires data such as equipment type, cost, and age. This information was requested as part of the Radiographic Equipment Inventory Survey conducted by the Quebec government (Anon., 1972a). Unfortunately the equipment cost and age part of the Inventory, data set D.3, is completely riddled with errors (Renard, 1974). In fact, accompanying the D.3 tape was a 100 page computer print-out indicating the errors and omissions of this particularly crucial section (Thuret, 1974). As data set D.3 is the only one of the three which contains capital cost data, all hope of using a monetary approach was abandoned.

Some usable data is still available. Data set D.3 provides the number of x-ray rooms per department, as well as information on the number of hours utilized, floor space, and type (one of nine different room classes) per room. D.1 provides the number of pieces of equipment in each of 13 different classes per department. At present there is no way to merge D.1 and D.3 in order to know which machines are in which rooms. The different room and equipment classes are shown in Table 5.2.

All three room oriented measures of input, i.e. total room-hours and total floor space (square feet) per room and the total number of x-ray room-years per department, have a certain degree of heterogeneity because of the varying types of services that are provided in different types of rooms. In none of these three cases can the varying service factor be accounted for explicitly in the production model we shall pursue.

X-Ray Room Categories

1 General
2 Angiography
3 Craniography
4 Mammography
5 Neuroradiography
6 Scopic
7 Tomography
8 Urography
9 Others

Radiographic Equipment Categories	Not Equipped for Fluouroscopy	Equipped for Fluouroscopy	
		without spot film device	with spot film device
Stationary Machines	1	2	3
Mobile Machines	4	5	6
Miniature Chest Machines			7
Image Intensifier - viewing only			8
Image Intensifier - viewing & recording			9
Mobile Image Intensifier			10
Video Tape Recorders			11
Image Amplifiers			12
Others			13

Table 5.2. The nine x-ray room categories and the thirteen equipment classes provided on data sets D.3 and D.1, respectively.

Heterogeneity can also be introduced through technical change as discussed above. Lindheim wrote in 1971:

> "The greatest technological advance to affect the practice of diagnostic radiology occurred a decade ago when automatic processing was introduced. Since then, the x-ray machine itself has remained relatively unchanged."

Hence the state of radiographic technology may be considered to be relatively stable for at least the period 1961-71.

In addition to the room data described above, we are provided, per room per department, with the year in which the room was established. This does not preclude the possibility of having newer equipment in a room than is indicated by the year in which it was established. These years then, represent an upper bound for the age of the fixed capital within the x-ray rooms. This assumes of course, that none of the rooms ever established in a radiology department, are presently being used in the hospital by other departments. This might require transferring older equipment, from a room no longer in use by diagnostic radiology, to a newer room.

Although data set D.3 does not provide complete room data for all 139 departments, we can estimate from the 93 departments that are available, that: 44% of all diagnostic x-ray rooms in acute hospitals in the province were established from 1966 to 1971; 34% from 1960 to 1965; 20% from 1954 to 1959; and only 2% before 1954. Thus, approximately 78% of all rooms were established during the decade preceding the year upon which this study is based. Because of this, we assume that functional degradation is not an overriding concern in terms of deriving a model. In the next chapter we reinvestigate this by plotting efficiency versus mean room age per department.

Both total room-hours and floor space are not significantly affected by technological change, yet their requisite data is provided for only 47% and 50%, respectively, of the possible 139 departments.

Room-years data, on the other hand, is available on 93 departments; a data base size increase of at least 34%.

Our criteria of maximizing sample size in the choice of production variables is important because construction of a production model requires the merging of various input and output samples. The resulting model sample can at best be as large as the smallest production variable sample included in the model. In fact, experience with different models has shown that a complete model sample will typically be 10 to 30% less than the size of the smallest individual production variable sample. This results from the many holes that exist in the data.

Thus, one measure of the fixed capital input to departments will be room-years. The same argument as above, with respect to technological change and sample size, can be used to rationalize the adoption of total equipment-years as another measure of capital input. The equipment data is provided on D.1 for 136 departments.

In Table 5.2 we have indicated thirteen different equipment classes. Only the first 6 contribute directly to the production of diagnostic services, whereas the latter 7 may be considered as supportive or peripheral machinery. This is verified by the fact that not one of the 136 departments had a zero volume for all of the first 6 pieces, but many, approximately 32%, had a zero volume for all of the remaining seven. Hence the equipment input is taken to be the number of pieces in the first six classes of equipment. The same type of heterogeneity that exists by aggregating rooms in different classes, is also present when combining the different equipment types. The following question then arises: why have we combined equipment and room types, but acknowledged a degree of disaggregation in defining the three types of non-medical manpower?

The answer depends on the definition of output. In the next section we discuss two measures of output; the total volume of services and a weighted sum of services. Each type of non-MD labour provides

a different productive input per unit of output, whereas it is possible to produce a volume of output in only one room (regardless of type), or with one of the six types of equipment. Therefore, inputs are defined in terms of their contribution or role in the production process.

Admittedly, neither equipment-years nor x-ray room-years would be the optimal choice for a capital input to a production model, yet they not completely void of redeeming features. First, as the production model is meant to provide information to an equipment investment decision it is necessary to include some representative index of equipment to the model. Second, while investment decisions can obviously be considered in value terms, they may also be regarded as a change in the number of machines and possibly in the number of rooms. And finally, the solution of the linear programming model is associated with a maximization of output per different inputs, thus maximizing the output per total number of equipment-years (room-years) or the average output per piece of equipment (room), which in turn, is the type of production improvement we recognize as desirable.

Ultimately, the difference between using a value term or the physical inventory for a capital input, lies with the interpretation of the model output, or in the case of this work, the technical efficiency. As discussed in section 4.5.5, the non-homogeneity of an input variable affects the meaning of the technical efficiency, and may even go so far as to be determinant in its value. The homogeneity of the dollar as a unit of measure has the advantage of allowing a depreciated monetary input. This is contrasted with ascribing an average quality to an input, or specifically an "average equipment type" or "average room type". This implies that when using an input such as equipment or room-years the resulting technical efficiency cannot be taken as all encompassing but must be tempered by a re-examination of the raw data.

Various schemes to quantitatively account for the non-homogeneity of these capital inputs in the interpretation of the technical

efficiency are suggested in the next chapter.

5.2.1.5 Definition of Inputs. Regardless of which output measure is chosen, the definition of inputs is important to the interpretation of the technical efficiency. Two distinct avenues are available. The first approach would be to define the inputs in terms of what is produced; that is, the volume of labour-hours directly involved in production, only those materials that are actually used and not just the inventory of supplies, and the volume of machine-hours dedicated to producing output. The resulting technical efficiency would then reflect solely upon the internal environment of a department in terms of the organization, labour and technological efficiencies, that is, a localized concept.

Secondly, one could define the inputs to reflect what is available for production within the period of consideration. Such a set of input production variables would be defined at the beginning of the production period. In this instance the technical efficiency would reflect both on the extent to which these inputs are utilized and the effectiveness with which they are utilized. Hence, the technical efficiency would have two components, one related to the radiographic service demand from the hospital, and the other corresponding to the internal (department) technological efficiency. In Chapters 3 and 4 it was argued that this second approach is preferred.

The four types of production inputs we have recognized satisfy the requirements of this latter point of view. This is explicitly the case for radiologist man-years, the non-labour budget and, either room or equipment years.

Initially, one may have thought that non-MD labour hours is really an output reflecting the extent to which the non-MD labour input has been involved in production. In fact, the hours and salaries of the non-MD staff are stipulated at the beginning of each fiscal year in the departmental budget (Anon., 1974a). In addition, the existence

of salary contracts for the technical and clerical staff, and their hourly wage scale, would make it politically inopportune to vary the utilization of these groups with the demand for services over the period encompassed by their contracts. Therefore, the non-MD labour hours can be taken as a measure of input. As with the other inputs, this manpower input can be utilized, i.e. involved in the production of services, to varying degrees over the production period.

5.2.2 Production Outputs. In section 4.5.1 we introduced two measures of output for the radiographic subsystem of diagnostic radiology; diagnostic examinations and diagnostic radiology professional units.

Examinations require the utilization of all the input resource types discussed above. Different types of examinations demand varying levels of each input. In 1972 the Quebec government recognized 155 different types of examinations, ranging from procedures as simple as a finger x-ray to the more complicated ones such as a specific encephalography: posterior fossa pneumogram, or a major chest and bone survey: lungs, skull, spine, pelvis, thorax and shoulder (Anon., 1972b).

In this way, examinations are very similar to other types of output measures used in health services research, such as patient visits or cases treated. That is, they mark only the occurrence of an event, where the volume of resources and type of resources used may very dramatically. Therefore, the total volume of examinations is, in fact, a non-homogeneous measure of output.

The number of examinations provided in each department in 1971 is available on data set D.1. A non-zero value for this measure is indicated for 136 departments, or approximately 98% of the maximum sample size.

The volume of diagnostic radiology professional units (P.U.) produced in a department over a period of one year represents a weighted sum of the number of examinations produced during the same period. Each type of examination is assigned a fixed number of

professional units, where all P.U. are equivalent. Radiologists are paid according to the volume of units they produce.

The number of P.U. per examination is meant to reflect the radiologist time and skill required for the examination. This ratio can be interpreted as a measure of the complexity of each procedure and hence, to a large extent, the required utilization of input resources per examination. The ratio of professional units associated with the most and least complex examinations is 12:1 (Anon., 1972b). Therefore, diagnostic radiology professional units are a relatively homogeneous unit of measure for output.

The volume of P.U. provided in each department in 1971 is available from data set D.2. A non-zero value is indicated for 110 departments, or approximately 79% of the maximum sample size.

5.2.2.1 Choice of Model Output. In section 5.2.1 we established the pattern of trying to maximize each production variable sample size in the choice of variables. At this point we reverse the trend and choose professional units as the measure of output. The reason for doing so is two-fold: i) diagnostic examinations is the relatively non-homogeneous output and; ii) the volume of professional units is a standardized measure of output, and as such, would be expected to more closely obey the constant returns to scale assumption upon which the production model is premised (see section 4.5.1).

In Figs. 5.2 to 5.4 we have plotted both the total number of diagnostic examinations, and the number of P.U. per department in 1971, versus various indicators of department size.

Figs. 5.2a and b depict examinations and professional units respectively, versus the total radiologist salary, in dollars. All value measures are taken from data set D.1. Two trends are visible here; first, professional units are related to M.D. salaries in a more linear fashion than are examinations, and second, the variability in output at any salary level is reduced in going from examinations to P.U. These of course are to be expected, as a result of the fee-

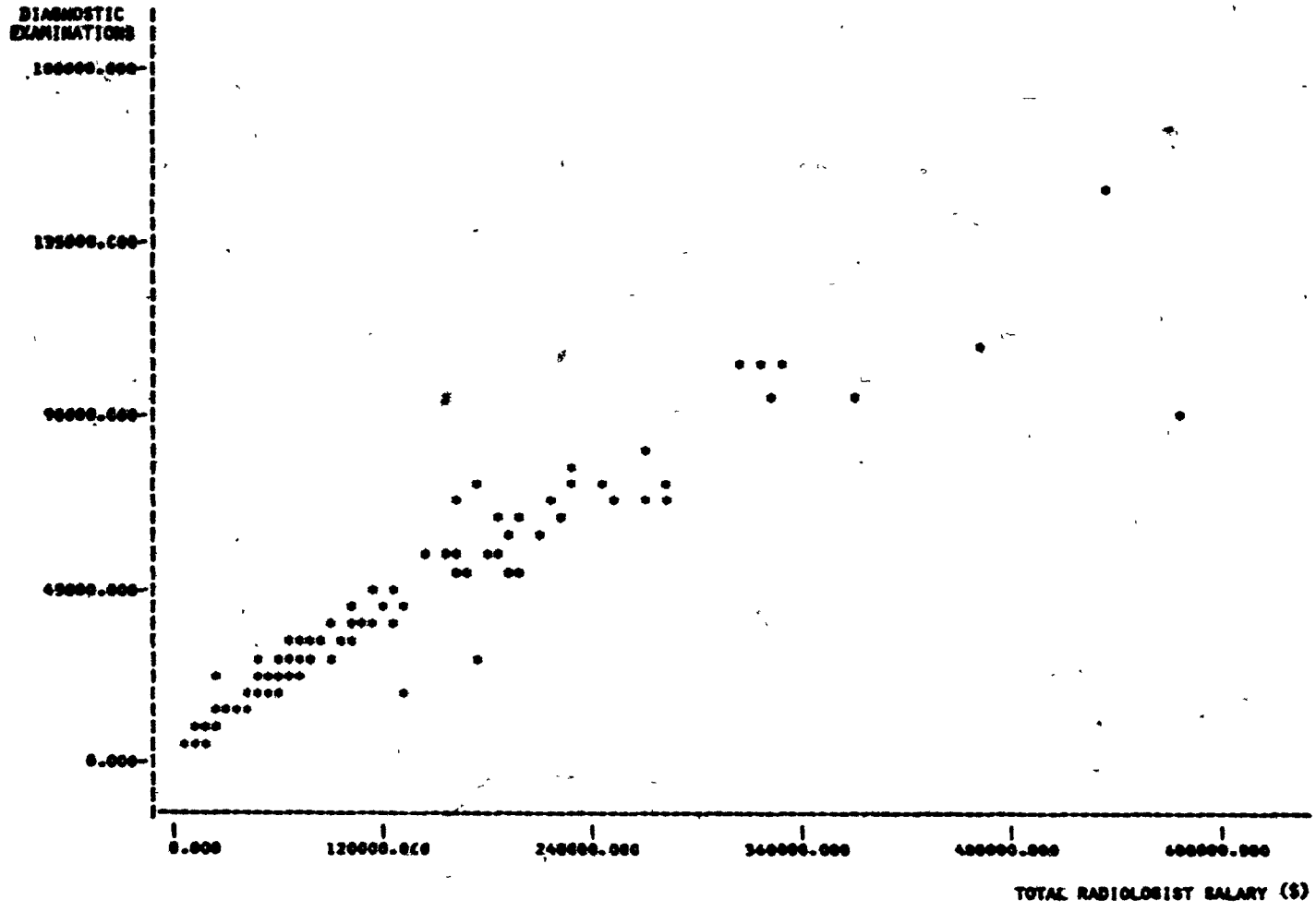


Fig. 5.2a. The number of diagnostic examinations provided versus the total radiologist salary in diagnostic radiology departments for 1971.

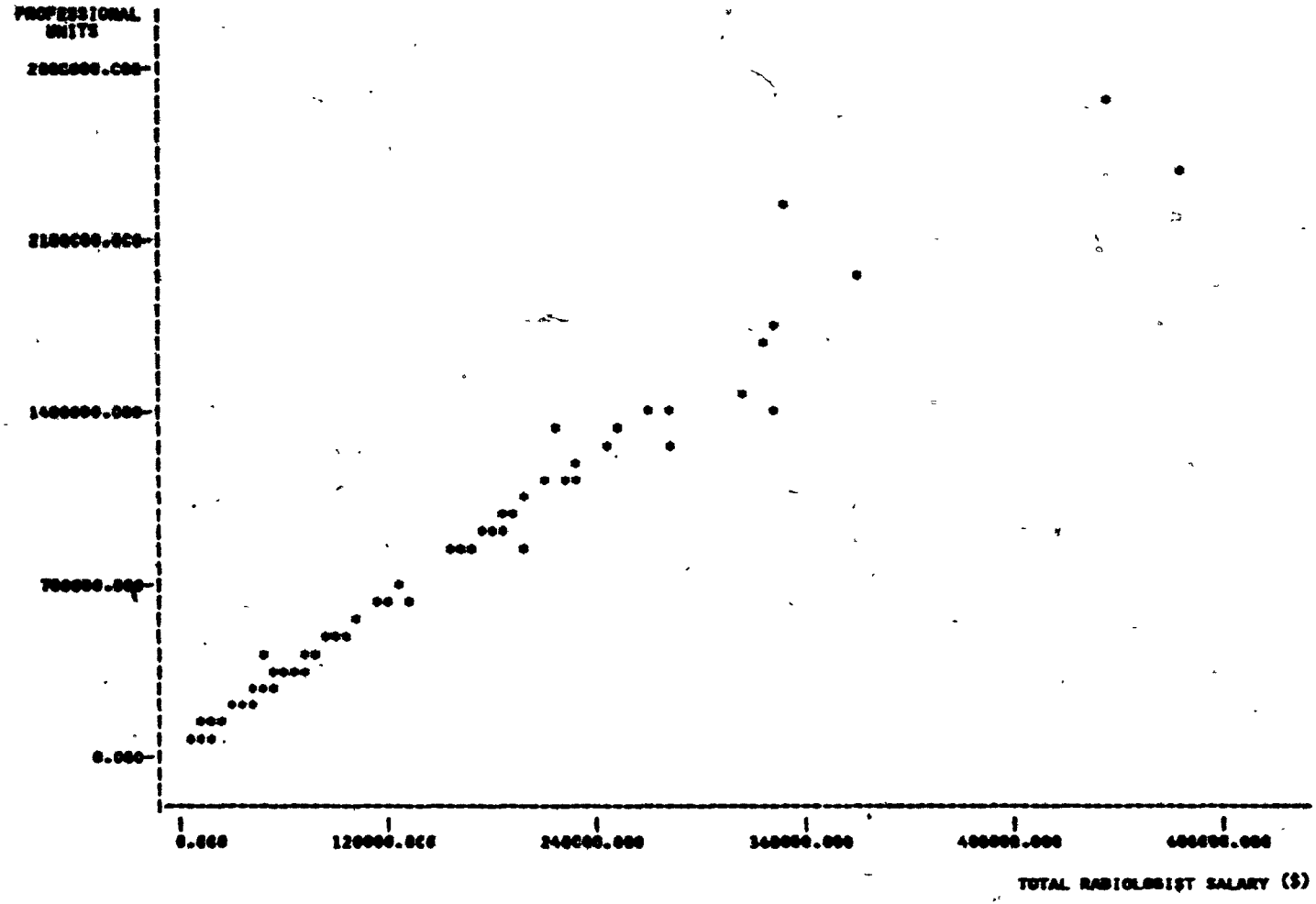


Fig. 5.2b. The number of professional units of service provided versus the total radiologist salary in diagnostic radiology departments for 1971.

for-service basis upon which radiologists are paid. The variability that does arise in Fig. 5.2b, results from some radiologists holding staff positions in their hospitals for which they are directly salaried.

In Figs. 5.3 and 5.4 we have plotted these same output measures versus the input departmental budget (excluding MD salaries) and the input non-labour expenses respectively. The same trends cited above emerge. They are attributed to the homogeneity of the P.U. measure, and the existence of constant returns to scale at least in terms of this output. Admittedly, there appears to be a marginal decrease in the slope of the professional units plot for large values. It may be possible to associate this with either the sparsity of the data in these regions, or the existence of slightly decreasing returns.

The production model that was developed in the previous chapter deals with capacity as the output measure. Hence our assumption of constant returns to scale applies strictly to the capacity output, and not necessarily to the observed output. In section 4.5.1 we cited the existence of specialized rooms in departments as a major source for decreasing returns in the observed output. Specialized rooms are established to respond to a service need from the patient population in a hospital. This does not imply that the service demand for these rooms will be as great as for general rooms, and hence, idle capacity can sometimes be observed in specialized rooms. As discussed below, the proportion of rooms in a department that are recognized as specialized, increases with department size, which in turn, explains the decreasing returns in observed output, however slight. Table 5.2 lists the nine types of x-ray rooms that can be situated in a radiology department as indicated on data set D.3; one general and eight specialized.

By combining the room data of D.3 with the output data of D.2, we estimate that: i) 97% of all departments producing more than 1 million professional units have at least one specialized x-ray room, and of those departments in this output range with specialized rooms, 100% have more specialized than general rooms; and; ii) of the departments

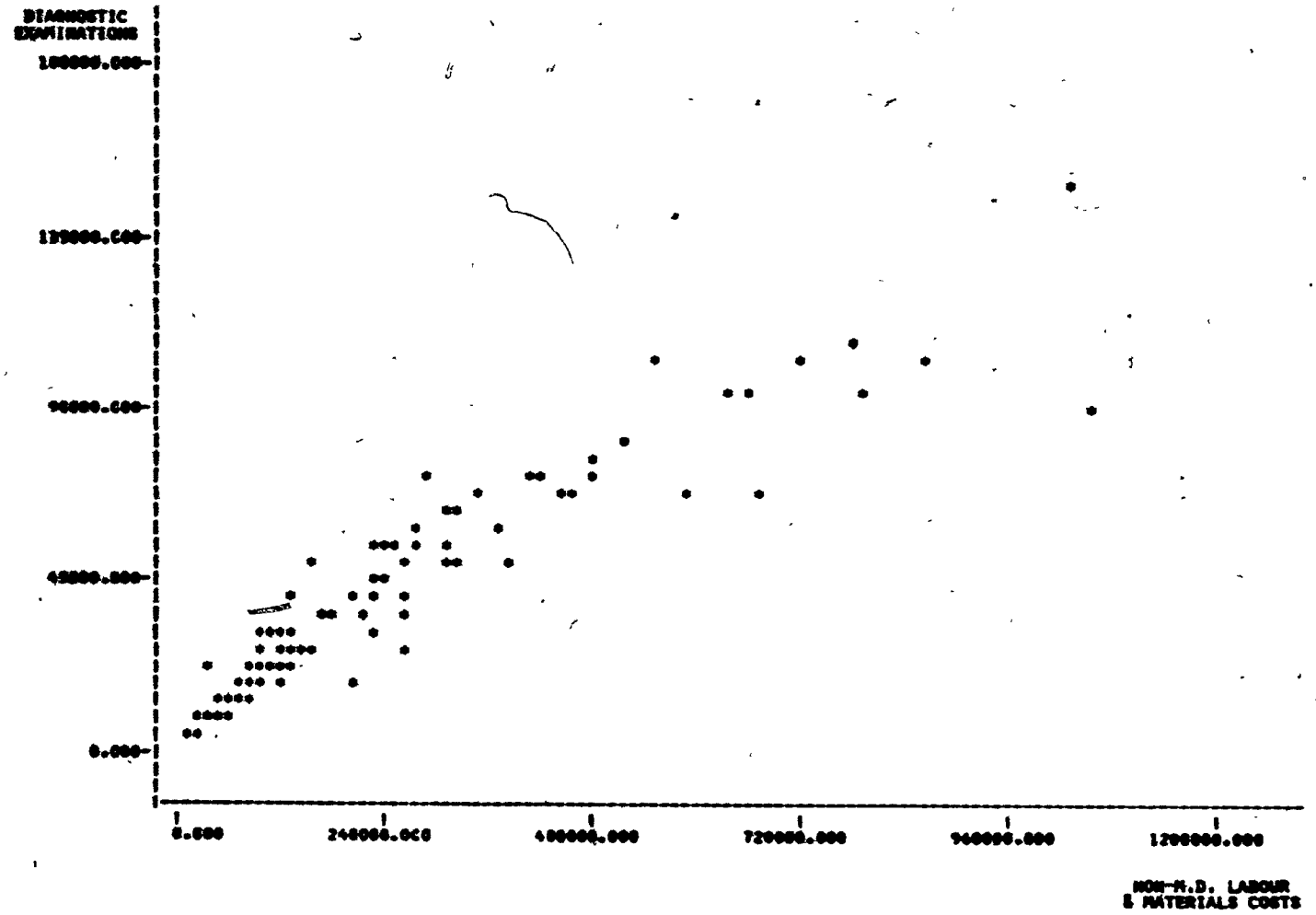


Fig. 5.3a. The number of diagnostic examinations provided versus the departmental operating budget in diagnostic radiology departments for 1971.

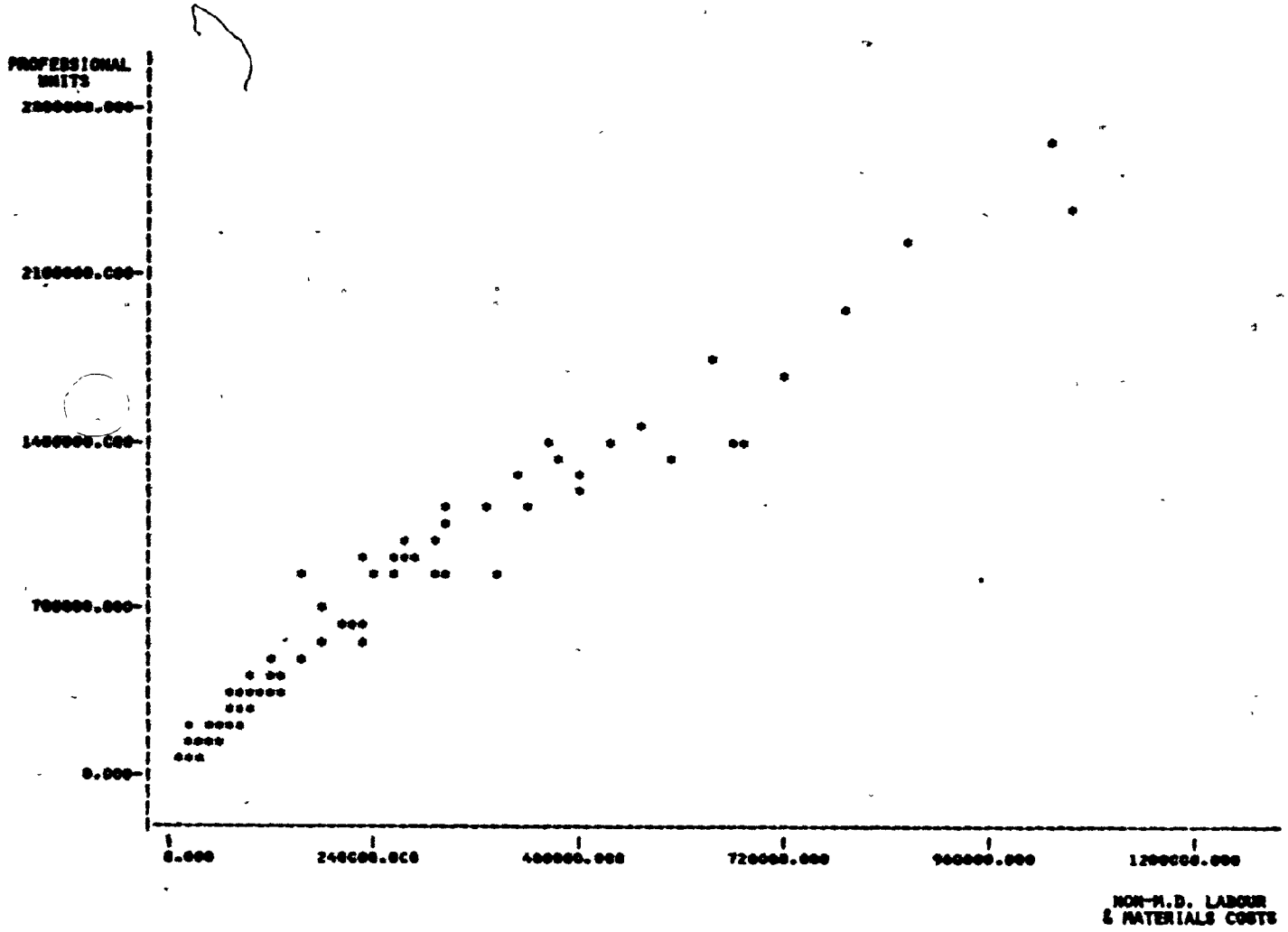


Fig. 5.3b. The number of professional units of service provided versus the departmental operating budget in diagnostic radiology departments for 1971.

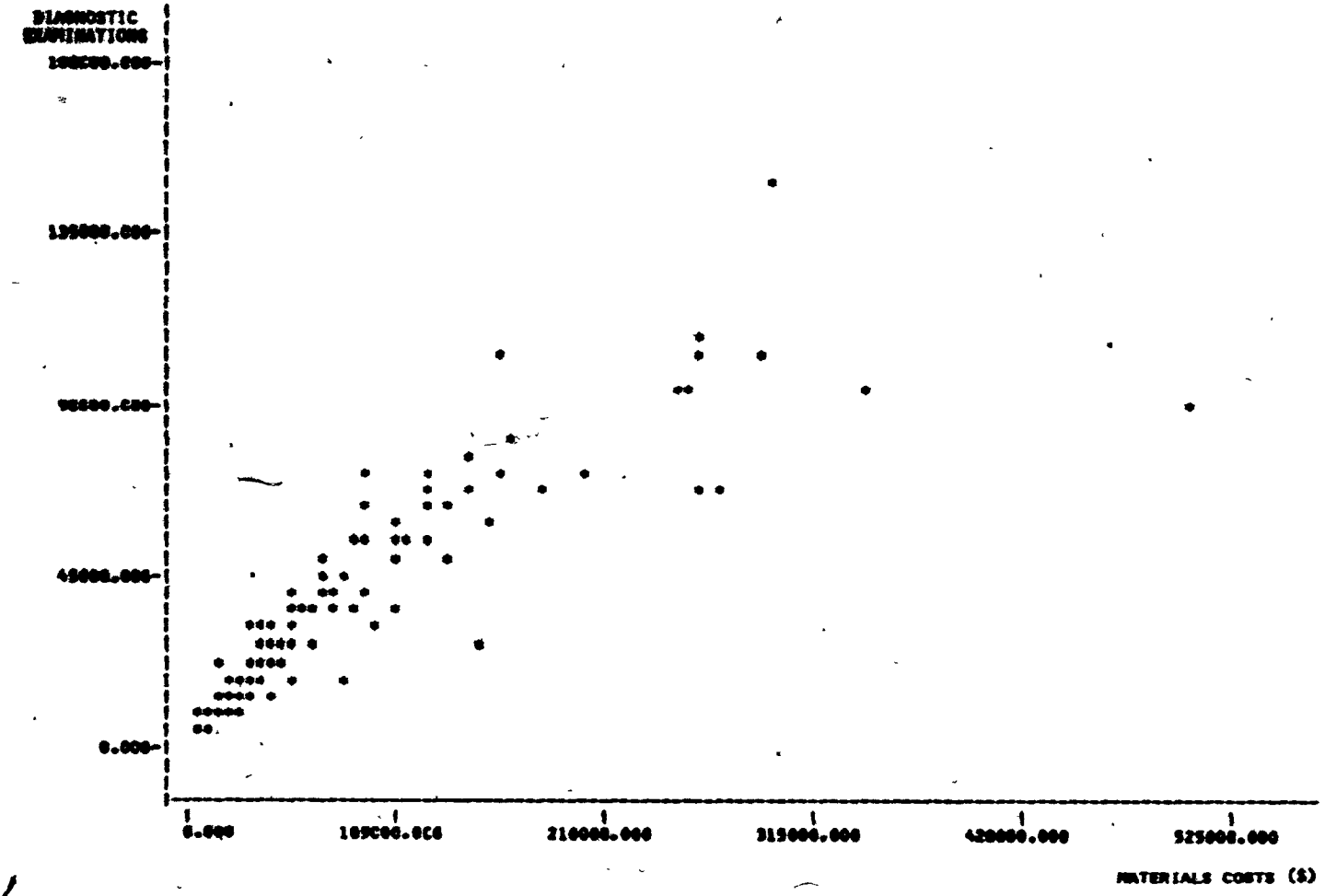


Fig. 5.4a. The number of diagnostic examinations provided versus the non-labour expenses in diagnostic radiology departments for 1971.

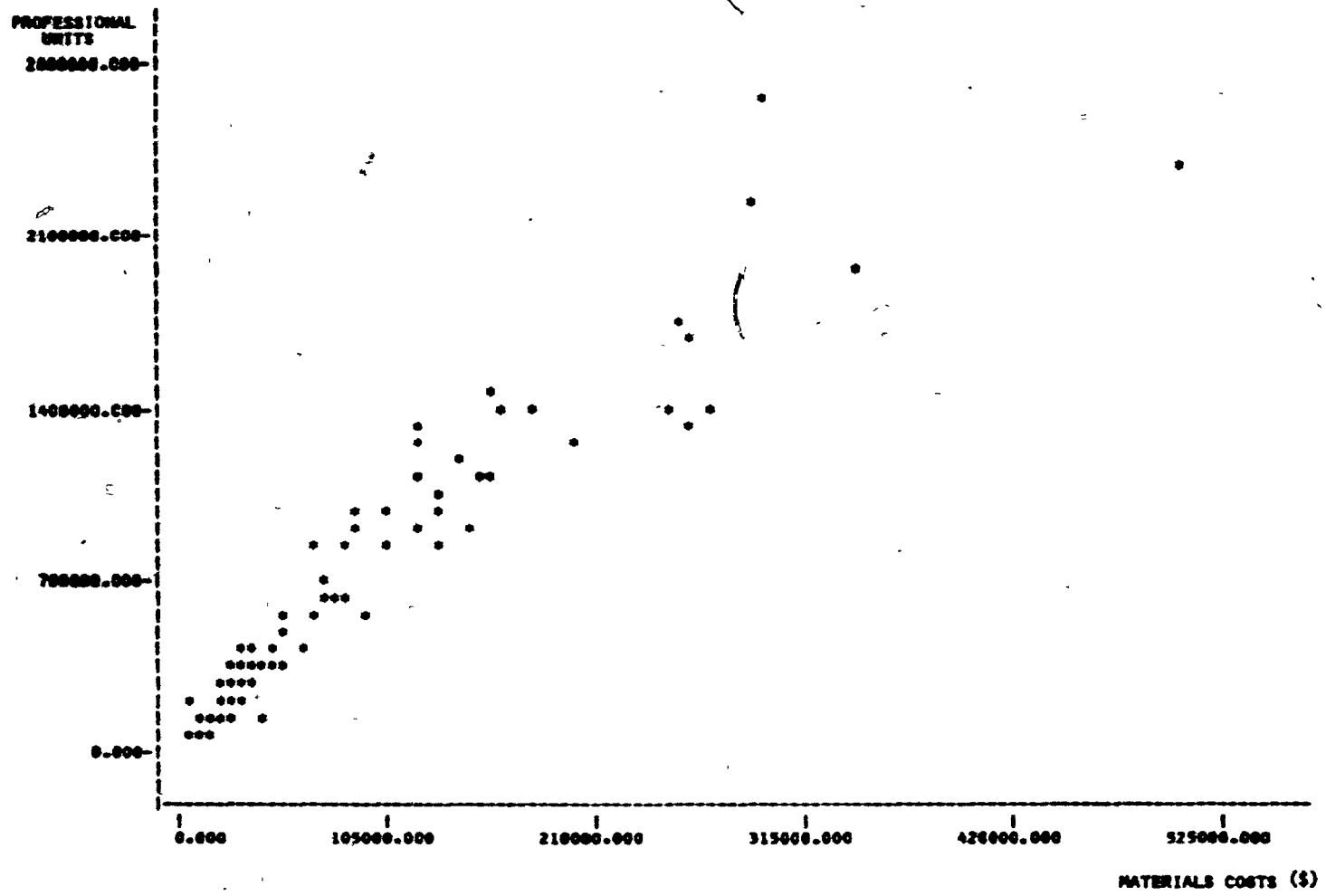


Fig. 5.4b. The number of professional units of service provided versus the non-labour expenses in diagnostic radiology departments for 1971.

producing less than 1 million professional units, less than 30% have any specialized x-ray rooms, and of these departments, less than 50% have more specialized than general rooms.

This implies that on average, more specialized procedures are produced in larger departments. Figs. 5.5 and 5.6, which show plots of the professional unit output versus examinations, and the ratio of professional units to examinations for each department versus the number of hospital beds (Anon., 1972d) respectively, graphically demonstrate this. In both cases, the P.U. to examinations ratio, reflecting the average complexity of each examination in a department, increases slightly with output and hospital size (beds being an index of hospital size).

Therefore, the trend towards decreasing returns, in consideration of examinations, is concluded to be derived from the demand for more specialized procedures, which may lead to an idle specialized room capacity. Introduction of professional units as the output measure, is seen to have a normalizing effect. Although the hospital environment may impose a decreasing returns situation in some departments, in terms of the observed volume of P.U., the assumption of constant returns to scale for the capacity (P.U.) output remains likely. This hypothesis is supported by the fact that a small number of specialized examinations is compensated by a proportionately large volume of P.U., and further, because the vast majority of departments, over a significant range of observed output, appear to obey a constant returns to scale rule.

The above discussion has been solely in terms of x-ray rooms. It is apparent that when situating a piece of equipment in a specialized room, it will be utilized, both in terms of the type of services and volume of services, in the same manner as the room itself. Hence our argument is equally valid with respect to equipment.

The output measure that will be used to identify the production function for diagnostic radiology facilities is the volume of professional units, which is seen to closely adhere to the requisite homogeneity and returns to scale assumptions of the production model.

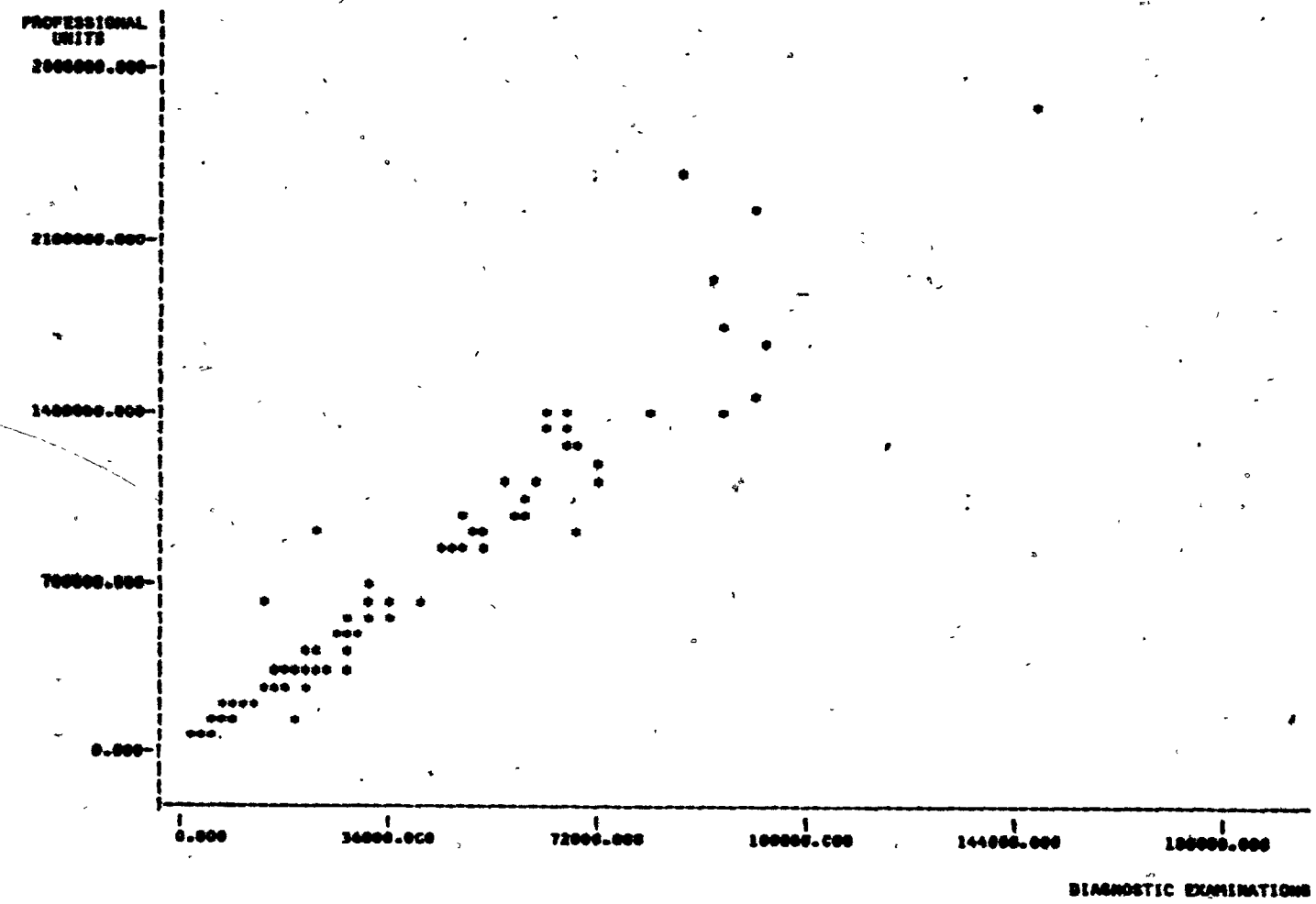


Fig. 5.5. The number of professional units of service provided versus the number of diagnostic examinations provided in diagnostic radiology departments for 1971.

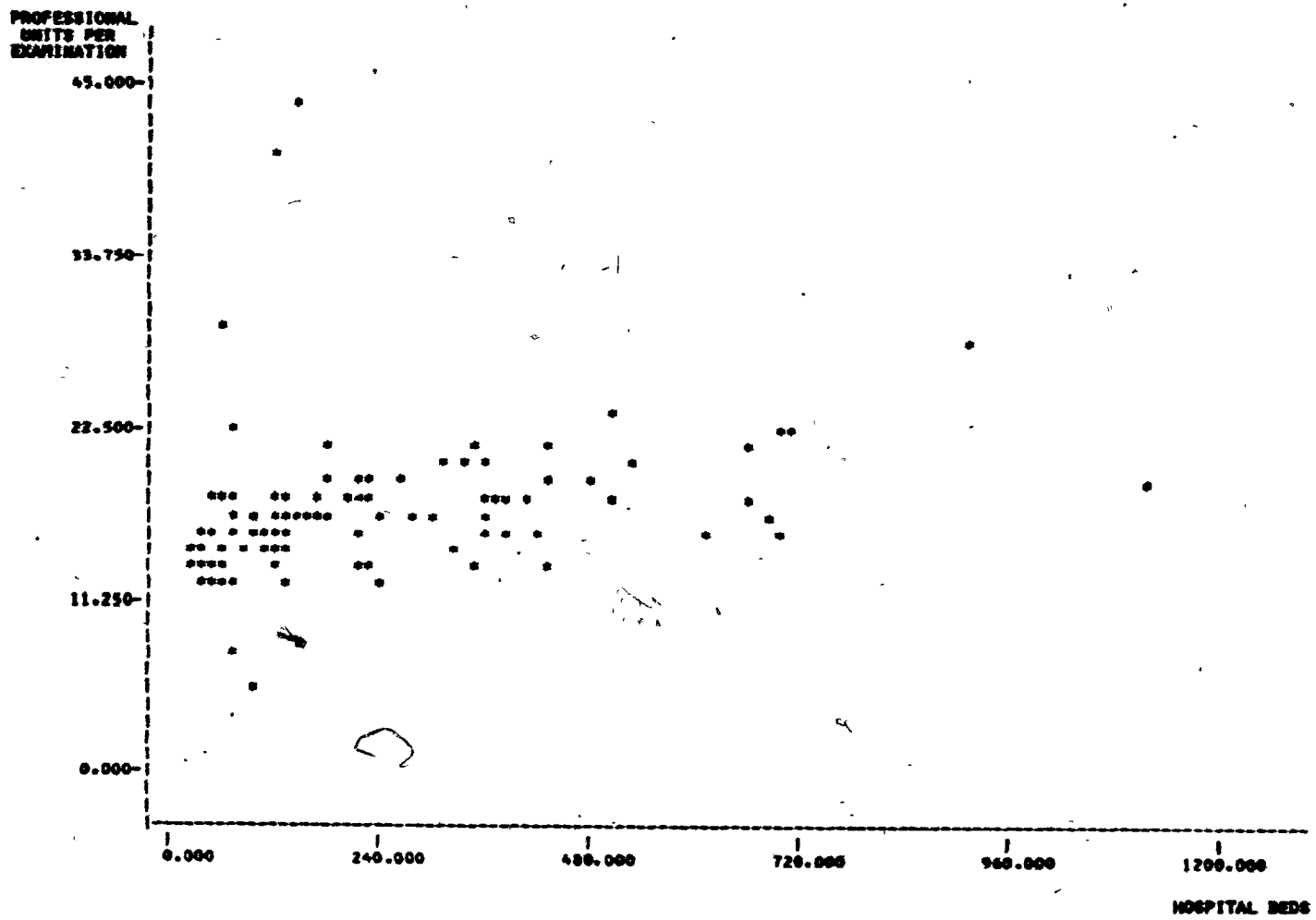


Fig. 5.6. The average number of professional units per diagnostic examination in diagnostic radiology departments versus the number of hospital beds for the acute hospitals in which these departments are located (1971).

5.3 Method for Determining the "Best" Model

In this section we investigate an approach to determining which of a series of production models is most suitable for defining the technical efficiency of radiology departments.

If we consider one possible output measure and a set of m possible input production variables, then, excluding the situation of the dependent variable being a function of only one independent variable, the maximum number of different production models that can be constructed is,

$$\sum_{r=2}^{m} \frac{m!}{r!(m-r)!}$$

Of course the actual number of feasible models applicable to a particular production environment is somewhat less. This is because of an *a priori* specification that one or more of the independent variables (though less than m) are particularly crucial to the process under examination, and hence must be included in any and all possible models.

How does one decide which of the remaining possible production models is the "best" one?

The answer to this question is dependent on what is meant by "best", which in turn, is related to the object of the analysis. For example, consider the estimation of a continuous production function by means of multiple regression. The goal of this type of process is to explain, or account for, as much of the variation in the dependent variable as possible. Typically, regression analysis results in indices which: i) measure the combined and individual importance of the independent variables in explaining the differences in the value of the dependent variable, and; ii) the precision of the estimate of the dependent variable, i.e. the level of confidence one can have in the empirical function and its parameters.

The selection of the "best" model, or which combination of

independent variables to include in the production function, would seem to be indicated by the variable set that maximizes both explanatory power and functional confidence. Yet the fixed sample size, i.e. the fixed number of observations on each variable, often dictates that these be opposing trends. For example, if the number of observations is "much greater than the number of potential X (independent) variables under consideration, then the addition of a new variable will always increase R^2 (*multiple correlation coefficient - index of explanatory power*) but it will not necessarily increase the precision of the estimate of the response" (Draper, 1967; Kovner, 1968).

If the sample size increases, one can easily introduce new variables to the function. For a fixed number of observations, the point of the exercise is to choose the k best variables, where k is usually less than m.

A similar approach to that which has been described for regression analysis will be used to determine the most suitable set of input production variables for the linear programming production model. The construction of this model is achieved through non-statistical means, and hence the type of indices mentioned above are not part of the analysis. Therefore, we are forced to rely on subjective means in deciding on the most plausible set of production variables.

Before proceeding to examine the suitability of different models, it is important to note that for the linear programming production formulation developed in section 4.5, "the introduction of a new factor of production into the analysis cannot lower, and in general raises the technical efficiency of any state (*department*). Thus, the more factors that are considered, the more are apparent differences in efficiency explained as being due to differing inputs of these factors". (Farrell, 1957). This phenomenon is illustrated in Fig. 5.7.

We consider only two departments in a space where the axes represent the input per unit output for each input. If we examine

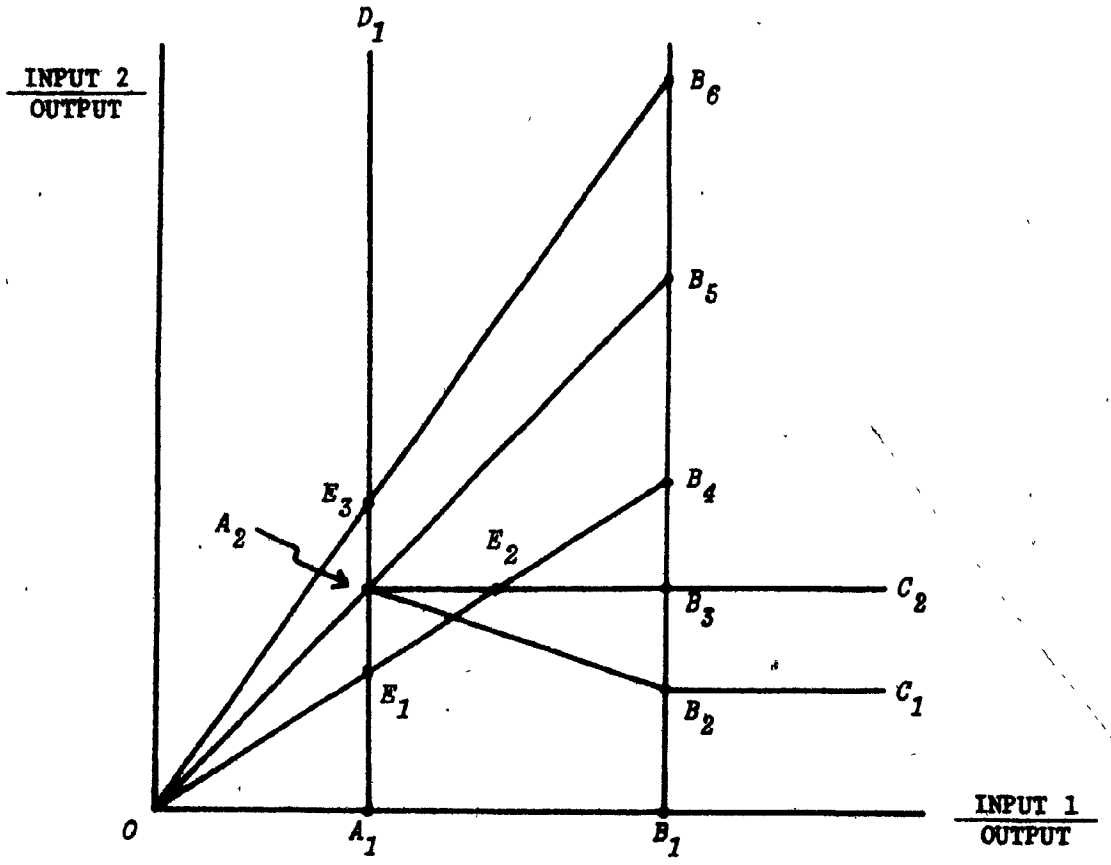


Fig. 5.7. Examples of one input-one output and two input-one output production processes illustrating how increasing the number of production factors in a model cannot lower, and will in general raise, the technical efficiency of any production process relative to the sample of producers.

only one factor of production (input *1*), these departments are situated at points A_1 and B_1 in Fig. 5.7. Point A_1 is considered to be 100% efficient (relative to the sample), and the technical efficiency of point B_1 is $OA_1/OB_1 \times 100\%$ (see section 4.5.3).

Assume that the introduction of a second factor (input *2*) moves the department situated at A_1 to A_2. If the department at B_1 then moves to a point B_2, where line segment $B_1B_2 \leq$ line segment B_1B_3, then the efficient production boundary for this two point sample is mapped by the points D_1, A_2, B_2 and C_1 (see section 4.5.3). Let *TE-X* represent the technical efficiency of point *X*, then,

$$TE\text{-}B_1 = \frac{OA_1}{OB_1} \times 100\% \quad < 100\% = TE\text{-}B_2$$

If the department at B_1 moves to a point B_4, where line segment $B_1B_3 <$ line segment $B_1B_4 <$ line segment B_1B_5, and B_5 is on the ray OA_2, then the efficient production boundary is mapped by the points D_1, A_2 and C_2, and

$$TE\text{-}B_1 = \frac{OA_1}{OB_1} \times 100\% = \frac{OE_1}{OB_4} \times 100\% < \frac{OE_2}{OB_4} \times 100\% = TE\text{-}B_4$$

Finally, if the department at B_1 moves to a point B_6, where line segment $B_1B_5 \leq B_1B_6$, then the efficient production boundary is still mapped by the points D_1, A_2 and C_2, and

$$TE\text{-}B_1 = \frac{OA_1}{OB_1} \times 100\% = \frac{OE_3}{OB_6} \times 100\% = TE\text{-}B_6$$

This geometric exercise can be generalized to the situation of many points and hence the technical efficiency of efficient departments (A_1) remains the same, while the efficiency of non-efficient departments (B_1) either increases or stays the same, with

an increase in the number of production factors.

In the previous chapter it was noted that the linear programming model is solved for all departments in the sample upon which it is based, and so, results in a technical efficiency for each point in the sample. We adopt the procedure described by Farrell (1957) to decide on the acceptability of a model, i.e. a particular input combination. This approach examines the frequency distribution or histogram of technical efficiencies per model.

> "It is to such frequency distributions that one must look for a measure of the success of the analysis, corresponding to the multiple correlation coefficient in regression analysis. It might seem at first that the obvious counterpart of the correlation coefficient was simply the average level of efficiency, being a measure of the extent to which differences have been explained in causal terms. But whereas in regression analysis the object is to explain as much of the variation as possible, here one does not wish to explain away all differences. In other words, one regards as ideal not an analysis that makes all states (*departments*) perfectly efficient, but one that leaves only 'genuine' differences in efficiency,... As any direct test of whether the differences in efficiency are genuine or not is impossible one is reduced to a consideration of the plausibility of the distribution of efficiencies. This is necessarily a highly subjective matter, and one hesitates to lay down any objective criteria of plausibility."
> (Farrell, 1957)

Farrell elaborates two basic criteria for plausibility, that

one can use in examining the frequency distribution of efficiencies: i) histogram shape and; ii) the change in histogram shape introduced by changing the set of input production variables. In regard to distribution shape, we reject bimodal distributions, such as illustrated in Fig. 5.8, as acceptable. "The reason why this is regarded as implausible is perhaps that one feels that there must be some neglected input accounting for the apparently low efficiency of such a large group" (Farrell, 1957), or proportion of the sample. Having rejected bimodal histograms, it would be difficult to conclude that some shapes, such as shown in Fig. 5.9a, are more plausible than others, such as shown in Fig. 5.9b. Hence, criteria other than mere shape must be introduced (Farrell, 1957).

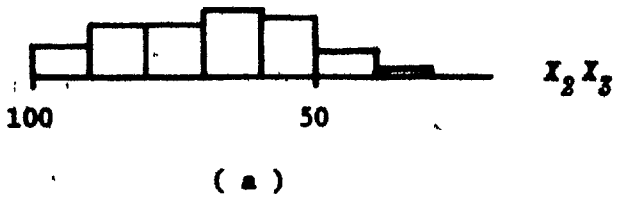
The X_i in Fig. 5.9 indicate which inputs comprise the input production variable set upon which a model, and hence the histogram is based. The output is the same for both models. In going from model X_2X_3 to model $X_1X_2X_3$, "one has clearly explained a good deal of the apparent variation in efficiency" (Farrell, 1957). The introduction of input X_1 to the production variable set of the first model has resulted in an enlarged independent variable set which appears, by the shift in the histogram, to account for a greater proportion of the differences in the value of the independent variable or efficiency. Therefore model $X_1X_2X_3$ is clearly the better, more plausible representation.

One has to check against carrying this procedure to an extreme. Consider the example of Fig. 5.7 with a two-input, two-observation sample, where it is possible that the "entire sample" is recognized as 100% efficient - a clearly implausible situation.

In the next section we pursue the identification of a production function for diagnostic radiology facilities. The choice of most appropriate model, from a series of models defined by various combinations of inputs, all from the same input set, will rely on the procedure outlined above.



Fig. 5.7. An example of a bimodal efficiency frequency distribution or histogram - number of producers versus technical efficiency intervals - resulting from the application of the linear programming production model to a sample of producers (Farrell, 1957, p. 271).



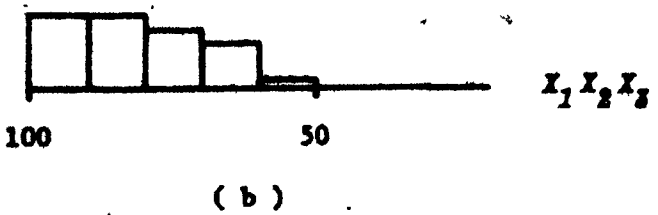


Fig. 5.8. The effect of increasing the number of production factors (in going from a to b) on the resulting efficiency histograms for the same sample of producers where X_i denotes input i (Farrell, 1957, p. 271).

5.4 Different Models

Once a technique has been established by which a production function can be identified, then for each distinct set of production variables a unique relationship is defined. In this section we define two possible sets of production variables. From each, we derive a series of feasible models, based on varying combinations of the inputs within each set. We then choose the most plausible model from each series. In all instances, we apply the linear programming production formulation derived in the previous chapter to establish the production models.

5.4.1 Feasible Input Combinations. Regardless of which inputs are included in any particular model, the output measure to be used is the total volume of diagnostic radiology professional units per department. Table 5.3 provides a list of the input production variables described in section 5.2.1. The actual data per department for the listed inputs and the output, as found on data sets D.1, D.2, and D.3, is provided in Appendix C.

The non-medical manpower input may be viewed three different ways, as shown in Table 5.4. Only one of these groups can be considered representative of the non-medical labour input to a radiology department in any one particular model. If we consider these three different types of non-medical manpower input plus the four other possible inputs, and examine only those models which have at least two distinct inputs, we are then confronted with well over 250 different possible models - a highly undesirable situation. Fortunately, we may significantly reduce this number by recognizing certain features of both the system under investigation and the aims of the analysis.

By introducing an input to a production model, we are implicitly stating that this production variable is more important to the conceptualization of the production process imbedded in the model,

Input Category	Actual Inputs	Number of Distinct Inputs
Medical Manpower*	Full-time Radiologist Man-years Part-time Radiologist Man-years	2
Non-Medical Manpower**	Technicians(dipl) & Non-M.D. Specialist Man-hours Technicians(no dipl) Man-hours Others Man-hours	3
Materials	Non-Labour Expenses ($)	1
Fixed Capital	Diagnostic Equipment-years X-Ray Room-years	2

* both medical manpower inputs must be introduced to a model at the same time

** the three non-medical manpower inputs will be denoted: Tech(dipl), Tech(ndipl), and Others

Table 5.3. The set of feasible diagnostic radiology department production model inputs.

Non-Medical Manpower Group	Actual Inputs	Number of Distinct Inputs
I	Tech(dipl) Tech(ndipl) Others	3
II	Tech(dipl)+Tech(ndipl) Others	2
III	Total Staff = Tech(dipl)+Tech(ndipl) +Others	1

Table 5.4. The three possible non-medical manpower input groups for a production model.

than those variables which have been excluded.

Consider the non-medical manpower groups listed in Table 5.4. The aggregations within each group acknowledge the different roles the various types of non-medical manpower can have in the production of diagnostic services. The distinction between inputs in any one non-medical labour group, is based on the assumption of a correspondence between level of training and assigned tasks. It then follows, that the higher the level of training for any defined input within a group, the more important is its productive role relative to all other inputs in the same group. Thus, if non-medical manpower is to be introduced to a model, then for each of the recognized groups of this type of input, only certain combinations of the member inputs are allowed. These combinations are shown in Table 5.5.

The quantification of the site for the production of services has been achieved by introducing diagnostic equipment years and x-ray room-years. Either one, or both of these inputs should be used as a production variable(s) in all of the models we will consider because: i) of the critical role they play in the production process; ii) they represent the type of resource change with which this work is concerned and; iii) they embody the bulk of the hard technology in radiology departments, and as such, should be explicitly determinant in a measure of technical efficiency.

We have decided not to consider models involving the x-ray room-year input. The reason for doing so are three-fold: i) a change in the equipment input to a department does not necessarily imply a change in the room input; ii) total room-years is a less homogeneous measure (an aggregation of 9 classes), than equipment-years (an aggregation of 6 classes), and; iii) room data is available for 93 departments as opposed to 136 departments for equipment (see section 5.2.1).

Group	Non-Medical Manpower Combinations For A Production Model		
	1 st	2 nd	3 rd
I	[Tech(dipl)]	[Tech(dipl) Tech(ndipl)]	[Tech(dipl) Tech(ndipl) Others]
II	[Tech(dipl)+Tech(ndipl)]	[Tech(dipl)+Tech(ndipl) Others]	
III	[Total Staff]		

Table 5.5. The various possible combinations of non-medical manpower inputs within each of the non-medical manpower groups defined in Table 5.4.

Two basic groups of manpower have been discussed; medical and non-medical labour. At first glance, it might appear difficult to decide which of the two to introduce first in a model, or whether both should be considered at the same time.

The radiologist is either in charge of, or has a supervisory role in the department, and is primarily involved with the emergency, fluouroscopy and special procedures, whereas the members of the non-medical labour force are responsible for most of the film-taking (Lindheim, 1971). The technician and clerk input support the ultimate function of the radiologist, namely interpretation and consultation (Drui, 1970). Hence, the non-medical manpower input is required for all procedures in the radiographic subsystem of diagnostic radiology (see section 3.2.2). A medical input is explicitly required for only some procedures, although it is implicitly present for all through its management role. Since our measure of technical efficiency is meant to reflect on the performance of the radiographic subsystem of departments, we shall introduce the non-medical labour input first. The medical labour input is introduced only in the presence of the other manpower type. The possible manpower group inputs to a production model are shown in Table 5.6.

The non-labour expenses or materials input, while important, is difficult to weigh relative to the other types of inputs. Thus, we examine models where this input is either present or absent, independent of the other production variables in the models.

In Table 5.4 we have described three different groupings for the non-medical manpower input. As the total non-MD staff aggregation provides the least information, and is the most heterogeneous of the three, we choose to exclude it from consideration.

In terms of the discussion throughout this section we can delineate two possible production variable sets, and a series of feasible models for each. These two series of models are listed in Tables 5.7 and 5.8, where each set of X_i corresponds to a different model.

Manpower Combinations For A Production Model

1 st	2 nd
Non-Medical Manpower (one of the combinations from Table 5.5)	Non-Medical Manpower (one of the combinations from Table 5.5) Medical Manpower (from Table 5.3)

Table 5.6. Possible combinations of input manpower variables for a production model.

Production Variable Set A	Denoted
Equipment-years	X_1
Non-labour expenses ($)	X_2
Tech(dipl) man-hours	X_3
Tech(ndipl) man-hours	X_4
Others man-hours	X_5
Full-time radiologist man-years Part-time radiologist man-years	X_6

Model Series A (MS.A)*

1A	X_1X_2		
2A	$X_1X_2X_3$	5A	X_1X_3
3A	$X_1X_2X_3X_4$	6A	$X_1X_3X_4$
4A	$X_1X_2X_3X_4X_5$	7A	$X_1X_3X_4X_5$
8A	$X_1X_2X_3X_6$	11A	$X_1X_3X_6$
9A	$X_1X_2X_3X_4X_6$	12A	$X_1X_3X_4X_6$
10A	$X_1X_2X_3X_4X_5X_6$	13A	$X_1X_3X_4X_5X_6$

* The output for all of the above models is measured in professional units.

Table 5.7. A series of production models (production variable sets) derived from the set of variables described in Tables 5.3 & 5.6, and denoted as series A.

Production Variable Set B	Denoted
Equipment-years	X_1
Non-labour expenses ($)	X_2
Tech(dipl)+Tech(ndipl) man-hours	X_{3+4}
Others man-hours	X_5
Full-time radiologist man-years Part-time radiologist man-years	X_6

Model Series B (MS.B)*

1B	X_1X_2		
2B	$X_1X_2X_{3+4}$	4B	X_1X_{3+4}
3B	$X_1X_2X_{3+4}X_5$	5B	$X_1X_{3+4}X_5$
6B	$X_1X_2X_{3+4}X_6$	8B	$X_1X_{3+4}X_6$
7B	$X_1X_2X_{3+4}X_5X_6$	9B	$X_1X_{3+4}X_5X_6$

* The output for all of the above models is measured in professional units.

Table 5.8. A series of production models (production variable sets) derived from the set of variables described in Tables 5.3 & 5.6, and denoted as series B.

We have reduced the problem of comparing over 250 different models to an examination of two independent series of 13 and 9 feasible models each. It is apparent that some of the same forces which focused our attention to the set of production variables depicted in Table 5.3, i.e. the unavailability of certain data for some or all departments and the quality of that which is available, played a large part in this reduction process.

Admittedly then, the constructs we shall pursue are not optimal. Yet they are acceptable in terms of the goals of this work. First, as evidenced by the discussion throughout this chapter the models represent at least a first order approximation to the production process of diagnostic radiology. As such they may be usefully integrated into the decision process for equipment investment by government. And second, though equally important, their viability in this decision process will implicitly encourage an improvement in data collection, in terms of both quantity and quality, from which "better" models may be derived.

5.4.2 Solving the L.P. Production Model. Data sets D.1, D.2, and D.3 were made available to this project on 9 track IBM magnetic tapes - one data set per tape. Each tape is Fortran readable with a fixed format. There is one record per department on the tapes corresponding to D.1 and D.3, and one record per hospital on the D.2 tape. All programs were written in Fortran and programmed on the IBM 360/75 computer available at McGill University. Due to the high cost of mounting a tape on this system, punched cards were produced from the tapes for the data required by both the production models, as well as all other facets of the project.

All programs were first debugged with the WATFIV compiler which is especially designed for Fortran program development, and then run with the Fortran G1 compiler which is the standard IBM production compiler. Whenever possible programs were planned on a

modular basis, and after being debugged, were converted to object modules with the Fortran H compiler which "does a thorough analysis of program loops and uses the fastest coding techniques" (Ratzer, 1974). These object decks were also run with the Fortran G1 compiler.

Although a description of the computer program used to solve problem P4 (see section 4.5.4) and calculate the technical efficiencies of each department in a sample is provided in Appendix D, we will discuss three of its more important features in this section. Before proceeding, we repeat problem P4;

Let s — number of departments in the model

m — number of inputs in the model

n — $m + s$

d_{ij} — amount of input i used in process (department) j

c_j — amount of output produced from process (department) j

X_{ik} — amount of input i used in department k

$$\text{P4:} \quad \max \sum_{1}^{s} c_i \lambda_i$$

$$\text{s.t.} \quad \left[\begin{array}{ccc:cc} d_{11} & d_{12} \cdots & d_{1s} & 1 & 0 \\ d_{21} & & & & 1 \\ \vdots & & \vdots & & \\ d_{m1} & & d_{ms} & 0 & 1 \end{array}\right] \left[\begin{array}{c} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{array}\right] - \left[\begin{array}{c} X_{1k} \\ X_{2k} \\ \\ X_{mk} \end{array}\right]$$

$$\lambda_i \geq 0 \quad i = 1,2,\ldots,n$$

or in matrix notation,

$$\text{P4:} \quad \max \; (\mathbf{c}^t, 0)\,\boldsymbol{\lambda}$$

$$\text{such that} \quad [\mathbf{D}\ \mathbf{I}]\,\boldsymbol{\lambda} = \mathbf{X}_k \qquad \boldsymbol{\lambda} \geq 0$$

Vector $\mathbf{X}_k$ is merely the k-th column of matrix $\mathbf{D}$, i.e. $\mathbf{X}_k = \mathbf{d}_k$. Thus to find the technical efficiency for each of the s departments in the sample upon which this model is based, requires solving problem P4 for $k = 1,2,\ldots,s$. If $\boldsymbol{\lambda}^k$ is the optimal solution to P4, then the technical efficiency of department k is given by (see section 4.5.4),

$$TE\text{-}k = \frac{o_k}{(\mathbf{c}^t, 0)\,\boldsymbol{\lambda}^k}$$

We first note that problem P4 can be solved by using the Simplex method (Dantzig, 1963). The application of this technique, or any of its variants, requires the presence of an initial basic feasible solution of the linear program. Sometimes this is not immediately available and so the two-phase method was introduced (Dantzig, 1963). The first phase merely finds a basic feasible solution to the linear program, while the second phase actually solves the program. As seen above, a basic feasible solution to P4 is immediately available, i.e.,

$$\lambda_i = 0 \qquad i = 1,2,\ldots,s$$

$$\lambda_j = X_{lk} \qquad j = s+1,\ldots,n \quad l = j-s$$

and therefore the original problem can be solved without prior manipulation.

Secondly, experience with the production models has shown that while the number of departments in any model is in the order of 100, the number of inputs is at most 7, that is $z >> m$ or $n >> m$. This feature of the program promoted adoption of the Revised Simplex method (Luenberger, 1973) for solving problem P4.

> "Extensive experience with the simplex procedure applied to problems from various fields, and having various values of m and n, has indicated that the method can be expected to converge to an optimum solution in about m, or perhaps $3m/2$ pivot operations. Thus, particularly if m is much smaller than n..., pivots will occur in only a small fraction of the columns during the course of optimization. Since the other columns are not explicitly used, it appears that the work expended in calculating the elements in these columns after each pivot, is, in some sense, wasted effort. The revised simplex method is a scheme for ordering the computations required of the simplex method so that unnecessary calculations are avoided." (Luenberger, 1973)

More sophisticated forms of the Revised Simplex method (e.g. product form of the inverse, LU decomposition) were not used because they only become more profitable when applied to programs having unique properties, such as large size or sparsity.

Thirdly, we recall that if the medical manpower input is included in the production variable set of a model, then it is mathematically represented as two distinct inputs; that is, full-time and part-time radiologists (see section 5.4.1). It is possible then, that the solution of problem P4 for a department with a non-zero value for both of these inputs, will label it as 100% efficient and hence one of the departments defining the production function. In sections 4.4.2 and 4.5.4 this was shown to contradict the production theory of section 4.2. Hence we exclude as plausible any model which recognizes

this type of department as efficient.

This does not exclude estimating the efficiency of a department with both full and part-time radiologists. If in problem P4, vector $\mathbf{X}_k$ has a non-zero value for both full and part-time radiologists, and the columns of matrix **D** have non-zero values for only one of these inputs but not both, it is easily seen that solutions to the matrix equation still exist. Thus, the capacity output for this department ($\mathbf{X}_k$) is found by simultaneously operating some of the production processes represented by those departments with only a full-time radiologist man-year input and some of those with only a part-time radiologist man-year input.

5.4.3 Model A. The series of 13 feasible models listed in Table 5.7, that is MS.A, are examined in this section to determine which represents the most plausible production function for diagnostic radiology facilities. Recall that these models were established by first defining a set of possible production variables, and then defining a series of subsets of these variables (see section 5.4.1). A different model is associated with each distinct subset of production variables.

The best model of MS.A will be denoted M.A. Because M.A. is chosen from a series, which is based upon a particular set of production variables, it represents the most plausible production function relative to this set. Defining another set of production variables will of course result in another "best model" relative to the new set. In fact, the next section below is dedicated to investigating such a different series of models.

Although we examine only two different production variable sets in this work, it is possible, that with the appropriate data, several distinct sets could be defined. This in turn, would result in several production functions. The advantage of having more than one production function, is that it allows a degree of latitude to those individuals in government who will eventually apply the present analysis. The measurement of a technical efficiency of a department may be pursued

with different types of data, assuming of course, that a model is available which corresponds to the data. The introduction of more than one model may be viewed as an effort to compensate for the data problems endemic to any large institution.

The computer program described in Appendix D was used to analyze the 13 different models, except that output sensitivities and the implementation of resource changes were not produced. Fig. 5.10 shows the histograms for each of the models in MS.A.

The list of production variables upon which MS.A is based, their notation, and the number of departments for which their data is available, is presented in Fig. 5.10a. Associated with each histogram in Fig. 5.10b is the input production variable set for the model from which the histogram is derived, and which is denoted by a set of X_i corresponding to the notation in Fig. 5.10a, plus the number of departments in each model sample and the resultant mean efficiency per model.

We first note, that as expected, all model sample sizes are less than the minimum production variable sample size for those variables included in each model. In addition, increasing the number of production variables in a model, increases the explanatory capabilities of a model (see section 5.3).

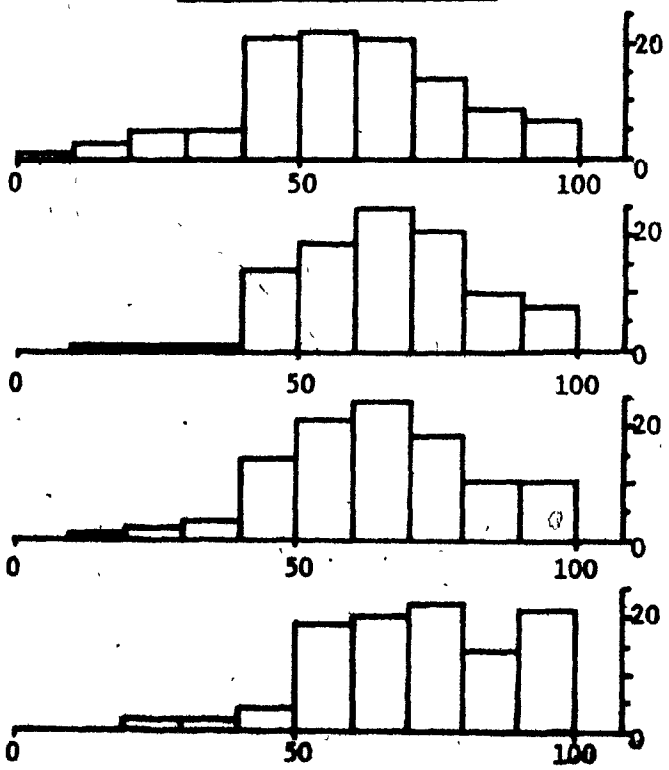
The introduction of non-labour expenses (denoted as X_2) to a model with equipment (X_1) and some non-medical manpower inputs (X_3, X_4, & X_5), appears to have a more dramatic effect than the introduction of the medical labour input (X_6) for these same models: although the latter change is seen to effect an improvement as well. This can be seen by comparing histogram 5A with histograms 2A and 11A, histogram 6A with histograms 3A and 12A, and, histogram 7A with histograms 4A and 13A, all of which are shown in Fig. 5.10b While we justifiably differentiated between medical and non-medical manpower in terms of their respective roles in the provision of diagnostic services, it is apparent that as labour inputs, they have much in common in terms of production. Non-labour expenses represents a significantly different type of input, and as such, may be expected

Production Variable Set A	Denoted	Sample Size
Equipment-years	X_1	136
Non-labour expenses ($)	X_2	137
Tech(dipl) man-hours	X_3	131
Tech(ndipl) man-hours	X_4	
Others man-hours	X_5	
Full-time radiologist man-years	X_6	133
Part-time radiologist man-years		
Professional units produced		110

Notation is not provided for the output measure as it is common to all models.

(a)

Model	Sample Size	Inputs	Number of Departments versus Efficiency
1A	108	X_1X_2	
2A	100	$X_1X_2X_3$	
3A	103	$X_1X_2X_3X_4$	
4A	104	$X_1X_2X_3X_4X_5$	



(b)

Fig. 5.10. The (a) production variables and (b) efficiency histograms for the series of models described in Table 5.7.

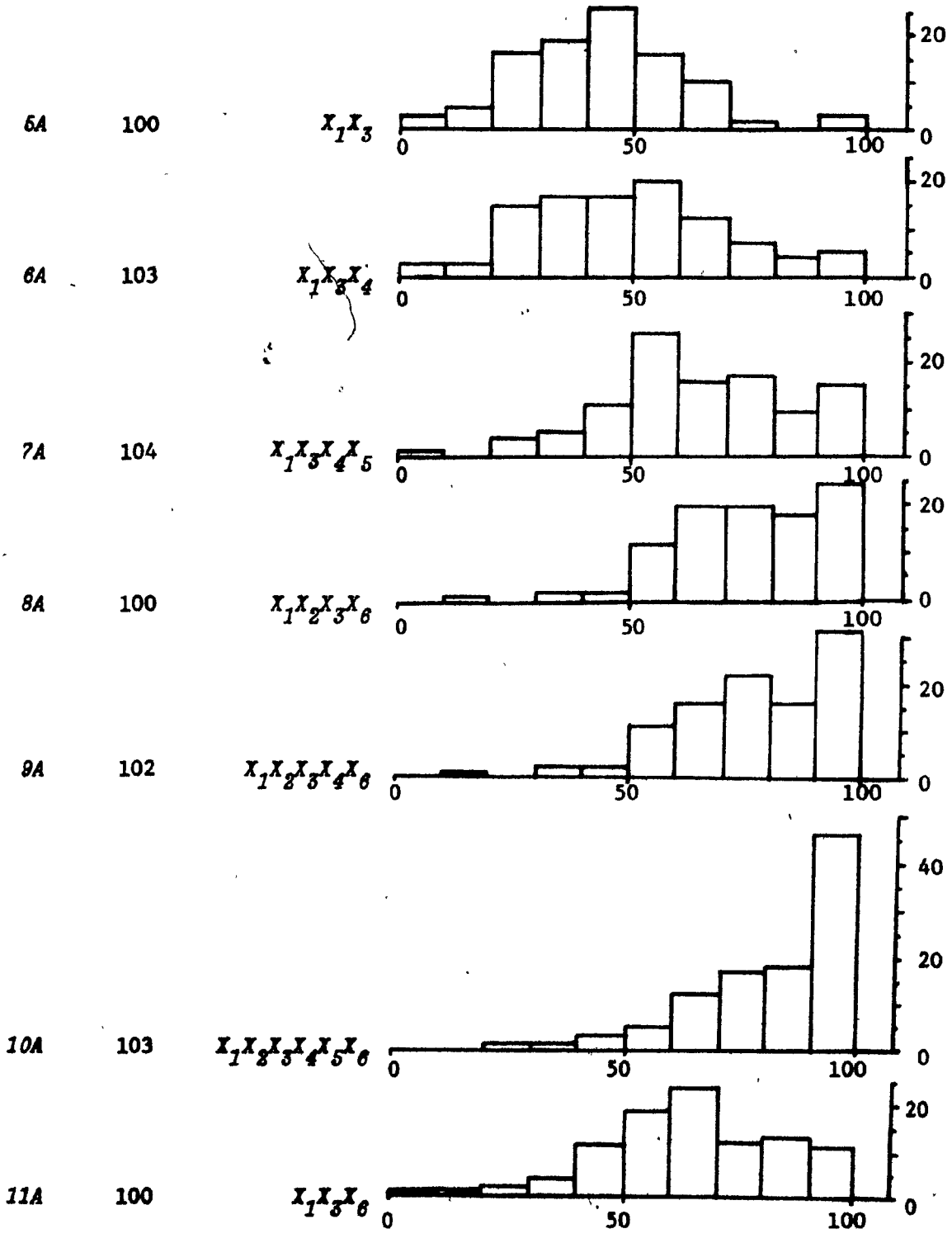


Fig. 5.10b. Continued.

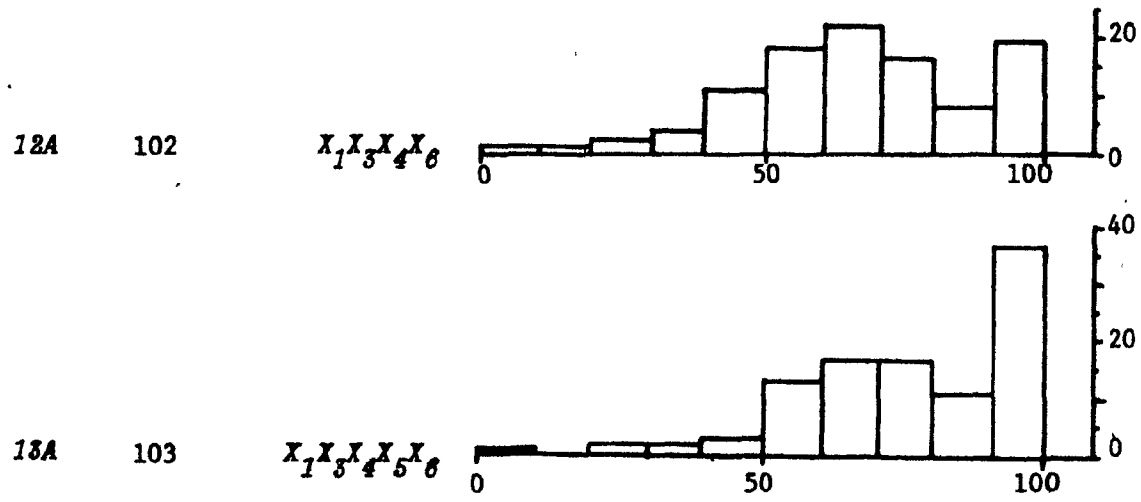


Fig. 5.10b. Continued.

to provide more information on the status of production when introduced to a model. This may account for the greater explanatory effect the inclusion of non-labour expenses appears to have relative to the medical manpower input in the models.

The 13 histograms of Fig. 5.10b may be grouped, as to shape, into three broad categories. In order of increasing explanatory power, they are: i) normal distributions (1*A*, 2*A*, 3*A*, 5*A*, 6*A*, 7*A*, 11*A*, & 12*A*); ii) rectangular distributions (4*A* & 8*A*) and; iii) skewed distributions (9*A*, 10*A*, & 13*A*). The models included in the first group are easily seen to admit greater variance in departmental efficiencies than those of the remaining two groups. In terms of the procedure outlined in section 5.3 for determining a "best" model, we can conclude that the first group of models are the least plausible set of representations.

In section 5.3 we warned against introducing "too many" factors of production to a model. This was seen to have the effect of obscuring genuine differences in efficiency. In addition, it was shown that increasing the number of factors would generally increase the efficiencies of all departments in a sample. Hence, we attribute the skewed shape of distributions 9*A*, 10*A* and 13*A*, to an over specification of the production process; that is, the number of input factors is too large relative to the fixed sample size. Therefore the two models in the second group are seen as the most plausible of the thirteen.

Interestingly, models 4*A* and 8*A* have the same number of inputs. The equipment (X_1), materials (X_2), and "Tech (dipl)" (X_3) inputs are common to both. Model 4*A* also includes the two other non-MD manpower inputs (X_4 and X_5). Model 8*A* has, in addition to the three inputs mentioned, a radiologist input which actually represents two inputs; full-time and part-time radiologists. Therefore both models have five distinct inputs.

Based purely on the shapes of the frequency distributions for models 4*A* and 8*A*, there is no rational reason for choosing one as superior to the other. Yet examination of the actual production variables in each model indicates that model 8*A* is the model of choice.

This decision is indicated for two reasons. First, although both

models include the equipment and non-labour expenses inputs, model 8A has medical and non-medical manpower inputs, whereas model 4A includes a representation of only the latter type. Thus, for the same number of distinct inputs, more information is included in model 8A. And second, radiologist data is much more readily available to the government than a complete breakdown of the non-medical staff. If data is present on the "Tech(ndipl)" and "Others" inputs then it will surely be available on the "Tech (dipl)" input, whereas the reverse situation is not necessarily true. This is illustrated with data set D.3 (the government survey), where the radiologist input is provided, but the non-MD staff input encompasses only the "Tech (dipl)" input.

Fig. 5.11 shows the complete description of model 8A, which henceforth will be denoted M.A. Included in this figure are the departments composing the model sample (see Appendix C).

In Fig. 5.12a we list those departments identified as 100% efficient (relative to the sample), and which therefore map the production function. Using the columns of matrix **D** and the corresponding elements of vector **c** for these efficient departments, the radiographic facilities production function for this set of production variables can be constructed. This program is shown in Fig. 5.12b, with the departments partitioned according to whether they have full or part-time radiologists.

From Eqn. 4.1 we have the general mathematical representation for a production function,

$$Y = f(X_1, X_2, \ldots)$$

where Y is the output, the X_i are the inputs over the period encompassed by the production function, and f represents the relationship between inputs and output determined by the technology. This equation

MODEL A

DESCRIPTION: INPUT 1 = MAJOR DIAGNOSTIC EQUIPMENT YRS
INPUT 2 = FULL-TIME RADIOLOGIST MAN-YRS
INPUT 3 = PART-TIME RADIOLOGIST MAN-YRS
INPUT 4 = TECH(DIPL) & NON-PD SPEC HRS
INPUT 5 = NON LABOUR EXPENSES ($)
OUTPUT = DIAGNOSTIC RADIOLOGY PROFESSIONAL UNITS

SAMPLE: 100 OF 139

2	3	4	5	8	9	10	11	16	17	18	19	20	21	22	23	25	26	28	29
30	31	32	34	35	37	38	39	40	41	42	43	44	46	50	51	52	53	54	56
57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	77	79	80
81	84	85	86	87	88	89	90	91	92	94	95	96	97	98	99	100	101	102	103
107	109	111	112	113	116	117	118	122	125	126	127	128	130	132	134	135	136	137	139

Fig. 5.11. The production variables of model M.A as well as a list of the diagnostic radiology departments (see App. C) comprising the sample upon which this model is based.

Departments Defining the Efficient Frontier of Model A

20 23 32 38 41
46 56 57 68 89
96 113 137

(a)

$$\max\ (e^t, 0)\,\lambda$$

such that $[D\ I]\,\lambda = b$

$$D = \begin{bmatrix}
4 & 2 & 5 & 5 & 7 & 6 & 2 & 6 & 1 & 2 & 3 & 2 & 7 \\
3 & 2 & 1 & 3 & 2 & 1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 4 & 1 \\
22295 & 9532 & 14548 & 4639 & 19536 & 16036 & 7311 & 24617 & 6296 & 8768 & 6089 & 4573 & 28342 \\
116112 & 45149 & 70499 & 63712 & 81887 & 46002 & 20856 & 77227 & 6561 & 18346 & 28038 & 32083 & 113184
\end{bmatrix}$$

$$e = \begin{bmatrix}
866634 \\
441542 \\
564707 \\
754426 \\
813394 \\
479198 \\
264168 \\
753416 \\
288849 \\
253087 \\
263112 \\
229062 \\
1078844
\end{bmatrix}$$

$$\lambda_i \geq 0 \quad i = 1,2,\ldots,18$$

$$\left.\begin{aligned}
b_1 &= X_1 \\
b_2 \\
b_3 &\Big\} = X_6 \\
b_4 &= X_3 \\
b_5 &= X_2
\end{aligned}\right\} \text{ from Fig. 5.10a}$$

(b)

Fig. 5.12. (a) The radiology departments from the model A sample described in Fig. 5.11 which are recognized as the most efficient producers and which therefore define the production function, and, (b) the linear programming formulation of model M.A.

states that Y is the maximum amount of output y which can be produced with X_i units of input x_i, $i = 1, 2, \ldots$ etc.

The solution of the linear program in Fig. 5.12b represents the identification of such a function for radiology departments and may be restated (droping the k subscript for the departmental inputs) as follows:

$$P = f(X_1, X_2, X_3, X_6) \qquad (5.1)$$

In constructing the linear program production function formulation we have established a previously unavailable, comprehensive standard, against which the performance of any department can be measured if the appropriate inputs and observed output are available.

This standard provides an estimate of the capacity output for a department relative to the production behaviour of a sample of departments in 1971. It represents an attainable best for 1971 and not a theoretical upper bound. Yet, there is no reason why one cannot compare the behaviour of departments in more recent years to this same standard, as long as our interpretation of the result is so tempered. This is particularly true if the department we wish to examine has not undergone a major technological change during the intervening years. In addition, as new data becomes available, we are open to the possibility of identifying new production functions and hence, to realize a standard which is more relevant in time to the present.

5.4.4 Model B. In this section we examine the models listed in series B of Table 5.8 in order to determine the most plausible production function for radiographic facilities relative to MS.B. Rather than duplicate the discussion of the previous section, we merely present the results of having applied the same analysis to this series.

In Fig. 5.13a is shown a list of the production variables for MS.B

and their notation. The individual production variable sample sizes are omitted as they are the same as those provided in Fig. 5.10a. Fig. 5.13b provides the efficiency distributions for the nine possible models comprising the series which were listed in Table 5.8.

The same three histogram groups as discussed for series A can be observed here. They are the: i) normal distribution (1*B*, 2*B*, 3*B*, 4*B*, 5*B*, & 8*B*); ii) rectangular distribution (6*B*) and; iii) the skewed distribution (7*B*). The histogram for model 9*B* may be considered bimodal (at least more so than 7*B*) and hence is immediately excluded from consideration. Thus, in terms of our previous analysis, model 6*B* appears the most plausible, and is therefore chosen as the best model of series B. It is henceforth denoted M.B.

Models M.A and M.B both operate on the same production variable set, excluding the non-medical manpower input. Yet even for this input group, they are similar in that only one type of non-medical manpower input is recognized, although they are defined differently. This might imply that the two models we have recognized as most plausible are not in fact unique, but result from some optimal ratio between the number of production variables and the model sample size, and at least some representative input from each of the four input classes we have defined.

To examine this hypothesis a model with the same inputs as M.A & M.B, except that the non-medical manpower input was chosen to be the total non-MD staff man-hours, was run. The resulting efficiency frequency distribution for this model is shown in Fig. 5.14. The bimodal nature of this histogram relative to those of M.A (8*A* in Fig. 5.10b) and M.B (6*B* in Fig. 5.13b) indicates that the actual definition of the non-medical manpower input is in fact important, and hence reinforces the plausibility of M.A & M.B.

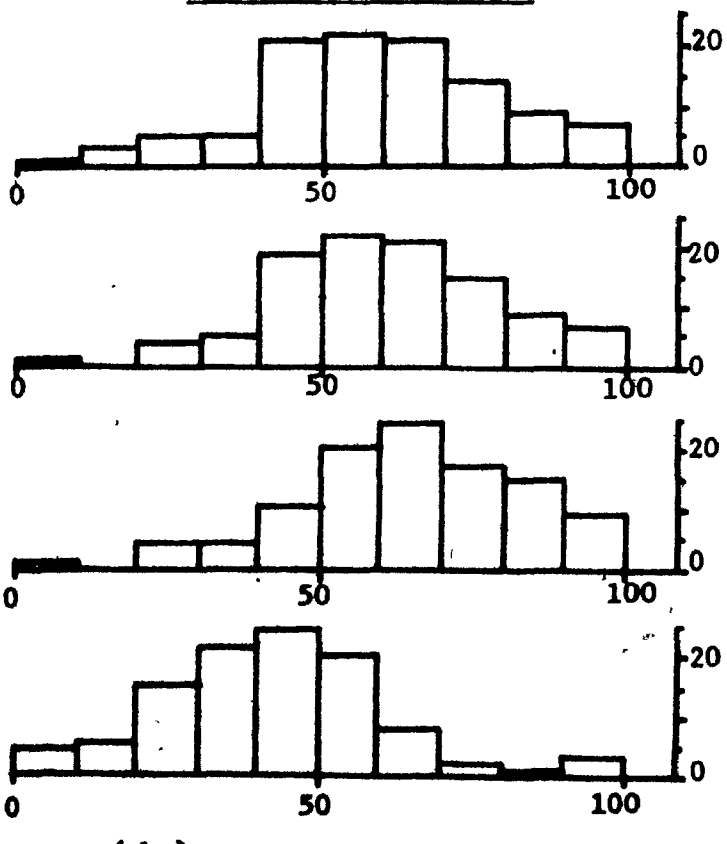
The complete model description for M.B is provided in Fig. 5.15. The departments defining the production function are listed in Fig. 5.16a.

Production Variable Set B	Denoted
Equipment-years	X_1
Non-labour expenses ($)	X_2
Tech(dipl)+Tech(ndipl) man-hours	X_{3+4}
Others man-hours	X_5
Full-time radiologist man-years	X_6
Part-time radiologist man-years	
Professional units produced	

Notation is not provided for the output measure as it is common to all models.

(a)

Model	Sample Size	Inputs	Number of Departments versus Efficiency
1B	108	X_1X_2	
2B	103	$X_1X_2X_{3+4}$	
3B	104	$X_1X_2X_{3+4}X_5$	
4B	103	X_1X_{3+4}	



(b)

Fig. 5.13. The (a) production variables and (b) efficiency histograms for the series of models described in Table 5.8.

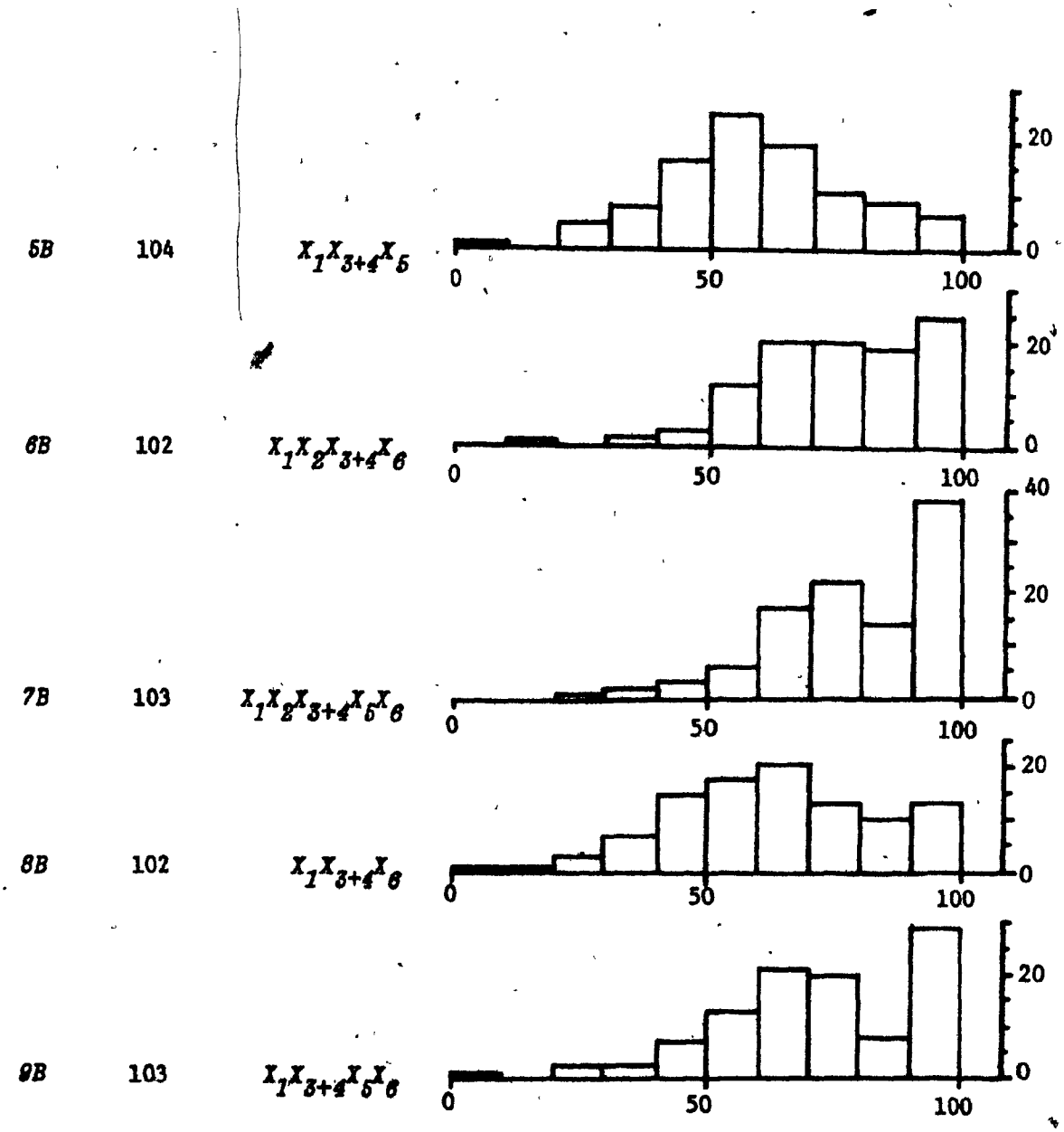


Fig. 5.13b. Continued.

Production Variables

Equipment-years

Non-labour expenses ($)

Total non-MD staff man-hours
(=Tech(dipl)+Tech(ndipl)+Others man-hours)

Full-time radiologist man-years

Part-time radiologist man-years

Professional units produced

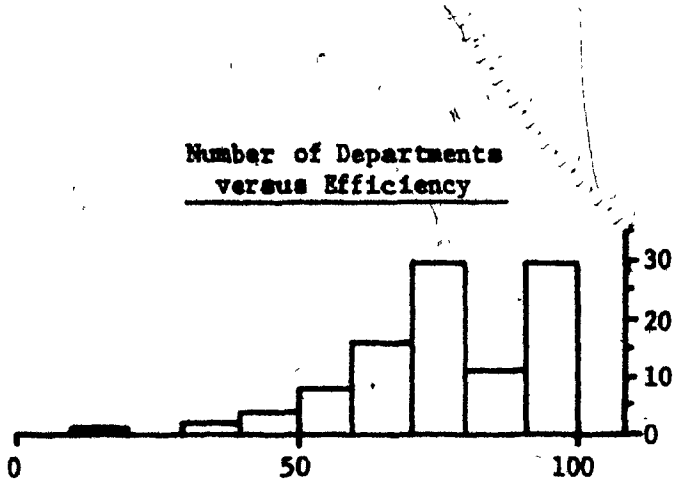


Fig. 5.14. The efficiency histogram of a production model with the same inputs as models M.A and M.B, except that the non-medical manpower input is equal to the total non-MD staff man-hour input.

MODEL B

DESCRIPTION: INPUT 1 = MAJOR DIAGNOSTIC EQUIPMENT YRS
INPUT 2 = FULL-TIME RADIOLOGIST MAN-YRS
INPUT 3 = PART-TIME RADIOLOGIST MAN-YRS
INPUT 4 = TECH(DIPL), NON-MD SPEC & TECH(NDIPL) HRS
INPUT 5 = NON LABCUR EXPENSES ($)
OUTPUT = DIAGNOSTIC RADIOLOGY PROFESSIONAL UNITS

SAMPLE: 102 OF 139

2 3 4 5 8 9 10 11 16 17 18 19 20 21 22 23 25 26 28 29
30 31 32 34 35 37 38 39 40 41 42 43 44 46 47 50 51 52 53 54
56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 75 77
79 80 81 84 85 86 87 88 89 90 91 92 94 95 96 97 98 99 100 101
102 103 107 109 111 112 113 116 117 118 122 125 126 127 128 130 132 134 135 136
137 139

Fig. 5.15. The production variables of model M.B as well as a list of the diagnostic radiology departments (see App. C) comprising the sample upon which this model is based.

Departments Defining
the Efficient Frontier of Model B

20	23	32	38	44
46	47	56	57	65
68	96	113	137	

(a)

$$\max\ (e^t, 0)\,\lambda$$

such that $[D\ I]\,\lambda = b$

$$D = \begin{bmatrix}
4 & 2 & 5 & 5 & 7 & 6 & 2 & 6 & 1 & 2 & 3 & 3 & 2 & 7 \\
3 & 2 & 1 & 3 & 2 & 1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 & 1 & 3 & 1 \\
30718 & 13400 & 26929 & 4639 & 20147 & 17224 & 9639 & 31144 & 6396 & 10502 & 10022 & 1827 & 2946 & 28242 \\
116112 & 45169 & 70499 & 63712 & 81887 & 44002 & 20058 & 77227 & 6561 & 18246 & 45443 & 14159 & 13223 & 113134
\end{bmatrix}$$

$$e = \begin{bmatrix} 860634 \\ 441542 \\ 564707 \\ 754426 \\ 813334 \\ 479198 \\ 264168 \\ 735416 \\ 188048 \\ 232017 \\ 392030 \\ 113513 \\ 122969 \\ 1070844 \end{bmatrix}$$

$$\lambda_i \geq 0 \quad i = 1,2,\ldots,19$$

$$\left.\begin{array}{l} b_1 = X_1 \\ b_2 \\ b_3 \end{array}\right\} = X_6$$

$$\left.\begin{array}{l} b_4 = X_{3+4} \\ b_5 = X_2 \end{array}\right\} \text{ from Fig. 5.13a}$$

(b)

Fig. 5.16. (a) The radiology departments from the model B sample described in Fig. 5.15 which are recognized as the most efficient producers and which therefore define the production function, and, (b) the linear programming formulation of model M.B.

Note that 11 of these 14 efficient departments also defined the production function of M.A, and that two of the efficient departments of M.A are not present in M.B. This would seem to indicate that the difference in the definition of the non-medical manpower input between these models has only a marginal effect. This is verified by an examination of Table 5.9 which provides a list of the departments common to both models and their technical efficiencies as measured by each.

Fig. 5.16b shows the resulting linear program which describes the most plausible production function for radiographic facilities that can be identified for production variable set B.

5.5 Summary

This chapter began with an examination of the data on production variables available to this project, and implicitly described the problems posed in using a real data base. Subsequently, a procedure was advanced for determining the most plausible production model from a series of such models, where all models are constructed using the technique developed in the previous chapter. Finally, two series were investigated, and from each, a "best" model was chosen for which the complete results were presented.

Having developed an index of performance for each department, called the technical efficiency, we should now like to find the sources for this index. Why do some departments have a high efficiency, while others do not? This is the specific question to which we address ourselves in the next chapter.

DEPT	M.A	M.B
2	74.51	74.51
3	82.55	87.87
4	62.84	63.21
5	46.09	46.13
8	73.57	73.57
9	86.25	86.25
10	71.66	71.66
11	82.30	82.30
16	61.34	61.34
17	61.33	64.60
18	60.74	62.49
19	56.93	56.82
20	100.00	100.00
21	53.22	53.62
22	73.70	73.95
23	100.00	100.00
25	65.81	70.00
26	90.25	95.69
28	70.16	70.16
29	87.70	88.58
30	67.06	65.73
31	65.29	66.05
32	100.00	100.00
34	64.81	64.55
35	78.76	73.03
37	56.25	56.89
38	100.00	100.00
39	74.84	74.82
40	89.22	89.22
41	100.00	92.75
42	91.59	91.59
43	95.03	95.04
44	97.72	100.00
46	100.00	100.00
50	71.46	74.99
51	87.18	83.52
52	95.99	95.99
53	74.04	73.01
54	92.87	85.33
56	100.00	100.00
57	100.00	100.00
58	79.30	81.58
59	19.24	19.24
60	62.39	62.39
61	84.03	83.96
62	69.48	69.53
63	72.12	72.44
64	59.01	59.01
65	98.35	100.00
66	84.76	87.65
67	67.92	73.19
68	100.00	100.00
69	86.45	90.49
70	88.31	82.50
71	87.24	79.19
72	93.30	98.52
73	94.47	95.47
77	75.22	75.43
79	68.85	68.85
80	86.49	84.05
81	66.65	70.16
84	70.26	70.28
85	70.79	62.13
86	59.44	59.44
87	35.60	37.42
88	62.06	57.55
89	100.00	72.90
90	86.48	86.51
91	76.78	69.93
92	87.37	75.88
94	59.27	63.18
95	73.24	73.98
96	100.00	100.00
97	66.66	66.77
98	79.01	80.46
99	52.17	52.38
100	65.03	65.27
101	58.74	58.81
102	87.12	86.73
103	92.65	92.65
107	72.89	72.89
109	93.44	94.26
111	59.20	59.40
112	46.56	46.54
113	100.00	100.00
116	53.26	44.62
117	57.77	57.44
118	32.09	34.23
122	64.13	64.13
125	73.94	69.24
126	84.98	88.00
127	67.27	67.43
128	67.55	62.83
130	83.51	87.32
132	89.25	87.69
134	93.89	97.64
135	65.62	62.99
136	72.15	54.86
137	100.00	100.00
139	55.91	54.80

Table 5.9. A list of the departments common to the samples of models M.A and M.B and their technical efficiencies as measured by these models.

CHAPTER 6

APPROACHES TO THE INVESTIGATION OF PRODUCTIVE INEFFICIENCY IN RADIOLOGY DEPARTMENTS

6.1 Introduction

The two production functions established in the previous chapter, M.A and M.B, allow one to estimate the technical efficiency of a diagnostic radiology department. This assumes, of course, the availability of the appropriate departmental model data. We recall that in the process of identifying either model from their respective samples, we defined the technical efficiency of each department in both samples.

In this chapter we attempt to discover why some departments are more efficient than others. The rationale behind this analysis is easily appreciated.

Our stated goal throughout this work has been to define a mechanism to support the decision making process for investment in radiographic equipment by government. For the most part this has been accomplished through our models. The technical efficiency of a department describes the extent to which a department has utilized its productive capabilities or its capacity output. In addition, this maximum output is defined by the best producers in a sample, and as such, represents an attainable standard of production behaviour rather than some theoretical upper bound.

An application for new equipment or an expansion of facilities by a department is, in effect, a statement of a need for an increase in productive capacity as perceived by the planners, i.e. radiologists, of the department. An important question which enters into the evaluation of such a proposal is: how productive is the department with its present manpower and technological resources? Is an expansion

of capacity really warranted? To a large extent, the answers to these questions can be found in the technical efficiency of a department.

It is apparent that one would initially favour expanding the facilities of a department which is currently utilizing a large part of its productive capacity, i.e. has a high technical efficiency. This is because more efficient departments have less freedom to increase output at their given resource level. Hence examining the technical efficiency of a department supplies a first order evaluation of either the departments application for funds, or, with more than one department, a comparison between applications.

In addition, if forecasts of the demand for services are available, either from departmental or government data, then an estimation of the technical efficiency with present resources over one or several years can be provided. This would be accomplished by comparing the forecasted outputs with the approximated capacity output. Although we have introduced a time factor, the process remains valid as the technological capabilities of a department with unchanged resources will remain the same as in the year for which the technical efficiency was estimated. If a department is predicted to rapidly approach the efficient production function, this too could justify an investment at the present time, regardless of its initial (current) efficiency.

The above discussion should not be interpreted to necessarily exclude investment possibilities in departments which presently have low efficiency. The circumstances for such a decision however, would have to take into account more information. For example, if a hospital decides to expand its capabilities to treat patients within a particular diagnostic category, then it may follow that one or more of the hospital departments, such as diagnostic radiology, will be called upon to provide the types of services these patients require. In this instance, one could justifiably de-emphasize consideration of the technical efficiency in favour of the health care needs of

a particular hospital patient population. Yet even in this case, the department should be called upon to demonstrate the validity of this new demand for services and its inability to provide them with existing resources.

In any event, it is obvious that increasing the productive efficiency of all departments is desirable. How can this be instigated? The major deterent to achieving this goal, at present, is the lack of an explicit incentive to pursue this end at the departmental level. The global budget system of financing hospitals (Anon, 1972h, 1974a) discourages hospitals and their departments to decrease their facilities (Lee, 1975), regardless of the degree of resource utilization.

The sole remaining impetus for increased efficiency in radiology departments would be the gradual increase in the demand for services with a constant resource level. This would result in a department slowly increasing its efficiency and might, if pushed far enough, promote a change in the productive operation of a department towards a more efficient utilization of the available resources. An example of such a change could be the adoption of a new, and presumably better, scheduling system for diagnostic examinations. Yet this process of gravitating towards more efficient production behaviour is slow, unsure, and may in the end be self-defeating by allowing unnecessary consumption of services (see section 3.3.1).

It may be possible, however, for the government to take direct action for improved departmental efficiency. Specifically, by delineating the causes of efficient and inefficient behaviour, it may occur that the investment decisions for which the government is responsible can be used to explicitly encourage departments towards increased technical efficiency.

We define four basic sources of inefficient behaviour as measured by the production models: i) the assumption of constant returns to scale may be invalid; ii) the possible non-homogeneity or quality

differences in the measured variables; iii) an insufficient demand for services, i.e. an imbalance between the resources of the radiology department and the structure of the hospital, and; iv) the effect of department management, i.e. an internal imbalance between the manpower and technological resources. The first two sources involve details of model construction, while the latter two are related to a description of the production environment which could not be accounted for in the model.

In this chapter we investigate each of the above possible sources of inefficient behaviour. Before doing so, we first extend our discussion of the efficiency measures themselves by examining a sensitivity analysis of the linear programming models.

6.2 Sensitivity Analysis

In section 4.3.1 we introduced two ways of observing input changes that one could effect in a production function: i) an incremental resource change as seen in the marginal productivities and substitution ratios described in section 4.2.4, and; ii) a resource change in units appropriate to the acquisition of new inputs. We have previously indicated that the level of interpretation of this analysis is dependent on the rigour with which the production function is originally estimated.

6.2.1 Qualifying the Effect of Input Changes. The methodology we pursued to identify a production function results in a piecewise continuous envelope to the sample of observations represented in a production variable space. As such, a relatively small number of extreme points (best producers) ultimately define the production function. In resolving model M.A, only 13 departments from a sample of 100 specify the linear programming model. In the case of M.B, only 14 of a possible 102 departments are used.

The output change we observe, corresponding to either incremental

or absolute input changes in a department, are seen as a change in the capacity output and not in the technical efficiency. Since the input variations are positive, the capacity output must respond by increasing. In this way we abstain from making any statement as to how the observed output, what is actually produced, will change. If one were to examine the technical efficiency instead, i.e. the same observed output with an increased capacity this would imply that increasing an input would result in decreased efficiency. Rather than suggest how increased productive capabilities will be used, it will suffice in terms of investment decisions to investigate how, and to what extent, the capacity output changes.

A sample of departments portrays the breadth of possible production processes in a particular technology. The production function is identified from the sample and the technical efficiency, marginal productivities, and other input change effects, per department, are measured relative to this production function.

As discussed in sections 4.5.3 and 4.5.4, the technique used to identify the function constrains the application of the production model to the analysis and comparison of production behaviour. Only 10% of the points in a model sample actually define the production function. The linear programming formulation was adopted as the means for identifying the production function of diagnostic radiology as a purely technological concept (see sections 4.2 and 4.4). The resulting function represents a piecewise continuous approximation to this theoretical concept. The calculation of marginal productivities and implementable resource change capacity responses are totally dependent on the specific shape of a production function. Since our formulation is only an approximation to the "real" production function, little or no meaning can be ascribed to the actual values in the sensitivity analysis that are derived from the linear programming model, with respect to the expected production behaviour of the departments.

Although the specific shape of the production function defined

by the linear program is acknowledged to be different from that of the actual production function, we assume that the general curvature, i.e. the degree of convexity, of these surfaces are similar. In section 4.2.4 we demonstrated that the substitution ratio, which is equal to the ratio of the marginal productivities of two different inputs, defines the slope of a tangent to the production surface at the point where these ratios are measured.

In Fig. 6.1 we show a continuous curve SS' and a piecewise continuous approximation to it, TT', such as could be produced by a linear program. While the slopes of these curves at some points, such as A and A', may be equal, there are other points where they are dissimilar, such as B and B'. Yet it appears that the relative values of the slopes at points on both curves and the same ray from the origin (representing the same production process, see Fig. 4.15) remain consistent. That is, while their actual values may differ, the slopes at corresponding points are generally of the same magnitude.

This implies that the relative effects of different input changes in the linear programming formulation should not differ significantly from those of the continuous case, and in particular, that the input production variable having the greatest effect would be the same. Therefore, we will compare the effects of incremental and absolute input changes only in terms of which variable appears to have the most dramatic effect on the departmental capacity. This approach allows the least demanding interpretation of the production models with respect to the effect of input changes.

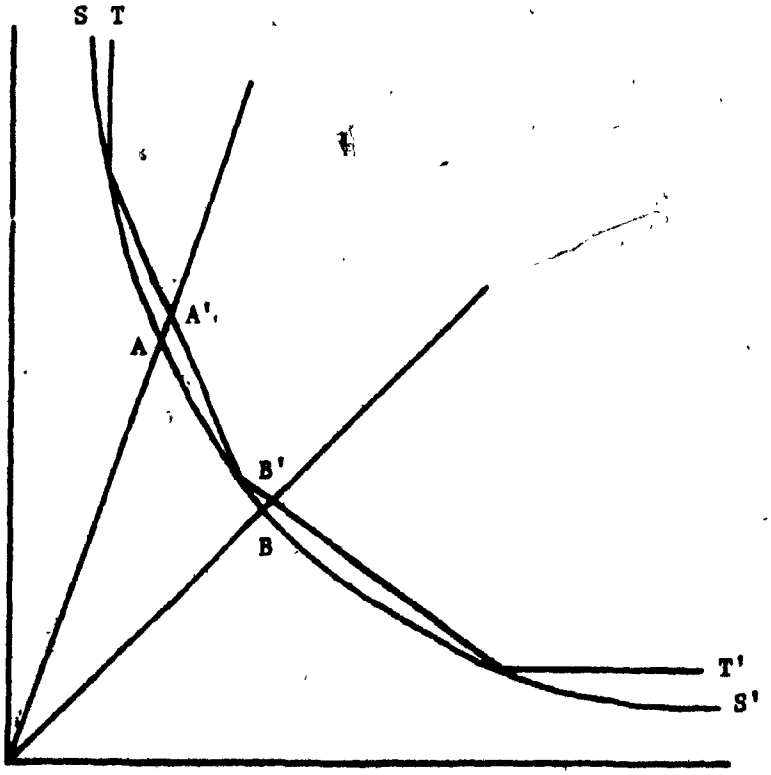


Fig. 6.1. A continuous curve (SS') and a piecewise linear approximation to it (TT') such as could be represented by the linear programming production model.

6.2.2 Quantifying the Effect of Input Changes. The solution of a linear program readily admits estimation of the marginal productivities. Consider the following linear program,

$$\max P$$

where $P = \mathbf{c}^t \boldsymbol{\lambda}$

$$\mathbf{A}\boldsymbol{\lambda} = \mathbf{b} \qquad \boldsymbol{\lambda} \geq 0$$

and $\mathbf{c} = n \times 1$ vector

$\boldsymbol{\lambda} = n \times 1$ vector

$\mathbf{b} = m \times 1$ vector

$\mathbf{A} = m \times n$ matrix, $n > m$

Using the production function notation from Eqn. 4.1, the solution of the above program is equivalent to,

$$P = f(b_1, b_2, \ldots, b_m) \tag{6.1}$$

and from Eqn. 4.2 we have that,

$$\frac{\partial P}{\partial b_i} = \text{marginal productivity of input factor } i \tag{6.2}$$

The simplex method solution of the above program requires choosing a set of m basic feasible variables (i.e. m non-zero λ_i's) which which maximizes P (Luenberger, 1973). Denote by $\mathbf{B}$ the $m \times m$ submatrix of $\mathbf{A}$ corresponding to a set of basic feasible variables. Then partitioning $\mathbf{A}$, $\boldsymbol{\lambda}$, and $\mathbf{c}$ as,

$$A = [B\ D] \qquad \lambda = \begin{bmatrix} \lambda_B \\ \lambda_D \end{bmatrix} \qquad c = \begin{bmatrix} c_B \\ c_D \end{bmatrix}$$

the linear program can be rewritten as,

$$\max P$$

where $$P = c_B^t \lambda_B + c_D^t \lambda_D$$

$$B\lambda_B + D\lambda_D = b \qquad \lambda_B,\ \lambda_D \geq 0$$

Solving the matrix equation for λ_B we have,

$$\lambda_B = B^{-1}b - B^{-1}D\lambda_D$$

from which

$$P = c_B^t(B^{-1}b - B^{-1}D\lambda_D) + c_D^t\lambda_D$$

$$P = c_B^t B^{-1}b + (c_D^t - c_B^t B^{-1}D)\lambda_D$$

Assuming that λ is a basic feasible solution, then,

$$\lambda = \begin{bmatrix} \lambda_B \\ 0 \end{bmatrix} \quad \text{i.e. } \lambda_D = 0 \qquad (6.3)$$

and therefore,

$$P = \mathbf{c}_B^t \mathbf{B}^{-1} \mathbf{b} \tag{6.4}$$

If vector $\mathbf{A}$ in Eqn. 6.3 is the optimal solution to the linear programming problem then Eqn. 6.4 is equivalent to Eqn. 6.1. Incrementing the inputs by $\Delta\mathbf{b}$ and denoting the corresponding change in the objective function as ΔP then from Eqn. 6.4 we have,

$$P + \Delta P = \mathbf{c}_B^t \mathbf{B}^{-1} (\mathbf{b} + \Delta\mathbf{b})$$

$$P + \Delta P = \mathbf{c}_B^t \mathbf{B}^{-1} \mathbf{b} + \mathbf{c}_B^t \mathbf{B}^{-1} \Delta\mathbf{b} \tag{6.5}$$

from which,

$$\Delta P = \mathbf{c}_B^t \mathbf{B}^{-1} \Delta\mathbf{b}$$

Let $\Delta b_j > 0$, $\Delta b_i = 0$ $i \neq j$ $i = 1, 2, \ldots, m$

and $(\mathbf{c}_B^t \mathbf{B}^{-1})_j = j^{\text{th}}$ element of the $1 \times m$ vector $\mathbf{c}_B^t \mathbf{B}^{-1}$

(6.6)

Taking the difference between Eqn. 6.5 and 6.4, dividing by Δb_j and taking the limit of the resulting expression as Δb_j goes to zero, we have from Eqn. 6.6 that,

$$\frac{\partial P}{\partial b_j} = (\mathbf{c}_B{}^t \mathbf{B}^{-1})_j = \text{marginal productivity of input factor } j \tag{6.7}$$

In finding the optimal solution to the linear program, $(\mathbf{\Lambda}_B, 0)^t$, by the revised simplex method, we are provided with the corresponding elements of vector $\mathbf{c}_B$ and the inverse of matrix $\mathbf{B}$ (Luenberger, 1973). Therefore, the marginal productivities for all input factors may be calculated by solving Eqn. 6.7 for $j = 1, 2, \ldots, m$ after the linear program itself has been solved. The $(\mathbf{c}_B{}^t \mathbf{B}^{-1})_j$ are sometimes called the dual prices of a linear program (Luenberger, 1973).

Simulating implementable resource changes requires solution of a new linear program for each change. Suppose input i must be acquired in units of Δb_i. The effect of increasing input i on the capacity output can be observed by solving the same linear program except that now,

$$\mathbf{b} = (b_1, \ldots, b_{i-1}, b_i + \Delta b_i, b_{i+1}, \ldots, b_m)^t$$

This results in a new value for the objective function, or capacity output. If $P(\Delta b_i)$ denotes this new solution and P, the original solution, then of course,

$$P(\Delta b_i) \geq P$$

and we quantify the effect of changing the i-th input as

$$\frac{P(\Delta b_i)}{P} \tag{6.8}$$

Calculation of the marginal productivities in Eqn. 6.7 and the effect of an implementable resource change in Eqn. 6.8 is built into the computer program which identifies the production function from a sample of departments (see Appendix D).

6.2.3 An Example. In models M.A and M.B there are usually four, though sometimes five, distinct inputs per department. They are the equipment, non-MD labour, non-labour expenses, and either a full-time or part-time radiologist input or possibly both. Non-labour expenses are measured in dollars while all other inputs are quantified in some unit of time.

In deriving the linear programming production model in section 4.5 we assumed that each department in a sample defines its own production process; that is, a fixed quantitative ratio between the inputs. The production function was found by maximizing the output for each process, given the departmental inputs. Changing only one of the inputs would result in a change in the ratio of this input to all others, and in effect defines a new, though not radically different, production process.

For both models, marginal productivities were calculated for all inputs present in a particular department. Hence the absence of an input, such as part-time radiologist man-years, implies that the corresponding marginal productivity was not found. In addition, only the equipment and radiologist inputs were in units conducive to an implementable change. In all departments the number of equipment-years, full-time radiologist man-years and part-time radiologist man-years were independently incremented by one. This was meant to test the effect of purchasing a piece of equipment or hiring radiologists.

The method used to construct the production functions dictates that the relative effectiveness of changing different inputs for a department will depend on the positional relationship of its

production process to the function itself (see Fig. 4.15). The sensitivity results of four departments of model M.A are shown in Table 6.1: the first column of figures lists the amount of each input that was present; the second column provides the result of incrementing selected inputs by one, as indicated in Eqn. 6.8; and the third column lists the marginal productivities as calculated in Eqn. 6.7.

In these four departments, as with all departments in the model sample, the first three inputs were separately incremented by 1. For example, in the first department in Table 6.1, we resolved the linear program with 12 equipment-years, the other four resources being the same as in the input column, with 6 full-time radiologist man-years, the four other resources being the same as in the input column, and, with 1 part-time radiologist man-year, the four other resources being the same as in the input column. These three simulated resource acquisitions resulted in a 3.1%, 6.1% and 15.3% increase in capacity respectively. The marginal productivities can only be calculated for an input that is initially non-zero.

With very few exceptions, the marginal productivities for all departments, in both models, indicate that the major sources for increasing the productive capacity of a department are either the equipment input or one of the radiologist inputs. If both full and part-time radiologists are present in a department the marginal productivities and implemented resource changes for the first three inputs are in agreement in terms of which input has the greatest effect on expanding the capacity - see the third department in Table 6.1. If either one of the medical manpower inputs is absent, then the relative effects of either absolute or incremental input changes for the remaining inputs (of the first three) will coincide, yet the effect of an absolute change on the omitted medical input

Input Variables (M.A)	Value	$\frac{P(\Delta b_i)}{P}$	$\frac{\partial P^*}{\partial b_i}$
equipment-yrs	11	1.031	51.3721
full-time radiologist man-yrs	5	1.061	108.1148
part-time radiologist man-yrs	0	1.153	-
tech(dipl) man-hrs	47358	-	0.0011
non-labour expenses ($)	191867	-	0.0026
equipment-yrs	2	1.020	7.2398
full-time radiologist man-yrs	0	1.234	-
part-time radiologist man-yrs	1	1.000	0.0000
tech(dipl) man-hrs	6172	-	0.0126
non-labour expenses ($)	24860	-	0.0059
equipment-yrs	4	1.138	106.7885
full-time radiologist man-yrs	2	1.078	59.4196
part-time radiologist man-yrs	1	1.000	0.0000
tech(dipl) man-hrs	19095	-	0.0114
non-labour expenses ($)	91194	-	0.0000
equipment-yrs	5	1.000	0.0000
full-time radiologist man-yrs	1	1.224	156.4678
part-time radiologist man-yrs	0	1.161	-
tech(dipl) man-hrs	6881	-	0.0097
non-labour expenses ($)	36244	-	0.0038

* All values of the partial derivative are multiplied by 10^3.

Table 6.1. Sensitivity analysis results for four different diagnostic radiology departments as performed with model M.A.

can provide additional information. In the first two examples of Table 6.1, increasing the omitted medical input by 1 would appear to have a greater effect on the capacity output than a similar change in either the included medical or the equipment inputs. In the fourth example this is not the case; that is, increasing the number of full-time radiologists to 2 has a greater effect than increasing the number of part-time radiologists to 1.

The first department in Table 6.1 provides a particularly interesting example. One could reasonably ask, how is it possible that an additional full-time radiologist is less effective with respect to capacity expansion than a part-time radiologist?

Suppose the i-th element of the input vector $\mathbf{X}$ (see problem P4 in Chap. 4) in a linear program is zero. It follows that in all solutions to P4, with matrix equation $(\mathbf{D}\ \mathbf{I})\lambda = \mathbf{X}$, the non-zero elements of λ must correspond to columns of $\mathbf{D}$ in which the i-th elements are zero. Hence, a department with only full-time radiologists will be represented in the linear program by operating one or more of those production processes for which the part-time radiologist input is absent. Similarly, departments with only part-time radiologists are analyzed with respect to the behaviour of those production processes without a full-time radiologist input. Only those radiology departments with both a full- and part-time radiologist input are represented by simultaneously operating one or more processes with only a full-time input and one or more departments with only a part-time input.

Hence the first department in Table 6.1 is initially examined with respect to the set of processes (submatrix of $\mathbf{D}$) that utilize only full-time radiologists. On introduction of the 1 part-time radiologist, the analysis is pursued with respect to the complete set of processes; that is, those with full-time radiologists and those with part-time radiologists.

It was found on examining the $\mathbf{D}$ matrices in models M.A (Fig. 5.12b) and M.B (Fig. 5.16b) that the efficient departments with only part-time radiologists generally utilize a proportionately greater amount of the

technician input then the efficient departments with only full-time radiologists, Therefore, an increase in the part-time radiologist input will result in a greater capacity increase than that of a full-time radiologist increase when there is a relatively large underutilization (slack) in the present technician input. This implies that little or no capacity change will result from increasing the technican input to this department. Hence, variation in the medical inputs can shed light on the contribution of the technician input to production.

The point of pursuing sensitivity analyses as described above, in terms of evaluating a proposal for new equipment, is to generate questions. If the capacity output is most affected by a medical manpower input change, then, is the percieved need for an expansion of facilities by a department in reality a relection of poor manpower utilization? If the services that are to be provided by the desired equipment are not different from those that are presently available, will an increase in some part of the staff or a greater utilization of the present staff yield similar benefits? Even where an equipment change produces the largest capacity expansion, does the projected demand for services justify such an expansion in terms of the present level of efficiency?

These questions should be confronted by the government planners and, in turn, passed on to the applying radiology departments.

A major difficulty in problem solving is simetimes to merely find the appropriate questions. If found, the solution may readily follow. The sensitivity analysis we have described is addressed to this process.

6.3 The Technical Efficiency and Previous Assumptions

In this section we examine the relationship between the departmental efficiencies of models M.A and M.B, and several "explanatory" variables. This exercise was pursued in an effort to explain differences in efficiency between departments. The

variables were chosen as proxies for the sources of inefficient production outlined initially in this chapter.

6.3.1 Returns to Scale. The fundamental assumption of our linear programming production models is the existence of constant returns to scale in the relationship between the input variables and the capacity output. In section 5.2.2 we recognised that if constant returns were not present, then decreasing returns to scale would be. How then can one empirically resolve the true nature of the system?

We recall that with decreasing returns to scale, multiplying each of the inputs to a department by a positive constant k would result in the capacity output increasing by some positive factor less than k. With constant returns to scale the capacity output would change in the same proportion as the inputs. Any hypotheses testing on the observed input and output rates of the departments would be inappropriate, as this would neither confirm nor reject any assumed relationship between the inputs and the capacity output. In addition, if radiology departments obey decreasing returns in terms of their capacity output, it then follows that the observed output would be constrained to do likewise. If this in fact is the case, then the technical efficiencies, measured with a construct presuming constant returns, would generally decrease as departments became larger. Figs. 6.2a and b show plots of efficiency versus department output for the samples of models M.A and M.B respectively.

In both Figs. 6.2a and b three features emerge: i) as output increases the samples become increasingly sparse, i.e. there are more small departments than large ones; ii) smaller departments are generally not the most efficient but appear to exhibit the greatest variation in efficiency and; iii) large departments, i.e. those with output greater than 1 million professional units, are not as efficient as some of the smaller departments.

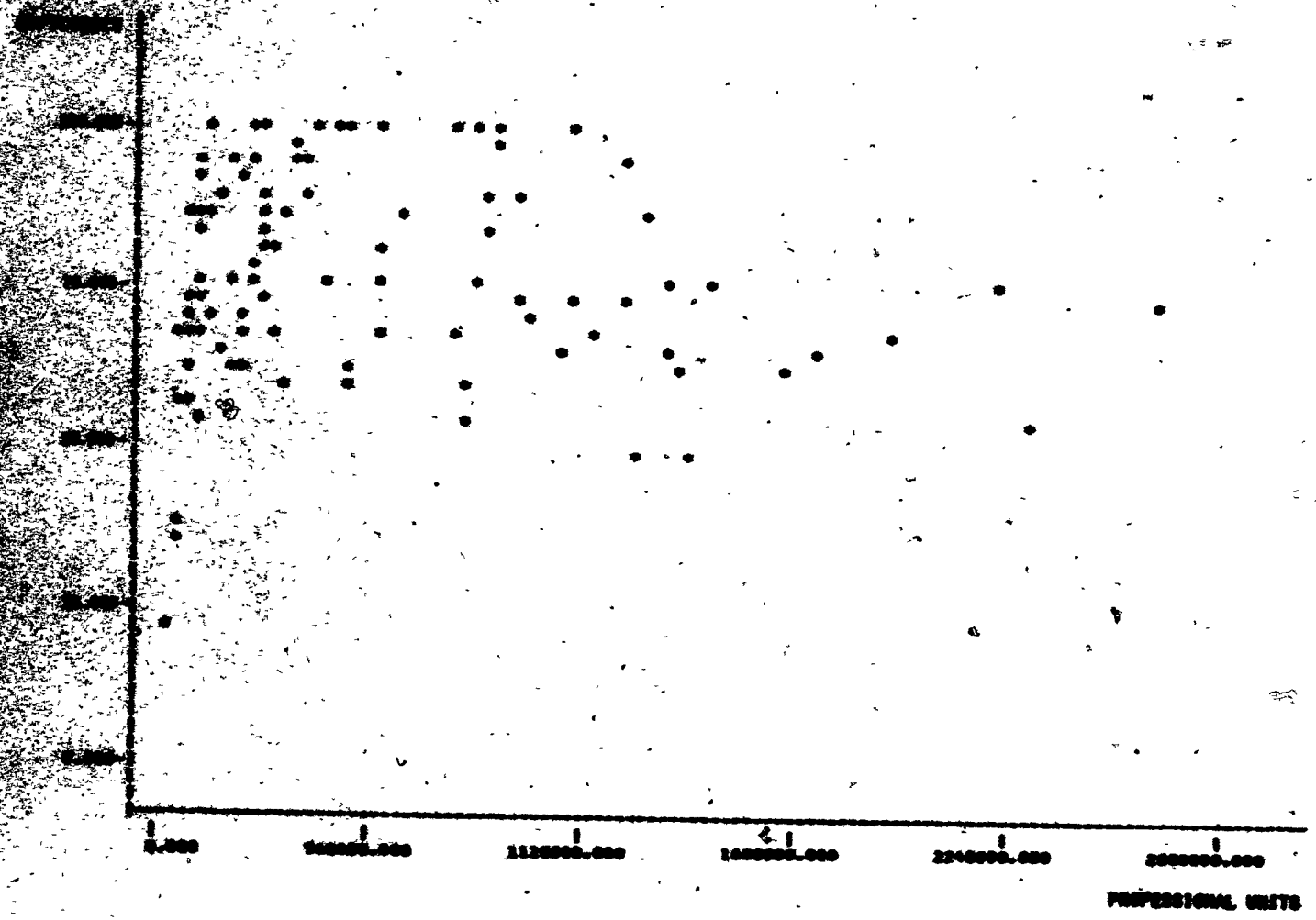


Fig. 6.2a. The technical effiency of radiology departments as measured by model M.A versus the number of professional units of service provided by these departments.

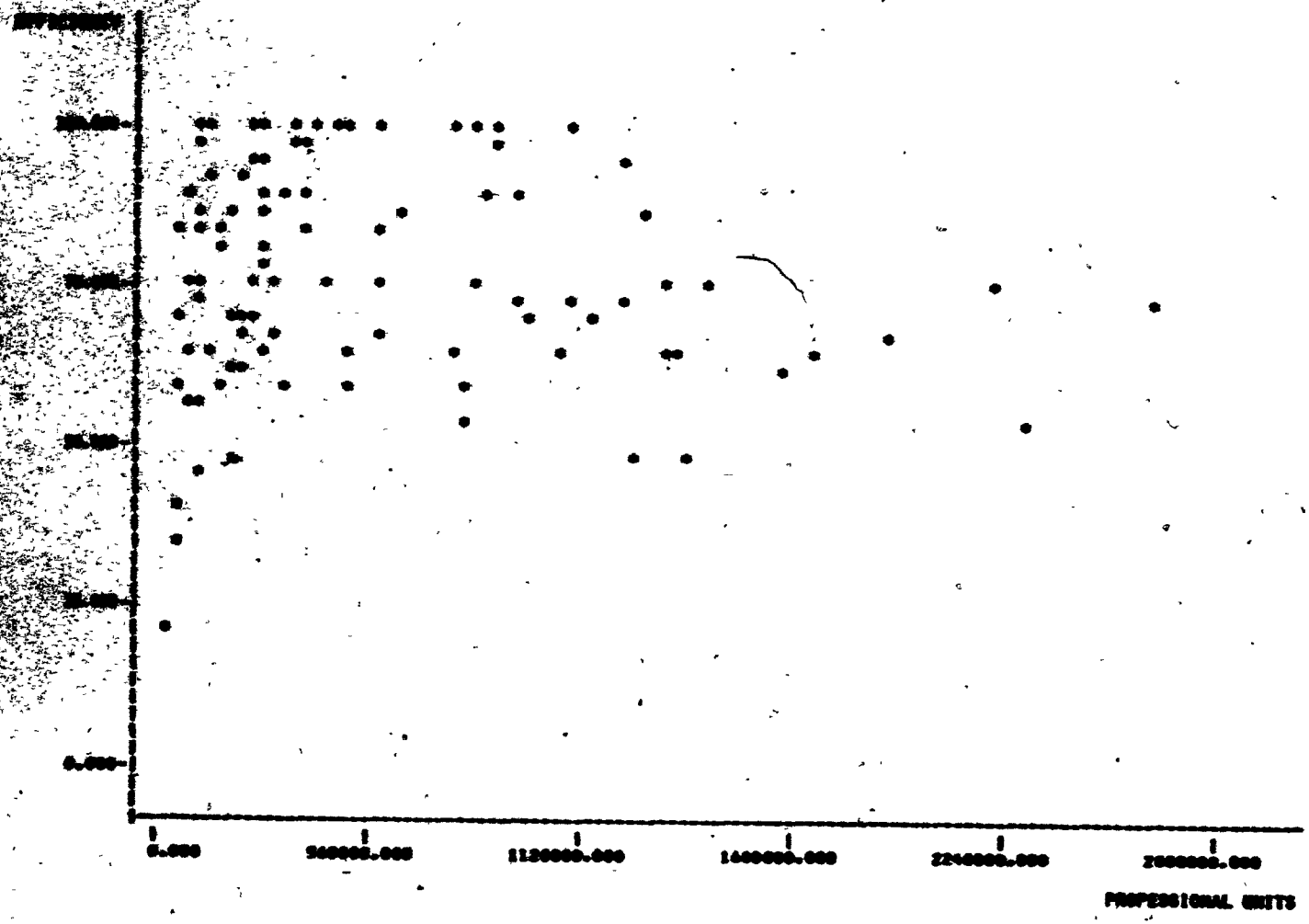


Fig. 6.2b. The technical efficiency of radiology departments as measured by model M.B versus the number of professional units of service provided by these departments.

Assuming there are enough observations from which to draw a conclusion, one might initially decide from the evidence of Fig. 6.2 that larger departments are being unduly penalized under a constant returns assumption. To check this result the sample was divided into three subsets; those departments producing less than ½ million P.U., those producing between ½ and 1 million P.U., and those departments producing over 1 million P.U. The mean efficiencies for each of these subsets is provided in Table 6.2.

From this table we see that: i) the mean efficiencies of these groups do not decrease with output, and in fact are highest for the middle subset, and; ii) combining the second two groups, i.e. over ½ million units of output, results in a mean efficiency of approximately 75%, which is not too different from that of either the first group or the entire sample. Both of these facts, plus the wide efficiency range of the smaller departments contradict the existence of decreasing returns in the capacity output. Therefore we can safely regard our initial assumption of constant returns to scale as appropriate.

6.3.2 <u>The Technical Efficiency in the Larger Radiology Departments.</u> Why are most of the largest departments not high efficiency producers? In section 5.2.2 we established that the larger departments are: i) in larger hospitals, i.e. with a greater number of beds; ii) have a greater proportion of specialized x-ray rooms than do smaller departments, and; iii) on average produce more complex procedures, i.e. have a higher mean number of P.U. per examination ratio (see section 5.2.2). In addition, many, if not all the largest radiology departments in the province are in university teaching hospitals. Hence they are involved with some research and training activities aside from the provision of diagnostic services. These academic activities are resource consuming though are not recognized as productive in our model (see section 3.2.2).

Figs. 6.3a and b show plots of efficiency versus P.U./examination

	M.A		M.B	
	Sample size	Mean efficiency	Sample size	Mean efficiency
Complete sample	100	76.2	102	75.8
Departments with: 500,000 > OUTPUT	62	76.3	64	75.9
Departments with: 500,000 < OUTPUT < 1,000,000	19	81.1	19	81.4
Departments with: OUTPUT > 1,000,000	19	68.2	19	68.3

Table 6.2. Classification of the radiology departments analyzed by models M.A and M.B as to their output volume (P.U.) and the corresponding average technical efficiency of the departments in the resulting groups.

for the samples of models M.A and M.B respectively. It appears that the level of departmental efficiency is not directly related to the average complexity of the procedures that are produced; that is, we have recognized average procedure complexity to be a differentiating feature of radiology departments, but not an explicit source of technical inefficiency.

Examining plots of efficiency versus number of hospital beds, as in Fig. 6.4, is hardly illuminating. As a result of the direct correlation between departmental output and hospital size, it is not surprising that Figs. 6.2 and 6.4 are similar. Thus our initial question may be rephrased slightly: why are the radiology departments in the largest hospitals less efficient? Admittedly, the sizes of a department and its hospital are directly correlated. The difference between the previous question and the one we address now, is the difference between attributing departmental inefficiency to the complexity of the internal production environment or the service needs of the external hospital environment. Two possible sources of departmental inefficiency in larger hospitals emerge: i) large (university) hospitals are engaged in activities other than patient care, and; ii) the types of patients that are treated in larger hospitals may require different kinds of radiographic services than those in smaller hospitals.

Although we have omitted the teaching and research activities a radiology department may pursue, and which would make it appear less productive, we have also omitted another important factor. In these institutions there are radiology residents and interns who can be as productive as a radiologist, but which are not included as part of the medical manpower input, or for that matter, any input. They provide services which we include in the departmental output measure, but are not recognized as a resource to be utilized. Therefore

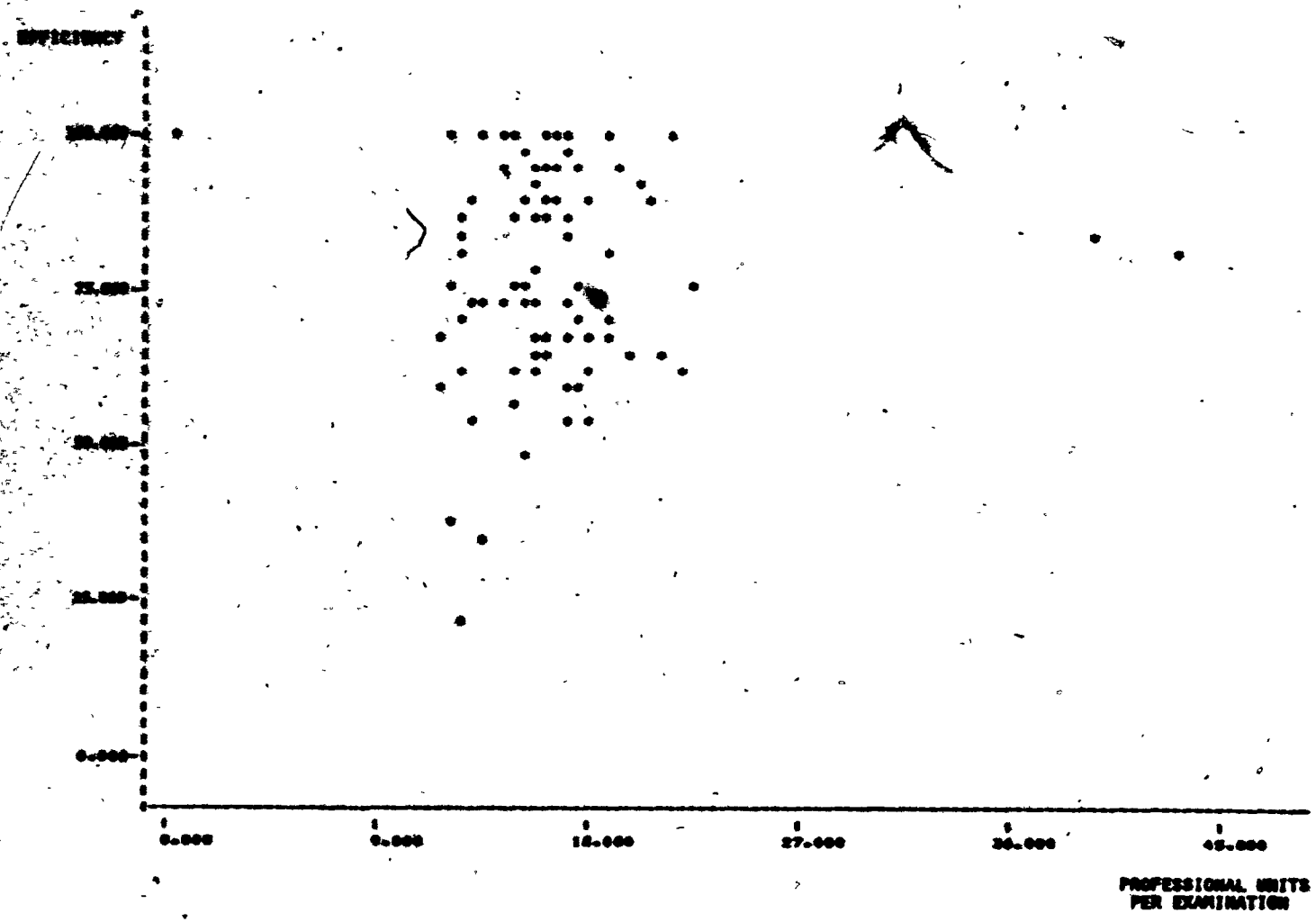


Fig. 6.3a. The technical efficiency of radiology departments as measured by model M.A versus the average number of professional units per diagnostic examination in these departments.

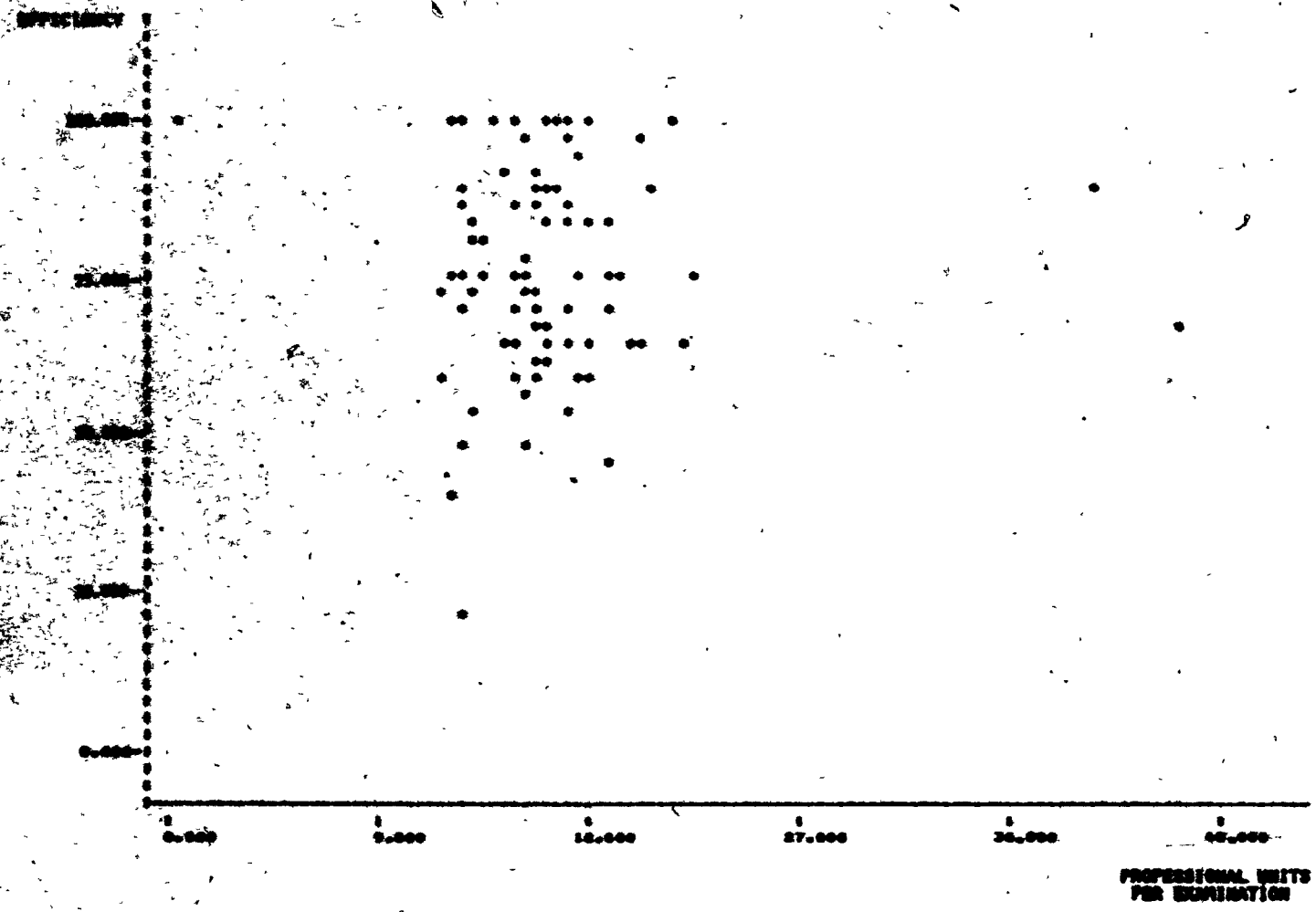


Fig. 6.3b. The technical efficiency of radiology departments as measured by model M.B versus the average number of professional units per diagnostic examination in these departments.

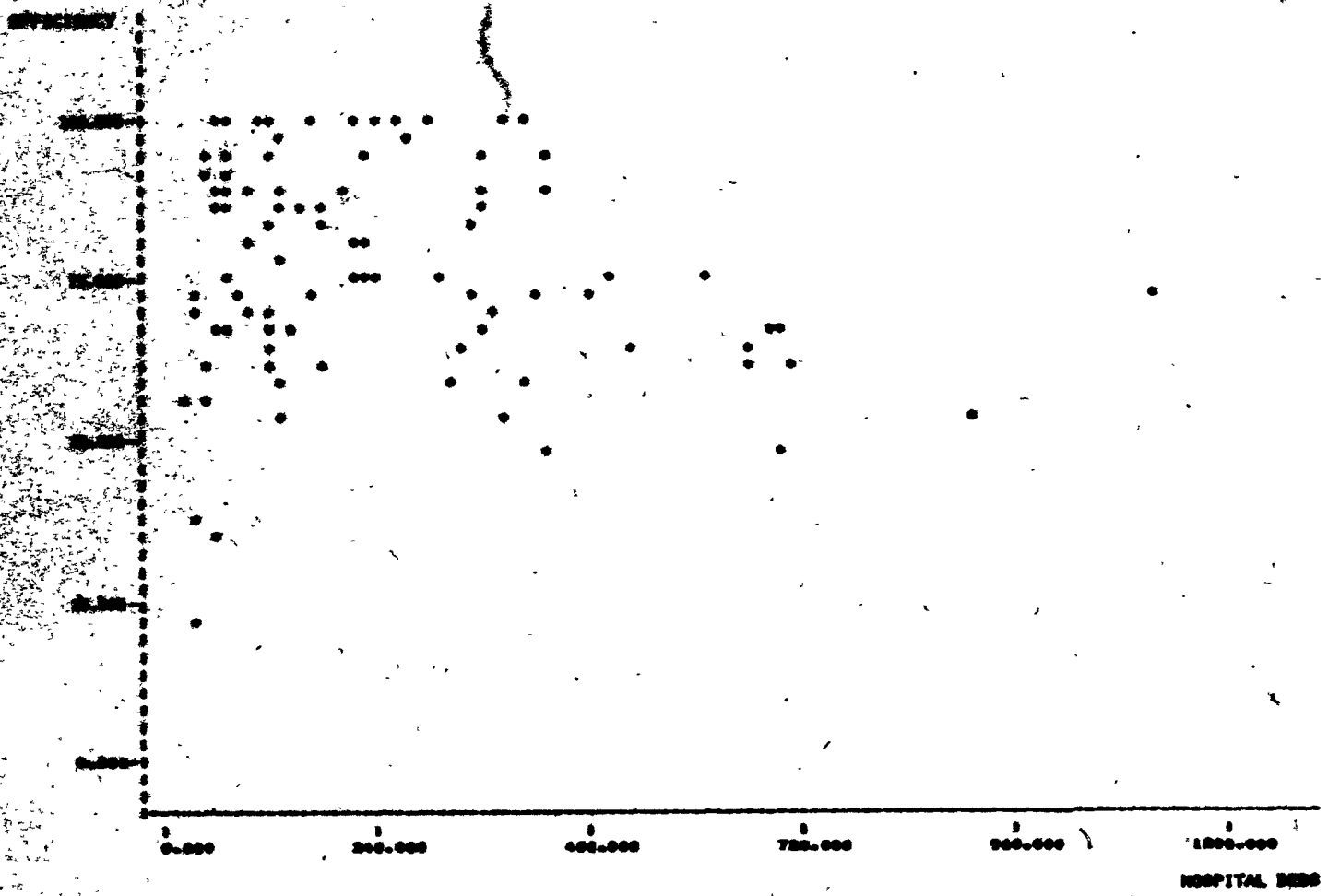


Fig. 6.4a. The technical efficiency of radiology departments as measured by model M.A versus the number of hospital beds for the hospitals in which these departments are located.

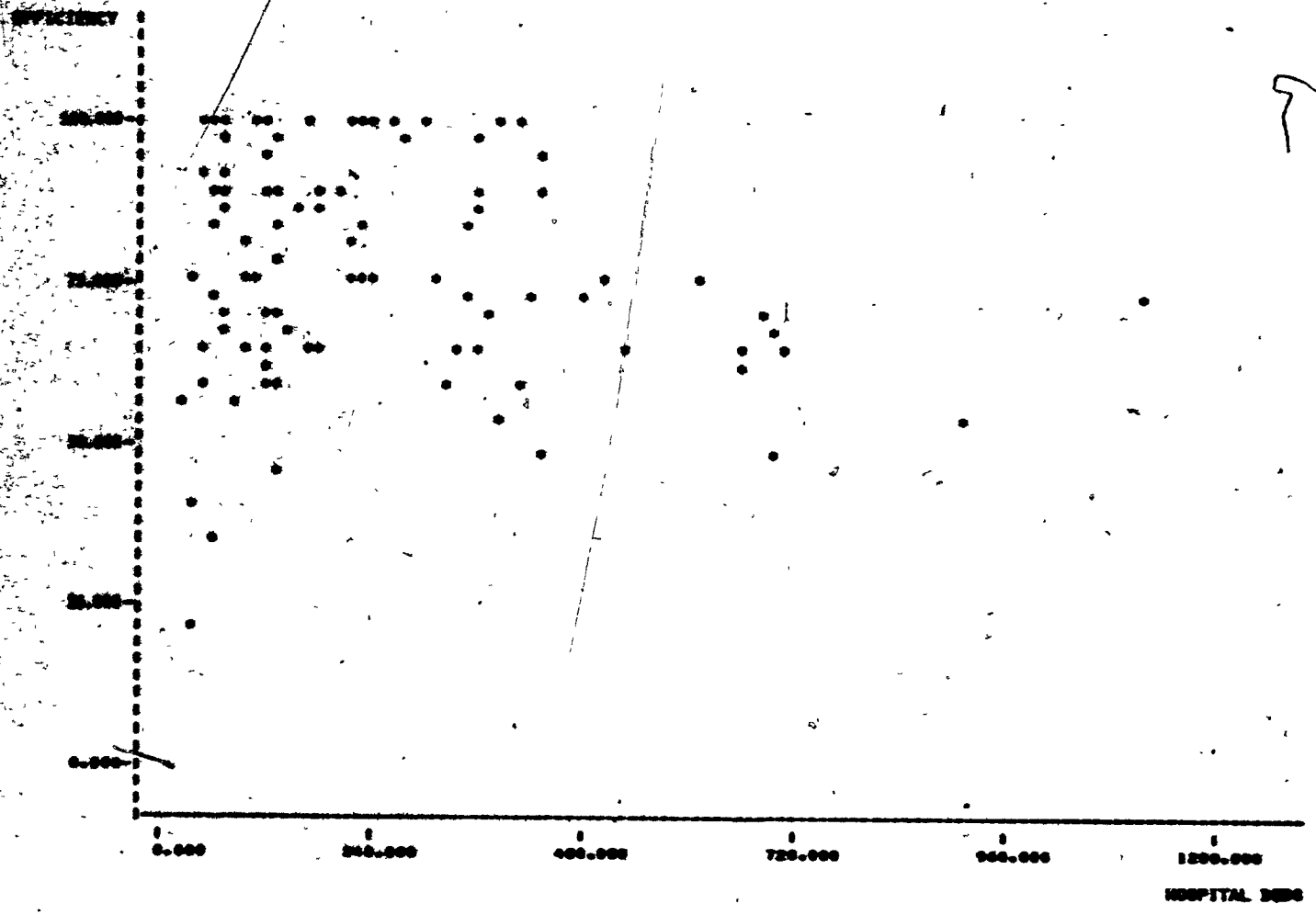


Fig. 6.4b. The technical efficiency of radiology departments as measured by model M.B versus the number of hospital beds for the hospitals in which these departments are located.

interns and residents providing services in a department would make it appear more efficient, in terms of the resources included in the models, than it actually is. Thus we assume that the productivity of the residents balances the loss of productivity taken up by research activities, and if not exactly balanced, we assume that the balance is not a major determinant of the efficiencies of large departments.

As the requisite data on interns and residents was not made available to this project, this assumption cannot be tested, and hence, is a matter of convenience more than anything else. Therefore, subsequent data gathering efforts by government should include information on the productive input to radiology of interns and residents so that our hypothesis can be tested.

Differences in hospitals can be observed in the relative number of inpatients and outpatients. These two patient populations generally require different radiographic services. The number of outpatient visits was not available from our data base. Yet data set D.1 does provide the number of diagnostic examinations each group received. Fig. 6.5 shows a plot of inpatient versus outpatient examinations. We see that the ratio of outpatient examinations to inpatient examination decreases as output (hospital size) increases.

In Figs. 6.6a and b we examine efficiency versus the ratio of outpatient to inpatient examinations for M.A and M.B respectively. If a correlation between the efficiency of large departments and low values of this ratio should exist, we would expect that Fig. 6.6 would look like Fig. 6.2 turned around. In fact this is not the case. Hence the patient population as represented by this crude measure, does not appear to be an explanatory variable for the efficiencies of large departments.

There remains one factor which separates the largest departments from the others, and which may account for the difference in efficiencies;

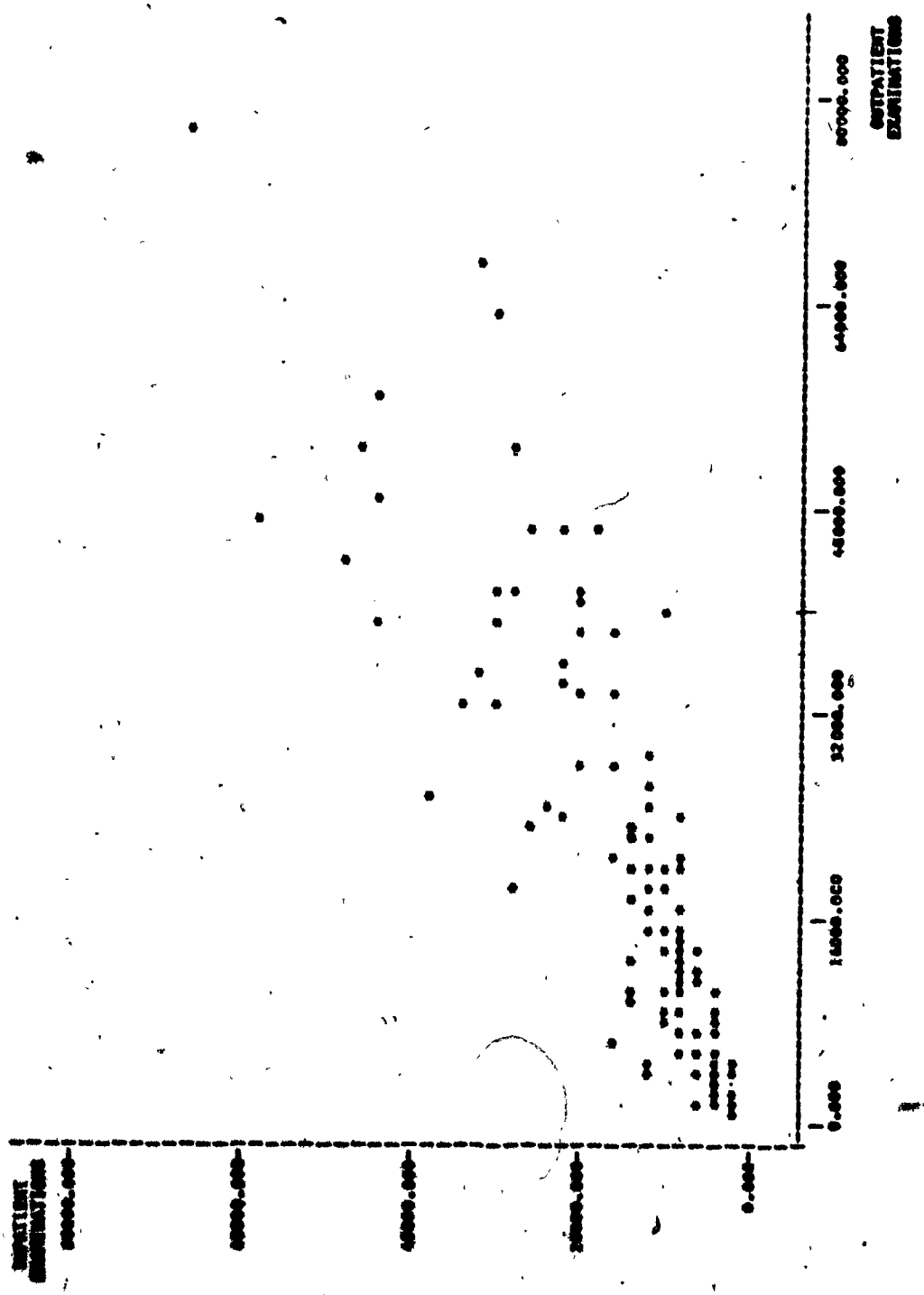


Fig. 6.5. The number of diagnostic examinations provided to inpatients versus the number of diagnostic examinations provided to outpatients in diagnostic radiology departments for 1971.

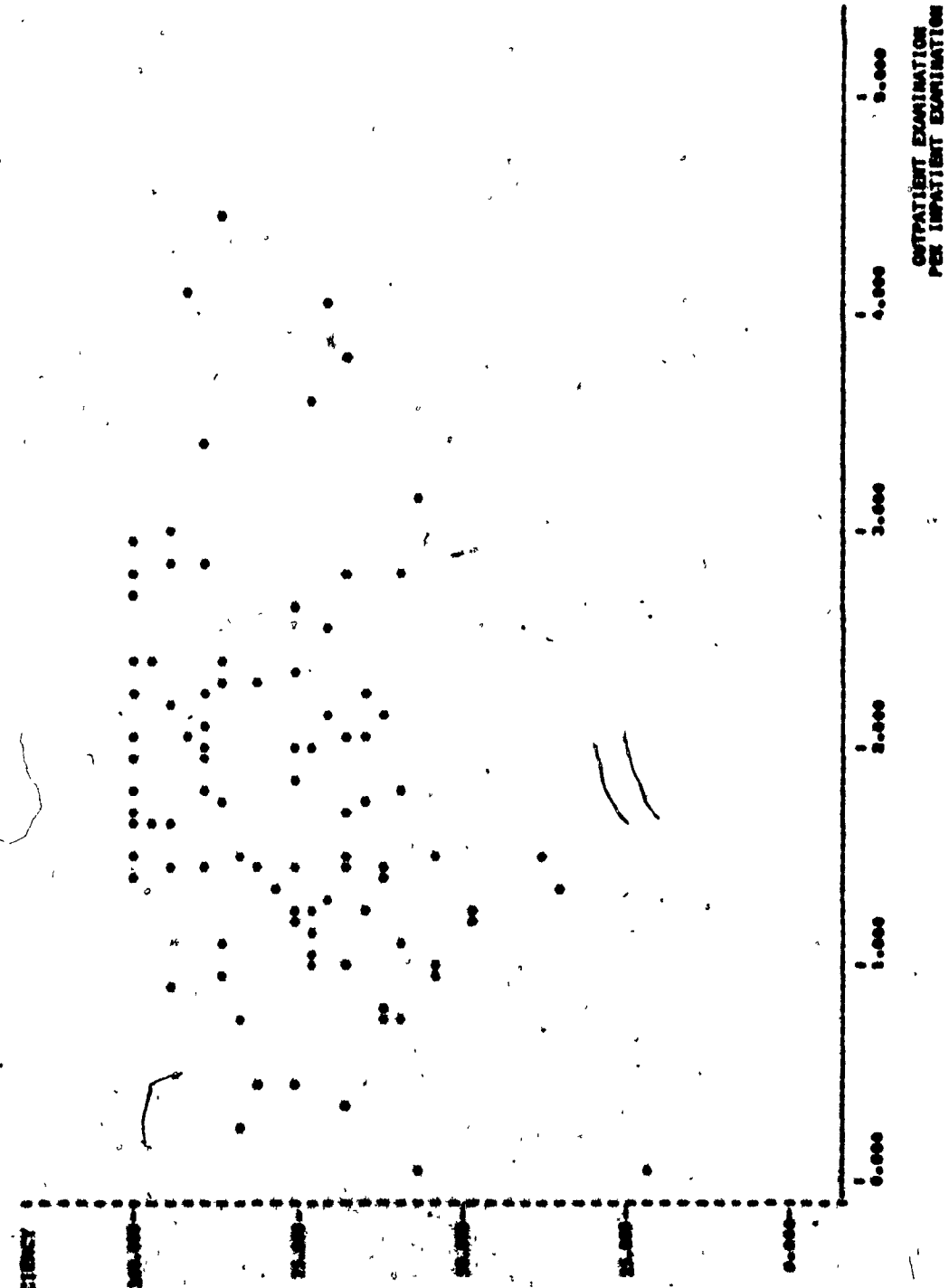


Fig. 6.6a. The technical efficiency of radiology departments as measured by model M.A versus the ratio of outpatient examinations to inpatient examinations provided by these departments.

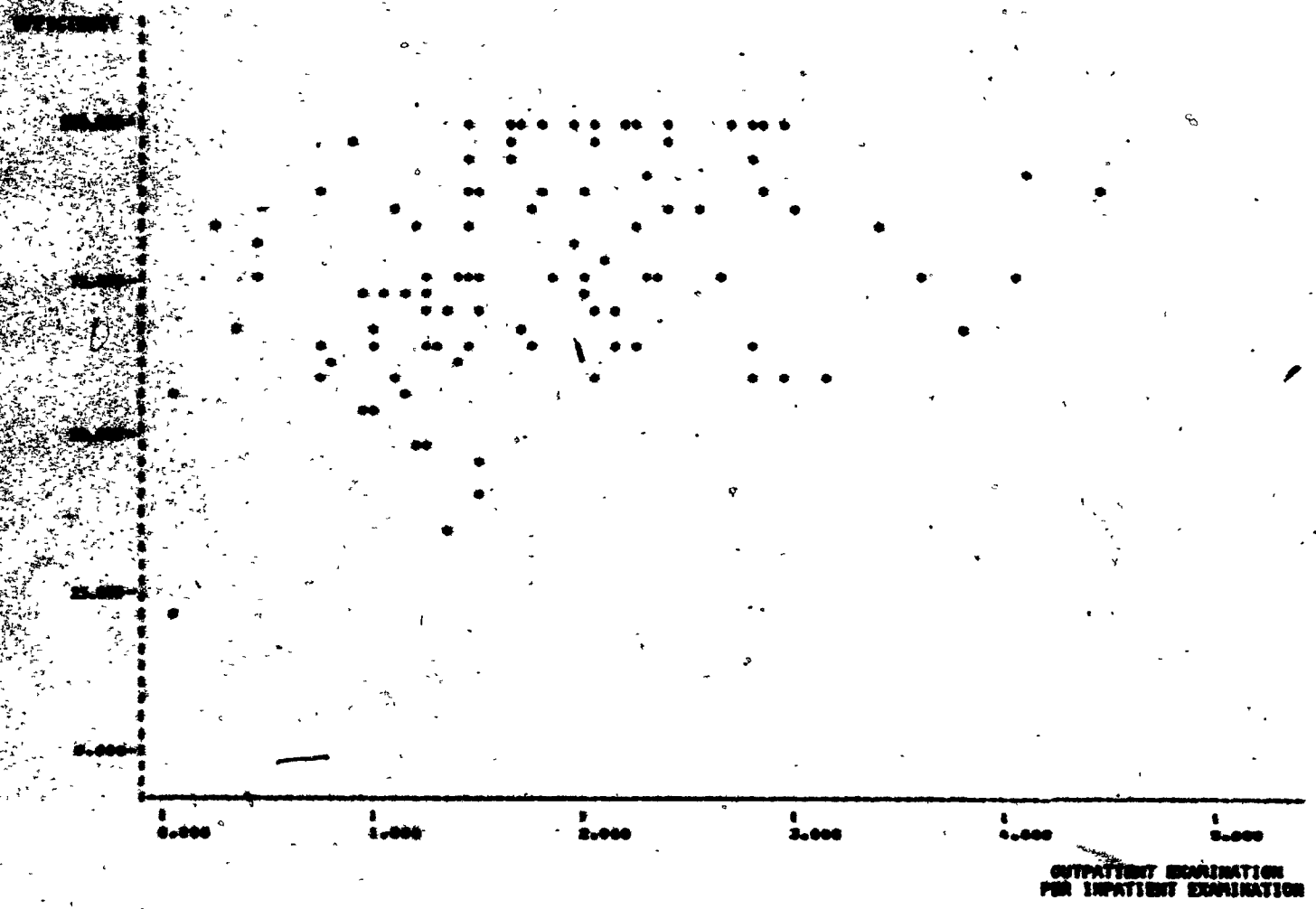


Fig. 6.6b. The technical efficiency of radiology departments as measured by model M.B versus the ratio of outpatient examinations to inpatient examinations provided by these departments.

that is, the high proportion of specialized rooms. Although the ratio of specialized to general rooms in a department can be explained in terms of hospital size, patient treatment needs, and is definitely correlated to the production of more complex procedures in large institutions, it can also be an important source of inefficiency.

In section 5.2.2 we discussed in detail how the idle capacity of specialized rooms, and hence of the equipment in them, can be recognized as an inefficiency. This does not necessarily imply an imbalance between the facilities of a department and its hospital. In all likelihood, there will be a genuine demand for these specialized facilities, yet the demand may not be great enough to fully utilize these resources. As data is not available on the utilization of particular rooms or machines, it is presently impossible to check this hypothesis.

What is most important to note, is that the largest departments are observed to be less than maximally efficient, not solely because they are large, but in part because of the demand-supply relationship with their hospitals and possibly as a result of pursuing some "nonproductive" activities. The broad spectrum of efficiencies observed for the smaller departments would indicate that there are more fundamental factors coming to play as sources of efficiency/inefficiency which may be determinant in all departments regardless of size. In addition, it is clear that the small sample size, particularly for the larger departments, makes it exceedingly difficult to draw firm conclusions except where an outstanding trend exists.

6.3.3 Choice of Explanatory Variables. One feature is common to all the "explanatory" variables that we have examined in this section; that is, a trend relationship with departmental size. Professional units, hospital beds, and the average complexity of services are directly correlated with department size. On the

other hand, the ratio of outpatient to inpatient examinations appears to be inversely correlated to size. It would seem then, that by examining similar measures we could at best be able to differentiate only the larger radiology departments in terms of technical efficiency. In addition, except for hospital beds, the remaining variables are really some function of output or what was produced.

By recognizing internal departmental and department-hospital imbalances as sources of inefficiency, we have implicitly stated that the efficiency of a department is conditioned by what it is and where it is, and that levels of output and efficiency follow as a result. In the following sections we investigate measures and indices which can be high or low independent of department size but which are based on a detailed examination of the input resources to the department and hospital. It is hoped that by this approach one can explain the differences in efficiency for all departments in a sample.

6.3.4 Technical Change. The model based sources of inefficiency have been described in terms of the assumptions underlying the models. In this section we investigate an effect of non-homogeneity in the production variables on efficiency. As described previously (see section 5.2.1), equipment years, representing the fixed capital model input to diagnostic radiology, is the greatest source of heterogeneity in the models, and as such, we shall focus our attention on this particular input.

The heterogeneity of the equipment input has two facets: i) that introduced by combining different types of equipment, and; ii) that resulting from the presence of technical change. In this section we investigate the latter variable.

We have previously outlined two components of technical change (see section 5.2.1). First, we recognized that equipment of varying age may result in dissimilar productive capabilities, both in terms of the quality and quantity of services a unit can be made to

provide. And second, there may be significant differences in the technology acquired from radiographic equipment suppliers between departments; that is, for similar types of equipment there may be some new innovation available from some manufacturers or possibly a new type of equipment altogether. Since the development of radiographic technology had stabilized within the decade preceding the year of this study (Lindheim, 1971), the age of the equipment alone will be used as representative of both of these facets.

Data set D.1, which was used as the source for the equipment data, does not provide information on the age of the machinery. Yet from data set D.3 we know the year in which the x-ray rooms were established in a department. As the diagnostic equipment is, for the most part, situated in these rooms, the mean room age per department will be substituted as a proxy for equipment age.

The room data sample was constructed from those departments on D.3 for which the room data was provided on all rooms in a department. This resulted in a sample of 61 departments. All of these were present in the sample from model M.B (consisting of 102 departments), while the room data sample corresponding to M.A (consisting of 100 departments) was reduced to 59 observations. Figs. 6.7a and b provide plots of departmental efficiency, from M.A and M.B respectively, versus the mean room age per department.

If equipment vintage is a significant factor in determining the productive efficiency of a department, we would expect that because this variable was excluded from the model and implicitly homogenized by aggregating equipment to a single measure, then the trend would be towards decreasing efficiency with increasing age. In fact this is not observed. While the merged model and room age samples comprise only 60% of their original model samples, there are a sufficient number of points present to clearly indicate the lack of correlation between efficiency and age. Although the samples are sparse, the distribution of efficiencies at all mean room

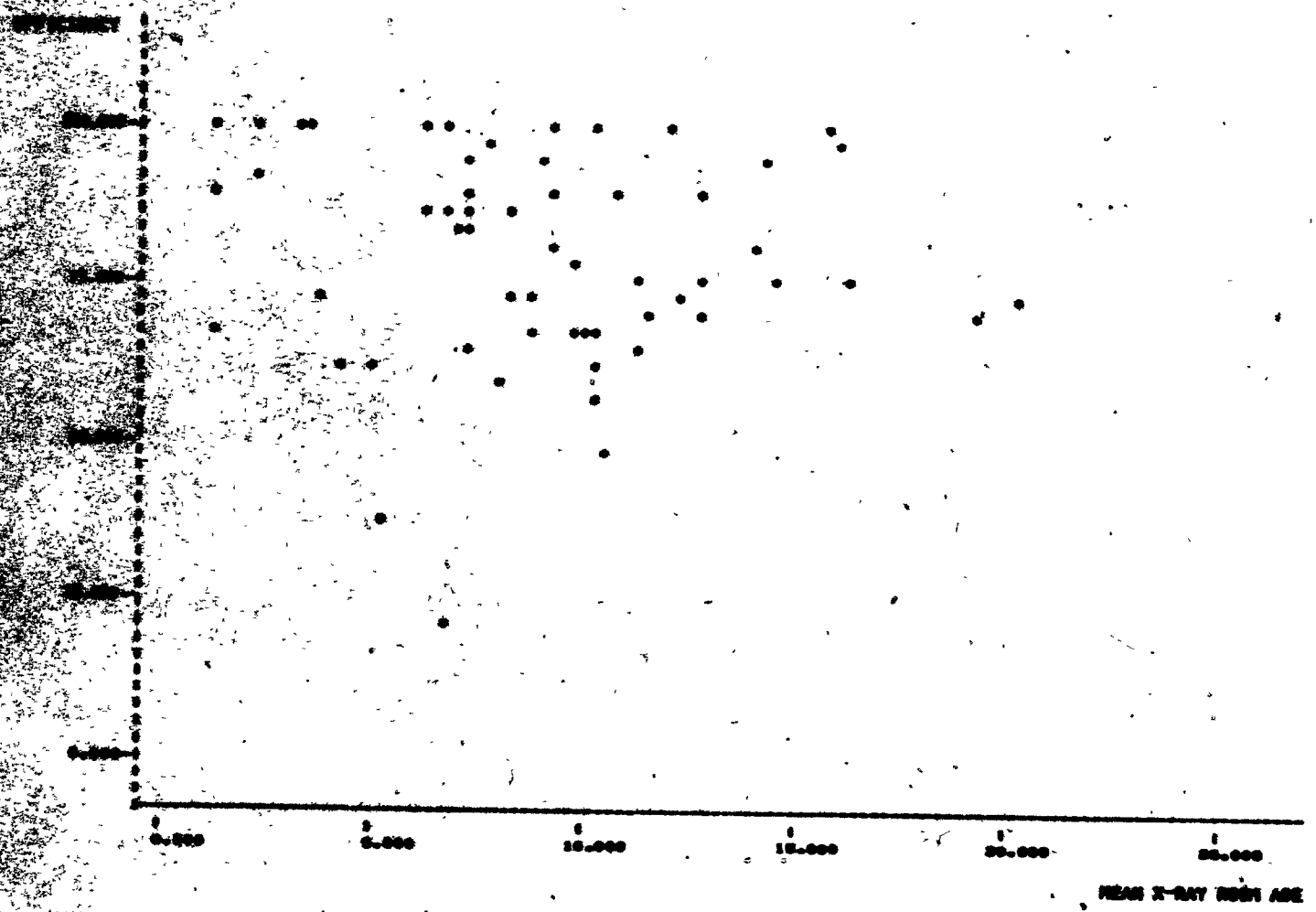


Fig. 6.7a. The technical efficiency of radiology departments as measured by model M.A versus the mean x-ray room age of those departments.

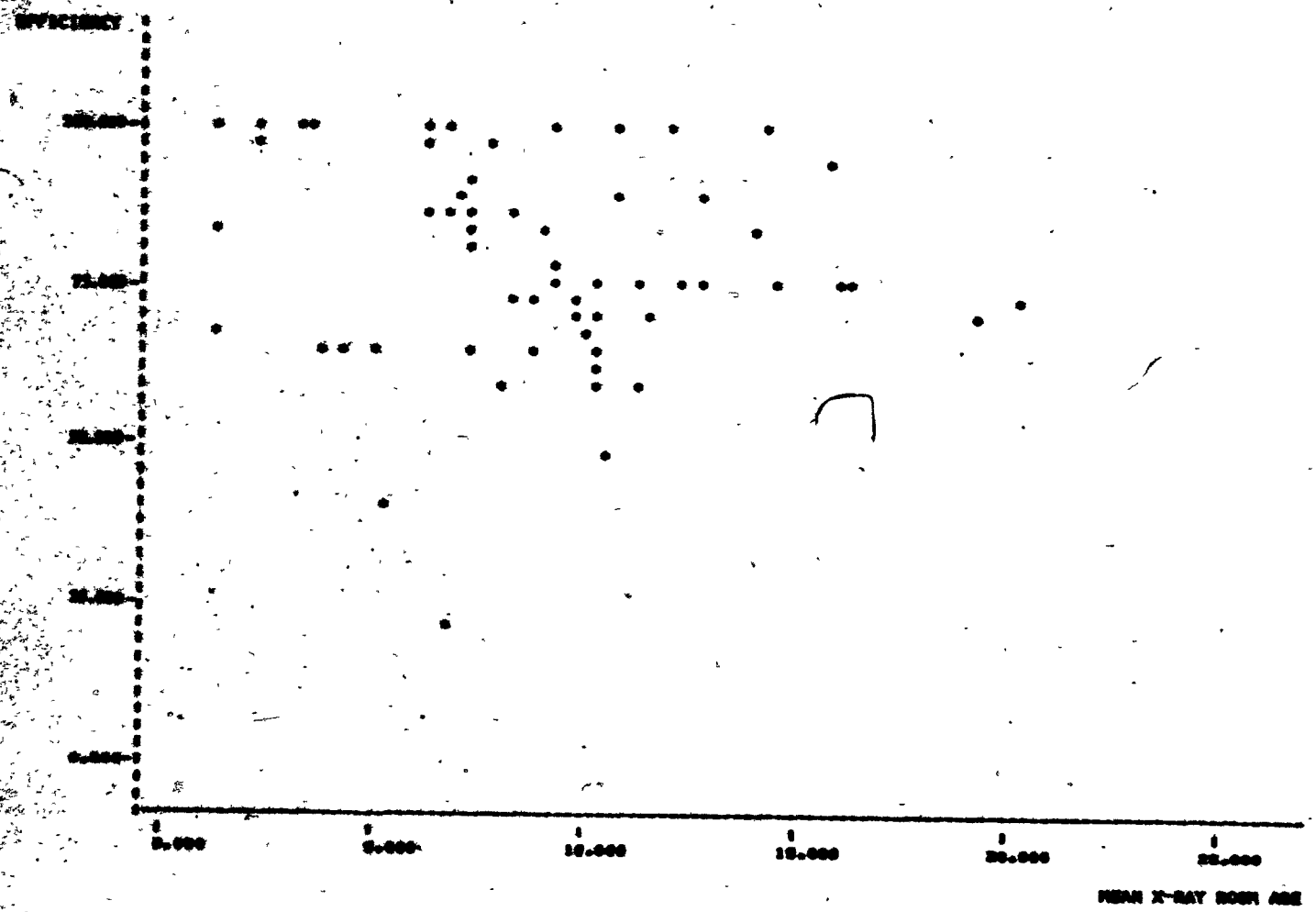


Fig. 6.7b. The technical efficiency of radiology departments as measured by model M.B versus the mean x-ray room age of those departments.

age levels appear to remain uniform, or at least lacking in any visible trends.

Thus technical change does not appear to be an explicit source of inefficiency in our models. This would imply that the heterogeneity of technical change is distributed evenly over the sample of producers.

In a later section of this chapter we investigate quantitative representations of the first facet of equipment input heterogeneity, i.e. the aggregation of different equipment types, and subsequently examine their correlation with technical efficiency.

6.4 The Need for a New Approach to Examining the Data

We have hypothesized that the technical efficiency of a radiology department is largely conditioned by both the resources of the department and the relationship between the department and its surrounding hospital environment. Hence we attribute the technical (in)efficiency of a department to an (im)balance in the demand-supply relationship for radiographic services and the (non) coordination of departmental resources in production. Easily said - now what of the actual quantification of these variables?

We will focus our attention on only the inputs to a radiology department and hospital. The reason for doing so is three-fold: i) the relationship between the resource inputs and the output for a department was explicitly examined in the production models; ii) the idea of departmental indices is proposed to help explain the technical efficiency (which is a function of the output), and so it would be fruitless to explain an output measure as a function of the output, and; iii) we have previously agreed that the technical efficiency is determined by the inputs to the system (department-hospital), as is the case for the observed output.

Consider the problem of developing an index of a radiology department. A first approach would naturally be to use the same inputs as were introduced to the production models, and investigate

whether some input combinations result in higher efficiency than others. Examining the technique used to derive the production models would indicate this to be a poor choice.

Fig. 6.8 shows a typical production function (RR') derived from a sample of producers and the corresponding frontiers of 80% (SS') and 60% (TT') efficiency. Any ray from the origin, such as OA or OB, represents a different quantitative combination of inputs, regardless of output. It is easily seen that all possible combinations of inputs can be observed at any level of efficiency. Hence departments which have significantly different input profiles can be observed to have the same level of technical efficiency. In point of fact, this is one of the major strengths of our technique to identify a production function. Thus examining the inputs used in the models can be expected to provide little or no explanatory information with respect to the technical efficiencies.

Recognizing that the model inputs were constructed as aggregate measures, the next logical step would be to examine their component parts. Consider only the equipment input. We recall that this input was based on 6 different types of radiographic machinery, and that the total number of different types of equipment that we can observe per department on D.1 is 13 (see section 5.2.1). Given that equipment is but one of the resources we would want to quantize, and even if we focused our attention to the original six types, there would in all be to many variables to consider individually relative to the model samples - approximately 100 observations per model. Thus we must reduce the number of explanatory variables, but in a different way than simply aggregating.

If there were only one type of diagnostic machinery in radiology departments, it would be a simple matter to compare the equipment input between departments. As the number of different equipment types increases, a comparison of departmental equipment profiles becomes increasingly difficult. Consider just two departments and thirteen equipment types. Even when the total volume of machines is the same, it is unlikely that the numbers in

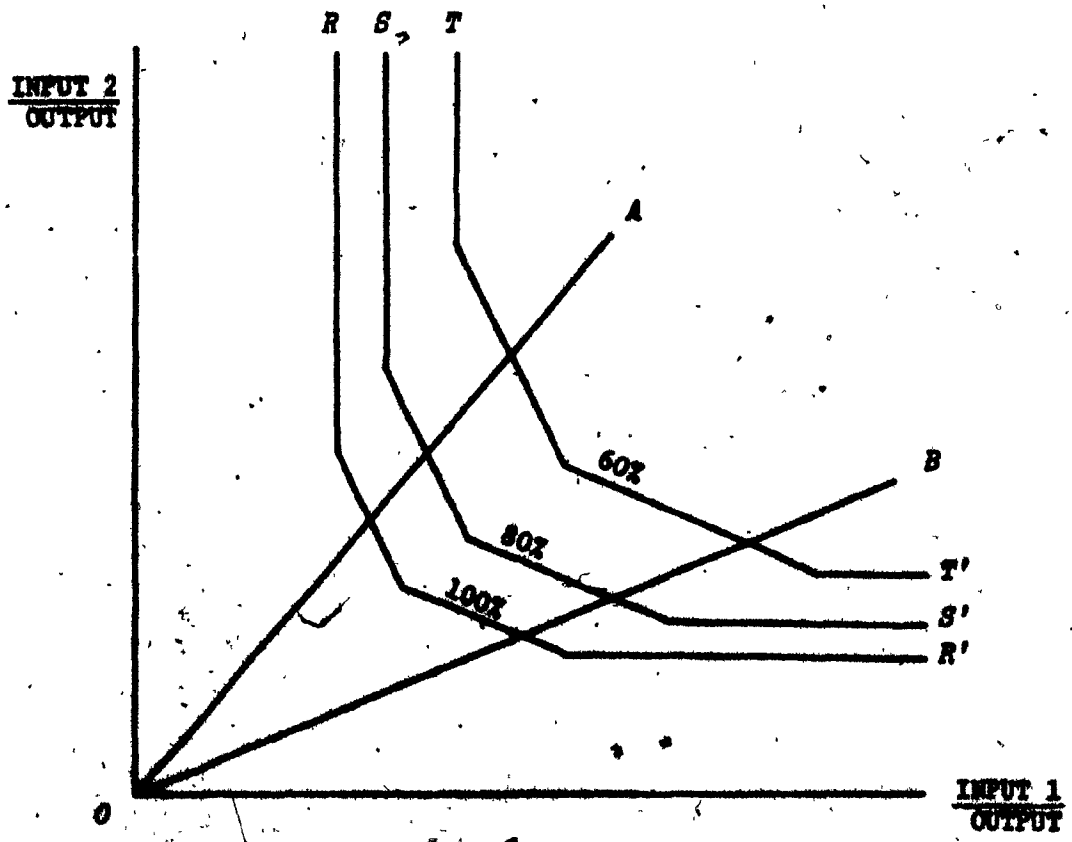


Fig. 5.1. A typical production function derived from a sample of producers using the linear programming identification technique, as well as the frontiers at 80% and 60% efficiency. This illustration demonstrates that the technical efficiency of decision-making is independent of quantity input combinations.

each category will be equal. In fact it is plausible that not one category of equipment would be common to both departments.

The fundamental question is one of interpretation. Given the number of machines in each of 13 categories in a department, what meaning can be attached to this profile, either in the context of that department or relative to all departments in the province? By examining the resources of a department we receive information, but what is the nature and value of that information, and is it relevant to explaining the technical efficiency?

6.5 An Information Theoretic Approach to Quantizing the Resource Data

The indices we propose in this section were originally developed in the field of communications to monitor the transmission of information. The idea of quantizing information was first proposed by Shannon (1964). Theil (1967) used a similar mathematical framework in an economic context for the measurement of income inequality and industrial concentration. Evans and Walker (1972) applied information measures to the diagnostic case mix in hospitals to help explain hospital costs.

The exposition that follows is alternatively based on the developments of Theil (1967, Chap. 2) and Evans & Walker (1972, pp. 399-402). After presenting a theoretical introduction we propose: i) a set of five indices based on the input resources to a department which is derived from the work of Evans & Walker, and which are used to explain the technical efficiency of a radiology department in a later section, and; ii) an application of the information theoretic approach to the monitoring of technological resources allocation in the health sector, and which is derived from the work of Theil. The reader who is familiar with information theory can proceed directly to section 6.5.2.

6.5.1 The Entropy Concept. Consider a set of n events,

$$E_1, E_2, \ldots, E_n$$

where we are certain that exactly one event will occur. Corresponding

to each event we know the probability that it will occur. We denote these probabilities as,

$$p_i \qquad i = 1, 2, \ldots n$$

and

$$\sum_{i=1}^{n} p_i = 1 \quad p_i \geq 0$$

What is the information content of a message which states that E_i has occurred. If p_i is small, we would obviously be surprised, and thus the message will provide a large amount of information. Conversely, if p_i is large, we would expect E_i to occur, and hence the information content of the message is small. We define the information content of the message that E_i has occurred as $h(p_i)$, where

$$h(p_i) = -\log p_i = \log \frac{1}{p_i} \tag{6.9}$$

This expression demonstrates the desired properties of an information measure, i.e.,

(a) $h(p_i) \geq 0$ for $0 \leq p_i \leq 1$

(b) $p_i \to 0 \Rightarrow h(p_i) \to \infty$

(c) $p_i \to 1 \Rightarrow h(p_i) \to 0$

Before the message is received we do not know how much information it will provide, yet we can compute an expected information content. The message that a particular event will occur has the same probability

as the event itself, and therefore the expected information content is,

$$H(p) = \sum_{i=1}^{n} p_i h(p_i) = \sum_{i=1}^{n} p_i \log \frac{1}{p_i} = -\sum_{i=1}^{n} p_i \log p_i \qquad (6.10)$$

We define $p_i \log p_i$ to be identically zero if $p_i = 0$, and therefore $H(p)$ is non-negative. The $H(p)$ of a distribution is frequently called the entropy of that distribution (Theil, 1967).

If $p_i = 1$ and $p_j = 0$ for $j \neq i$, then we know that the i-th event will occur with certainty, and hence the expected information from a message about which event has occurred would be nil. Substituting these values in Eqn. 6.10 results in $H(p)$ being zero. If $p_i = 1/n$ for all i, then we are faced with a situation of maximum uncertainty; that is, all possible outcomes are equally likely. In this instance $H(p)$ is equal to $\log n$, which can be shown to be the maximum value of $H(p)$ for a distribution of n entities (Theil, 1967, p. 25). This is quite reasonable, as any message which describes what has occurred will provide more information if all events are equiprobable than if some are more probable than others, and therefore,

$$0 \leq H(p) \leq \log n$$

Consider a message which does not indicate which event has occurred, but implies that the probabilities of event occurrence have changed. If we label the p_i as prior probabilities, then this message will transform them into posterior probabilities q_i, where,

$$\sum_{i=1}^{n} q_i = 1 \qquad q_i \geq 0 \qquad i = 1, 2, \ldots n$$

If $q_i = 1$ and $q_j = 0 \quad j \neq i$, then this is equivalent to the message that the i-th event has occurred, or at least, will occur with certainty, and is the same as the type of message we initially introduced.

The expected information content of the message providing the q_i's is easily derived if we first consider a single event, E_i.

if p_i = probability of E_i before the message is received

and q_i = probability of E_i after the message is received

then $h(p_i) - h(q_i)$ = information received with the message

$$h(p_i) - h(q_i) = \log \frac{q_i}{p_i} \qquad (6.11)$$

If the message is that event E_i has occurred, i.e. $q_i = 1$, then as expected, Eqns. 6.9 and 6.11 are identical. The message which transforms the prior probabilities into posterior probabilities concerns n events. Therefore the expected information of this message is:

$$H(q:p) = \sum_{i=1}^{n} q_i \log \frac{q_i}{p_i} \qquad (6.12)$$

It can be proved that the expression in Eqn. 6.12 is always non-negative (Theil, 1967, p. 28). If the message states that the probabilities have not changed, i.e. if $q_i = p_i$ for all i, then we are provided with no information and appropriately $H(q:p)$ equals zero. When an event that was labelled impossible becomes possible, i.e. $q_i > p_i = 0$ for some i, we are provided with a large amount of information and in fact would be "infinitely surprised by the message" (Theil, 1967). Correspondingly the value of $H(q:p)$ becomes infinitely large.

One final example may prove illuminating. If we are completely ignorant initially then we can assume that all events are equally likely,

$$p_i = 1/n \qquad i = 1, 2, \ldots n$$

from which, $H(q:p) = \sum_{i=1}^{n} q_i \log q_i n = \sum_{i=1}^{n} q_i \log q_i + \sum_{i=1}^{n} q_i \log n$

$$= \log n - \sum_{i=1}^{n} q_i \log \frac{1}{q_i}$$

$$H(q:p) = \log n - H(q)$$

Therefore the expected information of receiving a message on the probability of events, given that nothing is known *a priori* is equal to the difference between the maximum and actual entropies of the distribution described by the message.

6.5.2 Accounting for Resource Mix Differences. Evans & Walker (1972) extended Theil's (1967) use of information theory to develop five different hospital measures defined on the hospital patient population diagnostic proportions, as an approach to adjusting for differences in diagnostic mix between hospitals. In this section we focus our attention on the equipment mix in diagnostic radiology departments and investigate a similar set of five indices. It will become apparent that this information theoretic approach is applicable to almost any entity which can be broken down as to type and location.

Let n = number of diagnostic radiology departments in the province

m = number of diagnostic radiology equipment classifications

e_{ij} = number of pieces of equipment of type j in department i

E_i = total number of pieces of equipment in department i

E_j = total number of pieces of equipment type j in all departments

E = total number of pieces of equipment in all departments

therefore, $E_i = \sum_{j=1}^{m} e_{ij}$ $\qquad$ $E_j = \sum_{i=1}^{n} e_{ij}$

$$E = \sum_{i=1}^{n} \sum_{j=1}^{m} e_{ij}$$

We can now define,

$q_{ij} = \frac{e_{ij}}{E_j}$ = the proportion of the total provincial volume of type j equipment situated in department i

$p_{ij} = \frac{e_{ij}}{E_i}$ = the proportion of the total volume of equipment in department i of the j-th type

where $\sum_{i=1}^{n} q_{ij} = 1$ $\qquad$ $\sum_{j=1}^{m} p_{ij} = 1$

$q_{i\cdot} = \frac{E_i}{E}$ = proportion of all machines in the province situated in department i

$q_{\cdot j} = \frac{E_j}{E}$ = proportion of all machines in the province [illegible]

In section 5.2.1 we hypothesized a relationship between the resources of a radiology department and its output capabilities. The smaller departments have less extensive facilities and are seen to concentrate on more general procedures and possibly some specialized ones. The larger departments can produce a greater volume of general procedures, in addition to a proportionately larger volume of specialized procedures. Hence, smaller departments can be seen to "specialize" in the simpler types of examinations, whereas the large departments, being more versatile, are less specialized in their orientation with respect to all types of procedures.

In this section we use information theory to derive measures of the resource specialization of departments. In terms of the above comments, the smaller departments will be seen as more specialized in having lesser volumes of only some types of machines, while the departments with more extensive facilities, both in the spectrum of types and their volumes, will be less specialized. Hence an almost inverse relationship exists between the specialization of resources and the volume and/or breadth of services.

$$\text{Let } H_j = \sum_{i=1}^{n} q_{ij} \log \frac{1}{q_{ij}} \qquad j = 1, 2, \ldots m \qquad (6.13)$$

Then H_j is the entropy of the distribution of a particular type of equipment amongst all departments. This entropy measure has two interesting properties:

i) if $q_{ij} = 1/n \quad i = 1, 2, \ldots n$

This implies complete equality in the distribution of the j-th type of equipment, i.e. all departments have the same volume of machines of type j, and,

$H_j = \log n$

ii) if $q_{ij} = 1 \quad q_{kj} = 0 \quad \forall k \neq i \quad k = 1, 2, \ldots n$

This implies complete inequality in the distribution of the j-th type of machine, i.e. all machines of this type are concentrated in one department, and, $H_j = 0$

Therefore H_j is a measure of the equality in the distribution of the j-th type of diagnostic equipment amongst all departments.

6.5.2.1 Departmental Indices Derived from the Inter-Department Resource Distribution. In order to quantize the specialization of facilities in a department, one approach would be to first examine the specialization of the individual resources from a global perspective. On the assumption that more specialized types of equipment will be concentrated in a few departments, while less specialized equipment will be more evenly distributed, it follows that the above entropy (see Eqn. 6.13) is a measure of non-specialization.

$$\text{Let} \quad H_j^1 = \log n - H_j = \sum_{i=1}^{n} q_{ij} \log nq_{ij} = \sum_{i=1}^{n} q_{ij} \log \frac{q_{ij}}{1/n} \tag{6.14}$$

H_j^1 is a measure of inequality. We shall defer a detailed discussion of H_j^1 as a measure of equipment specialization to the next section. To see that H_j^1 is a valid index of inequality we note the following three properties described by Theil (1967, p. 92-93).

i) We require that the measure should not change when all departments increase their volume of the j-th type of equipment by the same proportion. This results because the q_{ij} will not change.

ii) It is desirable that the upper limit of H_j^1, i.e. $\log n$, should increase with the number of departments. Consider the following examples:

a) If there are only two departments and all equipment of the j-th type is in one of them, then,

$$H_j^1 = \log 2$$

b) If there are 200 departments and all equipment of the j-th type is located in only one of them, then,

$$H_j^1 = \log 200$$

This second case is obviously more unequal than the first, and this is reflected in the difference in the H_j^1's.

c) If a set S of n' departments, $0 < n' \leq n$, have all the equipment of the j-th type, and each department in S has the same share, i.e. $1/n'$ then

$$H_j^1 = \sum_{i=1}^{n} q_{ij} \log nq_{ij} = \sum_{i \in S} q_{ij} \log nq_{ij} = \log n - \sum_{i \in S} q_{ij} \log \frac{1}{q_{ij}}$$

$$\sum_{i \in S} q_{ij} \log \frac{1}{q_{ij}} = \log n'$$

therefore $$H_j^1 = \log \frac{n}{n'}$$

This case is as unequal as example a) above when the same proportion of departments equally share the total volume of the j-th type of equipment.

$$\text{if} \qquad n = 200 \qquad \text{and} \quad n' = 100 \text{ then}$$

$$\log \frac{200}{100} = \log \frac{2}{1} = \log 2$$

iii) A third desired property can be described as follows: if $q_{kj} > q_{lj}$ and suppose that q_{lj} increases at the expense of q_{kj} such that $q_{kj} + q_{lj}$ is constant, i.e. equipment is transferred from one department to another, where all other q_{ij} are unchanged, then H_j^1 will show a decrease until $q_{kj} = q_{lj}$ and increase thereafter. This result is proved by Theil (1967, p. 93-94). Thus H_j^1 will increase the more concentrated is equipment of type j among departments. In order to meaningfully compare entropies for different types of equipment we can standardize all measures (Evans & Walker, 1972). The expected or mean value of H_j^1 is,

$$\sum_{i=1}^{n} H_j^1 \frac{E_j}{E} = \sum_{j=1}^{m} H_j^1 Q_j$$

It follows that the standardized form of H_j^1 is then,

$$H_{jN}^1 = \frac{H_j^1}{\sum_{j=1}^{m} H_j^1 Q_j} \qquad j = 1, 2, \ldots m$$

and thus the average equipment distribution inequality over all departments is,

$$\sum_{j=1}^{m} \bar{H}_j^{I} Q_j = 1$$

We now define the mean equipment or resource mix inequality for department i as the sum of the entropies of each type of equipment weighted by the proportion of department i's equipment load falling into the corresponding equipment category, that is,

$$X_i^{1} = \sum_{j=1}^{m} \bar{H}_j^{1} p_{ij} \qquad i = 1, 2, \ldots n \qquad (6.15)$$

By examining Eqn. 6.14 we see that H_j^{1} will be large if there are departments of very different sizes in the sample and the j-th type of equipment is distributed across all departments in exactly the same way as the total volumes. From a theoretical perspective the above entropies are based on prior probabilities of $1/n$ and posterior probabilities of q_{ij}. To account for differences in departmental size we introduce the proportion of all equipment in the i-th department, i.e. P_i, as the prior probabilities. We then have that,

$$\sum_{i=1}^{n} q_{ij} = \sum_{i=1}^{n} P_i = 1$$

$$\bar{H}_j^{1} = \sum_{i=1}^{n} q_{ij} \log \frac{q_{ij}}{P_i} \qquad j = 1, 2, \ldots m$$

(6.16)

H_j^2 is "sensitive to deviations in the q_{ij} away from P_i in either direction, more or less concentrated" (Evans & Walker, 1972). "If some case (*equipment*) types are more concentrated than average, it follows that others will be less concentrated" (Evans & Walker, 1972), and hence H_j^2 is a measure of inequality relative to the size (total machine volume) distribution. By a similar manipulation as above we can derive a measure per department which reflects the mean resource mix inequality after accounting for size differences. That is,

$$X_i^2 = \sum_{j=1}^{m} \overline{H}_j^2 \; p_{ij} \qquad i = 1, 2, \ldots, n \tag{6.17}$$

If the equipment for a radiology department were chosen in a random fashion, then X_i^1 and X_i^2 could be very different, yet because there must be some pattern to the growth of a department we might expect them to be similar. An example is provided in the next section.

6.5.2.2 Departmental Indices Derived from the Intra-Department Resource Distribution. As yet, we have only used the q_{ij} structure to define two different measures per equipment category, reflecting inequality in the distribution of equipment across all departments, i.e. the specialization of equipment. These were then used to develop indices of the mean equipment mix inequality or equipment specialization per department. Using the rows of the p_{ij} matrix we can directly derive three indices on the specialization of equipment in a department by utilising the following three assumptions (Evans & Walker, 1972):

i) there are equal numbers of machines in each category of equipment,

$$\text{prior distribution} = 1/m$$

ii) the proportion of machines in the j-th category in a department is the same as the proportion of all machines of the j-th type,

$$\text{prior distribution} = Q_j$$

iii) the proportion of machines of the j-th category in a department is the same as the proportion of the same type of machine in the "average" department,

$$\text{prior distribution} = Q_j^1 = \sum_{i=1}^{n} \frac{p_{ij}}{n}$$

In analogy to Eqns. 6.14 and 6.16 we present the following indices per department,

$$G_i^1 = \sum_{j=1}^{m} p_{ij} \log p_{ij} m \qquad i = 1, 2, \ldots n$$

$$G_i^2 = \sum_{j=1}^{m} p_{ij} \log \frac{p_{ij}}{Q_j} \qquad i = 1, 2, \ldots n \qquad (6.18)$$

$$G_i^3 = \sum_{j=1}^{m} p_{ij} \log \frac{p_{ij}}{Q_j^1} \qquad i = 1, 2, \ldots n$$

The five equipment measures from Eqns. 6.15, 6.17 and 6.18, have been interpreted as reflecting the specialization of equipment in each department. If we re-examine how the equipment input to the production models was constructed then another interpretation is admitted. Consider only two departments. If the volume of machines in both departments (for the first six equipment categories) are the same then then their model equipment inputs will be equal. From Eqns. 6.15, 6.17 and 6.18 we see that the information indices are ultimately dependent on the p_{ij} structure, and hence on the mix of equipment per department. Therefore these information measures for the two departments will be different unless their mixes are identical. And thus these indices, particularly the G_i^k , reflect the degree of heterogeneity of the equipment input.

Recall that if the production model input variables differ by a positive constant, then they are both members of the same production process. If the volumes of each type of machine in these departments differ by the same constant, then their p_{ij} will be the same, and hence their indices will be equal. Yet it is possible that the mixes of equipment will not be in the same proportion as the total equipment inputs, in which case the indices would not be equal. Hence the information based measures provide a means of differentiating between departments on the basis of the departmental technology mix, at a level that could not be integrated into the development of the production models.

6.5.3 An Example. In order to illustrate calculation of the information measures and further interpret their meaning, we have investigated an example. The above scheme is applied to the equipment mix of departments as represented on data set D.1. We have chosen to consider only the three categories of stationary diagnostic equipment (see Table 5.2), rather than the complete spectrum of thirteen types. This was done solely to make the final interpretation of results more manageable.

A sample of 35 departments was chosen. Each department and the corresponding number of machines in each of the three categories is provided in Table 6.3. Department 33 is absent as its equipment input on D.1 was indicated to be zero for all types. This, once again, reflects a data collection error, rather than a production characteristic. Therefore $n = 35$ and $m = 3$. Tables 6.4 and 6.5 respectively provide the q_{ij} and p_{ij} matrices for this sample and Table 6.6 lists the prior distributions used in Eqns. 6.15, 6.17 and 6.18.

The unstandardized and standardized values of H_j^1 and H_j^2 for each type of equipment are presented in Table 6.7. In both instances, standardization results from dividing all terms by a constant, and therefore the relative values for each category of equipment remain the same. The values of H_j^1 and $\overline{H}_j^1$ indicate that the greatest distribution inequality is for the second type of equipment, followed by the first and then the third. From Table 6.4 we see that 72% of the total volume of equipment of type 2 for all 35 departments is contained in only 8 departments, or 23% of the sample. The largest 8 departments with respect to equipment type 1 contain 65% of the total volume of that type of equipment and similar analysis for equipment type 3 indicates that 59% of the total volume is located in 8 departments.

We have previously ascribed a specialization interpretation to the H_j^1 entropy measures. From Table 5.2 we know that the first equipment category encompases general stationary machines, whereas the machines in the other two categories are equipped for fluouroscopy. This would seem to indicate that the first type of equipment is less specialized than the others, which in turn contradicts our results. Hence, the real question is: what is meant by specialized, is it a concept relating to function or distribution?

It should be remembered that although we have recognised three categories of equipment, this does not imply that each can provide only a unique set of services. Rather, some are more

DEPT	$^{\circ}_{11}$	$^{\circ}_{12}$	$^{\circ}_{13}$
1	0.0	0.0	1.000
2	0.0	1.000	0.0
3	1.000	5.000	0.0
4	6.000	10.000	5.000
5	13.000	1.000	6.000
6	0.0	0.0	2.000
7	1.000	1.000	0.0
8	7.000	2.000	5.000
9	3.000	1.000	2.000
10	12.000	10.000	1.000
11	0.0	1.000	0.0
12	5.000	0.0	1.000
13	2.000	0.0	2.000
14	10.000	3.000	7.000
15	1.000	0.0	1.000
16	0.000	1.000	5.000
17	0.0	2.000	0.0
18	6.000	4.000	1.000
19	0.000	1.000	4.000
20	1.000	2.000	0.0
21	4.000	0.000	0.0
22	10.000	2.000	2.000
23	0.0	0.0	1.000
24	2.000	0.0	2.000
25	0.0	0.0	1.000
26	0.0	1.000	0.0
27	0.0	1.000	0.0
28	1.000	0.0	2.000
29	3.000	3.000	2.000
30	1.000	0.0	1.000
31	1.000	0.0	2.000
32	1.000	1.000	0.0
33	4.000	3.000	2.000
34	2.000	1.000	0.0
35	2.00	0.0	1.000

Table 4.3. A count of 35 diagnostic radiology departments and the corresponding values of equipment in each of the three categories of auxiliary radiographic machinery.

DEPT	q_{i1}	q_{i2}	q_{i3}
1	0.0	0.0	0.016
2	0.0	0.015	0.0
3	0.009	0.075	0.0
4	0.053	0.149	0.049
5	0.114	0.015	0.098
6	0.0	0.0	0.033
7	0.009	0.015	0.0
8	0.041	0.030	0.082
9	0.026	0.015	0.033
10	0.105	0.149	0.016
11	0.0	0.015	0.0
12	0.044	0.0	0.049
13	0.018	0.0	0.033
14	0.088	0.045	0.115
15	0.009	0.0	0.016
16	0.070	0.015	0.082
17	0.0	0.030	0.0
18	0.070	0.060	0.016
19	0.053	0.015	0.066
20	0.009	0.030	0.0
21	0.035	0.119	0.0
22	0.088	0.030	0.033
23	0.0	0.0	0.016
24	0.018	0.0	0.033
25	0.0	0.0	0.016
26	0.0	0.015	0.0
27	0.0	0.015	0.0
28	0.026	0.0	0.033
29	0.026	0.045	0.033
30	0.009	0.0	0.049
31	0.009	0.0	0.033
32	0.009	0.015	0.0
34	0.035	0.075	0.033
35	0.009	0.015	0.0
36	0.0	0.0	0.066

Table 6.4. The q_{ij} matrix corresponding to the e_{ij} structure in Table 6.3.

DEPT	p_{i1}	p_{i2}	p_{i3}
1	0.0	0.0	1.000
2	0.0	1.000	0.0
3	0.167	0.833	0.0
4	0.316	0.526	0.158
5	0.650	0.050	0.300
6	0.0	0.0	1.000
7	0.500	0.500	0.0
8	0.500	0.143	0.357
9	0.500	0.167	0.333
10	0.922	0.435	0.043
11	0.0	1.000	0.0
12	0.625	0.0	0.375
13	0.500	0.0	0.500
14	0.500	0.150	0.350
15	0.500	0.0	0.500
16	0.571	0.071	0.357
17	0.0	1.000	0.0
18	0.615	0.308	0.077
19	0.545	0.091	0.364
20	0.333	0.667	0.0
21	0.333	0.667	0.0
22	0.714	0.143	0.143
23	0.0	0.0	1.000
24	0.500	0.0	0.500
25	0.0	0.0	1.000
26	0.0	1.000	0.0
27	0.0	1.000	0.0
28	0.600	0.0	0.400
29	0.375	0.375	0.250
30	0.250	0.0	0.750
31	0.333	0.0	0.667
32	0.500	0.500	0.0
34	0.364	0.455	0.182
35	0.500	0.500	0.0
36	0.0	0.0	1.000

Table 6.5. The p_{ij} matrix corresponding to the e_{ij} structure in Table 6.3.

$1/n = 0.029 \quad 1/m = 0.333$

j	1	2	3
Q_j	0.471	0.277	0.252
Q_j^1	0.338	0.331	0.332

$(DEPT_i, P_i) \quad i = 1,2,\ldots,n$

1 0.004	2 0.004	3 0.025	4 0.079	5 0.083
6 0.008	7 0.008	8 0.058	9 0.025	10 0.095
11 0.004	12 0.033	13 0.017	14 0.083	15 0.008
16 0.058	17 0.008	18 0.054	19 0.045	20 0.012
21 0.050	22 0.058	23 0.004	24 0.017	25 0.004
26 0.004	27 0.004	28 0.021	29 0.033	30 0.017
31 0.012	32 0.008	34 0.045	35 0.008	36 0.004

Table 6.6. The prior distributions corresponding to the information measures found in Eqns. 6.15 ($1/n$), 6.17 (P_i), and 6.18 ($1/m$, Q_j, & Q_j^1) for the sample of departments described in Table 6.3.

EQUIPMENT TYPE	H^1	H^2	
1	0.656	0.068	UNSTANDARDIZED
2	0.790	0.438	
3	0.365	0.356	
1	0.979	0.350	STANDARDIZED
2	1.179	1.734	
3	0.843	1.409	

Table 6.7. The unstandardized and standardized values of H_j^1 and H_j^2 for each of the three types of machinery equipment. (Table 6.4).

versatile than others. On examining Table 6.3 we see that many of the smaller departments (in terms of the total number of machines) do not have any of the first type of equipment, whereas without exception, all departments which have some machines in the first category have at least one machine of either or both of the latter two types. Hence it appears that for this sample of departments, having equipment of the first type is only profitable in conjunction with other types of equipment, and thus are situated in larger departments.

We should note that this sample of 35 departments is not representative of the complete sample of 139 departments because of the large proportion of larger departments. If many of the departments which have been excluded from this sample did have equipment of the first type, then increasing the sample size would tend to introduce more intermediate and smaller departments, in which case we would expect H_1^1 to decrease relative to H_2^1 and H_3^1. If the same pattern exists for the complete sample as for this subset of departments, then H_1^1 would be expected to increase. Thus, the H_j^1 index surely provides a distributional concept of specialization, though not necessarily a functional one.

Examining the H_j^2 , which take into account variations in department size, we see the expected pattern emerges in terms of a functional concept of specialization, but with a distributional quality present as seen by the difference between H_2^2 and H_3^2.

Table 6.8 provides the values of X_i^1, X_i^2, G_i^1, G_i^2, and G_i^3, for each of the 35 departments in the sample. By comparing these results with Table 6.3, it might appear that small departments have more specialized resource mixes than larger ones. Yet as indicated previously, these indices are independent of the actual volume of machines or department size, but rather, are dependent on the mix, i.e. the p_{ij} . Hence the trend we observe is that as departments increase in size, resources are acquired in a more balanced manner. This would seem reasonable. Producing

DEPT	X_i^1	X_i^2	a_i^1	a_i^2	a_i^3
1	0.843	1.489	1.099	1.378	1.104
2	1.179	1.734	1.099	1.284	1.106
3	1.146	1.503	0.648	0.745	0.652
4	1.063	1.245	0.105	0.138	0.106
5	0.948	0.737	0.308	0.176	0.301
6	0.843	1.409	1.099	1.378	1.104
7	1.079	1.042	0.405	0.325	0.403
8	0.959	0.926	0.106	0.060	0.103
9	0.967	0.934	0.087	0.038	0.084
10	1.060	0.998	0.261	0.173	0.258
11	1.179	1.734	1.099	1.284	1.106
12	0.928	0.747	0.437	0.324	0.431
13	0.911	0.880	0.405	0.372	0.402
14	0.961	0.928	0.100	0.053	0.097
15	0.911	0.880	0.405	0.372	0.402
16	0.949	0.827	0.223	0.138	0.218
17	1.179	1.734	1.099	1.284	1.106
18	1.030	0.857	0.268	0.186	0.235
19	0.948	0.861	0.182	0.112	0.178
20	1.112	1.272	0.462	0.471	0.463
21	1.112	1.272	0.462	0.471	0.463
22	0.988	0.699	0.302	0.122	0.295
23	0.843	1.409	1.099	1.378	1.104
24	0.911	0.880	0.405	0.372	0.402
25	0.843	1.409	1.099	1.378	1.104
26	1.179	1.734	1.099	1.284	1.106
27	1.179	1.734	1.099	1.284	1.106
28	0.925	0.774	0.426	0.330	0.420
29	1.020	1.134	0.816	0.826	0.814
30	0.877	1.144	0.536	0.659	0.537
31	0.888	1.096	0.462	0.533	0.461
32	1.079	1.042	0.405	0.325	0.403
33	1.048	1.171	0.862	0.872	0.862
34	1.079	1.042	0.405	0.325	0.403
35	0.843	1.409	1.099	1.378	1.104

Table 6.6. The values of X_i^1 (Eqn. 6.15), X_i^2 (Eqn. 6.17), a_i^1, a_i^2, & a_i^3 (Eqn. 6.18) based on three equipment categories for each of the 35 departments in the sample (Table 6.3).

a large output volume would allow delegating many of the procedures to less versatile machinery. This is a luxury which smaller production units could not afford in attempting to maximize the utilization of all their machinery.

To illustrate the interpretation of the resulting information measures, we shall focus on departments 1 to 5 in Table 6.8. The X_i^1, and X_i^2 are calculated as weighted sums of $\overline{H}_j^1$ and $\overline{H}_j^2$ respectively (see Table 6.5). The independence of these first two indices from the total volume of machines (see Table 6.3), is easily seen by comparing the X_i^1 and X_i^2 values of departments 1 and 2 and of departments 4 and 5. In addition, we see that by comparing departments 1 and 3, an imbalance in the mix of a large volume of equipment can lend for a greater mean resource mix inequality than a smaller, more concentrated, equipment volume. Hence, these indices account for both the degree of imbalance between resources, and the nature of the imbalance in terms of the scarcity of the resources composing the mix.

The G_i^1 are strictly a function of the imbalance of resources. From Table 6.5 we see that G_1^1 must equal G_2^1, and it is not surprising that G_4^1 is the lowest of the first five G_i^1 (Table 6.3). The G_i^3 values are very similar to the G_i^1 values because the mix of the average department is almost the same as the prior distribution of equivalent proportions in each class. Compare the Q_j^1 and $1/m$ in Table 6.6. Since the proportions of all machines of the j-th type, i.e. the Q_j, are quite different from either the Q_j^1 or $1/m$ as shown in Table 6.6, the values of G_i^2 are dissimilar to its counterpart G_i^1 or G_i^3 values. Yet the overall pattern between departments is still the same. Of particular interest is the comparison of the G_i^1, G_i^2, and G_i^3 values of departments 1 and 3. All three indicate an increased balance in the equipment mix in going from 1 to 3, but a significant change in the nature of the mixes as demonstrated by the corresponding X_i^1 and X_i^2 values.

It's apparent that G_i^1, G_i^2, and G_i^3 are easily interpreted as measures of the specialization for the equipment mix in departments, and hence of the "quality" of the equipment inputs to the production models. The mean values for these three indices (for this particular sample) are 0.538, 0.576, and 0.538 respectively. The homogeneity assumption of the models assigned an average quality to the production variables. Hence by comparing these indices to their means we can, in effect, compare the actual quality to the "average quality" used in the models.

The scheme we have outlined can be applied to any entity which is distributed as to category and location. In a later section we apply it to three sets of variables; the 13 classes of equipment in 139 radiology departments, the manpower mix of these departments, and the diagnostic case mix in their hospitals.

6.5.4 Monitoring the Allocation of Technological Resources.

Before proceeding to examine the relationship between a set of information measures and the technical efficiency of a department, we shall first investigate a possible use of information theory in resource allocation.

In section 2.2.2 we described the basic philosophy vis-a-vis the allocation of resources in the health sector that was adopted by the Quebec Government. Two related goals which were described as important are: the centralization of specialized resources and the decentralization of non-specialized resources. The actual utilization of these criteria in decision making is difficult due to the fuzzy nature of the terms involved. That is, what do we mean by specialized resources or centralization?

The H_j^1 entropies developed in section 6.5.2 were demonstrated to be measures of specialization in terms of the distribution of resources. Even if we reject this concept of specialization as meaningful, it is apparent that these measures of the inequality in the distribution of resources can be used as indices of the concentration or centralization of resources.

If a particular resource is concentrated in a small number of institutions, i.e. a large proportion of the total volume of the resource is situated in a small proportion of the possible locations, then we can say that this resource is centralized in these institutions. This would result in a relatively high value for the corresponding value of H_j^1. Conversely, if this resource were more evenly distributed among the possible locations, the corresponding value of H_j^1 would be relatively small and the resource may be considered less centralized.

This notion is independent of a distance or geographic concept, and relies solely on the recognition of different resource repositories, regardless of their proximity. This in fact is desirable. Imagine two institutions, one very large and the other very small. The acquisition of a resource that is plentiful in the larger institution, by the smaller one, should be regarded as a decentralized allocation independent of whether they are next door to each other or hundreds of miles apart.

One would be hard pressed to indicate exactly what level of centralization or decentralization of a particular resource is desirable. Given the prior distribution of $1/n$ for H_j^1, and the fact that a sample may encompass a broad spectrum of institution sizes, it follows that one would not want to guide allocations by trying to set the H_j^1 to a prescribed value. Hence the actual value of these indices are not important. What can be observed as important, is the change in H_j^1 with an allocation of the j-th resource. If H_j^1 increases, this would indicate a centralized allocation, whereas if it should decrease, this would imply a decentralized allocation. In addition, the extent of (de)centralization can be observed by investigating the relative change in this entropy measure.

Without having made any statement on the desirability of

centralizing or decentralizing an allocation, we have proposed a method whereby decision-makers can monitor the effects of their decisions, with respect to what has been professed to be an important allocation criterion.

In Tables 6.9, 6.10 and 6.11 we show the equipment entropies which result from independently incrementing each of the three types of equipment by 1, in departments 1, 10 and 20 respectively, of the example of section 6.4.3. The results are as expected: for department 1, adding any type of equipment is a decentralized allocation; incrementing equipment categories 1 and 2 in department 10 are centralized, whereas an addition to the volume of equipment type 3 is a decentralized allocation; all allocations to department 20 are decentralized.

Several facets of these allocations are of particular note: i) of the three allocations to department 1, that of equipment type 2 is the most decentralized; ii) the addition of equipment type 1 to department 1 is more decentralized an allocation than the same equipment in department 20; iii) although the allocation of equipment type 3 to all three departments is seen as a decentralized allocation, the effect on the distribution of this resource is the same in departments 1 and 10, and most dramatic in department 20, and; iv) although the addition of a machine in category 2 in department 20 is a decentralized allocation, the effect is sufficiently small that for a department originally having 4 or possibly 3 pieces of this type of equipment, the same allocation would be a centralized one.

In evaluating an application from a radiology department or in comparing two or more applications, the H_j^1 indices provide a means of telling the decision-makers the exact nature of the proposed allocations.

We now extend the use of the H_j^1 measures for the above purpose by using one of Theil's developments (1967, p. 93-95). This is

Initial entropy values from Table 6.7	H_1^1	H_2^1	H_3^1
	0.656	0.790	0.565

DEPARTMENT	EQUIPMENT TYPE 1	2	3	H_1^1	H_2^1	H_3^1
1	1	0	1	0.631	0.790	0.565
1	0	1	1	0.656	0.754	0.565
1	0	0	2	0.656	0.790	0.553

Table 6.9. Equipment entropies (H_j^1) that result from incrementing each of the three types of equipment in department 1 by 1.0 .

DEPARTMENT	EQUIPMENT TYPE 1	2	3	H_1^1	H_2^1	H_3^1
10	13	10	1	0.662	0.790	0.565
10	12	11	1	0.656	0.804	0.565
10	12	10	2	0.656	0.790	0.553

Table 6.10. Equipment entropies (H_j^1) that result from incrementing each of the three types of equipment in department 10 by 1.0 .

DEPARTMENT	EQUIPMENT TYPE 1	2	3	H_1^1	H_2^1	H_3^1
20	2	2	0	0.643	0.790	0.565
20	1	3	0	0.656	0.783	0.565
20	1	2	1	0.656	0.790	0.531

Table 6.11. Equipment entropies (H_j^1) that result from incrementing each of the three types of equipment in department 20 by 1.0 .

pursued only to indicate the possibilities inherent in this type of index, and hence we merely provide a discussion. From Eqn. 6.13 we have the following measure of equipment-distribution equality:

$$H_j = \sum_{i=1}^{n} q_{ij} \log \frac{1}{q_{ij}} \qquad j = 1, 2, \ldots m \qquad (6.19)$$

As the j subscript is common to all variables in this expression, and because we do not introduce any operation over it below, we shall drop it from the derivation with the understanding that the following expression is equivalent to that in Eqn. 6.19 and that all terms derived from either H_j or q_{ij} implicitly have a j subscript.

$$H = \sum_{i=1}^{n} q_i \log \frac{1}{q_i} \qquad (6.20)$$

We now devide the n departments into g subsets, S_1 , $S_2 \ldots S_g$ where each department belongs to one and only one subset. Typically, these subsets might represent different geographic regions or classes of hospitals, by bed volume for example. Let n_k = number of departments in S_k, $k = 1, 2, \ldots g$ where,

$$n = \sum_{k=1}^{g} n_k \qquad (6.21)$$

Eqn. 6.20 can now be written as,

$$H = \sum_{k=1}^{g} \left[\sum_{i \in S_k} q_i \log \frac{1}{q_i} \right] \qquad (6.22)$$

The term in brackets then becomes,

$$\sum_{i \in S_k} q_i \log \frac{1}{q_i} = Q'_k \sum_{i \in S_k} \frac{q_i}{Q'_k} \left[\log \frac{1}{q_i/Q'_k} + \log \frac{1}{Q'_k} \right]$$

$$= Q'_k H'_k + Q'_k \log \frac{1}{Q'_k} \qquad (6.23)$$

where $Q'_k = \sum_{i \in S_k} q_i \qquad k = 1, 2, \ldots g$

$$H'_k = \sum_{i \in S_k} \frac{q_i}{Q'_k} \log \frac{1}{q_i/Q'_k} \qquad k = 1, 2, \ldots g$$

(6.24)

Combining Eqns. 6.22 and 6.23, we have,

$$H = \sum_{k=1}^{g} Q'_k \log \frac{1}{Q'_k} + \sum_{k=1}^{g} Q'_k H'_k \qquad (6.25)$$

The first term on the right is the between-set entropy, Q'_k being the proportion of all machines (of the type being considered) in S_k. The second term is the weighted average of the within-set entropies H'_k. Each of these involves departmental equipment loads measured as fractions of the corresponding set loads rather than the total volume of machines. As indicated in section 6.5.2 the entropy of Eqn. 6.25 is a measure of distribution equality and hence we can convert it to a measure of inequality as was accomplished in Eqn. 6.14, that is,

$$H^1 = \log n - H = \log n - \sum_{k=1}^{g} Q'_k \log \frac{1}{Q'_k} - \sum_{k=1}^{g} Q'_k H'_k$$

$$H^1 = \log n - \sum_{k=1}^{g} Q'_k \log n_k - \sum_{k=1}^{g} Q'_k \log \frac{1}{Q'_k} + \sum_{k=1}^{g} Q'_k \left[\log n_k - H'_k\right]$$

$$- \sum_{k=1}^{g} Q'_k \log \frac{Q'_k}{n_k/n} + \sum_{k=1}^{g} Q'_k \left[\log n_k - H'_k\right]$$

Combining the above with Eqns. 6.14 and 6.24 we have,

$$H^1 = \sum_{i=1}^{n} q_i \log \frac{q_i}{1/n} = \sum_{k=1}^{g} Q'_k \log \frac{Q'_k}{n_k/n} + \sum_{k=1}^{g} Q'_k \left[\sum_{i \in S_k} \frac{q_i}{Q'_k} \log \frac{q_i/Q'_k}{1/n_k} \right]$$

(6.26)

The first term on the right measures the between-set distribution inequality based on the prior probabilities of n_k/n for each set and the observed proportions of Q'_k. The second term is the weighted average of the within-set inequalities and is similar to the X_i^1 in Eqn. 6.13. In the separate within-set expressions the prior probabilities correspond to equal shares within each set, i.e. $1/n_k$, and the observed proportions q_i/Q'_k are also relative to the k-th set. Re-introducing the j subscript we have,

$$H_j^1 = \sum_{k=1}^{g} Q'_{kj} \log \frac{Q'_{kj}}{n_k/n} + \sum_{k=1}^{g} Q'_{kj} \left[\sum_{i \in S_k} \frac{q_{ij}}{Q'_{kj}} \log \frac{q_{ij}/Q'_{kj}}{1/n_k} \right]$$

(6.27)

Obviously all allocations will result in equivalent values for H_j^1 from Eqns. 6.14 and 6.27, yet the latter equation allows a more detailed analysis. Thus, an allocation that is viewed as centralized overall, may be seen as decentralized in the context of a particular subset of departments, and vice-versa. For example, if one of the sets encompassed all university hospitals with more than 500 beds, one could observe a decentralized allocation within

this group by examining the corresponding within-set inequality measure, and an overall centralized allocation by examining the total inequality measure.

We can see that adding one piece of the j-th type of equipment to any radiology department will change all the q_{ij} terms $i = 1...n$ and the Q'_{kj} terms $k = 1...g$ as well. Hence the effect of an allocation is really spread around to all locations, and so one might argue against the merit of examining any particular within-set entropy. In Table 6.12 we show the columns of the q_{ij} matrix corresponding to the addition of one machine to each of the three categories in department 1 of the example from section 6.5.3. Comparing these results to the original q_{ij} matrix in Table 6.4 we see that except for the department where the equipment change occurs, most other q_{ij} values change by 0.001 or less. If we consider the relatively small number, both of departments and total machine volumes in this example, it is easily appreciated that the effect of an equipment addition to a department would almost totally be concentrated in that department with a realistically sized sample. Therefore we can justifiably examine the change in both the within-set entropy of interest and the total distribution inequality measure H_j^1, for a test allocation.

Our original intent was to present measures that would reflect variations in the mix of certain quantities at different locations and so to at least partially account for differences in the technical efficiencies of departments. As a by product, we have proposed indices to monitor the allocation of technological resources in the health sector. In terms of our goal to develop a scheme to support the decision making process for the allocation of resources by government, this secondary result must be viewed as equally important and deserving of further investigation.

DEPT	q_{11}	q_{12}	q_{13}
1	0.009	0.015	0.032
2	0.0	0.015	0.0
3	0.009	0.074	0.0
4	0.052	0.147	0.048
5	0.113	0.015	0.097
6	0.0	0.0	0.032
7	0.009	0.015	0.0
8	0.061	0.029	0.081
9	0.026	0.015	0.032
10	0.104	0.147	0.016
11	0.0	0.015	0.0
12	0.043	0.0	0.048
13	0.017	0.0	0.032
14	0.087	0.044	0.113
15	0.009	0.0	0.016
16	0.070	0.015	0.081
17	0.0	0.029	0.0
18	0.070	0.059	0.016
19	0.052	0.015	0.065
20	0.009	0.029	0.0
21	0.035	0.118	0.0
22	0.087	0.029	0.032
23	0.0	0.0	0.016
24	0.017	0.0	0.032
25	0.0	0.0	0.016
26	0.0	0.015	0.0
27	0.0	0.015	0.0
28	0.026	0.0	0.032
29	0.026	0.044	0.032
30	0.009	0.0	0.048
31	0.009	0.0	0.032
32	0.009	0.015	0.0
33	0.035	0.074	0.032
34	0.009	0.015	0.0
35	0.0	0.0	0.016

Table 6.12. The columns of the q_{ij} matrix (Table 6.4) corresponding to the addition of one machine to each of the three categories of equipment in department 1.

6.6 Classifying Departments as a Means of Explaining Efficiencies

As evidenced by the discussion in section 6.3, there are no easy solutions or answers to the problem of explaining the technical efficiency of radiology departments. In fact, there are probably as many approaches to quantitatively explaining the results of a model as there are schemes for constructing a model in the first place.

In section 6.1 we described four basic sources of inefficiency as measured by the production models. To repeat then, they are: i) the constant returns to scale assumption; ii) the homogeneity assumption for the variables of a model; iii) an external, department-hospital, imbalance reflecting an insufficient demand for radiographic services, and; iv) an internal, departmental resource, imbalance reflecting the production organization of a department.

In section 6.3 we saw that: i) the assumption of constant returns to scale is appropriate for the system under investigation, and hence, not a source of inefficiency; ii) the technical change component of the equipment variable heterogeneity, as measured by the mean x-ray room age per department, is not a determining factor of technical efficiency, and; iii) the latter two non-model sources of inefficiency, could not be satisfactorily accounted for by department/hospital size related variables. The other major component of heterogeneity, which affects both the technological and manpower inputs to the model, results from aggregation.

In the previous section we introduced an information theoretic approach to quantifying the resource mix of a department. The resulting indices were seen to also reflect the heterogeneity of the aggregated variables. It is felt that by applying these measures to different facets of the department-hospital system, we can partially account for both the heterogeneity of some of the production variables and the internal/external resource imbalances. In this section we lay the groundwork for introducing these information indices to a quantitative explanation of efficiency.

6.6.1 Alternative Approaches to the Analysis of Multiple Sources of Influence. A large part of the research in the health field is concerned with determining the influence of health system and patient population characteristics on the utilization of health services. Typical examples would be: annual physician visits per capita as a function of population age group, economic class and residence (Reinke & Baker, 1967); hospital length of stay as a function of patient age, sex and race (Ro, 1973); percentage of population with physician visits in a 2 week period as a function of population residence location, monthly per capita income, age, insurance benefit status and years of education (Reinke, 1973), and; the economic efficiency of private practices as a function of practice facilities, regional location, speciality composition of staff and type of physician compensation (Kimbell & Lorant, 1973b).

We, of course, are interested in the utilization of the production capacity of radiology departments, i.e. the technical efficiency, as a function of both departmental and hospital resources. Demographic characteristics of the patient population are omitted as contributing factors in the technical efficiency because diagnostic radiology, as a component part of the larger hospital system, is only indirectly interfaced with the patient population through a referral process.

The first step in determining the nature of a causal relationship between variables, as in the above examples, is to identify and quantitatively define the dependent variable or measure of utilization, and the corresponding set of independent or explanatory variables. Next, we would choose a technique for analyzing this data which is: i) most adaptable to the peculiarities of the data set, and; ii) capable of providing the kind of answers the research requires Examples of frequently used techniques are multiple regression (Feldstein & German, 1965), dummy variable regression (Ro, 1973), analysis of

variance (Rosenthal, 1964), automatic interaction detection (Anderson *et al.*, 1971), and the multisort methods (Reinke & Baker, 1967).

Each of these approaches to data analysis have certain strengths and weaknesses. The two regression techniques provide the strongest quantification of a causal relationship between the set of independent variables and the dependent variable. Their application requires an *a priori* specification of the mathematical relationship expected to be present. Detailed expression of a regression model is "often incompatible with the uncertainty concerning the nature of interactions and nonlinearities in the effects" (Reinke & Baker, 1967) and is frequently "based on arbitrary formulations that result in naive analysis" (Reinke, 1973).

The latter three techniques essentially use a block-splitting principle to identify factors of possible importance to the dependent variable and the interactions of these factors in terms of this variable. The analysis of variance is "extremely productive in uncovering effects and interactions... whenever each combination of independent variables contains the same number of observations, although this condition is seldom encountered except in controlled experiments" (Reinke & Baker, 1967). The multisort technique is "an approximation to analysis of variance, retaining the block-sorting features of the latter but sacrificing some mathematical rigor in the interests of computational simplicity" (Reinke & Baker, 1967). The automatic interaction detection technique is a branching process, which because of its sequential nature "causes later identification to be dependent upon the particular population classifications initially produced" (Reinke & Baker, 1967), and as a result can be very misleading. In addition, all three methods appear better suited to categorical data rather than ordinal or continuous variables (Reinke & Baker, 1967; Sonquist *et al.*, 1971; Kimball & Lorant, 1973b).

It is usually most profitable to combine these techniques. One

would first apply a block-sorting method to uncover the principal determinants of utilization and their interactions, and subsequently incorporate these results into a regression formulation (Reinke & Baker, 1967; Reinke, 1973). From the discussion of section 6.3, it is clear that we are faced with a large number of possible "explanatory" variables per department, most of which are highly inter-correlated and are not readily associated with either high or low efficiency. It would thus seem preferable to adopt the two stage process mentioned above, rather than proceed directly to a regression model formulation.

6.6.1.1 The Need for a Re-examination of the Radiology Departments. Two major problems arise in applying any one of the block-sorting techniques to our problem. First, the data set is really quite small; approximately 100 observations per production model. This problem would be further aggravated with the introduction of a sizeable number of variables into the analysis. Typical numbers that have been used in previous utilization studies are; 45,315 observations on 17 independent variables (Reinke, 1973), 66,000 observations on 16 independent variables (Reinke & Baker, 1967), and 9,000 observations on 6 independent variables (Ro, 1973). The second problem involves the choice of variables. As a result of the small number of observations that are available we would have to choose some subset of the possible variables to include in the analysis. Yet it would be difficult to decide *a priori* as to which would be the superior subset. In light of these two problems it was decided that the radiology departments should be re-investigated independent of their efficiencies. Two distinct approaches for examining the departments come to mind, both of which may be considered as classification procedures.

First, one could reduce a large set of independent variables to a smaller set representing most of the information in the original set. This is sometimes called dimensionality reduction and is a common

procedure in socio-economic research. It is usually accomplished by a principal components (Pidot, 1969) or a factor analysis (Adelman & Morris, 1965). These methods are not without their problems. For example, in factor analysis the "factors will not always be easily interpreted... Moreover the number of factors required to explain a set of attributes may be relatively high. The application of factor analysis thus does not always guarantee reduction in the size of the opportunity set" (Kimbell & Lorant, 1973b).

Second, one could categorize radiology departments into groups based on some measure of similarity between departments. Specifically, we have implied that departmental efficiency is related to the internal departmental resource balance and the external department-hospital match, and thus would attempt to classify departments on this basis. This type of classification would then be introduced, as one of a set of variables, to the kind of two-stage scheme described above for quantitatively explaining the utilization of health services.

6.6.2 A Procedure for Departmental Classification. "The purpose of a classification has an important bearing on the efficiency of a classification scheme and should influece the choice of... techniques used to select this scheme" (Kimbell & Lorant, 1973). As indicated in the previous section, our goal in classifying radiology departments is not to explain the technical efficiencies, but to provide a general description of departments, presumably from a new perspective, which could ultimately serve that purpose. The problem then becomes one of deciding on the extent to which we will let the final purpose of this analysis colour the choice of classification methodology.

Kimbell & Lorant (1973b) argue that "the seemingly innocent task of running breakdowns or cross tabulations... using intuitively 'obvious' classifications is less efficient than a sequence of more systematic steps,... Much of the potential information embodied in the 'raw' data has simply been submerged by casual choices of variables and attributes. These choices may have appeared efficient, viewed from the general perspective, but they are often distressingly

inefficient for testing formalized behavioral propostions."

The process of classifying or grouping departments involves two steps, which in turn, require two decisions. First, one must decide on which variables to include in the set upon which the classifications will be based. And second, one must decide on the actual technique to be used. If we require that this technique should be independent of any *a priori* problem knowledge, i.e. the technical efficiencies, and be able to effectively define subclasses within the sample set, then a degree of latitude is introduced in the choice of variables. Since the technique should search for groups based on a degree of homogeneity or consistency for the departments within each group, in terms of this variable set, then, whether some or all the variables are believed to be related to the technical efficiency is of little consequence.

In this section we present a procedure for classifying the points in a sample from a general perspective, and leave the actual definition of variables and application of this scheme to the next section.

Generally a classification "allocates entities to initially undefined classes so that individuals in a class are in some sense close to one another" (Cormack, 1971). Hence, the object of classification is "to obtain classes such that any member of a class can be treated as if it possessed certain properties" (Jones, 1970). In the broader sense of data analysis, classifications are pursued "to arrive at a useful description of the sample and to discover unsuspected clusterings which may prove to be important" (Fleiss & Zubin, 1969).

Similarity between points in a class or the internal cohesion of a class is important to defining subclasses within a sample. Equally important is the relationship between classes; that is, we would like the classes to be different or isolated from each other. These two basic classification criteria, the internal cohesion

and external isolation of a group of points in a sample, are endemic to the idea of a cluster of points and are used by the set of techniques known as clustering procedures to partition data into subsets (Cormack, 1971).

Clustering is a well established area in the literature (Sokol & Sneath, 1963; Williams & Dale, 1964; Ball, 1965; Cormack, 1971; Ling, 1971), yet because most of these techniques have developed "without formal basis, as algorithms" (Cormack, 1971), one cannot easily demonstrate that any one of these methods is uniquely superior to any other in all instances. Since our goal is to demonstrate that this type of approach is of value to studies of health services utilization, we are not required to necessarily find the "best" algorithm, but merely one that is workable in the present context. Rather than provide the reader with a detailed introduction to clustering techniques, as this is readily available from the sources cited above, we discuss three criteria that were felt to be important, and subsequently present a scheme which appears to satisfy them.

In that two of these criteria are concerned with the concept of a cluster, we will first elaborate somewhat on this idea. To do so, we must define some terminology. Ball (1965) provides the following definitions:

> *Measurement* - By measurement we mean a component of the pattern vector; that is, it is one of several numerical values (related to the property of the pattern) used to define a pattern - for example, one out of several answers on a questionnaire.
>
> *Pattern* - By pattern we mean the collection of measurements considered to be a single entity in the clustering program, for example, the answers to a set of questions on a questionnaire.
>
> *Cluster* - A cluster of patterns is ...a set of patterns contained in a high dimensional space where the density of patterns is large compared to the surrounding volumes.

A department is represented in a multi-dimensional space as a

pattern vector composed of observations, equal in number to the dimension of the space. Thus a cluster of departments generally exhibits a common "pattern". The term density in the definition of a cluster connotes a distance concept, yet the proximity of points need not be measured in a Euclidean sense. Although we shall concentrate on Euclidean distance measures in this work because of their simplicity, the reader should note that this is but one of many accepted measures that can be used (Ball, 1965). Having adopted the measure of similarity between two patterns as the distance between them, we are then confronted with the problem of evaluating the partitioning of patterns into clusters; that is, how similar must patterns be for membership in the same cluster?

If we choose a similarity criterion that is highly stringent for a sample of patterns, then many very consistent clusters will result. If we should lower our sights somewhat, then both the number of clusters and the cohesion of the individual clusters will decrease. The two extreme examples of this process are; first, where all points in a sample define their own cluster, and second, where there is only one cluster encompassing the entire sample. Figure 6.9a shows a two dimensional sample of patterns while Fig. 6.9b provides, what appears to be, the obvious set of clusters for this sample. In recognizing these four clusters we have implicitly (visually) decided on a particular criterion for disimilarity. Fig. 6.10 shows the effect of gradually lowering this criterion, Fig. 6.10a to c, until finally only one cluster is recognized.

The results of a clustering will be dependent on the particular choice of Euclidean distance measure. Although this type of measure will yield clusters that are invariant to translations or rotations, they will be dependent on other types of transformations that distort the distance relationship (Duda & Hart, 1973). Fig. 6.11 shows the effect of scaling on the resulting clusters. Since the clustering technique should be able to find natural

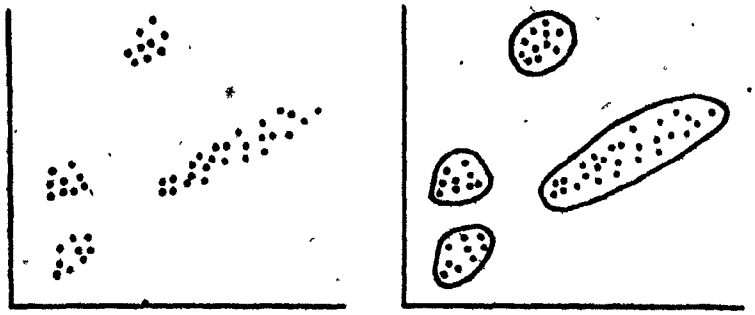
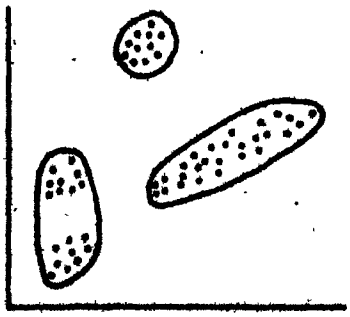


Fig. 6.9. (a) A sample of patterns in a two dimensional space and (b) the resulting set of "obvious" clusters.



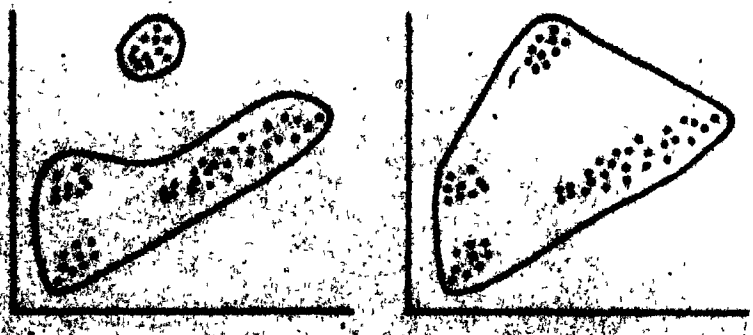


Fig. 6.10. Pairs of clusters resulting from various criteria of dissimilarity for the sample of patterns from Fig. 6.9.

groupings of points, it is desirable that the technique be independent of whether the patterns are widely spread or tightly positioned. In other words, the clusters themselves may vary in size, i.e. as to the volume they occupy, but not in terms of which departments are to be included in any one cluster.

Clusters may be divided into two categories with respect to shape; globular and nonglobular. These are illustrated in Fig. 6.12. It is apparent that if a sample is represented by the configurations in Fig. 6.12b or c, then there are really two distinct clusters as opposed to only one cluster of the entire sample; that is, each cluster has different properties. Hence, in addition to the scaling criterion cited above, we would also want the clustering technique to be able to recognize and differentiate nonglobular structures.

The third goal we would want to impose on the choice of technique is that of computational simplicitly. Almost all methodological research in this area is, at least implicitly, geared towards this end. In all likelihood we will want to apply this method to more than one data set, and therefore would also want the technique to be versatile in terms of sample size and dimensionality.

The algorithm we have chosen, which appears to satisfy these three criteria, was proposed by Jarvis & Patrick (1973). It is an agglomerative procedure in that it involves a series of successive fusions of the patterns into groups (Cormack, 1971), and may further be considered as a "nearest neighbour" algorithm which is the simplest type of agglomerative sorting procedure.

"When speaking of clustering as a partitioning of data sets into subsets whose in-class members are 'similar' in some sense and whose cross-class members are 'dissimilar' in a corresponding sense, we are putting the whole weight of any computational method that reflects this loose definition on a suitable mathematical

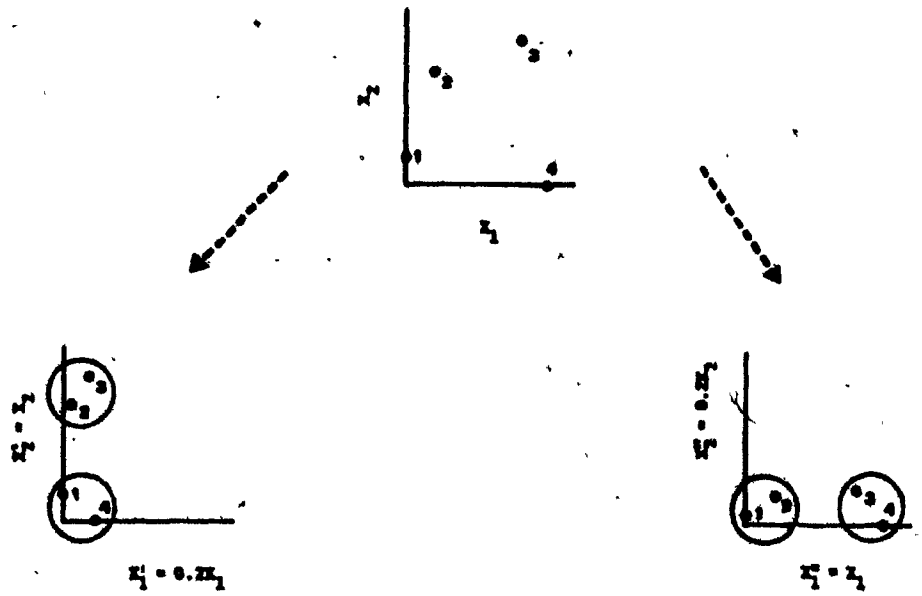
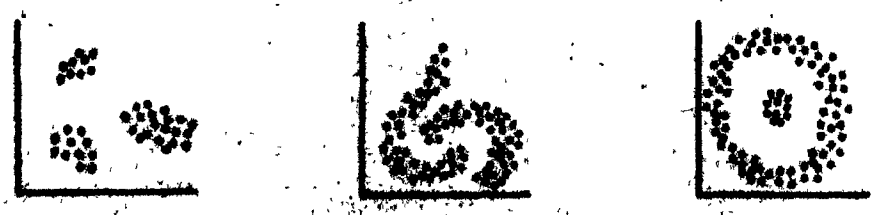


Fig. 6.11. The effect of scaling on the apparent clustering of a sample of patterns (Duda and Hart, 1973, p. 215).



(a) (b) (c)

Fig. 6.12. Examples of (a) globular clusters and (b) & (c) non-globular clusters.

description of similarity" (Jarvis & Patrick, 1973). The concept of similarity presented by Jarvis & Patrick is based on the idea of shared near neighbours. If x_i and x_j are any two points of the pattern set or data vectors (x_1, x_2,.... x_n) in an m dimensional Euclidean vector space, then the following two statements (Jarvis & Patrick, 1973) represent the basic ideas in the paper:

i) If x_i and x_j are sufficiently similar, then the probability that they come from the same cluster approaches unity. If then, a suitable point pair similarity measure can be found, a clustering technique is easily devised.

ii) Data points are similar to the extent that they share the same neighbours; in particular, two data points are similar to the extent that their respective k nearest neighbour list match. In addition, for this similarity measure to be valid, it is required that the tested points themselves belong to the common neighbourhood. This avoids the possibility of combining a small relatively isolated number of points with a high density group.

The clustering algorithm developed from the above concept of similarity consists of four steps (Jarvis & Patrick, 1973):

I. For each of the n points in the data set list the k nearest neighbours (e.g. using Euclidean distance measure) in order of their proximity to the point under examination, that point being its own nearest neighbour. The resulting nearest neighbour list per point is placed in a neighbourhood table (an $n \times (k + 1)$ integer matrix). The raw data can now be discarded.

II. Set up a label table (an integer column vector) of length n with the i-th element set equal to i for $i = 1, 2,...n$. The results of these first two steps are shown by the label and neighbourhood tables in Fig. 6.13.

III. All possible pairs of neighbourhood rows are tested as follows: replace both label entries by the smaller of the two existing entries if,

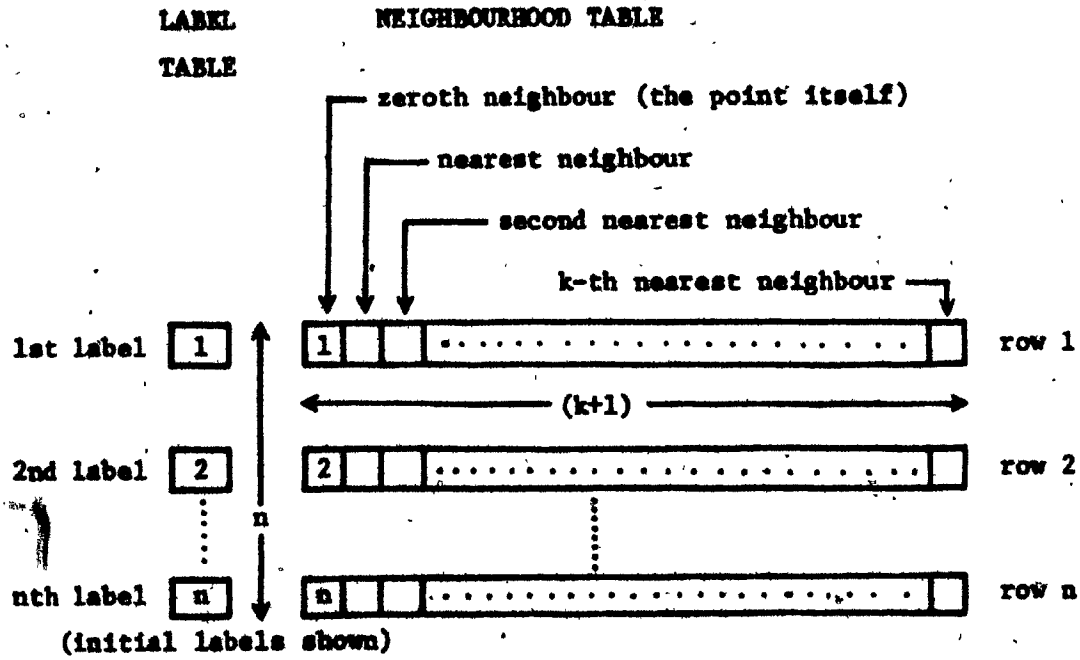


Fig. 6.13. Label and neighbourhood tables for a sample of n patterns (Jarvis and Patrick, 1973, p. 1026).

a) both zeroth neighbours (the points being tested) are found in both neighbourhood rows, and,

b) at least k_t neighbour matches exist between the rows.

If this test is successful replace all appearances of the higher label, throughout the label table, by the lower label.

The parameter k_t is referred to as the similarity threshold.

IV. The clusters are now indicated by identical labelling of the points belonging to a cluster in the label table.

This particular set of clusters is conditioned by the parameters k and k_t. We can find the set of clusters corresponding to a new k and k_t by returning to step II as long as this new value of k is less than or equal to the old value.

Assuming that the first condition in step III is satisfied, the second condition states that if a match is found between two neighbourhood rows then,

$$N \leftarrow N + 1$$

After having checked all possible matches, the two points are in the same cluster only if,

$$N \geq k_t$$

"A degree of refinement can be added to the clustering scheme by simply weighting the vote of shared neighbours according to their positions in the ordered neighbourhood row associated with each point" (Jarvis & Patrick, 1973). If a match is found between the i-th and j-th neighbourhood table rows, and r and q are the respective positions of the common point in these two rows (see Fig. 6.14) then,

$$N \leftarrow N + f(k, r, q)$$

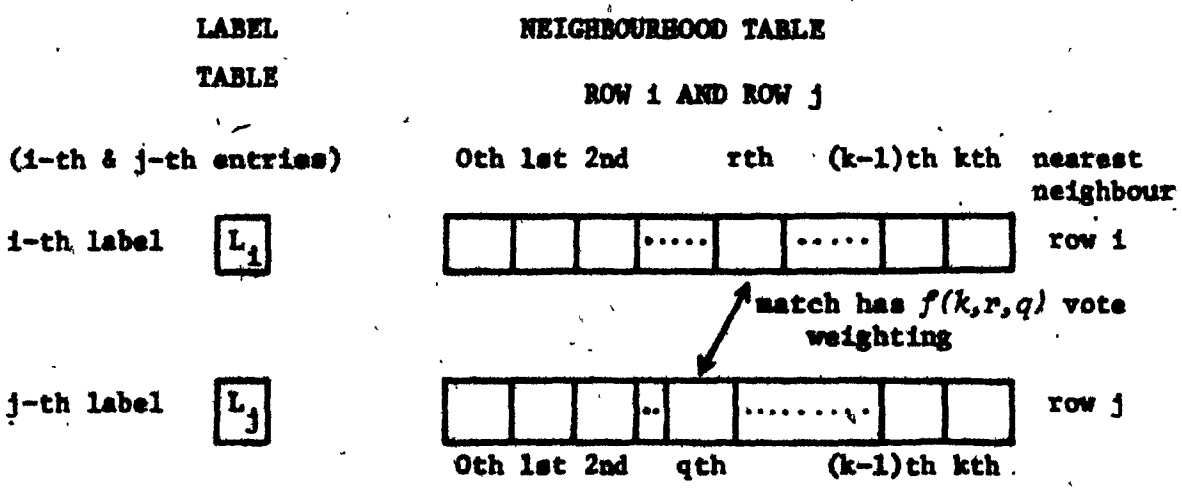


Fig. 6.14. Illustration of the weighted voting scheme for clustering patterns (Jarvis and Patrick, 1973, p. 1028).

Two different forms of $f(k, r, q)$ are provided (Jarvis & Patrick, 1973),

$$a) \quad f(k, r, q) = (k - r + 1) \times (k - q + 1)$$

$$b) \quad f(k, r, q) = (k - r + 1) + (k - q + 1)$$

In the first case "if the nearest neighbour of both points are the same, the match gets $k \times k$ votes, while if the furthest neighbours match, the match gets only 1 vote, and, all in-between matches (vertical and diagonal) get votes between $k \times k$ and 1" (Jarvis & Patrick, 1973). If this product weighting is felt to be too severe, the additive weighting of the second case can be used. Both of these modifications are consistent with the concept of sharing near neighbours. The condition that N must be greater than or equal to k_t is still applied in these latter two voting schemes to test whether points are in the same cluster. Hence the choice of k_t will depend both on the value of k and which of the three schemes is used.

Consider the set of 20 patterns shown in Fig. 6.15. Figs. 6.16 and 6.17 show the results of applying the above clustering algorithm with unweighted and weighted multiplicative votes respectively. The computer program used to cluster these points follows the four steps outlined above.

It is apparent that increasing the value of k, for any of the three methods, will allow a finer choice of k_t, yet at the same time, this increase in neighbourhood size will tend to result in globular clusters (Jarvis & Patrick, 1973). Hence in applying this procedure, a degree of subjective judgement will be introduced to the choice of k, k_t, and the voting scheme, with respect to what is viewed as the "best" cluster analysis for the problem at hand. In addition, we should note that because this clustering technique is dependent on the relative position of points and not the absolute distances, the "pair point similarity measure has its own built in automatic scaling" (Jarvis & Patrick, 1973).

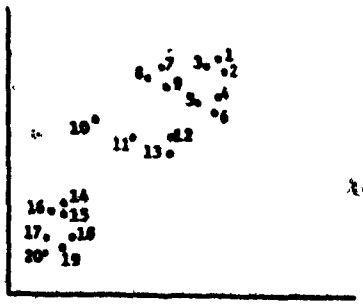


Fig. 6.15. A sample of 20 patterns in a two-dimensional space (Jarvis and Patrick, 1973, p. 1028).

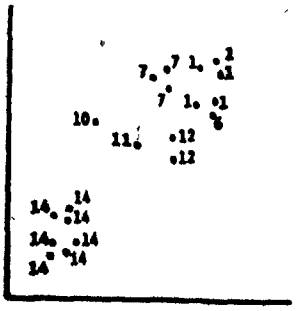


$k = 5 \quad k_t = 4$ (a)

$k = 5 \quad k_t = 3$ (b)

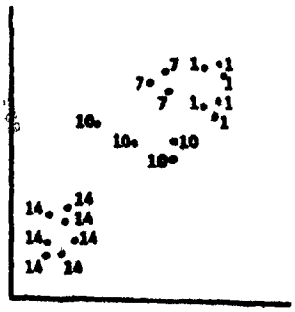
$k = 5 \quad k_t = 2$ (c)

Fig. 6.16. The result of applying the clustering algorithm of Jarvis and Patrick (1973) to the sample in Fig. 6.15 with unweighted voting, $k = 5$, and (a) $k_t = 4$, (b) $k_t = 3$, (c) $k_t = 2$.



$k = 5 \quad k_t = 30$

(a)



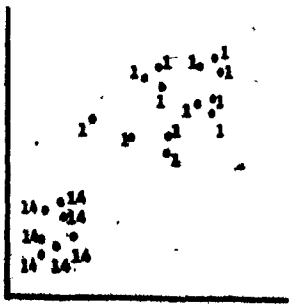
$k = 5 \quad k_t = 27$ to 15

(b)



$k = 5 \quad k_t = 12$ to 10

(c)



$k = 5 \quad k_t = 9$ to 6

(d)

Fig. 6.17. The result of applying the clustering algorithm of Jarvis and Patrick (1973) to the sample in Fig. 6.15 with weighted multiplicative voting, $k = 5$, and (a) $k_t = 30$, (b) $k_t = 27$ to 15, (c) $k_t = 12$ to 10, (d) $k_t = 9$ to 6.

6.6.3 A Source of Departmental Efficiency. In section 6.3 we saw that while the larger departments were not the most efficient producers, they were at least as good, if not better, than many of the smallest departments. From this we concluded that one could not explain the differences in efficiency solely in terms of variables that were correlated to department size.

We have hypothesized two sources of technical inefficiency in radiology departments that are unrelated to the assumptions of the production models and independent of the size of a department. They are the production organization of a department, as reflected by its internal resource balance, and, the radiographic services demand-supply relationship between the hospital and the radiology department, as reflected in the external, hospital-department, resource balance. The rest of this chapter is devoted to quantitatively defining a resource balance, and subsequently determining if some level or (im)balance is correlated with technical (in)efficiency.

In section 6.5 we presented a set of five information theoretic indices which, when applied to the equipment profile of a department, were seen to be measures of the technological resource mix in a radiology department. If similar measures are derived for the manpower mix in a department, then on comparing these technology and manpower measures, one could, in effect, be examining at least one facet of the technology-manpower balance (production organization) in a department. This type of resource comparison is very different from that which was effected in the production models, where aggregated inputs are compared to define different production processes.

The equipment and manpower data used to construct the information based measures is found on data set D.1. For equipment, we recognise 13 possible classes per department as listed in Table 5.2. To construct manpower measures reflecting the entire labour force in radiology departments, we need a means of reducing radiologists

and technicians to a common unit of measure. Since the information indices will operate on the distributions of different labour types we have no hesitation in combining these two groups. Tables 5.1 lists seven mutually exclusive and collectively exhaustive possible groups of technicians in a radiology department. Data set D.1 provides the number of full and part-time individuals in each of these classes. We therefore recognize 14 (2×7) technician groups which, when combined with the two (full and part-time) radiologist groups results in 16 manpower types. With these breakdowns for equipment and manpower, Eqns. 6.15, 6.17 and 6.18 were used to produce five measure per department for each of these two resources.

Table 6.13 provides a list of the five equipment information measures for the same 35 departments examined in section 6.5.3, except that these are derived for 13 categories of equipment and from a sample of 135 departments. In addition, the equipment inputs to a production model, based on the first 6 equipment categories in Table 5.2 corresponding to each of these departments is also provided. It is easily seen that the information indices: i) are uncorrelated with department size, as measured by the total equipment input to the models, and; ii) provide a means of differentiating between departments with the same or similar fixed capital inputs. The manpower indices were derived from 16 categories and 130 locations (departments).

Assuming that a comparison between these two sets of indices adequately accounts for the internal resource mix in a department, the problem then arises of introducing some hospital related measure to portray the relationship between the department and hospital. If we were to take the same approach as above and concentrate on the input resources to a hospital, we are confronted with two problems:

DEPT	X_i^1	X_i^2	a_i^1	a_i^2	$X_i^{a^2}$
1	0.607	0.784	1.872	1.299	0.964
2	0.700	1.175	2.565	1.977	1.489
3	1.158	1.138	0.519	0.508	0.747
4	1.001	1.150	0.456	0.210	0.194
5	1.125	1.042	0.565	0.317	0.555
6	0.718	0.791	1.288	0.824	0.682
7	0.770	0.839	1.179	0.618	0.526
8	1.044	0.937	0.383	0.085	0.291
9	0.990	0.864	0.622	0.299	0.499
10	1.058	0.984	0.462	0.204	0.414
11	1.180	1.658	1.872	2.077	1.881
12	0.971	0.702	1.300	0.677	0.939
13	1.420	1.686	0.830	1.030	1.228
14	1.006	0.902	0.574	0.203	0.385
15	1.178	1.075	0.619	0.518	0.819
16	0.972	0.841	0.548	0.144	0.304
17	0.892	0.951	1.446	1.031	0.796
18	1.176	1.154	0.367	0.223	0.462
19	1.086	1.014	0.467	0.225	0.433
20	0.918	0.922	0.817	0.521	0.587
21	0.876	0.965	0.545	0.432	0.385
22	0.924	0.741	1.038	0.198	0.581
23	0.869	1.093	2.565	2.104	1.843
24	0.861	0.698	1.179	0.598	0.715
25	0.869	1.093	2.565	2.104	1.843
26	0.866	1.190	1.872	1.562	1.354
27	0.762	1.046	1.925	1.285	0.974
28	0.823	0.655	1.199	0.572	0.671
29	0.941	1.147	0.860	0.494	0.396
30	0.967	0.841	0.869	0.517	0.669
31	1.233	1.246	0.540	0.611	0.865
32	0.852	0.838	1.872	1.104	1.093
34	1.039	1.190	0.544	0.354	0.385
35	0.779	0.839	1.179	0.618	0.526
36	0.607	0.784	1.872	1.299	0.964

DEPT	EQUIP	DEPT	EQUIP	DEPT	EQUIP	DEPT	EQUIP
1	2	11	2	21	16	31	5
2	1	12	9	22	16	32	2
3	11	13	6	23	1	34	13
4	26	14	28	24	6	35	3
5	24	15	3	25	1	36	2
6	5	16	19	26	1		
7	3	17	4	27	4		
8	19	18	16	28	8		
9	11	19	14	29	10		
10	29	20	4	30	6		

Table 6.13. The five equipment information measures for the 35 departments examined in section 6.5.3, except that these were derived for 13 categories of equipment and from a sample of 135 departments. In addition the equipment inputs to the production models for these same 35 departments are shown.

i) it would be difficult to categorize these inputs in a common unit of measure, and; ii) the only hospital variables we have are the manpower and non-labour expenses in the different hospital departments (data set D.2) and none of the fixed resources except hospital beds.

From this data set, D.2, we know the number of inpatients in each of 98 diagnostic categories that were treated in each hospital in 1971. These are as much a reflection on a hospital as its input resources, and in actuality, are a superior representation in terms of a radiographic services demand. Evans & Walker (1972) argue that the indices in Eqns. 6.15 and 6.17 when applied to diagnostic proportions represent measures of hospital complexity, while the three measures of Eqn. 6.18 reflect hospital specialization.

> "If we hypothesize that complex cases tend to be handled in a few hospitals with more extensive facilities and more specialized staff, while relatively straightforward cases tend to be distributed more evenly over the hospital system, then we might use the degree of concentration of case type across hospitals as a measure of its complexity. As polar examples, a diagnosis might be of such complexity that it could be handled by only one hospital in the province, or, alternatively, it could be sufficiently straightforward that any hospital in the province could handle it.
>
> In general we expect small hospitals to be more specialized as they are geared to handle a smaller range of cases." (Evans & Walker, 1972).

To reduce the amount of computation and enhance the difference between diagnostic categories these 98 case types were aggregated into 18 groups (Lavasseur, 1972).. The complete case mix sample from data set D.2 consisted of 139 hospitals.

These three sets of five indices representing the diagnostic radiology equipment mix, the diagnostic radiology manpower mix, and the hospital inpatient mix, are used as the measurements which comprise departmental patterns. These patterns are then grouped with the shared near neighbour algorithm described in the previous section. Our goal in this analysis is to classify departments in terms of their resource mixes and departmental-hospital balance.

6.6.3.1 Choice of Variables and Clustering Parameters. It is obviously unrealistic to cluster 100 points in a 15 dimensional space. Two steps were taken to reduce this problem to a manageable size. First, we decided to not investigate the information measures based on uniform prior distributions, i.e. X_t^1 (Eqn. 6.15) and G_t^1 (Eqn. 6.18). This was done because of their relative naivety; that is, they assume maximum uncertainty in the inter-departmental and intra-departmental equipment distributions, $1/n$ and $1/m$ respectively. Second, we chose to examine departmental patterns consisting of only one measurement from each of the three series of indices per department. Hence we have reduced the analysis to a 3 dimensional space where the axes each represent some measure of the equipment mix, manpower mix and diagnostic mix. There are a total of 27 different combinations of variables taking one variable from each of the three representative types, each of these being a series of three different measures.

Recall that we have implied an efficiency interpretation of the resulting clusters.

> "... in a clustering (unsupervised) operation, the question of labelling as a naming procedure cannot and should not be brought up directly. In general, custering cannot provide naming. Only the external interpretation of the clusters can invest them with 'meaning' as related to the problem. If the investigator wants to assign names to the clusters because he feels that the cluster results do reflect the state-of-nature classification, he is free to do so; but the clustering algorithm itself cannot automatically do so with any guarantee that the results are meaningful." (Jarvis & Patrick, 1973).

Although we have imposed an interpretation of the clusters in terms of the radiology department/hospital system, we do not expect that each and every analysis should be meaningful with respect to technical efficiency. In choosing a particular combination of variables (one of 27), the k and k_t parameters, and the method for examining shared neighbours (whether weighted voting or not), we are effectively investigating the same system from a different point of view. It would be foolish to assume that each and every perspective can have the same meaning. As discussed further below, only a few of the many viewpoints we examine are truly meaningful in an efficiency context.

We sometimes forget when reading published research, that the chosen variables and presented technique, are often only the most productive of a set of variables and techniques, and not necessarily deduced *a priori* as the "best" means of solving the problem.

The first decision we confront in applying the clustering algorithm is the choice of parameters and the method for examining shared neighbours that are most appropriate for the sample of points under investigation. It was decided that only those clusters with

5 or more points would be taken as meaningful groups. The reason for this was two-fold. First, because of the automatic scaling feature of the algorithm, clusters with less than five points or departments, and in particular singletons (clusters consisting of only one point), can be interpreted as stray points. Although the number five may appear somewhat arbitrary, it was found upon investigating the clustering results that the vast majority of clusters consisted of either five or more points, or, two or less points. Second, the model samples, and hence the technical efficiency samples, are smaller than the clustering variables samples (100 or 102 points versus 129 points). Therefore, it is possible that even when a cluster does contain five points, the number of departmental efficiencies represented in the cluster could be less than five. Hence it would be difficult to associate an efficiency label with a smaller cluster which could possibly have only one or two departmental efficiencies represented.

In selecting the clustering parameters and voting method we want to: i) maximize the number of sample points contained in clusters (with at least five points); ii) minimize the number of resulting clusters, short of the point where the complete sample defines one cluster, i.e. 3 to 5 relatively large clusters are preferred over a large number of smaller clusters, and; iii) minimize the amount of computation required in clustering a sample, i.e. minimize the value of k.

The results of applying various parameter and methodology combinations to the sample of patterns whose measurements consist of the X_i^2 information index (see Eqn. 6.17) on the equipment, labour and diagnostic mixes are shown in Appendix E. The complete results of applying the clustering scheme with $k = 10$, $k_t = 6$ and unweighted voting is provided, as well as a listing of each department in the resulting clusters with at least five points for

the following parameter combinations, all with unweighted voting: $k = 15$ and $k_t = 12, 10, 8$; $k = 10$ and $k_t = 7, 6, 5$; $k = 7$ and $k_t = 5, 4, 3$.

Three interesting features emerge from these results: i) as k_t is reduced for a fixed value of k, the total number of clusters (including those with less than five points) decreases, while the clusters themselves increase in size; ii) for any particular value of k_t, its clusters are formed by either the merging of two or more of the clusters found from the next largest value of k_t, the merging of those clusters which had previously consisted of less than five points each, or both, and; iii) for any two values of k, essentially the same clusters are found with appropriate values of k_t, i.e. compare $k = 15$, $k_t = 12$ with $k = 10$, $k_t = 7$, or $k = 10$, $k_t = 6$ with $k = 7$, $k_t = 4$ (all in Appendix E). With respect to the three criteria outlined above for choosing parameters, it appears that $k = 7$, $k_t = 3$ with unweighted voting, is the best combination for this particular data set.

In order to find the best parameter combination for weighted multiplicative voting we used $k = 7$ only. This was done for three reasons: i) to reduce the amount of computation; ii) because it was the best for unweighted voting, and; iii) the optimality of the k parameter appears more dependent on the subsequent choice of k_t than on the value of k itself. The clustering results for $k = 7$ and $k_t = 55, 50, 45, 40, 35$ and 30 are also provided in Appendix E. Rather than choose a particular value for k_t that is best, it was decided that because of the large range that is admitted, we would settle on $k_t = 45$, 40 and 35 as the best subset of these results.

6.6.3.2 The Results of Clustering the Information Indices.

The above parameter analysis was applied to only one of a possible 27 combinations of variables. It was decided that the computational expense involved in examining each of these combinations in a similar manner was not appropriate to the immediate goals of this work. Hence we applied these four schemes, unweighted voting with $k = 7$ & $k_t = 3$, and weighted multiplicative voting with

$k = 7$, $k_t = 45, 40, 35$ to the 27 different combinations of variables, for a total of 108 cluster analyses.

A technical efficiency, derived from the production models, is associated with most of the points or radiology departments in the cluster samples. As a result of omissions and errors in some parts of our data base, some of the departments included in clusters are excluded from the production model samples, and vice versa. For each cluster containing at least five points, in each of the 108 cluster analyses, we constructed an efficiency histogram based on the departments included in the cluster.

A cluster analysis will divide a sample of departments into a set of mutually exclusive groups. Thus, for any one cluster analysis, an efficiency histogram can be constructed for each cluster based on the departments included in it and their efficiency results from a particular production model. Hence the set of histograms corresponding to the resulting set of clusters for this analysis is essentially a decomposition of the efficiency histogram for the complete production model sample. Per cluster analysis, the cluster histograms were compared to the model histogram. This was done to see whether the particular grouping of departments resulting from this analysis, corresponds to a decomposition of the model histogram which explains part of the difference in efficiency between departments.

Two points are worth repeating. First, although the weighted multiplicative schemes are relatively flexible, the cluster parameters were actually attuned to only one of the 27 data sets. Second, this classification technique operates independent of any efficiency consideration. Hence it is not surprising that only a handful of the 108 different ways of viewing a radiology department actually appear to explain the difference in efficiency between departments.

Our initial hypothesis was that there are some combinations of the information measures that are conducive to higher efficiencies and others to lower efficiencies. This does not necessarily imply

that we expect some continuous relationship between these indices and the technical efficiencies in the clustering space. If we label clusters with respect to the mean departmental efficiency of the departments in each cluster, we can then compare these means and the corresponding histogram shapes to the mean efficiency and histogram shape of the entire model sample. This will indicate whether the clusters do in fact divide the sample into departmental subsets of high and low efficiency.

Consider the example shown in Fig. 6.18. Here we have a two dimensional data set clustered into 5 groups where the letter inside of each cluster indicates whether the mean efficiency of departments in a cluster are above (H) or below (L) the mean efficiency of the complete sample. Clusters 1 to 4 have lower than average efficiency, whereas cluster 5 has above average efficiency. If a department is included in any one of clusters 1 to 4, we can expect it to have lower efficiency, and if in cluster 5, to have higher efficiency. This implies that the five different clusters in Fig. 6.18, which each exhibit some common property relating to the measurements upon which their pattern vectors are based, can be combined in terms of an efficiency interpretation into 2 groups; that is, cluster 5 and the union of clusters 1 to 4. Our interpretation of these two groups is that there is some relationship or balance between the variables represented by the axes that is conducive to more efficient production (cluster 5), whereas other combinations of variables (clusters 1, 2, 3 or 4) represent an imbalance with respect to efficiency.

In Figs. 6.19 to 6.24 we present the results of three clustering analyses for the sample of efficiencies from model M.A (see section 5.4.3). Figs. 6.19, 6.21, and 6.23 provide: the clustering parameters and voting method used; the equipment, manpower and diagnostic mix variables (information indices) upon

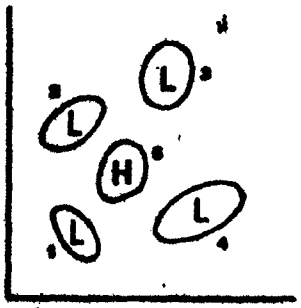


Fig. 6.18. A two-dimensional data set clustered into 5 groups where the letter inside each cluster indicates whether the mean technical efficiency of departments in the cluster are above (H) or below (L) the mean technical efficiency of the complete sample.

Information Measures
Mean Values

X_i^2	X_i^2	X_i^2	Sample Size	Mean Eff	
			100	76.23	Complete Sample
0.93	1.78	0.94	40	82.70	Cluster 1
1.11	0.83	0.98	9	72.36	Cluster 2
0.98	0.57	1.07	11	69.53	Cluster 3
0.87	0.55	0.95	15	71.86	Cluster 4
1.07	4.06	1.00	6	63.35	Cluster 5

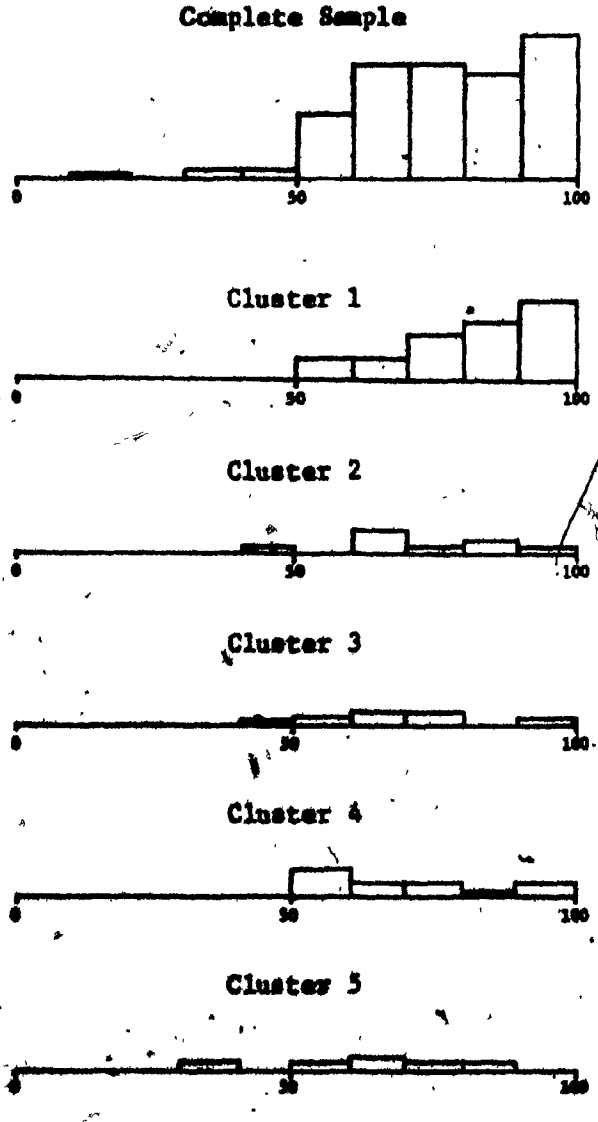


Fig. 6.19. The efficiency histograms derived from the efficiencies of experiments (as measured by model M.A) in each cluster with 5 or more points that resulted from applying the clustering algorithm of Jarvis and Patrick (1973) with $k = 7$, $k_s = 3$ and unweighted voting to information measures based on the radiographic equipment mix (X_i^2), radiology manpower mix (X_i^2), and hospital diagnostic mix (X_i^2).

Sample Size	Mean Eff	
100	76.23	Complete Sample
40	82.70	Cluster 1
51	70.07	Cluster 2+3+4+5

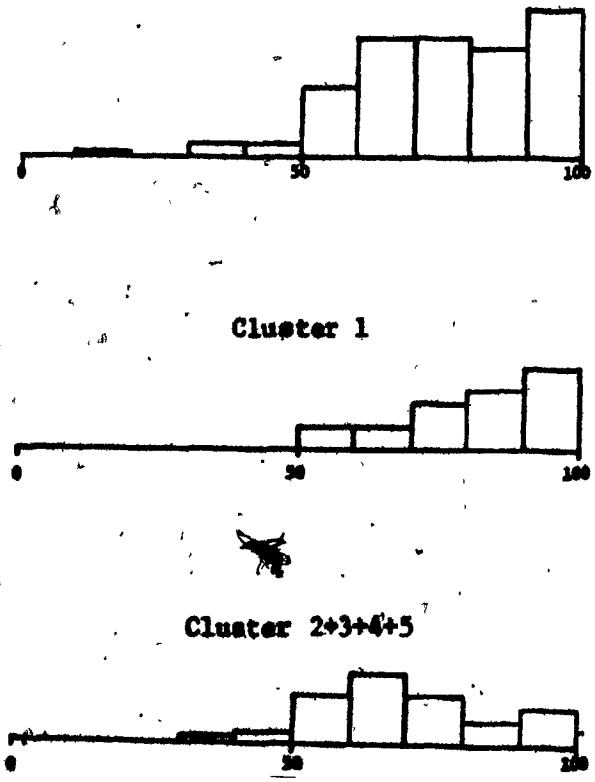


Fig. 6.20. The result of combining the efficiency histograms corresponding to those numbers from Fig. 6.19 with similar mean technical efficiencies.

Information Measures
Mean Values

G_i^3	G_i^2	G_i^2	Sample Size	Mean Eff	
			100	76.23	Complete Sample
0.53	0.22	0.10	43	73.52	Cluster 1
1.17	0.72	0.20	44	81.73	Cluster 2
0.84	1.43	0.11	4	71.86	Cluster 3

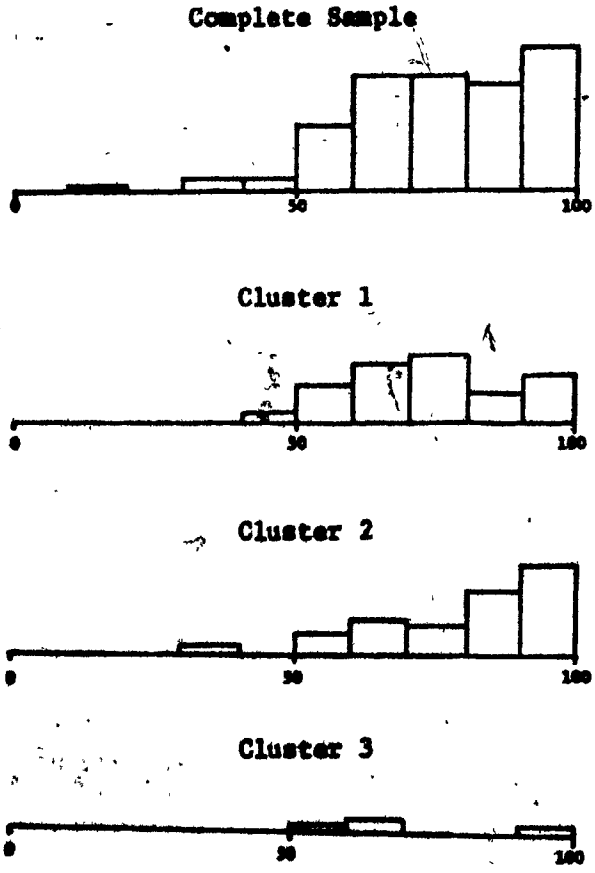


Fig. 6.21. The efficiency histograms derived from the efficiencies of departments (as measured by model M.A) in each cluster with 5 or more points that resulted from applying the clustering algorithm of Jarvis and Patrick (1973) with $k = 7$ $k_t = 40$ and weighted multiplicative voting to information measures based on the radiographic equipment mix (G_i^3), radiology manpower mix (G_i^{*2}), and hospital diagnostic mix (G_i^2).

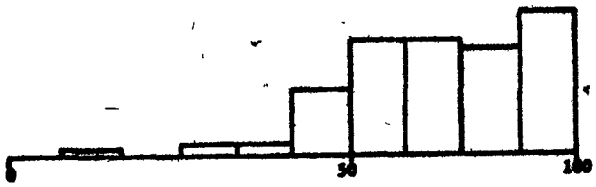
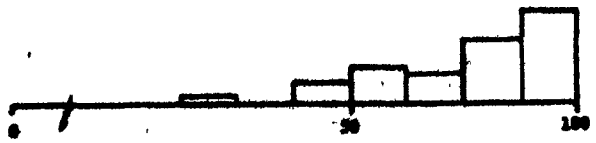
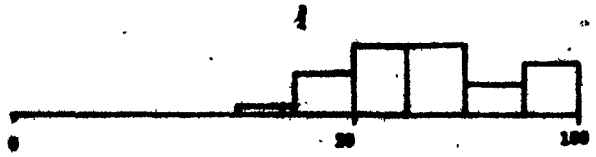
Sample Size	Mean Eff	
100	76.23	Complete Sample 
44	81.73	Cluster 2 
47	73.38	Cluster 1+3 

Fig. 6.22. The result of combining the efficiency histograms corresponding to those clusters from Fig. 6.21 with similar mean technical efficiencies.

Information Measures Mean Values					
G_i^3	X_i^2	G_i^2	Sample Size	Mean Eff	
			100	76.23	Complete Sample
1.65	2.48	0.38	18	75.69	Cluster 1
0.38	0.69	0.07	11	68.58	Cluster 2
0.60	0.90	0.09	36	75.58	Cluster 3
0.91	2.11	0.09	28	82.32	Cluster 4

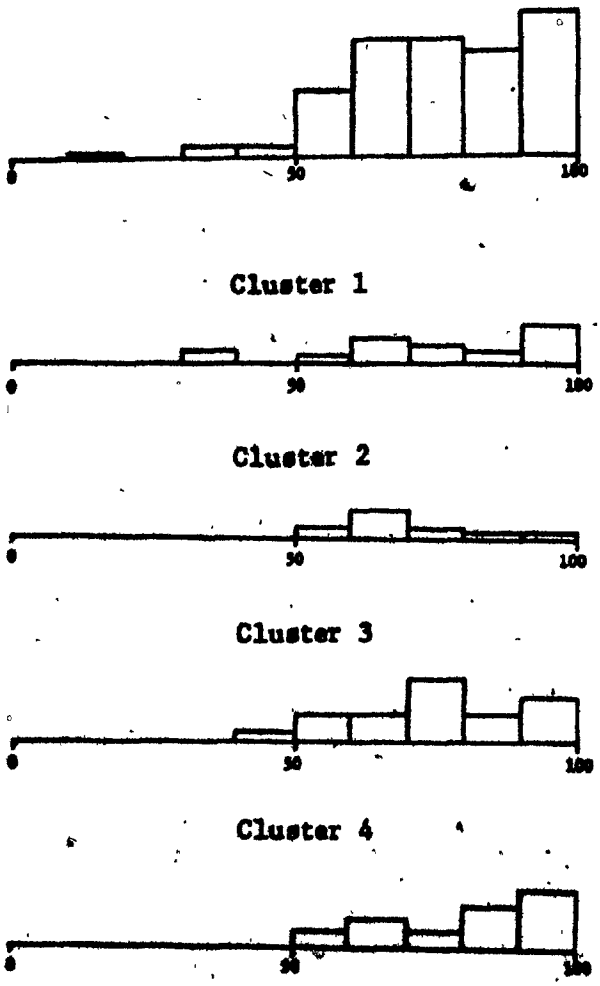


Fig. 6.23. The efficiency histograms derived from the efficiencies of departments (as measured by model M.A) in each cluster with 5 or more points that resulted from applying the clustering algorithm of Jarvis and Patrick (1973) with $k = 7$, $k_t = 40$ and weighted multiplicative voting to information measures based on the radiographic equipment mix (G_i^3), radiology manpower mix (X_i^2), and hospital diagnostic mix (G_i^2).

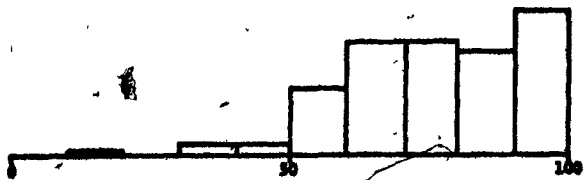
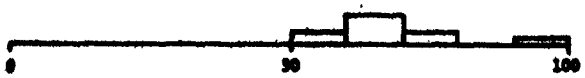
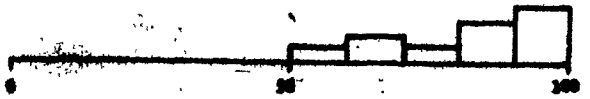
Sample Size	Mean Eff	
		Complete Sample
100	76.23	
		Cluster 2
11	68.58	
		Cluster 1+3
54	75.61	
		Cluster 4
28	82.32	

Fig. 6.24. The result of combining the efficiency histograms corresponding to those clusters from Fig. 6.23 with similar mean technical efficiencies.

which the clusters are based; the complete M.A efficiency histogram (see Fig. 5.10b); the efficiency histogram for each cluster with more than 5 points, and; the mean efficiencies and number of included departments for the model and each cluster. Figs. 6.20, 6.22 and 6.24, show the results of combining clusters and the corresponding histograms in terms of their mean efficiencies as described above (Fig. 6.18) for the examples in Figs. 6.19, 6.21 and 6.23, respectively. In the first two cases the samples are divided into two groups, one of high efficiency and one of low efficiency, whereas in the third example, the model sample is broken into groups of low, average and high efficiency.

The initial choice of 5 points for recognition of a cluster is appropriate. If the measurements comprising the departmental vectors are indicated as (equipment index, manpower index, diagnostics index), then we see that for variable set (X_i^2, X_i^2, X_i^2), 81% of the model sample is contained in 5 clusters, for (G_i^3, G_i^3, G_i^2), 91% is contained in 3 clusters and for (G_i^3, X_i^2, G_i^2), 93% is contained in 4 clusters. This is not surprising as the parameter set for the unweighted voting scheme was derived for the first example. The latter two examples resulted from the flexibility of the multiplicative voting technique when applied with the three different values of k_t.

In examining all three of these examples, one important trend consistently emerges; that is, the technology-manpower mix balance appears to be the overriding factor in explaining departmental efficiency. This is in contrast to the relatively uniform values for either the hospital complexity (X_i^2) or specialization (G_i^2) between clusters. Departments are apparently classified in these cluster analyses as a function of their internal resources, almost independent of the hospital diagnostic mix measures. The subsequent

correlation of these clusters with the departmental efficiencies, seemingly unrelated to the diagnostic mix, is in part explained by our initial choice for the production model output measure. The volume of professional units, as opposed to the number of diagnostic examinations, effectively normalizes for different examination mixes, which in turn, is dependent on the diagnostic case mix. Although the clustering of information measures appears adequate for quantifying what we have labelled the production organization of a department, it would seem inappropriate for dealing with the radiographic services demand-supply factor. The fault lies with the hospital measure and not the general approach.

To summarize then, we have classified departments based essentially on their input resources in a manner that is dissimilar to that of the production process classifications of the models (see section 6.5). For at least a few of these classification schemes (three are shown), we see that a correlation exists between the departmental groupings and a level of departmental efficiency. It is apparent that the analysis is, as yet, incomplete. In the next chapter we suggest different ways in which to improve on this clustering process.

Two important questions must now be addressed. First, do the clusters from the above analyses divide the sample of departments into subsets that are different from those that could have been derived by using somewhat simpler variables, as were introduced in section 6.3? In other words, does the cluster analysis really provide new information on radiology departments? Table 6.14 presents the cluster results for the example in Fig. 6.19, with each cluster of departments divided with respect to their output volume (professional units), as in Table 6.2

From this table we see that the cluster analysis essentially isolated departments that are small (cluster 1 and cluster 5),

	Number of Depts	Mean efficiency	Number of Depts with OUTPUT: < 500,000	> 500,000 & < 1,000,000	> 1,000,000
Model M.A	100	76.2	62	19	19
Cluster 1	40	82.7	34	3	3
Cluster 2	9	72.4	2	2	5
Cluster 3	11	69.5	3	3	5
Cluster 4	15	71.9	6	6	3
Cluster 5	6	63.4	6	0	0

Table 6.14. The cluster results for the example in Fig. 6.19 with each cluster of departments divided with respect to departmental output volume (professional units).

intermedfate, though covering a broad range (cluster 4), and large (cluster 2 and cluster 3). Of particular note, is the isolation of clusters 1 and 5, both of which essentially consist of producers with less than ½ million P.U., but which result in significantly different technical efficiencies. While we previously recognized that larger departments were easily differentiated from the complete sample in terms of their efficiencies (see Fig. 6.2a), we could not at the same time make any conclusive statements about the smaller departments. Although the resulting clusters of departments may appear obvious (see Fig. 6.2a), this is only so in terms of the efficiency results, whereas they were constructed completely independent of these results. It would appear then, that the information theoretic-clustering approach we have used is at least partially capable of providing a unique vantage point for investigating departments that is meaningful in a production context.

The next question is: how much of the technical efficiency of a department is explained by virtue of its membership in a particular cluster or set of clusters? The answer to this can be found by returning to one of the techniques described in section 6.6.1 for analyzing multiple sources of variation. One possible approach to introducing the results of the clustering program to these analyses would be in terms of a binary variable. For example, the results in Fig. 6.20 would indicate that if a department is in the above average efficiency cluster, it would be assigned a value of 1 and 0 otherwise. Another possibility is to use an inverse distance measure from the center of mass of the above average efficiency cluster.

The histograms in Figs. 6.20, 6.22 and 6.24 indicate that the cluster analysis does not provide a complete explanation

of departmental efficiency differences; that is the complete model efficiency histogram has not been dissected. There are some departments in the high efficiency clusters with below average efficiency, and similarly, in the lower efficiency clusters there are some departments with above average efficiency. Thus, the efficiency results of the cluster analysis, as measured by a dummy variable or an inverse distance measure per department, should be only one of a set of variables provided to the multivariate analysis. Typically, one might also introduce the number of hospital beds, which as we saw in section 6.3.1, appears to isolate university hospitals in terms of efficiency. The point then, is that we have provided previously unavailable information on departments which partially removes the burden of mathematically forcing an "explanation" of efficiency on the more readily available, though not necessarily relevant, data.

The effect of a technological investment, a manpower change, or both on the efficiency of a department could be monitored by the change of the cluster in which a department resides, if any, though more specifically quantified through a change in the relevant inputs to the multivariate analysis.

We make no pretence that the present analysis is either uniquely appropriate or all encompassing, nor do we apologize because it is not. There are no easy solutions. Kimball & Lorant (1973b) argue that the "obvious" may be far from that, from which they conclude that descriptive approaches, such as we have proposed, may ultimately be the more substantive effort towards understanding system behaviour.

6.7 Summary

In this chapter we have investigated a broad spectrum of topics under the general heading of a post-model analysis. Our starting point was a consideration of the production model in terms of technological investment decisions. We subsequently moved on to an

examination of a sensitivity analysis of the model. The primary conclusion from both of these investigations is that this type of production model can serve a dual purpose by defining problems and raising issues, as well as by providing answers. We then described the desirability of being able to quantitatively account for the behaviour of radiology departments as observed in the model. This was seen as particularly relevant to understanding the effect of technological resource allocations on the behaviour of the system.

The attempt to account for the technical efficiency of departments as a function of the "obvious" variables did not produce the desired result. It was therefore concluded that a systematic procedure for the definition of significant variables is required. Towards that end we took two steps. First, we presented a means to quantify the resources of a department which included much of the information hidden in the production model variables and which is based on information theory. A further development of the information measures is a scheme to monitor the allocation of technological resources in the health sector in terms of the question of centralization versus decentralization. Second, we described a detailed framework in which a quantitative analysis of departmental behaviour could be pursued.

It became apparent on investigating the analysis of departmental behaviour that the choice of inputs to this analysis was ill-defined. We decided that a viable means to improving the situation lay in a detailed examination of the departments themselves. This was accomplished by classifying departments using a clustering technique applied to indices on the equipment, manpower and diagnostic case mixes of the department and hospital. These indices were calculated using the proposed information measures. The results of this preliminary departmental classification effort indicate that this type of approach can provide valuable input to a more established framework, as mentioned above, for the analysis of health services utilization.

CHAPTER 7

DISCUSSION

In this final chapter the major features of the work covered in this thesis are summarized. Extensions of the present work and possible future studies are indicated. A primary catalyst for the developments in this project has been the necessity of utilizing a predetermined data base. Hence, it is hardly surprising that the majority of the studies we propose will require the collection of more extensive data.

7.1 The Diagnostic Radiology Department Model

The major focus of this study has been the development and analysis of a mathematical model of a diagnostic radiology department. The model that was adopted falls within the general category of economic models known as production functions. These functions describe the technical relationship between production inputs and outputs, and indicate the maximum amount of output capable of being produced by each and every set of specified inputs or factors of production.

It was argued in sections 4.5.1, 5.2.1 and 5.2.2, that with an appropriately chosen set of inputs and output, the production of diagnostic services is represented by a linear homogeneous production function; that is, a production function exhibiting constant returns to scale. Recognizing the theoretical difficulties inherent to the estimation of a continuous form of this function, a piecewise linear approximation to it was identified by solving a linear program (see sections 4.3.2 and 4.5.4).

The resulting production function is itself represented by a

linear program, and it is this program which we have labelled as a model of a diagnostic radiology department.

Twenty-two different models of a radiology department were estimated in Chapter 5. Each model is identified with the same linear programming technique, though is different in terms of the production variables comprising their input sets. From this set of models, two were chosen as the most credible representations of a radiology department. This was accomplished by comparing the efficiency histograms resulting from the sample of departments used in the identification of each model in terms of a set of credibility criteria. The two models are denoted as M.A and M.B.

The discussion which follows is of a general nature. Unless explicitly stated otherwise, when describing "the model" of a diagnostic radiology department we refer to either of the two models cited above.

7.1.1 The Role of the Model in Decision Making. The strength, and in turn, the significance of the diagnostic radiology department model is that it satisfies the set of model building criteria developed in Chapters 2 and 3. Specifically the model provides a unique description of this health care delivery facility. This description is attuned to the decision environment controlling the implementation of new facilities and the types of decisions that must be pursued therein.

The linear programming production model maps a set of production inputs (equipment-years, full-time radiologist man-years, part-time radiologist man-years, technician man-hours and non-labour expenses) into an output capacity (diagnostic radiology professional units). This capacity output can be estimated for any combination of input production variables, and hence the model can be applied to any diagnostic radiology department in an acute hospital in the province. By comparing the observed output of a department to its capacity output, one can calculate the technical efficiency of a department; that is, the extent to which its productive capabilities are actively utilized.

In addition, by changing one or more inputs to a department the allocation of resources to that department may be simulated, and the model can then estimate a capacity output for the "new" set of inputs. This can then be compared to the original capacity output to approximate the effect of a resource change on a department's ability to produce increased output.

Changing only one of the input resources often represents an unrealistic radiographic production situation. For example, a change in the equipment input would normally be accompanied by a change in non-labour expenses and possibly the technician input as well. Although simultaneously increasing more than one input can and should be pursued with the model, changing individual resources still represents an important analysis as it will provide information on the present balance of resources in a department. For example, a small capacity change corresponding to an increase in the equipment input would indicate that one or both of the manpower groups are being fully utilized (relative to the other resources), whereas a large capacity increase would indicate that they are presently being under utilized.

Having recognized the impracticality of estimating a theoretical and unattainable capacity output, it was decided that a department's performance can reasonably be measured relative to the "best" or most efficient departments in the province. Although these "best" departments may not be actively using 100% of their theoretical capacities, they at least represent an attainable level of production. A linear programming scheme was used to identify the most efficient producers or departments in the province, which were then used to form the linear programming production model of a radiology department.

Therefore, in examining a department (a specific combination of input production variables) with the model, the resultant output capacity is that which could be produced by combining the efforts of the most efficient production processes using the same input levels as the department under investigation. Hence, the model describes an output level which is attainable by using modes of production that have

actually been observed. The resultant technical efficiency i.e. the observed output divided by the capacity, is once again, an efficiency measure relative to the best producers.

As the identification methodology is non-statistical and ultimately only 10% of the sample of departments actually define the production functions of models M.A and M.B, it is felt that the effect of input changes on the capacity output is subject to a qualitative rather than a quantitative interpretation. It is of little consequence whether the increase in the equipment input results in a 10, 20 or 30% increase in the capacity. What is important, is whether the capacity change is greater or less than the capacity changes corresponding to one of the manpower inputs or the material input, for this provides information on departmental resource utilization. As with the technical efficiency, the result of changing an input is relative.

The question which naturally arises is: what does the diagnostic radiology department model do for planners?

An application for new radiographic equipment is in actuality a request for an increase in the capacity output of a radiology department. The model supplies a measure of a realizable capacity, the utilization of that capacity output, and the potential ways of increasing that capacity both by equipment allocations and otherwise. This information has previously been unavailable to planners.

One would initially favour providing equipment funds to those departments which have a high technical efficiency; that is, departments currently using all or most of their capacity output. However, as the model is only a rough approximation to the behaviour of a department, certain qualifications are necessary.

Consider a radiology department which makes an application for new equipment and is observed to be operating at 70% efficiency with model M.A, where the average departmental efficiency for the sample upon which this model was based is 76%. This would not necessarily imply the refusal of the application, although the department should be called

upon to justify its inefficiency. Are there highly specialized x-ray rooms which, though necessary in terms of the patient population in the hospital, are used only part-time? Is the department centralized in one location in the hospital hence requiring a large effort for transportation of patients, requisitions and reports, or is the department decentralized, having x-ray rooms attached to emergency rooms, outpatient clinics, etc.? Is there a forecasted increase in the demand for diagnostic services from the radiology department to the extent that a large part of the "excess" 30% of capacity will be consumed in the near future? Is the hospital expanding its capacity to treat patients in a particular diagnostic category such that this patient group requires services from diagnostic radiology which are presently unavailable?

If, on the other hand, the applying radiology department is observed to operate at 90% efficiency, this would not necessarily imply the immediate acceptance of their equipment application. For example, testing various input increases in the model might indicate that a change in one of the manpower inputs would result in the most effective increase in capacity. This in turn reflects on the production setup of the department and is particularly important in light of the fact that the cost of fixed capital investment is generally large.

At present, medical equipment planners establish a <u>priority listing</u> of those departments whose equipment applications have been accepted in a particular year. This is necessary as budgetary limitations might allow funding only a subset of accepted applications. The rank ordering of departments in this list is prescribed by the planners' perceptions of the need for equipment in these departments, which in turn, is the result of highly subjective evaluations. The model can be used to rank order departments in a more objective manner. Two examples that come to mind immediately would be to list the approved departments in descending order of: i) their technical efficiencies, or; ii) the effect of an equipment change on their capacity outputs.

The model can also be used to evaluate the productivity of individual x-ray rooms. This is accomplished by simulating a department with one machine, one full-time radiologist and varying technician and material inputs. If one knows the type of examinations that are performed in a particular room, then by calculating the average number of professional units per examination in that room and using the output capacity (in professional units) from the model, one can estimate the x-ray room capacity for the type of examinations pursued in the room. This analysis can be applied to either general or highly specialized x-ray rooms, both of which would result in equivalent professional unit capacities, but significantly different diagnostic examination capacities.

Government planners have indicated that 12,000 examinations per year from a room would represent a highly productive general room. Using an average of 14 professional units per examination (see Fig. 5.6), this output is equivalent to 168,000 professional units. From the model (model M.A specifically) we have estimated an output capacity for an x-ray room, i.e. one piece of equipment, one full-time radiologist and the combination of technician man-hours and materials dollars which results in the maximum output, to be 166,000 professional units. This would indicate that our model is very similar to the equipment planners' own perceptions of the system yet at the same time provides a previously unavailable multi-dimensional viewpoint.

Another possible application of the model is for individuals in Finance, rather than Planning, in the Ministry of Social Affairs. Their interest is in the price efficiency of radiology departments. The principal measure used to monitor the system is the investment per unit output ratio; that is, the input budget for a radiology department, in dollars, divided by the number of professional units produced in the year encompassed by the budget. In Fig. 7.1 we show a plot of the technical efficiency versus the investment per unit output ratio for each department analyzed by model M.A.

Two features of particular interest can be seen in Fig. 7.1. First, the results of the model correspond in a very general fashion to

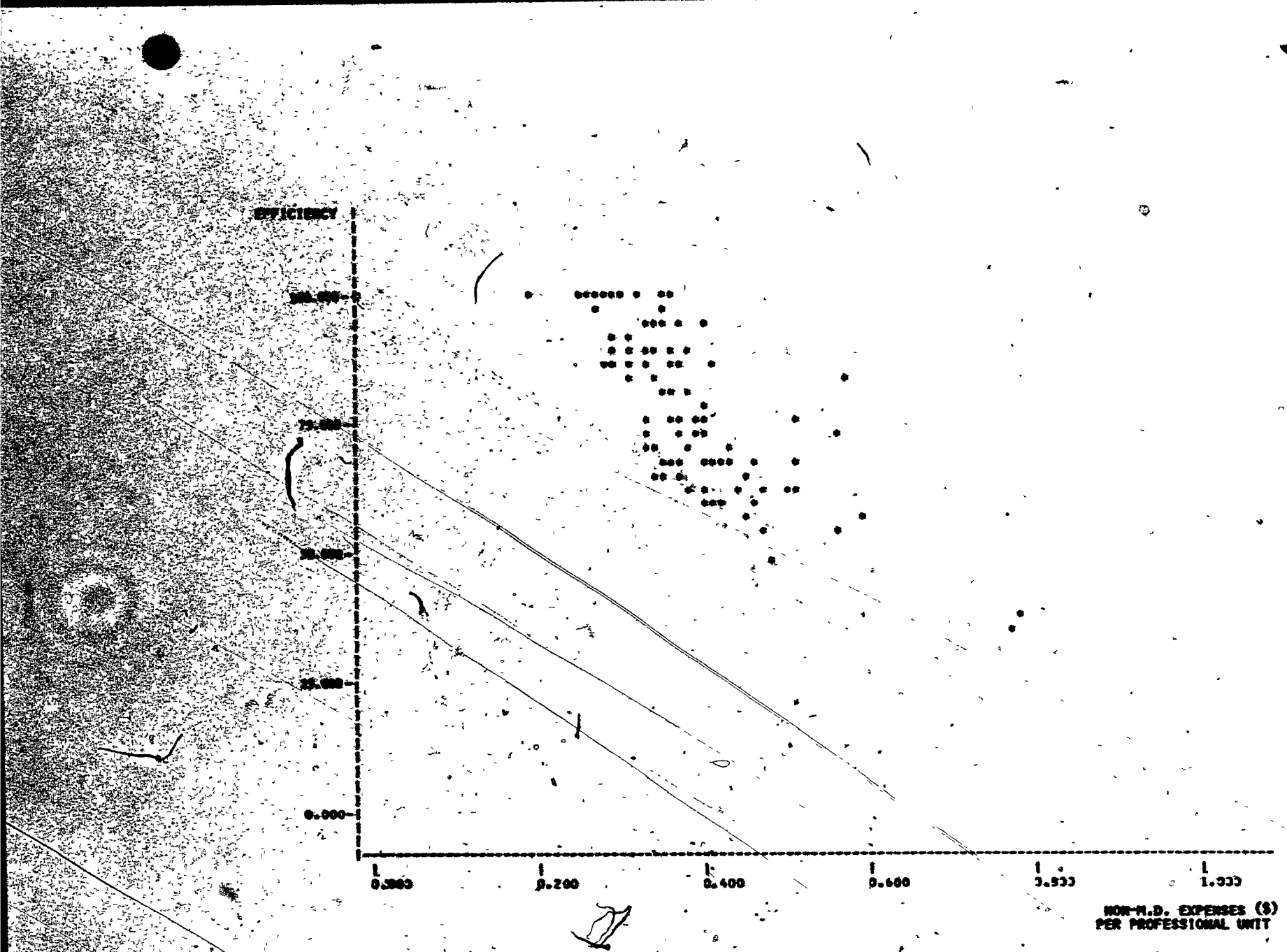


Fig. 7.1. The technical efficiency of radiology departments as measured by model M.A versus the departmental budget divided by the number of professional units produced.

what people in Finance see as important. That is, there is a strong correlation between the technical efficiency of a department and the price efficiency; the lower the dollar per output ratio, the more price efficient is a radiology department. Second, we see that at most dollar per unit output levels there exists a range of possible technical efficiencies. For example, at $0.35/professional unit the technical efficiencies range from 60 to 100 percent. This would indicate that departmental resources can be actively utilized to varying degrees yet have the same price efficiency. Hence the model can provide valuable information by differentiating between departments at the same price efficiency level, and at the same time, demonstrate that some departments can have an above average technical efficiency but below average price efficiency and vice versa.

7.1.2 Advantages of the Present Model. The significance of the diagnostic radiology department model is that it admits a new perspective from which to evaluate applications for equipment, and one that is particularly relevant to the types of decisions that must be made by government planners regarding medical equipment investment.

Of particular importance in terms of implementing this construct in the decision making environment in government is its conceptual simplicity. Consider a sample of radiology departments with observations on only one input production variable and one output variable per department. If we calculate the input-output ratio for each department, it is a simple matter to isolate the most "efficient" department; that is, the one with the lowest input per unit output ratio. The efficiency of all the other departments in the sample, relative to the most efficient one, can be found by comparing their input per unit output ratios to that of the most efficient department. In effect, the linear programming production model pursues the same exercise except with more than one input.

The model is highly adaptive; that is, new information is easily integrated into its structure. This information must of course be in terms of the variables included in the present model, and as such,

would be acquired as observations on departments not included in the sample upon which the model is based. An expanded sample of departments would not require the re-identification of the model. Rather, each of the new observations could be tested with the present linear programming production function. If the calculated efficiency for one of these departments is 100 percent, then that department would be added in the form of another column of the **D** matrix and a corresponding addition to the **c** vector in the objective function of the linear program (see Chapter 5). In this manner, a more refined piecewise linear approximation to the continuous production function for a diagnostic radiology department can be achieved.

Admittedly, the model does not represent a detailed picture of a radiology department. Yet the model is comprehensive in that it includes three important macro-features: i) each of the four principle production inputs are present; ii) the model exhibits the "law" of diminishing marginal productivity for each of these inputs, and; iii) the model allows substitution between inputs. It is apparent that had we pursued a highly detailed description of a diagnostic radiology department, the resulting model would have exhibited properties unique to the production of radiographic services. As it stands now, and this is a major advantage of the model, the identical modeling technique can be applied to other hospital departments, such as laboratories for example.

Our requirement of only using data available to this project in constructing the model has resulted in several advantageous model features: i) the very "simplicity" of the model which was discussed above; ii) in its present form the model is readily applicable to real decision problems (see Figs. 5.12b and 5.16b), and; iii) the model is geared to the utilization of information which is readily accessed from the government's regular data collection efforts.

The diagnostic radiology department model has been formally presented to medical equipment planners in the Ministry of Social Affairs. Their response was very positive and the actual process of fine-tuning

this construct to integrate it into their decision making process has begun. Naturally it would be premature at this stage to claim that our model will be a complete success and that it will actually be used to support decision making.

Nevertheless, we may conclude, that the derivation of model building guidelines from an examination of both the system under investigation and the decision environment in which the construct is to be implemented is basically sound. We believe it to be representative of the type of approach necessary for the development of quantitative tools aimed at improving the management of the health care delivery system.

7.1.3 Models and Health Planning. Mathematical planning techniques and models have as yet met with little success in aiding planners. Shuman *et al.*, (1974) attribute this to the fact that "the health planning problems that have been defined and solved (mathematically) do not resemble the problems for which rational decisions must be made." They claim that "the problems that have been attempted have not been well formulated," and in addition, "the usual output measures such as quantity, time or dollars are inadequate for decisions." Although we concur with these observations, at the same time, it is felt that they are symptomatic of a more fundamental problem.

The rejection of planning models by decision makers is not unique to the health sector. In section 2.1 we discussed this same phenomenon in the context of corporate planning. We believe that the basic problem has been one of conceptualizing mathematical models and techniques independent of the decision environment in which they are to be implemented. The term decision environment encompasses both "the types of decisions that must be made and the types of constraints imposed upon the planning system" (Shuman *et al.*, 1974). Yet equally important, it should include the actual decision making process and how it is affected by the particular decisions and constraints of an organization.

In Chapter 2 we introduced the model of the decision making process proposed by Morton (1971). This model consists of three

successive phases (intelligence, design, and choice), each of which are divided into three successive subphases (generation, manipulation, and selection). The decisions pursued by planners in the government and the legislative constraints precipitating these decisions were associated with the various phases and subphases of the decision making process model. Subsequently, we deduced that the manipulation subphase of the design phase of the planners' decision process is, at this time, the area for which implementation of a quantitative decision making aid is apt to be the easiest and most productive.

By integrating the function of this part of the decision process with certain health care policy goals of the government and features of diagnostic radiology, a set of guidelines were proposed for the development of a quantitative tool. Morton pursued a similar approach in a corporate environment for deriving the design criteria of a computer-based support for management decision making.

It is apparent that this approach can be applied to any health planning agency, and that for each such agency, there will be a different description of the various phases comprising its decision making process. Once a particular facet of the decision process has been delineated, its features and requirements will provide a unique vantage point for examining the health care system. At that point, it should be possible to describe the most appropriate quantitative decision aid with respect to the decision making needs of an agencies decision environment and the realities of the health delivery system of interest.

In effect, we are advocating a <u>detailed investigation of health planning decision environments</u> as prerequisite to an examination of the health system for which they are responsible. At present this approach is all but absent in the health planning literature. We suggest that this at least partially accounts for the general lack of implementable models for aiding planners.

The procedure that has been outlined above is not a panacea for the problems of developing credible health system planning techniques

and models, and in particular, for the difficulty in quantifying the many qualitative variables in this type of system. Yet as demonstrated in this thesis, an *a priori* investigation of the decision process and mandate of a health planning agency can result in the delineation of a particular facet of the health care delivery system for which quantitative tools are readily applicable. We therefore conclude that this is the most pragmatic approach to improving the "real world" decision making problems in the health sector.

7.1.4 What the Model Does Not Do. The linear programming production model of a diagnostic radiology department describes the purely technical facet of diagnostic radiology; that is, the transformation of physical inputs into a physical output. Although this output is associated with patients, it does not really contribute to their treatment process. Rather, it is the diagnostic information that can be derived from radiographic examinations that will affect a patient's treatment program. The model associates a measure of productive efficiency for the provision of services to a radiology department which excludes, and is independent of, the effectiveness of these services for the patient population in the hospital. Thus, we have examined diagnostic radiology as an isolated component of the larger hospital system.

In section 3.3.1 we discussed a growing concern with the overconsumption of radiographic services. This is a patient related concept in that it assumes some diagnostic service standard per patient or per patient type which is currently being surpassed. The production model cannot differentiate between those services, as measured in professional units, which are medically necessary and those which are not. Yet the overconsumption of services is a medical problem relating to the practice of radiology and the system by which physicians requisition radiographic services for their patients. As such it is independent of the technical efficiency of the production environment in which these services are produced.

In addition, two points are of particular note. First, the output

service capacity of a department is the same regardless of whether a part of this capacity is utilized for medically unnecessary services. Second, because of the voiced concern with overconsumption, we assume it to be a generally widespread phenomena. In this case, the model, wherein all measurements are relative, effectively eliminates this facet of a medical situation from the analysis of productivity.

It is apparent though, that studies aimed at directly relating a hospital's patient population with the provision of radiographic services is necessary in order to determine the nature and extent of overconsumption of services. Yet as indicated above, we believe this is essentially, in a broad sense, a medical problem, and as such, the instruments necessary to alleviate it are not available to medical equipment planners in the government.

7.1.5 Extensions. The particular type of mathematical model that we have chosen to represent the production of services in a radiology department was adopted, in part, because of the limited data that was available for this project. On the other hand, the data base was sufficiently diverse that more than one possible candidate existed for most of the production variables in the model. The delineation of a suitable data subset from which a model could be quantitatively identified was based on the fundamental assumptions underlying the model; that is, homogeneity of the production variables and constant returns to scale in production. These two assumptions, in turn provide the primary sources for possible extensions of the present methodology.

Two avenues are available for increasing the homogeneity of a production variable: i) disaggregation into more than one new variable based on its component parts, and; ii) a less heterogeneous unit of measure for the same production variable. For the present study, either one of these efforts would require the collection of more data. Reducing production variable heterogeneity is seen as important

because "when there are differences between firms in the average quality (or, more strictly, in the distribution of qualities) of a factor, then a firm's technical efficiency will reflect the quality of its inputs as well as the efficiency of its management" (Farrell, 1957).

Consider the non-medical manpower input to the model which is measured in man-hours. Recall that we have aggregated the seven non-MD labour classes available from data set D.1, each of which can be considered to contribute differently to the production of diagnostic services, into three manpower categories (see Chapter 5). Model M.A uses only one of these new labour classes, whereas model M.B uses an aggregation of two of these classes to form the technician input. This input, in either model, can be made more homogeneous by using its original components. In the case of model M.A this would result in two non-MD labour inputs rather than one, and for model M.B, six instead of one. Yet with the present data base such changes would lead to meaningless results.

As discussed in Chapter 5, there is a direct correlation between the credibility of a model, i.e. a model resulting from a particular set of production variables, and the relationship between the number of inputs to the model and the number of points in the sample upon which the model is based. That is, for a fixed sample size there appears to be an optimal number (and mix) of inputs, below or above which credibility is lost in terms of the shape of the resulting efficiency histogram for the model sample. Thus in order to increase the number of non-M.D. manpower inputs to a model, a larger sample of departmental observations is required.

In terms of the present work, and the finite number of radiology departments in the province, this would necessitate gathering information on those departments that were eliminated from the model samples. It should be remembered that they were not used as a result of either omissions or errors in their data files.

Two important input variable candidates for decreasing

heterogeneity by choosing a new unit of measure are the radiologist and equipment inputs. At present, they are quantified as full and part-time radiologist man-years and equipment-years. It is apparent that radiologist man-years in different departments can represent vastly different productive inputs. The advantages of measuring this input in man-hours are two-fold: i) the distinction between full and part-time radiologists could be eliminated by using their total aggregated hourly input, which in turn would eliminate the difference between departments having only one of these inputs with respect to the linear program (see Fig. 5.12b), and; ii) departmental productivity differences which result solely from the radiologist unit of measure, and this is particularly acute for the part-time category which can be defined as anything from a half day to four days a week, would be reduced.

As radiologists are not hourly wage earners this man-hour information would be tremendously difficult to acquire. In addition, radiologist-hours really represent an output measure of the extent to which this input (radiologists) is actively used in production. This is opposed to the other model inputs which are fixed at the beginning of each fiscal year, and consumed to varying degrees during the course of the year. Nevertheless, as the non-medical inputs more fully portray the production setting with the radiologists representing a management role, it is felt that if the requisite man-hour information were available the advantages of introducing it to a model outweigh the disadvantages.

The equipment input production variable resulted from the aggregation of six different categories of equipment taken from data set D.1. Unlike the non-medical manpower input, each of the component members of this input perform an identical function in terms of the production of output as measured in diagnostic radiology professional units. Hence, the simultaneous introduction of more than one equipment input to a model is not possible. Yet an improvement can be effected by changing

the unit of measure to a value term. This would be measured as the sum of individual equipment values. These would typically be either the discounted cost (over-time) of a machine or the resale/scrap value of such an asset.

The advantages associated with such a unit change are two-fold. First, the technical differences between machines of different vintage and category would implicitly be included in the model, and hence could not be a major source of measured departmental inefficiency. The second advantage relates to using a model derived from observations taken during a particular year to analyze radiology departments in some future period. The manpower inputs, both medical and non-medical, are not affected by time. Yet materials are measured in dollars, and if this were the case for equipment as well, then both could be updated by an appropriate discounting scheme.

At present, it would be difficult to introduce time related degradation to the equipment input of the model. This is because its unit of measure, i.e. equipment-years, has the same meaning for past, present and future periods, unlike dollars for example. Yet the model (based on 1971 data) can currently be used to analyze a department if either one of the following two has occurred. First, the department has not undergone a major equipment change since the year for which the model was estimated. Second, the department has acquired equipment of the same capabilities as those that were available on the market in 1971. Although we believe that most departmental "equipment changes" over the intervening years will fall within one of these categories, we should note that equipment value information is accessible to the government as evidenced by its presence for some departments in the Radiographic Equipment Inventory (App. B).

The assumption of <u>constant returns to scale</u> between the capacity output of a radiology department, as measured in professional units, and the input production variables to our model cannot be verified empirically. This is because the capacity output is not observed

but estimated. It has been demonstrated in Chapter 6 that this assumption is at least approximately correct for the system under investigation. Yet normalized units, i.e. the professional units, are not always available, and in particular, if the same methodology is to be applied to hospital departments other than diagnostic radiology. In fact, from Figs. 5.2 to 5.5 it would appear that if diagnostic examinations were taken as the output measure of radiology departments for our model, then decreasing returns would be expected.

Farrell (1957) describes one possible scheme for introducing non-constant returns to scale with the present modeling methodology.

> "The only practical way of dealing with this problem (*non-constant returns*) seems to be that of dividing the observations into groups of roughly equal output, and applying the method to each of these groups separately, the assumption being that returns to scale are constant within a group to a sufficient degree of approximation. This will yield a different isoquant for each level of output, and comparison of these isoquants will show the extent and nature of the economies of scale.
>
> This device should work satisfactorily if there are a sufficient number of observations, but it clearly needs many more observations than the original method."

The introduction of the above technique involves a trade-off. Whereas before, all departments could be compared with respect to their technical efficiencies, now only those departments within any one group can be compared. Yet at the same time, the use of more than one production function may more accurately reflect the mode of radiographic production.

Two data collection efforts relevant to the present model have been started. First, recall that the data available for this project was for 1971. The type of data used to identify the model is now being

sought for all diagnostic radiology departments in acute hospitals in Quebec for 1972 and 1973. By estimating a model for each of these years, with the appropriate discounting, we will be able to determine whether there has been a shift in radiographic productivity over the period 1971-73. If the shift is minimal, then we can assume that radiographic equipment, as quantified for the model, has not been significantly affected by technical change. In this instance, we will have a larger sample (at least 200% larger) from which to identify a more refined approximation to the continuous production function for diagnostic radiology. It will also be of interest to investigate whether any of those producers that were the "best" in 1971 are better or equivalent to the most efficient departments, or are in fact the most efficient departments, in the two succeeding years.

Second, we have also decided to collect data on radiology departments in other provinces in Canada from 1971 onwards. This will allow both inter-provincial productivity comparisons as well as expansion of our data base. The diagnostic radiology professional units are unique to Quebec. The only output measure we will have for the other provinces is the number of diagnostic examinations. Examinations and all the model inputs are in equivalent units of measure for all provinces as the source for this data is the federal government.

The technique described above can be used to introduce the non-constant returns expected with the examinations output measure. Yet the problem of non-comparability between groups of departments, i.e. efficiencies measured with different production functions will arise. It may be possible though, to convert examinations to professional units. One means of pursuing this would be to estimate the following equation:

$$Y = A \cdot X^{\beta} \qquad (7.1)$$

where Y — output volume in professional units
X — output volume in diagnostic examinations
A & β — parameters of the equation

This equation would be estimated by linear regression for a sample of radiology departments from Quebec. The plot of Y versus X from Eqn. 7.1 for different values of A and $\beta > 1$ is shown in Fig. 7.2. A strong similarity exists between the shape of the curves in Fig. 7.2 and the plot of professional units versus examinations based on observations for Quebec (1971) shown in Fig. 7.3. Equation 7.1 is presented merely as an example; other more sophisticated relationships are possible and should be investigated. It is apparent that estimating the output in professional units is not as reliable with respect to productivity measurement as using an observed output measure directly. It is felt though, that the advantage of having one common framework by which all departments, and hence all provinces, can be analyzed will outweigh the disadvantages of using an estimated output.

Both of the above studies are being pursued by myself at Scidem Inc. (Montreal) as part of a project to introduce the results of this thesis to the health planning environment in the Ministry of Social Affairs (Quebec).

7.2 Analyzing Sources of Departmental Efficiency

The work in this thesis is divided in two parts; first, a mathematical model of a diagnostic radiology department was derived, and second, an analysis of the primary output from the model, i.e. the technical efficiency of a radiology department, was initiated. Admittedly, the bulk of this project is devoted to the former pursuit. The latter effort is included because of its relevance to the evaluation of departments applying to the government for new equipment in light of the information provided to planners by the model.

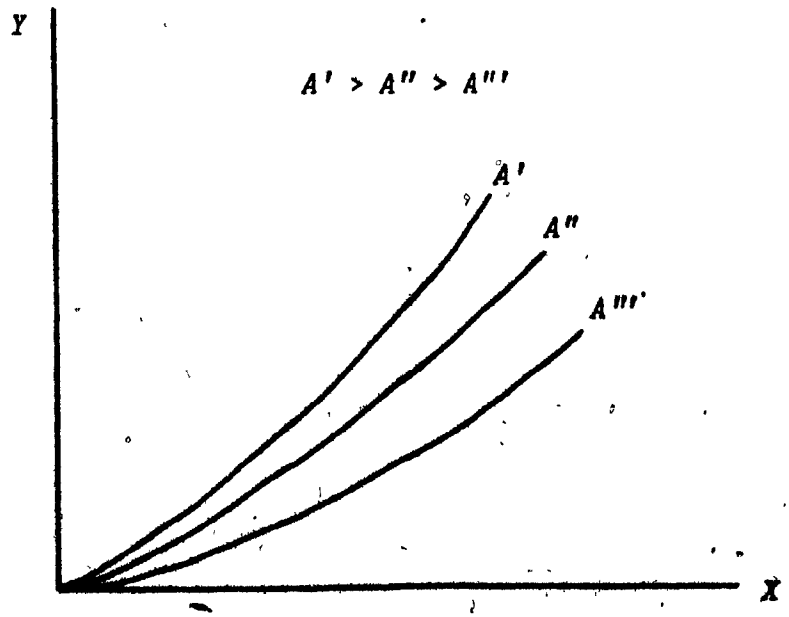


Fig. 7.2. A plot of Y versus X from the equation $Y = A \cdot X^{\beta}$ for different values of A and $\beta > 1$.

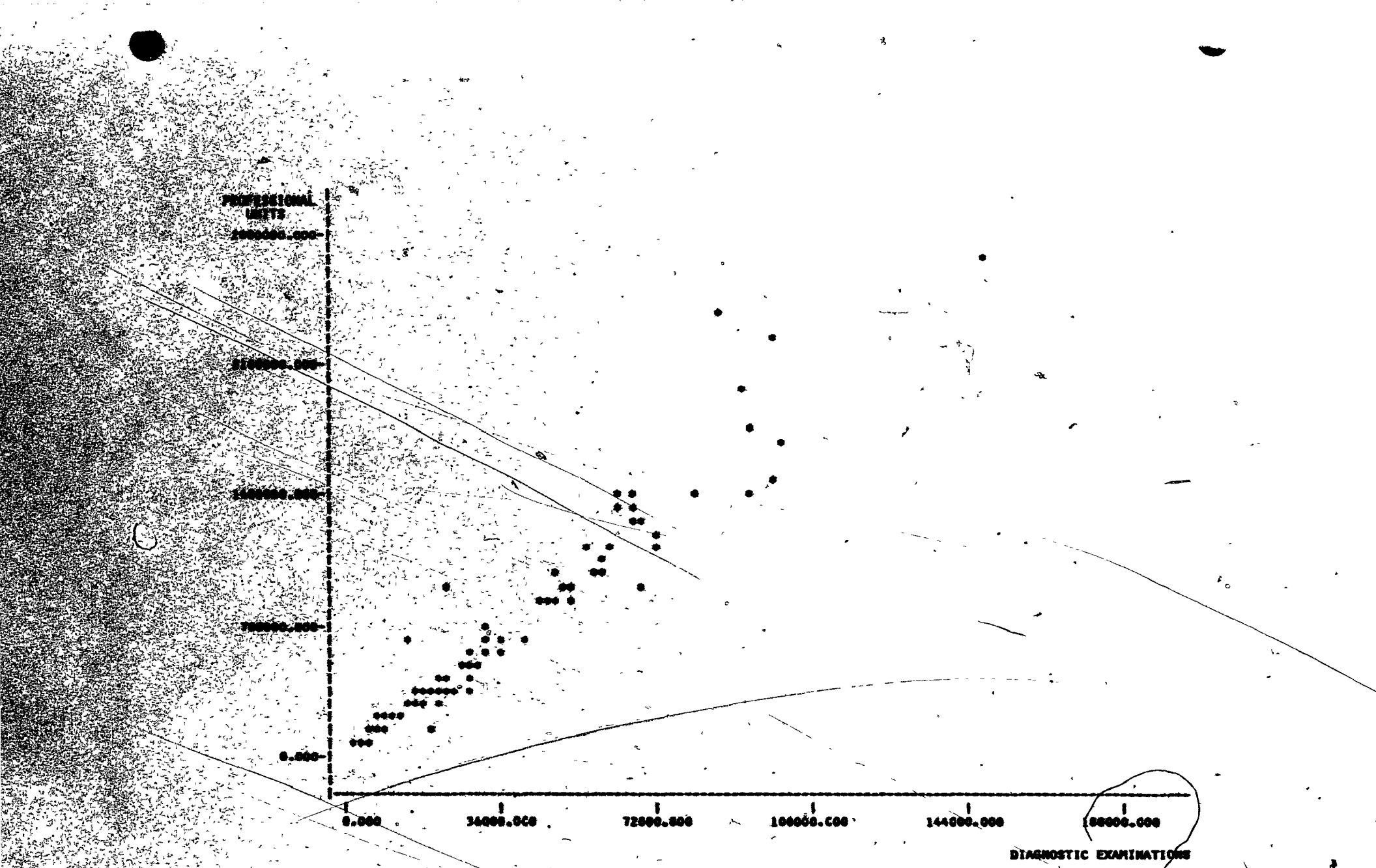


Fig. 7.3. The number of professional units of service versus the number of diagnostic examinations provided in diagnostic radiology departments for 1971.

Departments having above average or high technical efficiencies are seen as superior to those departments with below average efficiencies from a productivity perspective. It is desired that a department should, if possible, improve its technical efficiency. This goal is realistic. Our measure of efficiency is relative, and therefore, we would in fact be encouraging departments to produce at the level achieved by the best producers in the province, rather than 100 percent of their theoretical maximum.

An analysis of the sources of departmental efficiency explicitly addresses the following question: why are some departments more efficient than others? This analysis is important to medical equipment planners in terms of the following questions: i) is the efficiency of a radiology department a function of controllable factors, such as the production organization of the department, or uncontrollable factors, such as the degradation of equipment as a result of ageing, and; ii) will the allocation of equipment by planners to a radiology department affect its technical efficiency, and can these allocations be used to promote increased efficiency in a department?

Examining the quantitative determination of sources of technical (in)efficiency in a radiology department revealed that an indepth analysis was required that was beyond the scope of the present thesis. However, in recognition of the importance of this type of analysis, it was decided that at least a preliminary investigation should be presented. This investigation established the framework for a continuing study.

Our investigation consisted of three parts: i) a description of the sources of (in)efficiency in a department as well as a graphical examination of the relationship between some representative measures of these sources and the efficiency of departments; ii) the elaboration of a two stage process for systematically quantifying the extent to which each of the efficiency sources accounts for differences in efficiency between departments, and; iii) the presentation of a

scheme to classify departments and the subsequent examination of the resulting departmental groupings as an explanatory variable of the technical efficiency.

As the second part of this analysis was based on existing studies in the literature (see section 6.6.1), it will not be re-introduced at this stage.

7.2.1 Sources of Technical (In)Efficiency in Diagnostic Radiology. Two sources of diagnostic radiology department (in)efficiency as measured by the production model have been described (see Chapter 6):
i) those relating to the assumptions underlying the model, and;
ii) those resulting from departmental or hospital features which are relevant to the production of radiographic services, but which could not be included in the present production model format.

7.2.1.1 Model Based Sources. Within the first category, it is apparent that some part of the measured efficiency of a department would result solely from the measuring instrument if either one or both of the basic model assumptions were not valid. If the assumption of constant returns is not appropriate for the radiographic system we have described, then many departments, and in particular the larger ones, would in reality have (lower) higher efficiencies than observed if (increasing) decreasing returns to scale is in fact obeyed by this system. Since the returns to scale cannot be tested and because it appears, from the available evidence (see section 6.3.1), to be correct to an acceptable degree of approximation, we did not pursue this source of efficiency further.

The other basic assumption of the radiology model is the homogeneity of the production variables. Admittedly, this assumption will never be absolutely valid. No two departments will ever be qualitatively identical in each of their corresponding variables, regardless of their unit of measure or how finely the variables are defined. Consider one of the seven non-medical manpower groups quantified in man-hours. Although the formal education of individuals

in this group will be the same from one department to the next, it is apparent that for equivalent man-hours, the quality of their contribution to the production of services can be vastly different. This is simply because the same non-medical manpower input will represent different people working in different environments. As a result, the technical efficiency of a radiology department must always to some extent reflect the quality of its inputs.

In addition, any variable in the present model which was derived by aggregation must introduce some heterogeneity. Yet, heterogeneity is not an important factor as long as it is distributed evenly over all departments in the sample. For example, in constructing the equipment input to the model we have aggregated three types of stationary equipment and three types of mobile equipment. Mobile machinery is generally used less frequently than stationary machinery. Hence, one might expect that the most efficient producers in our sample would have only stationary machines, whereas in fact, most of the best producers have both stationary and mobile equipment. Thus, it would appear that the heterogeneity resulting from aggregating these two equipment groups is evenly distributed over departments.

We should note that this is not necessarily the case when considering the six different equipment types comprising the equipment input. "When there are differences between firms in the average quality, (or more strictly, in the distribution of qualities) of a factor, then a firm's technical efficiency will reflect the quality of its inputs" (Farrell, 1957).

An additional means by which heterogeneity can be introduced to the production model is through technical change in the fixed capital input; that is, the equipment variable. In order to examine the relationship between the efficiency of a department and the vintage of its assets, we plotted the technical efficiency versus the mean x-ray room age of a department (see Fig. 6.7). Our assumption

was that the age of a room represents the maximum possible age for the equipment in the room. From the distinct lack of a correlation between the efficiency and mean room age we conclude that the heterogeneity introduced by the age of equipment is, as with the stationary and mobile equipment, spread evenly over the sample of departments.

To further check this hypothesis we suggest collecting data on the actual age of machinery as well as the maintenance costs in each room in a department. Admittedly this information is listed as part of data set D.3 (see App. B). Yet as the number of departments which actually supplied this data was far less than the model sample size, it was decided that plotting these variables versus efficiency would be inconclusive.

7.2.1.2 Non-Model Based Sources. Within the second category of efficiency sources, i.e. variables excluded from the model, we have described two basic types. The first encompasses the production environment in diagnostic radiology, while the second involves the relationship between the radiology department and its surrounding hospital environment.

In the context of the model, we describe a radiology department solely in terms of the production process it defines; that is, a department is identified by the ratio between its model inputs. It is apparent that two or more departments can provide diagnostic services with very similar or identical production processes, yet at the same time, exhibit vastly different production environments.

The term production environment for diagnostic radiology includes those features which will affect a department's productivity, but which cannot be explicitly recognized as part of our production model. Typical examples would be: the scheduling procedure for examinations; the location of x-ray rooms in the hospital; the mix, distribution and balance of technological and manpower resources; as well as the quality of the department's management.

In constructing the diagnostic radiology model we have considered the radiology department as a system isolated from its surrounding environment; that is, the hospital in which it is located. As a result, we concluded that some part of a department's technical efficiency must be attributed to the properties of this hospital environment. Specifically, it has been hypothesized that there exists some balance or demand/supply equilibrium between a radiology department and its hospital which is conducive to more efficient production of services, i.e. a greater utilization of the capacity output. Assuming the internal organization of a department is such that this capacity output could be achieved, then the resulting efficiency will depend on the demand for radiographic examinations, which in turn, will depend on the medical needs of the patient population in the hospital. We graphically examined the correlation between three different measures representative of the hospital-department relationship and the efficiency of departments.

One feature of the service demand imposed on a department is the average complexity of the resulting examinations. A representative index of this complexity is the ratio of the total volume of professional units produced to the total volume of diagnostic examinations. A second measure characterizing the radiographic demand is the ratio of outpatient examinations to inpatient examinations. This is because each of the recipient groups generally require different types of diagnostic procedures. From the plots of these two ratios versus the technical efficiency, Figs. 6.3 and 6.6 respectively, we conclude that the efficiency of a radiology department as measured by our model is independent of the type or mix of procedures produced. This in fact was expected as our choice of professional units as the output measure was based on a desire to normalize for differences in the examinations output between departments.

The third hospital index examined was the number of hospitals beds. Figure 6.4, which shows a plot of efficiency versus hospital

beds, revealed that those radiology departments in hospitals with more than 500 beds were distributed over a smaller range of efficiencies, 50 to 80 percent, than departments in hospitals with less than 500 beds, 50 to 100 percent. Three hypotheses were proposed to explain these results. First, although the demand for specialized diagnostic procedures is greater in larger hospitals, it may not be sufficient to fully utilize all the special (vs. general) x-ray rooms which are proportionately more abundant in these larger hospitals. This results in some idle room capacity, which in turn, is measured by the model as inefficient production. Second, the larger hospitals are university teaching hospitals, and hence their diagnostic radiology departments are called upon to pursue teaching and research. These activities are resource consuming yet are not recognized as productive in the model. The question is whether the academic activities are compensated by the productive contributions of interns and residents which were excluded from either of the manpower groups in the model. At present, neither one of these hypotheses can be tested because of insufficient data.

Those departments in hospitals with less than 500 beds comprise the majority of our model sample. From their distribution of efficiencies, we hypothesized that there are some fundamental sources of inefficiency at work relating to the production organization of a department and its relationship to its hospital, that is independent of hospital size and academic orientation. The results of our efforts at confirming this hypothesis, using the available data, are presented in section 7.2.3 below.

7.2.2 A Preliminary Analysis. The basic problem in a quantitative analysis of departmental efficiency is that those variables which initially appear "obvious" will not necessarily be the most productive in terms of the goals of the total analysis. We have suggested that an investigation of the data independent of the dependent variable to be explained can serve to alleviate this problem.

Most often some classification procedure is used for examining the initial data. The technique adopted for this project was developed by Jarvis & Patrick (1973), and falls within the general category known as nearest neighbour clustering algorithms. The result of applying this scheme to a sample of observations on a set of variables is to define mutually exclusive groups of observations based on a criterion of similarity for membership in a group. Our goal in pursuing this analysis was to define groups of departments with similar production environments and hospital-departmental balances. Obviously these two phrases and the notion of similarity will have dissimilar interpretations when the clustering technique is applied to different sets of departmental variables. A full discussion of the variables that were actually used, the results, and their implication in terms of explaining the technical efficiency of departments is presented in section 7.2.3.

At this point though, we note that there are two advantages to using the type of classification scheme mentioned above over the more frequently use multi-variate tools, such as principal components, for classifying health systems.

First, a clustering algorithm is normally initialized by a set of parameters, which in turn allows freedom in the choice of groupings to the extent that a complete hierarchy of groups can be defined for a sample. The trade-off between different parameter sets will be in terms of the number of resulting clusters versus the consistency or internal cohesion of these clusters. In this way one can gear the technique to the unique properties of a particular sample with respect to what is seen as appropriate results for the long term goals of the analysis; that is, explaining efficiency in terms of departmental clusters.

Second, our clustering technique can delineate nonglobular clusters which we recognized as important for revealing unique system properties. Statistical techniques, on the other hand, are based on means and covariances and hence force a globular structure on the data. This can mask important system features.

7.2.3 An Information Theoretic Approach. It was decided that two of the efficiency sources we have cited, i.e. the heterogeneity of the production variables and at least one facet of departmental production organization, could be quantified by comparing the technological and manpower resource mixes of departments. Both because of the limited size of the sample of efficiencies (approximately 100 observations), and the fact that the above efficiency sources represent only part of the possible set of sources, it became apparent that the resource mix comparisons must be quantified by only one or two variables. It was for this reason that information measures were presented in Chapter 6 as a means of quantifying resource mixes.

If there are two categories of a particular resource, then the mix of these categories comprising the total resource volume is easily measured by taking the ratio of the amount of the first category to that of the second. Yet if there are n categories, where $n > 2$ then a similar approach would require $n-1$ measures, i.e. the fraction of the total for $n-1$ categories, to quantify this mix. The problem of multiple measures is further compounded when comparing the mixes of two resources, where each resource consists of three or more categories.

The fundamental question we have addressed is as follows: how much information is provided with knowledge of the volume of each category of a resource in a department? The answer is of course dependent on the distribution of categories we expect *a priori*. If the actual distribution is very different from that which was expected, no doubt we are surprised and hence provided with a large amount of information. Conversely, little or no information is provided when the two distributions are similar or identical.

Based on this simple idea, we used the entropy index developed by Shannon (1964) for measuring the information content of an indirect message to derive resource mix indices for a diagnostic radiology

department or its hospital. These measures are in no way unique to health systems, but can be applied to any entity that is distributed as to type and location.

7.2.3.1 Government Allocation Policy and Information Measures.

It is demonstrated in the thesis that Shannon's index can be used to measure the inequality in the distribution of a particular resource type over a set of locations (index H_j^1, see section 6.5.4). Though not directly related to the problem of quantifying efficiency sources, this result has interesting implications for medical resource allocation decision making in the government. In particular it was demonstrated that this measure can be used to monitor the allocation of medical resources with respect to the governments policy of (de)centralizing (non)specialized resources in the health sector.

Two interpretations of the inter-departmental distribution inequality measure for a particular category of a resource have been put forth in the thesis. First, we noted that the inequality in the distribution of a resource is equivalent to a measure of the concentration of that resource from a global perspective. Hence, if a resource is allocated to a location, such as a particular type of radiographic equipment to a particular diagnostic radiology department, then if the inequality measure (decreases) increases, the concentration of that particular resource will have (decreased) increased, and therefore the resource will be more (de)centralized than before this allocation. Thus H_j^1 will, for the first time, let government planners know whether their decisions are consistent with the Ministry's global allocation policy. This index is also consistent with our original mandate of providing information relevant to the government's decision environment without actually making decisions; that is, at no time do we indicate what level of (de)centralization of a particular resource is desirable.

The second interpretation involves the definition of specialization when applied to resources. In particular, the inequality measure

corresponds to a distributional, rather than functional, concept of specialization. For example, we know that functionally medical specialists are more specialized than general practitioners. Yet in 1972 there were 3,392 general practitioners and 4,331 medical specialists in Quebec (Anon., 1973). Hence G.P.s are less common, and in a global sense represent a more unique (specialized) type of physician. In a similar manner, those resources which are more evenly distributed are less specialized than those resources that are centralized in a few locations, independent of their functional role or cost. The relative speciality of two types of resource, categories j and k, can be found by comparing H_j^1 and H_k^1.

This type of analysis was applied to the three types of stationary radiographic equipment (without fluouroscopy, with fluouroscopy but without a spot device, and with fluouroscopy with a spot device) for a sample of 35 diagnostic radiology departments in Chapter 6. Interestingly, it was found that machines without fluouroscopy are more specialized, i.e. more concentrated based on maximum uncertainty in the *a priori* distribution, than machines with fluouroscopy and with a spot device (see Table 6.7). In addition, machines without fluouroscopy are generally concentrated in the larger departments. Two possible explanations are: i) as machines without fluouroscopy are functionally less versatile, it only becomes profitable to use them in larger departments (which are few in number) where the demand is sufficient to have machines devoted to less specialized procedures, or; ii) fluouroscopic machines with a spot device represent a more recent technological development than machines without fluouroscopy, and hence one would expect the latter type of machine to be concentrated in the older, and generally larger, radiology departments. Either one of these hypotheses could be validated by examining the equipment applications made by different types of radiology departments over the past several years.

7.2.3.2 Quantifying Departmental Resource Mixes and Hospital/Departmental Balances. In Chapter 6, five different information theoretic indices were presented, i.e. X_i^1, X_i^2, G_i^1, G_i^2, and G_i^3, where i denotes the i-th location. The first two are weighted sums of the inter-departmental distribution inequality measure for each category of a resource (see section 6.5.2.1). The difference between X_i^1 and X_i^2 is the result of using different *a priori* distributions in constructing their corresponding resource category inequality measures. The last three information indices are intra-departmental distribution inequality measures (see section 6.5.2.2), once again, differing solely with respect to their *a priori* distributions.

Two properties of the information measures makes them particularly useful as efficiency source variables. First, of course, a mix of n resource categories is reflected in a single measure. Second, they are dependent on the proportions of the total resource volume for each resource category at a location, rather than the total volume of the resource.

Most of the efficiency source variables described in section 7.2.1 are correlated with either the radiology department or hospital size. The information measures for two departments or hospitals will be identical only if the resource category proportions are the same, independent of volumes. Normalized indices such as these are required to quantify what we have labelled as the fundamental sources of inefficiency in departments. In the production model, two departments will be identical if the total volume of each of their model inputs are the same, and will use the same production process if the ratios of their inputs are identical. Applying the information measures to the resource categories comprising a model input allows one to differentiate between equivalent departments as well as between equivalent production processes.

Index G_i^3, in particular, can reflect the heterogeneity of a

production variable as it is based on an *a priori* resource mix which is equivalent to that of the "average" department or location. The *a priori* assumption for this index is the same as assuming that the heterogeneity of an input is evenly spread over a sample of departments. Hence G_i^3 measures the deviance of a department's resource mix relative to the sample in a manner which reflects a model based source of inefficiency.

The technology mix of a radiology department was quantified by applying each of the five information measures to the thirteen different classes of equipment on data set D.1. The manpower mix of a department was quantified by using 16 manpower classes; that is, seven technician categories plus the radiologists, each in full and part-time classes, also from data set D.1. As pursued in a study by Evans & Walker (1972), a hospital was represented by information measures applied to a set of eighteen mutually exclusive and collectively exhaustive patient diagnostic categories. That is, for every diagnostic radiology department in an acute hospital, five information measures were calculated in each of three "resource" categories for a total of fifteen measures per department.

Recognizing that a cluster analysis on approximately 100 departments in a space of dimension fifteen would produce meaningless results, we decided to introduce only one technology, manpower and diagnostic mix measure at a time. In addition, information measures X_i^1 and G_i^1 were eliminated for all three mix categories because of their naive *a priori* distributions (see section 6.5.2). This resulted in three different information measures for each of three mix categories per department. Taking only one measure from each of the three categories at a time, produces 27 combination of three measures per department. Hence the problem was reduced to a space of three dimensions.

The amount of computation necessary for deriving an optimal clustering parameter set for each of the 27 samples would be tremendous

(see App. E). Hence, a range of parameters was chosen from a detailed examination of only one of these samples, which was then used for that sample as well as the remaining twenty-six. The assumption here was that the remaining samples were not significantly different from the particular sample from which the parameter range was derived.

The decision as to whether the results of a cluster analysis are acceptable was based on an examination of the efficiency histograms corresponding to the departments in each cluster. Our goal was to find the particular set(s) of variables and parameter set(s), such that the cluster analysis divides the complete model efficiency histogram into groups of departments with significantly different efficiencies.

Two features of the cluster results (see section 6.6.3.2) are of particular importance. First, although this analysis has generally divided the sample of departments into groups of above average and below average efficiencies, concurrently, there are some departments in the (below) above average clusters with (above) below average efficiencies. This would indicate that the particular facet of the production organization of a department and the hospital/department relationship that is quantified by our information theoretic indices does not completely account for differences in efficiency between departments. In fact, it was not supposed to. Our original intent in pursuing this analysis was to derive only one of a possible set of efficiency source variables to introduce to a complete efficiency analysis.

Second, the cluster analysis divided the sample of departments into groups which were most definitely not obvious *a priori* that is, before calculating the technical efficiency of radiology departments. As indicated by Fig. 6.2a and Table 6.14, these groups are in fact quite natural in terms of the resulting efficiencies, yet were derived independent of these efficiencies. This is significantly different from classifying departments by relating a dependent (efficiency) variable to a set of independent (efficiency source) variables. In

this case, the resulting groups are valid only in the context of the specific dependent variable. We would claim that our analysis has resulted in more universally reliable departmental groupings.

We therefore conclude that the investigation of information theoretic indices and clustering analysis is justified, and is representative of the type of systematic data examination requisite to accounting for differences in technical efficiency among diagnostic radiology departments.

7.2.4 Extensions. Our effort at classifying diagnostic radiology departments was presented as preliminary to a complete quantitative determination of the sources of technical (in)efficiency in these departments. Hence, the primary extension of the present work will be the pursuit of such an analysis using the procedure described in section 6.6.1.

For those cluster analyses which have produced groups of departments with above average and below average efficiencies, their results can be introduced to an efficiency determination scheme in the form of a binary variable; that is, associating a (zero) one with those departments in the (below) above average efficiency groups. Yet before doing so, it may be advisable to further investigate the classification analysis itself, for which three avenues exist.

First, although twenty-seven different variable sets were independently examined with the clustering technique, the parameter range used for all sets was the same. It is felt that the unsatisfactory departmental groupings derived from many of these variable sets resulted from dissimilarities between their samples and the sample for which the parameters were actually chosen. By unsatisfactory groupings we refer to the production of either a tremendously large number of small clusters or a single cluster of the complete sample. Hence, an improvement in the results would be expected if clustering parameters were established for each and every variable set.

Second, for some of the clustering analyses it appears that

the introduction of the diagnostic mix information measures had a minimal effect on defining the clusters (see Figs. 6.19 and 6.21). This hypothese can easily be tested by re-clustering departments in terms of just the technology mix and manpower mix variables. If the diagnostic mix is in fact superfluous to this classification scheme, then two principal courses exist: i) some other hospital measure could be introduced to this analysis, such as the cost per patient, average length of stay, or the ratio of inpatients to outpatient visits, or; ii) exclude a hospital related measure from this classifying process, but introduce the diagnostic mix measures, which represent the complexity or speciality of a hospital depending on which of the five indices is used (Evans & Walker, 1972), as an efficiency source variable to the efficiency quantification process.

Third, on examining each of the three series of five information measures, it was found that some of the information measures within any one series appear related (see Table 6.13 for example). If this is the case, then at least some of the 27 combinations of information measures are somewhat repetitive. We would suggest that the correlation matrix (5 by 5) be calculated for each set of mixes. If within any one series the information measures are highly interrelated, then a dimensionality reduction scheme, such as a factor analysis, should be applied to these measures. This would produce a smaller set of variables, i.e. less than five, each of which will provide more "information" on the mixes than any of the original variables. These new variables can then be introduced to the cluster analysis in the same fashion as the original variables. In this way, far less computation would be required to produce as good or better departmental classifications.

7.3 Conclusion

Summarizing the work in this thesis, the following results are claimed:

i) A procedure was established for developing quantitative tools to support planning and decision making in the health sector which is based on an examination of the health system in terms of the needs of the health agency's decision environment for which the tool is to be designed;

ii) A linear programming production model of a diagnostic radiology department was identified which is applicable to any such department in an acute hospital, estimates a feasible departmental capacity output, allows investigation of the effect of various resource changes on this capacity output, and can be used to calculate the technical efficiency of a department relative to the most efficient departments in the province;

iii) An information theoretic index was proposed to indicate, from a global perspective, whether the allocation of a specific type of medical resource to a particular location is consistent with the government's policy of (de)centralizing (non)specialized resources in the health sector, and;

iv) It was demonstrated, by clustering radiology departments in terms of information measures reflecting their technology, manpower, and hospital diagnostic mixes, that the technical efficiency of departments is correlated with, if not conditioned by, a balance of these mixes.

Picasso once said that art is a lie which makes us realize the truth. A representation or interpretation of a specific reality can never exactly replicate that entity. And yet, through the eyes of the artist and the medium of the work, a viewer can be shown an explicit facet of that reality which hitherto may have been sensed, but was unseen. Mathematical modeling is much the same. The analyst abstracts certain properties and relationships from a system for quantification in order to produce an intellectual construct

which portrays a particular system behaviour. The goal of this exercise is to "see" previously unobservable system structures and parameters. In this thesis we have developed such a picture of diagnostic radiology through which government planners can more effectively resolve their radiographic equipment allocation decisions.

APPENDIX A

RADIOGRAPHIC EQUIPMENT APPLICATION FORM

This appendix contains a copy of the application form submitted by diagnostic radiology departments for new radiographic equipment.

PROVINCE DE QUÉBEC
MINISTÈRE DE LA SANTÉ
SERVICE DE L'ASSURANCE HOSPITALISATION

DEMANDE POUR L'OBTENTION D'EQUIPEMENT
RADIOLOGIQUE (SECTION DIAGNOSTIC)

A.- RENSEIGNEMENTS GENERAUX

Hôpital _____

Localité _____

Population environnante (approx.) _____

Nombre de lits (excluant bassinettes) _____

Nombre de bassinettes _____

Accréditation par la Commission canadienne d'accréditation :

oui _____ Non _____

B.- ENSEIGNEMENT

Affiliation universitaire? Oui _____ Non _____

Si oui, nom de l'université _____

Enseignement organisé pour les étudiants en Médecine non gradués? Oui _____ Non _____

Nombre d'internes en rotation _____

Nombre de résidents à l'entrainement post-gradué (excluant la radiologie) _____

C.- PERSONNEL DU DEPARTEMENT DE RADIOLOGIE

Nombre de radiologistes certifiés: a) à plein temps _____

b) à temps partiel _____

Nombre de résidents à l'entrainement en radiologie _____

1ère année ___ 2ème année ___ 3ème année ___ 4ème année ___

Nombre de techniciens gradués _____

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Nombre d'étudiants-techniciens à l'entrainement _____

D.- TRAVAIL RADIOLOGIQUE (Section diagnostic)

Nombre d'examens enregistrés l'an dernier: Internes _____

Externes _____

Nombre d'examens enregistrés depuis le 1er janvier à cette date (_____) Internes _____
date

Externes _____

Nombre d'examens de chaque catégorie enregistrés depuis l'an dernier:

a) Repas barytés _____

b) Lavement barytés _____

c) Angiocardiographies _____

d) Artériographies (périphériques _____
(cérébrales _____
(autres _____

e) Pneumo-encéphalographies et ventriculographies _____

f) Tomographies _____

g) Examens portatifs approximatifs: sur les étages _____

à la salle d'opération _____

E.- AUTRES DEPARTEMENTS

Les disciplines suivantes sont-elles pratiquées de façon active dans votre hôpital?

Neurochirurgie:	Oui _____	Non _____
Chirurgie thoracique:	Oui _____	Non _____
Chirurgie cardiaque:	Oui _____	Non _____
Chirurgie vasculaire: (autre que varices)	Oui _____	Non _____

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SERVICE DE L'ASSURANCE HOSPITALISATION

Nombre de lits réservés aux patients tuberculeux _____

Nombre de lits réservés aux patients de psychiatrie_____

F.- EQUIPEMENT RADIOLOGIQUE EN OPERATION (énumérer chaque pièce d'équipement. Annexer pages supplémentaires s'il y a lieu.

DESCRIPTION	ANNÉE D'INSTALLATION	ESTIME DE LA VALEUR ACTUELLE
1.-		
2.-		
3.-		
4.-		
5.-		

Dépréciation allouée sur l'équipement actuel au Service de Radiologie par le SAHQ _____%

G.- DEMANDE ACTUELLE D'EQUIPEMENT (énumérer en détail chaque pièce d'équipement, sans mentionner le nom du manufacturier, sauf si l'équipement est manufacturé par un seul fournisseur. Annexer pages supplémentaires s'il y a lieu).

1.- _____

_____

Installation additionnelle___de remplacement___coût approx.__

S'il s'agit de remplacement, indiquer de quel item mentionné dans la section F il s'agit.

_____

2.- _____

_____

Installation additionnelle___de remplacement___coût approx.__

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S'il s'agit de remplacement, indiquer de quel item mentionné dans la section F il s'agit.

_____

3.-_____

_____

Installation additionnelle___de remplacement___coût approx.__

S'il s'agit de remplacement, indiquer de quel item mentionné dans la section F il s'agit.

_____

4.-_____

_____

Installation additionnelle___de remplacement___coût approx.__

S'il s'agit de remplacement, indiquer de quel item mentionné dans la section F il s'agit.

_____

H.- L'équipement sera-t-il installé dans un nouvel hôpital?
Oui ____ Non _____

Si non, des altérations à la bâtisse actuelle seront-elles nécessitées par la ou les nouvelles installations? Oui ___ Non___

Si oui, estimé du coût _____

NOTE Un mémoire motivant la présente requête devra accompagner cette formule, en particulier si les détails ne peuvent être adéquatement inclus dans le questionnaire ci-haut.

Il y aura également lieu d'annexer les copies de soumissions provenant d'au moins trois fournisseurs d'installations radiologiques. Il n'est pas nécessaire que les raisons de votre choix d'un fournisseur plutôt que d'un autre, soient motivées par la soumission la plus basse, mais elles pourront l'être par les facilités du service de l'espace actuel et par préférence personnelle.

PROVINCE DE QUÉBEC
MINISTÈRE DE LA SANTÉ
SERVICE DE L'ASSURANCE HOSPITALISATION

A tout événement, les raisons de votre choix définitif devront être clairement établies dans la formule de demande initiale présentée au Service de l'Assurance-Hospitalisation du Québec.

DATE

Signature du Chef de Service

PROVINCE DE QUÉBEC
MINISTÈRE DE LA SANTÉ
SERVICE DE L'ASSURANCE HOSPITALISATION

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ESPACE RESERVE AU COMITE
D'EQUIPEMENT RADIOLOGIQUE

Commentaires du Comité d'équipement radiologique.

Recommandations du Comité d'Equipement radiologique.

Président.

APPENDIX B

DATA BASE DESCRIPTION

This appendix provides a listing of the information available from the three data sets described in Table 3.1 . The following descriptions are extracted directly from the 'Fiche de Description des Zones' accompanying the computer tape corresponding to each of the data sets.

Data Set D.1

1 année
2 numéro hôpital
3 radiographie pulmonaire de dépistage - hospitalisés de la tuberculose (petit film)
4 " - malades externes
5 " - examens courants
6 " - total examens
7 " - à l'hôpital
8 " - autre organismes
9 radiographie pulmonaire - hospitalisés (film ordinaire)
10 " - malades externes
11 " - examens courant
12 " - total examens
13 " - à l'hôpital
14 " - autre organismes
15 examens fluouroscopique - hospitalisés
16 " - malades externes
17 " - examens courant
18 " - total examens
19 " - à l'hôpital
20 " - autre organismes
21 examens cinéradiographies - hospitalisés
22 " - malades externes
23 " - examenes courant
24 " - total examens

25 examens cinéradiographies - à l'hôpital
26 " - autre organismes
27 autre examens radiologiques - hosp.
28 " - ext.
29 " - ex. cour.
30 " - total
31 " - à l'hôp.
32 " - autres org.
33 total partiel - hosp.
34 " - ext.
35 " - ex. cour.
36 " - total
37 " - à l'hôp.
38 " - autres org.
39 autres - hosp.
40 " - ext.
41 " - ex. cour.
42 " - total
43 " - à l'hôp.
44 " - autres org.
45 total examens radiologiques - hosp.
46 " - ext.
47 " - ex. cour.
48 " - total
49 " - à l'hôp.
50 " - autres org.
51 interprétations écrites de radiographies - radiologue - débitées à l'hôpital
52 " - " - non débitées
53 " - autres - débitées à l'hôpital
54 " - " - non débitées
55 appariels fixes sans fluouroscopie
56 " " avec fluour. sans petit film
57 " " " " avec petit film

58 appariels mobiles sans fluouroscopie
59 " " avec fluour. sans petit film
60 " " " " avec petit film
61 appareils clichés pulmonaire
62 amplificateurs d'image visionnement
63 amplificateurs d'image visionnement enregistrable
64 amplificateurs d'image mobiles
65 magnétoscopes
66 nombre d'amplificateurs branchés ou magnétique
67 autres appareils diagnostique
68 radiologie - temps plein
69 " - temps partiel
70 autres personnel (radiologie) - temps plein
71 " " " - temps partiel
72 personnel spécialisé non medicale - temps plein
73 " - temps partiel
74 " - départ
75 " - heures rénumérées
76 techniciens diplômés - temps plein
77 " - temps partiel
78 " - départ
79 " - heaures rénumérées
80 techniciens non diplômés - temps plein
81 " - temps partiel
82 " - départ
83 " - heures rénumérées
84 techniciens de laboratoire et radiologie - temps plein
85 " - temps partiel
86 " - départ
87 " - heures rénumérées
88 autres personnel techniques - temps plein
89 " - temps partiel
90 " - départ

91 autres personnel techniques - heures rénumérées
92 infirmières diplômées - temps plein
93 " - temps partiel
94 " - départ
95 " - heures rénumérées
96 autre personnel de radiologie - temps plein
97 " - temps partiel
98 " - départ
99 " - heures rénumérées
100 total personnel de radiologie - temps plein
101 " - temps partiel
102 " - départ
103 " - heures rénumérées
104 traitment et salaries brut - personnel médicale
105 " - personnel non médicale
106 fournitures et dépenses ($)
107 total ($) de radiologie diagnostique

Data Set D.2 :

1	numéro d'hopital
2 - 99	nombre de patients pour chacun des 98 diagnostics
100 - 197	nombre de jours/lits pour chacun des 98 diagnostics
198	la region de hopital
199	le numéro du groupe de l'hopital
200 - 287	les 4 renseignements pour les 22 centres d'activités

Les 22 centres d'activités sont:

600 605 615 620 623 626 630 641 660 676
677 678 679 680 683 684 687 688 730 750
755 760

Les 4 renseignements sont:

1 le total des heures de main-d'oeuvre
2 le total des coûts de main-d'oeuvre en argent
3 les autres dépenses directes en argent
4 activités du centre, unités de mesure

Liste des centres d'activité:

No. du centre	Désignation	Unité de mesure
600	Administration des soins infirmiers	Heures rémunérées
605	Soins infirmiers - maladies aigues	Malade traités
615	Soins infirmiers - maladies de longue durée	Jours-malades
620	Pouponnière	Nouveau-nés
623	Salles d'accouchement	Accouchements
626	Salles d'operation	Durée totale en heures des interventions
630	Service ambulatoire	Visites
641	Clinique psychiatrique	Visites
660	Laboratoire	Unités techniques

No. du centre	Désignation	Unité de mesure
676	Electrocardiographie E.C.G.	Examens
677	Electroencéphalograohie E.E.G.	Examens
678	Services de radioisotopes (Médecine nucléaire)	Unités professionelles
679	Hémodialyse	Traitements
680	Pharmacie	Prescriptions
683	Radiologie diagnostique	Unités professionelles
684	Radiologie thérapeutique	Unités professionelles
687	Physiothérapie	Unités
688	Thérapie de réadaption occupationnelle et ergothérapie	Moyenne quotidienne de personnes ayant participé aux activités du centre
730	Administration	Heures rémunérées
750	Archives médicales	Admissions
755	Alimentation	Jours-repas ou équivalents
760	Buanderie - lingerie	Nombre de jours-malades plus nombre d'accouchements et nombre d'operations

Data set D.3 :

1 numéro d'hopital
2 region socio-sanitaire
3 propriete
4 nombre de médicins
5 nombre de médicins - specialistes
6 nombre de radiologistes - plein temps
7 nombre de radiologistes - temps partiel
8 nombre de techniciens
9 nombre de salles - generale
10 " " " - angiographie
11 " " " - craniographie
12 " " " - mammographie
13 " " " - neuroradiographie
14 " " " - scopie
15 " " " - tomographie
16 " " " - urographie
17 " " " - autres spec.
18 nombre total de salles
19 nombre total d'examens - 1968
20 " " " - 1969
21 " " " - 1970
22 " " " - 1971
23 " " " - 1972
24 nombre de films - 1968
25 " " " - 1969
26 " " " - 1970
27 " " " - 1971
28 " " " - 1972
29 nombre d'appareils rayon-x mobiles
30 nombre d'intensificateurs TV mobile
31 cycle de developpement

Pour chaque salle:

1 code d'identification
2 année d'installation
3 nombre moyen d'examens/jour
4 nombre d'heures en usage/jour
5 surface du plancher
6 frais d'entretien - 1968
7 " " - 1969
8 " " - 1970
9 " " - 1971
10 " " - 1972
11 générateur - phase
12 " - puissance
13 " - nombre d'impulsions
14 " - année d'installation
15 " - fournisseur
16 table d'examens - inclination
17 " - fluouroscopie
18 " - sériographie
19 " - année d'installation
20 " - fournisseur
21 intensificateur d'image - code
22 " " - année d'installation
23 système de TV - type
24 " " " - année d'installation
25 ciné-caméra - dimension image
26 " - année d'installation
27 magnétoscope - code
28 " - année d'installation
29 chargeur de films - code
30 " " " - année d'installation
31 sériographie - code
32 " - année d'installation

33 injecteur automatique - simple
34 " " - code
35 mammographie - code
36 " - année d'achat
37 " - dénomination
38 tomographie - fonction
39 " - année d'installation
40 " - dénomination

APPENDIX C

PRODUCTION MODEL DATA

This appendix provides the numerical values of the production variables used for the identification of the linear programming production model of a diagnostic radiology department. For each of the 139 radiology departments in our sample, the four types of input and the output produced are given in the following format:

	DEPARTMENT NUMBER
Medical Manpower	FULL-TIME RADIOLOGIST MAN-YEARS
	PART-TIME RADIOLOGIST MAN-YEARS
Non-Medical manpower	TECHNICIAN(DIPL) & NON-M.D. SPECIALIST MAN-HOURS
	TECHNICIANS(NO DIPL) MAN-HOURS
	OTHERS MAN-HOURS
Materials	NON-LABOUR EXPENSES IN DOLLARS
Fixed Capital	EQUIPMENT-YEARS
Output Produced	DIAGNOSTIC RADIOLOGY PROFESSIONAL UNITS

1	2	3	4	5
0.	0.	3.	5.	9.
3.	1.	0.	0.	2.
4027.	5396.	14925.	40392.	71940.
0.	1759.	0.	0.	15072.
1828.	2117.	17572.	21378.	48042.
5792.	8736.	140769.	146608.	242010.
2.	1.	11.	26.	24.
0.	84643.	827379.	1038455.	1361526.

6	7	8	9	10
2.	3.	11.	6.	18.
2.	0.	0.	0.	0.
14584.	16064.	87492.	44428.	135443.
1820.	6757.	45012.	7384.	11483.
9026.	7678.	68889.	45350.	128378.
37847.	50429.	282203.	116525.	287435.
5.	3.	19.	11.	29.
0.	0.	2176346.	1265842.	2608648.

11	12	13	14	15
0.	0.	3.	13.	1.
1.	5.	0.	3.	0.
10251.	20393.	15653.	88570.	11171.
1884.	1783.	2255.	18282.	0.
3912.	30597.	11839.	66669.	4310.
5931.	82053.	59304.	251902.	27151.
2.	9.	6.	28.	3.
74433.	0.	0.	1.	0.

16	17	18	19	20
9.	2.	6.	4.	3.
0.	1.	0.	0.	0.
63561.	19095.	59744.	27267.	22295.
17852.	0.	1805.	1836.	8423.
62866.	18038.	56189.	29170.	10531.
252002.	91194.	263529.	125369.	116112.
19.	4.	16.	14.	4.
1637868.	468958.	1354103.	772723.	860654.

21	22	23	24	25
4.	10.	0.	5.	0.
1.	0.	1.	0.	1.
46932.	63945.	6296.	22736.	1995.
955.	3753.	0.	0.	0.
27851.	54821.	2734.	34765.	0.
125063.	153270.	6561.	138988.	5001.
16.	16.	1.	6.	1.
795928.	1430567.	100049.	0.	38453.

26	27	28	29	30
0.	0.	3.	3.	1.
1.	1.	1.	0.	0.
3391.	11228.	28715.	27487.	10764.
0.	3194.	6847.	0.	1820.
0.	5362.	13185.	30526.	4477.
9068.	46773.	128116.	82801.	39922.
1.	4.	8.	10.	6.
93026.	0.	920452.	842312.	274786.

31	32	33	34	35
1.	2.	0.	3.	0.
0.	0.	2.	1.	3.
8520.	9532.	650.	31863.	8975.
0.	3868.	0.	9024.	3260.
3680.	5562.	0.	24653.	5138.
22601.	45149.	86.	100343.	36534.
5.	2.	0.	13.	3.
182213.	441542.	9001.	769871.	275165.

36	37	38	39	40
0.	0.	0.	2.	0.
1.	1.	1.	0.	1.
0.	2620.	8768.	21067.	8747.
0.	0.	1814.	10775.	0.
4739.	0.	1599.	10392.	6652.
8202.	3115.	18246.	74265.	29150.
2.	2.	2.	6.	2.
72412.	25620.	232017.	555784.	258120.

41	42	43	44	45
0.	0.	3.	0.	1.
1.	1.	0.	4.	0.
6089.	12911.	29327.	10022.	9316.
2745.	0.	0.	0.	4771.
3532.	2224.	27783.	8445.	3744.
28038.	16755.	85268.	45445.	42898.
3.	2.	7.	3.	3.
263112.	197077.	879800.	392030.	0.

46	47	48	49	50
1.	0.	0.	0.	0.
0.	1.	2.	0.	2.
14548.	0.	0.	0.	2495.
12381.	1827.	0.	0.	0.
10117.	11391.	0.	0.	0.
70499.	14159.	1815.	305.	7371.
5.	3.	1.	0.	1.
564707.	113515.	16820.	0.	57511.

51	52	53	54	55
1.	1.	0.	0.	0.
0.	1.	1.	1.	0.
10430.	7831.	10013.	4654.	0.
4411.	0.	1894.	2210.	1734.
5494.	8265.	2041.	3344.	0.
39543.	27426.	28415.	17187.	1315.
5.	4.	6.	2.	3.
353405.	333229.	230975.	160620.	131156.

56	57	58	59	60
3.	2.	2.	0.	9.
0.	0.	0.	1.	0.
4639.	19856.	15636.	720.	65218.
0.	591.	0.	0.	9547.
28511.	17831.	19428.	0.	45405.
63712.	81887.	76980.	360.	173421.
5.	7.	5.	1.	15.
754426.	813334.	549659.	1956.	1322146.

61	62	63	64	65
2.	4.	5.	3.	0.
0.	0.	0.	0.	3.
16119.	30560.	47358.	24218.	2946.
5949.	1839.	1943.	5963.	0.
5708.	27024.	39765.	10184.	1885.
70746.	124141.	191867.	64197.	13223.
6.	18.	11.	10.	2.
611104.	951892.	1199885.	475855.	122969.

66	67	68	69	70
1.	0.	1.	0.	0.
0.	1.	0.	2.	4.
6881.	2898.	16038.	4271.	4749.
0.	0.	1186.	0.	1885.
4280.	0.	17810.	4779.	1820.
36244.	6239.	46002.	13194.	13270.
5.	1.	6.	3.	2.
305502.	53024.	479198.	122415.	130826.

71	72	73	74	75
0.	0.	1.	0.	0.
2.	2.	0.	1.	1.
5597.	12340.	9003.	1822.	0.
2781.	0.	0.	2412.	1833.
1856.	12127.	15110.	0.	0.
14930.	31695.	39408.	10396.	1963.
2.	3.	3.	2.	2.
148221.	338912.	385265.	0.	24427.

76	77	78	79	80
0.	3.	2.	1.	0.
1.	0.	0.	0.	3.
6818.	27842.	21462.	9395.	2201.
2595.	0.	0.	2681.	0.
1715.	21297.	16640.	572.	0.
23261.	101232.	61038.	25040.	15199.
2.	9.	10.	5.	2.
0.	803890.	0.	206516.	91682.

81	82	83	84	85
0.	0.	1.	5.	1.
1.	1.	0.	0.	0.
3952.	5786.	6668.	37835.	6533.
0.	1696.	5738.	8848.	5218.
4202.	7897.	2195.	32544.	3910.
11741.	30222.	36551.	154424.	46377.
3.	3.	4.	10.	4.
85260.	0.	0.	1053007.	263057.

86	87	88	89	90
1.	0.	0.	0.	1.
0.	1.	1.	4.	0.
6316.	1947.	6172.	4573.	5000.
1926.	0.	2213.	4101.	1800.
2150.	2023.	2571.	2540.	6093.
26464.	10561.	24860.	32883.	40307.
2.	1.	2.	2.	2.
169934.	31174.	147846.	223042.	261875.

91	92	93	94	95
0.	0.	0.	0.	2.
1.	1.	1.	1.	0.
8047.	6762.	2488.	3268.	14618.
3918.	2752.	0.	0.	0.
3995.	4549.	0.	1835.	15130.
26968.	33524.	11386.	8051.	48933.
4.	3.	2.	2.	9.
214540.	249531.	0.	56178.	426715.

96	97	98	99	100
0.	9.	0.	16.	3.
1.	0.	1.	0.	0.
28242.	76795.	7571.	83637.	16380.
0.	15201.	0.	18214.	0.
30852.	71845.	5772.	86004.	14080.
113124.	335791.	33459.	500114.	75202.
7.	18.	3.	24.	7.
1070844.	1911697.	246758.	2280636.	865112.

101	102	103	104	105
1.	3.	3.	3.	3.
2.	0.	3.	0.	0.
14192.	25712.	45433.	22176.	21965.
0.	5762.	9784.	10807.	646.
8047.	16650.	31620.	10599.	21283.
41574.	100251.	114494.	106463.	100333.
4.	10.	9.	5.	7.
303294.	934447.	1212158.	0.	0.

106	107	108	109	110
3.	6.	0.	1.	0.
0.	1.	4.	0.	0.
27815.	43580.	0.	4209.	0.
0.	21814.	0.	0.	0.
23360.	36897.	0.	10316.	0.
61056.	156929.	3075.	19637.	38086.
12.	12.	3.	4.	2.
0.	1316517.	25554.	224727.	55212.

111	112	113	114	115
1.	5.	1.	0.	0.
0.	1.	0.	1.	0.
5956.	70456.	7511.	0.	0.
0.	20609.	2128.	0.	0.
4054.	42865.	1899.	0.	0.
27575.	251504.	20858.	268.	0.
3.	20.	2.	1.	0.
184445.	1229376.	264168.	3000.	0.

116	117	118	119	120
0.	2.	0.	0.	1.
1.	0.	1.	1.	1.
4344.	9028.	1843.	2110.	11605.
4040.	3483.	0.	0.	8613.
1880.	10139.	0.	1945.	10165.
19253.	43186.	3585.	7711.	37294.
2.	5.	1.	2.	4.
97082.	302650.	14833.	0.	0.

121	122	123	124	125
0.	6.	0.	0.	0.
1.	1.	0.	1.	1.
2088.	54396.	0.	1251.	5667.
0.	4038.	0.	0.	2060.
2184.	54684.	0.	2203.	1979.
8205.	248347.	0.	3898.	20962.
1.	18.	0.	1.	2.
0.	1701646.	1216016.	0.	154513.

126	127	128	129	130
0.	6.	0.	0.	0.
1.	0.	1.	0.	1.
2063.	52952.	5118.	1510.	9181.
0.	6172.	2141.	210.	0.
1937.	64911.	883.	425.	6539.
7509.	136395.	15874.	3471.	27509.
2.	15.	2.	2.	4.
64532.	1130877.	114904.	0.	247430.

131	132	133	134	135
0.	1.	1.	0.	0.
1.	0.	1.	1.	1.
2116.	8605.	11462.	1828.	1949.
0.	6479.	2350.	0.	0.
0.	4017.	6387.	2002.	2164.
13665.	55354.	48208.	10017.	10353.
2.	3.	9.	1.	2.
0.	351668.	0.	77393.	57206.

136	137	138	139
0.	2.	0.	0.
1.	0.	3.	2.
2530.	24817.	6640.	2356.
2053.	6327.	0.	478.
4198.	13508.	5648.	0.
14890.	77227.	31085.	9999.
2.	6.	3.	1.
82176.	755416.	0.	80319.

APPENDIX D

DESCRIPTION OF COMPUTER PROGRAM WHICH IDENTIFIES A PRODUCTION MODEL

The computer program which identifies the production function for radiographic facilities consists of two phases which are shown in Fig. C.1: i) the establishment of the linear program (L.P.) which may be considered to have 3 parts and; ii) the solution of the linear program which may be considered to have 6 parts. If the number of input production variables is m and the resulting department sample consists of s observations, then the linear program for the k-th department can be written as;

$$\max\ (\ c^t,\ 0)\ \lambda$$

$$\text{s.t.} \qquad [\mathbf{D}\ \mathbf{I}]\lambda = \mathbf{X} \qquad \lambda \geq 0$$

where

$\mathbf{D}$ — m x s matrix

$\mathbf{c}$ — s x 1 vector

λ — $(s + m)$ x 1 vector

$\mathbf{X}$ — m x 1 vector

and

d_{ij} — amount of input i used in dept. j

a_j — amount of output produced from j

$\mathbf{X}$ — $\mathbf{d}_k$, a column of matrix $\mathbf{D}$

Therefore the establishment of a linear program for any and all departments in a sample merely requires the construction of vector $\mathbf{c}$ and matrix $\mathbf{D}$ for the sample.

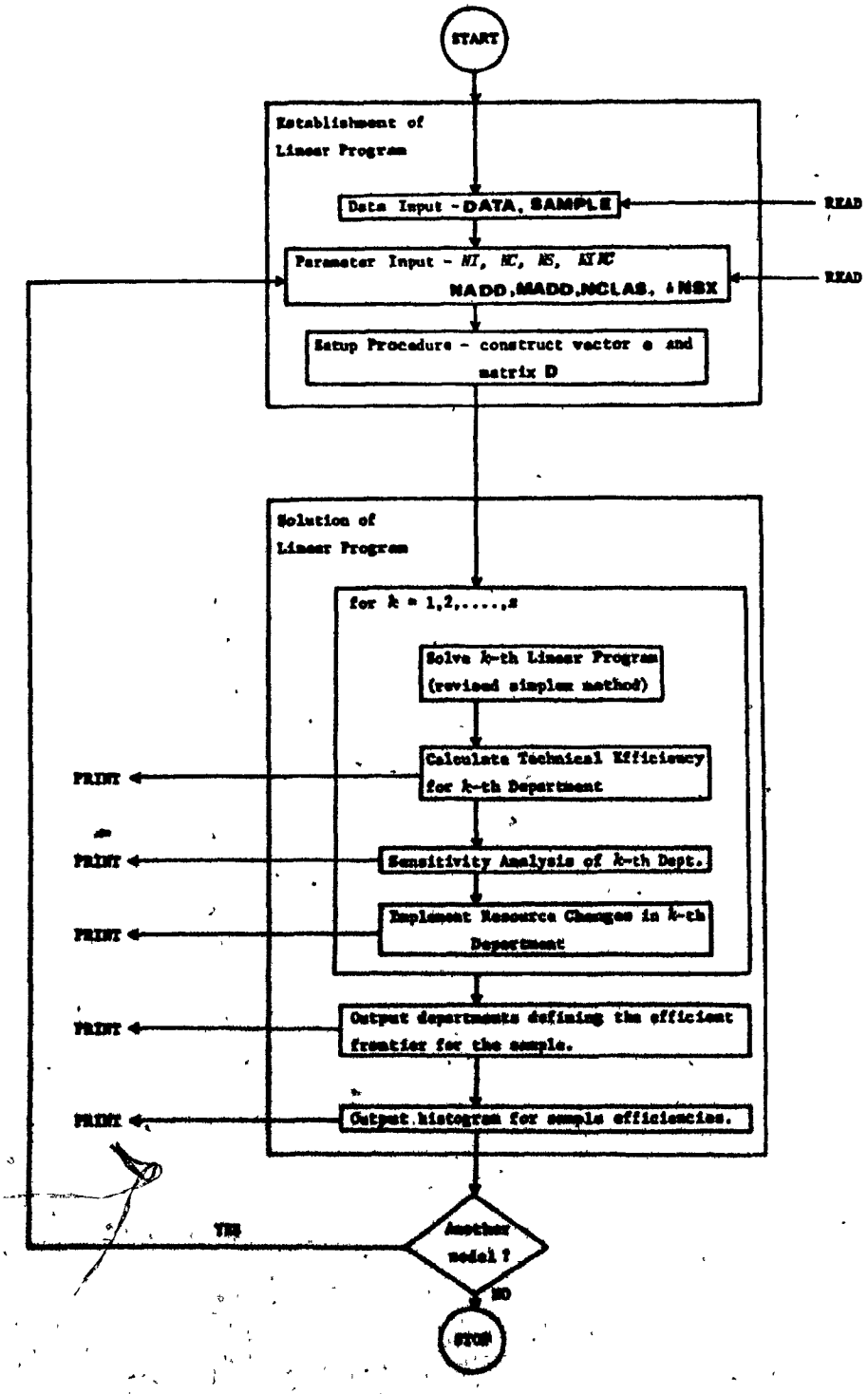


Fig. C.1. The two-phase computer program which identifies a linear programming production model from a sample of producers.

I. Establishment of the Linear Program

i) Data Input - two matrices are read, **DATA** & **SAMPLE**

Let n — maximum number of departments available from data sets D.1, D.2, D.3

m_1 — maximum number of possible production variables in a model, including the output

m_2 — number of possible production variable classes, e.g. non-MD labour represents one class with as many as three different inputs whereas expenses represents one class and one input

then **DATA** — $m_1 \times n$ matrix, $(\mathbf{DATA})_{ij}$ — amount of variable i in department j (App. C)

SAMPLE — $m_2 \times n$ matrix, $(\mathbf{SAMPLE})_{ij}$ — 1 if data is available (non-zero) on a variable in class i in department j,
— 0 otherwise

ii) Parameter Input - a series of parameters (*NI*, *NC*, *NS*, *NINC*) and vectors (**NADD**, **MADD**, **NCLAS**, **NSX**) are read which completely define a particular model (see example below),

NI — number of different models inputs

NC — number of model production variable classes

NS — number of model input classes in which at least one

NINC — of the inputs of each classes must be non-zero, e.g., medical manpower is one class, having two distinct inputs - full and part-time radiologists, in which at least one of these must be non-zero. number of different inputs to be incremented by +1.0 to simulate an implementable resource change - corresponds to first *NINC* elements of vector **X** .

NADD – $(NI + 1) \times 1$ vector, contains how many variables from **DATA** are to be added to form each model production variable.

MADD – $\begin{pmatrix} NI+1 \\ \sum_{1} NADD_i \times 1 \end{pmatrix}$ vector, contains indices corresponding to rows of **DATA** of which inputs are to be added to form model inputs

NCLAS – $NC \times 1$ vector, contains indices corresponding to rows of **SAMPLE** which describe model production variable classes.

NSX – $NC \times 1$ vector, contains the number of model inputs in each model input class in which at least one of these model inputs must be non-zero.

Example: if the rows of matrix **DATA** correspond to the following variables,

1. equipment-years
2. full-time radiologist man-years
3. part-time radiologist man-years
4. Tech (dipl) man-hrs
5. Tech (ndipl) man-hrs
6. "Others" man-hrs
7. non-labour expenses ($)
8. diagnostic radiology professional units

and the rows of matrix **SAMPLE** to the followong variable classes,

1. equipment
2. medical manpower
3. non-medical manpower
4. non-labour expenses
5. professiona units

and we wish to construct a model with the following inputs (in this order)

1. equipment-years
2. full-time radiologist man-years
3. part-time radiologist man-years
4. Tech (dipl) + Tech (ndipl) man-hrs
5. "Others" man-hrs

and the output = professional units, then,

$NI = 5 \quad NC = 4 \quad NS = 3$ & $NINC = 3$

$$\mathbf{NADD} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{MADD} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 8 \end{bmatrix} \quad \mathbf{NCLAS} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 5 \end{bmatrix} \quad \mathbf{NSX} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

iii) Setup Procedure - construct vector **c** and matrix **D** for linear program

a) multiply corresponding elements of rows indicated in **NCLAS** of matrix **SAMPLE**- the result is a $1 \times n$ row vector whose zero elements correspond to departments which do not have enough data to be included in the model.

b) for those departments corresponding to a non-zero value in the above operation construct the input and output sets as indicated in **NADD** & **MADD** & store in matrix **D1** and vector **c1** respectively.

c) scan through the rows of matrix **D1** as indicated by **NSX** searching for zero values, and eliminate these departments.

Note: step c) is necessary - for example; a model is constructed with a non-medical manpower input consisting solely of Tech (dipl). A department with a zero value of Tech (dipl), but with a non-zero value for either Tech (ndipl) or Others or both, will have been included in the model after step b) above, but must in reality be excluded as the non-medical manpower for this department in this particular model is zero.

d) the remaining departments can now be appropriately stored in a vector **c** and a matrix **D** as described initially.

II. Solving the Linear Program

i) actual solution of the linear program - if s is the number of departments included in the model, then for $k = 1, 2, \ldots, s$ do a) to f) below;

a) let $\mathbf{X} = \mathbf{d}_k$ where $\mathbf{d}_k$ = k-th column of matrix $\mathbf{D}$

b) if some element of $\mathbf{X}$ is zero, eliminate all columns of matrix $\mathbf{D}$ (and corresponding elements of vector $\mathbf{c}$) which have a non-zero value corresponding to these elements, i.e. construct a reduced problem. This procedure will avoid degeneracy in the solution of the linear program.

c) solve the resulting linear program using the revised simplex method.

d) calculate and print the technical efficiency for department k

e) calculate and print sensitivities from linear program solution, i.e. the dual prices (Luenberger, 1973).

f) for $i = 1, 2, \ldots . NINC$ increment the i-th element of vector $\mathbf{X}$ by +1.0 and solve new L.P., calculate and print the ratio of the output from the new (input incremented) linear program to the original linear program output - i.e. the capacity change corresponding to an implimentable resource change.

after solving all s linear program problems

ii) print departments defining the efficient production function, i.e. those departments which are 100% by the analysis above.

iii) print the frequency distributions of efficiencies for all s departments in the sample.

APPENDIX E

SOME CLUSTERING RESULTS

The results of applying the clustering scheme of Jarvis and Patrick (1973) to a sample of 129 diagnostic radiology departments is shown below. The departments are represented in a three dimensional Euclidean space by their departmental equipment mix, departmental labour mix, and hospital diagnostics mix. Each of the three departmental vector components was constructed by using the information measure described in Eqn. 6.15 . The various cluster analyses which follow are the result of using different cluster parameters and membership voting techniques. One such set, for k = 10, k_t = 6, with unweighted voting (sharing of near neighbours), is shown in complete detail.

RUN SUMMARY

SAMPLE SIZE=129
DIMENSION OF SPACE= 3
NO. OF CLUSTERS= 28
NEIGHBOURS(K)= 10
THRESHOLD(KT)= 6
METHOD= SHARING OF NEAR NEIGHBOURS
DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE

cluster 1

DEPT	EQUIP X2	LABOR X2	HOSP X2
1	0.784	3.005	0.948
35	0.839	2.616	0.953
37	0.784	3.054	0.928
54	0.895	2.669	0.884
65	0.824	3.445	0.918
71	0.924	2.963	0.913
93	0.824	2.882	1.025
131	0.824	2.882	0.943
132	0.887	2.571	1.021

2

DEPT	EQUIP X2	LABOR X2	HOSP X2
2	1.175	1.787	1.411

3

DEPT	EQUIP X2	LABOR X2	HOSP X2
3	1.138	0.313	1.295
9	0.844	0.483	1.294
22	0.741	0.395	1.249
100	0.903	0.229	1.361

4

DEPT	EQUIP X2	LABOR X2	HOSP X2
4	1.150	0.666	0.903
5	1.042	0.886	0.982
18	1.154	0.676	0.960
29	1.147	0.821	0.976
34	1.190	1.226	0.985
61	1.109	0.721	0.997
82	1.131	0.855	0.954
97	1.165	0.803	1.034
98	1.092	0.833	0.951
122	1.125	1.058	1.076

5

DEPT	EQUIP X2	LABOR X2	HOSP X2
6	0.791	1.278	0.890
12	0.702	1.209	0.934
52	0.766	1.051	0.890
101	0.790	0.963	0.836

6

DEPT	EQUIP X2	LABOR X2	HOSP X2
7	0.839	1.205	0.963

7

DEPT	EQUIP X2	LABOR X2	HOSP X2
8	0.936	1.207	1.072

8

DEPT	EQUIP X2	LABOR X2	HOSP X2
10	0.984	0.588	1.055
21	0.965	0.377	1.006
60	0.984	0.722	1.028
63	0.969	0.576	1.079
73	0.887	0.195	1.078
77	1.083	0.263	1.041
106	1.051	0.320	1.020
127	0.992	0.678	1.063

9

DEPT	EQUIP X2	LABOR X2	HOSP X2
11	1.657	1.800	1.690

10

DEPT	EQUIP X2	LABOR X2	HOSP X2
13	1.686	0.592	0.925
38	2.238	1.317	0.944
72	1.591	1.241	0.975
74	2.084	3.696	0.954
89	2.254	3.213	0.991
91	1.489	1.373	0.940

11

DEPT	EQUIP X2	LABOR X2	HOSP X2
14	0.902	0.898	0.968
16	0.861	0.601	0.958
17	0.951	0.601	0.909
19	1.014	0.502	0.947
28	0.655	0.825	1.006
30	0.861	0.470	0.932
32	0.838	0.915	0.986
57	0.751	0.561	1.012
58	0.807	0.557	0.964
62	0.911	0.353	1.010
78	0.910	0.479	0.940
84	0.771	0.799	0.961
86	0.824	0.683	0.924
95	0.925	0.493	0.948
96	0.787	0.983	1.022
99	0.881	0.779	0.981
105	0.972	0.512	0.954
109	0.900	0.221	0.987
111	1.007	0.225	0.893
117	0.822	0.599	0.958
130	0.784	1.002	0.959
137	0.828	0.920	1.010

12

DEPT	EQUIP X2	LABOR X2	HOSP X2
15	1.075	0.786	1.144
44	0.901	1.155	1.213
112	1.068	1.178	1.132

13

DEPT	EQUIP X2	LABOR X2	HOSP X2
20	0.922	1.093	0.947
51	0.838	1.237	0.905
61	0.941	1.252	0.868
83	0.900	1.247	0.954
102	0.946	1.170	0.912
103	0.958	1.248	0.997
133	0.963	1.265	0.904

14

DEPT	EQUIP X2	LABOR X2	HOSP X2
23	1.093	1.924	0.797
27	1.066	1.756	0.876
39	1.062	1.632	0.948
41	1.380	1.955	0.935
88	1.289	1.801	0.886
119	1.417	2.339	0.921
121	1.093	1.986	0.869
134	1.093	2.339	0.912
135	1.173	1.986	0.944

15

DEPT	EQUIP X2	LABOR X2	HOSP X2
24	0.697	0.331	0.928
43	0.778	0.367	0.981
66	0.727	0.225	1.000

16

DEPT	EQUIP X2	LABOR X2	HOSP X2
25	1.093	2.882	1.562
70	1.192	3.247	1.273
118	1.175	2.882	1.073

17

DEPT	EQUIP X2	LABOR X2	HOSP X2
24	1.190	1.957	1.163

18

DEPT	EQUIP X2	LABOR X2	HOSP X2
31	1.266	3.754	0.917
45	0.950	4.705	0.978
50	1.175	3.807	0.954
75	0.824	4.299	1.093
79	1.111	5.329	0.946
80	0.824	4.270	1.003
124	1.175	3.083	0.950
139	1.175	4.375	1.075

19

DEPT	EQUIP X2	LABOR X2	HOSP X2
36	0.784	2.032	0.925
40	0.824	1.832	0.981
42	0.824	1.490	0.864
44	0.887	2.384	0.949
47	0.941	1.434	0.949
53	0.786	1.423	1.028
69	0.716	2.000	1.024
76	0.951	1.824	1.010
90	0.824	1.583	0.965
94	0.784	1.988	0.932
104	0.928	1.554	0.947
107	0.927	1.391	1.053
116	0.951	1.991	0.984
120	0.831	1.591	0.996
126	0.784	1.984	0.897
128	0.824	2.142	0.940
134	0.784	1.933	0.883
138	0.818	1.870	1.026

20

DEPT	EQUIP X2	LABOR X2	HOSP X2
46	1.135	1.344	0.925

21

DEPT	EQUIP X2	LABOR X2	HOSP X2
56	1.369	0.237	0.958

22

DEPT	EQUIP X2	LABOR X2	HOSP X2
59	0.474	4.528	1.944

23

DEPT	EQUIP X2	LABOR X2	HOSP X2
67	0.501	2.882	1.223

24

DEPT	EQUIP X2	LABOR X2	HOSP X2
68	0.982	0.692	0.936

25

DEPT	EQUIP X2	LABOR X2	HOSP X2
85	1.275	1.479	1.037
87	1.175	1.986	1.047

26

DEPT	EQUIP X2	LABOR X2	HOSP X2
92	0.941	1.941	0.854

27

DEPT	EQUIP X2	LABOR X2	HOSP X2
113	1.019	0.950	0.949

28

DEPT	EQUIP X2	LABOR X2	HOSP X2
125	0.824	1.914	0.892

```
**********************************************************
*                                                        *
*  RUN SUMMARY                                           *
*                                                        *
*          SAMPLE SIZE=129                               *
*   DIMENSION OF SPACE=  3                               *
*      NO. OF CLUSTERS= 79                               *
*        NEIGHBOURS(K)= 15                               *
*        THRESHOLD(KT)= 12                               *
*              METHOD= SHARING OF NEAR NEIGHBOURS        *
*     DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE  *
*                                                        *
**********************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 35 37 44 54 71 93 131 132
2	4 18 29 61 82 98
3	16 17 30 78 95
4	24 43 57 58 66 86 117
5	31 45 50 75 79 80 139

```
RUN SUMMARY

          SAMPLE SIZE=129
  DIMENSION OF SPACE=  3
     NO. OF CLUSTERS= 22
       NEIGHBOURS(K)= 15
       THRESHOLD(KT)= 10
              METHOD= SHARING OF NEAR NEIGHBOURS
   DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE
```

Clusters with 5 or more points

Cluster	Departments
1	1 31 35 37 44
	45 50 54 65 70
	71 75 79 80 93
	118 124 131 132 139
2	3 4 5 9 15
	18 22 29 34 61
	82 97 98 100 112
	113 122
3	6 7 10 14 16
	17 20 21 24 28
	30 32 36 40 42
	43 47 51 52 57
	58 60 62 63 66
	68 69 73 76 77
	78 81 83 84 86
	90 94 95 96 99
	101 102 104 105 106
	109 111 116 117 120
	125 126 127 128 130
	133 136 137 138
4	23 27 39 41 87
	88 119 121 134 135

```
**********************************************************
*                                                        *
*  RUN SUMMARY                                           *
*                                                        *
*          SAMPLE SIZE=129                               *
*   DIMENSION OF SPACE=  3                               *
*      NO. OF CLUSTERS=  4                               *
*        NEIGHBOURS(K)= 15                               *
*        THRESHOLD(KT)=  8                               *
*              METHOD= SHARING OF NEAR NEIGHBOURS        *
*    DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE   *
*                                                        *
**********************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 2 3 4 5
	6 7 8 9 10
	12 13 14 15 16
	17 18 19 20 21
	22 23 24 25 26
	27 28 29 30 31
	32 34 35 36 37
	38 39 40 41 42
	43 44 45 46 47
	50 51 52 53 54
	56 57 58 60 61
	62 63 64 65 66
	68 69 70 71 72
	73 74 75 76 77
	78 79 80 81 82
	83 84 85 86 87
	88 89 90 91 92
	93 94 95 96 97
	98 99 100 101 102
	103 104 105 106 107
	109 111 112 113 116
	117 118 119 120 121
	122 124 125 126 [illegible]
	128 130 131 132 [illegible]
	134 135 136 137 138
	139

```
**********************************************************
*                                                        *
*  RUN SUMMARY                                           *
*                                                        *
*          SAMPLE SIZE=129                               *
*   DIMENSION OF SPACE=  3                               *
*      NO. OF CLUSTERS= 62                               *
*        NEIGHBOURS(K)= 10                               *
*        THRESHOLD(KT)=  7                               *
*              METHOD= SHARING OF NEAR NEIGHBOURS        *
*     DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE  *
*                                                        *
**********************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 35 37 54 65 71 93 131 132
2	4 5 18 29 34 61 82 97 98 122
3	10 63 77 106 127
4	16 58 86 99 117
5	17 30 78 95 105
6	20 51 81 83 102 103 133
7	23 27 88 121 135
8	31 45 50 75 79 80 139

```
**********************************************************
*                                                        *
*  RUN SUMMARY                                           *
*                                                        *
*          SAMPLE SIZE=129                               *
*   DIMENSION OF SPACE=  3                               *
*      NO. OF CLUSTERS= 28                               *
*        NEIGHBOURS(K)= 10                               *
*        THRESHOLD(KT)=  6                               *
*              METHOD= SHARING OF NEAR NEIGHBOURS        *
*     DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE  *
*                                                        *
**********************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 35 37 54 65 71 93 131 132
2	4 5 18 29 34 61 82 97 98 122
3	10 21 60 63 73 77 106 127
4	13 38 72 74 89 91
5	14 16 17 19 28 30 32 57 58 62 78 84 86 95 96 99 105 109 111 117 130 137
6	20 51 81 83 102 103 133
7	23 27 39 41 88 119 121 134 135
8	31 45 50 75 79 80 124 139
9	36 40 42 44 47 53 69 76 90 94 104 107 116 120 126 128 136 138

```
*****************************************************************
*                                                               *
*  RUN SUMMARY                                                  *
*                                                               *
*          SAMPLE SIZE=129                                      *
*   DIMENSION OF SPACE=  3                                      *
*      NO. OF CLUSTERS= 12                                      *
*        NEIGHBOURS(K)= 10                                      *
*        THRESHOLD(KT)=  5                                      *
*              METHOD= SHARING OF NEAR NEIGHBOURS               *
*     DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE         *
*                                                               *
*****************************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 6 7 10 12
	14 16 17 19 20
	21 24 28 30 31
	32 35 36 37 40
	42 43 44 45 47
	50 51 52 53 54
	57 58 60 62 63
	65 66 68 69 71
	73 75 76 77 78
	79 80 81 83 84
	86 90 93 94 95
	96 99 101 102 103
	104 105 106 107 109
	111 116 117 120 124
	125 126 127 128 130
	131 132 133 136 137
	138 139
2	3 4 5 9 15
	18 22 29 34 61
	64 82 97 98 100
	112 113 122
3	13 23 26 27 38
	39 41 72 74 85
	87 88 89 91 119
	121 134 135

```
**********************************************************
*                                                        *
*  RUN SUMMARY                                           *
*                                                        *
*          SAMPLE SIZE=129                               *
*   DIMENSION OF SPACE=  3                               *
*      NO. OF CLUSTERS= 92                               *
*        NEIGHBOURS(K)=  7                               *
*        THRESHOLD(KT)=  5                               *
*              METHOD= SHARING OF NEAR NEIGHBOURS        *
*    DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE   *
*                                                        *
**********************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 35 37 54 65 71 93 131
2	16 30 58 86 117
3	17 19 78 95 105 111
4	18 29 61 82 97 98

```
*****************************************************************
*                                                               *
*  RUN SUMMARY                                                  *
*                                                               *
*          SAMPLE SIZE=129                                      *
*   DIMENSION OF SPACE=  3                                      *
*     NO. OF CLUSTERS= 48                                       *
*       NEIGHBOURS(K)=  7                                       *
*       THRESHOLD(KT)=  4                                       *
*              METHOD= SHARING OF NEAR NEIGHBOURS               *
*    DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE          *
*                                                               *
*****************************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 35 37 54 65 71 93 131 132
2	4 5 18 29 61 82 97 98
3	10 21 60 63 73 77 106 127
4	16 17 19 24 30 43 57 58 66 68 78 86 95 105 111 117
5	20 81 83 102 103 133
6	23 27 41 72 88 91 119 121 134 135
7	28 32 96 130 137
8	36 40 42 69 94 126 128 138
9	45 75 79 80 139

```
**********************************************************
*                                                        *
*  RUN SUMMARY                                           *
*                                                        *
*           SAMPLE SIZE=129                              *
*   DIMENSION OF SPACE=  3                               *
*      NO. OF CLUSTERS= 17                               *
*        NEIGHBOURS(K)=  7                               *
*        THRESHOLD(KT)=  3                               *
*                METHOD= SHARING OF NEAR NEIGHBOURS      *
*     DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE  *
*                                                        *
**********************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 6 7 12 20
	23 27 28 32 35
	36 37 39 40 41
	42 44 47 51 52
	53 54 65 69 71
	72 76 81 83 88
	90 91 92 93 94
	96 101 102 103 104
	107 116 119 120 121
	125 126 128 130 131
	132 133 134 135 136
	137 138
2	4 5 18 29 61
	82 97 98 113 122
3	10 15 21 60 62
	63 64 73 77 106
	109 112 127
4	14 16 17 19 24
	30 43 57 58 66
	68 78 84 86 95
	99 105 111 117
5	31 45 50 75 79
	80 118 124 139

```
**********************************************************
*                                                        *
*  RUN SUMMARY                                           *
*                                                        *
*         SAMPLE SIZE=129                                *
*   DIMENSION OF SPACE=  3                               *
*      NO. OF CLUSTERS= 28                               *
*        NEIGHBOURS(K)=  7                               *
*        THRESHOLD(KT)= 55                               *
*               METHOD= WEIGHTED VOTING(MULTIPLICITIVE)  *
*    DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE   *
*                                                        *
**********************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 35 37 44 54 65 71 93 131 132
2	4 5 18 29 61 82 97 98 113
3	6 12 42 51 52 101 125 136
4	10 21 60 62 63 73 77 106 109 127
5	14 16 17 19 24 28 30 32 43 57 58 66 68 78 84 86 95 96 99 105 111 117 130 137
6	20 81 83 102 103 133
7	23 27 41 72 88 91 92 119 121 134 135
8	31 45 50 75 79 80 124 139
9	36 39 40 47 53 69 76 90 94 104 116 120 126 128 138

```
*************************************************************
*                                                           *
*  RUN SUMMARY                                              *
*                                                           *
*          SAMPLE SIZE=129                                  *
*   DIMENSION OF SPACE=  3                                  *
*      NO. OF CLUSTERS= 20                                  *
*        NEIGHBOURS(K)=  7                                  *
*        THRESHOLD(KT)= 50                                  *
*              METHOD= WEIGHTED VOTING(MULTIPLICITIVE)      *
*     DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE     *
*                                                           *
*************************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 35 37 44 54 65 71 93 131 132
2	4 5 18 29 61 82 97 98 113
3	6 7 8 12 20 42 51 52 81 83 101 102 103 107 125 133 136
4	10 15 21 60 62 63 64 73 77 106 109 112 122 127
5	13 23 27 41 72 88 91 92 119 121 134 135
6	14 116 17 19 24 28 30 32 43 57 58 66 68 78 84 86 95 96 99 105 111 117 130 137
7	31 45 50 75 79 80 124 139
8	36 39 40 47 53 69 76 90 94 104 116 120 126 128 138

```
**********************************************************************
*                                                                    *
*  RUN SUMMARY                                                       *
*                                                                    *
*          SAMPLE SIZE=129                                           *
*   DIMENSION OF SPACE=  3                                           *
*      NO. OF CLUSTERS= 17                                           *
*        NEIGHBOURS(K)=  7                                           *
*        THRESHOLD(KT)= 45                                           *
*              METHOD= WEIGHTED VOTING(MULTIPLICITIVE)               *
*    DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE               *
*                                                                    *
**********************************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 35 37 44 54 65 71 93 131 133
2	4 5 18 29 61 82 97 98 113
3	6 7 8 12 14 16 17 19 20 24 28 30 32 42 43 51 52 57 58 66 68 78 81 83 84 86 95 96 99 101 102 103 105 107 111 125 130 133 136 137
4	10 15 21 60 62 63 64 73 77 106 109 112 122 127
5	13 23 27 41 72 88 91 92 119 121 134 135
6	31 45 50 75 79 80 124 139
7	36 39 40 47 53 69 76 90 94 104 116 120 126 128 138

```
**********************************************************************
*                                                                    *
*  RUN SUMMARY                                                       *
*                                                                    *
*           SAMPLE SIZE=129                                          *
*   DIMENSION OF SPACE=  3                                           *
*      NO. OF CLUSTERS= 13                                           *
*        NEIGHBOURS(K)=  7                                           *
*        THRESHOLD(KT)= 40                                           *
*                 METHOD= WEIGHTED VOTING(MULTIPLICITIVE)            *
*     DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE              *
*                                                                    *
**********************************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 35 37 44 54
	65 71 93 131 133
2	4 5 18 29 61
	82 97 98 113
3	6 7 8 10 12
	14 15 16 17 19
	20 21 24 28 30
	32 36 39 40 42
	43 47 51 52 53
	57 58 60 62 63
	64 66 68 69 73
	76 77 78 81 83
	84 86 90 94 95
	96 99 101 102 103
	104 105 106 107 109
	111 112 116 117 120
	122 125 126 127 128
	130 133 136 137 138
4	13 23 27 34 41
	72 85 88 91 92
	119 121 134 135
5	31 45 50 75 79
	80 124 139

```
**********************************************************************
*                                                                    *
*  RUN SUMMARY                                                       *
*                                                                    *
*          SAMPLE SIZE=129                                           *
*   DIMENSION OF SPACE=  3                                           *
*      NO. OF CLUSTERS= 11                                           *
*        NEIGHBOURS(K)=  7                                           *
*        THRESHOLD(KT)= 35                                           *
*              METHOD= WEIGHTED VOTING(MULTIPLICITIVE)               *
*     DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE              *
*                                                                    *
**********************************************************************
```

Cluster with 5 or more points

Cluster	Departments
1	1 6 7 8 10
	12 14 15 16 17
	19 20 21 24 28
	30 32 35 36 37
	39 40 42 43 44
	47 51 52 53 54
	57 58 60 62 63
	64 65 66 68 69
	71 73 76 77 78
	81 83 84 86 90
	93 94 95 96 99
	101 102 103 104 105
	106 107 109 111 112
	117 120 122 125 126
	127 128 130 131 132
	133 136 137 138
2	4 5 18 29 61
	82 97 98 113
3	13 23 27 34 41
	46 72 85 88 91
	92 119 121 134 135
4	31 45 50 75 79
	80 124 139

```
**********************************************************
*                                                        *
*  RUN SUMMARY                                           *
*                                                        *
*          SAMPLE SIZE=129                               *
*   DIMENSION OF SPACE=  3                               *
*      NO. OF CLUSTERS=  9                               *
*        NEIGHBOURS(K)=  7                               *
*        THRESHOLD(KT)= 30                               *
*              METHOD= WEIGHTED VOTING(MULTIPLICITIVE)   *
*    DISTANCE MEASURE= EUCLIDEAN STANDARDIZED BY RANGE   *
*                                                        *
**********************************************************
```

Clusters with 5 or more points

Cluster	Departments
1	1 4 5 6 7
	8 10 12 14 15
	16 17 18 19 20
	21 24 28 29 30
	32 35 36 37 39
	40 42 43 44 47
	51 52 53 54 57
	58 60 61 62 63
	64 65 66 68 69
	71 73 76 77 78
	81 82 83 84 86
	90 93 94 95 96
	97 98 99 101 102
	103 104 105 106 107
	109 111 112 113 116
	117 120 122 125 126
	127 128 130 131 132
	133 136 137 138
2	2 11 25 26 70
	87 118
3	13 23 27 34 41
	46 72 85 88 91
	92 119 121 134 135
4	31 45 50 75 79
	80 124 139

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