

Distribution feeder reduction for dispersed generation applications



Master's thesis

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Abstract

Electric power systems today are undergoing a paradigm shift in operational and market philosophy through technologies like distributed generation and the “smart grid.” Decentralizing the power system and allowing users to inject power into the grid, however, introduces a wide array of problems, and much research has gone towards implementing a growing number of small generation sources throughout the existing electric power network infrastructure. This thesis describes the issues involved in reducing a typical rural distribution feeder to a model that can be used for distributed generation interconnection studies, particularly for islanding studies.

Résumé

Nos systèmes de puissance électrique procèdent présentement à un changement de paradigme autant dans leurs philosophies opérationnelles que dans celles du marché grâce à des technologies telles que la génération distribuée et la plateforme «smart grid». Décentraliser le système d'énergie et permettre aux usagers d'injecter de l'énergie dans le réseau présente néanmoins de nombreux problèmes, et beaucoup de nouvelles études cherchent à établir un nombre croissant de petites sources de génération dans le cadre du réseau d'infrastructure d'énergie électrique existant présentement. Cette thèse décrit les questions liées à la réduction d'une artère de distribution rurale typique d'un modèle qui peut être utilisé pour des études d'interconnexion distribués génération, en particulier pour les études îlotage.

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Abbreviations and Symbols

DSAP	Distribution system analysis program
DG	Distributed generation
EMTP	Electromagnetic transients program
MV	Medium voltage
R_1	Positive-sequence series resistance
X_1	Positive-sequence series reactance
R_0	Zero-sequence series resistance
X_0	Zero-sequence series reactance
B_1	Positive-sequence shunt susceptance
B_0	Zero-sequence shunt susceptance
R_{neg}	Negative-sequence series resistance
X_{neg}	Negative-sequence series reactance
L	Length of a line
P	Real power
Q	Reactive power
U	Voltage magnitude
Z	Impedance
I	Current
S	Complex power
V	Voltage (expressed as line-to-neutral RMS, unless otherwise noted)
I_{LLL}	Three-phase fault current
I_{LLG}	Line-to-line-to-ground fault current
I_{LL}	Line-to-line fault current
I_{LG}	Line-to-ground fault current

1 Introduction

Economic and technological trends have shifted the very concept of the electric power system from tightly controlled, top-heavy unidirectional energy pipelines to systems that are more distributed – both in terms of actual generation capability and in terms of market control. This has posed a huge challenge to the control and protection schemes of these vast networks, most of which are designed and coordinated for conventional power systems with large-scale, centralized generation. With the electric power system becoming more and more decentralized, many researchers and engineers have been working to manage the implementation of distributed energy resources into the existing power grid. Technologies like distributed generation, local energy storage, demand-side response, and the associated means for coordination and communication are being developed in an effort for conventional power systems to evolve into “smart grids” that will handle a variety of multidirectional power flows between many independent parties.

Because distributed generation (DG) sources must comply with dedicated interconnection guidelines, it is necessary for DG impact studies to be conducted in order to assess the effects the DG’s will have on the area electric power system upon connection. The goal of this thesis is to outline a methodology to simplify a utility’s distribution feeder into a representative model that is as simple as possible but with the characteristics preserved that are relevant for power flow studies and transient analysis for events that occur in faults, fault protection actions, and DG response to faults.

1.1 Distributed Generation

Large-scale generation projects are less likely nowadays to have the political and financial resources to see fruition. The increasing prevalence of environmental concerns associated with CO₂ emissions, safety and sustainability of nuclear power, environmental effects of large-scale hydroelectric projects, etc. provide a realm of uncertainty that lingers over any generation company trying to secure capital for a large power plant. In addition, the expansion of transmission networks has been slowed by the financial uncertainties associated with deregulation.

In the meantime, a variety of technologies have focused the spotlight of power systems research and development efforts to local power systems (i.e. distribution level). Distributed generation, energy storage systems, and advanced metering have the potential to give consumers more control over their consumption – and perhaps production – thereby granting a more active role to consumers in the electricity marketplace. The trend is towards the development of self-reliant local networks that depend less on both the transmission systems and large-scale power producers. It is hoped that this decentralization will not only make power systems more resilient and make electricity markets more competitive, but also defer or

eliminate the need for capital-intensive, politically sensitive, large-scale generation and transmission projects in the future.

A conventional power system is composed of three distinct levels of operation – generation, transmission, and distribution. The generation system is responsible for producing electric power from a particular energy source like coal, natural gas, nuclear power, or hydroelectric power. Because these generation facilities are generally very large and environmentally intrusive, they are typically located in remote areas far from urban centers. Transmission lines provide the connection between these large, centralized generation facilities and load centers. Spawning from these transmission lines are distribution networks that handle low-voltage power and connect to loads like factories, businesses, and homes in order to provide power from the transmission line directly to the consumers as needed. Because of the delicate balance necessary to constantly match supply to demand, these power systems are highly centralized and strictly coordinated.

Distributed generation (DG) is a paradigm of electric power systems placing power generation capability at the distribution level. Individually, these generators are small in power output, compared to those used in the conventional system. Because of this difference in scale, they do not have the significant negative environmental impact of larger, conventional power generators; thus, it is possible to place these sources closer to load centers where they are needed. The small size and modularity of these sources enable widespread accessibility of potential generation to consumers, who can then sell their own generated power into a market on the power system. This improved competition gives consumers more choice on where their power comes from and how much they are willing to pay for it. DG also decreases reliance on the conventional centralized power sources for electricity, which has the potential to improve access for remote areas where bulky and costly transmission lines are unable to reach. In addition, because power near a given DG source is generated locally, less of the power delivered to the load from the DG source is wasted by transmission losses and other conversion losses. Figure 1-1 illustrates the difference between a conventional power system and one containing several distributed generators throughout.

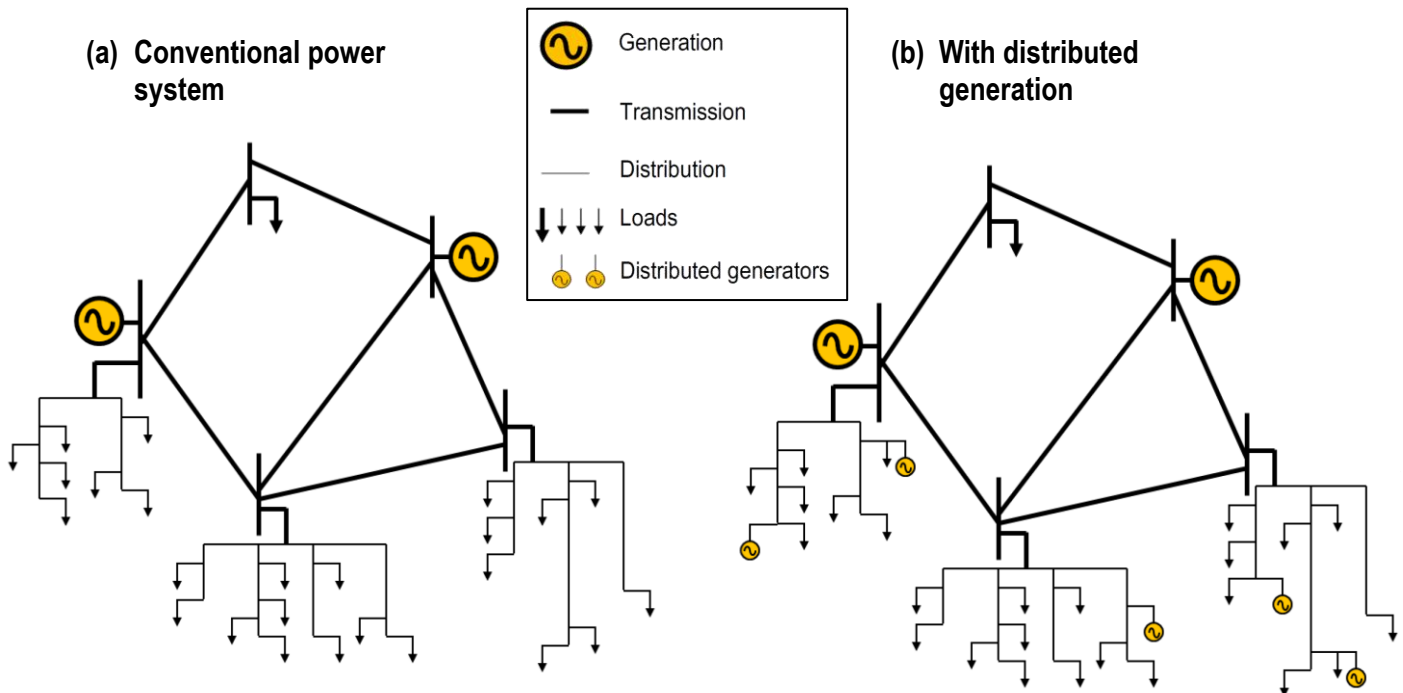


Figure 1-1: Conventional power system structure vs. system with distributed generation

1.2 Distribution Systems

Distribution systems deliver electrical power from the high-voltage transmission system to individual buildings and other consumption sites. A substation will interface the distribution system with the transmission system. Primary feeders radiate outward from substations to load centers; distribution transformers reduce the voltage from distribution voltage to utilization voltage; and secondary networks distribute energy from the distribution transformer to individual customers [1]. Figure 1-2 illustrates these parts of a distribution system. This paper focuses on benchmarking specifically at the primary feeder level.

Since nearly all electricity consumers are connected to the distribution network as their means of receiving electrical power, the distribution system is the most expansive level of a power system. Topologies vary widely among each other and are a function of their geographic environment and consumer demand profiles. However, most distribution networks share several common features, many of which are useful in the development of the benchmark feeder model.

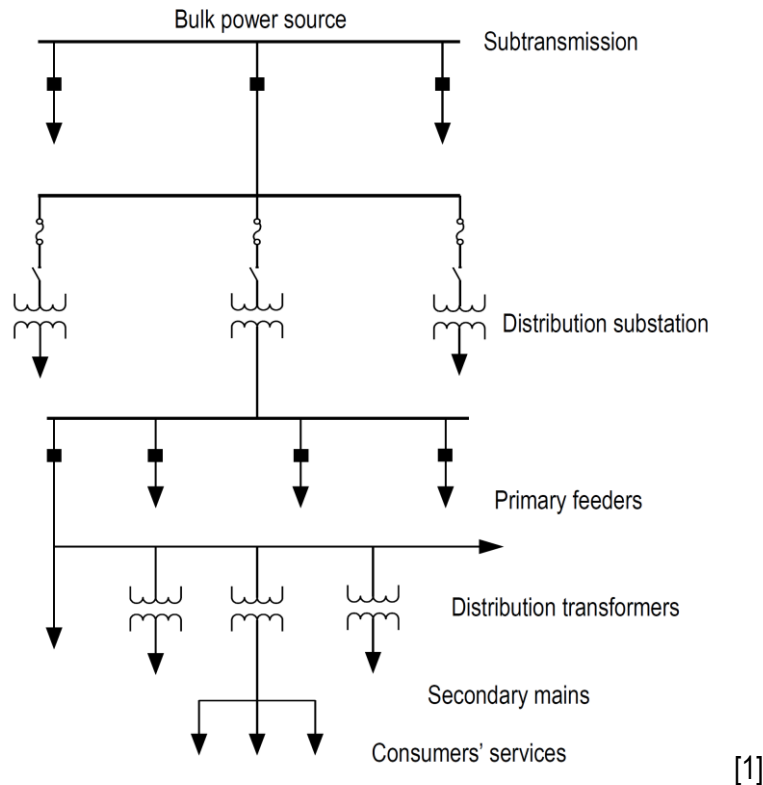


Figure 1-2: One-line diagram of a typical distribution system [1]

Most distribution feeders share many common characteristics, such as voltage classes, loading capabilities, feeder lengths, and more. The primary voltage classes are 5, 15, 25, and 35 kV. The 15 kV class voltage level is the most popular, comprising more than 80% of all distribution circuits in the US. These networks generally spawn feeders ranging from 5 to 25 km in length with three-phase branches and single-phase lateral lines branching from a three-phase main line. Typical loading is 4-6 MVA on most 15 kV circuits. Higher-voltage circuits handle correspondingly higher loading; for instance, 35 kV feeders typically carry 10-16 MVA. Only large power consumers (e.g. large businesses and factories) are connected to the primary network. Most consumers are served by secondary networks whose voltage is stepped down by distribution transformers from the primary network. Common secondary voltages for three-phase, grounded-wye services are 277 V or 120 V (phase-to-neutral) [2]. Table 1-1 below highlights some of the parameters for a typical distribution system.

Table 1-1: Typical distribution system parameters

	Most common value	Other common values
<u>Substation characteristics</u>		
Voltage (line-to-line)	12.47 kV	4.16, 4.8 kV 13.2, 13.8 kV 24.94 kV 34.5 kV
Number of station transformers	2	1 – 6
Substation transformer size	21 MVA	5 – 60 MVA
Number of feeders per bus	4	1 – 8
<u>Feeder characteristics</u>		
Peak current	400 A	100 – 600 A
Peak load	7 MVA	1-15 MVA
Power factor	0.98 lagging	0.8 lagging – 0.95 leading
Number of customers	400	50 – 5000
Length of feeder mains	4 mi	2 – 15 mi
Length including laterals	8 mi	4 – 25 mi
Area covered	25 mi ²	0.5 – 500 mi ²
Mains wire size	500 kcmil	4/0 AWG – 795 kcmil
Lateral tap wire size	1/0 AWG	#4 – 2/0 AWG
Lateral tap peak current	25 A	5 – 50 A
Lateral tap length	0.5 mi	0.2 – 5 mi
Distribution transformer size (single-phase)	25 kVA	10 – 150 kVA

[2]

Other distribution network characteristics may depend more heavily on the specific configuration and geographical features present. For three-phase balanced systems, impedances in overhead lines range from 0.11 - 0.76 Ω per thousand feet. The current capacity for these lines typically ranges from 60 - 1500 A, depending on the conductor material, cross-sectional area, stranding, and temperature. The substations range in size from 5 MVA for small rural substations to beyond 200 MVA for urban substations [2].

Power systems supply a broad range of loads, whose quantities can vary according to urban density, customer type (i.e. residential, commercial, or industrial), usage patterns, etc. Rural areas might have a load density of 10 kVA/mi², whereas a dense downtown core may demand around 300 MVA/mi². A house's power consumption may peak in the realm of 10-20 kVA, and a nearby factory might peak at around 5 MW. Proper perspective on power quantities is important to consider, especially considering the relative power contributions of the proposed DG units.

Because distribution systems, by nature, cover a large geographical area, they are exposed and vulnerable to faults. Most faults involve a short circuit between phases or between phase and ground. In order to protect the distribution system from damages associated with large fault currents, a number of protective devices are installed in a coordinated fashion to form a protection scheme for the network. The main

objectives of these protection schemes are to minimize the duration of a fault and to minimize the impact of these faults on consumers. Secondary objectives are as follows:

- Eliminate safety hazards as quickly as possible.
- Limit service outages to the smallest possible segment of the system.
- Protect consumers' equipment.
- Protect the system from unnecessary service interruptions and disturbances.
- Disconnect lines, transformers, and other equipment that are faulted.

Protective devices applied to distribution systems include relay-controlled circuit breakers, automatic circuit reclosers, fuses, and automatic line sectionalizers [1]. These devices are coordinated with each other in order to provide backup protection (in case of the failure of a protective device to interrupt the fault current) and to minimize the area affected by faults in the network.

The majority of Canadian primary distribution systems operate as a radial network [3]. Radial networks allow for easy fault detection and clearing by the protection system. Because power flow to any given load is constricted to only one path at any time, the line impedance provides a natural limiter to fault currents, especially when they are located far from the substation. Radial systems also help voltage control and ease the analysis and prediction of power flows throughout the distribution network. To ensure a higher degree of reliability for critical loads, tie points are often installed that connect the load to alternate feeder paths, in the case of a fault in the connected feeder. Alternate feeder paths can be constructed that provide parallel means for loads to receive power from the substation, as long as these paths remain open (disconnected), except in the case of a contingency. In order for a system to be radial, all loads must be connected to the substation with only one path.

1.3 Integrating Distributed Generation

One consequential aspect of distributed generation arises from the ownership and control of DG sources. Most often, DG sources are not owned by the utility; yet, they may introduce any combination of positive and adverse effects on the local electric power system. As the penetration levels of DG increase, so do the probability of such events, as well as the extent of their impact on the power system.

The transformer configurations used throughout the primary distribution system, as well as the configuration of the transformer interfacing the DG with the system, affects the interaction between the DG and the distribution system. This is most evident during faults and imbalance conditions, during which improper transformer configurations may cause significant overvoltage, ferroresonance, and compromises in the sensitivity of fault protection. Choosing the correct transformer configuration for the DG interconnection depends on the distribution system characteristics, as well as the size and type of DG being implemented.

DG can have a number of effects on power quality throughout the distribution system and nearby consumers. Undesired harmonics can be both generated and absorbed by DG's, depending on the circumstances. These harmonics can damage both the distribution system and the DG itself through overheating of transformers or heating in the generator. Several types of DG can contribute significantly to voltage flicker, which can lead to irritating fluctuations in light in lamps, televisions, computer monitors, etc. at loads near fluctuating DG sources.

Because power systems have traditionally contained all generators at the generation level, these systems are designed on the basis of one-way power flow – from generators, through transmission lines, to distribution networks, and then out to loads. DG disrupts this one-way convention, because it introduces power sources at the distribution and load levels.

The voltage regulation scheme of the distribution system can be affected by these changes in power flow directionality. Voltage regulation equipment, such as transformer tap changers, in-line regulators, and switched capacitor banks, are placed and controlled in the system under the assumptions of radial power flow and, subsequently, a steady voltage drop that is a function of the distance along the feeder from the substation. The changes in voltage profile along the feeder that result from DG complicate the manner in which voltage regulation equipment operate. In addition, the interaction of regulating equipment and the DG's own voltage control mechanisms may cause conflicting compensation measures between the control devices that lead to undesirable cycling of regulation devices and further impacts on power quality as a result [4].

Changes in power flow directionality also affect power system protection schemes, which are also designed on the basis of one-way power flow from the substation. When a fault occurs on a distribution feeder, a coordinated system of fuses, reclosers, and relays works together to locate the fault on the feeder, determine whether or not to isolate it, isolate it, and restore severed connections after the fault has been cleared. This coordination is well-established and is based on the radial layout of the system.

The introduction of DG sources on these feeders complicates this procedure of fault response. When a lasting fault occurs in a feeder, the system's protection mechanisms isolate it by closing the fuse that is upstream (towards the transmission system) and closest to the fault, thereby leaving as much as possible of the feeder upstream from the fault unaffected. Conventionally, the faulted section is left isolated and without power until it is cleared, either automatically or by service crews. However, problems arise if DG sources are located on this faulted section. Islanding is such a power system state, in which part of a distribution network is isolated from the rest of the network, yet continues to be supplied with power from DG sources located within the isolated network itself. Figure 1-3 shows an example of an island situation, in which the section highlighted in red is isolated from the rest of the system by the open protective switch, yet continues to receive power from the local DG.

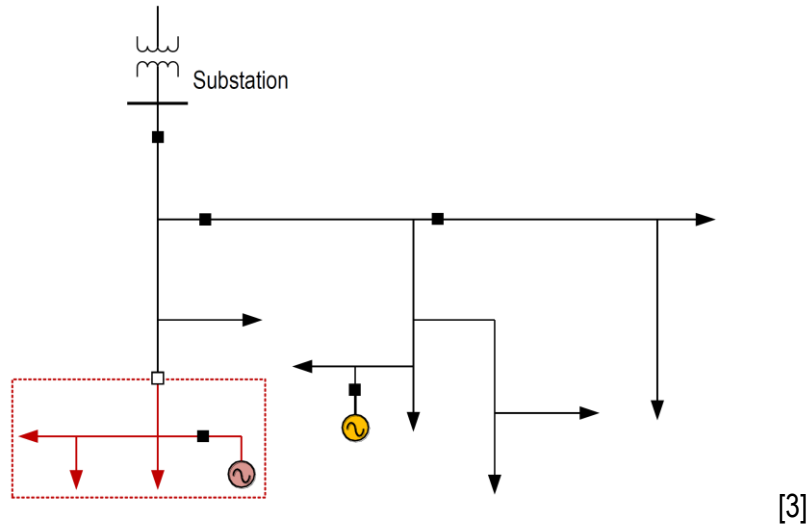


Figure 1-3: Islanding in a distribution network

This condition brings about several problems, including the following [5]:

- Lack of regulation of voltage and frequency, which is usually provided by the rest of the power system.
- Danger to utility line workers who are trying to repair line that continues to be energized by DG.
- Danger to the public due to the utility's inability to easily de-energize damaged lines.
- Potential damage to DG units if the island is no longer synchronized with the power system at the instant of reconnection.
- Interference with manual or automatic service restoration procedures for neighbouring loads.

Islanding can sometimes be done intentionally in order to facilitate microgrids, particularly in cases where increased reliability and backup power are desired precisely at the facility where the DG unit is installed. However, the considerable amount of engineering effort, control functionality, and communications infrastructure necessary for this type of operation do not yet exist beyond the scope of the DG/load bus, and further consideration must be given when trying to operate local islands comprising multiple DG's. Therefore, current IEEE and state standards seek to minimize the possibility of island formation and to immediately dispel these islands by disconnecting all DG units on the islanded portion of the distribution system [5]. In most cases, DG's are expected to detect an island and disconnect themselves from it within 500-1000 ms of the island occurrence in order to avoid out-of-phase reconnection by the acting protection devices. Such a loss of synchronism between the DG and the system can result in large currents to the generators and adverse impacts on the protection scheme elsewhere in the system [3].

Because of the risks mentioned above posed by inadvertent islanding, it is crucial for a DG-penetrated system to be able to detect the occurrence of such an island and respond appropriately within the given

timeframe. A variety of islanding detection mechanisms has been developed that seek such indications in order to detect and respond to the occurrence of an island.

1.4 Existing Distribution Feeder Benchmarks

Because of the numerous challenges associated with DG integration, there are a variety of research topics devoted to reimagining distribution-level planning and operation with more system intelligence, system control, and coordination schemes. In order to provide a consistent platform with which to simulate and verify the proposed techniques for DG integration, benchmark distribution systems have been developed.

IEEE published a series of benchmark distribution feeders in order to make available a common data set with which to compare the performance of different distribution simulation programs [6]. These programs were designed for steady-state analysis of unbalanced three-phase radial feeders. Because this work focused on steady-state analysis, applications of this feeder modeling work and the associated simulation programs were focused on planning, reliability and security, and economic analyses.

The proliferation of DG has shifted the motivations of recent benchmarking efforts towards interconnection of these sources into existing networks, along with the complications posed in planning, coordination, and control. These call for a set of benchmark feeders that is better tailored to capture the phenomena associated with DG interconnection studies.

CIGRE (International Council on Large Electric Systems) Task Force C6.04.02 proposed a set of benchmarks for distribution networks specifically geared towards DG integration. It distinguishes between three different types of networks to be looked at – low-voltage urban distribution systems, medium-voltage rural distribution systems, and high-voltage transmission networks. Because many potential DG units are in the form of wind turbines, solar cell arrays, and other renewable energy projects that are often located in rural communities, this thesis will focus on a benchmark that models a medium-voltage (MV) rural distribution system. CIGRE's MV rural distribution network benchmark is derived from a German MV distribution network that supplies a small town and surrounding rural area. The network is rated at 20 kV and is fed from a 110 kV transformer station; however, the benchmark's parameters (e.g. voltage rating, load sizes) may be adapted according to regional standards. The benchmark network was designed in order to study the impact of DG on the following [7]:

- Power flow in MV distribution lines
- Voltage profile throughout the MV distribution network
- Power quality issues, including harmonics, flicker, frequency fluctuations, and voltage fluctuations
- Small-signal stability
- Voltage stability
- Protection against faults

In Canada, Natural Resources Canada's CANMET Energy Technology Centre is developing its own set of benchmarks for application in Canadian systems. It classifies distribution systems into three types – urban, suburban, and rural – according to the length of the feeder main, types of protection devices used, types of laterals, load density, and voltage levels [8]. These benchmarks are valid for North American systems, which differ from European systems in the structure of the primary and secondary distribution systems, distribution transformer sizing, and grounding practices [1].

1.5 Modeling Tools

This project utilizes two different commercial software platforms – a distribution system analysis program (DSAP) and an electromagnetic transients program (EMTP) – in order to construct and validate the distribution system models for simulation and analysis. Each of the programs is designed for analysis of the system within a specified context. Comparison of the same power system between these programs allows for a more complete understanding of the system, as well as cross-validation of the feeder reduction and benchmarking procedure.

The DSAP is designed for planning studies and for the steady-state simulation of electrical distribution network behaviour under different operating conditions and scenarios. It performs analyses such as load flow calculations, short-circuit studies, and network optimizations on balanced or unbalanced systems built with any combination of phases and configurations. The data gathered on the feeder system described in this thesis is in the format of a DSAP model [9].

The EMTP is a program designed to simulate electromagnetic, electromechanical, and control system transients in multiphase power systems. It is capable of modeling oscillations from these types of phenomena ranging in duration from microseconds to seconds. It is typically used in switching and lightning surge analysis, insulation coordination, shift torsional oscillations, ferroresonance, and power electronics applications in power systems. Simulation options include frequency scans, steady-state solutions, time-domain solutions, and statistical analysis [10]. Because it performs simulations in the time domain, it can be used to portray and analyze phenomena that a standard DSAP does not capture.

1.6 Objectives and Scope

The objective of this thesis is to describe a methodology that can be used to obtain a simplified rural distribution feeder model credible enough to accommodate DG impact studies and, more specifically, power flow, short circuit, and islanding studies. The relevant timescale of these studies is defined by the time required for DG's to react to an islanding event, which is typically on the order of 0.5 to 1 second, assuming the presence of high-speed reclosing in the local power system [23]. High-speed reclosing is assumed to be present, since it does not only impose constraints on the islanding protection scheme but

may also have adverse effects on synchronous DG's because of the possibility of out-of-phase reclosing. If high-speed reclosing is not present, islanded DG disconnection times may be extended to the range of 2 seconds. This electromechanical type of transient analysis is called for here in order to accurately portray the relevant effects from the dynamics of the DG's and their interaction with the feeder's protection scheme.

This work is concerned with the characteristics of the distribution network itself, not the details of the DG itself. It deals with rural distribution feeders and is confined to analyzing the primary feeder level of the distribution network, including laterals. This encompasses lines and equipment at the distribution level between the substation transformer and distribution transformers.

It should be noted that this thesis does not aim to establish a universal benchmark, but rather to provide the tools for utilities and researchers to generate their own benchmark that is more specifically applicable to an existing system.

1.7 Outline of Topics

This thesis discusses the following topics, as related to the benchmarking efforts in distribution feeders:

- Characteristics and components of a benchmark feeder
- Simplification techniques to aggregate a section of lines and loads into a single equivalent
- Development of a methodology that can be applied universally to rural distribution feeders in order to obtain a benchmark
- Description of a real feeder that will be used as a case study to demonstrate the methodology
- Application of feeder reduction methodology to obtain an equivalent benchmark for the feeder described
- Modeling of benchmark components for transient analysis
- Validation of feeder reduction methodology and benchmark modeling

2 Characteristics of Typical Distribution Feeders

2.1 Characterization and Classification

Distribution systems vary among geographic areas. The biggest differences in topology and operational philosophies are between systems in North America and those in Europe. These differences are related to grounding configuration, the share of lines comprised by the primary and secondary levels of the distribution system, phasing, and nominal frequency (50 Hz vs. 60 Hz systems).

In Canada, distribution systems are classified into three distinct categories – urban, suburban, and rural. These categories are distinguishable by their voltage levels, topology, loading, and protection scheme. Table 2-1 highlights these characteristics. Distribution feeders can be further classified into overhead and underground feeders, depending on the type of layout used. Overhead feeders are typically used for rural distribution, while underground construction is more common in high-density urban systems.

Table 2-1: General features of different distribution system types

	Urban	Suburban	Rural
System voltage (line-to-line)	12.5 or 13.8 kV	25 or 27.6 kV	27.6 kV
Feeder rating	6-10 MVA	12-20 MVA	10-30 MVA
Feeder construction	Little/no overhead	Mostly overhead	Overhead lines
Backbone	Shorter backbones, Fewer laterals	Longer backbone, Large number of laterals	Much longer backbone, Large number of laterals
Load density	High	Medium	Low
Voltage regulators	Not used	May be used	Used
Protection	Feeder head end overcurrent relay, Lateral fuses, No recloser	Feeder head end overcurrent relay, Lateral fuses, Recloser	Feeder head end overcurrent relay, Lateral fuses, Recloser

[8]

Rural feeders have a number of features that distinguish them from the urban and suburban types. Because they serve a relatively large geographical area, their topologies feature long backbones with a large number of laterals. Because power is traditionally delivered over a greater length of overhead lines, higher voltage levels are used in order to mitigate losses. The associated voltage drop in these long lines

is typically compensated by at least one in-line voltage regulator. The geographical vastness and the sparseness of loads of such feeders justify the design and operational differences in such feeders.

Because of the fundamental differences between each type of distribution system, the benchmark feeders generated by the methodology presented in this thesis will be confined to rural feeders consisting of overhead lines in a radial layout. Rural feeders are suitable candidates for DG integration for several reasons. Many DG's are located at sites away from major population centers in order to avoid problems with pollution (in the case of fuel-burning microturbines) or noise pollution (in the case of wind turbines). Additionally, reclosing protective devices are found exclusively on rural overhead feeders and pose a risk to DG out-of-phase reclosing in the case of a temporary fault and temporary island scenario. Because rural feeders are typically radial and do not redundant service paths during contingencies, DG integration provides a great deal of potential improvement in the reliability of service provided to customers. For these reasons, rural feeders will be the focus of these benchmarking efforts.

2.2 Elements of a Distribution Feeder

Most rural feeders can be described by a set of components that is universal to this type of feeder arrangement. Table 2-2 lists these components, as well as the parameters that are essential to describing them for the benchmarking work. This framework derives from that used in describing urban and suburban distribution systems for their respective benchmarks [17]. This set of components will form a template from which parameters and other details can be obtained to complete an appropriately descriptive benchmark for the rural feeder in this paper.

Table 2-2: Elements common in rural feeders

Element	Important parameters
Source representation	<ul style="list-style-type: none">➤ Voltage at source➤ Equivalent impedance of source
Substation transformer	<ul style="list-style-type: none">➤ Nominal power➤ Configuration➤ Winding voltages➤ Winding impedance
Overhead lines	<ul style="list-style-type: none">➤ Phasing➤ Series impedance➤ Current rating
Loads	<ul style="list-style-type: none">➤ Locations➤ Phasing➤ Complex power drawn
Voltage regulator	<ul style="list-style-type: none">➤ Nominal voltage➤ Desired regulated voltage➤ Number of taps➤ Voltage step per tap➤ Voltage deadband➤ Transformer nominal power➤ Transformer winding impedance
Shunt capacitor	<ul style="list-style-type: none">➤ Reactive power injected
Protection devices	<ul style="list-style-type: none">➤ Recloser ratings➤ Sectionalizer ratings➤ Fuse ratings

Chapter 5 describes the rural feeder that serves as a case study for this benchmarking work. The feeder is described in terms that are similar to the template outlined above in Table 2-2.

2.3 Uses of a Distribution Feeder

Of fundamental consideration in selecting a benchmark model for a given distribution feeder is the type of studies it is to be used for and, more specifically, the timescale of interest for the study. Table 2-3 lists some of the different types of studies used when assessing DG interconnection options, distinguishing them by the time range over which they are most useful.

Table 2-3: DG interconnection studies

Timescale	Relevant DG integration studies
Steady-state analysis	<ul style="list-style-type: none">➤ Impact of DG on feeder power flow➤ Impact of DG on feeder voltage profile➤ Impact of DG on reliability➤ Economic impact of DG in electricity markets
Transient stability	<ul style="list-style-type: none">➤ Study of small-signal stability➤ Study of voltage stability
Transient analysis (less than 1 sec)	<ul style="list-style-type: none">➤ Power quality issues (harmonics, flicker, frequency variations, voltage variations)➤ Interaction of DG with feeder protection scheme

[7,12]

The timescale also an important factor in determining the degree of modeling complexity necessary for the benchmark and its components. For example, in steady state, it may be more appropriate to model loads not as a fixed complex power injection but rather as a value that fluctuates over time in the form of a daily load profile. Likewise, in transient analysis addressing electromechanical types of transients, the effects of temporary or permanent switching actions of the protective devices are more visible and relevant, calling importance to how these devices are modeled over this time range.

2.4 Obtaining a Representative Distribution Feeder Benchmark

One of the goals of this work is to represent the benchmark feeder in EMTP for transient analysis. The data available for this feeder benchmark comes in the form of a DSAP model, which is a portrayal of the distribution feeder for purposes of planning and steady-state operation studies. Each load and each line section are represented individually, comprising a feeder model of hundreds of components. There is very little practicality in transferring this data directly into EMTP and trying to execute a detailed, nonlinear simulation with hundreds of these components. A feeder reduction technique is sought in order to yield a simpler network that will require less computational effort and data for repetitive “what-if” sorts of DG impact studies.

2.5 Applicability of Benchmarks

The potential value of the feeder reduction methodology described below is that it may provide some guidance towards obtaining a simple distribution feeder from a larger one encountered in reality. This

simplified feeder would be used for DG impact studies of the type mentioned above. Some guidance is also provided for the eventuality in which data is missing.

2.6 Summary

The key features of a rural distribution feeder were defined in order to form the foundation for the benchmark feeder. These features help distinguish rural distribution feeders from urban and suburban types. Rural distribution feeders share a set of common components that can be used as a template to help classify and characterize any given feeder, based on the data available. This benchmark is obtained through a process that simplifies the feeder's numerous, sprawling branches into a more concise, aggregate model that can better be utilized in steady-state and transient DG impact studies. The next chapter develops and explains this process of feeder simplification.

3 Feeder Reduction Techniques

Distribution systems are a challenge to simulate and require a large amount of data for detailed representation, because they contain a large number of individual loads and are unbalanced. If every load was included in the model, the system model would become unnecessarily large and difficult to simulate, particularly for transient analysis. Thus, it is common practice to follow a set of simplifications and assumptions in order to reduce the complexity of the distribution system models to suit a particular purpose.

The purpose of this benchmark feeder model, as stated previously, is to facilitate simulations for islanding and related interconnection studies for proposed DG unit. This limits the scope of analysis to transient dynamics of the electromechanical type, particularly those involving switching actions from protective devices, other DG's already in the system, and the DG under test. The overlying simplifications will be made with these objectives in mind, neglecting components that have no significant effect on the power system within this scope.

An important step in this simplification process is lumping the loads that are distributed throughout the feeder and its laterals. To minimize the number of nodes necessary in the resulting feeder model, nodes will only be designated at the locations of retained components (see Section 4.6). All loads will be lumped, and their equivalents will be placed at the retained nodes, such that the reduced network exhibits the same characteristic as the original network in respect to the following properties:

- Voltage drop along the length of the feeder
- Through real and reactive power
- Short-circuit current

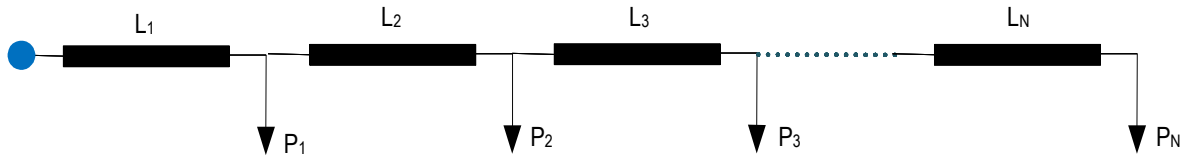
Several such techniques have already been developed and reported in the literature, having been applied mostly to distribution load flow solving algorithms. They are briefly discussed in the following sections.

3.1 Voltage Drop and Line Loss Models

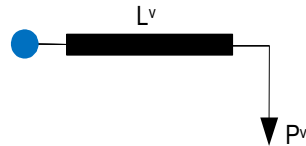
The voltage drop and line loss models were developed as means to reduce laterals and feeders to a simplified equivalent model. The voltage drop model sets the length of the equivalent line such that its total voltage drop matches that seen at the end of the original section. The line loss model sets its length such that its line losses match that seen throughout the original section. The hybrid model combines the voltage drop and line loss models in order to match both the voltage drop and losses between the equivalent line and the original section [13]. Figure 3-1 shows each of these models, based on the following parameters:

L_i	Length of section i [km]
P_i	Active power consumption of load at section i [kW]
L^v	Length of equivalent section [km], voltage drop model
P^v	Active-power consumption of equivalent load [kW], voltage drop model
L^l	Length of equivalent section [km], line loss model
P^l	Active-power consumption of equivalent load [kW], line loss model
L_A	Length of equivalent section A [km], hybrid model
L_B	Length of equivalent section B [km], hybrid model
P_A	Active-power consumption of equivalent load at section A [kW], hybrid model
P_B	Active-power consumption of equivalent load at section B [kW], hybrid model

(a) Original section



(b) Voltage drop model



(c) Line loss model



(d) Hybrid model

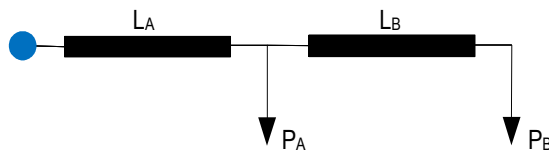


Figure 3-1: Lumping lines and loads, using the voltage drop, line loss, and hybrid models

The voltage drop model's length can be determined using the following formula:

$$L^v = \frac{\sum_{i=1}^N (L_i \sum_{j=1}^i P_j)}{\sum_{i=1}^N P_i} \quad (\text{Eq. 3-1})$$

The equivalent load placed at the end of the voltage drop model is simply the sum of the individual loads on the original section, as follows:

$$P^v = \sum_{i=1}^N P_i \quad (\text{Eq. 3-2})$$

The line loss model makes an adjustment to the length of the equivalent model in order to better reflect the line losses experienced by the original model. The length for the equivalent line loss model is calculated as follows:

$$L^l = \frac{\sum_{i=1}^N \left[\left(\sum_{j=1}^i P_j \right)^2 L_i \right]}{\left(\sum_{i=1}^N P_i \right)^2} \quad (\text{Eq. 3-3})$$

The equivalent load for the line loss model is equal to that for the voltage drop model.

$$P^l = P^v \quad (\text{Eq. 3-4})$$

The hybrid model splits the equivalent line into two sections in order to reconcile the disparities between the voltage drop and line loss seen in their respective equivalent models. Figure 3-1d illustrates how the hybrid model derives from the voltage drop and line loss models. The procedure below, outlined by the following formulae, yields the equivalent length and load values for the hybrid model.

First, calculate the total length of the original section, as follows:

$$L_{tot} = \sum_{i=1}^N L_i \quad (\text{Eq. 3-5})$$

The total load of the original section has already been calculated above in the generation of the voltage drop and line loss models, as follows:

$$P_{tot} = P^v = \sum_{i=1}^N P_i \quad (\text{Eq. 3-6})$$

The coefficients k_1 and k_2 will determine how the total load and total line length are split between sections A and B of the equivalent hybrid model. Those coefficients are calculated as follows:

$$k_1 = \frac{L^v - L^l}{L_{tot} - L^v} \quad (\text{Eq. 3-7}) \quad \quad k_2 = \frac{L_{tot} \times L^l - (L^v)^2}{L_{tot}^2 - 2L_{tot} \times L^v + L_{tot} \times L^l} \quad (\text{Eq. 3-8})$$

Finally, the load and length for the equivalent hybrid model are determined as follows:

$$P_A = (1 - k_1)P \quad (\text{Eq. 3-9}) \qquad L_A = k_2 L_{tot} \quad (\text{Eq. 3-11})$$

$$P_B = k_1 P \quad (\text{Eq. 3-10}) \qquad L_B = (1 - k_2) L_{tot} \quad (\text{Eq. 3-12})$$

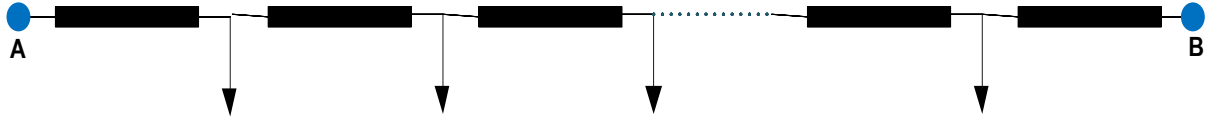
This method assumes single-phase sections – or, at best, a balanced three-phase section that can be conceived of as a single-phase equivalent. It also assumes that the sections are at the end of a feeder or lateral, with only one point at which it is connected to the rest of the feeder. It is most suitable for reductions in which an entire distribution feeder – rather than discrete sections of it – is reduced to a simple equivalent model.

3.2 Equivalent Load Model

The equivalent load model is a technique proposed by [14] that uses an iterative method to obtain the parameters for a simplified model, using inputs obtained from the measured voltages and through power at both ends of the section. Point K is defined in the equivalent load model illustrated in Figure 3-2 as the estimated location where the equivalent load is placed. Throughout numerous iterations of this calculation, the location of point K is moved along the equivalent model between points A and B until the error seen as the voltage of point K is below a satisfactory threshold. The following parameters are used to describe this model and perform the necessary calculations:

U_A	Voltage magnitude measured at point A
P_A	Active power measured flowing through point A [kW]
Q_A	Reactive power measured flowing through point A [kVar]
U_B	Voltage magnitude measured at point B
P_B	Active power measured flowing through point B [kW]
Q_B	Reactive power measured flowing through point B [kVar]
R_A	Resistance of equivalent section A
X_A	Reactance of equivalent section A
R_B	Resistance of equivalent section B
X_B	Reactance of equivalent section B
U_{KA}	Voltage magnitude of point K, calculated with respect to point A
U_{KB}	Voltage magnitude of point K, calculated with respect to point B
U_{Ke}	Error defined by disparity between U_{KA} and U_{KB}

(a) Original section



(b) Equivalent load model

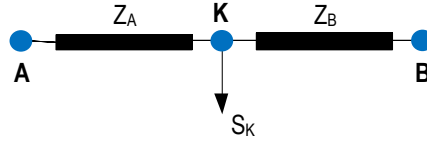


Figure 3-2: Lumping lines and loads, using the equivalent load model

First, a guess on the location of point K is made, providing initial values for the impedances of sections A and B. Then, the voltage is calculated for this projected location of point K based on two parallel calculations – one using point A as the voltage reference and the other using point B, as follows:

$$U_{KA} = \sqrt{\left(U_A - \frac{P_A R_A + Q_A X_A}{U_A}\right)^2 + \left(\frac{P_A X_A - Q_A R_A}{U_A}\right)^2} \quad (\text{Eq. 3-13})$$

$$U_{KB} = \sqrt{\left(U_B + \frac{P_B R_B + Q_B X_B}{U_B}\right)^2 + \left(\frac{P_B X_B - Q_B R_B}{U_B}\right)^2} \quad (\text{Eq. 3-14})$$

If both of these voltage estimates for point K are equal (within a specified threshold), then the estimated location and associated impedance values can be deemed valid, ending the iteration routine. Otherwise, the error term is calculated as follows:

$$U_{Ke} = U_{KA} - U_{KB} \quad (\text{Eq. 3-15})$$

If $U_{Ke} > 0$, then the impedance of section A is increased proportionally (i.e. keeping the X/R ratio constant) at the next iteration. Otherwise, if $U_{Ke} < 0$, then the impedance of section A is decreased proportionally for the next iteration. This iteration routine is repeated until the magnitude of the error term U_{Ke} is below an acceptable threshold, after which the electrical location of point K between A and B will be determined.

Once the location of K is decided, the equivalent complex load at point K can be calculated. First, the complex current I_K drawn by the equivalent load is calculated as follows:

$$I_A = \frac{S_A^*}{V_A^*} \quad (\text{Eq. 3-16})$$

$$I_B = \frac{S_B^*}{V_B^*} \quad (\text{Eq. 3-17})$$

$$I_K = I_A - I_B \quad (\text{Eq. 3-18})$$

Next, the complex voltage V_K is calculated by incorporating the impedance of section A determined iteratively in the above steps, as follows:

$$V_K = V_A - I_A Z_A \quad (\text{Eq. 3-19})$$

Finally, the complex power drawn at the equivalent load at point K is calculated as follows:

$$S_K = V_K I_K^* \quad (\text{Eq. 3-20})$$

Because this method relies on voltages to obtain appropriate impedance values for the equivalent model, it does not favor accuracy with regard to line losses, similarly to the voltage drop model discussed in Section 3.1. Furthermore, it requires information from the feeder terminal units (FTU) located at each end. This information consists of values for the voltages and through complex power at both ends of the feeder section, requiring either measurement at each of these points in a real-world system or a load flow analysis in a simulated model. Plus, being an iterative model, it is more computationally intensive.

3.3 Equivalent Load Density Model

Building upon the equivalent load model described in the previous section, the authors of [14] discuss the equivalent load density model. However, instead of using a single lumped load at a determined point to characterize the feeder section, a set of six triangular load distribution patterns is used to characterize the loading. These distribution patterns are illustrated in Figure 3-3. Each pattern is described mathematically by its own voltage drop and line loss properties that are normalized on the basis of the largest load. By scaling and summing different combinations of these six patterns, one can form a large variety of load distribution patterns, making it likely that any real-world load configuration is representable by a unique combination of these six load patterns.

Because the voltage drop and line loss factors are already determined and known for each of these six patterns, these parameters for the equivalent section can be calculated without the need to execute a power flow.

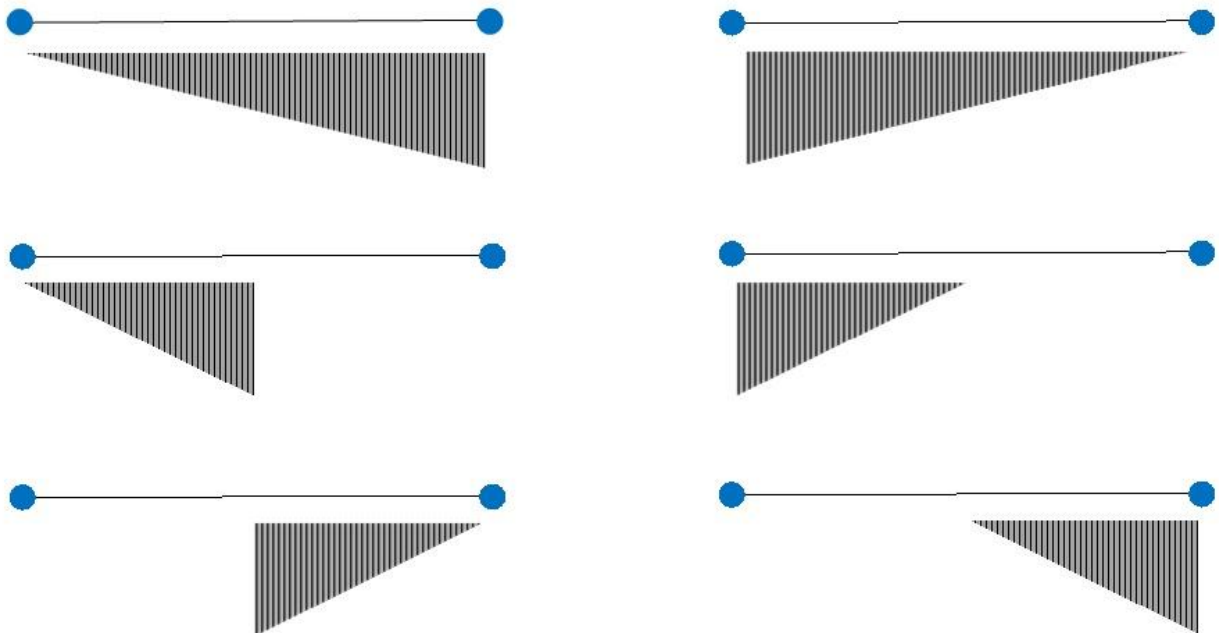


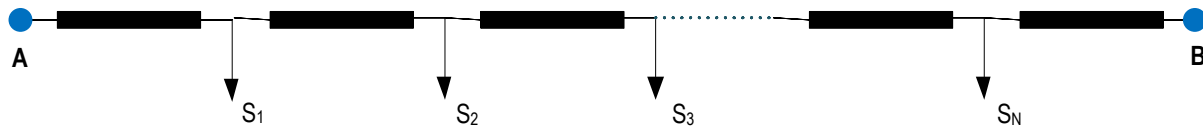
Figure 3-3: Load distribution patterns for equivalent load density model

This model accurately conforms to the voltage drop and line loss characteristics of the original section and is flexible in handling a wide variety of arbitrary load configurations that may exist in the real world. However, something simpler is desired in order to represent a larger number of equivalent sections that may exist in a distribution feeder, given the reduction criteria. Plus, the case of unbalanced three-phase loads is not discussed in this model, limiting the applicability of this method in the feeder being analyzed in this thesis.

3.4 Evenly Distributed Loading

A common case of distributed loading found in the literature is the uniformly distributed load case described in [15] and [16]. This method takes a section of uniformly distributed loads and lumps them at both ends of the equivalent section, as illustrated in Figure 3-4, such that the voltage magnitude drop through the section is the same as the original section. The authors of [15] find that the best way to allocate the load among the two nodes is simply to split the total load in half and place each half at one of the two endpoints. In fact, even throughout a range of values for endpoint bus voltages, the optimal proportion for which to split the total load among the two endpoints remains very close to 0.5.

(a) Original section



(b) Equivalent load model



Figure 3-4: Lumping a section of evenly distributed loads

$$S_A = 0.5 \sum_{i=1}^N S_i \quad (\text{Eq. 3-21})$$

$$S_B = 0.5 \sum_{i=1}^N S_i \quad (\text{Eq. 3-22})$$

This model is hampered by the fact that it assumes the very ideal case of a uniformly distributed load. However, it is desirable in that it allows a great simplification of distributed lines and loads to be expressed in simple terms – a single impedance and two loads. In the context of the feeder reduction, it can be applied in a manner that minimizes the number of nodes remaining in the reduced network, such that only nodes need to be present that explicitly contain retained components. Thus, it provides a promising framework that can be built upon in order to develop a technique that can be applied to an arbitrary series of unbalanced loads.

3.5 Heuristic Reduction Technique

The proposed technique for lumping loads builds upon the uniformly distributed load model discussed in Section 3.4, except that it allows for an arbitrary configuration and allocation of distributed loads within a section. By placing the lumped loads at the retained nodes such that the effects on the feeder's voltage profile and power flows are unchanged, the number of nodes necessary to portray the distribution feeder can be minimized. The feeder section to be reduced consists of several three-phase spot loads that are distributed throughout the section at varying distances along the way.

Often, the data available for analyzing the distribution feeder is limited and contains gaps in information. Several simplifying assumptions are asserted in order to simplify the model and subsequent calculations, as well as to accommodate a variety of feeders with varying levels of data available. The following assumptions are made:

- The loads are constant-PQ loads, meaning that their respective power consumption is not a function of voltage.
- The three-phase line sections are assumed to be transposed, yielding line impedances that are balanced in all three phases.
- For a given line section, $\frac{Z_{pi}}{Z_{0i}} = \frac{Z_{pEQ}}{Z_{0EQ}}$; that is, all retained lines must be three-phase and symmetric as prescribed above.
- The X/R ratio of the line is constant throughout the section.

Figure 3-5 illustrates the lumping of a generic feeder section, relating it to the notation used in the calculations below. The following are definitions for the variables used in Figure 3-5:

L_i	Length of section i [km]
Z_{pi}	Positive-sequence impedance within the three-phase line of section i [Ω]
Z_{0i}	Zero-sequence impedance within the three-phase line of section i [Ω]
S_{ai}	Complex phase-a load at section i [kVA]
S_{bi}	Complex phase-b load at section i [kVA]
S_{ci}	Complex phase-c load at section i [kVA]

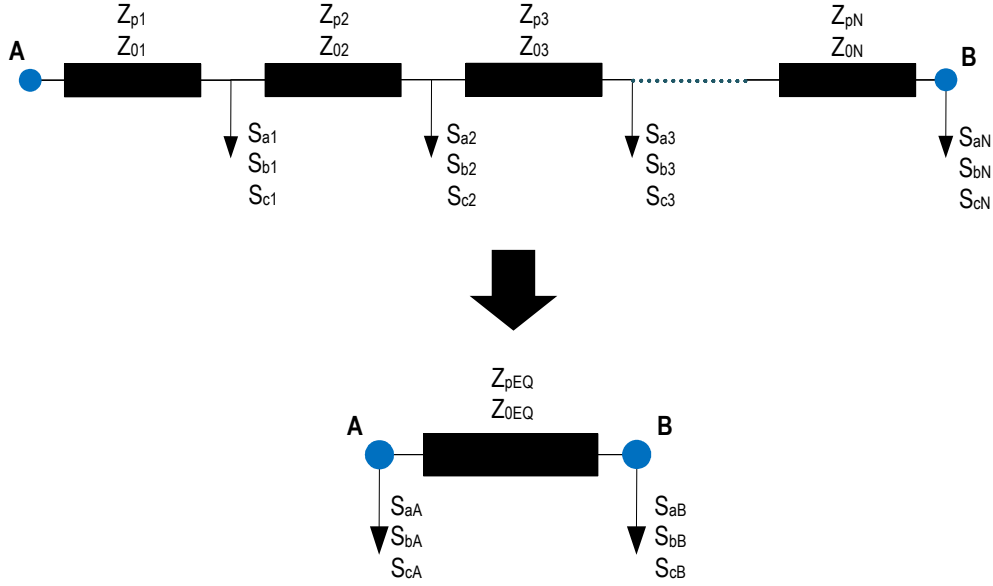


Figure 3-5: Lumping lines and loads in a feeder section

The equivalent impedance of the reduced section can be calculated by summing the impedances of the individual component sections in series, as follows:

$$Z_{pEQ} = \sum_{i=1}^N Z_{pi} \quad (\text{Eq. 3-23})$$

$$Z_{0EQ} = \sum_{i=1}^N Z_{0i} \quad (\text{Eq. 3-24})$$

Intuitively, the length of the reduced section is calculated by summing the lengths of the individual components sections in series, similarly to the manner in which the equivalent impedance of the section was calculated above:

$$L_{EQ} = \sum_{i=1}^N L_i \quad (\text{Eq. 3-25})$$

In order to allocate the total section loading among nodes A and B in a way that preserves the character of the section with respect to voltage drop and through power, the individual loads in all three phases are considered individually in determining their contribution to nodes A and B. Each phase is handled individually, wherein all loads within a phase are aggregated and allocated along the respective phase at nodes A and B.

For each phase, each individual load's contribution to node A and B depends on its distance from the nodes. The closer a load is to a node, the greater the proportion is in its contribution to the respective node. A linear function of the line's resistance can be used to determine the weighting of each load's contribution. This function is utilized in the following equation to solve for the equivalent load at nodes A and B:

$$S_{aA} = \sum_{i=1}^N S_{ai} \left(1 - \frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \quad (\text{Eq. 3-26})$$

$$S_{aB} = \sum_{i=1}^N S_{ai} \left(\frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \quad (\text{Eq. 3-27})$$

The above equations obtain load quantities for phase a at nodes A and B. The same relationships apply for the other phases.

$$S_{bA} = \sum_{i=1}^N S_{bi} \left(1 - \frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \quad (\text{Eq. 3-28})$$

$$S_{cA} = \sum_{i=1}^N S_{ci} \left(1 - \frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \quad (\text{Eq. 3-30})$$

$$S_{bB} = \sum_{i=1}^N S_{bi} \left(\frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \quad (\text{Eq. 3-29})$$

$$S_{cB} = \sum_{i=1}^N S_{ci} \left(\frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \quad (\text{Eq. 3-31})$$

3.6 Validation of Reduction Technique

The proposed reduction technique was validated on the feeder section illustrated in Figure 3-6. This section is taken from the feeder described in Chapter 5. The line and load parameters for this section are given in Table 3-1.

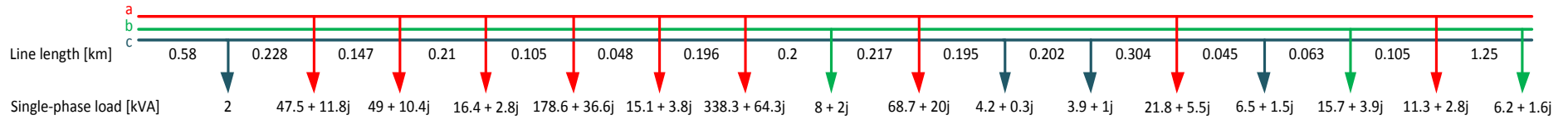


Figure 3-6: Line section load aggregation example

Table 3-1: Sample section line and load parameters

Sec	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
		Z_p	Z_0	S_a	S_b	S_c
1	0.580	$0.067 + 0.229j$	$0.223 + 0.767j$			2
2	0.228	$0.026 + 0.090j$	$0.088 + 0.302j$	$47.5 + 11.8j$		
3	0.147	$0.017 + 0.058j$	$0.056 + 0.194j$	$49.0 + 10.4j$		
4	0.21	$0.024 + 0.083j$	$0.081 + 0.278j$	$16.4 + 2.8j$		
5	0.105	$0.012 + 0.041j$	$0.040 + 0.139j$	$178.6 + 36.6j$		
6	0.048	$0.006 + 0.019j$	$0.018 + 0.064j$	$15.1 + 3.8j$		
7	0.196	$0.023 + 0.077j$	$0.075 + 0.259j$	$338.3 + 64.3j$		
8	0.200	$0.023 + 0.079j$	$0.077 + 0.265j$		$8 + 2j$	
9	0.217	$0.025 + 0.086j$	$0.083 + 0.287j$	$68.7 + 20.0j$		
10	0.195	$0.023 + 0.077j$	$0.075 + 0.258j$			$4.2 + 0.3j$
11	0.202	$0.023 + 0.080j$	$0.078 + 0.267j$			$3.9 + 1.0j$
12	0.304	$0.035 + 0.120j$	$0.117 + 0.402j$	$21.8 + 5.5j$		
13	0.045	$0.005 + 0.018j$	$0.017 + 0.060j$			$6.5 + 1.5j$
14	0.063	$0.007 + 0.025j$	$0.024 + 0.083j$		$15.7 + 3.9j$	
15	0.105	$0.012 + 0.041j$	$0.040 + 0.139j$	$11.3 + 2.8j$		
16	1.250	$0.145 + 0.494j$	$0.480 + 1.654j$		$6.2 + 1.6j$	

Appendix B outlines the calculations for the equivalent impedance and spot loads for the feeder section described above in Table 3-1. These lumped parameters are incorporated into the reduced model for this section as follows:

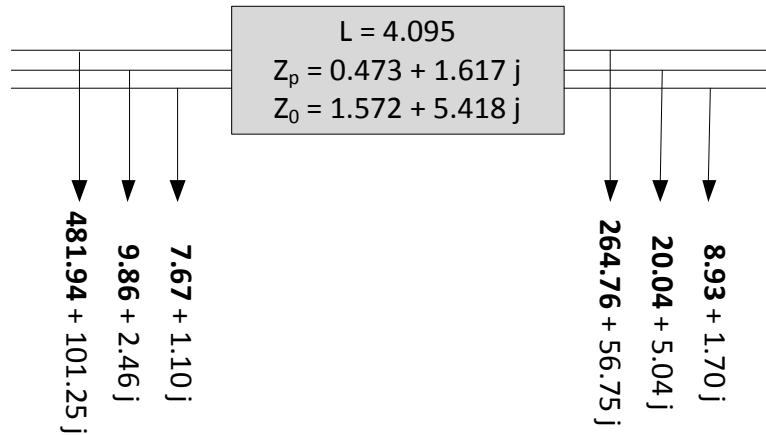


Figure 3-7: Line section after reduction

This reduction technique is verified by analyzing the voltage drop and through power flows at both ends of the section in steady state. To set up a test for this analysis, the section is connected on one end with a voltage source and connected on the other end with a three-phase load. The load represents the real and reactive power drawn by lines and loads in other sections downstream of the tested section. Table 3-2 below contains the values used to represent the external load applied to the section, as well as the magnitude of the voltage source. In order to verify that the simplified feeder section is bilateral and reflects accurately upon the original feeder section, regardless of directionality of power flow, two test cases are set up, alternating the node at which the load is connected and the node at which the voltage source is connected.

Table 3-2: Source and external load parameters for line section reduction testing

Source voltage (line-to-line)	25 kV
External load [kVA]	(a) 2614.5 + 276.3 j
	(b) 3280.0 + 400.3 j
	(c) 4904.1 + 2065.3 j

The reduction technique will be deemed satisfactory if the reduced section imitates the voltage and power profile of the original section according to the following criteria:

- Voltage within 1% of that seen in the respective point
- Difference in through active power from that seen in the respective point within 2% of the total feeder load
- Difference in through reactive power from that seen in the respective point within 2% of the external load's total reactive power (see Table 3-2)

For the first test case, the source and load are connected to the feeder section, as illustrated below in Figure 3-8. The source is connected to node A, and the external load is connected to node B.

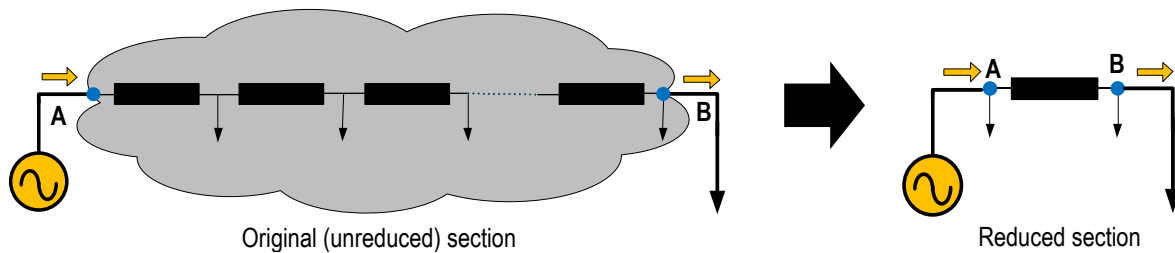


Figure 3-8: Section reduction validation test case #1

The results of the first test case are shown below in Tables 3-3 and 3-4. Table 3-3 contains the voltage and power measurements at nodes A and B for the original section, and Table 3-4 contains that for the reduced section.

Table 3-3: Power flows and node voltages for original (unreduced) section, test case #1

	Node voltages [p.u.]			Downstream power flow, by phase						Downstream power flow, total	
	V _a	V _b	V _c	P _a [kW]	Q _a [kVar]	P _b [kW]	Q _b [kVar]	P _c [kW]	Q _c [kVar]	P _{total} [kW]	Q _{total} [kVar]
A	1.042	1.042	1.038	3545.9	537.7	3504.0	465.3	5013.1	2345.1	12063.0	3348.1
B	1.045	1.030	0.998	2805.8	296.5	3438.2	419.7	4891.5	2060.3	11135.5	2776.5

Table 3-4: Power flows and node voltages for reduced section, test case #1

	Node voltages [p.u.]			Downstream power flow, by phase						Downstream power flow, total	
	V _a	V _b	V _c	P _a [kW]	Q _a [kVar]	P _b [kW]	Q _b [kVar]	P _c [kW]	Q _c [kVar]	P _{total} [kW]	Q _{total} [kVar]
A	1.042	1.042	1.038	3584.1	544.8	3505.0	465.7	5014.0	2344.8	12103.1	3355.3
B	1.045	1.030	0.998	2805.1	296.4	3438.7	419.8	4891.5	2060.2	11135.3	2776.4

For the second test case, the source and load are reversed, as illustrated in Figure 3-9, i.e. the source is connected to node B, and the external load is connected to node A.

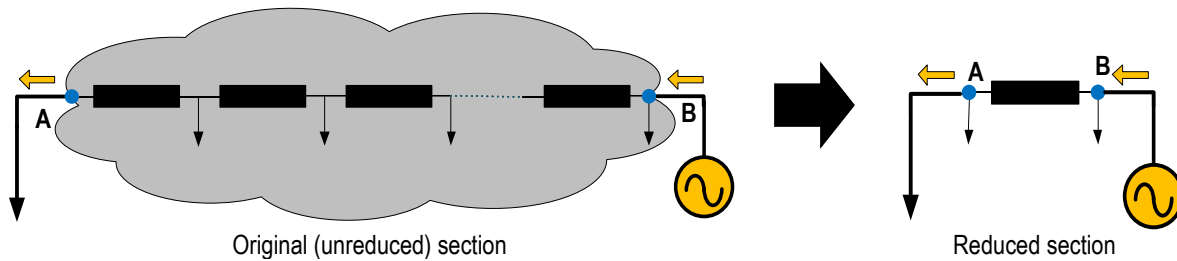


Figure 3-9: Section reduction validation test case #2

The results of the second test case are shown below in Tables 3-5 and 3-6. Table 3-5 contains the voltage and power measurements at nodes A and B for the original section, and Table 3-6 contains that for the reduced section.

Table 3-5: Power flows and node voltages for original (unreduced) section, test case #2

	Node voltages [p.u.]			Downstream power flow, by phase						Downstream power flow, total	
	V _a	V _b	V _c	P _a [kW]	Q _a [kVar]	P _b [kW]	Q _b [kVar]	P _c [kW]	Q _c [kVar]	P _{total} [kW]	Q _{total} [kVar]
A	1.043	1.031	0.998	2799.1	295.8	3445.8	420.7	4886.7	2058.3	11131.6	2774.8
B	1.042	1.042	1.038	3540.7	551.7	3507.0	465.4	5014.3	2338.6	12062.0	3355.7

Table 3-6: Power flows and node voltages for reduced section, test case #2

	Node voltages [p.u.]			Downstream power flow, by phase						Downstream power flow, total	
	V _a	V _b	V _c	P _a [kW]	Q _a [kVar]	P _b [kW]	Q _b [kVar]	P _c [kW]	Q _c [kVar]	P _{total} [kW]	Q _{total} [kVar]
A	1.043	1.032	0.998	2798.1	295.7	3446.7	420.8	4886.5	2058.2	11131.3	2774.7
B	1.042	1.042	1.038	3578.6	559.5	3508.3	465.9	5015.1	2338.0	12102.0	3363.4

By comparing Tables 3-3 and 3-4 with one another and doing the same with Tables 3-5 and 3-6, one can gather a sense how closely the reduced section mimics the steady-state character of the original network. Table 3-7 below summarizes the disparity between the two models, highlighting the largest discrepancies in both cases. Overall, these discrepancies are quite small, and even the largest are not significant enough to

violate the thresholds set forth to evaluate the reduction method evaluation criteria. Thus, this method can be applied with a good level of confidence in order to aggregate several serial lines, with their loads, into large sections within the feeder reduction methodology outlined in the next chapter.

Table 3-7: Steady-state evaluation of line section reduction

		Largest disparity from original model	Location of disparity	Threshold	Pass?
1	Node voltage	0.001 p.u.	A (phase b) Test case #2	0.01 p.u.	YES
2	Active power flow	38.2 kW	A (phase a) Test case #1	222 kW	YES
3	Reactive power flow	7.8 kVar	B (phase a) Test case #2	54 kVar	YES

3.7 Summary

This chapter reviews several feeder reduction techniques encountered in the literature in order to explore ways to simplify a large, sprawling distribution network. Based on the notions upon which these techniques reside, a similar heuristic technique is proposed in order to handle a wide array of load placement possibilities. It handles loads and lateral sections of any phasing – balanced three-phase loads, unbalanced three-phase loads, single-phase laterals and loads, etc. It does not require any sort of load flow or live measurement data, such as voltages or currents. Furthermore, the equivalent model is a simple one that lumps the effective loads at the ends of the newly formed equivalent section, keeping minimal the number of nodes necessary to portray the reduced feeder sections.

For validation purposes, the technique is applied to a single feeder section in order to generate an equivalent model. This equivalent model is then compared to the unreduced section using a load flow with a representative voltage source connected at one end and a three-phase load connected at the other end. It is shown that the simplification technique outlined in this section sufficiently preserves the steady-state character of the specimen feeder section.

It is this simplification technique that is used from this point on throughout the distribution feeder reduction procedure.

4 Feeder Benchmarking Methodology

4.1 Reduction Criteria

The power system components that are retained in the feeder model and the assumptions allowed to be made will be determined by the reduction criteria and the intended applications for the model. For the scope discussed in this thesis, the model shall preserve characteristics of the feeder that will impact switching of protection devices and the DG, as well as the resulting transient phenomena.

4.2 Simplifying Assumptions

In order to simplify the feeder model and treat all of the varieties of feeders in a consistent manner, the following assumptions are made:

- Three-phase lines are perfectly symmetrical, so that there is no mutual coupling between sequences.
- Loads are constant-power loads that do not fluctuate within the time window of interest. The interconnecting distribution transformers are not included in the model.
- Interconnected feeders and distribution spot networks are excluded.

4.3 Retained Components

Based on the reduction criteria and simplifications stated above, the following components shall be retained in the feeder model:

- Sources of real and reactive power (substation source, DG, shunt capacitor)
- All transformers along the primary feeder (including voltage regulators)
 - NOTE: This does not include distribution transformers that connect the primary feeder to either loads or secondary mains. These transformers have already been truncated from the load modeling of the feeder (see Figure 5-2).
- Three-phase switching devices (breakers, reclosers, relay-controlled breakers, sectionalizers, switches) that following either of the following criteria shall be retained:
 - Service a load area downstream totalling at least 1 MVA (or 10% of the network's total load for networks with a total load of less than 10 MVA)
 - Admit at least 1 MVA (or 10% of network total load for networks with a total load of less than 10 MVA), as determined by a load flow calculation

4.4 Defining the Feeder Backbone

The feeder backbone shall be defined by those existing lines connecting together the retained components. All lines in this backbone shall be retained. All other lines shall be clipped from the point at which it meets the feeder backbone.

4.5 Clipping of Laterals

A lateral, as distinct from the feeder backbone described above, is defined within this thesis as a contiguity of lines, loads, and equipment that connect to the retained backbone at a single point. Each of these laterals shall be replaced at its junction with the feeder backbone by a three-phase spot load equal to the apparent power absorbed by the lateral during operation.

Figure 4-1 below illustrates an example of how a lateral along the feeder backbone is clipped. The feeder backbone between two retained components (indicated in blue) is shown, which contains one section of laterals (highlighted in red). The values for the representative three-phase load used to replace the lateral section on the feeder backbone are derived from the power flow analysis of the respective section (indicated by the yellow box). That way, the actual real and reactive power consumed by the loads, as well as that power dissipated in losses within the lateral section, are accounted for in the representative three-phase load used to replace the lateral.

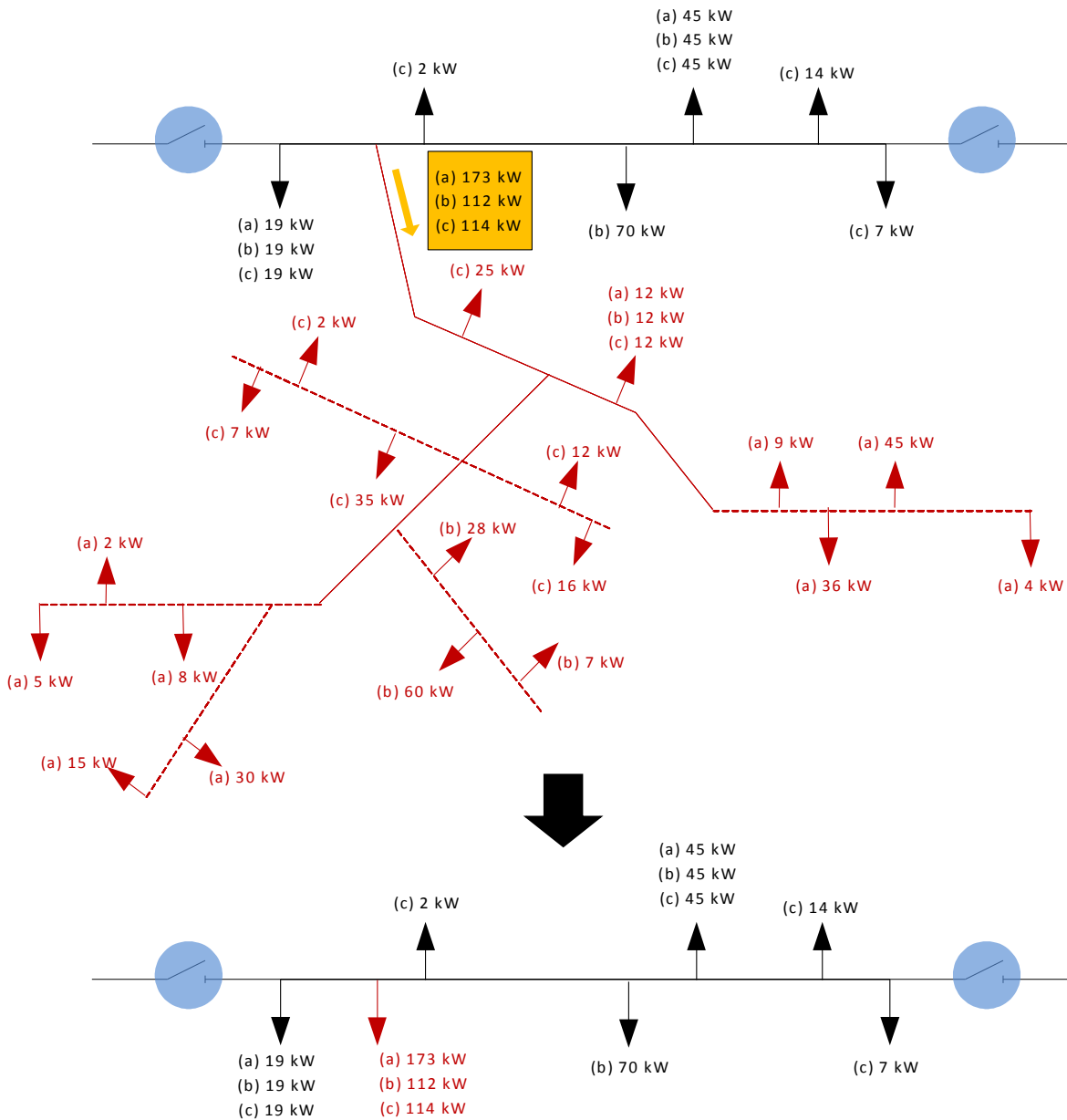


Figure 4-1: Clipping of laterals

4.6 Defining Nodes and Line Sections

Nodes are defined at points along the feeder backbone at which (1) retained components are located, or (2) the backbone diverges to multiple paths. Line sections are defined in this thesis as the contiguity of lines and loads between two nodes.

In a few cases, there is a significant disparity in the locations of loads along a line section. This happens, in particular, when a switch (or other retained component) is located at the end of a long, unloaded section of line. In such a case, the long, unloaded line on which the switch is located is designated its own line section, with the remaining lines and loads comprising another section, thus resulting in two line sections between nodes. Figure 4-2 illustrates this case, showing (a) a line section that would not be considered for further split (normal case) and (b) a line section with an uneven load distribution that would be considered for further split into separate sections.

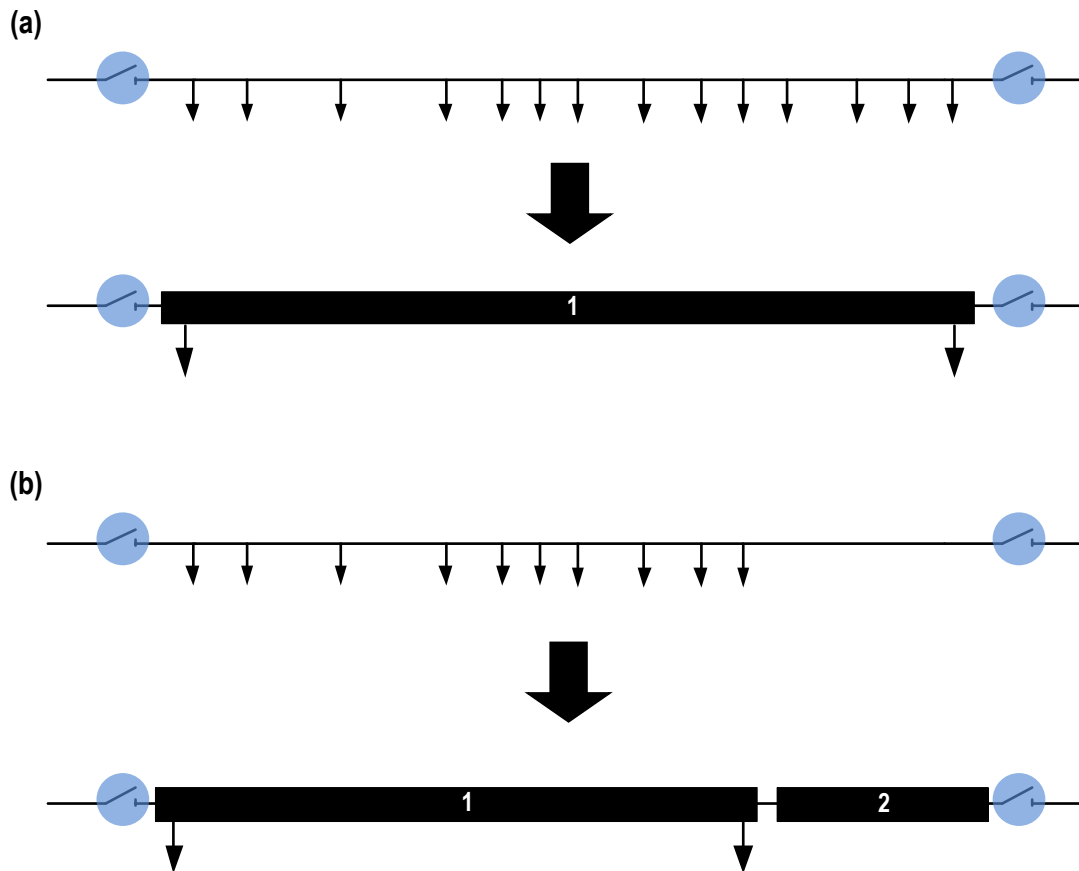


Figure 4-2: Multiple line sections used for equalizing load distribution among each section

This provision to split a line further is made if all of the following conditions exist:

- The section consists of more than one line.
- There are one or more consecutive unloaded lines adjacent to an endpoint.
- The total length of this unloaded line series is greater or equal to 20% of the total length of the entire line section.

This additional line section created in this process is a compromise to model simplicity made for the sake of improving the accuracy of the how the loads are aggregated in the next step. The driving rationale is that a more uniform distribution of loads along a line within a section will improve the accuracy of the load aggregation performed in the next step.

4.7 Aggregation of Loads

Each line section will be characterized by an equivalent series impedance and two lumped loads – one at each node on the ends of the section. These aggregate parameters define a line section that yields identical behaviour to that of the respective section of lines and loads in the original network, without aggregation.

Refer to Section 3.5 for a detailed procedure on how to obtain the aggregate values for the equivalent series impedance and lumped loads for each section.

Each line section can be modeled as follows. The example section below in Figure 4-3 is based on the reduction and aggregation calculations outlined above in Section 4.6.

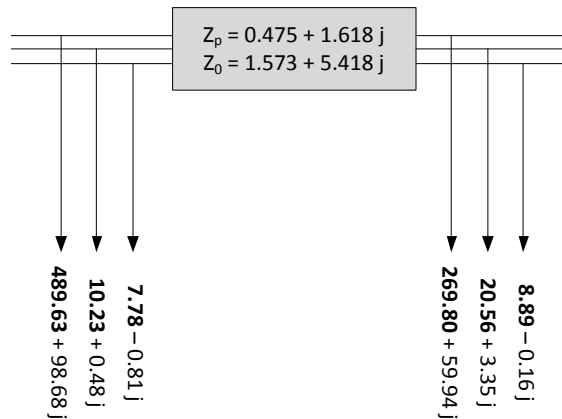


Figure 4-3: Line section model with unbalanced loads

These impedance and load calculations are performed at all equivalent line sections throughout the reduced network.

4.8 Balancing of Loads

The final step is to balance the aggregated loads among the three phases, such that all loads in the reduced network are three-phase balanced loads, as seen in Figure 4-4.

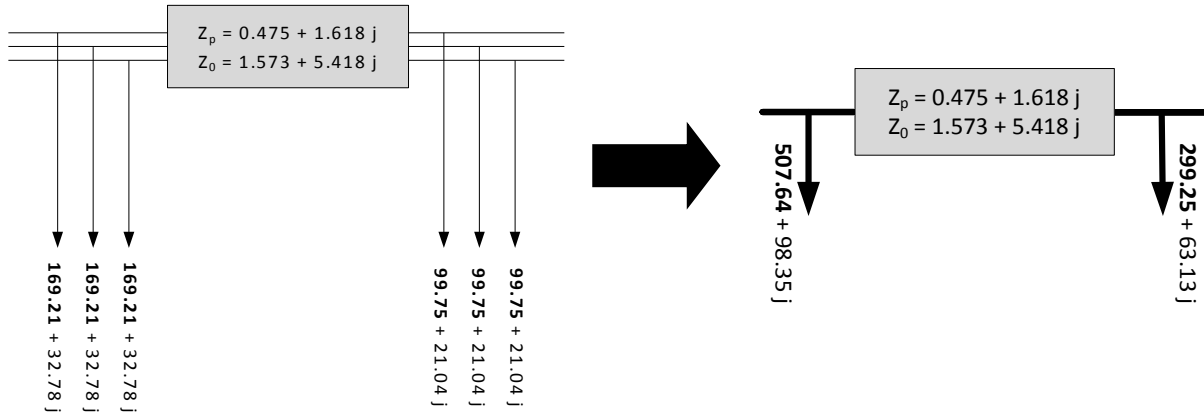


Figure 4-4: Balanced line section model

At this point, the reduced network consists entirely of the main three-phase feeder sections, which are represented by balanced three-phase impedances and balanced three-phase loads.

4.9 Summary

A sequence of steps is built around the feeder simplification technique described in the previous chapter in order to systematically reduce a given rural distribution feeder into a simple equivalent model. Specific steady-state performance criteria have been observed in this reduction process. The result is a reduced network comprised of a manageable number of balanced three-phase impedances and loads, along with a set of relevant distribution system components that have been retained, that can be used for DG interconnection studies.

5 Feeder System Description

This thesis focuses on a specific application example pertaining to a distribution feeder typical for rural Quebec. The feeder supplies about 11 MW of active power. Its nominal distribution line-to-line voltage is 25 kV at 60 Hz. The feeder contains a distributed generator (DG) in the form of a hydroelectric synchronous generator that supplies a constant power of 16 MW. The most distant load is connected to the substation through about 39.8 km of overhead line. The feeder contains an in-line regulator and shunt capacitor bank that provide compensation against voltage drops along the feeder length. The network is unbalanced, comprising a number of both three-phase and single-phase loads, as well as single-phase laterals. Because the DG is generating 16 MW of power and the network is consuming about 11 MW, it can be assumed that, even after considering line losses, it exports power back to the substation and subsequently back to the transmission network.

Figure 5-1 shows a DSAP geographical representation of the distribution feeder, indicating the locations of the substation, DG, regulator, and shunt capacitor. Both the substation and DG are connected to the 25 kV network through their own transformer. The substation transformer is delta-connected at the 120 kV source side and wye-connected on the 25 kV feeder side. The DG transformer is delta-connected at the 4.17 kV generator side and wye-connected on the 25 kV feeder side. Three-phase lines are represented by solid lines, while single-phase lines are represented by dashed lines.

The feeder contains several open switches along the main trunk that allow for interconnection with adjacent feeders (not shown). Some of these switches can be remotely operated, allowing for quick reconfiguration of the distribution system during maintenance operations or in cases of failure within a section.

Data for this feeder is extracted from the DSAP feeder model file. Any additional information that is needed for the benchmark that cannot be found in the DSAP steady-state analysis tool has been obtained from typical values found in analogous distribution systems throughout the literature.

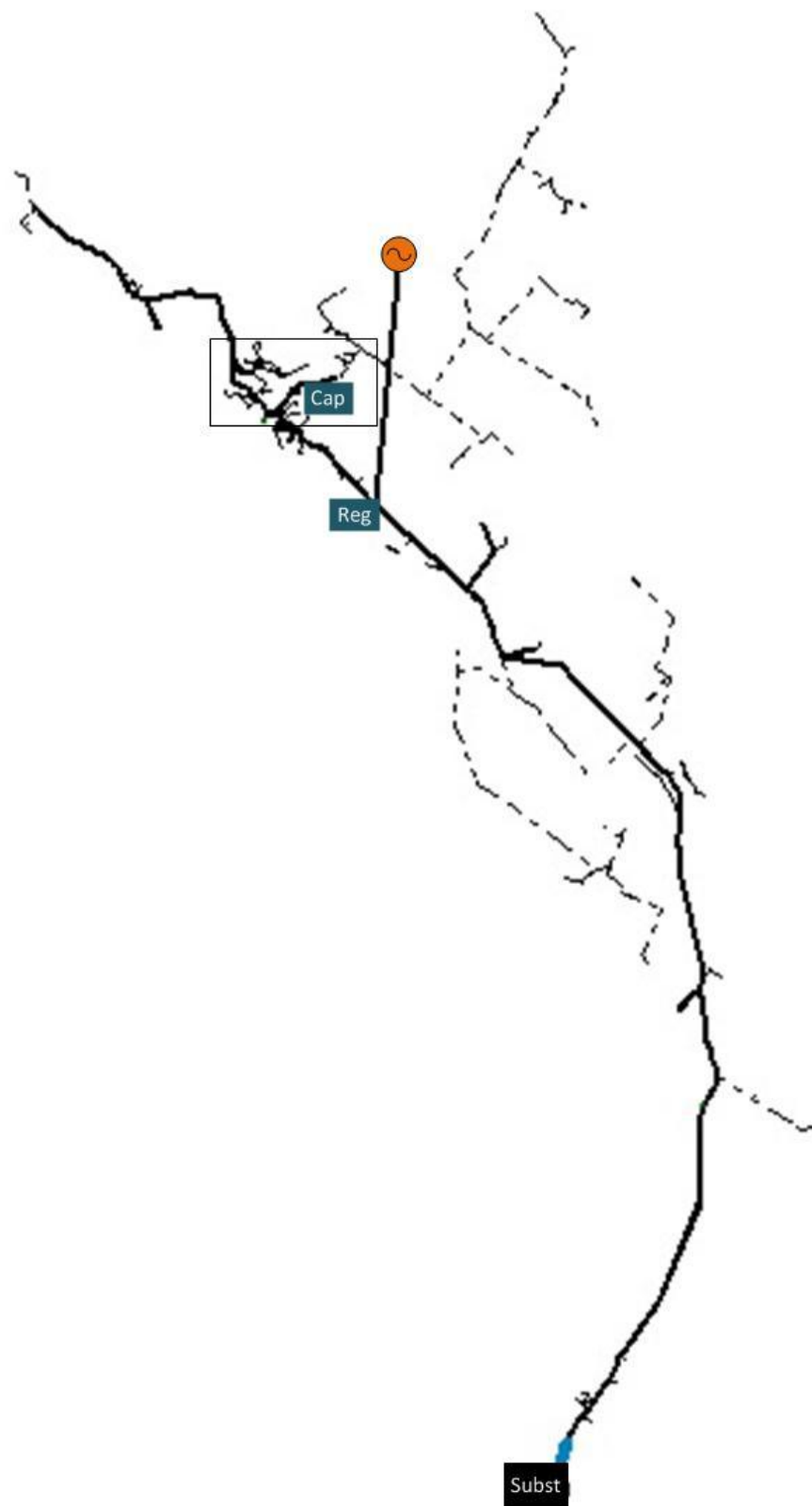


Figure 5-1: Geographical representation of rural distribution feeder

5.1 Loads

Because this feeder is located in a rural area, the distribution of loads tends to be sparse along the length of the feeder. However, the feeder does contain a relatively densely populated area of three-phase and single-phase loads in this system, denoted by the box in Figure 5-1, presumably a small town. This area comprises over 5 MW of loads, about 45% of the total loads serviced by this distribution feeder. Thus, a great deal of load is concentrated within this small area.

Because this model is concerned with the primary feeder of the distribution network, it does not explicitly include the individual distribution transformers and secondary mains that would appear at each of the loads indicated in the model. These components are lumped into their respective constant-power load at their point of interconnection on the medium-voltage side of the distribution transformer, as illustrated in Figure 5-2. Because the time scale of interest in this benchmark is on the order of seconds, the loads can be considered constant values.

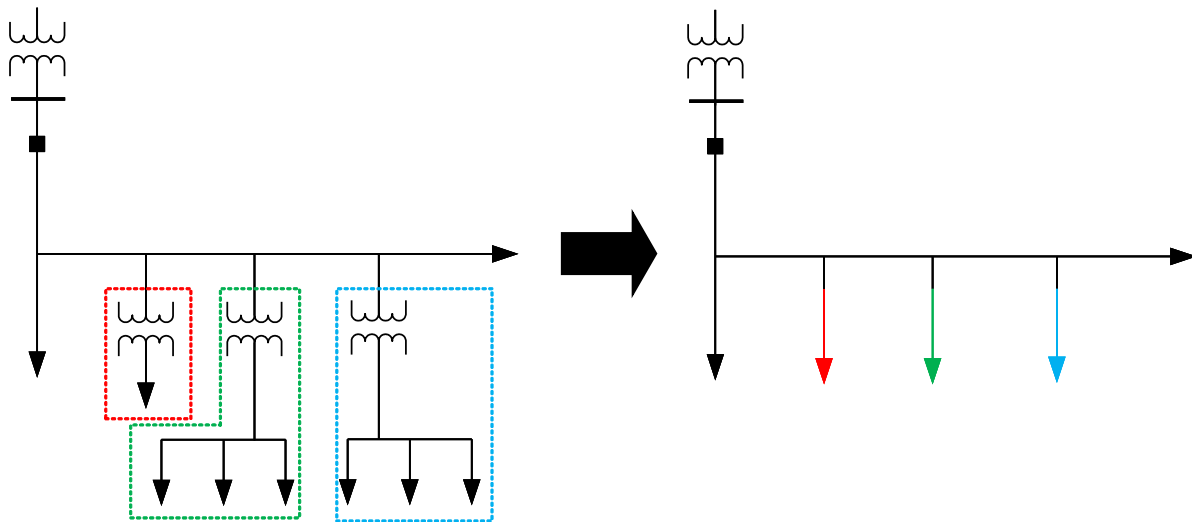


Figure 5-2: Scope of load modeling

Table 5-1 breaks down the loading of the feeder according to the phase onto which they are connected.

Table 5-1: Feeder loading, by phase

Phase	Real power [kW]	Reactive power [kVar]
A	3,297	745
B	3,052	671
C	4,425	987
Total	10,774	2,403

5.2 Feeder Lines

This rural feeder consists entirely of overhead lines. Table 5-2 below shows the different conductor types used along the feeder and their respective impedances. They are all used for the main three-phase backbone of the feeder and are assumed to have balanced impedances.

Table 5-2: Overhead line parameters (per unit length)

Conductor name	Series impedance [Ω/km]				Shunt admittance [$\mu\text{S}/\text{km}$]		Current rating [A]
	R_1	X_1	R_0	X_0	B_1	B_0	
477AL	0.116	0.395	0.384	1.323	4.227	1.814	640
30AL	0.326	0.439	0.5939	1.3669	3.761	1.722	315
20AR	0.429	0.475	0.6969	1.4029	3.729	1.715	255
2AR	0.851	0.506	1.211	1.5659	3.512	1.66	170

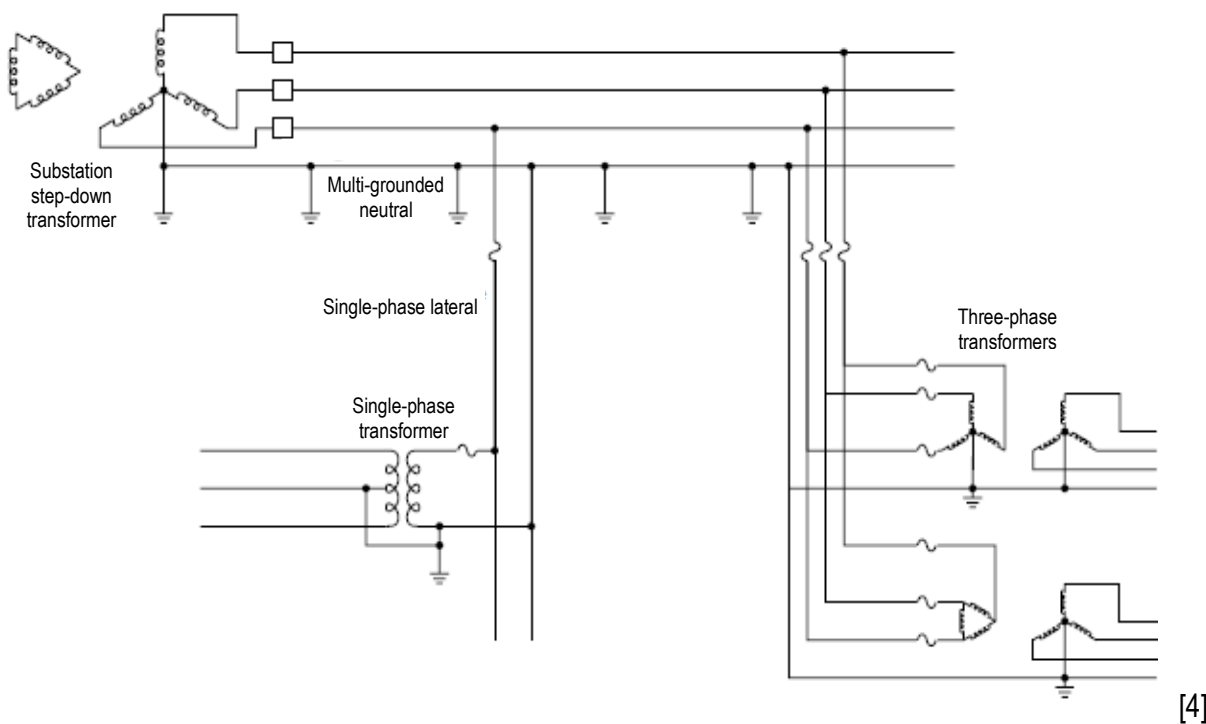
Appendix C.1 illustrates the placement of the different conductor types throughout the feeder model.

The 477AL conductor comprises about 41.4 km of the overhead lines in this feeder and forms most of the backbone of the primary feeder. Table 5-3 below shows how much of the feeder is composed of each type of conductor.

Table 5-3: Overhead line lengths along feeder

Conductor name	Length [km]
477AL	41.42
30AL	4.36
20AR	29.04
2AR	58.25

Like on most distribution systems in North America, the feeder lines are configured as a four-wire multi-grounded neutral system. All three-phase sections throughout the feeder contain the neutral as a fourth wire running alongside the primary feeder lines. This configuration has implications for the interconnection requirements of the DG, as well as the behaviour of the system during fault events. Figure 5-3 below illustrates such a grounding configuration.

**Figure 5-3: Grounding configuration of a four-wire multi-grounded distribution system**

[4]

5.3 Substation Source

For this particular system, the substation is strong enough to be represented as a voltage source with an internal impedance in series with the stepdown transformer. This may not always be sufficient, particularly in weak systems that interact with synchronous DG's. Table 5-4 shows the impedance data of the substation source, as described in the DSAP model.

Table 5-4: Substation source equivalent impedance data

Parameter	Value
R_0 [Ω]	0
X_0 [Ω]	0
R_1 [Ω]	6.4438
X_1 [Ω]	19.3314

5.4 Transformers

5.4.1 Substation transformer

The transformer at the substation is a three-phase step-down transformer connecting the distribution feeder with other feeders at the substation and the rest of the electric power system. It is delta-connected at the 120 kV supply side and grounded-wye-connected at the 25 kV feeder side. The delta-wye configuration causes a 30° phase shift from the high-voltage bus to the medium-voltage (feeder) bus. The parameters of the transformer relevant to the benchmark model are outlined below in Table 5-5.

Table 5-5: Step-down transformer parameters

Parameter	Value
Nominal power [MVA]	15
Nominal frequency [Hz]	60
Configuration	Delta/Gnd-Wye (+30°)
Winding 1 voltage [kV RMS-LL]	120
Winding 2 voltage [kV RMS-LL]	26.4
Winding R [p.u.]	2.8876×10^{-3}
Winding X [p.u.]	0.07219

5.4.2 Voltage regulator

The in-line voltage regulator is a tap-changing autotransformer with independent phase voltage control. Table 5-6 below outlines the parameters of the autotransformer for the voltage regulator. The impedance information for the transformer comes from typical values obtained from literature [11], whereas all other parameters come from the DSAP model. Details of the voltage regulator control settings can be found in Section 5.5.1.

Table 5-6: Voltage regulator autotransformer parameters

Parameter	Value
Nominal power [MVA]	15
Nominal frequency [Hz]	60
Winding 1 voltage [kV RMS-LL]	25
Winding 2 voltage [kV RMS-LL]	25
Winding R [p.u.]	9.5×10^{-3}
Winding X [p.u.]	0.0204

5.4.3 DG interconnection

The local DG is interconnected to the feeder by a three-phase delta/wye-grounded transformer. The DG itself is connected to a 4.16 kV bus. The delta-wye configuration causes a 30° phase shift from the low-voltage DG bus to the medium-voltage bus on the feeder. Table 5-7 below outlines the parameters of the DG interconnection transformer.

Table 5-7: DG interconnecting transformer parameters

Parameter	Value
Nominal power [MVA]	19.5
Nominal frequency [Hz]	60
Configuration	Delta/Gnd-Wye (+30°)
Winding 1 voltage [kV RMS-LL]	4.16
Winding 2 voltage [kV RMS-LL]	24.9
Winding R [p.u.]	8.955×10^{-3}
Winding X [p.u.]	0.089553

5.5 Voltage Regulation

Because many of the loads on the feeder are located far away from the substation through several kilometres of overhead line, the voltage drop through the length of the feeder must be considered and compensated for. This compensation is provided by the in-line voltage regulator and a shunt capacitor bank farther downstream (away from the substation).

5.5.1 Voltage regulator

The voltage regulator operates on each phase individually, modifying the turns ratio so that the secondary winding voltage is at the desired level for all three phases. Table 5-8 highlights the control parameters outlining the performance of the voltage regulator.

Table 5-8: Voltage regulator control parameters

Parameter	Value
Nominal LL voltage [kV]	25
Number of taps	32
Voltage step per tap [p.u.]	0.00625
Initial tap position	0
Desired regulated voltage [p.u.]	1.0292
DeadBand [p.u.]	0.01

5.5.2 Shunt capacitor

The shunt capacitor provides about 400 kVar of reactive power on each phase. This reactive power injection provides additional compensation against voltage drops along the feeder length. It is a wye-connected component.

5.6 Local Generation

The feeder contains a distributed generator (DG) in the form of a hydroelectric synchronous generator that supplies a constant power of 16 MW. The detailed physical and electrical characteristics of the DG are outside the scope of this study. However, because the power injection from this source will impact the steady-state and transient performance throughout the feeder, those relevant aspects of the DG must be included into the feeder model. Thus, for the purposes of arriving at a steady-state distribution feeder equivalent model, the DG is modeled as an ideal voltage source with a series impedance that interfaces the feeder at the DG interconnection transformer. Table 5-9 shows the impedance data used in this model.

Table 5-9: DG impedance data

Parameter	Value
R_0 [Ω]	0.1
X_0 [Ω]	0.1
R_1 [Ω]	0.1
X_1 [Ω]	155.1733
R_{neg} [Ω]	0.1
X_{neg} [Ω]	155.1733

5.7 Protection Devices

In order to respond to faults and mitigate their effects on the power system, the feeder contains a number of protective switching devices. For this study, protection devices are classified according to their location on the feeder – those on the primary backbone of the feeder and those on the laterals. The definitions and distinction between the backbone and laterals are made clearer as part of the network reduction methodology employed upon the line, as discussed in Section 4.4.

The main backbone of the feeder comprises automatic reclosers that implement a fuse-saving scheme designed to preclude immediate operation of the lateral fuses. Figure 6-3 shows the locations of the feeder backbone's protection devices. The laterals contain hundreds of protective devices that provide supplementary protection to users throughout these minor sections, mostly consisting of current-limiting fuses. Also along these laterals are a host of low-voltage circuit breakers and sectionalizing switches. Table 5-10 highlights some of the key parameters of the protective devices throughout the feeder, as obtained from the DSAP file of the feeder. Figures C-5 and C-6 in Appendix C shows the locations of these protection devices among the feeder's laterals.

The objective of this thesis is not the operating performance characteristics of the protective devices but rather their locations; it is the locations of these components that need to be retained in the equivalent feeder model, since these locations represent prospective opening points along the feeder.

Table 5-10: Protective device parameters

Device	Key parameters
Recloser	Phase pickup current: 420 A Groupd pickup current: 160 A Rated voltage: 24.9 V
Sectionalizers	Current rating: 600 A Rated voltage (line-to-line): 24.9 V Reversible
Fuses	Feeder contains fuses of different rating classes. Those classes are shown below. Phase trip rating (A):25, 40, 65, 100 Ground trip rating (A): 25, 40, 65, 100

5.8 Load Flows

An unbalanced load flow calculation, executed in the DSAP, yields the following overall results for the network, shown below in Table 5-11:

Table 5-11: Power injections and losses

Total generation	15,596 kW
Total load	11,100 kW
Shunt capacitor	1,257 kVar
Real power losses	868 kW
Power exported to substation	3,628 kW

5.9 Summary

This chapter describes a specific rural distribution feeder that will be used as a case study in order to demonstrate and validate the benchmarking and feeder reduction methodology outlined later in this paper. The feeder features a sparse allocation of unbalanced loads over a large geographical area, including along several three-phase and single-phase lateral branches. The particularity of the feeder is that it is not passive, i.e. it contains a DG in the form of a synchronous machine that produces more power than what is consumed throughout the entire feeder. The feeder is composed of overhead lines in a four-wire multi-grounded configuration. Voltage regulation is provided by an in-line voltage regulator along with supporting reactive power from a shunt capacitor. Protection is implemented by a recloser, several sectionalizers, and a number of fuses throughout the feeder.

6 Application Example of Methodology to Feeder

6.1 Methodology Execution

This section outlines how each step of the methodology described in Chapter 4 was executed specifically in the feeder described in the previous chapter.

6.1.1 Retained components

Table 6-1 highlights the retained components selected based on the criteria set forth in Section 4.3. Figure 6-1 on the next page highlights the location of each of these retained components.

Table 6-1: Retained components in feeder

A	Substation + switch
B	Switch
C	Switch
D	Voltage regulator
E	Switch
F	Switch
G	Switch
H	DG + interconnecting transformer
I	Capacitor

6.1.2 Defining the feeder backbone

The feeder backbone is defined by “connecting the dots” between the retained components along the feeder’s existing lines, as highlighted in Figure 6-1. Consult Appendix C.2 for a more detailed account of each of the individual lines and loads that comprise the main backbone defined here.

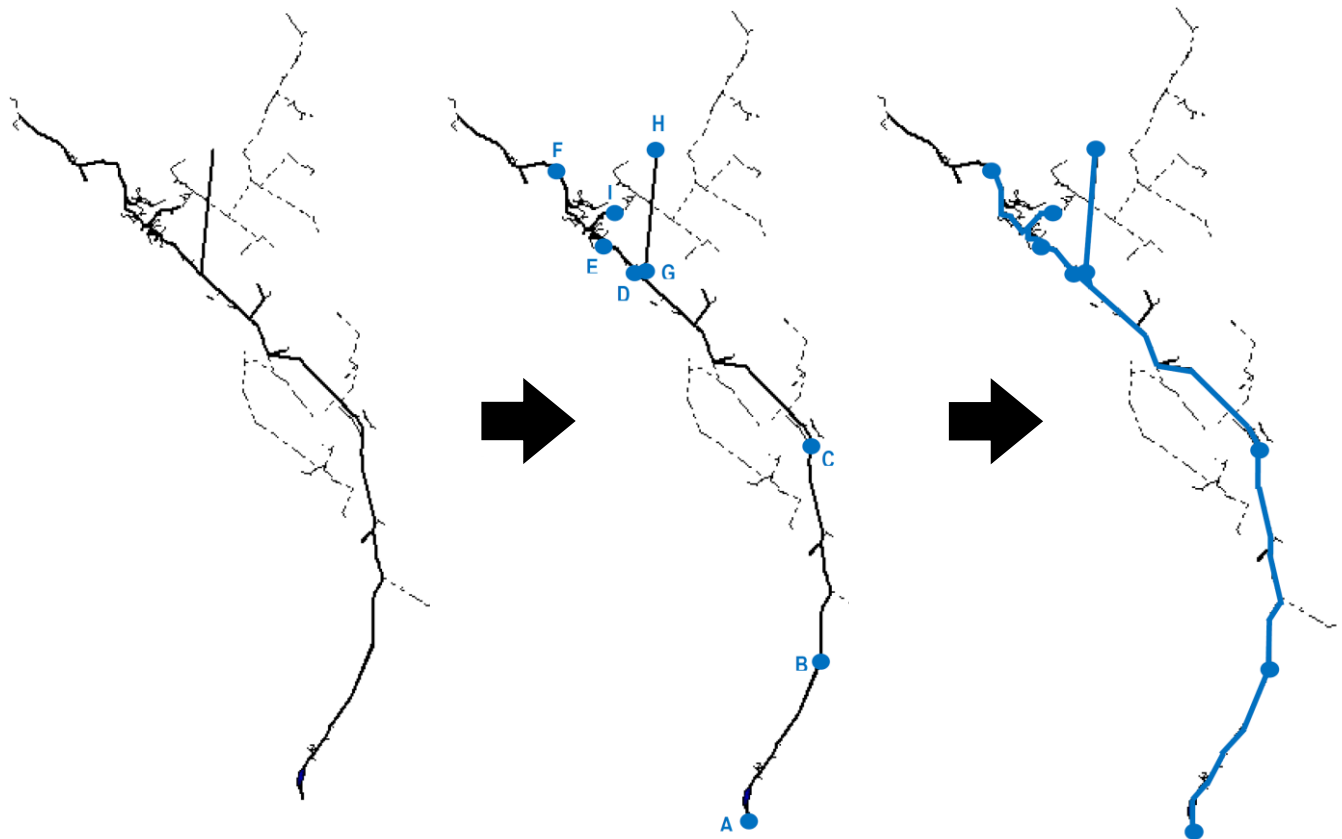


Figure 6-1: Selecting retained components and defining the feeder backbone

6.1.3 Clipping of laterals

All lines and loads not included within the feeder backbone highlighted in Figure 6-1 are now considered laterals. It is apparent that the laterals can be quite extensive, consisting of a large share of the feeder's overhead lines and loads. The top diagram in Figure 6-2 (not drawn to scale) illustrates the extent of the laterals seen in the feeder. In the manner in which the feeder backbone was designated in the previous step, this feeder system consists of 45 laterals. Consult Appendix C.3 for a more detailed account of each of the laterals and their electrical characteristics (e.g. impedance, loads).

The bottom diagram of Figure 6-2 shows the clipped laterals. As specified in Section 4.5, a load flow analysis of the entire feeder system was executed, and the power flow into each lateral from its junction with the backbone was recorded. These values were used as a designated three-phase load to represent the current consumed by the loads and line losses of these laterals and to retain their effects on the feeder backbone in the simplified model. Appendix D.1 contains the values for each of the loads that served to replace their respective laterals.

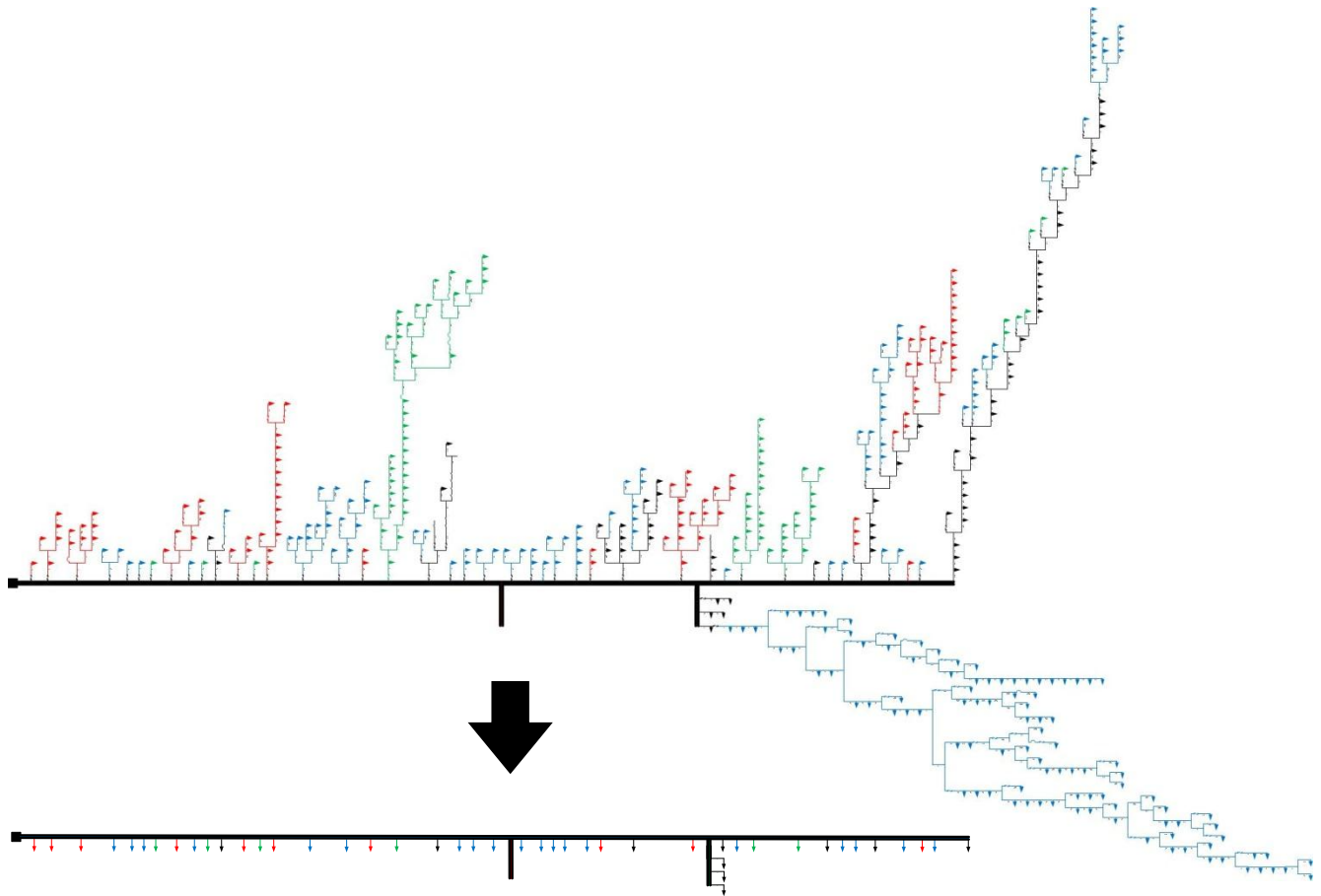


Figure 6-2: Feeder laterals, before and after clipping

6.1.4 Defining nodes and line sections

At this point, the feeder is comprised of one long section, composed of a chain of lines and loads, with two offshoot sections, with their respective lines and loads. Table D-2 in Appendix D.2 lists out the relevant electrical characteristics of each of the individual line pieces.

The feeder backbone is sectioned off by the retained components. Each resulting line section is defined as all adjacent lines and loads between any pair of retained components. Figure 6-3 shows where these distinctions have been made between line sections, showing how each of the individual line pieces has been grouped.

Two of the laterals from the feeder are significantly extensive and contain a large quantity of load, compared to the rest of the laterals throughout the feeder. In addition to their size relative to the other laterals, they are not located within a series of other line/load sections, as the other laterals are; rather, they are located beyond the last retained components at their respective ends of the retained backbone of the feeder. Thus, each of them serves as a terminus for the backbone and will be portrayed simply as a spot

three-phase load, i.e. no impedance considered, at the location of the respective retained component. These two laterals can be seen distinctively in Figure 6-3 below as the rightward-pointing arrows; one of them is located adjacent to the shunt capacitor, and the other is downstream of the recloser at the end of the main backbone section.

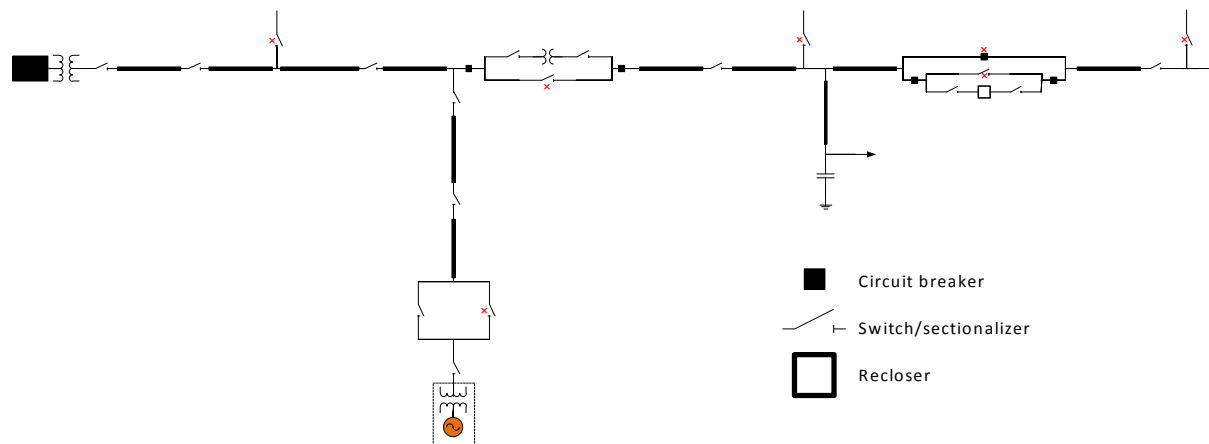


Figure 6-3: Grouping of backbone sections, according to retained components

There are a couple instances along the feeder in which sections are further split up in order to equalize the load distribution within each section, as described in Section 4.6. One such case is in the first section, between the first two retained components. Appendix D.2 explains which sections were split according to these criteria.

6.1.5 Aggregation of loads

With the feeder now split into distinct sections, the individual pieces of each section are now aggregated in order to calculate the equivalent impedance and load values for each section as a whole. Section 3.5 shows how this is done. An example is provided for illustration and validation in Appendix B that walks through this step for the first line section of the feeder.

Figure 6-4 in Section 6.2 shows the layout for the outcome of this methodology, the reduced feeder model. Table 6-2 shows the equivalent impedance and load values calculated throughout each line section within this step.

6.1.6 Balancing of loads

The loads must be balanced in order to achieve compliance with negative-sequence voltage requirements anywhere along the equivalent feeder model. Table 6-3 in Section 6.2 contains the equivalent impedance and load values for the reduced feeder model after the loads are balanced.

6.2 Reduced Feeder Model

Figure 6-4 on the next page is the reduced feeder model developed according to the methodology described in Chapter 4 and illustrated previously in this chapter. Each of the equivalent section's lines and loads are characterized in Table 6-2 for the unbalanced load case and Table 6-3 for the balanced load case. L1 and L2 on the model represent the laterals that were clipped from the termini of the feeder backbone, described in the third paragraph of Section 6.1.4. All laterals are absorbed at their connection points, as described in Section 6.1.3.

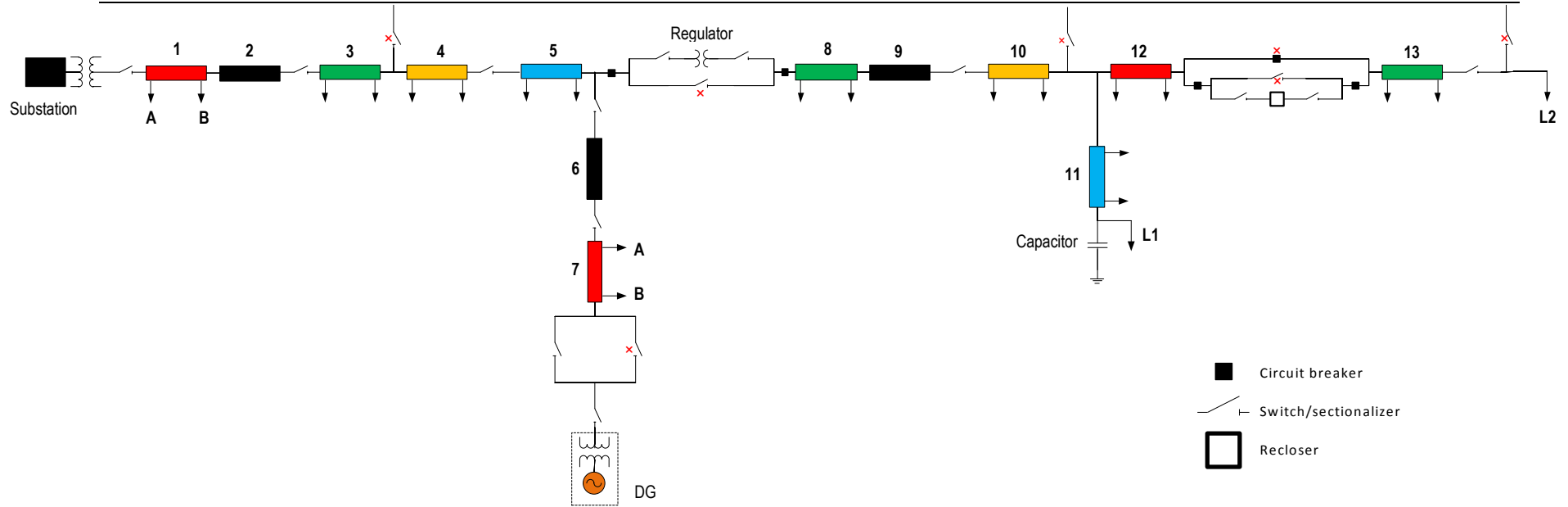


Figure 6-4: Reduced feeder model line sections

Table 6-2: Line section equivalent impedance and lumped load parameters, without load balancing

Sec	Length [km]	Series impedance [Ω]		Upstream load A [kVA]			Downstream load B [kVA]		
		Z_p	Z_0	A_a	A_b	A_c	B_a	B_b	B_c
1	4.11	$0.474 + 1.621j$	$1.576 + 5.431j$	$480.92 + 101.04j$	$9.84 + 2.45j$	$7.65 + 1.09j$	$265.78 + 56.96j$	$20.06 + 5.05j$	$8.95 + 1.71j$
2	2.29	$0.266 + 0.905j$	$0.880 + 3.031j$						
3	2.04	$0.236 + 0.805j$	$0.783 + 2.697j$	$5.32 + 1.50j$	$5.59 + 1.48j$	$6.04 + 1.68j$	$51.08 + 10.00j$	$52.81 + 9.92j$	$71.86 + 15.22j$
4	6.49	$0.755 + 2.562j$	$2.491 + 8.585j$	$217.81 + 40.80j$	$25.61 + 5.61j$	$97.60 + 24.41j$	$278.39 + 53.11j$	$7.39 + 2.39j$	$21.70 + 4.89j$
5	8.33	$0.969 + 3.293j$	$3.198 + 11.028j$	$27.20 + 7.16j$	$161.38 + 28.97j$	$175.24 + 39.34j$	$53.90 + 13.74j$	$208.72 + 38.63j$	$148.56 + 41.26j$
6	10.50	$1.218 + 4.148j$	$4.032 + 13.892j$						
7	0.07	$0.009 + 0.027j$	$0.026 + 0.092j$	$2.13 + 0.53j$	$2.13 + 0.53j$	$2.13 + 0.53j$	$17.77 + 4.27j$	$17.77 + 4.27j$	$17.77 + 4.27j$
8	1.50	$0.173 + 0.593j$	$0.576 + 1.987j$	0	0	$39.28 + 7.84j$	0	2.00	$34.92 + 5.86j$
9	0.45	$0.052 + 0.179j$	$0.174 + 0.598j$						
10	1.05	$0.120 + 0.413j$	$0.402 + 1.384j$	$230.04 + 46.35j$	$82.12 + 19.36j$	$297.53 + 60.26j$	$774.76 + 152.35j$	$216.49 + 51.04j$	$523.97 + 109.24j$
11	1.40	$0.346 + 0.479j$	$0.641 + 1.497j$	$110.95 + 24.56j$	$73.68 + 15.65j$	$107.75 + 26.01j$	$220.45 + 59.54j$	$179.52 + 49.65j$	$161.65 + 46.69j$
12	0.40	$0.104 + 0.170j$	$0.212 + 0.545j$	$0.29 - 0.10j$	$-0.10j$	$62.47 + 13.95j$	1.71	0	$2.73 + 0.75j$
13	2.36	$0.772 + 1.039j$	$1.404 + 3.231j$	$243.63 + 55.26j$	$998.11 + 188.48j$	$457.08 + 90.87j$	$217.77 + 45.44j$	$409.39 + 77.32j$	$430.32 + 86.73j$
L1				0	$19.27 + 3.71j$	$1180.28 + 218.41j$			
L2				$216.50 + 77.70j$	$592.50 + 148.80j$	$604.90 + 151.10j$			

Table 6-3: Line section equivalent impedance and lumped load parameters, after balancing loads

Sec	Length [km]	Series impedance [Ω]		Lumped three-phase load [kVA]	
		Z_p	Z_0	A	B
1	4.11	$0.474 + 1.621 j$	$1.576 + 5.431 j$	$498.42 + 104.59 j$	$294.78 + 63.71 j$
2	2.29	$0.266 + 0.905 j$	$0.880 + 3.031 j$		
3	2.04	$0.236 + 0.805 j$	$0.783 + 2.697 j$	$16.95 + 4.66 j$	$175.75 + 35.14 j$
4	6.49	$0.755 + 2.562 j$	$2.491 + 8.585 j$	$341.02 + 70.82 j$	$307.48 + 62.28 j$
5	8.33	$0.969 + 3.293 j$	$3.198 + 11.028 j$	$363.82 + 75.47 j$	$411.18 + 93.63 j$
6	10.50	$1.218 + 4.148 j$	$4.032 + 13.892 j$		
7	0.07	$0.009 + 0.027 j$	$0.026 + 0.092 j$	$6.40 + 1.60 j$	$53.30 + 12.80 j$
8	1.50	$0.173 + 0.593 j$	$0.576 + 1.987 j$	$39.28 + 7.84 j$	$36.92 + 5.86 j$
9	0.45	$0.052 + 0.179 j$	$0.174 + 0.598 j$		
10	1.05	$0.120 + 0.413 j$	$0.402 + 1.384 j$	$609.69 + 125.97 j$	$1515.21 + 312.63 j$
11	1.40	$0.346 + 0.479 j$	$0.641 + 1.497 j$	$292.38 + 66.23 j$	$561.62 + 155.87 j$
12	0.40	$0.104 + 0.170 j$	$0.212 + 0.545 j$	$62.76 + 13.75 j$	$4.44 + 0.75 j$
13	2.36	$0.772 + 1.039 j$	$1.404 + 3.231 j$	$1698.81 + 334.61 j$	$1057.49 + 209.49 j$
L1				$1199.55 + 222.12 j$	
L2				$1413.90 + 377.60 j$	

6.3 Validation of Feeder Reduction

In order to validate the reduction methods used to simplify the distribution feeder, the steady-state performance of the reduced model is compared with that of the original feeder model. This is done by looking at the power flow and short circuit behaviour of the models at a common set of observation points, using the DSAP. Figure 6-5 shows the location of these test points, where steady-state values are recorded of voltages, power flows, and fault currents.

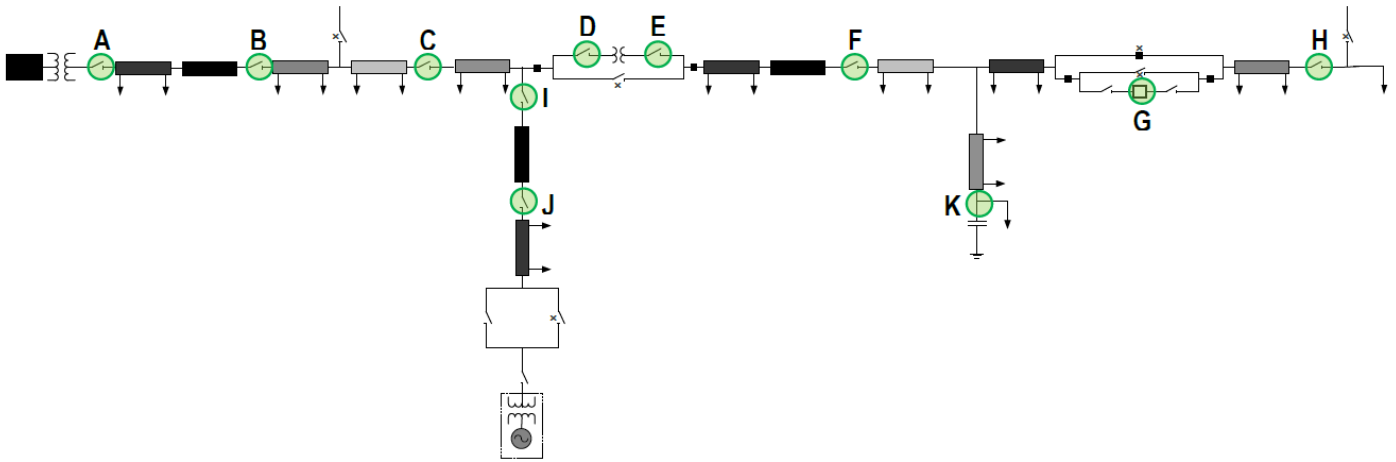


Figure 6-5: Test points for model validation

The feeder reduction will be deemed satisfactory in achieving the desired simplification without altering the feeder's steady-state characteristics if the reduced model satisfies all of the following criteria at each measurement point upon testing:

1. Voltage within 2% of that seen in the respective point in the DSAP model
2. Difference in through active power from that seen in the respective point in the DSAP model within 5% of the total feeder load
3. Fault current within 7% of that seen in the respective point in the DSAP model, for the fault types outlined in Section 6.3.2

6.3.1 Load flow of unbalanced feeder

Even though a balanced feeder model is sought, power flows are also simulated with the original unbalanced loads using the DSAP package in order to further ascertain the quality of the reduction procedure in preserving the feeder's equivalent electrical characteristics. Unbalanced power flows are conducted using the DSAP application software utilising both the original and reduced feeder model, with unbalanced loads.

The power flow results for the original network can be found in Table 6-4, and those for the reduced network can be found in Table 6-5. Figure 6-6 shows the voltage profile along each phase of the orange highlighted portion in the feeder diagram, based on the data in Tables 6-4 and 6-5. It also illustrates the real and reactive profile along each phase of the feeder, in terms of power flow (in the direction away from the substation) through the respective measurement point.

All power flows and node voltages at all test points in the reduced network match very closely with those in the original network. The largest deviation in voltage is 0.26%, and the largest deviation in through active power is 137.8 kW, or 1.24% of the total feeder active power loading (11,100 kW). The largest deviation in through reactive power is 42.4 kVar.

Table 6-4: Power flows and node voltages in original network

	Node voltages [p.u.]			Downstream power flow, by phase						Downstream power flow, total	
	V _a	V _b	V _c	P _a [kW]	Q _a [kVar]	P _b [kW]	Q _b [kVar]	P _c [kW]	Q _c [kVar]	P _{total} [kW]	Q _{total} [kVar]
A	1.042	1.043	1.041	-1763.3	1648.6	-1787.0	1140.6	-433.6	2090.7	-3983.9	4879.9
B	1.045	1.029	1.012	-2577.9	1421.0	-1833.4	1061.6	-440.5	2034.5	-4851.8	4517.1
C	1.055	1.011	0.977	-3242.6	1187.2	-1941.4	939.4	-620.9	1925.8	-5804.9	4052.4
D	1.072	0.994	0.947	2045.0	57.5	2628.1	169.9	3975.1	534.8	8648.2	762.2
E	1.029	1.029	1.029	2045.0	57.5	2628.0	169.9	3975.0	534.7	8648.0	762.1
F	1.032	1.023	1.021	2050.3	44.7	2612.0	157.8	3878.7	449.2	8541.0	651.7
G	1.033	1.018	1.018	686.2	178.8	2008.5	419.7	1498.9	335.2	4193.6	933.7
H	1.032	1.012	1.013	216.5	77.7	592.5	148.8	604.9	151.1	1413.9	377.6
I	1.072	0.994	0.947	-5482.3	942.9	-4950.5	582.2	-4897.7	1250.6	-15330.5	2775.7
J	1.080	1.019	0.963	-5594.9	399.0	-5110.6	133.7	-5100.2	653.2	-15805.7	1185.9
K	1.035	1.020	1.015	116.0	-395.7	132.6	-380.1	1299.0	-156.3	1547.6	-932.1

Table 6-5: Power flows and node voltages in reduced network

	Node voltages [p.u.]			Downstream power flow, by phase						Downstream power flow, total	
	V _a	V _b	V _c	P _a [kW]	Q _a [kVar]	P _b [kW]	Q _b [kVar]	P _c [kW]	Q _c [kVar]	P _{total} [kW]	Q _{total} [kVar]
A	1.042	1.043	1.041	-1625.5	1666.2	-1745.8	1141.5	-373.3	2095.3	-3744.6	4903.0
B	1.044	1.030	1.011	-2453.3	1463.4	-1789.3	1089.8	-385.9	2056.2	-4628.5	4609.4
C	1.054	1.012	0.975	-3162.9	1210.7	-1903.7	945.0	-562.0	1927.5	-5628.6	4083.2
D	1.069	0.995	0.944	2115.9	73.5	2672.2	177.8	4039.9	545.9	8828.0	797.2
E	1.029	1.029	1.029	2115.9	73.4	2672.2	177.8	4039.8	545.8	8827.9	797.0
F	1.032	1.023	1.021	2120.1	62.7	2659.2	168.1	3945.4	474.7	8724.7	705.5
G	1.033	1.019	1.018	713.4	185.9	2056.6	428.5	1530.2	341.3	4300.2	955.7
H	1.032	1.013	1.013	227.5	81.6	604.4	151.8	617.7	154.2	1449.6	387.6
I	1.069	0.995	0.944	-5478.0	961.0	-4956.6	587.3	-4887.9	1249.1	-15322.5	2797.4
J	1.078	1.020	0.961	-5592.6	408.6	-5117.3	134.0	-5093.3	646.5	-15803.2	1189.1
K	1.036	1.020	1.013	0.1	-429.5	20.0	-413.1	1207.1	-187.8	1227.2	-1030.4

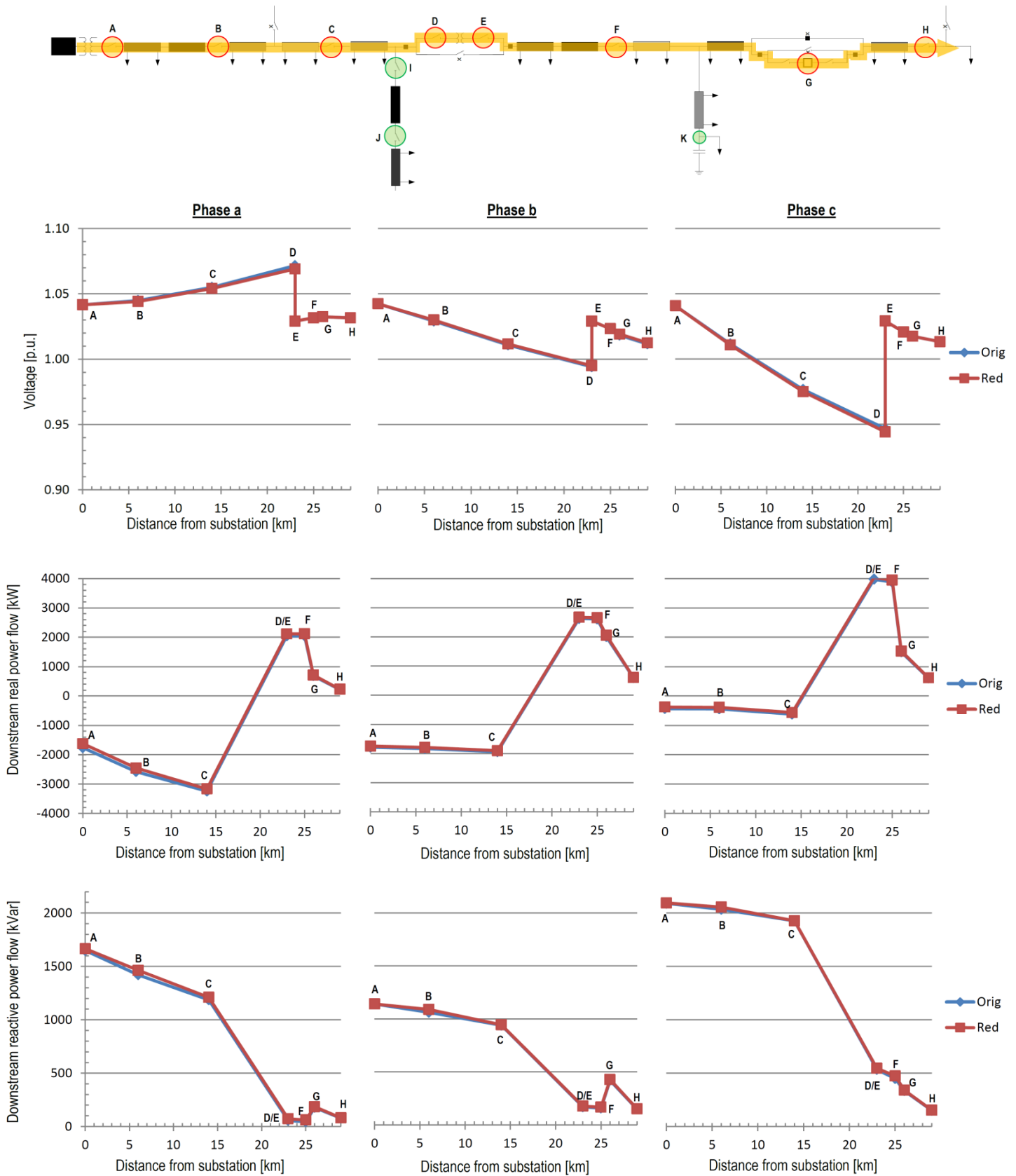


Figure 6-6: Voltage and power flow profile for each phase of highlighted section, reduction validation

6.3.2 Fault currents

In order to test the short circuit behaviour of both the original and reduced networks, each of the test points is inflicted with one of four different types of faults considered. The four types of faults looked at are as follows:

- Three-phase faults (LLL)
- Line-to-line-to-ground fault (LLG)
- Line-to-line fault (LL)
- Line-to-ground fault (LG)

Appendix E illustrates each of these faults and explains the manner in which the fault current is recorded at each test point along the distribution feeder. Table 6-6 shows the fault current values at each of the test points in the original feeder, and Table 6-7 shows the fault current values at each of the test points in the reduced feeder. The fault currents at all test points in the reduced feeder are nearly identical to those in the original network. The feeder reduction method appears to allow simplification of this distribution feeder without any noticeable compromise in portraying the short-circuit properties of the original network.

Table 6-6: Fault currents in original network

	Fault current [A]			
	LLL	LLG	LL	LG
A	3522	3769	3050	3836
B	2240	2126	1940	1903
C	1502	1393	1301	1192
D	1137	1064	985	912
E	1137	1064	984	912
F	1075	1001	931	843
G	1032	957	894	797
H	957	886	829	725
I	1135	1063	983	911
J	870	839	753	740
K	1000	928	866	767

Table 6-7: Fault currents in reduced network

	Fault current [A]			
	LLL	LLG	LL	LG
A	3522	3769	3050	3836
B	2238	2124	1938	1901
C	1501	1392	1300	1191
D	1136	1064	984	912
E	1136	1064	984	911
F	1075	1000	931	843
G	1033	958	895	798
H	958	887	830	726
I	1134	1062	982	910
J	870	839	753	740
K	1000	927	866	767

6.3.3 Performance criteria evaluation

The criteria for the comparison of the steady-state behaviour between the original feeder model and the reduced feeder model are as follows:

1. Voltage within 2% of that seen in the respective point in the DSAP model
2. Difference in through active power from that seen in the respective point in the DSAP model within 5% of the total feeder load
3. Fault current within 7% of that seen in the respective point in the DSAP model, for the fault types outlined in Section 6.3.2

The figures and tables shown for the load flow and short circuit calculations above indicate close resemblance between the two models. Table 6-8 below highlights this, in terms of the outlined validation criteria. Only if the largest disparity in all three relevant steady-state features (voltage, active power flow, fault current) between the two models is less than the threshold value can the reduced network be said to be true to the original. As expected, the reduced feeder model passes this test, validating the steady-state fidelity of the network methodology.

Table 6-8: Evaluation of steady-state behaviour of reduced network

		Largest disparity from original model	Location of disparity	Threshold	Pass?
1	Node voltage	0.003 p.u.	D (phase c)	0.02 p.u.	YES
2	Power flow	137.8 kW	A (phase a)	555 kW	YES
3	Fault current	0.105% (2 A)	B (LG fault)	7%	YES

6.4 Construction of EMTP Model

The reduced model is synthesized in the EMTP environment. Each of the components described in Chapter 5 is modeled in the program, using the information from the DSAP data available.

The line sections determined by the feeder reduction are constructed in EMTP using the device models for a pi-section overhead line and those for three-phase PQ loads. The shunt capacitor is seen in this model as a reactive power injection, modeled as a balanced three-phase PQ load, with $P = 0$ and a negative Q value.

The distributed generator is modeled as an ideal voltage source interconnected to the 25 kV distribution network by a three-phase delta/wye-grounded transformer. Because the scope of this analysis is limited to the feeder network itself, a simple ideal voltage source is used, rather than a more complex machine model.

Tables 6-9 and 6-10 highlight the pertinent details of the EMTP model's retained components. The EMTP schematic of the feeder model is shown in Appendix D.3.

Table 6-9: DG voltage and angle settings

Parameter	Value
Magnitude [kV line-to-line RMS]	4.2348
Phase [deg]	77.366

Table 6-10: EMTP parameters for retained components

Component	Key parameters
Substation source	120 kV $\theta = 0$ $Z_1 = 6.4438 + 19.3314j \, \Omega$ $Z_0 \approx 0$
Substation transformer	15 MVA 120/26.4 kV $Z = 0.28876 + 7.219j \, \%$
DG load flow	$P_{in} = 16 \text{ MW}$
DG source	4.2348 kV $\theta = 0$ $Z_1 = 0.1 + 155.1733j \, \Omega$ $Z_0 = 0.1 + 0.5j$
DG transformer	19.5 MVA 4.16/24.9 kV $Z = 0.8955 + 8.9553j \, \%$
Voltage regulator	15 MVA $Z = 0.95 + 2.04j \, \%$
Shunt capacitor	$Q_{in} = 1200 \text{ kVar}$

6.5 Validation of EMTP Model

In order to validate the modeling of the lines and components in EMTP, the steady-state behaviour of the EMTP model was compared to that of the original (unreduced) model in the DSAP. This is similar to the comparative analysis done in Section 6.3 in order to validate the feeder reduction methodology. A power flow calculation and short circuit analysis are carried out, and the values are compared between a common set of points between both the DSAP model and the EMTP model constructed as described in the previous section.

The same set of criteria is used to compare the node voltages, through active power, and fault currents through each measurement point as was used before in Section 6.3. Since a balanced feeder model is sought, only the performance of the balanced feeder model was analyzed using EMTP.

6.5.1 Load flow of balanced feeder

In order to make a fair comparison between the EMTP model, which has balanced loads, and the original feeder model, the loads in the DSAP model are considered also to be balanced. This is done by executing a balanced power flow calculation in the DSAP and using these results as the basis on which to compare the EMTP model's power flow behaviour.

The results of the DSAP balanced power flow analysis of the feeder model are shown in Table 6-11, and those for the EMTP model are shown in Table 6-12. Figure 6-7 shows the voltage profile along each phase of the orange highlighted portion of the feeder diagram, based on the data in Tables 6-11 and 6-12. The same figure also illustrates the real and reactive power profile along each phase of the feeder, in terms of power flow (in the direction away from the substation) through the respective measurement point.

The agreement between the two models is seen to be quite satisfactory. The voltage profile in the EMTP model correlates much more closely with that of the balanced DSAP model. The same can be said about the active power profile, since the through active power measured in the EMTP model does not deviate any more than 336 kW from that of the feeder model in DSAP.

Table 6-11: Power flows and node voltages in original network, DSAP model, balanced loads

	Node voltages [p.u.]			Downstream power flow, by phase						Downstream power flow, total	
	V _a	V _b	V _c	P _a [kW]	Q _a [kVar]	P _b [kW]	Q _b [kVar]	P _c [kW]	Q _c [kVar]	P _{total} [kW]	Q _{total} [kVar]
A	1.042	1.042	1.042	-1343.8	1526.7	-1343.8	1526.7	-1343.8	1526.7	-4031.4	4580.1
B	1.030	1.030	1.030	-1626.8	1423.9	-1626.8	1423.9	-1626.8	1423.9	-4880.4	4271.7
C	1.017	1.017	1.017	-1931.0	1295.8	-1931.0	1295.8	-1931.0	1295.8	-5793.0	3887.4
D	1.007	1.007	1.007	2888.7	245.5	2888.7	245.5	2888.7	245.5	8666.1	736.5
E	1.029	1.029	1.029	2888.7	245.5	2888.7	245.5	2888.7	245.5	8666.1	736.5
F	1.025	1.025	1.025	2854.4	213.4	2854.4	213.4	2854.4	213.4	8563.2	640.2
G	1.023	1.023	1.023	1403.2	311.0	1403.2	311.0	1403.2	311.0	4209.6	933.0
H	1.018	1.018	1.018	473.8	126.1	473.8	126.1	473.8	126.1	1421.4	378.3
I	1.007	1.007	1.007	-5112.2	904.2	-5112.2	904.2	-5112.2	904.2	-15336.6	2712.6
J	1.024	1.024	1.024	-5268.5	381.5	-5268.5	381.5	-5268.5	381.5	-15805.5	1144.5
K	1.023	1.023	1.023	407.5	-342.8	407.5	-342.8	407.5	-342.8	1222.5	-1028.4

Table 6-12: Power flows and node voltages in EMTP model, balanced loads

	Node voltages [p.u.]			Downstream power flow, by phase						Downstream power flow, total	
	V _a	V _b	V _c	P _a [kW]	Q _a [kVar]	P _b [kW]	Q _b [kVar]	P _c [kW]	Q _c [kVar]	P _{total} [kW]	Q _{total} [kVar]
A	1.032	1.032	1.033	-1254.1	1849.4	-1249.1	1623.2	-1051.2	1739.3	-3554.4	5211.9
B	1.018	1.018	1.018	-1548.6	1750.4	-1565.9	1507.4	-1342.4	1613.3	-4456.9	4871.2
C	1.002	1.002	1.003	-1861.0	1625.6	-1896.0	1363.4	-1647.3	1464.4	-5404.4	4453.4
D	0.990	0.990	0.991	2992.8	157.9	3224.0	437.9	2862.1	494.6	9078.9	1090.4
E	1.024	1.023	1.025	2932.5	58.7	3167.4	326.6	2815.9	393.0	8915.7	778.3
F	1.019	1.019	1.020	2893.7	25.8	3127.8	288.3	2781.9	356.8	8803.5	670.9
G	1.016	1.016	1.017	1400.9	218.6	1535.9	328.8	1372.6	389.2	4309.4	936.6
H	1.012	1.011	1.013	471.8	94.8	519.5	129.9	465.2	153.1	1456.5	377.8
I	0.990	0.990	0.991	-5143.4	1333.7	-5421.1	771.8	-4787.8	816.5	-15352.3	2922.0
J	1.011	1.011	1.012	-5377.4	782.5	-5596.4	184.1	-4956.4	297.6	-15930.3	1264.2
K	1.017	1.016	1.018	455.4	-368.3	456.6	-299.1	395.5	-332.9	1307.4	-1000.2

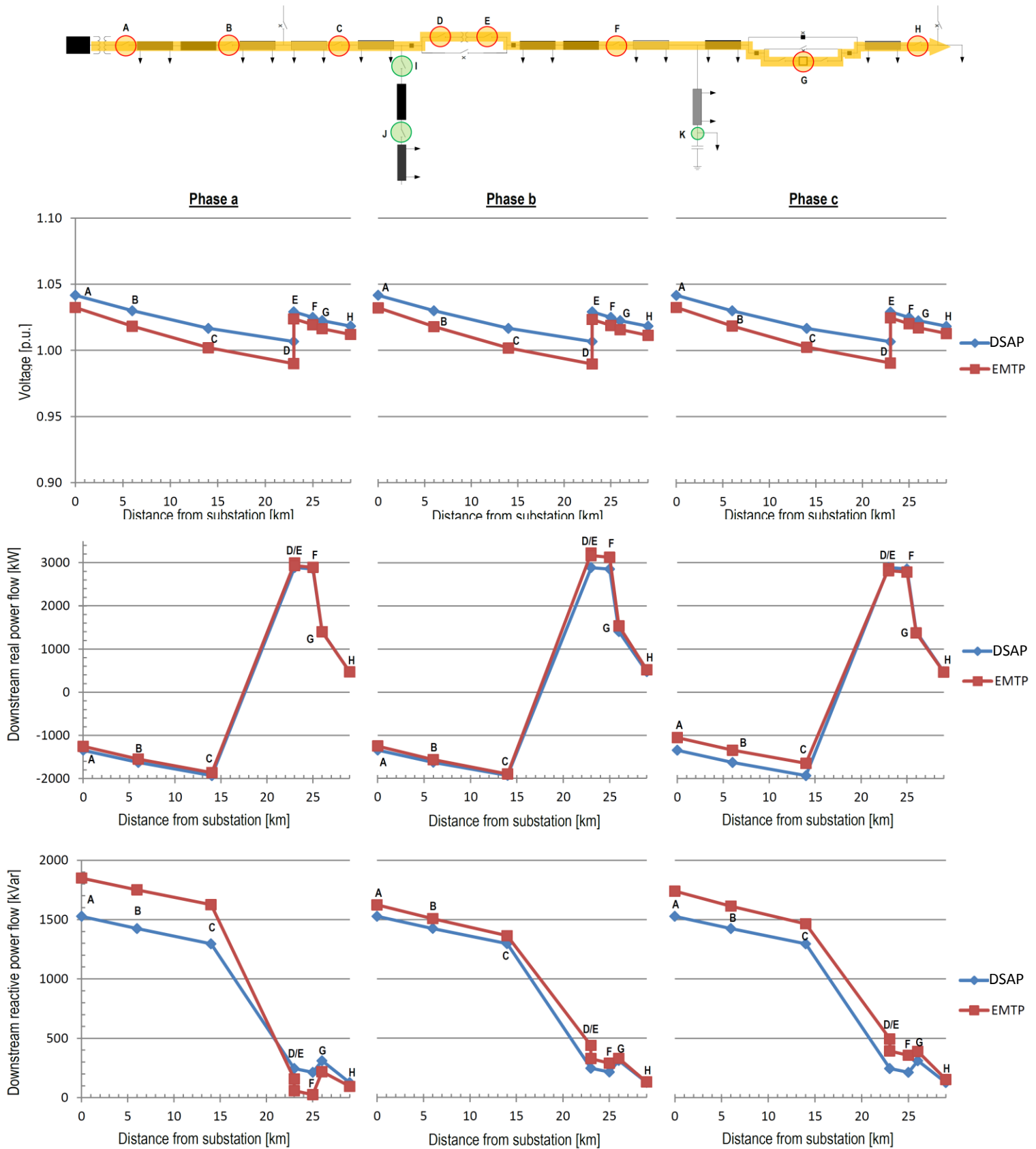


Figure 6-7: Voltage and power flow profile for each phase of highlighted section, after balancing loads

6.5.2 Fault currents

The short-circuit behaviour of the EMTP model is compared to that of the original DSAP model using the same method described in Section 6.3.2. Again, the four types of faults considered are as follows:

- Three-phase faults (LLL)
- Line-to-line-to-ground fault (LLG)
- Line-to-line fault (LL)
- Line-to-ground fault (LG)

All generators (including the DG) are disabled, with the fault current contribution coming from only the substation.

The RMS magnitude of the fault current is recorded for each type of fault inflicted at each of the test points A through K along the distribution feeder. Table 6-13 shows the fault current values at each of the test points in the DSAP feeder model, and Table 6-14 shows those at each of the test points in the EMTP feeder model. Figure 6-8 illustrates each of the fault current profiles in the DSAP model and EMTP model, allowing for visual comparison of the fault currents between the two models.

The fault current profiles for the EMTP feeder model correlate well with those for the DSAP model. The largest deviation, in percentage terms, is 6.52% (49 A), which is seen in the line-to-line fault current at point J.

Table 6-13: Fault currents in DSAP model

	Fault current [A]			
	LLL	LLG	LL	LG
A	3519	3746	3048	3801
B	2237	2103	1937	1806
C	1499	1373	1298	1056
D	1134	1029	982	752
E	1134	1029	982	752
F	1073	972	929	705
G	1030	931	892	672
H	955	860	827	620
I	1132	1026	980	750
J	867	782	751	551
K	998	901	865	651

Table 6-14: Fault currents in EMTP model

	Fault current [A]			
	LLL	LLG	LL	LG
A	3492	3371	2881	3772
B	2177	2182	1886	1788
C	1444	1453	1251	1080
D	1085	1094	940	791
E	1085	1094	940	791
F	1025	1034	888	744
G	982	992	851	711
H	904	913	783	650
I	1085	1094	940	791
J	811	821	702	553
K	955	965	827	688

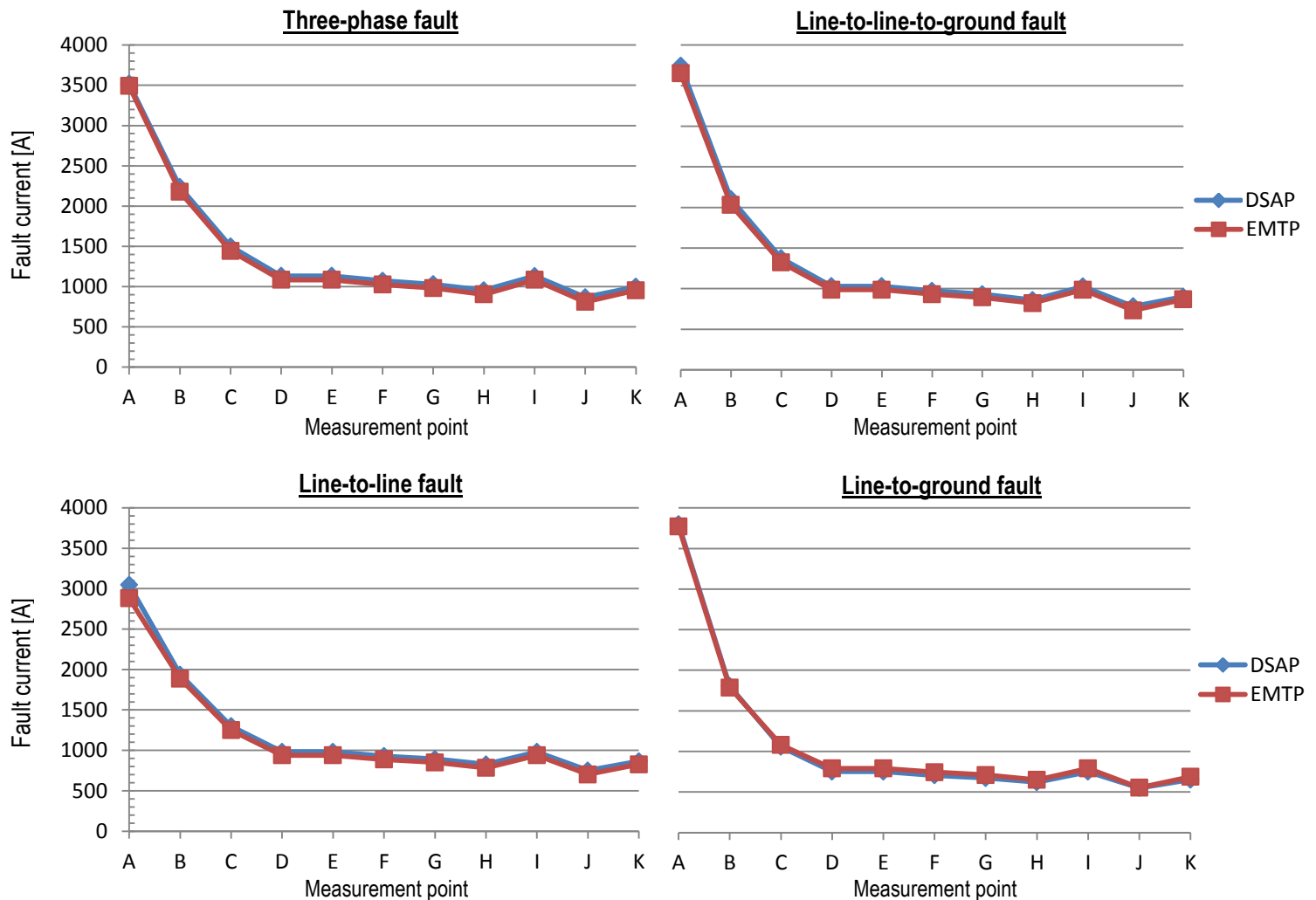


Figure 6-8: Fault current profile for all measurement points

6.5.3 Performance criteria evaluation

The criteria for the comparison of the steady-state behaviour between the original feeder modeled in the DSAP and the EMTP model are as follows:

1. Voltage within 2% of that seen in the respective point in the DSAP model
2. Difference in through active power from that seen in the respective point in the DSAP model within 5% of the total feeder load
3. Fault current within 7% of that seen in the respective point in the DSAP model, for the fault types outlined in Section 6.3.2

Simulating the two models with balanced loads greatly improves the correlation between the two models. Table 6-15 on the next page shows that each of the measures used to assess the EMTP model's steady-state character falls within the accepted threshold. Thus, the EMTP model passes the test, validating the steady-state fidelity of the EMTP model for transient analysis.

Table 6-15: Evaluation of steady-state behaviour of EMTP model, balanced loads

		Largest disparity from original model	Location of disparity	Threshold	Pass?
1	Node voltage	0.017 p.u.	D (phase a)	0.02 p.u.	YES
2	Power flow	335.3 kW	D (phase b)	555 kW	YES
3	Fault current	6.52% (49 A)	J (LL fault)	7%	YES

6.6 Summary

An application example of the feeder reduction methodology is undertaken on the rural distribution feeder described in Chapter 5, articulating in detail each step of the reduction. The reduced feeder is then outlined, highlighting the salient features of the benchmark rural feeder that had been developed in this process. The reduction technique is validated by comparing the steady-state character of the original feeder with that of the reduced model within the DSAP environment. The reduced feeder model is then recreated within the EMTP environment and tested against the original (unreduced) model in DSAP. Power flow simulations and short-circuit current calculations confirm the validity of the reduced model against the stated performance criteria.

7 Conclusions

7.1 Summary

Distributed generation resources are proliferating and comprise an increasing share of the electric power system's generating capability. This holds great promise for alleviating transmission constraints, curtailing environmentally harmful emissions, and providing effective demand-side management capability. This thesis described the procedure to obtain a distribution feeder model compatible for use with currently standing DG interconnection guidelines.

The benchmarking methodology is used to obtain a balanced three-phase distribution feeder model, given an unbalanced distribution feeder model comprising a large number of laterals and unbalanced spot loads. The need for a balanced feeder model arose from the necessity for the ability to connect a DG source at any point along the feeder in order to perform impact studies, particularly those concerning DG islanding and related feeder protection issues.

The necessary analytical steps were described in detail, and a literature review was performed to summarize the available distribution feeder reduction techniques. This circuit simplification methodology would be used to serve the needs of typical system studies. Furthermore, criteria were set on assessing the quality of the resulting feeder reduction, based on voltage profiles and losses. The proposed reduction methodology serves a range of impact studies comprising typical power flow and voltage drop analyses, fault analysis, and classical electromechanical-type transient analysis that are pertinent in the simulation of islanding events.

The proposed reduction methodology was validated against commercial-grade distribution analysis packages and an electromagnetic transients program. The results were found to be in satisfactory agreement.

7.2 Future Work

The work presented here can be extended in several ways, with the aims of providing suitable platforms for testing new distributed generation solutions and associated interconnection issues.

The most apparent extension of this work would be in articulating guidelines for that suitably retain some imbalance in the benchmark feeder upon reduction. This would accommodate the needs for more specialized protection coordination analysis, drawing better focus on the actual characteristics of the protective devices themselves.

This methodology can be further demonstrated and verified by applying it on a great variety of rural distribution feeders. In addition to validating upon individual feeders, this work can be expanded to include

application on multiple feeders that share the same substation or on neighboring feeders with switchable interconnections between them. DG islanding issues sometimes involve interactions between multiple feeders with a shared substation, giving potential importance to this direction of work.

8 References

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Appendix A: Terminology Used

Conventional power systems	Electric power systems in which power is produced in large quantities at centralized facilities and flows from these facilities through high-voltage transmission lines and then through lower-voltage distribution lines in order to reach consumers.
Distributed generation	Placement of power generation capability at the distribution level of the electric power system.
Distribution feeder	Individual distribution network that begins at a single point at a substation and radiates outward towards loads.
Distribution system	Local level of the electric power system that delivers power from the high-voltage transmission system (via a substation) to individual buildings and other consumption sites.
Distribution system analysis program	Analysis software designed for planning studies and for steady-state simulation of electrical distribution system behaviour under different operating conditions and scenarios.
Downstream	In the direction away from the substation along a distribution feeder.
Electromagnetic transients program	Software designed to simulate electromagnetic, electromechanical, and control system transients in multiphase power systems.
Islanding	State in which part of a distribution network is isolated from the rest of the electric power system, yet continues to be supplied with power from a local generation source within the isolated part of the network.
Protection	Coordinated scheme of switching devices that protects the distribution system from damages from faults by isolating them and minimizing their duration.
Transmission system	Meshed network of high-voltage power lines that transport electric power over great distances, usually connecting remote generation facilities with load centers.
Upstream	In the direction towards the substation along a distribution feeder.

Appendix B: Line Reduction Technique Example

This appendix outlines the execution of the reduction technique proposed in Section 3.5 upon the feeder section illustrated on the next page in Figure B-1. This section is one of several from the rural Quebec feeder described in Chapter 5. The line and load parameters for this section are given on the next page in Table B-1.

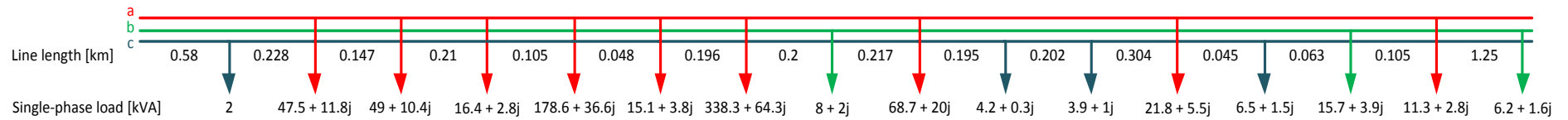


Table B-1: Sample section line and load parameters

Sec	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
		Z_p	Z_0	S_a	S_b	S_c
1	0.580	$0.067 + 0.229j$	$0.223 + 0.767j$			2
2	0.228	$0.026 + 0.090j$	$0.088 + 0.302j$	$47.5 + 11.8j$		
3	0.147	$0.017 + 0.058j$	$0.056 + 0.194j$	$49.0 + 10.4j$		
4	0.21	$0.024 + 0.083j$	$0.081 + 0.278j$	$16.4 + 2.8j$		
5	0.105	$0.012 + 0.041j$	$0.040 + 0.139j$	$178.6 + 36.6j$		
6	0.048	$0.006 + 0.019j$	$0.018 + 0.064j$	$15.1 + 3.8j$		
7	0.196	$0.023 + 0.077j$	$0.075 + 0.259j$	$338.3 + 64.3j$		
8	0.200	$0.023 + 0.079j$	$0.077 + 0.265j$		$8 + 2j$	
9	0.217	$0.025 + 0.086j$	$0.083 + 0.287j$	$68.7 + 20.0j$		
10	0.195	$0.023 + 0.077j$	$0.075 + 0.258j$			$4.2 + 0.3j$
11	0.202	$0.023 + 0.080j$	$0.078 + 0.267j$			$3.9 + 1.0j$
12	0.304	$0.035 + 0.120j$	$0.117 + 0.402j$	$21.8 + 5.5j$		
13	0.045	$0.005 + 0.018j$	$0.017 + 0.060j$			$6.5 + 1.5j$
14	0.063	$0.007 + 0.025j$	$0.024 + 0.083j$		$15.7 + 3.9j$	
15	0.105	$0.012 + 0.041j$	$0.040 + 0.139j$	$11.3 + 2.8j$		
16	1.250	$0.145 + 0.494j$	$0.480 + 1.654j$		$6.2 + 1.6j$	

For the feeder section described by the data above in Table B-1, the equivalent impedance is calculated as follows:

$$\begin{aligned}
 Z_{pEQ} &= \sum_{i=1}^{16} Z_{pi} \\
 &= [0.067 + 0.229j] + [0.026 + 0.090j] + [0.017 + 0.058j] \\
 &\quad + [0.024 + 0.083j] + [0.012 + 0.041j] + [0.006 + 0.019j] + [0.023 + 0.077j] \\
 &\quad + [0.023 + 0.079j] + [0.025 + 0.086j] + [0.023 + 0.077j] + [0.023 + 0.080j] \\
 &\quad + [0.035 + 0.120j] + [0.005 + 0.018j] + [0.007 + 0.025j] + [0.012 + 0.041j] \\
 &\quad + [0.145 + 0.494j] \\
 &= 0.473 + 1.617j
 \end{aligned}$$

$$\begin{aligned}
 Z_{0EQ} &= \sum_{i=1}^{16} Z_{0i} \\
 &= [0.223 + 0.767j] + [0.088 + 0.302j] + [0.056 + 0.194j] \\
 &\quad + [0.081 + 0.278j] + [0.040 + 0.139j] + [0.018 + 0.064j] + [0.075 + 0.259j] \\
 &\quad + [0.077 + 0.265j] + [0.083 + 0.287j] + [0.075 + 0.258j] + [0.078 + 0.267j] \\
 &\quad + [0.117 + 0.402j] + [0.017 + 0.060j] + [0.024 + 0.083j] + [0.040 + 0.139j] \\
 &\quad + [0.480 + 1.654j] \\
 &= 1.572 + 5.418j
 \end{aligned}$$

The loads on the feeder section for each phase are lumped and allocated between the two ends of the section, according to the following calculations:

$$\begin{aligned}
 S_{aA} &= \sum_{i=1}^{16} S_{ai} \left(1 - \frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \\
 &= S_{a1} \left(1 - \frac{R_{p1}}{R_{pEQ}} \right) + S_{a2} \left(1 - \frac{R_{p1} + R_{p2}}{R_{pEQ}} \right) + S_{a3} \left(1 - \frac{R_{p1} + R_{p2} + R_{p3}}{R_{pEQ}} \right) \\
 &\quad + S_{a4} \left(1 - \frac{R_{p1} + R_{p2} + R_{p3} + R_{p4}}{R_{pEQ}} \right) + \dots + S_{a16} \left(1 - \frac{\sum_{j=1}^{16} R_{pj}}{R_{pEQ}} \right)
 \end{aligned}$$

$$\begin{aligned}
 &= [47.5 + 11.8j] \left(1 - \frac{0.093}{[0.473]}\right) + [49 + 10.4j] \left(1 - \frac{0.11}{[0.473]}\right) + [16.4 + 2.8j] \left(1 - \frac{0.134}{[0.473]}\right) \\
 &\quad + [178.6 + 36.6j] \left(1 - \frac{0.146}{[0.473]}\right) + [15.1 + 3.8j] \left(1 - \frac{0.152}{[0.473]}\right) \\
 &\quad + [338.3 + 64.3j] \left(1 - \frac{0.175}{[0.473]}\right) + [68.7 + 20j] \left(1 - \frac{0.223}{[0.473]}\right) \\
 &\quad + [21.8 + 5.5j] \left(1 - \frac{0.304}{[0.473]}\right) + [11.3 + 2.8j] \left(1 - \frac{0.328}{[0.473]}\right) \\
 &= 481.94 + 101.25j
 \end{aligned}$$

$$\begin{aligned}
 S_{aB} &= \sum_{i=1}^N S_{ai} \left(\frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \\
 &= S_{a1} \left(\frac{R_{p1}}{R_{pEQ}} \right) + S_{a2} \left(\frac{R_{p1} + R_{p2}}{R_{pEQ}} \right) + S_{a3} \left(\frac{R_{p1} + R_{p2} + R_{p3}}{R_{pEQ}} \right) \\
 &\quad + S_{a4} \left(\frac{R_{p1} + R_{p2} + R_{p3} + R_{p4}}{R_{pEQ}} \right) + \dots + S_{a16} \left(\frac{\sum_{j=1}^{16} R_{pj}}{R_{pEQ}} \right) \\
 &= [47.5 + 11.8j] \left(\frac{0.093}{[0.473]} \right) + [49 + 10.4j] \left(\frac{0.11}{[0.473]} \right) + [16.4 + 2.8j] \left(\frac{0.134}{[0.473]} \right) \\
 &\quad + [178.6 + 36.6j] \left(\frac{0.146}{[0.473]} \right) + [15.1 + 3.8j] \left(\frac{0.152}{[0.473]} \right) \\
 &\quad + [338.3 + 64.3j] \left(\frac{0.175}{[0.473]} \right) + [68.7 + 20j] \left(\frac{0.223}{[0.473]} \right) \\
 &\quad + [21.8 + 5.5j] \left(\frac{0.304}{[0.473]} \right) + [11.3 + 2.8j] \left(\frac{0.328}{[0.473]} \right) \\
 &= 264.76 + 56.75j
 \end{aligned}$$

The above calculations acted upon the loading of phase a. Phases b and c are dealt with in the same fashion, using the spot loads and impedance information for those respective sections.

$$\begin{aligned}
 S_{bA} &= \sum_{i=1}^{16} S_{bi} \left(1 - \frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \\
 &= [8 + 2j] \left(1 - \frac{0.198}{[0.473]} \right) + [15.7 + 3.9j] \left(1 - \frac{0.316}{[0.473]} \right) + [6.2 + 1.6j] \left(1 - \frac{0.473}{[0.473]} \right) \\
 &= 9.86 + 2.46j
 \end{aligned}$$

$$\begin{aligned}
 S_{bB} &= \sum_{i=1}^{16} S_{bi} \left(\frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \\
 &= [8 + 2j] \left(\frac{0.198}{[0.473]} \right) + [15.7 + 3.9j] \left(\frac{0.316}{[0.473]} \right) + [6.2 + 1.6j] \left(\frac{0.473}{[0.473]} \right) \\
 &= 20.04 + 5.04j
 \end{aligned}$$

$$\begin{aligned}
 S_{cA} &= \sum_{i=1}^N S_{ci} \left(1 - \frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \\
 &= [2] \left(1 - \frac{0.067}{[0.473]} \right) + [4.2 + 0.3j] \left(1 - \frac{0.246}{[0.473]} \right) + [3.9 + j] \left(1 - \frac{0.269}{[0.473]} \right) \\
 &\quad + [6.5 + 1.5j] \left(1 - \frac{0.309}{[0.473]} \right) \\
 &= 7.67 + 1.10j
 \end{aligned}$$

$$\begin{aligned}
 S_{cB} &= \sum_{i=1}^N S_{ci} \left(\frac{\sum_{j=1}^i R_{pj}}{R_{pEQ}} \right) \\
 &= [2] \left(\frac{0.067}{[0.473]} \right) + [4.2 + 0.3j] \left(\frac{0.246}{[0.473]} \right) + [3.9 + j] \left(\frac{0.269}{[0.473]} \right) + [6.5 + 1.5j] \left(\frac{0.309}{[0.473]} \right) \\
 &= 8.93 + 1.70j
 \end{aligned}$$

The results of the impedance and load lumping calculations outlined in this appendix are outlined as follows:

$$\begin{aligned}
 Z_{pEQ} &= 0.473 + 1.617j \\
 Z_{0EQ} &= 1.572 + 5.418j
 \end{aligned}$$

$$\begin{aligned}
 S_{aA} &= 481.94 + 101.25j \\
 S_{aB} &= 264.76 + 56.75j
 \end{aligned}$$

$$\begin{aligned}
 S_{bA} &= 9.86 + 2.46j \\
 S_{bB} &= 20.04 + 5.04j
 \end{aligned}$$

$$\begin{aligned}
 S_{cA} &= 7.67 + 1.10j \\
 S_{cB} &= 8.93 + 1.70j
 \end{aligned}$$

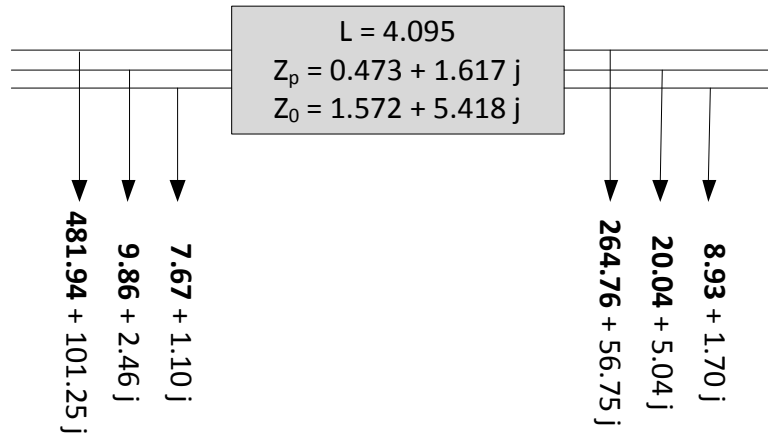


Figure B-2: Line section after reduction

Appendix C: Feeder System Data

This appendix states the data from each of the individual lines and loads of the rural feeder described in Chapter 5, as seen in the DSAP model and considered in the network reduction methodology.

C.1 Conductor Properties

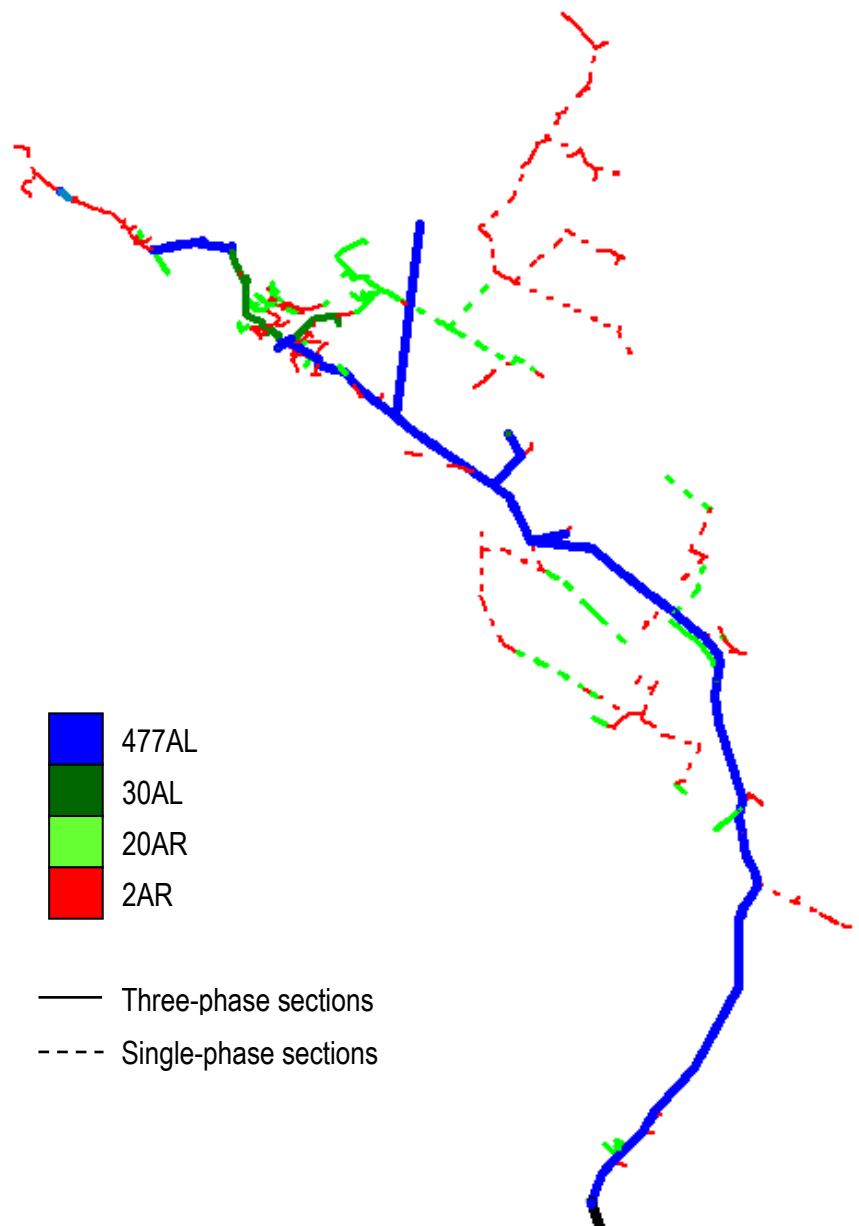


Figure C-1: Overhead line placement in feeder

The DSAP feeder model consists mainly of four types of conductors. As a simplifying measure, the three-phase conductors are assumed in the model to have balanced impedances. The parameters for these lines are shown in Table C-1 below. The series impedance and shunt admittance parameters indicated below are expressed in sequence format, assuming overhead lines with balanced impedances among the three phases.

Table C-1: Overhead line parameters (per unit length)

Conductor name	Series impedance [Ω/km]				Shunt admittance [$\mu\text{S}/\text{km}$]		Current rating [A]
	R_1	X_1	R_0	X_0	B_1	B_0	
477AL	0.116	0.395	0.384	1.323	4.227	1.814	640
30AL	0.326	0.439	0.5939	1.3669	3.761	1.722	315
20AR	0.429	0.475	0.6969	1.4029	3.729	1.715	255
2AR	0.851	0.506	1.211	1.5659	3.512	1.66	170

C.2 Main Backbone

The backbone of the feeder is composed of all line sections that connect together the retained components of the feeder. This feeder's backbone can be broken up into three zones, labelled A, B, and C as seen in Figure C-2 below.



Figure C-2: Feeder backbone sections

Table C-2 lists the characteristic parameters of each individual line section comprising the feeder backbone. Figure C-3 illustrates the convention relating the parameters to the physical attributes of the line sections. The parameters relevant to the analysis in the network reduction methodology are as follows:

- x Distance of section from substation [km]
- L Length of section [km]
- Z_p Positive-sequence impedance of section [Ω]
- Z_0 Zero-sequence impedance of section [Ω]
- S_a Complex phase-a load at section [kVA]
- S_b Complex phase-b load at section [kVA]
- S_c Complex phase-c load at section [kVA]

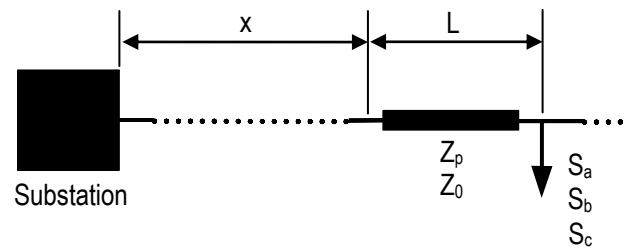


Figure C-3: Line parameter relationships

Table C-2: Main backbone line and load parameters

Zone	Dist. from substation [km]	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
			Z_p	Z_0	S_a	S_b	S_c
A	0.071	0.580	$0.067 + 0.229j$	$0.223 + 0.767j$			2
A	0.651	0.228	$0.026 + 0.090j$	$0.088 + 0.302j$			
A	0.879	0.147	$0.017 + 0.058j$	$0.056 + 0.194j$	$49.0 + 10.4j$		
A	1.026	0.210	$0.024 + 0.083j$	$0.081 + 0.278j$	$16.4 + 2.8j$		
A	1.236	0.105	$0.012 + 0.041j$	$0.040 + 0.139j$			
A	1.341	0.048	$0.006 + 0.019j$	$0.018 + 0.064j$	$15.1 + 3.8j$		
A	1.389	0.196	$0.023 + 0.077j$	$0.075 + 0.259j$			
A	1.585	0.200	$0.023 + 0.079j$	$0.077 + 0.265j$			
A	1.785	0.217	$0.025 + 0.086j$	$0.083 + 0.287j$	$68.7 + 20.0j$		
A	2.002	0.195	$0.023 + 0.077j$	$0.075 + 0.258j$			
A	2.197	0.202	$0.023 + 0.080j$	$0.078 + 0.267j$			
A	2.399	0.304	$0.035 + 0.120j$	$0.117 + 0.402j$	$21.8 + 5.5j$		
A	2.703	0.045	$0.005 + 0.018j$	$0.017 + 0.060j$			
A	2.748	0.063	$0.007 + 0.025j$	$0.024 + 0.083j$		$15.7 + 3.9j$	
A	2.811	0.105	$0.012 + 0.041j$	$0.040 + 0.139j$	$11.3 + 2.8j$		
A	2.916	1.250	$0.145 + 0.494j$	$0.480 + 1.654j$		$6.2 + 1.6j$	
A	4.166	2.291	$0.266 + 0.905j$	$0.880 + 3.031j$			
A	6.457	1.735	$0.201 + 0.685j$	$0.666 + 2.295j$	$34.6 + 10.0j$	$34.6 + 10.0j$	$34.6 + 10.0j$
A	8.192	0.027	$0.003 + 0.011j$	$0.010 + 0.036j$			
A	8.219	0.210	$0.024 + 0.083j$	$0.081 + 0.278j$			$21.5 + 5.4j$
A	8.429	0.051	$0.006 + 0.020j$	$0.020 + 0.067j$	$21.8 + 1.5j$	$21.8 + 1.5j$	$21.8 + 1.5j$
A	8.480	0.016	$0.002 + 0.006j$	$0.006 + 0.021j$			
A	8.496	0.031	$0.004 + 0.012j$	$0.012 + 0.041j$		$23.6 + 5.5j$	
A	8.527	0.355	$0.041 + 0.140j$	$0.136 + 0.470j$	$10.8 + 3.1j$		
A	8.882	0.260	$0.030 + 0.103j$	$0.100 + 0.344j$	$0.6 + 0.1j$	$0.6 + 0.1j$	$0.6 + 0.1j$
A	9.142	0.058	$0.007 + 0.023j$	$0.022 + 0.077j$	$12.7 + 3.6j$		
A	9.200	0.068	$0.008 + 0.027j$	$0.026 + 0.090j$			
A	9.268	0.077	$0.009 + 0.030j$	$0.030 + 0.102j$			
A	9.345	0.172	$0.020 + 0.068j$	$0.066 + 0.228j$	$59.3 + 15.4j$		
A	9.517	0.125	$0.015 + 0.049j$	$0.048 + 0.165j$			$27 + 4.7j$
A	9.642	0.264	$0.031 + 0.104j$	$0.101 + 0.349j$	62.7		
A	9.906	0.062	$0.007 + 0.024j$	$0.024 + 0.082j$			2
A	9.968	0.333	$0.039 + 0.132j$	$0.128 + 0.441j$	2		
A	10.301	0.326	$0.038 + 0.129j$	$0.125 + 0.431j$			
A	10.627	0.472	$0.055 + 0.186j$	$0.181 + 0.624j$	$7.2 + 1.8j$		
A	11.099	0.310	$0.036 + 0.122j$	$0.119 + 0.410j$	2		
A	11.409	0.101	$0.012 + 0.040j$	$0.038 + 0.133j$			
A	11.510	2.196	$0.255 + 0.867j$	$0.843 + 2.905j$	$2.9 + 0.7j$		
A	13.706	0.890	$0.103 + 0.352j$	$0.342 + 1.177j$	$6.3 + 1.6j$		
A	14.596	0.123	$0.014 + 0.049j$	$0.047 + 0.163j$			
A	14.719	0.267	$0.031 + 0.105j$	$0.103 + 0.353j$			

Zone	Dist. from substation [km]	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
			Z_p	Z_0	S_a	S_b	S_c
A	14.986	0.250	$0.029 + 0.099j$	$0.096 + 0.331j$	$2.7 + 1.1j$	$2.7 + 1.1j$	$2.7 + 1.1j$
A	15.296	0.250	$0.029 + 0.099j$	$0.096 + 0.331j$			$14.6 + 3.6j$
A	15.546	0.250	$0.029 + 0.099j$	$0.096 + 0.331j$			
A	15.796	0.095	$0.011 + 0.038j$	$0.036 + 0.126j$		1	
A	15.891	0.650	$0.075 + 0.257j$	$0.250 + 0.860j$			$8.2 + 3.3j$
A	16.541	0.110	$0.013 + 0.043j$	$0.042 + 0.146j$			$14.6 + 3.7j$
A	16.651	0.560	$0.065 + 0.221j$	$0.215 + 0.741j$			$4.3 + 1.1j$
A	17.211	0.200	$0.023 + 0.079j$	$0.077 + 0.265j$			$2.7 + 0.7j$
A	17.411	0.679	$0.079 + 0.268j$	$0.261 + 0.898j$	2		
A	18.090	0.431	$0.050 + 0.170j$	$0.166 + 0.570j$			$6.8 + 2.7j$
A	18.521	0.350	$0.041 + 0.138j$	$0.134 + 0.463j$			
A	18.871	0.680	$0.079 + 0.269j$	$0.261 + 0.900j$			
A	19.551	0.550	$0.064 + 0.217j$	$0.211 + 0.728j$		2	
A	20.101	0.095	$0.011 + 0.038j$	$0.036 + 0.126j$			2
A	20.196	0.250	$0.029 + 0.099j$	$0.096 + 0.331j$			$9.3 + 3.7j$
A	20.446	0.325	$0.038 + 0.128j$	$0.125 + 0.430j$			$5 + 2j$
A	20.771	0.195	$0.023 + 0.077j$	$0.075 + 0.258j$			$19.1 + 7.5j$
A	20.966	0.350	$0.041 + 0.138j$	$0.134 + 0.463j$			$6.2 + 1.6j$
A	21.316	0.050	$0.006 + 0.020j$	$0.019 + 0.066j$			$10.7 + 2.7j$
A	21.366	0.069	$0.008 + 0.027j$	$0.026 + 0.091j$			
A	21.435	0.110	$0.013 + 0.043j$	$0.042 + 0.146j$			
A	21.545	0.475	$0.055 + 0.188j$	$0.182 + 0.628j$			
A	22.020	1.050	$0.122 + 0.415j$	$0.403 + 1.389j$			
A	23.070	0.300	$0.035 + 0.119j$	$0.115 + 0.397j$			$6.9 + 1.7j$
A	23.370	0.010	$0.001 + 0.004j$	$0.004 + 0.013j$			
A	23.417	0.028	$0.003 + 0.011j$	$0.010 + 0.037j$			$8.1 + 2j$
A	23.445	0.675	$0.078 + 0.267j$	$0.259 + 0.893j$			$13.2 + 3.4j$
A	24.120	0.056	$0.006 + 0.022j$	$0.022 + 0.074j$			
A	24.176	0.325	$0.038 + 0.128j$	$0.125 + 0.430j$			
A	24.501	0.363	$0.042 + 0.143j$	$0.139 + 0.480j$			
A	24.864	0.055	$0.006 + 0.022j$	$0.021 + 0.073j$		2	
A	24.919	0.452	$0.052 + 0.179j$	$0.174 + 0.598j$			
A	25.371	0.133	$0.015 + 0.053j$	$0.051 + 0.176j$			$17.0 + 2.4j$
A	25.504	0.150	$0.017 + 0.059j$	$0.058 + 0.198j$			
A	25.654	0.139	$0.016 + 0.055j$	$0.053 + 0.184j$			$29.9 + 6.0j$
A	25.793	0.084	$0.010 + 0.033j$	$0.032 + 0.111j$			
A	25.877	0.070	$0.008 + 0.028j$	$0.027 + 0.093j$			$32.5 + 6.1j$
A	25.947	0.046	$0.005 + 0.018j$	$0.018 + 0.061j$	$46.6 + 9.2j$		
A	25.993	0.137	$0.016 + 0.054j$	$0.053 + 0.181j$			
A	26.130	0.046	$0.005 + 0.018j$	$0.017 + 0.061j$	$41.5 + 10.9j$		
A	26.176	0.035	$0.004 + 0.014j$	$0.013 + 0.046j$			
A	26.211	0.153	$0.018 + 0.060j$	$0.059 + 0.202j$			$29.3 + 8.5j$

Zone	Dist. from substation [km]	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
			Z_p	Z_0	S_a	S_b	S_c
A	26.364	0.054	$0.006 + 0.021 j$	$0.021 + 0.071 j$			
A	26.419	0.130	$0.015 + 0.051 j$	$0.050 + 0.172 j$			
A	26.549	0.224	$0.073 + 0.098 j$	$0.133 + 0.306 j$	2		
A	26.773	0.046	$0.015 + 0.020 j$	$0.027 + 0.063 j$			
A	26.826	0.233	$0.076 + 0.102 j$	$0.139 + 0.318 j$			$18.9 + 2.7 j$
A	27.059	0.040	$0.013 + 0.018 j$	$0.024 + 0.055 j$			
A	27.099	0.029	$0.009 + 0.013 j$	$0.017 + 0.040 j$			
A	27.128	0.118	$0.038 + 0.052 j$	$0.070 + 0.161 j$			$47.6 + 12.1 j$
A	27.246	0.060	$0.020 + 0.026 j$	$0.036 + 0.082 j$			
A	27.306	0.060	$0.020 + 0.026 j$	$0.036 + 0.082 j$			
A	27.366	0.103	$0.034 + 0.045 j$	$0.061 + 0.141 j$	$77.2 + 22.4 j$		
A	27.469	0.143	$0.047 + 0.063 j$	$0.085 + 0.195 j$	$15.8 + 4.6 j$		
A	27.612	0.063	$0.021 + 0.028 j$	$0.037 + 0.086 j$			
A	27.675	0.288	$0.094 + 0.126 j$	$0.171 + 0.394 j$			
A	27.963	0.073	$0.024 + 0.032 j$	$0.043 + 0.100 j$		$4.3 + 1.1 j$	
A	28.036	0.185	$0.060 + 0.081 j$	$0.110 + 0.253 j$			$11.5 + 3.3 j$
A	28.221	0.165	$0.054 + 0.072 j$	$0.098 + 0.226 j$		$71.7 + 20.6 j$	
A	28.386	0.077	$0.025 + 0.034 j$	$0.046 + 0.105 j$			$33.5 + 8.4 j$
A	28.463	0.075	$0.025 + 0.035 j$	$0.046 + 0.104 j$			
A	28.538	0.110	$0.036 + 0.048 j$	$0.065 + 0.150 j$	$35.2 + 7.0 j$		
A	28.648	0.040	$0.013 + 0.018 j$	$0.024 + 0.055 j$			
A	28.688	0.077	$0.025 + 0.034 j$	$0.046 + 0.105 j$			
A	28.765	0.014	$0.005 + 0.006 j$	$0.008 + 0.019 j$		2	
A	28.779	0.154	$0.050 + 0.068 j$	$0.091 + 0.211 j$	$21 + 4 j$		
A	28.933	0.147	$0.048 + 0.065 j$	$0.087 + 0.201 j$			$27.5 + 6.1 j$
A	29.080	0.108	$0.035 + 0.047 j$	$0.064 + 0.148 j$		$13.2 + 3.3 j$	
A	29.188	0.050	$0.006 + 0.020 j$	$0.019 + 0.066 j$			
B	23.480	10.500	$1.218 + 4.148 j$	$4.032 + 13.892 j$			
B	33.980	0.058	$0.007 + 0.023 j$	$0.022 + 0.077 j$			
B	34.038	0.006	$0.001 + 0.002 j$	$0.002 + 0.008 j$	$19.2 + 4.8 j$	$19.2 + 4.8 j$	$19.2 + 4.8 j$
B	34.044	0.005	$0.001 + 0.002 j$	$0.002 + 0.007 j$	0.7	0.7	0.7
C	26.418	0.055	$0.006 + 0.022 j$	$0.021 + 0.073 j$			$85.6 + 19.6 j$
C	26.473	0.138	$0.045 + 0.061 j$	$0.082 + 0.189 j$		$28.4 + 5.5 j$	
C	26.611	0.066	$0.022 + 0.029 j$	$0.039 + 0.090 j$	$46.3 + 8.2 j$		
C	26.677	0.128	$0.042 + 0.056 j$	$0.076 + 0.175 j$		$37.6 + 6.2 j$	
C	26.805	0.146	$0.048 + 0.064 j$	$0.087 + 0.200 j$	$93.4 + 21 j$		
C	26.951	0.099	$0.032 + 0.043 j$	$0.059 + 0.135 j$	0.7	0.7	0.7
C	27.050	0.035	$0.011 + 0.015 j$	$0.021 + 0.048 j$	$19.6 + 5.7 j$	$19.6 + 5.7 j$	$19.6 + 5.7 j$
C	27.085	0.080	$0.026 + 0.035 j$	$0.048 + 0.109 j$			
C	27.165	0.050	$0.016 + 0.022 j$	$0.030 + 0.068 j$		2	
C	27.215	0.300	$0.098 + 0.132 j$	$0.178 + 0.410 j$			
C	27.515	0.194	$0.165 + 0.098 j$	$0.235 + 0.304 j$			

C.3 Laterals

Laterals are defined in this analysis as all line sections and components that are not included within the main backbone of the feeder described above. In the scheme used on this feeder as a result of the network reduction guidelines prescribed in Chapter 5, there are 44 laterals.

Table C-3 lists the characteristic parameters of each individual line section throughout the 44 laterals of the feeder. Figure C-4 defines the parameters used in Table C-3 to characterize each section. The parameters used in Table C-3 to describe the lateral sections are as follows:

x_m	Distance from substation of the point where the lateral of the section splits from the main backbone [km]
L	Length of section [km]
Z_p	Positive-sequence impedance of section [Ω]
Z_0	Zero-sequence impedance of section [Ω]
S_a	Complex phase-a load at section [kVA]
S_b	Complex phase-b load at section [kVA]
S_c	Complex phase-c load at section [kVA]

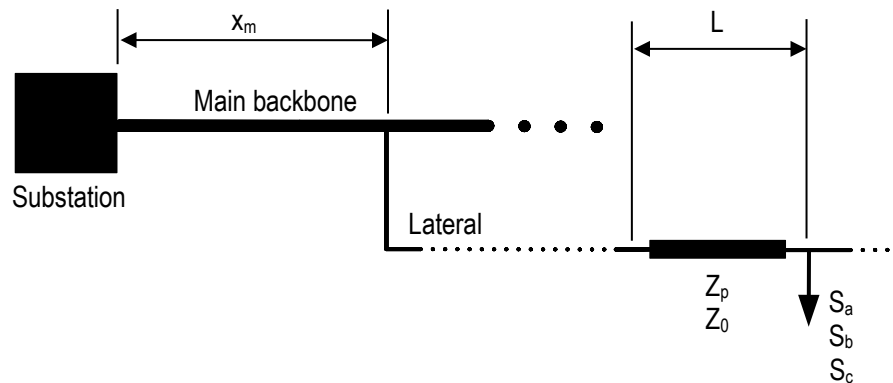


Figure C-4: Line parameter relationships for lateral sections

Figures C-5 and C-6 map the layout of the feeder's laterals, illustrating the relative location of each of the sections described in Table C-3.

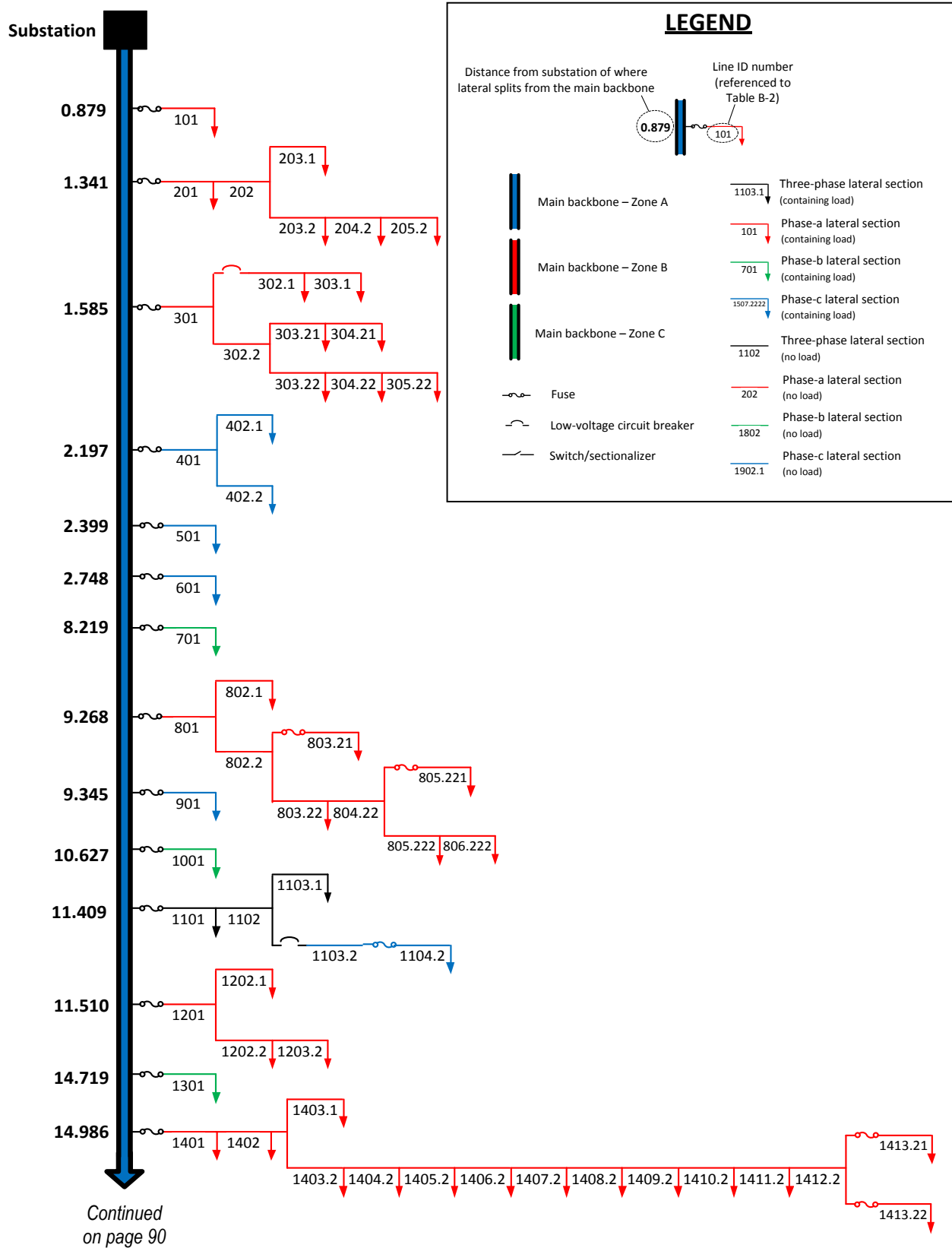


Figure C-5: Laterals along feeder zone A

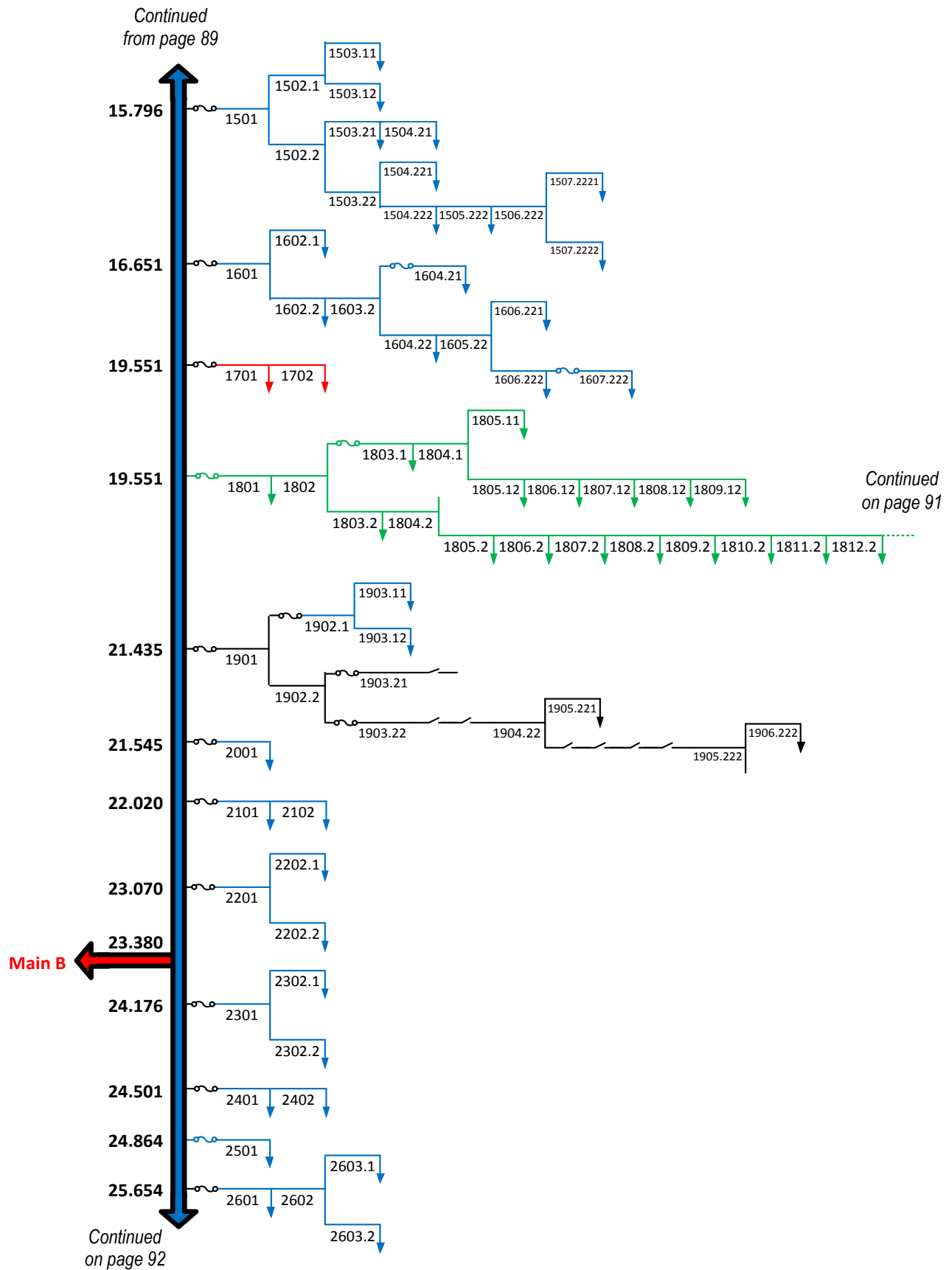


Figure C-5b: Laterals along feeder zone A

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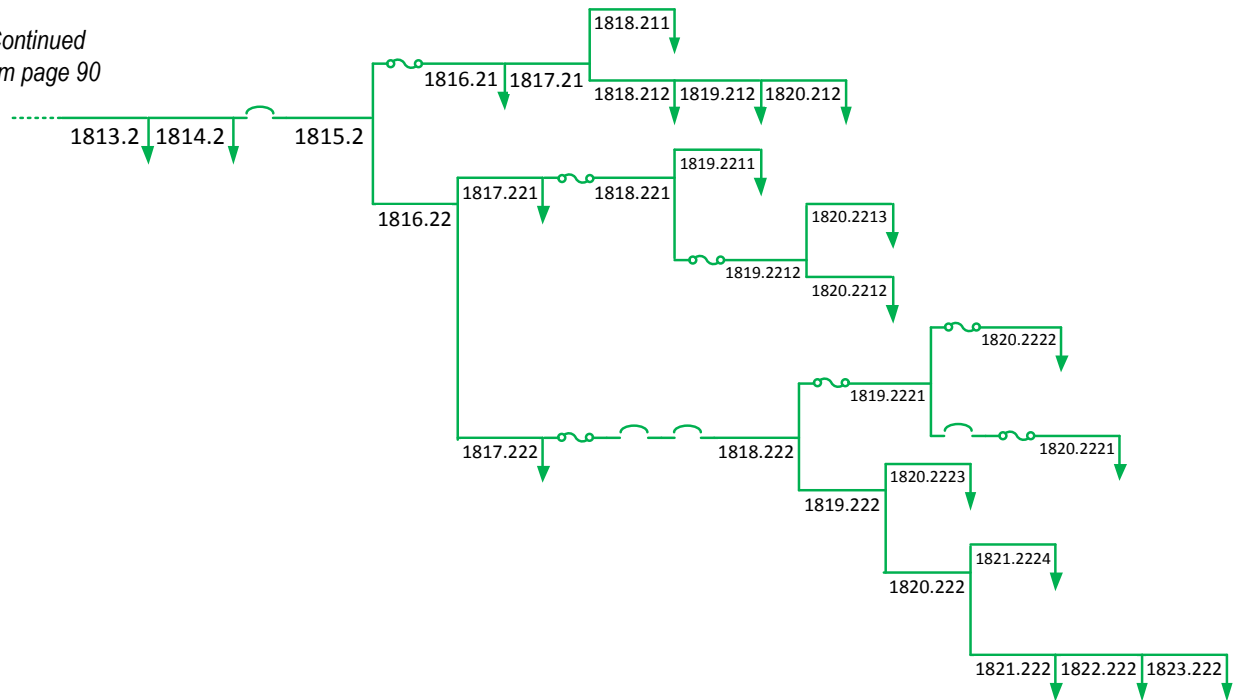
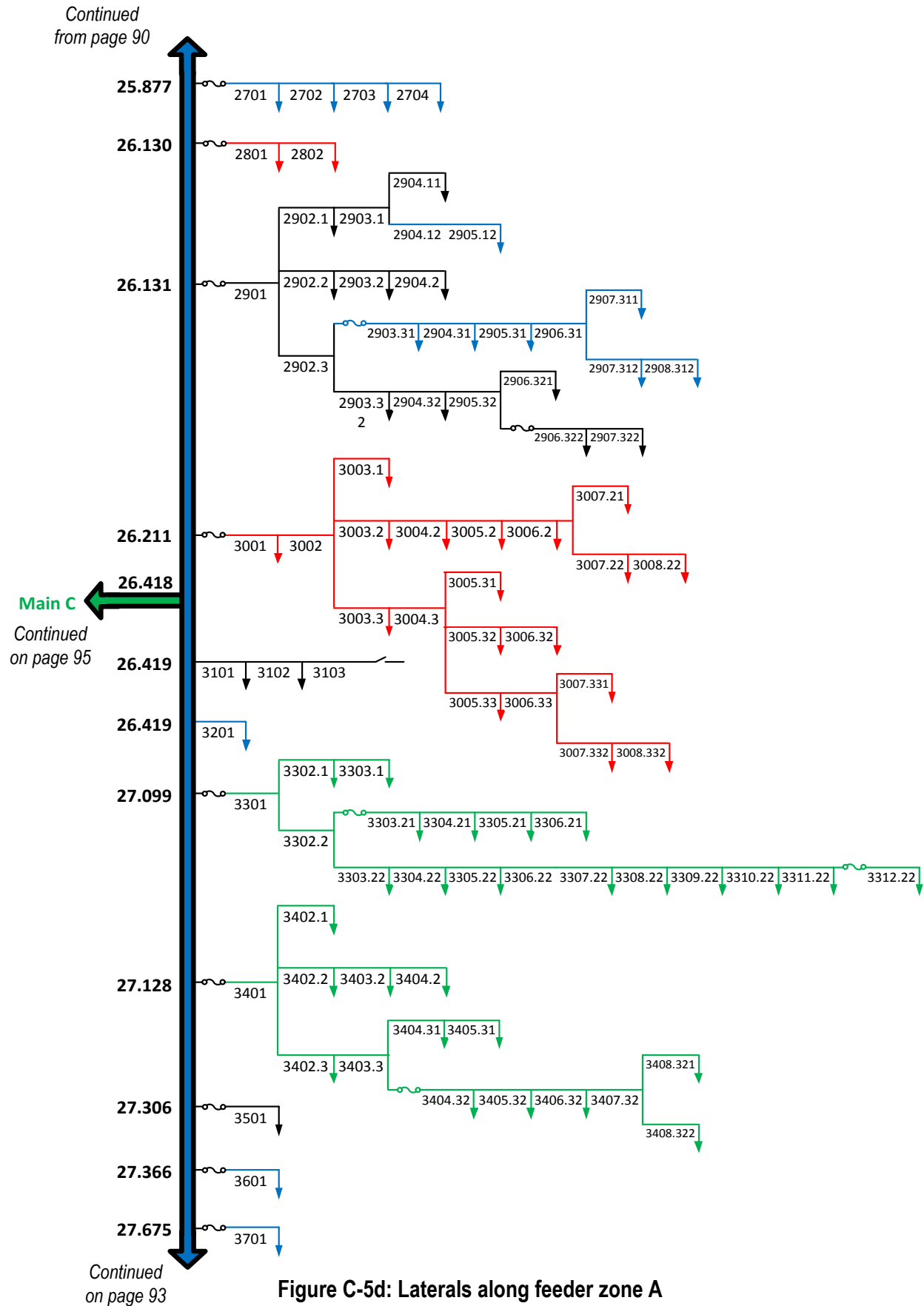
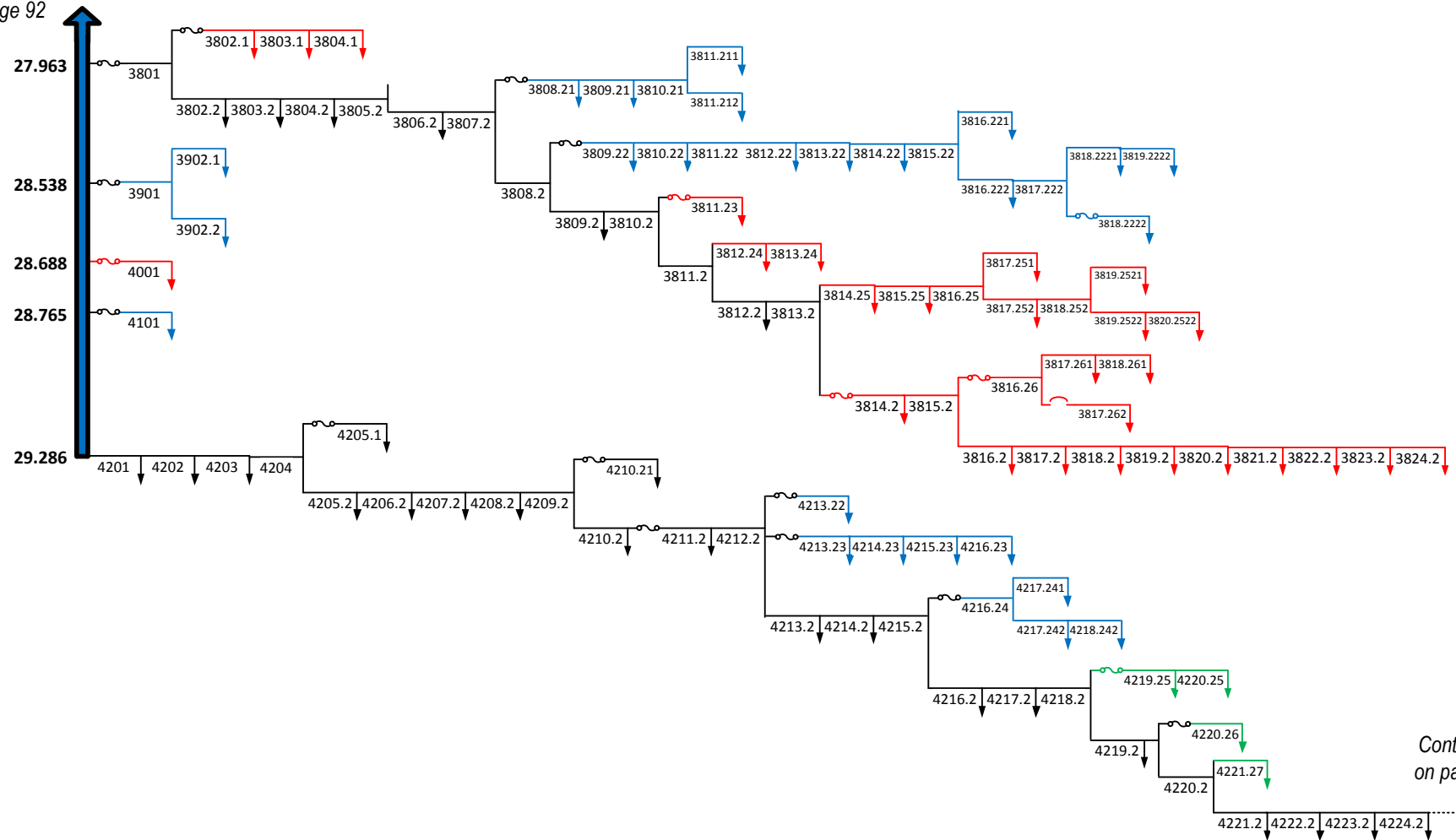


Figure C-5c: Laterals along feeder zone A



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Figure C-5e: Laterals along feeder zone A

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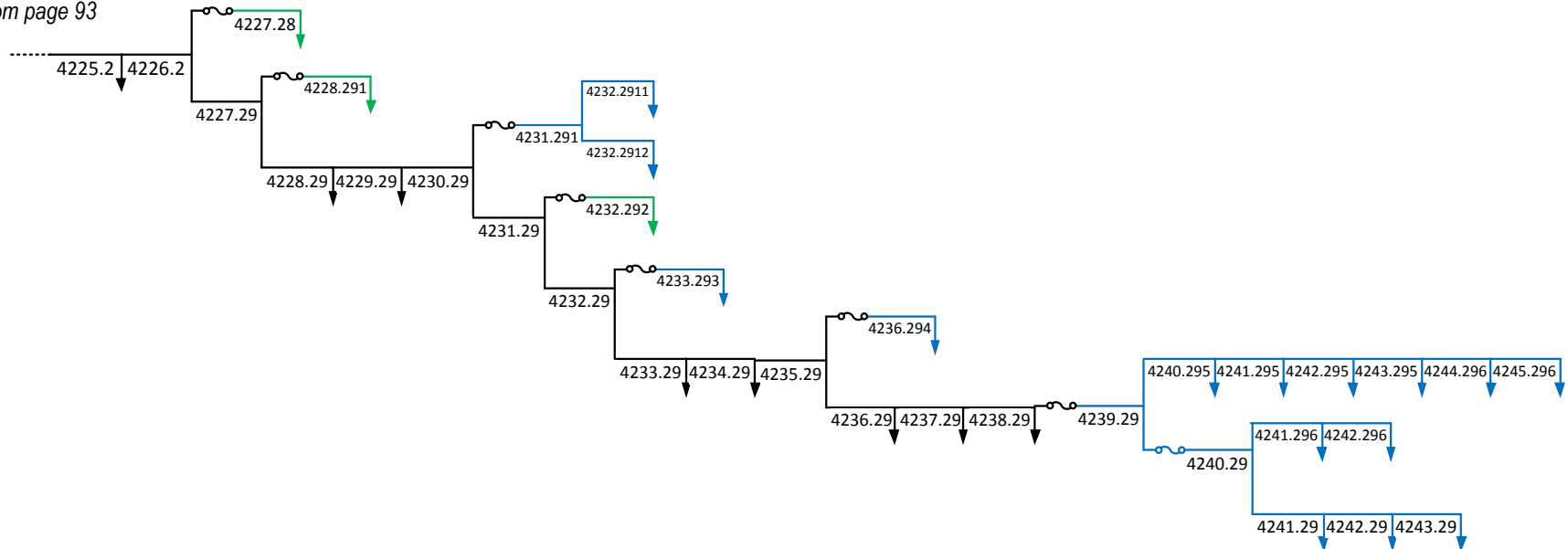


Figure C-5f: Laterals along feeder zone A

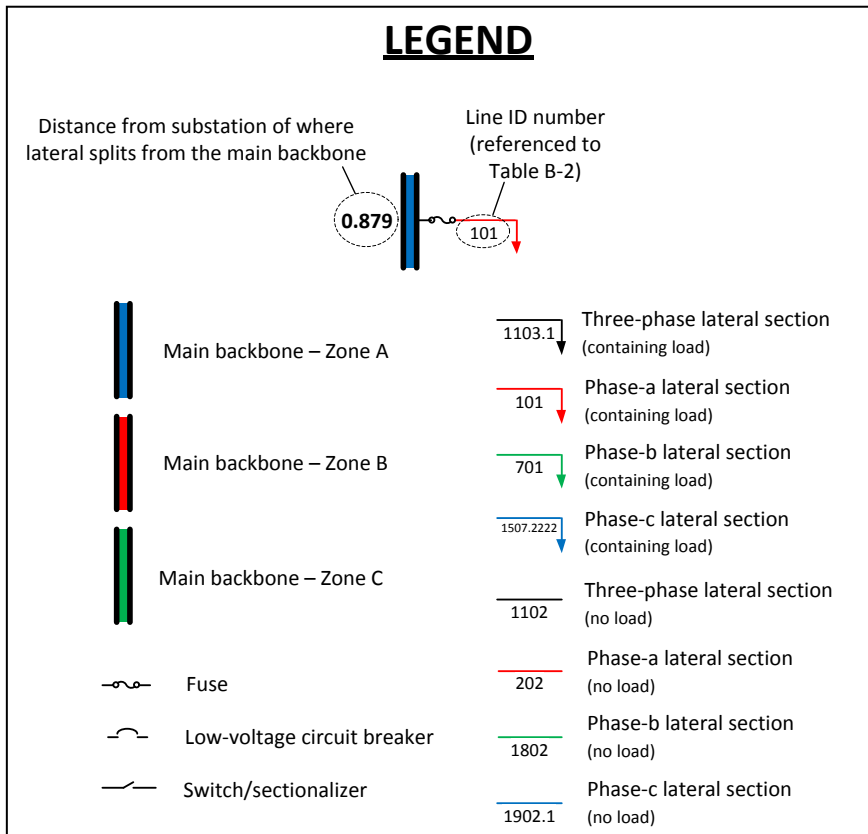
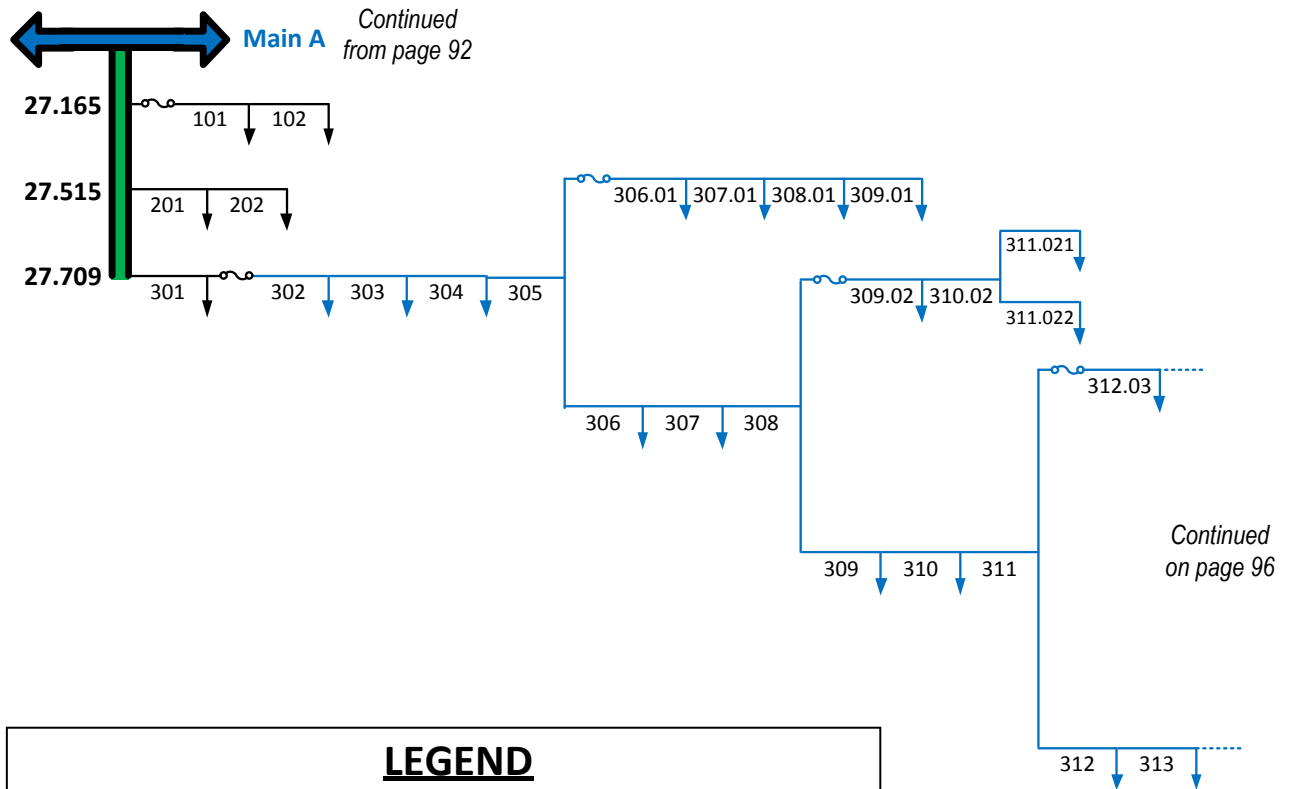


Figure C-6: Laterals along feeder zone C

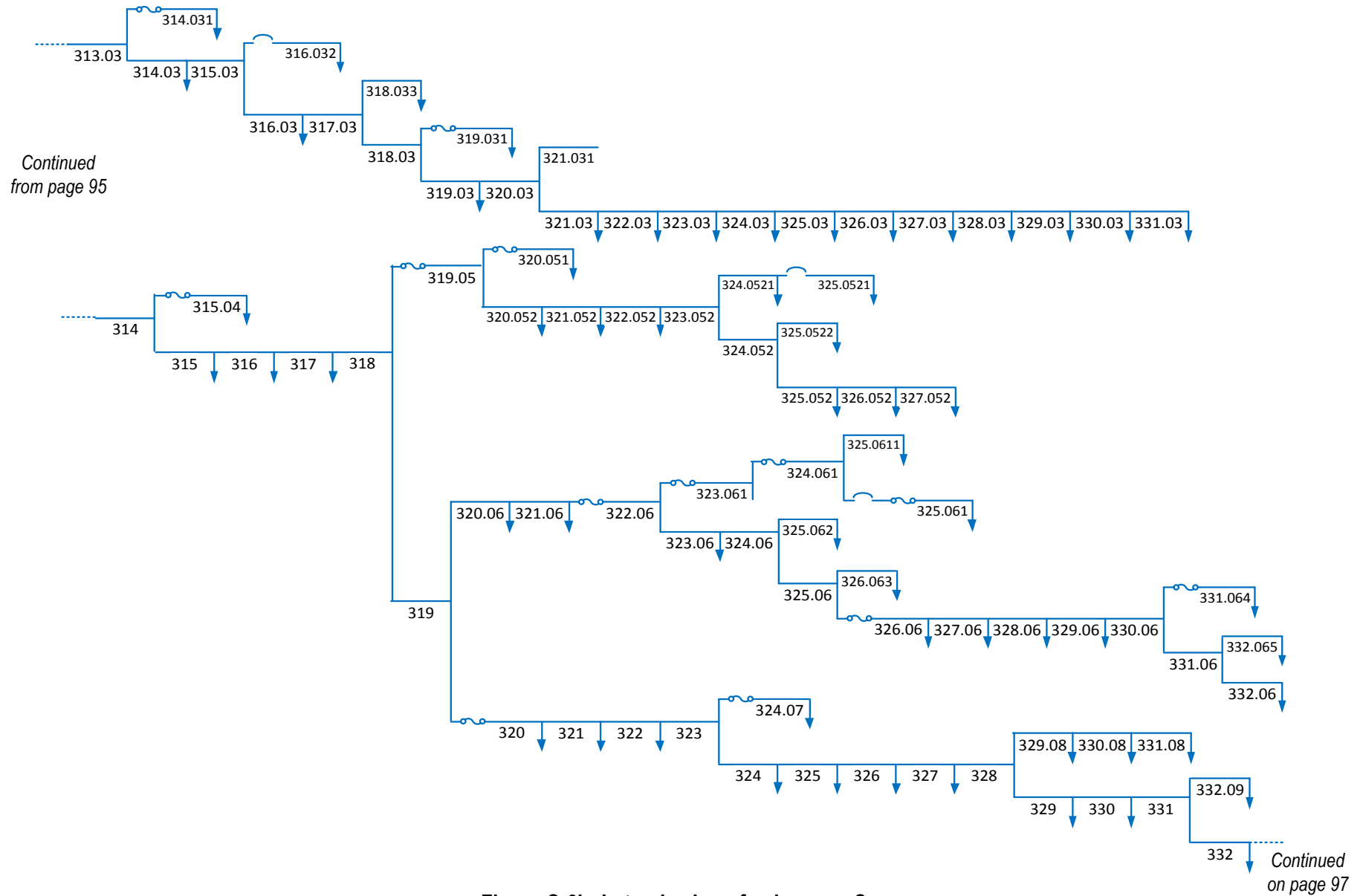


Figure C-6b: Laterals along feeder zone C

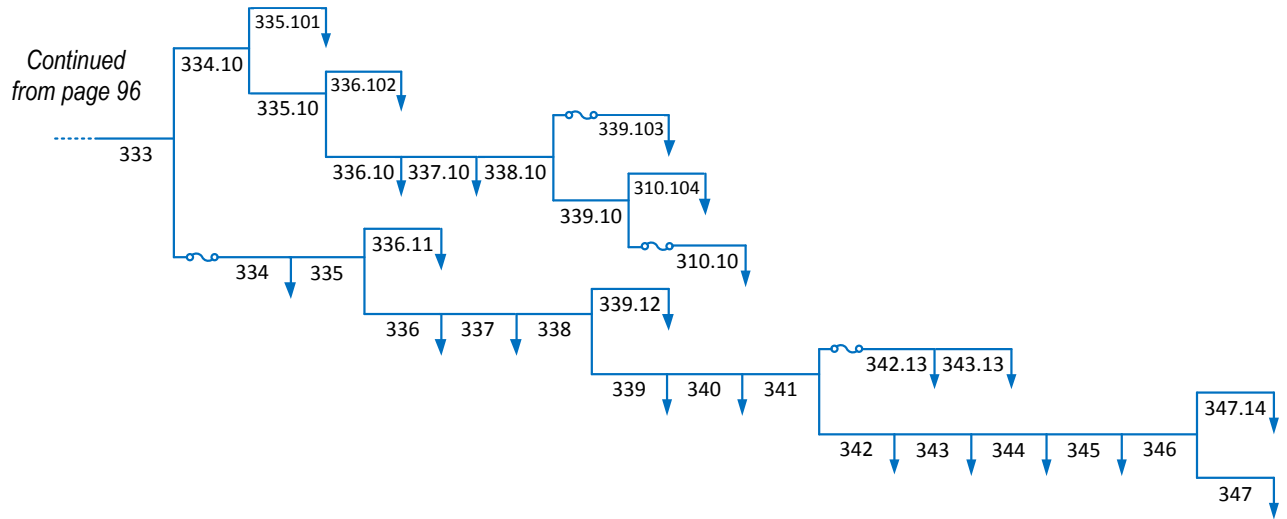


Figure C-6c: Laterals along feeder zone C

Table C-3: Lateral line and load parameters

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	101	0.879	A	0.082	$0.035 + 0.039j$	$0.035 + 0.039j$	$47.5 + 11.9j$		
A	201	1.341	A	0.074	$0.063 + 0.037j$	$0.063 + 0.037j$	$28.9 + 5.0j$		
A	202	1.341	A	0.070	$0.060 + 0.035j$	$0.060 + 0.035j$			
A	203.1	1.341	A	0.033	$0.028 + 0.017j$	$0.028 + 0.017j$	$47.4 + 13.3j$		
A	203.2	1.341	A	0.063	$0.054 + 0.032j$	$0.054 + 0.032j$	$32.2 + 6.4j$		
A	204.2	1.341	A	0.049	$0.042 + 0.025j$	$0.042 + 0.025j$	$35.8 + 6.6j$		
A	205.2	1.341	A	0.052	$0.044 + 0.026j$	$0.044 + 0.026j$	$23.1 + 3.3j$		
A	301	1.585	A	0.048	$0.021 + 0.023j$	$0.021 + 0.023j$			
A	302.1	1.585	A	0.086	$0.037 + 0.041j$	$0.037 + 0.041j$	$50.1 + 8.8j$		
A	302.2	1.585	A	0.082	$0.035 + 0.039j$	$0.035 + 0.039j$			
A	303.1	1.585	A	0.179	$0.077 + 0.085j$	$0.077 + 0.085j$	$32.2 + 7.4j$		
A	303.21	1.585	A	0.052	$0.022 + 0.025j$	$0.022 + 0.025j$	$28.9 + 5.3j$		
A	303.22	1.585	A	0.030	$0.013 + 0.014j$	$0.013 + 0.014j$	$45.4 + 7.7j$		
A	304.21	1.585	A	0.077	$0.033 + 0.037j$	$0.033 + 0.037j$	$51.6 + 10.6j$		
A	304.22	1.585	A	0.162	$0.069 + 0.077j$	$0.069 + 0.077j$	$59.9 + 8.6j$		
A	305.22	1.585	A	0.153	$0.066 + 0.073j$	$0.066 + 0.073j$	$48.9 + 12.7j$		
A	401	2.197	C	0.001	$0.001 + 0.001j$	$0.001 + 0.001j$			
A	402.1	2.197	C	0.246	$0.209 + 0.124j$	$0.209 + 0.124j$			$2.2 + 0.6j$
A	402.2	2.197	C	0.032	$0.014 + 0.015j$	$0.014 + 0.015j$			2
A	501	2.399	C	0.124	$0.014 + 0.049j$	$0.014 + 0.049j$			$3.8 + 1.1j$
A	601	2.748	C	0.156	$0.133 + 0.079j$	$0.133 + 0.079j$			$6.2 + 1.6j$
A	701	8.219	B	0.068	$0.008 + 0.027j$	$0.008 + 0.027j$		2	
A	801	9.268	A	0.001	$0.001 + 0.001j$	$0.001 + 0.001j$			
A	802.1	9.268	A	0.033	$0.028 + 0.017j$	$0.028 + 0.017j$	$43.8 + 10.2j$		
A	802.2	9.268	A	0.243	$0.207 + 0.123j$	$0.207 + 0.123j$			
A	803.21	9.268	A	0.132	$0.112 + 0.067j$	$0.112 + 0.067j$	$4.4 + 1.8j$		
A	803.22	9.268	A	0.424	$0.361 + 0.215j$	$0.361 + 0.215j$	$13.4 + 3.4j$		
A	804.22	9.268	A	0.103	$0.088 + 0.052j$	$0.088 + 0.052j$			
A	805.221	9.268	A	0.300	$0.255 + 0.152j$	$0.255 + 0.152j$	2		
A	805.222	9.268	A	0.358	$0.305 + 0.181j$	$0.305 + 0.181j$	$5.4 + 1.3j$		
A	806.222	9.268	A	0.425	$0.362 + 0.215j$	$0.362 + 0.215j$	$8.8 + 2.2j$		
A	901	9.345	C	0.070	$0.060 + 0.035j$	$0.060 + 0.035j$			$75.3 + 22.0j$
A	1001	10.627	B	0.022	$0.019 + 0.011j$	$0.019 + 0.011j$		$1.3 + 0.4j$	

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	1101	11.409	ABC	0.185	$0.079 + 0.088j$	$0.129 + 0.260j$			$9.8 + 2.4j$
A	1102	11.409	ABC	0.048	$0.021 + 0.023j$	$0.033 + 0.067j$			
A	1103.1	11.409	ABC	0.542	$0.233 + 0.257j$	$0.378 + 0.76j$	0.7	0.7	0.7
A	1103.2	11.409	C	0.041	$0.018 + 0.019j$	$0.018 + 0.019j$			
A	1104.2	11.409	C	0.315	$0.268 + 0.159j$	$0.268 + 0.159j$			$4.3 + 1.1j$
A	1201	11.510	A	0.060	$0.051 + 0.030j$	$0.051 + 0.030j$			
A	1202.1	11.510	A	0.440	$0.374 + 0.223j$	$0.374 + 0.223j$	$7.7 + 3.0j$		
A	1202.2	11.510	A	0.150	$0.128 + 0.076j$	$0.128 + 0.076j$	$12. + 2.9j$		
A	1203.2	11.510	A	0.040	$0.034 + 0.020j$	$0.034 + 0.020j$	$6.2 + 1.8j$		
A	1301	14.719	B	0.080	$0.033 + 0.036j$	$0.033 + 0.036j$		$6.6 + 2.6j$	
A	1401	14.986	A	0.035	$0.015 + 0.017j$	$0.015 + 0.017j$	2		
A	1402	14.986	A	0.400	$0.172 + 0.190j$	$0.172 + 0.190j$	$26.9 + 4.3j$		
A	1403.1	14.986	A	0.065	$0.028 + 0.031j$	$0.028 + 0.031j$	$11.6 + 2.9j$		
A	1403.2	14.986	A	0.100	$0.043 + 0.048j$	$0.043 + 0.048j$	$19. + 4.5j$		
A	1404.2	14.986	A	0.150	$0.064 + 0.071j$	$0.064 + 0.071j$	$23.9 + 4.0j$		
A	1405.2	14.986	A	0.200	$0.086 + 0.095j$	$0.086 + 0.095j$	$3.9 + 0.6j$		
A	1406.2	14.986	A	0.120	$0.051 + 0.057j$	$0.051 + 0.057j$	$3.0 + 0.8j$		
A	1407.2	14.986	A	0.130	$0.056 + 0.062j$	$0.056 + 0.062j$	$3.4 + 0.8j$		
A	1408.2	14.986	A	0.275	$0.118 + 0.131j$	$0.118 + 0.131j$	$8.7 + 2.2j$		
A	1409.2	14.986	A	0.100	$0.043 + 0.048j$	$0.043 + 0.048j$	$26.5 + 7.8j$		
A	1410.2	14.986	A	0.200	$0.086 + 0.095j$	$0.086 + 0.095j$	$30.6 + 6.4j$		
A	1411.2	14.986	A	0.500	$0.215 + 0.238j$	$0.215 + 0.238j$	$7.8 + 2.0j$		
A	1412.2	14.986	A	0.375	$0.319 + 0.190j$	$0.319 + 0.190j$			
A	1413.21	14.986	A	0.072	$0.061 + 0.036j$	$0.061 + 0.036j$	$5.9 + 2.3j$		
A	1413.22	14.986	A	0.098	$0.083 + 0.050j$	$0.083 + 0.050j$	$26.4 + 3.8j$		
A	1501	15.796	C	0.160	$0.136 + 0.081j$	$0.136 + 0.081j$			
A	1502.1	15.796	C	0.199	$0.169 + 0.101j$	$0.169 + 0.101j$			
A	1502.2	15.796	C	0.001	$0.001 + 0.001j$	$0.001 + 0.001j$			
A	1503.11	15.796	C	0.285	$0.243 + 0.144j$	$0.243 + 0.144j$			$9.9 + 2.5j$
A	1503.12	15.796	C	0.200	$0.170 + 0.101j$	$0.170 + 0.101j$			2
A	1503.21	15.796	C	0.147	$0.125 + 0.074j$	$0.125 + 0.074j$			$3.0 + 0.8j$
A	1503.22	15.796	C	0.199	$0.085 + 0.095j$	$0.085 + 0.095j$			
A	1504.21	15.796	C	0.113	$0.096 + 0.057j$	$0.096 + 0.057j$			$8.6 + 1.2j$
A	1504.221	15.796	C	0.118	$0.051 + 0.056j$	$0.051 + 0.056j$			$30.4 + 8.8j$
A	1504.222	15.796	C	0.091	$0.077 + 0.046j$	$0.077 + 0.046j$			2

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	1505.222	15.796	C	0.172	$0.146 + 0.087 j$	$0.146 + 0.087 j$			$3.0 + 0.8 j$
A	1506.222	15.796	C	0.001	$0.001 + 0.001 j$	$0.001 + 0.001 j$			
A	1507.2221	15.796	C	0.188	$0.160 + 0.095 j$	$0.160 + 0.095 j$			$15.2 + 3.6 j$
A	1507.2222	15.796	C	0.187	$0.159 + 0.095 j$	$0.159 + 0.095 j$			2
A	1601	16.651	C	0.589	$0.253 + 0.280 j$	$0.253 + 0.280 j$			
A	1602.1	16.651	C	0.043	$0.018 + 0.020 j$	$0.018 + 0.020 j$			2
A	1602.2	16.651	C	1.170	$0.502 + 0.556 j$	$0.502 + 0.556 j$			$8.3 + 1.2 j$
A	1603.2	16.651	C	0.077	$0.066 + 0.039 j$	$0.066 + 0.039 j$			
A	1604.21	16.651	C	0.830	$0.706 + 0.420 j$	$0.706 + 0.420 j$			$5.8 + 1.5 j$
A	1604.22	16.651	C	0.299	$0.254 + 0.151 j$	$0.254 + 0.151 j$			2
A	1605.22	16.651	C	0.745	$0.634 + 0.377 j$	$0.634 + 0.377 j$			
A	1606.221	16.651	C	0.093	$0.079 + 0.047 j$	$0.079 + 0.047 j$			$3.0 + 0.4 j$
A	1606.222	16.651	C	0.832	$0.708 + 0.421 j$	$0.708 + 0.421 j$			$1.7 + 0.2 j$
A	1607.222	16.651	C	1.474	$0.632 + 0.700 j$	$0.632 + 0.700 j$			$13.8 + 2.5 j$
A	1701	19.551	A	0.040	$0.034 + 0.020 j$	$0.034 + 0.020 j$	$10.2 + 1.5 j$		
A	1702	19.551	A	0.200	$0.170 + 0.101 j$	$0.170 + 0.101 j$	$11.7 + 4.6 j$		
A	1801	19.551	B	0.095	$0.081 + 0.048 j$	$0.081 + 0.048 j$		$6.6 + 1.7 j$	
A	1802	19.551	B	0.850	$0.723 + 0.430 j$	$0.723 + 0.430 j$			
A	1803.1	19.551	B	0.091	$0.039 + 0.043 j$	$0.039 + 0.043 j$		2	
A	1803.2	19.551	B	0.655	$0.557 + 0.331 j$	$0.557 + 0.331 j$		$8.2 + 3.2 j$	
A	1804.1	19.551	B	0.232	$0.100 + 0.110 j$	$0.100 + 0.110 j$			
A	1804.2	19.551	B	0.450	$0.383 + 0.228 j$	$0.383 + 0.228 j$			
A	1805.11	19.551	B	0.119	$0.051 + 0.057 j$	$0.051 + 0.057 j$		$4.9 + 1.2 j$	
A	1805.12	19.551	B	1.012	$0.434 + 0.481 j$	$0.434 + 0.481 j$		$5.9 + 1.5 j$	
A	1805.2	19.551	B	1.200	$1.021 + 0.607 j$	$1.021 + 0.607 j$		1	
A	1806.12	19.551	B	0.814	$0.349 + 0.387 j$	$0.349 + 0.387 j$		$8.3 + 3.3 j$	
A	1806.2	19.551	B	0.237	$0.202 + 0.120 j$	$0.202 + 0.120 j$		$3.6 + 0.9 j$	
A	1807.12	19.551	B	0.075	$0.032 + 0.036 j$	$0.032 + 0.036 j$		$16.5 + 2.4 j$	
A	1807.2	19.551	B	0.388	$0.330 + 0.196 j$	$0.330 + 0.196 j$		$2.4 + 0.9 j$	
A	1808.12	19.551	B	0.230	$0.099 + 0.109 j$	$0.099 + 0.109 j$		$9.8 + 3.9 j$	
A	1808.2	19.551	B	0.650	$0.553 + 0.329 j$	$0.553 + 0.329 j$		$8.6 + 3.4 j$	
A	1809.12	19.551	B	0.972	$0.417 + 0.462 j$	$0.417 + 0.462 j$		$4.6 + 0.7 j$	
A	1809.2	19.551	B	0.150	$0.128 + 0.076 j$	$0.128 + 0.076 j$		2	
A	1810.2	19.551	B	0.250	$0.213 + 0.127 j$	$0.213 + 0.127 j$		$20.6 + 2.9 j$	
A	1811.2	19.551	B	0.510	$0.219 + 0.242 j$	$0.219 + 0.242 j$		$5.9 + 1.5 j$	

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	1812.2	19.551	B	0.506	$0.217 + 0.240 j$	$0.217 + 0.240 j$		2	
A	1813.2	19.551	B	0.351	$0.151 + 0.167 j$	$0.151 + 0.167 j$		$17.9 + 3.4 j$	
A	1814.2	19.551	B	0.171	$0.146 + 0.087 j$	$0.146 + 0.087 j$		$5.8 + 2.3 j$	
A	1815.2	19.551	B	0.690	$0.587 + 0.349 j$	$0.587 + 0.349 j$			
A	1816.21	19.551	B	0.275	$0.234 + 0.139 j$	$0.234 + 0.139 j$		2	
A	1816.22	19.551	B	0.079	$0.067 + 0.040 j$	$0.067 + 0.040 j$			
A	1817.21	19.551	B	0.056	$0.048 + 0.028 j$	$0.048 + 0.028 j$			
A	1817.221	19.551	B	0.001	$0.001 + 0.001 j$	$0.001 + 0.001 j$		$24.0 + 3.4 j$	
A	1817.222	19.551	B	0.185	$0.157 + 0.094 j$	$0.157 + 0.094 j$		$7.5 + 1.9 j$	
A	1818.211	19.551	B	0.055	$0.047 + 0.028 j$	$0.047 + 0.028 j$		$22.3 + 3.4 j$	
A	1818.212	19.551	B	0.123	$0.105 + 0.062 j$	$0.105 + 0.062 j$		$4.8 + 1.2 j$	
A	1818.221	19.551	B	0.700	$0.596 + 0.354 j$	$0.596 + 0.354 j$			
A	1818.222	19.551	B	0.099	$0.084 + 0.050 j$	$0.084 + 0.050 j$			
A	1819.212	19.551	B	0.087	$0.037 + 0.041 j$	$0.037 + 0.041 j$		2	
A	1819.2211	19.551	B	0.250	$0.325 + 0.130 j$	$0.325 + 0.130 j$		$7.7 + 1.9 j$	
A	1819.2212	19.551	B	0.299	$0.254 + 0.151 j$	$0.254 + 0.151 j$			
A	1819.222	19.551	B	0.879	$0.748 + 0.445 j$	$0.748 + 0.445 j$			
A	1819.2221	19.551	B	0.001	$0.001 + 0.001 j$	$0.001 + 0.001 j$			
A	1820.212	19.551	B	0.286	$0.123 + 0.136 j$	$0.123 + 0.136 j$		$27.7 + 3.9 j$	
A	1820.2212	19.551	B	0.125	$0.106 + 0.063 j$	$0.106 + 0.063 j$		$7.0 + 2.8 j$	
A	1820.2213	19.551	B	0.099	$0.084 + 0.050 j$	$0.084 + 0.050 j$		$9.2 + 2.3 j$	
A	1820.222	19.551	B	0.957	$0.814 + 0.484 j$	$0.814 + 0.484 j$			
A	1820.2221	19.551	B	0.119	$0.101 + 0.060 j$	$0.101 + 0.060 j$		$13.1 + 5.2 j$	
A	1820.2222	19.551	B	0.331	$0.282 + 0.167 j$	$0.282 + 0.167 j$		$4.6 + 1.8 j$	
A	1820.2223	19.551	B	0.059	$0.050 + 0.030 j$	$0.050 + 0.030 j$		2	
A	1821.222	19.551	B	0.105	$0.089 + 0.053 j$	$0.089 + 0.053 j$		$23.4 + 3.3 j$	
A	1821.2224	19.551	B	0.150	$0.128 + 0.076 j$	$0.128 + 0.076 j$		$20.6 + 2.9 j$	
A	1822.222	19.551	B	0.111	$0.048 + 0.053 j$	$0.048 + 0.053 j$		2	
A	1823.222	19.551	B	0.122	$0.052 + 0.058 j$	$0.052 + 0.058 j$		2	
A	1901	21.435	ABC	1.012	$0.117 + 0.400 j$	$0.389 + 1.339 j$			
A	1902.1	21.435	C	0.095	$0.081 + 0.048 j$	$0.081 + 0.048 j$			
A	1902.2	21.435	ABC	0.761	$0.088 + 0.301 j$	$0.292 + 1.007 j$			
A	1903.11	21.435	C	0.230	$0.196 + 0.116 j$	$0.196 + 0.116 j$			$10.7 + 2.3 j$
A	1903.12	21.435	C	0.086	$0.073 + 0.044 j$	$0.073 + 0.044 j$			$3.7 + 1.5 j$
A	1903.21	21.435	ABC	0.075	$0.033 + 0.012 j$	$0.103 + 0.043 j$			

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	1903.22	21.435	ABC	0.075	$0.033 + 0.012j$	$0.103 + 0.043j$			
A	1904.22	21.435	ABC	0.001	$0.001 + 0.001j$	$0.001 + 0.001j$			
A	1905.221	21.435	ABC	0.001	$0.001 + 0.001j$	$0.001 + 0.001j$	$46.4 + 13.5j$	$46.4 + 13.5j$	$46.4 + 13.5j$
A	1905.222	21.435	ABC	0.001	$0.001 + 0.001j$	$0.001 + 0.001j$			
A	1906.222	21.435	ABC	0.001	$0.001 + 0.001j$	$0.001 + 0.001j$	0.7	0.7	0.7
A	2001	21.545	C	0.060	$0.051 + 0.030j$	$0.051 + 0.030j$			$32.4 + 12.8j$
A	2101	22.020	C	0.325	$0.277 + 0.164j$	$0.277 + 0.164j$			1
A	2102	22.020	C	0.175	$0.149 + 0.089j$	$0.149 + 0.089j$			2
A	2201	23.070	C	0.298	$0.254 + 0.151j$	$0.254 + 0.151j$			
A	2202.1	23.070	C	0.360	$0.306 + 0.182j$	$0.306 + 0.182j$			$9.4 + 2.4j$
A	2202.2	23.070	C	0.058	$0.049 + 0.029j$	$0.049 + 0.029j$			$3.8 + 0.90j$
A	2301	24.176	C	0.159	$0.135 + 0.080j$	$0.135 + 0.080j$			
A	2302.1	24.176	C	0.037	$0.031 + 0.019j$	$0.031 + 0.019j$			$17.4 + 2.5j$
A	2302.2	24.176	C	0.075	$0.064 + 0.038j$	$0.064 + 0.038j$			$26.8 + 5.2j$
A	2401	24.501	C	0.070	$0.060 + 0.035j$	$0.060 + 0.035j$			$3.7 + 0.90j$
A	2402	24.501	C	0.287	$0.244 + 0.145j$	$0.244 + 0.145j$			1
A	2501	24.864	C	0.232	$0.10 + 0.11j$	$0.10 + 0.11j$			2
A	2601	25.654	C	0.079	$0.067 + 0.040j$	$0.067 + 0.040j$			$21.3 + 6.2j$
A	2602	25.654	C	0.154	$0.131 + 0.078j$	$0.131 + 0.078j$			
A	2603.1	25.654	C	0.229	$0.195 + 0.116j$	$0.195 + 0.116j$			$15.1 + 2.1j$
A	2603.2	25.654	C	0.155	$0.132 + 0.078j$	$0.132 + 0.078j$			$13.4 + 1.9j$
A	2701	25.877	C	0.085	$0.072 + 0.043j$	$0.072 + 0.043j$			$52. + 10.8j$
A	2702	25.877	C	0.165	$0.14 + 0.083j$	$0.14 + 0.083j$			$28.5 + 7.2j$
A	2703	25.877	C	0.057	$0.049 + 0.029j$	$0.049 + 0.029j$			$29.5 + 7.4j$
A	2704	25.877	C	0.157	$0.134 + 0.079j$	$0.134 + 0.079j$			$6.5 + 0.9j$
A	2801	26.130	A	0.040	$0.034 + 0.020j$	$0.034 + 0.020j$	$50.0 + 14.2j$		
A	2802	26.130	A	0.038	$0.032 + 0.019j$	$0.032 + 0.019j$	$28.5 + 5.6j$		
A	2901	26.131	ABC	0.060	$0.051 + 0.030j$	$0.073 + 0.094j$			
A	2902.1	26.131	ABC	0.074	$0.063 + 0.037j$	$0.090 + 0.116j$			2
A	2902.2	26.131	ABC	0.012	$0.010 + 0.0060j$	$0.015 + 0.019j$			$45.0 + 9.6j$
A	2902.3	26.131	ABC	0.200	$0.17 + 0.101j$	$0.242 + 0.313j$			
A	2903.1	26.131	ABC	0.014	$0.012 + 0.0070j$	$0.017 + 0.022j$			
A	2903.2	26.131	ABC	0.067	$0.057 + 0.034j$	$0.081 + 0.105j$	$96.7 + 28.2j$	$96.7 + 28.2j$	$96.7 + 28.2j$
A	2903.31	26.131	C	0.050	$0.043 + 0.025j$	$0.043 + 0.025j$			$28.9 + 6.5j$
A	2903.32	26.131	ABC	0.045	$0.038 + 0.023j$	$0.054 + 0.070j$	$8.7 + 1.5j$	$8.7 + 1.5j$	$8.7 + 1.5j$

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	2904.11	26.131	ABC	0.011	0.0090 + 0.0060 j	0.013 + 0.017 j	12.4 + 3.1 j	12.4 + 3.1 j	12.4 + 3.1 j
A	2904.12	26.131	C	0.001	0.0010 + 0.0010 j	0.0010 + 0.0010 j			
A	2904.2	26.131	ABC	0.032	0.027 + 0.016 j	0.039 + 0.050 j		65.8 + 14. j	
A	2904.31	26.131	C	0.093	0.079 + 0.047 j	0.079 + 0.047 j			93.9 + 18.1 j
A	2904.32	26.131	ABC	0.123	0.105 + 0.062 j	0.149 + 0.193 j		72.8 + 18.9 j	
A	2905.12	26.131	C	0.044	0.037 + 0.022 j	0.037 + 0.022 j			40.1 + 9.3 j
A	2905.31	26.131	C	0.136	0.116 + 0.069 j	0.116 + 0.069 j			36.8 + 7.2 j
A	2905.32	26.131	ABC	0.102	0.087 + 0.052 j	0.124 + 0.16 j			
A	2906.31	26.131	C	0.055	0.047 + 0.028 j	0.047 + 0.028 j			
A	2906.321	26.131	ABC	0.039	0.033 + 0.020 j	0.047 + 0.061 j			32.5 + 5.6 j
A	2906.322	26.131	ABC	0.178	0.076 + 0.085 j	0.124 + 0.250 j			2
A	2907.311	26.131	C	0.058	0.049 + 0.029 j	0.049 + 0.029 j			33.8 + 5.2 j
A	2907.312	26.131	C	0.078	0.066 + 0.039 j	0.066 + 0.039 j			39.4 + 7.3 j
A	2907.322	26.131	ABC	0.066	0.028 + 0.031 j	0.046 + 0.093 j	33.9 + 3.4 j	33.9 + 3.4 j	33.9 + 3.4 j
A	2908.312	26.131	C	0.250	0.213 + 0.127 j	0.213 + 0.127 j			21.1 + 3.0 j
A	3001	26.211	A	0.075	0.064 + 0.038 j	0.064 + 0.038 j	16.7 + 4.2 j		
A	3002	26.211	A	0.030	0.026 + 0.015 j	0.026 + 0.015 j			
A	3003.1	26.211	A	0.125	0.106 + 0.063 j	0.106 + 0.063 j	50.6 + 11.6 j		
A	3003.2	26.211	A	0.050	0.043 + 0.025 j	0.043 + 0.025 j	53.1 + 9.3 j		
A	3003.3	26.211	A	0.065	0.028 + 0.031 j	0.028 + 0.031 j	67.3 + 12. j		
A	3004.2	26.211	A	0.150	0.128 + 0.076 j	0.128 + 0.076 j	37.5 + 5.8 j		
A	3004.3	26.211	A	0.099	0.042 + 0.047 j	0.042 + 0.047 j			
A	3005.2	26.211	A	0.100	0.085 + 0.051 j	0.085 + 0.051 j	79.4 + 12.6 j		
A	3005.31	26.211	A	0.090	0.077 + 0.046 j	0.077 + 0.046 j	26.9 + 4.4 j		
A	3005.32	26.211	A	0.175	0.149 + 0.089 j	0.149 + 0.089 j	19.3 + 2.8 j		
A	3005.33	26.211	A	0.001			53.7 + 10.6 j		
A	3006.2	26.211	A	0.050	0.043 + 0.025 j	0.043 + 0.025 j	3.0 + 0.8 j		
A	3006.32	26.211	A	0.075	0.064 + 0.038 j	0.064 + 0.038 j	18.4 + 4.6 j		
A	3006.33	26.211	A	0.070	0.060 + 0.035 j	0.060 + 0.035 j			
A	3007.21	26.211	A	0.079	0.067 + 0.040 j	0.067 + 0.040 j	45.2 + 8.0 j		
A	3007.22	26.211	A	0.044	0.037 + 0.022 j	0.037 + 0.022 j	15.4 + 2.2 j		
A	3007.331	26.211	A	0.103	0.088 + 0.052 j	0.088 + 0.052 j	42.6 + 7.1 j		
A	3007.332	26.211	A	0.051	0.043 + 0.026 j	0.043 + 0.026 j	61.1 + 8.9 j		
A	3008.22	26.211	A	0.110	0.094 + 0.056 j	0.094 + 0.056 j	20.1 + 4.1 j		
A	3008.332	26.211	A	0.115	0.098 + 0.058 j	0.098 + 0.058 j	32.4 + 7.1 j		

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	3101	26.419	ABC	0.074	$0.024 + 0.032 j$	$0.044 + 0.101 j$			$27.8 + 5.0 j$
A	3102	26.419	ABC	0.055	$0.018 + 0.024 j$	$0.033 + 0.075 j$			$21.3 + 5.3 j$
A	3103	26.419	ABC	0.042	$0.014 + 0.018 j$	$0.025 + 0.057 j$			
A	3201	26.419	C	0.070	$0.060 + 0.035 j$	$0.060 + 0.035 j$			$14.2 + 4.1 j$
A	3301	27.099	B	0.061	$0.052 + 0.031 j$	$0.052 + 0.031 j$			
A	3302.1	27.099	B	0.061	$0.052 + 0.031 j$	$0.052 + 0.031 j$		$61.6 + 9.2 j$	
A	3302.2	27.099	B	0.092	$0.078 + 0.047 j$	$0.078 + 0.047 j$			
A	3303.1	27.099	B	0.058	$0.049 + 0.029 j$	$0.049 + 0.029 j$		$28.3 + 7.1 j$	
A	3303.21	27.099	B	0.052	$0.044 + 0.026 j$	$0.044 + 0.026 j$		2	
A	3303.22	27.099	B	0.046	$0.039 + 0.023 j$	$0.039 + 0.023 j$		$32.4 + 6.8 j$	
A	3304.21	27.099	B	0.170	$0.145 + 0.086 j$	$0.145 + 0.086 j$		$27.1 + 3.9 j$	
A	3304.22	27.099	B	0.105	$0.089 + 0.053 j$	$0.089 + 0.053 j$		$33.2 + 6.5 j$	
A	3305.21	27.099	B	0.050	$0.043 + 0.025 j$	$0.043 + 0.025 j$		$27.1 + 5.4 j$	
A	3305.22	27.099	B	0.055	$0.047 + 0.028 j$	$0.047 + 0.028 j$		$40.6 + 7.9 j$	
A	3306.21	27.099	B	0.051	$0.043 + 0.026 j$	$0.043 + 0.026 j$		$21.9 + 4.0 j$	
A	3306.22	27.099	B	0.028	$0.024 + 0.014 j$	$0.024 + 0.014 j$			
A	3307.22	27.099	B	0.071	$0.060 + 0.036 j$	$0.060 + 0.036 j$		$42. + 6.0 j$	
A	3308.22	27.099	B	0.063	$0.054 + 0.032 j$	$0.054 + 0.032 j$		$7.5 + 1.9 j$	
A	3309.22	27.099	B	0.037	$0.031 + 0.019 j$	$0.031 + 0.019 j$		$29.6 + 6.7 j$	
A	3310.22	27.099	B	0.070	$0.060 + 0.035 j$	$0.060 + 0.035 j$		$15.2 + 2.2 j$	
A	3311.22	27.099	B	0.030	$0.026 + 0.015 j$	$0.026 + 0.015 j$		$40.2 + 7.7 j$	
A	3312.22	27.099	B	0.054	$0.046 + 0.027 j$	$0.046 + 0.027 j$		$17.7 + 3.0 j$	
A	3401	27.128	B	0.060	$0.020 + 0.026 j$	$0.020 + 0.026 j$			
A	3402.1	27.128	B	0.067	$0.057 + 0.034 j$	$0.057 + 0.034 j$		$87.4 + 16.8 j$	
A	3402.2	27.128	B	0.047	$0.040 + 0.024 j$	$0.040 + 0.024 j$		$16.3 + 4.6 j$	
A	3402.3	27.128	B	0.080	$0.068 + 0.040 j$	$0.068 + 0.040 j$		$18.4 + 2.6 j$	
A	3403.2	27.128	B	0.114	$0.097 + 0.058 j$	$0.097 + 0.058 j$		$84.9 + 20. j$	
A	3403.3	27.128	B	0.080	$0.068 + 0.040 j$	$0.068 + 0.040 j$			
A	3404.2	27.128	B	0.155	$0.132 + 0.078 j$	$0.132 + 0.078 j$		$24.3 + 7.1 j$	
A	3404.31	27.128	B	0.051	$0.043 + 0.026 j$	$0.043 + 0.026 j$		$3.0 + 0.8 j$	
A	3404.32	27.128	B	0.066	$0.056 + 0.033 j$	$0.056 + 0.033 j$		$28.6 + 6.1 j$	
A	3405.31	27.128	B	0.055	$0.047 + 0.028 j$	$0.047 + 0.028 j$		$15.2 + 3.2 j$	
A	3405.32	27.128	B	0.090	$0.077 + 0.046 j$	$0.077 + 0.046 j$		$22.2 + 4.3 j$	
A	3406.32	27.128	B	0.198	$0.168 + 0.100 j$	$0.168 + 0.100 j$		2	
A	3407.32	27.128	B	0.059	$0.025 + 0.028 j$	$0.025 + 0.028 j$			

Zone	Line ID No.	Mains location [km] X_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	3408.321	27.128	B	0.287	$0.123 + 0.136j$	$0.123 + 0.136j$		$36.2 + 6.5j$	
A	3408.322	27.128	B	0.123	$0.053 + 0.058j$	$0.053 + 0.058j$		2	
A	3501	27.306	ABC	0.059	$0.019 + 0.026j$	$0.035 + 0.081j$	$19.5 + 4.9j$	$19.5 + 4.9j$	$19.5 + 4.9j$
A	3601	27.366	C	0.048	$0.016 + 0.021j$	$0.016 + 0.021j$			$16.5 + 4.8j$
A	3701	27.675	C	0.177	$0.058 + 0.078j$	$0.058 + 0.078j$			$39.5 + 8.0j$
A	3801	27.963	ABC	0.161	$0.069 + 0.076j$	$0.112 + 0.226j$			
A	3802.1	27.963	ABC	0.047	$0.020 + 0.022j$	$0.020 + 0.022j$	2		
A	3802.2	27.963	ABC	0.099	$0.042 + 0.047j$	$0.069 + 0.139j$			$24.5 + 6.1j$
A	3803.1	27.963	ABC	0.117	$0.050 + 0.056j$	$0.050 + 0.056j$	$12. + 3.0j$		
A	3803.2	27.963	ABC	0.115	$0.049 + 0.055j$	$0.080 + 0.161j$			$23.8 + 5.5j$
A	3804.1	27.963	ABC	0.081	$0.035 + 0.038j$	$0.035 + 0.038j$	$9.0 + 2.3j$		
A	3804.2	27.963	ABC	0.069	$0.030 + 0.033j$	$0.048 + 0.097j$			$16.0 + 3.0j$
A	3805.2	27.963	ABC	0.048	$0.021 + 0.023j$	$0.033 + 0.067j$			
A	3806.2	27.963	ABC	0.178	$0.076 + 0.085j$	$0.124 + 0.250j$			$16.9 + 4.2j$
A	3807.2	27.963	ABC	0.071	$0.030 + 0.034j$	$0.049 + 0.100j$			
A	3808.2	27.963	ABC	0.103	$0.088 + 0.052j$	$0.125 + 0.161j$			
A	3808.21	27.963	C	0.062	$0.027 + 0.029j$	$0.027 + 0.029j$			$45.4 + 9.3j$
A	3809.2	27.963	ABC	0.123	$0.053 + 0.058j$	$0.086 + 0.173j$		$41.1 + 5.9j$	
A	3809.21	27.963	C	0.120	$0.051 + 0.057j$	$0.051 + 0.057j$			$30.1 + 5.4j$
A	3809.22	27.963	C	0.091	$0.077 + 0.046j$	$0.077 + 0.046j$			$37.9 + 6.0j$
A	3810.2	27.963	ABC	0.119	$0.051 + 0.057j$	$0.083 + 0.167j$			
A	3810.21	27.963	C	0.023	$0.010 + 0.011j$	$0.010 + 0.011j$			
A	3810.22	27.963	C	0.063	$0.054 + 0.032j$	$0.054 + 0.032j$			$19.3 + 3.2j$
A	3811.2	27.963	ABC	0.001	$0.001 + 0.001j$	$0.001 + 0.002j$			
A	3811.211	27.963	C	0.099	$0.042 + 0.047j$	$0.042 + 0.047j$			$44.5 + 7.2j$
A	3811.212	27.963	C	0.062	$0.027 + 0.029j$	$0.027 + 0.029j$			$47.9 + 7.5j$
A	3811.22	27.963	C	0.122	$0.104 + 0.062j$	$0.104 + 0.062j$			
A	3811.23	27.963	A	0.070	$0.060 + 0.035j$	$0.060 + 0.035j$	$54.6 + 11.3j$		
A	3812.2	27.963	ABC	0.117	$0.100 + 0.059j$	$0.142 + 0.183j$		$6.0 + 1.5j$	
A	3812.22	27.963	C	0.123	$0.105 + 0.062j$	$0.105 + 0.062j$			$42.3 + 7.3j$
A	3812.24	27.963	A	0.054	$0.046 + 0.027j$	$0.046 + 0.027j$	$32.6 + 7.2j$		
A	3813.2	27.963	ABC	0.118	$0.100 + 0.060j$	$0.143 + 0.185j$			
A	3813.22	27.963	C	0.096	$0.082 + 0.049j$	$0.082 + 0.049j$			$68.6 + 13.5j$
A	3813.24	27.963	A	0.086	$0.073 + 0.044j$	$0.073 + 0.044j$	$14.0 + 2.4j$		
A	3814.2	27.963	ABC	0.050	$0.021 + 0.024j$	$0.021 + 0.024j$		$45.7 + 9.5j$	

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	3814.22	27.963	C	0.047	$0.040 + 0.024 j$	$0.040 + 0.024 j$			$8.7 + 2.2 j$
A	3814.25	27.963	A	0.133	$0.057 + 0.063 j$	$0.057 + 0.063 j$	$34.7 + 8.2 j$		
A	3815.2	27.963	ABC	0.150	$0.128 + 0.076 j$	$0.128 + 0.076 j$			
A	3815.22	27.963	C	0.017	$0.014 + 0.009 j$	$0.014 + 0.009 j$			
A	3815.25	27.963	A	0.108	$0.046 + 0.051 j$	$0.046 + 0.051 j$	$27.8 + 7.0 j$		
A	3816.2	27.963	ABC	0.065	$0.055 + 0.033 j$	$0.055 + 0.033 j$		$23.8 + 3.4 j$	
A	3816.221	27.963	C	0.055	$0.047 + 0.028 j$	$0.047 + 0.028 j$			$45.6 + 7.4 j$
A	3816.222	27.963	C	0.002	$0.002 + 0.001 j$	$0.002 + 0.001 j$			$29.5 + 5.1 j$
A	3816.25	27.963	ABC	0.020	$0.009 + 0.009 j$	$0.009 + 0.009 j$			
A	3816.26	27.963	ABC	0.124	$0.106 + 0.063 j$	$0.106 + 0.063 j$			
A	3817.2	27.963	ABC	0.060	$0.051 + 0.030 j$	$0.051 + 0.030 j$		$59.9 + 9.7 j$	
A	3817.222	27.963	C	0.027	$0.023 + 0.014 j$	$0.023 + 0.014 j$			
A	3817.251	27.963	ABC	0.058	$0.049 + 0.029 j$	$0.049 + 0.029 j$	$7.0 + 1.8 j$		
A	3817.261	27.963	ABC	0.060	$0.051 + 0.030 j$	$0.051 + 0.030 j$		$37.9 + 5.4 j$	
A	3817.252	27.963	ABC	0.063	$0.027 + 0.030 j$	$0.027 + 0.030 j$	$13.6 + 1.9 j$		
A	3817.262	27.963	ABC	0.159	$0.135 + 0.080 j$	$0.135 + 0.080 j$		2	
A	3818.2	27.963	ABC	0.060	$0.051 + 0.030 j$	$0.051 + 0.030 j$		$56.1 + 10.5 j$	
A	3818.2221	27.963	C	0.182	$0.155 + 0.092 j$	$0.155 + 0.092 j$			$25.1 + 3.6 j$
A	3818.2222	27.963	C	0.083	$0.036 + 0.039 j$	$0.036 + 0.039 j$			$15.1 + 3.3 j$
A	3818.261	27.963	ABC	0.094	$0.023 + 0.025 j$	$0.023 + 0.025 j$			
A	3818.252	27.963	ABC	0.053	$0.080 + 0.048 j$	$0.080 + 0.048 j$		$34.0 + 4.8 j$	
A	3819.2	27.963	ABC	0.060	$0.051 + 0.030 j$	$0.051 + 0.030 j$		$68.4 + 11.2 j$	
A	3819.2222	27.963	C	0.179	$0.077 + 0.085 j$	$0.077 + 0.085 j$			$45.6 + 8.9 j$
A	3819.2521	27.963	ABC	0.061	$0.026 + 0.029 j$	$0.026 + 0.029 j$	2		
A	3819.2522	27.963	ABC	0.070	$0.030 + 0.033 j$	$0.030 + 0.033 j$	$36.0 + 5.6 j$		
A	3820.2	27.963	ABC	0.060	$0.051 + 0.030 j$	$0.051 + 0.030 j$		$54.5 + 7.8 j$	
A	3820.2522	27.963	ABC	0.134	$0.057 + 0.064 j$	$0.057 + 0.064 j$	$31.5 + 6.4 j$		
A	3821.2	27.963	ABC	0.137	$0.117 + 0.069 j$	$0.117 + 0.069 j$		$46.2 + 9.7 j$	
A	3822.2	27.963	ABC	0.251	$0.214 + 0.127 j$	$0.214 + 0.127 j$		$9.5 + 1.4 j$	
A	3823.2	27.963	ABC	0.112	$0.095 + 0.057 j$	$0.095 + 0.057 j$		$8.0 + 1.9 j$	
A	3824.2	27.963	ABC	0.073	$0.031 + 0.035 j$	$0.031 + 0.035 j$		$7.8 + 1.1 j$	
A	3901	28.538	C	0.001					
A	3902.1	28.538	C	0.066	$0.022 + 0.030 j$	$0.022 + 0.030 j$			$31.5 + 6.6 j$
A	3902.2	28.538	C	0.091	$0.030 + 0.040 j$	$0.030 + 0.040 j$			$34.7 + 9.9 j$
A	4001	28.688	A	0.089	$0.076 + 0.045 j$	$0.076 + 0.045 j$	2		

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	4101	28.765	C	0.114	$0.097 + 0.058j$	$0.097 + 0.058j$			$4.1 + 1.0j$
A	4201	29.286	ABC	0.049	$0.0060 + 0.019j$	$0.019 + 0.065j$			2
A	4202	29.286	ABC	0.316	$0.037 + 0.125j$	$0.121 + 0.418j$	2		
A	4203	29.286	ABC	0.418	$0.048 + 0.165j$	$0.161 + 0.553j$		$18.9 + 2.7j$	
A	4204	29.286	ABC	0.050	$0.0060 + 0.020j$	$0.019 + 0.066j$			
A	4205.1	29.286	ABC	0.075	$0.0090 + 0.030j$	$0.029 + 0.099j$	$4.3 + 1.7j$	$4.3 + 1.7j$	$4.3 + 1.7j$
A	4205.2	29.286	ABC	0.057	$0.0070 + 0.023j$	$0.022 + 0.075j$		$19.3 + 3.4j$	
A	4206.2	29.286	ABC	0.286	$0.033 + 0.113j$	$0.110 + 0.378j$		$48.6 + 12.2j$	
A	4207.2	29.286	ABC	0.140	$0.016 + 0.055j$	$0.054 + 0.185j$		$31.9 + 6.0j$	
A	4208.2	29.286	ABC	0.123	$0.014 + 0.049j$	$0.047 + 0.163j$		$40.1 + 8.1j$	
A	4209.2	29.286	ABC	0.021	$0.002 + 0.008j$	$0.008 + 0.028j$			
A	4210.2	29.286	ABC	0.027	$0.003 + 0.011j$	$0.010 + 0.036j$		$18.7 + 3.0j$	
A	4210.21	29.286	ABC	0.652	$0.280 + 0.310j$	$0.454 + 0.915j$	$78.4 + 55.3j$	$78.4 + 55.3j$	$78.4 + 55.3j$
A	4211.2	29.286	ABC	0.089	$0.076 + 0.045j$	$0.108 + 0.139j$	$45.4 + 8.8j$		
A	4212.2	29.286	ABC	0.085	$0.072 + 0.043j$	$0.103 + 0.133j$			
A	4213.2	29.286	ABC	0.091	$0.077 + 0.046j$	$0.110 + 0.142j$		$37.7 + 6.6j$	
A	4213.22	29.286	C	0.090	$0.117 + 0.047j$	$0.117 + 0.047j$			2
A	4213.23	29.286	C	0.038	$0.032 + 0.019j$	$0.032 + 0.019j$			$43.8 + 9.1j$
A	4214.2	29.286	ABC	0.075	$0.064 + 0.038j$	$0.091 + 0.117j$		$26.9 + 5.5j$	
A	4214.23	29.286	C	0.089	$0.076 + 0.045j$	$0.076 + 0.045j$			$29.4 + 7.4j$
A	4215.2	29.286	ABC	0.025	$0.021 + 0.013j$	$0.030 + 0.039j$			
A	4215.23	29.286	C	0.121	$0.103 + 0.061j$	$0.103 + 0.061j$			$30.5 + 7.6j$
A	4216.2	29.286	ABC	0.074	$0.063 + 0.037j$	$0.090 + 0.116j$		$58.1 + 10.9j$	
A	4216.23	29.286	C	0.075	$0.064 + 0.038j$	$0.064 + 0.038j$			$24.8 + 4.2j$
A	4216.24	29.286	C	0.047	$0.040 + 0.024j$	$0.040 + 0.024j$			
A	4217.2	29.286	ABC	0.081	$0.069 + 0.041j$	$0.098 + 0.127j$		$12. + 2.1j$	
A	4217.241	29.286	C	0.051	$0.043 + 0.026j$	$0.043 + 0.026j$			$22.9 + 5.8j$
A	4217.242	29.286	C	0.087	$0.037 + 0.041j$	$0.037 + 0.041j$			$58.9 + 11.7j$
A	4218.2	29.286	ABC	0.046	$0.039 + 0.023j$	$0.056 + 0.072j$			
A	4218.242	29.286	C	0.145	$0.062 + 0.069j$	$0.062 + 0.069j$			$46.1 + 7.4j$
A	4219.2	29.286	ABC	0.075	$0.064 + 0.038j$	$0.091 + 0.117j$	$48. + 9.5j$		
A	4219.25	29.286	B	0.047	$0.040 + 0.024j$	$0.040 + 0.024j$		$51.6 + 9.0j$	
A	4220.2	29.286	ABC	0.061	$0.052 + 0.031j$	$0.074 + 0.096j$			
A	4220.25	29.286	B	0.125	$0.106 + 0.063j$	$0.106 + 0.063j$		$5.3 + 1.3j$	
A	4220.26	29.286	B	0.076	$0.065 + 0.038j$	$0.065 + 0.038j$		$20.8 + 4.6j$	

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	4221.2	29.286	ABC	0.115	$0.098 + 0.058 j$	$0.139 + 0.18 j$			$12.7 + 3.2 j$
A	4221.27	29.286	B	0.111	$0.094 + 0.056 j$	$0.094 + 0.056 j$		$20. + 3.7 j$	
A	4222.2	29.286	ABC	0.114	$0.097 + 0.058 j$	$0.138 + 0.179 j$			2
A	4223.2	29.286	ABC	0.053	$0.045 + 0.027 j$	$0.064 + 0.083 j$			$18.1 + 3.0 j$
A	4224.2	29.286	ABC	0.104	$0.089 + 0.053 j$	$0.126 + 0.163 j$		$16.3 + 2.3 j$	
A	4225.2	29.286	ABC	0.076	$0.065 + 0.038 j$	$0.092 + 0.119 j$			1
A	4226.2	29.286	ABC	0.175	$0.149 + 0.089 j$	$0.212 + 0.274 j$			
A	4227.28	29.286	B	0.060	$0.051 + 0.030 j$	$0.051 + 0.030 j$		$22.4 + 4.5 j$	
A	4227.29	29.286	ABC	0.065	$0.055 + 0.033 j$	$0.079 + 0.102 j$			
A	4228.29	29.286	ABC	0.127	$0.108 + 0.064 j$	$0.154 + 0.199 j$		$15.9 + 2.3 j$	
A	4228.291	29.286	B	0.015	$0.013 + 0.008 j$	$0.013 + 0.008 j$		$6.6 + 1.6 j$	
A	4229.29	29.286	ABC	0.098	$0.083 + 0.050 j$	$0.119 + 0.153 j$		2	
A	4230.29	29.286	ABC	0.105	$0.089 + 0.053 j$	$0.127 + 0.164 j$			
A	4231.29	29.286	ABC	0.180	$0.021 + 0.071 j$	$0.069 + 0.238 j$			
A	4231.291	29.286	C	0.001	$0.001 + 0.001 j$	$0.001 + 0.001 j$			
A	4232.29	29.286	ABC	0.043	$0.037 + 0.022 j$	$0.052 + 0.067 j$			
A	4232.2911	29.286	C	0.054	$0.046 + 0.027 j$	$0.046 + 0.027 j$			$30. + 4.3 j$
A	4232.2912	29.286	C	0.068	$0.058 + 0.034 j$	$0.058 + 0.034 j$			$15.3 + 2.7 j$
A	4232.292	29.286	B	0.067	$0.087 + 0.035 j$	$0.087 + 0.035 j$		$21.3 + 3.0 j$	
A	4233.29	29.286	ABC	0.054	$0.046 + 0.027 j$	$0.065 + 0.085 j$		$5.7 + 0.8 j$	
A	4233.293	29.286	C	0.077	$0.100 + 0.040 j$	$0.100 + 0.040 j$			$25.1 + 3.6 j$
A	4234.29	29.286	ABC	0.077	$0.066 + 0.039 j$	$0.093 + 0.121 j$			$18.9 + 2.7 j$
A	4235.29	29.286	ABC	0.092	$0.078 + 0.047 j$	$0.111 + 0.144 j$			
A	4236.29	29.286	ABC	0.050	$0.043 + 0.025 j$	$0.061 + 0.078 j$	$23.2 + 3.3 j$		
A	4236.294	29.286	C	0.049	$0.064 + 0.025 j$	$0.064 + 0.025 j$			2
A	4237.29	29.286	ABC	0.264	$0.225 + 0.134 j$	$0.320 + 0.413 j$	$8.6 + 2.1 j$		
A	4238.29	29.286	ABC	0.060	$0.051 + 0.030 j$	$0.073 + 0.094 j$	0.7	0.7	0.7
A	4239.29	29.286	C	0.107	$0.091 + 0.054 j$	$0.091 + 0.054 j$			
A	4240.29	29.286	C	0.147	$0.125 + 0.074 j$	$0.125 + 0.074 j$			
A	4240.295	29.286	C	0.045	$0.038 + 0.023 j$	$0.038 + 0.023 j$			$9.1 + 2.7 j$
A	4241.29	29.286	C	0.148	$0.126 + 0.075 j$	$0.126 + 0.075 j$			2
A	4241.295	29.286	C	0.460	$0.391 + 0.233 j$	$0.391 + 0.233 j$			$31.9 + 4.5 j$
A	4241.296	29.286	C	0.340	$0.289 + 0.172 j$	$0.289 + 0.172 j$			2
A	4242.29	29.286	C	0.220	$0.187 + 0.111 j$	$0.187 + 0.111 j$			2
A	4242.295	29.286	C	0.094	$0.080 + 0.048 j$	$0.080 + 0.048 j$			$25.8 + 5.6 j$

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
A	4242.296	29.286	C	0.056	$0.048 + 0.028j$	$0.048 + 0.028j$			$22.5 + 5.7j$
A	4243.29	29.286	C	0.094	$0.080 + 0.048j$	$0.080 + 0.048j$			2
A	4243.295	29.286	C	0.176	$0.150 + 0.089j$	$0.150 + 0.089j$			2
A	4244.295	29.286	C	0.166	$0.141 + 0.084j$	$0.141 + 0.084j$			$16.2 + 2.9j$
A	4245.295	29.286	C	0.170	$0.145 + 0.086j$	$0.145 + 0.086j$			$11.3 + 2.8j$
C	101	27.165	ABC	0.069	$0.059 + 0.035j$	$0.084 + 0.108j$	2		
C	102	27.165	ABC	0.069	$0.059 + 0.035j$	$0.084 + 0.108j$	$69.1 + 20.1j$	$69.1 + 20.1j$	$69.1 + 20.1j$
C	201	27.515	ABC	0.001	$0.001 + 0.001j$	$0.001 + 0.001j$	$36.5 + 10.7j$	$36.5 + 10.7j$	$36.5 + 10.7j$
C	202	27.515	ABC	0.001	$0.001 + 0.001j$	$0.001 + 0.001j$	$73.1 + 21.3j$	$73.1 + 21.3j$	$73.1 + 21.3j$
C	301	27.709	ABC	0.106	$0.090 + 0.054j$	$0.128 + 0.166j$		$18.7 + 3.7j$	
C	302	27.709	C	0.018	$0.008 + 0.009j$	$0.008 + 0.009j$			$29.6 + 5.5j$
C	303	27.709	C	0.124	$0.053 + 0.059j$	$0.053 + 0.059j$			$27. + 6.5j$
C	304	27.709	C	0.126	$0.054 + 0.060j$	$0.054 + 0.060j$			$7.1 + 1.8j$
C	305	27.709	C	0.033	$0.014 + 0.016j$	$0.014 + 0.016j$			
C	306	27.709	C	0.168	$0.072 + 0.080j$	$0.072 + 0.080j$			$2.9 + 0.7j$
C	306.01	27.709	C	0.062	$0.027 + 0.029j$	$0.027 + 0.029j$			$15.5 + 2.2j$
C	307	27.709	C	0.066	$0.028 + 0.031j$	$0.028 + 0.031j$			$11.6 + 2.7j$
C	307.01	27.709	C	0.131	$0.056 + 0.062j$	$0.056 + 0.062j$			$18.4 + 3.6j$
C	308	27.709	C	0.056	$0.024 + 0.027j$	$0.024 + 0.027j$			
C	308.01	27.709	C	0.112	$0.048 + 0.053j$	$0.048 + 0.053j$			$9.4 + 1.3j$
C	309	27.709	C	0.103	$0.044 + 0.049j$	$0.044 + 0.049j$			$3.0 + 0.8j$
C	309.01	27.709	C	0.091	$0.039 + 0.043j$	$0.039 + 0.043j$			$17.8 + 2.5j$
C	309.02	27.709	C	0.083	$0.036 + 0.039j$	$0.036 + 0.039j$			$17.7 + 2.9j$
C	310	27.709	C	0.095	$0.041 + 0.045j$	$0.041 + 0.045j$			$19.8 + 4.4j$
C	310.02	27.709	C	0.082	$0.035 + 0.039j$	$0.035 + 0.039j$			
C	310.1	27.709	C	0.083	$0.071 + 0.042j$	$0.071 + 0.042j$			$3.6 + 0.90j$
C	310.104	27.709	C	0.183	$0.156 + 0.093j$	$0.156 + 0.093j$			$19.4 + 2.8j$
C	311	27.709	C	0.105	$0.045 + 0.050j$	$0.045 + 0.050j$			
C	311.021	27.709	C	0.121	$0.052 + 0.057j$	$0.052 + 0.057j$			$37.5 + 9.2j$
C	311.022	27.709	C	0.092	$0.039 + 0.044j$	$0.039 + 0.044j$			$18.2 + 4.0j$
C	312	27.709	C	0.151	$0.065 + 0.072j$	$0.065 + 0.072j$			$2.5 + 0.60j$
C	312.03	27.709	C	0.105	$0.045 + 0.050j$	$0.045 + 0.050j$			$29.3 + 4.2j$
C	313	27.709	C	0.318	$0.136 + 0.151j$	$0.136 + 0.151j$			$8.6 + 1.9j$
C	313.03	27.709	C	0.073	$0.031 + 0.035j$	$0.031 + 0.035j$			
C	314	27.709	C	0.041	$0.018 + 0.019j$	$0.018 + 0.019j$			

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
C	314.03	27.709	C	0.147	$0.063 + 0.070 j$	$0.063 + 0.070 j$			2
C	314.031	27.709	C	0.082	$0.035 + 0.039 j$	$0.035 + 0.039 j$			$17.2 + 4.3 j$
C	315	27.709	C	0.387	$0.166 + 0.184 j$	$0.166 + 0.184 j$			$15.7 + 3.9 j$
C	315.03	27.709	C	0.001	$0.001 + 0.001 j$	$0.001 + 0.001 j$			
C	315.04	27.709	C	0.151	$0.196 + 0.078 j$	$0.196 + 0.078 j$			$19.0 + 4.4 j$
C	316	27.709	C	0.420	$0.180 + 0.200 j$	$0.180 + 0.200 j$			$13.7 + 4.5 j$
C	316.03	27.709	C	0.058	$0.025 + 0.028 j$	$0.025 + 0.028 j$			$16.7 + 3.1 j$
C	316.032	27.709	C	0.083	$0.036 + 0.039 j$	$0.036 + 0.039 j$			$13.4 + 1.9 j$
C	317	27.709	C	0.137	$0.059 + 0.065 j$	$0.059 + 0.065 j$			$21.4 + 5.4 j$
C	317.03	27.709	C	0.053	$0.023 + 0.025 j$	$0.023 + 0.025 j$			
C	318	27.709	C	0.195	$0.084 + 0.093 j$	$0.084 + 0.093 j$			
C	318.03	27.709	C	0.177	$0.076 + 0.084 j$	$0.076 + 0.084 j$			
C	318.033	27.709	C	0.111	$0.048 + 0.053 j$	$0.048 + 0.053 j$			$7.1 + 1.8 j$
C	319	27.709	C	1.599	$0.686 + 0.760 j$	$0.686 + 0.760 j$			
C	319.03	27.709	C	0.023	$0.010 + 0.011 j$	$0.010 + 0.011 j$			$8.5 + 2.1 j$
C	319.031	27.709	C	0.162	$0.069 + 0.077 j$	$0.069 + 0.077 j$			$16.3 + 2.7 j$
C	319.05	27.709	C	0.195	$0.084 + 0.093 j$	$0.084 + 0.093 j$			
C	320	27.709	C	0.448	$0.381 + 0.227 j$	$0.381 + 0.227 j$			$29.7 + 4.2 j$
C	320.03	27.709	C	0.070	$0.030 + 0.033 j$	$0.030 + 0.033 j$			
C	320.051	27.709	C	0.089	$0.116 + 0.046 j$	$0.116 + 0.046 j$			$10.2 + 4.0 j$
C	320.052	27.709	C	0.036	$0.015 + 0.017 j$	$0.015 + 0.017 j$			2
C	320.06	27.709	C	0.038	$0.032 + 0.019 j$	$0.032 + 0.019 j$			$22.7 + 7.1 j$
C	321	27.709	C	0.107	$0.091 + 0.054 j$	$0.091 + 0.054 j$			$6.4 + 1.6 j$
C	321.03	27.709	C	0.113	$0.048 + 0.054 j$	$0.048 + 0.054 j$			$6.3 + 1.6 j$
C	321.031	27.709	C	0.498	$0.214 + 0.237 j$	$0.214 + 0.237 j$			$7.6 + 3.0 j$
C	321.052	27.709	C	0.529	$0.227 + 0.251 j$	$0.227 + 0.251 j$			$15.7 + 3.3 j$
C	321.06	27.709	C	0.095	$0.081 + 0.048 j$	$0.081 + 0.048 j$			2
C	322	27.709	C	0.447	$0.380 + 0.226 j$	$0.380 + 0.226 j$			$6.4 + 2.5 j$
C	322.03	27.709	C	0.291	$0.125 + 0.138 j$	$0.125 + 0.138 j$			2
C	322.052	27.709	C	0.384	$0.165 + 0.182 j$	$0.165 + 0.182 j$			$12.1 + 4.8 j$
C	322.06	27.709	C	0.087	$0.074 + 0.044 j$	$0.074 + 0.044 j$			
C	323	27.709	C	0.073	$0.062 + 0.037 j$	$0.062 + 0.037 j$			
C	323.03	27.709	C	0.123	$0.053 + 0.058 j$	$0.053 + 0.058 j$			1
C	323.052	27.709	C	0.153	$0.066 + 0.073 j$	$0.066 + 0.073 j$			
C	323.06	27.709	C	0.779	$0.663 + 0.394 j$	$0.663 + 0.394 j$			$7.1 + 1.0 j$

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
C	323.061	27.709	C	1.269	$1.080 + 0.642 j$	$1.080 + 0.642 j$			
C	324	27.709	C	0.265	$0.226 + 0.134 j$	$0.226 + 0.134 j$			$25.0 + 3.6 j$
C	324.03	27.709	C	0.220	$0.094 + 0.105 j$	$0.094 + 0.105 j$			$42.7 + 9.3 j$
C	324.052	27.709	C	0.126	$0.054 + 0.060 j$	$0.054 + 0.060 j$			
C	324.0521	27.709	C	0.433	$0.562 + 0.225 j$	$0.562 + 0.225 j$			$5.4 + 1.4 j$
C	324.06	27.709	C	0.371	$0.316 + 0.188 j$	$0.316 + 0.188 j$			
C	324.061	27.709	C	0.219	$0.186 + 0.111 j$	$0.186 + 0.111 j$			
C	324.07	27.709	C	0.078	$0.066 + 0.039 j$	$0.066 + 0.039 j$			$4.3 + 0.6 j$
C	325	27.709	C	0.400	$0.340 + 0.202 j$	$0.340 + 0.202 j$			$16.7 + 4.2 j$
C	325.03	27.709	C	0.221	$0.095 + 0.105 j$	$0.095 + 0.105 j$			$32.7 + 6.3 j$
C	325.052	27.709	C	0.442	$0.190 + 0.210 j$	$0.190 + 0.210 j$			$7.2 + 2.8 j$
C	325.0521	27.709	C	0.317	$0.270 + 0.160 j$	$0.270 + 0.160 j$			$5.8 + 1.5 j$
C	325.0522	27.709	C	0.087	$0.037 + 0.041 j$	$0.037 + 0.041 j$			2
C	325.06	27.709	C	0.414	$0.352 + 0.209 j$	$0.352 + 0.209 j$			
C	325.061	27.709	C	0.180	$0.153 + 0.091 j$	$0.153 + 0.091 j$			2
C	325.0611	27.709	C	0.914	$0.778 + 0.462 j$	$0.778 + 0.462 j$			$7.3 + 1.8 j$
C	325.062	27.709	C	0.080	$0.068 + 0.040 j$	$0.068 + 0.040 j$			$4.2 + 0.6 j$
C	326	27.709	C	0.518	$0.441 + 0.262 j$	$0.441 + 0.262 j$			2
C	326.03	27.709	C	0.172	$0.074 + 0.082 j$	$0.074 + 0.082 j$			$26.8 + 4.1 j$
C	326.052	27.709	C	0.077	$0.033 + 0.037 j$	$0.033 + 0.037 j$			$10.7 + 2.7 j$
C	326.06	27.709	C	0.364	$0.310 + 0.184 j$	$0.310 + 0.184 j$			$14.1 + 2.8 j$
C	326.063	27.709	C	0.068	$0.058 + 0.034 j$	$0.058 + 0.034 j$			$7.6 + 1.9 j$
C	327	27.709	C	0.128	$0.109 + 0.065 j$	$0.109 + 0.065 j$			$17.7 + 3.6 j$
C	327.03	27.709	C	0.350	$0.150 + 0.166 j$	$0.150 + 0.166 j$			$7.3 + 1.1 j$
C	327.052	27.709	C	0.253	$0.328 + 0.131 j$	$0.328 + 0.131 j$			$3.6 + 1.4 j$
C	327.06	27.709	C	0.397	$0.338 + 0.201 j$	$0.338 + 0.201 j$			$8.6 + 2.1 j$
C	328	27.709	C	1.127	$0.959 + 0.570 j$	$0.959 + 0.570 j$			
C	328.03	27.709	C	0.135	$0.058 + 0.064 j$	$0.058 + 0.064 j$			$5.2 + 0.8 j$
C	328.06	27.709	C	0.235	$0.200 + 0.119 j$	$0.200 + 0.119 j$			$8.0 + 2.0 j$
C	329	27.709	C	0.273	$0.232 + 0.138 j$	$0.232 + 0.138 j$			$10.6 + 1.5 j$
C	329.03	27.709	C	0.064	$0.027 + 0.030 j$	$0.027 + 0.030 j$			$8.4 + 2.1 j$
C	329.06	27.709	C	0.107	$0.091 + 0.054 j$	$0.091 + 0.054 j$			$9.8 + 1.4 j$
C	329.08	27.709	C	0.087	$0.074 + 0.044 j$	$0.074 + 0.044 j$			1
C	330	27.709	C	0.060	$0.051 + 0.030 j$	$0.051 + 0.030 j$			$15.9 + 2.3 j$
C	330.03	27.709	C	0.061	$0.026 + 0.029 j$	$0.026 + 0.029 j$			$17.2 + 2.5 j$

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
C	330.06	27.709	C	0.030	$0.026 + 0.015j$	$0.026 + 0.015j$			
C	330.08	27.709	C	0.179	$0.152 + 0.091j$	$0.152 + 0.091j$			$11.1 + 4.4j$
C	331	27.709	C	0.189	$0.161 + 0.096j$	$0.161 + 0.096j$			
C	331.03	27.709	C	0.108	$0.046 + 0.051j$	$0.046 + 0.051j$			$13.9 + 2.0j$
C	331.06	27.709	C	0.093	$0.079 + 0.047j$	$0.079 + 0.047j$			
C	331.064	27.709	C	0.300	$0.255 + 0.152j$	$0.255 + 0.152j$			$2.9 + 0.7j$
C	331.08	27.709	C	0.090	$0.077 + 0.046j$	$0.077 + 0.046j$			1
C	332	27.709	C	0.089	$0.076 + 0.045j$	$0.076 + 0.045j$			2
C	332.06	27.709	C	0.209	$0.178 + 0.106j$	$0.178 + 0.106j$			$9.4 + 3.7j$
C	332.065	27.709	C	0.094	$0.080 + 0.048j$	$0.080 + 0.048j$			2
C	332.09	27.709	C	0.190	$0.162 + 0.096j$	$0.162 + 0.096j$			$8.6 + 2.2j$
C	333	27.709	C	0.413	$0.351 + 0.209j$	$0.351 + 0.209j$			
C	334	27.709	C	0.556	$0.473 + 0.281j$	$0.473 + 0.281j$			$9.6 + 1.4j$
C	334.10	27.709	C	0.499	$0.425 + 0.252j$	$0.425 + 0.252j$			
C	335	27.709	C	0.095	$0.081 + 0.048j$	$0.081 + 0.048j$			
C	335.10	27.709	C	0.001	$0.001 + 0.001j$	$0.001 + 0.001j$			
C	335.101	27.709	C	0.101	$0.086 + 0.051j$	$0.086 + 0.051j$			$6.5 + 1.6j$
C	336	27.709	C	0.077	$0.066 + 0.039j$	$0.066 + 0.039j$			2
C	336.10	27.709	C	0.700	$0.596 + 0.354j$	$0.596 + 0.354j$			$11.1 + 1.6j$
C	336.102	27.709	C	0.258	$0.220 + 0.131j$	$0.220 + 0.131j$			$10.3 + 4.1j$
C	336.11	27.709	C	0.091	$0.077 + 0.046j$	$0.077 + 0.046j$			$9.8 + 2.5j$
C	337	27.709	C	0.683	$0.581 + 0.346j$	$0.581 + 0.346j$			2
C	337.10	27.709	C	0.152	$0.129 + 0.077j$	$0.129 + 0.077j$			2
C	338	27.709	C	0.138	$0.117 + 0.070j$	$0.117 + 0.070j$			
C	338.10	27.709	C	0.090	$0.077 + 0.046j$	$0.077 + 0.046j$			
C	339	27.709	C	0.028	$0.024 + 0.014j$	$0.024 + 0.014j$			$24.3 + 3.5j$
C	339.10	27.709	C	0.132	$0.112 + 0.067j$	$0.112 + 0.067j$			
C	339.103	27.709	C	0.476	$0.405 + 0.241j$	$0.405 + 0.241j$			$14.2 + 2.0j$
C	339.12	27.709	C	0.060	$0.051 + 0.030j$	$0.051 + 0.030j$			$4.2 + 0.6j$
C	340	27.709	C	0.287	$0.244 + 0.145j$	$0.244 + 0.145j$			$4.8 + 1.2j$
C	341	27.709	C	0.045	$0.038 + 0.023j$	$0.038 + 0.023j$			
C	342	27.709	C	0.240	$0.204 + 0.121j$	$0.204 + 0.121j$			$7.5 + 1.1j$
C	342.13	27.709	C	0.041	$0.035 + 0.021j$	$0.035 + 0.021j$			$13.1 + 1.9j$
C	343	27.709	C	0.118	$0.100 + 0.060j$	$0.100 + 0.060j$			$6.2 + 0.9j$
C	343.13	27.709	C	0.269	$0.229 + 0.136j$	$0.229 + 0.136j$			$25.4 + 3.6j$

Zone	Line ID No.	Mains location [km] x_m	Phase	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
					Z_p	Z_0	S_a	S_b	S_c
C	344	27.709	C	0.088	$0.075 + 0.045j$	$0.075 + 0.045j$			2
C	345	27.709	C	0.164	$0.140 + 0.083j$	$0.140 + 0.083j$			$14.8 + 2.1j$
C	346	27.709	C	0.255	$0.217 + 0.129j$	$0.217 + 0.129j$			
C	347	27.709	C	0.362	$0.308 + 0.183j$	$0.308 + 0.183j$			2
C	347.14	27.709	C	0.060	$0.051 + 0.030j$	$0.051 + 0.030j$			$31.6 + 4.7j$

Appendix D: Execution of Reduction Methodology onto Feeder

D.1 Clipped Lateral Equivalents

Figure D-1 illustrates the clipping of the feeder's laterals. In the manner in which the feeder backbone was designated in Section 6.1.2, this feeder system consists of 45 laterals.

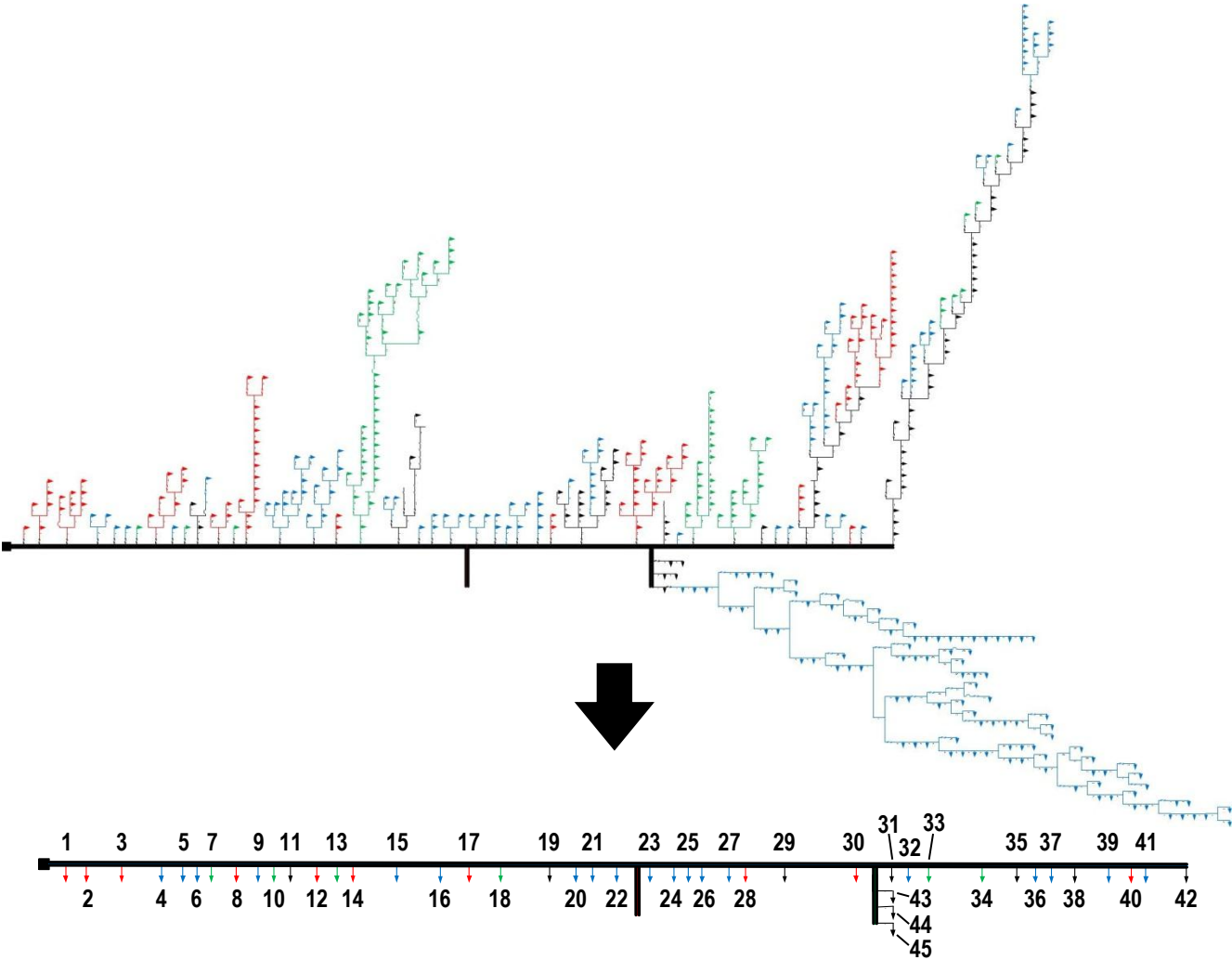


Figure D-1: Clipping of feeder laterals

Table D-1 contains the values for each of the loads representing their respective clipped laterals, as labelled in Figure D-1.

Table D-1: Load equivalents to replace clipped laterals

Lateral No.	Dist. from substation [km]	Equivalent load [kVA]		
		S _a	S _b	S _c
1	0.879	47.5 + 11.8 j		
2	1.341	178.6 + 36.6 j		
3	1.585	338.3 + 64.3 j		
4	2.197			4.2 + 0.3 j
5	2.399			3.9 + 1.0 j
6	2.748			6.5 + 1.5 j
7	8.219		2.0 – 0.1 j	
8	9.268	83.7 + 18.7 j		
9	9.345			75.2 + 21.9 j
10	10.627		1.4 + 0.4 j	
11	11.409	0.7 – 0.7 j	0.7 – 0.6 j	14.5 + 2.6 j
12	11.510	28 + 7.9 j		
13	14.719		6.7 + 2.6 j	
14	14.986	217.3 + 43.6 j		
15	15.796			73.2 + 15.5 j
16	16.651			35.3 + 1.3 j
17	19.551	24.2 + 6.5 j		
18	19.551		317.5 + 54.6 j	
19	21.435	52.2 + 13.3 j	46.9 + 11.9 j	57.0 + 14.3 j
20	21.545			30.0 + 11.8 j
21	22.020			3.0 – 0.3 j
22	23.070			12.2 + 2.6 j
23	24.176			46.0 + 7.8 j
24	24.501			4.9 – 0.7 j
25	24.864			2.0 – 0.2 j
26	25.654			51.4 + 10.1 j
27	25.877			120.2 + 26.8 j
28	26.130	82.7 + 20.8 j		
29	26.131	157.2 + 36.9 j	298.6 + 70.4 j	541.2 + 109.6 j
30	26.211	676.9 + 120.9 j		
31	26.419	- 0.1 j	- 0.1 j	50.6 + 10.5 j
32	26.419			14.6 + 4.2 j
33	27.099		438.0 + 79.6 j	
34	27.128		347.4 + 72.3 j	
35	27.306	19.5 + 4.8 j	19.5 + 4.8 j	19.5 + 4.8 j
36	27.366			16.9 + 4.9 j
37	27.675			40.4 + 8.0 j
38	27.963	290.7 + 58.0 j	511.4 + 84.1 j	599.9 + 109.5 j
39	28.538			67.6 + 16.8 j
40	28.688	2.0 – 0.1 j		
41	28.765			4.1 + 1.0 j

Lateral No.	Dist. from substation [km]	Equivalent load [kVA]		
		S_a	S_b	S_c
42	29.286	$216.5 + 77.7 j$	$592.5 + 148.8 j$	$604.9 + 151.1 j$
43	27.165	$75.0 + 21.2 j$	$71.3 + 20.7 j$	$70.8 + 20.5 j$
44	27.515	$116.0 + 33.7 j$	$113.2 + 32.9 j$	$112.3 + 32.6 j$
45	27.709		$18.7 + 3.7 j$	$1184.7 + 221.6 j$

D.2 Aggregation of Main Backbone

Table D-2 lists out the relevant electrical characteristics of each of the individual line pieces comprising the feeder backbone. Figure D-2 shows how these line pieces have been grouped to form aggregated line sections, according to their position among the retained components.

There are a couple instances along the feeder in which sections are further split up in order to equalize the load distribution within each section, as described in Section 4.6. One such case is in the first section, consisting of lines 1-17 in Table D-2. Line 17 is the last line in the section that leads to the downstream node, and its length is 2.291 km, or about 35.9% of the section's length of 6.386 km. This qualifies line 17 to be separated from lines 1-16 and formed into a new unloaded section. Similar treatment is given to line 73, whose length is 0.452 km, which is 23.1% of the section's length of 1.954 km. Note that the section lengths in these calculations include the length of the lines considered for splitting off the respective section.

Figure D-3 shows the layout for the outcome of the feeder reduction methodology. Figure D-4 relates the line sections of this reduced network to the original feeder diagram, showing what lines, loads, laterals, etc. were encapsulated into each section.

Table D-2: Line and load parameters of main backbone pieces, including clipped laterals

Sec	Dist. from substation [km]	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
			Z_p	Z_0	S_a	S_b	S_c
1	0.071	0.580	$0.067 + 0.229 j$	$0.223 + 0.767 j$			2
2	0.651	0.228	$0.026 + 0.090 j$	$0.088 + 0.302 j$	$47.5 + 11.8 j$		
3	0.879	0.147	$0.017 + 0.058 j$	$0.056 + 0.194 j$	$49.0 + 10.4 j$		
4	1.026	0.210	$0.024 + 0.083 j$	$0.081 + 0.278 j$	$16.4 + 2.8 j$		
5	1.236	0.105	$0.012 + 0.041 j$	$0.040 + 0.139 j$	$178.6 + 36.6 j$		
6	1.341	0.048	$0.006 + 0.019 j$	$0.018 + 0.064 j$	$15.1 + 3.8 j$		
7	1.389	0.196	$0.023 + 0.077 j$	$0.075 + 0.259 j$	$338.3 + 64.3 j$		
8	1.585	0.200	$0.023 + 0.079 j$	$0.077 + 0.265 j$		$8 + 2 j$	
9	1.785	0.217	$0.025 + 0.086 j$	$0.083 + 0.287 j$	$68.7 + 20.0 j$		
10	2.002	0.195	$0.023 + 0.077 j$	$0.075 + 0.258 j$			$4.2 + 0.3 j$
11	2.197	0.202	$0.023 + 0.080 j$	$0.078 + 0.267 j$			$3.9 + 1.0 j$
12	2.399	0.304	$0.035 + 0.120 j$	$0.117 + 0.402 j$	$21.8 + 5.5 j$		

Sec	Dist. from substation [km]	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
			Z_p	Z_0	S_a	S_b	S_c
13	2.703	0.045	0.005 + 0.018 j	0.017 + 0.060 j			6.5 + 1.5 j
14	2.748	0.063	0.007 + 0.025 j	0.024 + 0.083 j		15.7 + 3.9 j	
15	2.811	0.105	0.012 + 0.041 j	0.040 + 0.139 j	11.3 + 2.8 j		
16	2.916	1.250	0.145 + 0.494 j	0.480 + 1.654 j		6.2 + 1.6 j	
17	4.166	2.291	0.266 + 0.905 j	0.880 + 3.031 j			
18	6.457	1.735	0.201 + 0.685 j	0.666 + 2.295 j	34.6 + 10.0 j	34.6 + 10 j	34.6 + 10.0 j
19	8.192	0.027	0.003 + 0.011 j	0.010 + 0.036 j		2.0 – 0.1 j	
20	8.219	0.210	0.024 + 0.083 j	0.081 + 0.278 j			21.5 + 5.4 j
21	8.429	0.051	0.006 + 0.020 j	0.020 + 0.067 j	21.8 + 1.5 j	21.8 + 1.5 j	21.8 + 1.5 j
22	8.480	0.016	0.002 + 0.006 j	0.006 + 0.021 j			
23	8.496	0.031	0.004 + 0.012 j	0.012 + 0.041 j		23.6 + 5.5 j	
24	8.527	0.355	0.041 + 0.140 j	0.136 + 0.470 j	10.8 + 3.1 j		
25	8.882	0.260	0.030 + 0.103 j	0.100 + 0.344 j	0.6 + 0.1 j	0.6 + 0.1 j	0.6 + 0.1 j
26	9.142	0.058	0.007 + 0.023 j	0.022 + 0.077 j	12.7 + 3.6 j		
27	9.200	0.068	0.008 + 0.027 j	0.026 + 0.090 j	83.7 + 18.7 j		
28	9.268	0.077	0.009 + 0.030 j	0.030 + 0.102 j			75.2 + 21.9 j
29	9.345	0.172	0.020 + 0.068 j	0.066 + 0.228 j	59.3 + 15.4 j		
30	9.517	0.125	0.015 + 0.049 j	0.048 + 0.165 j			27.0 + 4.7 j
31	9.642	0.264	0.031 + 0.104 j	0.101 + 0.349 j	62.7		
32	9.906	0.062	0.007 + 0.024 j	0.024 + 0.082 j			2
33	9.968	0.333	0.039 + 0.132 j	0.128 + 0.441 j	2		
34	10.301	0.326	0.038 + 0.129 j	0.125 + 0.431 j		1.4 + 0.4 j	
35	10.627	0.472	0.055 + 0.186 j	0.181 + 0.624 j	7.2 + 1.8 j		
36	11.099	0.310	0.036 + 0.122 j	0.119 + 0.410 j	2.7 – 0.7 j	0.7 – 0.6 j	14.5 + 2.6 j
37	11.409	0.101	0.012 + 0.04 j	0.038 + 0.133 j	28.0 + 7.9 j		
38	11.510	2.196	0.255 + 0.867 j	0.843 + 2.905 j	2.9 + 0.7 j		
39	13.706	0.890	0.103 + 0.352 j	0.342 + 1.177 j	6.3 + 1.6 j		
40	14.596	0.123	0.014 + 0.049 j	0.047 + 0.163 j		6.7 + 2.6 j	
41	14.719	0.267	0.031 + 0.105 j	0.103 + 0.353 j	217.3 + 43.6 j		
42	14.986	0.250	0.029 + 0.099 j	0.096 + 0.331 j	2.7 + 1.1 j	2.7 + 1.1 j	2.7 + 1.1 j
43	15.296	0.250	0.029 + 0.099 j	0.096 + 0.331 j			14.6 + 3.6 j
44	15.546	0.250	0.029 + 0.099 j	0.096 + 0.331 j			73.2 + 15.5 j
45	15.796	0.095	0.011 + 0.038 j	0.036 + 0.126 j		1	
46	15.891	0.650	0.075 + 0.257 j	0.250 + 0.860 j			8.2 + 3.3 j
47	16.541	0.110	0.013 + 0.043 j	0.042 + 0.146 j			49.9 + 5.0 j
48	16.651	0.560	0.065 + 0.221 j	0.215 + 0.741 j			4.3 + 1.1 j
49	17.211	0.200	0.023 + 0.079 j	0.077 + 0.265 j			2.7 + 0.7 j
50	17.411	0.679	0.079 + 0.268 j	0.261 + 0.898 j	2		
51	18.090	0.431	0.050 + 0.170 j	0.166 + 0.570 j			6.8 + 2.7 j
52	18.521	0.350	0.041 + 0.138 j	0.134 + 0.463 j			
53	18.871	0.680	0.079 + 0.269 j	0.261 + 0.900 j	24.2 + 6.5 j	317.5 + 54.6 j	
54	19.551	0.550	0.064 + 0.217 j	0.211 + 0.728 j		2	

Sec	Dist. from substation [km]	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
			Z_p	Z_0	S_a	S_b	S_c
55	20.101	0.095	$0.011 + 0.038 j$	$0.036 + 0.126 j$			2
56	20.196	0.250	$0.029 + 0.099 j$	$0.096 + 0.331 j$			$9.3 + 3.7 j$
57	20.446	0.325	$0.038 + 0.128 j$	$0.125 + 0.430 j$			$5 + 2 j$
58	20.771	0.195	$0.023 + 0.077 j$	$0.075 + 0.258 j$			$19.1 + 7.5 j$
59	20.966	0.350	$0.041 + 0.138 j$	$0.134 + 0.463 j$			$6.2 + 1.6 j$
60	21.316	0.050	$0.006 + 0.020 j$	$0.019 + 0.066 j$			$10.7 + 2.7 j$
61	21.366	0.069	$0.008 + 0.027 j$	$0.026 + 0.091 j$	$52.2 + 13.3 j$	$46.9 + 11.9 j$	$57.0 + 14.3 j$
62	21.435	0.110	$0.013 + 0.043 j$	$0.042 + 0.146 j$			$30.0 + 11.8 j$
63	21.545	0.475	$0.055 + 0.188 j$	$0.182 + 0.628 j$			$3.0 - 0.3 j$
64	22.020	1.050	$0.122 + 0.415 j$	$0.403 + 1.389 j$			$12.2 + 2.6 j$
65	23.070	0.300	$0.035 + 0.119 j$	$0.115 + 0.397 j$			$6.9 + 1.7 j$
66	23.370	0.010	$0.001 + 0.004 j$	$0.004 + 0.013 j$			
67	23.417	0.028	$0.003 + 0.011 j$	$0.010 + 0.037 j$			$8.1 + 2 j$
68	23.445	0.675	$0.078 + 0.267 j$	$0.259 + 0.893 j$			$13.2 + 3.4 j$
69	24.120	0.056	$0.006 + 0.022 j$	$0.022 + 0.074 j$			$46.0 + 7.8 j$
70	24.176	0.325	$0.038 + 0.128 j$	$0.125 + 0.430 j$			$4.9 - 0.7 j$
71	24.501	0.363	$0.042 + 0.143 j$	$0.139 + 0.480 j$			$2.0 - 0.2 j$
72	24.864	0.055	$0.006 + 0.022 j$	$0.021 + 0.073 j$		2	
73	24.919	0.452	$0.052 + 0.179 j$	$0.174 + 0.598 j$			
74	25.371	0.133	$0.015 + 0.053 j$	$0.051 + 0.176 j$			$17 + 2.4 j$
75	25.504	0.150	$0.017 + 0.059 j$	$0.058 + 0.198 j$			$51.4 + 10.1 j$
76	25.654	0.139	$0.016 + 0.055 j$	$0.053 + 0.184 j$			$29.9 + 6 j$
77	25.793	0.084	$0.010 + 0.033 j$	$0.032 + 0.111 j$			$120.2 + 26.8 j$
78	25.877	0.070	$0.008 + 0.028 j$	$0.027 + 0.093 j$			$32.5 + 6.1 j$
79	25.947	0.046	$0.005 + 0.018 j$	$0.018 + 0.061 j$	$46.6 + 9.2 j$		
80	25.993	0.137	$0.016 + 0.054 j$	$0.053 + 0.181 j$	$239.9 + 57.7 j$	$298.6 + 70.4 j$	$541.2 + 109.6 j$
81	26.130	0.046	$0.005 + 0.018 j$	$0.017 + 0.061 j$	$41.5 + 10.9 j$		
82	26.176	0.035	$0.004 + 0.014 j$	$0.013 + 0.046 j$	$676.9 + 120.9 j$		
83	26.211	0.153	$0.018 + 0.060 j$	$0.059 + 0.202 j$			$29.3 + 8.5 j$
84	26.364	0.054	$0.006 + 0.021 j$	$0.021 + 0.071 j$	$-0.1 j$	$-0.1 j$	$65.2 + 14.7 j$
85	26.419	0.130	$0.015 + 0.051 j$	$0.050 + 0.172 j$			
86	26.549	0.224	$0.073 + 0.098 j$	$0.133 + 0.306 j$	2		
87	26.773	0.046	$0.015 + 0.020 j$	$0.027 + 0.063 j$			
88	26.826	0.233	$0.076 + 0.102 j$	$0.139 + 0.318 j$			$18.9 + 2.7 j$
89	27.059	0.040	$0.013 + 0.018 j$	$0.024 + 0.055 j$		$438.0 + 79.6 j$	
90	27.099	0.029	$0.009 + 0.013 j$	$0.017 + 0.040 j$		$347.4 + 72.3 j$	
91	27.128	0.118	$0.038 + 0.052 j$	$0.070 + 0.161 j$			$47.6 + 12.1 j$
92	27.246	0.060	$0.020 + 0.026 j$	$0.036 + 0.082 j$	$19.5 + 4.8 j$	$19.5 + 4.8 j$	$19.5 + 4.8 j$
93	27.306	0.060	$0.020 + 0.026 j$	$0.036 + 0.082 j$			$16.9 + 4.9 j$
94	27.366	0.103	$0.034 + 0.045 j$	$0.061 + 0.141 j$	$77.2 + 22.4 j$		
95	27.469	0.143	$0.047 + 0.063 j$	$0.085 + 0.195 j$	$15.8 + 4.6 j$		
96	27.612	0.063	$0.021 + 0.028 j$	$0.037 + 0.086 j$			$40.4 + 8.0 j$

Sec	Dist. from substation [km]	Length [km]	Series impedance [Ω]		Downstream-end load [kVA]		
			Z_p	Z_0	S_a	S_b	S_c
97	27.675	0.288	0.094 + 0.126 j	0.171 + 0.394 j	290.7 + 58.0 j	511.4 + 84.1 j	599.9 + 109.5 j
98	27.963	0.073	0.024 + 0.032 j	0.043 + 0.100 j		4.3 + 1.1 j	
99	28.036	0.185	0.060 + 0.081 j	0.110 + 0.253 j			11.5 + 3.3 j
100	28.221	0.165	0.054 + 0.072 j	0.098 + 0.226 j		71.7 + 20.6 j	
101	28.386	0.077	0.025 + 0.034 j	0.046 + 0.105 j			33.5 + 8.4 j
102	28.463	0.075	0.025 + 0.035 j	0.046 + 0.104 j			67.6 + 16.8 j
103	28.538	0.110	0.036 + 0.048 j	0.065 + 0.150 j	35.2 + 7 j		
104	28.648	0.040	0.013 + 0.018 j	0.024 + 0.055 j	2.0 - 0.1 j		
105	28.688	0.077	0.025 + 0.034 j	0.046 + 0.105 j			4.1 + 1.0 j
106	28.765	0.014	0.005 + 0.006 j	0.008 + 0.019 j		2	
107	28.779	0.154	0.050 + 0.068 j	0.091 + 0.211 j	21 + 4 j		
108	28.933	0.147	0.048 + 0.065 j	0.087 + 0.201 j			27.5 + 6.1 j
109	29.080	0.108	0.035 + 0.047 j	0.064 + 0.148 j		13.2 + 3.3 j	
110	29.188	0.050	0.006 + 0.020 j	0.019 + 0.066 j	216.5 + 77.7 j	592.5 + 148.8 j	604.9 + 151.1 j
111	23.480	10.500	1.218 + 4.148 j	4.032 + 13.892 j			
112	33.980	0.058	0.007 + 0.023 j	0.022 + 0.077 j			
113	34.038	0.006	0.001 + 0.002 j	0.002 + 0.008 j	19.2 + 4.8 j	19.2 + 4.8 j	19.2 + 4.8 j
114	34.044	0.005	0.001 + 0.002 j	0.002 + 0.007 j	0.7	0.7	0.7
115	26.418	0.055	0.006 + 0.022 j	0.021 + 0.073 j			85.6 + 19.6 j
116	26.473	0.138	0.045 + 0.061 j	0.082 + 0.189 j		28.4 + 5.5 j	
117	26.611	0.066	0.022 + 0.029 j	0.039 + 0.090 j	46.3 + 8.2 j		
118	26.677	0.128	0.042 + 0.056 j	0.076 + 0.175 j		37.6 + 6.2 j	
119	26.805	0.146	0.048 + 0.064 j	0.087 + 0.200 j	93.4 + 21 j		
120	26.951	0.099	0.032 + 0.043 j	0.059 + 0.135 j	0.7	0.7	0.7
121	27.050	0.035	0.011 + 0.015 j	0.021 + 0.048 j	19.6 + 5.7 j	19.6 + 5.7 j	19.6 + 5.7 j
122	27.085	0.080	0.026 + 0.035 j	0.048 + 0.109 j	75.0 + 21.2 j	71.3 + 20.7 j	70.8 + 20.5 j
123	27.165	0.050	0.016 + 0.022 j	0.030 + 0.068 j		2	
124	27.215	0.300	0.098 + 0.132 j	0.178 + 0.410 j	116.0 + 33.7 j	113.2 + 32.9 j	112.3 + 32.6 j
125	27.515	0.194	0.165 + 0.098 j	0.235 + 0.304 j		18.7 + 3.7 j	1184.7 + 221.6 j

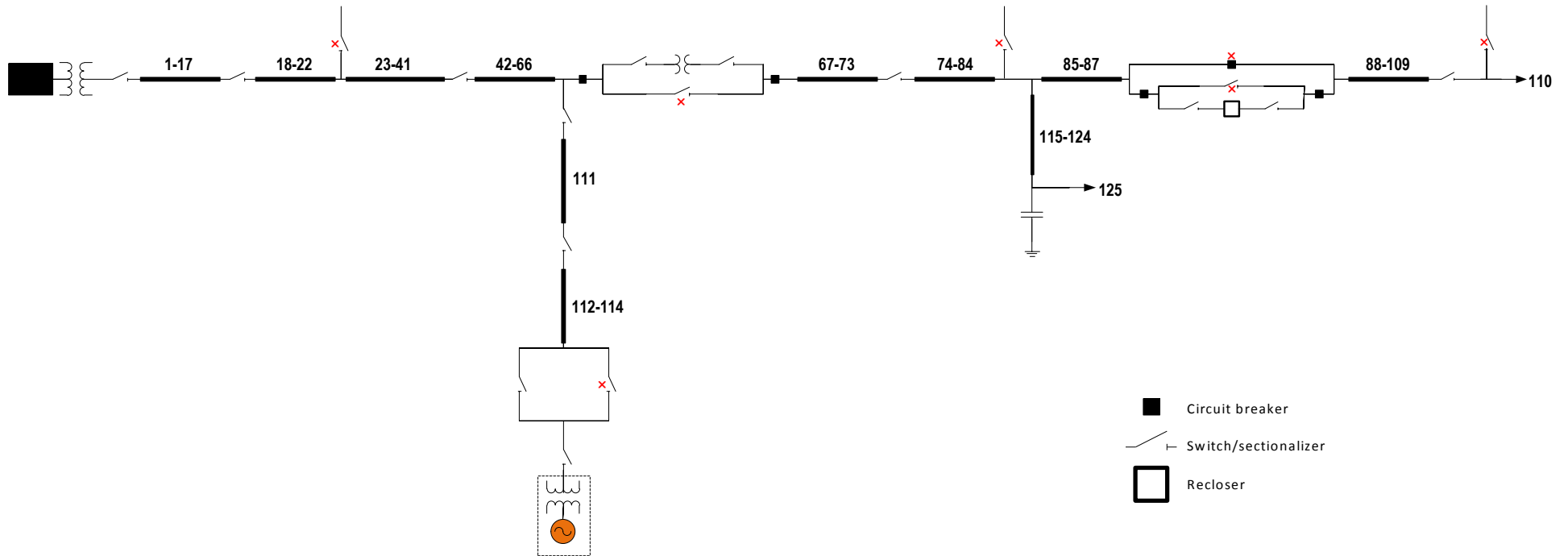


Figure D-2: Grouping of backbone sections, according to retained components

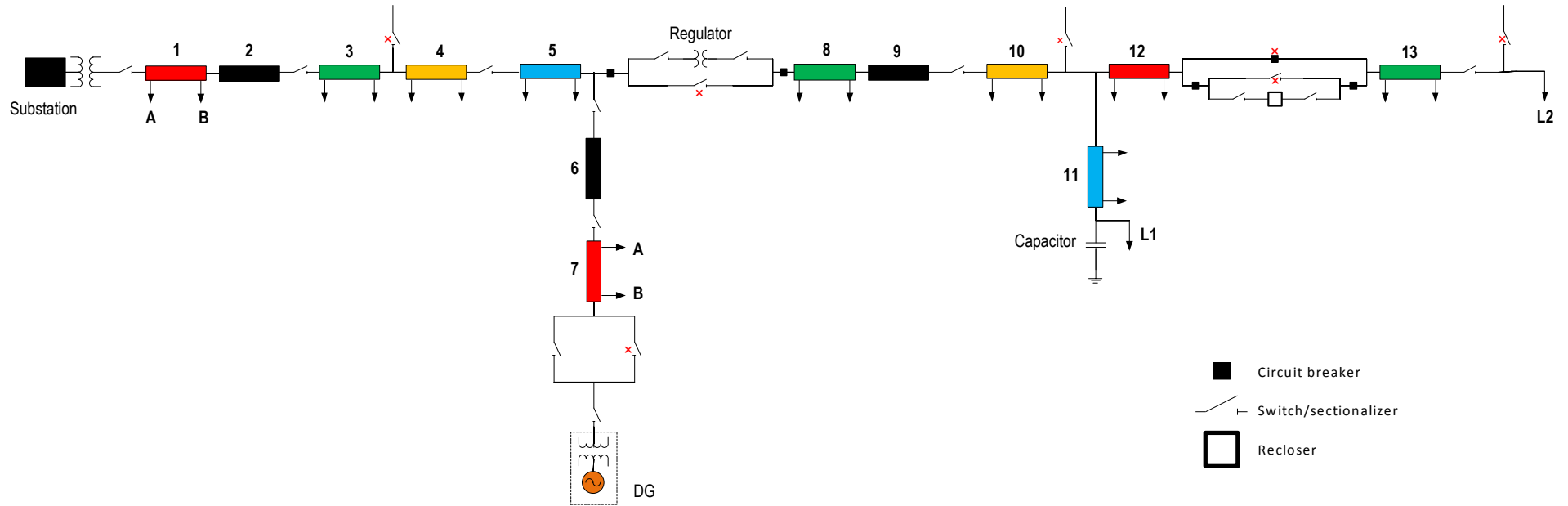


Figure D-3: Reduced feeder model line sections

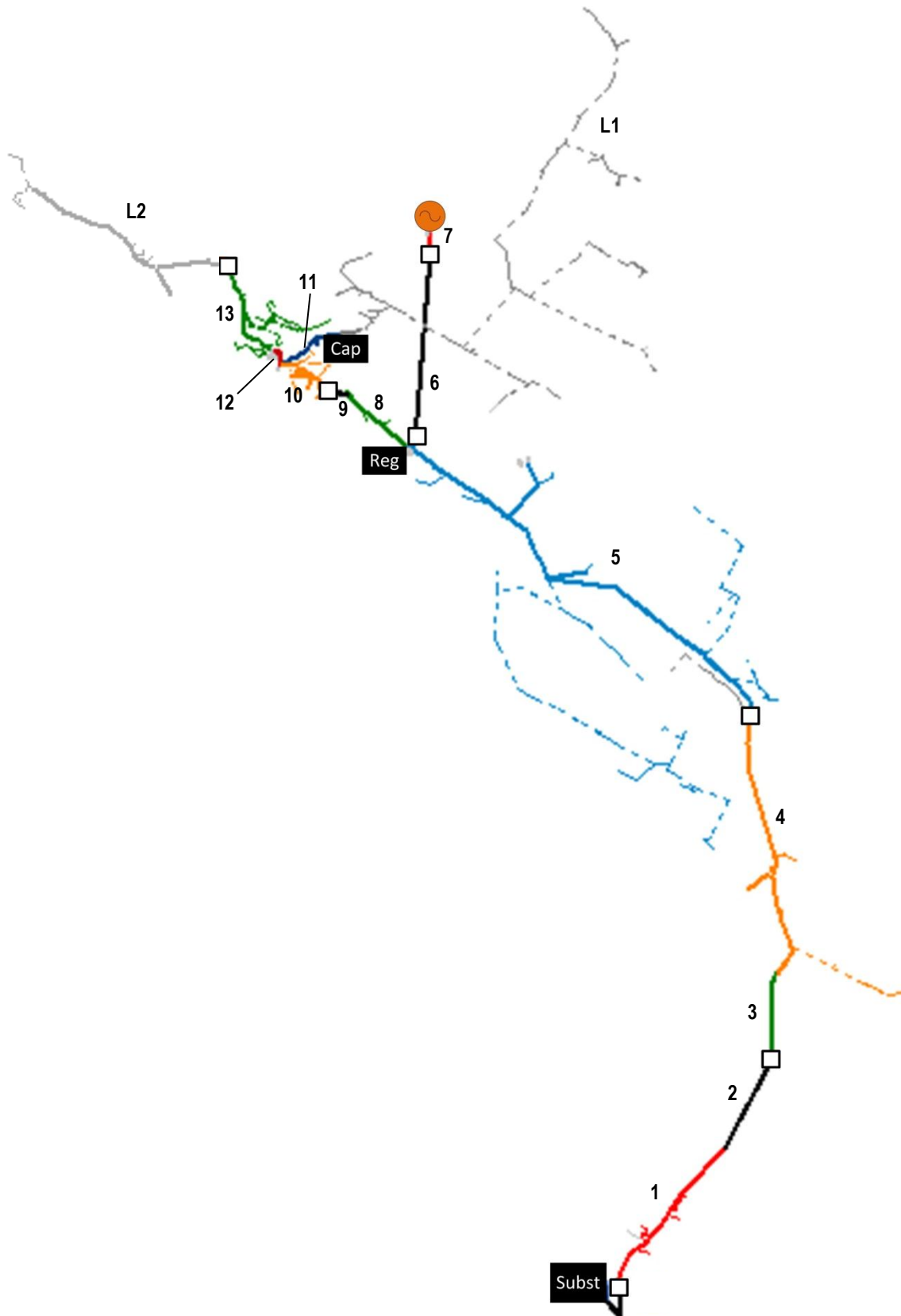


Figure D-4: Domain of each section within original feeder diagram



Appendix E: Fault Current Calculation

Figure E-1 below illustrates each of these faults, as they occur among the three phases at a given point along the feeder. The fault current measure used in the short circuit analysis is portrayed as I_f . For the line-to-line-to-ground fault, the fault current I_f is calculated by averaging the magnitudes of I_a and I_b , which are nearly identical to one another. The faults are assumed to be bolted, i.e. the fault impedance and ground impedance are assumed to be zero. In addition, all generators (including the DG) are disabled, with the fault current contribution coming from only the substation. Because the impedance is assumed to be balanced in all line sections of the feeder, the fault current values obtained can be assumed to be the same, regardless of which phases were shorted during fault analysis.

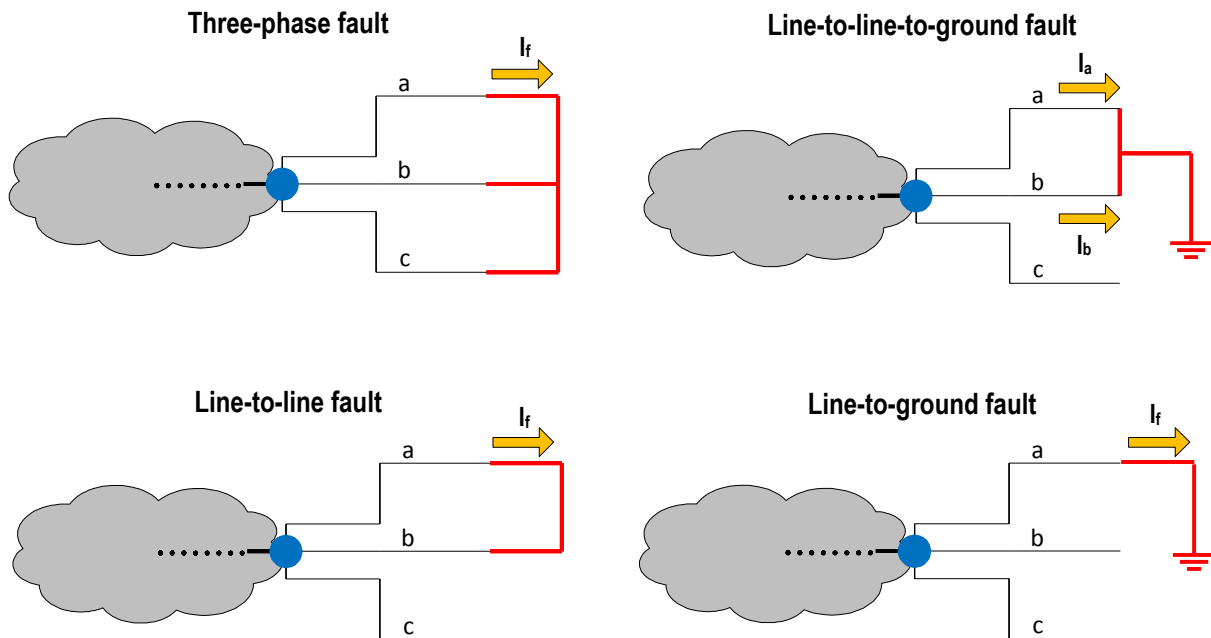
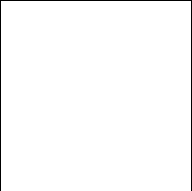


Figure E-1: Fault current types

The RMS magnitude of the fault current I_f is recorded for each type of fault inflicted at each of the test points along the distribution feeder.



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