SHAPE OF THE TANDEM BOOM

OF LOGS

by

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A lone tug hauls gigantic log booms
down the Saguenay River.

S U M M A R Y .

A theory has been developed to predict a shape of a towed boom of logs, where the logs are treated as a continuum which transmits only normal stresses. It is assumed , that the wood is sufficiently moved by the water that it does not support any shearing stresses. It is found that if the skin friction between the bottom of the raft and the water is assumed to be constant, the shape of the boom is one of the single-parameter family of elasticas. The solution can be obtained in terms of elliptical integrals. Shapes of symmetrical single boom, tandem of booms joined at one point and symmetrical cinched boom have been succesfully obtained and now the theory has been extended to predict the shape of an unsymmetrical tandem of booms joined at two points.

An attempt has been made to compare theory with experimental results and the response of the shape of the boom to a change in its parameters has been studied.

CONTENTS

N O T A T I O N

a	: horizontal distances between points on the booms
A	: whole or partial area of raft
b	: vertical distances between points on the booms
C	: constant of integration
E (m,ϕ)	: elliptic integral of the second kind
F (m,ϕ)	: elliptic integral of the first kind
F	: towing force
k	: $= (\frac{T}{\tau})^{1/2}$
m	: $= (\frac{1+\sin\alpha}{2})^{1/2}$
P	: the total length of the boom, also opposing end force bending a thin beam
s	: length of an elastica
T	: tension in the boom
x,y	: coordinates
X,Y	: coordinates
α	: angle between the boom and the direction of tow at the towing point
θ	: local angle between tangent to the boom and the direction of tow
σ	: normal compressive force per unit length in plan view
τ	: hydrodynamic skin friction beneath the raft
ϕ	: such that $\sin^2\phi = \frac{1+\sin\theta}{1+\sin\alpha}$

1. INTRODUCTION

1.1. History of Related Work.

Transportation of pulpwood on the lakes and along the seashore has received much less attention then the much more spectacular river drive. Very few refferences to this subject can be found. The huge rafts must be moved over long stretches of water by mechanical means. On shallow lakes the rafts are winched across by special winch boats. Large powerful tugs are used on Great lakes or when moving the rafts along the seashore. One of the best works on different types of rafts used in Canada is (2).

However, it is mostly descriptive, goes into considerable detail in spots, but is very sketchy elswhere. It provides a useful information about how the booms are constructed, studies different types of rafts as used in different parts of the country and provides a method for determining the volume of the wood in the raft. Windforce on the raft and the loss of wood during the tow are also considered.

In Canada, especially in eastern part, the wood is often transported as a loose floating raft, surrounded by a boom of large timbers, up to 30 feet long. Sometimes, several timbers must be bolted together to achieve a sufficient diameter. These are then joined by chaines to form the complete boom.

The rafts are usually made up at the mouth of the river, where it empties into the lake. The wood flows freely downstream into the boom thet is anchored to the banks. The boom is then closed and forms a circular raft, which is joined by a cable to the towing boat. When the towing starts, the wood moves to the rear and piles up three or four logs deep at the end with open water at the front. The boom now assumes the characteristic oval shape.

The Forest Work Studies Section of the Central Association of Finish Woodworking Industries conducted a research on timber raft drag in 1950.

Study of factors affecting the capacity of the rafting vessel has been included. It is shown that the towage expenses can be reduced by 25% by using larger bundles if the constant speed and a constant traction power is employed. The great advantage of large size rafts lies in the fact that the resistance of the raft increases more slowly than its size grows.

At too high speeds (implying that the size of the raft is inadequate) the increase in costs may easily total 50 %. However, the boat can be so powerful that adequate raft cannot be built because the waterway may not allow the pasage of so big a raft. This can be avoided if the raft is either cinched, or tandem of rafts is used, in which case the same amount of wood can be transported but the width of the raft is considerably reduced.

1.2. Reasons for Investigation.

As stated before, cost of transportation can be reduced by using larger rafts, but this presents problems if the water-passage is narrow and then the exact knowledge of the shape and of the dimensions of the raft is necessary to determine how much wood in what manner can be transported.

1.3. Range of Investigation.

A theory has been developed in (1) to predict the shape of a single boom. Also an approximate relation for the geometry of the boom in terms of the maximum number of logs a boom can hold was obtained. The theory then has been extended to predict the distortion of the boom if a second raft is attached and to give the shape of a boom that was cinched by an auxiliary boom. In the present paper the theory developed in (1) is further extended to predict the shape of an unsymmetrical tandem of booms when the trailing boom is attached at two points.

Then different symmetrical cases are studied to find the general behaviour of the tandem of booms, when some of the parameters are changed.

2. THEORY

2.1. Assumptions Made about the Problem.

If a raft is towed in a straight line at a constant speed, the following assumptions are utilized as stated in (1):

a) The aerodynamic, as distinct from the hydrodynamic forces, on the boom may be neglected. (The typical towing speed is 1 m.p.h.).

b) The Froude Number $\frac{U}{gL}$, which is typically .01, is sufficiently small for wave drag to be neglected. This is somewhat difficult to justify experimentally since there are few available measurements on wide, flatbottomed, hulls; However the Froude number is very low, and aerial photographs of tows do not show any waves emanating from the boom.

c) The logs are so numerous (there are typically 10^5 to 10^6 logs in a full scale boom of pulp wood) that they may be replaced by a continuum.

d) The shearing stresses transmitted from one group of logs to another are continually changing in magnitude and direction due to the movement of water around them so that the continuum has no shearing stresses within it.

e) The boom, which in reality is formed by joining together boom sticks with chains, may be treated as a flexible rope, which itself does not experience any hydrodynamic force.

f) The hydrodynamic skin friction on the bottom of the raft is constant and independent of position. This assumption is justified on the grounds that the logs probably present a fully-rough condition to the turbulent boundary layer which grows on the underside of the raft. Moreover since the logs pile up towards the rear of the boom, the roughness appropriately increases with x.

2.2. Equation of an Elastica.

The following is an outline of the theory which was developed for a single boom in (1).

Consider a small element dx . dy of the continuum towed in the water in -X -direction.

To achieve equilibrium

$$\frac{\partial \sigma}{\partial X} = \tau \qquad \frac{\partial \sigma}{\partial Y} = 0$$

where σ is the compressive force between the logs per unit length.

Solving the equation :

$$\sigma = \tau X + C \qquad -(2.2.1)$$

and evaluating the constant

$$\sigma = \tau x \qquad -(2.2.2)$$

where x is measured from the leading edge of the wood as opposed to X which is measured from the point of tow.

Considering the boom itself :

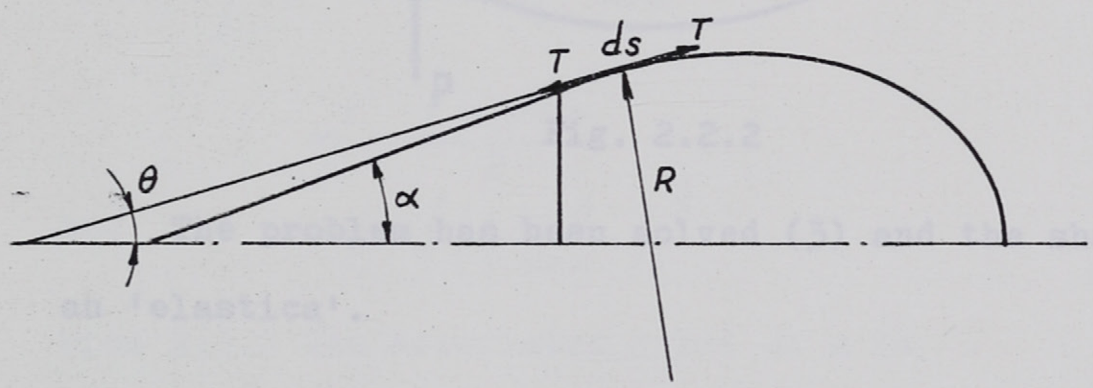

Fig 2.2.1

$$\sigma ds = -T d\theta$$

$$\sigma = \tau x = \frac{T}{R} \qquad -(2.2.3)$$

where $R = -\frac{ds}{d\theta}$ is local radius of curvature of the boom.

It follows that the shape of the boom is given by an equation of the form

$$x \propto \frac{1}{R} \qquad -(2.2.4)$$

which turns out to be the same as the equation governing a shape of a thin beam subjected to opposing loads at the ends - Fig. 2.2.2.

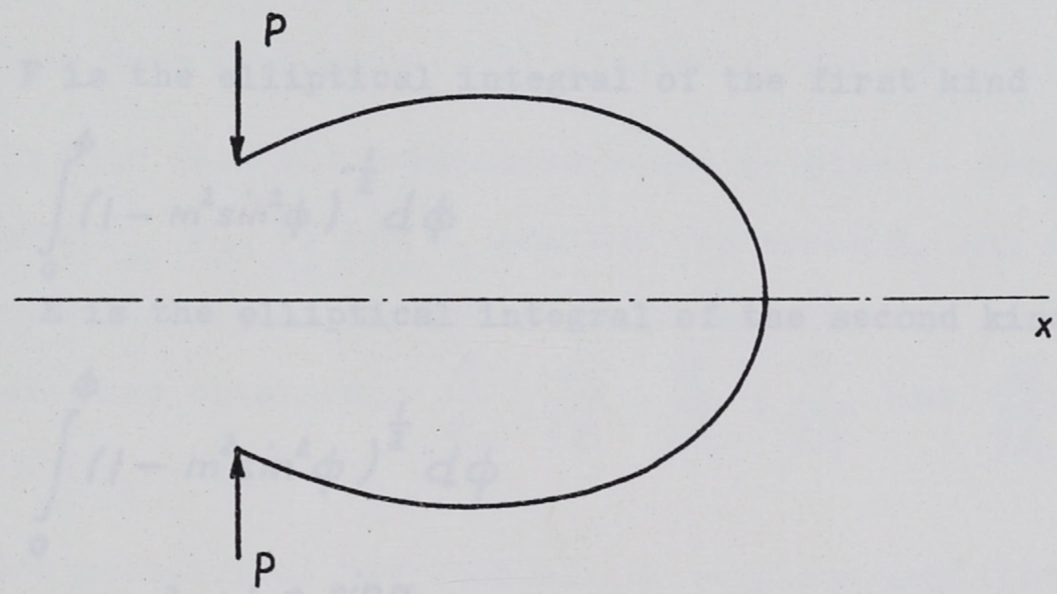

Fig. 2.2.2

The problem has been solved (3) and the shape was called an 'elastica'.

2.3. Length and Shape of an Elastica.

Integrating the equation 2.2.3 following expression is obtained :

$$\tau \frac{x^2}{2} = T\,(\sin\alpha - \sin\theta) \qquad - (2.3.1)$$

or $$x = \sqrt{2k^2(\sin\alpha - \sin\theta)} \;; \quad k^2 = \frac{T}{\tau} \qquad - (2.3.2)$$

$$dy = \frac{-k\sin\theta\, d\theta}{\sqrt{2(\sin\alpha - \sin\theta)}} \qquad - (2.3.3)$$

since $\frac{dx}{ds} = \sin\theta$

after some manipulation it is found that

$$y = k\left[2E(m,\phi) - F(m,\phi)\right] \qquad - (2.3.4)$$

where F is the elliptical integral of the first kind

$$\int_0^{\phi} (1 - m^2\sin^2\phi)^{-\frac{1}{2}}\, d\phi$$

E is the elliptical integral of the second kind

$$\int_0^{\phi} (1 - m^2\sin^2\phi)^{\frac{1}{2}}\, d\phi$$

$$m^2 = \frac{1 + \sin\alpha}{2}$$

and

$$\sin\phi = \left(\frac{1 + \sin\theta}{1 + \sin\alpha}\right)^{\frac{1}{2}}$$

From 2.3.2 the associated value of x is

$$x = 2mk\cos\phi \qquad - (2.3.5)$$

The length of the elastica measured from the end to the general point θ is :

$$s = k\left[F(m,\tfrac{\pi}{2}) - F(m,\phi)\right]$$

Thus the length of any part of the curve and the coordinates of points lying on it can be obtained if α , θ and k are known.

2.4. Unsymmetrical Tandem of Booms Joined at Two Points.

It is quite common to attach a second boom, which trails the first one. They can either be joined at one point or the trailing boom can be attached at two places. This distorts the first boom in a manner illustrated in Fig. 2.4.1.

In this case six parameters would be given - lengths of the booms P_1, P_2, P_3 and P_4 and the two areas A_1 and A_2. Hence five non-dimensional parameters which specify the geometry of the boom are obtained : $\frac{A_1}{A_2}$, $\frac{P_1}{\sqrt{A}}$, $\frac{P_2}{\sqrt{A}}$, $\frac{P_3}{\sqrt{A}}$ and $\frac{P_4}{\sqrt{A}}$ where $A = A_1 + A_2$.

The geometry of the boom may be obtained implicitly from the resolution of forces at three points : A, M and N.

Since the booms experience only normal forces, the tension in each boom is constant.

Junction A :

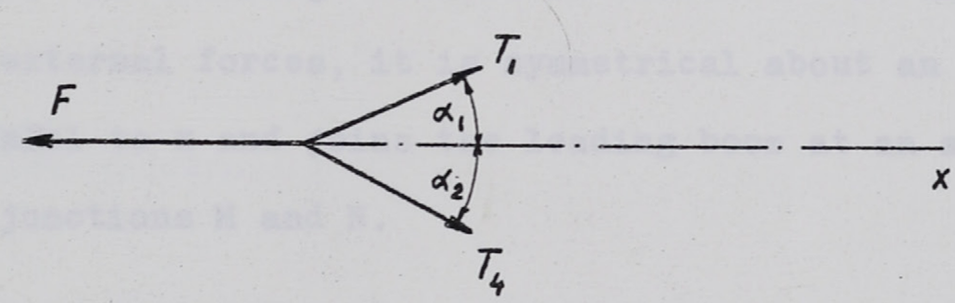

Fig. 2.4.2

Resolution of forces into their x and y components gives the following equations :

$$F = T_1 \cos\alpha_1 + T_4 \cos\alpha_2 \qquad - (2.4.1)$$

$$T_1 \sin\alpha_1 = T_4 \sin\alpha_2 \qquad - (2.4.2)$$

Also the towing force must be equal to the total area, multiplied by the skin friction τ :

$$F = (A_1 + A_2)\tau \qquad - (2.4.3)$$

or, in terms of k's :

$$A_1 + A_2 = k_1^2\left[\cos\alpha_1 + \frac{k_4^2}{k_1^2}\cos\alpha_2\right] \qquad - (2.4.4)$$

$$k_1^2 \sin\alpha_1 = k_4^2 \sin\alpha_2 \qquad - (2.4.5)$$

where $k_1^2 = \frac{T_1}{\tau}$ and $k_4^2 = \frac{T_4}{\tau}$

Junction M :

Since the trailing boom is not distorted in any way by some external forces, it is symmetrical about an offset axis parallel to x and joins the leading boom at an angle θ_2 at both junctions M and N.

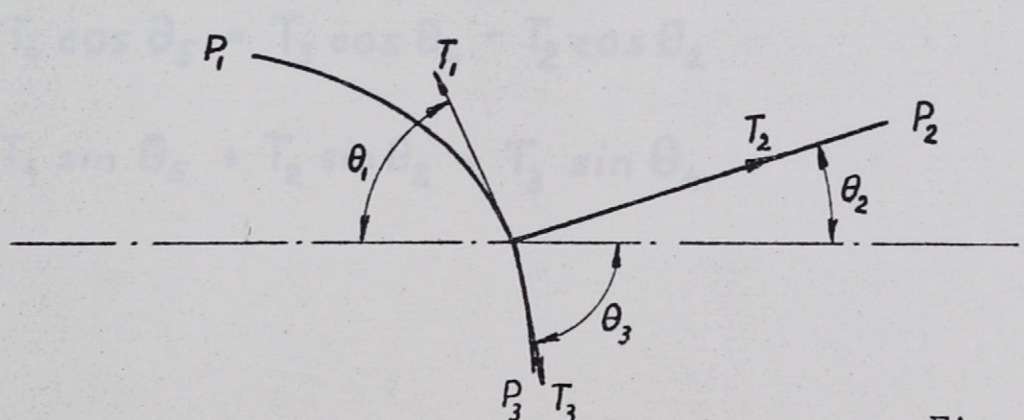

Fig. 2.4.3

Again, resolution of forces into x and y components yields :

$$T_1 \sin\theta_1 + T_2 \sin\theta_2 = T_3 \sin\theta_3 \qquad - (2.4.6)$$

$$T_1 \cos\theta_1 = T_2 \cos\theta_2 + T_3 \cos\theta_3 \qquad - (2.4.7)$$

or : $$k_1^2 \sin\theta_1 + k_2^2 \sin\theta_2 = k_3^2 \sin\theta_3 \qquad - (2.4.8)$$

$$k_1^2 \cos\theta_1 = k_2^2 \cos\theta_2 + k_3^2 \cos\theta_3 \qquad - (2.4.9)$$

where $k_2^2 = \frac{T_2}{\tau}$; $k_3^2 = \frac{T_3}{\tau}$

Junction N :

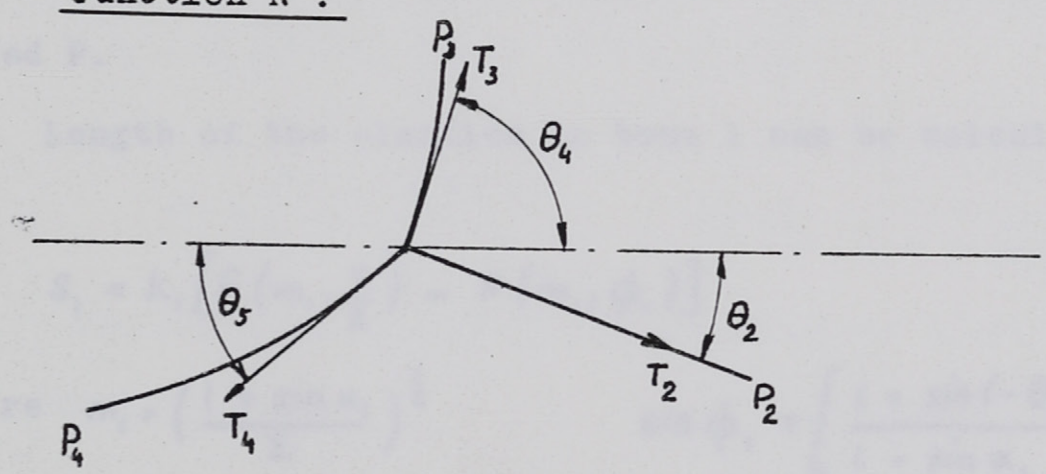

Fig. 2.4.4

By the same procedure :

$$T_4 \cos\theta_5 = T_3 \cos\theta_4 + T_2 \cos\theta_2 \qquad - (2.4.10)$$

$$T_4 \sin\theta_5 + T_2 \sin\theta_2 = T_3 \sin\theta_4 \qquad - (2.4.11)$$

or $$k_4^2 \cos\theta_5 = k_3^2 \cos\theta_4 + k_2^2 \cos\theta_2 \qquad - (2.4.12)$$

$$k_4^2 \sin\theta_5 = k_3^2 \sin\theta_4 - k_2^2 \sin\theta_2 \qquad - (2.4.13)$$

In relating lengths of the booms and areas of the rafts to the angles and tensions involved each boom has to be considered separately and the x and y distances (a's and b's) of the respective portions as indicated in Fig. 2.4.5 have to be found.

The portion of the boom that is an elastica will be called s while the complete length of the boom will be denoted P.

Length of the elastica in boom 1 can be calculated as :

$$s_1 = k_1\left[F(m_1, \tfrac{\pi}{2}) - F(m_1, \phi_1)\right] \qquad - (2.4.14)$$

where $m_1 = \left(\dfrac{1 + \sin\alpha_1}{2}\right)^{\frac{1}{2}}$ $\qquad \sin\phi_1 = \left[\dfrac{1 + \sin(-\theta_1)}{1 + \sin\alpha_1}\right]^{\frac{1}{2}}$

Length of the elastica in boom 2 :

$$s_2 = 2k_2\, F(m_2, \tfrac{\pi}{2}) \qquad - (2.4.15)$$

where $m_2 = \left(\dfrac{1 + \sin\theta_2}{2}\right)^{\frac{1}{2}}$

Length of the elastica in boom 3 :

$$s_3 = k_3\left[F(m_3,\phi_3) + F(m_3,\phi_4)\right] \qquad - (2.4.16)$$

where $m_3 = \left(\dfrac{1+\sin\alpha_3}{2}\right)^{\frac{1}{2}}$ $\qquad \sin\phi_3 = \left[\dfrac{1+\sin(-\theta_3)}{1+\sin\alpha_3}\right]^{\frac{1}{2}}$

$$\sin\phi_4 = \left[\frac{1+\sin(-\theta_4)}{1+\sin\alpha_3}\right]^{\frac{1}{2}}$$

Length of the elastica in boom 4 :

$$s_4 = k_4\left[F(m_4,\tfrac{\pi}{2}) - F(m_4,\phi_5)\right] \qquad - (2.4.17)$$

$$m_4 = \left(\frac{1+\sin\alpha_2}{2}\right)^{\frac{1}{2}} \qquad \sin\phi_5 = \left[\frac{1+\sin(-\theta_5)}{1+\sin\alpha_2}\right]^{\frac{1}{2}}$$

The distance a_2 and b_2 can be obtained from considering elastica s_3

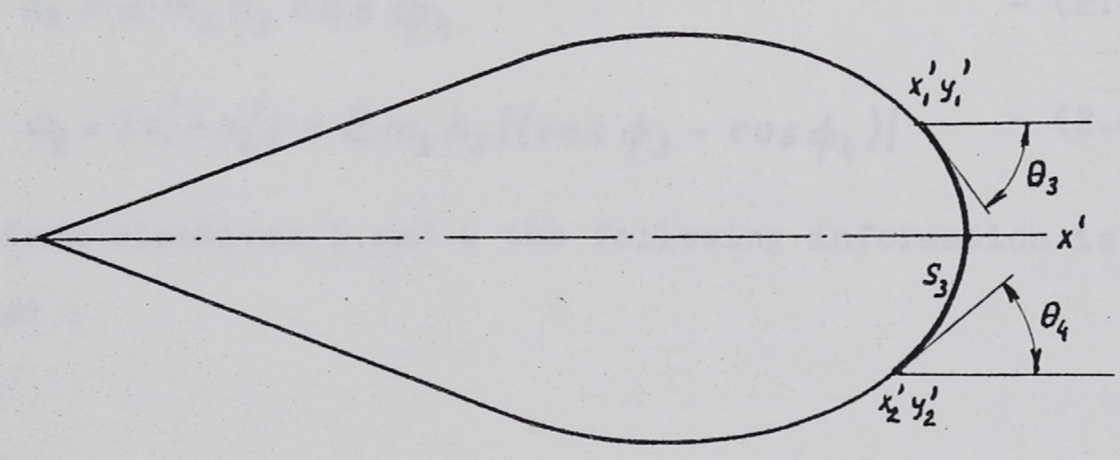

Fig. 2.4.6

$$y_1' = k_3[2E(m_3,\phi_3) - F(m_3,\phi_3)] \qquad - (2.4.18)$$

$$y_2' = k_3[2E(m_3,\phi_4) - F(m_3,\phi_4)] \qquad - (2.4.19)$$

Here it is assumed that the distortion of the boom is not such that the part of the elastica in consideration would completely lie above or below the axis x'.

If that was the case, than in the next equation the y' coordinates would have to be subtracted from each other.

$$b_2 = y_1' + y_2' = k_3\{2[E(m_3,\phi_3) + E(m_3,\phi_4)] -$$

$$-[F(m_3,\phi_3) - F(m_3,\phi_4)]\} \qquad - (2.4.20)$$

$$x_1' = 2\, m_3 k_3 \cos\phi_3 \qquad - (2.4.21)$$

$$x_2' = 2\, m_3 k_3 \cos\phi_4 \qquad - (2.4.22)$$

$$a_2 = |x_1' - x_2'| = 2\, m_3 k_3 |(\cos\phi_3 - \cos\phi_4)| \qquad - (2.4.23)$$

Now from elasticas 1 and 4 the following information is obtained :

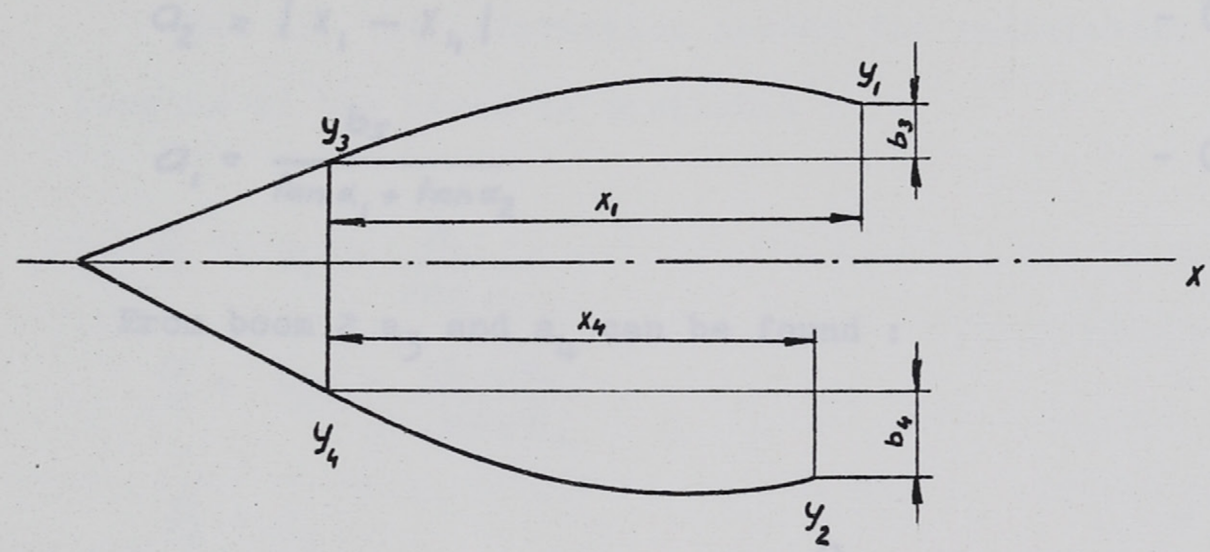

Fig. 2.4.7

$$y_1 = k_1\left\{2E(m_1,\phi_1) - F(m_1,\phi_1)\right\} \qquad - (2.4.24)$$

$$y_3 = k_1\left\{2E(m_1,\tfrac{\pi}{2}) - F(m_1,\tfrac{\pi}{2})\right\} \qquad - (2.4.25)$$

$$y_2 = k_4\left\{2E(m_4,\phi_5) - F(m_4,\phi_5)\right\} \qquad - (2.4.26)$$

$$y_4 = k_4\left\{2E(m_4,\tfrac{\pi}{2}) - F(m_4,\tfrac{\pi}{2})\right\} \qquad - (2.4.27)$$

$$x_1 = 2\,m_1 k_1 \cos\phi_1 \qquad - (2.4.28)$$

$$x_4 = 2\,m_4 k_4 \cos\phi_5 \qquad - (2.4.29)$$

$$b_3 = y_1 - y_3 \qquad - (2.4.30)$$

$$b_4 = y_2 - y_4 \qquad - (2.4.31)$$

$$b_5 = b_2 - b_3 - b_4 \qquad - (2.4.32)$$

$$a_2 = | x_1 - x_4 | \qquad - (2.4.33)$$

$$a_1 = \frac{b_5}{\tan\alpha_1 + \tan\alpha_2} \qquad - (2.4.34)$$

From boom 2 a_3 and a_4 can be found :

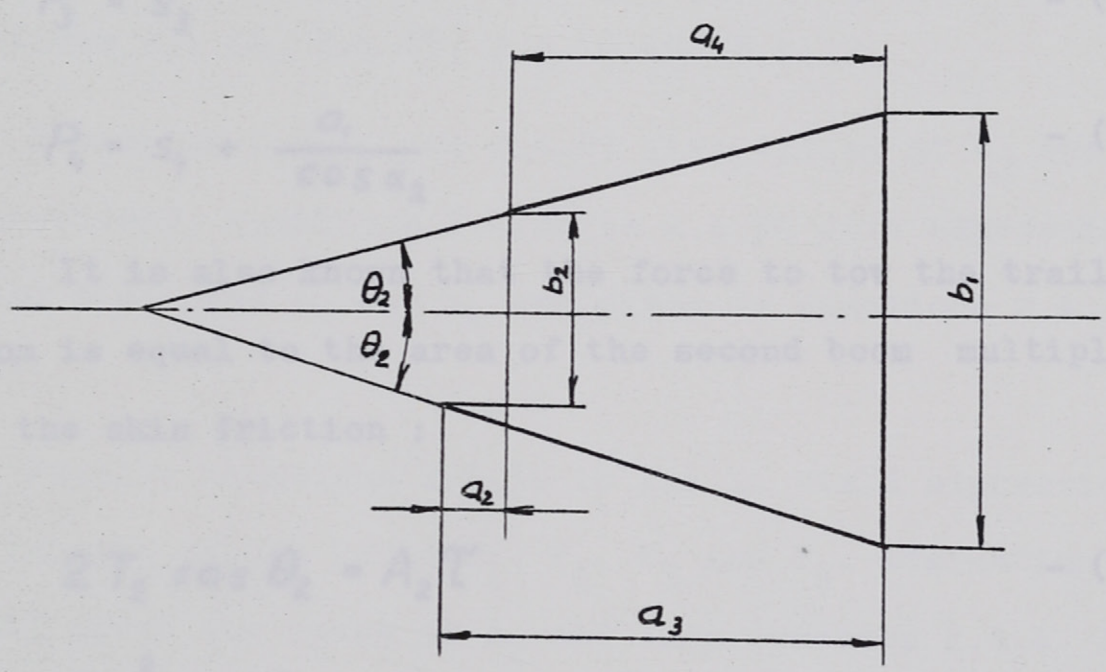

Fig. 2.4.8

$$b_1 = 2k_2\left[2E(m_2, \tfrac{\pi}{2}) - F(m_2, \tfrac{\pi}{2})\right] \qquad - (2.4.35)$$

Two equations of two unknowns are obtained :

$$a_3 = a_2 + a_4 \qquad - (2.4.36)$$

$$b_1 - a_3 \tan\theta_2 - a_4 \tan\theta_2 = b_2 \qquad - (2.4.37)$$

Hence a_3 and a_4 can be solved for.

Now all the information needed for calculating the lengths of the booms is available :

$$P_1 = s_1 + \frac{a_1}{\cos \alpha_1} \qquad - (2.4.38)$$

$$P_2 = s_2 + \frac{a_3 + a_4}{\cos \theta_2} \qquad - (2.4.39)$$

$$P_3 = s_3 \qquad - (2.4.40)$$

$$P_4 = s_4 + \frac{a_1}{\cos \alpha_2} \qquad - (2.4.41)$$

It is also known that the force to tow the trailing boom is equal to the area of the second boom multiplied by the skin friction :

$$2 T_2 \cos \theta_2 = A_2 \tau \qquad - (2.4.42)$$

$$\text{or: } 2 k_2^2 \cos \theta_2 = A_2 \qquad - (2.4.43)$$

The total area can be calculated using the equation 2.4.4. Hence the relationship between the geometry of the boom and the five nondimensional parameters $\frac{P_1}{\sqrt{A}}$, $\frac{P_2}{\sqrt{A}}$, $\frac{P_3}{\sqrt{A}}$, $\frac{P_4}{\sqrt{A}}$ and $\frac{A_1}{A_2}$ has been determined.

It does not seem possible to solve for the particular angles from these equations. However, if the procedure is reversed, for assumed values of α_1, α_2, α_3, θ_1 and θ_3 the five nondimensional parameters can be obtained in the following manner :

From equations 2.4.5, 2.4.8, 2.4.9, 2.4.12, 2.4.13 and from equating expressions for a_2 from equations 2.4.23 and 2.4.33 the values of $\frac{k_2}{k_1}$, $\frac{k_3}{k_1}$, $\frac{k_4}{k_1}$, θ_2, θ_4 and θ_5 are calculated. Then lengths of elasticas s_1, s_2, s_3 and s_4 are obtained from equations 2.4.14, 2.4.15, 2.4.16 and 2.4.17.

Equations 2.4.20 and 2.4.23 give values of b_2 and a_2 respectively, equations 2.4.24 to 2.4.34 give expressions for a_1 and distances a_3 and a_4 are given by equations 2.4.35 to 2.4.37.

Finally equations 2.4.38 to 2.4.41 yield expressions for lengths of the booms P_1, P_2, P_3 and P_4. Evaluation of 2.4.43 gives the value of A_2 and from equation 2.4.4 A_1 is obtained. Hence values of $\frac{A_1}{A_2}$, $\frac{P_1}{\sqrt{A}}$, $\frac{P_2}{\sqrt{A}}$, $\frac{P_3}{\sqrt{A}}$ and $\frac{P_4}{\sqrt{A}}$ are determined.

As can be seen from equation 2.4.4, the total area $A_1 + A_2$ varies proportionally with k_1^2. Therefore $\sqrt{A}$ is proportional to k_1. Also from equations 2.4.14, 2.4.15, 2.4.16 and 2.4.17 s_1, s_2, s_3, s_4 and therefore also p_1, p_2, p_3 and p_4 are proportional to k_1 since they can be written as

$$S_2 = 2k_1\left(\frac{k_2}{k_1}\right)F\left(m_2, \frac{\pi}{2}\right) \qquad \text{etc}$$

then

$$\frac{A_1}{A_2} = \frac{k_1^2(\cdots)}{k_1^2(\cdots)} \; ; \quad \frac{P_1}{\sqrt{A}} = \frac{k_1(\cdots)}{k_1(\cdots)} \; ; \quad \frac{P_2}{\sqrt{A}} = \frac{k_1(\cdots)}{k_1(\cdots)} \qquad \text{etc}$$

therefore if $\frac{k_2}{k_1}$, $\frac{k_3}{k_1}$ and $\frac{k_4}{k_1}$ are calculated, the value of k_1 itself does not change any of the five nondimensional parameters. In the computer program it is not carried along and values are calculated as if k_1 is equal to 1.

3. DISCUSSION

The theory presented in this paper was developped for an unsymmetrical case. However, in the first part of the computer program a system of nonlinear equations has to be solved. To obtain the solution, the external subprogram 'XUX' is used. It employs the Brown's method to give the results. Initial values of the unknowns have to be guessed. They are then compared and new values are assigned to the unknowns by the computer untill the solution is obtained.

Unfortunately, two of the unknowns appear as parts of the argument of the arcsin function. The subprogram chooses the new values of the unknowns regardless of the fact that the argument of arcsin must be less or equal to one. When the program was run it was always terminated prematurely because the argument of arcsin was greater than one. No other method for solving a system of nonlinear equations was available on the computer and the subprogram presently used could not have been modified. In the future surely some other method can be made available, which avoids this problem or the 'XUX' (actually listed as 'AUX') subprogram can be modified by someone with enough expertise in computer programing.

Since the unsymmetrical solution could not have been obtained, the symmetrical cases are studied. The angle α_2 is put equal to α_1 and a little modification in the subprogram 'XUX' is needed.

Cases that are studied are listed in the table 3.1. One 'standard' case is picked and then the given parameters are changed one by one to obtain the response of the shape to these changes. Since the procedure is reversed, i.e. the values of the angles are assumed, when in reality the lengths of the boom and areas of the rafts would be given, some of the chosen values are not possible in reality. In this case the program is terminated prematurely and a message explaining the problem is printed out.

The shape of the boom when $\alpha_1 = 0.4$ rad., $\alpha_3 = 0.5$ rad., $\theta_1 = 0.733$ rad. and $\theta_3 = 1.025$ rad. is chosen as a 'standard' case and is presented in Fig. 1. The area A_2 of the trailing raft is about one half of the area A_1 and the leading edge of the wood is fairly close to the towing point (or in the case of the second raft it follows quite close behind the boom of the first raft). The length of the boom P_3 (spacing between points M and N) is about 1/3 of P_1. This would be the most probable way of attaching the trailing boom.

The first angle that is varied is θ_3. The shape proves to be very sensitive to even very slight changes. Especially the distance by which the second raft trails the first one i.e. the length of the boom P_2 varies greatly. It is very closely related to changes in the angle θ_1, as is found in latter cases. When the angle θ_1 increases, the trailing boom moves farther and farther back as can be seen from figures 3.2 and 3.3 when θ_3 was chosen 1.03 rad. and 1.1 rad., respectively. Physically, increasing the angle θ_3 means that the tension in the boom P_2 takes on larger value of the reaction to the tension in the boom 1, since the direction of the tension in the boom 3 moves closer to being perpendicular to the direction of the tension in the boom 1. This could be done only by making angle θ_2 smaller, while keeping the area A_2 unchanged, which is indeed the case.

When θ_3 is made smaller, the second raft moves closer to the first one, till it actually becomes embedded in it, which is obviously impossible in reality.

Interesting results are obtained, when the angle α_3 is varied. The shapes of the booms when α_3 assumes values of 0.6 radians and 1.05 radians are indicated on Fig. 3.4 and 3.5, respectively. The shape is almost insensitive to the changes in the range from 0.5 radians to 1.05 radians. The leading edges of the wood are almost in the same place and points of attachement of the second raft remain approximately the same. The shape only lengthens very slightly as α_3 increases.

The ratio of the areas A_1 and A_2 proves to be sensitive to changes in θ_1 as indicated in Fig. 3.6 and 3.7 where θ_1 is changed to 0.73 rad. and 0.72 rad., respectively. However, this change has to be accompanied by appropriate changes in θ_3, otherwise solutions which are not realistic are obtained in most cases. It is found, that as θ_1 decreases, θ_3 must be made larger and vice versa.

Physically, an increase in the tension in the boom P_2 causes the angle θ_1 to decrease. Since the tension in the boom is directly proportional to the square root of the area of the raft, it is expected that A_2 would also increase. That is indeed the case, as indicated in Fig. 3.6 and 3.7, where A_2 increases to 2/3 of A_1 in the first case and approximately equals A_1 in the latter.

Finally the same values are assumed as in Fig. 3.7 but α_1 is changed to 0.39 radians - Fig. 3.8. The areas remained in approximately the same ratio, only the boom P_3

became longer. The leading edge of the wood in the leading raft moved back, which is exactly what we would expect, since a decrease in value of α_1, is caused either by smaller area of the leading raft or by greater distance between the raft and the point of towing.

4. CONCLUSION

The general theory has been extended to predict the shape of the tandem of booms, towed by a single boat. With six assumptions, the shape of the rafts is shown to consist of a single-parameter family of elasticas joined by straight lines to the point of towing or to the attachement.

Four of these assumptions are probably acceptable without questioning. The remaining two, however, are more doubtful. Neglecting the shear stresses within the continuum and the constancy of the hydrodynamic skin friction have not yet been checked experimentally. If the skin friction of the second raft is less due to turbulent conditions, then compared to the reality the theory would tend to predict smaller area of the trailing raft for the same tension in the boom.

However, two fullscale measurements and the model tests showed that the theory is capable of predicting the shape of a boom with a good accuracy. The model tests were made using the facilities of the Department of Civil Engineering at Queen's University, Ontario. The logs were 3/8 ins. in diameter and 2 3/8 ins. long cut from rough oak dowel. The boomsticks were 6 7/10 ins. long. The details of the tests are described in (1). The fullscale test was made in the Baie de Chaleur in very light winds. The geometry of a single boom very closely agreed with that predicted by the theory. The details of the survey can be found in (4).

Unfortunately, no accurate survey has been made for a tandem of booms. The only accessible survey of the two rafts is Fig. 4.1. However, the surveying was done in a manner that did not allow good accuracy. Apparatus was held in hand by men stationed on the booms that were continually moving. The resulting shape can hardly be very accurate, in fact, it is quite doubtful. The best method to find suitable fullscale comparisons is by taking aerial photographs. In the future it would be desirable to do so as a more stringent test of the assumptions.

In the future the theory can be further extended to predict the shape of the rafts, when wood is transported in bundles - usually five or six bundles joined together. If the assumption of a constant skin friction is dropped, then the theory can be utilized in predicting the shape when collecting oil slicks on the water surfaces.

ACKNOWLEDGEMENTS

I would like to thank professor B.G.Newman for his helpful suggestions and discussions concerning this work. I also want to thank Mrs L.R.Prasil for her help with typing this paper.

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3. Timoshenko, S.P. and Gere, J.M. - "Theory of Elastic Stability", Mc Graw Hill, p. 76-82, 1961.

4. Lazier, S.S. and Kennedy, R.J. - "The Water Transportation of Pulpwood IV. Moving Wood over Slack and Shallow Water", Pulp and Paper Research Institute of Canada, Technical Report 509, April 1967.

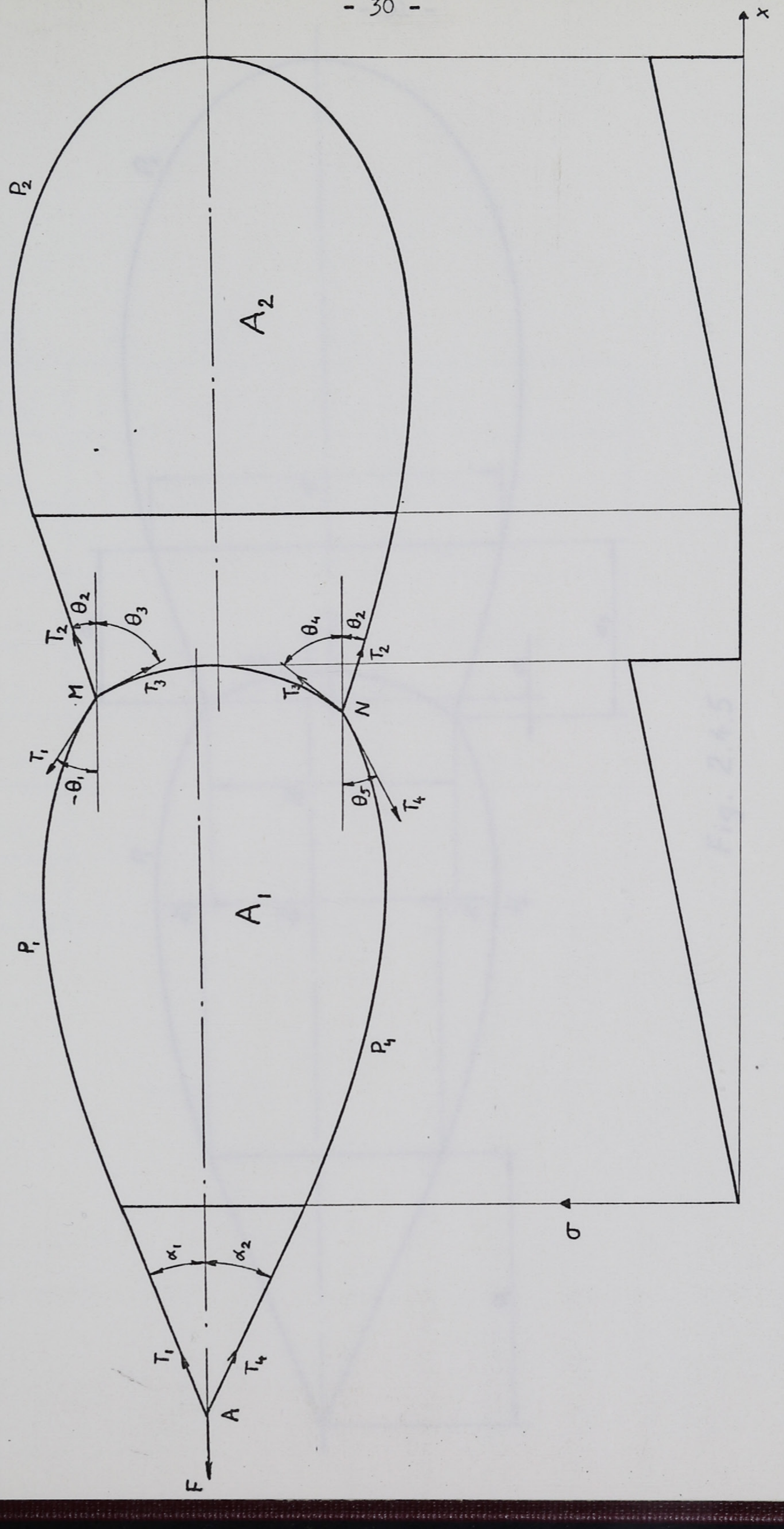
x
σ
F
A
P_1
P_2
P_4
A_1
A_2
M
N
T_1
T_2
T_3
T_4
$-\theta_1$
θ_2
θ_3
θ_4
θ_5
α_1
α_2

Fig. 2.4.1

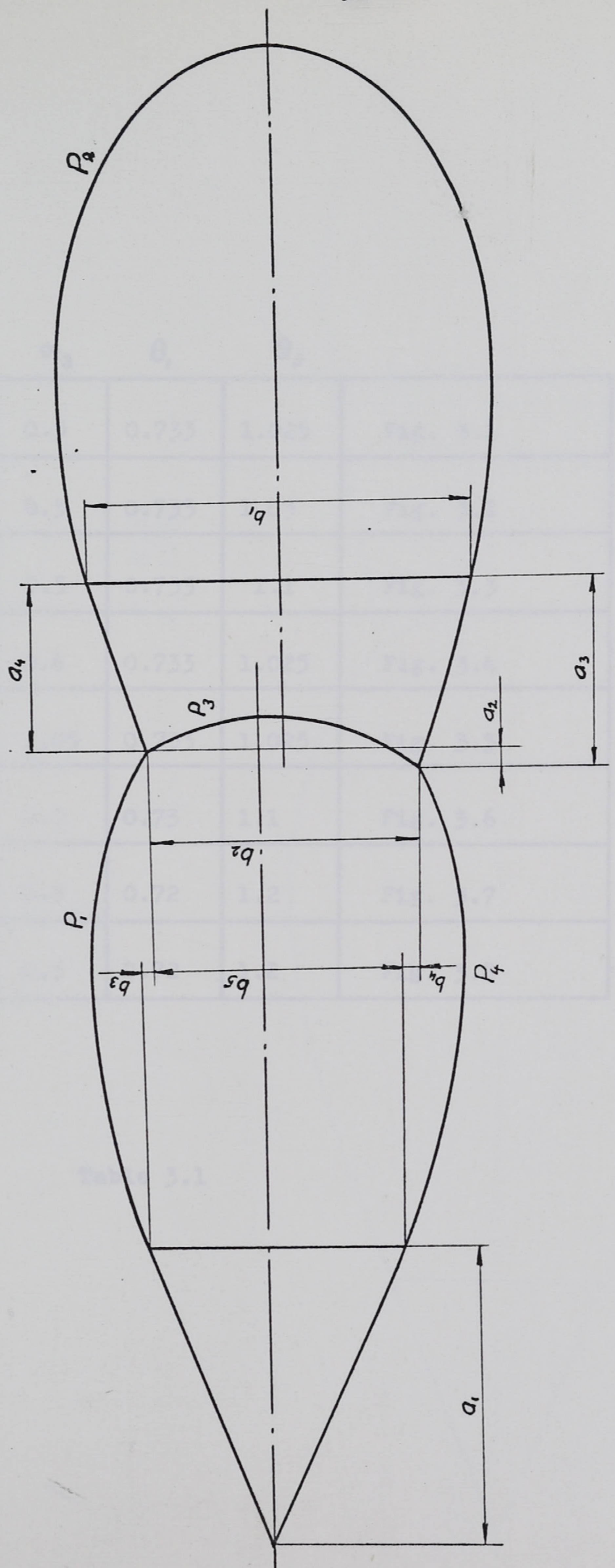

Fig. 2.4.5

α_1	α_3	θ_1	θ_3	
0.4	0.5	0.733	1.025	Fig. 3.1
0.4	0.5	0.733	1.03	Fig. 3.2
0.4	0.5	0.733	1.1	Fig. 3.3
0.4	0.6	0.733	1.025	Fig. 3.4
0.4	1.05	0.733	1.025	Fig. 3.5
0.4	0.5	0.73	1.1	Fig. 3.6
0.4	0.5	0.72	1.2	Fig. 3.7
0.39	0.5	0.72	1.2	Fig. 3.8

Table 3.1

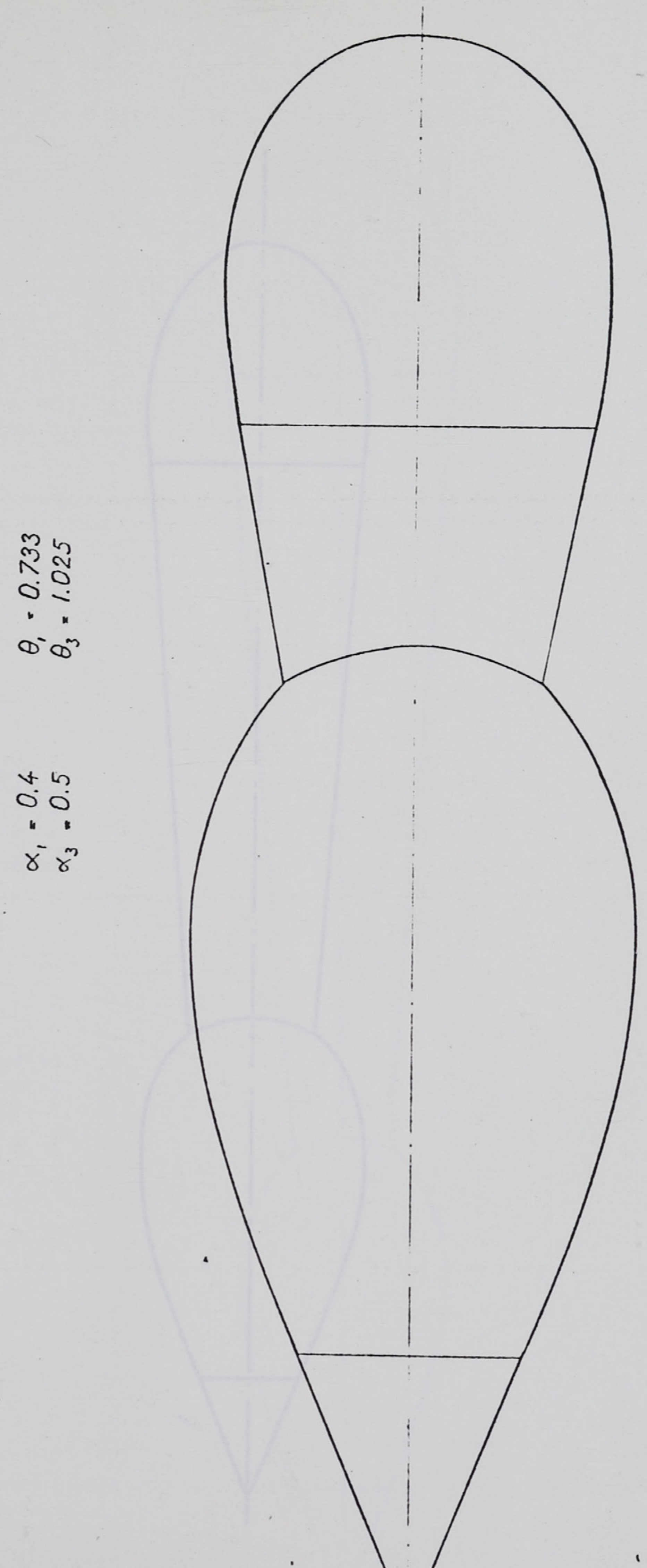

Fig. 3.1

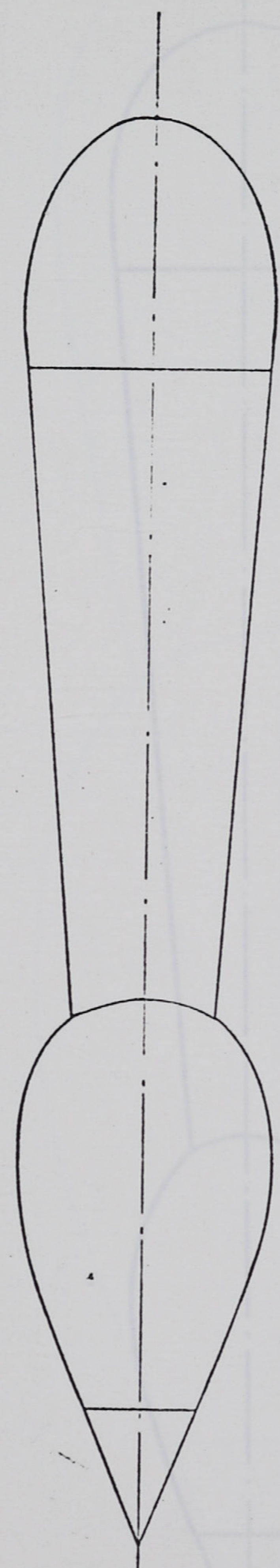

Fig. 3.2

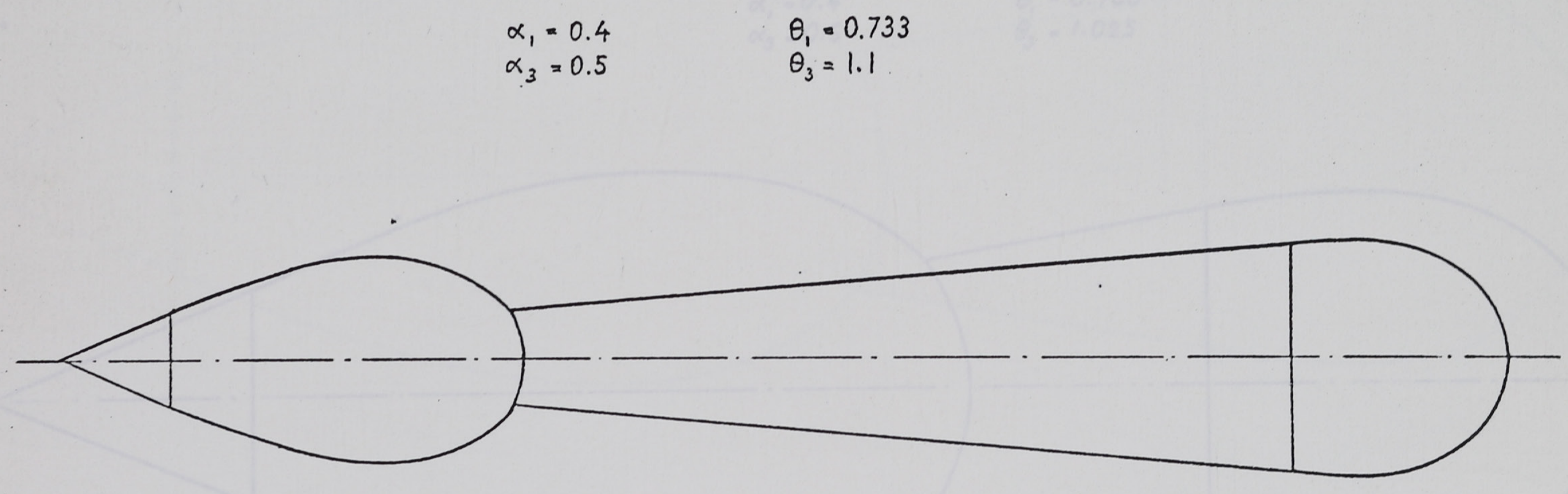

Fig. 3.3

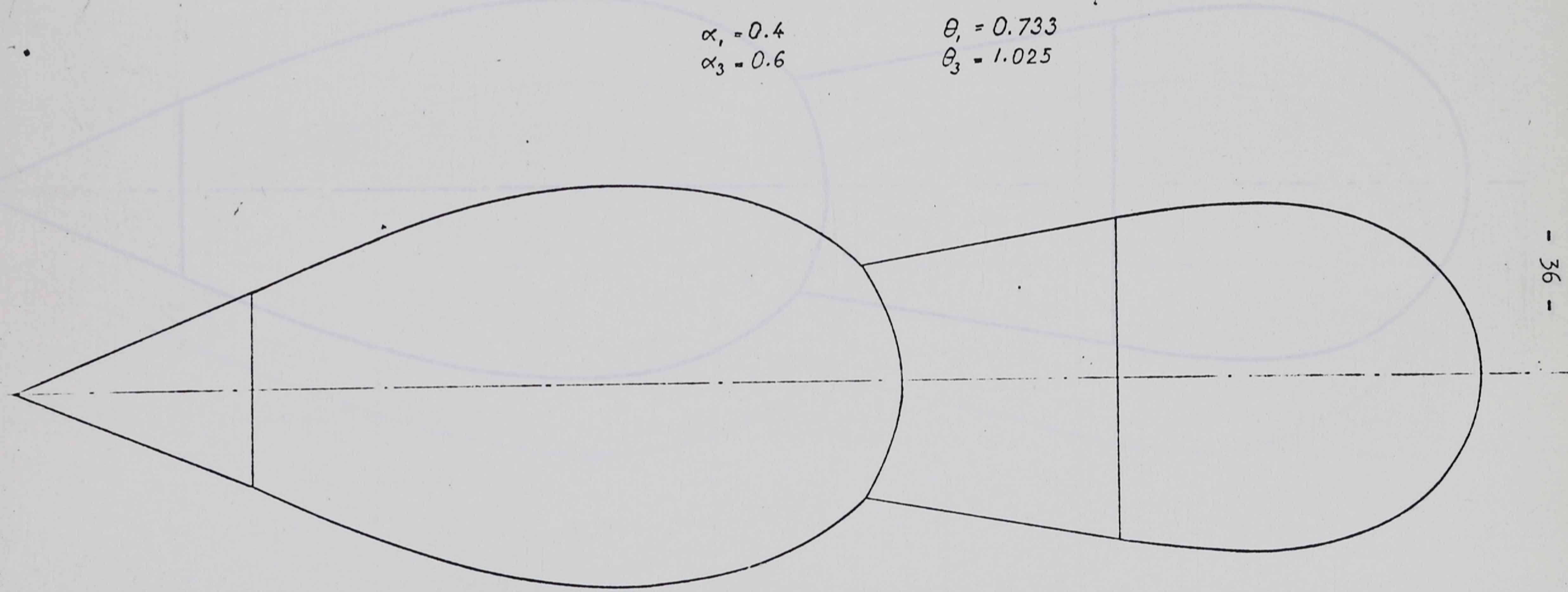

Fig. 3.4

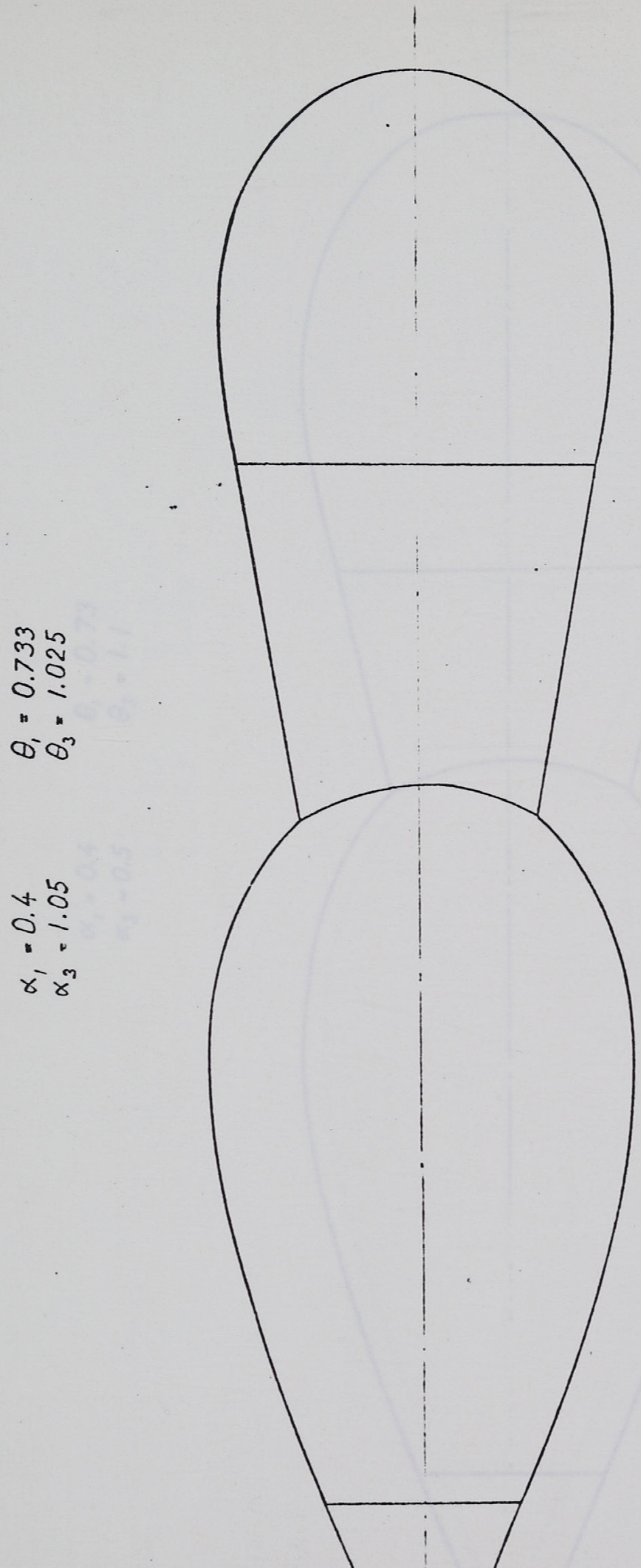

Fig. 3.5

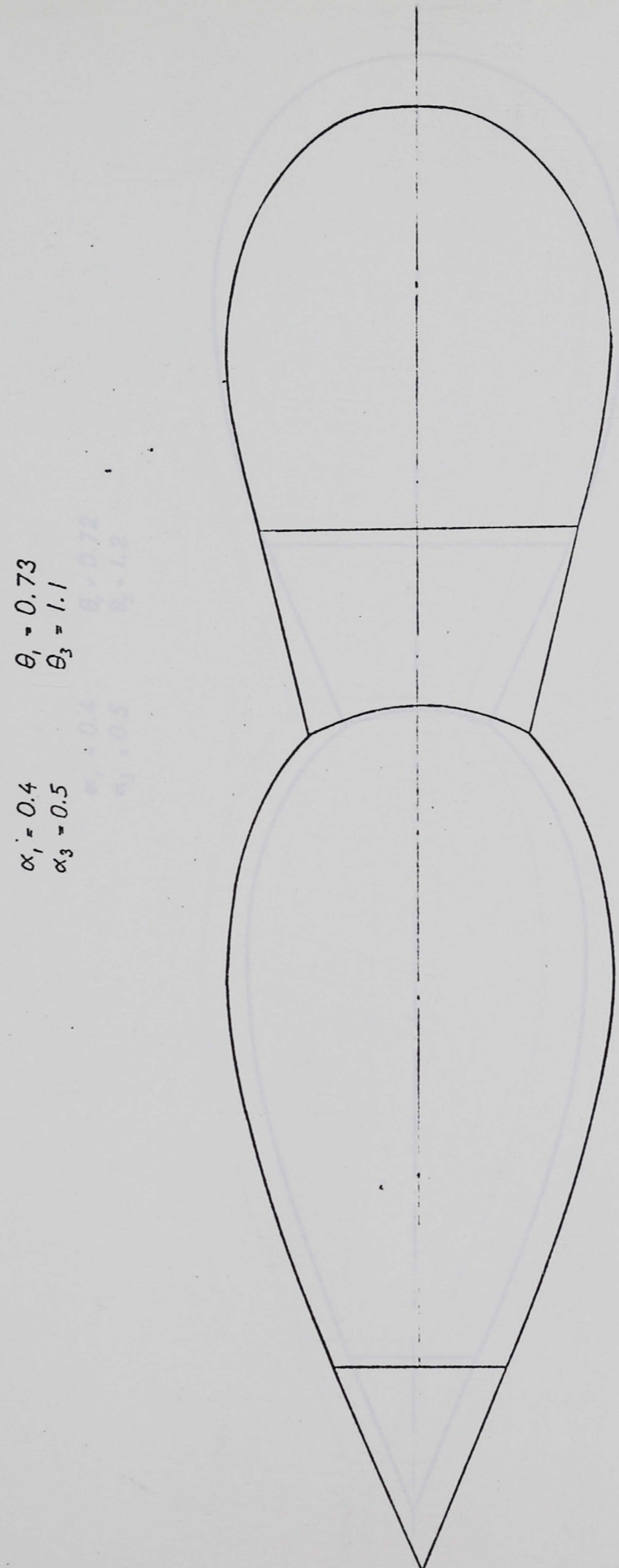

Fig. 3.6

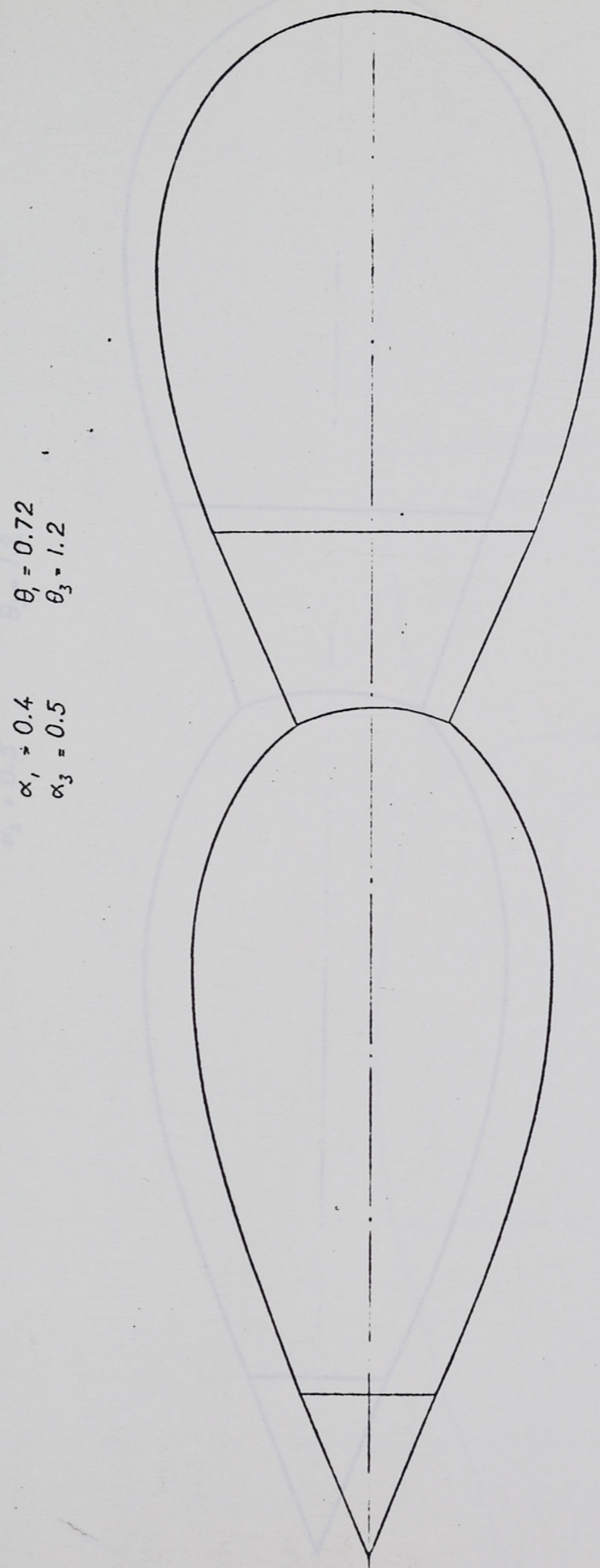

Fig. 3.7

$\alpha_1 = 0.39$ $\theta_1 = 0.72$

$\alpha_3 = 0.5$ $\theta_3 = 1.2$

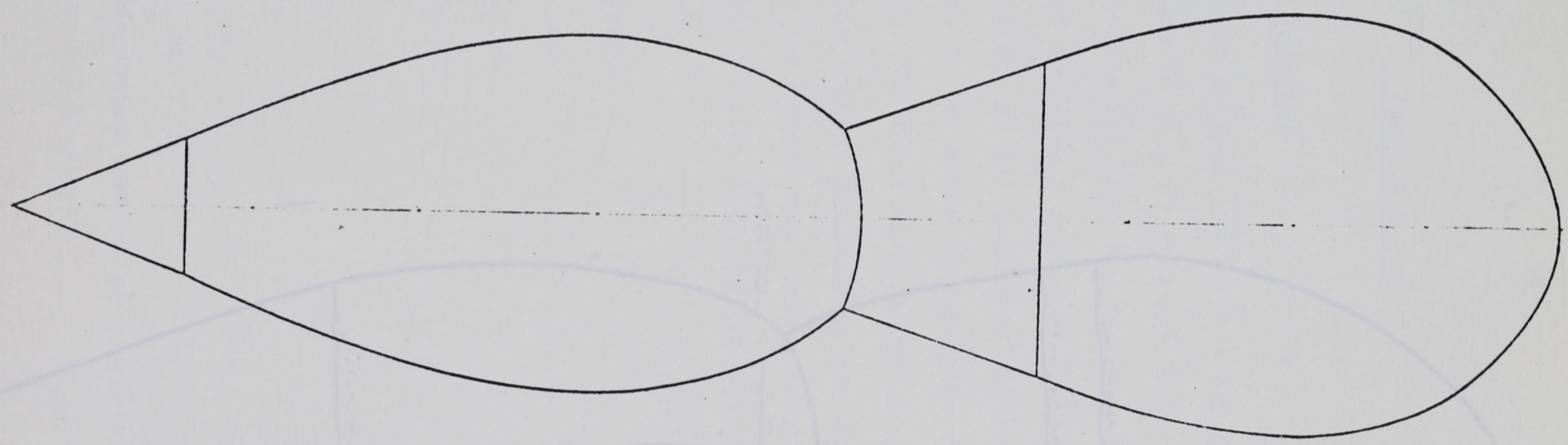

Fig. 3.8

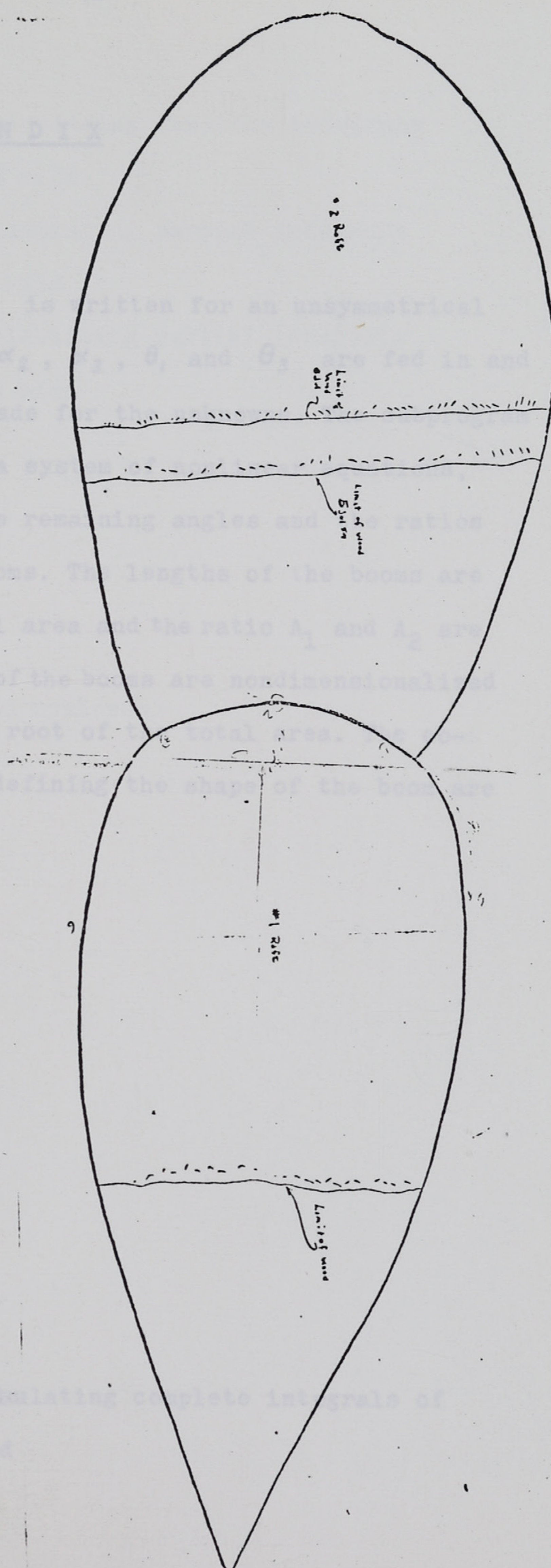

Fig. 4.1

APPENDIX

Computer Program.

The computer program is written for an unsymmetrical case. The values of α_1 , α_2 , α_3 , θ_1 and θ_3 are fed in and the initial guesses are made for the unknowns. The subprogram 'XUX' is called to solve a system of nonlinear equations, yielding the values of the remaining angles and the ratios of the tensions in the booms. The lengths of the booms are calculated next. The total area and the ratio A_1 and A_2 are obtained and the lengths of the booms are nondimensionalized by dividing by the square root of the total area. The coordinates of the points, defining the shape of the boom are calculated last.

Nomenclature :

A1 : α_1 etc

AD1 : a_1 etc

AM1 : m_1 etc

AR : A

AR1 : A_1

AR2 : A_2

ARR : A_1/A_2

B1 : b_1 etc

CEL1 : subroutine calculating complete integrals of the first kind

CEL2	:	subroutine calculating complete integrals of the second kind
ELI1	:	subroutine calculating partial integrals of the first kind
ELI2	:	subroutine calculating partial integrals of the second kind
P1	:	P_1 etc
PH1	:	ϕ_1 etc
S1	:	s_1
SA	:	$\sqrt{A}$
TH1	:	θ_1 etc
U(i)	:	the calculated unknowns
X(i)	:	initial values
X,Y	:	coordinates

```
$BATCH WATFIV Y400000 700971-PRASIL(TIME,PAGES)
*****************************************************************************************

$WATFIV Y400000 700971 PRASIL,TIME=10,PAGES=10
C
C
C      ***THE SHAPE OF THE TANDEM BOOM OF LOGS***
C
C
C
C      ******************************************************************
C
C      THIS PROGRAM PREDICTS THE SHAPE OF AN UNSYMMETRICAL TANDEM OF
C      BOOMS. FIRST, IT USES THE EXTERNAL SUBPROGRAM IXUXI TO SOLVE
C      A SYSTEM OF NONLINEAR EQUATIONS TO OBTAIN THE VALUES OF THE
C      REMAINING ANGLES; THEN IT CALCULATES THE RATIO OF THE TWO AREAS
C      AND GIVES THE RATIOS OF THE LENGTHS OF THE BOOMS TO THE SQUARE
C      ROOT OF THE TOTAL AREA. FINALLY IT PROCEEDS TO CALCULATE AND PRINT
C      OUT THE COORDINATES OF THE POINTS OF THE BOOMS.
C
C      ******************************************************************
C
C
       IMPLICIT REAL*8(X,W)
       EXTERNAL XUX
       DIMENSION X(7),WA(38),U(7)
100    FORMAT(2F12.3)
C
C
C      INITIAL VALUES
C
C
       X(1)=1.D0
       X(2)=5.D0
       X(3)=7.D0
       X(4)=7.1D0
       X(5)=3.3D-1
       X(6)=0.7D0
       XPS=1.D-3
       NSIG=3
       N=6
       ITMAX=100
C
C
C      SOLUTION OF SYSTEM OF NONLINEAR EQUATIONS
C
C
       CALL ZSYSTM(XUX,XPS,NSIG,N,X,ITMAX,WA,IER)
       U(2)=X(2)
       U(3)=X(3)
       U(4)=X(4)
       U(5)=X(5)
       U(6)=X(6)
       U(7)=X(1)
       A1=0.41
       A2=0.39
       A3=.5
       TH1=0.733
       TH3=1.025
C
C
C      CALCULATION OF PARAMETERS
C
```

```
C
      AM1=SQRT((1.+SIN(A1))/2.)
      AM2=SQRT((1.+SIN(U(5)))/2.)
      AM3=SQRT((1.+SIN(A3))/2.)
      AM4=SQRT((1.+SIN(A2))/2.)
      PH1=ARSIN(SQRT((1.+SIN(-TH1))/(1.+SIN(A1))))
      PH3=ARSIN(SQRT((1.+SIN(-TH3))/(1.+SIN(A3))))
      PH4=ARSIN(SQRT((1.+SIN(-U(7)))/(1.+SIN(A3))))
      PH5=ARSIN(SQRT((1.+SIN(-U(6)))/(1.+SIN(A2))))
      CALL CEL1(FC1,AM1,IE1)
      ZT1=TAN(PH1)
      ZT3=TAN(PH3)
      ZT4=TAN(PH4)
      ZT5=TAN(PH5)
      CM11=(1.-SIN(A1))/2.
      CM1=SQRT(CM11)
      CM44=(1.-SIN(A2))/2.
      CM4=SQRT(CM44)
      CM33=(1.-SIN(A3))/2.
      CM3=SQRT(CM33)
      CM22=(1.-SIN(U(5)))/2.
      CM2=SQRT(CM22)
C
C
C     CALCULATION OF THE LENGTHS OF BOOMS
C
C
      CALL ELI1(F1,ZT1,CM1)
      S1=FC1-F1
      CALL CEL1(FC2,AM2,IE2)
      S2=U(2)*2.*FC2
      CALL CEL1(FC4,AM4,IE4)
      CALL ELI1(F4,ZT5,CM4)
      S4=U(4)*(FC4-F4)
      CALL ELI1(F31,ZT3,CM3)
      CALL ELI1(F32,ZT4,CM3)
      P3=U(3)*(F31+F32)
      CC=1.
      CALL ELI2(E31,ZT3,CM3,CC,CM33)
      CALL ELI2(E32,ZT4,CM3,CC,CM33)
      Y13=U(3)*(2.*E31-F31)
      Y23=U(3)*(2.*E32-F32)
      B2=Y13+Y23
      AD2=2.*AM3*U(3)*ABS(COS(PH3)-COS(PH4))
      CALL ELI2(E1,ZT1,CM1,CC,CM11)
      CALL ELI2(E4,ZT5,CM4,CC,CM44)
      CALL CEL2(EC1,AM1,CC,CM11,IE3)
      CALL CEL2(EC4,AM4,CC,CM44,IE5)
      Y11=2.*E1-F1
      Y31=2.*EC1-FC1
      Y24=U(4)*(2.*E4-F4)
      Y44=U(4)*(2.*EC4-FC4)
      B3=Y11-Y31
      B4=Y24-Y44
      B5=B2-B3-B4
      AD1=B5/(TAN(A1)+TAN(A2))
      P1=S1+AD1/COS(A1)
      P4=S4+AD1/COS(A2)
      CALL CEL2(EC2,AM2,CC,CM22,IE6)
      B1=2.*U(2)*(2.*EC2-FC2)
```

```
      AD4=(B1-B2-AD2*TAN(U(5)))/2./TAN(U(5))
   11 AD3=AD2+AD4
      P2=S2+(AD3+AD4)/COS(U(5))
C
C
C     CALCULATION OF THE AREAS
C
C
      AR2=2.*U(2)*U(2)*COS(U(5))
      AR=COS(A1)+U(4)*U(4)*COS(A2)
      AR1=AR-AR2
      SA=SQRT(AR)
      SP1=P1/SA
      SP2=P2/SA
       SP3=P3/SA
      SP4=P4/SA
      ARR=AR1/AR2
      PRINT,' '
      PRINT,'         REL. LENGTH OF BOOM P1  ',SP1
      PRINT,'         REL. LENGTH OF BOOM P2  ',SP2
      PRINT,'         REL. LENGTH OF BOOM P3  ',SP3
      PRINT,'         REL. LENGTH OF BOOM P4  ',SP4
      PRINT,'         RATIO OF AREAS  ',ARR
      PRINT,' '
      PRINT,'         X-COORD.     Y-COORD.'
C
C
C     PLOTTING OF BOOM 1
C
C
      PRINT,'         BOOM 1 '
      Y=Y31/SA
      D=Y-AD1/SA*TAN(A1)
      Y=Y-D
      SX=AD1/SA
      WRITE(6,100)SX,Y
      T=A1-0.1
   10 PH=ARSIN(SQRT((1.+SIN(T))/(1.+SIN(A1))))
      ZT=TAN(PH)
      CALL ELI1(P1,ZT,CM1)
      CALL ELI2(P2,ZT,CM1,CC,CM11)
      PY=(2.*P2-P1)/SA-D
      PX=2.*AM1*COS(PH)/SA+AD1/SA
      WRITE(6,100)PX,PY
      T=T-0.1
      IF(T.LT.-TH1) GO TO 20
      GO TO 10
   20 Y1=Y11/SA-D
      SX1=2.*AM1*COS(PH1)/SA+AD1/SA
      WRITE(6,100)SX1,Y1
C
C
C     PLOTTING OF BOOM 2
C
C
      PRINT,'         UPPER BRANCH OF SYMMETRICAL BOOM 2  '
      CALL CEL2(EC2,AM2,CC,CM22,IE7)
      Y2P=U(2)*(2.*EC2-EC2)/SA
      D=Y2P-Y1-AD4*TAN(U(5))/SA
      SY2=Y2P-D
```

```
      SX2=SX1+AD4/SA
      WRITE(6,100)SX2,SY2
      T=U(5)-0.1
 30   PH=ARSIN(SQRT((1.+SIN(T))/(1.+SIN(U(5)))))
      ZT=TAN(PH)
      CALL ELI1(P1,ZT,CM2)
      CALL ELI2(P2,ZT,CM2,CC,CM22)
      PY=U(2)*(2.*P2-P1)/SA-D
      PX=2.*AM2*U(2)*COS(PH)/SA+SX2
      WRITE(6,100)PX,PY
      T=T-0.1
      IF(T.LT.-1.57) GO TO 40
      GO TO 30
 40   PY2=-D
      PX2=2.*AM2*U(2)/SA+SX2
      WRITE(6,100)PX2,PY2
C
C
C     PLOTTING OF BOOM 4
C
C
      PRINT,'          BOOM 4  '
      Y=Y44/SA
      D=Y-AD1/SA*TAN(A2)
      Y=-Y+D
      SX=AD1/SA
      WRITE(6,100)SX,Y
      T=A2-0.1
 50   PH=ARSIN(SQRT((1.+SIN(T))/(1.+SIN(A2))))
      ZT=TAN(PH)
      CALL ELI1(P1,ZT,CM4)
      CALL ELI2(P2,ZT,CM4,CC,CM44)
      PY=-U(4)*(2.*P2-P1)/SA+D
      PX=2.*AM4*U(4)*COS(PH)/SA+AD1/SA
      WRITE(6,100)PX,PY
      T=T-0.1
      IF(T.LT.-U(6)) GO TO 60
      GO TO 50
 60   Y4=-Y24/SA+D
      SX4=2.*AM4*U(4)*COS(PH5)/SA+AD1/SA
      WRITE(6,100)SX4,Y4
C
C
C     PLOTTING OF BOOM 3
C
C
      D=Y13/SA-Y1
      AD=2.*AM3*U(3)*COS(PH3)/SA-SX1
      T=-TH3-0.1
      PRINT,'          BOOM 3  '
 70   PH=ARSIN(SQRT((1.+SIN(T))/(1.+SIN(A3))))
      ZT=TAN(PH)
      CALL ELI1(P1,ZT,CM3)
      CALL ELI2(P2,ZT,CM3,CC,CM33)
      PY=U(3)*(2.*P2-P1)/SA-D
      PX=2.*AM3*U(3)*COS(PH)/SA-AD
      WRITE(6,100)PX,PY
      T=T-0.1
      IF(T.LT.-1.57) GO TO 80
      GO TO 70
```

```
 80     PY3=-D
        PX3=2.*AM3*U(3)/SA-AD
        WRITE(6,100)PX3,PY3
        D=-Y23/SA-Y4
        AD=2.*AM3*U(3)*COS(PH4)/SA-SX4
        T=-U(7)-0.1
 90     PH=ARSIN(SQRT((1.+SIN(T))/(1.+SIN(A3))))
        ZT=TAN(PH)
        CALL ELI1(P1,ZT,CM3)
        CALL ELI2(P2,ZT,CM3,CC,CM33)
        PY=-U(3)*(2.*P2-P1)/SA+D
        PX=2.*AM3*U(3)*COS(PH)/SA-AD
        WRITE(6,100)PX,PY
        T=T-0.1
        IF(T.LT.-1.57) GO TO 95
        GO TO 90
 95     CONTINUE
        STOP
        END
C
C
C       ****************************************************************
C
C       THIS IS AN EXTERNAL SUBPROGRAM FOR SOLVING A SYSTEM OF NONLINEAR
C       EQUATIONS USING BROWN'S METHOD.
C
C       ****************************************************************
C

        DOUBLE PRECISION FUNCTION XUX(X,K)
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION X(1)
        A1=0.41D0
        A2=0.39D0
        A3=5.D-1
        TH1=0.733D0
        TH3=1.025D0
        AM1=DSQRT((1.D0+DSIN(A1))/2.D0)
        AM4=DSQRT((1.D0+DSIN(A2))/2.D0)
        AM3=DSQRT((1.D0+DSIN(A3))/2.D0)
        PH1=DARSIN(DSQRT((1.D0+DSIN(-TH1))/(1.D0+DSIN(A1))))
        PH3=DARSIN(DSQRT((1.D0+DSIN(-TH3))/(1.D0+DSIN(A3))))
        GO TO (1,2,3,4,5,6),K
 1      XUX=AM3*X(3)*DABS(DCOS(PH3)-DCOS(DARSIN(DSQRT((1.D0+DSIN(-X(1)))/
       1(1.D0+DSIN(A2))))))-DABS(AM1*DCOS(PH1)-AM4*X(4)*
       2DCOS(DARSIN(DSQRT((1.D0+DSIN(-X(6)))/(1.D0+DSIN(A2))))))
        RETURN
 2      XUX=DSIN(A1)-X(4)*X(4)*DSIN(A2)
        RETURN
 3      XUX=DSIN(TH1)+X(2)*X(2)*DSIN(X(5))-X(3)*X(3)*DSIN(TH3)
        RETURN
 4      XUX=DCOS(TH1)-X(2)*X(2)*DCOS(X(5))-X(3)*X(3)*DCOS(TH3)
        RETURN
 5      XUX=X(3)*X(3)*DSIN(X(1))-X(2)*X(2)*DSIN(X(5))-X(4)*X(4)*DSIN(X(6))
        RETURN
 6      XUX=X(3)*X(3)*DCOS(X(1))+X(2)*X(2)*DCOS(X(5))-X(4)*X(4)*DCOS(X(6))
        RETURN
        END
```