# Brane World Cosmology

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# Abstract

Recently, the ideas of extra dimensions and brane world scenarios have captured considerable interest. The potential ability of these brane-based models to explain the gauge hierarchy problem and the cosmological constant problem has been examined in numerous recent papers (on these subjects). In the first part of this thesis we examine many aspects of warped brane world scenarios, focusing on the Randall-Sundrum model. We begin with the stability issue of the Randall-Sundrum model and its cosmological implications. We will specifically verify that the Randall-Sundrum scenario, once stabilized, will reproduce the late-time conventional cosmology. In further studies, the possibility of providing a very small cosmological constant, in accordance with recent observations, is examined in the context of the warped brane world scenario. We also show that the "self-tuning" mechanism of the cosmological constant problem does not improve in the brane world models and the unnatural fine-tuning mystery of the cosmological constant problem remains unexplained. In the second part of this thesis some cosmological implications of the tachyon are examined. In particular, a mechanism is presented to take advantage of a time-varying tachyonic background to convert the energy of the tachyon to radiation at the end of inflation.

# Résumé

Récemment, l'idée de dimensions supplémentaires et de scénarios de « brane world » a capté un intérêt considérable. La capacité potentielle de ces modèles, reposant sur le concept de branes, à expliquer le problème de la hiérarchie de jauge et le problème de la constante cosmologique a été examinée dans de nombreux articles parus récemment sur le sujet. Dans la première partie de cette thèse, plusieurs aspects des scénarios de « warped brane world » sont examinés, en mettant l'emphase sur le modèle de Randall-Sundrum. On débute par la question de stabilité de ce modèle et ses implications cosmologiques. On vérifiera spécifiquement que le scénario de Randall-Sundrum, une fois stabilisé, reproduira la cosmologie traditionnelle aux temps récents. Dans une étude subséquente, la possibilité de produire une très faible constante cosmologique, en accord avec les observations récentes, est examinée dans le contexte de scénarios de « warped brane world ». On démontre aussi que le mécanisme de « self-tuning » du problème de la constante cosmologique ne se voit pas amélioré dans les modèles à branes, et le mystère du « fine-tuning », lié au même problème, reste inexpliqué. En deuxième partie de cette thèse, quelques implications cosmologiques de la corde tachyonique sont examinées. En particulier, on présente un mécanisme qui tire avantage d'un arrière-plan tachyonique variant dans le temps pour convertir, à la fin de l'inflation, l'énergie du tachyon en radiation.

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To begin I would like to thank my supervisor, Professor Jim Cline for his continual support and encouragement during my PhD studies at McGill University. He always respected my independence in research and provided me with many new ideas. The fact that I began my PhD program with no background in high energy physics will certainly indicate how patient and supportive he was.

I am also indebted to Professor Clifford Burgess. Certainly all my knowledge of the Standard Model and Supersymmetry, if any, has come from his excellent courses. I have also learned many new ideas in collaboration with him. I also am grateful to Professors Harry Lam and Guy Moore, who were open to all of my questions and requests, scientific or otherwise. I also have benefited from very useful conversations with Professor Robert Myers. Finally I would like to thank Professor Marc Grisaru who was always kind to me and who was always open to my questions in Supersymmetry. He is a person devoted to science and is my spiritual encouragement in pursuing physics as a professional scientist.

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# Statement of Original Contributions

This thesis contains materials that were previously published in refs.[1, 2, 3, 4, 5, 6]. Chapter 1 is a review of the literature and material presented in the following chapters. Chapters 2, 3, 4, and 5, respectively, are based on refs. [1], [2], [3] and [4], which were published by J. M. Cline and myself. Chapter 6 is based on ref. [5], which was published by C. P. Burgess, J. M. Cline, N. R. Constable and myself. Finally, chapter 7 is based on ref. [6] which was published by J. M. Cline, P. Martineau and myself. In each of these papers, in parallel to other author(s), I performed all of the calculations. The initial motivation in [1] is provided by me and further developed by J. M. Cline. In [2], I constructed the linearized Einstein equations and found the gauge invariant quantities. In [4], I discovered the no-go theorem. In [5], I performed the radion stability analyses in detail and examined the effect of the boundary conditions and the form of the brane stress-energy tensor. In [6], some of the central analytical solutions for the tachyonic background and the gauge field were provided by me.

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# Chapter 1

# Introduction

Recently, the idea of the extra dimensions and brane-based scenarios have captured a considerable level of interest in the high energy physics community. The brane world scenarios have the ability to provide a novel interpretation of interactions between gravity and the Standard Model of particle physics. In this regard they may provide a unique solution to some of the longstanding problems in the high energy physics. Among these are the gauge hierarchy problem and the cosmological constant problem. Also, the brane world models may open a new window toward the cosmology of the very early universe.

In one class of these brane world scenarios proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD) [7], space-time is the product of a four-dimensional Minkowski space and a compact factorizable manifold. The Standard Model of particle physics is trapped on the brane, while gravity can propagate in the higher dimensions. An observer on the 3-brane measures the effective gravitational scale, $M_{Pl}$, to be $M_{Pl}^2 = M^{d+2}V_d$, where $M$ is the fundamental gravity scale in higher dimensions and $V_d$ is the volume of the extra dimensions. For a very large $V_d$ ($\text{mm}^3$), $M$ can be as low as the weak scale. This provides a partial solution of the gauge hierarchy problem;

i.e. the question of why $M_{Pl}$ is 16 orders of magnitude larger than the weak scale. On the other hand this opens up the new question of why $R$, the typical radius of the extra dimensions, is so much larger than the expected scale $1/M$.

Assuming that $M \sim$ TeV, we find $R \sim 10^{32/d} \times 10^{-17}$ cm. For $d = 1$ the radius of the extra dimension is unacceptably large. For $d = 2$ one obtains the approximate value $R \sim$ mm, just on the edge of the table-top experimental bound for gravity, due to the fact that the four-dimensional Newtonian gravity has been experimentally verified down to a distance of about 0.2 mm [8]. However the most severe experimental constraints come from light Kaluza-Klein (KK) graviton excitations. The typical KK graviton excitation has mass $m_n = n/R$ and interacts with SM fields by $M_{Pl}^2$ suppression instead of $M^2$. On the other hand the total emission rate of KK gravitons is large for energies comparable to $M$ due to the small mass gap between KK excitations. Consequently the bound from collider experiments on $M$ could be as low as few TeV [9]. However, astrophysics imposes even more severe constraints. For example, requiring the light graviton emission to not cool down the supernova 1987A too quickly implies that $M > 30$ TeV.
For $d > 2$, yet smaller values for $R$ are obtained. For example when $d = 6$, we obtain $R \sim 10^{-12}$ cm, where the search for any deviation from conventional gravity is impossible.

Randall and Sundrum (RSI) proposed a different brane world scenario to explain the gauge hierarchy problem [10]. In their model, the geometry along the extra dimension is not factorizable and the space-time is "warped" due to a negative cosmological constant which fills the bulk. The Hierarchy problem is now explained by the curvature of the extra dimension. The physical mass scale depends on the position of the observer in the bulk, similar to the Doppler effect.

The RS scenarios, RSI and RSII, have proved to have even richer properties and physical applications than the ADD model. It does not suffer from the light KK excitations and is compatible with cosmological requirements. Also, it might have a plausible string theoretic origin. In this regard, we will mainly focus on the warped brane world scenarios and their cosmological implications.

Interestingly, the Kaluza-Klein picture of the brane world scenario, either in the form of the ADD or RS model, is not the only physical application of branes motivated by string theory. Recently applications of the tachyon to the cosmology of the very early universe have been extensively studied. The tachyon appears in some formal brane world models, either in brane-antibrane collisions or in the decay of unstable branes to lower dimensional branes. It indicates another example of the richness and vast number of applications of string theory motivated branes in current theoretical high energy physics. We will also devote some space to study the tachyon cosmology in this thesis.

This thesis will be divided up as follows. We begin with a review of the Randall-Sundrum model and how it can explain the gauge hierarchy problem. A discussion of its excitation modes and their phenomenological implications is presented. The idea of "self-tuning" as a potential solution to the cosmological constant problem in the context of the brane world scenario is presented in the section following. A review of the cosmological applications of the RSI and RSII models is the last topic concerning the warped brane world scenario in this introduction. The last section of the introduction is devoted to a brief study of the tachyon.

Chapters 2-6 are devoted to many aspects of warped brane world scenarios. In chapters 2 and 3 some cosmological aspects of the stabilized RSI model via the Goldberger-Wise (GW) scalar field are studied. In chapter 2 we study the radion potential in the GW mechanism. In chapter 3 we will verify perturbatively that the RSI model reproduces the standard Friedmann-Roberston-

Walker (FRW) cosmology.

Chapters 4 and 5 are devoted to the cosmological constant problem in the context of the brane world scenarios. In chapter 4 we use an RSI-like model and demonstrate how a very small cosmological constant can be generated, in accordance with recent observations. In chapter 5 we present a no-go theorem on how the self-tuning mechanism in brane world scenarios fails to explain the cosmological constant problem. While the theorem is easily derived, it is quite robust and model independent.

In chapter 6 we present a six-dimensional brane world model, the AdS soliton. This has the interesting property of being a hybrid of the ADD and RSI models. While it solves the gauge hierarchy problem in the same way as RSI, it does not have any negative tension brane. The different excitation modes and the stability of the model are extensively studied.

Chapter 7 is devoted exclusively to tachyon cosmology. We demonstrate a reheating mechanism for the universe at the end of inflation, based on a time-varying tachyonic background.

## 1.1 The RS Models

### 1.1.1 The Set-Up

In the RSI model, the space-time is five dimensional and the metric is given by the ansatz

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - b^2 dy^2 \; . \tag{1.1}$$

The space-time is compactified on $S_1/Z_2$. Two branes are located at the fixed points $y = 0, 1$. For reasons that shall be made clear later, the brane located at $y = 0$ is referred to as the "hidden" or "Planck brane", while the brane located at $y = 1$ is called the "visible" or "TeV brane". The action for this model is

$$
\begin{aligned}
S &= \int d^5x\sqrt{-g}\left(-\frac{1}{2\kappa^2}R-\Lambda\right)+\int d^4x\sqrt{g}\left(\mathcal{L}_{m,0}-V_0\right)|_{y=0} \\
&+ \int d^4x\sqrt{g}\left(\mathcal{L}_{m,1}-V_1\right)|_{y=1}\ . \qquad (1.2)
\end{aligned}
$$

where $\kappa^2$ is related to the 5-D Planck scale $M$ by $\kappa^2 = 1/(M^3)$ and $\Lambda$ is the bulk cosmological constant. The sign of $\Lambda$ will be determined by the solution. On each brane there exists a vacuum energy, $V_0$ and $V_1$, as a gravitational source even in the absence of particle excitations. In the following discussion we will not be concerned about the existence of the matter fields, $\mathcal{L}_m$. In the cosmological discussion of the next chapters, we will consider the existence of matter on the branes. Also in what follows, the 5-D indices are denoted by capital Roman indices, $M$, $N$,... while the 4-D indices for the observer on the branes are denoted by Greek indices, $\mu$, $\nu$,....

We would first like to investigate the solution of the classical Einstein equations where the space-time is static. Written explicitly, the Einstein equations are

$$
\left(R_{MN}-\tfrac{1}{2}g_{MN}R\right)=\kappa^2 g_{MN}\Lambda+\tfrac{\kappa^2}{b}g_{\mu\nu}\delta^{\mu}_{M}\delta^{\nu}_{N}\left(V_0\delta(y)+V_1\delta(y-1)\right)\ . \qquad (1.3)
$$

The (00) and (55) components of the Einstein equation, respectively, are

$$
2A'^2-A''=-\frac{\kappa^2\Lambda}{3}b^2-\frac{\kappa^2}{3}bV_0\delta(y)-\frac{\kappa^2}{3}bV_1\delta(y-1)\ , \qquad (1.4)
$$

$$
A'^2=-\frac{\kappa^2\Lambda}{6}b^2\ . \qquad (1.5)
$$

From Eq (1.5) it is clear that $\Lambda \leq 0$, i.e. the bulk is an $AdS$ space-time. Parameterizing the bulk cosmological constant as $\Lambda \equiv -6k^2/\kappa^2$, the solution of Eq (1.5) with orbifold symmetry, $y \to -y$, is

$$
A = kb|y|\ . \qquad (1.6)
$$

The boundary conditions can be read off directly from the Dirac delta function in Eq (1.4). More explicitly they are $[A_i'] = (-1)^i\,\frac{\kappa^2}{3}bV_i$, where $i = 0, 1$

and $[X(y_i)] \equiv X(y_i{}^+) - X(y_i{}^-)$. Imposing the boundary condition on each brane, the following constraint is obtained

$$V_0 = -V_1 = \frac{6k}{\kappa^2} \ . \tag{1.7}$$

This fine-tuning between the bulk cosmological constant and the brane tensions must be satisfied in order to obtain a solution that respects the four-dimensional Poincare invariance. As a consistency check we can directly show that the total action, $S$, is zero when the constraint (1.7) is satisfied, meaning there is no remnant energy left over on the 3-branes. To see this, note that

$$R = \tfrac{1}{b^2}\left(20A'^2 - 16A'\delta(y) + 16A'\delta(y)\right) \ . \tag{1.8}$$

The delta functions in the above expression come from the discontinuity of the warp factor at the position of each brane. Using this expression for the Ricci scalar, the bulk part of the action is

$$\int_{-1}^{1} dy\sqrt{-g}\left(-\frac{1}{2\kappa^2}R - \Lambda\right) = \frac{6k}{\kappa^2}(1 - e^{-4kb}) \ . \tag{1.9}$$

This exactly cancels the last two terms in the action (1.2) coming from branes. In chapter 4 we will slightly relax the fine-tuning (1.7) to produce a very small effective 4-D cosmological constant.

In order to trust this solution one must assume $k/M \leq 1$, which means that the bulk curvature is smaller than the fundamental mass scale and the higher curvature terms are negligible in the action.

### 1.1.2 Physical Implications

To see how the gauge hierarchy problem can be addressed in the RS model, first we need to find an expression for the four-dimensional gravitational strength, $M_{Pl}$. The four-dimensional graviton zero mode is determined by replacing $\eta_{\mu\nu}$ in (1.1) by $\bar{g}_{\mu\nu}$. The corresponding 4-D effective action is

$$S_{eff} \supset 2 \times \frac{M^3}{2} b \int d^4x \int_0^1 dy\sqrt{-\bar{g}}e^{-4A}e^{2A}\bar{R} \ , \tag{1.10}$$

where the factor 2 is due to the orbifold symmetry and the second exponential factor, $e^{2A}$, comes from the rescaling relation between the 5-D Ricci scalar, $R$, and the 4-D Ricci scalar $\bar{R}$ in metric (1.1). We find that

$$M_{Pl}{}^2 = \frac{M^3}{k}(1 - e^{-2kb}) \; . \tag{1.11}$$

The above relation between $M_{Pl}$ and $M$ indicates that $M_{Pl}$ depends only weakly on $b$ for relatively large $kb$, in sharp contrast to ADD model.

It is assumed that the SM particles are confined to the brane located at $y = 1$. For a typical scalar field on this brane, the action is given by

$$S = \int d^4x e^{-4A(y)} \left[ e^{2A(y)} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 \right] |_{y=1} \; . \tag{1.12}$$

The bare mass, $m$, is of the order of the fundamental mass scale, $M \sim M_{Pl}$. After normalizing the kinetic part by the rescaling $e^{-A}\phi \to \phi$, we find

$$m_{phys} = e^{-kb} m \; . \tag{1.13}$$

This is the main result of the RS model. To get a hierarchy of $10^{16}$ between TeV and Planck scales, one only requires that $kb \sim 35$.

### 1.1.3 Excitation Modes

It is important to verify that the RS model is consistent with four-dimensional gravity. This requires an investigation of the behavior of the modes appearing in the four-dimensional effective theory. There are two kinds of physical excitations around the static background of metric (1.1). The first one is the graviton excitations, corresponding to perturbing $\eta_{\mu\nu}$ by $h_{\mu\nu}$. The second one is the radion, corresponding to the fluctuation in the brane separations. It can be shown that these two modes do not couple to one another and each can be excited separately[11, 12].

## Gravitons

To be consistent with the four-dimensional gravitational expriments, the graviton excitations should have a zero mode, corresponding to a long distance gravitational attraction for massive objects on the TeV brane. On the other hand, the massive KK graviton excitations should not modify the Newtonian $1/r$ potential for distances larger than $0.2mm$, as was verified in the table-top gravity experiment [8].

Consider a linearized fluctuation of the metric in the form

$$\begin{aligned} ds^2 &= e^{-2A(z)}[(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + dz^2] \\ &\equiv e^{-2A(z)}d\tilde{s}^2 \ . \end{aligned} \tag{1.14}$$

In the above expression, the new coordinate, $z$, is defined by the relation $kz + 1 = e^{kby}$ for $y > 0$ and $A(z) = ln(kz+1), z > 0$. Also, for later applications we define a new line element, $d\tilde{s}^2$, conformally related to $ds^2$.

In order to find the equations for the linearized perturbations, we can compute the effective action up to $\mathcal{O}(h^2)$ and vary it to find the equations of motion. To begin with, we choose the Randall-Sundrum gauge[10, 11]

$$\partial_\mu h^\mu_\nu = 0 \qquad , \qquad h^\mu_\mu = 0 \ . \tag{1.15}$$

This gauge has the advantage that all components of $h_{\mu\nu}$, regardless of the indices, satisfy the same equation of motion, as we will see later. Using the conformal relation $ds^2 = e^{-2A(z)}d\tilde{s}^2$, one can show that

$$R = e^{2A}(\tilde{R} - 8\tilde{\nabla}^2 A + 12A'^2) \ , \tag{1.16}$$

where $\tilde{R}$ is the curvature scalar calculated purely by the $d\tilde{s}^2$ line element, given by

$$\tilde{R} =^{(4)} \tilde{R} - h^{\alpha\beta}h''_{\alpha\beta} - \frac{3}{4}h'^{\alpha\beta}h'_{\alpha\beta} \ , \tag{1.17}$$

and ${}^{(4)}\tilde{R}$ is the four-dimensional curvature on each slice of the brane located at constant position $z$,

$$ {}^{(4)}\tilde{R} = \frac{1}{4}h^{\alpha\beta,\mu}h_{\alpha\beta,\mu} - \frac{1}{2}\partial^{\mu}(2h^{\alpha\beta}h_{\alpha\beta,\mu} - h^{\alpha\beta}h_{\beta\mu,\alpha}) \ . \tag{1.18} $$

Plugging equations (1.16), (1.17) and (1.18) into the action for $R$ and by defining $\hat{h} \equiv e^{\frac{-3}{2}A}h$, we find, after performing some tedious but straightforward algebra, that

$$ \begin{aligned} S &= \frac{1}{2\kappa^2}\int d^4x dz\Bigg[\frac{1}{4}(h^{\alpha\beta,\mu}h_{\alpha\beta,\mu} - h'^{\alpha\beta}h'_{\alpha\beta}) \\ &- \left(\frac{15k^2}{16(kz_c+1)^2} - \frac{3}{4}k\delta(z) + \frac{3}{4(kz_c+1)}k\delta(z-z_c)\right)\hat{h}^{\alpha\beta}\hat{h}_{\alpha\beta}\Bigg] \end{aligned} \tag{1.19} $$

The corresponding equation of motion is

$$ \frac{1}{2}\partial_{\mu}\partial^{\mu}\hat{h}_{\alpha\beta} - \frac{1}{2}\hat{h}''_{\alpha\beta} + \left[\frac{15k^2}{8(kz_c+1)^2} - \frac{3}{2}k\delta(z) + \frac{3}{2(kz_c+1)}k\delta(z-z_c)\right]\hat{h}_{\alpha\beta} = 0 \ . \tag{1.20} $$

To analyze these modes, we impose the separation of variables, $\hat{h}_{\alpha\beta}(x,z) = \psi(z)e^{ip.x}$, where $p^2 = m^2$ is understood as the mass of each excitation mode. The final mode equation is

$$ \left[-\frac{1}{2}\partial_z{}^2 + V(z)\right]\psi = \frac{m^2}{2}\psi \ , \tag{1.21} $$

where

$$ V(z) = \frac{15k^2}{8(kz_c+1)^2} - \frac{3}{2}k\delta(z) + \frac{3}{2(kz_c+1)}k\delta(z-z_c) \ . \tag{1.22} $$

Much information can be obtained from the general shape of this analog nonrelativistic potential. The delta function supports a normalizable zero mode. Its wave function is given by

$$ \psi_0(z) = N_0(kz+1)^{-\frac{3}{2}} \ , \tag{1.23} $$

where $N_0$ is a normalization constant. Unlike the ordinary Kaluza-Klein theories with factorizable geometry, the zero mode wave function is a nontrivial function of the extra dimensional coordinate and is decreasing toward $z = z_c$, the position of the second brane. This indicates that the coupling of the zero mode to the TEV brane is much weaker than its coupling to the Planck brane. Consequently the zero mode is peaked near the Planck brane.

The massive KK graviton wave function is given by a linear combination of $(kz+1)^{\frac{1}{2}} J_2\left(\frac{m}{k}(kz+1)\right)$ and $(kz+1)^{\frac{1}{2}} Y_2\left(\frac{m}{k}(kz+1)\right)$, where $J_2$ and $Y_2$ are Bessel functions. Imposing the boundary condition $\psi'_m = -\frac{3}{2}k\psi_m$ at $z=0$, we find

$$\psi_m(z) = N_m(kz+1)^{\frac{1}{2}}\left[J_1(\tfrac{m}{k})Y_2\left(\tfrac{m}{k}(kz+1)\right) - Y_1(\tfrac{m}{k})J_2\left(\tfrac{m}{k}(kz+1)\right)\right] , \tag{1.24}$$

where $N_m$ are the normalization constants.
Further, imposing the boundary condition $\psi'_m = -\frac{3}{2}\frac{k}{(kz_c+1)}\psi_m$ at $z = z_c$, the following quantization condition on $m_n$ is obtained

$$J_1(\tfrac{m_n}{k})Y_1\left(\tfrac{m_n}{k}(kz_c+1)\right) - Y_1(\tfrac{m_n}{k})J_1\left(\tfrac{m_n}{k}(kz_c+1)\right) = 0 . \tag{1.25}$$

Phenomenologically interesting are the lightest excitations, corresponding to the creation of massive gravitons accessible to an observer on the TeV brane. In the limit $\frac{m_n}{k} \ll 1$, for example with KK gravitons of mass of order TeV, the approximate solution of $m_n$ is given by $J_1\left(\frac{m_n}{k}(kz_c+1)\right) = 0$. The first few $m_n$ are $m_n = 3.8ke^{-kb}, 7ke^{-kb}, 10.2ke^{-kb}, \ldots$.
We see that the massive KK are approximately quantized in units of $ke^{-kb}$ and consequently the tower of massive KK excitations is split by the TeV scale. This gives another sharp distinction between the RS and ADD models, since in the ADD scenario with two extra dimensions the mass gap in the KK excitations is around $10^{-3}$ eV, wheras RS has a mass gap of TeV.

We would also like to compare the couplings of $\psi_m$ and $\psi_0$ to the TeV brane. For large $mz$ one can use the plane wave approximation for $\psi_m$

$$\sqrt{z}J_2(mz) \sim \sqrt{\tfrac{2}{\pi m}}cos(mz - \tfrac{5}{4}\pi)$$
$$\sqrt{z}Y_2(mz) \sim \sqrt{\tfrac{2}{\pi m}}sin(mz - \tfrac{5}{4}\pi) \ . \tag{1.26}$$

Consequently

$$\psi_m(z) \simeq N_m J_1(\tfrac{m}{k})\sqrt{\tfrac{2k}{\pi m}}\left[sin(mz - \tfrac{5}{4}\pi) + \frac{4k^2}{\pi m^2}cos(mz - \tfrac{5}{4}\pi)\right] \ . \tag{1.27}$$

The normalization constants $N_0$ and $N_m$, can be fixed by the following orthogonality condition

$$\tfrac{1}{2z_c}\int_{-z_c}^{z_c} dz\psi_m\psi_{m'} = \delta_{mm'} \ . \tag{1.28}$$

Fixing $N_0$ and $N_m$ in this manner, we find that $\psi_m(z_c)/\psi_0(z_c) \sim kz_c$. This implies that $\psi_m$ couples to the TeV brane $kz_c \sim 10^{16}$ times stronger than does the $\psi_0$. In other words the massive KK gravitons couple with TeV strength to the TeV brane. This is quite interesting from a phenomenological point of view. The excitations of KK gravitons with mass around a few TeV and with interactions suppressed by the TeV scale can potentially be detected in future collider experiments at the LHC.

One can simply remove the TeV brane from the above picture. In this model, RSII, the SM particles are confined to the Planck brane and the hierarchy problem can not be explained. Since the four-dimensional gravitational strength, $M_{Pl}$, is weakly related to the position of the second brane in Eq (1.11), the gravity is still localized even if the second brane is moved to infinity. This can also be understood from the fact that the zero mode in Eq (1.23) is still normalizable if $z_c \to \infty$. In this model the KK excitations are continuous and thus have all possible $m^2 > 0$. To find the wave functions of the KK excitations, we can still use Eq (1.24). Now $N_m$ must be normalized by

$$\int_{-\infty}^{\infty} dz\psi_m(z)\psi_{m'}(z) = \delta(m - m') \ . \tag{1.29}$$

Using the orthogonality properties of the Bessel functions we find

$$N_m = \sqrt{\tfrac{m}{k}}\left[J_1(\tfrac{m_n}{k})^2 + Y_1(\tfrac{m_n}{k})^2\right]^{-\frac{1}{2}} . \tag{1.30}$$

The gravitational potential between two particles $m_1$ and $m_2$ on the Planck brane will now be given by

$$\begin{aligned}\Delta V(r) &= -G_N m_1 m_2 \left(\tfrac{1}{r} + \int_0^\infty \tfrac{dm}{k} |\psi_m(0)|^2 \tfrac{e^{-mr}}{r}\right) \\ &= -G_N m_1 m_2 \left(\tfrac{1}{r} + \int_0^\infty \tfrac{dm}{k} \tfrac{m}{k} \tfrac{e^{-mr}}{r}\right) \\ &= -G_N \tfrac{m_1 m_2}{r}(1 + \tfrac{1}{r^2 k^2}) \ . \end{aligned} \tag{1.31}$$

In the final expression, the first term comes from the exchange of a massless graviton between $m_1$ and $m_2$. The second term is the sum of the Yukawa potentials from continuum KK graviton exchanges. Also in the step before the final expression, the factor $\frac{dm}{k}$ comes from the measure of the plane wave for large $z$. The extra factor $\frac{m}{k}$ is due to the suppression of the KK excitations at $z = 0$: by using Eq (1.30) for $N_m$ in Eq (1.24), for small $\frac{m}{k}$ we find that $\psi_m(0) \simeq \sqrt{\frac{m}{k}}$.

This correction is extremely small and the deviation from usual Newtonian gravity is beyond experimental observations. For example, taking the current bound on the precision of the ordinary gravity $r = 0.2mm \sim 10^{16}(TeV)^{-1}$, we find a correction of order $10^{-64}$.

**The Radion**

In order to explain the gauge hierarchy problem in RSI, the combination $kb$ must be approximately 35, so that $e^{-kb} \sim 10^{-16}$. Yet in the original proposal, the value of $b$, which is the size of the extra dimension, was completely undetermined. It is a modulus with no potential, which is phenomenologically unacceptable. As we will see below, the particle associated with 4-D fluctuations of $b$, the radion, would couple to matter on the TeV brane in a manner similar to the gravitons, but stronger by a factor of $e^{kb}$ [13]. This would lead to a fifth force that should easily have been detected. Furthermore, a massless radion leads to problems with cosmology: our brane universe would have to have a negative energy density to expand at the expected rate, assuming that energy densities on the branes are tuned to give a static extra dimension[28, 29]. It was shown in refs. [30, 31] that this problem disappears when the size of the extra dimension is stabilized.

In [14, 15] the radion excitation of the metric has been found to be

$$ds^2 = e^{-2A(y)-2F(x,y)}\eta_{\mu\nu}dx^\mu dx^\nu - (1+2F(x,y))\,b^2dy^2 \ , \tag{1.32}$$

where $F(x,y) = e^{2k|y|}T(x)$ .

In order to identify the radion's coupling to the branes, we need a Kaluza-Klein reduction of the five-dimensional Einstein-Hilbert action for this perturbation. The kinetic part of the 5-D action corresponding to this perturbation is $S_{kin,r} = -\frac{1}{2}M^3 \int d^5x\sqrt{g}R_{kin}$, where $R_{kin}$ corresponds to the part of the 5-D curvature scalar containing the time derivatives. A straightforward calculation lead us to

$$\begin{aligned} S_{kin,r} &= 3M^3 \int d^4x(\partial T(x))^2 \int dy e^{-2A} e^{4k|y|} \\ &= \frac{3M^3}{k}(e^{2kb}-1)\int d^4x(\partial T(x))^2 \ . \end{aligned} \tag{1.33}$$

Correspondingly, using the relation $M_{Pl}^2 = \frac{M^3}{k}$, the normalized radion, $\phi(x)$, is given by $\phi(x) = \sqrt{6}M_{Pl}e^{kb}T(x)$ .

On the other hand, the induced metric on the brane, $\bar{g}_{\mu\nu}$, is given by the relation $g_{\mu\nu} = e^{-2A(y)-2F(x,y)}\ \bar{g}_{\mu\nu}$. The coupling of the radion to the matter field is

$$\begin{aligned} \phi(x)\frac{\delta S_{SM}}{\delta\phi} &= \phi(x)\frac{\delta S_{SM}}{\delta g^{\mu\nu}}\frac{\delta g^{\mu\nu}}{\delta\phi} \\ &= \phi(x)(-\tfrac{1}{2}\sqrt{-g}T_{\mu\nu})(\tfrac{2}{\sqrt{6}M_{Pl}}e^{kb}g^{\mu\nu}) \\ &\simeq -\frac{\phi(x)}{\sqrt{6}M_{Pl}e^{-kb}}\sqrt{-\bar{g}}(e^{-4A}T^\mu_\mu) \\ &= -\frac{\phi(x)}{\sqrt{6}M_{Pl}e^{-kb}}\sqrt{-\bar{g}}\ T_{phys}{}^\mu_\mu \ . \end{aligned} \tag{1.34}$$

In the penultimate relation above, we have used the approximation $F \ll A$, and in the final expression we have identified $e^{-4A}T^\mu_\mu$ as the physical stress energy tensor, in light of the resolution of the hierarchy problem in the RS model as given by Eq (1.13).

As indicated in Eq (1.34), the massless radion couples to the trace of the stress energy tensor on the TeV brane with TeV suppression. It produces a long range gravitational force $10^{32}$ times stronger than the usual gravitational force. Radion stabilization is therefore a crucial ingredient of the Randall-Sundrum idea. Goldberger and Wise (GW) have presented an elegant mechanism for accomplishing this [16], using a bulk scalar field. Self-interactions of the field on the branes forces it to take non-vanishing vacuum expectation values, $v_0$ and $v_1$ respectively, which are generally different from each other. The field thus has a gradient in the extra dimension, and the competition between the gradient and potential energies gives a preferred value for the size of the extra dimension. In other words, a potential for the radion is generated that has a nontrivial minimum. It is easy to obtain the correct brane separation using natural values of the parameters in the model.

The stabilized radion in the GW mechanism has a mass somewhat lighter than the TeV scale and therefore lighter than the KK modes of the bulk fields[17] and the KK excitations of the graviton. In this regard the detection of the radion field might be the first detectable signal of the RS scenario.

In chapter 2, we will investigate the radion potential for a stabilization mechanism closely related to the GW method.

## 1.2 The Self-tuning Mechanism of the Cosmological Constant Problem

Recent observations of high redshift Type 1a supernovae [18] indicate that the universe is accelerating with a large fraction of the energy density in the form of a cosmological constant, $\Lambda$. Combined with observations of the cosmic microwave background(CMB), an approximately flat FRW cosmological model with total energy density $\Omega_m + \Omega_\Lambda \simeq 1$ and $\Omega_\Lambda \simeq 0.7$ is suggested. This corresponds to a vacuum energy density of $\rho_V \sim 10^{-47}$ GeV$^4$.

On the other hand, from a basic field theoretical point of view, the energy density of the vacua is expected to be much larger than $10^{-47}$ GeV$^4$. Adding the zero-point energies of a typical field with mass $m$ produces a vacuum energy density

$$\rho_V \sim \int_0^M dk k^2 \sqrt{k^2 + m^2} \simeq M^4, \tag{1.35}$$

where M is a cutoff scale of the theory.
The fundamental mass scale of general relativity is $M_{Pl} \sim 10^{18}$ GeV. This would imply a vacuum energy density $\rho_V \sim 10^{73}$ GeV$^4$, which is $10^{120}$ times bigger from the observed value. This is one of the biggest discrepancies between theoretical prediction and experimental observation in physics. Stated another way, the cosmological constant problem requires a cancellation of 120 decimal places between an adjustable classical cosmological constant in the gravitational action and the one loop vacuum fluctuation of the SM fields. The extreme smallness of the cosmological constant leads one to the belief that the true vacuum has a zero cosmological constant. In a review article Weinberg [19] presented a sort of no-go theorem stating that any mechanism attempting to adjust the cosmological constant dynamically will suffer from another fine-tuning, probably no better than original cosmological constant fine-tuning.

Recently, it has been argued that in the context of the brane world scenario the 4-D no-go argument of Weinberg can be evaded. In [20, 21] it was shown that choosing the higher dimensional gravitational dynamics carefully, one can convert the SM vacuum energy into the bulk as a current. The gravitational back-reaction of this current warps the higher-dimensional space-time while maintaining the Poincare symmetry on the brane. These models have the same geometric set-up as the RSII and the SM fields are confined to the brane while a scalar field along with gravity can propagate in the bulk. Choosing the coupling of the scalar field to the brane in a specific conformal form, for a given value of the brane tension there exists a flat solution on the brane. This flat solution remains unaffected by one loop quantum corrections on the brane. However, in these solutions there is a naked curvature singularity in the bulk at a finite physical distance from the brane. To solve the cosmological constant problem in this method, a satisfactory study of the naked singularity must be undertaken. In this section we briefly review the self-tuning idea.

The 5-D gravitational action coupled to the scalar field $\Phi$ is

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{2\kappa^2}R + \tfrac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - V(\Phi)\right) - T\int d^4x\sqrt{g}V_0(\Phi)|_{y=0} \ . \tag{1.36}$$

In this expression $T$ is the brane tension or the vacuum energy density on the brane and contains the SM quantum loops and $V_0(\Phi)$ indicates the coupling of the scalar field to the brane. The coupled Einstein equations with the ansatz (1.1) and $b \equiv 1$ are

$$2A'^2 - A'' = -\tfrac{\kappa^2}{3}V(\Phi) - \tfrac{\kappa^2}{6}\Phi'^2 - \frac{\kappa^2}{3}TV_0\delta(y) \tag{1.37}$$

$$A'^2 = -\tfrac{\kappa^2}{6}V(\Phi) + \tfrac{\kappa^2}{12}\Phi'^2 \tag{1.38}$$

$$\Phi'' = 4A'\Phi' + V'(\Phi) + TV_0'(\Phi)\delta(y) \ . \tag{1.39}$$

Here the primes on the potentials $V$ and $V_0$ denote $\frac{\partial}{\partial\Phi}$.

Following[20, 21], we first assume $V(\Phi) \equiv 0$ and that the brane coupling will have the conformal form $V_0(\Phi) = e^{\frac{2\kappa}{\sqrt{3}}\Phi}$. The solution is then easily found to be

$$A(y) = -\tfrac{1}{4}ln|1 - \tfrac{r}{r_0}|$$

$$\Phi(y) = \Phi_0 - \tfrac{\sqrt{3}}{2\kappa}ln|1 - \tfrac{r}{r_0}| \ , \tag{1.40}$$

where $r_0$ and $\Phi_0$ are constants of integration. Intuitively speaking, the idea of self-tuning is based on having more constants of integration than constraining equations from the boundary conditions.

Imposing the boundary conditions $A' = \frac{\kappa^2}{6} T V_0(\Phi)$ and $\Phi' = \frac{T}{2} V_0'(\Phi)$, we find

$$T = \frac{3}{2\kappa^2 r_0}\, e^{-\frac{2\kappa}{\sqrt{3}}\Phi_0}\,. \tag{1.41}$$

Interestingly enough, there is no fine-tuning of $T$ to the microscopic parameters of the theory. Given a solution for one value of $T$, a shift $T + \Delta T$ in $T$ is simply compensated by a shift in $\Phi_0$.

As can be seen from the solution, there is a naked singularity at $r_0 = \frac{3}{2\kappa^2 T}\, e^{-\frac{2\kappa}{\sqrt{3}}\Phi_0}$. This indicates that the 5-D effective field theory breaks down near the singularity, and the solution for $r > r_0$ can not be trusted since the short distance physics of quantum gravity becomes important. One might hope that the fundamental theory, i.e. string theory, will resolve this singularity. Also one might argue that placing a brane in front of the singularity and implementing orbifold symmetry will resolve the singularity problem. It is easy to verify that in order to restore the 4-D Poincare invariant solution the tension of the regulator brane $T_c$ must be exactly fine-tuned to $T = T_c$ [10, 22], which destroys the whole idea of self-tuning.

One interesting property of this solution is the translational symmetry $\Phi \to \Phi +$ constant in the bulk. The associated conserved current

$$J^M = -\frac{1}{\sqrt{g}} \frac{\delta S_{bulk}}{\delta \partial_M \Phi} \tag{1.42}$$

has non-zero component $J_y$ due to 4-D Poincare invariance. The conformal coupling of $\Phi$ to the brane explicitly breaks the shift symmetry. This corresponds to a localized source on the brane given by

$$J_y = \frac{\kappa}{\sqrt{3}}\, T e^{\frac{2\kappa}{\sqrt{3}}\Phi_0}\,. \tag{1.43}$$

This implies that the SM vacuum energy is transferred to the bulk via this current. There is no 4-D curvature due to the vacuum energy on the brane, but the back-reaction of this current warps the bulk.

The general feature of the self-tuning solution with non-zero bulk potential $V(\Phi)$ and with an arbitrary brane coupling $V_0(\Phi)$ was studied in[23]. In this paper the authors proved a no-go theorem demonstrating that the self-tuning solution with localized 4-D gravity will always produce a naked singularity at a finite distance from the brane.

Up to now, it was assumed that the bulk is globally Poincare invariant. An interesting loophole for avoiding the no-go theorem in [23] is to break the global Poincare invariance in the bulk while maintaining that a brane slice at a constant position in the bulk retains the Poincare invariance [24, 25]. As a result, depending on the field content of the model, the singularity in the bulk can be shielded by a horizon, making it physically tolerable like the ordinary Schwarzchild black hole. The metric ansatz in this warped asymmetric model is [24]

$$ds^2 = h(r)dt^2 - a(r)d\vec{x}^2 - h(r)^{-1}dr^2 \ . \tag{1.44}$$

A gauge field, accompanied by gravity, will propagate in the bulk. A generalization of Birkhoff's theorem implies that this metric has the geometry of an AdS-Reissner-Nordström black hole with

$$h(r) = \frac{r^2}{l^2} - \frac{\mu}{r^2} + \frac{Q^2}{r^4} \qquad , \qquad a(r) = r^2 \tag{1.45}$$

where $\mu$ and $Q$ are proportional to the black hole mass and charge, respectively, and $l^{-2} = -\frac{1}{6}\kappa^2\Lambda$ is the AdS curvature radius.
Imposing the boundary conditions at the brane position $r_b$ will determine the brane tensions in term of the black hole mass and charge:

$$\begin{aligned} \mu &= 3(l^{-2} + \tfrac{1}{36}\kappa^4\omega\rho^2){r_b}^4 \\ Q^2 &= 2\left(l^{-2} + \tfrac{1}{72}\kappa^4(1+3\omega)\rho^2\right){r_b}^6 \ . \end{aligned} \tag{1.46}$$

In this expression $\rho$ indicates the $-T_0^0$ component of the brane tension while $p = \omega\rho$ corresponds to the $T_i^i$ component of the brane tension. As a comparision, in the RS model $\omega = -1$, which corresponds to a homogeneous brane.

A careful investigation of the above relations indicates that a self-tuning solution can be found while the singularity is hidden behind a horizon located between the brane and the singularity[24]. This requires $\omega < -1$ in the desired domain of the parameter solutions, corresponding to an exotic matter field on the brane which violates the weak energy condition. One might argue that the existence of such an exotic matter content can be cured if a complete model of asymmetrically warped background with more fields in the bulk is considered. In chapter 5 a no-go theorem is presented showing that the situation does not improve and the existence of this exotic matter is actually generic.

Besides providing a new manner of dealing with the cosmological constant problem, the asymmetrically warped brane scenario incorporates the interesting new idea of Lorentz violation on the brane from the gravitational sector. The induced geometry on each brane slice at constant $r$ is still Poincare invariant, however different 4-D sections of the metric (1.44) have different Poincare symmetry. The local speed of light is a function of the brane position in the extra dimension given by $c(r) = \sqrt{h(r)/a(r)}$. Due to globally broken Poincare symmetry, the speed of gravity propagation will be different from the speed of light. Intuitively, if the local speed of light is an increasing function in the bulk, then the gravitational wave can slightly bend in the bulk and therefore arrive earlier than electromagnetic waves confined to the brane. In this regard, gravitational signals can move faster than light. This intuitive picture of Lorentz violation from the gravitational sector was explicitly verified in [24] by studying the light-like geodesic behaviour in the bulk and also by studying the graviton zero mode perturbatively around the

RS solution.

The future gravitational wave experiments like LIGO, VIRGO or LISA may test this phenomenon. Also the apparent Lorentz violation may provide a mechanism other than inflation to explain the horizon problem of the FRW cosmology[26].

### 1.2.1 Cosmology of the RS model

The cosmological implications of the RS model have been extensively studied [27]-[37], [1, 2]. In the early works, it was realized that the expansion of the brane universe could be significantly different from the standard FRW cosmology[27, 28, 29].

In [27] a two-brane world universe with the same topology as RSII was studied. The bulk was empty and the branes contained matter energy density and pressure without any brane tension. It was shown that the cosmological evolution on the brane is quadratically related to the energy density, in contrast to the conventional linear expansion in late time cosmology. The sensitivity of Big Bang Nucleosynthesis (BBN) to the expansion rate will exclude any such dramatic change in late time FRW cosmology.

The cosmology of the RSI was studied soon after and it was shown that there is a wrong sign in the Friedmann equation for the observer on the TeV brane[28, 29]. Also matter on the TeV brane, $\rho$, and the matter on the Planck brane, $\rho_*$, are correlated to each other by $\rho_* = -\Omega^2\rho$, where $\Omega \equiv e^{-kb}$. As was argued before, this pathology is due to the lack of radion stabilization and it was then shown that once the radion is stabilized the conventional FRW cosmology is recovered[30, 31].

The cosmological implications of the stabilized RSI model were extensively studied in[30]. Either by averaging the Einstein equations in the bulk to obtain the 4-D equations or by averaging the 5-D action over the bulk to obtain the effective 4-D action, the following picture was concluded: for general matter states without a radion potential the radion runs off to infinity. This can be avoided by tuning the energy densities on the two branes as mentioned before. Once the radion is stabilized this constraint is removed and the conventional cosmology is recovered. The constraint for matter fields on the two branes is now replaced by an expression of the radion displacement around its vev

$$\frac{\Delta b}{b} \sim \frac{[\Omega^4(\rho - 3p) + \Omega^2(\rho_* - 3p_*)]}{kb\ {m_r}^2 {M_{Pl}}^2 \Omega^2} \ . \tag{1.47}$$

On the other hand, both Newton's constant and the SM particle masses depend on the vev of the radion. A substantial change in these quantities before BBN would result in a different prediction for the abundance of light particles. The success of the standard BBN imposes severe constraints on the changes in these quantities. An estimation of change since BBN therefore could be insightful. Here we assume that there is no matter on the Planck brane, i.e. $\rho_* = 0$.

From Eq (1.11) we find $\Delta G/G \sim k\Delta b e^{-2kb}$, which in combination with Eq(1.47), implies

$$\frac{\Delta G}{G} \sim e^{-2kb} \frac{\rho_{NR}}{{m_r}^2 (TeV)^2} \ , \tag{1.48}$$

where $\rho - 3p \sim \rho_{NR}$. The energy density of the non-relativistic particles just before the start of BBN is $\rho_{NR} \sim (T_{BBN}/T_0)^3 \rho_{c,0} \sim 10^{20}$ eV$^4$, where $T_{BBN} \sim 10MeV$ and $\rho_{c,0} \sim 10^{-10}$ eV$^4$. Substituting this into the above expression one finds

$$\frac{\Delta G}{G} \sim 10^{-30} \times 10^{-28} (\frac{TeV}{m_r})^2 \ . \tag{1.49}$$

The fact that this should not be more than 10% will impose the lower bound $m_r > 10^{-17}$ eV. Also, requiring that the weak scale has not changed more than 10% since the BBN implies from Eq (1.13) that $k\Delta b < 1/10$. Using Eq (1.47), this gives the upper bound

$$m_r > 10^{10} \tfrac{(eV)^2}{TeV} \sim O(10^{-2}) eV \ . \tag{1.50}$$

Previously we showed that the radion couples to the SM particles with TeV suppression. This indicates that the above cosmological constraints are extremely weaker than the constraints from collider experiments.

In chapter 3 a systematic investigation of the stabilized RSI cosmology is presented. In the rest of this section we will review the cosmology of the RSII model in detail.

Due to the fact that there is no stabilization problem in the RSII model, the cosmology of the RSII model does not suffer any of the above pathologies. Also, an analytical investigation of the model can be performed exactly. There are two methods of studying the cosmology in the RSII scenario, the brane-based approach and the bulk-based approach. In the brane-based approach the brane is located at a fixed point in space-time while the bulk is dynamic [33, 32]. The Gauss-Codazzi equations and the Israel junction conditions[38] are used to find the Einstein equation on the brane. The effect of the bulk geometry is encoded in an undetermined tensor, $E_{\mu\nu}$, corresponding to the projection of the 5-D Weyl tensor on the brane: $E_{\mu\nu} \equiv^{(5)} C_{\mu\alpha\nu\beta} n^{\alpha} n^{\beta}$, where $n^{\alpha}$ is vector normal to the brane [33]. The brane-based method has the advantage of using a metric ansatz similar to the FRW metric but with the disadvantage of losing the information from the bulk hiding in $E_{\mu\nu}$. On the other hand, in the bulk-based method, the bulk has the geometry of the static AdS-Schwarzschild black hole and the brane is a moving boundary[34, 35, 36]. The Israel junction conditions give the energy and pressure of the brane as a function of its trajectory. The beauty of this approach is that there is no hidden information in the bulk and the cosmological solution translates into a wall moving in an AdS space-time. The only disadvantage of this method is that this is perhaps more abstract and less intuitive compared to the brane-based approach. It was explicitly shown in [36] that these two approaches are equivalent. In this section we will use the bulk based approach following [34] and [36].

In this method the metric in the bulk is given by

$$ds^2 = h(r)dt^2 - r^2 d\Sigma_k^2 - \frac{dr^2}{h(r)} \ , \tag{1.51}$$

where $h(r) = k - \frac{\Lambda}{6}r^2 - \frac{\mu}{r^2}$ and $d\Sigma_k^2$ is the metric for a homogeneous 3-D space of constant curvature with $k = 0, \pm 1$. In this expression $\mu$ is interpreted as the bulk black hole mass originating from $E_{\mu\nu}$.

The induced metric on the wall is given by

$$ds_{in}^2 = d\tau^2 - R(\tau) d\Sigma_k^2 \ . \tag{1.52}$$

For an observer comoving with the wall, $\tau$ is the proper time. The condition that the wall is moving in the bulk will imply

$$\dot{t}^2 h(r) - \dot{R}^2 h^{-1}(r) = 1 \ , \tag{1.53}$$

where $t = t(\tau)$ and the derivatives are with respect to the proper time $\tau$. By definition $t$ is the time coordinate in the bulk and may not agree with the proper time on the brane.

Let $n^\mu$ be the unit normal vector on the brane such that $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$ and $n^\mu n_\mu \equiv g_{\mu\nu} n^\mu n^\nu = -1$, where $h_{\mu\nu}$ indicates the induced metric on the brane. Also define $u^\mu$ to be the wall velocity vector, which by construction is given by $n^\mu u_\mu \equiv g_{\mu\nu} n^\mu u^\nu = 0$ and for an observer on the brane $u_\tau = u^\tau = 1, \vec{u} = 0$. Using the identities $u^t = \frac{\partial t}{\partial \tau} u^\tau = \dot{t}$ and $u^r = \frac{\partial r}{\partial \tau} u^\tau = \dot{R}$, the observer in the bulk finds

$$u_\mu = (\dot{t}, \vec{0}, \dot{R}). \tag{1.54}$$

Furthermore, the orthogonality condition between $u$ and $n$ implies

$$n_\mu = \pm(\dot{R}, \vec{0}, -\dot{t}). \tag{1.55}$$

The $\pm$ sign reflects the choice of which part of the space-time we wish to keep, $r < R$ or $r > R$. This in general depends on the sign of the brane energy density: positive(negative) energy brane matches across two interior(exterior) space-times with $-(+)$ sign choice and with outward(inward) normal vector.

The Israel junction conditions imply

$$\Delta K_{ab} = -\kappa^2 (S_{ab} - \tfrac{1}{3} S h_{ab}) \ , \tag{1.56}$$

where $K_{ab}$ are the components of the extrinsic curvature and

$$S_{ab} = -p_T h_{ab} + (\rho_T + p_T) u_a u_b \ . \tag{1.57}$$

In the above expression $\rho_T$ and $p_T$ are the total energy density and the total pressure on the brane, respectively. More explicitly

$$\rho_T = T + \rho \qquad ; \qquad p_T = -T + P \ , \tag{1.58}$$

where now T represents the pure brane tension in the static RS background and $\rho$ and p represent the ordinary matter energy density and pressure, respectively.

The components of the extrinsic curvature in terms of the intrinsic brane coordinates $\xi^a = (\tau, \vec{x})$ are explicitly given by

$$K_{ab} = \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^a} \nabla_\mu n_\nu \ . \tag{1.59}$$

The non-zero components of the extrinsic curvature are $k_{00}$ and $k_{ii}$. Imposing the $Z_2$ symmetry between the interior $(-)$ and the exterior $(+)$ regions, they are given by

$$k_{00}^{\pm} = \pm \frac{\ddot{R} + \frac{1}{2} h'}{\dot{t} h} \ , \tag{1.60}$$

and

$$k_{ii}^{\pm} = \mp R \dot{t} h \ . \tag{1.61}$$

The (ii) component of Eq (1.56), using Eq (1.61), will result in $\rho_T = \mp \frac{6}{R} \dot{t} h$. Using Eq (1.53) to replace $\dot{t}$ in favour of $\dot{R}$ in this expression of $\rho_T$, we arrive at

$$(\frac{\dot{R}}{R})^2 = (\frac{\kappa^4}{36} {\rho_T}^2 + \tfrac{\kappa^2}{6} \Lambda) - \frac{k}{R^2} + \frac{\mu}{R^4} \ . \tag{1.62}$$

The other non-zero component of the extrinsic curvature will give the energy conservation condition on the brane

$$\dot{\rho}_T + 3\frac{\dot{R}}{R}(\rho_T + p_T) = 0 \ . \tag{1.63}$$

Equations (1.62) and (1.63) can be used to investigate the cosmological wall solutions. The chosen form of $\rho_T$ and $p_T$ distinguishes various different cases.

The most straightforward choice is when the bulk is empty and the only energy source on the brane is that of ordinary matter, i.e. $\rho_T = \rho$ and $p_T = p$. In this case we find a non-conventional cosmology with $H \sim \rho$, which is in contradiction with the late-time matter dominated cosmology[27].

The second possibility is the "domain wall" when $\rho_T = -p_T$, while $\rho = p = 0$. Following Eq(1.63) we find $\rho_T$=constant and Eq (1.62) is simplified to

$$(\frac{\dot{R}}{R})^2 = \Lambda_{eff} - \frac{k}{R^2} + \frac{\mu}{R^4} \ , \tag{1.64}$$

where $\Lambda_{eff} \equiv (\frac{\kappa^4}{36}{\rho_T}^2 + \frac{\kappa^2}{6}\Lambda)$. The background RSII model is the one with $\Lambda_{eff} = k = \mu = 0$. As was mentioned before in Eq (1.7), the fine-tuning between the brane tension, $\rho_T$ here, and the bulk cosmological constant is to enforce the vanishing vacuum energy on the brane. The analytical solution of Eq (1.64) for the case of a planar domain wall, $k = 0$, can be found [36] with no physically relevant applications.

The most physically interesting case is when the matter energy density and the pressure are added to the brane on top of the the background RSII tension. This corresponds to $\rho_T = T + \rho$ and $p_T = -T + P$ with the equation of state $p = \omega\rho$. Eq (1.63) implies $\rho = \rho_0 R^{-3(1+\omega)}$ and Eq (1.62) is transformed to

$$(\frac{\dot{R}}{R})^2 = \Lambda_{eff} - \frac{k}{R^2} + \frac{\kappa^4 T}{18}\rho + \frac{\kappa^4}{36}\rho^2 + \frac{\mu}{R^4} \ . \tag{1.65}$$

Several interesting conclusions can be drawn from Eq (1.65). First we note that in the low energy limit when $\rho \ll T$ and taking $\mu = 0$, the late time standard FRW cosmology is recovered with $8\pi G = \frac{\kappa^4}{6}T$. In the case of vanishing $\Lambda_{eff}$ this reduces to $M_{Pl}^2 = \frac{M^3}{k}$, in exact agreement with Eq (1.11). The fact that the conventional FRW cosmology is recovered in this case should be expected, since we already verified that both the zero mode and the KK graviton in RS background have well-defined behaviour, and the low energy 4D gravity is indeed recovered for the observer on the brane. On the other hand, the $\rho^2$ correction in Eq (1.65) will decay rapidly in the early radiation era as $R^{-8}$. This is not likely to have a significant effect on the BBN predictions and is quite negligible in the late-time matter dominated era given that $\rho \ll M \sim M_{Pl}$. The term corresponding to $\mu$, however, is of considerable interest. In this term the scale factor changes as in the radiation era with constant $\mu$, and in this regard it is called the dark radiation. Mathematically, both positive and negative $\mu$ are possible. If $\mu$ is positive, then it behaves like extra relativistic particles and increases the expansion rate, whereas for negative $\mu$ the argument is reversed. Due to its effect on BBN and CMB, the sign and magnitude of $\mu$ are restricted. An analysis of dark radiation by the BBN constraints is presented in [39]. It is convenient to compare the dark radiation to $\rho_\gamma$, the energy density of the photon in the BBN epoch at $T =$ 1MeV. In this terminology $\rho_{DR}/\rho_\gamma \equiv 3\mu/8\pi G R_N^4 \rho_\gamma$, as can be read off from Eq(1.65). Furthermore, $\rho_{DR}/\rho_\gamma$ can be interpreted as an effective number of extra neutrino species, $\frac{7}{8}\Delta N_\nu$. For positive dark radiation the upper bound $\Delta N_\nu \leq 0.13$ is obtained from primordial $^4$He and D/H abundances in the BBN epoch. For a negative dark radiation, a wider range of dark radiation density is allowed and a lower limit of $\Delta N_\nu > -1.41$ is obtained.

### 1.2.2 Tachyon Cosmology

We have seen that the brane world scenarios have drastically changed our traditional intuition about the interaction of SM particle physics with gravity. It is reasonable that other, more formal, brane-based models may have novel applications to the cosmology of the very early universe, especially inflation.

Inflation has become a central facet of the cosmology. Not only is it capable of addressing the shortcomings of standard cosmology, i.e. flatness and the horizon problems, but it also provides an interesting explanation of structure formation in the universe. The expansion due to inflation is usually driven by a slow rolling scalar field. In some brane world scenarios, a natural candidate for this rolling scalar field is provided by the inter-brane separations. In particular in brane-antibrane collisions the distance between two branes is the field which plays the role of the inflaton, while the onset of tachyon condensation after the branes reach some critical separation provides the unstable direction of hybrid inflation[90, 91, 92].

The cosmological investigation of the tachyon was triggered by the recent work of Sen[96]. It was shown that at the end of tachyon condensation the tachyon field can be described by a pressureless gas with non-zero energy density, known as tachyon matter. In this regard it is very similar to non-relativistic cold dark matter. Its cosmological relevance and shortcomings were studied in [99], [100] and [101]. Since the tachyon field does not oscillate around a minimum of the potential, it appears that the universe will become immediately dominated by cold dark matter and never resemble the hot big bang. This is not acceptable because it is well-known that at some early stage our universe was radiation dominated. It is important to convert very nearly 100% of the energy stored in tachyon matter into radiation because the former redshifts more slowly than the latter.
In this section we briefly study the basic properties of the tachyon, closely following [99].

The tachyon is described by the effective action

$$\mathcal{L} = -V(T)\sqrt{1 - \eta^{\mu\nu}\partial_\mu T \partial_\nu T}, \tag{1.66}$$

where in bosonic string theory $V(T) = V_0 e^{-T/T_0}$, and in superstring theory $V(T) = V_0 e^{-T^2/T_0{}^2}$. $T_0$ is close to the fundamental length scale, $T_0 \sim l_s$, and $V_0$ is the brane tension.

The interaction of the tachyon with gravity is described by the action

$$S = \int d^4x\sqrt{-g}\left(-\frac{R}{16\pi G} - V(T)\sqrt{1 - \eta^{\mu\nu}\nabla_\mu T \nabla_\nu T}\right). \tag{1.67}$$

The corresponding Einstein equations following from the above action are

$$G_{\mu\nu} = 8\pi G\left(\frac{V}{\sqrt{1 - \nabla_\alpha T \nabla^\alpha T}}\nabla_\mu T \nabla_\nu T + g_{\mu\nu} V\sqrt{1 - \nabla_\alpha T \nabla^\alpha T}\right), \tag{1.68}$$

while the field equation for the tachyon is

$$\nabla_\mu \nabla^\mu T + \frac{\nabla_\mu \nabla_\nu T}{1 - \nabla_\alpha T \nabla^\alpha T}\ \nabla^\mu T\ \nabla^\nu T + \frac{V_{,T}}{V} = 0\ . \tag{1.69}$$

We are interested in applying these equations to the homogeneous FRW cosmological model

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2\ . \tag{1.70}$$

The energy-momentum tensor of the tachyon matter is given by $T^\mu{}_\nu = \mathrm{diag}(-\rho, p, p, p)$, where the energy density and the pressure, respectively, are

$$\begin{aligned} \rho &= \frac{V(T)}{\sqrt{1-\dot{T}^2}} \\ p &= -V(T)\sqrt{1-\dot{T}^2}\ . \end{aligned} \tag{1.71}$$

The equations of motion for the evolution of the scale factor and the rolling tachyon are

$$\begin{aligned} \frac{\dot{a}^2}{a^2} &= \frac{8\pi G}{3}\frac{V(T)}{\sqrt{1-\dot{T}^2}} \\ \frac{\ddot{T}}{1-\dot{T}^2} &+ 3\frac{\dot{a}}{a}\dot{T} + \frac{V_{,T}}{V} = 0\ . \end{aligned} \tag{1.72}$$

After a small $T_0$, the tachyon will be fast rolling. In units of $T_0$, where both $t$ and $T$ are dimensionless, the approximate solutions of the above equations are

$$\begin{aligned} T(t) &= t + \tfrac{1}{4}a(t)^6 e^{-2t} \\ a(t) &\sim t^{\frac{2}{3}}. \end{aligned} \tag{1.73}$$

As argued before, the universe is expanding like in a matter dominated era and the tachyon behaves like cold dark matter. More explicitly, from Eq (1.71) we find

$$\rho = \frac{V_0\, e^{-t/T_0}}{a^3\, e^{-t/T_0}} \to \frac{V_0}{a^3} , \tag{1.74}$$

while

$$p = -V_0\, e^{-t/T_0}\, a^3 e^{-t/T_0} \sim -a^3 e^{-2t/T_0} \to 0 . \tag{1.75}$$

In this regard, inflationary models which incorporate the tachyon field should be treated carefully. The energy density of the tachyon field dominates the total energy density of the universe at the end of inflation with an equation of state $\rho \sim a^{-3}$. The energy density of radiation $\rho_r \sim a^{-4}$ is decreasing faster than the energy of the tachyon. In ordinary inflationary models the matter is created through the reheating mechanism around the minimum of the potential at the end of inflation. However, the effective potential of the tachyon field does not have any minimum at a finite $T$, so reheating can not occur. In chapter 7 a mechanism is presented to take advantage of the time-varying tachyonic background in order to convert the energy of the tachyon into radiation *via* the Bogoliubov procedure, by coupling the tachyon field to a massless gauge field in the Dirac-Born-Infeld action.

# Chapter 2

# Cosmological Implications of the Radion Potential

## 2.1 Introduction

As explained in the introductory chapter, one of the most striking proposals in current elementary particle theory is the existence of extra dimensions. In particular, the Randall-Sundrum proposal involves a "Planck brane" located at a position $y = 0$ in a single additional dimension, and a second "TeV brane" located at $y = 1$, in our conventions. The extra dimension is permeated by a negative bulk vacuum energy density, so that the space between the branes is a slice of anti-de Sitter space. Solving the 5-D Einstein equations results in the line element

$$ds^2 = e^{-2\sigma(y)}(dt^2 - d\vec{x}^2) - b^2\, dy^2 \ , \tag{2.1}$$

$$\sigma(y) = kby; \qquad y \in [0, 1] \ . \tag{2.2}$$

The constant $k$ is related to the 4-D and 5-D Planck masses, $M$ and $M_p$, respectively, by $k = \frac{M^3}{M_p^2}$. The warp factor $e^{-\sigma(y)}$ determines the physical masses of particles on the TeV brane: even if a bare mass parameter $m_0$ in the TeV brane Lagrangian is of order $M_p$, the physical mass gets rescaled by $m_{\rm phys} = e^{-\bar{\sigma}} m_0$, where $\bar{\sigma} \equiv \sigma(1) = kb$.

In order for $m_0$ to be of order 100 GeV, the combination $kb$ must be approximately 36, so that $e^{-\bar{\sigma}} \sim 10^{-16}$. On the other hand, in the original RSI model no stabilization mechanism to fix the value of $b$, the size of the extra dimension, is proposed. In the introductory chapter the phenomenological and cosmological problems of an unstable radion were discussed and it was argued that radion stabilization is a crucial ingredient of the RSI scenario.

One elegant mechanism for radion stabilization is presented by Goldberger and Wise [16] by using a bulk scalar field. It was shown in [16] that the presence of a bulk scalar propagating on the RSI background can generate a potential that stabilizes the radion. The minimum of the potential can be arranged to yield the desired value of $kb$ without extreme fine-tuning of the parameters.

Roughly speaking, the radion potential in [16] has the form

$$V(\phi) \cong \lambda\phi^4 \left(\left(\frac{\phi}{f}\right)^{\epsilon} - \frac{v_1}{v_0}\right)^2 . \tag{2.3}$$

In the above expression $v_0$ and $v_1$, respectively, are the vacuum expectation values (VEV's) on the Planck brane and TeV brane and $\epsilon \equiv m^2/4k^2$, where $m$ is the bulk scalar field mass.

A nontrivial minimum is located at $\phi = f(v_1/v_0)^{1/\epsilon}$. However, there is another minimum at $\phi = 0$, which represents an infinitely large extra dimension. This could not describe our universe, because then $e^{-\bar{\sigma}}$ would be zero, corresponding to vanishing particle masses on the TeV brane. In the more exact expression for the potential, we will show that $\phi = 0$ is actually a false vacuum—it has higher energy than the nontrivial minimum. Nevertheless, it is quite likely that the metastable state could be reached through classical evolution in the early universe. The question then naturally arises whether tunneling or thermal transitions to the desired state occurs. This is the subject of our study.

In section 2.2 we derive the Goldberger-Wise potential for the radion in a slightly simplified model which allows for the exact solution of the potential. The classical evolution of the radion field is considered in section 2.3, where we show that for generic initial conditions, the universe reaches a state in which the radion is not stabilized, but instead the extra dimension is expanding. This is a metastable state however, and in section 2.4 the rate of transitions to the minimum energy state, with finite brane separation, is computed. Conclusions are given in section 2.5.

## 2.2 Radion Potential

Let $\psi(y)$ be the bulk scalar field which will be responsible for stabilizing the radion. Its action consists of a bulk term plus interactions confined to the two branes, located at coordinate positions $y = 0$ and $y = 1$, respectively. Using the variable $\sigma$ of Eq (2.2) in place of $y$, the 4-D effective Lagrangian for a static $\psi$ configuration can be written as

$$\begin{aligned} \mathcal{L} &= -\frac{k}{2}\int_{-\bar{\sigma}}^{\bar{\sigma}} e^{-4\sigma}\left((\partial_\sigma\psi)^2 + \hat{m}^2\psi^2\right) d\sigma \\ &- m_0(\psi_0 - v_0)^2 - e^{-4\bar{\sigma}} m_1(\psi_1 - v_1)^2 \,, \end{aligned} \tag{2.4}$$

where $\hat{m}$ is the dimensionless mass $m/k$, $\psi_i$ are the values of $\psi$ at the respective branes, and the orbifold symmetry $\psi(\sigma) = \psi(-\sigma)$ is to be understood. In 5-D, the field $\psi$, as well as the VEV's $v_i$ on the two branes, have dimensions of $(\text{mass})^{3/2}$, while the parameters $m_i$ have dimensions of mass. Denoting $\partial_\sigma\psi = \psi'$, the the Euler-Lagrange equation for $\psi$ is

$$\begin{aligned} ke^{-4\sigma}\left(\psi'' \right. &- \left. 4\psi' - \hat{m}^2\psi\right) = 2m_0(\psi_0 - v_0)\delta(\sigma) \\ &+ 2e^{-4\bar{\sigma}} m_1(\psi_1 - v_1)\delta(\sigma - \bar{\sigma}) \,. \end{aligned} \tag{2.5}$$

The general solution has the form

$$\psi = e^{2\sigma}\left(Ae^{\nu\sigma} + Be^{-\nu\sigma}\right);$$
$$\nu = \sqrt{4+\hat{m}^2} \equiv 2 + \epsilon\ . \quad (2.6)$$

To get the correct hierarchy between the Planck and weak scales, it is necessary to take $\hat{m}^2$ to be small, hence the notation $\epsilon$.

The brane terms induce boundary conditions specifying the discontinuity in $\psi'$ at the two branes. Imposing the orbifold symmetry $\psi(-\sigma) = \psi(\sigma)$, this implies that

$$\psi'(0) = \hat{m}_0(\psi_0 - v_0)$$
$$\psi'(\bar{\sigma}) = -\hat{m}_1(\psi_1 - v_1)\ , \quad (2.7)$$

where we defined $\hat{m}_i = m_i/k$. Substituting the solution (2.6) into (2.7) allows us to solve for the unknown coefficients $A$ and $B$ exactly. In this respect, the present model is simpler than that originally given in ref. [16]. There the brane potentials were taken to be quartic functions, so that $A$ and $B$ could only be found in the approximation where the field values $\psi_i$ were very strongly pinned to their minimum energy values, $v_i$. In our model this would occur in the limit $m_i \to \infty$. However, we can easily explore the effect of leaving these parameters finite since $A$ and $B$ can be determined exactly. Let us denote

$$\hat{\phi} = e^{-kb} = e^{-\bar{\sigma}}\ , \quad (2.8)$$

which will be convenient in the following, because $\hat{\phi}$ is proportional to the canonically normalized radion field. Then $A$ and $B$ are given by

$$A = \left(-C_1\hat{\phi}^{\nu} + C_2\hat{\phi}^2\right)\hat{\phi}^{\nu}/D(\hat{\phi})$$
$$B = \left(C_3\hat{\phi}^{-\nu} - C_4\hat{\phi}^2\right)\hat{\phi}^{\nu}/D(\hat{\phi})\ , \quad (2.9)$$

where

$$C_1 = \hat{m}_0 v_0(\hat{m}_1 - \epsilon)$$

$$
\begin{aligned}
C_2 &= \hat{m}_1 v_1(\hat{m}_0 + \epsilon) \\
C_3 &= \hat{m}_0 v_0(\hat{m}_1 + 4 + \epsilon) \\
C_4 &= \hat{m}_1 v_1(\hat{m}_0 - 4 - \epsilon) \\
D(\hat{\phi}) &= \frac{\left(C_2 C_3 - C_1 C_4 \hat{\phi}^{2\nu}\right)}{(\hat{m}_0 \hat{m}_1 v_0 v_1)} \,.
\end{aligned}
\tag{2.10}
$$

It can be checked that in the limit $\hat{m}_i \to \infty$, the field values on the branes approach $\psi_i \to v_i$. For finite $\hat{m}_i$, the competing effect of minimizing the bulk energy causes departures from these values, however.

The solution for $\psi$ can be substituted back into the Lagrangian (2.4) to obtain the effective 4-D potential for the radion, $V(\hat{\phi})$. However, rather than substituting directly, one can take advantage of the fact that $\psi$ is a solution to the Euler-Lagrange equation. Doing a partial integration and using the boundary terms in (2.5) gives a simpler expression for $V(\hat{\phi})$. In the general case where the brane potentials are denoted by $V_i(\psi)$, one can easily show that

$$
\mathcal{L} = -\sum_i e^{-4\sigma_i} \left(V_i(\psi_i) - \tfrac{1}{2}\psi_i V_i'(\psi_i)\right) \,. \tag{2.11}
$$

In the present case, we obtain

$$
\begin{aligned}
V(\hat{\phi}) &= -\mathcal{L} \\
&= m_0 v_0(v_0 - \psi_0) + \hat{\phi}^4 m_1 v_1(v_1 - \psi_1) \\
&= m_0 v_0(v_0 - (A + B)) \\
&+ \hat{\phi}^4 m_1 v_1(v_1 - \hat{\phi}^{-2}(A\hat{\phi}^{-\nu} + \hat{\phi}^{\nu} B)) \,.
\end{aligned}
\tag{2.12}
$$

In the following, we will be interested in the situation where $V(\hat{\phi})$ has a nontrivial minimum for very small values of $\hat{\phi} \sim 10^{-16}$, as needed to address the weak scale hierarchy problem. It is therefore a good approximation to expand $V(\hat{\phi})$ near $\hat{\phi} = 0$, keeping only the terms which are larger than $\hat{\phi}^{2\nu}$. This is accomplished by expanding the denominator $D(\hat{\phi})$ in eqs. (2.9), after which the potential can be written in the simple form

$$V(\hat{\phi}) = \Lambda\hat{\phi}^4\left((1+\epsilon_4-\epsilon_1)\,\hat{\phi}^{2\epsilon} - 2\eta\,(1+\epsilon_4)\,\hat{\phi}^{\epsilon} + \eta^2\right) . \tag{2.13}$$

where we introduce the notation

$$\epsilon_4 = \frac{\epsilon}{4}\ ; \qquad \epsilon_0 = \frac{\epsilon}{\hat{m}_0}\ ; \qquad \epsilon_1 = \frac{\epsilon}{\hat{m}_1}\ , \tag{2.14}$$

$$\eta = (1+\epsilon_0)\,\frac{v_1}{v_0}\ , \tag{2.15}$$

and

$$\Lambda = 4kv_0^2\,\frac{(1+\epsilon_4)\,(1+\epsilon_0)^2}{\left(1+\frac{4}{\hat{m}_1}+\epsilon_0\right)} . \tag{2.16}$$

In the following it will be convenient for us to rewrite $V(\hat{\phi})$ in the form

$$V(\hat{\phi}) = \Lambda'\hat{\phi}^4(\hat{\phi}^{\epsilon}-c_+)(\hat{\phi}^{\epsilon}-c_-)\ , \tag{2.17}$$

where $c_\pm$ are given by

$$c_\pm = \eta\left(\frac{(1+\epsilon_4)\pm\sqrt{(1+\epsilon_4)^2-(1+\epsilon_4-\epsilon_1)}}{(1+\epsilon_4-\epsilon_1)}\right)\ , \tag{2.18}$$

and $\Lambda' = \Lambda(1+\epsilon_4-\epsilon_1)$.

## 2.3 Phenomenology and Early Cosmology of the Model

The warp factor which determines the hierarchy between the weak and Planck scales can be found by minimizing the potential (2.17). Expanding in $\epsilon$, it has a global minimum and a local maximum at the respective values $\hat{\phi}_+$ and $\hat{\phi}_-$:

$$\begin{aligned}\hat{\phi}_\pm &= \left(\frac{v_1}{v_0}\right)^{1/\epsilon}\left(\frac{1+\epsilon_0}{1+\epsilon_4-\epsilon_1}\right)^{1/\epsilon}\left(1\pm\sqrt{\epsilon_1+\epsilon_4-\tfrac{1}{2}\epsilon\epsilon_1}\right)^{1/\epsilon}\\ &\cong \left(\frac{v_1}{v_0}\right)^{1/\epsilon}\exp\left(\pm\sqrt{\tfrac{1}{\epsilon}(\tfrac{1}{\hat{m}_1}+\tfrac{1}{4})}+\tfrac{1}{\hat{m}_0}+\tfrac{1}{\hat{m}_1}-\tfrac{1}{4}\right) .\end{aligned} \tag{2.19}$$

The last approximation holds in the limit of small $\epsilon$, $\epsilon_0$ and $\epsilon_1$; it is not always an accurate approximation for the parameter values of interest, so we will use the exact expression in any computations which might be sensitive to the actual value. The positions of the zeroes of $V$, $\hat{\phi} = c_{\pm}^{1/\epsilon}$, are slightly greater than $\hat{\phi}_{\pm}$, by the factor $(1+\epsilon_4)^{1/\epsilon} \cong e^{1/4}$, as can be seen by comparing (2.19) with (2.18). The large hierarchy of $\hat{\phi}_+ \sim 10^{-16}$ is achieved by taking a moderately small ratio $v_1/v_0 < 1$ and raising it to a large power,[1] $1/\epsilon$. This leads to the mass scale which functions like the cutoff on the TeV brane,

$$\hat{\phi}_+ M_p \equiv M_{\rm TeV} \ , \tag{2.20}$$

where $M_{\rm TeV}/(1\ {\rm TeV})$ is a number of order unity, which we will take to be one of the phenomenological free parameters of the model. The choice of $M_{\rm TeV}$ specifies precisely where we want our cutoff scale to be. In ref. [16] the exponential corrections to $(v_1/v_0)^{1/\epsilon}$ in (2.19) were not considered; these change somewhat the values one might choose for $v_1/v_0$ and $\epsilon$ to get the correct hierarchy. The factor $e^{\pm\sqrt{(1/\hat{m}_1+1/4)/\epsilon}}$ in particular can be significant.

Refs. [30] and [13] showed that the canonically normalized radion field is $\phi = f\hat{\phi}$, where $f \equiv \sqrt{6}M^3/k = M_p^2$ (a method of deriving the canonically normalized radion field is presented in the introduction chapter). The 4-D effective action for the radion and gravity is

$$\begin{aligned} S &= \frac{M^3}{2k}\int d^4x\,\sqrt{-g}\left(1-\hat{\phi}^2\right)R \\ &+ \int d^4x\,\sqrt{-g}\left(\tfrac{1}{2}f^2\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - V(\hat{\phi})\right) \ , \end{aligned} \tag{2.21}$$

where $V(\hat{\phi})$ is the potential (2.17) and $R$ is the Ricci scalar.

The radion mass is $f^{-2}$ times the second derivative of $V$ evaluated at its

[1]An alternative possibility, taking $v_1/v_0 > 1$ and $\epsilon < 0$, corresponding to a negative squared mass in the bulk Lagrangian (2.4), does not work. Although the negative $m^2$ does not necessarily lead to any instability, since the field is prevented from going to infinity by the potentials on the branes, the radion potential has no nontrivial minimum in this case.

minimum. We find the value

$$m_\phi^2 = 4\frac{\Lambda}{f^2}\left(1+\tfrac{\epsilon}{4}-\tfrac{\epsilon}{\hat{m}_1}\right)^{-1}\left(\sqrt{1+\tfrac{4}{\hat{m}_1}}+2\tfrac{\sqrt{\epsilon}}{\hat{m}_1}\right)\eta^2\,\hat{\phi}_+^2\,\epsilon^{3/2}\ , \tag{2.22}$$

which implies that $m_\phi$ is of typically of order $\epsilon^{3/4}$ times the TeV scale. The factor of $\epsilon^{3/4}$ leads to the prediction that the radion will be lighter than the Kaluza-Klein excitations of the graviton, which would also be a signal of the new brane physics [13]. However, we see that the corrections due to finite $\hat{m}_1$, which were not explicitly considered in [13], can possibly compensate this and make the radion heavier, if $\hat{m}_1 \sim \epsilon$.

Now let us turn to the evolution of $\hat{\phi}$ in the early universe. For this purpose it is important to understand that the depth of the potential at its global minimum, as well as the height of the bump separating the minimum from $\hat{\phi}=0$, is set by the TeV scale. The values of the potential at these extrema are approximately (to leading order in $\epsilon_4$, but exact in $\epsilon_1$)

$$V(\hat{\phi}_\pm) \cong \mp 2\eta^2\Lambda\hat{\phi}_\pm^4\frac{\epsilon_4\sqrt{\epsilon_1+\epsilon_4}}{1+\epsilon_4-\epsilon_1}(1\pm\sqrt{\epsilon_1})\ . \tag{2.23}$$

Since $\Lambda \sim M_p^4$, the depth at the minimum is $V(\hat{\phi}_+) \sim -\epsilon^{3/2}O(\mathrm{TeV})^4$—suppressed slightly by the factor of $\epsilon^{3/2}$. The height of the bump at $\hat{\phi}_-$ can be considerably smaller because of the exponential factors in (2.19). In fact

$$\left|\frac{V(\hat{\phi}_-)}{V(\hat{\phi}_+)}\right| = \left(\frac{\hat{\phi}_-}{\hat{\phi}_+}\right)^4 \equiv \Omega^4\ , \tag{2.24}$$

where

$$\Omega \equiv \left(\frac{1-\sqrt{\epsilon_1+\epsilon_4}}{1+\sqrt{\epsilon_1+\epsilon_4}}\right)^{1/\epsilon} \sim \exp\left(-\sqrt{\tfrac{1}{\epsilon}(1+\tfrac{4}{\hat{m}_1})}\right)\ . \tag{2.25}$$

For example, if $\epsilon = 0.01$ as suggested by [16], $\Omega^4$ is less than $10^{-17}$. If the brane potential parameter $m_1$ is not large, so that $\hat{m}_1 \lesssim 1$, the suppression will be much greater. Figure 1 illustrates the flatness of the potential for the case $\epsilon = 0.2$, where the barrier is not so suppressed. The new mass scale $\Omega$ TeV $\ll$ 1 TeV is due to the small curvature of the radion potential at the top of the barrier, and its smallness will play an important role in the following. Thus the barrier separating the true minimum at $\hat{\phi}_+$ from the false one at $\hat{\phi} = 0$ is extremely shallow. Moreover a generic initial condition for the radion is a value like $\hat{\phi} \sim 1$, quite different from the one we want to end up with, $\hat{\phi} \sim 10^{-16}$. Clearly, the shape of the potential is such that, if we started with a generic initial value for $\hat{\phi}$, it would easily roll past the local minimum and the barrier, hardly noticing their presence. The point $\hat{\phi} = 0$ toward which it rolls is the limit of infinite brane separation, phenomenologically disastrous since gravity no longer couples at all to the visible brane in this limit.

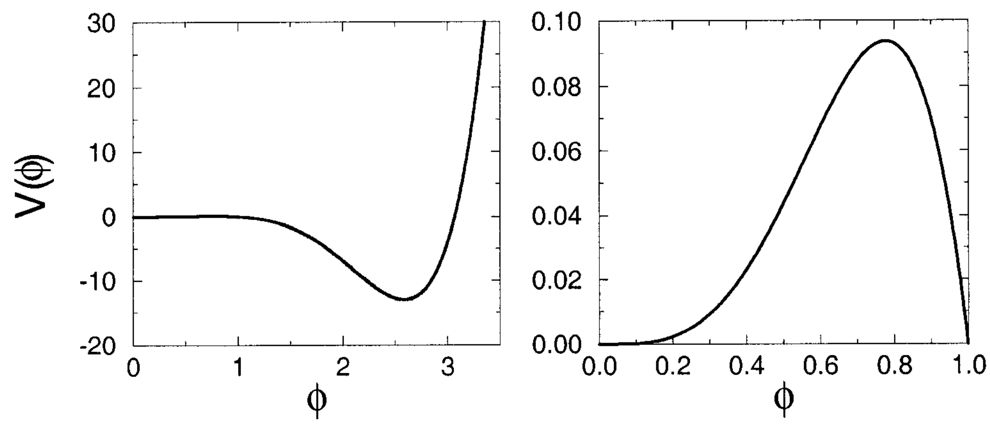



Figure 1: $V(\phi)$ versus $\phi$ for $\epsilon = 0.2$, showing the smallness of the barrier (right) relative to the minimum (left). Notice the difference in vertical scales.

One might wonder whether inflation could prevent this unwanted outcome, since then there would be a damping term in the $\phi$ equation of motion, possibly causing it to roll slowly:

$$\ddot{\phi} + 3H\dot{\phi} + V'_{\text{eff}}(\phi) = 0 \ . \tag{2.26}$$

Indeed, with sufficiently large Hubble rate $H$, the motion could be damped so that $\phi$ would roll to its global minimum. The condition for slow-roll is that

$$V''_{\text{eff}} \ll 9H^2 \ . \tag{2.27}$$

However, inflation is a two-edged sword in this instance, because the effective potential $V_{\rm eff}$ for the radion gets additional contributions from the curvature of the universe during inflation. From Eq (2.21) one can see that

$$\begin{aligned} V_{\rm eff}(\hat{\phi}) &= V(\hat{\phi}) + \frac{M^3}{2k} R\hat{\phi}^2 \\ &= V(\hat{\phi}) + 6\frac{M^3}{k} H^2\hat{\phi}^2 \\ &= V(\hat{\phi}) + H^2\phi^2 \ , \end{aligned} \tag{2.28}$$

using the relations $M_p^2 = M^3/k$ and $R = 12H^2$ which applies for de Sitter space. The new term tends to destroy the nontrivial minimum of the radion potential. One can estimate the relative shift in the position of the minimum as

$$\frac{\delta\hat{\phi}_+}{\hat{\phi}_+} = -\frac{\delta V'(\hat{\phi}_+)}{\hat{\phi}_+ V''(\hat{\phi}_+)} = -\frac{H^2}{m_\phi^2} \ . \tag{2.29}$$

This should be less than unity to avoid the disappearance of the minimum altogether.

Combining the requirement that the global minimum survives with the slow-roll condition (2.27), evaluated near the minimum of the potential, we find the following constraint on the Hubble rate:

$$\tfrac{1}{9} m_\phi^2 \ll H^2 < m_\phi^2 \ . \tag{2.30}$$

This is a narrow range, if it exists at all. In fact, one never expects such a large Hubble rate in the Randall-Sundrum scenario since the TeV scale is the cutoff: $H$ should never exceed $T^2/M_p \sim 10^{-16}$ TeV if the classical equations are to be valid. The problem of radion stabilization might also be exacerbated because contributions to the energy density of the universe which cause inflation can give additional terms of the type $\hat{\phi}^2$ to $V_{\rm eff}$ which are not considered in the above argument. For example, a field in the bulk which does not have its endpoints fixed on the branes has a 5-D energy density which is peaked on the visible brane [17], $\rho_5(y) \sim \rho_0 e^{2kby}$, and gives a contribution to $V_{\text{eff}}$ of

$$\begin{aligned} \delta V &\sim \rho_0 \int_0^1 dy\, b\, e^{-4kby} \rho_0 e^{2kby} \\ &= b\rho_0 \hat{\phi}^2 \,, \end{aligned} \tag{2.31}$$

remembering that $e^{-kb} = \hat{\phi}$. Such a contribution could destroy the nontrivial minimum even if (2.30) is satisfied. In any case, it does not appear to be natural to tune the Hubble rate during inflation to try to solve the stabilization problem.

## 2.4 Phase Transition to the True Vacuum

Since the barrier of the radion potential is too small to prevent the radion from rolling into the false minimum, perhaps we can take advantage of this smallness to get tunneling or thermal transitions back into the true vacuum. The situation is quite similar to that of "old inflation" [43], except that in the latter, the transition was never able to complete because the universe expanded too rapidly compared to the rate of nucleation of bubbles of the true vacuum. In the present case this problem can be avoided because we are not trying to use the radion for inflation. Indeed, a small amount of inflation may take place before the tunneling occurs, since the radion potential is greater than zero at $\phi = 0$, but we will not insist that this be sufficient to solve the cosmological problems inflation is intended to solve—otherwise we would be stuck with the problems of old inflation. Instead we will assume that inflation is driven by some other field, and consider the transitions of the radion starting from the time of reheating. The criterion that the phase transition completes is that the rate of bubble nucleation per unit volume, $\Gamma/V$, exceeds the rate of expansion of the universe per Hubble volume:

$$\frac{\Gamma}{V} \gtrsim H^4 \,. \tag{2.32}$$

The reason is that the bubbles expand at nearly the speed of light, so the relevant volume is determined by the distance which light will have traveled by a given time, which is of order $1/H$.

### 2.4.1 The Euclidean Bounce

To compute the nucleation rate $\Gamma/V$, one must construct the bounce solution which is a saddle point of the Euclideanized action [44, 45], in other words, with the sign of the potential reversed. This is a critical bubble solution with the boundary conditions

$$\begin{aligned} \phi(r)|_{r=0} &= \phi_0 \, ; \qquad \phi'(r)|_{r=0} = 0 \, ; \\ \phi(r)|_{r\to\infty} &= 0 \, ; \qquad \phi'(r)|_{r\to\infty} = 0 \, . \end{aligned} \tag{2.33}$$

The value of $\phi_0$ which ensures the desired behavior as $r \to \infty$ cannot be computed analytically because the motion of the field is damped by the term $\phi'/r$ in the equation of motion. We will consider bubble nucleation at finite temperature in the high $T$ limit, where the bounce solutions are three dimensional. The equation of motion is

$$\frac{1}{r^2}\left(r^2\phi'\right)' = +V'_{\rm eff}(\phi) \, , \tag{2.34}$$

where now $V_{\rm eff}$ includes thermal corrections, which are much larger than the $H^2\phi^2$ term considered in Eq (2.28):

$$\begin{aligned} V_{\rm eff}(\phi) \cong V(\phi) &+ \frac{T^2}{24}m_{\rm th}^2(\phi) - \frac{T}{12\pi}m_{\rm th}^3(\phi) \\ &+ \frac{c_b}{64\pi^2}m_{\rm th}^4(\phi) - V_0 \end{aligned} \tag{2.35}$$

$$m_{\rm th}^2(\phi) = V''(\phi) + \tfrac{1}{24}T^2V^{(4)}(0) \tag{2.36}$$

where $c_b \cong 3.9$ if the renormalization scale is taken to be equal to $T$. The leading thermal correction is of order $T^2\phi^2$, whereas $H^2\phi^2$ is suppressed by $T^2/M_p^2$ relative to this. The cubic term $m_{\rm th}^3$ becomes imaginary if $V''(\phi) +$

$\frac{T^2}{24}V^{(4)}(0)$ becomes negative; we take the real part only. The term $\frac{1}{24}T^2V^{(4)}(0)$ represents the thermal mass, which appears in the cubic and quartic terms when ring diagrams are resummed [46]. We subtract a constant term $V_0$ from $V_{\rm eff}$ so that $V_{\rm eff}(0) = 0$, as is needed to properly compute the action associated with the bounce solution.

The thermal corrections to the effective radion potential cause the bounce solution to fall exponentially at large $r$:

$$\phi \sim \frac{c}{r}e^{-r/r_0} \ , \tag{2.37}$$

where $1/r_0 \sim \eta\sqrt{\Lambda/f^4}\,T$ if $\Lambda/f^4 \ll 1$ [the exact expression is $1/r_0 = \sqrt{\lambda U_*}T$, in terms of quantities to be defined below, in eqs. (2.40) and (2.52)]. Once the bounce solution is known, it must be substituted back into the action, which can be written as

$$S = \frac{4\pi}{T}\int_0^\infty dr\, r^2\left(\tfrac{1}{2}\phi'^2 + V_{\rm eff}(\phi)\right) \ . \tag{2.38}$$

The nucleation rate is given by

$$\frac{\Gamma}{V} = \frac{|\omega_-|}{2\pi}\left(\frac{S}{2\pi}\right)^{3/2}|\mathcal{D}|^{-1/2}\, e^{-S} \ , \tag{2.39}$$

where $\omega_-$ is the imaginary frequency of the unstable mode of fluctuations around the the bounce solution, and $\mathcal{D}$ is the fluctuation determinant factor, to be described below. A typical profile for the bounce solution is shown in figure 2.

For the numerical determination of the bounce solution and action, as well as understanding their parametric dependencies, it is convenient to rescale the radius and the field by

$$r = \frac{\tilde{r}}{\sqrt{\lambda}T}; \qquad \lambda = \frac{\Lambda'}{f^4}c_-^2, \tag{2.40}$$

$$\phi = ZT\tilde{\phi}; \qquad Z = \frac{fc_-^{1/\epsilon}}{T} \ . \tag{2.41}$$

Then the action takes the form

$$
\begin{aligned}
S &= \frac{Z^2}{\sqrt{\lambda}}\tilde{S}(\epsilon, \hat{m}_1, \lambda, Z); & (2.42)\\
\tilde{S} &= 4\pi\int_0^\infty d\tilde{r}\,\tilde{r}^2\Bigg\{\tfrac{1}{2}\left(\tilde{\phi}'^2+\tilde{\phi}^2 f_2\right)+Z^2\tilde{\phi}^4 f_0 \\
&- \frac{1}{12\pi}\frac{\sqrt{\lambda}}{Z^2}\left[\left(\tfrac{c_\pm}{c_-}+12Z^2\tilde{\phi}^2 f_2\right)^{3/2}-\left(\tfrac{c_\pm}{c_-}\right)^{3/2}\right] \\
&+ \frac{3c_b}{8\pi^2}\lambda f_2\tilde{\phi}^2\left(\tfrac{c_\pm}{c_-}+6Z^2\tilde{\phi}^2 f_2\right)\Bigg\}, & (2.43)
\end{aligned}
$$

where

$$
\begin{aligned}
f_0(\tilde{\phi}) &= (\tilde{\phi}^\epsilon - 1)(\tilde{\phi}^\epsilon - \tfrac{c_\pm}{c_-}) \\
f_{n+1}(\tilde{\phi}) &= \left(1+\tfrac{1}{4-n}\phi\partial_{\tilde{\phi}}\right) f_n(\tilde{\phi})\ . & (2.44)
\end{aligned}
$$

In the following, it will be helpful to keep in mind that $Z$ can be extremely small, of order $\Omega$ in (2.25) when $\epsilon$ is small, whereas $\sqrt{\lambda}$ tends to be closer to unity, depending on the mass of the radion and the definition of the TeV scale (2.20):

$$
\sqrt{\lambda} = \frac{\epsilon^{-3/4}}{2\sqrt{6}}\frac{m_\phi}{M_{\rm TeV}}\frac{(1-\sqrt{\epsilon_1+\epsilon_4})}{\left(\sqrt{1+\frac{4}{\hat{m}_1}}+2\frac{\sqrt{\epsilon}}{\hat{m}_1}\right)^{1/2}} \qquad (2.45)
$$

In the rescaled variables, the value $\tilde{\phi} = 1$ corresponds to the first zero of the potential, which would be the starting point of the bounce if energy was conserved in the mechanical analog problem, *i.e.,* if there was no viscous damping term $\phi'/r$ in the equation of motion. The actual starting point turns out to have a value in the range $\tilde{\phi}_0 \sim 1.5 - 3$ because of this. The rescaled action $\tilde{S}$ depends mainly on the model parameters $\epsilon$ and $\hat{m}_1$, for the parameter values which are of interest to us. All the sensitive exponential dependence on $\epsilon$, namely the factor $c_-^{1/\epsilon}$, is removed from $\tilde{S}$. Numerically we find that

$$
\tilde{S} \cong \tfrac{2}{3}\hat{m}_1(\epsilon\hat{m}_1)^{-3/4}\ , \qquad (2.46)
$$

except when $\hat{m}_1$ becomes close to $\epsilon$. For $\hat{m}_1$ slightly smaller than $\epsilon$, $\epsilon_1$ starts to exceed 1, and $c_-$ becomes negative, signaling the onset of an instability in the radion potential toward coincidence of the two brane positions. Figure 3 shows the dependence of $\log(\tilde{S}/\hat{m}_1)$ versus $\log(\epsilon\hat{m}_1)$ .

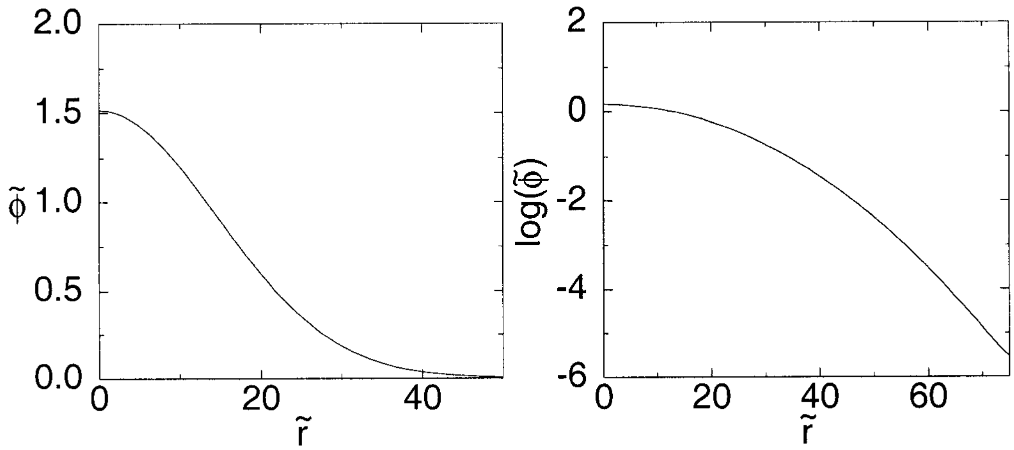



Figure 2: The bounce solution, for the parameters $\epsilon = 0.01$, $\hat{m}_1 = 0.1$, $m_\phi = T = 100$ GeV and $M_{\rm TeV} = 1$ TeV. The rescaling $\phi \to \tilde{\phi}$ and $r \to \tilde{r}$ is given in eqs. (2.40-2.41).

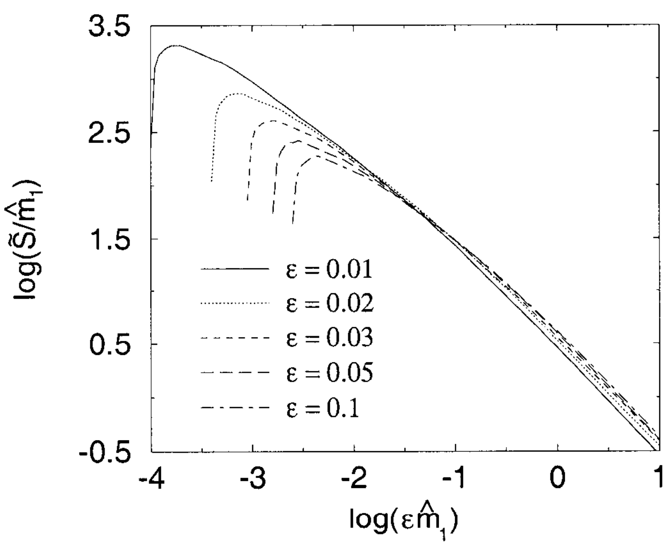



Figure 3: $\log_{10}(\tilde{S}/\hat{m}_1)$ versus $\log_{10}(\epsilon\hat{m}_1)$, where $\tilde{S}$ is the rescaled bounce action, Eq (2.43). The other parameters are $m_\phi = T = 100$ GeV and $M_{\rm TeV} = 1$ TeV.

We have computed the bounce solution and the corresponding action for a range of parameters $\epsilon$ and $\hat{m}_1$ which can be consistent with the solution to the hierarchy problem (*i.e.*, that $\hat{\phi} \sim 10^{-16}$ at the global minimum). The size of the bounce in position space, measured as the width at half-maximum, is small near $\epsilon = \hat{m}_1$, and reaches a larger constant value as $\hat{m}_1 \to \infty$. Using the rescaled radial variable $\tilde{r} = r\sqrt{\lambda}T$, the dependence of the width on $\epsilon$ and $\hat{m}_1$ is shown in figure 4.

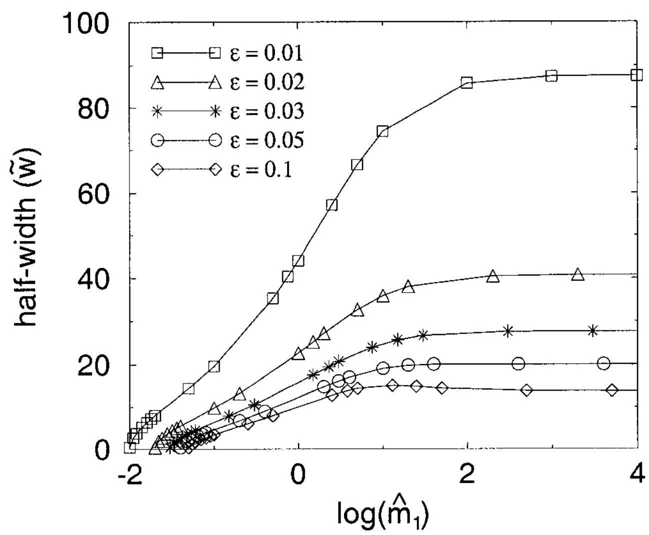



Figure 4: Half-width $\tilde{w}$ of the bounce solution, in terms of the rescaled radial distance $\tilde{r} = r\sqrt{\lambda}T$, versus $\log_{10}(\hat{m}_1)$, for the same parameters as in figure 3.

The action of the bounce can be much greater than or much less than 1, depending on the parameters: for $\epsilon \sim \hat{m}_1 \ll 1$, $S \ll 1$, while for larger values of $\epsilon$ and $\hat{m}_1$, $S \gg 1$. Where the crossover occurs ($S \sim 1$) depends on $m_\phi$, $T$ and $M_{\rm TeV}$. This behavior can be inferred from figure 3 (showing $\tilde{S}$) and the dependences of the coefficient in the relation $S = (Z^2/\sqrt{\lambda})\tilde{S}$. Rather than presenting further results for $S$ directly however, we will turn to the more relevant quantity, the rate of bubble nucleation. For this we need to determine the prefactor of $e^{-S}$ in the rate.

### 2.4.2 Prefactor of Bubble Nucleation Rate

The bounce action is the most important quantity determining the rate of tunneling, since it appears in the exponent of the rate (2.39). Since we do not have a model for the inflation and reheating of the universe which must occur prior to the bubble nucleation, hence an exact prediction for the reheating temperature which enters the rate, it would not be worthwhile to compute the prefactors in Eq (2.39) very accurately; however we can estimate their size.

First, consider the frequency $\omega_-$ of the unstable mode. $\omega_-^2$ is the negative eigenvalue of the Schrödinger-like equation for small fluctuations $\delta\phi$ around the bounce solution, which we will denote by $\phi_b(r)$:

$$-\left(\delta\phi'' + \frac{2}{r}\delta\phi'\right) + \left.\frac{\partial^2 V_{\rm eff}}{\partial\phi^2}\right|_{\phi_b(r)} \delta\phi = \omega_-^2\,\delta\phi\ . \tag{2.47}$$

Rescaling the radius and background field exactly as in eqs. (2.40-2.41), Eq (2.47) becomes

$$-\left(\delta\phi'' \ + \ \frac{2}{\tilde r}\delta\phi'\right) + U(\tilde r)\,\delta\phi = \frac{\omega_-^2}{\lambda T^2}\,\delta\phi\ , \tag{2.48}$$

$$\begin{aligned} U(\tilde r) &= \frac{1}{\lambda T^2}\frac{\partial^2 V_{\rm eff}}{\partial\phi^2}(\tilde\phi_b(r)) \\ &= f_4 + X - \frac{3\sqrt{\lambda}}{\pi}\left(f_4\sqrt{\tfrac{c_+}{c_-}+X} + \frac{12(Z\tilde\phi f_3)^2}{\sqrt{\tfrac{c_+}{c_-}+X}}\right) \\ &+ \frac{6c_b}{8\pi^2}\lambda\left(\tfrac{c_+}{c_-}f_4 + X(f_4+2f_3^2/f_2)\right)\ , \end{aligned} \tag{2.49}$$

where primes now denote $\frac{d}{d\tilde r}$, $X = 12Z^2\tilde\phi^2 f_2$, and the $f_i$ are defined in Eq (2.44). Except when the radion mass is significantly larger than 100 GeV, $\lambda$ is much smaller than 1, and $U$ is dominated by the first two terms in (2.48). Of these, the first ($f_4$) always dominates if $Z \ll 1$, while the second ($X$) can be important near $r = 0$ if $Z \gtrsim 1$. The two different cases are illustrated in figure 5. Since $1 < \tilde{\phi}_0^\epsilon < \frac{c_+}{c_-}$, both $f_4$ and $X$ are negative at $r = 0$, so that

$$\begin{aligned} U_0 \equiv U(0) &\cong -\epsilon^{3/2}\sqrt{1+\tfrac{4}{\hat{m}_1}} \\ &\times \left(2 + \ln\tilde{\phi}_0 + 12Z^2\tilde{\phi}_0^2(\tfrac{1}{2} + \ln\tilde{\phi}_0)\right) \ , \end{aligned} \tag{2.50}$$

and thus the smallest eigenvalue of Eq (2.47) is negative. This is the unstable mode of the saddle point solution, with imaginary frequency of order

$$\omega_-^2 \sim U_0 \lambda T^2 \ . \tag{2.51}$$

Recall that $|\omega_-|$ appears in the prefactor of the nucleation rate $\Gamma/V$.

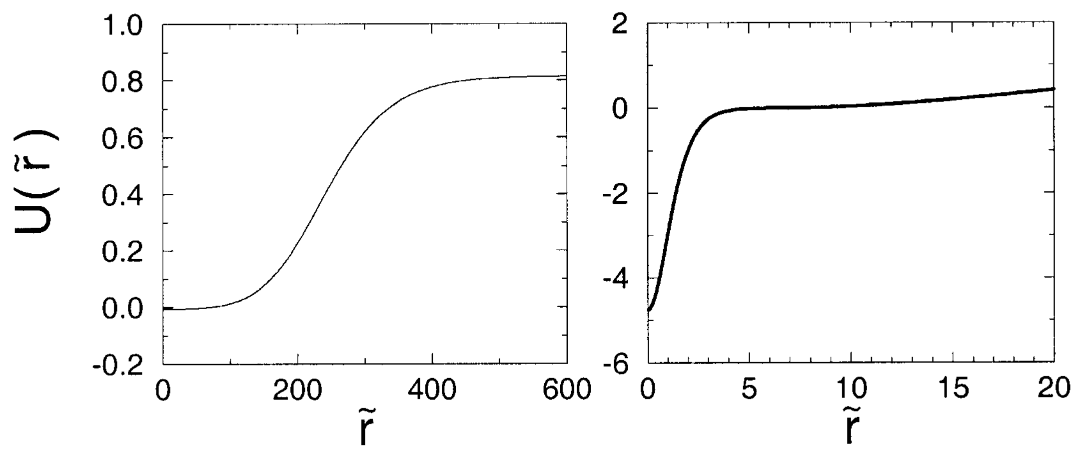



Figure 5: The potential for small fluctuations around the bounce solution, $U(\tilde{r}) = X^2\, V''(\phi_b(\tilde{r}))$, as a function of $\tilde{r}$, for the parameters $\epsilon = 0.02$, $\hat{m}_1 = 5$ (left) and $\epsilon = 0.1$, $\hat{m}_1 = 25$ (right).

As $\tilde{r} \to \infty$, $U(\tilde{r})$ approaches a maximum value

$$U_* \equiv U(\infty) = \tfrac{c_+}{c_-}\left(1 - \tfrac{3}{\pi}\sqrt{\lambda\tfrac{c_+}{c_-}} + \tfrac{3c_b}{4\pi^2}\lambda\tfrac{c_+}{c_-}\right) \ , \tag{2.52}$$

which determines the asymptotic behavior of the fluctuations at large $\tilde{r}$: $\delta\phi \sim e^{-\sqrt{U_0}\tilde{r}}$. The fluctuations around the false vacuum state ($\phi = 0$) thus have a mass given by

$$m^2 = V''_{\phi=0} = U_* \lambda T^2 \ , \tag{2.53}$$

which will be relevant for the following.

Next we must estimate the functional determinant factor,

$$\mathcal{D} = \frac{\det'(-\partial_\tau^2 - \nabla^2 + V''(\phi_b))}{\det(-\partial_\tau^2 - \nabla^2 + V''(0))} , \tag{2.54}$$

where $\tau$ is imaginary time ($\tau \in [0, 1/T]$), $\nabla^2$ is the three-dimensional Laplacian, $\phi_b$ is the bounce solution, and the prime on $\det'$ means that the three translational zero-mode eigenvalues must be omitted from the determinant for fluctuations around the bounce. These zero modes correspond to spatial translations of the bubble solution. Because of their removal, $\mathcal{D}$ has dimensions of (energy)$^{-6}$, as is required to get a rate per unit volume in Eq (2.39).

Ref. [47] has given a thorough account of how to compute $\mathcal{D}$ by a method which was discussed for one-dimensional systems in [48]. In 3-D one should classify the eigenvalues of the fluctuation operators by the quantum numbers $n$ and $l$, denoting Matsubara and angular momentum excitations, respectively. Then $\mathcal{D}$ can be written as a product, $\mathcal{D} = \prod_{n,l} \mathcal{D}_{n,l}$.

Ref. [47] shows that the contribution to $\mathcal{D}$ from the $l$th partial wave can be expressed, to leading order in a perturbative expansion in the potential $U(r)$, as

$$\mathcal{D}_{n,l} \cong \left(1 + h_l^{(1)}\right)^{2l+1} . \tag{2.55}$$

The quantity $h_l^{(1)}$ has the Green's function solution

$$\begin{aligned} h_l^{(1)} &= 2\int_0^\infty dr\, r\, I_{l+1/2}(\kappa r)\, K_l(\kappa r)\, \left(V''(\phi_b(r)) - m^2\right) \\ &= 2\int_0^\infty d\tilde{r}\, \tilde{r}\, I_{l+1/2}(\tfrac{\kappa\tilde{r}}{\sqrt{\lambda}T})\, K_l(\tfrac{\kappa\tilde{r}}{\sqrt{\lambda}T})\, (U(\tilde{r}) - U_*) \end{aligned} \tag{2.56}$$

using the modified Bessel functions $I$ and $K$, and the mass $m$ of the field in the false vacuum, Eq (2.53). For general Matsubara frequencies, $\nu = 2\pi n T$, one defines $\kappa = \sqrt{m^2 + \nu^2}$.

The subdeterminant for the $n = 0$ (zero-temperature) sector of the theory has the usual ultraviolet divergences of quantum field theory, namely the vacuum diagram  (the dot represents one insertion of $V(\phi)$), which should be absorbed by renormalization of the zero of energy for the radion potential. Since we are not attempting to solve the cosmological constant problem here, we are going to ignore all of this and compute only the factor $\mathcal{D}_{0,1}$, which contains the translational zero modes—or more precisely, which has the zero modes removed. This removal is accomplished by replacing

$$1 + h_1^{(1)} \to \frac{dh_1^{(1)}}{d\kappa^2} \; . \tag{2.57}$$

Notice that this quantity has dimensions of $(\text{mass})^{-2}$, and there are $2l+1 = 3$ such factors, so that $|\mathcal{D}|^{-1/2}$ has dimensions of $(\text{mass})^3$, as required. From Eq (2.56) one can show that

$$\begin{aligned} \frac{dh_1^{(1)}}{d\kappa^2} &= \frac{1}{\lambda T^2 U_*^2} I_U \; ; \\ I_U &\equiv \int_0^\infty dy\, y^2 \left(I_{3/2}(y) K_1(y)\right)' \left(U(\tfrac{y}{\sqrt{U_*}}) - U_*\right) \; . \end{aligned} \tag{2.58}$$

We have numerically evaluated the integral $I_U$ for each set of parameters. Our estimate for the fluctuation determinant factor in the nucleation rate can then be written as

$$|\mathcal{D}|^{-1/2} \sim \left(\frac{\lambda T^2 U_*^2}{I_U}\right)^{3/2} \; . \tag{2.59}$$

### 2.4.3 Results for Nucleation Rate

Putting the above ingredients together to find the rate of bubble nucleation per unit volume, $\Gamma/V$, we see that the latter depends on five undetermined parameters: $\epsilon$, $\hat{m}_1$, $m_\phi$, $M_{\rm TeV}$ and the temperature $T$. Ref. [30] showed that, as long as the energy density on the TeV brane is much less than $M_{\rm TeV}^4$, the usual 4-D effective theory governs the Hubble rate:

$$H^2 = \frac{\rho}{3M_p^2} \; , \tag{2.60}$$

where $\rho$ is the total energy density,

$$\begin{aligned}
\rho &= g_*\frac{\pi^2}{30}T^4 + \rho_\phi\ , \\
\rho_\phi &= \tfrac{1}{2}\dot{\phi}^2 + V(\phi) - V(\phi_+) \\
&\cong \tfrac{3}{8}M_{\rm TeV}^2 m_\phi^2 \frac{\sqrt{1+\frac{4}{\hat{m}_1}}(1+\sqrt{\epsilon_1})}{\sqrt{1+\frac{4}{\hat{m}_1}}+2\frac{\sqrt{\epsilon}}{\hat{m}_1}}\ .
\end{aligned} \tag{2.61}$$

We take the number of relativistic degrees of freedom, $g_*$, to be 100. The kinetic energy of the radion is zero since $\dot{\phi} = 0$ in the false vacuum, so $\rho_\phi$ is essentially the potential energy of the radion in the false vacuum, assuming the 4-D cosmological constant is zero: $V(\phi) - V(\phi_+) = |V(\hat{\phi}_+)|$, which is given by Eq (2.23). Depending on the parameters, this can be comparable in size or dominate over the energy density of radiation. Using our estimates for the prefactor of the tunneling rate, the logarithm of the ratio of $\Gamma/V$ to $H^4$ can be written as

$$\ln\frac{\Gamma}{VH^4} \cong \ln\left(\frac{\lambda^2\sqrt{U_0}\,U_*^3}{(2\pi)^{5/2}}T^4\left(\frac{3M_p^2}{\rho}\right)^2\left(\frac{S}{I_U}\right)^{3/2}\right) - S \tag{2.62}$$

where $S$ is the action of the bounce solution. The criterion for completion of the phase transition to the true vacuum state is that $\ln(\Gamma/VH^4) > 0$. The saddle point approximation leading to Eq (2.39) is only valid if the action $S$ is not much less than 1. Otherwise, the barrier is not effective for preventing the field from rolling to the true minimum, as in a second order phase transition. This situation occurs in the vicinity of $\ln(\Gamma/VH^4) \sim 150$ in the following results; thus the transition region where $\Gamma/VH^4 = 1$ is well within the realm of validity of the approximation.

In figure 6 we show the contours of constant $\ln(\Gamma/VH^4)$ in the plane of $\log_{10}(\hat{m}_1)$ and $\log_{10}(\epsilon)$, starting with the fiducial values $T = m_\phi = 100$ GeV, $M_{\rm TeV} = 1$ TeV for the other parameters, and showing how the results change when any one of these is increased. The dependencies can be understood from the prefactor $Z^2/\sqrt{\lambda}$ in the action, Eq (2.42):

$$\frac{Z^2}{\sqrt{\lambda}} \sim \epsilon^{3/4} \frac{M_{\rm TeV}^3\, \Omega^2}{T^2\, m_\phi} \,, \tag{2.63}$$

where $Z$ and $\lambda$ are given by (2.40-2.41) and $\Omega$ by (2.25). The factor $\Omega$ is responsible for suppressing the bounce action when $\epsilon \ll 1$ or $\epsilon \hat{m}_1 \ll 1$, explaining the shape of the allowed regions in each graph. Nucleation of bubbles containing the true minimum becomes faster when the temperature or the radion mass is increased, but slower if the definition of the TeV scale in increased. These dependences are dictated not only by the size of the barrier between the two minima in the effective potential, but also by the size of the bubbles.

Interestingly, the borderline between allowed and forbidden regions of parameter space falls within the range which is relevant from the point of view of building a model of radion stabilization. That is, some choices which would otherwise have been natural and acceptable are ruled out by our considerations. We see furthermore that the choice of $\hat{m}_1 \to \infty$, as was effectively focused on in ref. [16], is not the optimal one for achieving a large nucleation rate.

It might be thought that our analysis is rendered less important by the fact that one can always obtain fast nucleation simply by going to high enough temperatures. However it must be remembered that the TeV scale functions as the high-energy cutoff in the Randall-Sundrum scenario: the whole semiclassical description breaks down at super-TeV scales, where quantum gravity effects start to become important. From this point of view, the temperatures of $100 - 300$ GeV which we are discussing are already rather high, and a fairly efficient mechanism of reheating at the end of inflation will be needed to generate them.

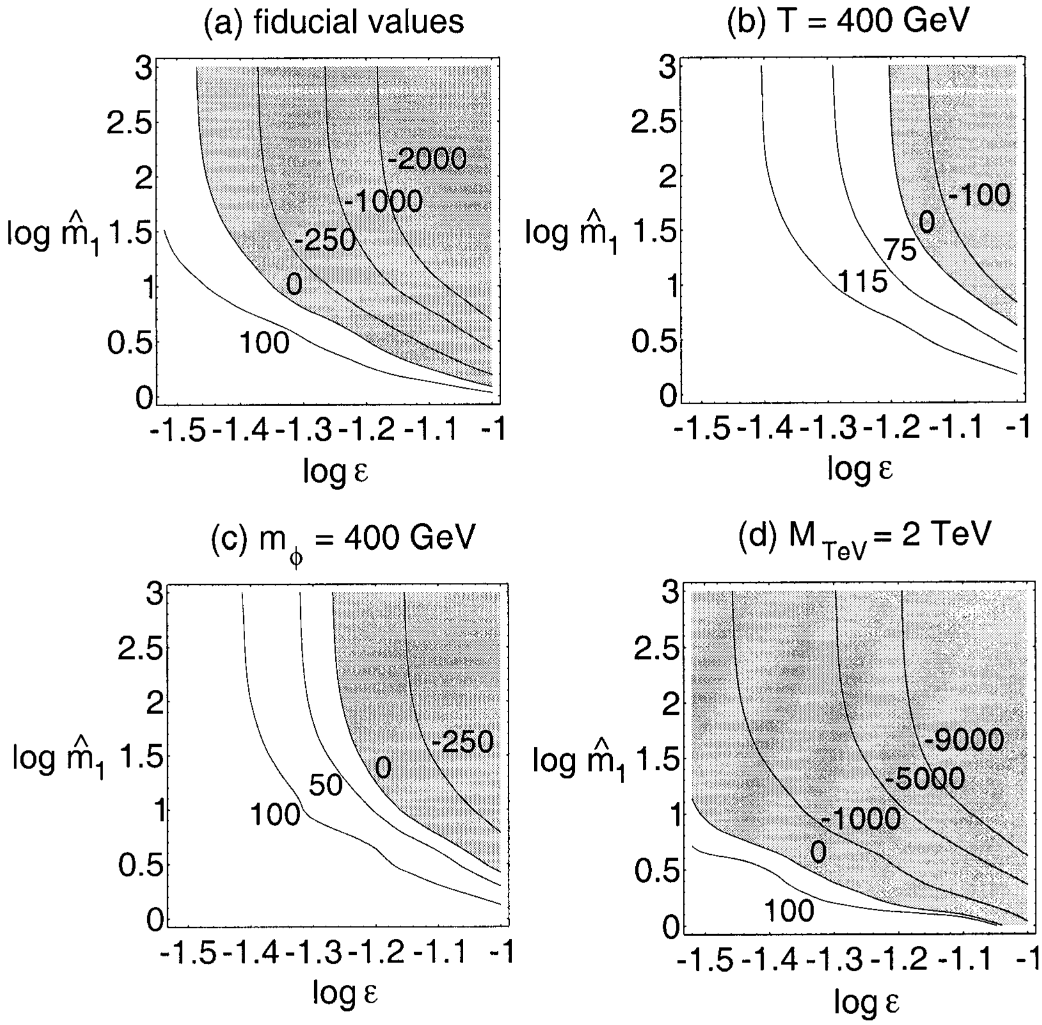



Figure 6: Contours of $\ln(\Gamma/VH^4)$ in the plane of $\log_{10}(\hat{m}_1)$ versus $\log_{10}(\epsilon)$. The shaded regions are where the tunneling rate is too small for the phase transition to complete. Figure (a) has $T = m_\phi = 100$ GeV, $M_{\rm TeV} = 1$ TeV. The other figures are the same except for the following changes: (b) $T = 400$ GeV; (c) $m_\phi = 400$ GeV; (d) $M_{\rm TeV} = 2$ TeV.

## 2.5 Discussion

In this chapter we have presented a somewhat simpler model of radion stabilization by a bulk field ($\psi$) than that of Goldberger and Wise [16]; although the physics is qualitatively identical, we are able to write the radion potential exactly, and thus explore the effect of letting the stabilizing field's VEV's on the branes be pinned more or less strongly to their minimum energy values. One such effect is that the mass of the radion can be significantly increased for small values of the parameter $m_1$, which is the coefficient of the potential for $\psi$ on the TeV brane. Moreover if $m_1/k \equiv \hat{m}_1$ is accidentally close to $\epsilon$, approximately the minimum value consistent with a stable potential, the radion mass can start to diverge, by the factor $(1 - \frac{\epsilon}{4} - \frac{\epsilon}{\hat{m}_1})^{-1/2}$. This modifies somewhat the expectation expressed in ref. [13] that the radion mass will be small relative to the TeV scale, due to a factor of $\epsilon^{3/4}$.

Our main focus was on the problem that the radion potential has a local minimum at infinite brane separation, and that the barrier between the true and false minima is so small that for generic initial conditions, one would expect the true minimum to be bypassed as the radion field rolls through it. We showed that for a large range of parameters, the high-temperature phase transition to the true minimum is able to complete, thus overcoming the problem. There are however significant constraints on the model parameters, and the initial temperature after inflation, to insure this successful outcome.

There remain some outstanding issues. The form of the radion effective potential is such that the field is able to reach $\phi = 0$ in a finite amount of time; yet $\phi = 0$ represents infinite brane separation in the extra dimension. This paradoxical situation may be due to the assumption that the stabilizing field, $\psi$, is always in its minimum energy configuration at any given moment. In reality $\psi$ must require a finite amount of time to respond to changes in the radion. Thus one should solve the coupled problem for time-varying $\phi$ and $\psi$ to do better. This is probably a difficult problem, which we leave to future study.

A related question is whether it is correct to treat thermal fluctuations of the radion field $\phi$ analogously to a normal scalar field with values in the range $(-\infty, \infty)$. Since $\phi$ is related to the size of the extra dimension by

$\phi = fe^{-kb}$, its range is $[0, f]$. We have not studied what effect this might have on the thermal part of the effective potential; instead we assumed that the usual treatment suffices.

Another approximation we made was to ignore the back-reaction of the stabilizing field on the geometry. Ref. [49] has given a method of finding exact solutions to the coupled equations for the warp factor $a(y)$ and the stabilizing field $\psi(y)$. This method cannot be applied in the present case because it works only for bulk scalar potentials with a special form that, among other things, requires them to be unbounded from below.[2] Moreover, since the method of [49] generates only static solutions to the equation of motion, it cannot be used to deduce the radion potential, which is a probe of the response of the geometry when it is perturbed away from a static solution. On the other hand, [49] does show that the neglect of the back-reaction is justified for the parameter values which most closely resemble the Goldberger-Wise model.

[2] In [49] this is asserted to be allowed because of special properties of anti-de Sitter space; however we believe that the real reason the bulk potential can be unbounded from below is that the potentials on the branes have the correct sign to prevent an instability.

# Chapter 3

# Cosmology of the Stabilized Randall-Sundrum Model

## 3.1 Introduction

As explained in the introductory chapter, the RSI idea as originally proposed was incomplete due to the lack of any mechanism for stabilizing the brane separation, $b$. This was a modulus, corresponding to a massless particle, the radion, which would be ruled out because of its modification of gravity: the attractive force mediated by the radion would effectively increase Newton's constant at large distance scales. An attractive model for giving the radion a potential energy was proposed by Goldberger and Wise (GW) [16]; they introduced a bulk scalar field with different VEV's, $v_0$ and $v_1$, on the two branes. If the mass $m$ of the scalar is small compared to the scale $k$ which appears in the warp factor $e^{-kby}$, then it is possible to obtain the desired interbrane separation. One finds the relation $e^{-kb} \cong (v_1/v_0)^{4k^2/m^2}$.

An important benefit of stabilizing the radion is that cosmology is governed by the usual Friedmann equations, up to small corrections of order $\rho/(\mathrm{TeV})^4$ [30]. Even with stabilization, as we showed in chapter 2, there may be a problem with reaching a false minimum of the GW radion potential [1], but without stabilization, there is a worse problem: an unnatural tuning of the energy densities on the two branes is required for getting solutions where the extra dimension is static [28, 29], a result which can be derived using the (5,5) component of the Einstein equations. However when there is a nontrivial potential for the radius, $V(b)$, the (5,5) equation serves only to determine the shift $\delta b$ in the radius due to the expansion, and there is no longer any constraint on the matter on the branes. Although this point is now well appreciated [31, 41, 50, 51], it has not previously been explicitly demonstrated by solving the full 5-dimensional field equations using a concrete stabilization mechanism. Indeed, it has been claimed that such solutions are not possible with an arbitrary equation of state for the matter on the branes [53]-[54], and also that the rate of expansion does not reproduce normal cosmology on the negative tension brane despite stabilization [55]. Our purpose is to present the complete solutions, to leading order in an expansion in the energy densities on the branes, thus refuting these claims.

## 3.2 Preliminaries

The action for 5-D gravity coupled to the stabilizing scalar field $\Phi$ and matter on the branes (located at $y = 0$ and $y = 1$, respectively) is

$$\begin{aligned} S &= \int d^5x\sqrt{g}\left(-\frac{1}{2\kappa^2}R - \Lambda + \tfrac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - V(\Phi)\right) \\ &+ \int d^4x\sqrt{g}\left(\mathcal{L}_{m,0} - V_0(\Phi)\right)|_{y=0} + \int d^4x\sqrt{g}\left(\mathcal{L}_{m,1} - V_1(\Phi)\right)|_{y=1} \end{aligned} \tag{3.1}$$

where $\kappa^2$ is related to the 5-D Planck scale $M$ by $\kappa^2 = 1/(M^3)$. The negative bulk cosmological constant needed for the RS solution is parametrized as $\Lambda = -6k^2/\kappa^2$ and the scalar field potential is that of a free field, $V(\Phi) = \frac{1}{2}m^2\Phi^2$. The brane potentials $V_0$ and $V_1$ can have any form that will insure nontrivial VEV's for the scalar field at the branes, for example $V_i(\Phi) = \lambda_i(\Phi^2 - v_i^2)^2$

[16]. In chapter 2 we pointed out that the choice $V_i(\Phi) = m_i(\Phi - v_i)^2$ is advantageous from the point of view of analytic calculability (see also [49]).

We will take the metric to have the form

$$\begin{aligned} ds^2 &= n^2(t,y)dt^2 - a^2(t,y)\sum_i dx_i^2 - b^2(t,y)dy^2 \\ &= e^{-2N(t,y)}dt^2 - a_0(t)^2 e^{-2A(t,y)}\sum_i dx_i^2 - b(t,y)^2 dy^2 \ , \end{aligned} \tag{3.2}$$

where a perturbative expansion in the energy densities of the branes will be made around the static solution:

$$\begin{aligned} N(t,y) &= A_0(y) + \delta N(t,y); \qquad A(t,y) = A_0(y) + \delta A(t,y) \\ b(t,y) &= b_0 + \delta b(t,y); \qquad\qquad \Phi(t,y) = \Phi_0(y) + \delta\Phi(t,y)\ . \end{aligned} \tag{3.3}$$

The perturbations are taken to be linear in the energy densities $\rho_*$ and $\rho$ of matter on the Planck and TeV branes, located at $y = 0$ and $y = 1$, respectively.

This ansatz is to be substituted into the Einstein equations, $G_{mn} = \kappa^2 T_{mn}$, and the scalar field equation

$$\partial_t\left(\frac{1}{n}ba^3\dot{\Phi}\right) - \partial_y\left(\frac{1}{b}a^3 n\Phi'\right) + ba^3 n\left[V' + V_0'\delta(by) + V_1'\delta(b(y-1))\right] = 0\ . \tag{3.4}$$

Here and in the following, primes on functions of $y$ denote $\frac{\partial}{\partial y}$, while primes on potentials of $\Phi$ will mean $\frac{\partial}{\partial\Phi}$. The nonvanishing components of the Einstein tensor are

$$\begin{aligned} G_{00} &= 3\left[(\frac{\dot a}{a})^2 + \frac{\dot a}{a}\frac{\dot b}{b} - \frac{n^2}{b^2}\left(\frac{a''}{a} + (\frac{a'}{a})^2 - \frac{a'b'}{ab}\right)\right] \\ G_{ii} &= \frac{a^2}{b^2}\left[(\frac{a'}{a})^2 + 2\frac{a'}{a}\frac{n'}{n} - \frac{b'}{b}\frac{n'}{n} - 2\frac{b'}{b}\frac{a'}{a} + 2\frac{a''}{a} + \frac{n''}{n}\right] \\ &+ \frac{a^2}{n^2}\left[-(\frac{\dot a}{a})^2 + 2\frac{\dot a}{a}\frac{\dot n}{n} - 2\frac{\ddot a}{a} + \frac{\dot b}{b}(-2\frac{\dot a}{a} + \frac{\dot n}{n}) - \frac{\ddot b}{b}\right] \\ G_{05} &= 3\left[\frac{n'}{n}\frac{\dot a}{a} + \frac{a'}{a}\frac{\dot b}{b} - \frac{\dot a'}{a}\right] \\ G_{55} &= 3\left[\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - \frac{b^2}{n^2}\left(\frac{\dot a}{a}\left(\frac{\dot a}{a} - \frac{\dot n}{n}\right) + \frac{\ddot a}{a}\right)\right] \end{aligned} \tag{3.5}$$

and the stress energy tensor is $T_{mn} = g_{mn}(V(\Phi)+\Lambda)+\partial_m\Phi\partial_n\Phi-\frac{1}{2}\partial^l\Phi\partial_l\Phi g_{mn}$ in the bulk. On the branes, $T^n_m$ is given by

$$\begin{aligned} T^n_m &= \delta(by)\,\mathrm{diag}(V_0+\rho_*, V_0-p_*, V_0-p_*, V_0-p_*, 0) \\ &+ \delta(b(y-1))\,\mathrm{diag}(V_1+\rho, V_1-p, V_1-p, V_1-p, 0) \end{aligned} \tag{3.6}$$

At zeroth order in the perturbations, the equations of motion can be written as

$$\begin{aligned} A_0'^2 &= \frac{\kappa^2}{12}\left(\Phi_0'^2 - m^2 b_0^2 \Phi_0^2\right) + k^2 b_0^2; \qquad A_0'' = \frac{1}{3}\kappa^2 \Phi_0'^2 \\ \Phi_0'' &= 4A_0'\Phi_0' + m^2 b_0^2 \Phi_0\ , \end{aligned} \tag{3.7}$$

and the solutions are approximately

$$\Phi_0(y) \cong v_0 e^{-\epsilon k b_0 y}\ ; \qquad A_0(y) \cong k b_0 y + \frac{\kappa^2}{12} v_0^2 (e^{-2\epsilon k b_0 y} - 1) \tag{3.8}$$

where we have normalized $A_0(0) = 0$, and introduced

$$\epsilon = \sqrt{4 + \frac{m^2}{k^2}} - 2 \cong \frac{m^2}{4k^2}\ . \tag{3.9}$$

The above approximation is good in the limit $\epsilon \ll 1$, which is the same regime in which the Goldberger-Wise mechanism naturally gives a large hierarchy without fine-tuning the scalar field VEV's on the branes: $e^{-kb_0} = (v_1/v_0)^{1/\epsilon}$. For small $\epsilon$, the GW solution coincides with an exact solution of the coupled equations that was presented in ref. [49].

## 3.3 Perturbation Equations

We can now write the equations for the perturbations of the metric, $\delta A$, $\delta N$, $\delta b$, and the scalar field, $\delta\Phi$. The equations take a simpler form when expressed in terms of the following combinations:

$$\Psi = \delta A' - A_0'\frac{\delta b}{b_0} - \frac{\kappa^2}{3}\Phi_0'\delta\Phi; \qquad \Upsilon = \delta N' - \delta A' \tag{3.10}$$

Further simplification comes from realizing that the perturbations will have the form, for example, $\Psi = \rho_*(t)g_0(y) + \rho(t)g_1(y)$, so that their time derivatives are proportional to $\dot{\rho}$ and $\dot{\rho}_*$. Below we will confirm that $\dot{\rho} = -3H(\rho+p)$, where $H \sim \sqrt{\rho}, \sqrt{\rho_*}$ is the Hubble parameter. Therefore time derivatives of the perturbations are higher order in $\rho$ and $\rho_*$ than are $y$ derivatives, and can be neglected at leading order (except in the (05) Einstein equation, where $\rho^{3/2}$ *is* the leading order). Using this approximation, we can write the combinations (00), (00)+(ii), (05) and (55) of the Einstein equations as

$$4A_0'\Psi - \Psi' = \left(\frac{\dot{a}_0}{a_0}\right)^2 b_0^2 e^{2A_0} \tag{3.11}$$

$$-4A_0'\Upsilon + \Upsilon' = 2\left(\left(\frac{\dot{a}_0}{a_0}\right)^2 - \frac{\ddot{a}_0}{a_0}\right) b_0^2 e^{2A_0} \tag{3.12}$$

$$-\frac{\dot{a}_0}{a_0}\Upsilon + \dot{\Psi} = 0 \tag{3.13}$$

$$A_0'(4\Psi + \Upsilon) + \frac{\kappa^2}{3}\left(\Phi_0''\delta\Phi - \Phi_0'\delta\Phi' + \Phi_0'^2\frac{\delta b}{b_0}\right) = \left(\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{\ddot{a}_0}{a_0}\right) b_0^2 e^{2A_0} \tag{3.14}$$

In addition, there is the scalar field equation,

$$\begin{aligned}\delta\Phi'' &= (4\Psi + \Upsilon)\Phi_0' + \left(\frac{4\kappa^2}{3}\Phi_0'^2 + b_0^2 V''(\Phi_0)\right)\delta\Phi \\ &+ 4A_0'\delta\Phi' + \left(2b_0^2 V'(\Phi_0) + 4A_0'\Phi_0'\right)\frac{\delta b}{b_0} + \Phi_0'\frac{\delta b'}{b_0}\end{aligned} \tag{3.15}$$

Assuming $Z_2$ symmetry (all functions symmetric under $y \to -y$), the boundary conditions implied by the delta function sources at the branes are

$$\Psi(t,0) = +\frac{\kappa^2}{6}b_0\rho_*(t); \qquad \Psi(t,1) = -\frac{\kappa^2}{6}b_0\rho(t) \tag{3.16}$$

$$\Upsilon(t,0) = -\frac{\kappa^2}{2}b_0(\rho_* + p_*)(t); \qquad \Upsilon(t,1) = \frac{\kappa^2}{2}b_0(\rho + p)(t) \tag{3.17}$$

$$\delta\Phi'(t,y) = \frac{\delta b(t,y_i)}{b_0}\Phi_0'(t,y_i) + (-1)^i(\frac{b_0}{2})V_i''(\Phi_0(t,y_i))\,\delta\Phi(t,y_i) \tag{3.18}$$

where in (3.18) $i = 0, 1, y_0 = 0$ and $y_1 = 1$.

## 3.4 Solutions

Naively, it would appear that we have five equations for four unknown perturbations, but of course since gravity is a gauge theory, this is not the case. First, we have the relation $\frac{\partial}{\partial t}$[Eq 3.11] $+\frac{\dot{a}_0}{a_0}$[Eq 3.12] = [Eq 3.13]. Furthermore, the (55) Einstein equation and the scalar equation can be shown to be equivalent, using (00),(ii) and the zeroth order relations (3.7): [Eq (3.14)]$'$ $- 4A_0'\times$[Eq (3.14)] $= \Phi_0'\times$[Eq (3.15)]. So our system is actually underdetermined because of unfixed gauge degrees of freedom. To see this more directly, consider an infinitesimal diffeomorphism which leaves the coordinate positions of the branes unchanged: $y = \bar{y} + f(\bar{y})$, where $f(0) = f(1) = 0$. The metric and scalar perturbations transform as

$$\begin{aligned} \delta A &\rightarrow \delta A + A_0' f; \qquad \delta N \rightarrow \delta N + A_0' f \\ \delta b &\rightarrow \delta b + b_0 f'; \qquad \delta\Phi \rightarrow \delta\Phi + \Phi_0' f \end{aligned} \tag{3.19}$$

If desired, one can form the gauge invariant combinations

$$\delta A' - A_0'\frac{\delta b}{b_0} - \frac{\kappa^2}{3}\Phi_0'\delta\Phi; \qquad \delta N' - \delta A'; \qquad \Phi_0''\delta\Phi - \Phi_0'\delta\Phi' + \Phi_0'^2\frac{\delta b}{b_0}\,, \tag{3.20}$$

where the first two are precisely our variables $\Psi$ and $\Upsilon$ and the last one appears in (55) equation. In terms of these gauge invariant variables, the system of equations closes.

It is now easy to verify the following solution from the (00) and (00)-(ii) equations, *i.e.,* eqs. (3.11-3.12). Denoting the warp factor $\Omega = e^{-A_0(1)}$, we find

$$\Psi = \frac{\kappa^2 b_0}{6(1-\Omega^2)} e^{4A_0(y)}\left[F(y)(\Omega^4\rho + \rho_*) - (\Omega^4\rho + \Omega^2\rho_*)\right] \tag{3.21}$$

$$\begin{aligned} \Upsilon &= \frac{\kappa^2 b_0}{2(1-\Omega^2)} e^{4A_0(y)}\Big[-F(y)(\Omega^4(\rho+p) + \rho_* + p_*) \\ &+ (\Omega^4(\rho+p) + \Omega^2(\rho_* + p_*))\Big] \end{aligned} \tag{3.22}$$

where

$$F(y) = 1 - (1-\Omega^2)\frac{\int_0^y e^{-2A_0}dy}{\int_0^1 e^{-2A_0}dy} \cong e^{-2kb_0 y} \quad (3.23)$$

and the Friedmann equations are

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{8\pi G}{3}\left(\rho_* + \Omega^4 \rho\right) \quad (3.24)$$

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 - \frac{\ddot{a}_0}{a_0} = 4\pi G\left(\rho_* + p_* + \Omega^4(\rho + p)\right) \quad (3.25)$$

$$8\pi G \equiv \kappa^2\left(2b_0\int_0^1 e^{-2A_0}dy\right)^{-1} \cong \kappa^2 k(1-\Omega^2)^{-1}. \quad (3.26)$$

The approximations in eqs. (3.23) and (3.26) hold when the back reaction of the scalar field on the metric can be neglected.

In the Friedmann equations (3.24-3.25), we note that $\rho$ is the bare value of the energy density on the TeV brane, naturally of order $M_p^4$, while $\Omega^4\rho$ is the physically observable value, of order $(\mathrm{TeV})^4$. Since $\rho_*$ has no such suppression, it seems highly unlikely that $\rho_*$ should be nonzero today; otherwise it would tend to vastly dominate the present expansion of the universe. We also point out that these equations are consistent only if energy is separately conserved on each brane: $\dot{\rho} + 3H(\rho + p) = 0$ and $\dot{\rho}_* + 3H(\rho_* + p_*) = 0$. This can be derived directly by considering the (05) Einstein equation, evaluated at either of the branes. The equations of state on the two branes are completely independent; there is no relation between $p/\rho$ and $p_*/\rho_*$.

## 3.5 Stiff Potential Limit

The above solutions are quite general, but they are not complete because we have not yet solved for the scalar field perturbation, $\delta\Phi$. This would generically be intractable, but there is a special case in which things simplify, namely, when the brane potentials $V_i(\Phi)$ become stiff. In this case, the boundary condition for the scalar fluctuation becomes $\delta\Phi = 0$ at either brane.

There is no information about the derivative $\delta\Phi'$ in this case; although $\delta\Phi \to 0$, at the same $V''(\Phi) \to \infty$ in such a way that the product $\delta\Phi V''(\Phi)$ remains finite, and Eq (3.18) is automatically satisfied.

Notice that the shift in $\delta\Phi$, Eq (3.19), respects the boundary conditions on $\delta\Phi$. Moreover, $\Phi_0'$ is always nonzero for our solution. It is therefore always possible, given some solution $\delta\Phi$ which vanishes at the branes, to choose an $f$ such that $\delta\Phi$ becomes zero. This is a convenient choice of gauge because it simplifies the equations of motion, and we will make it for the remainder of this letter.[1] Thus far we have satisfied the (00), (ii) and (05) Einstein equations. As noted above, eqs. (3.14) and (3.15) are equivalent, so either one just determines the shift in the radius. Using the former, and defining

$$W(y) \equiv \left[\tfrac{1}{2}e^{2A_0(y)} + A_0' e^{4A_0(y)} \int_0^y e^{-2A_0} dy\right] / \int_0^1 e^{-2A_0} dy \cong \frac{kb_0 e^{4kb_0 y}}{1-\Omega^2}\,, \quad (3.27)$$

we find that

$$\begin{aligned} \frac{\delta b}{b_0} &= \frac{b_0}{2\Phi_0'^2}\left[\Omega^4(\rho - 3p)W + (\rho_* - 3p_*)(W - A_0' e^{4A_0})\right] \\ &\cong \frac{kb_0^2 e^{4kb_0 y}}{2\Phi_0'^2(1-\Omega^2)}\left[\Omega^4(\rho - 3p) + \Omega^2(\rho_* - 3p_*)\right] \;; \end{aligned} \quad (3.28)$$

the last expression is found by approximating $A_0 = kb_0 y$ everywhere, which means neglecting the back reaction. Using the zeroth order solution (3.8) for $\Phi_0$, and integrating over $y$, we can obtain the shift in the size of the extra dimension,

$$\int_0^1 \delta b\, dy \cong \frac{[\Omega^4(\rho - 3p) + \Omega^2(\rho_* - 3p_*)]}{8(\epsilon k v_0)^2 \Omega^{4+2\epsilon}} \quad (3.29)$$

We can compare this to the result of ref. [15] by using their result for the radion mass, $m_r^2 \cong (4/3)\kappa^2(\epsilon v_0 k)^2 \Omega^{2+2\epsilon}$, and the relation $k\kappa^2 \cong 1/M_p^2$. Then

[1]The above argument is strictly true only for diffeomorphisms which are constant in time, while for our problem we need $f(t,y) \sim \rho(t), \rho_*(t)$. However, the time variation of such an $f$ is of higher order in $\rho$ and $\rho_*$, so we can neglect it to leading order in the perturbations.

$$\frac{\int_0^1 \delta b\, dy}{b_0} \cong \frac{[\Omega^4(\rho - 3p) + \Omega^2(\rho_* - 3p_*)]}{6kb_0 m_r^2 M_p^2 \Omega^2} \tag{3.30}$$

which agrees with ref. [15], except for small corrections of order $(1+\Omega^2)$.[2] As is well known, the shift in the radion vanishes when the universe is radiation dominated, because the radion couples to the trace of the stress energy tensor, which vanishes if the matter is conformally invariant.

## 3.6 Implications

Above we focused on the shift in the size of the extra dimension due to cosmological expansion, but the more experimentally relevant quantity is the shift in the lapse function, $n(t,1)$, evaluated on the TeV brane. As emphasized in ref. [52], the change in $n(t,1)$ between the present and the past determines how much physical energy scales on our brane, like the weak scale, $M_W$, have evolved. The time dependence of $M_W$ is given by $M_W(t)/M_W(t_0) = e^{-\delta N(t,1)+\delta N(t_0,1)+\delta N(t,0)-\delta N(t_0,0)}$. In terms of the variables of the previous section, $\delta N' = \Psi + \Upsilon + A_0'\delta b/b_0$. We find that

$$\int_0^1 \delta N'(t,y)dy \cong \frac{\kappa^2 b_0}{24kb_0}\Big((2\rho + 3p) - e^{2kb}(2\rho_* + 3p_*)\Big) + \int_0^1 A_0' \frac{\delta b}{b_0} dy \tag{3.31}$$

Interestingly, the new non-$\delta b$ contribution is present even during radiation domination, and is parametrically smaller than the radion part only in the matter dominated era, and then only if the back reaction is small ($\epsilon \ll 1$). If $M_p\Omega \sim 1$ TeV, the shift in the energy scale since nucleosynthesis is negligible, and this is the only cosmological constraint on $\delta N$. However, near the weak scale, the effect could be more interesting. To first order in the physical energy density $\rho_p = \Omega^4\rho$, at early times,

$$M_W(t) \cong M_W(t_0)\left(1 - \frac{\rho_p(t)}{8\Omega^4 M_p^2 k^2}\right), \tag{3.32}$$

[2]Our correction factor is $(1-\Omega^{4+2\epsilon})/(1-\Omega^2)$, while that of ref. [15] is $(1-\Omega^2)$.

assuming that $\rho_* = p_* = 0$. It is conceivable that $k \sim M_p/30$—in fact, the RS model requires $k < M_P$ for consistency, so that higher dimension operators in the gravitational part of the action do not become important. With these parameters and assuming $g_* \sim 100$ relativistic degrees of freedom and $\Omega M_p =$ 1 TeV, the correction to $M_W$ becomes of order unity at a temperature of 130 GeV.

# Chapter 4

# A Small Cosmological Constant from Warped Compactification with Branes

## 4.1 Introduction

There is mounting evidence that we live in a universe with a vacuum energy density $\Lambda$ which is about 0.7 of the critical density [18]. This value is close to 120 orders of magnitude less than the Planck density, $M_p^4$, which would be the natural expectation for the size of $\Lambda$ from quantum field theory. If there exists some mechanism to make $\Lambda$ small, it would seem much simpler if its value was zero than $10^{-120} M_p^4$. Our motivation in this chapter is to present a possible explanation for this enormous hierarchy of scales, taking advantage of Randall-Sundrum ideas involving 3-branes embedded in an extra dimension. Several attempts to use one or more extra dimensions to get a small $\Lambda$ have been made [20, 21, 23, 56, 57]. We will suggest that the RS idea can, with only minor changes, account for the smallness of the observed $\Lambda$. Our model relies heavily on stabilization of the distance between two branes using a bulk scalar field, as was suggested by Goldberger and Wise [16].

Our explanation only partially addresses the cosmological constant problem, in that we assume there is some unknown mechanism which forces the ground state of the universe to have vanishing 4-D vacuum energy. We show that there can be a false vacuum state with $\Lambda$ nonzero and exponentially small, as illustrated in figure 1. Its size is determined by the separation between the two branes, which in turn depends on the potential of the bulk scalar field $\phi$. The degree of freedom which is varying between the true and false vacua is the radion, the dynamical field associated with fluctuations in the size of the extra dimension.

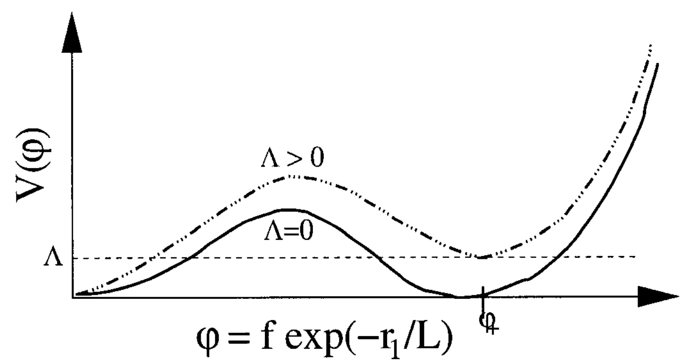



Figure 1: Qualitative form of the radion potential, for the two cases $\Lambda > 0$ and $\Lambda = 0$. The false vacuum state at $\varphi = \varphi_+$ naturally has $\Lambda \sim 10^{-120} M_p^4$.

We borrow the notation of reference [49] [1]. We will be working in a 5-D theory with the extra-dimensional coordinate $r$, whose metric is parametrized by

$$ds^2 = e^{2A(r)} \left( dt^2 - e^{2\sqrt{\bar{\Lambda}}t} \sum_i dx_i^2 \right) - dr^2. \tag{4.1}$$

We note that slices of constant $r$ represent 4-D deSitter spaces with vacuum energy $\Lambda = 3\bar{\Lambda}/(8\pi G)$, where $G$ is the 4-D Newton constant. There are two 3-branes, located at $r = 0$ and $r = r_1$ respectively. There is also a scalar field

[1]To recover the notation of chapter 1, change $A$ to $-A$ and $\kappa^2$ to $\frac{1}{2}\kappa^2$.

$\phi$ with potential $V(\phi)$ in the bulk, and separate potentials $\lambda_0(\phi)$ and $\lambda_1(\phi)$ on the two branes. $V(\phi)$ contains a negative bulk cosmological constant which induces the behavior $A(r) \propto -r$, so that $e^{2A(r)}$ is exponentially decreasing. The action for the model is

$$S = \int d^5x\sqrt{g}\bigg(-\frac{1}{4\kappa^2}R + \frac{1}{2}(\nabla\phi)^2 - V(\phi)$$
$$-\lambda_0(\phi)\delta(r) - \lambda_1(\phi)\delta(r-r_1)\bigg)\,. \tag{4.2}$$

Here $\kappa^2 = \frac{1}{2}M^{-3}$ defines the 5-D gravity scale $M$, which is assumed to be of order $M_p$. To simplify the analysis, we will take the brane potentials to have the form

$$\lambda_i(\phi) = T_i + \gamma_i(\phi^2 - \phi_i^2)^2 \tag{4.3}$$

with $\gamma_i \to \infty$, *i.e.*, the brane potentials are stiff. This will make the boundary conditions for the scalar field simply

$$\phi(0) = \phi_0, \qquad \phi(r_1) = \phi_1, \tag{4.4}$$

and the potentials will have the values $\lambda_i(\phi_i) = T_i$, where $T_i$ denotes the tension of the respective brane.

We will show that for the same simple choice of bulk potential used in the exact solution of ref. [49], it is possible to find a metastable solution to the equations of motion whose false vacuum energy (in the 4-D effective theory) is exponentially close to that of the true vacuum. By assuming the latter has vanishing cosmological constant, we can thus explain the small observed value of $\Lambda$ in our universe by imagining that we are stuck in the false vacuum.

## 4.2 Superpotential Method

To construct solutions to the coupled equations for 5-D gravity and the scalar field, we will take advantage of the superpotential method discussed in ref. [49]. One introduces a superpotential $W(r)$ and a function

$$\gamma(r) = \sqrt{1 + \frac{9\bar{\Lambda}}{\kappa^4 W(r)^2}\, e^{-2A(r)}} \tag{4.5}$$

such that the desired potential $V(\phi)$ is given by

$$V(\phi(r)) = \frac{1}{8\gamma^2}\left(\frac{\partial W}{\partial \phi}\right)^2 - \frac{\kappa^2}{3} W^2. \tag{4.6}$$

Then solutions to the coupled equations for $A$ and $\phi$ can be generated from the first order equations

$$A' = -\frac{\kappa^2}{3} W\gamma; \qquad \phi' = \frac{1}{2\gamma}\frac{\partial W}{\partial \phi}. \tag{4.7}$$

For flat branes (when $\Lambda = 0$), $W$ can be regarded as a function of $\phi$ alone, but for bent branes ($\Lambda \neq 0$), it must be considered as a function of $r$, and in that case one interprets $\frac{\partial W}{\partial \phi} = \frac{1}{\phi'}\frac{dW}{dr}$. The boundary conditions at the brane positions $r = 0$ and $r = r_1$ are

$$\lambda_0(\phi_0) = W\gamma|_{r=0}\,; \qquad \phi(0) = \pm\phi_0 \tag{4.8}$$

$$\lambda_1(\phi_1) = -W\gamma|_{r=r_1}; \qquad \phi(r_1) = \pm\phi_1\,. \tag{4.9}$$

## 4.3 Solutions with Vanishing $\Lambda$

Let us first present a ground state solution in which the branes are flat and the 4-D cosmological constant $\Lambda$ is tuned to be zero, hence $\gamma(r) = 1$. We will later perturb this solution with a small positive value of $\Lambda$. The unperturbed functional form for the superpotential is taken to be

$$W_0(\phi) = \frac{3}{\kappa^2 L} - b\phi^2, \tag{4.10}$$

where $L$ is a length scale that turns out to be the curvature radius of the 5-D anti-deSitter space in the limit where the back reaction of the scalar on the geometry is small. The resulting scalar potential is simply a polynomial in

$\phi$ of degree 4. It is easy to integrate the equations of motion (4.7) to obtain the solutions [49]

$$\begin{aligned} \phi &= \phi_0 \, e^{-br} \\ A &= \hat{A} - \frac{r}{L} - \frac{\kappa^2}{6}\phi_0^2 \, e^{-2br}, \end{aligned} \tag{4.11}$$

where $\hat{A}$ is a constant of integration (we have already chosen $\phi_0$ so as to satisfy the boundary condition on $\phi$ at $r = 0$.) The value of $\hat{A}$ is physically irrelevant, so we can for convenience choose it such that $A(0) = 0$. Applying the boundary condition on $\phi$ at the second brane determines $r_1$, the size of the extra dimension:

$$e^{-br_1} = \frac{\phi_1}{\phi_0} \tag{4.12}$$

This solution is essentially the same as the ones found in references [16] and in particular [49], differing from the latter only by our choice of stiff brane potentials. The stability of this solution against small perturbations has been demonstrated in [15] and [58]. The large hierarchy of energy scales between the two branes is generated by assuming that $b$ is parametrically small compared to $1/L$. In this case, although $e^{-br_1} \lesssim 1$, the warp factor will be exponentially small, $e^{-r_1/L} \ll 1$.

The remaining boundary conditions are satisfied by the constraints

$$T_0 = \frac{3}{\kappa^2 L} - b\phi_0^2 \tag{4.13}$$

$$T_1 = -\frac{3}{\kappa^2 L} + b\phi_1^2 \tag{4.14}$$

on the brane tensions.

## 4.4 Nearby Solutions with Nonzero $\Lambda$

Let us suppose that the tuning of brane tension $T_0$, eq. (4.13), is enforced, so that the solution where the second brane removed still exists, with vanishing 4-D cosmological constant. Consider what happens to the other solution if the tension $T_1$ is no longer tuned according to condition (4.14). We expect that $\Lambda$ is no longer zero in this case. The equations of motion with nonzero $\Lambda$ are difficult to solve exactly, so we will instead solve them perturbatively, to first order in $\Lambda$.

The first step in this procedure is to realize that, although $V(\phi)$ should still have the same functional form as in the $\Lambda = 0$ solutions, the superpotential need not be the same. In general, it must be corrected in such a way that $V(\phi)$ is unchanged. Let us denote the new solutions and superpotential by

$$\begin{aligned} \phi_\Lambda &\cong \phi + \delta\phi; \qquad A_\Lambda \cong A + \delta A \\ W_\Lambda(r) &\cong W(\phi_\Lambda(r)) + \delta W(r) \end{aligned} \tag{4.15}$$

where the quantities $\phi(r)$ and $A(r)$ refer to the bulk solutions with $\Lambda = 0$ found in the previous section, and $W(\phi)$ is the corresponding superpotential (4.10). By taking the variation of Eq (4.6) and keeping terms which are first order in $\Lambda$ weget a differential equation for $\delta W$,

$$\delta W' - \frac{4}{3}\kappa^2 W \delta W = \frac{9}{4\kappa^4}\bar{\Lambda} e^{-2A}\left(\frac{1}{W}\frac{\partial W}{\partial \phi}\right)^2 \tag{4.16}$$

which by using the equations of motions (4.7) can be written as

$$(e^{4A}\delta W)' = \bar{\Lambda} e^{2A}\left(\frac{\phi'}{A'}\right)^2 . \tag{4.17}$$

This can be formally integrated to solve for $\delta W(r)$,

$$\delta W = \bar{\Lambda} e^{-4A}\int_0^r e^{2A}\left(\frac{\phi'}{A'}\right)^2 dr + Ce^{-4A} \tag{4.18}$$

To determine the constant of integration $C$, we should impose the boundary condition (4.8) on the tension of the first brane. Recall that the value $T_0$ was already fixed by demanding the existence of the solution with only one brane and $\Lambda = 0$. Since the brane potentials are assumed to be stiff, $\lambda_0(\phi_0) = T_0$ regardless of whether $\Lambda = 0$ or not. This leads to the requirement that the variation of Eq (4.8) vanishes:

$$\delta W(0) + \frac{9\bar{\Lambda} e^{-2A(0)}}{2\kappa^4 W(\phi_0)} = 0. \tag{4.19}$$

Recalling the definition $A(0) = 0$, this fixes $C$ to be

$$C = -\frac{9\bar{\Lambda}}{2\kappa^4 W(\phi_0)} \tag{4.20}$$

In fact, because of our assumption that $b \ll 1/L$, needed to get a large hierarchy of scales, the $C$ term is the dominant one in $\delta W$. The first term in (4.18) is of order $\phi'^2 \sim b^2$ and can therefore be neglected.

Once $\delta W$ is known, it is straightforward to find the perturbations $\delta\phi$ and $\delta A$. In particular, the equation for $\delta\phi$ is

$$\delta\phi' = \frac{\delta W'}{2\phi'} - \frac{9\bar{\Lambda} e^{-2A}}{2\kappa^4 W(\phi)^2}\phi' + \frac{1}{2}\frac{\partial^2 W}{\partial\phi^2}\delta\phi \tag{4.21}$$

Again, since $b \ll 1/L$, the first term dominates. We thus find the approximate solution

$$\delta\phi \cong \frac{3\bar{\Lambda} L}{4\kappa^2 b\phi_0}\left(e^{4r/L} - e^{-br}\right). \tag{4.22}$$

To obtain this result, we have assumed that $1/L \gg b, b\phi^2$. (The fact that $b\phi_0$ appears in the denominator of this result might seem to upset our perturbation expansion; we will address this potential problem presently.) The constant of integration was chosen so that the boundary condition $\phi(0) = \phi_0$ is respected by the perturbed solution.

All that remains is for us to impose the two boundary conditions at the second brane. These will determine the new position of this brane, $r_\Lambda$, and the 4-D cosmological constant $\Lambda$. The b.c. on $\phi$ is

$$\phi_1 = \phi_0\, e^{-br_\Lambda} + \frac{3\bar{\Lambda} L}{4\kappa^2 b\phi_0}\left(e^{4r_\Lambda/L} - 1\right) \tag{4.23}$$

and the condition for the brane tension is

$$T_1 = -W(\phi_1)\left(1 + \frac{9\bar{\Lambda}e^{-2A(r_\Lambda)}}{2\kappa^4 W(\phi_1)^2}\right) - \delta W(r_\Lambda) \ , \tag{4.24}$$

Using the hierarchy $e^{-4A(r_\Lambda)} \gg e^{-2A(r_\Lambda)} \gg 1$, this can be approximately solved for $\bar{\Lambda}$ to give

$$\bar{\Lambda} \equiv \frac{8\pi G}{3}\Lambda \cong \frac{2\kappa^2}{3L}\, e^{-4r_\Lambda/L}(T_1 + W(\phi_1)) \tag{4.25}$$

This is our main result: if the interbrane distance is around $r_\Lambda \sim \ln(10^{30})L \sim 70L$, then the physical 4-D cosmological constant can be the observed value even if $(T_1 + W(\phi_1)$ is of order the Planck energy density. (Recall that $T_1 = -W(\phi_1)$ is the fine-tuned value that gave us $\Lambda = 0$ in the previous solutions.) To see if such a value of $r_\Lambda$ is natural, we must solve the $\phi$ boundary condition (4.23):

$$e^{-br_\Lambda} = \frac{\phi_1}{\phi_0} - \frac{(T_1 + W(\phi_1))}{2b\phi_0^2} \tag{4.26}$$

However, we must consider the fact that the second term on the right hand side of this equation is of order $1/b$. A small amount of tuning is required so that $T_1 + W(\phi_1)$ is of order $b^2$, so that the new term can be treated perturbatively. Once this is done, however, there is no great difficulty in achieving the enormous hierarchy of 120 orders of magnitude for $\Lambda$. For example if $bL = 0.01$, the r.h.s. of (4.26) need only be as small as 0.5 to achieve the required separation of $r_\Lambda = 70L$. We have therefore succeeded ameliorating the cosmological constant hierarchy problem in the same way as the original Randall-Sundrum model addressed that of the weak scale. The mild fine-tuning we demanded for the second brane tension might be merely a technical requirement for the convenience of analytically demonstrating the mechanism. Possibly it still works even without this small amount of tuning, though the solutions are harder to find in that case.

Ref. [59] pointed out that any warped compactification solution involving a scalar field must satisfy certain consistency conditions, notably that $\sum_i T_i +$

$\int \phi'^2 dr \cong 0$ in the case where $\Lambda \ll M_p^4$. The present solution has already been shown in [59] to satisfy this relation when $\Lambda = 0$, so we should verify that our perturbed solution also satisfies it. To first order in the perturbation, we should find that $\delta T_1 + 2\int \phi' \delta\phi' dr = 0$, where $\delta T_1$ is the mistuning of $T_1$ away from its $\Lambda = 0$ value, $\delta T_1 = T_1 + W(\phi_1)$. It is straightforward to show that in the regime where $b \ll 1/L$, $2\int_0^{r_1} \phi' \delta\phi' dr = \delta W|_0^{r_1} \cong Ce^{-4Ar_1}$, and using (4.20) and (4.25) that the consistency condition is satisfied.

To complete our argument we should demonstrate that the 4-D Newton constant $G$ can take its observed value without requiring any fine tuning. By integrating over the extra dimension in the 5-D action (4.2) to obtain the 4-D effective action, one finds the relation

$$\frac{1}{16\pi G} = \frac{1}{4\kappa^2} \int_{-r_\Lambda}^{r_\Lambda} dr\, e^{2A} \cong \frac{L}{4\kappa^2}\ , \tag{4.27}$$

Thus it is natural to assume that $G$, $\kappa^2$ and $L$ are all of the order $M_p$ to the appropriate power, and no additional tuning is needed to localize gravity.

Although we have succeeded in constructing the approximate solution corresponding to the local minimum of the radion potential, one might wonder why we are not able to find the unstable solution at the local maximum. Evidently this is nonperturbative in $\Lambda$. Recall that we needed to do some mild tuning, $T_1 + W(\phi_1) = O(b^2)$ to keep our perturbation expansion under control. Yet the unstable solution exists even when $T_1 + W(\phi_1)$ is exactly zero. It thus seems plausible that the barrier height is large, although we expect in order of magnitude that its size is governed by the same warp factor $e^{-r_\Lambda/L}$ which suppresses the perturbative solution value.

## 4.5 Lifetime of the False Vacuum

Our proposal is only viable if the false vacuum has a sufficiently large lifetime that we could still be in it at the present time. To study this we should construct the bounce solution of the Euclideanized radion action and check that the lifetime $\tau \sim \Lambda^{-1/4} e^{S_b}$ exceeds the age of the universe, where $S_b$ is the bounce action and $\Lambda^{-1/4} \sim 10^{-4}$ eV is the typical energy scale that will appear in the prefactor in the saddle point approximation for the path integral for the rate of false vacuum decay. We will take the following ansatz for the radion potential:

$$V(\varphi) = \frac{\lambda}{4}\varphi^4 \left[ ((\varphi/f)^\epsilon - \eta)^2 + (\alpha\eta\epsilon)^2 \right] \tag{4.28}$$

Note that $\varphi = f e^{-r_1/L}$ is the radion field, not to be confused with the bulk scalar $\phi$. In this notation, the small parameter $\epsilon$ is related to our superpotential parameters by $\epsilon = bL$, $\eta = \phi_1/\phi_0$, the quartic coupling is $\lambda \cong 4\phi_0^2/(9LM_p^4)$, and $f \cong \sqrt{6}M_p$ [1]. When $\alpha = 0$, this agrees with the approximate form found by ref. [16] in the case where $\Lambda = 0$. It has two degenerate minima at $\varphi = 0$ and $\varphi = f\eta^{1/\epsilon}$, corresponding to the solutions with infinite and finite interbrane distance, respectively, which we found above. (This can be seen through the relation $\varphi/f = e^{-r_1/L}$ between the radion field and the brane separation.) By adding the term $(\alpha\eta\epsilon)^2$, we have lifted the minimum near $\varphi \cong f\eta^{1/\epsilon}$ to a nonzero value of the 4-D cosmological constant:

$$\Lambda = \frac{\lambda}{32}\,\varphi_+^4 \eta^2 \epsilon^2 \delta \ , \tag{4.29}$$

where in the limit that $\epsilon \ll 1$, the value of $\varphi$ at the metastable minimum is given by

$$\varphi_+ \cong f\eta^{1/\epsilon} e^{-\delta/4}; \qquad \delta \equiv 1 - \sqrt{1-(4\alpha)^2} \ . \tag{4.30}$$

To estimate the bounce action, we use the thin wall approximation of ref. [44]. For a bubble of radius $R$ in 4-D Euclidean space, with false vacuum energy density $\Lambda$,

$$S_b = -\frac{\pi^2}{2}\Lambda R^4 + 2\pi^2 R^3 S_1 \ , \tag{4.31}$$

where $S_1$ is the action for the 1-D instanton corresponding to the $\alpha = \Lambda = 0$ limit of the radion potential,

$$S_1 = \int_0^{\varphi_+} d\varphi\, \sqrt{2V} = \frac{1}{9}\sqrt{\frac{\lambda}{2}}\, \epsilon\eta(f\eta^{1/\epsilon})^3, \tag{4.32}$$

and the radius of the bounce solution which minimizes (4.31) is $R = 3S_1/\Lambda$. Substituting this value into the (4-D) Euclidean action, we obtain

$$S_b \cong \frac{17\pi^2 e^{3\delta}}{\epsilon^2\eta^2\lambda\delta^3}\,, \tag{4.33}$$

which is so large for the parameters of interest for getting a large hierarchy of scales ($\epsilon \sim 0.01$) that the false vacuum state is easily more long-lived than the present age of the universe.

The 1-D instanton obeys the equation of motion $\ddot{\varphi} = \frac{dV}{d\varphi}|_{\alpha=0}$, which can be solved in terms of the Lerch transcendental function $\Phi$,

$$\epsilon\eta\sqrt{\lambda/2}\, t = \frac{1}{\varphi}\Phi\left(\eta^{-1}(\varphi/f)^{\epsilon},\, 1,\, -1/\epsilon\right), \tag{4.34}$$

as illustrated in figure 2. (We take $\epsilon = 0.011$ since $\Phi(x, 1, -1/\epsilon)$ is singular when $1/\epsilon$ is a positive integer.) This has the desired behavior that $\varphi \to 0$ as $t \to -\infty$ and $\varphi \to \varphi_+$ as $t \to \infty$.

We can verify that the thin-wall approximation is valid when $\delta \ll 1$. From figure 2, the width is $\Delta R = \Delta t \cong 10(\epsilon f\eta^{1+1/\epsilon}\sqrt{\lambda/2})^{-1}$. Comparing to the bounce radius $R = 3S_1/\Lambda$, we find that $\Delta R/R \cong \delta$.

In chapter 2 we studied a similar problem, that of transitions from a false to a true minimum of the radion potential in the original Goldberger-Wise mechanism. There it was necessary to consider thermally-induced transitions because the radion would have been in thermal equilibrium in the early universe. In the present case, assuming that inflation is driven by an inflaton on the observable brane, the radion is so weakly coupled that it does not come into thermal equilibrium. This is also true of the variant model we propose below in which the observable brane is at the TeV rather than the Planck scale.

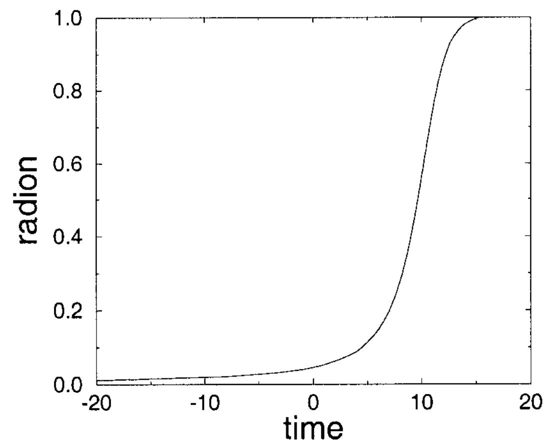



Figure 2: 1-D instanton solution for $\epsilon = 0.011$ and $\delta = 0$. The axes are scaled such that "time" $= \epsilon f \eta^{1+1/\epsilon}\sqrt{\lambda/2}\, t$ and "radion" $= (\varphi/f)\eta^{-1/\epsilon}$.

## 4.6 Physical Consequences

In the false vacuum state, the radion will have a very small mass, somewhat below the milli-eV range, since it has been shown that [15], [58]

$$m_r \cong \tfrac{8}{3} b e^{-r_\Lambda/L}. \qquad (4.35)$$

Since $M_p e^{-r_\Lambda/L}$ is supposed to be the meV scale and $b \sim 0.01/L \lesssim 0.01 M_p$, this is a dangerously small mass if the radion were to couple to matter as strongly as does gravity. We have investigated the corrections to this formula due to the $O(\Lambda)$ perturbations and found that they are of the same order as the unperturbed value if $\Lambda \sim b^2$. Hence there is the possibility that corrections to the radion mass which we cannot compute by perturbing in $b$ make it somewhat larger than the value (4.35).

However, if we live on the positive tension (Planck) brane, the radion's exponentially small couplings to matter make it impossible to observe directly. Let us denote the renormalized trace of the stress energy tensor by $(\tilde{T}^\mu_\mu)_i = e^{-4r_i/L}(T^\mu_\mu)_i$ at the two branes located respectively at $r = r_0 = 0$ and $r = r_1 = r_\Lambda$. Because the radion wave function grows like $e^{2r/L}$ away from the Planck brane [14], its coupling to matter on the respective branes goes like [30, 13]

$$\mathcal{L}_{\text{radion}} = \frac{\varphi}{\Lambda^{1/4}} \left( (\tilde{T}^\mu_\mu)_1 + e^{-2r_1/L} (\tilde{T}^\mu_\mu)_0 \right) . \tag{4.36}$$

(In the original RS model, the TeV scale appeared in place of the milli-eV scale $\Lambda^{1/4}$.) If we are living on the Planck brane and $(\tilde{T}^\mu_\mu)_0$ represents standard model physics, then the radion coupling to the standard model is negligible since $e^{-2r_1/L}\Lambda^{-1/4} \sim \Lambda^{-1/4}/M_p^2$.

There is a slightly better chance of detecting the effects of such a radion through cosmology. The shift in size of the extra dimension due to physical energy density $\rho$ on the Planck brane is [2, 15, 30, 58]

$$\frac{\Delta r_1}{r_1} \cong \frac{L\rho}{6 r_1 m_r^2 M_p^2} , \tag{4.37}$$

which becomes of order unity at temperatures $T \sim \sqrt{m_r M_p} \sim 1$ TeV. However this is still such a high temperature that it is difficult to imagine any surviving remnant of the changes to physical scales and the Hubble expansion rate which would arise from a variation in $r_1$ during this era.

A more testable and interesting situation is to imagine that we are living on a third brane located approximately halfway between the original two, instead of on the positive tension brane. This is desirable apart from considerations of the cosmological constant problem, in that it preserves the natural resolution of the weak scale hierarchy which was the original motivation of Randall and Sundrum. An intermediate brane can be inserted into our solution if it has zero tension and a potential of the form $\gamma_{\frac{1}{2}}(\phi - \phi_{\frac{1}{2}})^2$, with $\gamma_{\frac{1}{2}} \to \infty$ for ease of analysis. The position of this new TeV brane can be adjusted to the desired value by satisfying the approximate boundary condition

$e^{-br_{1/2}} \cong \phi_{\frac{1}{2}}/\phi_0$. There are now two radions, one at the TeV scale associated with fluctuations in the distance between the Planck and TeV branes, and the original milli-eV radion.

If there does exist a TeV brane, the coupling of the light radion to particles there will be significantly larger than on the Planck brane:

$$\mathcal{L}_{\substack{\text{radion}-\\ \text{TeV brane}}} \cong \frac{\varphi}{\varphi_+} e^{2(r_{1/2}-r_1)/L} (\tilde{T}^\mu_\mu)_{\frac{1}{2}} \sim \frac{\varphi}{M_p} (\tilde{T}^\mu_\mu)_{\frac{1}{2}}, \tag{4.38}$$

which means the radion is coupled with approximately the same strength as ordinary gravity. This is precisely the range of scalar masses and couplings which is presently being probed by measurements of the gravitational force at submillimeter range [8]. The new contribution to the potential energy for two masses $m_1$ and $m_2$ separated by a distance $r$ is

$$\Delta V = -\frac{4\pi G_N m_1\, m_2\, e^{-m_r r}}{3r}\, e^{(4r_{1/2}-2r_1)/L} \,. \tag{4.39}$$

The exponential factor should be of order $(\text{meV})^2 M_p^2/(\text{TeV})^4 \sim 6$ for the warp factors to naturally explain the TeV and meV scales of the standard model and cosmological constant, respectively. To evade the constraints of submillimeter gravity tests, this number must be made somewhat smaller, or the radion mass must be made somewhat larger than 1 meV. The model is therefore tightly constrained by present tests of the gravitational force.

In addition, the Kaluza-Klein gravitons have a mass gap similar in size to the meV radion, and to the extent that they are nearly massless, their effects will be like those of the KK gravitons in the noncompact RSII scenario. We find that the correction to the Newtonian gravitational potential from these modes is given approximately by

$$\Delta V \cong -G_N L^2 m^2 \frac{m_1\, m_2}{r} \frac{e^{-5mr/4}}{1-e^{-mr}} \left(\frac{1}{4} + \frac{1}{1-e^{-mr}}\right) \tag{4.40}$$

where $m \equiv \pi e^{-r_1/L}/L$, and we have approximated the KK masses by $m_n \cong (n+1/4)m$ using the large-$n$ behavior of the exact eigenvalues [60]. Moreover, the existence of the TeV brane would imply the usual TeV radion, whose phenomenology has been widely studied in connection with the compact Randall-Sundrum scenario [9, 13, 30, 61].

## 4.7 Conclusions

We have presented a warped compactification model which, at the expense of assuming there is some unknown solution to the first cosmological constant problem—the question of why the ultimate vacuum energy of the universe is zero—naturally resolves the second one: it explains how the observed value can be 120 orders of magnitude below the Planck energy density without requiring additional fine tuning. The hypothesis is that there exists a brane at such a distance ($\sim 70/M_p$) from the Planck brane that the mistuning of its tension from the flat-brane value contributes to $\Lambda$ at the $10^{-120}M_p^4$ level, due to the smallness of the warp factor.

Our idea has several shortcomings. Unlike quintessence models, it does not try to solve the coincidence problem of why $\Lambda$ happens be a significant fraction of the critical density *now*. In the version where we are assumed to live on the Planck brane, we sacrifice any new understanding of the weak scale hierarchy problem, and furthermore there seems to be no experimental signatures that would test the idea. These difficulties are overcome if we live instead on a zero-tension TeV brane between the two original branes, for then the light radion is in the right range for current tests of gravity at submillimeter distances, but then a new fine-tuning problem is introduced: why is the tension exactly (or so nearly) zero? We find it intriguing that this explanation of the cosmological constant, whose presence is revealed by gravity over cosmological distance scales, might be corroborated by table-top gravity experiments, as well as collider searches.

# Chapter 5

# No-Go Theorem for Horizon-Shielded Self-Tuning Singularities

## 5.1 Introduction

As argued in the introductory chapter, the braneworld scenario has created the hope of somehow circumventing Weinberg's no-go theorem for solving the cosmological constant problem using an adjustment mechanism, by virtue of introducing an extra dimension. Some attempts along these lines were made by [20, 21], in which a scalar field in the bulk adjusted itself to yield a static solution to Einstein's equations, for a range of values of the brane tension. These solutions relied upon singular behavior of the scalar somewhere in the bulk, which was shown by ref. [63] to be simply a way of hiding the fine-tuning problem, since a proper treatment required insertion of a new brane at the singularity, whose tension must be tuned with respect to that of the visible brane.[1] In essence, the original self-tuning idea was

[1]Moreover the original self-tuning solutions were shown to require a fine-tuning of the

pretending to gain extra free parameters by no longer requiring boundary conditions to be satisfied at the boundary of the bulk where the singularity was appearing, and where a brane would normally have appeared.

A significant attempt to improve on this situation was made in ref. [24, 25]. Their idea was to render the singularity more physical by introducing a horizon between it and the visible brane, in the same way that the Schwarzschild black hole singularity is hidden. In fact the bulk geometry is the AdS-Schwarzschild (or AdS-Reissner-Nordström in the case of a charged black hole) generalization of AdS, in which a singularity appears for all of 3-space at some position in the bulk. The singularity could thus be described as a black brane, though we will follow common usage and call it a black hole (BH).

The significance of the AdS-Schwarzschild solutions has become apparent in a number of works that deal with braneworld cosmology. In ref. [34] it was shown that this is the bulk solution which gives rise to the dark radiation term that was shown to be a possible addition to the Friedmann equation for the expansion of the brane. Ref. [64] subsequently identified the dark radiation as being identical to the thermal excitations of the CFT degrees of freedom in the context of the AdS/CFT correspondence. In ref. [65] it was shown that the bulk black hole must form in the early universe, since gravitational radiation emanating from the hot visible brane becomes infinitely dense as it falls toward the AdS horizon, for any cosmologically relevant initial brane temperature.

In contrast to these cosmological solutions, where the brane is moving away from the BH and thus seeing a bulk which becomes increasingly AdS-like (or alternatively, the dark radiation term in the brane Friedmann equation is redshifting away), ref. [24] finds a class of static solutions, so that the

initial conditions in order to avoid motion of the singularity with respect to the brane [56].

effect of the BH on the brane can be felt at arbitrarily late times. A very interesting application is that Lorentz invariance is broken, and gravitational signals travel with an average speed different from that of light on the brane. Most germane for the present work is that ref. [24] also finds self-tuning solutions with a horizon in the case of the charged (RN) black hole, where the mass and the charge of the BH adjusts itself to the energy density $\rho$ on the brane; but self-tuning and the horizon can coexist only if the positive energy condition is violated on the brane: $\rho < -p$.

Our motivation for the present work was to try to remove this seemingly unphysical restriction on the solutions and allow for positivity of the brane stress energy tensor. After various failed attempts we realized that the Einstein equations can be manipulated to show in a simple way why it is impossible to improve the situation by adding extra matter fields to the Lagrangian.

This is our no-go theorem, which is given in the next section. In section 3 we show that the theorem also holds when the gravitational part of the action is supplemented with a particular higher-derivative correction, the Gauss-Bonnet term. In section 4 we discuss a way of evading the theorem: giving positive curvature to the 3-D spatial hypersurfaces parallel to the brane. We generalize a solution of this type which was derived for the AdS-Schwarzschild case to the case of nonvanishing charge and show the range of parameters where self-tuning and a horizon coexist. In section 5 we show that it is not possible to put the curvature into extra dimensions instead of the usual 3-D space, which would have been desirable for describing our flat universe. Conclusions are given in section 6.

## 5.2 The No-Go Theorem

We begin with the following general action:

$$S = \int d^5x\sqrt{-g}\left(\frac{R}{2\kappa^2} + \mathcal{L}_B\right) + \int d^4x\sqrt{-g}\mathcal{L}_b \ , \tag{5.1}$$

where $\mathcal{L}_B$ and $\mathcal{L}_b$ are the Lagrangian densities in the bulk and on the brane respectively, and $\kappa^2 = 1/M_5^3$ in terms of the 5-D Planck mass $M_5$. The ansatz for the metric, which includes the AdS, AdS-Schwarzschild or AdS-RN geometries, is

$$ds^2 = -h(r)dt^2 + a(r)d\Sigma_k^2 + h(r)^{-1}dr^2 \tag{5.2}$$

where $d\Sigma_k^2$ is the line element for a homogeneous 3-D space of constant spatial curvature with $k = 0, \pm 1$. The 5-D generalization of Birkhoff's theorem guarantees that this form for the bulk metric is a general solution (in the appropriate coordinate system) when there is only a cosmological constant [34, 36] or a U(1) gauge field [24] in the bulk. However we will also take it to be our ansatz for the metric when there are more general sources of stress-energy in the bulk. Since we are interested in static solutions, a coordinate system can always be found which puts the metric into the form (5.2). For definiteness we will write the 3-D part of the metric as

$$\begin{aligned} d\Sigma_k^2 &= \frac{dx^2 + dy^2 + dz^2}{\left(1 + \frac{1}{4}k(x^2 + y^2 + z^2)\right)^2} \\ &\equiv \Sigma_k^2(x, y, z)(dx^2 + dy^2 + dz^2). \end{aligned} \tag{5.3}$$

The nonzero components of the Einstein tensor are

$$\begin{aligned} G_{00} &= -\frac{3}{4}h\left(\frac{a'}{a}h' + 2h\frac{a''}{a} - 4\frac{k}{a}\right) \\ G_{ii} &= a\Sigma_k^2\left(\frac{a'}{a}h' + h\frac{a''}{a} - \frac{h}{4}\left(\frac{a'}{a}\right)^2 + \frac{h''}{2} - \frac{k}{a}\right) \\ G_{55} &= \frac{3}{4}\left(\left(\frac{a'}{a}\right)^2 + \frac{h'}{h}\frac{a'}{a} - 4\frac{k}{a}\right) \end{aligned} \tag{5.4}$$

Next we will rewrite,

$$a(r) = a_0 e^{-A(r)}, \tag{5.5}$$

where $a_0$ is an arbitrary constant with dimensions of (length)$^2$, and consider the following linear combination of the Einstein tensor components: $2G_{00}/h + 2G_{11}/(a\Sigma_k^2)$. Using the Einstein equations,

$$G_{mn} = \kappa^2 \left( T^B_{mn} + T^b_{\mu\nu}\sqrt{h}\delta^\mu_m \delta^\nu_n \delta(r - r_0) \right) \ , \tag{5.6}$$

where $\delta^\mu_r \equiv 0$, we obtain

$$\begin{aligned} &(h' + hA')' - \frac{3}{2}A'(h' + hA') + 4\frac{k}{a_0}e^A \\ &= 2\kappa^2 \left( \frac{T^B_{00}}{h} + \frac{T^B_{11}}{a\Sigma_k^2} \right) + 2\kappa^2 \left( \frac{T^b_{00}}{h} + \frac{T^b_{11}}{a\Sigma_k^2} \right) \sqrt{h}\delta(r - r_0) \ . \end{aligned} \tag{5.7}$$

This can be integrated once since $(h' + hA')' - \frac{3}{2}A'(h' + hA')$ is proportional to $(e^{-3A/2}(h' + hA'))'$. Using $Z_2$ symmetric boundary conditions at the brane to interpret the contribution of the delta function, we have

$$\begin{aligned} (h' + hA')|_r &= -2\kappa^2 e^{\frac{3}{2}A} \int_r^{r_0} \left( \frac{T^B_{00}}{h} + \frac{T^B_{11}}{a\Sigma_k^2} \right) e^{-\frac{3}{2}A} dr \\ &- \kappa^2 e^{\frac{3}{2}(A(r) - A(r_0))} \left( \frac{T^b_{00}}{h} + \frac{T^b_{11}}{a\Sigma_k^2} \right) \sqrt{h} \Bigg|_{r_0} \\ &+ 4\frac{k}{a_0} e^{\frac{3}{2}A} \int_r^{r_0} e^{-\frac{1}{2}A} dr \ . \end{aligned} \tag{5.8}$$

Let us for the moment consider the cases of vanishing or negative spatial curvature, $k = 0$ or $-1$. Then at the horizon $r = r_H$, $h = 0$ and since the right hand side of Eq (5.8) is not positive—assuming that $-T^0_0 + T^1_1 \geq 0$, in accordance with positivity of the stress energy tensor—we conclude that $h' \leq 0$ at the horizon. But the brane is located at a value $r_0 > r_h$ beyond the horizon, by construction, and $h(r_0)$ must be positive for $t$ to be a timelike coordinate. This implies that $h'(r_h) > 0$ at the horizon, as a shown in figure 1, and in contradiction to Eq (5.8).

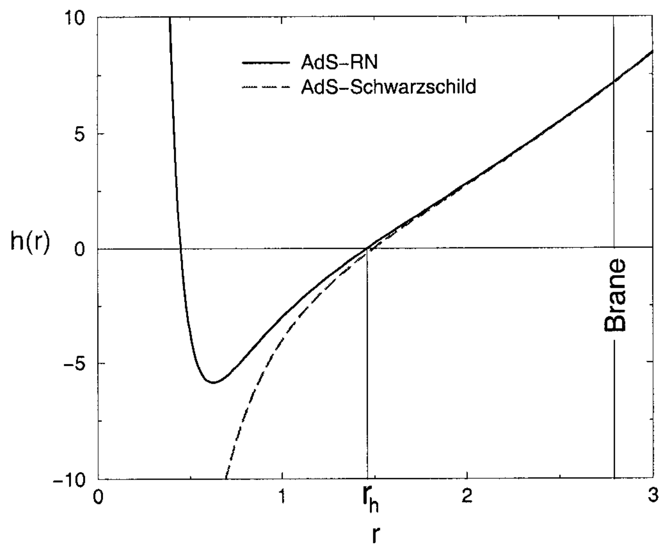



Figure 1: Required behavior for $h(r)$ near the horizon, $r = r_h$, for charged and for neutral black holes. Since the brane is at values of $r$ greater than $r_h$ and $h$ is supposed to be positive in this region, $h'(r_h)$ must be positive.

From the argument above, we can discern two ways of evading the no-go result. (1) Violate positivity of the stress-energy tensor in the bulk or on the brane, which would change the sign of either of the first two integrals in Eq (5.8). (2) Let the 3-D curvature be positive, making the third integral in (5.8) positive. In section IV we will consider the second possibility. For the moment, let us assume that neither (1) nor (2) is fulfilled, and moreover that the 3-D curvature is zero, which is the most favorable case for horizon formation, apart from the positive curvature case.

To demonstrate the usefulness of our no-go theorem, we will now study two examples. Inclusion of a scalar field in the bulk along with gravity is the simplest one. In this case the stress-energy tensor contributions are

$$\begin{aligned} T^B_{mn} &= -g_{mn}V + \partial_m\Phi\partial_n\Phi - \frac{1}{2}g_{mn}\partial^l\Phi\partial_l\Phi \\ (T^b)^m_n &= \mathrm{diag}(-V_0, -V_0, -V_0, -V_0, 0) \ , \end{aligned} \tag{5.9}$$

where $V$ and $V_0$ are the potentials in the bulk and on the brane, respectively.

Static solutions which respect Lorentz symmetry on the brane will be of the form $\Phi = \Phi(r)$, *i.e.,* dependent on the bulk coordinate only. Then from the above expressions we see that $\frac{T_{00}}{h} + \frac{T_{11}}{a} = 0$ both in the bulk and on the brane and hence $(h' + hA') = 0$ everywhere in the bulk, as a consequence of Eq (5.8). In this case $h$ must be of the form $e^{-A}$, just like the metric component $a$. Thus the metric is explicitly Lorentz invariant, regardless of the choice of bulk or brane potentials, and there can be no horizon, with the possible exception of the usual AdS horizon at $r = \infty$, where $h$ and $a$ vanish together. In this case the situation is the same as was investigated in ref. [23], which found that self-tuning solutions where gravity is localized have a naked singularity.

As a second example, we consider the AdS-RN solution which ref. [24] investigated in depth. Here one introduces a U(1) gauge field in addition to the negative vacuum energy in the bulk, giving the stress-energy tensor

$$T^B_{mn} = -g_{mn}\Lambda - \frac{1}{4}g_{mn}F_{ab}F^{ab} + F_{mc}F^c{}_n \ . \tag{5.10}$$

The solution to the equations of motion is

$$\begin{aligned} h(r) &= \frac{r^2}{l^2} - \frac{\mu}{r^2} + \frac{Q^2}{r^4} \\ a(r) &= r^2 \\ F_{tr} &= \frac{\sqrt{6}Q}{\kappa r^3} \ , \end{aligned} \tag{5.11}$$

where $\mu$ and $Q$ are proportional to the black hole mass and charge, respectively, and $l^{-2} = -\frac{1}{6}\kappa^2\Lambda$. Substituting this solution into the stress energy tensor we find

$$\frac{T^B_{00}}{h} + \frac{T^B_{11}}{a} = \frac{6Q^2}{\kappa^2 r^6} > 0 \ . \tag{5.12}$$

From Eq (5.8) it is then clear that there is no possibility of having a horizon unless the positivity of the stress energy tensor on the brane is violated, *i.e.,* $\rho + p < 0$, where $\rho = -(T^b)_0{}^0$ and $p = (T^b)_1{}^1$. We note in passing that the jump conditions at the brane are

$$\frac{[a']}{a} = -\frac{2}{3}\kappa^2\rho\sqrt{h^{-1}}; \qquad \frac{[h']}{h} = \frac{2}{3}\kappa^2(2\rho + 3p)\sqrt{h^{-1}}\ , \tag{5.13}$$

where [ $'$] denotes the discontinuity in the derivative across the brane. Since $h' > 0$ at the brane, its discontinuity assuming $Z_2$ symmetry is negative. Hence the brane tension must be positive; nevertheless $\rho + p$ is negative and the parameter $\omega \equiv p/\rho$ must be less than $-1$.

It was recently proposed that adding a dilatonic coupling to the gauge field will improve the situation in such a way that the horizon could be outside of the brane [66], and the interior region containing the singularity is cut away when the $Z_2$ symmetry around the brane is imposed. In this situation, to satisfy the jump conditions (5.13), $\rho$ must be negative and $\omega \equiv p/\rho$ must be positive. This is in contrast to the AdS-RN situation where $\omega < -1$ was required, in contradiction to the positive energy condition. The authors of ref. [66] find that $\omega > 0$ in their new solution, which may at first look like an improvement. However, when $\rho < 0$, positivity of the stress-energy tensor actually requires that $\omega \leq -1$ (so that $\rho + p > 0$), so we see that the problem still persists in their solution.[2]

In fact we can easily extend our no-go theorem to the case where the brane is placed between the singularity and the horizon to show that no improvement is provided by this variation. Let us suppose there exists a bulk solution which is qualitatively like one of those shown in figure 2; these are the negatives of the normal AdS-Schwarzschild or AdS-RN solutions. In this case the brane should not be placed at $r > r_h$ because in this region $r$ is the timelike coordinate, and such a brane would not be static, as we would like for self-tuning, but instead would represent a time-dependent solution.

[2]Ref. [66] does however remark upon the possibility of overcoming this problem when the curvature is $k = 1$.

Repeating the steps that led to Eq (5.8), we obtain

$$
\begin{aligned}
(h' + hA')|_r &= 2\kappa^2 e^{\frac{3}{2}A} \int_{r_0}^{r} \left( \frac{T_{00}^B}{h} + \frac{T_{11}^B}{a\Sigma_k^2} \right) e^{-\frac{3}{2}A} dr \\
&+ \kappa^2 e^{\frac{3}{2}(A(r)-A(r_0))} \left( \frac{T_{00}^b}{h} + \frac{T_{11}^b}{a\Sigma_k^2} \right) \sqrt{h} \Big|_{r_0} \\
&- 4\frac{k}{a_0} e^{\frac{3}{2}A} \int_{r_0}^{r} e^{-\frac{1}{2}A} dr. \qquad (5.14)
\end{aligned}
$$

The new condition (5.14) is identical to the old one (5.8) as far as the bulk contributions are concerned, but the sign of the brane contribution is changed because of the fact that the space is being cut away for $r < r_0$ rather than for $r > r_0$. Now when we apply (5.14) at the horizon, $r_h$, we find that $h'(r_h)$ gets only positive contributions unless the 3-D curvature is $k = 1$, or positivity of $T_{\mu\nu}$ is violated. But figure 2 makes clear that $h'(r_h)$ should be negative in this case, thus giving a contradiction.

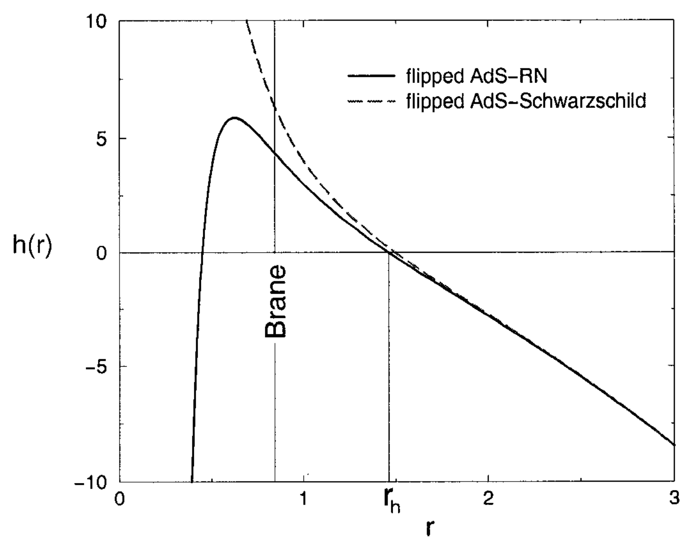



Figure 2. Qualitative behavior of hypothetical solutions which would require the brane to be placed between the horizon and the singularity.

## 5.3 Relaxation of $Z_2$ Symmetry, and Higher Derivative Corrections

We have seen that nonvanishing 3-D curvature $k$ can provide a way out of our no-go result, but one may wonder whether there are other loopholes. In this section we continue to leave $k = 0$ and explore two possibilities which, as it turns out, do not provide any additional loophole. The first is to relax the $Z_2$ symmetry imposed at the brane, at $r = r_0$. We consider possible solutions in which

$$h(r) = \begin{cases} h_1(r), & r < r_0 \\ h_2(r), & r > r_0 \end{cases} \tag{5.15}$$

and similarly for $a_i(r) = a_0 e^{-A_i(r)}$. The solution would have singularities on both sides of the brane, at positions $r = 0$ and $r = r_s > r_0$, say. The hope would be to obtain horizons on both sides before the singularities are reached, at $r = r_{h1}, r_{h2}$, as illustrated in figure 3.

Integrating Eq (5.7) in the bulk we get

$$\begin{aligned} (h_i' + h_i A_i')\big|_r &= 2\kappa^2 e^{\frac{3}{2}A_i} \int_{r_0}^{r} \left( \frac{T_{00}^B}{h_i} + \frac{T_{11}^B}{a_i} \right) e^{-\frac{3}{2}A_i} dr \\ &+ c_i e^{\frac{3}{2}A_i} , \end{aligned} \tag{5.16}$$

where $c_i$ are the constants of integration determined by the jump conditions (5.13). Integrating Eq (5.7) across the brane gives

$$(c_2 - c_1) e^{\frac{3}{2}A(r_0)} = \kappa^2 \left( \frac{T_{00}^b}{h} + \frac{T_{11}^b}{a} \right) \sqrt{h} \bigg|_{r_0} , \tag{5.17}$$

which implies $(c_2 - c_1) > 0$. But at the first horizon, $r_{h_1}$, we need $h' > 0$ and Eq (5.16) thus requires $c_1 > 0$. Similarly getting $h' < 0$ at $r_{h_2}$ requires $c_2 < 0$. These two conditions give rise to the contradictory relation $(c_2 - c_1) < 0$; hence nothing is gained by relaxing $Z_2$ symmetry.

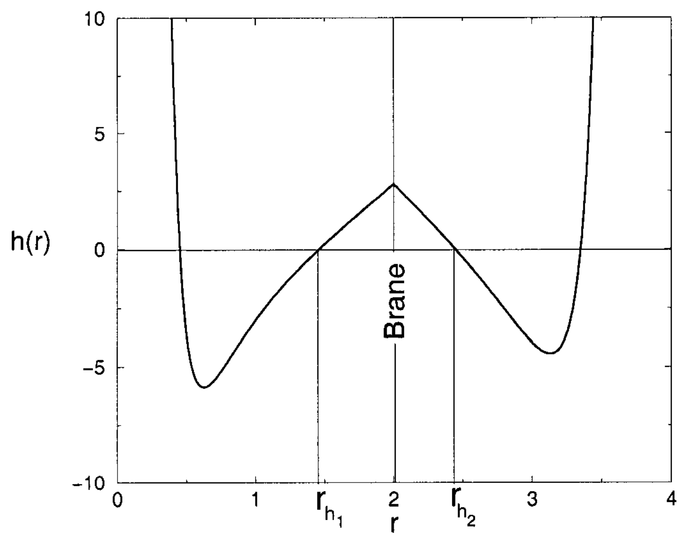



Figure 3. Qualitative behavior of possible solutions without $Z_2$ symmetry across the brane.

One might alternatively hope that adding higher derivative corrections to the action might circumvent the no-go theorem. The simplest such correction to the Einstein-Hilbert action is a Gauss-Bonnet term [67, 68], since this introduces extra powers of derivatives in the equations of motion without increasing the order of the equations. The action is

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R + \lambda(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd})\right) , \tag{5.18}$$

where $\lambda$ is the coefficient of the new Gauss-Bonnet term. The modified Einstein equation is $\overline{G}_{mn} = \kappa^2 T_{mn}$ where $\overline{G}_{mn} = G_{mn} + G^{(\lambda)}_{mn}$ and

$$\begin{aligned} G^{(\lambda)}_{mn} &= -\frac{\lambda}{2}g_{mn}(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}) \\ &+ 2\lambda(RR_{mn} - 2R_{mc}R^c_n + R_{mcde}R^{cde}_n + 2R^{cd}R_{mcdn}) \; . \end{aligned} \tag{5.19}$$

The (00) and (ii) components of the new contribution to Einstein's tensor are

$$
\begin{aligned}
G_{00}^{(\lambda)} &= +\frac{3}{4}h\lambda\left(2\left(\frac{a'}{a}\right)^2\frac{a''}{a}h^2-\left(\frac{a'}{a}\right)^4h^2+\left(\frac{a'}{a}\right)^3h'h\right)\\
G_{ii}^{(\lambda)} &= -\frac{\lambda}{2}\left(2\frac{a''}{a}\frac{a'}{a}hh'-\left(\frac{a'}{a}\right)^3hh'+\left(\frac{a'}{a}\right)^2h'^2+\left(\frac{a'}{a}\right)^2hh''\right)\ .
\end{aligned}
\tag{5.20}
$$

As before, adding the (00) and (55) components of Einstein's equations gives a differential equation similar to Eq (5.7), but $(h'+hA')$ must be replaced with $(h'+hA')(1-\lambda hA'^2)$. Integrating this gives

$$
\begin{aligned}
&(h'+hA')(1-\lambda hA'^2)\Big|_r = \\
&\quad -2\kappa^2 e^{\frac{3}{2}A}\int_r^{r_0}\left(\frac{T_{00}^B}{h}+\frac{T_{11}^B}{a}\right)e^{-\frac{3}{2}A}dr' \\
&\quad -\kappa^2 e^{\frac{3}{2}(A(r)-A(r_0))}\left(\frac{T_{00}^b}{h}+\frac{T_{11}^b}{a}\right)\sqrt{h}\Big|_{r_0}\ .
\end{aligned}
\tag{5.21}
$$

The new factor does not change anything with respect to achieving a horizon since $h$ vanishes there. Just as before, the theorem shows that at the putative horizon $h'<0$, in contradiction to the required behavior.

## 5.4 Solutions with 3-D Curvature

In contrast to the negative results described above, it is possible to have both self-tuning and a horizon, with no violations of stress-energy positivity, when the spatial curvature is nonvanishing and positive. In fact, such a solution has already been obtained in ref. [69] in the case of the chargeless black hole with $k=1$. The authors of [69] did not identify their solution as being self-tuning, but if one regards the position of the brane along with the black hole mass as the properties of the solution which adjust to compensate for the brane tension, then it should indeed be considered as self-tuning. It is straightforward to apply the jump conditions to show that a static solution with

$$h(r) = k + \frac{r^2}{l^2} - \frac{\mu}{r^2}; \qquad a(r) = r^2, \tag{5.22}$$

exists if $k = 1$, if the brane is placed at the position satisfying

$$\begin{aligned} \kappa^4 \rho^2 &= \frac{18}{r_0^2} + \frac{36}{l^2} \\ \rho &> 0; \qquad p = -\rho \end{aligned} \tag{5.23}$$

(again using $l^{-2} \equiv -\frac{1}{6}\kappa^2\Lambda$) and if the black hole mass parameter is

$$\mu = \frac{1}{2} r_0^2. \tag{5.24}$$

Thus self-tuning works for the range of brane tensions $\rho > 6\kappa^{-2}l^{-1}$. The horizon is located at

$$r_h^2 = \frac{1}{2}\left(-l^2 + \sqrt{l^4 + 2r_0^2 l^2}\right) , \tag{5.25}$$

which can be shown to be always between the singularity and the brane.

We can easily generalize the above solution to the case of a charged black hole, where

$$h(r) = k + \frac{r^2}{l^2} - \frac{\mu}{r^2} + \frac{Q^2}{r^4} , \tag{5.26}$$

The jump conditions determine the mass and charge of the black hole to be[3]

$$\begin{aligned} \mu &= r_0^4 \left(\frac{3}{l^2} + \frac{2}{r_0^2} - \frac{\kappa^4\rho^2}{12}\right) \\ Q^2 &= r_0^6 \left(\frac{2}{l^2} + \frac{1}{r_0^2} - \frac{\kappa^4\rho^2}{18}\right) . \end{aligned} \tag{5.27}$$

The additional constant of integration, $Q$, introduces some freedom in the position of the brane, which now can have some range of values for a given brane tension. The condition for existence of the horizon becomes complicated because $h(r_h) = 0$ is a cubic equation. It has real roots (hence a

[3]We follow ref. [24] in adopting the following $Z_2$ parity assignments for the gauge field: $A_r \to +A_r$, $A_t \to -A_t$, so that no new jump condition arises for it.

horizon) only if the following inequality coming from the discriminant of the cubic equation is satisfied:

$$\left(\frac{2}{27}l^4 + \frac{1}{3}\mu l^2 + Q^2\right)^2 < \frac{4}{27}l^2\left(\frac{1}{3}l^2 + \mu\right)^3 . \tag{5.28}$$

If we define

$$\epsilon \equiv l^2/r_0^2; \qquad \eta \equiv \frac{1}{36}\kappa^4\rho^2 l^2 \tag{5.29}$$

and use the expressions (5.27), this can be rewritten as $[1-\eta+\epsilon(1-\frac{1}{2}\eta)+\frac{1}{3}\epsilon^2+\frac{1}{27}\epsilon^3]^2 < [1-\eta+\frac{2}{3}\epsilon+\frac{1}{9}\epsilon^2]^3$. Although this is hard to solve analytically, the results are shown numerically in figure 4. The darkened region is where the horizon exists for positive values of $Q^2$. The region above the wedge ($\eta > 1+\epsilon/2$) corresponds to $Q^2 < 0$. The lower region has $Q^2$ exceeding the critical value beyond which the horizons are lost, resulting in a naked singularity. The boundary of this region can be approximated by the line $\eta \cong 1+\frac{12}{27}\epsilon$ (the approximation becoming exact as $\epsilon \to \infty$). Thus the allowed region for self-tuning with a horizon is given approximately by

$$1+\frac{12}{27}\epsilon \lesssim \eta \leq 1+\frac{1}{2}\epsilon . \tag{5.30}$$

These solutions correspond to a brane with positive tension since the discontinuity of $h'$ at $r_0$ is negative.

We thus see that generalizing to $Q^2 > 0$ does not significantly relax the relation between the brane's tension and its position relative to the original chargeless solution, unless $r_0 \ll l$. However the small $r_0$ regime is not very physical, because at distances much shorter than $l$, the AdS curvature scale, one expects higher derivative corrections to the gravitational action to alter the solution, so that one should not trust it in detail for $r \ll l$. Moreover, our universe would have to have very large values of $r_0$ in order to be nearly spatially flat.

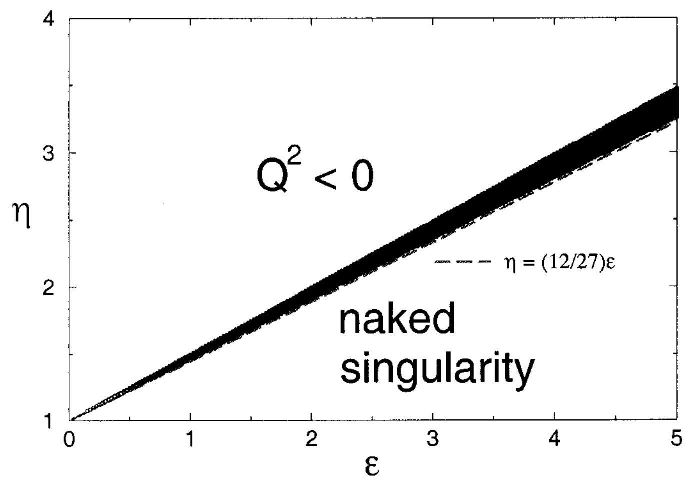



Figure 4: The dark wedge is the range of parameters (see Eq (5.29)) for which self-tuning with a horizon and with $Q^2 > 0$ occurs for the $k = +1$ AdS-RN solution. The upper limit corresponds to $Q = 0$, and the lower one to the critical charge for which the horizons disappear.

We have also searched for static solutions with negative tension branes and $k = 1$ in the AdS-RN case, where the brane is between the singularity and the horizon. These could in principle exist because of the inner horizon and the positivity of $h(r)$ in this region. However we do not find any such solutions. All those illustrated in figure 4 have positive tension branes located outside of the horizons. To arrive at this conclusion, we numerically evaluated the positions of the horizons, $r_{h_i}$, on a fine grid in the $\eta$-$\epsilon$ plane and checked whether $r_0$ was less than or greater than these values. The solutions of $h(r_h) = 0$ are given by

$$\frac{r_h^2}{l^2} = -\frac{1}{3} + 2\sqrt{\frac{A}{3}}\cos\theta\ , \tag{5.31}$$

where

$$\begin{aligned} A &= \frac{1}{3} + \frac{\mu}{l^2}; \qquad B = \frac{2}{27} + \frac{\mu}{3l^2} + \frac{Q^2}{l^4}; \\ \theta &= \frac{1}{3}\left(n\pi + \tan^{-1}\sqrt{\frac{4A^3}{27B^2} - 1}\right), \quad n = 0, 1, \ldots, 5\ . \end{aligned} \tag{5.32}$$

(The three extraneous roots of the six given by this procedure were identified by substituting back into the original equation.) There are always two roots with positive $r_h^2$ and one unphysical one with negative $r_h^2$, except in the regions $\eta > 1 + \epsilon/2$ ($Q^2 < 0$) and $\eta \lesssim 1 + (12/27)\epsilon$ (where $Q^2$ exceeds the critical value for having any horizon). The physical values of $r_h$ are always less than that of the brane position, $r_0$.

## 5.5 Curved Extra Dimensions

Since we live in a universe that is nearly flat, it would be better if the effect of a large brane tension could be counteracted by the curvature of some small extra dimensions rather than that of the usual three. To explore whether this is possible, we consider the following ansatz for the metric, in which two extra dimensions with the geometry of a two-sphere of radius $\sqrt{b(r)}$ are introduced:

$$ds^2 = -h(r)dt^2 + a(r)d\Sigma_k^2 + h(r)^{-1}dr^2 + b(r)(d\theta^2 + \sin^2\theta d\phi^2) \ . \tag{5.33}$$

The (00) and (ii) components of the Einstein tensor are

$$\begin{aligned}
G_{00} &= -\frac{3}{4}h\left(\frac{a'}{a}h' + 2h\frac{a''}{a}\right) \\
&- \frac{h}{4}\left(6\frac{a'}{a}\frac{b'}{b}h + 2\frac{b'}{b}h' - h\left(\frac{b'}{b}\right)^2 + 4\frac{b''}{b}h - 4\frac{a}{b}\right) \\
G_{ii} &= a\left(\frac{a'}{a}h' + h\frac{a''}{a} - \frac{h}{4}\left(\frac{a'}{a}\right)^2 + \frac{h''}{2}\right) \\
&+ \frac{a}{4}\left(4\frac{a'}{a}\frac{b'}{b}h + 4\frac{b'}{b}h' - h\left(\frac{b'}{b}\right)^2 + 4\frac{b''}{b}h - 4\frac{a}{b}\right) \ .
\end{aligned} \tag{5.34}$$

Although the last terms in $G_{00}$ and $G_{ii}$ show the effect of the positively curved extra dimensions, this effect cancels out of the relevant linear combination

$\frac{G_{00}}{h} + \frac{G_{ii}}{a}$. Repeating the same steps that led to our previous no-go result gives

$$(h' + hA')|_r = -2\frac{\kappa^2}{b}e^{\frac{3}{2}A}\int_r^{r_0} b\left(\frac{T_{00}^B}{h} + \frac{T_{11}^B}{a}\right)e^{-\frac{3}{2}A}dr$$
$$-\kappa^2\frac{b(r_0)}{b(r)}e^{\frac{3}{2}(A(r)-A(r_0))}\left(\frac{T_{00}^b}{h} + \frac{T_{11}^b}{a}\right)\sqrt{h}\bigg|_{r_0} \quad . \tag{5.35}$$

Again we need $h'(r_h) > 0$, whereas the above expression shows that $h'(r_h) \leq 0$, regardless of the curved extra dimensions. The latter thus do not provide any new loophole in our theorem.

## 5.6 Discussion

By integrating a certain linear combination of the (00) and (ii) components of the Einstein equations, we have derived an enlightening constraint on the $g_{00}$ component of the metric when there is a black hole in a five-dimensional bulk, which is reminiscent of other consistency conditions that have been deduced for brane/bulk solutions [59, 70]. Our theorem explains why previous attempts to hide self-tuning singularities behind a horizon have had to resort to a brane equation of state which violates positivity of the stress-energy tensor. It shows that one could alternatively achieve the same effect by violating positivity in the bulk rather than on the brane. Indeed, in our search for self-tuning solutions with a horizon and with $p = -\rho$ on the brane, prior to deriving this theorem, we discovered that it is possible if the black hole charge has unphysical values with $Q^2 < 0$.

In our quest for loopholes to this constraint, we found that augmenting the gravitational action with higher powers of the curvature was unsuccessful, as was relaxing the $Z_2$ orbifold symmetry that is often assumed when cutting the bulk space off at the brane. Including positive spatial curvature for the 3-D hypersurfaces provided a more successful way of evading the no-go theorem.

We noted that the previously discovered solution of ref. [69] was an example of self-tuning with a horizon, and we generalized it by allowing the black hole to have a charge.

Unfortunately these positive curvature metrics do not provide a realistic solution to the cosmological constant problem because the curvature is related to the brane tension $\rho$ by $1/r_0 = C\sqrt{\rho}/M_p$, where $C$ is a number which lies within a narrow range of values of order 1. In our universe, which is nearly flat, the same relation exists, where $\rho$ is the critical energy density. Therefore these "self-tuning" solutions can describe our universe only if the brane tension is on the order of the presently observed cosmological constant; this therefore constitutes fine-tuning after all. Precisely the same conclusion would hold if one tried to counteract the effect of a positive cosmological constant with positive curvature in a purely 4-D solution. This is nothing other than Einstein's static solution, which is known to be unstable against perturbations of the scale factor away from the special static value. It seems likely that the same problem will afflict the 5-D solutions as well. Moreover, we found that it was not possible to cancel the effect of the brane tension by shifting the positive curvature into small extra dimensions.

# Chapter 6

# Dynamical Stability of the AdS Soliton in RS Model

## 6.1 Introduction

Most of the brane world scenarios have focused on 5 space-time dimensions, making it difficult to ascertain which features are generic to warped geometries and which are artifacts of five dimensions. Indeed, some features of these models are very likely to be specific to five dimensions. For instance, the requirement of negative-tension branes – which are generic in five dimensions – does not arise in six dimensions [59, 70]. It is also unlikely that the Friedmann equations are modified in six dimensions in the same way as in five. Moreover, the absence of Kaluza-Klein excitations of the metric's radion mode is also a 5-dimensional artifact, due to the trivial geometrical nature of 1-D manifolds.

There have been numerous proposals for higher-dimensional generalizations of the RS idea. One of the earliest was to consider intersections of codimension-1 branes as the 3-branes [71]. Others involved modeling the 3-brane where we are supposed to live, or in some cases where gravity is localized, as a cosmic string or higher-dimensional defect [72]. Other relevant work warped higher dimensional spaces include [73]. A particularly attractive six-dimensional warped model has been considered in various contexts by several authors [70], [74]–[77]. This model is related to the AdS soliton [78], a double analytic continuation of a planar AdS Black hole metric, and involves two compact dimensions having the topology of a disc with a conical singularity at its center. The boundary of the disc occurs at a (Planck) 4-brane and a (TeV) 3-brane is placed at the conical defect. The stress energy of the 4-brane requires an anisotropic form which could arise from the smearing of 3-branes around the 4-brane, as suggested in ref. [70], or from Casimir energy of light particles confined to the 4-brane [74].

Since all observable consequences of this (or any other) geometry only involve the theory's low-energy degrees of freedom, essential to understanding its physical implications is a determination of its low energy spectrum. While this has been partially done in previous references, it is the purpose of this chapter to provide a complete accounting of the metric modes, especially as regards the elusive radion mode.

We find results which differ interestingly from what obtains for five-dimensional RS models. Instead of a massless radion, we find that generically the mass squared is nonzero, and possibly negative, depending on details of the 4-brane stress tensor. For a special case involving Casimir energy on the 4-brane, the radion is an exactly massless modulus at the classical level. Quantum corrections (which we do not here calculate) might stabilize the radion in this case. If the radion mode is stable, the magnitude of the mass squared is of order $(10^{-3}\ \text{eV})^2$. Both the stable and loop-stabilized cases might therefore make this mode of interest for table-top tests of gravity. The tachyonic instability, if it occurs, does so regardless of the value of the radial size of the extra dimensions in the static solution. We show that it is straightforward to cure this problem in the manner of Goldberger and Wise

[16], by adding a bulk scalar field which couples to the branes. The mass of the radion, once stabilized, is suppressed relative to the Planck scale by an additional fractional power of the warp factor, which puts it in the MeV rather than TeV range. Although this could potentially have been problematic, we find that the coupling of the radion is similar to that of gravity, because its wave function is not strongly peaked on the TeV brane. Therefore, although there are cosmological constraints on this model, it is not ruled out by constraints from supernova cooling or radion production in colliders.

We organize our presentation as follows. In section 2 we will introduce the model at the static level. In section 3 we will find the dynamical perturbations for 4-D modes which transform as tensors (gravitons), vectors, and scalars (the radion). We will show that, whereas the tensor and vector modes have a massless ground state, the radion mass squared is generically nonzero and possibly tachyonic, although its magnitude is exponentially small. In section 4 we will show how a bulk scalar field can stabilize the radion mode, and discuss the phenomenology of the model. A summary is given in section 5.

## 6.2 The AdS Soliton in Randall-Sundrum Models

In this section we present the AdS soliton [78] and its key properties relevant for braneworld applications. A more detailed description of this space-time including its role in terms of the AdS/CFT correspondence can be found in ref. [78].

The $(p+2)$-dimensional AdS soliton first arose in connection with the AdS/CFT correspondence [79] as a double-analytic continuation of a $(p{+}2)-$dimensional planar AdS black hole metric. We will be interested in the six dimensional (*i.e.,* $p = 4$) AdS soliton for which the line element may be written

$$ds^2 = a(r)\left(f(r)d\theta^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right) + a^{-1}(r)f^{-1}(r)dr^2 \ , \qquad (6.1)$$

where the metric functions are given by

$$f(r) = \frac{\rho^2}{L^2}\left(1 - \frac{\rho^5}{r^5}\right) \quad \text{and} \quad a(r) = \frac{r^2}{\rho^2} \qquad (6.2)$$

and $\eta_{\mu\nu}$ is the four dimensional Minkowski metric.[1] This is a solution to six dimensional Einstein gravity with negative cosmological constant

$$\Lambda = -\frac{10}{\kappa^2 L^2} \equiv -\frac{8}{5}k^2, \qquad (6.3)$$

where $\kappa^2 \equiv M_6^{-4}$, in terms of the 6-D gravitational scale $M_6$. The space-time is asymptotically locally AdS as $r \to \infty$; below we will cut off the radial extent by inserting a 4-brane at a finite value of $r$. For convenience we have normalized $a(\rho) = 1$, since we will be interested in placing the standard model on a 3-brane situated at that position.

The range of the $r$ coordinate is $\rho \leq r < \infty$, with the geometry smoothly ending at $r = \rho$ provided that the $\theta$ coordinate is periodic, with period

$$\beta = \frac{4\pi L^2}{5\rho}. \qquad (6.4)$$

This also requires that, in the context of supergravity, fermions are antiperiodic in the $\theta$ direction. It is important to note that this geometry is everywhere smooth and nonsingular including at $r = \rho$ where the circle parameterized by $\theta$ smoothly shrinks to a point and the geometry ends. An attractive feature of this geometry is that it ends in a natural and nonsingular fashion, allowing constructions similar to those proposed in refs. [80], but which are free of uncontrollable and likely unphysical curvature singularities.

[1]Here our metric's signature is 'mostly plus' and we adopt MTW [81] curvature conventions.

To construct a brane-world model, we will want to imagine that we are living on a (TeV) 3-brane at $r = \rho$, thereby introducing a conical defect there of size $\delta = \kappa^2 \tau_3$, where $\tau_3$ is the 3-brane tension. This modifies Eq (6.4) to become

$$\beta = \frac{2L^2}{5\rho} (2\pi - \delta). \tag{6.5}$$

The extra dimensions are compactified by terminating the space at a 4-brane at $r = R$.

In the horospheric coordinate system, the proper distance from $\rho$ to $r$ along the $r$-direction is given by

$$\tilde{r} \equiv \int_\rho^r \frac{dr}{\sqrt{af}} = k^{-1} \cosh^{-1} \left[ (r/\rho)^{5/2} \right], \tag{6.6}$$

and so $r/\rho = \cosh^{2/5}(k\tilde{r}) \sim e^{\tilde{r}/L}$ if $r \gg \rho$.

We will often find it enlightening to express the solution in polar coordinates, $\tilde{r}$, where the line element has the form

$$ds^2 = a(\tilde{r}) \eta_{\mu\nu} dx^\mu dx^\nu + b(\tilde{r}) d\tilde{\theta}^2 \; . + d\tilde{r}^2 \; . \tag{6.7}$$

Here the metric coefficients are given by

$$a(\tilde{r}) = \cosh^{4/5}(k\tilde{r}); \qquad b(\tilde{r}) = b_0 \frac{\sinh^2(k\tilde{r})}{\cosh^{6/5}(k\tilde{r})} \; , \tag{6.8}$$

where $b_0 = k^{-2}$ if the point $\tilde{r} = 0$ is regular, and $\tilde{\theta} \in [0, 2\pi]$. In general we will suppose the 3-brane has nonvanishing tension located at this point. Then the conical singularity at $\tilde{r} = 0$ introduces the deficit angle given by $2\pi(k\sqrt{b_0} - 1)$. In these coordinates the proper distance between two radii $\tilde{r}_1$ and $\tilde{r}_2$ is simply their difference, $\tilde{r}_2 - \tilde{r}_1$. We denote the radial position of the 4-brane by $\tilde{r} = \tilde{R}$.

We close this section with some comments regarding the stability of the AdS soliton. Since the AdS soliton is constructed from multiple analytic continuations of a black hole space-time one might worry about dynamical stability of the solution. In general such analytically continued space-times are not always well behaved. For example beginning with the Reissner-Nordstrom black hole in asymptotically flat space one can analytically continue the metric in such a way as to allow for arbitrarily large negative values for the mass parameter. Solutions such as this are inherently unstable *i.e.* small perturbations around the background are tachyonic—see ref. [78] for a detailed discussion.

One of the key results of ref. [78] was that (for $p = 3$) the AdS soliton was found to be perturbatively stable to such linearized fluctuations. Further, in ref. [82] this proof was extended to arbitrary $p \geq 2$ —see also ref. [83] for a recent discussion. One consequence of this proof is that at least locally within the space of solutions to the Einstein equations with asymptotically locally AdS boundary conditions the AdS soliton represents the minimum energy solution.

It is one of the purposes of this chapter to investigate whether the perturbative stability of this space-time persists when the geometry is cut off by the introduction of the 4-brane discussed above.

### 6.2.1 The Gauge Hierarchy

To understand how this model solves the gauge hierarchy problem, let us imagine that all the fundamental scales $M_6$, $k$, and $1/R$ are of order TeV. Then the standard reduction of the gravitational action from 6 to 4 dimensions (using polar coordinates) gives the 4-D Planck mass as

$$\begin{aligned} M_p^2 &= M_6^4 \int d\tilde{r}\, d\tilde{\theta}\, a(\tilde{r})\sqrt{b(\tilde{r})} \\ &\sim \frac{M_6^4}{k^2} a^{3/2}(\tilde{R}) = \frac{M_6^4}{k^2} e^{6k\tilde{R}/5} \,, \end{aligned} \tag{6.9}$$

Thus by taking $e^{3k\tilde{R}/5} \sim 10^{16}$, corresponding to $k\tilde{R} \cong 60$, we can explain the largeness of the Planck scale.

Notice that the relation $M_p^2 \sim a^{3/2}(\tilde{R})(\mathrm{TeV})^2$ differs from the analogous relation in the 5-D RSI model, $M_p^2 \sim a(\tilde{R})(\mathrm{TeV})^2$. The additional factor of $a^{1/2}$ is coming from the $b^{1/2}$ part of the measure, which gives the size of the extra compact dimension that was not present in the 5-D model. This shows that the present model is a hybrid of the RS and ADD scenarios, in that the hierarchy is due to a combination of warping and having a large extra dimension.

This difference can also be seen by considering the physical mass of a 4-D scalar field which is confined to a 3-brane at a fixed position $(\tilde{r}, \tilde{\theta})$ in the bulk. Since we are taking the fundamental scale to be TeV, we should assume that its bare mass parameter $m$ is of this order. But the physical mass is determined by the usual argument of canonically normalizing its kinetic term:

$$
\begin{aligned}
S_3 &= -\frac{1}{2}\int d^4x \, a^2 \left[a^{-1}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right] \\
&\to -\frac{1}{2}\int d^4x \left[\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + a(\tilde{r})m^2\phi^2\right] . \qquad (6.10)
\end{aligned}
$$

Thus the physical mass is given by $m_3 = m\sqrt{a(\tilde{r})}$. If we take the particle all the way to the 4-brane, its mass does not reach the Planck scale, but rather a smaller one, $a(\tilde{R})^{-1/4}M_p \sim 10^{-32/6}M_p \sim 10^{13}$ GeV. This reflects the fact that the strength of gravity is still diluted for a 4-D observer on the 4-brane, by the large extra dimension. We should refer to it as the "$10^{13}$ GeV brane" rather than the Planck brane.

### 6.2.2 Properties of the Branes

The 4-brane we need for compactifying the AdS soliton can be constructed by the standard cutting and pasting procedure. Here, the metric in Eq (6.2) will be cut along the surface $r = R$ and then pasted onto a mirror image of itself. The resulting space-time is then a solution of

$$\left(R_{MN} - \frac{1}{2}Rg_{MN} + \Lambda g_{MN}\right) = \kappa^2 S_{ab}\delta^a_M\delta^b_N\delta\left(\frac{r-R}{\sqrt{fa}}\right), \tag{6.11}$$

where $g_{MN}$ is the metric given in eqn.(6.2) and the induced metric on the 4-brane is $g_{ab} = g_{MN}(R)\delta^M_a\delta^N_b$.

$S_{ab}$ is the stress tensor of an infinitely thin brane located at the cutting surface. The stress-tensor so defined may be obtained from the Israel matching conditions [38]. A straightforward calculation yields,

$$S_{\mu\nu} = -\kappa^{-2}\sqrt{fa}\left(4\frac{a'}{a} + \frac{f'}{f}\right) g_{\mu\nu} \tag{6.12}$$

$$S_{\theta\theta} = -\kappa^{-2}\sqrt{fa}\left(4\frac{a'}{a}\right) g_{\theta\theta}. \tag{6.13}$$

Here and in the following uppercase Latin indices indicate six dimensional coordinates, while lower case Greek indices specify the coordinates parameterizing the directions along the 3-brane. Lower-case Latin indices similarly label directions parallel to the 4-brane *i.e.*, $x^a = (x^\mu, \theta)$.

A crucial point for this model is that the extra term $f'/f$ in Eq (6.12) relative to (6.13), though small, is nonvanishing, and therefore it is impossible to interpret the 4-brane stress tensor as being due to a pure tension. Were we to do this, thus making $S_0{}^0 = S_\theta{}^\theta$, the 4-brane would be forced to go to $r = \infty$, and we would lose the compactification of the extra dimensions and the localization of gravity.

There are several kinds of physics on the 4-brane which would naturally involve the required difference in $S_0{}^0 - S_\theta{}^\theta$. One is to imagine that the gluing surface is composed of multiple branes. As discussed in ref. [70], one could consider the superposition of the stress-energy tensors of a four-brane wrapping the internal circle and a three-brane which is smeared over the internal circle. Indeed eqs. (6.12,6.13) then take the form [70],

$$\begin{aligned} S_{\mu\nu} &= \left(T_4 + \frac{T_3}{L_\theta}\right) g_{\mu\nu} \\ S_{\theta\theta} &= T_4 g_{\theta\theta} \ , \end{aligned} \tag{6.14}$$

where $L_\theta$ is the proper period of the circle parameterized by $\theta$ at $r = R$.

Another very physical possibility is that the difference between $T_0$ and $T_\theta$ is due to the Casimir energy of any massless fields which are confined to the 4-brane [74]. For these the stress-energy tensor will take the form

$$\begin{aligned} S_{\mu\nu} &= \left(T_4 + \frac{c_0}{L_\theta^5}\right) g_{\mu\nu} \\ S_{\theta\theta} &= \left(T_4 - \frac{c_\theta}{L_\theta^5}\right) \ , \end{aligned} \tag{6.15}$$

with some dimensionless coefficients $c_0$ and $c_\theta$. To the extent that the trace anomaly vanishes (which is the case at one loop, since the 4-brane is odd-dimensional), the Casimir energy satisfies the condition $g^{ab}S_{ab} = 0$, which implies $c_\theta = 4c_0$.

In a static background, either of these stress-energy tensors are trivially conserved on the 4-brane. But when we discuss dynamical perturbations of the static space, conservation of stress-energy will yield a nontrivial constraint on the components of $S_{\mu\nu}$. This will be discussed in section 3.3, where we show that the ground state of the radion modes is tachyonic for the smeared 3-brane model, but massless for the Casimir model.

The issue of stabilization is closely related to another potential problem with the above models. This concerns the order of magnitude of the difference ${S_0}^0 - {S_\theta}^\theta$, which is required in order to achieve $a(R) \sim 10^{21}$ as is needed to solve the hierarchy problem. This requires

$$\frac{{S_0}^0 - {S_\theta}^\theta}{{S_0}^0 + {S_\theta}^\theta} \sim a(R)^{-5/2} \sim 10^{-53}, \tag{6.16}$$

which appears to be extremely fine-tuned. From this standpoint, only the Casimir effect can be considered to be natural, since its $L_\theta^{-5}$ dependence scales precisely like $a(R)^{-5/2}$. However the Goldberger-Wise stabilizing field makes it unnecessary to have nonvanishing ${S_0}^0 - {S_\theta}^\theta$, as was shown by [75]: with the scalar it becomes possible to achieve an exponential hierarchy even when ${S_0}^0 = {S_\theta}^\theta$. It is interesting that we are able to both determine the size of the extra dimension and stabilize the radion using the same scalar field. In the 5D RS1 model, the two phenomena are necessarily tied together, but not so in 6D. The fact that the size of the extra dimension is determined by the value of ${S_0}^0 - {S_\theta}^\theta$ does not prevent the instability we will demonstrate, so it is not obvious that introducing a new effect to determine the size of $R$ should stabilize the system. Nevertheless we shall show that it is true.

## 6.3 Stability Analysis

In the original model of Randall and Sundrum with two branes, fluctuations of the metric were decomposable into Kaluza-Klein modes. Most notably the spectrum included a zero mode which was bound to the brane and served as the graviton in a four dimensional world. The remaining excitations formed a tower of massive modes which were fully five dimensional and had very little support near the brane. In the AdS soliton model presented here we will find a very similar story emerging with a few differences. As in RSI, the space-time constructed in the previous section is finite and one can view the graviton fluctuations as linearized gravity in a box. This implies that the spectrum of gravitons will again be discrete. Another difference from RSI is that the fluctuations of the metric are now more complicated owing to the greater complexity of the background metric. With only a single extra dimension, the only degree of freedom for the radion mode is the distance between the two branes, since any apparent ripples in the $dr^2$ metric component can be gauged away by a coordinate transformation. With two or more extra dimensions this is no longer the case, and the radion too has a KK tower of excitations.

By virtue of the symmetries of the geometry at least four of the metric modes must be exactly massless. First, there are two massless states which correspond to the massless 4-D graviton which is ensured by the model's unbroken Lorentz invariance in the directions parallel to the 3-brane. Second there are two states making up the massless 4-D spin-one particle, which is a consequence of the isometry $\theta \to \theta + c$ of the extra dimensions.

The counting of massless modes may be further sharpened as follows. If gravity is indeed localized on the 4-brane, we would expect to find a total of five zero modes appropriate to the five independent fluctuations of a massless spin-two particle in 4+1 dimensions. Since we will find below that the radion generically has a nonvanishing mass in this theory (either tachyonic or real by adding the appropriate scalar), there are in fact only four zero modes bound to the brane. These will have a natural interpretation, at energies below the mass of the radion, as a $3+1$ dimensional graviton and a $3+1$ dimensional massless vector field.

The analysis will proceed by linearizing Eq (6.11) around the background of the AdS soliton. In particular we will consider fluctuations of the six dimensional metric which are given by $g_{MN} \to g_{MN} + h_{MN}$. A feature here is the fact that $h_{MN}$ is a tensor and there is thus a variety of polarizations, or graviton modes, that need to be considered. Following refs. [82, 84] we can divide the various polarizations of the six dimensional graviton into three categories. **(i)** Transverse traceless polarizations. These are modes which are polarized in directions parallel to the Lorentz invariant hypersurface spanned by the coordinates $x^i$. **(ii)** Vector polarizations. These are gravitons whose polarization is of the form $\epsilon_{i\theta}$, *i.e.*, modes polarized along the circle and in the flat piece of the brane. **(iii)** Scalar Polarizations. These are modes which are diagonal but not traceless. It is this last mode which is related to the radion field.

For the cases **(i)** and **(ii)** above we may write the metric fluctuations as $h_{MN} = H_{MN}(r)e^{ik\cdot x}$ where $H_{MN}(r)$ is the radial profile tensor and $k^\mu$ is a 4-dimensional momentum vector with $k^2 = \eta^{\mu\nu}k_\mu k_\nu = -M^2$. Further there are ambiguities in the metric perturbations arising from diffeomorphism invariance, which we (partially) handle by imposing a "transverse gauge:" $H_{M\mu}k^\mu = 0$. For massive excitations we may always, via the appropriate Lorentz boost, choose to work in the rest frame[2] so that the momentum can be written as $k^\mu = \rho\,\delta^\mu_t$. In this case, the transversality condition becomes,

$$H_{a\mu}k^\mu = 0 \quad \Rightarrow \quad H_{at} = 0 \quad \forall a \tag{6.17}$$

Our implicit notation for the $3+1$-dimensional Minkowski space coordinates is $x^\mu = (t, x^i)$ with $i = 1, \ldots, 3$.

We do not consider here the Kaluza-Klein modes[3] corresponding to angular excitations, *i.e.*, around the large extra dimension. One might at first have thought that these had a mass gap of order 10 eV since they are modes which are localized on the 4-brane (assuming they are radially unexcited) and the circumference of the 4-brane is of order $L_\theta \sim (10 \text{ eV})^{-1}$. However, if we imagine integrating out only the radial dimension to obtain the effective theory of these modes, we find that the fundamental scale is no longer TeV, but rather $\sqrt{a(R)}$ TeV $\sim 10^{13}$ GeV, because of the effect of warping. (The kinetic term of these excitations gets the same exponential factors as does the angular gradient term: $\mathcal{L} \sim (\dot{\phi})^2/a + (\partial_\theta\phi)^2/b$.) This effect of warping was alluded to in section 2.2. From this point of view, the size of the compact dimension looks like $(\text{TeV})^{-1}$, whose smallness compared to $\sim 10^{13}$ GeV is how the largeness of the extra dimension is manifested. We will leave aside

[2]Of course when searching for zero modes we are constrained to work with a null momentum vector.

[3]For a discussion of these KK modes as they relate to the stability of the AdS soliton see ref. [82].

these angular KK modes and instead consider the radial excitations. To determine the spectrum, we must solve Eq (6.11) with the ansatz (6.17) as an eigenvalue problem for the mass $M$.

The metric fluctuations for case (**iii**) are considerably more complicated and will be dealt with separately in section 3.3.

### 6.3.1 Transverse Traceless Modes

As explained above, these modes are polarized parallel to the Lorentz invariant directions on the brane and correspond to

$$H_{\theta M} = H_{rM} = 0 = H_{M\mu}k^{\mu} \qquad \forall M \ , \tag{6.18}$$

where the last equality is a restatement of Eq (6.17).

A consistent solution to Eq (6.11) linearized around the AdS soliton is provided by the following ansatz,

$$H_{MN} = \varepsilon_{MN} a(r) H(r) \ , \tag{6.19}$$

where $\varepsilon_{MN}$ satisfies the conditions in Eq (6.18) and $a(r)$ is the metric function appearing in the static solution above.

Solving the equations of motion, which come from linearizing eqn. (6.11) around our ansatz, imposes that the polarization, must also be traceless, $\eta^{\mu\nu}\varepsilon_{\mu\nu} = 0$. Thus Eq (6.18) describes five independent modes, which can be described as three off-diagonal polarizations, *e.g.*,

$$\varepsilon_{12} = \varepsilon_{21} = 1 \ , \qquad \text{otherwise } \varepsilon_{MN} = 0 \tag{6.20}$$

and two traceless diagonal polarizations, *e.g.*,

$$\varepsilon_{11} = -\varepsilon_{22} = 1 \ , \qquad \text{otherwise } \varepsilon_{MN} = 0 \ . \tag{6.21}$$

For all of these independent polarizations, the radial profile $H(r)$ satisfies the same differential equation. Substituting the above ansatz, (6.18) and (6.19), into Eq (6.11) and linearizing around the AdS soliton background one finds that the radial profile must satisfy

$$\frac{d^2H(r)}{dr^2} + \left(3\frac{a'(r)}{a(r)} + \frac{f'(r)}{f(r)}\right)\frac{dH(r)}{dr} + \frac{M^2}{f(r)a(r)^2}H(r) = 0 \ , \tag{6.22}$$

where primes denote differentiation with respect to $r$. Here the $\delta$-function coming from the right hand side has been canceled exactly by similar terms on the left hand side. One should note that this is *exactly* the equation for the transverse traceless modes originally obtained in ref. [82, 84]—see also ref. [85]. Again, this is independent of the form we choose for the stress tensor, since by exciting these transverse traceless modes we are not perturbing the size of the circle. It is interesting to note that this is precisely the equation describing the propagation of a minimally coupled massless scalar on the AdS soliton background [82, 84].

Here we will determine the eigenvalues numerically using a shooting technique (see ref. [86]). For the purposes of numerical calculation we will henceforth restrict to $r < R$ and replace the brane by effective boundary conditions on the gravitons at $r = R$. Obtaining the correct solution to this problem is equivalent to determining the correct boundary conditions that the radial profile $H(r)$ must obey at $r = \rho$ and $r = R$. At the brane the absence of $\delta$-functions in Eq (6.22) implies that $H(r)$ and its first derivative are continuous so requiring an even function of $r$ gives the boundary condition $H'(r) = 0$. The boundary at $r = \rho$ is more subtle since the metric function $f(r)$ vanishes there, *i.e.*, $f(r = \rho) = 0$. This is reflected in the fact that this point is a regular point of Eq (6.22). So requiring that $H(r)$ be regular at this boundary gives the condition

$$\left.\frac{dH(r)}{dr}\right|_{r=\rho} = -\frac{L^2M^2}{5}\, H(r)|_{r=\rho} \ . \tag{6.23}$$

It is now straightforward to see that this polarization of the six dimensional graviton indeed has a zero mode. This can be done by setting $M^2 = 0$ in Eq

(6.22) and integrating directly. After the first integration we find

$$\frac{dH(r)}{dr} = \text{const.} \times a(r)^{-3} f(r)^{-1} , \tag{6.24}$$

which can be seen by inspection to violate the boundary condition at the brane unless the integration constant is forced to vanish. Performing the second integration just leaves the constant solution obeying the above boundary conditions. So we see, referring back to the ansatz in Eq (6.19), that the physical zero mode for these polarizations is

$$h_{MN} = \varepsilon_{MN} a(r) , \tag{6.25}$$

where $\varepsilon_{MN}$ obeys the conditions in Eq (6.19) and we have used our freedom to perform one overall rescaling of the solution to set $H(r) = 1$.

In order to obtain the spectrum of nonzero modes we use the numerical shooting technique with the above boundary conditions. The mass eigenvalues are a function of the relative size of the extra dimension $R/\rho$, but in the limit that $R/\rho$ becomes exponentially large, as desired to solve the hierarchy problem, the masses quickly approach their asymptotic values. We give these values for the first few KK modes in the following table. We emphasize that there are no modes with $M^2 < 0$ and hence no instabilities in this sector.

| Mode Number | $M^2L^2$ |
|---|---|
| 0 | 0 |
| 1 | 16.494 |
| 2 | 44.731 |
| 3 | 85.545 |
| 4 | 138.92 |
| 5 | 204.85 |

Table 1. Mass squared of the radial graviton KK modes, in units of the AdS curvature radius, in the limit of large warp factor.

Unlike the zero mode, which is localized on the 4-brane, the KK modes are peaked at the TeV brane, a phenomenon which is familiar from the 5D RS model. This behavior is illustrated for the first three modes in figure 1, where the wave functions are plotted.

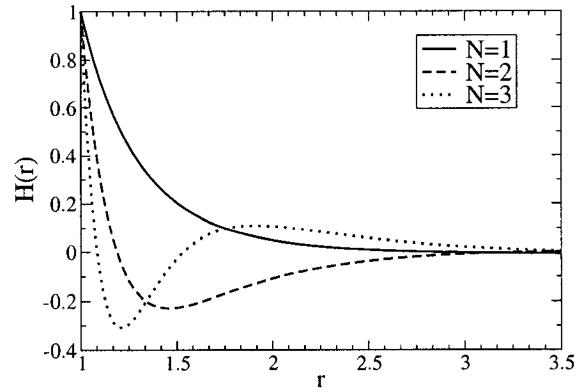



Figure 1. Wave functions for the first three radial KK modes of the graviton.

### 6.3.2 Vector Modes

The next set of polarizations comes from the same ansatz as in Eq (6.19); however in this case the polarization tensor is such that it has one leg in the Lorentz invariant directions of the four-brane and another on the circle. It has the form

$$\varepsilon_{\theta\mu} = \varepsilon_{\mu\theta} = v_\mu \qquad \text{with } k \cdot v = 0 \text{ and } v \cdot v = 1 \ , \tag{6.26}$$

Polarizations of this form contain three independent modes. Substitution into the equation of motion (6.11) and linearizing as before yields

$$\frac{d^2H(r)}{dr^2} + 3\frac{a'(r)}{a(r)}\frac{dH(r)}{dr} + \frac{M^2}{f(r)a(r)^2}H(r) = -2\frac{f'(r)}{f(r)}\delta(r-R)H(r) \ . \tag{6.27}$$

For these modes there is a net contribution from the $\delta$-function source term on the right hand side of Eq (6.11). This can be understood from the fact that the metric perturbation we are considering is a fluctuation in the $g_{\theta x}$ components of the metric. The corresponding variation of the stress-energy tensor only involves the pieces proportional to $T_4$ and not those proportional to $T_3$. In other words for these modes we have *effectively* that $\delta T_4 = \delta T_0$ under the fluctuations considered in this section. In our formalism this will manifest itself as a nontrivial boundary condition at $r = R$,

$$\left.\frac{dH(r)}{dr}\right|_{r=R} = \left.\frac{f'(r)}{f(r)}H(r)\right|_{r=R} , \tag{6.28}$$

while enforcing regularity at the point $r = \rho$ requires that $H(\rho) = 0$. As with the transverse traceless modes of the previous section, we can obtain an analytic solution for the zero mode of this equation by setting $M^2 = 0$ and performing the integration directly. We find an exact solution of the form

$$H(r) = -\frac{c}{5}\frac{L^6}{r^5} + b , \tag{6.29}$$

where $c$ and $b$ are arbitrary (dimensionful) constants of integration. Imposing the boundary condition in Eq (6.28) gives

$$b = \frac{c\,L^6}{5\rho^5} , \tag{6.30}$$

and from this it is straightforward to see that the zero mode is given by

$$H(r) = \frac{cL^8}{5\rho^7} f(r) . \tag{6.31}$$

Fortuitously, this solution also satisfies the boundary condition at $r = \rho$ for all values of $c$ since this is precisely where $f(r) = 0$. Choosing to normalize the wavefunction so that $H(R) = 1$ amounts to choosing $c = 5\frac{\rho^7}{L^8}f(R)^{-1}$. Returning to the ansatz in Eq (6.26) we see that the physical zero mode takes the simple form

$$H_{\mu\theta} = \varepsilon_{\mu\theta}a(r)\frac{f(r)}{f(R)} , \tag{6.32}$$

which is indeed peaked at $r = R$. In order to analyze the nonzero modes we again turn to numerics and find a positive definite spectrum which implies that no instabilities are caused by exciting these vector modes. In the table below we again present, in the limit of large warp factor, the first few eigenvalues in this spectrum.

Like the spin-2 modes, the vector KK modes are also peaked near the TeV brane, although their wave function vanishes precisely there. The first three modes are shown in figure 2.

| Mode Number | $M^2L^2$ |
|---|---|
| 0 | 0 |
| 1 | 25.001 |
| 2 | 59.752 |
| 3 | 106.91 |
| 4 | 166.61 |
| 5 | 238.80 |

Table 2. Mass squared of the radial graviphoton KK modes, in units of the AdS curvature radius, in the limit of large warp factor.

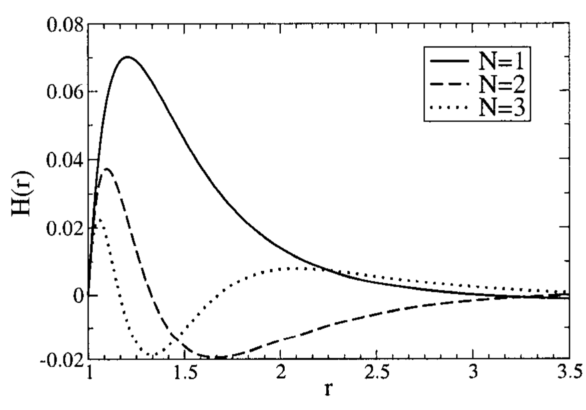



Figure 2. Wave functions for the first three radial KK modes of the graviphoton.

### 6.3.3 Scalar Modes

Here we will consider modes which fall into category **(iii)** above. There is in fact only a single mode (at the lowest KK level), although it corresponds to simultaneous fluctuations of different components of the metric. Most importantly this mode includes fluctuations of both $g_{\theta\theta}$ and $g_{rr}$ and thus couples fluctuations of the radial size to fluctuations in the size of the compact circle. For the radion, it is convenient to switch to the polar coordinates introduced in Eq (6.7), where the equations take a somewhat simpler form. We start by writing the perturbed metric as

$$ds^2 = a(\tilde{r},t)\eta_{\mu\nu}dx^\mu dx^\nu + b(\tilde{r},t)d\tilde{\theta}^2 + c(\tilde{r},t)d\tilde{r}^2 \ , \tag{6.33}$$

where the perturbation around the static background corresponds to

$$\begin{aligned} a(\tilde{r},t) &= e^{-A_0(r)-A_1(r,t)} \equiv a_0(\tilde{r})e^{-A_1(\tilde{r},t)} \\ b(\tilde{r},t) &= e^{-B_0(\tilde{r})-B_1(\tilde{r},t)} \equiv b_0(\tilde{r})e^{-B_1(\tilde{r},t)} \\ c(\tilde{r},t) &= 1 + C_1(\tilde{r},t) \ , \end{aligned} \tag{6.34}$$

and we have now included a subscript '0' to denote the metric functions of the background around which we are expanding, *i.e.*, the AdS soliton. Here we have gone to the 4-D rest frame of the fluctuation

$$A_1(\tilde{r},t) = A_1(\tilde{r})\mathrm{Re}(e^{-i\omega_r t}) \ , \tag{6.35}$$

and similarly for $F_1$ and $G_1$. This is generically valid since for arbitrary values of the parameters there will be no zero mode. We will find however that there is a special case in which there is a zero mode and hence no rest frame. For this case we have checked that the procedure outlined here gives the correct result.

We can unify the models of the 4-brane stress energy at $\tilde{r} = \tilde{R}$ which were discussed in section 2.3 by writing

$$
\begin{aligned}
S_{\mu\nu} &= \left(T_4 + \frac{T_3}{L_\theta{}^\alpha}\right) g_{\mu\nu} \equiv V_0\, g_{\mu\nu} \\
S_{\theta\theta} &= \left(T_4 - \frac{T_3'}{L_\theta{}^\alpha}\right) g_{\theta\theta} \equiv V_\theta\, g_{\theta\theta}\ ,
\end{aligned} \tag{6.36}
$$

where $L_\theta = \int d\theta \sqrt{b}|_{\tilde{r}=\tilde{R}}$ is the circumference of the compact 4-brane dimension. The smeared 3-brane model has $\alpha = 1$ and $T_3' = 0$, while the Casimir effect model has $\alpha = 5$ and $T_3, T_3' \neq 0$. Notice that the metric tensor elements appear within the definition of the circumference $L_\theta$. Also $T_3$ really has the dimensions of a 3-brane tension only when $\alpha = 1$.

We can now write the Einstein equations for this metric ansatz. Since it will later be necessary to add a bulk minimally-coupled scalar field, we include it here, although at first we shall carry out the analysis with no scalar. Ignoring terms like $\dot{a}^2$ which would be higher order in the perturbations, the $(tt)$, $(tt)+(ii)$, $(rr)$, $(\theta\theta)$ and $(tr)$ components of the Einstein equations are

$$
\begin{aligned}
&\frac{3}{2}\frac{a''}{a} + \frac{3}{4}\frac{a'}{a}\frac{b'}{b} - \frac{3}{4}\frac{a'}{a}\frac{c'}{c} + \frac{1}{2}\frac{b''}{b} - \frac{1}{4}\left(\frac{b'}{b}\right)^2 - \frac{1}{4}\frac{b'}{b}\frac{c'}{c} \\
&= -\kappa^2\left(c[\Lambda + V(\phi)] + \tfrac{1}{2}\phi'^2 + V_0\sqrt{c}\,\delta(\tilde{r}-\tilde{R}) + \ T_b\sqrt{\frac{c}{b}}\delta(\tilde{r})\right)
\end{aligned} \tag{6.37}
$$

$$
2\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} = 0 \tag{6.38}
$$

$$
\frac{3}{2}\left(\frac{a'}{a}\right)^2 + \frac{a'}{a}\frac{b'}{b} - \frac{1}{2a_0}\left(\frac{\ddot{b}}{b} + 3\frac{\ddot{a}}{a}\right) = -\kappa^2\left(c[\Lambda + V(\phi)] - \tfrac{1}{2}\phi'^2\right) \tag{6.39}
$$

$$
\begin{aligned}
&2\frac{a''}{a} + \frac{1}{2}\left(\frac{a'}{a}\right)^2 - \frac{a'}{a}\frac{c'}{c} - \frac{1}{2a_0}\left(3\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c}\right) \\
&\qquad = -\kappa^2\left(c[\Lambda + V(\phi)] + \tfrac{1}{2}\phi'^2 + V_\theta\sqrt{c}\,\delta(\tilde{r}-\tilde{R})\right)
\end{aligned} \tag{6.40}
$$

$$
6\frac{\dot{a}'}{a} - 6\frac{\dot{a}}{a}\frac{a'}{a} - \frac{\dot{b}}{b}\frac{a'}{a} - 3\frac{\dot{c}}{c}\frac{a'}{a} + 2\frac{\dot{b}'}{b} - \frac{\dot{b}}{b}\frac{b'}{b} - \frac{\dot{c}}{c}\frac{b'}{b} = 4\kappa^2\dot{\phi}\phi'\ , \tag{6.41}
$$

where $T_b$ is the tension of the 3-brane at $\tilde{r} = 0$ and primes denote $\partial_{\tilde{r}}$.

The next step is to linearize the field equations in the dynamical perturbations. We use the relation $\ddot{A}_1 = -m_r^2 A_1$, and similarly for the other perturbations, where $m_r$ is the sought-for radion mass. Expanding the $(tt)+(ii)$, $(tr)$ and $(rr)$ Einstein equations, respectively, to first order gives

$$m_r^2(2A_1 + B_1 - C_1) = 0 \tag{6.42}$$

$$4(A_1' - \frac{1}{2}A_0' C_1) + \left[B_1' - A_1' - \frac{1}{2}(B_0' - A_0')(B_1 + C_1)\right] - 2\kappa^2 \phi_0' \phi_1 = 0 \tag{6.43}$$

$$(3A_0' + B_0')A_1' + A_0' B_1' - A_0'\left(\frac{3}{2}A_0' + B_0'\right)C_1 - \frac{m_r^2}{2a_0}(3A_1 + B_1) \\ + \frac{\kappa^2}{2}\left[\phi_0'^2 C_1 + 2\frac{\partial V}{\partial \phi}\phi_1 - 2\phi_0'\phi_1'\right] = 0\ , \tag{6.44}$$

and similarly for the scalar field we find

$$\phi_1'' - (2A_0' + B_0')\phi_1' - \frac{1}{2}(4A_1' + B_1' + C_1')\phi_0' - C_1\frac{\partial V}{\partial \phi} - \frac{\partial^2 V}{\partial \phi^2}\phi_1 + \frac{m_r^2}{a_0}\phi_1 = 0. \tag{6.45}$$

We note that even for a massless mode, the energy is nonvanishing, so that Eq (6.42) would still provide a constraint among the components of the perturbation (we would then have to consider its spatial momentum too). Moreover (6.43) is the time-integrated form of the $(tr)$ equation, which we have organized in a form whose purpose will become clear momentarily. We have not written the $(tt)$ nor $(\theta\theta)$ components since these are not independent equations: $(tt)$ can be obtained from $(tr)'$ and $(tt) + (ii)$, which follows from the Bianchi identities; and $(\tilde{\theta}\tilde{\theta})$ follows from combining $(tr)$, $(rr)$, $(rr)'$ and $\phi$ equations in a manner which is not obvious, but which could be anticipated, since first order constraint equations, like $(rr)$ and $(tr)$, must be consistent with the second-order dynamical equations, $(tt)$, $(ii)$ and $(\tilde{\theta}\tilde{\theta})$.

For the remainder of this section, we will assume there is no bulk scalar field present. Effects of the bulk scalar will be considered in the next section.

The boundary conditions at the 4-brane come from integrating over the delta functions to find the discontinuity in the first derivatives, and using $Z_2$

symmetry across the brane to identify, *e.g.*, $\Delta a'/a = -2a'/a$ at $\tilde{r} = \tilde{R}$. At zeroth order in the perturbations, this gives the jump conditions (6.12,6.13) for the static solution. Expanding to first order, and assuming that $T_3$ is a constant (more about this below) we find

$$A_1' = \frac{1}{2}A_0'C_1 - \frac{\alpha}{8}\left(\frac{T_3'}{T_3+T_3'}\right)(B_0'-A_0')B_1 \tag{6.46}$$

$$B_1' - A_1' = \frac{1}{2}(B_0'-A_0')(C_1+\alpha B_1). \tag{6.47}$$

In addition, we must consider the boundary condition at the 3-brane at the center, $\tilde{r} = 0$. With two extra dimensions, the effect of such a point-like source is to introduce a conical singularity and a corresponding deficit angle, as we have discussed in section 2.1. Since the defect is unchanged by perturbations around the static solution, we insist that the deficit angle does not vary. If we consider a circle with $\tilde{r} = \epsilon$ around the 3-brane, with circumference $L$ and physical radius $D$, we therefore demand that $L/D$ be invariant in the limit that $\epsilon \to 0$:

$$\begin{aligned}\lim_{\epsilon\to 0}\delta\left(\frac{L}{D}\right) &= \lim_{\epsilon\to 0}\delta\left(\frac{\int d\tilde{\theta}\sqrt{b}}{\int_0^\epsilon \sqrt{c}}\right)\\ &= -\frac{L}{2D}\left(B_1+C_1\right)\big|_{\tilde{r}=0}, \end{aligned}\tag{6.48}$$

in other words, $B_1 + C_1 = 0$ at $\tilde{r} = 0$. In addition, we expect $A_1'$ to vanish at $\tilde{r} = 0$. The latter can be shown to be satisfied from $B_1 + C_1 = 0$ combined with the bulk equations of motion, so it gives no additional constraint.

We must pause to discuss an important point, concerning the counting of boundary conditions versus independent differential equations. Using the algebraic constraint (6.42) to eliminate $C_1$, we have two first order o.d.e.'s for $A_1$ and $B_1$. On the face of it, our system looks overconstrained: there are three boundary conditions! But in fact the system is not overconstrained. Rather, there is a constraint on the stress-energy tensor on the 4-brane, due to its conservation. By computing $\sum_{a\neq\tilde{r}} S^{0a}{}_{;a} \equiv 0$, where $S_{ab}$ is the surface stress energy on the 4-brane, we obtain

$$\frac{dT_3}{dt} = \frac{\dot{B}_1}{2}\left(T_3(1-\alpha) + T_3'\right) \ . \tag{6.49}$$

Integrating this, we see that unless the constraint

$$T_3' = (\alpha - 1)T_3 \tag{6.50}$$

is satisfied, then $T_3$ must have had extra hidden dependence on $B$ (hence the circumference of the circle $L_\theta$) beyond that which was explicitly assumed. If $T_3$ is truly constant, then any physical model of the stress-energy must satisfy (6.50). This is true for the model which corresponds to delocalizing (smearing) a 3-brane around the circular dimension of the 4-brane, since there $\alpha = 1$ and $T_3' = 0$. And it tells us that the Casimir energy model with $\alpha = 5$ must satisfy $T_3' = 4T_3$, as we saw earlier was indeed the case for massless particles. With any such choice, it is easy to see that there are not really two boundary conditions at the 4-brane; rather, imposing the first b.c. (6.46), together with the $(tr)$ equation (6.43), implies the second b.c., (6.47). The result of this discussion is that it suffices to impose just one b.c. at $\tilde{r} = \tilde{R}$, say (6.46), which using (6.50) can be written more simply as

$$A_1' = \frac{1}{2}A_0'C_1 - \frac{(\alpha-1)}{8}(B_0' - A_0')B_1. \tag{6.51}$$

We solved the above system of equations for the radion numerically, for the case of $\alpha = 1$ (the general result for arbitrary values of $\alpha$ will be given below) and we find that it has a negative value of $m_r^2$—it is a tachyon. The graph of its dependence on the ratio of warp factors between the 4-brane and the 3-brane is shown in figure 3a. If we normalize $a(0) = 1$ at the 3-brane, then in the regime where the hierarchy is large (right hand side of the graph), the radion mass squared depends on $a(\tilde{R})$ ($a$ evaluated at the 4-brane) as

$$m_r^2 \cong -20\, L^{-2}\, a(\tilde{R})^{-3/2} \qquad (\alpha = 1 \text{ case only}), \tag{6.52}$$

where we recall that $L$ is the AdS curvature radius. Since $L \sim 1/\text{TeV}$ and $a(\tilde{R})^{3/2} \cong 10^{32}$ to solve the hierarchy problem, we obtain

$$\sqrt{-m_r^2} \sim 10^{-3}\text{eV}. \tag{6.53}$$

This is well above the present Hubble scale, so we would have noticed the expansion or contraction of the extra dimension due to the change in the strength of gravity, if Eq (6.53) were true.

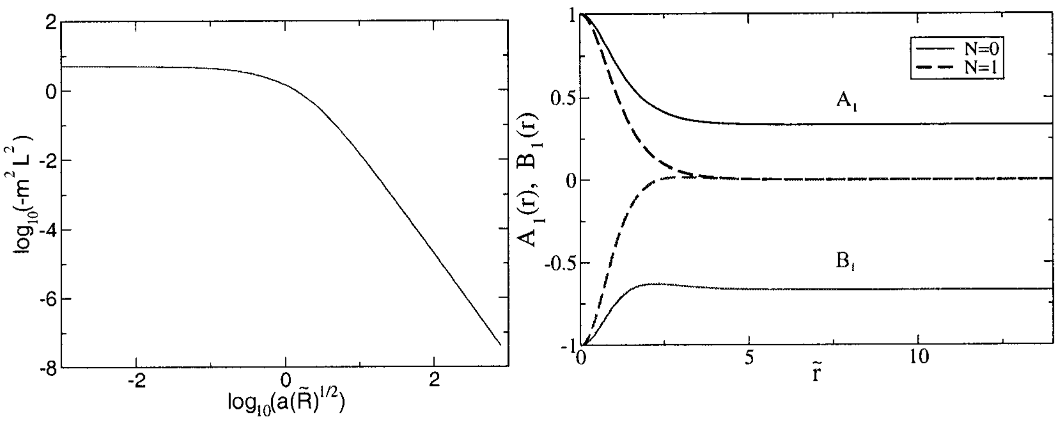



Figure 3. (a) Log of minus the radion mass squared versus log of the warp factor, for the case $\alpha = 1$. (b) Outer (solid) lines: the radion wave function for a large value of the warp factor; inner (dashed) lines: wave function of the first KK excitation of the radion.

The radion wave function in the large warp factor regime is shown in figure 3b. If the 4-brane is moved even farther away (larger warp factor), the wave function retains the same form, since it stays flat in the region of large $\tilde{r}$. Although these plots were made for the case $\alpha = 1$, we find that the wave function looks essentially the same for all values of $\alpha$. We see from its functional form that near the 3-brane $A_1 \cong -B_1$, and since $C_1$ is constrained to be $3A_1 + B_1$, therefore $C_1 \cong -2B_1$ in this region. For larger values of $\tilde{r}$ we have $A_1 = 1/3$ and $B_1 = -2/3$, so $C_1$ is extremely small throughout most of the bulk. Nevertheless its integral is nonvanishing, so the radial size of the extra dimension, which is given by $\int_0^{\tilde{R}} d\tilde{r}\, C(\tilde{r})$, changes in response to the instability, and it does so in the same sense as the size of the compact dimension, because of our choice of signs in the definitions of $B_1$ and $C_1$. That is, the instability is a simultaneous growth or shrinking of the radius together with the circumference. Either direction is a possibility, since the static solution is analogous to sitting on the top of a hill: the ball can roll down in either direction. The situation is illustrated in figure 4. Not shown there is the fact that the relative sizes of the brane directions, $x^\mu$, grow or shrink in the opposite sense relative to the extra dimensions. In the case where the extra dimensions grow, the endpoint must be the AdS soliton solution with no 4-brane, since this has been demonstrated to be the minimum energy solution which is stable [82]. In the case where they shrink, the 2-D surface presumably degenerates into a point.

In comparing the radion in this model to that of the 5-D Randall-Sundrum model, we can notice several similarities and differences. Similarly to the 5-D model [14, 15], in 6-D the radion is an admixture of the radial and brane metric components, such that oscillations of the radial size are accompanied by fluctuations in the scale factor of the 4-D universe which are 180° out of phase. But in 5-D, the radion was exactly massless in the absence of stabilization, whereas in 6-D it is a tunable parameter. Another difference is that, whereas in 5-D the radion has no tower of KK excitations, in 6-D it does. The mass gap is of order $1/L$, *i.e.,* the TeV scale. The first few eigenvalues, for large values of the warp factor, are given in table 3. One notices that these masses are systematically smaller than those of the graviton and vector modes. Thus the radion excitations would be the first signs of new physics from this model (once we have made it viable by stabilizing the radion) to appear in accelerator experiments. The wave functions of the second and third excited states are shown in figure 5, while that of the first excited state appears in figure 3b. Similarly to the spin 2 excitations, the modes above the ground state have wave functions which are exponentially strongly peaked on the TeV brane. The wave function of the radion ground state, on the other hand is relatively flat throughout the bulk. This plays an important role in its couplings, as we will discuss in the next section.

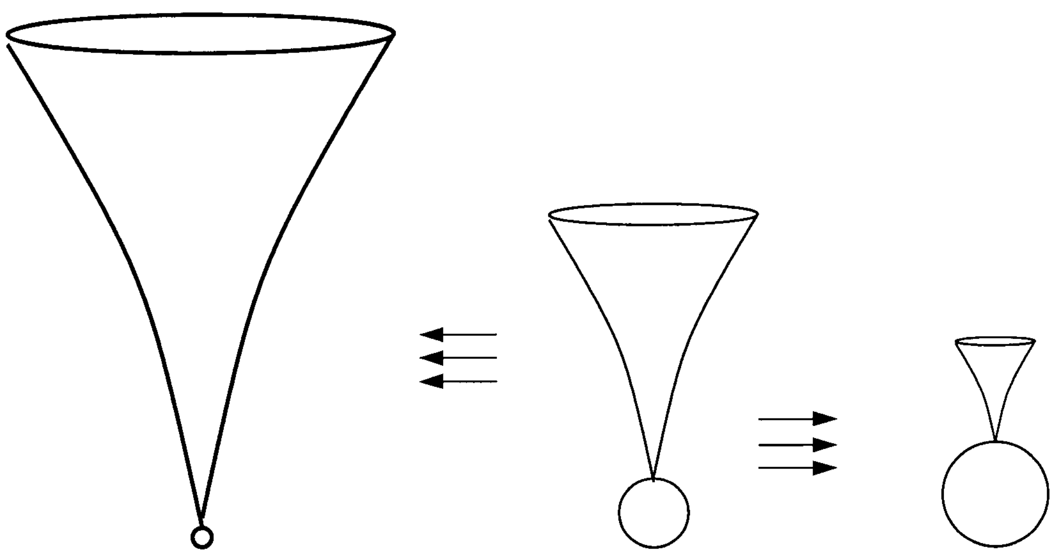


Figure 4. Illustration of the instability. A given static solution, shown in the center, is unstable toward growth (left) or shrinkage (right) of the extra dimensions (funnel), accompanied by the opposite behavior of the directions $x^{\mu}$ within the 3-brane (shown as a sphere).

| Mode Number | $m_r^2 L^2$ |
|---|---|
| 1 | 1.0188 |
| 2 | 6.1512 |
| 3 | 12.748 |
| 4 | 21.311 |
| 5 | 44.437 |
| 6 | 75.588 |

Table 3. Mass squared of the radion KK modes, in units of the AdS curvature radius, in the limit of large warp factor.

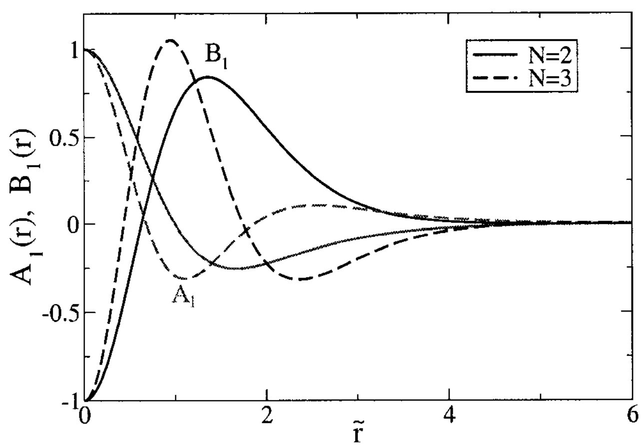



Figure 5. Solid lines: wave function of second excited state of the radion; dashed lines: the third excited state.

We can understand the preceding results for the radion ground state mass and wave function analytically, and generalize them to arbitrary values of the 4-brane stress-energy parameter $\alpha$. Toward this end, we first convert the coupled first order equations into a single second order equation. The form of the equations suggests that a natural dependent variable to consider is the linear combination $H = 3A_1 + B_1$. The variables $A_1$, $B_1$ and $C_1$

can be expressed in terms of $H$ using eqs. (6.42) and (6.43), where for later convenience we continue to show the effect of a bulk scalar field, even though we set it to zero for the present:

$$\begin{aligned}
A_1 &= \frac{1}{2B_0'}\left[-H' + (A_0' + B_0')\,H - 2\kappa^2\phi_0'\phi_1\right] \\
B_1 &= \frac{3}{2B_0'}\left[H' - \left(A_0' + \frac{1}{3}B_0'\right) H + 2\kappa^2\phi_0'\phi_1\right] \\
C_1 &= \frac{1}{2B_0'}\left[H' + (B_0' - A_0')H - 2\kappa^2\phi_0'\phi_1\right] . \qquad (6.54)
\end{aligned}$$

Substituting these into the remaining field equation (6.44), we obtain

$$\begin{aligned}
H'' &+ \frac{6A_0'^2 - 3B_0'^2 - 6A_0'B_0' - 2\kappa^2\phi_0'^{\,2}}{2B_0'}H' \\
&+ \left(\frac{(B_0' - A_0')(3A_0'^2 + 2B_0'A_0' - \kappa^2\phi_0'^{\,2})}{B_0'} + \frac{m_r^2}{a_0}\right) H \\
&= 4\kappa^2\left(\frac{\partial V}{\partial\phi} - \kappa^2\frac{\phi_0'}{B_0'}V\right)\phi_1 \ , \qquad (6.55)
\end{aligned}$$

and the boundary conditions at $\tilde{r} = \tilde{R}$ and $\tilde{r} = 0$, respectively, are

$$\begin{aligned}
\left(A_0' + \frac{1-\alpha}{4}B_0' + \frac{2\kappa^2\phi_0'^{\,2}}{3(B_0' - A_0')}\right) H' &= \\
\left((A_0' + \frac{1}{3}B_0')(A_0' + \frac{1-\alpha}{4}B_0') \right. &- \left. \frac{2}{3}\kappa^2\phi_0'^{\,2} + \frac{2B_0'm_r^2}{3(B_0' - A_0')a_0}\right) H \ , \\
H' &= A_0'H = 0 \ , \qquad (6.56)
\end{aligned}$$

The radion mass squared comes into the boundary condition because, in the process of eliminating $A_1$, $A_1'$, $B_1$ and $B_1'$ in favor of $H'$ and $H'$, it is necessary to use the bulk equation (6.44), evaluated at the 4-brane.

Now it happens that the important features of the radion ground state can be deduced from approximating the above equations by the form which they take in the asymptotic region of $\tilde{r} \sim \tilde{R}$ when $\tilde{R}$ is large. In this region, we have

$$A_0' \cong B_0' \cong -\frac{4}{5}k; \qquad B_0' - A_0' \cong -8ke^{-2k\tilde{r}} \equiv \delta A' \ , \qquad (6.57)$$

and the equations in the bulk and on the brane, respectively, simplify to:

$$H'' - \frac{3}{2}A'H' + \left(5A'\delta A' + \frac{m_r^2}{a_0}\right) H = 0 \ , \tag{6.58}$$

$$\left(\frac{\kappa^2\phi_0'^2}{A'} + \frac{3}{8}(5-\alpha)\delta A'\right) H' = \left(\frac{m_r^2}{a_0} + (\frac{5-\alpha}{2}A' - \frac{\kappa^2\phi_0'^2}{A'})\delta A'\right) H \ , \tag{6.59}$$

where now we take $A'$ to have the constant value $-4k/5$. The terms $\delta A'$ and $m_r^2/a_0$ are both of order $e^{-2k\tilde{r}}$ in the large $\tilde{r}$ region (as we will verify self-consistently), so that in the bulk equation (6.58) they can be ignored compared to the other terms. The solution in the bulk has the form

$$H(\tilde{r}) \cong c_1 + c_2 e^{-(6k/5)\tilde{r}} + \delta H, \tag{6.60}$$

where $\delta H$ represents the small effect of the parenthetical terms we have ignored. The latter give rise to a negligible effect on the bulk solution, $\delta H \ll H$. However, the small terms proportional to $\delta A'$ and $m_r^2/a_0$ *cannot* be neglected when applying the b.c. at the 4-brane. In fact, this equation, (6.59), can be used to solve for the radion mass, which in the absence of the scalar field gives

$$\begin{aligned} m_r^2 &= \frac{5-\alpha}{4} a_0\, \delta A' \left(\frac{3}{2}\frac{H'}{H} - 2A'\right)\Bigg|_{\tilde{r}=\tilde{R}} \cong (\alpha-5)\frac{20}{2^{4/5}} L^{-2} e^{-(6k/5)\tilde{R}} \\ &= (\alpha-5)\frac{5}{L^2} a_0^{-3/2}(R) + O(a_0(\tilde{R})\delta A'^2) \\ &= \frac{\alpha-5}{2}\frac{\Lambda}{a_0^{3/2}(R)} + O\left(\frac{\Lambda}{a_0^4(R)}\right) \ . \end{aligned} \tag{6.61}$$

The final expression assumes that $a_0$ is normalized to unity at the TeV brane, and uses the relation (6.3) between $\Lambda$ and $L$. (The intermediate factor of $2^{4/5}$ comes from $a_0(\tilde{R}) = \cosh^{4/5}(k\tilde{R})$.) Interestingly, the value for $m_r^2$ which we so obtain is completely insensitive to the details of $\frac{H'}{H} \sim e^{-(6k/5)\tilde{R}}$, much less $\delta H$, since all of these are much smaller than $A'$. We are therefore able to give a very accurate analytic estimate for the radion mass squared, when the warp factor is large. The small magnitude of $m_r^2$ is seen to be a direct consequence of the value of $B_0' - A_0'$ in the static solution. This expression agrees with our previous numerical results for $\alpha = 1$ (and we have also checked it numerically for other values of $\alpha$). In the limit that the 4-brane goes to infinity, so that the full AdS soliton is recovered, the radion becomes massless, but is not normalizable. Thus it does not contradict the fact that the uncut AdS soliton is a stable solution.

Interestingly, the radion mass vanishes almost exactly in the case where the anisotropy of the 4-brane stress tensor is provided by the Casimir energy and pressure of fields living on the compact extra dimension. In this case the relevant energy density scales like $L_\theta^{-5}$, *i.e.*, $\alpha = 5$, as expected from dimensional analysis. This is the unique case where no dimensionful parameter is introduced in the anisotropic part of the 4-brane stress tensor, which is the part that also controls the position of the 4-brane, and hence how large the extra dimensions are. Curiously, the mass does not vanish *exactly* when $\alpha = 5$ because of the $O(\delta A'^2)$ correction, whose coefficient turns out to be $(11 + \alpha)/8$. However, the natural size of this contribution to $m_r$ is of order $10^{-10}$ eV, which is far below experimental limits on scalar-tensor theories of gravity.

## 6.4 Radion Stabilization and Phenomenology

In this section we show how to increase the mass of the radion through using a bulk scalar field, and discuss the implications of the model for collider experiments, tests of the gravitational force, and cosmology.

### 6.4.1 Stabilization by a Bulk Scalar Field

We have found that the radion can be massive, massless, or tachyonic, depending on the value $\alpha$ which controls the dependence of the 4-brane stress-energy on its circumference. In the latter two cases ($\alpha \leq 5$), it is certainly necessary to increase the radion mass squared so that we have a stable universe, with Einstein gravity rather than scalar-tensor gravity at low energies. In the 5-D RSI model, this was achieved by Goldberger and Wise [16] by adding a bulk scalar field, whose VEV's at the two branes were constrained by potentials on the branes to take certain values. The bulk scalar then acts like a spring between the branes, whose gradient energy becomes repulsive if the branes get too close, and whose potential energy (from $m^2\phi^2$) causes attraction if the branes separate too much. We expect that the same mechanism should work in 6-D.

Scalar fields in AdS have solutions which are exponentially growing or decaying toward the ultraviolet cutoff brane. Ref. [75] studied these solutions and found the approximate behavior

$$\phi(\tilde{r}) = \phi_+ e^{\sigma_+ \tilde{r}} + \phi_- e^{\sigma_- \tilde{r}} \ , \tag{6.62}$$

where $\sigma_\pm = -k \pm \sqrt{k^2 + m^2}$. Near the 3-brane, where the space does not look like AdS, the behavior is different; $\phi(\tilde{r}) \cong \phi_0(1 + m^2\tilde{r}^2/4)$, but this will not be very important for understanding the effect of the scalar since most of the volume of the extra dimensions is near the 4-brane. For generic boundary conditions, the growing solutions dominate, and it is a good approximation to neglect the decaying ones. The main point is that the most natural configurations are ones where $\phi(0) < \phi(\tilde{R})$.

Before doing any analytic estimates, we solved the entire system of Einstein equations numerically, to find the effect of the scalar field on the radion mass. There are three kinds of corrections to consider. First, the scalar field induces a small back-reaction on the static solutions, $A_0$, $B_0$, determined by the zeroth order truncation of the Einstein equations (6.37-6.41). This effect has been analytically computed in [75]. Second, the background scalar configuration couples to the fluctuations of the metric. This arises solely through the term $\kappa^2 {\phi_0'}^2 C_1$ of the perturbed $(rr)$ component of the Einstein equations, (6.44). We will see that this is the really important effect for stabilizing the radion. The third kind of correction is from fluctuations of the scalar field, which can mix with the radion. These are governed by the perturbed scalar field equation (6.45).

$$\phi_1'' - \frac{1}{2}(4A_0' + B_0')\phi_1' - \phi_0' H' - \frac{1}{2B_0'}\frac{\partial V}{\partial \phi}\left((B_0' - A_0')H + H' - 2\kappa^2\phi_0'\phi_1\right) - \frac{\partial^2 V}{\partial \phi^2}\phi_1 + \frac{m_r^2}{a_0}\phi_1 = 0 \;, \tag{6.63}$$

where $V = \frac{1}{2}m^2\phi^2$ is the bulk potential.

Our numerical results demonstrating the stabilization of the radion are shown in figure 6. We considered scalar field configurations with $\phi = 0$ at the 3-brane and varied the value of $\phi$ at the 4-brane, showing that $m_r^2$ (in the tachyonic case $\alpha = 1$) becomes positive for sufficiently large values of $\phi(\tilde{R})$. (Treating $\phi(\tilde{R})$ as an adjustable parameter can be justified by imagining that we have stiff potentials for $\phi$ on the branes, fixing their boundary values to whatever we desire.) We checked that these results are quite insensitive to whether the fluctuations of the scalar are included. The mixing between the radion and $\phi_1$ was found to be negligible.

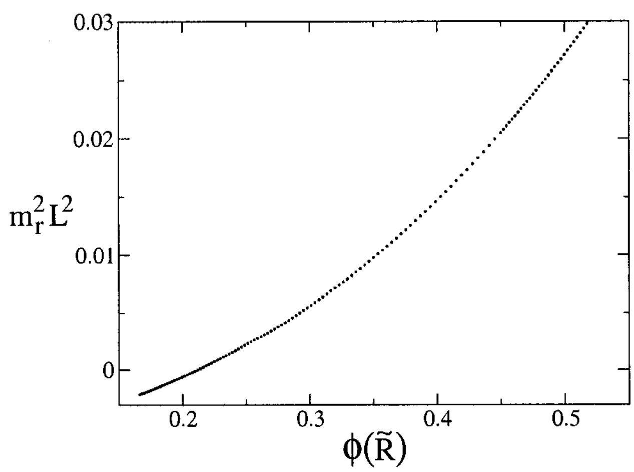



Figure 6. Dependence of $m_r^2$ on the value of $\phi$ at the 4-brane.

The behavior shown here can be easily understood by generalizing our previous derivation of the radion mass to include the effect of the scalar. The equation of motion and boundary condition for $H$, in the large-$\tilde{r}$ region, were given in (6.58-6.59). We can take $m^2$ of the scalar to be small, so that its effect can be neglected in the bulk equation and the approximate solution (6.60) is still valid. Now when we solve the b.c. for the radion mass, we obtain the previous expression plus a new term,

$$\frac{m_r^2}{a_0} = \frac{5-\alpha}{4}\delta A'\left(\frac{3}{2}\frac{H'}{H} - 2A'\right) + \frac{H'}{H}\frac{\kappa^2\phi_0'^2}{A'} , \tag{6.64}$$

whose origin can be traced to the extra term in the $(rr)$ Einstein equation. The first term is also changed by the presence of the scalar field, because of its back-reaction on the static metric. However, using the results of ref. [75] who computed this back-reaction, we find that $\delta A'$ is still of order $e^{-2k\tilde{R}}$. Therefore, since $\frac{H'}{H}$ is of order $e^{3k\tilde{r}/2}$, the new term on the r.h.s. of (6.64) is the dominant one. The fact that $\frac{H'}{H}$ has the correct sign (negative) to insure that the radion mass squared is positive is not obvious, but by numerically solving for $H(r)$ we have verified that indeed $\frac{H'}{H}(\tilde{R}) < 0$, as we show in figure 7. We thus find that for large enough values of $\phi(\tilde{R})$, the radion mass is

$$m_r \sim \frac{m^2\phi(\tilde{R})}{kM_6^2}e^{-k\tilde{R}/5} \sim \text{ MeV} , \tag{6.65}$$

independently of the details of the 4-brane stress-energy.

It is remarkable that the stabilized mass is not of order the TeV scale, as was the case in 5-D RSI [15, 16, 58]. The mass squared is suppressed by the fractional power of the warp factor left in the product $a_0\frac{H'}{H} \sim a_0^{-1/2}$. Recalling that the Planck scale hierarchy was set by $a_0^{3/2} \cong 10^{32}$, we see that the stabilized $m_r$ is suppressed by the factor $10^{16/3}$, giving $m_r \sim 10$ MeV. This is precisely the same factor by which a TeV mass particle, transported from $\tilde{r} = 0$ to $\tilde{r} = \tilde{R}$, falls short of the Planck scale, as we noted in section 2.2. Thus the smallness of the radion mass seems to be associated with the additional dilution of the strength of gravity that comes from the large extra dimension.

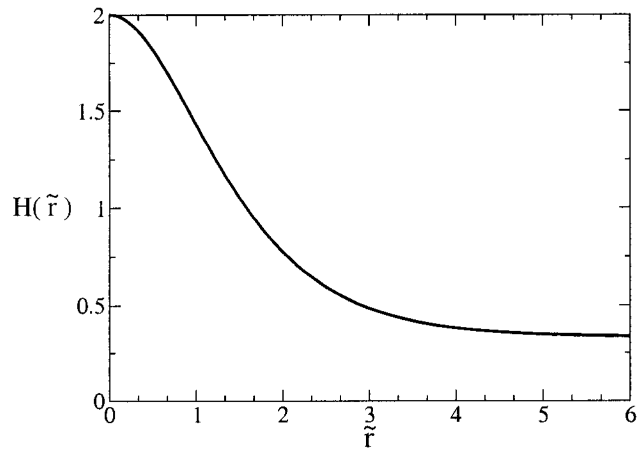



Figure 7. The radion wave function $H(r)$ for the ground state, showing that $H'/H < 0$.

In ref. [75], one advantage of having a bulk scalar field was already pointed out: because of the back-reaction of the scalar on the metric, the jump conditions at the 4-brane can generically be satisfied for large values of $\tilde{R}$, as desired for achieving a large hierarchy, without much sensitivity to the model of stress energy on the 4-brane. In particular, choices like $\alpha = 0$ (pure tension) or $\alpha = 1$ (smeared 3-brane), which by themselves could not have yielded a satisfactory value of $\tilde{R}$, become viable in the presence of the bulk scalar. The only requirement for getting sufficiently large $\tilde{R}$ is that the scalar mass should be somewhat light since $\tilde{R} \sim \frac{k}{m^2}$.
Hence one needs $k/m \sim 8$ to get the desired hierarchy. A similar relation occurs in the 5-D realization of Goldberger and Wise [16]. This requirement is quite compatible with the parameters we need for generating the radion mass, as in retrospect one would have expected.

### 6.4.2 Couplings of the Radion and Its Excitations

In the 5-D RSI model, the stabilized radion has a TeV scale mass and TeV suppressed couplings to standard model matter [13, 30]. Were the couplings of our MeV-scale radion so large, it would easily be observable in low-energy experiments and possibly affect the cooling of supernovae. Here we show that the couplings are actually Planck scale suppressed.

Computing the 4-D effective Lagrangian for the gravitational fluctuations at quadratic order, we obtain

$$\mathcal{L}_0 = \frac{1}{3\kappa^2} \int d\theta \, dr \, a_0(r) \sqrt{b_0(r)} \left[ \dot{H}^2(r,t) + 2\dot{B}_1^2(r,t) \right] + A_1(0,t) T^\mu_{\ \mu} \ , \quad (6.66)$$

where $T^\mu_{\ \mu}$ is the trace of the 3-brane stress-energy tensor, representing the standard model, $H(r,t) = H(r)\varphi_0(t)$, $B_1(r,t) = B_1(r)\varphi_0(t)$, *etc.* Here $\varphi_0(t)$ represents the 4-D ground state radion field, and $H(r)$, $B_1(r)$, $A_1(r)$ are the corresponding wave functions found in the previous section. Since they are nearly constant throughout the bulk, we can take them out of the integral and perform it to obtain

$$\mathcal{L}_0 \sim M_p^2 \dot{\varphi}_0^2 + \varphi_0 T^\mu_{\ \mu} \ . \quad (6.67)$$

This shows that the canonically normalized radion field ground state has Planck-suppressed couplings to TeV-brane matter. This differs from the behavior of the radion in the 5-D RS1 model. There, the wave function of the radion is exponentially peaked at the Planck brane, which overcomes the exponential warp factor in the measure to give $\mathcal{L}_0 \sim (\mathrm{TeV})^2 \dot{\varphi}_0^2 + \varphi_0 T^\mu_{\ \mu}$ instead. The flatness of the radion wave function in the present case accounts for its weak couplings to the TeV brane.

The KK excitations of the radion *are* exponentially peaked on the TeV brane, on the other hand. We can understand this from the asymptotic form of the bulk equation of motion (6.58); since the mass is no longer negligible, the solutions behave like $H(r) \cong c_2 e^{-6k\tilde{r}/5}$, with the constant piece $c_1$ equal to zero. The integrand of (6.66) behaves like $e^{-12k\tilde{r}/5+12k\tilde{r}/5} = O(1)$, so we obtain

$$\mathcal{L}_n \sim \frac{M_6^4}{k}\tilde{R}\,\dot{\varphi}_n^2 + \varphi_n T^\mu_{\ \mu} \qquad \rightarrow \qquad \dot{\varphi}_n^2 + M_6^{-2}\sqrt{\frac{k}{\tilde{R}}}\,\varphi_n T^\mu_{\ \mu}\,. \tag{6.68}$$

Hence the coupling of the radion excited state is suppressed only by the small factor $(M_6\tilde{R})^{1/2} \sim \sqrt{60}$ relative to the TeV scale. The radial KK gravitons have similar couplings, but larger masses (compare Tables 1 and 3), so the radion excitations would be the first signal of new physics in collider experiments. Heavy radions could be copiously produced in the $s$-channel at the LHC, through gluon-gluon fusion events due to the QCD trace anomaly contribution to $T^\mu_{\ \mu}$.

### 6.4.3 Gravity and Cosmology

Although perhaps less physically motivated, models of the 4-brane stress-energy with $\alpha > 5$ predict that the radion mass squared will be positive even without a bulk scalar, and that its magnitude is in the $10^{-3}$ eV regime. This is within the reach of Cavendish-type tests of submillimeter gravity [8], and will be even more accessible to upcoming versions of the experiment which will have improved sensitivity.

It may also be possible to achieve this situation without appealing to exotic forms of matter on the 4-brane. The radion mass will get radiative corrections from its couplings to matter. Since the radion couples to the trace of the stress energy tensor on either brane, the heaviest particles will contribute the most strongly. Considering matter which is on the TeV brane, we can estimate the size of the one-loop correction as

$$m^2_{r,1-\text{loop}} \sim \frac{\text{TeV}^4}{M_p^2}\,. \tag{6.69}$$

The numerator comes from the fact that the TeV scale is the cutoff on the 3-brane, and the heaviest particles will have masses of this order, whereas the denominator is due to the fact that the lowest mode of the radion has Planck-suppressed couplings. This argument could be upset if the 4-brane has massive particles which are much heavier than the TeV scale, since by the same argument these could apparently make the radion very heavy and presumably would destabilize the hierarchy which we have achieved. It may be necessary to assume that there are only massless particles on the 4-brane to avoid this.

On the other hand, if we allow heavy particles to exist on the 4-brane, their natural mass scale is $\sqrt{a(R)}$ TeV $\sim 10^{13}$ GeV. It is interesting that this is the right order of magnitude for generating the observed primordial density fluctuations from the simplest model of chaotic inflation. This is an advantage of the present model over the 5-D RS model, where a $\sim 10^{13}$ GeV particle would look unnaturally light were it living on the Planck brane, and of course too heavy to exist on the TeV brane.

## 6.5 Summary

We have focused on the simplest and most direct generalization to six dimensions of the 5-D Randall-Sundrum two-brane model: the AdS soliton model, with the TeV 3-brane at the center of the azimuthally symmetric extra dimensions, and a 4-brane cutting the space off at some finite radius. The model has many features in common with its 5-D predecessor: the geometry is highly warped and very close to AdS in the region far from the 3-brane, the graviton zero-mode is localized on the hidden brane, while the radial KK excitations are localized on the TeV brane and have a TeV mass gap. In both models, the radion can easily be stabilized by the Goldberger-Wise mechanism, using a bulk scalar field.

However, there are also some quite distinctive differences. The hierarchy between the Planck and weak scales, while generated mostly (2/3) by warping, is also partly (1/3) due to the exponentially large size of the compact extra dimension [87], giving it some features in common with the large extra dimension proposal. The mass scale at the 4-brane is not the Planck scale, but it is suppressed by the size of the large compact dimension to the $10^{13}$ GeV scale. There is a tower of relatively light ($\sim$ TeV) KK gravitons corresponding to this large dimension. In the absence of stabilization by a scalar field, the 6-D model requires some mildly exotic form of stress energy on the hidden brane in order to have a finite volume. The 4-brane stress tensor generically depends on the size of the extra dimension as $L_\theta^{-\alpha}$ with some model-dependent number $\alpha$.

Most of these features were already known; in the present work we computed the spectrum of metric perturbations, including the graviton and graviphoton modes, and we found the unexpected new result that the radion is not necessarily massless, but has a mass squared which depends linearly on $\alpha$ and the negative bulk cosmological constant: $m_r^2 \sim (5-\alpha)\Lambda$ TeV$/M_p$. Only for the special case of Casimir energy on the 4-brane ($\alpha = 5$) is it massless. For smaller values of $\alpha$ it is tachyonic, and the space-time is unstable. Its couplings to the TeV brane are Planck suppressed rather than TeV suppressed, due to the different behavior of its wave function relative to the 5-D case. Once stabilized by a bulk scalar field, the radion mass is not TeV scale, as in 5-D, but rather at the MeV scale. This suppression is related to the presence of the large extra dimension which does not feature in 5-D.

The 6-D model has similar phenomenology to the 5-D model, since the Kaluza-Klein excitations of the radion behave much like the ground state of the stabilized 5-D radion. However, there is a new possibility that the radion is stabilized not by the Goldberger-Wise mechanism, but by some form of stress energy on the 4-brane which has $\alpha > 5$, or perhaps by radiative corrections from standard model particles on the TeV brane. In this case the radion mass is in the milli-eV range, which is just right for being accessible in experiments which test gravity below 1 millimeter.

The latter possibility would seem to require the absence of massive particles on the 4-brane, since radiative effects there should induce much larger corrections to the radion mass. In fact it might be necessary to forbid heavy particles on the 4-brane just to maintain the large hierarchy we set out to achieve. This is a question which deserves further study. But if it is consistent to have heavy fields on the 4-brane, then the fact that their mass is naturally of order $10^{13}$ GeV is intriguing for inflation, since this is the right scale for getting density perturbations of order $10^{-5}$ in chaotic inflation.

# Chapter 7

# Reheating from Tachyon Condensation

## 7.1 Introduction

Because of the difficulty of finding direct laboratory probes of string theory, it is interesting to look for possible evidence from cosmology. Most notably, inflation may be tested more sensitively in the near future by the MAP [88] and PLANCK [89] observations of the cosmic microwave background radiation.

Recently there has been significant progress in constructing stringy inflation models which make use of naturally occuring potentials between D-branes to provide the false vacuum energy [90, 91, 92]. However, these attempts have not adequately addressed the question of how reheating occurs after inflation. In fact, there are reasons to fear that reheating may be generically difficult to achieve in D-brane inflation. For this reason, we aim to propose a generic mechanism for reheating in such models, which is qualitatively different from reheating in ordinary field-theory inflation models, and which has the hope of being fairly robust. It is based on particle production in a time-varying background, which will occur even if the background motion is not oscillatory.

Let us begin by describing the difficulty with reheating. In the simplest version of D-brane inflation, a parallel brane and antibrane begin with some separation between them in one of the extra dimensions required by string theory. Although parallel branes are supersymmetric and have no force between them, the brane-antibrane system breaks supersymmetry so that there is an attractive force and hence a nonvanishing potential energy. It is the latter which drives inflation. Once the branes have reached a critical separation, they become unstable to annihilation. The instability is described by condensation of a tachyonic mode [93]. Its low energy effective description is a field theory of a peculiar kind, whose Lagrangian has the form [94, 95][1]

$$\mathcal{L} = -\mathcal{T} e^{-T^2/a^2} F[-(\partial_\mu T)^2] \tag{7.1}$$

where

$$F(x) = \frac{4^x x \Gamma(x)^2}{2\Gamma(2x)}, \tag{7.2}$$

which is determined for the superstring from the boundary string field theory (BSFT). Here $\mathcal{T}$ is the sum of the original brane tensions and $a$ is of order the string length, $a \sim \sqrt{\alpha'}$ (the precise value to be used in the model we adopt will be discussed in section IV). The tachyon starts from the unstable maximum $T = 0$ and rolls to $T \to \infty$. This process requires an infinite amount of time, during which the tachyon fluid has an equation of state identical to that of pressureless dust as $\dot{T} \to 1$ [96, 97].

(There have been numerous recent attempts to make use of the tachyon fluid for cosmology [98], either as the inflaton or as quintessence. Although these ideas might work if one had the freedom to change the form of the tachyon potential, the action which arises from string theory is not suitable

[1]we use the metric signature $(1, -1, \ldots, -1)$

for either purpose. Since the late-time equation of state is $p = 0$, the string theory tachyon does not provide accelerated expansion in the recent universe. At early times, its potential is too steep to satisfy the requirements of inflation. Constraints on tachyon cosmology and its shortcomings have been discussed in [100] and [101].)

Returning to our description of the endpoint of D-brane inflation, the situation is similar to that of hybrid inflation [102], where the tachyon plays the role of the unstable direction in field space which allows for inflation to quickly end. The important difference is that in a normal hybrid inflation model, $T$ would have a minimum at some finite value, *e.g.*, due to a potential like $\lambda(|T|^2 - a^2)^2$, and the oscillations of $T$ around its minimum could give rise to reheating in the usual way, or the more efficient tachyonic preheating [103]. But with the exponential potential in (7.1) there can be no such oscillations. It thus appears that the universe will become immediately dominated by the cold tachyon fluid, and never resemble the big bang [101]. It is important to convert very nearly 100% of the energy stored in tachyon matter into radiation because the former redshifts more slowly than the latter. For example if we assume that the universe started initially with a temperature of at least 1 TeV for the purposes of baryogenesis, while matter domination begins at a temperature near 0.1 eV, then the ratio of energy density in tachyon matter versus that in radiation must have been no more than $10^{-13}$.

To emphasize the difficulty of getting such efficient reheating in the present situation, let us contrast the rolling tachyon with an inflaton which is decaying through its oscillations into massless fermions via a coupling $g\phi\bar{\psi}\psi$. The probability of particle production in the background $\phi = \phi_0 \sin(Mt)$ is proportional to the square of the transition matrix element $\int dt \sin(Mt) e^{2i\omega t} \sim \delta(M - 2\omega)$, where $\omega$ is the energy of the fermion or antifermion. The square of the delta function is understood in the usual fashion to be $\delta(M - 2\omega)$ times the total time; in other words, oscillations lead to a constant *rate* of particle production. By simply waiting long enough, the energy stored in the oscillations will naturally be reduced to an exponentially small level. In the case of the rolling tachyon however, $\phi$ is replaced by $T$ which has the asymptotic behavior $T \sim t$, and the whole action is multiplied by a factor $e^{-T/a}$ or $e^{-T^2/a^2}$. The corresponding transition matrix element $\int dt\, te^{-t/a+2i\omega t}$ is finite, with no delta function. As a result, it gives not a constant rate of particle production, but rather a finite number density of produced particles. It is not obvious that the energy stored in the tachyon fluid can be sufficiently reduced.

Nevertheless, it is known that the tachyon couples to massless gauge fields; one form that has been suggested for the low energy theory is the Dirac-Born-Infeld action [104, 105],[2]

$$\mathcal{L} = -\mathcal{T} e^{-T^2/a^2} \sqrt{|\det(g_{\mu\nu} + F_{\mu\nu} - \partial_\mu T \partial_\nu T)|}\,. \tag{7.3}$$

Because of its time dependence, we expect that some radiation will be produced by the rolling of the tachyon. It then becomes a quantitative question: can this effect be efficient enough to strongly deplete the energy density of the tachyon fluid, so that the universe starts out being dominated by radiation rather than cold dark matter? There is one immediate problem with this idea, however; the fact that the entire action is multiplied by the factor $e^{-T^2/a^2}$ means that the massless particles which are produced will not act like ordinary radiation [106]. Recent work has shown that these excitations have the same equation of state as the tachyon itself [107]. This is related to

[2]This form is not the same as what one derives from BSFT. However it has the right qualitative features, besides being simpler to work with. Ignoring for a moment the gauge field, it can be shown that $\sqrt{|\det(g_{\mu\nu} - \partial_\mu T \partial_\nu T)|} = \sqrt{1 - \dot{T}^2}$ for time-dependent configurations, so that the action vanishes as $\dot{T} \to 1$. The same is true for the BSFT-derived function $F(-\dot{T}^2)$ in (7.2). Moreover, both functions behave like $\sqrt{(\partial_x T)^2}$ for spatially dependent profiles in the limit that $\partial_x T \to \infty$, whose relevance will be explained in the next section.

the fact that the system is annihilating to the closed string vacuum, which does not support any open string states like the gauge fields.

An obvious way to circumvent the above difficulty is to assume that the branes annihilate not to the vacuum, but rather to branes of lower dimension [108]. This is a natural possibility since any initial fluctuations of the tachyon field where it changes sign will lead to the production of topological defects at the points where $T = 0$, via the Kibble mechanism [91]. In the effective description, the lower dimensional branes are represented by solitonic configurations of the tachyon field. A $D(p-2)$ brane is a vortex of the complex tachyon field, whereas a $D(p-1)$ brane is a kink [94, 95, 109] . These possibilities are described by the descent relations of Sen [108]. The stable descendant branes are able to support open string excitations of gauge fields, including those of the standard model. They will no longer decouple as a consequence of the rolling tachyon because the topological defect which prevents pins $T = 0$ at the origin.

In this chapter we will set up a simplified model of particle production by tachyon condensation, which we hope captures the essential features of a more realistic Lagangian. The computational method developed here should carry over straightforwardly to more complicated situations. In section II we motivate an ansatz for the space- and time-dependent tachyon background which describes condensation to a brane of one dimension lower than the initial configuration. In section III we present the solutions for a gauge field in this classical tachyon background. Section IV describes how we compute the spectrum of particles produced during the early phase of the tachyon motion, and presents numerical results. We conclude in section V with the interpretation of these results and a discussion of how the calculation can be improved in future work.

## 7.2 Tachyon background

There are two kinds of tachyonic solutions which have been described in the literature: (1) static solutions which are topological defects and represent lower dimensional branes, and (2) dynamical solutions which are spatially homogeneous and describe a cosmological fluid with vanishing pressure at late times. The mechanism we have in mind combines these two pictures by supposing that at $t = 0$ the tachyon starts from (or very near to) the unstable equilibrium configuration $T = 0$ throughout some number $d$ of extra dimensions which will be transverse to the final descendant brane. We denote the spatial coordinates of these extra dimensions by $x_1, \ldots, x_d$. Starting at $t = 0$, the tachyon starts rolling towards $T \to \infty$ for $|x_i|$ sufficiently far from the center, whereas it is pinned to $T = 0$ at $x_i = 0$. By charge conservation, there must be at least one other position where $T = 0$ also. The pair of defects represents a brane and an antibrane. For simplicity we will focus our attention on the half of the internal space near $x_i = 0$ which contains the brane, since by symmetry the physics near the antibrane will be identical, for the purposes of particle production. At any given time, this half of the extra dimensional space will contain an interior region where the defect resides, and an exterior one where the tachyon is still rolling. We might expect the tachyon profile to evolve with time similarly to fig. 1(a).

To find the real behavior of $T(t, x_i)$ we should solve the equation of motion for $T$ which follows from (7.1). This is already a difficult numerical task in itself, and its results would surely complicate our next step, which will be to solve the gauge field equation of motion in the background $T(t, x_i)$. Therefore we are going to satisfy ourselves with a simplified ansatz for the background and defer more detailed investigations to future work. Namely, we will consider a linear profile for the defect, and a purely time-dependent one in the exterior:

$$\begin{aligned} T(x) &= qx, & |x| < t/q \quad &\text{(region III)} & (7.4) \\ T(t,x) &= a\ln(\cosh(t/a))\mathrm{sgn}(x), & |x| > t/q \quad &\text{(region II)} & (7.5) \\ T &= 0, & t < 0 \quad &\text{(region I)} & (7.6) \end{aligned}$$

We have specialized to the case of a single codimension for the descendant brane ($p = 1$) and this dimension has been compactified by identifying the points $x = 2L$ and $x = -2L$, where an antikink is assumed to reside. We show only the half of the space $-L < x < L$ which surrounds the kink. A parameter $q$ describing the steepness of the tachyon kink profile has been introduced. The spacetime regions I-III are illustrated in figure 1(b).

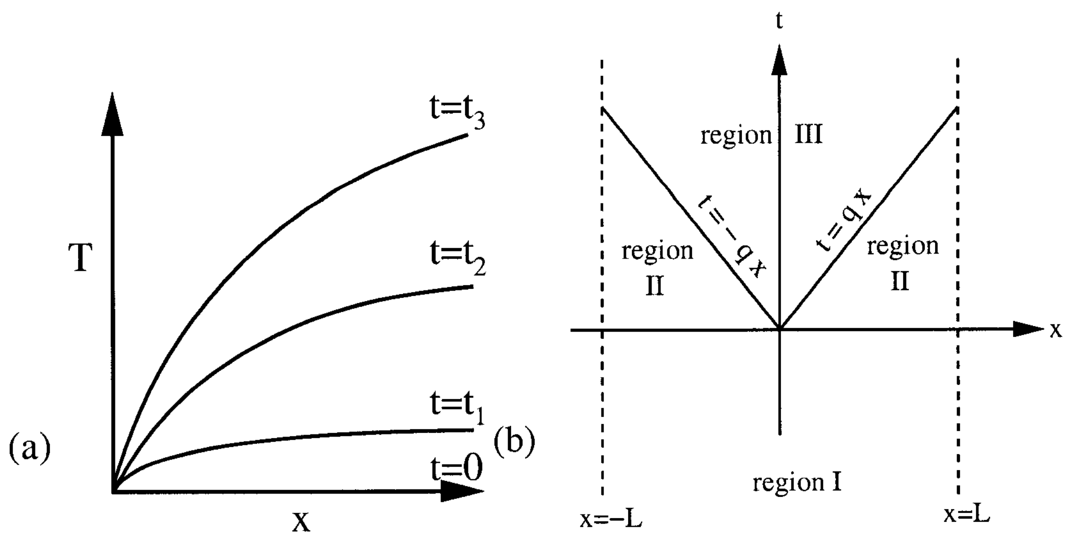



Figure 1. (a) Possible time evolution of the tachyon field $T$ during condensation. (b) Our ansatz for the tachyon evolution in time and in the half of the extra dimensional space containing the defect representing the final state brane: region III contains a linear kink, II contains the homogeneous rolling field, and I is the unstable vacuum configuration.

One may wonder to what extent the ansatz (7.4-7.6) reproduces a more realistic tachyon background. Let us first consider the rolling tachyon region (II). Eq (7.5) is in fact an exact solution not for the action (7.1) but rather for a mutilated version in which the argument of the exponential is linear:

$$\mathcal{L} = -\mathcal{T} e^{-|T|/a} \sqrt{|\det(g_{\mu\nu} - \partial_\mu T \partial_\nu T)|} \ . \tag{7.7}$$

This closely resembles the BSFT result for the effective action from the bosonic string, except for having $|T|$ rather than $T$ in the exponent. The bosonic string has an unphysical instability as $T \to -\infty$ which we artificially remove by taking the absolute value. The behavior of the solutions to (7.7) is similar to that coming from a potential with a quadratic argument, which is the appropriate one for the superstring. For example, the late-time behavior corresponding to the Lagrangian $\mathcal{L} = \mathcal{T} e^{-T^2/a^2}(1 - \dot{T}^2)^b$ is

$$T(t) \cong t + \frac{a^2(1-b)}{4t}(2b)^{1/(1-b)} e^{-t^2/(a^2(1-b))} + O(e^{-2t^2/(a^2(1-b))}) \ , \tag{7.8}$$

assuming that $b < 1$. (This can most easily be derived by computing the corresponding Hamiltonian density and using energy conservation to get a first integral of the motion.) Similarly, if we expand (7.5) for large times, we get

$$T(t) \cong t + a e^{-2t/a} + O(e^{-4t/a}) \ , \tag{7.9}$$

which resembles (7.8). Moreover if we compute the relevant factor $\sqrt{1 - \dot{T}^2}$ in the two cases (now taking $b = 1/2$),

$$\sqrt{1 - \dot{T}^2} \quad = \quad e^{-t^2/a^2} + O(e^{-2t^2/a^2}), \quad e^{-t/a} + O(e^{-2t/a}) \ , \tag{7.10}$$

respectively. In other words $\sqrt{1 - \dot{T}^2}$ is just equal to the other prefactor $e^{-t^2/a^2}$ or $e^{-t/a}$, as the case may be. This can easily be understood from the fact that the conserved Hamiltonian is $V(T)(1 - \dot{T}^2)^{-1/2}$, hence $V(T)$ and $\sqrt{1 - \dot{T}^2}$ must be proportional to each other. It will turn out that the simplified form $e^{-t/a}$ makes it easier to solve for the gauge field in the next section, which is one of our motivations for adopting the action (7.7).

Next we discuss the ansatz for the kink profile, $T = qx$, in region III. This static profile is not a solution to the equations of motion[3] except in the limit that $q \to \infty$. If we consider a generalized action of the form $\mathcal{L} = V(T)(1-T'^2)^b$, then the linear profile is a solution only if $q^2 = 1/(2b-1)$. The limit $q \to \infty$ has been discussed in [94, 95], where it was noted that the descendant brane resulting from tachyon condensation has the right tension to agree with string theory when $q \to \infty$. In this limit the descendant brane looks like a genuinely lower dimensional object, whereas it would have a nonvanishing thickness (revealed by the energy density of the profile) for finite values of $q$. However a different method, that of level-truncation [110, 111], leads to tachyon defect profiles which do have a nonzero width. In any case, there is no reason to believe that the kink thickness goes to zero immediately; it seems quite reasonable to assume that it will have some nonzero value initially, and possibly tend toward zero only as $t \to \infty$. We will assume that $q$ remains approximately constant for $0 < t < qL$ since this is the time during which particles are produced. If $q \to \infty$ at later times, it will not significantly change our conclusions.

A further issue is the shape of the spacetime boundary separating the static kink from the time-dependent solution. We have taken it to be linear, $t = q|x|$, but this implies that $T$ is not even continuous at the interface except for $t \gg a$. It might seem more reasonable to deform the

[3]An exact solution can be obtained from that of region II by analytically continuing $t \to ix$, giving

$$T = a\,\mathrm{sgn}(x)\ln\left(\frac{\cos((L-x)/a)}{\cos(L/a)}\right), \tag{7.11}$$

where we have imposed continuity of $T'$ at $x = \pm L$ and assumed that $L/a \leq \pi/2$. The solutions for the gauge field are complicated in this background, so we prefer to use the linear kink profile here.

boundary such that $e^{q|x|/a} = e^{t/a} + e^{-t/a}$. We could do so, but it would needlessly complicate the ansatz without even solving the problem of the tachyon Lagrangian being discontinuous at the interface, because the factor $\sqrt{|\det(g_{\mu\nu} - T_{\mu\nu})|} = \sqrt{1 - (\partial_\mu T)^2}$ is still not smooth: $(\partial_\mu T)^2$ changes sign as well as magnitude across the interface:

$$(\partial_\mu T)^2 \cong \begin{cases} 1 - e^{-2t/a}, & \text{region II} \\ -q^2, & \text{region III} . \end{cases} \tag{7.12}$$

This is an indication that our ansatz (7.4-7.6) is defective; the true solution should have a smooth Lagrangian density at the transition. Nevertheless, it would greatly complicate the solution for the gauge fields in the next section if we tried to smooth out this behavior. We will instead compensate for the difficulties which arise from this oversimplification in another way, as will be explained.

We found it simpler to consider condensation to a kink instead of a vortex configuration of the tachyon. The latter would be more realistic because of the fact that, assuming the parent branes are supersymmetric BPS states, a brane of only one dimension less is not, and consequently not stable, whereas a brane whose dimensionality differs by an even number *is* BPS. To reduce the dimensionality by two, the tachyon should be a complex field which condenses to a codimension-two defect, a vortex. Although we originally wanted to treat this case, it is not clear how to write the action of the complex tachyon in a way which extends to the time-dependent configurations of region II. BSFT calculations show that for a winding configuration of the form

$$T = q_1 x_1 + i q_2 x_2 \ , \tag{7.13}$$

the Lagrangian factorizes as [95]

$$\mathcal{L} \propto -F(q_1^2) F(q_2^2) \ , \tag{7.14}$$

where $F(x) = \frac{4^x x\Gamma(x)^2}{2\Gamma(2x)}$. It is not immediately obvious how to rewrite (7.14) in a Lorentz and gauge invariant way which would allow us to deduce the equations of motion for time-dependent homogeneous configurations. Explicit constructions involve the use of independent tensors like $\partial_\mu T^* \partial_\nu T$ and $\partial_\mu T \partial_\nu T$ in matching powers of derivatives, so that a compact expression like (7.7) does not seem to emerge. We leave the consideration of these complications for future work.

That being said, the kink configuration may still be physically relevant because of the possibility of descending from the original $Dp$-$\overline{Dp}$ pair in two steps, with the unstable $D(p-1)$ brane being a resonance through which the system passes on its way to the stable $D(p-2)$ endpoint [112]. The transition from $D(p-1)$ to $D(p-2)$ would be described by the formation of a kink.

Finally, let us consider the fact that the rolling phase of the tachyon field ends in a finite time within our ansatz. Fig. 1(b) shows that at late times we have eliminated the homogeneous condensate by fiat since at $t = qL$ the entire bulk has been replaced by the static kink. We don't know whether this is the actual behavior, or if the homogeneous region persists, which could be the case if $q$ grows with time:

$$T(t,x) = q(t)|x|, \qquad |x| < t/q(t) \quad \text{(region III)} \tag{7.15}$$

$$T(t,x) = a\ln(\cosh(t/a))\mathrm{sgn}(x), \qquad |x| > t/q(t) \quad \text{(region II)} \tag{7.16}$$

If $q(t)$ grows with $t$ faster than linearly, then region II survives and the tachyon fluid coexists with the final state brane at arbitrarily late times. We believe that the present calculation could give a reasonable approximation to the efficiency of particle production even in this case. The compactification length $L$ will be replaced by the size of region III at the characteristic time scale when the fast roll phase ends (*i.e,* when $\dot{T} \cong 1$ in the bulk). On the other hand, if $q(t)/t \to 0$ as $t \to \infty$, then as long as the bulk is compact, region II disappears completely. In this case it is still important to consider particle production on the brane since otherwise the energy that was stored in the rolling tachyon might go into invisible closed string modes in the bulk, namely gravitons, which would not be an acceptable form of reheating.

## 7.3 Gauge field solutions

Our aim is to find out whether the energy stored in the homogeneous tachyon fluid can be efficiently converted into radiation, so that the universe at least has a long period of radiation domination before possibly giving way to the cold dark matter of the rolling tachyon condensate. We will do this by quantizing the gauge field in the tachyon background and computing the Bogoliubov coefficients that quantify the mismatch between the vacuum states of regions I and III (see fig. 1(b)); see for example [113, 114]. That is, if we start in the vacuum state appropriate for region I, we find that it is no longer the vacuum in region III, and therefore radiation must be produced.

The first step is to find the action for the gauge fields to quadratic order in the fields. Expanding (7.7) in the tachyon background described in the previous section, we obtain

$$S = \frac{1}{2}\mathcal{T}\int dt\,dx\,d^3y \begin{cases} (\partial_t\vec{A})^2 - (\nabla_y\vec{A})^2 - (\partial_x\vec{A})^2, & \text{I} \\ (\partial_t\vec{A})^2 - e^{-2t/a}\left((\nabla_y\vec{A})^2 + (\partial_x\vec{A})^2\right), & \text{II} \\ \sqrt{1+q^2}\,e^{-q|x|/a}\left((\partial_t\vec{A})^2 - (\nabla_y\vec{A})^2 - \frac{(\partial_x\vec{A})^2}{1+q^2}\right), & \text{III} \end{cases} \tag{7.17}$$

where $y_i$ are the coordinates of the large 3 dimensions, and we have absorbed the volume of any other compact dimensions which are merely spectators into the brane tension $\mathcal{T}$. We employed the radiation gauge ($A_0 = \nabla_y\cdot\vec{A} = 0$) and projected out the extra polarization by setting $A_x = 0$, which is consistent since this state turns out to have a mass gap in region III, unlike the massless components among the large three dimensions, $\vec{A}$. (The factor $e^{-2t/a}$ in region

II should really be $(\cosh(t/a))^{-2}$, but this not an important difference, in the spirit of the other approximations we have made.) In the following we will drop the polarization indices of the gauge field and write simply $A$ instead of $\vec{A}$.

To make our analysis more tractable, we have ignored the time dependence of the metric due to the expansion of the universe in the above action. This neglect can be justified if the initial fast-roll regime of the tachyon, during which most of the particle production occurs, does not take more than approximately one Hubble time. We expect that this will be true if the string scale is somewhat below the Planck scale and strings are weakly coupled, since then $H \sim \sqrt{2L\mathcal{T}}/M_p^2 \sim g_s M_s/(2\pi M_p)$ [using the relation $g_s^2 M_p^2 = M_s^8 V/\pi(2^\pi)^6$ [92] and Eq (7.34)], whereas the time scale for the tachyon roll is of order the string scale.

In the previous section we gave detailed motivations for our choice of the ansatz for the classical tachyon background. It is worth emphasizing one further criterion: by choosing $T(t,x)$ to depend only on $t$ or $x$ in each region, we insure that the gauge field equations of motion can be solved using separation of variables, which greatly simplifies the task.

### 7.3.1 Solutions in each region

The solutions in region I are trivial since the background tachyon configuration is simply $T = 0$:

$$A_I = \frac{1}{\sqrt{4L}} \sum_m \frac{1}{\sqrt{2\omega_m}} \Big[ (a_m e^{-i\omega_m t} + a_m^\dagger e^{i\omega_m t}) \cos(k_m x) + (\tilde{a}_m e^{-i\omega_m t} + \tilde{a}_m^\dagger e^{i\omega_m t}) \sin(k_m x) \Big] \qquad (7.18)$$

where $k_m = m\pi/L$, $\omega_m^2 = \vec{p}^2 + k_m^2$, and $\vec{p}$ is the momentum in the three large dimensions. We have split the modes according to their parity in the extra dimension for later convenience.

In region II, the equation of motion for $A$ is

$$\ddot{A} + \omega_m^2 e^{-2t/a} A = 0, \tag{7.19}$$

which has the solutions

$$\begin{aligned} A_{II} &= \frac{1}{\sqrt{4L}} \sum_m \Big[ \Big( b_m J_0(a\omega_m e^{-t/a}) + c_m Y_0(a\omega_m e^{-t/a}) \Big) \cos(k_m x) \\ &+ \quad \text{same with } b_m \to \tilde{b}_m,\ c_m \to \tilde{c}_m,\ \cos(k_m x) \to \sin(k_m x) \Big] \end{aligned} \tag{7.20}$$

Near $t = 0$, these oscillate just like the region I solutions, but at large $t$ the oscillations freeze and the solutions grow linearly with time.

Region III is the important one at late times, since this is where the descendant brane and the standard model are supposed to reside. Here the equation of motion is

$$(1+q^2)\left(-\ddot{A} + \nabla_y^2 A\right) + A'' - \frac{q}{a}\mathrm{sgn}(x) A' = 0\ , \tag{7.21}$$

using primes to denote $\partial_x$. The solutions can be written as

$$A_{III} = \sum_n \frac{1}{\sqrt{2\bar{\omega}_n}} \Big[ (d_n e^{-i\bar{\omega}_n t} + d_n^\dagger e^{i\bar{\omega}_n t}) f_n(x) + (\tilde{d}_n e^{-i\bar{\omega}_n t} + \tilde{d}_n^\dagger e^{i\bar{\omega}_n t}) \tilde{f}_n(x) \Big] \tag{7.22}$$

with

$$\begin{aligned} \bar{\omega}_n &= \begin{cases} \sqrt{\vec{p}^2}, & n = 0 \\ \left[\vec{p}^2 + \frac{1}{1+q^2}\left(\frac{q^2}{4a^2} + k_n^2\right)\right]^{1/2}, & n \geq 1 \end{cases} \\ f_n(x) &= \begin{cases} N_0, & n = 0 \\ N_n e^{q|x|/2a}\left(\cos(k_n x) - \frac{q}{2k_n a}\sin(k_n |x|)\right), & n \geq 1 \end{cases} \\ N_n &= \begin{cases} \sqrt{\frac{q}{2a}}(1 - e^{-qL/a})^{-1/2}(1+q^2)^{-1/4}, & n = 0 \\ \frac{1}{\sqrt{4L}}\left(1 + \left(\frac{q}{2k_n a}\right)^2\right)^{-1/2}(1+q^2)^{-1/4}, & n \geq 1 \end{cases} \\ \tilde{f}_n(x) &= \left\{ \frac{1}{\sqrt{4L}} e^{q|x|/2a} \sin(k_n x), \right. \qquad n \geq 1 \end{aligned} \tag{7.23}$$

and $k_n = n\pi/L$.

These solutions have the desirable property, from the point of view of string theory, that there is a zero mode accompanied by a tower of heavy states [109, 112]. This is qualitatively similar to the spectrum of excited states of the open string, though we have sacrificed some of the similarity by taking the tachyon potential $e^{-|T|/a}$ instead of $e^{-T^2/a^2}$. With the latter choice one gets a more realistic spectrum of the form $\bar{\omega}_n^2 \sim \vec{p}^2 + n/a^2$, which has the correct $n$-dependence to match string theory. The disadvantage is that the solutions in region II cannot be found analytically since $\ddot{A} + \omega_m^2 e^{-2t^2/a^2} A = 0$ is not a standard differential equation. We have therefore given up some of the quantitative similarities with the real theory for the sake of being able to go as far as possible analytically.

### 7.3.2 Matching at interfaces

To complete our task, we must relate the solutions in each neighboring region to each other at the interfaces $t = 0$ and $t = q|x|$. This will impose relations between $a_m, d_m^\dagger$ and $b_m, c_m$ and between $b_m, c_m$ and $d_n, d_n^\dagger$. At $t = 0$ the procedure is straightforward; $A$ and $\partial_0 A$ must be continuous, leading to the relations

$$\begin{pmatrix} b_m \\ c_m \end{pmatrix} = \frac{\pi a \sqrt{\omega_m}}{2\sqrt{2}} \begin{pmatrix} -Y_1 - iY_0 & -Y_1 + iY_0 \\ J_1 + iJ_0 & J_1 - iJ_0 \end{pmatrix} \begin{pmatrix} a_m \\ a_m^\dagger \end{pmatrix}, \tag{7.24}$$

where the Bessel functions are all evaluated at $a\omega_m$. We used the Wronskian of $J_0$ and $Y_0$ to obtain (7.24). Notice that there is no mixing between different $m$ values at this point.

Matching at the $t = q|x|$ interface is more difficult. In this case we also demand continuity of $A$, but the fact that the prefactor $\sqrt{1 - (\partial_\mu T)^2}$ in the action is discontinuous means that derivatives of $A$ are discontinuous as well. By integrating the partial differential equation for $A$ along an infinitesimal path which crosses the interface perpendicularly, we can show that the following linear combination of derivatives must be continuous at the interface:

$$\Delta(F_t\dot{A} + qF_xA') = 0 \text{ from II to III }, \tag{7.25}$$

where $F_t$ is the coefficient of $\dot{A}^2$ and $F_x$ is that of $A'^2$ in the action. Namely, $F_t = 1$ in region II and $\sqrt{1+q^2}e^{-q|x|/a}$ in III, while $F_x = e^{-2t/a}$ in II and $e^{-q|x|/a}/\sqrt{1+q^2}$ in III. This gives the rather cumbersome matching conditions

$$\begin{aligned}\frac{1}{\sqrt{4L}}\sum_m \left(b_m J_0(a\omega_m e^{-q|x|/a}) + c_m Y_0(a\omega_m e^{-q|x|/a})\right)\cos(k_m x)& \\ = \sum_n \frac{1}{\sqrt{2\bar{\omega}_n}}\left(d_n e^{-i\bar{\omega}_n q|x|} + d_n^\dagger e^{i\bar{\omega}_n q|x|}\right) f_n(x)&\end{aligned} \tag{7.26}$$

for $A$ itself and

$$\begin{aligned}&\frac{1}{\sqrt{4L}}\sum_m \Big[\omega_m\left(b_m J_1(a\omega_m e^{-q|x|/a}) + c_m Y_1(a\omega_m e^{-q|x|/a})\right)\cos(k_m x)\\ &- \; qk_m e^{-q|x|/a}\left(b_m J_0(a\omega_m e^{-q|x|/a}) + c_m Y_0(a\omega_m e^{-q|x|/a})\right)\sin(k_m x)\Big]\\ &= \; \sum_n \frac{1}{\sqrt{2\bar{\omega}_n}}\Big[-i\omega_n\sqrt{1+q^2}\left(d_n e^{-i\bar{\omega}_n q|x|} - d_n^\dagger e^{i\bar{\omega}_n q|x|}\right) f_n(x)\\ &+ \; \frac{q}{\sqrt{1+q^2}}\left(d_n e^{-i\bar{\omega}_n q|x|} + d_n^\dagger e^{i\bar{\omega}_n q|x|}\right) f_n'(x)\Big]\end{aligned} \tag{7.27}$$

for the derivatives of $A$. Similar conditions hold among the odd parity modes $[a, d \to \tilde{a}, \tilde{d},\ f_n \to \tilde{f}_n,\ \cos(k_m x) \to \sin(k_m x)]$, which do not mix with those of even parity.

### 7.3.3 Solution of matching conditions

The technical challenge is now to solve for $d_n, d_n^\dagger$ in terms of $b_n, c_n$. Normally this would be done by taking the inner product of each equation (7.26, 7.27) with some function which is orthogonal to all but one of the functions multiplying $d_n$ or $d_n^\dagger$. But because of the diagonal nature of the II/III interface, the latter functions are *products* of orthogonal functions, which are no longer orthogonal. This makes it impossible to solve for $d_n, d_n^\dagger$ analytically.

We should therefore choose some set of basis functions $g_i(x)$ and integrate them against the matching conditions (7.26-7.27) to transform the latter into the discrete form

$$\sum_n L_{in} \begin{pmatrix} d_n \\ d_n^\dagger \end{pmatrix} = \sum_m R_{im} \begin{pmatrix} b_m \\ c_m \end{pmatrix} . \tag{7.28}$$

We were not able to identify any set of $g_i(x)$ that makes the matrix $L_{in}$ even approximately diagonal (which would facilitate an analytic approximation), making a numerical solution necessary. After some experimentation, one realizes that the most efficient way to do this is to discretize the system on a spatial lattice at positions $x_i$, so that $g_i(x) = \delta(x - x_i)$. This allows us to compute the matrices $L_{in}$ and $R_{im}$ without having to perform any integrals. An ultraviolet cutoff is thus introduced. Let $i = 0, \ldots, N$ so that $x_i = iL/N$. It makes sense to let the mode number of the spatial eigenfunctions $f_n(x)$ also range from 0 to $N$; then (7.28) gives exactly as many equations as unknowns. The system can be solved by numerically inverting the matrix $L_{in}$ [86]. The results are presented in the next section.

## 7.4 Particle Production

Our solutions for the gauge field in regions I and III are normalized so that $a_m$, $a_m^\dagger$, $d_m$, $d_m^\dagger$ are the correct creation and annihilation operators for particles in the distant past and future once the gauge field is quantized. We assume the universe starts in the vacuum state $a_n|0\rangle = 0$. But this state is not annihilated by $d_n$, since the latter is a superposition of $a$, $a^\dagger$ determined by the Bogoliubov coefficients $\alpha$ and $\beta$,

$$\begin{pmatrix} d_n \\ d_n^\dagger \end{pmatrix} = \sum_m \begin{pmatrix} \alpha_{nm} & \beta_{nm} \\ \beta_{nm}^* & \alpha_{nm}^* \end{pmatrix} \begin{pmatrix} a_m \\ a_m^\dagger \end{pmatrix} . \tag{7.29}$$

Therefore observers in the future will see a spectrum of particles in the final state given by

$$\mathcal{N}_n = \langle 0|d_n^\dagger d_n|0\rangle = \sum_m |\beta_{nm}|^2 \, . \tag{7.30}$$

In this section the numerical results for $\mathcal{N}_n$ and the total energy density of produced radiation will be presented.

### 7.4.1 UV sensitivity

Ideally one should obtain convergent results in the limit that $N \to \infty$, where we recall that $N$ is the number of sites of the spatial lattice in the extra dimension, introduced in the previous section. However we do not observe this from our numerical results; rather there is a steady growth with $N$. Our conjecture is that this is related to the discontinuity in the action which arises from the sign change in $(\partial_\mu T)^2$ across the II/III interface. In a simpler situation where particles are produced due to a (spatially constant) time-varying background, the spectrum is proportional to the square of the Fourier transform of the background. If the latter has sharp features, the spectrum falls only as a power of energy, whereas a smooth background leads to exponential suppression of high energies. For example, the Fourier transform of $e^{-t\Lambda}$ times a step function of time is $(\Lambda + i\omega)^{-1}$ whereas the Fourier transform of $(1 + t^2\Lambda^2)^{-1}$ is proportional to $e^{-|\omega|/\Lambda}$. In the former case, summing over high frequency modes could lead to nonconvergent results. We expect the high frequency contributions to be suppressed by a factor like $e^{-|\omega|/\Lambda}$ if we had a more realistic tachyon profile whose derivative changed smoothly over a time $1/\Lambda$. We have not yet found a way of altering $T(t, x)$ to incorporate this behavior while still allowing us to solve for $A(t, x)$ analytically.

To remove sensitivity to the lattice spacing, we therefore try to model the expected effects of smoothing out the background tachyon solution by inserting a convergence factor by hand, so that (7.28) becomes

$$\sum_n L_{in} \begin{pmatrix} d_n \\ d_n^\dagger \end{pmatrix} = \sum_m R_{im}\, e^{-\omega_m/\Lambda} \begin{pmatrix} b_m \\ c_m \end{pmatrix} . \tag{7.31}$$

Once this is done, the limit $N \to \infty$ is well-behaved. We have thus essentially traded the original ultraviolet cutoff $N/L$ for a new one, $\Lambda$, whose physical meaning is more transparent. Figure 2(a) shows the dependence on $N$ of the low-momentum zero-mode production, $\mathcal{N}_0(p_0)$, at $p_0 = 0.05/a$, for several values of $\Lambda$ and $q$ (recall that the latter parameter determines the slope of the interface between spacetime regions II and III). The convergence with $N$ is faster for smaller values of $q$; numerical limitations therefore prevent us from accurately studying large values of $q$. Fig. 2(b) shows the dependence of zero-mode production on moderate values of $q$; large values of $q$ at fixed $N$ give exponentially increasing results due to the term $Y_1(a\omega_m e^{-qx/a})$ in (7.27), but these nevertheless become well-behaved again as $N$ is increased to sufficiently large values. On physical grounds one might anyway expect $q \leq 1$ due to the finite speed of propagation of the kink.

### 7.4.2 Spectra

We note that each state in region III is distinguished not only by its mode number $n$, but also its momentum $\vec{p}$ in the large dimensions, which we have until now suppressed except for its appearance in the energies $\bar{\omega}_n$, Eq (7.23). In figure 3(a) we show the dependence of $\mathcal{N}_n(p)$ on these two variables. The spectrum of zero modes falls monotonically with $p$ as expected. Although the nonzero mode spectra temporarily rise with $p$, their contributions to the total energy density are smaller than that of the zero mode. We have taken $q = 1$ and $\Lambda = 1/a$ for the parameters controlling the steepness of the kink in spacetime and the suddenness with which it forms. The dependence on the size of the bulk $L$ can be seen in fig. 3(b).

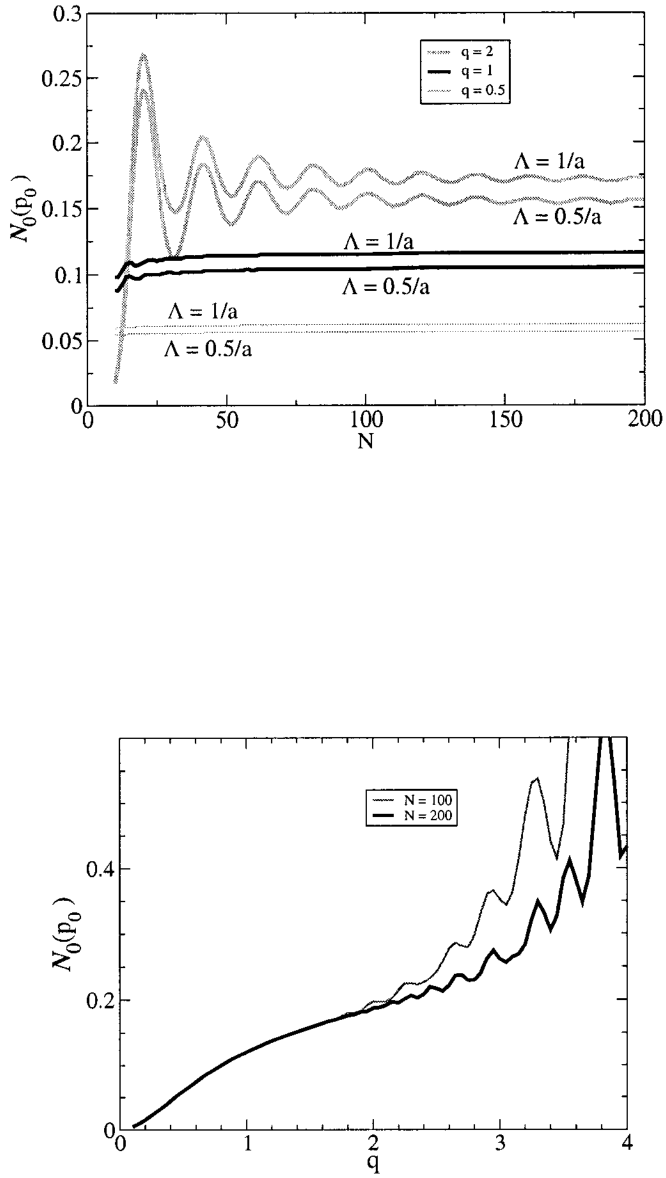
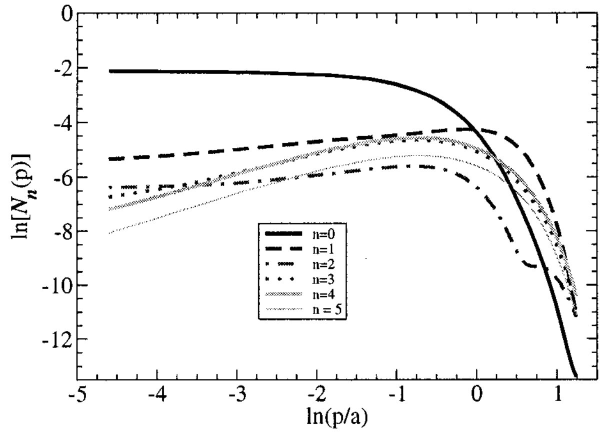



Figure 2. (a) Dependence of $\mathcal{N}_0(p_0)$ on $N$ for $p_0 = 0.05/a$, $L = 2a$, $\Lambda = 0.5/a,\ 1/a$ and $q = 0.5,\ 1,\ 2$. b) Dependence of $\mathcal{N}_0(p_0)$ on $q$ for $N = 100,\ 200$, at $p_0 = 0.01/a$, $L = 2a$ and $\Lambda = 1/a$. Physically meaningful results correspond to the $N \to \infty$ limit.



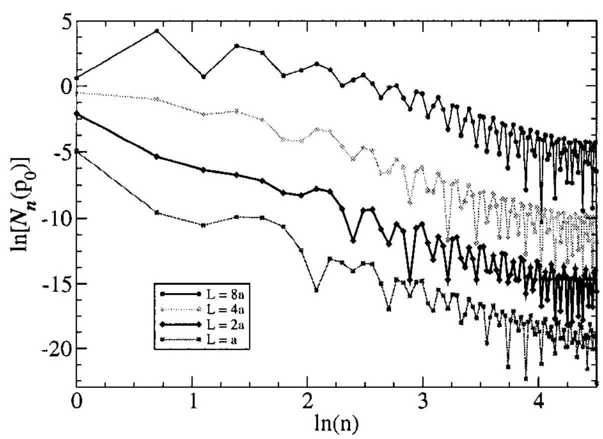




Figure 3.(a) Momentum dependence of the spectrum of produced particles, $\mathcal{N}_n(p)$ for the string excitation quantum numbers $n = 0, \ldots, 5$. (b) $\mathcal{N}_n(p)$ as a function of $n$ for $L = 8a,\ 4a,\ 2a,\ a$ and $p = p_0 \equiv 0.01/a$. Both graphs are for $q = 1$, $\Lambda = \frac{1}{a}$.

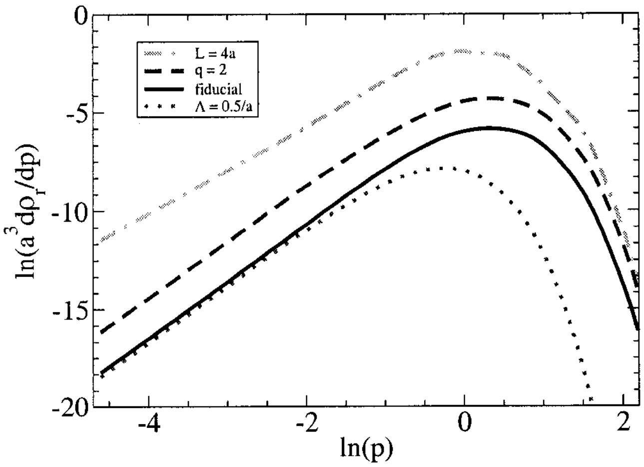



Figure 4. Differential energy density of produced radiation as a function of $p$. Fiducial values of parameters are $L = a$, $\Lambda = 1/a$ and $q = 1$.

Here and in the remainder we have made the simplifying approximation of ignoring the odd-parity modes and keeping only the even ones, which include the dominant zero mode. We expect that this gives a slight underestimate of the actual efficiency of particle production, by no more than a factor of order unity.

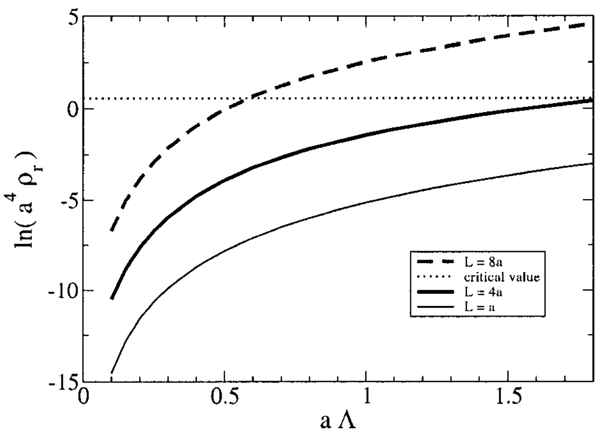
To find the total energy density of produced radiation, we should sum over both $n$ and $p$:

$$\rho_r = \int dp \frac{d\rho_r}{dp} \equiv \sum_n \int \frac{d^3p}{(2\pi)^3} \mathcal{N}_n(p) \ . \tag{7.32}$$

The heavier modes are counted because they will presumably decay very quickly into massless standard model particles. In figure 4 we show the differential energy density, $\frac{d\rho_r}{dp}$, as a function of momentum, for several values of the other parameters.

### 7.4.3 Energy density produced

We turn now to the main results, the total energy density $\rho_r$ of produced radiation, obtained from integrating Eq (7.32). In figures 5(a-c) we graph the dimensionless combination $\ln(a^4\rho_r)$ as a function of the main unknown parameters, $q$, $\Lambda$ and $L$. It is clearly an increasing function of all three parameters. The "critical value" shown in these figures, which we would like $\rho_r$ to exceed, is derived in the next subsection. We will argue that this is the value at which reheating starts to significantly deplete the energy stored in the rolling tachyon fluid.



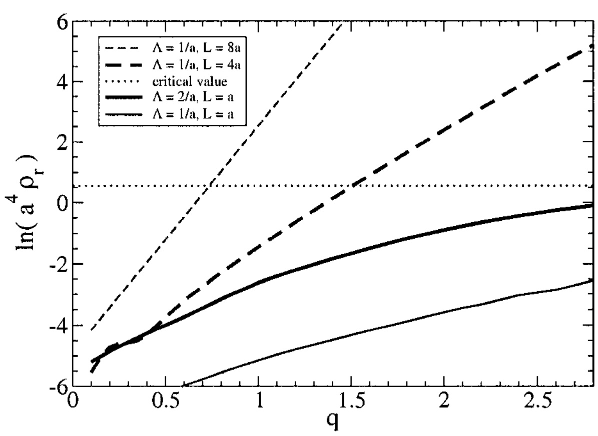




Figure 5.(a) $\ln(a^4\rho_r)$ versus $a\Lambda$ for $L = a, 4a, 8a$. (b) $\ln(a^4\rho_r)$ versus $q$ for several values of $L$ and $\Lambda$.

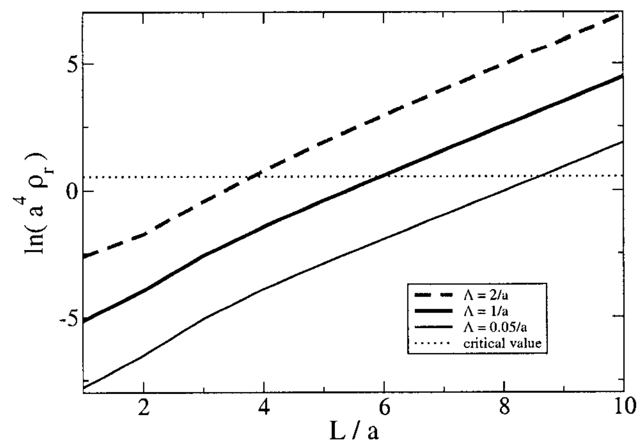



Figure 5(c). $\ln(a^4\rho_r)$ versus $L/a$ for $\Lambda = 2/a$, $1/a$ and $0.5/a$, with $q = 1$.

### 7.4.4 Efficiency of reheating

Our goal is to see if the produced $\rho_r$ is a large enough fraction of the energy density which is available from the homogeneous rolling tachyon fluid, $\rho_T$. We thus need to specify the value of $\rho_T$ which is predicted by string theory. In terms of the effective tension of the original brane and the size of the extra dimension,

$$\rho_T = 2L\mathcal{T} \ . \tag{7.33}$$

By the effective tension, $\mathcal{T}$, we mean the energy density in the $(4+1)$-dimensional spacetime which includes $x$. If we started from a nonBPS D$p$-brane, then $\mathcal{T} = \sqrt{2}\mathcal{T}_p V$ where $V$ is the volume of compact dimensions within the brane excluding $x$, and $\mathcal{T}_p$ is the tension of a BPS D$p$-brane,

$$\mathcal{T}_p = \frac{1}{g_s}(2\pi)^{-p}(\sqrt{\alpha'})^{-(p+1)} \ . \tag{7.34}$$

Therefore $\rho_T$ depends on $g_s$, $V$, $L$ and $\alpha'$. However, not all of these are independent; they are related to the fine structure constant of the gauge coupling evaluated at the string scale $M_s = (\alpha')^{-1/2}$ [92]:

$$\alpha(M_s) = \frac{g_s(2\pi\sqrt{\alpha'})^{p-3}}{2(2LV)} \ . \tag{7.35}$$

Interestingly, if we use this to eliminate the $V$ dependence from (7.33) we find that the dependence on the string coupling and on $L$ is also gone:

$$\rho_T = \frac{M_s^4}{\sqrt{2}(2\pi)^3\alpha(M_s)} \ . \tag{7.36}$$

To find out how much energy is available for reheating, we must also consider the descendant brane's 4-D energy density, $\rho_f$. This is obtained by integrating over the extra dimension in (7.7) for the kink profile $T = qx$ and taking the limit $q \to \infty$. The result is simply $\rho_f = 2a\mathcal{T}$. Therefore the excess energy density which can be used for reheating is

$$\Delta\rho = \rho_T - \rho_f = \rho_T\left(1 - \frac{a}{L}\right) \ . \tag{7.37}$$

Hence we should demand that $L > a$ to get any reheating at all.

In order to compare $\Delta\rho$ to the energy density of produced radiation, we need to know the parameter $a$ in terms of the string length. In our simplified model of tachyon condensation, this can be determined by demanding that the tension of the descendant $(p-1)$-brane match the string theoretic value for a BPS state, given by (7.34) with $p \to p-1$. In the limit $q \to \infty$, the ratio of the initial and final brane tensions in our field theory model is $(\lim_{q\to\infty}\sqrt{1+q^2}\int_{-\infty}^{\infty} e^{-q|x|/a}dx)^{-1} = 1/2a$, whereas the string theoretic value is $\sqrt{2}M_s/(2\pi)$. We therefore obtain

$$a = \pi\sqrt{\alpha'/2} = \frac{\pi}{\sqrt{2}M_s} \ . \tag{7.38}$$

Now we can quantify the efficiency of reheating. Our results for $\rho_r$ are expressed in units of $a^{-4}$. Using (7.38) we can convert the available energy density $\Delta\rho$ into the same units to find the critical value of $\rho_r$, call it $\rho_c$, for which the conversion into radiation would be 100% efficient:

$$\rho_c = \frac{\pi}{32\sqrt{2}\,\alpha(M_s)}a^{-4} \cong 1.7a^{-4} \ . \tag{7.39}$$

We take the fine structure constant to be 1/25, and we omit the factor $(1 - a/L)$ since it would require fine tuning of $L$ to make it very relevant.

The figure of merit for reheating is therefore $\rho_r/\rho_c$. The critical value $\rho_c$ appears as a dotted line in figures 5(a-c). We see that for large enough values of $\Lambda$, $q$, or $L$, reheating can be efficient.

## 7.5 Conclusion

We have made a case for reheating the universe after brane-antibrane inflation by production of massless gauge particles, due to their coupling to the fast-rolling tachyon field which describes the instability of the initial state. Our results are encouraging, indicating that if the extra dimensions within the original branes but transverse to the final one are large enough compared to the string length scale $\sqrt{\alpha'}$, reheating can be efficient enough so that radiation dominates over cold dark tachyon matter. Depending on details of the tachyon background and the compactification, an extra dimension of size $\sim 10\sqrt{\alpha'}$ could be sufficient. (For example, with $L = 4a$ in an orbifolded compactification, the size of the extra dimension would be $4\pi/\sqrt{2}M_s^{-1} \cong 9\sqrt{\alpha'}$). In the present analysis we counted only a single polarization of one $U(1)$ gauge boson and ignored the odd parity modes in the extra dimension, so the real situation could be less constrained.

The calculation is complicated, and we have made it tractable by invoking a number of simplifying assumptions. We considered formation of a kink in the tachyon field rather than a vortex. We used a simplified version of the tachyon action rather than the one which has been derived in BSFT. We used an ansatz for the background tachyon field which is a good approximation to the actual solutions in the regions where it depends only on space (the kink) or on time (the homogeneous roll), but we do not know how suddenly it makes the transition between these two regimes, hence we parameterized this uncertainty by introducing a cutoff $\Lambda$ on the tachyon field's derivative. We have also left the slope $q$ of the tachyon kink profile $T = qx$, during kink formation, as a free parameter. Moreover, we have ignored the expansion of the universe, which is only correct if the brunt of the tachyon roll completes in a time not much exceeding the Hubble time. We have also ignored the back-reaction of the produced particles on the tachyon background, so our criterion of requiring the produced energy density to meet or exceed that which is initially available is a crude one.

To do a better job, more numerical computations will probably be necessary since it is hard to solve the gauge field equations of motion in a tachyon background that depends on both space and time. One check that might be carried out relatively easily is to numerically obtain the time- and space-dependent solution $T(x,t)$ for the more realistic action and compare its features to those we have assumed. This is in progress. Very recent work on how to generate time-dependent tachyon solutions in ref. [115] might prove to be helpful here.

Finally, in the related work [116], the authors assume that there is a constant rate of decay of the tachyon fluid into radiation. We have presented arguments to the contrary. If such an assumption could be justified, it would greatly enhance the viability of inflation via brane-antibrane annihilation.

# Chapter 8

# Conclusions

In this thesis the cosmological implications of many brane world scenarios and the tachyon were studied.

In chapter 2, we pointed out that the potential of Goldberger and Wise for stabilizing the distance between two 3-branes, separated from each other along an extra dimension with a warp factor, has a metastable minimum when the branes are infinitely separated. The classical evolution of the radion (brane separation) will place it in this false minimum for generic initial conditions. In particular, inflation could do this if the expansion rate is sufficiently large. We presented a simplified version of the Goldberger-Wise mechanism in which the radion potential can be computed exactly, and we calculated the rate of thermal transitions to the true minimum, showing that model parameters can be chosen to ensure that the universe reaches the desired final state. Finiteness of bulk scalar field brane potentials can have an important impact on the nucleation rate, and it can also significantly increase the predicted mass of the radion.

In chapter 3, we presented the 5-dimensional cosmological solutions in the Randall-Sundrum warped compactification scenario with the Goldberger-Wise scalar field. The back-reaction of the scalar field on the metric was taken into account. Matter on the Planck and TeV branes is treated perturbatively, to first order. We identified the appropriate gauge-invariant degrees of freedom, and showed that the perturbations in the bulk scalar can be gauged away. We confirmed previous [15, 30], less exact computations of the shift in the radius of the extra dimension induced by matter. We pointed out that the physical mass scales on the TeV brane may have changed significantly since the electroweak epoch due to cosmological expansion, independently of the details of radius stabilization.

In chapter 4, we presented a possible explanation for the smallness of the observed cosmological constant using a variant of the RSI mechanism with a bulk scalar field. In contrast to RSI, we imagined that we are living on the positive tension Planck brane, or on a zero-tension TeV brane. In our model there were two solutions for the scalar field in the bulk and the corresponding brane separations, one of which is tuned to have zero cosmological constant. We showed that in the other solution, which is a false vacuum state, the 4-D cosmological constant can be naturally small, due to exponential suppression by the warp factor. The radion is in the milli-eV mass range, and if we live on a TeV brane its couplings are large enough that it can measurably alter the gravitational force at submillimeter distances. In this case the Kaluza-Klein excitations of the graviton can also contribute to submillimeter deviations from Newtonian gravity, and we have in addition the collider phenomenology of the usual TeV-scale radion.

In chapter 5, we derived a simple no-go theorem relating to self-tuning solutions to the cosmological constant for observers on a brane, which rely on a singularity in an extra dimension. The theorem showed that it is impossible to shield the singularity from the brane by a horizon, unless the positive energy condition ($\rho + p \geq 0$) is violated in the bulk or on the brane. The result holds regardless of the kinds of fields which are introduced in the bulk or on the brane, whether $Z_2$ symmetry is imposed at the brane, or whether higher derivative terms of the Gauss-Bonnet form are added to the gravitational part of the action. However, the no-go theorem can be evaded if the three-brane has spatial curvature. We discussed explicit realizations of such solutions which have both self-tuning and a horizon shielding the singularity.

In chapter 6, we studied a generalization of the Randall-Sundrum mechanism for generating the weak/Planck hierarchy, which uses two rather than one warped extra dimension, and which requires no negative tension branes. A 4-brane with one exponentially large compact dimension plays the role of the Planck brane. We investigated the dynamical stability with respect to graviton, graviphoton and radion modes. The radion was shown to have a tachyonic instability for certain models of the 4-brane stress-energy, while it is stable in others, and massless in a special case. If stable, its mass is in the milli-electron volt range, for parameters of the model which solve the hierarchy problem. The radion was shown to couple to matter with gravitational strength, so that it is potentially detectable by millimeter-range gravity experiments. The radion mass can be increased using a bulk scalar field in the manner of Goldberger and Wise, but only to order MeV, due to the effect of the large extra dimension. The model predicts a natural scale of $10^{13}$ GeV on the 4-brane, making it a natural setting for inflation from the ultraviolet brane.

In chapter 7, we argued that it is possible to reheat the universe after inflation driven by D-brane annihilation, due to the coupling of massless fields to the time-dependent tachyon condensate which describes the annihilation process. This mechanism can work if the original branes annihilate to a stable brane containing the standard model. Given reasonable assumptions about the shape of the tachyon background configuration and the size of the relevant extra dimension, the reheating can be efficient enough to overcome the problem of the universe being perpetually dominated by cold dark tachyon matter. In particular, reheating is most efficient when the final brane codimension is large, and when the derivatives of the tachyon background are large.

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