# Essays in Risk Modeling, Asset Pricing and Network Measurement in Finance

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# Contents

# Abstract

Modelling financial interconnections and forecasting extreme losses are crucial for risk management in financial markets. This thesis studies multivariate risk spillovers at the high-dimensional market network level, as well as univariate extreme risk modelling at the asset level. The first chapter proposes a novel time series econometric method to measure high-dimensional directed and weighted market network structures. Direct and spillover effects at different horizons, between nodes and between groups, are measured in a unified framework. Using a similar network measurement framework, the second chapter investigates the relationship between stock illiquidity spillovers and the cross-section of expected returns. I find that central industries in illiquidity transmission networks earn higher average stock returns (around 4% per year) than other industries. The third chapter proposes a new Dynamic Stable GARCH model, which involves the use of stable distribution with time-dependent tail parameters to model and forecast tail risks in an extremely high volatility environment. We can differentiate extreme risks from normal market fluctuations with this model.

La modélisation des interconnexions financières et la prévision des pertes extrêmes sont essentielles pour la gestion des risques sur les marchés financiers. Cette thèse étudie les retombées multivariées du risque á des niveaux de réseau de marché à haute dimension, ainsi que la modélisation de risque extrême univariée au niveau des actifs. Le premier chapitre propose une nouvelle méthode économétrique en série temporelle pour mesurer les structures de réseaux de marché dirigées et pondérées de grande dimension. Les effets directs et indirects à différents horizons, entre nuds et entre groupes, sont mesurés dans un cadre unifié. En utilisant un cadre de mesure de réseau semblable, le deuxième chapitre étudie la relation entre les retombées de l'illiquidité des actions et la section transversale des rendements attendus. Je constate que les industries centrales des réseaux de transmission d'illiquidité gagnent un rendement moyen plus élevé (environ 4% par an) par rapport aux autres industries. Le troisième chapitre propose un nouveau modèle GARCH Dynamic Stable qui implique l'utilisation d'une distribution stable avec des paramètres de queue dépendant du temps pour modéliser et prévoir les risques de queue dans un environnement de volatilité extrêmement élevée. Nous pouvons différencier les risques extrêmes des fluctuations normales du marché par ce modèle.

# Dedication and Acknowledgements

This thesis is dedicated to my parents, who raise me and teach me to think independently. They always respect and support every decision I made. It is also dedicated to my wife, Miao, and my son, William. This thesis would not be finished without their love and emotional support.

I am extremely indebted to my supervisor, Jean-Marie Dufour. Throughout my PhD study at McGill he is an excellent mentor and teacher. This thesis would not have happened without his support. I would like to thank John Galbraith and Victoria Zinde-Walsh for their numerous comments on this thesis, and thank Francisco Alvarez-Cuadrado, Ruslan Goyenko, Laura Lasio and Theodore Papageorgiou for their advice, guidance and support. I would also like to thank my friend and colleague, Chan Ying Tung, for his valuable suggestions, helpful comments and our many interesting discussions about economics, statistics, society and theology.

# Preface and Contribution of Authors

This thesis contains three papers. The first chapter of this thesis is written in collaboration with my supervisor Jean-Marie Dufour (Professor of Economics at McGill University). He contributed to revising the manuscript and discussions of the model. I contributed to coming up with the research idea, working with the data, estimating the model and writing the manuscript.

All three studies are original scholarship and distinct contributions to Financial Econometrics, Risk Management and Empirical Asset Pricing.

# Introduction

Since the financial crisis of 2007-09, academic researchers and financial regulators have a growing interest in investigating interconnections in financial markets and in revisiting extreme losses prediction methods. This thesis is composed by three papers and studies both multivariate risk spillovers at the high-dimensional market network level and univariate extreme risk modelling at the asset level.

Financial market components (markets, banks, products, etc.) are connected with each other, and these interconnections can be represented by financial network structures. However, many network structures are latent and not readily available in databases. For instance, the relationships between entities (e.g., detailed information on intra-bank asset and liability exposures) in a financial network are usually unknown. To empirically study a market network from financial data, we need an econometric measurement framework to identify and quantify the underlying network structure. Chapter 1 proposes a novel network econometric measurement framework to better measure directed and weighted network structures using financial time series data in a high-dimensional context. Direct and spillover effects, between nodes and between groups, are measured in a unified framework. Causality at different horizons in the network is measured through a causality measure at different horizons. With this framework at hand, We provide our estimated market networks with new econometric connectedness measures. The market systemic risk that is quantified by our connectedness measures has an intrinsic network foundation. We investigate the S&P 100 implied volatility network in the US stock market to illustrate the usefulness of our method in network analysis. We find that 7 out of the 10 most influential firms in the S&P 100 belong to the financial sector. Top investment banks (Morgan Stanley, Goldman Sachs and Bank of America) have the greatest influence in the financial sector. Market connectedness is especially strong during the recent global financial crisis, and this is mainly due to the high connectedness within the financial sector and the spillovers from the financial sector to other sectors.

In Chapter 2, I estimate the illiquidity interconnections among different industries in the US stock markets and investigate the relationship between stock illiquidity spillovers and the cross-section of expected returns. I study industry-level illiquidity spillovers in a directed network that describes the interconnections among stocks' bid-ask spreads, where the interconnections are latent and are estimated by a Granger-type measure. In the directed illiquidity transmission network, the illiquidity of high sensitive centrality (SC) industries, i.e., those active at receiving illiquidity from others, as well as high influential centrality (IC) industries, i.e., those active at transferring illiquidity to others, tends to covary with that of their neighbours and neighbours' neighbours across different horizons due to illiquidity spillovers. As a result, long run returns of the portfolios that contain stocks of central (high SC or high IC) industries may be more volatile because of weak diversification of the liquidity risk across different horizons. Thus, investors would require compensations for holding these central stocks. I confirm this conjecture and find that central industries in illiquidity transmission networks do earn higher average stock returns (around 4% per year) than other industries. Market-beta, size, book-to-market, momentum, liquidity and idiosyncratic volatility effects cannot account for the high average return earned by central industries.

Chapter 3 studies extreme risk measurement and prediction for univariate time series. I propose a new Dynamic Stable GARCH model, which involves the use of stable distribution with time-dependent tail parameters to model and forecast tail risks in an extremely high volatility environment. We can differentiate extreme risks from normal market fluctuations with this model. Asymptotic inference methods in high volatility environments are unreliable, as standard regularity conditions may not apply or may hold only weakly. I apply a Monte Carlo test inference procedure to construct the confidence interval of the tail parameter. Empirical analysis on the Nikkei 225 index shows that the Dynamic Stable GARCH model provides the best in-sample and out-of-sample one-day Value-at-Risk fittings and forecasts at levels above 99% across different model specifications.

# Chapter 1

# Multiple Horizon Causality in Network Analysis: Measuring Volatility Interconnections in Financial Markets


## Abstract

Existing literature does not provide economic and financial networks with a unified measure to estimate network spillovers for empirical studies. In this paper, we propose a novel time series econometric method to measure high-dimensional directed and weighted market network structures. Direct and spillover effects at different horizons, between nodes and between groups, are measured in a unified framework. We infer causality effects in the network through a causality measure based on flexible VAR models specified by the LASSO approach. (Non-sparse) network structures can be estimated from a sparse set of model parameters. To summarize complex estimated network structures, we also proposed three connectedness measures that fully exploit the flexibility of our network measurement method. We apply our approach to investigate the daily implied volatility interconnections among the S&P 100 stocks over the period of 2000 - 2015 as well as its subperiods. We find that 7 out of the 10 most influential firms in the S&P 100 belong to the financial sector. Top investment banks (Morgan Stanley, Goldman Sachs and Bank of America) have the greatest influence in the financial sector. Market connectedness was especially strong

during the recent global financial crisis, and this is mainly due to the high connectedness within the financial sector and the spillovers from the financial sector to other sectors.

## 1.1 Introduction

Since the financial crisis of 2007-09, academic researchers and financial regulators have a growing interest in investigating interconnections in financial markets. Network models have become increasingly popular to study economic interdependence by looking into the market architecture. Allen and Babus (2008) provide a survey showing a wide range of applications of network analysis in economics and finance. For example, bankruptcy contagion, volatility spillovers, risk propagation and amplification can all be studied in economic and financial network frameworks.[1] As Andersen, Bollerslev, Christoffersen and Diebold (2012) mention, modern network theory can provide a unified framework for systemic risk measures.

In macroeconomics, theoretical literature usually takes market structures as given, and then studies the roles of market architecture in the relationship between idiosyncratic risk and market-wide risk. In finance, economic links between firms may serve as the channel of gradual information diffusion. Individual firm's returns, return volatilities and credit spreads can be predicted via firms' linkages, while these empirical studies require identification of the underlying network structures, such as those from the Input-Output Surveys of the Bureau of Economic Analysis, the reported consumer-supplier relationships by public business enterprises or the international trade flows data from the International Monetary Fund (IMF) Direction of Trade Statistics.[2] In fact, many network structures are latent and not readily available in databases. For instance, the relationships between entities (e.g., detailed information on intra-bank asset and liability exposures) in a financial network are usually unknown. To empirically study a market network from financial data, we need an econometric measurement framework to identify and quantify the underlying network structure. A growing econometric literature is responding to this demand.[3] Perhaps surprisingly,

[1]See Buraschi and Porchia (2012), Elliott, Golub and Jackson (2014), Acemoglu, Ozdaglar and Tahbaz-Salehi (2015a), Acemoglu, Akcigit and Kerr (2015c) and Acemoglu, Ozdaglar and Tahbaz-Salehi (2015b) among others.

[2]See Cohen and Frazzini (2008),Hertzel, Li, Officer and Rodgers (2008), Menzly and Ozbas (2010), Aobdia, Caskey and Ozel (2014), Gençay, Signori, Xue, Yu and Zhang (2015), Albuquerque, Ramadorai and Watugala (2015) and Gençay, Yu and Zhang (2016) among others.

[3]See Billio, Getmansky, Lo and Pelizzon (2012), Hautsch, Schaumburg and Schienle (2015), Diebold and Yilmaz (2014), Demirer, Diebold, Liu and Yilmaz (2015), Bianchi, Billio and Casarin (2015), Barigozzi and Brownlees (2016)

however, none of the studies appear to provide a satisfactory tool to measure high-dimensional market networks for general empirical purposes.

In this paper, we propose a novel network econometric measurement framework to better measure directed and weighted network structures using financial time series data in a high-dimensional context. Direct and spillover effects, between nodes and between groups, are measured in a unified framework. Causality at different horizons in the network is measured through a causality measure at different horizons. With this framework at hand, we provide estimated market networks with new econometric connectedness measures. The market systemic risk that is quantified by our connectedness measures has an intrinsic network foundation.

More concretely, we apply the short run and long run Granger causality measures[4] as the basic econometric framework to quantify the strengths of directed edges in a market network. We go beyond the simple Granger noncausality testing, i.e. whether an edge exists between two nodes, but explicitly measure the degree of the multiple horizon causality to obtain the strength of interconnections between two sets of nodes. Following Dufour and Taamouti (2010), we estimate the multiple horizon causality in the Vector Autoregressive model (VAR) settings. To overcome high-dimensionality problems in estimation, we use and extend the Least Absolute Shrinkage and Selection Operator (LASSO) techniques in the VAR estimations, which are similar to those developed by Barigozzi and Brownlees (2014) and Barigozzi and Brownlees (2016). Actually, (non-sparse) network structures, which are measured by our causality measures table, can be estimated from a sparse set of autoregressive coefficients and errors concentration matrices. Under mild conditions, we prove the asymptotic consistency of the estimators of our directed and weighted edge measures.

Our network measurement method has the following 7 appealing features:

1. The network edges we measure are directed. Allowing directed network structures provides us with important insights into the direction of network spillovers, since spillovers and relationships in economic and financial networks are generally asymmetric.

2. The network edges we measure are weighted. We do not merely identify the edges between two sets of nodes, but explicitly quantify their economic strengths.

and Giudici and Spelta (2016) among others.

[4]See Dufour and Renault (1998), Dufour, Pelletier and Renault (2006) and Dufour and Taamouti (2010).

3. In contrast to correlation-based measures, the directed edges we measure have causality implications. This is an important feature for theory verifications, model predictions and policy making.

4. Spillovers at different horizons in an economic network can be identified and measured by analyzing causality measures at different horizons. The multiple horizon causality measures gauge the net effects while simultaneously taking direct and indirect effects into account.

5. Our network measurement method overcomes the high-dimensionality problems in estimations. Note that economic and financial network theories usually study the cases in which the size (number of nodes) of a network is large or even goes to infinity (see, e.g., Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012), Elliott et al. (2014) and Acemoglu et al. (2015b)).

6. Our network measures provide underlying market network structures with clear graphical representations. Eichler (2007) shows that the multiple horizon causality in Dufour and Renault (1998), the base of the multiple horizon causality measures, is well matched to path diagrams in the multivariate time series context. Thus our network measurement framework is also consistent with the network analysis in graph theory.

7. Point-wise edges, $(i \to j)$, as well as group-wise edges, $([i_1, i_2, ..., i_n] \to [j_1, j_2, ..., j_m])$, can be simultaneously analyzed by our unified network econometric framework. In empirical applications, for example, we can not only measure the relationship between firms, but also measure the relationship between sectors[5] by the same data observations at firm level and the same type of econometric measures.

We argue that a satisfactory econometric framework for studying market networks should at least satisfy Features 1 - 5: the network measurement method should be able to estimate directed and weighted network structures with causality implications, and it can be applied to study network spillover effects in a high-dimensional context. Feature 6 and Feature 7 are the extra advantages of our network measurement method. Moreover, Feature 7 provides us with a new angle to study

[5]A sector can be viewed as a group of firms.

market network connectedness. It is intuitive to decompose market connectedness by the interconnections between different sectors and the connectedness within each sector. This decomposition is straightforward for economic and financial network analysis. However, the group-wise edges measurement method for measuring sectors' interconnections is missing in existing econometric literature. Our network measurement method can exactly fill this blank with our Feature 7.

Considering the economy of interest, which is modelled by a market network, as an $N$-dimensional Euclidean space, we use the causality measures table to provide the coordinates of each firm's location in the multi-dimensional economic space. The interconnectedness of a firm to the network can be characterized by the firm's location in the economic space. Total market connectedness is measured by the mean of the interconnectedness measures of each firm to the economic space. Similar to Billio et al. (2012) and Diebold and Yilmaz (2014), our market connectedness measures are built on underlying market network structures, and thus the market systemic risk quantified by these measures has a market network foundation. Since an economic network can be viewed as a network connected by firms (firm-wise market), whose interconnections are measured by our point-wise edges method ($i \to j$), or a network connected by sectors (sector-wise market), whose interconnections are measured by our group-wise edges method ($[i_1, i_2, ..., i_n] \to [j_1, j_2, ..., j_m]$), we have three types of connectedness measures to gauge network interconnections: i) firm-wise connectedness, which measures the interconnectedness of a firm-wise market; ii) firm-wise connectedness within a sector, which measures the interconnectedness within a given sector in a firm-wise market; and iii) sector-wise connectedness, which measures the interconnectedness of a sector-wise market. These three types of connectedness measures fully take advantage of the flexibility of our network measurement method, so they can be applied to study market network connectedness in more flexible ways than those connectedness measures proposed by Billio et al. (2012) and Diebold and Yilmaz (2014).

Our network measurement methods have a wide range of applications and can be applied in a variety of research areas, including identifying and quantifying economic relationships between firms, between sectors and between areas; measuring market connectedness; predicting financial risks; guiding asset allocations in large portfolios; etc. Note that many latent economic and financial network structures can be estimated by our flexible network measurement method with varieties of panel databases, and observing that explicit identified economic network centrality and consumer-supplier linkage have been shown to be new risk factors in asset pricing and new determinants to predict financial variables, e.g., stock return, return volatility, and credit spread[6], we expect more pricing factors and financial and macroeconomic variables drivers are to be discovered by network econometric measurement methods.[7]

To illustrate the usefulness of our method in network analysis, we investigate the S&P 100 implied volatility network in the US stock market. Volatility network in financial markets has been studied in Diebold and Yilmaz (2014), Demirer et al. (2015) and Barigozzi and Brownlees (2016), but they mainly focus on realized volatility. For financial practitioners, the VIX index, calculated from the implied volatilities of S&P 500 index option contracts, is the most popular volatility measure to gauge market turbulences, and it is also known as a "market fear" index. Our implied volatility network among the S&P 100 stocks[8] can thus be naturally viewed as an "individual fear" network. To the best of our knowledge, implied volatility network has not yet been studied in the financial literature.

We first look at the static network with the full sample (2000 - 2015). We identify the most influential firms in the firm-wise market network, the most influential firms in the financial sector, and the most influential sectors in the sector-wise market network. Using rolling subsamples, we estimate the time-varying firm-wise market connectedness before, during and after the recent financial crisis of 2007-09, and compare it with the dynamic patterns of the firm-wise connectedness within each sector and the sector-wise connectedness among different sectors. In particular, we also examine the dynamic interconnections between the financial sector and other sectors.

We find that: i) 7 out of the 10 most influential firms in the S&P 100 belong to the financial sector, and top investment banks (Morgan Stanley, Goldman Sachs and Bank of America) have the greatest influence in the financial sector; ii) market connectedness was especially strong during the recent global financial crisis; iii) the high market connectedness was mainly due to the high connectedness within the financial sector and the spillovers from the financial sector to other sectors; iv) the financial sector had the highest firm-wise connectedness from 2008 to 2010, while the

[6]See Cohen and Frazzini (2008), Hertzel et al. (2008), Menzly and Ozbas (2010), Ahern (2013), Aobdia et al. (2014), Gençay et al. (2015) and Gençay et al. (2016).

[7]For example, Jian (2016) uses a Granger-type method to identify the illiqiudity network in stock markets and finds centralities in illiquidity networks are priced in the cross-section of expected returns.

[8]To be included in the S&P 100, the companies should be among the larger and more stable companies in the S&P 500, and *must have list options*.

connectedness of other sectors also reaches relatively high level during this period; v) the causality effects between the financial sector and other sectors were asymmetric and displayed considerable variation over time, which stresses the importance of directed and weighted edges settings in market network analysis.

This paper is motivated by the econometric literatures on the analysis of financial networks and contributes to different strands of literature. The topic of this paper is related to recent econometric literature on financial networks (see Billio et al. (2012), Diebold and Yilmaz (2014), Demirer et al. (2015), Bianchi et al. (2015), Barigozzi and Brownlees (2016), Hautsch et al. (2015), Ahelegbey, Billio and Casarin (2015), and Giudici and Spelta (2016) among others). We differ from the social network econometrics literature, e.g., Bramoulle, Djebbari and Fortin (2009), in the sense that the nodes in our network setting are represented by time series financial variables (e.g., return and volatility). The most closely related econometric literature to this paper includes: Billio et al. (2012), Diebold and Yilmaz (2014), Demirer et al. (2015) and Barigozzi and Brownlees (2016). Billio et al. (2012) detect the edge of a pair of nodes via testing bilateral Granger noncausality without taking into account other nodes in the network, and thus may find misleading "spurious" causality edges and tend to overestimate the number of linkages. Diebold and Yilmaz (2014) and Demirer et al. (2015) overcome the spurious relation problem. They measure the directed and weighted network structure by generalized forecast error variance decompositions in a VAR representation. The generalized forecast error variance decomposition technique is closely related to our multiple horizon causality measures. Unfortunately, Diebold and Yilmaz (2014) neglect the high-dimensionality problem in their study, Demirer et al. (2015) fail to provide the theoretical validity for their estimations and they both require the joint Gaussian innovation assumption in the econometrics model. These drawbacks inevitably limit their applications in market network analysis for general purposes. The time series network estimation settings in Barigozzi and Brownlees (2016) are similar to what we apply in this paper. Yet, their network structure is assumed to be sparse and their edges, measured by long run partial correlations, are basically undirected. Among recent literature[9], only the empirical model proposed in Demirer et al. (2015) is able to study a high-dimensional directed and weighted network structure, and none of them is able to estimate point-wise edges and group-wise edges in a unified framework.

[9] Ahelegbey (2015) provides a recent review on the network econometrics in the context of time series analysis.

We apply the short run and long run Granger causality measures as our basic network econometric measurement framework. The concept of the noncausality testing introduced by Granger (1969) and Sims (1972) has been widely used to study dynamic relationships between time series in economics and finance. Dufour and Renault (1998) and Dufour et al. (2006) extend this notion to multiple horizon cases to study indirect causality effects. Eichler (2007) connects the short run and long run Granger causality with path diagram in multivariate time series analysis. Based on Geweke (1982), Dufour and Renault (1998) and Dufour et al. (2006), Dufour and Taamouti (2010) propose the multiple causality measures to quantify the causality at any forecast horizon $h \geq 1$. Dufour, Garcia and Taamouti (2012) apply this tool in studying the relationship among returns, realized volatility and implied volatility. Dufour and Zhang (2015) further study the multiple horizons second-order causality. In this paper, we show that market networks, with directed and weighted edges, can be modelled and measured by the well-developed econometrics framework of the multiple horizon causality measures. Moreover, unlike Dufour and Taamouti (2010) and Dufour et al. (2012) who only deal with low-dimensional situations, we estimate the multiple horizon causality measures with the LASSO approach to better fit the multiple horizon causality measure framework into high-dimensional network analysis.

One of the motivations of this paper, identifying and quantifying the degree of interconnections between nodes and between groups in market networks, is to provide a new way to measure market-based systemic risk. Similar to Billio et al. (2012) and Diebold and Yilmaz (2014), our market connectedness measures are also built upon the underlying network structure and contribute to the strand of literature on market-based systemic risk measurement (see Acharya, Pedersen, Philippon and Richardson (2010), Brownlees and Engle (2015), Adrian and Brunnermeier (2011), Billio et al. (2012), Diebold and Yilmaz (2014), Hautsch et al. (2015) and Demirer et al. (2015) among others). Benoit, Colliard, Hurlin and Perignon (2015) provide a comprehensive survey on measurement methods for systemic risk.

Our key contribution is that we propose a novel time series econometrics network measurement framework, which can be applied to measure high-dimensional directed and weighted market network structures, without sparsity assumptions on network structures or the Gaussian assumption on econometric models. We successfully connect the causality literature with the LASSO approach in application to network measurement. Moreover, to the best of our knowledge, our econometric framework is the first one in the network econometric literature to explicitly allow point-wise edges and group-wise edges to be measured in a unified framework.

The rest of this paper is organized as follows. In section 1.2, we provide a brief description of general directed and weighted network structures and discuss the criteria of a satisfactory network econometric framework in economic and financial network analysis. In section 1.3, we show that directed and weighted network structures and network spillovers can be measured by the multiple horizon causality measures table. In section 1.4, we estimate the causality table with the LASSO approach in a high-dimensional context and provide asymptotic consistency results. In section 1.5, we propose new market network connectedness measures for systemic measurement. In section 1.6, we investigate the static structure and the time-varying characteristics of the implied volatility network in the US stock market. Finally, in section 1.7 we provide a short conclusion.

## 1.2 General Economic and Financial Network

A network is composed by two basic elements: nodes and edges. Financial institutions, for instance, represented by different nodes, are linked through networks of different types of financial contracts, such as derivatives, credits and securities. These contracts or business relationships, between any pair of financial institutions, are represented by their edges in the financial network. While nodes are given and known as they are always referred to some specific institutions, modelling edges is always an elusive part in financial network analysis. Edges represent some implicit economic relationships between nodes. The relationship among financial institutions in many cases are unknown or difficult to specify. When we study the systemic risk in a financial network, edges could be the position of banks' loans to each other in their balance sheets, or whether they hold a large bilateral position of some securities (e.g., credit default swap (CDS)). Without a prior specific definition of the systemic risk, which financial contract should be selected as the edge to study a financial network is a difficult decision to make, since loan's edges and CDS's edges are both theoretically important but their existences can be independent. Moreover, detailed information of the financial contracts that financial institutions are holding and their counterparties is usually unavailable to public. Therefore, what we can measure for the edges from data is at most a proxy of what we are interested in. This provides a broad space for econometricians to develop different statistical network measures for different research objectives. One of the main aspects of research papers differing from each other in the financial econometrics network literature is in their rationales of how to construct a statistical measure to quantify the edges in a network.

Despite it, all networks have basic structures in common. A simple static network has a mathematical notation: $G = \{V, E\}$, where $V = \{1, 2, ..., N\}$ is the set of nodes and $E = \{e^{ij} : (i, j) \in V \times V\}$ is the set of edges. Usually, the size of the network, $N$, is large. Any pair of nodes in $V$, $(i, j)$, may be linked by an edge in the edge set, $E$. When $e^{ij} = e^{ji}$ is assumed, the network is undirected; otherwise, the network is directed. If $e^{ij}$ is assumed to be indexed by $\{0, 1\}$, the network is unweighted; if $e^{ij}$ is continuous with certain degree of strength, the network is weighted.

The directed edges setting is crucial in economic and financial network analysis. Economic relations are usually directed and the directed structures play an important role in network analysis. For instance, the presence of directed intersectoral input-output linkages can explain why single idiosyncratic shocks may lead to market-wide aggregate fluctuations (see Acemoglu et al. (2012)). Economic effects and information flows have directions. We use causal relationships to describe such directed relationships in a economic network. Causality interpretations are required for economic networks because it is the foundation for theory verifications, model predictions and policy makings. Intuitively, if two firms have no business relationship, we do not expect there is a causal relationship between them and vice versa. We notate the directed edges in our network with arrows, $(i \rightarrow j)$, which indicates $i$ causes $j$. Figure 1.1 shows four simple possible relations between node A and node B in a unweighted setting. If the strength of the edges $e^{AB} = e^{BA} = 0$, we say node A and node B are unlinked (fig. 1.1a); if they are linked, then either $e^{AB} = 1$ or $e^{BA} = 1$ or $e^{AB} = e^{BA} = 1$ (fig. 1.1b, fig. 1.1c and fig. 1.1d).

The weighted edges setting is also important. Effects in an economic network are weighted. In social networks, knowing how well agents know each others is much more informative than merely knowing whether they know each others, since the probability of information transmissions is highly correlated with their familiarity. In financial networks, when we say a bank is "too big to fail", it implies that this bank has "big" impacts on others. When studying shock propagations or risk amplifications in a market network, we would be especially interested in quantifying spillover effects. Since spillovers may grow (or disappear) through edges in a network, unweighted edges setting is not able to model the quantitative change in spillover processes. Figure 1.2 shows three

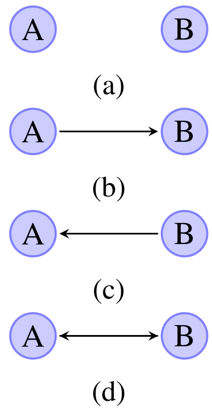



Figure 1.1: Directions of the edges between node A and node B

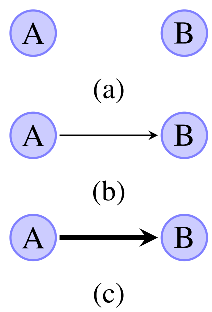



Figure 1.2: Strengths of the edge from node A to node B

possible strengths of edge from node A to node B. The strength of the edge could be zero, which implies there is no relation from node A to node B (see fig. 1.2a). The strength of the edge could be small and it is represented by a light arrow (see fig. 1.2b); the strength of the edge could also be large and it is represented by a thick arrow (see fig. 1.2c). The thickness of the edge ($i \to j$) is weighted scaled by $e^{ij}$.

Economic and financial network literature usually reports some graphs of the network they study. The graph representation of a static network does provide us a broad and concise picture of the underlying network structure. A static network, however, only tells us direct effects. The indirect effects, a central part in risk spillover analysis, is nontrivial to be revealed from the direct effects. For instance, suppose there are relations from node A to node B indirectly via two different paths in an unweighted static network, we may naively say that the risk from node A could cascade to node B. However, it is possible that node A has no effect on node B if the indirect effects in those two paths are just cancelled out by each other. Hence, a network graph drawn from a static network structure may mislead us to a wrong implication about spillover effects in the true economic network. Surprisingly, econometric literature on financial networks have not yet realized this important issue. Most of them just focus on estimating static network structures without directly measuring spillover effects.

In summary, the size of an economic network is usually large; some plausible causality interpretation for nodes' relationships in an economic network is desire; a directed and weighted edges setting is required to uncover the effects in the underlying economic network structures; network spillover effects need to be measured directly. Therefore, a satisfactory network econometric framework should be able to estimate directed and weighted network structures with causality implications, and it can be applied to study spillover effects in a high-dimensional context.

## 1.3 Multiple Horizon Causality and Networks

In this section, we model a complex network structure by causality relations, and apply the short run and long run Granger causality measures, introduced by Dufour and Taamouti (2010), to identify and quantify the edges between two sets of nodes in the underlying network structure. We demonstrate that the multiple horizon causality measures satisfy the criteria of a satisfactory network econometric framework. It is able to estimate directed and weighted network structures with causality implications and can be applied to study spillover effects in a high-dimensional context Moreover, our network measurement framework has some other important features.

Suppose we observe a data sample from a jointly strictly stationary process $X = \{X_{1t}, X_{2t}, ..., X_{Nt}\}_{t=1}^{T}$. $N$ is the number of nodes and $T$ is the observable sample size. In context of economic network analysis, the number of nodes, $N$, is large. The process of interest, $X$, can be divided by three sub-processes as $X = \{X_t^W, X_t^Y, X_t^Z\}_{t=1}^{T}$, such that $X_t^W = [X_{1t}, ..., X_{m_1 t}]$, $X_t^Y = [X_{(m_1+1)t}, ..., X_{(m_1+m_2)t}]$ and $X_t^Z = [X_{(m_1+m_2+1)t}, ..., X_{(m_1+m_2+m_3)t}]$, where $m_1, m_2, m_3 \geq 0$ and $m_1 + m_2 + m_3 = N$. $I$ denotes the full information set and $I_{-W}$ denotes the full information set without the information generated by $X^W$. If we further assume the full information set is generated only by $X$ itself, $I$ and $I_{-W}$ can be denoted by $I_{WYZ}$ and $I_{YZ}$ respectively, where $I_{WYZ}(t)$ denotes the information set generated by the process $X = \{X^W, X^Y, X^Z\}$ up to time $t$, and $I_{YZ}(t)$ denotes the information set generated by the sub-process $\{X^Y, X^Z\}$ up to time $t$.

**Definition 1.3.1.** Mean-square Causality Measure at forecast horizon $h$ relative to an information set $I$

For $h \geq 1$, where by convention $\ln(0/0) = 0$ and $\ln(x/0) = +\infty$ for $x > 0$,

$$C_L(X^W \underset{h}{\longrightarrow} X^Y|I) := \ln\left[\frac{\det\{\Sigma[X^Y(t+h)|I_{(-W)}(t)]\}}{\det\{\Sigma[X^Y(t+h)|I(t)]\}}\right] \tag{1.1}$$

is the mean-square causality measure from $X^W$ to $X^Y$ ($Y = W$ is allowed) at horizon $h$, given information set $I$.

Since we only consider the mean-square measures in this paper, we will just call it as short run and long run Granger causality measures or multiple horizon causality measures hereafter.

The multiple horizon causality measure, $C_L(X^W \underset{h}{\longrightarrow} X^Y|I)$, gauges the predictive power of $X^W$ to $X^Y$ conditional on $I$. We say $X^W$ causes $X^Y$ at forecast horizon $h$ if and only if $X^W$ helps to predict $X^Y$ at forecast horizon $h$. The value of $C_L(X^W \underset{h}{\longrightarrow} X^Y|I)$ measures the degree of the causal effect from $X^W$ to $X^Y$ at forecast horizon $h$. Consequently, the identified edge, $(W \to Y)$, has causality implications.

There are some important properties of this type of measures.

First, generally speaking, $C_L(X^W \underset{h}{\longrightarrow} X^Y|I) \neq C_L(X^Y \underset{h}{\longrightarrow} X^W|I)$. The effect from $W$ to $Y$ is not presumed to be equal to the effect from $Y$ to $W$. The edges between $W$ and $Y$, $(W \to Y)$ and $(Y \to W)$, are directed.

Second, $C_L(X^W \underset{h}{\longrightarrow} X^Y|I)$ is always nonnegative as $I_{(-W)}(t) \subseteq I(t)$. $C_L(X^W \underset{h}{\longrightarrow} X^Y|I) = 0$ if and only if there is no causal effect from $X^W$ to $X^Y$ at forecast horizon $h$. The value of $C_L(X^W \underset{h}{\longrightarrow} X^Y|I)$ is increasingly monotone to the the predictive power of $X^W$ to $X^Y$. Thus, the strength of the edge, $(W \to Y)$, measured by the value of $C_L(X^W \underset{h}{\longrightarrow} X^Y|I)$, is weighted.

Third, $C_L(X^W \underset{h}{\longrightarrow} X^Y|I)$ measures the indirect effect from $W$ to $Y$ at horizon $h$, while $C_L(X^W \underset{1}{\longrightarrow} X^Y|I)$ measures the direct effect as there is only one step to be considered. For example, suppose $C_L(X^W \underset{1}{\longrightarrow} X^Y|I) = 0$ and $C_L(X^W \underset{h}{\longrightarrow} X^Y|I) > 0$ for a $h > 1$, it implies there is no direct effect from $W$ to $Y$, but there is an indirect effect from $W$ to $Y$ via other node(s) in the network. The spillover effect from $W$ to $Y$ at any step $h$ can thus be directly measured by $C_L(X^W \underset{h}{\longrightarrow} X^Y|I)$.

Fourth, the dimensions of $X^W$ and $X^Y$ are arbitrary. To measure the edge from $W$ to $Y$, we only require the dimensions of the processes $X^W$ and $X^Y$, $m_1$ and $m_2$, such that $m_1, m_2 \geq 1$ and $m_1 + m_2 \leq N$. We can let $W$ and $Y$ represent a single node or a set of nodes. The point-wise edges, where $X^W$ and $X^Y$ are univariate variables ($m_1 = m_2 = 1$), and the group-wise edges, where $X^W$ and $X^Y$ are multivariate variables ($m_1, m_2 > 1$), can be analysed in this unified econometric framework. For instance, we can not only measure the edges between firms (firm-wise edges), where $X^W$ and $X^Y$ represent firms, but also measure the edges between sectors (sector-wise edges), where $X^W$ and $X^Y$ represent sectors and $m_1$ and $m_2$ are the number of firms in the sectors. Therefore, we can use the same data observations at firm's level and the same type of econometric measures defined in Definition 1.3.1 to study firm-wise edges and sector-wise edges in a unified framework. In the past, weighted aggregation is usually required if we want to study the sector-wise spillover effect with firm-wise data. However, it would inevitably come to a cost of losing information in firm-wise interconnections. The econometrics approach proposed in this paper overcomes this limitation.

**Remark 1.3.2.** If $X^W$ and $X^Y$ are univariate processes denoted by $X_i$ and $X_j$ respectively, then for $h \geq 1$

$$C_L(X_i \underset{h}{\longrightarrow} X_j|I) := \ln\left[\frac{\sigma^2[X_j(t+h)|I_{(-i)}(t)]}{\sigma^2[X_j(t+h)|I(t)]}\right]. \tag{1.2}$$

The variances of the forecast errors of $X_j(t+h)$, $\sigma^2[X_j(t+h)|I_{(-i)}(t)]$ and $\sigma^2[X_j(t+h)|I(t)]$, are both positive, and $\sigma^2[X_j(t+h)|I_{(-i)}(t)] \geq \sigma^2[X_j(t+h)|I(t)]$. $\sigma^2[X_j(t+h)|I_{(-i)}(t)] = \sigma^2[X_j(t+h)|I(t)]$ if the information generated by node $i$ does not help to decrease the forecast error variance of node $j$. $C_L(X_i \underset{h}{\longrightarrow} X_j|I)$ measures the causality strength from node $i$ to node $j$. For notation convenience, we hereafter let $C_{ij}^h := C_L(X_i \underset{h}{\longrightarrow} X_j|I)$ and $C_{ij} := C_{ij}^1$.

For any given forecast horizon $h \geq 1$, we have the multiple horizon causality measures for each pair of nodes in a network as Table 1.1 shows. Point-wise edges in the network are measured by the values of $C_{ij}^h$, $i = 1, ..., N$ and $j = 1, ..., N$. Table 1.1 is exactly corresponding to a static network structure. The $i$th row and $j$th column element in Table 1.1 is the strength of the directed edge from node $i$ to node $j$. $C_{ij}$ measures the direct effect from node $i$ to node $j$: $S(i \to j)$, where $S(i \to j)$ denote the effect from node $i$ to node $j$ via the path $(i \to j)$. For $h > 1$, $C_{ij}^h$ measures the total indirect effect from node $i$ to node $j$ via every possible path with length $h$: $S(i \to k_1 \to k_2 \to ... \to k_{h-1} \to j)$ for any $k_i \in V, i = 1, ..., h-1$, where $S(i \to k_1 \to k_2 \to ... \to k_{n-1} \to j)$ denote the indirect effect from node $i$ to node $j$ via the path $(i \to k_1 \to k_2 \to ... \to k_{n-1} \to j)$. In other words, $C_{ij}^h$ measures the indirect effect from node $i$ to node $j$ with taking into account all the interconnections in the network. Intuitively, the forecast horizon $h$ can be interpreted as the effect-radius when considering

Table 1.1: Causality table (given forecast horizon $h$)

| nodes | 1 | 2 | $\cdots$ | $i$ | $\cdots$ | $j$ | $\cdots$ | $N$ |
|---|---|---|---|---|---|---|---|---|
| 1 | $C_{11}^h$ | $C_{12}^h$ | $\cdots$ | $C_{1i}^h$ | $\cdots$ | $C_{1j}^h$ | $\cdots$ | $C_{1N}^h$ |
| 2 | $C_{21}^h$ | $C_{22}^h$ | $\cdots$ | $C_{2i}^h$ | $\cdots$ | $C_{2j}^h$ | $\cdots$ | $C_{2N}^h$ |
| $\vdots$ | | | | | | | | |
| $i$ | $C_{i1}^h$ | $C_{i2}^h$ | $\cdots$ | $C_{ii}^h$ | $\cdots$ | $C_{ij}^h$ | $\cdots$ | $C_{iN}^h$ |
| $\vdots$ | | | | | | | | |
| $j$ | $C_{j1}^h$ | $C_{j2}^h$ | $\cdots$ | $C_{ji}^h$ | $\cdots$ | $C_{jj}^h$ | $\cdots$ | $C_{jN}^h$ |
| $\vdots$ | | | | | | | | |
| $N$ | $C_{N1}^h$ | $C_{N2}^h$ | $\cdots$ | $C_{Ni}^h$ | $\cdots$ | $C_{Nj}^h$ | $\cdots$ | $C_{NN}^h$ |

the effect between any pair of nodes. For example, when $h = 1$, we only measure the direct effect (1-step effect); when $h = 100$, the effect between any pair of nodes could "walk" via as many as 99 different other nodes in the network.[10] Another way to understand the difference between $C_{ij}^1$ and $C_{ij}^h$ ($h > 1$) is to consider the difference among standard network centrality measures (e.g., Degree, Closeness, Betweenness and Eigenvector). These centrality measures differ from each others mainly in how to weight the importance of the nodes that a node connected to to measure this node's importance in the network. For instance, the degree centrality only calculate how many nodes that a node directly connected to to characterize this node's importance, while the eigenvector centrality assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes, thus the degree centrality is a "local" measure, and the eigenvector centrality is a "global" measure. Similarly, $C_{ij}^1$ is 1-step locally measuring the direct effect, and $C_{ij}^h$ is $h$-steps globally measuring the indirect effect.

In terms of mathematical definitions, group-wise edges are the generalization of point-wise edges. They are equivalent when the sizes of the groups equal 1. For any pair of nodes, $i$ and $j$, in a node set $V$, we say $i \underset{C,h}{\nrightarrow} j$ if and only if $C_{ij}^h = 0$. For any pair of groups of nodes

[10]The forecast time horizon and the length of path are equivalent if we assume that direct effects take and only take an unit of time to happen. If A does not affect B at horizon one but can affect B at horizon $N$, which is greater than one, this implies A affects some other nodes at horizon one and then affects B directly. This is also implied by the VAR specification using in this paper. Without the above assumption, the length of path depends on the time scale we use to define time horizon. 1-step in path could represent one day or two days.

$(i_1,...,i_{n_1})$ and $(j_1,...,j_{n_2})$, where $(i_1,...,i_{n_1}) = (j_1,...,j_{n_2})$ or $(i_1,...,i_{n_1}) \cap (j_1,...,j_{n_2}) = \emptyset$ for $(i_1,...,i_{n_1}),(j_1,...,j_{n_2}) \subset V$, we say $(i_1,...,i_{n_1}) \underset{C,h}{\nrightarrow} (j_1,...,j_{n_2})$ if and only if $C^h_{WY} = 0$, where $W = (i_1,...,i_{n_1})$ and $Y = (j_1,...,j_{n_2})$.

**Remark 1.3.3.** Let $V_1 = (i_1,...,i_{n_1})$ and $V_2 = (j_1,...,j_{n_2})$. For any $i \in V_1$ and $j \in V_2$, because of $I_{-V_1} \subset I_{-i}$, $C^h_{V_1V_2} = 0$ $[(i_1,...,i_{n_1}) \underset{C,h}{\nrightarrow} (j_1,...,j_{n_2})]$ implies $C^h_{iV_2} = 0$ $[i \underset{C,h}{\nrightarrow} (j_1,...,j_{n_2})]$, and $C^h_{V_1j} = 0$ $[(i_1,...,i_{n_1}) \underset{C,h}{\nrightarrow} j]$ implies $C^h_{ij} = 0$ $[i \underset{C,h}{\nrightarrow} j]$.

Remark 1.3.3 says if a set of node(s) has no effect on some other node(s), any element of this set of node(s) also has no effect on those node(s). It is worth to emphasize here that $C^h_{ij} = 0$ $[i \underset{C,h}{\nrightarrow} j]$ for any $i \in V_1$ and for any $j \in V_2$ does NOT necessarily imply $C^h_{V_1V_2} = 0$ $[(i_1,...,i_{n_1}) \underset{C,h}{\nrightarrow} (j_1,...,j_{n_2})]$. In other words, the strength of $[(i,j) \underset{C,h}{\rightarrow} k]$ may be strong even if the strengths of $[i \underset{C,h}{\rightarrow} k]$ and $[j \underset{C,h}{\rightarrow} k]$ are weak. This circumstance is analogous to the difference between pairwise independence and mutually independence. When $X_i$ and $X_j$ are contemporaneously highly correlated, $X_i$'s marginal effect on $X_k$, conditional on $X_j$, will be very small since all relevant information in $X_i$ that helps to predict $X_k$ has been captured by $X_j$.

In fact, studying the role of a group of nodes in a network is an important topic. In social network literature, for instance, just as looking into who is the center node in a network, which can be measured by standard centrality measures[11] (see, e.g., Freeman (1978) and Jackson et al. (2008)), we also may want to find which group of nodes is center in a network, which can be measured by the generalizations of the standard centrality measures (see Everett and Borgatti (1999)). From measurement perspective, the importance of a group of node(s) has to be based on the interconnections of this group to other nodes in the network (and all other interconnections in the network). To the best of our knowledge, surprisingly, measuring the effects of a group of nodes on other nodes in a network is still missing in the network econometric literature. Our network measurement method can exactly fill this blank. Moreover, our group-wise edges measurement method is compatible with the classic network literature. Remark 1.3.3 suggests our generation of pair-wise edges by group-wise edges is in line with the generation of the node centralities in Freeman (1978) by the group centralities in Everett and Borgatti (1999).

From the discussion in this section, we have seen that the multiple horizon measures in Definition 1.3.1 have causality implications for the edges it measures. It is also sufficiently flexible

[11] In network theory indicators of centrality identify the most important vertices within a graph.

to be applied to study indirect effects in directed and weighted network structures. Properties of network analysis, which can be applied to study complex interconnections in an economic system, have been studied in mathematics and computer science as graph theory. As Eichler (2007) shows, the multiple horizon causality in Dufour and Renault (1998), the base of the multiple horizon causality measures, is also well matched to path diagrams in the multivariate time series context. Thus our network measurement framework is also in line with the network analysis in graph theory. Lastly, point-wise edges, $(i \to j)$, and group-wise edges, $([i_1, i_2, ..., i_n] \to [j_1, j_2, ..., j_m])$, can be analysed by our multiple horizon causality measures framework.

## 1.4 LASSO Estimation of Causality Measure

In this section, we estimate the multiple horizon causality measures $C_L(X^W \underset{h}{\longrightarrow} X^Y|I)$ and $C_L(X^W \underset{h}{\longrightarrow} X^W|I)$ in a high-dimensional context. Given the network's nodes processes $X = \{X_{1t}, X_{2t}, ..., X_{Nt}\}_{t=1}^T$, following Dufour and Taamouti (2010) we use the VAR framework in our econometric analysis. Network estimation under the VAR representation is desirable since the VAR models are naturally developed to investigate the pairwise effect in a complex linear structure. Unlike Dufour and Taamouti (2010) who only deal with low-dimensional situations, we estimate the multiple horizon causality measures with the LASSO approach to better fit the multiple horizon causality measures framework into high-dimensional network analysis.

**Assumption 1.4.1.** Processes VAR Representations

The unrestricted process $X = \{X_t^W, X_t^Y, X_t^Z\}_{t=1}^T = \{X_{1t}, X_{2t}, ..., X_{Nt}\}_{t=1}^T$ is strictly stationary and has a VAR($\infty$) representation,

$$X(t) = \sum_{k=1}^{\infty} A_k X(t-k) + u(t), \tag{1.3}$$

where $X(t) = [X_{1t}, X_{2t}, ..., X_{Nt}]'$ is a $N \times 1$ vector, $A_k$ is $N \times N$ matrix and $u(t) \sim w.n.(0, \Sigma_u)$. $\Sigma_u$ is a $N \times N$ positive definite matrix.

The restricted process $X_0 = \{X_t^Y, X_t^Z\}_{t=1}^T$ is strictly stationary and has a VAR($\infty$) representation,

$$X_0(t) = \sum_{k=1}^{\infty} \bar{A}_k X_0(t-k) + \varepsilon(t), \tag{1.4}$$

where $X_0(t) = [X_t^Y, X_t^Z]'$ is a $(N-m_1) \times 1$ vector, $\bar{A}_k$ is $(N-m_1) \times (N-m_1)$ matrix and $\varepsilon(t) \sim w.n.(0, \Sigma_\varepsilon)$. $\Sigma_\varepsilon$ is a $(N-m_1) \times (N-m_1)$ positive definite matrix.

The restricted process has the following expanded representation,

$$X(t) = \sum_{k=1}^{\infty} \bar{A}_k^{\phi} J_2 X(t-k) + \nu(t) \tag{1.5}$$

where $\bar{A}_k^{\phi} = \begin{bmatrix} \bar{A}_k^W \\ \bar{A}_k \end{bmatrix}_{N \times (N-m_1)}$, $\bar{A}_k$ is defined in (1.4) and $\bar{A}_k^W$ is the expanded coefficients for $X^W$. $J_2 = [0_{(N-m_1)\times m_1},\ I_{(N-m_1)\times(N-m_1)}]_{(N-m_1)\times N}$ and $\nu(t) \sim w.n.(0, \Sigma_\nu)$. $\Sigma_\nu$ is a $N \times N$ positive definite matrix.

**Remark 1.4.2.** Under the Assumption 1.4.1, the covariance matrix of the forecast error at horizon $h$ for the unrestricted model (1.3) is

$$\Sigma[X(t+h)|\mathscr{F}(t)] \equiv \sum_{q=0}^{h-1} \varphi_q \Sigma_u \varphi_q', \tag{1.6}$$

where $\varphi_q = \sum_{k=1}^{q} A_k \varphi_{q-k}$ and $\varphi_0 = I_N$. The covariance matrix of the forecast error at horizon $h$ for the restricted model (1.4) is

$$\Sigma[X_0(t+h)|\mathscr{F}_{-W}(t)] \equiv \sum_{q=0}^{h-1} \bar{\varphi}_q \Sigma_\varepsilon \bar{\varphi}_q', \tag{1.7}$$

where $\bar{\varphi}_q = \sum_{k=1}^{q} \bar{A}_k \bar{\varphi}_{q-k}$ and $\bar{\varphi}_0 = I_{N-m_1}$.

**Definition 1.4.3.** Under the Assumption 1.4.1 and by the Remark 1.4.2, the multiple horizon causality measure, from $W$ to $Y$, at forecast horizon $h$ is

$$C_L(X^W \underset{h}{\longrightarrow} X^Y|I) := \ln\left[\frac{\det\{J_0 \Sigma[X_0(t+h)|\mathscr{F}_{-W}(t)] J_0'\}}{\det\{J_1 \Sigma[X(t+h)|\mathscr{F}(t)] J_1'\}}\right] \tag{1.8}$$

where $J_0 = [I_{m_2},\ 0_{m_2\times m_3}]_{m_2\times(N-m_1)}$ and $J_1 = [0_{m_2\times m_1},\ I_{m_2},\ 0_{m_2\times m_3}]_{m_2\times N}$. $\Sigma[X(t+h)|\mathscr{F}(t)]$ and $\Sigma[X_0(t+h)|\mathscr{F}_{-W}(t)]$ are defined in (1.6) and (1.7) respectively.

**Remark 1.4.4.** Under the Assumption 1.4.1, it can be easy to observe that $\Sigma_\varepsilon = J_2 \Sigma_\nu J_2'$ and $X_0(t) =$

$J_2X(t)$. Then the forecast error covariance of $X^W$ at horizon $h$, without its past information, is

$$\Sigma_W[X^W(t+h)|\mathscr{F}_{-W}(t)] \equiv J_3\left(\sum_{q=0}^{h-1}\phi_q\Sigma_v\phi_q'\right)J_3', \tag{1.9}$$

where $\phi_q = \sum_{k=1}^{q} A_k^{\phi}\phi_{q-k}$, $A_k^{\phi} = \bar{A}_k^{\phi}J_2$, $\phi_0 = I_N$, $J_3 = [I_{m_1\times m_1},\ 0_{m_1\times(N-m_1)}]_{m_1\times N}$.

**Definition 1.4.5.** Under the Assumption 1.4.1 and by the Remark 1.4.4, the multiple horizon causality measure, from $W$ to $W$, at forecast horizon $h$ is

$$C_L(X^W \underset{h}{\longrightarrow} X^W|I) := \ln\left[\frac{\det\{\Sigma_W[X_0(t+h)|\mathscr{F}_{-W}(t)]\}}{\det\{J_3\Sigma[X(t+h)|\mathscr{F}(t)]J_3'\}}\right] \tag{1.10}$$

where $\Sigma_W[X_0(t+h)|\mathscr{F}_{-W}(t)]$ and $\Sigma[X(t+h)|\mathscr{F}(t)]$ are defined in (1.9) and (1.6) respectively.

In order to obtain $C_L(X^W \underset{h}{\longrightarrow} X^Y|I)$ and $C_L(X^W \underset{h}{\longrightarrow} X^W|I)$, we just need to know the autoregressive matrices, $[A_1, A_2, ..., A_{h-1}]$ and $[\bar{A}_1^{\phi}, \bar{A}_2^{\phi}, ..., \bar{A}_{h-1}^{\phi}]$, and the contemporaneous covariance matrices, $\Sigma_u$ and $\Sigma_v$. To estimate these parameters, we consider the truncated models of the unrestricted process (1.3) and the expanded restricted process (1.5) as

$$X(t) = \sum_{k=1}^{p} A_k^p X(t-k) + u^p(t), \tag{1.11}$$

$$X(t) = \sum_{k=1}^{p} \bar{A}_k^p X_0(t-k) + v^p(t). \tag{1.12}$$

where $u^p(t) \sim w.n.(0, \Sigma_u^p)$ and $v^p(t) \sim w.n.(0, \Sigma_v^p)$. $A_k^p$ and $\Sigma_u^p$ are $N$ by $N$ matrices for $k = 1, 2, .., p$. $\bar{A}_k^p$ is a $N$ by $N - m_1$ matrix and $\Sigma_v^p$ is a $N$ by $N$ matrix for $k = 1, 2, .., p$.

While the dimensions for two groups in group-wise edge analysis, $m_1$ and $m_2$, are fixed, we assume that the number of nodes, $N$, and the lag $p$ can be functions of $T$ (i.e., $N_T = O(T^{c_1})$ and $p_T = O(T^{c_2})$ for constant $c_1, c_2 > 0$), but for notation simplicity we do not write the subscript $T$ explicitly. Under mild assumptions in Barigozzi and Brownlees (2014), the truncated bias is asymptotically negligible such that $||A_k^p - A_k||_\infty = o(1)$ for $k = 1, 2, ..., p$ and $||\Sigma_u^p - \Sigma_u||_\infty = o(1)$ as $T \longrightarrow \infty$. We can therefore estimate the parameters of interest with the truncated models. Similar arguments can be applied to the expanded restricted truncated model. The unrestricted model and the expanded restricted model basically share the same estimation procedure.

The main estimation challenge in a network context is the high-dimensionality problem. We have $N \times N \times p$ unknown parameters in the autoregressive matrices $A_k^p$, $k = 1, ..., p$, as well as $\frac{N(N+1)}{2}$ unknown parameters in the contemporaneous covariance matrix $\Sigma_u^p$, but we only have $N \times T$ observations. For a market network, the number of nodes, $N$, can be large. Traditional estimation methods are not reliable when $N \times p$ is close to $T$ or even infeasible when $N \times p > T$. One of the popular ways to solve the high-dimensional problem in statistics is by assuming sparsity such that the effective dimension of the parameter space keeps tractable. The sparsity assumption is convenient to estimate high-dimensional networks. But one thing we need to emphasize here is that we do not assume the network structure, measured by the multiple horizon causality measures table, is sparse. Instead, we only need to assume the autoregressive matrices, and the error concentration matrix are sparse. Since the multiple horizon causality measures are nonlinear functions of the autoregressive matrices and the concentration matrices, the causality table is generally nonsparse. The estimation technique in this section is called the Least Absolute Shrinkage and Selection Operator (LASSO) (see, e.g., Tibshirani (1996)). The sparsity assumption helps us estimate high-dimensional network and the empirical conclusions in this paper rely on this technical assumption.

Under sparsity assumptions, the autoregressive coefficients and the error concentration matrices could be estimated simultaneously (see Barigozzi and Brownlees (2016)). As the dimension of the unknown parameter space is huge, however, this estimation procedure could be time intensive. Note that the multiple horizon causality measures requires estimating as many as $N+1$ models (one unrestricted model and $N$ restricted models) to quantify the effects from one to others. The estimation efficiency, in terms of computational time, is also an important issue to be concerned when $N$ is large.

For empirical convenience, we apply a faster two-stage estimation procedure. At stage one, we use the Adaptive LASSO regression (see, e.g., Zou (2006)) to estimate the autoregressive coefficients. At stage two, the error concentration matrix can be estimated by the residuals from the stage one. It comes a cost that the rate of convergence of the estimator in the second stage will depend on the estimator in the first stage (see Barigozzi and Brownlees (2014)), and thus this method is theoretically less desirable than the joint estimation method in Barigozzi and Brownlees (2016). Nonetheless, we do get interesting results even using estimation procedure provided by Barigozzi and Brownlees (2014). The investigation utilizing the newly-proposed one-step method is left for future research.

### 1.4.1 Autoregressive Matrix Estimation

Each of the $N$ equation of the unrestricted VAR($p$) model can be written as

$$X_i(t) = \alpha_i' z(t) + u_i^p(t), \tag{1.13}$$

where $X_i(t)$ is the $i$th univariate time series in $X(t)$. $z(t) = (X'(t-1); X'(t-2); \ldots; X'(t-p))'$ is the $Np \times 1$ vector of lagged observations. $\alpha_i = (\alpha_{1i1}, \ldots, \alpha_{1iN}, \ldots, \alpha_{pi1}, \ldots, \alpha_{piN})'$ is a $Np \times 1$ parameter vector, such that $\text{vec}(\alpha_1'; \ldots; \alpha_N') = \text{vec}([A_1^p; A_2^{;}, \ldots; A_p^p]')$. For each of the $N$ equation of the expanded restricted VAR($p$) model, similarly,

$$X_i(t) = \bar{\alpha}_i' z_0(t) + \varepsilon_i^p(t), \tag{1.14}$$

the unknown autoregressive coefficient vector $\bar{\alpha}_i = (\bar{\alpha}_{1i1}, \ldots, \bar{\alpha}_{1iN-m_1}, \ldots, \bar{\alpha}_{pi1}, \ldots, \bar{\alpha}_{piN-m_1})'$ is a $(N-m_1)p \times 1$ vector, such that $\text{vec}(\bar{\alpha}_1'; \ldots; \bar{\alpha}_N') = \text{vec}([\bar{A}_1^p; \bar{A}_2^p; \ldots; \bar{A}_p^p]')$. $z_0(t) = (X_0'(t-1); X_0'(t-2); \ldots; X_0'(t-p))'$ is the $(N-m_1)p \times 1$ vector of lagged observations.

The Adaptive LASSO estimators of $\alpha_i$ and $\bar{\alpha}_i$ are defined respectively as

$$\hat{\alpha}_{Ti} = \underset{\alpha}{\text{argmin}} \frac{1}{T} \sum_{t=1}^{T} [X_i(t) - \alpha_i' z(t)]^2 + \frac{\lambda_T}{T} \sum_{j=1}^{Np} w_{Tij} |\alpha_{ij}| \quad \text{for } i = 1, \ldots, N, \tag{1.15}$$

$$\hat{\bar{\alpha}}_{Ti} = \underset{\bar{\alpha}}{\text{argmin}} \frac{1}{T} \sum_{t=1}^{T} [X_i(t) - \bar{\alpha}_i' z_0(t)]^2 + \frac{\lambda_T}{T} \sum_{j=1}^{(N-m_1)p} \bar{w}_{Tij} |\bar{\alpha}_{ij}| \quad \text{for } i = 1, \ldots, N \tag{1.16}$$

where $\lambda_T$ is an appropriate pre-selected value controlling the overall estimated sparsity level in the autoregressive models. If $\lambda_T$ equals 0, then the LASSO estimation is simply the OLS estimation and every element in $\alpha_i$ has to be estimated; if $\lambda_T \longrightarrow \infty$, the estimates of the parameter $\alpha_i$ are all zeros, which means the estimated autoregressive coefficients are perfectly sparse. The choice of $\lambda_T$ can be made by selecting it by the BIC criterion or by Cross-Validations. $w_{Tij}$ and $\bar{w}_{Tij}$ are pre-estimator weighted penalties to the sparse structures of $\alpha_i$ and $\bar{\alpha}_i$. They help to separate zero coefficients from nonzero coefficients when regressors are highly correlated. Following Zou (2006) we use $w_{Tij} = \frac{1}{|\hat{\alpha}_{ij}^{LASSO}|}$ and $\bar{w}_{Tij} = \frac{1}{|\hat{\bar{\alpha}}_{ij}^{LASSO}|}$ as the weighted penalties to $|\alpha_{ij}|$ and $|\bar{\alpha}_{ij}|$ respectively, where $\hat{\alpha}_{ij}^{LASSO}$ and $\hat{\bar{\alpha}}_{ij}^{LASSO}$ are the standard LASSO estimators when $w_{Tij} = 1$ for $\alpha_{ij}$ and $\bar{w}_{Tij} = 1$ for $\bar{\alpha}_{ij}$.

In order to maintain asymptotic properties in estimating $\alpha_i$ and $\bar{\alpha}_i$ in a high-dimensional context, sparsity assumptions are required. We denote the sets of nonzero entries in $\alpha_i$ and in $\bar{\alpha}_i$ as $\mathscr{A}_i$, which has $q_{Ti}^{\mathscr{A}}$ elements, and $\bar{\mathscr{A}}_i$, which has $q_{Ti}^{\bar{\mathscr{A}}}$ elements. $\mathscr{A}_i^C$ and $\bar{\mathscr{A}}_i^C$ are the sets of zero entries in $\alpha_i$ and in $\bar{\alpha}_i$, respectively. $q_{Ti}^{\mathscr{A}}$ and $q_{Ti}^{\bar{\mathscr{A}}}$ are functions of $T$. Since the estimation of $\alpha_i$ is similar to the estimation of $\bar{\alpha}_i$, we here only discuss the sparsity of $\alpha_i$. Following Barigozzi and Brownlees (2014), the key assumptions on the number of nonzero entries in the autoregressive coefficients and the pre-selected penalty constant controlling the overall estimated sparsity level are $q_{Ti}^{\mathscr{A}} = o\left(\sqrt{\frac{T}{\log T}}\right)$, $\frac{\lambda_T}{T}\sqrt{q_{Ti}^{\mathscr{A}}} = o(1)$, $\lim_{T\to\infty} \frac{\lambda_T}{T}\sqrt{\frac{T}{\log T}} = \infty$, $\sqrt{\frac{q_{Ti}^{\mathscr{A}} \log T}{T}} = o\left(\frac{\lambda_T}{T}\right)$ and $\frac{\lambda_T}{T^{1-c_1}}\sqrt{q_{Ti}^{\mathscr{A}}} = O(1)$ for $i = 1,...,N$. These assumptions provide the restrictions among the the underlying true sparsity level ($q_{Ti}^{\mathscr{A}}$), pre-selected penalty constant controlling the overall estimated sparsity level ($\lambda_T$) and rate of number of nodes $N$ as $T$ grows to infinity ($c_1$). To identify the zero entries in $\alpha_i$, we also need the following assumption on the signal strength: For all $i = 1,...,N$, there exists a sequence of positive real numbers $\{s_{Ti}^{\mathscr{A}}\}$ such that $|\alpha_{ij}| > s_{Ti}^{\mathscr{A}}$ and $\lim_{T\to\infty} \frac{s_{Ti}^{\mathscr{A}}}{\frac{\lambda_T}{T}\sqrt{q_{Ti}^{\mathscr{A}}}} = \infty$ for all $\alpha_{ij} \in \mathscr{A}_i$.

**Proposition 1.4.6.** *Under the Assumption 1 - 6 in Appendix A.1, as* $T \longrightarrow \infty$,

1. *if* $\alpha_{ij} \in \mathscr{A}_i^C$, $Prob\{\hat{\alpha}_{Tij} = 0\} \to 1, \quad i = 1,...,N$
2. *if* $\bar{\alpha}_{ij} \in \bar{\mathscr{A}}_i^C$, $Prob\{\hat{\bar{\alpha}}_{Tij} = 0\} \to 1, \quad i = 1,...,N$
3. $\hat{\alpha}_{Ti} \xrightarrow{p} \alpha_i$, *and thus* $\hat{A}_{Tk}^p \xrightarrow{p} A_k$ *for* $k = 1,...,p$
4. $\hat{\bar{\alpha}}_{Ti} \xrightarrow{p} \bar{\alpha}_i$ *and thus* $\hat{\bar{A}}_{Tk}^p \xrightarrow{p} \bar{A}_k^\phi$ *for* $k = 1,...,p$

*Proof.* See in Appendix A.2. □

Proposition 1.4.6 states that the Adaptive LASSO estimators in (1.15) and (1.16) correctly select the nonzero coefficients asymptotically, and the estimators are consistent. Even if the dimension of the network is large, this estimation procedure can still safely concentrate on estimating the nonzero coefficients using the limited information from the observable sample, given the sparsity assumption on the true coefficients vector.

### 1.4.2 Contemporaneous Covariance Matrix Estimation

The contemporaneous covariance matrix can be estimated by the sparse concentration matrix via the sparse errors partial correlations. We use the estimation strategy in Peng, Wang, Zhou and Zhu (2009) and Barigozzi and Brownlees (2014). The errors partial correlations matrix $\rho$ has generic component $\rho_{ij}$. The concentration matrix in the unrestricted model, $S_u^p \equiv [\Sigma_u^p]^{-1}$, and the concentration matrix in the expanded restricted model, $S_\nu^p \equiv [\Sigma_\nu^p]^{-1}$, have the following relationship with their respective errors correlations:

$$\rho_{ij}^u \equiv \mathrm{Corr}(u_i^p, u_j^p) = -\frac{s_{ij}^u}{\sqrt{s_{ii}^u s_{jj}^u}} \tag{1.17}$$

$$\rho_{ij}^\nu \equiv \mathrm{Corr}(\nu^p, \nu^p) = -\frac{s_{ij}^\nu}{\sqrt{s_{ii}^\nu s_{jj}^\nu}} \tag{1.18}$$

where $s_{ij}^u$ is the $(i,j)$ component of $S_u^p$ and $s_{ij}^\nu$ is the $(i,j)$ component of $S_u^\nu$. Moreover, the errors correlations can be also expressed as the coefficients of the linear regressions (see Lemma 1 in Peng et al. (2009)):

$$u_{ti}^p = \sum_{j \neq i}^N \rho_{ij}^u \sqrt{\frac{s_{ii}^u}{s_{jj}^u}} u_{tj}^p + \eta_{ti}^u, \quad i = 1, ..., N, \quad t = 1, ..., T \tag{1.19}$$

$$\nu_{ti}^p = \sum_{j \neq i}^N \rho_{ij}^\nu \sqrt{\frac{s_{ii}^\nu}{s_{jj}^\nu}} \nu_{tj}^p + \eta_{ti}^\nu, \quad i = 1, ..., N, \quad t = 1, ..., T \tag{1.20}$$

We assume the concentration matrices as well as the correlations matrices are sparse and denote the sets of nonzero entries in the unrestricted and restricted errors correlation matrices as $\mathscr{Q}_u$ and $\mathscr{Q}_\nu$ respectively. $\mathscr{Q}_u^C$ and $\mathscr{Q}_\nu^C$ are the sets of zero entries in the unrestricted and restricted errors correlation matrices. The LASSO estimator of the errors partial correlations in the unrestricted model (1.3) and the one in the restricted model (1.5) are defined respectively as

$$\hat{\rho}_T^u = \underset{\rho^u}{\text{argmin}} \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}(\hat{u}_{ti} - \sum_{j\neq i}^{N}\rho_{Tij}^u\sqrt{\frac{\hat{s}_{Tii}^u}{\hat{s}_{Tjj}^u}}\hat{u}_{tj})^2 + \frac{\gamma_T}{T}\sum_{i=2}^{N}\sum_{j=1}^{i-1}|\rho_{ij}^u|, \quad (1.21)$$

$$\hat{\rho}_T^v = \underset{\rho^v}{\text{argmin}} \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}(\hat{v}_{ti} - \sum_{j\neq i}^{N}\rho_{Tij}^v\sqrt{\frac{\hat{s}_{Tii}^v}{\hat{s}_{Tjj}^v}}\hat{v}_{tj})^2 + \frac{\gamma_T}{T}\sum_{i=2}^{N}\sum_{j=1}^{i-1}|\rho_{ij}^v| \quad (1.22)$$

where $\hat{u}_{ti} = X_i(t) - \hat{\alpha}_i' z(t)$ and $\hat{v}_{ti} = X_i(t) - \hat{\bar{\alpha}}_i' z_0(t)$. $\gamma_T$ is the tuning parameter controlling the model sparsity level as $\lambda_T$ in (1.15) and in (1.16). The estimator of the unrestricted concentration matrix $S_u^p$, denoted as $\hat{S}_T^u$, and the the estimator of the expanded restricted concentration matrix $S_v^p$, denoted as $\hat{S}_T^v$, have entries $\hat{s}_{Tij}^u = -\hat{\rho}_{Tij}^u\sqrt{\hat{s}_{Tii}^u\hat{s}_{Tjj}^u}$ and $\hat{s}_{Tij}^v = -\hat{\rho}_{Tij}^v\sqrt{\hat{s}_{Tii}^v\hat{s}_{Tjj}^v}$. The estimators $\hat{s}_{Tii}^u$ and $\hat{s}_{Tii}^v$ are given respectively by

$$\hat{s}_{Tii}^u = \left[\frac{1}{T-1}\sum_{t=1}^{T}(\hat{\eta}_{ti}^u)^2\right]^{-1}, \quad (1.23)$$

$$\hat{s}_{Tii}^v = \left[\frac{1}{T-1}\sum_{t=1}^{T}(\hat{\eta}_{ti}^v)^2\right]^{-1} \quad (1.24)$$

where $\hat{\eta}_{ti}^u = \hat{u}_{ti} - \sum_{j\neq i}^{N}\rho_{Tij}^u\sqrt{\frac{\hat{s}_{Tii}^u}{\hat{s}_{Tjj}^u}}\hat{u}_{tj}$ and $\hat{\eta}_{ti}^v = \hat{v}_{ti} - \sum_{j\neq i}^{N}\rho_{Tij}^v\sqrt{\frac{\hat{s}_{Tii}^v}{\hat{s}_{Tjj}^v}}\hat{v}_{tj}$

The estimator of the unrestricted errors concentration matrix, $\hat{S}_T^u$, can be obtained by iterating between (1.21) and (1.23). The estimator of the expanded restricted errors concentration matrix, $\hat{S}_T^v$, can be obtained by iterating between (1.22) and (1.24). For more discussions on the assumptions to estimate the correlation matrices, we refer readers to see Peng et al. (2009) and Barigozzi and Brownlees (2014).

**Proposition 1.4.7.** *Under the Assumption 1 - 9 in Appendix A.1, as $T \to \infty$,*

1. *if $\rho_{ij}^u \in \mathscr{Q}_u^C$, $Prob\{\hat{\rho}_{Tij}^u = 0\} \to 1, \quad i,j = 1,...,N$*
2. *if $\rho_{ij}^v \in \mathscr{Q}_v^C$, $Prob\{\hat{\rho}_{Tij}^v = 0\} \to 1, \quad i,j = 1,...,N$*
3. *$\hat{\rho}_{Tij}^u \xrightarrow{p} \rho_{ij}^u$, and thus $\hat{S}_T^u \xrightarrow{p} S_u \equiv \Sigma_u^{-1}$*
4. *$\hat{\rho}_{Tij}^v \xrightarrow{p} \rho_{ij}^v$, and thus $\hat{S}_T^v \xrightarrow{p} S_v \equiv \Sigma_v^{-1}$*

*Proof.* See in Appendix A.3. □

Proposition 1.4.7 states that the LASSO estimators in (1.21) and (1.22) correctly select the nonzero coefficients in the errors correlation matrics asymptotically and the estimators are consistent. By the relationships between errors correlations and the concentration matrix in (1.17) and (1.18), we obtain the consistent estimators of the concentration matrices for the unrestricted model (1.3) and the concentration matrices for the expanded unrestricted model (1.5).

### 1.4.3 Granger Causality Measures Estimation

Note that each of the multiple horizon causality measures under the Assumption 1.4.1 is mainly composed by two parts (see Definition 1.4.3 and Remark 1.4.2): i) autoregressive coefficients in the unrestricted model (1.3) and in the expanded restricted model (1.5); ii) contemporaneous covariances in the unrestricted model (1.3) and in the expanded restricted model (1.5). We have already shown consistency of the estimators in Proposition 1.4.6 and Proposition 1.4.7.

Finally, the estimator of the multiple horizon causality measure, from $X^W$ to $X^W$, is defined as

$$\hat{C}^h_{TWW} := \ln\left[\frac{\det\{\hat{\Sigma}_W[X^W(t+h)|\mathscr{F}_{-W}(t)]\}}{\det\{J_3\hat{\Sigma}[X(t+h)|\mathscr{F}(t)]J_3'\}}\right], \tag{1.25}$$

where

$$\hat{\Sigma}[X(t+h)|\mathscr{F}(t)] = \sum_{q=0}^{h-1}\hat{\varphi}_q(\hat{S}^u_T)^{-1}\hat{\varphi}_q', \tag{1.26}$$

$$\hat{\Sigma}_W[X^W(t+h)|\mathscr{F}_{-W}(t)] = J_3\left[\sum_{q=0}^{h-1}\hat{\phi}_q(\hat{S}^v_T)^{-1}\hat{\phi}_q'\right]J_3', \tag{1.27}$$

$$\hat{\phi}_q = \sum_{k=1}^{q}(\hat{\bar{A}}^p_{Tk}J_2)\hat{\phi}_{q-k}, \tag{1.28}$$

$$\hat{\varphi}_q = \sum_{k=1}^{q}\hat{A}^p_{Tk}\hat{\varphi}_{q-k}, \tag{1.29}$$

$$\hat{\varphi}_0 = I_N,\ \hat{\phi}_0 = I_N. \tag{1.30}$$

The estimator of the multiple horizon causality measures, from $X^W$ to $X^Y$, is defined as

$$\hat{C}^h_{TWY} := \ln \left[ \frac{\det\{J_0 \hat{\Sigma}[X_0(t+h)|\mathscr{F}_{-W}(t)] J_0'\}}{\det\{J_1 \hat{\Sigma}[X(t+h)|\mathscr{F}(t)] J_1'\}} \right], \tag{1.31}$$

where

$$\hat{\Sigma}[X(t+h)|\mathscr{F}(t)] = \sum_{q=0}^{h-1} \hat{\varphi}_q (\hat{S}^u_T)^{-1} \hat{\varphi}_q', \tag{1.32}$$

$$\hat{\Sigma}[X_0(t+h)|\mathscr{F}_{-W}(t)] = \sum_{q=0}^{h-1} \hat{\bar{\varphi}}_q (\hat{S}^\varepsilon_T)^{-1} \hat{\bar{\varphi}}_q', \tag{1.33}$$

$$(\hat{S}^\varepsilon_T)^{-1} = J_2 (\hat{S}^\nu_T)^{-1} J_2', \tag{1.34}$$

$$\hat{\bar{\varphi}}_q = \sum_{k=1}^{q} (J_2 \hat{\bar{A}}^p_{Tk})' \hat{\bar{\varphi}}_{q-k}, \tag{1.35}$$

$$\hat{\varphi}_q = \sum_{k=1}^{q} \hat{A}^p_{Tk} \hat{\varphi}_{q-k}, \tag{1.36}$$

$$\hat{\varphi}_0 = I_N, \ \hat{\bar{\varphi}}_0 = I_{N-m_1}. \tag{1.37}$$

**Theorem 1.4.8.** *Under the Assumptions 1 - 9 in Appendix A.1, for any given h, $h = 1, 2, ...$, as $T \longrightarrow \infty$,*

1. $\hat{\Sigma}[X(t+h)|\mathscr{F}(t)] \xrightarrow{p} \Sigma[X(t+h)|\mathscr{F}(t)]$;
2. $\hat{\Sigma}[X_0(t+h)|\mathscr{F}_{-W}(t) \xrightarrow{p} \Sigma[X_0(t+h)|\mathscr{F}_{-W}(t)]$;
3. $\hat{\Sigma}_W[X^W(t+h)|\mathscr{F}_{-W}(t) \xrightarrow{p} \Sigma_W[X^W(t+h)|\mathscr{F}_{-W}(t)]$;
4. $\hat{C}^h_{TWY} \xrightarrow{p} C_L(X^W \underset{h}{\longrightarrow} X^Y|I)$;
5. $\hat{C}^h_{TWW} \xrightarrow{p} C_L(X^W \underset{h}{\longrightarrow} X^W|I)$.

*Proof.* See in Appendix A.4. □

Now, we have the consistent estimators of the multiple horizon causality measures for any given network. An estimation procedure similar to the one proposed above can be applied to measuring the point-wise edge strength, $i \underset{C,h}{\rightarrow} j$, and the group-wise edge strength, $(i_1, ..., i_{n_1}) \underset{C,h}{\rightarrow} (j_1, ..., j_{n_2})$, for arbitrary horizon $h \geq 1$, and has similar limit properties.

## 1.5 Network Connectedness Measures

The world is not flat. While the relationships of entities in an economy can be modelled by 2-dimensional network representations, the economy itself, however, is multi-dimensionally structured. Different firms play different roles. Some of them are alike: insurances companies sell insurances, even though a wide range of insurance product; some of them are distinctive: restaurants serve cuisines while Space X provides space transportation services. We do not assume we have the prior knowledge of their exact roles, but we have their interconnection structures that can be measured by our causality table.

We consider an economy of interest as a $N$-dimensional Euclidean space. For any given firm $i$ we associate with it a vector in the Euclidean space with coordinates given the row (or column) entries from the causality table. Since the $i$th row (column) in the causality table measures the effects of firm $i$ to (from) others, the direction of a firm's vector can be interpreted as "what the firm's role is": firm $i$'s vector direction tends to point to the companies that firm $i$ has more relationships to; the norm of a firm's vector can be interpreted as "how strong the firm's role is": the norm of firm $i$'s vector measures the extent of the firm $i$' relationships to all companies in the economy. If we use the interconnection relationships of a firm to others as a proxy of the role of the firm in an economy, the causality table gives us the firms' coordinates of their roles in the space of a multi-dimensional economy. It helps us to study the structure and strength of the interconnectedness of an economic network with geometric illustrations, which shares the advantage of using network to study economic and financial interconnections.

Following this logic, we define our new connectedness measures in the market network base on the multi-dimensional economy setting. We hereafter take the estimated causality measures table as given. Note that a network can be divided into several subgroups, the network can be viewed as a combination of its sub-networks. In a stock market, for example, the market index can be viewed as the weighted average of the prices of individual stocks as well as the weighted average of different sector indices. Since an economic network can be viewed as a network among firms (firm-wise market), whose interconnections are measured by our point-wise edges method ($i \to j$), as well as a network among sectors (sector-wise market), whose interconnections are measured by group-wise edges method ($[i_1, i_2, ..., i_n] \to [j_1, j_2, ..., j_m]$), we have three types of connectedness measures to gauge network interconnections: i) firm-wise market connectedness, which measures the interconnectedness of a firm-wise market; ii) firm-wise connectedness within a sector, which measures the interconnectedness within a given sector in a firm-wise market; and iii) sector-wise market connectedness, which measures the interconnectedness of a sector-wise market. These three types of connectedness measures fully take advantage of the flexibility of our network measurement method, so they can be applied to study market network connectedness in more flexible ways than Billio et al. (2012) and Diebold and Yilmaz (2014).

### 1.5.1 Firm-wise Market Connectedness Measures

Market network connectedness can be decomposed by each firm's connectedness to the market. Firms' roles in an economy determine the firms' connectedness to the market network. As firm $i$'s vector direction represents firm $i$'s connectedness in the economy, we will use firm $i$'s vector direction as the foundation to measure the firm $i$'s connectedness to the market network.

The connectedness, in term of economic role, of firm $i$ to the economy can be measured by the angle of firm $i$'s vector to the subspace of the economy composed by all other firms. In Figure 1.3, we use a simple 3-dimensional economy space to illustrate this idea. The economy has only three firms: $i$, $k_1$ and $k_2$. We want to study firm $i$'s role connectedness to this market. From the causality table, we choose the vector of $i$, ($C_{ii}$,$C_{ik_1}$,$C_{ik_2}$). It measures the relationships from $i$ to $i$, $k_1$ and $k_2$. The direction of ($C_{ii}$,$C_{ik_1}$,$C_{ik_2}$) in the 3-dimensional space determines firm $i$'s economic role in this market. $\theta_i$ is the angle of $i$'s vector to the subspace of the economy composed by $k_1$ and $k_2$. If we take $k_1$ and $k_2$ as a unit, $\theta_i$ exactly measures the economic connectedness of firm $i$ to $k_1$ and $k_2$. When $\theta_i = \frac{\pi}{2}$, $i$ has no impact on $k_1$ and $k_2$; when $\theta_i = 0$, $i$ is fully accounted for by $k_1$ and $k_2$.

Given forecast horizon $h$, the $i$th row of the causality table measures the directed and weighted edges from the node $i$ to all nodes in an N-dimensional market network, which has a node set

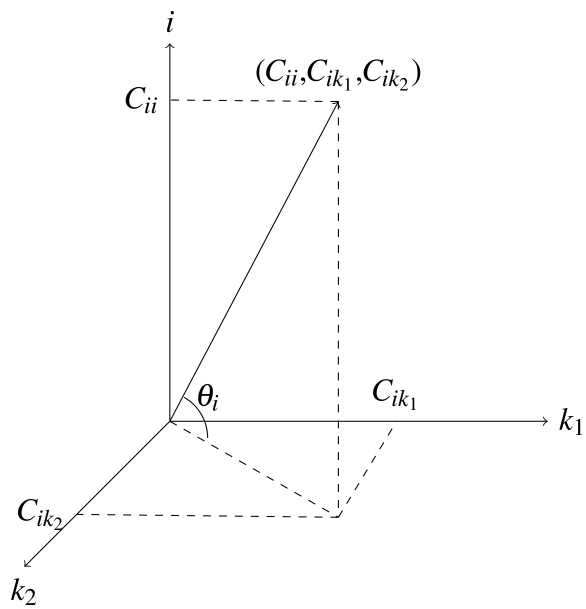



Figure 1.3: Relative connectedness between $i$ and network

$V = \{1,...,N\}$. Let $OUT_i^h = [C_{i1}^h, C_{i2}^h, ..., C_{iN}^h]'$, $OUT_i^h$ contains information about all the firms influenced by $i$. They are the "out" effects from $i$ to all firms in the market. For $i = 1,...,N \in V$, we define the "out" connectedness angle of the firm $i$ to the economy as $\theta_{i,V}^{out}(h)$,

$$\theta_{i,V}^{out}(h) = \arcsin \frac{C_{ii}^h}{\|OUT_{i,V}^h\|_2} \tag{1.38}$$

where we assume $\|OUT_{i,V}^h\|_2 > 0$, which is equivalent to say there exists $j \in \{1,...,N\}$ such that $C_{ij}^h \neq 0$. If $\|OUT_{i,V}^h\|_2 = 0$, we let $\theta_{i,V}^{out}(h) = 0$ .

$\theta_{i,V}^{out}(h)$ measures the "out" connectedness from firm $i$ to the economy and is a relative connectedness strength since it has been rescaled in $[0, \pi/2]$. The connectedness angle $\theta_{i,V}^{out}(h) = \pi/2$ if and only if $C_{ii}^h > 0$ and $C_{ij}^h = 0$ for any $j \in \{1,...,i-1,i+1,...,N\}$, which implies firm $i$ is isolated with the economy in the sense that it has no impact on other companies. If $C_{ii}^h = 0$ and thus the projection angle $\theta_{i,V}^{out}(h) = 0$, it implies all relationships from firm $i$ to the economy are all from its impacts to other firms in the economy.

The relative connectedness strength of firm $i$ to the economy, measured by $\theta_{i,V}^{out}(h)$, considers the economic role of firm $i$ to the economy. It is a directional measure, and it is more related to relative economic connectedness. The extent of how strong the economic connectedness, however, is not simply captured by $\theta_{i,V}^{out}(h)$. Besides the connectedness angle, we are also interested in the absolute connectedness strength. We define a function $K_{out}\left(\|OUT^h_{i,V}\|, \theta^{out}_{i,V}(h)\right)$ as a general formula of the absolute connectedness strength of firm $i$ to the economy. $K_{out}\left(\|OUT^h_{i,V}\|, \theta^{out}_{i,V}(h)\right)$ is a function of firm $i$'s connectedness angle, $\theta^{out}_{i,V}(h)$, and its causation strength to the economy, $\|OUT^h_{i,V}\|$. $K_{out}\left(\|OUT^h_{i,V}\|, \theta^{out}_{i,V}(h)\right)$ should at least satisfy the following properties:

$$K_{out}\left(\|OUT^h_{i,V}\|, \frac{\pi}{2}\right) = 0 \tag{1.39}$$

$$K_{out}\left(0, \theta^{out}_{i,V}(h)\right) = 0 \tag{1.40}$$

$$\frac{\partial K_{out}\left(\|OUT^h_{i,V}\|, \theta^{out}_{i,V}(h)\right)}{\partial \|OUT^h_{i,V}\|} \geq 0 \tag{1.41}$$

$$\frac{\partial K_{out}\left(\|OUT^h_{i,V}\|, \theta^{out}_{i,V}(h)\right)}{\partial \theta^{out}_{i,V}(h)} \leq 0 \tag{1.42}$$

The firm $i$ has no connectedness to the economy if has no impact on all other firms in the economy. The absolute connectedness strength between firm $i$ to the economy, should be an non-decreasing function of its causation strength to the economy and an nonincreasing function of its connectedness angle to the economy.

A simple functional specification of the absolute connectedness strength of firm $i$ to the economy we use in this paper is

$$K_{out}\left(\|OUT^h_{i,V}\|, \theta^{out}_{i,V}(h)\right) = \|OUT^h_{i,V}\| \cos\theta^{out}_{i,V}(h). \tag{1.43}$$

This absolute connectedness strength can be easily decomposed in to the causation strength, $\|OUT^h_{i,V}\|$, and the connectedness angle $\theta^{out}_{i,V}(h)$. In geometric terms, $K_{out}\left(\|OUT^h_{i,V}\|, \theta^{out}_{i,V}(h)\right)$ just measures the norm of the projection of $OUT^h_{i,V}$ on the subspace spanned by the all other firms in the economy. We again use a simple 3-dimensional economy space to illustrate this idea. The economy has only three firms: $i$, $k_1$ and $k_2$. The absolute connectedness strength of firm $i$ to this economy is the projection of the vector ($C_{ii}$,$C_{ik_1}$,$C_{ik_2}$) on the subspace spanned by $k_1$ and $k_2$, which is shown in Figure 1.4.

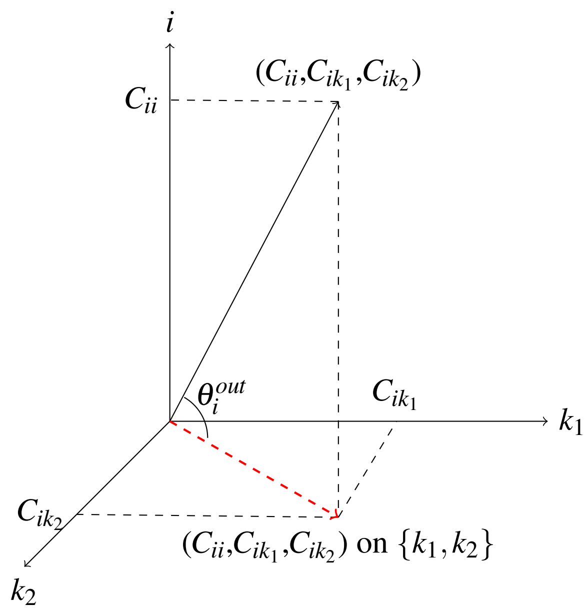



Figure 1.4: Absolute connectedness between $i$ and network

In summary, our absolute connectedness strength of firm $i$ to the economy simultaneously takes the firm $i$'s economic role structure and the economic role strength into account. In addition, the absolute connectedness strength can be easily decomposed into these two parts. Moreover, it has nice geometric interpretations in an N-dimensional economic space as illustrated by Figure 1.4. Therefore, our market connectedness measures, the means of all firms' connectedness measures to the economy, will also enjoy these features.

We define our firm-wise market network connectedness as the sum of firms' connectedness weighted by number of firms in the market. A common practise in the network literature is to use the number of nodes to measure the size of a network. This ensures that the connectedness of a sector will not simply grow with market size and be comparable across different markets.

**Definition 1.5.1.** Given a market network with node set $V = \{1, 2, ..., N\}$, the Firm-wise Market Network Relative Connectedness Structure Measure of "out" effects at horizon $h$, $MRC_{V_C}^{out}(h)$, is defined as following:

$$MRC_{V_C}^{out}(h) = \frac{1}{N} \sum_{i=1}^{N} \cos \theta_{i,V}^{out}(h) \tag{1.44}$$

**Definition 1.5.2.** Given a market network with node set $V = \{1, 2, ..., N\}$, the Firm wise Market Network Absolute Connectedness Strength Measure of "out" effects at horizon $h$, $MAC_{V_C}^{out}(h)$, is defined as following:

$$MAC_{V_C}^{out}(h) = \frac{1}{N}\sum_{i=1}^{N} \|OUT_{i,V}^{h}\| \cos\theta_{i,V}^{out}(h) \tag{1.45}$$

**Remark 1.5.3.** If the edges in a network are unweighted, the connection of node $i$ to the network can be solely characterized by the connectedness angle, $\theta_{i,V}^{out}(h)$, irrespective to its unweighted absolute connectedness magnitude to the network, $\|OUT_{i,V}^{h}\|$. Therefore, the Firm-wise Market Network Relative Connectedness Structure Measure is basically equivalent to the Firm-wise Market Network Absolute Connectedness Strength Measure in the context of unweighted network.

Note that the edges in our network are directed. Following similar procedures, we can also define market connectedness measures at "in" direction. Given a forecast horizon $h$, the $i$th column of the causality table measures the directed and weighted edges to the firm $i$ from all firms in the N-dimensional market network, which has node set $V = \{1,...,N\}$. Let $IN_i^h = [C_{1i}^h, C_{2i}^h, ..., C_{Ni}^h]'$, $IN_i^h$ contains all the directed edges information pointed from $i$. They are the "in" effects to $i$ from all firms in the market. For $i = 1,...,N \in V$, the "in" connectedness angle of the firm $i$ to the economy, $\theta_{i,V}^{in}(h)$, and the "in" absolute connectedness strength of the the firm $i$ to the economy, $K_{in}\big(\|IN_{i,V}^{h}\|, \theta_{i,V}^{in}(h)\big)$, are defined respectively as

$$\theta_{i,V}^{in}(h) = \arcsin\frac{C_{ii}^{h}}{\|IN_{i,V}^{h}\|_2} \tag{1.46}$$

and

$$K_{in}\big(\|IN_{i,V}^{h}\|, \theta_{i,V}^{in}(h)\big) = \|IN_{i,V}^{h}\| \cos\theta_{i,V}^{in}(h) \tag{1.47}$$

**Definition 1.5.4.** Given a market network with node set $V = \{1,2,...,N\}$, the Firm-wise Market Network Relative Connectedness Structure Measure of "in" effects at horizon $h$, $MRC_{V_C}^{in}(h)$, is defined as following:

$$MRC_{V_C}^{in}(h) = \frac{1}{N}\sum_{i=1}^{N} \cos\theta_{i,V}^{in}(h) \tag{1.48}$$

**Definition 1.5.5.** Given a market network with node set $V = \{1,2,...,N\}$, the Firm-wise Market Network Absolute Connectedness Strength Measure of "in" effects at horizon $h$, $MAC_{V_C}^{in}(h)$, is defined as following:

$$MAC_{V_C}^{in}(h) = \frac{1}{N}\sum_{i=1}^{N} \|IN_{i,V}^{h}\| \cos\theta_{i,V}^{in}(h) \tag{1.49}$$

### 1.5.2 Firm-wise Sector Connectedness Measures

The structure of an economic market can be viewed as a sort of network of sectors. Furthermore, there is also a sub-network for each sector in an economy. In this section, we discuss the firm-wise connectedness within each sector. Without loss of generality, we consider an economic network with node set $V$ composed by two sectors, $V_1$ and $V_2$, where $V = \{1,...,N\}$, $V_1 = \{i_1,...,i_{n_1}\}$, $V_2 = \{j_1,...,j_{n_2}\}$, $V_1 \cap V_2 = \emptyset$, $V_1 \cup V_2 = V$ and $n_1 + n_2 = N$. $V_1$ and $V_2$ are disjoint and complete sub-network elements of $V$.

Within a sector $V_z$, $z = 1$ or 2, our sector connectedness measures are defined in a similar manner as the firm-wise market connectedness measures in section 1.5.1. Sector connectedness is the sum of firms' connectedness weighted by sector size. This ensures that the connectedness of a sector will not simply grow with its size and be comparable across different sectors in an economy. Of course, sector size can be measured by different ways, such as by the number of firms in the sector or by the market value of the sector. In this paper, we use the number of firms for simplicity. This is also in line with the network literature that usually uses the number of nodes to describe the size of a network.

**Definition 1.5.6.** Given a sector node set $V_z$, $z = 1,2$, the Firm-wise Sector Relative Connectedness Structure Measure of "out" effects within sector $z$ at horizon $h$, $MRC_{V_z}^{out}(h)$, is defined as following:

$$MRC_{V_z}^{out}(h) = \frac{1}{n_z}\sum_{i=1}^{n_z} \cos\theta_{i,V_z}^{out}(h), \tag{1.50}$$

where $n_z = |V_z|$ is the number of nodes in the sector node set $V_z = \{i_1,...,i_{n_1}\}$ , $i \in V_z$, $\theta_{i,V_z}^{out}(h) = \arcsin \frac{C_{ii}^h}{\|OUT_{i,V_z}^h\|_2}$, and $OUT_{i,V_z}^h = [C_{ii_1}^h, C_{ii_2}^h, ..., C_{ii_{n_1}}^h]'$.

**Definition 1.5.7.** Given a sector node set $V_z$, $z = 1,2$, the Firm-wise Sector Absolute Connectedness Strength Measure of "out" effects within sector $z$ at horizon $h$, $MAC_{V_z}^{out}(h)$, is defined as following:

$$MAC_{V_z}^{out}(h) = \frac{1}{n_z}\sum_{i=1}^{n_z} \|OUT_{i,V_z}^{h}\| \cos\theta_{i,V_z}^{out}(h) \tag{1.51}$$

where $OUT^h_{i,V_z}$ and $\theta^{out}_{i,V_z}(h)$ are defined as above.

**Definition 1.5.8.** Given a sector node set $V_z$, $z = 1,2$, the Firm-wise Sector Relative Connectedness Structure Measure of "in" effects within sector $z$ at horizon $h$, $MRC^{in}_{V_z}(h)$, is defined as following:

$$MRC^{in}_{V_z}(h) = \frac{1}{n_z}\sum_{i=1}^{n_z} \cos\theta^{in}_{i,V_z}(h) \tag{1.52}$$

where $n_z = |V_z|$ is the number of nodes in the sector node set $V_z = \{i_1, ..., i_{n_1}\}$ , $i \in V_z$, $\theta^{in}_{i,V_z}(h) = \arcsin\frac{C^h_{ii}}{\|IN^h_{i,V_z}\|_2}$, and $IN^h_{i,V_z} = [C^h_{i_1 i}, C^h_{i_2 i}, ..., C^h_{i_{n_1} i}]'$.

**Definition 1.5.9.** Given a sector node set $V_z$, $z = 1,2$, the Firm-wise Sector Absolute Connectedness Strength Measure of "in" effects within sector $z$ at horizon $h$, $MAC^{in}_{V_z}(h)$, is defined as following:

$$MAC^{in}_{V_z}(h) = \frac{1}{n_z}\sum_{i=1}^{n_z} \|IN^h_{i,V_z}\| \cos\theta^{in}_{i,V_z}(h) \tag{1.53}$$

where $IN^h_{i,V_z}$ and $\theta^{in}_{i,V_z}(h)$ are defined as above.

The sector connectedness measures are just the blocked firm-wise market network connectedness measures for each sector in a firm-wise market .

**Remark 1.5.10.** For any given $h$, if we have $C^h_{ij} = C^h_{ji} = 0$ for any $i \in V_1$ and any $j \in V_2$, then

1. $(n_1+n_2)MRC^{out}_{V_C}(h) = n_1 MRC^{out}_{V_1}(h) + n_2 MRC^{out}_{V_2}(h)$;
2. $(n_1+n_2)MAC^{out}_{V_C}(h) = n_1 MAC^{out}_{V_1}(h) + n_2 MAC^{out}_{V_2}(h)$;
3. $(n_1+n_2)MRC^{in}_{V_C}(h) = n_1 MRC^{in}_{V_1}(h) + n_2 MRC^{in}_{V_2}(h)$;
4. $(n_1+n_2)MAC^{in}_{V_C}(h) = n_1 MAC^{in}_{V_1}(h) + n_2 MAC^{in}_{V_2}(h)$.

The market network connectedness can be obtained by the sector connectedness if the sectors are the disjoint and complete decomposition elements of the market network and if there is no point-wise edge between different sectors. Intuitively speaking, the market connectedness is simply the weighted sum of sectors' connectedness when there is no causality edge between firms across different sectors.

### 1.5.3 Sector-wise Market Connectedness Measures

Similar to the firm-wise market connectedness measures, the sector-wise market connectedness measures also measure market interconnectedness. But the sector-wise market connectedness measures gauge the interconnectedness among different sectors instead of different firms.

In a sector-wise market, nodes are groups of firms. We assume that any firm can only belong an unique sector. Suppose we have $M$ sectors: $V_i$ for $i = 1, 2, ..., M$. Then we have $V = \bigcup_{i=1}^{M} V_i$ and $V_S = \{V_1, V_2, ..., V_M\}$. In this case, the causality table is an $M$ by $M$ matrix. The $i$ row of the causality table, $(C_{V_iV_1}, C_{V_iV_2}, ..., C_{V_iV_M})$, measures the effects from sector $i$ to other sectors. The sector-wise market connectedness measures are defined in a similar manner as the firm-wise market connectedness measures in section 1.5.1.

**Definition 1.5.11.** The Sector-wise Market Relative Connectedness Structure Measure of "out" effects at horizon $h$, $MRC_{V_S}^{out}(h)$, is defined as following:

$$MRC_{V_S}^{out}(h) = \frac{1}{M}\sum_{i=1}^{M} \cos\theta_{V_i,V_S}^{out}(h), \tag{1.54}$$

where $\theta_{V_i,V_S}^{out}(h) = \arcsin\frac{C_{V_iV_i}^h}{\|OUT_{V_i,V_S}^h\|_2}$, and $OUT_{V_i,V_S}^h = [C_{V_iV_1}^h, C_{V_iV_2}^h, ..., C_{V_iV_M}^h]'$.

**Definition 1.5.12.** The Sector-wise Market Absolute Connectedness Strength Measure of "out" effects at horizon $h$, $MAC_{V_S}^{out}(h)$, is defined as following:

$$MAC_{V_S}^{out}(h) = \frac{1}{M}\sum_{i=1}^{M} \|OUT_{V_i,V_S}^h\| \cos\theta_{V_i,V_S}^{out}(h), \tag{1.55}$$

where $\theta_{V_i,V_S}^{out}(h) = \arcsin\frac{C_{V_iV_i}^h}{\|OUT_{V_i,V_S}^h\|_2}$, and $OUT_{V_i,V_S}^h = [C_{V_iV_1}^h, C_{V_iV_2}^h, ..., C_{V_iV_M}^h]'$.

**Definition 1.5.13.** The Sector-wise Market Relative Connectedness Structure Measure of "in" effects at horizon $h$, $MRC_{V_S}^{in}(h)$, is defined as following:

$$MRC_{V_S}^{in}(h) = \frac{1}{M}\sum_{i=1}^{M} \cos\theta_{V_i,V_S}^{in}(h), \tag{1.56}$$

where $\theta_{V_i,V_S}^{in}(h) = \arcsin\frac{C_{V_iV_i}^h}{\|IN_{V_i,V_S}^h\|_2}$, and $IN_{V_i,V_S}^h = [C_{V_1V_i}^h, C_{V_2V_i}^h, ..., C_{V_MV_i}^h]'$.

**Definition 1.5.14.** The Sector-wise Market Absolute Connectedness Strength Measure of "in" effects at horizon $h$, $MAC_{V_S}^{in}(h)$, is defined as following:

$$MAC_{V_S}^{in}(h) = \frac{1}{M}\sum_{i=1}^{M} \|IN_{V_i,V_S}^{h}\| \cos\theta_{V_i,V_S}^{in}(h), \tag{1.57}$$

where $\theta_{V_i,V_S}^{in}(h) = \arcsin\frac{C_{V_iV_i}^{h}}{\|IN_{V_i,V_S}^{h}\|_2}$, and $IN_{V_i,V_S}^{h} = [C_{V_1V_i}^{h}, C_{V_2V_i}^{h}, \ldots, C_{V_MV_i}^{h}]'$.

## 1.6 Application to Implied Volatility Network Structures

In previous sections, we have proposed a flexible network econometric measurement framework, a reliable estimation procedure designed for high-dimensional contexts and new market network connectedness measures. In this section, we illustrate the wide range of applications of our market network measurement methods by investigating a high-dimensional volatility network in the US equity market. We would like to study how the volatility network is structured and how it changes over time. Fruitful information extracted from the empirical exercises can be easily visualized by our reporting figures.

More specifically, we study the static volatility network with the full sample from 2000 to 2015 to see how firms and sectors connect to each other. We investigate the dynamics of the network structures to see how the interconnections among firms and the interconnections among sectors varied in the past 15 years. The market connectedness measures proposed in this paper are designed for measuring market systemic risk. It is a common wisdom that the systemic risk played an important role in the 2007-2009 financial crisis. Thus we examine dynamic market connectedness with our measures, and compare it with market indices (i.e. VIX index) before, during and after the crisis period. Our market connectedness measures are constructed based on the directed and weighted edges in the market network, and the superiority of the "directed" and "weighted" edges analysis against the "undirected" and "unweighted" edges analysis is demonstrated by the asymmetric effects between the financial sector and other sectors in the volatility network.

### 1.6.1 Data

Firms and sectors are connected with trade links or business relationships. It is an impossible mission to collect all qualitative and quantitative business information at firm-level to reveal their interconnections. As Diebold and Yilmaz (2014) argue, however, stock markets, which reflect forward-looking assessments of many thousands of smart, strategic and often privately-informed agents, provide us with feasible information that is close to the true business conditions and interconnections. For instance, there are numerous investment opportunities in the world, and using the S&P 500 index as a benchmark is almost a convention when evaluating excess returns in asset management. Therefore, we will study the crisis-sensitive volatility network in the US stock market. In addition, we are also interested in examining whether our volatility connectedness measures can reflect the underlying market systemic risk that plays an important role in the recent global financial crisis.

The volatility in stock markets is latent, so we need an volatility proxy. The well-known VIX, which has been widely accepted as a market volatility index by financial practitioners, is calculated from implied volatilities of the S&P 500 index options. It is sensitive to market turmoils. For each firm, we also exploit the information in their respective option contracts. We use implied volatility in our volatility network analysis, rather than using realized volatility estimated from stock intraday prices (see Diebold and Yilmaz (2014) and Barigozzi and Brownlees (2016)), for the quantities we are dealing with are more comparable to market indices (e.g., VIX). Similar to the VIX index known as a "market fear" index, our implied volatility network connectedness can also be viewed as "individual fear" connectedness. Volatility or implied volatility is sensitive to "terrifying news" in financial markets. For instance, the 9/11 attack terrified the people in the stock market and lead implied volatilities to jump up rapidly. Although the 9/11 event had very minor impacts on most firms' real business conditions and their interconnections, its shocks would spill over from firms to firms and from sectors to sectors in stock markets, just because of liquidity concerns and other risk issues faced by investors. The stock implied volatilities are inevitably contaminated by shocks in financial markets since risks are traded on markets. Nevertheless, implied volatility is still an excellent proxy to study the high-dimensional market volatility network. We hope that the underlying market network structure can be at least partially uncovered by its implied volatility network.

We estimate the volatility network of the S&P 100 components stocks quoted on 06/30/2015. Similar to the VIX index for the S&P 500 stock composite, in this paper the S&P 100 components[12] implied volatilities are constructed with their respective at-the-money option contracts with 30-day maturity. This implied volatility measures the expected volatility of the underlying stock over the next 30 days. We hereafter only consider the option contracts with 30-day maturity. Generally speaking, an at-the-money call (put) option usually has a delta[13] at approximately 0.5 (-0.5). A simple way to get the at-the-money implied volatility is to take the simple arithmetic mean of the interpolated implied volatility of the call option with delta 0.5 and the interpolated implied volatility of the put option with delta -0.5:

$$IV_{i,t} = \frac{1}{2}(IV_{i,t}^{C_{0.5}} + IV_{i,t}^{P_{-0.5}}). \tag{1.58}$$

where $IV_{i,t}^{C_{0.5}}$ is firm $i$'s interpolated implied volatility of the call option with delta 0.5 at time $t$, and $IV_{i,t}^{P_{-0.5}}$ is firm $i$'s interpolated implied volatility of the put option with delta -0.5 at time $t$. The data information of the daily implied volatility with different delta levels are provided in the OptionMetrics - Volatility Surface database. As the firms' implied volatilities measure the expected volatility of their stock prices over the next 30 days, the daily sequence of $\{IV_{i,t}\}_t$ is a highly persistent process. In other words, $IV_{i,t-1}$ would have a strong predictive power to forecast $IV_{i,t}$. To remove such self-effect that merely comes from the overlapping of measuring periods, we analyze the innovation processes by taking daily first differences on each implied volatility series:

$$\Delta IV_{i,t} = IV_{i,t} - IV_{i,t-1}. \tag{1.59}$$

This manipulation procedure is simple and easy to replicate [14]. We will hereafter use $\Delta IV_{i,t}$ to estimate our implied volatility network.

The date range of the database is from 01/01/1996 to 08/31/2015. The companies whose IPO

[12] To be included in the S&P 100, the companies should be among the larger and more stable companies in the S&P 500, and *must have list options*.

[13] Delta measures the degree to which an option is exposed to shifts in the price of the underlying asset.

[14] Ang, Hodrick, Xing and Zhang (2006) use this manipulation approach to deal with the VIX index to test whether the VIX index is market risk factor

dates are after 01/01/2000 will be dropped off, so that we can examine the two most important crises in the US stock market (i.e., the IT Bubble Burst and the Financial Crisis of 2007-09). The remaining full sample is from 20/08/1999 to 31/08/2015. There are missing values on some dates for some companies and we take linear interpolations to impute the missing values to get completed time series processes for estimations. We have 90 companies in the final sample, $N = 90$. Appendix B provides the ticker symbol list of nodes and their respective sectors in our implied volatility network. The Industry Group classification for each node is from the North American Industry Groups database from MorningStar, LLC.

As Diebold and Yilmaz (2014) point out, latent market network structures may vary with business circles or may shift abruptly with market environment (e.g., crisis and noncrisis). Whether and how much it varies is ultimately an empirical matter and there is no point to just simply assume it is constant. Hence, we allow the network structure to be time-varying, and thus the elements in the causality measures table are also allowed to be time-varying.[15] To capture time variations, we will estimate the dynamic implied volatility network structures with rolling samples.

Throughout the empirical exercise, We set the lag $p = 1$ and apply the VAR(1) model to approximate the unconstrained and the constrained models. Setting the same lag makes the conditional variance comparable between the unconstrained models and the constrained models. We will first estimate the static implied volatility network structure with full sample observations (20/08/1999 - 31/08/2015). As mentioned before, market connectedness can be decomposed into the connectedness within each sector and the interconnections among different sectors. Firm-wise interconnections within each sector and sector-wise interconnections are certainly of interest. For example, studying how financial firms connected to each other and how financial industry affects other industries is important to understand the recent financial crisis. To investigate the dynamic patterns of the volatility network structures, there is always a trade-off between estimation accuracies and more current conditional estimates when choosing the width of estimation windows. To examine market connectedness dynamics, we set the width of the moving window to be 2 years and update measures every one month. For example, the estimates on December 2008 are estimated based on the data from January 2007 to December 2008. By moving the estimation windows forward every

[15]This assumption does not contradict the constant parameters setting we made in estimating the multiple horizon causality measures. The "calendar time" for time-varying measures and the "sampling time" to estimate the measures are conceptually different. We just require the processes to be estimated are locally stationary.

month, we can obtain the dynamic pictures of the implied volatility network.

In robustness check, we compare our results with those setting the lag $p = 2$ and those using moving estimation windows of 1 year ($T = 252$). We find our results are robust to these different pre-selected modelling settings.

### 1.6.2 Empirical Results

Market network econometric analysis can be worked under two types of network representations: i) firm-wise market structure ($V_C$), under which the nodes in the market are the 90 companies; and ii) sector-wise market structure ($V_S$) under which the nodes in the market are the 8 sectors that the 90 companies belong to. We will apply the point-wise edge analysis technique in the firm-wise market structure and apply the group-wise analysis technique in the sector-wise market structure.

#### Static Implied Volatility Network Structures

Firm-wise market network structures give us a broad picture of how firms connect to each other. Sub-market network structures zoom in firms' interconnections within specific sector. Sector-wise market structures merge the firms in the same sector and give us a simple picture of how different sectors connect to each other.

In Figure 1.5, we show the firm-wise S&P 100 implied volatility network structure. To examine this big network (90 nodes and $90^2$ edges), we only keep the directed and weighted edge ($i \to j$) if its strength is greater or equal to 90% percentile of the strengths of the edges ($i \to \cdot$) and 90% percentile of the strengths of the edges ($\cdot \to j$). In other words, we only keep an edge if and only if this edge is important to the pair of nodes being connected by it. If $i \to j$ and $j \to i$ are both kept, we only plot the one with greater strength without confusions. At the first glance, edges are denser around the firms in the financial sector. A majority of the edges being shown in the figure comes from financial firms. Moreover, the financial firms have more interconnections due to the recent financial crisis. It is also documented by Barigozzi and Brownlees (2016) in the S&P 100 realized volatility network. Interestingly, we observe that GE (a major industrial goods company) and SLB (a major supplier to the oil and gas exploration and production industry) have relatively strong interconnections with the financial firms. GE was almost bankrupt in 2009 and 2010. The oil price is very volatile in the past ten years. As major credit suppliers, financial firms are sensitive to these economic shocks. Figure 1.5 has reflected some special market situations in the US economy in the past 15 years.

We identify the 10 most influential firms in Table 1.2. In Table 1.2, we report the minimum value, the maximum value, the mean value and the quantiles (25%, 50% and 75%) of the entries in each firm's "OUT" vector, $[C_{i1}, C_{i2}, ..., C_{iN}]$. The mean for almost every firm is greater than its median and is close to the 75% quantile; the discrepancy within the first 25% quantile is very small, but the discrepancy within the last 25% (75% - Max) is much larger. This represents strong evidence for right skewness of the distribution of firms' weighted edges in the "OUT" direction. Jackson et al. (2008) also documents right skewness in distributions in social networks. We select the median, rather than the mean, to describe the central tendency of the distributions of firms' edges[16].

We sort the firms' tickers by their medians. The most influential firm in the static network is the BAC (Bank of America). BAC helps to increase the forecast precision of the next-day implied volatility by 0.07% for more than a half of the firms in the S&P 100, and by at high as 5.33% for the firm that it affects most. Seven financial firms (BAC, C, BK, AIG, MET, F, JPM and MS) are listed in the top 10 influential firms at Table 1.2. In Figure 1.5, we have seen that many prominent edges are from financial firms. The firms in the financial sector have great influence in the S&P 100 network. On the other side, Table 1.3 reports the summary statistics of the entries in the "IN" vector, $[C_{1i}, C_{2i}, ..., C_{Ni}]$. Among the top 10 sensitive firms, only the firm (C) belongs to the financial sector and the other nine firms belong to the basic materials sector or to the Industrial goods sector. Therefore, the influential firms in the S&P 100 network are not those that will easily be affected. The "influential" and "sensitive" we mentioned so far are in the sense of direct effects, in which the causality measures are at forecast horizon $h = 1$. In Table 1.4, we report the top 10 influential firms at different forecast horizons, $h = 1, 2, 3, 4, 5$, to take spillover effects into account. The firms and their orders in the list of top 10 influential firms are slightly different at different forecast horizons. For instance, in the case of only taking direct effects into account ($h = 1$), the most influential financial firm is BAC and 7 out of 10 most influential firms are from the financial

[16]The firm's centrality described by our median measures is in alignment with the "Degree Centrality" in Freeman (1978) and Jackson et al. (2008)

sector; in the case of taking direct and indirect effects into account ($h = 5$), the most influential financial firm becomes AIG and only 4 out of 10 most influential firms is from the financial sector. The technology firms are actually influential. In the case of $h = 5$, 4 out of 10 most influential firms belong to the technology sector and the top 2 influential firms are from the technology sector, if the Apple Inc. is considered as a technology firm. In short, measuring a static network that only characterizes direct effects in an economic network is far from enough to fully understand all interconnections and indirect effects. In contrast, jointly measuring direct and indirect effects with the causality tables at different forecast horizons can provide us with "dynamic" pictures of interconnections in the S&P 100 network with different effect-radius. In many cases, what is truly important is the firm's total effect (direct effect and indirect effect) rather than just its direct effect.

Next, we zoom in the financial sector and investigate the interconnections within the financial sector. The firm-wise S&P 100 implied volatility network within the financial sector can be visualized by Figure 1.6. In this figure, we only keep the directed and weighted edges with the strength greater or equal to the 50% percentile of the strengths of edges in this financial network. In other words, only the "big" edges in this financial sector network will be kept. Again, if both $i \to j$ and $j \to i$ are kept, we only show the one with greater strength. We find that the most influential firms in the financial sector, in the sense of the out-degree (number of edges pointing from the firms), are the top investment banks: Morgan Stanley (MS), Goldman Sachs (GS) and Bank of America(BAC). In Table 1.2, Morgan Stanley and the Bank of America are both listed in the top 10 influential firms and Goldman Sachs is the 16th influential firm. The summary statistics of the entries in the "OUT" vector in the financial network in Table 1.5 confirms their great influence in the financial sector. Similar to the one in Table 1.2, the edges distributions in the financial sector are also skewed to right. We again use the median to describe the central tendency of these distributions. The top 3 influential firms in the financial sector are in order as: BAC (median = 0.42), MS (median = 0.30) and GS (median = 0.25), compared with the 4th influential firms: BK (median = 0.06). Roughly speaking, we could say that the financial sector is actually controlled by the top investment banks in the past 15 years. It is also interesting to look at who are the most sensitive firms in this financial sector. In Table 1.6, we sort the firms by their sensitivities. The top 3 sensitive financial firms are in order as: C (median = 0.33), ALL (median = 0.32) and BAC (median = 0.28). C is the only financial firm that is listed in the top 10 sensitive firms in the S&P

100 network, and it is also the most sensitive firms in the financial sector. BAC not only is the most influential firms in the S&P 100 network, but also has strong interconnections with other firms in the financial sector since it is the most influential firm as well as the 3rd most sensitive firm in the financial sector.

Lastly, Figure 1.7 shows the sector-wise S&P 100 implied volatility network structure. In this network, the nodes are the sectors that group together their respective firms as $V_i$. We only keep the directed and weighted edges with the strength greater or equal to the 50% percentile of the strengths of edges in this sector-wise network. In other words, only the "big" edges in this network will be kept. Once again, if both $V_i \to V_j$ and $V_j \to V_i$ are kept, we only show the one with greater strength. An important observation is that all sectors are strongly self-affected. It is in line with our common wisdom. Four most influential sectors, in the sense of the out-degree (number of edges pointing from the sectors), are Technology, Consumer Goods, Industrial Goods and Financial. They are also the key industries that support the growth of the US economy in these 15 years. In Table 1.7, we sort the sectors by their influences and obtain the top 4 influential sectors: Technology (median = 3.27), Industrial Goods (median = 1.55), Consumer Goods (median = 0.90) and Financial (median = 0.48). It is similar to what we have found in Figure 1.7. Moreover, Technology, Consumer Goods and Financial are also on the list of four least sensitive sectors, as reported in Table 1.8. Overall, the relationships among different sectors in the S&P 100 network are very asymmetric. There are two groups in this network: the influential sectors (Technology, Industrial Goods, Consumer Goods and Financial) and the sensitive sectors (Services, Basic Materials, Industrial Goods and Healthcare). Interestingly, the most influential sector in the sector-wise network (see Table 1.7) is the technology sector, rather than the financial sector that has the most influential firms in the frim-wise S&P 100 network found in Table 1.2. Note that the causality we measure is based on the marginal effect on prediction. When firms' marginal effects are small, their total (sector) marginal effect is not necessarily small, especially if the component marginal effects are positive correlated. Even though the technology firms, as single components, are not as influential as the financial firms, the technology sector, as a whole, can be more influential than the financial sector. This circumstance is also discussed theoretically in the Section 3. Therefore, the group-wise network measurement technique is an important complement for the point-wise network measurement technique to help us understand underlying market network structures.

### Connectedness Dynamics in Firm-wise Market

In Figure 1.8, we show the dynamic patterns of the market relative connectedness structure measures and the market absolute connectedness strength measures in the firm-wise market structure, at forecast horizon 1, $h = 1$, and at forecast horizon 10, $h = 10$. We only report the "out" connectedness measures as the "out" measures and their respective "in" measures are highly correlated. This is not out of surprise, because one's "out" causality measures are just someone's "in" causality measures, and thus their market connectedness measures will have a similar dynamic pattern.

If our market connectedness measures are truly able to measure the market systemic risk in the US stock market, they will vary with market conditions that can be reflected by market indices like the VIX index or the S&P 500 index. The market absolute connectedness strength measures indeed have significant variations across different periods. Prior to 2007, the absolute connectedness strength measures are close to zero, while the VIX index is relatively high before 2003 due to the IT Bubble Burst. Starting from 2007, both the market connectedness strength and the VIX index start to soar and become fluctuate more at relatively high levels until 2011. This is exactly the period of the recent global financial crisis. From 2011 to 2015, the market connectedness strength has a new "normal" level that is lower than the level during the crisis but higher than the level before the crisis, while the VIX index decreases to the pre-crisis level. Overall, there is an apparent synchronization between our market connectedness strength measures and the VIX index, except in the IT Bubble Burst period. It is actually in alignment with our common wisdom that the major difference of the financial crisis of 2007-09 from other crises is the recent global financial crisis is driven and amplified by the systemic risk in financial markets. Our absolute connectedness strength measure ("individual fear connectedness") looks to be positive correlated with the "market fear" level (VIX), but our measures do concentrate more on the systemic risk that comes from the connectedness in financial markets.

Unlike the absolute connectedness strength measure, the relative connectedness structure measure concerns more about the network connectedness structure instead of the connectedness strength. We first look at the relative market connectedness structure at forecast horizon 1 and discuss it in four periods (2000-2003, 2003-2006, 2006-2009, 2009-2015). During 2000-2003, the level of the relative connectedness structure measure is relatively high (0.90-0.95) and the stock market slides due to the IT Bubble Burst. The S&P 500 gets to a bottom in early 2003 and starts to recover, and the VIX index also starts to decrease. In this period (2003-2006), the relative connectedness structure goes down. During the pre-crisis and crisis period (2006-2009), the relative connectedness structure climbs up rapidly, and touches a historical record ($> 0.95$) at the end of 2008 when is the also the most fearful moment in financial markets as shown by the VIX index touching the historical peak and the S&P 500 touching the bottom. During 2009-2015, the VIX index goes down to normal and the S&P 500 fully recovers from the crisis. Interestingly, however, the relative connectedness structure measure still remains at the crisis level ($> 0.95$). Our conjecture is that the financial market is still remaining at a "crisis zone" that can be characterized by the high level of the market connectedness structure.

When comparing the relative connectedness at different forecast horizons, we find the market relative connectedness structure measures at forecast horizon 10 are much closer to the upper bound 1, than at forecast horizon 1. Note that longer forecast horizon allows every node in the network to have more steps of paths to connect each other, the relative connectedness structure measure will thus be larger at greater forecast horizons. Hence, we do not expect to find big time variations for the relative connectedness structure measure at long horizons (e.g., $h = 10$), while we still can see that the market connectedness structure measures at horizon 10 has a dynamic pattern similar to the one at horizon 1.

In Figure 1.8, the market relative connectedness structure measures and the market absolute connectedness strength measures have striking different dynamic patterns across our sample period. Absolute connectedness strength measures can be decomposed into relative connectedness structure measures and causation strengths. The difference of the relative measures and the absolute measures is totally accounted for by the time-varying causation strengths. By comparing these two types of measures at different periods, we find the causation strengths are relative large during the financial crisis. It again confirms our assertion that our causality measures can capture elements of the market systemic risk.

Also, we provide the 90% bootstrap confidence intervals for the absolute market connectedness strength measures on some specific dates[17] (2004-01-20, 2005-01-20, 2006-01-20, 2007-01-20,

[17] We do not report the confidence intervals every month in our sample period because the bootstrapping procedure is time costly.

2008-01-20, 2009-01-20, 2010-01-20, 2011-01-20, 2012-01-20, 2013-01-20 and 2014-01-20) in Figure 1.9. We use the bootstrapping procedure that is similar to the one described in Dufour and Zhang (2015). The raise of the market absolute connectedness strength during the financial crisis period is statistically significant.

While our market connectedness measures do show dynamic patterns corresponding to different major market conditions (before crisis, during crisis, and after crisis), it still seems to be counterintuitive that our dynamic connectedness measures are "too volatile". For instance, one may find the market connectedness strength measures jump up and down frequently[18], but the underlying market structures has no way to change at this rate even though the market structures may change abruptly because of crisis. In fact, our estimated implied volatility network not only measures the underlying market structures, but also captures the market effects in the stock market and in the option market. As we have discussed before, firm's implied volatility is sensitive to special events in financial markets. One of the regular important events in the equity market is the quarter earnings announcements. Publicly-traded companies have to release their earning reports every three months regarding their financial conditions, earning forecasts, etc. It means the firm's detailed information is only revealed to the public every three months. This kind of information is crucial for firm's credit grade and firm's stock price target evaluated by equity analysts in the market. If an earning report beats market expectations, the firm's stock price could jump up overnight and vice versa. As a result, option trading will become much more active during earnings seasons, and thus the implied volatilities are usually more volatile during this period. Moreover, different firms could release their earnings reports on different dates during a earnings season. Some investors would bet on some companies based on others' released performances, especially when these firms are in the same sector where they face a similar business environment. The high leverage and large possible payoffs of the option trading make a large proportion of active investors choose to bet on the option market[19]. Therefore, it is very likely that the connectedness measures of the implied volatility network would become more volatile during earnings seasons. It is mainly due to shocks in the financial market, rather than changes in the underlying market structures. Thus, the dynamics of our implied volatility market connectedness measures can be decomposed

[18]From 2007 to 2011, for example, we find about 10 spikes in the figure.

[19]Donders, Kouwenberg, Vorst et al. (2000) find firm's implied volatility increases before announcement days and drops afterwards.

into long-run stable market connectedness changes and short-run financial fluctuations, and this is exactly what we observe in Figure 1.8.

### Connectedness Dynamics Within Single Sector in Firm-wise Market

Taking the diagonal block that contains companies in a sector in the firm-wise market causality measures table, we have the sub-network structure for this sector. We do so for each sector, and then obtain the sector connectedness measures within every single sector in the firm-wise market.

Figure 1.10 reports the absolute connectedness strength measures within each of the 7 sectors in our implied volatility network[20]. As expected, the financial sector has the highest and the most persistent absolute connectedness strengths during the financial crisis. Other sectors also have higher connectedness strengths in this period, but they are very minor when compared with the financial sector. During the crisis, investors would be more sensitive to news comings, so the implied volatility connectedness could become more fluctuated. Since financial shocks (e.g., quarter earnings releases) to implied volatilities more easily spill over within a sector, at most of the times when there are major spikes in the market connectedness strengths in Figure 1.8, we can find their corresponding ones in one of the sector connectedness strengths in Figure 1.10.

### Connectedness Dynamics in Sector-wise Market

As has been emphasized before, the econometric framework proposed in this paper provides the first unified method to estimate point-wise effects and group-wise effects. The nodes in the sector-wise network structure ($V_S$) in this empirical exercise are the 8 sectors[21] that the S&P 100 components belong to.

In Figure 1.11, we report the dynamic patterns of the market absolute connectedness strengths and the market relative connectedness structures in the sector-wise market network at forecast horizon 1 ($h = 1$) and at forecast horizon 10 ($h = 10$). The sector-wise market absolute connectedness strength measure at forecast horizon 1 has a sharp peak at the end of 2008. However, it does not persistently remain at a high level compared with the absolute connectedness strength measures in the company-wise market structure during the crisis period shown in Figure 1.8. In other words,

[20] The "Utility" industry is not included as it only contains one company.

[21] Basic Materials, Consumer Goods, Financial, Healthcare, Industrial Goods, Services, Technology and Utilities

even if there is a high persistent market systemic risk during the financial crisis, it is not due to the connectedness among different sectors. The relative connectedness structure measures and the absolute connectedness strength measures are positively correlated before 2009, while again, the market connectedness structure does not decrease with the market connectedness strength after the crisis. The sector-wise absolute connectedness structure strengths in Figure 1.11 are generally lower than the firm-wise strengths in Figure 1.8. It is because sector-wise nodes have weaker interconnections than firm-wise nodes in an economic network. Connected firms usually have close business relationships and they tend to be in the same sector.

#### Directed and Weighted Edges Dynamics in Sector-wise Market

We now look at the sector-wise network interconnections in more details. In particular, we concentrate on the financial sector, which has the greatest influence on the US stock market in the past 10 years due to the global financial crisis, to see how the interconnections between the financial sector and other sectors looks like.

In Figure 1.12 and Figure 1.13, we report the time-varying direct effects from the financial sector to other sectors and from other sectors to the financial sector. As expected, the financial sector has the strongest influence on itself during the financial crisis. At the end of 2008, the magnitude of the financial sector affecting itself soars to a historical peak with the VIX index soaring, and it keeps at a relatively high level until 2011. We find the financial sector has an relatively strong effect on itself from 2009 to 2011, which matches the crisis period of financial crisis of 2007-09 as our estimates utilize 2 years rolling samples. The financial crisis is actually not yet over in the global financial market after 2009. Fore instance, the US financial crisis triggers the European debt crisis in early 2010. From 2011 to 2013, the financial sector still has a relative strong effect on itself.

The effect from the financial sector on other sectors also increases at the beginning of the crisis, while the raise only lasts for a few months. In contrast, all other sectors only have negligible effects on the financial sector, compared with the striking magnitude of the financial sector affecting itself. The effects between the financial sector and other sectors are quite asymmetric: the financial sector has a strong effect on others but the reverse is not true. The asymmetry and the time variations in effects between the financial sector and other sectors confirm the importance of directed and weighted edges setting in economic network analysis.

### 1.6.3 Robustness Check

Finally, we conclude this section with checking the robustness of our market connectedness results to the choice of lags $p$ in the VAR($p$) approximation to the causality estimation models and to the width of the estimation sample windows. In fact, different lags, $p$, correspond to different information sets using in causality estimations; different widths of estimation windows correspond to different sample market conditions. The estimates of given edges will change with different choices of them. Therefore, we do not expect that our estimated connectedness measures would be invariant to different lags and to different widths of estimation windows. Instead, if the underlying market systemic risk in the volatility network can truly be measured by our market connectedness measures and the our econometric models are good approximation to the real world, the measures, under different pre-selected model settings, should have similar dynamic patterns over time following the changes in the underlying market systemic risk.

In particular, we compare our estimated results with those estimated with VAR(2) models and with those estimated with 1-year estimation windows. Figure 1.14 reports the market absolute connectedness strength measures under three different model settings: i) VAR(1) and 2-year estimation windows (Benchmark); ii) VAR(2) and 2-year estimation windows; and iii) VAR(1) and 1-year estimation windows. They are estimated at forecast horizon 1 and at forecast horizon 10 in the firm-wise market network. They all have a similar dynamic pattern (low before 2007, soar up from 2008, resume pre-crisis level in 2011 and has a mild increasing trend from 2012 to present). Figure 1.15 shows the robustness of the relative connectedness structure measures with the same model settings comparison. All of the three relative connectedness structure measures at forecast horizon 1 have a similar dynamic pattern (relatively high from 2001 to 2003, decline from 2003 to 2006, soar up from 2006 to 2009 and remain at the financial crisis level from 2009 to present). For the connectedness structure measures at forecast horizon 10, they keep at a high level all the time.

To summarize, our robustness check shows that the time-varying characteristics of our market connectedness measures for the implied volatility network investigated in this paper are robust to the choices of $p$ in the VAR($p$) approximation, and also robust to the choices of the widths of the estimation windows for the modelling settings we examine above.

## 1.7 Conclusion

Economic and financial network analysis requires a well developed time series econometric framework for empirical studies. Weaker restrictions on network settings, fewer assumptions on the time series identification models and more empirical flexibility of the measurement framework would be favoured. In this paper, we propose a novel time series econometric method to measure high-dimensional directed and weighted market network structures. Direct and spillover effects at multiple horizons, between nodes and between groups, are measured in a unified framework. We argue that a satisfactory network econometric framework to study market networks should be able to estimate directed and weighted network structures with causality implications, and it can be applied to study network spillover effects in a high-dimensional context. Indeed, our network estimation method not only satisfies all these criteria, but also enjoys other appealing features.

We measure causality at different horizons in a network through the multiple horizon causality measures based on flexible VAR models specified by the LASSO approach. (Non-sparse) network structures can be estimated from a sparse set of autoregressive coefficients and concentration matrices. Asymptotic consistency results of the estimators of our directed and weighted edges measures are also provided in this paper. We do not require sparsity assumptions on network structures or the Gaussian assumption on econometric models. We successfully connect the causality literature with the LASSO approach in application to economic and financial network measurement. Moreover, to the best of our knowledge, our econometric framework is the first one, in the network econometric literature, to explicitly allow point-wise edges (relationships between firms) and group-wise edges (relationships between sectors) to be measured in a unified framework.

With this framework at hand, we also provide the estimated market network with new connectedness measures that are built upon the underlying network structures. Since an economic network can be viewed as a network among firms as well as a network among sectors, we propose three types of connectedness measures to gauge network interconnections. These types of connectedness measures fully take advantage of the flexibility of our network measurement method, so they can be applied to study market network connectedness in flexible ways

Our network measurement methods have a wide range of applications and can be applied in a variety of research areas, including identifying and quantifying economic relationships between firms, between sectors and between areas; measuring market connectedness; predicting financial risks; guiding asset allocations in large portfolios; etc. Note that many latent economic and financial network structures can be estimated by our flexible network measurement method with varieties of panel databases. Specifically, observing that explicitly identified economic network centrality and consumer-supplier linkage have been shown to be new risk factors in asset pricing and new determinants to predict financial variables, we expect more pricing factors and financial and macroeconomic variables drivers are to be discovered by our network econometric measurement methods.

To illustrate the usefulness of our method in network analysis, we investigate the S&P 100 implied volatility network in the US stock market, which can be viewed as a "individual fear" network and has not yet been studied in existing literature. We find that: i) 7 out of the 10 most influential firms in the S&P 100 belong to the financial sector, and top investment banks (Morgan Stanley, Goldman Sachs and Bank of America) have the greatest influence in the financial sector; ii) technology firms are influential when we consider indirect effects in the S&P 100 implied volatility network; iii) market connectedness was especially strong during the recent global financial crisis; iv) the high market connectedness was mainly due to the high connectedness within the financial sector and the spillovers from the financial sector to other sectors; v) the financial sector had the highest firm-wise connectedness from 2008 to 2010, while the connectedness of other sectors also reach relatively high level during this period; vi) the causality effects between the financial sector and other sectors were asymmetric and displayed considerable variation over time, which stresses the importance of directed and weighted edges settings in market network analysis.

Table 1.2: Summary statistics of causality measures from each firm to other firms. This table reports the summary statistics of each row of the firm-wise causality table $[C_{i\rightarrow\cdot}]$. The causality table is estimated by the full data sample (20/08/1999 - 31/08/2015). Nodes are the firms of selected S&P 100 components. For each firm $i$, we report the minimum value, the maximum value, the mean value and the quantiles (25%, 50% (median) and 75%) of the entries in its "OUT" vector. The reported values are 100 times of the raw values, and are kept with two digits. We sort the tickers by their median values and identify the top 10 influential firms.

| Sector* | Ticker | Median | Mean | Min | 25% | 75% | Max | Sector* | Ticker | Median | Mean | Min | 25% | 75% | Max |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **F** | **BAC** | 0.07 | 0.43 | 0.00 | 0.00 | 0.51 | 5.33 | I | LMT | 0.00 | 0.03 | 0.00 | 0.00 | 0.01 | 1.80 |
| C | AAPL | 0.07 | 0.16 | 0.00 | 0.02 | 0.20 | 1.03 | C | KO | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 1.34 |
| T | CSCO | 0.07 | 0.27 | 0.00 | 0.00 | 0.31 | 2.63 | H | AGN | 0.00 | 0.03 | 0.00 | 0.00 | 0.01 | 2.27 |
| **F** | **C** | 0.07 | 0.20 | 0.00 | 0.00 | 0.21 | 2.77 | B | CVX | 0.00 | 0.04 | 0.00 | 0.00 | 0.01 | 1.59 |
| **F** | **BK** | 0.06 | 0.33 | 0.00 | 0.00 | 0.33 | 5.75 | T | HPQ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 |
| **F** | **AIG** | 0.03 | 0.15 | 0.00 | 0.00 | 0.15 | 2.28 | B | OXY | 0.00 | 0.08 | 0.00 | 0.00 | 0.02 | 3.81 |
| **F** | **MET** | 0.03 | 0.11 | 0.00 | 0.00 | 0.07 | 1.32 | H | BAX | 0.00 | 0.10 | 0.00 | 0.00 | 0.02 | 4.88 |
| C | F | 0.03 | 0.12 | 0.00 | 0.00 | 0.10 | 4.04 | B | DVN | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.10 |
| **F** | **JPM** | 0.03 | 0.11 | 0.00 | 0.00 | 0.08 | 0.83 | H | BMY | 0.00 | 0.01 | 0.00 | 0.00 | 0.02 | 0.08 |
| **F** | **MS** | 0.02 | 0.38 | 0.00 | 0.00 | 0.17 | 4.83 | S | CMCSA | 0.00 | 0.30 | 0.00 | 0.00 | 0.14 | 6.38 |
| H | GILD | 0.02 | 0.09 | 0.00 | 0.00 | 0.10 | 1.39 | F | ALL | 0.00 | 0.20 | 0.00 | 0.00 | 0.07 | 4.67 |
| I | GE | 0.02 | 0.17 | 0.00 | 0.00 | 0.08 | 3.63 | F | USB | 0.00 | 0.16 | 0.00 | 0.00 | 0.04 | 2.62 |
| F | WFC | 0.02 | 0.15 | 0.00 | 0.00 | 0.12 | 3.96 | B | SLB | 0.00 | 0.09 | 0.00 | 0.00 | 0.00 | 3.28 |
| S | TGT | 0.02 | 0.06 | 0.00 | 0.00 | 0.05 | 0.37 | T | TXN | 0.00 | 0.09 | 0.00 | 0.00 | 0.01 | 4.93 |
| T | IBM | 0.02 | 0.07 | 0.00 | 0.00 | 0.06 | 1.02 | S | SBUX | 0.00 | 0.07 | 0.00 | 0.00 | 0.01 | 6.14 |
| F | GS | 0.02 | 0.12 | 0.00 | 0.00 | 0.08 | 1.70 | T | MSFT | 0.00 | 0.06 | 0.00 | 0.00 | 0.02 | 2.96 |
| T | VZ | 0.02 | 0.07 | 0.00 | 0.00 | 0.03 | 4.11 | S | DIS | 0.00 | 0.06 | 0.00 | 0.00 | 0.01 | 5.11 |
| F | SPG | 0.02 | 0.14 | 0.00 | 0.00 | 0.07 | 5.63 | T | ACN | 0.00 | 0.06 | 0.00 | 0.00 | 0.01 | 3.49 |
| S | TWX | 0.01 | 0.07 | 0.00 | 0.00 | 0.05 | 2.47 | H | UNH | 0.00 | 0.06 | 0.00 | 0.00 | 0.01 | 2.78 |
| B | DOW | 0.01 | 0.09 | 0.00 | 0.00 | 0.03 | 2.52 | U | EXC | 0.00 | 0.06 | 0.00 | 0.00 | 0.01 | 4.19 |
| I | BA | 0.01 | 0.11 | 0.00 | 0.00 | 0.03 | 7.36 | H | CVS | 0.00 | 0.05 | 0.00 | 0.00 | 0.01 | 2.97 |
| T | EMC | 0.01 | 0.04 | 0.00 | 0.00 | 0.03 | 0.78 | S | AMZN | 0.00 | 0.05 | 0.00 | 0.00 | 0.03 | 0.97 |
| F | AXP | 0.01 | 0.16 | 0.00 | 0.00 | 0.07 | 2.78 | H | LLY | 0.00 | 0.05 | 0.00 | 0.00 | 0.01 | 3.86 |
| T | ORCL | 0.01 | 0.05 | 0.00 | 0.00 | 0.02 | 3.26 | H | ABT | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 3.48 |
| H | PFE | 0.00 | 0.06 | 0.00 | 0.00 | 0.03 | 2.51 | B | APC | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 1.71 |
| S | COST | 0.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.98 | T | QCOM | 0.00 | 0.03 | 0.00 | 0.00 | 0.01 | 0.74 |
| H | CELG | 0.00 | 0.03 | 0.00 | 0.00 | 0.01 | 1.14 | B | XOM | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.86 |
| T | INTC | 0.00 | 0.04 | 0.00 | 0.00 | 0.03 | 0.68 | I | HON | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.29 |
| U | SO | 0.00 | 0.09 | 0.00 | 0.00 | 0.01 | 7.63 | S | WMT | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.37 |
| F | COF | 0.00 | 0.10 | 0.00 | 0.00 | 0.04 | 4.63 | H | BIIB | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.13 |
| S | UNP | 0.00 | 0.04 | 0.00 | 0.00 | 0.01 | 1.93 | C | NKE | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.31 |
| S | MCD | 0.00 | 0.08 | 0.00 | 0.00 | 0.01 | 6.30 | S | HD | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.14 |
| B | HAL | 0.00 | 0.10 | 0.00 | 0.00 | 0.02 | 6.68 | B | MON | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 |
| T | T | 0.00 | 0.05 | 0.00 | 0.00 | 0.02 | 3.70 | C | CL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 |
| H | MRK | 0.00 | 0.02 | 0.00 | 0.00 | 0.01 | 0.69 | I | GD | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 |
| S | FOXA | 0.00 | 0.10 | 0.00 | 0.00 | 0.01 | 6.94 | H | AMGN | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 |
| I | CAT | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.45 | I | UTX | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 |
| S | V | 0.00 | 0.11 | 0.00 | 0.00 | 0.04 | 7.98 | B | COP | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| I | EMR | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.27 | B | DD | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| S | EBAY | 0.00 | 0.17 | 0.00 | 0.00 | 0.08 | 3.26 | S | FDX | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| S | WBA | 0.00 | 0.02 | 0.00 | 0.00 | 0.01 | 0.70 | C | MO | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| I | MMM | 0.00 | 0.03 | 0.00 | 0.00 | 0.03 | 0.44 | S | LOW | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| H | JNJ | 0.00 | 0.07 | 0.00 | 0.00 | 0.01 | 4.87 | H | MDT | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| I | RTN | 0.00 | 0.07 | 0.00 | 0.00 | 0.01 | 5.79 | C | PEP | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| S | NSC | 0.00 | 0.12 | 0.00 | 0.00 | 0.01 | 8.78 | C | PG | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |

* B: Basic Materials; C: Consumer Goods; F: Financial; H: Healthcare; I: Industrial Goods; S: Services; T: Technology; U: Utilities.

Table 1.3: Summary statistics of causality measures to each firm from others firms. This table reports the summary statistics of each column of the firm-wise causality table $[C_{\cdot\to i}]$. The causality table is estimated by the full data sample (20/08/1999 - 31/08/2015). Nodes are the firms of selected S&P 100 components. For each firm $i$, we report the minimum value, the maximum value, the mean value and the quantiles (25%, 50% (median) and 75%) of the entries in its "IN" vector. The reported values are 100 times of the raw values, and are kept with two digits. We sort the tickers by their median values and identidy the top 10 sensitive firms.

| Sector* | Ticker | Median | Mean | Min | 25% | 75% | Max | Sector* | Ticker | Median | Mean | Min | 25% | 75% | Max |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| B | OXY | 0.02 | 0.19 | 0.00 | 0.00 | 0.17 | 3.81 | H | BIIB | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.04 |
| I | RTN | 0.02 | 0.09 | 0.00 | 0.01 | 0.03 | 5.79 | S | WMT | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.11 |
| I | LMT | 0.01 | 0.02 | 0.00 | 0.00 | 0.02 | 0.22 | U | SO | 0.00 | 0.09 | 0.00 | 0.00 | 0.01 | 7.63 |
| B | SLB | 0.01 | 0.25 | 0.00 | 0.00 | 0.10 | 3.26 | H | JNJ | 0.00 | 0.20 | 0.00 | 0.00 | 0.01 | 4.88 |
| I | GD | 0.01 | 0.05 | 0.00 | 0.00 | 0.05 | 0.59 | C | NKE | 0.00 | 0.02 | 0.00 | 0.00 | 0.03 | 0.25 |
| F | C | 0.01 | 0.18 | 0.00 | 0.00 | 0.07 | 3.34 | S | EBAY | 0.00 | 0.04 | 0.00 | 0.00 | 0.02 | 1.14 |
| S | DIS | 0.01 | 0.17 | 0.00 | 0.00 | 0.03 | 5.11 | S | AMZN | 0.00 | 0.10 | 0.00 | 0.00 | 0.01 | 4.93 |
| F | WFC | 0.01 | 0.15 | 0.00 | 0.00 | 0.07 | 3.52 | H | CELG | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 1.14 |
| B | COP | 0.00 | 0.09 | 0.00 | 0.00 | 0.09 | 1.32 | B | HAL | 0.00 | 0.11 | 0.00 | 0.00 | 0.02 | 6.68 |
| I | EMR | 0.00 | 0.04 | 0.00 | 0.00 | 0.04 | 0.51 | F | BK | 0.00 | 0.17 | 0.00 | 0.00 | 0.01 | 5.75 |
| I | GE | 0.00 | 0.18 | 0.00 | 0.00 | 0.04 | 2.13 | S | FOXA | 0.00 | 0.15 | 0.00 | 0.00 | 0.01 | 6.94 |
| F | ALL | 0.00 | 0.26 | 0.00 | 0.00 | 0.07 | 3.96 | S | UNP | 0.00 | 0.15 | 0.00 | 0.00 | 0.02 | 2.87 |
| B | DD | 0.00 | 0.05 | 0.00 | 0.00 | 0.03 | 0.98 | S | CMCSA | 0.00 | 0.13 | 0.00 | 0.00 | 0.01 | 6.38 |
| I | BA | 0.00 | 0.14 | 0.00 | 0.00 | 0.04 | 7.36 | S | NSC | 0.00 | 0.13 | 0.00 | 0.00 | 0.02 | 8.78 |
| T | T | 0.00 | 0.10 | 0.00 | 0.00 | 0.03 | 3.70 | F | BAC | 0.00 | 0.13 | 0.00 | 0.00 | 0.04 | 5.33 |
| F | MET | 0.00 | 0.09 | 0.00 | 0.00 | 0.03 | 1.52 | C | F | 0.00 | 0.11 | 0.00 | 0.00 | 0.01 | 4.04 |
| H | MDT | 0.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.18 | B | CVX | 0.00 | 0.11 | 0.00 | 0.00 | 0.08 | 1.06 |
| S | WBA | 0.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.20 | B | DVN | 0.00 | 0.09 | 0.00 | 0.00 | 0.05 | 1.84 |
| I | HON | 0.00 | 0.02 | 0.00 | 0.00 | 0.01 | 0.40 | S | V | 0.00 | 0.09 | 0.00 | 0.00 | 0.00 | 7.98 |
| F | SPG | 0.00 | 0.15 | 0.00 | 0.00 | 0.03 | 5.63 | S | MCD | 0.00 | 0.09 | 0.00 | 0.00 | 0.01 | 6.30 |
| F | COF | 0.00 | 0.08 | 0.00 | 0.00 | 0.01 | 4.63 | C | KO | 0.00 | 0.09 | 0.00 | 0.00 | 0.01 | 3.36 |
| H | UNH | 0.00 | 0.12 | 0.00 | 0.00 | 0.03 | 4.22 | H | PFE | 0.00 | 0.07 | 0.00 | 0.00 | 0.02 | 2.63 |
| B | APC | 0.00 | 0.11 | 0.00 | 0.00 | 0.07 | 1.71 | S | TWX | 0.00 | 0.07 | 0.00 | 0.00 | 0.02 | 1.11 |
| T | MSFT | 0.00 | 0.07 | 0.00 | 0.00 | 0.02 | 2.96 | B | XOM | 0.00 | 0.05 | 0.00 | 0.00 | 0.04 | 0.62 |
| F | AXP | 0.00 | 0.12 | 0.00 | 0.00 | 0.04 | 2.75 | B | MON | 0.00 | 0.05 | 0.00 | 0.00 | 0.01 | 0.70 |
| H | ABT | 0.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.30 | T | EMC | 0.00 | 0.05 | 0.00 | 0.00 | 0.00 | 2.17 |
| H | CVS | 0.00 | 0.06 | 0.00 | 0.00 | 0.01 | 2.97 | T | ORCL | 0.00 | 0.05 | 0.00 | 0.00 | 0.01 | 3.26 |
| F | USB | 0.00 | 0.08 | 0.00 | 0.00 | 0.02 | 2.23 | S | HD | 0.00 | 0.05 | 0.00 | 0.00 | 0.04 | 0.63 |
| B | DOW | 0.00 | 0.05 | 0.00 | 0.00 | 0.01 | 2.52 | T | ACN | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 3.49 |
| F | GS | 0.00 | 0.08 | 0.00 | 0.00 | 0.02 | 1.70 | I | CAT | 0.00 | 0.04 | 0.00 | 0.00 | 0.01 | 0.76 |
| S | SBUX | 0.00 | 0.17 | 0.00 | 0.00 | 0.04 | 6.14 | S | TGT | 0.00 | 0.04 | 0.00 | 0.00 | 0.04 | 0.41 |
| C | CL | 0.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.39 | S | LOW | 0.00 | 0.04 | 0.00 | 0.00 | 0.02 | 0.50 |
| S | FDX | 0.00 | 0.03 | 0.00 | 0.00 | 0.03 | 0.46 | F | JPM | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 1.21 |
| F | MS | 0.00 | 0.17 | 0.00 | 0.00 | 0.05 | 3.63 | H | AGN | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 2.27 |
| T | QCOM | 0.00 | 0.04 | 0.00 | 0.00 | 0.02 | 0.93 | F | AIG | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 1.86 |
| U | EXC | 0.00 | 0.13 | 0.00 | 0.00 | 0.02 | 4.19 | T | VZ | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.96 |
| S | COST | 0.00 | 0.13 | 0.00 | 0.00 | 0.03 | 2.59 | C | PEP | 0.00 | 0.02 | 0.00 | 0.00 | 0.01 | 0.64 |
| I | MMM | 0.00 | 0.03 | 0.00 | 0.00 | 0.02 | 0.59 | H | MRK | 0.00 | 0.02 | 0.00 | 0.00 | 0.01 | 0.28 |
| H | AMGN | 0.00 | 0.03 | 0.00 | 0.00 | 0.02 | 0.66 | I | UTX | 0.00 | 0.02 | 0.00 | 0.00 | 0.01 | 0.45 |
| C | AAPL | 0.00 | 0.02 | 0.00 | 0.00 | 0.01 | 0.20 | T | HPQ | 0.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.18 |
| T | INTC | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.15 | T | TXN | 0.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.24 |
| H | BAX | 0.00 | 0.09 | 0.00 | 0.00 | 0.01 | 3.48 | T | CSCO | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.20 |
| H | BMY | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.09 | C | PG | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.29 |
| H | LLY | 0.00 | 0.08 | 0.00 | 0.00 | 0.02 | 3.86 | H | GILD | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.13 |
| T | IBM | 0.00 | 0.03 | 0.00 | 0.00 | 0.02 | 1.17 | C | MO | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.08 |

* B: Basic Materials; C: Consumer Goods; F: Financial; H: Healthcare; I: Industrial Goods; S: Services; T: Technology; U: Utilities.

Table 1.4: Top 10 influential firms at different forecast horizons. This table reports the top 10 influential firms and their respective sector at different forecast horizons, $h = 1,2,3,4,5$. Given the forecast horizon $h$, we obtain the summary statistics of each row of the firm-wise causality table $[C^h_{i\rightarrow\cdot}]$. The causality table is estimated by the full data sample (20/08/1999 - 31/08/2015). Nodes are the firms of selected S&P 100 components. For each firm $i$, we have the median value of the entries in its "OUT" vector. For each given forecast horizon $h$, we sort the tickers by their median values and identify the top 10 influential firms.

| | h=1 | | h=2 | | h=3 | | h=4 | | h=5 | |
|---|---|---|---|---|---|---|---|---|---|---|
| Rank | Sector* | Ticker | Sector* | Ticker | Sector* | Ticker | Sector* | Ticker | Sector* | Ticker |
| 1 | F | BAC | T | CSCO | T | CSCO | T | CSCO | T | CSCO |
| 2 | C | AAPL | C | AAPL | C | AAPL | C | AAPL | C | AAPL |
| 3 | T | CSCO | F | C | F | AIG | F | AIG | F | AIG |
| 4 | F | C | F | AIG | F | C | F | C | F | C |
| 5 | F | BK | F | GS | F | GS | F | GS | F | GS |
| 6 | F | AIG | I | GE | I | GE | I | GE | I | GE |
| 7 | F | MET | F | MS | F | JPM | F | JPM | F | JPM |
| 8 | C | F | F | JPM | C | F | C | F | C | F |
| 9 | F | JPM | F | MET | T | IBM | T | IBM | T | IBM |
| 10 | F | MS | C | F | F | MET | T | EMC | T | EMC |

* B: Basic Materials; C: Consumer Goods; F: Financial; H: Healthcare; I: Industrial Goods; S: Services; T: Technology; U: Utilities.

Table 1.5: Summary statistics of causality measures from each financial Firm to other financial Firms. This table reports the summary statistics of the firm-wise causality table blocked by the financial sector $[C_{i\to j}]$, where $i, j \in$ Financial Sector. The causality table is estimated by the full data sample (20/08/1999 - 31/08/2015). Nodes are the firms of selected S&P 100 components. For each financial firm $i$, we report the minimum value, the maximum value, the mean value and the quantiles (25%, 50% (median) and 75%) of the entries in its "OUT" vector truncated within the financial sector. The reported values are 100 times of the raw values, and are kept with two digits. We sort the tickers by their median values and identify the top 3 influencial firms in the financial sector.

| Ticker | Median | Mean | Min | 25% | 75% | Max |
|---|---|---|---|---|---|---|
| **BAC** | **0.42** | 1.21 | 0.00 | 0.04 | 1.66 | 5.33 |
| **MS** | **0.30** | 1.19 | 0.00 | 0.04 | 2.21 | 4.83 |
| **GS** | **0.25** | 0.43 | 0.00 | 0.00 | 0.82 | 1.70 |
| BK | 0.06 | 0.91 | 0.00 | 0.00 | 0.98 | 5.75 |
| WFC | 0.03 | 0.49 | 0.00 | 0.00 | 0.38 | 3.96 |
| ALL | 0.01 | 0.30 | 0.00 | 0.00 | 0.37 | 1.46 |
| SPG | 0.01 | 0.54 | 0.00 | 0.00 | 0.08 | 5.63 |
| AXP | 0.00 | 0.43 | 0.00 | 0.00 | 0.24 | 2.78 |
| C | 0.00 | 0.42 | 0.00 | 0.00 | 0.53 | 2.77 |
| AIG | 0.00 | 0.14 | 0.00 | 0.00 | 0.12 | 0.74 |
| COF | 0.00 | 0.39 | 0.00 | 0.00 | 0.12 | 4.63 |
| JPM | 0.00 | 0.10 | 0.00 | 0.00 | 0.15 | 0.51 |
| MET | 0.00 | 0.31 | 0.00 | 0.00 | 0.58 | 1.32 |
| USB | 0.00 | 0.11 | 0.00 | 0.00 | 0.03 | 0.55 |

Table 1.6: Summary statistics of causality measures to each financial firm from other financial firms. This table reports the summary statistics of the firm-wise causality table blocked by the financial sector $[C_{j \to i}]$, where $i, j \in$ Financial Sector. The causality table is estimated by the full data sample (20/08/1999 - 31/08/2015). Nodes are the firms of selected S&P 100 components. For each firm $i$, we report the minimum value, the maximum value, the mean value and the quantiles (25%, 50% (median) and 75%) of the entries in its "IN" vector truncated within the financial sector. The reported values are 100 times of the raw values, and are kept with two digits. We sort the tickers by their median values and identify the top 3 sensitive firms in the financial sector.

| Ticker | Median | Mean | Min | 25% | 75% | Max |
|---|---|---|---|---|---|---|
| **C** | **0.33** | 0.65 | 0.00 | 0.03 | 0.94 | 3.34 |
| **ALL** | **0.32** | 1.02 | 0.00 | 0.02 | 2.21 | 3.96 |
| **BAC** | **0.28** | 0.64 | 0.00 | 0.00 | 0.63 | 5.33 |
| SPG | 0.22 | 0.85 | 0.00 | 0.13 | 0.64 | 5.63 |
| MET | 0.07 | 0.36 | 0.00 | 0.00 | 0.49 | 1.52 |
| MS | 0.01 | 0.38 | 0.00 | 0.00 | 0.62 | 1.75 |
| WFC | 0.01 | 0.66 | 0.00 | 0.00 | 1.00 | 3.52 |
| BK | 0.00 | 0.96 | 0.00 | 0.00 | 0.75 | 5.75 |
| AXP | 0.00 | 0.44 | 0.00 | 0.00 | 0.31 | 2.75 |
| AIG | 0.00 | 0.04 | 0.00 | 0.00 | 0.01 | 0.30 |
| COF | 0.00 | 0.46 | 0.00 | 0.00 | 0.18 | 4.63 |
| GS | 0.00 | 0.23 | 0.00 | 0.00 | 0.24 | 1.70 |
| JPM | 0.00 | 0.06 | 0.00 | 0.00 | 0.01 | 0.47 |
| USB | 0.00 | 0.23 | 0.00 | 0.00 | 0.00 | 2.23 |

Table 1.7: Summary statistics of causality measures from each sector to other sectors. This table reports the summary statistics of each row of the sector-wise causality table $[C_{V_i \to V.}]$. The causality table is estimated by the full data sample (20/08/1999 - 31/08/2015). Nodes are the sectors whose firms are selected in the S&P 100 components. For each sector $V_i$, we report the minimum value, the maximum value, the mean value and the quantiles (25%, 50% (median) and 75%) of the entries in its "OUT" vector. The reported values are 100 times of the raw values, and are kept with two digits. We sort the sectors by their median values and identify the top 4 influential sectors in the economy.

| Sector | Median | Mean | Min | 25% | 75% | Max |
|---|---|---|---|---|---|---|
| **Technology** | **3.27** | 5.75 | 0.00 | 0.00 | 7.40 | 18.84 |
| **Industrial Goods** | **1.55** | 3.28 | 0.00 | 0.01 | 3.16 | 15.04 |
| **Consumer Goods** | **0.90** | 1.07 | 0.00 | 0.43 | 1.47 | 3.02 |
| **Financial** | **0.48** | 9.61 | 0.00 | 0.05 | 5.46 | 57.89 |
| Utilities | 0.08 | 1.59 | 0.00 | 0.00 | 0.27 | 11.83 |
| Services | 0.00 | 7.74 | 0.00 | 0.00 | 2.67 | 52.77 |
| Healthcare | 0.00 | 3.15 | 0.00 | 0.00 | 0.25 | 24.58 |
| Basic Materials | 0.00 | 3.07 | 0.00 | 0.00 | 0.43 | 22.82 |

Table 1.8: Summary statistics of causality measures to each sector from other sectors. This table reports each column of the summary statistics of the sector-wise causality table $[C_{V. \to V_i}]$. The causality table is estimated by the full data sample (20/08/1999 - 31/08/2015). Nodes are the sectors whose firms are selected in the S&P 100 components. For each sector $V_i$, we report the minimum value, the maximum value, the mean value and the quantiles (25%, 50% (median) and 75%) of the entries in its "IN" vector. The reported values are 100 times of the raw values, and are kept with two digits. We sort the sectors by their median values and identify the top 4 sensitive sectors in the economy.

| Sector | Median | Mean | Min | 25% | 75% | Max |
|---|---|---|---|---|---|---|
| **Services** | **1.12** | 10.91 | 0.00 | 0.00 | 16.09 | 52.77 |
| **Basic Materials** | **1.11** | 4.43 | 0.00 | 0.00 | 3.63 | 22.82 |
| **Industrial Goods** | **0.69** | 2.72 | 0.00 | 0.06 | 2.28 | 15.04 |
| **Healthcare** | **0.57** | 3.79 | 0.00 | 0.00 | 1.84 | 24.58 |
| Technology | 0.36 | 2.64 | 0.00 | 0.00 | 0.77 | 18.84 |
| Consumer Goods | 0.09 | 0.73 | 0.00 | 0.00 | 0.84 | 3.02 |
| Utilities | 0.06 | 2.08 | 0.00 | 0.00 | 1.25 | 11.83 |
| Financial | 0.00 | 7.96 | 0.00 | 0.00 | 1.45 | 57.89 |

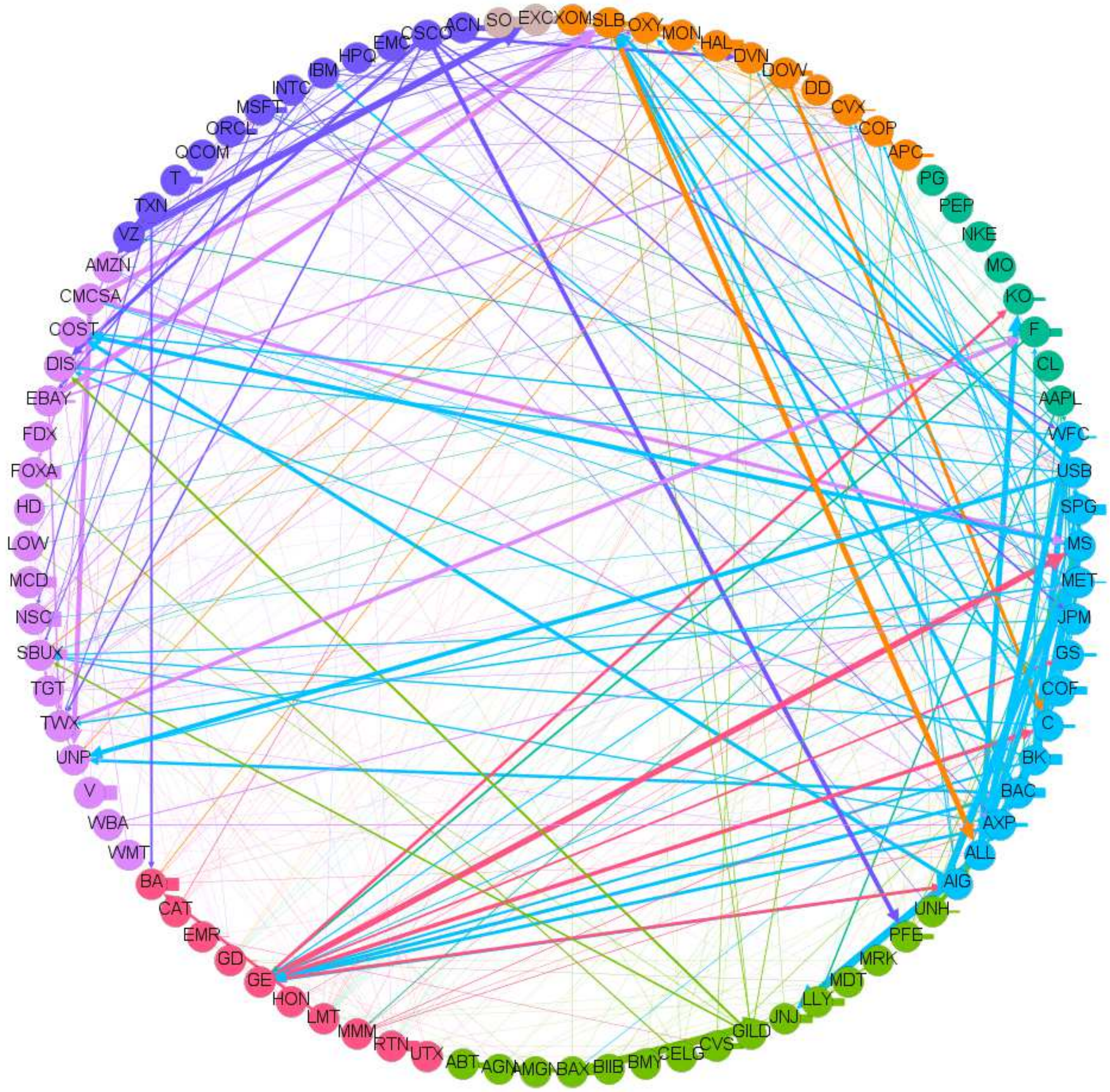



Figure 1.5: Firm-wise S&P 100 implied volatility network. This is a direct effect network corresponding to the causality table, $[C^1_{ij}]$. The causality measures table is estimated by the full data sample (20/08/1999 - 31/08/2015). Nodes are the firms of selected S&P 100 components. Different colors of the nodes correspond to different sectors that the nodes belong to (skyblue: financial; lawn green: healthcare; pink: industrial goods; purple: services; blue: technology; plum: utilities; orange: basic materials forest green: consumer goods). We only keep the directed and weighted edges ($i \to j$) if $C_{ij}$ is greater or equal to the 90% percentile element in $OUT^1_i$ ($C_{i\cdot}$) and the 90% percentile element in $IN^1_j$ ($C_{\cdot j}$). When $i \to j$ and $j \to i$ are both kept, only the edge with greater strength will be shown in this figure. The colors of the edges correspond to the colors of the source nodes. The thickness of the edges are weight rescaled.

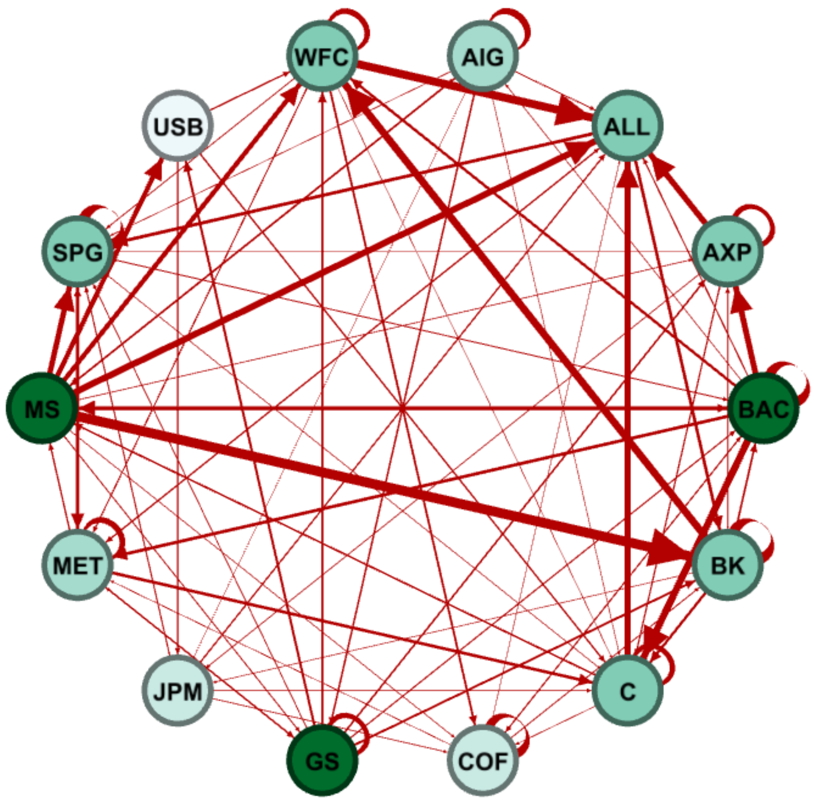



Figure 1.6: Firm-wise S&P 100 implied volatility network within financial sector. This is a direct effect network corresponding to the blocked causality table, $[C^1_{ij}]$ where $i, j \in$ Financial (both node $i$ and node $j$ are the firms that selected from S&P 100 components and belong the financial sector). The causality measures table is estimated by the full data sample (20/08/1999 - 31/08/2015). Nodes are the firms that selected from S&P 100 components and belong to the financial sector. We only keep the directed and weighted edges $(i \to j)$ if $C_{ij}$ is greater or equal to the 50% percentile element in the blocked causality measures table, $[C^1_{ij}]$ where $i, j \in$ Financial. The darkness of the nodes corresponds to the out-degree of the nodes in this filtered network (e.g., MS, BAC and GS have higher out-degree). When $i \to j$ and $j \to i$ are both kept, only the edge with greater strength will be shown in this figure. The thickness of the edges are weight rescaled.

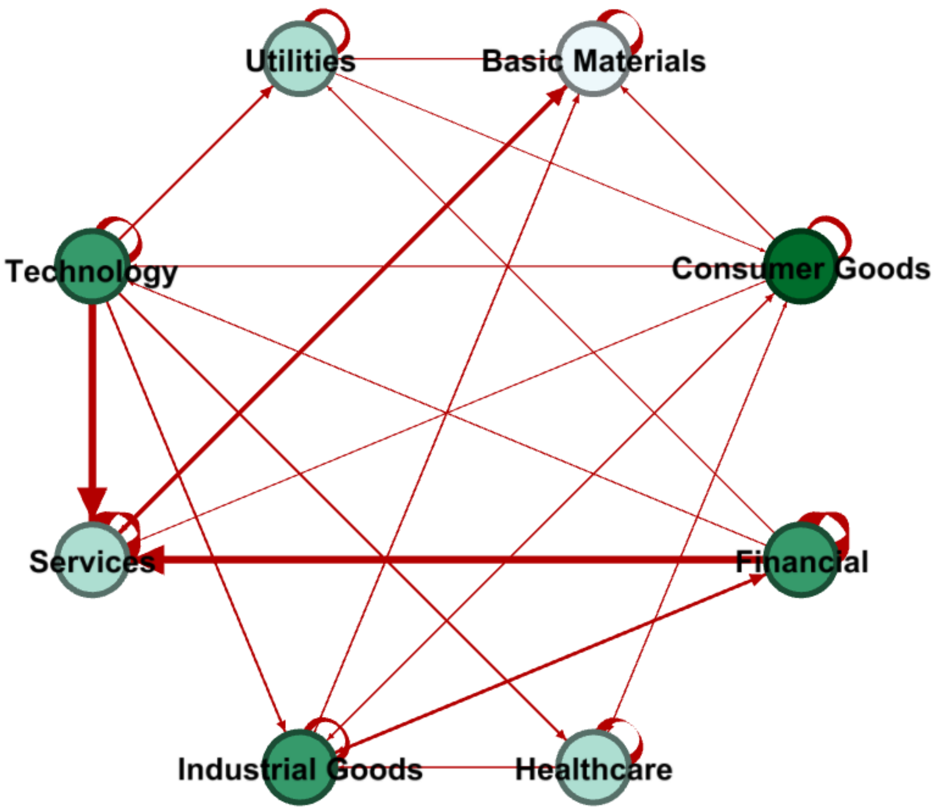



Figure 1.7: Sector-wise S&P 100 implied volatility network. This is a direct effect network corresponding to the causality table, $[C^1_{V_iV_j}]$. The causality table is estimated by the full data sample (20/08/1999 - 31/08/2015). Nodes are the sectors of the firms selected from S&P 100 components. We only keep the directed and weighted edges ($V_i \to V_j$) if $C_{V_iV_j}$ is greater or equal to the 50% percentile element in the causality measures table, $[C^1_{V_iV_j}]$. The darkness of the nodes corresponds to the out-degree of the nodes in this filtered network (e.g., Consumer Goods, Financial, Industrial Goods and Technology have higher out-degree). When $V_i \to V_j$ and $V_j \to V_i$ are both kept, only the edge with greater strength will be shown in this figure. The thickness of the edges are weight rescaled.

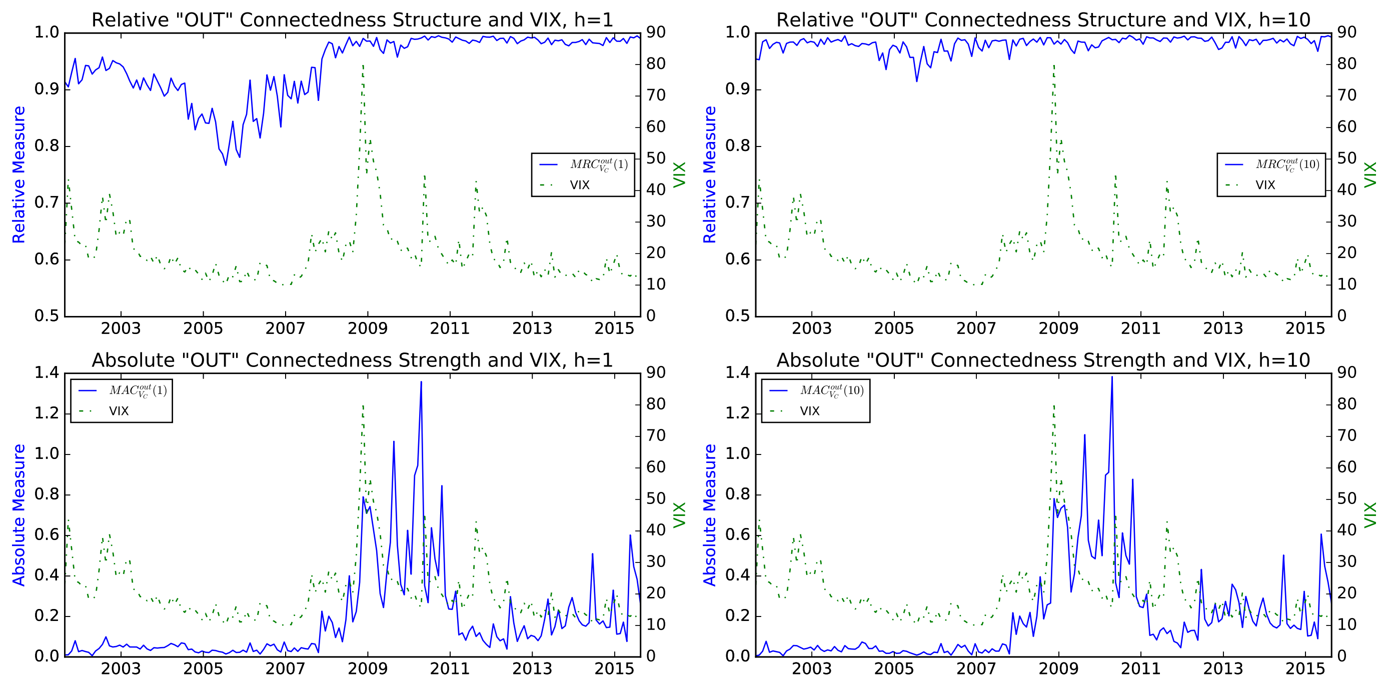



Figure 1.8: Firm-wise market connectedness "out" measures and the VIX index. The blue solid lines are our "out" connectedness measures (relative connectedness structure: upper row; absolute connectedness strength: bottom row) and the green dash lines are the VIX index. The reported connectedness measures are estimated at forecast horizon 1 (left column) and at forecast horizon 10 (right column). All measures are estimated every 1 month by the VAR(1) models with 2-year rolling estimation windows. The nodes of the underlying market are the companies that selected from the S&P 100 components.

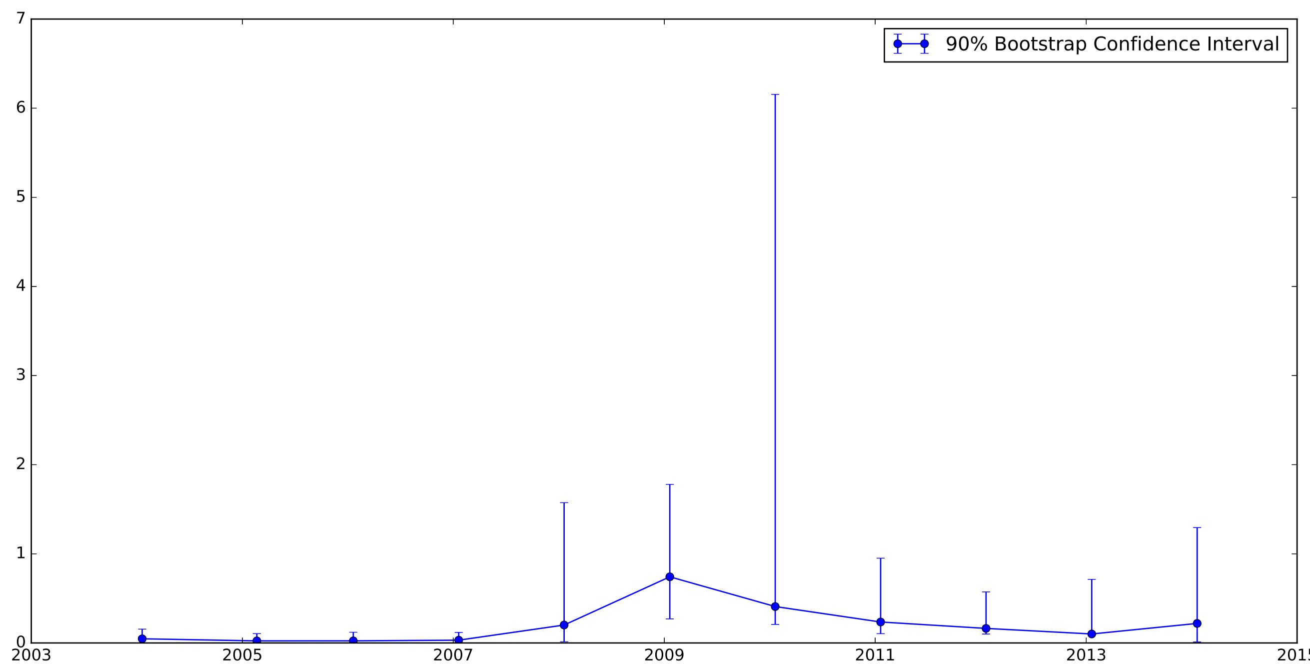



Figure 1.9: 90% bootstapping confidence intervals of market absolute connectedness strength “out” measures. For each date (20/01/2004, 20/01/2005, 20/01/2006, 20/01/2007, 20/01/2008, 20/01/2009, 20/01/2010, 20/01/2011, 20/01/2012, 20/01/2013, 20/01/2014), we construct a simple resampling bootstrap confidence interval of the market absolute connectedness strength “OUT” measure. The confidence level is set to be 90%. All measures are estimated by the VAR(1) models with 2-year rolling estimation windows. The nodes of the underlying market are the companies selected from the S&P 100 components.

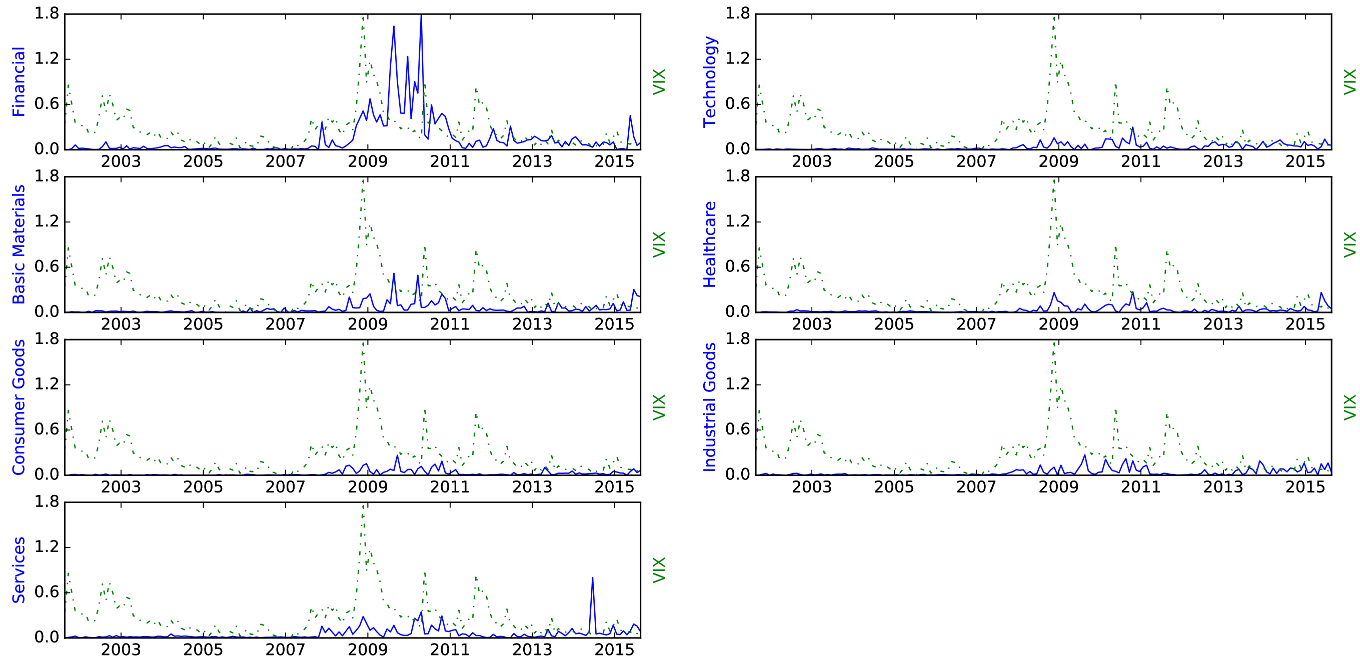



Figure 1.10: Sector connectedness “out” measures and the VIX index. The sector connectedness measures are defined as the connectedness measures within each of the 7 sectors (Financial, Technology, Basic Materials, Healthcare, Consumer Goods, Industrial Goods and Services). The solid blue lines are our absolute sector connectedness “out” strength measures and the green dash lines are the VIX index. All measures are estimated every 1 month by the VAR(1) models with 2-year rolling estimation windows. The nodes of the underlying market are the companies selected from the S&P 100 components.

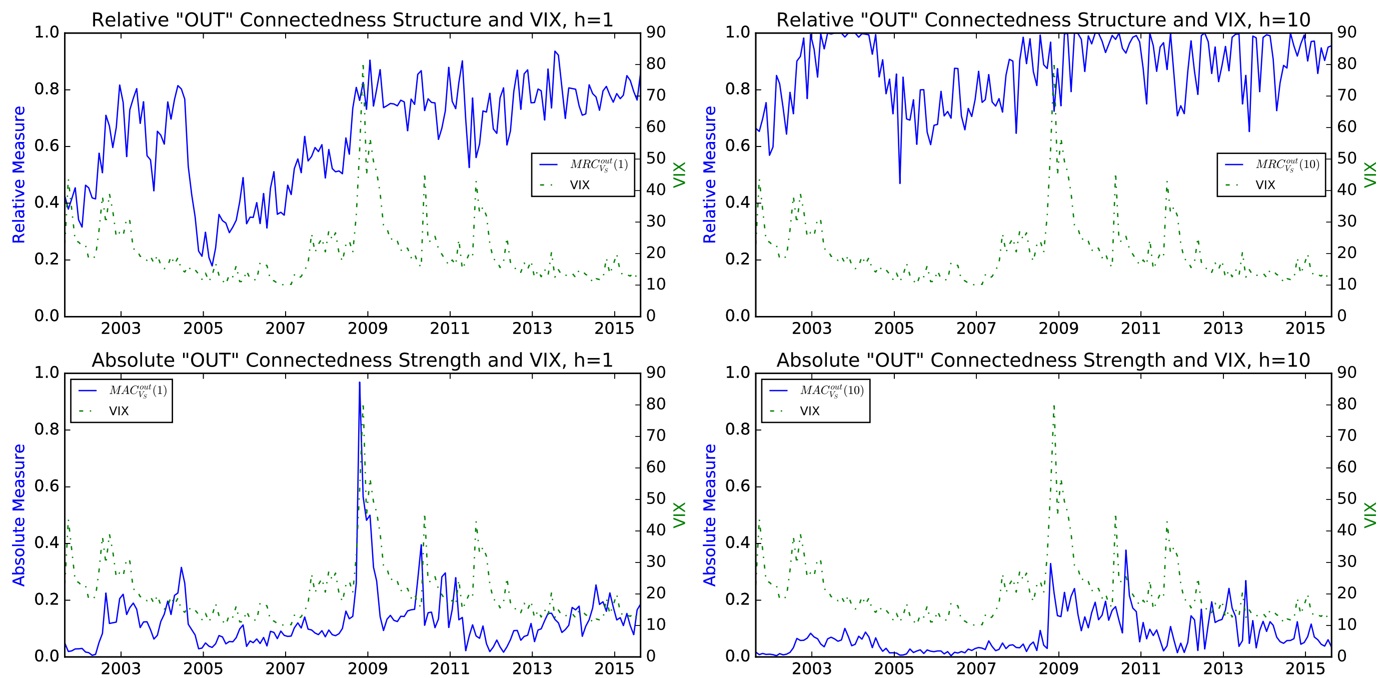



Figure 1.11: Sector-wise market connectedness “out” measures and the VIX index. The blue solid lines are our “out” connectedness measures (relative connectedness structure: upper row; absolute connectedness strength: bottom row) and the green dash lines are the VIX index. The reported connectedness measures are estimated at forecast horizon 1 (left column) and at forecast horizon 10 (right column). All measures are estimated every 1 month by the VAR(1) models with 2-year rolling estimation windows. The nodes of the underlying market are the sectors whose companies are selected from the S&P 100 components.

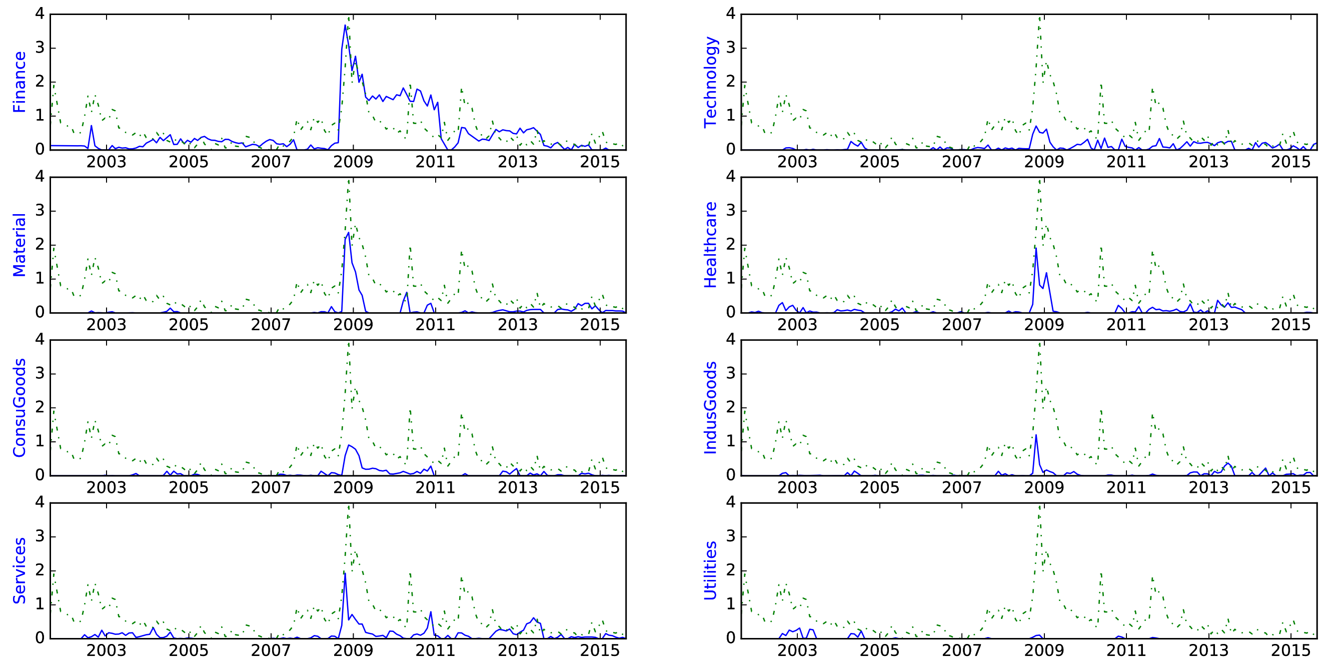



Figure 1.12: Sector-wise causality measures from the financial sector to other sectors. The blue solid line is our causality measures and the green dash line is the VIX index. This figure reports the direct effects from the financial sector on other sectors. The measures are estimated at forecast horizon $h = 1$ ($C^1_{\text{Fin}\to\cdot}$). All measures are estimated every 1 month by the VAR(1) models with 2-year rolling estimation windows. The nodes of the underlying market are the sectors whose companies are selected from the S&P 100 components.

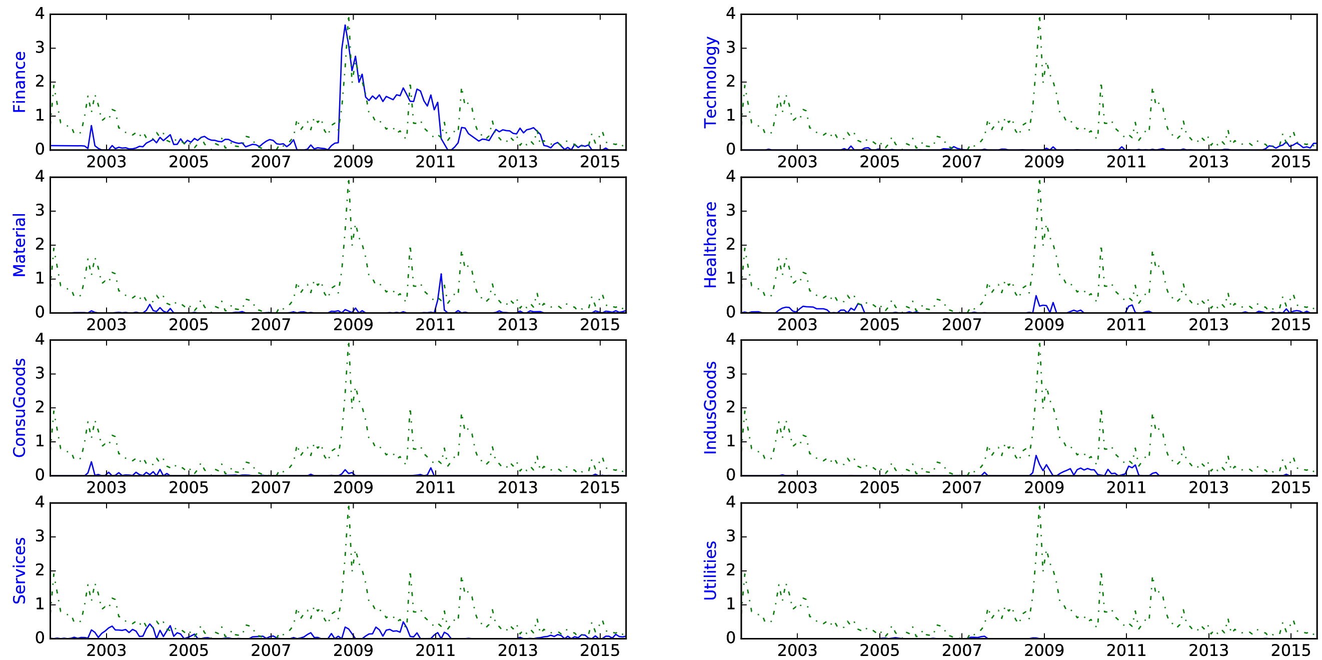



Figure 1.13: Sector-wise causality measures to the financial sector from other sectors. The blue solid line is our causality measures and the green dash line is the VIX index. This figure reports the direct effects from other sectors to the financial sector. The measures are estimated at forecast horizon $h = 1$ ($C^{1}_{\cdot\to\mathrm{Fin}}$). All measures are estimated every 1 month by the VAR(1) models with 2-year rolling estimation windows. The nodes of the underlying market are the sectors whose companies are selected from the S&P 100 components.

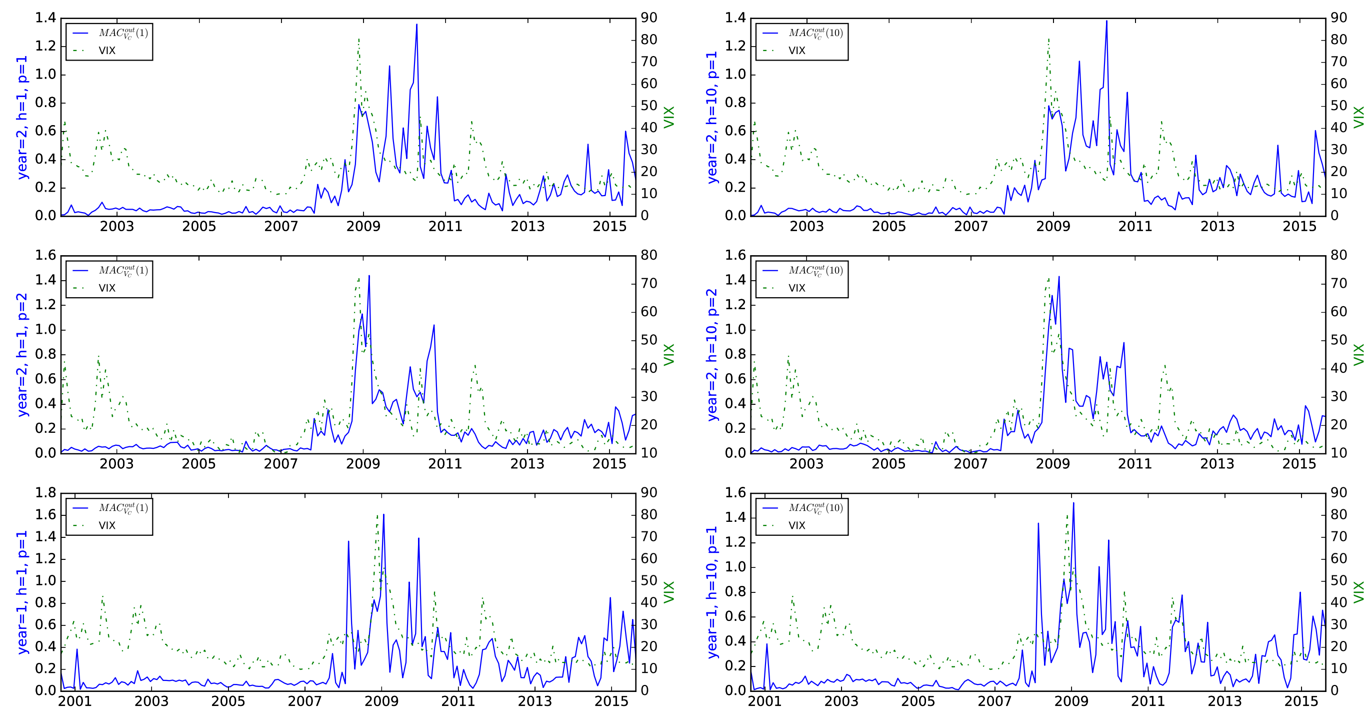



Figure 1.14: Robustness of firm-wise absolute market connectedness strength “out” measures. The blue solid lines are our absolute market connectedness strength “out” measures and the green dash lines are the VIX index. All measures are estimated every 1 month. The nodes of the underlying market are the companies selected from the S&P 100 components. The upper row reports the measures estimated with the VAR(1) model (p=1) and with 2-year rolling estimation windows (year=2). The middle row reports the measures estimated with the VAR(2) model (p=2) and with 2-year rolling estimation windows (year=2). The bottom row reports the measures estimated with the VAR(1) model (p=1) and with 1-year rolling estimation windows (year=1). The left column reports the measures estimated at forecast horizon 1 (h=1) and the right column reports the measures estimated at forecast horizon 10 (h=10).

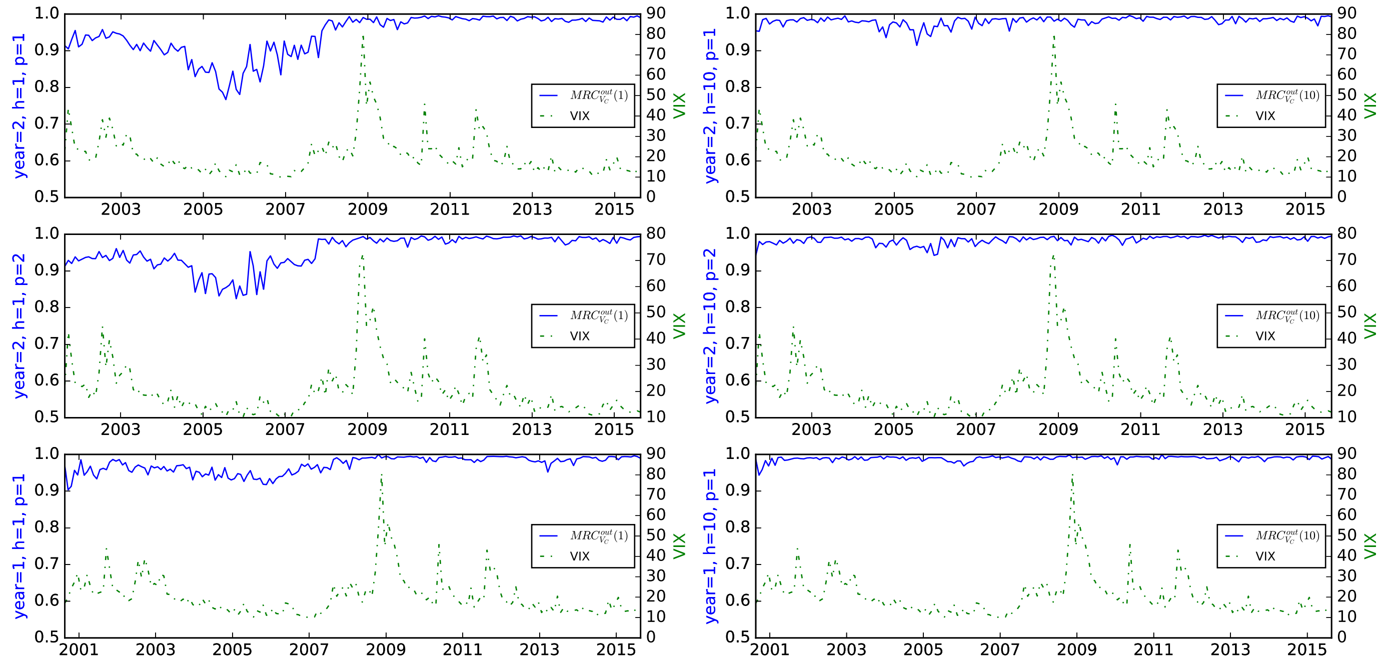



Figure 1.15: Robustness of firm-wise relative market connectedness structure "out" measures. The blue solid lines are our firm-wise relative market connectedness structure "out" measures and the green dash lines are the VIX index. All measures are estimated every 1 month. The nodes of the underlying market are the companies selected from the S&P 100 components. The upper row reports the measures estimated with the VAR(1) model (p=1) and with 2-year rolling estimation windows (year=2). The middle row reports the measures estimated with the VAR(2) model (p=2) and with 2-year rolling estimation windows (year=2). The bottom row reports the measures estimated with the VAR(1) model (p=1) and with 1-year rolling estimation windows (year=1). The left column reports the measures estimated at forecast horizon 1 (h=1) and the right column reports the measures estimated at forecast horizon 10 (h=10).

# Chapter 2

# Centralities in Illiquidity Transmission Networks and the Cross-Section of Expected Returns

## Abstract

This paper investigates the relationship between stock illiquidity spillovers and the cross-section of expected returns. I study industry-level illiquidity spillovers in a directed network that describes the interconnections among stocks' bid-ask spreads, where the interconnections are latent and are estimated by a Granger-type measure. In the directed illiquidity transmission network, the illiquidity of high sensitive centrality (SC) industries, i.e., those active at receiving illiquidity from others, as well as high influential centrality (IC) industries, i.e., those active at transferring illiquidity to others, tends to covary with that of their neighbours and neighbours' neighbours across different horizons due to illiquidity spillovers. As a result, long run returns of the portfolios that contain stocks of central (high SC or high IC) industries may be more volatile because of weak diversification of the liquidity risk across different horizons. Thus, investors would require compensations for holding these central stocks. I confirm this conjecture and find that central industries in illiquidity transmission networks do earn higher average stock returns (around 4% per year) than other industries. Market-beta, size, book-to-market, momentum, liquidity and idiosyncratic volatility

effects cannot account for the high average return earned by central industries.

## 2.1 Introduction

Liquidity plays a central role in the functioning of financial markets. Stock market liquidity is documented as being closely related to business cycles (Ns, Skjeltorp and degaard (2011)), stock market returns (Amihud (2002)) and cross-sectional returns (Pastor and Stambaugh (2003)). In a financial market where everyone is probably connected to everybody else, the illiquidity risk exposure for a firm is not only related to its idiosyncratic liquidity level and its correlation to market liquidity conditions, but also closely related to the properties of the connected individual firm. For example, a firm's poor liquidity condition could be a result of drops in liquidity of its connected firms due to illiquidity transmissions (see Oh (2013) and Cespa and Foucault (2014) among others). Current literature on illiquidity transmissions is mainly focusing on undirected commonality and aggregated contagion in liquidity,[1] and on directed illiquidity spillovers between two firms, two stocks and two markets.[2] In the recent financial crisis, however, we observe that a major market-wide liquidity problem could be a result of illiquidity spillovers originated from "important" industries, e.g., the financial industry. Not much attention is paid to understanding the heterogeneity in market-wide illiquidity spillovers. To better understand this issue, this paper investigates the spillover risk of illiquidity through modeling the market-wide illiquidity spillovers in a directed network that describes the interconnections among industries' idiosyncratic illiquidity risks.[3] Then I examine the relationship between the heterogeneous roles of industries in illiquidity spillovers and the cross-section of expected returns.

When studying illiquidity spillovers in network analysis, we can explore the architecture of the spillovers as a mechanism of how individual illiquidity evolves within an "illiquidity network". This exploration involves looking into the underlying illiquidity transmission structure, rather than just superficially treating the aggregated market illiquidity as a given outcome. In network analysis, centrality is a concept referring to a node's position in the functioning of network spillovers.

[1] See Cifuentes, Ferrucci and Shin (2005), Brockman, Chung and Pérignon (2009), Hameed, Kang and Viswanathan (2010), Karolyi, Lee and van Dijk (2012), Koch, Ruenzi and Starks (2016) among the most recent studies.

[2] See, e.g., Goyenko and Ukhov (2009), Oh (2013) and Cespa and Foucault (2014).

[3] Hameed et al. (2010) document inter-industry spillover effects in liquidity, which are likely to arise from capital constraints in the market making sector.

Actually, a directed network assumption is straightforward but implicit when considering network spillovers as any financial spillover must have a direction with a source and a target. In this regard, I study network centrality in two directions: i) sensitive centrality (SC), which measures the degree of an industry being affected by others, and ii) influential centrality (IC), which measures the degree of an industry affecting others. In an illiquidity transmission network, high SC industries are the ones whose illiquidity can easily be affected by the illiquidity of other industries, while high IC industries are the ones whose illiquidity can easily affect others' illiquidity. As a result, central (high SC or high IC) industries tend to play a major role in network spillovers, compared to those that are isolated from others.[4] I also assume a neighbour effect: being affected by high SC industries makes an industry more likely to be a high SC industry, and affecting high IC industries makes an industry more likely to be a high IC industry as well. Thus, an industry's centrality also takes its connected industries' centralities into account, sharing the characteristics of what kind of neighbours it is connected to in terms of the role in network spillovers. Implications of influential centrality in network analysis have drawn growing attention in the literature on financial systemic risk. For example, Acemoglu et al. (2012) and Acemoglu et al. (2015b) use asymmetric network structures to show the possibility that aggregate fluctuations may originate from idiosyncratic shocks to high IC firms. However, research on sensitive centrality is missing in the existing literature on financial network. I argue that SC is least as important as IC in terms of asset pricing. In this paper, I provide a comprehensive analysis of sensitive centrality and influential centrality simultaneously in a directed illiquidity network context.

Intuitively speaking, illiquidity spillovers would lead the illiquidity of a central industry to covary with that of its connected neighbours and neighbours' neighbours across different horizons due to illiquidity spillovers, thus long run returns of the portfolios that contain these central stocks may be more volatile due to weak diversification of the liquidity risk across different horizons. Since a high SC industry's illiquidity is easily affected by the illiquidity of other industries, investors will demand a premium for holding this high SC stock as agents demand compensation for not being able to use this stock to diversify the liquidity risk of others. Similarly, since it is difficult to find other stocks to diversify a high IC industry's liquidity risk as the high IC industry's illiquidity would easily affect others' illiquidity, the high IC stocks should also earn a premium.

[4] An industry is isolated in a network means it is not connected to anybody in this network.

The goal of this paper is to investigate whether such illiquidity centralities (SC and IC) are risk factors in asset pricing where industries are connected in an illiquidity network. I resolve this issue by examining the cross-sectional relationship between the illiquidity centralities and expected returns. Based on the argument stated above, my conjecture is that central stocks will earn higher average returns. The IC measured from other economic networks has already been documented as a risk factor in recent literature on network and asset pricing (see, e.g., Buraschi and Porchia (2012) and Ahern (2013)), but the result about SC is still missing. Indeed, the empirical result in this paper provides strong evidence to support my conjecture that both SC and IC industries do earn higher average returns. Interestingly, my robustness check suggests the effects of SC are even more robust than IC.

In this paper, illiquidity spillovers, network centralities and cross-sectional expected returns are to be explored together. To verify my previous conjecture, we need a new analytical procedure that includes four main steps: i) measuring industry's illiquidity, ii) estimating the illiquidity transmission network among different industries, iii) calculating centralities in the illiquidity network, and iv) examining the cross-sectional relationship between illiquidity centralities and expected returns.

First, liquidity has many dimensions; this paper focuses on a dimension associated with bid-ask spreads in stock markets, which reflects the difficulty (cost) of stocks' transactions. I use Corwin and Schultz (2012)'s bid-ask spreads estimate to measure firms' daily illiquidity. Industry's illiquidity is measured by the simple average of the individual bid-ask spreads estimates of the firms that belong to this industry.

Then adapting the financial network estimation technique suggested by Billio et al. (2012) and Dufour and Jian (2016), I use a Granger-type measure to estimate the directed relationships between every pair of industries in the stock market.[5] I identify the directed illiquidity spillover from industry A to industry B by testing whether the marginal effect of industry A's past illiquidity on industry B's current illiquidity is positive. The estimated illiquidity transmission network can be represented by an adjacency matrix.

Once we have the estimate of the adjacency matrix of the illiquidity transmission network, I take it as given and use Bonacich (1987)'s generalized eigenvector centrality measure, which is built on the neighbour effect assumption, to calculate industries' sensitive centralities and influ-

[5] Actually, I focus on the industry level just for feasibility of implementation.

ential centralities in the illiquidity network. I re-estimate industries' centralities each year by the subsample in that year, then I obtain the annual series (1963 - 2015) of industries' centralities (SC and IC). In fact, high SC and high IC tend to coexist and are persistent in an industry. I find that industries' illiquidity sensitive and influential centralities are positively correlated in time-series and in cross-section.

Following the classic procedure used by Fama and French (1992), I examine the cross-sectional relationship between the illiquidity centralities and expected returns at portfolio level as well as at industry level. Sorting industries by their respective SC and IC at the beginning of each year, I form portfolios in 10 deciles based on SC and IC, respectively. I find that with the portfolios rebalanced annually, average return differences between industries in the highest and lowest SC deciles and average return differences between industries in the highest and lowest IC deciles exceed 4% per year. The corresponding Fama-French-Carhart four-factor alphas also exceed 4% per year. Both the return differences and the four-factor alpha differences are economically and statistically significant at all standard significance levels. Not surprisingly, industries' centralities have relation with some well-known risk factors. For example, high SC industries tend to be those industries with small average firm size and high average book-to-market and low liquidity. To ensure that it is not these characteristics, but the illiquidity centralities (SC and IC), that drive the return differences documented in this paper, I perform a battery of bivariate sorts and re-examine the raw return and alpha differences. These results are robust to controls for market-beta, size, book-to-market, momentum, liquidity and idiosyncratic volatility. Results from cross-sectional regressions corroborate this evidence. The risk premium between the highest and the lowest deciles of SC and the premium of IC estimated by the Fama-MacBeth two-step procedure are approximately 9% per year and 12% per year, respectively. A robustness check for different subperiods (1970 - 2015, 1980 - 2015, 1990 - 2015 and 2000 - 2015) suggests the effects of SC are even more robust than IC. In short, the illiquidity centralities (SC and IC) do earn premiums in the cross-section of expected returns.

The rest of this paper is organized as follows. Section 2.2 discusses the contributions of this paper relative to related literature. Section 2.3 proposes a new analytical framework for empirical studies. Section 2.4 provides the univariate portfolio-level analysis, the bivariate analysis and industry-level cross-sectional regressions that examine a comprehensive list of control variables.

Section 2.5 makes a short conclusion.

## 2.2 Related Literature

This paper contributes to four strands of the literature: i) financial systemic risk with network analysis and its asset pricing implications, ii) commonality in liquidity, illiquidity contagions and illiquidity spillovers, iii) gradual information diffusion, and iv) financial network estimation.

The first stream studies financial systemic risk with network analysis and its asset pricing implications. As Andersen et al. (2012) mention, modern network theory can provide a unified framework for systemic risk measures. For example, Acemoglu et al. (2012), Elliott et al. (2014) and Acemoglu et al. (2015b) show that market architectures may function as a potential propagation mechanism of idiosyncratic shocks throughout the economy. Many of the efforts in this stream are concentrated on studying the effect of influential centrality because high IC firms (or sectors) are very likely to be a source of market turbulences. Motivated by this intuition, Buraschi and Porchia (2012) and Ahern (2013) conduct empirical analysis on firms' fundamentals networks and on input-output networks, respectively, and find evidence that supports the theory implications. They document that high IC firms do earn higher expected returns. This paper differs from theirs in two aspects. First, I stress that sensitive centrality is at least as important as influential centrality in terms of asset pricing. Sensitive centrality and influential centrality can be seen as twin concepts that built on directed network structures, but respectively characterize nodes' importance in a network in distinct directions. As discussed before, both high SC and high IC firms should earn risk premiums according to their network implications. In this paper, I provide a comprehensive analysis on high SC and high IC industries. The result related to IC is consistent with the implication of Acemoglu et al. (2012) and Acemoglu et al. (2015b)'s theory in asset pricing, while SC turns out to be a more robust risk factor than IC in explaining cross-sectional returns and is thus of great importance as well. Second, I focus on a well-known risk, illiquidity risk, and its transmission structures. The illiquidity network structure is directly identified by illiquidity spillovers. Thus the interpretation of the network effects in terms of risk spillovers is more straightforward.

The second stream of literature studies commonality in liquidity, illiquidity contagions and illiquidity spillovers in financial markets. Liquidity has been shown to covary strongly across stocks[6] and commonality in liquidity can influence expected returns[7]. Both illiquidity comovements and illiquidity spillovers may describe the phenomenon of covaried illiquidity across stocks. But illiquidity comovements characterizes the contemporaneous relationship among cross-sectional illiquidity, while illiquidity contagions and spillovers focus more on the relationships across different horizons. Cifuentes et al. (2005) explore liquidity risk in a system of interconnected financial institutions and find contagious failures can result from small shocks. Oh (2013) presents a model in which the contagion of a liquidity crisis between two nonfinancial institutions occurs because of learning activity within a common creditor pool. Cespa and Foucault (2014) show that cross-asset learning generates a self-reinforcing positive relationship between price informativeness and liquidity, which can lead when a small drop in the liquidity of one security can, through a feedback loop, spill over and result in a large drop in market liquidity. Longstaff (2010) conducts an empirical investigation into the pricing of subprime asset-backed collateralized debt obligations (CDOs) and finds that strong evidence of contagion in financial markets was propagated primarily through liquidity and risk-premium channels. These studies provide theoretical and empirical evidences of why illiquidity can spill over and cause contagions in financial markets across different horizons. In fact, illiquidity spillovers can happen even if there is no contemporaneous illiquidity comovement, and vice versa. The main departure of this paper from this literature is primarily in the emphasis on the network structure of illiquidity transmissions. Specifically, I focus on the asset pricing implications of the heterogeneity of illiquidity spillovers.

The third stream of the literature studies gradual information diffusion in financial markets. It has been documented that economic links between firms can serve as the channel of gradual diffusion of information. Individual firm's returns, return volatilities and credit spreads can be predicted via firms' linkages (see Cohen and Frazzini (2008),Hertzel et al. (2008), Menzly and Ozbas (2010), Aobdia et al. (2014), Gençay et al. (2015), Albuquerque et al. (2015) and Gençay et al. (2016) among others). This literature implies potential effects of network structures on asset pricing, since they find that firm's returns can be predicted by the returns of the firms it is connected to. Actually, gradual information diffusion may also provide a channel for risk spillovers.

The fourth strand of the literature studies the estimation on financial network structures. After

[6] See Brockman et al. (2009), Hameed et al. (2010), Karolyi et al. (2012), Koch et al. (2016) among the most recent studies.

[7] See, e.g., Pastor and Stambaugh (2003) and Acharya and Pedersen (2005).

all, most of financial relationships in financial markets are latent and need to be estimated from an appropriately identified model. Billio et al. (2012) use the Granger noncausality testing to meausre connectedness in financial markets. Hautsch et al. (2015) measures the downside risk relationship from A to B by estimating the marginal effect of the Value-at-Risk (VaR) of A's returns on B's returns. Diebold and Yilmaz (2014) and Dufour and Jian (2016) propose general network measurement frameworks to measure directed financial relationships. In this paper, illiquidity networks are estimated by a Granger-type procedure that identifies illiquidity transmissions by measuring the illiquidity prediction among industries. This method is in line with Billio et al. (2012) and Dufour and Jian (2016). We share the same estimation logic: if industry A's illiquidity transmits to industry B, then industry B's illiquidity can be predicted by industry A's illiquidity.[8] However, measuring network centrality also requires positive spillovers: if industry A's illiquidity transmits to industry B, a higher current illiquidity of industry A should increase the future illiquidity of industry B. Therefore in this paper, I estimate the direct effect in the illiquidity transmission network by testing positive prediction effects. Causality at multiple horizons could measure the illiquidity spillovers from one industry to another while simultanenously considering direct and indirect effects (see, e.g., Dufour and Jian (2016)). But the adjacency matrix representing the underlying network structure in terms of all bilateral direct effects is sufficient to calculate network centralities when we use eigenvector centrality measure that will be discussed in the next section. So I estimate the direct effect that is measured by forecasting at horizon one: if industry A's illiquidity transmits to industry B, a higher today's illiquidity of industry A should increase tomorrow's illiquidity of industry B.

## 2.3 Analytical Framework

In this section, I provide an analytical framework to formalize and quantify illiquidity centrality for empirical analysis. I use an adjacency matrix to represent a general illiquidity transmission network. Since any illiquidity transmission has direction, I categorize network centrality by: i) sensitive centrality, which measures how sensitive is a node to a random shock in a network; ii) in-

[8]Goyenko and Ukhov (2009) also use a Granger-type procedure to study the illiquidity spillovers between stock and bond markets.

fluential centrality, which measures how influential is a node's shock affecting others in a network. Given directed network structures represented by an adjacency matrix, I use Bonacich (1987)'s generalized eigenvector centrality to measure nodes' network sensitive centrality and influential centrality. Note that illiquidity transmission networks are latent, I use Corwin and Schultz (2012)'s bid-ask spreads estimate to measure firms' daily illiquidity and apply a specification method that is similar to Granger causality measures to empirically identify directed illiquidity network structures.

### 2.3.1 Illiquidity Transmission Network

Network analysis can be used to model and explain financial contagions. For example, Allen and Gale (2000) show that the possibility of contagion depends strongly on the completeness of the underlying network structure. For the complete network shown in Figure 2.1a, individuals can be insured by each others following Lucas (1977)'s diversification argument, such that microeconomic shocks would average out and thus have negligible aggregate effects. For the incomplete network shown in Figure 2.1b, idiosyncratic shocks may propagate throughout the entire system and an individual problem can cause a systemic failure.

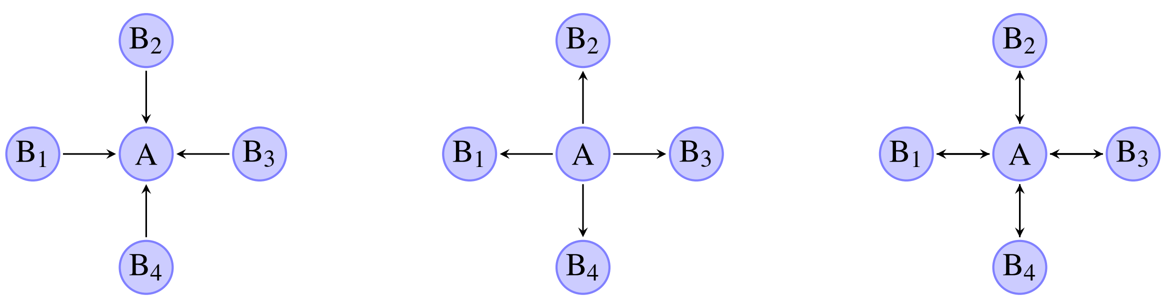

(a) Complete Network

(b) Incomplete Network

Figure 2.1: Financial Contagion and Network Structures

In this paper, I focus on illiquidity spillovers. Industries' illiquidity may transmit to other industries via an illiquidity network. I examine financial network structures in at industry level and focus on industries' centralities in their illiquidity network. Sensitive centrality (SC) measures the degree of a node being affected by others: how sensitive is a industry to a random shock in a network. In Figure 2.2a, industry A is a high SC firm as illiquidity from other industries can directly transmit to it. Influential centrality (IC) measures the degree of a node affecting others:

how influential of the shock of an industry affecting others in this network. In Figure 2.2b, industry A is a high IC industry as its illiquidity can directly transmit to all other industries. Note that a high SC industry is not necessarily low IC. Figure 2.2c shows a case where industry A is both high SC and high IC. I call it absolute centrality (AC). In Figure 2.2c, illiquidity from any other industries can directly transmit to industry A, meanwhile, industry A's illiquidity can also directly transmit to all other industries in this network. Intuitively speaking, an industry being affected by a high SC industry tends to be sensitive central as well. In Figure 2.3a, industry C is a high SC industry and it affects industry A. Illiquidity can easily transmit to industry C and then spillovers to industry A via industry C. Thus industry A is also a high SC industry due to industry C being sensitive central. Likewise, an industry affecting a high IC industry also tends to be influential central. In Figure 2.3 industry A's illiquidity can transmit to every industry in this network: directly to industry C and indirectly via industry C. Industry A is high IC since industry C is relatively influential central in the rest of the network. In this sense, our illiquidity centrality (SC and IC) has simultaneously taken directed direct effects and directed indirect effects into account.




(a) Sensitive Centrality (b) Influential Centrality (c) Absolute Centrality

Figure 2.2: Network Centrality


(a) Affected by a sensitive central industry (b) Affecting a influential central industry

Figure 2.3: Neighbour to a high sensitive (influential) central industry

In view of asset pricing, both high SC and IC industries' stocks are not desirable assets to hedge against a deterioration in investment opportunities. High SC stocks tend to have low liquidity once others experience illiquidity during bad times. For high IC stocks, their illiquidity may spread to the whole financial network and cause market-wide illiquidity and aggregate turbulences. Influential centrality could also be viewed as a source of market beta (see Ahern (2013)). Thus, as a "victim" of the illiquidity of others and a "villain" of market turbulences, high SC stocks and high IC stocks should both earn higher expected returns. In this paper, I will empirically examine whether illiquidity network centralities (SC and IC) are risk factors priced in cross-sectional stock returns. For now, I use a simple network setting to further illustrate the intuition of why high SC and high IC firms should earn premiums, even if there is no risk or return comovement.

**Example 2.3.1.** Suppose there are only three assets $(i, j, k)$ in the market where investors are risk-averse. Asset $i$'s illiquidity transmits to asset $j$, but they are independent from asset $k$. In this network as shown in Figure 2.4, asset $i$ and asset $j$ are connected, and asset $k$ is isolated. Thus, asset $i$ is a high IC asset as it affects asset $j$; asset $j$ is a high SC asset as it is affected by asset $i$; asset $k$ is neither high IC nor high SC asset as it is isolated with the spillovers from asset $i$ to asset $j$. I compare asset $k$ with asset $i$ and asset $j$. Asset $k$ is a benchmark asset in this example.

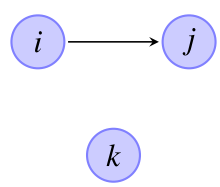



Figure 2.4: Simple network with high SC, high IC and isolated nodes

I assume the return of asset $k$ at time $t$, $R_{kt}$, is independently drawn from the standard normal distribution $N(0,1)$. For asset $i$, its return at time $t$, $R_{it}$, is also drawn independently from $N(0,1)$. The return of asset $j$ at time $t$, $R_{jt}$, is correlated to $R_{it-1}$ as asset $i$'s illiquidity at time $t-1$ can transmit to asset $j$'s illiquidity at time $t$. Without loss of generality, I simply let $R_{jt} = R_{it-1}$. In this case, they all have the same expected return: $\mathrm{E}(R_{it}) = \mathrm{E}(R_{jt}) = \mathrm{E}(R_{kt}) = 0$; and the same variance of returns: $\mathrm{Var}(R_{it}) = \mathrm{Var}(R_{jt}) = \mathrm{Var}(R_{kt}) = 1$. Moreover, there exists a financial spillover, $(i \rightarrow j)$, but no contemporaneous comovement of illiquidity or returns: $\mathrm{Cov}(R_{it}, R_{jt}) = \mathrm{Cov}(R_{kt}, R_{jt}) = \mathrm{Cov}(R_{it}, R_{kt}) = 0$. Given the size (number of chosen assets) of portfolios, all of these equal-weighted portfolios seem to be equivalent to investors. However, this is not true because the spillover from asset $i$ to asset $j$ does play a big role in affecting long run returns.

Suppose investors can only update their portfolio ($p$) every two periods and let's assume the interest rate is zero for simplicity; investors will be concerned about the average return over two periods, $\frac{1}{2}(R_t^p + R_{t+1}^p)$, instead of the current return, $R_t^p$. Now, we consider the cases when investors have to hold a given asset $z$, $z = i, j, k$, and are randomly assigned another asset with equal probability of 0.5 at the beginning of day $t$. I denote this random two-asset portfolio as $(z, \cdot)$. Investors hold the realized portfolio of $(z, \cdot)$ over day $t$ and day $t+1$.

There are three possible portfolios with two assets: $(i, j)$, $(i,k)$ and $(j,k)$, whose corresponding returns on day $t$ are denoted by $R_t^{ij}$, $R_t^{ik}$ and $R_t^{jk}$, respectively, where $R_t^{ij} = \frac{1}{2}(R_{it} + R_{j_t})$, $R_t^{ik} = \frac{1}{2}(R_{it} + R_{k_t})$ and $R_t^{jk} = \frac{1}{2}(R_{jt} + R_{k_t})$. For example, $(i, \cdot)$ implies investors have to hold a random portfolio composed by asset $i$ with probability 1 and either asset $j$ or asset $k$ with equal probability 0.5. At the beginning of day $t$, the realized portfolio could be $(i, j)$ or $(i,k)$, then investors hold the realized portfolio over 2 periods: day $t$ and day $t+1$ and obtain the average return $\frac{1}{2}(R_t^{i\cdot} + R_{t+1}^{i\cdot}) = \frac{1}{2}\big(\frac{1}{2}(R_t^{ij} + R_{t+1}^{ij}) + \frac{1}{2}(R_t^{ik} + R_{t+1}^{ik})\big) = \frac{1}{8}(2R_{it} + R_{jt} + R_{kt} + 2R_{it+1} + R_{jt+1} + R_{kt+1})$. Similarly, the average return of holding asset $j$ for sure is $\frac{1}{2}(R_t^{j\cdot} + R_{t+1}^{j\cdot}) = \frac{1}{8}(2R_{jt} + R_{it} + R_{kt} + 2R_{jt+1} + R_{it+1} + R_{kt+1})$ and the average return of holding asset $k$ for sure is $\frac{1}{2}(R_t^{k\cdot} + R_{t+1}^{k\cdot}) = \frac{1}{8}(2R_{kt} + R_{it} + R_{jt} + 2R_{kt+1} + R_{it+1} + R_{jt+1})$.

- The random portfolio $(k, \cdot)$ is superior to the random portfolio $(j, \cdot)$:
  The expected average return of $(k, \cdot)$ over two periods and the expect average return of $(j, \cdot)$ over two period are equal: $\mathrm{E}\big(\frac{1}{2}(R_t^{k\cdot} + R_{t+1}^{k\cdot})\big) = \mathrm{E}\big(\frac{1}{2}(R_t^{j\cdot} + R_{t+1}^{j\cdot})\big) = 0$. But the variance of the two-period average return of $(k, \cdot)$ is less than $(j, \cdot)$: $\mathrm{Var}\big(\frac{1}{2}(R_t^{k\cdot} + R_{t+1}^{k\cdot})\big) = \frac{7}{4}$, while $\mathrm{Var}\big(\frac{1}{2}(R_t^{j\cdot} + R_{t+1}^{j\cdot})\big) = 2$. Thus, the high SC asset $j$ is less attractive to investors than the isolated asset $k$, because asset $j$ will carry the shock from asset $i$ on day $t$ to day $t+1$; this results in the portfolios with asset $j$ having higher positive correlation across different periods, which increases the variance of return of holding asset $j$ in their portfolios.

- The random portfolio $(k, \cdot)$ is superior to the random portfolio $(i, \cdot)$:
  The expected average return of $(k, \cdot)$ over two periods and the expect average return of $(i, \cdot)$ over two period are equal: $\mathrm{E}\big(\frac{1}{2}(R_t^{k\cdot} + R_{t+1}^{k\cdot})\big) = \mathrm{E}\big(\frac{1}{2}(R_t^{i\cdot} + R_{t+1}^{i\cdot})\big) = 0$. However, the variance

of the two-period average return of $(k,\cdot)$ is less than $(i,\cdot)$: $\mathrm{Var}\big(\frac{1}{2}(R_t^{k\cdot}+R_{t+1}^{k\cdot})\big)=\frac{7}{4}$, while $\mathrm{Var}\big(\frac{1}{2}(R_t^{i\cdot}+R_{t+1}^{i\cdot})\big)=2$. Thus, the high IC asset $i$ is less attractive to investors than the isolated asset $k$, because asset $i$ will transmit its shock on day $t$ to other(s) on day $t+1$; this results in the portfolios with asset $i$ having higher positive correlation across different periods, which increases the variance of return of holding asset $i$ in their portfolios.

In summary, the isolated asset $k$ is more attractive to investors than the high SC asset $j$ and the high IC asset $i$, thus investors would demand compensations for holding high SC and high IC assets. In fact, the risk diversification argument in classic portfolio theory requires weakly correlated assets, such as the isolated asset $k$ in this example. Therefore, high SC or high IC assets may not be considered as desirable components in a portfolio in a network environment to diversify financial risks in the long run. □

To model network structures mathematically, I use an adjacency matrix to model all the direct relationships in a network. Suppose there are $N$ industries in an illiquidity network, $A=[A_{ij}]_{i,j=1,\dots,N}$ is an $N$ by $N$ matrix indicating which pairs of industries have direct illiquidity transmission. We let $A_{ij}=1$ if and only if industry $i$'s illiquidity will directly transmit to industry $j$; otherwise, $A_{ij}=0$ if industry $i$'s illiquidity does not directly transmit to industry $j$. For example, the network structures in Figure 2.1a and in Figure 2.1b can be represented by the matrices in Table 2.1 and in Table 2.2 respectively.

Table 2.1: adjacency matrix and the network structure in Figure 2.1a

| | A | B | C | D |
|---|---|---|---|---|
| A | 0 | 1 | 1 | 1 |
| B | 1 | 0 | 1 | 1 |
| C | 1 | 1 | 0 | 1 |
| D | 1 | 1 | 1 | 0 |

### 2.3.2 Eigenvector Centrality Measure

In network literature, there are some centrality measures to gauge a node's "central importance" in a network from different aspects. Among them, I use the generalized eigenvector centrality

Table 2.2: adjacency matrix and the network structure in Figure 2.1b

| | A | B | C | D |
|---|---|---|---|---|
| A | 0 | 1 | 0 | 0 |
| B | 0 | 0 | 1 | 0 |
| C | 0 | 0 | 0 | 1 |
| D | 1 | 0 | 0 | 0 |

measures proposed by Bonacich (1987) to better measure illiquidity spillovers centrality in stock markets.

Given an adjacency matrix of a directed network, $A = [A_{ij}]_{i,j=1,\ldots,N}$, where $N$ is the size of the network. $A_{ij} = 1$ if and only if industry $i$ affects industry $j$, otherwise, $A_{ij} = 0$. Following Bonacich (1987), we define industry $i$'s sensitive centrality, $SC_i$, as the sum of values of a linear function of the sensitive centralities of all the other industries that affect industry $i$:

$$SC_i = \sum_{j:A_{ji}=1} (\alpha + \frac{1}{\lambda} SC_j) = \sum_{j=1}^{N} A_{ji}(\alpha + \frac{1}{\lambda} SC_j), \tag{2.1}$$

where $\alpha \geq 0$, $\lambda > 0$. Being affected by a high SC industry $j$ ($SC_j$ is large) can increase industry $i$'s sensitive centrality ($SC_i$) in this network. $1/\lambda$ is the weight of one's sensitive centrality measure on others'. A smaller $\lambda$ means the influence of the neighbour effect is greater. In a given network, we say industry $i$ is more sensitive central than industry $j$ if and only if $SC_i > SC_j$.

In matrix notation, let $SC = [SC_1, \ldots, SC_N]'$, we have

$$\left(I - \frac{1}{\lambda} A'\right) SC = \alpha A' l, \tag{2.2}$$

where $I$ is an $N \times N$ identity matrix and $l$ is a $N \times 1$ column vector of ones.

When $\alpha = 0$, we have $\left(I - \frac{1}{\lambda} A'\right) SC = 0$ then $SC$ is an eigenvector of the transpose of the adjacency matrix $A$ with its eigenvalue $\lambda$. If $A$ is an irreducible non-negative matrix, Perron-Frobenius theorem states that the only eigenvector whose components are all positive is the one associated with the biggest eigenvalue $\lambda_{\max}$. In practice, we do require positive centrality measures in order to determine which nodes are more central in a network. Hence, the eigenvector sensitive centrality is the eigenvector associated with the biggest eigenvalue of $A'$.

When $\alpha > 0$, it is simply the scale of the centrality vector. Without loss of generality, we could let $\alpha = 1$. If $A$ is an irreducible non-negative matrix and $\lambda$ is greater than the biggest eigenvalue of $A'$ in magnitude, the sensitive centrality vector has the following representation,

$$\begin{aligned} SC &= \left(I - \frac{1}{\lambda}A'\right)^{-1} A'l \\ &= A'l + \frac{1}{\lambda}(A')^2 l + \left(\frac{1}{\lambda}\right)^2 \left(A'\right)^3 l + .... \end{aligned} \tag{2.3}$$

All elements in the sensitive centrality vector $SC$ are positive as all the elements in equation (2.3) are nonnegative and $A$ is irreducible. Moreover, the parameter $1/\lambda$ can be interpreted as a probability and $SC$ as the expected number of directed paths in a network activated directly or indirectly to each individual.

To obtain a positive sensitive centrality vector from equation (2.3), the weight of one's sensitive centrality measure on others', $1/\lambda$, is at most $1/\lambda_{\max}$, where $\lambda_{\max}$ is the biggest eigenvalue of $A'$.[9] If we wish to put more weight on considering the effect of being a neighbour to a high SC (IC) industry in a network, a greater weight parameter $1/\lambda$ should be selected. Therefore, in order to capture the neighbour effect as much as possible I will focus on the eigenvector centrality measure in empirical analysis hereafter.

Similar arguments apply to defining a industry's influential centrality (IC). We define industry $i$'s influential centrality, $IC_i$, as the sum of linear functions of the influential centralities of all the other industries who are affected by industry $i$:

$$IC_i = \sum_{j:A_{ij}=1} (\alpha + \frac{1}{\lambda} IC_j) = \sum_{j=1}^{N} A_{ij}(\alpha + \frac{1}{\lambda} IC_j). \tag{2.4}$$

Affecting a high IC industry $j$ ($IC_j$ is large) can increase industry $i$'s influential centrality ($IC_i$) in this network. In matrix notation, let $IC = [IC_1, ..., IC_N]'$, we have $\left(I - \frac{1}{\lambda}A\right) IC = \alpha A l$. The eigenvector influential centrality is the eigenvector associated with the biggest eigenvalue of the adjacency matrix $A$.

**Example 2.3.2.** In Figure 2.5, I show a small but complex network to illustrate how the eigenvec-

[9] Given $\alpha > 0$, if $1/\lambda \geq 1/\lambda_{\max}$, the equation (2.3) does not converge and $SC$ is not well defined in this case.

tor centrality measures, sensitive centrality (SC) and influential centrality (IC), can point out the central components in this network and quantify their degrees.

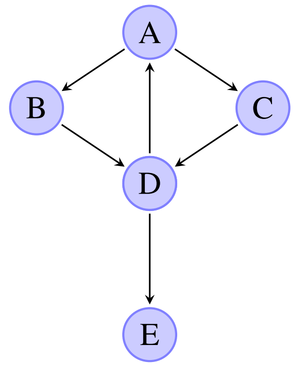



Figure 2.5: Eigenvector centrality of a small but complex network

The network shown in Figure 2.5 can be represented by the adjacency matrix in Table 2.3. This table also presents the calculated values of their respective eigenvector sensitive centrality and eigenvector influential centrality. The most sensitive central node is D (0.57) because it is affected by two main nodes B and C. Node A and node E are equally second sensitive central (0.45) as they are both only affected by node D. Node B and Node C are the least sensitive central (0.36) as they are only affected by node A. In terms of influential centrality, node A is the most central (0.64) because its effect can spillover to everyone in this network. Node D is second most central (0.51) as it can transmit node A's effect spilling via node B and node C to node E and back to node A. The influential centralities of node B and node C equal (0.40) as they only affect node D. Interestingly, the influential centrality of node E is zero, because it affects no one in this network.

Table 2.3: adjacency matrix and eigenvector centrality measures

| | A | B | C | D | E | IC |
|---|---|---|---|---|---|---|
| A | 0 | 1 | 1 | 0 | 0 | 0.64 |
| B | 0 | 0 | 0 | 1 | 0 | 0.40 |
| C | 0 | 0 | 0 | 1 | 0 | 0.40 |
| D | 1 | 0 | 0 | 0 | 1 | 0.51 |
| E | 0 | 0 | 0 | 0 | 0 | 0.00 |
| SC | 0.45 | 0.36 | 0.36 | 0.57 | 0.45 | |

□

### 2.3.3 Bid-Ask Spreads Measure for Illiquidity Risk

In general, illiquidity risk in financial markets is a financial risk that a given financial asset or security cannot be traded quickly enough in the market without impacting the market price. Liquidity has many dimensions. This study focuses on a dimension associated with bid-ask spreads. In stock markets, the spread is the difference between the bid and ask prices for a particular stock. The bid price corresponds to the highest price the demand side is willing to pay; the asking price corresponds to the lowest price the supply side is willing to sell. In other words, the bid-ask spread reflects the divergence of the demand side and the supply side for a stock. Wider divergence makes the transactions more difficult to make, since investors have to pay more "spread cost" to buy or sell a stock.

Thus, the level of the illiquidity risk of a stock increases with the size of its bid-ask spreads. The interconnections of industries' bid-ask spreads can be interpreted as industries' illiquidity risk transmission network.

In this paper, I use Corwin and Schultz (2012)'s bid-ask spreads estimate, which only requires stock' daily high and low prices, to measure firms' illiquidity risk. Moreover, since tick data is not available before 1990s but We assume that there is a spread of S%. Because of the spread, observed prices for buys are higher than the actual values by (S/2)%, and observed prices for sells are lower than the actual values by (S/2)%. If we further assume that the daily high price is buyer-initiated and the daily low price is seller-initiated, then we will have $H^O = H^A(1+S/2)$ and $L^O = L^A(1-S/2)$, where $H^O(L^O)$ is the observed high (low) price and $H^A(L^A)$ is the actual high (low) price. Following Corwin and Schultz (2012), the bid-ask spread estimate on day $t$ is

$$S_t = \frac{2(e^{\alpha}-1)}{1+e^{\alpha}}, \tag{2.5}$$

where $\alpha = \frac{\sqrt{2\beta}-\sqrt{\beta}}{3-2\sqrt{2}} - \sqrt{\frac{\gamma}{3-2\sqrt{2}}}$, $\beta = \left[\ln\left(\frac{H^O_{t-1}}{L^O_{t-1}}\right)\right]^2 + \left[\ln\left(\frac{H^O_t}{L^O_t}\right)\right]^2$ and $\gamma = \left[\ln\left(\frac{\max\{H^O_{t-1},H^O_t\}}{\min\{L^O_{t-1},L^O_t\}}\right)\right]^2$.

This bid-ask spread estimate has several advantages for our empirical analysis. First, this estimate is very easy to compute. No optimization problem needs to be solved. Second, this estimate only requires the daily observations of high price and low price. High price and low price are available in almost all stock databases. Third, the daily bid-ask spreads $S_t$ for any given stock can be estimated from low-frequency (daily) sample observations and high-frequency tick data is not available before 1990s. Fourth, Corwin and Schultz (2012) document that their bid-ask spread estimate provides the best approximation to the bid-ask spreads computed by high-frequency tick data. This liquidity measure enable us to avoid many problems (e.g., limited time-series observations in datasets[10] and incomparable liquidity measure across different sectors[11]) if we other alternative measures.

### 2.3.4 Granger Causality and Network Estimation

Once we have firms' estimates of their respective daily bid-ask spreads, we want to uncover the underlying network structures of how firms' bid-ask spreads spill over to each other. Following Billio et al. (2012) and Dufour and Jian (2016), this paper uses a Granger-type procedure (see, e.g., Granger (1969) and Sims (1972)) to identify the existence of directed relationships between every pair of nodes in the illiquidity risk network.

To identify the dynamic structures of the underlying illiquidity transmission network, I divide the whole daily panel sample into annual subsamples. Suppose in year $y$ we have $\tau_y$ days in this annual subsample, and we have $N_y$ firms' estimates of their respective daily bid-ask spreads: $[S_{1t}, S_{2t}, ..., S_{N_k t}]_{t=1}^{\tau_y}$. I assume the illiquidity risk network structure is fixed in each given year but can vary year by year. In year $y$, the network structure can be represented by an $N_y$ by $N_y$ adjacency matrix: $A^y = [A_{ij}^y]_{i,j=1,...,N_y}$. where $A_{ij}^y = 1$ if and only if firm $i$'s bid-ask spreads can affect firm $j$'s bid-ask spreads; otherwise, $A_{ij}^y = 0$.

To estimate the directed relationship from firm $i$ to firm $j$, $A_{ij}$, I use the following regression model,

$$S_{jt} = \beta_0 + \beta_i S_{it,p} + \beta_j S_{jt,p} + \beta_Z Z_{t,p} + \varepsilon_{jt}, \qquad t = 1, ..., \tau_k, \tag{2.6}$$

where $S_{jt}$ is firm $j$'s spread on day $t$, $S_{it,p} = [S_{it-p}, ..., S_{it-1}]'$ is the past recent $p$ days' observations of firm $i$'s spreads, and $S_{jt,p} = [S_{jt-p}, ..., S_{jt-1}]'$ is the past recent $p$ days' observations of firm $j$'s spreads. $Z_{t,p} = [Z_{t,p}^1, ..., Z_{t,p}^S]'$ is the past recent $p$ days' observations of $S$ state variables, $Z_{t,p}^s = [Z_{t-p}^s, ..., Z_{t-1}^s]$ for $s = 1, ..., S$. $\beta_0$ is a scalar parameter, $\beta_i$ is a row vector correspond to $S_{it,p}$, $\beta_j$ is a row vector correspond to $S_{jt,p}$ and $\beta_Z$ is a row vector correspond to $Z_{t,p}$. Then the general

[10] High-frequency trading data.
[11] Liquidity measures that computed by trading volume.

Granger-type procedure for identifying network structures becomes a testing problem ($H_0 : \beta_i = 0$, $H_1 : \beta_i \neq 0$):

$$A_{ij}^k = \begin{cases} 1, & \text{reject } H_0 \\ 0, & \text{can not reject } H_0 \end{cases} \tag{2.7}$$

Some notes of caution are needed here. First, selecting state variables $Z$ is important. One of the drawbacks of using the bilateral Granger noncausality testing in network estimation comes from spurious effects. If the regression model in equation (2.6) does not include the common factor(s) that are orthogonal to firm $j$'s past spreads but correlated to firm $i$'s past spreads and firm $j$'s current spread, we may reject $H_0$ even if there is no effect from firm $i$ to firm $j$.

Second, the choice of day lag $p$ is somewhat arbitrary, however, I suggest $p = 1$ for the network analysis in this paper. Setting $A_{ij}^k = 1$ implies we expect to see firm $i$'s spread yesterday will affect firm $j$'s spread today. When $p = 1$, the noncausality implication is in line with the direct effect interpretation in network adjacency matrix. Moreover, note that we only have one year daily observations in each subsample, thus small $p$ can increase the estimation precision, especially when we add some state variables in the regression model. Furthermore, measuring network centrality requires positive spillovers: if firm $i$'s illiquidity transmits to firm $j$, a higher today's illiquidity of firm $i$ should increase tomorrow's illiquidity of firm $j$. When $\beta_i$ is univariate, a more appropriate way to identify network structures is by testing whether $\beta_i > 0$.

Third, in order to ensure the adjacency matrix to be irreducible, the underlying illiquidity risk network should be strongly connected and not too sparse. Thus, the significance level selected for testing cannot be too low, otherwise the estimated network may be too sparse.

Fourth, illiquidity is unobservable and I use the bid-ask estimate to measure it. In the regression both the regressand and regressors are estimated with error; this may lead to biased estimates. To deal with this measurement error issue we need to verify our data for some econometric assumptions that ensure valid statistical inferences, and this is beyond the scope of this paper and I leave it for future research.

## 2.4 Illiquidity Network Centrality and the Cross-Section of Expected Returns

In the previous section I have discussed how to estimate illiquidity network structures by daily bid-ask spread estimates and how to apply eigenvector centrality measure to measure nodes' centralities in the network. This section explores the empirical relation between the cross-section of expected returns and the illiquidity centrality (SC and IC). For feasibility of implementation, the illiquidity network and the cross-section of expected returns are examined at industry level.

### 2.4.1 Data

The first dataset includes all the stock information from the Center for Research in Securities Prices (CRSP) for stocks traded in New York Exchange (NYSE), American Stock Exchange (AMEX), and NASDASQ with share codes 10 or 11 from January 1963 through December 2015. I use daily stock high prices and low prices to calculate daily bid-ask spread estimates. I use share prices and shares outstanding to calculate market capitalization. The first 3 digits of the Standard Industry Classification (SIC) code indicate the industry level. Industry's returns and bid-ask spreads are defined as the simple average of the returns and bid-ask spreads for the stocks belong to the industry. The second dataset is COMPUSTAT, which is used to obtain the equity book values for calculating the book-to-market ratios of individual firms and the book-to-market ratios of industry defines as the simple average of the book-to-market ratios of individual firms belong to the industry. The third dataset is from the Kenneth French's data library to obtain risk-free rates and four-factor portfolios returns. These variables are defined in detail in the Appendix C and will be discussed when they are used in the analysis.

### 2.4.2 Illiquidity Network Centralities

Using daily stock high prices and low prices I calculate daily bid-ask spreads estimates $S_{i_k t}$ for individual stock $i_k$ that belongs to industry $i$ on date $t$, with the adjustments suggested in Corwin and Schultz (2012). For the purpose of this exercise I assume industry $i$'s bid-ask spread estimate on date $t$, $S_{it} = \frac{1}{n_i}\sum_{i_k \in i} S_{i_k t}$, where $n_i$ is the number of stocks belong to industry $i$ on date $t$. In year $y$, we have $\tau_y$ daily observations of $N_y$ industries' daily bid-ask spreads estimates: $[S_{1t}, S_{2t}, ..., S_{N_y t}]_{t=1}^{\tau_y}$.

To identify $N_y$ industries' illiquidity network structure $A^y = [A^y_{ij}]_{i,j=1,...,N_y}$ in year $y$, I use the following regression equation:

$$S_{jt} = \beta_0 + \beta_i S_{it-1} + \beta_j S_{jt-1} + \beta_Z Z_{t-1} + \varepsilon_{jt}, \qquad t = 2, ..., \tau_y. \tag{2.8}$$

The directed relationship from industry $i$ to industry $j$ is specified as: $A^y_{ij} = 1$ if and only if $\beta_i > 0$; otherwise, $A^y_{ij} = 0$. The state variable $Z_{t-1}$ includes: 1) average bid-ask spreads estimates of the stocks belong to the major group of industry $j$ on day $t-1$, where the major group is indicated by the first two digits of SIC codes; 2) average bid-ask spreads estimates of all stocks on day $t-1$. By controlling major industry average illiquidity and market average illiquidity of industry $j$, a positive marginal effect ($\beta_i > 0$) of the illiquidity of industry $i$ on day $t-1$ on the illiquidity of industry $j$ on day $t$ can be safely interpreted as the illiquidity spillover from industry $i$ to industry $j$: increase in the illiquidity of industry $i$ today leads the illiqudity of industry $j$ up tomorrow. I use the simple t-statistic on one tail test at significance level 0.1 to test whether $\beta_i > 0$ in equation (2.8). I repeat this procedure for every pair of industries. After implementing $N_y \times N_y$ OLS regressions and testings on equation (2.8), we find all the directed relationships in the network and uncover the underlying illiquidity transmission network structure in year $y$, $A^y$.

Given each year $y$, we already have its adjacency matrix $A^y$ by the procedure described above. I calculate sensitive centralities and influential centrality for each industry by the eigenvector centrality measure. More central industry will have a higher centrality measure in cross-section, however note that the eigenvector is set to have unit norm, thus eigenvector centrality measures are not comparable directly across different years. To fix this problem, I rescale the industries' centrality measures for each year, such that the sum of squares of industries' centrality measures in year $y$ equals the size of the network in this year ($N_y$). After rescaling, more central industries given a year will still have higher centrality measures in cross-section as they are rescaled by the same weight. In addition, centrality measures in different years are comparable in terms of relative centrality in their respective networks. If the centrality measure of an industry is greater than 1, which is the root mean square of all industries' centrality measures in the network, it implies the industry is a relatively central industry, and vice versa. In any given year $y$, we have industry $i$' sensitive centrality measure ($SC_{iy}$) and its influential centrality measure ($IC_{iy}$) such that $\frac{1}{N_y}\sum_i SC_{iy}^2 = 1$ and $\frac{1}{N_y}\sum_i IC_{iy}^2 = 1$. $SC_{iy} = 1$ ($IC_{iy} = 1$) means (approximately) that industry $i$ does not have an unusually large or small degree of centrality in year $y$, irrespective of the number of industries in the illiquidity network in year $y$ ($N_y$), and I call these industries as "middle-industry".

Table 2.4 presents summary statistics of the empirical distributions of illiquidity network centralities in cross-section across different years from 1963 to 2015. The network centralities are estimated for every year from January 1963 to December 2015. There are 53 years from 1963 to 2015. In these years, there are 310 industries in illiquidity networks on average. Panel A presents summary statistics for sensitive centrality. The yearly median of the medians of cross-sectional sensitive centrality measures is 0.96, which is close to 1 of middle-industry. In contrast, for influential centrality in panel B the yearly median of the medians of cross-sectional influential centrality measures is only 0.73, but the yearly median of the 75% quantiles of cross-section influential centrality measures is 1.09, which is close to 1 of middle-industry. It implies high influential industry ($IC_i > 1$) is in the minority in illiquidity networks in average years. Compared to the sensitive centrality median 'max-min' spread (1.29 = 1.71 - 0.42), the influential centrality has a wider median 'max-min' spread (2.54 = 2.71 - 0.17). In cross-section, illiquidity influential centralities have a wider spread than illiquidity sensitive centralities. For both sensitive centrality and influential centrality, most of their cross-sectional empirical distributions from 1963 to 2015 are right-skewed and have heavier tails than normal distribution. Right-skewed network distribution is often documented in economic and social network literature (see e.g., Jackson et al. (2008)).

To investigate the empirical relation between sensitive centrality and influential centrality of a given industry in illiquidity networks, Table 2.5 presents the descriptive statistics of industries' time-series correlations. I only calculate the time-series correlations between sensitive centrality and influential centrality for those industries have more than 10 years centralities observations in sample. Then we have 395 industries' sensitive centralities and influential centralities time-series correlations and their respective $p$-values to null hypothesis of no correlation. The average sensitive centrality and influential centrality correlation is 0.42; the 25% quantile of the sensitive centrality and influential centrality correlations is 0.32; for most ($> 75\%$) industries the $p$-values are less than 0.1. It means the changes of illiquidity sensitive centrality for most industries tends to go with the direction of their changes in influential centrality. If an industry gets more connections to others, its illiquidity will have more chances to affect others as well as being affected by others. High SC (or IC) industries in the illiquidity network tend to be high absolute centrality.

### 2.4.3 Univariate Portfolio-Level Analysis

Table 2.6 presents the equal-weighted and value-weighted average monthly returns of decile portfolios that are formed by sorting the industries based on the illiquidity network centralities (SC and IC) respectively estimated in past calendar year. Centrality measures are estimated every year from January 1963 to December 2014. Industry's returns are calculated by the equal-weighted returns of stocks belonging to the industry, and value-weighted portfolios are the average industry returns weighted by industry's total market capitalizations. For example, I estimate industries' centrality measures in 2000 with the sample from January 2000 to December 2000, and form the portfolios from January 2001 to December 2001 based on the industries' centrality measures in 2000. Portfolios are rebalanced yearly. Portfolio 1 (Low SC (IC)) is the portfolio of industries with the lowest SC (IC) in the past calendar year, and portfolio 10 (high SC (IC)) is the portfolio of industries with the highest SC (IC) in the past calendar year.

In Panel A sorted by sensitive centrality, the equal-weighted raw return difference between decile 10 (high SC) and decile 1 (low SC) is 0.36% per month (4.32% per year) with a corresponding Newey-West (1987) t-statistics of 3.66. In addition to the raw returns, Table 2.6 also presents the intercepts (Fama-French-Carhart 4-factor alphas) from the regression of the equal-weighted portfolio returns on a constant, the excess market return, the size factor, the book-to-market factor, and the momentum factor, following Fama and French (1993) and Carhart (1997). The difference in alphas between the high SC and low SC equal-weighted portfolios is 0.45% per month (5.40% per year) with a Newey-West t-statistic of 3.72. This difference is economically significant and statistically significant at all conventional levels. Similar significant results also apply to value-weighted portfolios. The value-weighted raw return difference between decile 10 (high SC) and decile 1 (low SC) is 0.38% per month (4.56% per year) with a corresponding Newey-West t-statistics of 2.15; the difference in alphas between the high SC and low SC value-weighted portfolios is 0.49% per month (5.88% per year) with a Newey-West t-statistic of 2.65.

Taking a closer look at the value-weighted average returns and alphas across deciles, it is clear that they are not strictly monotonic increasing as SC increases. The average returns of decile 1 to 9 are very close, in the range of 1.24% to 1.47% per month, but decile 10 (high SC) average return jumps significantly to 1.82% per month. The alphas for the first 9 decile are close too, from 0.61% to 0.84%, but again the alpha for the decile 10 jumps up to 1.23%. A similar pattern also exists for equal-weighted average returns and alphas. The average return and alpha for the high SC decile portfolio are significantly higher than those in decile 1 to 9. It implies investors dislike the high SC portfolio industries' stocks especially. The most sensitive central industries are the most exposed to idiosyncratic illiquidity spillovers from other industries, thus investors may demand a premium to hold these high SC portfolio due to with they too sensitive to others' illiquidity.

In Panel B sorted by influential centrality, the equal-weighted raw return difference between decile 10 (high IC) and decile 1 (low IC) is 0.40% per month (4.80% per year) with a Newey-West t-statistic of 3.19. The difference in alphas between the high IC and low IC equal-weighted portfolio is 0.48% per month (5.67% per year) with a t-statistic of 3.35. Similar significant results also apply to value-weighted portfolios. The value-weighted raw return difference between decile 10 (high SC) and decile 1 (low SC) is 0.31% per month (3.27% per year) with a corresponding Newey-West t-statistics of 2.33; the difference in alphas between the high SC and low SC value-weighted portfolios is 0.31% per month (3.27% per year) with a Newey-West t-statistic of 2.31. The difference of average returns and alphas between high IC and low IC portfolios are economically and significant and statistically significant.

Again, the average returns and alphas across deciles for the equal-weighted and value-weighted portfolios are not strictly monotonic increasing as IC increases. But the high (low) IC portfolio still has the highest (lowest) average return and alpha across deciles. The highest influential centrality industries transfer their idiosyncratic illiquidity risk to many others and leave investors no place to hide in the stock market. Therefore, the illiquidity risk with holding the high IC portfolio is the most difficult to be hedged. The high IC stocks should earn a premium.

Comparing Panel A and Panel B, we can see that the average returns and alphas spreads between high SC and low SC and the spreads between high IC and low IC are close. Moreover, the patterns of average returns and alphas across deciles sorted by SC and by IC are similar. Note that we have already found the changes of illiquidity sensitive centrality for an industry tends to go with the direction of its change in influential centrality across different years in Table 2.5. Even though we find high SC and high IC portfolios earn significantly higher average returns and alphas compared with low SC and low IC portfolios respectively, these spreads may be generated from similar portfolios components. Table 2.7 presents the distribution of industries across deciles sorted by SC and sorted by IC. The $i$th row and $j$th column element in the table is the time-series average of the percentage ratios of the number of the industries in portfolio $j$ sorted by IC, as well as in portfolio $i$ sorted by SC, over the total number of the industries in portfolio $i$ sorted by SC. We can find from the table that industries in high (low) decile portfolios sorted by SC are more likely to be in high (low) decile portfolios sorted by IC. The table entries around the diagonal are clearly greater than those in off-diagonal positions. On average, 23.02 percent of decile 1's industries sorted by SC belong to decile 1 portfolio sorted by IC; 28.89 percent of decile 10's industries sorted by SC belong to decile 10 portfolio sorted by IC. In other words, about 3/4 of the industries that belong to the decile 1 (10) portfolio sorted by SC do not belong to the decile 1 (10) portfolio sorted by IC. These industries could help to separate the risk associated with SC from the risk associated with IC in their respective 10-1 portfolio[12]. The return and alpha differences of the 10-1 portfolios sorted by SC and IC respectively are not generated from similar portfolio components.

The 10-1 portfolios are constructed to capture the risk premium associated with sensitive centrality and influential centrality in the illiquidity network. In Table 2.6, we have found solid evidence that the 10-1 portfolios sorted by SC and sorted by IC are respectively both statistically and economically significant, however, it is still possible that we may just by "luck" pick up the well-performed industries in decile 10 and poor-performed industries in decile 1 as our portfolio formations are rebalanced annually. It is desirable for a trading strategy to utilize annually rebalanced portfolio as its transaction cost will be much lower than the strategies rebalanced monthly or even daily. But annually rebalancing does not provide many opportunities for changes in portfolio components. It would cast doubt on the reliability of the statistical properties for a trading strategy with low turnovers.

To examine this issue more carefully, we look at the transition matrix of industries in portfolios sorted by SC and sorted by IC. Table 2.8 presents the probability transition matrix of industries in different decile portfolios in successive two years. The $i$th row and $j$th column element in the 10 by 10 table is the time-series average of the percentage ratios of the number of the industries

[12] Long the high decile portfolio and short the low decile portfolio.

in portfolio $i$ in year $y$ shifting to portfolio $j$ in year $y+1$ over the number of the industries in portfolio $i$ in year $y$. If portfolio formations are purely random, industries are equally distributed in different deciles; all the entries in the transition matrix should equal 10(%). The range of the table entries is from 7.22 to 12.44 for deciles sorted by SC in Panel A; the range of the table entries is from 5.64 to 15.44 for deciles sorted by IC in Panel B. The maximum probability of an industries stay at the same decile in two successive years is only 12.44 (15.44) for decile sorted by SC (IC). In other words, it is quite unlikely that we pick up the well-performed industries consistently in decile 10 and poor-performed industries in decile 1 just by "luck" in Table 2.6 because most of industries do not stay at the same decile in two successive years and go to other different deciles with approximately equal probability. Taking a closer look at the tables, we can find the table entries around diagonal are a little bit greater than those in off-diagonal positions. In Panel A, for example, the probability of an industry in decile 10 (High SC) shifting to decile 1 (Low SC) next year has the lowest value of 8.25% for industries from decile 10, while the probability of an industry in decile 10 (High SC) staying at decile 10 (High SC) next year has the highest value of 12.44%. It is in line with our intuition since we expect a relatively low (high) SC industry in this year will be more likely to be relatively low (high) SC industry in next year. Similar arguments also apply to deciles sorted by IC in Panel B. In conclusion, the results documented in Table 2.6 appear to be trustworthy in term of statistics since industries in different deciles reshuffle enough in each year, even though our annually rebalancing does not provide many opportunities for changes in portfolio components.

In finance literature, market beta, book-to-market, illiquidity, momentum and idiosyncratic volatility are well-known risk factors of pricing returns in the cross-section at firm level (see Fama and French (1992), Fama and French (1993), Amihud (2002), Pastor and Stambaugh (2003), Jegadeesh and Titman (1993), Ang et al. (2006) among others). Though I study illiquidity network centralities at industry level, it would be important to investigate whether industries' sensitive centrality measures and influential centrality measures have relation with these well-know risk factors. To get a clearer picture of the component in portfolios sorted by sensitive centrality and influential centrality, Table 2.9 presents summary statistics for the industries in the deciles sorted by SC in Panel A and those sorted by IC in Panel B. Specifically, the table reports for each decile the simple average across the years and across the industries of various characteristics for the industries:

the average firm market capitalization (in millions of dollars, labeled FSIZE), the industry market capitalization (in millions of dollars, labeled ISIZE), the market beta (labeled BETA), the book-to-market (labeled BM), the average stock bid-ask spreads estimate (in percent, labeled SPREAD), the average Amihud (2002) illiquidity measure (scaled by $10^6$, labeled RTV), the average industry monthly return in the past calendar year prior to portfolio formation (in percent, labeled MOM), and the industry idiosyncratic volatility over the past calendar year prior to portfolio formation (labeled IVOL). Definitions of these variables are given in the Appendix.

In Panel A sorted by sensitive centrality, as SC increases across deciles industry market capitalization increases but firms' average market capitalization exhibits little change in a range from 1.13 millions of dollars to 1.22 millions of dollars with less than 10% in variation. In others words, an industry's sensitive centrality appears to be irrelevant to its average firm size, but a bigger industry, which has more firms and has bigger market capitalization, tends to have a higher sensitive centrality measure. It can be partially explained by the fact that an industry with more firms would have greater exposure to illiquidity spillovers in stock market. In contrast to the conjecture that sensitive centrality may serve as a source of market beta in financial network analysis (see Ahern (2013)), industries' market betas are almost the same across different deciles in our illiquidity network. Momentum and idiosyncratic volatility are also almost the same across deciles. As SC increases across the deciles, firms' average book-to-market ratio increases slightly. The value industries, which have higher average firms' book-to-market ratios, tend to have higher sensitive centrality measure. In additon to Corwin and Schultz (2012)'s bid-ask spreads estimate to measure illiquidity, I also consider a more widely used illiquidity measure proposed by Amihud (2002), which measures firm's illiquidity as the sensitivity of firm's absolute returns to its trading volume in dollars. Not surprisingly, those industries with higher sensitive centrality measures tend to have greater bid-ask spreads and return-to-volume (RTV). These results may provide an explanation of the value-premium known at least since Fama and French (1992). A motivation of the value-premium is that value firms are consistent bad performers in periods of systemic downturns. It may be because in the periods of systemic downturns value firms are more sensitive to market illiquidity thus poor liquidity make their returns further lower during these periods.

In Panel B sorted by influential centrality, as IC increases across deciles firms' average market capitalization decreases. Industries with small firms are more suitable distress vehicles than industries with large firms whose relatively large trading volumes could serve as temporary buffers to slow down illiquidity propagation.[13] Interestingly, the industry market capitalization exhibits an U-shape across deciles. A bigger industry, which has more firms and small caps on average, tends to has a higher influential centrality measure. As IC increases across deciles, industries' market betas decrease slightly (influential industries are less correlated to market returns); industries' book-to-market increases slightly (high book-to-market industries may be a source of systemic distress). Similar to the pattern across deciles sorted by SC, as IC increases across deciles illiquidity measures (SPREAD and RTV) are higher. Momentum and idiosyncratic volatility are also almost the same across deciles.

Given these differing characteristics, there is some concern that the 4-factor model used in Table 2.6 to calculate alphas is not adequate to capture the true difference in risk and expected returns across the portfolios sorted by SC and the portfolios sorted by IC. The 4-factor model does not control for the differences in expected returns due to differences in industry size or illiquidity. In the following two subsections I provide different ways to deal with the potential interaction of the illiquidity centrality measures with industry size, book-to-market and liquidity.

### 2.4.4 Bivariate Portfolio-Level Analysis

In this section I examine the relation between illiquidity causality measures and future industry returns after controlling for average firm market capitalization, industry market capitalization, market beta, book-to-market, illiquidity measured by return-to-volume, average industry monthly return in the past calendar year prior to portfolio formation, and industry idiosyncratic volatility over the past calendar year prior to portfolio formation. For example, I control for industry capitalization by first forming 5 decile portfolios ranked based on industry capitalization. Then, within each industry size decile, I sort industries into portfolio ranked based on sensitive centrality and portfolio ranked based on influential centrality so that decile 1 (decile 10) contains industries with lowest (highest) centrality measures.

Table 2.10 presents average industry return across the 5 control deciles to produce decile portfolio with dispersion in SC but with similar levels of the control variables. For each column

[13] Buraschi and Porchia (2012) find small firms have higher influential centrality in a network connecting firms' fundamentals.

controlling variables, the equal-weighted average return difference between the high SC and low SC portfolios are still all economically and statistically significant. After controlling for firms' average size, industry size, market beta, book-to-market, momentum and idiosyncratic volatility, the equal-weighted average return differences between the high SC and low SC portfolios are 0.29% (3.14%), 0.32% (3.84%), 0.28% (3.36%), 0.28% (3.36%), 0.29% (3.48%), and 0.30% (3.60%) per month (per year), with Newey-West t-statistics of 2.83, 3.27, 2.78, 2.83, 3.19 and 3.17, respectively. The corresponding values for the equal-weighted average risk-adjusted return differences are 0.40% (4.80%), 0.39% (4.68%), 0.31% (3.72%), 0.37% (4.44%), 0.35% (4.20%) and 0.40% (4.80%) per month (per year), with t-statistics of 2.71, 2.97, 2.93, 2.76, 3.46 and 2.67, which are also highly significant. Note that the absolute return to trading volume in dollars (RTV) illiquidity measure proposed by Amihud (2002) is a much more popular way to measure illiquidity in literature, for brevity hereafter I only use Amihud (2002)'s RTV measure to control the illiquidity risk to make the results in this paper comparable to existing studies.[14] I find that industries sensitive centralities are positively correlated with industry size, book-to-market and illiquidity (SPREAD and RTV)in Panel A of Table 2.9. After controlling each of these variables (ISIZE, BM and RTV), the average returns and alphas of the 10-1 portfolios sorted by SC remain significant. But the average return and alpha of the 10-1 portfolios decrease most after controlling RTV. After controlling RTV, the average return of the 10-1 portfolios decreases to 0.20% per month (2.4% per year) with a Newey-West t-statistic of 2.20; the alpha of the 10-1 portfolios decreases to 0.29% per month (3.48% per year) with a t-statistic of 2.11. Nevertheless, these results of high-low spread of the portfolios sorted by SC are still economically and statistically significant. For the double sorted value-weighted decile returns portfolios exhibit very similar significant results, except after controlling industry size the average returns of the 10-1 portfolios decrease to 0.21% per month (2.52% per year) with a t-statistic of 1.54, which is insignificant for conventional significance levels.

Table 2.11 presents average industry return across the 5 control deciles to produce decile portfolio with dispersion in IC but with similar levels of the control variables. For each column controlling variables, almost all the equal-weighted average returns and alphas of 10-1 IC portfolios are economically and statistically significant, and are close to those sorting only by SC in Table 2.6. After controlling for firms' average size, industry size, market beta, book-to-market, momentum

[14]The results of using SPREAD to control illiquidity risk are very similar.

and idiosyncratic volatility, the equal-weighted average return differences between the high SC and low SC portfolios are 0.35% (4.2%), 0.36% (4.32%), 0.31% (3.72%), 0.34% (4.08%), 0.35% (4.20%), 0.25% (3.00%) per month (per year), with t-statistics of 3.16, 2.99, 2.60, 2.85, 3.37, 2.11, respectively. The corresponding 10-1 alphas are 0.45% (5.40%), 0.45% (5.40%), 0.34% (4.08%), 0.44% (5.28%), 0.38% (4.56%) and 0.34% (4.08%) per month (per year), with t-statistics of 3.41, 3.11, 2.59, 3.21, 3.51 and 2.67, which are also both economically and statistically significant. The only exception is the average return of the 10-1 portfolio after controlling RTV, which is 0.16% per month with a t-statistic of 1.42. But the 10-1 alpha after controlling RTV is 0.27% per month (3.24% per year) with a t-statistic of 2.24, which is also significant. However, the 10-1 IC portfolios are not always significant for the value-weighted portfolio returns, even though their averages returns and alphas are all positive.

In summary, these results indicate that for both the equal-weighted and value-weighted portfolios, the well-known cross-sectional effects at firm level such as size, market beta, book-to-market, liquidity, momentum and idiosyncratic volatility can not explain the high returns to high SC industries, while similar robust results do not apply to the high returns to high IC industries except for the case of equal-weighted portfolios sorted by IC. So that the constructed centrality measures here have some valid interpretation.

### 2.4.5 Industry-Level Cross-Section Regressions

So far we have tested the significance of illiquidity sensitive centrality (SC) and illiquidity influential centrality (IC) as determinants of the cross-section of future returns at portfolio-level. The portfolio-level analysis has the advantage of being non-parametric in the sense that we do not impose a functional form on the relation between illiquidity centrality measures and future return. But the portfolio-level analysis misses a large amount of information in the cross-section via aggregation. Moreover, it fails to control for multiple effects simultaneously. In this section, I examine the cross-sectional relation between the centrality measures (SC and IC) and expected returns at the industry level using Fama and MacBeth (1973) two-step regressions.

I present the time-series averages of the slope coefficients from the regression of industry returns on sensitive centrality (SC), influential centrality (IC), market beta (BETA), average of logs of firms' market capitalizations (FSIZE), log of industry market capitalization, average of logs of firms' book-to-market (BM), illiquidity (RTV), momentum (MOM), and idiosyncratic volatility (IVOL). The average slopes provide standard Fama-MacBeth tests for determining which explanatory variables on average have non-zero premiums. Monthly cross-sectional regressions are run for the following econometric specification and nested versions:

$$\begin{aligned} R_{i,t,y+1} =& \lambda_{0,t,y} + \lambda_{1,t,y}\mathrm{SC}_{i,y} + \lambda_{2,t,y}\mathrm{IC}_{i,y} + \lambda_{3,t,y}\mathrm{BETA}_{i,y} + \lambda_{4,t,y}\mathrm{FSIZE}_{i,y} + \lambda_{5,t,y}\mathrm{ISIZE}_{i,y} \\ &+ \lambda_{6,t,y}\mathrm{BM}_{i,y} + \lambda_{7,t,y}\mathrm{RTV}_{i,y} + \lambda_{8,t,y}\mathrm{MOM}_{i,y} + \lambda_{9,t,y}\mathrm{IVOL}_{i,y} + \varepsilon_{i,t,y+1} \end{aligned}$$

where $R_{i,t,y+1}$ is the realized return on industry $i$ in month $t$ in year $y+1$, the predictive cross-section regression are run on the lagged values of SC, IC, BETA, FSIZE, ISIZE, BM, RTV, MOM, and IVOL, which are all calculated or estimated with the sample from January to December in year $y$. This setting assures the associated trading strategy is rebalanced annually.

Table 2.12 reports the time-series average of the slope coefficients $\lambda_{i,t,y}$ $(i = 1,...,9)$ over the 624 months from January 1964 to December 2015 for all industries in the illiquidity networks that are estimated annually from 1963 to 2014. The Newey-West adjusted t-statistics are given in parentheses. The univariate regressions show a positive and statistically significant relation between illiquility sensitive centrality and the cross-section of future industry returns; and a positive and statistically significant relation between illiquility influential centrality and the cross-section of future industry returns. The average slope, $\lambda_{1,y}$, from the monthly regressions of realized returns on SC alone is 0.82 with a t-statistic of 2.05. The economic magnitude of the associated effect is higher than that documented in Table 2.6 and Table 2.10 for the univariate and bivariate sorts. The spread in average SC between decile 10 and decile 1 is 0.93 (1.50 - 0.57). Multiplying this spread by the average slope yields an estimate of the monthly risk premium of 0.76% per month (9.12% per year). The average slope, $\lambda_{2,y}$, from the monthly regressions of realized returns on IC alone is 0.69 with a t-statistic of 1.70. The economic magnitude of the associated effect is also higher than that documented in Table 2.6 and Table 2.11. The spread in average IC spread between decile 10 and 1 is 1.64 (1.96 - 0.32). Multiplying this spread by the average slope yields an estimate of the monthly risk premium of 1.13% month (13.56% per year).

Conditional on 6 other variables (BETA, FSIZE, BM, RTV, MOM and IVOL), the economic magnitudes and the significance levels of $\lambda_{1,y}$ and $\lambda_{2,y}$ remain almost unchanged. The average slope coefficient on SC, $\lambda_{1,y}$, conditional on the 6 control variables, is 0.88 with a t-statistic of 2.14; the average slope coefficient on IC, $\lambda_{2,y}$, conditional on the 6 control variables, is 0.79 with a t-statistic of 1.95. Since we have found in Table 2.7 that SC and IC are cross-sectional positively correlated, our primary interest is the full specification with SC, IC, and the 6 control variables. In this specification, the average slope coefficient on SC is 0.83 with a t-statistic of 2.00; the average slope coefficient of IC is 0.62 with a t-statistic of 1.92.[15] Theses results are very similar to those in the univariate regressions.

In the last specification in Table 2.12, I exclude SC and IC in the full specification regression to investigate the effect of dropping SC and IC to other control variables in explaining the cross-section returns at industry level. In the last specification, the average slope coefficient on RTV is 0.81 and significant, while those average slope coefficient on RTV in the specification with either SC or IC or both are smaller than 0.81 and statistically insignificant. It implies the illiquidity risk premium associated with RTV can be captured by SC and IC but the illiquidity risk premium associated with SC and IC is not captured by RTV.

The table shows that only SC, IC and MOM are consistently significant under the regressions of all specifications in the table. Many well-known cross-sectional effects at firm level such as market-beta, size, book-to-market, liquidity, and idiosyncratic volatility are not robustly significant in explaining the cross-section returns at industry level. The size effect measured by ISIZE is significantly positive with a t-statistic of 2.10 only in the full specification; the book-to-market effect measured by BM is significantly positive only in the full specifications excluding either SC or IC; the liquidity effect measured by RTV is significantly positive only in the specification without SC and IC. The signs of these effects are in line with those documented in literature. Note that these variables in this paper are measured at industry level and renewed annually, return dispersions associated with these variables could be small due to firms' aggregations into industry level. The momentum effect, however, is surprisingly robust at industry level.

As a robustness check for the significant effects of SC and IC, Table 2.13 presents the cross-sectional regression results of the full specification model under different subperiods (1970 -2015,

[15]Controlling SPREAD instead of RTV in the full specification has little effect on the results. In such specification, the average slope coefficient on SC is 0.94 with a t-statistic of 2.18; the average slope coefficient of IC is 0.76 with a t-statistic of 2.27.

1980 - 2015, 1990 - 2015 and 2000 - 2015). SC is positive and statistically significant at the level of 0.1 in all subperiods. IC is also positive and statistically significant at the level of 0.1 in all subperiods except the most recent and shortest subsample period of 2010 - 2015, while the mean of coefficients for IC of 2010 - 2015 is still positive. Another observation is the effects of SC and IC measured by their respective mean coefficients are even larger in recent decades.

The clear conclusion is that the cross-sectional regressions provide strong corroborating evidence for an economically and statistically significant positive relation between the illiquidity centrality measures (SC and IC) and future returns, consistent with our conjecture that illiquidity centralities (sensitive centrality and influential centrality) are an important idiosyncratic risk that should be priced in financial markets, and they indeed earn risk premiums in the cross-sectional stock returns at industry level. Moreover, SC is a more robust risk factor than IC in explaining cross-sectional returns.

## 2.5 Conclusion

This paper proposes a new analytical framework to study centralities in an illiquidity transmission network and its asset pricing implication in the cross-section of expected stock returns. I document a statistically and economically significant relation between lagged illiquidity centralities (sensitive centrality and influential centrality) and future returns. This result is robust to controls for numerous other potential risk factors. The result related to influential centrality is consistent with the asset pricing implication of Acemoglu et al. (2012) and Acemoglu et al. (2015b)'s theory, while I find sensitive centrality is an even more robust risk factor than influential centrality in explaining cross-sectional returns. In summary, I find strong evidence that the illiquidity network centralities (SC and IC) may be important risk factors in asset pricing with network structures of securities.

This paper differs from the existing literature studying commonality in liquidity, illiquidity spillovers and contagions in that I consider illiquidity spillovers in a network environment with focus on industries' illiquidity interconnections, instead of basing it on simple two-agents settings or on contemporaneous correlation-based analysis. Moreover, I consider network centrality in two directions: i) sensitive centrality (SC), which measures the degree of a node being affected by others; and ii) influential centrality (IC), which measures the degree of a node affecting others.

The existing literature related to financial network centrality is mostly motivated by the systemic risk studies that suggest idiosyncratic shocks to an influential firm may cause aggregate market failures, so it tends to only consider influential centrality. I argue that sensitive centrality is at least as important as influential centrality in terms of asset pricing. Indeed, I find strong evidence in illiquidity network to support this conjecture. I find that SC and IC are positively correlated in time-series and in cross-section and each adds to the explanation of cross-sectional returns even given the other measure.

The approach used in this paper can be applied to study many other financial networks, such as return network, volatility network, and credit-spread network. An interesting direction for further research may be studying direct and indirect network effects in a unified framework with the general network measurement method proposed by Dufour and Jian (2016). After all, the adjacency matrix can only tell us about direct effects. If we want to study financial spillovers and propagations in depth, measuring indirect effects is also necessary. In this paper I assume the illiquidity network is unweighted. But weighted economic effects of financial spillovers could provide us more insights to understand the strength of underlying financial networks. Of course, different network centrality measures have to be selected accordingly. I leave a detailed analysis of these issues to future work.

Table 2.4: Summary statistics of illiquidity network centralities panels. Centrality measures are estimated every year from January 1963 to December 2015. Column descriptive statistics provide characteristics of the empirical distribution of cross-section centrality measures in a given year. Row descriptive statistics provide characteristics of each column's descriptive statistics across different years (1963 - 2015). Skewness is unbiased skew, for those are greater than 0 are right-skewed; kurtosis is unbiased kurtosis using Fisher's definition of kurtosis (kurtosis normal = 0). Panel A presents summary statistics for sensitive centrality; Panel B presents summary statistics for influential centrality.

Panel A: Sensitive Centrality

| | Cross-section Centrality Measures | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | count | mean | std | min | 25% | 50% | 75% | max | skewness | kurtosis |
| count | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| mean | 310.28 | 0.95 | 0.28 | 0.39 | 0.75 | 0.90 | 1.12 | 1.83 | 0.51 | 0.21 |
| std | 42.87 | 0.05 | 0.13 | 0.14 | 0.15 | 0.12 | 0.04 | 0.30 | 0.40 | 0.59 |
| min | 222 | 0.81 | 0.15 | 0.15 | 0.35 | 0.54 | 1.01 | 1.44 | -0.26 | -0.74 |
| 25% | 282 | 0.94 | 0.19 | 0.25 | 0.67 | 0.89 | 1.10 | 1.60 | 0.24 | -0.14 |
| 50% | 311 | 0.98 | 0.21 | 0.42 | 0.83 | 0.96 | 1.11 | 1.71 | 0.45 | 0.08 |
| 75% | 332 | 0.98 | 0.34 | 0.52 | 0.86 | 0.97 | 1.14 | 2.09 | 0.80 | 0.43 |
| max | 393 | 0.99 | 0.59 | 0.62 | 0.88 | 1.00 | 1.24 | 2.47 | 1.69 | 2.99 |

Panel B: Influential Centrality

| | Cross-section Centrality Measures | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | count | mean | std | min | 25% | 50% | 75% | max | skewness | kurtosis |
| count | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| mean | 310.28 | 0.85 | 0.49 | 0.18 | 0.52 | 0.72 | 1.05 | 2.84 | 1.38 | 2.34 |
| std | 42.87 | 0.10 | 0.14 | 0.11 | 0.17 | 0.17 | 0.14 | 0.62 | 0.53 | 2.22 |
| min | 222 | 0.41 | 0.22 | 0.00 | 0.05 | 0.11 | 0.26 | 1.62 | 0.27 | -1.14 |
| 25% | 282 | 0.82 | 0.41 | 0.12 | 0.41 | 0.63 | 1.01 | 2.46 | 1.09 | 1.16 |
| 50% | 311 | 0.87 | 0.49 | 0.17 | 0.51 | 0.73 | 1.09 | 2.71 | 1.37 | 1.78 |
| 75% | 332 | 0.91 | 0.57 | 0.23 | 0.65 | 0.84 | 1.12 | 3.18 | 1.54 | 2.55 |
| max | 393 | 0.98 | 0.91 | 0.44 | 0.84 | 0.97 | 1.35 | 4.92 | 3.57 | 11.94 |

Table 2.5: Summary statistics of the time-series correlations of sensitive and influential centralities of given industries. Centrality measures are estimated every year from January 1963 to December 2015. I only calculate the time-series correlations between sensitive centrality and influential centrality for those industries have more than 10 years centralities observations in sample. Column statistics provide time-series correlations of any given industry and its *p*-value to null hypothesis of no correlation. Row descriptive statistics provide characteristics of each column's statistics across different industries.

| | corr | *p*-value |
|---|---|---|
| count | 395 | 395 |
| mean | 0.42 | 0.09 |
| std | 0.19 | 0.19 |
| min | -0.37 | 0.00 |
| 25% | 0.32 | 0.00 |
| 50% | 0.45 | 0.00 |
| 75% | 0.55 | 0.07 |
| max | 0.82 | 0.96 |

Table 2.6: Return and alpha on portfolios of stocks sorted by illiquidity network centralities. Decile portfolios are formed every year from January 1964 to December 2014 by sorting industries based on the sensitive centrality (SC) in Panel A and based on the influential centrality (IC) in Panel B. Centrality measures are estimated every year from January 1963 to December 2014. Industry returns are calculated by the equal-weighted returns of stocks belong to the industry. Portfolio 1 (10) is the portfolio of industries with lowest (highest) centralities in the past calendar year. The tables reports the equal-weighted and value-weighted average monthly returns, the 4-factor Fama-French-Carhart alphas on the equal-weighted and value-weighted portfolios, and the average centrality of industries in the past calendar year. The last two rows present the differences in monthly returns and the differences in alphas with respect to the 4-factor Fama-French-Carhart model between portfolios 10 and 1 and the corresponding t-statistics. Average raw and risk-adjusted returns are given in percentage terms. Newey-West (1987) adjusted t-statistics are reported in parentheses.

Panel A: Sorted by sensitive centrality

| | Equal-Weighted | | Value-Weighted | | |
|---|---|---|---|---|---|
| Decile | Average Return | 4-factor Alpha | Average Return | 4-factor Alpha | SC |
| Low SC | 1.06 | 0.31 | 1.44 | 0.74 | 0.57 |
| 2 | 1.11 | 0.42 | 1.35 | 0.64 | 0.68 |
| 3 | 1.18 | 0.42 | 1.36 | 0.67 | 0.75 |
| 4 | 1.26 | 0.52 | 1.44 | 0.76 | 0.81 |
| 5 | 1.24 | 0.53 | 1.47 | 0.82 | 0.87 |
| 6 | 1.07 | 0.33 | 1.32 | 0.61 | 0.94 |
| 7 | 1.28 | 0.59 | 1.42 | 0.80 | 1.02 |
| 8 | 1.26 | 0.62 | 1.26 | 0.72 | 1.12 |
| 9 | 1.12 | 0.47 | 1.43 | 0.84 | 1.26 |
| High SC | 1.42 | 0.76 | 1.82 | 1.23 | 1.50 |
| 10-1 | 0.36 | 0.45 | 0.38 | 0.49 | |
| | (3.66) | (3.72) | (2.15) | (2.65) | |

Panel B: Sorted by influential centrality

| | Equal-Weighted | | Value-Weighted | | |
|---|---|---|---|---|---|
| Decile | Average Return | 4-factor Alpha | Average Return | 4-factor Alpha | IC |
| Low IC | 1.05 | 0.33 | 1.26 | 0.62 | 0.32 |
| 2 | 1.20 | 0.52 | 1.55 | 0.89 | 0.44 |
| 3 | 1.18 | 0.47 | 1.42 | 0.74 | 0.52 |
| 4 | 1.20 | 0.48 | 1.37 | 0.76 | 0.60 |
| 5 | 1.22 | 0.53 | 1.50 | 0.83 | 0.68 |
| 6 | 1.18 | 0.45 | 1.45 | 0.82 | 0.77 |
| 7 | 1.17 | 0.49 | 1.54 | 0.87 | 0.89 |
| 8 | 1.19 | 0.48 | 1.41 | 0.76 | 1.06 |
| 9 | 1.16 | 0.45 | 1.35 | 0.72 | 1.33 |
| High IC | 1.44 | 0.81 | 1.57 | 0.93 | 1.96 |
| 10-1 | 0.40 | 0.48 | 0.31 | 0.31 | |
| | (3.19) | (3.35) | (2.33) | (2.31) | |

Table 2.7: Distribution of industries across deciles sorted by sensitive centrality and sorted by influential centrality. Centrality measures are estimated every year from January 1963 to December 2014. Decile portfolios are formed every year from January 1964 to December 2015 by sorting industries based on the sensitive centrality (SC) and based on the influential centrality (IC). Portfolio 1 (10) is the portfolio of industries with lowest (highest) centralities in the past calendar year. The *i*th row and *j*th column element in the table is the time-series average of the percentage ratios of the number of the industries in portfolio *j* sorted by influential centrality, as well as in portfolio *i* sorted by sensitive centrality, over the total number of the industries in portfolio *i* sorted by sensitive centrality.

| By sensitive centrality | By influential centrality | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Low IC | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High IC |
| Low SC | 23.02 | 16.29 | 12.07 | 11.18 | 8.32 | 7.95 | 5.66 | 5.61 | 4.53 | 3.48 |
| 2 | 16.90 | 13.53 | 12.94 | 12.58 | 10.57 | 7.98 | 8.18 | 6.04 | 5.11 | 4.28 |
| 3 | 14.32 | 12.15 | 12.86 | 10.99 | 11.21 | 9.54 | 9.57 | 7.24 | 6.21 | 4.01 |
| 4 | 9.84 | 11.61 | 12.89 | 11.64 | 12.13 | 10.87 | 10.05 | 7.19 | 7.29 | 4.60 |
| 5 | 8.64 | 12.20 | 9.82 | 10.54 | 10.57 | 11.52 | 10.15 | 11.29 | 7.29 | 6.10 |
| 6 | 8.15 | 9.22 | 10.90 | 11.18 | 10.69 | 10.36 | 12.57 | 10.84 | 8.36 | 5.84 |
| 7 | 5.25 | 8.49 | 8.45 | 11.13 | 10.81 | 10.91 | 11.42 | 11.77 | 11.28 | 8.60 |
| 8 | 4.84 | 6.23 | 6.97 | 7.22 | 10.87 | 10.54 | 12.21 | 12.82 | 12.95 | 13.47 |
| 9 | 4.34 | 4.77 | 6.91 | 5.94 | 7.34 | 8.51 | 10.90 | 12.98 | 17.38 | 19.05 |
| High SC | 3.01 | 3.88 | 4.72 | 6.12 | 6.06 | 6.87 | 7.94 | 12.68 | 17.94 | 28.89 |

Table 2.8: Transition matrix of industries in portfolios sorted by illiquidity network centralities. Centrality measures are estimated every year from January 1963 to December 2014. Decile portfolios are formed every year from January 1964 to December 2015 by sorting industries based on the sensitive centrality (SC) in Panel A and based on the influential centrality (IC) in Panel B. Portfolio 1 (10) is the portfolio of industries with lowest (highest) centralities in the past calendar year. The $i$th row and $j$th column element in the table is the time-series average of the percentage ratios of the number of the industries in portfolio $i$ in year $y$ shifting to portfolio $j$ in year $y+1$ over the number of the industries in portfolio $i$ in year $y$.

Panel A: Sorted by sensitive centrality

| | To | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| From | Low SC | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High SC |
| Low SC | 11.39 | 10.58 | 11.05 | 10.56 | 7.22 | 8.75 | 8.78 | 9.12 | 8.91 | 8.25 |
| 2 | 10.90 | 9.04 | 9.96 | 9.78 | 9.79 | 9.81 | 8.19 | 8.77 | 8.96 | 8.96 |
| 3 | 9.95 | 8.98 | 10.60 | 8.89 | 10.30 | 9.62 | 9.82 | 8.72 | 9.33 | 8.64 |
| 4 | 9.35 | 11.01 | 8.65 | 9.99 | 10.19 | 8.13 | 9.58 | 9.56 | 9.33 | 9.36 |
| 5 | 9.05 | 9.65 | 8.43 | 10.24 | 10.27 | 9.78 | 10.41 | 9.10 | 9.13 | 9.04 |
| 6 | 10.00 | 9.79 | 9.42 | 9.01 | 9.06 | 8.87 | 10.30 | 9.80 | 9.24 | 9.13 |
| 7 | 8.63 | 9.01 | 10.28 | 9.55 | 10.43 | 8.28 | 10.06 | 9.18 | 10.50 | 9.67 |
| 8 | 8.78 | 8.43 | 8.37 | 9.51 | 9.44 | 10.63 | 9.20 | 10.86 | 9.87 | 10.44 |
| 9 | 7.78 | 9.33 | 9.06 | 8.14 | 10.26 | 9.67 | 9.46 | 10.33 | 10.52 | 10.99 |
| High SC | 8.25 | 8.57 | 8.57 | 9.22 | 8.53 | 8.61 | 10.13 | 11.33 | 10.19 | 12.44 |

Panel B: Sorted by influential centrality

| | To | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| From | Low IC | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High IC |
| Low IC | 10.86 | 11.20 | 11.28 | 10.77 | 9.83 | 9.86 | 10.07 | 7.41 | 6.52 | 7.54 |
| 2 | 10.87 | 11.64 | 10.74 | 9.74 | 11.71 | 7.47 | 9.95 | 8.11 | 7.69 | 7.15 |
| 3 | 12.92 | 9.85 | 10.43 | 9.17 | 10.58 | 8.45 | 8.89 | 9.39 | 7.62 | 8.07 |
| 4 | 9.42 | 10.82 | 11.33 | 10.55 | 9.73 | 8.94 | 8.64 | 8.60 | 8.47 | 7.94 |
| 5 | 11.27 | 9.99 | 9.91 | 9.62 | 8.24 | 8.90 | 8.76 | 9.17 | 9.61 | 8.90 |
| 6 | 8.70 | 8.84 | 8.19 | 8.42 | 9.91 | 10.49 | 9.50 | 11.03 | 9.80 | 9.81 |
| 7 | 8.69 | 11.38 | 8.51 | 8.49 | 9.48 | 9.58 | 9.90 | 8.58 | 11.21 | 9.63 |
| 8 | 8.05 | 7.76 | 8.56 | 9.71 | 8.42 | 10.91 | 10.14 | 10.58 | 11.72 | 10.16 |
| 9 | 6.27 | 7.55 | 8.64 | 9.32 | 8.36 | 8.40 | 11.50 | 10.85 | 11.34 | 13.09 |
| High IC | 7.05 | 5.64 | 7.08 | 8.17 | 9.00 | 9.31 | 9.23 | 12.28 | 12.13 | 15.14 |

Table 2.9: Summary statistics for decile portfolios sorted by illiquidity network centralities. Centrality measures are estimated every year from January 1963 to December 2014. Decile portfolios are formed every year from January 1964 to December 2015 by sorting industries based on the sensitive centrality (SC) in Panel A and based on the influential centrality (IC) in Panel B. Portfolio 1 (10) is the portfolio of industries with lowest (highest) centralities in the past calendar year. The table reports for each decile the simple average across the years and across the industries of various characteristics for the industries: the average stock market capitalization (in millions of dollars, labeled FSIZE), the industry market capitalization (in millions of dollars, labeled ISIZE), the market beta (labeled BETA), the book-to-market (labeled BM), the average stock bid-ask spreads estimate (in percent, labeled SPREAD), the average Amihud (2002) illiquidity measure (scaled by $10^6$, labeled RTV), the average industry monthly return in the past calendar year prior to portfolio formation (in percent, labeled MOM), and the industry idiosyncratic volatility over the past calendar year prior to portfolio formation (labeled IVOL).

Panel A: Sorted by sensitive centrality

| Decile | FSIZE($\$10^6$) | ISIZE($\$10^6$) | BETA | BM | SPREAD(%) | RTV($10^{-6}$) | MOM(%) | IVOL |
|---|---|---|---|---|---|---|---|---|
| Low SC | 1.22 | 14.86 | 0.89 | 2.59 | 1.84 | 4.82 | 0.08 | 0.28 |
| 2 | 1.13 | 16.05 | 0.87 | 3.17 | 2.09 | 6.63 | 0.08 | 0.29 |
| 3 | 1.18 | 15.51 | 0.87 | 3.19 | 2.16 | 6.56 | 0.07 | 0.28 |
| 4 | 1.25 | 19.02 | 0.87 | 2.89 | 2.25 | 7.11 | 0.08 | 0.27 |
| 5 | 1.19 | 18.00 | 0.87 | 3.41 | 2.11 | 7.70 | 0.08 | 0.28 |
| 6 | 1.17 | 20.97 | 0.88 | 3.08 | 2.21 | 7.25 | 0.08 | 0.27 |
| 7 | 1.13 | 20.06 | 0.88 | 2.77 | 2.32 | 7.85 | 0.09 | 0.28 |
| 8 | 1.28 | 25.64 | 0.86 | 3.82 | 2.41 | 7.51 | 0.08 | 0.28 |
| 9 | 1.18 | 20.25 | 0.85 | 3.28 | 2.55 | 6.02 | 0.08 | 0.26 |
| High SC | 1.18 | 30.09 | 0.87 | 4.36 | 2.64 | 7.53 | 0.08 | 0.27 |

Panel B: Sorted by influential centrality

| Decile | FSIZE($\$10^6$) | ISIZE($\$10^6$) | BETA | BM | SPREAD(%) | RTV($10^{-6}$) | MOM(%) | IVOL |
|---|---|---|---|---|---|---|---|---|
| Low IC | 1.37 | 21.20 | 0.94 | 2.93 | 1.59 | 4.63 | 0.08 | 0.26 |
| 2 | 1.29 | 23.67 | 0.89 | 2.76 | 1.75 | 6.74 | 0.08 | 0.28 |
| 3 | 1.34 | 20.01 | 0.90 | 3.10 | 1.76 | 4.78 | 0.08 | 0.27 |
| 4 | 1.27 | 17.25 | 0.88 | 4.06 | 1.92 | 6.64 | 0.08 | 0.27 |
| 5 | 1.15 | 15.75 | 0.87 | 2.92 | 1.88 | 6.06 | 0.08 | 0.28 |
| 6 | 1.12 | 16.97 | 0.87 | 3.32 | 2.12 | 6.58 | 0.07 | 0.28 |
| 7 | 1.26 | 18.67 | 0.85 | 3.23 | 2.43 | 7.35 | 0.08 | 0.28 |
| 8 | 1.00 | 18.96 | 0.84 | 2.88 | 2.57 | 8.14 | 0.08 | 0.28 |
| 9 | 1.16 | 19.12 | 0.84 | 3.72 | 3.06 | 8.70 | 0.07 | 0.28 |
| High IC | 0.94 | 28.72 | 0.82 | 3.68 | 3.48 | 9.34 | 0.07 | 0.28 |

Table 2.10: Returns on portfolios of industries sorted by sensitive centrality after controlling for FSIZE, ISIZE, BETA, BM, RTV, MOM, and IVOL. Centrality measures are estimated every year from January 1963 to December 2014. Double-sorted, equal-weighted and value-weighted decile portfolios are formed every year from January 1964 to December 2015 by sorting industries based on sensitive centralities after controlling for average firm market capitalization, industry market capitalization, market beta, book-to-market, return-to-volume, industry momentum, and industry idiosyncratic volatility. In each case, I first sort the industries in to 5 deciles using the control variable, then within each decile, I sort industries into 10 decile portfolios based on the sensitive centralities over the previous calendar year so that decile 1 (10) contains industries with the lowest (highest) SC. This table presents average industry returns across the 5 control deciles to produce decile portfolio with dispersion in SC but with similar levels of the control variable. "10-1 Return" is the difference in average monthly returns between the High SC and Low SC portfolios. "10-1 Alpha" is the difference in 4-factor alphas on the High SC and Low SC portfolios. Newey-West (1987) adjusted t-statistics are reported in parentheses.

| | Equal-Weighted Returns | | | | | | | Value-Weighted Returns | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Decile | FSIZE | ISIZE | BETA | BM | RTV | MOM | IVOL | FSIZE | ISIZE | BETA | BM | RTV | MOM | IVOL |
| Low SC | 1.10 | 1.04 | 1.05 | 1.03 | 1.13 | 1.04 | 1.07 | 1.57 | 1.50 | 1.29 | 1.28 | 1.40 | 1.27 | 1.47 |
| 2 | 1.12 | 1.15 | 1.15 | 1.18 | 1.17 | 1.09 | 1.13 | 1.54 | 1.54 | 1.38 | 1.45 | 1.41 | 1.30 | 1.31 |
| 3 | 1.18 | 1.19 | 1.22 | 1.19 | 1.12 | 1.27 | 1.14 | 1.57 | 1.57 | 1.31 | 1.33 | 1.37 | 1.43 | 1.45 |
| 4 | 1.23 | 1.19 | 1.24 | 1.25 | 1.23 | 1.21 | 1.28 | 1.55 | 1.62 | 1.48 | 1.51 | 1.43 | 1.49 | 1.51 |
| 5 | 1.13 | 1.20 | 1.12 | 1.09 | 1.31 | 1.17 | 1.18 | 1.57 | 1.64 | 1.31 | 1.30 | 1.59 | 1.37 | 1.44 |
| 6 | 1.08 | 1.04 | 1.13 | 1.21 | 1.11 | 1.16 | 1.12 | 1.39 | 1.35 | 1.38 | 1.33 | 1.35 | 1.32 | 1.33 |
| 7 | 1.22 | 1.32 | 1.18 | 1.19 | 1.19 | 1.13 | 1.28 | 1.59 | 1.64 | 1.34 | 1.27 | 1.34 | 1.45 | 1.59 |
| 8 | 1.25 | 1.19 | 1.16 | 1.28 | 1.15 | 1.29 | 1.13 | 1.63 | 1.61 | 1.20 | 1.40 | 1.31 | 1.34 | 1.35 |
| 9 | 1.21 | 1.24 | 1.36 | 1.27 | 1.22 | 1.25 | 1.28 | 1.62 | 1.63 | 1.55 | 1.58 | 1.48 | 1.37 | 1.49 |
| High SC | 1.39 | 1.36 | 1.33 | 1.31 | 1.33 | 1.33 | 1.37 | 1.84 | 1.71 | 1.55 | 1.50 | 1.68 | 1.51 | 1.75 |
| 10-1 Return | 0.29 | 0.32 | 0.28 | 0.28 | 0.20 | 0.29 | 0.30 | 0.27 | 0.21 | 0.26 | 0.23 | 0.29 | 0.24 | 0.29 |
| | (2.83) | (3.27) | (2.78) | (2.83) | (2.20) | (3.19) | (3.17) | (2.05) | (1.54) | (1.94) | (1.71) | (2.41) | (1.95) | (1.85) |
| 10-1 Alpha | 0.40 | 0.39 | 0.31 | 0.37 | 0.29 | 0.35 | 0.40 | 0.38 | 0.29 | 0.34 | 0.33 | 0.39 | 0.31 | 0.40 |
| | (2.71) | (2.97) | (2.93) | (2.76) | (2.11) | (3.46) | (2.67) | (2.41) | (1.96) | (2.46) | (2.26) | (2.41) | (2.57) | (2.10) |

Table 2.11: Returns on portfolios of industries sorted by influential centrality after controlling for FSIZE, ISIZE, BETA, BM, RTV, MOM, and IVOL. Centrality measures are estimated every year from January 1963 to December 2014. Double-sorted, equal-weighted) and value-weighted decile portfolios are formed every year from January 1964 to December 2015 by sorting industries based on influential centralities after controlling for average firm market capitalization, industry market capitalization, market beta, book-to-market, return-to-volume, industry momentum, and industry idiosyncratic volatility. In each case, I first sort the industries in to 5 deciles using the control variable, then within each decile, I sort industries into 10 decile portfolios based on the sensitive centralities over the previous calendar year so that decile 1 (10) contains industries with the lowest (highest) SC. This table presents average industry returns across the 5 control deciles to produce decile portfolio with dispersion in SC but with similar levels of the control variable. “10-1 Return” is the difference in average monthly returns between the High SC and Low SC portfolios. “10-1 Alpha” is the difference in 4-factor alphas on the High SC and Low SC portfolios. Newey-West (1987) adjusted t-statistics are reported in parentheses.

| | Equal-Weighted Returns | | | | | | | Value-Weighted Returns | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Decile | FSIZE | ISIZE | BETA | BM | RTV | MOM | IVOL | FSIZE | ISIZE | BETA | BM | RTV | MOM | IVOL |
| Low IC | 1.10 | 1.12 | 1.15 | 1.12 | 1.14 | 1.07 | 1.15 | 1.53 | 1.54 | 1.29 | 1.27 | 1.43 | 1.37 | 1.42 |
| 2 | 1.17 | 1.19 | 1.14 | 1.14 | 1.22 | 1.24 | 1.08 | 1.60 | 1.59 | 1.36 | 1.38 | 1.35 | 1.42 | 1.34 |
| 3 | 1.23 | 1.12 | 1.15 | 1.22 | 1.19 | 1.15 | 1.15 | 1.75 | 1.57 | 1.41 | 1.41 | 1.49 | 1.42 | 1.37 |
| 4 | 1.20 | 1.19 | 1.23 | 1.14 | 1.15 | 1.11 | 1.18 | 1.61 | 1.47 | 1.40 | 1.32 | 1.37 | 1.37 | 1.51 |
| 5 | 1.12 | 1.23 | 1.13 | 1.21 | 1.20 | 1.24 | 1.21 | 1.53 | 1.77 | 1.35 | 1.38 | 1.46 | 1.41 | 1.59 |
| 6 | 1.04 | 1.19 | 1.21 | 1.16 | 1.19 | 1.16 | 1.36 | 1.34 | 1.58 | 1.42 | 1.38 | 1.43 | 1.38 | 1.58 |
| 7 | 1.15 | 1.12 | 1.16 | 1.15 | 1.22 | 1.17 | 0.95 | 1.67 | 1.58 | 1.46 | 1.43 | 1.51 | 1.38 | 1.29 |
| 8 | 1.29 | 1.25 | 1.26 | 1.20 | 1.12 | 1.19 | 1.36 | 1.61 | 1.74 | 1.43 | 1.46 | 1.42 | 1.47 | 1.68 |
| 9 | 1.14 | 1.12 | 1.15 | 1.17 | 1.25 | 1.22 | 1.21 | 1.59 | 1.52 | 1.30 | 1.37 | 1.42 | 1.50 | 1.52 |
| High IC | 1.45 | 1.47 | 1.46 | 1.45 | 1.30 | 1.42 | 1.40 | 1.68 | 1.69 | 1.57 | 1.47 | 1.53 | 1.48 | 1.51 |
| 10-1 Return | 0.35 | 0.36 | 0.31 | 0.34 | 0.16 | 0.35 | 0.25 | 0.14 | 0.15 | 0.27 | 0.20 | 0.09 | 0.11 | 0.09 |
| | (3.16) | (2.99) | (2.60) | (2.85) | (1.42) | (3.37) | (2.11) | (1.36) | (1.26) | (2.41) | (1.82) | (0.92) | (1.03) | (0.65) |
| 10-1 Alpha | 0.45 | 0.45 | 0.34 | 0.44 | 0.27 | 0.38 | 0.34 | 0.21 | 0.24 | 0.30 | 0.24 | 0.21 | 0.13 | 0.18 |
| | (3.41) | (3.11) | (2.59) | (3.21) | (2.24) | (3.51) | (2.67) | (1.85) | (1.62) | (2.38) | (2.07) | (1.90) | (1.12) | (1.38) |

Table 2.12: Industry-level cross-sectional return regressions. Each month from January 1964 to December 2015 I run an industry-level cross-section regression of the return in that month on subsets of lagged predictor variables including sensitive centrality (SC), influential centrality (IC), FSIZE, ISIZE, BETA, BM, RTV, MOM, and IVOL. Centrality measures (SC and IC) are estimated every year from January 1963 to December 2014. Industry returns are calculated by the equal-weighted returns of stocks belong to the industry. For example, the industry return of each month in 2001 are regressed on the the lagged predictor variables estimated with the sample from January 2000 to December 2000. In each row, the table reports the time-series averages of the cross-sectional regression slope coefficients and their associated Newey-West (1987) adjusted t-statistics (in parentheses).

| SC | IC | BETA | FSIZE | ISIZE | BM | RTV | MOM | IVOL |
|---|---|---|---|---|---|---|---|---|
| 0.82 | | | | | | | | |
| (2.05) | | | | | | | | |
| | 0.69 | | | | | | | |
| | (1.70) | | | | | | | |
| 0.88 | | 0.48 | 0.47 | -0.52 | 0.91 | 0.50 | 0.66 | 0.04 |
| (2.14) | | (1.41) | (1.02) | (-1.30) | (2.18) | (1.32) | (2.57) | (0.06) |
| | 0.79 | 0.49 | 0.47 | -0.52 | 0.90 | 0.50 | 0.63 | 0.13 |
| | (1.95) | (1.43) | (1.02) | (-1.3) | (2.18) | (1.32) | (2.38) | (0.20) |
| 0.83 | 0.62 | 0.49 | -0.58 | 0.87 | 0.57 | 0.68 | 0.65 | 0.27 |
| (2.00) | (1.92) | (1.02) | (-1.46) | (2.10) | (1.52) | (1.45) | (2.52) | (0.39) |
| | | 0.65 | 0.45 | 0.51 | -0.44 | 0.81 | 0.64 | 0.50 |
| | | (1.52) | (1.41) | (1.10) | (-1.09) | (1.93) | (2.44) | (0.71) |

Table 2.13: Industry-level cross-sectional return regressions in subperiods (1970 -2015, 1980 - 2015, 1990 - 2015 and 2000 - 2015). Each month from January in each starting year (1970, 1980, 1990 and 2000) to December 2015 I run an industry-level cross-section regression of the return in that month on lagged predictor variables including sensitive centrality (SC), influential centrality (IC), FSIZE, ISIZE, BETA, BM, RTV, MOM, and IVOL. Centrality measures (SC and IC) are estimated every year from January 1963 to December 2014. Industry returns are calculated by the equal-weighted returns of stocks belong to the industry. For example, the industry return of each month in 2001 are regressed on the the lagged predictor variables estimated with the sample from January 2000 to December 2000. In each row, the table reports the subsample time-series averages of the cross-sectional regression slope coefficients and their associated Newey-West (1987) adjusted t-statistics (in parentheses).

| Subperiods | SC | IC | BETA | FSIZE | ISIZE | BM | RTV | MOM | IVOL |
|---|---|---|---|---|---|---|---|---|---|
| 1970 - 2015 | 0.89 | 0.68 | 0.50 | -0.62 | 0.97 | 0.65 | 0.75 | 0.70 | -0.09 |
| | (1.89) | (1.87) | (0.93) | (-1.36) | (2.05) | (1.50) | (1.39) | (2.74) | (-0.14) |
| 1980 - 2015 | 0.90 | 0.70 | 0.46 | -0.83 | 0.87 | 0.53 | 0.68 | 0.67 | -0.11 |
| | (1.69) | (1.75) | (0.75) | (-1.69) | (1.70) | (1.14) | (1.14) | (2.49) | (-0.16) |
| 1990 - 2015 | 1.24 | 0.94 | 0.81 | -0.89 | 1.16 | 0.84 | 0.88 | 0.50 | -0.29 |
| | (1.94) | (1.80) | (1.03) | (-1.49) | (1.78) | (1.45) | (1.12) | (1.83) | (-0.36) |
| 2000 - 2015 | 1.93 | 0.96 | 1.15 | -0.32 | 1.43 | 1.46 | 1.26 | 0.83 | -1.13 |
| | (2.10) | (1.34) | (0.91) | (-0.52) | (1.51) | (1.76) | (1.11) | (2.14) | (-0.91) |

# Chapter 3

# Dynamic Stable GARCH Model with Time-Dependent Tails


## Abstract

Predicting volatility conditional on current observables is crucial for financial risk management, from trading desks to financial institutions. This paper proposes a new Dynamic Stable GARCH model, which involves the use of stable distribution with time-dependent tail parameters to model and forecast tail risks in an extremely high volatility environment. We can differentiate extreme risks from normal market fluctuations with this model. Asymptotic inference methods in high volatility environments are unreliable, as standard regularity conditions may not apply or may hold only weakly. This paper applies a Monte Carlo test inference procedure to construct the confidence interval of the tail parameter. Empirical analysis on the Nikkei 225 index shows that the Dynamic Stable GARCH model provides the best in-sample and out-of-sample one-day Value-at-Risk fittings and forecasts at levels above 99% across different model specifications.


## 3.1 Introduction

To measure and predict volatility conditional on current observables is a crucial issue for financial institutions, from the desk level to the firm level. For instance, a reliable risk model for trading desks has to be based on well-developed volatility models. From the financial crisis in 2008, we learned that the risk models used by the industry may significantly underestimate the risks we are supposed to be dealing with. A correct risk model cannot prevent us from facing occasional losses, but it can provide management with a sense of how much risk we are exposed to. Managing tail risk, corresponding with possible extreme events, is necessary to avoid unexpected sudden margin calls and insufficient capital reserve against large losses in the trading book. In discussions of risk management issues, we are now more and more concerned about the extreme risk which corresponds to the tail part of the underlying distribution, instead of the central part.

In this paper, I propose a new type of GARCH model, whose innovations are driven by the stable distributions with time dependent tail parameters, to model tail risk dynamics. The family of stable distributions is a rich class and includes the Gaussian distributions, the Cauchy distributions and the Levy distributions as subclasses. Moreover, it is the only class of distribution to which we can apply the general central limit theorem, an appealing feature which is of great practical use in portfolio allocations. Even in high volatility environment, the non-normal stable distribution can easily model the cases with nonexistence of the second moment.

Two of the most important stylized facts of financial return data are heavy tails and volatility clustering. Extreme events occur more often than predicted by simple models, and they usually occur successively. The generalized autoregressive conditional heteroskedasticity (GARCH) model is the most commonly used financial econometric model to model these stylized facts. However, fitted standardized returns in the Normal-GARCH model are still usually found to be heavy-tailed. A flexible heavy-tailed distribution is needed to model the GARCH innovations. In high volatility environments, the tail could be so heavy that the second moment of the underlying innovation distribution does not even exist. In such cases even the traditional heavy-tailed distributions (e.g., student's t distributions and generalized error distributions) in the existing literature[1] may not be well defined and the misspecification could lead to underestimation of the tail risk.

Furthermore, the probability of extreme events is controlled by the tail parameter in my model. Modelling the dynamics of the tail parameter is useful for tail risk prediction since extreme events usually occur successively. The traditional volatility models with a single volatility measure (i.e., variance) cannot differentiate the usual market fluctuations (central risk) and unusual extreme

[1] For theoretical convenience, many empirical studies on financial heavy-tail distribution remain assuming that the second moment exists.

events (tail risk). To the best of my knowledge, existing literature has not investigated the time dependence of tail parameters in the stable GARCH framework. Actually, our model can also easily be extended to characterize the skewness dynamics.

The asymptotic distribution of the maximum likelihood estimator in our model is unknown, and the difficulty of inference in the context of the stable model is a long standing problem. However, the stable distribution can be easily simulated and our model is fully specified, and this permits implementation of Monte Carlo simulations and Monte Carlo testing. The test procedure is valid even under a small sample size. In practice, we always only have a limited sample and should avoid asymptotic inference procedure if possible. Moreover, asymptotic inference methods in high volatility environments are unreliable, as standard regularity conditions may not apply or may hold only weakly. This paper applies a Monte Carlo test inference procedure to construct the confidence interval of the tail parameter.

Based on this dynamic model, I investigate the performance of our risk model by empirical studies in the Japanese stock markets. I use the Dynamic Quantile test to examine the performance of the proposing model in modelling and predicting Value-at-Risk at extreme levels. I find that our model provides the best out-of-sample prediction for the VaR at low quantiles, and the stable distribution is better than the normal distribution and the t-distribution in terms of in-sample VaR fittings and out-of-sample VaR predictions at levels above 99%.

## 3.2 Related Literature

It's often found that the standardized GARCH residuals distribution are still heavy-tailed. GARCH models with heavy tail distributions have been proposed in many literatures. One of the most common used heavy-tailed distribution is the student's t distribution. Bollerslev (1987) first applies student's t distribution in GARCH model for empirical analysis [also see Baillie and Bollerslev (1989), Beine, Laurent and Lecourt (2002) and Franses, van der Leij and Paap (2008)]. McDonald and Newey (1988) propose the generalized student's t distribution which includes the power exponential or Box-Tiao, normal, Laplace, and t distributions as special cases. Skewed generalized student's t distribution proposed by Theodossiou (1998) is a skewed extension of the generalized t distribution. Several other skewed extension of student's t distribution have been proposed for financial and other applications [see for example Fernández and Steel (1998), Branco and Dey (2001), Jones and Faddy (2003), Bauwens and Laurent (2005) and Aas and Haff (2006)]. The other important class of distributions incorporated heavy tail skewness properties for financial applications is the generalized error distribution (GED) which includes normal as a special case. Many financial applications of the GED as well as its skew extensions have been considered in Hsieh (1989), Nelson (1991), Theodossiou (2000), Ayebo and Kozubowski (2003), Christoffersen, Dorion, Jacobs and Wang (2010), Komunjer (2007) and others. To allow separate parameters to control skewness and the thickness of each tail, Zhu and Zinde-Walsh (2009), Zhu and Galbraith (2011) and Zhu and Galbraith (2010) propose asymmetric exponential power distribution and asymmetric Student's t distribution and find evidence for the usefulness of these general distributions in improving fit and prediction of downside market risk.

Volatility dynamics in financial market is usually modelled by ARCH Engle (1982) type models and GARCH Bollerslev (1986) type models, which are mainly focused on how to model variance dynamics. However, a single volatility measure (i.e., variance) cannot differentiate the usual market fluctuations (central risk) and unusual extreme events (tail risk). It's natural to extend the (G)ARCH frameworks to model the tail risk separately in a time-vary setting. Hansen (1994) proposes Autoregressive Conditional Density (ARCD) model in GARCH framework and studies the time dependent skewness by skewed student's t distribution. After that, a substantial amount of empirical studies have discussed econometric specifications in modeling time dependent skewness and kurtosis. Harvey and Siddique (1999, 2000) use the conditional mean and skewness to compute noncentrality parameter in concentral-t distribution. Some other researches directly model the dynamics of time dependent Skewness and kurtosis in autoregressive form, see Jondeau and Rockinger (2003), León, Rubio and Serna (2005), Brooks, Burke, Heravi and Persand (2005), Bali, Mo and Tang (2008), Cheng and Hung (2011) and Lin, Changchien, Kao and Kao (2014). Almost all these empirical studies assume t class distributions as the underlying distribution and the second moment exists. They find significant time dependence of skewness, or kurtosis, or both. But very few people has ever investigate the time dependence of tail and skewness for the non-normal stable distribution in the GARCH framework. Stable GARCH model with time dependent tail and skewness is a blank in previous studies. In this paper, I will study the Stable GARCH model with time-varying tail .

Even thought student's t class distributions and GED class distribution embed flexibility to model skewness and tail, they are generally not closed under summation, a appealing feature which is of great practical use in portfolio allocation; see e.g., Doganoglu, Hartz and Mittnik (2007) and Giacometti, Bertocchi, Rachev and Fabozzi (2007). If a sum of independently identically distribution random variables has a limiting distribution, then it must be a stable distribution. In other words, stable distribution, which includes normal as a special case, is the only class of distributions to which the generalized central limit theorem applies. Mandelbrot (1963) and Fama (1965) first apply the stable distribution to financial time series to model unconditional heavy tail property.

A main critique of the use of the non-normal stable distribution is that it has infinite variance. This seems to contradict empirical studies [see, for example, Hols and De Vries (1991), Loretan and Phillips (1994) and Pagan (1996)]. However, these findings are almost always based on inference of the Hill and related tail estimators. McCulloch (1997) points out this inference is invalid and tail index estimates greater than 2 are to be expected for stable distributions with $\alpha$ as low as 1.65. The misleading problem for the tail index estimates with stable distributions has been discussed in depth by [McCulloch (1997), Mittnik, Paolella and Rachev (1998a) and Weron (2001)]. On the other hand, Mittnik, Rachev and Paolella (1998b) fit a return distribution using a number of parametric distribution and find the stable Paretian law is a more realistic assumption, which holds for unconditional, conditional homoskedastic and the conditional heteroskedastic distributions; Mittnik and Paolella (2003) demonstrate the effectiveness of stable GARCH in VaR. Whether stable distribution is suitable for financial studies, is still an open question.

Estimation of stable distribution is relatively difficult since it only has a characteristics function definition without a closed form of density function except for some special cases: Normal, Cauchy and Levy. It's well known that Maximum Likelihood Estimation method is the most efficient estimation method under certain conditions. Even though the stable density function is not known in closed form in general, DuMouchel (1973) shows that the nice properties ($\sqrt{n}$ asymptotic normality and Cramer-Rao lower bounds) of maximum likelihood method are still valid under looser restrictions. Actually, maximum likelihood estimation can be still implemented by numerical methods. Zolotarev (1966) gives the integral representation of the standard stable distribution function. Nolan (1997) provides a more convenient computational formula for the stable density by Zolotarev's (M) parameterization. McCulloch (1985) studies the adaptive conditional heteroskedastic (ACH) model and estimates it with symmetric stable maximum likelihood method. Panorska, Mittnik and Rachev (1995) and Mittnik, Paolella and Rachev (2002) discuss necessary and sufficient conditions of stationarity of stable GARCH processes. Liu and Brorsen (1995) estimate the Stable GARCH model using Zolotarev (1966)'s integral representation formula, but they assume the conditional stable innovations are identically independently distributed. Broda, Haas, Krause, Paolella and Steude (2013) go further and study the stable mixture GARCH model.

Several alternative methods, (quantile-based estimator (McCulloch (1986)), generalised method of moment with a finite set (see Hansen (1982), Feuerverger and McDunnough (1981a), Feuerverger and McDunnough (1981b) and Besbeas and Morgan (2008)) or continuum of moment conditions (see Carrasco and Florens (2000), Carrasco and Florens (2002) and Carrasco, Chernov, Florens and Ghysels (2007)) , the iterative Koutrouvelis regression method (see Koutrouvelis (1980) and Koutrouvelis (1981)), constrained indirect inference estimation method (see Gourieroux, Monfort and Renault (1993) and Garcia, Renault and Veredas (2011)) and the exact confidence sets and goodness-of-fit methods (see Dufour and Kurz-Kim (2010) and Beaulieu, Dufour and Khalaf (2014))), have been proposed to estimate the stable distribution without using its likelihood function.

## 3.3 Models

Extreme risk measures (e.g., VaR) are sensitive to tail parts of the underlying distribution. Misspecification may lead to incorrect inference and incorrect forecasting. In high volatility environments, the second moment of financial variables of interest could be not well-defined. In this section, I develop a stable GARCH model with time dependent tails to measure and forecast the extreme risks where the second moment is infinite to model such high volatility cases.

### 3.3.1 Stable Distribution

The most common used parameterization of stable distribution is the one in Samoradnitsky and Taqqu (1994) which have the characteristic function $\phi(t)$ with the form:

$$\phi(t) = E[\exp(itX)] = \begin{cases} \exp\{-\sigma^{\alpha}|t|^{\alpha}[1 - i\beta \text{sign}(t)\tan(\frac{\pi\alpha}{2})] + i\mu t\} & ,\text{for } \alpha \neq 1 \\ \exp\{-\sigma|t|[1 + i\beta(\frac{2}{\pi})\text{sign}(t)\ln|t|] + i\mu t\} & ,\text{for } \alpha = 1 \end{cases}$$

where $X$ is the stable random variable, $X \sim S(\mu, \sigma, \alpha, \beta)$, $\alpha \in (0,2]$ and $\beta \in (-1,1)$, and sign$(t)$ is the sign function. $\mu$ is the location parameter, $\sigma$ is the scale parameter, $\alpha$ is the tail parameter and $\beta$ is the skewness parameter. A standard stable random variable $Z$ takes the form: $Z = \frac{X-\mu}{\sigma} \sim S(0, 1, \alpha, \beta)$.

### 3.3.2 Model Specification

The random variable asset returns $r_t$ follow stable distribution $S(\mu, \sigma_t, \alpha_t, \beta)$, where $\mu$, $\sigma_t$, $\alpha_t$ and $\beta$ represent location, scale, tail and skewness parameters at each period $t$, $t = 1, ..., T$. In addition, I assume the processes are demeaned ($\mu = 0$) and the skewness parameter is constant ($\beta_t = \beta$). Then the dynamic Stable GARCH model could take the general form:

$$r_t = \sigma_t z_t \tag{3.1}$$

$$z_t \sim S(0, 1, \alpha_t, \beta) \tag{3.2}$$

$$\sigma_t = g_{\sigma}(z_{t-1}, z_{t-2}, ..., z_{t-p_{\sigma}}, \sigma_{t-1}, \sigma_{t-2}, ..., \sigma_{t-q_{\sigma}}) \tag{3.3}$$

$$\alpha_t = g_{\alpha}(z_{t-1}, z_{t-2}, ..., z_{t-p_{\alpha}}, \alpha_{t-1}, \alpha_{t-2}, ..., \alpha_{t-q_{\alpha}}) \tag{3.4}$$

where $g_{\sigma}$ and $g_{\alpha}$ are deterministic functions which govern the scale and the tail dynamics respectively. They are the functions of the past stable innovations with lags $p_{\gamma}$ and the past parameter values with lags $q_{\gamma}$ ($\gamma = \sigma, \alpha$). In high volatility environment, the tail parameter $\alpha_t$ could be smaller than 2.

For traditional GARCH models with time-varying moments, innovations $z_t$ follow generalized t type distributions, in which $E(z_t)^2 < \infty$. Here, $z_t \sim S(0, 1, \alpha_t, \beta_t)$ and in general, $E(z_t)^2 = \infty$ except for $\alpha_t = 2$. In the stable GARCH models proposed by Liu and Brorsen (1995), the tail and the skewness parameter are assumed to be constant, $\alpha_t = \alpha$ and $\beta_t = \beta$. In such case, $z_t \overset{iid}{\sim} S(0,1,\alpha,\beta)$. The volatility dynamics are accounted entirely by the the time dependent scale parameter $\sigma_t$. Scale parameter is a well known measure to describe the central spread of a distribution. It's most common to model variance to study volatility dynamics as almost all the GARCH type models do so. However, variance (or scale parameter) is just a proxy of volatility, not the volatility itself. Variance is a good index to measure the spread of a distribution for its central part, but it is relatively insensitive to tail changes. We are used to taking variance (or scale parameter) as volatility measure since the volatility of normal distributions is fully determined by its second moment. For stable distributions, volatility is characterized by the scale parameter $\sigma$, the tail parameter $\alpha$ and the skewness parameter $\beta$. The study of the tail parameter, $\alpha_t$, is at least as important as the scale parameter, $\sigma_t$, to model the tail risks. It helps to better capture the dynamics of extreme movements, which is crucial for prediction of the tail risk conditional on current observables. In our model, the tail parameter can also be a general function of the past innovations and their respective lagged values. The innovations, $z_t$, are no longer independent and identically distributed. We would like to model the tail dynamics ($\alpha_t$) to model and predict the extreme risks.

There is usually a trade-off between in sample fitting and forecasting precision. Note that the purpose to develop this model is for better modelling and predicting the tail risk. The desired model specification for the dynamic Stable GARCH model should be simple enough while allowing to characterize standard stylized facts of financial data as well as the tail dynamics. In this paper, the model specification takes the following functional form:

$$r_t = \sigma_t z_t, \qquad t = 1, ..., T \tag{3.5}$$

$$z_t \sim S(0,1,\alpha_t,\beta) \tag{3.6}$$

$$\sigma_t = b_0 + b_1 |r_{t-1} - \mu| + b_2(r_{t-1} - \mu) + b_3 \sigma_{t-1} \tag{3.7}$$

$$\alpha_t = \frac{2-m}{m}[\frac{2}{\pi}\arctan(\tilde{\alpha}_t) + 1] + m \tag{3.8}$$

$$\tilde{\alpha}_t = c_0 + c_1 |z_{t-1}| + c_2 \tilde{\alpha}_{t-1} \tag{3.9}$$

The scale parameter $\sigma_t$ keeps the asymmetric power 1 GARCH dynamics. The domain of the tail parameter $\alpha_t$ is in (m,2), when $m$ is a constant in (1,2) given by econometricians. The choice of $m$ is necessarily constrained by the regularity conditions discussed in Panorska et al. (1995) and Mittnik et al. (2002) for the special case of $c_1 = 0$. In this paper, I set $m = 1.6$ based on some preliminary simulation studies.[2] It's small enough to allow heavy tails and big enough to guarantee the stationarity of the underlying GARCH process. $\tilde{\alpha}_t$ is the unrestricted tail parameter and it could be unbounded in general for computation convenience. Transformation between restricted parameters and the unrestricted parameters are monotone and continuous mappings as in equation(3.8). I let the skewness parameter $\beta$ be a constant in (-1,1). Of course, it's easy to extend $\beta$ to be time-varying as $\alpha$ as well. My model nests the Normal-GARCH model $(c_0 = +\infty, c_1 = 0, c_2 = 0)$ and the Stable-GARCH model $(c_1 = 0, c_2 = 0)$.

Note that the conditional volatility in this model is determined by two parameters, the scale parameter $\sigma_t$ and the tail parameter $\alpha_t$, $b_2$ captures the leverage effect and $(b_3, c_2)$ captures the volatility clustering effect.

The fixed and unknown parameter is $\theta = (b_0, b_1, b_2, b_3, c_0, c_1, c_2, \beta) \in \Theta$. The whole parameter space is $\Theta$. Given $\theta$, the data generating process of the dynamic Stable GARCH is fully determined, then we can easily compute and predict the tail risk by Monte Carlo simulation method.

## 3.4 Estimation

The main difficulty to estimate stable distribution is the lack of closed form of density function. Several alternative methods[3] have been proposed to estimate the stable distribution without using its likelihood function. These methods are good enough to estimate the four parameters of a stable distribution but none of them can be easily extended to estimate the a dynamic model with dependent stable innovations. For instance, it's very difficult to derive a closed form expression of the characteristic function for the model specification of our interest; the slackness restriction for

[2]I simulate processes with different $m$ given a range of scale dynamic parameters around the values of those documented in literature.

[3]Quantile-based estimator (McCulloch (1986)), generalised method of moment with a finite (see Hansen (1982), Feuerverger and McDunnough (1981a), Feuerverger and McDunnough (1981b) and Besbeas and Morgan (2008)) or continuum moment conditions (see Carrasco and Florens (2000), Carrasco and Florens (2002) and Carrasco et al. (2007)) , the iterative Koutrouvelis regression method (see Koutrouvelis (1980) and Koutrouvelis (1981)), constrained indirect inference estimation method (see Gourieroux et al. (1993) and Garcia et al. (2011)) and the exact confidence sets and goodness-of-fit methods (see Dufour and Kurz-Kim (2010) and Beaulieu et al. (2014))

constrained indirect inference in Garcia et al. (2011) would become very complicated because I introduce time dependent tail in this paper. Capturing the dynamics of the tail parameter in the conditional distribution is very difficult by these methods but is fairly easy once we have conditional density for stable distribution. Actually, it's still feasible to compute the density value of the stable distribution with numerical methods. Nolan (1997) applies the (M) parameterization of Zolotarev (1986) and derives numerical formulas for the computation of the stable density. The performances of this numerical MLE method for stable distribution have been examined by Nolan (1997), Nolan (2001) and Calzolari, Halbleib and Parrini (2014). The criticism of this numerical method is about its accuracy. The density function only has a integral expression and thus it's computationally intensive to make it accurate. In addition, the numerical optimization routine searching the optimum point over the whole parameter space for a large sample size could be even more time consuming. However, these difficulties will be partially overcome in this section to make the numerical MLE method be a suitable candidate to estimate the complicated dynamic model.

### 3.4.1 Numerical Density Computation for Stable Distribution and Simulation Method for Stable Distribution

The main difficulty to apply MLE method to stable distribution is its lack of closed expression of density function. Nolan (1997) provides computational formulas for the stable density by Zolotarev's (M) parameterization. A detailed discussion about different parameterizations of stable distribution and a simple relation between the stable distribution with parameterization as described above and the one with Zolotarev's (M) parameterization can be found in Nolan (2015). In this paper, our stable innovations follow the parameterization in Samoradnitsky and Taqqu (1994).

For a standard stable random variable under M parameterization $Z \sim S(0,1,\alpha,\beta;0)$, its density function is defined as $f_Z(z;\alpha,\beta)$. Following Nolan (1997), define

$$\zeta = \zeta(\alpha,\beta) = -\beta \tan\frac{\pi\alpha}{2} \tag{3.10}$$

$$\phi_0 = \phi_0(\alpha,\beta) = \frac{1}{\alpha}\arctan(\beta\tan\frac{\pi\alpha}{2}) \tag{3.11}$$

$$V(\phi;\alpha,\beta) = (\cos\alpha\phi_0)^{\frac{1}{\alpha-1}}\left(\frac{\cos\phi}{\sin\alpha(\phi_0+\phi)}\right)^{\frac{\alpha}{\alpha-1}}\frac{\cos(\alpha\phi_0+(\alpha-1)\phi)}{\cos\phi}, \tag{3.12}$$

then when $\alpha \in (1,2)$ the standardized stable density has the following calculation formula,

$$f_Z(z;\alpha,\beta) = \begin{cases} \frac{\alpha(x-\zeta)^{\frac{1}{\alpha-1}}}{\pi(\alpha-1)}\int_{-\phi_0}^{\frac{\pi}{2}} V(\phi;\alpha,\beta)\exp\left(-(z-\zeta)^{\frac{\alpha}{\alpha-1}}V(\phi;\alpha,\beta)\right) & ,\text{for } z > \zeta \\ \frac{\Gamma(1+\frac{1}{\alpha})\cos(\phi_0)}{\pi(1+\zeta^2)^{1/(2\alpha)}} & ,\text{for } z = \zeta \\ f_Z(-z;\alpha,-\beta) & ,\text{for } z < \zeta. \end{cases}$$

The stable random variable under the parameterization in Samoradnitsky and Taqqu (1994) $r \sim S(\mu,\sigma,\alpha,\beta;1)$ has the density

$$f_r(r;\mu,\sigma,\sigma,\beta) = f_Z\left(\frac{r-\mu}{\sigma} - \beta\tan(\frac{\pi\alpha}{2});\alpha,\beta\right) \tag{3.13}$$

Even though computing density for stable distribution may be a challenging task, simulating stable distribution is relatively much easier. Simulation of stable distribution formulas can be found in Chambers, Mallows and Stuck (1976) and Weron (1996). Based on them, we can simulate the stable random variable under the parameterization in Samoradnitsky and Taqqu (1994).

### 3.4.2 Maximum Likelihood Estimation Method

Let's assume that we observe daily prices, i.e., $p_1, p_2, \ldots$. The daily return is defined as $r_t = \frac{p_t - p_{t-1}}{p_{t-1}}$. We would like to model the conditional distribution of the daily returns. The maximum likelihood estimator is given as

$$\hat{\theta}_T^{MLE} = \underset{\theta\in\Theta}{\text{argmax}}\frac{1}{T}\sum_{t=1}^{T}\ln f_r(r_t|r_{t-1},r_{t-2},\ldots,r_1;\theta). \tag{3.14}$$

### 3.4.3 Confidence Interval

The asymptotic distribution of the maximum likelihood estimator in our model is basically unknown. Moreover, asymptotic inference methods are unreliable in high volatility environment, for standard regularity conditions may not apply or may hold only weakly. In this section, I describe a

Monte Carlo procedure to construct confidence interval for the parameters of interest.[4]

Once the unknown parameter is fixed and a test statistic, which is a function of data, can be generated by Monte Carlo simulation method. An exact test based on this idea is called Monte Carlo test method. Inverting the test we can obtain an exact confidence set for unknown parameter at arbitrary levels in finite sample.

Let $S = S(Y_1, ..., Y_T)$ be a continuous test statistic for testing an hypothesis $H_0$, with critical region of the form $S \geq c$, then the test has level $\alpha$ if $P(S_0 \geq c) \leq \alpha$, where $S_0$ is the test statistic based on the observed data. Suppose we can generate by simulation $N$ iid replications of $S$ under $H_0$, $S_1, ..., S_N$. If $N$ is chosen so that $\alpha(N+1)$ is an integer, under $H_0$, $P[\frac{\sum_{i=1}^{T} 1(S_i \geq S_0)+1}{N+1} \leq \alpha] = \alpha$. Let $\hat{p}(x) = \frac{\sum_{i=1}^{N} 1(S_i \geq x)+1}{N+1}$, then the test which rejects $H_0$ when $\hat{p}(S_0) \leq \alpha$ has level $\alpha$ exactly. We can always control the size of test by picking the suitable number of simulation replications, $N$. The validity of this inference approach is independent from the sample size $T$, thus it's a finite sample method.

Suppose we have a test for hypothesis $H_0(\theta_0) : \theta = \theta_0$ with level $\alpha$, $\theta \in \Theta$, where $\Theta$ is a fixed parameter space. By the definition of test level, we have $P[\text{reject } H_0(\theta_0)|\theta = \theta_0] \leq \alpha$. Let $C_\alpha = \{\theta_0 \in \Theta : \text{ can not reject } H_0(\theta_0)\}$. $P[\theta \in C_\alpha] = 1 - P[\theta \notin C_\alpha] \geq 1 - \alpha$. Hence, $C_\alpha$ is the confidence set of $\theta$ with level $1 - \alpha$.

## 3.5 Empirical Analysis

In this section, I use the Nikkei 225 returns series to investigate the performance of my risk models, and compare with the traditional GARCH models with the Normal distribution and the T distribution. Specially, I will focus on the tail part.

I collect the daily adjusted closed prices ($p_t$) of the Nikkei 225 from 04/01/1984 - 07/03/2017 from Yahoo Finance. Daily returns are calculated as $r_t = \frac{p_t}{p_{t-1}}$ and also demeaned.

[4] For those who want to know more about the Monte Carlo test methods, Dufour (2006) provides a comprehensive discussion.

### 3.5.1 Tail Parameter

In this section, I study the thickness of tails in distributions of returns and return innovations. Specifically, I fit the returns processes ($r_t$) and estimated innovations processes ($\hat{z}_t = \frac{r_t}{\hat{\sigma}_t}$) using the stable distribution and construct the confidence intervals of their respective tail parameters:

- $r_t$. Suppose $r_t \sim S(\alpha, \beta, \mu, \sigma)$, then following Beaulieu et al. (2014) $\hat{r}_t = \frac{r_t - r[50]}{r[75] - r[25]}$ does not depend on $\mu$ and $\sigma$, where $r[x]$ refers to the $x$th quantile of $r$.

- $\hat{z}_{1t}$, where $\hat{z}_{1t} = r_t/\hat{\sigma}_{1t}$ and $\hat{\sigma}_{1t}$ is estimated by the asymmetric power 1 GARCH model with iid normal innovations [$r_t = \sigma_t z_t$, $\sigma_t = b_0 + b_1 |r_{t-1} - \mu| + b_2(r_{t-1} - \mu) + b_3 \sigma_{t-1}$, $z_t \sim N(0,1)$]. If this model is true, then $\hat{z}_t \sim S(2, 0, 0, \sqrt{2})$.

- $\hat{z}_{2t}$, where $\hat{z}_{2t} = r_t/\hat{\sigma}_{2t}$ and $\hat{\sigma}_{2t}$ is estimated by the asymmetric power 1 GARCH model with iid stable innovations [$r_t = \sigma_t z_t$, $\sigma_t = b_0 + b_1 |r_{t-1} - \mu| + b_2(r_{t-1} - \mu) + b_3 \sigma_{t-1}$, $z_t \sim S(\alpha, \beta, 0, 1)$]. If this model is true, then $\hat{z}_t \sim S(\alpha, \beta, 0, 1)$.

In order to construct confidence intervals for the tail parameters of $r_t$, $\hat{z}_{1t}$ and $\hat{z}_{2t}$, I consider quantile-base criterion: $\phi = \frac{\hat{y}[95] - \hat{y}[5]}{\hat{y}[75] - \hat{y}[25]}$, $y = r_t, z_{1t}, z_{2t}$, where $\hat{y}[x]$ refer to the $x$th quantile of $\hat{y}$. I construct confidence intervals using the following Monte Carlo test inference procedure:

1. Fix $(\alpha, \beta)$ to be $(\alpha_0, \beta_0)$.

2. Draw 999 iid samples of size $T$ from a stable distribution imposing step 1.

3. For each sample drawn, construct the quantiles which appear in the formulas for $\phi$; these yield 999 realizations of the measures under consideration.

4. Then average across the 999 simulated values $[\phi_i(\alpha_0, \beta_0), i = 1, ..., 999]$ to yield $\bar{\phi}(\alpha_0, \beta_0)$.

5. Compute 1,000 test statistics $\hat{\phi}_i(\alpha_0, \beta_0) = |\phi_i(\alpha_0, \beta_0) - \bar{\phi}(\alpha_0, \beta_0)|$, $i = 0, 1, ..., 999$, where $\hat{\phi}_0(\alpha_0, \beta_0) = \phi$.

6. Reject $(\alpha_0, \beta_0)$ if $\hat{\phi}_0(\alpha_0, \beta_0) > \hat{\phi}[95]$, where $\hat{\phi}[95]$ is the 95th quantile of $\{\hat{\phi}_i(\alpha_0, \beta_0), i = 0, ..., 999\}$.

7. Repeat steps 1-6 and consider all values of $(\alpha_0, \beta_0) \in (1,6,2) \times (-1,1)$, then the 95% confidence interval of $(\alpha, \beta)$ is all the $(\alpha_0, \beta_0)$ that cannot be rejected: $C_{95\%}(\alpha, \beta) = \{(\alpha_0, \beta_0) : (\alpha_0, \beta_0) \text{ cannot be rejected by step 6}\}$.

8. The 95% confidence interval of $\alpha$, $C_{95\%}(\alpha) = \{\alpha_0 : \text{there exists } \beta_0 \text{ such that } (\alpha_0, \beta_0) \in C_{95\%}(\alpha, \beta)\}$.

Using the procedure described above, Table 3.1 reports the 95% confidence intervals of the tail parameters of $r_t$, $\hat{z}_{1t}$ and $\hat{z}_{2t}$ with fitting in iid stable distributions. Not surprisingly, the return distribution $r_t$ shows heavy tail, whose tail parameter's 95% confidence interval is $[1.40, 1.52]$. GARCH models can in part explain the heavy tails in unconditional return distribution. Conditional on estimated scale parameters and assuming innovation terms follow the iid normal distribution and iid stable distribution, the 95% confidence interval of the tail parameters of the distributions of the estimated innovations $\hat{z}_{1t}$ and $\hat{z}_{2t}$ are $[1.68, 1.80]$ and $[1.69, 1.82]$ respectively. Yet, note that the tail parameters are statistically smaller than 2, the estimated innovations still exhibit strong heavy tail pattern. General volatility clustering may be well modelled by conditional scales models, but the dynamics of extreme events still lacks an appropriate explanation, and this is crucial for risk management conditional on current observables.

Table 3.1: Confidence Intervals

| | NIKKEI 225 |
|---|---|
| $r_t$ | [1.41, 1.54] |
| $\hat{z}_{1t}$ | [1.68, 1.80] |
| $\hat{z}_{2t}$ | [1.69, 1.82] |

### 3.5.2 In-Sample VaR Estimation

I use the Dynamic Quantile test to examine the performance of the proposed model in modelling and predicting Value-at-Risk at extreme levels. The Dynamic Quantile test is a joint test of unconditional coverage and conditional independence. Let $I_t$ be the hitting function

$$I_t = \begin{cases} 1 & ,\text{if } -r_t < VaR_q \\ 0 & ,\text{if } -r_t \geq VaR_q. \end{cases}$$

$I_t = 0$ if the loss $-r_t$ exceeds the given level that is measured by $VaR_q$ and $I_t = 1$ if the loss $-r_t$ is under the reserved level $VaR_q$, where $VaR_q$ is the q quantile of the loss function $-r_t$. The large quantile level $q \geq 0.95$ is of interest. If our risk model can correctly characterize the underlying tail risks, then $I_t'$ is a martingale process, which implies 1) $E(I_t') = 0$, 2) $I_t'$ is uncorrelated with variables included in the past information set, where $I_t' = I_t - q$. The joint test can be done using the artificial regression $I_t' = XB + \varepsilon_t$ where $X$ is a $T \times k$ matrix whose first column is a column of ones, and the remaining columns are additional explanatory variables. Some risk models fail as they predict clustering of exceeding to the $VaR$ level while they satisfy the unconditional requirement. I include five lags of $I_t'$, current $VaR$ and four lags of $VaR$. Under the null, $B = 0_{10\times 1}$, the test statistic $DQ = \frac{\hat{B}'X'X\hat{B}}{q(1-q)} \sim \chi^2(11)$, where $\hat{B}$ is the OLS estimate of $B$. Specifically, I check the tail parts in distributions: $q = 0.01, 0075, 0.005, 0.0025, 0.001$.

Table 3.2 provides the in-sample $DQ$ test statistics for the four competing models: 1) the GARCH model with iid Normal innovations (NGARCH), 2) the GARCH model with iid T innovations (TGARCH), 3) the GARCH model with iid Stable innovations, and 4) the GARCH model with Stable innovations with time-dependent tail parameters (DSGARCH).

As expected, the NGARCH model is always significantly rejected. Heavy-tailed models provide better in-sample fit in the tails. Since the SGARCH model is nested by the DSGARCH model, the DSGARCH model is always superior to the SGARCH model in terms of fitting at all quantiles. For $q = 0.0075, 0.005$, the SGARCH model is rejected while DSGARCH model cannot be rejected. An important observation is that both TGARCH and SGARCH models cannot fit these small quantiles very well and in contrast, the DSGARCH model provides a very good in-sample fit to the low quantiles. It implies the usefulness of applying time-dependent tail parameters in modelling processes. However, the better fit in sample may be explained by the fact that there are more parameters. We need to conduct out-of-sample analysis to further examine the performance of our model.

### 3.5.3 Out-of-Sample VaR Forecasting

A precise estimation of time-dependent tails requires large sample to ensure extreme events occur. The Dynamic Quantile test also requires large sample as we expect to observe certain extreme re-

Table 3.2: In-Sampel DQ

| q | NGARCH | TGARCH | SGARCH | DSGARCH |
|---|---|---|---|---|
| 1% | 28.5** | 32.3** | 23.3* | 21.6* |
| 0.75% | 33.7** | 26.8** | 22.5* | 15.6 |
| 0.5% | 43.4** | 24.4* | 18.9* | 11.8 |
| 0.25% | 64.0** | 26.3** | 12.7 | 5.8 |
| 0.1% | 114.8** | 4.7 | 5.7 | 3.9 |

* Denote significance at the 5% level.
** Denote significance at the 1% level.

turns would exceed VaR at certain levels. In this section, I examine the out-of-sample performance of the four competing models in VaR prediction. In Table 3.3, I use the daily observations from 01/01/1984 - 31/12/1998 to estimate the models and use the estimated coefficients to predict the daily VaRs from 01/01/1999 - 07/03/2017. This strategy ensures that the size of the training set is approximately equal to the size of the test set, and the sizes of them are as large as possible.

Table 3.3 shows that the NGARCH model is always rejected again, since it fails to model the heavy-tail property with the standard normal distribution. The TGARCH model cannot well predict the VaR at low quantiles (it is rejected at $q = 0.005, 0.0025$). In contrast, the SGARCH model and the DSGARCH model can well predict the VaR at low quantiles (they are not rejected at $q = 0.005, 0.0025, 0.001$), but they cannot well predict the VaR at relative high quantiles (they are rejected at $q = 0.01, 0.0075$). At $q = 0.0075$, the SGARCH is rejected at the significance level of 1% while the DSGARCH is rejected at the significance level of 5%. Overall, the dynamic Stable GARCH proposed in this paper provides the best out-of-sample prediction for the VaR at low quantiles, and the stable distribution is better than the normal distribution and the t-distribution in terms of in-sample VaR fittings and out-of-sample VaR predictions.

Table 3.3: Out-of-Sampel DQ

| q | NGARCH | TGARCH | SGARCH | DSGARCH |
|---|---|---|---|---|
| 1% | 87.2** | 19.6 | 36.0** | 40.2** |
| 0.75% | 84.4** | 18.0 | 25.2** | 22.3* |
| 0.5% | 89.7** | 30.7** | 17.8 | 16.5 |
| 0.25% | 117.1** | 60.1** | 6.0 | 4.5 |
| 0.1% | 131.2** | 1.2 | 2.9 | 2.9 |

* Denote significance at the 5% level.
** Denote significance at the 1% level.

## 3.6 Conclusion

Many financial econometric models are unable to measure correctly the very heavy-tail return distributions that are used to model high volatility environments. In this paper, I propose a dynamic stable GARCH model with dynamic tail parameters to measure and forecast extreme risks. Using this model, we can differentiate the tail risk dynamics from normal market fluctuations. Using a Monte Carlo test inference method, I construct confidence intervals for the tail parameters of returns and estimated innovations and find these distribution exhibit strong heavy-tail property. In terms of one-day VaR modelling at low quantiles $q = 0.01, 0075, 0.005, 0.0025, 0.001$, I find our dynamic Stable GARCH model provides the best in-sample fit and out-of-sample forecasting across different model specifications, and our model performs well in capturing both the rate of occurrence and the extent of extreme events in the Japanese stock markets.

# Appendix A

# Proof

We apply an assumptions set that is similar to the one used in Barigozzi and Brownlees (2014). The proofs of the Proposition 1.4.6 and the Proposition 1.4.7 can thus follow their results. The proof of Theorem 1.4.8 is based on Proposition 1.4.6 and Proposition 1.4.7.

## A.1 Assumptions

1. The N-dimensional random vector process $X(t)$ is non-deterministic, has zero mean, and is covariance stationary. Moreover,

   (a) here exist constants $M_1$ and $M_2$ such that for each $N$, $0 < M_1 < \mu_{\min}(\Gamma_X) \leq \mu_{\max}(\Gamma_X) < M_2 < \infty$, where $\Gamma_X$ is the covariance matrix of $X$ and $\mu_{\min}(\cdot)$ and $\mu_{\max}(\cdot)$ are the smallest and the largest eigenvalues operators respectively.

   (b) there exists constants $M_3(\omega)$ and $M_4(\omega)$ such that, for each $N$ and for any $\omega \in [-\pi, \pi]$, we have $0 < M_3(\omega) \leq \mu_{\min}(s_X(\omega)) \leq \mu_{\max}(s_X(\omega)) \leq M_4(\omega) < \infty$, where $s_X(\omega)$ is the spectral density matrix of (1.3).

   (c) define $\beta = \sup\{c : \sum_{h=1}^{\infty} h^c \sup_{i,j} |E[X(t)_i X(t-h)_j]|\}$, then $\beta > 0$.

   (d) the process has three representation forms (1.3), (1.4) and (1.5) as stated in Assumption 1.4.1.

2. There exist constants $c_1 > 0$ and $c_2 > 0$ such that $N = O(T^{c_1})$ and $p = O(T^{c_2})$. $\beta > \frac{4c_1}{c_2}$. The dimension of the two parties $W$ and $Y$ of analysis, $m_1$ and $m_2$, is fixed.

3. (a) The set of nonzero entries in $\alpha_i$, $\mathscr{A}_i$, has $q_{Ti}^{\mathscr{A}}$ elements, and $q_{Ti}^{\mathscr{A}}$ satisfies the following conditions:
$q_{Ti}^{\mathscr{A}} = o\left(\sqrt{\frac{T}{\log T}}\right)$, $\frac{\lambda_T}{T}\sqrt{q_{Ti}^{\mathscr{A}}} = o(1)$, $\lim_{T\to\infty}\frac{\lambda_T}{T}\sqrt{\frac{T}{\log T}} = \infty$, $\sqrt{\frac{q_{Ti}^{\mathscr{A}}\log T}{T}} = o\left(\frac{\lambda_T}{T}\right)$ and $\frac{\lambda_T}{T^{1-c_1}}\sqrt{q_{Ti}^{\mathscr{A}}} = O(1)$ for $i = 1,...,N$.

   (b) The set of nonzero entries in $\bar{\alpha}_i$, $\bar{\mathscr{A}_i}$, has $q_{Ti}^{\bar{\mathscr{A}}}$ elements, and $q_{Ti}^{\bar{\mathscr{A}}}$ satisfies the following conditions:
$q_{Ti}^{\bar{\mathscr{A}}} = o\left(\sqrt{\frac{T}{\log T}}\right)$, $\frac{\lambda_T}{T}\sqrt{q_{Ti}^{\bar{\mathscr{A}}}} = o(1)$, $\lim_{T\to\infty}\frac{\lambda_T}{T}\sqrt{\frac{T}{\log T}} = \infty$, $\sqrt{\frac{q_{Ti}^{\bar{\mathscr{A}}}\log T}{T}} = o\left(\frac{\lambda_T}{T}\right)$ and $\frac{\lambda_T}{T^{1-c_1}}\sqrt{q_{Ti}^{\bar{\mathscr{A}}}} = O(1)$ for $i = 1,...,N-m_1$.

4. (a) For all $i = 1,...,N$, there exists a sequence of positive real numbers $\{s_{Ti}^{\mathscr{A}}\}$ such that $|\alpha_{ij}| > s_{Ti}^{\mathscr{A}}$ and $\lim_{T\to\infty}\frac{s_{Ti}^{\mathscr{A}}}{\frac{\lambda_T}{T}\sqrt{q_{Ti}^{\mathscr{A}}}} = \infty$ for all $\alpha_{ij}\in\mathscr{A}_i$.

   (b) For all $i = 1,...,N-m_1$, there exists a sequence of positive real numbers $\{s_{Ti}^{\bar{\mathscr{A}}}\}$ such that $|\alpha_{ij}| > s_{Ti}^{\bar{\mathscr{A}}}$ and $\lim_{T\to\infty}\frac{s_{Ti}^{\bar{\mathscr{A}}}}{\frac{\lambda_T}{T}\sqrt{q_{Ti}^{\bar{\mathscr{A}}}}} = \infty$ for all $\bar{\alpha}_{ij}\in\bar{\mathscr{A}_i}$.

5. (a) For each $i = 1,...,N$, $|\hat{\alpha}_{Tij}^{LASSO} - \alpha_{ij}| = O_p(T^{-\theta})$ with $\theta\in[\frac{1}{4},\frac{1}{2}]$ for any $j = 1,...,N$.

   (b) For each $i = 1,...,N$, $|\hat{\bar{\alpha}}_{Tij}^{LASSO} - \bar{\alpha}_{ij}| = O_p(T^{-\theta})$ with $\theta\in[\frac{1}{4},\frac{1}{2}]$ for any $j = 1,...,N$.

6. (a) The set of nonzero entries in $\rho^u$, $\mathscr{Q}_u$, has $q_T^{\mathscr{Q}_u}$ elements, and $q_T^{\mathscr{Q}_u}$ satisfies the following conditions:
$q_T^{\mathscr{Q}_u} = o\left(\sqrt{\frac{T}{\log T}}\right)$, $\frac{\gamma_T}{T}\sqrt{q_T^{\mathscr{Q}_u}} = o(1)$, $\lim_{T\to\infty}\frac{\gamma_T}{T}\sqrt{\frac{T}{\log T}} = \infty$ and $\sqrt{\frac{q_T^{\mathscr{Q}_u}\log T}{T}} = o\left(\frac{\gamma_T}{T}\right)$.

   (b) The set of nonzero entries in $\rho^\nu$, $\mathscr{Q}_\nu$, has $q_T^{\mathscr{Q}_\nu}$ elements, and $q_T^{\mathscr{Q}_\nu}$ satisfies the following conditions:
$q_T^{\mathscr{Q}_\nu} = o\left(\sqrt{\frac{T}{\log T}}\right)$, $\frac{\gamma_T}{T}\sqrt{q_T^{\mathscr{Q}_\nu}} = o(1)$, $\lim_{T\to\infty}\frac{\gamma_T}{T}\sqrt{\frac{T}{\log T}} = \infty$ and $\sqrt{\frac{q_T^{\mathscr{Q}_\nu}\log T}{T}} = o\left(\frac{\gamma_T}{T}\right)$.

7. (a) For all $\rho_{ij}^u\in\mathscr{Q}_u$, there exists a sequence of positive real numbers $\{s_T^{\mathscr{Q}_u}\}$ such that $|\rho_{ij}^u| > s_T^{\mathscr{Q}_u}$ and $\lim_{T\to\infty}\frac{s_T^{\mathscr{Q}_u}}{\frac{\gamma_T}{T}\sqrt{q_T^{\mathscr{Q}_u}}} = \infty$.

   (b) For all $\rho_{ij}^\nu\in\mathscr{Q}_\nu$, there exists a sequence of positive real numbers $\{s_T^{\mathscr{Q}_\nu}\}$ such that $|\rho_{ij}^\nu| > s_T^{\mathscr{Q}_\nu}$ and $\lim_{T\to\infty}\frac{s_T^{\mathscr{Q}_\nu}}{\frac{\gamma_T}{T}\sqrt{q_T^{\mathscr{Q}_\nu}}} = \infty$.

8. (a) Let $D_t^u$ be a $\frac{N(N-1)}{2}\times 1$ vector such that it has generic component $d_{tij}^u = \sqrt{\frac{\hat{s}_{Tii}^u}{\hat{s}_{Tjj}^u}}\hat{u}_{ti}$ and let $\Gamma_D = E[(D_t^u)'D_t^u]$, then there exists a constant $M_u < 1$ such that for any $\rho_{ij}^u\in\mathscr{Q}_u^C$,

$|\Gamma''_{Dij\mathscr{Q}_u}(\rho^u)[\Gamma''_{D\mathscr{Q}_u\mathscr{Q}_u}(\rho^u)]^{-1}sign(\rho^u_{\mathscr{Q}_u}))| < M_u$, where $\Gamma''_{Dijsq}(\rho^u) := \frac{\partial^2 \Gamma_D}{\partial d^u_{tij} \partial d^u_{tsq}}|_{d^u_{tij}=\rho^u_{ij}, d^u_{tsq}=\rho^u_{sq}}$.

(b) Let $D^v_t$ be a $\frac{N(N-1)}{2} \times 1$ vector such that it has generic component $d^v_{tij} = \sqrt{\frac{\hat{s}^v_{Tii}}{\hat{s}^v_{Tjj}}}\hat{v}_{ti}$ and let $\Gamma_D = E[(D^v_t)' D^v_t]$, then there exists a constant $M_v < 1$ such that for any $\rho^u_{ij} \in \mathscr{Q}^C_v$, $|\Gamma''_{Dij\mathscr{Q}_v}(\rho^v)[\Gamma''_{D\mathscr{Q}_v\mathscr{Q}_v}(\rho^v)]^{-1}sign(\rho^v_{\mathscr{Q}_v}))| < M_u$, where $\Gamma''_{Dijsq}(\rho^v) := \frac{\partial^2 \Gamma_D}{\partial d^v_{tij} \partial d^v_{tsq}}|_{d^v_{tij}=\rho^v_{ij}, d^v_{tsq}=\rho^v_{sq}}$.

9. (a) For any $\delta > 0$, there exists a constant $K$ such that for $T$ large enough, we have $P\Big(\max_{1 \le i \le N_T} |\hat{s}^u_{Tii} - s^u_{ii}| \le K$ $1 - O(T^{-\delta})$.

   (b) For any $\delta > 0$, there exists a constant $K$ such that for $T$ large enough, we have $P\Big(\max_{1 \le i \le N_T} |\hat{s}^v_{Tii} - s^v_{ii}| \le K$ $1 - O(T^{-\delta})$.

## A.2 Proof of the Proposition 1.4.6

*Proof.* Under assumption 5a, the weighted penalty $w_{Tij} = \frac{1}{|\hat{\alpha}^{LASSO}_{Tij}|}$ in (1.15) satisfies the condition 1 for the pre-estimator in Barigozzi and Brownlees (2014). Then under assumptions 1, 2, 3a, 4a and 5a, using the result in the Theorem 1 in Barigozzi and Brownlees (2014), we have

1. for $T$ large enough and for any $\delta > 0$, $\hat{\alpha}_{Ti} = 0$ for $\alpha_i \in \mathscr{A}^C_i$ with at least probability $1 - O(T^{-\delta})$, where $\hat{\alpha}_{Ti}$ is defined in (1.15), and

2. for $T$ large enough and for any $\delta > 0$, there exist a constant $\kappa_u$ such that $\|\hat{\alpha}_{Ti} - \alpha_i\|_2 \le \kappa_u \frac{\lambda_T}{T}\sqrt{q^{\mathscr{A}}_{Ti}}$ with at least probability $1 - O(T^{-\delta})$.

From assumption 3a, we know $\frac{\lambda_T}{T}\sqrt{q^{\mathscr{A}}_{Ti}} = o(1)$. Thus we have Prob$\{\hat{\alpha}_{Tij} = 0 \text{ if } \alpha_{ij} \in \mathscr{A}^C_i\} \to 1$ and $\hat{\alpha}_{Ti} \xrightarrow{p} \alpha_i$ for $i = 1, ..., N$. Note also that vec$(\alpha'_1, ..., \alpha'_N) =$ vec$([A^p_1, A^p_2, ..., A^p_p]')$, and by the Lemma 2 in Barigozzi and Brownlees (2014) the truncated bias $\left\|A^p_k - A_k\right\|_\infty = o(1)$. Therefore, $\hat{A}^p_{Tk} \xrightarrow{p} A_k$ for $k = 1, ..., p$ .

Similarly, for the expanded restricted process, under the assumptions 1, 2, 3b, 4b and 5b, we have Prob$\{\hat{\bar{\alpha}}_{Tij} = 0 \text{ if } \bar{\alpha}_{ij} \in \bar{\mathscr{A}}^C_i\} \to 1$, $\hat{\bar{\alpha}}_{Ti} \xrightarrow{p} \bar{\alpha}_i$ for $i = 1, ..., N$ and thus $\hat{\bar{A}}^p_{Tk} \xrightarrow{p} \bar{A}^\phi_k$ for $k = 1, ..., p$. ∎

## A.3 Proof of the Proposition 1.4.7

*Proof.* For the $\hat{\rho}_T^u$ considered in (1.21), under the assumptions 1, 2, 3a, 4a, 5a, 6a, 7a, 8a and 9a, using the result in Theorem 2 in Barigozzi and Brownlees (2014), we have

1. for $T$ large enough and for any $\delta > 0$, $\hat{\rho}_{Tij}^u = 0$ for $\rho_{ij}^u \in \mathscr{Q}_u^C$ with at least probability $1 - O(T^{-\delta})$, and

2. for $T$ large enough and for any $\delta > 0$, there exists a constant $\kappa_q$ such that $\|\hat{\rho}_T^u - \rho^u\|_2 \leq \kappa_q \frac{\gamma_T}{T}\sqrt{q_T^{\mathscr{Q}_u}}$, or equivalently, $\left\|\hat{S}_T^u - S^u\right\| \leq \kappa_q \frac{\gamma_T}{T}\sqrt{q_T^{\mathscr{Q}_u}}$with at least probability $1 - O(T^{-\delta})$

For assumption 6a, we know $\frac{\gamma_T}{T}\sqrt{q_T^{\mathscr{Q}_u}} = o(1)$. Then we have Prob$\{\hat{\rho}_{Tij}^u = 0 \text{ if } \rho_{ij}^u \in \mathscr{Q}_u^C\} \to 1$ and $\hat{\rho}_{Tij}^u \xrightarrow{p} \rho_{ij}^u$ for $i, j = 1, ..., N$. Therefore, we also have $\hat{S}_T^u \xrightarrow{p} S_u \equiv \Sigma_u^{-1}$.

Similarly, for the $\hat{\rho}_T^v$ considered in (1.22), under the assumptions 1, 2, 3b, 4b, 5b, 6b, 7b, 8b and 9b, we have Prob$\{\hat{\rho}_{Tij}^v = 0 \text{ if } \rho_{ij}^v \in \mathscr{Q}_v^C\} \to 1$ and $\hat{\rho}_{Tij}^v \xrightarrow{p} \rho_{ij}^v$ for $i, j = 1, ..., N$. Therefore, we also have $\hat{S}_T^v \xrightarrow{p} S_v \equiv \Sigma_v^{-1}$. □

## A.4 Proof of the Theorem 1.4.8

*Proof.* Under the assumptions 1, 2, 3a, 3b, 4a, 4b, 5a, 5b, 6a, 6b, 7a, 7b, 8a, 8b, 9a, and 9b, which have been used in Proposition 1.4.6 and 1.4.7, and by these propositions, we have the consistent estimators, $\hat{A}_{Tk}^p$, $\hat{\bar{A}}_{Tk}^p$, $\hat{S}_T^u$, $\hat{S}_T^v$ for $A_k$, $\bar{A}_k^\phi$, $S^u$, $S^v$ respectively.

Note that from the Remark 1.4.2 and from the Remark 1.4.4, we have

1. The covariance matrix of the forecast error at horizon $h$ for the unrestricted model is

$$\Sigma[X(t+h)|\mathscr{F}(t)] = \sum_{q=0}^{h-1} \varphi_q \Sigma_u \varphi_q', \tag{A.1}$$

   where $\varphi_q = \sum_{k=1}^q A_k \varphi_{q-k}$ and $\varphi_0 = I_N$.

2. The covariance matrix of the forecast error at horizon $h$ for the restricted model is

$$\Sigma[X_0(t+h)|\mathscr{F}_{-W}(t)] = \sum_{q=0}^{h-1} \bar{\varphi}_q \Sigma_\varepsilon \bar{\varphi}_q', \tag{A.2}$$

where $\bar{\varphi}_q = \sum_{k=1}^{q} \bar{A}_k \bar{\varphi}_{q-k}$ and $\bar{\varphi}_0 = I_{N-m_1}$

3. The forecast error covariance of $X^W$, without its past information, at horizon $h$ is

$$\Sigma_W[X^W(t+h)|\mathscr{F}_{-W}(t)] = J_3 \left( \sum_{q=0}^{h-1} \phi_q \Sigma_\nu \phi_q' \right) J_3', \tag{A.3}$$

where $\phi_q = \sum_{k=1}^{q} A_k^{\phi} \phi_{q-k}$, $A_k^{\phi} = \bar{A}_k^{\phi} J_2$, $\phi_0 = I_N$, $J_3 = [I_{m_1 \times m_1},\ 0_{m_1 \times (N-m_1)}]_{m_1 \times N}$.

4. $\Sigma_\varepsilon = J_2 \Sigma_\nu J_2'$ and $\bar{A}_k = (J_2 \bar{A}_k^{\phi})'$, where $J_2 = [0_{(N-m_1)\times m_1},\ I_{(N-m_1)\times(N-m_1)}]_{(N-m_1)\times N}$

As $\hat{A}_{Tk}^{p} \xrightarrow{p} A_k$ and $(\hat{S}_T^u)^{-1} \xrightarrow{p} (S^u)^{-1} = \Sigma_u$, $\hat{\varphi}_q$ is iteratively defined as $\hat{\varphi}_q = \sum_{k=1}^{q} \hat{A}_{Tk}^{p} \hat{\varphi}_{q-k}$ for $q = 1, ..., h-1$, then $\hat{\varphi}_q \xrightarrow{p} \varphi_q$ and thus

$$\hat{\Sigma}[X(t+h)|\mathscr{F}_{-W}(t)] \xrightarrow{p} \Sigma[X(t+h)|\mathscr{F}_{-W}(t)], \tag{A.4}$$

where $\hat{\Sigma}[X(t+h)|\mathscr{F}_{-W}(t)] := \sum_{q=0}^{h-1} \hat{\varphi}_q (\hat{S}_T^u)^{-1} \hat{\varphi}_q'$ and $\hat{\varphi}_0 = I_N$.

As $\hat{\bar{A}}_{Tk}^{p} \xrightarrow{p} \bar{A}_k^{\phi}$ and $(\hat{S}_T^\nu)^{-1} \xrightarrow{p} (S^\nu)^{-1} = \Sigma_\nu$, $\hat{\phi}_q$ is iteratively defined as $\hat{\phi}_q = \sum_{k=1}^{q} (\hat{\bar{A}}_{Tk}^{p} J_2) \hat{\phi}_{q-k}$ for $q = 1, ..., h-1$, then $\hat{\phi}_q \xrightarrow{p} \phi_q = \sum_{k=1}^{q} (\bar{A}_k^{\phi} J_2) \phi_{q-k}$, and thus

$$\hat{\Sigma}_W[X^W(t+h)|\mathscr{F}_{-W}(t)] \xrightarrow{p} \Sigma_W[X^W(t+h)|\mathscr{F}_{-W}(t)], \tag{A.5}$$

where $\hat{\Sigma}_W[X^W(t+h)|\mathscr{F}_{-W}(t)] := J_3 \left( \sum_{q=0}^{h-1} \hat{\phi}_q (\hat{S}_T^\nu)^{-1} \hat{\phi}_q' \right) J_3'$ and $\hat{\phi}_0 = I_N$.

As $\hat{\bar{A}}_{Tk}^{p} \xrightarrow{p} \bar{A}_k^{\phi}$, $(\hat{S}_T^\nu)^{-1} \xrightarrow{p} (S^\nu)^{-1} = \Sigma_\nu$, $\Sigma_\varepsilon = J_2 \Sigma_\nu J_2'$ and $\bar{A}_k = (J_2 \bar{A}_k^{\phi})'$, then $\hat{\Sigma}_\varepsilon \xrightarrow{p} \Sigma_\varepsilon$ and $(J_2 \hat{\bar{A}}_{Tk}^{p})' \xrightarrow{p} (J_2 \bar{A}_k^{\phi})'$, where $\hat{\Sigma}_\varepsilon = J_2 (\hat{S}_T^\nu)^{-1} J_2'$. Also $\hat{\bar{\varphi}}_q$ is iteratively defined as $\hat{\bar{\varphi}}_q = \sum_{k=1}^{q} (J_2 \hat{\bar{A}}_{Tk}^{p})' \hat{\bar{\varphi}}_{q-k}$, then $\hat{\bar{\varphi}}_q \xrightarrow{p} \bar{\varphi}_q = \sum_{k=1}^{q} \bar{A}_k \bar{\varphi}_{q-k}$, and thus

$$\hat{\Sigma}[X_0(t+h)|\mathscr{F}_{-W}(t)] \xrightarrow{p} \Sigma[X_0(t+h)|\mathscr{F}_{-W}(t)], \tag{A.6}$$

where $\hat{\Sigma}[X_0(t+h)|\mathscr{F}_{-W}(t)] := \sum_{q=0}^{h-1} \hat{\bar{\varphi}}_q \hat{\Sigma}_\varepsilon \hat{\bar{\varphi}}_q'$ and $\hat{\bar{\varphi}}_0 = I_{N-m_1}$.

Finally, we have

$$\hat{C}^h_{TWY} = \ln\left[\frac{\det\{J_0\hat{\Sigma}[X_0(t+h)|\mathscr{F}_{-W}(t)]J_0'\}}{\det\{J_1\hat{\Sigma}[X(t+h)|\mathscr{F}(t)]J_1'\}}\right]$$
$$\xrightarrow{p} \ln\left[\frac{\det\{J_0\Sigma[X_0(t+h)|\mathscr{F}_{-W}(t)]J_0'\}}{\det\{J_1\Sigma[X(t+h)|\mathscr{F}(t)]J_1'\}}\right]$$

and

$$\hat{C}^h_{TWW} = \ln\left[\frac{\det\{\hat{\Sigma}_W[X^W(t+h)|\mathscr{F}_{-W}(t)]\}}{\det\{J_1\hat{\Sigma}[X(t+h)|\mathscr{F}(t)]J_1'\}}\right]$$
$$\xrightarrow{p} \ln\left[\frac{\det\{\Sigma_W[X^W(t+h)|\mathscr{F}_{-W}(t)]\}}{\det\{J_1\Sigma[X(t+h)|\mathscr{F}(t)]J_1'\}}\right].$$

Therefore,

$$\hat{C}^h_{TWY} \xrightarrow{p} C_L(X^W \underset{h}{\longrightarrow} X^Y|I), \tag{A.7}$$

$$\hat{C}^h_{TWW} \xrightarrow{p} C_L(X^W \underset{h}{\longrightarrow} X^W|I), \tag{A.8}$$

□

# Appendix B

# S&P 100 components (selected)

| Ticker | Company | Sector | Ticker | Company | Sector |
|---|---|---|---|---|---|
| AAPL | Apple Inc. | Consumer Goods | HPQ | Hewlett-Packard Co | Technology |
| ABT | Abbott Laboratories | Healthcare | IBM | Intl Business Machines Corp | Technology |
| ACN | Accenture plc | Technology | INTC | Intel Corp | Technology |
| AGN | Allergan plc | Healthcare | JNJ | Johnson & Johnson | Healthcare |
| AIG | American Intl Group Inc | Financial | JPM | JP Morgan Chase & Co | Financial |
| ALL | Allstate Corp | Financial | KO | Coca-Cola Co | Consumer Goods |
| AMGN | Amgen Inc | Healthcare | LLY | Lilly Eli & Co | Healthcare |
| AMZN | Amazon.com Inc | Services | LMT | Lockheed Martin | Industrial Goods |
| APC | Anadarko Petroleum Corp | Basic Materials | LOW | Lowe's Cos Inc | Services |
| AXP | American Express Co | Financial | MCD | McDonald's Corp | Services |
| BA | Boeing Co | Industrial Goods | MDT | Medtronic plc | Healthcare |
| BAC | Bank of America Corp | Financial | MET | Metlife Inc | Financial |
| BAX | Baxter Intl Inc | Healthcare | MMM | 3M Co | Industrial Goods |
| BIIB | Biogen Inc | Healthcare | MO | Altria Group Inc | Consumer Goods |
| BK | The Bank of New York Mellon Corp | Financial | MON | Monsanto Co. | Basic Materials |
| BMY | Bristol-Myers Squibb | Healthcare | MRK | Merck & Co Inc | Healthcare |
| C | Citigroup Inc | Financial | MS | Morgan Stanley | Financial |
| CAT | Caterpillar Inc | Industrial Goods | MSFT | Microsoft Corp | Technology |
| CELG | Celgene Corp | Healthcare | NKE | NIKE Inc B | Consumer Goods |
| CL | Colgate-Palmolive Co | Consumer Goods | NSC | Norfolk Southern Corp | Services |
| CMCSA | Comcast Corp | Services | ORCL | Oracle Corp | Technology |
| COF | Capital One Financial | Financial | OXY | Occidental Petroleum | Basic Materials |
| COP | ConocoPhillips | Basic Materials | PEP | PepsiCo Inc | Consumer Goods |
| COST | Costco Wholesale Corp | Services | PFE | Pfizer Inc | Healthcare |
| CSCO | Cisco Systems Inc | Technology | PG | Procter & Gamble | Consumer Goods |
| CVS | CVS Health Corporation | Healthcare | QCOM | QUALCOMM Inc | Technology |
| CVX | Chevron Corp | Basic Materials | RTN | Raytheon Co | Industrial Goods |
| DD | E. I. du Pont de Nemours and Company | Basic Materials | SBUX | Starbucks Corp | Services |

| | | | | | |
|---|---|---|---|---|---|
| DIS | Walt Disney Co | Services | SLB | Schlumberger Ltd | Basic Materials |
| DOW | Dow Chemical | Basic Materials | SO | Southern Co | Utilities |
| DVN | Devon Energy Corp | Basic Materials | SPG | Simon Property Group | Financial |
| EBAY | eBay Inc. | Services | T | AT&T Inc | Technology |
| EMC | EMC Corp | Technology | TGT | Target Corp | Services |
| EMR | Emerson Electric Co | Industrial Goods | TWX | Time Warner Inc | Services |
| EXC | Exelon Corp | Utilities | TXN | Texas Instruments Inc | Technology |
| F | Ford Motor Co | Consumer Goods | UNH | Unitedhealth Group Inc | Healthcare |
| FDX | FedEx Corp | Services | UNP | Union Pacific Corp | Services |
| FOXA | Twenty-First Century Fox, Inc | Services | USB | US Bancorp | Financial |
| GD | General Dynamics | Industrial Goods | UTX | United Technologies Corp | Industrial Goods |
| GE | General Electric Co | Industrial Goods | V | Visa Inc | Services |
| GILD | Gilead Sciences Inc | Healthcare | VZ | Verizon Communications Inc | Technology |
| GS | Goldman Sachs Group Inc | Financial | WBA | Walgreens Boots Alliance Inc | Services |
| HAL | Halliburton Co | Basic Materials | WFC | Wells Fargo & Co | Financial |
| HD | Home Depot Inc | Services | WMT | Wal-Mart Stores | Services |
| HON | Honeywell Intl Inc | Industrial Goods | XOM | Exxon Mobil Corp | Basic Materials |

# Appendix C

# Variable Definitions

1. SPREAD: SPREAD is the average of daily Corwin and Schultz (2012)'s bid-ask spreads estimates within a year for the firms belong to the same industry specified by the first three digits of SIC codes.

2. RTV: RTV is the average of daily Amihud (2002)'s illiquidity estimates ($\text{RTV}_{i,t}$) within a year for the firms belong to the same industry specified by the first three digits of SIC codes.

$$\text{RTV}_{i,t} = \frac{|R_{i,t}|}{VOLV_{i,t}} \tag{C.1}$$

   where $\text{RTV}_{i,t}$ is firm $i$'s illiquidity estimate on day $t$. $R_{i,t}$ is firm $i$'s return on day $t$. $VOLV_{i,t}$ is firm $i$'s trading volume in dollars on day $t$.

3. ISIZE: ISIZE is the average of daily sum of market capitalizations within a year for the firms belong to the same industry specified by the first three digits of SIC codes:

$$\text{ISIZE}_{i,t} = \sum_{i_k \in i} MC_{i_k,t} \tag{C.2}$$

   where $MC_{i_k,t}$ is firm $i_k$'s market capitalization (stock's price times shares outstanding in millions of dollars) on day $t$, and firm $i_k$ belongs to industry $i$.

4. FSIZE: FSIZE is the average of daily average of market capitalizations within a year for the

firms belong to the same industry specified by the first three digits of SIC codes:

$$\text{FSIZE}_{i,t} = \frac{1}{n_i} \sum_{i_k \in i} MC_{i_k,t} \tag{C.3}$$

where $MC_{i_k,t}$ is firm $i_k$'s market capitalization (stock's price times shares outstanding in millions of dollars) on day $t$, and firm $i_k$ belongs to industry $i$. $n_i$ is the number of firms belong to industry $i$.

5. BM: Following Fama and French (1992), I compute a firm's book-to-market ration in month $t$ using the market value of its equity at the end of December of the previous year and the book value of common equity plus balance-sheet deferred taxes for the firm's latest fiscal year ending in prior calendar year.

$$\text{BM}_{i,t} = \frac{1}{n_i} \sum_{i_k \in i} \text{BM}_{i_k,t} \tag{C.4}$$

where $\text{BM}_{i,t}$ is industry $i$'s book-to-market in month $t$. $\text{BM}_{i_k,t}$ is firm $i_k$'s book-to-market in month $t$, for firm $i_k$ belongs to industry $i$. $n_i$ is the number of firms belong to industry $i$. Industry's book-to-market in year $y$ is the simple average of monthly industry's book-to-market in year $y$.

6. BETA: To take into account nonsynchronous trading, I follow Scholes and Williams (1977) and Dimson (1979) and use the lag and lead of the market portfolio as well as the current market when estimating beta:

$$R_{i,d} - r_{f,d} = \alpha_i + \beta_{1,i}(R_{m,d-1} - r_{f,d-1}) + \beta_{2,i}(R_{m,d} - r_{f,d}) + \beta_{3,i}(R_{m,d+1} - r_{f,d+1}) + \varepsilon_{i,d}, \tag{C.5}$$

where $R_{i,d}$ is the average return of the stocks belong to industry $i$ on day $d$, $r_{f,d}$ is the risk-free rate on day $d$ and $R_{m,d}$ is the market return on day $d$. I use simple OLS to estimate equation C.5 for each industry using daily returns within a year. The market beta of industry $i$ in year $y$ is defined as $\hat{\beta}_i = \hat{\beta}_{1,i} + \hat{\beta}_{2,i} + \hat{\beta}_{3,i}$.

7. IVOL: I use a simple CAPM model specification to estimate the yearly idiosyncratic volatil-

ity of a firm:

$$R_{i,d} - r_{f,d} = \alpha_i + \beta_i(R_{m,d} - r_{f,d}) + \varepsilon_{i,d}, \tag{C.6}$$

where $\varepsilon_{i,d}$ is the firm $i$' idiosyncratic return on day $d$. The idiosyncratic volatility of firm $i$ in year $y$ is defined as the standard deviation of daily OLS residuals in year $y$:

$$\text{IVOL}_{i,t} = \sqrt{\widehat{\text{var}(\hat{\varepsilon}_{i,d})}}. \tag{C.7}$$

The idiosyncratic volatility of an industry in year $y$ is the average of the idiosyncratic volatilities of the firms belong to that industry in year $y$.

8. MOM: The momentum variable of firm $i$ for every months in year $y+1$ is the simple average of firm $i$' daily returns in year $y$. The momentum of an industry is the simple average of the momentums of the firms belong to that industry.

# Appendix D

# Data and Code

All data and python codes related to this thesis are shared on the Google Drive with the following URL:

https://drive.google.com/drive/folders/0B9eEne9KGIv_YVNoSWkwM1hYNDg?usp=sharing

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