PRECISION MICROWAVE MEASUREMENTS

USING RESONANCE CURVES

by

Kimball J. Keeping

A THESIS

submitted in partial fulfillment of

the requirements for the degree of

Doctor of Philosophy.

McGill University

August, 1952

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF PLATES (or full page illustrations)

PREFACE

The construction of apparatus and experiments described in this thesis were carried out in the Communications Engineering Laboratory of McGill University, under the direction of Dr. R.A. Chipman.

The author wishes to thank Dr. Chipman for his interest and assistance in carrying out this work, and the National Research Council for a Fellowship and financial assistance during the summer months.

The author wishes to thank also Mr. Paul Boire who constructed a part of the chain of frequency multipliers. A few circuit diagrams drawn by Mr. Boire are used in this thesis for the sake of completeness.

Mr. Noel Montagnon constructed a regulated klystron power supply which was used in this project. The author wishes to thank Mr. Montagnon for this, and Mr. Paul Lorimer of the "Radiation Laboratory" for several very excellent pieces of glass blowing.

The author wishes to thank also Mr. V. Avarlaid of the "Eaton Electronics Laboratory" for the construction of the circular wave guide cavity, and Mr. F. Corrick of the Mechanical Engineering Department for machining large tubing and constructing the vacuum chamber for the high frequency cavity.

Kimball J. Keeping

PRECISION MICROWAVE MEASUREMENTS USING RESONANCE CURVES

Abstract

An apparatus has been constructed in which the output of a 5 mc/s crystal-controlled oscillator of short-time frequency stability 1 in 10^8 is frequency multiplied to frequencies of 270 mc/sec and 2970 mc/sec. The oscillator frequency is variable by ± 600 cps by varying the capacity across the crystal, the frequency deviation from 5 mc/sec being measured within 0.1 cps by comparison with a 5 mc/s frequency standard and standard tuning forks.

By obtaining resonance-curves of a high-Q resonant cavity at 270 mc/s measurements have been made of the dielectric constant and loss tangent of dry air and three dry non-polar gases. The loss tangent of each of the gases at N. T. P. is shown to be less than 5×10^{-6}.

Preliminary similar measurements on dry air have been made at 2970 mc/s.

The application of the apparatus to measurements on liquids and solidd is discussed in detail.

CHAPTER I

Theoretical Background for Microwave Dielectric Measurements using Resonance Curves - lines and cavities completely filled by the dielectric being measured.

The parameters used to describe the behaviour of materials at high frequency are the complex dielectric constant $\epsilon = \epsilon' - j\epsilon''$

or $\epsilon = \epsilon'(1 - j\tan\delta) = \epsilon_0 K_e (1 - j\tan\delta)$

and the complex permeability $\mu = \mu' - j\mu''$. The subject matter of this thesis is concerned with the complex dielectric constant ϵ, and μ is taken to be μ_0 the permeability of free space for all materials dealt with. K_e is called the "Specific Inductive Capacity" and $\tan\delta$ is the loss tangent. These quantities are obtained by many different experimental procedures, but this work is concerned only with those involving resonance curves such as the slotted line and resonant cavity methods.

The material constants are obtained from the real and imaginary parts of the propagation constant when the losses are in any way appreciable as pointed out in W.B. Westphal's report.[1] This is the procedure used when the unknown dielectric sample fills the short circuited end of a wave guide. In this case it is found that the two material constants K_e and $\tan\delta$ cannot be separated easily, since both affect the real and imaginary parts of the propagation constant. For gaseous dielectrics where $\tan\delta$ is a very small quantity the situation is somewhat simpler - in this case to a high degree of accuracy it is true that $\tan\delta$ has negligible effect on the phase constant, and the dielectric constant may be found very much more directly than is possible for materials having higher losses. Gaseous dielectrics of course completely fill the measuring line or cavity. The remainder of this chapter is devoted to the mathematical analyses required for completely filled lines and cavities. The case of partial cavity filling will be considered in the next chapter.

The mathematical background for all methods of measurement considered are the transmission line equations representing the distribution of fields along the guide,

$$E_x = E_1 e^{-Px} + E_2 e^{+Px} \qquad \text{for the transverse electric field} \qquad \text{I}$$

$$H_x = H_1 e^{-Px} + H_2 e^{+Px} \qquad \text{for the transverse magnetic field} \qquad \text{II}$$

where P is the complex propagation constant per wave length and x is the distance in wave lengths measured from the generator.

The reflection coefficient for the electric field is defined as follows:

$$\frac{\text{Reflected wave at termination}}{\text{Incident wave at termination}} = k_t$$

$$\therefore \; E_2 = E_1 k_t e^{-2Pl} \qquad \text{where } l \text{ is the line length.}$$

Equation I may be written

$$E_x = E_1 \left[e^{-Px} + k_t e^{-2Pl} e^{+Px} \right] \qquad \text{III}$$

when the infinitely long line is considered the positive exponents in equation I and II must be omitted.

In that case $E_x = E_1 e^{-Px}$ and $H_x = H_1 e^{-Px}$

$$\therefore \; \frac{E_x}{H_x} = \frac{E_1}{H_1} = Z_0 \qquad \text{(a constant)}$$

This constant is known as the "characteristic impedance".

Equation II can now be written since $\frac{\partial E}{\partial x} = -HZ$

$$H_x = \frac{1}{Z_o}\left[E_1 e^{-Px} - E_2 e^{+Px}\right]$$

when $x = l$ the expression for impedance becomes

$$Z_T = \frac{E_T}{H_T} = Z_o\left[\frac{E_1 e^{-Pl} + E_2 e^{+Pl}}{E_1 e^{-Pl} - E_2 e^{+Pl}}\right] = Z_o\left[\frac{1+k_t}{1-k_t}\right]$$

$$\therefore k_t = \frac{Z_T - Z_o}{Z_T + Z_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{IV}$$

This is another definition for the reflection coefficient for electric fields.

Equation III may be conveniently transformed as follows: put $k_t = e^{-2(U+jV)}$ $P = \alpha + j\beta$ and $y = l - x$

$$\therefore E_y = \frac{E_1}{e^{U+jV} e^{+Pl}}\left[e^{(U+\alpha y)} e^{j(V+\beta y)} + e^{-(U+\alpha y)} e^{-j(V+\beta y)}\right]$$

$$= E_3 \cosh\left[(U+\alpha y) + j(V+\beta y)\right] \quad \text{V} \quad \text{where} \quad E_3 = \frac{2E_1}{e^{U+jV} e^{+Pl}}$$

This gives the distribution of the transverse electric field along the line as a function of the distance y from the termination. The discussion which follows is limited to the case of low loss lines and terminal impedances which are very much less than the characteristic impedance.

That is $\frac{|Z_T|}{|Z_o|} \ll 1 \qquad \therefore |k_t| \doteq 1 \qquad \therefore U \doteq 0$

The termination is also taken to be nearly lossless. This is consistent with precision measurements where losses must be kept small if undistorted resonance curves are to be avoided.

Thus $(U + \alpha y)$ is a very small quantity and equation V becomes

$$E_y \doteq E_3\left[\cos(V+\beta y) + j(U+\alpha y)\sin(V+\beta y)\right]$$

$$\therefore \frac{|E_y|^2}{|E_3|^2} = \cos^2(V+\beta y) + (U+\alpha y)^2 \sin^2(V+\beta y) \qquad \text{VI}$$

Since U and αy are small this leads to very high values of standing wave ratio. The detectors commonly used will not operate over such a wide range. Thus it is customary to move the probe slightly from the position of minimum response until the response is twice the minimum value to find the standing wave ratio.

From equation VI we see that $|E_y|^2$ will pass through a series of maxima for $V+\beta y = n\pi$ and a series of minima for $V+\beta y = (2n+1)\frac{\pi}{2}$. The distance of the first minimum from the termination $= y_{E_{min}}$

$$\therefore y_{E_{min}} = \frac{\lambda}{4}\left(1 - \frac{2V}{\pi}\right) \qquad \text{VII}$$

The increment in y i.e. δy required to make $|E_y|^2$ twice its minimum value is determined from equation VI thus

$$2(U+\alpha y)^2 = \sin^2\beta\delta y + (U+\alpha y)^2\cos^2\beta\delta y$$
$$= (U+\alpha y)^2 + \left[1-(U+\alpha y)^2\right]\sin^2\beta\delta y$$

$$\therefore \sin^2\beta\delta y = \frac{(U+\alpha y)^2}{1-(U+\alpha y)^2}$$ thus when $U+\alpha y$ is very small

$$U+\alpha y = \beta\delta y \qquad \text{VIII}$$

Now β may be determined from the relation $\beta = \frac{2\pi}{\lambda}$. λ is twice the distance between successive minima. U and V are quantities which are easily found for an air filled short circuited coaxial line or guide. (See the next chapter). With this information α

may be determined from equation VIII since U will be known and δy is measured. This system of measurement is used by many workers in a somewhat different way i.e. α and β are known quantities but U and V are to be measured to obtain the terminal reflection coefficient.

This method has many objections for precise work on gas measurements, e.g. the slotted line itself must be placed in the vacuum system and must be accurately controlled in temperature since thermal changes in the probe displacement must be avoided. If $|Ke - 1|$ must be determined to one part in 10^6, length measurements must be made accurately to one part in 10^7, since the dielectric susceptibility $|K_e - 1| = \frac{2\Delta\lambda}{\lambda}$. The frequency of the generator must be known to the order of one part in 10^8 over periods long enough to make measurements. The accuracy in length measurement required in say 10 cms. wave length is 10^{-5} mm. this is almost prohibitively difficult, moreover thermal effects are very large and troublesome in this range. There is always some radiation from the slot in such a line and the moving probe is subject to numerous errors which are avoided when other methods are used.

This method of measurement is properly regarded as a resonance method as is any method based upon reflection when standing waves are set up. The standing wave of electric field near the minimum (see equation VI) is quite accurately parabolic in shape for circuits of low loss $|E_y|^2 = |E_3|^2\left[\sin^2(\beta\delta y) + (U+\alpha y)^2\cos^2(\beta\delta y)\right]$ near the minimum when the circuit has low loss and $\beta\delta y$ is small. This equation may be quite accurately approximated

$$|E_y|^2 = |E_3|^2\left[(U+\alpha y)^2 + (\beta\delta y)^2\right]$$

That is $|E_y|^2$ plotted against δy is a parabola having its minimum value for $\delta y = 0$.

The resonant cavity methods to be discussed in the following paragraphs use detecting loops or probes permanently fixed at some convenient locations on the cavity wall or end plates, and either frequency or cavity length is varied.

There the quantities found by measurement are (a) for variable frequency, the resonant frequency and Q factor, and (b) for variable length, the resonant length and Q factor. For systems with low losses as previously explained the effects of the dielectric constant and loss tangent are easily separable. The resonant frequency or length defines the dielectric constant and the Q factors with and without the test dielectric define the loss tangent. The presence of the lossy dielectric broadens the resonance curve.

The mathematical background is again the transmission line equations:-

$$H_x = H_1 e^{-Px} + H_2 e^{+Px} \quad \ldots\ldots \quad \text{IX}$$

$$E_x = Z_0\left[H_1 e^{-Px} - H_2 e^{+Px}\right] \quad \ldots\ldots \quad \text{X}$$

for the transverse fields, where x is measured from the generator.

The reflection coefficient for the magnetic field follows from the definition

$$\frac{\text{reflected wave at termination}}{\text{incident wave at termination}} = k_t$$

$$\therefore\ H_2 = H_1 k_t e^{-2Pl}$$ where l is the line length.

Now $$Z_T = \frac{E_T}{H_T} = Z_0 \frac{H_1\left[e^{-Pl} - k_t e^{-Pl}\right]}{H_1\left[e^{-Pl} + k_t e^{-Pl}\right]} = Z_0 \frac{1 - k_t}{1 + k_t}$$

$$\therefore k_t = \frac{Z_0 - Z_T}{Z_0 + Z_T} = |k_t| e^{j\phi_T}$$

Now if Z_G is the impedance of the termination at $x = 0$

then $$k_G = \frac{Z_0 - Z_G}{Z_0 + Z_G} = |k_G| e^{j\phi_G}$$

Since reflections from the two terminations must be considered in this case the most direct way to obtain the useful equation which follows is to consider the multiple reflections of a transmitted wave and add up all the components[2] e.g. let the first wave of electric field proceeding from the input have a magnitude E_G it sees Z_0 and Z_G in series

$$\therefore H_1 = \frac{E_G}{Z_0 + Z_G}$$

Now adding up all the waves produced by reflection

$$H_x = \frac{E_G}{Z_0 + Z_G} \left\{ \frac{e^{-Px} + k_t e^{-2Pl} e^{+Px}}{1 - k_G k_t e^{-2Pl}} \right\}$$

The magnetic field at the termination i.e. for $x = l$ is then

$$H_l = \frac{E_G}{Z_0 + Z_G} (1 + k_t) \left\{ \frac{e^{-Pl}}{1 - k_t k_G e^{-2Pl}} \right\} \qquad \text{XI}$$

This equation may be transformed as follows $\quad \dfrac{1}{Z_G + Z_0} = \dfrac{1 + k_G}{2 Z_0}$

$$k_G k_t = e^{-2(p + jq)} \qquad P = \alpha + j\beta$$

$$\therefore H_l = \frac{E_G}{4 Z_0} \cdot \frac{(1 + k_G)(1 + k_t)}{\sqrt{k_G k_t}} \frac{1}{\sinh[(\alpha l + p) + j(\beta l + q)]} \qquad \text{XII}$$

where $$p = \ln \frac{1}{\sqrt{|k_G||k_t|}} \qquad \text{and} \qquad q = -\frac{\phi_t + \phi_G}{2}$$

Since square law detectors are used equation XII is written in the more useful form

$$|H_\ell|^2 = \left|\frac{E_G}{4Z_0}\ \frac{(1+k_G)(1+k_t)}{\sqrt{k_G k_t}}\right|^2 \left\{\frac{1}{\sinh^2(\alpha\ell+p)+\sin^2(\beta\ell+q)}\right\} \cdots \quad \text{XIII}$$

This equation is quite general applying to coaxial line and wave guide cavities - it may be simplified as follows for measurements on low loss systems

$$|H_\ell|^2 = \text{constant}\cdot\left[\frac{1}{\sinh^2(\alpha\ell+p)+\sin^2(\beta\ell+q)}\right] \cdots\cdots \quad \text{XIV}$$

This equation is fundamental in the treatment of cavities. The simplification of equation XIII is justified for high Q cavities since the change in frequency required to reduce $|H_\ell|^2$ to one half of its maximum value is so small compared to the resonant frequency. The reflection coefficients as well as Z_0 are frequency dependent but over the width of a resonance curve they can be taken as constant with negligible error.

We discuss first the case where the cavity length is fixed and the frequency is varied to delineate the resonance curves. From equation XIV it is evident that as $\beta\ell$ varies due to changes in β introduced via frequency variation, $|H_\ell|^2$ passes through a series of maxima for values of $\beta\ell + q = n\pi$ XV

i.e.
$$\frac{|H_\ell|^2}{|H_\ell|^2_{max}} = \frac{\sinh^2(\alpha\ell+p)}{\sinh^2(\alpha\ell+p)+\sin^2[(\beta+\Delta\beta)\ell+q]} \cdots \quad \text{XV(a)}$$
$$= \frac{\sinh^2(\alpha\ell+p)}{\sinh^2(\alpha\ell+p)+\sin^2\Delta\beta\ell}$$

since $\sin(\beta\ell+q)=0$ for resonance.

Now let the increment in β i.e. $\Delta\beta$ be such that

$\dfrac{|H_\ell|^2}{|H_\ell|^2_{max}} = \dfrac{1}{2}$ i.e. $\Delta\beta$ is the change in β required for the half power points then

$$\sinh^2(\alpha l + p) = \sin^2 \Delta\beta l$$

For low loss systems $\Delta\beta l$ will be very small since α and p are then very small quantities. Therefore the above expression may be approximated with negligible error

$$\alpha l + p = \Delta\beta l$$

$$\text{or} \quad \Delta\beta = \alpha + \frac{p}{l} \quad \text{- - - - - - - -} \quad \text{XVI}$$

The resonant value of β is found from equation XV when the resonant frequency for the evacuated cavity has been experimentally determined.

q is a very small quantity approaching zero for short circuiting metal plates. This measurement really serves to establish the electrical length of the cavity or it measures the velocity of propagation in free space if the cavity length is accurately known. Equation XV may be written for the coaxial line cavity in the form

$$\beta_o l = \frac{2\pi l}{\lambda_o} = \frac{2\pi f_o}{c} l = 2\pi\sqrt{\mu_o \epsilon_o}\, f_o l = n\pi$$

$$\therefore \sqrt{\epsilon_o} = \frac{n\pi}{2\pi\sqrt{\mu_o}\, f_o l} = \frac{n}{2\sqrt{\mu_o}\, f_o l}$$

it will be observed that l is proportional to n. Next when a gas whose dielectric constant $\epsilon = K_e \epsilon_o$ differs only slightly from ϵ_o is measured, we have

$$\sqrt{K_e}\sqrt{\epsilon_o} = \frac{n}{2\sqrt{\mu_o}\, f_1 l}$$

$$\therefore \sqrt{K_e} = \frac{f_1}{f_o} \qquad \text{let } f_1 = f_o - \Delta f$$

$$\therefore \sqrt{K_e} = 1 - \frac{\Delta f}{f_o} \qquad \therefore K_e = 1 - \frac{2\Delta f}{f_o}$$

$$\therefore |K_e - 1| = \frac{2\Delta f}{f_o} \quad 3 \quad \text{. . . . .} \quad \text{XVII}$$

Thus the change in the resonant frequencies between the evacutated and filled conditions gives a measure of the dielectric constant. This measurement can be carried out with a very high degree of accuracy - the frequency standard used holds its frequency to one part in 10^8, and Δf as determined with the 100 cycle General Radio tuning fork (which holds its frequency accurate to one part in 10^6) can be determined with an accuracy dependent upon the Q factor of the cavity. Δf may be determined within $\pm 500\ \nu/sec.$ for Q factors of 5000 at frequencies of the order of 270 megacycles per second, i.e. the value of $|K_e - 1|$ can be determined within four parts in 10^6. To maintain this accuracy experimental conditions such as humidity of the gas and cavity temperature must be very carefully controlled.

Equation XV is true also for wave guide cavities.

In this case $\dfrac{1}{\lambda_g} = \dfrac{n}{2l}$

From the cut off equation $\dfrac{1}{\lambda_g} = \sqrt{K_e \epsilon_o \mu_o f^2 - \left(\dfrac{1}{\lambda_o}\right)^2}$

$$\therefore K_e \epsilon_o \mu_o f^2 = \left(\frac{n}{2l}\right)^2 + \left(\frac{1}{\lambda_o}\right)^2$$

Here again $K_e \propto \dfrac{1}{f^2}$ and as in the case for coaxial cavities

$$|K_e - 1| = \frac{2\Delta f}{f_o} \quad ③ \qquad \text{XVIII}$$

Equation XVI must next be discussed for this is the relation which is used to determine the loss tangent of the material filling the cavity.

The Q factor for a cavity is defined by the relation $Q = \dfrac{f_o}{2\Delta f}$ ④ ; where f_o is the resonant frequency and Δf is the increment (or decrement) in frequency necessary to bring the detected power to one half of its value at resonance.

For coaxial line cavities

$$Q = \frac{f_0}{2\Delta f} = \frac{\beta}{2\Delta\beta} = \frac{\beta}{2\left(\alpha + \frac{p}{\ell}\right)} \quad \text{using equation XVI} \qquad \text{XIX}$$

For wave guide cavities a relationship between $\frac{f_0}{2\Delta f}$ and β and $\Delta\beta$ is required, since in this case β does not depend on frequency in the same linear manner which holds for coaxial cavities.

$$\beta^2 = \omega^2\epsilon\mu - k^2 = 4\pi^2 f^2\epsilon\mu - k^2$$

$$\therefore 2\beta\frac{d\beta}{df} = 8\pi^2\epsilon\mu f \qquad \therefore \Delta f = \frac{\beta\Delta\beta}{4\pi^2\epsilon\mu f}$$

but $$f^2 = \frac{\beta^2 + k^2}{4\pi^2\epsilon\mu} \qquad \therefore \Delta f = \frac{\beta\Delta\beta}{\sqrt{\beta^2+k^2}\sqrt{4\pi^2\epsilon\mu}}$$

$$\therefore \frac{f}{2\Delta f} = \frac{\sqrt{\beta^2+k^2}}{2\sqrt{4\pi^2\epsilon\mu}} \cdot \frac{\sqrt{\beta^2+k^2}\sqrt{4\pi^2\epsilon\mu}}{\beta\Delta\beta} = \frac{\beta^2+k^2}{2\beta\Delta\beta}$$

The Q factor for wave guide cavities is then

23

$$Q = \frac{\beta^2+k^2}{2\beta\Delta\beta} = \frac{\beta^2+k^2}{2\beta\left(\alpha + \frac{p}{\ell}\right)} \quad \text{again using equation XVI} \qquad \text{XX}$$

The reason why this complication did not arise in connection with equation XVIII is simply that the cavity is exactly resonant at the two frequencies considered in that connection. In equation XX however the cavity is exactly resonant at one of the frequencies considered but not at the other.

Equation XIX must now be converted into a more useful form. Using formulae developed for the coaxial line

$$\beta = \frac{2\pi f\sqrt{K_e}}{c} \quad \text{radians per unit length}$$

$$\alpha_c = \frac{\sqrt{\pi\epsilon_0}}{2}\sqrt{K_e}\sqrt{\frac{f}{\sigma}}\,\frac{\left(\frac{1}{a}+\frac{1}{b}\right)}{\ln\frac{b}{a}} \quad \text{nepers per unit length due to conductor loss}$$

$$\alpha_D = \frac{\pi f}{c}\sqrt{K_e}\ \tan\delta$$ nepers per unit length due to dielectric loss. (6)

The resistance of the end plate is $\dfrac{\ln\frac{b}{a}}{2\pi\delta\sigma}$ ohms

where σ = the conductivity of the metal plate in mhos per meter cube

a and b are the radii of the outer and inner conductors respectively

and δ is the skin depth in meters. Now $p = \ln\dfrac{1}{\sqrt{|k_G|\,|k_t|}}$

But $k_G = k_t$ when both ends of the cavity are closed by short circuiting metal plates

$$\therefore p = \ln\frac{1}{|k_t|} \quad \text{but} \quad k_t = \frac{Z_o - Z_T}{Z_o + Z_T}$$

$$\text{and} \quad Z_o = \frac{1}{2\pi}\sqrt{\frac{\mu_o}{K_e\epsilon_o}}\ln\frac{b}{a} \ \ \text{ohms}$$

$$\therefore k_t = \frac{1 - \frac{Z_T}{Z_o}}{1 + \frac{Z_T}{Z_o}} = \frac{1 - \frac{\sqrt{K_e}}{\sigma\delta\sqrt{\mu_o/\epsilon_o}}}{1 + \frac{\sqrt{K_e}}{\sigma\delta\sqrt{\frac{\mu_o}{\epsilon_o}}}}$$ on substituting the values for Z_T and Z_o

$$\therefore p = \ln\frac{1 + \frac{\sqrt{K_e}}{\sigma\delta\sqrt{\mu_o/\epsilon_o}}}{1 - \frac{\sqrt{K_e}}{\sigma\delta\sqrt{\frac{\mu_o}{\epsilon_o}}}}$$

Now $\ln\dfrac{1+x}{1-x} = 2\left(x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \ \cdots\cdots\right)$ when x is positive and less than unity

$$\therefore p = \frac{2\sqrt{K_e}}{\sigma\delta\sqrt{\frac{\mu_o}{\epsilon_o}}}$$ since $\sigma\delta\sqrt{\dfrac{\mu_o}{\epsilon_o}}$ is of the order of $\dfrac{10^9}{\sqrt{f}}$. It is therefore a very large quantity at all frequencies below 10^{10} ~/sec. Thus the above approximation is justified.

Using this expression for p and the phase and attenuation constants for the coaxial line equation XIX becomes

$$Q = \frac{1}{\frac{c}{\pi f}\left[\frac{\sqrt{\pi\epsilon_o}}{2}\sqrt{\frac{f}{\sigma}}\,\frac{\left(\frac{1}{a}+\frac{1}{b}\right)}{\ln\frac{b}{a}} + \frac{2}{l\sqrt{\frac{\mu_o}{\epsilon_o}}\,\delta\sigma}\right] + \tan\delta} \qquad \text{XXI}$$

This equation reveals $\tan\delta$ clearly

thus $$\tan\delta = \left[\frac{1}{Q_{filled}} - \frac{1}{Q_{evacuated}}\right] \quad (3) \qquad \text{XXII}$$

Equations XVII and XXII give the information required to determine dielectric constant and loss tangent from completely filled coaxial cavity data.

Equation XX likewise must be transformed to produce a useful result. Transverse magnetic modes in circular wave guide will be considered. Transverse electric modes may be treated in a somewhat similar manner. However the expressions for attenuation are much more complicated - the final result corresponding to equation XXII is the same for both TE and TM modes.

$$\beta = \frac{2\pi}{\lambda}\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}$$ radians per unit length

$\lambda =$ the T.E.M. wavelength of the frequency of propagation in the dielectric filling the cavity

$\lambda_0 =$ The cut off wavelength for the particular mode considered

$$\alpha_c = \frac{R_s}{\eta b\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}}$$ nepers per unit length due to conductor loss.

η is the intrinsic impedance of the dielectric filling the cavity

R_s is the skin effect resistance of the metal forming the guide walls

is the radius of the guide

b is the radius of the guide

$$\alpha_D = \frac{\pi \tan\delta}{\lambda\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}}$$ nepers per unit lengths due to dielectric loss

$\tan\delta$ – the loss tangent of the dielectric filling the cavity ⑦

For the evaluation of the factor p which appears in equation XX it is necessary to find the intrinsic impedance of the metal plates termination the cavity. This impedance is $R_s(1+j)$. ⑧

$$Z_0 = \eta\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}$$ for T.M. modes in wave guide

$$\therefore k_t = k_G = \frac{Z_0 - Z_T}{Z_0 + Z_T} = \frac{\eta\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2} - R_s(1+j)}{\eta\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2} + R_s(1+j)}$$

$$\doteq \frac{1 - \frac{R_s}{\eta\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}}}{1 + \frac{R_s}{\eta\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}}}$$

$$\therefore p = \ln_\epsilon \frac{1}{|k_t|} = \frac{2R_s}{\eta\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}}$$

using the same arguments as for the coaxial cavity in this respect.

Substitution in equation XX now yields

$$Q = \frac{\left(\frac{2\pi}{\lambda}\right)^2\left[\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}\right]^2 + \left(\frac{2\pi}{\lambda_0}\right)^2}{\frac{4\pi}{\lambda}\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}\left[\frac{R_s}{\eta b\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}} + \frac{\pi\tan\delta}{\lambda\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}} + \frac{2R_s}{l\eta\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}}\right]}$$

$$\therefore Q = \frac{1}{\frac{\lambda R_s}{\pi\eta}\left(\frac{1}{b} + \frac{2}{l}\right) + \tan\delta} \qquad \text{XXIII}$$

$$\therefore \tan\delta = \left[\frac{1}{Q_{filled}} - \frac{1}{Q_{evacuated}}\right] \quad ③ \qquad \text{XXIV}$$

Equations XVIII and XXIV are the ones required to obtain the material constants from circular wave guide cavity data when the cavity is completely filled.

When temperature and humidity can be carefully controlled, and when pressure can be accurately determined the limitation on the accuracy of determination of K_e depends upon the accuracy with which Δf may be determined. Experimentally this is found to depend upon the sharpness of the resonance curve and the overall amplitude stability of the apparatus. The accuracy of the determination of frequency is of a very high order (about one part in 10^8) but the resonant point on the Q curve may be in doubt to the extent of say 20 to 30 cycles per second in 10^7 cycles per second using Q factors as low as 4000. This would amount to an error of at most 60 cycles per second in Δf measured at 10 megacycles per second. This would give an accuracy of six parts in a million and permit the determination of $|K_e - 1|$ to five significant figures.

The difficulty arising here is that with the narrow band width of the apparatus both of the resonance curves cannot be depicted adequately within the available frequency range, which is about 2400 cycles per second in 10 megacycles per second. Higher cavity Q factors will give much more accurate results since the indeterminacy of the resonant frequency will be smaller.

The factors affecting the accuracy of the determination of $\tan\delta$ are much the same as those mentioned above, since the determination of Q depends upon the location of two frequencies accurately on the resonance curve. The fact that $\tan\delta$ is the difference between the reciprocals of two such quantities doubles the probable error, so that with Q factors as before $\tan\delta$ may be determined to an accuracy of about 12 parts in a million. Here again higher Q factors will greatly increase accuracy and conserve band width.

Referring to equation XV(a) the form of the resonance curve becomes apparent - when losses are small for small deviations from resonance

$$\frac{|H_l|^2}{|H_l|^2_{max}} = \frac{(\alpha l+p)^2}{(\alpha l+p)^2+(\Delta\beta l)^2} = \frac{1}{1+\frac{1}{(\alpha+\frac{p}{l})^2}(\Delta\beta)^2}$$

Writing this expression in terms of Δf for the coaxial cavity

$$\frac{|H_l|^2}{|H_l|^2_{max}} = \frac{1}{1+\left(\frac{2Q}{f}\right)^2(\Delta f)^2}$$

and for the wave guide cavity

$$\frac{|H_l|^2}{|H_l|^2_{max}} = \frac{1}{1+(2Q)^2\left(\frac{\beta}{f}\right)^2(\Delta f)^2}$$

For high Q cavities where the change in f or β is negligible compared to the quantity itself, it is clear that for cavities whether coaxial or circular wave guide $\frac{|H_l|^2}{|H_l|^2_{max}}$ is an inverted parabola having its maximum value for Δf equal to zero. This form is retained by the response over a range of frequency much greater than is needed to depict the resonance curve. For coaxial cavity having a Q factor as low as 3000, having a resonant frequency of about 300 megacycles per second, the deviation of the curve from the above mentioned form is about 0.1% for a frequency deviation of one megacycle per second from the resonant frequency.

The discussion of the variable length system depends again upon equation XIV. Again considering low loss systems the change in l required to depict the complete resonance curve is very small and $|H_l|^2$ will pass through a series of maxima for $\beta l_0 + g = n\pi$ XXV

Let l_0 be a particular resonant length, then

$$\frac{|H_l|^2}{|H_l|^2_{max}} = \frac{\sinh^2(\alpha l_0 + p)}{\sinh^2[\alpha(l_0+\Delta l) + p] + \sin^2[\beta(l_0+\Delta l) + g]}$$

$$= \frac{\sinh^2(\alpha l_0 + p)}{\sinh^2[\alpha(l_0+\Delta l) + p] + \sin^2(\beta \Delta l)} \qquad \text{XXVI}$$

Since low loss systems are being considered the Q curve is delineated well beyond the half power points for very small values of Δl. The above equation then may be approximated

$$\frac{|H_l|^2}{|H_l|^2_{max}} = \frac{(\alpha l_0 + p)^2}{[\alpha(l_0+\Delta l) + p]^2 + (\beta \Delta l)^2}$$

and Δl is so small compared to l_0, that $l_0 + \Delta l = l_0$ for all practical purposes

$$\therefore \frac{|H_l|^2}{|H_l|^2_{max}} = \frac{(\alpha l_0 + p)^2}{(\alpha l_0 + p)^2 + (\beta \Delta l)^2} \qquad \text{XXVII}$$

The value of Δl required to bring the response to the half power point is obtained as follows

$$\frac{1}{2} = \frac{|H_l|^2}{|H_l|^2_{max}} = \frac{(\alpha l_0 + p)^2}{(\alpha l_0 + p)^2 + (\beta \Delta l)^2}$$

$$\therefore \beta \Delta l = \alpha l_0 + p \qquad \text{XXVIII}$$

Equation XXV reveals the dielectric constant of the material filling the cavity. Let l_1 be the resonant length of a coaxial cavity when filled with gas. Then

$$\beta l_1 = \frac{2\pi}{\lambda} l_1 = 2\pi \sqrt{\mu_o \epsilon}\, f l_1 = 2\pi \sqrt{K_e}\sqrt{\mu_o \epsilon_o}\, f l_1 = n\pi$$

$$\therefore \sqrt{K_e} \propto \frac{1}{l_1}$$

Now let l_2 be the resonant length of the evacuated cavity and let

$l_2 = l_1 + \Delta l$

$$\therefore \sqrt{K_e} = \frac{l_1 + \Delta l}{l_1} = 1 + \frac{\Delta l}{l_1}$$

$$\therefore |K_e - 1| = \frac{2\Delta l}{l_1} \quad . \; . \; . \; . \; . \; . \; . \; . \qquad \text{XXIX}$$

where Δl is the change in resonant length produced by the introduction of the dielectric.

Equation XXV must now be discussed for the case of the wave guide cavity. The resonant condition is $\beta l_o = n\pi$. If and $\beta_1 l_1$ and $\beta_2 l_2$ are the corresponding quantities for the filled and evacuated conditions

$$\frac{\beta_2}{\beta_1} = \frac{l_1}{l_2} \qquad \text{put} \quad l_2 = l_1 + \Delta l$$

then $$\frac{\beta_1}{\beta_2} = \frac{l_1 + \Delta l}{l_1} = 1 + \frac{\Delta l}{l_1} = \sqrt{\frac{\omega^2 \epsilon_o \mu_o K_e - k^2}{\omega^2 \epsilon_o \mu_o - k^2}}$$

$$\therefore \frac{K_e - \left(\frac{\lambda}{\lambda_o}\right)^2}{1 - \left(\frac{\lambda}{\lambda_o}\right)^2} = 1 + \frac{2\Delta l}{l_1}$$

$$\therefore \frac{K_e - 1}{1 - \left(\frac{\lambda}{\lambda_o}\right)^2} = \frac{2\Delta l}{l_1} \quad . \; . \; . \; . \; . \; . \; . \; . \qquad \text{XXX}$$

where λ is the free space wave length corresponding to the frequency of excitation.

The Q factor of a cavity is defined to be $\dfrac{f_o}{2\Delta f}$

In a coaxial cavity $\beta = \dfrac{2\pi}{\lambda} = \dfrac{n\pi}{l} = \dfrac{2\pi f}{c}$

$\therefore f l =$ a constant $\qquad \therefore \dfrac{l_o}{2\Delta l} = \dfrac{f_o}{2\Delta f}$

$\therefore$ The Q factor for a coaxial cavity

$$= \frac{l_0}{2\Delta l} = \frac{\beta l_0}{2(\alpha l_0 + \beta)} = \frac{\beta}{2\left(\alpha + \frac{\beta}{l_0}\right)}$$ using equation XXVIII.

Thus the behaviour of a coaxial cavity is identical for frequency and length variation and

$$\tan \delta = \left[\frac{1}{Q_{filled}} - \frac{1}{Q_{evacuated}}\right] \qquad \text{XXXI}$$

For the wave guide cavity the Q factor is not $\frac{l_0}{2\Delta l}$ but rather

$$Q = \frac{1}{\left[1 - \left(\frac{\lambda}{\lambda_0}\right)^2\right]} \frac{l_0}{2\Delta l}$$ as shown below.

For wave guides

$$\omega^2 \epsilon \mu = \beta^2 + k^2$$

$$\therefore (2\pi)^2 \epsilon \mu f^2 = \left(\frac{n\pi}{l}\right)^2 + \left(\frac{2\pi}{\lambda_0}\right)^2$$

$$\therefore \epsilon \mu f^2 = \left(\frac{n}{2l}\right)^2 + \left(\frac{1}{\lambda_0}\right)^2$$

$$\therefore 2\epsilon\mu f \frac{df}{dl} = 2\left(\frac{n}{2l}\right)\left(-\frac{n}{2l^2}\right)$$

$$\therefore \frac{f}{df} = \frac{\left(\frac{n}{2l}\right)^2 + \left(\frac{1}{\lambda_0}\right)^2}{-\frac{2}{l}\left(\frac{n}{2l}\right)^2 \frac{dl}{2}}$$

$$= \frac{\epsilon\mu f^2}{-\left[\epsilon\mu f^2 - \left(\frac{1}{\lambda_0}\right)^2\right]} \frac{l}{dl}$$

$$= \frac{\left(\frac{1}{\lambda}\right)^2}{-\left[\left(\frac{1}{\lambda}\right)^2 - \left(\frac{1}{\lambda_0}\right)^2\right]} \frac{l}{dl}$$

$$\therefore \left|\frac{f}{df}\right| = \frac{1}{\left[1 - \left(\frac{\lambda}{\lambda_0}\right)^2\right]} \frac{l}{dl} \qquad (5)$$

$$\therefore Q = \frac{f}{2\Delta f} = \frac{1}{\left[1 - \left(\frac{\lambda}{\lambda_0}\right)^2\right]} \frac{l}{2\Delta l} \quad \ldots\ldots\ldots\ldots \text{XXXII}$$

With this method therefore the same relation holds as for variable frequency with completely filled wave guide cavities i.e.

$$\tan\delta = \left[\frac{1}{Q_{filled}} - \frac{1}{Q_{evacuated}}\right] \qquad \text{XXXIII}$$

Equations XXX and XXXII are on the whole not as useful as the corresponding formulae (i.e. XVIII and XX). They depend explicitly on λ_o the cut off wavelength - thus the cavity diameter must be known to a very high degree of accuracy.

Equation XXVII clearly shows that the form of the resonance curve for length variation will be an inverted parabola as in the case of frequency variation.

The requirements on accuracy of length measurement demanded by equation XXX are very stringent, e.g. at a wave length of 10 cms length must be measured accurately to 10^{-4} mm. if $|K_e - 1|$ should be known to one part in 10^6. Frequency stability is required to about one part in 10^8. This method would present great mechanical difficulty since it is necessary to measure small displacements to such a high degree of accuracy.

Frequency variation appears to be by far the most desirable of these methods when very high accuracy is desired. When lower accuracy may be tolerated variable length systems could conceivably prove quite useful but variable frequency systems seem to be just as useful. There is of course a great limitation to the frequency variation method. That is crystal controlled oscillators cannot be shifted to any considerable extent from the natural frequency of the crystal before they become unstable. Thus perhaps several crystals may be necessary to cover the required frequency range.

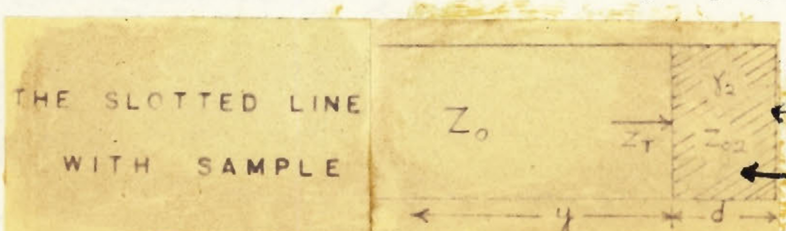
CHAPTER II

Theoretical Background for Microwaive Dielectric Measurement using Resonance Curves - lines and cavities containing dielectric samples not completely filling the line or cavity.

The same three methods that were considered in the last chapter will again be considered here to illustrate the principles involved and to indicate how the dielectric constant and loss tangent of the material under test may be obtained from the quantities actually measured. All three methods measure in effect the reflection coefficient of the portion of the line or cavity containing the dielectric. From this reflection coefficient the complex propagation constant of the dielectric is found. The material constants are then derived from the real and imaginary parts of the propagation constant. This will be treated in a later chapter. Much of the mathematical analysis of the last chapter is relevant here and only where necessary will further derivations be developed.

The slotted line technique.

This method has often been successfully employed. [7]



←short circuiting plate

←test dielectric

The terminal reflection coefficient at $y = 0$ was defined as

$$k_t = \frac{Z_T - Z_o}{Z_T + Z_o} = e^{-2(u+jv)}$$

$$\therefore \frac{Z_T}{Z_0} = \frac{1}{\tanh(U+jV)} \qquad \text{I}$$

U and V are quantities derived from measurement and Z_0 will be known for the air filled portion of the line.

$$Z_T = Z_{02} \tanh \gamma_2 d$$

when end plate losses may be neglected, which is assumed as a first approximation.

$$Z_{02} = \frac{j\omega\mu_0}{\gamma_2}$$ for T.E. and T.E.M. modes and non ferro-magnetic substances

$$\therefore Z_T = \frac{Z_0}{\tanh(U+jV)} = \frac{j\omega\mu_0}{\gamma_2} \tanh \gamma_2 d$$

$$\therefore \frac{\tanh \gamma_2 d}{\gamma_2 d} = \frac{\frac{Z_0}{j\omega\mu_0 d}}{\tanh(U+jV)} = \frac{1}{\gamma_1 d} \cdot \frac{1}{\tanh(U+jV)} \qquad \text{II}$$

where γ_1 is the propagation constant for the air space of the line. Equation II contains only one unknown, i.e. γ_2 , which is therefore obtainable from the measured quantities.

For T.M. modes $$Z_{02} = \frac{\gamma_2}{\sigma + j\omega\epsilon}$$

but $$\gamma_2^2 = j\omega\mu_0(\sigma + j\omega\epsilon) + k^2$$

$$\therefore \sigma + j\omega\epsilon = \frac{\gamma_2^2 - k^2}{j\omega\mu_0}$$

$$\therefore Z_{02} = \frac{j\omega\mu_0\gamma_2}{\gamma_2^2 - k^2}$$

$$\therefore Z_T = \frac{Z_0}{\tanh(U+jV)} = \frac{j\omega\mu_0\gamma_2}{\gamma_2^2 - k^2} \tanh \gamma_2 d$$

$$\therefore \left[\frac{\gamma_2 d}{(\gamma_2 d)^2 - (kd)^2}\right] \tanh \gamma_2 d = \frac{\dfrac{Z_0}{j\omega\mu_0 d}}{\tanh(U+jV)}$$

$$= \frac{\gamma_1}{d(\gamma_1^2 - k^2)} \frac{1}{\tanh(U+jV)} \qquad \text{III}$$

The technique adopted in practice is to make measurements on the line without the sample in place to obtain the end plate impedance and the attenuation caused by metal losses in the line. The positions of the successive minima are located and δy the half widths of the resulting resonance curves are obtained. Then since $\beta \delta y = \alpha y + U$ if $\beta \delta y$ is plotted as a function of y a line of slope α is obtained and U is revealed as the intercept on the $\beta \delta y$ axis.

β is obtained as $\frac{2\pi}{\lambda}$ where λ is the difference in y for two adjacent minima. The value of U obtained from this graph enables the end plate impedance to be calculated since $|k_T| = e^{-2U}$ and the characteristic impedance of the line is known. If this end plate impedance is significant as it will be if accurate results are desired the fundamental equation $Z_T = \frac{Z_0}{\tanh(U+jV)} = Z_{02} \tanh \gamma_2 d$ must be altered to $Z_T = \frac{Z_0}{\tanh(U+jV)} = Z_{02} \tanh(\gamma_2 d + \rho)$

where $\rho = \tanh^{-1} \frac{Z_{end\ plate}}{Z_{02}}$

Equations II and III will then have to be altered in accordance with the above and they will represent a much more involved function of γ_2.

The attenuation factors of lines in general are larger than the theoretical values - therefore attenuation and end plate impedance must be measured if any accuracy is to be obtained for the loss tangent of the dielectric.

To obtain γ_2 it will be observed that U and V must be obtained with the sample in place (see equations II or III). It was shown in Chapter I that the position of the first minimum in the standing wave pattern provides a value for V thus

$$V = \frac{\pi}{2}\left[1 - \frac{4 y_{E_{min}}}{\lambda}\right]$$

and if δy is the change in probe position necessary to get twice the minimum response with a square law detector (a technique applicable to low loss systems)

$$U + \alpha y = \beta \delta y$$

$$\therefore \; U = \beta \delta y - \alpha y$$

These values of U and V then, combined with the data obtained from measurement on the line without a sample enables γ_2 to be calculated for the sample dielectric.

γ_2 Real is partly due to wall loss in the dielectric filled portion but this point will be discussed later.

The partially filled cavity involving variable frequency technique presents problems which appear to make it of limited usefulness unless certain steps are taken such as the use of several different oscillators or the use of a number of cavities of various lengths. (9) The variable frequency method makes possible extremely accurate delineations of Q curves and changes of resonant frequency, but this accuracy will suffer when length measurements must be combined with frequency measurements to obtain the desired information. It is therefore desirable to use a cavity for this purpose without moveable end plates. Assume at first that a single oscillator is to be used to make the determination of the properties of a medium of which the dielectric constant and loss tangent are known approximately. For a given cavity radius, cavity length and oscillator frequency - the required length of dielectric sample may be calculated which will cause the cavity to resonate in the desired mode. The required length of sample must not introduce losses so large that the response curve will be distorted. If distortion does occur the exact point of resonance may be difficult to locate, since the curve is so wide and the zero reactance point may not correspond with the peak of the curve. One or preferably more extra resonators should now be available which are exactly an integral number of half air guide wavelengths long of the same mode at the operating frequency so that attenuation and end plate impedance may be accurately determined. This assumes similar surface conditions in all the resonators used which has been found to be justified. Alternatively a number of oscillators of widely different frequency may be used,[9] the cavity and sample length being so chosen that the cavity can be resonated with the sample in and at several frequencies with the sample removed. Either of these techniques involves a lot of apparatus, and has the disadvantage that the properties of the sample must be quite accurately known before the length of the sample is chosen. Also the required length may be such that the losses are too great for accurate measurements to be made. Extremely thin samples might possibly be a solution to this problem. Then the problem of accurate machining for solids arises.

Small errors in sample thickness become very important sources of error. Such problems as how to keep the sample in contact with the end plate at all points must be considered - or if not in contact how can its position be accurately known. Thin layers of liquid dielectric might prove very suitable media to measure using this method - because liquid could be added until resonance occurred and the depth of the liquid may be determined by volume or weight methods. Sufficient has been said to show that this may be a difficult method to use experimentally. Mathematically this method is similar to the others. It depends upon the measurement of the reflection coefficient of the termination.

← test dielectric

Measurements on the air filled cavity give the value of the reflection coefficient for the metal end plates and the air guide wave length. If several resonant lengths of air filled cavity are available a more accurate value can be found for the guide wave length, the wall attenuation and end plate impedance. The resonant frequency is noted in each case and the response curve is delineated by frequency variation. Thus various values of $\alpha + \frac{k}{l} = \frac{\beta^2 + k^2}{2\beta\left(\frac{f_0}{2\Delta f}\right)}$ are available, since k, β, f_0, and Δf are known from the measurements. (The guide radius and mode define k, since air is the dielectric the value of β will be known, and l will be almost exactly an integral number of half guide wave lengths since $\beta l + g = n\pi$. g is the phase angle of the reflection coefficient for the metal end plates and is extremely small. From any two of these measurements α and p may be determined. From this value of p the end plate impedance may be calculated since $p = \ln_e \frac{1}{|k_t|}$ and the characteristic impedance for the cavity is known.

Next measurements on the cavity containing the specimen dielectric are required. Here l indicates the length of the air filled portion and d the length of the dielectric sample. The resonant condition is

$$\beta l + g = n\pi$$

$$\therefore 2\pi l \sqrt{\left(\frac{f}{c}\right)^2 - \left(\frac{1}{\lambda_0}\right)^2} + g = n\pi$$

where l, c, λ_0, and n are known and f the resonant frequency is measured. But $g = -\frac{\phi_t + \phi_G}{2}$. ϕ_G may be obtained from measurements on the empty cavity (but in practical cases may be too small to be observed).

The important fact is that ϕ_t is determined from the condition for resonance. The Q factor for the cavity was shown to be

$$Q = \frac{f}{2\Delta f} = \frac{\beta^2 + k^2}{2\beta\left(\alpha + \frac{p}{l}\right)}$$

$$\therefore p = l\left[\frac{\beta^2 + k^2}{\beta\left(\frac{f}{\Delta f}\right)} - \alpha\right]$$

where Δf is the increment in frequency from resonance required to bring the response curve to half of its resonant value when using a square law detector. l, β, k, and α will now all be known and Δf is measured. Therefore p is determined via a Q measurement.

The definition of the reflection coefficients must next be examined.

$$k_G k_t = e^{-2(p+jq)} \qquad k_G = \frac{Z_o - Z_G}{Z_o + Z_G} \qquad k_t = \frac{Z_o - Z_T}{Z_o + Z_T}$$

$$= |k_t|\,|k_G|\,e^{j(\phi_t + \phi_G)}$$

$$\therefore p = \ln_e \frac{1}{\sqrt{|k_G|\,|k_t|}} \quad \text{and} \quad q = -\frac{\phi_t + \phi_G}{2}$$

For the air filled cavity with metal short circuiting plates at each end $k_G = k_t$

$$\therefore e^{-2(p_o + jq_o)} = |k_G|^2 e^{j2\phi_G}$$

$$\therefore e^{-(p_o + jq_o)} = |k_G|\,e^{j\phi_G} = k_G$$

From measurements on the cavity containing the dielectric

$$e^{-2(p+jq)} = k_G k_t = e^{-(p_o + jq_o)}\left(\frac{Z_o - Z_T}{Z_o + Z_T}\right)$$

$$\therefore \frac{Z_o - Z_T}{Z_o + Z_T} = e^{-2\left[\left(p - \frac{p_o}{2}\right) + j\left(q - \frac{q_o}{2}\right)\right]}$$

$$\therefore Z_T = Z_o \tanh\left[\left(p - \frac{p_o}{2}\right) + j\left(q - \frac{q_o}{2}\right)\right] \ldots \text{IV}$$

Thus the terminal impedance is evaluated by the measurements on the air filled line along with the measurements on the cavity containing the dielectric specimen. The effect of the end plate impedance may prove to be a significant part of Z_T and remarks exactly similar to those made for the slotted line technique apply here. That is Z_T may not be adequately represented by $Z_{o2} \tanh \gamma_2 d$ but perhaps should be written in the form $Z_T = Z_{o2} \tanh(\gamma_2 d + \rho)$

where $\rho = \tanh^{-1} \dfrac{Z_{end\ plate}}{Z_{o2}}$.

Using the simpler expression

$$Z_{o2} \tanh \gamma_2 d = Z_o \tanh\left[\left(p - \frac{p_o}{2}\right) + j\left(q - \frac{q_o}{2}\right)\right]$$

For T.E. and T.E.M. modes $\qquad Z_{o2} = \frac{j\omega\mu_o}{\gamma_2}$

for non ferromagnetic substances

$$\therefore \frac{\tanh \gamma_2 d}{\gamma_2 d} = \frac{Z_o}{j\omega\mu_o d} \tanh\left[\left(p - \frac{p_o}{2}\right) + j\left(q - \frac{q_o}{2}\right)\right]$$

$$= \frac{1}{\gamma_1 d} \tanh\left[\left(p - \frac{p_o}{2}\right) + j\left(q - \frac{q_o}{2}\right)\right] \quad \text{V}$$

For T.M. modes $\qquad Z_{o2} = \frac{j\omega\mu_o\gamma_2}{\gamma_2^2 - k^2}$

$$\therefore \frac{\gamma_2 d}{(\gamma_2 d)^2 - (kd)^2} \tanh \gamma_2 d = \frac{Z_o}{j\omega\mu_o d} \tanh\left[\left(p - \frac{p_o}{2}\right) + j\left(q - \frac{q_o}{2}\right)\right]$$

$$= \frac{\gamma_1}{d(\gamma_1^2 - k^2)} \tanh\left[\left(p - \frac{p_o}{2}\right) + j\left(q - \frac{q_o}{2}\right)\right] \quad \text{VI}$$

These expressions are more complicated when end plate impedance must be considered in Z_T . The important fact however is that γ_2 is now determined and the properties of the unknown dielectric may be found.

The technique employing partially filled cavities and variable lengths does not present serious difficulties as regards apparatus,[10] in the same way that the variable frequency technique does. The accuracy of the method depends upon the measurement of small increments in length which cannot be done with nearly the same precision as can measurements of increments in frequency unless very great care is taken to make the measuring screw free of backlash and its temperature is accurately controlled. In frequency measurement using WWV or a Secondary standard both the frequency and frequency increments are known to about one part in 10^8. Such accuracy in length measurement would be very difficult to obtain. This technique requires a generator of very high frequency stability over long periods of time. For accurate determination of loss tangents this method may not be very suitable because of indeterminate and possibly variable losses at the edge of the plunger. Steps may be taken to offset this difficulty e.g. the use of $TE_{0,n}$ modes does not require current to pass from end to side walls. This method has been used successfully by Lamb and others. The mathematical analysis of this problem is the same as for the variable frequency technique. By measurements on the completely air filled cavity $\beta l_0 + g = n\pi$ yields a series of resonant lengths corresponding to integral values of n i.e.

$n =$ 1, 2, 3 etc. Twice the difference in two successive values of l_0 is the guide wave length in the air portion of the cavity. From the various determinations of Q on the completely air filled cavity,

$$Q = \left(\frac{l_0}{2\Delta l}\right)\left[\frac{1}{1-\left(\frac{\lambda}{\lambda_0}\right)^2}\right] = \frac{\beta^2 + k^2}{2\beta\left(\alpha + \frac{p}{l_0}\right)}$$

$$\therefore \alpha + \frac{p}{l_0} = \frac{\beta^2 + k^2}{\beta\frac{l_0}{\Delta l}}\left[1 - \left(\frac{\lambda}{\lambda_0}\right)^2\right]$$

$$\therefore \alpha l_0 + p = K\Delta l \quad \text{where} \quad K = \frac{\beta^2 + k^2}{\beta}\left[1 - \left(\frac{\lambda}{\lambda_0}\right)^2\right]$$

When $K\Delta l$ is plotted against l_0 a straight line is obtained whose slope is α and whose intercept on the $K\Delta l$ axis is p.

Thus end plate impedance may be calculated from p as in the variable frequency case.

The technique next requires measurements on the cavity containing the dielectric sample. As before $\beta l_o + q = n\pi$

$$\therefore \phi_t = 2(\beta l_o - n\pi) - \phi_G$$

where ϕ_G is usually negligibly small being the phase angle of the reflection coefficient for the end plate.

ϕ_t is the phase angle of the reflection coefficient of the end of the cavity containing the dielectric sample.

From the expression for Q then

$$p = l\left\{\left[\frac{\beta^2 + k^2}{\beta \frac{l_o}{\Delta l}}\right]\left[1 - \left(\frac{\lambda}{\lambda_o}\right)^2\right] - \alpha\right\}$$

From this point the analysis is exactly similar to the previous case as it must be since the only mathematical difference in the two methods is the independent variable chosen to delineate the resonance curves.

CHAPTER III

Determination of the Material constants from the Complex Propagation Factor.

The propagation factor for wave guides is shown in many text books to be for non ferromagnetic dielectrics

$$\gamma^2 = -\left[\omega^2\mu_o\epsilon - \left(\frac{2\pi}{\lambda_o}\right)^2\right]$$

$$\epsilon = \epsilon_1^*\epsilon_o = \epsilon_1\left(1 - j\tan\delta\right) =$$ the complex dielectric constant

$$= \epsilon_1\left(1 - j\frac{\sigma}{\epsilon_1\omega}\right)$$

$K_e = \dfrac{\epsilon_1}{\epsilon_o}$ is the relative dielectric constant of the material filling the guide.

$$\gamma^2 = -\left[\omega^2\epsilon_1^*\epsilon_o\mu_o - \left(\frac{2\pi}{\lambda_o}\right)^2\right]$$

$$= -\left[\left(\frac{2\pi}{\lambda_a}\right)^2\epsilon_1^* - \left(\frac{2\pi}{\lambda_o}\right)^2\right]$$

$$\therefore \gamma^2 = -\left(\frac{2\pi}{\lambda_a}\right)^2\left[\epsilon_1^* - \left(\frac{\lambda_a}{\lambda_o}\right)^2\right]$$

note $\epsilon_1^* = \dfrac{\epsilon}{\epsilon_o} = K_e\left(1 - j\tan\delta\right) = \dfrac{\epsilon_1}{\epsilon_o}\left(1 - j\tan\delta\right)$

$\lambda_a =$ the T.E.M. wavelength is free space corresponding to the frequency used.

$$\therefore \gamma = j\frac{2\pi}{\lambda_a}\sqrt{\epsilon_1^* - \left(\frac{\lambda_a}{\lambda_o}\right)^2} \quad \cdots\cdots\cdots \quad \mathrm{I}$$

Now $\left(\dfrac{1}{\lambda_a}\right)^2 = \left(\dfrac{1}{\lambda_g}\right)^2 + \left(\dfrac{1}{\lambda_o}\right)^2$

$$\therefore \gamma = j\frac{2\pi}{\lambda_g}\sqrt{\epsilon_1^* + (\epsilon_1^* - 1)\left(\frac{\lambda_g}{\lambda_o}\right)^2} \quad \cdots \textcircled{1} \cdots \quad \mathrm{II}$$

λ_g is the guide wave length in the air portion of the guide.

γ is a complex number which may be expressed in polar coordinates as follows

$$\gamma = \frac{2\pi}{\lambda_g}\sqrt{K_e}\,\sqrt[4]{\left[\left(1+\left(\frac{\lambda_g}{\lambda_o}\right)^2\right)-\left(\frac{\lambda_g}{\lambda_o}\right)^2\frac{1}{K_e}\right]^2+\tan^2\delta\left[1+\left(\frac{\lambda_g}{\lambda_o}\right)^2\right]^2}$$

at an angle $\dfrac{\pi}{2} - \dfrac{1}{2}\tan^{-1}\dfrac{\tan\delta}{1-\dfrac{\lambda_g^2}{\lambda_o^2+\lambda_g^2}\dfrac{1}{K_e}}$ III

In rectangular form γ may be expressed

$$\gamma = \frac{2\pi}{\lambda_g}\sqrt{\frac{K_e\left(1+\left(\frac{\lambda_g}{\lambda_o}\right)^2\right)-\left(\frac{\lambda_g}{\lambda_o}\right)^2}{2}}\left(\sqrt{1+\left(\frac{\tan\delta}{1-\frac{\lambda_g^2}{\lambda_o^2+\lambda_g^2}\frac{1}{K_e}}\right)^2}-1\right)^{\frac{1}{2}}$$

$$+j\frac{2\pi}{\lambda_g}\sqrt{\frac{K_e\left(1+\left(\frac{\lambda_g}{\lambda_o}\right)^2\right)-\left(\frac{\lambda_g}{\lambda_o}\right)^2}{2}}\left(\sqrt{1+\left(\frac{\tan\delta}{1-\frac{\lambda_g^2}{\lambda_o^2+\lambda_g^2}\frac{1}{K_e}}\right)^2}+1\right)^{\frac{1}{2}} \qquad \text{IV}$$

Solving either the polar or rectangular forms for the material constants in terms of the real and imaginary parts of $\gamma = \gamma_r + j\,\gamma_i$ leads to

$$\frac{\tan\delta}{1-\frac{\lambda_g^2}{\lambda_o^2+\lambda_g^2}\frac{1}{K_e}} = \frac{2\,\gamma_r\gamma_i}{\gamma_i^2-\gamma_r^2} \quad . \; . \; . \; . \qquad \text{V}$$

and

$$K_e\left[1+\left(\frac{\lambda_g}{\lambda_o}\right)^2\right]-\left(\frac{\lambda_g}{\lambda_o}\right)^2 = \frac{\gamma_i^2-\gamma_r^2}{\left(\frac{2\pi}{\lambda_g}\right)^2} \quad . \; . \; . \qquad \text{VI}$$

These may be separately solved for K_e and $\tan\delta$

$$\tan\delta = \frac{2\,\gamma_r\gamma_i}{\left(\frac{2\pi}{\lambda_o}\right)^2+\gamma_i^2-\gamma_r^2} \quad . \; . \; . \; . \; . \; . \qquad \text{VII}$$

$$K_e = \frac{\left(\frac{1}{\lambda_o}\right)^2+\frac{\gamma_i^2-\gamma_r^2}{4\pi^2}}{\left(\frac{1}{\lambda_g}\right)^2+\left(\frac{1}{\lambda_o}\right)^2} \quad . \; . \; . \; ① \; . \; . \; . \qquad \text{VIII}$$

These then are the equations required to obtain the material constants from the value of γ determined as shown in the previous chapter. The equations above apply to any wave guide mode. The formulae for coaxial lines are the same except that here $\lambda_o = \infty$ for the principal mode.

Wall Loss in Wave Guides (1)

For low loss samples contained in wave guides the wall losses may be important changing equation VII to

$$\tan\delta_s + \tan\delta_{ws} = \frac{2\left(\gamma_{rs} + \gamma_{rw}\right)\gamma_i}{\left(\frac{2\pi}{\lambda_o}\right)^2 + (\gamma_i)^2} \quad . \quad . \quad . \quad . \quad . \qquad \text{IX}$$

where γ_r^2 has been omitted from the denominator because the sample has low losses.

$\tan\delta_s$ = the loss tangent of the sample dielectric

$\tan\delta_{ws}$ = the effective increase in $\tan\delta$ due to losses ocurring in the guide walls with the sample in place.

γ_{rs} = the attenuation factor due to sample losses.

γ_{rw} = the attenuation factor due to wall loss.

The attenuation factor γ_{rw} due to wall loss has been calculated. This value together with the value of γ_i for $\tan\delta = 0$ have been substituted into equation IX to give for the T.E 11 mode in circular wave guide for the ratio

$$\frac{\tan\delta_{ws}}{\tan\delta_{wa}} = \frac{1}{\mu'}\left[\frac{0.42 + \left(\frac{\lambda_q}{\lambda_o}\right)^2\frac{1}{\mu'\epsilon'}}{0.42 + \left(\frac{\lambda_q}{\lambda_o}\right)^2}\right] \quad . \quad . \quad . \quad . \quad . \qquad \text{X}$$

$$= \frac{\text{the effective } \tan\delta \text{ for wall loss with sample in}}{\text{effective } \tan\delta \text{ for wall loss with air}}$$

Now $\tan\delta$ for wall loss with air will be known from measurements on the air filled cavity or line. Then $\tan\delta_{ws}$ may be calculated using equation X and on substituting this result into equation IX $\tan\delta_s$ is obtained.

A somewhat similar approach may be taken to this problem as follows. Consider T.M. modes for which the formula for conductor attenuation provides the relationship

$$\alpha_c \propto \frac{1}{\eta\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}}$$

$$\therefore \quad \frac{\alpha_c \text{ air portion of the line}}{\alpha_c \text{ dielectric portion of the line}} = \frac{\eta_{dielectric}\sqrt{1-\left(\frac{\lambda_{g\,dielectric}}{\lambda_0}\right)^2}}{\eta_{air}\sqrt{1-\left(\frac{\lambda_{g\,air}}{\lambda_0}\right)^2}}$$

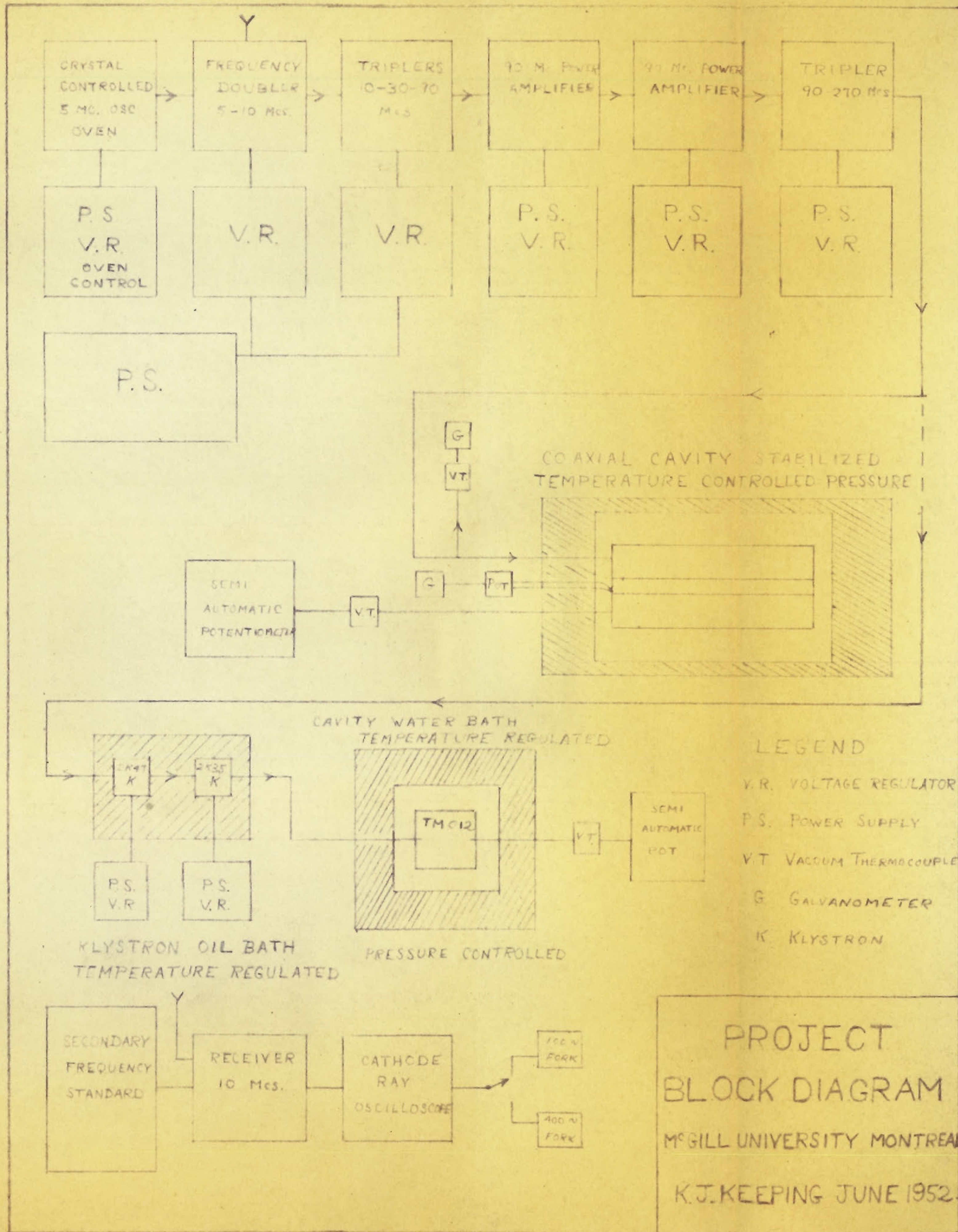
where $$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

This ratio will be known approximately if the dielectric constant is obtained from equation VIII on the basis of $\gamma r = 0$. Thus the attenuation due to the walls becomes known for the dielectric filled portion of the line or cavity. This may then be subtracted from the apparent value of γr to obtain the true value of $\tan\delta$ and Ke. In the above calculation the attenuation due to the walls in the air filled portion of the line is determined experimentally by measurements on the empty line or cavity as explained in the last chapter.

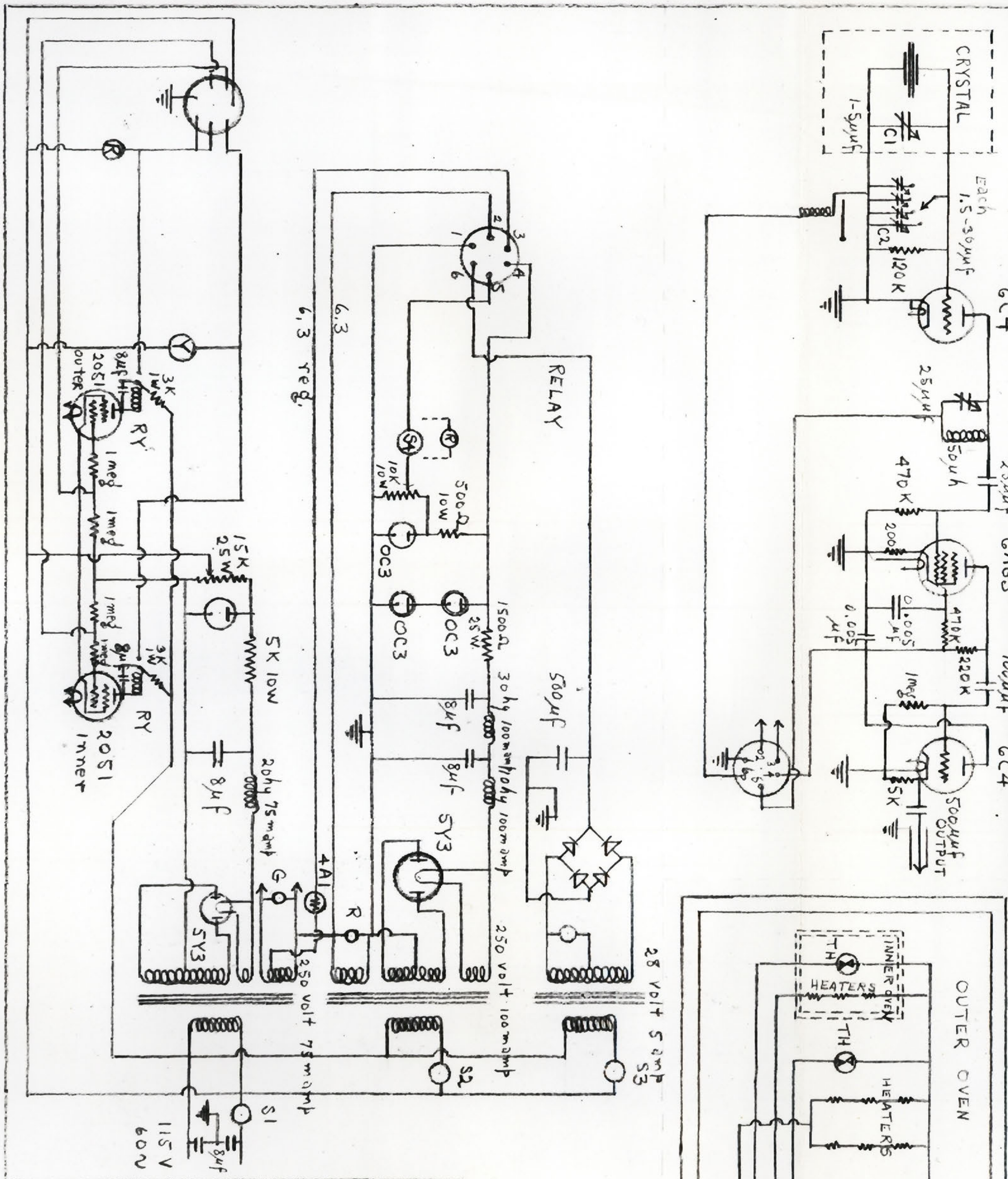
The difficulty in allowing for wall loss in the dielectric filled portion of the line or cavity arises from the fact that the properties of the unknown dielectric affect this attenuation. Calculation of dielectric constant on the basis of zero loss has been used [9] but in this method it is used only as a first approximation to obtain a more accurate result. It is often found that conductor losses are larger than the values calculated on a theoretical basis. It has been pointed out that in making dielectric measurements it is undesirable to use theoretical values of attenuation along with other quantities which are measured to derive the material constants.

The question of just what factors affect skin effect impedance of a metallic surface are not completely understood. A high polish does not seem to greatly reduce the metal losses. Minute scratches in the direction of current flow have much less effect on losses than scratches across the direction of the current. [11] A. C. Vivian has written a paper on superficial conductivity of metallic [12] conductors for wave guides and A. P. Pippard has [13] examined the question of metallic conductivity at high frequency and at very low temperatures, where under certain conditions it is fould that skin effect conductivity becomes independent of D.C. conductivity. This appears to be due to the mean free path of the electrons becoming very much greater than the skin depth.

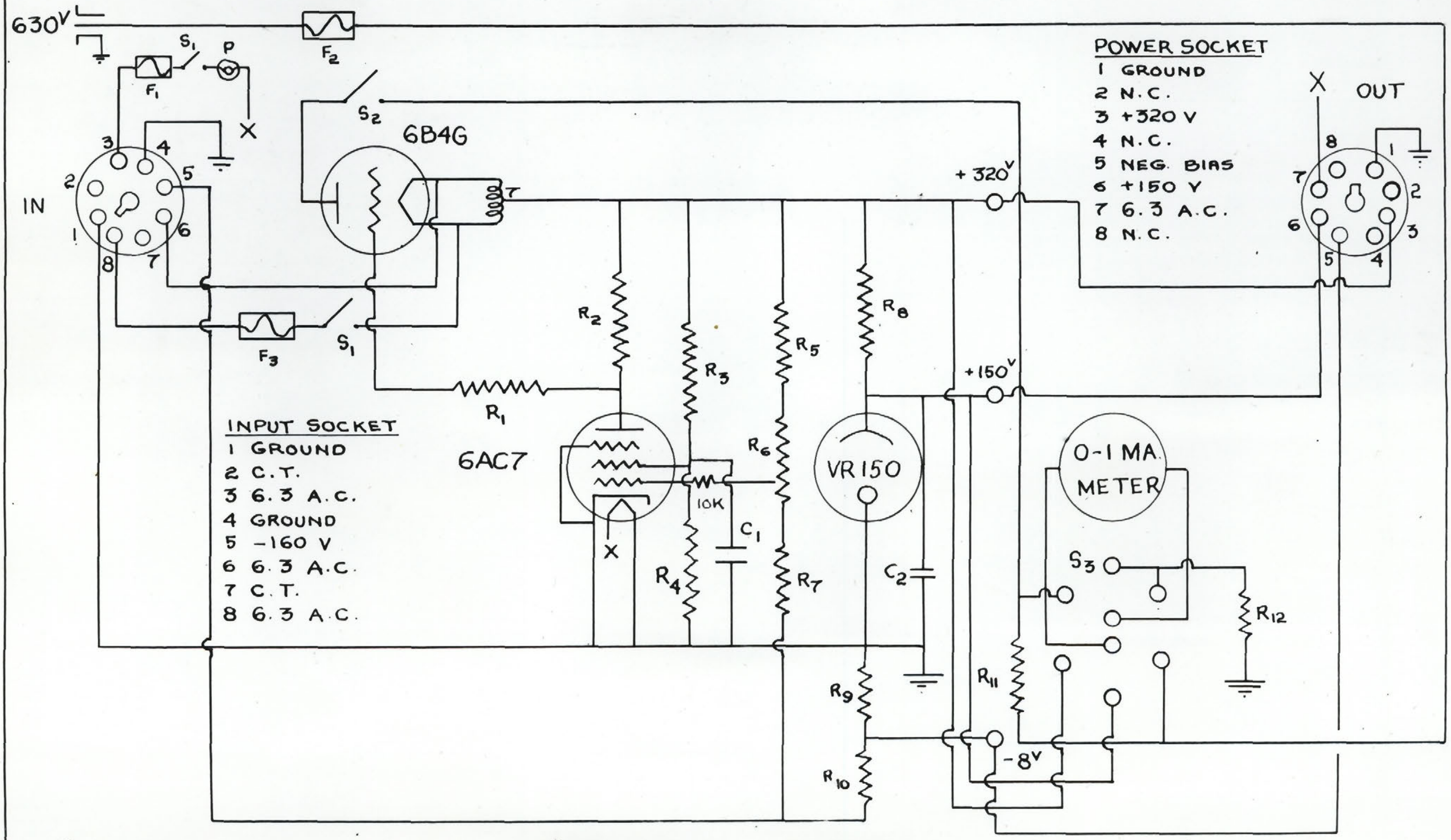
The experimental techniques employed at high frequency make it possible to obtain experimental values for attenuation due to metal losses and these values are used rather than the theoretical ones in making dielectric measurements.



PROJECT
BLOCK DIAGRAM
McGILL UNIVERSITY MONTREAL
K.J. KEEPING JUNE 1952



5 M.C. OSCILLATOR THERMOSTATS
HEATERS AND POWER SUPPLIES
MCGILL UNIVERSITY MONTREAL
K.J. KEEPING · JULY 1950.



RESISTORS

R_1 – 500 OHMS
R_2 – 250K
R_3 – 60K
R_4 – 40K
R_5 – 300K
R_6 – 100K POT.
R_7 – 100K
R_8 – 6K 5 WATT
R_9 – 20K POT.
R_{10} – 75K
R_{11} – 0.25 OHMS
R_{12} – 500K

CAPACITORS

C_1 – 0.1 uf
C_2 – 0.1 uf

FUSES

F_1 – 3A
F_2 – 1/8 A
F_3 – 1A

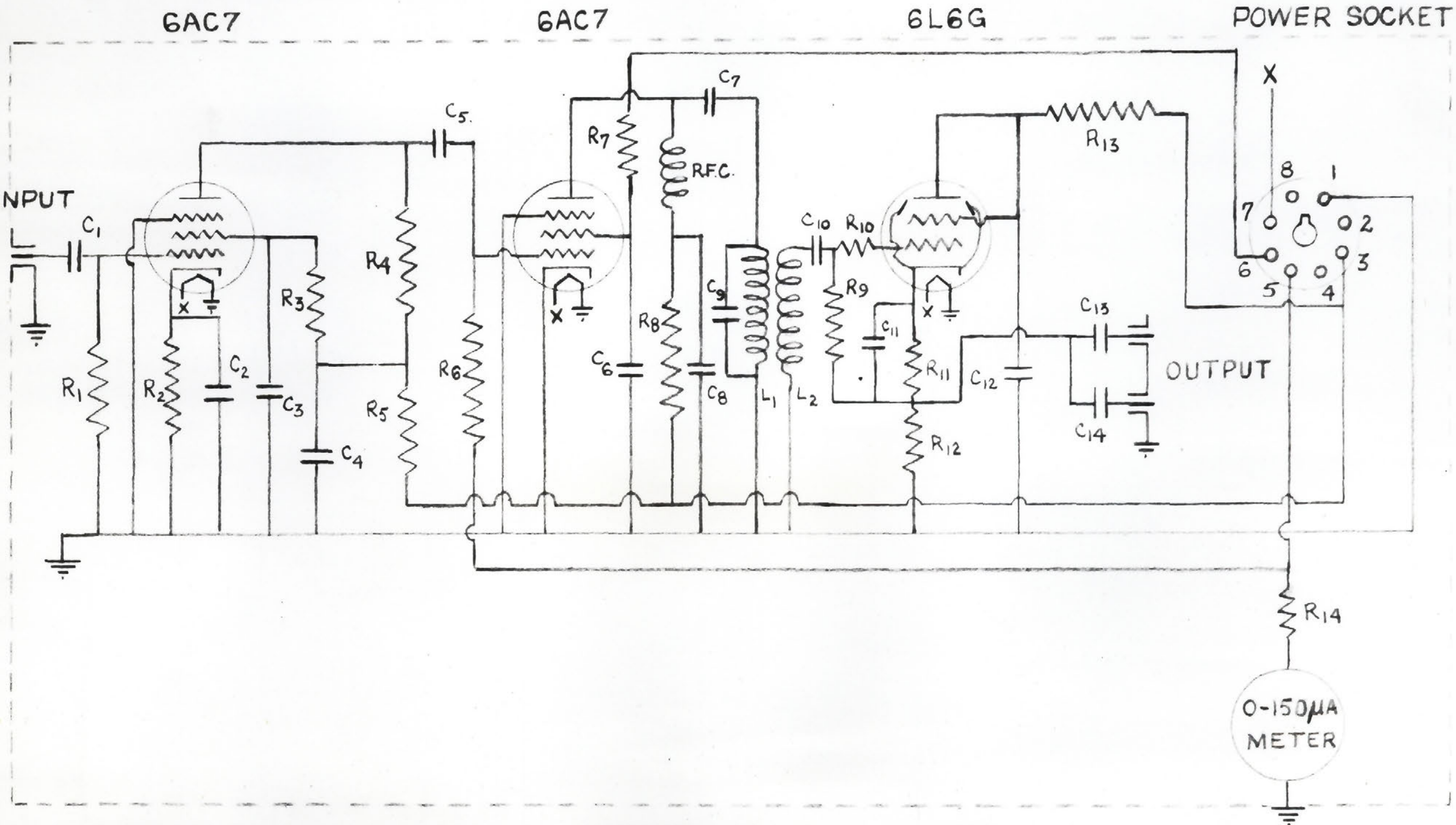
SWITCHES

S_1 – D P S T
S_2 – S. P. S. T.
S_3 – D. P. 6 CONT.

VOLTAGE REGULATOR FOR 5-10MC. DOUBLER

McGILL UNIVERSITY MONTREAL

JAN. 1948 P.C. BOIRE



RESISTORS

R1 - 0.25 meg.
R2 - 160 ohms
R3 - 60k ohms
R4 - 2k ohms
R5 - 2k ohms
R6 - 0.2 megs.
R7 - 2k ohms
R8 - 2k ohms
R9 - 390k ohms
R10 - 2k ohms
R11 - 1500 ohms
R12 - 1000 ohms
R13 - 2k ohms
R14 - 90k ohms

L1 and L2 - 16.6 uh

CAPACITORS

C1 - 0.006 uf
C2 - 0.01 uf
C3-- 0.01 uf
C4 - 0.01 uf
C5 - 400 uuf
C6 - 0.01 uf
C7 - 400 uuf
C8 - 0.01 uf
C9 - 15 uuf
C10 - 0.01 uf
C11 - 0.01 uf
C12 - 0.01 uf
C13 - 400 uuf
C14 - 400 uuf

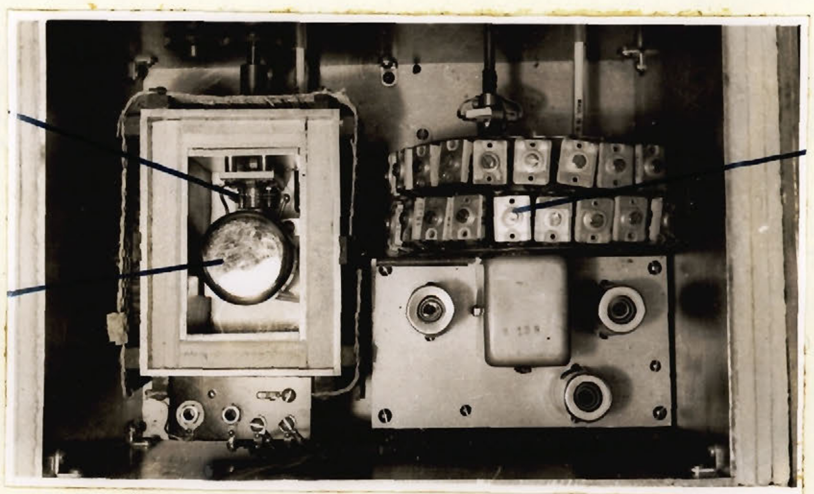
5-10 MC. AMPLIFIER
AND
FREQUENCY DOUBLER

McGILL UNIVERSITY MONTREAL
JAN 1948 P.C. BOIRE

CHAPTER IV

The Signal Generating Apparatus - Basic Oscillator and Frequency Multipliers.

The basic five megacycles per second signal is generated in a tuned plate tuned grid oscillator, using a 6C4 minature tube. The tuned circuit in the grid consists of a piezo electric quartz crystal along with several fixed plus one variable trimmer condensers.



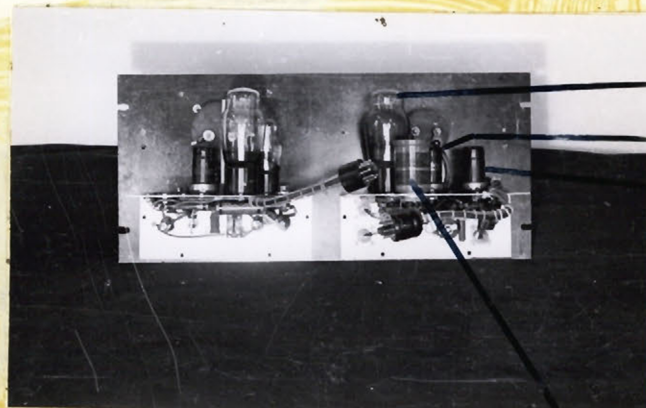
Crystal Oven Covers Removed.

This unit and its power supply were manufactured by the Piezo Products Company of Framingham, Massachusetts. The temperature of operation is carefully regulated by thermostatically controlled heaters. It will hold its frequency to one part in a million over extended periods and to one part in a hundred million over short periods of one minute or less. The generated frequency can be set to any frequency between $5 \times 10^6 + 600$ to $5 \times 10^6 - 600$ cycles per second. This is done by means of pre set padding condensers of graduated capacity which are switched in one by one by means of a magnetically operated switch. When a push push buttom is operated on the front panel the magnetic switch jumps the connection from one fixed padding condenser to another. The small variable trimmer condenser makes it possible to select any frequency between the fixed steps.

The whole oscillator circuit excluding the power supplies is housed in a temperature regulated oven. Within this oven there is a smaller temperature regulated oven containing the piezo electric crystal.

It would be desirable to increase the band width available at this stage since the changes in resonant frequency caused by gaseous dielectrics are sufficiently large that the measurements must be made at reduced pressures. The results must then be extrapalated to normal conditions on the basis of some gas law. It would be desirable to be able to measure the dielectric constant up to pressures on one atmosphere. (15) This problem may be overcome by employing more than one temperature regulated crystal. Using a series of say three crystals differing by about one kilocycle per second in frequency the band width would be quite sufficient for most purposes where gases are concerned. These crystals could be arranged so that any one could be selected at a time by means of a selector switch on the front of the instrument.

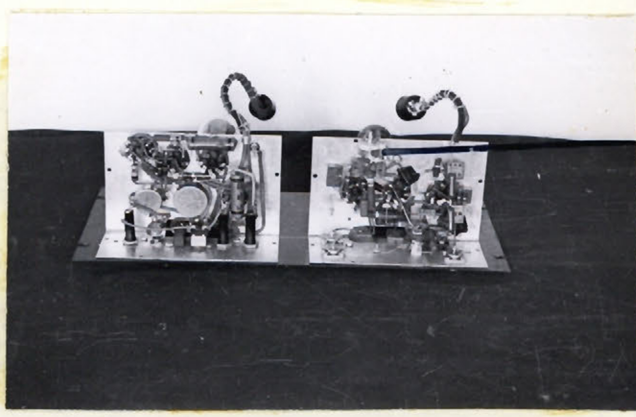
The five megacycle per second signal is amplified and the output is taken via cathode follower and coaxial line to the 5 to 10 me/sec. frequency doubler and amplifier



Cathode follower 6L6G
doubler 6AC7
amplifier 6AC7

5 - 10 Megacycle per second frequency doubler

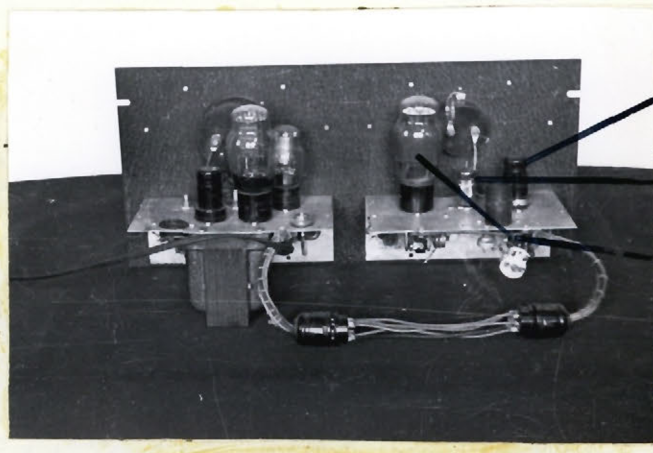
Tuned Plate circuit Resonant at 10 megacycles per second



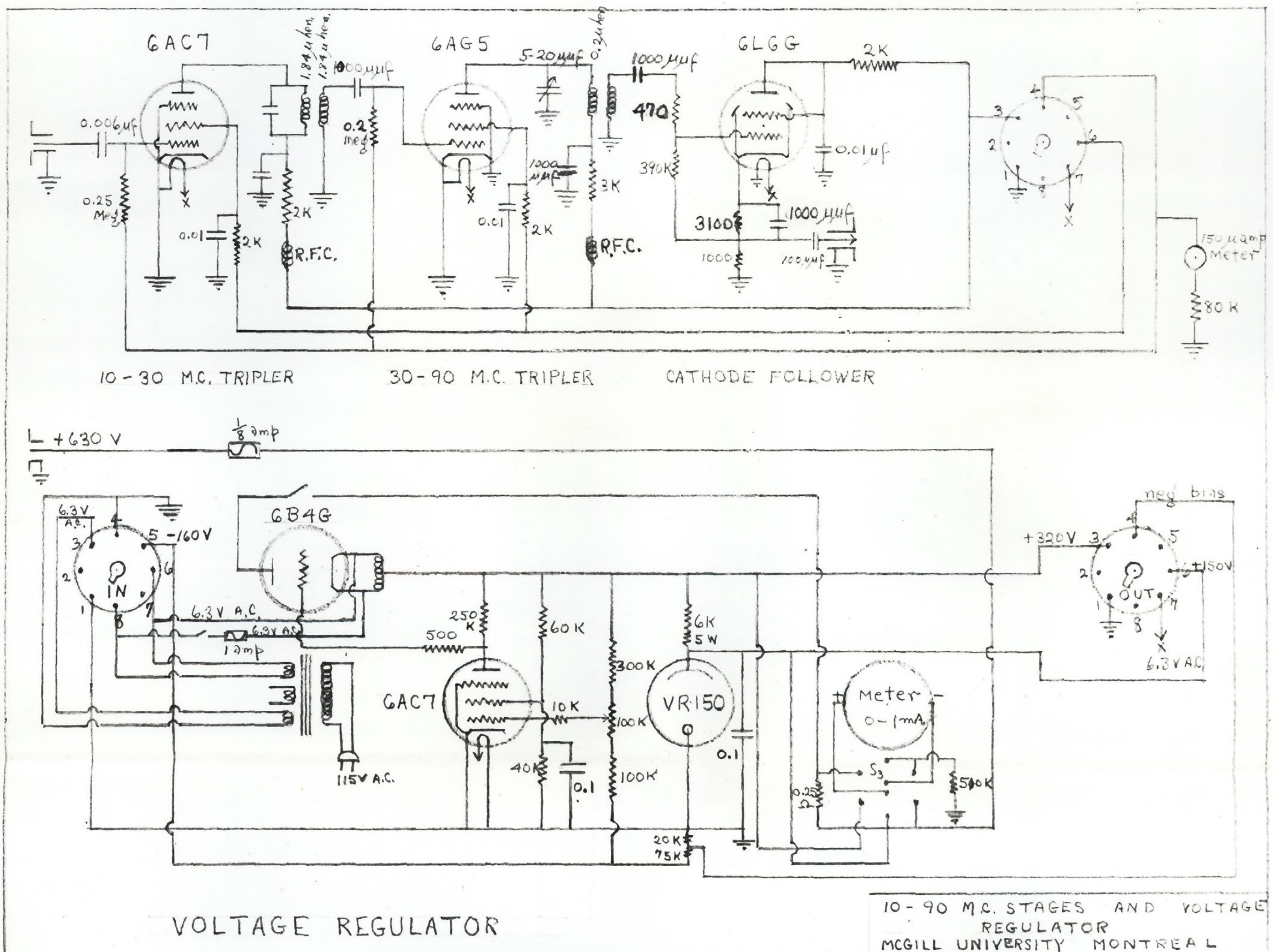
5 - 10 Megacycle per second frequency doubler lower view

The frequency doubler is a class C operated 6AC7 with the tuned circuit of the plate, tuned to twice the input frequency. This output is transformer coupled to the grid circuit of a 6L6G cathode follower and from there to a coaxial line.

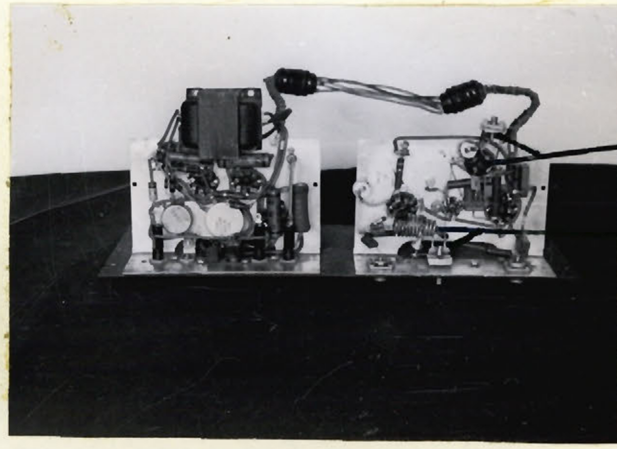
This leads to the 30 - 90 megacycle per second tripler circuits - all of these panels have separate voltage regulators. The 10 - 30 megacycle per second tripler uses a 6AC7 pentode and the 30 - 90 megacycle per second tripler uses a minature 6AG5. These frequency multipliers are similar in principle to the one already described.



10 - 30 - 90 Megacycle stages Upper View



10 - 90 M.C. STAGES AND VOLTAGE REGULATOR
MCGILL UNIVERSITY MONTREAL
K.J. KEEPING JULY 1950

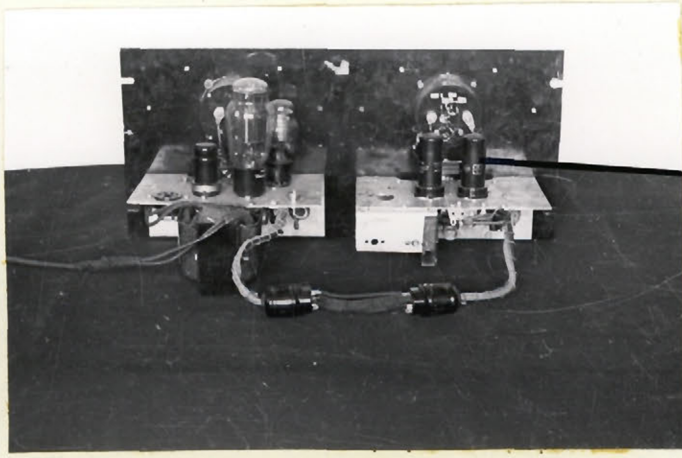


Tuned circuit resonant at 30 megacycles per second.

Tuned circuit resonant at 90 megacycles per second

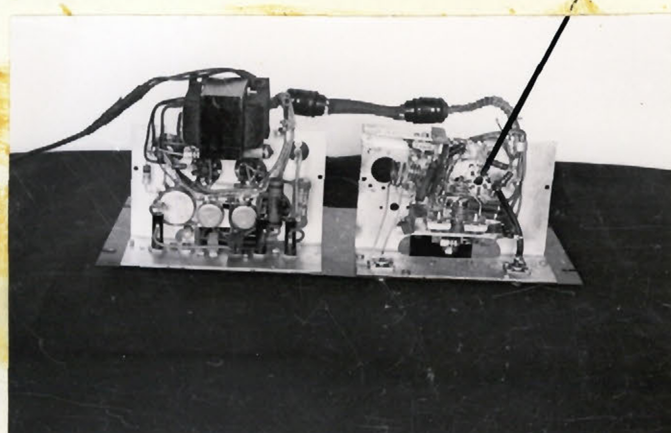
10 - 30 - 90 Megacycle stages   Lower View

At this stage the signal level is quite low (less than one volt from the cathode follower). Since it is desirable to have considerable output power at 270 megacycles per second both for measurement at that frequency and for the input signal to a 2K47 klystron, it was found desirable to raise the power level of the 90 megacycle per second signal. This is done in two stages involving a 6AG7 pentode and finally a push pull power amplifier using an 829 double beam tetrode. 14



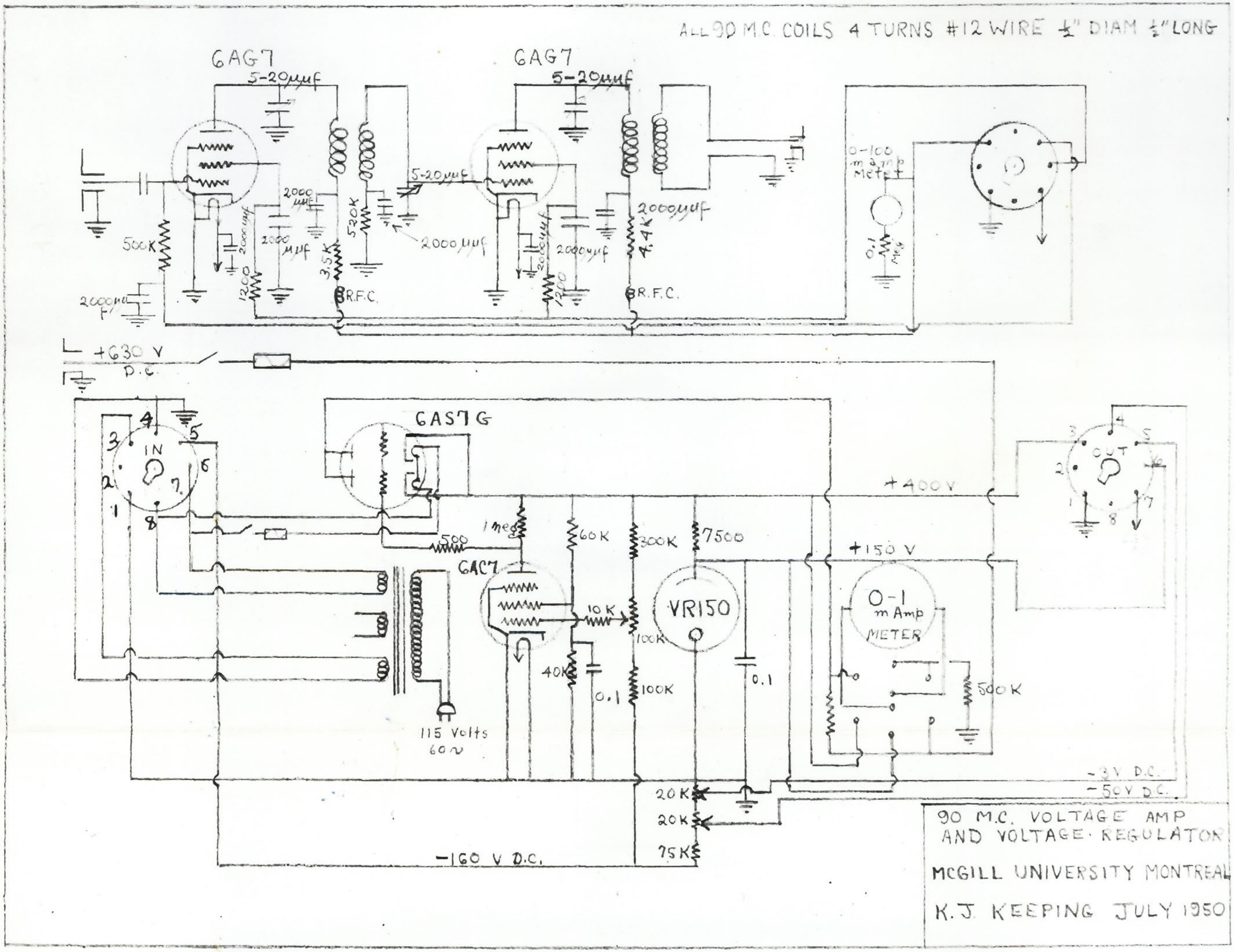
6AG7 power amplifier

90 Megacycle amplifiers upper view



90 Megacycle amplifiers lower view

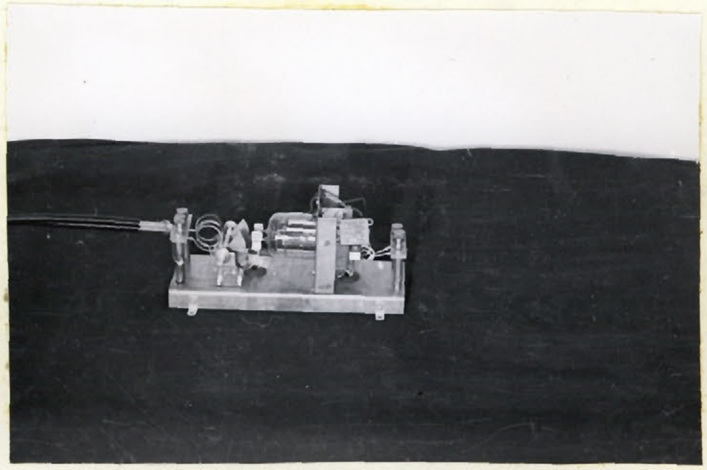
This circuit has been modified since the above photographs were taken and the circuit diagram was drawn up. The change consists in the removal of the second 6AG7 since it was found to be overdriven by the previous stage. The output from this circuit is taken via coaxial line to the 829 stage.



90 M.C. VOLTAGE AMP AND VOLTAGE REGULATOR

McGILL UNIVERSITY MONTREAL

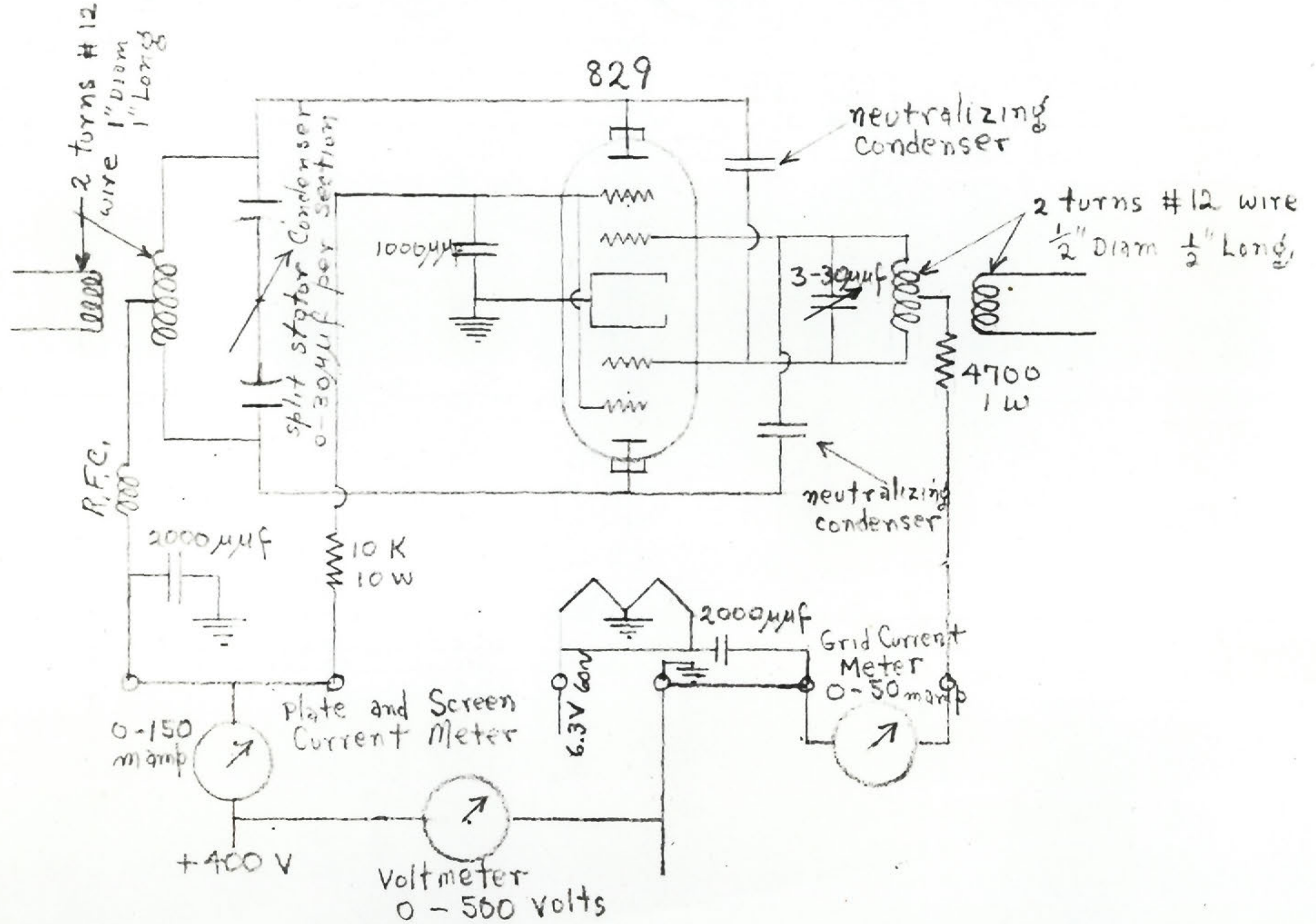
K.J. KEEPING JULY 1950



829 Stage

This apparatus i.e. up to but not including the 829 stage had been built by Mr. P. Boire at McGill University during the years 1947 and 1948. The apparatus was taken over by the writer in 1949. Minor changes were made to increase the level of output power but the present form of the chassis containing the 6AG7 90 megacycle per second power amplifier is very greatly changed from the original model. This chassis originally contained a 1614 transmitting tube to give the required output power. This however did not work too satisfactorily and the output power was still too small. It was then decided to use the 829 stage to obtain large power output. The power output of this stage will light a 30 watt bulb when placed across its output terminals.

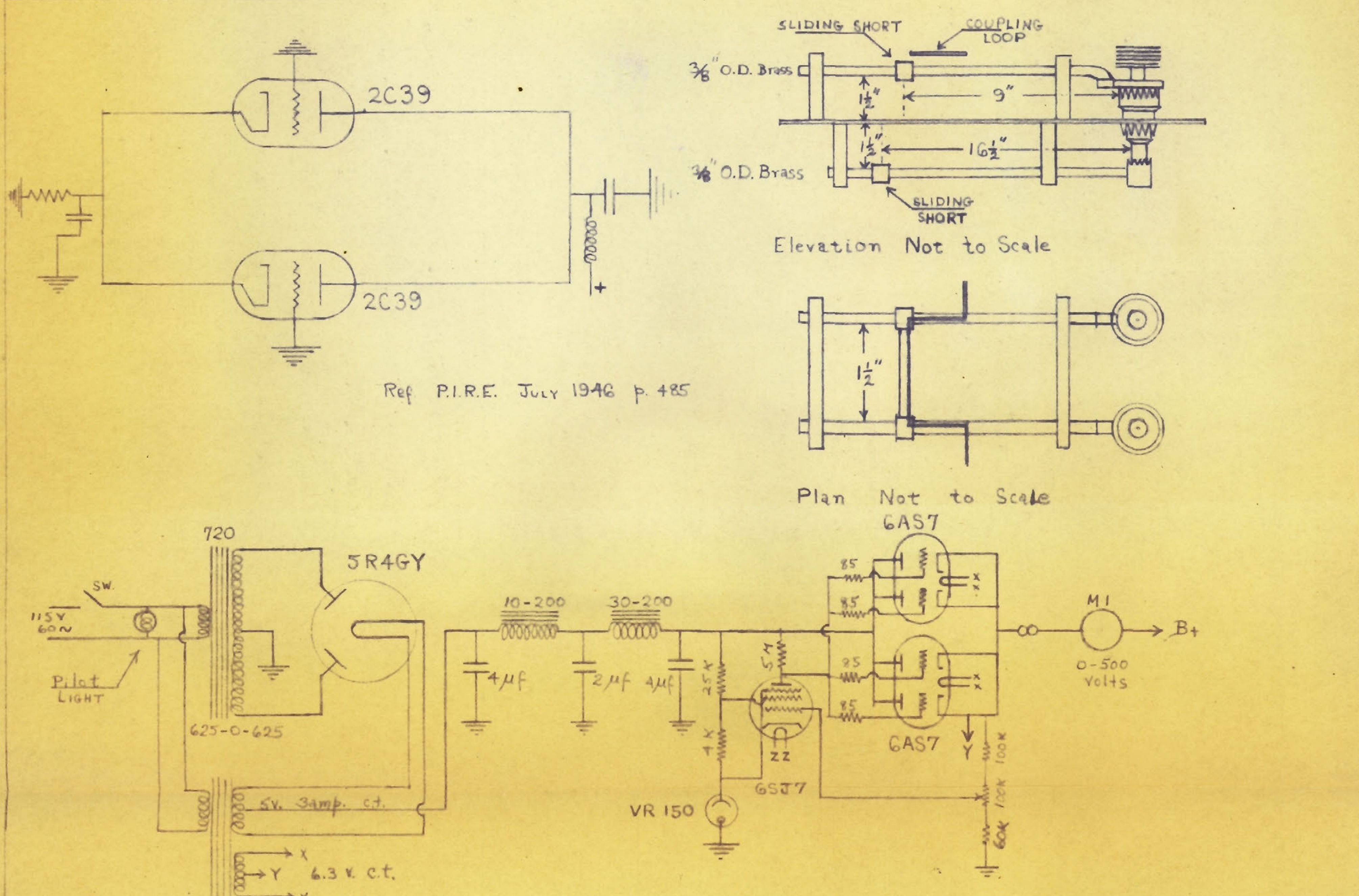
The output from the 829 stage is taken via balanced lines to the 90 - 270 megacycle per second frequency tripler. This



90 M.C. POWER AMPLIFIER
AND POWER SUPPLY
MCGILL UNIVERSITY MONTREAL
K.J. KEEPING JULY 1950

See Page 417 ARRL HANDBOOK 1949

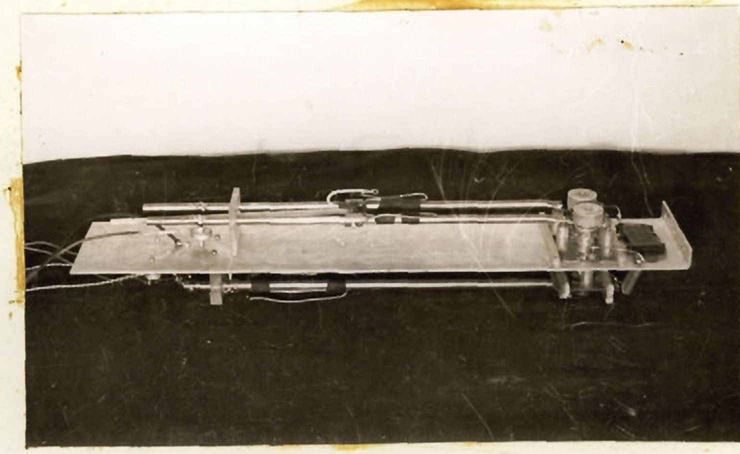
circuit employs two 2C39 lighthouse triodes in push pull.(16) These tubes have their grids grounded and the cathode circuit consists of a tuned line. There is no cathode connection on these tubes which is separate from the heater. Therefore the heater power is brought to the tube terminals by means of wires fed down the centre of the hollow brass tubes making up the cathode line. The brass tubes themselves forming one side of the sixty cycle heater circuit. The 2C39 needs special connectors to provide contact at many points around the various tube structures. These small connectors are not too readily constructed but after discarding a few attempts the contacts finally devised proved quite satisfactory. Tuning the cathode line is accomplished by calculating the input impedance to the tubes at 90 megacycles per second as seen between the two cathodes. The length of line required is then such as to produce at the cathodes an impedance the same as is seen there. If it is assumed that there should be a quarter wave length of line from the terminating short circuit to produce a resonant condition at the tubes, then about one eighth of a wave length is apparently contained within the tube structure. A similar approach is used in tuning the plate line. The input lines drive a coupling loop which excites the tuned line between the cathodes. The tuned plate line excites an output loop which feeds a coaxial line connected to the 2K47 frequency multiplying klystron.



90-270 Mc FREQUENCY TRIPLER AND REGULATED POWER SUPPLY

McGILL UNIVERSITY MONTREAL

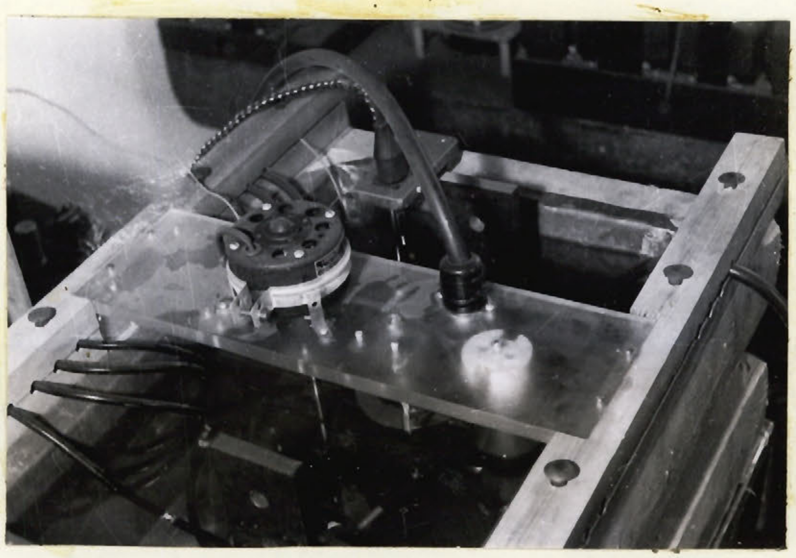
K.J. KEEPING JUNE 1952



90 - 270 Megacycle per second power amplifier

The output at 270 megacycles per second is sufficient to successfully operate a 2K47 Sperry frequency multiplier klystron, or to make measurements at 270 megacycles per second. It was found to be necessary to regulate the filament voltage of the 2C39's since the output showed a small dependence on line voltage until this was done.

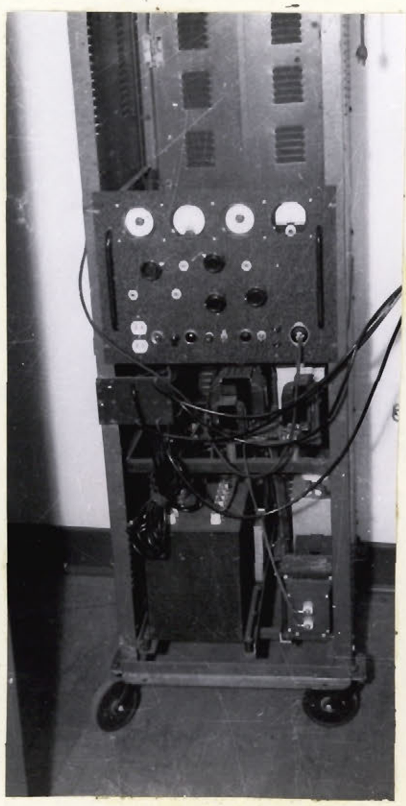
A temperature regulated oil bath was built for the 2K47 and 2K35 klystrons. This was done to avoid the thermal detuning action due to changes in temperature of the klystrons. The bath was constructed from groove and tongue lumber and lined with sheet copper - the seams being soldered. The bath is filled with Imperial Volt Esso oil and is kept at 60°C by means of thermostatically controlled heaters. The thermostat is the mercury in glass type which controls the bias of a 2051 gas filled thyratron.



Klystron oil bath

The operating coil of a relay forms the plate load of this tube. The relay contacts open and close the heater circuit. To ensure good temperature control a motor driven stirrer is used.

The power supplies used with the klystrons are voltage regulated having the positive side grounded. The 2K47 is a frequency multiplying tube which will give from ten to one hundred milliwatts of output power when the input power ranges from a fraction of a watt to several watts. The lower cavity may be tuned over the range 250 - 280 megacycles per second by means of three thermally compensated tuning screws. The upper cavity which is very much smaller than the lower one may then be tuned to obtain either the ninth, tenth, eleventh or twelvth harmonics of the input frequency. The high harmonic content of the election bunches makes this order of frequency multiplication possible. The particular harmonic used is the eleventh giving the output a frequency of 2970 megacycles per second. Putting this tube into operation involves tuning the input resonator to resonance with the input signal using a crystal and galvanometer to indicate resonance. (Each resonator of this tube has two coaxial terminals - therefore the detector system is connected to one and the input to the other). The second cavity is then carefully adjusted over a small range of the tuning screws, varying the beam voltage and control electrode voltage until the desired harmonic is detected. This procedure may be shortened somewhat by pretuning the output cavity if a suitable signal generator is available. The 2K35 klystron was next put into operation by pretuning each of its three cavities to the output signal from the 2K47. The 2K35 is a cascade amplifier which may be tuned over the range 2730 to 3330 megacycles per second. Each cavity of the 2K35 has only one coaxial line connector so that the cavities are tuned as absorption wavemeters, that is the detector is attached to the spare terminal on the high frequency cavity of the 2K47. The 2K35 is operated from a voltage regulated power supply very similar to the type 801A Universal Klystron Power Supply manufactured by the Polytechnic Research and Development Company of Brooklyn N.Y. The author wishes to express his thanks to Mr. Noel Montagnon for the construction of this power supply which was rather a large project in itself.



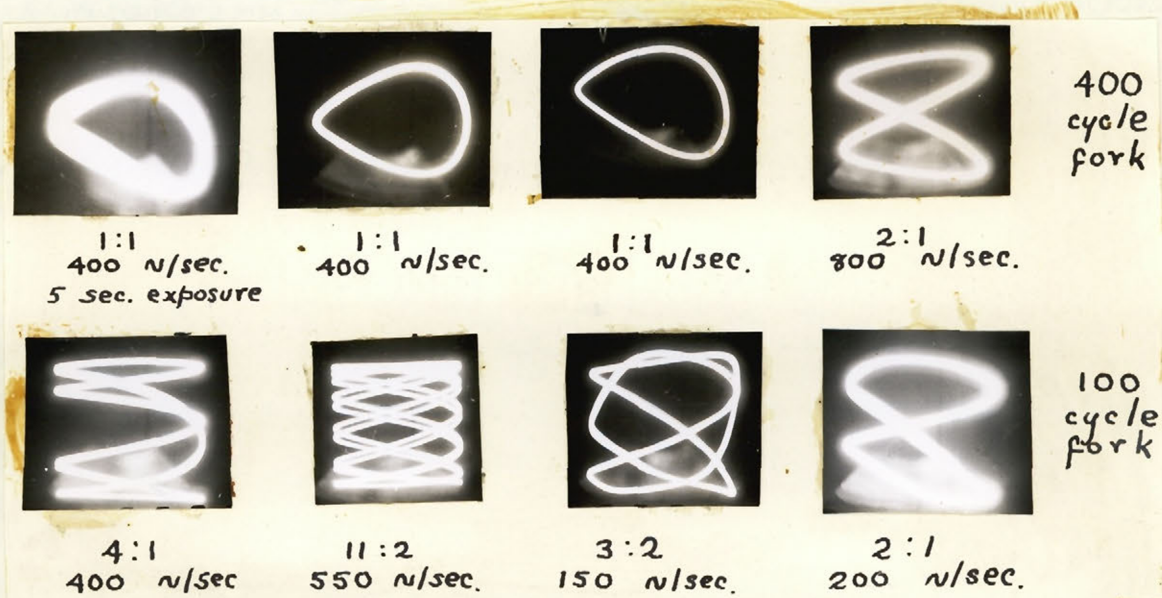
Klystron Power Supply used with 2K35

The power output from the 2K35 is taken by coaxial line to the high frequency cavity via coaxial line. The power output was found to be sufficient for the purposes of exciting the high frequency cavity, that is at resonance full scale indicator deflections could be obtained with a high degree of decoupling of the probes in the cavity.

CHAPTER V

Measurement of Frequency and Amplitude, and Experimental Procedure.

The principle of the frequency measuring system is as follows. The output of the ten megacycle per second frequency doubler is detected along with a standard frequency at ten megacycles per second by means of a receiver tuned to ten megacycles. The resulting audio beat frequency is compared to standard audio frequencies by means of Lissajou figures on a Cathode Ray Oscillograph.



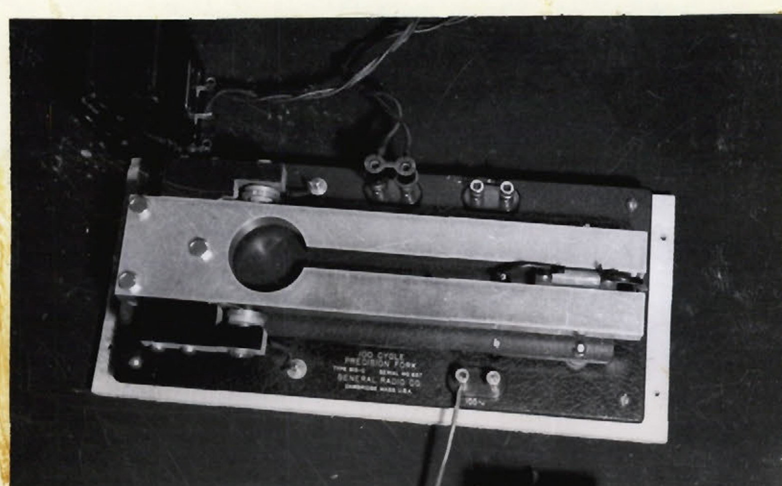
Typical Lissajou Patterns

It was thought that the standard frequency broadcast at 10 mc./sec. by the transmitter of the National Bureau of Standards at Washington D.C.(17) could be used as the standard against which to beat the local signal. Measurements were made using this system but reception of the signal was very sporadic also the signal was very noisy and of very low level. The greatest drawback however was that measurements could be made for only 20% of the time, since the standard carrier frequencies are modulated with audio frequencies of 440 and 600 cycles per second alternately in five minute intervals. In only one minute of this five minute interval is the modulation removed, even then code signals and noise announcements are made. In the presence of modulation many beat frequencies are obtained and it was possible to obtain several different values for the beat frequency during the period of modulation. Before discarding this plan a special antenna was erected between the roofs of two buildings on the university campus. It was directed so as to have its greatest sensitivity in the general direction of Washington. This is an untuned antenna whose performance was very much better than the antenna previously used.

A type 1100 - AQ Secondary Frequency Standard was purchased from the General Radio Company of Boston, Massachusetts to circumvent these difficulties. The specifications for this instrument state that after one months operation the frequency drift is less than five parts in 10^8 per day - this decreases to about 0.5 parts in 10^8 after a year's operation. Ordinary changes in air pressure, ambient temperature and line voltage have practically no effect on its frequency. The temperature coefficient of frequency of the quartz bar is less than one part in 10^7 per degree C and the temperature is controlled to within $\pm$ 0.01°C. Over short periods of time the frequency fluctuations are less than one part in 10^6. The overall accuracy of the standard is more than sufficient for the purposes of measurement.

1100 - AQ Secondary Frequency Standard

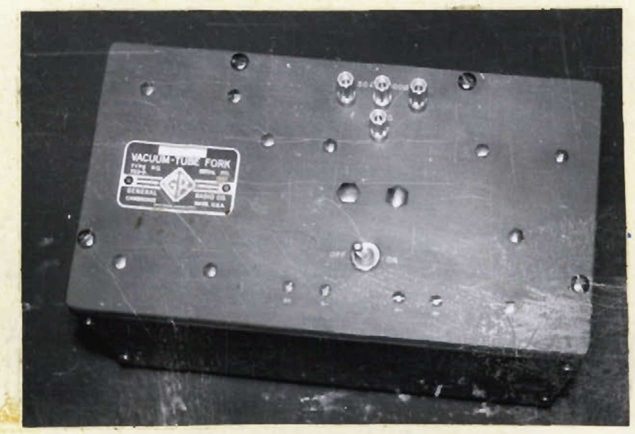
The standard audio signal is provided by a type 815 - C precision tuning fork made by the General Radio Company.



815C General Radio Precision tuning fork

The calibration data gives the frequency as 99.9998 cycles per second $\pm$ 0.001% with a temperature coefficient of frequency of 9.3 parts in 10^6 per degree Fahrenheit, negative. For steady Lissajou figures any error due to variations in the frequency of this fork are quite negligible. This fork is battery operated.

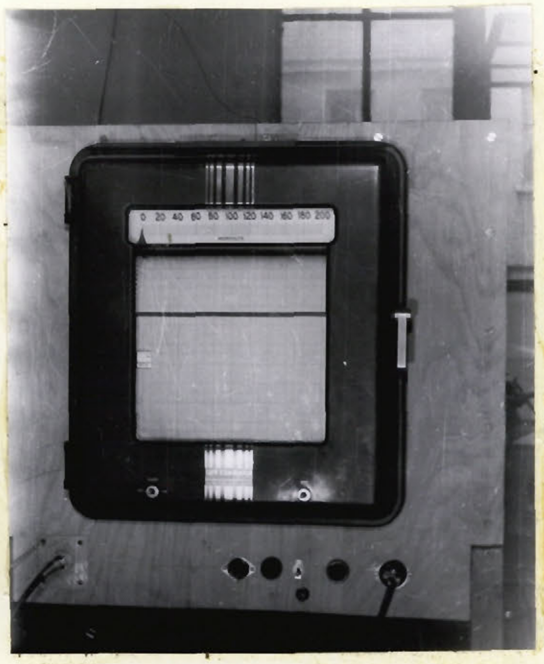
A second tuning fork, a type 723 Vacuum tube Fork - frequency 400 cycles per second - made by the General Radio Company was used in conjunction with the one mentioned above.



Type 723 General Radio Vacuum Tube Fork.

The two forks are connected to a D.P.D.T. switch so that either could be connected to the cathode ray oscillograph. This is not a precision instrument but it is very valuable as an aid in identification of the more complicated Lissajou figures obtained with the lower frequency fork.

The measurement of amplitude was accomplished as follows: a Speedomax type G Recording Semi Automatic Potentiometer manufactured by the Leeds and Northrup Company of Philadelphia Pennsylvannia was used to measure and record the output of a type 93L insulated vacuum thermocouple. This thermocouple is made by the American Thermo Electric Company.



The type G Speedomax Recording Potentiometer

The output probe or loop in the cavity drives the heater of this thermocouple to which it is connected by coaxial line. A monitor thermocouple and galvonometer is employed as a check in the input power to the cavity. The monitor indicated a very high degree of amplitude stability and it was found that the more sensitive test of stability was to see whether the amplitude as indicated by the Recording Potentiometer was accurately repeated over successive resonance curves. Deviations of not more than one mm. in about 170 mm. are found to be quite easily obtainable in successive resonance curves - the greatest part of this deviation was found to be due to drift in the zero position of the recording meter itself and not to actual alterations of the detected signal.

The experimental points whose coordinates are amplitude squared and frequency do not follow one another in systematic order due to the fact that the padding condensers in the five megacycle per second oscillator circuit are not set up in a perfectly graduated succession, so that as one padding condenser follows another the corresponding frequencies while tending to go from higher to lower frequencies are actually out of order at places. This needs mentioning only to explain the somewhat irregular appearance of the array of points as displayed on the record of the Speedomax recording potentiometer.

The procedure followed is to replot the points on a linear frequency scale - the resonance curve is then found to have a truly symmetrical inverse parabolic shape about a centre line. The location of the resonant frequency is found by the following procedure. Two points on the curve at the same height above the base line are selected and the point mid way between them is found. This is done for a succession of different pairs of points and the centre frequency is easily found by connecting the mid points with a straight line and projecting this to cut the curve. Since the curve is a symmetrical this process can be done by simply folding the curve back on itself until the two sides match. It is estimated that the resonant frequency can thus be found accurately to about $\pm$ 5 cycles per second in 10^7 cycles per second. This figure depends very largely upon the width of the resonance curve. The above figure is obtained from experimental observations on the low frequency cavity used in this project which has a Q factor of approximately 4000. Since two resonance curves are required for the determination of the dielectric constant the band width of the apparatus should be wide enough that the two resonance curves may be delineated down to their half power points on both sides of resonance. Thus high Q factors are very desirable, but are difficult to obtain practically especially at frequencies of the order of three hundred megacycles per second where the wavelength is about one meter. The reason for this is that the cavities having high Q factors in the frequency range are so very large. Skin effect resistance is actually lower at these wavelengths than it is at the shorter wavelengths, so that higher Q factors are possible at longer wavelengths. A simple calculation shows that for a cubical cavity resonator in the TE_{011} made at 270 megacycles per second the Q factor amounts to 67,500 when the walls are silver plated. (18) The great difficulty with this is that the edge of the cube is 78.5 centimeters long, to say nothing of the practical difficulties in getting perfect joins along the whole length of the twelve edges of the cube. The principal mode in a half wavelength coaxial line cavity was used therefore, to reduce size since the cavity must be placed in a vacuum chamber and to obtain a reasonable Q factor with very much greater practical ease.

The determination of the loss tangent of gases is difficult since it is so very small - but it too becomes much more definite the higher the Q factor. The reason for this is that the higher the Q factor used the larger will the dielectric losses be in relation to the metal losses. Thus the loss tangent becomes the difference between two quantities which do not differ too greatly from the loss tangent itself. The determination is thus much more accurate.

Several writers have published articles in the literature describing microwave gas dielectric measurements using Pound stabilized klystrons as their signal sources.[19][21][24] This system is limited to microwave frequencies for which klystrons are available and the use of the Pound stabilizer does not provide for the evaluation of the loss tangent of the dielectric under test.

It is true that the Pound stabilized generator does enable a very accurate determination of resonant frequency (about ± 100 cycles per second in 9000 mc./sec. - under the best conditions [25]). However these signal sources are not entirely free of frequency modulation - since their operation depends upon such action.

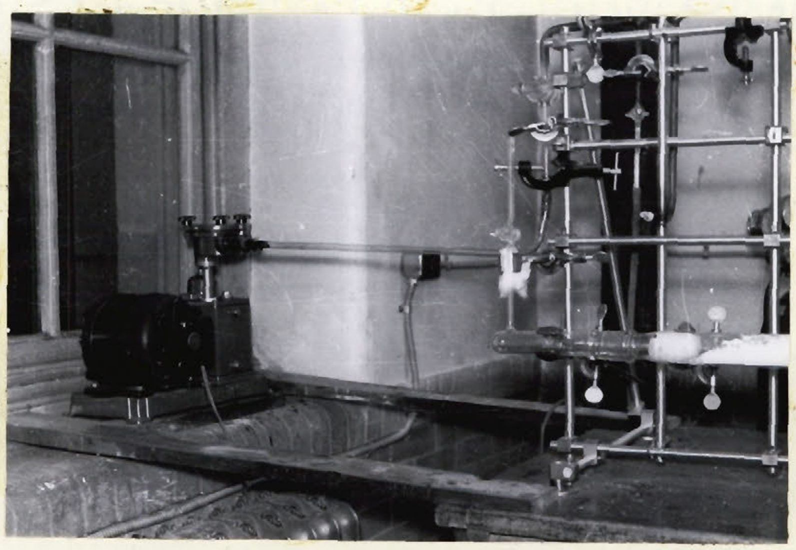
In this project very little was done at frequencies where the two systems may be compared.

For measurements at 270 megacycles per second the use of a crystal controlled oscillator and frequency multipliers appears to be the most satisfactory system available. At frequencies for which the two systems can be compared, the system used appears to be superior to the Pound stabilized generator in that it may be used for measurements of lossy dielectrics which cannot be done with the Pound system. The author claims that the method of measurement used in this project is original in that no references in the literature have been found of its use for measurements at frequencies of the order of 300 megacycles per second. Dielectric measurements at these frequencies are difficult because of the large sizes of the required components and the difficulty of obtaining high Q factors as pointed out elsewhere in this thesis.

CHAPTER VI

The Vacuum System

When dielectric measurements are made on gases using cavities it is necessary to be able to remove the gaseous dielectric so that resonant frequency and Q factor may be determined in the absence of the dielectric. These determinations are important since the dielectric properties depend upon the difference in these quantities between evacuated and filled conditions. Consequently a vacuum system was constructed as shown in the photograph below.



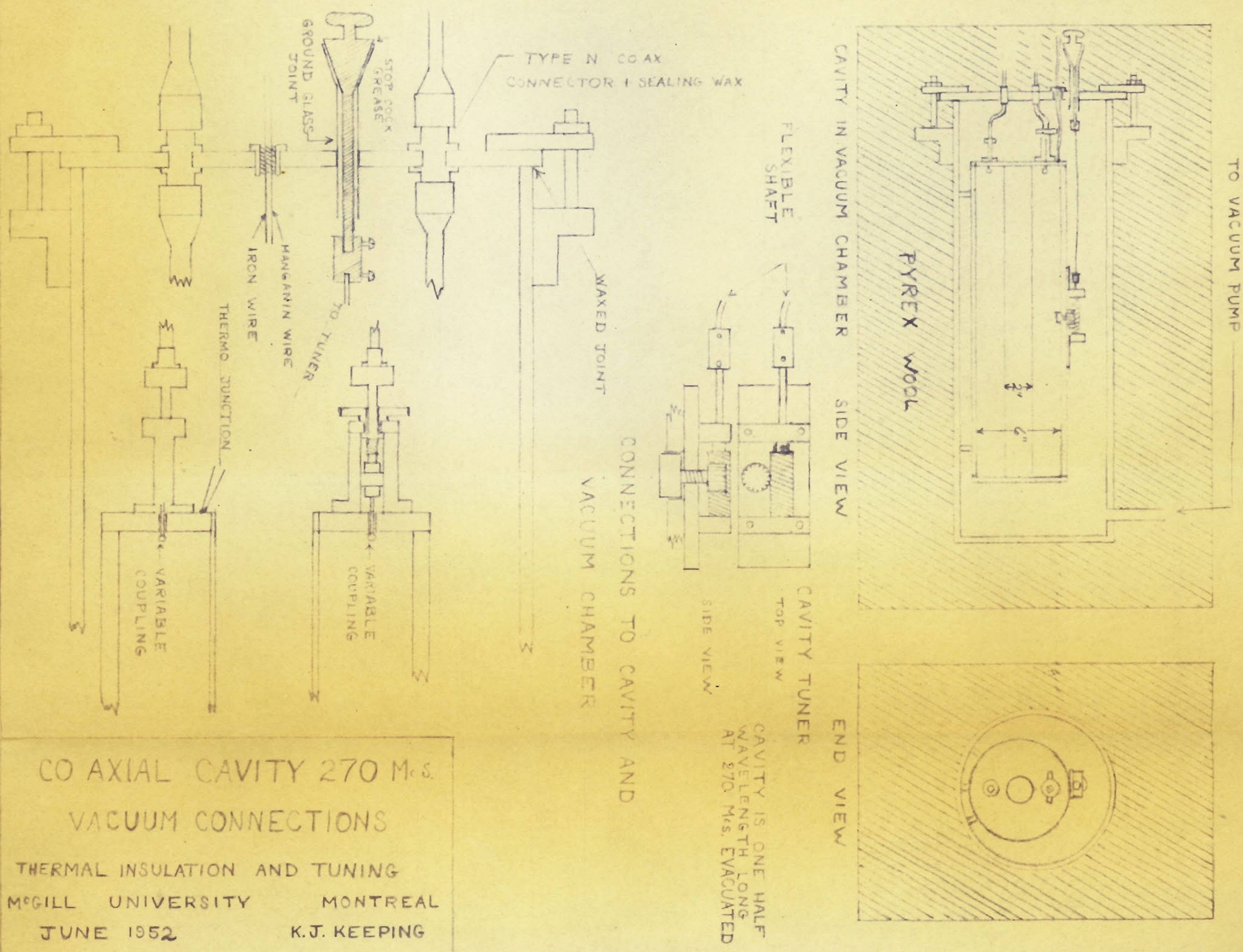
The vacuum pump and a portion of the vacuum system

A "Speedivac" rotary oil pump (Type 2S20), manufactured by W. Edwards and Company, London, England, was employed in conjunction with a phosphorous pentoxide moisture trap. This can be seen at the left of the above photograph. The pump has an ultimate vacuum of

0.0001 mm. Hg. as measured with a McLeod Gauge. The lower pressures were measured by means of a type 710 thermocouple gauge manufactured by the National Research Corporation of Cambridge, Mass. This instrument indicated ultimate pressures of one micron in the glass system itself and of about 30 microns for the coaxial cavity and glass system combined. This order of vacuum is sufficiently high so that the residual gas pressure can introduce only about 0.01% error in the measurement of $|K_e - 1|$ and is therefore negligible when compared with the 1% accuracy in $|K_e - 1|$ actually obtained.

Water vapour is very undesirable in the vacuum system due to its relatively large dielectric constant as compared to dry air [19] (1.02705 as compared to 1.000572 for dry air). Therefore in addition to the P_2O_5 moisture trap on the pump a drying tube was constructed from pyrex glass tubing of about 2 inches diameter about five feet long. This was made in two sections and was constructed in such a way that it can easily be detached and cleaned. It is packed with phosphorus pentoxide and pyrex glass wool. The gas being measured is slowly admitted to the system through the drying tube taking up to an hour to raise the gas pressure from 30 microns to 25 cms. Hg. At first the drying tube was only 2½ feet long and it was decided to see what difference if any resulted from doubling its length. There was no apparent effect in the measured results. Thus it is assumed that the gases admitted to the system are sufficiently dry that any remaining water vapour is completely negligible. Mr. Paul Lorimer of the Radiation Laboratory of McGill University made the drying tubes.

The higher gas pressures were measured with an "Absolute and Differential Mercury Manometer" connected to the vacuum system. This manometer is manufactured by the Emil Grenier Company of New York. It has a vernier which makes it possible to read the pressure to 0.1 mm. Hg. The two vacuum chambers which complete the vacuum equipment will be separately described in the following chapters. Pyrex glass was used throughout in the vacuum system. The pump is connected to the glass by means of a ground glass to metal joint which is treated with stop cock grease. The cavity is connected to the glass by means of a glass to metal joint sealed with sealing wax.



CO AXIAL CAVITY 270 Mcs.

VACUUM CONNECTIONS

THERMAL INSULATION AND TUNING

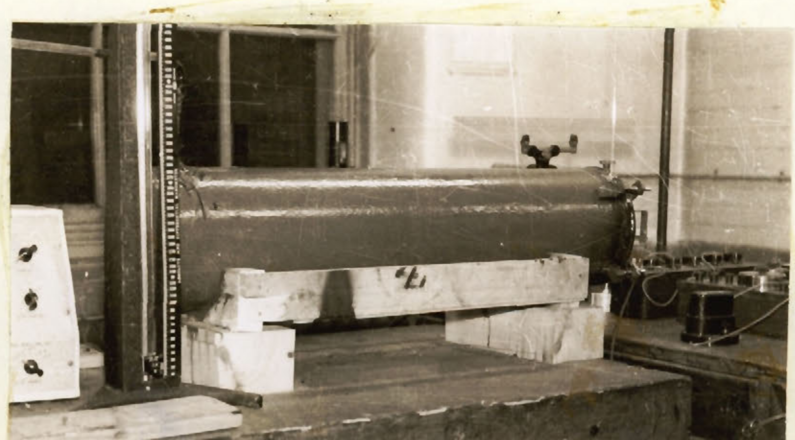
McGILL UNIVERSITY MONTREAL

JUNE 1952 K.J. KEEPING

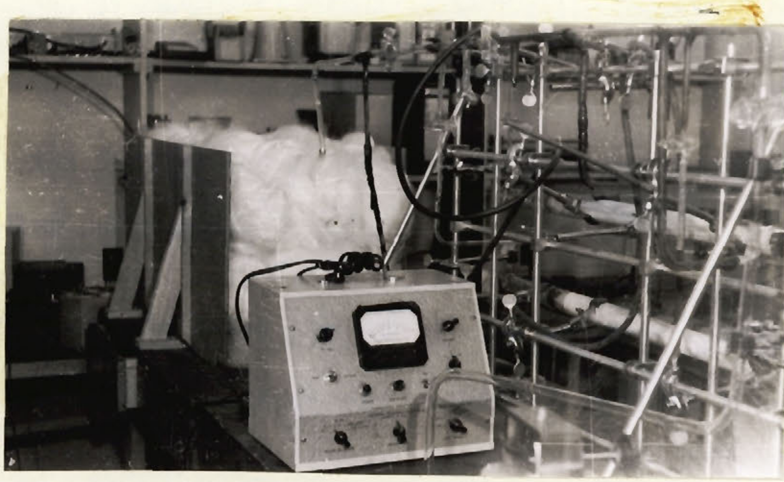
CHAPTER VII

The Coaxial Cavity, Vacuum Chamber and Gas Measurements at 270 megacycles per second.

Due to large size of the coaxial line cavity at 270 megacycles per second (about $6\frac{3}{4}$ inches outside diameter and about 28 inches overall length including coaxial connectors and probe adjustors,) a large vacuum chamber is necessary. A piece of steel pipe of 8 inch inside diameter and 3/8 inch wall thickness was located and a 3 foot length was cut off and machined by Mr. F. Corrick of the Mechanical Engineering Department. He welded a cover plate to one end and machined a removeable cover plate for the other end, both from 3/8 inch steel plate. It was not new pipe due to difficulty in obtaining pipe of this large diameter and proved to be slightly porous. This was overcome very largely by painting the outside of the vacuum chamber liberally with Glyptal paint. The cover plate seal is rendered vacuum tight by means of sealing wax. All of the leads passing through the cover plate are similarly sealed.



The vacuum chamber for the coaxial cavity - (Pyrex wool) lagging removed.



The vacuum chamber for the coaxial cavity with lagging.

The vacuum chamber is thoroughly lagged on the outside with Pyrex glass wool for thermal insulation. This insulation was increased until it was found that in the time necessary to take a set of resonance curves the temperature drift was negligible, being much less than a tenth of one degree Centigrade. This factor is very important because the linear coefficient of expansion of brass is about 16 parts in 10^6 per degree Centigrade. The change in resonant frequency brought about by thermal effects can be very objectionable unless precautions are taken to render such changes negligible.

The thermal detuning of a coaxial cavity is easily illustrated as follows $f = \frac{c}{\lambda}$ where $\lambda = 2l$ for a half wave length cavity $\therefore \frac{df}{f} = -\frac{dl}{l} = -\alpha \Delta t$

where f is the resonant frequency of the cavity

λ is the resonant wavelength

l is the length of the resonating cavity

α is the linear thermal coefficient for the metal walls

Δt is the increase of temperature.

But $|K_e - 1| = \frac{2\Delta f}{f}$ already proved in Chapter I. Therefore $\alpha\Delta t$ must be kept small compared to $|K_e - 1|$. Thus temperature changes kept smaller than one tenth of a degree Centigrade cannot prove serious since this can effect only the sixth place of decimals and is therefore well under one percent of $|K_e - 1|$ for all gases measured.

The effect of temperature drift was minimized by the following procedure. After filling the cavity with air (or gas) the temperature was periodically checked until the thermocouple showed no change in successive readings. Several resonance curves were then taken. When it was observed that the curves were accurately repeated it was assumed that any small temperature change due to the air or gas which had just been admitted had ceased. The chamber was then evacuated and resonated again - there is much less cause for temperature drift here - but the same procedure was used to obtain the best possible results.

A summary of experimental results follows which gives the dielectric constant for dry air and several dry gases. The thermocouple used to measure the cavity temperature was made from iron and manganin. The cold junction was held at 0° C using crushed ice. The thermal e.m.f. of this couple was measured with a No. 55 Potentiometer (D.C.) made by Elliott Bros. of London, England. A Weston standard cell was used for calibration purposes.

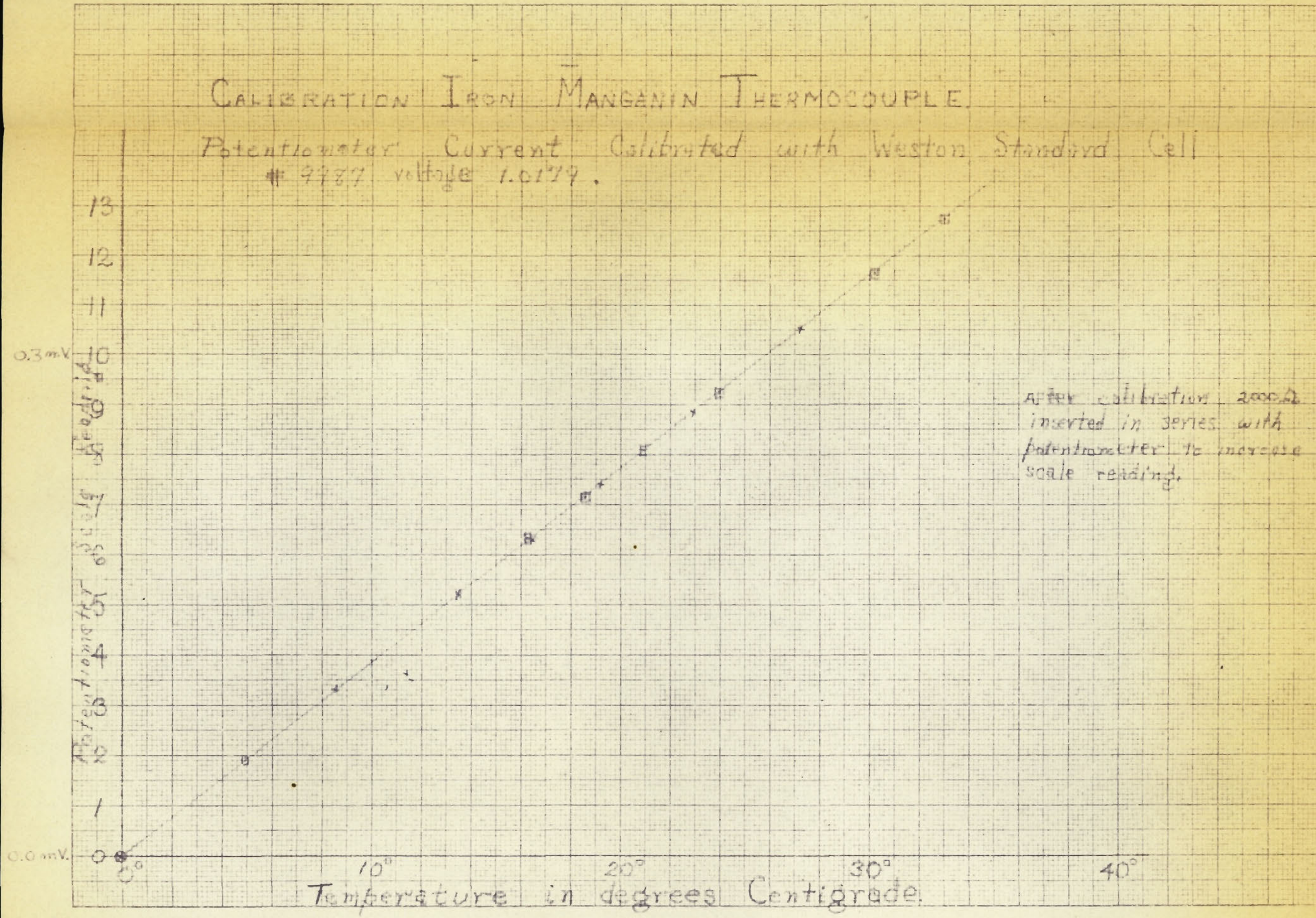
The indicator used was a sensitive galvanometer. The calibration of this thermocouple showed that it was linear over the range 0° C to 50°C and temperature changes as small as 0.1°C were easily read on the potentiometer scale.

A tuning arrangement was found to be essential since the evacuated resonant frequency depends upon the temperature of the cavity which for the low frequency cavity very slowly follows room temperature as it rises and falls. Thus with no tuning arrangement it is not possible to resonate the cavity when the temperature differs by ± 5°C from the value which causes the cavity to resonate at the centre of the band. The situation is somewhat worse than this however, since when gas is admitted the resonant frequency falls and it is desired to get the second resonance curve within the fixed band width. Therefore temperature changes are restricted to less than half of the above quoted ± 5°C when no arrangement is made to offset the thermal lengthening and shortening of the cavity.

The tuning mechanism consists of a brass plug which projects slightly through the surface of the outer conductor. The amount of penetration of the plug can be controlled by means of screw and pinion gears which are controlled by means of flexible cable from the exterior of the vacuum chamber.

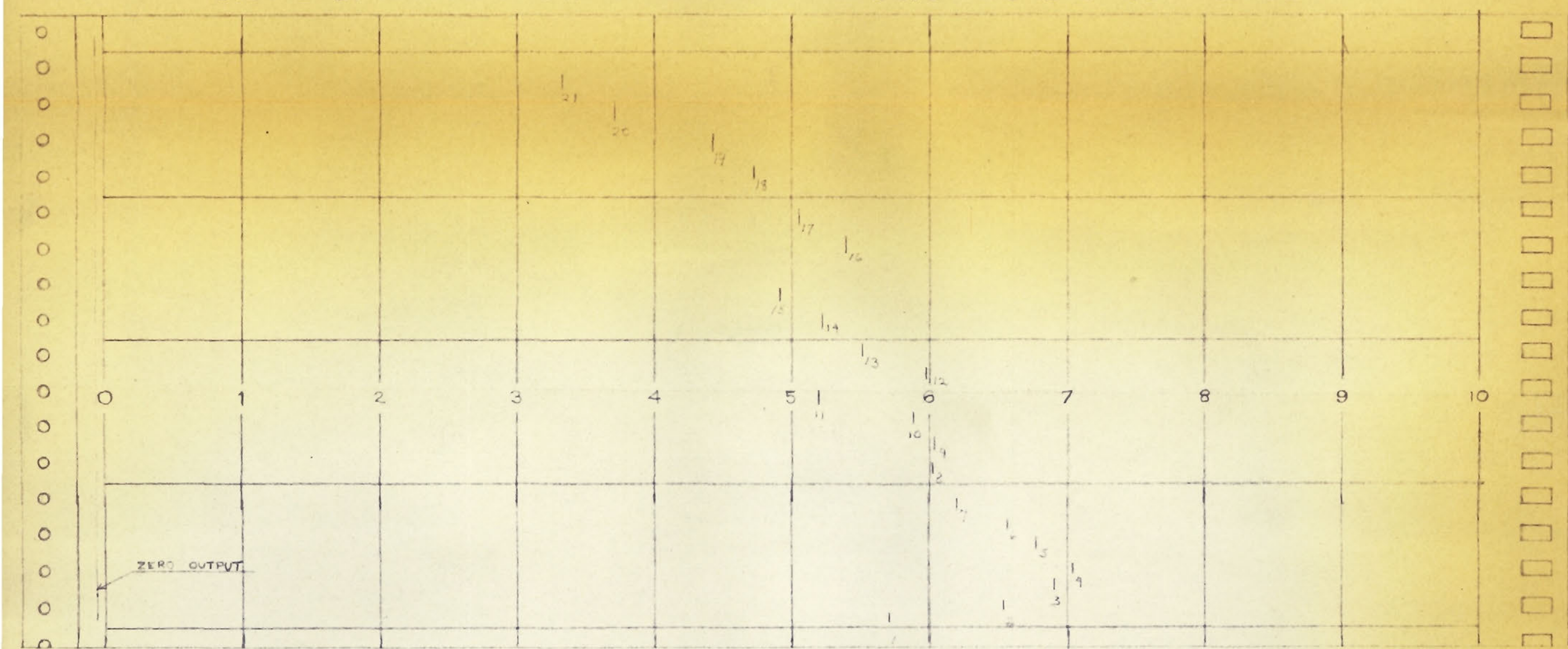
In the following section a detailed example is given showing how the original data is used to obtain the dielectric constant and loss tangent. Following this is a summary of experimental results giving original data in tabular instead of graphical form.

359-14 KEUFFEL & ESSER CO.
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U. S. A.



Typical Record of Vacuum Thermocouple Output as Obtained From The Recording Potentiometer

DRY AIR PRESSURE 31 MICRONS CAVITY TEMPERATURE 22.6 DEGREES CENTIGRADE

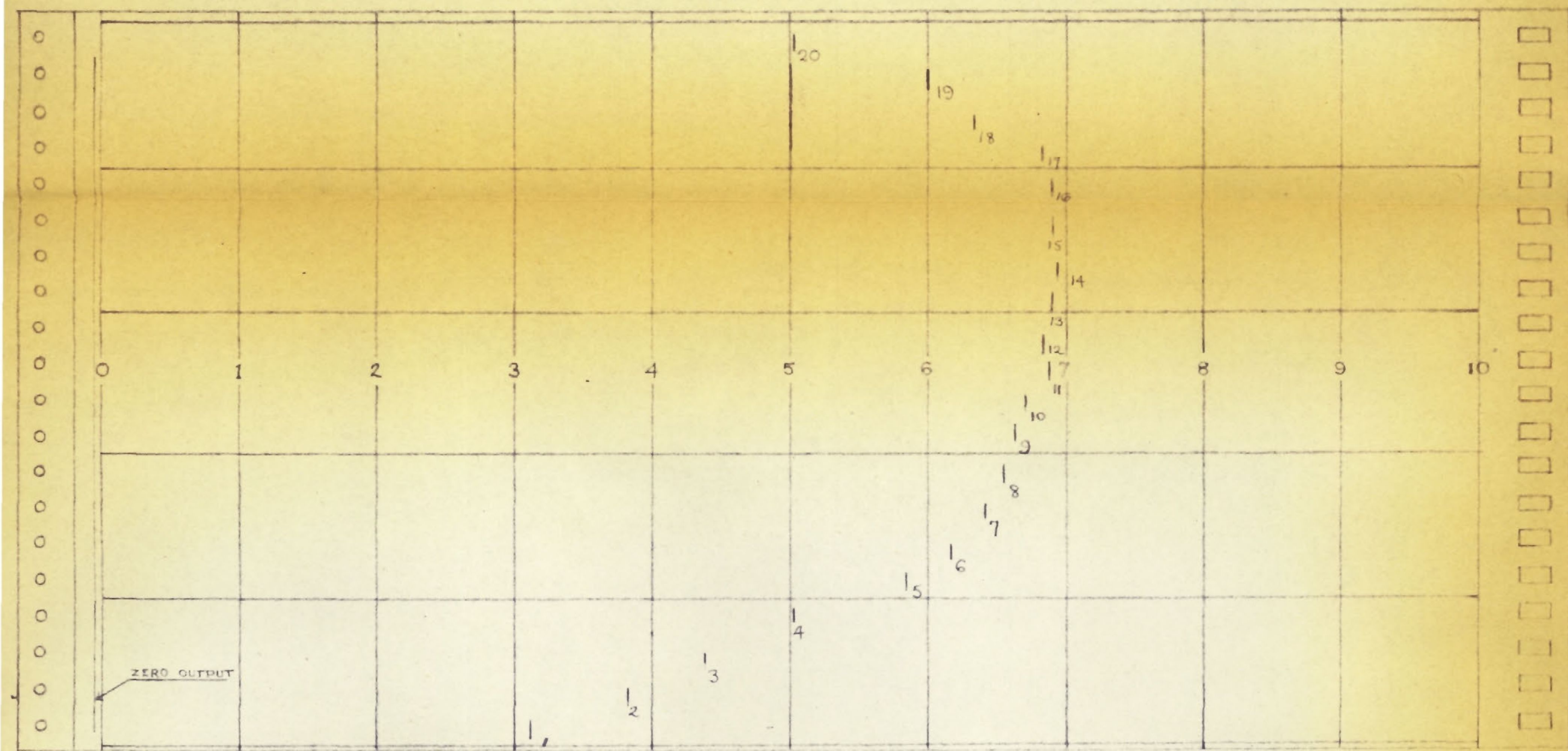


FREQUENCY CORRESPONDING TO EACH OF THE ABOVE EXPERIMENTAL POINTS USING 100 CYCLE FORK

Point No.	Freq ~/sec	Point No.	Freq. ~/sec	Point No.	Freq. ~/sec.	Point No.	Freq. ~/sec	Point No.	Freq. ~/sec
1	$10^7 + 1200$	6	$10^7 + 150$	11	$10^7 - 300$	16	$10^7 - 250$	21	$10^7 - 933$
2	$10^7 + 900$	7	$10^7 + 0$	12	$10^7 - 50$	17	$10^7 - 350$	22	
3	$10^7 + 700$	8	$10^7 - 50$	13	$10^7 - 200$	18	$10^7 - 450$	23	
4	$10^7 + 500$	9	$10^7 - 50$	14	$10^7 - 300$	19	$10^7 - 550$		
5	$10^7 + 250$	10	$10^7 - 100$	15	$10^7 - 400$	20	$10^7 - 800$		

THE ACTUAL POTENTIOMETER RECORD OF THE TYPICAL VACUUM THERMOCOUPLE OUTPUT—CAVITY EVACUATED.

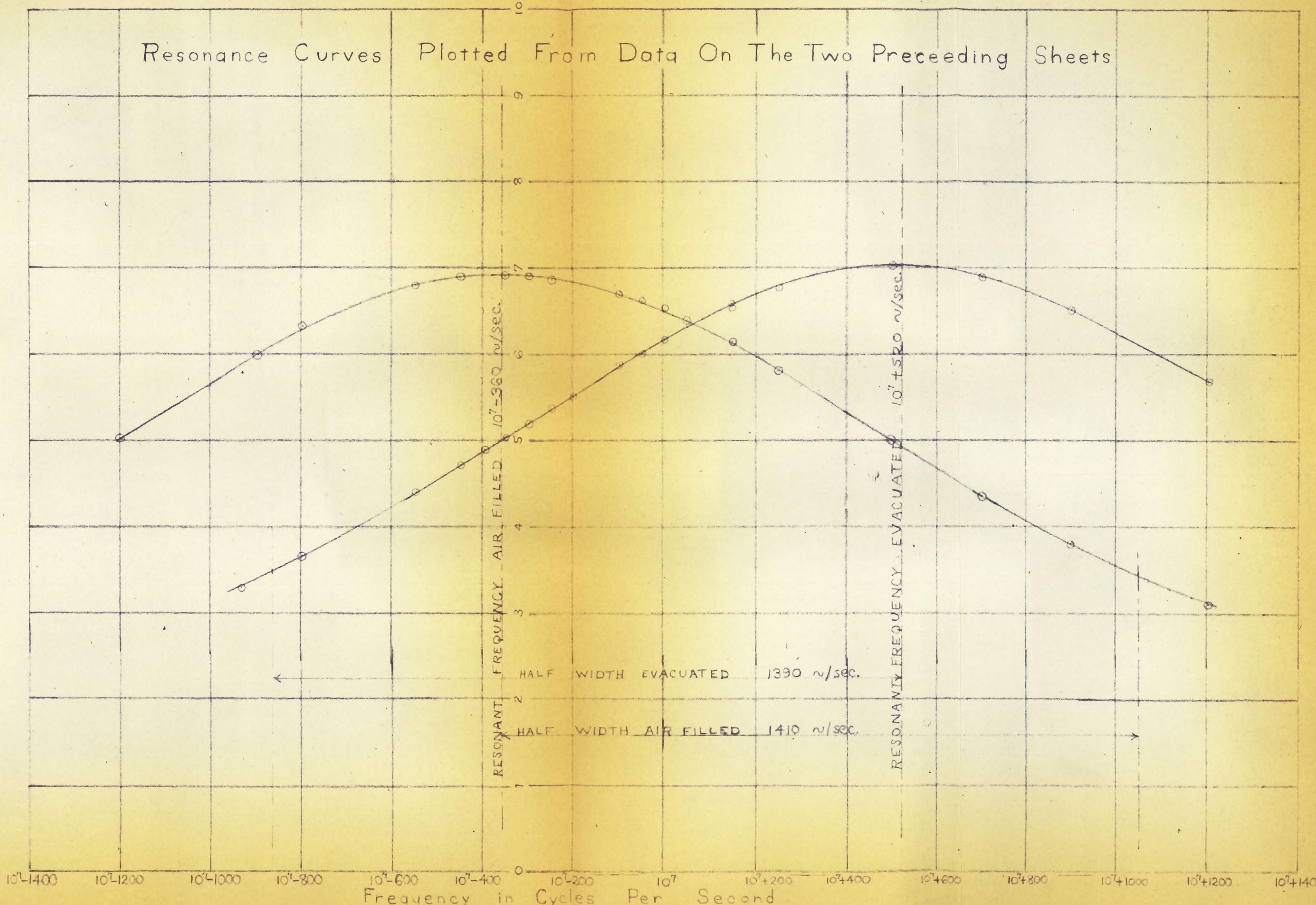
DRY AIR PRESSURE 25.11 CMS. MERCURY CAVITY TEMPERATURE 22.6 DEGREES CENTIGRADE



FREQUENCY CORRESPONDING TO EACH OF THE ABOVE EXPERIMENTAL POINTS USING 100 CYCLE FORK

Point No.	Freq. ~/sec.	Point No.	Freq. ~/sec.	Point No.	Freq. ~/sec.	Point No.	Freq. ~/sec.	Point No.	Freq. ~/sec.
1	10^7+1200	5	10^7+250	9	10^7-50	13	10^7-300	17	10^7-550
2	10^7+900	6	10^7+150	10	10^7-100	14	10^7-400	18	10^7-800
3	10^7+700	7	10^7+50	11	10^7-250	15	10^7-350	19	10^7-900
4	10^7+500	8	10^7+0	12	10^7-200	16	10^7-450	20	10^7-1200

THE ACTUAL POTENTIOMETER RECORD OF THE TYPICAL VACUUM THERMOCOUPLE OUTPUT — CAVITY FILLED



Resonance Curves Plotted From Data On The Two Preceeding Sheets
RESONANT FREQUENCY AIR FILLED 10^7-360 ~/sec.
RESONANT FREQUENCY EVACUATED 10^7+520 ~/sec.
HALF WIDTH EVACUATED 1390 ~/sec.
HALF WIDTH AIR FILLED 1410 ~/sec.
0
1
2
3
4
5
6
7
8
9
10
10^7-1400
10^7-1200
10^7-1000
10^7-800
10^7-600
10^7-400
10^7-200
10^7
10^7+200
10^7+400
10^7+600
10^7+800
10^7+1000
10^7+1200
10^7+1400
Frequency in Cycles Per Second

Calculations of dielectric constant and loss tangent of dry air based on information given on preceding graph. $|K_e - 1| = \frac{2\Delta f}{f_o}$

where Δf is the change in resonant frequency due to introduction of the dielectric, and f_o is the resonant frequency evacuated

$\therefore |K_e - 1| \times 10^6 = \frac{2 \times 880 \times 10^6}{10^7} = 176$ for dry air at a pressure of 25.11 cms. Hg. and a temperature of 22.6°C.

The index of refraction $\eta = \sqrt{K_e}$ for lossless media

$\therefore |\eta - 1| \times 10^6 = \frac{880 \times 10^6}{10^7} = 88.0$ for dry air at a pressure of 25.11 cms. Hg and a temperature of 22.6°C.

To reduce the measured index of refraction to standard conditions it is not strictly correct to use the ideal gas law. The formula used should allow for the fact that air deviates slightly from this law. (21)

$$\therefore (\eta_{0,760} - 1) \times 10^6 = (\eta_{t,p} - 1) \times 10^6 \times \frac{760.606(1 + 0.003661\,t)}{p[1 + p(1.049 - 0.0157\,t) \times 10^{-6}]}$$

p is the barometric pressure in mm.

t is the temperature in °C.

$$\therefore (\eta_{0,760} - 1) \times 10^6 = 88.0 \times \frac{760.606\,(1 + 0.003661 \times 22.6)}{251.1 \quad 1 + 251.1(1.049 - 0.0157 \times 22.6) \times 10^{-6}}$$

$$= \frac{88.0 \times 760.606 \times 1.08274}{251.1 \times 1.000174}$$

$$= 288.567$$

neglecting the term 1.000174 in the denominator

$$(\eta_{0,760} - 1) \times 10^6 = 288.61$$

Since the experimental accuracy to which $\Delta \bar{\rho}$ is known is only to the third significant figure, this term may be dropped and air to this degree of approximation follows the ideal gas law. The formula required then to reduce the refractive index to standard conditions of temperature and pressure is

$$(\eta_{0,760} - 1) \times 10^6 = (\eta_{t,p} - 1) \times 10^6 \times \frac{760.606\ (1 + 0.003661\,t)}{p}$$

The value of the index of refraction found in this experiment for dry air under standard conditions is $1 + 288 \times 10^6$. The value quoted by Essen and Froome is $1 + (288.15 \pm 0.1) \times 10^{-6}$, for dry air at 24,000 Mc/Sec.

The loss tangent is calculated from the formula

$$\tan\delta = \frac{1}{Q\ \text{filled}} - \frac{1}{Q\ \text{evacuated}} = \frac{2 \times 1410}{10^7} - \frac{2 \times 1390}{10^7}$$

$$= \frac{2 \times 20}{10^7} = 4 \times 10^{-6}$$

Summary of Experimental Results Dry Air.

Res. Freq. evac. cy./sec.	Res. Freq. filled cy./sec.	(Ke - 1) x 10^6	(n - 1) x 10^6	$\frac{1}{Q \text{ evac.}}$	$\frac{1}{Q \text{ filled}}$	$\tan\delta$	Pressure mm. Hg	temp. 0°C.	(n - 1) x 10^6 N.T.P.	(Ke - 1) x 10^6 N.T.P.
$10^7 + 520$	$10^7 - 360$	176.0	88.0	$\frac{2 \times 1390}{10^7}$	$\frac{2 \times 1410}{10^7}$	4×10^{-6}	251.1	22.61	288	576.0
$10^7 + 635$	$10^7 - 245$	176.0	88.0	$\frac{2 \times 1405}{10^7}$	$\frac{2 \times 1420}{10^7}$	3×10^{-6}	250.0	22.02	289.4	578.8
$10^7 + 455$	$10^7 - 436$	178.2	89.1	$\frac{2 \times 1400}{10^7}$	$\frac{2 \times 1420}{10^7}$	4×10^{-6}	254.0	24.25	290.0	580.0
$10^7 + 580$	$10^7 - 460$	177.0	88.5	$\frac{2 \times 1400}{10^7}$	$\frac{2 \times 1420}{10^7}$	4×10^{-6}	248.0	24.75	290.0	580.0

Mean value $|Ke - 1| \times 10^6 = 578.7$ Maximum Deviation $= \frac{2.7 \times 100}{578.7} = 0.4\%$

The values shown for $\tan\delta$ cannot be taken to indicate its actual value since occasionally much smaller values were obtained and at times the values were even slightly negative. However it may be concluded that for dry air at a pressure of 25 cms. of Mercury and a temperature of 22°C, the loss tangent is less than 5×10^{-6}. The quantity is so small that it cannot be resolved by the apparatus.

Summary of Measured Results for Oxygen

The oxygen used for the measurements was of 99.98% purity as supplied by the Dominion Oxygen Company.

	Res. Freq. evac. cy./sec.	Res. Freq. filled cy./sec.	$(Ke - 1) \times 10^6$	$(n - 1) \times 10^6$	$\frac{1}{Q \text{ evac.}}$	$\frac{1}{Q \text{ filled}}$	$\tan \delta$	Pressure mm. Hg.	temp. $0^oC.$	$(n - 1) \times 10^6$ N.T.P.	$(Ke - 1) \times 10^6$ N.T.P.
I	$10^7 + 263$	$10^7 - 558$	164.2	82.1	$\frac{2 \times 1390}{10^7}$	$\frac{2 \times 1400}{10^7}$	2.0×10^{-6}	260.0	27.4	264	528
II	$10^7 + 535$	$10^7 - 273$	161.6	80.8	$\frac{2 \times 1400}{10^7}$	$\frac{2 \times 1410}{10^7}$	2.0×10^{-6}	251.5	23.25	266	532
III	$10^7 + 773$	$10^7 - 70$	168.6	84.3	$\frac{2 \times 1380}{10^7}$	$\frac{2 \times 1395}{10^7}$	3.0×10^{-6}	262.5	23.00	264.2	528.5
IV	$10^7 + 595$	$10^7 - 275$	174	87.0	$\frac{2 \times 1370}{10^7}$	$\frac{2 \times 1380}{10^7}$	2.0×10^{-6}	272.1	25.71	266	531.5

Mean value 530 for $|Ke - 1|$ Maximum Deviation = 0.38%

The value obtained for $\tan \delta$ here is also not sufficiently consistent to form a definite answer as to its true value. It may be said however that $\tan \delta$ for dry oxygen is less than 5×10^{-6} when the pressure is 25 cms. of Mercury and the temperature is 25^oC. The values of $(n - 1)$ and $(Ke - 1)$ were reduced to standard conditions using the ideal gas law.

Summary of Measured Results for Carbon Dioxide

The carbon dioxide used was of 99.5% or better purity as supplied by Liquid Carbonic Canadian.

Res. Freq. evac. cy./sec.	Res. Freq. filled cy./sec.	$(Ke - 1) \times 10^6$	$(n - 1) \times 10^6$	$\frac{1}{Q \text{ evac.}}$	$\frac{1}{Q \text{ filled}}$	$\tan \delta$	Pressure mm. Hg.	Temp. $0^\circ C.$	$(n - 1) \times 10^6$ N.T.P.	$(Ke - 1) \times 10^6$ N.T.P.
$10^7 + 370$	$10^7 - 625$	198	99.5	$\frac{2 \times 1370}{10^7}$	$\frac{2 \times 1380}{10^7}$	2×10^{-6}	170.86	23.75	494.7	989.5
$10^7 + 320$	$10^7 - 645$	193	96.5	$\frac{2 \times 1380}{10^7}$	$\frac{2 \times 1380}{10^7}$	Zero	160.01	23.0	497.5	995
$10^7 + 320$	$10^7 - 324$	128.8	64.4	$\frac{2 \times 1380}{10^7}$	$\frac{2 \times 1395}{10^7}$	3×10^{-6}	113.58	23.0	493.5	987
$10^7 + 383$	$10^7 - 518$	180.2	90.1	$\frac{2 \times 1380}{10^7}$	$\frac{2 \times 1385}{10^7}$	1×10^{-6}	155.78	23.5	495.5	991

Mean value (Ke - 1) 990.63 Maximum deviation 0.44%

Carbon dioxide deviates from the ideal gas law and Von der Waal's equation must be used. The volume of the vacuum chamber + glass system must be known to use this equation. This was calculated to be 27.8 litres from the dimensions of the apparatus making allowances for the volume occupied by the cavity - connection - tuning apparatus etc. Then using the constants given by Maas and Steacie the formula required to convert (Ke - 1) is derived as follows:-

$$\left(P + \frac{7.06}{V^2}\right)(V - 0.166) = \frac{w}{m} \times R.T.$$

Where P is the pressure of CO_2 in atmosphere

V " " volume " " " litres = 27.8 litres

w " " mass " " " grams

m " " molecular weight of CO_2 in grams

R " " gas constant in litre - atmosphere

T " " temperature in degrees Absolute.

7.06 and 0.166 are the appropriate constants for Carbon Dioxide as given by Maas and Steacie.[22]

Since (Ke - 1) depends on the number of molecules per unit volume and the volume is a constant under the conditions of the experiment, (Ke - 1) depends upon the mass of the enclosed gas.

$$\text{thus } \frac{(Ke - 1)_{n.t.p.}}{(Ke - 1)} = \frac{w_1}{w_2} = \frac{(1 - 0.00914)}{(P - 0.00914)} \times \frac{273.18 + t}{273.18}$$

$$\therefore (Ke - 1)_{n.t.p.} = (Ke - 1)_{measured} \times \frac{0.99086}{273.18} \times \frac{(273.18 + t)}{(P - 0.00914)}$$

where t is temperature in degrees C. at which the experiment is conducted and P is the pressure in atmospheres.

Summary of Experimental Results for Argon

The argon used was of 99.92% purity as supplied by Dominion Oxygen Company.

Res. Freq. evac. cy./sec.	Res. Freq. filled cy./sec.	(Ke - 1) x 10^6	(n - 1) x 10^6	$\frac{1}{Q \text{ evac.}}$	$\frac{1}{Q \text{ filled}}$		Pressure mm. Hg.	Temp. $0^\circ C$	(n - 1) x 10^6 N.T.P.	(Ke - 1) x 10^6 N.T.P.
$10^7 + 438$	$10^7 - 438$	175.2	87.6	$\frac{2 \times 1390}{10^7}$	$\frac{2 \times 1400}{10^7}$	$+2 \times 10^{-6}$	258.90	23.01	279.0	558
$10^7 + 427$	$10^7 - 418$	169.0	84.5	$\frac{2 \times 1380}{10^7}$	$\frac{2 \times 1380}{10^7}$	0.0	251.30	23.23	277.5	555
$10^7 + 327$	$10^7 - 605$	186.4	93.2	$\frac{2 \times 1395}{10^7}$	$\frac{2 \times 1405}{10^7}$	$+2 \times 10^{-6}$	275.46	24.05	280	560
$10^7 + 327$	$10^7 - 495$	164.4	82.2	$\frac{2 \times 1385}{10^7}$	$\frac{2 \times 1390}{10^7}$	$+1 \times 10^{-6}$	243.00	24.11	280	560

Mean value (Ke - 1) x 10^6 558.25 Maximum deviation 0.58%

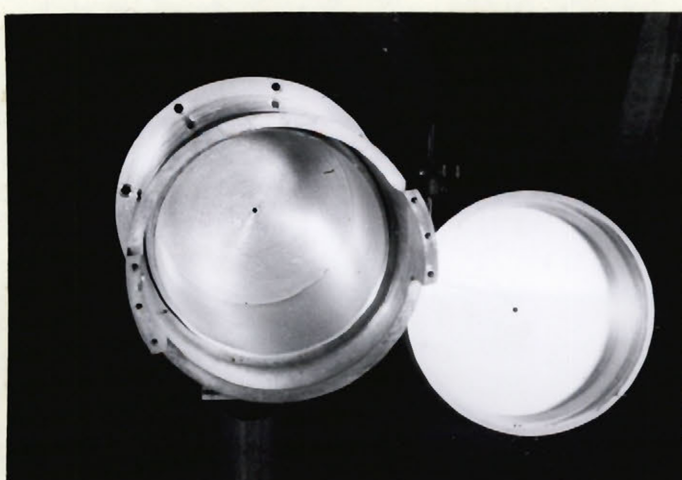
The measured values of (Ke - 1) for dry air and the gases considered are consistent to within 0.6% in all cases. The results being better in some cases. The consistency of these results gives a very good illustration of the stability of the apparatus both with respect to frequency and amplitude. The values obtained for the loss tangent are not sufficiently consistent to give an experimental value for this quantity but it may be deduced that the loss tangents for all of these gases are less than 5×10^{-6}.

A better way to express the consistency of the results than by quoting the per cent consistency is to express it as $\pm$ 5 parts in 10^6. Consistency expressed as a percentage is dependent upon the value of (Ke - 1) whereas the figure $\pm$ 5 parts in 10^6 is a value which indicates the accuracy of measurement actually attained. It has already been pointed out that this accuracy will be improved by using cavities with higher Q factors. The ability to determine the resonant frequency is not wholly dependent upon the frequency stability of the generator. It depends also upon the amplitude stability but even more so upon the sharpness of the resonance curve. The sharper the curve the more accurately can the resonant frequency be determined.

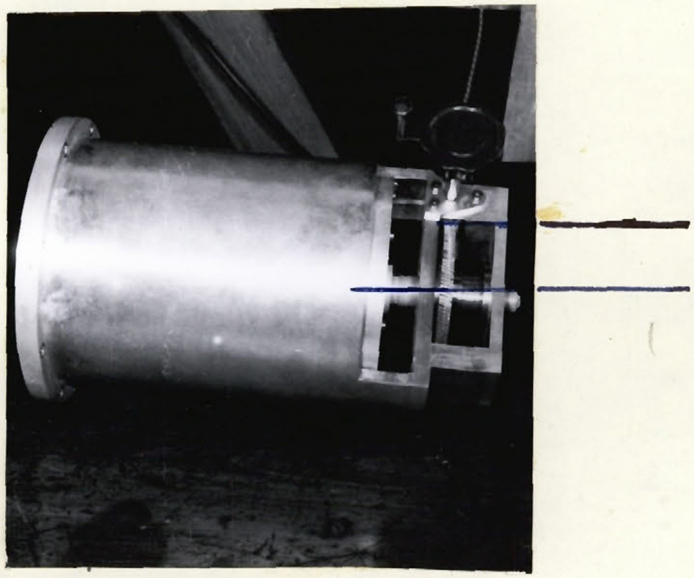
CHAPTER VIII

The Right Cylindrical Cavity and Vacuum Chamber operating in the TM_{012} mode at 2970 megacycles per second.

This cavity was constructed by Mr. V. Avarlaid of the Eaton Electronics Laboratory of McGill University. The cavity diameter is 10.419 cms. with a tolerance of less than $\frac{1}{100}$ of a mm. in the total length. The end plates have a tolerance in flatness of less than $\frac{1}{10\,000}$ of an inch. The cavity length may be adjusted by means of a worm pinion drive operated through a flexible shaft from outside the vacuum chamber wall. The minimum length of the cavity is 14.950 cms. long the shorter section being 3.740 cms. long and the longer 11.210 cms. in length. The coupling holes in the centre of the end plates are 3/32 inches in diameter and very fine wire probes which project not more than a fraction of a mm. beyond the surface of the end plate form the input and output probes.



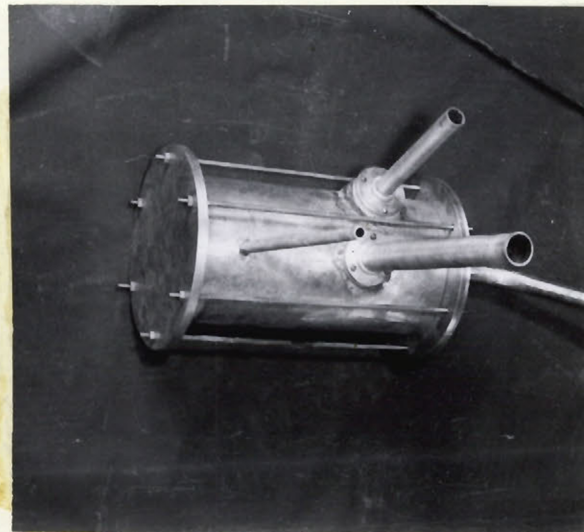
The TM_{012} cavity resonant at 2970 megacycles per second



The TM_{012} cavity assembled

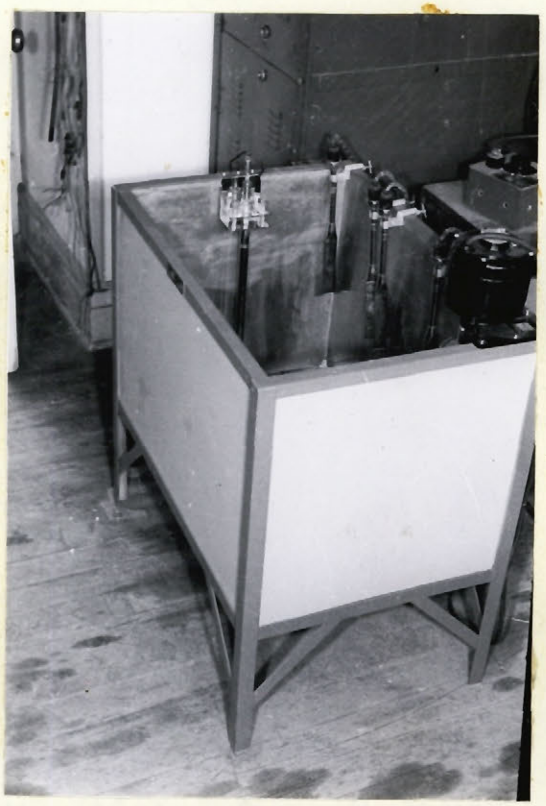
The cavity was machined from 4 inch inside diameter brass tubing and 1/4 inch wall thickness. The jacket which surrounds the cavity was made from $4\frac{1}{2}$ inch inside diameter tubing also of 1/4 inch wall thickness. The inside diameter of the jacket was machined to 4 5/8 inches to allow a clearance between the cavity and jacket. The purpose of the jacket is to provide a base on which to mount the worm and pinion assembly for tuning.

The vacuum chamber was made by Mr. F. Corrick of the Mechanical Engineering Department. It was machined from a piece of brass tube 6 1/8 inches inside diameter and 1/4 inch wall thickness. The vacuum chamber itself is 12 inches long and is provided with two cover plates machined from 1/2" brass plate. Six steel bolts 1/4 inch diameter equally spaced about the circumference hold the cover plates securely on the vacuum chamber, tightly compressing two "neoprene" gaskets to provide a vacuum tight seal between end plates and side wall of the vacuum chamber.



The Vacuum chamber for the TM_{012} cavity

The input coaxial line passes through the centre of one end plate by means of a glass to metal seal. The screened twin line in the output similarly is passed through the side wall of the chamber. At all points where connections are made to the vacuum chamber water jackets are provided to keep the surrounding water (i.e. the thermostatted medium for temperature control), away from the electrical connections to the vacuum chamber. A water jacket is also provided to keep the wax seal between the cavity and ground glass joint dry. These jackets consist of brass tubing fitted with a flange which may be bolted to the vacuum chamber wall in such a way as to compress a neoprene gasket making the arrangement water tight.



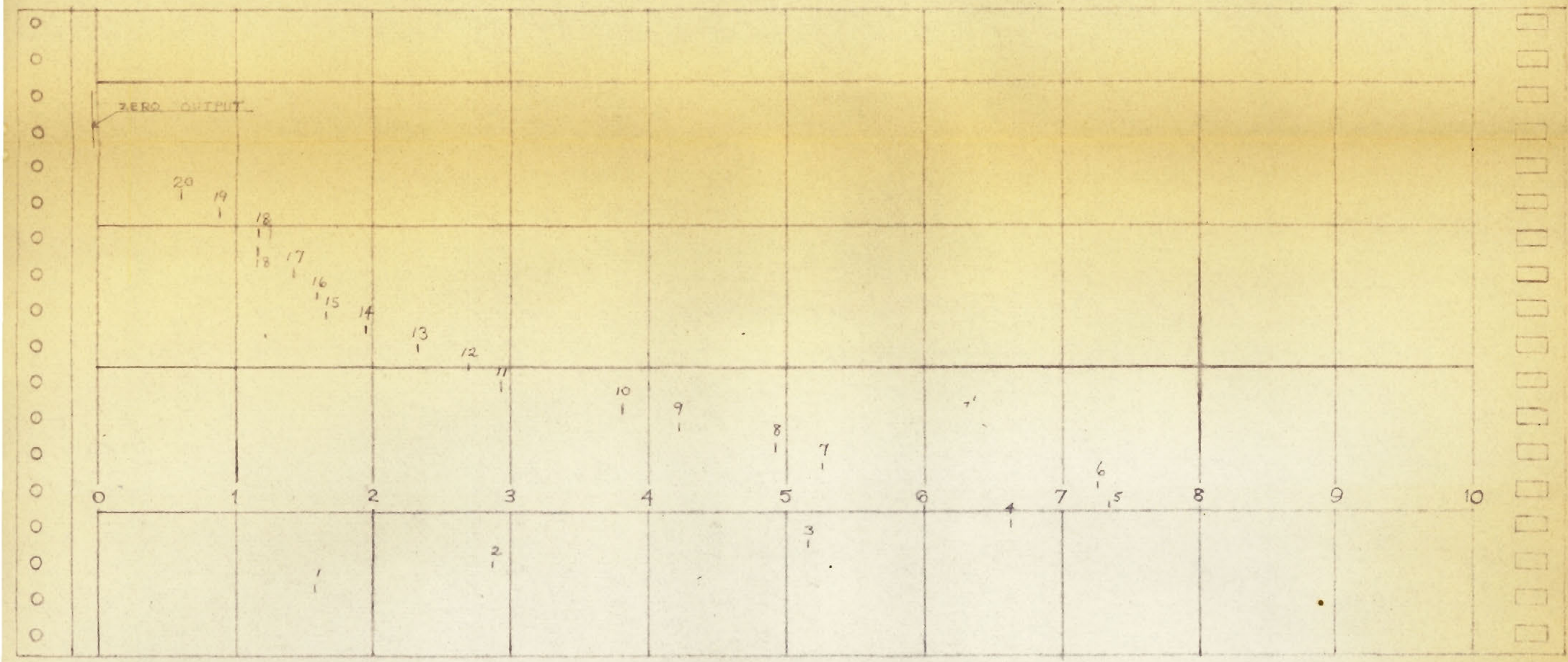
Temperature controlled water bath for the TM_{012} vacuum chamber and cavity

A type 97200 thermostat bath made by the Central Scientific Company is used to control the temperature of the high frequency cavity. The thermoregulator with which it is provided gives very excellent temperature regulation to within 1/20 of one degree Centigrade.

It was found that there were air leaks in the soldered joints to the vacuum chamber - which made it impossible to reach a pressure much lower than one millimeter of Mercury, and quite impossible to hold the pressure at a fixed value for any length of time. After finding and waxing these leaks the pressure could be reduced to 10 microns. At this point it was discovered that due to capillary action oil from the klystron bath was passing over from the oil bath to the water bath through the coaxial line. This was quite a rapid process and time was lacking to modify the output coaxial line from the final klystron. The oil leak plus the fact that the brass water shield around the output line could not be put in place due to the waxing process made it impractical to submerge the vacuum chamber in water. Several resonance curves however were obtained with the chamber evacuated to 10 microns and filled with air to about 25 cms. pressure. These curves may be taken only as an indication of how the cavity was operating and do not represent an accurate determination of the dielectric constant due to the impossibility of knowing the cavity temperature accurately. These resonance curves are given in the following set of plates.

RECORD OF VACUUM THERMOCOUPLE OUTPUT AS OBTAINED FROM THE RECORDING POTENTIOMETER

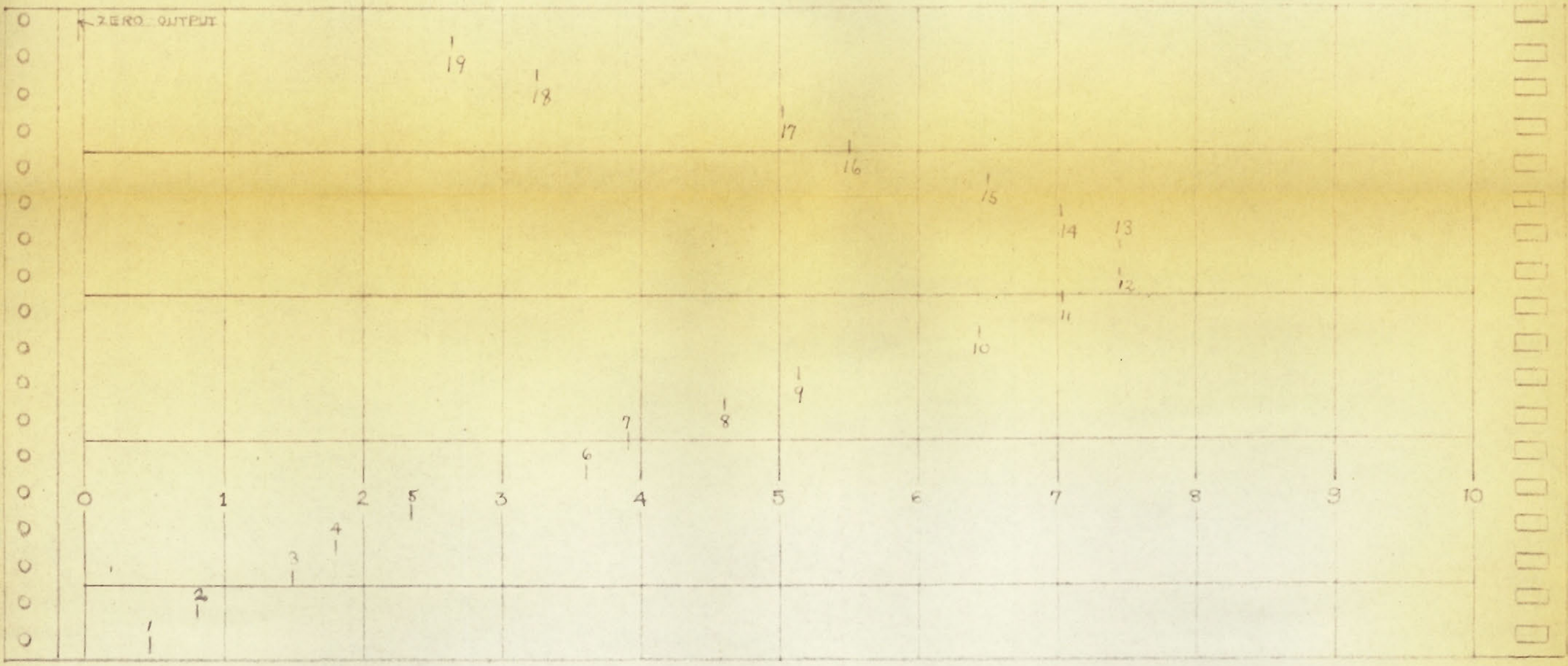
DRY AIR PRESSURE 10 MICRONS TEMPERATURE 26.5 °C



FREQUENCY CORRESPONDING TO EACH OF THE ABOVE EXPERIMENTAL POINTS USING 100 CYCLE FORK

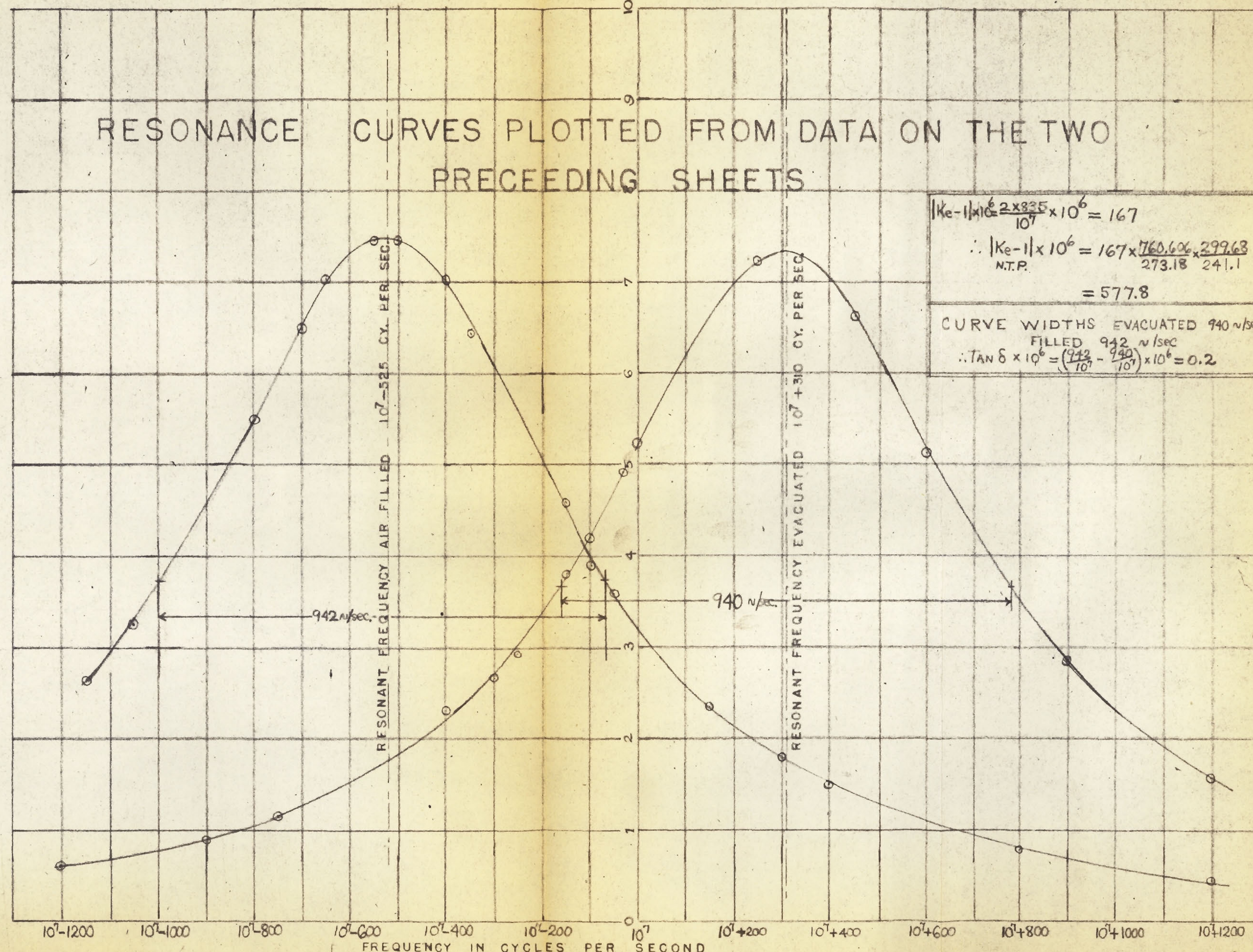
POINT NO.	FREQ. CY. PER SEC.	POINT NO.	FREQ. CY. PER SEC.	POINT NO.	FREQ. CY. PER SEC.	POINT NO.	FREQ. CY. PER SEC.	POINT NO.	FREQ. CY. PER SEC.
1	10^7+1200	5	10^7+300	9	10^7-100	13	10^7-400	17	10^7-650
2	" + 900	6	" + 250	10	" − 150	14	" − 450	18	" − 750
3	" + 600	7	" + 0	11	" − 250	15	" − 550	19	" − 900
4	" + 450	8	" − 333	12	" − 300	16	" − 600	20	" − 1200

DRY AIR PRESSURE 2411 MM. HG. TEMPERATURE 26.5 °C



FREQUENCY CORRESPONDING TO EACH OF THE ABOVE EXPERIMENTAL POINTS USING 100 CYCLE FORK

POINT NO.	FREQ. CY. PER SEC.	POINT NO.	FREQ. CY. PER SEC.	POINT NO.	FREQ. CY. PER SEC.	POINT NO.	FREQ. CY. PER SEC.	POINT NO.	FREQ. CY. PER SEC.
1	10^7+1200	5	10^7+150	9	10^7−200	13	10^7−500	17	10^7−850
2	" + 800	6	" − 50	10	" −350	14	" −650	18	" −1050
3	" + 400	7	" −100	11	" −400	15	" −700	19	" −1150
4	" + 300	8	" −150	12	" −550	16	" −800		


RESONANCE CURVES PLOTTED FROM DATA ON THE TWO PRECEEDING SHEETS
$|K_e - 1| \times 10^6 = \frac{2 \times 835}{10^7} \times 10^6 = 167$
$\therefore \underset{N.T.P.}{|K_e - 1| \times 10^6} = 167 \times \frac{760.606}{273.18} \times \frac{299.68}{241.1}$
$= 577.8$
CURVE WIDTHS EVACUATED 940 ~/sec
FILLED 942 ~/sec
$\therefore \tan\delta \times 10^6 = \left(\frac{942}{10^7} - \frac{940}{10^7}\right) \times 10^6 = 0.2$
RESONANT FREQUENCY AIR FILLED $10^7 - 525$ CY. PER SEC.
RESONANT FREQUENCY EVACUATED $10^7 + 310$ CY. PER SEC.
942 ~/sec.
940 ~/sec.
0
1
2
3
4
5
6
7
8
9
10
10^7-1200
10^7-1000
10^7-800
10^7-600
10^7-400
10^7-200
10^7
10^7+200
10^7+400
10^7+600
10^7+800
10^7+1000
10^7+1200
FREQUENCY IN CYCLES PER SECOND

BIBLIOGRAPHY

1. W.B. Westphal.
Techniques and Calculations used in Dielectric Measurements on Shorted Lines.
National Defense Research Committee Contract OEMsr-191
Report IX

2. R.A. Chipman.
A Resonance Curve Method for the Absolute Measurement of Impedance at Frequencies of the order of 300 Megacycles per second.
Journal of Applied Physics, Jan. 1939, vol. 10, No. 1, pages 27 - 38.

3. H.M. Barlow, A.L. Cullen.
"Microwave Measurements", pages 276 - 277.
Constable and Company Ltd., London, Eng.

4. H.M. Barlow, A.L. Cullen.
"Microwave Measurements", page 75.
Constable and Company Ltd., London, Eng.

5. H.M. Barlow, A.L. Cullen.
"Microwave Measurements", pages 240 - 241.
Constable and Company Ltd., London, Eng.

6. A.B. Bronwell, R.E. Beam.
"Theory and Application of Microwaves", page 155.
McGraw-Hill Book Company Inc., New York and London.

7. A.B. Bronwell, R.E. Beam.
"Theory and Application of Microwaves", pages 354 - 355.
McGraw-Hill Book Company Inc., New York and London.

8. H.M. Barlow, A.L. Cullen.
"Microwave Measurements", pages 69 - 70 - 71.
Constable and Company Ltd., London, Eng.

9. F. Horner, J.A. Taylor, R. Dunsmuir, J. Lamb and W. Jackson.
Resonance Methods of Dielectric Measurements at Centimeter Wavelengths.
J.I.E.E., part III, Jan. 1946, vol. 93, No. 21, pages 53 - 68.

10. J. Lamb.
"Dielectric Measurement at Wavelengths around 1 cm. by means of an H_{01} Cylindrical Cavity Resonator."
J.I.E.E., part IIIA, March May 1946, pages 1447 - 1451.

11. I. Paghis.
"Surface Losses in Electromagnetic Cavity Resonators". Canadian Journal of Physics, May 1952, vol. 30, No. 3, pages 174 - 184.

12. A.C. Vivian.
"The Superficial Conductivity of Metallic Conductors for Wave Guides". T.R.E. Technical Note No. 45, UDC No. 621.317.332:621.392.029.64.

13. A.P. Pippard.
"The Surface Impedance of Superconductors and Normal Metals at High Frequencies". Proc. Royal Society, vol. 191, 1947, pages 370 - 415.

14. The Radio Amateurs Handbook.
Page 417. American Radio Relay League, 1949.

15. Dr. L. Essen, Dr. K.D. Froome.
The Dielectric Constant and Refractive Index of Air and its Principal Constituents at 24,000 mc./sec. Nature 31, March 1951, pages 512 - 513.

16. H.W. Jamieson, J.R. Whinnery.
"Power Amplifiers with Disc-Seal Tubes." Proc. Inst. Radio Engineers, vol. 34, 1946, pages 483 - 489.

17. "Reference Data for Radio Engineers".
Federal Telephone and Radio Corporation, New York, N.Y.

18. A.B. Bronwell, R.E. Beam.
"Theory and Application of Microwaves", page 374. McGraw-Hill Book Company Inc., New York and London.

19. C.M. Crain.
"The Dielectric Constant of Several Gases at a Wavelength of 3.2 cms." Physical Review, vol. 74, No. 6, Sept. 15, 1948.

20. J.C. Slater, N.H. Frank.
"Electromagnetism", page 112. McGraw-Hill Book Company Inc., New York and London.

21. Dr. L. Essen, Dr. K.D. Froome.
"The Dielectric Constant and Refractive Index of Air and its Principal Constituents at 24,000 mc./sec.". Nature, March 31, 1951, pages 512 - 513.

22. O. Maass, E.W.R. Steacie.
"An Introduction to the Principles of Physical Chemistry", pages 61 - 66. John Wiley and Sons, Inc., New York.

23. K.J. Keeping.
An Apparatus for Microwave Measurement using Resonant Cavities.
Thesis for the degree of Master Of Engineering, McGill University, August 1950.

24. W.F. Gabriel.
"A Frequency Stabilization System for Microwave Gas Dielectric Measurements".
Proc. I.R.E. vol. 40, August 1952, pages 940 - 945.

25. C.G. Montgomery.
"Technique of Microwave Measurements", pages 75 - 78.
McGraw-Hill Book Company Inc., New York and London.