AN ANALYSIS OF SOLUTION STRATEGIES AND PROCESSING TIMES IN RATIO AND PROPORTION PROBLEMS

© Stan Gajewski

Department of Educational
Psychology and Sociology

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McGill University
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ABSTRACT

This thesis is concerned with the processes used by high school students in solving ratio and proportion problems. The research had two main objectives. The first was to compare performance on problems with varying semantic content. The second was to investigate the changes in strategy that occur as a function of increasing grade level and exposure to scientific subjects.

Each protocol was encoded on the basis of solution paths obtained by a preliminary task analysis. The time of each step in the solution path of each protocol was then measured.

Results showed that meaningful word problems were more difficult than symbolic problems. Furthermore, an interaction between grade level and semantic content was found for purely symbolic content problems, but the interaction differed with other types of problems.

There was a relationship established between the type of strategy selected, solution path times, and the difficulty of the problem.

Processing methods seemed to differ with grade level. The results indicate that there is a systematic shift to a more optimal form of strategy.

RÉSUMÉ

L'idée principale de cette thèse est centrée sur les méthodes employées par les étudiants d'une école secondaire ayant pour but de résoudre des problèmes de proportion. Deux objectifs saillants dominent cette recherche. Dans la première, une comparaison du rendement sur les problèmes de contenus sémantiques variés. La seconde servait d'investigation des changements qui prennent lieu proportionellement en fonction du niveau scolaire accroissant et de prendre connaissance des objectifs scientifiques.

La classification de chaque procédé se faisait d'après une base de solution prédéterminée, obtenue grâce a une tâche préliminaire analysée. La durée de chaque étape de cette solution fût alors mesurée.

Les résultats obtenus démontrèrent que les problèmes verbaux significatifs fûrent plus difficiles que les problèmes à charactères d'ordre numéral. En plus, une intéraction entre le niveau scolaire et le contenu sémantique a été trouvé pour seulement les problèmes symboliques par contre l'intéraction différait avec d'autre genre de problèmes.

Une relation fue établie entre le genre de stratégie choisie, le temps requis pour arriver à une solution et le niveau de difficulté du problème.

Les méthodes de procédé semblèrent différer avec le

niveau scolaire. Les résultats indiquent qu'il y a une tendance systématique à une forme de stratégie plus optimale.

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CHAPTER I

INTRODUCTION

This thesis is concerned with the processes used by high school students in solving ratio and proportion problems. The research has basically two main objectives. The first is to compare performance on problems with varying semantic content. The second objective is to investigate the changes that occur as a function of increasing grade level as well as exposure to scientific subjects.

In reviewing the current research on the topic of ratio and proportion, the following areas of concentration will be reported. First, the science and mathematics literature will be reviewed in order to present ratio and proportion in the proper context. Also, this literature examines the developmental aspect of ratio and proportion problem solving as well as hinting at other possible explanations for the difficulties encountered in this area. The vast majority of literature does not deal with strategy analysis, but it does point clearly towards a need for such an analysis.

With the establishment of the need for the study, related literature on Task Analysis will be reviewed. Special emphasis will be given to the area of mathematics research because of the similarities found within the various sub-categories of mathematical thinking.

The technique which will be used specifically in this study is an analysis of subject protocols. A review of related literature in this area will be presented as well as studies which use response latencies to measure speed of processing, in particular, those dealing with arithmetic computation.

Finally, a review of the applied protocol analysis dealing with expert-novice variations will be presented. This will show how in-depth protocol analysis is able to explain processing steps with relation to time and strategy variation factors.

REVIEW OF THE LITERATURE

1. MATH-SCIENCE RESEARCH: Introduction

A substantial amount of research has been done in the area of ratio and proportion, but only in regard to the areas of teaching, and the developmental problems which ratio and proportion tasks seem to exhibit. Very little research has been done on the processes involved in ratio-proportion problem solving. It would seem, at first glance, that the methodology used in gathering information on ratio and proportion has not been effective in tapping areas of procedure, at least according to the mathematics and science literature.

Studies such as that done by Karplus _et al_. (1977) exhibit findings which show that a substantial amount of students between the ages of thirteen and fifteen years "lack the ability to articulate proportional reasoning and/or control of variables". These results stem from an international study involving 3600 boys and girls from seven different countries. Based on this data, Karplus _et al_. recommend that science and mathematics programs in all but the highest levels should take diversity of student reasoning into account before designing programs or selecting curriculum topics. Similar findings are reported by Rogers (1977) who, in a study of ratio and proportion problems involving sixteen year old students found that only about 12% to 15% of the students whom he tested (about one student in every seven), obtained correct answers. Other research differs, however. As has been mentioned, the main theory regarding the concept of ratio and proportion is that it is a developmental concept. Research in this area has produced significant data to support this theory.

Ratio and Proportion: Developmental Research

Piaget and Inhelder (1958) make the clear assertion that students are unable to use proportional logic until they are capable of formal thought. Formal thought, according to Piaget is Stage IV of his hierarchical theory of development; Formal Operations. This statement has received considerable support in various other studies, Elkind (1962) and Lovell (1971).

Piaget and Inhelder's (1956) research with geometry problems reveals how the development of proportional reasoning is thought to evolve. Their procedure was to show their subjects a rectangle and ask them to re-draw the rectangle in larger proportions. Students between the ages of four and eight (Stage II) were overly concerned with the length of the rectangle, increasing it out of proportion and increasing the width of the rectangle either slightly or not at all. Stage III students maintained a constant relationship between length and width and were eventually able to produce simple whole number ratios such as 2 to 1. Stage IV students were able to formulate proportionalities and were able to apply them to all cases regardless of whether the ratios were whole number ratios or not.

Further work by Piaget and Inhelder (1956) was done using duplication of triangles as the task. Stage II and III students were able, in some cases, to duplicate the triangles, but only through what Piaget and Inhelder refer to as a "grouping of operations", which is merely a qualitative re-structuring of the figure. Extensive quantification only appeared in Stage IV subjects.

Rogers' (1977) study of sixteen year old subjects, using tasks from Piaget and Inhelder (1958) and Karplus, Karplus and Wollman (1974) as well as his own tasks, upheld the theory that formal-operational schemas involved a quantitative as well as a qualitative phase which proceeded in a developmental fashion.

Wollman and Karplus (1974) examined the performance of 450 seventh and eighth grade students on four concrete proportional tasks and two completely abstract proportional tasks (numerical and geometrical items). The results showed inconclusive evidence for a strict developmental sequence for all students. In the concrete task area, more students answered the questions successfully than in the abstract task area. Only one fifth of the students applied proportional reasoning consistently over several tasks and only one fourth of the students applied formal reasoning on some tasks. Other students used incorrect additive strategies in their solution attempts.

These results can be explained in either of two ways. First, consistent with Piaget and Inhelder's theory, it could be maintained that only twenty per cent of the students had reached the Stage IV level of formal operations. Several studies have obtained results consistent with this view. Hoemann and Ross (1971) found that in easier concrete tasks, the child depended upon "magnitude discrimination", similar to Piaget and Inhelder's triangle study. The students obtain a correct answer but for the wrong reason from the standpoint of ratio and proportion. Magnitude discrimination is seen as a precursor of proportional thinking, not necessarily as formal thought. This could explain correct solutions of students who do not seem to apply proportional logic.

Wollman and Karplus (1974) seem to agree with this theory. While maintaining that the application of simple ratios is proportional reasoning, it was noted that many students used addition, estimation, or even guesswork when faced with more complex proportional problems.

Other research offers explanations for discrepancies in proportional thought over age groups. Karplus et al. (1977) found both socioeconomic status and selectivity of school affected the performance of students on proportional reasoning tasks.

Karplus and Peterson (1970) studied urban and suburban school children from nine to eighteen years of age. Suburban subjects achieved mastery of the proportional tasks by the end of high school whereas urban students showed little or no progress over the same period of time.

Of perhaps considerably more interest to the present study however, is the second reason for Wollman and Karplus' (1974) results mentioned earlier. This is the effect of both cognitive style and strategy.

Both Lunzer (1972) and Karplus, Karplus and Wollman (1974) maintain that many subjects use several alternate procedures depending upon cues which may be given to them by the task. This is viewed as a development in strategy rather than as a separate developmental level.

The present study will attempt to analyze the strategies of individuals doing proportional reasoning problems in order to assess more precisely the effects of various strategies on success-failure and response rates. Furthermore, if proportional reasoning is developmental, then perhaps choice of strategy may correlate in some way with the level of development.

Ratio and Proportion: Effect of Instruction

If proportional reasoning is indeed developmental, as some seem to imply, instruction should have little or no effect on success rates unless rote procedures for solving problems are taught.

Herron and Wheatley (1971) advocate using a unit factor method to teach ratio and claim significant success rates in proportion problems in chemistry. Brown and Kinney, (1966) advocate the use of the concept of per cent in teaching students to solve proportion problems. Karplus et al. (1977) in their international study of seven countries found small but significant differences that teaching methods seemed to have on the development of proportional reasoning.

These studies notwithstanding, there is evidence contrary to the above which must be dealt with. Wollman and Karplus (1974) maintained that success rates in the teaching of proportion to younger students are often tainted by the use of procedural techniques such as fractional equivalents as well as those mentioned previously which do not teach proportion but merely treat the symptoms.

Fischbein, Pampu and Mânzat (1970), using five to thirteen year old students found that instruction increased performance but only on simple tasks. This coincides with the studies of Hoemann and Ross (1971) and Piaget and Inhelder (1956) mentioned earlier. Boulanger (1976) found that intensive training on Piaget's Task 2 caused the student to mistrust his intuitive logic in deference to teacher-induced procedures. He concludes that the concrete-operational child cannot be taught to retain and transfer formal-operational schemas.

What seems to be needed, therefore, is an approach to the analysis of ratio and proportionality which looks at the processes and strategies involved in this type of problem solving. A task analysis using appropriate techniques would seem to add considerable light to this area.

2. TASK ANALYSIS: Introduction

Analysis of specific tasks in order to obtain information about procedure is not a new concept, but the task analysis literature reviewed here will primarily deal with the more recent research in the area of mathematics and related problem solving.

Task analysis is seen by Resnick (1976) as the "study of complex performances so as to reveal the psychological processes involved". This type of analysis referred to by Resnick emphasizes changing data collected on a specific task into a psychological description of behavior. Task analysis was seen as a fundamental tool in assessing how learning takes place. Glaser and Resnick (1972) emphasize that the analysis of the task itself and not only the processes of how learning takes place is also important. Glaser and Resnick perceive task analysis as characterized by the "description of tasks in terms of the demands they place on such basic psychological processes as attention, perception, and linguistic processing. Further, since the individual's capacities change over time, task analyses reflect current knowledge and assumptions on the part of psychologists concerning the processes available at different stages of learning and development".

Historical Overview of Mathematical Task Analysis

Task analysis in mathematics has its roots in S-R theory beginning with Thorndike's (1922) Psychology of Arithmetic. Thorndike began a process of sequencing mathematical problems in order of difficulty, depending upon such factors as time and number of steps involved in the solution path. This concept is continued with the work of Gagné (1962; 1968) and the development of his Learning Hierarchy. Gagné's hierarchy presupposes a knowledge of subordinate concepts in order to achieve a specific task. The concepts are arranged in order according to difficulty, and no task can be accomplished without a knowledge of subordinate concepts. Tasks, according to Gagné, are studied with relation to the hierarchical structure. By analyzing a person's solution path, the level of knowledge is determined. In effect, positive transfer from simple to more complex tasks is expected to occur.

Gagné does not deal with processes in his work, at least explicitly, yet in order to determine and develop a learning hierarchy it seems necessary that some awareness of the cognitive processes involved in each level of the hierarchy would be implicit in Gagné's theory. Process analysis does not appear in his work, however. In a study of mathematical concepts, Gagné (1963) supported his hypothesis that an individual is unable to learn new knowledge without a subordinate knowledge of other concepts yet he explains away inconsistencies in the data as measurement error rather than attempting a process analysis.

Somewhat contrary to the behavioristic approach of Thorndike, was the gestalt psychologists' view of task analysis. The work of Wertheimer (1959) dealt with comparing and contrasting different ways of solving problems; either by memory, using basically a mechanical approach to the problem, or by an actual understanding of the concept underlying the problem. Wertheimer's findings showed fairly conclusively that it was the efficient management of inner structures which helped solve the problem. The efficient management and application of appropriate mental sets seemed to be more significant than memorization of facts and operations.

His work stressed the necessity of analyzing the components of a task in order to see their true relationship to the problem. Merely the application of an algorithm which somehow worked was unsatisfactory. Only the structural and perceptual aspects of the problem, when properly understood, could be generalized to other tasks.

Structural aspects of specific tasks were studied by Piaget (1956) and examined in terms of an individual either having or not having structures of different kinds. This possession of structures was synthesized into a developmental theory (Stages I to IV). The main contribution of Piaget's theory to task analysis was that it was able to point out specific differences among various age levels in their approach to certain tasks. Piaget assumed different levels of knowledge as well as different processing abilities were brought to a task by different individuals. Piaget was able to confirm his hypotheses through protocol analyses whereby different structural levels could be detected. What Piagetian task analysis did not explain, however, was what subjects actually do in solving a particular problem. For this type of analysis, the area of Information Processing must be examined.

Resnick (1976) distinguishes information processing task analysis from that of Thorndike and Gagné in that information processing explicitly attempts to describe internal cognitive processes. It differs from the work of Wertheimer and Piaget insofar as it is also more interested in finding temporally organized sequences of action in individual structures rather than the presence of "logical operations".

Empirical Information Processing analyzes tasks based on the interpretation of the data received from individual human performance. From this data, the approach attempts to develop a process model that would account for the data.

Both Resnick (1976) and Gregg (1976) outline what a task analysis should do based on information processing methodology. First, it should identify the component skills which assure success on the task; secondly, it should specify a complete set of strategies suitable to the task; and thirdly, it should map feasible strategies into a process model.

Further work on task analysis has been done concerning the instructional implications (types of instruction such as conceptual or strategy teaching) on performance, (Larkin, 1977; Egan and Greeno, 1973; Mayer, 1974) but will not be developed here.

Strategy

The task analysis area which is most important to this study is the analysis of strategy. The source for most of the current work on strategy is not definite, yet there is some evidence that most research stems from either Gagné's (1962) Learning Hierarchy or, more recently, from Newell, Simon and Shaw's (1958) General Problem Solver (GPS). GPS used a means-ends type of analysis which classified things in terms of the functions which they served. The system formed a basic heuristic for problem solving. Strategies were developed by GPS and formed the basis for further study. The means-ends analysis was of particular interest to the area of task analysis since it was capable of duplicating many of the strategies used in human performance.

When means-ends analysis is used, both the initial and terminal objects are known. Searching processes using the technique of generating and removing differences have the general effect of limiting the search to objects that lie between, or close to the initial and terminal objects respectively. It also finds paths between start and finish that are relatively short and direct. Working-backwards methods start from the goal object, using information about it to reduce the problem space. GPS on the other hand, uses the relationship between the initial and terminal object, which imposes a far more efficient search system on the program.

Both types of strategies are used by human subjects but similarities among individuals exist depending upon situational cues. Paige and Simon (1966) and Newell and Simon (1972) both emphasize the importance of qualitative similarities among individuals and stress that if there were no such similarities and each subject and each task was completely idiosyncratic, there could be no theory of problem solving whatsoever.

Newell and Simon's (1972) work with protocol analysis, strategies and production systems stands as perhaps the most significant achievement in problem solving in the past decade. The present study, however, differs from Newell and Simon's analysis in that the development of a production system is not a part of the study. A more detailed protocol analysis coupled with precise solution path times should reveal more exacting data needed for the analysis of the strategies involved in the present problem solving area of ratio and proportion.

Further work concerning strategies in a problem-solving task has been done by Simon and Reed (1976). In their investigation, a computer simulation model was fitted to human laboratory data for the Missionaries and Cannibals task in an attempt to explain 1) the effects upon problem performance of giving a hint, and 2) the effects of solving the problem a second time after one successful solution has been achieved. The subjects who were given a hint averaged 20.3 moves in solving the problem; subjects who did not receive the hint averaged 30.6 moves in solving the problem. Simon and Reed explain the difference with a strategy shift model involving a shift from a "balance strategy" to a "means-ends strategy". Similarly, a significant reduction of moves occurred after a successful completion of the problem because of the same strategy shift.

Strategy shifts and the processes involved in specific types of strategies have also been studied extensively by

Greeno (1978, 1973), Mayer and Greeno (1972) and Schoenfeld (1978), but will not be discussed in detail here.

A somewhat different area of research uses sub-goals as the focus of study in strategies used in problem solving. Simon (1976), after studying student' performance on the Tower of Hanoi problem limits strategy types to four: a rote method, a recursive method (storing a solution strategy in terms of his or her own analysis), a perceptual method, and a pattern method. The establishment of sub-goals and sub-routines can be an integral part of all of these four types of strategies.

Similar sub-goal creating strategies have also been found by Greeno (1976) in the area of Geometry and by Ericsson (1975) in the 8's puzzle. What seems to be involved in all of these studies is a procedure which replaces unknowns by knowns, thus creating sub-goals which aid in the solution of the final goal.

The tree searching techniques of Alderman (1978) indicate yet another system of sub-goals which seems to be consistent with both GPS and Gagné's hierarchical structure.

Variables Affecting Strategy Selection

In the review of the literature on strategies, mention was made of cues which seemed to have an effect on the choice of strategy on a specific task. Task analysis studies report various factors which affect choice of strategy in solving problems.

Mayer and Greeno (1975) compared "meaningful" format with "equation" format and concluded that meaningful format allowed individuals to assimilate information and connect it with other systems of knowledge. Equation format, on the other hand, resulted in adding information to memory in a way that retained original detail.

Rosenthal and Resnick (1974) experimented with the processes involved in addition and subtraction using verbal expression and processing as the variable. Using two groups of third grade children, a series of problems were presented in which the identity of the unknown set as well as the order of mention of chronological events was varied. Findings of the study indicated that both backward order of mention problems as well as problems with unknown starting sets produced more errors than did other types of problems. Problems with an unknown starting set also produced longer latencies than did other types of problems.

Simon and Hayes (1976) tried to predict human problem solver processes using the UNDERSTAND program. They found that human problem solvers are able to interpret relevant sentences in word problems by mapping them onto ~~list~~ structures previously stored in LTM.

In a later study, Mayer (1978) used four data statements describing quantitative relations among the elements.

A further statement asks a question concerning either a numeric or comparative question. The results showed that making correct inferences in quantitative reasoning is much more difficult than in comparative reasoning, probably due to the additional processing required in a quantitative answer. Further results suggested that there were differences in the amount of information stored by subjects with different problem-solving sets. Numeric set subjects encoded material in a way that retained more detailed information but performed poorer on comparative reasoning tasks relative to the comparative set subjects. The reverse regarding the comparative set subjects was also apparent.

To what extent different cues and sets promote different strategy selections is not specifically known. The present study may shed some light on the area.

Mapping Strategies

In order to set up an algorithm to follow the step by step procedure of a particular strategy, some researchers have used flowcharts. Paige and Simon (1966), in an extensive protocol analysis of students' performance on algebra word problems used a flowchart procedure, developed by Bobrow (1964), to analyze the strategies used on the problems. In doing so, they were able to compare contrasting strategies easily, using the same coding system.

Similarly, Groen and Parkman (1972) used a flowchart model in an analysis of addition problems and were able to chart response latencies accurately because of incrementing procedures used by the students.

In the present study, a flowchart will plot the possible strategies used for the ratio and proportion problems given to the students. This will enable easier encoding and analysis of the individual strategies and processes used therein.

3. REACTION TIMES AND RESPONSE LATENCIES: Introduction

In the present study, processing speed and the attainment of sub-goals and steps in a specific procedure may prove to be a significant variable in the protocol analysis presented. Early efforts to measure the speed of mental processes were undertaken by Donders (1868) and many years later revised and re-instituted by Sternberg (1969). The Donders method involved a subtractive technique which first partitioned reactions into separate variables and then subtracted the time for the variable under study from the other reactions. Chase (1978) gives a brief outline of the subtractive technique as well as an historical overview of early reaction time studies involving various areas of mental processing.

Mathematics and Reaction Time Rates

Perhaps the most extensive diagnostic work in the area of arithmetic has been done by Buswell and John (1926). An extensive time analysis investigation, using thirty elementary school students ranging in age from eight to twelve (levels three to six), was done using standard talking aloud techniques. An analysis of the four fundamental operations of addition, subtraction, multiplication and division showed mean times required for each of the subordinate operations. Extensive tables on all four mathematical operations are provided by Buswell and John showing percentage distributions of required time. The results show an irregularity in the percentage of time required by the different subjects for the various sub-processes. Obviously, one does not expect perfect mathematical regularity in the intervals required for the operations involved, but extreme variation is seen by Buswell and John as an indicator of inadequate mastery of some fundamental number combination.

Buswell and John also show many techniques for remedial treatment of fundamental processes.

It seems evident that reaction times can be used as an effective indicator of actual speed of mental processing and retrieval of stored information. This study of simple arithmetic problems can probably be considered the forerunner of modern arithmetic processing research.

Little had been done since this study until Groen and his associates started extensive analysis of arithmetic processes based primarily upon reaction time information.

Simple addition problems were given to first grade children by Suppes and Groen (1967) and reaction times were taken for the solution paths. A choice-type model of incrementing was developed from this data. Similar work by Groen and Parkman (1972) involving first grade children and their performance on addition problems was also measured using response latencies. Groen and Parkman found that these latencies were best explained either through an incrementing algorithm, or through a process of direct search of memory. A study by Parkman and Groen (1971), using subjects who were adults, seemed to favor the "direct memory look-up process" as Chase (1978) has referred to it.

A study by Groen and Poll (1973) tested children on open sentence problems which are structurally similar to subtraction problems. Again, using response latencies, Groen and Poll were able to establish a process involving a mixture of incrementing and decrementing which seemed to fit the data. A later study by Woods, Resnick and Groen (1975), concerning process models for subtraction, found the same process to be used by most subjects.

Reaction time techniques were also used in an analysis of childrens' solution processes in arithmetic word problems studied by Rosenthal and Resnick (1974). Once again, the type of process involved in correctly solving a problem was inferred directly from the response latencies measured by the researchers.

A somewhat different area of research was studied by Klahr and Wallace (1976) using quantification latency measures. They grouped all quantitative judgements into three categories: subitizing, counting, and estimation. The first two refer to mechanisms dealing with numerical quantity whereas estimation can be used either numerically or for continuous quantity.

Klahr and Wallace went on to develop a computer simulation of the quantification task which uses subitizing (immediate apprehension) as the basic mechanism, and beyond that range (Broadbent's (1975) estimate is about four "chunks"), a more complex process of grouping and adding takes over. Other experimental evidence (Akin and Chase, 1976; Beckwith and Restle, 1966) seems to be consistent with Klahr and Wallace's model.

Further research has been done on subitizing. Chi and Klahr (1975) found that when adults and five year olds were asked to state the number of dots in a randomly arranged display, the adults reported that they primarily grouped and added whereas the five year olds merely counted. Chi and Klahr noted that within the subitizing range, latencies were fast and error free, but latencies outside of this range increase linearly with number of objects and error also increases with N.

It seems evident from the research cited in this section that reaction times and response latencies were able to help isolate processes which appear within specific strategies.

The main problem with reaction time data, however, is that the processes inferred from this data do not necessarily reflect only those specific processes. Processes such as memory search cannot be inferred from such data. Long reaction times, in particular, can hide subordinate processes which can interfere with those processes which are being measured. It would seem, however, that even those steps in a procedure with long reaction times provide valuable information on type of strategy used in the solution processes.

4. PROTOCOL ANALYSIS: Newell and Simon's Analysis

The analysis of protocols is not a new procedure for gathering data about mental processes (Buswell and John, 1926; Duncker, 1945; Johnson, 1964). However, as a systematic technique, protocol analysis is a more recent investigation. The most extensive work in this area has been done by Newell and Simon (1972) who have presented a detailed theory of mental processing. An analysis of the verbal protocols of their subjects in the areas of cryptarithmetic, symbolic logic, and chess lead Newell and Simon to the development of production systems.

The protocol analysis in the cryptarithmetic tasks consisted of trying to infer rules of operation that the subject used in the solution of a problem. In analyzing these protocols, Newell and Simon summarized the thought processes in terms of rules or productions of the form P $\Rightarrow$A, where P is a predicate and A is an action which is executed if the predicate is true. The set of these rules is called a production system.

Extensions of the production system approach have been developed by Newell (1973) to include many of the more elementary processes which were not tapped by Newell and Simon earlier. Klahr and Wallace's (1976) model of quantification, mentioned earlier, was also based upon Newell and Simon's work.

Newell (1977) outlines four steps for making protocol analysis a useful tool. This series of steps attempts to point out regularities in the protocols which serve to construct process models of an individual's solution paths. The four steps include: a) dividing the protocol into phrases which includes encoding the various steps in the protocol; b) constructing a problem space which serves as an hypothesis about the subject's behavior; c) plotting the Problem Behavior Graph (PBG) which is an application of the operators of the problem space; d) creating a production system which is a synthesis of the regularities which appear in the protocols.

Paige and Simon's Analysis

Paige and Simon (1966) used protocol analysis in an empirical study of the solution paths used by students solving algebra word problems. They compared their findings to Bobrow's (1968) processing system STUDENT, which represents problems as equation sets. STUDENT was able to translate English statements into algebraic equations by parsing the language into simple statements, applying function tags to these statements, and then applying an appropriate equation to the statement. Using "cues", STUDENT was also able to handle sufficient auxiliary information to enable it to solve a wide range of algebra problems. These cues were structured by the rules which determined the order of the levels of operators used in the program.

Paige and Simon also investigated the algebra word problems within the context of verbal processes and found significant differences between STUDENT and human problem solvers. They noted individual differences in subject protocols in: a) the use of direct transformations of information contained in a problem in contrast to the use of auxiliary information that went beyond the information presented; and b) in primary reliance on either physical-spatial or verbal representations of the problem situation.

Paige and Simon's research stands as one of the major studies in mathematical problem solving, insofar as the extensive analysis of individual protocols and the steps within these protocols enabled them to arrive at significant new findings in problem solving research.

Their work differs substantially from that of Newell and Simon primarily in their degree of emphasis on artificial intelligence. Newell and Simon have, as their goal, the development of production systems which attempt to duplicate human performance. Paige and Simon, on the other hand, are more concerned with individual processes which appear in the protocols. Paige and Simon develop a more in-depth treatment of the individual protocols than Newell and Simon, and as such, are more interested in strategies as individual problem-solving techniques. It could be maintained that Paige and Simon study protocols for differences in strategies whereas Newell and Simon attempt to limit strategies into a production system.

Protocols: Analysis and Strategy

Greeno (1976) used protocol analysis to show how human problem solvers generate and use an indefinite goal structure in the process of working on an initially well-structured problem. Greeno continues with the notion that goal structure may simply be a system of pattern recognition processes. This seems to find some support in the problem-solving procedures that Newell and Simon (1972) use in descriptions of the problems used in cryptarithmetic. Both Greeno and Newell and Simon use the production system formalism in establishing the theory of pattern recognition.

Other studies are cited in a review article by Larkin, Heller and Greeno (1978). An analysis of protocols collected on high school geometry problems was developed into a computer simulation model called Perdix (Greeno, 1976; Greeno, 1978; Greeno, Magone and Chaiklin, in press). The model solves problems using similar steps as those used by students. Perdix involves three kinds of knowledge: a) a set of propositions used for making inferences, b) a set of perceptual concepts that perform pattern recognition, and c) strategic knowledge, consisting of procedures for planning and setting goals. Current research is continuing using this type of computer simulation approach.

Reed, Ernst and Banerji (1974) studied the protocols of college students on the Jealous Husbands and Missionary-Cannibal problems in order to find out whether similar strategies were used for homomorphic problem states. It was found that strategies, determined from the protocols, were only similar if the students actually recognized the relationship which existed between the two types of problems.

A study by Kantowski (1977) using ninth grade students solving non-routine geometry problems also provides some insight into strategies derived from protocol analysis. When a problem was repeated, either of two things happened which seemed to be affected by prerequisite knowledge as well as by personality factors: a) some subjects tried new techniques if they had been unsuccessful previously (A strategy shift was employed); b) some subjects abandoned the search for a solution immediately if they recognized a problem as one being too difficult for them to solve.

Days, Kulm and Wheatley (1979) studied process differences between concrete and formal operational students using protocol analysis and a unique encoding system. It was found that problem structure plays a greater role in determining process use for formal students than for concrete students. Also, problem structure had a greater effect on problem difficulty for formal students than for concrete students.

A criticism of Newell and Simon's theoretical approach levelled by Chase (1978) is aimed at the protocol analysis which was used in their research. Chase states that their level of analysis and methodology were appropriate only for complex processes which took several seconds to perform. Chase continues that Newell and Simon's theory must be "constrained by the evidence in the protocols, and the level of analysis and the real time constraints are an order of magnitude higher than appropriate for elementary processes. There is little evidence that elementary processes are available to conscious introspection. Indeed, nowhere in Newell and Simon's protocols can any verbal description be found that describes a memory search process."(p. 83)

The research on protocol analysis is conspicuous by the absence of any extensive solution time data. It would seem that solution times would substantiate programs developed to simulate students' protocols. Likewise, very little reaction time data has used protocol analysis as a tool to support the results which have been presented in the earlier part of this review. This study intends to use the techniques of both protocol analysis and solution time data to analyze processes involved in simple ratio and proportion problems.

5. EXPERT-NOVICE COMPARISONS: Strategy Differences

Recently, Simon and Simon (1977) performed an extensive protocol analysis of the individual differences of expert and novice in solving physics problems. While these problems were much more complex than the ratio and proportion problems used in the current study, the patterns of sub-goals and the resulting strategies seem to be somewhat consistent.

Simon and Simon's analysis showed clear strategy delineations between the expert and the novice that did not seem necessarily developmental. The novice, for example, used two different methods to solve two problems of the same type, indicating a problem-solving approach based upon the cues initiated by the problem which in turn trigger retrieval of a strategy stored in LTM.

In the same study, Simon and Simon exhibited proof that a strategy which can only be labelled as "physical intuition" appears. The advantages of physical intuition seem to account for the superior ability of the expert.

The expert also exhibited a "working forward strategy" much like that encountered in GPS, which involved a sequential attack on the problem, leading directly to the intended goal. The novice, on the other hand, exhibited a "working backward strategy" encountered previously in the Logic Theorist (Newell, Simon and Shaw, 1958). This strategy was characterized by a somewhat erratic sub-goal problem solving, leading ultimately to the final goal.

The expert's approach to problems has been characterized by Larkin, Heller and Greeno (1978) as a "physical" approach whereby the expert moves from problem statements to physical situations and from there to equations. In other words, the expert must have the laws of kinematics organized and "indexed" in LTM. The novice's approach is "algebraic", going directly from problem statements to the equations.

Similar findings have appeared in the research of Larkin (1977a, 1977b) who also used verbal protocols to study solution strategies in physics problems. Again, as in Simon and Simon's findings the novices went directly from the problem statement to the application of equations. The sequence of equations evoked was similar to that of trial and error procedure insofar as when dead ends were reached, the novices tried other procedures until a final solution was obtained.

The experts in Larkin's study, however, were slightly dissimilar to the expert in Simon and Simon's work. The expert in Larkin's research used a planning stage before generating any mathematical equations. This planning stage was explained as being a result of difficult problems even for the experts, unlike those problems in Simon and Simon's study.

In all three studies, however, the experts appeared to have matched solution methods with generalized problem types previously stored in LTM.

"Chunking"

An extensive review article on expert-novice problem solving has been written by Larkin, McDermott, Simon and Simon (1979). As well as the differences between novice and expert already mentioned, experts seem to be able to "chunk" familiar stimuli and are able, by pattern recognition, to evoke stored information about strategies from memory (Simon, 1974).

The critical component of translating verbal statements into mathematical equations is also evidenced more often in the expert problem solver. The result of this is that the experts solve the problems in less than one quarter of the time required by the novice and with fewer errors.

An automation process is also noted as being a part of the expert's strategy. While the expert, in his verbal protocol, mentions only numerical results, the novice goes through the entire literal equation. This difference is explained by Larkin et al. (1979) in terms of the expert having stored an entire procedure while the novice stores the knowledge that particular equations can be used in certain circumstances. Thus, the novice verbalizes her recognition of the different equations while the expert verbalizes the solution of the entire procedure.

The process of LTM storage and retrieval is noticed most particularly in chess. Studies by Chase and Simon, (1973, 1974) and Simon and Gilmartin (1973) reveal that an expert chess player can reproduce from memory the positions of about twenty five chess pieces with 80% to 90% accuracy after seeing the pieces for only five seconds. The weaker player can reproduce only five or six pieces. The master chess player can also play at a speed of ten seconds per move with very little loss in playing strength.

This information supports Simon's (1974) theory that familiar stimuli are chunked and then stored and retrieved through a pattern recognition cue. Similar findings in speed of recognition and accuracy of placement of pieces have also been reported by DeGroot (1966).

The ability of the expert to chunk information enables him to recognize early patterns forming in a problem state and further enable the expert to prepare a strategy using the processes stored in LTM.

Other studies mentioned supporting these hypotheses have been reported in the areas of physical dynamics problems, (McDermott and Larkin, 1978, and Larkin, in press), and also in problems of chemical engineering thermodynamics (Bhaskar and Simon, 1977).

In conclusion, the expert's ability to use pattern-indexed schemas enables him to solve problems more efficiently. Simon and Simon's (1977) reference to "intuition" might be synonymous with this ability.

CHAPTER II

ANALYSIS OF THE TASK

In order to test the abilities of the subjects on ratio and proportion problems, ten problems were compiled on various task areas with varying degrees of difficulty. Four of the problems were "symbolic" problems where the position of the unknown was systematically varied. The remaining six problems were word problems. Two of these were "nonsense" problems where the names of the terms were irrelevant to the solution of the problems. The next two problems were conventional "story" problems where the terms were familiar objects known to the sample; and the last two were problems involving chemical terminology which, although meaningful, had nothing to do with the solution of the problem itself.

It is important to note here that, although the ten questions were grouped into four distinct sections or types of problems for the purpose of analysis, no attempt has been made to make the questions within each section linguistically equivalent as was done, for example, by Rosenthal and Resnick (1974). This was primarily because each of the questions was intended to be analyzed separately, with no direct comparisons between questions implied or attempted. The ten questions in their final form appear in Table 1 on the following page.

In an independent study, three mathematicians, (Senior mathematics teachers from the high school) were asked to identify all the possible solution methods for all ten problems. The flowchart notation was used in order to help identify individual steps within a specific strategy. These steps were identified, labeled, and numbered in order of occurrence in all possible strategies. The flowcharts were then checked by the mathematicians to ensure reliability. Each strategy developed by the mathematicians was then run through the flowcharts until the flowcharts were able to accomodate each strategy within the ordered steps provided.

Two flowcharts, labeled METHOD A and METHOD B were the result of the task analysis. Both of these structures follow with a detailed explanation of each.

> (It should be stated here that other possible solution paths were suggested by the mathematicians, but it was unanimously agreed to that the likelihood of high school students using such highly sophisticated mathematical computations was highly improbable. These methods, therefore, are omitted.)

Table 1

PROBLEMS

1. $\frac{4}{6} = \frac{9}{?}$

2. $\frac{8}{?} = \frac{10}{15}$

3. $\frac{?}{35} = \frac{40}{56}$

4. $\frac{12}{24} = \frac{?}{8}$

\- - - - - - - - - - - -

5. There are 10 bottles of cleaning fluid in a case. If 2500 square feet of flooring can be cleaned with 4 bottles of fluid, how many square feet of flooring can be cleaned with 3 cases of fluid?

6. A man buys some grain to feed his cattle. He pays a total of $700. for 200 bags of grain. The following year, prices remain the same and he buys 275 bags of grain. How much does the man pay for the 275 bags of grain?

\- - - - - - - - - - - -

7. If 2 Serbs can make 7 sets of Tods, and 35 sets of Tods are needed to make 1 Fot, how many Fots can be made from 30 Serbs?

8. If 7 Ergs make up 4 Zots, how many Ergs can make up 28 Zots?

\- - - - - - - - - - - -

9. In Chemistry, a mole of Nitrogen weighs 28 grams. If a mole of Nitrogen is also equal to about 22 liters in volume, what will be the volume of 35 grams of Nitrogen?

10. If 80 grams of NaOH and 98 grams of H_2SO_4 are needed to make 36 grams of water, how many grams of NaOH and H_2SO_4 will be needed to make 45 grams of water?

Method A, presented in Figure 1 is used for all problems using only four variables. These problems are numbers 1, 2, 3, 4, 6, and 8. Method B is used for the problems using six variables. These are problems 5, 7, 9, and 10. It should be noted that Method B is the same as Method A except that there is a complete duplication of Method A using new terms in the second part of Method B.

Method B is seen as independent, however, because of the necessity of having the student choose the correct variables in the solution strategy. A further explanation of these Methods is necessary before proceeding further.

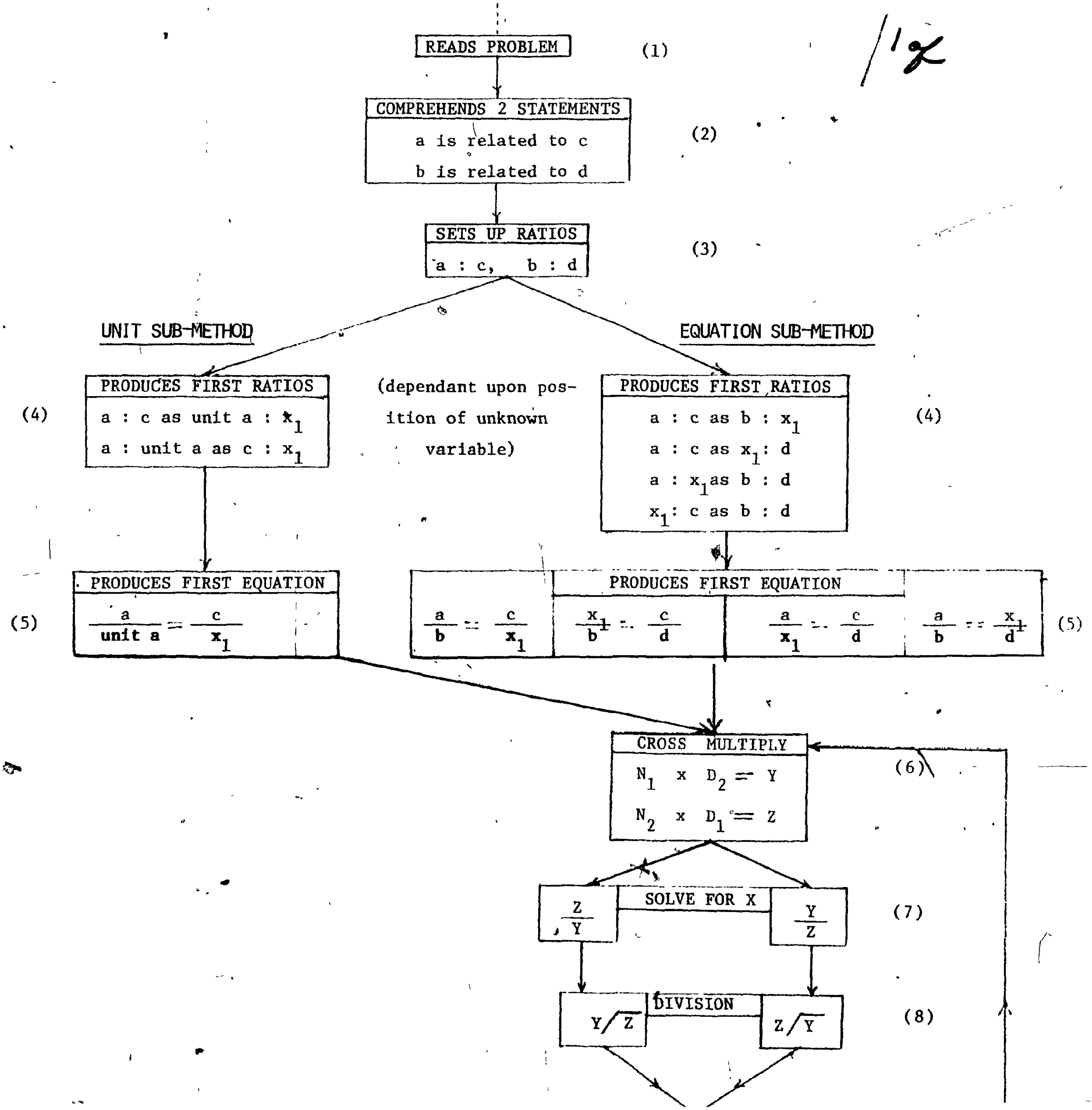
METHOD A

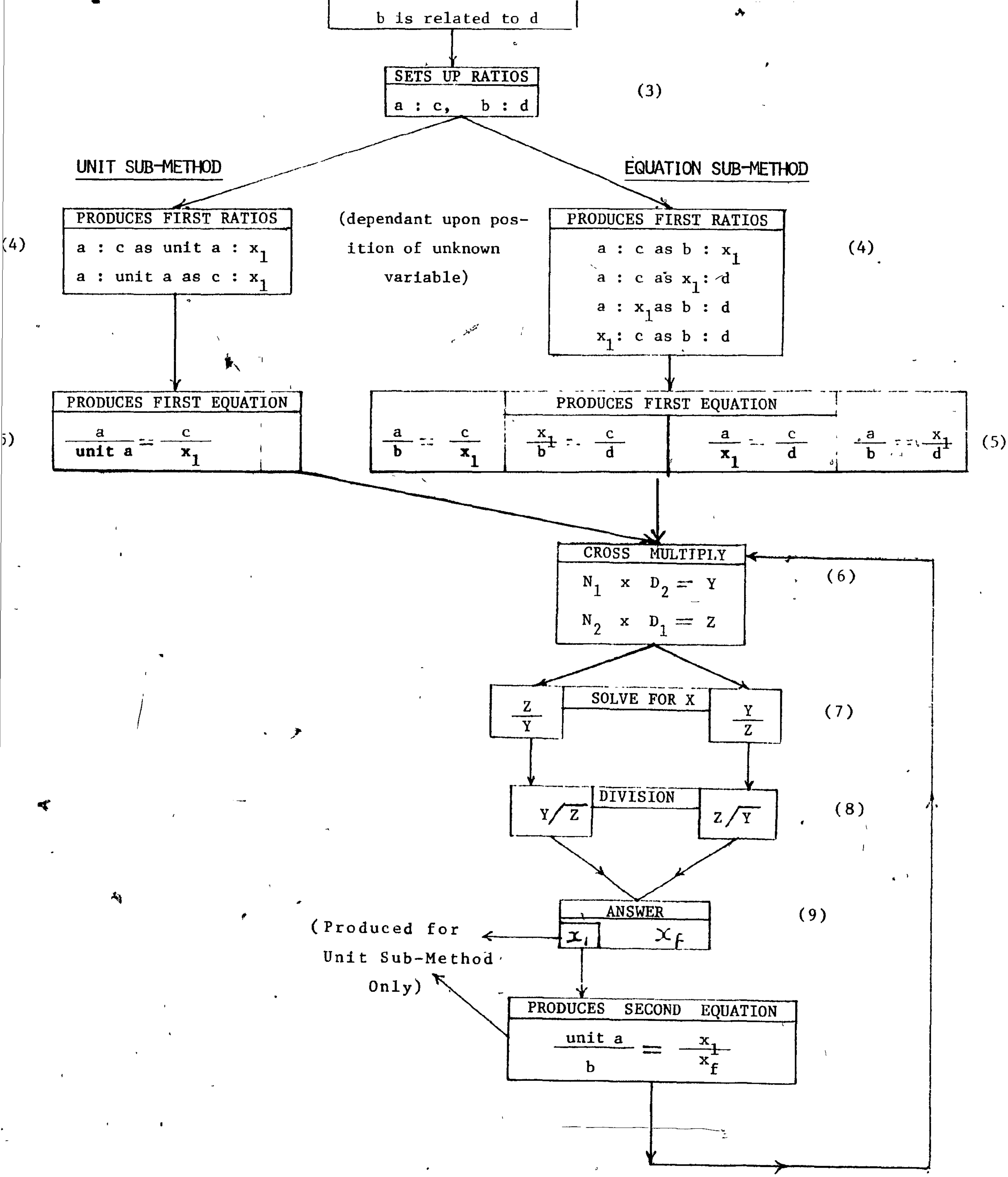
Method A enables the student to choose a number of solution paths starting with step 1 (READS PROBLEM). To explain Method A, problem number 8 will be explained according to the various steps used. Problem 8 reads: "If 7 Ergs make up 4 Zots, how many Ergs can make up 28 Zots?".

For purposes of this explanation, 7 Ergs will be term A, 4 Zots will be term C, the unknown term B, and finally 28 Zots will be term D. Following Method A, the student must first of all see a relationship between the two sets of terms and among all individual terms. This comprehension is represented as step 2 (COMPREHENDS 2 STATEMENTS). From this point, the student must produce

FIGURE 1

METHOD A




b is related to d
SETS UP RATIOS
a : c, b : d
(3)
UNIT SUB-METHOD
EQUATION SUB-METHOD
(4)
PRODUCES FIRST RATIOS
a : c as unit a : x_1
a : unit a as c : x_1
(dependant upon position of unknown variable)
PRODUCES FIRST RATIOS
a : c as b : x_1
a : c as x_1 : d
a : x_1 as b : d
x_1 : c as b : d
(4)
5)
PRODUCES FIRST EQUATION
$\frac{a}{\text{unit a}} = \frac{c}{x_1}$
PRODUCES FIRST EQUATION
$\frac{a}{b} = \frac{c}{x_1}$
$\frac{x_1}{b} = \frac{c}{d}$
$\frac{a}{x_1} = \frac{c}{d}$
$\frac{a}{b} = \frac{x_1}{d}$
(5)
CROSS MULTIPLY
$N_1 \times D_2 = Y$
$N_2 \times D_1 = Z$
(6)
SOLVE FOR X
$\frac{Z}{Y}$
$\frac{Y}{Z}$
(7)
DIVISION
Y/Z
Z/Y
(8)
ANSWER
x_1
x_f
(9)
(Produced for Unit Sub-Method Only)
PRODUCES SECOND EQUATION
$\frac{\text{unit a}}{b} = \frac{x_1}{x_f}$

2 of 2

a ratio, that is, A is to C as B is to D. This is step 3 (SET UP RATIOS). The student is then able to proceed to step 4, (PRODUCE FIRST RATIO), by introducing the unknown, X, into its correct position in the equation. Thus, A : C as X : D in this case. From here, the student proceeds to step 5 where he chooses either of two paths-- the UNIT SUB-METHOD or the standard EQUATION SUB-METHOD. The standard EQUATION SUB-METHOD will be explained first.

Step 5 of this method enables the student to produce an equation using the four terms. Step 5 is (PRODUCES FIRST EQUATION). In this problem the equation would be:

$$\frac{A}{X_1} = \frac{C}{D}$$

or 7 Ergs is to 4 Zots as how many Ergs is to 28 Zots?

Step 6 simplifies the problem by using the means-extremes postulate. (This is done by cross multiplying the four variables). Thus, N_1 or numerator A, multiplied by D_2 or denominator D, equals Y, and N_2 or numerator C, multiplied by D_1 or denominator X_1, equals Z. In this problem, Y equals 7 time 28 for an answer of 196, and Z equals 4 times X_1 or $4X_1$. This process completed, the student moves to step 7 (SOLVE FOR X). In this case, since the unknown is a part of Z, the division process of Y divided by Z takes place:

$$\frac{Y}{Z}$$

Thus, 196 divided by 4X is the procedure followed. Step 8 involves the actual division process which is performed, (DIVISION), which gives the final answer X_f in step 9 (ANSWER). Thus, 7 Ergs is to 4 Zots as 49 Ergs is to 28 Zots.

(Note: At any time during this process, the student could have reduced the variables to their lowest terms. Reducing the variables aids the student by shortening the length of multiplication and division process times.)

The second sub-method, the UNIT SUB-METHOD mentioned previously has more steps to the solution path but is very similar in many instances to the EQUATION SUB-METHOD. Using the same problem, Steps 1, 2, and 3 are identical. At step 4 however, (PRODUCES FIRST RATIO), the student produces as his ratio: A is to C as Unit A is to X. In this way, the student strives to find the number of Ergs needed for one Zot and once that number is found, the number of Ergs needed for 28 Zots is determined.

At step 5, the production of the first equation reads:

$$\frac{A}{\text{Unit } A} = \frac{C}{X_1} \qquad \text{or} \qquad \frac{4 \text{ Zots}}{1 \text{ Zot}} = \frac{7 \text{ Ergs}}{X}$$

Steps 6 to 9 are EQUATION SUB-METHOD equivalents, but the solution, X_1, is merely the first stage of a two stage process. Once X_1 is established, in this case, 1.75 Ergs, then a second equation is produced. This means a return to step 6 (PRODUCES "SECOND" EQUATION). The equation produced is:

$$\frac{\text{Unit A}}{\text{B}} = \frac{X_1}{X_f} \quad \text{or} \quad \frac{1 \text{ Zot}}{28 \text{ Zots}} = \frac{1.75 \text{ Ergs}}{X_f}$$

Step 7 continues and proceeds as in the EQUATION SUB-METHOD on to the final solution which would be 49 Ergs. Again, as in the EQUATION SUB-METHOD, the student can reduce the variables to their lowest terms during any step of the solution path to save calculation time.

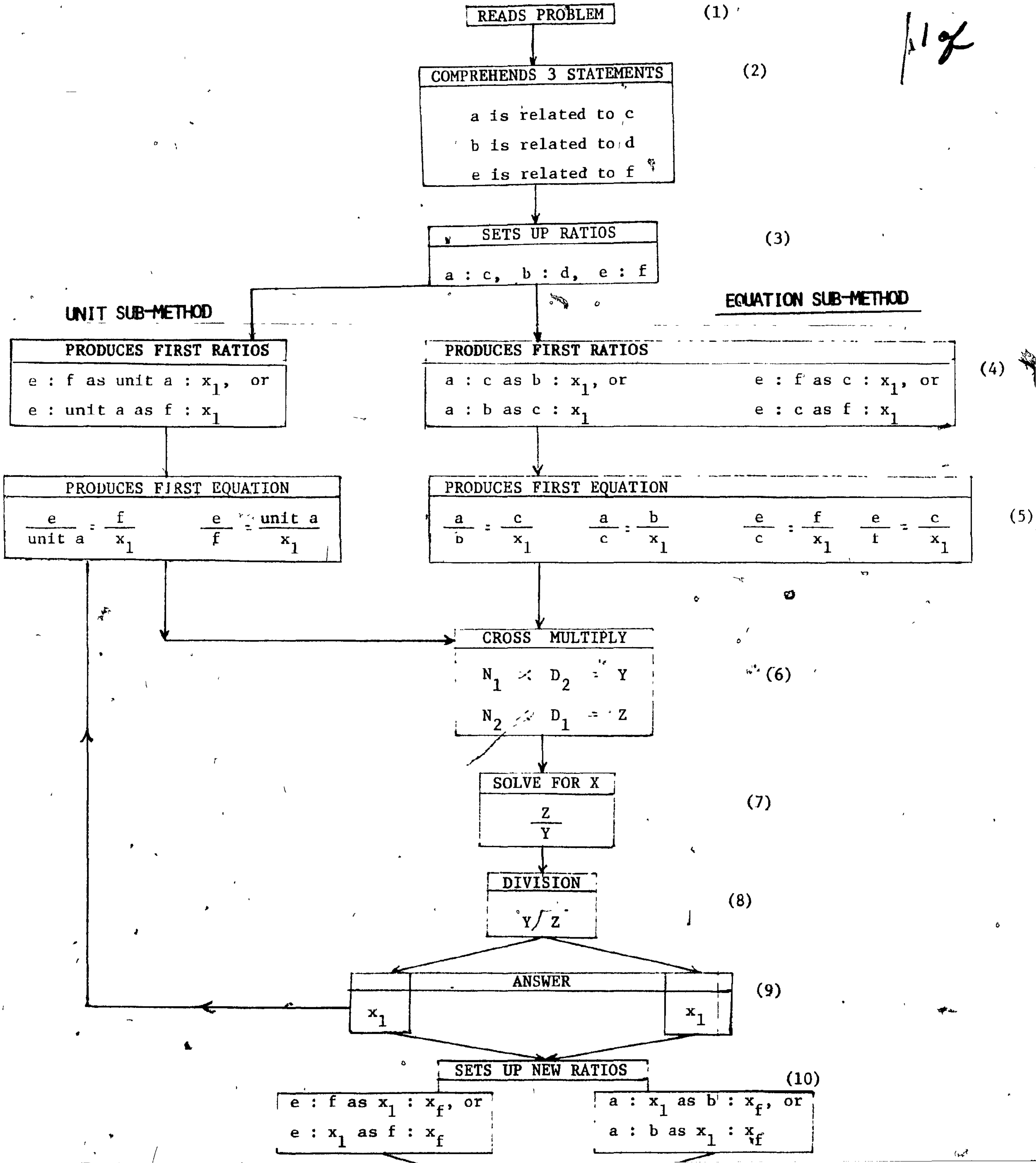
METHOD B

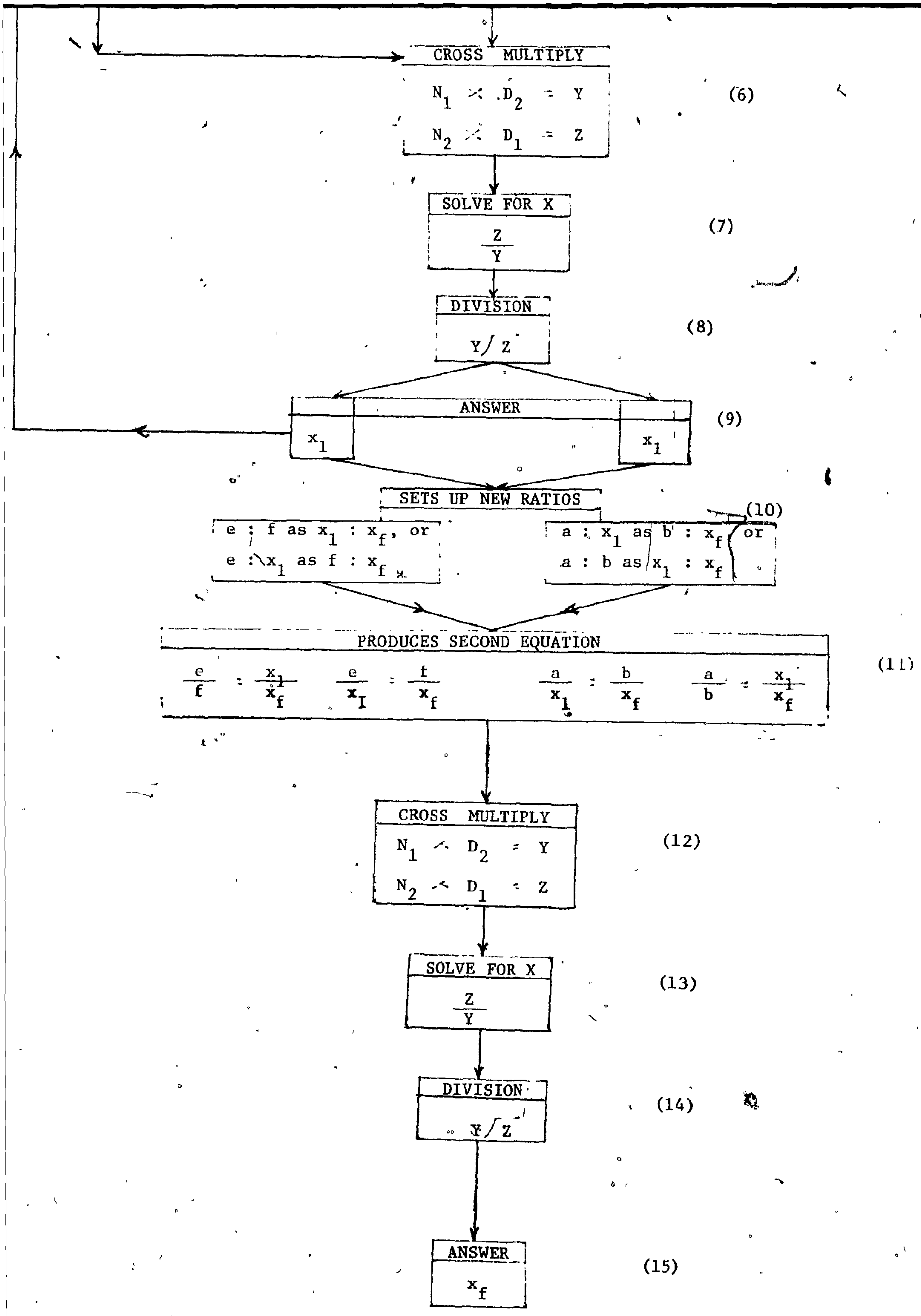
Method B, found in full on the following page is similar in type of steps to Method A, but it is more complex in that Method B is able to handle problems with six variables. The major difference in Method B is that after producing the first ratio in step 5, and following out the calculations to the first answer, a second set of ratios MUST be set up in order to complete the problem. Method A provided for this second equation only in the UNIT SUB-METHOD, and the arrangement of variables for that equation was not nearly as complex. Method B uses this second equation as an Integral Part of the EQUATION SUB-METHOD used in six variable problems.

Method B, because of its complexity, has much greater margin for error. The student must produce two sets of ratios at two different times during the problem set using new variables and confusion can occur precisely

FIGURE 2

METHOD B




CROSS MULTIPLY
$N_1 \times D_2 = Y$
$N_2 \times D_1 = Z$
(6)
SOLVE FOR X
$\frac{Z}{Y}$
(7)
DIVISION
Y/Z
(8)
ANSWER
x_1
x_1
(9)
SETS UP NEW RATIOS
(10)
e : f as $x_1 : x_f$, or
e : x_1 as f : x_f
a : x_1 as b : x_f or
a : b as $x_1 : x_f$
PRODUCES SECOND EQUATION
(11)
$\frac{e}{f} = \frac{x_1}{x_f}$
$\frac{e}{x_I} = \frac{f}{x_f}$
$\frac{a}{x_1} = \frac{b}{x_f}$
$\frac{a}{b} = \frac{x_1}{x_f}$
CROSS MULTIPLY
$N_1 \times D_2 = Y$
$N_2 \times D_1 = Z$
(12)
SOLVE FOR X
$\frac{Z}{Y}$
(13)
DIVISION
Y/Z
(14)
ANSWER
x_f
(15)

2f2

because of the inclusion of the two extra variables. It can be noted also, that Method B has a UNIT SUB-METHOD similar to that used in Method A. Again, the main difference between the two methods is the number of variables with which each deals. Method B's UNIT SUB-METHOD handles two extra variables and as such, is more complex although the process which is used for both methods is alike. Since the names of the steps are the same in both methods, a further example will not be used to indicate the use of Method B.

CHAPTER III

METHOD: SUBJECTS

Difficulties in ratio and proportion problems seem to manifest themselves primarily at the grade nine, ten, and eleven levels of high school education (Piaget and Inhelder, 1956). These difficulties seem to arise, not only in the area of mathematics, but also in related areas such as physics and chemistry. For this reason, a sample of students taken from these three levels were chosen as a somewhat representative group of the students who chose the more difficult subjects of physics, chemistry and higher mathematics.

Students were asked to participate in this study on a volunteer basis, and as a result, six students from grade 9, seven students from grade 10, and six students from grade 11 were chosen. No effort was made to completely randomize the sample, since the aim of the experiment was primarily to examine the protocols or solution paths which various students used at different levels. Students with learning disabilities or below average aptitudes in mathematics were automatically eliminated since these students were not a part of the population examined.

The students chosen came from a suburban high school with a total population of approximately 550 students. Those students who were selected ranged in age from 14 to 17 years and were considered by their teachers to be of between average and above average intelligence.

Background information on the students in the three grade levels was limited to sex, age, relevant school courses that were being taken during the current year, and any intelligence or aptitude tests which had been given to the students during the course of their high school careers. This information is provided in Tables 2, 3, and 4.

PROCEDURE

A test consisting of ten questions was administered to the students on an individual basis within a two week period.

The ten questions were presented to each student in written form identical to that in Table 1. This test paper was presented face down on the desk while the instructions were given. The instructions were standard

TABLE 2

BACKGROUND INFORMATION

GRADE 9

NAME	SEX	AGE	Day/Mo/Yr	RELEVANT SCHOOL COURSES Math 32	Mathmagic***	OTHER TESTS Otis Quick Scoring,	Alt.**
John K.	M	15	10/04/64	91%	Yes	114	131
Robert M.	M	14	30/10/64	88%	No	None	
John N.	M	15	31/12/63	84%	Yes	None	
Stan. N.	F	15	10/10/63	54%	No	108	
Vince R.	M	14	09/08/64	88%	No	None	
Paul Y.	M	15	10/10/63	99%	Yes	None	

** The alternate form of the Otis Quick Scoring Aptitude Test

*** An interest course in mathematics for those especially interested in rapid calculation. No mark is assigned to this course.

TABLE 3

BACKGROUND INFORMATION

GRADE 10

				RELEVANT SCHOOL COURSES		OTHER TESTS	
NAME	SEX	AGE	Day/Mo/Yr	Math 42	Chem 44	Otis	DAT**
Allan G.	M	16	22/03/63	77%	75%	None	
Paula L.	F	15	29/06/63	96%	84%	None	
Sharon P.	F	15	22/11/63	89%	75%	None	
Glenn R.	M	15	27/09/63	87%	75%	None	
Angela R.	F	15	28/09/63	91%	81%	None	
Ruth V.	F	15	10/06/63	60%	60%	None	
Leanna V.	F	16	03/10/62	77%	75%	115	70th. cent. Mathematics

** Differential Aptitude Test

TABLE 4

BACKGROUND INFORMATION

GRADE 11

				RELEVANT SCHOOL COURSES			OTHER TESTS	
NAME	SEX	AGE	Day/Mo/Yr	Math 52	Phys 51	Chem 56	Otis	MEAT**
Jackie D.	F	17	27/04/62	79%	77%	93%	None	
Tony G.	M	16	19/12/62	60%	60%	60%	None	
Karen H.	F	16	14/09/62	71%	79%	75%	113	683 96th. Cent.
Peter R.	M.	17	13/02/62	80%	76%	70%	None	
Sue W.	F	17	18/02/62	83%	85%	78%	None	
Diane Y.	F	16	20/05/62	91%	89%	81%	116	683 96th. Cent.

** Ministry of Education Aptitude Test (Province of Quebec) for each student with slight variation occurring only when a student questioned the instructions. If a question was asked, the instructions were repeated. In all nineteen cases, the instructions had to be repeated only three times in three individual cases.

The instructions were as follows:

> You will be given ten mathematical problems to solve. Instead of solving the problems in your head however, what you are asked to do is to think out loud as you are solving the problem. For example, if you are asked to multiply 12 by 37, by thinking out loud you might say: "2 times 7 is 14, carry the one, then 2 times 3 is 6 and 1 is 7. Then 1 times 7 is 7 and 1 times 3 is 3. Adding the two lines, we have 4, 7 plus 7 which is 14, carry the 1, 3 plus 1 is 4. The final answer is 444."
>
> Even when you are not doing calculations, I want you to tell me specifically what you are thinking about. You may use the pencil and paper provided on the desk in front of you, but remember to think out loud for all of the problems.
>
> If, for any reason, you can't do some of the problems, try to tell me why you think

> you can't do it. Likewise, if you can't finish a problem, try to tell me why.
>
> I'm going to tape record all of your thinking out loud and don't worry about time limits or the number of questions that you get right or wrong. Just do your best. Do you have any questions?
>
> O.K. turn over the question sheet and start. Please remember to think out loud when you are answering the questions.

Occasionally, during the time when the student was working on the problems, the observer would ask "What are you thinking about now?". The observer would not interrupt calculations, however, merely try to obtain thought processes during times of seeming inactivity.

The student's protocols were then tape recorded until the student had indicated that he/she had completed the problems. During the tests, questions were sometimes asked by the student, but no direct answers were given to indicate that one or another solution strategy should be followed. No further facts other than those presented in the test paper were given to the students at any time.

When all of the tapes were collected, all of the individual protocols were transcribed word for word onto paper. The individual protocols were than coded according to the steps indicated by the task analysis outlined in the previous chapter.

Once these individual steps were established, the tapes were re-played, and the reaction times for each of the steps were placed next to the appropriate phrases in the protocols. Reaction times were established using a stopwatch and recorded to the nearest two tenths of a second. The same person used the same timing mechanism for all the data in order to maintain some degree of consistency.

Reaction times were only measured after the student had read the question completely once. Any further referral to the question was included in the reaction times. As soon as a student reached the conclusion to a step of the strategy he/she had chosen, the time taken to accomplish that particular step was recorded and the time for any further steps was calibrated from that point. The use of a tape recorder with an instant stop-start mechanism proved invaluable for this type of reaction-time data.

The reliability of the reaction-time data should be good since the accomplishment of a particular step in the task is immediately recognizable from the subject's verbalizations. As soon as the goal "number" of a particular step is heard by the observer, the time of that step is recorded immediately. Thus, the beginning ~~and end~~ of each step is not a subjective estimation by the observer, but rather a precise timing of a definite and distinct period of processing.

The timing of the steps in the various strategies very often involves the process of what is often referred to as "chunking" (Miller, 1956). Two or more of the steps involved in a particular strategy are grouped and timed as a unit. This is done since it does not appear that these steps occur individually in the strategies of the sample in some cases. Since any attempt at breaking down the chunks into their components would be impossible, the chunks appear intact with a single time representing the entire chunk.

Total times for each problem were also calculated for further comparisons among grade levels and individuals.

All of these protocols are listed alphabetically according to grade level in Appendix II of this thesis. Incorrect answers to individual problems are indicated with a circle around the number of the problem. The precise step where the error occurred is indicated by a large arrow in the left hand margin. A summary of the number of errors made by each student, together with an error analysis will be found in Chapter IV. The encoded phrases together with the times of each step in any particular problem are found in Appendix I.

CHAPTER IV

RESULTS

The percentage of correct responses for the ten questions given on the test is presented in Table 5 on the following page.

From these results, some interesting patterns appear. There seems to be no difference among the three groups of students concerning the basic symbolic problems. Grade 9 students fared poorly compared with the other students in the two other grade levels on the story problems, and yet scored higher than the other groups on the nonsense problems. The grade 11 students scored higher than the other groups on the chemistry problems. It is worthy of mention here, that both the grade 10 and grade 11 students would be familiar with the terminology used in the chemistry problems, even though a knowledge of chemistry was unnecessary in arriving at solutions to the two problems.

A further breakdown of the ten problems appears in Table 6. In this table, a summary of the number of errors per problem per individual are detailed. It is interesting to note that although the same types of problems were given in the Symbolic Problem section of the test, (numbers 1 to 4), a practice effect seems to take place. Most errors were made on the first problem (8 errors) and least errors were made on the fourth problem (1 error), with four and five errors made on

Table 5

PERCENTAGE OF CORRECT RESPONSES AT EACH GRADE LEVEL

	Symbolic Problems	Story Problems	Nonsense Problems	Chemistry Problems
GRADE 9	79	42	92	33
GRADE 10	79	57	79	36
GRADE 11	71	58	75	75

N.B. These percentages are grouped according to correct responses alone. Method is not a factor in these figures.

Table 6

SUMMARY OF ERRORS

SUBJECT	QUESTION NUMBER										
	1	2	3	4	5	6	7	8	9	10	TOTAL
VINCE R.				X	X		X		X	X	5
ROBERT M.		X			X	X				X	4
JOHN K.					X	X					2
PAUL Y.										X	1
STAN. N.	X		X		X	X			X	X	6
JOHN N.	X								X	X	3
GLENN R.	X									X	2
PAULA L.			X			X					2
LEANNA V.	X	X			X	X	X		X	X	7
RUTH V.		X			X	X	X		X	X	6
ALLAN G.									X	X	2
ANGELA R.	X								X		2
SHARON P.					X		X			X	3
KAREN H.	X		X			X	X			X	5
TONY G.							X			X	2
JACKIE D.	X	X	X			X	X		X		6
DIANE Y.					X	X					2
SUE W.	X		X								2
PETER R.					X						1
TOTALS	8	4	5	1	9	9	7	0	8	12	63

problems two and three respectively.

Problems 5 and 6, the Story Problems, seem to have been difficult for all three grade levels with the grade 11 students having only two less errors than the grade 9 students and one less than the grade 10 students. In three cases, John K., Diane Y., and Peter R., the only mistakes which were made on the test were in this section. John K. and Diane Y. had both questions incorrect and Peter R. had one of the questions answered incorrectly.

In the third section, Nonsense Problems, the grade 9 students performed markedly better than the other two grade levels, having only one error in this section. It is interesting to note that question number 8 of this section was the only question which the entire sample computed correctly.

The last section, the Chemistry Problems, posed about eoual difficulty for the grade 9 and 10 levels, although the grade 10 students were familiar with the terms. Three students in both grade 9 and grade 10 solved both chemistry problems incorrectly while none of the grade 11 students had more than one mistake in the Chemistry Problem section.

The total number of problems answered by the 19 students was 190. Of the 190 problems, there were 63 problems solved incorrectly. This result clearly indicates the difficulty which students in all three grade levels experience with ratio and proportion problems.

The six grade 9 students had a total of 21 errors for an average error rate of 3.5; the seven grade 10 students had a total of 24 errors for an average error rate of 3.4, and; the six grade 11 students had a total of 18 errors for an average error rate of 3.0.

Median solution times for correct answers were taken for all problems and appear in summary form in Table 7. The initial reading of the problem was excluded from the median solution times.

From the results in Table 7 it seems evident that there is a definite interaction between semantic content and grade level. For the Symbolic Problems, the solution times decrease as grade level increases. On the other hand, there does not seem to be any evidence of change as a function of grade level for the Nonsense Problems. Response times for the non-trivial sections (Story Problems and Chemistry Problems) increase as a function of grade level.

Table 8 shows individual reaction times according to problem as well as the median times of correct and incorrect responses. It is worthy of note here that, except for one particular case, (the grade 10 median scores for problems 9 and 10), the median solution times follow the same pattern as that established in Table 7 which uses only correct responses as median times.

The number of problems having incomplete solutions

Table 7

MEDIAN SOLUTION TIMES FOR CORRECT RESPONSES (SECONDS)

	Symbolic Problems	Story Problems	Nonsense Problems	Chemistry Problems
GRADE 9	42	67	74	132
GRADE 10	24	98	56	157
GRADE 11	19	125	61	172

Table 8

REACTION TIMES

	Question Number									
SUBJECT	1	2	3	4	5	6	7	8	9	10
Vince R.	42.0	32.8	93.2	33.6*	141.4*	135.2	241.6*	33.6	166.6*	373.0*
Robert M.	46.8	24.6*	69.8	35.0	81.6*	86.4*	103.0	78.6	167.8	122.8**
John K.	28.8	9.8	80.4	15.2	42.6*	79.0*	70.0	40.0	62.0	97.2
Paul Y.	44.8	46.4	13.4	13.5	101.6	122.4	99.0	48.6	190.6	84.2**
Stan. N.	120.6*	33.4	123.2*	64.4	156.0**	142.6**	217.2	108.6	101.4**	45.2**
John N.	19.5*	68.6	62.0	13.0	61.6	66.6	66.8	18.4	82.4*	107.4
MEDIAN	43.4	33.1	75.1	24.4	81.6	86.4	101.0	44.3	166.6	107.4
Glenn R.	19.6*	19.8	73.0	18.6	109.0	124.4	94.4	65.0	243.6	522.2*
Paula L.	22.0	12.8	96.8*	15.6	86.4	99.6*	65.6	37.4	98.4	339.4
Leanna V.	37.6*	28.2*	58.4	40.6	75.2*	113.4*	106.0**	30.6	61.0**	113.0**
Ruth V.	24.3	64.1*	54.2	26.8	59.3*	117.4*	33.6**	39.2	56.0**	38.0**
Allan G.	41.2	25.0	62.6	31.0	121.2	68.2	56.2	55.6	131.0*	297.6*
Angela R.	9.0*	21.8	45.6	13.4	63.6	53.8	75.8	20.4	100.4*	107.2
Sharon P.	30.4	13.8	61.5	16.8	125.0*	112.2	140.4*	80.4	146.6	392.8**
MEDIAN	24.3	21.8	61.5	18.6	86.4	112.2	75.8	39.2	131.0	318.5
Karen H.	11.0*	23.0	64.4*	19.4	120.6	112.6*	218.2*	44.2	115.4	230.0*
Tony G.	111.4	21.4	50.6	12.2	178.4	112.4	165.4*	68.6	57.4	169.4*
Jackie D.	37.4*	21.0	78.2*	14.8	138.8	182.2*	84.8*	18.8	126.0*	281.6
Diane Y.	18.6	10.2	22.0	7.8	52.0*	66.4*	69.6	26.2	55.2	184.8
Sue W.	37.2*	16.0	64.0*	17.0	125.2	97.0	120.4	60.8	143.6	391.2
Peter R.	24.8	13.8	93.4	15.6	149.2*	126.2	142.6	45.4	171.6	192.6
MEDIAN	31.0	18.5	64.2	15.2	132.0	112.5	131.5	44.8	120.7	211.3

* Error **Incomplete Solution -- not included in MEDIAN calculation

indicates that the individual did not know how to arrive at the solution, either at the beginning of the problem or at some stage during the solution process. Of the thirteen incomplete solution processes, three individuals, one from grade 9 and two from grade 10, were responsible for 10 of the problems. None of the grade 11 students left a problem unfinished.

The types of solution strategies used by the nineteen students are broken down by individual problem in the following pages. All of the solution strategies include only those problems which were in the completion process. If the problem was started, but insufficient information was developed to determine a method, the strategy was completely omitted. The strategies involved are either:

a) Method A - Sub-Method (Equation)----Ae
b) Method A - Sub-Method (Unit)--------Au
c) Method B - Sub-Method (Equation)----Be
d) Method B - Sub-Method (Unit)--------Bu
e) A combination of the Sub-Methods (a - d)
e.g. Ae + Bu would mean that the first part of the problem was done using the A Method and the equation sub-method. The second part of the problem was done using Method B and the unit sub-method.

Questions 1, 2, and 3 of the Symbolic Problems evoked the same strategy, Method Ae for all students in all three grade levels. The tables on the following pages, Table 9, 10, and Table 11 indicate that six grade 9 students, seven

Table 9

Use of Method and Format by Grade Level

Problem 1 - - Symbolic Problem Area

METHOD Sub-Method	Grade 9		Grade 10		Grade 11	
	N	% of Errors	N	% of Errors	N	% of Errors
Ae	6	33	7	42	6	50

Table 10

Use of Method and Format by Grade Level

Problem 2 - - Symbolic Problem Area

METHOD Sub-Method	Grade 9		Grade 10		Grade 11	
	N	% of Errors	N	% of Errors	N	% of Errors
Ae	6	16	7	28	6	16

Table 11

Use of Method and Format by Grade Level

Problem 3 - - Symbolic Problem Area

METHOD Sub-Method	Grade 9		Grade 10		Grade 11	
	N	% of Errors	N	% of Errors	N	% of Errors
Ae	6	16	7	14	6	50

Table 12

Use of Method and Format by Grade Level

Problem 4 - - Symbolic Problem Area

METHOD Sub-Method	Grade 9		Grade 10		Grade 11	
	N	% of Errors	N	% of Errors	N	% of Errors
Ae	2	0	3	0	2	0
Ae + RLT	4	25	4	0	4	0

grade 10, and six grade 11 students all used Method Ae.

Two strategies were used in problem 4. Although the same method was used, Ae, the step RLT was included. RLT indicates that the variables were reduced to their lowest terms before Method Ae was applied. It can be seen in Table 12 that the split among the students is approximately equal. Half of the students used RLT and half did not. The only exception is the grade 11 students who did not use Method Ae and Ae + RLT to the same extent as the grade 9 and 10 students.

Three different strategies were used in problem 5. The grade 9 students used a combination of Methods A and B with different sub-methods whereas the grade 10 and 11 students used Method Be with only two exceptions. Table 13 clearly shows the discrepancy in methods used by the three groups.

The solution strategies for problem 6 also varied along the same lines as problem 5. The grade 10 and 11 students used one distinct method, Method Ae, and the grade 9 students used a unit sub-method of a totally different method, Bu. There was some crossover among the three groups, but there is a marked difference between the groups using the two strategies indicated in Table 14.

It is also worthy of note that the grade 9 students differed from the grade 10 and 11 students in a similar type of problem; both problems 5 and 6 were Story Problems. The strategies used by the groups of students for this

Table 13

Use of Method and Format by Grade Level

Problem 5 - - - Story Problem Area

METHOD Sub-Method	Grade 9		Grade 10		Grade 11	
	N	% of Errors	N	% of Errors	N	% of Errors
Ae + Bu	4	100	1	100	0	0
Ae + Be	2	0	1	0	0	0
Be	0	0	5	40	6	33

Table 14

Use of Method and Format by Grade Level

Problem 6 - - - Story Problem Area

METHOD Sub - Method	Grade 9		Grade 10		Grade 11	
	N	% of Errors	N	% of Errors	N	% of Errors
Bu	4	75	2	50	1	100
Ae	2	0	5	20	5	40

type of problem are obviously different whereas no such distinction was evident in the Symbolic Problem area, (Questions 1 to 4).

Four different strategies were used to solve problem 7 in the Nonsense Problem area of the test. As Table 15 indicates, there was no distinct method or strategy which any group of students favored at any grade level. Most of the students seemed to use a combination of Methods A and B to arrive at a solution. Only three students, one grade 10 student and two grade 11 students used one method totally (Be) to solve the problem.

In the second Nonsense Problem, Table 16 indicates that the same method is used by all of the students in the three grade levels, but all of the grade 9 students used RLT, reducing the variables to their lowest terms, in order to first simplify the problem. Half of the grade 10 students also found it necessary to simplify the terms, yet only one grade 11 student found this step necessary. The grade 11 students and half of the grade 10 students used a straight solution strategy (Ae) to solve the problem.

The solution strategies for question 9, the first Chemistry Problem are all non-combination strategies. Table 17 shows that equation methods, either Ae or Be are used by all of the grade 10 and 11 students and two of the grade 9 students while a unit sub-method Bu,

Table 15

Use of Method and Format by Grade Level

Problem 7 - - Nonsense Problem Area

METHOD Sub-Method	Grade 9		Grade 10		Grade 11	
	N	% of Errors	N	% of Errors	N	% of Errors
Au + Be	1	0	1	100	0	0
Ae + Be	3	33	1	0	3	66
Au + Bu	2	0	2	0	0	0
Be	0	0	1	0	2	0

*not completed - two grade 10 students, one grade 11 student

Table 16

Use of Method and Format by Grade Level

Problem 8 - - Nonsense Problem Area

METHOD Sub-Method	Grade 9		Grade 10		Grade 11	
	N	% of Errors	N	% of Errors	N	% of Errors
Ae + RLT	6	0	3	0	1	0
Ae	0	0	4	0	5	0

Table 17

Use of Method and Format by Grade Level

Problem 9 - - Chemistry Problem Area

METHOD Sub-Method	Grade 9		Grade 10		Grade 11	
	N	% of Errors	N	% of Errors	N	% of Errors
Bu	3	33	0	0	0	0
Ae	2	50	3	66	4	25
Be	0	0	2	0	2	0

*not completed - one grade 9 student, two grade 10 students

Table 18

Use of Method and Format by Grade Level

Problem 10 - Chemistry Problem Area

METHOD Sub-Method	Grade 9		Grade 10		Grade 11	
	N	% of Errors	N	% of Errors	N	% of Errors
Au × 3	2	50	0	0	0	0
Ae + $\frac{1}{2}$Be	1	0	2	0	1	0
Be	0	0	2	50	3	66
Ae + Be	0	0	0	0	1	0

*not completed - three grade 9, three grade 10, one grade 11, students.

is preferred by three of the grade 9 students.

Problem 10, presented in Table 18, shows four strategies being used. Similar to the strategies used in problem 9, grade 10 and 11 students use equation sub-methods or combinations of equation sub-methods, while, with one exception, grade 9 students use a unit sub-method (Au).

A further breakdown of methods and sub-methods is required, however, in order to show specific patterns of usage which develop. Table 19 provides a complete analysis of strategy type used by each of the 19 students for each of the 10 problems. Before analyzing the table, however, it is necessary to recall that the use of a unit sub-method in either Method A or Method B is an established procedure, but needs much less of an understanding of the ratio-proportion concept. Equation sub-methods, on the other hand, require the establishment of more sophisticated ratios and a clear understanding of the goal.

The overall pattern of strategies used to attack the problems differs markedly among grade levels. Grade 9 students use some form of unit sub-method in 15 individual cases. All of the grade 9 students used a unit sub-method at least once when a difficult problem was encountered. Grade 10 students used a unit sub-method in 5 cases and grade 11 students used a unit sub-method in only 1 individual case.

Table 19

Use of Method and Sub-Method by Subject

SUBJECTS	Problem Number 1	2	3	4	5	6	7	8	9	10
John K.	Ae	Ae	Ae	Ae + RLT	Ae + Bu	Bu	Au + Be	Ae + RLT	Bu	Au - 3X
Robert M.	Ae	Ae	Ae	Ae	Ae + Bu	Bu	Ae + Be	Ae + RLT	Bu	------
John N.	Ae	Ae	Ae	Ae + RLT	Ae + Be	Bu	Au + Bu	Ae + RLT	Bu	Au - 3X
Stan. N.	Ae	Ae	Ae	Ae	Ae + Bu	Bu	Ae + Be	Ae + RLT	----	------
Vince R.	Ae	Ae	Ae	Ae + RLT	Ae + Bu	Ae	Ae + Be	Ae + RLT	Ae	Ae + ½Be
Paul Y.	Ae	Ae	Ae	Ae + RLT	Ae + Be	Ae	Au + Bu	Ae + RLT	Ae	------
Allan G.	Ae	Ae	Ae	Ae	Ae + Be	Bu	Ae + Be	Ae	Ae	Be
Paula L.	Ae	Ae	Ae	Ae + RLT	Be	Ae	Be	Ae	Ae	Ae + ½Be
Sharon P.	Ae	Ae	Ae	Ae + RLT	Ae + Bu	Ae	Au + Be	Ae + RLT	Be	Ae + ½Be
Glenn R.	Ae	Ae	Ae	Ae + RLT	Be	Ae	Au + Bu	Ae	Be	* * *
Angela R.	Ae	Ae	Ae	Ae + RLT	Be	Ae	Au + Bu	Ae + RLT	Ae	Be
Ruth V.	Ae	Ae	Ae	Ae	Be	Ae	------	Ae	----	------
Leanna V.	Ae	Ae	Ae	Ae	Be	Bu	------	Ae + RLT	----	------
Jackie D.	Ae	Ae	Ae	Ae + RLT	Be	Bu	Ae + Be	Ae + RLT	Ae	Be
Tony G.	Ae	Ae	Ae	Ae + RLT	Be	Ae	Ae + Be	Ae	Ae	Be
Karen H.	Ae	Ae	Ae	Ae	Be	Ae	------	Ae	Be	Be
Peter R.	Ae	Ae	Ae	Ae + RLT	Be	Ae	Be	Ae	Ae	Be
Sue W.	Ae	Ae	Ae	Ae + RLT	Be	Ae	Ae + Be	Ae	Ae	Ae + ½Be
Diane Y.	Ae	Ae	Ae	Ae	Be	Ae	Be	Ae	Be	Ae + Be

* * * many sub-methods underlined - error ----- incomplete strategy

Strategies whereby combinations of methods and sub-methods are used also differ among the three grade levels. Combinations of method and sub-method at the grade 9 level were used in 15 cases, excluding the methods where RLT was used, since RLT was not an individual sub-method of either method, merely a single step to be used or rejected at any time during the solution of a problem. Grade 10 students used combination methods and sub-methods in 8 cases excluding RLT. Grade 11 students used combinations in only 5 individual cases.

A breakdown of reaction times according to method and sub-method is presented in Table 20. In all cases but problem 1, the median times for the incorrect problems were higher than the median times for the correct answers. This result seems to suggest that slower processing times seem to correlate positively with some degree of error. In all but two cases, (Problem 1 and 5, Method Be), median times for incorrect answers were higher than total median times for all methods and problems.

Table 20 also indicates that problems 7 and 10 both used the highest number of methods and sub-methods (4) to solve the problems. This seems to indicate that these two problems were considered more complex than the other eight. Although no one particular method and sub-method was preferred for these problems over the three grade levels, equation sub-methods for either Method A or Method B were used in 21 of the 28 solution attempts.

Table 20

Reaction Times for Method and Sub-Method

Problem Number	Method Sub-Method	Number of Students	Number of Errors	% of Errors	Median Time Correct Ans.	Median Time Incorrect Ans.	Median Time All Ans-
1	Ae	19	8	42.1	30.4	25.9	32.0
2	Ae	19	4	21.0	17.2	24.1	17.2
3	Ae	19	5	26.3	59.7	75.2	60.8
4	Ae + RLT	12	1	8.3	15.0	33.6	15.1
	Ae	7	0	0	29.2	----	29.2
5	Ae + Bu	5	5	100.0	----	85.6	85.6
	Ae + Be	2	0	0	76.6	----	76.6
	Be	11	5	45.4	83.1	84.1	88.1
6	Bu	7	5	71.4	69.0	111.0	97.4
	Ae	12	4	33.3	92.0	96.4	92.7
7	Au + Be	2	1	50.0	31.8	140.4	86.1
	Ae + Be	7	3	42.8	88.4	140.0	101.3
	Au + Bu	4	0	0	27.1	----	27.1
	Be	3	0	0	68.6	----	68.6
8	Ae + RLT	10	0	0	32.6	----	32.6
	Ae	9	0	0	41.2	----	41.2
9	Bu	3	0	0	75.2	----	75.2
	Ae	9	4	44.4	102.0	126.4	123.6
10	Au × 3	2	1	50.0	97.2	107.4	102.0
	Ae + ½Be	4	2	50.0	309.2	345.2	257.9
	Be	6	3	50.0	170.2	286.9	179.7
	Ae + Be	1	0	0	158.8	----	158.8

ERROR ANALYSIS

Types of errors for question number and individual are presented in Table 21. The types of errors made by the sample were divided into five specific categories. Multiplication errors (MULT.) indicate an error made in computing one number times another. Division errors (DIV.) indicate an error made in a long division process. A processing error (PROC.) occurs when an individual is either unable to set up correct ratios to enable a solution process to take plave, or the individual confuses the variables such that the ratios are incorrect. (RET.) indicates that an individual attempts the problem but gives up on the solution before establishing a strategy. Careless mistakes not covered by either arithmetic or processing errors such as copying a number down incorrectly are indicated by (C).

It is interesting to note that errors related to strategy account for only one third of the errors made in all cases. Strategy errors are either processing errors, (PROC.) or retiring from the problem, (RET.). Multiplication and division errors account for the remaining two thirds of the mistakes. Arithmetic errors are made up of either multiplication or division errors, with approximately 50% of each type appearing in the errors. Since both multiplication and division involve the simpler processes of addition and subtraction, it seems evident that errors in the ratio and proportion problems presented

Table 21

TYPES OF ERRORS

	Subject	Question Number									
		1	2	3	4	5	6	7	8	9	10
9	John K.					MULT.	MULT.				
	Robert M.		C			MULT.	MULT.				RET.
	John N.		PROC.							DIV.	PROC.
	Stan. N.	DIV.		DIV.		RET.	RET.			RET.	RET.
	Vince R.				DIV.	DIV.		PROC.		DIV.	DIV.
	Paul Y.										RET.
10	Allan G.									DIV.	MULT.
	Paula L.			DIV.			DIV.				
	Sharon P.					MULT.		MULT.			MULT.
	Glenn R.	MULT.									PROC.
	Angela R.	MULT.								DIV.	
	Ruth V.		PROC.			PROC.	MULT.	RET.		RET.	RET.
	Leanna V.	MULT.	PROC.			RET.	MULT.	RET.		RET.	RET.
11	Jackie D.	MULT.	PROC.	DIV.			MULT.	MULT.		DIV.	
	Tony G.							MULT.			DIV.
	Karen H.	MULT.		DIV.			DIV.	RET.			DIV.
	Peter R.					C					
	Sue W.	DIV.		DIV.							
	Diane Y.					DIV.	MULT.				

19	MULT.....error in multiplication
20	DIV.....error in division
8	PROC.....error in processing
14	RET.....student retires from the problem
2	C.......careless error not related to the above processes

in this test can be seen as computational errors in 66% of the cases.

It is also interesting to note that processing errors were responsible for incorrect answers in 4 of the 5 cases in problem 2 whereas the other three problems of that section have only computational errors. Problems 5, 6, 7, 9, and 10 all have both processing and computational errors, with problem 8 having no incorrect responses whatsoever. Problem 3 has division errors making up all of the 5 errors made in that problem, and problem 4 has a division error as its only mistake. Likewise, other than those students who retired from the problem before completion, problem 9 has division mistakes as the only errors.

The most important point is that no students of the nineteen students tested solved all ten problems correctly. Also, the fact that there were 63 errors out of a possible 190 correct answers indicates a substantial error rate in simple ratio and proportion problems. The hypothesis that ratio and proportion problems posed difficulties for high school students was not, apparently, unfounded. Difficulties in ratio and proportion were not merely limited to one grade level. All things being equal, grade 11 students should have had substantially less errors than either the grade 9 or grade 10 students. This was not the case. Average error rate per grade level was almost equal, although different problems posed particular difficulties for various grade levels as the results have shown.

The two areas which posed the most difficulty were the Story Problems and the Chemistry Problems. There seems to be a decrease in error in these two areas, however, with increased grade level. This decrease in error rate coupled with increased solution path time seems to be consistent with Larkin (1977) where a planning stage followed by equation methods is used. According to individual strategies, this seems to be the case with the grade 11 students.

ARITHMETIC ERRORS

The arithmetic errors are evenly distributed over the three grade levels and are equally distributed between multiplication and division errors. An examination of the protocols reveals some interesting facts concerning individual problems. Problem 1, for example, has five multiplication errors. Protocol analysis reveals that the same error was made in all five cases. The problem, 9 times 6, was calculated incorrectly, suggesting perhaps a lack of accurate retrieval from memory or interference on this particular problem. Since multiplication tables are a memory task, it would seem that either the tables were not practiced sufficiently or that some specific difficulty arises with the multiplication of these two numbers. The answers 36 and 63 were given twice each, and 48 was given once. Since this error was responsible for over a 25% error rate on a Symbolic Problem, it would seem to be an interesting area for further research.

Problem 3 has five errors and all of the errors are division miscalculations. Similar to Problem 1, the same division error was made in all five cases. The problem 56 divided into 1400 was given as 23 twice, 22.9, 25.6, and 2.5. Again, this same error was responsible for over 25% of the students getting the problem wrong. It would seem that there are a number of problem areas here. Since long division uses both subtraction and multiplication processes, an analysis of long division strategies at the high school level might reveal some interesting data.

Question 6 reveals, after examining the individual protocols, that of the six multiplication errors made on the problem, four errors were made on the multiplication of 275 times 350, giving various answers, and two errors were made on the multiplication of 275 times 700. Again, various elements of simple arithmetic could be responsible for these errors which did not show up in the protocols.

Question 8 was the only question which none of the nineteen students erred on. Question 8, as was stated previously, is the Word Problem which is most similar to the Symbolic Problems (Numbers 1 to 4). Since it was noted that a practice effect seemed to be occurring with the Symbolic Problems, it could be postulated that this same practice effect was responsible for the lack of error in Problem 8.

Finally, question 9 had five division errors. Three of the five errors occurred with the problem 28 divided into 770. In fact, all three students, when performing the first operation of dividing 28 into 77 said three and did not notice their error when multiplying 28 times 3 in the following step. All got less than 77 for the multiplication of 28 times 3. The other two division errors were 4 into 22, and 14 into 385.

Division and multiplication miscalculations were responsible for 39 of the 63 errors accounted for; almost 62%. Perhaps errors in basic arithmetic processes are responsible for a great deal of what is assumed to be processing errors in ratio and proportion problems. Still, however, 22 errors out of 63, or almost 35% of the errors were ratio-proportion processing errors. These errors were distributed over ten of the nineteen students who had taken the test. Thus, more than 50% of the students made an error in conceptualizing the variables in one or more of the problems.

RATIO-PROPORTION CONCEPT ERRORS (PROCESS ERRORS)

Of the 22 errors made by the students, four students, two from grade 9 and two from grade 10, were responsible for 16 of the errors. Thus it seems clear that four students, or more than 20% of the sample have great difficulty with ratio and proportion problems. Process errors were divided primarily among problems 2,7,9, and 10.

An analysis of the process errors by problem will shed some light on the strategy areas which were not clearly understood by the students in general. More will be mentioned of the individual students (four) at a later time.

Problem 2 was responsible for four processing errors. An analysis of the protocols reveals that in all four cases, the four variables of the Symbolic Problem were confused so that obviously incorrect answers were put down simply because the students followed a learned strategy and would not doubt their answers. Answers such as 8 over 15 is equal to 10 over 15 would not be challenged by the students. This phenomenon corresponds to Boulanger's (1976) findings on intensive training of formal operational concepts to concrete operational students. He found that the student mistrusted his own intuition and instead blindly followed a teacher-induced mechanical formula. The same results did not occur with the other three Symbolic Problems, however, which implies that the placement of the unknown variable could have been a determining factor. Rosenthal and Resnick (1974) found that there was a difference in error rate depending upon the position of the unknown set. The problem in this study had the unknown variable in an unusual position for a ratio problem. The three other types of Symbolic Problems are usually used more frequently. As a result, none of the other Symbolic Problems had any process errors whatsoever.

As can be seen in Table 21, none of the other 9 problems had as many process errors. Other problems had many RET.'s which involved retiring from the problem before a strategy was either started or completed. Specifically, this involved problems 5, 7, 9, and 10. Question 2 seemed, for some reason, to be a singular occurrence.

Of the fourteen cases where the student retired from the problem, three students, one grade 9 student and two grade 10 students were responsible for eleven of the cases. These three students were also three of the four students making the most concept errors. These three students will be discussed individually.

SN, RV, LV.

SN retired from both Story Problems and both Chemistry Problems, yet was able to calculate correctly both Nonsense Problems which were at least as difficult, if not more so, as the two Story Problems. SN had no difficulty with the Symbolic Problems except for two calculation errors, yet when she attempted the Story Problems, she could not maintain the variables of the problem in their correct relationship to each other. Contrary to Simon and Simon's (1977) study concerning the novice who kept creating equations out of the variables involved, she would attempt to unitize all the variables and eventually confuse the goal of the task.

For problems 9 and 10, SN gave up after merely reading the problems over a number of times with the statement: "I haven't the faintest idea.....you know that eh?"

This type of reaction is similar to that found by Kantowski (1977) whereby students would merely give up on the problem instead of shifting strategies. Kantowski attributes this giving up to either a lack of pre-requisite knowledge, or to personality factors. A review of SN's mathematics marks compared to the other students chosen from her grade level reveals that the former might be the case.

RV and LV both have difficulties similar to SN's. Problems 7, 9, and 10 pose what RV and LV see as insurmountable problems because they are unable to see the relationship between the placement of variables in former problems with the problems with which they have difficulty. LV, in problem 7, changes the variables from the nonsense syllables to horses and chickens in an effort to concretize the variables. This effort also failed. This seems to place LV in a concrete operational level of performance. That she was able to solve other problems correctly was probably due to learned procedures which she was able to apply. RV stated that she could not do problems 7, 9, and 10 and proceeded to state that she had trouble controlling the variables and their placement. Also, for problems 9 and 10, she admitted that the Chemistry terms confused her. This would seem to be an obvious lack of the concept of ratio and proportion. Again, she was able to work on the former Symbolic Problems successfully, but probably with the same learned strategy which she had used in similar situations. Similar results occurred with LV on problems 9 and 10, but she was not affected by interference from the chemistry terminology.

CHAPTER V

GENERAL DISCUSSION

It is clear from the previous chapter that one of the most salient features of the ratio and proportion problems is that they pose a great deal of difficulty for all levels of high school students, particularly grades 9, 10 and 11. The word problems seemed to be more difficult to solve than the symbolic problems, and the Story Problems as well as the Chemistry Problems seemed to cause the most confusion of all. Arithmetic errors were the cause of the majority of mistakes on the problems, yet process errors were still evident in more than half of the individual protocols.

Reaction time data directly linked to strategy type accounts for large discrepancies in solution path times among the three groups. While one might have expected the grade 11 students to process information much more rapidly than the grade 9 students, the reverse seemed to occur. An analysis of the more difficult word problems revealed strategy differences which accounted for a planning stage which the grade 11 students used which was not evident in the protocols of the other two grade levels.

It should be noted here, that conventional significance tests were not performed because of the small and highly variable number of observations in each cell which would somewhat negate adequate statistical conclusions. Also, the problems were not linguistically isomorphic nor mathematically equal in difficulty so a detailed analysis of pooled scores might be misleading. The information dealing with strategy type was considered of more immediate importance in this thesis and thus individual protocol analysis seemed to be more conducive to this type of study.

Strategy differences were the most interesting aspect of the research and most of the evidence collected from the protocols revealed relationships existing among the areas of error, reaction rate, and strategy type.

All three grade levels used a basic equation submethod of Method A for the first three Symbolic Problems. The fourth Symbolic Problem uses an identical strategy except for the fact that twelve of the nineteen students reduced the variables before applying the strategy. Thus, the same strategy is used for all four Symbolic Problems.

The strategies used in the word problems were not as precise, in many cases, insofar as there was not one single strategy used in the solution procedures. Question 5, a Story Problem, shows a distinct strategy differential appearing among the students of the three grade levels.

STRATEGIES: A FUNCTION OF GRADE LEVEL

The grade 9 students use combination strategies of Ae and either Be or Bu. Remember that Method Bu is a unit sub-method which is not as sophisticated as the equation sub-method for solving a problem. Method Bu requires less of an understanding of the concept of ratio and proportion and relies more upon a standard learned procedure of reducing the variables to units and then incrementing the appropriate unit to whatever number is required.

Grade 10 and 11 students use, for the most part, Method Be which is an equation sub-method using six variables and requiring a precise grasp of ratio and proportion as a concept. What seems evident, is a developmental trend from grades 9 to 11 whereby specific thought processes evolve either through practice and exposure or through intellectual maturity. This trend seems to be the most salient feature of the data which seems to be supported consistently throughout the range of questions on ratio and proportion as well as being strongly supported by grade level comparisons.

This hypothesis seems not inconsistent with Piaget's (1977) later writings. Piaget maintains that the stage of formal operations is reached somewhere between the ages of eleven and twenty, depending upon the student's aptitude or specialization (advanced studies or apprenticeship). Piaget continues/ however, that the manner in which the structures are used could be different in many cases. Thus, there is a possibility that grade 9 students and some grade 10 students have not adequately incorporated the ratio and proportion concept, yet they are still capable of achieving a satisfactory conclusion to a problem through a rote procedure which has served them in the past in similar situations.

An alternative developmental framework which seems to explain this difference in strategy more precisely is provided by Siegler (1979) in an extensive discussion on developmental sequences. According to Siegler, competence can be correlated with strategy type instead of relying totally on a Piagetian type of developmental maturity. Thus, processing conceptual variables in different ways can be seen as a norm for measuring developmental competence. The Piagetian framework does not seem to allow for initial, intermediate and final competencies of a specific concept, whereas Siegler views development as being linked consistently with the type of strategy used in these different areas of competence.

Question 6 exhibits the identical trend whereby a unit sub-method (Bu) is used more extensively by the grade 9 students and an equation sub-method (Ae) is favored by the grade 10 and 11 students. Curiosity about whether the more sophisticated equation procedure for ratio and proportion problems was taught in the later grades was aroused. After questioning the teachers and the mathematics department head at the high school, it was found that no such procedure was introduced. In fact, the topic of ratio and proportion as a specific area of concentration had been dropped from the mathematics curriculum four years previous to the present study.

STRATEGIES IN THE NONSENSE PROBLEM AREA

The types of methods used in the Nonsense Problem area differ greatly. Question 7, a problem involving the manipulation of six variables, sees a gradual progression in types of method. There seems to be a developmental trend taking place similar to that in the Story Problem area mentioned previously, but not to as great an extent. Grade 10 students seem to be vascillating between unit and equation sub-methods, similar to the grade 9 students. Grade 11 students, however, use only an equation sub-method of either methods A or B. This evidence seems to lend more support to the developmental hypothesis proposed earlier. The grade 10 students, in this case, seem to be moving towards an equation-type strategy such as that used by the grade 11 students, and away from a unit strategy predominantly favored by the grade 9 students.

STRATEGIES IN THE CHEMISTRY PROBLEM AREA

In the Chemistry Problem area, the unit sub-method is adopted primarily by the grade 9 students, and the equation sub-method by the grade 11 students with the grade 10 students using either equation sub-methods, unit sub-methods, or combinations thereof.

CONCLUSIONS ON STRATEGY VARIANCE

The evidence seems relatively clear. Something like Piaget's notion of a developmental structure evolving between adolescence and early adulthood, or Siegler's notion of a strategy shift seems to be supported. The use of a somewhat more sophisticated strategy, the equation sub-method, by the older students seems to indicate that these students are capable of abstracting the possible variables from a problem at the outset of the strategy and are capable of arranging those variables into workable equations. In effect, the older students seem to be able to set up subordinate goal structures while still keeping the ultimate goal in view. The younger students, using the unit sub-method, seem less capable of deducing what should be the final goal at the outset of the problem. As a result, subordinate goals are arranged in order to guide further thinking towards the final goal. This is very similar to Simon and Simon's (1977) findings in the area of expert-novice performance on physics problems. Subordinate goal structures in the problems worked on by the expert were set up in a similar fashion to the way the grade 11 students solved the ratio and proportion problems. Likewise, Newell and Simon's (1972) findings regarding subordinate goal structures in problems on cryptarithmetic and the Tower of Hanoi problem also provide almost identical strategy patterns. Thus, the most effective processes used in ratio and proportion problems seem to be more readily available for use in the older students than in the younger ones. The students using the unit sub-method seem to see ratio and proportion problems as being solved by a reduction process whereby the concept seems less formally defined in their thinking.

Further evidence along the lines of strategies used in this area is needed. If these strategies are truly developmental, then similar differences should arise in other problem-solving areas other than mathematical computation. It would be interesting to evaluate solution protocols in related areas of problem solving.

SOLUTION TIMES

The decision was made to conclude the research for this thesis at the strategy isolation stage. A process model for the strategies needs further detailed analysis of the individual protocols which will be concluded at a later time. Some in-depth analysis has been done, however, on these protocols in certain specific areas of interest which will be mentioned. It is useful to consider certain features of these protocols in order to explain specific discrepancies especially regarding solution times.

The median solution times for correct responses to all four types of problems indicate a direct relationship between semantic content and grade level. The mean times for the Symbolic Problems suggest that the grade 11 students can process the information and rapidly calculate an appropriate response much more quickly than the grade 9 students, as might be expected. Even though the same strategy is used for the Symbolic Problems, a practice effect seems evident whereby proficiency in this type of problem increases with age and grade level. Somewhat similar data occurs in the Nonsense Problem area. Speed increases from the grade 9 level to the grade 10 level but decreases slightly at the grade 11 level. Upon careful examination of the protocols, this decrease in speed is due directly to the strategy used by the grade 11 students in not using RLT, (reducing the variables) before applying the equation submethod. This accounts for the slight discrepancy seen in the Nonsense Problem area.

STORY PROBLEMS AND CHEMISTRY PROBLEMS

The two areas of specific interest are the Story Problems and the Chemistry Problems. These two areas decrease in reaction time according to grade level, directly contrary to the Symbolic and Nonsense Problem areas. An examination of the protocols in the Story Problem area reveals some interesting explanations for this somewhat unexpected turnabout. It would seem that the reaction times for the grade 9 students omit a total of 7 out of 12 reaction times because of error. It is also interesting to note that all of these errors occurred in a unit sub-method approach to the problem. Strategy Ae Be was used by the grade 9 students and only one grade 10 student for problem number 5. This strategy resulted in an accurate and time-saving procedure. The grade 10 and 11 students, but for the one exception mentioned, used a strategy with fewer steps (Be), but with a longer solution time. Similarly, problem 6 has parallel findings. More grade 9 and 10 students used a unit sub-method (Bu) than did the grade 11s (Ae). Again, Method Ae has fewer steps in the strategy, yet seems to have taken more time. This may be because the protocols revealed that in both cases, problems 5 and 6, the grade 11 students and the grade 10 students who used the same strategy, took much longer in setting up the variables before proceeding with the problem solution.

The discrepancies between grades 9 and 10 can be explained by the combinations of grade 9 and 11 strategies used by the grade 10 students. Grade 10 students who used the same strategies as the grade 11 students contributed to a higher reaction time for the entire grade level.

Similar reaction time differences occur in the Chemistry Problem area. In question 9, grade 9 students use predominantly a unit sub-method (Bu). An examination of the protocols reveals that the unit method used by the grade 9 students takes just over half the time that the other two methods used in this problem take, (Ae and Be). The time factor is again directly linked to the amount of time needed for the grade 11 students and some grade 10 students to set up the preliminary ratios. The first step of the unit sub-method used by the grade 9 students has a median time of less than half the time taken by the other two methods.

Problem 10 shows similar occurrences. The grade 9 students used a unit sub-method in 2 of 3 cases while the grade 10 and 11 students used an equation sub-method with more grade 11s using a shorter strategy (number of steps). The unit sub-method solution times were at the very least, 56 seconds faster than any of the other strategies used. This accounts for the low median solution times in Table 8 for the grade 9 students.

Similar discrepancies such as those pointed out for problem 9 also account for the difference between the grade 10 and 11 students.

These results are reminiscent of those of Simon and Simon (1977), and Larkin (1977) whereby a planning stage was used by the expert before generating mathematical equations. This planning stage in the present study, however, is not done by an expert as is the case in Larkin's and Simon and Simon's studies. This discrepancy could result in the grade 11 students taking more time to set up the ratios involved. Perhaps, with sufficient practice, this planning stage would be greatly reduced in time.

CONCLUSION

These reaction time results continue to support some form of developmental difference in strategy. Strategy type and reaction time seem to be inter-related both for the Symbolic and Nonsense areas on one extreme, and for the Story and Chemistry Problem areas on the other.

In this study, it may be evident that the strategy type used by students at various levels of performance is a useful indicator of conceptual attainment. It was noted that grade 9 students used a unit sub-method strategy; grade 10 students used half a unit sub-method and half an equation sub-method strategy; and the grade 11 students used almost entirely an equation sub-method strategy. Development of the concept of ratio and proportion can also be seen in the absence of process errors by the grade 11 students. In effect, there seems to be a direct relationship between strategy type and number of process errors.

The examination of the subject protocols has seemed to be a consistent and accurate method of determining both learning methods of students at various grade levels, as well as determining the types of strategies with which the students are familiar. It would seem that teaching methods adapted to the strategies for solving ratio and proportion problems which the students use, would be more effective in promoting development of the concept. Furthermore, conceptual development might be inferred from the type of strategy which the student uses in solving problems.

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APPENDIX I

Individual Solution Strategies
Encoded from the protocols in
Appendix II.

KEY TO ANALYSIS

SUR........Sets up a ratio

PFR........Produces the first ratio

PSR........Produces the second ratio

PTR........Produces the third ratio

PFR*.......Produces the fourth ratio

XM.........Cross Multiplies

Sx.........Solves for x

D..........Divides

x_1.........first subordinate goal

x_2.........second subordinate goal

x_3.........third subordinate goal

x_f.........main goal (final answer)

x_{f_1}........main goal (first of two final answers)

x_{f_2}........main goal (second of two final answers)

RLT.......Reduces to Lowest Terms

$\textcircled{x_f}$.......Indicates an incorrect answer

John K.

1

8,0	PFE
5,6	XM
	Sx
15,2	D
	xf

2

4,0	PFE
3,2	XM
	Sx
2,6	D
	xf

3

3,8	PFE
11,6	XM
	Sx
37,0	D
	xf
* *28,0	

4

7,8	PFE
4,6	RLT
	D
2,8	xf

8

	SUR
4,4	PFR
12,4	PFE ← r
	RLT
15,6	XM
	Sx
7,6	xf

METHOD

1	2	3	4	8
Ae	Ae	Ae	Ae RLT	Ae RLT

r Replaces original terms with simpler forms

* * Re-checks Calculations

Robert M.

1

2,6	PFE
11,4	XM
	Sx
32,8	D
	xf

2

4,0	PFE
14,0	XM
	Sx
6,6	D
	xf

3

6,2	PFE
28,8	XM
	Sx
34,8	D
	xf

4

3,0	RLT
9,6	PFE
12,6	XM
	Sx
9,8	D
	xf

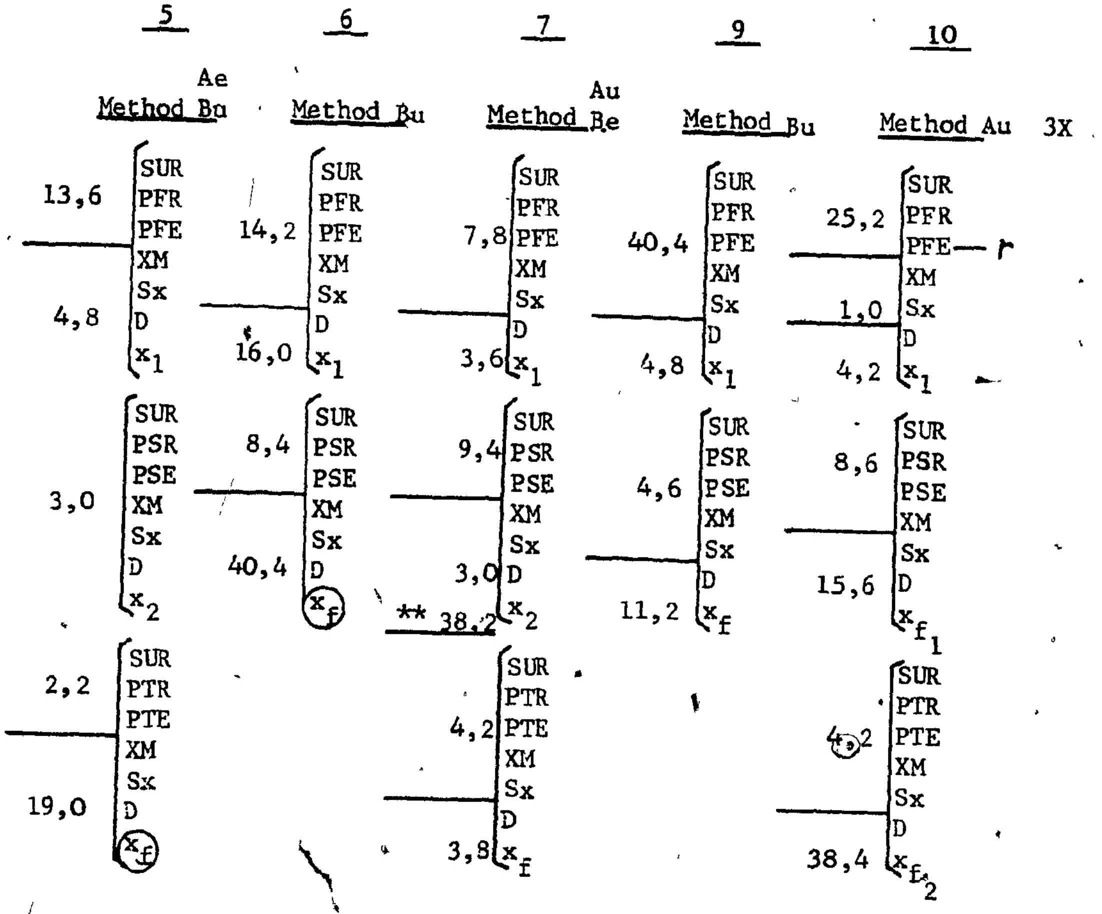
8

	SUR
34,8	PFR
	PFE
21,6	XM
14,8	RLT
	XM
7,4	Sx
	xf

METHOD

1	2	3	4	8
Ae	Ae	Ae	Ae	Ae RLT

John K.



** Re-checks calculations.

r Replacement of original terms with simpler forms

Robert M.

5

Method Bu Ae

7,6
SUR
PFR
PFE
XM
Sx
D
x_1

7,2
17,4
SUR
PSR
PSE
XM
Sx
D
x_2

3,4
46,0
SUR
PTR
PTE
XM
Sx
D
x_f

6

Method Bu

9,2
15,6
SUR
PFR
PFE
XM
Sx
D
x_1

11,8
49,8
SUR
PSR
PSE
XM
Sx
D
x_f

7

Method Be Ae

2,2
,8
**44,2
SUR
PFR
PFE
XM
Sx
D
x_1

6,2
7,2
SUR
PSR
PSE
XM
Sx
D
x_2

22,6
19,8
SUR
PTR
PTE
XM
Sx
D
x_f

9

Method Bu

18,2
24,2
SUR
PFR
PFE
XM
Sx
D
x_1

92,6
32,8
SUR
PSR
PSE
XM
Sx
D
x_f

10

Method –

Unable to SUR

122,8***

** Re-checks calculations

*** Time spent in attempting the problem

John N.

1		2		3		4		8	
		**57,6							
1,0	PFE	1,0	PFE	2,0	PFE	9,0	PFE		SUR
▶ 2,0	XM	7,0	XM	9,0	XM	1,0	D	10,8	PFR
7,5	D		Sx		Sx		D	4,0	RLT
	M	4,0	D	51,0	D	3,0	xf		XM
9,0	xf		xf		xf			3,6	Sx
									xf

METHOD

Ae	Ae	Ae	Ae RLT	Ae RLT

▶—— Changes procedures

** Interference from previous problem

Stan. N.

1		2		3		4		8	
15,0	PFE	3,0	PFE	5,8	PFE	8,6	PFE		SUR
51,2	XM	18,0	XM	31,4	XM	17,6	XM	25,4	PFR
**16,4	Sx		Sx		Sx		Sx		PFE
	D	12,4	D	86,0	D	38,2	D		RLT
38,0	xf		xf		xf		xf	53,4	XM
									Sx
									xf
								** 29,8	

** Re-checks Calculations

METHOD

Ae	Ae	Ae	Ae	Ae RLT

John N.

	5		6		7		9		10
	Method $\overset{\text{Ae}}{\text{Be}}$		Method Bu		Method $\overset{\text{Au}}{\text{Bu}}$		Method Bu		Method Au 3X
	SUR		SUR		SUR		SUR		SUR
	PFR	13,2	PFR		PFR		PFR	35,2	PFR
24,2	PFE		PFE		PFE		PFE		PFE
	XM		XM	50,0	XM	32,6	XM		XM
	Sx		Sx		Sx		Sx		Sx
	D		D		D		D	8,6	D
	x_1	20,0	x_1		x_1	**7,2	x_1		x_1
	SUR		SUR		SUR		SUR		SUR
	PSR	3,0	PSR		PSR		PSR	9,4	PSR
21,8	PSE		PSE		PSE		PSE		PSE
	XM	28,4	XM	1,0	XM	35,4	XM		XM
	Sx		Sx		Sx		Sx		Sx
	D		D		D		D	12,2	D
	x_2	2,0	x_f		x_2	7,2	x_f		x_2
	SUR				SUR				SUR
	PTR			12,4	PTR				PTR
	PTE				PTE			27,4	PTE
	XM				XM				XM
	Sx				Sx				Sx
6,4	D			3,4	D				D
	x_f				x_f			14,6	$\textcircled{x_f}$

** Re-checks Calculations

STAN. N.

5

Ae
Method Bu

23,2
SUR
PFR
PFE
XM
Sx
D
x_1

14,6
15,8
SUR
PSR
PSE
XM
Sx
D
x_2

No x_f

101,6****

6

Method Bu

61,4
13,8
SUR
PFR
PFE
XM
Sx
D
x_1

No x_f

67,4****

7

Ae
Method Be

68,4
SUR
PFR
PFE
XM
Sx
D
x_1

40,2
12,4
SUR
PSR
PSE
XM
Sx
D
x_2

**82,0

9,8
4,6
SUR
PTR
PTE
XM
Sx
D
x_f

9

Unable to
Start.

101,4****

10

Unable to
Start.

45,2****

**** Time spent in attempting the problem

** Re-checks calculations

Vince R.

1

3,2	PFE
9,4	XM
	Sx
29,4	D
	xf

2

6,4	PFE
23,2	XM
	Sx
3,2	D
	xf

3

4,8	PFE
20,6	XM
	Sx
67,8	D
	xf

4

3,0	PFE
4,8	RLT
	D
25,8	ⓧf

8

	SUR
18,8	PFR
	PFE
	RLT
	XM
14,8	Sx
	xf

METHOD

1	2	3	4	8
Ae	Ae	Ae	Ae RLT	Ae RLT

Paul Y.

1

6,6	PFE
16,6	XM
	Sx
22,2	D
	xf

2

3,2	PFE
11,0	XM
	Sx
32,2	D
	xf

3

3,0	PFE
8,4	XM
	Sx
2,0	D
	xf

4

5,0	PFE
2,2	RLT
	D
6,2	xf

8

	SUR
25,6	PFR
	PFE
	RLT
	XM
23,0	Sx
	xf

METHOD

1	2	3	4	8
Ae	Ae	Ae	Ae RLT	Ae RLT

Vince R.

5

Ae
Method Bu

SUR
22,0 PFR
PFE
XM
4,4 Sx
D
x_1

SUR
21,0 PSR
PSE
XM
Sx
37,0 D
x_2

SUR
4,2 PTR
PTE
XM
52,8 Sx
D
x_f

6

Method Ae

SUR
19,8 PFR
PFE
44,2 XM
Sx
71,2 D
x_f

7

Ae
Method Be

SUR
80,8 PFR
PFE
XM
10,0 Sx
D
x_1

SUR
8,2 PSR
PSE
XM
Sx
8,6 D
x_2

SUR
4,8 PTR
PTE
XM
28,8 Sx
D
x_3

**44,4

SUR
PFR*
PFE*
56,0 XM
Sx
D
x_f

9

Method Ae

SUR
79,2 PFR
PFE
21,4 XM
Sx
66,0 D
x_f

10

Ae
Method Be

SUR
90,8 PFR
PFE
37,2 XM
Sx
56,8 D
x_1

**118,8

SUR
9,0 PSR
PSE
22,6 XM
Sx
12,0 D
x_2

SUR
PTR
4,0 PTE
XM
Sx
D
21,8 $x_1 - x_2 = x_f$

** Re-checks Calculations

Paul Y.

5	6	7	9	10
Method Ae Be	Method Ae	Method Au Bu	Method Ae	Method -
37,2 4,2	8,6 3,2	14,6	23,8 56,6 15,4 96,8	66,2****
SUR PFR PFE XM Sx D x_1	SUR PFR PFE XM Sx D x_1	SUR PFR PFE XM Sx D x_1	SUR PFR PFE XM Sx D x_f	Unable to SUR
3,0 30,6	CHANGES **50,0 8,4 17,4 13,0 21,8	57,8 4,0		
SUR PSR PSE XM Sx D x_2	SUR PFR PFE XM RLT Sx D x_f	SUR PSR PSE XM Sx D x_2		
8,4 18,2		17,2 5,4		
SUR PTR PTE XM Sx D x_f		SUR PTR PTE XM Sx D x_f		

**** Time spent in attempting the problem

** Re-checks Calculations

Allan G.

1		2		3		4		8	
3,8	PFE	2,8	PFE	7,0	PFE	4,4	PFE		SUR
5,6	XM	12,2	XM	18,4	XM	9,0	XM	27,4	PFR
	Sx		Sx		Sx		Sx		PFE
23,6	D	4,0	D	37,2	D	17,6	D	11,2	XM
	xf		xf		xf		xf		Sx
								17,0	D
									xf
METHOD									
Ae		Ae		Ae		Ae		Ae	

Paula L.

1		2		3		4		8	
2,6	PFE	2,4	PFE	4,6	PFE	4,6	PFE		SUR
4,0	XM	8,4	XM	18,0	XM	10,0	RLT	17,4	PFR
	Sx		Sx		Sx		D		PFE
15,4	D	2,0	D	74,2	D	1,0	xf	11,0	XM
	xf		xf		xf				Sx
								9,0	D
									xf
METHOD									
Ae		Ae		Ae		Ae RLT		Ae	

Allan G.

5

Method Ae Be

57,4 SUR PFR PFE

2,0 XM Sx D x_1

**18,8 CHANGES

4,2 SUR PSR PSE

20,2 XM Sx D x_2

6,4 SUR PTR PTE

12,2 XM Sx D x_f

6

Method Bu

23,0 RLT

2,4 SUR PFR PFE

7,6 XM Sx D x_1

8,6 SUR PSR PSE

26,5 XM Sx D x_f

7

Method Ae Be

38,2 SUR PFR PFE XM Sx D x_1

1,8 SUR PSR PSE

5,0 XM Sx D x_2

9,6 SUR PTR PTE XM Sx

1,6 D x_f

9

Method Ae

47,2 SUR PFR PFE

21,2 XM Sx

55,4 D

7,2 ⓧx_f

10

Method Be

98,4 SUR PFR PFE

18,2 XM Sx

46,2 D

16,0 x_{f_1}

15,8 SUR PSR PSE

54,0 XM

49,0 Sx D x_{f_2}

** Re-checks Calculations

Paula L.

5

Method Be

SUR
PFR
29,8 PFE
XM
Sx
D
x_1

CHANGES

SUR
PFR
12,2 PFE
XM
Sx
D
x_1

SUR
12,8 PSR
PSE
3,8 XM
Sx
27,8 D
x_f

6

Method Ae

SUR
19,8 PFR
PFE
27,8 XM
Sx
52,0 D
x_f

7

Method Be

SUR
44,8 PFR
PFE
3,2 XM
Sx
2,0 D
x_1

SUR
10,6 PSR
PSE
XM
5,0 Sx
D
x_f

9

Method Ae

SUR
17,0 PFR
PFE
19,0 XM
Sx
62,4 D
x_f

10

Ae
Method ½Be

Add Totals 41,4

SUR
10,0 PFR
PFE
30,2 XM
Sx
51,2 D
x_1

SUR
150,2 PSR
PSE
35,4 XM
Sx
7,6 D
x_{f_1}

13,4 $x_1 - x_{f_1} = x_{f_2}$

Sharon P.

1	2	3	4	8
5,0 PFE 2,0 XM Sx 23,4 D xf	3,2 PFE 8,4 XM Sx 2,2 D xf	6,0 PFE 23,0 XM Sx 32,5 D xf	4,8 PFE 5,2 RLT D 6,8 xf	SUR 35,2 PFR PFE 29,4 RLT XM 15,8 Sx xf

METHOD

Ae	Ae	Ae	Ae RLT	Ae RLT

Glenn R.

1	2	3	4	8
4,0 PFE 9,8 XM Sx 5,8 D xf	4,8 PFE 10,0 XM Sx 5,0 D xf	8,4 PFE 18,0 XM Sx 36,6 D xf	6,6 PFE 5,8 RLT D 6,2 xf	27,2 SUR PFR < 13,8 XM Sx 24,0 D xf

METHOD

Ae	Ae	Ae	Ae RLT	Ae

< Insert....PFE

Glenn R.

5

Method Be

	SUR
	PFR
43,4	PFE
	XM
	Sx
	D
	x_1

	SUR
13,8	PSR
	PSE
15,0	XM
	Sx
36,8	D
	x_f

6

Method Ae

	SUR
34,0	PFR
	PFE
23,4	XM
	Sx
67,0	D
	x_f

7

Au
Method Bu

	SUR
18,6	PFR
	PFE
	XM
52,4	Sx
	D
	x_1

	SUR
	PSR
10,0	PSE
	XM
	Sx
	D
	x_2

	SUR
	PTR
	PTE
13,4	XM
	Sx
	D
	x_f

9

Method Be

	SUR
71,8	PFR
	PFE
8,4	XM
	Sx
74,0	D
	x_1

9,4	SUR
	PSR
25,4	PSE
	XM
54,6	Sx
	D
	x_f

10

Method X***

Interference from terms

168,6	SUR
	PFR
	PFE
85,0	XM
	Sx
	D
	x_1

	SUR
70,0	PSR
	PSE
7,0	XM
	Sx
37,6	D
	x_2

	SUR
41,4	PTR
	PTE
	XM
	Sx
40,6	D
	x_{f_1}

	SUR
8,8	PFR*
	PFE*
	XM
	Sx
63,2	D
	x_{f_2}

*** not a recognized method. Seems to be an assortment of various other procedures.

Sharon P.

5

Ae
Method Bu

40,8 SUR
PFR
PFE
XM
3,0 Sx
D
x_1

36,2 SUR
PSR
PSE
XM
8,0 Sx
D
x_2

SUR
10,2 PTR
BTE
XM
Sx
26,8 D
x_f

6

Method Ae

SUR
28,8 PFR
PFE
33,2 XM
11,6 RLT
Sx
19,4 D
x_f

7

Au
Method Be

SUR
PFR
PFE
79,2 XM
Sx
D
x_1

SUR
3,8 PSR
PSE
XM
15,0 Sx
D
x_2

10,4 SUR
PTR
PTE
XM
Sx

32,0 No x_f

9

Method Be

SUR
43,0 PFR
PFE
XM
Sx
55,4 D
x_1

SUR
3,6 PSR
PSE
XM
Sx
44,6 D
x_f

10

Ae
Method ½Be

Add Totals

SUR
235,6 PFR
PFE
45,8 XM
Sx
59,4 D
x_1

7,4 SUR
25,6 PSR
RLT

25,0 No x_f

Angela R.

1		2		3		4		8	
2,8	PFE	6,2	PFE	7,0	PFE	6,8	PFE		SUR
2,0	XM	11,8	XM	12,6	XM	4,0	RLT	17,4	PFR
	Sx		Sx		Sx		D		PFE
4,2	D	3,8	D	26,0	D	3,6	xf	1,6	RLT
	[✓ xf]		xf		xf			1,4	XM
									Sx
								,2	xf

METHOD

Ae	Ae	Ae	Ae RLT	Ae RLT

Ruth V.

1		2		3		4		8	
2,5	PFE	6,1	PFE	7,0	PFE	5,2	PFE		SUR
5,6	XM	11,0	XM	16,8	XM	6,0	XM	38,0	PFR
	Sx		Sx		Sx		Sx		PFE
16,2	D	45,2	D	30,4	D	15,6	D	9,6	XM
	xf		[✓ xf]		xf		xf		Sx
								10,6	D
									xf

METHOD

Ae	Ae	Ae	Ae	Ae

Angela R.

5

Method Be

	SUR
	PFR
33,0	PFE
	XM
	Sx
	D
	x_1

	SUR
2,0	PSR
	PSE
3,4	XM
	Sx
25,2	D
	x_f

6

Method Ae

	SUR
23,2	PFR
	PFE
16,6	XM
2,4	RLT
	Sx
11,6	D
	x_f

7

Method Au Bu

33,0	SUR
	PFR
10,2	PFE
	XM
	Sx
5,4	D
	x_1

	SUR
14,6	PSR
	PSE
2,6	XM
	Sx
3,2	D
	x_2

	SUR
	PTR
6,8	PTE
	XM
	Sx
	D
	x_f

9

Method Ae

	SUR
34,4	PFR
	PFE
19,0	XM
	Sx
47,0	D
	$\textcircled{x_f}$

10

Method Be

	SUR
28,0	PFR
	PFE
23,2	RLT
10,2	XM
	Sx
16,8	D
	x_{f_1}

	SUR
17,4	PSR
	PSE
4,2	XM
	Sx
7,4	D
	x_{f_2}

Ruth V.

5	6	7	9	10
Method Be	Method Ae	Method -	Method -	Method -
18,5 SUR PFR PFE 4,2 XM Sx 30,6 D x_f	SUR 56,4 PFR PFE 14,2 XM Sx 46,8 D x_f	Unable to SUR 33,7 ****	Unable to SUR 56,0 ****	Unable to SUR 38,0 ****

**** Time spent in attempting the problem

Leanna V.

	1		2		3		4		8
,2	PFE	5,6	PFE	7,2	PFE	5,0	PFE		SUR
1,8	XM	6,0	XM	6,6	XM	9,4	XM	7,8	PFR
	Sx		Sx		Sx		Sx		PFE
35,6	D	8,6	D	44,6	D	23,8	D		RLT
	xf		xf		xf		xf	3,8	XM
									Sx
									xf
								**19,0	

** Re-checks Calculations

METHOD

Ae Ae Ae Ae Ae, RLT

Leanna V.

5	6	7	9	10
Method Be	Method Bu	Method -	Method -	Method -
57,8 [SUR PFR PFE XM Sx D x_1	34,6 / 15,6 [SUR PFR PFE XM Sx D x_1	SUR PFR - ┌ Unable to PFE 106,0****	Unable to SUR 61,0****	Adds Totals 49,0 4,0 / 42,6 SUR PFR D Unable to PFE 17,4****
17,4 [SUR PSR PSE XM Sx D x_f	6,0 / 57,2 [SUR PSR PSE XM Sx D x_f			

┌ Replacement with simpler terms of original forms

**** Time spent in attempting the problem

Jackie D.

1		2		3		4		8	
2,0	PFE	2,8	PFE	4,4	PFE	8,0	PFE		SUR
6,2	XM	5,2	RLT	37,4	XM	6,6	RLT	9,0	PFR
	Sx		PFR		Sx		D		PFE
29,2	D	13,0	(xf)	36,4	D	,2	xf		RLT
	(xf)				(xf)			4,4	XM
									Sx
								**5,4	xf

METHOD

Ae Ae Ae Ae RLT Ae RLT

** Re-checks Calculations

Tony G.

1		2		3		4		8		
2,6	PFE	6,4	PFE	8,8	PFE	5,0	PFE		SUR	
4,4	XM	11,8	XM	21,6	RLT	2,0	RLT	3,4	PFR	
	Sx		Sx		XM		D		PFE	11,0
104,4	D	3,2	D	20,2	Sx	5,2	xf	13,2	XM ——▮	30,0
	xf		xf		D				Sx	
					xf			11,0	D	
									xf	

——▮ Corrects Error

METHOD

Ae Ae Ae Ae RLT Ae

Jackie D.

5

Method Be

17,4	SUR PFR PFE XM Sx D x_1
18,4	SUR PSR PSE
20,2	XM Sx
82,8	D x_f

6

Method Bu

70,0	SUR PFR RLT PFE
56,8	XM Sx D x_1
7,4	SUR PSR PSE
43,2	XM Sx D
12,8	x_f

7

Ae
Method Be

47,6	SUR PFR PFE XM Sx D x_1
15,4	SUR PSR PSE XM Sx D x_2
21,8	SUR PTR PTE XM Sx D x_f

9

Method Ae

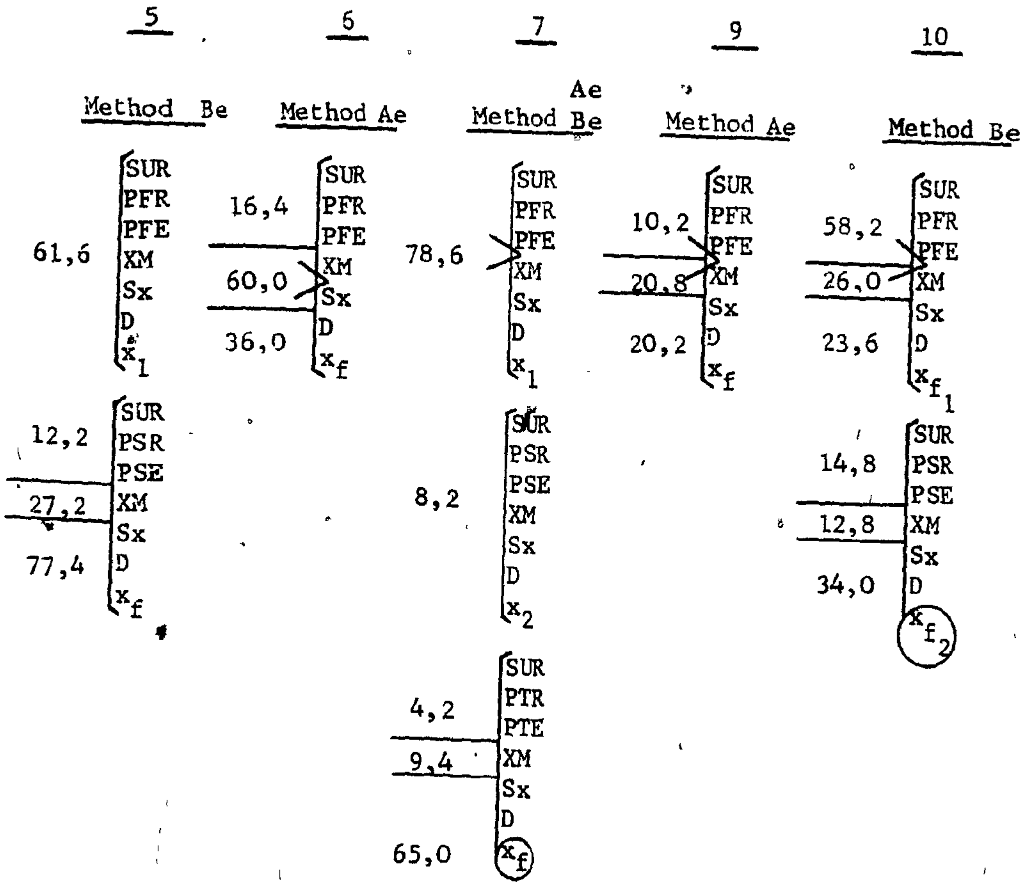
22,0	SUR PFR PFE
23,0	RLT
3,2	XM Sx
79,8	D x_f

10

Method Be

193,6	SUR PFR
13,2	PFE RLT XM
6,4	Sx D x_{f_1}
40,6	SUR PSR PSE RLT XM
14,6	Sx D
13,2	x_{f_2}

Tony G.



> Insert......RLT

Karen H.

1		2		3		4		8	
6,6	PFE	3,8	PFE	4,2	PFE	3,0	PFE		SUR
3,2	XM	18,2	XM	14,0	XM	11,2	XM	8,2	PFR
	Sx		Sx		Sx		Sx		PFE
1,2	D	1,0	D	46,2	D	5,2	D	12,4	XM
	[✓ xf]		xf		[✓ xf]		xf		Sx
								23,6	D
									xf

METHOD

1	2	3	4	8
Ae	Ae	Ae	Ae	Ae

Peter R.

1		2		3		4		8	
3,8	PFE	4,0	PFE	8,2	PFE	7,4	PFE		SUR
5,0	XM	7,4	XM	21,6	XM	5,0	RLT	10,0	PFR
	Sx		Sx		Sx		D		PFE
16,0	D	2,4	D	63,6	D	3,2	xf	15,4	XM
	xf		xf		xf				Sx
								10,0	D
									xf

METHOD

1	2	3	4	8
Ae	Ae	Ae	Ae RLT	Ae

Karen H.

5

Method Be

	SUR
	PFR
	PFE
	XM
25,6	Sx
	D
	x_1

	SUR
23,4	PSR
	PSE
11,6	XM
	Sx
60.0	D
	x_f

6

Method Ae

	SUR
13,2	PFR
	PFE
19,8	XM
	Sx
	D
79,6	RLT
	xf

7

Method-

61,4	SUR
	PFR

Unable to

PFE

56.8

9

Method Be

	SUR
42,2	PFR
	PFE
	XM
	Sx
46,4	D
	x_1

	SUR
	PSR
	PSE
26,8	XM
	Sx
	D
	x_f

10

Method Be

	SUR
	PFR
56,2	PFE
15,0	XM
	Sx
4,2	D
	x_{f_1}

	SUR
20,6	PSR
	PSE
57,2	XM
	Sx
76,8	D
	x_{f_2}

Peter R.

	5		6		7		9		10
	Method Be		Method Ae		Method Be		Method Ae		Method Be
	SUR		SUR		SUR		SUR		SUR
	PFR	40,2	PFR	29,6	PFR	42,0	PFR	27,6	PFR
	PFE		PFE		PFE		PFE		PFE
29,0	XM	14,6 >	XM	8,6	XM	17,6	XM	21,6	XM
	Sx		Sx		Sx		Sx		Sx
	D	64,0	D		D	112,0	D	12,4	D
	x_1		x_f		x_1		x_f		x_{f_1}
	SUR				SUR				SUR
11,8	PSR			85,0	PSR			13,2	PSR
	PSE				PSE				PSE
8,6	XM				XM			31,4	XM
	Sx			19,4	Sx				Sx
45,0	D				D			79,0	D
	x_f				x_f				x_{f_2}

\> Insert ..... RLT

Sue W.

1	2	3	4	8
4,4 PFE 6,2 XM Sx 26,6 D xf	7,0 PFE 5,4 XM Sx 3,6 D xf	3,2 PFE 8,0 XM Sx 52,8 D xf	11,0 PFE 2,2 RLT D 3,8 xf	SUR 34,0 PFR PFE 14,4 XM Sx 11,4 D xf

METHOD

Ae	Ae	Ae	Ae RLT	Ae

Diane Y.

1	2	3	4	8
,2 PFE 3,4 XM Sx 15,2 D xf	,2 PFE 9,0 XM Sx 1,0 D xf	,2 PFE 9,6 XM Sx 12,2 D xf	,2 PFE 4,4 XM Sx 3,2 D xf	SUR 6,0 PFR PFE 7,6 XM Sx 12,6 D xf

METHOD

Ae	Ae	Ae	Ae	Ae

Sue W.

5

Method Be

	SUR
42,6	PFR
11,0	PFE
2,2	XM
	Sx
2,4	D
	x_1

	SUR
21,0	PSR
	PSE
15,0	XM
	Sx
27,0	D
4,0	x_f

6

Method Ae

	SUR
24,4	PFR
	PFE
9,0	RLT
21,4	XM
	Sx
42,2	D
	x_f

7

Ae
Method Be

	SUR
	PFR
63,6	PFE
	XM
	Sx
	D
	x_1

	SUR
	PSR
22,0	PSE
	XM
	Sx
	D
	x_2

	SUR
26,8	PTR
	PTE
	XM
8,0	Sx
	D
	x_f

9

Method Ae

61,4	SUR
	PFR
10,0	PFE
12,6	RLT
4,2	XM
2,4	Sx
	D
53,0	x_f

10

Ae
Method ½Be

Adds Totals 70,8

	SUR
21,2	PFR
	PFE
34,2	XM
	Sx
81,8	D
	x_1

	SUR
94,0	PSR
	PSE
39,0	XM
	Sx
28,0	D
	x_{f_1}

22,2 $x_1 - x_{f_1} \doteq x_{f_2}$

Diane Y.

5 Method Be

23,8	SUR
	PFR
	PFE
7,0	XM
	Sx
	D
	x_1

	SUR
	PSR
	PSE
	XM
	Sx
21,2	D
	x_f

6 Method Ae

	SUR
22,4	PFR
	PFE
25,4	XM
10,0	Sx
	D
8,6	x_f

7 Method Be

	SUR
	PFR
46,0	PFE
	XM
	Sx
6,8	D
	x_1

	SUR
	PSR
	PSE
16,8	XM
	Sx
	D
	x_f

9 Method Be

	SUR
10,2	PFR
	PFE
	XM
23,4	Sx
	D
	x_1

	SUR
9,8	PSR
	PSE
	XM
11,8	Sx
	D
	x_f

10 Method Ae Be

Adds Totals 25,2

	SUR
4,8	PFR
	PFE
16,4	XM
	Sx
23,6	D
	x_1

	SUR
30,0	PSR
	PSE
14,4	XM
	Sx
20,6	D
	x_{f_1}

***** $\left[x_1 - x_{f_1} = x_{f_2}\right]$

	SUR
12,0	PTR
	PTE
16,0	XM
	Sx
21,8	D
	x_{f_2}

***** Step omitted in favor of further calculations.

> Insert.......RLT

APPENDIX II

Individual protocols of the 19 students are reported alphabetically by grade level.

KEY TO APPENDIX

BELOW ARE THE EQUATIONS NEEDED TO SOLVE THE 10 PROBLEMS. NOTE THAT THE PROBLEMS NEED NOT BE SET UP IN THE SAME MANNER, MERELY THAT THE NUMBERS ARE LABELED IN THIS WAY FOR RECOGNITION PURPOSES.

THE BASIC FORMAT FOR PROBLEMS WITH 4 POSSIBLE VARIABLES IS:

$$\frac{a}{b} = \frac{c}{d}$$

THE BASIC FORMAT FOR PROBLEMS WITH 6 POSSIBLE VARIABLES IS:

$$\begin{array}{ccc} a & & c \\ \hline b & = & d \\ \hline E & = & F \end{array}$$

IN THE FOLLOWING PROBLEMS, THE LETTER x_f WILL BE SUBSTITUTED FOR THE UNKNOWN VARIABLE. ANSWERS LEADING TOWARDS THE CORRECT ANSWER WILL BE LABELED AS $x_1, x_2, x_3 \ldots x_{10}$.

(1) $$\begin{array}{ccc} a & & c \\ \frac{4}{6} & = & \frac{9}{x} \\ b & & x_f \end{array}$$

(2) $$\begin{array}{ccc} a & & c \\ \frac{8}{x} & = & \frac{10}{15} \\ x_f & & d \end{array}$$

(3) $$\begin{array}{ccc} x_f & & c \\ \frac{x}{35} & = & \frac{40}{56} \\ b & & d \end{array}$$

(4) $$\begin{array}{ccc} a & & x_f \\ \frac{12}{24} & = & \frac{x}{8} \\ b & & d \end{array}$$

(5) $$\begin{array}{ccc} a & & c \\ \frac{1}{3} & = & \frac{10}{30} \\ b & & x_1 \\ \hdashline E & & F \\ \frac{4}{30} & = & \frac{2600}{x} \\ x_1 & & x_f \end{array}$$

(6) $$\begin{array}{ccc} a & & c \\ \frac{200}{275} & = & \frac{700}{x} \\ b & & x_f \end{array}$$

(7) $$\begin{array}{ccc} a & & c \\ \frac{2}{30} & = & \frac{7}{x} \\ b & & x_1 \\ \hdashline E & & F \\ \frac{35}{105} & = & \frac{1}{x} \\ x_1 & & x_f \end{array}$$

(8) $$\begin{array}{ccc} a & & c \\ \frac{7}{x} & = & \frac{4}{28} \\ x_f & & d \end{array}$$

KEY TO APPENDIX CONT'D.

(9)

$$\overset{a}{\underset{x_1}{\frac{1}{x}}} = \overset{c}{\underset{d}{\frac{28}{35}}} \quad \text{or} \quad \overset{a}{\underset{b}{\frac{28}{35}}} = \overset{c}{\underset{x_F}{\frac{22}{x}}}$$

$$\overset{E}{\underset{x_1}{\frac{1}{1.25}}} = \overset{F}{\underset{x_F}{\frac{22}{x}}}$$

(10)

$$\overset{a}{\underset{x_{F_1}}{\frac{80}{x}}} = \overset{c}{\underset{d}{\frac{36}{45}}} \qquad \overset{aE}{\underset{x_1}{\frac{178}{x}}} = \overset{c}{\underset{d}{\frac{36}{45}}}$$

$$\overset{E}{\underset{x_{F_2}}{\frac{98}{x}}} = \overset{F}{\underset{d}{\frac{36}{45}}} \qquad \overset{a}{\underset{x_{F_1}}{\frac{80}{x}}} = \overset{aE}{\underset{x_1}{\frac{178}{222{,}5}}}$$

$$\overset{E}{\underset{x_{F_2}}{\frac{98}{x}}} = \overset{aE}{\underset{x_1}{\frac{178}{222{,}5}}}$$

Step where error in problem was made. ⟹

Incorrect answer. ○

SECONDARY III

JOHN K.

QUESTION	TIME	STEP
1.	8,0	O.K. 4 over 6 equals 9 over what... $\frac{a}{b} = \frac{c}{x_f}$
	5,6	so I cross multiply...6 times 9 equals 54 $bc = z$
	1,0	4 $ax_f = y$
	14,2	into 54 goes....13,5...so the answer is 13,5 $\frac{z}{y} = x_f$
TOTAL	28,8 secs.	
2.	4,0	8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$
	3,2	again I cross multiply...get 120 $ad = z$
	2,6	so the answer is 12 $\frac{z}{y} = x_f$
TOTAL	9,8 secs.	
3.	3,8	x over 35 equals 40 over 56 $\frac{x_f}{b} = \frac{c}{d}$
	7,8	35 times 40....1400 $bc = z$

3,8 ...56

$$dx_f = y$$

37,0 ....so the answer is 25

$$\frac{z}{y} = x_f$$

28,0 check it..56 times 25....1400...O.K.

$$y \times x_f = z$$

TOTAL 80,4 secs.

4. 7,8 12 over 24 equals 4 over 8

$$\frac{a}{b} = \frac{x_f}{d}$$

4,6 reduce 12 over 24 to ½

$$\frac{a}{r}, \frac{b}{r}$$

2,8 equals to 4 over 8

$$x_f$$

TOTAL 15,2 secs.

⑤ 11,2 hmmm...so 10 bottles of cleaning fluid in a case...that's 4 tenths

$$\frac{E}{C}$$

2,4 or 2 fifths

$$\frac{E}{r}, \frac{C}{r}$$

4,8 3 cases is 30

$$bc = x_1$$

3,0 4 times 7½ equals 30

$$\frac{x_1}{E} = x_2$$

⟹ 21,2 2500 times 7,5 is....187500 square feet

$$f x_2 = x_f$$

TOTAL 42,6 secs.

⑥ 14,2 O.K. 200 into 700 goes about...3¼...

$$\frac{c}{a} = x_1$$

16,0 no...3½

$$\frac{c}{a} = x_2 \; (x_1)$$

⟹ 48,8 O.K. so 275 bags of grain times $3.50 a piece....275 times 3,50 equals....so it costs him 947,50...does that include tax?

$$b x_1 = x_f$$

TOTAL 79,0 secs.

7. 11,4 O.K....2 serbs....7 serbs....1 serb can make 3½ sets of tods

$$\frac{c}{a} = x_1$$

9,4 oh well that's sort of easy....30 serbs times 3½ is 115

$$b x_1 = x_2$$

3,0 ...105

$$b x_1 = x_2$$

38,2 uh...105 tods...105 sets of tods..right...so let's divide that by...hold it...3,5 times 30 ...105

$$\frac{x_2}{b} = x_3$$

8,0 that's right...105 divided by 35...3

$$\frac{x_2}{E} = x_f$$

TOTAL 70,0 secs.

8. 4,4 28 zots is 7 times that

$$\frac{d}{c} = x_1$$

12,4 7 apples makes up 4 bushels how many apples make 28 bushels

replacement

15,6 that makes,..so...apples...times 7

$$ax_1 = x_f$$

7,6 ...is 7 times as many apples too...49 apples or ergs.

$$ax_1 = x_f$$

TOTAL 40,0 secs.

9. 45,2 hmm...so that's 28 grams...so volume of 35 grams of Nitrogen...35 grams is a mole and ¼

$$\frac{d}{c} = x_1$$

7,0 so ¼ of 22 is 5,5

$$\frac{F}{\frac{1}{4}} = x_2$$

9,8 would give you 27½ liters

$$F + x_2 = x_f$$

TOTAL 62,0 secs.

10. 25,2 If 80 grams of...I don't know...80 grams of apples....98 grams of oranges make 36 grams of water, how many grams of apples and oranges make 45 grams of water

replacement

5,2 45 is ¼ more

$$\frac{d}{c} = x_1$$

24,2 so 36 grams is gonna be ¼ more of each so that'll be...100 grams of apples and NaOH

$$\frac{a}{x_1} = x_2 , \quad a + x_2 = x_{F_1}$$

23,4 and 98 over 4.....24,5

$$\frac{E}{x_1} = x_2$$

19,2 so that's 98,5....118,5...122½ of H_2SO_4..O.K.?

$$E + x_2 = x_{F_2}$$

TOTAL 97,2 secs.

ROBERT M.

QUESTION	TIME	STEP
1.	2,6	4 over 6 equals 9 over x $\frac{a}{b} = \frac{c}{x_f}$
	8,6	O.K. 9 times 6 is...54 $bc = z$
	2,8	so 4 $ax_f = y$
	32,8	times what equals 54.....13,5 $\frac{z}{y} = x_f$
TOTAL	46,8 secs.	
②	4,0	8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$
	9,4	8 times 15....120 $ad = z$
	4,6	um...10 times $ax_f = y$
⇒	6,6	12 would equal 120 so 8 over 120 ~~equals~~ 10 over 15 $\frac{z}{y} = x_f$, $\frac{a}{z} = \frac{c}{d}$
TOTAL	24,6 secs.	
3.	6,2	x over 35 equals 40 over 56 $\frac{x_f}{b} = \frac{c}{d}$

17,4 40 times 35.....1400

$$bc = z$$

11,4 so...I think I got it wrong...so 56

$$dx_f = y$$

34,8 into 1400...so it would be 25 over 35 equals 40 over 56

$$\frac{z}{y} = x_f$$

TOTAL 69,8 secs.

4. 3,0 ½ equals

$$\frac{a}{r}, \frac{b}{r}$$

1,8 x over 8

$$\frac{\frac{a}{r}}{\frac{b}{r}} = \frac{x_f}{d}$$

7,8 so that's 12 over 24 equals x over 8

$$\frac{a}{b} = \frac{x_f}{d}$$

8,4 96

$$ad = z$$

4,2 what...24

$$bx_f = y$$

9,8 into 96 is....so that's 12 over 24 equals 4 over 8

$$\frac{z}{y} = x_f$$

TOTAL 35,0 secs.

5. 7,6 so that's 3 times 10 is 30

$$bc = x_1$$

24,6 4 into 30 goes....7...7,5

$$\frac{x_1}{E} = x_2$$

⟹ 49,4 7,5 times 2500....7750 square feet

$$f x_2 = x_f$$

TOTAL 81,6 secs.

⑥ 24,8 hmm...200 into 700....goes 3,5

$$\frac{c}{a} = x_1$$

⟹ 61,6 3½...so 3½ times 275....$996.25..Is that right?

$$x_1 \times b = x_f$$

TOTAL 86,4 secs.

7. 3,0 2 into 30 goes 15

$$\frac{b}{a} = x_1$$

19,0 so...O.K....I have 14½ times 35

$$? \times E$$

25,2 I don't know..2 serbs makes 7 sets of tods so you have 30 serbs to start off with right? So if you have 30 serbs you divide it by ½...you get 15

$$\frac{b}{a} = x_1$$

13,4 cause you need to...and then 15 times 7...105

$$c x_1 = x_2$$

42,4 that's 105 tods...105 divided by 35...2...so that's 3..3 times 35 equals 105 so you can have 3 fots

$$\frac{x_2}{E} = x_f$$

TOTAL 103,0 secs.

8. 56,4 28....7 ergs makes 4 zots...hmmm...7 ergs ...7 times 28 would be...196

$$\frac{a}{x_f} = \frac{c}{d}, \quad ad = z$$

22,2 so 196 zots...how many ergs...7 times 7 is 49 so you need 49 ergs....don't ask me how I get that

$$\frac{c}{r}, \ \frac{d}{r}, \quad ad = x_f$$

TOTAL 78,6 secs.

9. 42,4 28 into 35 goes....1,25

$$\frac{d}{c} = x_1$$

125,4 if 1M of N_2....1,25 times 22....I did make a mistake....hmmm....1,25 times 22...would equal 27½ liters in volume

$$f x_1 = x_f$$

TOTAL 167,8 secs.

⑩ 27,2 36....45 minus 36 is 9

$$d - c = x_1$$

95,6 I don't know...I'm trying to figure out a way to figure out the corresponding numbers...I don't know if I can do this.....

RETIRES

TOTAL 122,8 secs.

JOHN N.

QUESTION	TIME	STEP
①	1,0	Just cross-multiply $\frac{a}{b} = \frac{c}{x_f}$
	2,0	for like 9 times 6 is 54 $bc = y$
	3,0	so...no that's wrong
	⇒ 3,5	the 4...how many times four can get into 9 $\frac{c}{a} = x$
	1,0	which is 2,5 x
	9,0	So I times 6 times 2,5 and get......15 $bx = x_f$
TOTAL	19,5 secs.	
2.	3,0	How many times 8 goes into 10 $\frac{b}{a} = x$
	4,6	which is ¼ x
	3,0	No...wait a minute...1¼ x
	47,0	No how many times 8 I'd get into 10 so... No that's not right either....

1,0 forget that one.......yeh...I'd cross-multiply...

$$\frac{a}{b} = \frac{x_f}{d}$$

1,0 10x

$$bx_f = y$$

6,0 times 8 times 15 is.......120

$$ad = z$$

2,0 cross out the zeros

$$\frac{y}{r}, \quad \frac{z}{r}$$

2,0 so my answer is 12

$$x_f$$

TOTAL 68,6 secs.

3. 2,0 Same thing with number 3 I'd cross-multiply

$$\frac{x_f}{b} = \frac{c}{d}$$

8,0 40 times 35 is ummm.....1400

$$bc = y$$

1,0 and 56

$$dx_f = z$$

51,0 goes into that.....so the answer is....25

$$\frac{y}{z} = x_f$$

TOTAL 62,0 secs.

4. 9,0 Number 4 is I just see how many times 8 goes into 24

$$\frac{b}{d} = x$$

1,0 which is 3

x

3,0 so times....divide...divide 3 into 12 which is 4

$$\frac{c}{x} = x_f$$

TOTAL 13,0 secs.

5. 46,0 Well I'm trying to find out how many square feet in one case...with the cleaning fluid, which is 6250

$$\frac{c}{E} = x_1, \quad F \times x_1 = x_2$$

6,4 and you times it by three to get the answer, that's 18750

$$x_2 \times b = x_f$$

9,2 So how many square feet of flooring can be done with one case so I just times it by 3.

x_f

TOTAL 61,6 secs.

6. 33,2 I divide 700 by 200 to see how many it costs for 1 bag.......that's $3.50

$$\frac{c}{a} = x_1$$

3,0 and I times it by 275 to get the answer

$$x_1 \times b = x_f$$

30,4 .....which is.....$962.50

x_f

TOTAL 66,6 secs.

7. 50,0 how that works.....figure out how many fots.... how many serbs will make....get 35 sets of tods

1,0 which is 10

$$\frac{E}{c} \times a = x_1$$

15,8 and which is 3 fots because 30 divided by 10 is 3

$$\frac{b}{x_1} = x_f$$

TOTAL 66,8 secs.

8. 10,8 You just times the one....you times the ergs by 7

$$\frac{d}{c} = x$$

4,0 to see how many zots are equal to 7 zots

$$x \times a = x_f$$

3,6 are equal to 28 zots because 4 times 7 is 28 zots

$$x_f$$

TOTAL 18,4 secs.

⑨ 32,6 well you just time the....just add ¼ of the 22 liters to the volume

$$\frac{d}{c} = x_1 , \qquad \frac{F}{x_1}$$

5,0 so....because 28 plus ¼ of the 28 is 7

$$\frac{F}{x_1} \Rightarrow c + x_1 = d$$

2,2 and that gives you 35 grams

$$d$$

$\Longrightarrow$ 35,4 so add ¼ of the 22....that's 8,8

$$x_1 + F = x_2 = \frac{F}{x_1}$$

7,2 so it would be.....so the volume would be 30,8 grams...uh...liters in volume

$$F + x_2 = x_f$$

TOTAL 82,4 secs.

⑩ 35,2 Uh....I'd figure out that I need 9 more grams

$$D - C = x_1$$

1,6 so ¼

$$\frac{C}{x_1} = x_2$$

7,0 so I divide by ¼ because that way it comes to NaOH but then you still need....

$$\frac{a}{x_2}$$

⟹ 9,4 then divide that in half (divided the ¼in half) to get the...split the two....the difference between 80 grams of NaOH and 98 grams of H_2SO_4

$$\frac{a}{\frac{x_2}{2}} = x_3$$

39,6 so that gives you 88 grams of NaOH and figuring out one eigth of 98 so.....12,25

$$a + x_3 = x_{f_1}, \quad E + x_3 = x_{f_2}, \quad \frac{E}{\frac{x_2}{2}}$$

14,6 so 88 grams of NaOH and 110,25 grams of H_2SO_4

$$x_{f_1}, \; x_{f_2}$$

TOTAL 107,4 secs.

STAN. N.

QUESTION	TIME	STEP
①	15,0	I forgot how to do that....Oh no...4 over 6 equals 9 over... $\frac{a}{b} = \frac{c}{d}(x_f)$
	51,2	I don't know...Oh..O.K....that doesn't make any sense...you picked a real stupid idiot for this...I'm telling you....sir I forgot how to do this....3 numbers....6 times 9 is 54 $bc = z$
	16,4	Is it?...I'm figuring out by cross-multiplying $\frac{a}{b} = \frac{c}{x_f}$
	1,8	I guess 4x $ax_f = y$
	1,2	equals 54 z
⟹	35,0	x equals 4 divided by 54... O.K.....that doesn't make any sense.....15,2 $\frac{z}{y} = x_f$
TOTAL 120,6 secs.		
2.	21,0	I must be doing something wrong around here... 8 times 15 is...120 $ad = z$
	1,8	divided by 10 $cx_f = y, \frac{z}{y}$
	10,6	.....12, that sounds more realistic x_f
TOTAL 33,4 secs.		

3. 37,2 that would be 35 times 40......1400

$bc = z$

4,6 divided by 56

$dx_f = y, \frac{z}{y}$

81,4 how many times does 56 go into 140......
23...3 would be 23

x_f

TOTAL 123,2 secs.

4. 9,8 24x

$bx_f = y$

16,4 equals 12 times 8 would be 96

$ad = z$

30,0 24 into 96....help....24 into 96.......4

$\frac{z}{y} = x_f$

8,2 my, we're brilliant today....it would be 4
right? yeh 4, so 4 is 4

x_f

TOTAL 64,4 secs.

5. 23,2 3 cases of cleaning fluid....so 3 cases would
have 30

$\frac{a}{b} = \frac{c}{x_1} = x_1$

31,4 each case...O.K.... so 3 cases would have...
30 cleaning bottles so that would be 4 into 30

$\frac{x_1}{E} = x_2$

would be 7,5

$$x_2$$

38,8 Oh, I did it wrong...Oh... what did I do now? ...2500 square feet ...well I just figured out how many bottles of fluid can... O.K. if 4 bottles of fluid can.....if 4 bottles O.K.....4 goes into 30, 7,5 times

$$\frac{x_1}{E} = x_2$$

⇒ 62,8 that means...ummm....I did something wrong around here...how do you figure it out.....if 2500..... <u>RETIRES</u>

<u>TOTAL 156,0 secs.</u>

⑥ 75,2 O.K. that's easy enough....200... no that's not ....that would be 200 into 700.....3,5

$$\frac{c}{a} = 3,5 = x_1$$

⇒ 67,4 so that would be 3,5....how would you....how in the world does one figure this out?...I can't think...O.K. let's go back to grade 7...let x be O.K......<u>RETIRES</u>

<u>TOTAL 142,6 secs.</u>

7. 68,4 What are tods?... laughs...this must be a polish one...O.K. this one's easier... 2 serbs into... 2 goes into...O.K. wait a second, this one's easy I know it is...really...2 serbs....2 fifteens... fifteen times... get it into pairs

$$\frac{b}{a} = x_1$$

52,4 why don't you times it out you jerk...7 times 15 105

$$x_1 \times c = x_2$$

82,0 so you have 105 sets of tods....is it...no... 35... so that's.... O.K. 105...I figured it out...I got one...no I don't...O.K. wait a second people...2 serbs make 7 tods...and 35 sets of tods make 1 fot...O.K...30 serbs... you get...you pair em off, so 15 serbs to make ...O.K. so you pair that mess off and you get 2, 15's makes 105 sets of tods

$$x_1 \times c = x_2$$

14,4 are needed to make 1 fot, therefore, so how many fots can be made from 30 serbs.......3... is it right?

$$\frac{x_2}{E} = x_f$$

TOTAL 217,2 secs.

8. 25,4 We're getting better, we're up to ergs and zot's. 7 ergs make 4 zots... 4 times that makes 28

$$ac = x, \quad \frac{d}{c} = a$$

53,4 4 times 7....O.K. wait a see that would make it 7 ergs....that would be 7.....7.....7...you need 49

$$a^2 = x_f$$

29,8 we figured out 7 ergs make 4 zots and we needed 28 zots in all, so we figured out is how many ergs, well....if 7 ergs make up 4 zots, there are 4 sets of zots in 28 zots...so that would make it like 7 times 7.

$$a^2 = x_f$$

TOTAL 108,6 secs.

9. → 101,4 .....A mole of Nitrogen weighs 28 grams..... 22 liters in volume.....22 liters, what will be the volume of 35 grams of Nitrogen. Oh, O.K. 35 grams...I hate this...22 liters...28 grams I don't know how to figure this out..RETIRES

TOTAL 101,4 secs.

⑩ → 45,2 .....Help....80 grams of NaOH....98 grams of H_2SO_4 are needed to make 36 grams of H_2O. I haven't the faintest idea....you know that eh?...RETIRES

TOTAL 45,2 secs.

VINCE R.

QUESTION	TIME	STEP
1.	3,2	4 over 6 equals 9 over x $\frac{a}{b} = \frac{c}{x_f}$
	7,2	6 times 9.....54 $bc = z$
	2,2	4 $a x_f = y$
	29,4	into 54 goes.....bring down the 4....13 and 2 quarters...13½ $\frac{z}{y} = x_f$
TOTAL	42,0 secs.	
2.	6,4	8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$
	4,0	2 goes into the 8 $\frac{a}{r}$
	19,2	....cross multiply..15 times 8....120 $ad = z$
	2,2	so that equals 120 and 10 $\frac{z}{y} = x_f$
	1,0	into that equals 12 x_f
TOTAL	32,8 secs.	

3. 4,8 x over 35 equals 40 over 56

$$\frac{x_f}{b} = \frac{c}{d}$$

18,4 35 times 40.....1400, so that's 1400

$$bc = z$$

2,2 56

$$dx_f = y$$

67,8 into 1400...56 goes...6 into there...O.K. that makes ....that's 25 so that's 25 over 35 equals 40 over 56

$$\frac{z}{y} = x_f$$

TOTAL 93,2 secs.

4. 3,0 12 over 24 equals x over 8

$$\frac{a}{b} = \frac{x_f}{d}$$

⟹ 4,8 O.K. that's lowest terms...so that's..uh... 3 eighths

$$\frac{b}{d} = x_f$$

10,0 24 divided by 8 is 3

$$\frac{b}{d} = x_f$$

15,8 and if you divide the bottom by 3 you gotta divide the top by 3 and it'll be the lowest term that's 12 twenty-fourths, 3 eighths

$$x_f$$

TOTAL 33,6 secs.

5. 6,6 O.K. that's 4 times...4 bottles over 10 bottles

$$\frac{E}{c}$$

19,8 so 4 bottles clean 2500...how many square feet of flooring can be cleaned with 3 cases so 3 cases...3 times 10 which is 30

$bc = x'$

21,0 4 into 30....O.K. 9...so that'd be 9 and 3 quarters

$\frac{x'}{E} = x_f'$

11,2 times the...so let's figure this out...9 times 4

$x_f' \times E$

25,8 wait...9 times 3 is twenty...that's 8...O.K. I did that wrong...so that's gonna be 8,5

x_f'

57,0 times 2500...Oh I made a mistake...O.K....so that makes 21,250 square feet of flooring

$x_f' \times F = x_f$

TOTAL 141,4 secs.

6. 19,8 O.K. for 200 bags...$700, for 275 what is it, so that's 275 equals x

$\frac{a}{b} = \frac{c}{x_f}$

44,2 O.K....so that's ...275 times 700...so that makes 192500

$bc = z$

71,2 so how many times does the 200 go into it...O.K. you don't need that...so that's...962 and the x is $962,50

$a x_f = y, \quad \frac{z}{y} = x_f$

TOTAL 135,2 secs.

⑦ 32,6 O.K. if 2 serbs....7 tods....35 sets of tods ....1 fot...so I gotta multiply this (7 tods) times 35....

$$C \times E$$

22,0 no wait...that's wrong cause that's 7 sets... so that's 5 times so that's 7 tods times 5 is 1 fot...O.K. so 7 times 5....35

$$C \times 5 = E$$

36,2 how many fots can be made from 30 serbs..O.K. let's see...so 2 equals 7..O.K. fots equals... O.K. 30 serbs...let's say we have 30.... O.K. if we multiply this by 15

$$\frac{b}{a} = x'$$

16,8 wait a minute you don't need that 7 fots...yes you do...O.K....7 makes 7 times 15 equals...105

$$C x' = x_1$$

33,6 so 105 times 5 would equal the fots...that is 525

$$x_1 \times 5 = x_f$$

44,4 that is 35 into 525...let's say this is a set so A is to A, B to B, C to C, the A works with A the B times 5 times 5...O.K. so that's right that's 525 fots

$$\frac{x_f}{E} = ?$$

26,0 let's see if it was this...2 over 35 which equals 1 fot....30 over 5

$$\frac{a}{E}, \quad \frac{b}{5},$$

30,0 times 15....yea the answer is 525 fots.

$$\frac{b}{5} \times \frac{b}{a} = x_f$$

TOTAL 241,6 secs.

8. 18,8 here we go again...7 ergs...let's write this down...7 ergs over 4 zots equals how many ergs are gonna make up 28 zots...let's see 4 into 28...7 times

$$\frac{a}{x_f} = \frac{c}{d}, \quad \frac{d}{c} = x_1 \text{ or } \frac{c}{r}, \frac{d}{r}$$

14,8 so ya gotta multiply the 4 by 7 and the 7 by 7 so that's 49 ergs make up 28 zots

$$x_1 \times a = x_f$$

TOTAL 33,6 secs.

⑨ 62,2 If 1 M of Nitrogen gas is also equal to about 221.....so that's 22 liters...what will be the volume of 28 grams of N_2...so that's 28 grams O.K. that equals that...so I've got...all we know is that 28 grams is equal to 22 liters in volume and we're trying to find out what 35 grams is in volume...and we know that 7 goes into 35 and 7 goes into 28...goes in there 4, goes in there 5..that's 4 fifths.

$$\frac{c}{r}, \frac{d}{r}$$

17,0 which is...28...that's 28 over 35 which is 4 fifths...that's 22 over x

$$\frac{a}{b} = \frac{c}{x_f}$$

21,4 all I've gotta do is cross multiply...O.K. 22 times 35 is...770

$$bc = z$$

7,8 28

$$a x_f = y$$

⟹ 27,8 into 770, 28x equals 770...so that's 3 and 10 28ths.

$$\frac{z}{y} = x_f$$

30,4 I forgot to mark this one down...which is 30... 30 and 10 twenty eighths.

$$\frac{z}{4} = x_F$$

TOTAL 166,6 secs.

⑩ 90,8 O.K. let's see...80 grams...that's 80 grams O.K....so you gotta add that plus 98 grams which is...H_2SO_4...that's how much it takes ...so that's equal to 36 grams of H_2O...let's see 45 grams over 3...that's 36 over 45... cross multiply so that you get the other side so that's 80 equals 36 grams of NaOH so that's 80 plus 98 which is...178

$$\frac{d}{c} = \frac{x_1}{a} ; \quad a + e = a_E$$

⟹ 37,2 let's cross multiply...let's see now 270

$$ad = z$$

3,0 divided by 36

$$\frac{z}{4}$$

⟹ 53,8 ...wait now...what times 36 equals 270?...oh that's right divide...28½..

$$\frac{z}{4} = x?$$

67,4 O.K. let's see does that work...36 times 7,5.., let's see now...so that means it made...I have a total of the 2 put together...wait now, I have 270 grams of NaOH and H_2SO_4

$$z = x?$$

51,4 so the 2 numbers of the grams equals 270..so so figure 2 separate ones...here we go again... I figured out what the total would be so all I gotta do is figure out what the 80 and 98 would be if they were cross multiplied...so how many grams of the NaOH would take and how many would take from the H_2SO_4...the 98 that I got...the total of the 2 would be equal to 270

9,0 O.K. so 36 over 45 equals 80 over x

$$\frac{c}{d} = \frac{a}{x_1}$$

22,6 wait...45 times 80.....3600

$$ad = z$$

12,0 so 36 into 3600...goes 100 times

$$\frac{z}{4} = x_{f_1}$$

4,0 so 80 over 100 all right...

$$x_{f_1}$$

21,8 heh that's right, it takes 100 grams of the NaOH and subtract 100 grams from 270 and that takes 170 grams of H_2SO_4

$$x? - x_{f_1} = x_{f_2}$$

TOTAL 373,0 secs.

PAUL Y.

QUESTION	TIME	STEP
1.	6,6	4 over 6 equals 9 over x $\frac{a}{b} = \frac{c}{x_f}$
	12,0	4x $a x_f = z$
	4,0	equals 6 times 9....54 $bc = y$ — — —
	22,2	x equals 54 divided by 4 equals....x equals 13½ $\frac{y}{z} = x_f$
TOTAL	44,8 secs.	
2.	3,2	x over 35 equals 40 over 56 $\frac{x_f}{b} = \frac{c}{d}$
	3,2	56x $d x_f = y$
	7,8	equals 35 times 40.....1400 $bc = z$
	32,2	x equals 1400 divided by 56....x equals 1400 divided by 56.....25 $\frac{z}{y} \cancel{= x_f}$
TOTAL	46,4 secs.	
3.	3,0	8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$

2,0 10x

$c x_f = 4$

6,4 equals 15 times 8.....120

$a d = z$

2,0 x equals 12

$\frac{z}{4} = x_f$

TOTAL 13,4 secs.

4. 5,0 12 over 24 equals.....½

$\frac{a}{r}, \frac{b}{r}$

2,2 ½ equals x over 8

$\frac{\frac{a}{r}}{\frac{b}{r}} = \frac{x_f}{d}$

6,2 4

x_f

TOTAL 13,4 secs.

5. 41,4 4 bottles equals 2500.....uh...uh...10 divided by 4 is 2½

$\frac{c}{E} = x_1$

33,6 2500 times 2,5.....6250

$F x_1 = x_2$

26,6 6250...1 case of fluid can clean 6250 square feet....3 cases is 6250 times 3....is 18750

$b x_2 = x_f$

TOTAL 101,6 secs.

6. 11,8 700 divided by 200.....is 3½

$$\frac{c}{a} = x_1$$

58,4 uh...200 bags of grain equals $700, 275 bags of grain....wait...x

$$\frac{a}{b} = \frac{c}{x_f}$$

3,4 200x

$$a x_f = y$$

14,0 equals 192500

$$bc = z$$

13,0 1925 divided by 2

$$\frac{y}{r} , \frac{z}{r}$$

21,8 is....962,5 bags of uh....$962.50

$$\frac{z}{y} = x_f$$

TOTAL 122,4 secs.

7. 14,6 O.K......1 serb would make 3½ sets of tods

$$\frac{c}{a} = x_1$$

61,8 and 35 sets of tods are needed to make 1 fot ....35 tods....it takes...35 uh..3½, uh 35 divided by 3,5 would be 10

$$\frac{E}{x_1} = x_2$$

17,2 it takes 10 serbs to make 35 tods which is 1 fot...so 10 serbs to make 1 fot

$$x_2 = F$$

5,4 so how many fots can be made from 30 serbs...3

$$\frac{b}{x_2} = x_f$$

TOTAL 99,0 secs.

8. 25,6 O.K....7 to make 4.....4 zots.....28 zots uh...28 divided by 4 is 7

$$\frac{d}{c} = x_1$$

23,0 ...it takes 49....it takes 49 ergs to make up 28 zots

$$a x_1 = x_f$$

TOTAL 48,6 secs.

9. 23,8 um....1 mole of Nitrogen gas....1 mole equals 28 grams, and 1 mole also equals 22 liters therefore 28 grams equals 22 liters in volume

$$a = c$$

56,6 what would be the volume of 35 grams of Nitrogen....28....28 grams....35 and x...O.K.....uh..28...uh..grams is the same as 22 liters so 35 grams will equal x

$$\frac{a}{b} = \frac{c}{x_f}$$

8,6 35 times 22 is.....770

$$bc = z$$

4,8 770 divided....770 equals 28x

$$a x_f = y$$

96,8 x equals 770 over 28....770 divided by 28 is ....29½....that's a 7...27½ would be the volume of 35 grams

$$\frac{z}{y} = x_f$$

TOTAL 190,6 secs.

10. 66,2 O.K. uh... 80 grams of NaOH and 98 grams of H_2SO_4 equals 36 grams of H_2O...it's uh...80 over 178...

$$\frac{a}{a\varepsilon}$$

⟹ 18,0 ....I don't think I understand how to do it....

RETIRES

TOTAL 84,2 secs.

SECONDARY IV

ALLAN G.

QUESTION	TIME	STEP
1.	3;8	O.K. 4 over 6 equals 9 over x $\frac{a}{b} = \frac{c}{x_f}$
	1,4	4x $ax_f = 4$
	4,2	equals 9 times 6 which is 54 $bc = z$
	23,6	54 divided by 4....so the answer is...9 over 13,5 $\frac{z}{4} = x_f$
	8,2	times 2 is 18 over 27 x_f (whole number)
TOTAL	41,2 secs.	
2.	2,8	8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$
	1,6	10x $cx_f = 4$
	10,6	equals 15 times 8....which is 120 $ad = z$
	4,0	x equals 12 $\frac{z}{4} = x_f$
	6,0	so 8 over 12 equals 10 over 15...is that right? x_f
TOTAL	25,0 secs.	

3. 7,0 x over 35 equals 40 over 56

$\frac{x_f}{b} = \frac{c}{d}$

3,0 56x

$d x_f = y$

15,4 equals 40 times 35...so the answer is 1400

$bc = z$

37,2 so then 56 divided into 1400...25, so 25 over 35 equals 40 over 56

$\frac{z}{y} = x_f$

TOTAL 62,6 secs.

4. 4,4 12 over 24 equals x over 8

$\frac{a}{b} = \frac{x_f}{d}$

2,2 24x

$b x_f = y$

6,8 equals 12 times 8......96

$ad = z$

17,6 x equals 24 divided into 96...x equals 4

$\frac{z}{y} = x_f$

TOTAL 31,0 secs.

5. 57,4 You got me in a weakness, I can't do problems.. Oh..so 4 bottles...I'm just thinking...so 25000 square feet..so that means 10 divided by 4

$\frac{C}{E} = x_1$

2,0 if it's 3 cases that means 30

$$\frac{a}{b} = \frac{c}{x_1} = x_1$$

18,8 Oh yea...that's all multiplication...so that means 4 divided into 10 is 2,5

$$\frac{c}{E} = x_1$$

24,4 that means 2,5 times 2500 that will equal... 6250

$$x_1 \times F = x_2$$

18,6 and then you have 3 cases so it's still times 3 so it's....18750 square feet....I hope... Is it right or wrong?

$$b \times x_2 = x_f$$

TOTAL 121,2 secs.

6\. 23,0 So then you'd take off the zeros

$$\frac{a}{r}, \frac{c}{r}$$

10,0 and you'd go 2 into 7 goes...that will equal 3,5 isn't it...yea 3,5

$$\frac{\frac{c}{r}}{\frac{a}{r}} = x_1$$

5,0 that's your ratio, well that's the price for 1 bag

$$x_1$$

30,2 so if you have 275 bags times 3,5....will give you....so it's $962,5

$$x_1 \times b = x_f$$

TOTAL 68,2 secs.

7\. 38,2 Hmmm...if 2 serbs can make 7 sets of tods.... well that means if it's 30 serbs...that's

15 times more

$$\frac{b}{a} = x_1$$

6,8 so 15 times 7 is.....105

$$x_1 \times c = x_2$$

11,2 that means 105 tods and 35 sets of tods are needed to make 1 fot..that means 105 divided by 35 will give me...uh....3

$$\frac{x_1}{E} = x_f$$

TOTAL 56,2 secs.

8. 27,4 O.K. that means 7 ergs make up 4 zots, so if 7 over 4 equals x over 28

$$\frac{a}{b} = \frac{x_f}{d}$$

2,0 4x

$$b x_f = y$$

9,2 equals....196

$$ad = z$$

4,6 x equals 4 divided by 196

$$\frac{z}{y}$$

12,4 x would give me 49 ergs

$$x_f$$

TOTAL 55,6 secs.

9. 47,2 Oh boy.....the volume.....that's a tough one.... that means 28 over 22 equals 35 over x

$$\frac{a}{b} = \frac{c}{x_f}$$

2,2 28x

$ax_f = y$

19,0 equals....770

$bc = z$

⟹ 55,4 x equals 28 divided into 770...so that'll go 31,07

$\frac{z}{y} = x_f$

7,2 maybe it's wrong...so 31,07 liters....... that's wrong....

x_f

TOTAL 131,0 secs.

⑩ 98,4 Hmmmm....that's all ratio again...so 80 grams ....do I have to break it down?...Oh I have to do two of them...80 grams of NaOH and 98 grams of H_2SO_4...that's in total the 36 grams... so 80....80 over 36 will make x over 45

$\frac{a}{c} = \frac{x_1}{d}$

6,2 36x

$cx_1 = y$

⟹ 12,0 equals 45 times 80.....3840

$da = z$

46,2 36 divided into 3840 will go....106

$\frac{z}{y} = x_1 \ (x_{f_1})$

16,0 106...you're kidding...yeh that's it, 106.6 is grams of NaOH

x_{f_1}

15,8 So now the other one,..so exactly the same thing...so this time it'll be...98 grams over 36 equals x over 45

$$\frac{{}^\circ E}{F} = \frac{x_2}{d}$$

2,4 so 36x

$$Fx_2 = 4$$

51,6 equals 45 times 98.....4410

$$Ed = Z$$

49,0 so x will equal 36 divided into 4410....122,5
so the H_2SO_4 will equal 122,5

$$\frac{Z}{4} = x_2 \ (x_{F2})$$

TOTAL 297,6 secs.

PAULA L.

QUESTION	TIME	STEP
1.	2,6	O.K. so this goes 4x $ax_f = y$
	4,0	equals 54 $bc = z$
	15,4	54 divided by 4 equals 13½ $\frac{z}{y} = x_f$
TOTAL	22,0 secs.	
2.	2,4	O.K. 8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$
	3,0	10x $cx_f = y$
	5,4	equals 120 $ad = z$
	2,0	x equals 12 $\frac{z}{y} = x_f$
TOTAL	12,8 secs.	
③	4,6	x over 35 equals 40 over 56 $\frac{a}{b} = \frac{x_f}{d}$
	3,0	56 x $bx_f = y$
	15,0	equals.......1400 $y, \ ad = z$

no...wait a minute...yeh...yeh I guess so...
x equals 56 into 1400..ummm......25,6

$$\frac{Z}{4} = x_f$$

TOTAL 96,8 secs.

4. 4,6 12 over 24 equals x over 8

$$\frac{a}{b} = \frac{x_f}{d}$$

10,0 well, it's just a half

$$\frac{a}{r}, \frac{b}{r}$$

1,0 4

$$x_f$$

TOTAL 15,6 secs.

5. 29,8 4 bottles give 2500 so 12 bottles to clean 7500

$$\underline{3 \times E} = \underline{3 X F}$$

12,2 O.K. so 4 bottles equals 2500 square feet
which means...no wait a minute....30 equals x

$$\frac{E}{x_1} = \frac{F}{x_f}$$

12,8 10 bottles in a case....3 cases, so 4x

$$c \times b = x_1, \quad E x_f = y$$

3,8 equals 75000

$$F x_1 = Z$$

27,8 x equals 18750

$$\frac{Z}{4} = x_f$$

TOTAL 86,4 secs.

6. 19,8 O.K......200 bags equals \$700., therefore
275 equals x

$$\frac{a}{b} = \frac{c}{x_f}$$

2,6 200x

$$a x_f = y$$

25,2 equals.......192500

$$bc = z$$

8,0 x equals......2 into 1925

$$\frac{y}{r}, \frac{z}{r}, \frac{z}{y} = x_f$$

44,0 x equals......\$975.

$$x_f$$

TOTAL 99,6 secs.

7. 44,8 O.K. so 2 serbs equals 7 tods...you need...
how many fots can be made from 30 serbs.....
O.K. so it's 30 serbs....how many tods I guess...
O.K. 2 serbs make 7 tods 30 serbs makes x

$$\frac{a}{b} = \frac{c}{x_1}$$

1,0 2x

$$a x_1 = y$$

2,2 equals 210

$$bc = z$$

2,0 x equals 105

$$\frac{z}{y} = x_1$$

15,6 then 35 tods can make 1 fot then divide by 35

$$\frac{x_1}{E} = x_f$$

TOTAL 65,6 secs.

....gives you 3.0.K. 3 fots.

x_f

8. 17,4 7 ergs make 4 zots, x ergs make 28 zots

$\frac{a}{x_f} = \frac{c}{d}$

3,2 O.K. so it's 4x

$cx_f = y$

7,8 equals 196

$ad = z$

9,0 x equals....49...O.K.?

$\frac{z}{y} = x_f$

TOTAL 37,4 secs.

9. 17,0 Ummmmm.......O.K. 28 grams equals 22 liters, 35 grams equals x

$\frac{a}{b} = \frac{c}{x_f}$

3,4 28x

$ax_f = y$

15,6 equals 22 times 35........770

$bc = z$

62,4 x equals......27,5 liters

$\frac{z}{y} = x_f$

TOTAL 98,4 secs.

10. 41,4 O.K. ....80 plus 98 makes 36, therefore how much would make 45...Ummmm...98 plus 80 equals 178

$$a + E = AE$$

10,0 178 equals 36, x equals 45

$$\frac{aE}{x_1} = \frac{c}{d}$$

3,0 cross multiply, 36x

$$cx_1 = y$$

27,2 equals 178 times 45.....8010

$$aE \times d = z$$

51,2 x equals 36 into 8010...that would mean 222,5...the total is 222,5

$$\frac{z}{y} = (x_2)\ x_1$$

45,2 this ratio...the ratio would be 98 to 80.... no that doesn't work...Umm..ratio would be... divide by 10

$$\frac{a}{r},\ \frac{E}{r}$$

105,0 no that doesn't work...Ummm...maybe a ratio... a ratio of say the NaOH to the total...Oh right, O.K., 80 to 178 is the same as x is to 222,5

$$\frac{a}{x_2} = \frac{aE}{x_1}$$

3,4 178 x

$$x_2\, aE = y$$

32,0 equals 222,5 times 80.......17800

$$ax_1 = z$$

7,6 O.K. 178 x equals 17800, x equals 100

$$\frac{z}{y} = x_{F_1}\ (x_2)$$

13,4 therefore there's 100g of NaOH and 122,5g of H_2SO_4

$x_1 - x_2 = x_f$

TOTAL 339,4 secs.

SHARON P.

QUESTION	TIME	STEP
1.	5,0	4 over 6 equals 9 over x $\frac{a}{b} = \frac{c}{x_f}$
	1,0	4x $ax_f = 4$
	1,0	equals 54 $bc = z$
	13,4	x equals... x equals... I hate this.... $\frac{z}{4}$
	10,0	13,5...so x equals 13,5 $\frac{z}{4} = x_f$
TOTAL	30,4 secs.	
2.	3,2	8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$
	1,0	10x $cx_f = 4$
	7,4	equals 8 times 15....120 $ad = z$
	2,2	x equals 12 $\frac{z}{4} = x_f$
TOTAL	13,8 secs.	
3.	6,0	x over 35 equals 40 over 56 $\frac{x_f}{b} = \frac{c}{d}$

2,0 56x

$dx_f = 4$

21,0 equals 35 times 40......1400, that's 1400

$bc = z$

30,0 x equals 56 divided into 1400.....

$\frac{z}{4}$

2,5 so x equals 25

x_f

TOTAL 61,5 secs.

4. 4,8 12 over 24 equals x over 8

$\frac{a}{b} = \frac{x_f}{d}$

5,2 4

x_f

3,8 4 eights equals 12 over 24

x_f

3,0 yeh... you reduce it......4

$\frac{a}{r}, \frac{b}{r} = x_f$

TOTAL 16,8 secs.

⑤ 40,8 .....10 bottles of cleaning fluid in a case...
O.K. so 2800 can equal 4 bottles so 10 times 3,

$b \times c = x_1$

3,0 30

x_1

36,2 equals 30.... is this all right? ...that I'm running?....(sure)....equals....so that's 2800 square feet.....so if 4.....how many 4's are in 30..

$$\frac{x_1}{E} = x_2$$

8,0 7,5

$$x_2$$

⇒ 10,2 ...so 7,5 times 250.....2500

$$x_2 \times F = x_f$$

26,8 .....so 25750 square feet.

$$x_f$$

TOTAL 125.0 secs.

6. 28,8 O.K. so 200...200 to 700 equals 275 to x

$$\frac{a}{b} = \frac{c}{x_f}$$

2,0 200 x

$$a x_f = y$$

31,2 equals 700 times 275.....192500

$$bc = z$$

2,6 192500 divided by 200

$$\frac{z}{y}$$

9,0 so that's 1925 divided by 2

$$\frac{z}{r}, \frac{y}{r}$$

19,4 .....so it's 962,5

$$\frac{z}{y} = x_f$$

19,2 that doesn't....that's not right...(confused)

TOTAL 112,2 secs.

⑦ 45,0 Ummmm....O.K. so we need 5 to make 7,..we need 5 tods to make 1 and if 2 can make 7....No

34,2 2 serbs can make 7 tods....35 make 1 fot, how many fots...then... if 2 can make 7 then 30 can make....1 can make 3,5

$$\frac{c}{a} = x_1$$

⟹ 3,8 then 30 can make 30 times 3,5

$$x_1 \times b = x_2$$

15,0 1050

$$x_2$$

10,4 1050?....1050 divided by 35...

$$\frac{x_2}{E} = x_3$$

32,0 Oh good grief, so 105....30...Oh that doesn't work. (Retires)

$$x_3$$

<u>TOTAL 140,4 secs.</u>

8. 35,2 ....If 7 ergs make up 4... if 7 ergs .. O.K. so that's 7 to 8...it's what to 28

$$\frac{a}{x_f} = \frac{(\ )}{d}$$

29,4 oops....that's wrong anyway...if 7 goes to 4 zots if 7 ergs make 4 zots how many ergs for 28 zots.. Oh.. x over 28

$$\frac{a}{b} = \frac{x_f}{d}$$

10,0 that's it ..O.K...so that's times 4 by 7

$$\frac{d}{b} \times a$$

1,6 49

x_f

4,2 you need 49 zots... 49 ergs

x_f

TOTAL 80,4 secs.

9. 32,0 You'll kill me if I don't get this one eh? O.K. I'll read it again....

11,0 28 equals...if 28 grams equals 1 mole then 35 grams equals x

$$\frac{c}{d} = \frac{a}{x_1}$$

55,4 So 35 divided by 28... 28 into 35....1,25, that's it

$$ad = z, \quad cx_1 = y, \quad \frac{z}{y} = x_1$$

3,6 So 22 times 1,25....

$$x_1 \times F = x_f$$

44,6 Ummm.....Oh.....so 27,5

x_f

TOTAL 146,6 secs.

10. 63,2 ....where'd you get this thing....O.K., Ummm... 80grams of NaOH and 98 grams of H_2SO_4 to make 36 grams of water....36 grams of H_2O...how many grams to make 45 grams of water...add 5 onto each one...

$$a + 5, \quad E + 5$$

3,0 85 and 103....

x_{f_1}, x_{f_2}

92,2 No... that isn't right...let's erase this... so... what time is it...NaOH and H_2SO_4... gives you water....some sort of H_2O... I have to figure out the whole thing...Oh he's getting us back you know...80 grams of NaOH and H_2SO_4 gives you water...Oh O.K. that's O.K..... 80 plus 98 give you 35 - Oh - 36

$a + E = C$

71,2 I need to know the uh...yes...but if I don't know....I can't do it...Oh am I ever dumb.... 98 plus that....Oh I know 80 plus 98 equals 178 over 36 to x over 45

$a + E = AE, \quad \frac{AE}{C} = \frac{x_1}{D}$

1,0 36x

$C x_1 = y$

4,2 equals 178 times 45

$AE \times D = Z$

40,6 so that's 7610

Z

59,4 so 36 into 7610 goes....so that's 211

$\frac{Z}{y} = x_1$

7,4 now my ratio... 80 is to 98

$a : E$

25,6 no... so I'll reduce it, 40 is to 49...48...49

$\frac{a}{r}, \frac{E}{r}$

8,0 divide by 40

$\frac{x_1}{\frac{a}{r}}$

2,0 40 into 49....

$\frac{\frac{E}{r}}{\frac{a}{r}}$

15,0 ....Retfres.

TOTAL 392,8 secs.

GLENN R.

QUESTION	TIME	STEP
1.	4,0	O.K. 4 over 6 equals 9 over x $\frac{a}{b} = \frac{c}{x_f}$
	6,8	so using the means-extremes postulate, so it's 4x $ax_f = y$
	3,0	equals 48 $bc = z$
	5,8	so x equals 12 $\frac{z}{y} = x_f$
TOTAL	19,6 secs.	
2.	4,8	8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$
	4,0	using the means-extremes postulate again, 10x $cx_f = y$
	6,0	equals......120 $ad = z$
	5,0	yeh...so x would equal....12 again $\frac{z}{y} = x_f$
TOTAL	19,8 secs.	
3.	8,4	x over 35 equals 40 over 56 $\frac{x_f}{b} = \frac{c}{d}$

4,8 using the means-extremes postulate, 56x

$$dx_f = y$$

13,2 equals.....14000

$$bc = z$$

36,6 so it's 14000 over 56 which equals.....so the answer is,...x equals 25

$$\frac{z}{y} = x_f$$

TOTAL 73,0 secs.

4. 6,6 12 over 24 equals x over 8

$$\frac{a}{b} = \frac{x_f}{d}$$

5,8 but the 12 over 24 can be reduced to 1 over 2 equals x over 8

$$\frac{a}{r} ; \frac{b}{r}$$

2,0 so 2x

$$bx_f = y$$

1,4 equals 8

$$ad = z$$

2,8 x equals 4

$$\frac{z}{y} = x_f$$

TOTAL 18,6 secs.

5. 43,4 O.K. there's 10 bottles in a case and if... so 2500 ft of floor can be cleaned with 4 bottles, so how many can be cleaned with 3 cases...30

$$\frac{a}{b} = \frac{c}{x_1} = x_1$$

13,8 O.K. so if 4 bottles equals 2500 square feet then 30 bottles equals x feet

$$\frac{E}{x_1} = \frac{F}{x_f}$$

3,2 so cross multiply, 4x

$$Ex_f = 4$$

11,8 equals 25 times 30....75000

$$Fx_1 = Z$$

36,8 is that right?...that's 75000 divided by 4 .....so the answer, x equals 18750 sq. ft.

$$\frac{Z}{4} = x_f$$

TOTAL 109,0 secs.

6. 34,0 O.K. same thing almost...200 bags equals $700. and 275 bags equals x

$$\frac{a}{b} = \frac{c}{x_f}$$

3,8 cross multiply, 200x

$$ax_f = 4$$

19,6 equals 275 times 700...so equals 192500

$$bc = Z$$

67,0 so you divide that by 200, 192500 divided by 200......so x equals $962.50 for 275 bags of grain.

$$\frac{Z}{4} = x_f$$

TOTAL 124,4 secs.

7. 81,0 O.K. if 2 serbs equals 7 tods and 35 tods equals 1 fot....then 30 serbs....equals x fots, Hmmmm....you can't turn it off if you want to think?...that means...that's...O.K. if 2 serbs equals 7 tods and 35 tods equals 1 fot, that

means 20 serbs equals 1 fot

$$E = x_1$$

13,4 so if 20 serbs equals 1 fot, then 30 serbs equals 3 fots...so the answer is 3 fots.

$$\frac{B}{x_1} = x_f$$

TOTAL 94,4 secs.

8. 27,2 O.K. so 7 ergs equals 4 zots...how many ergs are needed to make 28 zots... so 7 ergs over x ergs equals 4 zots over 28 zots

$$\frac{a}{x_f} = \frac{c}{d}$$

5,0 so cross multiply, 4x

$$cx_f = y$$

8,8 equals 196

$$ad = z$$

24,0 x equals...49, so the answer is 49 ergs are needed to make 28 zots.

$$\frac{z}{y} = x_f$$

TOTAL 65,0 secs.

9. 71,8 O.K., so 1M of N_2 equals 28 grams and 1M of N_2 gas is also equal to 22 liters so what would be the volume of 35 grams? Oh..O.K. so if 1M of N_2 equals 28 grams...then...Oh boy.. O.K. 1M of N_2 equals 28 ~~grams~~ then x Moles of N_2 equals 35 grams

$$\frac{a}{x_1} = \frac{c}{d}$$

6,2 cross multiply, 28x

$$cx_1 = y$$

2,2 equals 35

$ad = z$

74,0 28 goes into 350..goes into 35...so x equals 1,25

$\frac{z}{4} = x_1$

9,4 so 1,25M equals 35 grams of N_2

x_1

25,4 so 1M of N_2 equals x liters? O.K. I got it.. 1M of N_2 equals 22 liters therefore 1,25M of N_2 equals x liters

$\frac{E}{x_1} = \frac{F}{x_f}$

54,6 so x should equal.....so 27,5 liters of N_2 in 35 grams of N_2

$Fx_1 = z, \quad Ex_f = 4, \quad \frac{z}{4} = x_f$

TOTAL 243,6 secs.

⑩ 168,6 Hmmm...this is a toughy....NaOH and H_2SO_4 don't make water do they?..Oh O.K. plus something else. So...I think you have to make an equation ...I'm not sure but I'll try to make an equation so NaOH plus H_2SO_4 reacts to form Na_2SO_4 plus H_2O... Balancing the equation...you don't have to balance it...let's try something..Oh 80g equals NaOH ...Oh isn't this the thing we went over to find the ratio between....If I can remember how to do that now....O.K. Na equals 23 grams and O equals 16 grams and H equals 1, totalling up to...40, so that's half of what you need

$NaOH = \frac{1}{2}a$

85,0 so 2M of Na, 2M of O and 2M of H and for the next equation,...H_2 equals 2, S equals 32 and O times 4 equals 48, no 56, no 16 times 4.... 64...so total them all up..that equals...98, and that's it...so it's 1M of H_2SO_4 plus 2M of NaOH equals 2M of H_2O and what have you

$$H_2SO_4 = 1E$$

70,0 Oh shoot, I think I went the long way didn't I...36 grams of H_2O equals...so it's 2M of H_2O equals 36 so...shoot ...O.K. so that equals so 2M of NaOH and 1M of H_2SO_4 reacts to form Na_2SO_4 plus 2 M of H_2O so if 2M equals 36, x Moles equals 45

$$H_2O = 2C, \quad \frac{2C}{x_1} = \frac{C}{D}$$

5,2 so 36x

$$Cx_1 = 4$$

1,8 equals 90

$$2(C)D = Z$$

37,6 36 divided into 90....2,5...so 2,5 M equals 45 grams

$$\frac{Z}{4} = x_1$$

41,4 so all the grams are increased by...no I can't do that...O.K. so the difference in Moles from H_2O is ,5

$$x_1 - 2C = x_2$$

40,6 so I think...I think you have to...the difference in NaOH would be ,5 so 80 plus 20 equals 100 grams of NaOH

$$a \times x_2 \div 2 = x_{F_1}$$

⟹ 72,0 and for H_2SO_4 it's 1,5 times 98...so it's 1,5 times 98...147...so you need 147 grams of H_2SO_4

$$E + x_2 = x_{F_2}$$

TOTAL 522,2 secs.

ANGELA R.

QUESTION	TIME	STEP
1.	2,8	O.K. 4 over 6 equals 9 over x $\frac{a}{b} = \frac{c}{x_f}$
	1,0	4x $a x_f = y$
	⇒ 1,0	equals 36 $bc = z$
	4,2	x equals 9 $\frac{z}{y} = x_f$
TOTAL	9,0 secs.	
2.	6,2	8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$
	7,8	8 times 15 is......120 $ad = y$
	4,0	120 equals 10x $y = z$
	3,8	x equals 12
TOTAL	21,8 secs.	x_f
3.	7,0	x over 35 equals 40 over 56 $\frac{x_f}{B} = \frac{c}{d}$
	2,2	56x $d x_f = y$

10,4 equals 35 times 4.......1400

$bc = z$

25,0 oh God.....1400 over 56.....equals 25

$\frac{z}{4} = x_f$

1,0 x equals 25

x_f

TOTAL 45,6 secs.

4. 6,8 12 over 24 equals x over 8

$\frac{a}{b} = \frac{x_f}{d}$

4,0 24 divided by 3, 12 divided by 3

$\frac{b}{r}, \frac{a}{r}$

3,6 equals 4, x equals 4, 4 over 8

x_f

TOTAL 13,4 secs.

5. 33,0 O.K. so 4 bottles cleans...wait a minute...
10 bottles...4 bottles cleans 2500 square feet,
3 cases, if there's 10 in a case....then 3 times
10...if there's 30 in a case

$\frac{a}{f} = \frac{b}{x_{f_1}}, \quad x_1$

2,0 cleans x amount of square feet

$\frac{c}{x_1} = \frac{d}{x_f}$

1,0 4x

$c x_f = 4$

2,4 equals 75000

$dx_1 = z$

25,2 O.K. 4 goes into 75...O.K....once.....18750
x equals 18750

$\frac{z}{y} = x_f$

TOTAL 63,6 secs.

6. 23,2 O.K. so he's paying 700 dollars for 200 and how much, x, is he going to pay for 275

$\frac{a}{x_f} = \frac{c}{d}$

14,4 275 times 7 is.....192500

$ad = y$

2,2 equals 200x

$cx_f = z$

2,4 knock off the zeros

$\frac{y}{r}, \frac{z}{r}$

11,6 x equals 2 into 19........962,5

$\frac{y}{z} = x_f$

TOTAL 53,8 secs.

7. 33,0 O.K. 2 serbs equals 7 tods, 7 tods equals.... wait a minute...2 serbs equals 7 tods...35 tods equals 1 fot...how many fots can be made from 30 serbs

$a : b,\ c : d,\ E : F$

10,2 O.K. get this all the same...so for every 35, that's times 5

$\frac{E}{C} = x_1$

5,4 that's 10 to 35 to 1...they're all the same

$5a = E = F$

14,6 so that's serbs, this is tods, this is fots, so I'm going to use 30 serbs and I'm gonna have that's times 3

$$\frac{B}{5a} = x_2$$

1,8 so I'm gonna times this by 3

$$E \times x_2 = x_3$$

,8 that's gonna be 5

$$x_3$$

3,2 that's 105 times 3, is 3

$$x_3 \times x_3$$

6,8 what's the question?...how many fots can be made from 30 serbs...O.K. 3 fots can be made.

$$\frac{x_3}{E} = x_f$$

TOTAL 75,8 secs.

8. 17,4 O.K. 7 ergs make 4 zots, x ergs make 28 zots

$$\frac{a}{x_f} = \frac{c}{d}$$

1,6 that's 4 times 7 (reduced fraction to $\frac{1}{7}$ from $\frac{4}{28}$)

$$\frac{c}{r}, \frac{d}{r}$$

1,4 7-times 7 is 49

$$ad = x_f$$

TOTAL 20,4 secs.

9. 15,4 N equals 28..... N equals 28grams equals 22 liters

$$a = c$$

19,0 O.K. 28 is to 22 as x is equal to weight... how many grams?...35 grams...O.K. as 35 is to x

$$\frac{a}{b} = \frac{c}{x_f}$$

13,4 35 times 22 is.....770

$$bc = y$$

5,6 770 equals 28x

$$y = z$$

47,0 770 divided by 28....28 goes into 770...31,4 O.K. 31,4 liters

$$\frac{z}{y} = x_f$$

TOTAL. 100,4 secs.

10. 28,0 O.K. 80 plus 98 equals 36....O.K. how many grams of NaOH and H_2SO_4 are needed to make 45 grams of H_2O....This is going from 36 to 45....O.K. we'll cross-multiply

$$a + E = c$$

12,2 O.K......the ratio is 36 to 45...O.K. 9 goes into here 4 times and into there 5 times ... four fifths

$$\frac{c}{r}, \frac{d}{r}$$

11,0 O.K. 4 is to 5 as 98 is to x

$$\frac{\frac{c}{r}}{E} = \frac{\frac{d}{r}}{x_1}$$

6,0 98 times 5 is.....490

$$E \times \frac{d}{r} = z$$

4,2 490 equals 4x

$$x_1 \times \frac{c}{r} = y, \quad z = y$$

16,8 x equals...what am I doing?...O.K. x equals.... 122,5

$$\frac{z}{y} = x_{f_1} \; (x_1)$$

17,4 O.K. so 122,5 of the one I had 98.... so 122,5 of H_2SO_4 .....O.K. NaOH...80 over x equals 4 over 5

$$\frac{a}{x_2} = \frac{\frac{c}{r}}{\frac{d}{r}}$$

2,2 that's going to be 400

$$a \times \frac{d}{r} = Z$$

2,0 equals 4x

$$x_2 \times \frac{c}{r} = 4$$

7,4 x equals 100...so you need 100 grams of NaOH O.K.?

$$\frac{Z}{4} = x_{F_2}\,(x_2)$$

TOTAL 107,2 secs.

RUTH V.

QUESTION	TIME	STEP
1.	2,5	So 4x $ax_f = y$
	5,6	is equal to 54 $bc = z$
	3,2	O.K. so then 4 divided into 54 $\frac{z}{y} = x_f$
	13,0	that makes.....13,5 x_f
TOTAL	24,3 secs.	
②	6,1	8 over 10 times x over 15 $\frac{a}{b} = \frac{x_f}{d}$
	3,8	so 10x $bx_f = Z$
	7,2	is equal to 15 times 8 which is.....120 $ad = y$
⇒	3,8	so 120, divided by 15 $\frac{y}{d} = w$
	29,0	......which is 8 times w
	7,0	so it'll be 8, so 10x is equal to 8 $z = w$
	9,2	so that's ,8 $\frac{w}{z} = x_f$
TOTAL	64,1 secs.	

QUESTION	TIME	STEP
3.	7,0	x over 40 equals 35 over 56 $\frac{x_f}{a} = \frac{b}{d}$
	4,0	so that's 56x $cd = y$
	4,8	equals 35 times 40 $ab = z$
	8,0	which is 1400 z
	5,0	so then 56 divided into 1400 $\frac{z}{y} = x_f$
	25,4	gives.......that's 25 x_f

TOTAL 54,2 secs.

4.	5,2	x divided by 8 equals 12 divided by 24 $\frac{a}{x_f} = \frac{b}{d}$
	1,0	so 24x $bx_f = y$
	5,0	equals 96 $ad = z$
	4,0	equals 24 divided into 96 $\frac{z}{y} = x_f$
	11,6	so that's 4 x_f

TOTAL 26,8 secs.

QUESTION	TIME	STEP
⑤	12,7	O.K. so that's the same type $x_f \in \{a, b, c, d\}$
	5,8	so 4 over 2500 equals 3 over x $\frac{a}{b} = \frac{c}{x_f}$
	2,0	so 4x $a x_f = y$
⟹	2,2	equals 7500 $cb = z$
	3,8	4 divided into 7500 $\frac{z}{y} = x_f$
	26,8	is.........1875 x_f
	6,0	so my answer is 1875 square feet.

TOTAL 59,3 secs.

QUESTION	TIME	STEP
⑥	56,4	O.K. it's a bit different though - well if the price stays the same - well it's the same thing, we have to try and find the price - O.K. 200 over 700 times 275 over x $\frac{a}{c} = \frac{b}{x_f}$
	3,2	so 200x $a x_f = y$
⟹	11,0	equals.....equals 1925 $bc = z$
	3,4	so then if 200x equals 1925 $y = z$

4,0 then x equals 200 into 1925

$\frac{z}{y} = x_f$

17,6 (should I go to one decimal place?) Hmmmmm, something's wrong.....

x_f

21,8 <u>Question</u>: What is your answer? 9,6.......can I try the next one?

x_f

<u>TOTAL 117,4 secs.</u>

⑦ 1,5 O.K. so 2 is to 7

$a : b$

1,0 well I think I can't do this one

-- <u>Question</u>: What is throwing you off?

5,8 the fots....well....there's 3....you need 3 variables here....

3,2 look - you gotta do first 2 to 7, right?

$a : b$

3,0 then 35 to 1

$c : d$

4,4 and then you're mixing up these two

11,2 you see, this is to this and this is to this, but these and this don't go together anymore

3,6 I don't think it's impossible, but I don't think I can do it

<u>TOTAL 33,7 secs.</u>

QUESTION	TIME	STEP
8.	7,0	Hmmmm...O.K. this is an easier one
	21,2	O.K. first you do 7 to 4 and then x to 28 $\frac{a}{c} = \frac{x_f}{d}$
	9,8	O.K. so 7 divided by 4, times x divided by 28 $\frac{a}{c} = \frac{x_f}{d}$
	6,2	equals 7 times 28 $\underbrace{ad} = z$
	3,4	equals....196 z
	7,6	so x equals 196 divided by 4 $c x_f = 4 \qquad \frac{z}{4} = x_f$
	3,0	.........49 ergs x_f
TOTAL 39,2 secs.		
9.	56,0	I don't know how to do this one -- I don't know what you want me to find here -- I don't know what they're talking about here. $x_f = ?$
TOTAL 56,0 secs.		
10.	38,0	This is the same as the last one....... I can't do this one either. $x_f = ?$

LEANNA V.

QUESTION	TIME	STEP
①	1,0	O.K. 63 $bc = z$
	1,0	divided by 4 $ax_f = y$
	35,6	is.......so 4 over 6 equals 9 over 15,75 O.K.? $\frac{z}{y} = x_f$
TOTAL	37,6 secs.	
②	5,6	8 over question mark equals 10 over 15 $\frac{x_f}{b} = \frac{c}{d}$
	6,0	15 times 8 equals 120 $bc = z$
	8,6	120 divided by 8 equals 15 $dx_f = y$, $\frac{z}{y} = x_f$
	8,0	so 8 over 15 equals 10 over 15......O.K.? x_f
TOTAL	28,2 secs.	
3.	7,2	What over 35 equals 40 over 56 $\frac{x_f}{b} = \frac{c}{d}$
	6,6	35 by 4 equals.......1400 $bc = y$
	4,2	divided by 56 $dx_f = z$, $\frac{y}{z}$

40,4 .....is 25...so 25 over 35 equals 40 over 56

x_f

<u>TOTAL 58,4 secs.</u>

4. 5,0 O.K. 12 over 24 equals what over 8

$$\frac{a}{b} = \frac{x_f}{d}$$

9,4 12 by 8 equals...96

$$ad = y$$

23,8 96 over 24 is......4

$$bx_f = z, \quad \frac{y}{z} = x_f$$

2,4 so 12 over 24 equals 4 over 8

x_f

<u>TOTAL 40,6 secs.</u>

5. → 75,2 .....O.K. there are 10 bottles of cleaning fluid...if 2500 is 4 bottles....how many...3 cases....if 10 bottles...4 bottles...O.K. 4 equals...O.K. 3 cases....O.K....2500 times 30 equals 75000 square feet

$$\frac{a}{b} \times \frac{c}{x_1} = x_1, \quad x_1 \times F = x_f$$

<u>TOTAL 75,2 secs.</u>

6. 34,6 O.K. so $700....200 bags...O.K. 700 divided by 200

$$\frac{c}{a} = x_1$$

15,6 3 and a half

x_1

6,0 3½ times 275.

$x_1 \times b = x_f$

⟹ 57,2 wait a second....the following prices remain the same....the prices remain the same...O.K. so it's 900...one decimal place....$972.50

x_f

TOTAL 113.4 secs.

⑦ 25,8 O.K. 2 serbs equals 7 tods and 35 sets of tods....1 fot...how many fots can be made from 30 serbs....

⟹ 45,2 2 cows....7 sets of horses
35 sets of horses are needed to make 5 chickens.. how many chickens can be made from 30 cows

replacement

35,0 2 cows equals 7......Oh no....I don't believe this....I'm gonna go on to the next one..O.K.?

$a : c$

TOTAL 106.0 secs.

8. 7,8 O.K. 7 make up 4....how many ergs make up 28 zots

$\frac{a}{x_f} = \frac{c}{d}$

3,8 49

x_f

19,0 O.K.....7 ergs make up 4 zots, how many ergs are needed to make up 28 zots...you go 4 into 28..

$\frac{d}{c} \times a = x_f$

TOTAL 30.6 secs.

...goes 7 so you get...multiply 7 by 7 ergs and you get 49....49 ergs

x_f

⑨ ⇒ 61,0 O.K.....28 grams equals 1 mole.....22 liters ....35 grams.....In chemistry a mole...28 grams.. I'm gonna go on to the next one O.K.?

TOTAL 61,0 secs.

⑩ 41,6 .....O.K. 80 grams.....36 grams of water...how many grams.....H_2SO_4.....geez...80 grams to 36

7,4 80 plus 98 equals 178

$a + E = AE$

4,0 to 36

$AE : C$

⇒ 8,4 ....178 divided by 36

$\frac{AE}{C} = x_1$

34,2 4,9

x_1

17,4 .....that's hard......RETIRES

TOTAL 113,0 secs.

SECONDARY V

JACKIE D.

QUESTION	TIME	STEP
①	2,0	4 over 6 equals 9 over question mark $\frac{a}{b} = \frac{c}{x_f}$
	4,0	so you go 4 times x $ax_f = y$
⟹	2,2	is equal to 63 $bc = z$
	29,2	63 divided by 4 is equal to the x.....do I work it out?.......15 and three quarters $\frac{z}{y} = x_f$
TOTAL	37,4 secs.	
②	2,8	8 over question mark is equal to 10 over 15 $\frac{a}{x_f} = \frac{d}{d}$
	5,2	so it's a ratio of what...2 to 3.... $\frac{c}{r} , \frac{d}{r}$
⟹	13,0	so this would be a ratio of.....18 x_f
TOTAL	21,0 secs.	
③	4,4	x over 35 equals 40 over 56 $\frac{x_f}{b} = \frac{c}{d}$
	16,0	they're getting tough...well, I'd go 56x $dx_f = y$

15,2 is equal to 140...

$bc = z$

6,2 no wait....1400

$bc = z$

36,4 divided by 56....I'm really lousy at this you know....you picked the wrong person... that's about 22,9

$\frac{z}{4} = x_f$

TOTAL 78,2 secs.

4\. 8,0 12 over 24 equals question mark over 8

$\frac{a}{b} = \frac{x_f}{d}$

6,8 oh...it's 4....it's just a ratio of 1 to 2

$\frac{a}{r}, \frac{b}{r}, x_f$

TOTAL 14,8 secs.

5\. 17,4 10 bottles of fluid....2500 square feet.... oh no...oh well O.K. 3 cases is 30 bottles

$bc = x_1$

18,4 so 4 bottles can clean what...4 over 2500 so 30 can clean x

$\frac{E}{F} = \frac{x_1}{x_f}$

19,6 cross multiplying....so 7500 divided by

$Fx_1 = z$

,6 4

$Ex_f = 4$

68,6 uh.....so it's 187,5 or 1875 just a minute

$$\frac{z}{4} = x_f$$

9,2 it's 1875....no it isn't

$$\frac{z}{4} = x_f$$

5,0 this is stupid... it's 18750

$$\frac{z}{4} = x_f$$

<u>TOTAL 138,8 secs.</u>

⑥ ⇒ 16,0 O.K. so well 2 over 2 and ¼

$$\frac{a}{r}, \frac{b}{r}$$

20,0 so he increased by 150%...

$$b = 1\tfrac{1}{2}a$$

3,8 no...15% right?

$$b = 1.15a$$

21,2 Just a minute...so let's say... so it comes out to $7 for 2 bags

$$\frac{a}{r}, \frac{c}{r}$$

56,8 so 275 bags gives him......I've got it wrong... $7 for 2 bags so $3.50 for 1

$$\frac{\frac{c}{r}}{\frac{a}{r}} = x_1$$

⇒ 51,6 times 275....so 275 times $3.50 and that gives me. [illegible].50....

$$bx_1 = x_f$$

12,8 no...$9.52....so $9.52 is the answer

$$bx_1 \neq x_f$$

<u>TOTAL 182,2 secs.</u>

⑦ 47,6 Well 30 serbs give.....1,5 yes 15

$$\frac{b}{a} = x_1$$

⟹ 15,4 so 30 serbs make 150 sets of whatever

$$10x_1 = x_2$$

21,8 so 150 divided by 35 gives 4...yea 4...Is it right? ......Is it right?

$$\frac{x_2}{E} = x_f$$

TOTAL 84,8 secs.

8\. 9,0 Well 4 times 7.......

$$\frac{d}{c} = x_1$$

4,4 7 times 7......49

$$ax_1 = x_f$$

5,4 4 into 28 is 7 right?, so 7 times the 7 ergs

$$\frac{d}{c} = x_1, \quad ax_1 = x_f$$

TOTAL 18,8 secs.

⑨ 22,0 See I'm taking all this in Chemistry...I have 28 is equal to 22 and 35 is equal to x

$$\frac{a}{b} = \frac{c}{x_f}.$$

⟹ 11,8 so it's 14 over 11

$$\frac{\frac{a}{r}}{\frac{c}{r}}$$

11,2 equals 35 over x

$$\frac{\frac{a}{r}}{\frac{c}{r}} = \frac{b}{x_f}$$

1,2 so it's 385

$$b\frac{c}{r} = z$$

$\Longrightarrow$ 2,0 divided by 14

$$\frac{a}{r}x_f = 4$$

79,8 just a minute...oh shoot...all of a sudden I can't remember...see this is where my trouble is.. dividing...it's true, I have a real problem dividing...well let's say 27...it's 26,9....27

$$\frac{z}{4} = x_f$$

TOTAL 126,0 secs.

10. 62,4 Well...I'm just...well so far...well anyway... so it's....it should come out to about 17 (reducing)

$$\frac{a}{r}, \frac{E}{r}$$

131,2 or could I do it another way...I usually use a calculator....well look see you know it's a ratio of 40 to 49

$$\frac{a}{r} : \frac{E}{r}$$

13,2 it could be lower...I don't know and uh... I cross multiply 45 times 49....and that gave me 2205

$$\frac{d}{r} \times \frac{E}{r} = z$$

6,4 divided by 18 and that gave me 122

$$\frac{z}{\frac{c}{r}} = x_{f_1}$$

40,6 but now I'm gonna do it the other way...so 45 times 40 so that gives me 1800

$$d\frac{a}{r} = x_2 \; (z)$$

14,6 divided by 18 gives me 100

$$\frac{z}{\frac{E}{r}} = x_{f_2}$$

13,2 so the... I think it's 100 and 122

x_{f_1}, x_{f_2}

TOTAL 281,6 secs.

TONY G.

QUESTION	TIME	STEP
1.	2,6	4 Over 6 equals 9 over x $\frac{a}{b} = \frac{c}{x_f}$
	2,2	...so I go 4x $a x_f = y$
	2,2	equals 54 $bc = z$
	7,6	x equals 54 divided by 4 that's equal to 13... $\frac{z}{y} = x_f$
	6,8	I think...no 14 $\frac{z}{y} = x_f$
	90,0	hmm...something's wrong... I thought it came out evenly...wait a minute....this is bugging me ...oh, no wonder, O.K....I'm getting the fractions wrong...that's why I was wrong....13,5 $\frac{z}{y} = x_f$
TOTAL	111,4 secs.	
2.	6,4	O.K. 8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$
	3,0	10x $c x_f = y$
	8,8	equals 15 times 8...that's uh...120 $ad = z$

3,2 10x equals 120, x equals 12

$$\frac{z}{y} = x_f$$

TOTAL 21,4 secs.

3. 8,8 x over 35 equals 40 over 56

$$\frac{x_f}{b} = \frac{c}{d}$$

21,6 you can break that down to....divide by 2 so 20 over 28

$$\frac{c}{r}, \frac{d}{r}$$

20,2 you can still break it down.....I get x equals 25

$$\frac{c}{r}, \frac{d}{r}, \frac{d}{r}x_f = y, b\frac{c}{r} = z, \frac{z}{y} = x_f$$

TOTAL 50,6 secs.

4. 7,0 That's just ½

$$\frac{a}{r}, \frac{b}{r}$$

5,2 equals x over 8, x is equal to 4

$$\frac{\frac{a}{r}}{\frac{b}{r}} = \frac{x_f}{d}, \frac{d}{\frac{\frac{a}{r}}{\frac{b}{r}}} = x_f$$

TOTAL 12,2 secs.

5. 61,6 10 bottles...2500 square feet...find feet... 2500 square feet...O.K....how many square feet in 3 cases...3 cases is equal to 30 bottles

$$bc = x_1$$

12,2 2500 is to x as 4 is to 30

$$\frac{F}{x_f} = \frac{E}{x_1}$$

3,0 4x

$Ex_f = y$

24,2 equals 2500 times 30.....75000

$Fx_1 = z$

77,4 4x equals 75000, x equals....that doesn't look right.....x equals 18750

$\therefore \frac{z}{y} = x_f$

TOTAL 178,4 secs.

6. 16,4 O.K.....so it's 200 bags is $700 and 275 bags is to x

$\frac{a}{b} = \frac{c}{x_f}$

6,8 200x

$ax_f = y$

17,4 equals to....2x equals to 7......

$\frac{y}{r}, \; bc = \frac{z}{r}$

35,8 so 2x equals 1925

$\frac{y}{r} = \frac{z}{r}$

36,0 x equals to.....so x is equal to $962.50 ✓

$\frac{\frac{z}{r}}{\frac{y}{r}} = x_f$

TOTAL 112,4 secs.

⑦ 86,8 2 serbs equals 7 tods and 35 sets of tods... O.K. so.....all right we get....15 times 7... 2 is to 30, so 15 times 7.....105

$\frac{b}{a} = x_1, \; cx_1 = x_2$

4,2 so 2 is to 7 as 30 is to x

$$\frac{a}{b} = \frac{c}{x_f}$$

⟹ 9,4 so 215 tods

$$bc = x_3$$

65,0 divided by 35 sets....O.K....so 6,1 fots can be made

$$\frac{x_3}{E} = x_f$$

<u>TOTAL 165,4 secs.</u>

8. 11,0 7 ergs is to 4 zots as x is to 28 zots

$$\frac{a}{x_f} = \frac{c}{d}$$

8,2 so 4x

$$c x_f = 4$$

6,0 equals to 175

$$ad = z$$

15,8 x is equal to ...oops...I made a mistake... O.K.

3,4 7 is to 4 as x is to 28

$$\frac{a}{x_f} = \frac{c}{d}$$

2,0 4x

$$c x_f = 4$$

11,2 equals 196

$$ad = z$$

11,0 x equals to.....49

$$\frac{z}{4} = x_f$$

<u>TOTAL 68,6 secs.</u>

9. 10,2 So it's 28 grams is to 22 liters as 35 grams is to x

$$\frac{a}{b} = \frac{c}{x_f}$$

7,4 so the common factor here is 7...so that's four fifths

$$\frac{a}{r}, \frac{b}{r}$$

3,0 4 is to 22 as 5 is to x

$$\frac{\frac{a}{r}}{\frac{b}{r}} = \frac{c}{x_f}$$

6,0 so 4x

$$\frac{a}{r} x_f = y$$

4,4 equals to 22 times 5 which is 110

$$\frac{b}{r} c = z$$

20,2 that's right...x equals to 4 into 110... x equals to 27,5

$$\frac{z}{y} = x_f$$

6,2 so the volume of 35 grams of Nitrogen would be 27,5 liters

$$x_f$$

<u>TOTAL 57,4 secs.</u>

⑩ 13,0 So it would be...add 80 plus 98 which would be 178 grams

$$a + \epsilon = a\epsilon$$

28,8 are needed to make 36 grams....now we don't add them ...take them individually

$$a\epsilon = a, \epsilon$$

16,4 80 grams is to 36 grams as 40...as x is to 45

$$\frac{a}{x_{f_1}} = \frac{c}{d}$$

12,0 so the common factor here....4....

$$\frac{a}{r}, \frac{c}{r}$$

5,4 9x

$$\frac{c}{r}x_{F_1} = y$$

8,6 equals to 900

$$\frac{a}{r}d = z$$

23,6 x equals to 100...100 grams, O.K. it takes 100 grams of NaOH to make 45 grams of H_2O

$$\frac{z}{y} = x_{F_1}$$

14,8 and the next one, 98 grams is to 36 grams as x is to 45

$$\frac{E}{x_{F_2}} = \frac{F}{d}$$

7,4 98 times 45 is.....4410

$$Ed = z$$

5,4 36x

$$Fx_{f_2} = y$$

⇒ 34,0 equals 4410, x equals to.....122,3 grams, so it takes 122,3 grams of H_2SO_4 to make 45 grams of H_2O

$$\frac{z}{y} = x_{f_2}$$

TOTAL 169,4 secs.

KAREN H.

QUESTION	TIME	STEP
①	6,6	You just....cross multiply....O.K. 4 over 6 equals 9 over x $\frac{a}{b} = \frac{c}{x_f}$
	2,0	4x $a \cdot x_f = y$
	1,2	equals 36 $bc = z$
	1,2	x equals 9 $\frac{z}{y} = x_f$
TOTAL	11,0 secs.	
2.	3,8	Uh...then 8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{c}{d}$
	1,8	then 10x $c x_f = y$
	16,4	equals 15 times 8.....10x equals 120 $ad = z$
	1,0	x equals 12 $\frac{z}{y} = x_f$
TOTAL	23,0 secs.	
③	4,2	x over 35 equals 40 over 56 $\frac{x_f}{b} = \frac{c}{d}$

✓

7,0 um....56x

$$dx_f = y$$

7,0 equals to 40 times 35 which is.....1400

$$bc = z$$

⟹ 46,2 Oh...O.K....wait...x is equal to 2,5

$$\frac{z}{y} = x_f$$

TOTAL 64,4 secs.

4. 3,0 12 over 24 equals x over 8

$$\frac{a}{b} = \frac{x_f}{d}$$

9,8 24x

$$bx_f = y$$

1,4 equals 96

$$ad = z$$

5,2 x equals 4

$$\frac{z}{y} = x_f$$

TOTAL 19,4 secs.

5. 25,6 Oh...it's like a proportion problem...well you have 2500 square feet and that gives you 4 bottles and then which would they want to know how many square feet for three cases and there's 10 bottles in a case, so you have 30

$$bc = x_1$$

23,4 4 is to 2500 as 30 is to x

$$\frac{E}{x'} = \frac{F}{x_f}$$

5,2 so then you get 4x

$$Fx_1 = y$$

6,4 equals....75000

$$Ex_f = z$$

60,0 x equals...I lost it...O.K. 18750

$$\frac{y}{z} = x_f$$

TOTAL 120,6 secs.

6. 13,2 200 over 700 equals 275 over x

$$\frac{a}{c} = \frac{b}{x_f}$$

17,4 700x

$$a x_f = y$$

2,4 equals 55000

$$bc = z$$

14,2 so divide each side by 100 to cancel out the zeros to make things easier

$$\frac{y}{r}, \frac{z}{r}$$

65,4 ...OH...I think I've lost it....78,57

$$\frac{z}{y} = x_f$$

TOTAL 112,6 secs.

7. 45,4 Um....O.K. the ratio is like 2 serbs make 7 sets of tods and 35 sets of tods make 1 fot O.K. so 30, wait, so 35...35x

$$\frac{a}{E} = \frac{c}{F}$$

16,0 equals 30

$$Ex_f = b$$

60,2 O.K. it's like... so then...so they'd make 5

$$\frac{E}{C} = x_f$$

43,0 no it's wrong...I got them mixed up...O.K. can I just start that one all over again... O.K. 35 sets of tods in each...O.K. so you'd divide 7 into 35

$$\frac{E}{C} = x_f$$

53,6 ...O.K. like...you'd go 35 into 7 of those...

RETIRES

$$\frac{C}{E} = x_f$$

TOTAL 218,2 secs.

8. 8,2 O.K. it's just a ratio... you say 7 over 4 is equal to x over 28

$$\frac{a}{c} = \frac{x_f}{d}$$

3,2 4x

$$cx_f = y$$

9,2 equals to ...196

$$ad = z$$

23,6 so x is equal to...49

$$\frac{z}{y} = x_f$$

TOTAL 44,2 secs.

9. 42,2 O.K. 1M equals to 28 grams...O.K. so first, like you go...over...like 1 over 28 to find the molarity so like 1 over 28 is equal to x over 35

$$\frac{a}{c} = \frac{x_f}{d}$$

46,4 that's about ...1,2 moles

$$cx_f = y, \quad ad = z, \quad \frac{z}{y} = x_1$$

26,8 so you just multiply that by 22 to find out how many liters it'll come out to....... 26,4 liters

$$Fx_1 = x_f$$

TOTAL 115,4 secs.

⑩ 56,2 O.K. like you do them one at a time so 80 grams over 36 equals x over 45.

$$\frac{a}{c} = \frac{x_{f_1}}{d}$$

5,8 so 36x

$$cx_{f_1} = y$$

9,2 equals to 3600

$$ad = z$$

4,2 so x equals to ...you need 100 grams of NaOH

$$\frac{z}{y} = x_{f_1}$$

20,6 Um...and then to figure out the H_2SO_4, the ratio would be 98 over 36 equals x over 45

$$\frac{E}{F} = \frac{x_{f_2}}{d}$$

21,8 O.K. then...so the same thing, 36x

$$Fx_{f_2} = y$$

35,4 is equal to 98 times 45...is equal to 4410

$$Ed = z$$

⇒ 76,8 so x equals to 4410 divided by 36...so it would be about 122...ah...O.K. 122,3 liters

$$\frac{z}{y} = x_{f_2}$$

TOTAL 230,0 secs.

PETER R.

QUESTION	TIME	STEP
1.	3,8	4 over 6 equals 9 over x $\frac{a}{b} = \frac{c}{x_f}$
	2,0	4x $ax_f = y$
	3,0	equals to 54 $bc = z$
	16,0	x equals 54divided by 4.......13,5 $\frac{z}{y} = x_f$
TOTAL	24,8 secs.	
2.	4,0	8 over x equals 10 over 15 $\frac{a}{x_f} = \frac{d}{d}$
	1,4	10x $cx_f = y$
	6,0	equals 8 times 15.....120 $ad = z$
	2,4	x equals 12 $\frac{z}{y} = x_f$
TOTAL	13,8 secs.	
3.	8,2	x over 35 equals 40 over 56 $\frac{x_f}{b} = \frac{c}{d}$

11,2 56x

$$dx_f = y$$

10,4 equals 40 times 35......1400

$$bc = z$$

63,6 56 divided into 1400 goes.....25, x equals 25

$$\frac{z}{y} = x_f$$

TOTAL 93,4 secs.

4. 7,4 12 over 24 as x is to 8

$$\frac{a}{b} = \frac{x_f}{d}$$

5,0 ½ equals x over 8

$$\frac{a}{r}, \frac{b}{r}$$

2,0 2x equals 8

$$bx_f = y, \quad ad = z$$

1,2 ~~x~~ equals 4

$$\frac{z}{y} = x_f$$

TOTAL 15,6 secs.

5. 29,0 4 bottles.....2500 square feet, 3 cases, that's 30 bottles

$$\frac{a}{b} = \frac{c}{x_1}$$

11,8 criss-cross, 4 over 30 equals 2500 over x

$$\frac{E}{x_1} = \frac{F}{x_f}$$

2,2 that's 4x

$$Ex_f = y$$

6,4 equals 250 times 3.....750

$$Fx_1 = Z$$

⇒ 45,0 750 divided by 4...that's...187,5 square feet

$$\frac{Z}{4} = x_f$$

14,8 that's impossible...slightly impossible...3 cases...that's 30 bottles

$$\frac{a}{b} = \frac{c}{x_1} = x_1$$

40,0 what's coming off?...750 divided by 4...... 187,5 square feet.

$$\frac{Z}{4} = x_f$$

TOTAL 149,2 secs.

6. 40,2 200 bags of grain....\$700....0.K. x...
200 over 275 equals 700 over x

$$\frac{a}{b} = \frac{c}{x_f}$$

7,4 reduce by 5

$$\frac{a}{r}, \frac{b}{r}$$

4,6 40

$$\frac{a}{r} = 4$$

10,0 divided into 700 times 55

$$\frac{b}{r} \times c = Z$$

64,0 so 4 into 3850......\$962.50 for 275 bags...
962.50...that's what it is

$$\frac{Z}{4} = x_f$$

TOTAL 126,2 secs.

7. 29,6 2 serbs.....7 sets...35 sets....egad...70 serbs

$$a \times E = x_1$$

8,6 O.K. 70 serbs gives you 1 fot

$$x_1 = F$$

85,0 how many fots can be made from 30 serbs?...30 serbs gives you how many...Do it again...2 serbs 7 sets...35 sets...gives you 35 gives you 70...

$$x_1 = F$$

8,4 that's 35 sets is 1 fot

$$E = F$$

11,0 30 serbs you could get 3 fots

$$\frac{E}{c} \times a = x_2 , \quad \frac{b}{x_2} = x_f$$

TOTAL 142,6 secs.

8. 10,0 7 ergs make 4 zots...how many ergs make 28 zots

$$\frac{a}{x_f} = \frac{c}{d}$$

4,6 4x

$$cx_f = 4$$

10,8 equals 28 times 7.....196

$$ad = z$$

10,0 196 divided by 4........49

$$\frac{z}{4} = x_f$$

TOTAL 45,4 secs.

9. 42,0 .....O.K. what will be the volume of 35 grams? 28 grams equals 22, 35 grams equals what

$$\frac{a}{b} = \frac{c}{x_f}$$

4,2 so 28x

$ax_f = y$

13,4 equals....770

$bc = z$

9,4 28 divided by 770...no 770 divided by 28

$\frac{z}{y} = x_f$

102,6 that gives you....27,5...27,5 liters..you, that should be it

——— x_f

<u>TOTAL 171,6 secs.</u>

10\. 27,6 80 over 36 equals x over 45

$\frac{a}{c} = \frac{x_1}{d}$

7,4 36x

$cx_1 = y$

17,8 equals 80 times 45......3600

$ad = z$

4,8 3600 divided by 36 ...that goes 100grams

$\frac{z}{y} = x_1 \quad (x_{fr})$

11,4 Now NaOH...you would 100g of NaOH

(x_{f_1})

13,2 Now H_2SO_4 you would need..O.K. ...98 grams over 36 grams equals x over 45

$\frac{E}{F} = \frac{x_2}{d}$

4,0 36x

$Fx_2 = y$

27,4 equals 98 times 45......4410

$d_E = z$

79,0 4410 over 36........122,5 of H_2SO_4 and I'm finished.

$\frac{z}{4} = x_2 \; (x_{F2})$

TOTAL 192,6 secs.

SUE W.

QUESTION	TIME	STEP
①	4,4	O.K. I would cross-multiply, $\frac{a}{b} = \frac{c}{x_f}$
	6,2	So 6 times 9 would give me 54 $bc = y$
	6,0	and I divide the 4 into the 54 $\frac{y}{z}$
	20,6	and then my answer would be.....13,25 x_f
TOTAL	37,2 secs.	
2.	7,0	It's the same type of thing, I'd cross-multiply again $\frac{a}{x_f} = \frac{b}{d}$
	2,4	the 8 times the 15 $ad = y$
	3,0	and get 120 y
	1,6	and divide 10 into that $\frac{y}{z}$
	2,0	and so my answer would be 12 x_f
TOTAL	16,0 secs.	
③	3,2	Again we cross-multiply..... $\frac{x_f}{b} = \frac{c}{d}$

⟹ 8,0 cross-multiplying the 35 times the 40

$$bc = y, \quad \frac{y}{dx_f}$$

52,8 and then my answer would be 23

$$x_f$$

TOTAL 64,0 secs.

4. 9,6 O.K. it's 12 over 24

$$\frac{a}{b}$$

1,4 but instead of cross-multiplying,

2,2 I'd reduce the fraction first

$$\frac{a}{r}, \quad \frac{b}{r}$$

3,8 So my answer would be...uh...4

$$x_f$$

TOTAL 17,0 secs.

5. 42,6 O.K. well since you have 4 bottles of fluid to clean 2500 square feet of flooring, and how many...

$$E : F$$

11,0 O.K. ......so you want to know how many square feet of flooring can be cleaned with 3 cases....

2,2 you multiply the 3 cases times 10 bottles

$$\frac{a}{F} = \frac{b}{x_1} \quad \text{or} \quad b \times c = x_1$$

2,4 so I get 30 bottles

$$x_1$$

36,0 and that is again cross-multiplication because you want to get the number, so my answer is.... 2500 times 30, that will give you.....75000

$$\frac{c}{x_1} = \frac{d}{x_f} , \quad cx_f = y$$

1,6 and then you divide 4

$$dx_1 = z$$

,8 into 75000

$$\frac{z}{y}$$

28,6 so your answer will be 18750 square feet you can clean with 3 cases

$$x_f$$

TOTAL 125,2 secs.

6\. 24,4 O.K. well since the price remains the same, uh.... then it's again a cross multiplication

$$\frac{a}{x_f} = \frac{c}{d}$$

9,0 well first of all I can reduce the 200 over 700 to 2 over 7

$$\frac{a}{r} , \frac{c}{r}$$

4,2 so then I multiply 275 times 7

$$ad = y$$

17,2 and I get 1925

$$y$$

18,2 and I divide 275 into that....now... sorry, sorry, I divide 2 into that

$$\frac{y}{z}$$

24,0 so my answer is 962.5

$$x_f$$

TOTAL 97,0 secs.

QUESTION	TIME	STEP
7.	63,6	O.K. 2 serbs make 7 tods and 35 tods make 1 fot, then....O.K. if 30 serbs, that'll mean you can make 15 times 7 sets of tods.... $\frac{b}{a} \times c = x_1$
	22,0	2 is to 30 as 7 is to 105 $\frac{a}{b} = \frac{c}{x_1} = x_1$
	26,8	O.K. if you need 35 sets of tods to make 1 fot, then you divide your 105 by 35 sets to make 1 $\frac{x_1}{E} = x_f$
	8,0	so your answer is 3 fots can be made from 30 serbs x_f
TOTAL	120,4 secs.	
8.	34,0	O.K. if you need 7 ergs to make 4 zots, it's again cross-multiplication $\frac{a}{x_f} = \frac{c}{d}$
	11,4	to find out how many ergs are needed to make 28 zots, so you cross multiply and you get 196 $ad = y$
	4,0	and you divide 4 into 196 $\frac{y}{z}$
	11,4	so your answer is you need...49 ergs to make up 28 zots. x_f
TOTAL	60,8 secs.	
9.	30,6	1 mole of N_2 weighs 28 grams and if 1 mole is $a = (c)$

30,8 also equal to 22 liters

$$a = c$$

10,0 O.K. that means you have to put the grams over the liters on each side to figure out what the volume of 35 g of N_2 will be so

$$\frac{a}{b_1} = \frac{c}{x_F}$$

12,6 I reduce it first of all.....the 28 over 22, 14 over 11

$$\frac{a}{r} , \frac{b}{r}$$

4,2 and I multiply 35 times 11 and we get....385

$$bc = y$$

2,4 and divide 14 into 385

$$\frac{y}{z}$$

53,0 so then..... so the answer is 27,5 liters, the volume of 35 grams of N_2 is 27,5 liters

$$x_F$$

TOTAL 143,6 secs.

10. 70,8 O.K. 80 grams of NaOH and 98 grams of H_2SO_4 are needed to make 36 grams of H_2O, and to make 45 grams of H_2O..... well this is a stupid way of doing it.....I'm going to add up the grams of NaOH plus the grams of H_2SO_4 and I get 178

$$a + E = aE$$

6,2 over the grams of how much water you get, that's 36

$$\frac{aE}{c}$$

15,0 so then x over 45, O.K. so I'm going to cross multiply

$$\frac{aE}{c} = \frac{x_1}{D}$$

34,2 and then multiply 178 by 45, and I get 8010

$$Cx_1 = y, \quad D \times AE = Z$$

81,8 so then I divide the 36 into the 8010, and I will get my answer, well then, separately, O.K. so then I divide 36 into the 8010, so my answer would be 222,5

$$\frac{Z}{y} = x_1$$

94,0 then I want to find out the separate,...well... one way to work it out would be to work a percentage, ...O.K. so I'm going to put 80 grams over 178 grams which would equal x over 222,5

$$\frac{a}{aE} = \frac{x_2}{x_1}$$

14,0 I multiply 222,5 times 80

$$ax_1 = y$$

25,0 then I divide 178 into that number

$$aE \times x_2 = Z, \quad \frac{y}{Z}$$

28,0 so x....so one of the unknowns would be 100grams

$$\frac{y}{Z} = x_2, (x_{f_1})$$

12,2 and the other would be 222,5 minus 100 which equals 122,5

$$x_1 - x_{f_1} = x_{f_2}$$

10,0 so you'll need 122,5 grams of H_2SO_4 and 100 grams of NaOH.

$$x_{f_1}, x_{f_2}$$

TOTAL 391,2 secs.

DIANE . Y.

QUESTION	TIME	STEP
1.	3,4	6 times 9 is 54 $bc = z$
	6,2	divided by 4 is..... $ax_f = y$
	9,0	13,5 $\frac{z}{y} = x_f$
TOTAL	18,6 secs.	
2.	6,2	15 times 8 is 120 $ad = z$
	3,0	divided by 10 is $cx_f = y$
	1,0	12 $\frac{z}{y} = x_f$
TOTAL	10,2 secs.	
3.	7,8	40 times 35 is.....1400 $bc = z$
	2,0	divided by 56..... $dx_f = y$
	12,2	25 $\frac{z}{y} = x_f$
TOTAL	22,0 secs.	

QUESTION	TIME	STEP
4.	2,2	12 times 8 is 96 $ad = z$
	2,4	divided by 24..... is $bx_f = y$
	3,2	4 $\frac{z}{y} = x_f$
TOTAL	7,8 secs.	
⑤	23,8	2500 is to 4 as x is to 30 $\frac{E}{x_1} = \frac{F}{x_f}$
	3,0	4x $Ex_f = y$
	4,0	is equal to 75000 $Fx_1 = z$
⇒	21,2	x is equal to....umm.....18725 $\frac{z}{y} = x_f$
TOTAL	52,0 secs.	
⑥	22,4	200 bags is to $700. as 275 is to x. $\frac{a}{b} = \frac{c}{x_f}$
	3,6	200~~x~~ $ax_f = y$
⇒	21,8	equals 275 times 700 which is equal to 193500 $bc = z$

10,0 x is equal to 193500 divided by 200...which is equal to 1935 divided by 2

$$\frac{z}{r}, \frac{y}{r}, \frac{z}{4}$$

8,6 ....which is equal to $96.75

$$\frac{z}{4} = x_f$$

TOTAL 66,4 secs.

7. 46,0 2 serbs can make 7...35 sets of tods...35 sets can make 1, so we have to get 30 serbs...we have 7 times 30 is 210

$$bc = x_1$$

6,8 that's 105 sets of tods

$$\frac{x_1}{a} = x_2$$

16,8 and 105 divided by 35 is.....3

$$\frac{x_2}{E} = x_f$$

TOTAL 69,6 secs.

8. 6,0 7 is to 4 as x is to 28

$$\frac{a}{x_f} = \frac{c}{d}$$

1,2 4x

$$c x_f = 4$$

6,4 is equal to 7 times 28.....196

$$a,d = z$$

12,6 196 over 4 is equal to.....49

$$\frac{z}{4} = x_f$$

TOTAL 26,2 secs.

9. 10,2 So that's 1M of Nitrogen is equal to 28 grams and 22 liters and 35 grams......

$$a = c = F$$

23,4 28 into 35 is.....1,2

$$\frac{d}{c} = x_1$$

9,8 1 is to 22 as 1,2 is equal to x

$$\frac{E}{x_1} = \frac{F}{x_F}$$

1,0 x

$$E x_F = y$$

10,8 is equal to 1,2 times 22....which is 26,4

$$F x_1 = z, \quad \frac{z}{y} = x_F$$

TOTAL 55,2 secs.

10. 25,2 80 and 98 equal 36.....do they mean add it together?....so 98 plus 80 equals 178 is equal to 36

$$A+E = AE$$

4,8 178 is equal to 36, x equals 45

$$\frac{aE}{x_{F_{1+2}}} = \frac{c}{d}$$

1,0 36x

$$c x_{F_{1+2}} = y$$

15,4 is equal to 178 times 45.....8010

$$d\,aE = z$$

23,6 x is equal to 8010 divided by 36.....222

$$\frac{z}{y} = x_{F_{1+2}} (x_1) \rightarrow$$

30,0 80 is to 178 as x is equal to 222

$$\frac{a}{x_2} = \frac{aE}{x_1}$$

6,0 178x

$$aEx_2 = y$$

8,4 is equal to 222 times 80, ...178x equals 17760

$$ax_1 = z$$

20,6 x equals......99

$$\frac{z}{y} = x_{f1}$$

12,0 H_2SO_4......98 is equal to 178, x equals 222

$$\frac{E}{x_{f2}} = \frac{aE}{x_1}$$

1,2 178x

$$aEx_{f2} = y$$

14,8 equals 222 times 98.......21776

$$Ex_1 = z$$

21,8 x equals 178 into 21,776.....x goes into there
122.....that's it?

$$\frac{z}{y} = x_{f2}$$

TOTAL 184,8 secs