Insider Trading, Asymmetric Information, and Market Liquidity: Three Essays on Market Microstructure

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Abstract

This thesis comprises three essays on market microstructure, focusing on the issues of insider trading, asymmetric information and market liquidity. The first essay examines the effects of the mandatory disclosure regulations on the trading behavior of informed traders. Specifically, we compare the (perfect Bayesian) equilibrium when disclosure is mandatory to the equilibrium when insiders do not have to disclose their trades. We show that under mandatory disclosure the market becomes more efficient and more liquid, making the uninformed traders unambiguously better off. We also show that in order to conceal part of his information, under mandatory disclosure the insider may trade against his information, and, at the same time, add a random - "noise" - component to his trade order. As a result, insiders may end up buying (selling) when his information indicates the asset is overvalued (undervalued). This provides a rationale for contrarian trading.

The second essay examines trading behavior, price behavior and the informational efficiency and the informativeness of the price process in the equilibrium of a strategic trading game when some investors receive information before others. We show that the early informed investor may trade against his information to maintain his information superiority over the market. Under some conditions, subsequent price changes are positively correlated. We also find that the price process is less efficient and less informative than would be the case where there is no late-informed trader.

The third essay analyzes the intra-day behavior of market liquidity of the Toronto Stock Exchange which uses a computerized limit-order trading system. Along with previous studies, we show that the U-shaped intra-day pattern of spread does not depend on the market architecture. In addition, we confirm that bid-ask spread and market depth are two dimensions of market liquidity. Liquidity providers use both dimensions to deal with adverse selection problems. We also examine how price volatility and trading volume affect market liquidity. Price volatility is inversely related to market liquidity but trading volume is directly related to liquidity. High trading volume implies high liquidity trades and as a result, liquidity providers decrease (increase) ask (bid) price and/or increase depth at each quote.

Résumé

Cette thèse comprend trois essais traitant de la microstructure du marché : ils sont reliés aux problèmes de délit d'initié, d'information asymétrique et de liquidité du marché. Le premier essai examine les effets de la réglementation de la divulgation obligatoire de l'information sur les négociateurs informés. Plus précisément, nous comparons l'équilibre (parfait Bayésien) quand la divulgation est requise pour l'équilibre, quand des initiés ne sont pas obligés de divulguer leurs négociations. Nous montrons que dans le cadre d'une divulgation obligatoire, le marché devient plus efficient et plus liquide, poussant les négociateurs non informés à rester en dehors du marché et ce de façon non ambigue. Nous montrons aussi que, alors que la divulgation de l'information est obligatoire, de manière à cacher une partie de l'information, l'initié peut transiger *contre* son information et, en même temps peut ajouter un composant de « bruit » aléatoire à son ordre de négociation. Le résultat est que les initiés peuvent arrêter d'acheter (de vendre) quand leur information indique que l'actif est sur-évalué (sous-évalué). Ceci founit une preuve rationelle de la négociation contraire (« contrarian trading »).

Le second essai examine le comportement de négociation, de prix, l'efficacité informationnelle et le degré d'information attaché au processus de prix dans un équilibre de jeu de négociation stratégique quand des investisseurs recoivent l'information avant les autres. Nous montrons que l'investisseur le plus tôt informé peut négocier contre cette information pour maintenir sa supériorité de l'information par rapport au marché. Sous certaines conditions, des variations de prix postérieures sont positivement corrélées. Nous

trouvons aussi que le processus de prix est moins efficace et moins informatif que dans le cas où il n'y a pas de négociateur informé en retard.

Le troisième essai analyse, à l'intérieur d'une journée, le comportement de la liquidité du marché de la Bourse de Toronto (TSE) qui utilise un système de négociation d'ordres limites informatique. Comme dans les études précédentes, nous montrons que le schéma en U des écarts à l'intérieur d'une journée ne dépend pas de l'architecture du marché. De plus, nous confirmons que les écarts cours acheteur-cours vendeur et la profondeur du marché sont deux dimensions de la liquidité du marché. Les fournisseurs de liquidité utilisent les deux dimensions pour parer aux problèmes de sélection adverse. Nous examinons aussi comment la volatilité du prix et le volume de négociation affectent la liquidité du marché. La volatilité du prix est relié de façon inverse à la liquidité du marché mais le volume de négociation est directement relié à la liquidité. Un grand volume de négociation implique une grande liquidité des négociations; il en résulte une diminution (augmentation) du cours vendeur (du cours acheteur) par les fournisseurs de liquidité et/ou une augmentation de la profondeur du marché à chaque cotation.

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Contents

Chapter 1

Introduction

How is a price set in a market? In the standard economic paradigm, it is the intersection between supply and demand curves that determines the price. Indeed, this must be the case in equilibrium. However, how does the economy reach the equilibrium? In particular, what exactly coordinates supply and demand in the economy so that a price emerges and trades take place? Unfortunately, much of economics abstracts from this issue. For many economists, what matters is the properties of equilibrium price which is determined by solving for a market clearing price. The process by which the market reaches the equilibrium is not of interest. This approach implicitly assumes that the trading mechanism has no impact on the resulting equilibrium. Whatever the trading protocol is used, the same equilibrium would emerge. However, this may not be the case in financial markets where traders have differential information. Several economists raise this concern. Specifically, Demsetz (1968) who studies of the relationship between the spread and the volume of trades on the New York Stock Exchange, suggests that the market structure could affect the market behavior, including trading prices. Thus, if the trading mechanism is a determinant of equilibrium, then how such mechanism operates cannot be ignored. This sets the rationale for the formal study of market microstructure.

Market microstructure theory studies the process and outcomes of trading financial assets under specific trading protocols. It tries to characterize how trading mechanisms affect the price formation process, how insider trading affects prices, why prices exhibit certain time series properties, etc. Market microstructure has immediate applications in regulating markets and in designing new trading mechanisms. This thesis contains three essays on market microstructure which address the issues of insider trading, asymmetric information and market liquidity in financial markets.

To prevent the abuse of insider information, financial market regulations require corporate insiders to disclose their trades after the trades have been completed. Several economists raise the concern that such regulations could lead to market manipulation. Fishman and Hagerty (1995) show that the mandatory disclosure could create profitable trading opportunity for insiders even if they are uninformed about the asset value. This happens because the market cannot distinguish between informed and uninformed trades. John and Narayanan (1997) consider a model in which the insider is informed for sure and the value of the asset has two-state distribution. They show that if the probabilities of two states are different and the information the insider receives differs from what the market expects, then the insider may manipulate the market by trading against his information. Huddart, Hughes, and Levine (2001) examine an information monopolist model and propose an equilibrium in which the insider may play a mixed strategy to disguise this information. The result is that the insider can retain his information superiority over the market. Chapter 2 of this thesis introduces competition into the framework of Huddart, Hughes, and Levine (2001). We find that when there is competition between an informed insider and an informed outsider and under information asymmetry, not only does the insider add a random noise component into his order but he may also trade against his private information to prevent other market participants from learning his own information. As a result, less insider information is revealed than would be the case under no disclosure.

Since the insightful paper by Bagehot (1971), economists have begun to develop information-based models in market microstructure research. Those models use insights from the theory of adverse selection to explain the trading and price behavior in financial markets. Bagehot's starting point is the distinction between market gains and trading gains. The former refers to the comovement between market prices and the average investor's gain. When asset prices increase in general, most investors gain; when they fall, most investors lose. The latter, however, suggests that information asymmetry which arises because of informed traders, will make the average investor lose relative to the market return over time. Those informed traders trade to exploit their super information. The market maker knows that he would lose trading with informed traders; therefore, to offset these losses, he has to make gains from uninformed traders. Thus, bid-ask spreads arise. Hirshleifer, Subrahmanyam, and Titman (1994) examine a competitive equilibrium of a model in which some investors receive common private information before others. They make use of risk-aversion to generate speculative trading. The implication of the model is that under some conditions, investors focus on some assets while neglecting others even though those assets have identical exogenous characteristics. This provides important insights into herding effects in financial markets. In chapter 3 we analyze the strategic trading behavior when some investors receive information before others. We find that under some conditions, the early-informed investor manipulates the market by trading against his information first to move the price. Then, he exploits his information later to make money. We also show that under some conditions, price moves are positively correlated, and the price process is less efficient and less informative than would be the case in which there is no late-informed trader.

On the empirical side, interest in limit order trading has grown rapidly in recent years because it plays a very important role in providing liquidity to markets. Ann, Bae, and Chan (2001) investigate the role of limit orders in liquidity provision in the Stock Exchange of Hong Kong which is a pure order-driven market. They show that a rise in volatility is followed by a rise in depth, and an increase in depth is followed by a fall in volatility. Moreover, limit order traders submit more limit buy (sell) orders than market buy (sell) orders when there is a paucity of limit buy (sell) orders. This keeps the system in balance. Lee, Mucklow, and Ready (1993) examine the intraday patterns of spreads and depths and the impact of earning announcement on the New York Stock Exchange. They find that the specialists vary both spreads and depths in response to the changes in volume of trade. Lee, Mucklow, and Ready suggest that the specialists use trading volumes to infer the presence of informed traders and adjust both spread and depth to cope with the adverse selection problem. They also find that spreads widen and depths drop in anticipation of earning announcements. Overall, they interpret that liquidity providers are sensitive to information asymmetry risk and use both spreads and depths to manage it. Chapter 4 of this thesis examines the impacts of price volatility, trading activity, and trading volume on the liquidity of the Toronto Stock Exchange (TSE) which uses a computerized limit order trading system. Unlike Lee, Mucklow, and Ready (1993), we show that trading volumes do not reflect informed trades and therefore, liquidity providers do not use volumes to infer the presence of informed traders. Specifically, we find that volume is positively correlated to liquidity. This finding is inconsistent with the prediction of Easley and O'Hara (1992); however, it is consistent with the alternative interpretation, suggested by Harris and Raviv (1993), that because of the differences of opinion among investors, high volume may reflect mainly high liquidity trades and therefore the market is more liquid. In addition, we show that market liquidity is inversely related to price volatility. When there is a paucity of limit orders so that the price volatility increases, it is more beneficial for investors to provide liquidity to the market by placing limit orders. Those orders will help reduce the volatility. Furthermore, extending the result of Harris (1987) we show that the trading activity which is represented by the number of transactions is negatively related to market liquidity.

The thesis proceeds as follows. Chapter 2 examines the strategic insider trading under public disclosure regulations. Chapter 3 analyzes the strategic trading when some investors receive information before others. Chapter 4 investigates empirically the intraday behavior of market liquidity on the TSE.

Chapter 2

Strategic Insider Trading under Public Disclosure Regulations

2.1 Introduction

Corporate insiders routinely trade in the stock of the firm with which they are affiliated. From the empirical point of view, trading by corporate insiders appears to have become more profitable over time[1]. In order to diminish this unfair advantage, the US Congress enacted a law requiring insiders[2] associated with a firm to report any equity transactions they make in the stock of that firm to the Securities and

[1]Seyhun (1986) finds that insiders tend to buy before an abnormal rise in stock prices and to sell before an abnormal decline. Seyhun (1992a) presents compelling evidence that insiders earned over 5 percent abnormal returns on average. Seyhun (1992b) determines that insider trades predict up to 60 percent of the variation in returns.

[2]Insiders are defined as officers, directors, and beneficial owners of more than 10% of any classs of equity securities.

Exchange Commission (SEC)[3]. The reports are filed after the trade is completed, at which time they become publicly available. Congress's intent in enacting this law is reflected in the following statement: "The most potent weapon against abuse of inside information is full and prompt publicity."[4]

This chapter examines the effects of the mandatory disclosure regulations on the strategic trading of informed traders in financial markets. More specifically, we compare the (perfect Bayesian) equilibrium where disclosure is mandatory to the equilibrium where insiders do not have to disclose their trades. We show that the goal of the Congress is, indeed, achieved: compared to the benchmark case - where the insider is not required to disclose his trade, the market becomes more liquid. Because of that, uninformed traders unambiguously benefit from lower marginal trading costs. Moreover, more information about the stock is revealed to the public and therefore prices become more efficient.

We also find that in order to conceal part of his information, under mandatory disclosure the insider deliberately adds a random - "noise" - component to his trade order. This noise may be a buy or a sell and is independent of his insider information. As a result of this noise, the insider may end up buying (selling) when his information indicates that the asset is overvalued (undervalued). Following Huddart, Hughes and Levine (2001) we call it dissimulation. In contrast, in the benchmark case the insider's actions do not involve any random element (and he would never trade in a manner

[3]See Appendix A

[4]1934 House Report, p.13.

which is inconsistent with his private information).

In addition, we show, by a numerical example, that the insider might trade against his insider information at first to retain his super information and then reverse it in the last trading round. This behavior might be particularly important in markets with a few pivotal traders. It diminishes further the learning ability of other market participants.

Several recent papers discuss issues related to this research. Fishman and Hagerty (1995) examine a model in which an uninformed insider imitates an informed insider with good news, buys shares in order to move the price and then sells shares after the trade is publicly disclosed. The basic idea is that the uninformed insider exploits the inability of the market maker to distinguish uninformed trades from those of privately informed insiders.

John and Narayanan (1997) show that the mandatory disclosure regulations may create incentives for informed insiders to manipulate the market by trading against their information. They examine the case in which the insider is informed for sure and there is an asymmetry in the probability of good and bad news that the insider receives. A higher probability of good (bad) news may lead to a contrarian trading in which an insider with bad (good) news trades as if he had good (bad) news, and then unwinds his position in the following period to make money. However, the insider will never manipulate when (1) good news is less likely than bad news and the insider receives good news, or (2) bad news is less likely than good news and the insider receives bad news, or (3) good news and bad news are equally likely.

Huddart, Hughes and Levine (2001) introduce the mandatory disclosure rule to Kyle's (1985) framework and find that the insider would play a mixed strategy to preserve his information advantage for the future. They show that liquidity traders unambiguously benefit from lower expected trading cost by comparison to Kyle's results. Moreover, information is reflected more rapidly in price with disclosure of insider trades than without. We share with Huddart, Hughes and Levine those conclusions. In addition, we show that the insider may trade against his insider information to partially conceal this information. Furthermore, we show that less insider information is revealed to the public; however, the competition between informed traders makes more common information impounded into the price. Overall, more information is reflected in the price.

The remainder of the chapter is organized as follows. Section 2.2 presents the model. Section 2.3 provides a numerical example to illustrate the implications of the model. Section 2.4 characterizes the equilibrium in both cases: disclosure and non-disclosure. Section 2.5 gives some concluding remarks.

2.2 The Model

There are two assets in the economy: a risky stock and a riskless bond. Market participants include an informed insider trader, an informed outsider trader, a market maker, and a number of liquidity traders. These traders buy or sell the stock in two periods. The informed traders are assumed to be risk neutral.

We normalize the interest rate of the bond to zero. The liquidation value of the stock (i.e., its value in period 3) is a random variable[5] $\tilde{v}$. We assume that $\tilde{v} = \tilde{a} + \tilde{b}$, where the two random variables[6] are independently normally distributed with mean zero and variance σ^2. Thus, the liquidation value of the stock $\tilde{v}$ is normally distributed with mean zero and variance $2\sigma^2$.

Before the first trading takes place, both informed traders receive a signal, s_0, related to the liquidation value. This signal is the realization of $\tilde{b}$. Thus, $s_0 = b$. The insider, trader 1 receives, in addition, the value, v, of the realization of $\tilde{v}$. There is no new information in the second period. In period 3, the liquidation value of the stock is announced and holders of stock are paid accordingly. This information structure is common knowledge.

Liquidity traders buy or sell shares for reasons exogenous to the model. The quantity traded by liquidity traders in period t ($t = 1, 2$), denoted by $\tilde{u}_t$, is normally distributed with mean zero and variance σ_u^2, and is independent of all other random variables.

Denote the quantities traded in period t by the insider - trader 1, the outsider - trader 2 and liquidity traders by x_t, y_t, and u_t respectively. The aggregate order flow

[5]In this thesis, a tilde is used to distinguish a random variable from its realization.

[6]We assume that $\tilde{v}$ depends on two events A and B which could be, for example, a major contract with a new client, earning announcements, legal allegations, a new discovery, etc.

in period t is

$$w_t = x_t + y_t + u_t.$$

The market maker observes the aggregate order flow but does not know which orders come from which traders. We follow the tradition in the literature whereby the market maker is assumed to set informationally efficient prices; thus, his expected profit is zero. That is, the market maker sets the price equal to the expected value of the asset, conditioned on the history of orders received up to that time and trades the quantity necessary to clear the market at this price. Specifically,

$$p_1 = E\left(\widetilde{v}|\, w_1\right) \text{ and } p_2 = E\left(\widetilde{v}\ |w_1, w_2\right).$$

We are interested in deriving the resulting demands, x_t, y_t $(t = 1, 2)$, and the prices p_1 and p_2. To this end, we represent the above economy as an extensive form game with imperfect information, and employ the notion of Perfect Bayesian Equilibrium (PBE)[7]. We study this equilibrium notion because it captures the fact that the informed insider and outsider players are rational and forward-looking. That is, each informed trader takes into account that his demand will be used by the other traders to update their beliefs concerning the liquidation value of the stock.

More specifically, we will be interested in exploring the linear equilibria in this game because in addition to its appeal and tractability, given our normality assumption of all random variables, prices are linear functions of the history of aggregate

[7]See section 2.4 for precise definitions and derivation.

order flows[8]. As p_1 is a linear function of w_1, and p_2 is a linear function of w_1 and w_2, we have that prices fully reflect the aggregate demands. Thus, traders can infer the values of w_1 and w_2 from p_1 and p_2.

The information structure of the insider, trader 1, is not affected by the need to disclose his trade, x_1, at the end of period 1. (Of course, the equilibrium value of x_1 will depend on whether or not he has to disclose x_1.) By converting the price p_1, trader 1 can learn w_1. Since he also knows trader 2's information, s_0, he can computes trader 2's order y_2. Thus, the insider trader knows, at the end of period 1, the values of x_1, y_1, and u_1.

Trader 2's information structure does depend on whether or not the disclosure is enforced. In the case of no disclosure, trader 2 learns from the price p_1 the aggregate order flow w_1 and therefore $x_1 + u_1$ (since he knows his own order) and uses it to infer more information about the asset. His updated evaluation of $\tilde{v}$ after the first trading round is

$$s_1 = E\left(\widetilde{v} |\ s_0, x_1 + u_1\right).$$

We now turn to the case of disclosure, where trader 1's order in period 1 is publicly disclosed right after the trading is completed in period 1. To distinguish between the two cases, we use the superscript d to denote variables in the disclosure case. The outsider, trader 2, learns from the price p_1^d the aggregate order flow w_1^d and from the disclosure by trader 1, trader 2 also learns the value of x_1^d. Thus, (since he knows his

[8]This is a consequence of the Projection Theorem.

own order, y_1^d) he can also derive the value of u_1^d. His updated evaluation of $\tilde{v}$ after the first trading round is

$$s_1^d = E\left(\tilde{v}\,|s_0, x_1^d\right).$$

To sum up, we have the information structure of the non-disclosure case in Table 2.1 and the information structure of the disclosure case in Table 2.2.

Table 2.1
Information Structure of the Non-Disclosure Case

Player	Period 1	Period 2	Period 3
Trader 1	v, s_0	$v, s_0, u_1, s_1, x_1, y_1, u_1, p_1$	v
Trader 2	s_0	$s_0;\ s_1, y_1, x_1 + u_1,\ p_1$	v
Market maker	w_1	$w_2,\ w_1$	v

Table 2.2
Information Structure of the Disclosure Case

Player	Period 1	Period 2	Period 3
Trader 1	v, s_0	$v, s_0, u_1, s_1^d, x_1^d, y_1^d, p_1^d$	v
Trader 2	s_0	$s_0, u_1;\ s_1^d, y_1^d, x_1^d,\ p_1^d$	v
Market maker	w_1^d	$x_1^d, w_2^d,\ w_1^d$	v

2.3 An Example

This section presents a numerical example to illustrate some of the implications of the model. In particular, we are interested in how the mandatory disclosure regulations change the behavior of informed traders, how they affect the competition among informed traders and how the information is revealed to the public through time.

There are two parameters that fully describe the economic environment, namely, the variance σ^2 of the random variables[9] $\widetilde{a}$ and $\widetilde{b}$, and the variance σ_u^2 of the liquidity trades. We choose these values to be:

$$\sigma^2 = 1 \text{ and } \sigma_u^2 = 2.$$

If trader 1 (the insider) is not required to disclose his order after the fact, then the unique linear equilibrium is given by

$$x_1(v, s_0) = 0.62(v - s_0) + 0.63s_0, \tag{2.1}$$

$$x_2(v, s_1, p_1) = 1.35(v - s_1) + 0.9(s_1 - p_1), \tag{2.2}$$

$$y_1(s_0) = 0.58s_0, \tag{2.3}$$

$$y_2(s_1, p_1) = 0.9(s_1 - p_1), \tag{2.4}$$

$$p_1 = 0.48w_1 = 0.48(x_1 + y_1 + u_1),$$

$$p_2 = p_1 + 0.37w_2 = p_1 + 0.37(x_2 + y_2 + u_2).$$

We observe that trader 1's demands x_1 and x_2 are functions of both his insider information (the first terms in equations (2.1) and (2.2)) and the common information (the second terms in equations (2.1) and (2.2)) which is the difference between the common signal and the current price[10]. In the first period, he even trades more intensely on the common information; however, this is reversed in the second period.

[9]Recall that the liquidation value of $\widetilde{v}$ is a random variable with mean zero and variance $2\sigma^2$.
[10]The price is equal to zero before the first trading round.

Trader 2 (the outsider), on the other hand, trades on the common information only, and he trades more intensely on this information in period 2 than in period 1. The market maker learns more and more information as trades go on; as a result, he sets the prices such that the marginal trading cost[11] is decreasing. Specifically, in the first period, the marginal trading cost is 0.48, and in the second period, it is 0.37.

The unique linear equilibrium when trader 1 is required to disclose his order is given by

$$x_1^d(v, s_0) = -0.02(v - s_0) + 0.67 s_0 + z, \text{ where } z \sim N(0, 0.025), \tag{2.5}$$

$$x_2^d(v, s_1, p_1^d) = 1.47(v - s_1^d) + 0.98(s_1 - p_1^d),$$

$$y_1^d(s_0) = 1.05 s_0,$$

$$y_2^d(s_1, p_1) = 0.98(s_1 - p_1),$$

$$p_1^d = 0.34 w_1^d = 0.34(x_1^d + y_1^d + u_1).$$

$$p_2^d = p_1^* + 0.34 w_2^d = p_1^* + 0.34(x_2^d + y_2^d + u_2),$$

where p_1^* is the updated price in period 1 after the market maker sees the disclosure of trader 1 and is given by

$$p_1^* = 1.33 x_1^d + 0.04(y_1^d + u_1).$$

[11]Marginal trading cost in period i is defined as

$$MC_i = \frac{dp_i}{dw_i}.$$

We can observe that when disclosure is mandatory, trader 1 does not want his demand in the first period, x_1^d, to fully reveal his signal about the liquidation value, v. He obtains this goal by introducing noise - a random variable z into his first period demand (equation (2.5)). Because of it, other traders cannot infer from x_1^d the precise value of v. Following Huddart, Hughes and Levine (2001), we call it dissimulation. Dissimulation provides a novel rationale for contrarian trading. However, dissimulation is costly since sometimes it causes the insider to trade in a manner inconsistent with his private information.

Perhaps more surprising is the fact that, in this example the insider, when disclosure is mandatory, trades against his private information $(v - s_0)$ in the first trading round. This, coupled with dissimulation, helps trader 1 keep his insider information from being fully released to other market participants.

The behavior of trader 2 also changes tremendously. In period 2, while he trades more intensely on the common information in the non-disclosure case, he trades less intensely on that information in the disclosure case. Perhaps, this change in behavior is because in the disclosure case the competition between informed traders is more intense.

To investigate how information is revealed to the public via trading, we define the following variable to measure the remaining information at the end of period t. We use variance to measure the amount of information available to traders. In the non-disclosure case, we define

$$\Sigma_t = var(\widetilde{v}|\ w_1, ..., w_t) = var(\widetilde{v}\ - p_t).$$

$\Sigma_t(t = 0, 1, 2)$ is the variance of the liquidation value of the asset given the market maker's information after t period(s) of trading. It measures the total amount of information that has not been incorporated into price after t rounds of trading. A high value of Σ_t indicates that informed traders retain a large amount of information after t trading rounds.

Accordingly, in the disclosure case we define

$$\Sigma_t^d = var(\widetilde{v}|\ w_1^d, ..w_t^d, x_1^d, ..x_t^d) = var(\widetilde{v}\ - p_t^*).$$

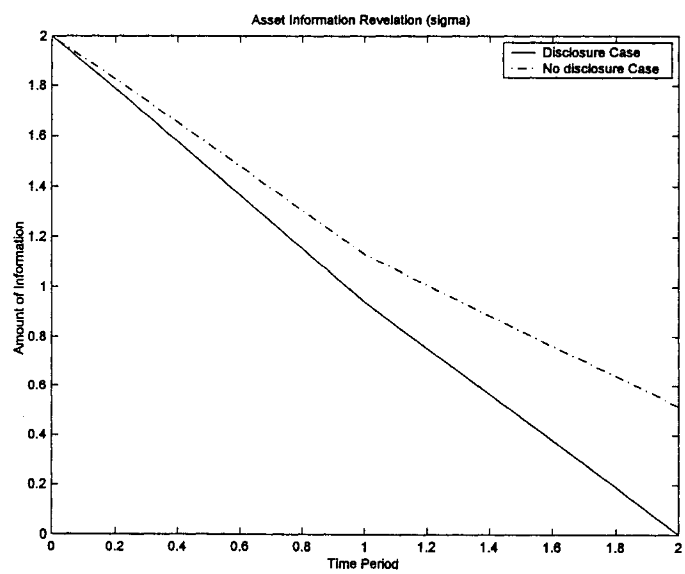

Figure 2.1: Revelation of Asset Information (Σ_t)
This figure contrasts the revelation of information of the asset under disclosure and non-disclosure cases. The solid line represents the amount of information that has not been released to the public yet in the case of disclosure. The dashed line represents the amount of information that has not been released to the public yet in the case of no disclosure.

Figure 2.1 plots the variances of the asset liquidation value given the market maker's information $\left(\Sigma_t, \Sigma_t^d\right)$. The solid line is Σ_t^d and the dashed line is Σ_t. The solid line is under the dashed line, indicating that the mandatory disclosure regulations make more information released to the public than would be the case without disclosure. Moreover, more information is revealed in the second period than in the first period in the disclosure case. This is because dissimulation only occurs in the first period. In contrast, more information is revealed in the first period than in the second period in the non-disclosure case.

The amount of insider information after t trading rounds in the non-disclosure case and disclosure case is measured by Λ_t and Λ_t^d respectively, where

$$\Lambda_t = var(\widetilde{v}|\ s_0, x_1 + u_1, ..., x_t + u_t) = var(\widetilde{v}\ - s_t),$$

$$\Lambda_t^d = var(\widetilde{v}|\ s_0, x_{1,..}^d x_t^d) = var(\widetilde{v}\ - s_t^d).$$

Λ_t, Λ_t^d are the variances of the asset liquidation value given trader 2's information after t trading periods. Thus, they measure the amount of information known only to trader 1 after t trading periods.

Figure 2.2 shows the revelation of trader 1's insider information. The solid line is Λ_t^d and the dashed line is Λ_t. It is interesting to note that in the disclosure case, less insider information is released after the first round of trading than would be the case of no disclosure. After the first trading round only 9 percent of the insider information is revealed in the disclosure case compared to 16 percent in the no disclosure case.

Since the second period is the last period market participants can trade, the disclosure after this trading round does not affect the information advantage of trader 1. As a result, trader 1 need not dissimulate his insider information as he did in period 1. This is the reason that all insider information is revealed after the second trading round.

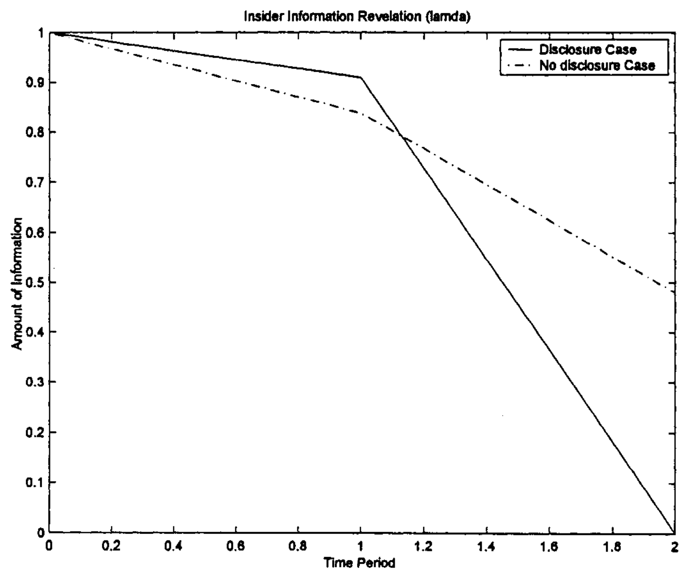

Figure 2.2: Revelation of Insider Information (Λ_t)
This figure contrasts the revelation of insider information under disclosure and non-disclosure cases. The solid line represents the amount of insider information that has not been released in the disclosure case. The dashed line represents the amount of insider information that has not been released yet in the case of no disclosure.

The amount of common information shared between traders 1 and 2 is measured by the difference between the total information and the insider information. Specifically,

$$\Pi_t = \Sigma_t - \Lambda_t = var(s_t - p_t),$$

$$\Pi_t^d = \Sigma_t^d - \Lambda_t^d = var(s_t^d - p_t^*).$$

Where p_t^* is the updated price in period t after trading and disclosure.

Π_t, Π_t^d are the variances (given the market maker's information) of trader 2's conditional expectation of the liquidation value of the asset, s_t, after t trading rounds. Therefore, they measure the amount of common information known to both traders 1 and 2 after t trading rounds.

Figure 2.3 shows how the common information is revealed. The solid line is Π_t^d and the dashed line is Π_t. It is apparent that more common information of informed traders is released in the disclosure case than would be the case of no disclosure because the competition between the two informed traders is more intense in the disclosure case. In addition, more common information can be learned from the public records in the disclosure case. As the amount of common information falls, the learning capability of trader 2 relative to the market maker's decreases. It is interesting to note that almost all common information is released after the first trading round in the disclosure case.

To examine the impacts of the mandatory disclosure regulations on liquidity trades we first define the marginal trading cost. Marginal trading cost in period t, MC_t is defined by

$$MC_t = \frac{\partial p_t}{\partial w_t}.$$

MC_t tells us by how much the price in period t increases when the aggregate order flow increases by 1 unit, ceteris paribus. Kyle (1985) defines $\frac{1}{MC_t}$ as market depth in period t. Thus, the term "market depth" refers to the ability of the market to absorb quantities without having a large effect on price. Lower values of MC_t mean that orders have a smaller impact on the price of the asset, which implies the market is deeper. In the disclosure case MC_t is constant and always lower than that in the non-disclosure case. This implies that the market is deeper in the disclosure case. Thus, liquidity traders are unambiguously better off.

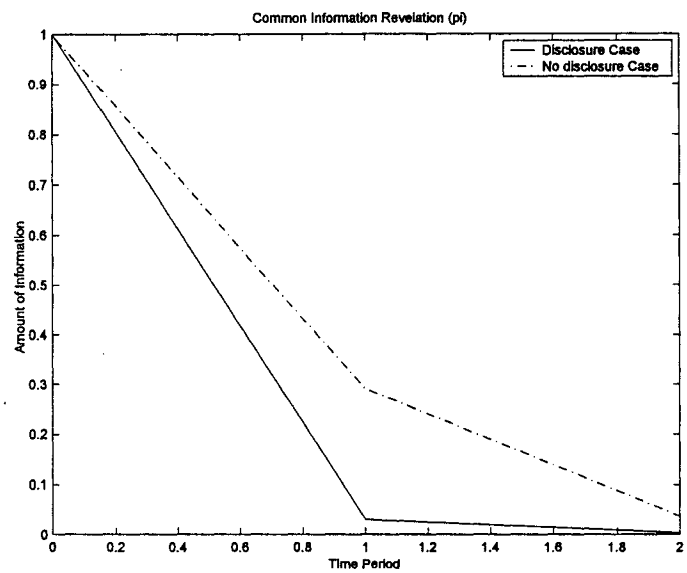

Figure 2.3: Revelation of Common Information (Π_t)
This figure contrasts the revelation of common information under disclosure and non-disclosure cases. The solid line represents the amount of common information that has not been released yet in the disclosure case. The dashed line represents the amount of insider information that has not been released yet in the case of no disclosure.

In short, this example shows that under the mandatory disclosure trader 1, the insider, conceals his super information by adding noise to his order and by trading against his private information in addition to trading very intensely on the common information. Because of that, less insider information is revealed after the first round of trading than would be the case of no disclosure. However, this trading strategy makes almost all common information released to the public after the first trading round. Overall, more information about the stock is impounded into the price; as a result, marginal trading costs reduce and liquidity traders are better off.

2.4 Characterization of Linear Equilibrium

The following definition defines formally the equilibrium in our model.

Definition 1 *A Perfect Bayesian Nash equilibrium of this trading game is given by a strategy profile* $\{[x_1(.), x_2(.)], [y_1(.), y_2(.)]\}$ *and a price system* $\{p_1(.), p_2(.)\}$ *such that the following conditions hold*

(1) *Profit maximization*

$$x_2 \in \arg\max_{x_2} E[x_2(\widetilde{v} - p_2) \mid I_1^2],$$

$$x_1 \in \arg\max_{x_1} E[x_1(\widetilde{v} - p_1) + x_2(\widetilde{v} - p_2) \mid I_1^1],$$

$$y_2 \in \arg\max_{y_2} E[y_2(\widetilde{v} - p_2) \mid I_2^2],$$

$$y_1 \in \arg\max_{y_1} E[y_1(\widetilde{v} - p_2) + y_2\left(\widetilde{v} - p_2\right) \mid I_2^1].$$

(2) *Market efficiency*

$$p_1 = E(\widetilde{v} \mid I_m^1),$$

$$p_2 = E(\widetilde{v} \mid I_m^2),$$

where I_1^t, I_2^t, I_m^t *are the information sets of trader 1, trader 2, and the market maker respectively in period* t*. The conditional expectations are derived using Bayes' rule to ensure that the beliefs are consistent with the equilibrium strategy.*

2.4.1 The Equilibrium in the Non-Disclosure Case

For comparison purposes, we are interested in deriving the equilibrium in the non-disclosure case[12]. The following proposition which follows from tedious calculations, characterizes a PBE when disclosure is not required.

Proposition 2 *A PBE in which all trading strategies and pricing rules are of linear form is given by*

$$x_1 = \beta_1(v - s_0) + \gamma_1 s_0, \tag{2.6}$$

$$x_2 = \beta_2(v - s_1) + \gamma_2(s_1 - p_1), \tag{2.7}$$

$$y_1 = \theta_1 s_0, \tag{2.8}$$

$$y_2 = \theta_2(s_1 - p_1), \tag{2.9}$$

$$p_1 = \lambda_1 w_1, \tag{2.10}$$

$$p_2 = p_1 + \lambda_2 w_2, \tag{2.11}$$

where $\beta_1, \beta_2, \gamma_1, \gamma_2, \theta_1, \theta_2, \lambda_1, \lambda_2$ *can be found by solving the following system of equations*

$$\beta_1 = \frac{2\lambda_2 - \phi + 2\lambda_2\mu(\phi - \lambda_1)}{2\lambda_2\rho}, \tag{2.12}$$

$$\beta_2 = \frac{1}{2\lambda_2}, \tag{2.13}$$

$$\gamma_1 = \frac{(1 - \lambda_1\theta_1)[1 + 2\xi(\phi - \lambda_1) - \mu\phi]}{2\rho}, \tag{2.14}$$

[12]The equilibrium in the non-disclosure case is based on the model of Foster and Viswanathan (1994)

$$\theta_2 = \gamma_2 = \frac{1}{3\lambda_2}, \tag{2.15}$$

$$\theta_1 = \frac{1 - 2\psi\lambda_1}{\lambda_1 \left[3 + 2\psi\left(\phi - 2\lambda_1\right)\right]}, \tag{2.16}$$

$$\lambda_1 = \frac{(\beta_1 + \gamma_1 + \theta_1)\sigma^2}{\left[\beta_1^2 + (\gamma_1 + \theta_1)^2\right]\sigma^2 + \sigma_u^2}, \tag{2.17}$$

$$\lambda_2 = \sqrt{\frac{9\Lambda_1 + 8\Pi_1}{36\sigma_u^2}}, \tag{2.18}$$

$$\xi = \psi = \frac{\mu}{3} = \frac{1}{9\lambda_2}, \tag{2.19}$$

$$\phi = \frac{\beta_1\sigma^2}{\beta_1^2\sigma^2 + \sigma_u^2}, \tag{2.20}$$

provided the second order conditions

$$\lambda_1 \left(1 - \psi\lambda_1\right) > 0,$$

$$\rho = \lambda_1 - \alpha\phi^2 - \xi\left(\phi - \lambda_1\right)^2 + \mu\phi\left(\phi - \lambda_1\right) > 0,$$

are satisfied.

The interested reader is referred to Appendix B for the complete proof of the proposition. For further discussions about this equilibrium, see Foster and Viswanathan (1994).

The expressions in Proposition 2 provides a benchmark against which to compare an equilibrium for the case where the insider's order is publicly disclosed on completion of trading.

2.4.2 The Equilibrium in the Disclosure Case

Since trader 1 discloses his order x_1^d after the first trading round, the price in period 2 depends not only on the history of aggregate order flow but also on trader 1's order in period 1. In addition, because trader 2 can extract some information from the disclosure, x_1^d also enters the second period demand function of trader 2.

In this setting, the pricing rules and the trading strategies presented in Proposition 2 are not an equilibrium anymore. Indeed, suppose that trader 1 followed the first period strategy indicated in (2.6). Then, the market maker as well as other market participants would learn more information about the asset in period 2 because of the disclosure. If the market maker kept the marginal trading cost λ_2 unchanged, then his expected profit in period 2 would be positive. This is a violation of the equilibrium conditions. Moreover, if trader 1 used the equilibrium trading strategy described in Proposition 2 then he would surrender his entire informational advantage the first time he is compelling to disclose the quantity he has traded. This is because trader 2 could deduce precisely his private information. Clearly, no invertible trading strategy in period 1 can be part of an equilibrium in which trader 1 still has information advantage over trader 2 in the second period.

Using the notion of dissimulation introduced by Huddart, Hughes and Levine (2001), we show in this section that there exists an equilibrium in which trader 1's order in period 1 consists of an information-based component and a random noise component z which is normally and independently distributed with mean zero and variance Σ_z. The random component may be a buy or a sell. Because of it, trader 1 sometimes buys (sells) when his information indicates that the asset is overvalued (undervalued). He may also buy or sell more aggressively than would be the case under no disclosure. This is to diminish the ability of trader 2 and the market maker to draw inference from the public records. However, as indicated above, dissimulation is costly because at times trader 1 has to trade in a manner inconsistent with his private information.

To derive the equilibrium, we first postulate that the demands of trader 1 and trader 2, and the market maker's pricing rule in period 1 take the form

$$x_1^d = \beta_1(v - s_0) + \gamma_1 s_0 + z \quad \text{where } z \sim N(0, \Sigma_z), \tag{2.21}$$

$$y_1^d = \theta_1 s_0, \tag{2.22}$$

$$p_1^d = E\left(\widetilde{v} \middle| w_1\right) = \lambda_1 w_1^d. \tag{2.23}$$

Based on the public disclosure of trader 1, the market maker updates his beliefs from those formed on the basis of the first period aggregate order flow. Specifically, denote by p_1^* the updated period 1 price after the public disclosure. Then,

$$p_1^* = E\left(\widetilde{v} \middle| x_1^d, y_1^d + u_1\right) = \vartheta x_1^d + \eta\left(y_1 + u_1\right). \tag{2.24}$$

Based on p_1^* and the aggregate order flow, the market maker sets the second period price. Thus,

$$p_2^d = E\left(\widetilde{v} \middle| p_1^*, w_2^d\right) = p_1^* + \lambda_2 w_2^d. \tag{2.25}$$

The public disclosure of trader 1 also allows trader 2 to update his information. His learning process can be written as

$$s_1^d = E\left(\widetilde{v} \middle| \, s_0, x_1\right) = s_0 + \phi x_1^d. \tag{2.26}$$

Since period 2 is the last period market participants can trade before the liquidation value of the stock becomes public, disclosure after this round does not affect trader 1's profit. Therefore, trader 1 need not hide his information by dissimulation. Thus, his demand consists of only information-based components. We hypothesize that the demand takes the following form

$$x_2 = \beta_2 \left(v - s_1^d\right) + \gamma_2 \left(s_1^d - p_1^*\right). \tag{2.27}$$

This form of trader 1's demand in period 2 is similar to that in the non-disclosure case except that the price p_1 is replaced by the updated price p_1^*.

Trader 2, after updating his signal from s_0 to s_1^d by (2.26), uses the updated signal s_1^d and price p_1^* to form his second period demand. We hypothesize that

$$y_2^d = \theta_2 \left(s_1^d - p_1^*\right). \tag{2.28}$$

To solve for the equilibrium, we use backward induction to obtain the informed traders' trading strategies and expected trading profits as a function of the price. Then, we use the market efficiency conditions to find the pricing rules of the market maker.

Lemma 3 *In equilibrium, the second period demand of trader 1 and the demands of trader 2 are*

$$x_2^d = \frac{1}{2\lambda_2}\left(v - s_1^d\right) + \frac{1}{3\lambda_2}\left(s_1^d - p_1^*\right),$$

$$y_1^d = \frac{9\lambda_2 - 2\lambda_1}{\lambda_1\left[27\lambda_2 + 2\left(\phi - 2\lambda_1\right)\right]} s_0,$$

$$y_2^d = \frac{1}{3\lambda_2}\left(s_1^d - p_1^*\right),$$

provided the second order conditions

$$\lambda_2 > 0,$$

$$\lambda_1\left(1 - \frac{\lambda_1}{9\lambda_2}\right) > 0$$

are satisfied.

The interested reader is referred to Appendix C for the complete proof of this lemma.

We observe that both traders 1 and 2 trade on the common information at the same intensity in period 2 and trader 1 trades more intensely on his insider information. This can be explained by the fact that trader 1 is a monopolist over the insider information and both traders 1 and 2 are duopolists for their common information.

Lemma 4 *In the equilibrium with dissimulation, the market depth (the reciprocal of* λ_t, $t = 1, 2$*) is the same in both periods, i.e.,*

$$\lambda_1 = \lambda_2 = \lambda,$$

and is given by

$$\lambda = \sqrt{\frac{9\Lambda_1 + 8\Pi_1}{36\sigma_u^2}}.$$

The complete proof is shown in Appendix C.

Unlike the non-disclosure case where the market depth (the reciprocal of λ_t) changes over time to reflect the fact that the market maker learns more and more information over time by observing the aggregate order flows, in the mandatory disclosure case the market depth is constant. This is a necessary condition to sustain a mixed strategy. Otherwise, traders would have an incentive to deviate from a mixed strategy in order to exploit the lower costs.

Using Lemmas 3, 4 and the zero profit conditions for the market maker, we can derive the linear equilibrium which is shown in the Proposition below.

Proposition 5 *A PBE in a setting with mandatory disclosure of insider trade is given by*

$$x_1^d = \beta_1(v - s_0) + \gamma_1 s_0 + z, \ \ \textit{where } z \sim N(0, \Sigma_z), \tag{2.29}$$

$$x_2^d = \frac{1}{2\lambda}\left(v - s_1^d\right) + \frac{1}{3\lambda}\left(s_1^d - p_1^*\right), \tag{2.30}$$

$$y_1^d = \theta_1 s_0, \tag{2.31}$$

$$y_2^d = \frac{1}{3\lambda}\left(s_1^d - p_1^*\right), \tag{2.32}$$

$$s_1^d = s_0 + \phi x_1, \tag{2.33}$$

$$p_1^d = \lambda w_1^d, \tag{2.34}$$

$$p_1^* = \vartheta x_1 + \eta \left(y_1 + u_1 \right), \tag{2.35}$$

$$p_2^d = \ p_1^* + \lambda w_2^d, \tag{2.36}$$

where $\beta_1, \gamma_1, \theta_1, \phi, \lambda, \vartheta, \eta, \Sigma_z$ *are the solution of the following equation system*

$$\beta_1 = \left[\frac{\Sigma_z + \sigma_u^2}{\sigma^2} + \beta_1^2 + (\gamma_1 + \theta_1)^2 \right] \lambda - \gamma_1 - \theta_1, \tag{2.37}$$

$$\gamma_1 = \frac{1}{\vartheta} - \frac{\eta}{\vartheta} \left(\theta_1 + \frac{\sigma_u^2}{\theta_1 \sigma^2} \right), \tag{2.38}$$

$$\theta_1 = \frac{7}{2\phi + 23\lambda}, \tag{2.39}$$

$$\phi = 6\lambda - 2\vartheta, \tag{2.40}$$

$$\lambda = \sqrt{\frac{9\Lambda_1 + 8\Pi_1}{36\sigma_u^2}}, \tag{2.41}$$

$$\vartheta = \frac{\left(\beta_1 + \gamma_1 - \eta \theta_1 \gamma_1 \right) \sigma^2}{\left(\beta_1^2 + \gamma_1^2 \right) \sigma^2 + \Sigma_z}, \tag{2.42}$$

$$\eta = \frac{3\lambda}{2} - \frac{1}{2\theta_1}, \tag{2.43}$$

$$\Sigma_z = \frac{\beta_1 \left(1 - \beta_1 \phi \right)}{\phi} \sigma^2. \tag{2.44}$$

The complete proof of Proposition 5 is in Appendix C. This proposition shows us conditions for a linear PBE that the dynamic game between trader 1 and trader 2 must satisfy in the case under mandatory disclosure.

2.5 Concluding Remarks

The aim of this chapter is to examine the effects of the mandatory disclosure regulations on the strategic trading of informed traders in financial markets. In order to conceal his private information under mandatory disclosure regulations, the insider adds a random noise to his order. This noise may be a buy or a sell and is independent of his insider information. In addition, the insider may trade against his private information. This, coupled with the random noise, diminishes the learning ability of other market participants. As a result, less insider information is revealed through trading than would be the case under no disclosure. However, more common information shared between the informed insider and the informed outsider is impounded into to price. This reduces the learning advantage of the informed outsider relative to the market maker. Overall, more information about the asset is released to the public than would be the case of no disclosure. As a result, the market is deeper and liquidity traders unambiguously benefit from lower marginal trading costs than would be the case of no disclosure.

Thus, the goal of the Congress is partly achieved. The mandatory disclosure regulations help to make the market more liquid and more efficient. However, they still fail to eradicate unfair enrichment by those with access to private information. The results in this chapter suggest that insiders continue to trade on private information after the public disclosure of trades from earlier rounds and before the public release of the information. More importantly, they can even keep more insider information than would be the case under no disclosure.

2.6 Appendix

Appendix A: US Securities Laws

Section 16(a) of the US Securities Exchange Act of 1934 imposes a disclosure requirement on corporate insiders, defined as the firm's officers and directors and shareholders who own 10 percent or more of the firm's stock. These insiders must report their trade within ten days following the end of the month in which the trade occurs. In addition, Section 13(d) added to the 1934 Act in 1968 by the William Act and amended in 1970, requires any individual who acquires 5 percent or more of a firm's stock to report it within ten days. Subsequent changes to the position must also be reported within ten days (as long as the shareholder has at least 5 percent of the stock). These reports are publicly available immediately upon receipt by the Securities and Exchange Commission (SEC). Antitrust law also mandates the disclosure of stock trades; specifically, Section 7 of the Clayton Act, as amended in 1976 by the Hart-Scott-Rodino Antitrust Improvements Act. For acquisition of stock with the intent to acquire control, the disclosure provision is triggered by the acquisition of either 15 percent of the firm's stock or 15 million dollars of the firm's stock. The disclosure is made to the Department of Justice, the Federal Trade Commission, and to the target firm. Though this disclosure is not directly available to the public, the information is typically publicly disclosed by the target (since this is a material event for the target). Acquisitions of less than 10 percent of a firm's stock without the intent to acquire control are exempted from this disclosure requirement.

Appendix B

Proof of Proposition 2

In period 2, trader 2 solves the following optimization problem

$$Max_{y_2} E\left[\left(\widetilde{v} - p_2\right) y_2 \mid s_1\right].$$

The first order condition gives

$$s_1 - p_1 - \lambda_2 x_2^e - 2\lambda_2 y_2 = 0,$$

$$\Longrightarrow y_2 = x_2^e = \frac{s_1 - p_1}{3\lambda_2},$$

where

$$x_2^e = E\left(x_2 \mid s_1\right).$$

$$\Longrightarrow \theta_2 = \frac{1}{3\lambda_2}. \tag{2.45}$$

Trader 2's value function is given by

$$V_2\left(s_1 - p_1\right) = \frac{1}{9\lambda_2}\left(s_1 - p_1\right)^2,$$

$$\Longrightarrow \psi = \frac{1}{9\lambda_2}.$$

Trader 1's problem is

$$Max_{x_2} E\left[\left(\widetilde{v} - p_2\right) x_2 \mid v, s_1\right].$$

The first order condition is

$$v - p_1 - 2\lambda_2 x_2 - \lambda_2 y_2 = 0,$$

$$\Longrightarrow x_2 = \frac{1}{2\lambda_2}(v - s_1) + \frac{1}{3\lambda_2}(s_1 - p_1),$$

$$\Longrightarrow \beta_2 = \frac{1}{2\lambda_2} \text{ and } \gamma_2 = \frac{1}{3\lambda_2}. \quad (2.46)$$

The second order condition is

$$\lambda_2 > 0.$$

Trader 1's value function is

$$V_1(v - s_1, s_1 - p_1) = \frac{1}{4\lambda_2}(v - s_1)^2 + \frac{1}{9\lambda_2}(s_1 - p_1)^2 + \frac{1}{3\lambda_2}(v - s_1)(s_1 - p_1),$$

$$\Longrightarrow \alpha = \frac{1}{4\lambda_2}, \xi = \frac{1}{9\lambda_2} \text{ and } \mu = \frac{1}{3\lambda_2}.$$

In period 1, trader 2 solves the following problem

$$Max_{y_1} E\left[\left(\tilde{v} - p_1\right) y_1 + V_2 \mid s_0\right].$$

The first order condition is

$$s_0 - \lambda_1 x_1^e - 2\lambda_1 y_1 - 2\psi\lambda_1 \left[s_0 + \phi x_1^e - \lambda_1 \left(x_1^e + y_1\right)\right] = 0, \quad (2.47)$$

where

$$x_1^e = E\left(x_1 \mid s_0\right).$$

The second order condition is

$$\lambda_1 \left(1 - \psi\lambda_1\right) > 0.$$

$$\Longrightarrow x_1^e = y_1 = \frac{1 - 2\psi\lambda_1}{\lambda_1 \left[3 + 2\psi\left(\phi - 2\lambda_1\right)\right]} s_0, \quad (2.48)$$

$$\Longrightarrow \theta_1 = \frac{1 - 2\psi\lambda_1}{\lambda_1 \left[3 + 2\psi\left(\phi - 2\lambda_1\right)\right]}.$$

Trader 1 solves

$$Max_{x_1}\left[\left(\widetilde{v}-p_1\right)x_1+V_1\mid v,s_0\right]$$

The first order condition is

$$v-2\lambda_1 x_1-\lambda_1 y_1-2\alpha\phi\left(v-s_0-\phi x_1\right)+2\xi\left(\phi-\lambda_1\right)\left[s_0+\left(\phi-\lambda_1\right)x_1-\lambda_1 y_1\right] \quad (2.49)$$

$$-\mu\phi\left[s_0+\left(\phi-\lambda_1\right)x_1-\lambda_1 y_1\right]+\mu\left(\phi-\lambda_1\right)\left(v-s_0-\phi x_1\right)=0.$$

The second order condition is

$$\rho=\lambda_1-\alpha\phi^2-\xi\left(\phi-\lambda_1\right)^2+\mu\phi\left(\phi-\lambda_1\right)>0.$$

Equations (2.48) and (2.49) imply

$$x_1=\frac{1-2\alpha\phi+\mu\left(\phi-\lambda_1\right)}{2\rho}\left(v-s_0\right)+\frac{\left(1-\lambda_1\theta_1\right)\left[1+2\xi\left(\phi-\lambda_1\right)-\mu\phi\right]}{2\rho}s_0,$$

$$\Longrightarrow \beta_1=\frac{1-2\alpha\phi+\mu\left(\phi-\lambda_1\right)}{2\rho}, \quad \text{and}$$

$$\gamma_1=\frac{\left(1-\lambda_1\theta_1\right)\left[1+2\xi\left(\phi-\lambda_1\right)-\mu\phi\right]}{2\rho}.$$

Trader 2's learning process can be described as

$$s_1=E\left(\widetilde{v}\middle|\, x_1+u_1,s_0\right)=s_0+\phi\left(x_1+u_1\right),$$

$$\Longrightarrow \phi=\frac{cov\left(a,x_1+u_1\mid b\right)}{var\left(x_1+u_1\mid b\right)}=\frac{cov\left(a,\beta_1 a+\gamma_1 b+u_1\mid b\right)}{var\left(\beta_1 a+\gamma_1 b+u_1\mid b\right)},$$

$$\Longrightarrow \phi=\frac{\beta_1\sigma^2}{\beta_1^2\sigma^2+\sigma_u^2}.$$

The market efficiency conditions imply

$$p_1=E\left(\widetilde{v}\middle|\, w_1\right)=\lambda_1 w_1,$$

$$\Longrightarrow \lambda_1 = \frac{(\beta_1 + \gamma_1 + \theta_1)\,\sigma^2}{\left[\beta_1^2 + (\gamma_1 + \theta_1)^2\right]\sigma^2 + \sigma_u^2},$$

$$p_2 = E\left(\widetilde{v}|\; w_1, w_2\right) = p_1 + \lambda_2 w_2,$$

$$\Longrightarrow \lambda_2 = \frac{cov\left(\widetilde{v} - p_1, w_2\right)}{var\,(w_2)}$$

$$\Longrightarrow \lambda_2 = \frac{\beta_2 \Lambda_1 + (\gamma_2 + \theta_2)\,\Pi_1}{\beta_2^2 \Lambda_1 + (\gamma_2 + \theta_2)^2\,\Pi_1 + \sigma_u^2}. \tag{2.50}$$

Substituting (2.45) and (2.46) into (2.50) yields:

$$\lambda_2 = \sqrt{\frac{9\Lambda_1 + 8\Pi_1}{36\sigma_u^2}},$$

$$\Sigma_1 = var\left(\widetilde{v}|\; w_1\right) = var\left(\widetilde{v}\right) - \frac{cov^2\left(\widetilde{v}, w_1\right)}{var\,(w_1)},$$

$$\Longrightarrow \Sigma_1 = 2\sigma^2 - \frac{(\beta_1 + \gamma_1 + \theta_1)^2\,\sigma^4}{\left[\beta_1^2 + (\gamma_1 + \theta_1)^2\right]\sigma^2 + \sigma_u^2} = \left[2 - \lambda_1\,(\beta_1 + \gamma_1 + \theta_1)\right]\sigma^2,$$

$$\Lambda_1 = var\left(\widetilde{v}|\; s_0, x_1 + u_1\right) = var\,(a \mid \beta_1 a + u_1) = (1 - \phi\beta_1)\,\sigma^2.$$

Appendix C

Proof of Lemma 3

We use backward induction. Trader 2's second period profit maximization problem given x_1 and p_1^* is

$$Max_{y_2} E\left[\left(\widetilde{v} - p_2\right) y_2 \mid s_1, x_1, p_1^*\right].$$

Using calculus we get

$$y_2 = x_2^e = \frac{s_1 - p_1^*}{3\lambda_2}. \tag{2.51}$$

Comparing (2.28) and (2.51), we have

$$\theta_2 = \frac{1}{3\lambda_2}. \tag{2.52}$$

The value function of trader 2 is

$$V_2\left(s_1 - p_1^*\right) = \frac{1}{9\lambda_2}\left(s_1 - p_1^*\right)^2 = \psi\left(s_1 - p_1^*\right)^2.$$

Where

$$\psi = \frac{1}{9\lambda_2}.$$

The second order condition is

$$\lambda_2 > 0.$$

Trader 1's second period profit maximization problem is

$$Max_{x_2} E\left[\left(\tilde{v} - p_2\right) x_2 \mid v, s_1, p_1^*\right]$$

$$\Longrightarrow x_2 = \frac{v - p_1^*}{2\lambda_2} - \frac{y_2}{2}. \tag{2.53}$$

Substituting (2.51) into (2.53) yields

$$x_2 = \frac{1}{2\lambda_2}\left(v - s_1\right) + \frac{1}{3\lambda_2}\left(s_1 - p_1^*\right). \tag{2.54}$$

Comparing (2.27) and (2.54) we have

$$\beta_2 = \frac{1}{2\lambda_2} \text{ and } \gamma_2 = \frac{1}{3\lambda_2} \tag{2.55}$$

The value function of trader 1 is

$$V_1\left(v - s_1, s_1 - p_1^*\right) = \frac{1}{4\lambda_2}\left(v - s_1\right)^2 + \frac{1}{9\lambda_2}\left(s_1 - p_1^*\right)^2 + \frac{1}{3\lambda_2}\left(v - s_1\right)\left(s_1 - p_1^*\right). \tag{2.56}$$

To determine his order in period 1, trader 2 solves

$$Max_{y_1} E\left[\left(\widetilde{v} - p_1\right) y_1 + V_2 \mid s_0\right].$$

Using calculus, we get

$$y_1 = \frac{9\lambda_2 - 2\lambda_1}{\lambda_1 \left[27\lambda_2 + 2\left(\phi - 2\lambda_1\right)\right]} s_0 \tag{2.57}$$

if the second order condition

$$\lambda_1 \left(1 - \frac{\lambda_1}{9\lambda_2}\right) > 0$$

is satisfied.

Proof of Lemma 4

Trader 1's first period optimization problem is

$$x_1 \in \arg\max_{x_1} E\left[x_1 \left(\widetilde{v} - p_1\right) + V_1 \mid v, s_0\right].$$

Substituting for p_1, p_1^* and V_1 from (2.23), (2.24) and (2.56) respectively, differentiating, and setting the result equal to zero lead to the following first order condition

$$\left[-2\lambda_1 + \frac{\left(\phi + 2\vartheta\right)^2}{18\lambda_2}\right] x_1 + \left[-\lambda_1\theta_1 + \frac{\eta\theta_1\left(\phi + 2\vartheta\right)}{9\lambda_2} + \frac{\phi + 2\vartheta}{18\lambda_2}\right] s_0 + \tag{2.58}$$

$$+ \left(1 - \frac{\phi + 2\vartheta}{6\lambda_2}\right) v = 0.$$

The second order condition is

$$-2\lambda_1 + \frac{\left(\phi + 2\vartheta\right)^2}{18\lambda_2} \leq 0.$$

If trader 1 uses the mixed strategy as indicated in (2.21), then he must be indifferent across all values of x_1. Thus, in expression (2.58) we must have

$$-2\lambda_1 + \frac{(\phi + 2\vartheta)^2}{18\lambda_2} = 0, \tag{2.59}$$

$$-\lambda_1\theta_1 + \frac{\eta\theta_1(\phi + 2\vartheta)}{9\lambda_2} + \frac{\phi + 2\vartheta}{18\lambda_2} = 0, \text{ and} \tag{2.60}$$

$$1 - \frac{\phi + 2\vartheta}{6\lambda_2} = 0. \tag{2.61}$$

Equations (2.59) and (2.61) imply

$$\phi + 2\vartheta = 6\lambda, \text{ and} \tag{2.62}$$

$$\lambda_1 = \lambda_2 = \lambda. \tag{2.63}$$

The zero profit condition for the market maker implies

$$p_2 = E\left(\widetilde{v} \middle|\, x_1, w_1, w_2\right) = p_1^* + \lambda w_2$$

$$\Longrightarrow \lambda = \frac{cov\left(\widetilde{v} - p_1^*, w_2\right)}{var\,(w_2)} = \frac{\beta_2\Lambda_1 - (\gamma_2 + \theta_2)\,\Pi_1}{\beta_2^2\Lambda_1 + (\gamma_2 + \theta_2)^2\,\Pi_1 + \sigma_u^2}.$$

Substituting $\beta_2, \gamma_2, \theta_2$ from (2.52) and (2.55) we get

$$\Longrightarrow \lambda = \sqrt{\frac{9\Lambda_1 + 8\Pi_1}{36\sigma_u^2}}.$$

Proof of Proposition 5

Expressions (2.30), (2.32), (2.41) follow directly from Lemmas 3 and 4. Substituting (2.63) and (2.62) into (2.60) we get

$$\eta = \frac{3\lambda}{2} - \frac{1}{2\theta_1}.$$

Equation (2.26) implies

$$s_1^d = E\left(\widetilde{v} |\, s_0, x_1^d\right) = b + E\left(a \mid s_0, x_1\right) = b + \phi x_1^d,$$

$$\Longrightarrow \phi = \frac{\beta_1 \sigma^2}{\beta_1^2 \sigma^2 + \Sigma_z}.$$

$$\Longrightarrow \Sigma_z = \frac{\beta_1 \left(1 - \beta_1 \phi\right)}{\phi} \sigma^2.$$

Expression (2.62) implied

$$\phi = 6\lambda - 2\vartheta$$

Applying the projection theorem into (2.24) we get

$$\vartheta = \frac{\left(\beta_1 + \gamma_1 - \eta\theta_1\gamma_1\right)\sigma^2}{\left(\beta_1^2 + \gamma_1^2\right)\sigma^2 + \Sigma_z},$$

$$\eta = \frac{\left(1 - \vartheta\gamma_1\right)\theta_1\sigma^2}{\theta_1^2\sigma^2 + \sigma_u^2}. \tag{2.64}$$

Equation (2.64) implies

$$\gamma_1 = \frac{1}{\vartheta} - \frac{\eta}{\vartheta}\left(\theta_1 + \frac{\sigma_u^2}{\theta_1\sigma^2}\right).$$

Applying the projection theorem into (2.23) we have

$$\lambda = \frac{\left(\beta_1 + \gamma_1 + \theta_1\right)\sigma^2}{\left[\beta_1^2 + \left(\gamma_1 + \theta_1\right)^2\right]\sigma^2 + \Sigma_z + \sigma_u^2},$$

which implies

$$\beta_1 = \left[\frac{\Sigma_z + \sigma_u^2}{\sigma^2} + \beta_1^2 + \left(\gamma_1 + \theta_1\right)^2\right]\lambda - \gamma_1 - \theta_1.$$

Chapter 3

Strategic Trading When Some Investors Receive Information Before Others

3.1 Introduction

Information plays a very important role in financial markets. Investors base their expectations about the payoff of an asset on their information. This information, therefore, affects their trading behavior and as a result the asset price. In a perfect market, all investors receive information on assets immediately and simultaneously. In practice, however, some investors, either due to the nature of their positions (e.g. corporate insiders and their favored analysts) or owing to superior skill, acquire pertinent information before others. By being first, an investor can exploit this information to great advantage. Investors who are uninformed or late-informed are aware of the fact that the actions of other traders are driven by their information. They try to collect and process their own information about the traded asset, and at the same time, they have to trade carefully and try to infer the private information of better and early informed investors. Better and early informed investors, of course, realize that their trading actions are followed closely, so they may want to use an optimal strategy that can help them keep their information advantage from being revealed too quickly via trading.

The chapter examines trading behavior, price behavior, and the informational efficiency and the informativeness of the price process in the equilibrium of a strategic trading game when some investors receive information about the liquidation value of the stock before others. Toward that end, we provide a theoretical framework for analyzing the investment choices of two risk neutral investors who investigate the prospect of a firm. The high-ability investor uncovers the payoff-relevant information early, while the low-ability investor uncovers less information and later. We show in this chapter that the sequential nature of information arrival has significant effects on the strategic trading decisions, the price behavior, and the informational efficiency and the informativeness of the price process. Instead of assuming that the rational expectations equilibrium is a competitive one, we analyze a Perfect Bayesian Nash equilibrium (PBE). We study this equilibrium notion because it captures the fact that the informed traders are rational and forward-looking. That is, each informed trader takes into account that his demand will be used by the other traders to update their beliefs concerning the liquidation value of the stock.

We show that after getting information, the early informed investor trades on both signals: his private signal and the one that he will shares with the late informed investor. Under some conditions, he may manipulate the market by trading against his information at first to move the price and then unwind his position in the next period. We also show that under some conditions, the price moves are always positively correlated with the private signals received by informed traders. More interesting, we discover that under some conditions, subsequent price changes are positively correlated. This is in favor of technical analysis. Furthermore, the analysis also shows that the price process is less informative and less efficient than would be the case in which there is no late informed trader.

This research is built on the existing literature on market microstructure. The seminal paper of Kyle (1985) investigates a model of speculative trading in which an informed trader chooses his trading strategy to maximize the value of his private information. The model also characterizes how information is incorporated into price when trades go on. Jennings, Starks, and Fellingham (1981) and Jennings and Barry (1983) consider models in which informed traders do not obtain information simultaneously. However, in those models, public information such as prices and order flows, which are potentially important sources of information about the asset, is not used by uninformed traders in forming their trading strategy. Brown and Jennings (1989), Grundy and McNichols (1989), Kim and Verrecchia (1991), and Wang (1993) overcome this limitation by analyzing the dynamic trading behavior where the uninformed investors condition their trades on all public information. However, in those models, all potentially informed investors receive information simultaneously. Later, Hirshleifer, Subrahmanyam and Titman (1994) analyze trading behavior and equilibrium information acquisition when some investors receive information before others. The model provides important insights into herding effects in financial markets. Nevertheless, they analyze the competitive equilibrium and rely on risk aversion to generate speculative trading.

In the literature on manipulation, Allen and Gale (1992) distinguish between trade-based, information-based and action-based stock price manipulation. In their trade-based manipulation model, there is a large trader who is either informed or uninformed. Other traders are price takers. Allen and Gale show that when the large trader is uninformed, it is optimal for him to manipulate the market by acting as if he received good news. Brunnermeier (2001) shows that a trader who receives a signal before the public announcement can exploit this information twice: when he receives the signal and after the public announcement. This happens because unlike other traders, he can infer how much the announced information has been incorporated in the current price.

The remainder of the chapter is organized as follows. Section 3.2 presents the model. Section 3.3 characterizes the linear equilibrium. Section 3.4 analyzes trading behavior and price behavior in the equilibrium. Section 3.5 examines the informational efficiency and the price informativeness. Section 3.6 provides concluding remarks.

3.2 The Model

There are two assets in the economy: a risky stock and a riskless bond. Market participants include an early informed trader, a late informed trader, a market maker, and a number of liquidity traders. These traders buy or sell the stock in two periods. The informed traders are assumed to be risk neutral.

We normalize the interest rate of the bond to zero. The liquidation value of the stock (i.e., its value in period 3) is a random variable $\widetilde{v}$ which is normally distributed with mean zero and variance Σ_v.

Before the first trading takes place, the early informed trader, denoted by trader 1, learns 2 signals[1] T and S related to the realization of $\tilde{v}$.

$$T = v + t \quad \text{where } t \sim N(0, \Sigma_t),$$

$$S = T + \epsilon \quad \text{where } \epsilon \sim N(0, \Sigma_\epsilon).$$

The late informed trader, denoted by trader 2, does not receive any information in period 1 but learns the signal S when the market opens in period 2. In period 3, the

[1]Trader 1 could be, for example, the insider who knows a great deal about the prospect of the firm. Trader 2 could be an investor with good research capability.

liquidation value of the stock is announced and holders of stock are paid accordingly. This information structure is common knowledge.

Without loss of generality, we take signal T as the conditional expectation of the liquidation value of the asset, given the first trader's information in period 1 and S as the conditional expectation of the liquidation value of the asset, given the second trader's information in period 2. Liquidity traders buy or sell shares for reasons exogenous to the model. The quantity traded by liquidity traders in period t ($t = 1, 2$), denoted $\tilde{u}_t$, is normally distributed with mean zero and variance σ_u^2, and is independent of all other random variables and serially uncorrelated.

Denote the quantities traded in period t by the early informed trader - trader 1, the outsider - trader 2 and liquidity traders by x_t, y_t, and u_t respectively. The aggregate order flow in period t is

$$w_t = x_t + y_t + u_t.$$

The market maker observes the aggregate order flow but does not know which orders come from which traders. The market maker is assumed to set informationally efficient prices; thus, his expected profit is zero. That is, the market maker sets the price equal to the expected value of the asset, conditioned on the history of orders received up to that time and trades the quantity necessary to clear the market at this price. Specifically,

$$p_1 = E\left(\tilde{v} \middle| w_1\right) \text{ and } p_2 = E\left(\tilde{v} \,|w_1, w_2\right).$$

Assume that the portfolio of trader 2 is optimal before the first trading round. Since he receives no information in this round, he does not trade when the market opens in period 1. Thus, $y_1 = 0$ and the aggregate order flow in period 1 is

$$w_1 = x_1 + u_1.$$

To simplify the notation, we omit the subscript in trader 2's order. Thus, trader 2's order in period 2 is y instead of y_2.

We are interested in deriving the resulting demands, x_t, y $(t = 1, 2)$, and the prices p_1 and p_2. To this end, we represent the above economy as an extensive form game with imperfect information, and employ the notion of Perfect Bayesian Equilibrium (PBE). The following definition defines formally a PBE of this trading game.

Definition 6 *A Perfect Bayesian Equilibrium of the trading game is defined as a strategy profile* $\{[x_1^*(.), x_2^*(.)], [0, y^*(.)]\}$ *and a price system* $\{p_1^*(.), p_2^*(.)\}$ *such that the following conditions hold:*

(1) *Profit maximization*

$$x_2^* \in \arg\max_{x_2} E[x_2(\widetilde{v} - p_2) \mid I_2^1],$$

$$x_1^* \in \arg\max_{x_1} E[x_1(\widetilde{v} - p_1) + x_2^*(\widetilde{v} - p_2) \mid I_1^1],$$

$$y^* \in \arg\max_{y} E[y(\widetilde{v} - p_2) \mid I_2].$$

(2) *Market efficiency*

$$p_1^* = E(\widetilde{v} \mid w_1^*),$$

$$p_2^* = E(\widetilde{v} \mid w_2^*, w_1^*).$$

where I_1^1 *and* I_2^1 *are the information set of trader 1 in periods 1 and 2 respectively, and* I_2 *is the information set of trader 2 in period 2. The conditional expectations are derived using Bayes' rule to ensure that the beliefs are consistent with the equilibrium strategy.*

This equilibrium concept is based on a dynamic programming argument. The strategies of informed traders are required to be optimal, not only when trader 1 plays his optimal strategy in period 1, but also when he plays any arbitrary strategy in period 1. However, there are no off equilibrium observations of order flow by the other informed trader in the model (even when trader 1 deviates from his optimal strategy) as liquidity trading makes every order flow is possible. Therefore, we need not concern with the issue of assigning off equilibrium beliefs.

3.3 Characterization of Linear Equilibrium

3.3.1 Equilibrium

We are interested in exploring the linear equilibria in this game because in addition to its appeal and tractability, given our normality assumption of all random variables, prices are linear functions of the history of aggregate order flows[2]. Thus, the prices fully reflect aggregate demand. To derive the linear equilibria, we begin by postulating that the price functions have the following linear forms

[2]This is a consequence of the Projection Theorem.

$$p_1 = \lambda_1 w_1,$$

$$p_2 = p_1 + \lambda_2 w_2. \tag{3.1}$$

In the following analysis, we derive a linear equilibrium in which this conjecture is confirmed to be correct.

To derive PBEs, we employ backward induction because a PBE requires equilibrium strategies to be optimal for each information set under the given Bayesian rational belief system.

Proposition 7 *A PBE in which all trading strategies and the market maker's pricing rule are of linear form is given by*

$$x_1 = \frac{9\lambda_2 - 3\lambda_1}{2\lambda_1(9\lambda_2 - \lambda_1)}T + \frac{1}{2(9\lambda_2 - \lambda_1)}S, \tag{3.2}$$

$$x_2 = \frac{T - S}{2\lambda_2} + \frac{S - p_1}{3\lambda_2}, \tag{3.3}$$

$$y = \frac{S - p_1}{3\lambda_2}, \tag{3.4}$$

$$p_1 = \lambda_1 w_1,$$

$$p_2 = p_1 + \lambda_2 w_2,$$

where λ_1 and λ_2 are given by

$$\lambda_1 = \frac{\alpha\Sigma_v}{\alpha^2\Sigma_v + \beta^2\Sigma_\epsilon + \gamma^2\Sigma_t + \Sigma_u}, \tag{3.5}$$

$$\lambda_2 = \sqrt{\frac{8var(\tilde{v} - p_1) + (2\lambda_1\beta - 1)\,\Sigma_\epsilon + 8\,(\lambda_1\gamma - 2)\,\Sigma_t}{36\Sigma_u}},$$

where

$$\alpha = \frac{9\lambda_2 - 2\lambda_1}{2\lambda_1(9\lambda_2 - \lambda_1)},$$
$$\beta = \frac{1}{2(9\lambda_2 - \lambda_1)},$$
$$\gamma = \frac{9\lambda_2 - 3\lambda_1}{2\lambda_1(9\lambda_2 - \lambda_1)},$$

if the second order conditions

$$\lambda_1, \lambda_2 > 0,$$

$$9\lambda_2 - \lambda_1 > 0 \tag{3.6}$$

are satisfied.

The interested reader is referred to the Appendix for a complete proof of the proposition.

3.3.2 Trading and Price Behavior in the Equilibrium

We investigate in this part some relevant relationships among price moves, trading behavior and information variables $\tilde{v}$, T and S.

Trading Behavior

In general, trading occurs for risk sharing purposes or for informational reasons. Risk sharing is excluded in this setting because all traders are risk-neutral. Thus, the only motive to trade is to exploit their informational advantage. Examining the demands of trader 1 (equations (3.2) and (3.3)) we can see that the demands depend on both signals T and S even though S is just an imprecise signal of T about the liquidation value of the stock. Trader 1's demand is positively related[3] to S in period 1 but negatively related to that signal in period 2. This is because the early informed trader knows at the beginning of the trading game that later, part of his private signal will be released to other traders[4] and this will affect his trading gain in the next period. Therefore, he tries to reap some profit in period 1 when the price is not affected by trader 2's action.

Another interesting feature of the model is that under some circumstance trader 1 may manipulate the price by trading against his private information in period 1 to move the price in his favor. This is shown in the first term in equation (3.2), $\frac{9\lambda_2-3\lambda_1}{2\lambda_1(9\lambda_2-\lambda_1)}T$. If the market is sufficiently deeper in period 2 than in period 1, in particular if $\lambda_2 < \frac{\lambda_1}{3}$, this term is negative[5]. In other words, trader 1 manipulates the price by trading against his private signal. If his signal is positive (negative), he sells (buys) the stock to push down (up) the price first since market is less liquid in period 1 and then trades according to the direction of the signal in the next period. He might incur a loss first but has higher expected future gain in period 2. This strategy intends to diminish the learning capability of other market participant. However, due to trader 1's incentive to reap some profit in period 1 on the information released to

[3]By the second order conditions for the equilibrium.
[4]Disclosure requirements by the exchange could be one reason.
[5]The denominator is positive by the second order conditions for the equilibrium.

trader 2 when he is an information monopolist, his period 1 order is always positively[6] related to trader 2's information (the second term in (3.2)).

Equation (3.4) outlines the order submission strategy for trader 2. He submits an order based on the difference between the expected true value of the stock known to him and the current price. The order submission strategy in period 2 of trader 1 is slightly different from that of trader 2 because he has an advantage of having better information. His demand consists of 2 parts. The first part is based on the difference between his more precise signal and the signal he shares with trader 2. The intensity at which he trades, based on his own private information, is $\frac{1}{2\lambda_2}$. The second part of trader 1's order is identical to trader 2's demand in period 2 which is related to the difference between the signal that he shares with trader 2 and the current stock price. If we compare the period 2 orders of both traders, we can see that the difference is the first term reflecting the better information trader 1 has over trader 2. In the limit case where both of them get the same information ($T = S$), they have the same order submission strategy in period 2 even though they get information at different times.

Because trader 1's trading strategies are functions of both signals T and S, if T and S are significantly different, trader 1 may have completely different trading strategies in period 1 and period 2. For example, it could be the case that trader 1 will sell (buy) the asset in period 1 and buy (sell) it back in period 2. This provides a possible explanation for contrarian trading that we observe in financial markets[7].

[6]By the second order condition.

[7]The public records of insider trades by SEC for the years 1994 to 1997 inclusive reveal 2,614

However, trader 1's period 1 order is always positively correlated to his information as shown in the following proposition.

Proposition 8 *The order of trader 1 in period 1 is positively correlated with his signals, i.e.,*

$$cov(x_1, T) > 0,$$

$$cov(x_1, S) > 0.$$

The proof of this proposition is given in the appendix.

At the first glance, this result looks odd. How is it that the correlations are positive and yet trader 1 may trade against his private signal T in period 1? Recall that the period 1 demand of trader 1 depends not only on his private signal T but also on the signal S that trader 2 will get. In addition, T and S are positively correlated and the covariance between T and S is equal to the variance of T. Therefore, if the intensity at which trader 1 trades on signal S is greater than the intensity at which he trades against his own signal T then his demand in the first period is still positively correlated with T even though he trades against T. This proposition confirms that if trader 1 trades against his own signal T then the intensity at which he trades against it is always less than the intensity at which he trades on the signal that he shares with trader 2. Moreover, it can be shown that in trader 1's demand function in period 1, the trading intensity on S is always greater than that on T. This result tells us that

cases in which insiders traded several times in a year. Of those, 15 percent engaged in both buying and selling in the same year.

even if trader 1 does not trade against his own signal, he still manipulates the price by trading more intensely on trader 2's signal and less intensely on his own. This is intended to minimize the amount of learning about his private information by other market participants.

Since trader 2 gets information late, he cannot manipulate the price as trader 1 might do. From his optimal demand (equation(3.4)), we can easily see that his order is positively related to the signal S he gets.

Price Behavior

This subsection describes some relevant relationships between prices, price moves and the private information variables.

Proposition 9 *The price moves*[8] *are always positively correlated with the private information of trader 1, i.e.,*

$$cov(p_1, T) > 0, \tag{3.7}$$

$$cov(p_2 - p_1, T) > 0. \tag{3.8}$$

Moreover, if the signal S shared between two informed traders is not very imprecise ($\Sigma_\epsilon \leqslant \Sigma_v + \Sigma_t$) then the price moves are also positively correlated with trader 2's signal, i.e.,

$$cov(p_1, S) > 0,$$

$$cov(p_2 - p_1, S) > 0.$$

[8] Before the market opens in period 1, the price is equal to zero.

The proof of this proposition is provided in the Appendix. The implication is that more and more information about the asset liquidation value is impounded into prices over time via trades. Our result is consistent with that of Hirshleifer, Subrahmanyam, and Titman (1994) in the context of competitive equilibrium.

Proposition 10 *Subsequent price changes are positively correlated if the information shared between two informed traders is sufficiently precise and the volatility of liquidity trades is sufficiently low, i.e.,*

$$cov(p_2 - p_1, p_1) > 0,$$

if Σ_ϵ *and* Σ_u *are sufficiently small.*

Proof: See appendix.

This result is in favor of technical analysis. However, if it is exploited, informed traders could change their behavior and the result may not hold anymore. Our result differs from that of Hirshleifer et al (1994). Indeed, Hirshleifer et al show that in a rational expectation equilibrium (where all traders behave competitively) successive price changes are uncorrelated and therefore technical analysis has no value. This happens because in their model equilibrium prices are unbiased conditional on all available public information.

3.4 Informational Efficiency and Price Informativeness in the Equilibrium

In the informational structure analyzed above, trader 1 receives in period 1 information about the liquidation value of the stock and part of this information is released to trader 2 in period 2. This section compares this to the benchmark case where no information is released to trader 2. That is trader 1 is the information monopolist throughout the game. In this section we address the issues of whether the information revelation makes the price more or less informationally efficient and how the information revelation affects the informativeness of the price process.

As it is usually defined, prices are strong-form efficient if they reflect all private information, semi-strong form efficient if they reflect all publicly available information, and weak-form efficient if they reflect the information in their own past values. In our model, since some investors trade to exploit their superior information while others trade for liquidity reasons, the price is, in general, not efficient. However, we can distinguish between more and less efficient markets.

Definition 11 *The reciprocal of the variance* $Var[E(\tilde{v} \mid \{p_\tau\}_{\tau\leq t}, T, S) \mid \{p_\tau\}_{\tau\leq t}]$ *measures the degree of informational efficiency at time t and the reciprocal of the conditional variance* $Var^{-1}[\tilde{v} \mid \{p_\tau\}_{\tau\leq t}]$ *measures how informative the price process is.*

Using these measures, we examine how the information released to trader 2 affects the informational efficiency and the informativeness of the price.

Proposition 12 *Information revelation makes the price process less informative and less informationally efficient if the information revealed to trader 2 is sufficiently imprecise, i.e.* Σ_ϵ *is sufficiently large.*

Proof: See Appendix.

The revelation of information to trader 2 makes the price p_1 less informative if $Var(\epsilon) = \Sigma_\epsilon$ is large. Recall that in period 1 trader 1 manipulates the price by placing an order which is positively related to the information revealed to trader 2. When this information (signal S) is very imprecise, i.e. Σ_ϵ is large, trader 1 exploits it by increasing his manipulative trading. His order, therefore, reflects more about signal S. Since the market maker sets a price on the basis of the aggregate order flow, the price will reflect more signal S. This is the reason that the price is uninformative. This happens again in period 2 when trader 1 unwinds his manipulative position in period 1 which is based on signal S. The result is that the price is uninformative since it reflects too much of the imprecise signal. On the other hand, if there is no information revealed to trader 2, trader 1 trades solely on his information T. There is no manipulative trading in this case. As a result, the market maker as well as the public can infer more information from the aggregate order flow w_1 and therefore, the price is more informative.

However, for small Σ_ϵ this might not be the case since it is more difficult for trader

1 to manipulate the stock price in the presence of trader 2 in period 2. Manipulative trading is small and the profit from manipulation is also small in this case. The price, therefore, does not reflect much the imprecise signal S. In the limit case when $\Sigma_\epsilon = 0$, trader 1 does not manipulate.

Information revealed also makes the market less informationally efficient. The reason is that if the information released to trader 2 is very imprecise, trader 1 would exploit the imprecision by trading heavily on that signal and therefore maintain his information superiority over the market maker. This explains the inefficiency of the price process in period 1. In period 2, the price is set based on the period 1 price and the order flow in period 2 therefore it is inefficient. Again we note that this might not be true if the released information is sufficiently precise.

3.5 Concluding Remarks

This chapter examines trading behavior, price behavior, and the informational efficiency and the informativeness of the price process in a PBE of a strategic trading game when some investors receive information before others. We shows that after getting information, the early informed investor trades on both the signals: his own signal about the liquidation of the asset and the noisier signal which will be revealed to the late informed investors. He may trade against his private information in the first period to move the price in his favor and unwinds his position in the next period. Moreover, if trader 2's information and trader 1's private information are significantly different then it could be the case that trader 1 has dramatically different trading strategies in period 1 and period 2 of the model. This could be an explanation for contrarian trading in financial markets.

We also shows that the price moves are always positively correlated with the private signals received by informed traders. More interestingly, we discover that under some conditions, subsequent price changes are positively correlated. This result is in favor of technical analysis: it is possible for a trader to profitably "trend chase", i.e., to systematically earn profit by trading based on earlier price moves alone. However, it may not be exploitable. Otherwise, informed traders may change their behavior and the result may not hold anymore. In addition, the analysis shows that information released to late-informed traders may make the price process less informative and less efficient.

3.6 Appendix

Proof of Proposition 7

In period 2, trader 2's problem is to maximize his expected profit given his information.

$$Max_y E[y(\widetilde{v} - p_2) \mid S, p_1].$$

The solution to this optimization problem is

$$y = \frac{S - p_1}{3\lambda_2} \tag{3.9}$$

if the second order condition

$$\lambda_2 > 0$$

is satisfied.

Trader 1 solves the following optimization problem to determine his demand in period 2.

$$Max_{x_2} E[x_2(\tilde{v} - p_2) \mid T, S, p_1].$$

Using calculus, the optimal period 2 demand of trader 1 satisfies

$$x_2 = \frac{T - p_1}{2\lambda_2} - \frac{y}{2}. \tag{3.10}$$

From (3.9) and (3.10) we have

$$x_2 = \frac{T - S}{2\lambda_2} + \frac{S - p_1}{3\lambda_2}.$$

The value function of trader 1 in period 2 is

$$V_2^1(p_1) = \frac{1}{4\lambda_2}(T - p_1 - \frac{S - p_1}{3})^2 = \frac{1}{4\lambda_2}[T - S + \frac{2(S - p_1)}{3}]^2.$$

In period 1, trader 1 chooses x_1 to solve the following problem

$$\max_{x_1} E[x_1(\tilde{v} - p_1) + V_2^1(p_1) \mid T, S].$$

$$\Longrightarrow x_1 = \frac{9\lambda_2 - 3\lambda_1}{2\lambda_1(9\lambda_2 - \lambda_1)}T + \frac{1}{2(9\lambda_2 - \lambda_1)}S.$$

The second order condition is

$$\lambda_1(1 - \frac{\lambda_1}{9\lambda_2}) > 0.$$

$$\Longrightarrow \lambda_1 > 0 \text{ and } 9\lambda_2 - \lambda_1 > 0. \tag{3.11}$$

In period 1, the pricing rule of the market maker is

$$p_1 = \lambda_1 w_1 = E(\tilde{v}|\, w_1) = \frac{cov(\tilde{v}, w_1)}{var(w_1)} w_1.$$

This implies

$$\lambda_1 = \frac{cov(\tilde{v}, w_1)}{var(w_1)}. \tag{3.12}$$

$$\Longrightarrow \lambda_1 = \frac{\alpha \Sigma_v}{\alpha^2 \Sigma_v + \beta^2 \Sigma_\epsilon + \gamma^2 \Sigma_t + \Sigma_u}. \tag{3.13}$$

where $\alpha = \frac{9\lambda_2 - 2\lambda_1}{2\lambda_1(9\lambda_2 - \lambda_1)}$, $\beta = \frac{1}{2(9\lambda_2 - \lambda_1)}$ and $\gamma = \frac{9\lambda_2 - 3\lambda_1}{2\lambda_1(9\lambda_2 - \lambda_1)}$.

In period 2, the pricing rule is

$$p_2 = E(\tilde{v}|\, w_1, w_2) = E(p_1 + \lambda_2 w_2 \mid w_1, w_2).$$

Therefore,

$$\lambda_2 = \frac{cov(\tilde{v} - p_1, w_2)}{var(w_2)}.$$

$$\Longrightarrow \lambda_2 = \frac{\frac{2}{3\lambda_2} var(\tilde{v} - p_1) - \frac{\lambda_1 \beta}{6\lambda_2} \Sigma_\epsilon - \frac{2\lambda_1 \gamma}{3\lambda_2} \Sigma_t}{\frac{4var(\tilde{v} - p_1)}{9\lambda_2^2} + \frac{\Sigma_\epsilon}{36\lambda_2^2} + \frac{4}{9\lambda_2^2} \Sigma_t + \Sigma_u - \frac{2\lambda_1 \beta}{9\lambda_2^2} \Sigma_\epsilon - \frac{8\lambda_1 \gamma}{9\lambda_2^2} \Sigma_t}.$$

$$\Longrightarrow \lambda_2 = \sqrt{\frac{8var(\tilde{v} - p_1) + (2\lambda_1 \beta - 1)\, \Sigma_\epsilon + 8\, (\lambda_1 \gamma - 2)\, \Sigma_t}{36 \Sigma_u}}.$$

Proof of Proposition 8

$$x_1 = \frac{9\lambda_2 - 3\lambda_1}{2\lambda_1(9\lambda_2 - \lambda_1)} T + \frac{T + \epsilon}{2(9\lambda_2 - \lambda_1)} = \frac{9\lambda_2 - 2\lambda_1}{2\lambda_1(9\lambda_2 - \lambda_1)} T + \frac{\epsilon}{2(9\lambda_2 - \lambda_1)}. \tag{3.14}$$

Thus,

$$cov(x_1, T) = \frac{9\lambda_2 - 2\lambda_1}{2\lambda_1(9\lambda_2 - \lambda_1)} (\Sigma_v + \Sigma_t) = \alpha (\Sigma_v + \Sigma_t) .$$

Since $(9\lambda_2 - \lambda_1) > 0$ by (3.11) and (3.13), $cov(x_1, T) > 0$.

Similarly,

$$cov(x_1, S) = \frac{9\lambda_2 - 2\lambda_1}{2\lambda_1(9\lambda_2 - \lambda_1)} (\Sigma_v + \Sigma_t) + \frac{\Sigma_\epsilon}{2(9\lambda_2 - \lambda_1)} > 0.$$

Proof of Proposition 9

Using (3.14) we have

$$p_1 = \lambda_1(x_1 + u_1) = \frac{9\lambda_2 - 2\lambda_1}{2(9\lambda_2 - \lambda_1)} T + \frac{\lambda_1 \epsilon}{2(9\lambda_2 - \lambda_1)} + \lambda_1 u_1. \tag{3.15}$$

$$\Longrightarrow cov(p_1, T) = \frac{9\lambda_2 - 2\lambda_1}{2(9\lambda_2 - \lambda_1)} (\Sigma_v + \Sigma_t) .$$

Since $(9\lambda_2 - \lambda_1) > 0$ by (3.11) and (3.13), $cov(p_1, T) > 0$.

Similarly,

$$cov(p_1, S) = \frac{9\lambda_2 - 2\lambda_1}{2(9\lambda_2 - \lambda_1)} (\Sigma_v + \Sigma_t) + \frac{\lambda_1}{2(9\lambda_2 - \lambda_1)} \Sigma_\epsilon > 0.$$

For the second part, from (3.1) we have:

$$p_2 - p_1 = \lambda_2(x_2 + y + u_2) = \frac{T - p_1}{2} + \frac{S - p_1}{6} + \lambda_2 u_2 = \frac{2}{3} T - \frac{2}{3} p_1 + \frac{\epsilon}{6} + \lambda_2 u_2. \tag{3.16}$$

Substituting (3.15) into (3.16) we get

$$p_2 - p_1 = \frac{2}{3} T - \frac{9\lambda_2 - 2\lambda_1}{3(9\lambda_2 - \lambda_1)} T - \frac{\lambda_1 \epsilon}{3(9\lambda_2 - \lambda_1)} - \frac{2}{3} \lambda_1 u_1 + \frac{\epsilon}{6} + \lambda_2 u_2$$

which leads to

$$cov(p_2 - p_1, T) = \frac{9\lambda_2}{3(9\lambda_2 - \lambda_1)} (\Sigma_v + \Sigma_t) > 0.$$

Therefore,

$$cov(p_2 - p_1, S) = cov(p_2 - p_1, T + \epsilon) = \frac{9\lambda_2 (\Sigma_v + \Sigma_t)}{3(9\lambda_2 - \lambda_1)} - \frac{\lambda_1 \Sigma_\epsilon}{3(9\lambda_2 - \lambda_1)} + \frac{\Sigma_\epsilon}{6}. \quad (3.17)$$

If $\Sigma_\epsilon \leqslant \Sigma_v + \Sigma_t$ then

$$cov(p_2 - p_1, S) > \frac{9\lambda_2 - \lambda_1}{3(9\lambda_2 - \lambda_1)} \Sigma_v + \frac{\Sigma_\epsilon}{6} = \frac{\Sigma_v}{3} + \frac{\Sigma_\epsilon}{6} > 0. \quad (3.18)$$

Proof of proposition 10

Since

$$p_2 - p_1 = \lambda_2 w_2 = \frac{9\lambda_2}{3(9\lambda_2 - \lambda_1)} T - \frac{\lambda_1 \epsilon}{3(9\lambda_2 - \lambda_1)} - \frac{2}{3}\lambda_1 \tilde{u}_1 + \frac{\epsilon}{6} + \lambda_2 \tilde{u}_2,$$

$$p_1 = \frac{9\lambda_2 - 2\lambda_1}{2(9\lambda_2 - \lambda_1)} T + \frac{\lambda_1 \epsilon}{2(9\lambda_2 - \lambda_1)} + \lambda_1 \tilde{u}_1,$$

we have

$$cov(p_2 - p_1, p_1) = \frac{9\lambda_2(9\lambda_2 - 2\lambda_1)}{6(9\lambda_2 - \lambda_1)^2} (\Sigma_v + \Sigma_t) - \frac{\lambda_1^2 \Sigma_\epsilon}{6(9\lambda_2 - \lambda_1)^2} + \frac{\lambda_1 \Sigma_\epsilon}{12(9\lambda_2 - \lambda_1)} - \frac{2\lambda_1^2}{3} \Sigma_u.$$

Therefore,

$$cov(p_2 - p_1, p_1) = \frac{9\lambda_2(9\lambda_2 - 2\lambda_1)}{6(9\lambda_2 - \lambda_1)^2} (\Sigma_v + \Sigma_t) + \frac{\lambda_1(9\lambda_2 - 3\lambda_1)}{12(9\lambda_2 - \lambda_1)^2} \Sigma_\epsilon - \frac{2\lambda_1^2}{3} \Sigma_u.$$

Thus, $cov(p_2 - p_1) > 0$ if Σ_ϵ and Σ_u are sufficiently small.

Proof of proposition 12

First we state the following lemma which is a well-known application of the projection theorem for normally distributed random variables. Several results in this proof are based on this lemma.

Lemma 13 *Let $x_0, x_{1,}, ..., x_n$ be normally and independently distributed random variables with zero means and variances $t_0^{-1}, t_1^{-1}, ..., t_n^{-1}$. Defining t^* by*

$$t^* = var^{-1}(x_0 \mid x_0 + x_1, ..., x_0 + x_n)$$

we have

$$t^* = t_0 + t_1 + ... + t_n$$

and

$$E(x_0 \mid x_0 + x_1, ..., x_0 + x_n) = \sum_{i=1}^{n} \frac{t_i}{t^*}(x_0 + x_i).$$

First we examine the case where there is no information leaked to trader 2. This case is similar to Kyle's (1985) model. We use backward induction to calculate the informed trader's demand in both periods. We start by postulating that the price process has the following form.

$$p_1 = \bar{\lambda}_1 \, w_1,$$

$$p_2 = p_1 + \bar{\lambda}_2 \, w_2,$$

In period 2, the informed trader solves the following problem

$$Max_{x_2} E[x_2(\tilde{v} - p_2) \mid T]$$

$$\Rightarrow x_2 = \frac{T - p_1}{2\,\bar{\lambda}_2}.$$

The second order condition is

$$\bar{\lambda}_2 > 0.$$

The value function of the informed trader in period 2:

$$V = \frac{T-p_1}{2\,\bar{\lambda}_2}\left(T-p_1-\frac{T-p_1}{2}\right) = \frac{(T-p_1)^2}{4\,\bar{\lambda}_2}.$$

The informed trader's problem in period 1 is

$$Max_{x_1} E\left[x_1\left(\tilde{v} - p_1\right) + V \mid T\right]$$

$$\Rightarrow x_1 = \frac{2\,\bar{\lambda}_2 - \bar{\lambda}_1}{\bar{\lambda}_1\left(4\,\bar{\lambda}_2 - \bar{\lambda}_1\right)} T = \bar{\alpha}\,T \quad where\ \bar{\alpha} = \frac{2\,\bar{\lambda}_2 - \bar{\lambda}_1}{\bar{\lambda}_1\left(4\,\bar{\lambda}_2 - \bar{\lambda}_1\right)} \tag{3.19}$$

The second order condition is

$$0 < \bar{\lambda}_1 < 4\,\bar{\lambda}_2\,. \tag{3.20}$$

Market maker's pricing function is

$$p_1 = E\left(\tilde{v} \mid x_1 + u_1\right) = \bar{\lambda}_1\left(x_1 + u_1\right).$$

$$\Rightarrow \bar{\lambda}_1 = \frac{cov\left(\tilde{v}, x_1\right)}{var\left(x_1 + u_1\right)} = \frac{\bar{\alpha}\,\Sigma_v}{\bar{\alpha}^2\left(\Sigma_v + \Sigma_t\right) + \Sigma_u}. \tag{3.21}$$

From equations (3.19), (3.20) and (3.21), we get:

$$\bar{\alpha} > 0.$$

In period 2, we have

$$p_2 - p_1 = E\left(\tilde{v} - p_1 \mid w_2\right) = E\left(\tilde{v} - p_1 \mid \frac{T-p_1}{2\,\bar{\lambda}_2} + u_2\right) = \bar{\lambda}_2\,w_2$$

$$\Rightarrow \bar{\lambda}_2 = \frac{var(\tilde{v} - p_1)}{2\,\bar{\lambda}_2\left[\frac{var(\tilde{v} - p_1)}{4\bar{\lambda}_2^2} + \Sigma_u\right]} = \frac{2\,\bar{\lambda}_2\,var\left(\tilde{v} - p_1\right)}{var\left(\tilde{v} - p_1\right) + 4\,\bar{\lambda}_2^2\,\Sigma_u}$$

$$\Rightarrow \bar{\lambda}_2^2 = \frac{var\left(\tilde{v} - p_1\right)}{4\Sigma_u}.$$

For *price informativeness*, first we consider the case where there is no information leaked to trader 2. We have

$$var\left(\tilde{v}|\, p_1\right) = var\left(\tilde{v}|\, w_1\right),$$

$$\frac{w_1}{\bar{\alpha}} = T + \frac{u_1}{\bar{\alpha}} = \tilde{v} + t + \frac{u_1}{\bar{\alpha}},$$

$$var\left(t + \frac{u_1}{\bar{\alpha}}\right) = \Sigma_t + \frac{\Sigma_u}{\bar{\alpha}^2} \Rightarrow var^{-1}\left(t + \frac{u_1}{\bar{\alpha}}\right) = \frac{1}{\Sigma_t + \frac{\Sigma_u}{\bar{\alpha}^2}}.$$

Applying lemma 8 we get

$$var^{-1}\left(\tilde{v}|\, p_1\right) = \Sigma_v^{-1} + \frac{1}{\Sigma_t + \frac{\Sigma_u}{\bar{\alpha}^2}}. \tag{3.22}$$

If there is information leaked to trader 2 then

$$var\left(\tilde{v}|\, p_1\right) = var\left(\tilde{v}|\, w_1\right).$$

Equation (??) implies:

$$\frac{w_1}{\alpha} = \tilde{v} + \frac{\beta}{\alpha}\epsilon + \frac{\gamma}{\alpha}t + \frac{u_1}{\alpha}$$

$$\Longrightarrow var^{-1}\left(\frac{w_1}{\alpha} - \tilde{v}\right) = \frac{\alpha^2}{\beta^2\Sigma_\epsilon + \gamma^2\Sigma_t + \Sigma_u}.$$

Using lemma 9 we get

$$var^{-1}\left(\tilde{v}|\, p_1\right) = \Sigma_v^{-1} + \frac{\alpha^2}{\beta^2\Sigma_\epsilon + \gamma^2\Sigma_t + \Sigma_u}. \tag{3.23}$$

Using (3.22) and (3.23), if Σ_ϵ is large, we have

$$\left[var^{-1}\left(\tilde{v}|\, p_1\right)\right]_{\text{leakage}} < \left[var^{-1}\left(\tilde{v}|\, p_1\right)\right]_{\text{Non leakage}}. \tag{3.24}$$

Thus, if the information leaked to trader 2 is sufficiently imprecise then p_1 is less informative if there is information leaked to trader 2.

For *informational efficiency*, in period 1, we have

$$var\left[E\left(\widetilde{v}|\, p_1, T, S\right) \mid p_1\right] = var\left(T \mid p_1\right) = var\left(\widetilde{v}|\, p_1\right) + var\left(t \mid p_1\right). \qquad (3.25)$$

From equations (3.22) and (3.23), we can see that $var\left(\widetilde{v}|\, p_1\right)_{\text{leakage}}$ is positively related to Σ_ϵ while $var^{-1}\left(\widetilde{v}|\, p_1\right)_{\text{non leakage}}$ is independent of Σ_ϵ. Therefore, using (3.24), if Σ_ϵ is sufficiently large, we have

$$var\left[E\left(\widetilde{v}|\, p_1, T, S\right) \mid p_1\right]_{\text{leakage}} > var\left[E\left(\widetilde{v}|\, p_1, T\right) \mid p_1\right]_{\text{non leakage}}.$$

This implies

$$var^{-1}\left[E\left(\widetilde{v}|\, p_1, T, S\right) \mid p_1\right]_{\text{leakage}} < var^{-1}\left[E\left(\widetilde{v}|\, p_1, T\right) \mid p_1\right]_{\text{non leakage}}.$$

In period 2 we have

$$var\left(\widetilde{v}|\, p_1, p_2\right) = var\left(\widetilde{v}|\, w_1, w_2\right) = var\left(\widetilde{v} - p_2\right). \qquad (3.26)$$

If there is information leakage, we have:

$$p_2 = p_1 + \lambda_2 w_2 = \frac{2}{3}\,\widetilde{v} + \frac{p_1}{3} + \frac{\epsilon}{6} + \frac{2t}{3} + \lambda_2 u_2,$$

$$p_1 = \lambda_1 w_1 = \lambda_1 \alpha\,\widetilde{v} + \lambda_1 \beta \epsilon + \lambda_1 \gamma t + \lambda_1 u_1,$$

Therefore,

$$\widetilde{v} - p_2 = \frac{1}{3}\left(1 - \lambda_1 \alpha\right)\widetilde{v} - \frac{1}{6}\left(1 + 2\lambda_1 \beta\right)\epsilon - \frac{1}{3}\left(2 + \lambda_1 \gamma\right) t - \frac{1}{3}\left(\lambda_1 u_1 + 3\lambda_2 u_2\right)$$

and

$$var\left(\widetilde{v}-p_2\right)=\frac{1}{9}\left(1-\lambda_1\alpha\right)^2\Sigma_v+\frac{1}{36}\left(1+2\lambda_1\beta\right)^2\Sigma_\epsilon+ \tag{3.27}$$
$$+\frac{1}{9}\left(2+\lambda_1\gamma\right)^2\Sigma_t+\left(\frac{\lambda_1^2}{9}+\lambda_2^2\right)\Sigma_u$$

If there is no information leakage, then

$$p_2=p_1+\bar{\lambda}_2\,w_2=\frac{T}{2}+\frac{p_1}{2}+\bar{\lambda}_2\,u_2,$$

$$p_1=\bar{\lambda}_1\,w_1=\bar{\lambda}_1\bar{\alpha}\,T+\bar{\lambda}_1\,u_1,$$

$$\widetilde{v}-p_2=\left(1-\frac{\bar{\lambda}_1\bar{\alpha}}{2}\right)\widetilde{v}-\frac{1}{2}\left(1+\bar{\lambda}_1\bar{\alpha}\right)t-\frac{\bar{\lambda}_1\,u_1}{2}-\bar{\lambda}_2\,u_2.$$

Therefore,

$$var\left(\widetilde{v}-p_2\right)=\left(1-\frac{\bar{\lambda}_1\bar{\alpha}}{2}\right)^2\Sigma_v+\frac{1}{4}\left(1+\bar{\lambda}_1\bar{\alpha}\right)^2\Sigma_t+\left(\frac{\bar{\lambda}_1^2}{4}+\bar{\lambda}_2^2\right)\Sigma_u. \tag{3.28}$$

From (3.27) and (3.28) we get

$$\left[var^{-1}\left(\widetilde{v}\,|\,p_1,p_2\right)\right]_{\text{leakage}}<\left[var^{-1}\left(\widetilde{v}\,|\,p_1,p_2\right)\right]_{\text{non leakage}}$$

if Σ_ϵ is sufficiently large.

For *informational efficiency*, in period 2, we have

$$var\left[E\left(\widetilde{v}\,|\,p_1,p_2,T,S\right)\,|\,p_1,p_2\right]=var\left(T\mid p_1,p_2\right)=var\left(\widetilde{v}\,|\,p_1,p_2\right)+var\left(t\mid p_1,p_2\right). \tag{3.29}$$

From equations From (3.26), (3.27) and (3.28), we can see that $var\left(\widetilde{v}|\ p_1, p_2\right)_{\text{leakage}}$ is positively related to Σ_ϵ while $var^{-1}\left(\widetilde{v}|\ p_1, p_2\right)_{\text{non leakage}}$ is independent of Σ_ϵ. Therefore, using (3.29), if Σ_ϵ is sufficiently large, we have

$$var\left[E\left(\widetilde{v}|\ p_1, p_2, T, S\right)\ |\ p_1, p_2\right]_{\text{leakage}} > var\left[E\left(\widetilde{v}|\ p_1, p_2, T\right)\ |\ p_1, p_2\right]_{\text{non leakage}}.$$

This implies

$$var^{-1}\left[E\left(\widetilde{v}|\ p_1, p_2 T, S\right)\ |\ p_1, p_2\right]_{\text{leakage}} < var^{-1}\left[E\left(\widetilde{v}|\ p_1, p_2, T\right)\ |\ p_1, p_2\right]_{\text{non leakage}}.$$

Chapter 4

Limit Orders and the Intraday Behavior of Market Liquidity: Evidence from the Toronto Stock Exchange

4.1 Introduction

Limit orders play a very important role in providing liquidity to the world stock exchanges of various market architectures. In an order-driven market, such as the Toronto Stock Exchange (TSE), the Paris Bourse, or the Tokyo Stock Exchange, limit orders provide all liquidity to the market. In a specialist market, such as the

New York Stock Exchange, a large amount of liquidity comes from limit orders[1]. In a dealership market, such as Nasdaq, some types of limit order trading have been used recently[2]. This chapter examines the impacts of price volatility, trading activity, and trading volume on the liquidity in an order-driven market where limit orders play a vital role in liquidity provision.

In a pure order-driven market, investors can submit limit orders or market orders. Limit orders are kept in a limit order book waiting for execution. If a trade takes place, a limit order is executed at a better price than a market order. However, there are risks associated with it. First, a limit order might not be executed. Second, as limit order prices are fixed, there is an adverse selection risk due to the arrival of informed traders. Market orders, on the other hand, are executed with certainty at the bid and ask prices established through previously placed limit orders. However, the execution price may not be favorable.

Glosten (1994) examines an equilibrium model in which there are 2 types of traders. The patient traders place limit orders and therefore supply liquidity to the market. The urgent traders, on the other hand, place market orders and consume liquidity. Informed traders are more likely to be urgent than patient because they want to exploit their super information[3]. Glosten shows that patient traders would not place limit orders unless the expected gains from trading with liquidity traders

[1]Harris and Hasbrouck (1996) report that limit orders account for 54 percent of all order submitted through SuperDot. Ross, Shapiro and Smith (1996) document that limit orders account for 65 percent of all executed orders and 75 percent of executed shares in SuperDot system.

[2]Market makers in Nasdaq are required to display limit orders.

[3]One of the reasons is the value of private information depreciates over time.

exceeded the expected loss from trading with informed traders. However, his model does not endogenize the traders' choice between market and limit orders. Handa and Schwartz (1996) extend Glosten's analysis by examining the investors' rational choice between market and limit orders. The choice depends on the investor's beliefs about the probability of his orders being executed against an informed or a liquidity trader. Handa and Schwartz show that in an order-driven market, if the price is very volatile investors submit more limit orders than market orders because the expected gains from providing liquidity to the market exceed the potential loss from trading with informed traders. Foucault (1999) shows that the price volatility is the main determinant of the mix between market and limit orders. Indeed, if asset price is very volatile, the probability of trading against informed investors and the expected loss to them are larger. Limit order traders have to post higher ask price and lower bid price. This establishes a direct relationship between bid-ask spread and price volatility. Moreover, when price is volatile, market orders become less attractive than limit orders; as a result, more limit orders than market orders are placed.

According to Harris (1990) liquidity is the willingness of some traders to take the opposite side of a trade that is initiated by someone at low cost. Thus, market liquidity has 2 dimensions: the price dimension, represented by spread, and the quantity dimension, represented by market depth. On the TSE, a complete quote comprises the bid price, the ask price and the depth which is the number of shares available at each price. If liquidity providers believe that there is an increase in the probability of informed trades in the market, they may respond by widening bid-ask spread and/or by quoting less depth at each price. This implies a negative relationship between spread and depth.

On the empirical side, Kavajecz (1999) compares the limit order book spread with the quoted spread of specialists. He finds that specialists play a vital role in narrowing the bid-ask spread, especially for less frequently traded stocks. Chung et al (1999) examine the roles of limit order traders and specialists in NYSE and find that the U-shaped intra-day pattern of spreads mostly reflects the intra-day behavior of spreads established by limit order traders. Lee, Mucklow and Ready (1993) show that in the NYSE wide spreads are accompanied by low depths and the liquidity falls in response to high volume and anticipation of earnings announcements.

In this chapter, we examine the empirical relation between bid-ask spreads and quoted depths, and the relations among market liquidity, trading activity, price volatility, and trading volume on the TSE, an order-driven market, where all liquidity is provided by limit order traders. We find that there is an inverse relationship between spread and depth. Thus, as in a specialist market, limit order traders in an order driven market use both dimensions of market liquidity to protect themselves from informed traders. We also show that the liquidity is directly related to the volume of trade. Our finding is inconsistent with the prediction of Easley and O'Hara (1992) who show that specialists use trading volume to infer the presence of informed traders. Thus, high volume should be accompanied by wide spread and low depth.

However, our finding is consistent with the alternative hypothesis, suggested by Harris and Raviv (1993), that because of the differences of opinion among investors, high volume may mainly reflect high liquidity trades and therefore the market is more liquid. In other words, there might be a positive relationship between market liquidity and trading volume.

In addition, we show that market liquidity is inversely related to price volatility. This finding is consistent with the prediction of Handa and Schwartz (1996). We also find that the trading activity which is represented by the number of transactions is negatively related to market liquidity. Harris (1987) shows that the number of trades could have an inverse relationship with price volatility if it reflects the rate of information flow and this extends to liquidity. However, Madhavan (1992) suggests that given trading volumes, the number of trades may be positively related to liquidity.

This chapter proceeds as follows. Section 4.2 describes the market and the dataset. Section 4.3 examines the intraday pattern of depth, spread and volume of TSE stocks. Section 4.4 presents the empirical relations among spread, depth, volume, price volatility, and trading activity. Section 4.5 provides some concluding remarks.

4.2 Description of the Market and the Dataset

The TSE has become a pure order-driven market since 1997 when it closed its trading floor. Bid and ask prices are determined by limit buy and sell orders in the absence of specialists. Limit order traders submit their orders to the electronic open book system, which is known as the Computer Assisted Trading System (CATS), through brokers. Limit orders are kept in the system and are executed using strict price and time priority. Trading is conducted on weekdays, Monday to Friday, excluding public holidays. Each trading day commences at 9:30 and ends at 16:00.

Information on the five best bid and ask prices and the corresponding depths is disseminated to the public on the real time basis. Large order traders have the option of not disclosing the part of the order which exceeds 5,000 shares. However, traders might want to make public their orders since the TSE gives priority to disclosed orders over undisclosed orders at the same price[4]. For each transaction, the identity of the buyer and the seller is also known to the public. The TSE is as transparent as the Hong Kong Stock Exchange and the Paris Bourse but more transparent than the Tokyo Stock Exchange where only the member of the lead offices can observe orders. This is probably because there is no consensus about the relation between transparency and liquidity.

The dataset is obtained from the Intraday Equity Trades and Quotes Record of the Toronto Stock Exchange. For each transaction, the dataset reports the execution time to the nearest second, the price and the quantity exchanged. For each quote, it reports the posting time, the best bid and ask prices and the quantity demanded or offered at those prices.

In order to have enough observations necessary for intraday time series analysis,

[4]This description of the TSE is drawn from Jiang and Kryzanowski (1998).

we focus on 31 most actively traded component stocks in the TSE 35 Index between September 1, 1999 and November 1, 1999. The list of those stocks is given in the appendix. Although it is the best available database for the current analysis, the dataset still has some limitations. In particular, we do not have order placement other than the best buying or selling limit orders. Therefore, the findings in this chapter need to be interpreted with some caution.

4.3 Intraday Patterns of Liquidity and Volume of TSE Stocks

4.3.1 The Microstructure Models for Intraday Variations in Liquidity

Several theoretical microstructure models attempt to explain the intraday variation of the bid-ask spread. In the inventory models[5], the spreads exist in order to compensate the specialists for the risk of holding undesired inventory. Specifically, the specialist adjusts bid and ask prices to go back to the optimal inventory position if the order imbalance moves him out of the desired position.

In the specialist market power models, Brock and Kleidon (1992) show that demand for transactions is less elastic and higher at the opening and closing than at the rest of the day. There are at least two reasons. First, the accumulation of overnight

[5] See Stoll (1978), Amihud and Mendelson (1980) (1982), Ho and Stoll (1981).

information may change investors' optimal portfolio. Second, at the closing, due to the imminent non-trading period, optimal portfolios can be different from the ones during the continuous trading periods. The specialist, therefore, can charge higher transaction price at those periods. This explains wide spreads at the opening and the closing of the trading day. This result can be extended to a pure order-driven market.

Information models[6] look at the adverse selection problem faced by the specialist who is at an informational disadvantage relative to informed traders. Therefore, the specialist must keep spreads wide enough so that the profits from trading with liquidity traders sufficiently compensate for the losses from trading with informed traders. In the model of Madhavan (1992), information asymmetry is gradually resolved during the trading day; therefore, spreads decline throughout the day.

While numerous studies have examined the intraday variations in bid-ask spreads during the last two decades or so, only recently have researchers begun to study the behavior and the determinants of quoted depths. Ye (1995) analyzes the optimal strategy of specialists and shows that when the probability of informed trades rises, specialists widen spreads and reduce depth at each quote. He also finds that specialists decrease depths in response to an increase in price volatility. Kavajecz (1999) shows that depths used by specialists as a strategic variable to reduce risk associated with information events.

A market maker in a specialist market and limit order traders in a pure order

[6]See Copeland and Galai (1983), Glosten and Milgrom (1985), Kyle (1985), Easley and O'Hara (1987), Madhavan (1992), Foster and Viswanathan (1994).

driven market supply liquidity and immediacy to the market. However, while the ultimate goal of the market maker is to provide an orderly and smooth market by continuously posting bid and ask quotes, limit order traders have much more freedom in their quotes. Intraday pattern of spreads and depths in a specialist market is the result of successive decisions by a single market maker while that in a pure order driven market is determined by many limit order traders. Nevertheless, due to the competition among limit order traders, we would expect that the pattern is the same in both market architectures.

4.3.2 Intraday Variation in Bid-Ask Spreads, Volumes, and Depths

In this section, we examine the intraday variation in spreads, depths, and volumes in a pure order-driven market. We partition each trading day into 13 successive 30-minute intervals and then calculate the average standardized spread, depth, and volume for each stock during each of the 30-minute intervals. The standardized value is obtained by subtracting the mean for the day from the value and dividing the difference by the standard deviation for the day for the respective stock. Table 4.1 reports the cross sectional mean values for standardized volumes, spreads, and depths for each 30-minute interval of the day.

Table 4.1

This table reports the mean values for the standardized trading volumes, bid-ask spreads and depths of 31 component stocks in the TSE35 index for each time interval during the day. The standardized variable is defined as $(X - \mu) / \sigma$, where X is the raw variable, μ is the mean of X for the day, and σ is the standard deviation of X for the day.

Time	SVolume	SSpread	SDepth
9:30-10:00	-0.031311	0.523855	-0.242232
10:01-10:30	0.006585	0.111237	-0.101693
10:31-11:00	-0.001707	0.005147	-0.009235
11:01-11:30	-0.003119	-0.088074	0.049991
11:31-12:00	-0.004632	-0.10714	0.038552
12:01-12:30	-0.007843	-0.119627	0.034728
12:31-13:00	-0.018975	-0.134378	0.005268
13:01-13:30	-0.024468	-0.127337	0.037791
13:31-14:00	-0.008385	-0.098485	0.056594
14:01-14:30	-0.008157	-0.129938	0.052364
14:31-15:00	-0.00025	-0.145834	0.076920
15:01-15:30	-0.001182	-0.133749	0.063579
15:31-16:00	0.0337	-0.084251	0.111489

The bid-ask spreads are highest at the beginning of the day, narrows until late morning and then increase very slightly during late afternoon. This result confirms the U-shaped pattern documented in many studies such as Chan et al (1995), Chung et al (1999) in a specialist market. Thus, along with other findings, our result suggests that the U-shaped pattern of spreads does not depend on the market architectures. Whether it is an order-driven market or a specialist market, bid-ask spreads display a U-shaped pattern . Indeed, Chung et al (1999) show that even though the market maker sets bid and ask prices in the NYSE, the U-shaped intraday pattern of spreads largely reflects the intraday variation in spreads established by limit order traders.

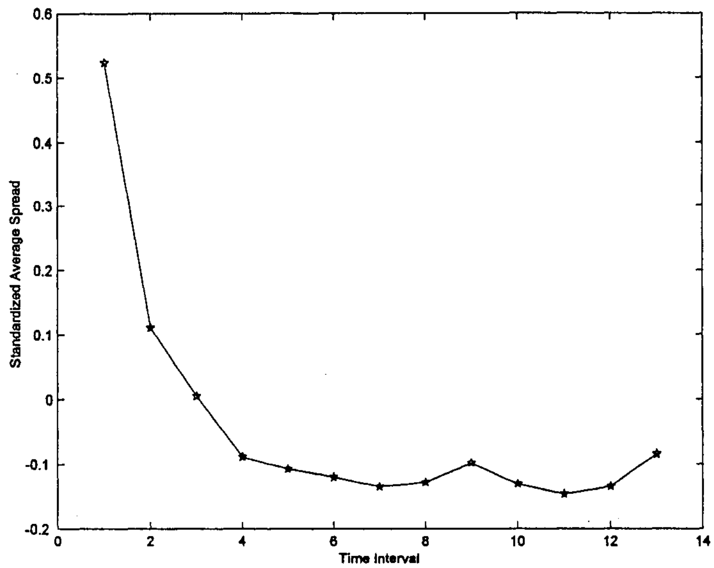

Figure 4.1: The Intraday Variation in the Standardized Spread. We define the standardized spread as $(S - \mu)/\sigma$, where S is the quoted spread, μ is the mean of quoted spread for the day, and σ is the standard deviation of the quoted spread for the day.

Trading volume intraday pattern differs from that of spreads. Volume is at the lowest when the market opens, increases significantly after half an hour, and then gradually decline until early afternoon. It increases for the rest of the day. However, the significant rise is in the last half hour of the trading day. Despite that, volume is rather stable relative to spread. Our result is different from that of Chan, Chung, and Johnson (1995) in the NYSE. Chan et al find that the intraday pattern of volume mimics that of spreads, i.e. U-shaped pattern. There are a couple of reasons for the lowest volume at the opening. First, uncertainty is high at the beginning of the day due to the overnight non-trading period. Second, trading costs are the highest at the beginning (since spread is widest when the market opens). Trading volume rises from early in the afternoon until the market closes.

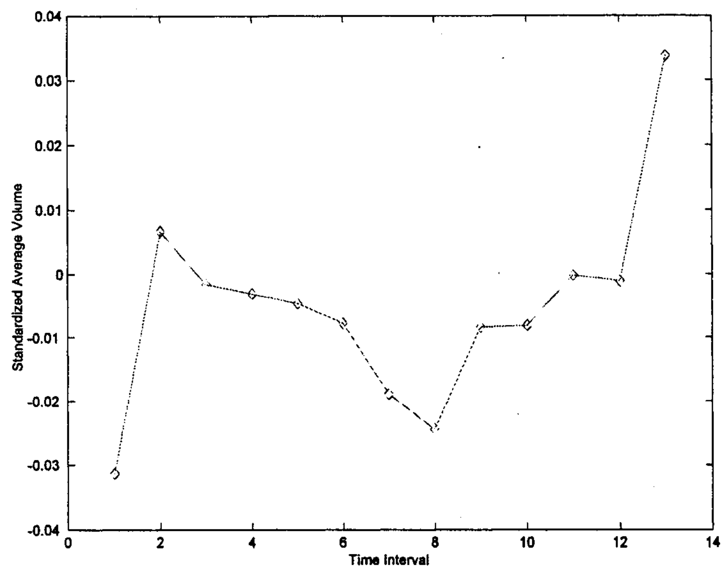

Figure 4.2: The Intraday Variation in the Standardized Volumes. We define the standardized volume as $(V - \mu)/\sigma$, where V is the volume of trade, μ is the mean of volume for the day, and σ is the standard deviation of the volume for the day.

Depths exhibit a completely different intraday pattern. The market is very thin at the open, then it becomes deeper and deeper in the next one and a half hour. After that it is rather stable. This pattern is consistent with information models which predict that the market is illiquid at the open (wide spread and low depth) due to high information asymmetry which results from the overnight non-trading period.

To formally examine the intraday behavior of bid-ask spreads, depths and volumes we use the following models.

$$STV_i = a_0 + a_1 D_1 + a_2 D_2 + a_3 D_3 + a_4 D_4 + a_5 D_5 + a_6 D_6 + \varepsilon_i, \qquad (4.1)$$

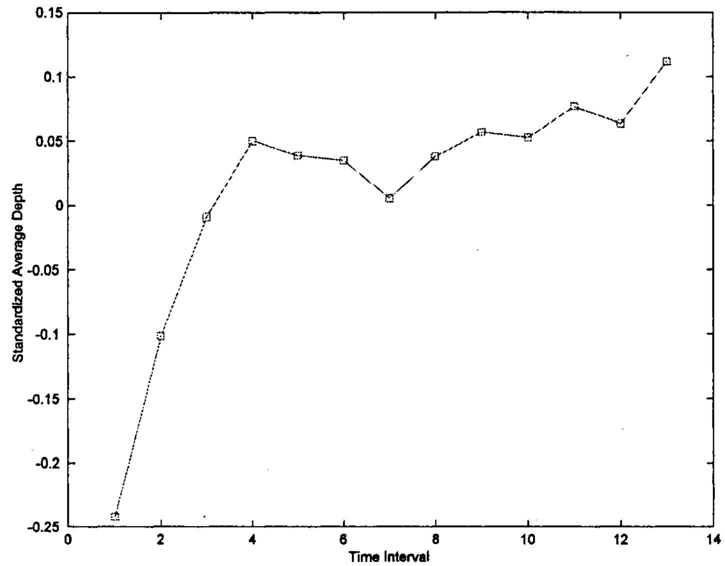

Figure 4.3: The Intraday Variation in the Standardized Depths. We define the standardized depth as $(D - \mu)/\sigma$, where D is the quoted depth, μ is the mean of quoted depth for the day, and σ is the standard deviation of quoted depth for the day.

where STV_i is the i^{th} observation of the standardized variable (bid-ask spread, depth or volume) of the stock and $D_1 - D_6$ are dummy variables.

$D_1 = 1$ if the interval is 9:30-10:00, and 0 otherwise;

$D_2 = 1$ if the interval is 10:01-10:30, and 0 otherwise;

$D_3 = 1$ if the interval is 10:31-11:00, and 0 otherwise;

$D_4 = 1$ if the interval is 14:31-15:00, and 0 otherwise;

$D_5 = 1$ if the interval is 15:01-15:30, and 0 otherwise;

$D_6 = 1$ if the interval is 15:31-16:00, and 0 otherwise.

α_0 measures the average of the standardized variable from 11:01 to 14:30 and $\alpha_1 - \alpha_6$ measure the difference between the mean of the standardized variable in the respective interval and the average of the standardized variable from 11:01 to 14:30.

We estimate equation (4.1) for each of 31 stocks using the Generalized Method of Moments (GMM) with Newey and West (1987) correction for serial correlation and heteroskedasticity. We obtain t-statistics which are robust to heteroskedasticity and autocorrelation. The results are reported in Table 4.2. For each dummy variable, we report the average coefficient from the regression of each individual stock and also the percentage of stocks with positive coefficient.

To test whether each coefficient is significantly greater than zero we use the procedure outlined in Meulbroek (1992). Specifically, assume that the individual stock regression t-statistics asymtotically follow a unit normal distribution, then the Z-statistic to test whether the mean regression coefficient for each dummy variable is greater than zero is given by

$$Z = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} t_i,$$

where N is the number of stocks. This test assumes that individual stocks are independent. If this assumption does not hold, our econometric specification is not perfect.

The results in Table 4.2 show that the bid-ask spreads are widest at the open, narrow during the day, and rise during the last 30-minute interval. During the two intervals before the last, the spreads are not significantly greater than the average spread during midday. Overall, our empirical results are inconsistent with the prediction of Madhavan (1992) in which private information is impounded into prices as trading continues; therefore, bid-ask spreads decline throughout the day. Our results appear consistent with the specialist market power models and the inventory models. However, since TSE is a pure order-driving exchange in which there are no market makers, the monopolistic behavior of specialists is not the explanation for the observed intraday patterns of the bid-ask spreads. An alternative explanation is that because of increased uncertainty at the open and the imminent non-trading period at the close, liquidity providers tend to increase (decrease) prices when submitting limit sell (buy) orders.

For transaction volumes, only α_1 is significantly less than zero, and α_6 is significantly greater than zero. $\alpha_2 - \alpha_5$ are not significantly different from zero. Thus, volumes are low during the first half hour at the open, then become stable during the day and rise in the last half hour of the trading day.

For market depths at quoted prices, $\alpha_1 - \alpha_3$ are significantly less than zero, suggesting that the market is thin at the open. Moreover, $\alpha_1 < \alpha_2 < \alpha_3$ tells us that the market becomes deeper as trading continues. However, the behavior at the end of the trading day is not clear. Even though $\alpha_6 > \alpha_5$ but $\alpha_5 < \alpha_4$. In addition, the percentage of stocks with positive coefficients is only 64.52 for α_4, 61.29 for α_5, and 67.74 for α_6. Thus, there is no clear evidence that the market depth at the end of the day differs from that at the middle of the day.

Table 4.2

This table presents the GMM estimates of the following model for each of the 31 component stocks in the TSE-35 index.

$$STV_i = a_0 + a_1 D_1 + a_2 D_2 + a_3 D_3 + a_4 D_4 + a_5 D_5 + a_6 D_6 + \varepsilon_i,$$

where STV_i is the i^{th} observation of the standardized variable (bid-ask spreads, depths or volumes) and $D_1 - D_6$ are dummy variables. $D_1 = 1$ if the interval is 9:30-10:00, and 0 otherwise; $D_2 = 1$ if the interval is 10:01-10:30, and 0 otherwise; $D_3 = 1$ if the interval is 10:31-11:00, and 0 otherwise; $D_4 = 1$ if the interval is 14:31-15:00, and 0 otherwise; $D_5 = 1$ if the interval is 15:01-15:30, and 0 otherwise; $D_6 = 1$ if the interval is 15:31-16:00, and 0 otherwise. Thus, α_0 measures the average of the standardized variables from 11:01 to 14:30 and $\alpha_1 - \alpha_6$ measure the difference between the mean spread in the respective interval and the average spread from 11:01 to 14:30.

		SSpread	SVolume	SDepth
D_1	Average	0.621264	-0.03172	-0.282695
	Positive coefficient (%)	96.77	22.58	9.68
	Z statistics	53.03	-7.6	-28.96
	p-value from Z statistic	0.0000	0.0000	0.0000
D_2	Average	0.217418	0.006175	-0.142156
	Positive coefficient (%)	96.77	45.16	12.9
	Z statistics	20.96	1.548198	-13.7093
	p-value from Z statistic	0.0000	0.060787	0.0000
D_3	Average	0.1135	-0.002116	-0.049697
	Positive coefficient (%)	87.1	48.39	32.26
	Z statistics	9.47	-0.301737	-4.41
	p-value from Z statistic	0.0000	0.38142	0.0000
D_4	Average	-0.038037	-0.000659	0.036457
	Positive coefficient (%)	32.26	58.06	64.52
	Z statistics	-4.01	-0.181401	3.05
	p-value from Z statistic	1.0000	0.428	0.00113
D_5	Average	-0.025375	-0.001591	0.023116
	Positive coefficient (%)	38.71	51.62	61.29
	Z statistics	-2.23	-0.720217	2.35
	p-value from Z statistic	0.9871	0.2357	0.01
D_6	Average	0.036085	0.033291	0.071026
	Positive coefficient (%)	64.52	83.87	67.74
	Z statistics	3.84	6.18	7.06
	p-value from Z statistic	0.0000	0.0000	0.0000

4.4 The Relation among Liquidity, Volume, and Price Volatility

4.4.1 The Theoretical Relation between Liquidity, Volume, and Price Volatility

Harris (1990) defines liquidity as follows. "A market is liquid if traders can buy or sell large number of shares when they want and at low transaction costs. Liquidity is the willingness of some traders (often but not necessarily dealers) to take the opposite side of a trade that is initiated by someone else, at low cost." This definition implies that spread and depth are two dimensions of market liquidity.

Lee Mucklow and Ready (1993), Ye (1995) argue that if the specialist believes that there is an increase in the probability of informed trades he would respond by increasing the bid-ask spread. Alternatively, he could reduce the depth at the quoted prices. Kavajecz (1999) shows that depth is used as a strategic variable by specialists to deal with risks associated with information events. Specifically, Kavajecz finds that liquidity providers, both market maker and limit order traders, reduce depths around earning announcements to decrease adverse selection costs. Logically, we can extend those reasonings to the case of order-driven market. On the TSE, a complete quote includes the bid and ask prices and the market depths which are the number of shares available at each quoted price. All liquidity in this exchange is provided by limit orders. If limit order traders believe that informed traders have arrived, they may respond by posting lower (higher) bid (ask) prices, and/or reducing the depth at each quoted price. This consideration leads us to the first hypothesis.

Hypothesis 1: *There is an inverse relationship between bid-ask spread and market depth.*

Handa and Schwartz (1996) argue that when transaction prices move solely in response to information, trading via limit order is suboptimal because investors who place a buy (sell) limit order have written a free put (call) option to the market. They show that if the price is very volatile, traders submit more limit orders than market orders because the expected gain exceed the expected loss from trading against informed traders. Foucault (1999) develops a model in which he endogenizes investors' decision to trade via limit orders or market orders. He finds that price volatility is a main determinant of limit orders. If the price volatility rises, the probability of trading against an informed trader increases; therefore, the expected losses to them are larger. To deal with the problem, limit order traders have to widen spreads by increasing (decreasing) ask (bid) prices and/or to reduce the quantity demanded / offered at each quoted price. Moreover, in this case, choosing market order is even a worse strategy because it is more likely that the order is executed at a poor price when the price volatility increases. Those considerations lead us to the second hypothesis.

Hypothesis 2: *There is an inverse relationship between price volatility and market liquidity.*

In Easley and O'Hara (1992) volume is a main determinant of spread. In particular, the spread exists because of the possibility of trading against informed traders. High trading volume is considered as an indication of the advent of informed traders. Initially, the market maker sets the spread based on the ex ante probability of informed trades and then increases (decreases) it if there is an abnormally high (low) volume of trade. Thus, this model predicts that high volume is accompanied by wide spread. The model does not consider market depth since a unit trade size is assumed. However, a logical extension of the model is that depth would decrease with volume. In addition, because the volume is positively related to the number of transactions, we will therefore have to control for the impact of the number of transaction in our empirical analysis. Although the model is developed in the context of a specialist market, we conjecture that the results also apply to a pure order-driven market. Those considerations lead us to the third hypothesis.

Hypothesis 3: *There is an inverse relationship between volume and market liquidity.*

4.4.2 Empirical Methodology

Time Interval

Each trading day commences at 9:30 and ends at 16:00 and will be partitioned into 13 half hour intervals. Since intraday observations are separated by overnight and weekend periods, the time series are not uniform in terms of interval length.

Price Volatility

The price volatility in interval t is computed as $VOLATI = \sum_{j=1}^{N} R_{j,t}^2$ where $R_{j,t} = \frac{P_{j,t}}{P_{j-1,t}} - 1$ is the return of the j^{th} transaction in the time interval t and N is the total number of transactions in interval t. Traditionally, the price volatility is calculated by $\frac{1}{N}\sum_{j=1}^{N}\left(R_{i,t} - \bar{R}\right)^2$. In this chapter, we assume that the mean return $\bar{R}$ in each interval is equal to zero and thus, we do not subtract it from $R_{i,t}$. This is a reasonable assumption given the fact that the interval is short (half hour). In addition, in computing the price volatility, we do not divide the sum of squared returns by the number of transaction since we would like to measure the cumulative price fluctuation rather than the average price fluctuation for each transaction.

Spread, Market Depth, and Volume

Spread is defined as the difference between the ask price and the bid price.

Market depth is measured by the total number of shares posted at the quoted prices.

$$DEPTH_t = DEPTH_t^{bid} + DEPTH_t^{ask}$$

where $DEPTH_t^{bid}$ and $DEPTH_t^{ask}$ are the number of shares posted at the bid price and ask price respectively.

Volume variable is measured by the total number of shares traded in interval t.

4.5 Empirical Results

The Relation between Spreads and Depths

To examine the relation between spreads and depths, we estimate the following regression for each stock.

$$SDEPTH_t = a_0 + a_1 SSPRD_t + a_2 SDEPTH_{t-1} + \sum_{i=1}^{11} \theta_i TIME_{i,t} + u_t, \qquad (4.2)$$

$SSPRD_t$ is the standardized spread in interval t, $SDEPTH_t$ is the standardized depth at the end of interval t, and $TIME_{i,t}$ is a dummy variable that takes the value of 1 if $i = t$ and 0 otherwise, u_t is the error term. We include $TIME_{i,t}$ and $SDEPTH_{t-1}$ on the right hand side of equation (4.2) in order to control for the intra-day variation and autocorrelation in the dependent variable. Even though there are 13 time intervals every day, there are only 12 observations since $SDEPTH_{t-1}$ is used as an explanatory variable. Moreover, to avoid multicollinearity we do not assign a dummy variable to one interval . Thus, we only have 11 dummy variables.

We estimate equation (4.2) for each stock using the GMM with the Newey and West (1987) correction. We obtain t statistics that are robust to heteroskedasticity and autocorrelation. As in the last section, to test whether each coefficient significantly differs from zero we use the procedure outlined in Meulbroek (1992). The regression results are reported in Table 4.3.

Table 4.3

This table reports the GMM estimates of the following regression model for each of 31 component stocks in the TSE-35 index.

$$SDEPTH_t = a_0 + a_1 SSPRD_t + a_2 SDEPTH_{t-1} + \sum_{i=1}^{11} \theta_i TIME_{i,t} + u_t,$$

where $SSPRD_t$ is the standardized spread in interval t, $SDEPTH_t$ is the standardized depth at the end of interval t, and $TIME_{i,t}$ is a dummy variable that takes the value of 1 if $i = t$ and 0 otherwise, u_t is the error term.

	a_1	a_2
Average Coefficient	-2.9232	0.2592
Negative Coefficients (%)	74.19	0
Z-statistic	-2.54	22.4938
p-Value from Z-Statistic	0.0017	0.0000

We find that the depth is significantly and negatively related to the spread. This result is consistent with hypothesis 1 and supports the view that limit order traders use both bid-ask spread and depth as means to respond to any indication that the probability of informed trades has risen.

The Impacts of Price Volatility, Trading Activity, and Transaction Volume on Market Liquidity

To investigate the impacts of price volatility, transaction volume, and trading activity on market liquidity, we estimate the following linear models for each stock.

$$SSPRD_t = \alpha_1 + \beta_1 VOLATI_{t-1} + \gamma_1 NT_t + \delta_1 SVOLUME_t \quad (4.3)$$

$$+ \sum_{i=1}^{11} \theta_i TIME_{i,t} + \varphi_1 SSPREAD_{t-1} + \epsilon_t,$$

$$SDEPTH_t = \alpha_2 + \beta_2 VOLATI_{t-1} + \gamma_2 NT_t + \delta_2 SVOLUME_t + \sum_{i=1}^{11} \theta_i TIME_{i,t} + \varphi_2 SDEPTH_{t-1} + u_t, \quad (4.4)$$

where $SSPRD_t$ is the standardized spread in time interval t, $SDEPTH_t$ is the standardized market depth at the end of time interval t, $VOLATI_{t-1}$ is the volatility during time interval $t-1$, NT_t is the number of transactions during time interval t, $SVOLUME_t$ is the standardized volume during time interval t, and $TIME_{i,t}$ is a dummy variable that takes the value of 1 if $i = t$ and 0 otherwise. The inclusion of $TIME_{i,t}$ and $SPREAD_{t-1}$ on the right hand side of equation (4.3), and $TIME_{i,t}$ and $SDEPTH_{t-1}$ on the right hand side of equation (4.4) is to control for the intra-day variation and autocorrelation in the dependent variables.

The results are reported in Table 4.4. For brevity, the estimates of θ_i are not reported here. However, these coefficients are significantly different from zero, indicating that it is necessary to control for the intra-day effects.

Theoretically, there are 2 effects of the number of transactions on market liquidity. On one hand, transactions consume the liquidity available in the market and therefore there should be an inverse relationship between market liquidity and the number of trades. In other words, spread (depth) should be positively (negatively) related to the number of trades. On the other hand, higher trading activities may capture market interest and induce investors to supply more liquidity to the market as shown in Admati and Pfleiderer (1988) and thus, there is a negative (positive) relationship between spread (depth) and the number of trades. Our empirical results show that the first effect dominates the second one. Ahn, Bae and Chan (2001) also find that there is a negative relationship between depth and number of transaction in the Stock Exchange of Hong Kong.

We find that the price volatility is positively (negatively) related to spread (depth). This result is consistent with the predictions of Handa and Schwartz (1996), and Foucault (1999) and therefore supports our hypothesis 2. Indeed, if the price is very volatile, the probability that a limit order is executed is higher even though bid - ask spread is wide. As a result, investors tend to submit limit orders with lower bid prices and higher ask prices. This leads to higher bid-ask spread and lower depth at the best quote.

The relationship between volume and spread is more complex. We find that there is a negative relationship between spread and volume and there is a positive relationship between depth and volume. In other words, volume is directly related to market liquidity. This result is inconsistent with the predictions of Easley and O'Hara (1992) and therefore does not support our hypothesis 3. However, this can be explained by the model of Harris and Raviv (1993). Harris and Raviv assume that traders share prior beliefs and receive common information but differ in the way they interpret the information. Thus, high volume could mean high liquidity trades and as a result spread should decrease and depth should increase. Those establish the direct relationship between volume and market liquidity.

Table 4.4

This table presents the GMM estimates of the following regression models for each of 31 component stocks in the TSE-35 index.

$$SSPREAD_t = \alpha_1 + \beta_1 VOLATI_{t-1} + \gamma_1 NT_t + \delta_1 SVOLUME_t$$

$$+ \sum_{i=1}^{11} \theta_i TIME_{i,t} + \varphi_1 SSPREAD_{t-1} + \epsilon_t,$$

$$SDEPTH_t = \alpha_2 + \beta_2 VOLATI_{t-1} + \gamma_2 NT_2 + \delta_2 SVOLUME_t$$

$$+ \sum_{i=1}^{11} \theta_i TIME_{i,t} + \varphi_2 SDEPTH_{t-1} + u_t,$$

where $SSPREAD_t$ is the standardized spread in time interval t, $SDEPTH_t$ is the standardized market depth at the end of time interval t, $VOLATI_{t-1}$ is the volatility during time interval $t-1$, NT_t is the number of transactions during time interval t, $SVOLUME_t$ is the standardized volume during time interval t, and $TIME_{i,t}$ is a dummy variable that takes the value of 1 if $i = t$ and 0 otherwise.

Panel A: Dependent variable is standardized spread				
	β_1	γ_1	δ_1	φ_1
Average Coefficient	0.192	0.0052	-0.0111	0.1742
Positive Coefficients (%)	64.52	77.42	32.26	100
Z-statistic	2.6923	4.5512	-2.5683	18.2461
p-value from Z-statistic	0.0035	0.0000	0.0051	0.0000

Panel B: Dependent variable is standardized depth				
	β_2	γ_2	δ_2	φ_2
Average Coefficient	-0.1893	-0.00028	0.02921	0.252
Negative Coefficients (%)	67.74	64.52	19.35	0
Z-statistic	-3.303	-1.227	3.856	21.488
p-value from Z-statistic	0.0005	0.11	0.000	0.0000

4.6 Concluding Remarks

This chapter examines the intraday behavior of the market liquidity in the Toronto Stock Exchange which uses a computerized limit order trading system. Along with previous studies, we confirm that the U-shaped intraday pattern of spread does not depend on the market architecture. Whether the exchange is order-driven, specialist, hybrid, or dealership, spread still displays a U-shaped intraday pattern. In addition, we show that the market is very thin at the opening but it becomes deeper and deeper as trades go on. After one and a half hours, it becomes stable. Our results also indicate that the volume of trade is low in the first half hour of the day and then stable until the last half hour when it rises.

Consistent with Harris (1990), our results show that limit order traders use both spread and depth to protect themselves from informed traders. If they believe that the probability that some traders possess superior information has increased, they would respond by widening spread and/or decreasing the depth at each quoted price. We find evidence that spread and depth are negatively correlated. Thus, as indicated by Harris (1990) and Lee, Mucklow and Ready (1993), spread and depth are two dimensions of market liquidity.

We find that price volatility is inversely related to market liquidity. When the price volatility increases, the probability of being bagged by informed traders also increases. Limit order traders have to post higher (lower) ask (bid) price and/or reduce the depth at each quote. This establishes the inverse relationship.

Finally, we show that there is a direct relationship between trading volume and liquidity. Harris and Raviv (1993) explain this result by postulating that traders receive the same information but they interpret the information in different ways. Thus, it could be the case that high volume implies high liquidity trade which leads to the increase in market liquidity.

4.7 Appendix

Below are the names and the corresponding tick symbols of the 31 firms in the dataset

Tick Symbol	Company Name
A	ABITIBI-CONSOLIDATED INC.
AL	ALCAN INC.
BMO	BANK OF MONTREAL
BNS	BANK OF NOVA SCOTIA
ABX	BARRICK GOLD CORPORATION
BCE	BCE INC.
BVF	BIOVAIL CORPORATION
BBD.B	BOMBARDIER INC. CL 'B' SV
CM	CANADIAN IMPERIAL BANK OF COMMERCE
CNR	CANADIAN NATIONAL RAILWAY CO.
CP	CANADIAN PACIFIC
CTR.A	CANADIAN TIRE CORP. LTD. CL 'A' NV
CLS	CELESTICA INC. SV
DFS	DOFASCO INC.
MG.A	MAGNA INTERNATIONAL INC. CL 'A' SV
N	INCO LIMITED
NA	NATIONAL BANK OF CANADA
NCX	NOVA CHEMICALS CORPORATION
NT	NORTEL NETWORKS CORPORATION
PCA	PETRO-CANADA
PDG	PLACER DOME INC.
RIM	RESEARCH IN MOTION LIMITED
RY	ROYAL BANK OF CANADA
SJR.B	SHAW COMMUNICATIONS INC. CL 'B' NV
SU	SUNCOR ENERGY INC.
TA	TRANSALTA CORPORATION
TD	TORONTO-DOMINION BANK
TEK.B	TECK COMINCO LIMITED CL 'B' SV
TLM	TALISMAN ENERGY INC.
TOC	THOMSON CORPORATION
TRP	TRANSCANADA PIPELINES LTD.

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