ASPECTS OF TIME DEPENDENCE IN STRING THEORY

by

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Abstract

The study of string theory has recently opened the way to many ground-breaking ideas in theoretical physics. An aspect that had been neglected until recently concerns the role played by time in this theory. It is an important subject because of its possible connections with the field of cosmology. In the first part of this thesis we study S(pacelike)D-branes which are objects arising naturally in string theory when Dirichlet boundary conditions are imposed on the time direction. SD-brane physics is inherently time-dependent. We set up the problem of coupling the most relevant open-string tachyonic mode to massless closed-string modes in the bulk, with back-reaction and Ramond-Ramond fields included. We find solutions (numerically) that are asymptotically flat in the future but plagued with a singularity in the past. The second part of the thesis is concerned with the study of important aspects related to the proposed duality between quantum gravity in de Sitter space and a Euclidean conformal field theory: the dS/CFT correspondence. First, we study solutions of Einstein gravity coupled to a positive cosmological constant and matter which are asymptotically de Sitter and homogeneous. These solutions are 'tall', meaning that the perturbed universe lives through enough conformal time for an entire spherical Cauchy surface to enter any observer's past light cone. Our main focus is on the implications of tall universes for the correspondence. Particular attention is given to the associated renormalization group flows, leading to a more general de Sitter c-theorem. We also discuss the conformal diagrams for various classes of homogeneous flows. Then, we consider the evolution of massive scalar fields in (asymptotically) de Sitter spacetimes of arbitrary dimension. Through the dS/CFT correspondence, our analysis points to the existence of new non-local dualities for the conformal field theory.

Résumé

Ces dernières années l'étude de la théorie des cordes a contribué à l'apport d'idées intéressantes en physique théorique. Jusqu'à récemment, le rôle du temps dans cette théorie n'avait presque pas été étudié. Il s'agit pourtant d'un sujet important à cause des applications potentielles en cosmologie. Dans la première partie de cette thèse, nous étudions la physique des S(patiales)D-branes qui sont des excitations obtenues lorsqu'une condition de Dirichlet est imposée sur la coordonnée temporelle d'une corde. Les SD-branes dépendent donc intrinsiquement du temps. Nous considérons le problème en couplant le mode corde ouverte tachyonique et les modes cordes fermées sans masse. Nous trouvons, numériquement, des solutions asymptotiquement plates dans le futur mais singulières dans le passé. La deuxième partie de cette thèse consiste en l'étude d'aspects importants de la correspondence dS/CFT, une dualité hypothétique entre la gravité quantique sur un espace de Sitter et une théorie de champs conforme euclidienne. Nous débutons par l'étude de solutions des équations d'Einstein qui sont asymptotiquement de Sitter et homogènes. Ces solutions sont 'vastes' en ce sens que l'univers perturbé existe durant une période de temps conforme assez longue pour qu'une surface de Cauchy entière soit inclue dans le passé du cone de lumière associé à n'importe quel observateur. Nous considérons les implications de ces univers pour la correspondence dS/CFT. Une attention particulière est portée au groupe de renormalisation menant à un théorème-c de Sitter généralisé. Nous présentons aussi les diagrammes conformes associés à différentes classes d'évolutions homogènes. Finalement, nous considérons l'évolution d'un champ scalaire massif dans des espaces (asymptotiquement) de Sitter de différentes dimensions. Dans le cadre de la correspondence dS/CFT, notre analyse implique l'existence de nouvelles dualités non-locales pour la théorie de champs conforme.

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Contributions of Authors

This thesis contains material that was previously published in refs. [1, 2, 3, 4]. Chapter 1 introduces the reader to basic concepts required for understanding the core of this work. In chapter 2, we present the results of refs. [1, 2] which were published in collaboration with Amanda Peet. My contribution to this work included performing most analytical calculations, all numerical simulations as well as writing half of the papers. Then, chapter 3 is a presentation of the work published in refs. [3, 4] by myself, Donald Marolf and Robert Myers. My contribution to these two projects involved performing most calculations, writing parts of the two papers and participating in discussions at every stage of development.

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CHAPTER 1

INTRODUCTION

This is an introductory chapter providing the reader with some notions necessary in order to assimilate the material contained in this thesis. Firstly in section 1.1 we introduce string theory by emphasizing mostly those aspects relevant to the physics of SD-branes which is the subject of chapter 2. Then in section 1.2 we present some elementary concepts of modern cosmology that will be used in chapter 3 where we study the physics of de Sitter space.

1.1 CONCEPTS OF STRING THEORY

Initially string theory was intended to describe the strong interactions. However the more conventional but extremely successful QCD led to a more appropriate description of the phenomena at play.[1] Subsequent studies of string theory led to ideas which are much more far reaching. In fact, a series of discoveries made in the 70's strongly support the idea that string theory, or rather its supersymmetric realization, has the potential to unify in a self-consistent manner the four interactions found in nature, *i.e.*, the electromagnetic, weak and strong forces as well as, quite remarkably, gravitation [7, 8, 9, 10, 11]. This remains one of the most compelling reasons explaining why the study of string theory is so fascinating.

The outline of this section is as follows: In section 1.1 we present the action from which the classical dynamics of strings is derived. Then in section 1.1.2 we provide a brief description of the perturbative spectrum of quantized strings. In

[1] It should be noted that there is convincing evidence around that gauge theories have, after all, a description in terms of strings [5] – see also ref. [6].

section 1.1.3 we discuss effective spacetime actions obtained from string theory. Finally in section 1.1.4 we present some aspects of the non-perturbative spectrum of the theory.

For simplicity we focus mostly on fermion-free, *i.e.*, non-supersymmetric, string theories. A more technical reason justifying this omission is that the existence of supersymmetric representations in a theory relies on the presence of a Casimir (conserved quantity) corresponding to the energy of the relevant states. This in turn implies there is a timelike Killing vector associated with the gravitational backgrounds for which the theory is defined. A Killing vector, say V_i, is one with vanishing Lie-derivative,

$$\mathcal{L}_V G_{ij} = \nabla_i V_j + \nabla_j V_i = 0, \tag{1.1}$$

where ∇_i is the covariant derivative derived from the metric tensor G_{ij}. For backgrounds with a time-dependent G_{ij}, which is the subject of this thesis, there are typically no global timelike Killing vectors.

1.1.1 Classical motion of fundamental strings

The classical motion of fundamental strings in a curved space endowed with the metric $G_{ij}(X)$ is governed by the Polyakov action

$$S_P = -\frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d\tau d\sigma\, (-\gamma)^{1/2}\, \gamma^{ab}\partial_a X^i \partial_b X^j G_{ij}(X), \tag{1.2}$$

where $\gamma = \det \gamma_{ab}$, $i, j = 0, \ldots d-1$ and $\mathcal{M}$ is the two-dimensional *worldsheet* manifold parametrized with the timelike variable τ (with the range: $-\infty < \tau < +\infty$) and the spacelike variable σ (our convention: $0 < \sigma \leq \pi$). The action (1.2) is that of a two-dimensional field theory where the dynamical variables are the d target space fields X^i. More precisely, this is a nonlinear sigma-model with couplings encoded in the metric components $G_{ij}(X)$ of the d-dimensional embedding spacetime. The tensor γ_{ab} is the two-dimensional metric of the Lorentzian worldsheet. It is a non-dynamical variable since the corresponding two-dimensional stress-energy tensor vanishes classically,[2]

$$T_{ab} = -\frac{1}{\alpha'}\left(\partial_a X^i \partial_b X^j - \frac{1}{2}\gamma_{ab}\partial_c X^i \partial^c X^j\right) G_{ij}(X) = 0. \tag{1.3}$$

The vanishing of this tensor is directly related to the trivial nature of the Einstein-Hilbert tensor in two-dimensions: $\mathcal{R}^{(2)}_{ab} - \frac{1}{2}\gamma_{ab}\mathcal{R}^{(2)} = 0$,[3] where $\mathcal{R}^{(2)}_{ab}$ and $\mathcal{R}^{(2)}$ are respectively the Ricci scalar and the Ricci tensor in two dimensions. Moreover the trace of the classical stress-energy tensor vanishes: $T^a_a = 0$. This implies (in any dimensions) invariance of the corresponding action under conformal transformations (see, *e.g.*, ref. [12]). The quantity $(2\pi\alpha')^{-1}$, which is the only arbitrary parameter in the theory, corresponds to the tension of a fundamental string. The so-called Regge slope (α') has dimensions of length-squared and is related to the fundamental 'string length' (l_s) via $\alpha' = l_s^2$.

The classical equations of motion governing the dynamics of classical strings is obtained by varying eq. (1.2) with respect to the target space fields,

$$\Box X^i + \Gamma^i_{jk}\partial_a X^j \partial^a X^k = 0, \tag{1.4}$$

where Γ^i_{jk} is the Christoffel symbol associated with the metric tensor G_{ij} and $\Box$ is the two-dimensional curved space d'Alembertian operator,[4]

$$\Box = \frac{1}{(-\gamma)^{1/2}}\partial_a\left[(-\gamma)^{1/2}\gamma^{ab}\partial_b\right]. \tag{1.5}$$

[2]The more geometric Nambu-Goto action is obtained by plugging the equation of motion for γ_{ab} back into eq. (1.2). String quantization using this action is difficult. In fact, the structure of the Nambu-Goto action is much more complicated than the Polyakov action since it involves square-roots of first-derivatives in the target space field.

[3]The action (1.2) is not the most general one. An Einstein-Hilbert term should be added for consistency. The role of this term will be explained in section 1.1.2.

[4]The expression for the operator $\Box$ is valid for any number of spacetime dimensions.

Classical string propagation must be supplemented with boundary conditions. These determine whether the strings are closed or open (see section 1.1.2 below). The propagation of classical strings in curved space has been considered in the literature. In particular, ref. [13] studies string evolution and back-reaction in time-dependent gravitational backgrounds.

1.1.2 String quantization and spectrum

In this section we summarize the quantization of bosonic strings in Minkowski space: $G_{ij} = \eta_{ij}$. The Polyakov action is then invariant under the SO$(1, d-1)$ Lorentz group supplemented with spacetime translations (in curved space the corresponding symmetry is invariance under d-dimensional diffeomorphisms). The theory is also invariant under worldsheet reparametrizations, *i.e.*, two-dimensional diffeomorphisms. Finally, the action is invariant under the Weyl transformation

$$\gamma_{ab} \longrightarrow \gamma'_{ab} = e^{2\omega}\gamma_{ab}, \tag{1.6}$$

which is an element of the two-dimensional conformal group. The challenge taken on by the pioneers of string theory was to quantize fundamental strings in such a way as to preserve these three classical symmetries (see refs. [7, 8, 9, 10] for details).

As mentioned at the end of section 1.1.1, the Polyakov action is minimized only if certain boundary conditions are satisfied. In flat space the boundary term is of the form

$$\frac{1}{2\pi\alpha'}\int_{-\infty}^{+\infty} d\tau\; X^{i\prime}\delta X_i|_{\sigma=0}^{\sigma=\pi} = 0. \tag{1.7}$$

Open strings correspond to Neumann boundary conditions being imposed on all target space fields,

$$X^{i\prime}(\tau, 0) = 0 \quad \text{and} \quad X^{i\prime}(\tau, \pi) = 0\,. \tag{1.8}$$

For closed strings, periodic boundary conditions must be imposed.

$$X^{i'}(\tau, 0) = X^{i'}(\tau, \pi)\,, \quad X^{i}(\tau, 0) = X^{i}(\tau, \pi) \quad \text{and} \quad \gamma_{ab}(\tau, 0) = \gamma_{ab}(\tau, \pi)\,. \tag{1.9}$$

Dirichlet boundary conditions,

$$X^{i}(0) = \text{const.} \quad \text{and/or} \quad X^{i}(\pi) = \text{const.}\,, \tag{1.10}$$

can instead be imposed on some or all of the target space fields for open strings. A Dirichlet boundary condition on a spacelike target space direction ($i \neq 0$) implies that the corresponding string endpoint is fixed along this direction. Imposing Dirichlet boundary conditions on the same endpoint for n spacelike target space fields means the string is fixed on a $(d-n)$-dimensional hyperplane.[5] It turns out that these planes are dynamical objects that must be included (for consistency) in the spectrum of string theory; these are called D-branes [14]. More recently, the investigation of open strings with a Dirichlet boundary condition on the timelike target space field ($i = 0$) has been initiated [15]. The corresponding objects have been dubbed SD-branes ('S' stands for 'spacelike'). They are topological defects that live only for an instant in time (a more appropriate description is provided in chapter 2).

In two dimensions there are exactly as many parameters needed to specify the local symmetries (two-dimensional diffeomorphisms and Weyl symmetry) as there are independent components in the worldsheet metric (*i.e.*, three). One can therefore choose a simple gauge in order to quantize the theory. The most common ones are the covariant and light-cone gauges,[6] each with its own advantages.[7] In

[5]More precisely the string endpoint can propagate on the n-dimensional plane but is not allowed to move along any of the $d-n$ other transverse directions.

[6]This amounts to setting $X^+ = \tau$, where X^+ is a spacetime light-cone coordinate.

[7]Note that other gauges might be more convenient in certain cases. For example, if the sigma-

light-cone gauge the physics is not covariant but it can be shown that all non-physical states (*i.e.*, negative norm ghosts) are automatically projected out. In the covariant gauge, *i.e.*, $\gamma_{ab} = \eta_{ab}$, there remains an infinite number of worldsheet symmetries corresponding to local Weyl (conformal) transformations. This gauge is explicitly covariant but there remain non-physical states in the spectrum. These are projected out if we impose that there are no residual symmetries (and anomalies) in the quantized theory. Among other things this implies that bosonic strings must propagate in a 26-dimensional spacetime ($d = 10$ for superstrings).

Let us sketch how the string spectrum is obtained in the covariant gauge. The equation of motion for the target space fields is simply

$$\Box X^i = 0. \tag{1.11}$$

The solutions to this wave equation with closed string boundary conditions are

$$X^i(\tau,\sigma) = x^i + \alpha' p^i \tau + \frac{i}{2}(\alpha')^{1/2} \sum_{n\neq 0} \frac{1}{n} \left(\alpha_n^i e^{-2in(\tau-\sigma)} + \tilde{\alpha}_n^i e^{-2in(\tau+\sigma)}\right), \tag{1.12}$$

where the couple $\{x^i, p^i\}$ are canonically conjugate variables corresponding to the center-of-mass position and momentum of the string. The Fourier coefficients $\{\alpha_n^i, \tilde{\alpha}_m^j\}$ are numbers which, along with x^i and p^i, are promoted to operators in the quantized theory. The following commutation relations are associated with these operators,

$$\begin{aligned} \left[x^i, p^j\right] &= i\,\eta^{ij}, \quad \left[\alpha_n^i, \alpha_m^j\right] = m\,\delta_{m+n}\,\eta^{ij}\,, \\ \left[\alpha_n^i, \tilde{\alpha}_m^j\right] &= 0, \quad \left[\tilde{\alpha}_n^i, \tilde{\alpha}_m^j\right] = m\,\delta_{m+n}\,\eta^{ij}\,, \end{aligned} \tag{1.13}$$

where $\delta_0 = 1$ and $\delta_{n\neq 0} = 0$. The coefficients α_n^i and $\tilde{\alpha}_n^i$ are respectively associated with right-moving and left-moving worldsheet degrees of freedom. The solution to

model couplings correspond to a time-dependent background it seems more natural to use the time gauge $X^0 = \tau$. However it should be said that using the time gauge in flat space does not lead to the simplifications found in light-cone gauge [9].

eq. (1.11) for open strings (Neumann boundary conditions) is

$$X^i(\tau,\sigma) = x^i + \alpha' p^i \tau + i(\alpha')^{1/2} \sum_{n\neq 0} \frac{1}{n} \alpha^i_n e^{-in\tau} \cos n\sigma. \tag{1.14}$$

with commutation relations imposed on $\{x^i, p^i\}$ and the α^i_n's in the quantized theory.

As pointed out above, there are too many states in the quantized spectrum. These are projected out by imposing the physical constraint

$$T^a_a|\text{physical state}>= 0. \tag{1.15}$$

In string theory, the multiplicity of elementary particles corresponds to different vibration modes of the string being excited. Every string theory has a one-string vacuum denoted $|0;k>$, where k is the center-of-mass momentum of the string. Acting on this state with worldsheet harmonic oscillators excites the string to a different quantum level. The mass-squared of a closed string is

$$M^2 = \frac{4}{\alpha'}\left(-a + \sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n\right), \tag{1.16}$$

where we have used the level-matching condition

$$\sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n = \sum_{n=1}^{\infty} \tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n, \tag{1.17}$$

which is obtained by imposing translation invariance along the coordinate σ. For open strings the mass equation is

$$M^2 = \frac{1}{\alpha'}\left(-a + \sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n\right). \tag{1.18}$$

The constant a is determined by requiring absence of ghosts. It can be shown that $a = 1$ and $d = 26$ lead to a spectrum composed entirely of physical states.

In bosonic string theory there is no supersymmetry and therefore no fermions. The massless spectrum is composed of a U(1) gauge field (photon) in the open string sector,

$$\eta_{ij}\xi^j\alpha^i_{-1}|0;k>_{\rm open}, \tag{1.19}$$

where ξ is a polarization vector. The photon state is gauge invariant as shown in ref. [7]. The massless spectrum[8] in the closed string sector includes the following three fields,

$$\left[\frac{a_{ij}+a_{ji}}{2}-\frac{1}{d-2}{\rm Tr}(a_{kl})\eta_{ij}\right]\alpha^i_{-1}\tilde{\alpha}^j_{-1}|0;k>_{\rm closed}\ :\ \text{graviton}$$
$$\left[\frac{a_{ij}-a_{ji}}{2}\right]\alpha^i_{-1}\tilde{\alpha}^j_{-1}|0;k>_{\rm closed}\ :\ \text{Kalb}-\text{Ramond field}$$
$$[{\rm Tr}(a_{kl})\eta_{ij}]\,\alpha^i_{-1}\tilde{\alpha}^j_{-1}|0;k>_{\rm closed}\ :\ \text{dilaton.}$$

An apparent negative feature of the bosonic string theory is that both the closed string and the open string vacuum contains a tachyon, *i.e.*, a state with negative mass-squared. The modern interpretation is not that the theory is inconsistent but rather that the perturbative vacuum considered is unstable. It is believed that when expanded around some yet undiscovered stable vacuum the bosonic string theory will make sense. Progress in this direction has been made in recent years through the use of open string field theory (see ref. [16] for a review).

We now discuss the special status of the dilaton field in string theory. The obvious generalization of action (1.2) consistent with worldsheet diffeomorphism invariance consists in adding to to it the term

$$\lambda\,\chi=\lambda\left[\frac{1}{4\pi}\int_{\mathcal{M}}d\tau d\sigma\sqrt{-\gamma}\mathcal{R}^{(2)}+\frac{1}{2\pi}\int_{\partial\mathcal{M}}ds\,K\right], \tag{1.20}$$

where χ is the Euler characteristic (a topological invariant) of the worldsheet, $\partial\mathcal{M}$ its boundary and K the corresponding extrinsic curvature. In string theory the parameter λ is the expectation value of the dilaton, $\lambda=<\Phi>$. Moreover, the

[8]Higher level excitations (more oscillators acting on the vacuum) correspond to massive modes which are negligible at low energy.

string coupling itself is fixed by the expectation value of the dilaton field,

$$g_s = e^{<\Phi>}. \tag{1.21}$$

Then the Euler characteristic fixes the order of a given diagram contributing to the path integral of string theory,

$$\mathcal{Z} = e^{-\chi<\Phi>}e^{-S_P}. \tag{1.22}$$

There are no fermions in bosonic string theory. More realistic theories are obtained by extending the local worldsheet symmetries to include supersymmetry. Then each physical worldsheet boson is associated with a fermion field. As will be discussed in section 1.1.3 string theory leads to interesting spacetime actions where worldsheet supersymmetry has become spacetime supersymmetry. It is a prediction of superstring theory that the world is superpersymmetric at high energy.[9] The spectrum of the superstring is richer than that obtained in the bosonic case. Quantization of the superstring led to the discovery of five consistent perturbative theories which are free of tachyons: Type IIa, Type IIb, Type I, Heterotic SO(32) and Heterotic $E_8 \times E_8$. Here we consider only the Type IIa and Type IIb theories. The bosonic sector of these superstrings (the so-called Neveu-Schwartz–Neveu-Schwartz (NS-NS) sector) contains the graviton (h_{ij}), the Kalb-Ramond (b_{ij}) and the dilaton (ϕ) fields. However, there are other bosonic degrees of freedom coming from the Ramond-Ramond (RR) sector of the theory.[10] These consist of a family of antisymmetric tensor fields (form fields):

$$\begin{aligned}\text{Type II a}: &\quad C^{(1)},\ C^{(3)},\\ \text{Type II b}: &\quad C^{(0)},\ C^{(2)},\ C^{(4)}.\end{aligned}$$

[9] The hunt for supersymmetry has begun fifteen years ago. Now the hope is that the first superparticle will be discovered at the Large Hadron Collider scheduled to go online in 2008.

[10] See refs. [7, 10] for a detailed account of the rationale behind the classification of string excitations between RR and NS-NS sectors.

Of course, there are extra fermion degrees of freedom (coming fron the NS-R and R-NS sectors) but we do not consider these here.

1.1.3 Effective spacetime actions

As pointed out in section 1.1.2 the string spectrum of the bosonic string and the Type IIa,b superstring theories contains three massless quantum fields: the graviton, the Kalb-Ramond and the dilaton fields. Both the open and the closed strings can act as a source for these fields. The Polyakov string action must be generalized in order to include these new couplings (see, for example, ref. [9]),

$$S = \frac{1}{4\pi\alpha'}\int_M (g)^{1/2}\left[\left(g^{ab}G_{ij}(X) + i\epsilon^{ab}B_{ij}(X)\right)\partial_a X^i \partial_b X^j\right] + \alpha' \mathcal{R}^{(2)}\Phi(X), \quad (1.23)$$

where G_{ij}, B_{ij} and Φ must be regarded as background fields, *i.e.*, respectively as coherent excitations of the quanta h_{ij}, b_{ij} and ϕ. The action (1.23) defines a consistent string theory only if the two-dimensional field theory is Weyl invariant at the quantum level. In bosonic string theory the quantum mechanical Weyl anomaly is [9]

$$T_a^a = -\frac{1}{2\alpha'}\beta^G_{ij}\, g^{ab}\partial_a X^i \partial_b X^j - \frac{i}{2\alpha'}\beta^B_{ij}\,\epsilon^{ab}\partial_a X^i \partial_b X^j - \frac{1}{2}\beta^\Phi \mathcal{R}, \quad (1.24)$$

where

$$\beta^G_{ij} = \alpha' \mathcal{R}_{ij} + 2\alpha' \nabla_i \nabla_j \Phi - \frac{\alpha'}{4} H_{ijk}H^{ijk} + \mathrm{O}(\alpha'^2), \quad (1.25)$$

$$\beta^B_{ij} = -\frac{\alpha'}{2}\nabla^k H_{kij} + \alpha' \nabla^k \Phi H_{kij} + \alpha' \nabla^k \Phi H_{kij} + \mathrm{O}(\alpha'^2), \quad (1.26)$$

$$\beta^\Phi = \frac{D-26}{6} - \frac{\alpha'}{2}\nabla^2\Phi + \alpha' \nabla_i \Phi \nabla^i \Phi - \frac{\alpha'}{24} H_{ijk}H^{ijk} + \mathrm{O}(\alpha'^2), \quad (1.27)$$

with $H_{ijk} = \partial_i B_{ij} + \partial_j B_{ki} + \partial_k B_{ij}$ being the Kalb-Ramond field strength. Then, string propagation is quantum mechanically consistent if and only if

$$\beta^G_{ij} = 0, \quad \beta^B_{ij} = 0 \quad \text{and} \quad \beta^\Phi = 0, \quad (1.28)$$

holds to *all orders* in α'. Apart from flat space. backgrounds satisfying these conditions are hard to come by. A much studied example consists in spacetimes involving a Calabi-Yau factor, *e.g.*. $\mathbf{R}^{n+1} \times \mathrm{CY}_{9-n}$, where n is odd [17]. Interesting time-dependent backgrounds corresponding to exact conformal field theories are the plane fronted wave spacetimes [18].

The beta-functions (1.25), (1.26) and (1.27) were obtained by studying one-loop string theory, *i.e.*, spherical worldsheet diagrams. There are anomalous contributions coming from higher genus (including loops) diagrams. The results presented thus far are therefore only valid perturbatively, *i.e.*, for small g_s. To lowest order in α', the equations of motion for the non-linear sigma model couplings of the bosonic theory can be derived from the 26-dimensional spacetime action

$$S_{bosonic} = \frac{1}{2\kappa_0^2} \int d^D X \, (-G)^{1/2} \, e^{-2\Phi} \left[\mathcal{R} + 4\nabla_i \Phi \nabla^i \Phi - \frac{1}{12} H_{ijk} H^{ijk} - \frac{2(D-26)}{3\alpha'} \right] . \tag{1.29}$$

By using a conformal transformation to bring this action to Einstein frame,[11] we find that the overall constant is related to the Newton's constant as follows,

$$\kappa_0 e^{<\Phi>} = (8\pi G_N)^{1/2} . \tag{1.30}$$

The ten-dimensional spacetime actions associated with the Type IIa and Type IIb superstring theories (the Type IIa and Type IIb supergravity actions) are presented in chapter 2.

1.1.4 Non-perturbative excitations

We have been discussing aspects of string theory related with its perturbative spectrum. A window into the non-perturbative regime of the theory has been opened a few years ago when it was found that all five perturbative descriptions

[11] The choice of a metric in the class $e^{\alpha\Phi} G_{ij}$ is known as the choice of conformal frame.

of superstring theory are related by non-perturbative dualities [19]. An essential ingredient of these dualities is that the string spectrum must be enlarged to include non-perturbative objects called D-branes. These are solitons in the sense that their mass (or tension) is proportional to the inverse coupling, *i.e.*, g_s^{-1}. These objects decouple in the perturbative limit because they acquire a large mass.

As alluded to earlier, the worldsheet description of D-branes is in terms of open strings with Dirichlet boundary conditions imposed on $(9-p)$ directions [14]. More concretely, Dp-branes are $(p+1)$-dimensional hyperplanes where string endpoints are confined. The massless excitations of these open strings correspond to a $(p+1)$-dimensional gauge field $A_\alpha(\mathbf{y})$ and $(9-p)$ massless scalar fields $\phi_I(\mathbf{y})$. The coordinates $\mathbf{y}$ correspond to directions tangent to the brane worldvolume, and, the range for the indices is: $\alpha = 0, \ldots, p$ and $I = p+1, \ldots 9$. The fields $\{A_\alpha, \phi_I\}$ are the degrees of freedom relevant to the low energy (with respect to $\alpha'^{-1/2}$) dynamics of D-branes. These massless modes (once their fermion partners are included) form a supersymmetric U(1) gauge theory. Ref. [20] shows that the effective action for the dynamics of a D-brane is of the Born-Infeld form

$$S_{BI} = -T_p \int d^{p+1}y\, e^{-\Phi} \sqrt{-det\left(\mathcal{P}[G+B]_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta}\right)}, \qquad (1.31)$$

where T_p is the brane tension and $F_{\alpha\beta}$ is the field-strength associated with the gauge field A_α. The scalar fields labelled ϕ_I correspond to the transverse displacements of the brane. Their contribution in the Born-Infeld action is implicit in the pull-back of the bulk spacetime tensors to the $(p+1)$-dimensional worldvolume,

$$\mathcal{P}\left[G_{\alpha\beta}\right] = G_{\alpha\beta} + 4\pi\alpha' \left(G_{I\alpha}\partial_\beta\phi^I + G_{I\beta}\partial_\alpha\phi^I\right) + 4\pi^2\alpha'^2\, G_{IJ}\partial_\alpha\phi^I\partial_\beta\phi^J. \qquad (1.32)$$

The action (1.31) incorporates the couplings of the worldvolume vector and scalars to the massless NS-NS bulk closed string modes, *i.e.*, the graviton, the dilaton and the Kalb-Ramond fields. D-branes also carry a Ramond-Ramond charge [21] so the effective action must include a coupling to these fields. For a single D-brane, the coupling is given by the Chern-Simons term

$$S_{CS} = \mu_p \int \mathcal{P}\left[C^{(p+1)}\right]. \tag{1.33}$$

where μ_p is the RR charge of the brane.

Up to now we have been describing the physics of a single D-brane. There are 'macroscopic' supersymmetric spacetime solutions in Type IIa and Type IIb supergravity[12] that correspond to a collection of N superposed D-branes,

$$ds^2 = H_p^{-1/2}\eta_{ij}dX^i dX^j + H_p^{1/2}\left(dr^2 + r^2 d\Omega_{8-p}^2\right) \tag{1.34}$$

$$e^{2\Phi} = g_s^2 H_p^{\frac{3-p}{2}} \tag{1.35}$$

$$C^{(p+1)} = -(H_p^{-1} - 1)g_s^{-1}dX^0 \wedge \cdots \wedge dX^p. \tag{1.36}$$

where the harmonic function H_p is

$$H_p = 1 + \frac{d_p(2\pi)^{p-2}\alpha'^{(7-p)/2}g_s N}{r^{7-p}}, \tag{1.37}$$

and

$$d_p = 2^{7-2p}\pi^{\frac{9-3p}{2}}\Gamma\left(\frac{7-p}{2}\right). \tag{1.38}$$

These string frame solutions are appropriate when the parameter $g_s N$ is large in which case the curvature remains small throughout the spacetime.

Because they are supersymmetric, the perturbative objects we presented are stable. There are objects in the string theory spectrum that are unstable. For example, the D-branes in bosonic string theory are unstable because there is an open string tachyon excitation on their worldvolume. We have seen earlier that in Type IIa(b) superstring theory there are RR fields $C^{(n)}$ with n odd (even).

[12]The part of the Type IIa,b supergravity actions relevant at this stage corresponds to eq. (2.14) from chapter 2.

Correspondingly, there are only stable D-branes with p even (odd). The D-branes with p odd (even) are not absent, they are unstable. Again, this instability can be traced down to the existence of a worldvolume open string tachyon. As shown in ref. [22], a Dp-brane with a worldvolume tachyon couples to closed RR fields with a topological term of the form

$$\mu_p \int f(T)\, dT \wedge \mathcal{P}C^{(p)}, \tag{1.39}$$

where $f(T)$ is a coupling function. This term has been used to study the presence of timelike soliton kinks on the worldvolume. More recently, evidence was provided that the spectrum of string theory must include other types of 'unstable' branes: the SD-branes [15]. From the worldsheet point of view these correspond to open strings with a Dirichlet boundary condition imposed on the target space time X^0. The spacetime realization of these objects should correspond to supergravity solutions corresponding to spacelike kinks. We describe these objects in more detail in chapter 2 taking the point of view that their low energy dynamics is governed by a modified Born-Infeld action supplemented with a Chern-Simons term of the form (1.39).

1.2 Elements of cosmology

A very active direction of research consists in studying the connections between string theory and cosmology. The latter has presented theoretical physicists with new challenges by providing them with many experimental data unexplained by current cosmological models. Cosmology is therefore an arena where string theory, which is the most promising candidate for a theory of quantum gravity, might have interesting things to say. The research projects presented in chapter 3 are based on an attempt to relate string theory to the fact that the universe appears to be evolving towards a phase where its evolution is controled by a constant positive vacuum energy [23].

In this thesis, the cosmological (or time-dependent) spacetimes we consider are isotropic and homogeneous. The large scale structure of our universe possesses these characteristics. Of course, this is true only on an average sense since there are inhomogeneities corresponding to stars, galaxies and galactical clusters.[13] More precisely, the spacetimes of interest are of the form $\mathbf{R} \times \Sigma$, where $\mathbf{R}$ is the time direction and Σ is a homogeneous and isotropic $(d-1)$-dimensional manifold. The most general metric foliated with maximally symmetric spacelike slices is called a Friedmann-Robertson-Walker (FRW) universe,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega_{d-1}^2\right], \tag{1.40}$$

where $k = -1$, $k = 0$ or $k = 1$ depending on whether the space is respectively open (hyperbolic), flat or closed (spherical). Section 3.1 provides the reader with more details concerning these different foliations. The function $a(t)$ is called the scale factor. It is a measure of how large the spacelike slice Σ is at time t. The metric (1.40) is written in co-moving coordinates. An observer staying at a constant point on Σ is said to be co-moving.

Different functions $a(t)$ will correspond to universes with different energy-momentum tensors (T_{ij}) as is seen through the Einstein equation

$$\mathcal{R}_{ij} - \frac{1}{2}\mathcal{R}\, G_{ij} = 8\pi G_N\, T_{ij}, \tag{1.41}$$

where G_N is the Newton constant. Vacuum solutions are those for which the stress-energy tensor vanishes, *e.g.*, Minkowski space. For isotropic and homogeneous universes, it is natural to consider energy-momentum tensors in the form of a

[13]An active area of research consists in studying how models inspired by string theory might have given rise to these inhomogeneities.

perfect fluid at rest in co-moving coordinates,

$$T^i_j = \mathrm{diag}\,(-\rho, p, \, ... \, , p), \tag{1.42}$$

where ρ is the density of energy and p the normal pressure. Many realistic cosmological models obey the equation of state

$$\omega = \frac{p}{\rho}, \tag{1.43}$$

where ω is assumed not to depend on time. Covariant conservation of the stress-energy tensor ($\nabla_i T^i_j = 0$) leads to the continuity equation

$$\frac{\dot{\rho}}{\rho} = -3(1+\omega)\frac{\dot{a}}{a}, \tag{1.44}$$

which can integrated to give

$$\rho \sim \frac{1}{a^{3(1+\omega)}}. \tag{1.45}$$

For example, if the pressure vanishes the corresponding cosmological fluid is called dust and has $\omega = 0$. This corresponds to a matter-dominated phase. If the universe is mostly composed of massless radiation then the equation of state is $\omega = 1/3$ (radiation-dominated phase). Of most interest to us is the case $\omega = -1$ which corresponds to the universe being vacuum-dominated. The source of gravity then corresponds to a stress-energy tensor of the form

$$T_{ij} = -\frac{\Lambda}{8\pi G_N} g_{ij}, \tag{1.46}$$

which corresponds to a distribution of energy constant in time. Λ is then called the cosmological constant (see chapter 3 for more details).

We conclude this section by describing the most common energy conditions. As pointed before, energy in a time-dependent background is not conserved. It is commonly believed that the energy conditions must be obeyed by the classical stress-energy tensor of any *reasonable* equation of state. The weak energy condition is the least constraining one. It states that $\rho + p \geq 0$ and

$$T_{ij}\xi^i\xi^j \geq 0, \tag{1.47}$$

for any timelike or null vector ξ. The dominant energy condition is

$$T_{ij}\xi^i\eta^j \geq 0, \quad \rho \geq |p|, \tag{1.48}$$

where η is a timelike or null vector. The strong energy condition, which is the most restrictive, requires that

$$T_{ij}\xi^i\xi^j - \frac{1}{d-2}\xi^k\xi_k T^i_i \geq 0, \tag{1.49}$$

and

$$\rho + p \geq 0, \quad (d-3)\rho + (d-1)p \geq 0. \tag{1.50}$$

The fact that gravity is always attractive (seen by studying the Raychaudhuri equations) depends on whether or not the weak energy condition is satisfied. Also, one of the assumptions of the Hawking-Penrose singularity theorems is that the energy conditions hold.

1.3 Outline

In the remainder of this thesis we investigate two aspects of time-dependence in string theory. In chapter 2 we obtain the supergravity solutions corresponding to SD-branes. These are associated with a time-dependent open string tachyon that excites massless closed string modes during its evolution. The resulting solutions are asymptotically flat and non-singular. The material of this chapter was presented earlier in ref. [1]. Chapter 3 is devoted to presenting our work on the dS/CFT correspondence [3, 4] which is a proposed duality between quantum gravity on a de Sitter (dS) spacetime and a Euclidean conformal field theory. Firstly we provide an introduction of dS space and the correspondence. Then, we present our work on the interpretation of asymptotically dS spacetimes in terms of renormalazation group flows. Finally, we provide evidence for the existence of new non-local field theory dualities in the context of the dS/CFT correspondence.

CHAPTER 2

SD-BRANE GRAVITY FIELDS

2.1 INTRODUCTION

SD-branes in string theory were first studied by Gutperle and Strominger in ref. [15]. They were introduced as objects arising when Dirichlet boundary conditions on open strings are put on the time coordinate, as well as on spatial coordinates. SD-branes are not supersymmetric objects, which makes them hard to handle but potentially very interesting. The boundary conditions for SD-branes imply that they live for only an "instant" of time, and so the worldvolume is purely spatial. SD-branes should not be confused with instantons, because they live out their lives in Lorentzian signature context. Recent discussion of the relation between SD-branes and instantons may be found in section 6 of ref. [24].

SD-branes are especially interesting objects to study in the context of tachyon condensation, which will be the arena of our investigation. SD-branes are indeed inherently related to the general study of time dependence in string theory. One of the original goals of ref. [15] was in fact to seek examples of gauge/gravity dualities where a time direction on the gravity side is holographically reconstructed by a Euclidean field theory.[1]

There have been several investigations of SD-branes since they were introduced [26, 27, 28, 29, 30, 31, 24]. Most of them involve taking the limit $g_s \to 0$, the regime in which perturbative string computations can be done. As the tachyon rolls down its potential hill, there is a divergence in production of higher mass open string

[1] An attempt to find a realization for such a duality is the dS/CFT correspondence [25] which we study in chapter 3.

modes [28, 32, 24]. This divergence occurs before the tachyon gets to the bottom of the potential well, as it must because there are no perturbative open string excitations around the true minimum of the tachyon potential [33]. Also, the time taken to convert the energy of the rolling tachyon into these massive open string modes is of order[2] $\mathcal{O}(g_s^0)$. This analysis was done for a single SD-brane using CFT methods; analysis for production of massive closed string modes was also done [34, 35].

One aspect of SD-brane physics has become clear: that the decoupling limit applied to SD-branes is not a smooth limit like it is for regular D-branes. In particular, as $g_s \to 0$ the brane tachyon becomes decoupled from bulk modes, which were however the most natural modes into which the initial energy of the unstable brane should decay. Then, the endpoint of the rolling tachyon must include a somewhat mysterious substance called "tachyon matter" [37]. Consideration of the full problem with g_s finite would presumably eliminate the need for mysterious tachyon matter; this was in fact part of our motivation for this work. Regarding production of closed string massive modes, at very small g_s, it seemed that there was some debate [34, 35] about the form of a divergence. The results from ref. [35] make clear that the divergence depends on the number of dimensions transverse to the decaying unstable brane: for unstable Dp-branes with $p < 2$ there was a need to invoke a cutoff to get a finite result. In any case, unstable brane decay should presumably be a physically smooth process for $\{g_s, \ell_s\}$ finite.

The point of view that we will be taking is to consider a system of N SD-branes, with g_sN large, study the overall centre-of-mass tachyon, and couple it to bulk massless closed string modes. Obviously, it would be nice to understand the full problem including coupling to all massive open and closed string modes, but

[2]Our calculation, which includes gravity backreaction, agrees with this in the sense that the time it takes for the tachyon to decay only slightly depends on the particular value of g_s.

this is a hard problem beyond our reach at this point. We will make a beginning here with a quantitative analysis involving only the lowest modes in each of the open and closed string sectors. We believe that our approach, while "lowbrow" by comparison to string field theory (see ref. [16] for a review) computations, already shows some very interesting physics.

We study (numerically) spacetime solutions for the evolution of the open string tachyon coupled to bulk supergravity modes. In the case of full-SD-branes, we find that a singularity develops during the evolution of the tachyon. This is consistent with the singularity theorem for full SD-branes given in [36]. This work showed analytically that full-SDp-branes with codimension one or greater must develop singularities either in the past or the future of of the tachyon evolution. However, we find that solutions associated with half-S-branes, which are closely related to unstable D-branes, are typically nonsingular. Our work can also be considered to shed light on the question of tachyon cosmology [38] including Ramond-Ramond fields, the effect of which was ignored in previous investigations. Tachyon cosmology itself may not yet provide realistic models for inflation, nonetheless, see recent work including, *e.g.*, refs. [39, 40]. One of the reasons is that, in the low-energy actions used to describe tachyon cosmology dynamics, there is only one length scale — the string length. It would certainly be interesting if a mechanism generating a lower scale for inflation were found within this context. Also, the behavior of inhomogeneities during the later stage of the roll of the tachyon may be a general problem [39]. Tachyon cosmology involving brane-antibrane annihilation may be relevant only to a pre-inflationary period, but it is interesting to analyze the dynamics from the "top-down" perspective in string theory.

The plan of this chapter is as follows. We begin by reviewing in section 2.2 the previous work on gravity fields of SD-branes, and commenting on the nature of singularities found in the past literature. In section 2.2.1 we discuss pathologies of the solutions found in refs. [26, 27], where only supergravity fields were considered. In section 2.2.2 we move to discussing ref. [29], in which an unstable brane probe was coupled to a $d = 4$ SD0-brane supergravity background; we generalized their arguments but still find generic singularities in the probe approximation (exceptions are considered in Appendix A). In section 2.3 we set up the full backreacted problem of interest. The equations are naturally highly nonlinear, and since backreaction is essential we have no desire to ignore it or treat it perturbatively. We need to use a particular homogeneous ansatz to facilitate solution of the equations of motion, and we discuss implications of the ansatz. In section 2.4 we demonstrate half-SD-brane numerical solutions of our backreaction-inclusive equations, and interpret qualitative features found. Then we briefly review the singularity theorem for full-SD-branes and give an alternate formulation. We then move to showing two situations of notable interest where the singularity theorem does not hold: cosmological applications and decaying unstable branes. We also give strong indications that space-filling SD8-branes may escape the theorem completely.

Lastly, in the Discussion section we summarize our conclusions, open issues, and directions of future work. More specifically, we remark on what may resolve the full-SD-brane singularity. We also point out several directions in which the singularity theorem could usefully be generalized.

2.2 Singular supergravity SD-branes: Review

In the paper [15], a small number of supergravity solutions, thought to be appropriate for a large number N of SD-branes, were presented. Subsequently, the class of supergravity solutions was widened considerably, simultaneously by two groups (refs. [26] and [27]). A later paper showed that these two sets of solutions were equivalent [30], by matching boundary conditions on asymptotic fields and finding the coordinate transformation explicitly.

2.2.1 Sourceless SD-brane supergravity solutions

The convention of ref. [15], which we will use, is that SDp-branes have $(p+1)$ worldvolume coordinates. We call these $\vec{y}$. There is also the time coordinate t, and the $(8-p)$ overall transverse coordinates $\vec{x}$.

The most general SDp-brane supergravity solutions of refs. [26] and [27] can then be written in string frame as follows.

$$\begin{aligned}
ds^2_{Sp} &= F(t)^{1/2}\beta(t)^G\alpha(t)^H\left(-dt^2+t^2dH^2_{8-p}(\vec{x})\right) \\
&\quad +F(t)^{-1/2}\left[\sum_{i=2}^{p+1}\left(\frac{\beta(t)}{\alpha(t)}\right)^{-k_i}(dy^i)^2+\left(\frac{\beta(t)}{\alpha(t)}\right)^{k_1+\tilde{k}}(dy^1)^2\right] , \\
e^{2\Phi} &= F(t)^{\frac{3-p}{2}}\left(\frac{\beta(t)}{\alpha(t)}\right)^{-\sum_{i=2}^{p+1}k_i} , \\
C^{(p+1)} &= \sin\theta\cos\theta\,\frac{C(t)}{F(t)}\,dy^1\wedge\cdots\wedge dy^{p+1} ,
\end{aligned} \tag{2.1}$$

where

$$C(t)\equiv\left(\frac{\beta(t)}{\alpha(t)}\right)^{k_1}-\left(\frac{\beta(t)}{\alpha(t)}\right)^{\tilde{k}} , \qquad F(t)\equiv\cos^2\theta\left(\frac{\beta(t)}{\alpha(t)}\right)^{\tilde{k}}+\sin^2\theta\left(\frac{\beta(t)}{\alpha(t)}\right)^{k_1} , \tag{2.2}$$

and

$$\alpha(t)\equiv 1+\left(\frac{\omega}{t}\right)^{7-p} , \quad \beta(t)\equiv 1-\left(\frac{\omega}{t}\right)^{7-p} . \tag{2.3}$$

The supergravity equations of motion will be satisfied when the exponents satisfy the constraints,

$$\begin{aligned}
\tilde{k}^2+\sum_{i=1}^{p+1}k_i^2+\frac{7-p}{4}(H-G)^2-4\frac{8-p}{7-p} &= 0 , \\
\tilde{k}+\sum_{i=1}^{p+1}k_i-\frac{7-p}{2}(H-G) &= 0 , \\
H+G-\frac{4}{7-p} &= 0 .
\end{aligned} \tag{2.4}$$

The general metric above is not isotropic in the worldvolume directions $\vec{y}$. However, from a microscopic point of view, one expects that the supergravity solution would have an isotropic worldvolume. Isotropy in the worldvolume will be restored in the above solution for the choice $-k_2 = \cdots = -k_{p+1} = k_1 + \tilde{k} \equiv n$. Unfortunately, the isotropy requirement excludes SD-brane solutions with regular Cauchy horizon. In fact, the curvature invariants associated to all isotropic solutions diverge at $t = \pm\omega$ and $t = 0$. Consequently, although they possess the right symmetries and "charge", these solutions do not appear to be well-defined.

Besides, SD-branes should be represented by solutions with, roughly, three distinct regions: the infinite past with incoming radiation only in the form of massless closed strings, an intermediate region with both open and closed strings, and finally the infinite future with dissipating outgoing radiation in the form of massless closed strings. But the supergravity solutions of refs. [15, 26, 27] cannot represent this process, because there are no rules for deciding how to go through the singular regions (see, however, Appendix A where we show that some anisotropic solutions avoid the pathology).

Nevertheless, the isotropic solutions have a positive feature worth noting: they have the correct asymptotics at large time. In the limit that the functions $\alpha(t)$ and $\beta(t)$ become trivial, part of the metric is simply the Milne universe: flat Minkowski space foliated by hyperbolic sections,

$$\lim_{t\to\pm\infty} ds^2_{s_p} = -dt^2 + t^2 dH^2_{8-p}(\vec{x}) + \sum_{i=1}^{p+2}(dy^i)^2. \tag{2.5}$$

On the other hand, there are (at least) two reasons to suspect that the above solutions are not the final word in the SD-brane supergravity story. The first is that there are one too many parameters in the solution, by comparison to expectations from microscopics of SD-branes [27]. A possible explanation for this may be as follows. In the rest of this chapter, we will consider the full coupling/backreaction between the open-string tachyon and the closed-string bulk modes. It is possible that the freedom in the supergravity solutions may correspond to a freedom in picking boundary conditions for the rolling tachyon — the coupling to which was not included in refs. [15, 26, 27]. We will see, however, that introducing a source associated with an open string tachyon is not sufficient to resolve the singularities.

The most noticeable negative feature of the above solutions is that the isotropic solutions are nakedly singular. Quite generally, nakedly singular spacetimes arising in low-energy string theory come under immediate suspicion, even though they are solutions to the supergravity field equations. No-hair theorems are usually what we rely on in order to be sure that we have the unique supergravity solution, but no-hair theorems are never valid for solutions with *naked* spacetime singularities. It is worth noting that it has been shown with an explicit counterexample [41] that even no-hair theorems themselves fail for black holes in $d = 5$ with mass and angular momentum — and hence the idea of no-hair theorems in all higher dimensions is under suspicion. (Nonetheless, with particular assumptions about field couplings, no-hair theorems can be proven for static asymptotically flat dilaton black holes [42]. Also, uniqueness of the *supersymmetric* rotating BMPV [43] black hole in $d = 5$ has been proven [44].) Even if a no-hair theorem appropriate to the supergravity theory involving SD-branes could be proven, however, the above solutions we have reviewed would be ruled out as candidates because their singularities are uncontrollably nasty. So we have to look elsewhere.

Let us take a brief sidetrip here to comment on the singularity story for supergravity fields of $N \gg 1$ regular D-branes with timelike worldvolume. Certainly, the geometry of BPS D3-branes is nonsingular, and there are several other pretty situations known in the literature where branes "melt" into fluxes: the sources are no longer needed. However, the disappearance of D-brane sources for supergravity fields only occurs when the branes are BPS. If any energy density above

BPS is added to these systems, singularities reappear: this certainly happens for the D3-brane system. Also, in the low-energy approximation to string theory, it is misleading to think of supergravity fields of Dp-branes as simple condensates of massless closed string modes. The reason is directly analogous to the fact that the Coulomb field of an electron cannot be a photon condensate because the photon is transverse. Similarly, Coulomb Ramond-Ramond fields of Dp-branes cannot be represented by supergravity fields alone.[3] This river runs deeper: in the decoupling limit, resolution of dilaton and curvature singularities for $p \neq 3$ Dp-branes is in fact *provided* by the gauge theory on the D-branes [45, 46].

Let us now get back to our SD-branes. The supergravity situation looks similar to that for non-BPS (ordinary) D-branes: it seems that brane modes will be required for singularity resolution. Therefore, we are motivated to try to solve all problems with prior candidate SD-brane spacetimes by solving the coupled system of brane tachyon plus bulk supergravity fields with full backreaction.

2.2.2 Unstable brane probes in sourceless SD-brane backgrounds

The first attempt towards the goal of singularity resolution in SD-brane systems was made in ref. [29]. We now briefly review what is, for our purposes, the most relevant point of that work.

Essentially, they take the reasonable point of view that the process of creation and subsequent decay of a SDp-brane must be driven by a single open string mode: the tachyon field denoted $T(t)$, which lives on the associated unstable D(p+1)-brane. They use the $p = 0$ *non*-dilatonic version of the worldvolume

[3] This is the case even though Dp-branes are "solitonic" in string theory while electrons are fundamental in QED. The straightforward argument we use here depends only on the couplings of the charge-carrying objects to the bulk gauge fields.

action [47, 48, 49, 50, 37, 51]

$$\begin{aligned} S_{brane} = & -T_{p+1} \int d^{p+2}y\, e^{-\Phi} V(T) \sqrt{-det\ (\mathcal{P}\,[G_{\alpha\beta}] + \partial_\alpha T \partial_\beta T)} \\ & + \mu_{p+1} \int f(T) dT \wedge C^{(p+1)}, \end{aligned} \tag{2.6}$$

to study the dynamics of the tachyon and its couplings with bulk (closed string) modes. The operation $\mathcal{P}$ is the pullback, and $\alpha, \beta = 1 \ldots (p+1)$. In section 2.3.1 we will comment both on the validity of this type of effective action, and on the expected form for the tachyon potential $V(T)$ and the Ramond-Ramond coupling $f(T)$. For now, we just use it.

The way we look at the calculation of ref. [29] is as follows. Supergravity SD-brane fields should be regarded as arising directly from a large number of unstable branes. Then, using the intuition gained from studying the enhançon mechanism [52], it is natural to use an unstable brane probe[4] to study more substantively the candidate supergravity solutions of refs. [26, 27]. Then, we look for problems arising in the probe calculation. The idea is that whenever the probe analysis goes wrong, it signals a pathology for the gravitational background. There are at least two ways the probe analysis can signal a problem: infinite energy or pressure density for the tachyon may be induced, ($\rho_{probe}, p_{probe} \rightarrow \pm\infty$), or there might not exist any reasonable solutions for $T(t)$ across the horizon.

So let us consider inserting an unstable brane probe in a background with fields corresponding to the sourceless SD-brane supergravity fields, eqs. (2.1), of the previous subsection. The equation of motion for the open string tachyon is, generally,

$$(-g_{tt})^{\frac{1}{2}} (g_{y^i y^i})^{\frac{p}{2}} (g_{y^1 y^1})^{\frac{1}{2}} e^{-\Phi} \frac{\partial V(T)}{\partial T} \Delta^{\frac{1}{2}}$$

[4]We refer to a probe as an object that has negligible backreaction on the fields composing the background in which it propagates. It can be regarded as a small perturbation.

$$+ \quad f(T)\, F^{(p+2)} + \frac{d}{dt}\left(\dot{T}\, V(T) \frac{(g_{y^1y^1})^{\frac{1}{2}} (g_{y^iy^i})^{\frac{p}{2}}}{e^{\Phi} \Delta^{\frac{1}{2}} (-g_{tt})^{\frac{1}{2}}} \right) = 0, \tag{2.7}$$

where $\dot{} \equiv d/dt$, our notation for the Ramond-Ramond field strength is $F^{(p+2)} = dC^{(p+1)}$, we have factored out T_{p+1} by including a factor of g_s in $C^{(p+1)}$, and we also defined the following expression,

$$\Delta = 1 + \frac{(\dot{T})^2}{g_{tt}}. \tag{2.8}$$

The question needing attention here is whether or not the tachyon field, regarded as a probe,[5] is well-behaved when inserted in the candidate supergravity backgounds (2.1). Ref. [29] provided a clear answer for the case of a $d = 4$ SD0-brane with dilaton field set to zero. Let us now see how this goes.

The $d = 4$ SD0-brane background introduced in ref. [15] has the form,

$$g_{tt} = -g^{y^1y^1} = -\frac{Q^2}{\omega^2} \frac{t^2}{t^2 - \omega^2}, \quad g_{y^iy^i} = 0, \quad g_{xx} = \frac{Q^2}{\omega^2} t^2, \tag{2.9}$$

and

$$F_2 = Q\epsilon_2\,. \quad \Phi = 0. \tag{2.10}$$

This spacetime metric has a regular horizon (a coordinate singularity) at $t = \omega$, and a genuine timelike curvature singularity at $t = 0$. The expressions for the energy density (T_t^t) and the pressure ($T_{y^1}^{y^1}$) associated with the probe are respectively,

$$\rho_{probe} \sim \frac{V(T)}{\Delta^{1/2}}, \quad p_{probe} \sim V(T)\Delta^{1/2}. \tag{2.11}$$

The only time (apart from $t = 0$) when the probe limit becomes ill-defined is around $t = \omega$. In fact, in the near horizon limit, the dynamical quantity Δ satisfies the simple ordinary differential equation

$$\Delta^2 - \Delta + (t - \omega)\dot{\Delta} = 0. \tag{2.12}$$

[5]The unstable brane will be a probe as long as its backreaction is small and can therefore be treated self-consistently as a perturbation.

This has the general solution

$$\Delta = \frac{t-\omega}{t-\omega+g}, \tag{2.13}$$

where g is a constant of integration. There are two possible solutions at $t = \omega$: $\Delta = 0$ ($g \neq 0$) or $\Delta = 1$ ($g = 0$). Clearly, the case for which $\Delta = 0$ corresponds to the probe limit breaking down since the energy density of the unstable brane diverges.[6] The other possibility, $\Delta = 1$, implies that the time-derivative of the tachyon diverges on the horizon. This last case is clearly pathological and cannot correspond to a physically relevant tachyon field solution.

The conclusion is that unstable brane *probes* are not well-defined in the SD0-brane background. Not only are they useless to resolve the timelike singularity at $t = 0$ but, worse, they appear to generically induce a spacelike curvature singularity on the horizon at $t = \omega$. That is, if we take the probe story to be a good indicator of the story for the full backreacted problem.

One of the first motivations for the work presented in this chapter was to check how restricted the conclusions of ref. [29] were. Did this above story work only for bulk couplings of the kind arising for SD0-branes in $d = 4$, where no dilaton field appears? Was it true only for the case of SD0-branes, which are a special case for SDp-branes since there can be no anistropy in a one-dimensional worldvolume? Are all timelike clothed singularities turned into naked spacelike singularities by probes? Could we even trust the probe approximation to tell us anything about the solution with full backreaction?

The first generalization we considered was to look at an unstable brane probe in the background of the isotropic SDp-brane solutions of ref. [27]. However, what we saw there was that the naked spacelike singularities remained naked spacelike

[6] This is true assuming that the tachyon is not close to the bottom of the potential in the region $t \simeq \omega$. It would in fact be natural to expect that in this region of higher curvature the tachyon is close to the top of the potential hill, *i.e.*, $V(T) \neq 0$.

singularities; the small effect of the probe could not undo that pathology. Next, we moved to analyzing anisotropic solutions of the form (2.1), those with regular horizons. Some of these are actually completely nonsingular; we analyzed the details of the probe computation in those backgrounds, and the specifics are recorded in Appendix A. The results there are simple to summarize: the solutions with singularities hidden behind horizons do not give rise to conclusions qualitatively different than what we have reviewed here for $d = 4$ SD0-branes. The picture therefore remains unsatisfactory.

The upshot, then, is that the probe story does not resolve singularities found for sourceless supergravity SD-brane solutions. So we now move to the full backreacted problem for SDp-branes in d=10, which is the main content of this chapter.

2.3 Supergravity SD-branes with a tachyon source

In this section we find, in the context of supergravity, the equations of motion associated with the real-time (formation and/or) decay of a clump of unstable D-branes. We begin this section by writing the form of the action which we will use in our analysis. We will concentrate on only the most relevant modes in both the open and closed string sectors; in other words, we keep in our analysis only the (homogeneous) tachyon and massless bulk supergravity fields. Potentially, the effect of massive open string modes could be encoded in a modified equation of state for the tachyon fluid on the unstable D-brane. We leave for future work the issue of non-homogeneous tachyonic modes, and of massive string modes in both the open and closed string sectors, for the coupled bulk-brane system with full backreaction. In order to use the supergravity approximation here self-consistently, we will take g_s small but g_sN large, and time-derivatives will be small compared to ℓ_s. Based on numerical analysis for unstable D-branes, we find that it is simple to choose boundary conditions such that these remain true for all time.

2.3.1 Preliminaries: action and equations of motion

For this section we will be able to suppress R-R Chern-Simons terms in writing the bulk action. This is a consistent truncation, to set the NS-NS two-form to zero throughout the evolution of the system of interest, as long as consistency conditions on the R-R fields are satisfied. *E.g.* for the SD2-brane system with R-R field $C^{(3)}$ activated, it is necessary to make certain that $dC^{(3)} \wedge dC^{(3)} = 0$ in order not to activate the NS-NS two-form and the accompanying Chern-Simons terms. Other cases are related to this one by T-duality. Therefore, we allow only electric-type coupling of the SDp-brane to $C^{(p+1)}$ (or equivalently magnetic-type coupling to $C^{(7-p)}$). Later we will show that this ansatz is physically consistent provided we assume that there is $ISO(p+1)$ symmetry along the worldvolume of the SDp-brane, the object we are interested in. This is equivalent to considering only the lowest-mass tachyon, *i.e.*, not allowing any excitations of the brane tachyon along the spatial worldvolume directions. Of course, the R-R field strengths are then very simple: $G^{(p+2)} = dC^{(p+1)}$, and the string frame bulk action takes the form [10]

$$S_{\text{bulk}} = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left\{ e^{-2\Phi} \left[\mathcal{R} + 4(\partial\Phi)^2 \right] - \frac{1}{2} \left| dC^{p+1} \right|^2 \right\} , \tag{2.14}$$

where $\mathcal{R}$ is the Ricci scalar. We use a mostly plus signature. In the above conventions, the R-R field solutions automatically get a factor of g_s (as we mentioned also in the previous subsection), and

$$16\pi G_{10} = (2\pi)^7 g_s^2 \ell_s^8 , \quad \tau_{Dp} = \frac{1}{g_s (2\pi)^p \ell_s^{p+1}} . \tag{2.15}$$

The equations we provide are in string frame but in section 2.5 we use the Einstein frame. We convert to the latter – with metric $\tilde{g}_{\mu\nu}$ – with canonical normalization of the metric and positive dilaton kinetic energy, by using the standard $d = 10$ conformal transformation

$$\tilde{g}_{\mu\nu} = e^{-(\Phi-\Phi_\infty)/2} g_{\mu\nu}. \tag{2.16}$$

Stress-tensors are defined in Einstein frame,

$$\tilde{T}_{\mu\nu} \equiv \frac{-1}{\sqrt{-\tilde{g}}} \frac{\delta S_{\text{matter}}}{\delta \tilde{g}^{\mu\nu}}, \tag{2.17}$$

with the usual

$$\tilde{T}_{\mu\nu}[\Phi] = \frac{1}{2}\left[\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}\tilde{g}_{\mu\nu}(\tilde{\partial\Phi})^2\right], \tag{2.18}$$

$$\tilde{T}_{\mu\nu}\left[C^{p+1}\right] = \frac{e^{(3-p)\Phi/2}}{2(p+1)!}\left[\tilde{G}_\mu^{\ \lambda_2\ldots\lambda_{p+2}} G_{\nu\lambda_2\ldots\lambda_{p+2}} - \frac{1}{2(p+2)}\tilde{g}_{\mu\nu}\tilde{G}^2\right]. \tag{2.19}$$

We can transform to string-frame "Einstein" equation using standard formulæ

$$\begin{aligned} &\tilde{\mathcal{R}}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\mathcal{R}} - \frac{1}{2}\left[\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}\tilde{g}_{\mu\nu}(\tilde{\partial\Phi})^2\right] \\ &= \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} + 2\left[\nabla_\mu\partial_\nu\Phi - g_{\mu\nu}\nabla^2\Phi + g_{\mu\nu}(\partial\Phi)^2\right]. \end{aligned} \tag{2.20}$$

For all matter fields except the dilaton, it is therefore obvious that string frame "stress-tensors" take the form

$$T_{\mu\nu} = \frac{-1}{\sqrt{-g}} \frac{\delta S_{\text{matter}}}{\delta g^{\mu\nu}}. \tag{2.21}$$

For the dilaton we find

$$T^\mu_{\ \nu}[\Phi] = 2\left[-\nabla^\mu\partial_\nu\Phi + g^\mu_{\ \nu}\nabla^2\Phi - g^\mu_{\ \nu}(\partial\Phi)^2\right], \tag{2.22}$$

and, obviously, familiar energy conditions for bulk fields are only satisfied in Einstein frame, not string frame.

For the bulk field equations, we must include coupling to the brane tachyon. Hence, we now turn to the brane action. The brane theory is that appropriate to unstable D$(p+1)$-branes, for SDp-branes. We consider N branes. At low energy, the overall $U(1)$ center-of-mass tachyon T couples as follows:

$$S_{\text{brane}} = \frac{\Lambda}{16\pi G_{10}} \left\{ \int dt d^{p+1}y \left[- e^{-\Phi}\sqrt{-A} V(T) \right] + \int f(T) dT \wedge C^{p+1} \right\} , \quad (2.23)$$

where the matrix $A_{\alpha\beta}$ is defined as

$$A_{\alpha\beta} = \mathcal{P}\left[G_{\alpha\beta} + B_{\alpha\beta}\right] + F_{\alpha\beta} + \partial_\alpha T \partial_\beta T \, , \quad (2.24)$$

where $\mathcal{P}$ stands for pullback. For the constants, our conventions are that T is normalized like $F_{\alpha\beta}$, and also

$$\Lambda \equiv \frac{N\mu_{p+1}}{g_s}(16\pi G_{10}) = (Ng_s)(2\pi\ell_s)^{6-p}. \quad (2.25)$$

Notice that Λ is proportional to $g_s N$. This will be the *sole* continuous[7] control parameter associated with the physics of our final solutions for the coupled tachyon-supergravity system.

It is important to know when we can expect to trust the action we use. Our approach consists in assuming that the kinetic terms of the open string tachyon field are re-summed to take a Born-Infeld–like form. Strictly speaking, this has only be shown to be a valid claim late in the tachyon evolution. We refer the reader to ref. [51] for more details on the limits in which this approximation holds (see also ref. [53]). The functions $V(T)$ and $f(T)$ are therefore not known exactly at all times. For definiteness in our numerical analysis, we will choose a specific form and assume that the dynamics of the tachyon is governed by eq. (2.23). For a SD-brane, we make the choice $V(T) = 1/\cosh(T/\sqrt{2}) = |f(T)|$ which has been shown to be the correct large-T behavior of the couplings. Our results turn out to be quite robust, in that their qualitative features do not depend on the precise form of $V(T)$ and $f(T)$.

[7] Λ is effectively continuous in the supergravity approximation since $g_s N$ is large and all derivatives small in string (ℓ_s) units.

For the remainder of our discussion it will be convenient to use static gauge,[8] which is an appropriate gauge choice for our problem of interest. Therefore, in the following, we will be rather cavalier about dropping pullback symbols.

When we get to solving the coupled brane-bulk equations, it will be convenient to allow for a density of branes, denoted $\rho_\perp$, in the transverse space:

$$\int_{\text{brane}} dt d^{p+1}y \longrightarrow \int_{\text{bulk}} dt d^{p+1}x d^{8-p}x \rho_\perp \,. \tag{2.26}$$

Therefore our brane action becomes

$$S_{\text{brane}} = \frac{\Lambda}{16\pi G_{10}} \int d^{10}x \rho_\perp \left\{ -V(T)\sqrt{-A}e^{-\Phi} + f(T)\epsilon^{\lambda_1\ldots\lambda_{p+2}} C_{[\lambda_2\ldots\lambda_{p+2}}\partial_{\lambda_1]}T \right\}, \tag{2.27}$$

where ϵ is the worldvolume permutation tensor with values $(0, \pm 1)$.

We are now ready to write down the coupled field equations. The simplest bulk equation to pick off is the dilaton. In string frame we see immediately that

$$\nabla^2\Phi - (\partial\Phi)^2 + \frac{1}{4}\mathcal{R} = \frac{1}{8}\left(\frac{\Lambda\rho_\perp}{\sqrt{-g}}\right) e^{+\Phi}V(T)\sqrt{-A}, \tag{2.28}$$

and for the Ramond-Ramond field

$$\nabla_\mu G^{\mu\lambda_2\ldots\lambda_{p+2}} = -\left(\frac{\Lambda\rho_\perp}{\sqrt{-g}}\right) f(T)\epsilon^{\mu\lambda_2\ldots\lambda_{p+2}}\partial_\mu T. \tag{2.29}$$

For the metric equation of motion (string frame "Einstein" equations), it is convenient to define

$$\mathcal{R}^\mu{}_\nu = \hat{T}^\mu{}_\nu \equiv T^\mu{}_\nu - \frac{1}{8}T^\lambda_\lambda. \tag{2.30}$$

Therefore, we have

$$\hat{T}^\mu{}_\nu[\Phi] = -2\nabla^\mu\partial_\nu\Phi - \frac{1}{4}g^\mu{}_\nu\nabla^2\Phi + \frac{1}{2}(\partial\Phi)^2 g^\mu{}_\nu. \tag{2.31}$$

---

[8]The static gauge consists in using the worldvolume and spacetime diffeomorphism invariances to align the worldvolume of the SD-brane with the first $(p+1)$ spacetime coordinates.

For the brane stress-tensor, we need to figure out the dependence of $\sqrt{-A}$ on $g_{\mu\nu}$ (the Wess-Zumino term clearly does not contribute). We find

$$T^{\mu}{}_{\nu}[T] = -\frac{1}{2}e^{\Phi}\left(\frac{\Lambda\rho_\perp}{\sqrt{-g}}\right)V(T)\sqrt{-A}(A^{-1})^{\alpha\mu}g_{\alpha\nu}\,, \tag{2.32}$$

$$\hat{T}^{\mu}{}_{\nu}[T] = -\frac{1}{2}e^{\Phi}\left(\frac{\Lambda\rho_\perp}{\sqrt{-g}}\right)V(T)\sqrt{-A}\left[(A^{-1})^{\alpha\mu}g_{\alpha\nu} - \frac{1}{8}g^{\mu}_{\nu}(A^{-1})^{\lambda\sigma}g_{\lambda\sigma}\right]\,. \tag{2.33}$$

The other object we need for the metric equation of motion is

$$\hat{T}^{\mu}{}_{\nu}[C] = \frac{1}{2(p+1)!}e^{2\Phi}G^{\mu\lambda\ldots\sigma}G_{\nu\lambda\ldots\sigma} - \frac{(p+1)}{16}e^{2\Phi}\frac{G^2}{(p+2)!}g^{\mu}{}_{\nu}\,. \tag{2.34}$$

Lastly, for the tachyon we find the equation of motion

$$\frac{dV}{dT}e^{-\Phi}\sqrt{-A} - \partial_\mu\left[V(T)e^{-\Phi}\sqrt{-A}(A^{-1})^{\mu\alpha}\partial_\alpha T\right] + f(T)\epsilon^{\mu\ldots\lambda}G_{\mu\ldots\lambda} = 0\,. \tag{2.35}$$

In the Discussion section we will make some remarks about the robustness of these equations.

2.3.2 THE HOMOGENEOUS BRANE SELF-CONSISTENT ANSATZ

As pointed out earlier, we are interested in time-dependent processes by which massless Type IIa or Type IIb supergravity fields are sourced by an open-string tachyon mode on the worldvolume of an unstable brane. A reasonable assumption is that the gravitational background generated by backreaction of the rolling tachyon is of the form

$$ds^2 = -dt^2 + a(t)^2 d\Sigma^2_{p+1}(k_\parallel) + R(t)^2 d\Sigma^2_{8-p}(k_\perp)\,, \tag{2.36}$$

where the n-dimensional Euclidean metric $d\Sigma^2_n(k)$ is

$$d\Sigma^2_n(k) = \begin{cases} d\Omega^2_n \text{ for } k = +1 \\ dE^2_n \text{ for } k = 0 \\ dH^2_n \text{ for } k = -1\,, \end{cases} \tag{2.37}$$

where $d\Omega_n^2$ is the unit metric on S^n, dE_n^2 the flat Euclidean metric, and dH_n^2 the 'unit metric' on n-dimensional hyperbolic space H^n. The corresponding symmetry groups are

$$\begin{cases} SO(n+1) & \text{for } k=+1 \\ ISO(n) & \text{for } k=0 \\ SO(1,n) & \text{for } k=-1\ . \end{cases} \tag{2.38}$$

For $k=\pm 1$ we obviously require that $n \geq 2$.

To be physically relevant, solutions should be asymptotically flat. For example, we would expect SD-brane gravity solutions to be such that

$$\lim_{t\to\pm\infty} \dot{a}(t) = 0\,, \quad \lim_{t\to\pm\infty} \dot{R}(t) = 1\,, \tag{2.39}$$

for $k_\parallel = 0$ and $k_\perp = -1$. Also, for the dilaton and R-R field

$$\lim_{t\to\pm\infty} \dot{C}(t) = 0\,, \quad \lim_{t\to\pm\infty} \dot{\Phi}(t) = 0. \tag{2.40}$$

By inspection of the tachyon equation of motion (2.35), we see that the electric- (or magnetic-) only ansatz referred to at the beginning of this section will be obviously consistent if we only allow worldvolume time-derivatives. This is tantamount to imposing an $ISO(p+1)$ symmetry on the worldvolume. Ref. [54] argues that spatial inhomogeneities of the tachyon field will play an important role in the decay (a view which is also supported, although using a different line of reasoning, by the results of refs. [55, 39]). It will be interesting to investigate the full importance of such effects in the context of our effective supergravity analysis. We will include a discussion of the nontrivial issues raised in the Discussion section.

It turns out that the equations for the combined bulk-brane evolution in the time-dependent system are complicated enough to require numerical solution. For this reason, we will not be able to accommodate the most natural ansatz[9] $\rho_\perp =$

[9]Strictly speaking, instead of being a delta-function distribution, the more general ansatz for the source should be extended (*e.g.*, a Gaussian) with its size of the order of the string length.

$\delta(\vec{x})$. Instead, we will use the "smeared" ansatz also used in ref. [29],

$$\rho_{\perp} = \rho_0 \sqrt{g_{\perp}} \,. \tag{2.41}$$

A smeared brane source does not contribute stress-energy perpendicular to the worldvolume, which is in the directions $t, \vec{y}$. It should be noted that the effect of using this ansatz will be minimized by using a small value for the density parameter ρ_0. Of course, the aim when using such an ansatz is to get rid of any brane action dependence on the transverse coordinates $\vec{x}$.

We should remark that supergravity solutions corresponding to unstable D-brane systems have been found before [56]. Their solutions are time-independent, a feature which might seem rather unreasonable since they are, after all, supposed to describe unstable objects. Typically, these solutions are nakedly singular; there is no horizon. For reasons discussed previously, these solutions would therefore justifiably be regarded with some level of suspicion. Possibly, we should really regard these solutions as fixed-time snap-shots of the unstable brane system during its evolution. They do however reflect one desirable feature: taking into account warping of space in the directions transverse to the unstable branes.

What we really want is a sort of hybrid of that approach – where transverse dependence is the only dependence – and what we are doing here – where time dependence is all there is. This is something we postpone to a future investigation [61]; remarks on this will be given in the Discussion section.

Let us now get back to the simplified ansatz, and just go ahead and solve it. We are therefore interested in the precise system of *ordinary* differential equations for our coupled tachyon-supergravity system. Using the form $C_{12\ldots p+1} \equiv C(t)$ (which is consistent with our ansatz) the equation of motion for the R-R field (2.29) becomes

$$\ddot{C} + \dot{C}\left[(8-p)\frac{\dot{R}}{R} - (p+1)\frac{\dot{a}}{a}\right] = \lambda \frac{a^{p+1} f(T) \dot{T}}{R^{8-p}} \,. \tag{2.42}$$

Now let us find the dilaton equation of motion. A useful identity is

$$\mathcal{R} = 5(\partial\Phi)^2 - \frac{9}{2}\nabla^2\Phi + \frac{(3-p)}{8(p+2)!}e^{2\Phi}G^2 + \frac{1}{8}\left(\frac{\Lambda\rho_\perp}{\sqrt{-g}}\right)e^{\Phi}V(T)\sqrt{-A}(A^{-1})^{\lambda\sigma}g_{\lambda\sigma}\,, \tag{2.43}$$

with which the dilaton equation of motion can be written,

$$2(\partial\Phi)^2 - \nabla^2\Phi = \frac{(p-3)}{4(p+2)!}e^{2\Phi}G^2 + \left(\frac{\Lambda\rho_\perp}{\sqrt{-g}}\right)e^{\Phi}V(T)\sqrt{-A}\left[1 - \frac{1}{4}(A^{-1})^{\lambda\sigma}g_{\lambda\sigma}\right]. \tag{2.44}$$

This last expression is simply the Einstein frame equation of motion. So the dilaton in our ansatz satisfies

$$\begin{aligned}&\ddot{\Phi} + \dot{\Phi}\left[(8-p)\frac{\dot{R}}{R} + (p+1)\frac{\dot{a}}{a}\right] - 2\dot{\Phi}^2 \\ &= \frac{(3-p)}{4}\left(\frac{e^{\Phi}\dot{C}}{a^{(p+1)}}\right)^2 + \frac{\lambda}{4R^{8-p}}e^{\Phi}V(T)\left[(3-p)\sqrt{\Delta} - \frac{1}{\sqrt{\Delta}}\right].\end{aligned} \tag{2.45}$$

This will be used whenever double time-derivatives of the dilaton need to be substituted for.

With $T = T(t)$ we find that the dynamics of the tachyon field is governed by

$$\ddot{T} = \Delta\left\{\dot{\Phi}\dot{T} - \dot{T}\left[(p+1)\frac{\dot{a}}{a}\right] - \frac{1}{V(T)}\frac{dV(T)}{dT} + \frac{f(T)}{V(T)}\dot{C}e^{\Phi}a^{-(p+1)}\sqrt{\Delta}\right\}, \tag{2.46}$$

where $\Delta = 1 - \dot{T}^2$. We will be assuming that $|f(T)| = V(T)$, a statement which has been shown to be correct only for large-$|T|$. However, we have also done numerical experiments which show that some breaking of this relation at intermediate times (near the hilltop) does not change the features of our solutions.

We now turn to the equations of motion for the metric components $a(t)$ and $R(t)$. For the stress-tensors, eliminating second order derivatives in matter fields, we have

$$\hat{T}^t{}_t = 4(\dot{\Phi})^2 - 2\dot{\Phi}\left[(p+1)\frac{\dot{a}}{a} + (8-p)\frac{\dot{R}}{R}\right] + \frac{(5-2p)}{4}\left(\frac{e^{\Phi}\dot{C}}{a^{p+1}}\right)^2$$

$$+\frac{1}{4}\lambda\frac{e^{\Phi}V(T)}{R^{8-p}}\left[(7-2p)\sqrt{\Delta}-\frac{4}{\sqrt{\Delta}}\right],$$

$$\hat{T}^{y}_{\ y}=2\dot{\Phi}\frac{\dot{a}}{a}-\frac{1}{4}\left(\frac{e^{\Phi}\dot{C}}{a^{p+1}}\right)^{2}-\frac{1}{4}\lambda\frac{e^{\Phi}V(T)\sqrt{\Delta}}{R^{8-p}},$$

$$\hat{T}^{x}_{\ x}=2\dot{\Phi}\frac{\dot{R}}{R}+\frac{1}{4}\left(\frac{e^{\Phi}\dot{C}}{a^{p+1}}\right)^{2}+\frac{1}{4}\lambda\frac{e^{\Phi}V(T)\sqrt{\Delta}}{R^{8-p}}. \tag{2.47}$$

The components of the Ricci tensor are easily evaluated:

$$\mathcal{R}^{t}_{\ t}=(p+1)\frac{\ddot{a}}{a}+(8-p)\frac{\ddot{R}}{R}, \tag{2.48}$$

$$\mathcal{R}^{y}_{\ y}=\frac{\ddot{a}}{a}+(8-p)\frac{\dot{a}}{a}\frac{\dot{R}}{R}+p\left[\left(\frac{\dot{a}}{a}\right)^{2}+\frac{k_{\parallel}}{a^{2}}\right], \tag{2.49}$$

$$\mathcal{R}^{x}_{\ x}=\frac{\ddot{R}}{R}+(p+1)\frac{\dot{a}}{a}\frac{\dot{R}}{R}+(7-p)\left[\left(\frac{\dot{R}}{R}\right)^{2}+\frac{k_{\perp}}{R^{2}}\right]. \tag{2.50}$$

For the (string-frame) "Einstein" equations, the time, longitudinal and transverse components are respectively

$$(p+1)\frac{\ddot{a}}{a}+(8-p)\frac{\ddot{R}}{R}=4(\dot{\Phi})^{2}-2\dot{\Phi}\left[(p+1)\frac{\dot{a}}{a}+(8-p)\frac{\dot{R}}{R}\right]+\frac{(5-2p)}{4}\left(\frac{e^{\Phi}\dot{C}}{a^{p+1}}\right)^{2}$$
$$+\frac{1}{4}\lambda\frac{e^{\Phi}V(T)}{R^{8-p}}\left[(7-2p)\sqrt{\Delta}-\frac{4}{\sqrt{\Delta}}\right], \tag{2.51}$$

$$\frac{\ddot{a}}{a}=-(8-p)\frac{\dot{a}}{a}\frac{\dot{R}}{R}-p\left[\left(\frac{\dot{a}}{a}\right)^{2}+\frac{k_{\parallel}}{a^{2}}\right]+2\dot{\Phi}\frac{\dot{a}}{a}-\frac{1}{4}\left(\frac{e^{\Phi}\dot{C}}{a^{p+1}}\right)^{2}-\frac{1}{4}\lambda\frac{e^{\Phi}V(T)\sqrt{\Delta}}{R^{8-p}}. \tag{2.52}$$

$$\frac{\ddot{R}}{R}=-(p+1)\frac{\dot{R}}{R}\frac{\dot{a}}{a}-(7-p)\left[\left(\frac{\dot{R}}{R}\right)^{2}+\frac{k_{\perp}}{R^{2}}\right]+2\dot{\Phi}\frac{\dot{R}}{R}+\frac{1}{4}\left(\frac{e^{\Phi}\dot{C}}{a^{p+1}}\right)^{2}+\frac{1}{4}\lambda\frac{e^{\Phi}V(T)\sqrt{\Delta}}{R^{8-p}}. \tag{2.53}$$

In the end we have a system of five second order coupled ordinary differential equations for $\{T,C,\Phi,a,R\}$ as functions of t. These are respectively equations (2.46), (2.42), (2.45), (2.52) and (2.53). For consistency this system of equations must be supplemented with the first-order constraint

$$\frac{R^{8-p}}{\lambda e^{\Phi}}\left\{2(p+1)(8-p)\frac{\dot{a}}{a}\frac{\dot{R}}{R}+p(p+1)\left[\left(\frac{\dot{a}}{a}\right)^2+\frac{k_{\parallel}}{a^2}\right]+(7-p)(8-p)\left[\left(\frac{\dot{R}}{R}\right)^2+\frac{k_{\perp}}{R^2}\right]\right.$$
$$\left.-4\dot{\phi}\left[(p+1)\frac{\dot{a}}{a}+(8-p)\frac{\dot{R}}{R}-\dot{\phi}\right]-\frac{1}{2}\left(\frac{e^{\Phi}\dot{C}}{a^{p+1}}\right)^2\right\}=\frac{V(T)}{\sqrt{\Delta}} \tag{2.54}$$

obtained by plugging eqs. (2.52) and (2.53) in eq. (2.51). Of course, if this last equation is satisfied at $t=0$ it will be for all times.

2.4 The roll of the tachyon

We refer to a supergravity SD-brane as the field configuration generated by a system composed of a large number, N, of microscopic SD-branes. As emphasized in section 2.3.2, there is a single continuous parameter that controls the dynamics of these fields, *i.e.*, $\lambda=\rho_0\Lambda$. This is the parameter that determines the relative importance of the unstable brane source. Clearly, for $\lambda\to 0$ the open string tachyon decouples from the bulk fields (no backreaction).

In this section we investigate solutions with the symmetries of SD-branes and a non-vanishing λ.

2.4.1 Tachyon evolution in flat space

The solutions relevant for SD-brane physics in Type II a,b superstring theory could be the ones corresponding to an open string tachyon evolving from

$$\lim_{t\to-\infty}T(t)=+\infty \tag{2.55}$$

to

$$\lim_{t\to+\infty}T(t)=-\infty \tag{2.56}$$

in a symmetric runaway potential of the form shown on figure 2.1. Solutions of this type correspond to the tachyon evolving between two different minima of the potential. Possible initial conditions (at $t = 0$) for these solutions are of the form

$$\dot{T}(0) = \text{const.}\,, \qquad T(0) = 0\,. \tag{2.57}$$

Another set of solutions corresponds to a tachyon evolution with

$$\lim_{t \to \pm\infty} T(t) = \alpha\, \infty\,, \tag{2.58}$$

where $\alpha = 1$ and $\alpha = -1$ would lead to equivalent solutions. Appropriate boundary conditions for the tachyon (at $t = 0$) are then of the form

$$\dot{T}(0) = 0\,, \qquad T(0) = \text{const.} \tag{2.59}$$

The open string tachyon evolutions associated to all these different boundary conditions would correspond to full-SD-branes. However, a singularity theorem [36] states that the corresponding tachyon evolutions, with couplings to supergravity modes, will *always* lead to a curvature singularity either in the past (big-bang) or the future (big-crunch). We explain these results and provide an alternate derivation of the theorem in section 2.5.

We show in section 2.5 that solutions associated with half-S-branes can be nonsingular. These are essentially the same as unstable D-branes since they correspond to a tachyon evolution that begins at or close to the top of the potential hill. In this section we characterize the supergravity solutions generated by a tachyonic source with this property.

In our analysis we use the potential

$$V(T) = \frac{1}{\cosh\left(T/\sqrt{2}\right)} \tag{2.60}$$

because it agrees with open string field theory calculations for large values of the tachyon for unstable D-brane systems in Type II a,b superstring theory.[10] It is

[10] In bosonic string theory the potential is asymmetric and unbounded from below as $T \to -\infty$.

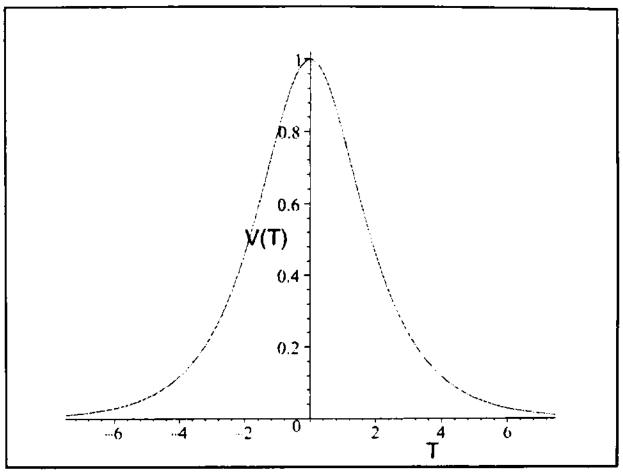

Figure 2.1: Possible form of the open string tachyon potential $V(T)$ on a SD-brane in Type II a,b superstring theory.

not known what the exact potential is for intermediate times but our solutions are only mildly affected by its particular form. The expression for the tachyon when $\lambda = 0$, *i.e.*, when there are no couplings to the bulk supergravity modes, is[11]

$$T(t) = -\sqrt{2}\,\text{arcsinh}\left(-\dot{T}(0)\sinh\frac{t}{\sqrt{2}}\right) \tag{2.61}$$

for the boundary conditions (2.57). When the boundary conditions are of the form (2.59), the analytic expression for the tachyon is

$$T(t) = -\sqrt{2}\,\text{arcsinh}\left(\sinh\left(-\frac{T(0)}{\sqrt{2}}\right)\cosh\frac{t}{\sqrt{2}}\right). \tag{2.62}$$

Figure 2.2 shows a tachyon profile for $T(0) = 0$ and $\dot{T}(0) = 0.1$. Homogeneous solutions such as eqs. (2.61) and (2.62), derived from a tachyonic action, correspond to a fluid that has a constant energy density and vanishing pressure asymptotically (tachyon matter). We will shortly see how these features are affected by the inclusion of couplings to bulk modes.

[11] We refer the reader to Appendix B for a derivation of this expression.

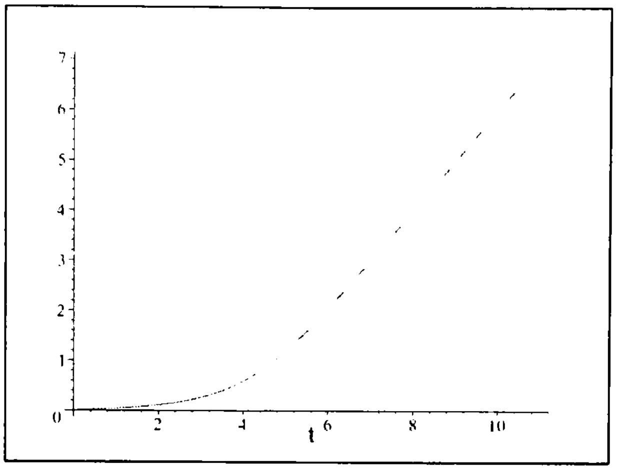

Figure 2.2: The tachyon field evolution for $\dot{T}(0) = 0.1$ and $T(0) = 0$.

2.4.2 R-SYMMETRY GROUP

Before presenting the numerical results we comment on the issue of R-symmetry. As pointed out in ref. [15], SD-brane solutions should be invariant under the transverse Lorentz transformations leaving the location of the brane fixed. This corresponds to an $SO(1, 8-p)$ R-symmetry.[12] This property is embodied in the supergravity solutions found in refs. [15, 26, 27] where the transverse space metric has a factor of the form: $R(t)^2 dH^2_{8-p}$. The embedding group of the hyperbolic space H^{8-p} is $SO(1, 8-p)$.[13] This explains why supergravity solutions with $k_{\|} = 0$ and $k_{\perp} = -1$ are usually considered in more details. However, the cases with $k_{\perp} = 0$ are also candidate solutions for SD-brane (and, more generally, unstable D-brane) supergravity solutions. We present an analysis of those and other cases in section

[12] The interpretation in terms of R-symmetry is relevant for the idea that SD-brane gravity fields are holographically dual to a worldvolume Euclidean field theory.

[13] The intuition for the nature of the R-symmetry group is inherited from the AdS/CFT correspondence. For example, the metric of a 3-brane is of the form: $ds^2 = f(r)\left[-dt^2 + d\vec{y}^2\right] + 1/f(r)\left[dr^2 + r^2 d\Omega_5^2\right]$. The R-symmetry group in this case is $SO(6)$, a statement that can be traced to the fact that the near horizon limit of the geometry is $\mathrm{AdS}_5 \times S^5$, *i.e.*, the gravitational background dual to the worldvolume field theory on an ensemble of D3-branes.

2.4.4.

It is suggested in ref. [15] that the spacelike naked singularities [15, 26, 27] associated with the supergravity SD-branes could be resolved by using a metric ansatz that allows for a breaking of the R-symmetry in the region around the core of the object ($t = 0$). The intuition from the AdS/CFT correspondence comes from ref. [57] which describes cases of spontaneously broken R-symmetry. Our ansatz as it is cannot accommodate such an R-symmetry breaking. Presumably, this would correspond to a time-dependent process by which the R-symmetry is broken down to $SO(8-p)$ in a finite region. The closest our ansatz can come to realizing this scenario is if we consider the cases with $k_\perp = 0$. Then, the transverse symmetry group is $ISO(8-p)$, the compactification of which is $SO(8-p)$. We find that solutions with $k_\perp = 0$ are *regular* and asymptotically flat. Moreover, as we will see in section 2.4.4, a generic feature of the $k_\perp = 0$ solutions is that the effective string coupling $g_s \exp(\Phi(t))$ vanishes asymptotically. We will also remark on the cases with $k_\perp$ and/or $k_\parallel$ equal to $+1$, which all develop big-crunch singularities in finite time.

2.4.3 Numerical results

The ansatz we consider for describing supergravity SDp-brane solutions is

$$\begin{aligned} ds^2_{Sp} &= -dt^2 + a(t)^2 d\vec{y}^2 + R(t)^2 dH^2_{8-p}, \\ C &= C(t)\,, \quad \Phi = \Phi(t), \end{aligned} \tag{2.63}$$

where we have taken $k_\parallel = 0$ and $k_\perp = -1$ in accord with the original paper ref. [15]. Again, these supergravity bulk modes will be excited by couplings to an homogeneous open string tachyon as described in section 2.3.1. We consider only homogeneous solutions (*i.e.*, depending only on time). As discussed earlier, it is possible that inhomogeneities might play an important role in the creation/decay process [54], but we are postponing this issue for now.

The resulting system of coupled differential equations for which we will find solutions is of course highly non-linear. Among other things, this implies that it is not possible to extract scaling behavior for the fields from their equations of motion. We could therefore expect that the behavior of these solutions depends in a physically important way on the initial conditions for the field components involved: $a(t)$, $R(t)$, $C(t)$, $\Phi(t)$ and the source $T(t)$.

The multiplicity of potential solutions could be large: one solution would be expected for each set of consistent initial conditions. It would appear natural to consider, as candidate solutions for SD-branes, those which are time-reversal symmetric around $t = 0$. In fact, precisely time-symmetric solutions re actually not allowed (see section 2.4.4 for details). That is, unless they have $k_{\|} = 0, k_{\perp} = +1$, in which case they have R-symmetry which is markedly in conflict with the proposals of earlier papers, *e.g.*, ref. [15]. As we said above, these solutions also develop a big-crunch singularity in finite time.

Bulk asymmetry at $t = 0$ appears reasonable if we consider particle/string production. We are of course neglecting such production in our analysis. It is nonetheless clear that, for a full SD-brane solution, backreaction will combine with particle production to make bulk fields naturally asymmetric. It is only in the approximation of zero backreaction that time-symmetry is possible when particle production occurs, but of course that approximation cannot be self-consistent.

Unfortunately we have been unsuccessful at finding analytical expressions for the bulk modes and tachyon field when their mutual coupling is non-vanishing ($\lambda \neq 0$). We therefore resorted to solving the corresponding system of differential equations numerically.[14]

In what follows we show the results associated with a half-SD2-brane, and

[14]Recall that this is the primary reason why it is so difficult to include $\vec{x}$-dependence in our ansatz.

present some interesting analysis of the effect of varying the initial conditions. We provide general comments for other half-SD-branes with $p < 7$ and explain how the half-SDp-branes with $p = 7, 8$ are different. Throughout our analysis we pay special attention to the impact of varying the initial condition $\Phi(0)$ on the dilaton, because this controls the string coupling close to the hilltop.

For finding the numerical solutions we fix the initial conditions at $t = 0$, and evolve this data forward in time. Then, we evolve the same data backward in time from $t = 0$. The result should be the solution associated with a full SD-brane, *i.e.*, the bulk fields sourced by the open string tachyon as it evolves from $t = -\infty$ to $t = +\infty$. In this section we present the results associated with a SD2-brane with boundary conditions

$$\begin{gathered} T(0) = 0\,, \quad \dot{T}(0) = 0.1\,, \quad a(0) = 0.1\,, \quad \dot{a}(0) = 0.079\,, \quad R(0) = 1\,, \\ \dot{R}(0) = 0\,, \quad \phi(0) = -1\,, \quad \dot{\phi}(0) = 0\,, \quad C(0) = 0\,, \quad \dot{C}(0) = 0\,, \end{gathered} \tag{2.64}$$

and $\lambda = 0.1$. Again, these solutions would correspond to the tachyon rolling up the potential from $T = +\infty$ ($t = -\infty$) up to $T = -\infty$ for $t = +\infty$. We find that this particular solution is plagued with a curvature singularity for $t < 0$. Numerical experiments suggest that this is a generic result: there is always a curvature singularity either in the past or the future of the tachyon evolution. This is supported by the singularity theorem of ref. [36] (see section 2.5 for details). Therefore in this section we only consider the half-SD-brane, *i.e.*, the $t > 0$ evolution of a full-SD-brane. These are interesting solutions because of their relation to unstable D-branes.

Figure 2.2 shows the evolution of the tachyon for the half-S2-brane. For $\lambda = 0$ (no coupling between the closed and the open string modes), we find

$$\lim_{t \to +\infty} T(t) = t. \tag{2.65}$$

This observation is suggestive that the tachyon field might play the role of time itself in cosmological models driven by brane decay (as proposed in ref. [51]). We find that this asymptotic behavior for the tachyon survives when couplings to bulk modes are introduced. In fact, for $\lambda \neq 0$ we find

$$\lim_{t \to +\infty} T(t) = t + \kappa_T. \tag{2.66}$$

The constant κ_T depends non-trivially on the boundary conditions at $t = 0$ and p. For example, as p is increased κ_T decreases. Also, for large values of $\Phi(0)$ the tachyon deformation from the flat space case becomes larger. As mentioned in Appendix B, for $\lambda = 0$ the state of the tachyon for large time ($t \to +\infty$) is that of a perfect fluid with constant energy density and vanishing pressure. This is the so-called tachyon matter. We find that for $\lambda \neq 0$, both the energy density and the pressure (physical quantities measured in the Einstein frame) vanish. In other words, the tachyon matter is clearly only an illusion of the $g_s \to 0$ limit.

We consider briefly the effect of varying the initial conditions $\dot{T}(0)$ and $T(0)$. For the half-SD-brane we find that the time it takes the tachyon to reach the bottom of its potential increases for smaller values of $\dot{T}(0)$ and $T(0)$. Not only that, for very small initial velocities the tachyon appears to stay perched at the top of the potential for a certain period of time. In general, we observe that it takes less time for the tachyon to reach the bottom of its potential when we increase λ. Now, we also observe that the difference between the curves associated with flat space ($\lambda = 0$) and $\lambda \neq 0$ tachyons decreases as $\dot{T}(0)$ and $T(0)$ increase. Also, for large negative values of $\Phi(0)$, κ_T becomes very small. This is simply a reflection of the fact that such cases correspond to a very small initial string coupling (see below for details).

There is another interesting feature of the tachyon when coupled to the massless closed string modes. Firstly, the time it takes the tachyon to reach the bottom of its potential is not significantly altered even when considering large values of the "coupling" λ. Finally, we find that as the coupling λ is increased, κ_T, or the deformation away from the flat space tachyon, increases correspondingly.

Time-dependent string coupling

The string coupling is given by the expression

$$g_s = e^{<\Phi_0>} , \tag{2.67}$$

where Φ_0 is the background dilaton field in the absence of sources, *i.e.*, strings and D-branes. Typically the presence of stringy excitations will modify the coupling of the theory,

$$g_s \longrightarrow g_s e^{\Phi(\eta)}, \tag{2.68}$$

where η is a spacelike variable for D-branes and is timelike for SD-branes.

For supergravity Dp-brane solutions the dilaton field is (see, for example, ref. [58]),

$$g_s(r) = g_s e^{\Phi(r)} = g_s \left(1 + \frac{c_p g_s N_p l_s^{7-p}}{r^{7-p}}\right)^{\frac{1}{4}(3-p)} , \tag{2.69}$$

where $c_p = (2\sqrt{\pi})^{5-p}\Gamma[\frac{1}{2}(7-p)]$. The string coupling is seen to vary according to whether test closed strings propagate close or far from the core of the Dp-branes. For all static supergravity solutions (including NS5-branes), the asymptotic string coupling is such that

$$\lim_{r \longrightarrow +\infty} g_s(r) = g_s. \tag{2.70}$$

The effect of supergravity D-branes is therefore to modify the coupling locally. For $p < 3$, it is large close to the horizon and decreases to g_s as $r \to +\infty$. For $p > 3$, the string coupling is small close to the horizon but increases to g_s for large r. The case $p = 3$ is special because the dilaton field sourced by the 3-brane is constant throughout the spacetime. Typically, the size of the region where the dilaton is not constant depends on the parameter $g_s N$. Large values of this parameter are associated with larger regions where dilaton perturbations associated with the brane are noticeable.

The solutions associated with supergravity SD-branes induce dilaton perturbations corresponding to a time-dependent string coupling,

$$g_s(t) = g_s e^{\Phi(t)}. \tag{2.71}$$

We find that the time dependence of the dilaton sourced by half-SD-branes is qualitatively different when compared to the radial dependence of the dilaton associated with regular D-branes.[15]

Numerical analysis show that typically the function $g_s^{-1} e^{\Phi}(t)$ decreases from $t = 0$ as $t \to +\infty$. We find that smaller values of p correspond to larger asymptotic string couplings. The dilaton field generated by half-SD-branes has at least two interesting properties. First, all solutions are such that the dilaton stabilizes to a constant asymptotically.

$$\lim_{t \to +\infty} \Phi(t) = \Phi_{+\infty}. \tag{2.72}$$

Secondly, the asymptotic value of the dilaton is always smaller than its initial value at $t = 0$,

$$\Phi_{+\infty} < \Phi(0). \tag{2.73}$$

[15]This is an other example where features of SD-branes are not simply those inherited by analytic continuation of D-branes. In fact, a double analytic continuation ($r \to ir$ and $t \to it$) of the supergravity Dp-brane solutions lead to objects with an imaginary R-R charge. This is unphysical.

This implies that the late string coupling ($t \to +\infty$) is always smaller than the coupling when the tachyon is at the top of its potential ($t = 0$), *i.e.*,

$$g_s e^{\Phi_{+\infty}} < g_s e^{\Phi(0)}. \tag{2.74}$$

We find that the qualitative features of the dilaton evolution are preserved when the boundary conditions on the various fields are changed. Nevertheless, we consider the effect of varying $\Phi(0)$ in some detail. An interesting quantity to study is the ratio

$$h = \frac{e^{\Phi_{+\infty}}}{e^{\Phi(0)}}, \tag{2.75}$$

which gives a quantitative measure of how much the initial string coupling is modified asymptotically. The tachyon profile is not altered significantly when the initial condition on $\Phi(t)$ is varied. Nevertheless, we observe that for large values of $\Phi(0)$ (large initial coupling) the tachyon field reaches the bottom of the potential well faster. A large initial coupling also means that the bulk fields relax faster to their stable asymptotic configuration compared to cases where $\Phi(0)$ is smaller.[16] The overall effect on the bulk fields is also enhanced for larger values of $\Phi(0)$. For example, as the initial coupling is increased the scale factor $a(t)$ stabilizes to significantly smaller values (see next section). As for the dilaton field itself, we find that the ratio h is large for larger values of $\Phi(0)$. For very small values of the initial string coupling (large negative values of $\Phi(0)$), the ratio h approaches unity.

In summary, we find that the parameter $\Phi(0)$, *i.e.*, the parameter determining the string coupling when the tachyon is close to the top of its potential, strongly determines the importance of the unstable brane source effect on the supergravity bulk modes.

[16] Obviously $\Phi(0)$ can be taken to have negative values.

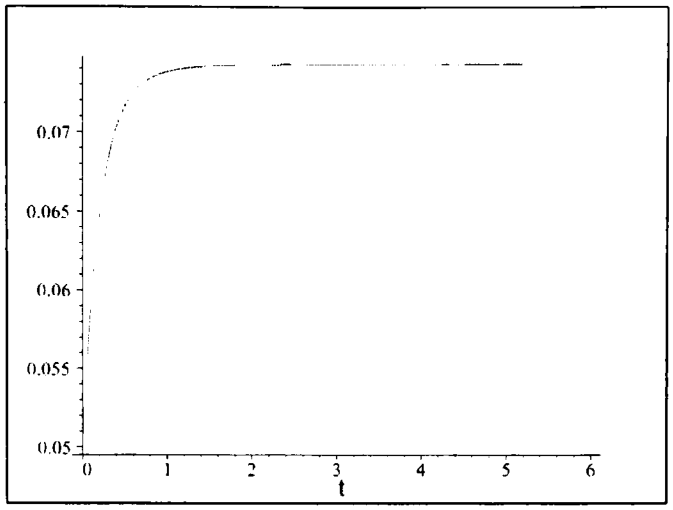

Figure 2.3: The scale factor $a(t)$ on the worldvolume of a supergravity half-SD2-brane with boundary conditions (2.64).

Gravitational field

We now describe the effect of the unstable brane source on the time-dependent metric components $a(t)$ and $R(t)$. Figure 2.3 shows the scale factor $a(t)$ on the worldvolume of a half-SD2-brane with the boundary conditions (2.64). A general feature is that away from $t = 0$ the scale factor increases from its initial value, $a(0)$, to a stable asymptotic value,

$$\lim_{t \to \pm\infty} a(t) = a_{+\infty} . \tag{2.76}$$

The time it takes for this scale factor to reach its asymptotic value corresponds roughly to the time it takes for the tachyon to reach the bottom of its potential. As pointed out above, for large initial values of the dilaton, $\Phi(0)$, the asymptotic values of the scale factor, $a_{+\infty}$, become smaller. Correspondingly, when the initial coupling is weak the effect of the probe on the bulk modes is small and $a(t)$ stabilizes to a value closer to its initial value $a(0)$.

Figure 2.4 shows the behavior of the metric function $R(t)$. A generic feature

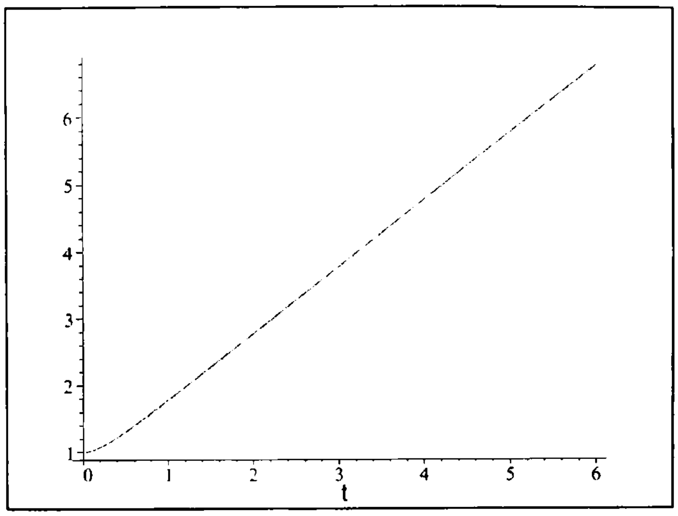

Figure 2.4: The SD2-brane transverse scale factor $R(t)$ with boundary conditions (2.64).

of the supergravity SD-brane solutions is that

$$\lim_{t \to +\infty} R(t) = t + \kappa_R. \tag{2.77}$$

The constant κ_R is generically larger for larger values of p.

Ramond-Ramond field

Figure 2.5 shows the time dependence of the Ramond-Ramond form field for a supergravity SD2-brane with boundary conditions (2.64). Again, the energy stored in this field (proportional to its time-derivative) goes to zero in approximately the time it takes for the tachyon to reach the bottom of its potential. A generic feature of the Ramond-Ramond field associated with a SD-brane is therefore,

$$\lim_{t \to +\infty} C(t) = C_{+\infty}, \tag{2.78}$$

where $C_{+\infty}$ is a constant. Typically we find that this constant is smaller for larger values of p.

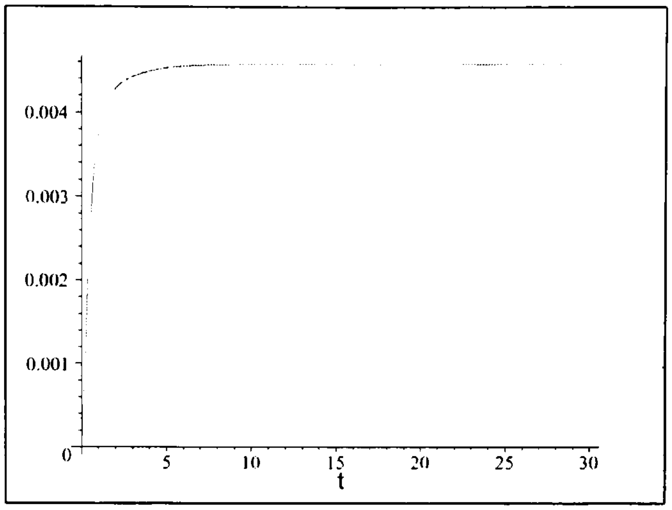

Figure 2.5: The SD2-brane Ramond-Ramond field $C(t)$ associated with the boundary conditions (2.64).

Curvature bounds and asymptotic flatness

The supergravity equations of motion are derived from a worldsheet calculation by requiring that, at a certain order in perturbation theory, the beta-functions associated with bulk fields vanish. Typically, there are higher order (in α') curvature corrections to the beta-functions. These corrections may be negligible only if the curvature involved is small when measured with respect to the string length, $l_s = \sqrt{\alpha'}$. This is why our solutions can, strictly speaking, be trusted only if the curvature involved is such that $|\mathcal{R}|$, $|\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}|$, $\left|\mathcal{R}_{\mu\nu\rho\lambda}\mathcal{R}^{\mu\nu\rho\lambda}\right|$ are small. We verify that this condition is satisfied by studying the behavior of the time-dependent Ricci scalar of the supergravity half-SD-branes,

$$\begin{aligned}\mathcal{R}(t) = \; & 2(p+1)\frac{\ddot{a}}{a} + 2(8-p)\frac{\ddot{R}}{R} + 2(p+1)(8-p)\frac{\dot{a}}{a}\frac{\dot{R}}{R} \\ & + p(p+1)\left(\frac{\dot{a}^2}{a^2} + \frac{k_{\parallel}}{a^2}\right) + (8-p)(7-p)\left(\frac{\dot{R}^2}{R^2} + \frac{k_{\perp}}{R^2}\right), \end{aligned} \tag{2.79}$$

where $k_{\perp} = -1$ and $k_{\parallel} = 0$ for the cases of interest here.

A property of the solutions, which is apparent from studying the evolution of bulk modes, is that of asymptotic flatness. In fact, we find

$$\lim_{t\to+\infty} ds^2_{Sp} = -dt^2 + a^2_{+\infty} d\vec{y}^2 + (t+\kappa_R)^2 dH^2_{8-p}\,, \tag{2.80}$$

$$\lim_{t\to+\infty} \Phi(t) = \Phi_{+\infty}\,, \qquad \lim_{t\to+\infty} C(t) = C_{+\infty}, \tag{2.81}$$

where $a_{+\infty}$, κ_R, $\Phi_{+\infty}$ and $C_{+\infty}$ are constants. Both the first- and second-derivative of the bulk modes vanish asymptotically. The resulting brane configuration is then clearly flat for $t \to +\infty$, as it should be.

An important question to answer at this stage is: *Which quantity in the problem sets an upper bound on the curvature for half-SD-branes ?* First, we have found that whatever the curvature is at $t = 0$, its absolute value will never exceed it significantly in the course of the evolution. This is true only for half S-branes with boundary conditions corresponding to positive derivatives of the bulk fields. More generally for half-SD-branes the acceptable solutions are those with a combination of the boundary conditions such that

$$\lim_{t\to 0} |\mathcal{R}(t)| \;\text{ small}\;. \tag{2.82}$$

These requirements are certainly attainable within the self-consistent supergravity approximation we are considering — along with the specific ansatz we introduced in order to be able to solve the equations numerically.

THE $p = 7$ AND SPACE-FILLING HALF-SD-BRANES

The case $p = 8$ should be special because there is no transverse space into which "energy" can be dissipated. The only bulk fields involved are then the metric component, $a(t)$, the dilaton, $\Phi(t)$, and the Ramond-Ramond field, $C(t)$. We find that the time-derivative of the scale factor, $\dot{a}(t)$, decreases to zero, as $t \to +\infty$, many orders of magnitude slower than for $p < 7$. The scale factor $a(t)$ also goes to zero after an infinite time (see section 2.4.3 for a physical interpretation) which is to be contrasted with the cases $p < 7$ where $a_{+\infty} \neq 0$. One would expect that to be the source of a curvature singularity in the future but it is not the case. The Ricci scalar for the space filling SD-brane is

$$\mathcal{R}(t) = -72\left(\frac{\dot{a}}{a}\right)^2 - \frac{9}{2}\left(\frac{e^{\Phi}\dot{C}}{a^9}\right)^2 + 36\dot{\Phi}\frac{\dot{a}}{a} + \frac{9}{2}\lambda e^{\Phi}\Delta^{1/2}V(T). \tag{2.83}$$

Curvature singularities are avoided because all time-derivatives in the problem go to zero faster than the scale factor as $t \to +\infty$. In particular, the quantity $e^{\Phi}\dot{C}$ goes to zero faster than $a(t)$ for large time. Also the string coupling slowly goes to zero asymptotically ($\Phi_{+\infty} \to -\infty$). For the Ramond-Ramond field we find the same behavior as for the cases $p < 7$. The only difference is that the relaxation time of the bulk modes is many orders of magnitude larger than in the other cases.

The functions $a(t)$, $\Phi(t)$ and $C(t)$ associated with the half-SD7-brane behave in the same way as the space-filling half-SD-brane. Of course in this case there is a transverse space and we find that the relaxation of $|R(t)|$ to its asymptotic form $t + \kappa_R$ takes an infinite amount of time. In other words, it relaxes to its asymptotic form much slower than for the cases $p < 7$.

Einstein frame

Let us end this section with a remark about the physics of our half-SD-branes in Einstein frame. This would be the more natural and more physical frame to use in discussions of potential uses of rolling tachyons in the context of cosmology.

The transformation from string frame to Einstein frame involves a multiplicative factor of $e^{-\Phi/2}$ for $d = 10$, which is the dimension in which we are working here. Now, we have already commented at length on the behavior of the time-dependent dilaton field in section 2.4.3. The essential physics there was that the dilaton is biggest at the top of the potential hill; in particular, it stabilizes in the infinite future at a constant value smaller than the initial condition at the hilltop. Dilaton derivatives also remain small at all times during the evolution. Therefore, most of the qualitative features of our solutions will be preserved upon transformation to Einstein frame; in particular, the half-SD-brane solutions remain completely nonsingular.

A case that deserves further comments is that of the space filling SD8-brane discussed earlier. In string frame the metric is written

$$ds_{S8}^2 = -dt^2 + a(t)^2 d\vec{y}^2. \tag{2.84}$$

Upon converting to Einstein frame we get the metric

$$ds_{ES8}^2 = -d\tau^2 + a_E(\tau)^2 d\vec{y}^2, \tag{2.85}$$

where we have used a change of coordinate such that $d\tau^2 = e^{-\Phi/2}dt^2$, and where

$$a_E(\tau) = e^{-\Phi(t)/4}a(t). \tag{2.86}$$

For the SD8-brane we found

$$\lim_{t\to+\infty} a(t) = 0\,, \qquad \lim_{t\to+\infty} e^{\Phi(t)} = 0. \tag{2.87}$$

In string frame this implies that the metric "closes off" at infinity but in the (more physical) Einstein frame the converse happens, *i.e.*, the limit $\tau \to +\infty$ corresponds to a constant scale factor,

$$\lim_{t\to+\infty} a_E(\tau) = \text{const.} \tag{2.88}$$

This is a very sensible result. We mentioned before that, because of the absence of a transverse space, there appeared to be no channel into which the energy could go. We therefore find that asymptotically the energy has gone into inflating the worldvolume of the SD-brane. In fact, the relaxation time for the gravitational field is essentially infinite.

2.4.4 Other classes of solutions

In this section we present a more general analysis of the family of solutions with parameters $\{k_\parallel, k_\perp\}$ associated with the metric ansatz (2.36).

Time-reversal symmetric solutions

We consider the time-reversal symmetric solutions, *i.e.*, those associated with bulk fields having vanishing time-derivatives at $t = 0$:

$$\dot{a}(0) = \dot{R}(0) = \dot{C}(0) = \dot{\Phi}(0) = 0. \tag{2.89}$$

The constraint equation (2.54) at $t = 0$ is then

$$\frac{V(T)}{\sqrt{\Delta}} = \frac{1}{\lambda e^{\Phi}} \left(p(p+1)\frac{k_\parallel}{a^2} + (7-p)(8-p)\frac{k_\perp}{R^2} \right). \tag{2.90}$$

The LHS being positive-definite, the constraint can be satisfied if and only if the metric ansatz contains a subspace of positive curvature. These consist in five categories of solutions, *i.e.*, $\{k_\parallel, k_\perp\} = \{0, 1\}, \{-1, 1\}, \{1, -1\}, \{1, 1\}, \{1, 0\}$. We have performed a detailed analysis of these solutions for $p = 4$ and $\dot{T}(0) \leq 1/10$. Typically, we find that when the tachyon has reached a point of its evolution where $|V(T)| \simeq 0$, labelled t_c, the solutions develop a curvature singularity. The behavior of the bulk fields is as follows. The time derivative of the scale factor is such that

$$\lim_{t \to t_c} \dot{a} = -\infty, \quad \lim_{t \to t_c} a(t) = 0, \tag{2.91}$$

which corresponds to a big-crunch singularity on the $(p+1)$-dimensional world-volume. The cases $\{k_\parallel = -1, k_\perp = 1\}$, $\{k_\parallel = 0, k_\perp = 1\}$ and $\{k_\parallel = 1, k_\perp = 0\}$ are such that

$$\lim_{t \to t_c} \dot{R}(t) = -\infty, \quad \lim_{t \to t_c} R(t) = 0, \tag{2.92}$$

corresponding to the transverse spherical space collapsing to zero-size in finite time. For $\{k_\parallel = 1, k_\perp = 1\}$ and $\{k_\parallel = 1, k_\perp = -1\}$, we find

$$\lim_{t \to t_c} \dot{R}(t) = +\infty, \quad \lim_{t \to t_c} R(t) = +\infty. \tag{2.93}$$

For $k_\perp = -1$, $R(t)$ goes to infinity faster than t. All solutions are such that

$$\lim_{t \to t_c} \dot{\Phi}(t) = -\infty. \tag{2.94}$$

Generically, the large time behavior of the R-R field is well-behaved, *i.e.*,

$$\lim_{t \to t_c} \dot{C}(t) \approx 0, \quad \lim_{t \to t_c} C(t) = C_c, \tag{2.95}$$

where C_c is finite but typically many orders of magnitudes larger than the constants C_∞ associated with the regular solutions presented earlier. The case $\{k_\parallel = 0, k_\perp = -1\}$ is special because then the time derivative of the R-R field diverges as well at finite time.

Our conclusions to be unaltered for other values of p. We found no evidence that the singularities are resolved when the time-reversal symmetry is broken. These results are in accord with the generalized version of the singularity theorem [36] presented in section 2.5.

More regular solutions

Another class of solutions we have studied are those with $k_\parallel = 0$ and $k_\perp = 0$. The results we present for these solutions are generic, *i.e.*, they hold for all p and reasonable boundary conditions. The corresponding solutions are regular for $t > 0$ and asymptotically flat. We find that

$$\lim_{t \to +\infty} \dot{a}(t) = 0, \quad \lim_{t \to +\infty} a(t) = \text{const.}, \tag{2.96}$$

and

$$\lim_{t \to +\infty} \dot{R}(t) = 0, \quad \lim_{t \to +\infty} R(t) = \text{const.} \tag{2.97}$$

For the dilaton we obtain

$$\lim_{t \to +\infty} \dot{\Phi}(t) = 0, \quad \lim_{t \to +\infty} \Phi(t) = -\infty, \tag{2.98}$$

i.e., the string coupling vanishes asymptotically. Finally the R-R field behaves like

$$\lim_{t\to+\infty} \dot{C}(t) = 0, \quad \lim_{t\to+\infty} C(t) = \text{const.} \tag{2.99}$$

We note that the relaxation time for these solutions is many orders of magnitude larger than for the $\{k_\parallel = 0,\ k_\perp = -1\}$ cases presented earlier.

The two remaining cases are $\{k_\parallel, k_\perp\}$ equal to $\{-1, -1\}$ and $\{-1, 0\}$. We found evidence that there are $t > 0$ regular and asymptotically flat solutions.

2.5 Comments on a no-go theorem for full-SD-branes

A singularity theorem for full SD-branes was given in ref. [36]. This work showed analytically that full-SDp-branes with codimension one or greater must develop singularities either in the past or the future of the tachyon evolution. This theorem applies to solutions with $p \leq 7$ and with $k_\parallel = 0$, $k_\perp = -1$.

In this section we briefly review the singularity theorem itself, and provide an alternate more geometric formulation that applies for all values of $k_\parallel$ and $k_\perp$. Then we explain why the no-go theorem does not hold for cosmological applications (tachyon cosmology) and decaying unstable branes (half-SD-branes). We also give strong indications that the space-filling SD8-branes may escape the no-go theorem completely. Finally we remark on what may resolve the singularities. We also point out several directions in which the no-go theorem could usefully be generalized. One of them is relaxing the restrictiveness of the ansatz for the bulk and tachyon fields; another is allowing tachyon inhomogeneity.

2.5.1 Reformulation of the singularity theorem

As pointed out before, there can be two types of open string tachyon evolutions that could correspond to full-SD-branes:

- The tachyon evolves from one side of the potential $V(T)$ $(T \rightarrow \pm\infty)$ up and over to the other side corresponding to $T \rightarrow \mp\infty$;
- the tachyon evolves from one side of the potential $(T \rightarrow \pm\infty)$ but never reaches the maximum at $T = 0$. There is then a turning point $(\dot{T} = 0)$ for some $|T| > 0$ and the future evolution of the SD-brane proceeds towards $T \rightarrow \pm\infty$.

The singularity theorem of ref. [36] states that the corresponding tachyon evolutions (for $k_{\parallel} = 0$ and $k_{\perp} = -1$) will always lead to a curvature singularity either in the past (big-bang) or the future (big-crunch) depending on the boundary conditions on the tachyon and the supergravity fields. This theorem assumes an homogeneous tachyon field and a metric ansatz as in eq. (2.36). Each constant time Cauchy surface of a SD-brane is associated with the volume function

$$V_{\mathrm{S}} = v\, a(t)^{p+1} R(t)^{8-p} = v\, A(t)^9, \tag{2.100}$$

where v is a constant and $A(t)$ is an average scale factor. The theorem in ref. [36] relies on the following combination of the field equations (from now on the equations are in Einstein frame),

$$9\frac{\ddot{A}}{A} = -\left[I_{\mathrm{sugra}} + I_{\mathrm{tachyon}}\right], \tag{2.101}$$

where

$$I_{\mathrm{sugra}} = \frac{1}{4}\left[2\dot{\Phi}^2 + \frac{(7-p)}{4} e^{\Phi(3-p)/2}\dot{C}^2\right] + \frac{(p+1)(8-p)}{9}\left(\frac{\dot{a}}{a} - \frac{\dot{R}}{R}\right)^2, \tag{2.102}$$

and

$$I_{\text{tachyon}} = \frac{\lambda V(T) e^{\Phi\left(\frac{p}{4}-\frac{1}{2}\right)}}{16R^{8-p}} \left(\frac{7}{\sqrt{\Delta}} - (p+1)\sqrt{\Delta}\right), \tag{2.103}$$

with $\Delta \equiv 1 - e^{-\Phi/2}\dot{T}^2$ in Einstein frame. The quantity Δ starts out at (or near) unity at $t = 0$, and becomes monotonically smaller with time.

The singularity theorem of ref. [36] implies that for full-SD-brane evolution a singularity will develop unless the gravitational sources introduced provide enough leeway that $\ddot{A}/A$ can become positive. Clearly, $I_{\text{sugra}} \geq 0$ so in the context of pure supergravity (no explicit introduction of extended sources) the volume of a full-SD-brane will never experience a period of positive acceleration. This could be related with failed earlier attempts at finding non-singular SD-brane solutions in the context of supergravity [15, 26, 27]. The hope was that the tachyon might be able to resolve this singularity. It can be easily shown that for $p < 7$ we have $I_{\text{tachyon}} \geq 0$ which implies that introducing a tachyon source cannot be used to circumvent the singularity theorem.

The most general supergravity solutions obtained in ref. [27] were associated with a worldvolume which is anisotropic while the ansatz used to derive the singularity theorem in ref. [36] is isotropic. Only in the anisotropic case were there solutions free of curvature singularities. Presumably the anisotropy modifies the RHS of eq. (2.101) allowing the volume to gain positive acceleration during its evolution in such a way as to avoid a big-crunch or a big-bang singularity. Also, tachyon inhomogeneity, and any effect on the bulk fields' ansatz, was not handled in the singularity theorem either. Work on this and other inhomogeneity questions is in progress [61].

However, for $p = 8$, I_{tachyon} can be negative[17] which means the tachyon can drive the volume of the SD-brane into periods of positive acceleration therefore

[17] For $p = 7$ I_{tachyon} can also be negative but ref. [36] showed that there is nevertheless a singularity theorem in this case.

potentially avoiding the big-crunch or big-bang singularities predicted by the theorem. We provide some remarks on this specific case in the next section.

In short, the singularity theorem implies that full-SD-branes are singular in one of two ways. Type I: full-SD-branes will evolve out of a big-bang singularity (the singularity is in the past of the tachyon evolution) or, Type II: full SD-branes lead to a big-crunch singularity in the future. The results of ref. [36] is that during the evolution of the tachyon the Hubble function

$$H = \frac{(p+1)}{9} H_{\parallel} + \frac{(8-p)}{9} H_{\perp}, \tag{2.104}$$

where $H_{\parallel} = \dot{a}/a$ and $H_{\perp} = \dot{R}/R$, will diverge in finite time. Inspection of the associated Ricci scalar expression

$$\begin{aligned} \mathcal{R}(t) = & -2(p+1)(8-p)H_{\parallel}H_{\perp} - p(p+1)\left[H_{\parallel}^2 + \frac{k_{\parallel}}{a^2}\right] - (8-p)(7-p)\left[H_{\perp}^2 + \frac{k_{\perp}}{R^2}\right] \\ & + \tfrac{1}{4}\left[(p+1)(p-7)P_{\parallel} - (8-p)P_{\perp} + 8(p+1)\rho\right] \end{aligned} \tag{2.105}$$

shows that, unless there is an unlikely conspiracy among terms, this corresponds to a curvature singularity. In the above expression we have introduced the notation

$$\rho = \frac{\lambda V(T) e^{\phi(p/4)-1/2}}{2R^{8-p}\sqrt{\Delta}} + \frac{1}{4}\dot{\Phi}^2 + \frac{1}{4}e^{a\Phi}\dot{C}^2, \tag{2.106}$$

$$P_{\parallel} = -\frac{\lambda V(T) e^{\phi(p/4)-1/2}\sqrt{\Delta}}{2R^{8-p}} + \frac{1}{4}\dot{\Phi}^2 - \frac{1}{4}e^{a\Phi}\dot{C}^2, \tag{2.107}$$

$$P_{\perp} = \frac{1}{4}\dot{\Phi}^2 + \frac{1}{4}e^{a\Phi}\dot{C}^2, \tag{2.108}$$

therefore writing the sources explicitly as a perfect fluid.

At this stage we would like to reformulate the singularity theorem of ref. [36] in a geometric form that is more closely related to a no-go theorem for full-SD-branes. Depending on the intrinsic geometry of the SD-brane of interest, *i.e.*, the value of $k_{\parallel}$ and $k_{\perp}$, the constant of proportionality v in (2.100) will vary but it is irrelevant for the following analysis. Let us consider in details the cases for which

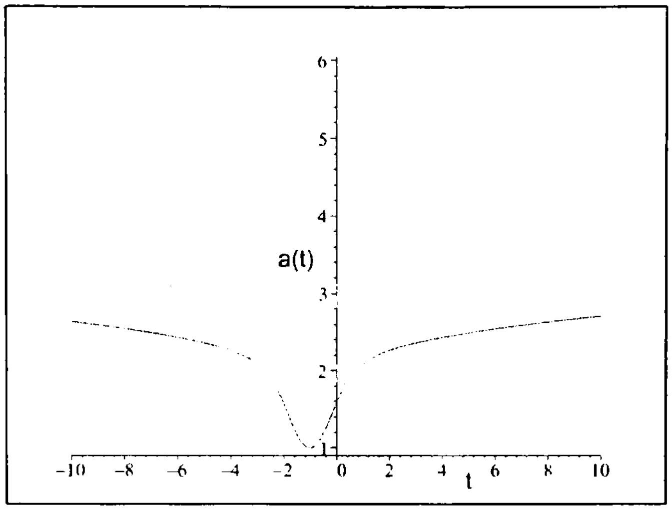

Figure 2.6: Examples of functions $a(t)$ that *would* be associated with full-SD-branes with $k_{\|} = 0$ and $k_{\perp} = -1$. From top to bottom these correspond to $p = 2$, $p = 4$ and $p = 6$.

$k_{\|} = 0$ and $k_{\perp} = -1$. Asymptotically flat boundary conditions for a full-SD-brane are then of the form

$$\lim_{t \to \pm\infty} a(t) = a_{\pm\infty}, \quad \lim_{t \to \pm\infty} \dot{a}(t) = 0, \tag{2.109}$$
$$\lim_{t \to \pm\infty} R(t) = \pm(t + \kappa_{\pm\infty}), \quad \lim_{t \to \pm\infty} \dot{R}(t) = \pm 1,$$

where $a_{\pm\infty}$ and $\kappa_{\pm\infty}$ are non-zero constants. We do not consider the boundary condition of the type

$$\lim_{t \to \pm\infty} R = +t \tag{2.110}$$

since this will most likely lead to a curvature singularity as $R \to 0$. The average scale factor $A(t)$ associated with the boundary conditions (2.109) behaves like

$$\lim_{t \to \pm\infty} A(t) \sim |t + k_{\pm\infty}|^{\frac{8-p}{9}}. \tag{2.111}$$

Figure 2.6 shows, for $p = 2$, $p = 4$ and $p = 6$, the example of a functions $A(t)$ with this asymptotic behavior. The region $t = 0$ could either correspond to the tachyon reaching the top of the potential (Type I full-SD-branes) or to a turning

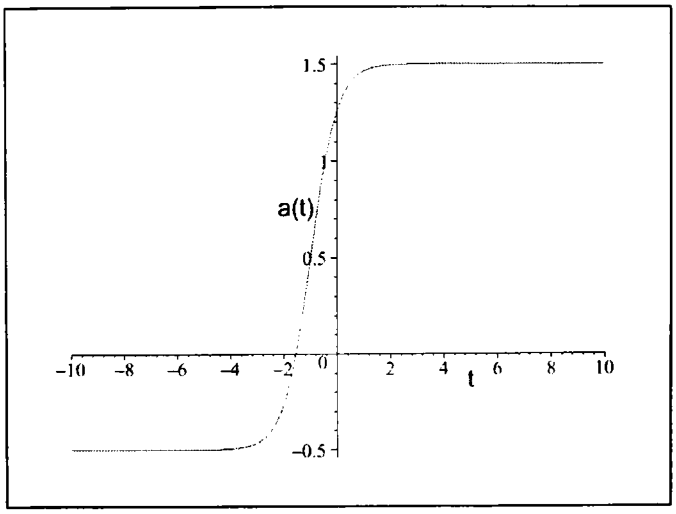

Figure 2.7: Example of a function $a(t)$ that *would* be associated with a full-SD-brane with $k_{\|} = 0$ and $k_{\perp} = 0$.

point (Type II full-SD-branes). For $t > 0$ we have $\ddot{A}/A < 0$, which is consistent with the equations of motion, and for $t < 0$ there is necessarily a region for which $\ddot{A}/A > 0$. Recall that the time-reversal symmetric solutions were excluded from the start for consistency reasons. The important point here is that a behavior as shown on figure 2.6 can only be obtained if there is a region in the evolution of $A(t)$ such that $\ddot{A}/A > 0$. In other words, there is no solutions unless the volume of the full-SD-brane is allowed to go through a phase of positive acceleration. We have seen before that this cannot be accomplished in pure supergravity and that adding the most obvious open string mode, the tachyon, does not help for $p \leq 7$. Presumably, adding more exotic matter or using a more general ansatz could be useful. The results in ref. [27] suggest that allowing for sources inducing worldvolume anisotropies is likely to provide positive results for full-SD-branes.

It therefore seems likely that full-SD-brane solutions with an homogeneous and isotropic worldvolume simply do not exist in pure supergravity and also when the extended sources are associated with a tachyon with dynamics governed by a Dirac-Born-Infeld-like action. This conclusion is of course restricted to the specific ansatz for the unstable branes density in the space transverse to their worldvolume, namely, they were smeared along the corresponding directions. It will be interesting to consider whether or not the no-go theorem can be circumvented by using a less restrictive ansatz [61]. We should also note that other effective actions for the tachyon dynamics are proposed in the literature (see, *e.g.*, refs. [62, 63]). It would certainly be interesting to check whether or not these actions would allow for a period of positive acceleration which is a pre-requisite for full-SD-brane solutions to exist.

As pointed out before, SD-branes can have different geometries depending on the curvature of the worldvolume and of the transverse space. For example for $k_{\|} = 0$ and $k_{\perp} = 0$ (asymptotically flat) full-SD-branes are such that

$$\begin{aligned} \lim_{t\to\pm\infty} a(t) &= \text{const.}, & \lim_{t\to\pm\infty} \dot{a}(t) &= 0, \\ \lim_{t\to\pm\infty} R(t) &= \text{const.}, & \lim_{t\to\pm\infty} \dot{R}(t) &= 0. \end{aligned} \tag{2.112}$$

In this case a typical function $A(t)$ is shown on figure 2.7. Again, this type of behavior cannot be achieved unless there is matter in the system allowing a period of constant positive acceleration for the overall volume of the geometry.

Geometric arguments of the type discussed here can be extended to all values of $k_{\|}$ and $k_{\perp}$. This leads to a no-go theorem for homogeneous full-SD-branes in pure supergravity and in the case where the supergravity modes evolution is driven by an open string tachyon. This is a reformulation of the singularity theorem in ref. [36] which says that that there will always be a curvature singularity (or, more conservatively, a region where $\dot{A}/A$ diverges) in the region $t < 0$ of the type of evolutions illustrated on figures 2.6 and 2.7. The geometric no-go theorem was here illustrated for Type I full-SD-branes which are those emerging from a big-bang singularity. Type II full-SD-branes will roughly be obtained by looking at the $t \to -t$ cases. The no-go theorem applies to those as well.

2.5.2 Space-filling SD-branes

In the space-filling case, $p = 8$, the conditions leading to the singularity theorem of ref. [36] are no longer satisfied. It is easy to see this by inspecting the form of I_{tachyon} and I_{sugra} in eq. (2.103). For $p = 8$ the tachyon contribution to the RHS of the equation for $\ddot{A}/A$ is

$$-\frac{1}{9} I_{\text{tachyon}} = +\frac{1}{9 \times 8} \frac{\lambda V(T) e^{3\Phi/2}}{\sqrt{\Delta}} \left[1 - \frac{9}{2} \dot{T}^2 e^{-\Phi/2} \right]. \tag{2.113}$$

At large times, this is negative (like the cases of larger codimension which possess this property for all time). However, the $p = 8$ tachyon contribution to the acceleration is *positive* until $|\dot{T}| = e^{\Phi/4}\sqrt{2}/3$.

For the bulk fields

$$-\frac{1}{9} I_{\text{sugra}} = -\frac{1}{2} \dot{\Phi}^2 + \frac{1}{16} e^{-5\Phi/2} \dot{C}^2. \tag{2.114}$$

Notice that the contribution of the Ramond-Ramond field is positive definite. The metric contribution is absent for $p = 8$.

Therefore, the interesting question to ask is whether there always exists some choice of initial conditions such that the big-crunch can be avoided for a negative initial Hubble parameter. (Note that we expect no singularity trouble for positive initial Hubble parameter; see next sections for details).

Let us consider the constraint carefully. For $p = 8$ we have

$$\frac{8 \times 9}{2} \left[H^2 + \frac{k^2}{A^2} \right] = \rho, \tag{2.115}$$

where $H = \dot{A}/A$ is the Hubble parameter and the energy density is

$$\rho = \frac{\lambda V(T) e^{3\Phi/2}}{2\sqrt{\Delta}} + \frac{1}{4} \dot{\Phi}^2 + \frac{1}{4} e^{-5\Phi/2} \dot{C}^2. \tag{2.116}$$

Let us specify to the case $k = 0$ for definiteness. Then we have for $t = 0$ the relation $H^2(t = 0) = \rho(t = 0)/36$. So the size of the initial Hubble parameter is set by

$$\left|\frac{\dot{A}(0)}{A(0)}\right| = \frac{1}{6}\sqrt{\rho(0)}. \tag{2.117}$$

The question is whether a small enough negative Hubble parameter can be chosen at $t = 0$, consistent with the constraint, such that the positive acceleration at $t = 0$ lifts it up to becoming a positive Hubble parameter before the era of positive acceleration runs out.

The point to notice here is that the answer depends crucially on what the Ramond-Ramond field is doing. On the other hand, the dilaton field cannot help sustain the period of positive acceleration, because as we see above in eq. (2.114), it always contributes negatively to the acceleration. The dilaton also makes the era of positive tachyon contribution to the acceleration shorter. This is because that era ends when $|\dot{T}| = e^{\Phi/4}\sqrt{2}/3$, and, as found for typical half-SD-brane evolutions, the string loop-counting parameter $g_s e^{\Phi}$ falls.

If there are no Ramond-Ramond fields turned on at $t = 0$, then it looks rather unlikely to us that the SD8-brane could remain nonsingular with a negative initial Hubble parameter. Let us see why, explicitly. For the purposes of this argument we can turn off the dilaton; as we saw above it only strengthens the likelihood of developing a singularity. So for the moment we are considering a situation with metric and tachyon only. Now, $\rho(0)$ sets the initial Hubble parameter, and the initial positive acceleration, and whether or not supergravity is actually valid. Explicitly, using eqs. (2.113) and (2.114) we have

$$\frac{\ddot{A}(0)}{A(0)} = \frac{1}{36}\rho(T(0)) \qquad \text{(for } g, T \text{ only)}. \tag{2.118}$$

Here, we used the fact that the tachyon derivative is negligible at $t = 0$.

Now, in order for supergravity to be a reasonable approximation to the physics, we require small curvature. Using eq.(2.105) for the Ricci scalar at $t = 0$ we find it to be of order $\rho(T(0))$ for the tachyon contribution, of order $[\dot{A}(0)/A(0)]^2$ for the gravity contribution, of order $\dot{\Phi}^2$ and of order $e^{-5\Phi/2}\dot{C}^2$ for the dilaton and Ramond-Ramond fields respectively. Therefore, each of these quantities, such as $\rho(T(0))$, must be small.

Since for $\rho(0)$ small, $\sqrt{\rho(0)}$ is significantly bigger, it looks unlikely that the era of positive tachyon contribution lasts long enough to turn the Hubble parameter around into positive territory.

On the other hand, this conclusion can change dramatically if we turn on a nonzero Ramond-Ramond field at $t = 0$, because in general

$$\frac{\ddot{A}(0)}{A(0)} = \frac{1}{36}\rho(T(0))\left[1 - \frac{9}{2}\dot{T}^2(0)e^{-5\Phi(0)/2}\right] + \frac{1}{16}e^{-5\Phi/2}\dot{C}^2(0) - \frac{1}{2}\dot{\Phi}^2 \qquad \text{(general)}. \tag{2.119}$$

The Ramond-Ramond contribution to the positive acceleration has to be small, in order for supergravity to remain a decent approximation, but it looks to us that it is quite possible for it to be big enough to lengthen the positive-acceleration period. In addition, although the Ramond-Ramond contribution will typically fall with time in an SD-brane evolution, we can see that it will persist in contributing positively to the acceleration.

The equations are sufficiently complex, even in this restricted ansatz of homogeneous tachyon,[18] that we cannot settle this question absolutely definitively here. However, it looks very likely to us that it is possible for the SD8-brane to escape the singularity theorem.

One effect that various approaches to the problem of SD-brane supergravity fields has ignored thus far is particle production. This will typically slow down the rolling tachyon, and therefore potentially prolong the period of positive acceleration. This may help the likelihood of finding a completely nonsingular SD8-brane.

[18]And, for $p < 8$, branes smeared in the transverse space

2.5.3 COSMOLOGICAL APPLICATIONS

Cosmological scenarios involving tachyon condensation have been an area of active investigation (see refs. [40, 39]). It is particularly interesting to look at what the singularity theorem of ref. [36] has to say about these situations, which are of considerable interest to the early universe cosmology community. As we will show, the theorem simply does not apply; if it did, it would point to a quite general inability to have nonsingular cosmologies for matter satisfying physically reasonable energy conditions. In particular, an interesting nonsingular cosmology involving plain ordinary $d = 4$ Einstein gravity coupled to the tachyon was actually first pointed out in ref. [38].

Let us briefly recap the cosmology story of ref. [38], changing the action to general dimension d (and renormalizing the tachyon potential to conform to the conventions used in this chapter). We have

$$S = \frac{1}{16\pi G_d}\int d^dx\sqrt{-g}\left[\mathcal{R} - \lambda V(T)\sqrt{1 - g^{\mu\nu}\nabla_\mu T\nabla_\nu T}\right]. \tag{2.120}$$

The metric ansatz is taken to be

$$ds^2 = -dt^2 + A^2(t)d\Sigma^2_{d-1}(k). \tag{2.121}$$

The tachyon equation of motion is not important for us here, except that it rolls fast (nearly at $\dot{T} = 1$) at large t.

The constraint equation is

$$\left(\frac{\dot{A}}{A}\right)^2 + \frac{k}{A^2} = \frac{1}{(d-1)(d-2)}\frac{\lambda V(T)}{\sqrt{1-\dot{T}^2}} \tag{2.122}$$

and can be considered as the initial condition at $t = 0$. For $k = 0$, $H = \dot{A}/A$ must be nonzero at $t = 0$ in order to satisfy the constraint. The cases $k = -1$ and $k = +1$ are presumably less interesting for cosmology since there is convincing experimental evidence that our universe is flat. We can choose either the positive or negative branch (for the initial Hubble parameter) for any given cosmology.

The bulk equation of motion is

$$\frac{\ddot{A}}{A}=\frac{1}{(d-1)(d-2)}\frac{\lambda V(T)}{\sqrt{1-\dot{T}^2}}\left[1-\frac{(d-1)}{2}\dot{T}^2\right]. \tag{2.123}$$

Notice that this is precisely the same result as we saw in the previous section for the case $p=8$, if the dilaton is negligible.

Following usual cosmological intuition, the positive Hubble parameter branch solution of the Friedmann equation is chosen in ref. [38]. Then, according to the bulk equation of motion the scale factor starts out concave up, but as the tachyon gets rolling fast it becomes concave down. But because the tachyon obeys the weak energy condition ($\rho > 0$), $\dot{A}/A$ always stays positive. The long-time behavior is $\ddot{A}/A \sim 0^-$, and $A(t)$ either approaches a constant as $t \to \infty$ for $k=0$ or it is linear in time for $k=-1$. These give perfectly nonsingular cosmologies. For $k=+1$, however, there is a big-crunch.

These conclusions hold for any $d>2$, and in particular any dimension d that is sensible for cosmology. All of this is very intuitive, based on previous experience with ordinary minimally coupled scalar fields in four-dimensional cosmology.

Comments on accelerating cosmology

Although they are singular, the SD2- and SM2-brane spacetimes found in ref. [15, 26, 27] have been studied with the aim of finding characteristics shared with our own universe [64]. The corresponding time-dependent 10- and 11-dimensional spacetimes have been shown to admit, for some values of the parameters in the solutions, periods of positive worldvolume acceleration [64]. There is convincing evidence that the universe is currently in such an accelerating phase. Experimental data also suggest that the dark energy component of the universe has $\Omega_D=0.7$.

The papers refs. [64] were considering $\Omega_D = 1$ which is unrealistic. However progress towards achieving more realistic models were made in ref. [65] where additional external sources were added to the supergravity system.

Certainly, our approach consisting in adding unstable brane sources follows the philosophy of ref. [65]. This approach is in fact a more general study than what was done so far in tachyon cosmology. It would be very interesting to see in details how adding the dilaton and a Ramond-Ramond field would affect the results obtained in tachyon cosmology in a more general way than has been done thus far [61].

As a first small step, we can ask whether the tachyon is likely to prefer positive or negative worldvolume acceleration. It is clear from the no-go theorem that

$$(p+1)\frac{\ddot{a}}{a} = -(8-p)\frac{\ddot{R}}{R} - I_{\rm sugra} - I_{\rm tachyon}. \qquad (2.124)$$

It was noted before that for $p = 2$ we have $I_{\rm tachyon} \geq 0$. Simply by inspection of eq. (2.124) we see that the tachyon contribution to the equations of motion appears to favor negative rather than positive worldvolume acceleration. Since $I_{\rm sugra} \geq 0$ the only way positive worldvolume acceleration can be achieved is if $\ddot{R}/R < 0$ during the tachyon evolution.

2.5.4 Half-SD-branes

In the supergravity approximation, the half-SD-branes, and the unstable D-branes, are morally equivalent to the cosmological case of ref. [38] shown above. In particular, small bulk kicks will be needed at $t = 0$, *i.e.*, we need at least one nonzero (small, positive) Hubble parameter at $t = 0$. For the numerical solutions presented in section 2.4, the $t \geq 0$ solutions are not ruled out by the singularity theorem of ref. [36].

Let us see this analytically. Typically we find that $a(t)$ evolves ever upwards when it starts at $t = 0$ with a positive derivative; $R(t)$ evolves ever upwards as well. In particular, the geometric average scale factor $A(t)$ starts out with positive derivative, it slows down with time (because $\ddot{A} < 0$), and eventually asymptotes to a constant. This is clearly consistent with the asymptotics: $a(t \to \infty) \sim$ const., $R(t \to \infty) \sim t$. This $t \geq 0$ behavior is also thoroughly consistent intuitively with the results of ref. [38]. As a simple example let us consider the type of evolutions illustrated on figures 2.6 and 2.7 where an half-SD-brane corresponds only to the $t \geq 0$ part of the evolution: either the tachyon starts at the top of the potential (perhaps with some initial kinetic energy although the initial conditions $T(0) = 0$ and $\dot{T}(0) = 0$ are consistent as long as there is kinetic energy in the bulk fields at $t = 0$) or the tachyon starts evolving away from the top of the potential. The behavior shown on the figures is such that $\dot{A}/A \geq 0$ and $\ddot{A}/A \leq 0$ throughout the evolution.

2.6 Discussion

Our primary motivation for this work was the general problem of seeking mechanisms for resolution of singularities in spacetimes of interest in string theory. SD-brane supergravity spacetimes presented in refs. [15, 26, 27] create somewhat of a supergravity emergency because they are not only singular but nakedly so.

A first attempt in this program was made in ref. [29], where some effects of unstable brane probes in these backgrounds were considered. In particular, the probe physics was also sick, and taking this analysis seriously led to an even more dire assessment of the likelihood of singularity resolution without resorting to the inclusion of massive string modes in both the open and closed sectors. In our work, then, we began by plumbing the depths of the probe approach. We found it to be generally insufficient for our purposes; one reason is that the probe approximation takes itself out of its regime of self-consistency.

We then launched into an investigation of the physics of the gravitational fields exerted by SDp-branes for general p by including backreaction. In order to get started on this problem, we had to make the approximation of considering only the most relevant open- and closed- string modes, with full gravitational backreaction taken into account. The equations we derived are highly nonlinear and couple brane with bulk, so did not lend themselves to solution analytically. We therefore resorted to numerical techniques to search for solutions. Because of this restriction, we had to use an ansatz which smeared the branes in the transverse space; this allowed us to turn the equations into ODE's and integrate them numerically. An essential step was to begin the numerical integration near the maximum of the potential hill, and then attempted to reconstruct asymptopia. We were only partly successful at doing that since we find that the solutions for full-SD-branes are plagued with a curvature singularity either in the past or the future. This is consistent with the singularity theorem proposed in ref. [36]. The only non-singular solutions we could find are for half-SD-branes, *i.e.*, the future or the past of a full-SD-brane. These are interesting solutions because they are related to the physics of unstable D-branes.

It is hard to know whether our results will survive refinement. Therefore, let us now make some specific remarks about technical roadblocks we encountered which forced us to make approximations, their physical consequences, and future outlook.

The most obvious refinement of our work here will be to attack the problem of relaxing the requirement of zero NS-NS B-field. Allowing $B^{(2)}$ to be turned on will allow us to break $ISO(p+1)$ on the worldvolume and allow inhomogeneous tachyonic modes — the importance of which is discussed in refs. [39, 54] — and also to turn on more components of R-R fields. Inhomogeneities would of course have to be included in initial conditions, because homogeneous on-shell tachyons do not couple to non-homogeneous tachyons [53]. We have postponed the non-homogeneous problem to the future; our work reported here should be considered a step in a larger program. It is conceivable that inhomogeneities might play a role in resolving the singularity for full-SD-branes.

An other important approximation we made in our work was in section 2.3.2, where we had to smear the SD-brane sources in the transverse space to facilitate integrating the equations numerically by turning them into ODE's. This limits our ability to fully probe the properties of the system in which we are interested. Here we would also like to record another physical consequence of this ansatz. Namely, this restriction has notable, negative, consequences for our ability to track whether black holes form as intermediate states during the time evolution of our coupled system including full backreaction. The issue of black holes was raised in the discussion section of ref. [27]. The essential point is that a black hole intermediate state may arise as an alternative to SD-brane formation and decay. Integrating partial differential equations of motion (including dependence on transverse coordinates) may be particularly difficult numerically. Or the obstruction to finding the full solution may yet turn out to be negotiable.

Let us now make comments related to the singularity theorem proposed in ref. [36]. We have pointed out that cosmology with rolling tachyons is of course not ruled out by the singularity theorem. In particular, half-SD-branes certainly exist as well-behaved supergravity solutions, provided that small (positive) bulk kicks are given to at least one bulk field at $t = 0$, such as $a(t)$. As we saw in section 2.5.3, this initial condition choice is in tune with textbook cosmological intuition.

We have already pointed out that the general question of inhomogeneities in tachyon cosmology is currently under further investigation [61]. In particular, in the current context it will be interesting to know whether the singularity theorem of ref. [36] can be extended to cover those cases. Intuition based on standard cosmology indicates that inhomogeneity will probably only strengthen the no-go theorem, but this must be checked, since inhomogeneities notably complicate all the field equations.

Another thing to check will be the role of *un*smearing the branes in the transverse space. This is important because the validity of the no-go theorem is limited by the restrictive nature of the supergravity ansatz used in deriving it. We feel this issue has particular importance because we have pointed out that the SD8-brane may actually avoid the no-go theorem. And the $p = 8$ case is really the only one *honestly* covered by the ansatz used.

It could be interesting to include in this investigation also the question of whether changes in the low-energy effective action for the tachyon, as suggested in refs. [62, 63], may change the outcome.

The details of how the tachyon rolls can be significantly affected by particle production when backreaction is included. In particular, if the rolling slows down because of particle production, there may be a longer time of positive acceleration for the space-filling case.

Finally, there is the question of whether quantum string theoretic ingredients are unavoidably necessary for description of SD-branes. It is possible that quantum stringy ingredients may turn out to be important for getting long periods of positive acceleration in models of this type. Clearly, an important issue is whether supergravity can remain a decent approximation to the physics in such a situation. Let us suppose for the sake of argument that this can be done. Our program consisted in solving the Einstein equations with sources in the form of a perfect fluid with stress-energy tensor components eqs. (2.106), (2.107) and (2.108). Each of the sources separately satisfies the dominant energy condition (see ref. [66] for definitions). Of course, this implies that the weak energy condition is satisfied as well. The strong energy condition,

$$7\rho + (p+1)P_{\|} + (8-p)P_{\perp} \geq 0, \tag{2.125}$$

is satisfied by eqs. (2.106)–(2.108) for $p < 7$. As shown in section 2.5.1, circumventing the no-go theorem would, as a minimum requirement, require that a period of positive acceleration (for the volume V_S of the spacetime) be allowed. This could be achieved by introducing new terms (*i.e.*, sources with a different equation of state) to the RHS of (2.101) that contribute positively, *i.e.*, in such a way as to violate the strong energy condition. Sources in the form of massive open or closed strings might achieve the desired behavior therefore resolving the singularity and circumventing the no-go theorem. Analysis of massive open string mode production during tachyon evolution were performed in refs. [24, 28, 32]. BCFT calculations for the emission of massive closed strings can be found in refs. [34, 35].

To illustrate our point let us use a simple example, *i.e.*, that of a scalar field in four dimensions, with the usual stress-energy tensor. An implication of the strong energy condition being satisfied is that

$$\left(T_{ab} - \frac{1}{2} g_{ab} T^c_c\right) \xi^a \xi^b \geq 0, \tag{2.126}$$

which becomes

$$(\nabla_a \phi \, \xi^a)^2 - \frac{1}{2} m^2 \phi^2 \geq 0 \tag{2.127}$$

for a massive scalar field. ξ^a is a timelike unit vector. After using the equation of motion and integrating over a region M of the spacetime the LHS of (2.127) becomes

$$\frac{1}{2} \int_M \left(g^{ab} + 2\xi^a \xi^b\right) \nabla_a \phi \nabla_b \phi \, d^4x - \frac{1}{2} \int_{\partial M} \phi \nabla_a g^{ab} dn_b, \tag{2.128}$$

where ∂M is a boundary and dn_b a normal vector. The first term in eq. (2.128) is always positive while the second term, which is negative-definite, will remain small as long as the region of integration M is large compare to the Compton wavelength

($\lambda = 1/m$) of the mode. The strong energy condition might therefore be violated in small regions of the spacetime in particular when the domain of existence (in time and space) of the modes is small. This simple line of reasoning suggest that the emission of massive modes (open and/or closed) might provide a loophole to the singularity theorem for full-S-branes. Of course, as we mentioned above, this assumes that the tachyon evolution, including emission of massless and massive open and closed strings is smooth and that the contribution of the massive modes does not overwhelm that of the massless modes. Consideration of this issue is also under investigation.

Let us end with some somewhat speculative remarks. Typically in the limit $g_s N \to 0$ the open and closed strings decouple. This is true in our effective lowest-modes analysis here, but also explicit in other worldsheet-inspired approaches. There should also exist a limit (in time) to be taken where only the open strings survive. From the worldsheet definition of a SD-brane, it is suggestive that the open string degrees of freedom would combine to form a Euclidean conformal field theory in $p+1$ dimensions. The same appears true when considering the effective action of massless open string degrees of freedom on an unstable D-brane [55]. In both approaches, however, it is not clear what the role of the tachyon could be. Physically, it is the source of a process by which energy is siphoned out of the open string sector and pumped into the closed string sector. So, in a sense, the decay of a D-brane through tachyon condensation corresponds to the decrease of a c-function-like quantity on the gauge theory side. Then, we can entertain the idea that time-evolution on the gravity side should really be regarded as a renormalization group (RG) flow on the gauge theory side. From this viewpoint, formation and decay of a SD-brane would be a process corresponding to first an *inverse* RG flow (integrating in degrees of freedom) followed by regular RG flow (integrating out degrees of freedom).[19] This might be related to the study of open string tachyon condensation using RG flow in the worldsheet theory [60].

[19] Similar ideas will be explored in chapter 3 in the context of the dS/CFT correspondence

Chapter 3

Quantum gravity in de Sitter space

Recent observations suggest that our universe is proceeding toward a phase where its evolution will be dominated by a small positive cosmological constant — see, *e.g.*, ref. [23]. This suggestion poses new challenges for string theory, which has seen much success in asymptotically flat spaces and in settings with an effective negative cosmological constant (*e.g.*, Freund-Rubin compactifications [67] producing asymptotically anti-de Sitter spaces). The impressive success of the AdS/CFT correspondence [6], which has provided fairly concrete realizations of a holographic duality between quantum gravity and a field theory in one dimension lower without gravity, has prompted speculations that it may be possible to describe string theory or quantum gravity in asymptotically de Sitter spaces by a similar dS/CFT correspondence [68, 70]. Like the AdS case, the symmetries of de Sitter space suggest that the dual field theory is conformally invariant.

Of course, the nature of de Sitter space is quite different from its AdS counterpart. In particular the conformal boundaries, which one expects to play a central role in any dS/CFT correspondence, are hypersurfaces of Euclidean signature. As a result, one expects the dual field theory to be a Euclidean field theory. Further in de Sitter space, there are two such hypersurfaces: the future boundary, I^+, and the past boundary, I^-. Hence one must ask whether the proposed duality will involve a single field theory [68, 70] or two [103]. Unfortunately at present, the most striking difference from the AdS/CFT duality is the fact that we have no rigorous realizations of the dS/CFT duality — some progress has been made with pure three-dimensional de Sitter gravity [104].

However, the idea of a dS/CFT correspondence is a powerful and suggestive one that could have fundamental implications for the physics of our universe. A present difficulty is that rather little is known about the Euclidean field theory which is to be dual to physics in the bulk. Our goal here is to explore further the requirements for a candidate dual field theory

3.1 De Sitter space basics

The simplest construction of the $(n+1)$-dimensional de Sitter (dS) spacetime is through an embedding in Minkowski space in $n+2$ dimensions, where it may be defined as the hyperboloid

$$\sum_{A,B=0}^{n+1} \eta_{AB} X^A X^B = \ell^2. \tag{3.1}$$

The resulting surface is maximally symmetric, *i.e.*,

$$\mathcal{R}_{ijkl} = \frac{1}{\ell^2}(g_{ik}\, g_{jl} - g_{il}\, g_{jk}) \ , \tag{3.2}$$

which also ensures that the geometry is locally conformally flat. Hence dS space solves Einstein's equations with a positive cosmological constant,

$$\mathcal{R}_{ij} = \frac{2\Lambda}{d-2} g_{ij} \qquad \text{with} \quad \Lambda = \frac{(d-1)(d-2)}{2\ell^2} \ , \tag{3.3}$$

where $i, j = 0, \ldots, n$. The topology of the space is $R \times S^n$ where $n \equiv d-1$. The Penrose diagram is represented by a square [66], as illustrated in Figure 3.2 (a). Any horizontal cross section of the figure is an n-sphere, so that any point in the interior of the diagram represents an $(n-1)$-sphere. At the right and left edges, the points correspond to the north and south poles of the n-sphere. The diagram is just tall enough for a null cone emerging from, say, the south pole at I^-, to re-converge on the north pole at I^+.

One may present the metric on dS space in many different coordinate systems, which may be particularly useful in different situations. Three simple choices for d-dimensional dS space come from foliating the embedding space above with flat hypersurfaces, $n_A X^A$ = constant. The three distinct choices correspond to the cases where the normal vector n_A is timelike, null or spacelike. With these distinct choices, a given hypersurface intersects the hyperboloid on a spatial section which has a spherical, flat or hyperbolic geometry, respectively. Following the standard notation for Friedmann-Robertson-Walker cosmologies, we denote these three cases as $k = +1, 0$ and -1, respectively. Then the three corresponding metrics on dS space can be written in a unified way as follows:

$$ds^2 = -dt^2 + a_k^2(t) d\Sigma_{k,n}^2 \ , \tag{3.4}$$

where the n-dimensional Euclidean metric $d\Sigma_{k,n}^2$ is

$$d\Sigma_{k,n}^2 = \begin{cases} \ell^2 d\Omega_n^2 & \text{for } k = +1 \\ \sum_{i=1}^{n} dx_i^2 & \text{for } k = 0 \\ \ell^2 d\Xi_n^2 & \text{for } k = -1 \ , \end{cases} \tag{3.5}$$

where $d\Omega_n^2$ is the unit metric on S^n. The 'unit metric' $d\Xi_n^2$ is the n–dimensional hyperbolic space (H^n) which can be obtained by analytic continuation of that on S^n. For $k = \pm 1$ we assume that $n \geq 2$.

The scale factor in each of these cases would be given by

$$a_k(t) = \begin{cases} \cosh(t/\ell) & \text{for } k = +1 \\ \exp(t/\ell) & \text{for } k = 0 \\ \sinh(t/\ell) & \text{for } k = -1 \ . \end{cases} \tag{3.6}$$

Hence we see that $k = +1$ corresponds to the standard global coordinates, in which the spatial n-sphere begins by shrinking from infinity to a minimum size (with radius ℓ) and then it re-expands. The choice $k = 0$ corresponds to the standard inflationary coordinates, where the flat spatial slices experience an exponential expansion (assuming a positive sign in the exponential). In this case, $t = -\infty$ corresponds to a horizon (*i.e.*, the boundary of the causal future) for a co-moving observer emerging from I^-. Hence these coordinates only cover half of the full dS space but, of course, substituting a minus sign in the exponential of eq. (3.6) yields a metric which naturally covers the lower half. The choice $k = -1$ yields a perhaps less familiar coordinate choice where the spatial sections have constant negative curvature. In this case, $t = 0$ again represents a horizon. However, this horizon is the future null cone of an actual point inside dS space. Figure 3.1 illustrates slices of constant t on a conformal diagram of dS space. It is straightforward to see that all of the above coordinate systems in fact are related by a local diffeomorphism. One might also note the exponential expansion that dominates the late time evolution of all three slicings independent of the spatial curvature, *i.e.*, $a_k(t) \sim e^{(t/\ell)}$ as $t \to \infty$ for all k.

These three different coordinate patches are displayed on the full Penrose diagram in figure 3.1. One comment on the $k = -1$ case is that the central diamond cannot be foliated by homogeneous spacelike hyperboloids. However, it is naturally foliated by timelike hyperboloids, *i.e.*, by copies of dS space. The metric in the central diamond region may be obtained by double analytic continuation of (3.4) and the result is

$$ds^2 = dt^2 + \ell^2 \sin^2(t/\ell) ds^2_{dS}, \tag{3.7}$$

where ds^2_{dS} is the metric for n-dimensional de Sitter space with unit radius of curvature. Here t is a spacelike coordinate that is naturally thought of as the analytic continuation of the t in (3.6) behind the horizon.

With the metric (3.4), we have apparently displayed dS_{n+1} with three different boundary geometries:

$$S^n, \quad \mathbb{R}^n \quad \text{and} \quad H^n\,. \tag{3.8}$$

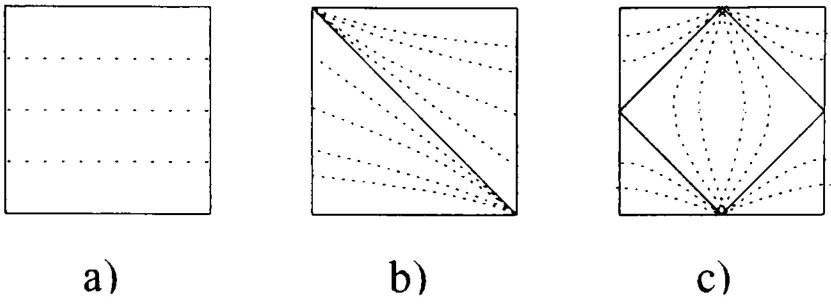

Figure 3.1: Constant t slices in a) the spherical slicing, b) the flat slicing, and c) the hyperbolic slicing of de Sitter space.

The dS/CFT correspondence would imply then an equivalence between, on the one hand, quantum gravity in dS space and a CFT on any of the above backgrounds (3.8). On the other hand, we know that the (past and future) timelike infinities of dS_{n+1} space have the topology S^n. Hence the correct observation is that the latter two boundary geometries are equivalent to (a portion of) S^n up to a conformal transformation. However, a singularity in the latter transformation effectively removes (*i.e.*, pushes off to infinity) certain points on the sphere to produce the resulting geometry. In the case where the boundary appears to be H^n, it is obvious from the Penrose diagram (see figure 3.1) that these coordinates cover only half of the boundary of dS space. Hence the conformal transformation has pushed out the equator of the boundary sphere to produce two copies of the hyperbolic plane.

A common feature of the three coordinate systems (3.4) above is that the spatial geometry is uniformly scaled in the time evolution. In Appendix C, we present some other metrics on dS space which have non-uniform scalings. In particular, the boundary metric takes the form of a direct product of two sub-manifolds and each of the latter sub-manifolds evolves with a different scale factor.

3.2 Generalities of gauge/gravity dualities

We wish to discuss the interpretation of scalar field theory in a de Sitter background in the context of the dS/CFT correspondence. However, much of the motivation for the dS/CFT duality, as well as the interpretation of the dS space calculations, comes from our understanding of the AdS/CFT duality [6]. Hence we begin with a brief review of the latter correspondence (section 3.2.1) and then present the most relevant aspects of dS/CFT (section 3.2.2). Much of this discussion is review, although our emphasis is somewhat different than in previous treatments. The reader interested in the mathematical details can consult the appendices to which we will refer in the following.

3.2.1 Brief AdS/CFT review

Consider probing anti-de Sitter space with a massive scalar field. We consider the following metric on $(n+1)$-dimensional AdS space,[1]

$$ds^2 = dr^2 + e^{2r/\tilde{\ell}} \eta_{\mu\nu} dx^\mu dx^\nu \ , \tag{3.9}$$

and the standard equation of motion for the scalar,

$$\left[\Box - M^2\right] \phi = 0 \ . \tag{3.10}$$

Then to leading order in the asymptotic region $r \to \infty$, the two independent solutions take the form [111]

$$\phi_\pm \simeq e^{-\Delta_\pm r/\tilde{\ell}} \phi_{0\pm}(x^\mu) \qquad \text{where} \qquad \Delta_\pm = \frac{n}{2} \pm \sqrt{\frac{n^2}{4} + M^2 \tilde{\ell}^2} \ . \tag{3.11}$$

[1]The essential feature for the following analysis is the exponential expansion of the radial slices with proper distance r. While we have chosen to consider pure AdS space in Poincaré coordinates for specificity, this expansion, of course, arises quite generally in the asymptotic large-radius region for any choice of boundary metric and for any asymptotically AdS spacetime.

Now the interpretation of these results depends on the value of the mass, and there are three regimes of interest:

$$\text{(i)}\;\; M^2 > 0\;, \quad \text{(ii)}\;\; 0 > M^2 > -\frac{n^2}{4\tilde{\ell}^2} \quad \text{and} \quad \text{(iii)}\;\; M^2 < -\frac{n^2}{4\tilde{\ell}^2}\;. \tag{3.12}$$

In case (i), Δ_- is negative and so the corresponding "perturbation" is actually divergent in the asymptotic regime. Hence in constructing a quantum field theory on AdS space, only the ϕ_+ modes would be useful for the construction of an orthogonal basis of normalizable mode functions [112]. In particular, the bulk scalar wave operator is essentially self-adjoint and picks out the boundary condition that the ϕ_- modes are not excited dynamically. In the context of the AdS/CFT then, the ϕ_{0-} functions are associated with source currents (of dimension Δ_-). These may then be used to generate correlation functions of the dual CFT operator of dimension Δ_+ through the equivalence [111, 113]

$$Z_{\text{AdS}}(\phi) = \int D\phi\, e^{iI_{\text{AdS}}(\phi_-,\phi_+)} = \left\langle e^{i\int \phi_{0-}\mathcal{O}_-}\right\rangle_{\text{CFT}}\,. \tag{3.13}$$

On the other hand, the boundary functions ϕ_{0+} are associated with the expectation value for states where the dual operator has been excited [112].

In case (ii), the lower limit corresponds precisely to the Breitenlohner-Freedman bound [114, 115]. While the scalar appears tachyonic, it is not truly unstable and it is still possible to construct a unitary quantum field theory on AdS space. Further, in this regime, both sets of solutions (3.11) are well-behaved in the asymptotic region. However, together they would form an over-complete set of modes. The theory must therefore be supplemented with a boundary condition at AdS infinity which selects out one set of modes to define a self-adjoint extension of the scalar wave operator (and thus the time evolution operator). For $0 > M^2\tilde{\ell}^2 > 1 - n^2/4$, there is a unique boundary condition which produces an AdS invariant quantization [114]. However, for

$$1 - n^2/4 > M^2\tilde{\ell}^2 > -n^2/4\;, \tag{3.14}$$

the boundary condition is ambiguous. The AdS/CFT interpretation is essentially the same as above. That is, the ϕ_{0+} and ϕ_{0-} functions may be associated with expectation values and source currents of the dual CFT operator, respectively. For the ambiguous regime (3.14), there is a freedom in this equivalence associated with a Legendre transformation of the generating functional [116].

Finally in case (iii), the mass exceeds the Breitenlohner-Freedman bound [114, 115] and the scalar field is actually unstable; no sensible quantization is possible. However, if one were to attempt an AdS/CFT interpretation analogous to those above, the dimension Δ_+ of the dual CFT operator would be complex, which might be interpreted as indicating that the corresponding theory is not unitary. Hence one still seems to have agreement on both sides of the correspondence as to the unsuitability of the regime $M^2\tilde{\ell}^2 < -n^2/4$.

3.2.2 Some dS/CFT basics

Given the brief overview of the AdS/CFT correspondence, we now turn to asymptotically de Sitter spaces, where one would like to study the possibility of a similar duality between quantum gravity in the bulk and a Euclidean CFT [68]. As in the previous review, we focus the present discussion on the case of a pure de Sitter space background:

$$ds^2 = -dt^2 + \cosh^2(t/\ell)\ d\Omega_n^2\ , \tag{3.15}$$

where $d\Omega_n^2$ is the standard round metric on an n-sphere. This metric solves Einstein's equations, $\mathcal{R}_{ij} = 2\Lambda/(n-1)\,g_{ij}$, in $n+1$ dimensions. The curvature scale ℓ is related to the cosmological constant by $\ell^2 = n(n-1)/(2\Lambda)$. Again the important feature of this geometry is the exponential expansion in the spatial metric in the asymptotic regions, *i.e.*, $t \to \pm\infty$. Much of the following discussion carries over to spacetimes that only resemble dS asymptotically (see section 3.3.1) and indeed, if the proposed dS/CFT duality is to be useful, it must extend to such spacetimes.

We will later explore certain aspects of the dS/CFT for such backgrounds.

Consider a free scalar field propagating on the above background (3.15), which we wish to treat in a perturbative regime where the self-gravity (back-reaction) is small. Hence, the equation of motion is

$$\left[\Box - M^2\right]\phi = 0 \, . \tag{3.16}$$

In general, the effective mass may receive a contribution from a non-minimal coupling to the gravitational field [117]. Therefore we write

$$M^2 = m^2 + \xi \mathcal{R} \, , \tag{3.17}$$

where m^2 is the mass squared of the field in the flat space limit and ξ is the dimensionless constant determining the scalar field's coupling to the Ricci scalar, $\mathcal{R}$. In the dS background (3.15), we have $\mathcal{R} = n(n+1)/\ell^2$. A case of particular interest in the following section will be that of the conformally coupled massless scalar field, for which $m^2 = 0$, $\xi = (n-1)/4n$ and hence $M^2 = (n^2-1)/4\ell^2$. With these parameters, the solutions of eq. (3.16) transform in a simple way under local conformal scalings of the background metric [117].

In parallel with the AdS case, scalar fields propagating in de Sitter space can have two possible behaviors near the boundaries. Let us for the moment think of defining these boundary conditions at past infinity (I^-). Equation (3.16) above is readily solved [68] near I^- to yield two independent solutions with the asymptotic form $\phi_\pm \sim e^{h_\pm t/\ell}$ where

$$h_\pm = \frac{n}{2} \pm \sqrt{\frac{n^2}{4} - M^2\ell^2} \, . \tag{3.18}$$

Note that this asymptotic time dependence is independent of the details of the spatial mode. In the pure dS background (3.15), the same exponents also govern the behavior of the fields at future infinity — see Appendix F for details.

The fact that the boundaries are spacelike in de Sitter space means that the 'boundary conditions' have a different conceptual status than in the AdS setting.

In particular, requiring that the bulk evolution is well-defined in dS space will not impose any restrictions on past or future boundary conditions. So in contrast to the AdS/CFT correspondence, in the dS/CFT correspondence, both the ϕ_+ and ϕ_- modes appear on an equal footing.[2] Certainly, a complete description of physics in the bulk must include both sets of modes as dynamical quantum fields. Following the analogy with the AdS/CFT correspondence and in accord with the preceding discussion, it is natural then to associate both modes $\phi_\pm$ with source currents for dual field theory operators $\mathcal{O}_\pm$, with conformal dimensions $h_\mp$ [68]. As we will discuss shortly, this matching of modes with dual operators is further supported by a bulk construction of a generating functional for correlation functions in the CFT.

As in the AdS case, one can classify the scalars displaying distinct types of boundary behavior in three different regimes:

$$\text{(i)}\ \ M^2 > \frac{n^2}{4\ell^2}\ , \quad \text{(ii)}\ \ \frac{n^2}{4\ell^2} \geq M^2 > 0 \quad \text{and} \quad \text{(iii)}\ \ M^2 < 0\ . \tag{3.19}$$

These three regimes also appear in discussions in the mathematics literature — see, *e.g.*, refs. [118, 119, 120, 121]. There the scalar field is classified according to M^2 regarded as its SO$(1, n+1)$ Casimir. A common nomenclature for the three possibilities delineated above is the (i) principal, (ii) complementary (or supplementary) and (iii) discrete series of representations of SO$(1, n+1)$. As is evident from eq. (3.18), the distinguishing feature of scalar fields in the principal series is that they are oscillatory near past (or future) infinity. In contrast, the exponents for fields in the complementary series are real and positive, and so their asymptotic behavior is a pure exponential damping near both boundaries.

Let us consider case (iii) $M^2 < 0$ in detail. While $h_\pm$ are both real, the modes $\phi_- \sim e^{h_- t}$ diverge as one approaches I^- since $h_- < 0$. One finds similar divergent

[2]Attempts have been made to distinguish these modes through energy considerations [110] but we disagree with their discussion, as described in Appendix G.

behavior for one of the modes at the future boundary I^+. The discrete series then corresponds to special values of the mass in this range where a subset of the modes display the convergent h_+ behavior at both $I^\pm$ — see Appendix F and refs. [118, 119, 120, 121]. However, we emphasize that even in these special cases, the full space of solutions still includes modes diverging at both asymptotic boundaries. In a physical situation then, the uncertainty principle would not allow us to simply set the amplitude of the divergent modes to zero. Hence the formal mathematical analysis of these fields is only of limited physical interest and we will not consider them further in the following. Of course, the divergence of the generic field configuration is simply an indication that treating the tachyonic fields as linearized perturbations is inappropriate. Nonlinear field theories with potentials including unstable critical points may play an important role in the dS/CFT correspondence, *e.g.*, in constructing models of inflationary cosmology. The essential point though is that one must study the full nonlinear evolution of such fields including their back-reaction on the spacetime geometry.

Considering the principal series, (i) $M^2 > n^2/4\ell^2$, in more detail, one expects to find a pair of dual operators $\mathcal{O}_\pm$ with *complex* conformal dimensions $h_\mp$. Having operators with a complex conformal weight suggests that the dual CFT is non-unitary [68]. We add the brief observation that, since in the quantum field theory, the two sets of independent modes $\phi_\pm$ correspond roughly to creation and annihilation operators (positive and negative frequency modes) in the bulk (see, *e.g.*, refs. [71, 122, 124, 125]), the corresponding operators in the dual theory should have non-trivial commutation relations.

Finally we consider the complementary series, (ii) $n^2/4\ell^2 \geq M^2 > 0$. In this case, the time dependence for both sets of modes is purely a real exponential decay near both $I^\pm$. In the bulk, the two linearly independent solutions may be chosen to be real, as is readily verified by explicit computations — see Appendix F. Because the $\phi_\pm$ solutions are real, they each have zero norm in the usual Klein-Gordon inner product, while a non-vanishing inner product arises from (ϕ_+, ϕ_-). It follows that upon quantization the corresponding operator coefficients are analogous to position and momentum operators, rather than creation and annihilation operators. That is, these degrees of freedom are canonically conjugate. In any event, both types of modes are again required to describe standard quantum field theory in the bulk.

As before, the dual CFT should contain a pair of operators $\mathcal{O}_\pm$ dual to the $h_\mp$ modes. In this case, the operators have real conformal weights and must be distinct as their weights are different. One can readily see that both $\mathcal{O}_\pm$ will have local correlation functions. One simply notes that the corresponding source currents are obtained from the bulk scalar field through

$$\begin{aligned} J_-(\Omega) &\equiv \lim_{t \to -\infty} e^{-h_- t/\ell} \phi(\Omega, t) \ , \\ J_+(\Omega) &\equiv \lim_{t \to -\infty} e^{-h_+ t/\ell} [\phi(\Omega, t) - e^{h_- t/\ell} J_-(\Omega)] \ , \end{aligned} \tag{3.20}$$

where Ω denotes a point on the n-sphere. As these constructions are local in position, their two-point functions will also be local. Note that the above discussion of inner products indicates that the operators $\mathcal{O}_\pm$ should have non-trivial commutation relations with each other but vanishing commutators amongst themselves.

Next we consider the generator of correlation functions in the dual field theory. A natural construction proposed in ref. [68] for a free bulk field theory is

$$\mathcal{F} = \lim_{t,t' \to -\infty} \int d\Sigma^i d\Sigma'^j \phi(x) \overleftrightarrow{\partial_i} G(x, x') \overleftrightarrow{\partial_j} \phi(x') \ . \tag{3.21}$$

In the original proposal of ref. [68], $G(x, x')$ was chosen as the Hadamard two-point function

$$G(x, x') = \langle 0 | \{\phi(x), \phi(x')\} | 0 \rangle \tag{3.22}$$

in the Euclidean vacuum, which is symmetric in its arguments. Generalizing this construction to other two-point functions was considered in refs. [71, 72]. These alternatives all provide essentially the same short-distance singularities discussed below.

One proceeds by evaluating the generating functional $\mathcal{F}$. First the boundary conditions (3.18) at I^- yield

$$\lim_{t\to-\infty} \phi(\Omega,t) \simeq \phi_{0+}(\Omega)\, e^{h_+ t} + \phi_{0-}(\Omega)\, e^{h_- t} , \tag{3.23}$$

where we imagine that $M^2 > 0$ so that the above shows no divergent behavior. Now the dS-invariant two-point function may also be expanded in the limit that $t, t' \to -\infty$ with the result being

$$G(x,x') \simeq c_+ \frac{e^{-h_+(t+t')}}{(w^a w'^a - 1)^{h_+}} + c_- \frac{e^{-h_-(t+t')}}{(w^a w'^a - 1)^{h_-}} , \tag{3.24}$$

where c_+ and c_- are constants and w^i denote direction cosines on S^n. Using the notation of ref. [72], one has

$$w^1 = \cos\theta_1,\ w^2 = \sin\theta_1 \cos\theta_2,\ \ldots,\ w^d = \sin\theta_1 \ldots \sin\theta_{n-1}\sin\theta_n . \tag{3.25}$$

Note in particular that with this choice of coordinates when the points on sphere coincide, one has $w^a w'^a = 1$, while for antipodal points, one has $w^a w'^a = -1$. Taking into account the measure factors, the final result for the generating functional reduces to

$$\mathcal{F} = -\frac{(h_+ - h_-)^2}{2^{2n}} \int d\Omega d\Omega' \left[c_+ \frac{\phi_{0-}(\Omega)\,\phi_{0-}(\Omega')}{(w^a w'^a - 1)^{h_+}} + c_- \frac{\phi_{0+}(\Omega)\,\phi_{0+}(\Omega')}{(w^a w'^a - 1)^{h_-}} \right] . \tag{3.26}$$

Note that the Klein-Gordon inner product has eliminated the cross-terms (which were potentially divergent). Further the coincidence singularities in eq. (3.26) are proportional to the Euclidean two-point function on a n-sphere, *i.e.*,

$$\Delta_{h_\pm} \simeq \frac{1}{(w^a w'^a - 1)^{h_\pm}} , \tag{3.27}$$

for operators with conformal weight $h_\pm$. Hence $\mathcal{F}$ appears to be a generating functional for CFT correlation functions with $\phi_{0\pm}$ acting as source currents for

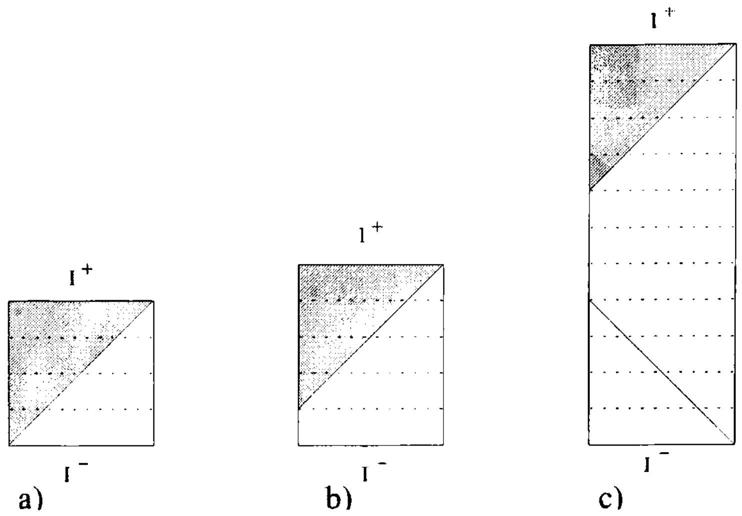

Figure 3.2: Conformal diagrams of a) de Sitter space, b) perturbed de Sitter space, and c) a very tall asymptotically de Sitter spacetime. The worldline of the 'central observer' is the right boundary of each diagram and various horizons related to her worldline are shown. Shaded regions cannot send signals to this observer.

operators with conformal dimensions $h_{\mp}$. The above relies on having a free field theory in the bulk dS space, but extending the construction to an interacting field theory was considered in ref. [72].

3.3 Renormalization group flows

The ideas presented in section 3.2.2 are naturally extended to include general solutions of Einstein gravity coupled to a positive cosmological constant with asymptotically de Sitter regions to the past and/or future. The time evolution of these solutions corresponds to a re-scaling of the boundary metric and so within the context of the dS/CFT duality, the evolution has a natural interpretation in terms of a renormalization group flow [59, 69].

An interesting property of de Sitter space is that it has compact Cauchy surfaces.[3] However, as illustrated in figure 3.2(a), the causal structure of pure de Sitter space is such that an observer can never see an entire compact Cauchy surface. Instead, the observer's light cone only expands to include the full Cauchy surface at I^-, asymptotically as the observer approaches I^+. This causal connection between I^+ and I^- plays an important role in understanding the role of both of these surfaces in the dS/CFT correspondence [68, 70, 71, 72]. However, a theorem (Corollary 1) of Gao and Wald [77] tells us that under a generic perturbation[4] of de Sitter space, the conformal diagram becomes taller so that an entire compact Cauchy surface now becomes visible at some finite time. This is shown in figure 3.2(b). Pushing this somewhat further, one can imagine that in certain circumstances asymptotically de Sitter spacetimes of the sort shown in figure 3.2(c) may arise. That is, in these spacetimes, a compact Cauchy surface lies in the intersection of the past *and* future of a generic worldline. In fact, we construct explicit examples of such spacetimes in section 3.3.4. These particular examples are of some interest with regard to general discussions of 'the number of degrees of freedom' in asymptotically de Sitter spaces [73, 79, 80], as the region open to experimental probing by an observer contains arbitrarily large spatial volumes.

The primary focus in this section is investigating renormalization group flows in the context of the dS/CFT duality. It is organized as follows: In section 3.1, we review the basics of de Sitter space, establish our conventions, and introduce the three types of foliations (spherical, flat, and hyperbolic) that will be most relevant in the following sections. Section 3.3.1 then generalizes to include a rolling scalar field, which may yield 'flows' which are asymptotically dS. In particular, we describe two solution generating techniques which may be used to construct explicit

[3]A Cauchy surface is a spacelike hypersurface which every non-spacelike curve intersects exactly once.

[4]Technically, any perturbation satisfying the null generic condition [78].

solutions. Section 3.3.2 is devoted to extending the 'c-theorem' for asymptotically de Sitter evolutions [59, 69]. We generalize the 'c-theorem' to include flows with spherical or hyperbolic spatial sections and also certain situations where the spatial geometries are anisotropic. We find that under these general circumstances, an 'effective' cosmological constant always decreases (increases) during an expanding (contracting) phase. Finally we consider the possibilities for transitions between phases of expansion and contraction. Because of our interest in global structures, we discuss the Penrose diagrams relevant to these flows in section 3.3.3. Section 3.3.4 describes the construction of a 'very tall' universe. Within the context of one model, we illustrate solutions with asymptotically de Sitter regions both to the future and past and that contain an arbitrarily long lived intermediate matter dominated phase in which the spatial volume is arbitrarily large. We conclude with a discussion of our results in section 3.4.5. Finally in appendices C, D and E we provide additional examples of asymptotically de Sitter renormalization group flows, and flesh out the construction of the conformal diagrams discussed in section 3.3.3.

3.3.1 Generating asymptotically de Sitter solutions

In this section, we set forth a framework in which to consider generalized dS flows or asymptotically dS solutions. In investigating new phenomena, it is useful to have a set of explicit solutions at one's disposal. Hence, we introduce two solution generating techniques below. We describe a method that allows for a variety of potentials but is useful mainly for the case of spatially flat sections ($k = 0$). This approach is a simple adaptation of the techniques developed in ref. [81] for the case of spatially varying solutions with a negative cosmological constant. This approach allows the equations of motion to be reduced to two first order ODE's. We then treat the special case of piece-wise constant potentials in a way that remains useful for any value of k. Throughout this section, our attention is restricted to spacetimes with homogeneous spatial sections.

In general we consider $(n+1)$-dimensional models of Einstein gravity coupled to a scalar field ϕ. We write the action as

$$S = \frac{1}{16\pi G_N} \int d^{n+1}x\sqrt{-g}\, \left[R - n(n-1)g^{ij}\partial_i\phi\partial_j\phi - n(n-1)V(\phi)\right] \,. \qquad (3.28)$$

Note that the scalar field terms have been normalized in an unconventional manner (including the fact that Newton's constant G_N appears in an overall factor in front of the total action) to simplify Einstein's equations in the following analysis. Einstein's equations may be written as

$$R_{ij} - \frac{1}{2}g_{ij}R = T_{ij}. \qquad (3.29)$$

where the stress-energy tensor is given (with a slightly unconventional normalization) as

$$T_{ij} = n(n-1)\left[\partial_i\phi\partial_j\phi - \frac{1}{2}g_{ij}\left((\partial\phi)^2 + V(\phi)\right)\right] \,. \qquad (3.30)$$

Note that if the scalar field sits at a critical point $\phi = \phi_0$ of the potential $V(\phi)$, the effective cosmological constant is given by $\Lambda = \frac{n(n-1)}{2}V(\phi_0)$.

Now we will consider the spatially homogeneous solutions of the form

$$\phi = \phi(t) \quad \text{and} \quad ds^2 = -dt^2 + a^2(t)d\Sigma^2_{k,n} \qquad (3.31)$$

with $d\Sigma^2_{k,n}$ defined in eq. (3.5). Given this ansatz, the scalar field equation reduces to

$$\ddot{\phi} + n\frac{\dot{a}}{a}\dot{\phi} = -\frac{1}{2}\frac{\partial V}{\partial\phi} \,, \qquad (3.32)$$

where a 'dot' denotes a derivative with respect to t. The dynamics of the scale factor $a(t)$ is governed by the Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \dot{\phi}^2 + V(\phi) \,. \qquad (3.33)$$

$$\frac{\ddot{a}}{a} = -(n-1)\dot{\phi}^2 + V(\phi) \ . \tag{3.34}$$

The second of these is redundant, and a complete solution may be determined from eqs. (3.32) and (3.33) alone.

PRE-POTENTIALS

One approach to producing explicit solutions is to adapt the technique of ref. [81] to the present case. This method considers potentials of a special form which allow us to simplify the equations of motion, (3.32) and (3.33), for both $a(t)$ and $\phi(t)$ to a system of two first order ODE's. It is easily verified that eqs. (3.32), (3.33) and (3.34) are satisfied when ϕ, $\dot{a}/a$, and $V(\phi)$ are related to a pre-potential $W(\phi)$ through

$$\dot{\phi} = \frac{1}{\gamma}\frac{\partial W(\phi)}{\partial \phi}, \qquad \frac{\dot{a}}{a} = -\gamma n W(\phi), \tag{3.35}$$

$$V(\phi) = -\frac{1}{\gamma^2}\left(\frac{\partial W(\phi)}{\partial \phi}\right)^2 + n^2 W^2(\phi), \tag{3.36}$$

where

$$\gamma = \left(1 - \frac{k}{n^2 W(\phi)^2 a^2}\right)^{1/2} . \tag{3.37}$$

For $k = \pm 1$, these expressions are of limited use as they are highly nonlinear and further the scale factor $a(t)$ appears in eq. (3.36) through the factor of γ^2. However, for $k = 0$, $\gamma = 1$ and the scalar potential is completely determined by the pre-potential $W(\phi)$. In this case, the equations (3.35) reduce to

$$\dot{\phi} = \frac{\partial W(\phi)}{\partial \phi}, \qquad \frac{\dot{a}}{a} = -nW(\phi) \ , \tag{3.38}$$

which can readily be solved analytically given a sufficiently simple $W(\phi)$. While this technique allows us to construct many interesting analytic flows, we will not consider any explicit examples here.

Step potentials

Another class of tractable models arises when $V(\phi)$ is piece-wise constant. On each constant piece, the system can be mapped to a particle in a one-dimensional potential so that the dynamics is conveniently summarized by the effective potential. One may then patch together the solutions at the boundaries of the steps. With some number of small steps, this approach may be useful to approximate a slowly varying potential. Alternatively, this technique may be used to simulate the effect of phase transitions in the matter sector. We will consider an interesting example based on these models in section 3.3.4.

Let us first consider the dynamics within one step with constant $V(\phi) = V_0$, which is assumed to be positive. Recall that in this phase of the evolution, the cosmological constant is given by $\Lambda = n(n-1)V_0/2$. From (3.32), the scalar field equation reduces to

$$0 = \ddot{\phi} + n\frac{\dot{a}}{a}\dot{\phi} = a^{-n}\frac{\partial}{\partial t}(a^n\dot{\phi}). \tag{3.39}$$

Thus, we introduce an integration constant $C_0 = a^n\dot{\phi}$. The familiar Friedmann constraint (3.33) may then be rewritten as

$$-k = \dot{a}^2 + w_{eff}(a) \qquad \text{where} \quad w_{eff}(a) = -\frac{C_0^2}{a^{2(n-1)}} - V_0 a^2\ . \tag{3.40}$$

Hence we may view the dynamics of the scale factor a as that of a classical particle moving in a potential w_{eff} with energy $-k$.

Note that if $C_0 \neq 0$, the effective potential is manifestly negative. However, for the flat and hyperbolic cases ($k = 0, -1$), the effective energy is non-negative. Hence for all such solutions, the scale factor reaches zero in either the past or future. Further with $C_0 \neq 0$, one finds a curvature singularity at this point: $a \sim t^{1/n}$ and $R \simeq -\frac{n}{n+1}t^{-2}$. Hence these solutions correspond either to a universe which begins in a contracting dS phase and for which there is enough energy density in the evolving scalar field to produce a big crunch singularity, or similarly to a universe

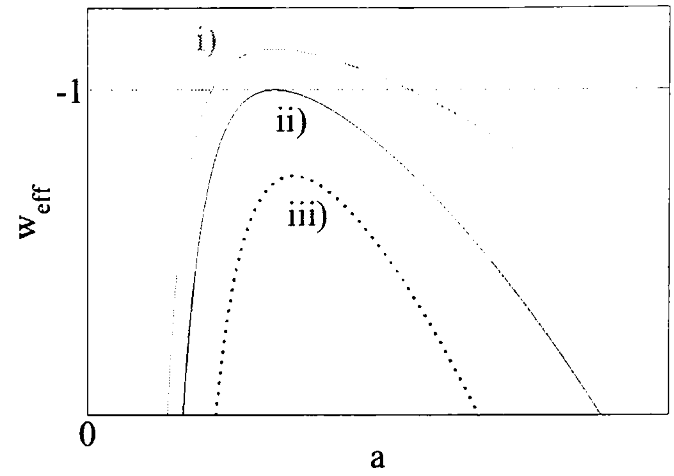

Figure 3.3: The effective potential w_{eff} with $n = 3, k = +1$ for three different cases: i) small $C_0^2V_0^2$, ii) $C_0^2V_0^2 = \frac{4}{27}$, and iii) large $C_0^2V_0^2$.

which emerges from a big bang to evolve into an expanding dS phase. Of course, with $C_0 = 0$, the scale factor a reaches zero in a non-singular way which simply corresponds to a horizon in dS space, as discussed above.

However, the spherical case ($k= +1$) is more interesting since the effective energy is negative. For $C_0^2 > V_0^{-(n-1)}\frac{1}{n-1}\left(\frac{n-1}{n}\right)^n$, one has $w_{eff} < -1$ for all values of a. Hence the solutions in this range have a similar interpretation as above with a big crunch or big bang singularity. With $C_0^2 = V_0^{-(n-1)}\frac{1}{n-1}\left(\frac{n-1}{n}\right)^n$, the peak of the effective potential is precisely -1, and so there are various classes of solutions (see figure 3.3): i) the scale factor is constant with $a^{2n} = V_0^{-n}\left(\frac{n-1}{n}\right)^n$ and the geometry corresponds to an Einstein static universe, ii) the universe emerges from a big bang and asymptotically evolves toward the previous static geometry, iii) the universe begins with a contracting dS phase and asymptotically approaches the static phase above, and iv) the time reversal of the solutions in either (ii) or (iii). For smaller values of C_0, the universe is confined either to small a (leading to solutions having both a big bang and a big crunch) or to large a (with dS phases both to the past and future). In all cases, ϕ rolls monotonically in one direction.

A piece-wise constant potential can now be dealt with by patching together solutions described by the above effective potentials. When ϕ crosses a jump in the potential V, the equations of motion show that a, $\dot{a}$, and ϕ are continuous but that $\dot{\phi}$ and $\ddot{a}$ suffer a discontinuity. The Friedmann constraint (3.33) then shows that this discontinuity is determined by the requirement that the scalar field energy density $\dot{\phi}^2 + V$ is conserved through the transition. That is, the two solutions are pasted together such that the effective potential is continuous across the join. As a result, the integration constants C_1, C_2 associated with potentials V_1, V_2, respectively satisfy the constraint:

$$C_2^2 + V_2 a^{2n} = C_1^2 + V_1 a^{2n} \ . \tag{3.41}$$

This analysis will be used in the construction of a 'very tall' universe in section 3.3.4.

3.3.2 A generalized de Sitter c-theorem

Having illustrated a framework in which we may study asymptotically dS spaces which are more general than the original dS spacetime, we would like to consider the role of such solutions in the context of the dS/CFT correspondence [68]. Much of the development of this duality relies on intuition developed in studying the AdS/CFT correspondence [6]. One of the interesting features of the latter is the UV/IR correspondence [82, 83]. That is, physics at large (small) radii in the AdS space is dual to local, ultraviolet (non-local, infrared) physics in the dual CFT. As was extensively studied in gauged supergravity — see, *e.g.*, ref. [84] — 'domain wall' solutions which evolve from one phase near the AdS boundary to another in the interior can be interpreted as renormalization group flows of the CFT when perturbed by certain operators. In analogy to Zamolodchikov's results for two-dimensional CFT's [85], it was found that a c-theorem could be established for such flows [86] — see also ref. [87] — using Einstein's equations. The c-function defined in terms of the gravity theory then seems to give a local geometric measure of the number of degrees of freedom relevant for physics at different energy scales in the dual field theory.

In the dS/CFT duality, there is again a natural correspondence between UV (IR) physics in the CFT and phenomena occurring near the boundary (deep in the interior) of dS space. In the context then of more general solutions which are asymptotically dS, one has an interpretation in terms of renormalization group flows, which should naturally be subject to a c-theorem [59, 69]. The original investigations [59, 69] considered only solutions with flat spatial sections ($k = 0$), and we generalize these results in the following to include spherical and hyperbolic sections ($k = \pm 1$). We also consider the flows involving anisotropic scalings of the boundary geometry, but our results are less conclusive in this case.

The c-function

The foliations of spacetimes of the form given in eq. (3.31) are privileged in that time translations a) act as a scaling on the spatial metric, and thus in the field theory dual and b) preserve the foliation and merely move one slice to another. In the context of the dS/CFT correspondence, these properties naturally lead to the idea that time evolution in these spaces should be interpreted as a renormalization group flow [59, 69]. Certainly, the same properties apply for time evolution independent of the curvature of the spatial sections, and in fact also apply (in the asymptotically dS regions) for any of the metrics presented in Appendix C. Hence if a c-theorem applies for the $k = 0$ solutions [59, 69], one might expect that it should extend to these other cases if properly generalized.

For $k = 0$, the proposed c-function [59, 69], when generalized to $n+1$ dimensions, is

$$c \simeq \frac{1}{G_N \left|\frac{\dot{a}}{a}\right|^{n-1}} . \tag{3.42}$$

The Einstein equations ensure that $\partial_t (\dot{a}/a) < 0$, provided that any matter in the spacetime satisfies the null energy condition [66]. This result then guarantees that c will always decrease in a contracting phase of the evolution or increase in an expanding phase.

For our general study, we wish to define a c-function which can be evaluated on each slice of some foliation of the spacetime. Of course, our function should satisfy a 'c-theorem', *e.g.*, our function should monotonically decrease as the surfaces contract in the spacetime evolution. Further, it should be a geometric function built from the intrinsic and extrinsic curvatures of a slice. Toward this end, we begin with the idea that the c-function is known for any slice of de Sitter space, and note that in this case, eq. (3.42) takes the form

$$c \sim \frac{1}{G_N \Lambda^{(n-1)/2}} . \tag{3.43}$$

Thus, if our slice can be embedded in some de Sitter space (as was shown to be the case for any isotropic homogeneous slice in section 3.1), the value of the c-function should be given by eq. (3.43). In other words, we can associate an effective cosmological constant Λ_{eff} to any slice that can be embedded in de Sitter space and we can then use this Λ_{eff} to define our c-function.

It is useful to think a bit about this embedding in order to express Λ_{eff} directly in terms of the intrinsic and extrinsic curvatures of our slice. The answer is readily apparent from the general form of the 'vacuum' Einstein equations with a positive cosmological constant: $G_{ij} = -\Lambda g_{ij}$. Contracting these equations twice along the unit normal n^i to the hypersurface gives the Hamiltonian constraint, which is indeed a function only of the intrinsic and extrinsic curvature of the slice.[5] The effective cosmological constant defined by such a local matching to de Sitter space is therefore given by

$$\Lambda_{eff} = G_{ij} n^i n^j . \tag{3.44}$$

For metrics of the general form (3.4), this becomes

$$\Lambda_{eff} = \frac{n(n-1)}{2} \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right] . \tag{3.45}$$

Taking the c-function to be a function of this effective cosmological constant, dimensional analysis then fixes it to be

$$c \sim \frac{1}{G_N \Lambda_{eff}^{(n-1)/2}} = \frac{1}{G_N} \left(G_{ij} n^i n^j \right)^{-(n-1)/2} . \tag{3.46}$$

For the $k = 0$ isotropic case, it is clear that this reduces to the c-function (3.42) given previously in refs. [59, 69]. For other isotropic cases, it is uniquely determined by the answer for the corresponding slices of de Sitter space. The same holds for an anisotropic slice that can be embedded in de Sitter (see, *e.g.*, Appendix C for examples). While the choice (3.46) is not uniquely determined by the constraints imposed thus far for any slice which cannot be so embedded, it does represent a natural generalization and, as we will see below, this definition allows a reasonable 'c-theorem' to be proven.

THE C-THEOREM

For any of the homogeneous flows as considered in the previous section, it is straightforward to show that our c-function (3.46) always decreases (increases) in a contracting (expanding) phase of the evolution. However, we would like to give

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[5]The momentum constraints vanish in a homogeneous universe, and time derivatives of the extrinsic curvature only appear in the dynamical equations of motion.

a more general discussion which in particular allows us to consider anisotropic geometries, as well as these isotropic cases.

To prove our theorem, we note that the Einstein equations relate our effective cosmological constant to the energy density ρ on the hypersurface,

$$\Lambda_{eff} = G_{ij}\, n^i n^j = T_{ij}\, n^i n^j = \rho. \tag{3.47}$$

Consider now the 'matter energy' $E = \rho V$ contained in the volume V of a small co-moving rectangular region on the homogeneous slice. That is, we take

$$V = \int_R \sqrt{g}\, d^n x \tag{3.48}$$

for some small co-moving region R of the form $R = \{x | x_a^i < x^i < x_b^i\}$ where x^i denote co-moving spatial coordinates. We also introduce $\delta x^i = x_b^i - x_a^i$, the co-moving size of R in the ith direction. Since R is small, each coordinate x^i can be associated with a scale factor $a^i(t)$ such that the corresponding physical linear size of R is $a^i(t)\delta x^i$.

Without loss of generality, let us assume that the coordinates x^i are aligned with the principle pressures P_i, which are the eigenvalues of the stress-tensor on the hypersurface. Let us also introduce the corresponding area A_i of each face. Note that a net flow of energy into R from the neighboring region is forbidden by homogeneity. As a result, energy conservation implies that $dE = -P_i A_i d(a\delta x^i)$ as the slice evolves. However, clearly $dE = \rho dV + V d\rho$, so that we have

$$d\Lambda_{eff} = d\rho = -\sum_i (\rho + P_i)\, d\ln a_i. \tag{3.49}$$

Now we will assume that any matter fields satisfy the weak energy condition so that $\rho + P_i \geq 0$. Thus, if all of the scale factors are increasing, we find that the effective cosmological constant can only decrease in time.

This result provides a direct generalization of the results of refs. [59, 69] to slicings that are not spatially flat. In particular, in the isotropic case (where all scale factors are equal, $a \equiv a_i = a_j$), it follows that $c(a)$ as given in eq. (3.45) is, as desired, a monotonically increasing function in any expanding phase of the universe.

Note, however, that the anisotropic case is not so simple to interpret. For example, it maybe that the scale factors are expanding in some directions and contracting in others. In this case our effective cosmological constant may either increase or decrease, depending on the details of the solution.

Complete Flows versus Bouncing Universes

The general flows are further complicated by the fact that they may 'bounce', *i.e.*, the evolution of the scale factor(s) may reverse itself. The simplest example of this would be the $k = +1$ foliation of dS space in section 3.1. In this global coordinate system, the scale factor (3.6) begins contracting from $a(t = -\infty) = \infty$ to $a(t = 0) = 1$, but then expands again toward the asymptotic region at $t = +\infty$. In contrast, we refer to the k=0 and –1 foliations as 'complete'. By this we mean that within a given coordinate patch, the flow proceeds monotonically from $a = \infty$ in the asymptotic region to $a = 0$ at the boundary of the patch — the latter may be either simply a horizon (as in the case of pure dS space) or a true curvature singularity.

For any homogeneous flows, such as those considered in section 3.3.1, it is not hard to show that the k=0 and –1 flows are always complete and that only the $k = +1$ flows can bounce. The essential observation is that for $a(t)$ to bounce the Hubble parameter $\dot{a}/a$ must pass through zero. Now the (tt)-component of the Einstein equations (3.29) yields

$$\left(\frac{\dot{a}}{a}\right)^2 = T_{tt} - \frac{k}{a^2} \,. \tag{3.50}$$

Now as long as the weak energy condition applies,[6] it is clear that the right-hand-side is always positive for $k = 0$ and -1 and so $\dot{a}/a$ will never reach zero. On the other hand, no such statement can be made for $k = +1$ and so it is only in this case that bounces are possible. Further one might observe that this analysis does not limit the number of bounces which such a solution might undergo. In certain cases with a simple matter content, *e.g.*, dust or radiation, one may show that only a single bounce is possible. However in (slightly) more complex models, multiple bounces are possible. We will illustrate this behavior in section 3.3.4, where a solution with a rolling scalar is constructed with multiple bounces — see also Appendix D. Finally in the case of anisotropic solutions — see, *e.g.*, Appendix C — the characterization of the flows as complete or otherwise is more complicated.

The renormalization group interpretation of bouncing universes is certainly less straightforward. Perhaps greater insight into this question can come from further study of renormalization group flows and the UV/IR correspondence in the AdS/CFT context for foliations of AdS space where the sections have negative curvature. Such AdS solutions show a similar bounce behavior — see, *e.g.*, ref. [127].

3.3.3 The global perspective

As pointed out in section 3, asymptotically de Sitter conformal diagrams are 'tall', *i.e.*, an entire compact Cauchy surface will be visible to observers at some finite time, and hence that perturbations of dS space may bring features that originally lay behind a horizon into an experimentally accessible region. Specifically, these results rely on Corollary 1 of ref. [77], which we paraphrase as follows:

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[6] Note that if $k = 0$ and the energy density is identically zero, it follows that a is a constant. Hence in this case, we will not have an asymptotically dS geometry.

Let the spacetime (M, g_{ij}) be null geodesically complete and satisfy the weak null energy condition and the null generic condition. Suppose in addition that (M, g_{ij}) is globally hyperbolic with a compact Cauchy surface Σ. Then there exist Cauchy surfaces Σ_1 and Σ_2 (of the same compact topology, and with Σ_2 in the future of Σ_1) such that if a point q lies in the future of Σ_2, then the entire Cauchy surface Σ_1 lies in the causal past of q.

This is sufficient to guarantee that the conformal diagram is 'tall' in the sense of figure 3.2 (b). While it need not necessarily be 'very tall' in the sense of figure 3.2 (c), the possibility is open that this may hold in certain cases so that a compact Cauchy surface may actually be found to lie within the intersection of the past *and* future of a generic worldline. An example of such a spacetime will be constructed in section 3.3.4 below.

The diagrams in figure 3.2 were not intended to represent generic asymptotically de Sitter spacetimes. Instead, these diagrams only illustrate what is meant by 'tall' and 'very tall' spacetimes. The purpose of the current section is to construct the general such conformal diagrams corresponding to our homogeneous flows. This may be of particular interest if in the end there is a meaningful dS/CFT correspondence in which (either distinct or isomorphic) field theories are associated with both I^+ and I^- (see section 3.4 for details). We note that two field theories are indeed of relevance to certain applications [130, 90] of the more developed AdS/CFT correspondence.

Although we have shown that the evolution of $c(a)$ is monotonic, certain unusual features of our flow become apparent when we study the global structure of the spacetimes dual to our field theory. Let us assume that the slices are isotropic and take each of the three possible cases (spheres, flat slices, and hyperbolic slices) in turn.

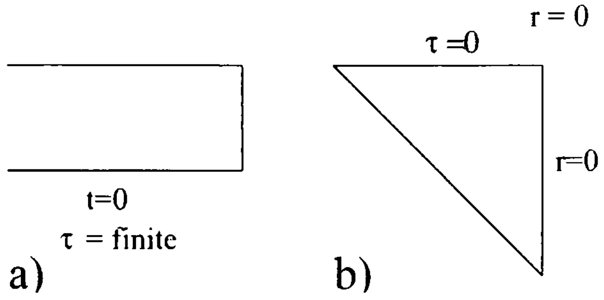

Figure 3.4: In the distant past τ is a) finite b) infinite.

FLAT SLICES ($k = 0$)

We now wish to construct the conformal diagram for flows with flat spatial sections. In order to draw useful two-dimensional diagrams, we shall use the common trick of studying rotationally symmetric spacetimes and drawing conformal diagrams associated with the 'r-t plane,' *i.e.*, associated with a hypersurface orthogonal to the spheres of symmetry.

For later use, we begin with an arbitrary $n+1$ dimensional spatially homogeneous and spherically symmetric metric in proper time gauge:

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + \hat{R}^2(r)d\Omega_{n-1}^2\right), \qquad (3.51)$$

where $d\Omega_{n-1}^2$ is the metric on the unit $n-1$ sphere and the form of $\hat{R}(r)$ depends on the spatial geometry: $\hat{R}(r) = \sin(r), r, \sinh(r)$ for spherical, flat, and hyperbolic geometries respectively. In fact, the function $\hat{R}(r)$ will not play a role below as our diagrams will depict the conformal structure only of the $(1+1)$-dimensional metric $ds_{1+1}^2 = -dt^2 + a^2(t)dr^2$. However, it will be important to note that r takes values only in $[0, \pi]$ for the spherical geometry but takes values in $[0, \infty]$ for the flat and hyperbolic cases. The usual change of coordinates to conformal time $\tau(t)$ defined by $d\tau = \frac{dt}{a}$ leads to the conformally Minkowski metric

$$ds_{1+1}^2 = a^2(t)(-d\tau^2 + dr^2). \qquad (3.52)$$

Let us assume that our foliation represents an expanding phase that is asymptotically de Sitter in the far future. That is, for $t \to +\infty$ the scale factor a diverges exponentially. There are now two possibilities. Suppose first that $a = 0$ at some finite t. If a^{-1} diverges as a small enough power of t then τ will only reach a finite value in the past and the spacetime is conformal to a half-strip in Minkowski space.

In contrast, if a vanishes more quickly or if it vanishes only asymptotically then τ can be chosen to take values in $[-\infty, 0]$. From (3.52) we see that the region covered by our foliation is then conformal to a quadrant of Minkowski space. We take this quadrant to be the lower left one so that we may draw the conformal diagram as in figure 3.4 (b).

We now wish to ask whether the region shown in figure 3.4 (b) is 'complete' in some physical sense. In particular, we may wish to know whether light rays can reach the null 'boundary' in finite affine parameter. A short calculation shows that the affine parameter λ of a radial null ray is related to the original time coordinate t by $d\lambda = adt$. The affine parameter is clearly finite if a vanishes at finite t. In the remaining case, we have seen that ρ is bounded below. As a result, a must vanish at least exponentially and the affine parameter is again finite.

Thus, this null surface represents either a singularity or a Cauchy horizon across which our spacetime should be continued. This statement is essentially a restricted version of the results of refs. [91, 92, 93, 94] (see also ref. [95] for other interesting constraints on the 'beginning' of inflation). Note that there is no tension between our possible Cauchy horizon and the claim of a "singularity" in these references, as their use of the term singularity refers only to the geodesic incompleteness of the expanding phase.

From (3.49) we see that unless $\rho + P_i$ vanishes as $a \to 0$, the energy density must diverge and a curvature singularity will indeed result. However, a proper tuning

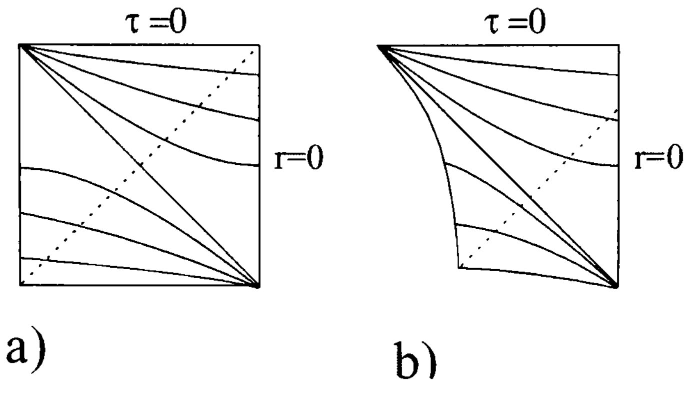

Figure 3.5: a) In a square diagram, a light ray reaches the antipodal point only at I^+. b) The generic conformal diagram for an asymptotically de Sitter space with flat surfaces of homogeneity. A light ray starting in the lower left corner reaches the antipodal point at a finite time.

of the matter fields can achieve a finite ρ at $a = 0$. It is therefore interesting to consider solutions which are asymptotically de Sitter near $t = 0$ so that a vanishes exponentially. In this case, the $a = 0$ surface represents a Cauchy horizon across which we should continue our spacetime. We will focus exclusively on such cases below.

Since the boundary is a Cauchy horizon, there is clearly some arbitrariness in the choice of extension. We make the natural assumption here that the spacetime beyond the horizon is again foliated by flat hypersurfaces. Although at least one null hypersurface ($t = -\infty$) will be required, it can be shown that the surfaces of homogeneity must again become spacelike across the horizon if the spacetime is smooth. The key point here is that the signature can be deduced from the behavior of $a^2(t)$, which gives the norm $|\xi|^2$ of any Killing vector field ξ associated with the homogeneity. We impose a "past asymptotic de Sitter boundary condition" so that the behavior of this quantity near the Cauchy horizon must match that of some de Sitter spacetime. Consider in particular the behavior along some null geodesic crossing the Cauchy horizon and having affine parameter λ. It is straightforward

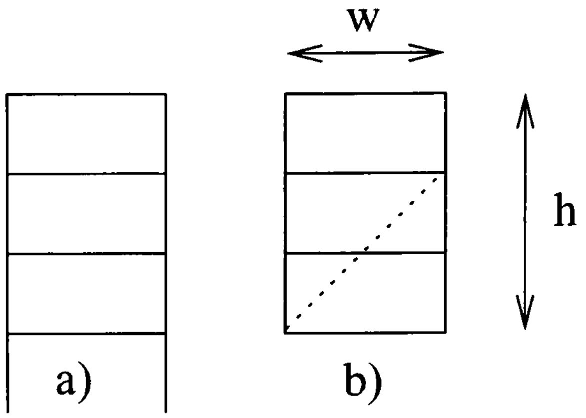

Figure 3.6: Conformal diagrams for spherical surfaces of homogeneity a) for the case where τ diverges in the past and b) for the case where τ converges in the past.

to verify that $\lambda \sim a$, so that matching derivatives of $|\xi|^2$ across the horizon requires ξ to again become spacelike beyond the horizon.

It follows that the region beyond the Cauchy surface is just another region of flat spatial slices, but this time in the contracting phase. It is therefore conformal to the upper right quadrant of Minkowski space. However, having drawn the above diagram for our first region we have already used a certain amount of the available conformal freedom. Thus, it may not be the case that the region beyond the Cauchy surface can be drawn as an isosceles right triangle. The special case where this is possible is shown in figure 3.5 (a). The exceptional nature of this case can be seen from the fact that it allows a spherical congruence of null geodesics to proceed from the upper right corner of I^+ (where it would have zero expansion) to the lower right corner of I^- (where it would also have zero expansion). Assuming as usual the weak energy condition, it follows that this congruence encountered no focusing anywhere along its path; *i.e.*, $\rho + P = 0$. Given the high degree of symmetry that we have already assumed, this can happen only in pure de Sitter space. The correct diagram for the general case is shown in figure 3.5 (b) (see Appendix E for a complete derivation).

SPHERICAL SLICES ($k = +1$)

The conformal diagrams in this case are relatively simple. Since the radial coordinate now takes values only in an interval, we see from (3.52) that the conformal diagram is either a rectangle or a half-vertical strip, depending on whether or not τ is finite at the past boundary.

All such rectangles with the same ratio h/w (see figure 3.6 (b)) can be mapped into each other via conformal transformations. For the case of pure de Sitter space we have h=w. On the other hand, for any spacetime satisfying the generic condition (so that null geodesics suffer some convergence along their trajectory), we know from ref. [77] that the region to the past of any point p sufficiently close to I^+ must contain an entire Cauchy surface. Thus [7], for such cases we have h > w.

HYPERBOLIC SLICES ($k = -1$)

Recall that the hyperbolic flows are complete, *i.e.*, a reaches 0 at finite t (say, $t = 0$), and vanish at least as fast as t. The asymptotically de Sitter boundary conditions also require that a diverge exponentially as $t \to +\infty$. Note that since a vanishes quickly, τ will diverge at $t = 0$ and the region is again conformal to a quadrant of Minkowski space. As usual, this may or may not be singular depending on the matter present.

Consider in particular the asymptotically de Sitter case where a vanishes linearly. One then finds that the affine parameter λ along a null ray near the horizon is asymptotically $\lambda \sim a^2$. The Killing vector field that implements spatial translations in the direction along this null ray has a norm given by $a^2 \sim \lambda$ along this null

[7]This conclusion may also be reached by considering the sphere of null geodesics that begins in, say, the lower left corner and progresses toward the upper right and using the non-increase of the expansion θ implied by the weak null energy condition.

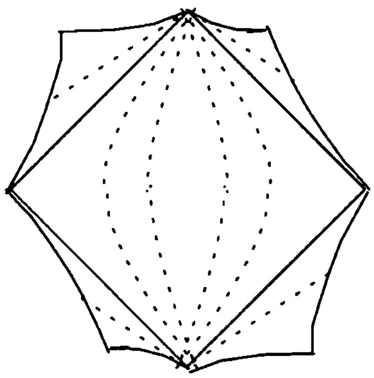

Figure 3.7: The general conformal diagram for an appropriately complete asymptotically de Sitter spacetime with hyperbolic surfaces of homogeneity. A conformal frame has been chosen such that the diagram has a Z_2 reflection symmetry through the center.

ray and so, if the spacetime is smooth, must become timelike beyond the horizon. Thus, the homogeneous surfaces must become timelike on the other side. As can be seen from (3.7), an asymptotically de Sitter region foliated by timelike hyperbolic slices (*i.e.*, copies of de Sitter space) has an 'r-t plane' that is conformal to a diamond in Minkowski space.

Assuming that no singularities are encountered within this diamond or on its boundaries, this provides three further Cauchy horizons across which we would like to extend our spacetime. A study of the norm of the Killing fields tells us that the foliation must again become spacelike beyond these horizons. Just as we saw for the flat foliations, we are therefore left with the task of attaching pieces conformal to various quadrants of Minkowski space. By the same reasoning as in Appendix E, the complete conformal diagram can be drawn as shown in figure 3.7.

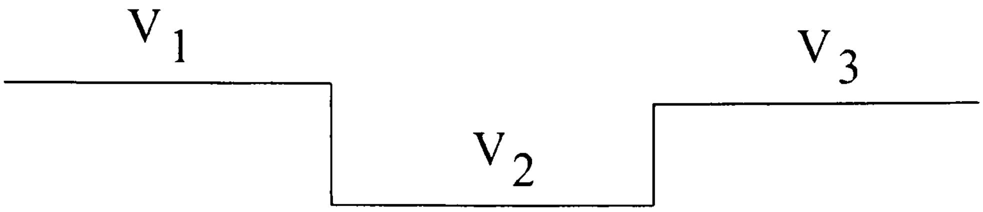

Figure 3.8: Our piece-wise constant potential.

3.3.4 'VERY TALL' UNIVERSES

This section is devoted to constructing an asymptotically de Sitter spacetime whose conformal diagram is 'tall' enough that a compact Cauchy surface can be experimentally probed by a co-moving observer. In particular, such a Cauchy surface will lie in the intersection of the observer's past and future. We refer to such spacetimes as 'very tall.' The solutions we construct below will all satisfy the dominant energy condition. Interestingly, we will discover spacetimes of this form for which the spatial volume of the visible Cauchy surface can be made arbitrarily large.

Our solution will again be built by considering a scalar field ϕ interacting with Einstein-Hilbert gravity as described by the action (3.28). We will be using the spatially homogeneous ansatz (3.31) with spherical sections ($k = +1$). The scalar potential is taken to be non-negative and piece-wise constant, and so the dynamics of this system are of the form discussed in section 3.3.1. In particular, we consider a potential of the form shown in figure 3.8, which consists of three steps of heights $V_{1,2,3}$. We will be interested in the case where V_1 and V_3 are comparable, though not necessarily equal, and $V_2 < V_{1,3}$. There will be three corresponding constants of integration $C_{1,2,3}$ for eq. (3.39) that will be related by the matching condition (3.41). Given these constants, the evolution of the scale factor on each of the three steps corresponds to that of a classical particle with energy -1 in the corresponding effective potential, $w_{1,2,3}$, as given in eq. (3.40).

Suppose that we choose $C_1^2 < V_1^{-(n-1)}\frac{1}{n-1}\left(\frac{(n-1)}{n}\right)^n$ so that the effective potential satisfies $w_1 > -1$ for some values of the scale factor. If we begin the universe in an asymptotically de Sitter contracting phase, the universe will reach some locally minimum size a_1 and then bounce off of the potential. That is, a is confined to values *larger* than those where $w_1 \leq -1$. Suppose also that we arrange the initial conditions so that ϕ rolls from V_1 down to V_2 when the universe is at some size $a_{12} > a_1$. We will imagine keeping V_1, C_1, and a_{12} fixed and then tuning V_2 to attain the desired behavior in the following.

Now the matching condition (3.41) yields $C_2^2 = C_1^2 + (V_1 - V_2)a_{12}^{2n} > C_1^2$. It is clear that by taking V_2 sufficiently small we will have $C_2^2 < V_2^{-(n-1)} \frac{1}{n-1} \left(\frac{n-1}{n}\right)^n$ so that the effective potential rises above -1. Furthermore, a study of (3.40) shows that w_2 rises above the value -1 only at some $a_2 > a_{12}$. Hence on the second step, the solution will continue expanding for a while, but eventually it bounces off the new effective potential to enter a contracting phase. Physically, the scalar field has attained so much kinetic energy by rolling from V_1 down to V_2 that this kinetic energy now dominates the dynamics and the spacetime will expand to a locally maximum size a_2 and then begin to re-collapse again, much as in a standard spherical matter-dominated FRW cosmology.

To reach a final expanding asymptotically de Sitter phase, we need only run the construction in reverse. In particular, we need only arrange the scalar field to roll up to V_3 when the scale factor takes some value a_{23} close to a_{12} and to take V_3 close to V_1. In this case the cosmological constant again takes over, creating another bounce at some locally minimum size a_3. After this point, the spacetime expands forever and asymptotically approaches a de Sitter solution.

It is also clear that one could in principle alternate de Sitter and FRW-like phases indefinitely. Note that by taking a_{23} *smaller* than a_{12} the same behavior can be obtained by choosing a V_3 that is larger than V_1. In this way, one can construct a spacetime that is asymptotically de Sitter to the past and future with arbitrary independent cosmological constants $\Lambda_{initial}$ and Λ_{final} and which nevertheless oscillates arbitrarily between large and small sizes.

While some amount of tuning is required in our construction, it is clear that the scenario is stable in the sense that there is an open set of parameter space that leads to the desired behavior. For the reader who may find our analysis with a piece-wise scalar potential lacking, we also present some numerical calculations illustrating analogous bouncing universes for a smooth potential in Appendix D. It should also be clear that we may choose the size a_2 and the duration of the internal FRW region as large as we wish. Hence these 'very tall' universes may also be 'big wide' universes.

ENTROPY IN 'BIG WIDE' UNIVERSES

The 'big wide' universes constructed above are particularly interesting in the light of recent discussions in which the fact that the cosmological horizon prohibits an observer in de Sitter space from accessing more than a fixed, finite spatial volume was used to motivate the idea that asymptotically de Sitter spaces contain only a finite number of degrees of freedom [73]. Given such arguments, the construction of big wide asymptotically dS universes satisfying the weak and dominant energy conditions may come as a surprise. Certainly, any standard field theory given access to a fixed energy and an arbitrarily large volume will exhibit an arbitrarily large number of degrees of freedom.

Scenarios of this kind may certainly be envisioned in a big wide universe and they are instructive to explore. Suppose for example that we add a single massive particle of small but fixed mass to such a spacetime, where the mass m is chosen small enough that the particle has negligible effect on the gravitational dynamics through the first de Sitter bounce (at a_{12}) and into the middle expanding phase. We may imagine that this particle is coupled to some massless field (say, another scalar field), into which it may decay when the universe is very large. This in principle supplies the scalar field with a fixed amount m of energy in a large volume. In some sense then this system does 'have' an arbitrarily large number of available states.

Let us investigate, however, what happens when those states are actually accessed. The particle decays and some small amount of massless radiation is excited in a universe much larger than the length scale l_{final} set by the final cosmological constant. Our big wide universe will eventually reach the maximum size (a_2) of its FRW-like expansion and begin to re-collapse. While the expansion or contraction of the spacetime had little effect on the massive progenitor particle, the contraction will significantly blueshift massless radiation in proportion to the inverse scale factor ($1/a$). For example, one might approximate this massless radiation by a thermal state, in which case the temperature will increase as the spacetime contracts. As a result, if this entropy survives to reach the second de Sitter bounce at a_{12} the matter energy density is now much greater than it was during the first bounce (roughly by a factor of a_2/a_{12}). It may therefore be large enough to dominate the gravitational dynamics and drive the universe into a big crunch instead of a second re-expansion.

That a big crunch would indeed be the outcome can be said with some confidence. Let us recall that Bousso's 'covariant entropy bound' [96] can be proven to hold [97] in the context of classical general relativity coupled to matter satisfying the restrictions

$$T_{ij}k^ik^j \geq c_1\ell_p^{-1}|k_is^i|^2, \qquad T_{ij}k^ik^j \geq c_2k^ik^j\nabla_is_j, \tag{3.53}$$

where k^i is an arbitrary null vector and T_{ij} and s^i are the matter stress-energy tensor and entropy current,[8] respectively. The constants c_1 and c_2 are coefficients

[8]While the fundamental status of such a notion may be unclear, we will see that it is sufficiently

of order 1 and we have explicitly indicated factors of the Planck length ℓ_p. The theorem is then that the integral $\oint k_i s^i \, dS$ over any non-expanding null surface S is bounded by $A/4\ell_p^{n-1}$, where A is the area of its initial cross-section. It was argued in ref. [97] that subject to a restriction on the number of species, eq. (3.53) holds for any matter below the Planck temperature whose entropy can be successfully modelled by a classical entropy current of the sort used in fluid dynamics. As noted above, it suffices to consider the evolution of the universe that would result if our scalar field is placed in a thermal state at the moment the massive particle decays.

Suppose that the entropy of this thermal state is more than[9] $2A/4\ell_p^{n-1}$, where A is the area of the de Sitter horizon associated with the final cosmological constant Λ_{final} in the original big wide universe. For the discussion below we will assume that all temperatures remain below the Planck scale so that we can make predictions with some confidence.

The re-collapse of the universe acts on the massless field just as if we studied a field inside a box that is being compressed. The compression is adiabatic as the gravitational field does not exchange heat with the thermally excited scalar field. As a result, the entropy of each co-moving volume element will remain constant.

Supposing that the universe has an asymptotically de Sitter final phase with the same final effective cosmological constant Λ_{final} as in our original (tall!) big wide universe will now lead to a contradiction. Consider the final bounce and in particular the spherical light front emitted from the equator of the sphere at this

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well-defined in the current context.

[9]The unusual factor of two is not important here. It may be of interest for other purposes, but most likely it is a product of our inefficient estimate below of the back reaction of the thermally excited field on the metric. In any case, adjusting the constants c_1, c_2 in (3.53), and thus restricting the temperature to be a factor of order one below the Planck scale, can easily be made to insert factors of 2 into the bound on the entropy flux.

time. There are two such light fronts, one emitted toward either pole. Let us arbitrarily choose the one emitted toward the north pole.

Such a null congruence necessarily begins with non-positive expansion since $\dot{a}$ vanishes on the slice and, within the given slice, there is no larger surface toward which to expand. The weak energy condition then guarantees that the congruence must continue to contract until it reaches the north pole. Thus, this null surface satisfies the conditions for the application of the covariant entropy bound [96].

Note that $\dot{a}$ vanishes at the final bounce and that the matter energy density is greater than in the case where the massive particle did not decay. As a result, the Friedmann constraint (3.33) guarantees that the minimal sphere is *smaller* and the initial area of our light sheet is less than $A/4\ell_p^{n-1}$. However, we see that half of the matter entropy must flow through this null surface, contradicting the theorem of ref. [97]. Note that it does not help to somehow arrange to heat only 'the other half of the universe' as we may simply choose the light front that contracts toward the south pole.

We see that an attempt to access an entropy larger than that commonly associated with the final de Sitter phase creates such a large perturbation that the final de Sitter phase is destroyed. If one requires the spacetime to have a final asymptotically de Sitter (expanding) phase with cosmological constant Λ_{final}, one finds that the observer is not in fact allowed to excite more than of order $A_{final}/4\ell_p^{n-1}$ degrees of freedom, where A_{final} is the area of a de Sitter cosmological horizon associated Λ_{final}.

At first sight, this focus on Λ_{final} might appear to lead to a new time asymmetry. However, this is really just the familiar thermodynamic arrow of time as we have allowed the entropy of the matter fields to increase (say, by decay of a massive particle into massless radiation) but not to decrease with time. Of course, we expect that number of degrees of freedom (or even the entropy $\mathrm{tr}(\rho \ln \rho)$ of a mixed quantum state) do not really change at all with the passage of time. Instead, the increase in entropy is an artifact of choosing a course-graining of the system in which the initial entropy appears small. Thus, barring unexpectedly large violations of unitary from possible quantum gravity effects, the time reverse of our argument suggests that one cannot prepare the FRW region in more than of order $A_{initial}/4\ell_p^{n-1}$ states if the spacetime is to pass through an initial (contracting) asymptotically de Sitter phase with cosmological constant $\Lambda_{initial}$. The number of states that can be accessed subject to both boundary conditions is therefore determined by the larger of $\Lambda_{initial}$ and Λ_{final}.

We believe that the terminology used here of 'accessible' versus 'available' degrees of freedom or entropy is an appropriate one. However, one may also take seriously the idea [73] that the fundamental Hilbert space associated to asymptotically de Sitter space should contain only those states that can in fact be accessed without destroying the asymptotically de Sitter behavior.

3.3.5 Discussion

Thus far our calculations focused on two aspects of asymptotically dS spacetimes interpreted as renormalization group flows in the dS/CFT correspondence. One aspect was establishing a generalized c-theorem for these flows. The other was the construction of conformal diagrams for various homogeneous flows.

For isotropic flows, we showed that Λ_{eff} defines a c-function (3.43) that is a locally increasing function of the scale factor a regardless of whether the foliation was by flat planes, spheres, or hyperbolic spaces. A similar result applies at least for certain anisotropic cases. In parallel with the results in the AdS/CFT duality, the present c-theorem essentially says that the effective cosmological constant is larger in the interior of the space than at the (conformal) boundary.

The flows associated with flat and hyperbolic spatial sections were seen to differ markedly from those on spheres. In particular, the k=0 and −1 flows are always *complete*, running monotonically between $a = 0$ and ∞. In contrast, we showed that the $k = +1$ flows can yield bouncing universes, in which the evolution of the scale factor may reverse its direction, and in particular produce two conformal boundaries. In the latter case, while the c-theorem still maintains that Λ_{eff} is larger in the interior of the space than at the boundaries, it does not establish any relationship between the $\Lambda_{initial}$ and Λ_{final} at I^- and I^+, respectively. In fact by analyzing simple models, it is clear that there can be no simple relation. For example, considering a single step potential as discussed in section 3.3.1, we find that for a fixed $\Lambda_{initial}$, the initial conditions of the scalar can always be chosen such that the universe succeeds in evolving to an asymptotically dS region as $t \to +\infty$ for an arbitrarily small Λ_{final}.

One feature of the $k = 0$ foliation which makes the renormalization group interpretation manifest is the existence of the Killing vector $\partial_t - \frac{x^i}{\ell}\partial_i$ in pure dS space [68]. While this Killing vector naturally acts to evolve from one time slice to another, it also acts as a global scale transformation on each slice, and in particular on the boundary I^+. The fact that this flow is a symmetry of dS space is in keeping with the conformal symmetry of the dual field theory. While the symmetry is lost for general k=0 flows, it is still natural to associate the time evolution with a flow in energy scales in the dual picture. For the $k = \pm 1$ coordinates on de Sitter space, there is no analogous Killing vector which preserves the foliation of the spacetime. However, the scaling of the spatial slices that arises in the time evolution is still suggestive of a renormalization group flow for such solutions.

As mentioned, the flows with spherical slices often reach a regime where $\dot{a}$ vanishes even though a remains finite. Beyond that point, $\dot{a}$ typically changes sign so that the derivative of our c-function does as well. That the renormalization group flow cannot go beyond a finite point may not be a surprise from the point of view of field theory on the sphere. In contrast to the flat or hyperbolic plane, any (simply connected) compact space has a longest length scale[10] and that, beyond that scale, no useful local effective description of the theory can be obtained. The surprise is perhaps that the flow does not just stop, but in fact continues with the c-function reversing the direction of its course. In the case where $\dot{a}$ vanishes at only one sphere of minimal size, it is natural to interpret the full spacetime as consisting of *two* renormalization group flows, each starting at $I^{\pm}$ and flowing to the same effective theory at the sphere where $\dot{a}$ vanishes. It is similarly possible that more complicated spacetimes which oscillate several times illustrate various complicated combinations of flows upward and downward in length scales.

Much the same interpretation can be made of the geodesically complete spacetimes discussed in section 3.3.3, in connection with k=0 flows. If the flow does not proceed to a singularity at $a = 0$, it can be patched across the Cauchy horizon to a second flow which produces the same result in the IR, *i.e.*, produces the same geometry at the Cauchy horizon. We then see two flows coming from $I^{\pm}$ and ending at the Cauchy horizon.

For the flows on hyperbolic spatial slices, we saw that it was impossible to patch one such flow directly to another across the Cauchy horizon. Instead, an intermediate region is required in which the surfaces of homogeneity are timelike, instead of spacelike, as in eq. (3.7) for example. The interpretation of this matching remains obscure to us and deserves further investigation.

Although anisotropic flows are more complicated to interpret, much of the above discussion of the isotropic case admits a clear extrapolation. If one is willing to sacrifice rotational invariance then one may course-grain a field theory dif-

[10]In the non-simply connected case, it is well known that Wilson lines may effectively expand the compact directions — see, *e.g.*, refs. [98, 99]. We expect this would play a role in $k= -1$ flows where the hyperbolic slices were compactified with appropriate identifications.

ferently in different directions. One may use this idea to construct anisotropic renormalization group flows. As in the isotropic case, we regard any surface on which $\dot{a}_i$ vanishes for one of the scale factors as representing the joining of two flows at some particular scale. The idea of a flow in which we move upward in scale in some directions but downward in scale in the others (*i.e.*, in which the $\dot{a}_i$ do not all have the same sign) may be unfamiliar, but is certainly allowed if we adopt the viewpoint advocated above that the coarse-graining described by our flows in fact keeps track of all the information in the more fine-grained theory but that the c-function describes an effective theory at the scale set by the a_i.

Above we considered joining flows by smoothly matching various asymptotically dS geometries at a Cauchy horizon. Recall from the discussion in section 3.1 that such horizons are naturally associated with a(n effective) boundary of the manifold on which the dual CFT is formulated. Such a boundary appears because a singular conformal transformation has been used to push off to infinity various points in the S^n naturally appearing at $I^{\pm}$. In smoothly matching geometries across the horizon, we are implicitly making a very precise selection for the CFT conditions or geometric data at these boundaries. This choice, of course, in not unique. For example, it would be straightforward to match geometries at a Cauchy horizon so that the derivatives of the metric are discontinuous even though the metric itself was continuous. One is also free to break homogeneity beyond a Cauchy horizon.

A related discussion arises for the singularity conjecture of ref. [69]. It was pointed out in ref. [75] that the negative mass Schwarzschild-dS solution seems to evade this conjecture in that, while the mass as defined by ref. [69] is greater than that of dS space, there is no 'cosmological singularity.' Rather observers may proceed from I^- to I^+ without ever encountering a singularity in this spacetime. On the other hand, if we consider situation in terms of evolving Einstein's equations forward from I^-, the maximally analytically continuation of the negative mass Schwarzschild-dS solution is certainly a very special solution requiring very precise boundary conditions at $t = \pm\infty$ on I^- (as well as all along the timelike singularity at $r = 0$ beyond the Cauchy horizon). Clearly for generic boundary conditions, there will be discontinuities, *i.e.*, impulsive gravitational waves, travelling along the Cauchy horizon. Hence in the generic situation, observers should be expected to encounter a 'cosmological singularity.' Therefore it seems that the conjecture of ref. [69] may still be valid if refined to include some sort of generic condition – or perhaps even just the existence of a single smooth Cauchy surface.

The c-theorem suggests that the effective number of degrees of freedom in the CFT increases in a generic solution as it evolves toward an asymptotically dS regime in the future. We would like to point out, however, that this does not necessarily correspond to the number degrees of freedom accessible to observers in experiments. Here we are thinking in terms of holography and Bousso's entropy bounds [96]. Consider a four-dimensional inflationary model with k=0 and consider also the causal domain relevant for an experiment beginning at t=$-\infty$ and ending at some arbitrary time t=t_o. For a sufficiently small t_o, it is not hard to show that the number of accessible states is given by $3\pi/G\Lambda_{initial}$. Naively, one expects that this number of states will grow to $3\pi/G\Lambda_{final}$ as $t_o \to \infty$. However, this behavior is not universal. It is not hard to construct examples[11] where in fact the initial cosmological constant still fixes the number of accessible states for arbitrarily large t_o. This behavior arises because the apparent horizon is spacelike in these geometries. Hence in such models, the number of degrees of freedom required to describe physical processes throughout a given time slice grows with time while the number of states that are accessible to experimental probing by a given physicist

[11] A particularly nice example to work with analytically is a model constructed as in section 3.3.1 with a pre-potential $W(\phi) = -4\beta_1 + 4\beta_2\cos^2(\alpha\phi)$ with $\beta_1 > \beta_2$.

remains fixed.

This discussion reminds us of the sharp contrast in the 'degrees of freedom' in the dS/CFT duality [68] and in the Λ-N correspondence [73]. In the Λ-N framework, the physics of asymptotically de Sitter universes is to be described by a finite dimensional space of states. This dimension is precisely determined as the number of states accessible to probing by a single observer. (The latter is motivated in part by the conjecture of black hole complementary [100, 101, 102].) In contrast, in the dS/CFT context, one would expect that a conformal field theory with a finite central charge should have an infinite dimensional Hilbert space[12] , and these states are all involved in describing physical phenomena across the entire time slices. Further as shown above, the central charge as a measure of number of degrees of freedom on a given time slice need not be correlated with the number of states experimentally accessible to observers on that slice.

A similar discrepancy between 'accessible' and 'available' states already appeared in the discussion of big wide universes in section 3.3.4. A related conceptual issue is the tension, alluded to above, between the unitary time evolution of the bulk dS theory and the variation in available number of degrees of freedom as manifest in the c-function. To be concrete, consider for example the case of a dS flow foliated by spherical spatial slices. Barring unforeseen quantum gravity effects, one expects that the bulk operators associated with each sphere are related to the bulk operators on any other sphere by some quantum version of the equations of motion. In some sense then, any information extracted from one slice should also be available on any of the other slices and hence there is no apparent variation in the 'number of degrees of freedom.' On the other hand, one wishes to interpret the flow between slices as a renormalization group flow in the dual

[12]Though this infinity might perhaps be removed if one imposes, as described in ref. [70], that the conformal generators vanish on physical states.

theory where 'degrees of freedom are integrated out (or in),' as indicated by the variation of the c-function. The synthesis of these apparently orthogonal points of view may be tied to some course-graining scheme. However, it is logically possible that the bulk unitarity implies that the renormalization group paradigm which was so successfully developed for the AdS/CFT is not in fact appropriate in a dS/CFT setting. An alternative paradigm, or perhaps a parallel feature, for the dS/CFT is the mapping between CFT's associated with I^- and I^+ in the $k = +1$ flows [68, 70]. A suitable generalization seems to describe an interesting 'duality' between different CFT's, involving a non-local mapping of operators.

3.4 FIELD THEORY DUALITIES

In this section we are exploring the dS/CFT correspondence under the assumption that the bulk physics must reproduce standard background quantum field theory in the low energy limit. In particular then, up to possible quantum gravity violations, it should be possible to express the observables[13] localized near any Cauchy surface in terms of the observables localized near any other Cauchy surface. Of course, this is what one usually refers to as 'unitarity,' though even in standard quantum field theory it may not represent unitary evolution in the technical sense [107, 108]. Below, we will find that this benign assumption about the bulk physics has rather extraordinary consequences for the dual CFT.

Our primary tool for exploring these consequences is Einstein gravity with a positive cosmological constant coupled to an otherwise free scalar field. For simplicity, we consider quantum field theory for the scalar linearized about fixed de Sitter or asymptotically de Sitter backgrounds. However, it will be clear that,

[13]Here we use the term 'observables' in the technical sense of 'gauge invariant operators' without direct concern for a sense in which these observables might be measurable [105, 106].

at least as far as the above property is concerned, linearized gravitons and even non-linear background quantum field theory must yield similar results. Before proceeding with a detailed discussion of the scalar field, let us briefly present our main observations.

Central to our investigations are the two conformal boundaries appearing in de Sitter space, and the claim [68, 70] that only a single dual field theory is needed. Recently, an alternate point of view has been advocated [103] in which the dual description of dS space involves an entangled state of two field theories associated with the two separate boundaries. We will comment on this idea in the discussion (section 3.4.5), but our present focus will be on the single CFT proposal where one does not have a 'separate' dual theory associated with I^+ and with I^-. Some intuition for this point of view can be found by combining the above assumption of bulk unitarity with the central postulate of the dS/CFT correspondence. That is, operators in the dual field theory may be associated with appropriate limits of bulk operators pushed to either I^- or I^+. So consider the full set of bulk operators localized near some arbitrary Cauchy surface. Now unitary time evolution in the bulk allows us to 'push' this complete set of operators to either I^- or I^+ and hence obtain corresponding sets of dual operators. Of course, since we began with a complete set of bulk operators, correlation functions for any bulk operators can then be computed in terms of those of the dual operators on either boundary. In particular, correlation functions in the dual theory associated with I^+ can be expressed in terms of correlation functions associated with I^- and vice versa.

The original discussion of refs. [68, 70] emphasized the causal connection between points on the two boundaries of dS space. In particular, a light cone emerging from a point on I^- expands into the space and re-converges at the antipodal point on the sphere at I^+. As a consequence, the singularity structure of certain boundary correlation functions is left invariant when, *e.g.*, a local operator on I^-

is replaced by a corresponding local operator at the antipodal point on I^+ [68]. Not only does this observation suggest that there is a single dual field theory, but further that dual operators associated with the two boundaries are simply related by the antipodal map on the sphere. The latter would be surprising as one expects that in general the time evolution map connecting the corresponding bulk operators is non-local. In fact, as will be discussed below, this latter intuition is correct. The mapping between dual operators on $I^\pm$ is highly non-local, as is readily revealed by explicitly examining the (retarded) Green's functions in dS space. While the singularities in the Green's function propagate along the light cone, generically there is also non-trivial support within the light cone. For certain special cases, however, the time evolution map does produce a simply antipodal mapping between I^+ and I^-. Given the lack of guidance coming from a working example of the proposed duality, one might interpret this result as a hint towards the specific types of fields that would appear in a successful realization of the dS/CFT. Unfortunately, however, this selection rule based on locality of the mapping between boundaries does not seem to bear up in more interesting applications, as follows.

The discussion of the dS/CFT correspondence has also been extended beyond pure dS space to more general backgrounds with asymptotically dS regions. Again, in analogy to the AdS/CFT duality, such backgrounds might have an interpretation in terms of 'renormalization group flows' in the dual field theory [59, 69, 3, 109, 110]. Now it is a straightforward consequence of a theorem of Gao and Wald [77] that in such a generic (nonsingular) background, observers are able to view an entire Cauchy surface at a finite time. The corresponding conformal diagrams may be described as 'tall' (see figure 3.2— see also ref. [3] for details).

The nonlocality in the relation between dual operators on the two boundaries becomes manifest when we consider such 'tall' backgrounds. The above discussion, in which the bulk evolution map relates the CFT operators associated with I^+ and

I^-, remains essentially unchanged. However, a key difference is that the causal connection between I^- and I^+ is now manifestly *not* local. In a tall spacetime, the light rays emerging from a point on I^- re-converge,[14] but this occurs at a finite time long before they reach I^+. After passing through the focal point, the rays diverge again to enclose a finite region on I^+. This observation precludes any intuition that the dual operators associated with the two boundaries could be related by a simple local map (*e.g.*, the antipodal map) on the sphere. That is, following the arguments presented above, one is still lead to conclude that in these tall backgrounds pushing the bulk operators to I^- and I^+ must yield equivalent dual physics, but the map that implements this equivalence can no longer be local. Hence in the context of the dS/CFT duality, it seems that non-locality will be an unavoidable aspect of the relation between field theory operators associated with two conformal boundaries. One can reproduce precisely the same boundary correlators or observables but through some non-local reorganization of the degrees of freedom within the dual field theory. It seems appropriate to refer to such relations as *non-local dualities* within the field theory.

The remainder of this chapter is organized as follows: In section 3.4.2, we explore in more detail the consequences of the bulk evolution map for the relationship between the dual field theories associated with the two boundaries, I^- and I^+. Section 3.4.5 provides a short discussion of our results. Various useful technical details about scalar field theory in de Sitter space are presented in appendices. Appendix F provides a detailed analysis of the evolution of massive scalar fields in a pure $(n+1)$-dimensional dS background, while Appendix G considers the stability of dS space with respect to perturbations by the scalar field modes.

[14] For simplicity, our description is restricted to spherically symmetric foliations. Generically the converging light rays would not be focussed to a single point.

3.4.1 One or two field theories?

As pointed out earlier, de Sitter space has two conformal boundaries and so one may ask the question as to whether the dS/CFT correspondence involves a single dual field theory or two. One simple argument in favor of one CFT is as follows [126]: The isometry group of $(n+1)$-dimensional dS space is $SO(1, n+1)$, which agrees with the symmetries of a single Euclidean CFT in n dimensions. Further note that the global Killing vector fields corresponding to these isometries in dS space act nontrivially on both $I^{\pm}$. Hence there is a simple correlated action on source currents or dual operators identified with each of the boundaries. Hence given the single symmetry group, it is natural to think that the dual description involves a single CFT.

Further we would recall our experience from the AdS/CFT correspondence. A central point in this context is that the CFT does not 'live' on the boundary of the AdS space. Usually one has chosen a particular foliation of AdS [127], and the bulk space calculations are naturally compared to those for the field theory living on the geometry of the surfaces comprising this foliation. Via the UV/IR correspondence, each surface in the bulk foliation is naturally associated with degrees of freedom in the CFT at a particular energy scale [128]. The boundary of AdS space plays a special role in calculations as this is a region of the geometry where the separation between operator insertions and expectation values is particularly simple. One notable exception where two CFT's seem to play a role is the eternal black hole [90, 129, 130]. In this case, however, the bulk geometry has two causally disconnected boundaries. In fact, one can show that for any solution of Einstein's equations with more than one asymptotically AdS boundary, the boundaries are all causally disconnected from each other [131]. In the case of dS space, the past and future boundaries are certainly causally connected and so it seems $I^{\pm}$ can be considered as two (special) slices in a certain foliation (3.15) of the spacetime.

Hence this reasoning suggests that one should only consider a single CFT in the dual description.[15]

3.4.2 NON-LOCALITY IN THE BOUNDARY MAP

Next we turn to the observation in ref. [68] that the generating functional (3.21) can in certain circumstances be extended to incorporate sources on I^+. Certainly the construction of the generating functional in the previous section produces essentially the same result if we replace both of the limits in eq. (3.21) with $t, t' \longrightarrow +\infty$. This would produce an analogous generating functional with source currents defined by the asymptotic behavior of the scalar near I^+, *i.e.*,

$$\lim_{t \to +\infty} \phi(\Omega, t) \simeq \tilde{\phi}_{0+}(\Omega)\, e^{-h_+ t} + \tilde{\phi}_{0-}(\Omega)\, e^{-h_- t} \, . \tag{3.54}$$

However, it is also interesting to consider the case where only one of the limits in eq. (3.21) is replaced with one approaching I^+,

$$\widetilde{\mathcal{F}} = \lim_{t \to +\infty, t' \to -\infty} \int d\Sigma^i d\Sigma'^j \phi(x)\, \overleftrightarrow{\partial}_i\, G(x, x')\, \overleftrightarrow{\partial}_j\, \phi(x') \, . \tag{3.55}$$

Now an essential observation [68, 70] is the causal connection between points on the two boundaries $I^\pm$. In particular, a null geodesics emerging from a point on I^- expands out into the dS spacetime and refocuses precisely at the antipodal point on the n-sphere at I^+. Hence the two-point function in eq. (3.55) (or any dS-invariant Green's function) will introduce singularities when the point on I^+ approaches the antipode to the point on I^-, as the proper separation of these points vanishes. In fact, in certain circumstances (see the details below), evaluating the

[15]See, however, ref. [123] where, in the context of the AdS/CFT correspondence, the correlation functions between two CFT's are used to describe physics hidden behind the horizon of a BTZ black hole.

above expression yields the simple result:

$$\tilde{\mathcal{F}} = -\frac{(h_+ - h_-)^2}{2^{2n}} \int d\Omega d\Omega' \left[\tilde{c}_+ \frac{\tilde{\phi}_{0-}(\Omega)\,\phi_{0-}(\Omega')}{(w^i w'^i + 1)^{h_+}} + \tilde{c}_- \frac{\tilde{\phi}_{0+}(\Omega)\,\phi_{0+}(\Omega')}{(w^i w'^i + 1)^{h_-}} \right] . \quad (3.56)$$

This expression incorporates the same Euclidean two-point function except that the singularities now arise as the sources $\tilde{\phi}_{0\pm}(\Omega)$ approach antipodes on the n-sphere.

These results suggest that one needs only consider a single copy of the CFT and that an operator on I^+ is identified with the same operator on I^- after an antipodal mapping. One finds further support for this interpretation by considering the isometries of dS space. For example, the isometry[16] which produces a dilatation around a point on I^-. On I^+, the same symmetry corresponds to a dilatation around the antipodal point on the n-sphere.

However, this suggestion for identifying operators at I^+ and I^- is easily seen to require some revision as follows. As discussed in the introduction, bulk correlators are naturally related by time evolution. The key ingredient is simply the free field evolution of the scalar, which given some configuration specified on a n-dimensional hypersurface is characterized by the formula

$$\phi(x') = \int d\Sigma^i\ \phi(x)\ \overleftrightarrow{\partial}_i\ G_R(x, x') \ , \quad (3.57)$$

where $G_R(x, x')$ is a retarded Green's function, *i.e.*, it vanishes for $t > t'$. Now as an example, the integral appearing in the generating functional (3.21) is covariant and so should be invariant when evaluated on any time slices t and t'. The advantage of pushing these slices to I^- (or I^+) lies in the fact that one can easily separate the source currents according to their conformal weights.

We can explicitly consider the relation between currents on the past and future boundaries by simply following the classical evolution (3.57) of the fields from I^-

[16]This isometry corresponds to the action of a time translation ∂_t in the static patch coordinates [132].

to I^+. Unfortunately, it is clear that generically there is no simple local relation between the currents on I^- and those on I^+. This remark comes from the observation that in general the retarded Green's function will have support throughout the interior of the light cone. This intuition is readily confirmed by explicit calculations. Ref. [118] presents explicit Green's functions for generic masses in four-dimensional de Sitter space. So, for example, for scalar fields in the principal series, the retarded Green's function becomes, for large timelike proper separation,

$$G_R(t,\Omega;t',\Omega') \propto \frac{\sin^{n/2}\tau \, \sin^{n/2}\tau'}{(w^i w'^i - \cos\tau \, \cos\tau')^{n/2}} \, \theta(\tau'-\tau) \; , \tag{3.58}$$

where τ is the conformal time coordinate, $\sin\tau = 1/\cosh(t/\ell)$ — see eq. (3.63) below. Here the θ-function ensures the proper time-ordering of the points. In any event, eq. (3.58) illustrates how the field 'leaks' into the interior of the light-cone with the classical evolution. Generically this leads to a non-local mapping between the currents on I^- and I^+. This complication will only be avoided in certain exceptional cases, for example, if the retarded Green's function has only support precisely on the light cone — a point to which we return below.

The non-local relation between the currents on I^- and those on I^+ can be made more explicit through the mode expansion of the fields on dS space — see Appendix F. A well-documented feature of cosmological spacetimes is mode-mixing or particle creation [117]. For the present case of dS space, this corresponds to the fact that a mode of the scalar field with a given boundary behavior on I^-, *e.g.*, having h_- scaling, will usually have a mixture of $h_\pm$ scaling components at I^+. Appendix F provides a detailed discussion of the mode expansions on dS space as well as the Bogolubov transformation relating the modes with a simple time dependence (scaling behavior) near I^- to those near I^+. Using these results, we may discuss the mapping between the currents on the conformal boundaries. Following the notation of Appendix F, we decompose the asymptotic fields in terms of spherical harmonics on the n-sphere

$$\phi_{0\pm}(\Omega) = \sum_{L,j} a_{\pm Lj} Y_{Lj} \ , \qquad\qquad \tilde{\phi}_{0\pm}(\Omega) = \sum_{L,j} \tilde{a}_{\pm Lj} Y_{Lj} \ . \tag{3.59}$$

Denoting the antipodal map on the n-sphere as $\Omega \to J\Omega$, one has[17] $Y_{Lj}(\Omega) = (-)^L Y_{Lj}(J\Omega)$. Now let us imagine that $\phi_{0\pm}$ and $\tilde{\phi}_{0\pm}$ are related by the antipodal map, *i.e.*, $\phi_{0\pm}(\Omega) = z\,\tilde{\phi}_{0\pm}(J\Omega)$ with some constant phase z. Then one must have

$$a_{\pm Lj} = z\,(-)^L\,\tilde{a}_{\pm Lj} \tag{3.60}$$

where in particular the constant z is independent of L.

However, in general, the Bogolubov transformation given in Appendix F gives a more complicated mapping. For example, from eq. (F.17) for the principal series, one finds

$$a_{\pm Lj} = C_-^-(\omega) e^{\pm 2i\theta_L} \tilde{a}_{\pm Lj} + C_-^+(\omega)\tilde{a}_{\mp Lj} \ . \tag{3.61}$$

Now given eq. (F.18) for n odd with both $C_-^-(\omega)$ and $C_-^+(\omega)$ non-vanishing, certainly eq. (3.60) is inapplicable. One comes closer to realizing the desired result with even n for which $C_-^-(\omega) = 1$ and $C_-^+(\omega) = 0$. However, for either n odd or even, the phase θ_L always introduces a non-trivial L dependence (beyond the desired $(-)^L$) as shown in eq. (F.19). Thus while the mapping between I^- and I^+ may look relatively simple in this mode expansion, it will clearly be non-local when expressed in terms of the boundary data $\phi_{0\pm}(\Omega)$ and $\tilde{\phi}_{0\pm}(\Omega)$.

The complementary series gives some more interesting possibilities with

$$\begin{aligned} a_{-Lj} &= \bar{C}_-^-(\mu)\,\tilde{a}_{-Lj} + \bar{C}_-^+(\mu)\,\tilde{a}_{+Lj} \ , \\ a_{+Lj} &= \bar{C}_+^-(\mu)\,\tilde{a}_{-Lj} + \bar{C}_+^+(\mu)\,\tilde{a}_{+Lj} \ . \end{aligned} \tag{3.62}$$

[17]This result becomes clear when the n-sphere is embedded in R^{n+1} with $(x^1)^2 + (x^2)^2 + \cdots + (x^{n+1})^2 = 1$. In this case, the spherical harmonics Y_{Lj} may be represented in terms of symmetric traceless tensors, $Z_{i_1 i_2 \cdots i_L} x^{i_1} x^{i_2} \cdots x^{i_L}$, and hence it is clear that the antipodal map, which takes the form $J : x^i \to -x^i$, produces an overall factor of $(-)^L$.

In particular for n odd and μ half integer, one finds $\bar{C}_-^+(\mu) = 0 = \bar{C}_+^-(\mu)$ and $\bar{C}_-^-(\mu) = (-)^{\frac{n}{2}+\mu}(-)^L = \bar{C}_+^+(\mu)$. Note that these special cases include $\mu = 1/2$, which corresponds to the conformally coupled massless scalar field to which we will return in the following section. Similarly for n even and μ integer: $\bar{C}_-^- = \bar{C}_+^+ = (-1)^{\frac{n}{2}+\mu+1}(-1)^L$ and $\bar{C}_-^+ = 0 = \bar{C}_+^-$. Hence the coefficients for these special cases give a precise realization of eq. (3.60). Further for these cases then, the generating functional considered in eq. (3.55) will take the simple form given in eq. (3.56).

Hence when considering the principal series or generic masses in the complementary series, it seems that non-locality will be an unavoidable aspect of the relation between field theory operators associated with two conformal boundaries. The essential point is that the time evolution of the scalar generically introduces non-locality in the mapping because the retarded Green's function smears a point-like source on I^- out over a finite region on I^+. However, note that one reproduces precisely the same boundary correlators but after some non-local reorganization of the degrees of freedom within the dual field theory. It seems appropriate to refer to such relations as *non-local dualities* within the field theory. On the other hand, the complementary series does seem to provide some situations where the mapping of the boundary data between I^- and I^+ is local. In the absence of a working example of the proposed dS/CFT duality, one might interpret these results as a hint towards the specific types of fields that would appear in a successful realization of the dS/CFT. Unfortunately, however, this selection rule based on locality of the mapping between boundaries does not seem to survive in more interesting applications, as we will see in the following.

3.4.3 Non-local dualities in 'tall' spacetimes

It is of interest to extend the application of the dS/CFT correspondence from dS space to more general spacetimes with asymptotically dS regions. As a consequence of a theorem of Gao and Wald [77], such a (nonsingular) background will be 'tall' (see section 3.3.1). That is, the conformal diagram for such spacetimes must be taller in the timelike direction than it is wide in the spacelike direction. Of course, this feature has important implications for the causal connection between the past and future boundaries, and hence for the relation between the dual field theory operators defined at these surfaces. In particular, the latter relation becomes manifestly non-local.

We may explicitly illustrate the causal structure of the tall spacetimes by working in conformal coordinates. For asymptotically dS spacetimes which are homogeneous on spherical hypersurfaces, the metric may be written

$$ds^2 = C(\tau)\left[-d\tau^2 + d\theta^2 + \sin^2\theta\, d\Omega_{n-1}^2\right] \ . \tag{3.63}$$

For pure dS space, $C(\tau) = \ell^2/\sin^2\tau$. Note that the proper time t in eq. (3.15) is related to the conformal time τ above by the coordinate transformation $\sin\tau = 1/\cosh(t/\ell)$.

In this case, the conformal time runs from $\tau = 0$ at I^- to $\tau = \pi$ at I^+. The angular coordinate θ on the n-sphere runs over the same range, *i.e.*, from $\theta = 0$ at the north pole to $\theta = \pi$ at the south pole. Hence it is clear that the conformal diagram for dS space is a square (see figure 3.2).

Now for a tall spacetime, the conformal time above would run over an extended range $0 \le \tau \le \pi + \Delta$ where $\Delta > 0$. The assumption that the background is asymptotically dS means that the conformal factor has the following behavior near $I^\pm$:

$$\begin{aligned}\lim_{\tau\to 0} C(\tau) &= \frac{\ell^2}{\sin^2\tau}\ , \\ \lim_{\tau\to\pi+\Delta} C(\tau) &= \frac{\tilde{\ell}^2}{\sin^2(\tau-\Delta)}\ ,\end{aligned} \tag{3.64}$$

where we have allowed for the possibility that the cosmological 'constant' is different at I^+ than at I^-. This possibility may be realized in a model where a

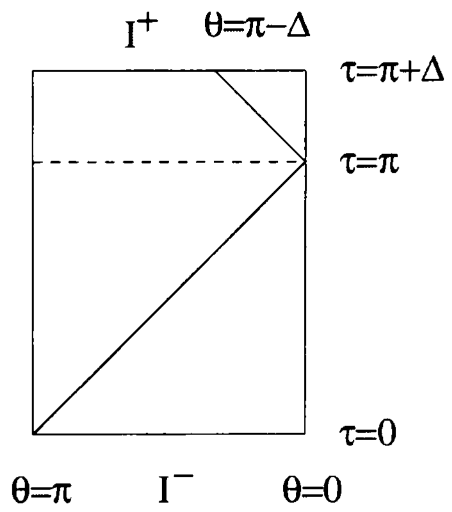

Figure 3.9: Conformal diagram of a perturbed de Sitter space. The excess height is represented by Δ.

scalar field rolls from one critical point of its potential to another (as shown in section 3.3.1). In any event, the corresponding conformal diagram will be a rectangle with height $\delta\tau = \pi + \Delta$ and width $\delta\theta = \pi$ (see figure 3.9).

This increase in the height of the conformal diagram modifies the causal connection between $I^{\pm}$ in an essential way. Consider the null rays emerging from the north pole ($\theta = 0$) at I^- ($\tau = 0$). This null cone expands out across the n-sphere reaching the equator ($\theta = \pi/2$) at $\tau = \pi/2$, and then begins to re-converge as it passes into the southern hemisphere. The null rays focus at the south pole ($\theta = \pi$) at $\tau = \pi$, however, in this tall spacetime, this event corresponds to a finite proper time for an observer at the south pole. Beyond this point, the null cone expands again and intersects I^+ ($\tau = \pi + \Delta$) on the finite-sized (n–1)-sphere at $\theta = \pi - \Delta$.

The discussion of the previous section made clear that an essential ingredient in finding a simple local mapping of boundary data on I^- to that on I^+ in dS space was the refocusing of the above null cone precisely at the future boundary. Even in that case, we pointed out that the time evolution of the scalar generically introduces non-locality in the mapping because the retarded Green's function smears a point-like source on I^- out over a finite region on I^+. Here we see that in a tall spacetime, a non-local map is inevitable since the causal connection between the past and future boundaries is itself non-local. So we should expect that even in the special cases found to have a local map for pure dS space, the mapping should become non-local for these same theories in a tall background. That is, for these more general asymptotically dS spacetimes, the relation between the dual field theory operators defined at each of the boundaries becomes non-local. Hence we are naturally led to consider a non-local self-duality of the CFT. Further we note that given the results of Gao and Wald [77] this would be the generic situation. For example, injecting a single scalar field quantum into dS space would actually lead to back-reaction effects which would produce a tall spacetime.

3.4.4 The conformally coupled massless scalar

We now turn to consider conformally coupled massless scalar field theory as an example which illustrates several of the points discussed above. In particular, it is an example where the mapping between the past and future boundaries is local in pure dS space, but becomes non-local in a tall background. Another useful feature is that one can perform explicit calculations in a tall spacetime without referring to the detailed evolution of the conformal factor $C(\tau)$. Rather a knowledge of the boundary conditions (3.64) is sufficient.

The conformally coupled massless scalar corresponds to the curvature coupling $\xi = \frac{n-1}{4n}$ and $m^2 = 0$ in eq. (3.17). Hence in pure dS space or in an asymptotically dS region, $M^2\ell^2 = (n^2 - 1)/4$ and the corresponding scaling exponents (3.18) become $h_\pm = (n \pm 1)/2$, independent of the value of the cosmological constant. As one might infer from the real exponents, this field lies in the complementary series for any value of the cosmological constant. The remarkable property of this scalar field theory is that the solutions of the wave equation (3.16) transform in a simple way under local conformal scalings of the background metric [117].

The backgrounds of interest (3.63) are conformally flat[18] and therefore the Green's function describing the evolution in the tall background is simply the flat space Green's function for a *massless* scalar field, up to some overall time-dependent factors. In particular then, for d even (n odd), the Green's function will have support precisely on the light cone. For example, in four-dimensional dS space, the retarded Green's function can be written as

$$G_R(\tau,\Omega;\tau',\Omega') = -\frac{\sin\tau\sin\tau'}{4\pi\ell^2}\,\delta\left(w^a w'^a - \cos(\tau'-\tau)\right)\theta(\tau'-\tau)\ . \qquad (3.65)$$

Similarly in higher even-dimensional dS spaces, the Green's function will contain δ-functions (and derivatives of δ-functions) with support only on the light cone [133]. Given this form of the retarded Green's functions, the evolution of the scalar field (3.57) from I^- to I^+ will produce precisely the antipodal mapping for all of these cases. Note that this result is confirmed by the mode analysis in the first part of this section. The conformally coupled massless scalar has $\mu = 1/2$ and we are considering even dimensions or n odd. This combination matches one of the special cases in which the modes transformed according to the antipodal mapping.

Using the conformal transformation properties of the field [117], the analogous Green's function for any spacetime of the form (3.63) is easily constructed. For $d=4$, it may be written as

$$G_R(\tau,\Omega;\tau',\Omega') = -\frac{1}{4\pi}\frac{1}{\sqrt{C(\tau)}\sqrt{C(\tau')}}\,\delta(w^a w'^a - \cos(\tau'-\tau))\,\theta(\tau'-\tau)\ . \quad (3.66)$$

For other values of even d, the corresponding Green's function has a similar form. For the conformally coupled scalar in such tall spaces, the delocalization of the

[18]Note that the coordinate transformation $T = e^\tau$ puts the metric (3.63) in the form of the flat Milne universe, up to a conformal factor.

boundary map does not depend on the detailed evolution, *i.e.*, the details of $C(\tau)$. Rather the non-locality is completely characterized by Δ, the excess in the range of the conformal time. For example, a source current placed at the north pole ($\theta = 0$) on I^- is smeared over an $(n-1)$-sphere centered at the south pole ($\theta = \pi$) and of angular radius $\delta\theta = \Delta$ on I^+.

Using eq. (3.66), we can make this discussion completely explicit for four dimensions. Consider an arbitrary tall space (3.63) with $n = 3$ satisfying the boundary conditions given in eq. (3.64). First with the conformal time coordinate, the asymptotic boundary conditions (3.23) for the scalar field at I^- become

$$\lim_{\tau\to 0} \phi(\Omega, \tau) \simeq \phi_{0+}(\Omega)\, \tau^{h_+} + \phi_{0-}(\Omega)\, \tau^{h_-}, \tag{3.67}$$

and similarly at I^+, we have

$$\lim_{\tau\to\pi+\Delta} \phi(\Omega, \tau) \simeq \tilde{\phi}_{0+}(\Omega)\, (\pi + \Delta - \tau)^{h_+} + \tilde{\phi}_{0-}(\Omega)\, (\pi + \Delta - \tau)^{h_-} . \tag{3.68}$$

These boundary conditions apply for a general scalar field theory. In the present case of a conformally coupled massless scalar with $n = 3$, we have $h_+ = 2$ and $h_- = 1$. Hence inserting (3.66) and (3.67) into (3.57), we may evaluate the result at a point $(\Omega', \tau' = \pi + \Delta - \epsilon)$ near I^+ and compare to eq. (3.68). The final result for the boundary fields on I^+ is

$$\begin{aligned} \tilde{\phi}_{0+}(\Omega') &= \frac{|\sin\Delta|}{\sin\Delta}\frac{\ell}{\tilde{\ell}}\Big\{ \sin\Delta\,\langle\phi_{0-}\rangle_\Delta(J\Omega') - \cos\Delta\,\langle\phi_{0+}\rangle_\Delta(J\Omega')\Big\} \\ \tilde{\phi}_{0-}(\Omega') &= \frac{|\sin\Delta|}{\sin\Delta}\frac{\ell}{\tilde{\ell}}\Big\{ \cos\Delta\,\langle\phi_{0-}\rangle_\Delta(J\Omega') + \sin\Delta\,\langle\phi_{0+}\rangle_\Delta(J\Omega') + \sin\Delta\,\partial_\theta\langle\phi_{0-}\rangle_\Delta(J\Omega')\Big\} . \end{aligned} \tag{3.69}$$

where J is the antipodal map on the two-sphere and $\langle\phi_{0\pm}\rangle_\Delta(J\Omega')$ denote the average of $\phi_{0\pm}$ on the two-sphere separated from $J\Omega'$ by an angle Δ. The factors $\frac{|\sin\Delta|}{\sin\Delta}$ are to be understood as being continuous from below; *i.e.*, this factor is -1 at $\Delta = 0$ and $+1$ at $\Delta = \pi$.

This expression simplifies tremendously in the case of dS space with $\Delta = 0$ (as well as $\tilde{\ell} = \ell$) to yield

$$\tilde{\phi}_{0+}(\Omega') = \phi_{0+}(J\Omega') \,, \qquad \tilde{\phi}_{0-}(\Omega') = -\phi_{0-}(J\Omega') \,. \tag{3.70}$$

Thus, in pure four-dimensional dS space the map from I^- to I^+ acts on the conformally coupled massless scalar field as simply the antipodal map on ϕ_{0+} and -1 times the antipodal map on ϕ_{0-}. Note that the time reflection symmetry of de Sitter allows solutions for the mode functions to be decomposed into even and odd parts and, furthermore, both even and odd solutions will exist. Thus, with our conventions and $h_\pm$ real, when evolution from I^- to I^+ leads to the antipodal map it will be associated with a phase $z = +1$ for one set of modes and the opposite phase $z = -1$ for the other.

3.4.5 Discussion

The dS/CFT correspondence is a striking proposal which carries the potential for extraordinary new insights into cosmology and the cosmological constant problem. Unfortunately, the outstanding problem remains to find a concrete example where the bulk gravity theory and the dual field theory are understood or at least known. Lacking the guidance that such a working model would provide, one is left to study various aspects of physics in (asymptotically) dS spacetimes from this new point of view and to determine properties which this correspondence implies for the dual Euclidean CFT.

Such investigations have yielded a number of unusual properties for the dual field theory. It is likely to be non-unitary, *e.g.*, if the bulk theory involves scalars in the principal series [68]. A non-standard inner product is required to reproduce ordinary quantum field theory in the bulk [71, 103]. One might also observe that this Euclidean field theory should not simply be a standard Wick rotation of a conventional field theory since attempting to 'un-Wick rotate' would produce a bulk theory with two time directions and all of the associated confusions. We may add to this list the observation of section 3.2.2 that, since bulk correlators are not symmetric in Lorentz signature quantum field theory, a straightforward duality would require non-symmetric correlation functions in the dual Euclidean theory. But correlators generated by functional differentiation of a partition function are always symmetric, so the Euclidean theory could have no definition through a partition sum. Finally, in the present paper, we have also inferred the existence of unusual non-local dualities within the field theory itself.

Our investigation focussed on the mapping of operators between I^+ and I^- provided by time evolution in the bulk spacetime. The essential point is that the time evolution of the scalar generically introduces non-locality in the mapping because the retarded Green's function smears a point-like source on I^- out over a finite region on I^+. However, despite this non-local reorganization of the degrees of freedom within the dual field theory, one reproduces the same boundary correlators. Hence we referred to this relation as a non-local duality within the field theory. While this non-locality already applies for many fields in pure dS space, it seems unavoidable in tall spacetimes because the causal connection between I^+ and I^- is inherently non-local. We emphasize that tall spacetimes are quite generic as a result of the theorem in ref. [77]. As soon as one perturbs dS even slightly by, *e.g.*, the introduction of matter fields or gravitational waves, the resulting background solution will have the property that its conformal diagram is taller than it is wide. As the inferred self-duality is non-local, *i.e.*, local operators are mapped to non-local operators, it seems that the underlying field theory does not have a unique concept of locality. That is, one has a specific dictionary whereby the same short-distance singularities can be reproduced by a set of local or non-local operators.

Faced with the daunting task of consolidating all of these unusual characteristics in a single Euclidean field theory, one is tempted to revise the interpretation of the dS/CFT correspondence. One suggestion [103] is that the duality should involve two CFT's but that dS spacetime is defined as a correlated state in Hilbert space of the two field theories. The correlated state is constructed so as to preserve a single $SO(1, n+1)$ symmetry group, which is then reflected in the isometries of the dS space. As discussed in section 3.4, we still feel that our experience with the AdS/CFT is highly suggestive that the two boundaries should not be associated with distinct field theories. Further, it is difficult to see how this framework could incorporate big bang or big crunch backgrounds with a single asymptotic dS region. Note that the latter spacetimes will still give rise to horizons, as well as the associated thermal radiation and entropy.

However, this approach with two CFT's remains an intriguing suggestion. Within this context, the mapping of the boundary data between $I^{\pm}$ would provide information about correlations in the field theory state. Hence our calculations would still find application in this context. The non-localities discussed here, while not unnatural, give an indication of the complexity of these correlations.

We should also remark that all of our investigations treated only the time evolution of a free scalar field theory. The mapping of boundary operators will become even more complex if one was to consider an interacting field theory. Of course, in accord with the discussion here, we would still expect that time evolution of the fields or operators in an interacting theory would still provide the basis for this mapping.

While it is amusing to speculate on such matters, we note that the central thesis of ref. [134] is that one cannot successfully understand the physics of dS space within the context of quantum field theory in curved spacetime. It is interesting to consider how their comments may relate our discussion. Essentially, they suggest

APPENDIX F

SCALAR FIELD MODES IN dS SPACE

In this appendix, we present a detailed analysis of the bulk physics of massive scalar fields propagating in a dS space of arbitrary dimension, emphasizing characteristics of their evolution which should be relevant to the proposed dS/CFT correspondence. Our aim is to characterize fully the mode-mixing phenomenon inherent to physics in dS space. While the details of this analysis are readily available in the literature for the modes of the principal series (see, for example, ref. [71]), we did not find explicit accounts of the complementary and discrete series.

F.1 FIELD EQUATION AND ASYMPTOTIC BEHAVIOR

The spherical foliation of $(n+1)$-dimensional dS space is given by the metric

$$ds^2 = -dt^2 + \cosh^2 t \; d\Omega_n^2, \tag{F.1}$$

where in this appendix we set the dS radius to unity ($\ell = 1$). We consider a massive scalar field propagating in this background according to

$$\left[\Box - M^2\right] \phi(x) = 0. \tag{F.2}$$

It is convenient to write the solutions to eq. (F.2) in the form

$$\phi(x) = y_L(t) \; Y_{Lj}(\Omega), \tag{F.3}$$

where the Y_{Lj}'s are spherical harmonics on the n-sphere satisfying

$$\nabla^2 Y_{Lj} = -L(L+n-1) Y_{Lj}, \tag{F.4}$$

of the form $u - v = R_0$.

If one desires, one can perform a transformation (a translation in u and v) to the conformal frame shown below in which the diagram has a Z_2 symmetry of inversion through the center. Again, focusing arguments imply that the figure must be 'taller than it is wide,' so that the dashed congruence of light rays in figure E.4 does not pass from I^- to I^+.

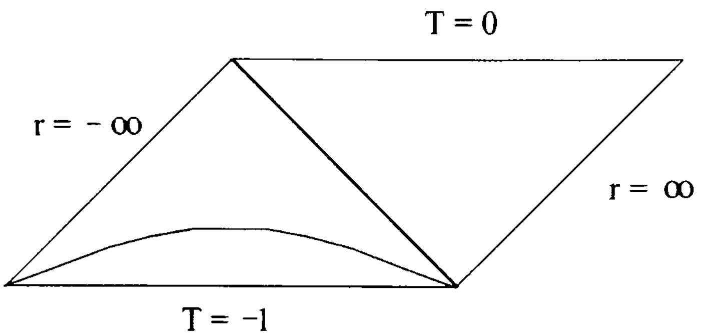

Figure E.3: The coordinate r is extended to range over the real line.

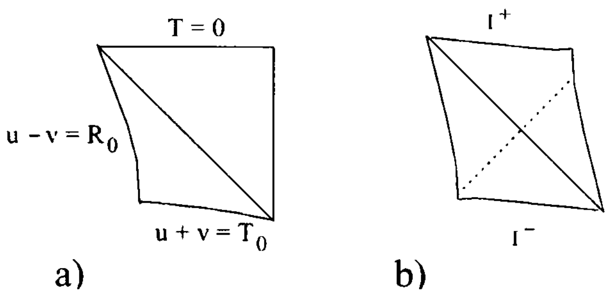

Figure E.4: The diagram for an asymptotically de Sitter spacetime with flat surfaces of homogeneity in a) the frame described above and b) a frame where the diagram has a reflection symmetry though the center.

diagram above. As a result, it must be of the form $f(U)\frac{\partial}{\partial U} + g(v)\frac{\partial}{\partial v}$ across the entire conformal diagram.

Now, in the upper triangle, the expression for $\frac{\partial}{\partial r}$ in terms of $\frac{\partial}{\partial U}$ and $\frac{\partial}{\partial v}$ is fixed and can be computed from the coordinate definitions. The same function $g(v)$ must therefore give the component of $\frac{\partial}{\partial r}$ along $\frac{\partial}{\partial v}$ in the lower part of the diagram. It remains only to determine the function $f(U)$ giving the component along $\frac{\partial}{\partial U}$ for $U < -1$. But this is fixed by the requirement that I^- be along the line $u + v = T_0$. Since I^- is a surface of homogeneity, $\frac{\partial}{\partial r}$ must be tangent to $\frac{\partial}{\partial u} - \frac{\partial}{\partial v}$ at $T = -1$. This suffices to determine $f(U)$ for $U < -1$. Without solving these equations in detail, it is clear that the result is simply that the left timelike boundary is a line

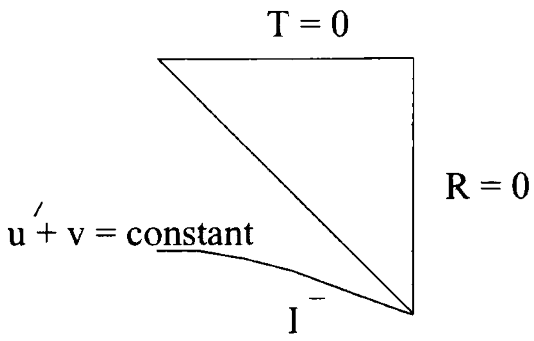

Figure E.2: The form of the line representing I^-.

Note that we retain the conformal freedom to replace U by any smooth function of U in the region $U < -1$. We may therefore use this freedom to place I^- on some convenient line, subject only to the constraint that the angle (in the Lorentzian geometry sense) between I^- and $R = 0$ is unchanged. A convenient choice is to use the analogue of a constant t line in the region beyond $U = -1$. That is, we define a new coordinate $u'(U) = \tanh(2 + U)$ in this region and use a line of the form $u' + v = T_0$.

It remains only to determine the location of the left timelike boundary of our diagram, which will represent the center of spherical symmetry for the flat slices beyond our Cauchy horizon. Note that this line must intersect each surface of homogeneity orthogonally. Thus, the shape of this boundary will be determined if we find the surfaces of homogeneity.

To do so, we simply extend the coordinate r to range over $[-\infty, \infty]$ on both sides of the Cauchy horizon. This corresponds to taking a slice all of the way across our original higher dimensional spacetime instead of truncating the slice at $r = 0$. For any such slice there is a translation that reduces to $\frac{\partial}{\partial r}$ along the slice, so that we may consider $\frac{\partial}{\partial r}$ to generate surfaces of homogeneity in the above spacetime. The important point is that, since $\frac{\partial}{\partial r}$ is a Killing field of the spacetime, it must be a conformal Killing field of the conformally re-scaled spacetime drawn in the

Appendix E

Conformal diagram of $k = 0$ de Sitter flows

In this appendix we finish the derivation of figure 3.5 (b), the conformal diagram for asymptotically de Sitter spaces with flat spatial slices. As described in section 3.4, the diagram will consist of two regions, each of which is conformal to one quadrant of Minkowski space and each of which has an interior that is foliated by flat spatial hypersurfaces. One of these regions may be represented by a quadrant of a diamond as shown in figure 3.4 (b), but this fixes part of the conformal freedom so that the representation of the second region is more constrained. To find its shape it is useful to choose explicit coordinates. Let us begin by introducing the null coordinates $u = \tau - r$ and $v = \tau + r$ and the corresponding $U = \tanh(u)$, $V = \tanh(v)$. We shall also introduce T, R by $U = T - R$ and $V = T + R$ and take the convention that our conformal diagrams are drawn in such a way that T, R appear as Cartesian coordinates. In particular, we take the boundaries of the triangle in figure E.1 to lie at $T = 0$, $R = 0$, and $U = -1$.

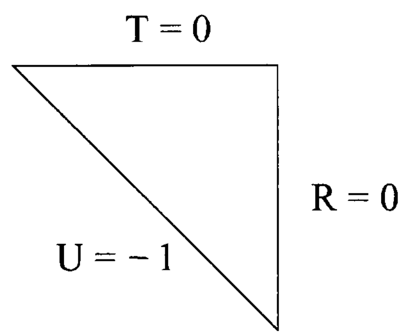

Figure E.1: Conventions for the R, T, U coordinates.

step which the scalar could climb with the given initial conditions, according the construction of section 3.3.1. Figures D.2-D.4 illustrate various aspects of the evolution for $\alpha = 0.9558$. The equations of motion being left unchanged when $t \to -t$, the scale factor from $t = -\infty$ to $t = 0$ can be deduced simply by using the reflection of figure D.3 across the $t = 0$ axis. Figure D.4 shows the evolution of the effective cosmological constant Λ_{eff}. Note that asymptotically Λ_{eff} is a constant (as is the scalar) indicating that the evolution reaches an asymptotically dS phase. Hence the full solution would begin in a dS phase, enter a matter-(scalar-)dominated bounce and finally return to a dS phase, just as described for the 'very tall' universe models in section 3.3.4.

Upon examining figures D.3 and D.4, we see that Λ_{eff} increases monotonically during the contracting phase of the evolution and decreases in the expanding phase, in accord with the c-theorem of section 3.3.2. It is interesting to note that for $\alpha > 0.9559$ (holding all other parameters fixed) the scalar continues to roll after climbing up the wall and the resulting kinetic energy forces the scale factor to contract into a big crunch. On the other hand, for $\alpha < 0.9557$ (again holding all other parameters fixed) the field fails to completely climb the wall and ϕ returns to zero, again resulting in a big crunch. Only in the (approximate) range $0.9557 < \alpha < 0.9559$ do the solutions reach an asymptotically de Sitter regime.

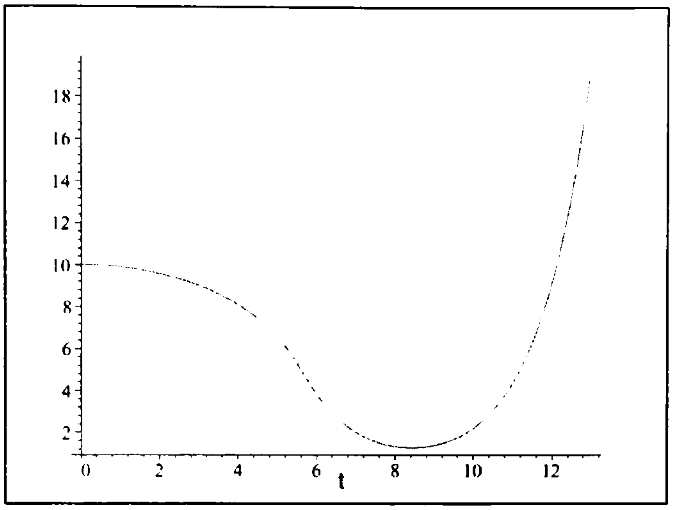

Figure D.3: The behavior of the scale factor is such that the universe contracts from $t = 0$ up to when the scalar field hits the wall of the potential. Then the universe re-enters an expansion phase. The equations of motion are symmetric when $t \to -t$ so the model of universe presented is in fact asymptotically dS on $I^{\pm}$.

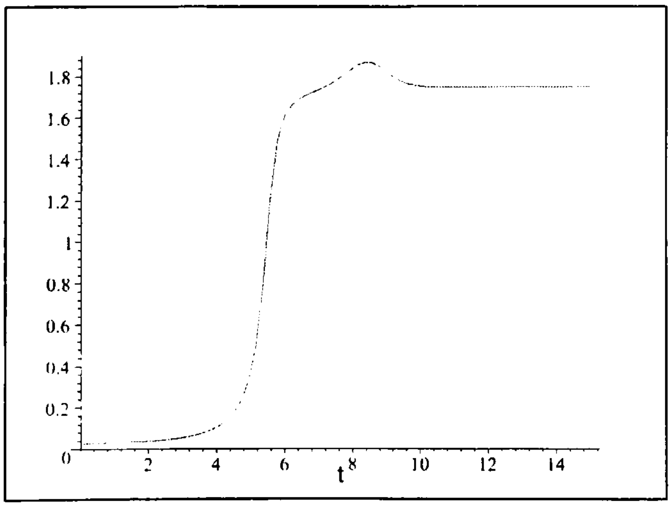

Figure D.4: The effective cosmological Λ_{eff} for the numerical bouncing universe is such that the spacetime is asymptotically de Sitter in the future.

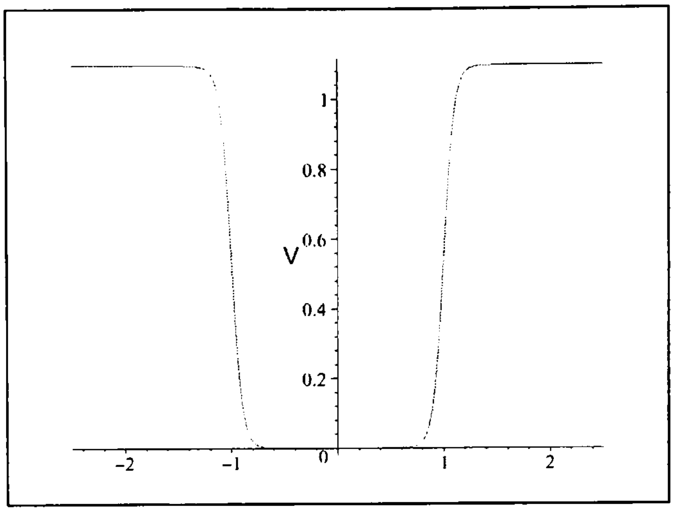

Figure D.1: The continuous 'step' potential with parameters $\omega = 10$, $\phi_0 = 1$ and $\alpha = 0.9958$.

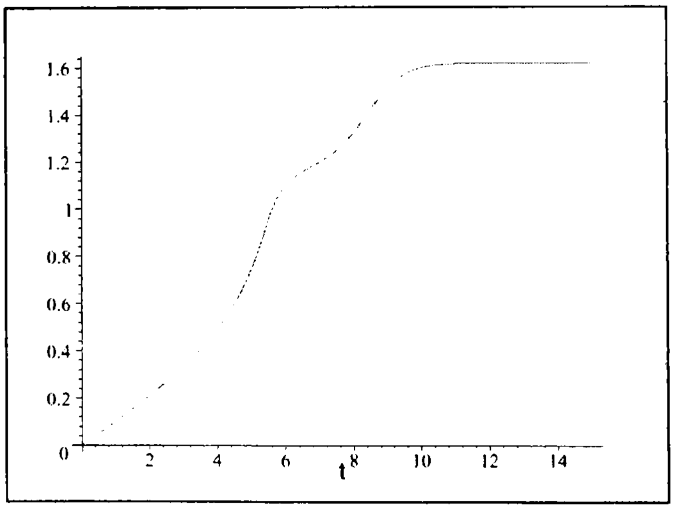

Figure D.2: The scalar field $\phi(t)$ increases steadily until it reaches its maximum value at the wall of the potential where it becomes frozen.

APPENDIX D

NUMERICAL BOUNCING UNIVERSE

In section 3.3.4, we constructed an asymptotically dS solution with a very large matter-dominated region in the middle. The potential used there was rather unrealistic (*i.e.*, discontinuous). To assure the skeptical reader that our results are not an artifact of this construction. we present an analogous numerical solution here using an everywhere smooth potential. While smooth, however, the potential has edges which can be made arbitrarily sharp (see figure D.1),

$$V(\phi) = \frac{1}{2L^2}\frac{\tanh w(\phi - \phi_0) - \tanh w(\phi + \phi_0) + 2\tanh w\phi_0}{\tanh w\phi_0}. \tag{D.1}$$

The parameter w characterizes the steepness of the potential steps and $\pm\phi_0$ are approximately the values of the scalar field where the jumps occur. This potential is chosen to be symmetric under $\phi \to -\phi$, so that we may consider a time-symmetric solution. Further the potential is constructed such that L corresponds to the cosmological scale in the asymptotic regions. That is, $\Lambda_{final} = 3/L^2$ as we are working in four dimensions ($n = 3$) in the following.

Solving the second order Friedmann equation (3.34) numerically using Maple yields the scalar field and a scale factor respectively shown on figures D.2 and D.3 given we use the initial conditions

$$a(0) = 1, \quad \phi(0) = 0. \quad \dot{a}(0) = 0, \quad \text{and} \quad \dot{\phi}(0) = 10. \tag{D.2}$$

The parameters in eq. (D.1) are chosen such that $w = 10$, $\phi_0 = 1$ and

$$L = \alpha\frac{a^3(0)\dot{\phi}(0)}{\cosh^{\frac{3}{2}}(2\,\phi(0))}. \tag{D.3}$$

The final parameter α was tuned to study various different classes of solutions. For $\alpha = 1$, the value of L given by eq. (D.3) corresponds to the maximum potential

Of course if $Q=0$, eq. (C.7) provides a vacuum solution and for $k=+1$, this reproduces the standard Schwarzschild-dS solution. For these vacuum solutions with μ positive, $k=+1$ is the only case for which there *may be* cosmological (and black hole) event horizons. For $k=0,-1$, the solutions have a spacelike or cosmological singularity at $\rho=0$. If μ is negative, all of the solutions have a cosmological horizon which separates the asymptotically dS regions from timelike singularities at $\rho=0$, as illustrated in figure C.1. The connection of these solutions to the dS/CFT duality were discussed in, *e.g.*, refs. [75, 76]. Similar comments apply about the causal structure for the full solution with $Q\neq 0$.

Interpreted as a renormalization group flow, this family of solutions (C.7) is interesting as the scaling of the τ direction is different from that for the remaining boundary directions. However, for the vacuum solutions with $Q=0$, the corresponding flows are trivial in that the c-function (3.46) is simply a fixed constant since $\Lambda_{eff}=\Lambda$. On the other hand, with the background Maxwell field (*i.e.*, $Q\neq 0$), we have a time-dependent Λ_{eff}. The c-function (3.46) for the flow under consideration has the form

$$c\sim\Lambda_{eff}^{-(n-1)/2}=|G^{\rho}{}_{\rho}|^{-(n-1)/2}\ . \tag{C.9}$$

From eq. (C.7), we find

$$\Lambda_{eff}=-G^{\rho}{}_{\rho}=\frac{n(n-1)}{2\ell^2}-\frac{(n-1)(n-2)Q^2}{2\rho^{2(n-1)}}\ . \tag{C.10}$$

Eq. (C.10) is a monotonically increasing function from $\rho\to+\infty$. It is finite both on I^- and at the cosmological horizon $\rho=\rho_+$, which is the largest root for $H(\rho_+)=0$. We find that for all possible values of n, Q and k, the c-function (3.46) is monotonically decreasing from I^- to the cosmological horizon at $\rho=\rho_+$. Hence this provides a nontrivial example of our c-theorem of section 3.3.2.

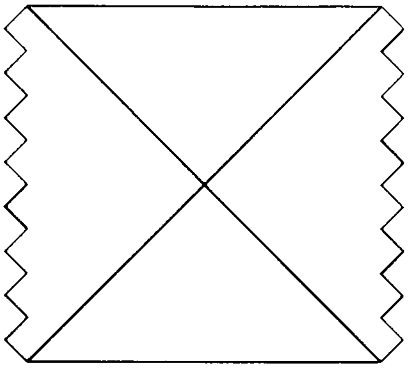

Figure C.1: The conformal diagram for negative μ.

of the S^p factors (with $p>1$) with any space satisfying $\widetilde{\mathcal{R}}_{ab}=(p-1)/\ell^2\,\tilde{g}_{ab}$. Similarly the H^p factors can be replaced by any space satisfying $\widehat{\mathcal{R}}_{ab}=-(p-1)/\ell^2\,\hat{g}_{ab}$, and the $\mathbb{R}^p$ factors can be replaced by any Ricci flat solution, *i.e.*, $\mathcal{R}_{ab}=0$. For example then, S^p can be replaced by a product of spheres $S^{p_1}\times\cdots\times S^{p_q}$ where $\sum_{i=1}^{q}p_i=p$ with $p_i>2$ and the radii of the individual spheres is scaled so $r_i^2=(p-1)/(p_i-1)\,\ell^2$. These generalized solutions will no longer be conformally flat or locally dS. Furthermore generically a true curvature singularity is introduced at the minimum radius, *e.g.*, $\mathcal{R}_{ijkl}\mathcal{R}^{ijkl}$ grows without bound as ρ approaches ℓ or 0.

Another simple extension of the solutions given in eq. (C.2) comes from introducing a 'charged black hole' into dS space. The corresponding metric may be written as

$$ds^2=-\frac{1}{H(\rho)}d\rho^2+H(\rho)d\tau^2+\frac{\rho^2}{\ell^2}d\Sigma^2_{k,n-1}\ , \tag{C.7}$$

where

$$H(\rho)=\frac{\rho^2}{\ell^2}-k+\frac{\mu}{\rho^{n-2}}+\frac{Q^2}{\rho^{2n-4}}, \tag{C.8}$$

and our notation is adapted to the asymptotic cosmological regions where the 'radial' variable ρ appears timelike. Of course, the full solution now contains a Maxwell field and the electrostatic potential will have the form: $\phi(\rho)\propto Q/\rho^{n-2}$. For $k=+1$, this reproduces the standard Reissner-Nordstrom-dS solution.

assume that both $\hat{m}, \tilde{m} \geq 2$. For $k{=}{+}1$, the boundary geometry is $H^{\hat{m}}{\times}S^{\tilde{m}}$, while for $k{=}{-}1$, we simply interchange the hyperbolic space and the sphere. However, in the latter case, the coordinate transformation $\tilde{\rho}^2{=}\rho^2{-}\ell^2$ puts the metric back in the $k{=}{+}1$ form with $\hat{m}{\leftrightarrow}\tilde{m}$. With $k{=}{+}1$, $\rho = \ell$ again corresponds to an horizon (*i.e.*, a coordinate singularity). Finally we note that as in the previous example for $k = \pm 1$, the scaling of the boundary metric is not homogeneous as the metric evolves in from $\rho = \infty$.

Thus with the metrics in eqs. (C.1), (C.2) and (C.3), we have displayed dS_{n+1} with a wide variety of boundary geometries:

$$\mathbb{R}^n,\ \mathbb{R}\times S^{n-1},\ \mathbb{R}\times H^{n-1},\ S^n,\ H^n,\ S^m\times H^{n-m}\ . \tag{C.4}$$

A dS/CFT correspondence would then imply an equivalence between quantum gravity in dS space and a CFT on any of the above backgrounds (C.4). Of course, as discussed in section 3.1, these geometries are all related to (a portion of) S^n by a singular conformal transformation.

All of the above dS metrics are maximally symmetric, *i.e.*, they satisfy

$$\mathcal{R}_{ijkl} = \frac{1}{\ell^2}(g_{ik}\,g_{jl} - g_{il}\,g_{jk})\ , \tag{C.5}$$

which ensures that the geometry is conformally flat. This condition also ensures the geometries are all locally dS. One could generate additional solutions by making additional discrete identifications of points on the spatial slices, however, this procedure would tend to introduce null orbifold singularities on the cosmological horizons [69].

However, eq. (C.5) is an extremely restrictive condition. If one is simply interested in solving Einstein's equations with a negative cosmological constant

$$\mathcal{R}_{ij} = \frac{n}{\ell^2}g_{ij}\ , \tag{C.6}$$

then the above metrics remain solutions when the spatial geometries are replaced by arbitrary Einstein spaces. In all of the metrics (C.1–C.3), one may replace any

APPENDIX C

THE MANY FACES OF DE SITTER

Note that we may re-cast the foliations of dS_{n+1} presented in eqs. (3.4)-(3.6) as follows:

$$ds^2 = -\frac{d\rho^2}{\left(\frac{\rho^2}{\ell^2} - k\right)} + \frac{\rho^2}{\ell^2} d\Sigma^2_{k,n} \ . \tag{C.1}$$

This coordinate choice provides an interesting basis for comparison with the following representations of dS space.

Consider the following three metrics for dS_{n+1},

$$ds^2 = -\frac{d\rho^2}{\left(\frac{\rho^2}{\ell^2} - k\right)} + \left(\frac{\rho^2}{\ell^2} - k\right) d\tau^2 + \frac{\rho^2}{\ell^2} d\Sigma^2_{k,n-1} \ , \tag{C.2}$$

where the $(n-1)$-dimensional metric $d\Sigma^2_{k,n-1}$ is defined in precisely the same way as in eq. (3.5). This metric is only really new for $k = \pm 1$, since for k=0 it simply reproduces the k=0 metric in eq. (C.1). For $k = +1$ and $\rho < \ell$, these are standard static coordinates on dS space where $\rho = 0$ corresponds to the position of the static observer's worldline while $\rho = \ell$ is a cosmological horizon. In eq. (C.2), our notation is adapted to the cosmological region (*i.e.*, $\rho > \ell$) where this 'radial' coordinate plays the role of the cosmological time, which parametrizes the renormalization group flow. Note that for $k = \pm 1$, the scaling of the boundary metric is not homogeneous along the ρ-flow.

One other metric on dS space which we will consider is

$$ds^2 = -\frac{d\rho^2}{\frac{\rho^2}{\ell^2} - k} + \left(\frac{\rho^2}{\ell^2} - k\right) d\widehat{\Sigma}^2_{-k,\hat{m}} + \frac{\rho^2}{\ell^2} d\widetilde{\Sigma}^2_{k,\tilde{m}} \ , \tag{C.3}$$

where again the metrics $d\widehat{\Sigma}^2_{-k,\hat{m}}$ and $d\widetilde{\Sigma}^2_{k,\tilde{m}}$ are defined in eq. (3.5), and $\hat{m} + \tilde{m} = n$. For k=0 we once again reproduce the k=0 metric in eq. (C.1). For k=± 1, we

The solution we presented are referred to as *tachyon matter*. The stress-energy components (which are independent of the number of dimensions in the theory) correspond to a conserved energy density ($\rho \sim V(T)/\sqrt{\Delta}$ = constant) and a pressure ($p \sim -V(T)\sqrt{\Delta}$) that vanishes as $t \to +\infty$ [37].

and taking $T(t) \simeq t$ leads to the large time formula

$$T(t) \simeq t - a^2\sqrt{2}e^{-\sqrt{2}t}, \tag{B.7}$$

where we have fixed the integration constants by imposing

$$a^2\sqrt{2}e^{-\sqrt{2}t_c} + b - t_c = 0. \tag{B.8}$$

B.2 Particular solutions

We consider solutions to the equation of motion (B.1) with the potential

$$V(T) = \frac{1}{\cosh\left(T/\sqrt{2}\right)}. \tag{B.9}$$

The tachyon equation of motion becomes

$$\ddot{T} + \frac{1}{\sqrt{2}}(1 - \dot{T}^2)\tanh\left(T/\sqrt{2}\right), \tag{B.10}$$

which has a solution of the form

$$T(t) = -\sqrt{2}\,\mathrm{arc\,sinh}\left(\frac{\sqrt{2}}{2}\left[c_1 e^{t/\sqrt{2}} - c_2 e^{-t/\sqrt{2}}\right]\right), \tag{B.11}$$

where c_1 and c_2 are constants of integration. We usually specify boundary conditions at $t = 0$.

$$T(0) = -\sqrt{2}\,\mathrm{arc\,sinh}\left(\frac{\sqrt{2}(c_1 - c_2)}{2}\right). \tag{B.12}$$

$$\dot{T}(0) = -\sqrt{2}\frac{c_1 + c_2}{(4 + 2(c_1 - c_2)^2)^{1/2}}. \tag{B.13}$$

The family of solutions characterized by $T(0) = 0$ ($c_1 = c_2$) corresponds to possible tachyon velocities at $t = 0$: $\dot{T}(0) = -\sqrt{2}c_1$. An other class of solutions are those for which $\dot{T}(0) = 0$ ($c_1 = -c_2$). Those correspond to allowing the tachyon to begin its evolution with $T(0) = -\sqrt{2}$ arcsinh $\sqrt{2}c_1$.

APPENDIX B

TACHYON IN FLAT SPACE

We consider solutions to the equation of motion for an open string tachyon when the massless closed string modes are decoupled. The relevant equation of motion is

$$\ddot{T} + (1 - \dot{T}^2)\frac{\partial \ln V(T)}{\partial T} = 0. \tag{B.1}$$

B.1 GENERAL SOLUTION

Eq. (B.1) is a second order differential equation with a general solution of the form

$$T(t) = \int dt\, \frac{1 + a^2\, V^2(t)}{1 - a^2\, V^2(t)} + b, \tag{B.2}$$

where a and b are constants of integration. To integrate this equation one needs the function $V(t)$ which would imply that we already know the solution $T(t)$. Open string field theory has taught us that for $t = t_c$ large we have

$$\lim_{t \to t_c} V(t) << 1, \tag{B.3}$$

which implies that

$$T(t) \simeq \int_{t_c}^{t} dt\, (1 + 2a^2 V(t)) + b. \tag{B.4}$$

Therefore at large time the tachyon behaves like

$$T(t) = t + (b - t_c) + 2a^2 \int_{t_c}^{t} dt\, V^2(t). \tag{B.5}$$

Using the string field theory result

$$\lim_{T \to +\infty} V(T) = e^{-T/\sqrt{2}}, \tag{B.6}$$

For $t \simeq \omega$, the dominant contribution to the equation of motion for the tachyon is

$$\Delta^2 - \Delta + \frac{2}{9}(t-\omega)\dot{\Delta} = 0. \tag{A.11}$$

This is solved for

$$\Delta(t) = \frac{(t-\omega)^{9/2}}{(t-\omega)^{9/2} - g}, \tag{A.12}$$

where g is a constant of integration. The solution $\Delta = 0$ ($g \neq 0$) clearly corresponds to the brane probe inducing a curvature singularity on the horizon. The only physical solution is the one for which $g = 0$ which corresponds to $\Delta = 1$. For the anisotropic backgrounds considered here the metric component g_{tt} neither vanishes nor blows up at $t = \omega$. The solution $\Delta = 1$ therefore corresponds to a tachyon field for which the time-derivative vanishes ($\dot{T} = 0$) at $t = \omega$. It therefore appears that there are solutions for the probe evolution that avoids the pathologies described earlier. We repeated the calculation around the regions $t = 0$ and $t = -\omega$. We find that for both p *odd* and *even* the unstable brane probe is not well-behaved at $t = 0$, *i.e.*, it induces a curvature singularity there.

ground could actually be built.

The calculations and results of ref. [29] were summarized in section 2.2.2. In this appendix we generalize this calculation by probing the $d = 10$ *anisotropic* backgrounds presented above, since these are the only ones which are either non-singular or have singularities (at $t = -\omega$) 'shielded' by a horizon ($t = \omega$). We felt this generalization to be necessary because ref. [29] did not, for example, address the issue as to how the inclusion of the dilaton might affect the brane probe calculation. The unstable brane action is a generalization of the case studied in ref. [29].

We investigate whether or not an unstable brane probe is a well-defined object in the vicinity of the region $t = \omega$. In Einstein frame, the energy density for the probe propagating in the anisotropic backgrounds is

$$\rho_{probe} = \frac{N\mu_{p+1}}{g_s} f(\Phi) \frac{V(T)}{\Delta^{1/2}}, \tag{A.8}$$

while the pressure corresponds to

$$p_{probe} = \frac{N\mu_{p+1}}{g_s} f(\Phi) V(T) \Delta^{1/2}. \tag{A.9}$$

The dilaton function $f(\Phi)$ was picked up during the transformation from the string frame to the Einstein frame. It plays no role in the upcoming analysis because the dilaton is well-behaved,

$$\lim_{t \to \omega} f(\Phi) = \text{const.} \tag{A.10}$$

As we saw in section 2.2.2. whenever the probe analysis goes wrong, it signals a pathology for the gravitational background. As we saw, there are at least two ways the probe analysis can go wrong: it may induce an infinite energy or pressure density ($\rho_{probe}, p_{probe} \to \pm\infty$), or, there might not exist any reasonable solutions for $T(t)$.

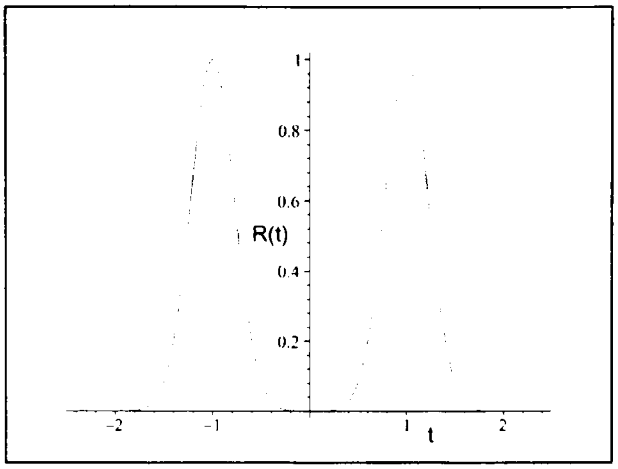

Figure A.1: This figure illustrates the Ricci scalar for the $p = 1$ anisotropic SD-brane solution with $\omega = 1$ and $\theta = \pi/4$. The other curvature invariants behave similarly.

This is finite except for $\theta = 0$ in which case the Ricci scalar diverges like $\mathcal{R} \sim 1/(t-\omega)^3$. We also found expressions for an other curvature invariant ($p = 1$),

$$\lim_{t\to\pm\omega} \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} = \frac{92^{3/4}}{2\omega^4}\frac{-272\cos^6\theta + 14 + 255\cos^4\theta + 101\cos^8\theta - 98\cos^2\theta}{\sin^{10}\theta}. \tag{A.7}$$

We found expressions with similar qualitative behavior for the other values of p *odd*.

For p *even* the solutions are not time-reversal symmetric. As pointed out previously, curvature invariants are finite at $t = \omega$ but there is a curvature singularity at $t = -\omega$. These are the timelike curvature singularities (surrounded by an horizon at $t = \omega$) described in ref. [27].

A.3 Unstable brane probe analysis

As mentioned in section 2.2.2, the motivation behind considering an unstable brane probe in a background with singularity problems is to ask if the singular back-where $\tau = \omega^2/t$. The expression (A.3) is simply flat space with part of it written in Milne coordinates. Not surprisingly, we find

$$\lim_{t\to 0} \dot{\Phi}(t) = 0, \quad \lim_{t\to 0} \dot{C}(t) = 0, \tag{A.4}$$

which shows that all stress-energy components vanish in the the region close to the origin.

A.2 HORIZON PHYSICS

We demonstrate that, contrary to previous claims, many of the anisotropic solutions are actually regular in the *full* range: $-\infty < t < +\infty$. The anisotropic solutions were already shown to be non-singular at $t = \omega$ and $t = 0$. We now investigate the region $t = -\omega$ further. Let us introduce the change of coordinates

$$T = \left(1 + \left(\frac{\omega}{t}\right)^{7-p}\right)^{2/(7-p)} t \tag{A.5}$$

in order to make comparison with the results of ref. [27] easier. For p even, $T = 0$ corresponds to $t = -\omega$ while for p odd we have $T = -2\omega$ when $t = -\omega$. A comment in ref. [27] is that $T = 0$ corresponds to a timelike (non-naked) curvature singularity. Actually, this is not always the case. For example, we considered the case $p = 1$ and computed the associated curvature invariants at all times. Figure A.1 shows the evolution of the Ricci scalar for the solution with $\omega = 1$ and $\theta = \pi/4$. Clearly, the $p = 1$ solution is symmetric under time-reversal and therefore has no curvature singularity. The qualitative behavior of all curvature invariants is similar to the Ricci scalar and is quite generic, *i.e.*, it is unchanged for all *odd* values of p, θ and ω. For $p = 1$ we obtain

$$\lim_{t\to\pm\omega} \mathcal{R} = -\frac{32^{1/3}}{\omega^2}\frac{19\cos^4\theta - 26\cos^2\theta + 7}{\sin^5\theta}. \tag{A.6}$$

APPENDIX A

REGULAR KMP S-BRANE SOLUTIONS

Among the supergravity solutions found by Kruczenski, Myers and Peet in ref. [27], some are regular in the region corresponding to $t = \omega$. This is only realized for the following values of the parameters,

$$\tilde{k} = 2, \quad H = \frac{4}{7-p}, \quad G = k_i = 0. \tag{A.1}$$

The corresponding metric is

$$\begin{aligned} ds^2 = \; & F(t)^{1/2}\alpha(t)^{4/(7-p)}\left(-dt^2 + t^2 dH^2_{8-p}\right) \\ & + F(t)^{-1/2}\left[\sum_{i=2}^{p+1}\left(dx^i\right)^2 + \left(\frac{\beta(t)}{\alpha(t)}\right)^2\left(dx^1\right)^2\right]. \end{aligned} \tag{A.2}$$

Because these solutions are anisotropic in the worldvolume directions, it is not clear that they are physically relevant. We will nevertheless study some interesting properties not considered in ref. [27].

A.1 THE REGION CLOSE TO THE ORIGIN

For the anisotropic solutions there is no curvature singularity at $t = \omega$. It is therefore interesting to consider the behavior of the metric components and curvature invariants close to the potentially problematic region around the origin, $t = 0$. We evaluated the curvature invariants: $\mathcal{R}$, $\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}$, and $\mathcal{R}_{\mu\nu\rho\lambda}\mathcal{R}^{\mu\nu\rho\lambda}$. They identically vanish for $t = 0$. The metric tensor there appears suspicious (for example, the component g_{tt} diverges) but we find

$$\lim_{t\to 0} ds^2 = -d\tau^2 + \tau^2 dH^2_{8-p} + \sum_{i=1}^{p+1}\left(dx^i\right)^2, \tag{A.3}$$

unstable objects. In the theory there are D-branes which are stable, *i.e.*, supersymmetric, and others which are not. The latter we referred to as SD-branes. They have many features of the ordinary D-branes but they typically exist for a very short time. In chapter 2 we performed a detailed study of the gravitational fields generated by the SD-branes. We found, numerically, that the gravitational backgrounds associated with full-SD-branes are singular while those associated with half-SD-branes (more closely related to unstable D-branes) can be nonsingular. Previous investigations had mainly focused on trying to describe these objects as vacuum solutions to the supergravity equations of motion, *i.e.*, neglecting the unstable brane source contribution. We solved the supergravity equations of motion taking into account the unstable brane source (with the instability being driven by an open string tachyon).

There have been many papers investigating the implication of the tachyons in inflationary cosmology. There are at least two aspects of our work which might be relevant to this topic. Firstly, we have shown that coupling gravity to the tachyon leads to a dissipation of the tachyon matter into massless closed string modes. Also, for large time the tachyon becomes proportional to time itself. This is interesting because it suggests that the origin of time might be the tachyon associated with the decay of a system of unstable branes. Secondly, in our analysis we have included couplings of the tachyon to other modes such as the dilaton and a Ramond-Ramond field. Most importantly, we find that the string coupling, which is related to the expectation value of the dilaton, becomes time-dependent and is typically driven to smaller values during the rolling of the tachyon. As pointed out in the Discussion section in chapter 2, there are at least two aspects of D-brane decay that should be taken under consideration in a complete analysis of the corresponding gravity fields. Firstly, the effect of worldvolume inhomogeneities is currently under investigation. Also, contributions from the emission of massive

CHAPTER 4

CONCLUSION AND SUMMARY

There has been many breakthroughs in string theory in the last decade. These include the discovery of D-branes [21] which had many far reaching consequences. Examples of these include: the entropy calculation for some black holes [142], the AdS/CFT correspondence [6] and. the realization that there are dualities relating the different consistent supersymmetric string theories [19]. These advances were made in a context where supersymmetry plays a crucial role. In particular. this implies that the systems under consideration are static (see section 1.1).

In the last few years the investigation of the role played by time in string theory was undertaken. This, of course, seems like a very natural thing to do yet, because of the absence of supersymmetry, the corresponding problems are much harder to solve. Perhaps the most compelling reason for studying time in string theory is the potential connections with the prolific field of cosmology. Recent experiments have provided cosmological data remaining to this date poorly explained by theory. This includes, for example, the measurement of a positive cosmological constant in the Universe (dark energy), the anisotropy of the cosmic microwave background and, the existence of the enigmatic dark matter. Of course, there is also no satisfying fundamental theory describing the physics of the early universe. These can be regarded as challenges for string theory which is a unification theory and a candidate for describing quantum gravity.

In this thesis we analyzed two aspects of time dependence in string theory, both of which might have important consequences for cosmology. We now summarize our findings and comment on some possible extensions of our work.

A natural place to look for time dependence in string theory is in the study of that the coarse graining procedure is unique. This fits well with the interpretation suggested in section 3.3.1 for 'renormalization group flow spacetimes' with spherical homogeneity surfaces. There, one naturally considers two UV regions (one at I^- and one at I^+) which both 'flow' to the same theory at some minimal sphere where the two parts of the spacetime join. One simply reads the flow as starting in the UV, proceeding toward the IR, but then reversing course. Interestingly, it is possible to arrive at a different UV theory from which one began. Such an odd state of affairs is more natural when one recalls that we have already argued that the theory must possess a non-local duality, so that it in fact has two distinct local descriptions.

AdS/CFT case and the interpretation of renormalization group flows. Recall that the primary assumption is that the relevant asymptotically AdS spacetime is in fact dual to the vacuum of some field theory. The important point is that one begins by placing the entire spacetime in correspondence with the vacuum of some single theory. One then uses the IR/UV connection to argue that different regions of the bulk spacetime are naturally related to different energy regimes in the dual theory. The suggestion that this description at differing energy scales is somehow connected to a renormalization group flow seems natural and, in that context, there was no evolution map relating the inner and outer regions to provide such an obvious tension.

In the present dS/CFT context, such a tension does exist. However, the more primitive association of different parts of the spacetime with the behavior of the field theory at differing energy scales still seems plausible. A more concrete version of this idea is suggested by the behavior of the bulk evolution map itself. As we have seen, the evolution map from t to t' 'coarse grains' the observables on t' in the sense that the theory is now presented in terms of variables (those that are local at time t) which are non-local averages over the intersection of past light cones from time t with the original hypersurface at t'. However, a sufficient number of overlapping coarse grainings are considered that no information is lost. Such a procedure can also be performed in a Euclidean field theory and one might speculate that keeping only the simplest terms in the resulting action might bare some similarity to those obtained from more traditional renormalization group methods. This would be in keeping with the identification of a c-function in which a spacetime region was associated with a copy of de Sitter space by considering only the metric and extrinsic curvature on a hypersurface.

Note that this interpretation readily allows us to run our flow both 'forward' (toward the IR) and 'backward' (toward the UV). However, it is far from clear that bulk properties of dS space should be analogous to the physics in a thermal system with a finite number of states and deduce from this that the evolution map of linearized quantum field theory should not be trusted in detail near the past and future boundaries. As a result, they suggest that a dual theory may not be as local as one might expect by studying limits of bulk correlation functions in background quantum field theory. The comments of ref. [135] raise further questions about correlation functions between points with a large separation in time. In particular, the problematic correlators would include precisely those between operators on I^- and I^+. Here the smearing observed in the tall spacetimes is likely to play a role since, if back-reaction is properly accounted for, even injecting a single scalar field quantum into dS should deform it to a (slightly) tall spacetime. It may be that the non-localities discussed here may be a hint that the 'correct physical observables' are themselves non-local[19] so that the boundary map would preserve the form of such operators.

Note that there is a certain tension between our strong reliance on time evolution, through which observables near any two Cauchy surfaces can be related, and the idea that the bulk evolution is related to a renormalization group flow in the dual theory [59, 69]. The point is that time evolution naturally produces a scaling of distances on Cauchy surfaces (at least in simple examples) and so these surfaces are naturally associated with different distance scales in the dual theory. However, the time evolution map relating different surfaces is invertible. In contrast, the usual notion of the renormalization group is actually that of a semi-group, in which different scales are related by integrating out modes, *i.e.*, by throwing away short-distance details so that the descriptions at two different scales are not fully equivalent.

To gain some perspective on this issue, we would like to return briefly to the

[19]Similar implications can be drawn from the finite time resolution discussed in ref. [136].

where ∇^2 is the standard Laplacian on the n-sphere. The differential equation for $y_L(t)$ is then

$$\ddot{y}_L + n \tanh t \, \dot{y}_L + \left[M^2 + \frac{L(L+n-1)}{\cosh^2 t} \right] y_L = 0. \tag{F.5}$$

As discussed in section 2.2, of particular relevance to the dS/CFT correspondence is the behavior of the scalar field near the boundaries I^+ and I^- ($t \to \pm\infty$). In these limits, eq. (F.5) becomes

$$\ddot{y}_L \pm n\dot{y}_L + M^2 y_L = 0, \tag{F.6}$$

which implies that

$$\lim_{t \to -\infty} y_L \sim e^{h_\pm t}, \qquad \lim_{t \to +\infty} y_L \sim e^{-h_\pm t}, \tag{F.7}$$

where the weights $h_\pm$ are defined by

$$h_\pm = \frac{n}{2} \pm \sqrt{\frac{n^2}{4} - M^2} \equiv \frac{n}{2} \pm \mu. \tag{F.8}$$

Formally, one may classify scalar fields according to the irreducible representations of SO($1, n+1$), the isometry group of de Sitter space, which are labelled by the eigenvalues associated with the Casimir operator[1] $Q = \Box$, which simply corresponds to the mass parameter M^2. The *principal series* is defined by the inequality $M^2 > n^2/4$. In this case, the weights $h_\pm$ have an imaginary part, and the corresponding modes, while still being damped near the boundaries, have an oscillatory behavior in the bulk. For the *complementary series*, the effective mass

[1] In fact, there are two coordinate invariant Casimir operators associated with the de Sitter isometry group but only one is relevant in characterizing massive scalar fields. The other Casimir operator automatically vanishes for all spin-zero fields but will play a role in the classification of higher spin representations [137]. Another interesting formal question is the behavior of these representations in the limit where the cosmological constant is taken to zero. A complete treatment of representation contraction in de Sitter space can be found in ref. [138].

falls in the range $0 < M^2 \leq n^2/4$. As will be made more explicit later, the modes are non-oscillatory asymptotically in this case since both $h_\pm$ are real quantities. The remaining *discrete series* corresponds to $M^2 < 0$. This last condition means that $h_- < 0$ (and $h_+ > n$), which implies that the tachyonic fields scaling like $y_L \sim e^{\mp h_- t}$ in approaching $I^\pm$ are growing without bound. Still, one is able to find 'normalizable' modes in a certain limited number of cases, as will be discussed below.

The case of $M^2 = 0$, *i.e.*, a massless scalar field, is interesting and deserves further comment. One finds that in this case it is impossible to construct a vacuum state which is invariant under the full de Sitter group $SO(1, n+1)$. A great deal of discussion about the peculiar nature of this quantum field theory can be found in the literature [139, 140, 122]. The weights associated with the $M^2 = 0$ field are $h_+ = n$ and $h_- = 0$. Dual to the latter should be a marginal operator in the CFT, *i.e.*, a deformation which does not scale under conformal transformations.

To fully solve eq. (F.5), we make the change of variables

$$y_L(t) = \cosh^L t \, e^{(L+\frac{n}{2}+\mu)t} \, g_L(t). \tag{F.9}$$

Setting $\sigma = -e^{2t}$, this equation for the time-dependant profile takes the form of the hypergeometric equation:

$$\sigma(1-\sigma)g'' + [c - (1+a+b)\sigma]\, g' - abg = 0, \tag{F.10}$$

where a 'prime' denotes a derivative with respect to σ and the coefficients are

$$a = L + \frac{n}{2}, \qquad b = L + \frac{n}{2} + \mu, \qquad c = 1 + \mu. \tag{F.11}$$

The two independent solutions can then be expressed in terms of hypergeometric functions:

$$y_{L+}(t) = N_+ \cosh^L t \, e^{(L+h_+)t} \, F(L + \frac{n}{2}, L + h_+; 1 + \mu; -e^{2t}), \tag{F.12}$$

$$y_{L-}(t) = N_- \cosh^L t\, e^{(L+h_-)t}\, F(L + \frac{n}{2}, L + h_-; 1 - \mu; -e^{2t}), \qquad \text{(F.13)}$$

where $N_\pm$ are normalization constants, which will be fixed below. More specifically, we have chosen here the two linearly independent solutions of eq. (F.10) in the neighborhood of the singular point $-e^{2t} = 0$ [141], which corresponds to one of the two limits of interest, *i.e.*, $t \to -\infty$. Following eq. (F.3), we denote the complete mode functions as $\phi_{L\pm} = y_{L\pm}(t) Y_{Lj}(\Omega)$.

One important aspect of time evolution of the scalar field in the bulk is the mode-mixing that occurs between the two boundaries. For example, this would be related to the particle production in the dS space [71, 122, 124, 125]. In the following, we emphasize the differences between the principal, complementary and discrete series.

F.2 Principal series

The principal series is frequently discussed in the physics literature, *e.g.*, refs. [71, 122, 124, 125], and would seem to be the most relevant case for the particle spectrum observed in nature. We review some of the salient points here for comparison with the other representations in the following section. For the principal series, it is useful to introduce $\omega \equiv -i\mu$. Then the above modes become

$$y_{L-}(t) = \frac{2^{L+(n-1)/2}}{\sqrt{\omega}} \cosh^L t\, e^{(L+\frac{n}{2}-i\omega)t}\, F(L + \frac{n}{2}, L + \frac{n}{2} - i\omega; 1 - i\omega; -e^{2t}), \quad \text{(F.14)}$$

$$y_{L+}(t) = \frac{2^{L+(n-1)/2}}{\sqrt{\omega}} \cosh^L t\, e^{(L+\frac{n}{2}+i\omega)t}\, F(L + \frac{n}{2}, L + \frac{n}{2} + i\omega; 1 + i\omega; -e^{2t}), \quad \text{(F.15)}$$

where $y^*_{L-}(t) = y_{L+}(t)$. Here the normalization constants have been fixed by imposing $(\phi_{L+}, \phi_{L+}) = 1 = (\phi_{L-}, \phi_{L-})$ as usual with the standard Klein-Gordon inner product [117]. As emphasized above, these solutions have the simple time dependence of eq. (F.7) in the asymptotic region $t \to -\infty$ near I^-. Because the differential equation (F.5) is invariant under $t \to -t$, one can easily define another pair of linearly independent solutions by applying this transformation to the above modes. We label the resulting modes: ${y_L}^-(t)$ and ${y_L}^+(t) = {y_L}^{-*}(t)$, where

$$y^-(t) = y_+^*(-t). \tag{F.16}$$

It readily follows that ${y_L}^- \sim e^{-h_- t}$ and ${y_L}^+ \sim e^{-h_+ t}$ near I^+. The two sets of modes $y_{L\pm}(t)$ and ${y_L}^\pm(t)$ can respectively be used to construct the 'in' and 'out' vacua with no incoming and outgoing particles. The Bogolubov coefficients relating these two sets of modes are defined through

$$y_{L-}(t) = C_-^-(\omega)\, e^{-2i\theta_L}\, {y_L}^-(t) + C_-^+(\omega)\, {y_L}^+(t), \tag{F.17}$$

with a similar expression for y_{L+} (with $C_+^+(\omega) = C_-^-(\omega)$ and $C_+^-(\omega) = C_-^+(\omega)$). When n is even [71], one finds that $C_-^-(\omega) = 1$ and $C_-^+(\omega) = 0$. This corresponds to the physical statement that there is no particle creation (no mode-mixing) in dS space for an odd number of spacetime dimensions. For n odd, there is nontrivial mode-mixing with,

$$C_-^-(\omega) = \coth \pi\omega \qquad C_-^+(\omega) = (-1)^{\frac{n+1}{2}} \frac{1}{\sinh \pi\omega}. \tag{F.18}$$

where $|C_-^-(\omega)|^2 - |C_-^+(\omega)|^2 = 1$ holds since the modes are properly normalized throughout their evolution. The expression for the phase in eq. (F.17) is

$$e^{-2i\theta_L} = (-1)^{L-\frac{n}{2}} \frac{\Gamma(-i\omega)\Gamma(L + \frac{n}{2} + i\omega)}{\Gamma(i\omega)\Gamma(L + \frac{n}{2} - i\omega)}. \tag{F.19}$$

It is clear that for large enough ω, the mixing coefficient $C_-^+(\omega)$ becomes negligible which is in accord with the intuition that there will be limited particle production in high energy modes. We will find that in the other two series there is no phase comparable to eq. (F.19). This complicates the expressions for mode-mixing between the boundaries and will lead to interesting features.

F.3 Complementary series and tachyonic fields

For the modes of both the complementary and the tachyonic series, the weights h_+ and h_- are real and so the above mode functions are entirely real,

$$y_{L+}(t) = \bar{N}_+ \cosh^L t\, e^{(L+\frac{n}{2}+\mu)t}\, F(L+\frac{n}{2}, L+\frac{n}{2}+\mu; 1+\mu; -e^{2t}), \qquad \text{(F.20)}$$

$$y_{L-}(t) = \bar{N}_- \cosh^L t\, e^{(L+\frac{n}{2}-\mu)t}\, F(L+\frac{n}{2}, L+\frac{n}{2}-\mu; 1-\mu; -e^{2t}). \qquad \text{(F.21)}$$

Hence with respect to the usual Klein-Gordon product these two solutions have zero norm, *i.e.*, $(\phi_{L+}, \phi_{L+}) = 0 = (\phi_{L-}, \phi_{L-})$.

Of course, this is not unnatural. One gains intuition by considering the usual plane wave decomposition in flat spacetime. There, one may choose between two bases, the one involving complex exponentials and the one involving cosines and sines. The latter basis in fact has the same characteristics as the present modes in the complementary series in terms of normalization with respect to the Klein-Gordon inner product. Consequently, to define a reasonable normalization for the mode functions (F.20) and (F.21), we require $(\phi_{L-}, \phi_{L+}) = i$

$$\bar{N}_+ = \frac{2^{L+\frac{n-1}{2}}}{\sqrt{\mu}} = \bar{N}_-, \qquad \text{(F.22)}$$

where we have resolved the remaining ambiguity by simply demanding that $\bar{N}_+ = \bar{N}_-$. With this choice of normalization, it is clear that upon quantizing the scalar field in the dS background the corresponding mode coefficients will have commutation relations analogous to those of coordinate and momentum operators, rather than raising and lowering operators.

As in the previous subsection, by substituting $t \to -t$, we define modes $y_L{}^{\pm}(t) \equiv y_{L\pm}(-t)$ which have the simple time dependence of eq. (F.7) in the asymptotic region approaching I^+. Using a simple identity of hypergeometric functions [141], one can relate the two sets of modes as

$$y_{L^-}(t) = \bar{C}_-^-(\mu)\, y_L{}^-(t) + \bar{C}_-^+(\mu)\, y_L{}^+(t),$$
$$y_{L^+}(t) = \bar{C}_+^-(\mu)\, y_L{}^-(t) + \bar{C}_+^+(\mu)\, y_L{}^+(t), \tag{F.23}$$

where the elements of the mixing matrix C (the Bogolubov coefficients) are given by

$$\bar{C}_-^+(\mu) = \frac{\Gamma(1-\mu)\Gamma(-\mu)}{\Gamma(\frac{2-n}{2}-\mu-L)\Gamma(\frac{n}{2}-\mu+L)}, \quad \bar{C}_-^-(\mu) = -(-1)^L \frac{\sin(\frac{\pi n}{2})}{\sin \pi\mu}, \tag{F.24}$$

$$\bar{C}_+^-(\mu) = -\frac{\Gamma(1+\mu)\Gamma(\mu)}{\Gamma(\frac{2-n}{2}+\mu-L)\Gamma(\frac{n}{2}+\mu+L)}, \quad \bar{C}_+^+(\mu) = -(-1)^L \frac{\sin(\frac{\pi n}{2})}{\sin \pi\mu}. \tag{F.25}$$

We now describe some features of the resulting mode-mixing for the complementary series. In this case, recall that $0 < M^2 \leq n^2/4$ which implies that $0 \leq \mu < n/2$. Of course, certain features depend on the spacetime dimension $n{+}1$ as before:

a) n odd: Generically for the case of an even spacetime dimension, there is non-trivial mode-mixing. An exception occurs for $\mu = (2m+1)/2$ with m a positive integer. For these special cases, there is no mixing since $\bar{C}_-^+ = 0 = \bar{C}_+^-$ and one finds that $\bar{C}_-^- = \bar{C}_+^+ = (-1)^{\frac{n}{2}+\mu}(-1)^L$.

b) n even: Generically for this case of an odd number of spacetime dimensions, one finds $\bar{C}_+^+ = 0 = \bar{C}_-^-$ and $\bar{C}_-^+\bar{C}_+^- = -1$ (where $\bar{C}_-^+$ and $\bar{C}_+^-$ both have a nontrivial dependence on L). This means that a mode that is scaling like $e^{h_\pm t}$ on I^- will have the 'opposite' scaling $e^{-h_\mp t}$ on I^+. We refer to this phenomenon as 'maximal-mixing'. This phenomenon is absent when μ is an integer. This case must be treated with some care as the solution for y_{L-} appearing in eq. (F.21) breaks down.[2] The correct solution [141] has an additional logarithmic singularity near I^-, *i.e.*, subdominant power law behavior in t. In any event, the final result for n even and μ integer is: $\bar{C}_-^- = \bar{C}_+^+ = (-1)^{\frac{n}{2}+\mu+1}(-1)^L$ and $\bar{C}_-^+ = 0 = \bar{C}_+^-$.

[2]Similar remarks apply for n odd and μ integer, but in that case one still finds nontrivial mode-mixing.

Finally we briefly consider the tachyonic or discrete series [119, 120, 121]. Recall that in this case with $M^2 < 0$, $h_- < 0$ so that the modes scaling as $e^{\pm h_- t}$ diverge as one approaches either I^- or I^+, respectively. Generically there is nontrivial mode-mixing and so even if a mode is convergent at one asymptotic boundary it will be divergent at the opposite boundary. However, an interesting exceptional case is when a y_{L+} mode (scaling like $e^{h_+ t}$ as $t \to -\infty$) evolves to the corresponding $y_L{}^+$ mode (with $e^{-h_+ t}$ behavior for $t \to \infty$). Such a mode would have convergent behavior both towards the future and past boundaries. This behavior would result when $\bar{C}_+^-$ vanishes. A brief examination of eq. (F.25) requires that $1 + |h_-| - L$ is zero or a negative integer. As above this constrains μ to be an integer or half-integer depending on the spacetime dimension. We express this constraint in terms of the (tachyonic) mass

$$-M^2 = \begin{cases} \frac{1}{4}((2m+1)^2 - n^2) & \text{for } n \text{ odd} \quad \text{with } m = (n-1)/2, (n+1)/2, \dots \\ m^2 - \frac{n^2}{4} & \text{for } n \text{ even} \quad \text{with } m = n/2, n/2+1, \dots \end{cases} \tag{F.26}$$

This constraint gives rise to the nomenclature that these modes are often referred to as the discrete series. However, the above constraint is not sufficient; rather we must also impose a constraint on the 'angular momentum' quantum number L, namely,

$$L \geq 1 + |h_-| \ . \tag{F.27}$$

Hence the completely convergent modes only appear for sufficiently large angular momenta. Note that it is still true that using the usual Klein-Gordon inner product these modes have a vanishing norm $(\phi_{L+}, \phi_{L+}) = 0$. However, in the mathematics literature (*e.g.*, ref. [121]) these modes are singled out by having finite norm in the sense given by the spacetime integral: $\int d^{n+1}x\sqrt{-g}\,|\phi_{L+}|^2 = 1$.

This construction shows that even in the tachyonic mass range, one can find certain normalizable modes (in the above sense) for special choices of parameters.

However, we reiterate that while these formal results for the *discrete series* may be interesting mathematically, they are not useful in understanding the physics of dS space. As emphasized above in the discussion of the dS/CFT correspondence, one must consider the full space of solutions, and presently even in the exceptional cases, the normalizable modes are accompanied by modes diverging at both asymptotic boundaries. Thus, the normalizable modes do not form a complete set of modes on a Cauchy surface. Such divergences, which occur in the generic case as well, are simply an indication that a linearized analysis of tachyonic fields is inappropriate. Of course, nonlinear field theories with potentials including unstable (or metastable) critical points may play an important role in the paradigm of inflationary cosmology, and such theories can produce interesting asymptotically dS spacetimes (see section 3.3.1). Our point here is simply that one should consider the full nonlinear evolution of such fields including their back-reaction on the spacetime geometry.

APPENDIX G

STABILITY OF SCALAR MODES

Here we discuss the stability of dS space with respect to the various scalar field modes introduced in appendix F. In ref. [110], an attempt is made to distinguish the modes associated with $\phi_{0\pm}$ boundary data on the basis of an 'energy functional'. However, one might find this result unsatisfactory given that the 'energy' is not conserved and so it does not give a covariant indicator by which we might measure back-reaction effects. So in the following we readdress the question of whether or not these scalar mode functions can successfully be regarded as fluctuations without taking into account any back-reaction effects. Our conclusion below is that for the principal or complementary series (*i.e.*, $M^2 > 0$), all of the modes can consistently introduce only a small perturbation throughout the entire time evolution, including the asymptotic dS regions. Therefore, the $\phi_{0\pm}$ modes cannot be distinguished on this basis. Similar arguments were previously sketched out in ref. [?].

To begin, consider a free scalar field propagating in a fixed dS background. In the asymptotic future, *i.e.*, $t \to +\infty$, the background metric (3.15) takes the form

$$ds^2 \simeq -dt^2 + \frac{1}{4}e^{2t/\ell}\, d\Omega_n^2. \qquad \text{(G.1)}$$

In this asymptotic region, the solutions of scalar field equation (F.2) take the form

$$\phi \simeq e^{-h_\pm t/\ell}\, \phi_{0\pm}(\Omega). \qquad \text{(G.2)}$$

Note that we are assuming that the spatial wave-functions $\phi_{0\pm}(\Omega)$ have reasonable behavior on the n-sphere, but the precise details will be unimportant.

Consider the contribution of the scalar to the right-hand-side of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \, T_{\mu\nu}(\phi) - \Lambda \, g_{\mu\nu} \ . \tag{G.3}$$

Taking ϕ to be a complex scalar field for simplicity, the scalar stress-tensor is

$$T_{\mu\nu} = 2\nabla_\mu \phi^* \nabla_\nu \phi - g_{\mu\nu} \left(g^{\sigma\rho} \nabla_\sigma \phi^* \nabla_\rho \phi + M^2 |\phi|^2 \right) \ . \tag{G.4}$$

Now to address the stability of the asymptotic dS geometry in eq. (G.1), we will compare the above source in Einstein's equations coming from the scalar field with that arising from the cosmological constant, which defines the background geometry.

In general, one might look at the components of the stress-tensor in an orthonormal frame so that they will correspond to the physical energy density and stresses observed by an inertial observer. Note, however, in the metric (G.1) above, the coordinate time t really is the proper time of co-moving observers and so by focussing on the energy density T_{tt}, one need not worry about transforming between coordinate and frame indices. Note that the analysis of the pressure components yields the same results as described for the energy density below. Proceeding from eq. (G.4), the relevant energy density is

$$T_{tt} = |\partial_t \phi|^2 + |\hat{\nabla}_k \phi|^2 + M^2 |\phi|^2 \ . \tag{G.5}$$

For comparison purposes, we also consider the contribution of the cosmological constant to Einstein's equations (G.3). The corresponding energy density is a constant:

$$T_{tt} = \Lambda = \frac{n(n-1)}{2\ell^2} \ . \tag{G.6}$$

Let us begin by considering scalar mass parameters corresponding to the complementary series, *i.e.*, $0 < M^2\ell^2 < n^2/4$, so that the exponents $h_\pm$ are real. With the wave-functions given in eq. (G.2), the energy density (G.5) becomes

$$T_{tt} \simeq \frac{1}{\ell^2} \left(h_\pm^2 + M^2 \ell^2 \right) e^{-2h_\pm t/\ell} \, |\phi_{0\pm}|^2 + O\left(e^{-2(h_\pm - 1)t/\ell} \right)$$

$$\simeq \frac{1}{\ell^2}\left(\frac{n^2}{2} \pm n\sqrt{\frac{n^2}{4} - M^2\ell^2}\right) e^{-2h_\pm t/\ell}\, |\phi_{0\pm}|^2\,. \tag{G.7}$$

Note that the spatial gradients in the stress-tensor (G.5) make sub-leading contributions above. Now this expression shows that the contribution of the scalar to the local energy density decays in the asymptotically dS region. Certainly this criterion only holds for the complementary series with $0 < M^2\ell^2 \le n^2/4$. When the mass is in this range, the perturbation which these modes introduce in the Einstein equations becomes diminishingly small in the asymptotic region. Of course, this result matches the naive intuition that one might derive from the simple observation that the mode functions are decaying as $t \to \infty$. In any event, if the scalar fluctuations begin as small perturbations, they remain a 'small' disturbance throughout the evolution of the spacetime (and the scalar field). Therefore this analysis confirms that the perturbative treatment of the scalar field is consistent and that the asymptotic dS geometry is stable against the introduction of such perturbations.

Similarly, we may consider mass parameters in the range of the principal series, *i.e.*, for $M^2\ell^2 > n^2/4$. Of course, in this case, the exponents are complex: $h_\pm = -n/2 \pm i\omega$. Now, the scalar energy density (G.5) becomes

$$\begin{aligned} T_{tt} &\simeq \left(\frac{|h_\pm|^2}{L^2} + m^2\right) e^{-nt/\ell}|\phi_{0\pm}|^2 + \cdots \\ &\simeq 2m^2 e^{-nt/\ell}\, |\phi_{0\pm}|^2\,. \end{aligned} \tag{G.8}$$

Hence we see that in this mass range, in accord with naive expectations, the disturbance of scalar modes to Einstein's equations is small. As above then, we conclude that the asymptotic dS geometry is stable against these perturbations.

For a tachyonic field with $M^2 < 0$, the exponents are real but h_- is negative. Therefore the corresponding scalar perturbations are growing exponentially in the asymptotic dS region. The asymptotic scalar energy density is again given by the expression in eq. G.7. Hence we see that the contribution of the ϕ_{0-} modes to the local energy density is growing without bound in this region of the spacetime. Therefore the energy density of the scalar field would quickly overwhelm that of the cosmological constant (G.6) irrespective of how small the perturbations began. Of course, all we can really conclude is that with the growth of these modes, the system will enter a nonlinear regime where the scalar field can no longer be consistently treated using a linearized perturbation analysis. In any event, we interpret this result as indicating that the exponential growth of these (or any tachyonic) fields produces an instability as one cannot expect the asymptotic spacetime geometry to resemble dS space (G.1). This confirms the naive expectations for tachyonic fields in dS space. This result is in complete agreement with the previous discussion stating that a successful analysis of such fields must consider the full nonlinear evolution of the scalar including its back-reaction on the spacetime geometry.

To reiterate our conclusions, let us note that the analysis here and in appendix F apply to global coordinates on dS space. Further the explicit mode functions for the principal and complementary series given in appendix F are well-behaved and bounded throughout the entire spacetime. Therefore in these cases with suitably 'small' mode coefficients, the above analysis indicates that the scalar energy density (G.5) will be negligible compared to that introduced by the background cosmological constant throughout the spacetime. Hence in these cases, the scalar field will provide a small perturbation throughout the entire evolution of the dS spacetime, not only in the asymptotic regions. It is also clear that this result applies for both the $\phi_{0\pm}$ modes and so these two distinct boundary data cannot be distinguished on this basis.

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