INVESTIGATION OF THE PRESSURE LOSSES
IN THE HOT HEAT EXCHANGER
OF THE
EXPERIMENTAL COAL BURNING GAS TURBINE

by

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PART I - ANALYSIS OF THE EXISTING EXCHANGER

1. INTRODUCTION

The following calculation of pressure losses in the hot heat exchanger is based on comparatively few data of mass flow and corresponding temperatures. Nevertheless it has been necessary to apply or develop no small number of formulae and rules in order to get near to the problem: how can the combined pressure loss in a baffled heat exchanger be computed? Unless one intends to adhere to some traditional formula and to multiply the result with a safety factor of say 2 or 2.5, there is hardly another way than to explore all possible sources of loss and to compute from the latter the overall probable loss by a process of weighing and comparing for which no theoretical background can be given.

The method developed here is, then, at least as much a trial and error run as it appears to a founded on theory on behalf of the sizeable number of formulae and deductions. Among a series of books and publications pertaining to the theme of cross flow pressure loss there was none dealing with flow over baffles; there was even none dealing with inclined cross flow and none which would aim at adapting the established equations for cross flow losses over the ideal rectangular bank of tubes to the circular arrangement in a tube and shell exchanger. Different spacing of tubes, different shapes like fins, blades, corrugated sheets present an unlimited variety pertaining to the coefficients in the equations for heat transfer and pressure loss. To consider, moreover, different angles of incidence, simultaneous axial and cross flow, sudden changes of direction together with the effects of edges, orifices, flow past a disc, seems a hopeless task. No coherent formula will ever be set up, it seems, which would cover at once all these effects and allow for numerical evaluation, under given flow conditions and apparatus dimensions. Through this is what is required for the calculation of the pressure drop in a daily life baffled exchanger operating under normal conditions.

The way of solving this problem could be purely deductive only. All sources of loss were investigated individually, compared with corresponding ones and, finally put together. In fact, there have been derived in section 9 the "normalized coefficients" which, upon multiplication with $G^2/2g$, will give directly the partial pressure loss, of a certain kind, comprising a number of consecutive compartments or stations. They might be an easy means to apply the derived relations for other mass flow, pressure and temperatures in the same heat exchanger. To the utmost they may be valid under special restrictions for a part of a similar exchanger.

If it is possible--and only the experiment will tell it--to present pressure loss, be it the partial or overall loss, in terms of "normalized coefficients", then the method developed in the following paragraphs can be said to be satisfactory. However, this is not the only criterion. There are other ways of cross checking assumptions. The heat balance can be arrived at from different directions. The relation between useful heat transfer and radiation losses through a composite wall flushed by cooling air can only be clarified by multiple checking. There still might be an axial heat transfer by conduction within the metal walls. The point to point temperature distribution in the whole of the exchanger is the fundament upon which any calculation of pressure changes has to be based. Therefore, two chapters, 3 and 5, are devoted to this problem. Chapter 2 gives an outline of the principal individual pressure loss equations to be applied later on in chapters 7 and 8. However, before being ripe for application they have to be shaped in order to fit the existing apparatus, under empirically assumed flow conditions. In chapter 10 the heat transfer coefficients are handled. However, they are but a means to arrive at a certain cross check, for the purpose of comparing the flow pattern at various mass velocities.

Evidently the Hot Heat Exchanger is designed like a plant process apparatus, not as a piece of equipment for a turbo-compressor station. Roughly speaking, half of the total pressure loss could be eliminated by internal streamlining without affecting its heat transfer capacity. This might be open to proof by way of calculation.

2. MATHEMATICAL AND EMPIRICAL RELATIONS RE. PRESSURE LOSSES

Pressure changes produced along the path of a fluid in motion may be due to three different causes:

A) Change of energy and momentum applied to an ideal fluid with streamline motion. Call the ensuing pressure variations

HYDRODYNAMIC CHANGES; They are brought about by

1) Elevation against gravity
2) Change of flow area
3) Change of velocity due to variation of density
4) Work supplied to or abstracted from the fluid
5) Any combination of these

B) Change of energy and momentum applied to a real fluid with vortices and turbulence set up. Call the ensuing pressure variations

LOCAL CHANGES; They are brought about by

1) Sudden enlargement
2) Sudden contraction
3) Change of direction
4) Flow over a weir
5) Flow through orifice
6) Flow past any surface or body
7) Any combination of these

C) Transfer of energy and momentum from a real fluid, be it in turbulent or laminar motion, to its boundary walls. Call the ensuing pressure variations

FRICTIONAL CHANGES; They are brought about by

1) Skin friction due to motion along walls, tubes, plates
2) Skin friction due to motion across tubes or other obstructions
3) Any combination of these

In almost any real flow problem losses under the headings A), B) and C) occur simultaneously and except for HYDRODYNAMIC CHANGES the established relations for pressure loss often include several effects at the time.

Example

Flow through a valve or a header compartment will produce losses acc. to B) 1-2-3 and probably other ones acc. to A) 2 and C) 2.

Flow over a bank of tubes will give rise to pressure changes acc. to C) 2 and B) 1-2.

It is not difficult to set up equations for each kind of pressure loss under B) and C) and to express the result in terms of flow geometry, flow velocity and property values of fluid and/or boundary wall.

It is much more difficult to present in one formula pressure losses due to similar kind, but different shape and surface quality of obstructions.

It is incomparably more difficult to express in an analytical way pressure losses produced by simultaneous action of several obstructions of different kind and shape, let alone those under simultaneous influence of effects acc. to B) and C). Imagine, e.g. the effect of a baffle in the path of a flow across tubes which has a longitudinal and perpendicular component. The art, then, is to estimate the degree by which the various losses have to be taken into account rather than to add them indiscriminately.

Rules

a) If the flow pattern--or the time average of the velocity field--will be uneffected by adiabatic* introduction of another cause for pressure change, the individual pressure losses may be added.

b) They may still be added if the relative flow pattern remains unchanged; that means, if the relative velocity distribution in each cross section remains the same as before.

c) Otherwise the different pressure losses may not be added quantitatively.

Examples

to a) A fluid under a certain hydrostatic pressure passes through a horizontal tube and leaves it with a certain velocity, into the open atmosphere. No friction is involved. If friction will be introduced slowly and the hydrostatic pressure increased in such a way that the average velocity of efflux remains the same, the total pressure drop is exactly equal to the sum of the velocity head and friction head. The only compromise to be accepted is that formerly it was the velocity of efflux and now it is the average velocity over the efflux cross area which shall be equal.

to b) A fluid flows frictionless through a gradually expanding duct. The flow is one dimensional.

$\frac{\partial v}{\partial t} > 0$ (v, specific volume; t, temperature)

At constant temperature there is a certain pressure rise due to change of flow area. If a temperature gradient be set up in the direction of flow, a non-isothermal pressure change is produced and the ensuing pressure variation is exactly equal the sum of both changes mentioned separately under A) 2 and 3. Gravitational changes may as well be incorporated.

* The change shall be so slow that the system is always in equilibrium. Reversing the change shall bring the system back into the previous state.

to c) A real fluid with a very low viscosity flows past a disc. Since there can be no infinite pressure gradient near the rim of the disc, form drag and a corresponding pressure loss will be present even if the wake is definitely at rest. The viscosity may then be increased, for example through change of temperature. The line of flow separation changes, vortices are set up along the line and surface-drag is produced in addition. The total pressure change can only be calculated if the flow pattern is known in detail. Another question is if this knowledge is deduced by experiment or analytically. But in either case the flow pattern will have so much changed that combination of the previous form drag and the additional surface drag is virtually impossible.

THE FANNING EQUATION

Limiting the considerations to cases fitting into Rule a) and b), the fanning equation expresses the combined pressure variation due to HYDRODYNAMIC and FRICTIONAL CHANGES.

1. $$-v\,dp = g\,dZ + \frac{V\,dV}{\alpha} + f\,\frac{V^2}{2}\,\frac{\Delta L}{r_{hy}}$$

It is valid for compressible and incompressible fluids.

Symbols

- ρ : density
- v : specific volume
- p : pressure
- T : absolute temperature
- R : gas constant per lb.
- Z : elevation
- V : velocity
- α : conversion factor, ≈ 1 for turbulent flow
- ε : abbreviation for $\frac{dZ}{dL}$
- κ : abbreviation for $\frac{dT}{dL}$
- g : gravitational constant
- M : mass flow
- G : mass velocity
- A : area
- L, ΔL : length
- d : diameter
- d_{hy} : hydraulic diameter
- r_{hy} : hydraulic radius
- f : frictional coefficients
- F : functions of L
- f_m-p_m-R_m : mean values

The integration of (1) for different conditions follows. The results are obtained in poundals per sq.ft. (p/ft^2). For derivation see Appendix 1.

Incompressible case: $\frac{\partial v}{\partial p} = 0$, $\frac{\partial v}{\partial T} = 0$

AREA CONSTANT -- TEMPERATURE CHANGES

2. $$p_1 - p_2 = \frac{1}{v}\left[g(Z_2 - Z_1) + f_m\,\frac{V^2}{2}\,\frac{(L_2 - L_1)}{r_{hy}}\right]$$

AREA -- TEMPERATURE CHANGE

3. $$p_1 - p_2 = \frac{1}{v}\left[g(Z_2 - Z_1) + \frac{1}{2\alpha}(V_2^2 - V_1^2) + f_m\,\frac{(V_1^2 + V_2^2)\,\Delta L}{4\,r_{hy}}\right]$$

Compressible Case: $\frac{\partial v}{\partial p} \neq 0 \qquad \frac{\partial v}{\partial T} \neq 0$

AREA CONSTANT -- TEMPERATURE CHANGES

4. $$p_1 - p_2 = \frac{gp}{R_m}\left(\frac{dZ}{dL}\right)_m \int_{L_1}^{L_2} F_1(L)dL + \frac{G^2 R_m}{\alpha\, p_m}\int_{L_1}^{L_2} F_2(L)dL + \frac{G^2 f_m R_m}{2 p_m r_{hy}}\int_{L_1}^{L_2} F_3(L)dL$$

$$F_1(L) = \frac{1}{T(L)}\ ; \qquad F_2 = \frac{dT}{dL}\ ; \qquad F_3 = T(L)$$

AREA CHANGES -- CONSTANT TEMPERATURE

5. $$p_1 - p_2 = g\rho(Z_2 - Z_1) - \frac{M^2}{8\pi^2\rho\alpha}\int_{L_1}^{L_2} F_4(L)dL + \frac{M^2 f}{32\pi^2\rho}\int_{L_1}^{L_2} F_5(L)dL$$

$$F_4(L) = \frac{1}{r_{hy}^5}\frac{dr_{hy}}{dL}\ ; \qquad F_5(L) = \frac{1}{r_{hy}^5}$$

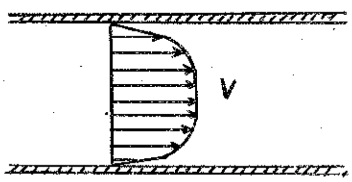
The case of changing temperature and area is too tedious. If there are no sudden enlargements, contractions, etc.--which are anyhow excluded from the range of the FANNING EQUATION--this case may be reduced either to a combination of the previous ones or solved by step to step evaluation.

AREA CONSTANT -- LINEAR TEMPERATURE CHANGE

$$\frac{dT}{dL} = \mathcal{K} \qquad \frac{dZ}{dL} = \mathcal{E} \qquad \mathcal{K}, \mathcal{E} \text{ constant}$$

6. $$p_1 - p_2 = \frac{g\mathcal{E}}{\mathcal{K}}\frac{T_1}{v_1}\ln\frac{T_2}{T_1} + \frac{1}{\alpha}(\rho_2 V_2^2 - \rho_1 V_1^2) + \frac{f_m}{4}(\rho_1 V_1^2 + \rho_2 V_2^2)\frac{\Delta L}{r_{hy}}$$

Neglecting the effect of gravity (6) becomes the familiar expression for non-isothermal pressure drop plus frictional losses. In a heat exchanger temperatures may not generally be approximated by a linear function and (4) may be applied for the tube side provided $T(L)$ is known and α be derived from the turbulent velocity profile. $T(L)$ is given in chapter 3, for the cold and the hot fluid as well as for parallel and counter flow.



$$\alpha = \frac{V_m^2}{(V^2)_m} = \frac{1}{A}\frac{(\int V dA)^2}{\int V^2 dA}$$

dA = area element

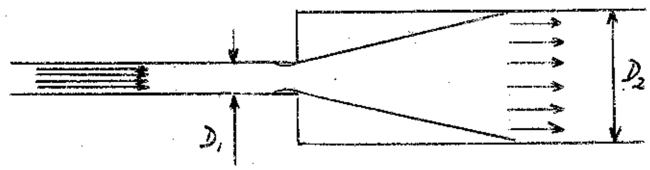
The more the velocity profile is uniform the more will α be near to unity. $\alpha \leq 1$. Strictly speaking this correction should be applied in <u>any</u> expression involving V^2 if V_m, only, is given. Usually it is neglected.

For the air side of the Hot Heat Exchanger none of the formula (2) to (5) can be applied since the flow area changes in a rather irregular manner. Pressure losses, here, have to be estimated apart from HYDRODYNAMIC CHANGES, in terms of

LOCAL CHANGE COEFFICIENTS

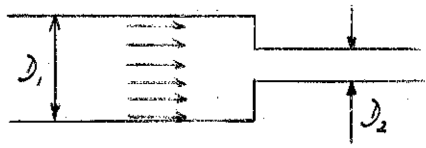
These are expressed as a multiple of the velocity head $\frac{\rho V^2}{2}$ or $\frac{G^2}{2\rho}$. Derivation or Reference: See Appendix 2.

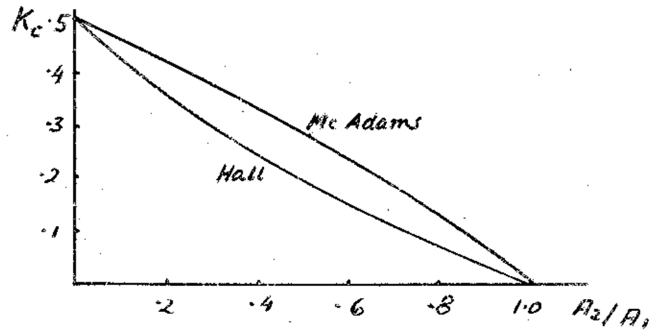
SUDDEN ENLARGEMENT



7. $$-\Delta p = \frac{\rho V_1^2}{2}\left(1-\frac{D_1^2}{D_2^2}\right)^2$$

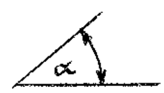
SUDDEN CONTRACTION





8. $$-\Delta p = K_c \frac{\rho V^2}{2}$$

CHANGE OF DIRECTION by angle α



9. $$-\Delta p = \frac{\sqrt{2}}{2}\sqrt{1-\cos\alpha}\;\frac{\rho V^2}{2}$$

RETURN LOSS for $\alpha = 180°$ the pressure drop will be

10. $$-\Delta p = 1/2\,\frac{\rho V^2}{2}.$$

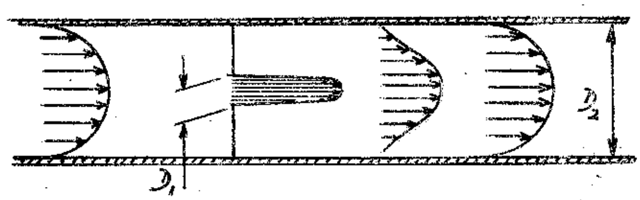
This is only correct for smooth pipes. For return through a header box the combined loss may be up to

10a. $$-\Delta p = 2\,\frac{\rho V^2}{2}.$$

For other flow pattern the loss will be in between.

FLOW THROUGH AN ORIFICE

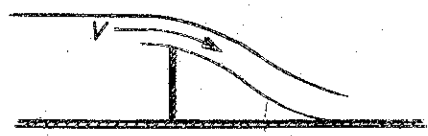
11. $$-\Delta p = .8\,\frac{.25}{(D_1/D_2)^2}\,\frac{\rho V^2}{2}$$



V to be taken at orifice. For $D_1/D_2 = .5$, the coefficient becomes .8. D_1/D_2 must not approach ONE.

FLOW OVER A WEIR, in analogy to orifice effect

11a. $$-\Delta p = .54 \frac{\rho V^2}{2}$$



FLOW PAST A DISC, in an infinitely broad stream

12. $$-\Delta p = 1.12 \frac{\rho V^2}{\rho} = C_D \frac{\rho V^2}{2}$$ V = Velocity of undisturbed stream

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FLOW PAST TANDEM DISCS

12a. D = Diameter of disc
L = Distance between discs

L/D	C_D
0	1.12
1	.93
2	1.04
3	1.54

For NON-INFINITELY BROAD STREAM the pressure loss is larger because of the higher velocity around the rim and the greater turbulence produced in the wake.

FRICTIONAL CHANGES

These are usually divided into axial flow losses and cross flow losses. Nothing was found in literature regarding other angles of incidence. Formulae and values for cross flow mentioned all refer to a laboratory set up with a well defined flow pattern, perpendicular to a rectangular bank of tubes. Nothing was found in literature with regard to the flow conditions in a usual heat exchanger, where the width of the rows changes continuously over the diameter of the shell, except one mention in KERN, Process Heat Transfer.

Two things have, therefore, to be done here:

1) Comparison of cross flow pressure loss equations of different authors.

2) Adaptation of the formulae for the ideal rectangular tube bank to the tube lay out in a real heat exchanger.

Comparison of the equations of different authors

The pressure loss may be presented in the form

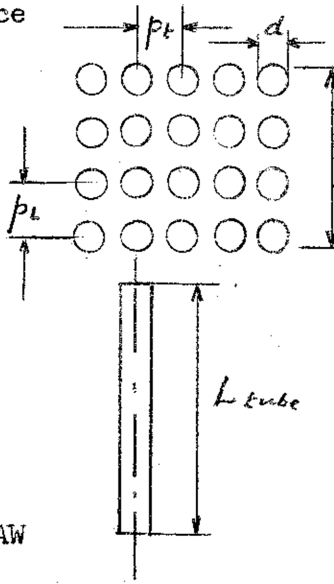
13. $$-\Delta p = 4fN \frac{G^2}{2\rho} \left[p/ft^2\right]$$ JACOB, MCADAMS p. 126, ASME, Vol. 60, p. 385

14. $$-\Delta p = \frac{f\ N\ G^2 \times 10^{-8}}{10.84 \times \rho} \left[''W\right]$$ HUGE, ASME, Vol. 59, 1937, p. 575

15. $$-\Delta p = f \frac{G^2}{2} \frac{L}{d_{hy}} \left(\frac{\mu}{\mu_w}\right)^{-.14} \left(\frac{d_{hy}}{p_t}\right)^{.4} \left(\frac{p_L}{p_t}\right)^{.6} \left[p/ft^2 \right]$$

GUNTER, SHAW, ASME, Vol. 67, 1945, P. 644

with
- G : <u>lb/hr in (14), lb/se in (13),(15)</u> mass velocity in narrowest place
- N : Number of rows in direction of flow
- p_L : Longitudinal pitch, center to center distance
- p_t : Transverse pitch
- μ : Viscosity of fluid at bulk temperature
- μ_w : Viscosity of fluid at wall temperature
- L : Length across rows



with regard to (13),(15) :

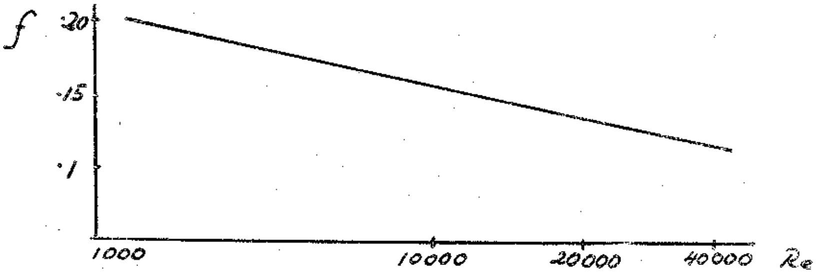
13a. $$f = \left[.25 + \frac{.1175}{\left(\frac{p_t}{d} - 1\right)^{1.08}} \right] \left(\frac{Gd}{\mu}\right)^{-.15}$$ acc. to JACOB

13b. $$f = \left[.23 + \frac{.11}{\left(\frac{p_t}{d} - 1\right)^{1.08}} \right] \left(\frac{Gd}{\mu}\right)^{-.15}$$ acc. to McADAMS

15a. $$f = 2 \times .96 \left(\frac{Gd_{hy}}{\mu}\right)^{-.145}$$ acc. to GUNTER, SHAW

HUGE and others -- comp. ASME, Vol. 59, p. 567 -- give f as function of R_e in a graph.

14a.



Actually f depends somewhat upon the longitudinal distance between the rows, but the various graphs are very near to each other. The given curve is correct down to $p_L = 1.25$ d. For triangular pitch p_L is still smaller, $= \sqrt{3}/2\ p_t = 1.083$ d and the resistance may increase <u>very rapidly</u> with decreasing p_L.

In ASME, Vol. 59, p. 587, GRIMISON reports values for f at "equal flow area in transverse and diagonal direction," that is for triangular pitch. His values, over the range of R_e = 8,000 to 40,000 are almost identical with those of HUGE.

(13a) and (13b) give no account of p_L. (15) represents a later development and considers wall temperature, p_L/p_t ratio, d_{hy}/p_t ratio and, most notably, the hydraulic diameter d_{hy} in R_e, as the more congenial characteristic length instead of d. Results of widely different tube arrangements and flow conditions are given uniformly by f vs R_e acc. to (15). Again, as these results are derived from laboratory set ups, even (15) will not yield a necessarily correct figure for an industrial exchanger.

Example

Air at 800°F, p = 26 psia, $G = 6 \frac{lb.}{ft^2\ se}$, triangular pitch,

$p_t = p_l = 1.25d$, 8 rows of tubes, d = 1"

$-\Delta p = \mathscr{C} \frac{G^2}{2\rho}$. Compare $\mathscr{C}$ in (13), (14), (15) and (16). f = .15 in (16) refers to formula (14).

$d = .0833$ ft, $\mu = 2.23 \times 10^{-5}$, $\left(\frac{\mu}{\mu}\right)^{-.14} \approx 1$

$d_{hy} = .06$ ft

$p = .104$ ft $\qquad G^2 = 6^2\left[\frac{lb.}{ft^2 se}\right]^2 \doteq 6^2 \times 13 \times 10^6 \left[\frac{lb.}{ft^2\ hr}\right]^2$

$p_t/p_l = 1$

$p_t/d = 1.25 \qquad R_e(d) = 2.24 \times 10^4$, $R_e(d_{hy}) = 1.61 \times 10^4$

Acc. to (13a):

$$\frac{Gd}{\mu} = 2.24 \times 10^4, \left(\frac{Gd}{\mu}\right)^{-.15} = .222$$

$$f = \left(.25 + \frac{.1175}{.25^{1.08}}\right) \times .222 = (.25 + .525) \times .222 = .172$$

$$\mathscr{C}_{(13a)} = 4 \times .172 \times 8 = \underline{5.5}$$

Acc. to (13b):

$$f = .222\left(.23 + \frac{.11}{.224}\right) = .222\ (.23 + .492) = .16$$

$$\mathscr{C}_{(13b)} = 4 \times .16 \times 8 = \underline{5.12}$$

Acc. to (14):

$$-\Delta p = \frac{G^2}{2\rho} \frac{f\ N \times 13 \times 10^6 \times 10^{-8}}{5.42} 167$$

f from (14a) at $R_e = 22{,}400$ equals .12

$$\mathscr{C}_{(14)} = \frac{8 \times .12 \times 13 \times 167 \times 10}{5.42} = \underline{3.84}$$

Acc. to (15):

$$\mathscr{C}_{(15)} = 2 \times .96 \times \left(\frac{6 \times .06}{2.23 \times 10^{-5}}\right)^{-.145} \frac{(8-1)\sqrt{3} \times 1.25\ d}{2 \times .72\ d}(.575)^{.4}$$

$$\mathscr{C}_{(15)} = 1.92 \times .245 \times 10.5 \times .801 = \underline{3.96}$$

For $G = 10 \frac{lb.}{ft^2\ sc}$ $\quad R_e(d) = 3.74 \times 10^4 \quad R_e(d_{hy}) = 2.69 \times 10^4$

$$\mathscr{C}_{(13a)} = 5.5\ \frac{.206}{.222} = \underline{5.1} \qquad \mathscr{C}_{(13b)} = 5.12\ \frac{.206}{.222} = \underline{4.75}$$

$$\mathscr{C}_{(14)} = 3.84 \frac{.114}{.12} = \underline{3.66}$$

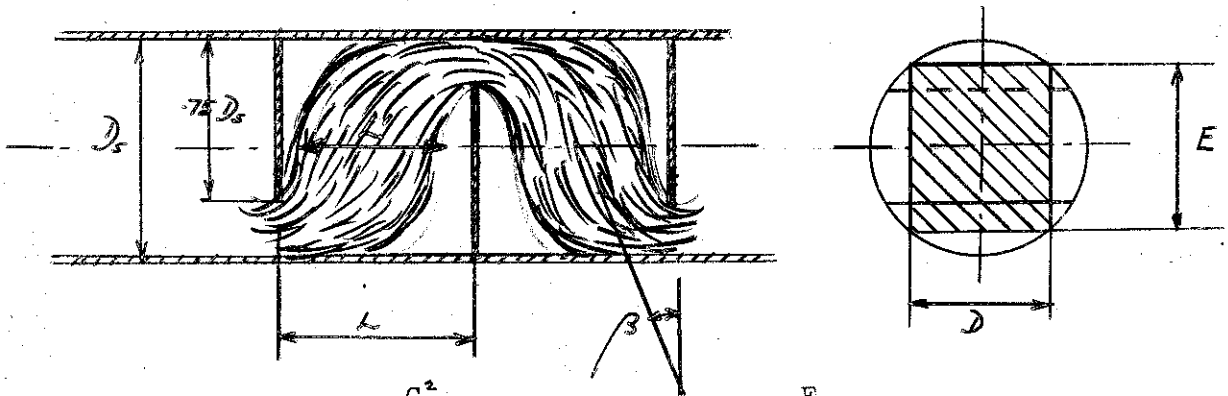
$$\mathscr{C}_{(15)} = 1.92 \times .228 \times 10.5 \times .801 = \underline{3.69}$$

It will be noticed that $\mathscr{C}$ decreases in about the same ratio for increasing R_e, in the various equations. $\mathscr{C}_{(15)}$ may be increased to 4.5 and 4.2 for G = 6 and 10, respectively, and thus brought nearer to McADAMS values, $\mathscr{C}_{(13b)}$, by multiplying it with 8/7; 8 is number of tube rows, 7 number of distances between rows. Regarding this point there is no very clear distinction in the litterature.

To go safe, $\mathscr{C}_{(13b)}$ should be applied.

Adaptation to the real heat exchanger

1. The 25% cut baffles exchanger:



$$\Delta p_{cross} = 4\, f_{cross} \times N \frac{G_{cr}^2}{2\rho} \qquad N = \frac{E}{p_L} \qquad p_L = p$$

$$G_{cr} = \frac{M}{DL'}$$

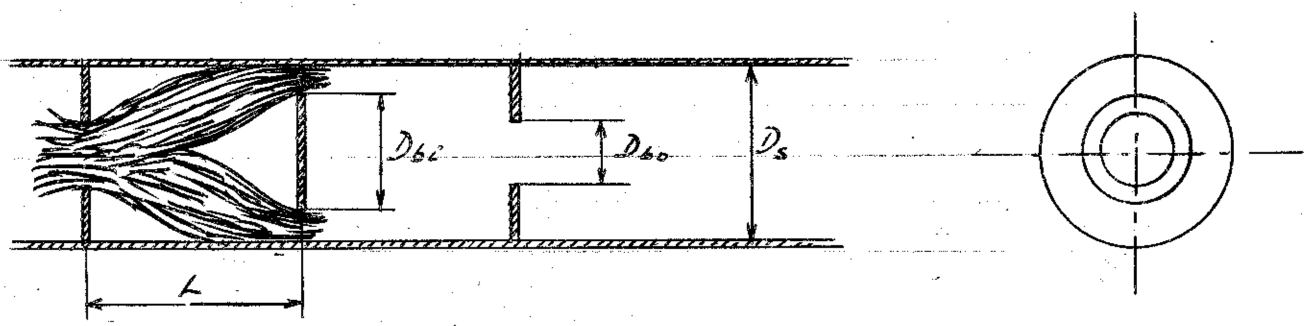
$$\Delta p_{axial} = f_{axial} \times \frac{G_{ax}^2}{2\rho} \frac{L}{r_{hy}}$$

$$\begin{cases} E = .71\, D_s \quad (D_s = D_{shell}) \\ D = .67\, D_s \\ L' = .75\, L \text{ for } \beta > 30^\circ \\ L' = L \text{ for } \beta < 30^\circ \\ G_{ax} = \frac{M}{DE} \end{cases}$$

16.

These equations do not consider either the constriction of flow area by tubes or the true average value for G_{cr} and G_{ax}. f_{ax} is known from tables; f_{cr} might be taken from either (13a), (13b), (14a) or (15a).

2. The disc and doughnut baffle exchanger:

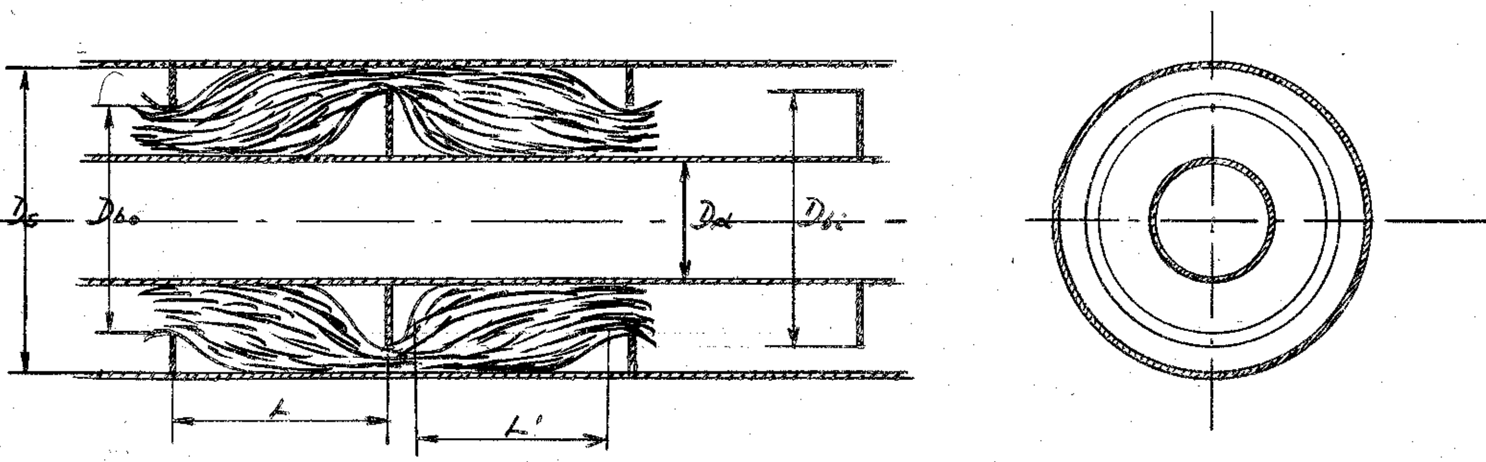


17.

$$\begin{cases} N = \dfrac{D_s + D_{bi} - D_{bo}}{4\,p_L} \\ G_{cr} = \dfrac{2M}{L\pi(D_{bi} + D_{bo})} \\ G_{ax} = \dfrac{12M}{\pi(2D_s^2 + D_{bo}^2 - D_{bi}^2)} \end{cases}$$

Remark: same as for (16)

3. The exchanger with central duct:



18.

$$\begin{cases} N = \dfrac{(D_s + D_{bi}) - (D_d + D_{bo})}{4\,p} \\ G_{cr} = \dfrac{2M}{L'\pi(D_{bi} + D_{bo})} \\ G_{ax} = \dfrac{12M}{\pi(2D_s^2 - 2D_d^2 + D_{bo}^2)} \end{cases}$$

Remark: same as for (16)

3. METHEMATICAL RELATIONS REGARDING THE TEMPERATURES IN A HEAT EXCHANGER

When handling a heat exchange problem the four temperatures, at inlet and outlet of the two fluids are usually given. The aim, then is to calculate the necessary area of transfer for the given temperature range and the flow conditions and property values pertaining to it.

Nothing can then be said of the actual temperature either of the two fluids or of the wall separating them at any distance from inlet or outlet.

For detailed calculation of heat transfer and pressure loss knowledge of the temperature at any place of the system is essential. Following are, therefore, given the functions for all temperatures, temperature differences and so on which will be present in a heat exchanger.

It is assumed that there exists a well defined area of transfer and that the transfer coefficients as well as the specific heat of both fluids are constant. The equations are built up in such a way that the inlet temperatures only of the fluids are supposed to be known and, of course, the initial temperature difference. The ensuing temperature changes down to the outlet temperatures may then be found analytically.

No derivations are given since no problems deserving special mathematical interest are involved. However, the limiting cases for the infinitely long exchanger, for equal and zero individual transfer coefficients and for equal heat flow values, are mentioned in detail.

In the last section the derivatives of the temperature functions are presented. They may be instrumental for proper integration of several of the pressure loss equations in chapter 2.

SYMBOLS

T : hot fluid temperature
t : cold fluid temperature
T_i : initial h. f. temp.
T_e : end h.f. temp.
t_i : initial c. f. temp.
t_e : end c.f. temp.
θ : temperature diff., $(T-t)$
θ_i : initial t.d., $(T_i - t_i)$
θ_e : end t.d., $(T_e - t_e)$
θ_1 : $T_w - t$
θ_2 : $T - T_w$
($\theta_1 + \theta_2 = \theta$)
T_w : wall temperature
ϑ_1 : $t - t_i$
ϑ_2 : $T_i - T$

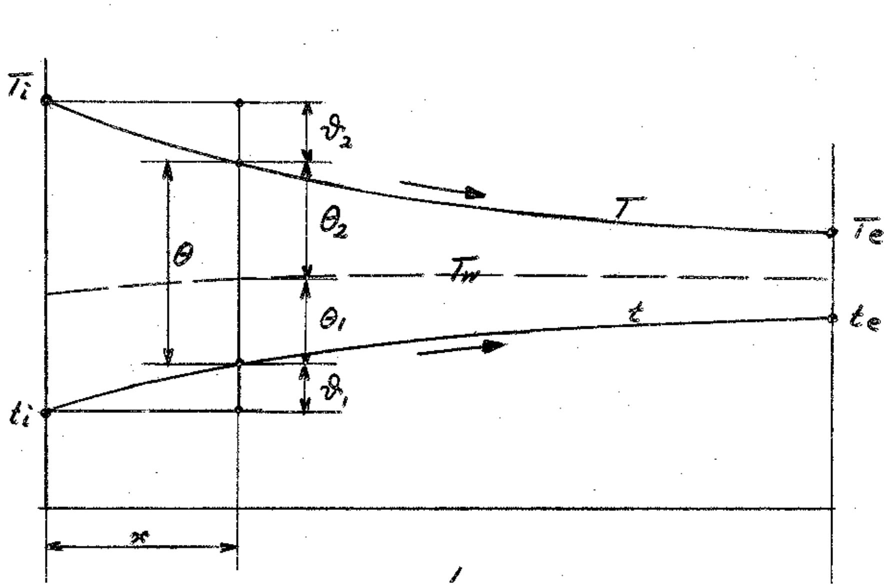
x : distance
L : length of exchanger
A : heat transfer area
a : A/L, area per unit length
M_1 : cold mass flow
M_2 : hot mass flow
C_{p_1} : cold fluid specific heat
C_{p_2} : hot fluid specific heat
W_1 : $M_1 c_{p_1}$, c.f. heat flow
W_2 : $M_2 c_{p_2}$, h.f. heat flow
h_1 : transf. coeff. wall → c. fluid
h_2 : transf. coeff. h. fluid → wall
U : overall heat transf. coeff.
C : abbreviation, $aU\left(\frac{1}{W_1} + \frac{1}{W_2}\right)$

These are employed for parallel flow. For counter flow

θ_i is replaced by θ_l (left hand) $\qquad \theta_l = T_i - t_e$
θ_e " " " θ_r (right hand) $\qquad \theta_r = T_e - t_i$
C " " " $aU\left[\frac{1}{W_2} - \frac{1}{W_1}\right]$

θ_i remains, otherwise, as $\Delta T = T_i - t_i$

PARALLEL FLOW



1. $\theta = \theta_i \; e^{-cx}$ $\qquad\qquad \theta_e = \theta_i \; e^{-cl}$

2. $\vartheta_1 = \theta_i \dfrac{W_2}{W_1 + W_2}(1-e^{-cx})$

3. $\vartheta_2 = \theta_i \dfrac{W_1}{W_1 + W_2}(1-e^{-cx})$

4. $t = t_i + \theta_i \dfrac{W_2}{W_1 + W_2}(1-e^{-cx})$

5. $T = T_i - \theta_i \dfrac{W_1}{W_1 + W_2}(1-e^{-cx})$

5a. For $x \rightarrow \infty$

$$t_\infty = t_i + \theta_i \frac{W_2}{W_1 + W_2}$$

$$T_\infty = T_i - \theta_i \frac{W_1}{W_1 + W_2}$$

$$t_\infty = T_\infty$$

6. $\theta_1 = \theta_i \dfrac{h_2}{h_1 + h_2} e^{-cx}$

7. $\theta_2 = \theta_i \dfrac{h_1}{h_1 + h_2} e^{-cx}$

8.

$$T_w = t_i + \theta_i\left(\frac{h_2}{h_1 + h_2} - \frac{W_2}{W_1 + W_2}\right)e^{-cx} + \theta_i \frac{W_2}{W_1 + W_2}$$

$$T_w = t_\infty + \theta_i\left(\frac{h_2}{h_1 + h_2} - \frac{W_2}{W_1 + W_2}\right)e^{-cx}$$

8a. for $\dfrac{h_2}{h_1 + h_2} = \dfrac{W_2}{W_1 + W_2}$ $\qquad T_w = t_\infty$, constant

8b. for $\dfrac{W_2}{W_1 + W_2} > \dfrac{h_2}{h_1 + h_2}$ $\qquad \dfrac{d\,T_w}{dx} > 0$, for <u>all</u> <u>$x$</u> and vice versa

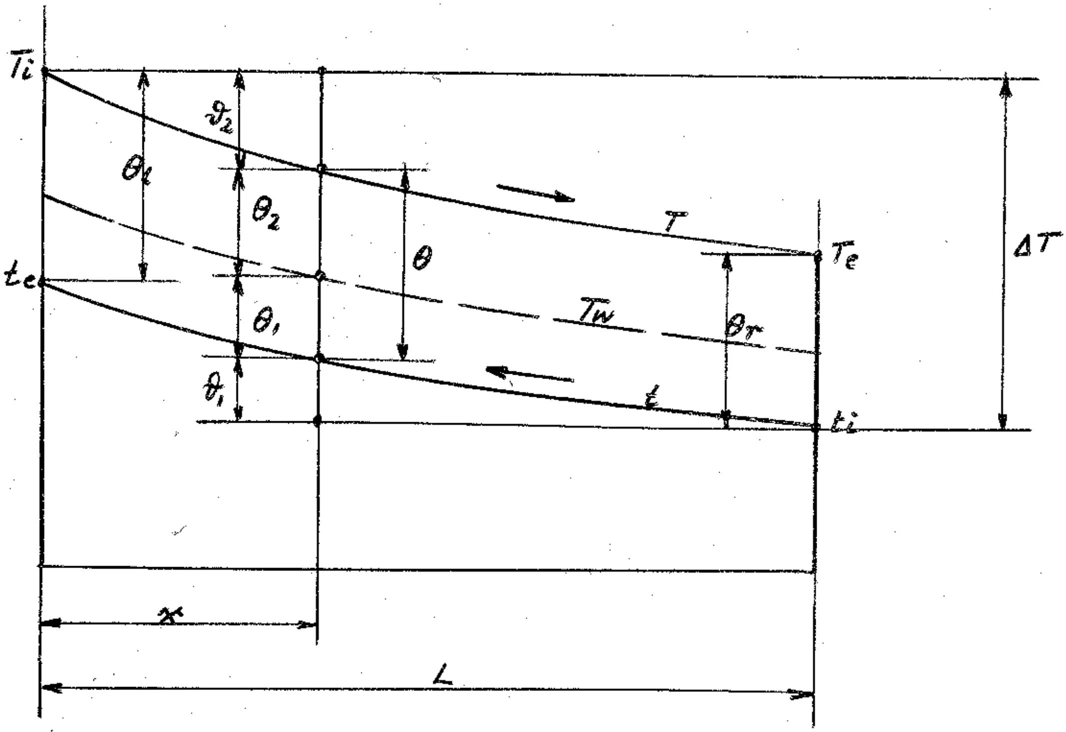
for $W_1 = \infty$ compared with W_2, as in the case of a boiler,

4a. $t = t_i + \dfrac{W_2}{W_2 + \infty}\theta_i\,(1-e^{-cx}) = t_i$

for $h_1 = \infty$ compared with h_2, as in the case of an economizer,

8c. $T_w = t + \theta_i = t + \theta_i \dfrac{h_2}{h_2 + \infty} e^{-cx} = t$

COUNTER FLOW



9. $\Theta = \Theta_L \, e^{-cx}$ $\qquad \Theta = \Theta_r \, e^{c(L-x)}$

10.
$$\Theta_L = \Delta T \, \frac{1 - \frac{W_2}{W_1}}{1 - \frac{W_2}{W_1} e^{-cL}}$$

$$\Theta_r = \Delta T \, \frac{1 - \frac{W_2}{W_1}}{e^{cL} - \frac{W_2}{W_1}}$$

9a. $$\Theta = \Delta T \, \frac{1 - \frac{W_2}{W_1}}{1 - \frac{W_2}{W_1} e^{-cL}} \, e^{-cx}$$

11.
$$\vartheta_{1L} = \Delta T \, \frac{1 - e^{-cL}}{\frac{W_1}{W_2} - e^{-cL}}$$

$$\vartheta_{2r} = \Delta T \, \frac{1 - e^{-cL}}{\frac{W_2}{W_1} - e^{-cL}}$$

for $W_1 = W_2$, the e = function has to be developed into a series and $\Theta_L, \Theta_r, \vartheta_{1L}, \vartheta_{2r}$ will become

10a. $$\theta_L = \Delta T \frac{1}{1 + \frac{UA}{W}} = \theta_r$$

11a. $$\vartheta_{1L} = \Delta T \frac{1}{1 + \frac{W}{UA}} = \vartheta_{2r}$$

9b. $$\theta = \Delta T \frac{1}{1 + \frac{UA}{W}}, \quad \text{constant}$$

12. $$\begin{cases} \vartheta_1 = \Delta T \dfrac{e^{c(L-x)} - 1}{\frac{W_1}{W_2} e^{cL} - 1} \\ \vartheta_2 = \Delta T \dfrac{e^{-cx} - 1}{\frac{W_2}{W_1} e^{-cL} - 1} \end{cases}$$

13. $$t = t_i + \Delta T \frac{e^{c(L-x)} - 1}{\frac{W_1}{W_2} e^{cL} - 1}$$

14. $$T = T_i - \Delta T \frac{e^{-cx} - 1}{\frac{W_2}{W_1} e^{-cL} - 1}$$

for $W_1 = W_2$

12a. $$\vartheta_1 = \Delta T \frac{Ua(L - x)}{UA + W} \qquad \vartheta_2 = \Delta T \frac{U a x}{UA + W}$$

13a. $$t = t_i + \Delta T \frac{Ua(L - x)}{UA + W}$$

14a. $$T = T_i - \Delta T \frac{U a x}{UA + W}$$

15. $$\theta_1 = \frac{h_2}{h_1 + h_2} \Delta T \frac{1 - \frac{W_2}{W_1}}{1 - \frac{W_2}{W_1} e^{-cL}} e^{-cx}$$

$$\theta_2 = \frac{h_1}{h_1 + h_2} \Delta T \frac{1 - \frac{W_2}{W_1}}{1 - \frac{W_2}{W_1} e^{-cL}} e^{-cx}$$

16. $$T_w = t_i + \Delta T \frac{\left(1 + \frac{h_2}{h_1 + h_2} \frac{W_1 - W_2}{W_2}\right) e^{c(L-x)} - 1}{\frac{W_1}{W_2} e^{cL} - 1}$$

for $h_1 = h_2$ θ_1 will be equal to θ_2, trivial.

for $W_1 = W_2$ θ_1 and θ_2 will become constant and T_w a linear function of x, just as t and T.

(In the parallel flow case t, T, T_w will never become linear.)

$W_1 = W_2$:

15a. $$\theta_1 = \frac{h_2}{h_1 + h_2} \Delta T \frac{1}{1 + \frac{UA}{W}} ; \qquad \theta_2 = \frac{h_1}{h_1 + h_2} \Delta T \frac{1}{1 + \frac{UA}{W}}$$

16a. $$T_w = t_1 + \Delta T \frac{\frac{h_2}{h_1 + h_2} + \frac{Ua}{W}(L - x)}{1 + \frac{UA}{W}}$$

If the temperatures t_i, t_e, T_i, T_e in a heat exchanger are given, (13), (14) and (16) may be used to calculate T_w and all intermediate temperatures.

DERIVATIVES OF THE TEMPERATURE FUNCTIONS

Parallel flow:

17. $$\frac{d\theta}{dx} = -\theta_i C e^{-cx}$$

18. $$\frac{dt}{dx} = \theta_i C \frac{1}{1 + \frac{W_1}{W_2}} e^{-cx}$$

19. $$\frac{dT}{dx} = -\theta_i C \frac{1}{1 + \frac{W_2}{W_1}} e^{-cx}$$

20. $$\frac{dT_w}{dx} = -C\left(\frac{h_2}{h_1 + h_2} - \frac{W_2}{W_1 + W_2}\right)e^{-cx}$$

Counter flow:

21. $$\frac{d\theta}{dx} = -C \frac{1 - \frac{W_2}{W_1}}{1 - \frac{W_2}{W_1} e^{-cL}} e^{-cx}$$

22. $$\frac{dt}{dx} = -C \frac{e^{cL}}{\frac{W_1}{W_2} e^{cL} - 1} \Delta T e^{-cx}$$

23. $$\frac{dT}{dx} = -C \frac{1}{1 - \frac{W_2}{W_1} e^{-cL}} \Delta T e^{-cx}$$

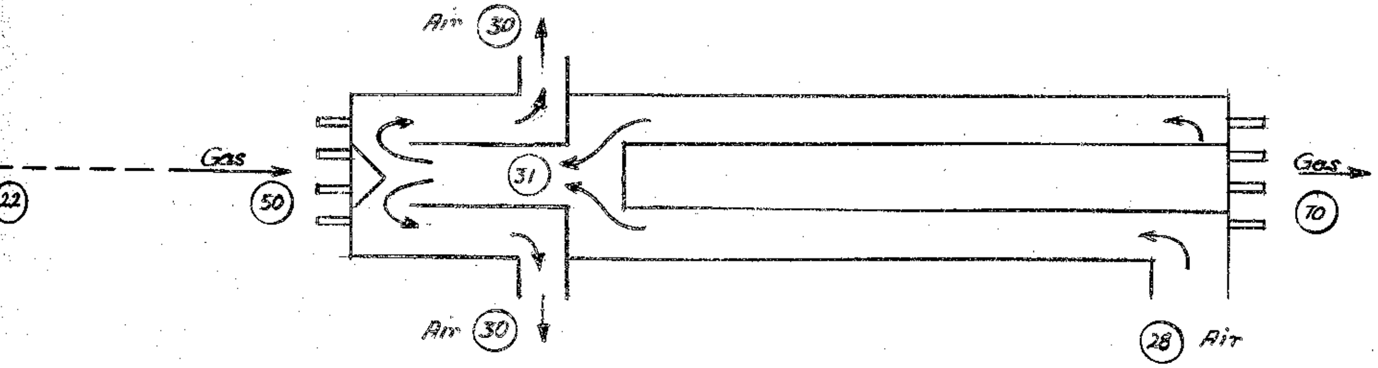
24. $$\frac{dT_w}{dx} = -C \frac{\left(1 + \frac{h_2}{h_1 + h_2}\,\frac{W_1 - W_2}{W_2}\right) e^{cL}}{\frac{W_1}{W_2}\, e^{cL} - 1}\, \Delta T\, e^{-cx}$$

for $W_1 = W_2$:

21a. $$\frac{d\theta}{dx} = 0$$

22a. $$\frac{dt}{dx} = -\frac{Ua}{W + UA}\,\Delta T = \frac{dT}{dx} = \frac{dT_w}{dx}$$

4. GIVEN DATA



HOT HEAT EXCHANGER

Observed data on January 22, 1954

Time	10:35	11:23	12:20
t_{28}	195°C	292°C	336°C
t_{31}	384	565	565
t_{30}	437	571	631
T_{50}	585	775	857
T_{70}	256	368	423
M_{air}	6.61 lb/se	8.2 lb/se	9.91 lb/se
M_{gas}	5.36 "	6.02 "	5.8 "
p_{22}	9.5 psig	18.5 psig	23.75 psig
$p_{22} - p_{28}$	-11.1 " W	-17 " W	-22 " W
$p_{22} - p_{31}$	- 2.2 " Hg	-3.5 " Hg	-4.35 " Hg
$p_{22} - p_{30}$	- 3.4 " Hg	-5.5 " Hg	-6.5 " Hg

Data as received

Time	10:35	11:23	12:20
t_{28}	388	560	638
t_{31}	730	950	1060
t_{30}	822	1060	1170
T_{50}	1190	1430	1580
T_{70}	495	697	795
M_{air}	6.61	8.2	9.1
M_{gas}	5.36	6.02	5.8
p_{22}	24.2	33.3	38.45
$p_{22} - p_{28}$	-.401	-.615	-.795
$p_{22} - p_{31}$	-1.08	-1.72	-2.14
$p_{22} - p_{30}$	-1.67	-2.71	-3.19

Data in °F, psia, psi

Fig. 1 shows temperatures and the pressures at the four stations 22, 28, 31, 30 as function of M_{air}.

Fig. 2 presents the heat exchanger with principal dimensions.

Fig. 3 may characterize the flow pattern.

Fig. 4 shows several cross sections of the exchanger

Fig. 5 gives a graph of the temperatures all over the length of the heat exchanger for M_a = 7 lb/sc.

Fig. 6 gives values for P_r and for ρ, at different pressure, for air vs. temperature.

Fig. 7 gives values for C_p, μ, K, for air vs. temperature.

Throughout the calculation, consistent units are used. Basic units are:

for	length	: foot	ft
	mass	: pound	lb
	force	: poundal	p
	time	: second	se
	temperature	: degree F	°F

Derived units:

viscosity	: μ	lb/se ft
conductivity	: K	BTU/°F ft se
density	: ρ	lb/ft^3
mass flow	: M	lb/se
mass velocity	: G	lb/ft^2 se
heat flow	: W	BTU/se °F

The pressure losses are first of all derived in p/ft^2 and then transformed into " W. 1" W ≑ 167 p/ft^2.

5. THE ACTUAL TEMPERATURES IN THE HEAT EXCHANGER

MASS FLOW AND ENERGY BALANCE FOR M = 7 lb/se

For the three trial runs at 10:35, 11:23 and 12:20, M may be assumed to increase steadily from 6.61 to 9.91 lb/se, at the air side; this is not true for the gas side.

For M_{air} = 7, M_{gas} is obtained as follows:

M_g, 10:35 = 5.36 lb/se	M_a, 10:35 = 6.61
M_g, 11:23 = 6.02	M_a, 11:23 = 8.20
ΔM_g = .66	ΔM_a = 1.59

ΔM_a between 7 lb/se and the value at 10:35 equals $7 - 6.61 = .39$

$\frac{.39}{1.59} = .245$. The same fraction, applied to the gas side gives a rise in M_g of $.245 \times .66 = .161$.

For $M_a = 7$, therefore, $M_g = 5.36 + .161 = 5.52$ lb/se

If t denotes the temperature of air and T that of the gas, given values are:

$t_{28} = t_A = 440^\circ F$	$T_{70} = T_A = 550^\circ F$
$t_{31} = t_G = 800^\circ F$	$T_G = ?$
$t_{30} = t_L = 895^\circ F$	$T_{50} = T_L = 1255^\circ F$

T_G has to be determined.

For the overall heat exchange

$$M_g C_{pg} \times (1255 - 550) = M_a C_{pa} (895 - 440)$$

$$5.52 \times .267 \times 705 = 7 \times .2526 \times 455 + \text{Radiation losses}$$

C_{pg} and C_{pa} from KEENAN--"Gas Tables"

Radiation losses = 1040 - 805 = 235 BTU/se

This amounts to 22.6%

The losses are partly given off from the outer skin to the ambient air, partly carried away by axially flowing coolant, on the outside of inner shell. They cannot be calculated in a simple manner; thus, their distribution has to be estimated. Assume: 80% of the losses occur in the parallel flow exchanger and 20% in the counter flow exchanger, along the first 3 ÷ 4 feet.

First of all $T_{G'}$ is calculated under the assumption that NO losses occur in the counter flow exchanger. The shell side and tube side temperatures, t and T', are then obtained from 3 - 13, 3 - 14.

As next step T_G is fixed as $T_{G'}$ plus 20% of the difference $(T_I - T_{G''})$, the total temperature loss of gas due to radiation. The final graph for the gas side (tube side) is then drawn through T_G, gradually approaching T'. No appreciable effect of this variation $T'(x) \rightarrow T(x)$ upon $t(x)$ is considered; however, since the actual temperature difference $T(x) - t(x)$ is greater than the theoretical one, $T'(x) - t(x)$, a smaller factor UA (overall heat transfer coefficient into area) is at work than in the ideal case where no losses occur.

Calculation of UA, the constant referring to the physical heat exchanger will, therefore, be made based on T_G, whereas calculation of C, referring to the process—under the assumption of constant UA—will be performed based upon $T_{G'}$ for the purpose of determining $t(x)$. This C, from now on, will be called C'.

<u>THE COUNTER FLOW EXCHANGER CALCULATION OF $T_{G'}$, T_G, $C'L$, C'</u>

$W_a = M_a C_{pa} = 1.77$ $\qquad \frac{W_a}{W_{g'}} = 1.20$

$W_{g'} = M_g C_{pg} = 1.475$

$\frac{W_{g'}}{W_a} = .833$

$$W_q (T_{G'} - T_A) = W_a (t_G - t_A)$$

$$T_{G'} = T_A + \frac{W_a}{W_q}(t_G - t_A) = 550 + 1.2\ (800 - 440) = 550 + 432$$

$$T_{G'} = 982^{\circ}F$$

The total temperature loss, due to radiation of the gas, equals $235/W_{q'} = 235/1.475 = 160^{\circ}F$. $T_G = 982 + .20 \times 160 = 982 + 32$

$$T_G = 1014^{\circ}F$$

From 3 - 10: $T_A - t_A = (T_{G'} - t_A)\dfrac{1 - .833}{e^{c'L} - .833}$

$$550 - 440 = (982 - 440)\frac{.167}{e^{c'L} - .833}$$

$$e^{c'L} - .833 = \frac{542}{110} \cdot 167 = .822$$

$$e^{c'L} = 1.655 \qquad C'L = .505 \qquad e^{-c'L} = .604$$

$$C' = \frac{.505}{14.75} = .0342$$

CALCULATION OF t, T', T

From 3 - 13: $t(x) = t_A + (T_{G'} - t_A)\dfrac{e^{c'(L-x)} - 1}{1.2\ e^{cL} - 1}$

$$t(x) = 440 + 542\ \frac{e^{c'x} - 1}{1.2 \times 1.655 - 1} = 440 + 550\ (1.655\ e^{-c'x} - 1)$$

$$t(x) = 440 + 910\ e^{-c'x} - 550$$

1. $$\boxed{t(x) = 910\ e^{-.0342x} - 110}$$

From 3 - 14: $T'(x) = 982 - 542\ \dfrac{e^{-cx} - 1}{.833 \times .604 - 1}$

$$T'(x) = 982 - 1090\ (1 - e^{-cx})$$

2. $$\boxed{T'(x) = 1090\ e^{-.0342x} - 108}$$

x	.0342x	$e^{-.0342x}$	910 e^{-} ...	1090 e^{-} ...	t	T'	T
0	0	1	910	1090	800	982	1014
3	.1025	.903	820	985	710	877	880
6	.2050	.815	741	889	631	781	781
9	.3075	.735	669	801	559	693	693
12	.410	.664	604	724	494	616	616

CALCULATION OF UA

With $C'L = .505$ and $AU = CL\frac{1}{\left[\frac{1}{W_q} - \frac{1}{W_a}\right]}$, from chapter 3 - SYMBOLS,

$$\frac{1}{W_q} = .678 \quad , \quad \frac{1}{W_a} = .565$$

$$(AU)' = .505 \frac{1}{.113} = 4.47$$

AU, the effective average term, is smaller than (AU)' by the ratio

$\frac{\text{Average of } (T' - t)}{\text{Average of } (T - t)}$. The averages shall be taken at $x = 0, 3, 6, 9, 12$, and 14.75 [ft]

x	0	3	6	9	12	14.75
T' - t	182	167	150	134	122	110
T - t	214	170	150	134	122	110

$$\Sigma(T' - t) = 865$$
$$\Sigma(T - t) = 900$$

$$AU = 4.47 \times \frac{865}{900}.$$

3. $AU = 4.30$

THE PARALLEL FLOW EXCHANGER

T(y) is more or less fixed by T(x) and T_I. For the curvature of t(y) in point K-L (Fig.5) a simple rule may be applied, in absence of exact calculation:

$$\left|\frac{dt}{dy}\right|_{K-L} : \left|\frac{dt}{dx}\right|_{G} = \Big(T(y) - t(y)\Big)_{K-L} : \Big(T(x) - t(x)\Big)_{G}$$

$$\Big(T(y) - t(y)\Big)_{K-L} = 140; \quad \Big(T(x) - t(x)\Big)_{G} = 214; \quad \left|\frac{dt}{dx}\right|_{G} = 35 \frac{^{o}F}{ft}$$

$$\left|\frac{dt}{dy}\right|_{K-L} = 35 \frac{140}{214} = 23 \frac{^{o}F}{ft}$$

Thus, t(y) may be drawn between I and K-L.

TEMPERATURES, DENSITIES AT DIFFERENT STATIONS

Fig. 5 shows t(x), t(y), T(x), T(y), so that all temperatures required for the calculation of the pressure losses at certain station and in certain compartments may be read from it. From Fig. 6 the corresponding values of ρ, at the appropriate pressure are obtained. From Fig. 7 those of μ are taken.

	t	ρ	$\mu \times 10^5$
A_o	440	.0785	1.80
A	440	.0785	1.80
B	466	.076	1.82
C	490	.074	1.85
D	515	.072	1.88
E	540	.071	1.91
C_1	600	.0663	2.00

	t	ρ	$\mu \times 10^5$
E_1	650	.0625	2.05
C_2	710	.0595	2.12
E_2	775	.0562	2.20
F	785	.0557	2.22
F_1	785	.0557	2.22
F_2	785	.0557	2.22
G	800	.0552	2.23

	t	ρ	$\mu \times 10^5$
H	800	.0552	2.23
I	800	.0552	2.23
J	850	.053	2.30
K	895	.0512	2.35
L	895	.0512	2.35

Between	t	ρ	$\mu \times 10^5$
B-C	475	.075	1.83
C-E	520	.072	1.89
E-C_1	560	.0685	1.94
C_1-E	620	.0645	2.01
E_1-C_2	675	.061	2.08
C_2-E_2	740	.058	2.16

In both tables ρ refers to 26 psia.

MASS FLOW AND ENERGY BALANCE FOR M = 10 lb/se

M_g 11:23 = 6.02
M_g 12:20 = 5.8
$\Delta M_g = -.22$

M_a 11:23 = 8.2
M_a 12:20 = 9.91
$\Delta M_a = 1.71$

ΔM_a between 12:20 and 10 lb/sc = .09

Corresponding ΔM_g equals $\Delta M_g = -.09 \frac{.22}{1.71} = -.013$

for $M_a = 10$, $M_g = 5.79$ lb/sc

$t_{28} = t_A = 640°F$
$t_{31} = t_G = 1060°F$
$t_{30} = t_L = 1170°F$

$T_{70} = T_A = 795°F$
$T_G = ?$
$T_{50} = T_I = 1585°F$

Determine T_G

$C_{pa} = .260$ from KEENAN & KAYE "Gas-Tables"
$C_{pg} = .276$

$M_g\ C_{pg}\ (1585 - 795) = M_a\ C_{pa}\ (1170 - 640) +$ R.-losses

$5.79 \times .276 \times 790 = 10 \times .26 \times 530 +$ R.-losses

R - losses = $1260 - 1380 = -120\ \frac{\text{BTU}}{\text{se}}$?

Obviously something is wrong with the given data or conditions were not yet

6. CALCULATION OF FLOW AREAS, TUBES, AREA RATIOS

GIVEN

	Total length for heat exchanger between	$A-I = 17.5$ ft
	Shell inside diameter	$D_s = 3$ "
	Duct outside diam.	$D_{do} = 1.25$ "
	Duct inside diam.	$D_{di} = 1.125$ "
	Flow area diam. at H	$D_{(H)} = .8$ "
	Outer baffle diam.	$D_{bo} = 2.06$ "
	Inner baffle diam.	$D_{bi} = 2.25$ "
	Tube diam.	$d = 1"$ or $1/12$ ft.
	Heat exchanger inlet diam.	$D_{(A_o)} = 1.07$ ft
Three ducts:	Heat exchanger outlet diam.	$D_{(L)} = 1.0$ "
	Radial flow diam. at F	$D_{(F)} = 2.0$ "
	Same at F_1	$A_{(F_1)} = 1$ "
	Effective diam. at H	$D_{(H)} = .8$ "
	Total number of tubes	$N = 504$
	Number of tubes at B	$N_{(B)} = 230$
	Number of tubes at C	$N_{(C)} = 185$
	Effective number of tubes at D	$N_{(D)} = 310$

Ratio $\dfrac{\text{Actual axial flow area}}{\text{Area without tubes}} = \dfrac{1.1\,S^2 - 1}{1.1\,S^2} = \dfrac{.72}{1.72}$ $R_{ax} = .418$

$S = 5/4$

Ratio $\dfrac{\text{Actual cross flow area}}{\text{Area without tubes}} = R_{cr} = .22 \approx \dfrac{1/4}{5/4}$

Ratio $\dfrac{\text{Actual flow area at A}}{\text{Possible flow area at A}} = R_{(A)}$

$R_{bo} = \dfrac{D_{bo}}{D_s} = .686$

$R_{(A)} = .5$, assumed
derived on next page

$R_{bi} = \dfrac{D_b}{D_s} = .75$

Flow area at F' $\quad A_{(F')} = 6 \times 4/12 \times 6/12 = 1\ ft^2$

Effective flow area at H $\quad A_{(H)} = 1\ ft^2$

CALCULATION OF CROSS AREAS at all stations

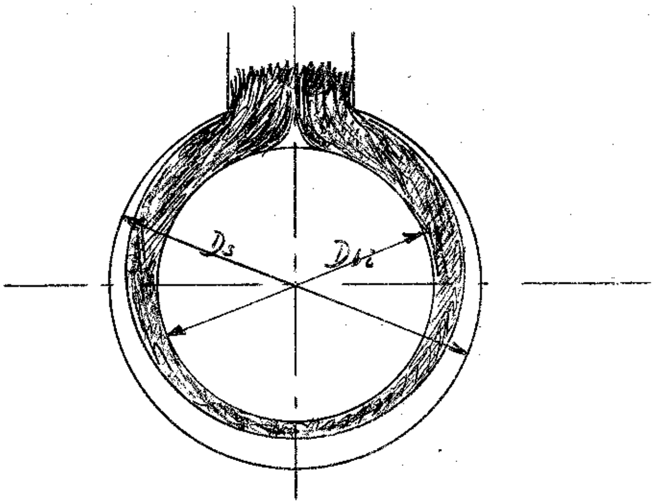
Stations: A_o, A, B, C, D, E, C_1, E_1, C_2, F, F_1, F_2, G, H, I, J, K, L

A_o $\quad A = D_{(A_o)}^2\ \pi/4 = 1.07^2\ \pi/4 \qquad = .9\ ft^2$

A $\quad A_{(A)} = \left[(D_s^2 - D_{bi}^2)\ \pi/4 - N_{(B)}\ \dfrac{d^2\pi}{4}\right] R_{(A)}$

$A_{(A)} = \pi/4\ (3^2 - 2.25^2 - 230 \times .00692) \times .5$

$A_{(A)} = \pi/4\ (9 - 5.08 - 1.59) \times .5 = \pi/8 \times 2.33 \qquad = .915\ ft^2$



$R_{(A)} = \dfrac{\text{Black Area}}{\text{Full Ring Area}} = .5$

<u>B</u> $A_{(B)} = \pi/4\ (D_s^2 - D_{bi}^2 - N_{(B)}\ d^2) \qquad = 1.83\ ft^2$

<u>C</u> $A_{(C)} = \pi/4\ (D_{bo}^2 - D_{do}^2 - N_{(C)}\ d^2)$

$= \pi/4\ (\ 2.06 - 1.25 - 185 \times .00692)$

$= \pi/4\ (4.25 - 1.56 - 1.28) = \pi/4\ 1.41 \qquad = 1.11\ ft^2$

<u>D</u> Assumed to be the average axial flow area, as computed in chapter 7. $A^* = A_{(D)}$ $= 1.53\ ft^2$

<u>E</u> alike B $= 1.83\ ft^2$

<u>C_1</u> alike C $= 1.11\ ft^2$

<u>E_1</u>, <u>E_2</u> alike B $= 1.83\ ft^2$

<u>C_2</u> alike C $= 1.11\ ft^2$

<u>F</u> $A_{(F)} = .5 \times \pi\ D_{(F)} \times R_{cr}$

$= .5 \times \pi \times 2.0 \times .22 \qquad = .69\ ft^2$

<u>F_1</u> $A_{(F_1)} = .5 \times \pi \times 1 \times .22 \qquad = .344\ ft^2$

<u>F_2</u> Six rectangular openings, $A_{(F_1)}$ $= 1.0\ ft^2$

<u>G</u> $A_{(G)} = D_{di}^2\ \pi/4$ minus several ducts

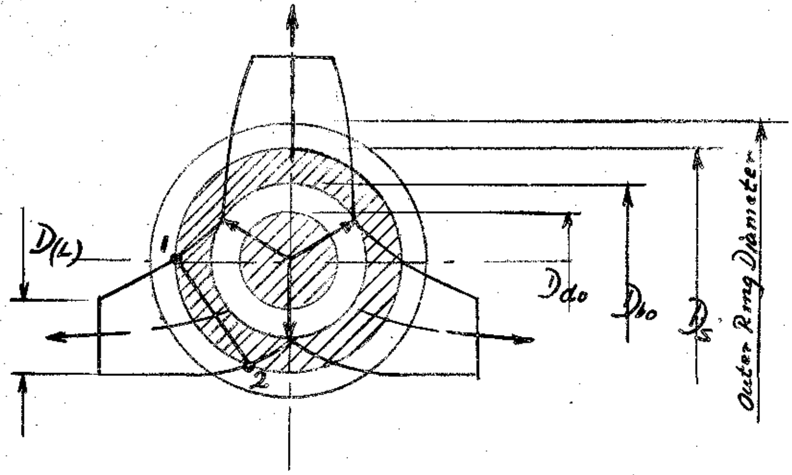
$= 1.125^2\ \pi/4$ minus $= .99$ minus $\sim \qquad = .95\ ft^2$

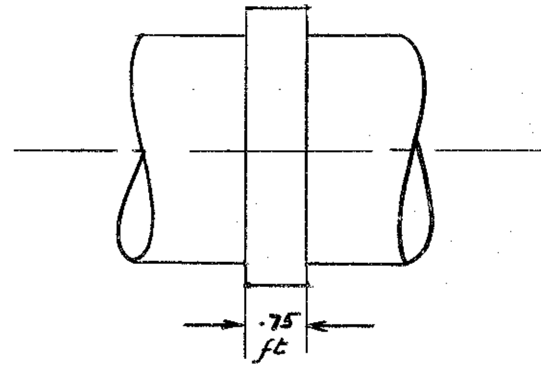
<u>H</u> $A_{(H)} = \pi\ D_{(H)} \times .4 = .4\pi \times .8 \qquad = 1.0\ ft^2$

<u>I</u> $A_{(I)} = .5\pi\ D_{do} \times R_{cr} = .5\pi \times 1.25 \times .22 \qquad = .433\ ft^2$

$\underline{J}$ $A_{(J)}$ alike $A_{(B)}$ $= 1.83\ ft^2$

$\underline{K}$ $A_{(K)} = 3 \times .75 \times (1 \div 2) \times R_{cr}$ $= .86\ ft^2$





Distance between 1 ÷ 2 = 1.75 ft

Air emerges from Ring $D_{bo} - D_{do}$ and passes across the tubes until D_s

$\underline{L}$ $A_{(L)} = 3\ \pi/4\ D_{(L)}^2 = 3\ \pi/4 \times 1$ $= 2.36\ ft^2$

THE HYDRAULIC DIAMETER

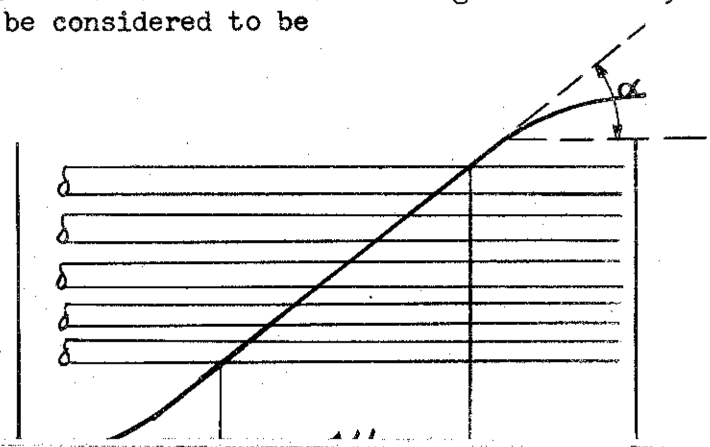
$d_{hy} = d(1.1\ S^2 - 1)$ $S = 5/4$

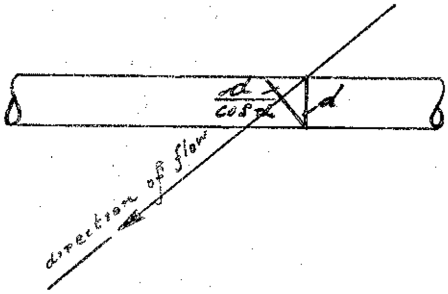
$d_{hy} = .72\ d$ or $.06\ ft$

This holds for all places where the air flows along the axis of the tubes, i.e. at B, C, E, C_1, E_1, C_2, E_2, J.

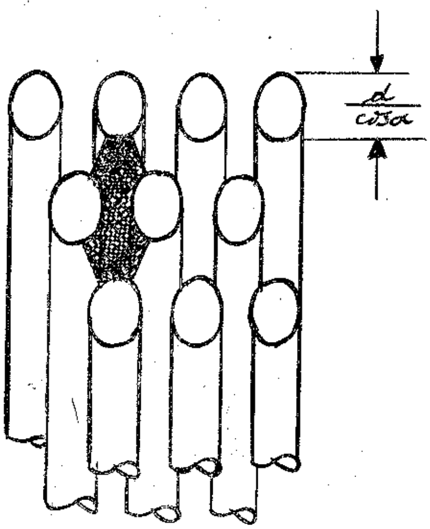
Between the baffles the flow is nearly in axial direction. Friction due to the cross-flow component is handled either according to 2-13 where no d_{hy} is required or according to 2-15, with d_{hy} as defined for axial flow.

At A and D as well as before and after J the flow crosses the tubes with the angle of incidence α. As long as $\Delta L' < \Delta L$, the length of the compartment, d_{hy} can be considered to be





$$d_{hy} = \frac{4\ \text{Area}}{\text{Wetted Perimeter}}$$



$$d_{hy} = \frac{4\,\frac{d^2\pi}{4}\,(1.1\,S^2 - 1)\,\frac{1}{\cos\alpha}}{\pi\left(\frac{d}{2} + \frac{d}{2\cos\alpha}\right)}$$

1 $$d_{hy} = \frac{d\,(1.1\,S^2 - 1)}{\frac{1}{2}\,(1 + \cos\alpha)}$$

for $\alpha > 60^\circ$, e.g., the formula will be void of sense.

For perpendicular incidence no d_{hy} is required for frictional loss. See 2-13. At F, I and K $\alpha = 90^\circ$, but there is no heat exchange assumed; neither d_{hy}, nor h have to be calculated.

For $S = 5/4$

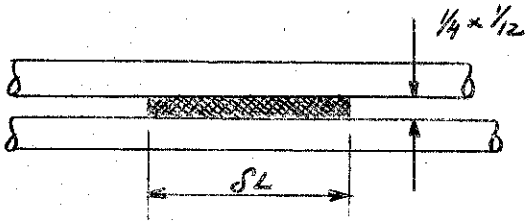
2. $$d_{hy} = \frac{1.44\,d}{1 + \cos\alpha}$$

At A, $\alpha = 60^\circ$ $$d_{hy} = \frac{1.44 \times 1}{1.5 \times 12} = .080 \text{ ft}$$

At D, $\alpha = 30^\circ$ $$d_{hy} = \frac{1.44}{1.876 \times 12} = .064 \text{ ft}$$

Before and after J, $\alpha = 45^\circ$ $$d_{hy} = \frac{1.44 \times 1}{1.707 \times 12} = .0705 \text{ ft}$$

Although of no direct concern, d_{hy} is derived for F, F_1 and K as follows:



3. $d_{hy} = \frac{4 \times 1/4 \times 1/12 \times \delta L}{2 \delta L} = .0416$

independent from δ L.

7. THE PRESSURE DROP IN THE COUNTER FLOW EXCHANGER

It will be attempted to analyze the total pressure drop as a variety of single, fundamental effects each being either known from recognized experimental data or derived from basic formulae by a process of adaptation to the actual geometrical dimensions of the heat exchanger.

Immediately two points of uncertainty will be noticed.

1) The geometrical dimensions of the heat exchanger on the air side are not a sufficient base to determine the flow pattern and, thus, terms like G and G^2 (G = mass velocity). The pattern depends upon M and ρ and has to be assumed, in the first instance.

2) Obviously it is not correct to add the various pressure losses imagined to occur independently from each other under various headings within one and the same compartment or at one station.

As long as there is no tool at hand to compute the actual pattern of flow with regard to tangential and normal component at each element of area of tubes, walls, baffles, containing implicite the effects of viscous shear and form resistance, the latter cannot be computed and expressed in terms of pressure drop in a strictly mathematical sense.

Unfortunately the two standard books about design of heat transfer equipment, KERN and McADAMS, give no account of observed pressure losses produced by the resistance of baffles.

The GRIMISON-JACOB equation 2-13 holds for perpendicular flow, the same is true for KERN's equation 1). which is but a very coarse empirical relationship.

In the Hot Heat Exchanger the flow is partly axial, partly under an incidence of about 12-20°, never pure cross flow as far as the baffles are concerned.

The way of attacking the problem "What is the total pressure drop?" has been chosen as follows:

A. Separate losses due to

1. Skin friction
2. Change of flow area or direction
3. Change of density
4. Change of final flow area

1). KERN, Process Heat Transfer, p 147

B. Compare the results obtained for similar kinds of loss and correct them, eventually, aiming at a well balanced overall picture.

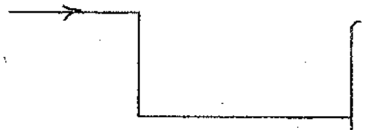
C. Try to apply the relations apparently valid after screening, for other mass flow and for the other part, the Parallel Flow Exchanger where similar problems are encountered.

With regard to A:

1. Skin friction — Axial flow friction / Cross flow friction

   Both depend on V^2; since $V^2 = V_{ax}^2 + V_{Tang}^2$ the combined pressure loss is obtained numerically correct, provided that the real velocity is composed of two <u>independent</u> components.

2. Change of flow area or direction



   - Sudden contraction
   - Sudden enlargement
   - Flow through orifice
   - Flow past disc
   - Change of direction and return flow

   These are local pressure losses and do not depend on any length of path or material contact, although area ratios or velocity ratios are involved. They, too, have been added in their effect. Since these effects overlap, the absolute minimum has been taken for each of them.

3. Change of density loss

   or non-isothermal pressure loss can be computed correctly, even if the flow area changes between the two stations considered.

   In this case the flow at the down stream station is reduced to the initial cross area and an additional pressure change computed, with unchanged density. This leads to

4. Change of final flow area.

With regard to B and C: nothing can be planned in advance.

For other mass flow the corresponding temperature distribution has to be calculated distinctly as done in chapter 5, based on the values of Fig. 1. Thereupon the value of UF can be compared with that obtained for $M = 7$ lb/se in chapter 5. Chapter 10 will give an estimate of how or if h_a and h_g change with temperature.

If $\frac{1}{h_a} + \frac{1}{h_g}$ is to remain constant, but UF has changed appreciably, F has changed in effect and the flow pattern, for another M, has changed. In this case the mean value of G and G^2, on which the skin friction losses are based, will have to be computed anew.

The wall temperature calculated in chapter 10 won't have any sensible effect upon the pressure loss. The temperature difference between wall and bulk will hardly be more than 150-200°F and the corresponding ratio $(\mu_b/\mu_w)^{.14}$, which is instrumental for skin friction will, in fact, remain the same.

However, no peculiar effect upon the temperature distribution of the air side, between stations I and K, can be expected at variance with that given in Fig. 5, since the whole rise is only 100°F. Therefore, the computation of the pressure loss in the parallel flow exchanger will be attempted following that of the counter flow exchanger.

THE AXIAL FLOW PRESSURE DROP

between B and E_2, i.e. between B-C, C-E, E-C_1, C_1-E_1, E_1-C_2, C_2-E_2

according to 2-2, $\Delta p = f_{ax} \frac{G^2}{2\rho} \frac{\Delta L}{r_{hy}}$

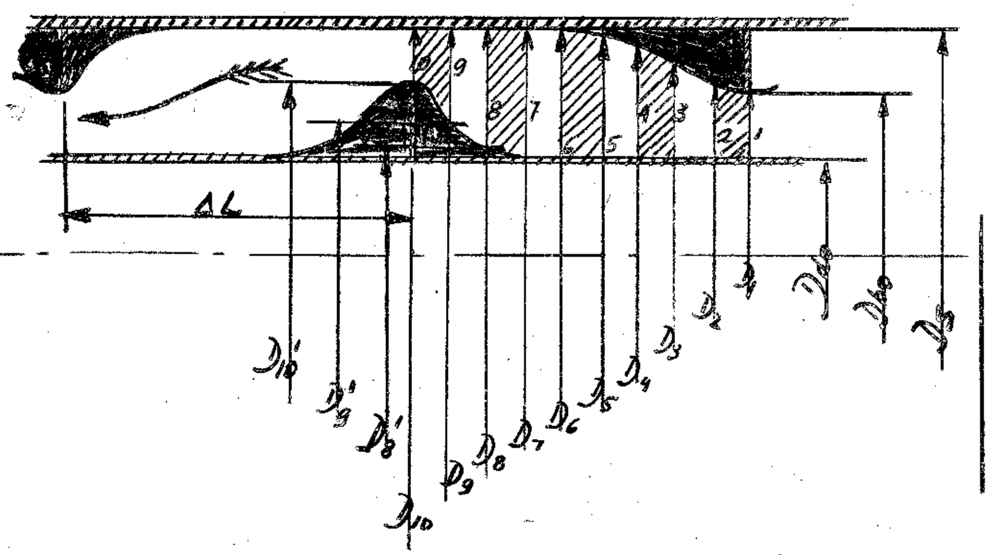
ΔL_{B-C} = 1.2 ft. Other ΔL, on the average, 11.4/5 = 2.28 ft

Although the direction of flow changes within each compartment d_{hy} is considered to be constant, for the axial flow component.

d_{hy} = .06 ft $\qquad$ r_{hy} = .015 ft

Since A and G change, $\overline{G^2}$ is derived as follows:

Assumed flow pattern for M = 7, V about 80 ft/se :



$$G = \frac{M}{A}$$

$$G^2 = \frac{M^2}{A^2}$$

$$\overline{G^2} = M^2\left[\overline{\frac{1}{A^2}}\right]$$

$$A_\lambda = \frac{\pi}{4}(D_\lambda^2 - D_\lambda'^2)$$

$$R_{c\,r\,x}^2\left[\overline{\frac{1}{A^2}}\right] = \frac{1}{\lambda}\sum_1^{10}\frac{1}{A_\lambda^2}$$

2 cm ≙ 1 ft

	1	2	3	4	5	6	7	8	9	10
D_λ	2.06	2.2	2.65	2.8	2.9	3.0	3.0	3.0	3.0	3.0
D'_λ	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.35	1.7	2.25
D_λ^2	4.25	4.84	7.00	7.80	8.40	9.00	9.00	9.00	9.00	9.00
D'^2_λ	1.56	1.56	1.56	1.56	1.56	1.56	1.56	1.82	2.88	5.09
$D_\lambda^2 - D'^2_\lambda$	2.69	3.28	5.44	6.24	6.84	7.44	7.44	7.18	6.12	3.91
A_λ	2.11	2.58	4.29	4.91	5.39	5.82	5.82	5.61	4.81	3.08
A_λ^{-2}	.225	.150	.0545	.0415	.0345	.0295	.0295	.0318	.0432	.1056
A_λ^{-1}	.475	.398	.233	.204	.185	.172	.172	.178	.208	.324

$$\sum_1^{10} A_\lambda^{-2} = .7451 \qquad \frac{1}{\lambda}\sum\frac{1}{A_\lambda^2} = .07451$$

$$\text{Call} \quad .07451 = \frac{R_{ax}^{\ 2}}{A^{*2}} \qquad \frac{R_{ax}}{A^*} = .273 \qquad \frac{A^*}{R_{ax}} = 3.66 \text{ ft}^2$$

$$A^* = 3.66 \times R_{ax} = 3.66 \times .418 = \underline{1.53 \text{ ft}^2}$$

This value depends only upon the flow pattern. It is assumed to be constant over all compartments, despite changing temperature and velocity.

A* will slightly decrease with increasing mass flow.

$$\overline{G^2} = \frac{M^2}{(A^*)^2} = \left(\frac{7}{1.53}\right)^2 = \underline{20.9} \left[\frac{\text{lb}^2}{\text{se}^2 \text{ ft}^4}\right]$$

$$G^* = \sqrt{\overline{G^2}} = \underline{4.57}$$

$$\bar{G} = \frac{1}{R_{ax}}\frac{M}{\lambda}\sum\frac{1}{A_\lambda} = \frac{7}{.418}\frac{1}{10}\, 2.549$$

$$\underline{\bar{G} = 4.26} \left[\frac{\text{lb}}{\text{se ft}^2}\right]$$

Unlike in the case of cross flow described in the following section, $\bar{G}$ and $\sqrt{\overline{G^2}}$ can be assumed the same for the distance between B and C, here.

$\frac{t}{^{o}F}$	ρ [lb/ft³]	$G^2/2\rho$ [p/ft²]	μ [lb/se ft]
475	.075	139	1.83×10^{-5}
520	.072	145	1.89
560	.0685	153	1.94
620	.0645	162	2.01
675	.061	171	2.08
740	.058	180	2.16

$\frac{G\, d_{hy}}{\mu}$	f_{ax}	$f_{ax}\frac{G^2}{2\rho}$ [p/ft²]
1.4×10^4	.007	.973
1.35	.0072	1.04
1.32	.0074	1.13
1.27	.0076	1.23
1.23	.0078	1.33
1.18	.008	1.44

$\frac{\Delta L}{r_{hy}}$	Δp [p/ft²]	
80	78	B - C
$\frac{2.28}{.015} = 152$	158	C - E
	172	E - C_1
	187	C_1 - E_1
	202	E_1 - C_2
	219	C_2 - E_2

$$\sum \Delta p = 1016 \ (p/ft^2) \ w$$

$$\Delta p_{ax} = \frac{1016}{167} = 6.1 \text{ "W}$$

THE CROSS FLOW PRESSURE DROP

According to 2.-13 , 2.-13 b

$$\Delta p = 4 \ f_{cr} \ N \frac{\rho V_{cr}^2}{2} = 4 \ f_{cr} \ N \frac{G^2}{2\rho}$$

considered paths: between B-C, C-E, E-C_1, C_1-E_1, E_1-C_2, C_2-E_2.

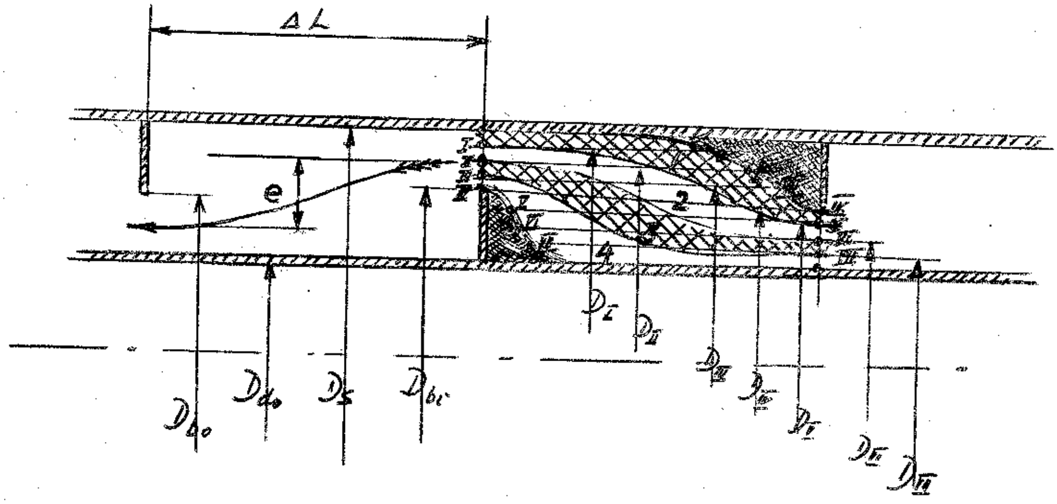
Average length of five latter distances equals 11.4/5 = 2.28 ft (L'). Path between B and C will be considered later.

For the average length of compartment L', G is constant. It has to be computed, however, as average over the different cylindrical layers of the compartments for evaluation of f_{cr}.

$$f_{cr} = f_{cr} \ (\bar{G}); \ \frac{N\ G^2}{2\rho} = \sum (N_\lambda \ G_\lambda^2) \ \frac{1}{2\rho} = N_\lambda (\sum G_\lambda^2) \ \frac{1}{2\rho}$$

Since ρ is constant for each compartment, $\frac{N\ G^2}{2\rho} \rightarrow \frac{N_\lambda}{2\rho} \ \sum G_\lambda^2$

$$G_\lambda = \frac{M_\lambda}{A_\lambda \ R_{cr}} \ ; \quad A_\lambda = (\lambda \div \lambda) \ \pi \ D_\lambda$$



2 cm ≑ 1 ft

$$e = \frac{(D_s + D_{bi}) - (D_{bo} + D_{do})}{4}$$

$$e = \frac{(3 + 2.25) - (1.25 + 2.06)}{4} = \frac{1.94}{4} = .485$$

$$N = \frac{e}{p} = \frac{.485 \times 12}{5/4} = \underline{4.66} \qquad N_\lambda = \frac{N}{4} = \underline{1.16}$$

	Shell I-I	II-II	III-III	IV-IV	V-V	VI-VI	VII-VII
M_λ	1/4 M	2/4 M	3/4 M	4/4 M	3/4 M	2/4 M	1/4 M
A_λ	(I-I)π D_I	(II-II)π D_{II}	17.5	14.1	11.4	10.2	(VII-VII)π D_{VII}
$G_\lambda R_{cr}$	.073	.171	.30	.498	.461	.344	.212
G_λ	.332	.78	1.36	2.28	2.1	1.56	.965
G_λ^2	.11	.54	1.85	7.12	4.41	2.44	.93

$\sum G_\lambda = 9.377$; $\underline{\bar{G} = 1.34 \frac{\text{lb}}{\text{ft}^2\ \text{se}}}$

$\sum G_\lambda^2 = 17.4$; $1/2\ N_\lambda \sum G_\lambda^2 = 10.1$

$$\Delta p = 4\ f_{cr}\ N \frac{G^2}{2\rho} \longrightarrow 4\ f_{cr}(\bar{G}) \times \frac{1}{\rho} \frac{N_\lambda}{2} \sum G_\lambda^2$$

For all stations between C-E, E-C_1, C_1-E_1, E_1-C_2, C_2-E_2 :

$\bar{G} = 1.34$, constant; $\frac{N_\lambda}{2} \sum G_\lambda^2 = 10.1$, constant

Call $\frac{N_\lambda}{N} \sum G_\lambda^2 = (G^*)^2$ $\qquad (G^*) = 1/4 \times 17.4 = 4.35$

$G^* = \sqrt{\overline{G^2}} = 2.08$ $\qquad\qquad$ $A^* = \frac{M}{G^*} = 3.36$

$$D^* = A/R_{cr} \times 1/\pi L' = \frac{3.36}{.22\pi \times 2.24}$$

$D^* = 2.13$

For distance between B-C, $L'_{B-C} = 1.2$ $\qquad$ $\frac{L'}{L'_{B-C}} = \frac{2.28}{1.2} = 1.9$

$\overline{G}_{B-C} = \overline{G} \times 1.9$ $\qquad$ $\frac{N_\lambda}{2} \sum_{B-C} G_\lambda^2 = \left(\frac{N_\lambda}{2} \sum G_\lambda^2\right) \times 1.9^2$

$\overline{G}_{B-C} = 2.55$ $\qquad$ $\frac{N_\lambda}{2} \sum_{B-C} G_\lambda^2 = 36.5$

2-(13b)
$$f_{cr} = \left(.23 + \frac{.11}{\left(\frac{p}{d} - 1\right)^{1.08}}\right)\left(\frac{dG}{\mu}\right)^{-.15}$$

$\frac{p}{d} = 5/4$ $\qquad$ $f_{cr} = \frac{.23 \times .223 + .11}{.223}\left(\frac{dG}{\mu}\right)^{-.15} = .723\left(\frac{dG}{\mu}\right)^{-.15}$

	t °F	ρ lb/ft³	$10.1/\rho$ lb/ft²	μ lb/ft se	$d\overline{G}$ lb/ft se	$\frac{d\overline{G}}{\mu}$	$\left(\frac{d\overline{G}}{\mu}\right)^{-.15}$	4 f_{cr}	Δp lb/ft²	Δp "W
B-C	475	.075	487	1.83 x 10	.212	11.7 x 10³	.246	.71	346	
C-E	520	.072	140	1.89	.1116	5.9 "	.272	.785	110	
E-C₁	560	.0685	148	1.94	.1116	5.75 "	.274	.791	118	970/167
C₁-E₁	620	.0645	157	2.01	.1116	5.55 "	.275	.794	125	= 5.8"W
E₁-C₂	675	.061	165	2.08	.1116	5.37 "	.276	.796	131	
C₂-E₂	740	.058	174	2.16	.1116	5.17 "	.278	.803	140	

$\sum \Delta p = 970$ lb/ft² or 5.8 "W

THE PRESSURE DROP OVER BAFFLES

The inner baffles, at B, E, E_1, E_2, act like a disc, perpendicular to the direction of flow. The outer ones, at C, C_1, C_2, have the effect of an orifice upon the stream. In addition, in both cases, there will be a certain pressure drop due to change of direction.

According to 2-13, due to a disc within the infinitely broad stream

$\Delta p = 1.12 \frac{\rho V^2}{2}$, $\qquad$ with V being the velocity before and further down stream, after the disc.

$$\Delta p = 1.12 \frac{G^2}{2\rho}$$

As the flow is not infinitely broad, the ensuing pressure drop equals at least this value. V_{min}, and G_{min} are between the baffles, over the flow area

$$\pi/4 \, (D_s^2 - D_{do}^2) \, R_{ax} = 2.46 \text{ ft}^2$$

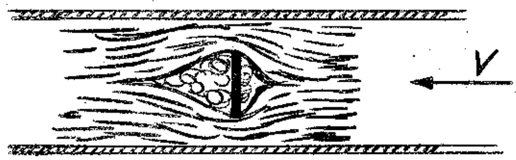
$$G = \frac{7}{2.46} = 2.84 \;;\quad G^2 = 8.1 \;;\quad 1/2\, G^2 = 4.05$$

Halfway before	t	ρ	$G^2/2\rho$	Δp
B	450	.077	52.5	59
E	515	.072	56.5	63.5
E_1	625	.0645	63	70.5
E_2	740	.0578	70	78

$\sum \Delta p = 271$ lb/ft²

$\sum \Delta p = 271$ lb/ft² or

$\Delta p = 1.6$ "W

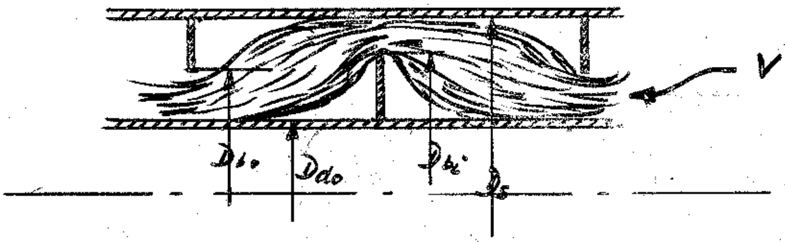


According to 2-11 the pressure drop over an orifice with the diameter ratio $D_1/D_2 = .5$

equals $\Delta p = .8 \frac{G^2}{2\rho}$

At C, C_1, C_2 the diameter ratio equals the square root of the area ratio

$$\frac{D_{bo}^2 - D_{do}^2}{D_s^2 - D_{do}^2} = \frac{4.24 - 1.56}{9 - 1.56} = \frac{2.68}{7.44} = .36$$



Area ratio for $D_1/D_2 = .5$ equals .25. In the case of the baffles, the ratio is not so extreme and, accordingly, the pressure drop will be smaller by $\frac{.25}{.36} = .696$. This is a very conservative estimate, as far as this pressure loss alone can be considered.

$$\Delta p = .8 \times .696 \frac{G^2}{2\rho} = .555 \frac{G^2}{2\rho}$$

Formal calculation results in

$$A = \pi/4\,(D_{bo}^2 - D_{do}^2)\, R_{ax} = \pi/4 \times 2.68 \times .418 = .88 \text{ ft}^2.$$

True A equals 1.11 ft² (Chapter 6) since there are no tubes near the inner duct.

At	t	ρ	A	G	G^2	$G^2/2\rho$	Δp
C	490	.074	1.11	6.3	39.6	268	149
C_1	600	.0663				300	167
C_2	710	.0595				333	185

$$\sum \Delta p = 501\ p/ft^2 \quad \text{or}$$

$$\Delta p = 3\ \text{"W}$$

PRESSURE DROP THROUGH CHANGE OF DIRECTION OVER BAFFLES

The minimum pressure drop over a 180° reversal of direction in a single duct equals, according to 2-10,

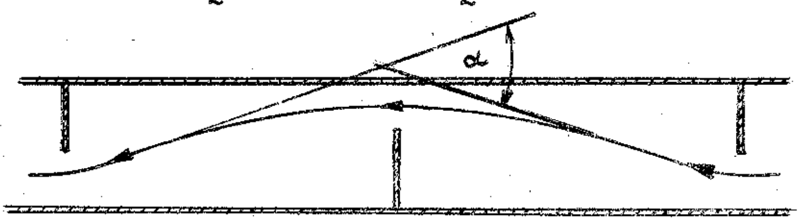
$$\Delta p = 1 \frac{\rho V^2}{2}$$

For the most adverse case, where the return flow passes through a header with several ducts going into and out of it, according to 2-10a,

$$\Delta p = 4 \frac{\rho V^2}{2}$$

This relationship must not be true for smaller angles of deviation, . The turbulence set up will, then, be proportionally smaller. However, within the heat exchanger the duct is neither single nor smooth. The ensuing pressure drop is, therefore, calculated according to 2-9, whose general form is

$$\Delta p = \frac{\sqrt{2}}{2} \sqrt{1 - \cos \alpha}\ \frac{\rho V^2}{2}$$



α about 36°, over each baffle

$$\frac{\sqrt{2}}{2} \sqrt{1 - \cos 36^\circ}$$

$$= .306$$

This appears to be much, since part of the lateral change in direction is already accounted for through the preceeding sections, "Flow past disc" and "Flow through orifice."

On the other hand, the pressure drop in the preceeding two sections refers to ONE disc and to ONE orifice. For tandem discs (ROUSE, P. 249) the drop is considerably larger, 1.54 in place of 1.12 velocity heads. In lieu of .306, a factor of .20 is applied. This drop occurs at stations B, C, E, C_1, E_1, C_2.

Station	$G^2/2\rho$	Δp
B	96	19.2
C	268	53.7
E	103	20.6
C_1	298	59.5
E_1	117	23.4
C_2	333	66.7

$$\sum \Delta p = 243 \text{ p/ft}^2 \quad \text{or} \quad \underline{1.45 \text{ "W}}$$

The total pressure drop between A and F amounts to

6.1	longitudinal loss
4.3	transversal loss
1.6	disc resistance
3.0	orifice resistance
1.45	change of direction loss
= 16.5 "W	

In order to compare this with the pressure drop on the air side in case no baffles would be present, pure axial flow over the area $\pi/4\ (D_s^2 - D_{do}^2)\ R_{ax}$ is considered

$$\pi/4\ (D_s^2 - D_{do}^2)\ R_{ax} = \pi/4 \quad 7.44 \times .418 = 2.44 \text{ ft}^2$$

$$G = \frac{7}{2.44} = 2.87 \text{ lb/ft}^2 \text{ se} \qquad \rho_{average} \text{ assumed as } .062$$

$$\Delta p = 4\ f_{ax} \frac{\rho V^2}{2} \frac{L}{d_{hy}} \qquad f_{ax_{average}} \text{ assumed as } .0075$$

$$\Delta p = 4 \times .0075 \frac{2.87^2}{2 \times .062} \frac{14}{.06} = 463 \text{ p/ft}^2 \quad \text{or} \quad \underline{2.77 \text{ "W}}$$

Now, if the area of 2.44 ft^2 is reduced to the root mean square area A* = 1.53, (p.30) the ensuing drop will become

$$2.77 \left(\frac{2.44}{1.53}\right)^2 = \underline{\text{about } 7 \text{ "W}}$$

Roughly speaking, more than half of the pressure drop actually encountered is due to the baffles proper; that means to the "disc"-"orifice"-"change of direction" effect, in addition to the effect of cross flow.

In fact, this figure of 7 "W compares well with 6.1 "W for axial pressure loss as computed previously.

PRESSURE DROP THROUGH CHANGE OF DIRECTION AT A, F, F_2

At A the flow swings gently around from perpendicular to lengthwise direction, with regard to the axis of the tubes. No loss is considered.

At F and F_2 there are abrupt 90° turns;

$$\Delta p = \frac{\sqrt{2}}{2}\sqrt{1 - \cos\alpha}\,\frac{\rho V^2}{2} = .707\,\frac{\rho V^2}{2}$$

Between F and F_1 there is a tremendous acceleration due to the reduced cross section at F, $A_{(F)} = .69\ ft^2$, and the still decreasing section down to $A_{(F_1)} = .344\ ft^2$.

The pure velocity head $\Delta\left(\frac{\rho V^2}{2}\right)$, however, is regained to a certain amount at F_2, with a cross section $A_{(F_2)} = 1.0\ ft^2$, compared with $A_{(E_2)} = 1.83\ ft^2$, when the acceleration started. At G, $A_{(G)} = 1.0\ ft^2$. This Bernoullian pressure head difference is part and parcel of the overall static change due to variation of flow area between A and G and will be included implicite when the latter is calculated later on.

Here, the velocity for which the equation $\Delta p = .707\,\frac{\rho V^2}{2}$ applies has to be chosen. It may be taken as $V_{(E_2)}$ at F and $V_{(F_2)}$ at F_2.

Station	A	G	ρ	$G^2/2\rho$	Δp
F	1.83	3.82	.0555	115	82
F_2	1.0	7	.0552	443	312

$\Sigma\Delta p = 394\ p/ft^2$ or 2.35 "W

PRESSURE DROP DUE TO "FLOW ACROSS TUBES", BETWEEN F AND F_1

There are about 6 rows of tubes the flow has to pass through radially, between F and F_1. Taking the smallest possible number for rows, i.e.

$$n = \frac{D_{bi} - D_{de}}{2\ p} = \frac{2.25 - 1.25}{2\ 5/4\ \ 1/12} = \frac{1 \times 12 \times 4}{5 \times 2} \approx 5,$$

and the smallest mass velocity, at the largest circumference, at F,

$$G_{min} = \frac{M}{A_{max}} = \frac{M}{A_{(F)}} = \frac{7}{.54 \times 2.25\,\pi}\,\frac{1}{R_{cr}}$$

$$G_{min} = \frac{7}{.54 \times 2.25\,\pi \times .22} = 8.35\ lb/ft^2 se$$

and applying 2-13,

$$-\Delta p = 4\ f_{cr}\ N\,\frac{G^2}{2\rho}.$$

there results a pressure drop of

$$4\ f_{cr}\ 5\,\frac{8.35^2}{2 \times .0552} \qquad f_{cr} = .723\left(\frac{8.35 \times .06}{2.23 \times 10^{-5}}\right)^{-.15}$$

$$f_{cr} = .723(2.24 \times 10^4)^{-.15} = .723 \times .222 = .161$$

$$4\ N\ f_{cr} = 4 \times 5 \times .161 = 3.22$$

$$\frac{G^2}{2\rho} = \frac{8.35^2}{2 \times .0552} = \frac{35}{.0552} = 635$$

$$\Delta p = 635 \times 3.22 = 2040\ p/ft^2 \quad \text{or} \quad 12.2\ \text{"W}$$

Taking into account the still higher mass velocity towards the centre, an average Δp of about 16 "W is obtained.

This is by far too large. No reason can be given at this moment for the excessive result except that formula 2-13 may not be applicable in its present shape.

The restriction of flow area by the last baffle, at E_2, is considerable. This piece of metal seems to be entirely unwarranted where it is; it should be moved 1/2 foot towards C_2. No appreciable loss in heat transfer will be incurred, but any pressure loss over the tubes to the 6 openings will be reduced to ONE QUARTER.

Assumed pressure loss between F and F_1, under present conditions:

$$\Delta p = 4 \text{ "W}$$

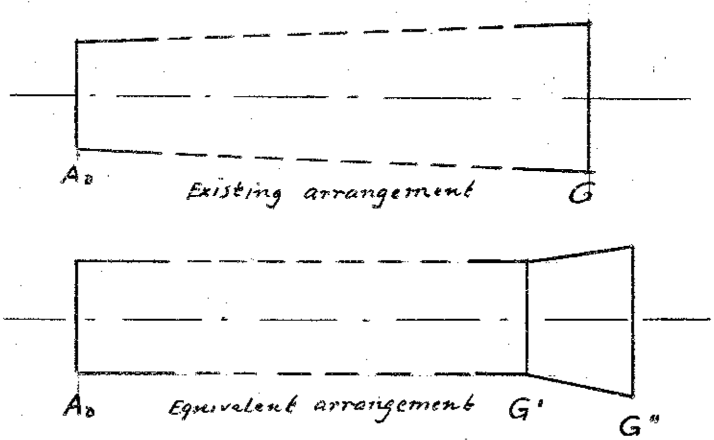
Between F_1 and F_2 there is a sudden enlargement from .344 to 1.0 ft^2. No additional pressure loss is associated herewith, since the loss between F and F_1 is due to a number of contractions and enlargements of flow area represented by the number of tube rows, N, in equation 2-13, and the enlargement from .344 to 1.0 ft^2 is nothing but the last of this series.

In the preceeding section THE CROSS FLOW PRESSURE DROP has been obtained as 5.8 "W. Possibly this is too high because of the same hitherto unknown reason which gave a result of 16 "W, as mentioned above. Let 5.8 be cut down to 3 "W.

These corrections do not effect the derived pressure losses due to "disc", "orifice", "change of direction" effect.

THE NON-ISOTHERMAL PRESSURE LOSS

This applies as a whole over the path A_o-G. As the areas in A_o and G are different, the arrangement has to be replaced as follows:



$A_{(A_o)} \neq A_{(G)}$; $\rho_{(A_o)} \neq \rho_{(G)}$

$A_o = .90\ ft^2$ $A_G = .95\ ft^2$

$A_{(A_o)} = A_{(G')}$; $\rho_{(A_o)} \neq \rho_{(G')}$

$A_{(G')} \neq A_{(G'')}$; $\rho_{(G')} = \rho_{(G'')}$

$A_{(G)} = A_{(G'')}$

The non-isothermal pressure loss A_o-G is that to be computed for A_o-G'.

	t	ρ	G	G^2/ρ
A_o	440	.0785	7.77	773
G'	800	.0552	7.77	1090

$$\Delta p = 1/\rho_2 \; G_2^2 - 1/\rho_1 \; G_1^2$$

$$\Delta p = 1090 - 773 = 317 \text{ p/ft}^2 \quad \text{or} \quad \underline{1.9 \text{ "W}}$$

THE BERNOULLIAN PRESSURE CHANGE

This effect is the last to be considered in this context. It equals, with regard to the whole counter flow exchanger,

$$\Delta p = \left(\frac{G^2}{2\rho}\right)_{G''} - \left(\frac{G^2}{2\rho}\right)_{G'}$$

	ρ	Area	G	G^2	$G^2/2\rho$
G'	.0552	.90	7.77	60.4	547
G"	.0552	.95	7.36	54.2	491

$$\Delta p = -56 \text{ p/ft}^2 \quad \text{or} \quad \underline{-.33 \text{ "W}}$$

The minus sign indicates a pressure gain.

THE TOTAL PRESSURE DROP

1) Axial flow pressure drop	:	very likely	6.10 "W
2) Cross flow pressure drop	:	probable	3.00
3) "Disc" pressure drop	:	probable	1.60
4) "Orifice" pressure drop	:	combined effect	3.00
5) Change of direction within baffles	:		1.45
6) Change of direction at other places	:	possibly too high	2.35
7) Constriction between F and F_1	:	possibly too low	4.00
8) Non-isothermal pressure drop	:	exact	1.90
9) Bernoullian pressure change	:	exact	-.33
		Total ≈	23.1 "W

The result obtained appears to be on a reasonable foundation, though details are questionable.

The observed pressure drop for M = 7 lb/se, interpolated from Fig. 1, is

$$\Delta p_{22\rightarrow 31} = .8 \text{ psi} \quad \text{or} \quad \frac{.8}{.0361} = \underline{22.1 \text{ "W}}$$

Calculation of the pressure drop for mass flow other than 7 lb/se as well as calculation of the pressure drop in the parallel heat exchanger, along the lines developed here, will indicate if the method is sound.

All individual pressure losses are proportional to G^2/ρ. The greater part of them is proportional, too, to either f_{ax} or $\left(\frac{Gd}{\mu}\right)^{-.15}$.

Assume that all losses were $\propto$ to $(G^2/\rho)\times$(Function of R_e).

$$G^2_{M=10} = G^2_{M=7}\,\frac{100}{49} = G^2_{M=7} \times 2.04,$$ for unchanged flow pattern.

$$\rho_{M=10} = \rho_{M=7}\,\frac{p_{M=10}}{p_{M=7}}\,\frac{(t_m + 460)_{M=7}}{(t_m + 460)_{M=10}} .$$

$$\rho_{M=10} = \rho_{M=7} \times \frac{37.3}{26}\,\frac{1095}{1310} = \rho_{M=7} \times 1.2$$

$$\mu_{t_m} = \mu_{630^\circ} = 2.03 \times 10^{-5} \quad \text{for } M = 7$$

$$\mu_{t_m} = \mu_{850^\circ} = 2.30 \times 10^{-5} \quad \text{for } M = 10$$

$$(G/\mu)^{-.15}_{M=10} = (G/\mu)^{-.15}_{M=7} \times \left(\frac{10}{7}\,\frac{2.3}{2.03}\right)^{-.15} = (G/\mu)^{-.15}_{M=7} \times .966$$

The axial flow friction coefficient decreases to approximately .96 of its former value when $\left(\frac{Gd}{\mu}\right)_{M=7}$ changes to the corresponding value $\left(\frac{Gd}{\mu}\right)_{M=10}$.

Total loss $\Delta P_{M=10} = \Delta P_{M=7} \times 2.04\,\frac{1}{1.2} \times .966 = \Delta P_{M=7} \times 1.64$

$\Delta P_{M=7}$ observed equals .8 psia

1.64 x .8 = 1.31 psia = $\Delta P_{M=10}$, calculated

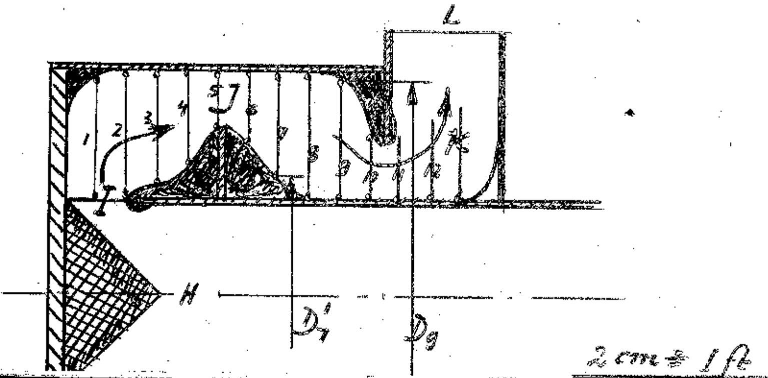
from Fig. 1: 1.2 psia = $\Delta P_{M=10}$, observed ; deviation about 9%

8. THE PRESSURE DROP IN THE PARALLEL FLOW EXCHANGER

The losses will be calculated as

1) Axial flow losses between I and K
2) Cross flow losses between I and J
3) Cross flow losses between J and K
4) Disc effect at J
5) Orifice effect at K
6) Change of direction at H, I, J, K
7) Cross flow pressure loss between K and L
8) Non-isothermal loss between G and K
9) Bernoullian pressure change, between G and L

THE AXIAL FLOW PRESSURE LOSS



In order to derive $\overline{\frac{G^2}{2\rho}}$ for the compartments I-J and J-K, the actual flow area has to be determined.

Constant ρ is assumed in each compartment.

Through Section 1 M/2 is passing.
Through Sections 2 to 11, M passes.
Through Section 12 M/2 passes.

Section	1	2	3	4	5	6	7	8	9	10	11	12
D_λ	2.8	3	3	3	3	3	3	3	2.9	2.1	2.2	1.9
D_λ'	1.25	1.3	1.6	1.9	2.3	2.1	1.6	1.4	1.25	1.25	1.25	1.25
D_λ^2	7.83	9	9	9	9	9	9	9	8.4	4.4	4.8	3.61
$D_\lambda'^2$	1.56	1.69	2.56	3.61	5.3	4.4	2.56	1.96	1.56	1.56	1.56	1.56
$D_\lambda^2 - D_\lambda'^2$	6.27	7.31	6.44	5.39	3.7	4.6	6.44	7.04	6.84	2.84	3.24	2.05
A_λ	2.06	2.4	2.11	1.77	1.22	1.51	2.11	2.31	2.24	.93	1.06	.67
A_λ^{-2}	.236	.174	.225	.32	.675	.44	.225	.189	.2	1.17	.89	2.24

Compartments I - J ← → Compartments J - K

$$A = (D_\lambda^2 - D_\lambda'^2)\,\pi/4 \times R_{ax} = (D_\lambda^2 - D_\lambda'^2) \times .328$$

<u>I - J:</u>

$$1/4 \sum_2^5 A_\lambda^{-2} = .349 = \frac{1}{{}_{2-5}A^{*2}}$$

$${}_{2-5}\overline{G^2} = \frac{M^2}{{}_{2-5}A^{*2}} = 7^2 \times .349$$

$${}_{2-5}\overline{G^2} = 17.1$$

$${}_1G^2 = (M/2)^2 \times .236$$

$${}_1G^2 = 12.2 \times .236 = 2.88$$

$$\text{True } \overline{G^2} = 1/5\left[(4 \times 17.1) + (1 \times 2.88)\right]$$

$$\text{True } \overline{G^2} = 14.3$$

$$\text{True } A^{*2} = \frac{M^2}{\text{True } \overline{G^2}} = \frac{49}{14.3} = 3.42$$

$$\text{True } A^* = \underline{1.85 \text{ ft}^2}$$

$$\text{True } G = \sqrt{\text{True } \overline{G^2}} = 3.78$$

t_m in compartments = 825°F

ρ_m in compartments = .054

μ_m in compartments = 2.27×10^{-5}

$$Re = \frac{Gd_L}{\mu} = 10^4$$

f_{ax} = .008

<u>J - K:</u>

$$1/7 \sum_5^{11} A_\lambda^{-2} = .54 = \frac{1}{{}_{5-11}A^{*2}}$$

$${}_{5-11}\overline{G^2} = \frac{M^2}{{}_{5-11}A^{*2}} = 7^2 \times .54$$

$${}_{5-11}\overline{G^2} = 25.4$$

$${}_{12}G^2 = (M/2)^2 \times 2.24$$

$${}_{12}G^2 = 12.2 \times 2.24 = 27.4$$

$$\text{True } \overline{G^2} = 1/12\left[(11 \times 25.4) + (1 \times 27.4)\right]$$

$$\text{True } \overline{G^2} = 25.6$$

$$\text{True } A^{*2} = \frac{M^2}{\text{True } \overline{G^2}} = \frac{49}{25.6} = 1.91$$

$$\text{True } A^* = \underline{1.38 \text{ ft}^2}$$

$$\text{True } G = \sqrt{\text{True } \overline{G^2}} = 5.07$$

t_m in compartments = 880°F

ρ_m in compartments = .052

μ_m in compartments = 2.33×10^{-5}

$$Re = \frac{Gd_{hy}}{\mu} = 1.3 \times 10^4$$

f_{ax} = .0075

$L = 1$ ft; $d_{hy} = .06$

$$\Delta p = f_{ax} \frac{\text{True } \overline{G}^2}{2\rho} \frac{L}{r_{hy}}$$

$$= .008 \frac{14.3 \times 1}{2 \times .054 \times .015}$$

$\Delta p = 70.5$ p/ft

or .42 "W

$L = 1.5$ ft; $d_{hy} = .06$

$$\Delta p = f_{ax} \frac{\text{True } \overline{G}^2}{2\rho} \frac{L}{r_L}$$

$$= .0075 \frac{25.6 \times 1.5}{2 \times .052 \times .015}$$

$\Delta p = 184$ p/ft

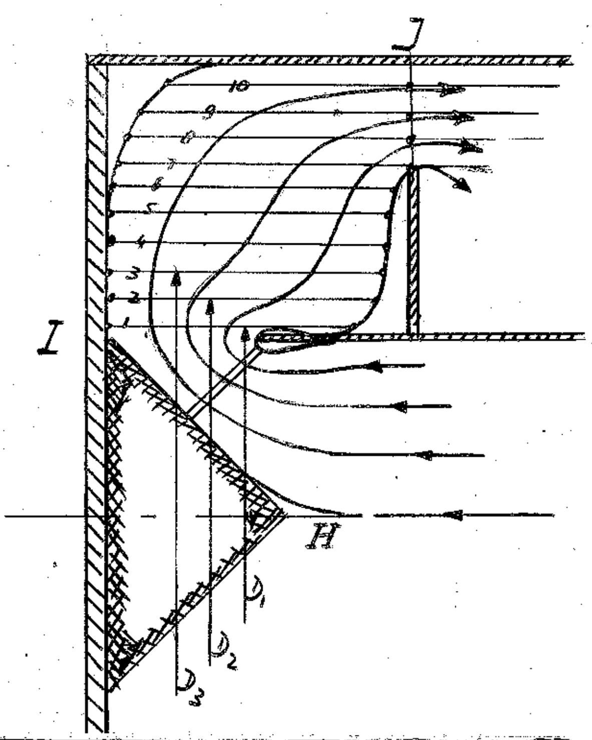
or 1.1 "W

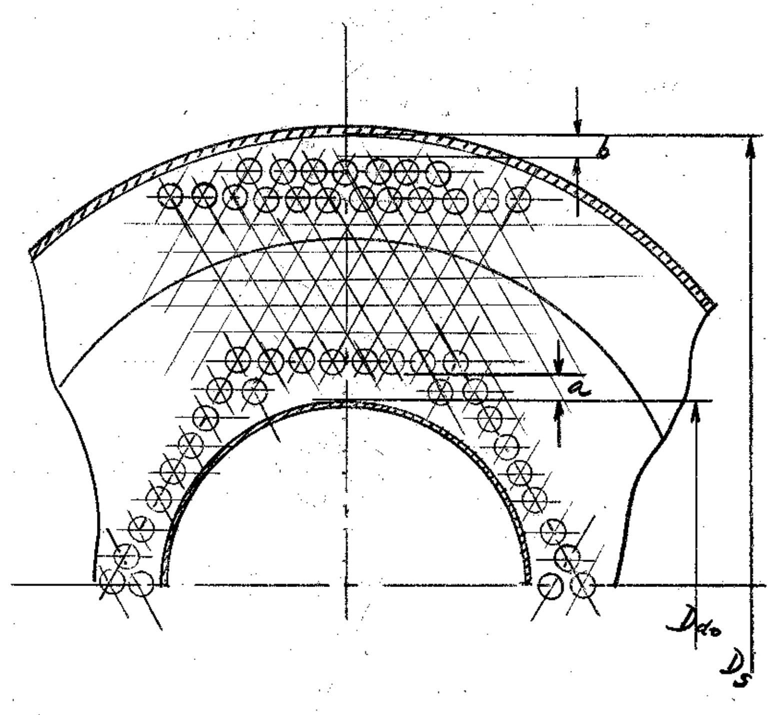
Total axial pressure drop = .42 + 1.1 = 1.52 "W

THE CROSS FLOW PRESSURE DROP

There is a sudden contraction at I deserving special attention. Notwithstanding the fact that in the previous chapter, section CROSS FLOW PRESSURE LOSS, the result of 5.8 "W over the counter flow exchanger was cut down to 3 "W, the same method is employed here, for compartments I-J and J-K, with a view to reduce the final result by the same ratio.

$D_s - D_{do}$ is divided in sketch below into 10 shells of diameters D_1, D_2 -- D_{10}; just outside D_{do} and just inside D_s are no tubes, thus the flow will rapidly expand on shell surface No. 1. Consequently L_2, L_3 etc. will, already, be comparatively large. Shell 1 and Shell 10 do not produce cross flow pressure drop. The shell diameters are chosen so that between each will be 1 row of tubes. n = 1.





4 cm ≙ 1 ft

	L	D	A	Mass Flow	G	G^2	$G^2/2\rho$
1	.82	1.26	*3.24	4/4 M	2.16	4.65	43
2	.9	1.44	.9	4/4	7.8	60.8	564
3	.92	1.62	1.03	4/4	6.8	46	426
4	.94	1.80	1.17	4/4	6	36	333
5	.96	1.98	1.31	4/4	5.35	28.5	264
6	.98	2.16	1.47	4/4	4.76	22.6	209
7	1.0	2.34	1.62	4/4	4.32	18.6	172
8	.98	2.52	1.71	3.5/4	3.58	12.8	128
9	.95	2.70	1.78	2.5/4	2.46	6.0	55
10	.86	2.82	*7.6	1.5/4	.35	.12	1

$\rho = .054$

$\sum G^2/2\rho = 2150$, without Nos. 1 and 10 since there are no tubes to be crossed.

$$\Delta p = 4\ f_{cr}\ N \frac{G^2}{2\rho}$$

$$\Delta p = 4\ f_{cr}\ n \sum_{1}^{10} \frac{G^2}{2\rho} \qquad n = 1\text{, as mentioned}$$

$\overline{G} = 4.36$
$\mu = 2.27 \times 10^{-5}$
$d = .0833$

$R_e = 1.6 \times 10^4$
$(R_e)^{-.15} = .234$

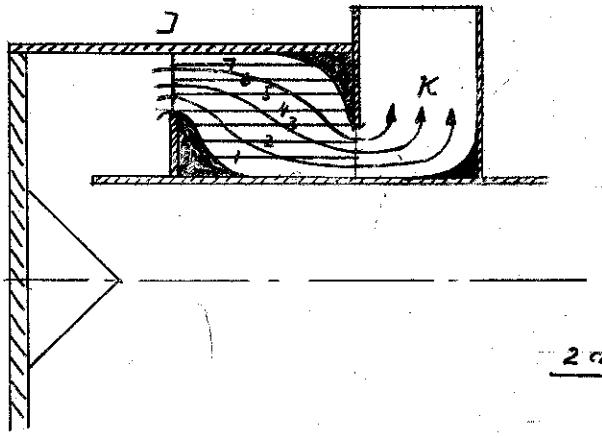
$4\ f_{cr} = 4 \times .723 \times .234$
$4\ f_{cr} = .676$

$\Delta p = .71 \times 2150 = 1450$ #/ft^2 or 8.7 "W

For compartments J-K, the analysis must not be rigorous.

t = 880 °F

$\rho = .052$



2 cm ≑ 1 ft

	L	D	πD	A	Mass Flow	G	G^2	$G^2/2\rho$
1	.85	1.5	4.72	.88	1.5/4	3.0	9.0	83.5
2	1.0	1.72	5.4	1.19	2.5/4	3.68	13.5	125
3	.9	1.95	6.13	1.21	3.5/4	5.06	25.6	238
4	1.0	2.18	6.85	1.51	4/4	4.63	21.5	200
5	1.05	2.4	7.55	1.75	3/4	3.0	9.0	83.5
6	1.0	2.62	8.25	1.81	2/4	1.93	3.72	40.2
7	.9	2.85	9.0	1.78	1/4	.98	.96	9

$\sum \frac{G^2}{2\rho} = 781$ p/ft^2.

$\bar{G} = 3.2$
$\mu = 2.33 \times 10^{-5}$
d = .0833

$\frac{Gd}{\mu} = 1.14 \times 10^4$

$\left(\frac{Gd}{\mu}\right)^{-.15} = .246$

$4 f_{cr} = 4 \times .723 \times .246 = .71$

$n \approx 8/7 \approx 1.1$

$\Delta p = 4 f_{cr} \; n \sum \frac{G^2}{2\rho}$

$\Delta p = .71 \times 1.1 \times 781$

$\Delta p = 611$ p/ft^2 or <u>3.66 "W</u>

The cross flow within K is best divided into two parts: from D_{do} to D_{bo} and from D_{bo} outwards to L. For the first part, the full circumference is involved; for the second, it reduces to 3 times distance between 1 and 2 in the cross section K, chapter 6. The first will be handled now.

	L	D	πD	A	Mass Flow	G	$G^2/2\rho$
1	.75	1.5	4.72	.78	1.5/4	3.37	111
2	.75	1.72	5.4	.89	2.5/4	4.92	235
3	.75	1.95	6.13	1.01	4/4	6.92	466

t = 895
$\rho = .0512$
$\mu = 2.35 \times 10^{-5}$
n = 1.1

$\bar{G} = 7.07$

$\frac{Gd}{\mu} = \frac{7.07 \times .0833}{2.35} 10^5$

$\frac{Gd}{\mu} = 2.5 \times 10^4$

$\left(\frac{Gd}{\mu}\right)^{-.15} = .219$

$4 f_{cr} \; n = 4 \times .723 \times .219 \times 1.1 = .695$

$\sum \frac{G^2}{2\rho} = 812$

$\Delta p = .695 \times 812 = 555$ p/ft^2

or <u>3.38 "W</u>

<u>THE PRESSURE DROP OVER THE BAFFLES J AND K</u>

The disc J:

$\Delta p = 1.12 \frac{G^2}{2\rho}$ assumed, with G halfway between I and J.

t in compartments I-J = 870°F
ρ in compartments I-J = .052
A_{max} in compartments I-J = $(D_s^2 - D_{do}^2)\ \pi/4\ R_{ax}$ = 2.46 ft²

$$\frac{G^2}{2\rho} = \frac{8.1}{.104} = 78 \text{ p/ft}^2 \qquad \Delta p = 1.12 \times 78 = 87.5 \text{ p/ft}^2$$

or .52 "W

The orifice K:

$\Delta p = .8 \frac{G^2}{2\rho}$, G to be taken at narrowest section, K.

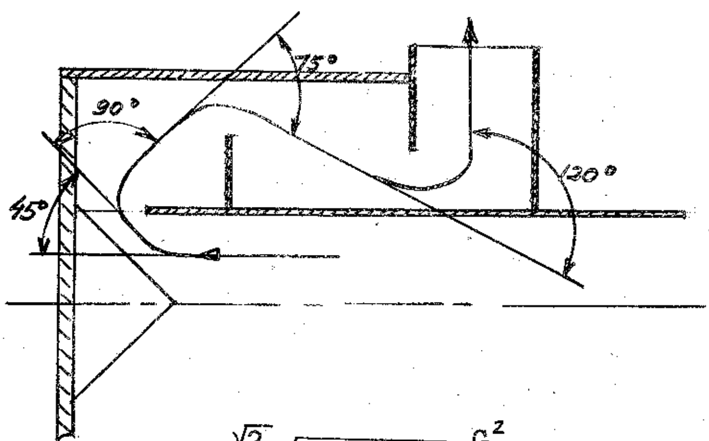
t at K = 895°F
ρ at K = .0512
A at K for axial flow = 1.11 ft² (compare with chapter 6, station C, C_1, C_2)

$$G = \frac{7}{1.11} = 6.3 \qquad \frac{G^2}{2\rho} = 387$$

The coefficient .8 still has to multiplied with .696 (see chapter 7, orifice loss).

Δp = .8 x .696 x 387 = 216 p/ft² or 1.3 "W

PRESSURE DROP THROUGH CHANGE OF DIRECTION



The directional change over J and K is here considered together with those at H and I.

$$\Delta p = \frac{\sqrt{2}}{2} \sqrt{1 - \cos\alpha}\ \frac{G^2}{2\rho}$$

	α	$\cos\alpha$	$1-\cos\alpha$	$\sqrt{1-\cos\alpha}$	$\sqrt{2}/2 \times \sqrt{1-\cos\alpha}$	$G^2/2\rho$	Δp
H	45	.707	.293	.54	.382	446	170
I	90	0	1	1	.707	135	95
J	75	.2	.8	.895	.632	138	87
K	120	-.5	1.5	1.22	.865	387	334

$\sum \Delta p$ = 686 p/ft², or 4.1 "W.

With a refined flow pattern a Δp of 4.8 "W is obtained.

THE PRESSURE DROP DUE TO THE CROSS FLOW BETWEEN K AND L

$$\Delta p = 4\ f_{cr}\ N \frac{G^2}{2\rho}$$

Compare cross section K in chapter 6.

for each of the 3 branches

$$G = 7/3 \frac{1}{.75 \times 1.75 \times .22}$$

$$G = 2.33 \frac{1}{.29} = 8 \text{ lb/ft}^2 \text{se}$$

$$\frac{G^2}{2\rho} = \frac{64}{.1024} = 624$$

$t = 895^\circ F$
$\rho = .0512$
$\mu = 2.35 \times 10^{-5}$

$$\frac{G\ d_{hy}}{\mu} = \frac{8 \times .06}{2.35} 10^5 = 2.04 \times 10^4$$

$$\left(\frac{G\ d_{hy}}{\mu}\right)^{-.15} = .226$$

$$4\ f_{cr} = 4 \times .723 \times .226 = .653 \qquad N = 4$$

$$\Delta p = .653 \times 4 \times 624 = 1630 \text{ p/ft}^2 \quad \text{or} \quad \underline{\underline{9.8 \text{ "W}}}$$

THE NON-ISOTHERMAL PRESSURE DROP BETWEEN H AND L

$t_{(H)} = 800^\circ$ $\qquad \rho_{(H)} = .0552$
$t_{(L)} = 895^\circ$ $\qquad \rho_{(L)} = .0512$ $\qquad \rho_{(L)} = \rho_{(L')}$

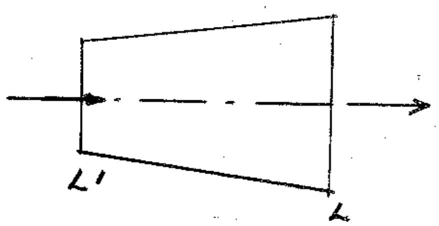
$A_{(H)} = 1.0 \text{ ft}^2$ $\qquad A_{(L)} = 2.36 \text{ ft}^2$
Reduce $A_{(L)}$ to $A_{(L')} = 1 \text{ ft}^2$

$G_{(H)} = 7.0 \text{ lb/ft}^2 \text{se}$ $\qquad G_{(L')} = 7 \text{ lb/ft}^2 \text{se}$

$$\Delta p = G^2 \left(\frac{1}{\rho_{(L)}} - \frac{1}{\rho_{(H)}}\right) = 49 \left(\frac{1}{.0512} - \frac{1}{.0552}\right) = 49 \times 1.4$$

$$\Delta p = 68.5 \text{ p/ft}^2 \quad \text{or} \quad \underline{\underline{.41 \text{ "W}}}$$

THE BERNOULLIAN PRESSURE CHANGE



$A_{(L')} = 1.0 \text{ ft}^2$
$A_{(L)} = 2.36 \text{ ft}^2$

$\rho_{(L')} = \rho_{(L)} = .0512$

$$\Delta p = \left(\frac{G^2}{2\rho}\right)_L - \left(\frac{G^2}{2\rho}\right)_{L'}$$

$$\Delta p = \frac{7^2 - 2.96}{2 \times .0512} = \frac{40.2}{.1024} = 393 \text{ p/ft}^2 \quad \text{or} \quad \underline{\underline{-2.35 \text{ "W}}}$$

THE TOTAL PRESSURE DROP

	Straight Calculation	Corrected
1) Axial flow pressure drop between I-K	1.52 "W	1.52 "W
2) Cross flow pressure drop between I-J	8.7	
3) Cross flow pressure drop between J-K	3.66	8.20
4) Cross flow pressure drop within K	3.38	
5) Disc effect at J	.52	.52
6) Orifice effect at K	1.3	1.30
7) Change of direction at H, I, J, K	4.1	4.80
8) Cross flow pressure loss between K-L	9.8	5.10
9) Non-isothermal loss between G-K	.41	.41
10) Bernoullian pressure change between G-L	-2.35	-2.35
2), 3), 8) reduced by $\frac{3}{5.8} = .52$	Total ≈	19.6 "W

Total pressure drop observed

$$\Delta p_{31 \to 30}, \text{ from Fig. 1, equals } \frac{.8}{.0361} = 22.1 \text{ "W}$$

The pressure on which the calculation of ρ has been based, throughout chapter 8, was assumed to be 26 psia. Actually Fig. 1 shows that the average pressure between curves P_{30} and P_{31} equals about 25 psia. ρ_m is correspondingly smaller and the calculated total pressure drop should be

$$\Delta p = 19.6 \times \frac{26}{25} = 20.4 \text{ "W}$$

PRESSURE LOSS FOR M = 10 lb/se

$$\Delta P_{M=10} = \Delta P_{M=7} \frac{(G^2/\rho)_{M=10}}{(G^2/\rho)_{M=7}} \times \frac{(Re)_{M=10}^{-.15}}{(Re)_{M=7}^{-.15}}$$

$$= \Delta P_{M=7} \frac{100}{49} \times \frac{25}{36} \times \frac{1570}{1310} \left(\frac{10/.258}{7/.23}\right)^{-.15}$$

$$= .8 \times 2.04 \times .695 \times 1.2 \times .965 = .8 \times 1.64$$

$\Delta P_{M=10} = 1.31$ psi, calculated

$\Delta P_{M=10} = 1.20$ psi, observed   *from Fig. 1

Deviation, about 9%

The argument is the same as at the end of chapter 7. For the parallel flow exchanger at M = 7 and M = 10, the pressures are 25 and 36 psia; the mean temperatures, 1310 and 1570°F; and the viscosities, .23 and .258 times 10^{-5}.

The deviation between calculated and observed pressure drop is as well the same. However, this coincidence does not lend additional weight for the correctness of the argument, it being merely a repetition.

GENERAL CONSIDERATIONS

Another factor will influence this conclusion from ΔP at low mass flow to ΔP at higher mass flow: the "Area of flow" which is about the same as the "Area of heat transfer." In chapter 10 the heat transfer coefficients, h_a and h_g, will be calculated in a cursory manner. From chapter 8 the values of UA are known. If U is computed from h_a and h_q, A may be fixed. $A_{M=7}$ and $A_{M=10}$ can thus be compared. The result will be considered valid only qualitatively.

If a turbulent fluid is forced through additional irregularily shaped passages, it might go on to loose available energy and pressure although it is already turbulence saturated. This partial contradiction can be resolved as follows:

1) Frictional losses upon boundary walls will go on as long as the fluid flows. There is no saturation with regard to transfer of energy and momentum to the adjacent walls.

2) The geometrical obstructions like curves, edges, faces give rise to large size vortices which dissipate by virtue of the viscosity of the fluid into turbulent lumps of semi-microscopical size and further degenerate into molecular energy, the least available form. Thus, additional formation and dissipation of vortices can take place almost continuously and the pressure will keep on decreasing.

Under certain flow conditions there is a marked drop in drag resistance with increasing velocity, i.e. when the laminar sublayer becomes turbulent. This would be an argument for relatively decreasing flow resistance with increasing mass velocity. However, this can hardly be assumed for the air side of the hot heat exchanger, since the flow there is already highly turbulent from the beginning. Such an effect, if present at all, could hardly be traced or defined as such.

There remains, as the only reason for a comparative smaller pressure loss at higher mass velocities, the decrease in f_{ax} and f_{cr}, as outlined above.

9. THE NORMALIZED PRESSURE LOSS COEFFICIENTS

Provided that there will not be any major change of flow area, the individual pressure losses for other mass flow may be expressed as products of certain numerical values, distinct for each kind of loss, with the term M^2/ρ. For the axial and cross flow pressure loss f, the frictional coefficient, will change also, depending on varied Re.

At the same time losses occurring within a number of consecutive compartments may be comprehended and expressed by means of their average coefficients as a single term. This will serve as well for calculation of and for comparison of the individual losses with figures obtained from observation.

The numerical values implying the choice of proper average temperature, density, viscosity are so defined as to give the calculated loss for M = 7.

THE COUNTER FLOW EXCHANGER

AXIAL FLOW $\Delta p = f_{ax}(\bar{G},\mu) \times 830 \frac{1}{\rho} \frac{M^2}{7^2}$ Valid for compartment B-C

1. $\Delta p = f_{ax}(\bar{G},\mu) \times 1586 \frac{1}{\rho} \frac{M^2}{7^2}$ Valid for compartments C-E to C_2-E_2

$\bar{G} = 4.26 \frac{M}{7}$; $A^* = 1.53\ ft^2$; $\frac{\sqrt{\bar{G^2}}}{\bar{G}} = 1.07$

$$\Delta p = f_{ax_{C_1-E_1}} \times 7940 \frac{1}{\rho_{C_1-E_1}} \frac{M^2}{7^2}$$

Combined loss for compartments C-E to C_2-E_2

CROSS FLOW

$$\Delta p = \left(\frac{.158\bar{G}}{\mu}\right)^{-.15} \times 55 \frac{1}{\rho} \frac{M^2}{7^2}$$

Valid for compartment B-C

$$\Delta p = \left(\frac{.0833\bar{G}}{\mu}\right)^{-.15} \times 15.2 \frac{1}{\rho} \frac{M^2}{7^2}$$

Valid for compartments C-E to C_2-E_2

2. $\bar{G}_{B-C} = 1.9\ \bar{G}$; $\bar{G} = 1.34 \frac{M}{7}$; $A^* = 3.36\ ft^2$; $\frac{\sqrt{\bar{G^2}}}{\bar{G}} = 1.5$

$$\Delta p = \left(\frac{.112\frac{M}{7}}{\mu_{C_1-E_1}}\right)^{-.15} \times 76 \frac{1}{\rho_{C_1-E_1}} \frac{M^2}{7^2}$$

Combined loss for compartments C-E to C_2-E_2 considering the term .52 = 3/5.8

DISC

$$\Delta p = 4.55 \frac{M^2}{7^2} \frac{1}{\rho}$$

ρ between stations

3. $$\Delta p = 18.2 \frac{1}{\rho_{C_1}} \frac{M^2}{7^2}$$

Combined for stations B, E, E_1, E_2

ORIFICE

$$\Delta p = 11 \frac{M^2}{7^2} \frac{1}{\rho}$$

ρ at station

4. $$\Delta p = 33 \frac{1}{\rho_{C_1}} \frac{M^2}{7^2}$$

Combined for stations C, C_1, C_2

CHANGE OF DIRECTION OVER BAFFLE

5.

$$\Delta p = 4.5 \frac{1}{\rho_E} \frac{M^2}{7^2}$$

Combined for stations B, E, E_1

$$\Delta p = 12 \frac{1}{\rho_{C_1}} \frac{M^2}{7^2}$$

Combined for stations C, C_1, C_2

CHANGE OF DIRECTION

$$\Delta p = 5.15 \frac{M^2}{7^2} \frac{1}{\rho}$$

at station F

6. $$\Delta p = 17.3 \frac{M^2}{7^2} \frac{1}{\rho}$$

at station F_2

$$\Delta p = 22.5 \frac{1}{\rho_F} \frac{M^2}{7^2}$$

FLOW ACROSS TUBES $F \rightarrow F_1$

7. $$\Delta p = 37 \frac{M^2}{7^2} \frac{1}{\rho_F}$$

NON-ISOTHERMAL DROP

8. $$\Delta p = 60.5 \frac{M^2}{7^2} \left(\frac{1}{\rho_{(G)}} - \frac{1}{\rho_{(A_0)}} \right)$$

BERNOULLIAN PRESSURE CHANGE

9. $$\Delta p = 27.1 \, (1 - 1.115) \frac{M^2}{7^2} \frac{1}{\rho}$$

$$\Delta p = -3.1 \frac{M^2}{7^2} \frac{1}{\rho_{(G)}}$$

THE PARALLEL FLOW EXCHANGER

AXIAL FLOW

10. $$\Delta p = f_{ax}\left(\frac{.264 \frac{M}{7}}{\mu_{(J)}} \right) \times 1750 \frac{1}{\rho_{(J)}} \frac{M^2}{7^2}$$

$f_{ax}(\ldots)$ means function of $(\ldots)$

CROSS FLOW

11. $$\Delta p = \left(\frac{.384 \frac{M}{7}}{\mu_{(J)}} \right)^{-.15} \times 311 \frac{1}{\rho_{(J)}} \frac{M^2}{7^2}$$

DISC

12. $$\Delta p = 4.5 \frac{1}{\rho_{I-J}} \frac{M^2}{7^2}$$

ORIFICE

13. $$\Delta p = 11 \frac{1}{\rho_{(K)}} \frac{M^2}{7^2}$$

CHANGE OF DIRECTION

14. $$\Delta p = 42.5 \frac{1}{\rho_{(J)}} \frac{M^2}{7^2}$$

CROSS FLOW $K \rightarrow L$

15. $$\Delta p = 43.5 \frac{1}{\rho_{(K)}} \frac{M^2}{7^2}$$

NON-ISOTHERMAL DROP

16. $$p = 49 \frac{M^2}{7^2} \quad \frac{1}{\rho(L)} - \frac{1}{\rho(H)}$$

BERNOULLIAN PRESSURE CHANGE

17. $$p = 20 \frac{1}{\rho(L)} \frac{M^2}{7^2}$$

10. THE HEAT TRANSFER COEFFICIENTS

These will be calculated without much detail, first of all for M = 7 lb/se. The aim is twofold:

1) to get an estimate for the wall temperature T_W.

2) to compute an average value for U and, thus, for that fraction of the total heat exchanger area which participates in the heat exchange.

Apply $Nu = .024\ R_e^{.8}\ Pr^{.31}$

Evaluate for x = 0, 3, 6, 9, 12, 14.75 ft.

THE TUBE SIDE COEFFICIENT h_g

Assume:

a) P_r for the gas side equals the P_r of air above 550°F, i.e., .66 (see Fig. 6),
b) K, virtually, is equal to K of air,
c) μ, virtually, is equal to μ of air,

Since the products of combustion with 400% theoretical air contain only about 5% CO_2.

$$\frac{h_g\ d_i}{K} = .024 \times P_r^{.31} \times Re^{.8} \times \frac{d_i}{d}$$

$$G = 5.52/Nd_i^2\ \pi/4 = 5.52/504 \times .0782^2\ \pi/4 = \frac{5.52}{2.42} = 2.28$$

$$P_r^{.31} = .879;\ \frac{d_i}{d} = .938; \qquad G\ d_i = 2.28 \times .0782 = .178$$

$$h_g = \frac{.024}{d_i}\ K \times .879\ \left(\frac{.178}{\mu}\right)^{.8} \times .938$$

$$h_g = .307 \times .879 \times .938\ \frac{K'}{3600} \times \left(\frac{.178}{\mu}\right)^{.8}$$

1. $$h_g = \frac{.253}{3600}\ K'\ \left(\frac{.178}{\mu}\right)^{.8}$$

x	T	K'	μ	$.178/\mu$	$(.178/\mu)^{.8}$	$K' \times (\ldots)^{.8}$	h_g
0	1014	.0362	2.48×10^{-5}	7.18×10^{3}	1.21×10^{3}	43.8	3.08×10^{-3}
3	880	.0335	2.33	7.64	1.28	42.9	3.02
6	781	.0312	2.21	8.05	1.32	41.2	2.90
9	693	.0292	2.10	8.47	1.38	40.3	2.83
12	616	.0275	2.01	8.85	1.41	38.8	2.72
14.75	550	.026	1.92	9.26	1.49	38.8	2.72

Values for K', as given in table, refer to one hour.
Values for h_g, as calculated, refer to one second.

THE SHELL SIDE COEFFICIENT h_a

Assumptions have to be made as to what mass velocity, in cross flow or axial flow, h_a shall refer. Since the direction of flow, at least in the counter flow exchanger, is definitely more axial, axial flow alone will be considered. For the sake of comparison with h_g, constant mass velocity and, thus, flow area has to be assumed.

A is the average between $A_{(C)}$ and $A_{(E)}$.

$$A = 1/2\ (1.11 + 1.83) = \underline{1.465\ ft^2}$$

$$G = 7/1.465 = \underline{4.77\ lb/ft^2\,se}$$

P_r, between 440 and 650°F, changes.

$$R_e \text{ is here } \frac{G\, d_{hy}}{\mu} = \frac{4.77 \times .06}{\mu} = \frac{.286}{\mu}$$

$$h_a = \frac{K}{d_{hy}} \times .024\ P_r^{.31} \left(\frac{.286}{\mu}\right)^{.8}$$

$$h_a = \frac{.024}{.06} K'\ P_r^{.31} \left(\frac{.286}{\mu}\right)^{.8} \times \frac{1}{3600}$$

2. $$h_a = \frac{.4}{3600} K'\ P_r^{.31} \left(\frac{.286}{\mu}\right)^{.8}$$

x	t	K'	P_r	$P_r^{.31}$	μ	$(.286/\mu)^{.8}$	$K' \times P_r^{.31} \times (.286/\mu)^{.8}$	h_a
0	800	.0316	.66	.879	2.24×10^{-5}	1.94×10^{3}	54.0	6.0×10^{-3}
3	710	.0296	.66	.879	2.13	2.0	52.0	5.77
6	631	.028	.66	.879	2.03	2.1	51.6	5.73
9	559	.0263	.662	.88	1.94	2.15	49.8	5.54
12	494	.025	.666	.882	1.85	2.2	48.5	5.39
14.75	440	.0235	.67	.883	1.79	2.3	47.7	5.30

CALCULATION OF U AND T_w

$$\frac{1}{U} = \frac{1}{h_a} + \frac{1}{h_g}$$, neglecting the resistance of the wall.

$$T_w = t + (T - t)\frac{h_g}{h_a + h_g}$$

x	$1/h_a$	$1/h_g$	$1/U$	U	$\frac{h_g}{h_a + h_g}$	$(T-t)$	$\frac{(T-t)h_g}{h_a + h_g}$	t	T_W
0	167	325	492	2.03×10^{-3}	.339	214	72.5	800	872
3	173	331	504	1.98	.344	170	58.5	710	768
6	174	345	519	1.93	.336	150	50.4	631	681
9	180	354	534	1.87	.338	134	45.3	559	604
12	185	368	553	1.81	.335	122	40.9	494	535
14.75	188	368	556	1.80	.339	110	37.3	440	477

Average $\frac{h_g}{h_a + h_g} = .338$; (T - t) is multiplied with this average value.

3. Average U $= \underline{\underline{1.91 \times 10^{-3}}} \frac{BTU}{ft^2 \text{ se } {}^\circ F}$

CALCULATION OF EFFECTIVE HEAT TRANSFER AREA

Referring to the last section of chapter 8 and to formula 5-3, the effective heat transfer area shall be

$$A = \frac{4.3}{U} = \frac{4.3}{1.91} \times 10^3 = \underline{\underline{2250 \text{ ft}^2}}$$

The total outside area of the tubes within the counterflow exchanger equals

$$A_{total} = 504 \pi \times .0833 \times 14.75 = 1945 \text{ ft}^2$$

Even if one adds to this the corresponding shell area, $A_{sh} = 3\pi \times 14.75 = 139$ ft², the combined total, 1945 + 139 = 2084 ft², is still somewhat below the observed effective value of 2250 ft², whereas the latter should be smaller than the geometrical value.

Additional measurement of pressure and temperature are necessary to bring these facts to light. Also, the temperature of the shell should be known at several stations, in order to determine the true radiation losses.

11. CONCLUSIONS

I - From observed data:

1. The pressure loss in the parallel flow exchanger is as great as that in the whole rest of the apparatus. This is out of any proportion.

2. This leads to the question whether a pure counter-flow exchanger would not do the job as well. T30 - T31 is small compared with T50 - T30 and the quick cooling effect to be produced by the parallel part is possibly brought about, in actual fact, by the tube plate cooling.

3. The gain in heat transfer capacity produced by the baffles is in all likelihood jeopardize right on the spot by the areas of wake formed after each baffle. Anyhow, the transfer coefficients h_a and h_g as calculated in Chapter 10 for a straight heat exchanger match more or less the existing amount of heat transferred.

4. The contradiction mentioned on P.53 together with that on P.22 makes it impossible either to assess the true heat transfer coefficients or to extend with certainty the results of this paper, i.e. the "Normalized Pressure Loss Coefficients" in their present form to other operating conditions.

5. The reasons for this are twofold: Uncertain temperature measurements, first of all at T50 - it is a question of special interest to estimate the error in measurement of temperature and mass flow - and, more fundamentally, an assumed temperature distribution as in Fig. 5, which does not consider in detail cooling of tube plate and shell. The cooling of the tube plate causes T_I (Fig. 5) to be considerably lower than T50. Thus, the actual temperature curves are, probably, more flat or more near to a straight line than those in Fig. 5.

6. Knowledge of the true temperature at any point of the exchanger is highly desirable, on the base of any calculation, be it for pressure loss, heat transfer coefficients, stress analysis or a problem of corrosion.

7. If the actual temperature distribution is fairly well known it may be attempted to reduce the number of the "Normalized Pressure Loss Coefficient" from 15 to about 4 or 5. Note that the temperature distribution is different for varying mass flow ratio at gas and air side.

8. Since the hot heat exchanger is no piece of equipment installed for the purpose of finding out pressure loss coefficients under varying conditions of flow, it is of no use, here, to develop comprehensive pressure loss equations with coefficients containing the given data in p and T, although it does not appear unreasonable to assume that this were, eventually, possible.

9. It would be more conceivable to work out an expression for the characteristic temperature in the term M^2T/P^2 which is $\propto$ to $\Delta P/P$ and, furthermore, to split the observed pressure loss into three parts, namely FRICTIONAL, LOCAL and HYDRODYNAMIC losses (comp. Chapter 2, page 2) and to apply for each term separately a suitable expression which still has to be set up. In Chapter 18 the simplest possible method has been applied to obtain a general expression for the pressure loss.

10. It is desirable to supplant the comparatively small number of data for a given moment of operation by further temperature and pressure readings, thus reducing the ratio between assumptions or derivations on the one hand and observed figures on the other hand to a more factual equilibrium.

II - Additional readings that might be taken with the existing arrangement:-

11. Determine pressure drop over one baffle, at two points of equal flow area, or at points where something like the total pressure may be observed.

    a) in the counterflow exchanger.
    b) in the parallel flow exchanger and
    c) between F - G
    d) between H - J
    e) between K - L

12. Try to establish additional temperature readings.
    a) at one or two places of the tubes, mainly opposite station 31.
    b) at three to four additional places on the air and gas side.
    c) at or near the tube plate and at inner and outer shell, in some characteristic points, in order to get a more detailed picture of the energy losses and boundary temperatures.

13. If there were any way to measure the extent of wake near the cold end of the heat exchanger, for example on the lee side of a baffle, their influence regarding the overall heat transfer coefficient would be cleared by one important step.

III - Changes in the Hot Exchanger with the aim to decrease the pressure loss:-

14. Shift the baffle E_2 by one foot to the right and enlarge the six ports. Shorten the inner duct by about .30 ft. at H. Increase D_{bo} at K, decrease D_{bi} at J by about .4 ft. each (see fig. 2).

15. Either produce additional through-passages between the tubes in baffle areas or

16. blow out the wake by opening a circular segment between inner duct and inside baffles and outer shell and outside baffles, respectively over a considerable portion of the circumference.

17. The heat transfer may be improved by providing flow limiting sheets in axial direction along the whole length of the counterflow exchanger. The pressure loss would thereby increase somehow, but this were a useful loss. On those places where the temperature of the shell is higher than that of the air the device must not be mounted. This would modify proposition 16 on those places where proposition 17 is applied.

PART II - ELEMENTS OF DESIGN OF A NEW EXCHANGER

12. GENERAL APPROACH

PART I of this Report has been computed based on a few observed data for temperature and pressure originating from the early days of life of the Experimental Coal Burning Gas Turbine, of which the Hot Heat Exchanger is an integral part.

The first trials were done in November and December 1953 and the data produced in Chapter 4 were observed during this period. In coarse language it might be said that the analysis as of Chapters 7 and 8 is not a theoretical deduction but, merely, a compilation of known facts which have been handled and figured until the final result was in fairly good accordance with the observed data for pressure losses.

This somehow exaggerated view is taken here in order to emphasize the importance of experience - above and beyond pure knowledge of the basic equations and phenomena of Fluid Dynamics - which is essential when attempting to analyze the performance of an existing component under certain operating conditions or, to make a performance prediction of such a part whilst still in the drawing board stage.

Between January and October 1954 the plant has been operating for more than a hundred hours and valuable experience has been gained by all those who were connected with the work. By his participating in these developments as well as by numerous discussions and exchange of opinions with the other staff of the Gas Dynamics Laboratory, where the author has been working during the summer 1954 he, too, has become more familiar with the problems in question.

Thus, the outlay of the following pressure loss analysis referring to the same type of heat exchanger but with number of new design features, differs in many ways from the approach taken in PART I.

Partly, this is due to physical changes of the exchanger and, partly, to revised methods of calculation. With respect to several other points more realistic assumptions have been put forth.

The new Hot Heat Exchanger shall have a lower pressure loss. In fact, it will be attempted to reduce the various losses to the minimum compatible with the overall design.

A "Pure Counterflow" Type and a "Parallel and Counterflow" Type Exchanger will be investigated, the latter being similar in shape to the existing exchanger. Calculation of heat transfer capacity (or length) and pressure loss will be based on Full Load conditions as given in Chap. 13.

The tube arrangement in both types is also very similar to that in the Existing Exchanger. Gas flows inside, air outside the tubes. The shell has to withstand the compressor delivery pressure. Again, 506 tubes of 1" O.D. are provided, arranged in triangular pitch around a central core.

The physical changes proposed below have not been tried yet in the existing plant. Some of them are well known elements, others might be a challenge compared with customary methods of construction.

The principal changes of design are:

1. Central introduction of air.

2. Guide vanes and guide rings whenever the air flow changes direction, except in the entrance region within the tube bundle.

3. Inner and outer tube bundle linings designed to constrict the air flow to the space immediately around the tubes.

4. Ladder-type tube suspension in place of the customary baffles, except at one place in the entrance region.

5. A p/d ratio of 1.30, in place of the previous value of 1.25.

6. Increased effective length and, total length, of the exchanger.

7. Additional insulating layer between Tube Shell and Main Shell.

8. Adjustable bypass for cold air to be diverted from the air entrance region directly to the surface of the hot tube plate, for the purpose of cooling; to be applied for the "Pure Counterflow" Type exchanger with its elevated tube wall temperature.

9. Generous cross flow area, especially for the "Parallel and Counterflow" Type Exchanger, in order to lower the cross flow pressure losses.

10. Curved vanes in the outlet ring surrounding the tube bundle which guide the air to the exit flanges being, at the same time, a structural element of the construction.

11. Thin metal diffusers at the gas outlet from the tubes, in addition to the ceramic diffusers at gas inlet.

Varied methods of calculation are exemplified by:

1. The assumption of linear temperature distribution, as first approximation on both the gas and air side.

2. Consideration of the initial energy losses on the gas side, thru' the intense tube plate cooling, leading to the design temperature $T\ 50_{eff}$ in place of T 50.

3. Consideration of the entrance pressure losses.

4. Reduction of the cross flow pressure loss coefficient f_{cr} to .70 f_{cr}, in accordance with more recent data in the literature; (Comp. pp. 38, 47).

5. Application, in a consistent manner, of a loss factor $\mathcal{L}$ in the expression for "pressure loss due to change of direction"; (Comp. 2 - 10, 2 - 10a).

6. Simplified method of calculating the average value of the heat transfer coefficients based on the equation $Nu = .023\ Re^{.8} \times Pr^{.4}$.

All in all this constitutes a more rigorous analysis. The resulting overall pressure loss, nevertheless, will be much smaller than that in the existing exchanger. Primarily, this is due to the elimination of the bottleneck passages, formerly $F - F_1$ and $H - I$ and, to the absence of baffles.

13. ENERGY BALANCE, HEAT TRANSFER COEFFICIENTS AND DESIGNED LENGTH OF THE HEAT EXCHANGER.

A. Given Data

In accordance with Gas Dynamics Laboratory Report, Note N 5, given data for full load conditions are:

M_{air} = 14.6 lb/sec.

T 28 = 380 °C ≑ 653 °K (≑ 715 °F)

T 30 = 726 °C ≑ 999 °K (≑1340 °F)

P_{air} = 64 psia ≑ 4.35 at

M_g: to be computed

T 50 = 1072 °C (≑ 1965 °F)

T 70 = 490 °C ≑ 763 °K (≑ 914 °F)

P_{gas} = 16 psia ≑ 1.09 at

Temperatures in °F are indicated in order to facilitate use of KEENAN and KAYE's Gas Tables.

$$\begin{aligned} h\ 28 &= 285 \quad \text{Btu/lb} \\ h\ 30 &= 449 \\ \hline \Delta h &= 165 \end{aligned}$$

$$\Delta H = \frac{165 \times 14.6}{1.8} = 1340 \text{ (CHU/sec)}$$

Assuming 6% losses through radiation and convection from the outer surface,

$$\Delta H \text{ required} = 1.06 \times 1340 = 1420$$

This is the amount of heat given off by the gas in the exchanger proper. Still, tube plate cooling losses have to be added. Assume 200 CHU/sec, being nominally about 20% of T 50.

The 6% radiation losses are radiated from the tube bundle to the outer tube bundle lining and the Tube Shell and further on conducted through the Main Shell and the two insulation layers. See Fig. 8. There is no interaction with the air side. Thus, the heat transfer coefficients will be calculated based upon

$$1340 \text{ (CHU/sec)} .$$

For the energy balance of the gas side, 1420 + 200 =

1620 (CHU/sec) have to be accounted for.

From KEENAN and KAYE, for "Products of Combustion with 400% Theoretical Air", M = 29.237

$$
\begin{aligned}
h\ 70 &= 9894 \quad \text{Btu/Mol} \\
h\ 50 &= 18481 \\
\hline
\Delta h &= 8587
\end{aligned}
$$

$$\Delta H = \frac{8587 \times M_g}{29.237 \times 1.8} = 1620$$

$$M_g = \frac{1620 \times 29.237 \times 1.8}{8587} = \underline{10} \text{ lb/sec}$$

The effective initial gas temperature is obtained from the Gas Tables as follows:

$$200 \text{ (CHU/sec) losses correspond to } \frac{200}{10} = 20 \text{ (CHU/lb)}$$

$$\text{or, } 20 \times 29.237 \times 1.8 = 1054 \ (\Delta \text{Btu/Mol})$$

$$18481 - 1054 = 17427 \text{ (Btu/Mol)}$$

$$
\begin{aligned}
T_{50\ eff} &= 1841\ °F \\
&\quad 1004\ °C \\
&\quad \underline{\underline{1277\ °K}}
\end{aligned}
$$

B. The "Pure Counterflow" Type Exchanger

Assuming Linear temperature change on gas and air side

$$T_{m-a} = \frac{380 + 726}{2} = 553\ °C, \quad 826\ °K, \quad 1025\ °F$$

$$T_{m-g} = \frac{490 + 1004}{2} = 747\ °C, \quad 1023\ °K, \quad 1374\ °F$$

The air-side Heat Transfer Coefficient h_a:

As indicated in Fig. 9

$$d_{hy} = .0715 \text{ ft.}$$

$$\text{Flow area } A_a = 2.45 \text{ ft.}^2$$

From Figs. 6 and 7, for 1025 °F, $Pr = .65$
$\mu = 2.51 \times 10^{-5}$
$K = .0367$

$$Re = \frac{G \times d_{hy}}{\mu} = \frac{14.6 \times .0715}{2.45 \times 2.51 \times 10^{-5}} = 17000$$

$$Re^{.8} = 2400$$
$$Pr^{.4} = .843$$

$$Nu = .023\ Re^{.8} \times Pr^{.4}$$

$$\frac{h_a\ d_{hy}}{K} = .023 \times 2400 \times .843 = 46.4$$

$$h_a = \frac{46.4 \times K}{d_{hy}} = \frac{46.4 \times .0367}{.0715} = \underline{23.8} \left(\frac{CHU}{ft^2\ h\ °C}\right)$$

The gas side Heat transfer coefficient h_g:

As indicated in Fig. 9
$d_i = .0782$ ft.
Flow area $A_g = 2.42$ ft.^{2}

Assuming for Pr, μ and K on the gas side the corresponding values for air, Figs. 6 and 7 yield

for 1374 °F $Pr = .65$
$\mu = 2.87 \times 10^{-5}$
$K = .0437$

$$Re = \frac{G\ d_i}{\mu} = \frac{10 \times .0782}{2.42 \times 2.87 \times 10^{-5}} = 11250$$

$$Re^{.8} = 1750$$
$$Pr^{.4} = .843$$

Again, $Nu = .023\ Re^{.8} \times Pr^{.4}$

$$\frac{h_g\ d_i}{K} = .023 \times 1750 \times .843 = 34$$

$$h_g = \frac{34 \times K}{d_i} = \frac{34 \times .0437}{.0782} = 19$$

Referring to the tube outside diameter.

$$h_{g'} = 19\ \frac{.938}{1} = \underline{17.8} \left(\frac{CHU}{ft^2\ h\ °C}\right)$$

Neglecting the tube wall resistance, the Overall Heat Transfer Coefficient is obtained as

$$\frac{1}{U} = \frac{1}{h_a} + \frac{1}{h_g} = \frac{1}{23.8} + \frac{1}{17.8} = .042 + .0562 = .0982$$

$$U = \underline{\underline{10.2}} \left(\frac{CHU}{ft^2\ h\ ^\circ C}\right) \quad \text{or} \quad \underline{\underline{2.84 \times 10^{-3}}} \left(\frac{CHU}{ft^2\ sec.\ ^\circ C}\right)$$

Since at both the air and gas side linear temperature change is assumed the mean temperature difference

$$\Delta T = T_{m-g} - T_{m-a} = 747 - 553 = 194\ ^\circ C$$

$$Q = A \times \mu \times \Delta T \qquad A = \frac{Q}{U \times \Delta T}$$

$$A = \frac{1340}{2.84 \times 194 \times 10^{-3}} = 2440\ ft^2$$

From Fig. 9 heat transfer area per ft of tube length equals 13.2 ft^2.

Necessary Length of exchanger = 2440/132 = <u>18.5</u> ft.

Adding a reserve of 5% for air pockets and 4% for scale, apart from the additional heating area of the tube bundle linings of $3.8 \left(\frac{ft^2}{ft\ of\ tube}\right)$ which contributes to the heat transfer to the air,

Effective Length of the exchanger = 1.09 x 18.5 = <u>20.2 ft</u>. As seen from Fig. 8 the entrance region is partly lost with regard to heat transfer; therefore, another 1.4 ft. have to be added.

Length of the exchanger between tube plates 20.2 + 1.4 = <u>21.6 ft</u>.

Length of tube bundle assumed as <u>22.0 ft</u>.

The wall temperature T_w is obtained as usual from the ratio $h_a : h_g$.
$h_a : h_g$ = 23.8 : 17.8 = 1.335 $\quad$ 1/1+1.335 = .428.
At air inlet: T 70 – T 28 = 490 – 380 = 110 °C

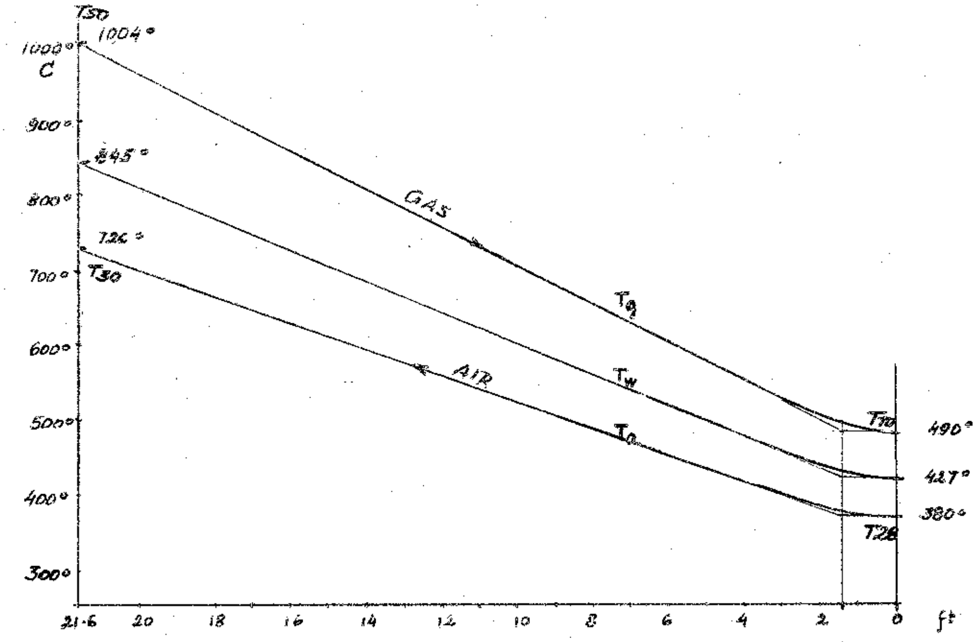
.428 x 110 = 47°

T_w at air inlet = 380 + 47 = <u>427</u> °C

Near gas inlet: T 50 eff – T 30 = 1004 – 726 = 278 °

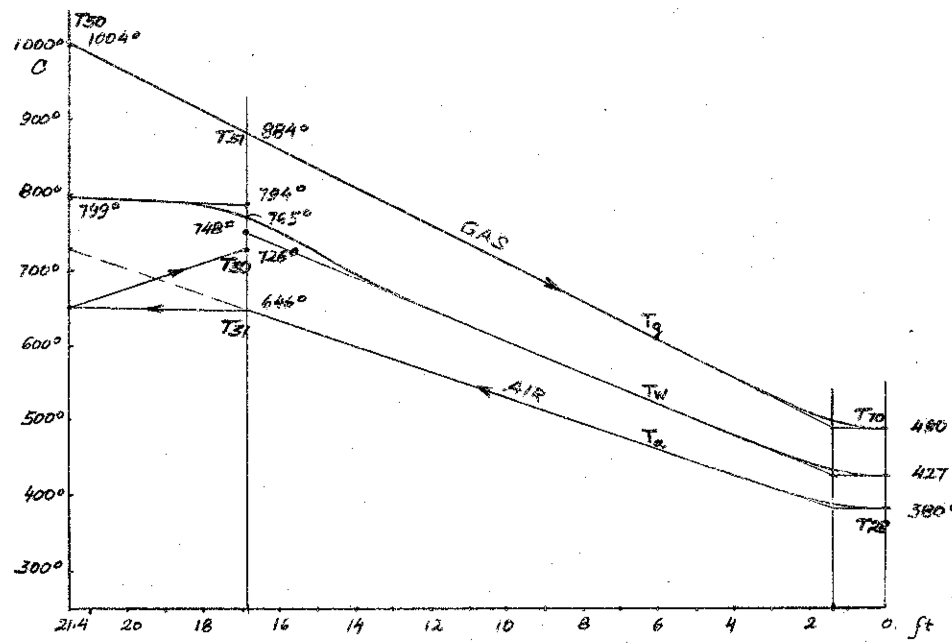
.428 x 278 = 119°
T_w near gas inlet = 726 + 119 = 845°

In actual fact, near the tube plates the influence of axial heat conduction and various other discontinuities are superimposed upon the design temperatures. These effects are not considered here.



The temperatures in the "PURE COUNTERFLOW" Type Exchanger

C. The "Parallel and Counterflow" Type Exchanger



The temperatures in the "PARALLEL AND COUNTERFLOW" Type Exchanger

With same mass flow and same temperatures T28, T30, T50 eff., T70 as before and the same active length of tubes, i.e. 20.2 ft. the air and gas temperature at the partition wall separating the parallel and counterflow part are obtained as

T31 = 646 °C and

T51 = 884 °C respectively;

these figures are based on a distance 16.9 ft. between the partitition wall and the R.H. end of the exchanger.

$$(16.9 - 1.4)/(21.6 - 1.4) = .768$$

$$\Delta T_a = T30 - T28 = 346$$

$$.768 \times 346 = 266$$

$$T31 = 380 + 266 = 646°$$

Similarly, T51 is obtained. The tube temperature at the partition wall may be assumed to be 765°C.

With these data given the amount of heat transferred has to be calculated separately for both the counterflow and the parallel part. The final length of each of them remains, for the time being, open.

I - <u>The Counterflow part:</u>

$$T_{m-a} = \frac{380 + 646}{2} = 513 \text{ °C} \quad \text{or } 956 \text{ °F}$$

$$T_{m-g} = \frac{400 + 884}{2} = 687 \text{ °C} \quad \text{or } 1268 \text{ °F}$$

$$\Delta T = 687 - 513 = \underline{174 \text{ °C}}$$

From Fig. 7 for 956 °F $\mu = 2.42 \times 10^{-5}$
$k = .035$

$$Re = \frac{G\, d_{hy}}{\mu} = \frac{5.96 \times .0715}{2.42 \times 10^{-5}} = 17600$$

$$Re^{.8} = 2500$$

$$Pr^{.4} = .843$$

$$Nu = .023 \times 2500 \times .843 = 48.5$$

$$h_a = \frac{48.5 \times k}{d_{hy}} = \frac{48.5 \times .035}{.0715} = \underline{23.75}$$

From Fig. 7 for 1268 °F $\mu = 2.75 \times 10^{-5}$
$K = .0415$

$$Re^{.8} = 1800$$

$$Pr^{.4} = .843$$

$$Nu = .023 \times 1800 \times .0843 = 34.9$$

$$h_{g'} = \frac{34.9 \times K}{d_i} \times .938 = \frac{34.9 \times .0415 \times .938}{.0782} = \underline{17.35}$$

$$\frac{1}{U} = \frac{1}{23.75} + \frac{1}{17.35} = .0421 + .0576 = .0997$$

$$U = 10 \left(\frac{CHU}{ft^2\ h\ ^oC}\right) \quad or \quad \underline{2.78 \times 10^{-3}} \left(\frac{CHU}{ft^2\ sec\ ^oC}\right)$$

Amount of heat absorbed by the air equals

$$\Delta H_a = \frac{M_a}{1.8}(h\ 31 - h\ 28)$$

$$\begin{aligned} h31 &= 410 \\ h28 &= 285 \\ \hline \Delta h &= 125 \text{ Btu/lb} \end{aligned}$$

$$\Delta H_a = \frac{14.6 \times 125}{1.8} = 1015 \left(\frac{CHU}{sec}\right)$$

Amount of heat given off by the gas equals

$$\Delta H_g = \frac{Mg}{1.8}(h\ 51 - h\ 70)$$

$$\begin{aligned} h51 &= 15\ 590 \\ h70 &= 9\ 894 \\ \hline \Delta h &= 5\ 696 \end{aligned}$$

$$\Delta H_g = \frac{10 \times 5696}{1.8 \times 28.237} = 1081 \left(\frac{CHU}{sec}\right)$$

The amount of heat available to cover radiation losses

$$\Delta H_g - \Delta H_a = 1081 - 1015 = 66 \left(\frac{CHU}{sec}\right)$$

The necessary length of the counterflow part shell be based upon ΔH_a.

$$Q = U \times \Delta T \times A = \quad \times \Delta T \times L \times 132$$

$$L = \frac{Q}{U \times \Delta T \times 132} = \frac{1015}{2.78 \times 10^{-3} \times 174 \times 132} = \underline{15.85}\ ft$$

The designed length of the counterflow section is

$$16.9 - 1.4 = 15.5\ ft$$

this should include about 10% reserve (9% in the previous case).

However, the mean temperature difference is only 174 °C, compared with 194 °C in the "Pure Counterflow" Type exchanger.

Things will be different with regard to the parallel flow part where the mean temperature difference is much larger than 194 °C.

The combined heat transfer capacity of both the counterflow and the parallel flow part is discussed in more detail in the following Section II.

II - THE PARALLEL FLOW PART:

$$T_{m-a} = \frac{646 + 726}{2} = 686\ ^\circ C \quad \text{or} \quad 1266\ ^\circ F$$

$$T_{m-g} = \frac{884 + 1004}{2} = 944\ ^\circ C \quad \text{or} \quad 1732\ ^\circ F$$

The arithmetic mean temperature difference

$$\Delta T = 944 - 686 = 258\ ^\circ C$$

The LMTD equals $\frac{358 - 158}{\lg e\ 2.265} = 245\ ^\circ C$

At 1266 °F $\quad \mu = 2.75 \times 10^{-5}$

$K = .0415$

$$Re = \frac{5.96 \times .0715}{2.75 \times 10^{-5}} = 15\ 500$$

$$Re^{.8} = 2230$$

$$Pr^{.4} = .843$$

$$Nu = .023 \times 2230 \times .843 = 43.3$$

$$h_a = \frac{43.3 \times .0415}{.0715} = \underline{25.1}$$

At 1732 °F $\quad \mu = 3.2 \times 10^{-5}$

$K = .0505$

$$Re = \frac{4.13 \times .0782}{3.2 \times 10^{-5}} = 10080$$

$$Re^{.8} = 1600$$

$$Pr^{.4} = .843$$

$$Nu = .023 \times 1600 \times .843 = 31$$

$$h_{g'} = \frac{31 \times .0505 \times .938}{.0782} = \underline{19}$$

$$\frac{1}{U} = \frac{1}{25.1} + \frac{1}{19} = .0398 + .0525 = .0923$$

$$U = 10.8 \left(\frac{CHU}{ft^2\ h\ ^\circ C}\right) \quad \text{or} \quad 3 \times 10^{-3} \left(\frac{CHU}{ft^2\ sec\ ^\circ C}\right)$$

$$\Delta H_a = \Delta H_{a,\ total} - \Delta H_{counterflow\ part} = 1340 - 1015 = 325$$

$$\Delta H_g = \Delta H_{g,\ total} - \Delta H_{counterflow\ part} = 1420 - 1081 = 339$$

$$A = (20.2 - 15.5) \times 132 = 620\ ft^2$$

Based on the arithmetic mean temperature difference of 258° the heat transferred in the parallel flow section becomes

$$Q = U \times \Delta T \times A = 3 \times 10^{-3} \times 258 \times 620 = 480 \text{ (CHU/sec)}$$

Based on the LMTD of 245 °C, Q = 466.

Both figures are by far in excess of the required amount 325 + 14 = 399 (CHU/sec), 14 being the difference between the total radiation of 1420 - 1340 = 80 and the reserve in the counterflow section of, 66 (CHU/sec).

There is, thus, a sufficient reserve in heat transfer capacity of the exchanger as a whole and the "Parallel and Counterflow" Type may retain the same dimension as the Pure Counterflow type, namely

Effective length	=	20.2 ft.
Length between tube plates	=	21.6 ft.
Average total length of tubes	=	22 ft,

including some 9% excess area for air pockets and scaling and, as an additional reserve, the area of the tube bundle linings.

In actual fact, because of the large cross flow area near the main tube wall, on account of the heat transferred to the air from the surface of the inner duct past the counterflow region as well as on account of the heat conducted through the partition wall there is neither a distinct geometrical boundary between both sections nor an accurately defined length of each of them.

Thus, the pressure loss calculations in the following Chapters will be based simply on the assumption of purely linear temperature change, i.e.

T 51 = 884 °C and
T 31 = 646 °C.

The effective length of tubes governing the axial pressure loss must be considered to be at variance with that governing heat transfer, due to the comparatively large areas of cross flow incorporated in this design. (Comp. Sketch in Chapter 16 and Fig. 10).

These complications which are characteristic for the "Parallel and Counterflow" Type Exchanger, its higher pressure losses and the higher capital outlay for its more elaborate construction are justified by only ONE reason: Lowered tube wall temperature T_W near the gas entry, namely, T_W = 800 °C, compared with 845° for the "Pure Counterflow" Type Exchanger, as of section B.

Again, T_W is obtained as previously:

$$h_a : h_{g'} = 25.1 : 19 = 1.32$$

$$\frac{1}{1 + 1.32} = .431$$

$$.431 \times (1004 - 646) = 154$$

$$T_W = 646 + 154 = \underline{800\ °C}$$

D. Critical Remarks

The lower T_w in the mixed type exchanger should enable a safer working, for long periods. Conversely, there are means at hand to lower T_w in the Counterflow Type. These possibilities and the ensuing effects upon Plant Net Power Output and Cycle Efficiency will be dealt with in a separate report. The influence of the gas side pressure loss, which is an equally important part of the overall losses of the exchanger, is considered in Chapter 18.

Temperatures and heat transfer coefficients have been derived in this chapter in a very cursory manner; by no means are they the result of a true analysis in which one would have to consider such factors as radiation losses, axial heat transfer and heat transfer under cross flow, apart from a more congenial expression for the temperature distribution itself.

Two reasons, however, lend justification to the concept of Linear Temperature Change on both the gas and air side:

1. The observed temperatures in the Existing Exchanger follow a near-linear pattern.
2. The object of this investigation is to compare different types of exchanger, primarily from the point of view of their pressure losses. The same simplification applied consistently to various types will not alter the results in this respect.

The Overall Heat Transfer Coefficients for the counterflow and the parallel flow part of the mixed Type exchanger differ only as much as 10.0 and 10.8, for a difference in design temperature of about 210°C. The difference in design temperature of anyone part or, of the whole exchanger based upon either linear temperature distribution or upon a non-linear distribution will be, to the utmost, some 15 - 20°C. The effect upon U is, therefore, minute and the designed effective length of 20.2 ft, referring to both Types is as real as Nusselt's equation

$$Nu = .023\ Re^{.8} \times Pr^{.4}$$

"Design temperature" is used here for the average of the air - and gas side mean temperatures, on which calculation of h_a and h_g have been based.

14. DETAILS OF CONSTRUCTION

The following sketches show the characteristic elements of construction of the new exchanger. No physical design was attempted at this stage. Expansion joints, welds, supports and all the other details which make the real exchanger were left aside. Evidently, several ideas concerning the physical shape of such parts as guide rings, inlet and outlet channels, tube suspension arrears a.o. had to be developed and incorporated in the design. The main goal, however, has been to produce a purely functional design.

Fig. 8: Shows the entrance region.

Fig. 9: gives a cross-sectional view of the exchanger and the flow area for air and gas, the axial and cross flow area ratios, the surface area for tube bundle and linings and some other pertinent data.

Fig. 10: shows the transition from the counterflow part, the parallel flow part in whole and the exit region.

Fig. 11: is a cross-sectional view of the latter; the air, after emerging from the tube bundle is divided into three sections the angular ratio 2 : 2 : 3. This ratio fits the seven inlet ports of the turbine manifold.

Figs. 8-9-10-11 together show the elements of construction of the "Parallel Counterflow" Type Exchanger, Fig. 9 and 11 being valid, also, for the "Pure Counterflow" Type.

Fig. 12: represents a slightly varied version of the entrance region for the latter, with an adjustable bypass for cold air to be flushed through the central duct towards the hot end tube plate, where it will tend to reduce the high tube wall temperature. Through enlarged openings in the curved tube sheet the cooling air emerges and mixes with the main air stream. Guide rings and some dimensions referring to the flow conditions at exit are also shown.

Fig. 13: shows the principal elements for supporting the tubes. Since their weight is partly rested upon the Tube Shell and, partly, upon the Central Core the weight of the latter is rested upon the Main Shell in such a fashion that will accommodate differential thermal expansion of tubes, Central Core and Main Shell and will produce a minor resistance, only, to the air flow. The ladder-type support of the tubes themselves is also viewed.

Fig. 14: gives the overall dimensions of the new Exchangers and, for comparison, those of the Existing Exchanger.

15. THE PRESSURE LOSS IN THE "PURE COUNTERFLOW"

TYPE EXCHANGER

The loss is composed of the following partial Pressure Losses:

A. Entrance Loss
B. Axial Flow Pressure Loss
C. Non-isothermal Pressure Loss
D. Bernoullian Pressure Change
E. Exit Loss
F. Other Losses

Pressure loss throughout this investigation refers to Loss in Total Pressure. Observations and computations, apparently, are concerned only with changes in static pressure. However, any appreciable change in impact pressure is taken care of separately under the heading "Bernoullian Pressure Change".

Three basic formulae will be employed in the following pressure loss calculations:

Losses due to	Reference
Axial flow	2 - 16
Change of direction	2 - 9 and Appendix 2
Cross flow over tubes	2 - 13 & 2-13 b

The "Entrance Loss" is investigated distinctly, the terms under "Other Losses" will be estimated.

A. Entrance Loss (Comp. Fig. 8)

This loss may be considered as the sum of 4 different partial losses

1. Loss in bend
2. Loss through change of direction of 50°, in guide rings
3. Loss through cross flow over tubes
4. Loss through change of direction of 50°, in tube bundle.

The entrance loss will be the same for both the pure counterflow and parallel&counterflow exchanger, neglecting the effect of the bypass for tube plate cooling air.

FIRST LOSS

Widely differing results are obtained in calculating the pressure loss in the bend acc. to different formulae.

a) Acc. to McAdams 1954, p. 161: For a 90° elbow, medium radius, $L_e/D = 26$. L_e is the equivalent length in terms of the diameter D, to be added to $L = \pi/2\ D_m$, the actual length of elbow, in order to get the effective length in the formula.

$$\Delta p = \frac{G^2}{2\rho} 4f \frac{L_{eff}}{D} = \frac{G^2}{2\rho} \zeta$$

$D = 13\text{-}1/2" \div 1.125'$
$R_m = 17\text{-}3/4" \div 1.48'$
$R_o = 24\text{-}1/2" \div 2.04'$ $\qquad \mu_{380°C} = 2.14 \times 10^{-5}$

$$G = \frac{14.6}{1.125^2\ \pi/4} = 14.6 \qquad \rho = .075\ \frac{64}{14.7}\ \frac{298}{653} = .149$$

$$\frac{G^2}{2} = \frac{14.6^2}{2 \times .149} = 716 \left(\frac{p}{ft^2}\right) \div 4.28\ "W$$

$$Re = \frac{14.6 \times 1.125}{2.4 \times 10^{-5}} = 768\ 000$$

$f = .0036$, acc. to McAdams, p. 156
$4f = .0144$

$$L_{eff} = 26 \times D + \pi/2\ R_m = 26 \times 1.125 + \pi/2\ 1.48 = 29.3 + 2.3$$

$$L_{eff} = 31.6'$$

$$\Delta p = 4.28 \times .0144\ \frac{31.6}{1.125} = 4.28 \times .404$$

Δp = <u>1.73 "W</u> $\qquad$ <u>C = .404</u>

b) Acc. to Crane Catalogue no. 53, p. 530-31:

For a "Long sweep elbow", 13-1/2" I.D. equivalent length of pipe is 22 x D

$$C = \frac{22}{31.6} \times .404 = \underline{.281}$$

c) Acc. to N. Hall, "Thermodynamics of Fluid flow", p. 34:

$$C = \pi/2\ \ f\ \frac{R_m}{D} + .016 \left(\frac{R_m}{D}\right)^{-2.5} + 2000\ f^{2.5}$$

$R_m = 1.48$
$D = 1.125$
$4f = .0144$

$$\frac{R_m}{D} = \frac{1.48}{1.125} = 1.314 \qquad \left(\frac{R_m}{D}\right)^{-2.5} = .506$$

$$f^{2.5} = .0144^{2.5} = 2.5 \times 10^{-5}$$

$$C = \pi/2 \times .0144 \times 1.314 + .016 \times .506 + 2000 \times 2.5 \times 10^{-5}$$

$$C = .0298 + .0081 + .050$$

<u>C = .0879</u>

There are still other values scattered in the literature. Their variance is explained by comparatively small differences in shape or construction of the bend which have a large bearing upon its performance. Assume C = .35, for a smooth drawn elbow with internal guide vanes and a ratio $R_{m/D} = 1.314$

$$\Delta p = 4.28 \times .35 = \underline{1.50 \text{ "W}}$$

Second Loss:

$$\Delta p = \frac{G^2}{2} \; \frac{\sqrt{2}}{2} \quad \sqrt{1 - \cos\alpha}\; \mathcal{L}$$

$\mathcal{L}$ is a loss factor depending upon the probable amount of turbulence to be expected in the region of

Within the guide rings as of Fig. 8 $\mathcal{L}$ may be assumed to be 1.2. G is based on a cross area of efflux

$$A = \frac{d\pi \; (5\text{-}1/2 + 4\text{-}1/2 + 2\text{-}1/2)}{144}$$

where d is derived from $15^2 - (1\text{-}1/4)^2 = 2\,(15^2 - d^2)$,
d = 10.56"

$$A = 12.5 \times 10.56\,\pi/144 = 2.88 \text{ ft}^2$$

$$G = \frac{14.6}{2.88} = 5.07 \qquad \frac{G^2}{2\rho} = \frac{5.07^2}{2 \times .149} = 86.2 \left[\frac{p}{ft^2}\right]$$

$$\alpha = 50^\circ \qquad \cos\alpha = .644 \qquad 1 - \cos\alpha = .356 \qquad \sqrt{.356} = .598$$

$$\Delta p = 86.2 \times .707 \times .598 \times 1.2 = 86.2 \times .507 = 43.8 \left[\frac{p}{ft^2}\right] \text{ or,}$$

$$\Delta p = \underline{.262 \text{ "W}}$$

Third Loss:

$$\Delta p = \frac{G^2}{2\rho} \; 4 \; f_{cr}\, N$$

In accordance with the procedure in Chapter 7, p. 31, $\overline{G^2}$ has to be computed. Air crosses tube rows Nos. 8 - 9 - 10 - 11 - 12 as of Fig. 9 in equal amount, whereupon the amount diminishes towards the periphery, due to the onset of axial flow

$$G = \frac{14.6}{A_{cr}} = \frac{14.6}{D\,L\,\pi\,R_{cr}}$$

assume constant L, R_{cr} . L = 24" or 2', approx.
$R_{cr} = .231$.

Compute $\sqrt{\overline{\left(\frac{1}{D^2}\right)}}$

Tube row No.	D (in.)	D (ft.)	D^2	$1/D^2$
8	18.0	1.5000	2.25	.444
9	20.25	1.690	2.83	.353
10	22.5	1.876	3.52	.284
11	24.75	2.06	4.24	.236
12	27	2.248	5.04	.199

$$\Sigma\, 1/D^2 = 1.516$$

$$\overline{\left(\frac{1}{D^2}\right)} = 1.516/5 = .303$$

$$\sqrt{\overline{\left(\frac{1}{D^2}\right)}} = .55$$

Equivalent Diameter $D^* = \frac{1}{.55} = 1.82$ ft.

or <u>21.8"</u>

This refers to the cross flow over 5 tube rows. In order to account for the partial flow over rows no. 13-14-15 take N = 5 + 2 = G being based for the whole of the problem on L = 2' and D* = 1.82 ft

$$\sqrt{\overline{G^2}} = \frac{14.6}{2 \times 1.82 \times .231} = \frac{14.6}{2.66} = 5.5$$

$$\frac{\overline{G^2}}{2\rho} = \frac{5.5^2}{2 \times .149} = 101.4 \left(\frac{p}{ft^2}\right) \quad \text{or } .607 \text{ "W.}$$

The frictional coefficient for cross flow, f_{cr} is composed of 2 terms, one depending upon the tube arrangement only and the other one on Re or flow conditions. The first one is constant in all following applications

$$f_{cr} = \left[.23 + \frac{.11}{\left(p_t/d - 1\right)^{1.08}}\right] \times Re^{-.15}$$

$$\frac{p_t}{d} = 1.3 \;; \qquad .3^{1.08} = .273 \;; \quad \frac{.11}{.273} = .404$$

$$f_{cr} = .634 \;\; Re^{-.15}$$

$$\mu_{380°C} = 2.14 \times 10^{-5}$$

$$Re = \frac{6.18 \times .0833}{2.14 \times 10^{-5}} = 24\,000$$

$$Re^{-.15} = .22$$

$$f_{cr} = .634 \times .22 = .14$$

Re in McAdams formula is based on Mass flow at narrowest cross-section, in this case on 2.36 ft² and, somehow illogically, upon the tube diameter d (<u>not</u> d_{hy}).

$$\Delta p = .607 \times 4 \times .14 \times 7 = .607 \times 3.92$$

$$\Delta p = \underline{2.38 \text{ "W}}$$

In the 1954 edition of McAdams, p. 163 notice is given that the expression for f_{cr} as used hitherto results in unduly high pressure losses. For $P_t/d = 1.25$ the observed pressure loss is about 50% smaller than the calculated one. This explains the unduly high figures for cross flow pressure loss and the applied corrections as of p.p. 37, 38, 46 of this Report.

B. ECK, "Technische Strömungslehre", 1954 cites some standard formulae for cross flow pressure loss, derived by two Russian authors early in 1937 which result likewise in about half the loss as the original McAdams equation. For $P_t/d = 1.3$ the reduction might be somewhat smaller.

Assume from now on: true f_{cr} = 70% of the computed value with $p_{t/d} = 1.30$. Thus, the preceding loss will be

$$\Delta p = .70 \times 2.38 = \underline{1.67 \text{ "W}}$$

<u>FOURTH LOSS</u>

$$\Delta p = \frac{G^2}{2\rho} \frac{\sqrt{2}}{2} \sqrt{1 - \cos\alpha}$$

$$\alpha = 50^\circ, \quad \sqrt{1 - \cos\alpha} = .598$$

$\mathcal{L} = 3$, for turn within tube bundle without guide rings;

G based on flow area:

$$A = \pi/_4 \left(3.125^2 - 2.167^2\right) - 4 \times 76 \times 1^2 \times \pi/4 \frac{1}{144}$$

(76 tubes in the quarter ring outside the baffle)

$$A = 2.3 \text{ ft}^2 \qquad G = \frac{14.6}{2.3} = 6.35$$

$$\frac{G^2}{2\rho} = \frac{6.35^2}{2 \times .149} = 135 \left(\frac{lb}{ft^2}\right) \quad \text{or} \quad \underline{.81 \text{ "W}}$$

$$\Delta p = .81 \times .707 \times .598 \times 3 = .81 \times 1.27$$

$$\Delta p = \underline{1.03 \text{ "W}}$$

	1.	1.50
	2.	.26
	3.	1.67
	4.	1.03
Total Entrance Loss:		4.46 "W

B. AXIAL FLOW PRESSURE LOSS

$$\Delta p = \frac{G^2}{2\rho} \, 4 \, f_{ax} \frac{L}{d_{hy}}$$

$$G = \frac{M_a}{A_a} = \frac{14.6}{2.45} = 5.95 \ (lb/ft^2 \ sec)$$

T_m air = 553°C ÷ 826°K
÷ 1028 °F

$$f_{ax} = f \ (Re)$$

$$\rho_m = .075 \, \frac{.298}{826} \, \frac{64}{14.7} = .1176$$

$$L = 19.5 \ ft$$

$$\mu_{1028\ °F} = 2.51$$

$$d_{hy} = .0715$$

$$Re = \frac{G \times d_{hy}}{\mu} = \frac{5.95 \times .0715}{2.51 \times 10^{-5}}$$

$$\frac{G^2}{2\rho} = \frac{5.95^2}{2 \times .1176} = 151 \ \left(\frac{lb}{ft^2}\right)$$

Re = 17000, $f_{ax} = .0075$

$4 \, f_{ax} = .030$

$$\Delta p = 151 \times .030 \, \frac{19.5}{.0715} = 1235$$

$$\Delta p = \underline{7.4 \ "W}$$

C. NON-ISOTHERMAL PRESSURE LOSS

$$\Delta p = \frac{G^2}{\rho_{ex}} - \frac{G^2}{\rho_{in}}$$

Consider constant G
G_{in} at L = 1.4 ft. from cold tube plate
G_{ex} at L = 1.4 + 19.5 ft from cold tube plate.

$$G = 5.95$$

$$\Delta p = 5.95^2 \left(\frac{1}{.096} - \frac{1}{.149}\right)$$

$$\rho_{in} = \rho_{380°C} = .149$$

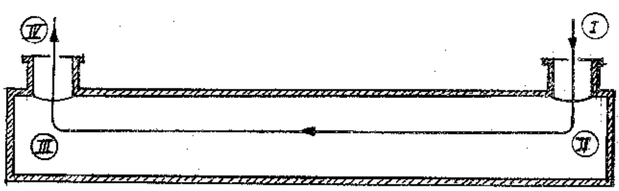
$$\rho_{ex} = \rho_{726°C} = .096$$

$$\Delta p = 35.4 \times (10.42 - 6.72)$$

$$\Delta p = 35.4 \times 3.7 = 131$$

$$\Delta p = \underline{.785 \ "W}$$

D. BERNOULLIAN PRESSURE CHANGE



$$\Delta p = \left[\left(\frac{G^2}{2\rho}\right)_{II} - \left(\frac{G^2}{2\rho}\right)_{I}\right] + \left[\left(\frac{G^2}{2\rho}\right)_{IV} - \left(\frac{G^2}{2\rho}\right)_{III}\right]$$

$$\rho_I = \rho_{II} = \rho_{in} = .149$$

$$\rho_{III} = \rho_{IV} = \rho_{ex} = .096$$

$$\Delta p = \frac{1}{2 \times .149} \left(5.95^2 - 14.6^2\right) + \frac{1}{2 \times .096} \left(6.18^2 - 5.95^2\right)$$

$$G_{II} = G_{III} = G_{ax} = 5.95$$

$$G_{I} = \frac{14.6}{1} = 14.6$$

$$G_{IV} = \frac{14.6}{3 \pi/4 \times 1^2} = \frac{14.6}{2.36} = 6.18$$

$$= 3.35 \left(35.4 - 214\right) + 5.21 \left(38.1 - 35.4\right)$$

$$= 3.35 \times 178.6 + 5.21 \times 2.7 = -598 + 14 = -584$$

$$\Delta p = \underline{-3.5} \text{ "W}$$

This is actually a pressure gain due, mainly, to the increase in flow area from entrance duct to area between tubes (from 1 ft^2 to 2.45 ft^2); One can hardly assume that the head is recovered in between the tubes. Therefore, discard the "Bernoullian Change".

E. EXIT LOSS

This loss is the sum of 2 different losses

1. Loss due to change of direction
2. Loss due to cross flow over tubes

First Loss: $$\Delta p = \frac{G^2}{2\rho} \frac{\sqrt{2}}{2} \sqrt{1 - \cos\alpha}\, \mathcal{L}$$

$\alpha = 90°$ $\cos\alpha = 0$

$\mathcal{L} = 2.2$, compared with $\mathcal{L} = 3$ without guide rings (Section A)

$$\rho = .075 \frac{298}{999} \frac{63}{14.7} = .096$$

$$G = \frac{14.6}{2.45} = 5.95$$

$$\frac{G^2}{2\rho} = \frac{5.95^2}{2 \times .096} = 184 \left(\frac{lb}{ft^2}\right) \quad \text{or } 1.10 \text{ "W}$$

$$\Delta p = 1.10 \times .707 \times 2.2$$

$$\Delta p = \underline{1.71 \text{ "W}}$$

Second Loss: $$\Delta p = \frac{G^2}{2\rho} 4 f_{cr} N$$

Based on the procedure as of Section A and the dimensions of Fig. 12 , G is calculated here assuming $L = 14"$, $D^* = 23"$, $N = 6$ and $R_{cr} = .231$

$$G = \frac{14.6}{L D^* \pi R_{cr}} = \frac{14.6 \times 144}{14 \times 23 \times \pi \times .231} = 9.0$$

$$\frac{G^2}{2\rho} = \frac{9^2}{2 \times .096} = 422 \left(\frac{\#}{ft^2}\right) \text{ or } \underline{2.53 \text{ "W}}$$

$$\mu_{726°C} = 2.83 \times 10^{-5}$$

$$Re = \frac{9.0 \times .0833}{2.83 \times 10^{-5}} = 26\ 450$$

$$Re^{-.15} = .217$$

$$f_{cr} = .634 \times .217 = .138$$

$$\text{true } f_{cr} = .70 \times .138 = .0966$$

$$\Delta p = 2.53 \times 4 \times .0966 \times 6 = 2.53 \times 2.32$$

$$\Delta p = \underline{5.87 \text{ "W}}$$

Total Exit Loss: 1.71
5.87

$$\Delta p = \underline{7.58 \text{ "W}}$$

Other Losses:

Assume for

Obstructions through baffles, partial baffles and other tube supports:	4.00
Skin friction on outside and inside linings. This figure is obtained as $R_L \times \Delta p_{ax} = .0288 \times 7.4$, R_L being the area ratio of linings and tube surface. See Fig. 9.	.21
Diffusion and change of direction in exit flanges	.50
TOTAL	4.71 "W

Total Loss of the Pure Counterflow Exchanger:

Entrance Loss	4.46
Axial Flow Loss	7.40
Non-isoth. Loss	.78
Exit Loss	7.58
Other Losses	4.71
GRAND TOTAL	24.93

Say 25 "W

16. THE PRESSURE LOSS IN THE "PARALLEL AND COUNTERFLOW" TYPE EXCHANGER

This loss is the sum of the combined pressure losses of both the Parallel and the Counterflow Part. Instead of computing the two Totals and add them up to the Grand Total, one may as well consider groups of losses of similar origin in both parts together.

This is the way things are done in this Chapter. The losses are:

A. Entrance Loss	- Counterflow Part
B. Axial Flow Pressure Loss	- both Parts
C. Loss due to Change of Direction	- both Parts
D. Non-isothermal Pressure Loss	- both Parts
E. Bernoullian Pressure Change	- both Parts
F. Other Losses	- both Parts

The Exit loss, at the end of the Parallel-flow Part, is contained within Section C. and equals the "Exit Loss" as of Chapter 15.

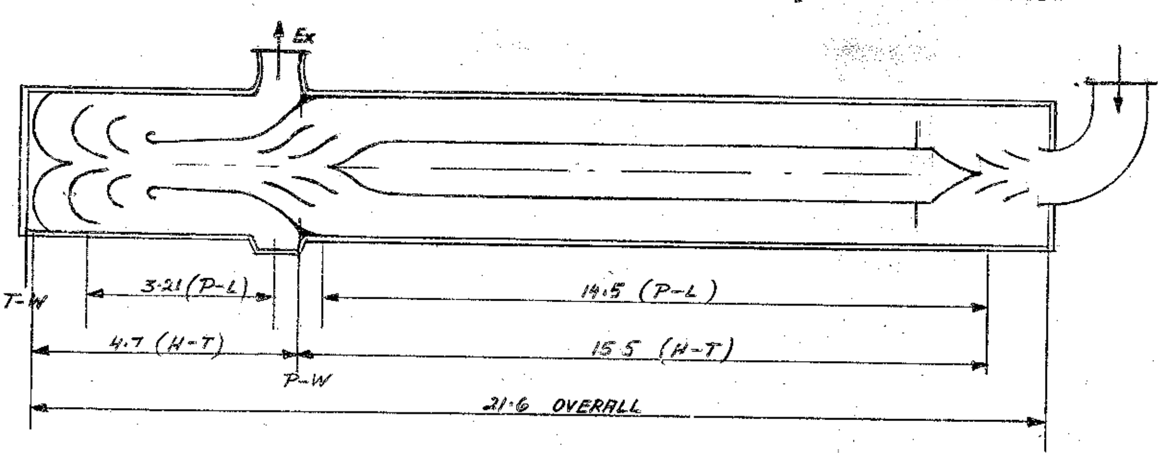
Likewise, the losses under A, D, E have already been computed in the previous Chapter and will be applied here without modification. The losses under F. are estimated.

Thus, Sections B. and C., only remain to be calculated.

A. Entrance Loss: 4.46 "W

B. Axial Flow Pressure Loss:

In order to facilitate calculation different lengths are introduced for heat transfer and pressure loss. Compare Sketch below



The effective lengths for Heat Transfer and Axial Pressure Loss in the "PARALLEL & COUNTERFLOW" HEAT EXCHANGER

All dimensions in ft. P-W : Partition Wall T-P : Tube Plate Ex: Exit

(H-T) Length for calculation of heat transfer
(P-L) Length for calculation of axial pressure loss.

a. Counter Flow Part:

Initial temperature,	at zero length	380 °C
Final temperature,	at P-W	646 °
Mean temperature (380 + 646)/2 =		513° or 786 °K

$$\mu_{513°C} = 2.42 \times 10^{-5}$$

$$\rho_{513° - 64\ psia} = .075 \frac{298}{785} \times \frac{64}{14.7} = .1235$$

$$\Delta p = \frac{G^2}{2\rho} 4\ f_{ax} \frac{L}{d_{hy}}$$

$$G = \frac{14.6}{2.45} = 5.95$$

$$L = 14.5$$

$$\frac{G^2}{2\rho} = \frac{5.95^2}{2 \times .1235} = 143.3 \left(\frac{lb}{ft^2}\right)$$

$$d_{hy} = .0715$$

$$Re = \frac{5.95 \times .0715}{2.42 \times 10^{-5}} = 17\ 550$$

$$\Delta p = 143.3 \times .030 \frac{14.5}{.0715} = 872''$$

$$f_{ax} = .0075$$

$$\Delta p = \underline{5.22\ ''W}$$

$$4f_{ax} = .030$$

b. Parallel Flow Part:

Initial temperature	646 °C
Final temperature	726°
Mean temperature	686° or 959 °K

$$\mu_{686°C} = 2.75 \times 10^{-5}$$

$$\rho_{686°C - 64\ psia} = .075 \frac{298}{959} \frac{64}{14.7} = .1014$$

$$\frac{G^2}{2\rho} = \frac{5.95^2}{2 \times .1014} = 174.5 \left(\frac{lb}{ft^2}\right)$$

$$L = 3.21$$

$$d_{hy} = .0715$$

$$Re = \frac{5.95 \times .0715}{2.75 \times 10^{-5}} = 15\ 500$$; again $f_{ax} = .0075$

$$4f_{ax} = .030$$

$$\Delta p = 174.5 \times .030 \frac{3.21}{.0715} = 235 \left(\frac{lb}{ft^2}\right)$$

$$\Delta p = \underline{1.41\ ''W}$$

TOTAL AXIAL FLOW
PRESSURE LOSS

	5.22
	1.41
	6.63 "W

C. Loss due to Change of Direction

There are three more stations where these losses occur:

a. Before Partition - Wall
b. Before Tube - Plate
c. At exit

At a. and b. there will be considered two losses due to change of direction and one loss due to cross flow.

At c. there will be considered one loss due to change of direction and one loss due to cross flow.

a. Before Partition-Wall:

First Loss $\Delta p = \frac{G^2}{2\rho} \frac{\sqrt{2}}{2} \sqrt{1 - \cos \alpha} \; \mathcal{L}$

$\alpha = 40^\circ$

G based on Flow area in between tubes, $5.95 \frac{lb}{ft^2 \; sec}$

ρ based on 646 °C $\div \rho = .075 \frac{298}{919} \times 435 = .1057$

$\cos 40^\circ = .765$, $1 - \cos 40^\circ = .235$, $\sqrt{1 - \cos 40^\circ} = .485$

$\mathcal{L} = 2.2$

$$\Delta p = \frac{5.95^2}{2 \times .1057} \; .707 \times .485 \times 2.2 = 167.5 \times .755 = 126.5 \; \left(\frac{p}{ft^2}\right)$$

$\Delta p = .76$ "W

Second Loss $\Delta p = \frac{G^2}{2\rho} \frac{\sqrt{2}}{2} \sqrt{1 - \cos 40^\circ} \; \mathcal{L}$

deviation 40°

G based on flow through surface obtained as

$$(2" + 3\text{-}3/4" + 6\text{-}1/4") \times 12\text{-}1/2" \times \frac{\#}{144} = 3.28 \text{ ft.}$$

Comp. sketch below.

$G = \frac{14.6}{3.28} = 4.45$

$\rho = .1057$, as previously
$\alpha = 40^\circ$ " "
$\mathcal{L} = 1.2$ assumed

$$\Delta p = \frac{4.45^2}{2 \times .1057} \times .707 \times .485 \times 1.2 = 93.8 \times .411 = 38.5 \; \left(\frac{p}{ft^2}\right)$$

$\Delta p = .23$ "W

Third Loss: $\Delta p = \frac{G^2}{2\rho} 4 f_{cr} N$

G based on L = 24" D* = 23" $A = \frac{24 \times 23 \times \pi \times .231}{144} = 2.78$

$G = \frac{14.6}{2.78} = 5.25$

$\frac{G^2}{2\rho} = \frac{5.25^2}{2 \times .1057} = 131 \left(\frac{p}{ft^2}\right)$

$\mu_{646^\circ C} = 2.68 \times 10^{-5}$

$Re = \frac{5.25 \times .0833}{2.68 \times 10^{-5}} = 16\,300$

$f_{cr} = .634 \times .234 = .148$

$Re^{-.15} = .234$

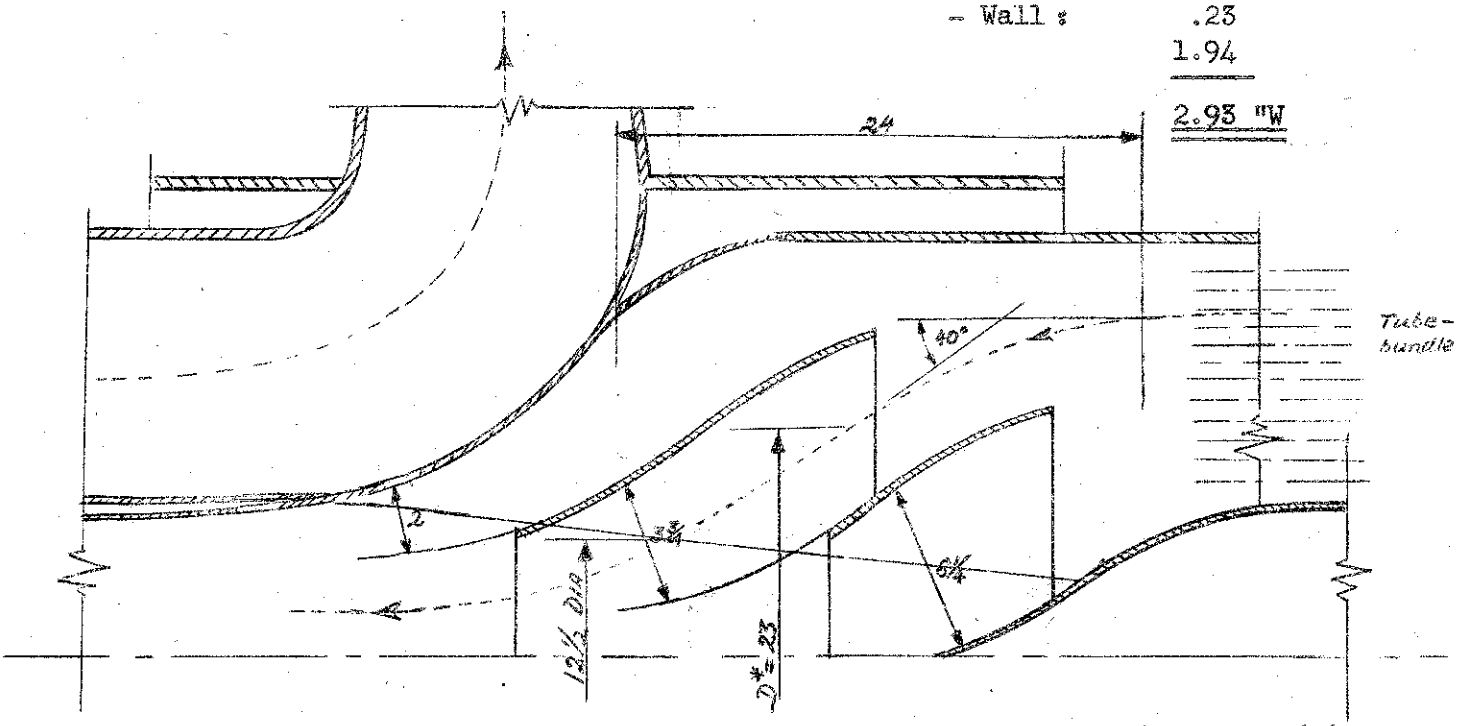
true $f_{cr} = .70 \times .148 = .103_2$

N = 6

$\Delta p = 131 \times 4 \times .103 \times 6 = 131 \times 2.48 = 325 \left(\frac{p}{ft^2}\right)$

Δp = 1.94 "W

Total Loss Before Partition	.76
- Wall :	.23
	1.94
	2.93 "W



All dimensions in inches.

Change of direction of flow near the Partition Wall. Not to scale

b. Before Tube Plate:

Since the 180° change of direction before T-P is a characteristic feature of the "Parallel & Counterflow" Exchanger the following losses are computed rather meticulously.

First Loss: $\Delta p = \frac{G^2}{2\rho} \frac{\sqrt{2}}{2} \sqrt{1 - \cos\alpha}\, \mathcal{L}$

$\rho = .1057$ as previously
$\mathcal{L} = 1.2$ assumed
$\alpha = 90^\circ$ $\cos\alpha = 0$

The governing term is G. Flow area changes from

A_1 = 12" Dia. at throat section to
A_2 = (5-1/2" + 2-1/4" + 2") x 10" x π sqi at some intermediate point to
A_3 = 16" x 21-1/2" x π sqi be-for the tubes

$A_1 = 1.23$ ft^2	$G_1 = 14.6/1.23 = 11.86$	$G^2 = 140$
$A_2 = 2.13$ "	$G_2 = 14.6/2.13 = 6.85$	$G^2 = 47$
$A_3 = 7.75$ "	$G_3 = 14.6/7.75 = 1.88$	$G^2 = 3.6$

$$\sum G^2 = 190$$

$$\overline{G^2} = 190/3 = 63.5$$

$$\frac{G^2}{2\rho} = \frac{63.5}{2 \times .1057} = 300$$

$$\Delta p = 300 \times .707 \times 1.2 = 254 \left(\frac{p}{ft^2}\right)$$

$$\Delta p = \underline{1.52 \text{ "W}}$$

Second Loss: $\Delta p = \frac{G^2}{2\rho} \frac{\sqrt{2}}{2} \sqrt{1 - \cos\alpha}\, \mathcal{L}$

$\rho = .1057$	as previously
$\mathcal{L} = 2.2$	assumed
$\alpha = 90°$	$\cos \alpha = 0$
$G = 5.95$	based on flow area in between tubes

$$\frac{G^2}{2\rho} = \frac{5.95^2}{2 \times .1057} = 167.5$$

$$\Delta p = 167.5 \times .707 \times 2.2 = 261 \left(\frac{p}{ft^2}\right)$$

$$\Delta p = \underline{1.56 \text{ "W}}$$

Third Loss: $\Delta p = \frac{G^2}{2\rho} = 4 f_{cr} N$

G based on flow area $A = : D^* \pi R_{cr} = 21\text{-}1/2" \times 23" \pi \times .231 \frac{1}{144}$
$A = 2.56$

$G = \frac{14.6}{2.56} = 5.70$ $\mu_{646°C} = 2.68 \times 10^{-5}$

$\rho = .1057$ $Re = \frac{5.70 \times .0833}{2.68 \times 10^{-5}} = 17\ 700$

$f_{cr} = .634 \times .232 = 1.47$ $Re^{-.15} = .232$

true $f_{cr} = .70 \times .147 = .103$
N = 6

$$\Delta p = \frac{5.70^2}{2 \times .1057} \times 4 \times .103 \times 5 = 154 \times 2.48 = 382 \left(\frac{lb}{ft^2}\right)$$

$$\Delta p = \underline{\underline{2.29 \text{ "W}}}$$

Total Loss Before Tube-Plate	1.52
	1.56
	2.29
	5.37 "W

C. At Exit:

Both the losses due to change of direction and due to cross flow over tubes equal those in the "Pure Counterflow" Exchanger, 15 - E.

Δp due to change of direction = 1.71 "W
Δp due to cross flow over tubes = 5.81 "W

Total Loss at Exit 7.58 "W

D. Non-isothermal Pressure Loss .78 "W

E. Bernoullian Pressure Change - to be discarded

F. Other Losses 6.0 "W

The last term is chosen, here, larger than in the previous Chapter, because of the more intricate design of the mixed type exchanger.

TOTAL LOSSES OF THE "PARALLEL AND COUNTERFLOW" EXCHANGER:

Entrance Loss	4.46
Total Axial Flow Pr. Loss	6.63
Non-Isothermal Pressure Loss	.78
Loss Before Partition-Wall	2.93
Loss Before Tube-Plate	5.37
Exit Loss	7.58
Other Losses	6.00
GRAND TOTAL	33.75 "W

FOR THE COUNTERFLOW PART ALONE THE LOSSES MAY BE SAID TO BE:

Entrance Loss	4.46
Axial Flow Loss	5.22
Non-isoth. Loss	.60
Loss Before P-W	2.93
Other Losses	4.00
	17.21 "W

FOR THE PARALLEL FLOW SECTION REMAINS, THUS 16.54 "W

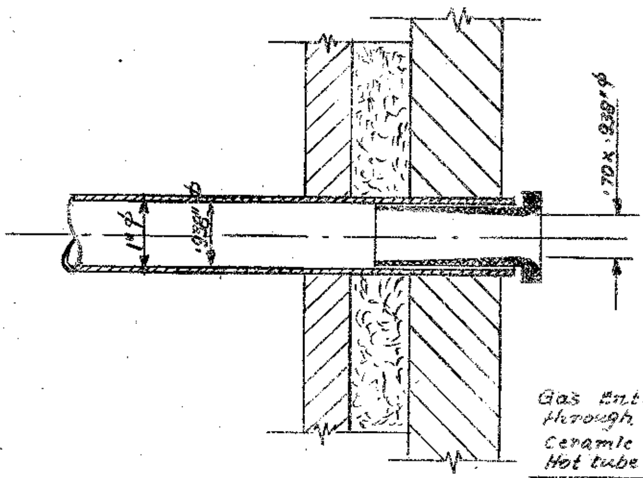
17. THE GAS SIDE PRESSURE LOSS

This loss is the sum of the following

1. Venturi entrance Loss
2. Axial flow Loss
3. Exit Loss
4. Non-isothermal pressure change
5. Bernoullian pressure Loss

First Loss: $\Delta p = \zeta \left[\left(\frac{G^2}{2\rho} \right)_{II} - \left(\frac{G^2}{2\rho} \right)_{I} \right]$

ζ is the Pressure Loss Coefficient; assume $\zeta = .20$
(Dubbel I, 1939 p. 226: $\zeta = .1 \div .3$)



Gas Entrance at tube plate through venturi like Ceramic diffusers. Hot tube plate is water cooled.

$A_I = 30^2 \, \pi/4 \, \frac{1}{144} = 4.90 \text{ ft}^2$

$A_{II} = 2.42 \left(\frac{.7}{.938} \right)^2 = 1.34 \text{ ft}^2$

$T = 1072 + 273 = 1345 \text{ °K}$

$G_{II} = \frac{M_g}{A_{II}} = \frac{10}{1.34} = 7.46 \qquad G_{II}^2 = 55.7$

$G_I = \frac{M_g}{A_I} = \frac{10}{4.90} = 2.04 \qquad G_I^2 = 4.16$

Assuming for gas, as "Product of Combustion with 400% Theoretical Air" the molecular weight of air.

$$\rho = .075 \times \frac{16}{14.7} \times \frac{298}{1345} = .075 \times 1.088 \times .221 = .018$$

$$\Delta p = .20 \, \frac{1}{2 \times .018} (55.7 - 4.16) = .555 \times 51.54 = 286 \left(\frac{p}{ft^2} \right)$$

$$\Delta p = \underline{1.72 \text{ "W}}$$

Second Loss: $\Delta p = \frac{G^2}{2\rho} \, 4 f \, \frac{L}{d}$ $\qquad T_m = \frac{490 + 1004}{2} + 273$

$G = \frac{10}{2.42} = 4.135 \qquad T_m = 747 + 273 = 1020 \text{ °K}$

$$\rho = .075 \times 1.088 \times \frac{298}{1020} = .0238$$

$$\frac{G^2}{2\rho} = \frac{(4.135)^2}{2 \times .0238} = 364 \left(\frac{lb}{ft^2}\right) \quad \text{or } 2.18 \text{ "W}$$

$$\mu_{747°C} = 2.87 \times 10^{-5}$$

$$Re = \frac{4.135 \times .0833 \times .938}{2.87 \times 10^{-5}} = 11\ 240$$

$$f = .008 \qquad 4f = .032$$

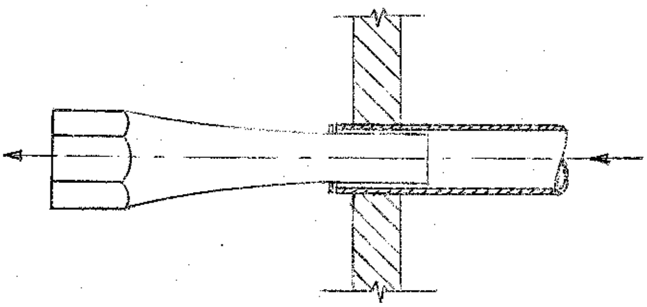
$$L = 22 \text{ ft.}$$
$$d = .078 \text{ ft}$$

I. Dia. of Tube:
.0833 x .938 = .078 ft

$$\Delta p = 2.18 \times .032 \times \frac{22}{.078}$$

$$\Delta p = \underline{19.7 \text{ "W}}$$

Third Loss: Applying the diffusers, 75% of the decrease in velocity head at exit may be gained in static pressure indicated by the pressure gauges. However, it is more convenient to account a loss of 25% of the decrease in velocity head in terms of total pressure loss.



At exit $\rho = .075 \times 1.088 \times \frac{298}{763} = .0319$

$T = 490 + 273$
$T = 763$ °K

$$\Delta\left(\frac{G^2}{2\rho}\right) = \frac{G^2}{2\rho}_{\text{in tubes}} - \frac{G^2}{2\rho}_{\text{outside}} = \frac{1}{2\rho}\left(G^2_{\text{in tubes}} - G^2_{\text{outside}}\right)$$

Assume $\text{Area}_{\text{outside tubes}}$ 4.90 ft². $A_{\text{inside}} = 2.42$ ft²

$G_{\text{inside}} = 4.135 \qquad G_{\text{outside}} = 2.04$

$G^2_{\text{inside}} = 17.1 \qquad G^2_{\text{outside}} = 4.16$

$$\Delta\left(\frac{G^2}{2\rho}\right) = \frac{1}{2 \times .0319}(17.1 - 4.16) = 15.7 \times 12.94 = 203 \left(\frac{lb}{ft^2}\right)$$

25% of this amount to 51 $\frac{lb}{ft^2}$ or

$$\Delta p = \underline{.305 \text{ "W}}$$

Fourth Loss: $\Delta p = \left(\frac{G^2}{\rho}\right)_{\text{exit}} - \left(\frac{G^2}{\rho}\right)_{\text{entr.}} = G^2\left(\frac{1}{\rho_{\text{exit}}} - \frac{1}{\rho_{\text{entr.}}}\right)$

$$G = \frac{10}{2.42} = 4.135 \qquad G^2 = 17.1$$

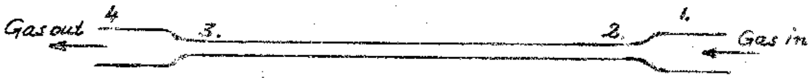
$$\rho_{\text{exit}} = \rho_{490°C} = .0319 \qquad \rho_{\text{entr.}} = \rho_{1072°C} = .018$$

$$\Delta p = 17.1 \left(\frac{1}{.0319} - \frac{1}{.018}\right) = 17.1\,(31.4 - 55.5)$$

$$\Delta p = -17.1 \times 24.1 = -413 \quad \left(\frac{lb}{ft^2}\right) \quad \text{or}$$

$\Delta p =$ <u>-24.7 "W</u> , actually a pressure gain.

Fifth Loss: $$\Delta p = \left[\left(\frac{G^2}{2\rho}\right)_2 - \left(\frac{G^2}{2\rho}\right)_1\right] + \left[\left(\frac{G^2}{2\rho}\right)_4 - \left(\frac{G^2}{2\rho}\right)_3\right]$$



$G_1 = G_4 \qquad \rho_1 = \rho_2$

$G_2 = G_3 \qquad \rho_3 = \rho_4$

$G_1 = 2.04 \qquad \rho_1 = .018$

$G_2 = 4.135 \qquad \rho_3 = .0319$

$$\Delta p = \frac{G_2^2}{2\rho_1} - \frac{G_1^2}{2\rho_1} + \frac{G_1^2}{2\rho_3} - \frac{G_2^2}{2\rho_3}$$

$$= \frac{1}{2}\left[\frac{1}{\rho_1}(G_2^2 - G_1^2) - \frac{1}{\rho_3}(G_2^2 - G_1^2)\right]$$

$$= \frac{1}{2}\left(\frac{1}{\rho_1} - \frac{1}{\rho_3}\right) \times (G_2^2 - G_1^2)$$

$$= \frac{1}{2}\left(\frac{1}{.018} - \frac{1}{.0319}\right)(17.1 - 4.16)$$

$$\Delta p = \frac{1}{2} \times 24.1 \times 12.94 = 156 \quad \left(\frac{lb}{ft^2}\right)$$

$\Delta p =$ <u>.935 "W</u>

This is a true loss, superimposed on the non-isothermal pressure gain.

The Third Loss without diffusers would have to be handled as pressure loss due to a sudden enlargment.

Acc. to 2-7,

$$\Delta p = \frac{G^2 \text{ inside tubes}}{2}\left[1 - \frac{2.42}{4.9}\right]^2 = \frac{17.1}{2 \times .0319}\,(.506)^2$$

$$\Delta p = 268 \times .256 = 68.6 \quad \left(\frac{lb}{ft^2}\right) \quad \text{or } \underline{.41 \text{ "W}}$$

The gain in pressure drop produced by the diffusers is, thus, not more than .41 - .305 <u>≈.10 "W</u>.

TOTAL PRESSURE LOSS ON THE GAS SIDE

1.	Venturi entrance loss	1.72
2.	Axial flow loss	19.70
3.	Exit loss	.305
5.	Bernoullian Pr. Loss	.935
	TOTAL	22.660
4.	Non-isothermal Pr. L.	- 2.47

With diffusers GRAND TOTAL ≈ 20.20 "W

Without diffusers GRAND TOTAL ≈ 20.30 "W

18. COMPARISON OF RESULTS

In order to compare the pressure loss produced by the new Type exchanger with that of the existing one, the same full load conditions as of Chapter 13 have to be applied to the latter, i.e.

M_a = 14.6 lb/sec
P = 64 psia.
T = 553°C ≙ 826 °K, as arithmetic mean of T 28 = 380°C and T 30 = 726°C.

Denote these with suffix II, whereas the operating conditions as of Chapter 5, are denoted with I. Then,

$$1.\quad \Delta P_{II} = \Delta P_I \; \frac{(M_{II})^2}{(M_I)^2} \; \frac{T_{II}}{T_I} \; \frac{P_I}{P_{II}}$$

P_I is either an observed or a calculated pressure loss which should be known in order to proceed to other plant operating conditions.

1. is usually written as relative pressure loss

$$2.\quad \frac{\Delta P}{P} = C \times \frac{M^2\,T}{P^2} ,$$

with C being considered a constant for all M, T, P. This representation is extremely simple insofar as the pressure loss functions are straight lines emerging from the origin, every type of exchanger or component being characterized by a certain C. Moreover, one set of values of ΔP, M, T, P enables the loss function or the factor C to be determined completely and, thus, to predict the pressure loss for any other operating conditions.

However, 1. and 2. are not sound theoretically. They are based on the assumption

$$3.\quad \Delta P_\lambda = C_\lambda \; \frac{M^2}{\rho} , \; \rho \text{ being } \propto \frac{P}{T} .$$

P_λ, C_λ refer to anyone of the individual pressure losses of which the total, ΔP is built up. In Chapter 9, on the other hand, it has been outlined how the individual losses may be represented based on the local temperature or, on the average temperature of the compartment. There is hardly one characteristic temperature at any ONE station which, applied to equation 2. would yield the same ΔP_{total} as that given by $\sum \Delta P_\lambda$, for varying operating conditions.

The non-isothermal pressure loss, e.g. does not follow at all equation 3. and, even, such a well defined term as the axial flow pressure loss

$$\Delta p = \frac{G^2}{2\rho} f \frac{d_{hy}}{L}$$

may be subjected to a variation of G due to changes of the boundary layer thickness with varying temperature with means, in effect, a non-constant C_λ.

Although none of the C_λ will be truly constant one may well adhere to equation 2. with T more or less arbitrarily selected and C based on a sufficiently large number of observations. In other words: the inherent inaccuracy of 2. is covered up or made invisible by the still larger inaccuracy of the readings leading to computation of M. T, P which is, usually, encountered. In the case of pressure loss prediction there is no argument about C. Selection of T remains somewhat arbitrary.

For operating conditions as of Chapter 5

M = 7 lb/sec
P = 26 psia
T = 667°F ≑ 627 °K, as arithmetic mean of
t28 = 440 °F and
t30 = 895 °F.

the combined pressure loss in the Existing Exchanger has been calculated to

ΔP_I = 23.1 + 20.4 = 43.5 "W or 1.575 psi. (p.p. 39, 47)

$$\left(\frac{\Delta P}{P}\right)_I = \frac{1.575}{26} = .0606 \qquad \text{or } \underline{\underline{6.06\%}}$$

The corresponding value of $\frac{M^2\ T}{P^2}$ is $\frac{7^2 \times 627}{26^2} = \underline{\underline{45.5}}$

By these both figures the Pressure Loss Line of the exchanger is defined.

Now, for operating conditions as of Chapter 13

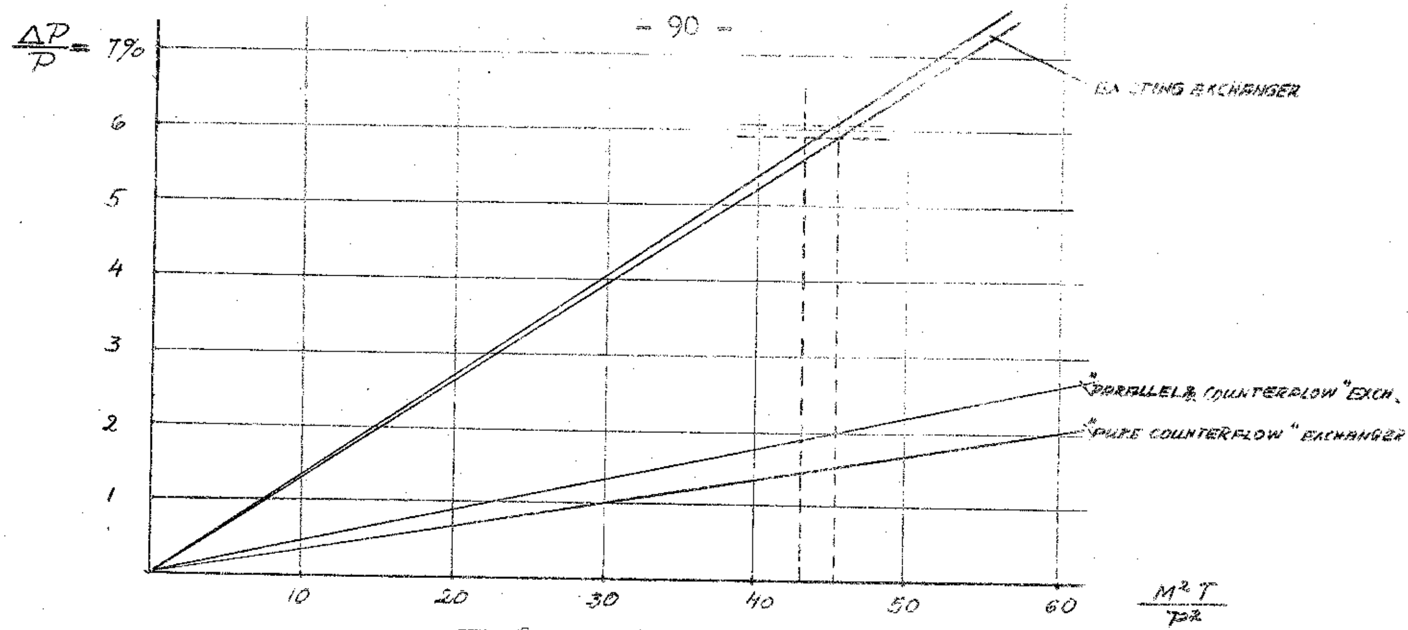
$$\frac{M^2\ T}{P^2} \text{ equals } \frac{14.6^2 \times 826}{64^2} = 43, \text{ and}$$

the relative pressure loss in the Existing Exchanger would amount to

$$\left(\frac{\Delta P}{P}\right)_{II} = 5.7\%$$

If the Pressure Loss Line is based on a large number of observations, with widely varying values of M, P, T the corresponding figure for conditions as of Chapter 5 would be $\left(\frac{\Delta P}{P}\right)_I$ = 6.3%, which would yield

$$\left(\frac{\Delta P}{P}\right)_{II} = 5.9\%$$



The Pressure Loss Line for the different Hot Heat Exchangers -- Air Side

For the new "Pure Counterflow" Type Exchanger

$$\Delta P = 25\ \text{"W or .905 psi.}$$

This is equivalent to a relative pressure loss

$$\frac{\Delta P}{P} = \frac{.905}{64} = .0141 \quad \text{or } \underline{1.41\%}$$

Corresponding figures for the new "Parallel and Counterflow" Type Exchanger are

$$\Delta P = 33.75\ \text{"W or 1.22 psi and}$$

$$\frac{\Delta P}{P} = \frac{1.22}{64} = .0191 \quad \text{or } \underline{1.91\%}$$

These results are directly comparable with the values 5.7 resp. 5.9%. The difference in pressure loss is striking and it can only be hoped that it will be possible to translate the elements of design as outlined in Part II of this paper into practical engineering language without loosing too much of their specific hydrodynamic features.

In order to get the overall picture of exchanger performance, also the gas side pressure loss has to be taken into account.

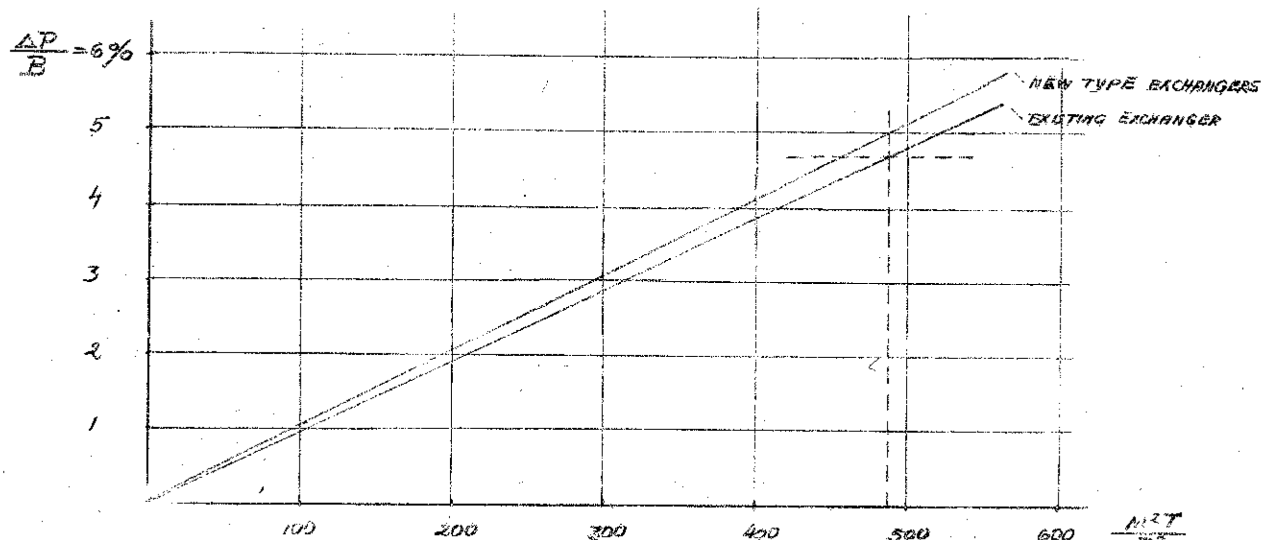
The gas side Pressure Loss Line of the Existing Exchanger, based on a large number of observations yields for

M = 10 lb/sec
P = 16 psia.
T = 781 °C ≑ 1054 °K, as arithmetic mean of
T = 1072 °C and
T 70 = 490 °C

or, $\frac{M^2\,T}{B^2} = \frac{10^2 \times 1054}{14.7^2} = 488,$ i.e. operating conditions as of Chapter 13,

a relative pressure loss of $\frac{\Delta P}{B} = 4.7\%$ (B = barometric pressure)

There are good reason to choose in place of T 50 = 1072°C the value T 50_{eff} = 1004 °C because all but the venturi entrance loss are bound to temperatures other than 1072 °C. But, since the gas side Pressure Loss Line has been based on T 50 as an observed temperature there is, practically, no other possibility than to keep to it when comparing performance of the difference exchangers.



The Pressure Loss Line for the different Heat Exchangers — Gas Side

Assuming that the new "Pure Counterflow" Type and "Parallel and Counterflow" Type Exchanger will exhibit the same gas side pressure loss one obtains for both, with the results of the previous Chapter,

With diffusers ΔP = 20.2 "W or .731 psi.

$$\frac{\Delta P}{B} = \frac{.731}{14.7} = .0497 \quad \text{or } 4.97\%$$

Without diffusers the figures are ΔP = 20.3 "W or .735 psi.

$$\frac{\Delta P}{B} = \frac{.735}{14.7} = .050 \quad \text{or } 5.0\%$$

Even with diffusers of 100% efficiency the pressure loss ratio $\frac{\Delta P}{B}$ would not be less than 4.90%. This means that the Existing Exchanger has a slightly better performance on the gas side. Application of the diffusers seems not to be warranted.

The higher pressure loss in the new Type exchangers is due the greater tube length of 22 ft., compared with about 18 ft. in the existing one. (See Fig. 2).

This comparatively large pressure loss is an inherent property of the gas side. Due to small ρ the terms $\frac{G^2}{2\rho}$ become large, since $\Delta P \propto \frac{G^2}{2\rho}$.

$\frac{\Delta P}{B}$ still becomes larger because B is much smaller than P, the corresponding value in the denominator for the air side.

Nothing can be done, unfortunately, to reduce this high percentage of gas side pressure losses.

With respect to Cycle Efficiency and Plant Net Power Output the overall pressure loss of the Heat Exchanger enters the calculation. It is most suitably expressed as

$$\Delta = \left(\frac{\Delta P}{P}\right)_{air} + \left(\frac{\Delta P}{B}\right)_{gas}$$

Summary: For operating conditions as of Chapter 13

	$\left(\frac{\Delta P}{P}\right)_a$	+	$\left(\frac{\Delta P}{B}\right)_g$	=	
Existing Exchanger based on a large number of observations	5.9	+	4.7	=	<u>10.6%</u>
New "Pure Counterflow" Type Exchanger	1.41	+	5.0	=	<u>6.4%</u>
New "Parallel and Counterflow" Type Exchanger	1.91	+	5.0	=	<u>6.9%</u>

The result, in terms of overall pressure loss of the new type exchanger is encouraging though it is far from what might be expected if one considers the reduced air side pressure loss alone.

APPENDIX I

The integration of Fannings Equation 2-1

FOR INCOMPRESSIBLE FLOW

The specific volume v remains constant.

$$\frac{\partial v}{\partial T} = 0 \qquad \frac{\partial v}{\partial p} = 0$$

As first case assume CONSTANT FLOW AREA, CHANGING TEMPERATURE. $\frac{\partial T}{\partial L}$ along the path L is given. This includes, too, the case where T = constant, $\frac{\partial T}{\partial L} = 0$.

$$-dp = \frac{1}{v}\left(g\,dZ + \frac{V\,dV}{\alpha} + f\,\frac{V^2}{2}\,\frac{dL}{r_{hy}}\right)$$

2-2: $$-\Delta p = p_1 - p_2 = \frac{1}{v}\left[g\,(Z_2 - Z_1) + f_m \frac{V^2}{2}\,\frac{L_2 - L_1}{r_{hy}}\right]$$

Since V = constant, f_m is the mean frictional coefficient between L_1 and L_2 incorporating changes of temperature as well as changes in the geometry of the boundary. Its choice is somewhat arbitrary.

As second case assume CHANGING FLOW AREA AND TEMPERATURE. V is no more constant, and the expression for the pressure loss becomes

2-3: $$-\Delta p = p_1 - p_2 = \frac{1}{v}\left[g\,(Z_2 - Z_1) + \frac{1}{2\alpha}(V_2^2 - V_1^2) + f_m \frac{(V_1^2 + V_2^2)(L_2 - L_1)}{4\, r_{hy}}\right]$$

provided that the average value of V^2 over the length $\Delta L = L_2 - L_1$ can be expressed as $\frac{V_1^2 + V_2^2}{2}$.

The correct value for the last term would be

$$\int_{L_1}^{L_2} f\,\frac{V^2}{2}\,\frac{dL}{r_{hy}} = f_m\,\frac{(v\,M)^2}{2\,r_{hy}}\int_{L_1}^{L_2}\frac{dL}{A(L)^2}, \quad \text{since } V = \frac{v\,M}{A}.$$

For V^2 being a linear function of L, e.g., $V^2 = m\,L + n$

$$A^2 = \frac{v^2\,M^2}{m\,L + n} \qquad \text{(m, n constant)}$$

$$A = \frac{v\,M}{\sqrt{m\,L + n}} \qquad \text{2-3 is fulfilled.}$$

FOR COMPRESSIBLE FLOW

$$\frac{\partial v}{\partial T} \neq 0 \qquad \frac{\partial v}{\partial p} \neq 0$$

As first case assume CONSTANT FLOW AREA, CHANGING TEMPERATURE. T(L) is given; furthermore, Z(L) is given.

$$v = \frac{R}{p}\,T(L) \qquad V = G\,v = G\,\frac{R\,T}{p}$$

$$dV = \frac{G\,R}{p}\,\frac{\partial T}{\partial L}\,dL, \text{ with L as independent variable}$$

$$\frac{1}{v} = \frac{p}{R\,T} \qquad V\,dV = \frac{G^2 R^2}{p^2}\,T\,\frac{\partial T}{\partial L}\,dL$$

$$-dp = \frac{g\,p}{R\,T}\,\frac{\partial Z}{\partial L}\,dL + \frac{1}{\alpha}\,\frac{p}{R\,T}\,\frac{G^2 R^2}{p^2}\,T\,\frac{\partial T}{\partial L}\,dL + f\,\frac{G^2 R^2 T^2}{2\,p^2}\,\frac{p}{R\,T}\,\frac{dL}{r_{hy}}$$

$$-dp = \frac{g\,p}{R}\,\frac{\partial Z}{\partial L}\,\frac{1}{T(L)}\,dL + \frac{1}{\alpha}\,\frac{G^2 R}{P}\,\frac{\partial T}{\partial L}\,dL + \frac{f}{2}\,\frac{G^2 R}{p\,r_{hy}}\,T(L)\,dL$$

2-4: $$-\Delta p = p_1 - p_2 = \frac{g\,p}{R_m}\left(\frac{\partial Z}{\partial L}\right)_m \int_{L_1}^{L_2}\frac{dL}{T(L)} + \frac{G^2 R_m}{\alpha\,p}\int_{L_1}^{L_2}\frac{\partial T}{\partial L}\,dL + \frac{f_m G^2 R_m}{2\,p\,r_{hy}}\int_{L_1}^{L_2} T(L)\,dL$$

$\Delta p \ll p$ has to be assumed. Having resolved 2-4 for p, p_m may be introduced as better approximation.

$\left(\frac{\partial Z}{\partial L}\right)_m$, f_m, R_m are arithmetic mean values over the path $L_2 - L_1$.

$$p_m = p + \Delta p/2$$

As second case assume CONSTANT TEMPERATURE, CHANGING AREA. This is a rather abstract case, since in a heat exchanger there will be hardly a change of flow area. In a duct there might be. Following, the area is expressed in terms of $r_{hy} = 1/4\ d_{hy}$.

$$r_{hy} = \frac{\text{flow area}}{\text{wetted perimeter}} \qquad \text{or}$$

$$\text{flow area} = r_{hy} \times \pi d$$

with d = tube outside diam. Without great error, $d = d_{hy}$. Thus will be obtained an equation for the pressure loss, valid for both flow inside tubes and flow outside tubes. The 1st term of 2-1 remains unchanged.

The 2nd term becomes

$$A = 4 \pi r_{hy}^2 \qquad V = G v = v M \frac{1}{A}$$

$$\frac{\partial A}{\partial L} = 2 \times 4 \pi r_{hy} \frac{\partial r_{hy}}{\partial L} \qquad d V = -v M \frac{\frac{\partial A}{\partial L} d L}{A^2}$$

$$V\, d V = -(v M)^2 \frac{\frac{\partial A}{\partial L}}{A^3} d L$$

$$\frac{V\, d V}{\alpha\, v} = -v M^2 \frac{8 \pi r_{hy}}{64 \pi^3 r_{hy}^6} \frac{\partial r_{hy}}{\partial L} d L$$

$$\boxed{\frac{V\, d V}{\alpha\, v} = -v M^2 \frac{\frac{\partial r_{hy}}{\partial L} d L}{8 \pi^2 r_{hy}^5}}$$

The 3rd term becomes

$$V^2 = v^2 M^2 \frac{1}{A^2} = v^2 M^2 \frac{1}{16 \pi^2 r_{hy}^4}$$

$$\frac{f}{2 v} V^2 \frac{d L}{r_{hy}} = \frac{f M^2 v}{32 \pi^2 r_{hy}^5} dL = \boxed{\frac{f M^2 v}{32 \pi^2} \frac{d L}{r_{hy}^5}}$$

2-5: with $v = \frac{1}{\rho}$

$$-\Delta p = p_1 - p_2 = g \rho (Z_2 - Z_1) - \frac{M^2}{8 \pi^2 \rho} \int_{L_1}^{L_2} \frac{\partial r_{hy}}{\partial L} \frac{1}{r_{hy}^5} d L + \frac{f_m M^2}{32 \pi^2 \rho} \int_{L_1}^{L_2} \frac{d L}{r_{hy}^5}$$

For flow inside tubes, replace $r_{hy} = 1/4\ d_{hy}$ by $1/4\ d$.

More important is the following case, CONSTANT FLOW AREA, LINEAR TEMPERATURE CHANGE, as well as linear function in L for elevation Z.

$$\frac{\partial T}{\partial L} = \kappa \qquad \frac{\partial Z}{\partial L} = \varepsilon \qquad \kappa \text{ and } \varepsilon \text{ constant.}$$

Taking equation 2-4, introduce κ and ε and transform the integrand,

<u>the 1st term</u> becomes

$$\frac{g\,p}{R_m}\,\varepsilon\int_{T_1}^{T_2}\frac{1}{\kappa}\frac{dT}{T} = \boxed{\frac{g\,p}{R_m}\frac{\varepsilon}{\kappa}\ln\frac{T_2}{T_1}}$$

the integration from L_1 to L_2 being replaced by the integration from corresponding T_1 to T_2.

<u>The 2nd term</u> becomes

$$\frac{G^2 R_m}{\alpha\, p}\int_{T_1}^{T_2} dT = \frac{G^2 R_m}{\alpha\, p}\,(T_2 - T_1)$$

This expression may remain that way, since T_1 and T_2 have to be given; it might, however, be transformed into a more familiar one:

$$\frac{R}{p} = \frac{v}{T} \qquad G^2 = \frac{V^2}{v^2}$$

$$\frac{G^2 R_m}{\alpha\, p} = \frac{V^2}{v^2}\frac{v}{T}\frac{1}{\alpha}(T_2 - T_1) = \frac{1}{\alpha}\left(V_2\,\rho_2\frac{T_2}{T_2} - V_1^2\,\rho_1\frac{T_1}{T_1}\right) = \boxed{\frac{1}{\alpha}\left(V_2^2\rho_2 - V_1^2\rho_1\right)}$$

<u>The 3rd term</u> becomes

$$\frac{f_m}{2\,r_{hy}}\frac{V^2}{v^2}\frac{v}{T}\int_{T_1}^{T_2}\frac{1}{\kappa}\,T\,dT = \frac{f_m}{4\,r_{hy}}\frac{\rho\,V^2}{T\,\kappa}\left(T_2^2 - T_1^2\right) = \frac{f_m}{4\,r_{hy}\,\kappa}\left(\rho_2 T_2 V_2^2 - \rho_1 T_1 V_1^2\right)$$

Again this expression may be transformed into a more familiar one:

$$\frac{f_m}{2\,r_{hy}}\frac{V^2}{v^2} \text{ remains,} \qquad \frac{v}{T}\int_{T_1}^{T_2}\frac{1}{\kappa}\,T\,dT = \frac{R_m}{p_m}\,T_m\,\Delta L$$

$$\frac{R_m T_m}{p_m} = v_m = \frac{v_1 + v_2}{2}$$

<u>The 3rd term</u> becomes

$$\frac{\Delta L\, f_m}{4\,r_{hy}}\frac{V^2}{v^2}(v_1 + v_2) = \boxed{\frac{f_m}{4\,r_{hy}}\left(V_1^2\rho_1 + V_2^2\rho_2\right)\Delta L}$$

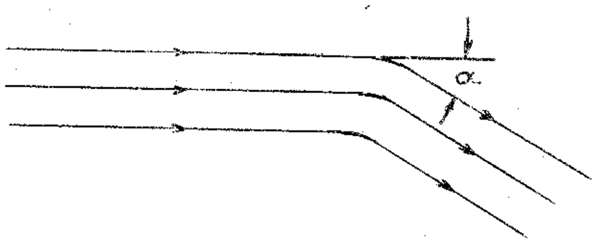
Thus 2-6 becomes

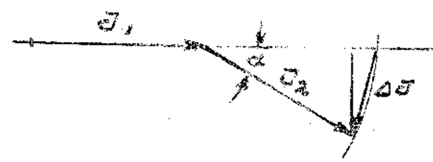
$$-\Delta p = p_1 - p_2 = \frac{g\,p}{R_m}\frac{\varepsilon}{\kappa}\ln\frac{T_2}{T_1} + \frac{1}{\alpha}\left(\rho_2 V_2^2 - \rho_1 V_1^2\right) + \frac{f_m}{4}\left(\rho_1 V_1^2 + \rho_2 V_2^2\right)\frac{\Delta L}{r_{hy}}$$

APPENDIX II

The pressure loss due to CHANGE OF DIRECTION, 2 - 9.

If an incompressible, frictionless fluid changes its direction of flow in an horizontal plane, the absolute velocity remaining constant, the ensuing centrifugal forces are perpendicular to the direction of flow; no work is expended, no pressure loss occurs. This is even true for a non-infinitely wide stream with any kinds of obstructions and boundaries provided that the flow pattern remains two-dimensional. In this case the mechanical property most suitable to describe the state of fluid is the impuls $\mathcal{J}$. The change of impuls, corresponding to a change of direction of flow by angle α, equals





$$\Delta\mathcal{J} = \mathcal{J}_2 - \mathcal{J}_1 \qquad \text{(vector relation)}$$

$$\Delta I = I\sqrt{(1-\cos\alpha)^2 + (\sin\alpha)^2} \qquad \text{(scalar relation)}$$

Referring the impuls to that mass crossing the unit area in unit time

$$I = M \times V = \rho V^2 \qquad \text{and}$$

$$\Delta I = \rho V^2 \sqrt{1 - 2\cos\alpha + \cos^2\alpha + \sin^2\alpha}$$

1. $$\Delta I = \rho V^2 \sqrt{2} \quad \sqrt{1-\cos\alpha}$$

In a real fluid undergoing a change of direction turbulence is set up and a pressure loss Δp produced. The amount of the latter may be said to depend upon

a. the geometry of obstructions, bends, boundaries
b. the internal friction of the fluid
c. the amount of the change of direction, α
d. the density and velocity of the fluid

Assuming as kind of a <u>working hypothesis</u> that the terms under c. and d. may be expressed by equation 1, the pressure loss will be of the form

2. $\Delta p = \rho V^2 \sqrt{2} \sqrt{1-\cos\alpha} \times C,$

C comprising the effect of the terms a. and b.

Experiments show that the return losses in a hairpin bend, duct or header with irregular shape are of the order of magnitude

$$\frac{1}{2}\frac{\rho V^2}{2} \quad \text{to} \quad 2\frac{\rho V^2}{2} \quad (\alpha = 180°)$$

Assume as an average value for such a case

$$\Delta p = 1 \times \frac{\rho V^2}{2}. \quad \text{This means } C = \frac{1}{4} \text{ and}$$

3. $$\Delta p = \frac{\rho V^2}{2} \frac{\sqrt{2}}{2} \sqrt{1 - \cos\alpha} \quad (\alpha \text{ from } 0 \text{ to } 180°)$$

As a refinement a loss factor $\mathcal{L}$ may be added taking care, primarily, of effects under term a., like shape and size of obstructions, local hydraulic diameter a.s.o. Then,

4. $$\Delta p = \frac{\rho V^2}{2} \frac{\sqrt{2}}{2} \sqrt{1 - \cos\alpha}\, \mathcal{L}.$$

ACKNOWLEDGEMENT

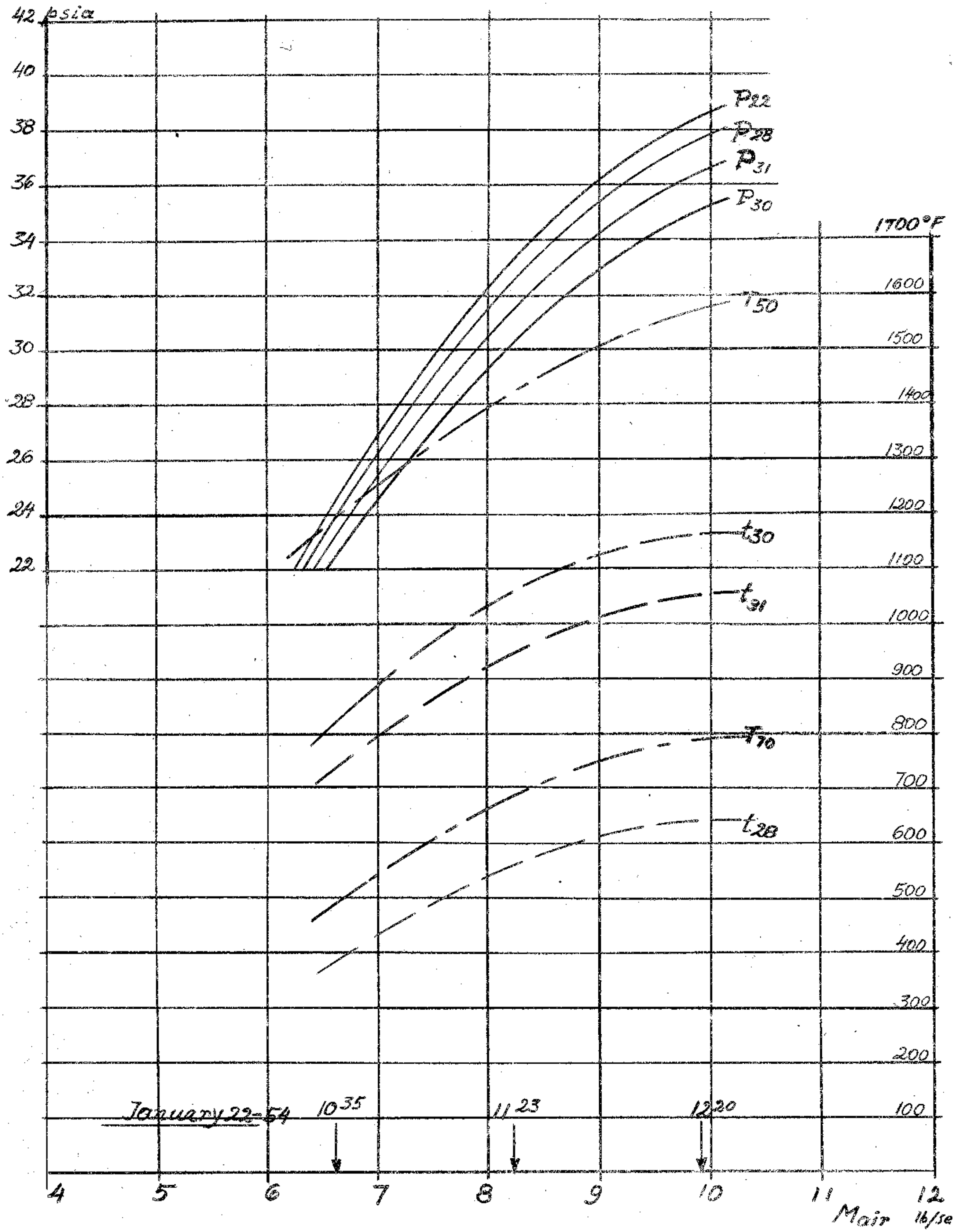
This investigation has been suggested by Prof. D. L. Mordell, Director of the Gas Dynamics Laboratory, to whom I owe, first of all, my basic knowledge in Heat-Transfer and Hydrodynamics and whose presence and interest were a constant stimulus for the work.

The main aspects of what should be the questions to be answered, the appropriate propositions and operating conditions and the various possibilities of design to be put on trial were frequently discussed with R.W. Foster-Pegg, in charge of the Experimental Coal Burning Gas Turbine Project.

An equally large part of suggestions and information I owe to the other staff members H. J. Tucker, T. Turczeniuk and, most notably, E. Juricek whose approach and forsightedness were very valuable for me whenever a problem called for clarification.

Books and publications which had been made use of are mentioned within the text.

Th. L.



42 psia
40
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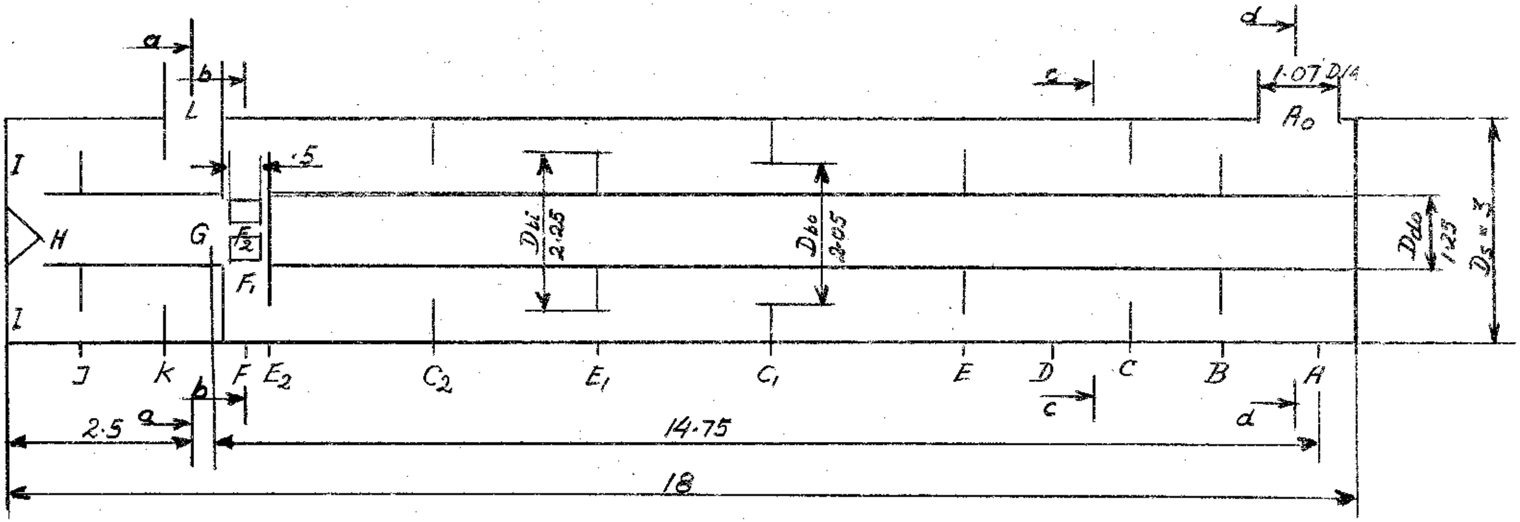
Fig. 1


a
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D_{bi} 2.25
D_{bo} 2.05
D_{do} 1.25
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14.75
18

Fig. 2

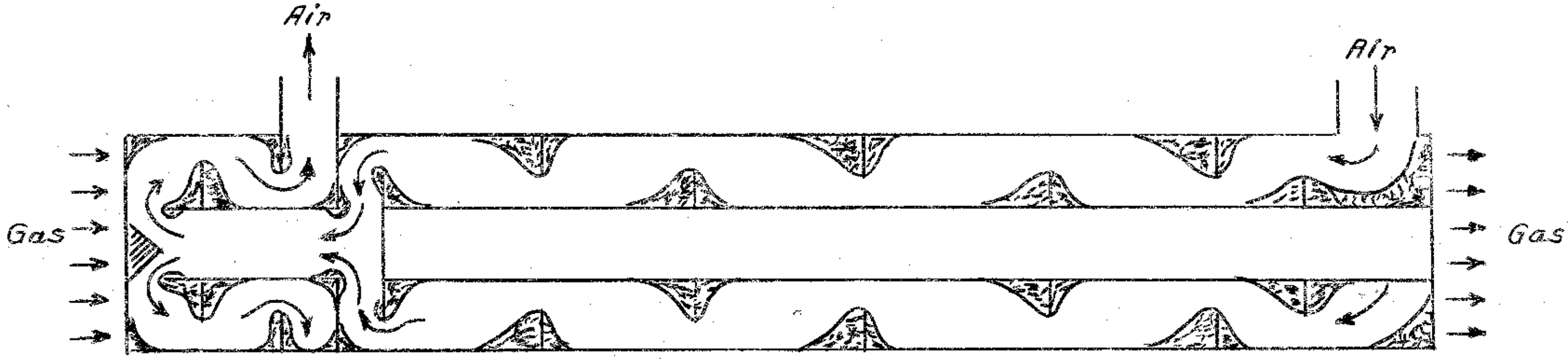

Air
Air
Gas
Gas

Fig. 3

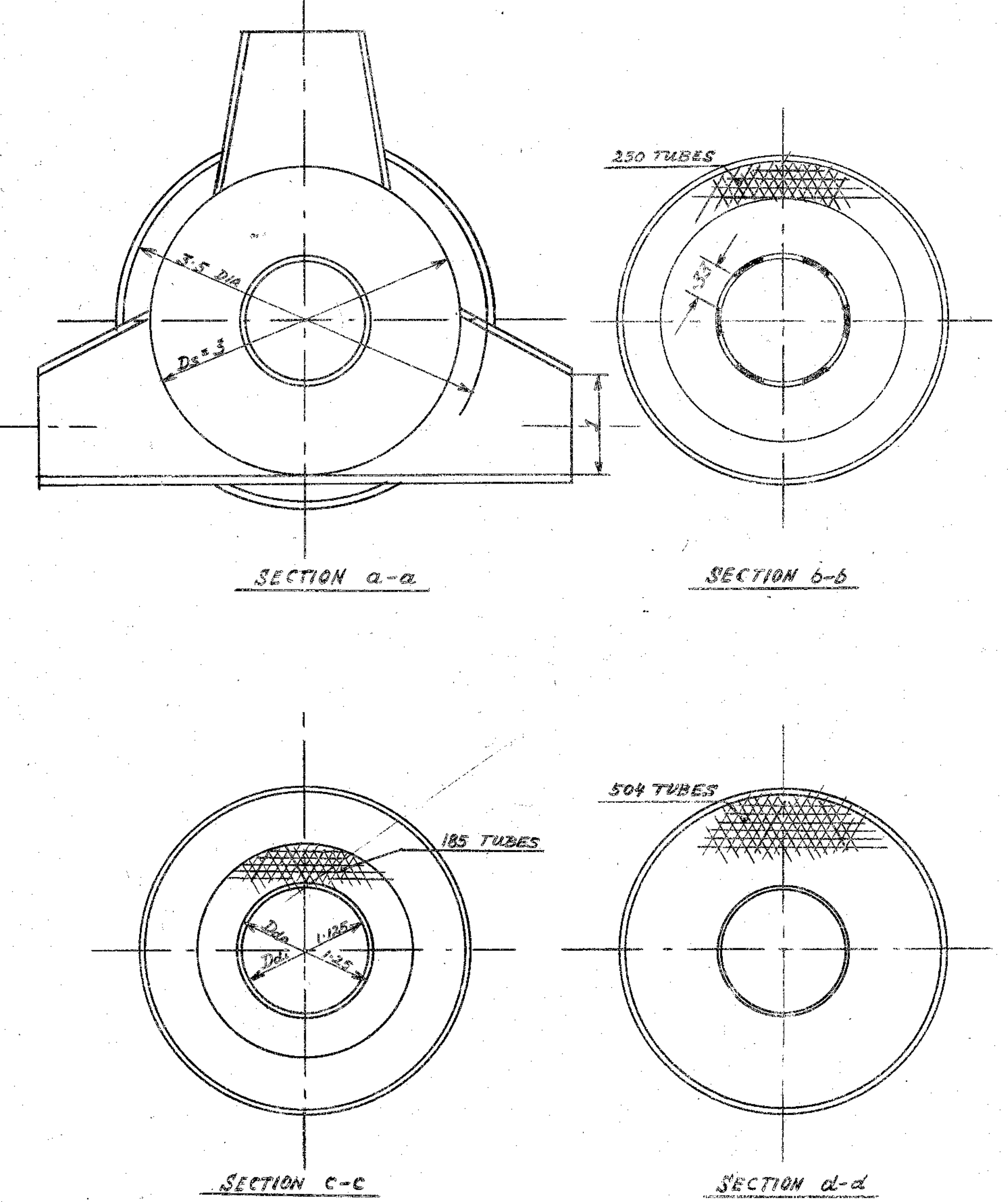
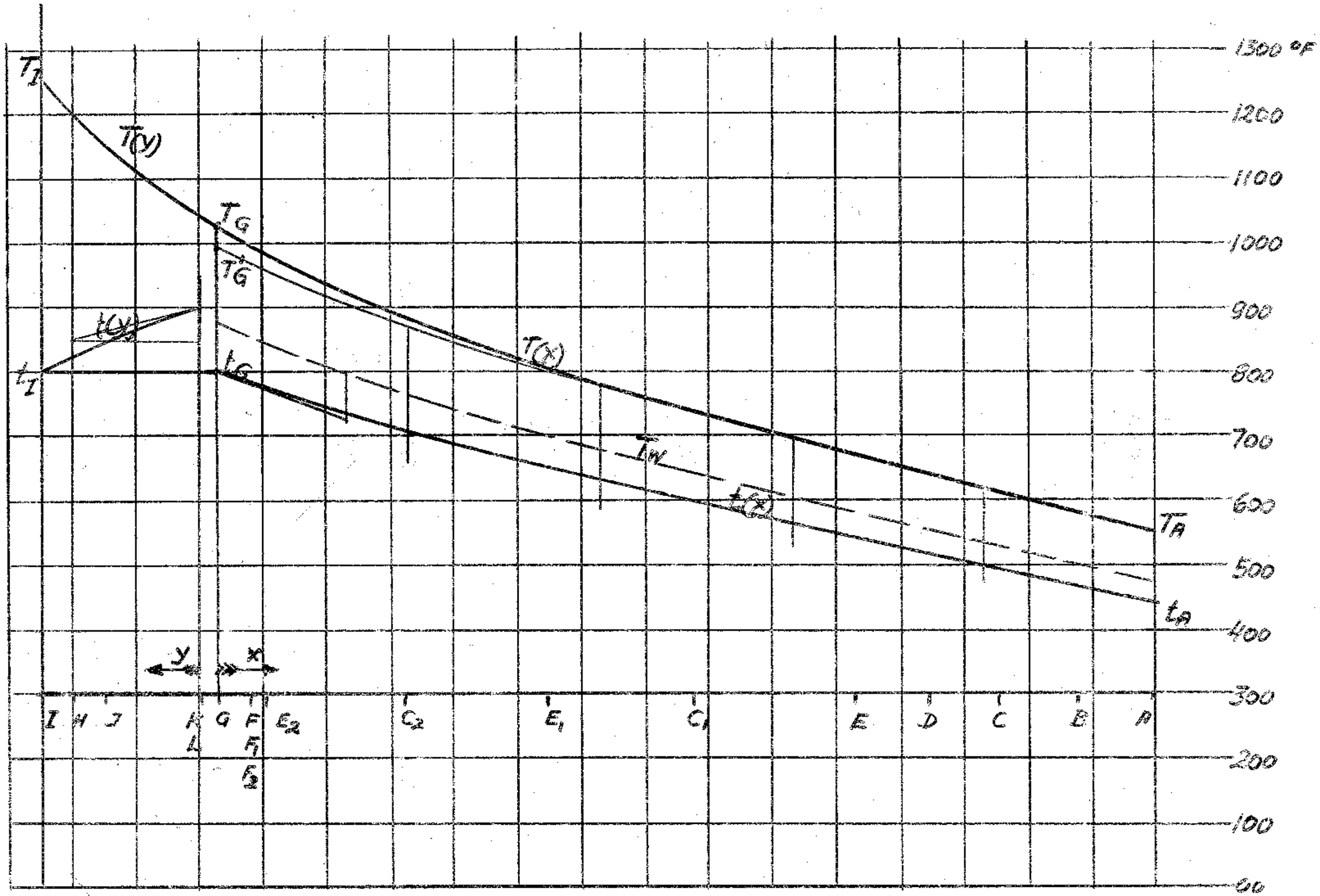

3.5 DIA
Ds = 3
1
SECTION a-a
250 TUBES
SECTION b-b
185 TUBES
Ddo
Ddi
1.125
1.25
SECTION c-c
504 TUBES
SECTION d-d

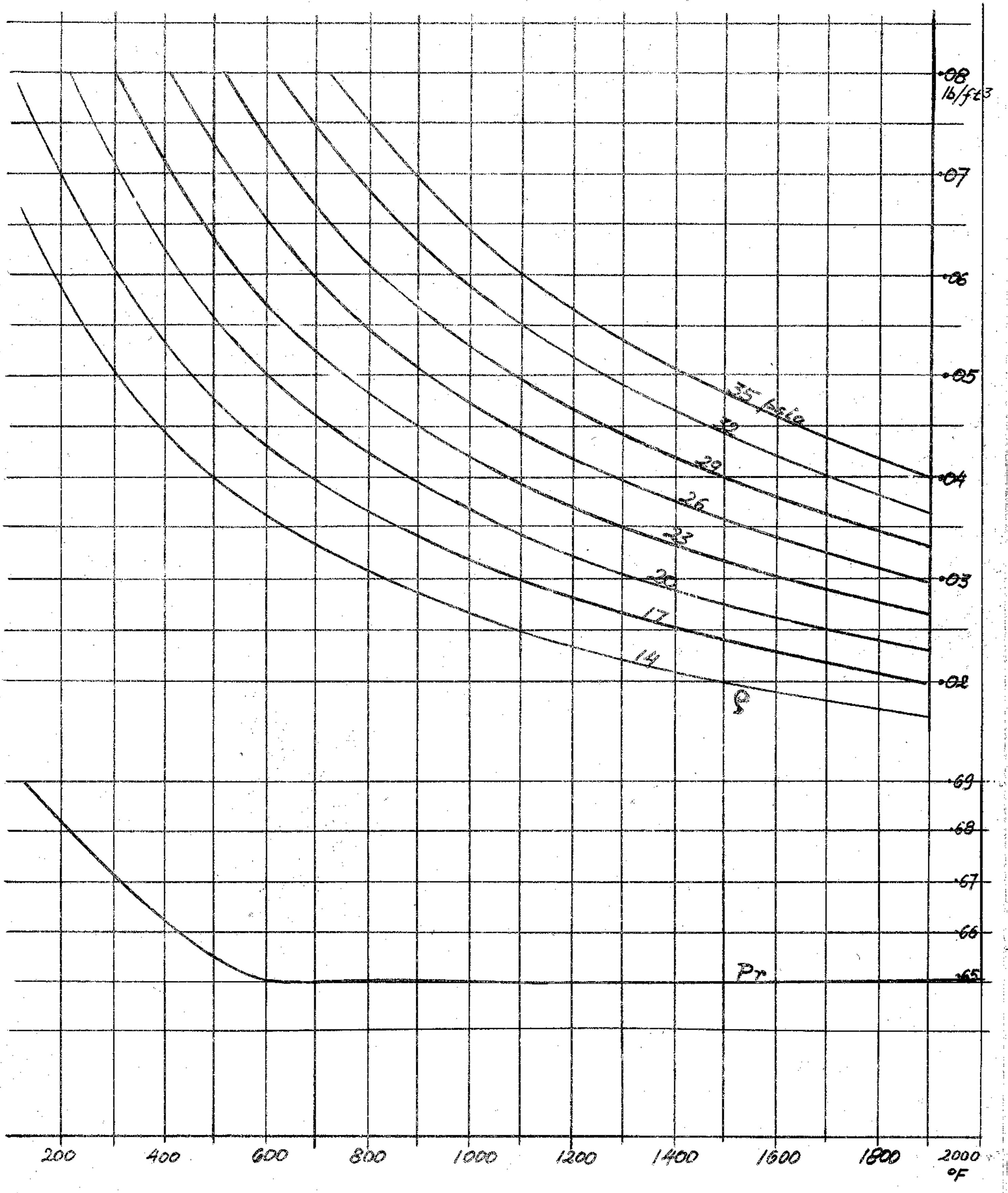
Fig. 4



1300 °F
1200
1100
1000
900
800
700
600
500
400
300
200
100
00
T(y)
t(y)
T(x)
t(x)
A
B
C
D
E
C1
E1
C2
E2
F
F1
F2
G
K
L
J
H
I
y
x

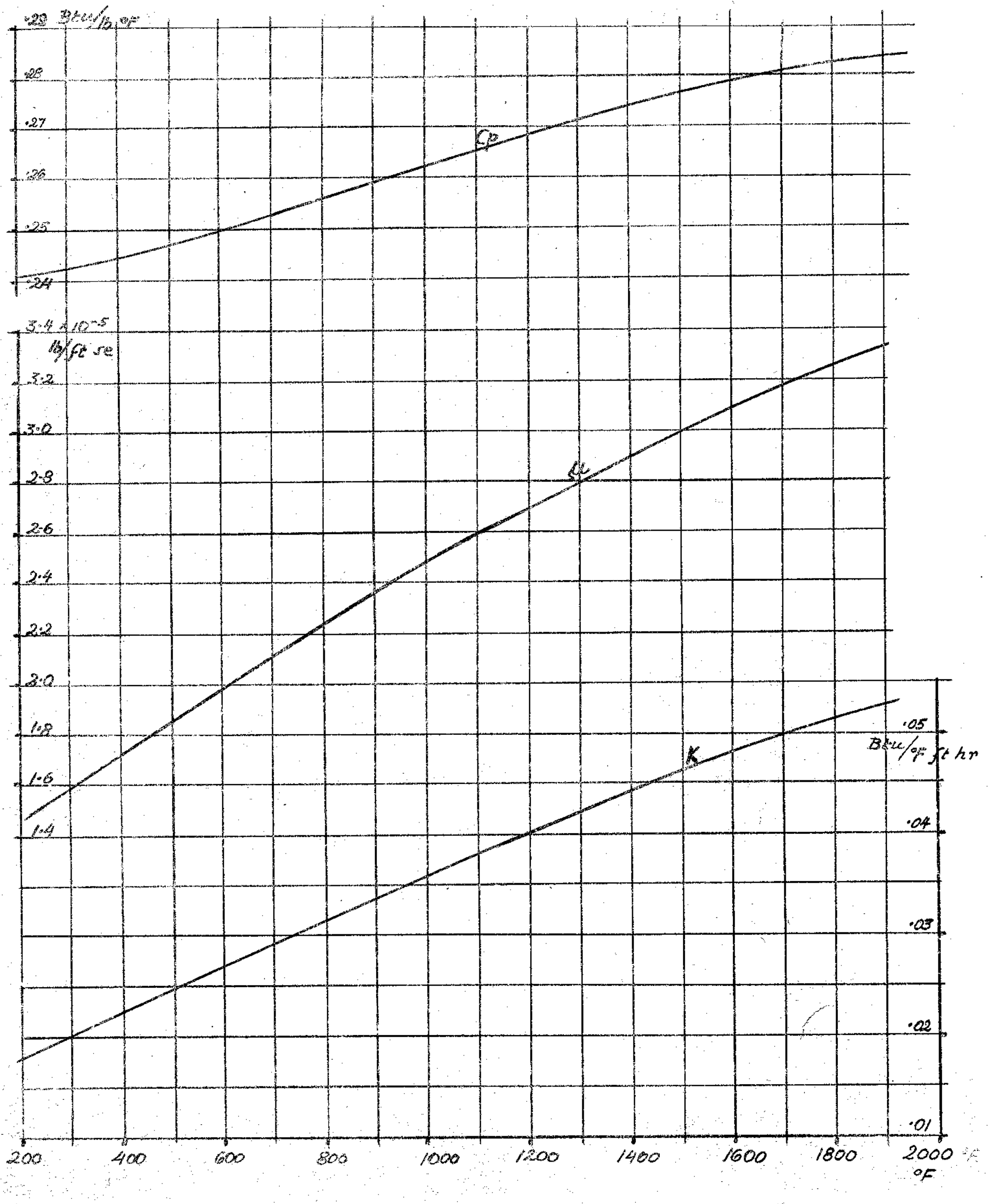
1cm ≙ 1 ft

Fig. 5



·08
lb/ft3
·07
·06
·05
·04
·03
·02
·69
·68
·67
·66
·65
35 psia
32
29
26
23
20
17
14
ρ
Pr
200
400
600
800
1000
1200
1400
1600
1800
2000
°F

Fig.6



·29 Btu/lb °F
·28
·27
·26
·25
·24
Cp
3·4 × 10⁻⁵ lb/ft sec
3·2
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
1·4
μ
K
·05
Btu/°F ft hr
·04
·03
·02
·01
200
400
600
800
1000
1200
1400
1600
1800
2000 °F

Fig. 7

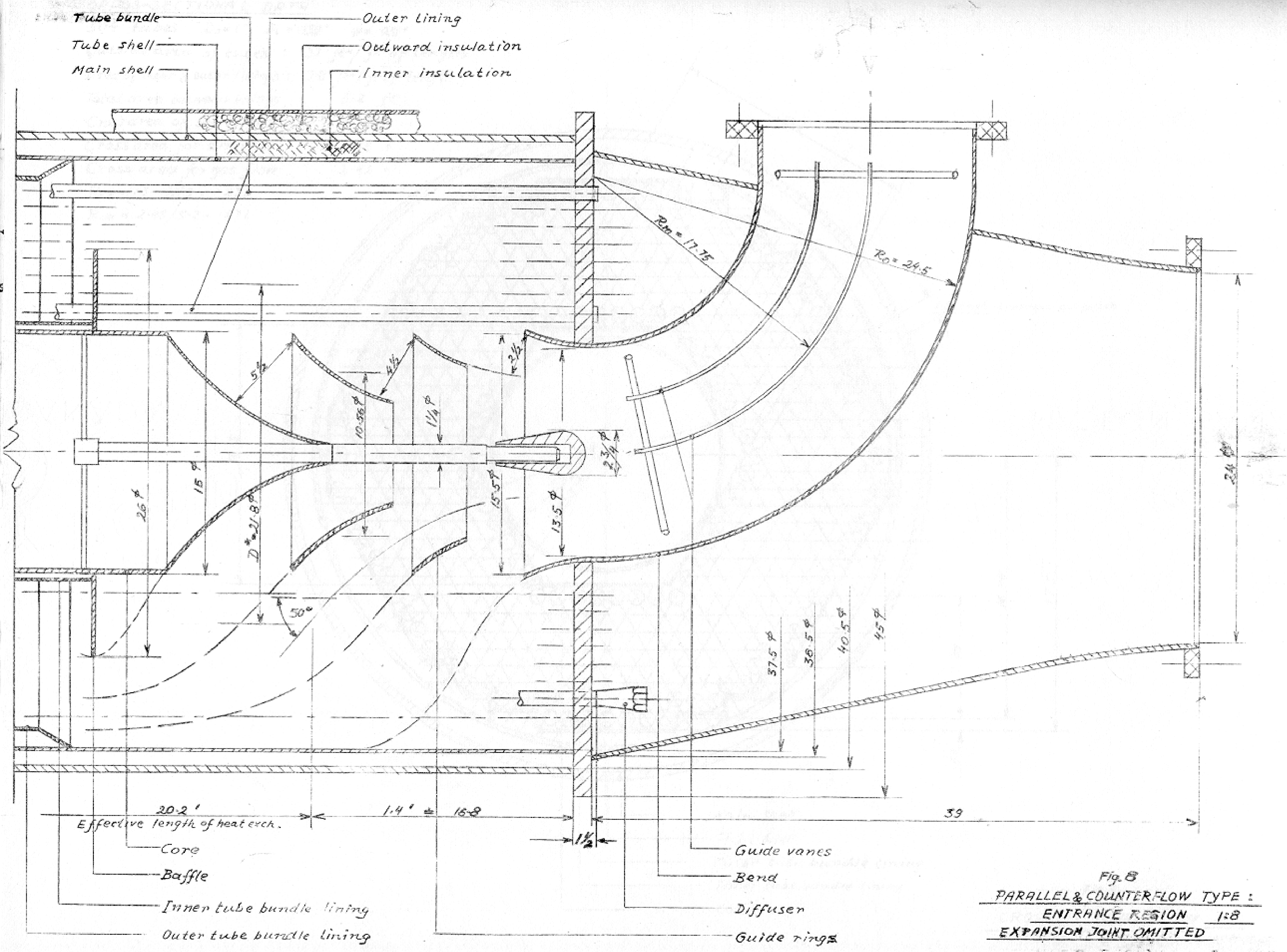


Fig. 8
PARALLEL & COUNTERFLOW TYPE :
ENTRANCE REGION 1:8
EXPANSION JOINT OMITTED

CROSS-SECTIONAL DATA

504 tubes $D_o = 1"$ $D_i = .938"$ $W = .031"$

Outside area of tubes : 132 ft²/ft of length

Area of inner & outer linings : 3·8 ft²/ft of length

Total area between linings	5·2 ft²
Cross area of 504 tubes	2·75 "
Cross area for air flow	2·45 "
Cross area for gas flow	2·42 "

$R_{cr} = (1{\cdot}3 - 1)/1{\cdot}3 = {\cdot}231$

$R_{ax} = 2{\cdot}45/5{\cdot}2 = {\cdot}471$

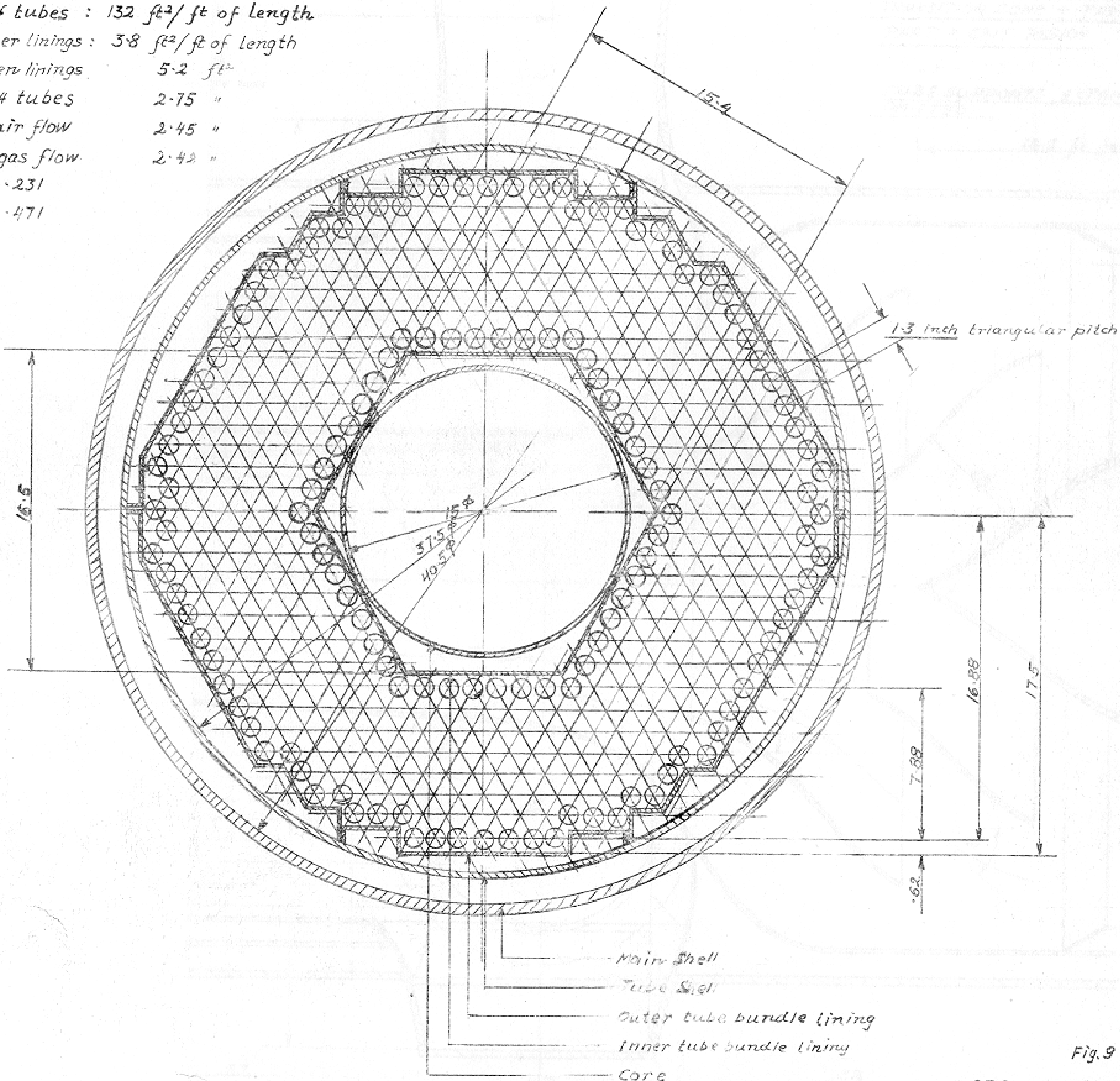


Fig. 9

CROSS SECTIONAL VIEW 1:8

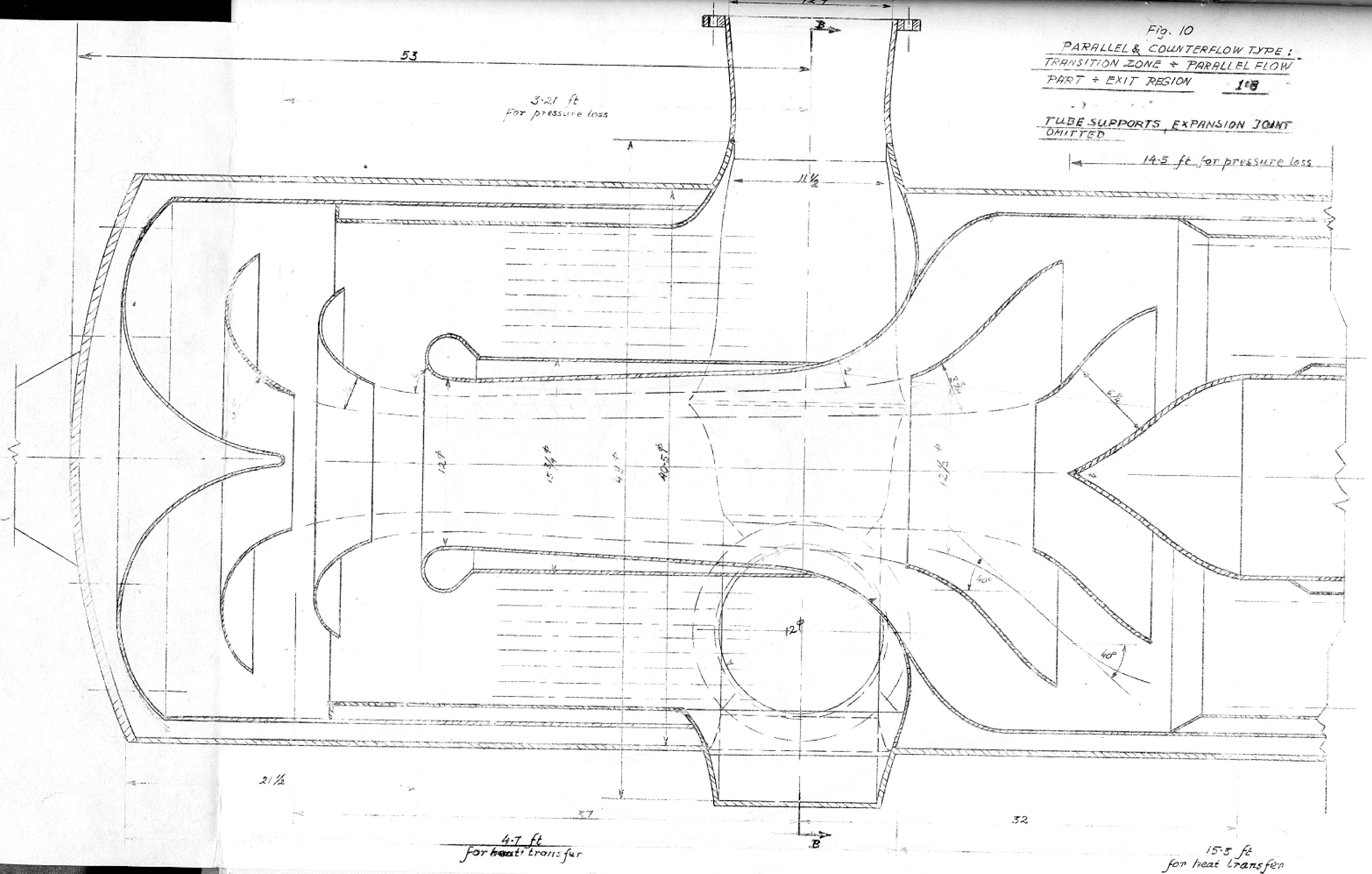


Fig. 10
PARALLEL & COUNTERFLOW TYPE:
TRANSITION ZONE + PARALLEL FLOW
PART + EXIT REGION 1:8

TUBE SUPPORTS, EXPANSION JOINT
OMITTED

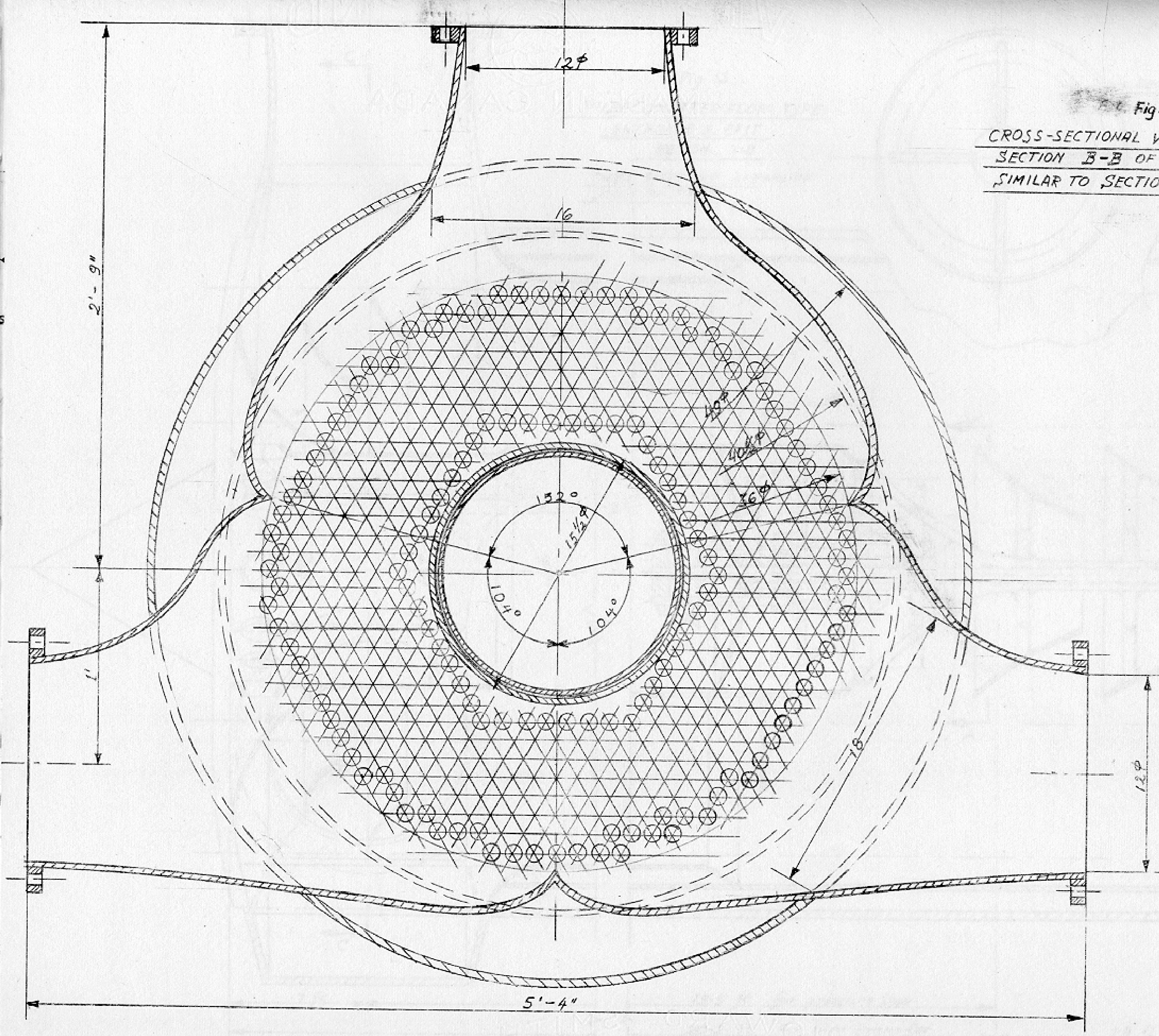


Fig. 11

CROSS-SECTIONAL VIEW OF EXIT REGION :
SECTION B-B OF Fig. 10,
SIMILAR TO SECTION C-C OF Fig. 12. 1:8

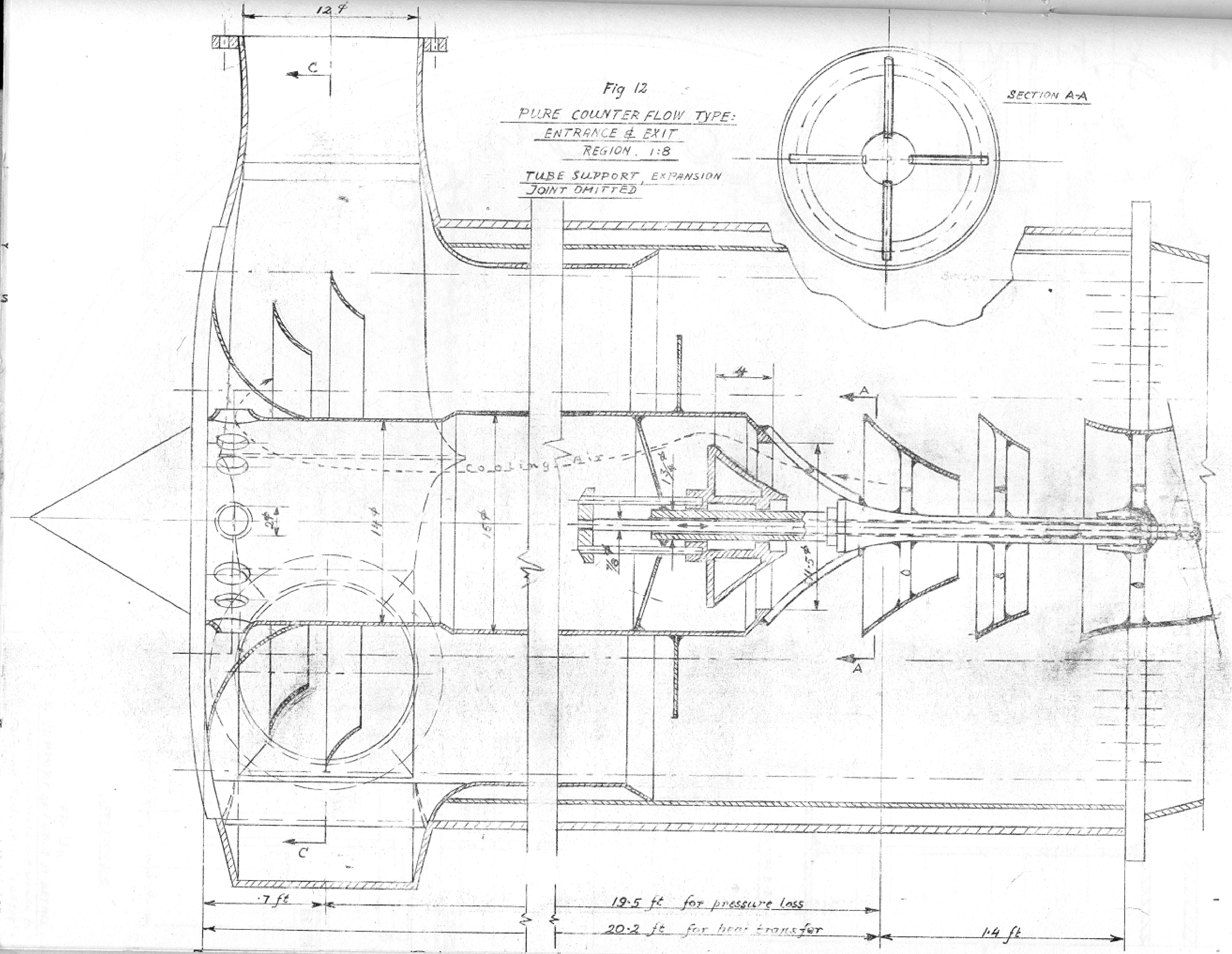


Fig 12
PURE COUNTER FLOW TYPE:
ENTRANCE & EXIT
REGION. 1:8
TUBE SUPPORT, EXPANSION
JOINT OMITTED
SECTION A-A
Cooling Air
C
A
4
·7 ft
19·5 ft for pressure loss
20·2 ft for heat transfer
1·4 ft

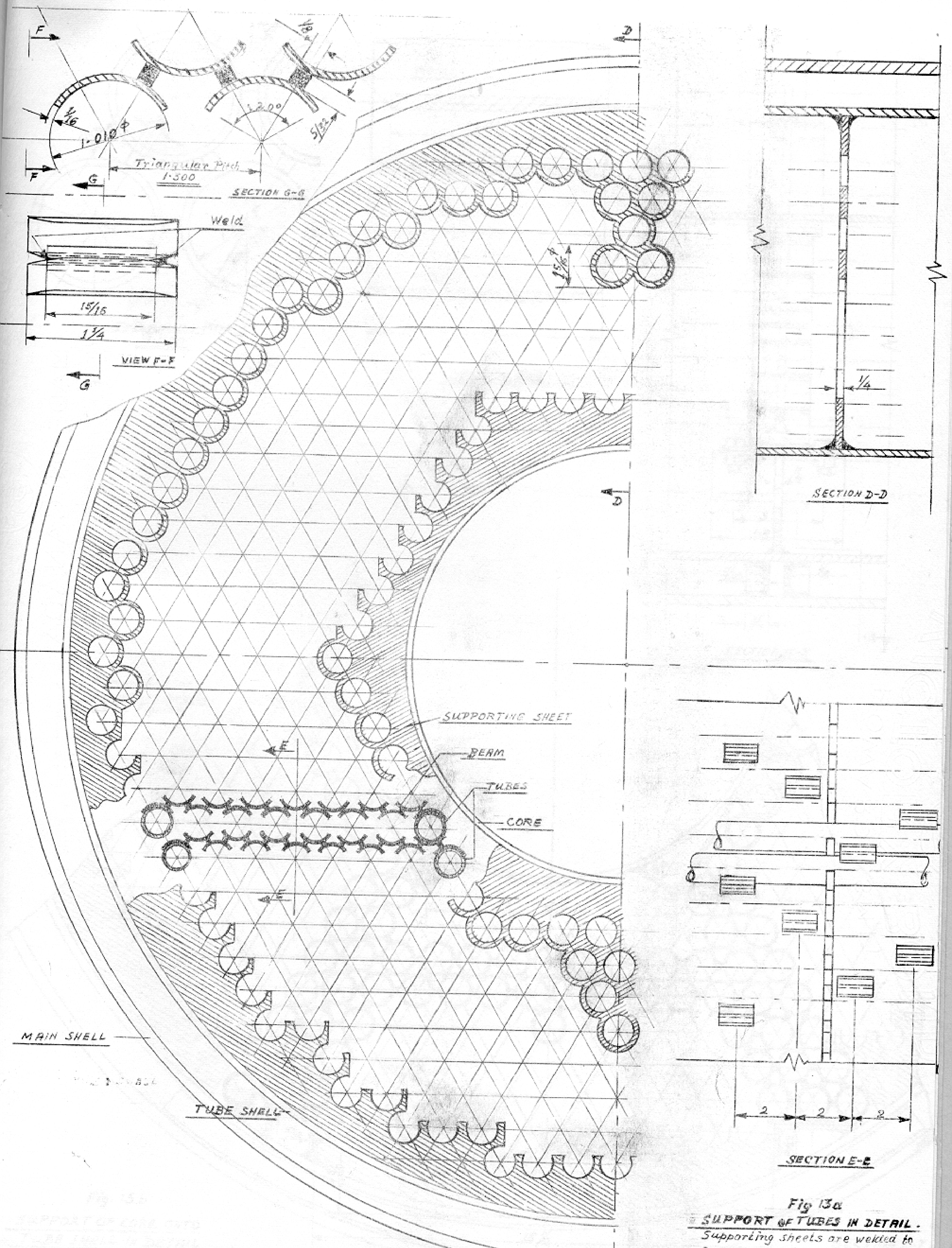


Fig 13a
SUPPORT OF TUBES IN DETAIL.
Supporting sheets are welded to Core and Tube shell - "Ladder" -

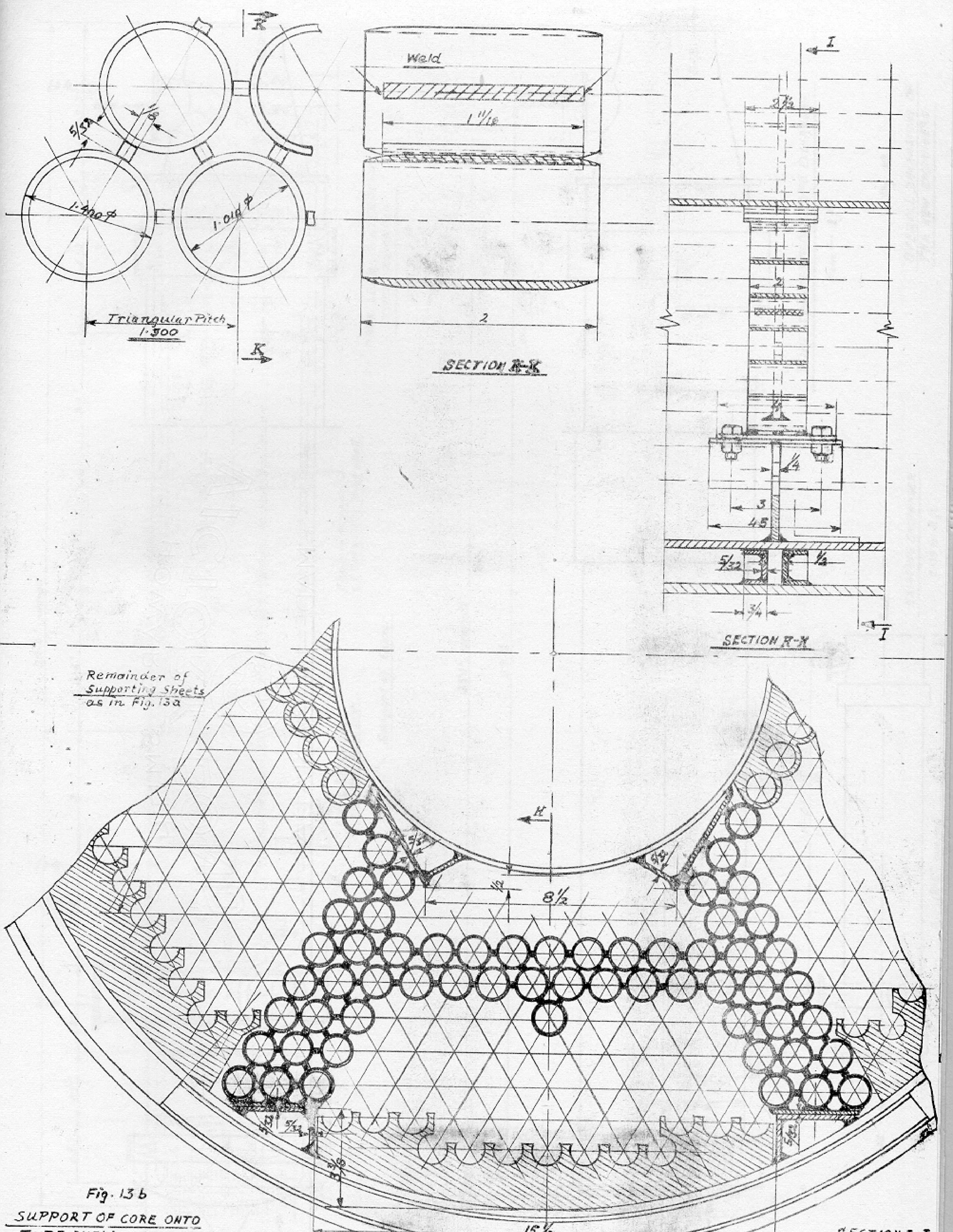


Fig. 13 b

SUPPORT OF CORE ONTO TUBE SHELL IN DETAIL

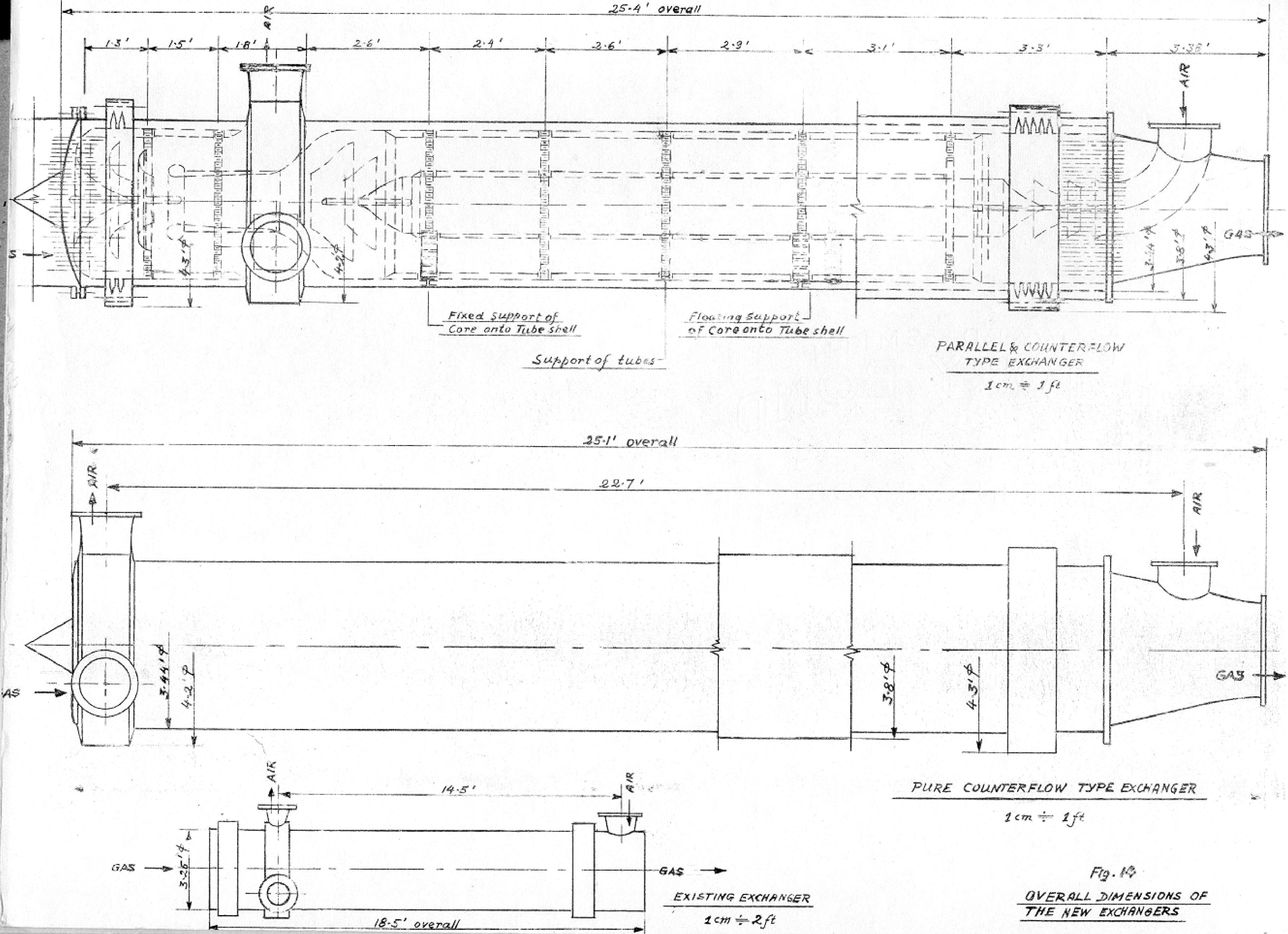


Fig. 14

OVERALL DIMENSIONS OF THE NEW EXCHANGERS