AN

ELECTRONIC SYNCHRONOUS SPEED REGULATOR

Submitted in partial fulfillment of the requirements for the degree Master of Engineering.

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PREFACE

This paper describes some of the development work undertaken by the author on an electronic synchronous speed regulating system for the National Research Council of Canada. Its purpose is to show how a control system of this type may be analysed by means of servomechanism theory, and the methods by which stable operation may be obtained. No originality is claimed for the principle of control or for the methods of analysis used here. However, the actual analysis of this control system and its proposed stabilization, has not, to the author's knowledge, been covered elsewhere in the literature. The bibliography contains those articles which were used by the author to obtain an understanding of the subject, as well as those referred to in the text.

CHAPTER ONE

INTRODUCTION

At the National Research Laboratories, Ottawa, Canada, a sixty cycle frequency standard for laboratory use is obtained from an electronic oscillator. The maximum power which may be drawn from this source is limited however, to about one kilowatt. In order to obtain more power, the development of a regulator which would maintain the output frequency of a D.C.motor-driven alternator in synchronism with this standard was undertaken.

Synchronous speed regulators have been developed before, notably in the paper mill industry, where electrical or mechanical differential devices are used to adjust the field rheostats of the various D.C. driving motors. The response of this type cannot be made fast enough, however, to take care of instantaneous load variations without introducing the danger of hunting, and, in modern high-speed paper machines, they have been superseded by electronic or electronic-amplidyne speed regulators, where adequate anti-hunt provision can be made. Although the latter are not synchronous in the strict sense of the word, the regulation was found to be sufficient for practical purposes.

A regulator for the use cited here, however, will have to maintain exact synchronism, and hence,no steady state speed error at any load can be permitted. In order to meet this stipulation, it is necessary to control the integral of the speed rather than the speed itself. The controlling force must be derived, therefore, from an angular displacement instead of an angular velocity. Load compensation will be obtained by steady state variations in angular position, which do not show up as steady state speed errors, since the speed is the time derivative or rate of change of angular position.

The method proposed here is to apply a voltage proportional to the angular displacement between the reference and alternate frequencies to buck the field current of the D.C.motor. Field current control is similar to armature voltage control, but has the advantage that smaller components can be used, with resultant saving in cost and space. It has the disadvantage of introducing a fairly large time constant, the effect of which must be considered.

DESCRIPTION OF THE SYSTEM.

The motor generator set for which the regulator described in this paper was designed, consists of an A.C. generator coupled directly to two identical D.C.motors. The rating of the machinesis as follows:-

A.C.generator - 30 KVA, 550 V, 31.5 A, 1200 RPM, 60 CY, 3 PH.

D.C.motor (each)20 H.P.,115 V, 147 A, 1200 RPM, (connected in series across 220 V).

The machines are coupled together by means of a flexible coupling. However, since this has a non-linear characteristic, a clamp was provided to alleviate the necessity of allowing for its action in the stability calculations. Most of the machine constants were obtained from manufacturer's data but are easily measured. The method of measuring a few of the less easily obtainable constants, however, is worth mentioning here.

1. <u>Moment of inertia of the rotating parts.</u> The retardation method was used to determine this constant. The set was driven at synchronous speed by the D.C.motors and the input to the machine at no load measured. The core, friction, and windage losses at synchronous speed were determined by subtracting other known losses. The speed was then raised about 10% above synchronous speed by inserting a resistance in the field. The armature was disconnected from the line,the additional resistance in the field shorted out so as to return the field current to normal value, and a decceleration curve obtained by means of a stroboscopic disc and watch. If the disc contains 16 segments and the stroboscopic light is set to flash n times per minute where n is synchronous speed in revolutions per minute, then the times at which the rotor speeds reach 17/16, 16/16, 15/16 ... of n revolutions per minute can be obtained. The moment of inertia is a function of the slope of the retardation curve at any speed, and the losses at that speed, and may be obtained from the following equation -

$$WR^2 = \frac{Kw}{rpm \frac{d}{dt}(rpm)} \times 2.165 \times 10^6 \ Lb(FT)^2 \qquad 1.1$$

The results obtained by this method were compared with manufacturer's data and found to be within 2%.

2. Inductance of the D.C.motor field. The set was driven at rated speed from the A.C. end, and field current for rated voltage applied to the D.C. machines. The field was then short circuited and the time required for the generated voltage to decay to 0.368 of its former value was determined. The inductance was obtained by multiplying the time constant thus obtained by the resistance of the circuit.

3. Inductance of the D.C. motor armature. The inductance of the armature was obtained approximately by the General Radio Impedance bridge. The time constant is very small compared with other circuits and therefore great accuracy in determining this constant was not deemed to be important.

The regulator consists in general of a full wave, grid controlled, thyratron rectifier. The plate voltage supply, and hence the control power, is derived from the alternator, and is at alternator frequency. The standard frequency voltage is applied to the grids and the output voltage of the thyratrons is therefore proportional to the phase displacement between the two frequencies. This voltage is applied to a resistor in the field circuit in such a way as to impede the flow of current in that circuit.

Thus, if the frequency of the alternator increases, the thyratron output and hence field bucking voltage will decrease, allowing more field current to flow. This will slow down the speed of the machines, decreasing the frequency.

It will be shown later that this form of control is unstable except at very small gain. The gain is a measure of the stiffness of the system and determines the value of the force developed to restore the system to normal after a disturbance. Furthermore, the steady state error of the control system is a function of the gain, and since the firing range of the thyratrons is limited to less than 180°, the field current variation required by the D.C. motor to compensate for a given load change may be unobtainable without exceeding this range. The gain is thus limited by stability on one hand and steady state error on the other. It will be necessary, therefore, to find some way of increasing the stability of the system, so that it will be capable of compensating for a reasonable steady state load variation. A method of accomplishing this by the addition of derivative or error-rate feedback is proposed in chapter four.

APPLICATIONS.

The application of this regulator, as stated previously, is to maintain synchronism between a D.C. motor driven alternator and a frequency source of low power. Another application however, would be in various mill drives such as in a paper mill, where maintenance of synchronism is an important factor. A further application, closely akin to the purpose here, is the speed control of D.C. motors by a low frequency oscillator.

CHAPTER TWO

ANALYTICAL METHODS AND DESIGN

In attempting to make a preliminary analysis of the operation of the regulator, advantage may be taken of servomechanism theory which has been developed rather extensively during the late war. Although this theory deals mainly with the response of a control system to a definite input function, it is equally applicable to regulators where the input is constant, and it is desired to maintain the corresponding output constant, or nearly so, during a temporary external disturbance. It is not intended here to give a detailed development of modern servomechanism theory but rather to outline briefly the methods by which a system may be studied in order to obtain the requirements for stability and desired response. Two methods, the transient analysis method and the frequency response method, are in present use but due to limitations in contemporary mathematical theory, these may be applied only to linear systems. A linear system is one in which the motion or variation is related to the cause by a linear differential equation with constant coefficients. Such a system is seldom encountered in practice. However, assumptions in the interests of producing linearity over the range of control required are usually made, and useful design criteria obtained therefrom. A third method of approach, that of electromechanical analogies, is at present being developed. The analogous electrical circuit for the system is set up and the response to various inputs viewed on a cathode ray screen. This method is very flexible and the effect of changing the parameters may easily be determined. The apparatus required is considerable however, and the expense is justified only if a fairly large amount of work is to be done on control systems.

A servomechanism may be defined as a control system wherein the controlling force is derived from the difference between the input and output quantities. Such a closed loop control system may be represented in block schematic as shown in figure 2.1. It is comprised generally of an error measuring device, an amplifier, and a power unit. In addition, differentiating or integrating networks and devices may be included between any two components.

THE TRANSIENT ANALYSIS METHOD

Closed loop control systems have a definite tendency to hunt or, at least, perform damped oscillations about the zero error point. The problem is to find methods of damping these oscillations sufficiently while still retaining a reasonable degree of response and static stiffness. In the transient analysis method, the differential equations of each component are set up and combined to form the complete differential or characteristic equation of the system. This equation will have the following form:-

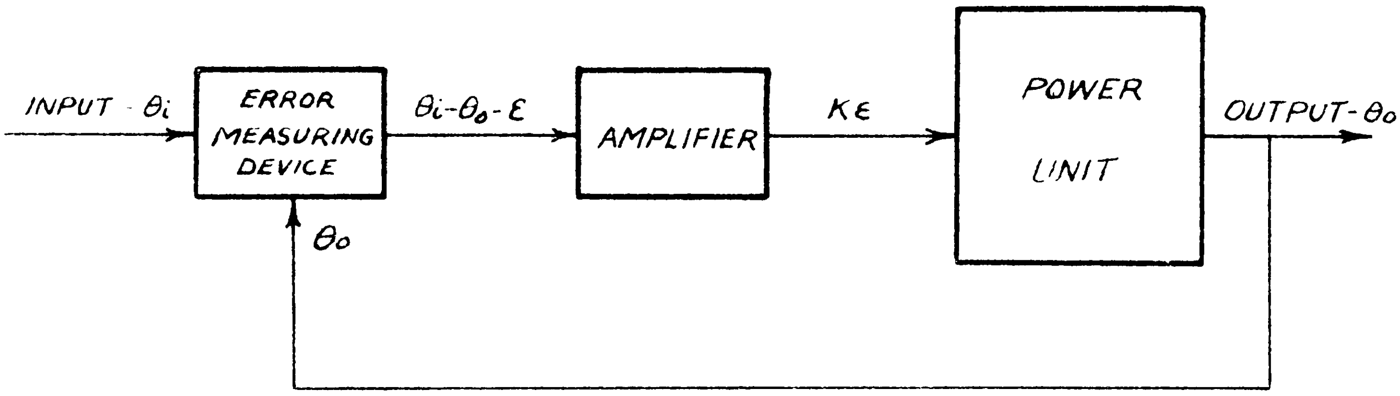

FIGURE 2.1

CLOSED LOOP CONTROL SYSTEM

$$a_n p^n + a_{n-1} p^{n-1} \; \ldots\ldots \; a_1 p + a_o = 0$$

where p represents the first derivative with respect to time, p^2 the second derivative, etc. The coefficients are proportional to the various system parameters such as inertia, friction, amplifier gain, and the electrical circuit constants. This equation is then solved, using any of the several classical, operational, or transform methods. The roots of the equation will either be real or in conjugate complex pairs, giving the complementary function with terms of the form -

$Ae^{at}(\cos wt \pm B)$ - for each complex root

Ae^{at} - for each real root.

There will also be a particular solution based on the input function from which may be obtained the degree of steady state error that exists. The key to the stability and response of a system analysed by this method however, lies in the complementary function solution of the characteristic equation. It is essential for stability that the roots of the characteristic equation contain no positive real parts. The value of these roots, which gives the frequency of oscillations, if any, and the degree of damping, determines the speed of response of the system.

The main disadvantage of this method is the labour involved in the solution of the characteristic equation, particularly if the degree is five or more. Furthermore once a solution has been obtained, there is no way of determining what parameters should be altered to improve system performance, since they are so completely intermixed in the coefficients. If it is merely desired however to determine whether the system is stable or not, the characteristic equation need not be solved. The application of Hurwitz' and Routh's criteria to the coefficients will reveal whether there are any roots with positive real parts. These criteria however, have no provision for determining the value of damping, hence the information obtained is of little use.

THE FREQUENCY RESPONSE METHOD

The frequency response or sinusoidal analysis method eliminates the necessity of solving the characteristic equation, and also shows clearly the effect of each parameter on system performance. The method consists in general of determining the output/input ratio of the system for inputs of different frequencies. In any stable system the transient response dies out after a given period of time, and, if a new similar disturbance occurs, the transient will repeat itself. Such periodic functions of time may be analysed in a fourier sine series of sinusoidal terms. For each term in the input, a corresponding term of the same frequency will appear in the output. Hence variations of the output/input ratio and phase angle with frequency, called the system response function, may be determined. Since the input and output functions each consist of the sum of their various frequency components, this system response function will contain all the information available from the characteristic equation. The actual interpretation of this function is rather difficult, due to intermixing of parameters. However, in automatic control systems where the input consists of the difference or error between the input and output, the loop transfer function based on the output/error ratio leaves most of the parameters independent.

THE SYSTEM RESPONSE FUNCTION.

Although control systems are not usually analysed by means of the system response function, a discussion of some of its properties may be included here. This function may be obtained by setting up the characteristic equation of the system, and solving for the output/input ratio in terms of the derivative operator p. The substitution p = jw is then made. From this expression, the magnitude and phase angle of the system response function may be determined for various frequencies. Typical system response functions are shown in figure 2.2, and the system response to a unit step input is shown in figure 2.3. The ideal function would have a value of unity and no phase angle variation over a range of frequencies. Actual functions however differ from the ideal in three ways:-

1) The frequency response is not linear and resonant frequencies sometimes appear which produce damped oscillations under transient conditions. The relation between the height of the resonant peak and the time constant of the oscillations is not expressible generally in mathematical form.

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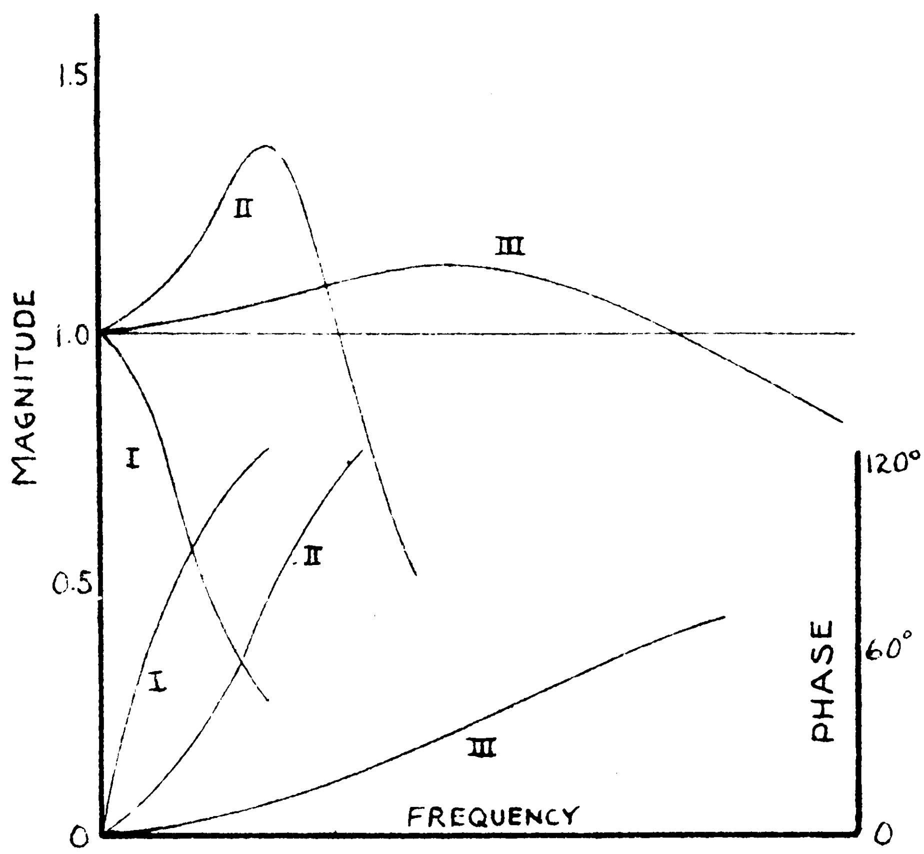


FIGURE 2.2 SYSTEM RESPONSE FUNCTIONS

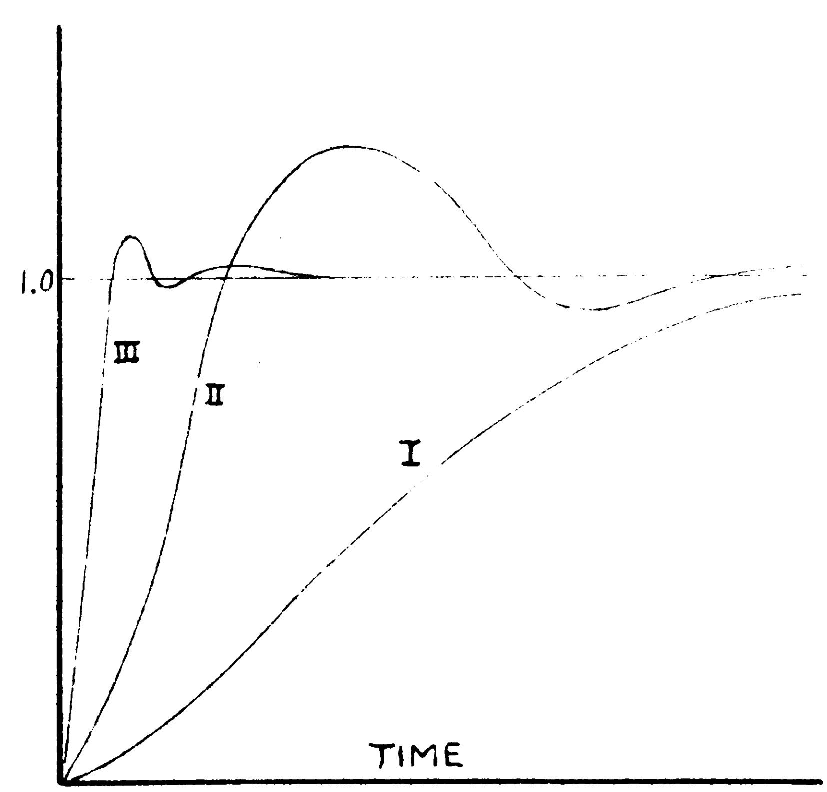


FIGURE 2.3 RESPONSE TO UNIT STEP INPUT

However, experience has shown that peaks up to a maximum of 1.3 are acceptable.

2) The higher frequencies are usually attenuated, resulting in sluggish motion of the system. The system which passes the higher frequencies has a faster motion, but since the lower frequencies predominate in most input motions, the value in this range is most important.

3) The phase variation is equivalent to a time delay, and the best performance is obtained with a system of least phase lag. It is a measure of the speed of response of a system and is especially important at frequencies near resonance.

<u>THE LOOP TRANSFER FUNCTION</u>.

It was pointed out previously that in certain types of automatic control systems, the control is based on the error between the input and output, and hence an investigation of the system response function in terms of the output/error ratio or loop transfer function is justified. Furthermore, stability, the fundamental requirement of all automatic control systems, is solely dependent on the output as a function of error and not of input. The system response function is related to this new function by the following equation:-

$$\theta_o/\theta_i = \frac{\theta_o/e}{1 + \theta_o/e} \qquad 2.1$$

where $e = \theta_i - \theta_o$. Equation 2.1 reveals a striking similarity to that of the feedback amplifier where θ_o/e

may be compared with the so called $\mu\beta$ characteristic. It is from the terminology of such amplifiers that the term loop transfer function is borrowed. The loop transfer function may be investigated following the methods outlined by MacColl and Bode, the Nyquist criterion applied, and the results referred to the more general system response function by use of a few simplerules developed by Hall.

Nyquist's criterion for stability, as originally stated in his article, "Regeneration Theory", is in rather complicated mathematical form involving the theory of functions. Although this must be resorted to in all questionable cases, a more practical form which is frequently used by control engineers may be stated as follows:-

> "For a control system to be stable, it is required that the Nyquist point -1 + j0 be always "seen" to the left when progressing along a complex frequency plot of the loop transfer function in the direction of increasing frequency".

A plot of typical loop transfer functions is shown in figure 2.4. The curves A,B,C, exhibited here are stable with various degrees of damping depending on their proximity to the Nyquist point. Curve D on the other hand is unstable, and its system will be subject to oscillations whose amplitude increases with time.

The frequency characteristic of the loop transfer function however, will reveal more information than this. In figure 2.5 the point O represents the origin, N the

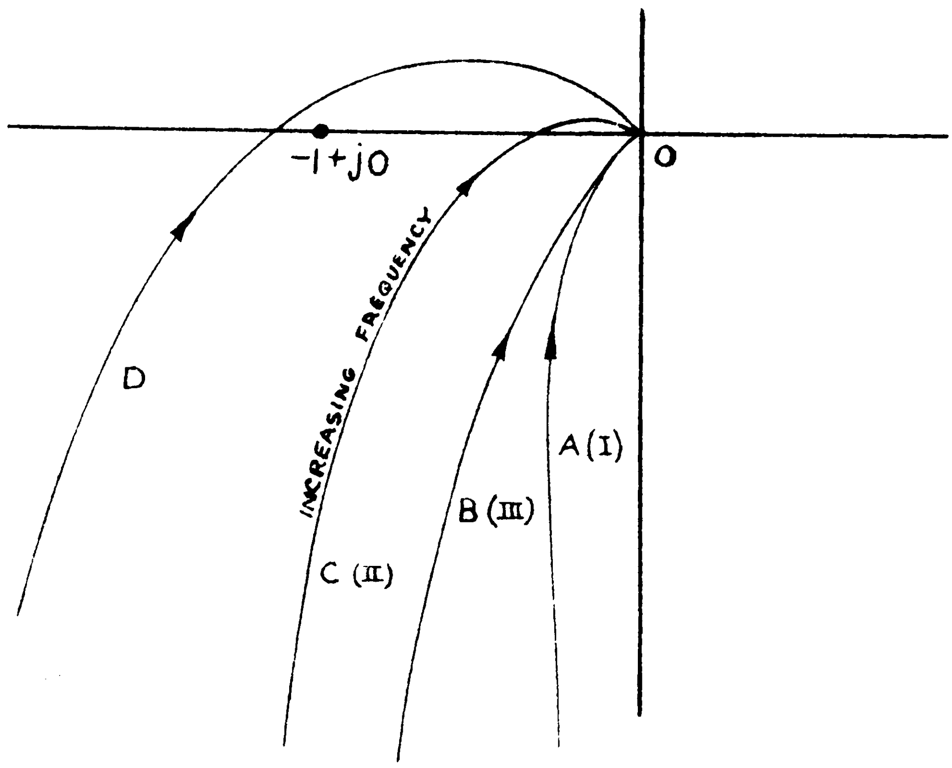


FIGURE 2.4 LOOP TRANSFER FUNCTIONS

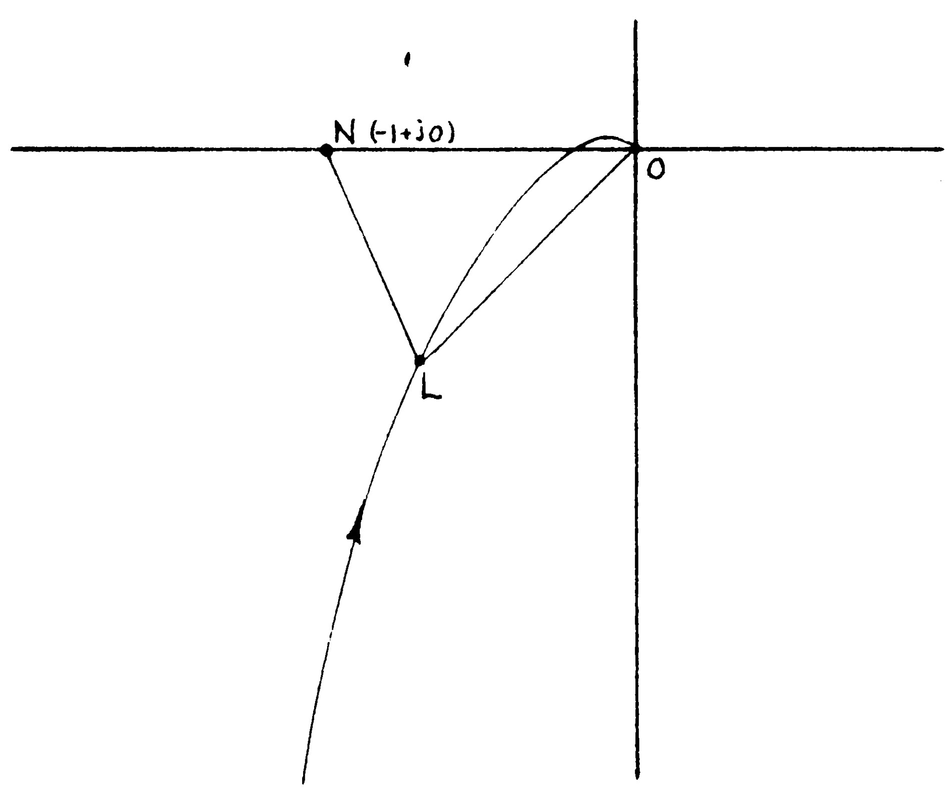


FIGURE 2.5

Nyquist point and L a point on the characteristic at some particular frequency. The vector OL is equal to the value of the loop transfer function θ_o/e at that frequency, and the vector NL is equal to $1 + \theta_o/e$. Referring back to equation 2.1 it is apparent that the magnitude M of the system response function is determined by

$$M = \frac{NL}{OL}$$

and that the phase angle at that frequency is $\angle NLO$. Thus from a plot of the loop transfer function, the system response function may easily be determined, plotted as in figure 2.2, and the degree of resonance noted.

THE INVERSE TRANSFER FUNCTION.

In certain cases, the inverse transfer function or error/output ratio simplifies the calculation. Inverting equation 2.1, the following equivalence results:-

$$\theta_i/\theta_o = \frac{e}{\theta_o} + 1 \qquad 2.2$$

If e/θ_o is plotted on the complex frequency plane, the same curve may be used for the θ_i/θ_o function by displacing the origin to -1+j0. The ratio $1/_M$ then is the distance from the new origin to a point on the curve corresponding to a particular frequency. Likewise, the phase angle of the system response function is the negative of the angle that the θ_i/θ_o vector makes with the real axis.

DESIGN CONSIDERATIONS.

In the design of automatic control systems, the first step is of course the selection of the type of control to be used and components required. It will be governed by such factors as cost, size, weight and availability. The next step consists mainly of predicting the values of the adjustable parameters which will give optimum performance. In some cases it will be easier to set the system up and use cut and try methods of adjustment. However, in more complicated systems an analysis is sometimes of use. This is especially so where the simple system does not meet the specifications and the addition of extra circuits and feedback is necessary. In making the adjustment, the following criteria should be adhered to:-

1) The system must be stable.
2) The resonant frequency or frequencies should be as high as possible.
3) The damping at each resonant frequency should be high.
4) The gain factor of the system should be high.

Unfortunately the above factors are not independent and hence any design will in general be a compromise. Furthermore, accurate prediction of adjustable parameters for optimum performance is hindered by the difficulty of obtaining mechanical and electrical constants accurately. However, the results obtained are usually sufficient to enable the engineer to set up the system, upon which final adjustment or "tuning up" can be made.

The method of obtaining values for the adjustable parameters of a system, which will be given here, is an interpretation of the loop transfer function and was developed by Hall. It will be apparent however that the process could also be applied to the inverse transfer function with slight modification.

The loop transfer function will consist of two parts, a frequency dependent portion and a frequency invariant portion or gain factor. The gain is usually the most easily adjustable and hence a method for obtaining optimum gain is desirable.

It has been stated previously that the gain of the system should be high. This is based on the premise that high gain -

1) reduces steady state and transient errors in the system,
2) increases the resonant frequency of the system in most cases,
3) increases the magnitudes of the real roots of the system in most cases.

High gain however also reduces the real parts of the complex roots or damping constants, and hence continued increase in gain will result in instability. The optimum value of gain may be determined from the transient response of the system if known. A transient analysis, except for simple systems is laborious, and therefore selection based on the amplitude of the resonant peak of the system response function is frequently used. In figure 2.6, some typical system response functions are shown. Curve A represents an overdamped system (no complex roots), curve B critically damped,

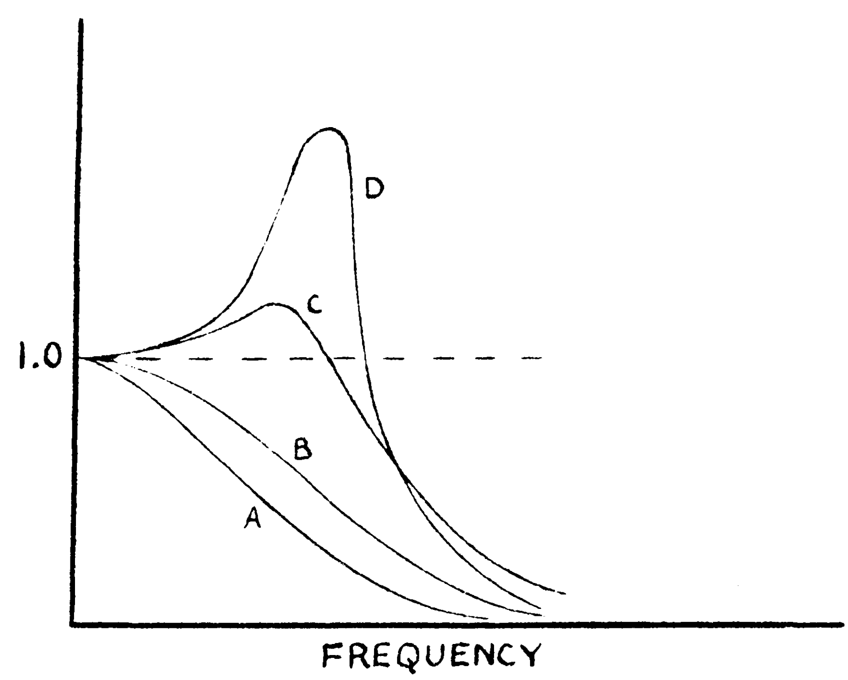


FIGURE 2.6

SYSTEM RESPONSE FUNCTIONS

curve C underdamped with large damping factor, and curve D, underdamped with small damping factor. Curve C is usually considered optimum for most applications and represents an amplitude response of approximately one and one-third. If the gain obtained for optimum transient performance of the system is not sufficient to meet other specifications, variations in the frequency dependent portion, either by adjusting parameters, adding feedback, or even extra components, must be considered.

SELECTION OF THE GAIN FACTOR.

Before giving the steps for determining optimum gain, the locus of constant amplitude M of the system response function will be derived.

Let the loop transfer function vector $\theta_o/e = X + jY$

Then
$$M = \frac{\theta_o}{\theta_i} = \frac{\theta_o/e}{1 + \theta_o/e} = \sqrt{\frac{X^2 + Y^2}{(1+X)^2 + Y^2}}$$

or
$$M^2 = \frac{X^2 + Y^2}{(1 + X)^2 + Y^2}$$

$$X^2 + \frac{2M^2}{M^2-1}X + Y^2 = \frac{-M^2}{M^2-1}$$

$$\left[X + \frac{M^2}{M^2-1}\right]^2 + Y^2 = \frac{M^2}{(M^2-1)^2} \qquad 2.3$$

$$\text{Center at } \frac{-M^2}{M^2-1}, 0 \qquad 2.4$$

and a further relation
$$\frac{\text{Center of circle}}{\text{intercept on real axis}} = \frac{M}{M-1} \qquad 2.5$$

Having now obtained the locus for constant M, it is merely required to select the value of gain which will place the locus of the transfer function in tangency with the locus of maximum amplitude response desired. The simplest procedure is to recognize that changes in gain are equivalent to scale changes in the complex frequency plane where the transfer function is plotted. This procedure is as follows:-

1) Plot the frequency dependent portion only of the loop transfer function.
2) Construct a circle that is both tangent to the locus of the transfer function and which has a radius and position that equation 2.5 holds for the desired M (usually M=1.3)
3) The value of gain then is the factor by which the location of the centre of this circle must be multiplied to agree with the value obtained from equation 2.4.

This method is illustrated in the next chapter where it is applied to the loop transfer function of the simple regulator.

CHAPTER THREE

ANALYSIS OF THE SIMPLE REGULATOR

The circuit of the first prototype of the electronic synchronous speed regulator, without anti-hunt feedback, is shown in fig. 3.1. The output frequency and voltage of the alternator is applied to the thyratron plates through transformer T_1 and is compared with the reference frequency which is applied to the thyratron grids through transformer T_2. The variable resistor R_1 is a device by means of which the amount of bucking voltage introduced into the field circuit may be varied without changing the effective resistance offered to the field supply voltage. Resistor R_2 is added to limit the thyratron current to a safe value. Resistor R_3 is the usual field rheostat for rough adjustment of the field current to a value which is in the range of thyratron control. Selection of the steady state firing angle of the thyratrons may be obtained by small adjustments of R_3.

For purpose of analysis, the regulator may be represented in block form as shown in fig.3.2. Before proceeding with the development of the transfer function however, a few assumptions must be made in the interests of preserving the linearity of the system. These are:-

1) The function $(1+\cos\theta)$ is assumed to be linear and of the form $(2-K_1\theta)$, in order that the voltage output

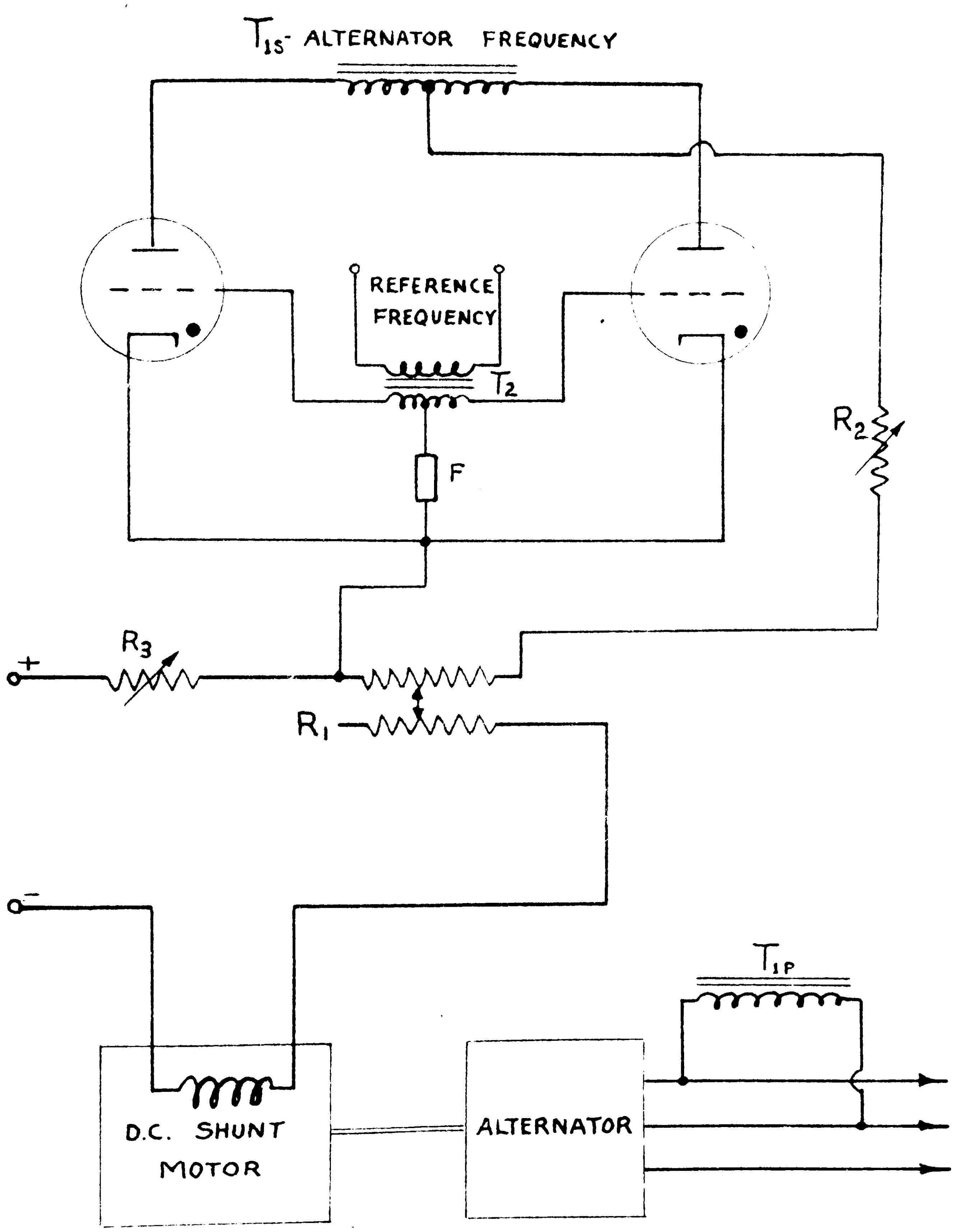


FIGURE 3.1

SYNCHRONOUS SPEED REGULATOR

(FEEDBACK APPLIED AT 'F')

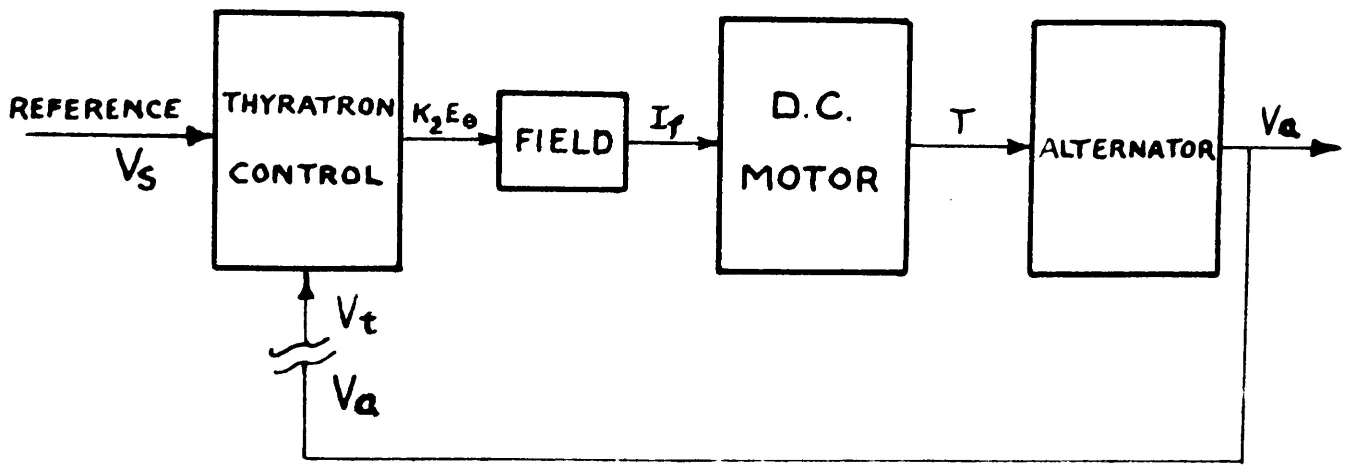


FIGURE 3.2

BLOCK DIAGRAM

of the thyratrons may be expressed as a direct linear function of the firing angle. Since the regulator must be stable over its entire operating range, the maximum value of K_1, unity, will be used in stability calculations. If the system is fairly "stiff" and the angular variation for a given load change is small, then the error obtained in making this assumption will not be large.

2) The field current-speed relationship for the shunt motor is also assumed to be linear over the range considered. That is:-

$$V' = \frac{A - BI_f'}{1+T_r p(1+T_a p)} \qquad 3.1$$

The numerator is the familiar equation of the straight line and the denominator contains inherent time delays which must be considered in any transient analysis. The derivation of this equation will be given later.

3) The variation of the thyratron plate voltage E_t, which is derived from the alternator output voltage and hence is a function of the speed, is assumed to be negligible. This is necessary to avoid the occurrence of a derivative squared term in the equation which would destroy the linearity.

DERIVATION OF THE LOOP TRANSFER FUNCTION.

In the study of closed cycle control systems, it is convenient to "break" the continuity of the control loop at some point, and determine the transfer function in terms of the input and output quantities at that point. In this case the break is made between the alternator and the thyratron control, as shown in figure 3.2. The firing angle of the thyratrons is the difference in phase between the plate and grid voltages, and may be expressed as the integral of the difference between the two angular velocities. That is:-

$$\theta = \frac{V_t - V_s}{p} \qquad 3.2$$

where V_t, V_s are the angular velocities of the two voltages expressed in electrical radians per second. Hence the voltage output of the thyratrons is:-

$$E_\theta = E_t(1+\cos\theta) - I_f' R_k$$

$$= E_t - I_f' R_k + E_t \cos\left(\frac{V_t - V_s}{p}\right) \qquad 3.3$$

Applying the cosine approximation:-

$$E_\theta = 2E_t - I_f' R_k - E_t K_1 \left(\frac{V_t - V_s}{p}\right) \qquad 3.4$$

To cause sufficient field current variation for a given load change, only part of the thyratron output, $K_2 E_\theta$, need be applied to the field bucking resistor. The field current is then:-

$$I_f' = \frac{E_s - K_2 E_\theta}{R_f(1+T_f p)}$$

$$= \frac{E_s - 2K_2 E_t + K_2 I_f' R_k + E_t K_1 K_2 \left(\frac{V_t - V_s}{p}\right)}{R_f(1+T_f p)} \qquad 3.5$$

Solving for I_f':-

$$I_f' = \frac{E_s - 2K_2 E_t + E_t K_1 K_2 \left(\frac{V_t - V_s}{p}\right)}{(R_f - K_2 R_k)(1+T_f p)} \qquad 3.6$$

For analysis, only the transient portion of equation 3.6 is required leaving

$$I_f = \frac{-E_t K_1 K_2 \left(\frac{V_t - V_s}{p}\right)}{R_{eq}\,(1 + T_f p)} \qquad 3.7$$

The derivation of the field speed relations for a shunt motor is given elsewhere by McCann, Osbon and Kirschbaum, but is reproduced here for completeness. Neglecting saturation effects, the motor air gap flux, for variations of field current I_f about the normal value I_{fo}, may be expressed by:-

$$\phi = \frac{\phi_o}{I_{fo}} (I_{fo} + I_f)$$

The motor voltage constant which is proportional to field flux can then be written:-

$$K_v' = \frac{K_v}{I_{fo}} (I_{fo} + I_f) \text{ volts/radian/sec.}$$

and the motor torque coefficient:-

$$K_t' = \frac{K_t}{I_{fo}} (I_{fo} + I_f) \text{ ft.lbs/ampere}$$

Applying these last two equations to the relationships which describe the action of a shunt motor, the following equations are obtained:-

1) <u>the voltage equation</u>

$$E_s = K_v'(V_o + V) + R_a(1 + T_a p)(I_{ao} + I_a)$$

$$= \frac{K_v}{I_{fo}}(I_{fo} + I_f)(V_o + V) + R_a(1 + T_a p)(I_{ao} + I_a)$$

of which the transient portion is:-

$$0 = \frac{K_v}{I_{fo}} (I_{fo}V + I_f V_o + I_f V) + R_a(1 + T_a p) I_a$$

2) <u>the torque equation</u>

$$K_t'(I_{ao}+I_a) = Jp(V_o+V)$$

$$\frac{K_t}{I_{fo}}(I_{fo}+I_f)(I_{ao}+I_a) = Jp(V_o+V)$$

of which the transient position is:-

$$\frac{K_t}{I_{fo}}(I_{fo}I_a+I_fI_{ao}+I_fI_a) = JpV \qquad 3.9$$

In any practical speed controlling system, the transient speed variation V will not be more than a few per cent of normal speed V_o and the field current variation I_f will be small compared to the normal value I_{fo}. The terms I_fV, I_fI_{ao}, and I_fI_a, in equations 3.8 and 3.9 may therefore be neglected without affecting the accuracy of the result to any great degree. The equations then become:-

$$\frac{-K_vV_o}{I_{fo}} I_f = R_a(1+T_ap)I_a+K_vV \qquad 3.10$$

$$K_tI_a = JpV \qquad 3.11$$

which may be combined to obtain:-

$$V = \frac{-\left(\frac{K_vK_t}{R_a}\right)\left(\frac{V_o}{I_{fo}}\right)I_f}{\frac{K_vK_t}{R_a} + Jp\,(1+T_ap)} \qquad 3.12$$

$$= \frac{-BI_f}{1+T_rp\,(1+T_ap)} \qquad 3.13$$

where $B = V_o/I_{fo}$

$$T_r = \frac{JR_a}{K_vK_t}$$

In general, the value of B obtained here will be somewhat greater than actual due to neglecting saturation effects. However, this represents the worst possible value and any tendency for it to be smaller will result in increased stability. Armature reaction would also affect the value of B in a manner opposite to saturation but since the machine is operated at something less than full load, it is assumed to be negligible.

Applying equation 3.7 to that just obtained, the transient speed variation of the rotor in electrical radians per second or alternator output frequency variation becomes:-

$$V_a = \frac{-BK_1K_2\ E_t \left\{\frac{V_t-V_s}{p}\right\}}{R_{eq}\left\{1+T_rp(1+T_ap)\right\}(1+T_fp)} \quad 3.14$$

$$= \frac{-G(V_t-V_s)}{p(1+T_fp)\left\{1+T_rp(1+T_ap)\right\}} \quad 3.15$$

$$= \frac{-G(V_t-V_s)}{Q(p)} \quad 3.16$$

If the break in the control loop is now closed, V_t becomes V_a. The system response function is obtained by solving for V_a in terms of V_s -

$$V_a/V_s = \frac{G/Q(p)}{1+G/Q(p)} \quad 3.17$$

The loop transfer function may be determined by solving for V_a in terms of the error V_s-V_a, or by inspection from equation 3.17.

$$\frac{V_a}{V_s-V_a} = G/Q(p) \quad 3.18$$

$$= \frac{BK_1K_2E_t/R_{eq}}{p(1+T_fp)\{1+T_rp(1+T_ap)\}} \quad 3.19$$

In accordance with the method set out in chapter two, the derivative operator p may now be replaced by jw and the frequency dependent portion of the loop transfer function $1/_{Q(p)}$ plotted in the complex plane. Inserting numerical values for the motor generator set described in chapter one, the following equation is obtained and plotted:-

$$\frac{1}{Q(p)} = \frac{1}{p(1+0.5p)\{1+0.9p(1+0.007)\}} \quad 3.20$$

$$= \frac{1}{p+1.4p^2+0.457p^3+0.00315p^4} \quad 3.21$$

Inserting p = jw

$$\frac{1}{Q(jw)} = \frac{1}{jw-1.4w^2-j0.457w^3+0.00315w^4} \quad 3.22$$

A plot of equation 3.22, for frequencies varying from 0.2 radians/second to 1.4 radians/second is shown in figure 3.3. The locus of M=1.3 is now drawn tangent to the transfer function, using the relation:-

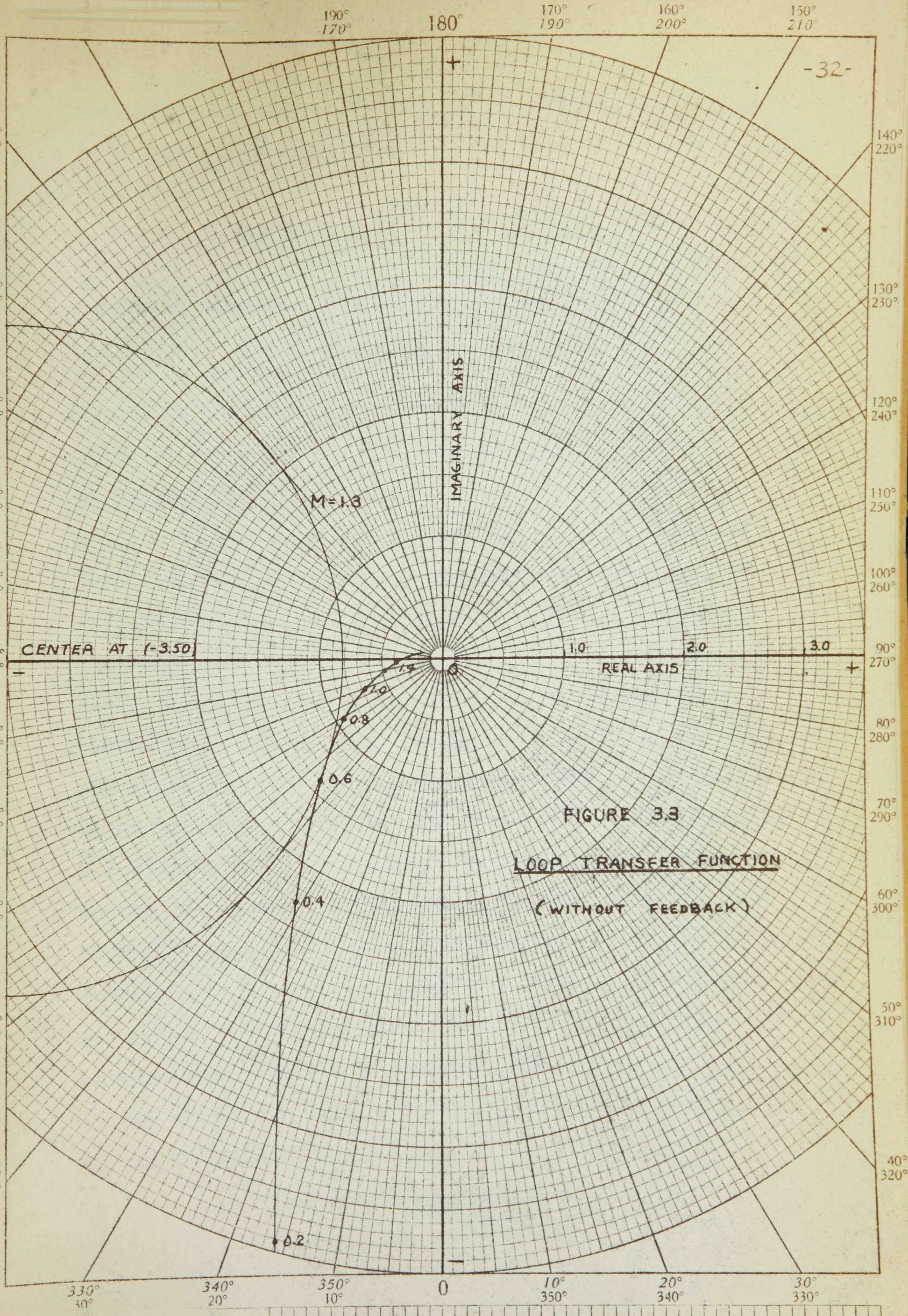
$$\frac{\text{center of circle}}{\text{real axis intercept}} = \frac{M}{M-1} = \frac{1.3}{0.3} = \quad 4.33$$

The center of this circle is apparently at -3.5, but according to equation 2.3, it should be at

$$-\frac{M^2}{M^2-1} = \frac{-1.69}{0.69} = -2.45$$

Hence the scale change or allowable gain is $\frac{2.45}{3.5} = 0.70$.

Having obtained the gain for optimum performance,



180°
IMAGINARY AXIS
REAL AXIS
CENTER AT (-3.50)
M=1.3
1.0
2.0
3.0
1.4
1.0
0.8
0.6
0.4
0.2
FIGURE 3.3
LOOP TRANSFER FUNCTION
(WITHOUT FEEDBACK)

the proper adjustment can now be determined. Referring to equation 3.19, the gain of the system is evidently equal to:-

$$G = \frac{BK_1K_2E_t}{R_{eq}}$$

Inserting numerical values, the equation becomes:-

$$G = \frac{(121)(1)(49.5)}{60} K_2 = 100K_2$$

To obtain the desired performance then, the value of K_2 should be adjusted to approximately 0.007.

EXPERIMENTAL RESULT.

The regulating system, with the adjustment just obtained, was set up in the laboratory and the transient performance found to be quite satisfactory. By making slight adjustments in R_3, it was found that stable operations could be obtained over the entire firing range of the thyratrons. However, the steady state load change over which regulation could be maintained was found to be only about 0.5 kw. This is due to the low value of K_2 which is a measure of the field current variation for a corresponding change in thyratron firing angle. If the change in field current required to maintain constant speed for a given load variation is known, the approximate value of K_2 required may be determined from a modification of equation 3.7. The modification consists of replacing the cosine approximation, by the actual function, and dropping the derivative term, giving the following equation:-

$$I_{f1}-I_{f2} = \frac{E_tK_2(\cos\theta_2-\cos\theta_1)}{R_{eq}}$$

from which

$$K_2 = \frac{R_{eq}(I_{f1}-I_{f2})}{E_t(\cos\theta_2-\cos\theta_1)} \qquad 3.23$$

The K_2 term in R_{eq} is small and may be neglected. From the motor generator set described here, the value of K_2 required for a 5 kw load change is 0.07. This is calculated assuming that the change in firing angle is from 135° to 45° so as to allow ample room for a slight overshoot during the transient.

Attempts to increase K_2 in the present system resulted first in hunting and finally in complete instability. If the control is to extend over a reasonable load range then some means of increasing the allowable value of K_2 will be required. Examination of the gain factor reveals that none of the other parameters are readily adjustable. Variation of E_t and R_{eq} will result in no net improvement whereas B is a machine constant and therefore fixed. The frequency dependent portion of the loop transfer function will therefore have to be modified in order to increase the allowable gain. Chapter four illustrates how this may be effected by means of additional feedback circuits.

CHAPTER FOUR

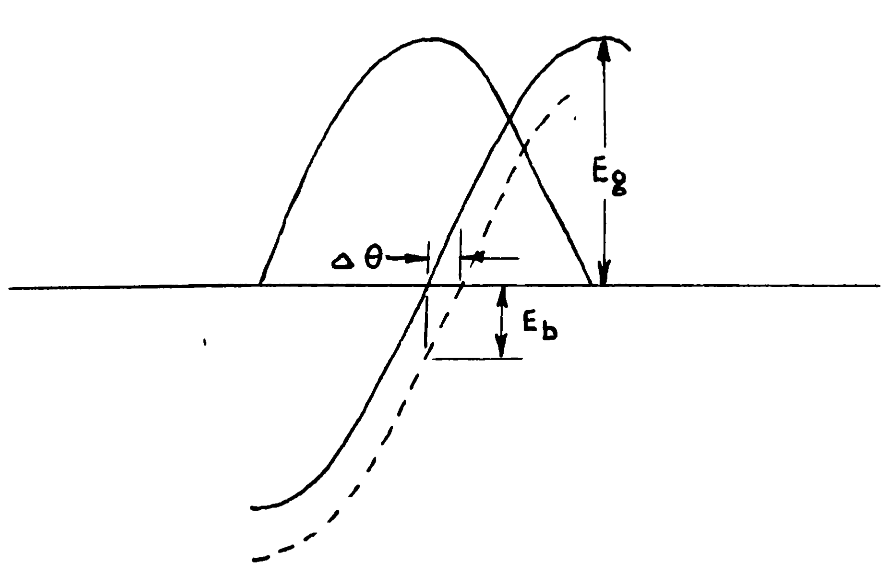
INCREASED STABILITY BY DERIVATIVE FEEDBACK

Although it is intended that this regulator should compensate for steady state load variations by similar variations in thyratron firing angle, it should be mentioned here that the problem could be solved in another way. A completely separate control, based on load variation, could be used in addition to the synchronous regulator. However, this type of control is not generally of the closed loop variety and hence it is rather sensitive to external disturbances. One such control, using a differential series field to maintain nearly constant speed under load, was added to the system adjusted as in chapter three, and was found capable of withstanding load variations up to 6 kw. Slight variations in line voltage and other parameters that were assumed constant, resulted in complete loss of synchronism however, and investigation of this type of load compensation was discontinued.

In the usual control system, several sources of feedback voltage exist, and it is the problem of the control engineer to decide which of these will be of the most use. In choosing a source of feedback voltage, the method suggested by Hanna, Oplinger, and Valentine in their paper on voltage regulators may be used as a guide. This method consists briefly of taking the derivative of the voltage obtained just following any time lag in the control system and feeding it back into the controller. It will be apparent that this is a powerful method of overcoming the effect of such time delays. In most cases, the feedback voltage will be too small for immediate use, but this difficulty may be overcome by use of suitable amplifiers.

In grid controlled rectifier circuits, the feedback voltage may be conveniently introduced in the form of a D.C. bias between the grid and the cathode. The effect of this voltage in controlling the output of the thyratrons is not generally expressible in linear mathematical form, especially where it is superimposed on phase shift control. However, by making certain assumptions in the interests of linearity as before, an equation can be formed and various feedback voltages investigated. Although accurate design data cannot be determined therefrom, an indication will be obtained as to whether or not the particular type of feedback will tend to improve the stability.

If the maximum variation of the bias voltage is kept to something less than 70% of the peak value of the applied grid voltage, the firing angle may be assumed to vary linearly with bias voltage without introducing a very large error.



Thus -

$$\theta = \frac{K_3 E_b}{E_g} \qquad 4.1$$

where the constant K_3 is of somewhat similar nature to K_1 and like K_1, has a maximum value of unity. Applying this to the voltage output equation of the thyratrons, 3.3, the following equation is obtained:-

$$E_\theta = 2E_t - I'_f R_k - E_t K_1 \left\{\frac{V_t - V_s}{p}\right\} \frac{-K_3 K_1 E_t}{E_g} E_b \qquad 4.2$$

By inserting various voltage functions for the value of E_b, the effect of different types of feedback may now be determined.

DERIVATIVE FEEDBACK PROPORTIONAL TO FIELD CURRENT.

In any circuit containing resistance and inductance, there will be a time delay between a variation in applied voltage and a consequent variation in current. Thus, a voltage proportional to the field current may be obtained directly from the field terminals as in figure 4.1 and may be expressed by:-

$$e = I'_f (R + L_f p) \qquad 4.3$$

where R is the internal resistance of the field only. If this voltage is applied to the terminals of a resistance-capacitance differentiating circuit, as shown, the voltage across the resistor alone will be:-

$$E_{bf} = i\ R_c$$

$$= \frac{e\ R_c}{R_c + \frac{1}{pC}}$$

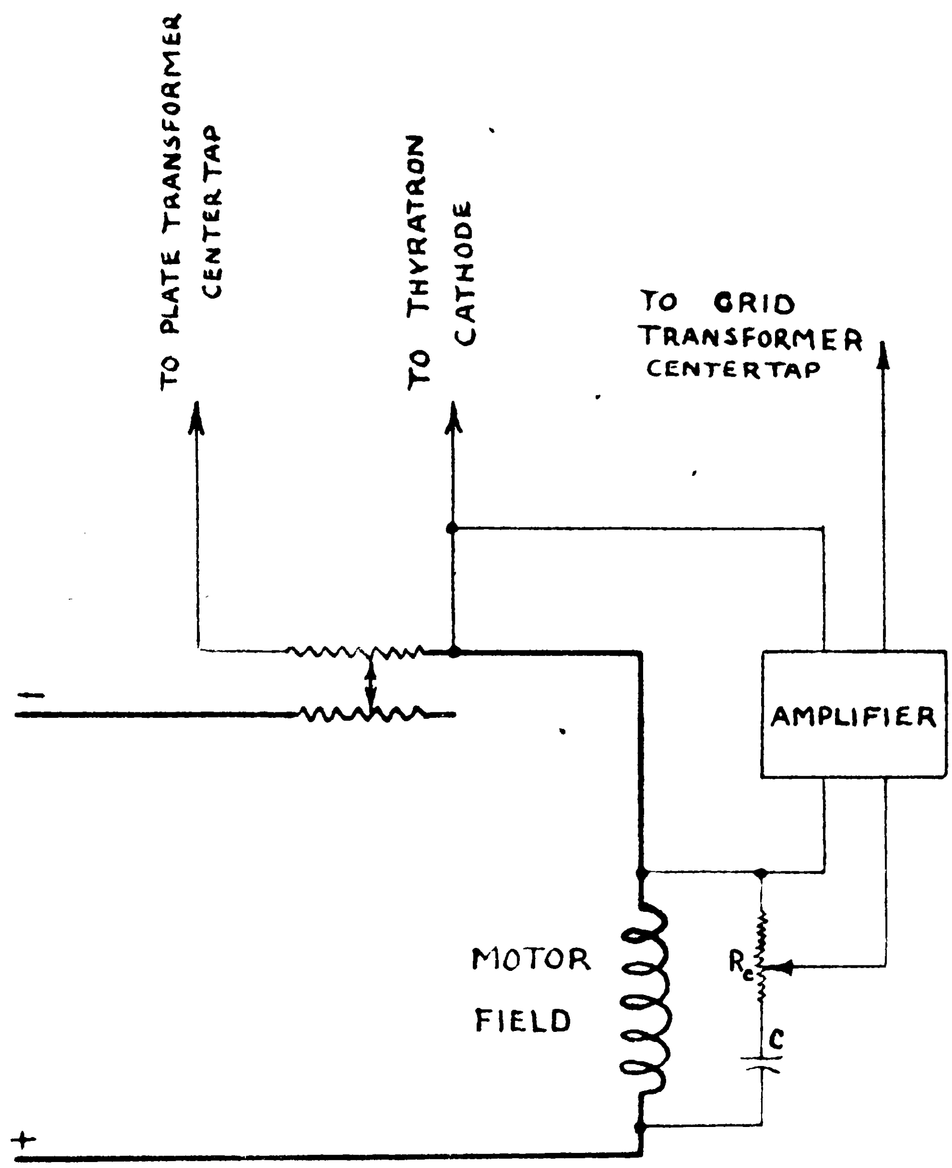


FIGURE 4.1

FIELD CURRENT FEEDBACK

$$E_{bf} = \frac{R_c Cp}{1 + R_c Cp} \times (R + L_f p) I_f' \qquad 4.4$$

If the time constant R_cC is made equal to that of the field L_f/R , then

$$E_{bf} = RR_c CpI_f \qquad 4.5$$

Inserting this voltage, amplified if necessary, in equation 4.2

$$E_\theta = 2E_t - I_f'R_k - E_tK_1 \frac{(V_t - V_s)}{p} - \frac{K_aK_1K_3E_tRR_cCI_fp}{E_g}$$

and solving as before:-

$$I_f = \frac{E_tK_1K_2\frac{(V_t - V_s)}{p}}{R_{eq}\ (1 + T_fp - A_fp)} \qquad 4.6$$

where

$$A_f = \frac{K_aK_1K_2K_3E_tRR_cC}{R_{eq}\ E_g}$$

Examination of equation 4.6 will reveal that with proper adjustment of amplifier gain K_a, the effect of the field time constant may be removed entirely. Fig. 4.2 shows the improvement in the loop transfer function resulting from this type of feedback. The allowable gain of the system has been increased from o.70 to 1.53 or over 100%. It will also be noted that the resonant frequency of the system is slightly higher, which will tend to increase the speed of response.

DERIVATIVE FEEDBACK PROPORTIONAL TO VELOCITY

The other major time lag in the control system described here occurs in the field current-speed relationship of the D.C.motor and is caused by the mechanical inertia of

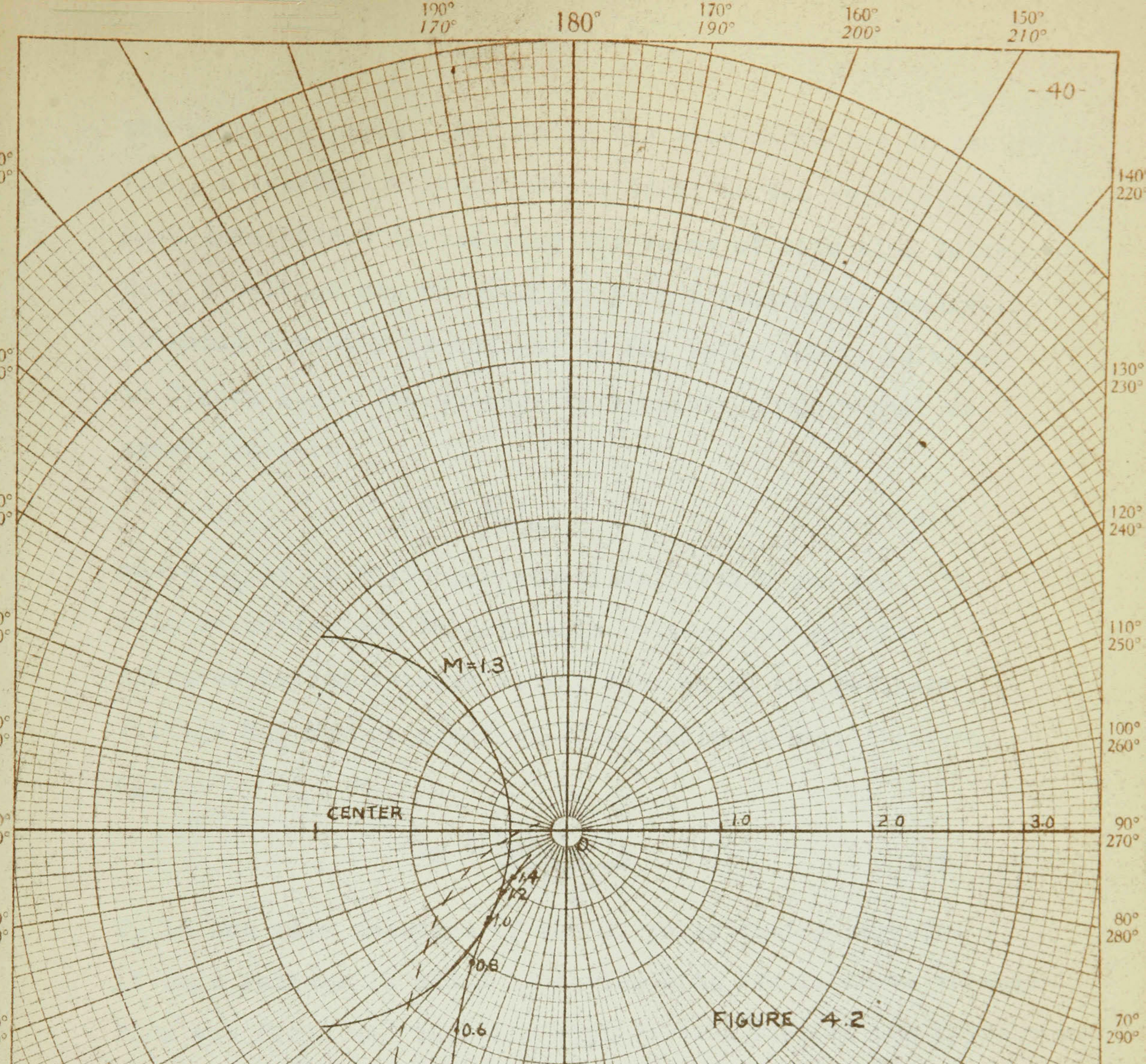


FIGURE 4.2

LOOP TRANSFER FUNCTION

the rotating parts. Following the same procedure as before then, a feedback voltage proportional to the velocity should have some effect in reducing the value of this time delay. Such a voltage may be obtained from a tachometer generator attached directly to the end of the rotor shaft, as shown in figure 4.3. The voltage is directly proportional to speed, hence:-

$$e = K_g V_a \qquad 4.7$$

where K_g is in volts per electrical radian per sec. It will be convenient to pass this voltage through the differentiating network shown, from which the feedback voltage E_{bg} may be obtained after suitable amplification K_a.

$$E_{bg} = \frac{K_a K_g T_g p V_a}{1+T_g p} \qquad 4.8$$

T_g is the time constant of the differentiating circuit. If this voltage is added in series with the feedback voltage from the field and they are applied as a bias to the thyratron grids, the following equation for the velocity may be obtained:-

$$V_a = \frac{\frac{-E_t K_1 K_2 B}{Req}\left(\frac{V_t - V_s}{p}\right)}{\left(1+T_f p - A_f p\right)\left\{1+T_r p(1-T_a p)\right\} - \frac{A_g p}{1+T_g p}} \qquad 4.9$$

where

$$A_g = \frac{K_1 K_2 K_3 K_a K_g \; B \; E_t T_g}{E_g \; R\,eq}$$

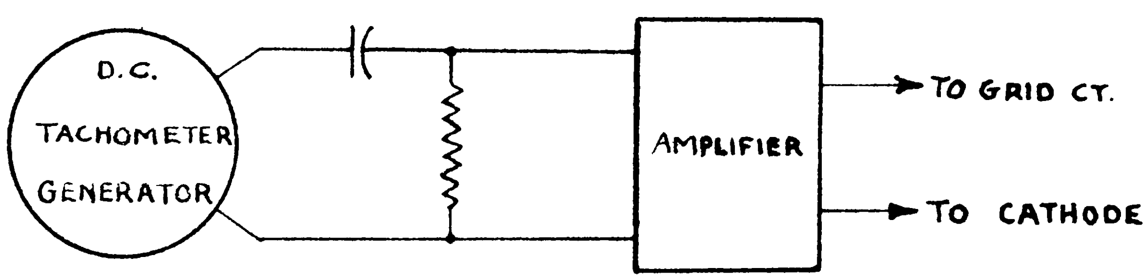


FIGURE 4.3

TACHOMETRIC FEEDBACK

Equating Af to Tf, as before, the denominator only of equation 4.9 becomes:

$$1 + TrP\,(1 - TaP) - \frac{AgP}{1 + TgP}$$

In this expression, the term Ta is quite small and by suitable circuit components, the term Tg can be made equally small also. If then the term Ag is made approximately equal to Tr, by suitable amplification, these two terms will tend to cancel each other. Thus the effect of the inertia of the rotor may be reduced, leaving only a few very small time constants wich have negligible effect on the stability of the system. The denominator of equation 4.9 will in fact approach the value of one, which is ideal since the locus of the loop transfer function will be coincident with the imaginary axis. (Note that in equation 4.9 there is still a 1/P term in the numerator.) The centre of the constant M circle is at the origin and hence the allowable gain is infinite.

CONCLUSION

The control system has been set up on a motor-generator set at McGill University and an attempt has been made to stabilize the regulator at higher values of gain by the methods just outlined. However, some difficulties arose which are not too apparent theoretically but which have to be considered before any practical result can be obtained. These difficulties are caused mainly by the operating characteristics of the thyratron tubes. The current output is of a pulsating nature and hence the voltage which appears across the field will also be pulsating. This is of little or no consequence as far as the field current is concerned due to the smoothing effect of the field inductance. However, the pulsating voltage is unsuitable for application as a D.C. bias control on the thyratrons and an averaging circuit had to be devised. For this purpose, a diode detector circuit, such as those used in radio, was found to be suitable. The time constant of the detector was about one-tenth of a second which is long compared to the pulse duration but fairly short compared to the overall oscillating frequency of the system.

A further difficulty arose when this voltage was applied between the cathode and grid transformer centre tap. Although the voltage appeared at this point, it was not evident on the grids themselves. It is thought that this may be caused by the protective resistances in the grid circuit. The addition of an entirely separate circuit for applying this voltage to the grids appears to solve this problem, but, due to time limitations, this has not been investigated. Thus, at the time of presentation, no experimental verification of the proposed stabilization was obtained and satisfactory operation of the regulator cannot be reported.

LIST OF SYMBOLS

(Representative values shown in brackets)

p - the Heaviside derivative operator

θ - firing angle of the thyratrons

V_o- normal angular velocity of the motor - elect. rad./sec. (377)

V - transient variation only of the angular velocity of the motor.

V'- $(V_o + V)$

V_t- angular velocity of voltage applied to thyratron plates - elect. rad./sec.

V_s- angular velocity of voltage applied to thyratron grids - elect. rad./sec.

V_a- angular velocity of alternator output voltage - elect. rad./sec.

E_s- D.C. motor supply voltage (200v)

E_θ- voltage output of thyratrons at firing angle θ.

E_t- voltage output of thyratrons at $\theta = 90^o$ (49.5v)

E_g- peak value of voltage applied to thyratron grids.

E_b- voltage applied as a bias to thyratron grids.

E_{bf}- feedback bias voltage proportional to field current.

E_{bg}- " " " " " velocity.

I_{fo}- normal field current of D.C. motot. (3.10a)

I_f- transient portion only the field current

I_f'- $(I_{fo} + I_f)$

I_{ao}- normal armature current.

I_a- transient portion only of armature current

R_f- total field circuit resistance

R_k- resistance of thyratron circuit.

K_2R_k- resistance across which bucking voltage is applied.

R_{eq}- $(R_t - K_2R_k)$ - (60 ohms)

R_a- total armature circuit resistance (0.26)

R - actual resistance of field alone

R_c- resistance in differentiating circuit

L_f- inductance of the field circuit

L_a- " " " armature circuit

J - moment of inertia of the rotating parts - ft. lbs./elect. rad./sec^2 (2.10)

C - capacitance of the differentiating circuit.

T_f- time constant of the field circuit - (0.5 secs)

T_a- " " " " armature circuit - (0.007 secs.)

T_r- " " " " rotating parts - (0.9 secs.)

T_g- time constant of the tachometer generator differentiating circuit.

ϕ_o - normal field flux

ϕ - transient variation of the field flux

K'_v- voltage constant proportional to flux and velocity

K_v- " " " " velocity only - volts/elect. rad./sec. (0.5)

K'_t- torque constant proportional to flux and armature current

K_t - " " " " armature current only - ft. lbs./ampere. (1.198)

B - V_o/I_{fo} - elect. rad./ampere (121)

K_1- cosine approximation constant

K_2- proportionality constant - see K_2R_k

K_3- proportionality constant between the D.C. bias voltage and firing angle of the thyratrons.

K_g- voltage constant of the tachometer generator - volts/elect. rad./sec.

K_a- amplifier gain

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