Strings, Effective Theories, and the Cosmological Constant

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In memory of my mother.

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Abstract

In this thesis we study the feasibility of realizing scale-separated compactifications of type IIB string theory to four-dimensional Anti-de Sitter or de Sitter space, the latter in particular being important for string theoretic approaches to cosmology, while maintaining a controlled effective theory description. We start by presenting the necessary elements of string theory and reviewing the existing proposals for constructing such compactifications as well as the challenges they face. We then present an approach to determining the regime of validity of an effective field theory description, using notions from asymptotic analysis. Combined with string theory's lack of dimensionless parameters, this approach allows us to establish whether the set of quantum corrections required to yield a particular type of solution leads to a breakdown of the effective field theory description. Applying this approach to scale-separated anti-de Sitter type IIB compactifications, we find no obstacle to having such solutions once suitable non-perturbative corrections are introduced, which are precisely the type used in existing proposals. We also find that for a similar ansatz describing de Sitter compactifications, the equations of motion can not be satisfied without leaving the effective theory regime. We then study various modifications to the initial ansatz and find that the problem persists unless one allows for additional low-energy effects combined with time-dependent internal field strengths.

Abrégé

Dans cette thèse on étudie la possibilité de réaliser des compactification de la théorie des cordes de type II à un espace Anti-de Sitter ou de Sitter avec séparation d'échelle, ce dernier étant important pour des approches de théorie des cordes à la cosmologie, en maintenant une description de théorie effective contrôlée. On commence par la présentation des éléments nécessaires de la théorie des cordes et une révision des constructions éxistantes de telles compactifications, ainsi que les défis auxquels elles font face. On présente ensuite une approche pour déterminer le régime de validité d'une description de théorie effective de champs, en utilisant des notions provenant de l'analyse asymptotique. En combinaison avec l'absence de paramètres sans dimensions dans la théorie des cordes, cette approche nous permet à établir si un ensemble de corrections quantiques nécessaires pour rendre une solution de type particulier mènent à un échec de la description de théorie effective de champs. En appliquant cette approche aux compactifications de type II avec séparation d'échelle à l'espace Anti-de Sitter, nous ne trouvons aucun obstacle pour ces solutions dès que des corrections non-perturbatives appropriées sont introduites, qui sont précisément du même type que celles qui sont utilisées dans les propositions existantes. Nous trouvons aussi que pour un ansatz similaire décrivant les compactifications à l'espace de Sitter, les équations de mouvement ne peuvent pas être satisfaites sans quitter le régime de validité de la théorie effective. Nous étudions ensuite des différentes modifications à l'ansatz initial et trouvons que le problème persiste, sauf si on permet des effets additionnels à basse énergie en combinaison avec des champs dépendants du temps.

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Preface

This monograph thesis contains the results of a research program that I took part in over the past two years as part of my doctoral studies, related to the problem of realizing de Sitter compactifications in string theory. The ideas, calculations and results presented here have appeared in a series of papers, [1, 2, 3, 4], co-authored with Keshav Dasgupta (my research advisor), Evan McDonough, Mir Mehedi Faruk and Radu Tatar, as well as a single-author paper [5]. Throughout these research projects, I have taken full participation in the discussions, performed and/or verified all the calculations, contributed the figures to [3], wrote parts of the manuscript for [1, 2], and edited the rest. The single-author paper [5] is entirely my own work.

This thesis aims at presenting the ideas, calculations and results appearing in these papers in a streamlined manner that I find most clearly explains our approach and its implications and reflects my most current understanding of the topic. For this reason, all the calculations have been presented in the notation and formalism resembling [5], which differs from that of the earlier papers in appearance, but not in substance. There are also minor differences in starting assumptions among the aforementioned papers, as these have evolved over the course of the research project. This leads to slight variations in some calculations and their interpretations, without significantly changing the general approach or the overall conclusions. To the extent that one must choose between these variations, the assumptions used in this thesis are those of [5], in order to maximally reflect my own views on the subject.

Other publications, produced over the course of my doctoral studies, that are not directly related to the topic of this thesis are:

1. M. Emelin and R. Tatar, "Axion Hilltops, Kahler Modulus Quintessence and the Swampland Criteria," Int. J. Mod. Phys. A **34**, no.28, 1950164 (2019) [arXiv:1811.07378 [hep-th]].

2. K. Dasgupta, J. Elituv, M. Emelin and A. K. Trinh, "Non-Kahler Deformed Conifold, Ultra-Violet Completion and Supersymmetric Constraints in the Baryonic Branch," Int. J. Mod. Phys. A **35**, no.25, 2050139 (2020) [arXiv:1805.03676 [hep-th]].

3. K. Dasgupta, M. Emelin, C. Gale and M. Richard, "Renormalization Group Flow, Stability, and Bulk Viscosity in a Large N Thermal QCD Model," Phys. Rev. D **95**, no. 8, 086018 (2017) [arXiv:1611.07998 [hep-th]].

4. K. Dasgupta, M. Emelin and E. McDonough, "Fermions on the antibrane: Higher order interactions and spontaneously broken supersymmetry," Phys. Rev. D **95**, no. 2, 026003 (2017) [arXiv:1601.03409 [hep-th]].

5. K. Dasgupta, M. Emelin and E. McDonough, "Non-Kahler resolved conifold, localized fluxes in M-theory and supersymmetry," JHEP **1502**, 179 (2015) [arXiv:1412.3123 [hep-th]].

Chapter 1

Introduction

Cosmological observations indicate that the late-time behavior of our universe will one of accelerated expansion [6, 7, 8]. Furthermore, the dominant paradigm for explaining observed features of the early universe, known as inflation [9, 10, 11, 12, 13], also involves a phase of even more drastic accelerated expansion. These facts motivate the search for solutions that exhibit such expansion within string theory, which is currently the most promising approach to quantum gravity as well as a unifying framework for all the known the types of interactions.

As string theory is naturally defined in 10 dimensions, we must compactify 6 of them to obtain a four-dimensional spacetime description. Furthermore, for spacetimes with non-zero curvature, we would usually like to require that the length scale of the compactification is small compared to the natural length scale of the four-dimensional spacetime, a feature known as scale-separation.

The maximally symmetric spacetime that exhibits accelerated expansion is de Sitter space, so one may be naturally inclined to look for compactifications of string theory to this spacetime. Unfortunately, explicit top-down constructions in string theory present many technical challenges at the present time. The existing proposals for constructing de Sitter space, or even scale-separated Anti-de Sitter space (the maximally symmetric negative curvature spacetime), rely on a subtle patchwork of ten-dimensional and four-dimensional phenomena arising from an interplay of classical and quantum effects [14, 15, 16, 17]. In particular, the inclusion of quantum effects is crucial in avoiding no-go results that arise at the classical level [18, 19]. How and whether the necessary ingredients combine in the correct way to produce the desired result remains a matter of some debate [20].

In parallel to this work on the application of string theory to cosmology, there is an active effort to establish the restrictions that string theory places on the possible low-energy effective theories that arise from it, known as the swampland program [21, 22, 23]. The study of string compactifications has revealed a vast "landscape" of solutions, leading one to believe that any desired low-energy behavior may be obtained, provided various effective theory consistency conditions are satisfied. The swampland program posits that this is not the case and that the "landscape" of legitimate string theory compactifications is surrounded by a "swampland" of seemingly consistent effective theories which do not have an embedding into string theory.

The technical challenges in constructing a full top-down de Sitter compactification has led to the idea that de Sitter compactifications may be part of this swampland, which would suggest that cosmological models in string theory may need to invoke alternative early universe scenarios (e.g. string gas cosmology [24, 25, 26]) and quintessence models [27] for the late-time behavior of the universe. This view has been formalized in a series of "de Sitter swampland conjectures" [28, 29, 30, 31, 32] which propose constraints on valid string theory models, which are in direct conflict with prolonged phases of accelerated expansion.

The swampland conjectures themselves, however, are generally based on what is known about effective theories in regimes of string theory where explicit top-down calculations are possible. They might therefore be missing out on some important and intricate effects, such as the quantum effects used in the proposed de sitter constructions, thus coming back full circle.

A systematic investigation of the possible quantum corrections that arise in string theory is therefore necessary in order to make progress on this question. The study of quantum corrections in string theory is complicated by the fact that explicit high order calculations in string theory are generally at least as difficult as those in field theory. Among the known quantum corrections, only some are the result of direct calculation of scattering processes, with most others then being inferred via constraints such as supersymmetry or anomaly cancellation. Thus even at the next-to-leading order, we do not have complete knowledge of the signs and coefficients of the corrections.

This incomplete knowledge of the quantum corrections even at a given order in perturbation theory means that for any proposed model or no-go result relying on an effective description based on a finite set of known ingredients, it's hard to rule out that some unknown ingredient may in fact be important and spoils the conclusion.

Thus, in this thesis, rather than working with a set of known corrections and look for solutions or principles prohibiting them, we will instead build on the approach of [19], where we start with an ansatz for a solution that fails to satisfy the classical equations of motion, and ask what properties the quantum corrections would have to have in order to make it into a solution. We may not know the exact corrections, but what we can know, is their general form, i.e. the degrees of freedom that these corrections are built out of, when working in an effective theory framework.

Naturally, once we find these properties it isn't guaranteed that the corrections appearing in string theory will have them, so finding a consistent set of criteria should not be viewed as proof that solutions of a given type exist. What we may find, however is that the set of corrections required to make the ansatz into a solution is inconsistent with the existence of the effective theory description that we started with. Thus while we may not completely rule out certain solutions, we can rule out their existence within certain regimes. Note that this approach is naturally better suited for finding no-go results, rather than proving existence of solutions. The advantage of the approach is that it does not require the full knowledge of the exact coefficients and index structures that appear in the corrections, only the fields that they are built out of.

The thesis is organized as follows: Chapter 2 is dedicated to a review of the relevant aspects of string theory, from its perturbative definition to the proposed construction of scale-separated anti-de Sitter and de Sitter solutions, as well as the technical challenges they face and recent conjectures that appear to exclude their existence.

The first half of the chapter 3 reviews important notions from perturbation theory and asymptotic analysis. The second half of that chapter connects these mathematical tools to the notion of effective theories and establishes an organizing principle for the possible corrections to an effective theory description, as well as a criterion for when a solution fails to exist within the regime of validity of that effective theory, which serves as the basis for the rest of our approach.

The next two chapters apply the notions described in chapter 3 to scale-separated compactifications with non-zero spacetime curvature. Chapter 4 examines a simple ansatz describing scale-separated compactifications with non-zero external spacetime curvature and finds that certain non-local effects, most naturally interpreted as non-perturbative corrections, allow for this ansatz to be made into a stable solution to the equations of motion, but only when the curvature is negative. Attempting to introduce additional corrections to change the sign of the curvature leads to a violation of the criteria laid out in the second chapter, indicating a breakdown of an effective theory description.

In chapter 5 we investigave the consistency of various modifications to the initial ansatz in an attempt to avoid this effective theory breakdown. We do so by introducing additional flux components as well as allowing for additional dependence of the internal fields on the coordinate that corresponds to time in the de Sitter ansatz. We find that these new ingredients can satisfy certain consistency conditions, which means they can consistently be added to the ansatz. However, they alone do not solve the problem of the effective theory breakdown. Finally we consider the option of interpreting the non-local effects not as non-perturbative effects but as effects that dominate the low-energy limit of the theory. When combined with suitable corrections, which involve the new time-dependent ingredients, we find that we get better control over the sign of the external curvature opening the way for de Sitter solutions.

We conclude the thesis in chapter 6 with a summary and discussion of our results and their relation to other work surrounding the issues of de Sitter compactification and scale-separation.

Chapter 2

Preliminaries

String theory is a framework for unifying the various types of interactions known in physics, including gravity. Originally proposed as an approach to understanding the strong nuclear force (before the development of QCD), it was eventually re-purposed as theory of quantum gravity, due to the inevitable presence of spin-2 states in its spectrum. The most rigorous definitions of string theory are only available perturbatively, however a patchwork of rigorous calculations, arguments, perspective shifts and consistency checks strongly suggest that string theory is a much richer theory than what the perturbative definition alone might suggest.

This chapter is dedicated to a brief overview of string theory, starting from the perturbative definition of string theory, and working our way to constructions of spacetimes with non-zero cosmological constant and parametric scale-separation, which is the main topic of this thesis. We will also discuss some of the technical challenges they face as well as the more principled objections that have been advanced against such constructions in recent years, within the scope of the swampland program.

Most of the contents of this chapter, with the exception of the end of section 2.4 and section 2.5, is standard material, which can be found in textbooks such as [33, 34, 35, 36, 37].

2.1 Perturbative String Theory: Worldsheet and Spacetime Physics

2.1.1 Bosonic String

The conceptual starting point for the perturbative definition is the action for a string propagating in an ambient spacetime, proportional to the worldsheet area of the string:

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{\det g_{MN}(X)\partial_a X^M \partial_b X^N}, \tag{2.1.1}$$

known as the “Nambu-Goto action”. The coordinates $\sigma^a, \quad a = 1, 2$ parametrize the string worldsheet and the determinant is taken with respect to these indices. We are therefore considering a two-dimensional field theory, where the “fields” X^M represent coordinates of the string in the ambient “target space” with metric g_{MN}, which depends on X. The prefactor $1/2\pi\alpha'$ is the tension of the string and defines a length scale called the string length, given by $l_s = \sqrt{\alpha'}$. The strings can *a priori* be closed, with periodic boundary conditions, or open, with Dirichlet or Neumann boundary conditions specified separately at each end point. For the moment we shall focus on the closed string.

The functional form of the action (2.1.1) presents technical problems for quantization, so instead one considers a different, but classically equivalent action:

$$S_P = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{h}h^{ab}g_{MN}(X)\partial_a X^M \partial_b X^N. \tag{2.1.2}$$

This action introduces a non-dynamical worldsheet metric h_{ab}, which one ought to be free to choose arbitrarily without changing the physics. Due to this freedom, combined with the parametrization invariance of the worldsheet, the action has diffeomorphism and Weyl symmetry in two dimensions, which is equivalent to conformal symmetry. Consistency of the theory requires that this symmetry is preserved upon quantization, which leads to non-trivial constraints. We are thus dealing with a two-dimensional quantum conformal field theory (CFT) consisting of D scalar fields, where D is the dimension of the target space.

The action can further be extended to include other parametrization invariant terms, producing the "Polyakov action"

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{h}\left(h^{ab}g_{MN}(X)\partial_a X^M\partial_b X^N + \epsilon^{ab}B_{MN}(X)\partial_a X^M\partial_b X^N + \alpha' R^{(2)}(X)\phi(X)\right), \tag{2.1.3}$$

where B_{MN} is an anti-symmetric tensor and ϕ is a scalar called the "dilaton", both functions of the target space coordinates X, and $R^{(2)}$ is the worldsheet Ricci scalar. This is the action that serves as the definition of perturbative (bosonic) string theory. The final term is not Weyl-invariant but it is also higher order in α', and therefore only contributes to the consistency conditions at higher order.

The consistency conditions for conformal invariance at the quantum level impose certain conditions, organized order by order in α', on the target space of the theory (see [38] for a review). The leading order condition fixes the central charge of the conformal field theory, which in turn requires the target-space dimensions to be $D = 26$.

On the one hand, a prediction of the dimensionality of space is a very interesting feature, absent in QFT. On the other hand, the required dimension does not match the observed dimensionality of the universe. This, however is not an immediately disqualifying outcome, as there are well-established ways to effectively hide the excess dimensions from view. This is the issue of compactification, which we will return to later in the chapter.

The next order condition for preserving conformal invariance on the worldsheet is more exciting. This condition is the vanishing of the one-loop β-function coefficients

$$\beta_{MN}(G) = \alpha'\left(R_{MN} + 2\nabla_M\nabla_N\phi - \frac{1}{4}H_{MPQ}H_N^{PQ}\right) \tag{2.1.4}$$

$$\beta_{MN}(B) = \alpha'\left(-\frac{1}{2}\nabla^P H_{PMN} + \nabla^P\phi H_{PMN}\right) \tag{2.1.5}$$

$$\beta(\phi) = \alpha'\left(-\frac{1}{2}\nabla^2\phi + \nabla_P\phi\nabla^P\phi - \frac{1}{24}H_{MNP}H^{MNP}\right), \tag{2.1.6}$$

which is equivalent to the equation of motion for a scalar field ϕ, the natural generalization for Maxwell's equations for the anti-symmetric tensor field strength $H_3 = dB_2$ and Einstein's equations for the target-space metric in the presence of these fields.

Moreover, computing the β-functions to higher order in α', one can systematically obtain higher derivative corrections to these equations. In other words, the requirement of worldsheet conformal invariance at the quantum level, not only correctly reproduces general relativity, but also provides a way to compute the higher derivative corrections to it!

Having quantized the worldsheet theory, we can study the spectrum of the theory and the properties of the various states of the string as viewed from the spacetime perspective. Specifically one can compute the mass and spin of these states. The calculation is the same as standard QFT. One expands the fields in modes and defines creation and annihilation operators for each mode, with the ground state being annihilated by all the annihilation operators. The rest of the states are built by acting on the ground state with various combinations of creation operators.

The ground state of the closed string is tachyonic! This is a problem, since it indicates that although the backgrounds satisfying (2.1.4) ensure the consistency of the worldsheet theory, they are nonetheless unstable toward creating condensates of closed strings in their ground states. The end-point of this instability is unknown and the tachyonic nature of the ground state is generally viewed as a fatal problem for the bosonic string.

If we press on, however, and look at the next excited states, obtained by acting with the lowest mode creation operators for the left-moving and right-moving modes, we find that they are massless[1] and carry two target-space indices (one from each creation operator).

$$|\psi\rangle = \tilde{\alpha}^\dagger_M \alpha^\dagger_N |0\rangle, \tag{2.1.7}$$

where $\alpha^\dagger$ is the creation operator for the left-moving sector, while $\tilde{\alpha}^\dagger$ is the creation operator for the right-moving sector. The fact that we must match excitation levels between these sectors is known as "level-matching" and is a consequence of worldsheet parametrization invariance.

This state decomposes into a trace, a traceless symmetric and an antisymmetric part, in perfect correspondence with what one expects from excitations of the dilaton, the metric and the antisymmetric tensor respectively.

Furthermore, one can compute the scattering amplitudes of these excitations. This is done using standard conformal field theory techniques, where conformal invariance allows

[1]This only holds when the dimension of the target space is the same as what is required by conformal invariance, as described previously.

one to replace asymptotic states on the worldsheet, by appropriate local operator insertions at isolated points and computing their correlation function. The results reproduce the scattering amplitudes expected from quantizing linearized gravity, providing a non-trivial consistency check.

It is worth stressing however, that the scattering amplitude computations can only be truly rigorously performed and compared for small excitations about flat space, while the equations coming from the β-functions allow for curved target-space backgrounds.

Note that string theory does not provide us with a spacetime action. Instead it provides us with a set of equations of motion, with corrections organized in a perturbative expansion, and a set of scattering amplitudes. A spacetime action can only be inferred, by essentially guessing and checking that it reproduces the results. The action that produces matching equations of motion and tree-level scattering amplitudes to those obtained from worldsheet computations, is referred to as the low-energy effective action for that string theory.

2.1.2 Fermions and the Superstring

The next step in the development of string theory is to include fermionic worldsheet degrees of freedom. This introduces fermionic creation and annihilation operators which satisfy anti-commutation relations and carry worldsheet spinor indices, but Lorentz spacetime indices. While it seems like an *ad hoc* addition, it actually provides several interesting features. The first is that the worldsheet theory becomes supersymmetric. Another is that the condition on the dimensionality of the ambient spacetime changes, from $D = 26$, to $D = 10$.

For closed strings, fermions can obey periodic or anti-periodic boundary conditions, referred to as "Ramond" or "Neveu-Schwarz" conditions, respectively. These can be chosen separately for the left- and right-moving modes, leading to a total of 4 different sectors, typically denoted by R-R, NS-NS, NS-R and R-NS. The states in the R-R and NS-NS sectors behave as spacetime bosons. In particular, the NS-NS sector houses the familiar massless excitations of the metric, dilaton and antisymmetric tensor fields as the bosonic string. The massless part of the R-R sector consists of excitations of antisymmetric p-form fields. The NS-R and R-NS sectors behave as spacetime fermions, since the different boundary conditions imply that a global rotation flips the total sign of the state. In particular they contain a massless spin-3/2 state, which implies that the theory must have spacetime supersymmetry! This feature plays a crucial role in restricting the form of the low-energy effective action.

The NS-NS ground state is still tachyonic, and moreover, does not have a supersymmetric partner, i.e. a fermionic state of the same mass. This seems to exacerbate a problem that was bad enough in the bosonic string. Fortunately, there is a small miracle that occurs: by studying the operator algebra, we find that the spectrum actually splits into non-interacting subsets, classified by the fermion number in each sector! This allows us to project away some of the states without affecting the consistency of the theory. We are thus free (in fact, obliged) to remove completely the subset that contains the tachyon, which, fortunately enough, resides in a different subset from the NS-NS massless states. This procedure is known as the "GSO projection" [39]. Its precise details are again not entirely relevant for our purposes, except that there are several consistent ways to carry out this projection, resulting in several different consistent theories.

The construction outlined above leads to to the "type IIA" and "type IIB" string theories, which will be the main setting in future chapters. These theories both have $\mathcal{N} = 2$ spacetime supersymmetry and their massless spectra contain the same metric, dilaton and anti-symmetric tensor that is present in the bosonic string. The massless fermion spectrum contains the spin-3/2 gavitino and consists of both left-handed and right-handed fermions for the type IIA case and only left-handed fermions for the type IIB case. In fact, the entire massless spectrum of both type II theories can be assembled into one extended supergravity multiplet whose two-derivative action is fixed by supersymmetry and gives "type IIA/B supergravity". The β-function calculations proceed in very much the same way as for the bosonic string and reproduce the equations of motion for the NS-NS fields, as well as provide higher-derivative corrections. The R-R fields, however do not appear in the worldsheet action, and so their equations of motion can only be deduced from the spacetime supersymmetry constraints in this approach.[2]

2.1.3 Type IIA/B Effective Action

Here we write down some basic facts about the type IIA/B field content and two-derivative effective action for future reference and to establish notation. The NS-NS fields are denoted

[2] There is a second approach to quantizing strings, known as the Green-Schwarz superstring [33, 34], which has spacetime supersymmetry built in from the beginning. In this approach, the R-R fields can be coupled to the string through a fermion bilinear term and the equations of motion can be found from physical state constraints in the worldsheet theory. In either approach, spacetime supersymmetry is an essential ingredient to determining the action for the R-R fields

$$\phi, \quad g, \quad B_2, \tag{2.1.8}$$

for the dilaton, the metric and the anti-symmetric 2-form, respectively. The NS-NS 3-form field strength is

$$H_3 = dB_2. \tag{2.1.9}$$

The R-R p-form potentials are denoted C_p and their field strengths are

$$F_{p+1} = dC_p. \tag{2.1.10}$$

Due to the different GSO projections, the type IIA/B theory only contains odd/even rank potentials and therefore even/odd rank field strengths, respectively.

The antisymmetric fields transform under a gauge symmetry $C_p \rightarrow C_p + d\Lambda_{p-1}$ and $B_2 \rightarrow B_2 + dA_1$ where the A_1 gauge parameters is not independent of the Λ_p. The gauge-invariant field combinations are

$$\tilde{F}_4 = F_4 + C_1 \wedge H_3 \tag{2.1.11}$$

in type IIA and

$$\begin{aligned} \tilde{F}_5 &= F_5 + \frac{1}{2} B_2 \wedge F_3 + \frac{1}{2} H_3 \wedge C_2 \\ \tilde{F}_3 &= F_3 + C_0 H_3 \end{aligned} \tag{2.1.12}$$

in type IIB.

Both theories have the same kinetic action for the NS-NS sector

$$S_{NS} = \frac{1}{2\kappa^2} \int d^{10}x e^{-2\phi} \left(R + 4\partial_M \phi \partial^M \phi - \frac{1}{12} H_{MNP} H^{MNP} \right) , \tag{2.1.13}$$

where $\kappa^2 = \frac{1}{4\pi}(2\pi l_s)^8$.

This is supplemented by the action for the R-R fields, which consists of the kinetic terms for each p-form field

$$S_R = -\frac{1}{4\kappa^2} \sum_p \int \tilde{F}_p \wedge \star \tilde{F}_p \ , \tag{2.1.14}$$

where the $\tilde{F}$ are the gauge invariant combinations described above[3]. There is also a topological interaction referred to as the "Chern-Simons" action, which takes the form

$$S_{CS} = -\frac{1}{4\kappa^2} \int B_2 \wedge F_4 \wedge F_4 \tag{2.1.15}$$

for type IIA and

$$S_{CS} = -\frac{1}{4\kappa^2} \int C_4 \wedge H_3 \wedge F_3 \tag{2.1.16}$$

for type IIB.

As a final remark for this discussion, we note that a similar story to what we described in the previous two subsections holds for strings, which only have fermionic degrees of freedom in the left-moving part of the spectrum. This gives rise to two more consistent string theories, known as $E8 \times E8$-heterotic and $SO(32)$-heterotic theories, commonly denoted by HE and HO, respectively, whose low energy limits are $\mathcal{N} = 1$ gauged supergravity theories with the corresponding gauge group. We will not be making use of these theories in our work.

2.1.4 Open strings and D-branes

So far we have discussed closed strings, which produce 10-dimensional supergravity theories as their low-energy limits. A similar discussion holds for open strings, however since

[3] F_2 and F_1 are already gauge invariant, so they simply have their canonical kinetic term

there is no longer a separate left- and right-moving sector, there is only one set of creation and annihilation operators for the bosonic degrees of freedom. We do, however, now have to fix the boundary conditions at the end points of the open string. We can choose between Dirichlet and Neumann boundary conditions for the bosonic fields, while for the worldsheet fermions, we must choose some relative signs between the boundary terms for each helicity at each endpoint. These produce what are again called "Ramond" or "Neveu-Schwarz" boundary conditions, depending on whether the relative sign choices are the same or different between the endpoints.

The rest of the story proceeds in similar fashion to the closed string. For purely Neumann boundary conditions, the ground state once again comes out tachyonic, while the first excited states are massless and in particular contain the state

$$|\psi\rangle = \alpha_M^\dagger|0\rangle, \tag{2.1.17}$$

which behaves as a spacetime gauge boson. We can once again project out the ground state via GSO projection and are left with a theory whose lowest states behave as massless gauge bosons. Thus, instead of producing a gravity theory, open strings with Neumann boundary conditions produce a gauge theory in the low energy limit.

If we instead choose Dirichlet boundary conditions for some of the bosonic fields, this corresponds to fixing the ends of the string on some hypersurfaces. Such hypersurfaces are called Dp-branes, where p denotes the number of spatial dimensions of the brane. The oscillation modes of the string that are longitudinal to the D-brane still produce a gauge theory just as before, only in lower dimensions, while the transverse oscillations can be interpreted as excitations of the D-brane itself. Indeed, the calculation involving a vertex operator in the worldsheet CFT that describes a coherent state of these transverse string modes is exactly the same as a calculation in which the D-brane profile itself is deformed. Thus from the spacetime perspective, the D-branes themselves can be thought of as dynamical objects.

In the same way that the closed string β-functions yield the spacetime equations of motion for the metric, dilaton and NS-NS 2-form, a similar similar open string calculation yields the equations of motion for the D-brane [40, 41]. The resulting equations of motion are those obtained from the Dirac-Born-Infeld action:

$$S_{DBI} = -T_{Dp} \int d^{p+1}\sigma e^{-\phi} \sqrt{\det(g_{ab} + B_{ab} + 2\pi\alpha' F_{ab})}, \tag{2.1.18}$$

where g_{ab}, B_{ab} are the pullbacks of the spacetime metric and antisymmetric tensor to the brane worldvolume and F_{ab} is the worldvolume gauge field strength $F = dA$.

A further very important property of D-branes is that they couple to the p-form fields coming from the closed string R-R sector [42]. Type IIA/B theories contain odd/even p-form potentials and thus even/odd p-form fluxes, respectively, as part of their massless spectrum. The superstring worldsheet action does not contain terms that couple to these potentials, which means that although there are string states corresponding to excitations of these fields, the strings do not act as sources for them. This is in contrast to the NS-NS antisymmetric 2-form, which appears explicitly in the worldsheet action (2.1.3). D-branes however contain an additional term in their effective action

$$S_{CS} = \mu_p \int C_{p+1} + (B + 2\pi\alpha' F) \wedge C_{p-1} + (B + 2\pi\alpha' F) \wedge (B + 2\pi\alpha' F) \wedge C_{p-3} + \ldots\ , \tag{2.1.19}$$

which allows them to couple directly to the p-form potentials. One way to realize that this coupling occurs is by considering worldsheets with boundaries. The boundary state can be represented by an appropriate vertex operator and one can compute a non-vanishing amplitude to produce a Ramond-Ramond state. Another is to notice that D-branes are BPS states that preserve half of the supersymmetries of the type II theory. Such states always carry a conserved charge, and their RR charges are precisely that. The absence of such a coupling would be inconsistent with supersymmetry.

Aside from D-branes, there are also objects called orientifold planes, or O-planes. These objects arise when the target spacetime is an orbifold, i.e. a smooth spacetime "modded out" by a discreet group. The fixed points of this action are called orbifold planes, and if one combines the orbifold action with worldsheet parity, they are called orientifold planes. From the worldsheet perspective they are described by "cross-cap" states, i.e. boundaries where points are identified with their antipode. These objects also turn out to source R-R fields, since the amplitudes involving R-R states and the cross-cap states are also non-vanishing, but have no further dynamics of their own[4], since there are no open strings living on them, whose collective excitations could describe the motion of the O-plane. These O-planes play an important role in compactifications, as they can help ensure global charge cancellation.

As a final comment, let us mention the fifth consistent string theory, knows as the "Type I" superstring. It can be obtained from Type IIB strings by including D9-branes filling the entire spacetime and orientifolding, i.e. projecting out all worldsheet parity-violating states of the strings. This, in particular, removes the NS-NS 2-form from the spectrum as well as the R-R 0-form and 4-form potentials, but keeps the dilaton, metric, the R-R 2-form, and introduces the gauge field coming from the D9-branes. This theory contains both closed and open strings.

2.2 String Dualities and M-theory

The procedure outlined in the previous sections results in the construction of several consistent, but different-looking theories, each with its own low-energy limit, in the form of a consistent 10D supergravity theory supplemented by higher derivative corrections. However, a series of developments known as the "second superstring revolution" has revealed that these theories are in fact all connected by a web of relations known as dualities.

Duality is a general phenomenon in theoretical physics, in which two possibly very different looking theories, turn out to in fact be alternate descriptions of the same physical system. That is, despite appearing to be defined in terms of different degrees of freedom with different actions, their physical observables are in one-to-one correspondence with each other. The two sides of the duality typically involve taking the underlying system to different limiting regimes, (e.g. strong vs weak coupling) and the different appearance of the theories can be attributed to different effects being dominant in these regimes.

The two kinds of dualities relating different string theories are referred to as T-duality and S-duality, which we now briefly describe.

2.2.1 S-duality

S-duality is arguably the more similar one to the dualities one finds in QFT. All five string theories contain a scalar degree of freedom, the dilaton ϕ, which appears in the third

[4] At least none that can be seen at the level of perturbative string theory

term of the action (2.1.3).

$$S \supset -\frac{1}{2\pi}\int d^2\sigma\sqrt{h}\ \phi R\ . \tag{2.2.20}$$

For constant ϕ, this term is proportional to the Euler characteristic of the worldsheet and therefore depends purely on the topology of the worldsheet and factors out of the path integral over worldsheets with fixed genus. The computation of scattering amplitudes in string theory involves a sum over all worldsheet topologies, so the contributions coming from higher genus worldsheets appear with higher powers of e^{ϕ}. In this sense, the expectation value of e^{ϕ} behaves like a perturbative expansion parameter and is called the string coupling, denoted by g_s.

In type IIB supergravity we can combine the 0-form potential and the dilaton combine to form a complex scalar $\tau = C_0 + ie^{-\phi}$. The action of IIB supergravity is then invariant under,

$$\begin{aligned} \tau &\to \frac{a\tau+b}{c\tau+d} \\ \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} &\to \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \end{aligned} \tag{2.2.21}$$

For any real a, b, c, d, with $ad - bc = 1$. In full string theory, the charges and fluxes must be quantized, so the symmetry gets reduced to $SL(2,\mathbb{Z})$. S-duality is the particular case when

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{2.2.22}$$

which relates type IIB at weak string coupling to type IIB at strong string coupling. Since the perturbative string calculations are only valid at weak coupling, this duality is remarkable, in that it allows us to get a handle on the strong coupling regime as well. Of course this requires S-duality to continue to be a symmetry once corrections to the two-derivative supergravity action are introduced, including non-perturbative corrections.

One compelling piece of evidence that the duality continues to hold at the non-perturbative level comes from looking at the tensions of various extended objects in the theory. Besides strings, the theory contains D1-branes, which couple naturally to C_2, while the string couples to B_2. Since the duality interchanges these two-forms, we expect it to interchange their sources as well. Both the fundamental string and the D1-brane are BPS objects, so the expressions for their tension are protected from corrections by supersymmetry.

The string tension is proportional to $1/\alpha'$, while the D1-brane tension is given by $1/(g_s\alpha')$. The Planck length is related to the string tension by

$$l_p^2 = g_s^{1/2}\alpha', \tag{2.2.23}$$

so when measured in Planck units, we see that the D1 and the string tension get interchanged. This means that as we move to strong coupling, the string tension exceeds the Planck scale, but the D1-branes become lighter and take on the role of fundamental strings! In a similar vein, the D3 tensions remains invariant, in line with the invariance of F_5, while the D5 brane changes places with the NS5-brane, a different kind of extended object in string theory that is magnetically charged under B_2. Other $SL(2,\mathbb{Z})$ transformations produce new objects called (p,q)-strings or -branes, which carry combinations of NS-NS and R-R charges.

The S-duality transformation (2.2.22) also maps the type I supergravity action to the heterotic $SO(32)$ supergravity action. Note that the B_2 and C_2 fields are only present in one of the two theories and map to each other, with similar non-perturbative checks involving the tensions of BPS objects suggesting that the duality holds at the non-perturbative level as well.

2.2.2 T-duality

While some field theories can also exhibit S-duality, in the sense that taking their coupling to its inverse produces a weakly coupled dual theory, which may even be itself, as is the case with $\mathcal{N} = 4$ super Yang-Mills, T-duality is an inherently stringy duality, which appears from a symmetry relating string momentum and winding modes.[5]

Consider a string theory where one of the spacetime directions is compactified on a circle

[5]Note, however that upon compactification and dimensional reduction, both dualities can be cast in a similar form from the lower-dimensional perspective.

of size R, as measured in string units. A closed string can wind around this circle and its Hilbert space will contain winding states. The energies of these states are proportional to nR where n is the "winding number", i.e. the number of times the string is wound around the circle. On the other hand, the values of the momentum of the string along the circle also become quantized, and the momentum states have energies proportional to m/R, where m is the momentum. There are also oscillator modes of the string, which are insensitive to the size of the circle.

The first piece if evidence for T-duality is the observation that the spectrum of the closed string remains invariant under $R \to 1/R$. Moreover, if X is the worldsheet field corresponding to this compact direction, then one can define a new worldsheet field Y, which has all the same OPE coefficients as X with other fields, but has periodicity $1/R$. These fields are related to each other by

$$\partial_\alpha X = \epsilon_{\alpha\beta}\partial^\beta Y, \tag{2.2.24}$$

where α, β are worlsheet indices. This relationship imples the following transformation rules for the NS-NS fields

$$\begin{aligned} \tilde{g}_{YY} = \frac{1}{g_{XX}}, \qquad \tilde{g}_{YM} = \frac{B_{XM}}{g_{XX}}, \qquad \tilde{g}_{MN} = g_{MN} + \frac{B_{XM}B_{XN} - g_{XM}g_{XN}}{g_{XX}}, \\ \tilde{B}_{YM} = \frac{g_{XM}}{g_{XX}}, \qquad \tilde{B}_{MN} = B_{MN} + \frac{g_{XM}B_{XN} - B_{XM}g_{XN}}{g_{XX}}, \end{aligned} \tag{2.2.25}$$

known as "Buscher's rules" [43], where X and Y denote the circle coordinate before and after duality and tildes denote the dual quantities. These rules can also be derived from the closed string path integral [44].

For the open string, (2.2.24) implies that Dirichlet boundary conditions become Neumann and vice-versa. This means that a Dp-brane becomes a D($p\mp1$)-brane, depending on whether the original brane was wrapped around the circle or not. This in turn means that the R-R fields that couple to these branes must also transform into each other as

$$C_{XMN...}dX \wedge dx^M \wedge dx^N \wedge ... \quad \to \quad C_{MN...}dx^M \wedge dx^N \wedge ... \tag{2.2.26}$$

$$C_{MN...}dx^M \wedge dx^N \wedge ... \quad \to \quad C_{YMN...}dY \wedge dx^M \wedge dx^N \wedge ... \tag{2.2.27}$$

More rigorous derivations of these rules for the R-R from various perspectives can be found in [45, 46, 47, 48, 49]. Note that it is important for the application of these rules that the circle coordinate has the correct periodicity, otherwise numerical coefficients will appear. Since T-duality maps even p-forms to odd p-forms, it must map type IIA string theory to type IIB and vice versa. It also connects the two heterotic theories, which do not have R-R fields, but with some additional subtleties relating to how their respective gauge groups transform into each other.

Note that both (2.2.25) and (2.2.26) preserve the isometry along y. On the other hand, the rule for open strings seems to suggest that when D-branes lose a dimension they must localize to some point on the circle and break the isometry. Furthermore T-duality is known to relate KK-monopoles (geometries where the circle radius is non-trivially fibered over the transverse manifold and smoothly vanishes at a point) to NS5-branes, which are the magnetic sources for the NS-NS 2-form [50].

Applying Buscher's rules in both these scenarios only yields smeared branes. While this is acceptable from a supergravity perspective, in the same sense that continuous charge distributions are fine in classical electrodynamics, from a string theory perspective this is unsatisfactory. The mechanism responsible for breaking the translation symmetry and localizing the NS5-brane dual to a KK-monopole was given in [51] and via S-duality can be mapped to the case of D-branes. Thus, in the presence of branes, Buscher's rules must be used with caution. Furthermore, these rules are only valid to leading order and receive additional corrections, suppressed by powers of the string length [52, 53, 54].

As a quick aside, although we presented type I strings as an orientifold of type IIB strings in the presence of D9-branes, historically the development was in the opposite direction. Type I string theory was first known as one of the five consistent string theories, and D-branes were discovered as the result of applying T-duality to the type I string [42].

2.2.3 M-theory and F-theory

We have seen that the five string theories are not all separate theories, but are related by S- or T-duality as well as orientifold projections. Yet another very important piece of this duality web is the observation that the HE and IIA string theories can both realized as the dimensional reduction of an 11-dimensional theory [55]. There is a unique 11-dimensional supergravity theory whose bosonic field content consists only of the metric, an antisymmetric 3-form potential, accompanied by their superpartner, the gravitino. At the two-derivative level, its bosonic action can be written as

$$S = M_p^9 \left(\int d^{11}x \sqrt{g} R - \int G_4 \wedge \star G_4 - \frac{1}{6} \int C_3 \wedge G_4 \wedge G_4 \right) , \tag{2.2.28}$$

where $G_4 = dC_3$, and M_p is the 11-dimensional Planck mass.

Type IIA supergravity can then be obtained by dimensionally reducing this theory on a circle, while the $E8 \times E8$ heteoritic theory is obtained by dimensionally reducing on an interval, where each $E8$ gauge group factor can then be viewed as living on the end-point of the interval.

Let us spell out the dimensional reduction of the bosonic sector to IIA in more detail. Compactifying on a circle parametrized by y, the 11-dimensional metric g_{MN} decomposes into a 10-dimensional metric g_{mn}, a scalar field, coming from the g_{yy} metric component, and a $U(1)$ gauge field $A_m = g_{my}$. Let us write this decomposition in the following way:

$$ds_M^2 = e^{-2\phi/3} g_{mn} dx^m dx^n + e^{4\phi/3} (dy + C_m dx^m)^2 \ . \tag{2.2.29}$$

This choice of parametrization will result in the correct normalization after dimensional reduction. Furthermore, the C_3 potential decomposes into several components as well depending on whether it has a y index or not. This results in $B_{mn} = C_{ymn}$ and C_{mnp} remains as is. Carrying out the dimensional reduction gives precisely the type IIA supergravity action given in section 2.1.3,

$$
\begin{aligned}
S &= \frac{1}{2\kappa^2}\int d^{10}x\sqrt{g}e^{-2\phi}\left(R+4(\nabla\phi)^2-\frac{1}{2}|H_3|^2\right)+ \\
&\quad -\frac{1}{4\kappa^2}\int F_2\wedge\star F_2-\frac{1}{4\kappa^2}\int F_4\wedge\star F_4-\frac{1}{4\kappa^2}\int B_2\wedge F_4\wedge F_4\ ,
\end{aligned}
\qquad (2.2.30)
$$

where $F_2 = dC_1$, $F_4 = dC_3$ and $H_3 = dB_2$, and we have $\kappa^{-2} \propto M_p^8 e^{\phi/3}$, which means $M_p^{-1} = e^{\phi/3} l_s$.

Note that e^{ϕ} appears precisely as the type IIA string coupling and becomes small when the y-circle becomes small, so the 11-dimensional theory really describes the strong-coupling regime of type IIA supergravity.

Of course, this exercise only establishes the relation between 11-D supergravity and 10-D type IIA supergravity. The important claim is that this correspondence extends to the full type IIA string theory. The 11-dimensional theory that describes the strong coupling regime of type IIA strings should then be some theory that reduces to 11D supergravity at low energies. This theory is referred to as M-theory, and at the present time, no complete top-down formulation of this theory is known.

Much like with the other dualities, the evidence that the relation holds at the string theory level comes from comparing extended BPS objects in the theory. Since 11-D supergravity contains the C_3 field, there should be objects that couple to it electrically and magnetically. These objects are known as $M2$- and $M5$-branes. Although their complete actions are not known, the minimal action of the $M2$ brane ought to contain terms of the form

$$
\int d^3\sigma\sqrt{h}\left(g_{MN}\partial_\alpha X^M\partial^\alpha X^N + C_{MNP}\epsilon^{\alpha\beta\gamma}\partial_\alpha X^M\partial_\beta X^N\partial_\gamma X^P\right), \qquad (2.2.31)
$$

where h is an auxiliary worldvolume metric, plus its supersymmetric completion. This action can also be dimensionally reduced, when one of the directions wraps the y circle, and yields precisely the Polyakov action for the string. Thus the fundamental strings of type IIA can be identified with M2-branes wrapping the extra dimension of 11-d supergravity.

Similarly, if the M2-brane is not wrapped around the y-circle, it should be identified with the D2-brane of type IIA. A fully non-perturbative correspondence is hard to establish since for neither objects the complete action is known, but it can be checked to leading order and supersymmetry pretty much requires this to be the case, since they are both BPS objects that couple to their respective p-form fluxes, which are related by the dimensional reduction. In a similar vein, the M5-brane can be argued to correspond to either a D4-brane or an NS5-brane depending on whether it wraps the circle direction or not.

Any theory of gravity compactified on a circle will give rise to two-more kinds of objects in the lower-dimensional theory. The first simply arises from Kaluza-Klein modes of the metric along the circle directions, which we truncated when obtaining (2.2.30). Keeping them in, results in a 10D term that describes a particle with electric coupling to the vector field A_m. These are identified with the D0-branes of type IIA theory.

The second kind of object arises when the y-circle has a non-trivial fibration over the 10-dimensional base manifold. In particular, there are solutions that are asymptotically of the form $\mathbb{R}^{1,6} \times \mathbb{R}^3 \times S_1$, where the S_1 has a non-trivial fibration over the $\mathbb{R}^3$ manifold. The $\mathbb{R}^3 \times S_1$ part of the manifold has a metric described by a "gravitational instanton" metric [56, 57].

These solutions can have the the y-circle degenerates at one of multiple points creating coordinate singularities, which describe various $6+1$-dimensional objects in the type IIA theory, depending on the exact nature of the singularity. The non-trivial fibration of the y-circle induces off-diagonal components of the metric, which upon reduction to 10-D produce exactly the gauge-field due to a magnetic source.

Two important examples of these solutions are the Taub-NUT solution, which corresponds to a single D6 brane in 10-dimensions, and the Atiyah-Hitchin metric [58], whose asymptotic behavior is actually $(\mathbb{R}^3 \times S_1)/\mathbb{Z}_2$, which describes the O6-plane. These relationships can be checked by the matching of the ADM masses and magnetic charges of these objects. Note that the magnetic charges of these objects appear as topological charges from the 11-dimensional perspective.

In curved and/or compact space, the exact uplifts of D6 and O6 planes are generally not known, but since both these objects carry charges coming from the topological properties of their uplifts, it is safe to assume that their geometries will maintain their general form, differing only by various warpings and deformations. We will return to these objects in chapter 4, since as we will see, they host an important set of quantum corrections to M-theory.

Thus we have completed the dictionary between the basic objects present in type IIA theory and M-theory. We can also use T-duality to relate type IIB theory to M-theory.

Recall that type IIB strings have an $SL(2,\mathbb{Z})$ symmetry, which acts on the axio-dilaton as if it were the modular parameter of a torus. There is a way to make this torus appear explicitly by considering type IIB compactified on a circle, with string frame metric.

$$ds_B^2 = e^{\phi/2}\tilde{R}(x)^2 dz_1^2 + g_{mn}dx^m dx^n, \tag{2.2.32}$$

where the metric g_{mn} is independent of z_1. We will also assume a string coupling e^{ϕ}, which may also depend on x. Since z_1 is an isometry, we can T-dualize this metric to obtain

$$ds_A^2 = e^{-\phi/2}R^2(x)dz_1^2 + g_{mn}dx^m dx^n, \tag{2.2.33}$$

with $R(x) = 1/\tilde{R}(x)$ and the type IIA string coupling is given by $Re^{-\phi}$ (we set the string length to 1). Lifting this to M-theory, we obtain

$$ds_M^2 = R^{4/3}(e^{-\phi}dz_1^2 + e^{\phi}dz_2^2) + R^{-2/3}g_{mn}dx^m dx^n, \tag{2.2.34}$$

and thus e^{ϕ} appears as the modular parameter of the z_1, z_2 torus. Furthermore, if we had a non-vanishing C_0 potential on the IIB side, it would become a $C_1 = C_0 dz_1$ on the IIA side and produce off-diagonal terms in the torus metric. Thus it is the whole axio-dilaton that gives the modular parameter of the torus. The $SL(2,\mathbb{Z})$ symmetry of type IIB manifests itself as the modular symmetry of this torus.

One can further check that the transformation rules for the 3-form fluxes in type IIB also map to analogous transformations for the G_4 fluxes of M-theory, with one leg along the torus directions. $SL(2,\mathbb{Z})$ transformations then alter which leg it is, converting between H_3 and F_3 exactly reproducing (2.2.21).

Note that this M-theory description of the type IIB geometry does not require the type IIB string coupling to be large or small. However to obtain the non-compact IIB limit, the total volume of the torus needs to shrink to zero. This M-theory description of this class of IIB geometries is called F-theory and offers a powerful handle on the strong coupling limit of type IIB strings. In particular, we will be using precisely this duality chain when examining scale-separated compactifications in chapters 4 and 5, where we will discuss more details.

2.3 Compactification

In order to obtain a 4-dimensional theory from the 10-dimensional string theories, one must compactify them on a 6-dimensional manifold and integrate out the massive Kaluza-Klein modes. The task of determining which compactifications are allowed and what the resulting 4-dimensional effective theory is complicated in full generality and carries a trade-off between tractability and phenomenological viability. Requiring that some supersymmetry is preserved after compactification leads to important constraints which simplify the analysis.

2.3.1 Calabi-Yau compactifications

For supersymmetry to be preserved, it must be the case that the variation of the gravitino under the corresponding supercharges vanishes.

$$\delta_\epsilon \Psi_M = (\nabla_M + \mathbb{F}_M)\epsilon = 0, \tag{2.3.35}$$

where $\mathbb{F}_M$ is an expression built out of the various p-form fluxes and gamma matrices that depends on the type of string theory we are working in. In the absense of internal fluxes, this amounts to requiring that the spinor ϵ is covariantly constant.

The existence of such a spinor requires that both the 4-dimensional and 6-dimensional submanifolds are Ricci-flat. Furthermore, bilinears constructed out of this spinor provide the internal manifold with a complex structure, making it a Kahler manifold, and a global 3-form that is holomorphic with respect to this complex structure. This in turn implies that the Ricci 2-form is exact (commonly stated as the vanishing of the first Chern class) and the manifold therefore has no non-trivial 1-cycles. Manifolds satisfying these properties are called Calabi-Yau manifolds, and an important theorem [59, 60] guarantees the existence and uniqueness of a Ricci-flat metric on these manifolds, despite it not being known explicitly in the general case.

A Calabi-Yau manifold can be characterized by its overall volume, which fixes the purely holomorphic 3-form, as well as the sizes of its various 2-cycles and 3-cycles. These are referred to as the Kahler moduli and complex structure moduli respectively. The unbroken supersymmetry contrains the 4-dimensional effective theory to be 4-dimensional supergravity coupled to a set of supermultiplets, whose lowest components correspond to these moduli. The type of superfield describing each of the modulus depends on the type of string theory.

In type IIB CY compactifications, which will be the most relevant case for our purposes, and which preserve $\mathcal{N} = 2$ SUSY in 4 dimensions, the complex structure moduli are described by (the lowest component of) the vector multiplets, while Kahler moduli are described by hypermultiplets.

The two-derivative action is then also completely constrained by supersymmetry as is the form of possible corrections to it (provided we demand SUSY is preserved). Thus simply by compactifying on a SUSY preserving manifold, we can obtain the low energy 4-dimensional theory without doing the dimensional reduction explicitly. An important detail is that the fields describing the CY moduli remain massless to all orders in perturbation theory.

This is a double edged sword. On the one hand, it means that we are free to work in a regime where we have control over stringy corrections to the SUSY conditions, which can change some of the analysis. On the other hand, this generates a large number of (perturbatively) massless fields in the lower-dimensional theory, which poses phenomenological problems. What we would like is for these moduli to be stabilized dynamically, but in such a way that we maintain control over the possible corrections to the effective theory. This is known as the problem of moduli stabilization.

2.3.2 Flux Compactifications

The way that partial moduli stabilization can be achieved is through compactifications that involve fluxes along the internal dimensions. At the pure supergravity level, without localized sources such as branes, there is a well-known no-go result that forbids non-trivial fluxes [61]. The way around this is to consider a *warped* product manifold, where the metric of the non-compact directions comes with a conformal factor that depends on the internal coordinates, combined with the addition of higher derivative corrections to the flux equations of motion,. Indeed, these corrections will generically make non-trivial fluxes not only possible, but necessary. For a comprehensive review of the topic see [62].

An illustrative example, and one that is similar in many ways to the spacetimes that we will be studying in later chapters, is a warped compactification of M-theory to 3-dimensional Minkowski space [63]. We will write an ansatz of the form

$$ds^2 = \frac{1}{h(y)^{2/3}}\eta_{ij}dx^i dx^j + h(y)^{1/3}g_{mn}(y)dy^m dy^n, \tag{2.3.36}$$

where x^i parametrize $\mathbb{R}^{2,1}$, while y^m parametrize an 8-dimensional manifold. The choice of powers on the warp factor $h(y)$ is for later comparison with type IIB compactifications.

Poincare invariance requires that fluxes with legs along the non-compact directions are proportional to the volume element of the non-compact space,

$$G_{ijkm} = \epsilon_{ijk} f_m(y), \tag{2.3.37}$$

where $f_m(y)$ is a set of functions of the internal coordinates only. The other flux components must be entirely along the internal directions. The two-derivative equation of motion is

$$d \star G = G \wedge G, \tag{2.3.38}$$

and we see that if the left hand side has the external flux, then the right hand side must contain the wedge product of the internal fluxes. If we integrate both sides of the equation over the 8-manifold, the left hand side vanishes, and we are seemingly forced into setting the internal fluxes to zero. However, there is a higher-derivative topological contribution to the Chern-Simons action of the form

$$\delta S_{CS} = -M_p^9 \int C_3 \wedge X_8, \tag{2.3.39}$$

where

$$X_8 = \frac{1}{192}(p_1(R)^2 - 4p_2(R)^2) = \frac{1}{(2\pi)^4}\left(\frac{1}{192}\mathrm{tr}R^4 - \frac{1}{768}(\mathrm{tr}R^2)^2\right) \tag{2.3.40}$$

is a topological invariant built out of the first and second Pontryagin classes (denoted p_1 and p_2) of the tangent bundle of the 8-manifold. For a (conformally) Calabi-Yau manifold the integral of X_8 gives the Euler characteristic of the manifold. This term is required in the action in order to ensure cancellations of gravitational anomalies in the presence of M5-branes [64, 65, 66], and thus represents an effect that is beyond supergravity alone. The integrated equation of motion then has the schematic form

$$\int G \wedge G + \int X_8 = 0, \tag{2.3.41}$$

and is referred to as the "tadpole cancellation" condition. In the presence of explicit M2-brane sources their total number would appear in the equation as well. Essentially, this condition is a Gauss-law type constraint, requiring that the total electric charge sourcing C_3 must vanish. Note that in the absence of M2-brane sources, non-trivial fluxes now are possible only when the internal manifold has non-zero $\int X_8$.

It is worth noting that X_8 is in fact the only topologically invariant correction to the flux equations of motion one could add. There can be other higher-derivative terms involving curvatures and fluxes, but they will be non-topological and therefore expressed in terms of the field strength G_4, rather than the potential C_3. This in turn means that these terms will only contribute total-derivatives to the flux equation of motion and thus not contribute to the tadpole cancellation.

A further set of constraints can be obtained by requiring supersymmetry, via the condition (2.3.35), now with a non-zero flux term. The constraints one obtains from it are that the external flux needs to be related to the warp factor as

$$f_m(y) = -\partial_m h(y)^{-1}, \tag{2.3.42}$$

or in other words, $C_{ijk} = h^{-1}\epsilon_{ijk}$. With this form of the external flux, both the Einstein's equations and the flux equation of motion involve $\Box h$ and can be combined to show that the internal flux must be self-dual.

$$G = \star G. \tag{2.3.43}$$

Furthermore if the internal manifold is conformally Calabi-Yau, the supersymmetry conditions further require that the internal fluxes are $(2,2)$ with respect to its complex structure (recall that CY-manifolds are Kahler and therefore complex) and *primitive*, meaning

$$J \wedge G = 0, \tag{2.3.44}$$

where J is the complex structure.

A somewhat similar story happens for flux compactifications of type IIB to $\mathbb{R}^{3,1}$. Here, the standard ansatz is

$$ds^2 = \frac{1}{\sqrt{h(y)}}\eta_{ij}dx^i dx^j + \sqrt{h(y)}g_{mn}dy^m dy^n, \tag{2.3.45}$$

where now x^i parametrizes $\mathbb{R}^{3,1}$ and the internal manifold is 6-dimensional. Once again, Poincaré symmetry requires that the only non-zero flux with external legs comes is

$$F_{0123m} = f_m(y), \tag{2.3.46}$$

while the internal fluxes come from the 3-forms F_3 and H_3, which one typically combines into $G_3 = F_3 - \tau H_3$, where $\tau = C_0 + ie^{-\phi}$ is the axio-dilaton.

A particular class of solutions once appears if we assume that the external 4-form potential is the warped volume element.

$$C_{0123} = \frac{1}{h(y)}, \qquad F_{0123m} = \partial_m \frac{1}{h(y)} \ . \tag{2.3.47}$$

A combination of Einstein's equations and the flux EOM leads to the 3-form flux G_3 being imaginary self-dual:

$$\star G_3 = iG_3 \ . \tag{2.3.48}$$

Requiring supersymmetry further constrains the fluxes to be $(2,1)$ and primitive with respect to the complex structure (assuming the internal manifold is conformally Calabi-Yau). This in principle leaves open the possibility of having non-supersymmetric solutions, by including a $(3,0)$ component of the flux, which is also imaginary self-dual, but breaks supersymmetry.

As in the M-theory case, there is a tadpole condition that needs to be obeyed on a compact manifold. In this case, the two-derivative flux equations in the absense of sources of motion read

$$d \star F_5 = G_3 \wedge G_3 \ , \tag{2.3.49}$$

and as before, the left hand side vanishes when integrated over the compact manifold. As in the M-theory case, we need to turn to higher derivative effects, which in this case come from the inclusion of D7-branes in the background. As we saw, the worldvolume action of a D7 brane consists of the DBI action (2.1.18) and the Chern-Simons action coupling it to various p-forms (2.1.19). The DBI action is exact to all orders in α', while the Chern-Simons action is only the leading order term and takes an α' correction from worldvolume curvature of the form

$$\delta S_{CS} = \int C_4 \wedge \mathrm{tr}(R \wedge R), \tag{2.3.50}$$

where R is the curvature 2-form of the 4-cycle wrapped by the D7-branes. This term modifies the tadpole condition to

$$\int G_3 \wedge G_3 + \sum_{\Sigma_i} \int \omega_2(y_\perp)\mathrm{tr}(R \wedge R) = 0, \tag{2.3.51}$$

where the second term is the sum over all 4-cycles wrapped by branes with $\omega_2(y_\perp)$ being a 2-form that is delta-function localized at the position of the branes.

An important feature of these solutions is that the addition of fluxes stabilizes some of the moduli [67, 68]. Recall that the dimensional reduction on a Calabi-Yau (including orientifolds) resulted in a set of massless scalar fields parametrizing the sizes of the cycles. The complex structure moduli parametrize the sizes of the 3-cycles, while the Kahler moduli parametrize the size of the 4-cycles. These scalars were the lowest components of their respective supermultiplets, and so it is convenient to describe their dynamics in supersymmetric language, in terms of a Kahler potential $\mathcal{K}$, which determines the kinetic terms for the scalars and a superpotential W, which gives the scalar potential via

$$V = e^{\mathcal{K}} \left(\mathcal{K}^{a\bar{b}} D_a W D_{\bar{b}} W - 3|W|^2 \right), \tag{2.3.52}$$

with $D_a = \partial_a + (\partial_a K)$, the indices a, b labelling the various chiral multiplets in the theory. Solutions to the equations of motion can be obtained by solving $D_a W = 0$.

The presence of the flux G_3 induces a superpotential in the 4-dimensional theory [69]

$$W = \int \Omega \wedge G_3 \ , \tag{2.3.53}$$

which induces a scalar potential for the complex structure moduli and dynamically fixes them! The axio-dilaton also gets fixed since it appears in the definition of G_3. Thus flux compactifications provide partial moduli stabilization. The Kahler moduli, usually denoted ρ, remain unfixed at this level. The superpotential is independent of these moduli and their Kahler potential, to leading order, is of a "no-scale" structure: $\mathcal{K} = -3\log(\rho - \bar{\rho})$, which results in a flat scalar potential. We will discuss their stabilization in the next section.

Finally, let us point out that there is a direct relationship between this type IIB scenario and the M-theory scenario. Performing the duality chain described in section 2.2.3 between type IIB and M-theory, the ingredients of both of these scenarios map to each other. The external F_5 flux maps to the external G_4 flux by losing a leg to T-duality. The internal fluxes F_3 and H_3 each gain a leg, the former by T-duality, the latter from the uplift to 11 dimensions, and become the internal G_4 flux. The D7 branes, get replaced by warped Taub-NUT solutions, as discussed in section 2.2.3, which contribute a nontrivial topological charge. Indeed, the $\int R \wedge R$ term on the D7-worldvolume can be seen as coming from the $\int X_8$ term after doing the integral over the directions transverse to the Taub-NUT solution.

A particularly interesting case that we will make use of in future chapters is the "constant coupling" scenario [70, 71], where the axio-dilaton charge of the D7 branes is cancelled in a local fashion, by placing them on top of O7-planes in a 4-to-1 ratio. This cancels the global D7 charge of the background, but also ensures that the axio-dilaton is constant everywhere except the D7/O7 loci. This means that apart from those loci, the M-theory dual has the internal manifold given by $M_6 \times T^2$, with the T^2 having constant modular parameter. All the non-trivial topology of the torus fibration is localized at the duals to the D7/O7 stacks, which become D-type ADE singularities. We will return to discussing the properties of this scenario in future chapters.

2.4 De Sitter Space in String Theory

The compactifications we have discussed so far have all been to flat space, partly as a result of relying on supersymmetry conditions to obtain solutions. Meanwhile, cosmological observations indicate that the eventual fate of our universe is one of accelerated expansion [6, 7, 8]. Furthermore, the dominant paradigm for explaining the origins of various features of the cosmic microwave background involves a phase of very rapid accelerated expansion known as "inflation" [9, 10, 11, 12, 13]. While alternative early-universe scenarios exist that do not involve such accelerated expansion (see [72, 73] for a review) the late-time accelerated expansion appears somewhat more certain. If the history of our universe really contains epochs of sustained accelerated expansion and string theory is to be a theory of nature, it must be able to produce cosmological solutions exhibiting such accelerated expansion.

The maximally symmetric spacetime exhibiting accelerated expansion is de Sitter space, and realizations of it in string theory have been the subject of a lot of recent debate. In this section we will review the main proposal for realizing four-dimensional de Sitter space in string theory, known as the KKLT scenario [14]. In the next section we will discuss some of the uncertainties surrounding this construction and various conjectures that in fact prohibit de Sitter space from being realized in string theory.

The starting point for the KKLT scenario is in fact the type IIB flux compactifications we have discussed in the previous section with the complex structure moduli stabilized. As we mentioned earlier, the Kahler potential for the Kahler moduli ρ appears as

$$\mathcal{K} = -3\log(\rho + \bar{\rho}). \tag{2.4.54}$$

Here we are assuming a single Kahler modulus for simplicity, but the story is similar when there are more.

In supersymmetric compactifications, the total superpotential vanishes and one obtains a flat potential for the Kahler moduli. As we mentioned in the previous section, introducing a $(3,0)$ part of the 3-form flux breaks supersymmetry and results in a non-zero contribution to the superpotential. This superpotential is still independent of ρ so upon fixing the complex structure moduli we simply have.

$$W = W_0. \tag{2.4.55}$$

Supersymmetry protects the superpotential W from receiving any perturbative corrections in ρ, but allows for non-perturbative corrections of the form

$$W_{NP} = Ae^{-a\rho}. \tag{2.4.56}$$

The physical origin of such a correction would come from Euclidean D-branes wrapping the 4-cycle whose volume is given by ρ. Such effects are called D-brane instantons. Alternatively, a similar term can arise from the gauge theory living on a stack of D7 branes wrapping that cycle, specifically gaugino condensation, which also induces a non-zero instanton density on the brane-worldvolume. In either case, including such a term to the superpotential results in a scalar potential of the form

$$V(\rho) = \frac{ae^{-2a\rho}}{6\rho^2}\left(A^2(3 + a\rho) + 3Ae^{a\rho}W_0\right), \tag{2.4.57}$$

which has a minimum at which $DW = 0$

$$W_0 = -Ae^{-a\rho}(1 + \frac{2}{3}a\rho), \tag{2.4.58}$$

giving a supersymmetric solution! This is despite the fact that we have broken the supersymmetry with the W_0 term. The potential at this solution is equal to

$$V_c = -\frac{a^2A^2}{\rho_c}e^{-2a\rho_c}, \tag{2.4.59}$$

and thus we have an anti-de Sitter vacuum, in which the ρ modulus is fixed and supersymmetry is restored. Note that the value of the potential is exponentially small for large ρ so the "cosmological constant" of the AdS space is parametrically separated from the compactification length scale, which is governed by ρ. Such solutions are called "scale-separated".

A similar scenario known as the "large volume scenario" (LVS) [16], includes the leading

α' corrections to the Kahler potential leads to a similar behavior for the potential, effectively dressing the non-perturbative terms with additional powers of ρ. This has the benefit of making the scale separation even more pronounced, while simultaneously relaxing the requirements on the value of W_0. In the KKLT scenario, W_0 needs to be of order 10^{-4}, while LVS allows for Kahler moduli stabilization for more typical values of W_0. These vacua, however are no longer supersymmetric. In both scenarios, the main physics is the same: Kahler moduli stabilization is obtained through a balance of perturbative and non-perturbative terms in the effective potential, leading to an AdS minimum.

To obtain de Sitter space, KKLT proposes to include one additional ingredient: an anti-D3-brane. The expected effect of this ingredient is to introduce an additive term to the total potential

$$\delta V = \frac{D}{(\rho + \bar{\rho})^3}, \tag{2.4.60}$$

which then lifts the AdS minimum to positive energy producing a de Sitter space. Originally, this change to the potential was argued for by noting that if we include a D3 brane instead, the energy shift would vanish, due to a cancellation between terms coming from its DBI and Chern-Simons action. Since an anti-D3 has the opposite sign for the Chern-Simons term it was argued that the contribution should double instead of cancelling, producing (2.4.60).

Later, this uplift term was realized as the contribution of a nilpotent supermultiplet [74]. Such supermultiplets arise in theories with spontaneously broken supersymmetry. Introducing this multiplet modifies the Kahler potential and superpotential to

$$\begin{aligned} \mathcal{K} &= -3\log(\rho + \bar{\rho}) + S\bar{S}, \\ W &= W_0 + Ae^{-a\rho} + bS, \end{aligned} \tag{2.4.61}$$

where S is the nilpotent multiplet satisfying $S^2 = 0$, so that no non-linear terms can appear in W. Computing the scalar potential and imposing the nilpotency leads to

$$\delta V = \frac{b^2}{(\rho + \bar{\rho})^3}, \tag{2.4.62}$$

i.e. precisely reproduces the uplift term, but now in a manifestly supersymmetric effective theory, with spontaneously broken supersymmetry, which is what one expects from an anti-brane. The existence of such nilpotent multiplets were eventually also derived from a top-down approach, showing that fermionic terms on the worldvolume of an anti-brane do indeed produce the necessary massless fermions, [75, 76, 77, 78, 79, 80, 81, 82], supporting the plausibility of (2.4.61) as the correct low-energy 4-d theory to describe the particular combination of ingredients that are used. It should be noted that several variations of this scenario exist, for example changing the number of non-perturbative contributions [83], or replacing the anti-brane by supersymmetry-breaking worldvolume fluxes on the 7-branes that are present in the flux-compactification [15]. However the general spirit of all these constructions are the same in that they consist of a flux-compactification with non-perturbative effects and a supersymmetry breaking "uplift" ingredient.

De Sitter space constructions, and KKLT specifically, have been under considerable scrutiny in recent years and we summarize some of the recent work in this direction in the remainder of this section.

Questions regarding de Sitter solutions in string theory begin with an important no-go theorem forbidding de Sitter compactifications [61, 18] at the supergravity level. This can be further extended to apply to compactifications with arbitrary fluxes and branes and even orientifold planes [19]. For all except the latter, the no-go theorem essentially amounts to these ingredients failing to violate the strong energy condition $T^{\mu}_{\mu} > T^{m}_{m}$, where μ and m are the external and internal spacetime indices respectively. Orientifold planes on their own do violate the SEC, but they also necessarily source additional flux, which eliminates the total SEC violation. Thus it seems that in order to obtain de Sitter solutions, one is forced to make use of the quantum corrections to the string theory equations of motion.

This is not immediately incompatible with the validity of KKLT-type constructions, since they do make use of non-perturbative effects. However the non-perturbative contribution is used for the Kahler moduli stabilization,while the positive cosmological constant is supposedly generated by an antibrane or similar ingredient, which would appear to fall under the no-go theorem conditions. Thus it is not clear from the formulation of the scenario as it currently stands, how exactly it evades the no-go result. To determine this a top-down approach is called for.

Various top-down analyses of the KKLT construction have been performed in recent years presenting objections to the steps in the KKLT construction. One concern is that of obtaining a small W_0. Since the origin of W_0 is from internal fluxes, which are constrained by the tadpole cancellation and flux quantization conditions, constructing small W_0 is a challenge (however see [84] for an example). This is a relatively minor point, since small W_0 is only required in KKLT, and scenarios like the large volume scenario work well with more typical values of W_0. Thus it could very well be that even if small W_0 were not achievable, the general spirit of the KKLT approach is correct.

A second set of potential problems relates to the validity of the effective theory description for the Kahler modulus. A series of investigations were prompted by [85, 86], which looked at the effects of the anti-brane backreaction on the non-perturbative term generated by gaugino condensate on D7 branes wrapping the 4-cycle governed by the Kahler modulus, and argued that the uplift of the AdS minimum is accompanied by a flattening of the potential to the point that no de Sitter minimum was possible. The main argument was presented from a 10-dimensional perspective, but the authors also provided a 4-dimensional interpretation.

The 4-dimensional interpretation is that the coefficient A of the non-perturbative term in (2.4.61), should also contain some dependence on the nilpotent multiplet S. The nilpotency means that the dependence can only be linear and computing the resulting potential gives the advertized flattening effect. This argument resulted in a series of papers [87, 86, 88, 89, 90] arguing for or against the validity of this new coupling.

The 10-dimesional version of the argument prompted a deeper investigation of the stress tensor generated by the non-perturbative effect, specifically gaugino condensation on D7-branes, from the 10-dimensional perspective. Following initial proposals of a new four-fermion coupling on the D7-brane worldvolume [91, 92], which was necessary to properly describe the effects of gaugino condensation in the 10-dimensional description, three simultaneous papers appeared [93, 94, 95], one finding disagreement with the KKLT effective theory [95], one finding agreement with it [93]. The difference in their results stems from a difference in how they treat the dependence of the gaugino condensate on the Kahler modulus. The third simultaneous paper [94] found agreement with KKLT in the absense of an uplift ingredient, but also revealed potential problems with the uplift procedure. Later works have also investigated this question in the absense of uplift [96], and by considering the non-perturbative and uplift ingredients separately [97], finding agreement with the KKLT effective theory.

Finally there is a separate argument, which casts doubt on the compatibility of a non-zero W_0 with the no-scale Kahler potential [98]. The argument claims that since tadpole cancellation involves higher derivative terms in the 10-dimensional theory, one must also include the effects of all other derivative corrections of the same order, which correct the Kahler potential, causing a runaway of the Kahler modulus and possibly undermining the calculation of non-perturbative effects, which one usually performs around static classical solutions.

Yet a third, slightly older but still relevant set of concerns has to do with the properties of the anti-brane responsible for the uplift. Before one even worries about the possibility of the flattening effects described above, there are potential problems with the stability of the anti-brane itself, explored in [99, 100, 101, 102, 103, 104, 105, 106]. The main takeaway from these investigations is that the anti-brane interacts with the surrounding flux and in particular gets polarized into various 5-branes via the Myers effect [107]. At least one such polarization channel is well understood in the non-compact case [108] and produces a meta-stable state, which is generally thought to be identified with the KKLT de Sitter vacuum upon compactification. However there are other polarization channels, whose eventual fate is not so clear, and may represent runaway instabilities.

Furthermore, [109, 110] also found that the introduction of an anti-brane can disrupt complex structure moduli stabilization, which is generally assumed to have been taken care of before one even begins the KKLT construction.

Thus, while KKLT-like constructions represent a very clever and reasonably motivated effective theory approach to arguing for de Sitter vacua, pretty much every step combination of steps has potential pitfalls and caveats when analyzed in full string theory. It is probably safe to say that at the current moment the ultimate fate of de Sitter space constructions in string theory remains uncertain.

2.5 The Landscape and the Swampland

The multitude of choices for the compactification manifold, combined with the various choices of flux one can put on them and the KKLT-style Kahler moduli stabilization leads to a picture of string theory, where there is a vast number of (meta-)stable solutions, with various particle spectra and even different values of the cosmological constant. This multitude of solutions is known as the “string landscape”.

In arriving at these scenarios, however, we have done a lot of description-hopping. In particular, by the time we reach flux compactifications we are very far from the original perturbative definition of string theory. This is in large part due to the fact that the effective action of the RR fields does not appear directly from the worldsheet description, but is inferred by supersymmetry. In fact, while string compactifications without flux can be explicitly realized as solutions to the two-dimensional worldsheet sigma model, no such realization is known for flux compactifications. Our confidence in the existence of these solutions arises because they solve the effective 10-D spacetime equations of motion, which are themselves partly inferred from spacetime supersymmetry constraints. The Kahler moduli stabilization of KKLT is even further removed from perturbative string theory due to its reliance on non-perturbative effects and because it uses 4-dimensional effective theory arguments.

The vastness of the string landscape might lead one to believe that just about any low-energy theory can be concocted with a clever enough arrangement of ingredients, and thus justify such effective theory approaches. On the other hand, the solutions must obey many consistency conditions that are absent in the effective theory perspective, and thus one must be careful when making genericity arguments in an effective field theory framework. A simple example that we already discussed is the tree-level superpotential W_0, in the KKLT scenario. In effective theory, taking small W_0 is not a problem, since W_0 appears as a free parameter. However, as it arises from the energies of the internal fluxes, one must do extra work to verify whether compactifications with small W_0 are even possible.

The fact that string theory is more restrictive than field theory, means that not every consistent-looking low-energy effective theory can arise as a solution to string theory. Another way of saying this, is that not all consistent effective theories remain consistent once coupled to quantum gravity. The set of effective theories that do not have a string theory realization has been called the "swampland" [21], and there is an extensive research program dedicated to establishing the criteria that an effective theory needs to satisfy to be in the string landscape rather than the swampland (see [22, 23] for reviews). These proposed criteria are known as the swampland criteria, or swampland conjectures and are inspired by a variety of considerations, from the common features of known explicit stringy solutions, to general properties of black hole physics and cosmological spacetimes, as well as properties of conformal field theories via gauge/gravity duality.

A very important swampland conjecture is known as the distance conjecture, which states that as a field in some effective field theory takes large expectation values (in Planck units), the effective field theory description should break down, due to a tower of light states entering the dynamics of the system. When this happens, one may be able to switch to a dual description in terms of these light fields.

As an example consider string theory compactified on a circle of radius R, in string units. Since this radius is controlled by one of the metric components, it is in fact a dynamical variable. The canonically normalized degree of freedom is the logarithm of the circle size $\phi = \log R$. If we start with a string-scale circle, corresponding to $\phi = 0$ we can work in an effective theory in one lower dimension. As ϕ deviates far from zero, the circle becomes large or small, and either Kaluza-Klein or string winding modes enter the system. We can always work switch to a duality frame where the circle becomes large, and thus the original effective theory breaks down and transforms into a higher-dimensional theory as the circle decompactifies.

One can also look at it in the reverse way. If one starts with a large circle, and makes it smaller, the winding modes enter the theory and one is forced to perform a T-duality to again obtain an effective theory description. The same thing happens with the expectation value of the dilaton and S-duality.

This restriction on the maximal value of field expectation values in a given effective theory means that one must be wary of solutions to a given effective theory which require large values or excursions in a field's expectation value. The distance conjecture implies that once embedded into string theory, these large expectation values will result in new degrees of freedom entering the dynamics and likely destroying the solution.

Another set of swampland conjectures directly concerns the feasibility of de Sitter space solutions. Motivated by the absense of explicit top-down de Sitter constructions and the properties of other known explicit top-down solutions, the following conjecture about the shape of the effective potential has been proposed [28, 29]:

$$|\nabla V| \geq cV \qquad \textit{or} \qquad \min \nabla^2 V \leq -aV, \tag{2.5.63}$$

where c and a are constants of order one, and $\min \nabla^2 V$ means the minimal eigenvalue of the Hessian of V, with the derivatives being with respect to the fields in the theory. These conditions explicitly forbid stable de Sitter solutions. These conjectures can be shown to follow from the distance conjecture [30] at least in certain regimes. Further physical motivation for these conjectures was given by proposing principles such as forbidding eternal inflation [31] or elevating the Transplanckian Problem of inflation [111] to the level of the Transplanckian Censorship Conjecture (TCC) [32], which states that there should be no solutions in string theory which would allow a subplanckian mode to grow to super-Hubble scales. The TCC is slightly weaker than (2.5.63), in that it allows for meta-stable de Sitter minima in the potential, provided the tunnelling time is sufficiently short.

Regardless, the overall common point of the de Sitter Swampland conjectures is that there can be no extended periods of accelerated expansion. This is, of course, in conflict with the inflationary picture of the early universe, which requires many e-folds of expansion. The late-time expansion of our universe would also have to be explained not by a cosmological constant but by a quintessence model. Current observations may be borderline compatible with a quintessence model obeying (2.5.63), largely depending on how generous one is with the definition of "order one constant" [112, 113, 114, 115].

In either case, KKLT-type constructions are certainly in conflict with the de Sitter swampland criteria in any of their forms, so something has to give. One option is that de Sitter space truly does not exist in string theory and in particular the KKLT construction does fall prey either to one of the existing caveats or perhaps a new problem altogether. Another possibility is that something like the KKLT construction can be realized and would then serve as a counterexample to the swampland criteria. In either case, studying KKLT-type constructions can provide valuable insight into the deeper questions of string theory and quantum gravity, either by understanding how it evades the swampland criteria in case of success, or by better understanding the physical origins of the swampland criteria in case of failure.

Finally, there are also conjectures related to the existence of compactifications to AdS as well. Specifically there is a swampland conjecture forbidding large scale separation between the AdS curvature and the size of the internal manifold [116]. This conjecture is motivated by the fact that the explicit constructions of AdS always involve a compact space which is of the same size as the AdS scale. It can also be related for using the distance conjecture, since the AdS scale is also a dynamical variable and making it large should bring in a tower of massless states, which in the known cases are the Kaluza-Klein modes of the compact manifold.

2.6 Chapter Summary and Discussion

In this chapter, we have presented and overview of some aspects of string theory, starting from its perturbative definition, effective theory limits and duality structure, working our way up to compactifications with and without flux and ending with an overview of proposals for constructing solutions with fully stabilized moduli, scale-separation and a non-zero cosmological constant. We have also mentioned the challenges these proposals face and discussed some ideas relating to the string swampland research program, which among other things suggest that these solutions should in fact not exist.

In the rest of this thesis, we will also tackle the problem of scale-separated AdS and dS solutions in string theory, using a slightly different approach from what we have discussed so far. Any dS or scale-separated AdS solution must avoid the corresponding classical no-go theorems [18, 117], which requires the use of quantum corrections. Rather than attempting to include some of the known corrections and search for solutions in the resulting EFT, we will instead first choose the ansatz for the solution and ask what properties must the quantum corrections, when evaluated on this ansatz, have in order for the ansatz to satisfy the resulting equations of motion.

In particular, our interest will be whether these solutions can be realized in a well-controlled regime of string theory, where an effective theory description is possible. Note that any EFT-based constructions of these solutions implicitly assume that the answer to this question is yes. In order to do verify this, we must have an organizing principle for classifying the possible corrections that appear in string theory, establish a clear criterion for what it means to have an EFT description and when it breaks down, then evaluate the quantum-corrected equations of motion on our desired ansatz and see whether the necessary quantum corrections are compatible with an EFT description. These are the topics we will tackle in the upcoming chapters.

Chapter 3

Transseries, Truncations and Effective Theories

In this chapter, we review some important mathematical properties of asymptotic series, which arise in perturbation theory, their extensions into objects known as *resurgent transseries*, which captures non-perturbative information. We will then use these notions to offer a definition of an effective theory as finite truncations of such transseries, and introduce the general method by which we may determine whether a particular ansatz may be realized within the regime of validity of any EFT limit of a larger theory, and particularly within a theory that lacks free dimensionless parameters, such as string theory. This will pave the way for us to apply this approach in chapters 4 and 5 to the study of scale-separated AdS and dS compactifications.

3.1 Asymptotic Series, Borel Summation and Resurgent Transseries

Many difficult problems in physics and mathematics can be tackled via perturbation theory, by introducing or identifying a parameter, say ϵ, and expressing the solution as a function of this parameter in some limit, say $\epsilon \to 0$ (see [118] for an extensive review of perturbation theory and related techniques). The result typically involves a divergent series in powers of the perturbative parameter. The divergent nature of the series indicates that the obtained result should not be regarded as being *equal* to the solution but rather *asymptotic* to it in our chosen limit, i.e.

$$\lim_{\epsilon\to 0} \frac{f(\epsilon) - f_N(\epsilon)}{a_{N+1}\epsilon^{N+1}} = 1, \tag{3.1.1}$$

where $f(\epsilon)$ is the true answer to our problem, a_n are the coefficients in the perturbative series and $f_N(\epsilon)$ is the N-th partial sum of the perturbative series. The divergence of the perturbative series is due to a rapid growth of the coefficients a_n, typically a factorial growth. In the context of quantum field theory, this growth can be understood in terms of the combinatorial growth of the number of higher order Feynman diagrams. Thus the typical behavior for an asymptotic series is that its terms start off decreasing, due to the higher powers of ϵ, but for some $n = \tilde{N}$, which depends on ϵ, the combinatorial growth of the coefficients takes over and the terms grow.

Since $a_{N+1}\epsilon^{N+1}$ measures the error between f and f_N, we see that taking larger $N > \tilde{N}$ is counterproductive, as the error on those partial sums will actually be larger. Thus $f_{\tilde{N}}(\epsilon)$ represents the best approximation to f, for that value of ϵ. This is known as the *optimal truncation* of the asymptotic series.

As we take the limit $\epsilon \to 0$, $\tilde{N}$ also grows and the error on the optimal truncation decreases, typically as e^{-1/ϵ^p}, for some power p.

Suppose we wish to do better than that. We can then augment our asymptotic approximation by a non-perturbative term, to obtain an expression of the form

$$f(\epsilon) \sim f_{\tilde{N}}(\epsilon) + e^{-s_1/\epsilon^p} g(\epsilon), \tag{3.1.2}$$

where $f_{\tilde{N}}$ is the optimal truncation of the perturbative series and s_1 is an ϵ-independent quantity, which we can determine. $g(\epsilon)$ is an unknown function of ϵ, which we can solve for perturbatively. This produces yet another asymptotic series for g, which we can optimally truncate again and receive an error of order e^{-s_2/ϵ^p}, with $s_2 > s_1$, which we can append to our asymptotic approximation again, dressed by a function $h(\epsilon)$, and so on. This approach is known as *hyper-asymptotics* [119].

Note however that since the optimal truncations depend on the value of ϵ, each optimal truncation has a different number of terms for different values of ϵ. We can represent the general asymptotic approximation to our function by a formal series

$$f_K(\epsilon) = \sum_{k}^{K} \sum_{n}^{\tilde{N}(\epsilon)} a_{k,n} e^{-s_k/\epsilon^p}, \tag{3.1.3}$$

where K is the maximal non-perturbative contribution we keep, the $a_{k,n}$ are the perturbative coefficients in the k-th asymptotic series, and the sum over n is always taken to be the optimal truncation for that value of ϵ. The error on this approximation is then of order e^{-s_{K+1}/ϵ^p}.[1]

As we move away from the $\epsilon \to 0$ limit, however, we see that $\tilde{N}$ decreases for all the asymptotic series in our expression, until the leading term is the optimal truncation, at which point perturbation theory breaks down. To make progress beyond that point we need to find a way to extract the true function $f(\epsilon)$ from the information contained in the various asymptotic series. For a convergent series, we would simply sum the series and analytically continue it beyond its radius of convergence. For a divergent series, we must make use of resummation techniques.

A technique that is well suited to the combinatorial growth in perturbative series is Borel summation. Consider a divergent series

$$f(\epsilon) = \sum_{n} a_n \epsilon^n, \tag{3.1.4}$$

where a_n grow as $n!$. We can then define a new convergent series, which we will call the *Borel transform* of the series

$$\mathcal{B}[f](s) = \sum_{n} \frac{a_n}{n!} s^n. \tag{3.1.5}$$

This convergent series can be summed up and we define the *Borel sum* of f as

$$\mathcal{S}[f](\epsilon) = \int_0^\infty ds e^{-s} \mathcal{B}[f](\epsilon s). \tag{3.1.6}$$

[1] More generally, there may be further non-perturbative exponential terms with different powers of ϵ appearing in the exponent.

Note that doing the integral term by term on the Borel transform would reproduce the $n!$ factors in the original series. More generally, the Borel transform can be defined by dividing by enough factorials to render (3.1.5) convergent. This allows it to handle more general types of combinatorial growth of coefficients.

When the integral in (3.1.6) exists, the original series is said to be *Borel summable* and this essentially solves the problem. We have found the function $f(\epsilon)$ and are free to push ϵ beyond the perturbative regime. More often than not, however, the Borel transform has singularities on the real axis. We can therefore define a *directional Borel transform*

$$\mathcal{S}_\theta[f](\epsilon) = \int_0^{e^{i\theta}\infty} ds e^{-s}\mathcal{B}[f](\epsilon s), \tag{3.1.7}$$

which avoids these singularities. Approaching the real axis from above or below, we obtain imaginary ambiguities coming from the singularities, whose size is, again, of order e^{-1/ϵ^p} for some power p.

The location of the Borel singularities is closely related to the asymptotic growth rate of the perturbative coefficients. A marvelous feature that is observed in pretty much every case where the calculation can be carried out explicitly is that the Borel singularities lie precisely the right places so that the ambiguity it generates is e^{-s_k/ϵ^p}, where the s_k are the same as those generated by hyper-asymptotics.

Thus, although it may seem like any perturbative terms beyond the optimal truncation are useless, they in fact contain information about the non-perturbative effects in our problem. This is rather famously how the existence of objects with tension proportional to $1/g_s$, rather than $1/g_s^2$ was predicted in string theory, due to the faster $(2n)!$ growth of the world-sheet genus expansion compared to the $n!$ growth of Feynman diagrams in field theory, years before the actual discovery of D-branes, which have that tension [120].

The appearance of quantities of order e^{-1/ϵ^p} is rather unsurprising to a physicists eye. Indeed, such terms appear in many contexts and represent corrections due to non-perturbative effects, such as instantons. The locations of the Borel singularities is then exactly in one-to-one correspondence with the actions of instanton saddles that contribute to the observable in question. The reason for this can be understood roughly by imagining performing the path-integral by level sets. The integral expression for the Borel sum (3.1.6) then becomes precisely equivalent to the integral over the action and the singularities correspond to discontinuities in the level sets, i.e. when new saddles appear.

Asymptotic approximations that include non-perturbative exponentials have an interesting property known as the *Stokes' phenomenon*, where changing the phase of the perturbative parameter leads to sudden discontinuities in the coefficients of these exponentials, even though the function itself is smooth. The iconic example of this is the Airy function, where the asymptotic approximation in the $x \to +\infty$ limit involves a single decaying exponential, while the $x \to -\infty$ limit is asymptotic to a sum of two complex exponentials, resulting in oscillatory behavior. The jumps in the coefficients happen at $arg(x) = \pm 2\pi/3$. The lines along which the coefficients of the non-perturbative exponentials in an asymptotic approximation have discontinuities are referred to as Stokes lines.[2]

In the case of the Airy function, the leading order solution already has a non-perturbative exponential. In quantum field theory applications, where we typically divide out the leading exponent, we obtain first a purely perturbative power series, possibly in the form of a Frobenius series where the leading power is shifted, followed by non-perturbative instanton-like corrections. The miracle that can occur is that the phases of ϵ, for which the Borel singularities lie on the real axis and therefore introduce the ambiguity in the Borel sum of the series, are precisely the Stokes lines, where the coefficients of the non-perturbative corrections are also ambiguous, and the two ambiguities cancel out! This phenomenon is indeed observed and well-established in a wide variety of contexts, from non-linear differential equations and $0+1$-dimensional quantum mechanics, where it can be shown rigorously [121, 122, 123], to some more limited but compelling signs of it in quantum field theory [124, 125, 126], topological strings [127, 128] and the theory of superconductors [129]. The aforementioned prediction of D-branes from the asymptotic growth of the genus expansion can also be regarded as evidence that this structure persists in string theory.

This phenomenon continues to higher non-perturbative orders as well. The first non-perturbative correction is also dressed with a divergent series of perturbative corrections, which again have Borel ambiguities, which cancel against the Stokes' ambiguity of the next non-perturbative correction and so on *ad infinitum*. We are thus left with the following expression

[2] In some conventions they are referred to as Anti-Stokes lines, while Stokes lines are the lines, along which one exponential contribution is most dominant.

$$\sum_k A_k(\epsilon)e^{-s_k/\epsilon^p}\left(1+\sum_n a_{k,n}\epsilon^n\right), \tag{3.1.8}$$

where each sum over n is to be understood as needing to be Borel summed, and any Borel ambiguities in it are cancelled by the Stokes ambiguity coming from terms at higher k. The expression (3.1.8) is an example of a *transseries*. This procedure of converting the asymptotic series to a transseries by including the necessary non-perturbative terms to cancel Borel ambiguities and then resumming the whole thing is called *Borel-Écalle summation*. The cancellations that occur place enough constraints on the coefficients A_k and $a_{k,n}$ that simply knowing any one of the sums over n is enough to reconstruct the rest of the transseries. This phenomenon is called *resurgence* and the transseries where it occurs are referred to as *resurgent transseries*, originally studied in [130, 131], while the functions they represent are called *resurgent functions*. We refer the reader to [132, 133, 134] for an overview of the mathematical properties of these objects, and to [121, 135, 136, 137] for discussions of their relevance in physical contexts.

As a final aside, the most general form of a transseries involves logarithmic terms, which also occur frequently in physical contexts typically from quasi-zero modes, as well as allows for nesting of exponentials and logarithms. With this general form, the set of transseries is in fact closed under all the operations one typically encounters in a physical context, including but not limited to arithmetic operations, differentiation, integration, composition and functional inversion. The set of functions that can be represented by a transseries are known as *analyzable* functions, which is the closure of analytic functions under the same operations [132]. We will restrict our attention to transseries consisting of only powers and non-perturbative exponential terms for the remainder of this thesis.

3.2 Effective Theories as Truncated Transseries

Having presented an overview of transseries and their relationship to perturbation theory, we will now present a transseries based view of effective theories and dualities, and how one may use transseries as an organizing principle for classifying corrections in string theory. Let us first consider a theory that is defined in terms of some action S.

An important distinction to keep in mind when discussing the effective action is between the Wilsonian effective action and the 1PI quantum effective action,. The Wilsonian effective action, S_μ, is obtained by integrating out higher energy modes. It is defined by

$$\int \mathcal{D}\phi_L e^{-S_\mu[\phi_L]} = \int \mathcal{D}\phi_H \mathcal{D}\phi_L e^{-S[\phi_L,\phi_H]}, \tag{3.2.9}$$

where S is the original UV-complete action of the theory, μ is some energy scale and ϕ_L and ϕ_H are the modes below and above that scale respectively.[3]

The 1PI effective action Γ_{eff} on the other hand is defined by

$$\Gamma_{eff} = W[J] - \int d^D x J\phi \tag{3.2.10}$$

$$e^{-W[J]} = \int \mathcal{D}\phi e^{-S[\phi]+J\phi}. \tag{3.2.11}$$

Note that in this case the path integral has been performed and all the quantum effects have been incorporated. This effective action has the property that

$$\left.\frac{\delta\Gamma_{eff}}{\delta\phi}\right|_{J=0} = 0 \tag{3.2.12}$$

and thus provides an equation of motion for the fully quantum behavior of the field expectation values, also known as the Schwinger-Dyson equations.

In practice, of course, computing the full Γ_{eff} is hard and one typically computes a perturbative expansion for it, by expanding $S[\phi]$ in powers of a small parameter, such as a coupling, which then typically produces an asymptotic series for Γ_{eff}. Note that nothing like Γ_{eff}, even to a finite order is known for string theory.[4] Some corrections are known of course [65, 66, 141, 142, 143], but the list is incomplete even for the leading order.

We can combine these two notions of effective actions, by considering the quantum effective action one obtains from a Wilsonian action at some scale μ. Let us define,

[3] We should also assume a low-energy cutoff to remove any IR divergences. This cutoff is the same on both sides of (3.2.9).

[4] The distinction between the Wilsonian and the 1PI effective actions and the lack of the latter in string theory is emphasized, for example in [138, 139, 140], as part of an argument against the existence of a string landscape altogether. While we do not adopt this view here, we do agree with the necessity to be careful with this distinction. It is also possible that ideas related to resurgent transseries can help in circumventing the arguments of those papers.

$$\mathcal{S}(\mu, g, \phi) = -\log\left(\int \mathcal{D}\phi e^{-S_\mu[\phi_L]}\right) - \int d^D x J\phi \ , \tag{3.2.13}$$

where g is the set of couplings that appear in the theory. We will refer to this quantity as the *quantum effective action* from now on, while Γ_{eff} will be called the UV quantum effective action. $\mathcal{S}$ is an action whose equations of motion govern the expectation values of the light fields in the theory. As $\mu \to \infty$ this should approach the UV quantum effective action.

The Wilsonian action can be expanded in powers of μ/M where M is some physical mass scale of the theory, such as a heavy particle mass. This translates into an asymptotic expansion for the in powers of μ/M on top of any other weak coupling asymptotic expansion we may have used. Thus for low energies and weak couplings we can have an asymptotic expansion for our quantum effective action

$$\mathcal{S}(\mu, g, \phi) \sim \sum_{m,n} a_{m,n} g^m (\frac{\mu}{M})^n \qquad \frac{\mu}{M} \to 0, \quad g \to 0. \tag{3.2.14}$$

Note that at this point we have completely eliminated the mathematical distinction between the low-energy limit and the weak-coupling limits, despite their different physical origin. Furthermore, rather than expanding in some arbitrary coupling parameters, we can also expand the action in powers of the expectation values of fields, and thus obtain asymptotic expansions in $\langle\phi\rangle$ as well. This is particularly important in string theory, since it has no coupling "constants", but all the expansion parameters come from VEVs of the fields. We will therefore include any VEVs in the set of g's, while the fields ϕ should be thought of as fluctuations around these VEVs.

The object $\mathcal{S}$ should be well-defined for all values of μ, g and ϕ, even though the asymptotic expansion in (3.2.14) only serves as a good approximation in a certain limit.

At this point we make the leap of faith and *conjecture that $\mathcal{S}$ is an analyzable function of μ, g and ϕ and can therefore be completed to a resurgent transseries* in these parameters, as described in the previous chapter. This transseries is then well-defined for *all* values of the parameters and is an exact representation of $\mathcal{S}$ for all energies and couplings.

The same can be said about the variation of $\mathcal{S}$ with respect to the degrees of freedom.

$$\mathcal{C}_\phi = \frac{\delta \mathcal{S}}{\delta \phi}, \tag{3.2.15}$$

i.e. its quantum corrected equation of motion. This equation is the Schwinger-Dyson equation which governs the expectation values of the fields ϕ, having accounted for all the quantum mechanical effects. These also take the form of a transseries.

Note that the quantum equations of motion are enough to derive all the on-shell observables of the theory, without the need to formulate it in terms of an action. This might be important in the case of string theory, since the worldsheet description provides us with spacetime equations of motion, rather than an action. In general, given a set of EOMs, it is not always possible to construct an action from which they are derived. Thus although we will often be talking in terms of a quantum effective action, it should be understood that it really need not exist and that the quantum equations of motion that are the important objects. As long as the quantum EOM exist the theory should be well-defined.

We will then define an *effective theory* that describes the dynamics in our asymptotic regime, as any finite truncation of that transseries, whose error is below some desired precision. Note that as we leave the asymptotic regime, our ability to truncate the transseries gets compromised, even though the full transseries remains well-defined! This is what we will refer to as the *breakdown of EFT*. This notion of effective theories and their regimes of validity resonates very strongly with the spirit of the Swampland distance conjecture, although it's not clear that the non-truncation of the transseries needs to always be interpreted as the introduction of an infinite tower of massless states.

3.3 Breakdown of EFT in String Theory

A special feature of string theory is that it has no dimensionless free parameters. Instead every expansion parameter that appears in any of the controlled limits of string theory arise from expectation values of some degree of freedom. There is of course also a derivative expansion, which can be expressed in powers of an inverse mass scale, such as the Planck mass M_p. We now apply the reasoning presented in the previous subsection to the EFT's that appear in such theories.

Rather than being expressed in a transseries in some fixed coupling constants, the equations of motion in string theory should be expressed as a transseries in the expectation values of the various low-energy fields, combined with a transseries expansion in $1/M_p$ to account for any higher derivative corrections arising from integrating out the high-energy modes. This is not unusual even in field theory, where couplings and masses can arise from the VEV of a scalar field. In string theory, the genus expansion is also of this type, since the string coupling is dynamical.

The asymptotic regimes of string theory are then defined by a combination of the low-energy limit and a limit for the expectation values of some fields. For example the non-compact 10D flat space limit in which serves as the perturbative definition of string theory, can be regarded as the limit of fully compactified string theory as the compactification size goes to infinity. The expansion parameters of the transseries are then the (inverse) metric components that govern the size of the compactification. Note that since we are interested in the 10D equations of motion, the expansion must really be in the *local* value of the metric, point by point, rather than the total size of the cycle. The latter would then naturally emerge as an expansion parameter if we decided to switch to a lower-dimensional effective description and integrated over the compact space. In the higher-dimensional description, volumes of cycles should only appear in the effective action as a result of non-local effects, as we will see in the next chapter.

Thus, suppose we wish to study string theory in a regime where the some of degrees of freedom ϕ_i, with , have expectation values near a certain limit. This limit can be defined by choosing a set of expansion parameters $g_a \to 0$, $a = 1, 2, ..., N$, and specifying some asymptotic behavior for $\langle \phi_i \rangle$ as a function of the g_i. Note that *a priori* we do not need to choose as many expansion parameters as we have fields. Having the number of parameters equal to the number of fields specifies the behavior of each field individually and essentially amounts to a field redefinition. If we choose to have fewer expansion parameters, this amounts either to specifying some relation between the fields in our asymptotic limit, (i.e. restricting our attention to a set of configuration ansatzes) or simply leaving the asymptotic behavior of some fields unspecified, which we can further restrict later if needed.

Let us define

$$\frac{\mu}{M_p} = e^{\alpha_0}$$
$$g_a = e^{\alpha_a}, \tag{3.3.16}$$

where μ is the energy scale at which we are studying the theory, and let us denote the set of α_a by $\vec{\alpha}$. The effects of all the heavy degrees of freedom will then be captured by the expansion in M_p^{-1}. We are interested in the equations of motion that govern the expectation values of ϕ_i, i.e. the Schwinger-Dyson equations. These can usually be seen as arising from a quantum effective action, which we can write as the following transseries expanded around our asymptotic limit.

$$\mathcal{S} = \mathcal{G}\left(\sum_{\vec{m},k} \exp(-s_k/e^{\vec{\alpha}\cdot\vec{n}_k})\ e^{\vec{\alpha}\cdot\vec{m}}\ S^{(k,\vec{m})}\right) \tag{3.3.17}$$

where $\vec{m}, \vec{n}_k$ are N-dimensional vectors, $\vec{m}$ labels the various combinations of the expansion parameters that apperas in the perturbative series, k labels the non-perturbative contributions, with $k = 0$ corresponding to the perturbative sector and $\vec{n}_k$ denotes the particular combination of expansion parameters that appears in the exponent of that non-perturbative term, which may be the same for several non-perturbative contributions.[5], s_k and $S^{k,\vec{m})}$ are all functionals of the light fields in the theory, including the fluctuations of ϕ_i around their expectation values.

With an appropriate choice of expansion parameters, we can choose the components of $\vec{n}$ and $\vec{m}$ to be integers, and we can furthermore ensure that they are positive by factoring out the dominant asymptotic behavior into the prefactor $\mathcal{G}$. Let us denote the dominant asymptotic contribution to the action as $\mathcal{S}_0 = \mathcal{G}\ S^{(0,0)}$.

Note that simply specifying a set of expansion parameters and their limiting behavior does not specify what the behavior of $\mathcal{S}_0$ is. Determining this requires additional information about the physical properties of the system in that limit. By determining the behavior of $\mathcal{S}_0$, we are also establishing the separation between perturbative and non-perturbative effects,

[5]For example, all the multi-instanton saddles in a Yang-Mills theory carry an e^{-s_k/g_{YM}^2} prefactor. The s_k are different, but the power of g_{YM} is the same.

which is very important for properly organizing the various corrections and determining the regime of validity of any EFT truncation of the theory.

Let us now look at the equation of motion arising from $\mathcal{S}$, which takes the form

$$\begin{aligned} \sum_{k,\vec{m}} C_i^{(k,\vec{m})} &= 0 \\ C_i^{(k,\vec{m})} &= e^{-s_k/e^{\vec{\alpha}\cdot\vec{n}_k}}\ e^{\vec{\alpha}\cdot\vec{m}}\ \left(\frac{\delta S^{(k,\vec{m})}}{\delta\phi_i} - e^{-\vec{\alpha}\cdot\vec{n}}\frac{\delta s_k}{\delta\phi_i}\right). \end{aligned} \tag{3.3.18}$$

Here ϕ_i need no longer be only the fields that define our asymptotic limit, but the full set of light degrees of freedom that are dynamical at our energy scale. The main focus of the upcoming chapters will be to determine whether particular kinds of solutions are possible within a given asymptotic regime of string theory. The fact that all the expansion parameters in string theory are always related to the expectation values of fields means that when we choose a solution ansatz, we simultaneously specify a set of expansion parameters, and every combination of fields in the theory will have a definite scaling with respect to the expansion parameters. The same goes for their scaling with respect to the Planck mass, which is simply dictated by the mass dimension of the fields. We thus end up with precisely a set of equations of the form, with every term having a definite scaling with respect to all expansion parameters.

Let us suppose we chose an ansatz parametrized by fewer parameters than there are degrees of freedom. It is possible that some combinations of these degrees of freedom, which we will refer to "operators", despite working in a field formalism,[6] may remain independent of our expansion parameters, i.e. they scale as g_a^0 with respect to all the parameters. Here one of two things can happen, depending whether the number of such operators is finite or infinite.

When operators have zero scaling with respect to g_a, combining other operators with this finite set simply produces more operators with the same scaling, which generates additional terms in the EOM at the same order in g_a. If the set of such operators is finite, which can happen for example is a symmetry prevents higher powers of an operator from existing,

[6] By "combinations" we mean any form of product, contraction, or even non-local terms involving integrals, built out of some set of fields. We will also use the term "powers of operators", to refer to any possible cross-contraction of several copies of the constituent fields, rather than simple repeated multiplication.

then we simply have more terms in the EOM than we may have originally anticipated. In particular the leading order EOM might change and lead to new solutions.

If the set of such operators is infinite then this generates an infinite set of new terms in the EOM and would therefore not admit a finite truncation. However, since these operators are disctinct, they must differ by powers of some degrees of freedom. In this case, we may include the expectation values of these degrees of freedom as an additional expansion parameter. In other words, the presence of an infinite family of non-scaling operators is an indication that the original set of expansion parameters is not sufficient to define a single EFT limit. One we include the new expansion parameter, our ability to truncate depends on its expectation value. For example, let us denote a set of non-scaling operators,

$$O_i \sim \psi_i^{\Delta}, \tag{3.3.19}$$

where ψ is some field whose powers can be used to distinguish between these operators. Then $\langle\psi\rangle$ can be used as a new expansion parameter. For $\langle\psi\rangle \ll 1$ this generates a suppression for terms containing higher powers of ψ and allow for a truncation of the EOM and thus define an effective theory. However as $\langle\psi\rangle = \mathcal{O}(1)$, we see that all the terms creep back into the EOM and the EFT description breaks down.

The case that we will be interested in is the case when a particular type of solution ansatz violates the EOM when $\langle\psi\rangle = 0$, but including O_1 into the EOM would allow for solutions of this type. Turning on O_1 at the EOM level means turning on an expectation for ψ, which in turn turns on all the other operators O_i. Whether the EFT description survives then depends on whether the solutions to the EOM require $\langle\psi\rangle$ to be $\mathcal{O}(1)$ or not. If so, then although the solution may still exist, it is no longer within the regime of validity of the original EFT. Note that this notion of the breakdown of EFT is very much in the spirit of the swampland distance conjecture, although it's not clear that the family of corrections that enters the EOM in our case can always be recast as a massive tower of states becoming light. In the case when the family of corrections consists of higher derivative corrections, however, we expect there to be a direct relationship between our approach and the distance conjecture, which would be interesting to flesh out.

We conclude this chapter by considering an alternative representation of perturbative and non-perturbative corrections. The presence of dualities in string theory, particularly ones where an expansion parameter gets sent to its inverse might tempt us to leave non-perturbative corrections as simply additive terms with inverse powers of some expansion parameter. If there were finitely many such "polar" terms, this simply provides a new dominant behavior to the series and is simply an indication that we chose $\mathcal{G}$ wrong. Typically, however there are infinitely many such corrections in ever increasing inverse powers of the expansion parameter, and we may be temped to look for a representation in the form of a two-sided Laurent series.

To illustrate why this gets us into trouble when dealing with the asymptotic regimes where string theory is well-understood, let's consider a toy action described by one-parameter transseries, with parameter g.

$$\frac{\mathcal{S}}{\mathcal{G}} = \sum_m S^{(0,m)} g^m + \sum_k e^{-s_k/g^{n_k}} \sum_m S^{(n,m)} g^m. \tag{3.3.20}$$

In this representation, it's clear that $S^{(0,0)}$ dominates in the asymptotic limit $g \to 0$, followed by subdominant perturbative corrections, which in turn dominate over the non-perturbative corrections, which are themselves hierarchically organized. However, the non-perturbative exponentials can be expanded in a Taylor series[7] to transform the transseries into a two-sided Laurent series. We have

$$e^{-s_k/g^{n_k}} = \sum_p \frac{(-s_k/g^{n_k})^p}{p!}, \tag{3.3.21}$$

so that we can, at least formally, rewrite it as a two-sided Laurent series:

$$\frac{\mathcal{S}}{\mathcal{G}} = \sum_{m=-\infty}^{\infty} c_m g^m \tag{3.3.22}$$

$$c_m = \sum_{k=0}^{\infty} \sum_{p=0}^{\infty} S^{(k,m+pn_k)} \frac{(-s_k)^p}{p!}. \tag{3.3.23}$$

[7]The Taylor series for the exponential has infinite radius of convergence, so this expansion is valid even as $1/g$ becomes large.

To make sense of this object, we remark that for negative m, $1/p!$ typically wins over the growth of the $S^{(k,m+pn_k)}$, so the coefficients should be finite, and in fact factorially suppressed for large and negative m. Thus, at least the negative power side of the Laurent series (3.3.22) seems well-behaved. For positive m, as p becomes large, the combinatorial growth of the $S^{(k,m+pn_k)}$ will overpower the $1/p!$ so even a single coefficient appears to be divergent and in need of a summation prescription. We also see that $S^{(0,0)}$ plays a special role in the series, serving as the boundary between terms with finite coefficients and formally divergent ones.

However, this feature is simply a remnant of the fact that we started from a transseries representation, with a definite asymptotic behavior. If there is a suitable summation prescription for the c_m with positive m, and we were handed the final answers, then we'd simply be faced with a two-sided Laurent series with finite coefficients and no way to tell where the "zeroth" order is (recall that we have an overall unknown shift in the powers of g coming from $\mathcal{G}$). Furthermore, even if we were also told where the "zeroth" term is, the physical meaning of the c_m is still rather obscure, since it is related to the usual perturbative coefficients by (3.3.23) and thus isn't related to the result of any single perturbative calculation, such as a worldsheet scattering amplitude at fixed genus, in any straightforward manner. Thus, although a two-sided Laurent series representation may be possible and sensible, its coefficients can not be easily inferred from what we know about string theory in any asymptotic regime.

It is worth pointing out, however that the two-sided Laurent series representation seems particularly well suited for dealing with dualities, as it puts terms g and $1/g$ on similar footing. Thus the relation (3.3.23) may be useful for investigations of potential interplay between the resurgent transseries structure in both the small and large g regimes, combined with a duality relation between these regimes. We do not pursue this question here.

3.4 Chapter Summary

In this chapter we have reviewed certain features of perturbation theory and asymptotic expansions and argued that the resurgent transseries are not only a natural way to organize corrections around an asymptotic regime, but also continues to make sense outside of that regime. We then defined effective theories as finite truncations of this transseries, which provides a criterion for the regime of validity of an EFT and whether or not a particular solution ansatz resides within that regime.

In the following chapters we will use apply this approach to the problem of scale-separated AdS or dS compactifications. We will adopt a metric ansatz for such a compactification and ask what kinds of corrections to the supergravity equations of motion would allow for such a solution. This will involve determining the scalings of all combinations of curvature and flux components, which will naturally organize into a transseries structure. Upon determining the scalings we will assess whether our ansatzes fall within the regime of validity of low-energy large volume type IIB compactifications.

Chapter 4

Scale-Separation and Effective Field Theory

4.1 The Ansatz

4.1.1 Metric, Fluxes and Duality Chain

The primary object of our study will be a type IIB metric ansatz of the form

$$ds^2_{(IIB)} = -\frac{e^{\phi_B/2}}{\sqrt{h(y)}}\frac{1}{\Lambda\tilde{x}^2}\left(d\tilde{x}^2 + dx_1^2 + dx_2^2 + dz_1^2\right) + e^{\phi_B/2}\sqrt{h(y)}\;\tilde{g}_{mn}dy^m dy^n, \quad (4.1.1)$$

which describes a warped compactification to a Euclidean continuation of either de Sitter or anti-de Sitter space, depending on the sign of Λ, in string frame (hence the factor of $e^{\phi_B/2}$). The portion of the full spacetime covered by the coordinates we've used corresponds to either the so-called "flat slicing" of dS (which is most commonly used in cosmology), or the "Poincaré patch" of AdS, (which notably appears in the description of near-horizon geometries of extremal black holes/branes). Note that in the case of de Sitter space, $\tilde{x}$ corresponds to a time coordinate, while in the AdS case, this role is played by either x_1 or x_2, while $\tilde{x}$ becomes a "radial" coordinate, such that $\tilde{x} = 0$ is the conformal boundary of AdS.

As is usual for warped compactifications, we wish for the internal metric $\tilde{g}_{mn}$ to be such that $\int d^6y\sqrt{\tilde{g}} = 1$. Of course we expect that supporting this geometry, if at all possible, will require some non-trivial configuration of fluxes and branes, similar to those appearing in compactifications to Minkowski space.

In particular we expect there to be a spacetime filling 4-form potential

$$C_4 = \frac{c}{h\Lambda^2\tilde{x}^4} d\tilde{x} \wedge dx_1 \wedge dx_2 \wedge dz_1 \ , \tag{4.1.2}$$

as well as 3-form fluxes H_3, F_3 wrapping various 3-cycles of the internal space. These internal fluxes will generically act as sources for the spacetime-filling C_4 field and compactification requires that the total charge vanishes. This means that just like in Minkowski compactifications, we generically expect to have some anti-D3 charge present in the compactification, which can arise from actual anti-branes, provided we can stabilize them, or from induced anti-brane charge on D7 branes coming from worldvolume fluxes or topological higher-curvature terms. We will return to these contributions later.

We will wish to work in the M-theory description of this metric, using the duality chain described in section 2.2.3. This means we must compactify one of the "spatial" directions, specifically we will choose the z_1 direction. Note that this means that the size of the z_1 circle becomes small for large values of $\tilde{x}$. In this limit corrections coming from string winding modes become important and the natural description for the system becomes in terms of type IIA fields. At small $\tilde{x}$ however, the dynamics are well described by type IIB SUGRA plus corrections, and it is this limit that we will be interested in.

The full theory, however, does not care which duality frame we use to describe it, and we will therefore proceed with the M-theory uplift, first using Buscher's rules (2.2.25), to obtain the following type IIA metric.

$$ds^2_{(IIA)} \ = \ -\frac{e^{\phi_B/2}}{H(y)^2}\frac{1}{\lambda^2}\left(d\tilde{x}^2 + dx_1^2 + dx_2^2\right) + e^{-\phi_B/2}H^2\lambda^2 dz_1^2 + e^{\phi_B/2}H(y)^2\ \tilde{g}_{mn}dy^m dy^n, \tag{4.1.3}$$

where we have defined $\lambda = \sqrt{-\Lambda}\tilde{x}$ and $H(y) = h^{1/4}$, which then lifts to the following M-theory metric

$$ds^2_{(M)} = H^{-8/3}\lambda^{-8/3}\left(d\tilde{x}^2 + dx_1^2 + dx_2^2\right) + H^{4/3}\lambda^{-2/3}\tilde{g}_{mn}dy^m dy^n + H^{4/3}\lambda^{4/3}\delta_{ab}dz^a dz^b. \tag{4.1.4}$$

Our conventions for the coordinate names and indices will be that the 0 index, refers to the $\tilde{x}$ direction, x_i, x_j are the other two spacetime directions, $y_m, y_n, ...$ are the 6-manifold directions. and z_a, z_b refer to the torus directions.

Duality chasing the fluxes, we obtain a spacetime-filling flux

$$C_3 = \frac{c}{H^4\lambda^4} d\tilde{x} \wedge dx_1 \wedge dx_2, \tag{4.1.5}$$

and the internal fluxes dualize to G_4 fluxes wrapping 3-cycles of the internal 6-manifold and a one-cycle of the z_1, z_2 torus.

Any D3-branes present in the ansatz dualize to M2-branes located at the same point on the internal 6-manifold. The remaining ingredient to describe are any D7 branes or O7 planes that may be present. As mentioned in our discussion of compactifications to Minkowski space, D7 branes will generically be present in order to satisfy tadpole cancellation on a compact manifold, if we wish to turn on any non-trivial internal fluxes. We will therefore also allow for these objects in our ansatz. In fact, the scenario we will consider will be similar to the "constant coupling" scenario [70], where the 7-branes for stacks consisting of an O7-plane and four D7-branes. These stacks have no net charge and allow for the axio-dilaton to remain constant.

Going from IIB to IIA, we need to T-dualize the D7 branes into D6 branes. At this point, as discussed in chapter 2 simply applying Buscher's rules transforms the D7-branes into smeared D6-branes. Non-perturbatively, however, we expect the D6 branes to localize to a some point along the z_1 direction. This expectation is further reinforced by the fact that smeared O6 planes are physically nonsensical beyond the supergravity approximation, where they can simply be treated as sources.

The M-theory description of D6-branes and O6-planes are in terms of localized solutions, which we will henceforth refer to as "lumps", whose transverse metric is similar to an ALE or ALF space, with appropriate topological properties, that glues smoothly onto the rest of the manifold.

This means that the type IIA dual of our ansatz as well as its uplift to M-theory must be modified to

$$\begin{aligned} ds^2_{(M)} = &= H^{-8/3}\lambda^{-8/3}\left(d\tilde{x}^2 + dx_1^2 + dx_2^2\right) + H^{4/3}\lambda^{-2/3}g_{mn}dy^m dy^n \\ &+ H^{4/3}\lambda^{1/3}h_{ma}dy^m dz^a + H^{4/3}\lambda^{4/3}g_{ab}dz^a dz^b, \end{aligned} \tag{4.1.6}$$

with

$$\begin{aligned} g_{mn} &= \tilde{g}_{mn}(y) + h_{mn}(y,z) \\ g_{ab} &= \delta_{ab} + h_{ab}(y,z), \end{aligned} \tag{4.1.7}$$

where h_{mn}, h_{ab}, h_{ma} are the components describing the lump deformations of the metric dual to the D7/O7 stacks in the IIB picture. In the constant coupling scenario, these deformations are localized to a point, forming a D-type ADE singularity, which obscures their physics. We will therefore really be considering a slight deformation from the constant coupling scenario in which the corrections to the metric can have some spatial extent, but fall off rapidly, so that on most of the compactification manifold the metric is still essentially given by (4.1.4). We will revisit these lump objects when considering the equations of motion for our ansatz.

Before that, however we will make some remarks about the various expansion parameters that we have introduced by choosing our ansatz and the exact limits of these expansion parameters that we are considering.

4.1.2 Limits, Energy Scales and Expansion Parameters

In choosing our ansatz we have introduced several parameters that can serve as expansion parameters and organize the various corrections that may appear. Specifically, we have introduced the functions λ and H, which are related to the curvature along the spacetime directions and the volume of the compactification manifold respectively. The limit that interests us is the limit of small cosmological constant and large compactification volume, so it is natural to organize the corrections to the equations of motion as a trans-series in λ and H^{-1}. Note however that these parameters are coordinate dependent and it is not guaranteed that they are everywhere small. For a truncated series this would be a problem, since it would invalidate the truncation, however as discussed in chapter 3 the full trans-series continues to exist and make sense for all values of the parameters. Moreoever, the equations of motion will still need to hold term-by-term in the trans-series expansion, as well as point-by-point in space. This fact is what will allow us to determine that some variants of our ansatz are not realizable, despite not knowing the presice form of the corrections.

Besides these two parameters, there is a third parameter which is always present, which is the energy scale μ, at which we are studying our theory. The limit we wish to take is such that the energy scale is always below the Kaluza-Klein scale of the IIB compactification.

$$\begin{aligned} \frac{\mu}{m_p} &\to 0 \\ \frac{\mu}{M_{KK}} &\to 0, \end{aligned} \tag{4.1.8}$$

where m_p is the four-dimensional Planck mass and M_{KK} is the mass of the lightest Kaluza-Klein states. Note that strictly speaking this limit needs to precede the $H^{-1} \to 0$ limit, as taking $H^{-1} \to 0$ will eventually bring down the Planck and KK scale below any fixed energy scale we choose. Of course physically it is H that will be stabilized at some presumably large value, and we must choose the energy scale to be below the 4D Planck and KK scales.

Since we will be working with the M-theory description, it would be convenient to express all the mass scales in terms of the 11-dimensional Planck mass, which we'll denote by M_p. This is related to the type IIB Planck mass by

$$m_s = (\lambda H)^{1/3} M_p \tag{4.1.9}$$

$$M_B = e^{-\phi_B/4} m_s = e^{-\phi_B/4} (\lambda H)^{1/3} M_p, \tag{4.1.10}$$

where m_s is the inverse string length and M_B is the 10-d Planck mass in the type IIB descriptions.

The above relations appear to mix the energy expansion with the λ and H expansion when moving from the M-theory description to the IIB description, due to the $(\lambda H)^{1/3}$ that appears when passing from M_p to m_s. However, we must remember that the dimensionless expansion parameter is not the Planck mass *per se*, but its ratio to the energy scale at which we are studying the theory. The metric coefficients on the time coordinate in the IIB picture and the M-theory picture are different, which means that local observers in the different duality frames have a different notion of energy scale, since they have a different notion of proper time. The proper times are related by

$$d\tau_M = (\lambda H)^{1/3} d\tau_B, \tag{4.1.11}$$

where τ_M and τ_B are the proper time coordinates for the M-theory and type IIB metrics respectively. This defines a relation between the natural notions of energy scales for observers in each duality frame. The corresponding energy scales in these frames are related by

$$\mu_M = (\lambda H)^{-1/3} \mu_B, \tag{4.1.12}$$

in the sense that a mode of energy μ_B in the type IIB description is dual to a mode of energy μ_M in the M-theory description. From this relation we see that

$$\frac{\mu_M}{M_p} = \frac{\mu_B}{M_B} e^{\phi_B/4}. \tag{4.1.13}$$

In this way, the expansion in μ_B/M_{IIB} is identical to the expansion in μ_M/M_p. Since we will always be working in the M-theory frame, we will set μ_M to one, and our expansion parameter will then simply be M_p^{-1} with the low-energy limit corresponding to $M_p \to \infty$.

So at the end of the day, we expect the full quantum equations of motion to be represented by a trans-series in the following three parameters

$$\lambda, \quad H^{-1}, \quad M_p^{-1}. \tag{4.1.14}$$

An EFT description of our ansatz is then possible if these equations of motion, evaluated on our ansatz, can be successfully truncated in the sense described in chapter 3.

4.2 Classical Equations of Motion

We now return to the analysis of the equations of motion evaluated on our ansatz. At the two-derivative level we will see that the introduction of a non-zero spacetime curvature immediately results in a problem with the Einstein's equations that is absent in compactifications to flat space, namely that one of the terms has a different H-scaling from any of the other terms.

Setting $e^{\phi} = 1$ for convenience, the Einstein tensor for the metric (4.1.4) is given by [19]

$$G_{ij} = -\frac{\delta_{ij}}{\lambda^2}\left(\frac{\tilde{R}}{2H^4} + \frac{4\tilde{g}^{pq}\partial_p H \partial_q H}{H^6} - \frac{\Box H^4}{H^8} + 3\Lambda\right) \tag{4.2.15}$$

$$G_{mn} = \tilde{G}_{mn} - \frac{8\partial_m H \partial_n H}{H^2} + \tilde{g}_{mn}\left(\frac{4(\partial H)^2}{H^2} - 6\Lambda H^4\right) \tag{4.2.16}$$

$$G_{ab} = \delta_{ab}\lambda^2\left(-\frac{\tilde{R}}{2} - 9\Lambda H^4 + \frac{4\tilde{g}^{pq}\partial_p H \partial_q H}{H^2}\right), \tag{4.2.17}$$

where $\tilde{G}_{mn}$ is the unwarped Einstein tensor for the internal space. Meanwhile the stress tensor for the fluxes is

$$T_{ij} = -\frac{\delta_{ij}}{\lambda^2}\left(\frac{4(\partial H)^2}{H^6} + \frac{\tilde{G}_{mnpa}\tilde{G}^{mnpa}}{4!H^8}\right) \tag{4.2.18}$$

$$T_{mn} = \frac{4\tilde{g}_{mn}(\partial H)^2}{H^2} - \frac{8\partial_m H \partial_n H}{H^2} + \frac{1}{4H^4}\left(\tilde{G}_{mpqa}G_n{}^{pqa} - \frac{1}{6}\tilde{g}_{mn}\tilde{G}_{pqra}\tilde{G}^{pqra}\right) \tag{4.2.19}$$

$$T_{ab} = \frac{\lambda^2}{12H^4}\left(\tilde{G}_{mnpa}\tilde{G}^{mnp}{}_b - \frac{1}{2}\delta_{ab}\tilde{G}_{mnpc}\tilde{G}^{mnpc}\right), \tag{4.2.20}$$

where $\tilde{G}^{mnpa}$ has its indices raised with the unwarped metric. We also included the 0 index in the i, j components of the above expressions. The exact coefficients and tensor contractions are in fact not very important for our main point. What is important is the scalings of each term with respect to λ, H and M_p. The latter is, of course, the same everywhere, since these are the two-derivative equations of motion, so all the terms scale as M_p^{-2}.

For the other scalings, more care is required, since we have not yet specified whether the functional forms of the fluxes G_{mnpa} contain any factors of λ or H. These are not arbitrary, but are constrained by the flux equation of motion, which is

$$d \star G = G \wedge G + X_8 \tag{4.2.21}$$

where X_8 is, as discussed in chapter 2, a topological quantity constructed out of the first and second Pontryagin classes of the 8-manifold. For the metric (4.1.4), it can be shown to vanish identically, due to the presence of the torus factor. The only non-vanishing contributions appear from the corrections to the metric in the vicinity of the lumps corresponding to the D7/O7 stacks in the IIB description. This is expected and is precisely what is needed to cancel the total charge over the compact manifold. Indeed, if we integrate over the 8-manifold, $d \star G$ integrates to zero and we require

$$\int G \wedge G + \int X_8 = 0 \tag{4.2.22}$$

It is imperative that both terms in this equation have the same λ-scaling for the tadpole cancellation to hold at all values of $\tilde{x}$ and we will see in the next section that this is indeed the case.[1] That being said, since X_8 only has support near the lumps it must be the case that everywhere else on the manifold we essentially have

$$d \star G \quad = \quad G \wedge G \tag{4.2.23}$$

Note that in order for this equation to actually provide a constraint, we need the left-hand side to not vanish. This happens when $\Box H \neq 0$, i.e. the geometry contains warped throats with non-trivial warping supported by fluxes [144]. This is in contrast to compactifications where the only source of warping is M2-branes and $\Box H$ vanishes everywhere except the M2-loci. Since warped throats are useful features to have for phenomenological purposes and are fairly generic, we will assume that $d \star G_{(ext)}$ does not identically vanish.

[1] One can also write down higher derivative non-topological terms contributing to the flux EOM, but their non-topological nature means they will only contribute total derivatives to (4.2.21) and therefore not contribute to tadpole cancellation. We will investigate such terms in more detail in the next chapter.

Denoting

$$\begin{aligned} G_{(1)} &= G_{0ijm}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dy^m \\ G_{(2)} &= G_{mnpa}\, dy^m \wedge dy^n \wedge dy^p \wedge dz^a \\ G_{(3)} &= G_{qrsb}\, dy^q \wedge dy^r \wedge dy^s \wedge dz^b \end{aligned} \tag{4.2.24}$$

we can determine their scalings as follows. Assuming the fluxes have generic scalings [2]

$$G_{(i)} \sim \lambda^{a_i} H^{b_i} \qquad i = 1, 2, 3 \tag{4.2.25}$$

$$\tag{4.2.26}$$

We have

$$G_{(i)} \wedge G_{(j)} \sim \lambda^{a_i + a_j} H^{b_i + b_j} \tag{4.2.27}$$

$$d \star G_1 \sim \lambda^{a_1 + 4} H^{b_1 + 8} \tag{4.2.28}$$

$$d \star G_2 \sim \lambda^{a_2 - 4} H^{b_2 - 4}. \tag{4.2.29}$$

The leading order flux equations of motion then read

$$\begin{aligned} d \star G_{(1)} &= G_{(2)} \wedge G_{(3)} \\ d \star G_{(2)} &= G_{(1)} \wedge G_{(3)}, \end{aligned} \tag{4.2.30}$$

which imposes the conditions

$$\begin{aligned} a_3 = 0 \quad b_3 &= 2 \\ a_2 &= a_1 + 4 \\ b_2 &= b_1 + 6 \end{aligned} \tag{4.2.31}$$

[2] Note that the scalings here should in principle be regarded as the leading order scalings in an expansion in λ and H^{-1}. For now we assume that the fluxes have a unique scaling behavior, and we will insist that this is always the case for the external flux G_{0ijm} throughout the rest of our analysis, since that is part of our ansatz. For the other flux components, however, we will consider the full expansion in the next chapter where we include additional time-dependences to all the ingredients.

and since $a_1 = b_1 = -4$, we have

$$G_{(2),(3)} \sim \lambda^0 H^2. \tag{4.2.32}$$

Looking back at the stress tensor we can see that for each individual component all the terms scale the same way once we take these scalings into account. Furthermore, we can see that the corresponding components of the Einstein tensor contain mostly terms that scale in the same way as well and so can therefore successfully cancel with the right choice of internal metric.

For example we see that T_{mn} scales overall as $\lambda^0 H^0$, as do most terms in G_{mn}. However there is one term in G_{mn} which carries an additional power of H^4 and can therefore not be cancelled by the available two-derivative stress tensor. This term is proportional to Λ and is absent in Minkowski compactifications. This is the reason Minkowski compactifications can be achieved almost at the two-derivative level, modulo global constraints coming from tadpole cancellation.

The appearance of this extra term, however, forces us to look for new terms that could cancel it. This requires an investigation of the scalings of the various possible quantum terms in the equations of motion, which is the topic of the next section.

4.3 Scaling Analysis

In this section, we perform a systematic analysis of the possible corrections to the 11-dimensional equations of motion, and specifically of the scalings these terms have with respect to λ, H and M_p when evaluated on our ansatz. We will start by revisiting the classical two-derivative terms to introduce some machinery and short-hand notation. Then we will consider higher-derivative corrections built out of bulk curvatures, fluxes and their derivatives, and show that they do not have the correct scalings to make our ansatz viable. We will then turn our attention to the localized curvatures and fluxes coming from the lump regions, where we will find a new set of terms with different scalings, including an infinite family of terms which all have the same λ scaling. In particular we will verify the scaling of the X_8 term that we mentioned in the previous section. Unfortunately these terms will also not be of immediate use, both due to their scalings and their localized nature. Finally we will study non-local terms, which take the form of integrals of fields at separate points joined by some non-locality functions. We will see that a suitable choice of non-local correction can give us precisely the scaling we require to cancel the spacetime curvature term in the EOM. These corrections will have a natural interpretation as coming from M5-instantons wrapping a four-cycle of the internal manifold as well as the torus.

4.3.1 Bulk Classical and Perturbative Terms

Although we have already written down the two-derivative Einstein's equations explicitly, let us calculate their scalings again to illustrate some generalities and introduce some shorthand notation that will be used through the rest of this chapter. We will only investigate the "bulk" equations of motion here, in the regions away from the localized objects, where the metric is given by (4.1.4). We will return to the corrections coming from localized objects further below.

The terms in Einstein's equations are obtained by taking completely contracted curvatures and fluxes and varying with respect to the metric. Due to the warped product structure of our ansatz, there are separate equation of motion components for each subspace with the other components vanishing identically. The resulting scaling is that of the corresponding term in the action, divided by the scaling of the metric component that is varied. Since the different subspaces have different scaling metrics, it is convenient to consider the equations of motion with one index raised back up, to cancel the scaling from this metric component.

$$G^M_N - T^M_N = \frac{1}{2\sqrt{g}} g^{MP} \frac{\delta S}{\delta g^{PN}} = 0 \tag{4.3.33}$$

This way, the scalings of the terms in the EOM are the same as the scalings of the corresponding fully contracted terms in the action from which they are derived. This means that for every flux or curvature component we simply need to calculate what its scaling will be after all its indices are contracted, either among themselves or with another field.

Let us introduce some notation to keep track of the different kinds of components we will encounter. For the fluxes, we have the spacetime-filling flux, as well as internal fluxes wrapping 3-cycles of the 6-manifold and a 1-cycle along the torus. We will use standard index notation when we wish to refer to the uncontracted components. In this way we have

$$G_{0ijm} \sim \lambda^{-4}H^{-4}M_p^{-1} \tag{4.3.34}$$
$$G_{mnpa} \sim \lambda^{0}H^{2}M_p^{-1} \tag{4.3.35}$$

If we wish to refer to these components when they appear inside fully contracted combinations, or as part of the Einstein's equations in the form (4.3.33), we will denote them by $G_{(ext)}$ and $G_{(int)}$ respectively. To calculate the contribution of any flux component to the total scaling of any fully contracted term, we simply need to multiply by the square root of the inverse metric scaling for each of its indices, i.e. by the inverse vielbein scaling. Doing so, yields

$$G_{(ext)} \sim \lambda^{1/3}H^{-2/3}M_p^{-1} \tag{4.3.36}$$
$$G_{(int)} \sim \lambda^{1/3}H^{-2/3}M_p^{-1} \tag{4.3.37}$$

For the curvatures, the analysis is a little bit more subtle. The general schematic form for the Riemann tensor with two upper indices is

$$\mathcal{R} = g^{-1}\partial g^{-1}\partial g + g^{-1}(g^{-1}\partial g)^2. \tag{4.3.38}$$

The warped product structure of our ansatz guarantees that at the two-derivative level the curvature components that contribute to the EOM will have indices on the derivatives that belong to the same subspace, otherwise the indices to not fully contract. At higher derivatives, when the Riemann tensor can contract with other factors we will also have contributions from components with mixed derivatives. We will denote these three types of curvature components by

$$R_{(00)}, R_{(yy)}, R_{(0y)}, \tag{4.3.39}$$

respectively. The subscripts in parentheses only denote the directions along which the derivatives act and are not to be confused with free indices of any curvature tensor.

Since the metrics have both $\tilde{x}$ and y dependence, we need to determine how the derivatives affect their scalings.

First note that all the $\tilde{x}$ dependences in our ansatz are power laws. For any quantity X that depends polynomially on $\tilde{x}$, we have

$$\partial_0 X \propto \frac{1}{\tilde{x}} X = \sqrt{-\Lambda}\ \lambda^{-1} X \tag{4.3.40}$$

The 0 index will eventually be contracted by an inverse metric, so the overall effect of each derivative on the scaling is to introduce a factor of $\sqrt{-\Lambda}\lambda^{1/3}H^{4/3}M_p^{-1}$. This gives,

$$R_{(00)} \sim -\Lambda\lambda^{2/3}H^{8/3}M_p^{-2}. \tag{4.3.41}$$

For the y dependence, some of it appears in the unwarped 6d metric, and outside of that, all other y-dependence is encoded in the H-dependence, which is also always polynomial. This means we have

$$\partial_m g_{MN} \propto \frac{\partial_m H}{H} g_{MN} + ... \tag{4.3.42}$$

where "..." denotes terms where the derivative hits the unwarped metric and thus doesn't alter the H scaling. By diffeomorphism invariance, we expect that the combinations of derivatives hitting the metric to assemble into powers of $\Box H$ or $|\nabla H|^2$ and other covariant combinations. So the overall effect of these derivatives will be to give additional factors of

$$\frac{\Box H}{H} \quad \text{or} \quad \frac{|\nabla H|^2}{H^2} \quad \text{etc.} \tag{4.3.43}$$

These quantities can be big or small in different locations on the internal manifold, even if H is always large, and the equations of motion will imply additional conditions on the y-dependence of the flux terms or on the functional form of $H(y)$ itself. This is similar to the conditions that relate the external flux in terms of the warp factor for Minkowski compactifications. Factoring out these quantities, the remaining H-scaling is the same as that of the term inside the derivative. Upon contraction, the derivative will pick up a factor of the inverse vielbein scaling, so the overall scaling contribution of y-derivatives is $\lambda^{1/3}H^{-2/3}$

Thus, up to a factor of some function $f(\frac{\nabla H}{H}, \frac{\Box H}{H}, ...)$, the curvatures with y_m derivatives scale as

$$R_{(yy)} \sim \lambda^{2/3}H^{-4/3}M_p^{-2}. \tag{4.3.44}$$

Thus, the overall scaling contribution from the curvature terms ends up simply being that of the inverse metric for the subspace along which the derivatives act. This remains the case for terms with mixed derivatives, from which we obtain

$$R_{(0y)} \sim \lambda^{2/3}H^{2/3}M_p^{-2}, \tag{4.3.45}$$

although as we pointed out, this curvature component does not contribute to the equations of motion at the two-derivative level.

Here we can once again clearly see the problem that arises due to non-zero Λ, which we described earlier. $R_{(yy)}$ has the same scaling as $G^2_{(int)}$ and $G^2_{(ext)}$, which in principle allows them to cancel against each other, while the $R_{(00)}$ term is non-vanishing and has a different H scaling, meaning it has nothing to cancel against. This forces us to go beyond the two-derivative equations of motion and look for corrections which can cancel $R_{(00)}$, while still allowing the series of corrections to be truncated.

Perturbative corrections will be composed of various contractions of the fluxes and curvatures as well as their derivatives, i.e. they will be of the form.

$$g^{MN}\frac{1}{2\sqrt{g}}\frac{\delta}{\delta g^{MN}}\left(\sqrt{g}(g^{-1})^{(p_1+p_2)/2+2m+2n}(\nabla)^{p_1}(G)^m(\nabla)^{p_2}(R)^n\right), \tag{4.3.46}$$

where the factors (G) and (R) can represent any of the fluxes and curvatures present in the ansatz, the derivatives are meant to be distributed among the factors in some fashion and the inverse metrics contract all the indices. The total scaling of any such term will simply be the product of all the scalings of the individual factors, since the scalings are insensitive to how the indices are contracted, and the scalings of $\sqrt{g}$ before and after the variational derivative cancel out. We have done all the work required to compute the scaling of any higher derivative correction to the equations of motion.

We summarize the scalings of all the ingredients that make up the perturbative terms in table 4.1.

Field	scaling
$R_{(00)}$	$-\Lambda\ \lambda^{2/3}\ H^{8/3}\ M_p^{-2}$
$R_{(0y)}$	$\sqrt{-\Lambda}\lambda^{2/3}\ H^{2/3}\ M_p^{-2}$
$R_{(yy)}$	$\lambda^{2/3}\ H^{-4/3}\ M_p^{-2}$
$G_{(ext)}$	$\lambda^{1/3}\ H^{-2/3}\ M_p^{-1}$
$G_{(int)}$	$\lambda^{1/3}\ H^{-2/3}\ M_p^{-1}$
∇_m	$\lambda^{1/3}\ H^{-2/3}\ M_p^{-1}$
∇_0	$\lambda^{1/3}\ H^{4/3}\ M_p^{-1}$

Table 4.1 Scaling contributions from bulk fields after complete contraction.

One very important observation here is that all the λ scaling contributions are positive. This means that global higher derivative terms, when evaluated on our ansatz, are always accompanied by additional powers of λ and can therefore never cancel $R_{(00)}$ in the equations of motion. Moreover, since we are looking at the $\lambda \to 0$ limit, all of these higher order corrections will become small. We are thus forced to look to the other corrections that may be present, or to modify the ansatz, in the hope that additional λ dependences in the metric or fluxes can produce terms with the correct scaling. We will explore the latter option in the next chapter. For now we turn to the study of the other corrections that are present in our ansatz.

4.3.2 Localized Terms

So far we were only concerned with the global curvatures and fluxes, that are present in the regions where the metric is well approximated by (4.1.4). In this section we turn to the curvatures and fluxes which are only present in the vicinity of the lumps that are dual to the D7/O7 stacks in the IIB compactification. In the strict "constant coupling" scenario, these lump solutions degenerate into a D-type ADE singularity. To get a handle on the physics we will deform slightly away from this strict limit [3], which resolves the singularity and gives us the lump solution that corrects the metric (4.1.4) into (4.1.6). We will denote the strict

[3] in general we expect quantum fluctuations to do this for us anyway

constant coupling scenario manifold by $\tilde{\mathcal{M}}$ and the manifold with the singularities resolved by $\mathcal{M}$.

The uplifts of separate D6 and O6 branes in asymptotically flat space are known. The transverse metrics are described by gravitational solitons, specifically the Taub-NUT and the Atiyah-Hitchin metric, for the D6 and O6 respectively. A metric for a D6/O6 stack in flat space can then be obtained via harmonic superposition. In our ansatz, the transverse space is not asymptotically flat, so the exact metric of the lump solutions will be deformed. Furthermore, the space is compact, so we will require that the characteristic size of the lumps is much smaller than the size of the transverse manifold[4], so that the corrections to the metric (4.1.4) fall off fast enough that most of the manifold is well-described by the uncorrected metric.

If we take the manifold $\tilde{\mathcal{M}}$, we can introduce a radial coordinate near the lumps. A natural choice would be the proper distance from the orbifold point in the strict orbifold limit, as computed by the metric on $\tilde{\mathcal{M}}$, which we will call ρ, given by

$$\rho = M_p \int_\gamma w, \tag{4.3.47}$$

where w is the vielbein tangent to the geodesic γ connecting our point to the orbifold point.

Far from the orbifold points, the metrics on $\tilde{\mathcal{M}}$ and $\mathcal{M}$ agree, and the radial coordinate is still a good coordinate. The resolution of the singularity has a characteristic size, which we can call ρ_0 where the the non-trivial fibration of the torus over the base manifold becomes convoluted and strongly dependent on the exact details of the singularity resolution, and ρ stops being a good coordinate. For $\rho > \rho_0$, however, we can reasonably expect that the difference between the metric on $\tilde{\mathcal{M}}$ and $\mathcal{M}$ is described at leading order by a power law dependence in $1/\rho$, by analogy with the asymptotically flat solutions.

The radial direction is related to our usual coordinates on $\tilde{\mathcal{M}}$ by

$$d\rho = \lambda^{-1/3} H^{2/3} M_p dy_{5,6}, \qquad \theta = \pi/2, \tag{4.3.48}$$

$$d\rho = \lambda^{2/3} H^{2/3} M_p dz_1, \qquad \theta = 0, \tag{4.3.49}$$

[4]This, in particular, requires small type IIB string coupling.

where $y_{5,6}$ represents the two y-coordinates transverse to the D6/O6 stack, while θ is the latitude coordinate on the S^2 surrounding the orbifold points in $y_\perp, z_1$ space, given by

$$\theta = \arctan \frac{z_1}{\sqrt{y_5^2 + y_6^2}}. \tag{4.3.50}$$

The z_2 circle is the M-theory circle, which becomes non-trivially fibered as we approach the stack and pinches off at the locations of the D6-branes in a smooth fashion.

Note that for power-law functions $f(\rho)$, we have

$$\partial_m f(\rho) = \frac{\partial \rho}{\partial y^m} \frac{df}{d\rho}, \tag{4.3.51}$$

and the extra scalings appearing in (4.3.48) cancel out, so the y_m derivatives acting on the localized functions behave the same way as on bulk fields, i.e. their scaling contribution comes only from contracting their indices. The new feature is the appearance of z_a derivatives, whose contribution also only comes from from their index contraction, but since the torus metric has a different scaling, we end up with a new set of contributions to the curvature tensor:

$$R_{(zz)} \sim \lambda^{-4/3} H^{-4/3} M_p^{-2} \tag{4.3.52}$$

$$R_{(yz)} \sim \lambda^{-1/3} H^{-4/3} M_p^{-2}, \tag{4.3.53}$$

where the subscripts, once again, denote the subspaces along which the derivatives act. Note that the scalings of these curvature terms are different from all other two-derivative terms, which means that their contributions to the two-derivative EOM must vanish independently. This is simply the statement that the lump geometry must be a solution to the equations of motion in its own right. Provided that is the case, only different contractions of these curvature components can appear within higher derivative corrections. Of particular interest are the combinations

$$R_{(zz)}R_{(yy)}R_{(yy)} \sim \lambda^0 H^{-4} M_p^{-6} R_{(yz)}R_{(yz)}R_{(yy)} \sim \lambda^0 H^{-4} M_p^{-6}, \tag{4.3.54}$$

which are λ-independent and thus do not vanish in the IIB limit of $\lambda \to 0$. Thus, these combinations of curvatures are precisely examples of the "non-scaling operators" that were discussed in chapter 3. We can combine them with any of the other curvature terms that are present at the two-derivative level, and obtain terms with the desired $\lambda^{2/3}$ scaling. The same goes for including higher powers of these terms. Unfortunately, in terms of the other expansion parameters, we face the disappointing observation that these terms still have negative H-scaling, while the $R_{(00)}$ we are trying to cancel requires an overall positive H-scaling. Nonetheless, we will keep these terms in mind, as we will see that they can be used to dress up other terms.

A further set of λ-independent terms can be obtained from transverse derivatives acting on localized fluxes. In addition to the localized metric corrections, the lumps admit localized 2-forms ω_{ma} which can be used to construct localized fluxes of the form:

$$G^{(loc)}_{mnpa} = F_{mn} \wedge \omega_{pa} \tag{4.3.55}$$

where the y_m, y_n coordinates are parallel to the D6/O6 worldvolume, and y_p and z_a are transverse to it.

The intrinsic scalings of these localized fluxes are constrained by the same equation of motion as the bulk fluxes and thus have the same H^2 scaling before contraction.[5]

The exact form of ω once again depends on the exact details of the singularity resolution. However, by looking at the known form of their asymptotically flat cousins [145], we can once again expect that it vanishes away from the lumps in a power-law fashion, so transverse z_a derivatives acting on it also provide new scalings.

In particular the combination

$$(\nabla_{(z)} G_{(loc)})^2 \sim \lambda^0 H^{-4/3} M_p^{-2} \tag{4.3.56}$$

[5] Although note that the functional dependence must now match only along the brane worldvolume. The transverse dependence of the localized fluxes is encoded in $f(\rho)$.

scales the same way as $R_{(zz)}$. So we can replace any $R_{(zz)}$ with $(\nabla_{(z)}G_{(loc)})^2$ without changing the scalings of the corrections. Similarly we can replace $R_{(yz)}$ by $\nabla_{(y)}G_{(loc)}\nabla_{(z)}G_{(loc)}$.

Let us remark that the terms that we have described here do not seem related to the expansion of the DBI action. Their distinct nature can be seen by observing that the terms in the DBI action are related to the derivative expansion along the worldvolume directions, whereas the terms we have described contain orthogonal derivatives. The presence of derivatives along the orthogonal directions in these terms appears to probe the effective thickness of the branes, which is related to the string coupling, leading us to interpret these corrections as loop corrections to the effective brane action.

At this point, we can complete the analysis of the tadpole cancellation conditions (4.2.22) by finding the scaling of X_8 in the vicinity of the brane stacks. The non-trivial fibration of the M-theory circle results in a vielbein for the form

$$w = f(dz_2 + g dy_n), \tag{4.3.57}$$

where f, g are functions of $z_1, y_{5,6}$. The non-vanishing of X_8 can be traced back to the fact that

$$dw = df \wedge (dz_2 + g dy_n) + f(dg \wedge dy_n), \tag{4.3.58}$$

which contains wedge products that don't include the z_2 coordinate. So the non-vanishing part of X_8 contains exactly the curvature terms coming from the transverse derivatives, and in particular must contain either one factor of $R_{(zz)}$ or two factors of $R_{(yz)}$ to form a non-vanishing wedge product.

The scaling of X_8 can then be computed by taking the fully contracted scalings of $R^3_{(yy)}R_{(zz)}$, or $R^2_{(yy)}R^2_{(yz)}$ and "uncontracting" 8 indices, by multiplying by the vielbein scaling for each metric component. This gives

$$(X_8)_{mnpqrsab} \sim M_p^{-8}\left((\lambda^{2/3}H^{-4/3})^3\lambda^{-4/3}H^{-4/3}\right)\left(\lambda^{-2/3}H^{16/3}\right) = \lambda^0 H^4 M_p^{-8}, \tag{4.3.59}$$

where the first parenthesis is the fully contracted scaling, and the second is the scaling of

$\sqrt{g_8}$. The result is in perfect accordance with the $\lambda^0 H^2$ scaling of the bulk and localized G_{mnpa} fluxes. The discrepancy in the powers of M_p corresponds to the fact that it is a higher derivative correction, and the magnitude of the resulting fluxes themselves must be suppressed by a factor of M_p^{-3}. This is in accordance with the flux-quantization condition, which requires

$$\frac{1}{2\pi}\int G = \frac{1}{2\pi}M_p^4 \int d^3y dz \sqrt{g_4}\epsilon^{mnpa}G_{mnpa} = n + \frac{p_1(R)}{2} \qquad n \in \mathbb{Z}, \tag{4.3.60}$$

where $p_1(R)$ is the first Pontryagin class of the 4-cycle. We see that the scaling of the components of G as M_p^{-3} combined with its intrinsic M_p^{-1} scaling cancels precisely the factor of M_p^4 coming from the integration measure, which allows the integral to take order one values.

4.3.3 Non-local Terms and the Transseries Structure

We now turn to the most exotic type of corrections that we will consider in this chapter, namely non-local corrections. Although we are interested in corrections at the EOM level, let us start by considering corrections to the effective action of the form

$$S^{(nloc)} = M_p^{22}\int d^{11}xd^{11}x'\sqrt{g(x)\ g(x')}O(x)G(x,x')O'(x'), \tag{4.3.61}$$

where O and O' are some completely contracted products of fields and $G(x,x')$ is some appropriate non-locality function, which depends on the physical nature of the correction. The exact form of the allowed operators O and functions G will of course be constrained by the various symmetries of the theory, as is the case with the local corrections. We can further generalize this construction, by allowing the operators O, O' to themselves be further non-local resulting in corrections of ever-increasing degree of non-locality, involving nested integrals of chains of operators connected by non-locality functions.

Taking G to be a normalized delta function, we recover the local term $(OO')(x)$, provided the operators O, O' are local themselves. If instead G is completely delocalized along certain directions, we obtain contributions that can be interpreted as configurations of extended objects, including spacelike configurations, such as brane-instantons. Note that the integral contributes positive powers of M_p, and unless the operators O, O' contain enough derivatives, the overall power of M_p in the non-local corrections will be positive. This would imply that these terms should dominate in the low energy limit. Moreover, the corrections with higher degrees of non-locality would appear to dominate over corrections with lower degrees of non-locality resulting in an infinite family of corrections entering in the far IR limit and thus fail to reproduce 11-dimensional supergravity as the low-energy limit of M-theory.

The way out of this conundrum is to demand that the positive powers of M_p should actually resum into the non-perturbative exponentials that appear in the action as

$$\begin{aligned} S^{(nloc)} &= M_p^{11} \int d^{11}x \sqrt{g(x)}\, O(x) \left(a_1 e^{-\mathcal{I}(x)} + a_2 e^{-2\mathcal{I}(x)} + ...\right) \\ \mathcal{I}(x) &= \int d^{11}x' \sqrt{g(x')} G(x,x') O'(x'). \end{aligned} \tag{4.3.62}$$

This has the desired behavior as $M_p \to \infty$ and results in the action having exactly the transseries structure described in chapter 3. One can view this requirement as a mathematical trick to obtain the desired behavior in our asymptotic regime, but it also has the usual physical interpretation as arising from a dilute gas of brane-instantons or other non-perturbative effects. Note that the exponential terms can be evaluated at any point, which is a reflection of the fact that despite the brane-instantons being partially localized, we must also perform the path integral over all its transverse positions, so their effect can be felt everywhere in the internal manifold, properly weighed by the amplitude of the extended object to appear in that location.

Note that in principle, we might wonder if we can allow for some of the non-local terms to remain un-exponentiated, which would correspond to some deep IR corrections to 11-dimensional supergravity. Provided we only leave finitely many such terms, or find some way to suppress or remove the higher degrees of non-locality, this may be allowed, and would result in a theory whose deep IR behavior can differ from 11-d SUGRA. For now we will assume that 11-d SUGRA is all there to M-theory at low energies, but we will return to this possibility in the next chapter.

The stress tensor coming from the leading non-perturbative contribution in (4.3.62) is

$$T^M{}_N(x) = g^{MP}\left(\left(\frac{\delta O(x)}{\delta g^{PN}(x)} - g_{PN}O(x)\right)e^{-\mathcal{I}(x)} - \int d^{11}x'\sqrt{g}\; O(x')\frac{\delta\mathcal{I}(x')}{\delta g^{PN}(x)}\; e^{-\mathcal{I}(x')}\right). \tag{4.3.63}$$

We see that the first term is simply a small non-perturbative shift to the perturbative stress tensor coming from $O(x)$, while the second term is new and has the same scaling as $O(x)\mathcal{I}(x)$, which is lower order in M_p^{-1} due to the positive scaling of $\mathcal{I}(x)$, which makes it the dominant non-perturbative contribution. The overall scaling of this new term depends on the scalings of the operators and non-locality functions that appear in $\mathcal{I}(x)$.

Thus we have determined the contributions from non-local corrections to the equations of motion. Requiring that the low-energy limit reduces to the supergravity equations, makes these corrections take on the trans-series structure above. We will now consider a particular example of such a correction and show that it is precisely the type of correction that we need to satisfy the equations of motion to get a scale-separated solution with non-zero Λ.

4.3.4 M5-instantons, de Sitter Uplift and EFT Breakdown

A particularly simple example of a non-local correction of the sort described above is:

$$\mathcal{I}(x') = M_p^{11}\int d^4y_{\parallel}d^2z\sqrt{g_6}\int d^3x d^2y_{\perp}\sqrt{g_3 g_2}\,\delta^{(3)}(x-x')\delta^{(2)}(y_{\perp}-y'_{\perp}), \tag{4.3.64}$$

which can be obtained from (4.3.62) by choosing $O' = 1$ and the function G to be extended along a 4-cycle of the internal manifold and the torus, parametrized by $y_{\parallel}$ and z respectively, and localized in the other directions. This correction is related to the worldvolume term in the action of an M5 instanton, which is dual to the D3 instantons used to generate the non-perturbative contributions in KKLT and similar scenarios. If instead of the torus, the function G extends along the non-trivial 2-cycles appearing in the multi-centered lump solutions describing our localized brane stacks, they become stuck at the lump locus and correspond to a worldvolume instanton density related to worldvolume gauge theory effects like gaugino condensation.

The reason for considering this correction is motivated by the observation that it scales as

$$I(x') \sim \lambda^0 H^4 M_p^6, \tag{4.3.65}$$

making it non-vanishing in the $\lambda \to 0$ limit. If we take fewer torus directions or more of any other direction, the resulting non-local term would have negative λ scaling and the resulting non-perturbative exponential would vanish in that limit. As an aside, the only other non-local term with $O' = 1$ that scales as λ^0 comes from wrapping one torus direction and two internal directions. We can recognize this as describing an M2-instanton, which is dual to type IIB worldsheet or D1-instantons wrapping 2-cycles of the internal manifold.

For the case at hand, however, we can now also consider corrections of the form

$$\int d^{11}x\sqrt{g}O(x)e^{-\mathcal{I}(x)}, \tag{4.3.66}$$

where $O(x)$ is any of the two-derivative terms in the action other than $R_{(00)}$. For this term, the interesting contribution to the stress tensor (i.e. the second term in (4.3.63)) will be

$$T^{(nloc)M}{}_N(x) = g^M{}_N(x)\int d^{11}x'\sqrt{g_6}O(x')e^{-\mathcal{I}(x')}, \tag{4.3.67}$$

whose scaling contribution is

$$T^{(nloc)M}{}_N(x) \sim \lambda^{2/3}H^{8/3}M_p^{-2} \times (M_p^6 e^{-H^4 M_p^6}). \tag{4.3.68}$$

The factor in parentheses is an exponentially small λ-independent term, while the pre-factor has exactly the correct H and λ scaling as $R_{(00)}$, provided

$$\Lambda = -M_p^6 e^{-\langle H^4\rangle M_p^6}. \tag{4.3.69}$$

Note the overall negative sign, which arises as a combination of the negative sign in the exponential and the sign of the prefactor of the non-perturbative action. The latter is not fixed *a priori*, but a requirement that the solution is stable with respect to H fixes its sign to be positive in the action, resulting in negative Λ. This also aligns well with the general expectation that instanton corrections lower ground state energies.

The above expression for Λ can also be rewritten as

$$\Lambda\langle H^4\rangle \sim -\left(\frac{m_p}{M_{IIB}}\right)^2 e^{-\left(\frac{m_p}{M_{IIB}}\right)^2}, \tag{4.3.70}$$

where we omitted numerical coeffcients in the pre-factor and the exponent. Recall that $\Lambda\langle H^4\rangle$ is precisely the parameter that measures the scale-separation in our ansatz, and our result indicates that it is indeed exponentially small for large compactification volumes, indicating a large scale separation hierarchy.

Thus it appears that non-local terms, corresponding to the non-perturbative effects of M5-instantons are exactly the effects one needs to obtain a scale separated type IIB AdS solution. We recognize this as essentially the same as the AdS vacuum in KKLT and similar constructions. What our approach additionally shows, however, is that these effects were really the only option from the very beginning. No other terms have the correct H-scaling to cancel the EOM-violating terms coming from the external curvature.

Note that the large scale-separation ultimately stems from the non-perturbative nature of the correction responsible for it and the fact that it exponentiates due to the standard dilute gas argument. It is noteworthy that our motivation to exponentiate the non-perturbative terms comes from requiring the correct behavior as $M_p \to \infty$. The fact that these terms are also non-perturbative in H^{-1} and are related to the Kahler moduli of the internal manifold appears as a happy coincidence. However, we could have also guessed that this might be the case by recognizing that to obtain the scaling of $R_{(00)}$ from the other terms, we needed to contract them with something that had positive H scaling. Since our EFT regime is defined by expanding in inverse powers of H, the necessary factor would have to be non-perturbative in H as well and we might have decided to exponentiate it on those grounds.

The fact that these factors also end up λ-independent is a further non-trivial feature. Note that corrections coming from M2-instantons, while also λ-independent, would not give the required H^4 scaling. We have therefore identified M5-instantons (and therefore D3-instantons, or their fractional versions in the form of worldvolume gaugino condensation on the IIB side) as essentially the unique type of correction that allows for non-zero curvature in the external directions, while maintaining scale-separation.

Unfortunately, it appears that for the correction (4.3.66), the stable solutions always have $\Lambda < 0$. We wish to know whether it is possible to obtain a de Sitter solution with $\Lambda > 0$. The negative sign of the leading contribution stems from an interplay of the negative sign inside the non-perturbative exponential, which can't be changed, and the overall positive sign of the contribution in the action, which is determined by demanding the stability of the solution with respect to H, so it appears our only hope is to introduce additional corrections where we dress the non-perturbative term with an appropriate operator $O(x)$. To preserve the λ-scaling of the term, we must dress the original contribution with one of the λ-independent localized terms described in section 4.3.2. This is not *a priori* ruled out. These terms are made up of combinations of brane worldvolume curvatures and fluxes and it is conceivable that an appropriate choice of worldvolume flux, for example, might give an overall positive contribution to Λ. This approach would be in the spirit of [15], which is similar to KKLT except that the anti-brane is replaced by D7-brane worldvolume fluxes.

However, since the original non-dressed contribution is still present, the dressing factors would have to be of order one just to cancel the original negative contribution to the cosmological constant. In other words, the equation of motion would contain a part that schematically looks like

$$\frac{\delta}{\delta g^{MN}}\sqrt{g}\left(R_{(00)} + O_c(x)e^{-\mathcal{I}(x)} + O_q O_c(x)e^{-\mathcal{I}(x)}\right) = 0, \tag{4.3.71}$$

where O_c are the bulk classical 2-derivative terms, and O_q are the localized λ-neutral factors. Recall that these terms had additional suppression by powers of $H^{-4}M_p^{-6}$, so in order for these corrections to successfully flip the overall sign of Λ, the curvatures and/or fluxes on the brane worldvolumes would have to be large enough that this suppression is no longer present[6], which switches on the entire family of λ-independent terms in the EOM and destroys the EFT description. Thus it appears that while scale-separated AdS solutions are possible, lifting them to dS solutions requires the introduction of corrections that destroy the EFT

[6] In fact, if the 4-cycle wrapped by the 7-branes has non-constant warp factor, the curvatures or fluxes would need to have a functional dependence that matches the missing factors of $H(y)$ along the 4-cycle directions. This is not necessary along the orthogonal directions, however, so the new "dressed" term still formally has a different H-scaling, which may have helped maintain the stability of the solution after the uplift, if it didn't lead to a breakdown of the EFT.

description, meaning that if classically stable dS solutions exist, they are not within the regime of validity of the EFT defined by our original asymptotic limit.

4.4 Chapter Summary

In this chapter we have systematically studied the corrections to M-theory evaluated on an ansatz dual to a type IIB compactification with non-zero external curvature. Our analysis of the two-derivative equations revealed that the external curvature term has a distinct scaling with respect to the warp factor $H(y)$, compared to all the other curvature or flux terms, meaning that it could not cancel from the equations of motion. Higher derivative corrections in the bulk have the wrong scaling as well, while corrections associated to localized objects dual to D7/O7 stacks, can yield the correct λ-scaling, but require large curvatures or worldvolume fluxes to have the right H scaling, which would destroy the EFT.

We then turned out attention to non-local corrections, and showed that one type of correction, which is naturally interpreted as coming from M5-instantons, has precisely the right scaling in order to cancel the spacetime curvature term and yield a scale-separated AdS solution.

As for de Sitter compactifications, we found that reversing the sign of the spacetime curvature is only possible by once again turning to the localized corrections and asking for large worldvolume fluxes or curvatures, which destroys the EFT description. This result does not necessarily invalidate the existence of time-independent de Sitter vacua *per se*, but does, however, put into question whether it is appropriate to view de Sitter vacua as states within the same EFT as the SUSY compactifications that serve as starting points for dS constructions.

Chapter 5

Generalizations and Additional Ingredients

In the previous chapter, we have studied scale-separated compactifications to Anti-de Sitter space and de Sitter space within the regime of large-volume, weakly-coupled type IIB theory. We found that non-perturbative corrections seem to allow for AdS solutions, but not dS. The problem ultimately stems from the necessity to flip the sign of the non-perturbative term, which requires dressing it with perturbative corrections of order one, thus breaking the effective theory description.

Since the scalings of all the terms in the equations of motion are determined by our ansatz, it may be possible that a suitable modification to the ansatz may help avert our result. In particular, modifying the ansatz can introduce additional suppression parameters, that will prevent the infinite series of corrections from appearing. In this chapter, we will make two modifications to the original ansatz: the introduction of isometry-violating flux components and a time-dependent internal manifold. We will show that while some forms of these modifications are consistent, they are not enough to eliminate the EFT breakdown for de Sitter space. However, their failure will pave the way towards the final possibility that we will consider, involving the introduction of dominant IR effects. This requires shifting our expectations of the low-energy behavior of M-theory, which will in turn shift the behavior of our trans-series of corrections in the appropriate way to potentially allow for de Sitter solutions.

5.1 Isometry-violating Fluxes

One modification that we can make to the ansatz from the previous chapter is to allow for additional flux components, specifically for G_{mnpq} and G_{mnab}. At first glance these fluxes do not respect the dS isometries upon duality chasing back to IIB, but become either a Wilson line or a winding mode condensate along z_2. Note that we have already singled out the z_2 direction by periodically identifying it, so the appearance of objects threading should not be a disqualifying feature at this point. However, as we will soon see, further consistency conditions will reveal that these fluxes must in fact be localized, rather than global.

The presence of a different set of indices means that higher derivative corrections containing these fluxes has a chance of producing different scaling terms. For example, if G_{mnab} were time-independent, for example, then upon index contraction, its contribution to the scaling would be $\lambda^{-2/3}H^{-8/3}M_p^{-1}$, a scaling which does not appear from the fields in the original ansatz.

Here we must tread carefully, however, since the fluxes will also have intrinsic scaling determined by the flux equation of motion. As before, we have

$$d \star G = G \wedge G + X_8, \tag{5.1.1}$$

where X_8 vanishes identically except near the lumps describing the uplift of the D6/O6 stacks. As before, if there is any warped throats in the internal geometry, where $\square H$ doesn't vanish, we will have a set of equations analogous to (4.2.27) governing the leading-order scaling of the internal fluxes.

We have two options for which flux shares a leg with G_{0ijm}. Let's first consider

$$G_{(1)} = G_{0ijm}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dy^m \tag{5.1.2}$$

$$G_{(2)} = G_{mnab}\, dy^m \wedge dy^n \wedge dz^a \wedge dz^b \tag{5.1.3}$$

$$G_{(3)} = G_{pqrs}\, dy^p \wedge dy^q \wedge dy^r \wedge dy^s, \tag{5.1.4}$$

where as before we have

$$G_{(i)} \sim \lambda^{a_i} H^{b_i} \qquad i = 1, 2, 3 \tag{5.1.5}$$

$$G_{(i)} \wedge G_{(j)} \sim \lambda^{a_i + a_j} H^{b_i + b_j} \tag{5.1.6}$$

$$d \star G_{(1)} \sim \lambda^{a_1 + 4} H^{b_1 + 8} \tag{5.1.7}$$

$$d \star G_{(2)} \sim \lambda^{a_2 - 4} H^{b_2 - 4}, \tag{5.1.8}$$

and away from the lumps we neglect X_8 as before so the flux EOM read

$$d \star G_{(1)} \quad = \quad G_{(2)} \wedge G_{(3)} \tag{5.1.9}$$

$$d \star G_{(2)} \quad = \quad G_{(1)} \wedge G_{(3)}. \tag{5.1.10}$$

Taking $a_1 = b_1 = -4$ we obtain a solution of the form.

$$G_{mnab} \quad \sim \quad \lambda^{-1} H^2 M_p^{-1} \tag{5.1.11}$$

$$G_{pqrs} \quad \sim \quad \lambda^1 H^2 M_p^{-1}. \tag{5.1.12}$$

In the second case, if we take

$$G_{(1)} = G_{0ijm}\ dx^0 \wedge dx^1 \wedge dx^2 \wedge dy^m \tag{5.1.13}$$

$$G_{(2)} = G_{mnpq}\ dy^m \wedge dy^n \wedge dy^p \wedge dy^q \tag{5.1.14}$$

$$G_{(3)} = G_{rsab}\ dy^r \wedge dy^s \wedge dz^a \wedge dz^b, \tag{5.1.15}$$

we obtain through a similar manipulation

$$G_{mnpq} \sim \lambda^{-1} H^2 M_p^{-1} \tag{5.1.16}$$

$$G_{rsab} \sim \lambda^1 H^2 M_p^{-1}. \tag{5.1.17}$$

These intrinsic scalings of the fluxes imply that *after full index contraction*, their scalings are

$$G_{mnpq} \sim \lambda^{1/3} H^{-2/3} M_p^{-1}, \qquad G_{mnab} \sim \lambda^{1/3} H^{-2/3} M_p^{-1}. \tag{5.1.18}$$

These scalings are, of course, exactly the same as the scalings of $G_{(int)}$ from the original ansatz. This means that even if the isometry violating fluxes can be consistently added to the AdS ansatz, they do not help in restoring the EFT description for the de Sitter case.

The question of whether these fluxes can consistently included is interesting in its own right, however. Note that in both cases $G_2 \wedge G_3 \sim \lambda^0$ which allows its integral to successfully cancel against the time-independent part of X_8, to satisfy the tadpole condition, just as in the original ansatz.

Individually, however each flux now has λ-dependence, which presents a potential conundrum for flux quantization and the Bianchi identity, since these fluxes appear to no longer be closed. In order to resolve this issue we need to look at potential higher derivative corrections to the Bianchi identity.

Recall that the Bianchi identity can be viewed as the equation of motion for the Hodge dual of the flux. In other words, defining

$$G_7 = \star G_4, \tag{5.1.19}$$

which we can also locally write as $G_7 = dC_6$. We can write the sum of all higher the terms in the action involving contractions of G_4 as

$$S_G = \int G_7 \wedge (G_4 + \mathbb{Y}_4), \tag{5.1.20}$$

$$\tag{5.1.21}$$

where the first term is the usual kinetic term for the fluxes, while the second term captures all the other higher-derivative corrections containing G_4. There can also be corrections of the form $C_6 \wedge \mathbb{Y}_5$, where $\mathbb{Y}_5$ is not closed (otherwise it can be recast as a $G_7\, wedge \mathbb{Y}_4$ coupling), although this would require a non-trivial 5-cycle in the internal space. These can come from explicit M5-sources in the geometry or from higher derivative couplings such as those studied in [146]. In the following we will assume that there is no such 5-cycle in the geometry.

In the absence of the higher derivative corrections, the equation of motion for C_6 simply gives

$$dG_4 = 0, \tag{5.1.22}$$

which is the usual Bianchi identity, which upon integration over a 5-dimensional manifold with boundary gives the flux quantization condition. In the presence of $\mathbb{Y}_4$ the Bianchi identity gets modified to

$$d(G_4 + \mathbb{Y}_4) = 0. \tag{5.1.23}$$

One contribution to $\mathbb{Y}_4$ consists of topological $R \wedge R$ terms of the sort we have already seen. These terms integrate to a constant and are responsible for a shift of the flux quantization condition. If the resulting shift in the quantization condition is not integral, then the lack of a constant term in G_4 would render the system inconsistent, so we will assume this is not the case.[1]

It is then the other contributions to $\mathbb{Y}_4$ that are responsible for cancelling the terms coming from the λ-dependence of G_4. In order for this to be possible their λ-scalings must match. To check this we must compute the λ-scaling of $\mathbb{Y}_4$. We can start with a generic fully contracted higher-derivative correction involving the G_4 flux, "uncontract" the flux indices and divide by the intrinsic scaling of the flux itself.

For example, consider a correction involving G_{mnab}, which has a total scaling of λ^a. Then we have

[1]For the cycle wrapped by G_{mnab} the topological shift vanishes identically, so the cycles wrapped by G_{mnpq} are the only one we must be careful with.

$$G_{mnab}\mathbb{Y}^{mnab} \sim \lambda^{a} \tag{5.1.24}$$

$$G_{mnab}\mathbb{Y}_{pqcd} \sim \lambda^{a+4/3} \tag{5.1.25}$$

$$\mathbb{Y}_{pqcd} \sim \lambda^{a+1/3}, \tag{5.1.26}$$

and in order to match the scaling of G_{mnab} we need $a = 2/3$. The same result is obtained by considering $G_{mnpq}\mathbb{Y}^{mnpq}$.

The scaling $\lambda^{2/3}$ is precisely the scaling of the two-derivative terms in the action. Any higher-derivative terms built out of the global curvatures and fluxes will have additional powers of λ and can therefore induce higher order terms in the flux, but won't cancel the leading order piece.

This is not necessarily the case for localized fluxes however. Recall that localized curvatures and fluxes can produce λ-independent factors, and since they are localized, they do not need to match the full functional form of the H-scaling, but only need to have the correct magnitude to match the correct power of H locally. For example a term in the action of the form

$$G^{(loc)}_{mnab}G^{(loc)}{}_{pq}{}^{ab}(R^{(loc)}_{(zz)}R^{(loc)}_{(yy)}R^{(loc)}_{(yy)})^{mnpq} \tag{5.1.27}$$

built out of the localized curvature corrections that we described in the previous chapter, can produce a correction with the right λ-dependence to satisfy the leading order Bianchi identity for G_{mnab}. It is not clear however, that this can be done without forcing the $(R^{(loc)})^3$ factor to be of order one, which would lead to a breakdown of the EFT description.

Note that the localized fluxes that we have studied in the previous chapter do not have the isometry violating index structure, because of the normalizable 2-forms near the D6/O6 locus having only one torus leg. There are however additional two-forms associated to various 2-cycles present inside the lump solution, which vanish in the constant-coupling limit and are associated to the non-abelian dynamics of the brane stack [147], which can provide the correct index structure. Thus it appears that the would-be isometry-violating fluxes should be interpreted as localized fluxes dual to the non-abelian gauge fields on the brane stacks.

Once we've matched the leading order terms in the Bianchi identity, we can allow for a more general form for the fluxes, involving an expansion in higher powers of λ. These will then also appear in the Bianchi identity, but will have exactly the right scaling to be cancelled by even higher order derivative-corrections. In the next section we consider a scenario where the fluxes have this general time-dependence as well as consider a time-dependent manifold.

5.2 Time-dependent Internal Manifold

Although the intrinsic scalings of the fluxes are determined by the flux equations of motion, the scaling after contraction also depends on the scalings of the internal metric. Moreover, the dependence on the metric also appears in the Hodge dual operation.

With this in mind, the next modification to try is introducing a time-dependence for the internal manifold. It is well known that even in supergravity, one can obtain transient phases of accelerated expansion when the total volume of the internal manifold changes. This essentially produces a class of quintessence models with a time-varying 4-dimensional Newton's constant. What we would like to explore, however, are models where the 4D Newton's constant remains time-independent, but different cycles in the internal manifold can vary with time.

An example of such an ansatz would be a IIB metric of the form

$$ds^2_{IIB} = -\frac{e^{\phi_B/2}}{H(y)^2}\frac{1}{\Lambda x_0^2}\left(dx_0^2+dx_1^2+dx_2^2+dz_1^2\right)+e^{\phi_B/2}H(y)^2\left(F_1(\lambda)\tilde{g}_{mn}dy^m dy^n+F_2(\lambda)\tilde{g}_{\alpha\beta}dy^\alpha dy^\beta\right), \tag{5.2.28}$$

where we require that $F_1^2F_2=1$, so that the overall volume of the internal manifold remains constant.

This metric dualizes to

$$\begin{aligned} ds^2_M &= H^{-8/3}\lambda^{-8/3}\left(dx_0^2+dx_1^2+dx_2^2\right)+H^{4/3}\lambda^{-2/3}\left(F_1(\lambda)g_{mn}dy^m dy^n+F_2(\lambda)g_{\alpha\beta}dy^\alpha dy^\beta\right)\\ &+ H^{4/3}\lambda^{4/3}\left(e^{-\phi_B}dz_1^2+e^{\phi_B}dz_2^2+h_{ab}dz^a dz^b\right) \end{aligned} \tag{5.2.29}$$

on the M-theory side, where as usual the difference between $\tilde{g}$ and g as well as h_{ab} is localized on a set of lumps dual to the D7/O7-stacks on the IIB side.

Let us consider a Frobenius series ansatz for the F_i

$$F_1 = \lambda^{-2\gamma}(1+\sum_n C_n\lambda^{n\Delta}), \qquad F_2 = \lambda^{4\gamma}(1+\sum_n \tilde{C}_n\lambda^{n\Delta}), \tag{5.2.30}$$

where Δ is a rational number, so that n is an integer, and the condition $F_1^2F_2 = 1$ allows us to express all the $\tilde{C}_n$ in terms of C_n, the first few of which are

$$\tilde{C}_1 = -2C_1 \tag{5.2.31}$$
$$\tilde{C}_2 = 3C_1^2 - 2C_2, \tag{5.2.32}$$

and so on. The rational number Δ is chosen so that the power of λ in the expansion of the curvature changes by the same amount as between the different order curvature corrections, so that the EOM provide meaningful constraints on them. Since the higher derivative terms differ by powers of G^2, R, ∇^2 or ∇G, which all scale as $\lambda^{2/3}$, the prudent choice for Δ might seem to be 2/3. However, judging by the fact that the first two sets of higher derivative corrections differ by powers of R^3 or equivalently scaling combinations, we should not be surprised to find that the appropriate choice is in fact $\Delta = 2$, or at least that the first non-zero coefficient appears at order λ^2 relative to the leading term.

Introducing these functions into the metric changes the scalings of the Hodge dual operator and so the constraints coming from the flux EOM need to be reconsidered. $G_{(ext)}$ now splits into two types of components, $G_{(ext,m)}$ and $G_{(ext,\alpha)}$, where the additional m or α subscript denotes the subspace of the internal leg. The scalings of their hodge duals can be written as

$$d\star G_{(ext,m)} \sim F_1(\lambda)^{-1}H^4M_p^{-2} \tag{5.2.33}$$
$$d\star G_{(ext,\alpha)} \sim F_2(\lambda)^{-1}H^4M_p^{-2}, \tag{5.2.34}$$

where we assume that the intrinsic scaling of $G_{(ext)}$ is $\lambda^{-4}H^{-4}M_p^{-1}$, as in the original ansatz. If we modify this scaling to be λ^a, both expressions above will pick up an extra factor of λ^{a+4}.

Note that the H and M_p scalings remain unchanged from the time-independent case, so we will stop keeping track of them from now on.

Since the intrinsic scaling of the internal fluxes are determined by the scaling of $d\star G_{(ext)}$, they must now also become λ-dependent. Furthermore, since F_i consist of many terms with different λ-scalings, the internal fluxes will also now take the form of Frobenius series of the form:

$$G_{MNPQ} = \lambda^a \left(1 + \sum_n \mathcal{G}^{(n)}_{MNPQ} \lambda^{n\Delta}\right), \tag{5.2.35}$$

where again, we chose $\Delta = 2/3$ for the power in the expansion parameter in order to get interesting interplay with the higher derivative corrections.

Note that the *leading order* scaling λ^a, is only sensitive to the leading order scaling of the F_i, which means it's depends only on γ and not the C_i in (5.2.30).

Thus to determine the leading order flux scalings, and therefore the dominant asymptotic behavior in the $\lambda \to 0$ limit of the fluxes, we can simply take $F_1 \sim \lambda^{-2\gamma}$ and $F_2 \sim \lambda^{4\gamma}$ and solve the constraints coming from the two-derivative flux EOM. For example, taking

$$G_{(1)} = G_{0ijm}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dy^m \tag{5.2.36}$$

$$G_{(2)} = G_{mnpa}\, dy^m \wedge dy^n \wedge dy^p \wedge dz^a \tag{5.2.37}$$

$$G_{(3)} = G_{\alpha\beta qb}\, dy^\alpha \wedge dy^\beta \wedge dy^q \wedge dz^b \tag{5.2.38}$$

where as usual we have $G_{(i)MNPQ} \sim \lambda^{a_i}$ to leading order, and

$$d \star G_{(1)} \sim \lambda^{a_1+4+2\gamma} \tag{5.2.39}$$

$$d \star G_{(2)} \sim \lambda^{a_2-4+6\gamma}. \tag{5.2.40}$$

The flux EOM imply

$$a_3 = 4\gamma \tag{5.2.41}$$

$$a_2 = a_1 + 4 - 2\gamma. \tag{5.2.42}$$

The scalings of these fluxes after contraction are then

$$G_{(2)} \sim \lambda^{1/3+\gamma+(a_1+4)} \tag{5.2.43}$$

$$G_{(3)} \sim \lambda^{1/3+\gamma}. \tag{5.2.44}$$

As a sanity check, we note that this reproduces the original time-independent scalings for $\gamma = 0$.

We can now proceed to repeat this computation for all the flux components, including the isometry-violating components introduced in the previous section. The results are tabulated in tables 5.1 and 5.2. Note that in the last column of each table we have set $a_1 = -4$ so that the external flux is still proportional to the spacetime volume element. Interestingly, upon full contraction, all the flux scalings assemble into only two families, one with scaling $\lambda^{1/3+\gamma}$ and another with $\lambda^{1/3-2\gamma}$.

Table 5.1 Isometry preserving Flux Scalings

$G_{(ext)}$ component	$d\star G_{(ext)}$ λ-scaling	$G_{(int)}$ component	Intrinsic $G_{(int)}$ λ-scaling	Contracted $G_{(int)}$ λ-scaling
G_{0ijm}	$a_1+4+2\gamma$	G_{mnpa}	$a_1+4-2\gamma$	$\frac{1}{3}+\gamma$
		$G_{q\alpha\beta b}$	4γ	
		$G_{mn\alpha a}$	$a_1+4+\gamma$	
		$G_{pq\beta b}$	γ	
		$G_{m\alpha\beta a}$	$a_1+4+4\gamma$	
		G_{npqb}	-2γ	
$G_{0ij\alpha}$	$a_1+4-4\gamma$	$G_{\alpha mna}$	$a_1+4-2\gamma$	$\frac{1}{3}-2\gamma$
		$G_{pq\beta b}$	-2γ	
		$G_{\alpha\beta ma}$	$a_1+4+\gamma$	
		G_{npqb}	-5γ	

The different scalings of the metric along the two submanifolds of the 6-manifold also affects the scalings of the Riemann tensors. The rule for computing their scalings remains the same: the scalings of a fully contracted Riemann tensor component is given by the inverse vielbeins corresponding to the directions, along which the derivatives act. All the other metric scalings will cancel out due to the warped product structure of the metric. With the new ansatz, we now have more combinations of derivatives, whose scalings are tabulated in

Table 5.2 Isometry violating Flux Scalings

$G_{(ext)}$ component	$d\star G_{(ext)}$ λ-scaling	$G_{(int)}$ component	Intrinsic $G_{(int)}$ λ-scaling	Contracted $G_{(int)}$ λ-scaling
G_{0ijm}	$a_1+4+2\gamma$	G_{mnab} $G_{pq\alpha\beta}$	$a_1+5-\gamma$ $-1+3\gamma$	$\frac{1}{3}+\gamma$
		$G_{m\alpha ab}$ $G_{npq\beta}$	$a_1+5+2\gamma$ -1	
		G_{mnpq} $G_{\alpha\beta ab}$	$a_1+3-3\gamma$ $1+5\gamma$	
		$G_{mnp\alpha}$ $G_{q\beta ab}$	a_1+3 $1+2\gamma$	
		$G_{mn\alpha\beta}$ G_{pqab}	$a_1+3+3\gamma$ $1-\gamma$	
$G_{0ij\alpha}$	$a_1+4-4\gamma$	$G_{\alpha mab}$ $G_{\beta npq}$	$a_1+5-\gamma$ $-1-3\gamma$	$\frac{1}{3}-2\gamma$
		$G_{\alpha\beta ab}$ G_{mnpq}	$a_1+5+2\gamma$ $-1-6\gamma$	
		$G_{\alpha mnp}$ $G_{\beta qab}$	$a_1+3-3\gamma$ $1-\gamma$	
		$G_{\alpha\beta mn}$ G_{pqab}	a_1+3 $1-4\gamma$	

table 5.3.

Table 5.3 Leading order curvatures scalings, time-dependent case.

Curvature component	Contracted scaling
$R_{(00)}$	$-\Lambda\ \lambda^{2/3}\ H^{8/3}\ M_p^{-2}$
$R_{(mn)}$	$\lambda^{2/3+2\gamma}\ H^{-4/3}\ M_p^{-2}$
$R_{(\alpha\beta)}$	$\lambda^{2/3-4\gamma}\ H^{-4/3}\ M_p^{-2}$
$R_{(m\alpha)}$	$\lambda^{2/3-\gamma}\ H^{-4/3}\ M_p^{-2}$
$R_{(0m)}$	$\sqrt{-\Lambda}\ \lambda^{2/3+\gamma}\ H^{2/3}\ M_p^{-2}$
$R_{(0\alpha)}$	$\sqrt{-\Lambda}\ \lambda^{2/3-2\gamma}\ H^{2/3}\ M_p^{-2}$

We see once again, the same basic pattern that arose in the time-independent case. Curvatures with purely internal derivative indices have precisely the kind of scaling that can cancel against the terms quadratic in G. Curvatures with two $\tilde{x}$ derivatives once again have the unique H-scaling, which will require the introduction of non-perturbative effects to cancel them. Note that this curvature scaling is completely insensitive to γ, so the problem of generating a non-zero Λ is insensitive to the time-dependence of the internal manifold.

The curvature terms with mixed derivatives seem to provide a new feature. Their λ scalings are completely distinct from the other terms. This would generate a new set of higher-derivative corrections, with a different set of scalings, and vastly complicate the analysis. As we will see shortly, however, tadpole cancellation will require us to take $\gamma = 0$, so these corrections will once again fall in line with the rest in terms of their scalings.

There is, however, a more important complication. Unlike the time-independent case, the Einstein tensor arising from these curvatures no longer vanishes, but rather gives:

$$G_{0n} = -2\left(\frac{\dot{F}_1}{F_1} + \frac{\dot{F}_2}{F_2}\right)\frac{\partial_n H}{H}, \quad G_{0\alpha} = -4\left(\frac{\dot{F}_1}{F_1}\right)\frac{\partial_\alpha H}{H} \tag{5.2.45}$$

and would need to get cancelled by some appropriate quantum terms, since they don't have the same H-scaling as any of the other two-derivative terms. Note that they differ by a positive power of H from the purely internal curvature and flux terms. When we encountered this positive relative power of H for the external curvature term in the previous chapter, we had to turn to non-local corrections. However, this was because we had the additional constraint that there must be no relative power of λ arising from the correction, which is not the case here.

The non-zero Einstein tensor comes from a subleading part of the $R_{(0M)}$ Riemann tensor, which has a different λ-scaling as well. Indeed, taking $\gamma = 0$, we can readily see that

$$\frac{\dot{F}}{F} \sim \lambda^{\Delta - 1}, \tag{5.2.46}$$

which, if we multiply by the inverse vielbein scalings, gives a "contracted" scaling of $\lambda^{\Delta+2/3}$, rather than the leading order $\lambda^{2/3}$ scaling of the full Riemann tensor. Thus we need not worry about acquiring additional powers of λ and can simply cancel this term against a higher derivative term. In fact, the λ-scaling of the higher derivative term will dictate the value of Δ.

The lowest order term that has the appropriate H-scaling has the form

$$R_{(0M)}R^2_{(MN)}R_{(00)} \sim \lambda^{8/3}H^{2/3}M_p^{-8}, \tag{5.2.47}$$

where M, N can be either m or α, or any similar term with curvature factors replaced by equivalently scaling factors of G^2, ∇G or ∇^2. Note the extra powers of M_p^{-1}, relative to the two-derivative term, indicate that the first non-zero coefficients $C_1, \tilde{C}_1$ must be small in magnitude, similar to how the tadpole condition suppresses the magnitudes of the fluxes. The fact that this term differs by a power of λ^2 from the leading scaling of $R_{(0M)}$ means that we ought to take $\Delta = 2$, as anticipated.[2]

Of course, $\dot{F}_i/F_i$ has a further expansion in powers of λ^Δ, and those terms need to be cancelled as well. However, once a leading correction is obtained, the rest of the series can easily be generated by balancing them against further higher derivative corrections that have the appropriate scaling. Thus, once a single C_i can be made non-zero, the mixed derivative equations of motion provide a relationship between the higher order coefficients and the higher derivative corrections.

Similarly, if we expand the rest of the two-derivative terms in the equations of motion, i.e. include all the C_n and the $\mathcal{G}^{(n)}$ dependent pieces, we will also obtain a series in λ^Δ. These new higher order terms must then successfully cancel either among themselves or against the higher derivative terms with the matching λ scaling.

Thus the quantum-corrected Einstein's equations form a complicated system of equations order by order in λ^Δ, where each new λ-dependent piece is determined by the lower order pieces combined with higher-derivative corrections. Note, however that at each order, we have a new C_n variable appearing from the expansion of the F_i functions, as well as an additional rank-4 antisymmetric tensor's worth of components $\mathcal{G}^{(n)}_{MNPQ}$. Thus the number of variables vastly exceeds the number of equations to any finite order, and generically we should expect there to be non-trivial solutions.

Let us now revisit the localized features of the metric and the tadpole cancellation condition and see how it leads us to impose the condition $\gamma = 0$ in the ansatz for F_i.

Just as in the time-independent case, there are additional non-zero curvature components in the vicinity of the localized lumps, containing derivatives along the z_a directions. These

[2] Alternatively, we can keep $\Delta = 2/3$, but the lack of necessary quantum terms in the EOM will then imply that only the C_{3n} will be non-vanishing.

are unaffected by the F_i and so their scalings remain unchanged. Recall that the time-independent part of X_8 came exclusively from these localized regions. In the time-dependent case, we now have the following contributions to X_8:

$$R^2_{(mn)}R_{(\alpha\beta)}R_{(ab)} \sim \lambda^{2/3} \tag{5.2.48}$$
$$R^2_{(m\alpha)}R_{(mn)}R_{(ab)} \sim \lambda^{2/3} \tag{5.2.49}$$

when fully contracted, with all other combinations being higher order in λ. Upon "uncontracting" one of each 8-manifold indices, we still get

$$X_8 \sim \lambda^0 \tag{5.2.50}$$

at leading order. This is, of course, by no means surprising. $\int X_8$ is a topological invariant of the 8-manifold, and therefore can not vary continuously with λ. This means that any λ-dependent part of X_8 must integrate to zero, although it need not vanish locally, while the part that enters the tadpole cancellation, should be insensitive to any additional λ-dependent warpings of the metric we may introduce as long as these do not alter the topology of the fixed λ slices.

On the other hand, the internal fluxes are sensitive to the additional λ-dependence. Looking at table 5.1 and 5.2, we see that $G_{(2)} \wedge G_{(3)} \sim \lambda^{(-1\pm 3)\gamma}$ are the only scalings that appear, both of which are sensitive to γ. More precisely, if we keep the λ-scaling of $G_{(ext)}$ unspecified, as $\lambda^{a_\pm}$, with the $\pm$ options depending on whether the internal leg is m or α, we have

$$G_{(int)} \wedge G_{(int)} \sim \lambda a_\pm + 4 + (-1 \pm 3)\gamma \tag{5.2.51}$$

Thus, in order to satisfy the tadpole cancellation, we require $a_\pm = (1 \pm 3)\gamma - 4$. If we insist on having $a_\pm = -4$, i.e that the external flux be proportional to the type IIB external spacetime volume, we are forced to take $\gamma = 0$.

Of course we could also have vanishing $\int X_8$, which would mean removing the D7/O7 stacks from the type IIB ansatz. This would bring us to a much less exciting family of ansatzes, corresponding to type IIB compactifications with the only sources of warping and F_5-flux being D3 and anti-D3 branes, which suffer from various stability issues even in the λ-independent case, so we will not pursue them here.

This is not the only reason for taking $\gamma = 0$. Recall that the non-perturbative term that we have identified as the contribution from M5-instantons involved an integral over the z-torus and a 4-cycle of the internal manifold. For non-zero γ, we see that there are no γ-independent 4-cycles, so the non-perturbative term would fail to cancel $R_{(00)}$ in the EOM. This is yet another reason to take $\gamma = 0$.

Another consequence of taking $\gamma \neq 0$ is a heavy constraint on the types of fluxes that are allowed. Note that many of the same flux components appear in both halves of the tables 5.1 and 5.2. However the flux equations of motion then impose different scalings for these fluxes. There are several ways out of this problem. One is to demand that at most one of G_{0ijm} or $G_{0ij\alpha}$ has non-vanishing divergence. This would effectively remove half of the conditions on the fluxes, resolving the problem. This in turn implies that the warp-factor H has non-trivial dependence on only one of the two sub-manifolds. Another option would be to make all the flux components that appear twice in the tables vanish identically. This would still leave a limited set of components that could be present. Finally, the solution that leaves all the fluxes intact is, once again, to take $\gamma = 0$.

The condition $\gamma = 0$ has a straightforward physical interpretation. It simply means that our solution, if it exists, asymptotes to the solution with λ-independent internal manifold. In the AdS case, this means as we approach the conformal boundary, our solutions become the same, and the λ-dependent solutions can be interpreted as the result of some relevant operators in the dual CFT. In the de Sitter case, the λ-dependent contributions C_n and $\mathcal{G}^{(n)}$ can be seen as perturbations in the far past, which get diluted by the accelerating expansion.

However, we now see that since the $\lambda \to 0$ limit of this scenario asymptotes to the original ansatz of the previous chapter, the problem with generating $\Lambda > 0$ that appeared in the λ-independent case still persists. Indeed, all the new terms in the EOM coming from switching on C_n and $\mathcal{G}^{(n)}$ only contribute at higher order in λ, which implies some non-trivial conditions on them, but does not affect the leading order part of the EOM. This means that while they do not ruin the EFT description in the AdS case, they also do not manage to cure the problem in the dS case.

5.3 Shifting the Transseries, Brane Instantons vs IR Effects

The reason for the failure of the time-dependent corrections to cure the de Sitter EFT problem is rather obvious in retrospect. The functions F_i did not introduce any fundamentally new ingredient, but simply dressed existing ingredients with additional powers of λ. In this sense, their role in the asymptotic expansion of the EOM is the same as that of the global higher derivative terms. Recall that the EFT problem came about from a need to include λ-independent terms with $\mathcal{O}(1)$ coefficients, whose higher powers also had this property. Turning on λ-dependence while maintaining the same $\lambda \to 0$ behavior of the solution, the best that additional λ dependence could have done was re-suppress these corrections, which would in turn eliminate their ability to flip the sign of the non-perturbative term.

We can describe this problem more generally from the transseries perspective of chapter 3. Recall that in order to define an EFT regime, we need to specify a set of expansion parameters, which we can take as approaching zero, as well as the expected asymptotic behavior of our theory in that limit. This latter condition determines which powers of the expansion parameters appear as the leading order term in the transseries. Any operators that have lower powers of the expansion parameters will appear inside non-perturbative exponentials. If they do not, then they will take over as the dominant part of the action.

Since our interest is in ansatzes that only satisfy the leading order EOM after some set of corrections is included, any sub-leading alterations to the ansatz will have no effect on which corrections to the leading order EOM are required to make our ansatz into a solution, or whether they preserve of break the EFT description.

Recall that the difference between the AdS and dS case ultimately stems from the fact that the M5-instanton correction results in a negative contribution to the external spacetime curvature. This sign ultimately stems from the sign inside the non-perturbative exponentials combined with the sign of the "one-loop determinant" that can be fixed by requiring a stable solution. An important detail is that the term that generates the curvature isn't dressed with any further perturbative terms. If it were, we could conceivably have much greater control over the sign of the perturbative dressing, through a judicious choice of internal fluxes and internal curvatures, and therefore of the correction as a whole.

With this in mind, consider a non-local term of the form (4.3.61), which we repeat here for convenience

$$\begin{aligned} S^{(nloc)} &= M_p^{22}\int d^{11}x d^{11}x' \sqrt{g(x)\; g(x')} O(x) G(x-x_{(1)}) O'(x') && (5.3.52)\\ &= M_p^{11}\int d^{11}x\sqrt{g(x)} O(x)\mathcal{I}(x), && (5.3.53)\end{aligned}$$

where we now take the non-locality function is to be localized along x and z, but delocalized along all six y directions. The scaling contribution of the integral measure is then

$$M_p^6\int d^6y\sqrt{g_6}\sim\lambda^{-2}H^4M_p^6. \quad (5.3.54)$$

If we take O or O' to be a term that scales as $\lambda^{8/3}H^{-4/3}M_p^{-8}$ then the scaling of the term (5.3.52) will have the right as that of the external curvature and it seems like such a term could be our ticket to controlling the sign of the external curvature. We need to simply find an O or O' with the right sign.

Examples of operators that have this scaling are

$$R_{(00)}R^3_{(mn)}, \qquad R^2_{(0m)}R^2_{(mn)}, \qquad (\nabla_0 G_{mnab})^2 R^2_{(mn)} \qquad (\mathcal{G}^{(1)})_{mnpa}(\mathcal{G}^{(1)})^{mnpa}\text{etc.} \quad (5.3.55)$$

In the last example, $\mathcal{G}^{(1)}_{mnpa}$ denotes the first time-dependent correction to G_{mnpa}, which as we saw scales as λ^2H^2.

Here we must proceed with caution. The term with such an operator insertion is actually subleading to a lower order non-local contribution with $O=O'=1$. Let us denote the x' integral $\mathcal{I}$ appearing in this leading correction by $\mathcal{I}_0$. If instead we take O' to be one of the above operators, we will denote the corresponding integral by $\mathcal{I}_1$. Their scalings are

$$\mathcal{I}_0\sim\lambda^{-2}H^4M_p^6 \qquad \mathcal{I}_1\sim\lambda^{2/3}H^{8/3}M_p^{-2} \quad (5.3.56)$$

Note that $\mathcal{I}_1$ has exactly the scaling of $R_{(00)}$ and may be used to balance it in the EOM. $\mathcal{I}_0$, on the other hand, dominates over the two-derivative terms in the low-energy limit as well as the small λ limit, so we must be careful with its physical interpretation.

Here we have several options. One is to insist that the low-energy limit of M-theory is 11-dimensional supergravity and the contribution of $\mathcal{I}_0$ must not manifest itself in the $M_p \to \infty$ limit. This is the approach we took so far, which led us to exponentiate the contributions of the non-local integrals of the form $e^{-\mathcal{I}_0}$. In this form, it is most natural to interpret that exponential as capturing the contribution of a non-perturbative saddle in the path integral. In this case, the expression would correspond to an M5-instanton wrapped around the 6-manifold. This reduces to an NS5-instanton in type IIA, also known as the BBS instanton [148] and dualizes to an interesting Euclidean solution with a Taub-NUT core where the z_1 circle is nontrivially fibered. The actions of all of these configurations diverge as $\lambda \to 0$, which is why we couldn't make use of them.

Now we must deal with $\mathcal{I}_1$ and its physical interpretation. The most straightforward interpretation is that $\mathcal{I}_1$ is in fact a perturbative correction to the instanton action. This means that rather than viewing $\mathcal{I}_0$ and $\mathcal{I}_1$ as describing unrelated physical effects, we should view the physical contribution as coming from the sum of all such effects

$$\int d^6y O(y)\mathcal{I}(y) \qquad \mathcal{I} = \mathcal{I}_0 + \mathcal{I}_1 + \mathcal{I}_2 + ... \tag{5.3.57}$$

Upon exponentiation, the entire series ends up in the exponent and the subleading terms can then be Taylor expanded as

$$\int d^6y O(y)e^{-(\mathcal{I}_0+\mathcal{I}_1+...)} = \int d^6y O(y)e^{-\mathcal{I}_0}(1+...) \tag{5.3.58}$$

This, unsurprisingly, has the form of a piece of a transseries, describing a single non-perturbative effect with all its perturbative corrections. This would appear to be the most natural physical interpretation of this family of corrections, as it is entirely consistent with the usual behavior of instanton corrections. Unfortunately, this means that their contribution vanishes in the $\lambda \to 0$ limit, and thus can't help us with our de Sitter EFT problem.

We may of course be tempted to only exponentiate the $\mathcal{I}_0$ piece and leave $\mathcal{I}_1$ and higher order terms on par with the other two-derivative terms and higher derivative corrections. As we argued at the end of chapter 3, this would be a mistake. The result such a computation would yield, would be related to a two-sided Laurent series representation of the action, not a transseries representation, which is ill suited to determining the existence of an EFT description.

A final option that we have so far ignored is to simply leave $\mathcal{I}$ as is in the action and accept that 11-dimensional supergravity is not the leading order term in the low-energy limit of M-theory. In that case, rather than being a non-perturbative effect, the expression (5.3.52) would represent some far IR effect that dominates over the supergravity action. Note that this does not preclude us from also having the instanton contributions on top of this. Instead, this simply amounts to shifting the $\mathcal{G}$ factor of our transseries expansion, to use the language of (3.3.17), by a factor of $\lambda^{-2}H^4M_p^6$. The instanton effects described above can then easily contribute on top of this.

The possibility of IR modifications to gravity is not a new idea. In fact there has been interesting work examining the cosmological consequences of IR terms in gravity [149]. This idea also has some resonance, although is not identical, with views expressed in, for example, [150], where the cosmological constant should be regarded as an IR boundary condition.

If such a contribution does exist, then it must satisfy the equations of motion on its own, since none of the familiar perturbative or even nonperturbative terms appear at this order. The physical nature of this effect must also be such that it can not combine with itself, as this would lead to another shift in the dominant IR behavior and so on *ad infinitum*.

Assuming such an effect exists, the contribution $\mathcal{I}_1$ would actually provide us with a somewhat tunable correction to the two-derivative equation of motion. Note, however, that when these corrections contain $R_{(00)}$ or $R_{(0m)}$ themselves, they also naturally come with powers of Λ as per table 4.1 and thus the additional curvature factors would have to be of order one to satisfy the EOM, leading again to an EFT breakdown.

To avoid this we should instead use one of the terms containing the $\tilde{x}$-dependent fluxes G_{mnab}, whose magnitude is not directly tied to Λ. This means that the time-dependent fluxes are no longer optional. Whether the $\tilde{x}$-dependent fluxes that give the desired sign of Λ can be achieved, while satisfying all the other constraints that we presented earlier in this chapter depends on all the exact details, and particularly signs, of the IR term as well as the flux-containing higher derivative corrections that can appear in M-theory, neither of which are known.

5.4 Chapter Summary

In this chapter we explored various modifications to the ansatz of the previous chapter. First we considered turning on additional flux components, whose index structure appears to violate the de Sitter isometries. Using a similar approach to the previous chapter we determined the leading order intrinsic scalings of these fluxes and showed that they are consistent with tadpole cancellation. On the other hand, satisfying the Bianchi identity required the use of localized quantum terms, indicating that rather than being isometry-violating these fluxes are in fact localized and are dual to the non-abelian fluxes on the D7/O7 stacks in the type IIB description.

We then considered a more general ansatz with a $\tilde{x}$-dependent, but volume preserving internal manifold and general $\tilde{x}$-dependent fluxes. Once again we can find flux scalings that are consistent with tadpole cancellation and the Bianchi identity, provided the internal manifold asymptotes to that of the original ansatz as $\tilde{x} \to 0$. The equations of motion for both the flux and the metric split into a consistent set of equations order by order in λ, with more variables than equations, suggesting that non-trivial solutions should be possible.

For both modifications, however, the new ingredients only affect the sub-leading parts of the equations of motion and do not help resolve the problem of maintaining an EFT description for a de Sitter ansatz. To get around this problem we explored the possibility of additional non-local effects. We discussed the distinction between brane-instanton corrections and non-local IR effects, which may both involve integrals over subspaces, but appear in different places in the transseries expansion. Finally we suggested that a possible way to get the desired type of corrections is to posit an IR contribution to the action that is lower order in M_p^{-1} than the supergravity action. Dressing this IR effect by higher derivative corrections would generate terms that enter at the same order as the supergravity equations of motion and with the right choice of “dressing” may allow for positive curvature along the type IIB spacetime directions. The time-dependent fluxes described earlier appear to play a crucial role in providing suitable dressing operators.

Chapter 6

Discussion and Conclusion

The problem of constructing scale-separated compactifications to spacetimes with non-vanishing spacetime curvature is a fascinating problem both in light of its relevance to cosmological model building as well as on purely conceptual level, as we try to understand the rich structure of string theory and its relationship to low energy effective field theories in light of the swampland program.

The classical no-go theorems related to such compactifications [61, 18, 117] indicate that if these solutions exist, they must involve quantum corrections. In this thesis, we approached the question of constructing these spacetimes, by positing an ansatz for such a compactification and asking what the requisite quantum corrections should be in order to make it into a solution. This approach has the benefit of not requiring the precise form of their corrections, but only the powers of the fields that appear within them. Our organizing principle for the corrections involves expressing the equations of motion by a transseries, which we argued is the correct mathematical object to use when dealing with asymptotic regimes. We then defined effective theories as truncations of the equations of motion, thus relating the existence of an EFT to our ability to truncate the transseries.

By choosing an ansatz with non-zero spacetime curvature, and working in its M-theory description, we were able to classify all the local and non-local corrections in the limit corresponding to low-energy large-volume type IIB compactifications. We found that upon including appropriate corrections, most naturally interpreted as M5-instantons, the equations of motion seem to allow for scale-separated AdS solutions, while maintaining the EFT description intact. Interestingly, these instanton corrections are precisely dual to those commonly used in the existing mechanisms of moduli stabilization.

This result appears to be in conflict with the proposed AdS swampland conjectures, which state that AdS compactifications can not have large scale separation. It is worth remembering that these conjectures are motivated by studying the flat space limit of non-scale-separated Freund-Rubin type AdS solutions, which solve the 2-derivative EOM. The solutions we studied here do not belong to this family, and indeed do not even solve the 2-derivative EOM, but require non-perturbative effects to be included.

The more careful statement should perhaps be that scale-separated and non-scale-separated AdS solutions lie far from each other in configuration space and are not part of the same EFT, in the sense of the distance conjecture. Thus if one starts in the Freund-Rubin-like regime, one can not reach the scale-separated solutions without leaving the regime of validity of that EFT. Similarly, if we start in our scale-separated regime, we could not reach the the non-scale-separated regime either. Moving between the two-regimes, should then be described by a duality transformation, rather than a motion within the field space of a single EFT. However, once we are in the scale-separated regime, our approach does not seem to find any fundamental obstacles to having AdS solutions and indeed shows us precisely which ingredients are required to make it work.

Note that our use of non-perturbative effects circumvents the no-go theorem of [117], which is formulated at the two-derivative level. On the other hand, although the examples we consider contain orientifold planes, which also circumvent the no-go theorem, it isn't clear if we if our arguments make any use of their specific properties in constructing the AdS solutions. Thus it is possible that scale-separated AdS compactifications are realizable if not in the absence of orientifolds (given the other functions they perform in flux compactifications in general), then at least without them being crucial to generating the non-zero spacetime curvature.

For positive spacetime curvature the situation turns out to be more complicated, since achieving it appears only possible by flipping the sign of the non-perturbative contribution. This requires dressing it with order one perturbative corrections, which destroys our ability to truncate the transseries expansion and signifies a breakdown of the EFT description. This behavior is in agreement with the swampland distance conjecture.

Relating the results in our example to de Sitter constructions more broadly and KKLT specifically, it is worth separating out two questions: i) does the uplift procedure actually uplift past zero curvature to de Sitter space and ii) does it ruin the EFT description in doing so? Our present analysis doesn't have much to say about the first question, as the answer depends on the specific dynamics within the any given scenario. The latter question, however, boils down to the whether introducing anti-branes or similar uplift ingredients (worldvolume curvatures and fluxes, in our example) to an AdS compactification can be regarded as a motion in field space of the original AdS EFT, without ever leaving its regime of validity. Our results indicate that the answer to the above question is in the negative and that the uplift procedure generically takes us outside the regime of the effective theory where the AdS vacuum resides. Thus, if time-independent de Sitter compactifications exist, the dynamics around these states would be described by some other EFT that is only related to the EFTs around SUSY compactifications by the introduction of new degrees of freedom and a duality transformation, rather than a simple shift in the EFT field-space.

This is also consistent with the more recent formulations of the KKLT construction in the presence of anti-branes in terms of "de Sitter supergravity" with constrained superfields [76, 77]. As there is compelling evidence that the necessary ingredients to realize such a theory are present in string theory [74, 79, 80, 81, 75, 78, 82], it is possible that one should regard the usual 4D supergravity regime that governs supersymmetric Minkowski compactifications and "de Sitter supergravity" as two different asymptotic regimes of string theory related by a duality transformation.

Finally, we also considered the possibility of shifting the asymptotic behavior of our effective theory, by including non-local effects that would manifest in the deep infra-red. We found that combining this IR effect with the higher derivative corrections can produce additional terms at the approriate order in the equations of motion, which may be tuned to give a spacetime curvature of either sign. Interestingly this approach appears to require time-dependent fluxes to provide the necessary corrections.

IR effects in gravity and their effects on cosmology have been considered in the literature with interesting implications [149], so perhaps their existence in string theory is not out of the question. If these IR effects exist, then AdS and dS compactifications may indeed be realized within the same EFT regime of string theory. Note however, that this EFT is by construction not the same as the type IIB supergravity regime that serves as the usual starting point for the usual de Sitter constructions, although it might contain it as a further sub-regime.

Outside of its application to the particular solutions we studied here, our approach based on scaling analysis and transseries expansions is rather powerful in the absense of explicit knowledge of the quantum corrections and can provides a useful map of the regimes of validity of various EFT limits of string theory. It would be interesting to explore further the interplay between these asymptotic expansions and the duality structure of string theory. We have also made no use of supersymmetry in our investigations, despite it having a very intricate interplay with the resurgent structure of asymptotic expansions [123, 122]. It would be interesting to see if this formalism may be adapted to keep track of supersymmetry breaking and its effects on the existence of effective field theory descriptions.

Bibliography

[1] Keshav Dasgupta, Maxim Emelin, Evan McDonough, and Radu Tatar. "Quantum Corrections and the de Sitter Swampland Conjecture". In: *JHEP* 01 (2019), p. 145. DOI: 10.1007/JHEP01(2019)145. arXiv: 1808.07498 [hep-th].

[2] Keshav Dasgupta, Maxim Emelin, Mir Mehedi Faruk, and Radu Tatar. "de Sitter Vacua in the String Landscape". In: (Aug. 2019). arXiv: 1908.05288 [hep-th].

[3] Keshav Dasgupta, Maxim Emelin, Mir Mehedi Faruk, and Radu Tatar. "How a four-dimensional de Sitter solution remains outside the swampland". In: (Nov. 2019). arXiv: 1911.02604 [hep-th].

[4] Keshav Dasgupta, Maxim Emelin, Mir Mehedi Faruk, and Radu Tatar. "de Sitter Vacua in the String landscape: La Petite Version". In: *11th International Symposium on Quantum Theory and Symmetries*. Nov. 2019. arXiv: 1911.12382 [hep-th].

[5] Maxim Emelin. "Effective Theories as Truncated Trans-Series and Scale Separated Compactifications". In: (May 2020). arXiv: 2005.11421 [hep-th].

[6] Adam G. Riess et al. "Observational evidence from supernovae for an accelerating universe and a cosmological constant". In: *Astron. J.* 116 (1998), pp. 1009–1038. DOI: 10.1086/300499. arXiv: astro-ph/9805201.

[7] S. Perlmutter et al. "Measurements of Ω and Λ from 42 high redshift supernovae". In: *Astrophys. J.* 517 (1999), pp. 565–586. DOI: 10.1086/307221. arXiv: astro-ph/9812133.

[8] P.A.R. Ade et al. "Planck 2015 results. XIII. Cosmological parameters". In: *Astron. Astrophys.* 594 (2016), A13. DOI: 10.1051/0004-6361/201525830. arXiv: 1502.01589 [astro-ph.CO].

[9] Alexei A. Starobinsky. “A New Type of Isotropic Cosmological Models Without Singularity”. In: *Adv. Ser. Astrophys. Cosmol.* 3 (1987). Ed. by I.M. Khalatnikov and V.P. Mineev, pp. 130–133. DOI: 10.1016/0370-2693(80)90670-X.

[10] Alan H. Guth. “The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems”. In: *Adv. Ser. Astrophys. Cosmol.* 3 (1987). Ed. by Li-Zhi Fang and R. Ruffini, pp. 139–148. DOI: 10.1103/PhysRevD.23.347.

[11] Andrei D. Linde. “A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems”. In: *Adv. Ser. Astrophys. Cosmol.* 3 (1987). Ed. by Li-Zhi Fang and R. Ruffini, pp. 149–153. DOI: 10.1016/0370-2693(82)91219-9.

[12] Alexander Vilenkin. “The Birth of Inflationary Universes”. In: *Phys. Rev. D* 27 (1983), p. 2848. DOI: 10.1103/PhysRevD.27.2848.

[13] Paul Joseph Steinhardt. “NATURAL INFLATION”. In: *Nuffield Workshop on the Very Early Universe*. July 1982, pp. 251–266.

[14] Shamit Kachru, Renata Kallosh, Andrei D. Linde, and Sandip P. Trivedi. “De Sitter vacua in string theory”. In: *Phys. Rev. D* 68 (2003), p. 046005. DOI: 10.1103/PhysRevD.68.046005. arXiv: hep-th/0301240.

[15] C.P. Burgess, R. Kallosh, and F. Quevedo. “De Sitter string vacua from supersymmetric D terms”. In: *JHEP* 10 (2003), p. 056. DOI: 10.1088/1126-6708/2003/10/056. arXiv: hep-th/0309187.

[16] Vijay Balasubramanian, Per Berglund, Joseph P. Conlon, and Fernando Quevedo. “Systematics of moduli stabilisation in Calabi-Yau flux compactifications”. In: *JHEP* 03 (2005), p. 007. DOI: 10.1088/1126-6708/2005/03/007. arXiv: hep-th/0502058.

[17] Alexander Westphal. “de Sitter string vacua from Kahler uplifting”. In: *JHEP* 03 (2007), p. 102. DOI: 10.1088/1126-6708/2007/03/102. arXiv: hep-th/0611332.

[18] Juan Martin Maldacena and Carlos Nunez. “Supergravity description of field theories on curved manifolds and a no go theorem”. In: *Int. J. Mod. Phys. A* 16 (2001). Ed. by Michael J. Duff, J.T. Liu, and J. Lu, pp. 822–855. DOI: 10.1142/S0217751X01003937. arXiv: hep-th/0007018.

[19] Keshav Dasgupta, Rhiannon Gwyn, Evan McDonough, Mohammed Mia, and Radu Tatar. "de Sitter Vacua in Type IIB String Theory: Classical Solutions and Quantum Corrections". In: *JHEP* 07 (2014), p. 054. DOI: 10.1007/JHEP07(2014)054. arXiv: 1402.5112 [hep-th].

[20] Ulf H. Danielsson and Thomas Van Riet. "What if string theory has no de Sitter vacua?" In: *Int. J. Mod. Phys. D* 27.12 (2018), p. 1830007. DOI: 10.1142/S0218271818300070. arXiv: 1804.01120 [hep-th].

[21] Cumrun Vafa. "The String landscape and the swampland". In: (Sept. 2005). arXiv: hep-th/0509212.

[22] T. Daniel Brennan, Federico Carta, and Cumrun Vafa. "The String Landscape, the Swampland, and the Missing Corner". In: *PoS* TASI2017 (2017), p. 015. DOI: 10.22323/1.305.0015. arXiv: 1711.00864 [hep-th].

[23] Eran Palti. "The Swampland: Introduction and Review". In: *Fortsch. Phys.* 67.6 (2019), p. 1900037. DOI: 10.1002/prop.201900037. arXiv: 1903.06239 [hep-th].

[24] Robert H. Brandenberger and C. Vafa. "Superstrings in the Early Universe". In: *Nucl. Phys. B* 316 (1989), pp. 391–410. DOI: 10.1016/0550-3213(89)90037-0.

[25] Robert H. Brandenberger. "String Gas Cosmology". In: Aug. 2008, pp. 193–230. arXiv: 0808.0746 [hep-th].

[26] Robert H. Brandenberger. "String Gas Cosmology: Progress and Problems". In: *Class. Quant. Grav.* 28 (2011), p. 204005. DOI: 10.1088/0264-9381/28/20/204005. arXiv: 1105.3247 [hep-th].

[27] Shinji Tsujikawa. "Quintessence: A Review". In: *Class. Quant. Grav.* 30 (2013), p. 214003. DOI: 10.1088/0264-9381/30/21/214003. arXiv: 1304.1961 [gr-qc].

[28] Georges Obied, Hirosi Ooguri, Lev Spodyneiko, and Cumrun Vafa. "De Sitter Space and the Swampland". In: (June 2018). arXiv: 1806.08362 [hep-th].

[29] Sumit K. Garg and Chethan Krishnan. "Bounds on Slow Roll and the de Sitter Swampland". In: *JHEP* 11 (2019), p. 075. DOI: 10.1007/JHEP11(2019)075. arXiv: 1807.05193 [hep-th].

[30] Hirosi Ooguri, Eran Palti, Gary Shiu, and Cumrun Vafa. “Distance and de Sitter Conjectures on the Swampland”. In: *Phys. Lett. B* 788 (2019), pp. 180–184. DOI: 10.1016/j.physletb.2018.11.018. arXiv: 1810.05506 [hep-th].

[31] Tom Rudelius. “Conditions for (No) Eternal Inflation”. In: *JCAP* 08 (2019), p. 009. DOI: 10.1088/1475-7516/2019/08/009. arXiv: 1905.05198 [hep-th].

[32] Alek Bedroya, Robert Brandenberger, Marilena Loverde, and Cumrun Vafa. “Trans-Planckian Censorship and Inflationary Cosmology”. In: *Phys. Rev. D* 101.10 (2020), p. 103502. DOI: 10.1103/PhysRevD.101.103502. arXiv: 1909.11106 [hep-th].

[33] Michael B. Green, John H. Schwarz, and Edward Witten. *Superstring Theory Vol. 1: 25th Anniversary Edition*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Nov. 2012. ISBN: 978-1-139-53477-2, 978-1-107-02911-8. DOI: 10.1017/CBO9781139248563.

[34] Michael B. Green, John H. Schwarz, and Edward Witten. *Superstring Theory Vol. 2: 25th Anniversary Edition*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Nov. 2012. ISBN: 978-1-139-53478-9, 978-1-107-02913-2. DOI: 10.1017/CBO9781139248570.

[35] J. Polchinski. *String theory. Vol. 1: An introduction to the bosonic string*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Dec. 2007. ISBN: 978-0-511-25227-3, 978-0-521-67227-6, 978-0-521-63303-1. DOI: 10.1017/CBO9780511816079.

[36] J. Polchinski. *String theory. Vol. 2: Superstring theory and beyond*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Dec. 2007. ISBN: 978-0-511-25228-0, 978-0-521-63304-8, 978-0-521-67228-3. DOI: 10.1017/CBO9780511618123.

[37] K. Becker, M. Becker, and J.H. Schwarz. *String theory and M-theory: A modern introduction*. Cambridge University Press, Dec. 2006. ISBN: 978-0-511-25486-4, 978-0-521-86069-7.

[38] Jr. Callan Curtis G. and Larus Thorlacius. “SIGMA MODELS AND STRING THEORY”. In: *Theoretical Advanced Study Institute in Elementary Particle Physics: Particles, Strings and Supernovae (TASI 88)*. Mar. 1989, pp. 795–878.

[39] F. Gliozzi, Joel Scherk, and David I. Olive. “Supersymmetry, Supergravity Theories and the Dual Spinor Model”. In: *Nucl. Phys. B* 122 (1977), pp. 253–290. DOI: 10.1016/0550-3213(77)90206-1.

[40] E.S. Fradkin and Arkady A. Tseytlin. “Nonlinear Electrodynamics from Quantized Strings”. In: *Phys. Lett. B* 163 (1985), pp. 123–130. DOI: 10.1016/0370-2693(85)90205-9.

[41] R.G. Leigh. “Dirac-Born-Infeld Action from Dirichlet Sigma Model”. In: *Mod. Phys. Lett. A* 4 (1989), p. 2767. DOI: 10.1142/S0217732389003099.

[42] Joseph Polchinski. “Dirichlet Branes and Ramond-Ramond charges”. In: *Phys. Rev. Lett.* 75 (1995), pp. 4724–4727. DOI: 10.1103/PhysRevLett.75.4724. arXiv: hep-th/9510017.

[43] T.H. Buscher. “A symmetry of the string background field equations”. In: *Physics Letters B* 194.1 (1987), pp. 59–62. ISSN: 0370-2693. DOI: https://doi.org/10.1016/0370-2693(87)90769-6. URL: http://www.sciencedirect.com/science/article/pii/0370269387907696.

[44] T.H. Buscher. “Path-integral derivation of quantum duality in nonlinear sigma-models”. In: *Physics Letters B* 201.4 (1988), pp. 466–472. ISSN: 0370-2693. DOI: https://doi.org/10.1016/0370-2693(88)90602-8. URL: http://www.sciencedirect.com/science/article/pii/0370269388906028.

[45] Igor A. Bandos and Bernard Julia. “Superfield T duality rules”. In: *JHEP* 08 (2003), p. 032. DOI: 10.1088/1126-6708/2003/08/032. arXiv: hep-th/0303075.

[46] Mirjam Cvetic, Hong Lu, C.N. Pope, and K.S. Stelle. “T duality in the Green-Schwarz formalism, and the massless / massive IIA duality map”. In: *Nucl. Phys. B* 573 (2000), pp. 149–176. DOI: 10.1016/S0550-3213(99)00740-3. arXiv: hep-th/9907202.

[47] S.F. Hassan. “T duality, space-time spinors and RR fields in curved backgrounds”. In: *Nucl. Phys. B* 568 (2000), pp. 145–161. DOI: 10.1016/S0550-3213(99)00684-7. arXiv: hep-th/9907152.

[48] Eric Bergshoeff, Christopher M. Hull, and Tomas Ortin. “Duality in the type II superstring effective action”. In: *Nucl. Phys. B* 451 (1995), pp. 547–578. DOI: 10.1016/0550-3213(95)00367-2. arXiv: hep-th/9504081.

[49] Raphael Benichou, Giuseppe Policastro, and Jan Troost. “T-duality in Ramond-Ramond backgrounds”. In: *Phys. Lett. B* 661 (2008), pp. 192–195. DOI: 10.1016/j.physletb.2008.01.059. arXiv: 0801.1785 [hep-th].

[50] Hirosi Ooguri and Cumrun Vafa. "Two-dimensional black hole and singularities of CY manifolds". In: *Nucl. Phys. B* 463 (1996), pp. 55–72. DOI: 10.1016/0550-3213(96)00008-9. arXiv: hep-th/9511164.

[51] David Tong. "NS5-branes, T duality and world sheet instantons". In: *JHEP* 07 (2002), p. 013. DOI: 10.1088/1126-6708/2002/07/013. arXiv: hep-th/0204186.

[52] Ghasem Exirifard. "Quadratic alpha-prime corrections to T-duality". In: *JHEP* 07 (2005), p. 047. DOI: 10.1088/1126-6708/2005/07/047. arXiv: hep-th/0504133.

[53] Ghasem Exirifard and Martin O'Loughlin. "Two and three loop alpha-prime corrections to T-duality: Kasner and Schwarzschild". In: *JHEP* 12 (2004), p. 023. DOI: 10.1088/1126-6708/2004/12/023. arXiv: hep-th/0408200.

[54] Diego Marques and Carmen A. Nunez. "T-duality and α-corrections". In: *JHEP* 10 (2015), p. 084. DOI: 10.1007/JHEP10(2015)084. arXiv: 1507.00652 [hep-th].

[55] Edward Witten. "String theory dynamics in various dimensions". In: *Nucl. Phys. B* 443 (1995), pp. 85–126. DOI: 10.1016/0550-3213(95)00158-O. arXiv: hep-th/9503124.

[56] Tohru Eguchi and Andrew J. Hanson. "Asymptotically Flat Selfdual Solutions to Euclidean Gravity". In: *Phys. Lett. B* 74 (1978), pp. 249–251. DOI: 10.1016/0370-2693(78)90566-X.

[57] Tohru Eguchi, Peter B. Gilkey, and Andrew J. Hanson. "Gravitation, Gauge Theories and Differential Geometry". In: *Phys. Rept.* 66 (1980), p. 213. DOI: 10.1016/0370-1573(80)90130-1.

[58] M.F. Atiyah and Nigel J. Hitchin. *THE GEOMETRY AND DYNAMICS OF MAGNETIC MONOPOLES. M.B. PORTER LECTURES.* 1988.

[59] E. Calabi. "On Kähler Manifolds with Vanishing Canonical Class". In: 1957.

[60] Shing-Tung Yau. "On the ricci curvature of a compact kähler manifold and the complex monge-ampére equation, I". In: *Communications on Pure and Applied Mathematics* 31.3 (1978), pp. 339–411. DOI: 10.1002/cpa.3160310304. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/cpa.3160310304. URL: https://onlinelibrary.wiley.com/doi/abs/10.1002/cpa.3160310304.

[61] G.W. Gibbons. "ASPECTS OF SUPERGRAVITY THEORIES". In: *XV GIFT Seminar on Supersymmetry and Supergravity*. June 1984.

[62] Mariana Grana. "Flux compactifications in string theory: A Comprehensive review". In: *Phys. Rept.* 423 (2006), pp. 91–158. DOI: 10.1016/j.physrep.2005.10.008. arXiv: hep-th/0509003.

[63] Katrin Becker and Melanie Becker. "M theory on eight manifolds". In: *Nucl. Phys. B* 477 (1996), pp. 155–167. DOI: 10.1016/0550-3213(96)00367-7. arXiv: hep-th/9605053.

[64] M.J. Duff, James T. Liu, and R. Minasian. "Eleven-dimensional origin of string-string duality: A One loop test". In: *Nucl. Phys. B* 452 (1995), pp. 261–282. DOI: 10.1016/0550-3213(95)00368-3. arXiv: hep-th/9506126.

[65] Edward Witten. "Five-brane effective action in M theory". In: *J. Geom. Phys.* 22 (1997), pp. 103–133. DOI: 10.1016/S0393-0440(97)80160-X. arXiv: hep-th/9610234.

[66] Dan Freed, Jeffrey A. Harvey, Ruben Minasian, and Gregory W. Moore. "Gravitational anomaly cancellation for M theory five-branes". In: *Adv. Theor. Math. Phys.* 2 (1998), pp. 601–618. DOI: 10.4310/ATMP.1998.v2.n3.a8. arXiv: hep-th/9803205.

[67] Keshav Dasgupta, Govindan Rajesh, and Savdeep Sethi. "M theory, orientifolds and G - flux". In: *JHEP* 08 (1999), p. 023. DOI: 10.1088/1126-6708/1999/08/023. arXiv: hep-th/9908088.

[68] Steven B. Giddings, Shamit Kachru, and Joseph Polchinski. "Hierarchies from fluxes in string compactifications". In: *Phys. Rev. D* 66 (2002), p. 106006. DOI: 10.1103/PhysRevD.66.106006. arXiv: hep-th/0105097.

[69] Sergei Gukov, Cumrun Vafa, and Edward Witten. "CFT's from Calabi-Yau four folds". In: *Nucl. Phys. B* 584 (2000). [Erratum: Nucl.Phys.B 608, 477–478 (2001)], pp. 69–108. DOI: 10.1016/S0550-3213(00)00373-4. arXiv: hep-th/9906070.

[70] Ashoke Sen. "F theory and orientifolds". In: *Nucl. Phys. B* 475 (1996), pp. 562–578. DOI: 10.1016/0550-3213(96)00347-1. arXiv: hep-th/9605150.

[71] Keshav Dasgupta and Sunil Mukhi. “F theory at constant coupling”. In: *Phys. Lett. B* 385 (1996), pp. 125–131. DOI: 10.1016/0370-2693(96)00875-1. arXiv: hep-th/9606044.

[72] Robert H. Brandenberger. “Alternatives to the inflationary paradigm of structure formation”. In: *Int. J. Mod. Phys. Conf. Ser.* 01 (2011). Ed. by Sang Pyo Kim, pp. 67–79. DOI: 10.1142/S2010194511000109. arXiv: 0902.4731 [hep-th].

[73] Robert H. Brandenberger. “Beyond Standard Inflationary Cosmology”. In: *Beyond Spacetime*. Ed. by Nick Huggett, Keizo Matsubara, and Christian Wüthrich. Apr. 2020, pp. 79–104. DOI: 10.1017/9781108655705.005. arXiv: 1809.04926 [hep-th].

[74] Renata Kallosh and Timm Wrase. “Emergence of Spontaneously Broken Supersymmetry on an Anti-D3-Brane in KKLT dS Vacua”. In: *JHEP* 12 (2014), p. 117. DOI: 10.1007/JHEP12(2014)117. arXiv: 1411.1121 [hep-th].

[75] Renata Kallosh, Fernando Quevedo, and Angel M. Uranga. “String Theory Realizations of the Nilpotent Goldstino”. In: *JHEP* 12 (2015), p. 039. DOI: 10.1007/JHEP12(2015)039. arXiv: 1507.07556 [hep-th].

[76] Eric A. Bergshoeff, Daniel Z. Freedman, Renata Kallosh, and Antoine Van Proeyen. “Pure de Sitter Supergravity”. In: *Phys. Rev. D* 92.8 (2015). [Erratum: Phys.Rev.D 93, 069901 (2016)], p. 085040. DOI: 10.1103/PhysRevD.93.069901. arXiv: 1507.08264 [hep-th].

[77] Marjorie Schillo, Ellen van der Woerd, and Timm Wrase. “The general de Sitter supergravity component action”. In: *Fortsch. Phys.* 64 (2016). Ed. by Riccardo Argurio et al., pp. 292–302. DOI: 10.1002/prop201500074. arXiv: 1511.01542 [hep-th].

[78] Eric A. Bergshoeff, Keshav Dasgupta, Renata Kallosh, Antoine Van Proeyen, and Timm Wrase. “$\overline{\mathrm{D3}}$ and dS”. In: *JHEP* 05 (2015), p. 058. DOI: 10.1007/JHEP05(2015)058. arXiv: 1502.07627 [hep-th].

[79] Bert Vercnocke and Timm Wrase. “Constrained superfields from an anti-D3-brane in KKLT”. In: *JHEP* 08 (2016), p. 132. DOI: 10.1007/JHEP08(2016)132. arXiv: 1605.03961 [hep-th].

[80] Renata Kallosh, Bert Vercnocke, and Timm Wrase. “String Theory Origin of Constrained Multiplets”. In: *JHEP* 09 (2016), p. 063. DOI: 10.1007/JHEP09(2016)063. arXiv: 1606.09245 [hep-th].

[81] Niccolò Cribiori, Christoph Roupec, Timm Wrase, and Yusuke Yamada. "Supersymmetric anti-D3-brane action in the Kachru-Kallosh-Linde-Trivedi setup". In: *Phys. Rev. D* 100.6 (2019), p. 066001. DOI: 10.1103/PhysRevD.100.066001. arXiv: 1906.07727 [hep-th].

[82] Keshav Dasgupta, Maxim Emelin, and Evan McDonough. "Fermions on the antibrane: Higher order interactions and spontaneously broken supersymmetry". In: *Phys. Rev. D* 95.2 (2017), p. 026003. DOI: 10.1103/PhysRevD.95.026003. arXiv: 1601.03409 [hep-th].

[83] Renata Kallosh and Andrei D. Linde. "Landscape, the scale of SUSY breaking, and inflation". In: *JHEP* 12 (2004), p. 004. DOI: 10.1088/1126-6708/2004/12/004. arXiv: hep-th/0411011.

[84] Mehmet Demirtas, Manki Kim, Liam McAllister, and Jakob Moritz. "Vacua with Small Flux Superpotential". In: *Phys. Rev. Lett.* 124.21 (2020), p. 211603. DOI: 10.1103/PhysRevLett.124.211603. arXiv: 1912.10047 [hep-th].

[85] Jakob Moritz, Ander Retolaza, and Alexander Westphal. "Toward de Sitter space from ten dimensions". In: *Phys. Rev. D* 97.4 (2018), p. 046010. DOI: 10.1103/PhysRevD.97.046010. arXiv: 1707.08678 [hep-th].

[86] F.F. Gautason, V. Van Hemelryck, and T. Van Riet. "The Tension between 10D Supergravity and dS Uplifts". In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800091. DOI: 10.1002/prop.201800091. arXiv: 1810.08518 [hep-th].

[87] Jakob Moritz, Ander Retolaza, and Alexander Westphal. "On uplifts by warped anti-D3-branes". In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800098. DOI: 10.1002/prop.201800098. arXiv: 1809.06618 [hep-th].

[88] Renata Kallosh, Andrei Linde, Evan McDonough, and Marco Scalisi. "dS Vacua and the Swampland". In: *JHEP* 03 (2019), p. 134. DOI: 10.1007/JHEP03(2019)134. arXiv: 1901.02022 [hep-th].

[89] Renata Kallosh, Andrei Linde, Evan McDonough, and Marco Scalisi. "de Sitter Vacua with a Nilpotent Superfield". In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800068. DOI: 10.1002/prop.201800068. arXiv: 1808.09428 [hep-th].

[90] Renata Kallosh, Andrei Linde, Evan McDonough, and Marco Scalisi. "4D models of de Sitter uplift". In: *Phys. Rev. D* 99.4 (2019), p. 046006. DOI: 10.1103/PhysRevD.99.046006. arXiv: 1809.09018 [hep-th].

[91] Yuta Hamada, Arthur Hebecker, Gary Shiu, and Pablo Soler. "On brane gaugino condensates in 10d". In: *JHEP* 04 (2019), p. 008. DOI: 10.1007/JHEP04(2019)008. arXiv: 1812.06097 [hep-th].

[92] Renata Kallosh. "Gaugino Condensation and Geometry of the Perfect Square". In: *Phys. Rev. D* 99.6 (2019), p. 066003. DOI: 10.1103/PhysRevD.99.066003. arXiv: 1901.02023 [hep-th].

[93] Yuta Hamada, Arthur Hebecker, Gary Shiu, and Pablo Soler. "Understanding KKLT from a 10d perspective". In: *JHEP* 06 (2019), p. 019. DOI: 10.1007/JHEP06(2019)019. arXiv: 1902.01410 [hep-th].

[94] Federico Carta, Jakob Moritz, and Alexander Westphal. "Gaugino condensation and small uplifts in KKLT". In: *JHEP* 08 (2019), p. 141. DOI: 10.1007/JHEP08(2019)141. arXiv: 1902.01412 [hep-th].

[95] F.F. Gautason, V. Van Hemelryck, T. Van Riet, and G. Venken. "A 10d view on the KKLT AdS vacuum and uplifting". In: *JHEP* 06 (2020), p. 074. DOI: 10.1007/JHEP06(2020)074. arXiv: 1902.01415 [hep-th].

[96] Iosif Bena, Mariana Graña, Nicolas Kovensky, and Ander Retolaza. "Kähler moduli stabilization from ten dimensions". In: *JHEP* 10 (2019), p. 200. DOI: 10.1007/JHEP10(2019)200. arXiv: 1908.01785 [hep-th].

[97] Shamit Kachru, Manki Kim, Liam McAllister, and Max Zimet. "de Sitter Vacua from Ten Dimensions". In: (Aug. 2019). arXiv: 1908.04788 [hep-th].

[98] Savdeep Sethi. "Supersymmetry Breaking by Fluxes". In: *JHEP* 10 (2018), p. 022. DOI: 10.1007/JHEP10(2018)022. arXiv: 1709.03554 [hep-th].

[99] Iosif Bena et al. "Persistent anti-brane singularities". In: *JHEP* 10 (2012), p. 078. DOI: 10.1007/JHEP10(2012)078. arXiv: 1205.1798 [hep-th].

[100] Iosif Bena, Mariana Grana, Stanislav Kuperstein, and Stefano Massai. "Anti-D3 Branes: Singular to the bitter end". In: *Phys. Rev. D* 87.10 (2013), p. 106010. DOI: 10.1103/PhysRevD.87.106010. arXiv: 1206.6369 [hep-th].

[101] Iosif Bena, Mariana Graña, Stanislav Kuperstein, and Stefano Massai. "Polchinski-Strassler does not uplift Klebanov-Strassler". In: *JHEP* 09 (2013), p. 142. DOI: 10.1007/JHEP09(2013)142. arXiv: 1212.4828 [hep-th].

[102] Iosif Bena, Mariana Graña, Stanislav Kuperstein, and Stefano Massai. "Giant Tachyons in the Landscape". In: *JHEP* 02 (2015), p. 146. DOI: 10.1007/JHEP02(2015)146. arXiv: 1410.7776 [hep-th].

[103] Iosif Bena and Stanislav Kuperstein. "Brane polarization is no cure for tachyons". In: *JHEP* 09 (2015), p. 112. DOI: 10.1007/JHEP09(2015)112. arXiv: 1504.00656 [hep-th].

[104] Iosif Bena, Johan Blåbäck, and David Turton. "Loop corrections to the antibrane potential". In: *JHEP* 07 (2016), p. 132. DOI: 10.1007/JHEP07(2016)132. arXiv: 1602.05959 [hep-th].

[105] Ben Michel, Eric Mintun, Joseph Polchinski, Andrea Puhm, and Philip Saad. "Remarks on brane and antibrane dynamics". In: *JHEP* 09 (2015), p. 021. DOI: 10.1007/JHEP09(2015)021. arXiv: 1412.5702 [hep-th].

[106] Joseph Polchinski. "Brane/antibrane dynamics and KKLT stability". In: (Sept. 2015). arXiv: 1509.05710 [hep-th].

[107] Robert C. Myers. "Dielectric branes". In: *JHEP* 12 (1999), p. 022. DOI: 10.1088/1126-6708/1999/12/022. arXiv: hep-th/9910053.

[108] Shamit Kachru, John Pearson, and Herman L. Verlinde. "Brane / flux annihilation and the string dual of a nonsupersymmetric field theory". In: *JHEP* 06 (2002), p. 021. DOI: 10.1088/1126-6708/2002/06/021. arXiv: hep-th/0112197.

[109] Iosif Bena, Emilian Dudas, Mariana Graña, and Severin Lüst. "Uplifting Runaways". In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800100. DOI: 10.1002/prop.201800100. arXiv: 1809.06861 [hep-th].

[110] Iosif Bena, Alex Buchel, and Severin Lüst. "Throat destabilization (for profit and for fun)". In: (Oct. 2019). arXiv: 1910.08094 [hep-th].

[111] Jerome Martin and Robert H. Brandenberger. "The TransPlanckian problem of inflationary cosmology". In: *Phys. Rev. D* 63 (2001), p. 123501. DOI: 10.1103/PhysRevD.63.123501. arXiv: hep-th/0005209.

[112] Lavinia Heisenberg, Matthias Bartelmann, Robert Brandenberger, and Alexandre Refregier. "Dark Energy in the Swampland". In: *Phys. Rev. D* 98.12 (2018), p. 123502. DOI: 10.1103/PhysRevD.98.123502. arXiv: 1808.02877 [astro-ph.CO].

[113] Lavinia Heisenberg, Matthias Bartelmann, Robert Brandenberger, and Alexandre Refregier. "Dark Energy in the Swampland II". In: *Sci. China Phys. Mech. Astron.* 62.9 (2019), p. 990421. DOI: 10.1007/s11433-019-9392-7. arXiv: 1809.00154 [astro-ph.CO].

[114] Yashar Akrami, Renata Kallosh, Andrei Linde, and Valeri Vardanyan. "The Landscape, the Swampland and the Era of Precision Cosmology". In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800075. DOI: 10.1002/prop.201800075. arXiv: 1808.09440 [hep-th].

[115] Marco Raveri, Wayne Hu, and Savdeep Sethi. "Swampland Conjectures and Late-Time Cosmology". In: *Phys. Rev. D* 99.8 (2019), p. 083518. DOI: 10.1103/PhysRevD.99.083518. arXiv: 1812.10448 [hep-th].

[116] Dieter Lüst, Eran Palti, and Cumrun Vafa. "AdS and the Swampland". In: *Phys. Lett. B* 797 (2019), p. 134867. DOI: 10.1016/j.physletb.2019.134867. arXiv: 1906.05225 [hep-th].

[117] F.F. Gautason, M. Schillo, T. Van Riet, and M. Williams. "Remarks on scale separation in flux vacua". In: *JHEP* 03 (2016), p. 061. DOI: 10.1007/JHEP03(2016)061. arXiv: 1512.00457 [hep-th].

[118] C.M. Bender and S.A. Orszag. *Advanced Mathematical Methods for Scientists and Engineers I: Asymptotic Methods and Perturbation Theory*. Springer New York, 2012. ISBN: 9781475730708. URL: https://books.google.ca/books?id=X09drgEACAAJ.

[119] M.V Berry and C. J. Howls. "Hyperasymptotics". In: *Proc. R. Soc. Lond.* A430 (1990), pp. 653–668. DOI: 10.1098/rspa.1990.0111.

[120] Stephen H. Shenker. "The Strength of nonperturbative effects in string theory". In: *Cargese Study Institute: Random Surfaces, Quantum Gravity and Strings*. Aug. 1990, pp. 191–200.

[121] Gerald V. Dunne and Mithat Ünsal. "Generating nonperturbative physics from perturbation theory". In: *Phys. Rev. D* 89.4 (2014), p. 041701. DOI: 10.1103/PhysRevD.89.041701. arXiv: 1306.4405 [hep-th].

[122] Can Kozçaz, Tin Sulejmanpasic, Yuya Tanizaki, and Mithat Ünsal. “Cheshire Cat resurgence, Self-resurgence and Quasi-Exact Solvable Systems”. In: *Commun. Math. Phys.* 364.3 (2018), pp. 835–878. DOI: 10.1007/s00220-018-3281-y. arXiv: 1609.06198 [hep-th].

[123] Gerald V. Dunne and Mithat Unsal. “Deconstructing zero: resurgence, supersymmetry and complex saddles”. In: *JHEP* 12 (2016), p. 002. DOI: 10.1007/JHEP12(2016)002. arXiv: 1609.05770 [hep-th].

[124] Aleksey Cherman, Daniele Dorigoni, Gerald V. Dunne, and Mithat Ünsal. “Resurgence in Quantum Field Theory: Nonperturbative Effects in the Principal Chiral Model”. In: *Phys. Rev. Lett.* 112 (2014), p. 021601. DOI: 10.1103/PhysRevLett.112.021601. arXiv: 1308.0127 [hep-th].

[125] Aleksey Cherman, Peter Koroteev, and Mithat Ünsal. “Resurgence and Holomorphy: From Weak to Strong Coupling”. In: *J. Math. Phys.* 56.5 (2015), p. 053505. DOI: 10.1063/1.4921155. arXiv: 1410.0388 [hep-th].

[126] Aleksey Cherman, Daniele Dorigoni, and Mithat Unsal. “Decoding perturbation theory using resurgence: Stokes phenomena, new saddle points and Lefschetz thimbles”. In: *JHEP* 10 (2015), p. 056. DOI: 10.1007/JHEP10(2015)056. arXiv: 1403.1277 [hep-th].

[127] Inês Aniceto, Ricardo Schiappa, and Marcel Vonk. “The Resurgence of Instantons in String Theory”. In: *Commun. Num. Theor. Phys.* 6 (2012), pp. 339–496. DOI: 10.4310/CNTP.2012.v6.n2.a3. arXiv: 1106.5922 [hep-th].

[128] Marcel Vonk. “Resurgence and Topological Strings”. In: *Proc. Symp. Pure Math.* 93 (2015). Ed. by Vincent Bouchard, Stefan Méndez-Diez, Callum Quigley, and Charles Doran, p. 221. arXiv: 1502.05711 [hep-th].

[129] Marcos Mariño and Tomás Reis. “Resurgence for superconductors”. In: (May 2019). DOI: 10.1088/1742-5468/ab4802. arXiv: 1905.09569 [hep-th].

[130] J. Ecalle. *Les Fonctions Resurgentes, Vol I, II.* Publ. Math. Orsay, 1981.

[131] J. Ecalle. *Les Fonctions Resurgentes, Vol III.* Publ. Math. Orsay, 1985.

[132] O. Costin. *Asymptotics and Borel Summability*. Monographs and Surveys in Pure and Applied Mathematics. CRC Press, 2008. ISBN: 9781420070323. URL: https://books.google.ca/books?id=HWmJyqqxmXYC.

[133] Inês Aniceto and Ricardo Schiappa. "Nonperturbative Ambiguities and the Reality of Resurgent Transseries". In: *Commun. Math. Phys.* 335.1 (2015), pp. 183–245. DOI: 10.1007/s00220-014-2165-z. arXiv: 1308.1115 [hep-th].

[134] Daniele Dorigoni. "An Introduction to Resurgence, Trans-Series and Alien Calculus". In: *Annals Phys.* 409 (2019), p. 167914. DOI: 10.1016/j.aop.2019.167914. arXiv: 1411.3585 [hep-th].

[135] Marcos Mariño. *Instantons and Large N: An Introduction to Non-Perturbative Methods in Quantum Field Theory*. Cambridge University Press, Sept. 2015. ISBN: 978-1-107-06852-0, 978-1-316-37154-1.

[136] Gerald V. Dunne and Mithat Ünsal. "What is QFT? Resurgent trans-series, Lefschetz thimbles, and new exact saddles". In: *PoS* LATTICE2015 (2016), p. 010. DOI: 10.22323/1.251.0010. arXiv: 1511.05977 [hep-lat].

[137] Inês Aniceto, Gokce Basar, and Ricardo Schiappa. "A Primer on Resurgent Transseries and Their Asymptotics". In: *Phys. Rept.* 809 (2019), pp. 1–135. DOI: 10.1016/j.physrep.2019.02.003. arXiv: 1802.10441 [hep-th].

[138] T. Banks. "Landskepticism or why effective potentials don't count string models". In: Dec. 2004. arXiv: hep-th/0412129.

[139] Tom Banks. "The Top 10^{500} Reasons Not to Believe in the Landscape". In: (Aug. 2012). arXiv: 1208.5715 [hep-th].

[140] Tom Banks. "On the Limits of Effective Quantum Field Theory: Eternal Inflation, Landscapes, and Other Mythical Beasts". In: (Oct. 2019). arXiv: 1910.12817 [hep-th].

[141] Yoshifumi Hyakutake. "Toward the Determination of R**3 F**2 Terms in M-theory". In: *Prog. Theor. Phys.* 118 (2007), p. 109. DOI: 10.1143/PTP.118.109. arXiv: hep-th/0703154.

[142] Thomas W. Grimm, Tom G. Pugh, and Matthias Weissenbacher. "The effective action of warped M-theory reductions with higher derivative terms — part I". In: *JHEP* 01 (2016), p. 142. DOI: 10.1007/JHEP01(2016)142. arXiv: 1412.5073 [hep-th].

[143] Thomas W. Grimm, Tom G. Pugh, and Matthias Weissenbacher. "The effective action of warped M-theory reductions with higher-derivative terms - Part II". In: *JHEP* 12 (2015), p. 117. DOI: 10.1007/JHEP12(2015)117. arXiv: 1507.00343 [hep-th].

[144] Igor R. Klebanov and Matthew J. Strassler. "Supergravity and a confining gauge theory: Duality cascades and chi SB resolution of naked singularities". In: *JHEP* 08 (2000), p. 052. DOI: 10.1088/1126-6708/2000/08/052. arXiv: hep-th/0007191.

[145] Keshav Dasgupta, Govindan Rajesh, Daniel Robbins, and Savdeep Sethi. "Time dependent warping, fluxes, and NCYM". In: *JHEP* 03 (2003), p. 041. DOI: 10.1088/1126-6708/2003/03/041. arXiv: hep-th/0302049.

[146] Jock McOrist and Savdeep Sethi. "M-theory and Type IIA Flux Compactifications". In: *JHEP* 12 (2012), p. 122. DOI: 10.1007/JHEP12(2012)122. arXiv: 1208.0261 [hep-th].

[147] Ashoke Sen. "A Note on enhanced gauge symmetries in M and string theory". In: *JHEP* 09 (1997), p. 001. DOI: 10.1088/1126-6708/1997/09/001. arXiv: hep-th/9707123.

[148] Katrin Becker, Melanie Becker, and Andrew Strominger. "Five-branes, membranes and nonperturbative string theory". In: *Nucl. Phys. B* 456 (1995), pp. 130–152. DOI: 10.1016/0550-3213(95)00487-1. arXiv: hep-th/9507158.

[149] Enis Belgacem, Yves Dirian, Andreas Finke, Stefano Foffa, and Michele Maggiore. "Gravity in the infrared and effective nonlocal models". In: *JCAP* 04 (2020), p. 010. DOI: 10.1088/1475-7516/2020/04/010. arXiv: 2001.07619 [astro-ph.CO].

[150] Tom Banks and Willy Fischler. "Why The Cosmological Constant is a Boundary Condition". In: (Oct. 2018). arXiv: 1811.00130 [hep-th].