TORSIONAL RESISTANCE OF A STEEL BEAM

HAVING STIFFENERS II

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Master of Engineering

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SYNOPSIS

When a beam carries a load, which does not act through its shear center, torsional stresses are introduced along with the flexural stresses. Torsion may be a serious factor in producing beam failures. Rib reinforcement can reduce torsional stresses and angles of twist. The purpose of this investigation was to determine the influence web stiffeners have on these stresses and twist angles.

Tests were made on a simple beam with torsionally free ends carrying a complete uniform eccentric load. Common web stiffeners with bolts were used as rib stiffeners. The number and position of these stiffeners were varied; three pairs at quarter points, two pairs at third points, one pair at center, and no stiffeners.

A 10WF25 fifteen feet long, with a plate welded to the bottom flange (typical of a spandrel), was used as the test beam. Four loads were applied, simulating brick walls resting on the bottom plate, producing flexural and torsional stresses. The beam was also loaded with a pure torque in order to determine a torsional constant for the test section.

The angle of twist, flange and web stresses were examined to find what effects were produced by the stiffeners.

The observed test data was compared with the theoretical based on the Lyse-Johnston method of design. An attempt to modify the Lyse-Johnston theory to suit the loading conditions was also made.

The experiment stopped when permanent yielding took place in the bottom of the web near each support.

NOTATION

The following symbols are used in this paper and conform as closely as possible to those found in recent literature on this subject:

Symbol	Definition	Unit	Source
a	torsional bending constant	in.	17
A	area	$in.^2$	33
A_1, B_1, etc.	coefficients of a differential equation		17
b	width of flange of beam	in.	12
B	torsional flange stress constant of beam	$in.^{-3}$	18
C	angular twist constant	$(in.lb.)^{-1}$	19
d	depth of beam	in.	33
D	diameter	in.	33
e	eccentricity of applied load	in.	20
E	Young's modulus of elasticity (29×10^6)	$lb/in.^2$	
f_t	longitudinal flange stress due to torsion	$lb/in.^2$	15
f_b	longitudinal flange stress due to vertical bending	$lb/in.^2$	15
f_h	longitudinal flange stress due to horizontal bending	$lb/in.^2$	50
G	modulus of elasticity in shear (11.5×10^6)	$lb/in.^2$	
h	center to center of flanges	in.	17
I	moment of inertia of area	$in.^4$	
J	polar moment of inertia	$in.^4$	6

Symbol	Definition	Unit	Source
K	torsional constant	$in.^4$	10
L	length	in.	20
M	bending moment	in.lb.	18
N	torsional flange shear constant of beam	$in.^{-3}$	19
r	radius	in.	6
s_t	web shear stress from torsion	$lb/in.^2$	16
s_v	web shear stress from vertical loads	$lb/in.^2$	16
s_f	flange shear stress from torsion	$lb/in.^2$	16
s_q	flange shear stress from lateral bending	$lb/in.^2$	16
t	thickness	in.	12
T	applied torque or twisting moment	in.lb.	6
V	horizontal load	lb/in.	20
V_q	flange shear from lateral bending	lb.	16
w	uniform vertical load per foot of beam	lb/ft.	20
x, y, z	cartesian co-ordinates		
Z	torsional web shear constant of beam	$in.^{-3}$	19
	GREEK SYMBOLS		
μ (mu)	Poisson's ratio		
ψ (psi)	angle of twist	degrees	19
θ (theta)	rate of twist	deg/ft.	10
τ (tau)	shear stress	$lb/in.^2$	6
σ (sigma)	principal tensile or compressive stress	$lb/in.^2$	
ϕ (phi)	stress function		9

CONTENTS

CHAPTER 1

INTRODUCTION

The majority of members of a structural steel framework are submitted to direct or bending loads only, or to a combination of these two, but occasionally a torque may also be applied. Such torques occur when a beam is loaded in a plane which does not pass through the shear center of the section, or when the beam is curved in a plane at right angles to the applied loads. The large areas of fenestration, and rounded buildings in our modern architecture, introduce such spandrel beams and bow girders.

In torsional design, member selection may be governed by maximum allowable stress or by maximum allowable deflection (rotation). In public buildings minimum deflections to prevent cracking of inside or outside finishes is most often the governing condition. In industrial buildings where appearance is of secondary nature, either criterion may govern.

Loads producing flexural stresses are most commonly resisted by WF or I sections. It has been the practice of structural engineers to use these sections where a combination of flexure and torsion is encountered, even if the torsional stresses are critical. The unfavorable feature is that structural sections are extremely inefficient in resisting torsion (consider two shafts: a 10WF25 and a solid circular shaft only one-quarter this weight; equal pure torques will produce equal maximum stresses in these two bars).

Naturally, therefore, it would be of considerable value if a method could be devised for increasing the torsional rigidity of a structural section while keeping additional weight and fabrication costs as low as possible.

It is the purpose of this experiment to determine the possible beneficial effects of web stiffeners on the torsional properties of a 10WF25. Simultaneously, a comparison of the experimental results with those predicted by existing formulae will be made, and an effort made to modify such formulae to suit test specimen and loading conditions.

Existing literature on the subject has been used frequently, and due acknowledgement is made in the Bibliography.

HISTORICAL NOTE

The history of the solution of the torsion problem can be found in any recent literature on the subject (16)*, however, to place this work in its proper perspective a brief summary is desirable.

The original theory of torsion is attributed to Coulomb (3) who in 1784 determined the torsional rigidity of a circular wire by torsional oscillations.

In 1820 A. Duleau verified Coulombs formulae for circular bars, but found they were not applicable to other cross-sections. Cauchy in 1830 improved the theory for rectangular cross-sections, but it was Saint-Venant (13) who, in 1855, presented a differential equation for the solution of the torsion problem applicable to any cross-section. However, this equation can only be solved mathematically for the simpler cross-sections, namely; rectangular, elliptical and triangular.

For rolled or extruded sections no formal solution of Saint-Venant's equation is possible. Therefore, approximate analytical solutions, the use of analogies, or experimental treatment must be employed for all complex cross-sections.

In 1903 Prandtl (12) introduced the analogy between a stretched membrane and the shearing stresses in a similar cross-section due to

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* Numbers in brackets refer to reference in the Bibliography.

torsion. The outstanding investigations using Prandtl's analogy were carried out by Griffith and Taylor (8) and Cassie and Dobie (1) in England, and in the United States by Trayer and March (18) and by Lyse and Johnston (10).

Timoshenko (15) shortened the pure torsional theory by slight modification of Saint-Venant's equations and by mathematical applications of membrane analogy. In 1905 he investigated the problem of twisting in an I-beam with a built-in end and found that not only Saint-Venant's torsional stresses, but also bending stresses in the flanges must be considered to get satisfactory values for the angle of twist. This was the first investigation of the effect produced by sections prevented from warping.

Lyse and Johnston made the most recent valuable contribution. The laboratory procedure was to subject the member to a shaft torque under uniform torsion conditions. The straight line portion of the torque vs unit angle of twist graph was used to calculate the torsional constant K. Prandtl's soap film analogy was also used to good advantage. The test results brought out the importance of torsionally free (free to warp) ends and torsionally fixed (warping prevented) ends. The tests covered a sufficient number of specimens to enable Lyse and Johnston to derive equations for K for I-beam and WF sections.

Subsequent theorectical work on torsion was done by Sourochnikoff (14) who considered the change in eccentricity of a vertical load on a beam after twisting, and by Goldberg (5) who introduced the angle of warping in an attempt to simplify calculations.

Experimentally Chang and Johnston (2) tested plate girders in torsion and studied the effects on rivets, plates, etc.

In 1952 Dr. Esslinger (4) presented a paper in France dealing with the effects of welded rib stiffeners on the torsional properties of I-beams.

In 1955 the first of a series of tests was begun at McGill University by Mr. Carl Goldman (6) to determine if pairs of stiffeners would increase the torsional resistance of an 8WF17 beam subjected to an eccentric loading. Tests were carried out on this spandrel beam with four combinations of stiffener positions. The present work is a continuation of this work using a 10WF25 beam.

CHAPTER II

THEORY

Limitations of the Simple Torsion Theory

A member is subjected to pure torsion if the resultant of all the forces acting on each cross-section is a couple, the plane of the couple being normal to the axis of the member.

In the development of the formula for the evaluation of

$$\tau = \frac{Tr}{J} \qquad \text{-----------------} (i)$$

shearing stresses (i)* in a shaft of circular cross-section several assumptions are made, namely:

1. Statics

   (a) The resultant of the external forces is a couple that lies in a plane perpendicular to the longitudinal axis of the shaft.

   (b) The shaft is in equilibrium.

2. Geometry

   (c) The axis of the shaft is straight.

   (d) The shaft is circular and free from changes in cross-section.

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*Roman numerals will be used for all formulae to avoid confusion with reference notes.

(e) Cross-sections normal to the axis of the shaft before twisting, remain plane after twisting and rotate as if absolutely rigid.

3. Properties of the Material

(f) The material is homogenous and isotropic.

(g) The stresses do not exceed the proportional limit of the material; angles of twist are small.

All assumptions, with the exception of (e), can be controlled to the point where the errors they induce are negligible. Experiments are necessary to prove the validity of the assumption that plane cross-sections before twisting remain plane after twisting. Tests on circular bars (Fig. 1) have shown that the assumption is valid.

To invalidate this assumption for cross-sections other than circular, consider a plane x, y, of a shaft of rectangular cross-section (Fig. 2) subjected to a torque. Three small elements 'a', 'b' and 'c' taken from the bar are also shown in Fig. 2. Since element 'a' can develop no shear on its outside face, $\tau_{xz} = \tau_{xy} = 0$. Similarly for element 'b' $\tau_{yz} = \tau_{yx} = 0$. Element 'c', on the corner of the cross-section has no shear forces on two outside planes; hence all shearing forces are zero.

It is reasonable to expect then, that the shearing stresses will vary from a maximum at the middle of the sides to zero at the corners of the cross-section. This variation will cause warping of the

FIG. I.

A SOLID CIRCULAR BAR IN PURE TORSION

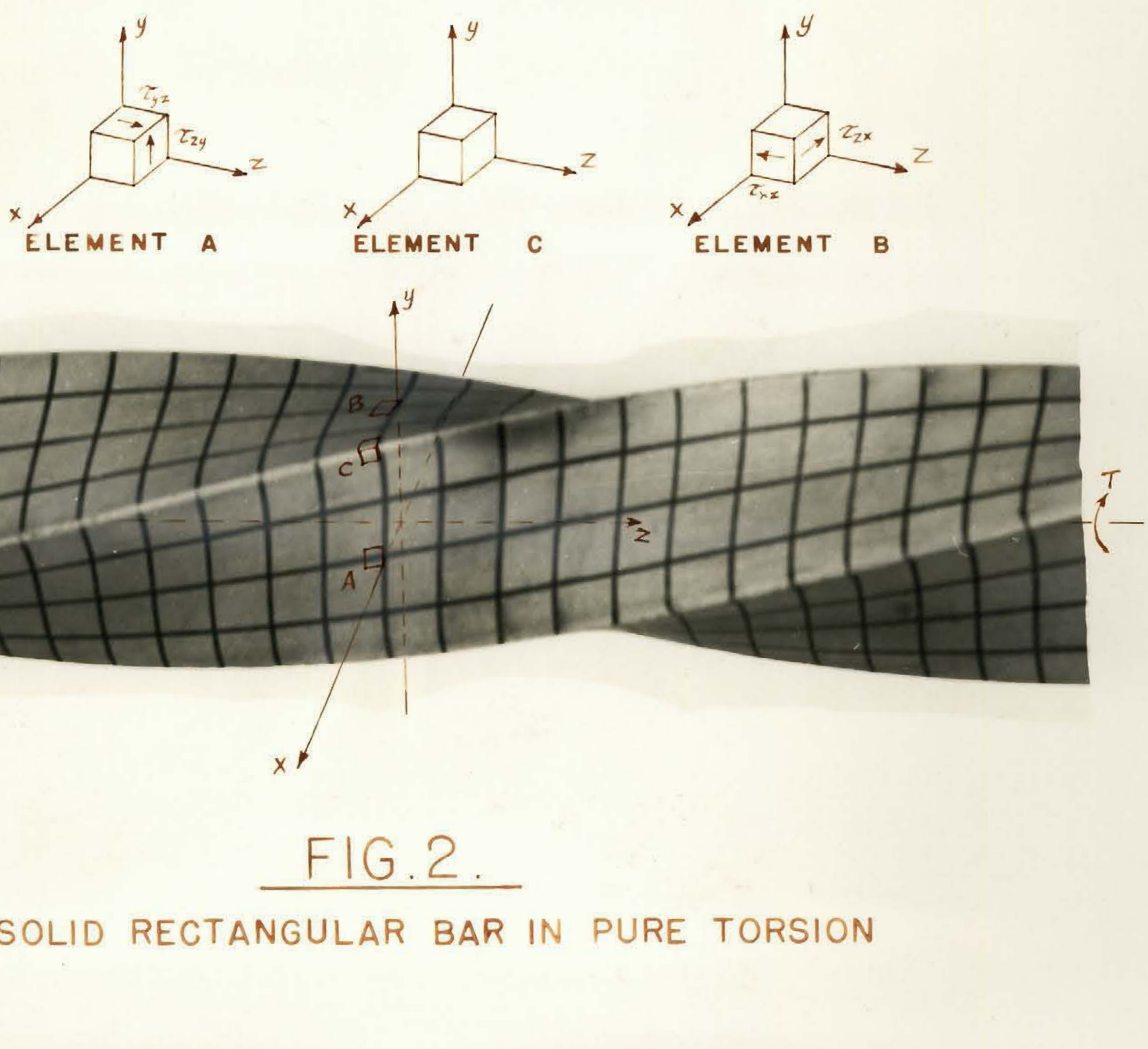


FIG. 2.

A SOLID RECTANGULAR BAR IN PURE TORSION

section during twist. This warping is shown experimentally (Fig. 2) with a rectangular bar of rubber on whose sides a net of small squares has been drawn. The figure shows that lines originally perpendicular to the axis of the bar have become curved. Note that the distortion of the squares (or shear) is a maximum at the middle and disappears at the corners. Equation (i) will not give this result, and consequently, it is not valid for cross-sections other than circular.

Saint-Venant's Torsion Theory

The solution of the torsion problem, like that of others, must satisfy the general stress equations of equilibrium, the equations of compatability of strain, and the boundary conditions of the problem. the assumption that direct stresses and strains are zero and that only two components of shear stress (τ_{xz} and τ_{yz}) need be evaluated simplifies the problem.

Saint-Venant's solution introduces a "stress function", ϕ , of x and y that defines the distribution of stress over a given cross-section. This "stress function" has the following relationships to stresses, angle of twist and applied moment:

$$\tau_{xz} = \frac{\partial \phi}{\partial y} \qquad \tau_{yz} = -\frac{\partial \phi}{\partial x}$$

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial x^2} = -2G\theta \qquad \text{-----------------(ii)}$$

$$T = 2\iint \phi \, dx \, dy$$

Saint-Venant showed how these relationships could give a solution for several of the simpler geometrical shapes, but the mathematical solution for complicated sections is impossible.

Methods of Evaluating the Torsion Constant K.

For a circular bar the following relationships hold for the shearing stress and the angle of twist per unit of length,

$$\tau = \frac{Tr}{J} \qquad \theta = \frac{T}{JG} \qquad \text{---------------(iii)}$$

where J is the polar moment of inertia. It has been shown that for other cross-sections these equations can be written,

$$\tau = \frac{TL}{K} \qquad \theta = \frac{T}{KG} \qquad \text{-----------------(iv)}$$

where L is some function depending on the shape of the cross-section and K is called the torsional constant. This constant has the same units as the polar moment J, but there is no direct relation between the two.

Many ingenious methods for the solution of the torsional constant have been made and some of these will now be discussed.

1. Direct Torsion Tests

When direct tests can be made the torsion constant can be calculated from equation (iv) after careful measurements of T, G, and θ.

There are several disadvantages to direct tests, namely:

(a) Rolled sections are subject to variations in dimensions and have high tolerances.

(b) The effect of the variation of any one dimension necessitates the testing of numerous special sections.

(c) Special twisting apparatus is required.

2. Membrane Analogy

Prandtl (12) introduced the concept of a thin homogeneous membrane laid across a sharp edged hole of the same shape as the section under consideration, and lightly stretched by air pressure from one side. The results of a study of this analogy gives the following relationships:

(a) The height of the membrane is proportional to the stress function of the twisted section.

(b) The volume under the membrane is proportional to the torsion constant.

(c) The maximum slope of the membrane at any point is proportional to the shear stress in the twisted section.

(d) The lines of action of the shear stresses in the twisted section follow the contours of the stretched membrane.

This analogy has been used to determine the torsion constant (see Historical Note), but its main value to an investigator, is that by visualizing what such a membrane would look like, he can easily imagine the distribution of shear stress and the effect on the torsion constant of changes in sectional shape or area.

3. Dissection Into Simple Shapes

Rolled sections can be considered as shapes built up of rectangles and trapeziums. The sum of the torsion constants of the individual sections will approximate the torsion constant of the whole.

Saint-Venant derived an accurate formula for the torsional constant of a rectangle

$$K = \frac{1}{3} bt^3 - 0.21 t^4 \qquad \text{(v)}$$

where b = long side and t = short side. The last term represents the "end loss" effect and can be neglected if b/t is greater than ten.

A WF beam with parallel sided flanges can be considered as an aggregation of three rectangles and its torsional constant can be written as,

$$K = \sum \frac{1}{3} bt^3 \qquad \text{(vi)}$$

This gives a good approximation, as the reductions due to "end losses" tend to offset the gain by "junction effect" (see Fig. 3).

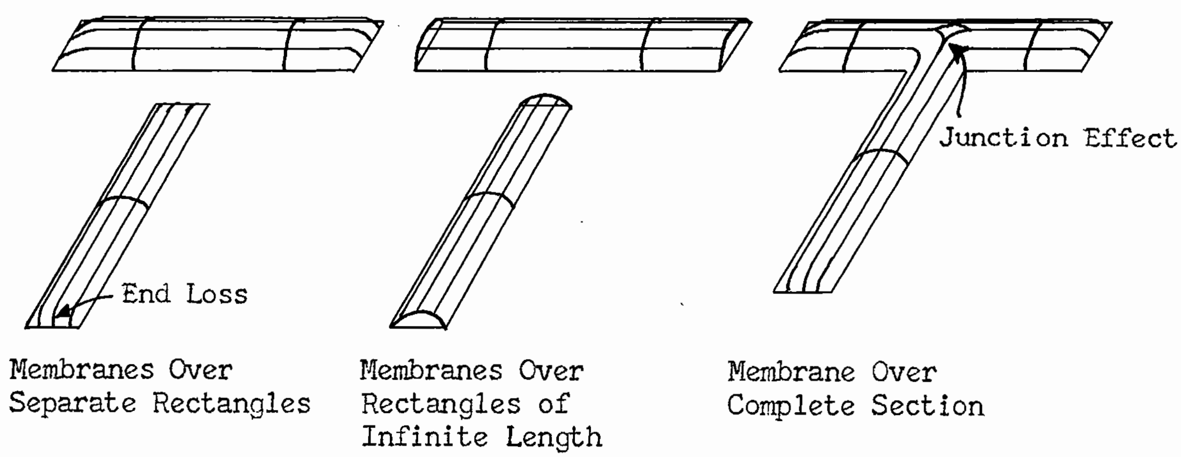


FIG. 3

Stretched Membranes Over a Tee Section

4. The Use of Emperical Formulae

The value of the torsion constant for WF and I sections can be found by using expressions derived by Lyse and Johnston (10). These formulae are based on numerous test results and can be considered as the correct solutions for K.

Sections Restrained from Warping

If all cross-sections in a bar subjected to twisting are free to warp, the longitudinal elements (lines) of the surface of the twisted bar remain practically straight lines with negligible change in length.

If, however, one or more sections of a channel or I beam subjected to a twisting moment is restrained from warping, then the elements of the surface become curved with marked changes in their lengths, and the accompanying longitudinal stresses in the outer elements of the flanges are not negligible. These longitudinal stresses combine numerically with other longitudinal stresses that might be present i.e., flexural, tension or compression stresses.

In Fig. 4 (a) an I beam is shown with a couple applied midway between supports. From symmetry it can be concluded that section a-b-c-d remains plane during twist. Timoshenko (15) solves this problem by showing that the applied twisting moment is resisted partly by torsional shearing stresses and partly by lateral shearing forces which accompany the lateral bending of the flanges. See Fig. 4 (b) and Fig. 4 (c).

(a) I Beam with Couple Mid-way Between Supports (b) Shearing Torsion (c) Flexural Torsion

FIG. 4

When a section is restrained from warping, a flexural torque is introduced along with the shearing torque. Thus, the torsional rigidity increases and the angle of twist decreases when one or more sections remain plane during twist.

Consequently, the type of end connections of a torsion member radically affects the beam behavior by allowing or preventing warping of the end section. In "torsionally free" end connections the desired effect is free warping of the cross-section, but no rotation. This is realized in practically all beam connections. The flanges and/or web are prevented from rotating, but the flanges cannot develop a moment at their ends. "Torsionally fixed" connections not only prevent rotation, but also warping. For structural sections it is difficult to construct an end connection which will prevent one hundred per cent warping.

Usually the end of the beam is welded to a transverse plate and short plates parallel to the web are welded to the extremities of the flanges, so as to prevent motion of the flanges relative to one another.

A beam may be simply supported with respect to vertical bending, and either free or fixed-ended in torsion; it may be fixed-ended in bending, and either free or fixed-ended in torsion.

<u>Definitions of Torsional Stresses</u>

The following stresses associated with torsion are clearly defined:

f_t is the longitudinal stress in the flange due to torsion. Across the width of the flange it is + at one edge, - at the other, and varies uniformly in between. It is constant throughout the flange thickness at any given point. In combining f_t with the longitudinal stress f_b due to vertical bending, the + and + are added to obtain the longitudinal stress on the outer corner of one flange, while - and - are added to obtain the same on the diagonally opposite outer corner of the other flange. The location of the maximum stress f_t along the beam is the same as that of f_b, in the case of a beam free ended in torsion and pin connected, as in this case. However, in the case of beams fixed-ended with respect to vertical bending but free ended in torsion, it is impossible a priori to assign the most highly stressed section. In the case of a shaft with pure torsion between ends f_t and f_b are both zero.

s_t is the shearing stress in the beam web due to torsion. It is a maximum at the support where the torque is greatest. The torsional shearing stress is a maximum and + on one surface of the web, a maximum and - on the other surface, but does not vary uniformly between. It is slightly greater at mid-depth of the web than elsewhere. The shearing stress from vertical bending s_v is greater at the neutral axis. In a vertically symmetrical section, the maximum combined web shearing stress will occur at mid-depth on that surface of the web upon which the shear due to torsion and that due to vertical bending have the same sign.

s_f is the transverse or shearing stress in the flange due to torsion. It is a maximum on the center line of the top surface of the flange. Along the span it varies with the torque.

s_q is the shearing stress due to lateral bending of the flange. This stress varies from zero at the edges to a maximum at the flange center line, being constant throughout the flange thickness. In the case of rectangular flanges the curve is a parabola, hence

$$s_q = 1.5 \frac{V_q}{bt}$$

where V_q = lateral bending shear.

The Lyse-Johnston Method of Design

The method of design proposed by Lyse and Johnston (10 and 17) is applicable to structural sections.with two axis of symmetry such as

WF or I sections. The design treats external loads and torques separately and assumes that the torsional stresses so found are unaffected by beam loads.

Fig. 5 shows a twisted I beam. The basic differential equation

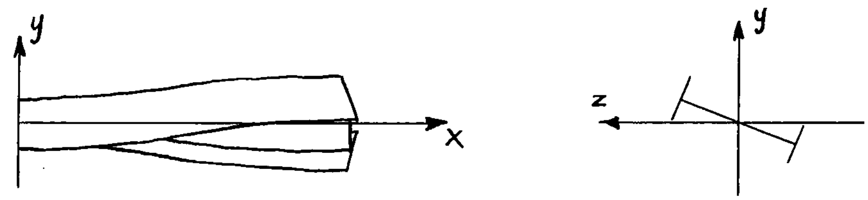


FIG. 5

of the center of a beam flange, as distorted by torsion only, and as proposed by Lyse-Johnston is

$$a^2 \frac{d^3y}{dx^3} - \frac{dy}{dx} = \frac{-2a^2 T_e}{h E I_y} \quad \text{-----------------(vii)}$$

where $a = \frac{h}{2}\sqrt{\frac{EI_y}{KG}}$

and T_e = external torque at point x,y from the end.

It is shown in Ref. 17 that the general solution for any case of loading and end condition is

$$y = \frac{2a^3T}{hEI_y}\left(A_1 \sinh\frac{x}{a} + B_1 \cosh\frac{x}{a} + C_1x^2 + D_1x + E_1\right) \quad \text{--------(viii)}$$

where T = total applied torque.

The solution (see Appendix 1) for the case of complete uniform load on a torsionally free ended simple beam gives the following values:

$$A_1 = -\frac{\tanh \frac{L}{2a}}{2\frac{L}{2a}}$$

$$B_1 = \frac{1}{2\frac{L}{2a}}$$

$$C_1 = -\frac{1}{2aL}$$

$$D_1 = \frac{1}{2a}$$

$$E_1 = -\frac{1}{2\frac{L}{2a}}$$

Denoting by (F) the function $\left(A_1 \sinh\frac{x}{a} + B_1 \cosh\frac{x}{a} + C_1 x^2 + D_1 x + E_1\right)$ it can be shown that:

Deflection: $$y = \frac{2a^3 T}{hEI_y}(F) \qquad \text{------------------ (ix)}$$

Slope: $$\frac{dy}{dx} = \frac{2a^3 T}{hEI_y} \cdot \frac{d(F)}{dx}$$

Flange Moment: $$M = \frac{EI_y}{2}\frac{d^2y}{dx^2} = \frac{a^3 T}{h} \cdot \frac{d^2(F)}{dx^2} \qquad \text{---------------- (x)}$$

Flange Stress From Torsional Bending: $$f_t = \frac{Mb}{I_y} = \frac{a^3 T}{h} \cdot \frac{b}{I_y} \cdot \frac{d^2(F)}{dx^2} = BTa^2 \frac{d^2(F)}{dx^2} \qquad \text{----- (xi)}$$

where $B = \frac{ab}{hI_y}$

Flange Shear From Torsional Flange Bending: $$V_g = \frac{dM}{dx} = \frac{a^3 T}{h} \cdot \frac{d^3(F)}{dx^3} \qquad \text{------------- (xii)}$$

Flange Unit Shearing Stress: $$S_g = 1.5\frac{V_g}{bt} \qquad \text{------------------- (xiii)}$$

Twist Angle (radians) $\psi = \frac{2y}{h} = \frac{4a^3 T}{h^2 E I_y}(F) = \frac{aT}{KG}(F) = CT(F)$ --------- (xiv)

where $C = \frac{a}{KG}$

Rate of Twist (radians/in.) $\frac{\partial \psi}{\partial x} = CT\frac{\partial (F)}{\partial x}$

Portion of Torque Producing Pure Torsion: $T_u = KG\frac{\partial \psi}{\partial x} = KG \cdot \frac{a}{KG} \cdot T\frac{\partial (F)}{\partial x} = Ta\frac{\partial (F)}{\partial x}$

Torsional Flange Shear: $S_f = N T_u = NTa\frac{\partial (F)}{\partial x}$ ----------------- (xv)

$N = \frac{D + t_2}{2K}$

Torsional Web Shear: $S_t = Z T_u = ZTa\frac{\partial (F)}{\partial x}$ ---------------- (xvi)

$Z = \frac{w + 0.3R}{K}$

The solution of the design problem by the Lyse and Johnston method of design is to so proportion the beam that none of the following stresses or angle of twist exceed the maximum allowable:

$(f_t + f_b)$ maximum

$(s_t + s_v)$ maximum

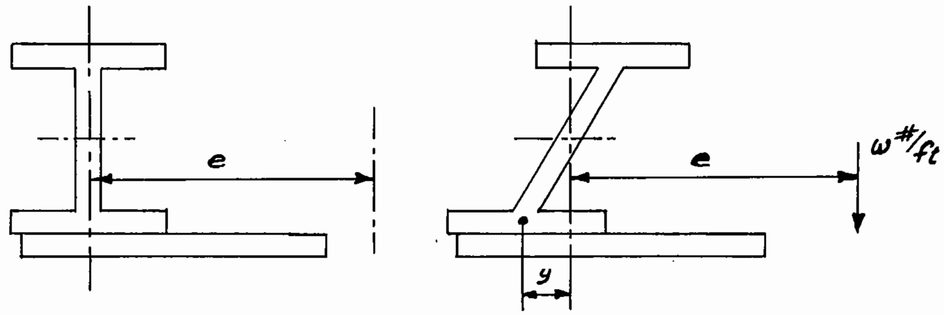
$(s_f + s_q)$ maximum

(ψ) maximum

Effect of the Bottom Plate

When a plate is welded to the bottom of the beam to carry an eccentric uniform load, the usual design practice is to design the beam as if the plate added no structural strength to the section.

The following procedure is proposed for a design which would take into consideration the fact that the bottom plate is part of the beam:



(a) Unloaded Beam (b) Loaded Beam

FIG. 6

Fig. 6 (a) shows a cross-section of a spandrel beam of length L which will carry a uniform load w at an eccentricity e. Fig. 6 (b) shows the movement y of the center of the bottom flange of this beam when the load is applied.

When this movement, y, takes place in the spandrel beam, the bottom plate must also move, and the resultant deformation will be less than that which would occur in the same beam without the bottom plate.

Let us assume that:

(a) The bottom plate undergoes no twisting; only bending in a horizontal plane.

(b) V is the load (lbs/in.) which causes this bending.

(c) V is a uniform load the length of the beam.

(d) The bending of the plate is caused by the bending of the bottom flange only (in other words, V causes no movement or stresses in the top flange of the beam).

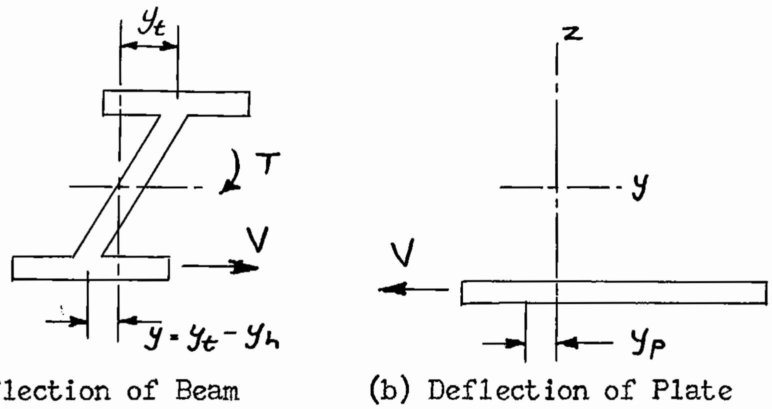


FIG. 7

Now then, the deflection of the bottom flange of the beam y is given by

$$y = y_t - y_h \qquad \text{(xvii)}$$

where y_t = deflection due to torque = $\dfrac{2a^3T(F)}{hEI_y}$

and y_h = deflection due to load V = $\dfrac{PV}{24E\,I_y/2}$

where P = $x(L^3 - 2Lx^2 + x^3)$

The deflection of the bottom plate is

$$y_P = \frac{PV}{24EI_p} \qquad \text{(xviii)}$$

where I_p = moment of inertia of the plate about the z axis. Equating the deflections and solving we have

$$V = \frac{48a^3T(F)}{hP(2 + I_y/I_p)} \qquad \text{(xix)}$$

P and (F) are variables of x. Solutions for V can be made at different x values to prove that V is a uniform load for all practical purposes.

For the particular case of $x = \frac{L}{2}$, a 10WF25 beam 15 feet long, and a 14 in. x 3/8 in. plate we have

$$V = \frac{.00249}{12} \; T \qquad \text{lbs/in.} \qquad \text{------------- (xx)}$$

where the units of T are (in.lbs.).

The twist angle in radians now becomes

$$\psi = \frac{2y_t}{h} - \frac{y_h}{h}$$

$$\psi = CT(F) - 4.78 \times 10^{-15} PT \qquad \text{----------------- (xxi)}$$

The top flange stresses in this design will be identical with those found by the ordinary Lyse-Johnston design. The bottom flange stresses will be changed due to the addition of the horizontal load V.

Effect of Stiffeners

Web stiffeners are most often used on structural beams to prevent web crippling at the points of concentrated loads. From the bending theory they do not influence the stresses caused by vertical bending. Web stiffeners do, however, have an effect on the torsional stresses in a structural beam. Stiffeners can reduce torsional stresses and/or the angle of twist in three ways, namely:

(1) By preventing twisting of one flange relative to the other. If the method of loading is such that one flange carries most of the load, that flange is apt to rotate through a greater angle than the unloaded flange.

(2) By preventing distortion of the web.

(3) By preventing warping of the cross-section.

By holding the flanges together, the stiffeners raise the torsional rigidity of the section slightly and greatly decrease the transverse fiber stresses. In a spandrel beam supporting a brick wall practically all the load enters the beam through the bottom flange, and hence, this fulfillment is the most important.

Distortion of the web is only important in the case of thin webs (7) where the distortion as in Fig. 8 is pronounced. The web in

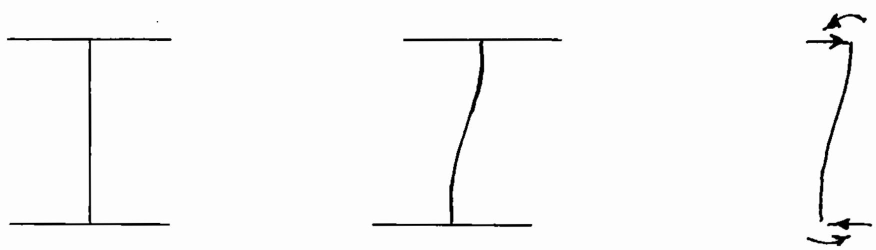


FIG. 8

this test is not thin i.e., 0.15 inches or less, and the effect of this distortion is negligible.

If the stiffeners prevented warping of the cross-section, the rigidity of the section would be increased considerably (4). However, to prevent warping a specially designed welded stiffener would have to be used. The bolted web stiffeners used in this investigation would not prevent warping of the cross-section.

The equations discussed in the theory are based on the assumption that the stresses do not exceed the proportional limit of the material. If the stresses do exceed the proportional limit, the actual stresses are less and the angle of twist greater than the values given by the formulae, and stress may no longer be an adequate criterion of safety.

CHAPTER III

DESCRIPTION OF APPARATUS

The Method of Loading

Since available testing equipment often limits the dimensions of the test specimen, the type and the method of loading used in this test will be examined first.

The most desirable loading in this investigation was one which would approximate the load of a brick or masonry wall resting on the bottom plate of a spandrel beam. A brick wall might give a triangular loading or a continuous uniform load. An eccentricity up to eight inches could be expected. When using a testing machine, only one or two point loads can be applied with accuracy. It was, therefore, necessary to use dead weight to obtain a continuous uniform load.

Since the beam would be one which would be found in common usage for a fifteen foot spandrel the maximum uniform live load per foot of beam at an eight inch eccentricity to produce failure might be as high as six hundred pounds. In the laboratory this load would be practically impossible to apply by dead weight method. It was necessary, therefore, to increase the eccentricity of the load. By increasing the eccentricity the load could be proportionately decreased to give the same torsional stresses. The accompanying decrease in flexural stresses can be accurately predicted by beam theory.

To accomplish the feat of increasing the eccentricity of the load without altering the width of the bottom plate, square bars were welded to the top of the plate at six inch intervals. These bars projected outwards from the beam, and it was on these that the load was applied.

Building bricks were used as dead weight. These bricks weighed 50.5 pounds per ten bricks. For test purposes a constant weight of five pounds per brick was assumed.

Loading Pan

This pan consisted of a plate one-quarter inch thick by fourteen inches wide by fifteen feet long (upon which the dead load rested) with vertical hangers every six inches connected to a one inch square bar running the length of the pan. In use, this bar rested on the projecting rods of the beam.

The pan was very flexible and when the beam deflected more at the center than at the ends, the loading pan simply sagged and did not bridge the center portion of the beam. This produced the desired effect of continuous uniform load.

The Beam

The beam used in the first investigation (6) was an 8WF17. It was decided to use a rolled section in the ten inch size in this test. A continuous variation in beam sizes may lead to a more qualitative analysis of the influence of the bottom plate on stresses and angles of twist.

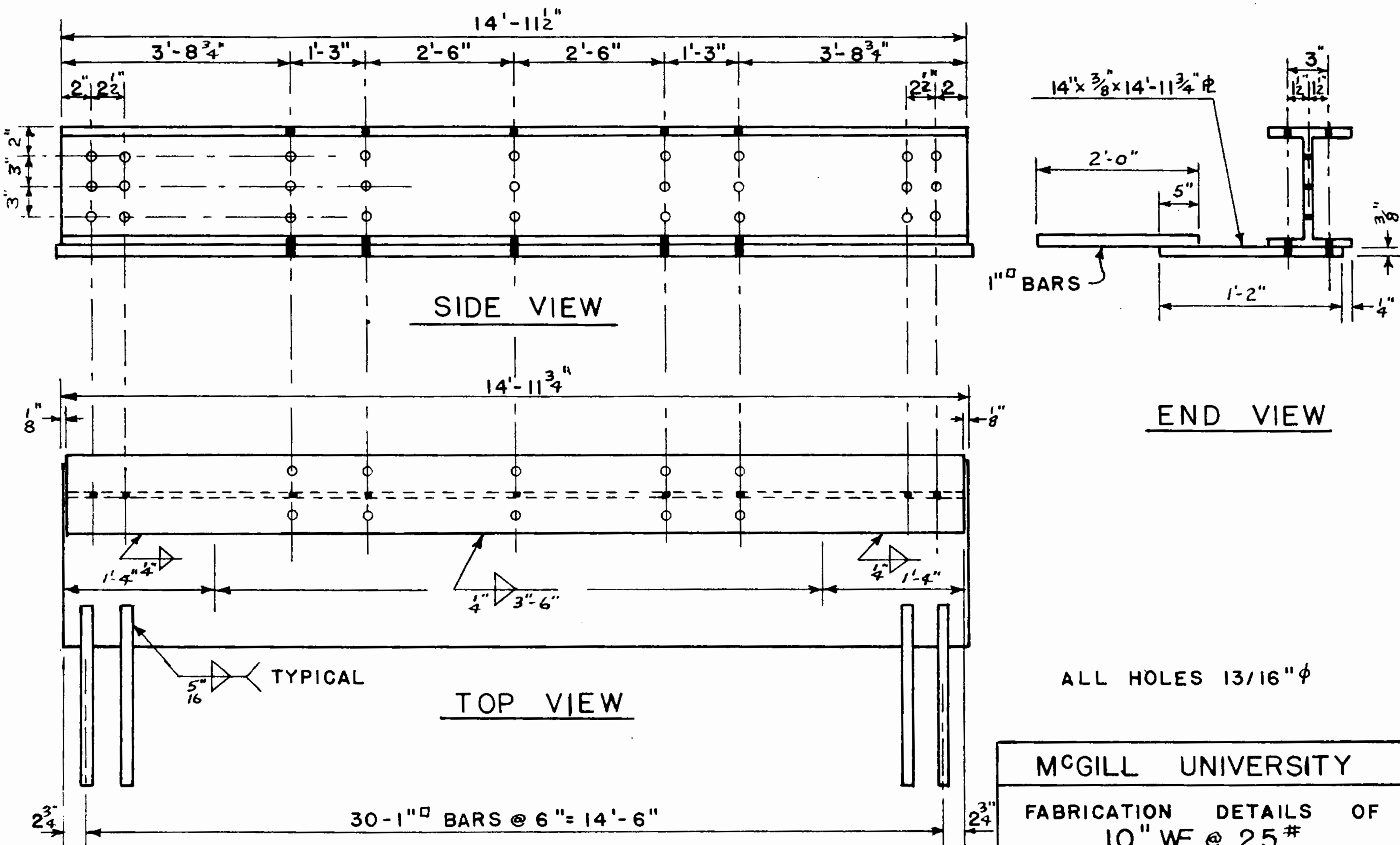
A 10WF25 was available in the fabricators stock pile so this was used.

Fig. 9 on Page 28 shows the fabrication details of the beam. Holes for mounting the stiffeners were punched at selected stations. It was assumed that these holes did not alter the capacity or characteristics of the beam.

The beam when delivered had significant waves or warps in the top flange. These may have been caused by stock piling other material on top of the beam. No estimate can be given regarding the effect of these waves on the test results.

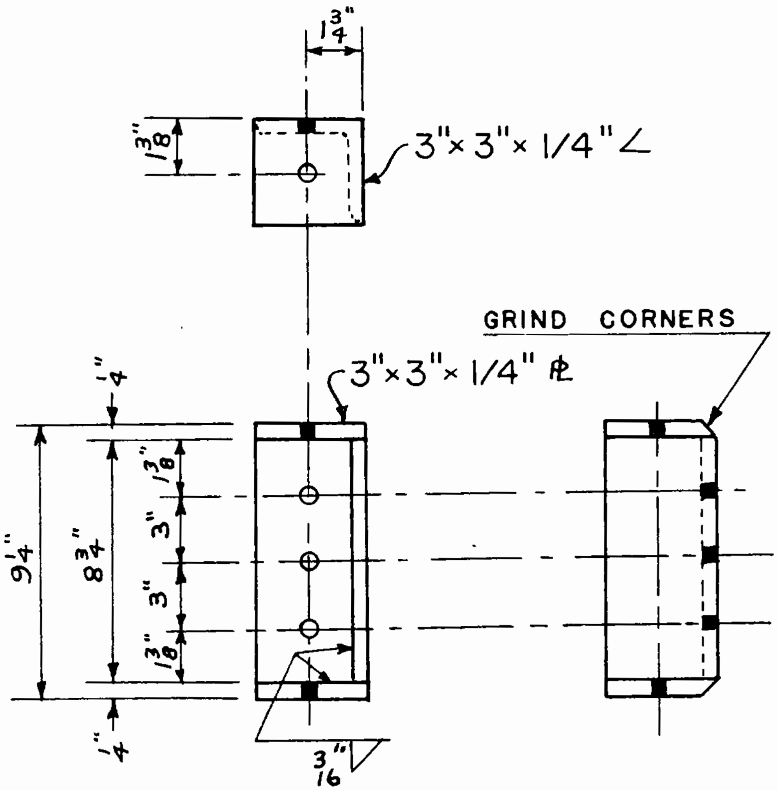
The Bottom Plate

A bottom plate of the same size as that which was used in the first investigation was used in the test, namely; a three-eighths inch thick plate fourteen inches wide. This size was originally selected so that it could be used with several beam sizes and loading conditions.

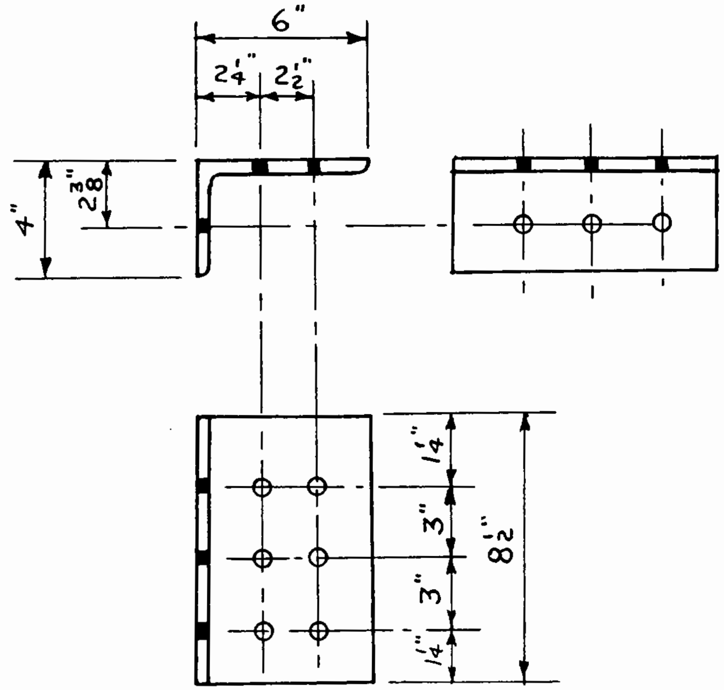


MᶜGILL UNIVERSITY	
FABRICATION DETAILS OF 10" WF @ 25# FOR TORSION TEST	
DWN V. NEIS	DATE NOV 10/55

FIG. 9.



STIFFENER FOR 10" WF @ 25#
6 STIFFENERS REQ'D

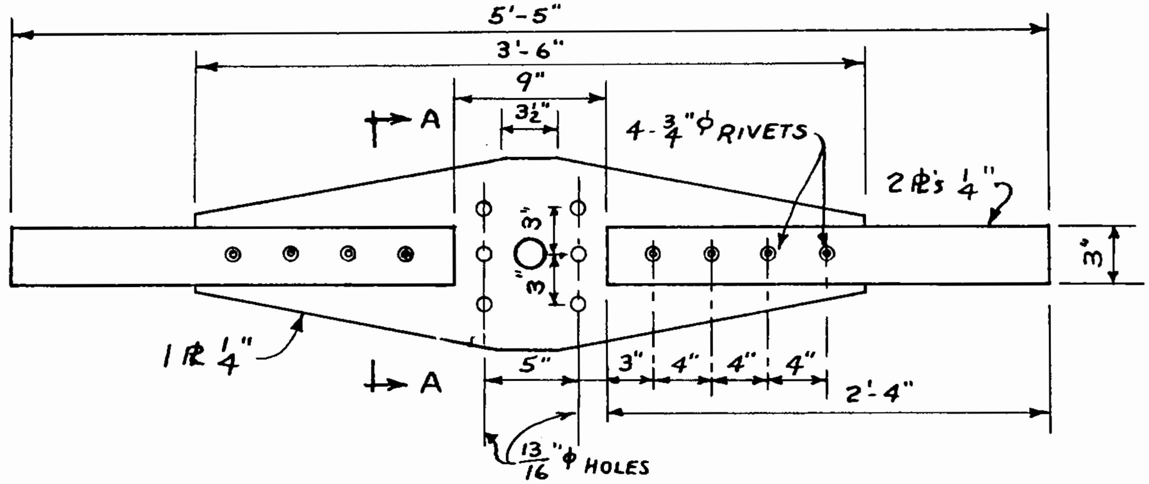


6"× 4"× 3/8" ANGLE
(WEB CONNECTION)
4 ANGLES REQ'D

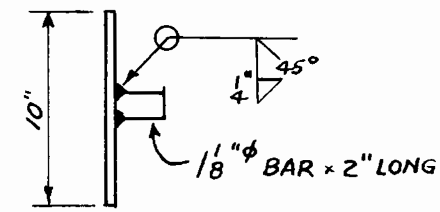
ALL HOLES 13/16" φ

FIG. 10.

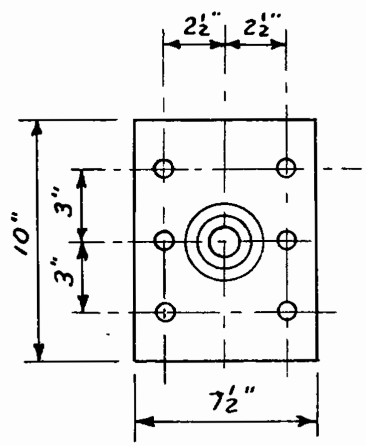
McGILL UNIVERSITY	
MISCELLANEOUS DETAILS FOR 10" WF @ 25# FOR TORSION TEST	
DWN V. NEIS	DATE NOV 10/55

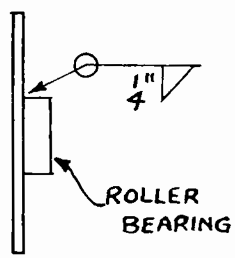


TORQUE ARM



SECTION A-A





BEAM SUPPORT PLATE

FIG. 11.

McGILL UNIVERSITY	
APPARATUS FOR LOADING THE BEAM WITH PURE TORQUE	
DWN V. NEIS	DATE NOV 10/55

The Rods

The projecting rods were one inch square and overlapped the top of the plate by five inches. There was considerable disalignment of the rods. To ensure that the loading pan produced equal loading on all the rods, and hence uniform loading on the beam, lead plugs and aluminum foil were used. These shims proved to be quite satisfactory.

The End Connections

Standard torsionally free end connections were used. These consisted of 6 in. x 4 in. x 3/8 in. angles bolted to the beam with six high-strength bolts, and to the stands with six similar bolts. Close inspection throughout the test showed no slipping of these angles on the stands.

The Stands

A stand or beam support (see Exhibits 8 to 11, Appendix 3) consisted of an 8WF31 column held vertical by two braces and standing on a triangular base. After the beam was in place the stands were positioned so that the column faces were vertical and lay in planes parallel to each other. The final tightening of the bolts was made and a check made on the column faces. Tapes and a Clinometer were used in these adjustments.

The stands rested on grout and steel shims placed on the concrete floor. To ensure no movement concrete blocks totalling over

1,000 pounds were placed on each stand.

Several checks of the position of the column faces were made under loaded conditions and there was no evidence of any movement.

Loading Devices for Pure Torque

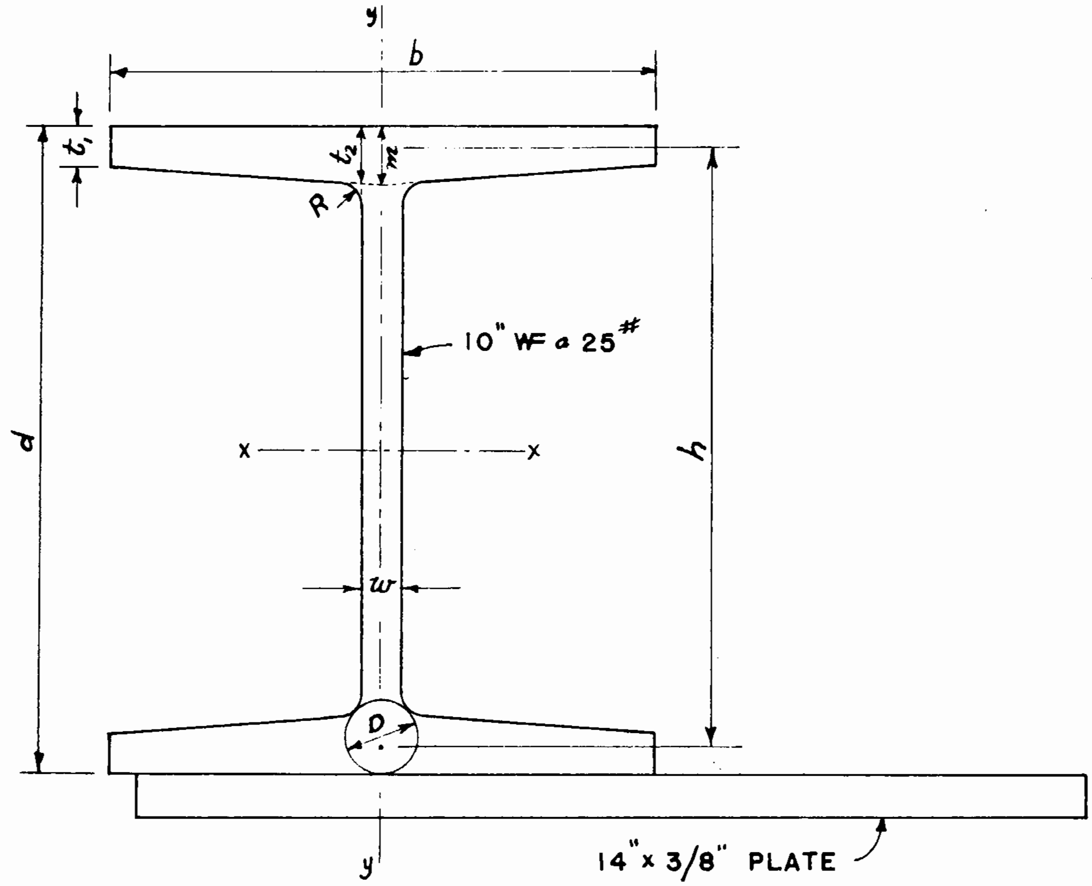
To determine a stiffness factor K for the test beam and plate an experimental determination of T and θ were necessary.

The beam was supported at one end of the stand as in the simple beam test. The other end was supported by a special connection. This connection consisted of a two inch long by one inch round pin centered in a roller bearing. The pin was welded to a loading arm which was bolted to the beam connection angles. The roller bearing was attached to an 8 in. x 3/4 in. x 10 in. plate which was bolted to the stand. Exhibit 6 shows these connection pieces.

The loading arm was sixty-five inches long and extended equally on each side of the pin connection. A small loading pan and a hydraulic jack were used to apply forces which would resolve into a moment about the pin connection. On one end of the loading arm was a pan with weights acting downward. The other end was pushed up by the hydraulic jack. The jack rested on a scale which measured the force that the jack exerted on the loading arm. The upward and downward forces were both at a lever arm of twenty-five inches from the pin connection.

FIG. 12

PROPERTIES OF THE TEST BEAM

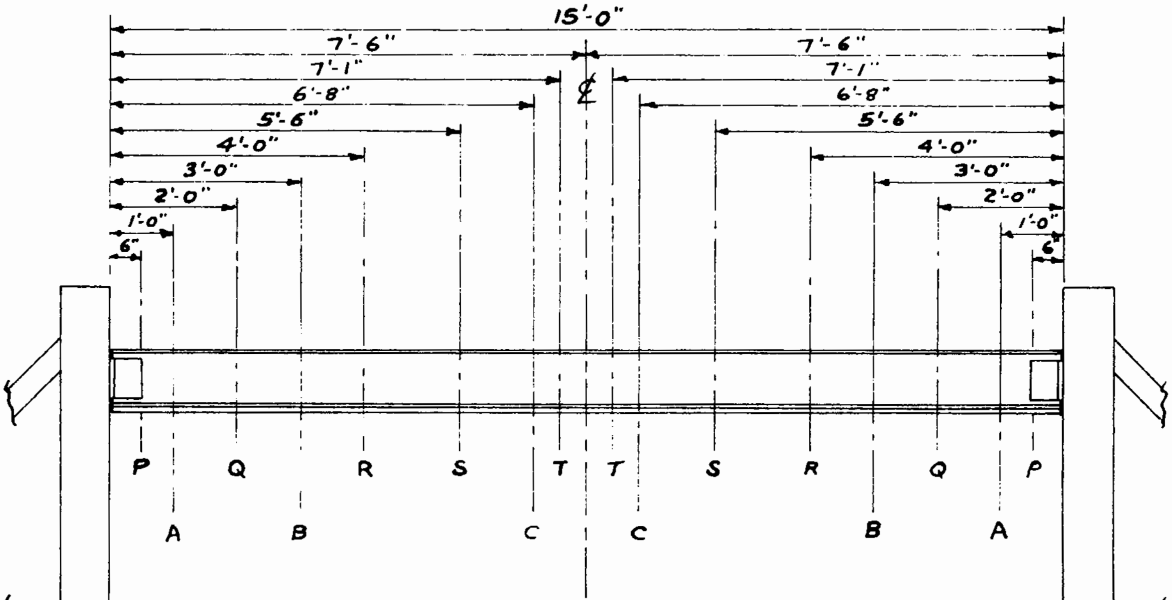


AREA	7.30 SQ IN
$_xI_x$	132.0 IN^4
$_yI_y$	12.9 IN^4
K	0.374 IN^4
a	45.7 IN
B	2.11 IN^{-3}
C	0.00001063 $\frac{1}{IN\ lb}$
N	1.426 IN^{-3}
Z	0.928 IN^{-3}

d	10.08 IN.
b	5.762 IN.
R	0.295 IN.
t_1	0.394 IN.
t_2	0.449 IN.
m	0.452 IN.
w	0.258 IN.
h	9.657 IN.
D	0.618 IN.

FIG. 13

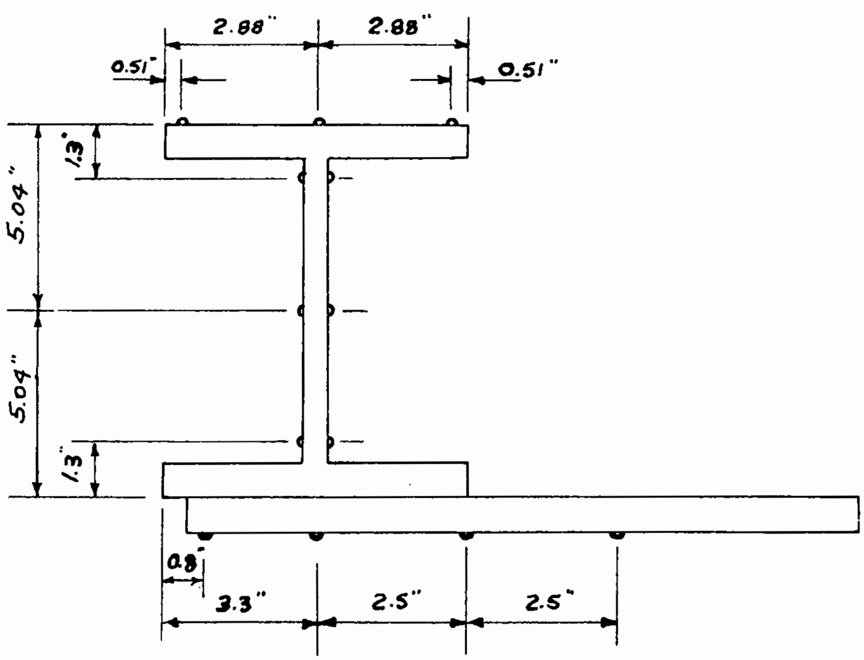
LOCATION OF MEASURING STATIONS



SIDE VIEW

STRAIN GAUGES LOCATED AT SECTIONS A, B + C

ANGLE OF TWIST READINGS TAKEN AT SECTIONS P, Q, B, R, S + T



LOCATION OF STRAIN GAUGES AT A TYPICAL CROSS-SECTION

<u>The Gauges for Strain Measurements</u>

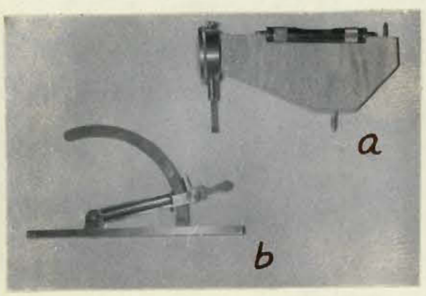
SR-4 electrical strain gauges, manufactured by the Baldwin-Lima Hamilton Corporation, were used to measure strains at six cross-sections (Fig. 13). A and C sections had rosette gauges at mid-depth of the beam on both sides of the web, and at the center-line of the flanges. Eight linear gauges were placed with their axis in the longitudinal direction on the outside corners of the flange, and on the web near the web-flange junction. B sections had rosette gauges exclusively. Thus, the number of gauges used was thirty-two linear A-3 gauges, and forty-two AR-1 rosette gauges giving one hundred and fifty-eight strain readings.

Two cross-sections of the beam, the end section, and the center section, are the sections of maximum theoretical stress. For example $(f_t + f_b)$ maximum is at the center-line, $(s_t + s_v)$ maximum is at the end section, $(s_f + s_q)$ maximum is at the end section. Thus the A and C sections for strain measurement were placed close to the end of the beam and the center-line of the beam respectively. B section was placed between A and C. Care was taken to avoid areas which would be subjected to high local stress caused by the shifting of the stiffeners. The six stations were made up of three symmetrical sections on each side of the center-line so that a check on each reading was available by comparing it with its "twin" on the other side of the beam.

When using the gauges, it is assumed that the material is isotropic and homogeneous, and strain gradients are so small that the strain can be considered as substantially uniform over the area covered by the gauge. The condition of isotropy assumes the modulus of elasticity, and Poisson's ratio to be constant within the elastic limit.

The Instruments for Measuring the Angle of Twist

The angle of twist of the beam was measured with two instruments. A six inch base Clinometer with a level bubble reading to five minute accuracy was used to measure the inclination of the top flange. An instrument which also contained a level bubble was devised for this experiment, which shall hereinafter be referred to as "a Dialometer".



(a) Dialometer

(b) Clinometer

FIG. 14

The Dialometer consisted of a wooden frame holding an Aimes dial and a level bubble. When in use, one end of the frame (a steel point) was brought in contact with the inclined surface. The shaft of the Aimes dial at the other end of the frame also touched the surface.

The frame was then raised or lowered until the level bubble indicated the frame was horizontal. As this movement was being made the shaft on the Aimes dial necessarily moved and the dial reading changed. A dial reading of zero indicated that the surface was horizontal. Other readings indicated a sloping surface. This instrument could be used on the bottom flange as well as on the top flange. Test results showed that the Dialometer gave readings which were identical to those obtained from the Clinometer - the only disadvantage being that a table had to be constructed giving a transfer from the dial reading to the equivalent in degrees.

The Stiffeners

Last to be discussed are the most important items of this test; the stiffeners.

For test purposes it would be advantageous to have removeable stiffeners in order to record the quantitative changes that different numbers of stiffeners produce on stress and strain. Therefore, it was necessary to select a bolted connection which would prevent twisting of one flange relative to the other and/or prevent distortion of the web. Common web stiffeners were used.

The number of stiffeners that might be used in practice is dependent on the material and fabrication costs of the stiffeners and also on the effectiveness of the stiffeners i.e., if a considerable reduction in beam weight is permissible with the use of more stiffeners. From the results of the previous investigation (6) it seemed reasonable to assume that designers would employ no more than three sets of stiffeners. Thus, this investigation used one pair of stiffeners at the center line, two pairs at one-third points, and three pairs at one-quarter points.

These stiffeners (Fig. 10) were made of 3 in. x 3 in. x 1/4 in. angles with 1/4 in. top and bottom plates. They were machined for perfect fit to the beam. Seven high strength bolts were used; three in the web and two on each flange. See (9) for a comparison of riveted and bolted connections.

CHAPTER IV

TEST PROCEDURE

Selection of the Simple Beam Loads

An unlimited number of combinations of loads and torques could be applied simply by varying the eccentricity of the loading pan. The practical number of test loads was restricted to three or four. Test data from three or four loadings would show similarities or differences resulting from changes in load only. The loads would range from the smallest which would give measurable strains to those which would produce stresses near the yield point of the beam material.

Consider a wall composed of two-thirds brick and one-third tile weighing 90 lbs/cu.ft. Let the wall be 10 feet high and rest on a lintel beam with an eccentricity of six inches from the center line of the web. Walls 2 in., 4 in., 6 in. and 8 in. thick produce 900, 1800, 2700 and 3600 in.-lb. torque per foot of beam respectively. By the Lyse-Johnston method of design the eight inch thick wall will produce a maximum stress of 27,800 p.s.i. in an ordinary 10WF25 beam. This flange stress will be reduced if a smaller load producing the same torque is applied. The test loads will produce the same torques as listed above and will be 50 lbs/lin.ft. at 18 inches eccentricity, 100 lbs/lin.ft. at 18 inches eccentricity, 150 lbs/lin.ft. at 18 inches eccentricity and 200 lbs/lin.ft. at 18 inches eccentricity.

All loadings have the same eccentricity. Applying the principle of superposition to the loadings as described, the stresses and strains due to the 100 lbs/ft. loading should be exactly double the corresponding ones for the 50 lbs/ft. loading and so on.

If the test data does not reveal such a relationship some explanation for variation must be made.

Beam Loading Procedure

Before actual testing was begun trial runs were made at varying loads to "shake down" the beam and to examine the operation of the strain gauge recorder. Erratic readings were encountered and after the gauges and wires were checked and found to be in good working order, a new Baldwin Recorder was obtained which produced satisfactory results.

A complete cycle of the loading procedure for any one load consisted of the following steps:

(1) A set of zero readings was recorded; one reading from each strain gauge, one reading of angle twist from the Clinometer and Dialometer at each selected station on top and bottom of the flanges. The loads acting on the beam during zero readings were the dead load of the beam and the weight of the loading pan. This pan was kept on the beam at the required

eccentricity at all times.

(2) The beam was loaded. Bricks were placed evenly on the loading pan until the required load was reached.

(3) With the load applied, another set of readings were recorded as in step (1) above. Differences of respective readings gave resultant strain.

(4) The load was removed.

(5) If testing was carried out continuously for a day steps (2), (3) and (4) were then repeated for one pair of stiffeners bolted to the beam at its center line; then for two pairs of stiffeners at the one-third points, then for three pairs of stiffeners at the one-quarter points. This procedure was reversed i.e., (3), (2) and (1), and then no stiffeners for the 200 lbs/ft. load for fear that buckling of the top flange might take place during the no stiffener test and void all subsequent readings.

(6) If testing was not continuous, steps (1) to (4) were repeated.

The results of trial runs prior to actual testing indicated that "loaded condition" readings included no "creep" errors if the readings were taken two or more hours after loading.

Selection of the Pure Torque Loads

By the Lyse-Johnston method of design a torque of 10,000 in.-lbs.

will produce a maximum stress of 14,250 p.s.i. in a 10WF25 when the torque is applied at the torsionally free ends of the beam.

The beam will be tested under four pure torque conditions; 4,000 in.-lbs., 6,000 in.-lbs., 8,000 in.-lbs., and 10,000 in.-lbs. The results of each will provide a check on the others. The torsional constant K can be computed from measurements of the angle of twist.

Pure Torque Loading Procedure

The test of pure torque conditions was completed between the beam loadings of 150 lbs/ft. and 200 lbs/ft. The beam was removed from the supports after the 150 lbs/ft. run was completed. One support was then moved so that the special end connection could be inserted.

When the beam was in place (supported at one end by the standard web connections and at the other end by the special pin-roller bearing joint), the following loading procedure was followed:

(1) The angles of twist at all stations were recorded under no load conditions.

(2) Weights were placed on the loading pan.

(3) The scale upon which the hydraulic jack rested was balanced. The jack was then raised till the force with which it pushed against the loading arm was recorded by the scale to be identical to the weight on the loading pan.

(4) The angles of twist at all stations were recorded under loaded conditions.

(5) The jack and the loads were removed.

The loading arm was such that the lever arm before loading from the pin (center of rotation) to either load was twenty-five inches. Thus, for a torque of 6,000 in.-lbs., the dead weight was 120 lbs., and the scale was set to read 120 lbs. above the weight of the hydraulic jack.

CHAPTER V

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

As stated in the test procedure, the test loads were of equal increments at a constant eccentricity of eighteen inches. Applying the principle of superposition, the strain readings for the 100 lb/ft. load should be twice those for the 50 lb/ft. load, etc. Test readings showed that this principle was valid for the 50 lb/ft. and 100 lb/ft. loadings, and also valid for most of the readings for the 150 lb/ft. load. Where permanent strain took place, the readings were naturally higher than those suggested by "superposition". For ease of presentation, a thorough examination of the stresses and twist angles for the 100 lb/ft. loading condition will be made presuming that the states of stress for other loadings, which do not cause permanent strain in the beam, are proportionate to the applied load.

Top Flange Stresses

Fig. 15 shows the distribution of longitudinal fiber stress in the top flange at stations A, B and C. This stress is the algebraic sum of the longitudinal torsion stress f_t and the vertical bending stress f_b. The maximum longitudinal fiber stress in the top flange is a compression stress along the outside edge furthest from the load, and is equal to the numerical sum of f_t and f_b. This stress is plotted against distance along the beam in Fig. 16. Only one-half of the beam is shown since by symmetry the stresses in the other half are the same.*

Figs. 17 and 18 are similar to the previous plots. However, these plots show the shearing stress in the top flange. This shearing stress is a combination of transverse shearing stress due to torsion s_f, and shearing stress due to lateral bending of the flange s_q. Again, the theoretical stress is that computed by the standard Lyse-Johnston theory.

Fig. 15 shows that the distribution of stress across the top flange very closely approximates the distribution given by the Lyse-Johnston theory. The variation along the beam as shown in Fig. 16 resembles the predicted variation but the actual stresses are always lower than the theoretical. The actual stress for no stiffeners is approximately twenty per cent lower and the stress continues to drop as stiffeners are added. Three stiffeners cause another ten per cent drop in stresses.

The plot at station B, Fig. 17, verifies the assumption that

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*Strain readings from symmetrical gauges showed a maximum of twenty microinches per inch difference. This was caused by irregularities in the beam, irregularities in loading and recording errors. In all cases the values from symmetrical gauges were averaged for greater accuracy.

FIG. 15

PLOT OF TORSIONAL AND BENDING STRESS f_t AND f_b ACROSS THE TOP FLANGE FOR LYSE-JOHNSTON AND OBSERVED VALUES USING 0, 1, 2 AND 3 STIFFENERS

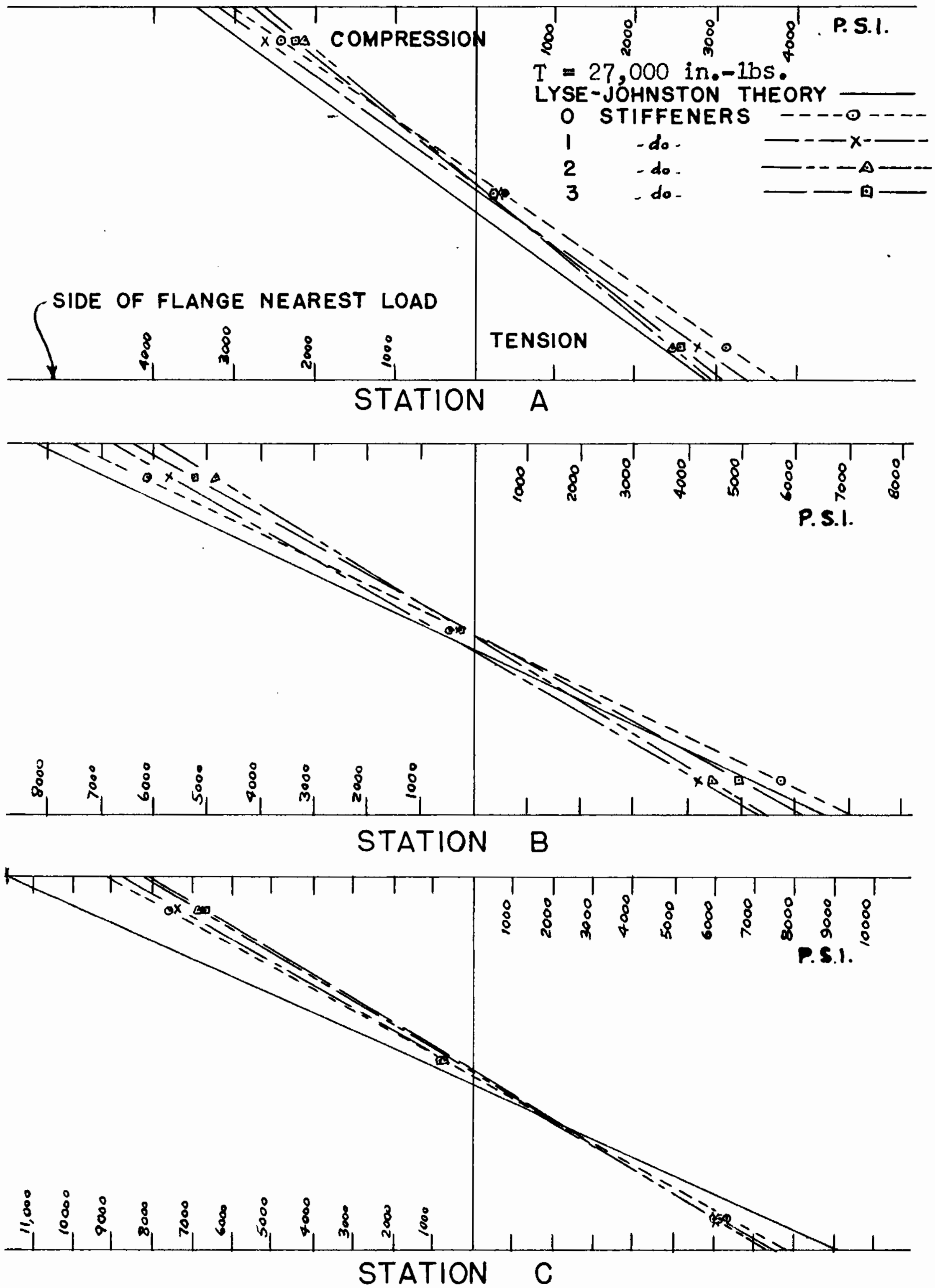
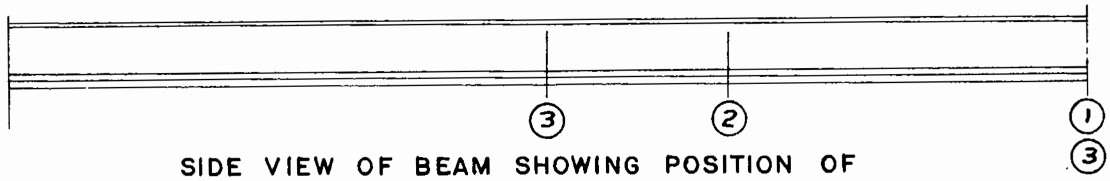


FIG. 16

PLOT OF TORSIONAL AND BENDING STRESS f_t AND f_b ON THE TOP FLANGE AT THE COMPRESSION EDGE ALONG LENGTH OF BEAM FOR LYSE-JOHNSTON AND OBSERVED VALUES USING 0, 1, 2 AND 3 STIFFENERS



SIDE VIEW OF BEAM SHOWING POSITION OF STIFFENERS

T = 27,000 in.-lbs.

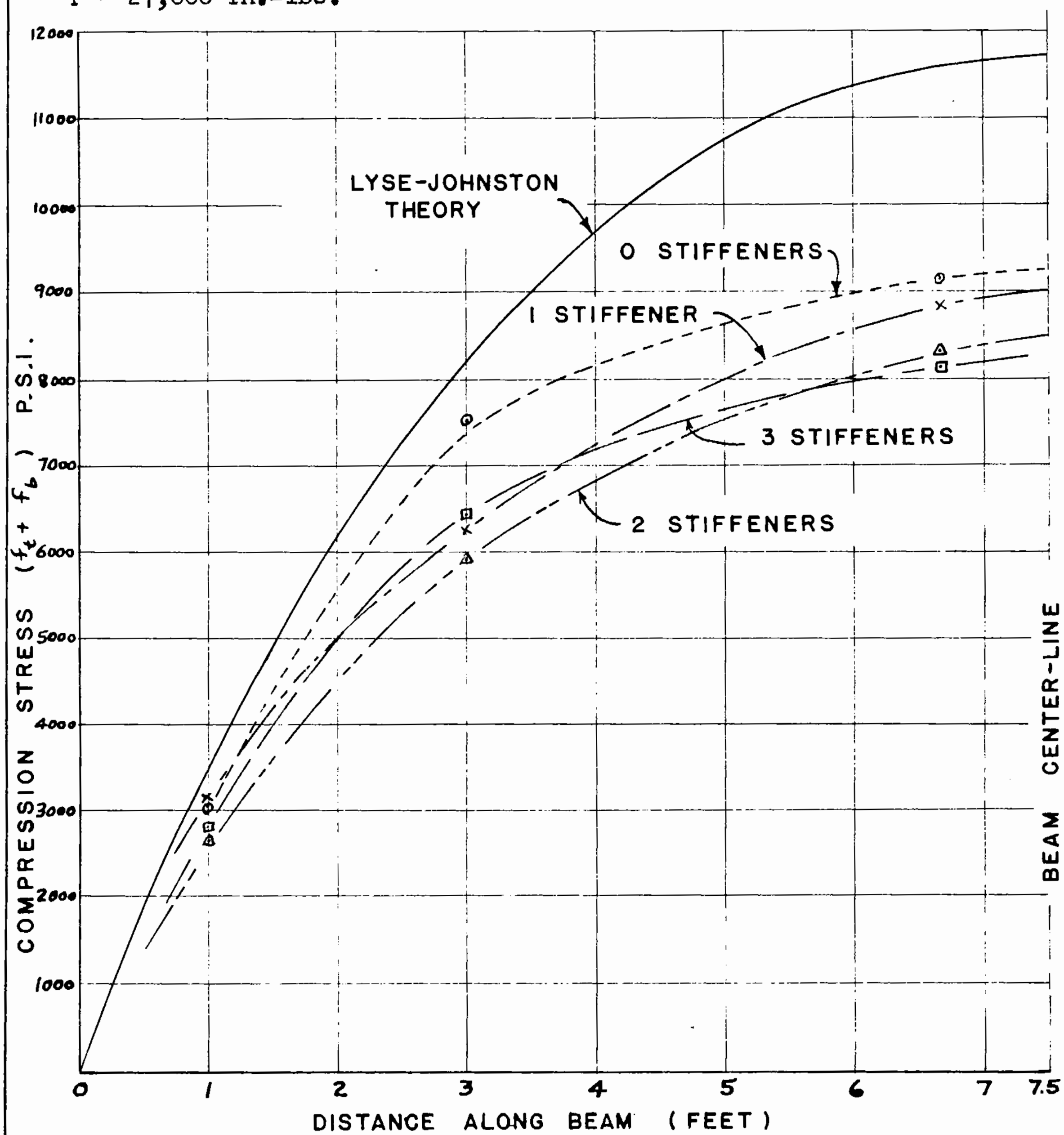


FIG. 17

PLOT OF SHEAR STRESS DUE TO TORSION AND LATERAL BENDING s_f AND s_q ACROSS THE TOP FLANGE FOR LYSE-JOHNSTON AND OBSERVED VALUES USING 0,1,2 AND 3 STIFFENERS

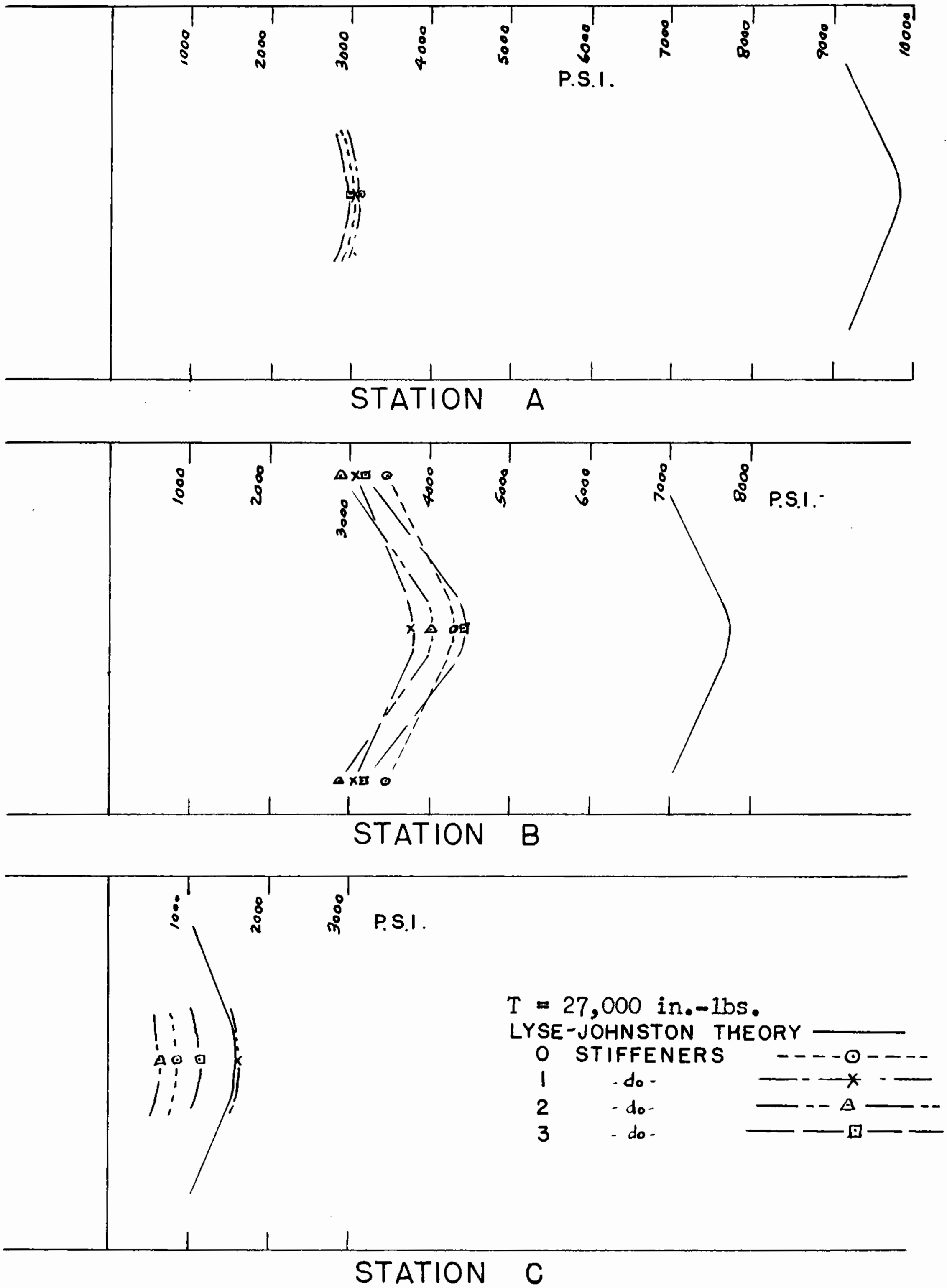
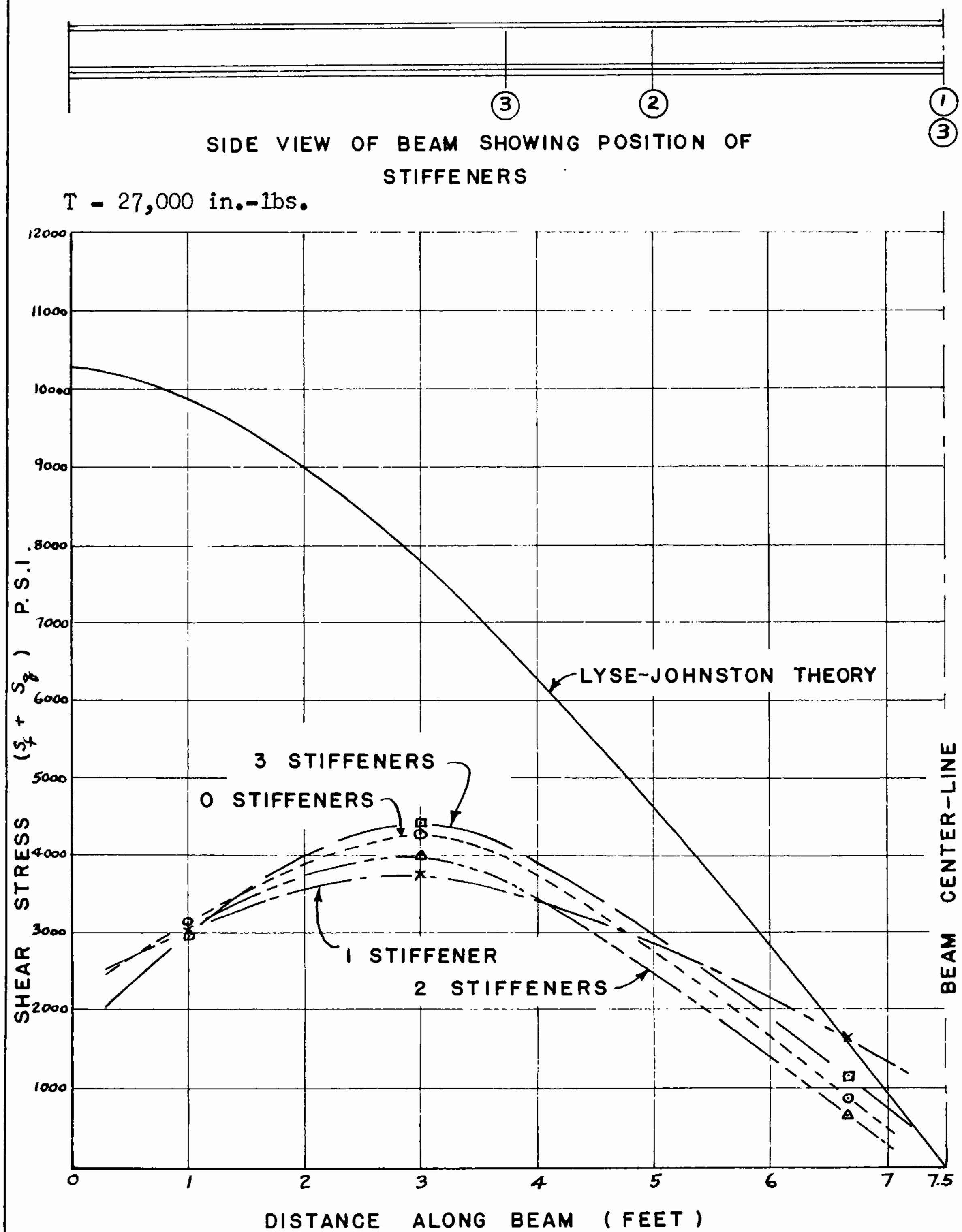


FIG. 18

PLOT OF SHEAR STRESS DUE TO TORSION AND LATERAL BENDING s_f AND s_g AT THE CENTER-LINE OF THE TOP FLANGE ALONG THE LENGTH OF THE BEAM FOR LYSE-JOHNSTON AND OBSERVED VALUES FOR 0, 1, 2 + 3 STIFFENERS



the flange shear stress is a maximum at the center-line of the flange. The important feature of Fig. 18 is the disagreement of the theoretical and actual curves for shear stress along the beam. The shear stress at the end of the beam is only one-third of that predicted by the theory. Since most of the shear stress near the end of the beam is torsional shearing stress, the conclusion is that the flange is not under torsion. The web of the beam is prevented from rotating at the support and consequently, must have torsional stresses, but the converse is true of the flange. The flange is not prevented from rotating except by its connection to the web. This rotation of the flanges will be discussed in later comments on the angle of twist. The maximum variation of shear stress with stiffeners is 1000 p.s.i., which is twenty-three per cent of the maximum actual stress.

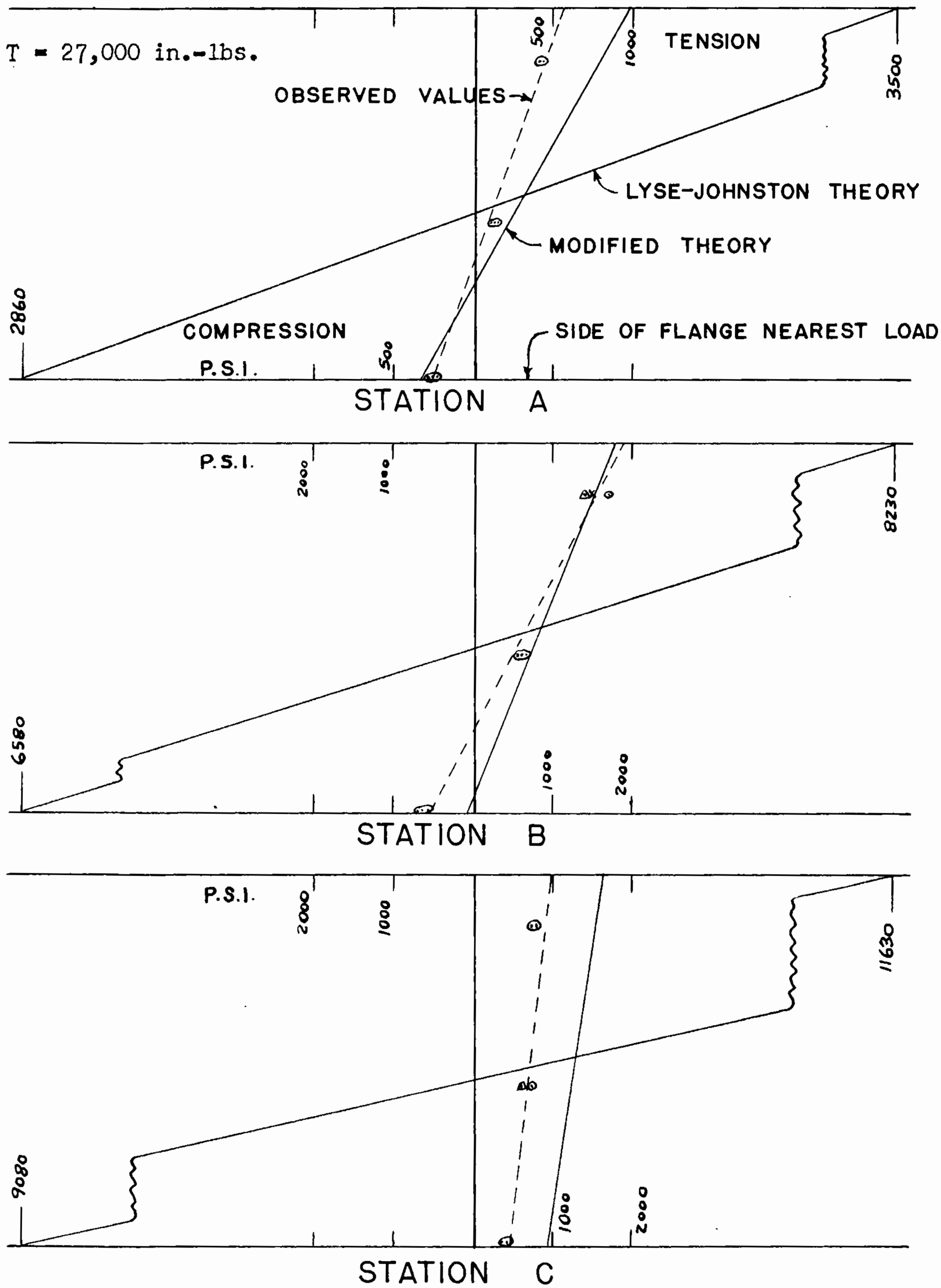
Note that since the transverse fiber stresses are zero, the state of stress at any point across the web can be defined by using the values of longitudinal fiber stress and shear stress as given by Figs. 15 and 17.

Bottom Flange Stresses

Fig. 19 shows the distribution of longitudinal fiber stresses in the bottom flange at stations A, B and C. By the standard Lyse-Johnston theory these stresses are similar to those of the top flange. By the "modified theory", however, the horizontal bending stress f_h is

FIG. 19

PLOT OF TORSIONAL AND BENDING STRESS f_t AND f_b ACROSS THE BOTTOM FLANGE FOR LYSE-JOHNSTON, MODIFIED AND OBSERVED VALUES USING 0,1,2 AND 3 STIFFENERS



added to the longitudinal torsion stress f_t, and vertical bending stress f_b. The numerical value of the maximum stress is thus much reduced. The "modified theory" stresses very closely approximate the actual stresses even though the test strains were measured on the bottom plate rather than the beam flange. The assumptions made in deriving the modified theory are validated by the agreement of these flange stresses.

Consider now the bottom plate as a cantilever beam as shown in Fig. 20. The weld material from the plate to the beam must produce some upward force on the plate. There will be some bearing force between the plate and the beam of unknown distribution to produce equilibrium.

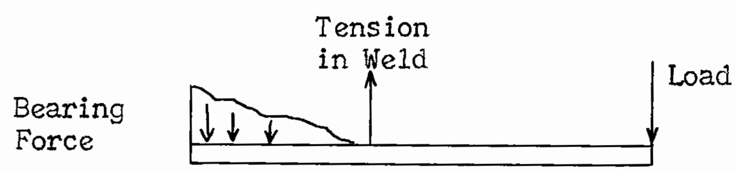


FIG. 20

Using the strain readings from the triaxial gauges at station B, it is possible to determine the bending moments at each gauge position. However, the gauges are on two and one-half inch centers and this is too far apart to allow an accurate bending moment diagram to be constructed. Hence, the distribution of load between the plate and the beam cannot be obtained. If the distribution could be determined, it could not be used for the theoretical determination of stresses in the beam unless the horizontal reaction between plate and beam was also determined. This is practically impossible, so it seems that the only procedure for finding the reactions between the plate and the beam is to find, by trial and error, what reactions will produce the observed stresses and angles of twist in the beam.

Web Stresses

1. Shear Stresses

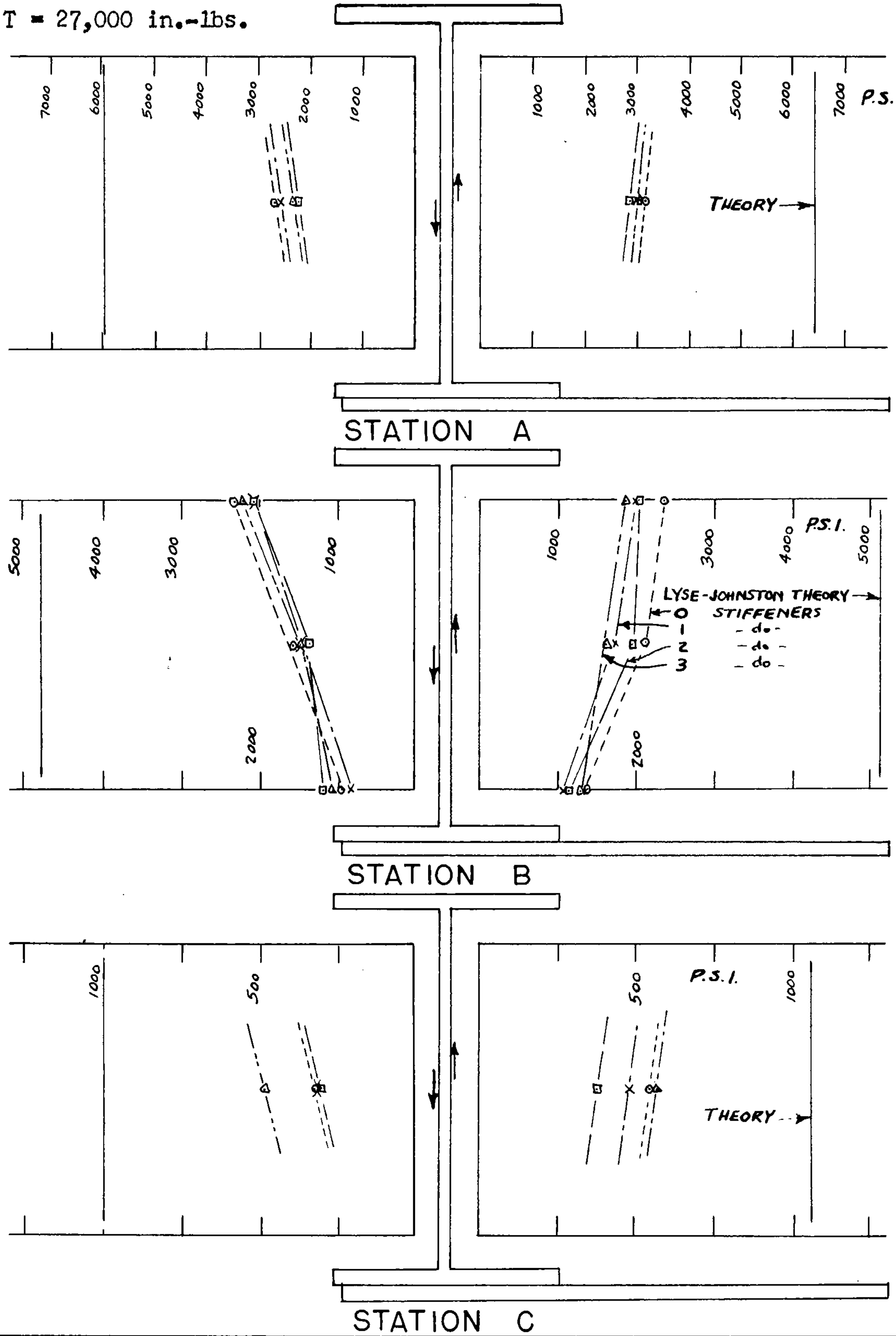
Fig. 21 shows the variation of web shear across the section at stations A, B and C. On the front face, the shearing stress due to vertical bending s_v, adds to the shearing stress due to torsion s_t. On the back face the shear is the difference between the two. The vertical load is small and consequently, s_v is only about four per cent of the total shear, so practically all the web shear is due to torsion.

The actual shear values are only one-quarter to one-half of the theoretical values. The bottom plate causes this decrease by resisting some of the torque and by decreasing the angle of twist. Since the torsional shear stresses are proportional to the rate of twist the actual stresses are less than the theoretical, since the rates of twist are smaller (see Angles of Twist).

FIG. 21

PLOT OF TORSIONAL AND VERTICAL WEB SHEAR S_t AND S_v ACROSS THE WEB FOR LYSE-JOHNSTON AND OBSERVED VALUES USING 0, 1, 2 + 3 STIFFENERS

T = 27,000 in.-lbs.



The shear stresses do not change appreciably when stiffeners are added. The maximum change in stress is 500 p.s.i. or twenty per cent of the observed values. The maximum observed shear stress in the web is at station A and equals 3200 p.s.i.

2. Transverse Fiber Stresses

Under pure torsion and pure bending, there would be no transverse fiber stresses, however, when an eccentric load is applied, the shape of the section and the method of getting the load "into" the beam may cause distortion of the cross-section and hence, transverse stresses will be present. Fig. 22 shows how distortion of the web occurs in the test set-up.

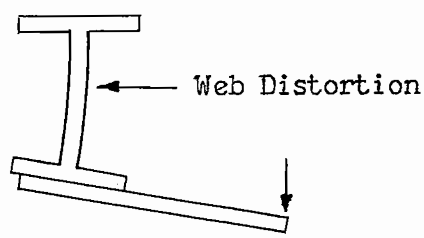


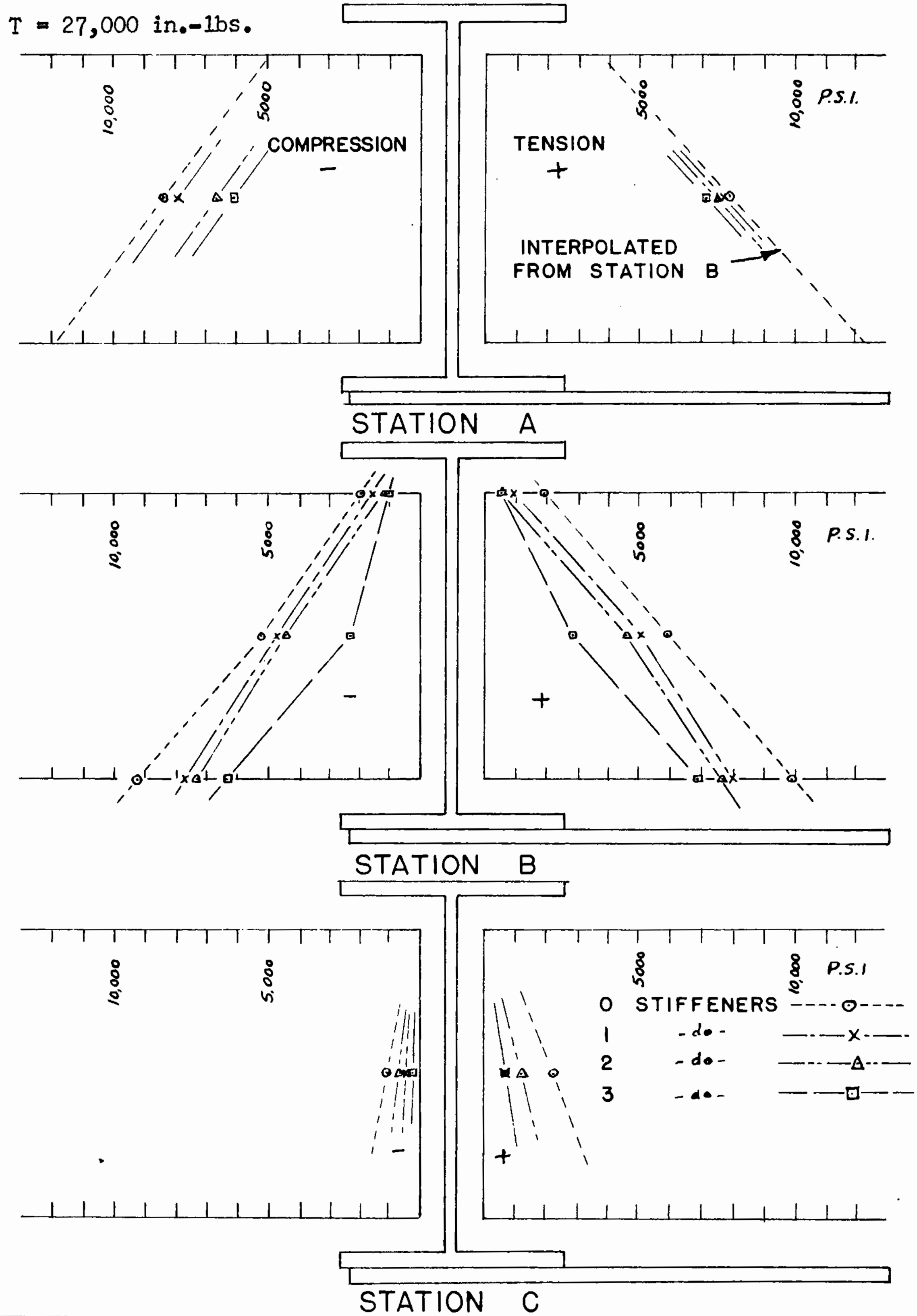
FIG. 22

Fig. 23 shows the transverse fiber stresses at stations A, B and C. There is a tension stress on the side of the web nearest the load and a compression stress on the other side of the web. When stiffeners are added to the beam, the distortion of the cross-section, as shown in Fig. 22, is reduced. The plots show that the transverse stresses are considerably reduced when stiffeners are used.

FIG. 23

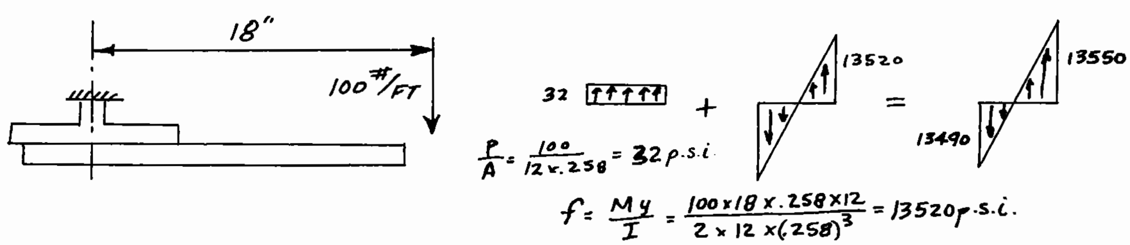
PLOT OF TRANSVERSE FIBER STRESS ACROSS THE FACE OF THE WEB FOR OBSERVED VALUES USING 0, 1, 2 AND 3 STIFFENERS

T = 27,000 in.-lbs.



At station B, three feet from the end, and eight and three-quarter inches from the nearest stiffener position, the maximum measured stress is 10,000 p.s.i. tension with no stiffeners. One stiffener reduces this nineteen per cent to 8100 p.s.i., two stiffeners reduce the 10,000 by twenty-three per cent to 7700 p.s.i., and three stiffeners cause a thirty-one per cent reduction to 6900 p.s.i. At station A, the maximum stress is 12,800 p.s.i. by interpolation, but the stiffeners do not reduce this value too much because they are not placed near the end supports. The stresses at station C are much lower than at the other measuring points, and it is interesting to note that the reduction of stress caused by one stiffener is practically identical to that caused by three stiffeners.

These transverse fiber stresses are the stresses which caused failure of the beam. The high tension stress on one side of the web, at the bottom fillet, and the compression stress on the other side of the web, caused yielding to take place and the bottom flange rotated with respect to the web. To calculate what these stresses might be at the end of the beam, let us consider the bottom flange as a cantilever beam with the web as a fixed support as in Fig. 24 (a).



(a) (b)

FIG. 24

By beam theory, the stress distribution across the web is as shown in Fig. 24 (b). The calculated stress of 13,550 p.s.i. is only 750 p.s.i. greater than the actual observed stress. This method of design gives a reasonable result for the end of the beam, but it cannot be applied to the rest of the beam length because the restraint offered by the web is unknown.

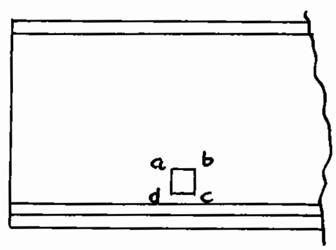
It should be noted that an end connection, consisting of clip angles from the flanges to the support, would prevent the flanges from rotating at the ends and would decrease the transverse fiber stresses considerably. A connection using two web angles plus two flange angles is recommended for all spandrel beams resisting torsion.

Load to Produce Yielding

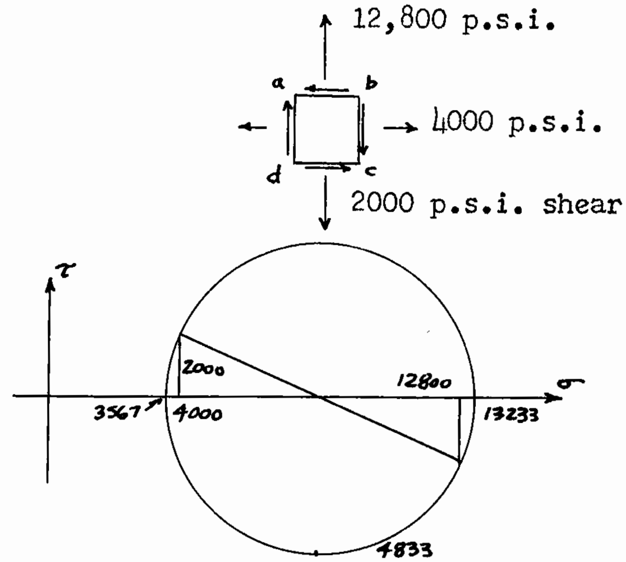
From the previous graphs it can be seen that any particular stress such as s_f, f_t, etc. changes from a minimum or maximum at the end of the beam to a maximum or minimum at the center of the beam.

To find the region where failure begins only the end and center sections A and C need be investigated, however, the readings from station B will be used indirectly. Only station B has triaxial gauges exclusively. The shear values and transverse fiber stress values obtained here enable an interpolation of the corresponding stresses at stations A and C. For an example of this interpolation see Fig. 23.

From an investigation of the observed stresses, the point of maximum stress is at station A on the loaded side of the web, near the bottom flange. The maximum stress occurs with no stiffeners and a stress diagram of this condition is shown in Fig. 25.



(a) Looking Toward the Beam from the Loaded Side



(b) Mohr Diagram

FIG. 25

Assuming $\mu = 0.3$, E = 29,000,000 p.s.i. and tensile elastic limit σ_e = 32,000 p.s.i. and that the stresses shown in Fig. 25 (b) increase in proportion to the load, the five theorys of failure give the following maximum loads:

(1) Maximum principal stress theory

$$\sigma_e = \frac{P}{A} \qquad \text{Maximum load} = \frac{32{,}000}{13{,}233} \times 100 = 242 \text{ lb/ft.}$$

(2) Maximum shearing stress theory

$\tau_e = \frac{1}{2}\sigma_e$ Maximum load $= \frac{16,000}{4,833} \times 100 = 330$ lb/ft.

(3) Maximum strain theory

$\epsilon_e = \frac{\sigma_e}{E}$ Maximum load $= \dfrac{\frac{32,000}{E}}{\frac{13,233}{E} - \frac{0.3 \times 3567}{E}}$

$= \frac{32,000}{12,163} = 263$ lb/ft.

(4) The total strain energy theory $w_e = \frac{1}{2}\frac{\sigma_e^2}{E}$

$$w = \frac{1}{2}\frac{\sigma_1^2}{E} + \frac{1}{2}\frac{\sigma_2^2}{E} - \frac{\mu\sigma_1\sigma_2}{E} = \frac{1}{2E}\left[(13233)^2 + (3567)^2 - 2 \times 0.3 \times 13233 \times 3567\right]$$

$$= \frac{1}{2E}\left[159,514,500\right]$$

Maximum load $= \frac{(32,000)^2}{159,514,500} \times 100 = 642$ lb/ft.

(5) The energy of distortion theory $w_{de} = \frac{1+\mu}{3}\frac{\sigma_e^2}{E}$

$$\omega_d = \frac{1+\mu}{6E}\left[(\sigma_1 - \sigma_2)^2 + \sigma_2^2 + \sigma_1^2\right] = \frac{1+\mu}{6E}\left[(9666)^2 + (3567)^2 + (13233)^2\right]$$

$$= \frac{1+\mu}{6E}\left[281,267,000\right]$$

Maximum load $= \frac{1}{2}\,\frac{(32,000)^2}{281,267,000} \times 100 = 180$ lb/ft.

The governing value is given by the energy of distortion theory, namely; 180 lb/ft.

Station A is one foot from the end of the beam and it is very likely that the transverse stresses increase near the support so that the allowable load per foot is slightly lower than 180 lb/ft. Yielding was indicated by the readings for 150 lb/ft. loading. The readings from gauges on the web and on the outside edges of the top flange at stations A and B showed that the strain for the 150 lb/ft. loading was twice that for the 100 lb/ft. loading. This indicated that yielding and an accompanying redistribution of stress took place when the 150 lb/ft. loading was applied.

Again, it should be emphasized that stiffeners are able to reduce the transverse stress to a negligible value. If the transverse stresses were zero, the governing stresses for failure would be found on the compression side of the top flange at the center of the beam.

Angle of Twist

The angles of twist vs the length along the beam are found in Fig. 26. The two curves of theoretical values are by the regular Lyse-Johnston theory and by the modified theory. The angle of twist of the top and bottom flange is shown four times, each time representing a different number of stiffeners.

Important features shown by the test result curves are as follows:

(a) The angle of twist increases from a minimum at the ends of the beam to a maximum at the center line.

(b) The top and bottom flanges undergo some twist at the ends of the beam; 0.25 degrees for the top flange and 1.6 degrees for the bottom flange. Also, the rate of twist of the flanges near the ends is much less than the theoretical.

(c) The minimum angle of twist of the top flange, and the maximum angle of twist of the bottom flange occurs when there are no stiffeners.

(d) When a stiffener is added, the top flange <u>near</u> the stiffener twists through a greater angle and conversely for the bottom flange.

(e) The maximum angle of twist is 2.85 degrees. The maximum reduction of this twist angle is caused by three stiffeners and is equal to 0.18 degrees or a percentage reduction of five per cent.

(f) The regular Lyse-Johnston theory gives a maximum value of twist angle which is two hundred per cent of the observed value. The modified theory which considers the effect of the bottom plate gives a maximum value of twist identical to the observed value.

Now, consider feature (b), namely; that the flange rotation at the ends is not zero. The theory is based on the assumption that the ends of the flanges do not rotate during loading. Consequently,

FIG. 26

PLOT OF ANGLE OF TWIST VS DISTANCE ALONG THE BEAM FOR LYSE-JOHNSTON, MODIFIED AND OBSERVED VALUES USING 0, 1, 2 AND 3 STIFFENERS

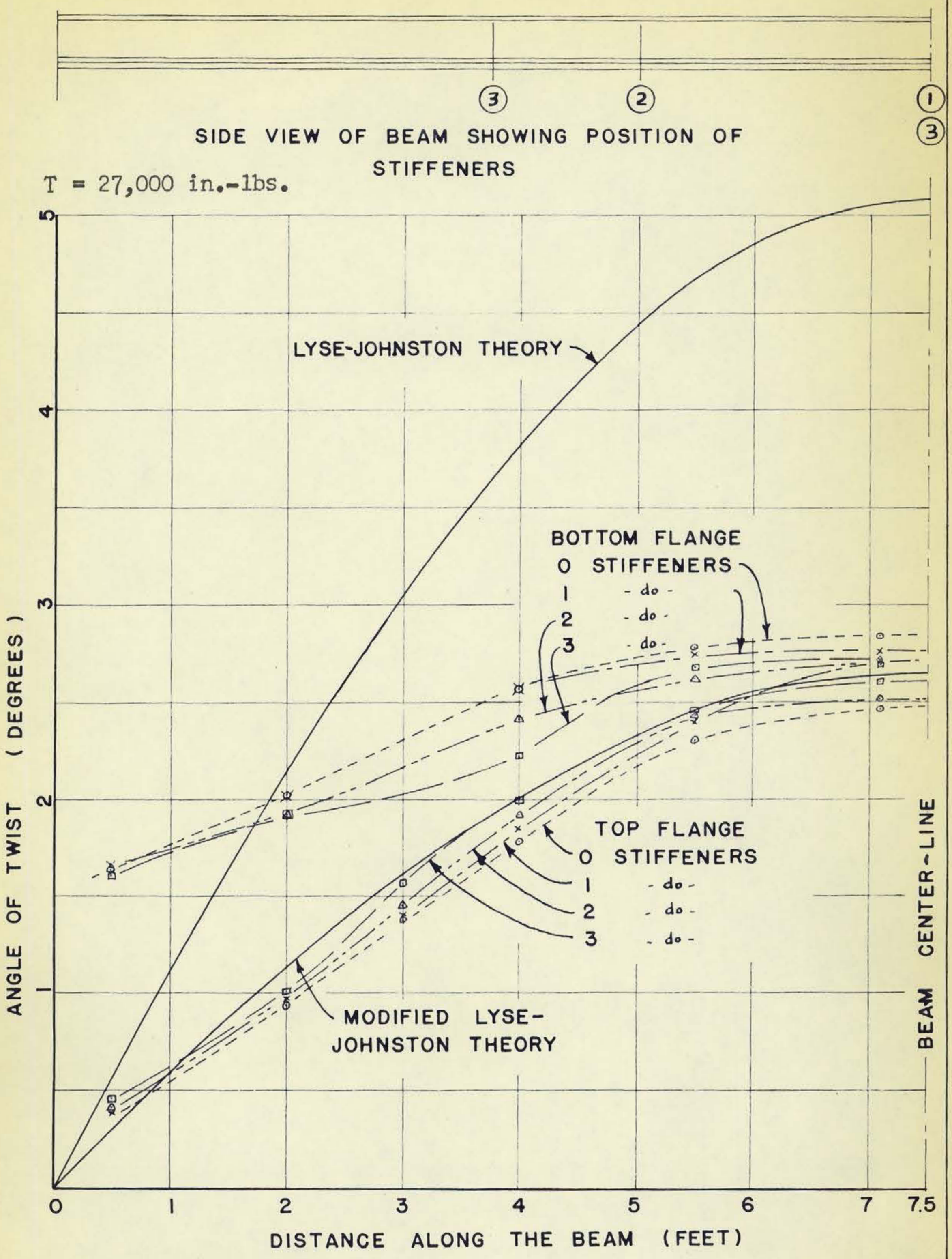
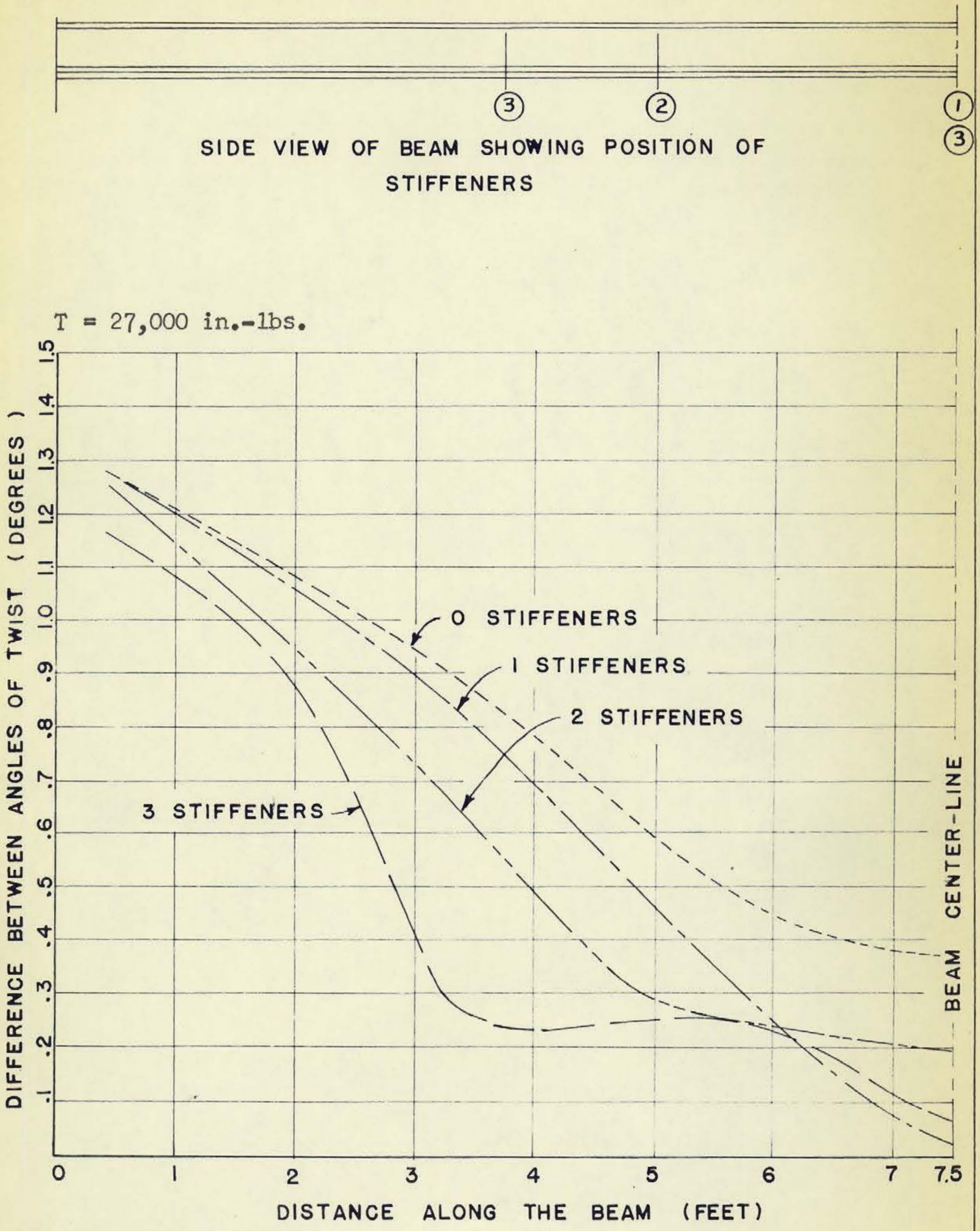


FIG. 27

PLOT OF DIFFERENCE BETWEEN ANGLES OF TWIST OF TOP AND BOTTOM FLANGES VS DISTANCE ALONG BEAM FOR OBSERVED VALUES



observed stresses may not conform to the theoretical (see Flange Shear Stresses), and the point of maximum stress as given by theory may not be the point of actual maximum stress (see Load to Produce Yielding).

The ideas expressed in features (c) and (d) are related to the web distortion as shown in Fig. 22, and can best be illustrated by Fig. 27. This is a plot of the difference of twist angle of the top and bottom flange vs beam length. The difference in twist of the two flanges is greatest at the ends of the beam where the torque is the greatest and the web is held vertical by the end connection. When one stiffener is used, the difference of the angle of twist drops considerably in the central portion of the span. When two and three stiffeners are used, the portion of the span over which the difference is reduced extends further toward the ends of the beam. (The minimum differences are 0.1 and 0.2 degrees. They probably never reach zero because the stiffeners are one hundred per cent effective in a very small area only.)

The small percentage reduction of angle of twist as given in feature (e) indicates that the stiffeners employed in this investigation did not increase the torsional rigidity of the section to any great extent.

Other test loadings produced plots similar to those shown for the 100 lb/ft. load, however, the principle of superposition could not be applied to obtain the maximum angles of twist.

The maximum angle of twist of the top flange for the following loads are as follows:

Actual Twist	Twist by "Superposition"
50 lb/ft. = 1.20 degrees	$\frac{1}{2}$ x 2.46 = 1.23 degrees
100 lb/ft. = 2.46 degrees	
150 lb/ft. = 4.56 degrees	$\frac{3}{2}$ x 2.46 = 3.69 degrees
200 lb/ft. = 6.30 degrees	2 x 2.46 = 4.92 degrees

The observed angles of twist for 150 lb/ft. and 200 lb/ft. loadings are greater than that predicted by "superposition" using the values obtained from the 50 lb/ft. and 100 lb/ft. loadings. This indicates that local yielding took place during the 150 lb/ft. loading.

Pure Torsion Test

The beam was loaded with four pure torques; 4000, 6000, 8000, and 10,000 in.-lbs., and the angles of twist were recorded at all stations. Fig. 28, which is a plot of the angle of twist vs distance along the beam, shows the results of these tests. From this plot the rate of twist (or slope of the straight lines) for each loading was found and a torque vs rate of twist graph was drawn - Fig. 29. The slope of the straight line so obtained was divided by the modulus of rigidity to give a value of the torsional constant K (see Equation iv) of 0.953 in ^{4}.

FIG. 28

PLOT OF ANGLE OF TWIST VS DISTANCE ALONG BEAM FOR PURE TORQUE LOADS

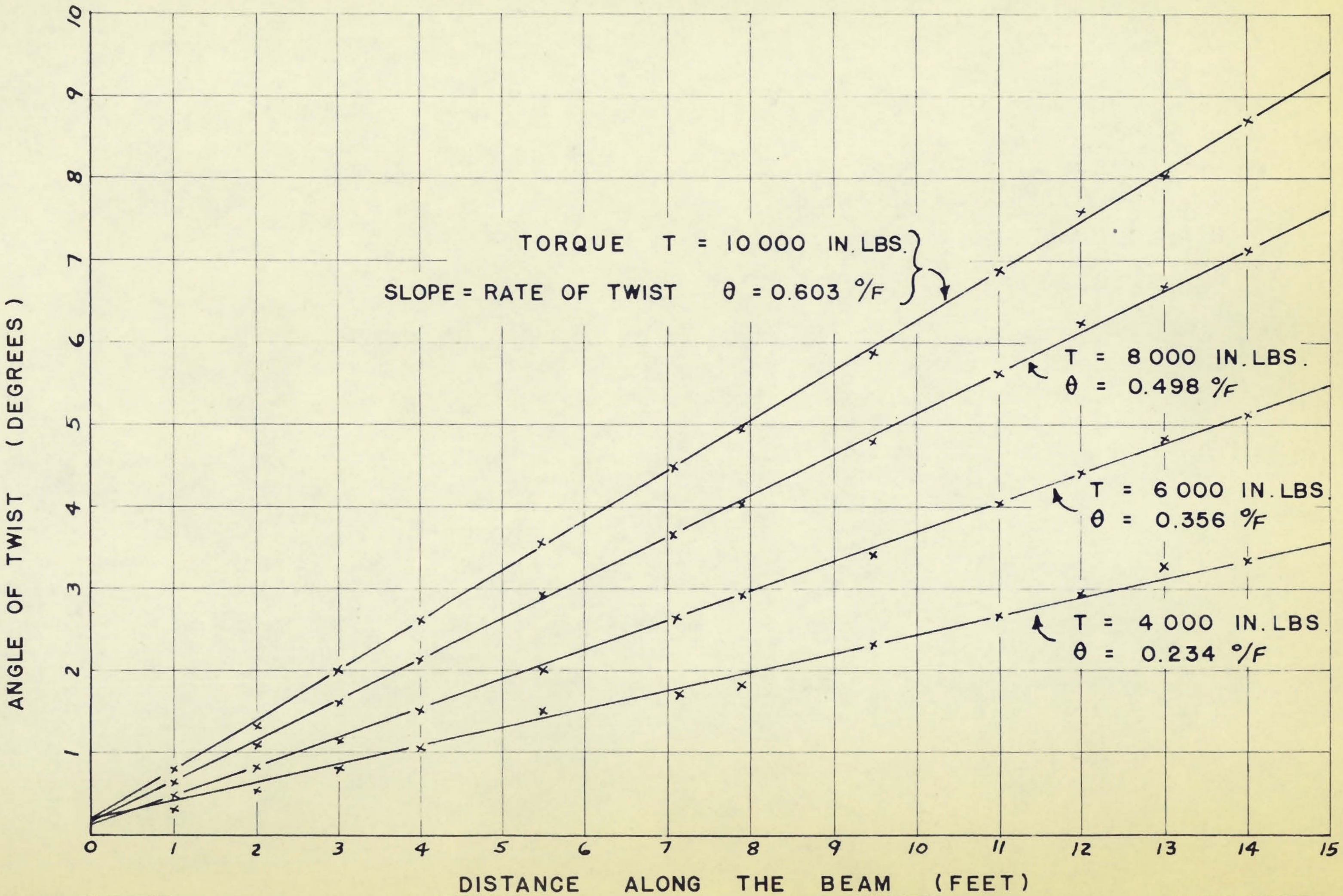
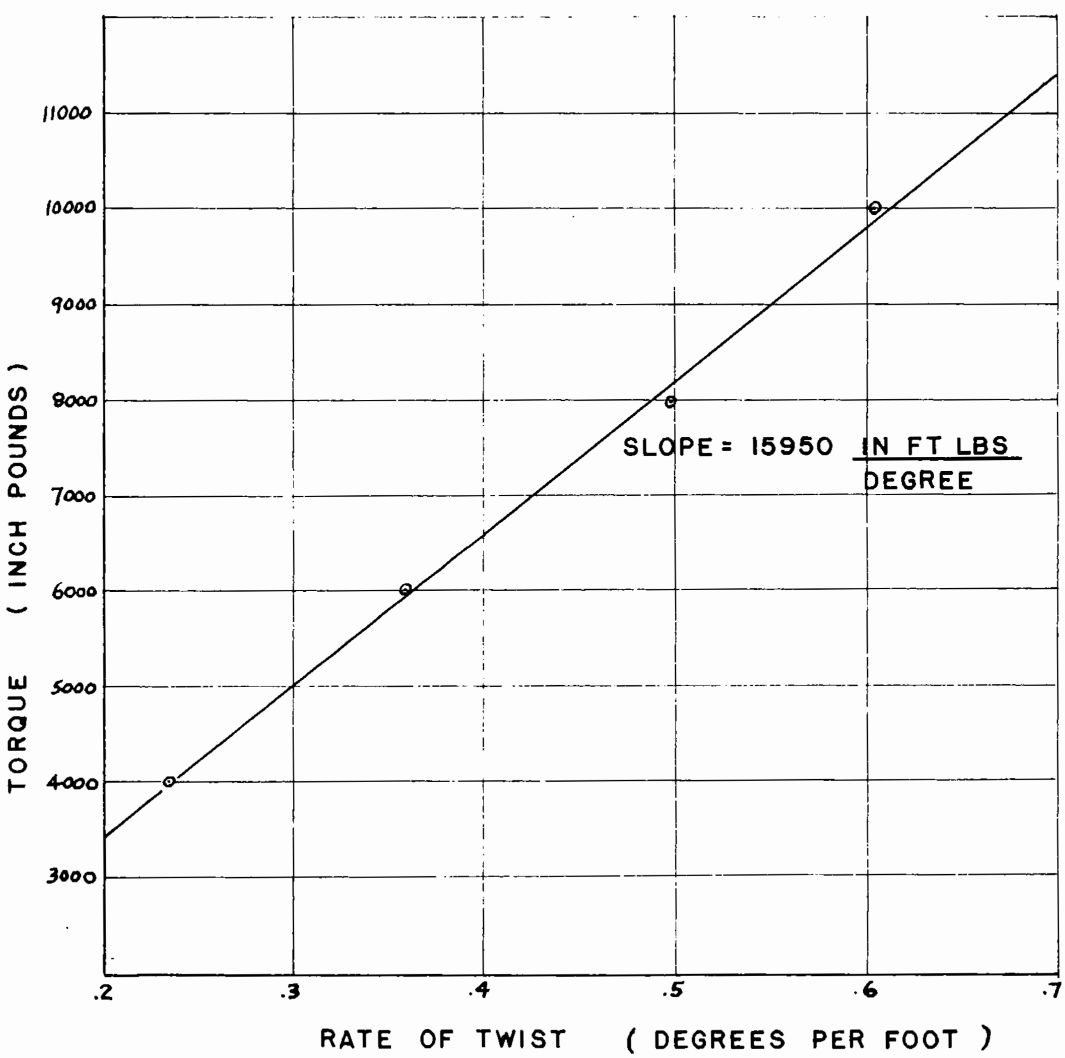


FIG. 29

PLOT OF PURE TORQUE VS RATE OF TWIST
TO OBTAIN A VALUE OF THE TORSION CONSTANT K

$$K = \frac{T}{\theta G} = \frac{15950 \times 57.3 \times 12}{11.5 \times 10^6} = 0.953 \ \text{IN.}^4$$



By "dissection into simple shapes" the torsional constant of the section can be said to be equal to that of the beam plus that of the plate. Thus we have

$$K \text{ of beam} = 0.374 \text{ in}^4.$$

$$K \text{ of plate} = \frac{1}{3} \times 14 \times (3/8)^3 = .246 \text{ in}^4.$$

$$K \text{ total} = 0.62 \text{ in}^4.$$

There are two reasons for the discrepancy between the observed and the theoretical values of the torsional constant. Firstly, the beam and the plate are welded together which will cause some unknown "junction effect", and secondly, the beam was loaded through its shear center, rather than the shear center of the total section i.e., beam and plate.

The agreement between the modified Lyse-Johnston theory and the test results indicate that the bottom plate does not resist torsional stresses when the beam has torsionally free ended connections such as used in this investigation. Hence, the larger K values of 0.953 or 0.62 in^4. can be used in this case. If, however, a type of stiffener was designed which would cause the plate, as well as the beam, to resist torsion, a larger K value would be applicable, and the spandrel would be more efficient (see Suggested Future Experiments).

Visual Observations

Unfortunately, the beam was painted in the shop with an iron oxide paint. This paint did not crack or show any Luder's lines until large strains occurred. Under the maximum load, there were cracks in the paint at the bottom web on the loaded side near each support.

Under the 150 lb/ft. and 200 lb/ft. loads a definite indication of yielding was noticeable at the supports.

A definite rotation of the bottom flange (as shown in Fig. 30) could be observed at the supports for the 150 lb/ft. load. This rotation increased considerably for the 200 lb/ft. load. The rotation was so great that the back edge of the bottom flange touched the support angle and this prevented any further rotation.

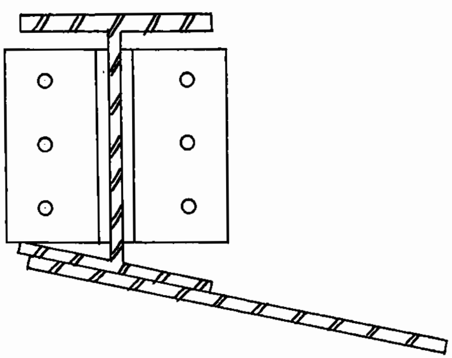


FIG. 30

CHAPTER VI

SUMMARY

Discussion

Electrical strain gauges were used to find the strains and hence the stresses in the beam. The gauges and recording apparatus are accurate to approximately ten microinches per inch strain. Under a certain load, say 100 lbs/ft. strain readings are required for different combinations of stiffeners. Often the change in strain at a particular gauge caused by a different stiffener combination is less than the possible accuracy of the strain reading. If the strain readings for one gauge for any two combinations of stiffeners differ by twenty microinches per inch it is difficult to determine whether this difference is an actual strain difference or if it is a reading error. The test readings in this experiment were assumed to be actual strain differences and plotted as such, even though they may be in error.

A final summary of all the results is made in the Tables on Page 72. It should be noted that the percentage reductions listed in Table 2 are only approximate and only apply to the stresses listed in Table 1. For instance, the maximum transverse stress which occurs at the end of the beam is only reduced ten per cent by the addition of

TABLE I Maximum Theoretical and Observed Stresses and Twist Angles

Load Per Foot of Beam	Max. Torsional & Vertical Bending Flange Stress $f_t + f_b$ p.s.i.		Max. Torsional & Vertical Bending Web Shear $s_t + s_v$ p.s.i.		Max. Torsional & Lateral Bending Flange Shear $s_f + s_q$ p.s.i.	
LBS.	L.-J. Theory	Actual	L.-J. Theory	Actual	L.-J. Theory	Actual
50	5,870	5,000	3,400	1,700	5,140	2,200
100	11,750	9,300	6,800	3,400	10,275	4,400
150	17,620	15,100	10,200	6,900	15,415	7,200
200	23,500	23,500	13,600	7,700	20,550	14,000

Load Per Foot of Beam	Max. Transverse Fiber Stress in Web p.s.i.		Max. Twist Angle ψ DEGREES		
LBS.	L.-J. Theory	Actual	L.-J. Theory	Modified L.-J. Theory	Actual
50	0	6,400	2.54	1.33	1.47
100	0	12,800	5.08	2.65	2.84
150	0	25,000	7.62	3.98	5.34
200	0	35,000	10.16	5.30	7.10

TABLE II Approximate Reduction of the Above Maximum Actual Stresses and Twist Angle Introduced by the use of Three Stiffeners

Feature	$f_t + f_b$	$s_t + s_v$	$s_f + s_q$	Transverse Fiber Stress	ψ
Per cent Reduction	11	10	0	10	5

three stiffeners. However, there is a much greater percentage reduction in this stress close to the stiffener.

The plots of test results in Chapter V show that the type of stiffener used in this experiment held the flanges together and prevented distortion of the web. They also show that the effectiveness of the beam to resist torsion can be increased in other ways, namely;

(a) By preventing the flanges from rotating at the supports.

(b) By insuring that the bottom plate undergoes torsion, and therefore, resists part of the torsional moment.

These two conditions could be realized by the use of a different type of stiffener. Such a stiffener would prevent rotation of the top flange and bottom plate at the support. These stiffeners would be located near the ends of the beam rather than the central portion of the beam.

Suggested Method of Design

In the light of the presented test results, the following method of design is proposed for spandrel beams having a bottom plate:

The Lyse-Johnston design method as presented in the Bethlehem Steel Booklet S-57 should be used for determining beam constants and selection of beam sizes for any particular loading.

To this design method should be added two important points, namely:

(1) Transverse fiber stresses (not considered in the Lyse-Johnston design procedure) may be very high and may actually govern the design. A rational approach to the determination of these stresses should be made as shown on Pages 57 and 58 of this paper.

(2) The actual angle of twist may be considerably reduced by the addition of the bottom plate. It should not be assumed that the bottom plate decreases any of the design stresses, but the effect of the plate in reducing the twist angle may be taken into consideration by the use of the modified design presented herein. Naturally, an investigation into what reduction the plate causes would only be limited to those cases where the twist angle controlled the design.

As to the importance or advantage of using web stiffeners, until a more comprehensive study is completed, it must be assumed that the reduction of stresses introduced by the use of web stiffeners is negligible. The one exception is the reduction of transverse fiber stress. If the method of loading is such that the transverse fiber stresses will be fairly high it is recommended that the web stiffeners be used.

Suggested Future Experiments

Future experimenters should use a different approach to the problem of designing a stiffener to increase the torsional rigidity of a WF beam. The stiffener should be designed to prevent distortion of the total cross-section. (Note that this does not mean warping of the cross-section which would be practically impossible to prevent by the use of bolted connections.) A suggested type of stiffener is shown in Fig. 31. A minimum of two of these stiffeners should be used, each located near one support. This minimum could be increased by adding more stiffeners to the central portion of the beam.

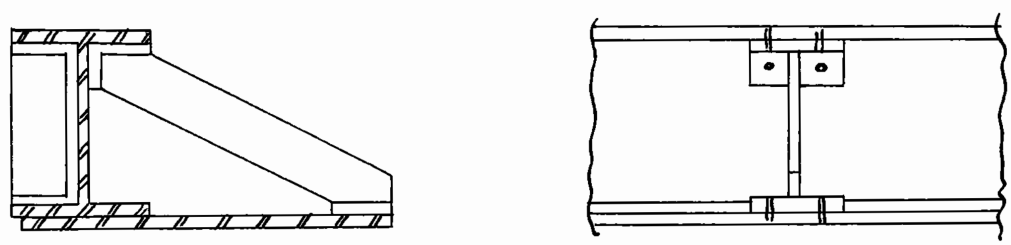


FIG. 31

The uniform load which was applied in this test with dead weight might well be applied with a rubber tube and air pressure.

The rosette strain gauges should be oriented in future tests so that readings as large as possible are obtained.

A bottom plate of the same size as used in this investigation should be used with a different beam size to determine if the modified Lyse-Johnston theory presented in this paper is applicable to other sizes of beams.

Conclusions

In this paper the general problem of a beam subjected to torsion and bending by a continuous uniform load was investigated. The effect of stiffeners on the torsional properties was also examined. An effort was made to discuss thoroughly those topics of particular interest to design engineers i.e., flange stresses, web stresses, angles of twist, and effects on each caused by the stiffeners. The observed results were compared with those predicted by the Lyse-Johnston method of design and an effort made to modify their design to fit the loading conditions.

It was found that the Lyse-Johnston method of design gave reasonable values for the flange stresses. Web and flange shears were somewhat lower than those predicted by theory. Lyse-Johnston theory considers no transverse stresses and these should always be considered. The modified theory gave good answers for the angle of twist whereas the Lyse-Johnston theory is too conservative.

The primary function of common web stiffeners is to hold the flanges together and decrease the large transverse stresses which would be present without such rib reinforcement. It is impossible to give an exact rule for the spacing of stiffeners to prevent such stresses from being the governing design criterion, but it is suggested that at least three stiffeners should be used on all spandrel beams.

In the opinion of the author, rib stiffeners should always be used on spandrel or lintel beams to assure unity of action of the flanges and to check the possibility of flange buckling, but it should not be assumed that they will eliminate excessive deflection or stresses due to torsion.

It should be pointed out that in this thesis, only the idealized condition was considered. The effects produced by the continuity and rigidity of connections and the restraints imposed by slabs and walls are left to the judgment of the engineer.

APPENDIX I

SOLUTION OF THE LYSE-JOHNSTON EQUATION

Solution of the Lyse-Johnston differential equation of the center of a beam flange for a simple span beam with torsionally free ends with a full uniform eccentric load.

Load per foot = w

Length (feet) = L

Eccentricity (inches) = e

The differential equation may be written

$$a^2 \frac{d^3y}{dx^3} - \frac{dy}{dx} = \frac{-2a^2 T_e}{hEI_y} \tag{1}$$

where $a = \frac{h}{2}\sqrt{\frac{EI_y}{KG}}$

and T_e = external torque at the point x, y from the end

$= \frac{wLe}{2} - wxe$

For a solution of (1) we may try

$$y = \frac{2a^3 T}{hEI_y} \left(A_1 \sinh \frac{x}{a} + B_1 \cosh \frac{x}{a} + C_1 x^2 + D_1 x + E_1\right) \tag{2}$$

where T = total external torque = wLe

$$\frac{dy}{dx} = \frac{2a^3 T}{hEI_y} \left(\frac{A_1}{a} \cosh \frac{x}{a} + \frac{B_1}{a} \sinh \frac{x}{a} + 2C_1 x + D_1\right) \tag{3}$$

$$\frac{d^2y}{dx^2} = \frac{2a^3T}{hEI_y}\left(\frac{A_1}{a^2}\sinh\frac{x}{a} + \frac{B_1}{a^2}\cosh\frac{x}{a} + 2C_1\right) \tag{4}$$

$$\frac{d^3y}{dx^3} = \frac{2a^3T}{hEI_y}\left(\frac{A_1}{a^3}\cosh\frac{x}{a} + \frac{B_1}{a^3}\sinh\frac{x}{a}\right) \tag{5}$$

Then $a^2\frac{d^3y}{dx^3} - \frac{dy}{dx} = -\frac{2a^3T}{hEI_y}(2C_1x + D_1)$ which can hold only if

$$-\frac{2a^3T}{hEI_y}(2C_1x + D_1) = -\frac{2a^2T_e}{hEI_y} \tag{6}$$

or $2aC_1x + aD_1 = \frac{1}{2} - \frac{x}{L}$

or $(2aC_1 + \frac{1}{L})x + aD_1 - \frac{1}{2} = 0$

It is evident that

$$2aC_1 + \frac{1}{L} \equiv 0 \qquad \text{or } C_1 = -\frac{1}{2aL} \tag{7}$$

$$\text{and } aD_1 - \frac{1}{2} = 0 \quad \text{or } D_1 = \frac{1}{2a} \tag{8}$$

The three independent sets of boundary conditions for the solution of constants A_1, B_1 and E_1 are ;

When $x = 0$ $\qquad y = 0$

$x = \frac{L}{2}$ $\qquad \frac{dy}{dx} = 0$

$x = 0$ $\qquad \frac{d^2y}{dx^2} = 0$

Then from (2) (3) and (4) since sinh 0 = 0 and cosh 0 = 1

$$0 = \frac{2a^3T}{hEI_y}\left(B_1 + E_1\right) \tag{9}$$

$$0 = \frac{2a^3T}{hEI_y}\left(\frac{A_1}{a}\cosh\frac{L}{2a} + \frac{B_1}{a}\sinh\frac{L}{2a} + 2C_1\frac{L}{2} + D_1\right)$$

$$\text{or } 0 = \left(\frac{A_1}{a}\cosh\frac{L}{2a} + \frac{B_1}{a}\sinh\frac{L}{2a}\right) \tag{10}$$

$$0 = \frac{2a^3T}{hEI_y}\left(\frac{B_1}{a^2} - \frac{2}{2aL}\right) \tag{11}$$

from (11) $B_1 = \frac{a}{L}$ or $\frac{1}{2\frac{L}{2a}}$ (12)

from (10) and (12) $A_1 = \frac{\tanh\frac{L}{2a}}{2\frac{L}{2a}}$ (13)

from (9) and (12) $E_1 = -\frac{1}{2\frac{L}{2a}}$ (14)

Thus equation (2) can be written

$$y = \frac{2a^3T}{hEI_y}\left(-\frac{\tanh\frac{L}{2a}}{2\frac{L}{2a}}\sinh\frac{x}{a} + \frac{1}{2\frac{L}{2a}}\cosh\frac{x}{a} - \frac{1}{2aL}x^2 + \frac{1}{2a}x - \frac{1}{\frac{2L}{2a}}\right)$$

APPENDIX II

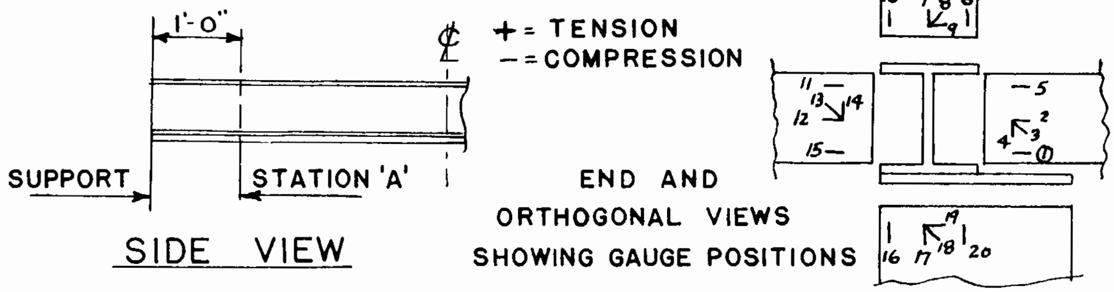
TEST DATA

The following test stresses are the observed strain readings in microinches per inch multiplied by Young's modulus which was assumed to be 30×10^6 lbs/sq.in. The tabulated stresses are not necessarily the actual stresses since most elements in the beam are in a state of biaxial or triaxial stress.

The actual stresses can be obtained by using the stress readings for triaxial gauges. After plotting a Mohr diagram, the observed principal stresses and Poisson's ratio are used to find the actual principal stresses. The actual principal stresses can then be used to plot a Mohr diagram representing the true state of stress at the point.

TEST STRESSES LB. PER SQ.IN. AT STATION 'A'

ECCENTRICITY OF ALL LOADS = 18"



SIDE VIEW

END AND ORTHOGONAL VIEWS SHOWING GAUGE POSITIONS

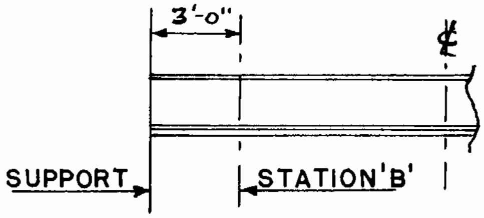
GAUGE	LOAD / FT. LBS.	NUMBER OF STIFFENERS			
		3	2	1	0
A1	50	+100	+100	+100	+100
	100	+210	+210	+210	+210
	150	+600	+600	+600	+600
	200	+6600	+6600		
A2	50	+360	+420	+450	+480
	100	+720	+840	+900	+960
	150	+900	+1050	+1200	+1350
	200	+900	+1000	+1000	+1100
A3	50	+3610	+3780	+3820	+4050
	100	+7230	+7560	+7640	+8100
	150	+10800	+11400	+12000	+12300
	200	+15000	+15000	+15200	+15500
A4	50	+3150	+3300	+3360	+3450
	100	+6300	+6600	+6720	+6900
	150	+9900	+9900	+10200	+10200
	200	+10000	+10000	+10200	+10100
A5	50	+150	+150	+150	+150
	100	+300	+300	+300	+300
	150	+600	+600	+600	+600
	200	+600	+600		
A6	50	+1250	+1200	+1350	+1600
	100	+2550	+2400	+2700	+3120
	150	+4500	+4800	+5000	+5100
	200	+7500	+7500		
A7	50	+135	+130	+140	+210
	100	+270	+270	+270	+420
	150	+390	+420	+480	+660
	200	+1200	+1200		
A8	50	+1900	+1900	+1980	+2000
	100	+3900	+3900	+3960	+4080
	150	+6000	+6300	+6300	+6300
	200	+10500	+10500		
A9	50	-150	-150	-180	-180
	100	-300	-300	-360	-360
	150	-450	-480	-600	-600
	200	-750	-900		
A10	50	-1050	-1050	-1380	-1440
	100	-2250	-2160	-2610	-2430
	150	-3600	-3600	-3690	-3690
	200	-4950	-4980		

GAUGE	LOAD / FT. LBS.	NUMBER OF STIFFENERS			
		3	2	1	0
A11	50	−	−	−	−
	100	-180	-180	-180	-180
	150	-240	-240	-240	-240
	200	-360	-450		
A12	50	-750	-900	-930	-1020
	100	-1530	-1770	-1890	-2010
	150	-2550	-2850	-3000	-3150
	200	-4710	-5100		
A13	50	-150	-300	-450	-480
	100	-300	-600	-900	-990
	150	-600	-900	-1200	-1200
	200	-1500	-1410		
A14	50	-2490	-2700	-3300	-3450
	100	-5010	-5490	-6660	-7020
	150	-7500	-8400	-8700	-9000
	200	-9900	-11340		
A15	50	-420	-390	-600	-450
	100	-840	-780	-1200	-930
	150	-1200	-1200	-1500	-1500
	200	-5790	-5100		
A16	50	+150	+210	+210	+210
	100	+300	+300	+300	+300
	150	+600	+600	+600	+600
	200	+1080	+810		
A17	50	+150	+150	+150	+150
	100	+300	+300	+300	+300
	150	+450	+450	+450	+450
	200	+690	+750		
A18	50	-1560	-1710	-1860	-1890
	100	-3150	-3420	-3750	-3780
	150	-5100	-5400	-5700	-5700
	200	-4620	-5340		
A19	50	-660	-720	-750	-750
	100	-1350	-1410	-1500	-1530
	150	-1950	-1950	-2100	-2250
	200	-2670	-2940		
A20	50	-90	-90	-90	-90
	100	-180	-210	-210	-180
	150	-450	-450	-450	-450
	200	-60	-270		

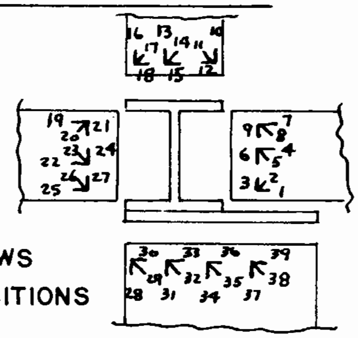
TEST STRESSES LB. PER SQ. IN. AT STATION 'B'

ECCENTRICITY OF ALL LOADS = 18"

+ = TENSION
− = COMPRESSION



SIDE VIEW



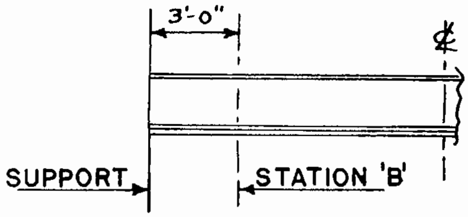
END AND ORTHOGONAL VIEWS SHOWING GAUGE POSITIONS

GAUGE	LOAD / FT. LBS.	NUMBER OF STIFFENERS 3	2	1	0
B1	50	+240	+210	+240	+150
	100	+510	+420	+480	+300
	150	+750	+630	+720	+450
	200	+1020	+930		
B2	50	+930	+960	+1200	+1440
	100	+1860	+1950	+2430	+2910
	150	+4020	+7380	+7800	+8250
	200	+6270	+10770		
B3	50	+3060	+3450	+3600	+4500
	100	+6150	+6900	+7200	+9000
	150	+11160	+19260	+20250	+21210
	200	+17340	+28920		
B4	50	−30	−30	−60	−90
	100	−90	−90	−120	−120
	150	−180	−120	−180	−180
	200	−90	−330		
B5	50	+1950	+2100	+2250	+2700
	100	+3900	+4200	+4500	+5400
	150	+7710	+8850	+10800	+11100
	200	+10800	+13950	+	
B6	50	+1260	+2100	+2310	+2700
	100	+2550	+4200	+4650	+5400
	150	+5100	+9900	+11100	+11400
	200	+7500	+13200		
B7	50	0	0	−60	−90
	100	+90	−90	−120	−180
	150	0	0	−180	−270
	200	−90	−120		
B8	50	+1440	+1350	+1500	+1950
	100	+2910	+2700	+3000	+3900
	150	+6300	+6450	+6900	+6900
	200	+7950	+8100		
B9	50	+210	+210	+450	+900
	100	+450	+450	+900	+1800
	150	+750	+900	+1350	+2700
	200	+1200	+1800		
B10	50	+2550	+3150	+3600	+4350
	100	+5100	+4500	+4200	+5700
	150	+11100	+11400	+11100	+11400
	200	+15900	+15000		

GAUGE	LOAD / FT. LBS.	NUMBER OF STIFFENERS 3	2	1	0
B11	50	−1110	−1080	−1050	−1050
	100	−2250	−2160	−2100	−2100
	150	−3300	−3450	−3600	−3750
	200	−5700	−4950		
B12	50	−960	−780	−630	−780
	100	−1950	−1560	−1260	−1590
	150	−3000	−2700	−2700	−2400
	200	−4350	−4200		
B13	50	−150	−210	−150	−210
	100	−300	−390	−300	−420
	150	−450	−600	−450	−630
	200	−750	−750		
B14	50	+2850	+2550	+2400	+2700
	100	+5700	+5100	+4800	+5400
	150	+12000	+11400	+10800	+10800
	200	+16500	+14400		
B15	50	−60	−60	0	−30
	100	−150	−90	0	−30
	150	0	0	0	0
	200	−150	−150		
B16	50	−2610	−2400	−2790	−3000
	100	−5250	−4800	−5580	−6600
	150	−10650	−10650	−10200	−10650
	200	−13500	−13200		
B17	50	+1260	+1050	+1120	+1320
	100	+2550	+2100	+2250	+2640
	150	+5100	+4800	+4500	+4500
	200	+9000	+8100		
B18	50	+750	+600	+600	+750
	100	+1500	+1200	+1200	+1500
	150	+2700	+2700	+2700	+2700
	200	+3900	+3600		
B19	50	−210	−300	−300	−300
	100	−450	−570	−600	−630
	150	−900	−1050	−1050	−1050
	200	−1650	−1530		
B20	50	−1650	−1800	−1800	−2100
	100	−3300	−3600	−3600	−4110
	150	−5700	−6000	−6000	−6300
	200	−7650	−7800		

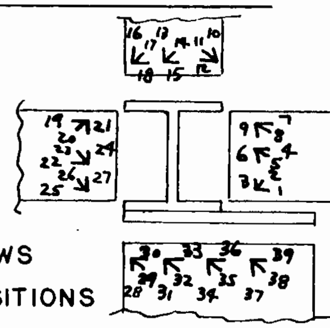
TEST STRESSES LB. PER SQ. IN. AT STATION 'B'

ECCENTRICITY OF ALL LOADS = 18"



SIDE VIEW

+ = TENSION
− = COMPRESSION



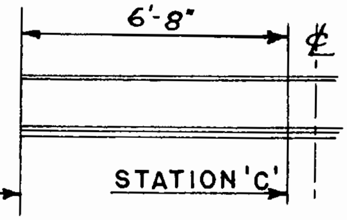
END AND ORTHOGONAL VIEWS SHOWING GAUGE POSITIONS

GAUGE	LOAD / FT. LBS.	NUMBER OF STIFFENERS 3	2	1	0
B21	50	−420	−450	−630	−810
	100	−840	−900	−1260	−1590
	150	−1260	−1500	−2100	−3000
	200	−1500	−2100		
B22	50	−360	−330	−300	−330
	100	−750	−660	−510	−630
	150	−1800	−1680	−1650	−1740
	200	−2250	−2100		
B23	50	+300	−300	−300	−300
	100	+600	−450	−510	−570
	150	+1050	−600	−900	−1050
	200	+1200	−1500		
B24	50	−1050	−1950	−2100	−2400
	100	−1800	−3750	−4050	−4500
	150	−4200	−6000	−6900	−7200
	200	−6600	−12900		
B25	50	0	0	0	0
	100	0	−30	−30	−30
	150	−120	−90	−150	−180
	200	−150	−150		
B26	50	−660	−990	−1200	−1500
	100	−1320	−1950	−2400	−3000
	150	−2700	−3600	−4800	−6000
	200	−6210	−10500	−	
B27	50	−2850	−3300	−3450	−4200
	100	−4700	−6600	−6900	−8400
	150	−9600	−10800	−11400	−12300
	200	−17100	−28800		
B28	50	+750	+750	+750	+900
	100	+1500	+1500	+1560	+1800
	150	+2550	+2850	+2850	+3000
	200	+3600	+4050		
B29	50	−900	−1200	−1260	−1350
	100	−1800	−2310	−2550	−7620
	150	−2550	−3600	−3900	−4050
	200	−3600	−4500		
B30	50	−270	−330	−330	−330
	100	−480	−780	−720	−870
	150	−900	−1200	−1260	−1290
	200	−1200	−1800	−	−

GAUGE	LOAD / FT. LBS.	NUMBER OF STIFFENERS 3	2	1	0
B31	50	+540	+540	+540	+540
	100	+1050	+1080	+1080	+1110
	150	+1800	+1680	+1680	+1680
	200	+2400	+2100		
B32	50	−1320	−1560	−1710	−1800
	100	−2670	−3210	−3450	−3600
	150	−3900	−4950	−5250	−5550
	200	−5250	−6450		
B33	50	−540	−600	−660	−690
	100	−1050	−1200	−1350	−1410
	150	−1800	−1800	−1850	−1800
	200	−2700	−2850		
B34	50	+210	+210	+240	+300
	100	+420	+450	+480	+600
	150	+750	+750	+900	+900
	200	+900	+900		
B35	50	−1680	−1950	−2100	−2190
	100	−3360	−3960	−4200	−4410
	150	−5100	−6000	−6600	−6600
	200	−7500	−8700		
B36	50	−1800	−1650	−1800	−1950
	100	−3450	−3300	−3420	−3900
	150	−5700	−5700	−5700	−5700
	200	−8100	−8400		
B37	50	−300	−330	−330	−300
	100	−600	−690	−690	−600
	150	−900	−900	−900	−900
	200	−1200	−1200		
B38	50	−1560	−1560	−1650	−1860
	100	−3210	−3150	−3330	−3810
	150	−5100	−5100	−5250	−5550
	200	−6900	−7050		
B39	50	−1500	−1380	−1500	−1590
	100	−3000	−2760	−3000	−3180
	150	−5100	−4800	−4500	−5100
	200	−7800	−6600		
	50				
	100				
	150				
	200				

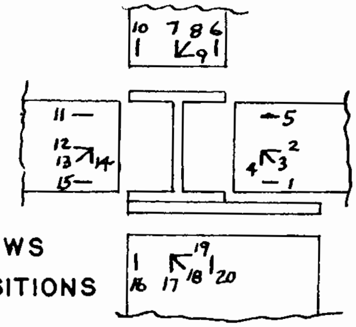
TEST STRESSES LB PER SQ IN AT STATION 'C'

ECCENTRICITY OF ALL LOADS = 18"



+ = TENSION
− = COMPRESSION

SIDE VIEW



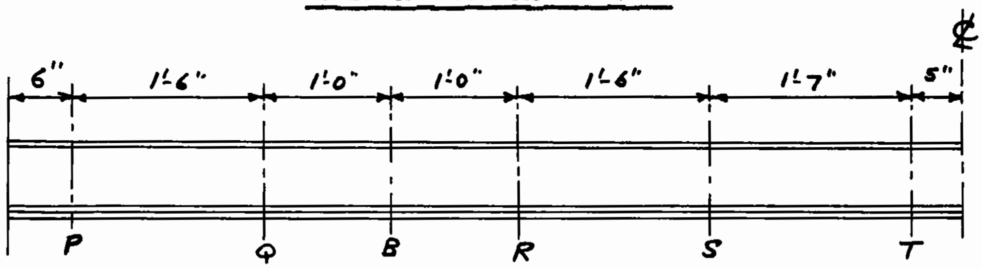
END AND ORTHOGONAL VIEWS SHOWING GAUGE POSITIONS

GAUGE	LOAD / FT LBS	NUMBER OF STIFFENERS 3	2	1	0
C1	50	+120	+180	+180	+210
	100	+240	+360	+240	+450
	150	+420	+600	+600	+750
	200	+900	+1080	+1200	+1350
C2	50	–	–	–	–
	100	+60	+180	+90	+240
	150	+60	+240	+180	+330
	200	+450	+840	+600	+900
C3	50	+390	+660	+480	+900
	100	+710	+1350	+960	+1800
	150	+1200	+1500	+1200	+2550
	200	+1800	+3060	+2010	+4830
C4	50	+300	+510	+300	+930
	100	+600	+1050	+600	+1950
	150	+900	+1380	+1170	+3450
	200	+810	+2400	+1080	+5700
C5	50	–	–	–	–
	100	−60	−60	−60	−90
	150	−150	−150	−150	−150
	200	−30	−90	−120	−180
C6	50	+3600	+3540	+2700	+3150
	100	+5850	+5250	+5700	+6150
	150	+9360	+10500	+10950	+10500
	200	+15750	+17100	+17100	+17100
C7	50	−360	−360	−360	−360
	100	−690	−690	−720	−750
	150	−1050	−1050	−1200	−1110
	200	−1380	−1320	−1320	−1320
C8	50	+600	+300	+900	+390
	100	+1230	+600	+1830	+810
	150	+1950	+900	+2850	+1350
	200	+2400	+1050	+3450	+2010
C9	50	+60	+60	+60	+60
	100	+150	+150	+120	+120
	150	+180	+270	+240	+210
	200	+360	+450	+480	+480
C10	50	−3450	−3600	−3900	−4200
	100	−6750	−6810	−7350	−7560
	150	−11130	−12420	−12780	−12300
	200	−18000	−19200	−19200	−19200

GAUGE	LOAD / FT LBS	NUMBER OF STIFFENERS 3	2	1	0
C11	50	−510	−510	−510	−510
	100	−1050	−1710	−1710	−900
	150	−1380	−1200	−1260	−1260
	200	−1740	−1680	−1710	−1740
C12	50	−90	−90	−90	−90
	100	−180	−210	−180	−180
	150	−300	−360	−330	−480
	200	−480	−600	−550	−690
C13	50	−300	−600	−450	−780
	100	−570	−1200	−900	−1620
	150	−900	−1650	−1800	−2550
	200	−1500	−2640	−2700	−4200
C14	50	−90	−450	−420	−1080
	100	−180	−960	−840	−2160
	150	−180	−1350	−1200	−3450
	200	−630	−2250	−2010	−3000
C15	50	+300	+300	–	+150
	100	+450	+300	+150	+330
	150	+360	+750	+330	+600
	200	+780	+1020	+780	+990
C16	50	+420	+210	+240	+300
	100	+840	+360	+450	+630
	150	+1350	+750	+900	+1200
	200	+2100	+2100	+2100	+2100
C17	50	+450	+450	+450	+450
	100	+930	+960	+900	+870
	150	+1440	+1560	+1440	+1380
	200	+1800	+1710	+1770	+1740
C18	50	−390	−480	−420	−420
	100	−780	−960	−750	−870
	150	−1170	−1440	−1170	−1320
	200	−1620	−2100	−1710	−1800
C19	50	−360	−420	−420	−450
	100	−750	−840	−810	−930
	150	−1110	−1260	−1110	−1440
	200	−1650	−1800	−1710	−2010
C20	50	+300	+300	+120	+180
	100	+600	+600	+300	+450
	150	+1050	+900	+840	+750
	200	+1110	+1110	+1050	+990

ANGLES OF TWIST OF THE TOP FLANGE (DEGREES)

SIMPLY SUPPORTED BEAM

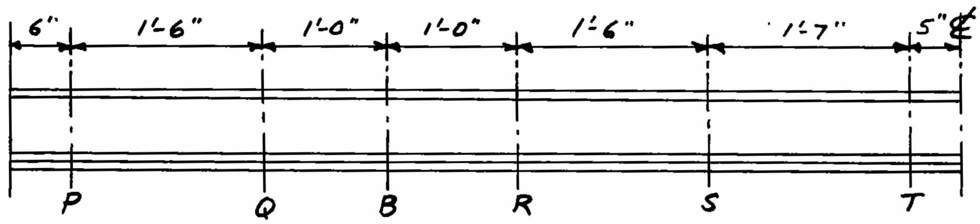


Location of Stations.

No. of Stiffeners	Station P	Station Q	Station B	Station R	Station S	Station T
LOADING - 50 lbs/lin.ft. at an 18 inch eccentricity						
3	0.21	0.48	0.76	1.00	1.21	1.26
2	0.20	0.48	0.69	0.92	1.23	1.26
1	0.20	0.48	0.66	0.90	0.21	1.32
0	0.21	0.48	0.65	0.82	1.10	1.20
LOADING - 100 lbs/lin.ft. at an 18 inch eccentricity						
3	0.46	1.05	1.56	1.99	2.43	2.60
2	0.42	0.96	1.45	1.92	2.45	2.51
1	0.39	0.96	1.40	1.85	2.40	2.70
0	0.38	0.94	1.39	1.78	2.30	2.46
LOADING - 150 lbs/lin.ft. at an 18 inch eccentricity						
3	0.87	1.81	2.73	3.74	4.40	4.69
2	0.94	1.77	2.68	3.68	4.57	4.86
1	0.90	1.77	2.58	3.51	4.31	4.96
0	0.87	1.72	2.61	3.37	4.21	4.56
LOADING - 200 lbs/lin.ft. at an 18 inch eccentricity						
3	1.16	2.41	3.68	4.84	5.78	6.38
2	1.23	2.40	3.56	4.89	6.26	6.46
1			3.50	4.70	5.92	6.56
0				4.60	5.72	6.30

ANGLES OF TWIST OF THE BOTTOM FLANGE (DEGREES)

SIMPLY SUPPORTED BEAM



Location of Stations.

No. of Stiffeners	Station P	Station Q	Station B	Station R	Station S	Station T
LOADING - 50 lbs/lin.ft. at an 18 inch eccentricity						
3	0.87	0.97		1.10	1.38	1.40
2	0.88	1.00		1.16	1.30	1.43
1	0.86	1.09		1.25	1.40	1.44
0	0.87	1.08		1.26	1.42	1.47
LOADING - 100 lbs/lin.ft. at an 18 inch eccentricity						
3	1.61	1.93		2.22	2.69	2.70
2	1.66	1.92		2.41	2.62	2.71
1	1.66	2.03		2.57	2.75	2.76
0	1.64	2.02		2.57	2.78	2.84
LOADING - 150 lbs/lin.ft. at an 18 inch eccentricity						
3	3.16	3.32		4.02	4.56	4.85
2	3.60	3.70		4.40	4.80	5.00
1	3.70	4.00		4.60	4.80	5.10
0	3.70	4.00		4.62	5.05	5.34
LOADING - 200 lbs/lin.ft. at an 18 inch eccentricity						
3					6.08	6.60
2					6.53	6.66
1					6.42	6.76
0					6.72	7.10

APPENDIX III

EXHIBITS

EXHIBIT 1

A general view of
the test set-up.

EXHIBIT 2

A side view showing
loading pan and bricks.

EXHIBIT 3

A view of the top
flange before failure.

EXHIBIT 4

A view of the top
flange after failure.

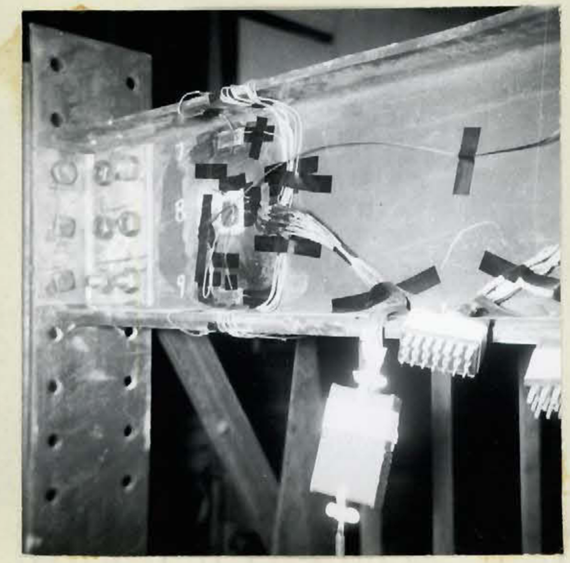


EXHIBIT 5

A view of the end connection.

EXHIBIT 6

The two test pieces for loading the beam in pure torque.

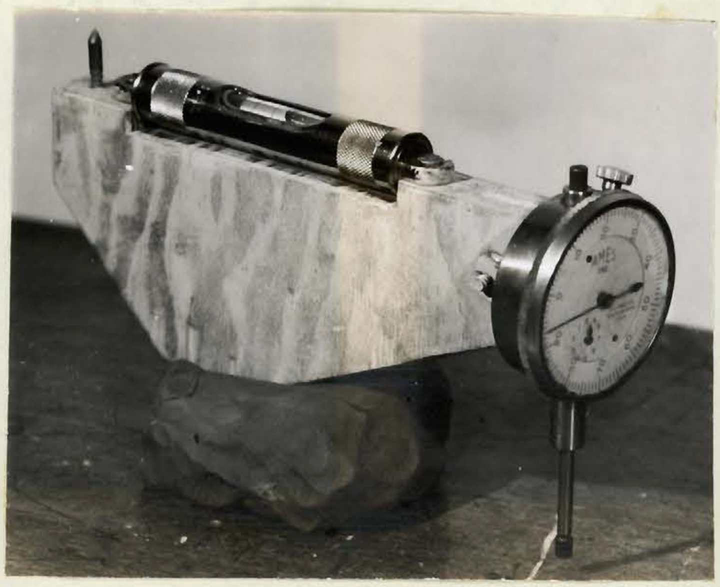


EXHIBIT 7

The "Dialometer". For measuring angle changes.

EXHIBIT 8

An end view of the beam loaded with pure torque.

EXHIBIT 9

A side view of the beam under pure torsion. Note the strain gauge sections.

EXHIBIT 10

A view of the pin and roller bearing end connection.

EXHIBIT 11

A view showing the method of applying the torque. Note jack resting on scale.

BIBLIOGRAPHY

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