Short title of Ph.D. thesis by Roger James BRAITHWAITE

AIR TEMPERATURE AND GLACIER ABLATION

AIR TEMPERATURE AND GLACIER ABLATION -
A PARAMETRIC APPROACH

by

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ABSTRACT

Two aspects of the glacier-climate problem are investigated by linear regression analysis of data from the White Glacier (Axel Heiberg Island, N.W.T.) and surroundings:

1) *the relationship between air temperatures over glaciers and at distant weather stations.*
2) *the relationship between short-term ablation and air temperature.*

Parameters in the two models are found to be reasonably consistent for different samples. Averaged parameters for the models are incorporated into a further model to simulate summer ablation on White Glacier for 1960-72 using summer temperatures at distant weather stations as input data. Results are tested by comparison with observed annual net balance data. Main sources of discrepancy are the effects of accumulation.

Analysis of data from twelve other glaciers suggests that relationships between mass balance and temperature are reasonably consistent for different glaciers and can be interpreted in terms of results from the White Glacier.

RESUME

Deux aspects du problème climatique du glacier sont examinés à l'aide d'analyses de régression faites avec des données du White Glacier (Axel Heiberg Island, NWT) et ses environs:

1) *Comparaison des températures ambiantes mesurées au-dessus du glacier et dans des stations météorologiques éloignées.*
2) *Relations entre l'ablation à court terme et la température ambiante.*

Il a été constaté que les paramètres des deux modèles appliqués à des échantillons différents sont à peu près analogues. Les moyennes des paramètres des deux modèles ont été incorporées dans un modèle ultérieur pour simuler l'ablation estivale du White Glacier de 1960 à 1972 en usant comme données de base les températures mesurées en été par des stations météorologiques éloignées. Les résultats ont été mis à l'épreuve avec les bilans annuels de masse. Les désaccords constatés résultent principalement de l'accumulation.

Une analyse des données de douze autres glaciers semble indiquer que les relations entre le bilan de la masse glaciaire et la température sont assez rapprochées pour les différents glaciers et peuvent être interprétées dans le cadre des résultats du White Glacier.

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PREFACE

i) Acknowledgements

I would like to acknowledge the encouragement and support given by Professor Fritz Müller, my thesis supervisor at McGill University, during the long years that it has taken for me to learn a little about glaciers. In particular, Professor Müller encouraged my participation in summer field work on the Devon Island Ice Cap in 1969 and on Axel Heiberg Island in 1971 in addition to putting a wealth of unpublished, and hard-won, data at my disposal for the purposes of this study. My study has greatly benefitted from the careful analyses of mass balance data from the White Glacier, Axel Heiberg Island, which were carried out by David Terroux, Jakob Weiss and Ludwig Braun under the supervision of Professor Müller. I would like to especially thank my friend and colleague Atsumu Ohmura who collected the greater part of the Axel Heiberg air temperature data that I use in the study in addition to showing me many kindnesses.

The work carried out on Axel Heiberg Island since 1959 by expeditions from McGill University, under the direction of Professor Müller, would not have been possible without the support of many organizations: especially financial support from the National Research Council of Canada and logistic support from the Polar Continental Shelf Project of the Department of Energy, Mines and Resources.

Dr C.M. Keeler (CRREL, Hannover), Dr Bjørn Holmgren (University of Uppsala) and Dr Gunner Østrem (NVE, Oslo) very kindly put unpublished data at my disposal. I received valuable comments and personal communications from Professor Walter Ambach and Dr Michael Kuhn (University of Innsbruck), Dr Herbert Lang (VAW, Zürich), Professor Dieter Steiner and Dr Theo Ginsburg (ETH, Zürich), Dr Olav Liestøl (Norsk Polarinstitutt, Oslo), Professor Valter Schytt (University of Stockholm), Professor Genadi Golubev (Moscow University), Dr W.L. Gutzman (CMC, Dorval) and Mr Simon Ommanney (Glaciology Division, Ottawa). In particular, I would like to acknowledge the useful criticisms of the study made by Professor Svenn Orvig and Professor Phil Langleben (McGill University) and Dr B. Boville (AES, Toronto) on the occasion of my Ph.D. Comprehensive Examination in Grenoble in 1975.

David Day, John Morrison and Bo Curtis cheerfully assisted in the fieldwork on Devon Island in 1969 and Axel Heiberg Island in 1971 respectively. Richard Hartley (ETH, Zurich) kindly provided me with translations from Norwegian into English, and Mrs Ursula Cooper (Zürich) typed the manuscript and attempted to detect my many mistakes of spelling, punctuation and syntax.

I was a resident Ph.D. candidate at McGill University for the academic sessions 1968/69 to 1970/71. During 1968/69 I was supported by a grant from the Arctic Institute of North America for which I am especially indebted to Mr Ken de la Barre, the Montreal Director of the Institute. During the 1969/70 and 1970/71 academic sessions I was in receipt of a Bursary from the National Research Council of Canada for which I shall always be grateful. Since 1971 I have been employed in the Geographical Institute of the Swiss Federal Institute of Technology (ETH) in Zürich as a *wissenschaftlicher Mitarbeiter*. Professor Müller, the director of the institute, always allowed me liberal use of institute facilities for the execution of this study, especially access to computing facilities and libraries.

ii) Publication of Data

Professor Müller has permitted me to use a large amount of unpublished data from Axel Heiberg Island for the purposes of this study. The data were collected as part of a long-term, and still continuing, research programme which will be the basis of further studies by Professor Müller and his collaborators. Accordingly, my use of the data does not constitute "publication", and I have refrained from quoting raw data or from making generalizations from them which are not directly pertinent to the problems that I set myself to study. I do, of course, quote all statistics and results that are necessary for assessing the validity, accuracy and shortcomings of the study.

iii) Symbols, Units and Terminology

For various reasons I have not been able to use a completely consistent set of symbols throughout the text nor have I followed fully the recommendations made by the *Journal of Glaciology* (Anonymous, 1969 & 1969) concerning units and terminology.

Symbols are redefined within the text whenever necessary. In particular, I have consistently used the overbar $\bar{\ }$ to denote the mean value of any quantity with respect to time-averaging, but the period of averaging is different in different contexts. For example, the symbol $\bar{T}$ denotes the time-average of the local air temperature with respect to (i) arbitrary periods of record or (ii) calendar months or (iii) summers of specified length. It is clear from context which is meant. I have avoided the use of the overbar $\bar{\ }$ to denote area-averages so that I denote "mean specific annual balance" by b_n rather than by $\bar{b}_n$ as recommended by Anonymous (1969, p.7).

The use of SI-Units in glaciology is recommended by Anonymous (1968). However, I do make use of energy and mass balance data that have been previously published in c.g.s. units, and I was reluctant to give them a "new" appearance by translating the units into SI equivalents. I have, accordingly, consistently used the Langley (Ly) and centimetre of water equivalent (cm H_2O) respectively for energy and mass balance quantities:

$$1 \text{ Ly} = 41\ 868 \text{ J m}^{-2}$$

$$1 \text{ cm } H_2O = 0.01 \text{ m } H_2O \text{ or } 10 \text{ kg m}^{-2}$$

For convenience in comparing computed specific ablation with observed specific balance quantities in the ablation area, I have expressed the latter in terms of "net ablation" in Chapters 9 & 10. This is consistent with previous usage in the Axel Heiberg literature and merely involves a change of sign so that a negative balance corresponds to a positive net ablation. I do differentiate between "specific" and "mean specific" quantities as referrring to values at a point and to values averaged over the whole glacier respectively, but it should be noted that the upper case letters A, B and C are also used for non-glaciological quantities, e.g. for constants in regression equations. Furthermore, I also use the symbol A to denote short-period specific ablation quantities which can be summed over time to give a specific ablation total denoted by the lower case letter a.

My use of the term "parameter" is consistent with the definitions given in the

Glossary of Meteorology (1959) and in the *International Glossary of Hydrology* (1969). The latter definition is as follows: "Any *variable* considered constant under certain circumstances".

iv) Contribution to Original Knowledge

I consider that the thesis contains several elements that are "contributions to original knowledge":

- *air temperature and ablation which are characteristics of the "local" climate of glaciers are shown to be responses to the "general" climate of the surrounding large-scale atmosphere.*
- *a method of expressing the "general" climate is developed using simple interpolation techniques.*
- *a new interpretation of the relationship between ablation and air temperature is proposed.*
- *numerical values of the various model parameters, evaluated by the use of statistics, are shown to be reasonably repeatable for different situations.*
- *wherever possible, physical interpretation of the statistical models is attempted.*

CHAPTER 1

INTRODUCTION

i) The Glacier-Climate Problem

Glaciers are a significant feature of the natural landscape under present climatic conditions and were even more extensive in the recent geological past. Even small variations in glacier behaviour and their extent could have profound effects upon human activity. The spatial distribution and time variations of glaciers are controlled by climate. The mechanisms are not yet fully understood, nor is the problem clearly defined.

The relationship between climate and glacier advance or retreat is often expressed in the form of a simplified flow diagram, e.g. by Meier (1965), Paterson (1969, p.226) and Müller (1965 and 1972) and in Figure 1.1 of the present work. A flow diagram such as Figure 1.1 expresses implicitly the notion that the glacier-climate system as a whole can be treated as a number of separate systems coupled together. The glacier-climate problem can then be conveniently broken down into two parts (Paterson, 1969, p.226):

1) *How does the large-scale climate control the glacier's mass balance ? This is primarily a meteorological/climatological problem.*
2) *How does the glacier respond to a change in mass balance ? This is primarily a problem of glacier dynamics.*

The present work only concerns itself with the first of these questions. The processes involved are the "... least understood and hardest to analyse of the several processes in the chain that links glaciers to climate" according to Meier (1965, p.796). In the following chapters hypotheses are advanced and tested in an attempt to develop a useful quantitative approach to the problem. In particular, the possibility of computing glacier mass balance/ablation from data at distant weather stations is investigated. This is useful because observed meteorological data are seldom available from glaciers themselves in parallel to mass-balance measurements. The problem of "explaining" the large-scale climate is not attempted. In principle, the glacier-climate system could be described by the usual hydrodynamic/thermodynamic equations of meteorology, but in practice the complexity of the system and the difficulty in specifying

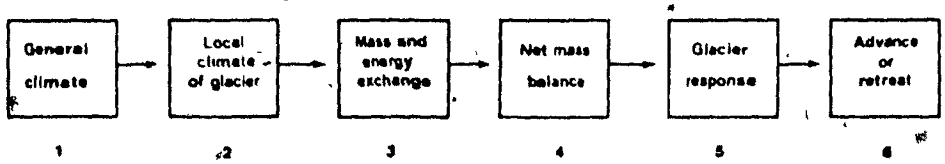


Fig 1.1 : Schematic Representation of the Glacier-Climate Problem. After Meier (1965), Müller (1965 & 1972) and Paterson (1969)

the state of the system necessitate an "empirical" approach. A heavy reliance is placed upon statistics for this approach. It is hoped that the scientific basis of the approach is adequately demonstrated.

The data used or analysed in the present study come mainly from Axel Heiberg Island, N.W.T., Canada, and nearby permanent weather stations. Glaciers in other areas, Arctic and Alpine, are briefly discussed for comparison.

ii) Brief Literature Review

There are no comprehensive textbooks or handbooks of glaciology in English. In German there is a series of books by Heim (1885), Hess (1904), Drygalski and Machatschek (1942), Klebelsberg (1948-49) and by Wilhelm (1975) which give a good overview of the historical development of glaciology. The most comprehensive textbook is by Lliboutry (1964-65) in French. An elementary introduction to glaciology in English is by Sharp (1960) whilst Paterson (1969) has written an admirable book on the physics of glaciers. Ahlmann (1948) and Wallén (1948) are of great historical interest for the study of the glacier-climate problem, and Orvig (1954) gives an extensive bibliography of early work in the field. Hoinkes (1964) and Meier (1965) present concise reviews of glacier-meteorology.

The relationship between glacier variations and climatic fluctuations has long been a subject of speculation, but monitoring of glacier mass balance on a systematic basis is relatively recent. Some of the longer records are from Aletschgletscher (Switzerland, starting in 1922/23), Storglaciären (Sweden, 1945/46), Glacier de Sarennes (France, 1948/49), Storbreen (Norway, 1948/49) and Hintereisferner (Austria, 1952/53). There are several long records from the Canadian Arctic: White Glacier and Baby Glacier (Axel Heiberg Island, 1959/60), Meighen Island Ice Cap (1959/60) and the Devon Island Ice Cap (1960/61). Data from these and other glaciers are published in summary form by the Permanent Service on the Fluctuations of Glaciers of the IUGG-FAGS/ICSU: for examples see Kasser (1967 and 1973).

The ablation process is probably better understood than the accumulation process. The relation between ablation and meteorological elements has often been studied by measurement of the various terms in the energy balance equation for short periods at a limited number of sites (Paterson, 1969, Chapter 4). Early examples of this approach in the Canadian Arctic (Baffin Island) are by Orvig

(1951 and 1954) whilst similar studies on Axel Heiberg Island and Devon Island have been reported by Andrews (1964), Havens (1964), Havens *et al* (1965), Müller and Keeler (1969), Keeler (1964) and Holmgren (1971 a-f). The applicability of the results is sometimes questioned: "energy balance studies have so far yielded detailed information only about the particular place where they were carried out" (Paterson, 1969, p.62). The method also involves theoretical problems (for example, the specification of the turbulent fluxes) and instrumental problems (especially measurement of long-wave radiation).

A new approach to the surface energy exchange by solution of a system of differential equations describing the boundary layer over glaciers is briefly reported by Ohmura (1972) and Müller *et al* (1973). Such methods are discussed by Kraus (1974). It is unlikely, however, that such approaches can ever be applied to the study of long series of mass balance data because of (a) lack of sufficiently detailed input data and (b) computing time. An alternative approach is to parameterize each term in the energy balance equation empirically in terms of routine meteorological data as in the Synoptic Energy Balance method of Vowinckel and Orvig (1968 and 1972). Taylor-Alt (1975) applied such an approach on the Meighen Ice Cap to study ablation over six summer seasons.

Actually long-term records of even simple meteorological elements on or very close to glaciers are rare. Analysis of long (10^1 year) series of mass balance data is often done (a) in terms of large-scale weather patterns or "Grosswetterlagen" (e.g. Hoinkes, 1968) or (b) in terms of data (usually temperature and precipitation) from distant (10^1 km) weather stations (e.g. Hoinkes and Steinacker, 1975 and Martin, 1974). Such approaches often involve statistical analyses. Paterson (1969, p.228) states that a statistical approach "may have some practical value" but that "a correlation established for one glacier will not necessarily hold in another area, or even on another glacier in the same area. Moreover, such analyses tell nothing about the physical factors which control accumulation and ablation". Kraus (1974, p.146-7) is also sceptical about such methods as is LaChapelle (1965) in his review of Marcus (1964). One of the objectives of the present work is to attempt justification of the "statistical approach".

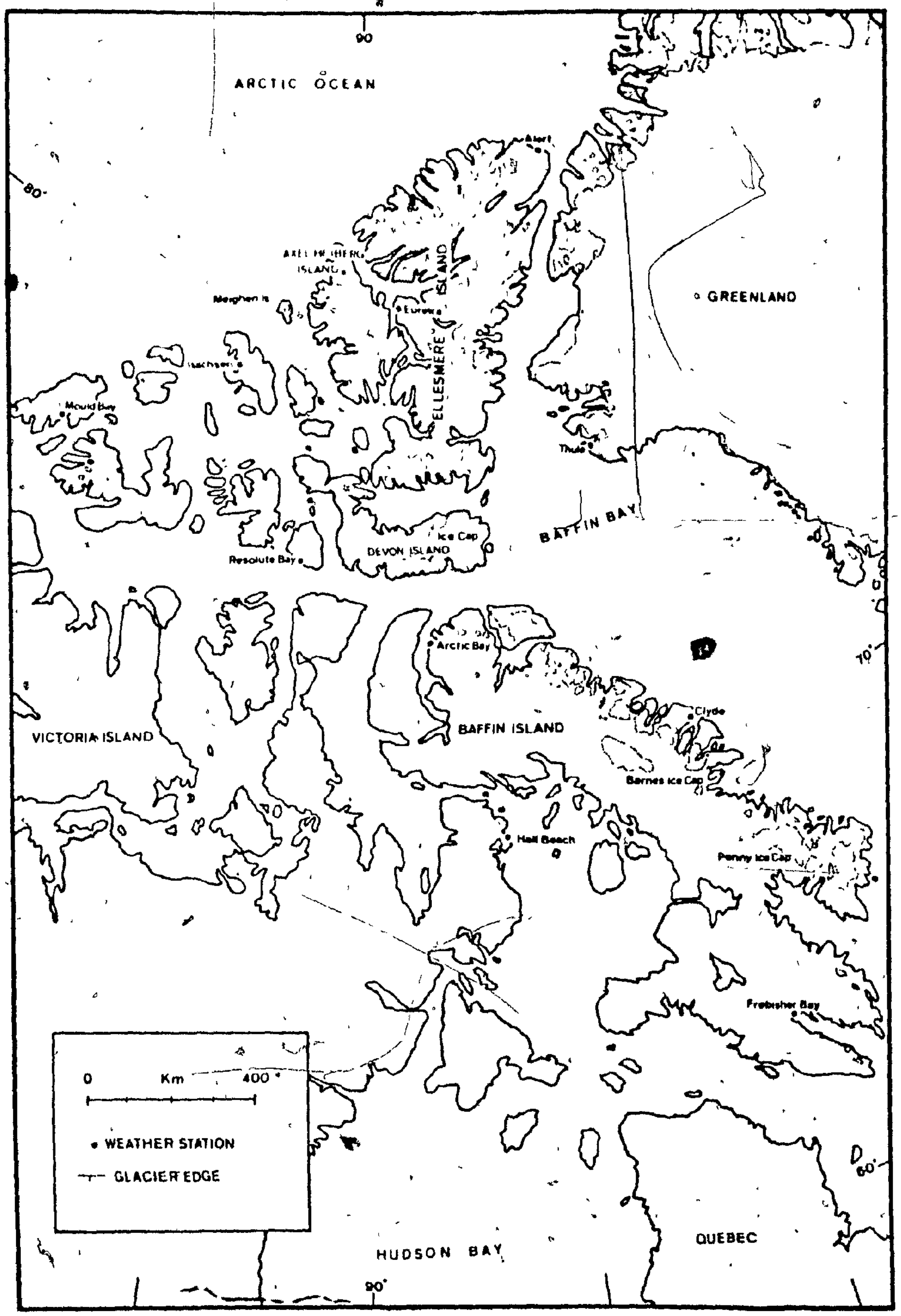


Fig 1.2: Map of Canadian Arctic Showing Locations of Axel Heiberg Island, Devon Island and Important Weather Stations

iii) Area of Study and Sources of Data

The field data used in the present study come mainly from White Glacier and its surroundings on Axel Heiberg, N.W.T., Canada. Some data from the Sverdrup Glacier and Ice Cap on Devon Island, N.W.T., are also analysed. Meteorological data from permanent weather stations in the Queen Elizabeth Islands, especially Eureka and Isachsen, are used in the analysis. Locations of the various places of interest are illustrated in Figure 1.2.

Finally, the results of the study are compared with results from other glaciers: 3 glaciers in the Canadian Arctic, 1 in Northern Sweden, 2 in the French Alps, 2 in the Austrian Alps and 4 in the Swiss Alps.

Much of the Axel Heiberg data used in the study is unpublished. Permission to use this data was kindly given by Professor Fritz Müller, leader of the McGill University Axel Heiberg Island Research Project. The efforts of many members of the research project must be gratefully acknowledged, particularly those of Atsumu Ohmura, Jakob Weiss, David Terroux and Ludwig Braun.

Unpublished meteorological data from the Sverdrup Glacier, Devon Island, and from the Devon Island Ice Cap were kindly made available by Dr C.M. Keeler and Dr Björn Holmgren respectively.

Axel Heiberg Island:

The expedition area on Axel Heiberg Island (see maps in Figures 1.3 and 1.4) has been visited every summer since 1959 by expeditions from McGill University. The main glacier under study is White Glacier - a medium sized sub-polar valley glacier approximately 14.5 km long, 1 km wide and extending from 75 m to about 1400 m a.s.l. (Adams, 1966, p. 1). A continuous record of annual net balance is available from 1959/60 to the present. However, the series are not completely homogeneous due to loss of stakes, changes of stake location and stake density etc. The climatic equilibrium line altitude (ELA) is approximately 900 m a.s.l. but varies widely from year to year (Müller, 1966) with a low of ca 400 m (1963/64) and a high of ca 1300 m (1961/62).

Adams (1966) studied ablation and run-off on White Glacier whilst Andrews (1964), Müller and Roskin-Sharlin (1967) and Havens *et al* (1965) present results

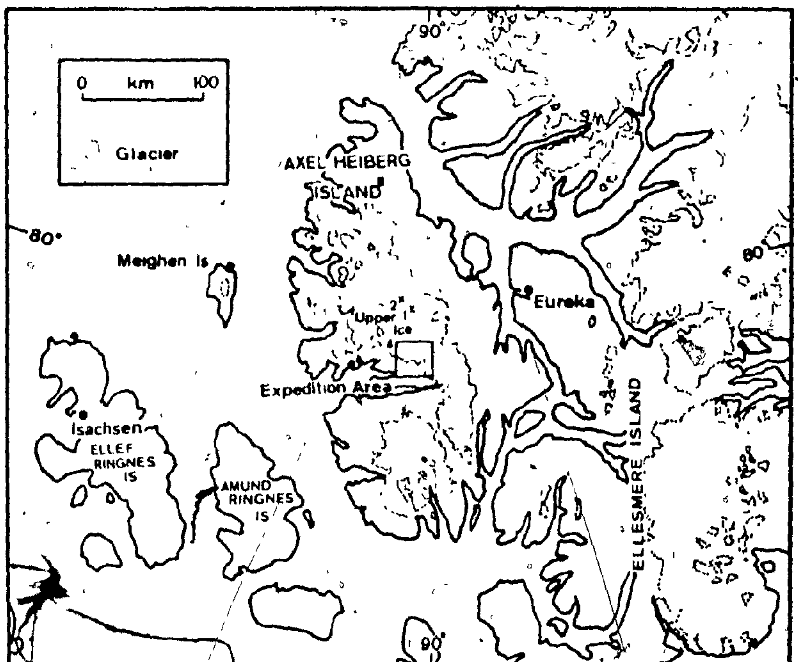


Fig 1.3: <u>The Expedition Area and Upper Ice Stations on Axel Heiberg Island in Relation to the Nearby Weather Stations at Eureka and Isachsen</u>

of summer heat balance studies carried out at "Lower Ice" at 210 m a.s.l. near the snout of White Glacier in the summer of 1960, 1961 and 1962 respectively. All the heat balance studies showed that net radiation was, on average, the major heat source for ablation.

Already during the period 1960-62 an emphasis was placed upon maintenance of a meso-scale network (1 km scale) of simple weather stations, mainly thermohygrographs/thermographs, to supplement the surface weather observations from Base Camp. Several automatic weather stations have also been incorporated into the network (Müller, 1969 and Müller & Schroff, 1976). Table 1.1 and Figure 1.4 show the station network as it existed in the period 1969-72. Despite great efforts the "summer" temperature record for the 13-year period 1960-72 is not complete: considering the glaciologically important months of June-August, the Base Camp record is 72% complete, Lower Ice 46% and Moraine Camp 44%. This is actually quite good compared to the situation for most glaciers, but there are no winter precipitation records at all.

Field data from Axel Heiberg Island which are used in the present study are as follows:

Daily Mean Temperature (Chapter 4): six summer temperature records comprising 564 days in total from three stations for each of the two summers 1960 and 1961. All data are published except for the 1960 Base Camp record, see Andrews (1964), Havens (1964) and Müller & Roskin-Sharlin (1967).

Daily Mean Temperature (Chapter 5): thirty-nine summer temperature records comprising 3427 days in total from nine to eleven stations for the four summers 1969-72. None of the data are published. The 1969-70 and 1972 data were collected by Atsumu Ohmura whilst Roger Braithwaite collected the 1971 data.

Short-Period Ablation and Energy Balance Data (Chapters 7 and 8): 16 periods (irregular duration) between 8 July and 19 August 1960 from Andrews (1964), 63 days between 12 June and 18 August 1961 from Müller and Keeler (1969) and 11 periods (irregular duration) between 16 July and 31 July 1962 from Havens *et al* (1965). In all three cases the measurements were made at the Lower Ice station at 210 m a.s.l. on White Glacier.

Annual Specific Net Balance (Chapters 9 and 10): 13 years 1959/60 to 1971/72 at three altitudes on White Glacier: at or near 210 m a.s.l. (Lower Ice), 370 m a.s.l. (Anniversary) and 870 m a.s.l. (Moraine). Summary data for 1959/60 to

Station	Altitude m a.s.l.	Location
Fjord	10	tundra, Expedition Fjord
Outwash	55	Outwash Plain, White Glacier
Valley	90	tundra, Expedition River
Base Camp*	190	tundra
Lower Ice	200	White Glacier
Anniversary	370	White Glacier
Phantom	450	tundra, Thompson Glacier
Gordon's	600	tundra
Ermine	800	rock tundra
Moraine	870	White Glacier
Baby	1050	Baby Glacier

* full surface weather records

Table 1.1: Altitudes and Locations of Weather Stations in Expedition Area, Axel Heiberg Island, in summers 1969-72.

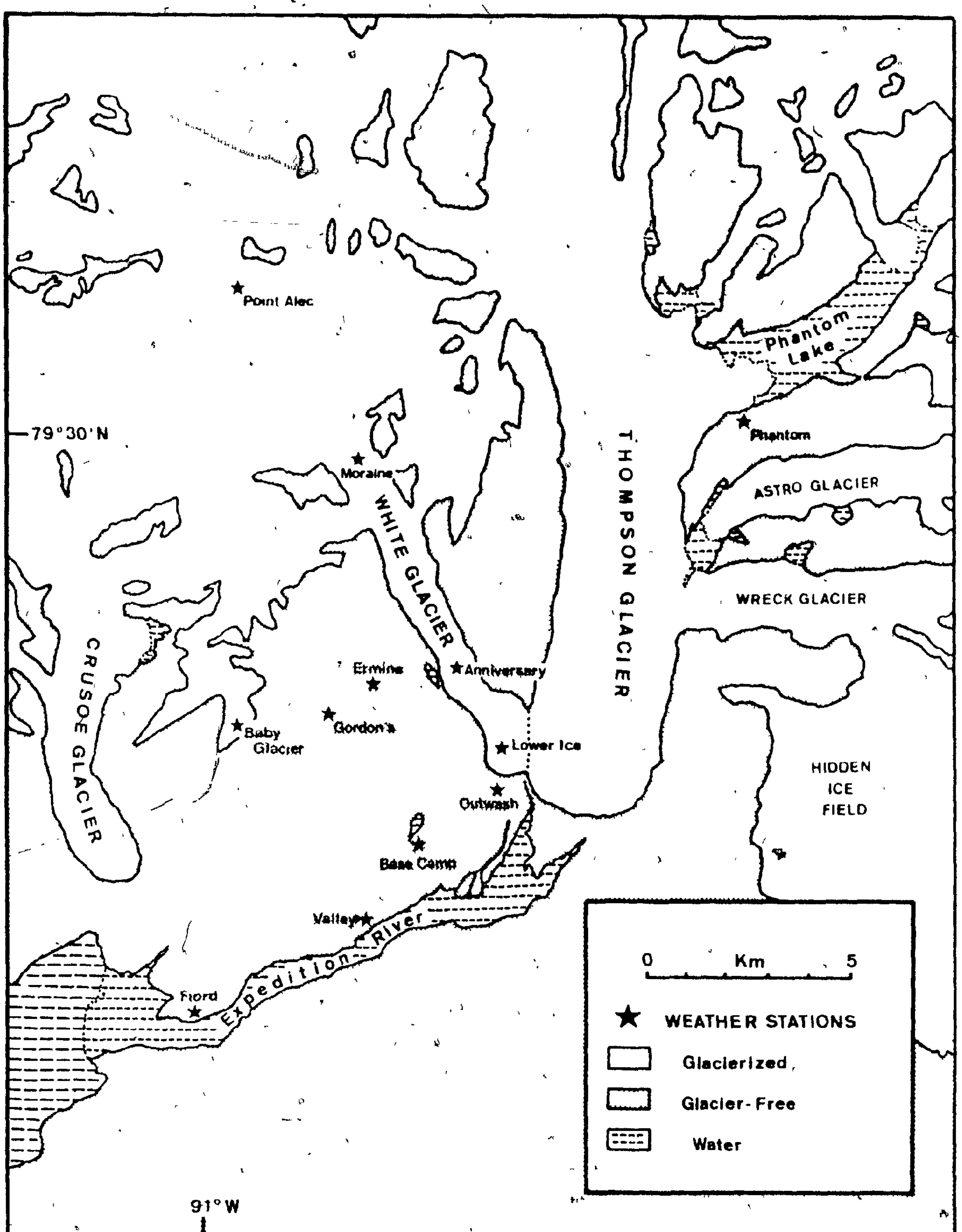


Fig 1.4: Part of Expedition Area, Axel Heiberg Island, Showing the Main Glaciers and Locations of Weather Stations During 1969-72

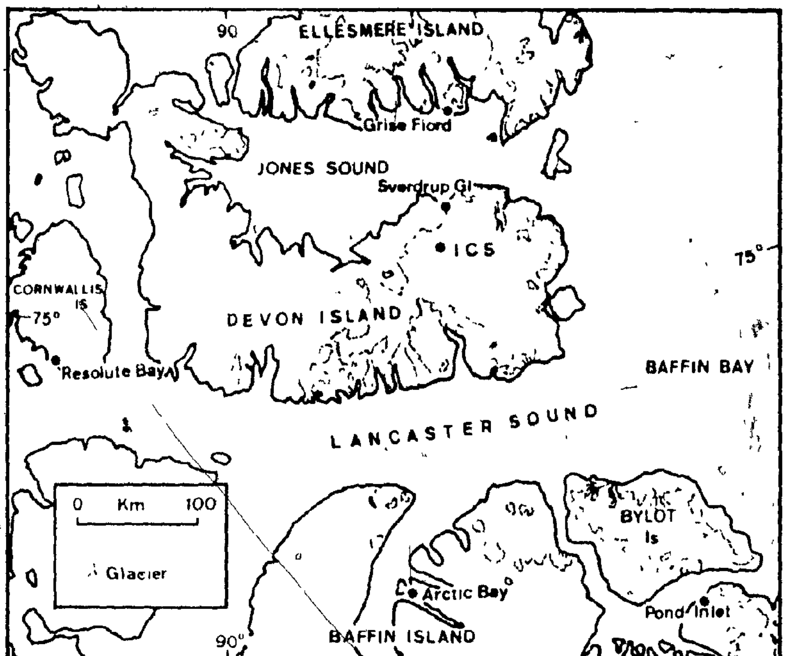


Fig 1.5: The Ice Cap Station (I.C.S.) and Sverdrup Glacier on Devon Island in Relation to the Weather Station at Resolute Bay

1961/62 are given by Muller (1966), and 1969/70 to 1971/72 data are given by Braun (1976): the data are otherwise unpublished. Data analysis was by David Terroux, Jakob Weiss and Ludwig Braun under the supervision of Professor Fritz Muller.

Devon Island:

The Devon Island Ice Cap (see Figure 1.5) has been ~~visited by~~ expeditions from the Arctic Institute of North America since 1961 and from Polar Continental Shelf Project (PCSP) since 1971. Mass balance data from various parts of the ice cap are reported by Koerner (1966 and 1970) and in *ICE* (1972-76). Meteorological observations were made at the Ice Cap Station (ICS) at 1320 m a.s.l. during the 1961-63 summers (Holmgren, 1971 A-F) and in 1969 (Braithwaite, 1970). Holmgren (1971 A-F) made a comprehensive study of the summer energy exchange on the ice cap whilst Keeler (1964) made micro-meteorological and run-off measurements on the Sverdrup Glacier at 300 m a.s.l. in summer 1963.

Field data from Devon Island used in the present study are as follows:
Daily Mean Temperature (Chapter 4): three temperature records comprising 232 days in total from the Ice Cap Station (ICS) in 1963 and 1969 and from Sverdrup Glacier in 1963. The unpublished data for 1963 were kindly made available by Dr Björn Holmgren and Dr C.M. Keeler.
Short-Period Ablation and Energy Balance Data (Chapters 7 and 8): 33 days between 9 July and 10 August 1963 from Keeler (1964). The measurements were made at 300 m a.s.l. on the Sverdrup Glacier.

Distant Weather Stations:

Regular meteorological observations have only been made in the Queen Elizabeth Islands since the 1940s (Rae, 1955). The climate of the Arctic has been discussed by Dorsey (1951), Hare (1951), Hare and Orvig (1958) and by Diem (1967) amongst others. Regular tabulations of meteorological and climatological data from Canadian Arctic weather stations are published by the Atmospheric Environment Service (formerly Meteorological Service of Canada) in various publications: "*Arctic Summary*" (up to 1971), "*Monthly Bulletin - Canadian Upper Air Data*" and "*Monthly Record - Meteorological Observations in Canada*". Monthly statistics for surface temperature, atmospheric pressure and precipitation for some Canadian Arctic stations are presented in "*World Weather Records 1951-60, Vol. 1*".

iv) The Hypotheses to be Tested

The hypotheses of the present study are:

1) *The atmosphere over the glacier acts as a spatially distributed system on the 1 to 10 km scale. The system is nearly deterministic. The inputs are provided by the large-scale atmosphere, and the outputs are the spatial distributions of local temperature and precipitation on the glacier (Chapters 3 to 6).*
2) *The inputs from the large-scale atmosphere can be reasonably accurately calculated by interpolation of temperature and precipitation data at distant (10^2 km scale) weather stations (Chapter 2).*
3) *Air temperature and precipitation at a site on the glacier are the main controlling factors for specific mass balance: summer temperature controls ablation, and winter precipitation controls accumulation, and summer precipitation inhibits ablation (Chapters 7 and 8).*

For the hypotheses to be useful the various models should be reasonably repeatable: a model developed for one situation should be valid for other situations. If this is true and if (1) to (3) are true then it should be possible to compute glacier mass balance series using data from distant weather stations. This further hypothesis is examined in Chapters 9 to 11.

v) Methodology of the Present Study

The hypotheses (1) to (3) are developed in the form of simple linear equations relating input variables to output variables. Parameters in the equations are evaluated for various situations by regression analysis (ordinary least-squares). Parameters, which are computed for one set of situations, are used to compute outputs (synthetic series) for other situations, which were not used for the computation of the parameters. Assessment of the validity of the approach is made by comparing these computed outputs to the corresponding observed outputs. If the discrepancies between these series are large the approach is not valid, and the models are not repeatable.

The approach may be termed a "grey box" or "parametric" approach and is a typical "systems" approach (Amorocho and Hart, 1964). The approach might also be termed a "statistical" approach because it uses statistical methods.

The hypothetical nature, initially, of the various models which are developed must be emphasised. However, they are tested for accord with reality as far as possible. The use of hypotheses and their testing in the present study attempts to follow the lines laid down by Jevons (1877, Chapter XXIII) and by Popper (1959).

CHAPTER 2

INTERPOLATION OF TEMPERATURE AND PRECIPITATION DATA FROM ARCTIC WEATHER STATIONS

i) Introduction

Suppose that temperature and precipitation data are available from a short-term station on or near a glacier at point (x,y,z). These data will be denoted by T(x,y,z,t) and P(x,y,z,t). They are influenced by the "local climate" of the glacier. It is assumed that the local climate is forced by the "general climate" which: "refers to average conditions over a large area, as indicated, for example, by data from the network of standard weather stations" (Paterson, 1969, p. 227). The problem is to establish some quantitative and objective way of expressing the temperature and precipitation characteristic of the general climate. This might be done by extraction of data from published weather maps, but the procedure would be cumbersome and inaccurate. For the purposes of the present study it is proposed to express the temperature and precipitation characterizing the general climate as interpolations of observed data at distant (10^2 km scale) stations around the glacier. Interpolations will be valid for the point (x,y,z), and interpolated values will be denoted by $T_{IN}(x,y,z,t)$ and $P_{IN}(x,y,z,t)$. That such interpolations are useful and meaningful is only, at this stage, a hypothesis (hypothesis 2 in Chapter 1(iv)). The testing of the hypothesis will be described in the following sections of the present chapter.

ii) Simple Interpolation Methods

It is desired to interpolate temperature and precipitation at a point (x_0,y_0,z_0). There are N permanent weather stations around the point of interest with coordinates (x_i,y_i) with i taking values of 1 to N. The problem of interpolating surface meteorological elements will be considered first, i.e. $z_0 = 0$. A linear interpolation scheme for some meteorological element F can be written:

$$F_{IN}(x_0,y_0,z_0,t) = \sum_{i=1}^{i=N} W(x_0,x_i,y_0,y_i)\, F(x_i,y_i,z_0,t) \qquad (2.1)$$

where $W(x_0,x_i,y_0,y_i)$ is a weighting factor.

According to Gandin (1965) all interpolation schemes used in meteorology are linear in the sense of Equation (2.1) but differ in the way in which the weighting factors are specified (in meteorology the interpolation of meteorological fields is usually termed "objective analysis"). He identifies three main kinds of scheme: polynomial (e.g. Panofsky, 1949), statistical-dynamic (e.g. Kruger, 1965) and optimum interpolation (e.g. Gandin, 1965). In general, development in the field has been very rapid, and operational objective analysis schemes have become very complex in order to take account of factors which are probably not relevant to the present problem.

In the present study three different methods were examined:

a) Distance Weighting - The weighting factors $W(x_o,x_i,y_o,y_i)$ are assumed to be inversely proportional to the scalar distance (d_{oi}) between (x_o,y_o) and (x_i,y_i) and to be "normalized" such that their sum is unity. Hence:

$$W(x_o,x_i,y_o,y_i) = \left(1 \Big/ \sum_{j=1}^{j=N} \frac{1}{d_{oj}}\right) \frac{1}{d_{oi}} \quad (2.2)$$

The number of weather stations used (N) can be prescribed arbitrarily or else a "nearest neighbour" approach can be used. This method is the simplest.

b) Polynomial Trend Surface - Spatial variations of the meteorological fields are assumed to be expressed by truncated power series of the spatial coordinates. The arbitrary coefficients in the polynomial expression are evaluated by least-squares fitting of station data to the model. This procedure has to be carried out separately for each time t and is very laborious. There is also the problem of physical interpretation of the various coefficients.

c) Optimum Interpolation - Equation (2.1) is assumed to be approximately true, and the weighting factors $W(x_o,x_i,y_o,y_i)$ are evaluated by least-squares so as to minimize the time-variance of the error involved in the approximation.

Methods (a) and (b) are formal methods in that they involve *a priori* assumptions whilst method (c) involves actual computed statistics for the fields under consideration. A simplified treatment of method (c) follows based upon Gandin (1965, Chapter 3).

For ease of notation $F(x_i,y_i,z_o,t)$ will be denoted by $F(i,t)$, and the deviation from the time-average at (x_i, y_i) will be denoted by $F(i,t)'$. The deviation of $F_{IN}(x_o, y_o, z_o, t)$ from its time-average can be written as:

$$F_{IN}(o,t)' = \sum_{i=0}^{i=N} W_{oi} F(i,t)' + e(o,t) \qquad (2.3)$$

where $e(o,t)$ is a random error and W_{oi} are the unknown weighting factors.

The weighting factors are chosen to minimize the variance of the random error. The condition for this, under simple least-squares assumptions, is:

$$R_{ok} = \sum_{i=1}^{i=N} R_{ik} W_{io} \qquad (k = 1, N) \qquad (2.4)$$

where R_{ik} is the correlation coefficient for deviations at the i th and k th stations. The standard deviations at the different stations are assumed constant. The standard deviation of the random error, after least-squares minimization, is E. This will be termed the Root Mean Square or RMS Interpolation Error. It is often expressed as a percentage of the standard deviation of the deviations $F(i,t)'$ and is denoted by η. The expression for η is:

$$\eta^2 = 1 - \sum_{i=1}^{i=N} R_{oi} W_{oi} \qquad (2.5)$$

η^2 is a measure of the "proportion of unexplained variance" associated with the interpolation model.

Equations (2.4) and (2.5) are analogous to expressions appearing in the multiple regression algorithm except that the correlation coefficients R_{ok} cannot be computed directly (because there are no data at the 0 th point). R_{ok} can however be estimated from the autocorrelation function. This, in turn, is usually estimated by fitting computed correlation coefficients R_{ik} to some function of the scalar distance d_{ik} between i th and k th stations.

The foregoing is valid for "optimum interpolation of exact values" according to Gandin (1965, p.63). He extends the method to take account of observation errors in the data (p.78) and to the problem of interpolation of absolute values of F rather than deviations F' (p.86).

For accurate interpolation it is important that η^2 should be small. As the correlation coefficients R_{ik} decay with increasing d_{ik} it would be desirable

Station		RE	TH	EU	IS	CL	HB	AL	MB	FB
Resolute	RE	0	749	628	502	1000	815	1123	699	1603
Thule	TH		0	547	862	676	954	679	1322	1422
Eureka	EU			0	384	1172	1257	486	871	1905
Isachsen	IS				0	1386	1310	851	474	2083
Clyde	CL					0	527	1351	1741	746
Hall Bch	HB						0	1613	1517	798
Alert	AL							0	1359	2097
Mould By	MB								0	2371
Frobisher	FB									0

Table 2.1: Interstation Distance Matrix for Upper Air Stations in the Eastern Arctic Region (all distances in kilometres)

Station:	Axel Heiberg Is.	Devon Island
Eureka	113 km	510 km
Isachsen	280 km	628 km
Resolute Bay	536 km	353 km
Alert	600 km	897 km
Thule	600 km	399 km
Mould Bay	760 km	991 km
Hall Beach	1120 km	747 km
Clyde	1186 km	731 km
Frobisher Bay	1908 km	1941 km

Table 2.2: Distances of Upper Air Stations in the Eastern Arctic Region from the Axel Heiberg Island Expedition Area and from the Devon Island Ice Cap Station (I.C.S.)

for the weather stations to be as close together as possible. For a quantitative examination of this problem it is necessary to examine the autocorrelation function of the various fields. This will be done in the following section for temperature and precipitation fields in the Canadian Arctic.

iii) Interpolation of Monthly Mean Temperature and Precipitation Totals in the Arctic

Data from eight weather stations were used in the study: Alert, Eureka, Isachsen, Resolute Bay, Clyde, Thule, Mould Bay and Arctic Bay (see map, Figure 1.2). Distances in kilometres between the stations are given in Table 2.1. Data for monthly mean surface temperature and monthly precipitation total were extracted for each month (January 1951 to December 1960) from *"World Weather Records 1951-60, Volume 1"* (Thule data - Volume 6). The data were divided into four samples: "Summer" (June-August) and "Winter" (September-May) samples for temperature and precipitation respectively. Statistics (mean and standard deviation for each station and correlation coefficients between data at different stations) were computed for each sample using the raw data. The computation was repeated after "cleaning" the data to remove seasonal trend. For temperature this involved expressing the monthly mean for each month as a deviation from the 1951-60 average (10-year norm) for that month whilst precipitation totals were expressed as the ratio of the deviation from the norm to the norm itself. It was found that statistics for the raw data were unduly "forced" by seasonal effects (particularly marked in the case of winter temperature), and further analysis was only carried out for cleaned data.

Correlation coefficients are plotted against inter-station distances in Figures 2.1 to 2.4 together with least-squares estimates of the autocorrelation functions (assuming exponential decay). In the case of temperature, the correlation coefficients are reasonably high for close stations and decay with increasing inter-station distance which indicates a high degree of spatial autocorrelation for temperature fields in the Arctic (at monthly mean level). By contrast, the precipitation fields do not appear to be strongly autocorrelated.

The consequences of the above findings for the problem of interpolation can be demonstrated by a simple example. It is proposed to interpolate temperature

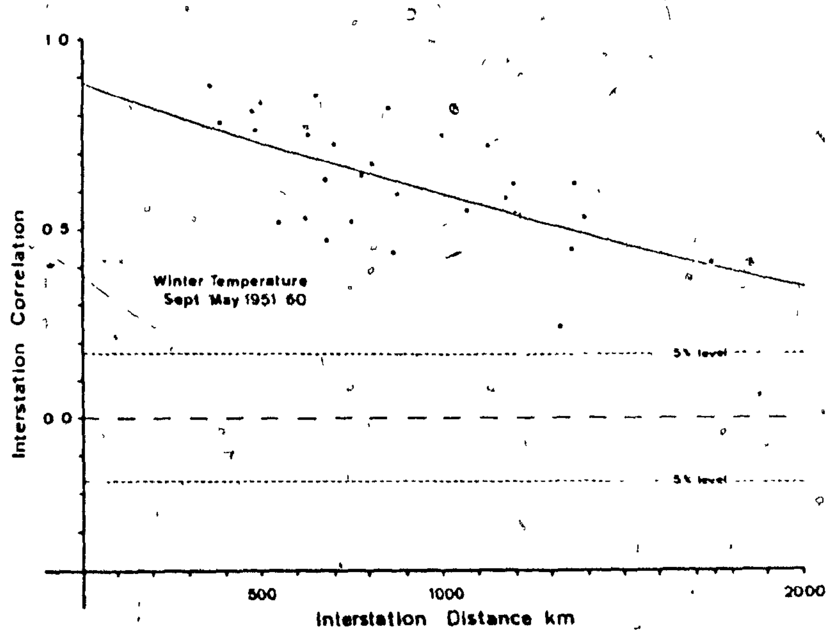


Fig 2.1: Correlation Coefficients Versus Interstation Distances for Winter (September-May) Temperatures at Arctic Weather Stations . Data are Deviations from Monthly Norms for 1951-60

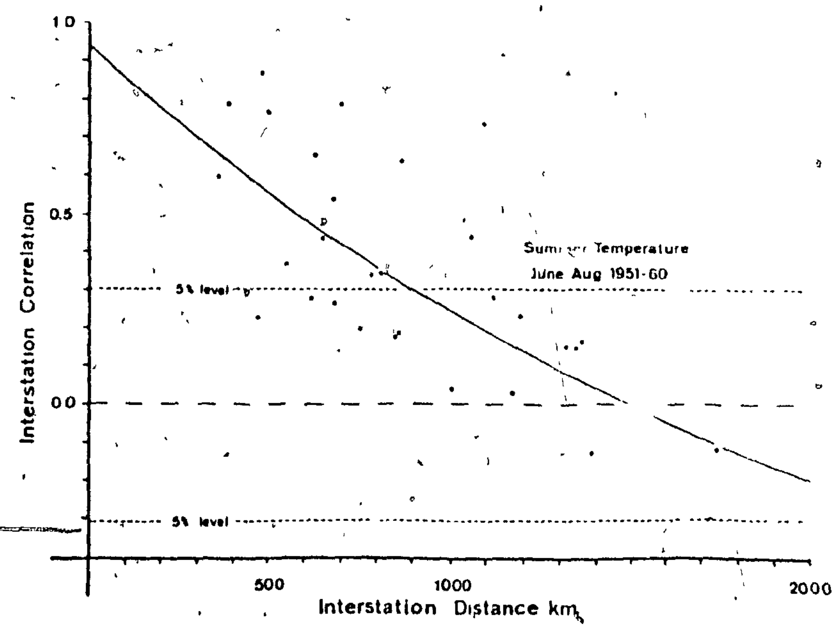


Fig 2.2: Correlation Coefficients Versus Interstation Distances for Summer (June-August) Temperatures at Arctic Weather Stations. Data are Deviations from Monthly Norms for 1951-60

and precipitation at Axel Heiberg Island Base Camp using data from the two closest weather stations (see Table 2.2) which are Eureka (113 km) and Isachsen (280 km). Taking Eureka as Station 1, Isachsen as Station 2 and Axel Heiberg Base Camp as Station 0, the values of R_{01}, R_{02} and R_{12} can be computed from the distances d_{01}, d_{02} and d_{12} by using the appropriate autocorrelation functions (Figures 2.1 to 2.4). Solution of the equations (2.4) and (2.5) yielded the following results:

	W_{01}	W_{02}	η^2
Summer Temperature	0.67	0.28	23 %
Winter Temperature	0.57	0.36	24 %
Summer Precipitation	0.19	0.14	94 %
Winter Precipitation	0.15	0.12	96 %

The relative errors of 23 % and 24 % of variance for summer and winter temperatures correspond to absolute errors in the interpolated monthly mean of $\mp 0.7^{\circ}C$ and $\mp 1.3^{\circ}C$ respectively (standard deviations $1.4^{\circ}C$ and $2.7^{\circ}C$). The weighting factors for temperature, if normalized to a sum of unity, are quite close to those for distance weighting method (a) which are 0.71 and 0.29. The relative error for interpolation of precipitation is hopelessly large, and interpolated precipitation would be useless.

The reason for the large error for interpolation of precipitation is, in the first place, due to the low autocorrelation of the precipitation data on the 10^2 km scale. This may be due to excessive measurement errors in the data (especially for snowfall measurement), due to different meso-scale effects at the various stations (exposure and topography) or due to the fact that the significant scale of autocorrelation of precipitation is actually at a smaller scale (e.g. 10^1 km scale). Probably all three effects are operating.

The magnitude of the interpolation error does decrease with the number (N) of stations used in the interpolation, but a law of "diminishing returns" seems to be operative. For example, for summer temperature η^2 is 100 % for N = 0 (no interpolation at all), 28 % with N = 1 (Eureka), 23 % with N = 2 (Eureka and Isachsen), 22 % with N = 3 (Eureka, Isachsen and Resolute) and 22 % with N = 4 (Eureka, Isachsen, Resolute and Alert). Details of the computed values are given below:

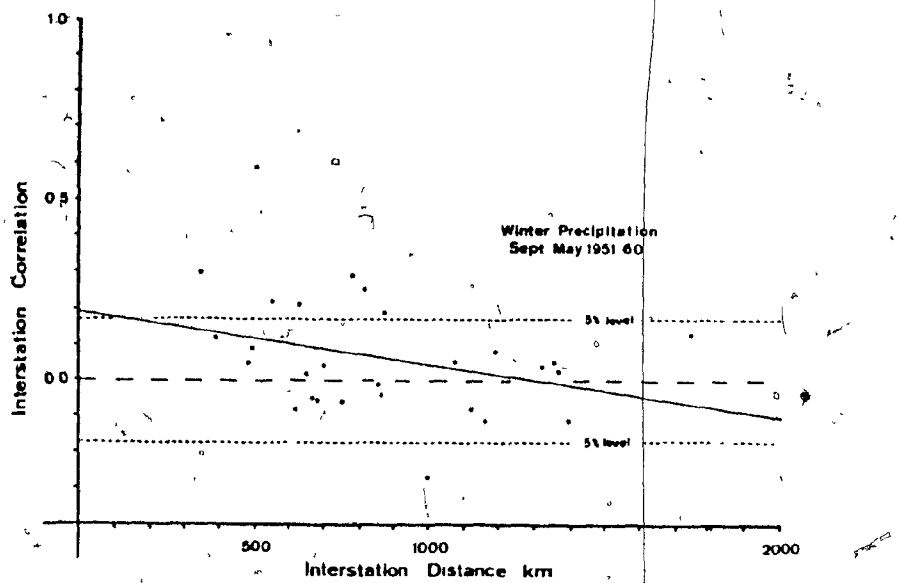


Fig 2.3: Correlation Coefficients Versus Interstation Distances for Winter (September-May) Precipitation at Arctic Weather Stations. Data are % Deviations from Monthly Norms for 1951-60

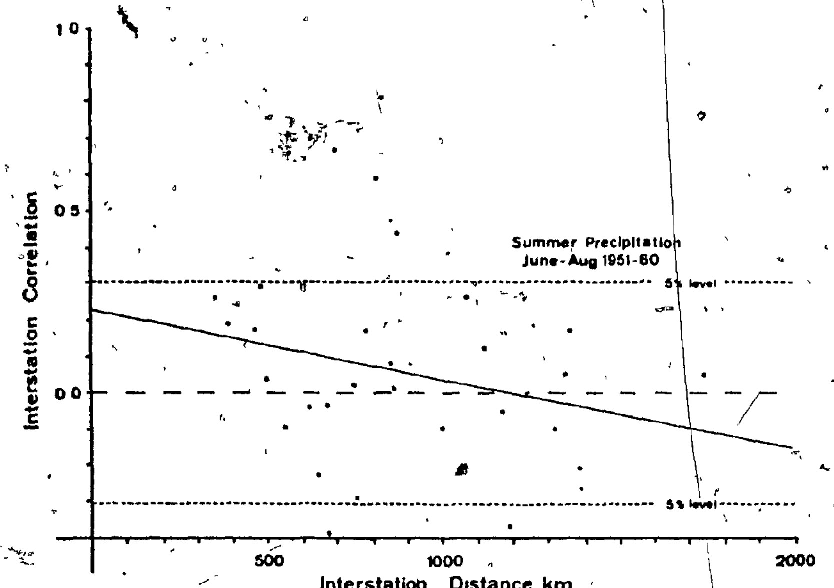


Fig 2.4: Correlation Coefficients Versus Interstation Distances for Summer (June-August) Precipitation at Arctic Weather Stations. Data are % Deviations from Monthly Norms for 1951-60

N	W_{01}	W_{02}	W_{03}	W_{04}	η^2
0	-	-	-	-	100 %
1	0.85	-	-	-	28 %
2	0.67	0.28	-	-	23 %
3	0.65	0.24	0.11	-	22 %
4	0.63	0.24	0.10	0.04	22 %

It can be readily seen that the weighting factors tend to get smoother, and their sum tends to unity with increase in N. Further increase in N will not decrease η^2 much but may ultimately increase it again due to ill-conditioning of the N by N system of equations that has to be solved to get solutions for $W_{01} \ldots W_{0N}$. For interpolation purposes it would be sufficient to use just data from Eureka and Isachsen for the Axel Heiberg (and Meighen Island) situation.

It is difficult to estimate the variance of the observation error in the monthly mean temperature, but it would almost certainly be less than 5 % of total variance. This value was adopted for computations using Gandin's more refined method which takes account of errors in the data (Gandin, 1965, p.78), but the results were only very slightly different from those shown above.

The discussion of interpolation of monthly mean temperature has been in terms of surface data. However, most glacier situations, for which interpolated values of temperature are required, are not at the surface with $z_0 = 0$ so that interpolation of upper air data is required. Some examples will be discussed in the following section.

iv) Interpolation of Daily Upper Air Temperature

For glacier-meteorological purposes it is desirable to interpolate temperatures at some given height z_0 (altitude of a glacier weather station). It is also desirable to interpolate daily values, rather than monthly values, and absolute values, rather than deviations from the norm. This can be done in a similar way to the previous case except that it is necessary to introduce a new condition, i.e. that the sum of the weighting factors must be unity (this was not required in the previous section although weighting factors for temperature were found to have sums close to unity). Upper air data for stations in the

Canadian Arctic are usually published with respect to standard isobaric levels (e.g. 1000 mb, 850 mb etc) with geopotential heights of z_1, z_2 etc. In the present study simple linear interpolation between suitable levels to compute temperature at altitude z_0 at each station was carried out before the horizontal interpolations.

Daily upper air data for four arbitrary summer periods (actually coinciding with some of the glacier weather station records which will be analysed later) were used for study. The periods were 89 days in 1960 (29 May - 25 August), 99 days in 1961 (19 May - 25 August), 93 days in 1963 (16 May - 16 August) and 58 days in 1969 (4 June - 31 July). Data were extracted for the 00 GMT and 12 GMT observations every day at two levels (1000 mb & 850 mb for 1960 & 1961 and 950 mb & 850 mb for 1963 & 1969). Four stations were used for 1960 & 1961 (Eureka, Isachsen, Resolute and Clyde), nine stations for 1963 (Eureka, Isachsen, Resolute, Clyde, Alert, Hall Beach, Mould Bay, Frobisher and Thule) and eight stations for 1969 (excluding Thule). Sources of data for the Canadian stations were *"Arctic Summary"* or *"Monthly Bulletin, Canadian Radio-Sonde Data"*. Thule data for 1963 were taken from *"Northern Hemisphere Data Tabulation, Daily Bulletin"* but were not available in this form for 1969 (actually Thule data are not, generally, easy to obtain). Missing data in tabulations were interpolated from neighbouring isobaric levels at the same station.

Correlation coefficients between data at the various stations were computed for each level in the same way as previously. Data were cleaned by least-squares fitting to an annual sine wave and expressed as deviations from the sine wave. It was found that the seasonal forcing effect was not so marked in the present case, and analysis was carried out in terms of the raw (uncleaned) data. It was once again found that the correlation coefficients were strongly dependent upon inter-station distance. A quadratic function of inter-station distance was used for computing the autocorrelation functions at each level for each year separately. It was found that the autocorrelation functions decreased strongly with distance and weakly with the square of the distance. No purpose is served here by reproducing all the results. However, it would be interesting to quote computed values of the weighting factors for interpolation at Axel Heiberg Base Camp using the computed autocorrelation functions. In each case, Eureka is Station 1, and Isachsen is Station 2.

Year	Level	W_{01}	W_{02}	η^2	U_{est}
[illegible]	1000 mb	0.75	0.27	6.3 %	∓ 1.3°C
1960	850 mb	0.72	0.28	9.4 %	∓ 1.[illegible]°C
1961	1000 mb	0.72	0.28	3.7 %	∓ 1.0°C
1961	850 mb	0.71	0.29	12.0 %	∓ 1.[illegible]°C
196[illegible]	[illegible] mb	0.72	0.28	4.6 %	∓ 1.[illegible]°C
1963	850 mb	0.78	0.22	1.1 %	∓ 0.6°C
[illegible]	[illegible] mb	0.69	0.31	10.0 %	∓ 1.[illegible]°C
1969	850 mb	0.73	0.27	7.6 %	∓ 1.4°C
Mean		0.73	0.28	6.8 %	1.3°C
St.Dvn		0.03	0.03	3.6 %	0.4°C

On average, W_{01} and W_{02} for daily upper air data are similar to the value computed for distance weighting method (a) (0.71 and 0.29). The relative error η^2 is much smaller than for the interpolation of summer monthly mean temperatures (about 7 % compared to 23 %), but the corresponding estimated absolute errors (denoted by U_{est}) are higher (about ∓ 1.3°C compared to ∓ 0.7°C). This seems reasonable and is, indeed, desirable. It would appear that U_{est} is relatively less variable than η^2 (with a standard deviation of 31 % of the mean compared to 53 %) which suggests that the method tends to conserve absolute error rather than relative error. This also seems reasonable.

The effect of increasing station distance on the relative error can be illustrated by the following examples. It has already been shown that the relative error for interpolation of daily temperature at Axel Heiberg Base Camp using data at Eureka (113 km) and Isachsen (280 km) is on the average 6.8 %. The corresponding figure for interpolation at the Devon Island Ice Cap Station (I.C.S.) using data at Resolute (353 km) and Thule (399 km) would be 16.4 %. If one were foolish enough to attempt interpolation at Axel Heiberg Base Camp using data from Resolute (536 km) and Clyde (1186 km) the relative error would be 35.8 %. Actually, this would be an extrapolation and, therefore, even more suspect.

v) Conclusions

Hypothesis (2) that "*Inputs from the large-scale atmosphere can be reasonably accurately calculated by interpolation of temperature and precipitation data from distant (100 km scale) weather stations*" has been examined. It is falsified with respect to precipitation (errors are too large), but it is not falsified with respect to temperature (errors are reasonably small).

It is concluded that interpolated temperatures can be computed for the Axel Heiberg situation just by using data from Eureka and Isachsen and that results using optimum interpolation are close to those using the simpler distance weighting method (weighting factors of 0.71 and 0.29 for Eureka and Isachsen respectively). Furthermore, it seems that, although there is considerable scatter in the plot of interstation correlations versus interstation distance (for examples see Figures 2.1 and 2.2), the weighting factors for optimum interpolation are quite consistent for different periods. The error of interpolating daily temperatures for the summer period is estimated to be about $\pm 1.3°C$ whilst errors for interpolating monthly mean temperature is estimated to be about $\pm 0.7°C$.

Falsification of the hypothesis with respect to precipitation was not unexpected. A number of authors have commented on the lack of synoptic control or coherence of precipitation, for example Andrews *et al* (1970, p.357) and Barry and Perry (1973, p.244-50). The modelling of precipitation fields in the Arctic is an urgent problem deserving more research, see for example Fogarasi (1972) and Barry (1974), but it probably requires an improved data base (improved instrumentation and denser station network).

From the point of view of the present study, the falsification of hypothesis (2) with respect to precipitation is regrettable as it means that accumulation on Axel Heiberg Island cannot be adequately modelled using data from distant weather stations. Accordingly, hypotheses (1) and (3) cannot be directly tested for precipitation.

With respect to the interpolation of temperature the results reported here might be improvable using the new Canadian multi-variant optimum interpolation method as it uses more information. However, in practice, especially over a limited region like the Arctic, it may not give better results because, although it uses both cross- and auto-correlations, they are averages for the whole hemisphere rather than computed locally (Dr W.L. Gutzman, CMC Dorval, personal communication). It is possible that an improvement to the interpolation could be made by applying a synoptic climatology approach, e.g. computing interstation correlations for samples drawn from different "weather types" rather than drawn from arbitrary periods. However, this would be very laborious and, at this stage, is not worthwhile for the temperature interpolation. The method of Gandin does not take account of autocorrelation in the temperature series which may be important for interpolation of daily temperatures."

CHAPTER 3

THERMAL MODIFICATION OF AIR BY A GLACIER

i) Introduction

Because of falsification of hypothesis (2) with respect to precipitation it is only possible to test hypothesis (1) with respect to temperature.

It is well known that glaciers exert a "cooling effect" on the air which is advected over the glacier from the surroundings (Müller and Roskin-Sharlin, 1967). If the local air temperature is indeed a major controlling factor for ablation then this cooling effect may be regarded as one of the mechanisms which help the glacier to maintain itself (Bonacina, 1947).

In principle, the thermal modification of air over a glacier may be studied by solution of the governing equations of the boundary layer over the glacier. Ohmura (1972) and Müller *et al* (1973) have presented preliminary results relating to the finite-difference integration (1 km scale) of a thermodynamic equation using field data from the Axel Heiberg Expedition Area. Such an approach does, however, involve major difficulties if it is ever to be used for relatively long-term (months to years) glacier-climate study. Firstly the equations, if realistically specified, are so complex that they cannot be solved analytically. Numerical solution of such equations is a relatively well established technique, discussed by Forsythe & Wasow (1960), Douglas (1961) and Emmons (1970), but computations are expensive in terms of computing time and storage. Secondly the method requires more detailed meteorological data, as input data, than are normally available close to glaciers. Thirdly when solutions are obtained it is difficult to isolate the effects of the different factors upon the solution.

A more economical, "empirical", approach is required. In the following section a parametric model of air temperature over glaciers will be proposed. Although simple, it is hoped that the model retains some basis of physical meaning. It will be regarded as a hypothesis and tested.

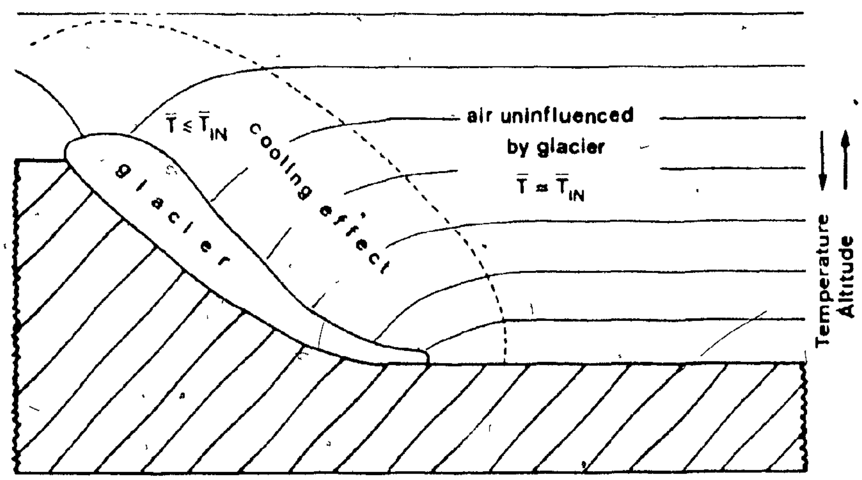


Fig 3.1: Conceptualization of Thermal Modification of Air by a Simple Glacier. The Large-Scale Atmosphere is Assumed Barotropic in this Case and Downward Bending of Isotherms Indicates Cooling Effect of the Glacier. Vertical and Horizontal Scales are Unspecified.

ii) A Parametric Model

The influence of the glacier upon the temperature fields in the large-scale atmosphere is schematically illustrated in Figure 3.1. There exists a point (x',y',z) which is just beyond the region of influence of the glacier. The temperature at time t is T(x',y',z,t). Under summer conditions this temperature will be greater than 0°C whilst the glacier surface will have temperature 0°C. As a parcel of air is carried by the wind from (x',y',z) into the region of influence of the glacier it suffers cooling. The lost heat will be mainly absorbed by the glacier surface through turbulent diffusion. The greatest heat loss will probably occur when the air parcel is within the Prandtl layer of the glacier (thickness of 10-40m), see for example Figure 3 in Müller *et al* (1973). By the time the air parcel arrives at a weather station at point (x,y,z) it will have temperature T(x,y,z,t). The basic hypothesis of the present discussion is that T(x,y,z,t) and T(x',y',z,t) are linearly related. There are two problems, however:

Firstly there are no measurements for T(x',y',z ,t). It will be assumed that this quantity can be approximated by the quantity $T_{IN}(x',y',z,t)$ which is computed by interpolation of upper air data from distant weather stations (see Chapter 2).

Secondly it is not known where the point (x',y',z) is. It will be assumed that temperature gradients in the large-scale atmosphere are small compared to gradients within the region of influence of the glacier so that $T_{IN}(x',y',z,t)$ may be replaced by $T_{IN}(x,y,z,t)$. This temperature is purely fictitious and may be understood to be an estimate of the temperature at (x,y,z) as it would be if the glacier were removed.

The hypothesis can be expressed:

$$T(x,y,z,t) = A + B\,T_{IN}(x,y,z,t) + U\,\epsilon(t) \qquad (3.1)$$

where A, B and U are parameters which are assumed constant with respect to time although they may be spatially variable. $\epsilon(t)$ is assumed to be a stationary stochastic process with zero mean and unit variance.

The reasoning and assumptions behind (3.1) are presented in Appendix 1. The parameter A expresses mainly the average effect of the glacier surface temperature, B parameterizes average wind and turbulence conditions whilst fluctuations from average conditions are absorbed into the $U\epsilon(t)$ term. The equation implies the belief that the main source of variation in T is due to variations in T_{IN} and that the dynamics of the air over the glacier, especially wind speed and turbulence regime, play a somewhat passive role in transforming T_{IN} into T. If this is not true, effects of variations in wind and turbulence regime will be "lumped" into the $U\epsilon(t)$ term which represents the "error" in the model.

For the purposes of data analysis T will be regarded as representing daily mean temperature at some point (x,y,z). The form of Equation (3.1) expresses the belief that the response of T to T_{IN} is essentially instantaneous on a sampling scale of one day. This is reasonable as the whole region of influence of the glacier, with horizontal dimensions of at most a few kilometres, will be completely ventilated in a period of only a few hours by winds of moderate speed. Alternatively, the local time derivative of temperature can be regarded as negligible compared to the advective derivative.

The requirements that must be satisfied by the parameters A, B and U for non-falsification of the hypothesis are discussed in the following section.

iii) Falsifiability of the Hypothesis

The first point to be made is that model (3.1) is meaningless unless some *a priori* requirements are placed upon the expected values of the various parameters A, B and U. This can be done partly by requiring that models should be repeatable and partly by application of conditions analogous to boundary conditions.

With respect to repeatability, it is clear that U should be as small as possible and that A and B parameters computed for different records at the same location should be very similar.

The lower limit upon the computed value of U will be set by the errors in the data for T and T_{IN} (mainly observational error for the former and interpolation errors in the latter). For daily temperatures the observational errors in T might have a standard deviation of about 0.5°C whilst the interpolation errors in T_{IN} will be of the order of 1.3°C (Chapter 2(iv)). For monthly temperatures the errors will be lower. If the model is to be a useful description of the data, for which it is computed, the variance (U^2) of the error term should be small compared to the variance of T.

In general, different values for A and B will be computed for different records at the same location; but if the model is to have any predictive power the values should not be too different. For example, suppose that values of A_1 & B_1 and A_2 & B_2 have been obtained from analysis of two records and it is required to compute the mean temperature during some third period using T_{IN} for that period and available information about A and B. In this case two different estimates of the temperature would be obtained and their difference should be small, e.g. at most $\mp$ 1.0°C. Small variations of A and B at the same location in different periods will arise if it is true that "weather" plays a passive role whilst large differences will arise if it is not true.

The expected range of A and B values, although not their actual values, can be deduced simply:

For a station outside the region of influence of the glacier, with essentially the same surface conditions as the distant weather stations whose data are used to compute T_{IN}, it may be expected that A = 0.0°C and B = 1.0. In this case $\overline{T} = \overline{T}_{IN}$ (the overbar ‾ denotes mean-values over at least a few days). Confirmation that this is the case would be a useful check on the accuracy of T_{IN}.

Under summer conditions within the region of influence of the glacier it is expected that $\overline{T}$ should increase with $\overline{T}_{IN}$ but always remain lower than it, i.e. the glacier does not become a heat source at some high value of $\overline{T}_{IN}$. This implies that $0.0 \leqslant B \leqslant 1.0$ under summer conditions. This inequality is certainly satisfied by the models in Appendix 1. However, B cannot be completely independent of $\overline{T}_{IN}$ as it might reasonably be claimed that the cooling condition $\overline{T} \leqslant \overline{T}_{IN}$ should also be satisfied with extrapolation to low values of $\overline{T}_{IN}$ so that B $>$ 1.0 for winter conditions.

From Appendix 1 it is clear that A is related to the surface temperature of the glacier and should satisfy the condition A $\leqslant$ 0°C. There will be a tendency for higher values of $\overline{T}_{IN}$ to favour more frequent cases with surface temperature equal to $0^{o}C$ which would imply $A = 0.0^{o}C$. Physically A is best regarded as a kind of boundary condition on T for the case when $T_{IN} = 0.0^{o}C$.

iv) Effects of Errors on Model Parameters

The values of A and B in the model will be computed using ordinary least-squares as, for example, implemented in IBM subroutine CORRE. The algorithm does formally require assumptions which will not be fulfilled by the data analysed. Some of these assumptions are: stationarity of the data with respect to mean and variance, zero autocorrelation of the error term $U\epsilon(t)$ and accurate values of the independent variable. Violation of these assumptions will affect the computed values of A, B and U.

The data will be non-stationary as T and T_{IN} will both contain seasonal trend. It might be claimed that a high correlation between T and T_{IN} arises solely on account of their common dependence upon astronomical factors without there being any causal link. This possibility can be tested by filtering out the seasonal trends and correlating the deviations from the trends.

Autocorrelation of the error term may arise on account of various factors such as incorrect specification of the form of the model, e.g. non-linear instead of linear, or by omission of an influential variable (Johnston, 1963, Chapter 8). The consequences would be unnecessarily large sampling variances for B, underestimation of those sampling variances using the usual least-squares formulae and inapplicability of the usual forms of t and F tests. Johnston (1963, Chapter 8) discusses this potentially serious but complex problem and proposes use of the Generalized Least-Squares method. However, this requires prescription of the autocorrelation of the error term which is not easy. The ideal, and physically most meaningful, solution would be to identify the sources of the error term and incorporate them into the model explicitly. This will be attempted. In the meanwhile it can be pointed out that some of the adverse effects of autocorrelation decrease with sample size. Bearing in mind the foregoing comments, only sparing use will be made of computed confidence intervals for B.

The data contain errors, both random and systematic. The random error in T_{IN}, the independent variable, is larger than that in T, the dependent variable.

If it can be assumed that random errors in T and T_{IN} are uncorrelated the main effect of errors in T_{IN} will be to cause underestimation of B which, in turn, will affect values of U and A. In Chapter 2(iv) the random error in T_{IN} due to interpolation error, was estimated to be about 7 % of the total variance, and this would cause underestimation of B by 7 %. This may be rather optimistic as it appears that absolute errors are conserved rather than relative errors so that the underestimation of B will increase with a decrease in the variance of T_{IN}. Random errors in T and T_{IN} combine to cause underestimation of the correlation coefficient. Systematic errors in T and T_{IN} will compensate or reinforce each other depending upon their relative magnitudes and signs to affect A. Correction of computed values of A, B and U for the effects of error would be difficult and will not be attempted.

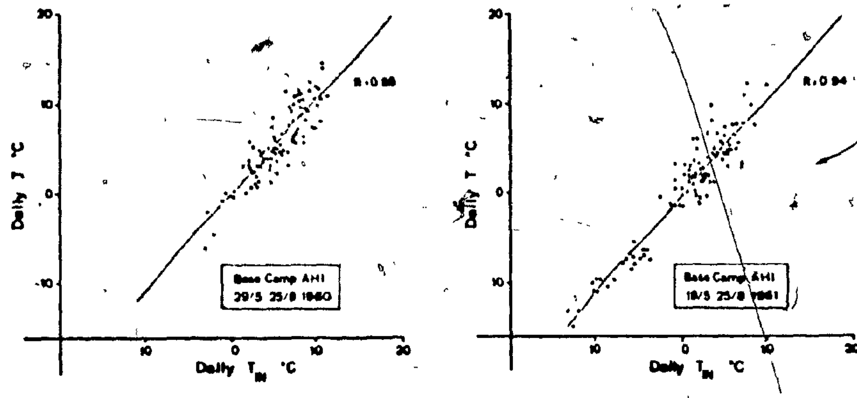
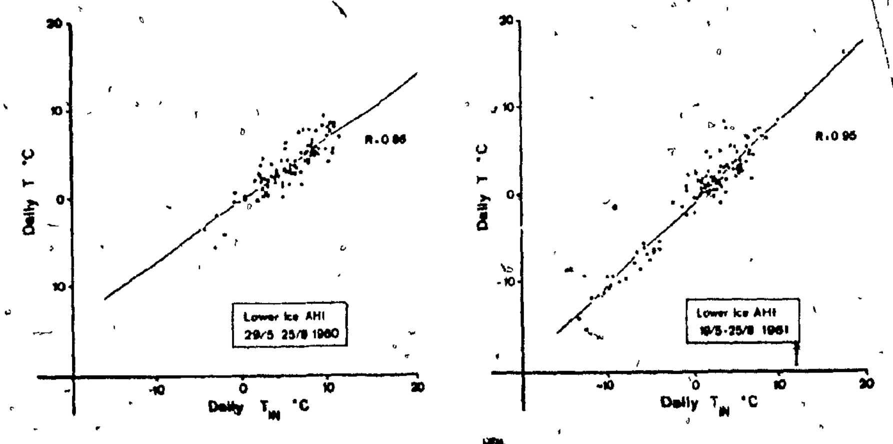


Fig 4.1: Daily Mean Local Temperature T Versus Interpolated Temperature T_{IN} at Base Camp Axel Heiberg Island for Summers 1960 and 1961



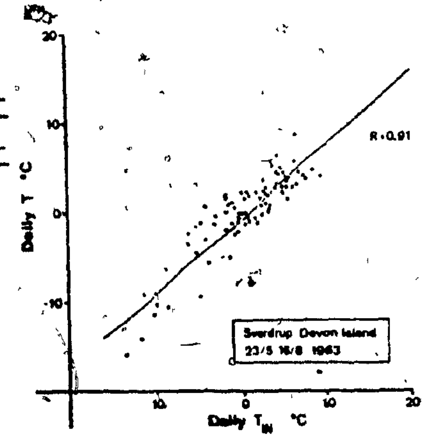


Fig 4.2: Daily Mean Local Temperature T Versus Interpolated Temperature T_{IN} at Lower Ice Axel Heiberg Island for Summers 1960 and 1961 and on Sverdrup Glacier Devon Island for Summer 1963

CHAPTER 4

THE FIRST TEST OF THE RELATION BETWEEN T AND T_{IN}

i) Introduction

For the first test of the hypothesis, outlined in the previous chapter, nine samples of daily mean air temperature for summer periods were chosen for analysis. The daily mean temperatures are in most cases computed as the average of the four six-hourly readings of mercury-in-glass thermometers in standard instrument shelters about 1.5 m above the surface. Care was taken to choose the samples from different kinds of location: glacier-free (2 samples), valley glacier (3 samples) and ice cap (4 samples). Details and sources of data are given in Table 4.1, and the locations of the stations are given in Figures 1.2 to 1.5. Base Camp A.H.I. is a relatively open location about 3 km from White Glacier and should be outside the region of influence of the glacier. Lower Ice is located in the snout of White Glacier but is rather exposed compared to Sverdrup Glacier station which is sheltered by high valley walls. Exposure of Lower Ice is southerly compared to Sverdrup which is northerly. Devon I.C.S., Upper Ice 1 and Upper Ice 2 are relatively open ice cap locations. It might be claimed that the order of arranging the stations in Table 4.1 - Base Camp A.H.I., Lower Ice, Sverdrup, Devon I.C.S., Upper Ice 1 and Upper Ice 2 - is in the direction of increasing "glacierness".

T_{IN} was computed for every day of each record at each station using the average of the 00 and 12 GMT radio-sonde observations at the surrounding upper air stations. For the Devon Island situations, Sverdrup and Devon I.C.S., the interpolation was made using a fitted quadratic polynomial (9 stations for 1963 and 8 stations for 1969) whilst Optimum Interpolation was used for the Axel Heiberg situations (four stations). Surprisingly, the interpolation errors for the Devon Island situations using the polynomial method were rather lower than those using Optimum Interpolation. This is contrary to Gandin (1965), but a careful study would have to be made before drawing any definite conclusions about the relative accuracies of the different methods.

Figures 4.1 to 4.4 are scatter diagrams of T versus T_{IN}. Although there appears to be considerable scatter in all the plots, a linear relationship is

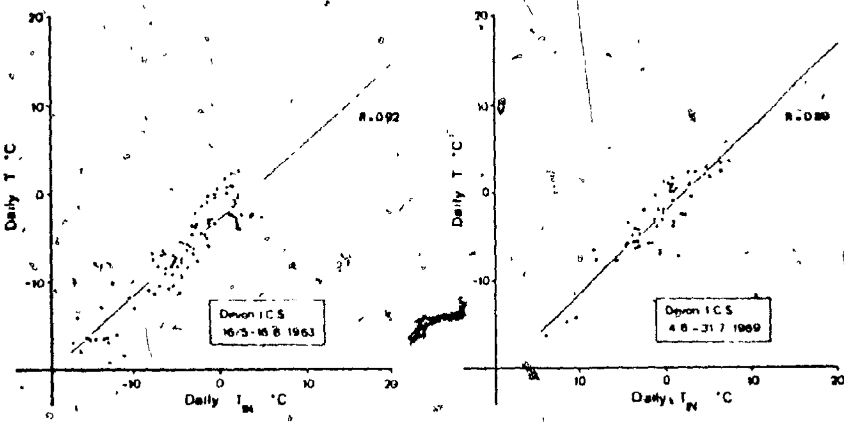


Fig 4.3: Daily Mean Local Temperature T Versus Interpolated Temperature T_{IN} at the Ice Cap Station (I.C.S.) Devon Island for Summers 1963 & 1969

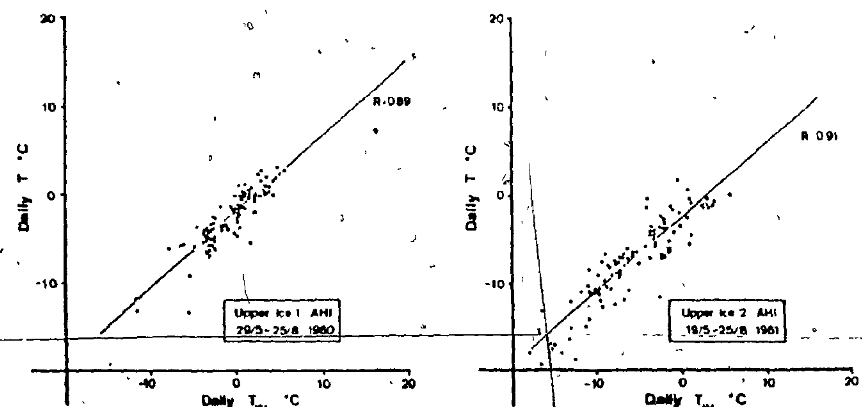


Fig 4.4: Daily Mean Local Temperature T Versus Interpolated Temperature T_{IN} at Upper Ice 1 and 2 on Axel Heiberg Island for Summers 1960 & 1961

apparent. However, some of the plots show a hint of non-linearity.

ii) The Computed Parameters

The data for T and T_{IN} for the nine samples were fitted to model (3.1) using ordinary least-squares (IBM SSP subroutine CORRE). The various statistics are given in Table 4.2: computed values of A, B and U together with the sample means $\bar{T}$ and $\bar{T}_{IN}$, standard deviations S_T and S_{IN} and correlation coefficient R. The mean and standard deviations of B, S_T, S_{IN} and R for the nine samples are given. This is to illustrate the sample-to-sample variability of the quantities and is not intended to suggest that they are drawn from common homogeneous populations (they are certainly not).

With respect to the hypothesis outlined in the previous chapter several comments can be made. In all cases T and T_{IN} are well correlated with an average R of 0.91 which would correspond to 83 % explanation of the variance of T by T_{IN}. The corresponding Root Mean Square Error U is about $\pm$ 1.8°C. This is somewhat larger than the interpolation error in T_{IN} (about $\pm$ 1.3) so that it can be concluded that the system linking T and T_{IN} contains "noise" with standard deviation of about $\pm$ 1.2°C. This is probably due to variable "weather" (wind speed, surface temperature etc) during the sample period. B is positive but less than unity for all the glacier situations whilst A is negative. B is slightly greater than unity for Base Camp A.H.I. although not significantly so. Actually, Base Camp A.H.I. does seem to have a slight heating effect of 0.1 or 0.2°C although the near agreement of $\bar{T}$ and $\bar{T}_{IN}$ is encouraging and suggests that the interpolation is quite accurate with respect to systematic error. The glacier situations do show cooling effects with less cooling for the valley glaciers than for the ice caps. This was expected and can be explained by the longer passage time of the air over the cool surface in the case of the ice caps. However, it is interesting in that it appears that A controls the cooling effect rather than B. On the whole, agreement between the two Base Camp models and between the four ice cap models is good. On the other hand, Lower Ice 1961 agrees better with Sverdrup Glacier 1963 than it does with Lower Ice 1960. The nine models are plotted in Figure 4.5 from which it can be seen that the spread of $\bar{T}$ values within each group (2 glacier-free, 3 valley glacier and 4 ice cap) is certainly less than 1°C for positive values of $\bar{T}_{IN}$.

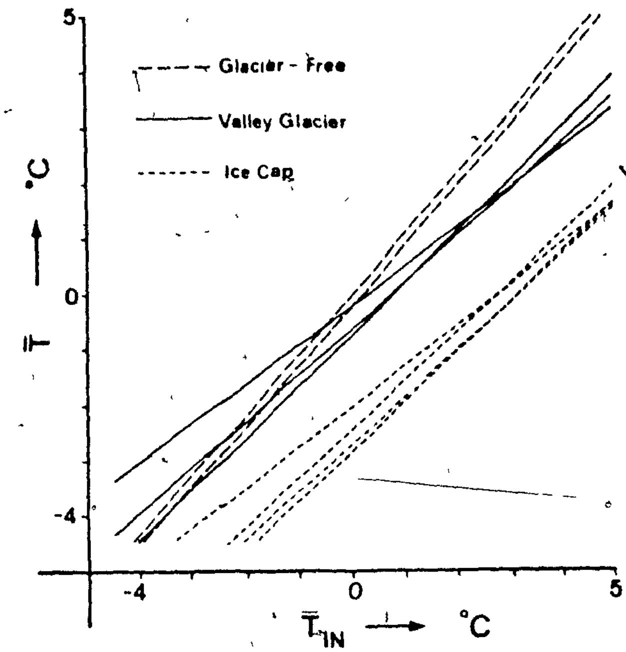


Fig 4.5: Plots of the Linear Regression Models for Local Temperature T Versus Interpolated Temperature T_{IN} for Nine Different Samples Representing Glacier-Free, Valley Glacier and Ice Cap Situations

Station	H	N	Period of Record	Data Source	Comment
Base Camp AHI	190	89	29/5 to 25/8 1960	Müller, unpub.	Glacier-free
Base Camp AHI	190	99	19/5 to 25/8 1961	§	Glacier-free
Lower Ice	210	89	29/5 to 25/8 1960	Andrews, 1964	Valley Glacier
Lower Ice	210	99	19/5 to 25/8 1961	§	Valley Glacier
Sverdrup Gla.	300	81	23/5 to 16/8 1963	Keeler, unpub.	Valley Glacier
Devon I.C.S.	1320	93	16/5 to 16/8 1963	Holmgren, unpub.	Ice Cap
Devon I.C.S.	1320	58	04/6 to 31/7 1969	Braithwaite, unpub.	Ice Cap
Upper Ice 1	1530	89	29/5 to 25/8 1960	Havens, 1964	Ice Cap
Upper Ice 2	1920	99	19/5 to 25/8 1961	§	Ice Cap

Altitude H in m a.s.l., N is number of days of record
§ = Müller and Roskin-Sharlin, 1967

Table 4.1 Stations and Periods for Data Analysed

Station	N	Year	A	B	U	$\overline{T}$	$\overline{T}_{IN}$	S_T	S_{IN}	R
Base Camp AHI	89	1960	-0.2	1.084	1.9	5.8	5.6	4.1	3.3	0.882
Base Camp AHI	99	1961	0.0	1.078	2.0	1.1	1.0	6.1	5.3	0.943
Lower Ice AHI	89	1960	-0.2	0.715	1.4	3.8	5.6	2.8	3.3	0.857
Lower Ice AHI	99	1961	-0.7	0.944	1.7	0.1	0.9	5.3	5.3	0.945
Sverdrup Gla.	81	1963	-0.6	0.844	2.0	0.1	0.9	4.8	5.1	0.907
Devon I.C.S.	93	1963	-2.8	0.889	2.0	-6.3	-3.9	5.2	5.4	0.924
Devon I.C.S.	58	1969	-2.0	0.942	2.0	-3.2	-1.2	4.6	4.3	0.893
Upper Ice 1	89	1960	-2.4	0.872	1.5	-2.6	-0.3	3.2	3.3	0.886
Upper Ice 2	99	1961	-2.7	0.857	2.1	-7.8	-6.0	5.0	5.3	0.906
Mean				0.914	1.8			4.6	4.5	0.905
Standard Dvn				0.116	0.3			1.0	1.0	0.029

Table 4.2 Statistics for Steady State Model $T = A + B.T_{IN}$ for Nine Situations: Data are Unfiltered.

Station	N	Year	A	B	U	$\bar{T}$	$\bar{T}_{IN}$	S_T	S_{IN}	R
Base Camp AHI	89	1960	0.0	1.042	1.9	0.0	0.0	3.1	2.3	0.775
Base Camp AHI	99	1961	0.0	0.834	1.9	0.0	0.0	2.6	2.3	0.711
Lower Ice AHI	89	1960	0.0	0.719	1.4	0.0	0.0	2.2	2.3	0.746
Lower Ice AHI	99	1961	0.0	0.723	1.9	0.0	0.0	2.4	2.0	0.593
Sverdrup Gla.	81	1963	0.0	0.622	2.0	0.0	0.0	2.4	2.1	0.529
Devon I.C.S.	93	1963	0.0	0.693	1.8	0.0	0.0	2.8	3.0	0.747
Devon I.C.S.	58	1969	0.0	0.738	1.8	0.0	0.0	2.9	3.1	0.773
Upper Ice 1	89	1960	0.0	0.767	1.4	0.0	0.0	2.5	2.7	0.818
Upper Ice 2	99	1961	0.0	0.735	1.9	0.0	0.0	3.7	4.4	0.866
Mean				0.764	1.8			2.7	2.7	0.729
Standard Dvn				0.119	0.2			0.5	0.7	0.106

Table 4.3: Statistics for Steady State Model $T = A + B.T_{IN}$ for Nine Situations. Data are High-Pass Filtered.

Station	N	Year	A	B	U	$\bar{T}$	$\bar{T}_{IN}$	S_T	S_{IN}	R
Base Camp AHI	89	1960	0.0	1.109	1.3	0.0	0.0	2.6	2.0	0.861
Base Camp AHI	99	1961	0.0	0.900	1.4	0.0	0.0	2.3	2.1	0.798
Lower Ice AHI	89	1960	0.0	0.765	1.0	0.0	0.0	1.9	2.0	0.831
Lower Ice AHI	99	1961	0.0	0.798	1.6	0.0	0.0	2.1	1.7	0.657
Sverdrup Gla.	81	1963	0.0	0.735	1.5	0.0	0.0	2.0	1.7	0.627
Devon I.C.S.	93	1963	0.0	0.738	1.4	0.0	0.0	2.5	2.7	0.812
Devon I.C.S.	58	1969	0.0	0.778	1.4	0.0	0.0	2.4	2.6	0.823
Upper Ice 1	89	1960	0.0	0.894	1.0	0.0	0.0	2.2	2.2	0.893
Upper Ice 2	99	1961	0.0	0.812	1.2	0.0	0.0	3.3	3.8	0.929
Mean				0.837	1.3			2.4	2.3	0.803
Standard Dvn				0.118	0.2			0.4	0.6	0.101

Table 4.4: Statistics for Steady State Model $T = A + B.T_{IN}$ for Nine Situations. Data are Band-Pass Filtered.

However, at lower temperatures the spread is greater: especially for the valley glacier group with a spread for T of 1.6°C at $\bar{T}_{IN}$ = -4.0°C.

It might plausibly be claimed that some of the results, especially the high correlation coefficients, may be due to the "forcing effect" of seasonal trend in the data and that T and T_{IN} are not meaningfully related. This possibility was tested by high-pass filtering of the data to remove the annual periodicity and repeating all the computations. Results obtained for the high-pass filtered data are given in Table 4.3. It is noteworthy that, although the standard deviations S_T and S_{IN} and the correlation coefficients R are reduced by filtering, the errors U remain the same. This supports the notion that T and T_{IN} are meaningfully related and that relationships have not been unduly forced by seasonal trend. The reduction in R is simply due to the fact that filtering reduces S_T without changing U. B is on average reduced by filtering although the spread of values remains about the same. It is interesting that filtering brings B for Lower Ice 1960 and 1961 into good agreement although a discrepancy for B between Base Camp 1960 and 1961 is introduced. The apparently anomalous B value for Lower Ice 1960, before filtering, may be explainable by an absence of seasonal trend in the 1960 record (August 1960 was the warmest month on record for Lower Ice) or by the fact that surface temperatures of 0°C were very frequent during this warm summer.

It should be noted that zero values for A in Table 4.3 arise simply on account of the filtering process.

If the error term in the regression equation, scaled by U, is approximately random the error could be reduced by low-pass filtering to remove periodicities of less than several days. The data, already high-pass filtered, were filtered with a simple Hanning Filter with centre weight 0.5 and side weights of 0.25 (Blackman & Tukey, 1959, p.171). The resulting data can be regarded as band-pass filtered with suppression of frequencies in the range of 0.50c dy^{-1} (the Nyquist Frequency for daily mean data) to about 0.22c dy^{-1} (the band-width of the filter). Computations were repeated for the band-pass filtered data, and results are given in Table 4.4. On average the standard deviations S_T and S_{IN} are further reduced by a small amount whilst U is reduced from 1.8°C to 1.3°C. B is increased somewhat by the filtering (from about 0.764 to 0.837). This is

Station	N	Year	A	f(0)	f(1)	f(2)	f(3)	f(4)	U	Θ
Base camp AHI	85	1960	0.1	1.041	0.079	-0.099	-0.125	0.144	1.9	1.040
Base Camp AHI	95	1961	0.1	0.620	0.386	-0.252	0.202	0.157	1.8	1.115
Lower Ice AHI	85	1960	0.4	0.556	0.258	-0.226	-0.038	0.084	1.3	0.634
Lower Ice AHI	95	1961	-0.6	0.509	0.327	-0.115	0.222	0.222	1.5	0.945
Sverdrup Gla.	77	1963	-0.5	0.548	0.025	0.196	0.182	0.182	1.9	0.873
Devon I.C.S.	89	1963	-2.8	0.568	0.258	0.102	0.006	0.006	1.8	0.865
Devon I.C.S.	54	1969	-1.6	0.710	0.077	0.238	0.098	0.098	1.4	1.180
Upper Ice 1	85	1960	-2.2	0.607	0.298	0.066	-0.041	-0.041	1.0	0.844
Upper Ice 2	95	1961	-1.9	0.531	0.354	0.026	0.021	0.021	1.5	0.989
Mean									1.6	0.943
Standard Dvn									0.3	0.163

Table 4.5: Statistics for General Linear Model for Nine Situations. Data are unfiltered.

no doubt due to partial suppression of errors in the data and of noise in the system linking T and T_{IN}.

From comparison of Tables 4.2 to 4.4 it would be unwise to claim frequency-dependence of the system. This question could only be answered by application of spectral methods to the data which is beyond the scope of the present study. It can, however, be concluded that the relationship between T and T_{IN} does not arise solely on account of seasonal effects. Furthermore, as T and T_{IN} are related in the high-pass and band-pass frequency ranges, it is plausible that the seasonal trends in T are put there by the relationship with T_{IN} rather than by an extraterrestrial source acting as a separate input. As it is in fact absolute values of T which are of glaciological interest, especially in relation to the $0^{o}C$ threshold, further analysis will be in terms of raw data rather than filtered data.

iii) A General Linear Model

It has been asserted that the response of the system is essentially instantaneous on the daily time-scale. It would be interesting to test this. If the response of the system were not instantaneous, the model (3.1) would have to be replaced with an equation containing lagged values of T_{IN}, i.e. $T_{IN}(t)$, $T_{IN}(t-1)$, $T_{IN}(t-2)$ etc, which would be a digital counterpart to a convolution integral. The regression coefficients of the various lagged terms would be analogous to the Green's Function of a differential equation containing a time derivative (previously neglected in comparison to the advective term in Appendix 1).

To test this possibility the data, analysed in the previous section, were fitted to a multiple regression model of the form:

$$T(t) = A + \sum_{n=0}^{n=4} f(n)\, T_{IN}(t-n) + U\epsilon(t) \qquad (4.1)$$

Results are given in Table 4.5 for unfiltered data. The quantity Θ is the "steady state response" of the system defined by:

$$\Theta = \sum_{n=0}^{n=4} f(n) \qquad (4.2)$$

Θ should satisfy the same conditions as B.

Comparing Tables 4.2 and 4.5 it can be seen that U is somewhat lower for the general linear model (4.1) than for the steady state model (3.1): on average 1.6°C compared to 1.8°C. However, the dispersion of Θ values is larger for the general linear model, and there is one value (Devon I.C.S. 1969) which is larger than unity in violation of the hypothesis. Many of the f(1) to f(4) coefficients are not statistically significant (at 5 % level).

It is concluded that the slightly better U values for the general linear model are probably fortuitous and that the greater inconsistency or dispersion of Θ makes use of the model inadvisable.

iv) Some Preliminary Conclusions

The results reported in Chapter 4(ii) are encouraging for the hypothesis advanced in Chapter 3. More results, from analyses of further samples, will be given in the next chapter. It will be noticed that little mention has been made of confidence intervals for the various computed statistics. They have been computed (at 5 % level), but it is felt that little reliance should be placed upon them because of the effects of autocorrelation in the data (see Figure 4.6 for plots of A versus B together with their corresponding confidence intervals). It is suggested that it is better to analyse many samples, if available, rather than to attempt to draw conclusions from possibly dubious confidence intervals for a few samples.

Some preliminary conclusions can already be drawn. On the basis of results in Table 4.2 it is suggested that temperature at Base Camp A.H.I. can be represented approximately by:

$$\overline{T} = -0.1 + 1.08\,\overline{T}_{IN} \qquad (4.3)$$

This is very close to $\overline{T} = \overline{T}_{IN}$. A valley glacier situation would be approximately:

$$\overline{T} = -0.5 + 0.83\,\overline{T}_{IN} \qquad (4.4)$$

and an ice cap situation would be approximately:

$$\overline{T} = -2.5 + 0.89\,\overline{T}_{IN} \qquad (4.5)$$

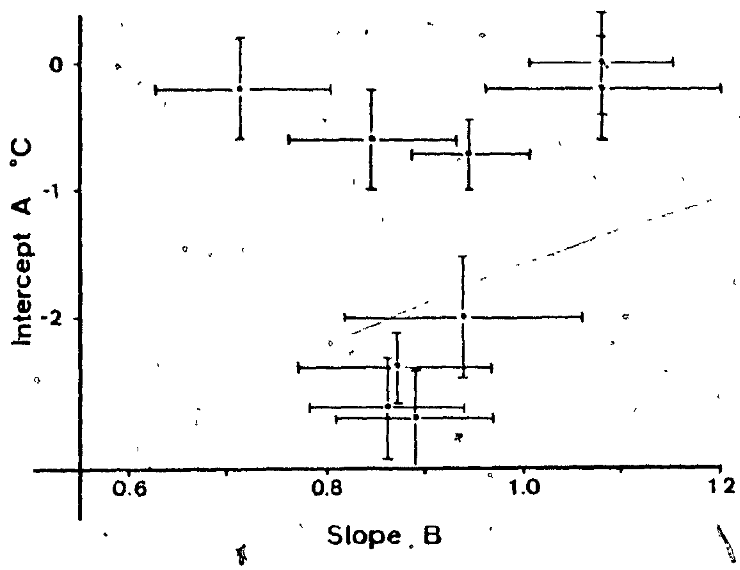


Fig 4.6: Intercept A Versus Slope B for the Nine Linear Regression Models Linking Local and Interpolated Temperature. Error Bars Denote 95% Confidence Intervals

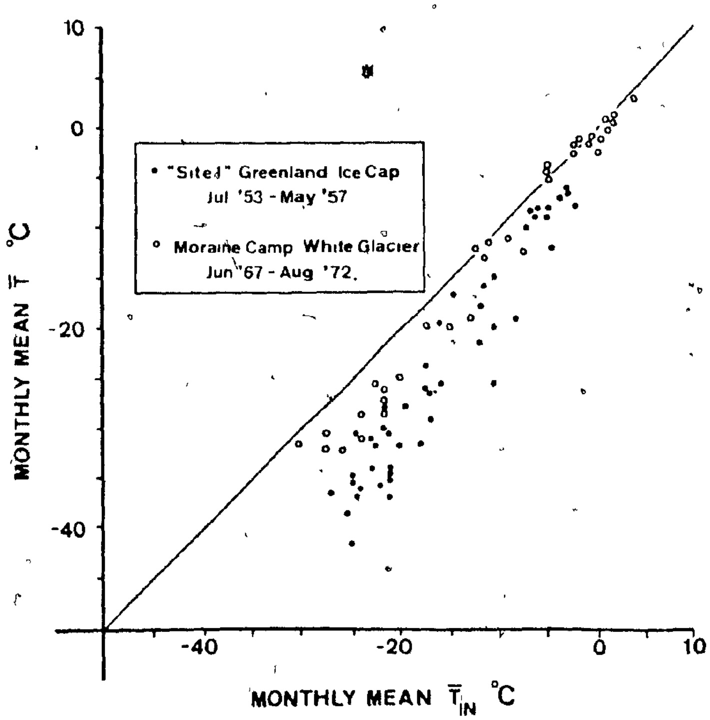


Fig 4.7: Monthly Mean Local Temperature $\overline{T}$ Versus Interpolated Temperature $\overline{T}_{IN}$ for Site 1 (Greenland Ice Cap) and Moraine Camp (White Glacier, Axel Heiberg Island). Data Include Winter Months

Analysis of further data will confirm, improve or falsify these preliminary conclusions.

Cooling effect is greater for an ice cap situation than for a valley glacier situation; this is due to lower A values rather than lower values of B. The greater B value for (4.5) compared to (4.4), i.e. 0.89 compared to 0.83, may have a physical explanation; for much of the time the temperature of the ice cap surface will be below $0^{o}C$ and in good adjustment with the air temperature except for a reasonably constant discrepancy. According to this explanation B should increase to unity as the mean value of T_{IN} decreases and the occurence of surface temperatures of $0^{o}C$ becomes rare. In Chapter 3(iii) it was suggested that B could become greater than unity to preserve cooling effect at low temperatures. Figure 4.7 is a scatter diagram of monthly mean values of $\overset{o}{T}$ versus $\overset{o}{T}_{IN}$ for Moraine Camp at 870 m a.s.l. on White Glacier (data from Muller, unpublished) and for Site 1 at 2128 m a.s.l. on the Greenland Ice Cap (data from Putnins, 1970, Table XVIII). Data at Eureka and Isachsen were used to compute T_{IN} for Moraine Camp whilst T_{IN} values for Site 1 were computed using data from Resolute, Alert and Egedesmind (Thule data would have been best but were unobtainable). From Figure 4.8 it is clear that cooling effect is preserved under winter temperature conditions, i.e. T_{IN} in the range -10 to $-30^{o}C$, and that B becomes greater than unity.

Some workers have compared temperature observations at glacier weather stations with observations at distant (10^1 to 10^2 km scale) weather stations. Some examples are Wallén (1948, p.504), Arnold (1968, p.56), Tvede (1973, p.97) and Marcus (1964, p.60). The results of the first three are difficult to compare with present results because their coefficients include effects of altitude differences between the glacier weather stations and distant weather stations. However, the models quoted by Marcus (1964) avoid this difficulty, and his coefficients do, in fact, lie in the range specified in Chapter 3(iii).

As a final comment it should be said that, although the statistical results appear to satisfy the hypothesis advanced in Chapter 3 and Appendix 1, the hypothesis has not been "proved". This is because there must be many plausible mechanisms which could have generated the various calculated statistics: the mechanism postulated in the present study would only be one of these.

CHAPTER 5

A FURTHER TEST OF THE RELATION BETWEEN T AND T_{IN}

i) Introduction

The purpose of the present chapter is to describe further testing of the model (3.1) and of conclusions reached in the previous chapter.

The data analysed comprise a total of 2491 daily mean temperatures observed during three summer field seasons (April/May to August) at a variety of stations in the Axel Heiberg Expedition Area (see Figure 1.4 for location of stations). In 1969 and 1970 ten stations were operated and nine stations in 1971. The data are, therefore, organized into 29 samples with each sample comprising the full available record, of length N days, for the station and summer in question. Few records are completely continuous, and statistics are computed for the full record so that they reflect the period of record as well as characteristics of the station.

With the exception of Base Camp, where full surface weather records are available, the temperature data are derived from thermograph/thermohygrograph records. Although great care was taken in calibrating the records by comparison with standard thermometers it is probable that they are less accurate than thermometer records. Some of the instruments were sheltered in standard Stevenson Screens (MSC type) whilst others were installed in home-made "economical" screens. The latter performed quite well (Ohmura, personal communication), but radiation errors cannot be completely excluded from consideration.

From the findings in Chapter 2 it was concluded that T_{IN} could be computed by simple distance weighting of temperature from Eureka and Isachsen alone (with weights of 0.71 and 0.29). Equivalent-altitude values of T_{IN} were computed at altitudes of T records by linear interpolation between the 1000 mb and 850 mb levels.

ii) Computed Parameters

Values of A, B and U in the model (3.1) were computed for the 29 samples. It had been expected that it would be possible to divide the models into two classes representing glacier-free and valley glacier situations (Classes 1 and

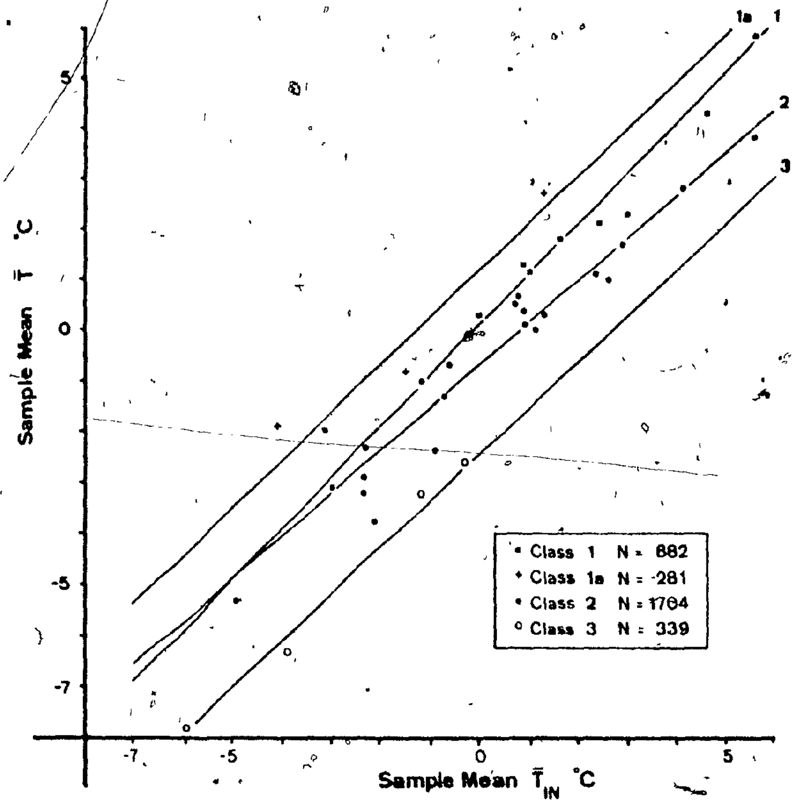


Fig 5.1: Sample Mean Values of Local Temperature $\overline{T}$ Versus Interpolated Temperature $\overline{T}_{IN}$ for 37 Different Samples Divided into Four Classes, N is Total Number of Days of Record for Each Class and Straight Lines Depict Average Regression Equations for Each Class

2). However, it was found that several glacier-free situations (Fjord, Outwash and Phantom) had models resembling those for glacier situations with A generally negative and B less than unity. The explanation is probably that Fjord station is still under the influence of the cold sea surface of Expedition Fjord whilst Phantom station is both under influence of the cold lake surface of Phantom Lake and of the surrounding glacierized area. Outwash station is probably within the region of influence of the White Glacier, particularly under down-glacier wind conditions, as well as being influenced by the cold surface of Expedition River. Ermine models were found to be sufficiently different from the other glacier-free situations (Base Camp, Gordon's and Valley) to merit creation of an extra class (Class 1a).

Tables 5.1 to 5.6 summarize the statistics for the models using both raw data and high-pass filtered data. Base Camp and Lower Ice situations f r 1960 and 1961 are repeated in the tables. The classification procedure is frankly subjective. However, the classification, once made, can be tested objectively. This is described in the following section. Figure 5.1 is a scatter diagram of $\overline{T}$ versus $\overline{T}_{IN}$ for the 37 situations so far analyzed (ice cap situations are Class 3). The lines are the averaged regression lines for $\overline{T}$ versus $\overline{T}_{IN}$ for each class. In only one case is a $(\overline{T},\overline{T}_{IN})$ pair separated from the regression line for its own class (Class 2) by the regression line for another class (Class 1).

With respect to actual values of the computed parameters, some comments can be made. Firstly, the Root Mean Square Error U is in all cases reasonably low although a trifle higher than previously found (Table 4.2). U is unchanged by high-pass filtering. The models are, therefore, useful representations of the data in the samples, and the hypothesis that T and T_{IN} are related is not falsified. Class 1 situations are close to $\overline{T} = \overline{T}_{IN}$ although Gordon's station can show substantial "heating effect". This may be related to sheltering and generally southerly exposure. Ermine shows substantial heating effect with values of A of 0.5 to 1.7°C although B tends to be less than unity. This seems a little surprising as Ermine station is rather exposed to wind but, on the other hand, the underlying surface is dry and rocky and probably enjoys a large income of solar radiation which would be the source of "heating effect"

Station	i	N	Year	A	B	U	$\overline{T}$	$\overline{T}_{IN}$	S_T	S_{IN}
Base Camp	2	89	1960	-0.2	1.084	1.9	5.8	5.6	4.1	3.3
Base Camp	2	99	1961	0.0	1.078	2.0	1.1	1.0	6.1	5.3
Base Camp	2	111	1969	0.3	0.950	2.5	0.3	-0.0	7.0	6.9
Base Camp	2	122	1970	-0.1	1.001	2.0	-3.1	-3.0	10.2	10.0
Base Camp	2	107	1971	-0.2	0.957	1.8	2.1	2.4	6.1	6.1
Valley	3	0	1969	-	-	-	-	-	-	-
Valley	3	0	1970	-	-	-	-	-	-	-
Valley	3	85	1971	0.1	0.906	2.2	4.3	4.6	4.9	4.8
Gordon's	5	71	1969	0.3	1.007	2.0	1.2	0.9	5.1	4.6
Gordon's	5	109	1970	0.9	0.903	2.0	-2.0	-3.1	7.2	7.7
Gordon's	5	89	1971	0.1	1.108	1.7	1.8	1.6	5.7	4.9
Mean				0.1	0.999	2.0				
Standard Dvn				0.3	0.077	0.2				

Table 5.1: Statistics for Steady State Models of Class 1 Situations in Axel Heiberg Island Expedition Area. Model is of Form $T = A + B.T_{IN}$, and Data are Unfiltered. N is the Number of Days in Each Sample (During Period May-August), U is R.M.S. Error, and S_T and S_{IN} are Standard Deviations of Daily Mean Temperature T and of T_{IN} Respectively.

Station	i	N	Year	A	B	U	$\bar{T}$	$\bar{T}_{IN}$	S_T	S_{IN}
Base Camp	2	89	1960	0.0	1.042	1.9	0.0	0.0	3.1	2.3
Base Camp	2	99	1961	0.0	0.834	1.9	0.0	0.0	2.6	2.3
Base Camp	2	111	1969	0.0	0.945	2.5	0.0	0.0	3.7	2.9
Base Camp	2	122	1970	0.0	0.883	2.0	0.0	0.0	2.9	2.3
Base Camp	2	107	1971	0.0	0.926	1.8	0.0	0.0	3.3	3.0
Valley	3	0	1969	-	-	-	-	-	-	-
Valley	3	0	1970	-	-	-	-	-	-	-
Valley	3	85	1971	0.0	0.714	2.2	0.0	0.0	2.9	2.8
Gordon's	5	71	1969	0.0	1.022	2.0	0.0	0.0	3.5	2.9
Gordon's	5	109	1970	0.0	0.942	2.0	0.0	0.0	2.9	2.2
Gordon's	5	89	1971	0.0	0.828	1.9	0.0	0.0	2.9	2.7
Mean				0.0	0.904	2.0				
Standard Dvn				0.0	0.102	0.2				

Table 5.2: Statistics for Steady State Models of Class 1 Situations in Axel Heiberg Island Expedition Area. Model is of Form $T = A + B.T_{IN}$, and Data are High-Pass Filtered. N is Number of Days in Each Sample (During Period May-August). U is R.M.S. Error, and S_T and S_{IN} are Standard Deviations of Daily Mean Temperature T and of T_{IN} Respectively.

Station	i	N	Year	A	B	U	$\bar{T}$	$\bar{T}_{IN}$	S_T	S_{IN}
Ermine	6	99	1969	0.5	0.870	2.2	-0.8	-1.5	5.4	5.7
Ermine	6	106	1970	1.7	0.896	2.0	-1.9	-4.1	6.9	7.3
Ermine	6	76	1971	1.4	1.032	2.0	2.7	1.3	5.4	4.9
Mean				1.2	0.933	2.1				
Standard Dvn				0.6	0.087	0.1				

Table 5.3: Statistics for Steady State Models of Class 1a Situations in Axel Heiberg Island Expedition Area. Model is of Form $T = A + B.T_{IN}$, and Data are Unfiltered. N is Number of Days in Each Sample (During Period May-August). U is R.M.S. Error, and S_T and S_{IN} are Standard Deviations of Daily Mean Temperature T and of T_{IN} Respectively.

Station	i	N	Year	A	B	U	$\bar{T}$	$\bar{T}_{IN}$	S_T	S_{IN}
Ermine	6	99	1969	0.0	1.155	1.9	0.0	0.0	4.1	3.2
Ermine	6	106	1970	0.0	0.998	2.0	0.0	0.0	3.1	2.4
Ermine	6	76	1971	0.0	0.864	2.3	0.0	0.0	3.3	2.7
Mean				0.0	1.006	2.1				
Standard Dvn				0.0	0.146	0.2				

Table 5.4: Statistics for Steady State Models of Class 1a Situations in Axel Heiberg Island Expedition Area. Model is of Form $T = A + B.T_{IN}$, and Data are High-Pass Filtered. N is the Number of Days in Each Sample (During Period May-August). U is R.M.S. Error and S_T and S_{IN} are Standard Deviations of Daily Mean Temperature T and of T_{IN} Respectively.

(Müller, personal communication).

Within Class 2 there is a situation with B greater than unity in violation of the hypothesis (Moraine 1969), but at the same time A appears to be rather low. These anomalies may have a common cause such as instrument error or calibration error or may represent some more subtle effect. It was suggested in Chapter 3(iii) that B could be greater than unity at very low values of T_{IN}, but this can hardly apply in this case as Moraine 1970 is much colder with a lower B value. Actually the B value for Moraine 1969 (B = 1.008) is not significantly greater than unity (at 5 % level), and it is chosen to ignore the problem.

The models for Baby Glacier deserve some discussion. The mean cooling effect for 1969 is $-0.1^{o}C$, for 1971 it is $-1.0^{o}C$, and it is actually positive ($+0.2^{o}C$) for 1970. This might be regarded as a falsification of the hypothesis. It could be claimed, on the basis of the small size of Baby Glacier, that the cooling effect should be weak and that the wrong sign arises on account of quite a small error in $\overline{T}$. This error might arise because Baby Glacier was less frequently visited than other stations for calibration of the recorder. However, A and B values for the three years are quite consistent. Although weak cooling could have been expected it is surprising that B is so low. With respect to this point it is interesting that high-pass filtering actually increases the value of B in two cases (1969 and 1971). This is unusual.

Clearly some of the results are difficult to explain. Undoubtedly some of the curiosities in Table 5.1 to 5.6 do arise on account of instrument errors as well as reflecting peculiarities of location and weather.

In the following section the usefulness of the model classification will be demonstrated.

iii) Test of the Classification of Models

In the previous sections it was suggested that the various situations which were analysed could be assigned to several different classes. This does not imply that the situations (periods and locations) do not have their own identities and peculiarities but rather that there are some overall patterns. This is a general principle of taxonomy. The main justification of the classification, as carried out, is that it should be useful.

Station	I	N	Year	A	B	U	$\bar{T}$	$\bar{T}_{IN}$	S_T	S_{IN}
Fjord	1	111	1969	-0.0	0.889	2.5	0.7	0.8	6.7	7.0
Fjord	1	102	1970	-0.7	0.894	2.0	-1.3	-0.7	7.9	8.6
Fjord	1	93	1971	-0.8	0.882	2.1	2.8	4.1	5.4	5.6
Baby Gla.	4	80	1969	-0.3	0.596	1.6	-0.7	-0.6	2.9	4.0
Baby Gla.	4	67	1970	-0.3	0.578	1.7	-1.0	-1.2	2.9	4.1
Baby Gla.	4	37	1971	-0.5	0.646	1.0	0.3	1.3	2.1	2.9
Outwash	7	102	1969	-0.1	0.872	2.1	0.5	0.7	6.3	6.8
Outwash	7	111	1970	-1.0	0.968	1.9	-3.2	-2.3	10.0	10.2
Outwash	7	96	1971	-0.5	0.904	2.0	2.3	3.0	6.1	6.4
Lower Ice	8	89	1960	-0.2	0.715	1.4	3.8	5.6	2.8	3.3
Lower Ice	8	99	1961	-0.7	0.944	1.7	0.1	0.9	5.3	5.3
Lower Ice	8	88	1969	-0.9	0.827	1.6	1.1	2.3	4.0	4.5
Lower Ice	8	109	1970	-0.8	0.905	1.8	-2.9	-2.3	8.6	9.3
Lower Ice	8	90	1971	-1.1	0.811	1.8	1.0	2.6	5.5	6.4
Anniversary	9	75	1969	-0.8	0.768	1.9	0.0	1.1	5.3	6.4
Anniversary	9	103	1970	-1.8	0.895	1.8	-3.8	-2.1	7.0	7.5
Anniversary	9	62	1971	-0.4	0.880	1.6	0.4	0.9	5.6	6.1
Moraine	10	52	1969	-1.5	1.008	1.9	-2.4	-0.9	5.9	5.6
Moraine	10	53	1970	-0.7	0.949	2.0	-5.3	-4.9	7.5	7.6
Moraine	10	0	1971	-	-	-	-	-	-	-
Phantom	11	53	1969	-0.8	0.876	1.5	1.7	2.9	2.9	2.8
Phantom	11	32	1970	-0.7	0.701	1.9	-2.3	-2.3	3.3	3.8
Phantom	11	0	1971	-	-	-	-	-	-	-
Mean				-0.7	0.834	1.8				
Standard Dvn				0.4	0.122	0.3				

Table 5.5: Statistics for Steady State Models of Class 2 Situations in Axel Heiberg Island Expedition Area. Model is of Form $T = A + B.T_{IN}$ and Data are Unfiltered. N is Number of Days in Each Sample (During Period May–August). U is R.M.S. Error and S_T and S_{IN} are Standard Deviations of Daily Mean Temperature T and of T_{IN} Respectively.

Station	I	N	Year	A	B	U	$\bar{T}$	$\bar{T}_{IN}$	S_T	S_{IN}
Fjord	1	111	1969	0.0	0.722	2.4	0.0	0.0	3.2	2.9
Fjord	1	102	1970	0.0	0.608	2.0	0.0	0.0	2.4	2.2
Fjord	1	93	1971	0.0	0.654	2.0	0.0	0.0	2.8	2.9
Baby Gla.	4	80	1969	0.0	0.679	1.6	0.0	0.0	2.9	3.5
Baby Gla.	4	67	1970	0.0	0.553	1.6	0.0	0.0	2.0	2.2
Baby Gla.	4	37	1971	0.0	0.751	1.0	0.0	0.0	2.0	2.2
Outwash	7	102	1969	0.0	0.643	2.2	0.0	0.0	2.9	2.8
Outwash	7	111	1970	0.0	0.73[illegible]	1.8	0.0	0.0	2.5	2.2
Outwash	7	96	1971	0.0	0.652	1.9	0.0	0.0	2.7	2.8
Lower Ice	8	89	1960	0.0	0.719	1.4	0.0	0.0	2.2	2.3
Lower Ice	8	99	1961	0.0	0.723	1.9	0.0	0.0	2.4	2.0
Lower Ice	8	88	1969	0.0	0.75[illegible]	1.6	0.0	0.0	2.6	2.8
Lower Ice	8	109	1970	0.0	0.738	1.8	0.0	0.0	2.4	2.1
Lower Ice	8	90	1971	0.0	0.682	1.8	0.0	0.0	2.6	2.7
Anniversary	9	75	1969	0.0	0.760	1.9	0.0	0.0	2.9	2.9
Anniversary	9	103	1970	0.0	0.796	1.9	0.0	0.0	2.6	2.2
Anniversary	9	62	1971	0.0	0.782	1.7	0.0	0.0	2.7	2.6
Moraine	10	52	1969	0.0	0.934	2.2	0.0	0.0	3.9	3.5
Moraine	10	53	1970	0.0	0.597	1.9	0.0	0.0	2.2	2.0
Moraine	10	0	1971	-	-	-	-	-	-	-
Phantom	11	53	1969	0.0	0.695	1.7	0.0	0.0	2.6	2.8
Phantom	11	32	1970	0.0	0.589	1.8	0.0	0.0	2.1	1.6
Phantom	11	0	1971	-	-	-	-	-	-	-
Mean				0.0	0.703	1.8				
Standard Dvn				0.0	0.085	0.3				

Table 5.6: Statistics for Steady State Models of Class 2 Situations in Axel Heiberg Island Expedition Area. Model is of Form $T = A+B.T_{IN}$ and Data are High-Pass Filtered. N is number of Days in Each Sample (During Period May-August). U is R.M.S. Error and S_T and S_{IN} are Standard Deviations of Daily Mean Temperature T and of T_{IN} Respectively.

It can be first pointed out that the variances of A and B within each class are less than the variances of A and B without regard to class.

Secondly the mean values of A and B for each class seem to be significantly different (5 % level) from those for other classes (except for B for Class 1a which overlaps with B for Class 1 and Class 3). The means and estimated confidence intervals for the different classes are:

	A	B	N
Class 1	0.1 ∓ 0.2	0.999 ∓ 0.050	9
Class 1a	1.2 ∓ 0.7	0.933 ∓ 0.098	3
Class 2	-0.7 ∓ 0.2	0.834 ∓ 0.052	21
Class 3	-2.5 ∓ 0.4	0.890 ∓ 0.036	4

The standard deviations of A and B without regard to class are 1.0 and 0.123 respectively for sample sizes of 37. A One-Way-Analysis of Variance computation confirms that variations between the classes are much larger than variations within the classes. The former may be regarded as "differences in kind" and the latter as "differences in degree".

If the values of A and B for each situation are replaced by the mean values of A and B for the appropriate class an estimate of $\overline{T}$, denoted by $\overline{T}_P$, can be computed from $\overline{T}_{IN}$. The error $(\overline{T}_P-\overline{T})$ which will be denoted by E should be small if the classification is useful. The mean values and standard deviations of E ($\overline{E}$ and S_E) are as follows:

	$\overline{E}$	S_E	$\overline{A}$	$\overline{B}$	N
Class 1	0.1	0.4	0.1	0.999	9
Class 1a	0.1	0.7	1.2	0.933	3
Class 2	0.0	0.5	-0.7	0.834	21
Class 3	0.1	0.3	-2.5	0.890	4
Unclassified	0.0	1.0	-0.5	0.880	37

The spread of errors using the mean values of A and B for the unclassified models (sample size 37) is relatively large compared to the spread within each class. For example, the error introduced in assuming that all Class 2 situations are described by the same model is on average 0.0 with standard deviat-

Station	N	$\overline{T}_P$	$\overline{T}$	$\overline{E}$
Fjord	102	-0.9	-0.3	-0.6
Base Camp	108	-0.9	-1.0	+0.1
Valley	98	-0.4	-0.5	+0.1
Baby Gla.	96	-2.5	-2.8	+0.3
Gordon's	86	-2.0	-1.5	-0.5
Ermine	106	-1.7	-0.9	-0.8
Outwash	99	-0.5	-0.8	+0.3
Lower Ice	98	-1.0	-1.5	+0.5
Anniversary	94	-1.5	-1.7	+0.2
Moraine	49	-5.1	-4.6	-0.5

Table 5.7:

Comparison of Predicted Temperature T_P and Observed Temperature T for N Days During Summer Field Season 1972 (May-August) for Stations in Axel Heiberg Expedition Area.

ion 0.5°C. Actually, this could be reduced by introducing a new class, Class 2a, for Baby Glacier which would have the model $T = -0.4 + 0.867\ T_{IN}$. However, the process of introducing new classes should not be carried to the extreme (until ultimately there were as many classes as models), and error within a class is the price to be paid for having only a limited, but manageable, number of classes. The efficiency of the classification can be assessed by comparison of the variances of the errors: for Class 1 the efficiency is 84 %, Class 1a 51 %, Class 2 75 % and Class 3 91 %.

The classification is primarily according to location (situations for the same location in different periods belong to the same class). It might be claimed that classification should be according to predominant weather; if all periods of record at the various stations were identical for a particular year the classification would then be simply according to year. The influence of weather upon the models is discussed in the following chapter. Meanwhile, the point can be illustrated by assuming that models in Tables 5.1 to 5.6 belong to one of three classes: Class 1969, Class 1970 and Class 1971. Using the 8 locations, for which there are records in all three years, the following results are obtained.

	A	B
Class 1969	-0.1 ± 0.4	0.847 ± 0.087
Class 1970	-0.3 ± 0.8	0.880 ± 0.089
Class 1971	-0.3 ± 0.6	0.903 ± 0.097

The models for the Classes 1969, 1970 and 1971 are not significantly different from one another (at 5 % level) and such a classification serves no useful purpose.

A rather convincing test of the classification would be to use the mean values of A and B to compute mean temperatures $\overline{T}_P$ at various stations from the $\overline{T}_{IN}$ values for a completely new year and compare results to the observed values of $\overline{T}$. Data for the 1972 field season were used for such a test. Base Camp, Gordon's and Valley stations were assumed to be Class 1 stations. Ermine was a Class 1a station. Fjord, Outwash, Lower Ice, Anniversary and Moraine were Class 2 stations, and Baby Glacier was assumed to be Class 2a. Results are

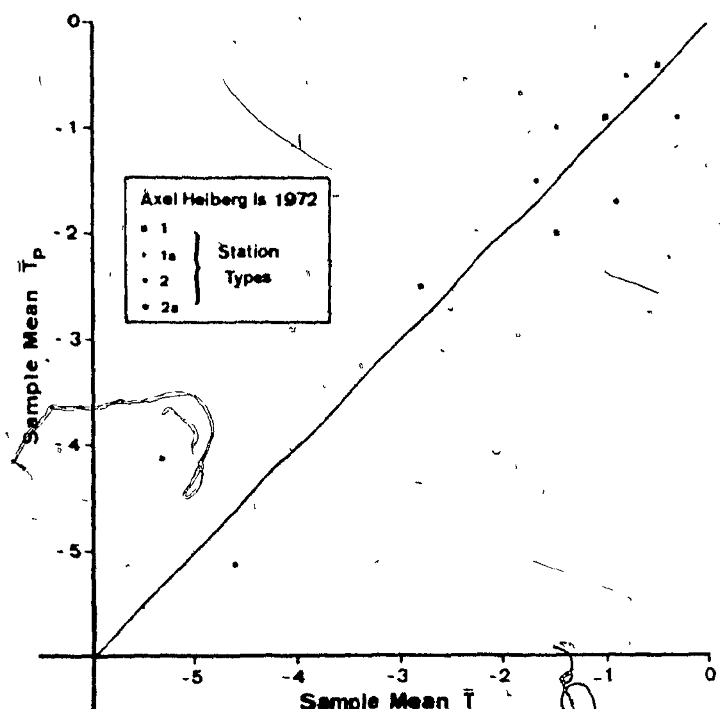


Fig 5.2: Comparison Between Sample Mean Values of Observed Local Temperature $\bar{T}$ and Computed Local Temperature $\bar{T}_P$ for 10 Stations in Expedition Area, Axel Heiberg Island, for Summer 1972. $\bar{T}_P$ is Computed from $\bar{T}_{IN}$ Using Appropriate Parameters Evaluated for Previous Summers. Errors are Denoted by Deviations from the 45° Line

given in Table 5.7 and Figure 5.2. $\bar{E}$ is defined as $(\bar{T}_p-\bar{T})$ and is represented by deviations from the straight line in Figure 5.2. N is the number of days of record at each station, and all mean values are computed for the period of length N. The results are quite good with $\bar{E}$ values well within the range of $\mp$1.0°C. The average of $\bar{E}$ over the 10 stations is only -0.1°C with 95 % confidence interval of $\mp$0.3°C. The mean errors for single months are a little worse: 0.3 $\mp$ 1.1°C (May), 0.0 $\mp$ 0.4°C (June), -0.3 $\mp$ 0.4°C (July) and -0.2 $\mp$ 0.3°C (August excluding Moraine for which there is no record). A Two-Way Analysis of Variance computation (Kreyszig, 1970, p.276) was carried out to try and detect significant differences in errors between months at the same station and between stations for the same month (see Appendix 2). Moraine was excluded from the calculation because data for August are missing, so that a 9 by 4 matrix was analysed. Both hypotheses, i.e. errors for months are different, and error for stations are different, were rejected at the 5 % significance level, and the monthly mean errors should be considered as random with respect to both month and station. This finding supports the validity of the classification and reinforces the suggestion that the influence of time (and possibly different prevailing weather types) is not significant.

It can also be shown that the models for the different classes are useful in describing the long-term records from Axel Heiberg stations. For the months of May to August in the period 1960-72 there are 35 months of record at Base Camp (months with at least 20 days of record), 21 months for Lower Ice and 14 months for Moraine (there are also several winters of record from the automatic weather station at Moraine). Monthly mean values of T_{IN} were computed by interpolation of Eureka and Isachsen data for every month (May-August) in the period 1960-72 for altitudes of 190, 210 and 870 m a.s.l. From these, values of $\bar{T}_p$ (synthetic series) were computed for the three stations. Base Camp was assumed to be Class 1 and Lower Ice and Moraine were assumed to be Class 2. The errors $(\bar{T}_p-\bar{T})$ at Base Camp have mean value -0.5°C with standard deviation 0.9°C for a sample size of 35 months. Corresponding values for Lower Ice are -0.2°C with standard deviation 0.9°C for 21 months and values for Moraine are +0.1°C with standard deviation 0.9°C for 14 months. As stated in Chapter 2(iii) the interpolation error should be about $\mp$0.7°C so that the standard deviations of the monthly mean errors are about as low as could be reasonably expected. The sy-

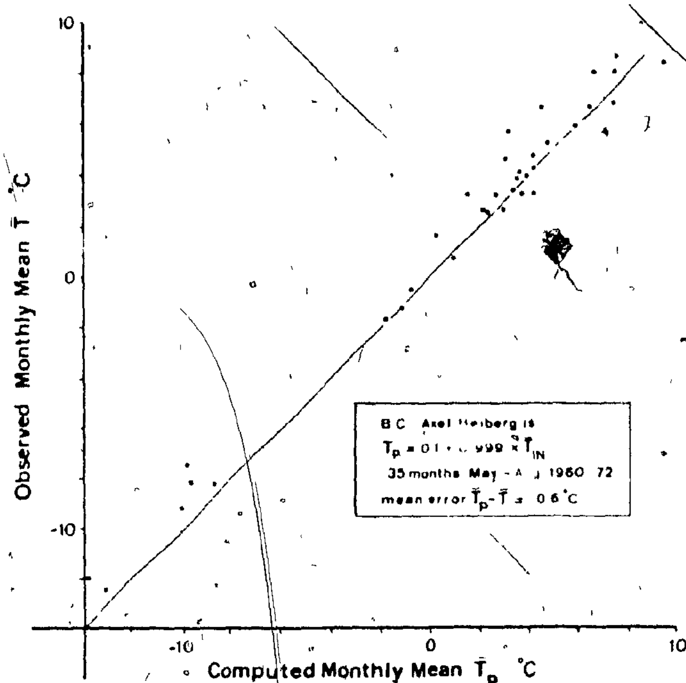


Fig 5.3: Comparison Between Observed $\overline{T}$ and Computed Monthly Mean Temperature $\overline{T}_p$ at Axel Heiberg Base Camp for 35 Months

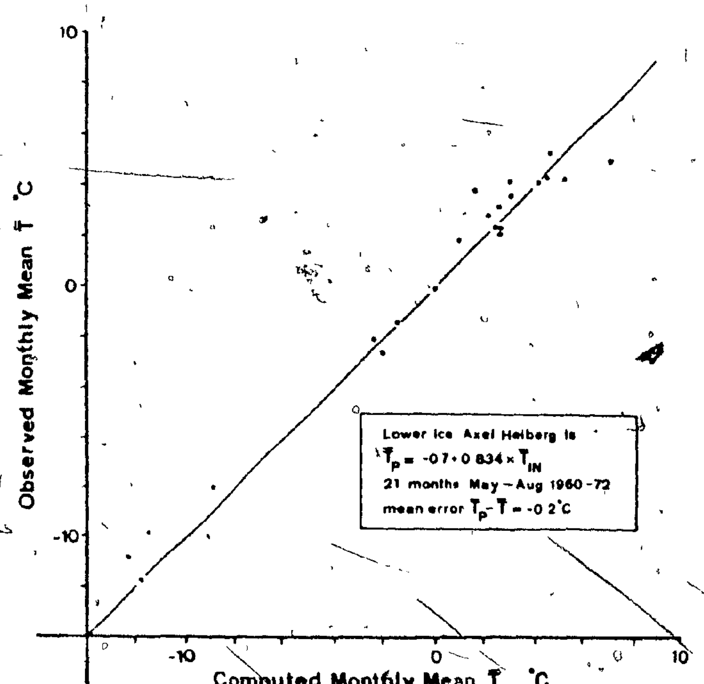


Fig 5.4: Comparison Between Observed $\overline{T}$ and Computed Monthly Mean Temperature $\overline{T}_p$ at Lower Ice, White Glacier, for 21 Months

stematic underestimation of $\overline{T}_p$ at Base Camp and Lower Ice is partly due to the fact that the $\overline{T}$ data include incomplete months of data (more than 20 days of record) with missing data mainly occurring in early May and late August. The very small error at Moraine is encouraging as the data mainly originated from a Rauchfuss Automatic Weather Station for which it was felt the calibration factors were problematic. The small systematic error at Moraine suggests that calibration was in fact correctly done (unless various errors compensated). The results are illustrated by scatter diagrams in Figures 5.3 to 5.5 where deviations from the $\overline{T}_p = \overline{T}$ line represent the monthly mean errors.

The previous two sections have mainly been concerned with discussion of what might be termed "low frequency response of the system relating T and T_{IN}". This has involved discussion of the variations of model parameters from situation to situation and the usefulness of classification of models.

In the following chapter attention will be directed to day-to-day errors involved in the models.

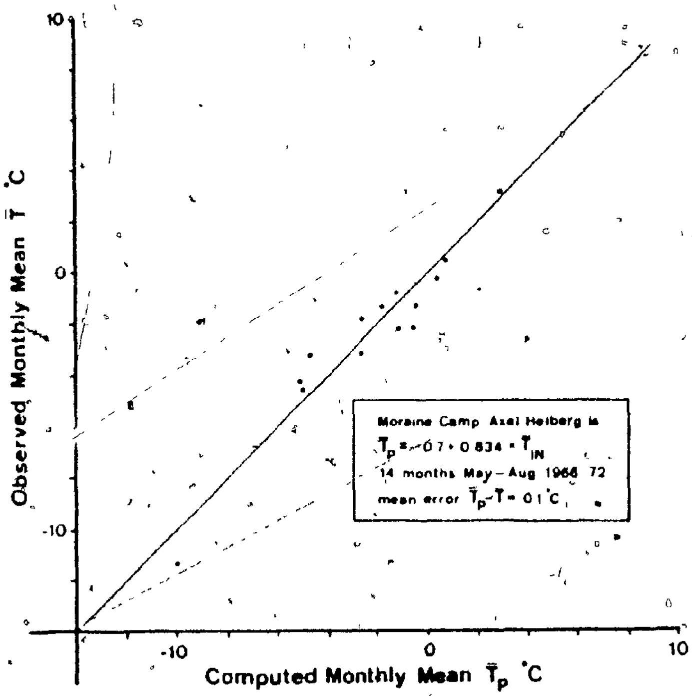


Fig 5.5 Comparison Between Observed $\bar{T}$ and Computed Monthly Mean Temperature $\bar{T}_p$ at Moraine Camp, White Glacier, for 14 Months

CHAPTER 6

THE RESIDUALS IN THE REGRESSION MODELS IN RELATION TO WEATHER

i) Introduction

The fitting of data for T(t) and $T_{IN}(t)$ to the model (3.1) involves an error (of standard deviation U) which will be termed the Residual of the regression equation. This error will have magnitude U*(t) on the t th day:

$$U^*(t) = U\epsilon(t) = T(t) - A - B\,T_{IN}(t) \qquad (6.1)$$

Clearly any time-dependent process which influences T(t) but is independent of $T_{IN}(t)$ will be "lumped" into U*(t). If it were possible to identify these processes and include them explicitly in the model an improvement in accuracy could be made. The formal assumption is made that U*(t) has zero mean value, but in fact the unknown "error processes" could have non-zero mean value and interact with $T_{IN}(t)$ so that A and B would be influenced.

Possible errors could have their origin in the large-scale atmosphere (e.g. interpolation error), in the local circulation around the glacier (fluctuations of wind speed, exchange coefficients etc, especially during Foehn events), in radiative processes (by way of the underlying surface) or in condensation. Although it is allowable to try and identify these errors using detailed local information (e.g. Base Camp Surface Weather Record) the exercise will only be useful if they can ultimately be expressed in terms of data at distant weather stations, weather maps, satellite images or astronomical factors.

ii) The Residuals at Base Camp

The patterns of residuals at the various stations in any particular year are remarkably similar. For example, U*(t) at Base Camp and Lower Ice can be quite accurately "matched" and, in general, residuals at any pair of stations are correlated with a correlation coefficient which is strongly influenced by the altitude difference between the stations. This suggests that the error processes are not too strongly localized and that the residuals at Base Camp can be treated as indices of residuals at other stations.

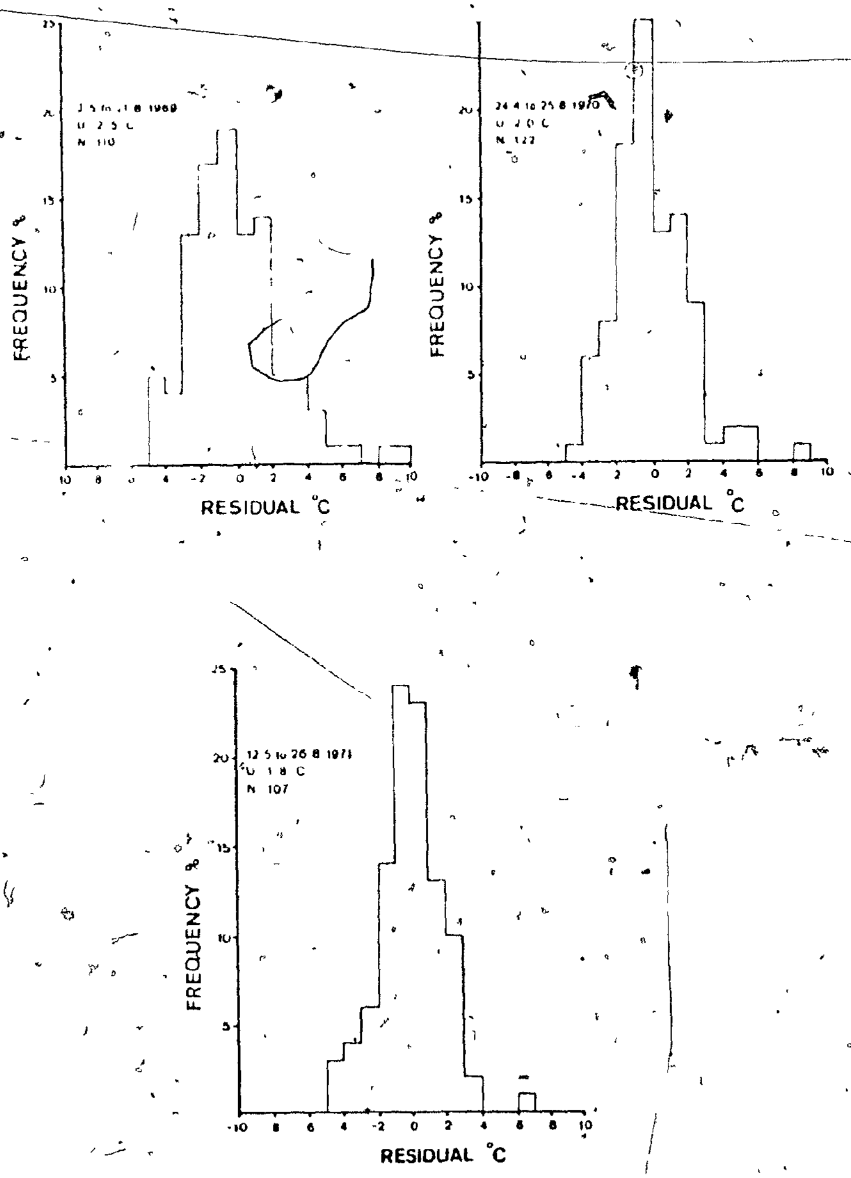


Fig 6.1: Distribution Histograms of the Residuals in the Linear Model Relating T(t) to $T_{IN}(t)$ at Base Camp, Axel Heiberg Island, for the Summers 1969-71. U is the Standard Deviation and N is the Sample Size

The residuals at Base Camp are strongly time-autocorrelated (autocorrelation coefficient of about 0.5 for a First-Order Process) and are inhomogeneous according to Abbe's Criterion (Conrad, 1944, Chapter XXII). The residuals do appear to be stationary at the monthly mean level. Distribution histograms are given in Figure 6.1. It can be seen that the distribution of residuals is somewhat skewed with some relatively large (5 to 10°C) positive values occuring. The distributions do not appear to be markedly bimodal unlike the temperature itself which often shows bimodality, e.g. Orvig (1951, Figure 11). Orvig (1954, Figure 18), Andrews (1964, Figure 8) and Müller and Roskin-Sharlin (1967, Figure 9).

The pattern of residuals was investigated to attempt to identify effects due to:

1) wind, particularly under Foehn conditions
2) stability of the atmosphere
3) precipitation
4) sunshine/sky cover as measures of short-wave radiation.

Figures 6.2 to 6.4 illustrate the courses of U*(t) and other elements throughout the three seasons: 1969, 1970 and 1971. Explanation of the various elements will be given in the following sections.

iii) Effects of Foehn, Wind Direction and Atmospheric Stability

Foehn is a precisely formulated condition (Defant, 1951). It is not directly observable in the Axel Heiberg Expedition area because of the wide separation (10^2 km scale) of weather stations. Periods of comparatively high temperature, low humidity and high gusty wind (usually north to north-easterly) occurring in the Expedition area are usually attributed to "Foehn" (Müller and Roskin-Sharlin, 1967, p.33).

Foehn periods for 1969-71 (Figures 6.2 to 6.4) were identified subjectively, mainly by rapid rises of temperature and humidity reflected in the daily mean values. It can be seen from Figures 6.2 to 6.4 that large positive residuals (greater than +2°C) do often occur during Foehn periods but not always. For example, in 1969 the Foehn periods F2, F7, F8, F10, F12, F13 and F14 are associated with large positive residuals whilst F3, F4, F5, F6, F9 and F11 are

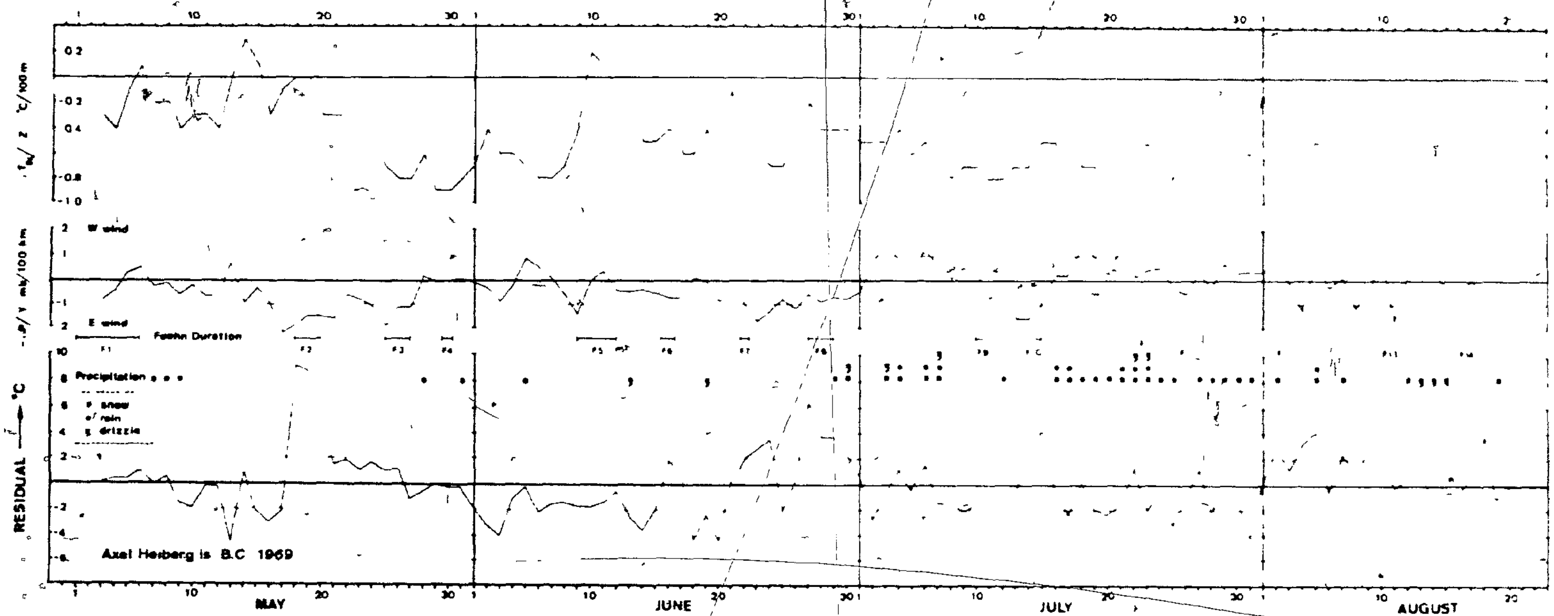


Fig 6.2 : Seasonal Course of Residuals in the Regression Equation for Daily Mean Air Temperature at Axel Heiberg Base Camp, Summer 1969, in Comparison to Other Elements: Precipitation Occurence, Foehn Occurence, North-South Pressure Gradient and Vertical Temperature Gradient in the Large-Scale Atmosphere

not. In the latter cases, the rapid rises in temperature T(t) are reasonably well explained by rises in $T_{IN}(t)$. This illustrates the fact that not all rapid changes in local temperature can be attributed to Foehn on the meso-scale and that some have their origin in the large-scale atmosphere, see Müller and Roskin-Sharlin (1967, p.60). On the other hand, there appears to be only one case in the three years when the residual was greater than +2°C without a Foehn, this occurred several days after Foehn F5 in 1971.

Daily mean geostrophic wind was computed for the point (80°N, 90°W) using surface pressure at Eureka, Isachsen, Resolute and Alert for every day of record in the three summers. The results are reasonably consistent with weather maps. The north-south pressure gradient at (80°N, 90°W), denoted by $-\Delta P/\Delta Y$ in units of mb 100 km^{-1}, is plotted in Figures 6.2 to 6.4. Negative values are associated with east geostrophic wind and positive values with west geostrophic wind. It can be seen that Foehn is often associated with east wind but not always, e.g. Foehn F5 in 1970.

The average vertical temperature gradient between 1000 mb and 850 mb was also computed for every day of record and is plotted in Figures 6.2 to 6.4 as $\Delta T_{IN}/\Delta Z$ °C 100 m^{-1}. This is a measure of the atmospheric stability. The stability changes from +ive values early in the season to -ive values for most of the record and does not appear to be related to the residuals. It is noteworthy that some of the "Foehn" events which are not accompanied by high positive residuals are accompanied by relatively large negative vertical temperature gradients. This would be consistent with an increase of T_{IN}, causing an increase in T, which is due to large-scale anticyclonic subsidence rather than subsidence in a genuine, smaller-scale, Foehn (Müller, personal communication).

Residuals for each summer were classified according to the computed geostrophic wind direction. The mean and standard deviations of residuals for the various geostrophic wind directions were as follows:

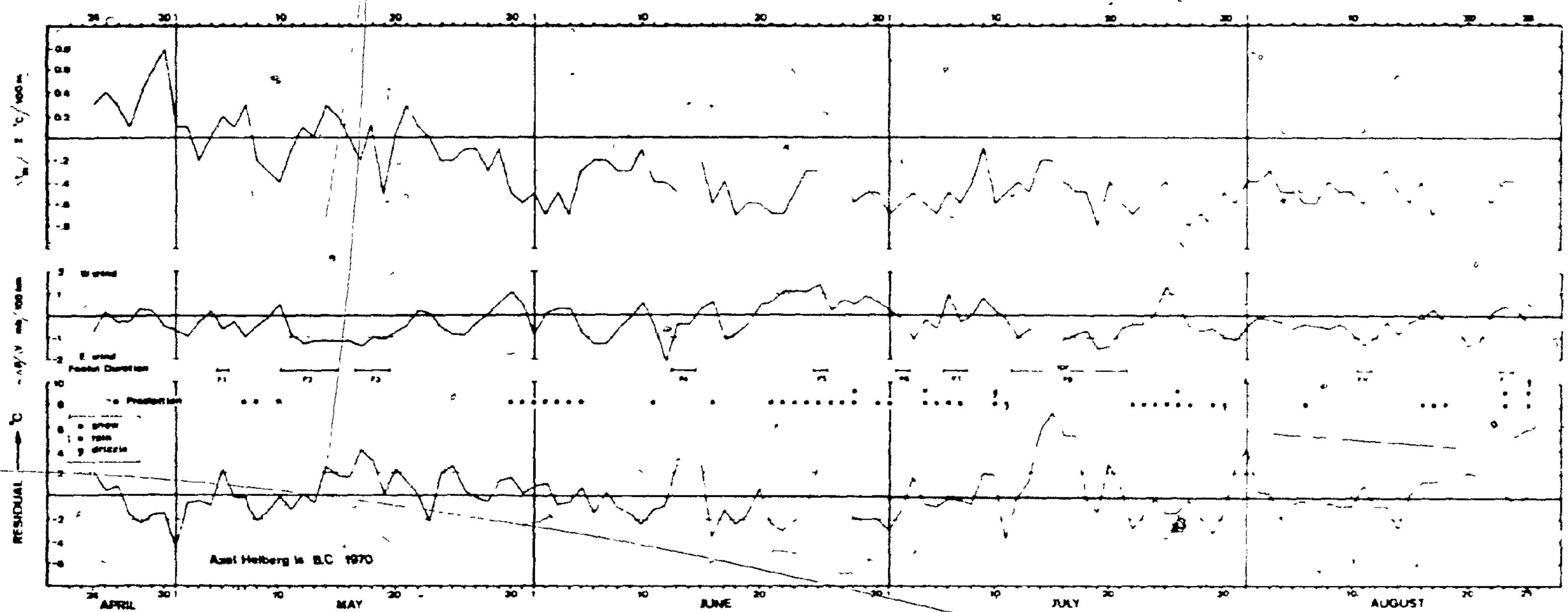


Fig 6.3: Seasonal Course of Residuals in the Regression Equation for Daily Mean Air Temperature at Axel Heiberg Base Camp, Summer 1970, in Comparison to Other Elements: Precipitation Occurence, Foehn Occurence, North-South Pressure Gradient and Vertical Temperature Gradient in the Large-Scale Atmosphere

[illegible] Camp 19[illegible]

	N	NE	E	SE	S	SW	W	NW
Mean	-1.8	-0.4	0.7	-0.8	0.1	-0.1	-0.7	-1.2
Standard Dev.	-	1.0	3.0	1.8	0.3	1.3	2.0	3
Sample Size	1	5	52	10	3	10	27	3

Base Camp 19[illegible]

	N	NE	E	SE	S	SW	W	NW
Mean	0.6	-1.3	0.4	-1.5	-	-0.7	-0.6	-0.3
Standard Dev.	1.0	1.9	2.2	-	-	1.3	1.7	1.7
Sample Size	[illegible]	4	75	1	0	5	29	6

Base Camp 19[illegible]

	N	NE	E	SE	S	SW	W	NW
Mean	0.7	0.6	0.0	0.7	-	-1.2	-0.4	-0.1
Standard Dev.	1.7	2.3	1.9	1.3	-	0.4	1.6	1.1
Sample Size	4	4	62	4	0	2	25	6

The predominance of east wind with fairly frequent west wind also is consistent between the years, but the relation between wind direction and mean residual is not clear except with respect to sign, with a tendency to positive residuals for east wind and negative with west wind. One-Way Analysis of Variance confirms that the residuals are best regarded as random with respect to wind direction (5% significance level).

In summary it seems that Foehn can cause quite large positive values of the residual such that the model (3.1) will substantially underestimate the temperature under Foehn conditions. However, not all Foehns have this effect, possibly this may help to identify genuine local Foehns as opposed to the large-scale atmospheric warmings. Foehn does not seem to be predictable in terms of geostrophic wind direction nor are the residuals significantly dependent upon wind direction. An effort to include geostrophic wind as a weighting factor of $T_{IN}(t)$ in a regression model gave very poor results.

In this discussion, little attention has been paid to large negative values of residuals. This is because the mean residual is zero by definition and the

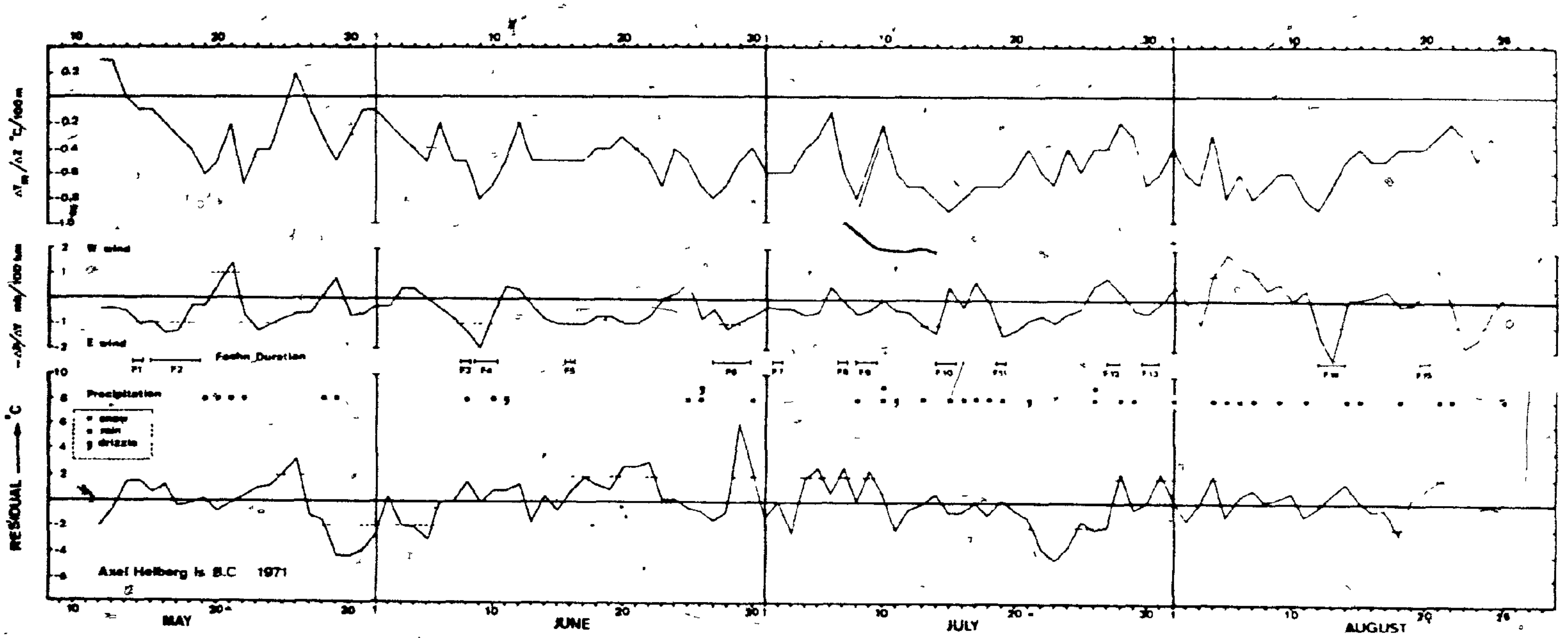


Fig 6.4: Seasonal Course of Residuals in the Regression Equation for Daily Mean Air Temperature at Axel Heiberg Base Camp, Summer 1971, in Comparison to Other Elements: Precipitation Occurence, Foehn Occurence, North-South Pressure Gradient and Vertical Temperature Gradient in the Large-Scale Atmosphere

$L^*(t)$ [illegible] "floating" with large negative errors partly forced in compensation to [illegible] positive errors which are in any case more frequent (Figure 6.1).

iv) Effect [illegible] Sunshine, Sky Cover and Precipitation

Variations of sunshine duration/sky cover (both related to global radiation) undoubtedly influence the temperature. For example, at Eureka the following correlations were found between monthly mean temperature (T_E), monthly mean sky cover in tenths (N_E) and monthly total precipitation in inches (P_E):

		T_E	N_E	P_E
Temperature	T_E	1.00	0.5[illegible]	-0.19
Sky Cover	N_E		1.00	0.39
Precipitation	P_E			1.00

The data were expressed as deviations from the monthly norms for 1961-70 and the computed statistics are valid for the months June-August in the period 1961-70 (sample size of 30). Similar values were found for Isachsen. T_{IN} which is interpolated between Eureka and Isachsen will, therefore, carry some information about Eureka and Isachsen sky cover. There are indications that sky cover is strongly correlated between Eureka and Isachsen (R = 0.80).

It was chosen to analyse the residuals at Base Camp in terms of daily sunshine duration as measured at Base Camp. The means and standard deviations of the residuals for 5 sunshine classes (0-4 hrs, 5-9 hrs, 10-14 hrs, 15-19 hrs and 20-24 hrs) are given below:

Base Camp 1969

Sunshine Class	00-04	05-09	10-14	15-19	20-24 hrs
Mean	-0.4	-0.7	-0.2	-0.6	1.6
Standard Dvn	2.1	1.6	2.3	2.0	3.5
Sample Size	38	19	17	12	23

Base Camp [illegible]

Sunshine Class	00-04	05-09	10-14	15-19	20-24 hrs
Mean	-0.[illegible]	-0.3	0.1	0.2	1.3
Standard Dvn	1.[illegible]	1.9	2.5	2.0	2.1
Sample Size	36	16	22	18	17

Base Camp [illegible]

Sunshine Class	00-04	05-09	10-14	15-10	20-24 hrs
Mean	[illegible]	0.2	1.3	-0.8	0.4
Standard Dvn	[illegible]	1.3	1.6	1.3	2.2
Sample Size	[illegible]	12	7	7	28

In two of the cases (1969 and 1970) there are certainly large mean positive residuals in the 20-24 hr sunshine class as one might have expected. But this is not the case for 1971. For the 1971 case the hypothesis that the average residuals are not significantly different cannot be rejected at 5% level (using One-Way Analysis of Variance). If the 20-24 hr sunshine class is excluded from consideration similar results are obtained for 1969 and 1970. It is concluded that, although sunshine duration may have an influence, the influence is hardly systematic and quantitatively useful. As already said, the residuals at Lower Ice are almost identical with those at Base Camp. This finding is, therefore, consistent with the finding of Havens et al (1965, Figure 7) that there is only weak relationship between insolation and cooling effect of White Glacier (defined as the temperature difference between Lower Ice and Base Camp).

Precipitation occurrences of three types (rain, drizzle and snow) are indicated in Figures 6.2 to 6.4. There does not appear to be any marked relationship with the residuals except in so far as precipitation does not usually occur during the Foehns which give large positive residuals.

v) Conclusions

With the exception of Foehn occurrences there does not appear to be the strong relationship between the residuals and various "weather" factors which might have been expected. Even Foehn does not appear to be quantitatively consistent in its effects. This all may seem rather surprising, but it should be remembered that T_{IN} is itself a "weather" factor controlled by the general circulation and external inputs to the atmosphere. Apparently T_{IN} carries much the same information as T does. The origins of U*(t) are, therefore, left unexplained. This is not very satisfactory. The main factor is probably interpolation error in T_{IN}, and the other factors such as Foehn, wind, radiation, precipitation etc probably do play roles, but the analysis has failed to isolate them. From the analyses in the previous chapter and in Appendix 2 it seems that the overall climatic influence of the residuals is small on the relatively long (10^1 day) time scale.

It is suggested that the hypothesis of a relationship between T(t) and $T_{IN}(t)$ has been adequately tested and not falsified. Furthermore, the parameters involved in the models do show certain consistencies. Finally, the method does allow, once parameters have been computed, the computation of reasonably accurate synthetic series of temperature.

It now remains to demonstrate glaciological applications of the computed air temperature. This will be done in two stages in the following four chapters.

A Note on the Autocorrelation of Residuals

As already stated on page 43, the residuals in the model for T(t) versus $T_{IN}(t)$ are autocorrelated and inhomogeneous. If the statistics in Tables 5.1 to 5.6 were to be used for computation of confidence intervals for individual B values, an adjustment would have to be made for this effect. Typical first-order autocorrelation coefficients are in the range 0.5 to 0.6 which would increase confidence intervals by about 30-110% (Johnston, p.247, 1963) above the values estimated according to the usual ordinary least-squares formula. Autocorrelation of the residuals should have little effect upon averaged values of B computed for each class, see also page 24.

CHAPTER

STATISTICAL ANALYSIS OF ENERGY BALANCE DATA FROM ARCTIC GLACIERS

1) Introduction

It has long been known that glacier mass balance and ablation are related to air temperature for example Agassiz (1840, p.201), Hobbs (1911, p.4), Ahlmann (1922, p.13 and p.50-51) and Lliboutry (1964-65, p.452 and p.835) amongst others. It has also been long known that the energy required for melting ice is governed by the energy balance at the glacier surface.

Early authors such as Heim (1885, p.235), Hess (1904, p.210) and Forel *et al* (1909, p.19) amongst others speculated that radiation should be the major heat source for ablation. Measurement of the energy balance at glacier surfaces, usually over short summer periods, have tended to confirm the importance of radiation as a heat source. For example, Paterson (1969, Table 4.2) quotes results from 32 energy balance studies, and in 24 cases (75 %) the net radiation is the major heat source. It is, of course, accepted that the relative importances of the various heat sources vary with season, latitude, altitude and "weather" etc in a complex way, and efforts to describe these variations in detail have not been very successful, for example see Andrews (1975, Chapter 2). The difficulty in interpretation of energy balance results has led Paterson (1969, p.62) to state: "However, energy balance studies have so far yielded detailed information only about the particular place where they were carried out."

The notion that radiation is "most important" has led many people to believe that ablation and radiation time-series should be strongly correlated. Studies of glacier run-off, related to the ablation, have often failed to show significant correlation between run-off and radiative factors such as cloudiness and sunshine duration although significant correlations with air temperature or humidity (and, naturally, precipitation) are found. Some examples are reported by Lang (1968), Østrem and Pytte (1968), Pytte (1969), Tvede (1971), Goodison (1972), Jensen and Lang (1974) and Østrem (1974) amongst others. On the other hand, Nakawo *et al* (1976) and Dosdov & Mosolova (1975) do report high correlations between glacier run-off and radiation for glaciers in the Himalayas and Pamirs respectively. However, the latter do also state that the correlation is negligible for glaciers in the Polar Urals. Also noteworthy is the fact that Goodison (1973) found very poor correlation between ablation and global radiation [illegible]-fields.

Despite the above results, a generally accepted point of view might be that radiation is [illegible] the "[illegible] important" factor in the ablation [illegible] that temperature [illegible] a "useful" factor because it is easily measured and is a function of other factors, particularly radiation. For example: "The temperature is the most important meteorological element in the study of ablation conditions, for it is a function of all other different meteorological factors, such as insolation, wind and humidity, which influence the ablation" - [illegible] (19[illegible]1, p.[illegible]). Or again: "As many of the meteorological parameters are interrelated, one cannot postulate that any one of them is more important than the others. However, experience has shown that air temperature (which in most cases is dependent upon incoming radiation) is closely correlated with the ablation, so that temperature observations should be considered important in cases in which only a limited observation-programme can be carried out" - Østrem and Stanley (1969, p.[illegible]). Similar statements are made or implied by Hoinkes (1955, p.501 and 1968a, p.254), Muller (1963b, p.60), Lliboutry (1964-65, p.357), Lang (1968) and Østrem (1974) amongst many others. The notion that temperature is "important" as an expression of radiation appears to be implicit in statements (which would be otherwise illogical) such as: "... solar radiation received at the ground or ice surface probably exerts the most important influence upon ablation. Thus there is a close relationship between regional temperature characteristics and glacierization" - Sugden and John (1976, p.85).

It is hereby suggested that this point of view does not completely solve the problem. In this the author follows Ahlmann (1922, p.13): "With regard to the melting factors, very great importance has of late been ascribed to insolation. The reasons which have been put forward for this, however, cannot be regarded as decisive by the present writer, on various grounds, which will be expounded in another treatise. Accordingly I still maintain that the factor which plays the greatest role in melting is the temperature of the air."

In the following sections a solution to the problem of the role of air temperature in the ablation process, particularly in comparison to radiation, will be proposed. The approach taken involves statistical analysis of time-series of measured energy balance data. This approach should combine "physics" and "statistics" in a synthesis.

Meanwhile it might be pointed out that the use of the term "most important heat source" for radiation has led to much confusion. What is meant is actually "on average the largest heat source". The notion that radiation is "most important" has led many people to assert that glacier ablation must vary in response to variations of radiation (e.g.Hoinkes, 1955, p.500-501) without testing that this is in fact the case. The notion that radiation is "on average the largest heat source" leads to no such conclusion and makes it clear that the variability of the various heat sources must also be considered. Clearly, the largest heat source need not be the most variable.

ii) Statistical Properties of the Energy Balance Equation

The energy balance approach in glacier-meteorology involves the measurement, or estimation, of the various heat sources and sinks, and computation of the heat required to melt ice as the residual in the energy balance equation. The ablation at time t computed by this method is denoted by $A_C(t)$. By definition:

$$A_c(t) = \frac{1}{L}\sum_{j=1}^{j=N} E_j(t) \qquad (7.1)$$

where $E_1(t) \ldots E_N(t)$ are measured values of N sources and sinks (sources are +ive and sinks -ive), and L is the Latent Heat of Fusion of ice. In fact, the various energy balance terms cannot be measured without error so that $A_C(t)$ is not identical to the true, but unknown, ablation A(t). The ablation can also be measured by direct glaciological methods to give $A_M(t)$. This cannot be done without error so that $A_M(t)$ is not identical to A(t) or $A_C(t)$. The discrepancy between $A_C(t)$ and $A_M(t)$ reflects the combined effects of errors in the two separate sets of measurements. This complex problem is discussed at length by Muller and Keeler (1969).

Under certain simple assumptions (Appendix 3) Eq. (7.1) gives rise to a set of N equations describing the correlations between A_C and the various terms

$E_1 \ldots E_N$

$$R(A_c, E_k) = \frac{1}{LS_c} \sum_{j=1}^{j=N} S_j R(E_j, E_k) \quad (k = 1, N) \tag{7.2}$$

where $R(A_C, E_k)$ is the correlation between A_C and the k th energy balance term E_k and $R(E_j, E_k)$ is the correlation between j th and k th energy balance terms and represents the interaction between the terms. S_C and S_k are the standard deviations of A_C and E_k respectively.

The magnitudes of the interactions between terms are in general unknown. They are not specified by energy balance "theory". If A_C and A_M are approximately equal then equation (7.2) will hold, as an approximation, for A_M also.

If it is assumed that the energy balance components are mutually independent of each other (orthogonal) then (7.2) reduces to a particularly simple form (Appendix 3):

$$R(A_c, E_k) = S_k / L S_c \tag{7.3}$$

In this case the correlation between A_C and E_k depends only upon the ratio of their standard deviations. This expresses mathematically the common-sense idea that large variations in A_C must be put there by large variations in one or other of the energy balance terms. Although the assumption of independence of energy balance components is possibly a dubious one, so that (7.3) cannot be perfectly true, the conclusion should be true in a qualitative sense at least. In any case, the correlations between A_C and any particular energy balance component E_k will have nothing to do with the mean value of E_k which is $\overline{E_k}$. When one says, for example, that radiation is "dominant" one means that it has larger mean value than other energy balance terms. According to Equation (7.2) or (7.3) there is no reason why, for example, radiation should not have the largest mean value amongst the energy balance components whilst some other energy balance component correlates highest with the ablation. This does not appear to have been stated anywhere in the literature, but Tvede (1974, p.81) comes very close: "Noen god korrelasjon mellom netto tilført stråling og ablasjon er det ikke mulig å finne hverken på Ålfotbreen eller Nigardsbreen. Det er åpenbart variasjon i energitilførselen fra konveksjon og kondensasjon some i det alt vesentlige bestemmer ablasjonsvariasjonene fra døgn. Både fig 70 og tilsvarende illustrasjoner i tidligere rapporter bekrefter at strålingen tilfører breene i Sør-Norge en noenlunde jevn "grunnsum" av ablasjonsenergi tran gjennom sommeren". (Richard Hartley kindly provided an English translation as follows: *"It is not possible to find good correlation between net radiation and ablation on either Ålfotbreen or Nigardsbreen. It is clearly variation in the energy supply from convection and condensation which essentially determines the variations in ablation from day to day. Both fig 70 and corresponding illustrations in earlier reports confirm that radiation supplies the glaciers in South Norway with a more or less constant base amount of ablation energy throughout the summer."*)

The regression equation for A_C in terms of E_k would be:

$$A_c(t) = A_o + B E_k + U_k \epsilon_k(t) \qquad (7.4)$$

Under the assumption (7.3) the values of B should be 1/L whilst A_o and $U_k \epsilon_k(t)$ will contain the mean values and deviations respectively of all the energy balance terms except E_k in a "lumped" fashion. The magnitude of U_k would be $\sqrt{S_c^2 - S_k^2/L^2}$

Several questions arise:

1) which energy balance term has the highest standard deviation (is the most variable) ?

2) does this term correlate best with A_C as implied by equation (7.3) ?

3) what are the interactions or intercorrelations between the various energy balance terms ?

4) with which of the energy balance terms is temperature related ?

5) how consistent or repeatable are the various regression coefficients for different situations ?

Some answers to these questions for four specific cases will be given in the following sections on the basis of statistical analysis of field data from the White Glacier, Axel Heiberg Island, and the Sverdrup Glacier, Devon Island.

iii) The Data Analysed

Four high quality series of energy balance data were analysed. The data comprise series of measurements for net radiation Q_R, sensible heat Q_S, latent heat Q_L, heat conduction into ice Q_I, heat supplied by precipitation Q_P, the computed ablation A_C and measured ablation A_M together with mean temperature for the same periods of measurement T. In all cases the ablation data refer to melting of ice. The samples are as follows:

a) White Glacier 1960: measurements at Lower Ice station at 210 m a.s.l. on White Glacier. Sample size 16 periods of irregular duration in the period 8/7 - 10/8/60. Data were extracted from Andrews (1964, Tables XIII and XVIII). Air temperature data at 170 cm above glacier surface were taken from Andrews (1964, Table I-C, Appendix A). Energy balance data were scaled to units of Ly dy^{-1}. It might be expected in this case that A_C and A_M should have a built-in correlation with one another as A_M was used to determine the exchange coefficients for the turbulent fluxes.

b) White Glacier 1961: measurements at Lower Ice. Sample size 63 periods of approximately daily duration in the period 12/6 - 18/8/61. Energy balance data given by Muller and Keeler (1969) and temperature data at 150 cm above the glacier surface from Muller and Roskin-Sharlin (1967, Table I-A, Appendix A).

c) White Glacier 1962: measurements at Lower Ice. Sample size 11 periods of irregular duration in period 16/7 - 31/7/62. Energy balance data given by Müller and Keeler (1969) and temperature data at 150 cm above glacier surface from Havens *et al* (1965, Table 9, Appendix B). Energy balance data scaled to units of Ly dy^{-1}.

d) Sverdrup Glacier 1963: measurements at 300 m a.s.l. on Sverdrup Glacier, Devon Island. Sample size 33 periods of approximately daily duration in period 9/7 - 10/8/63. Energy balance data given by Müller and Keeler (1969) and temperature data at 220 cm above glacier surface from Keeler (1964, Table 9).

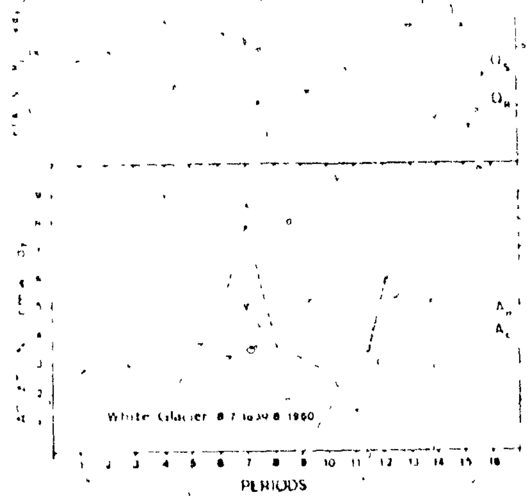


Fig 7.1: Comparison of Radiative and Sensible Heat Fluxes, Q_R & Q_S, With Measured and Computed Ablation, A_M & A_C, for 16 Periods in Summer 1960 on White Glacier, Axel Heiberg Island

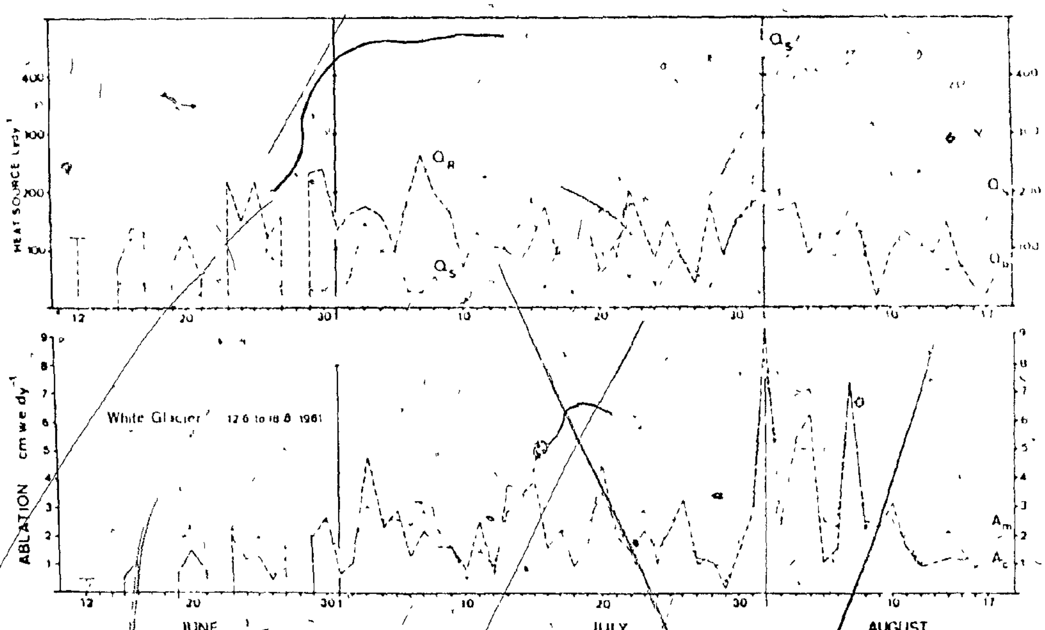


Fig 7.2: Comparison of Radiative and Sensible Heat Fluxes, Q_R & Q_S, With Measured and Computed Ablation, A_M & A_C, for 63 Days in Summer 1961 on White Glacier, Axel Heiberg Island

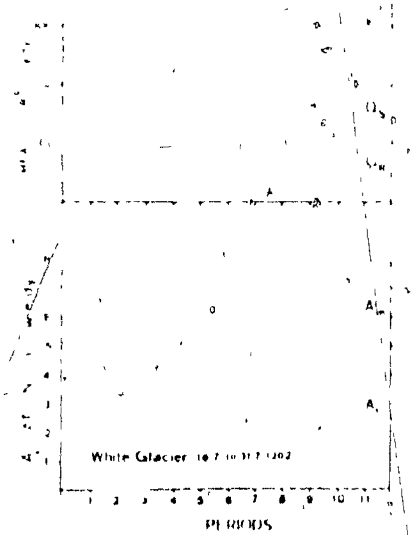


Fig 7.3: Comparison of Radiative and Sensible Heat Fluxes, Q_R & Q_S, With Measured and Computed Ablation, A_M & A_C, for 11 Periods in Summer 1962 on White Glacier, Axel Heiberg Island

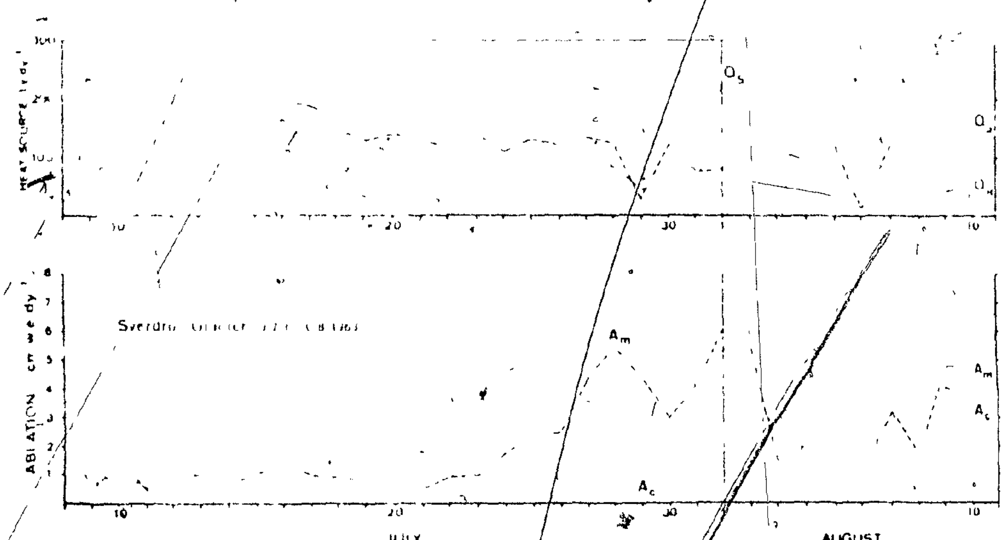


Fig 7.4: Comparison of Radiative and Sensible Heat Fluxes, Q_R & Q_S, With Measured and Computed Ablation, A_M & A_C, for 33 Days in Summer 1963 on Sverdrup Glacier, Devon Island

[illegible] two different estimates of ablation, i.e. $A_C(t)$ and $A_M(t)$, [illegible] equally [illegible] of the true, but unknown, ablation. [illegible] between the two estimates seems quite good, but their [illegible] $A_C(t)$ [illegible] significantly different from [illegible]) and are inhomogeneous in three ca[illegible] [illegible] criterion [illegible], 19[illegible], Chapter XII). These results [illegible] the following.

Situation	N	$\overline{d}$	S_d	K_d	AC	$1/\sqrt{N}$
[illegible]	16	-0.56	0.99	0.[illegible]	1.29	0.25
[illegible]	63	0.21	0.71	0.18	1.17	0.13
[illegible]	11	-0.56	1.67	0.99	0.85	0.30
Sverdrup Glacier 1963	33	0.01	1.24	0.42	3.35	0.17

where [illegible] sample size, $\overline{d}$ and S_d are the sample mean and standard deviation of the difference [illegible]$(t)-A_M(t)$) in cm H_2O, K_d is the 95 % confidence interval associated with [illegible] is Abbe's Statistic. It should be recalled that the condition for a homogeneous series is that $1-\frac{1}{\sqrt{N}} \leqslant AC \leqslant 1+\frac{1}{\sqrt{N}}$ with 67 % probability.

Despite the near perfect agreement of $\overline{A}_C$ and $\overline{A}_M$ for Sverdrup Glacier, with d = 0.01 cm, it is best to regard even this as accidental on account of the extreme inhomogeneity of the series of differences. Comparison of the traces for the two ablation estimates in Figures 7.1 to 7.4 support this view, for example in the case of Sverdrup 1963 A_C consistently overestimates A_M up to July 26 and then the situation is reversed.

Actually, Figures 7.1 to 7.4 are very instructive. Even without a mathematical analysis it can be seen that Q_R does not generally control ablation, A_C or A_M. In Figures 7.1, 7.2 and 7.4 it is clear that maxima in ablation do not occur together with maxima in Q_R.

iv) Results of the Statistical Analysis

Means, standard deviations and correlation coefficients for the four situations are given in Tables 7.1 to 7.4. Regression models of ablation in terms of various factors, i.e. radiation, sensible heat and temperature, are given in Table 7.5. Models for A_C in terms of sensible heat, observed ablation

WHITE GLACIER 1960

	Q_R	Q_S	Q_L	Q_P	Q_I	A_C	A_M	$\bar{T}$
	Ly/Dy	Ly/Dy	Ly/Dy	Ly/Dy	Ly/Dy	cm	cm	°C
Mean	162.4	129.0	77.0	0.0	-26.7	4.3	3.8	5.9
Standard Dvn	40.6	89.2	57.8	0.0	3.2	2.0	1.7	2.0

Intercorrelation Coefficient Matrix

Q_R	1.000	0.110	0.105		-0.330	0.348	0.271	0.116
Q_S		1.000	0.959		-0.377	0.965	0.858	0.8[illegible]3
Q_L			1.000		0.325	0.953	0.811	0.735
Q_P								
Q_I					1.000	0.268	0.189	0.356
A_C						1.000	0.868	0.781
A_M							1.000	0.669
T								1.000

Table 7.1: Summary Statistics of Energy Balance Study Carried Out at Lower Ice Station on White Glacier, Axel Heiberg Island, July 8 to August 19 1960. Sample Size = 16 Sampling Periods. Correlation Coefficients Greater Than 0.426 are Significant at 5 % Level. A_M was Used in Estimation of Q_S, Q_L and A_C.

WHITE GLACIER [illegible]

	Q_R	Q_S	Q_L	Q_P	Q_I	A_C	A_M	$\overline{T}$
	Ly/Dy	Ly/Dy	Ly/Dy	Ly/Dy	Ly/Dy	cm	cm	°C
Mean	[illegible]8.8	99.4	0.0	0.6	-45.5	2.3	2.1	3.2
Standard Dev.	54.9	104.7	42.2	2.1	18.3	1.6	1.7	2.2

Intercorrelation Coefficient Matrix

Q_R	1.000	-0.061	-0.124	-0.174	-0.352	0.284	0.105	0.059
Q_S		1.000	0.124	0.029	0.136	0.857	0.867	0.791
Q_L			1.000	0.326	0.316	0.429	0.364	0.432
Q_P				1.000	0.126	0.095	0.231	0.191
Q_I					1.000	0.211	0.162	0.229
A_C						1.000	0.907	0.803
A_M							1.000	0.757
$\overline{T}$								1.000

Table 7.2: Summary Statistics of Energy Balance Study Carried Out at Lower Ice Station on White Glacier, Axel Heiberg Island, June 12 to August 18 1961. Sample Size = 63 Days. Correlation Coefficients Greater Than 0.210 are Significant at 5 % Level.

WHITE GLACIER 1962

		Q_S	Q_L	Q_P	Q_I	A_C	A_M	$\overline{T}$
		Ly/Dy	Ly/Dy	Ly/Dy	Ly/Dy	cm	cm	°C
Mean	[illegible]	91.6	18.9	0.0	-37.0	3.5	4.1	5.5
Standard Dev	85.1	75.9	15.7	0.0	7.2	1.7	2.0	2.1

Intercorrelation Coefficient Matrix

Q_R	1.000	0.533	-0.056		-0.[illegible]6	0.883	0.185	0.371
Q_S		1.000	-0.050		-0.341	0.861	0.878	0.810
Q_L			1.000		-0.744	0.011	0.065	0.337
Q_P								
Q_I					1.000	-0.462	-0.236	-0.487
A_C						1.000	0.595	0.692
A_M							1.000	0.842
$\overline{T}$								1.000

Table 7. Summary Statistics of Energy Balance Study Carried Out at Lower Ice Station on White Glacier, Axel Heiberg Island, July 16 to July 31 1962. Sample Size = 11 Sampling Periods. Correlation Coefficients Greater Than 0.521 are Significant at 5 % Level.

SVERDRUP GLACIER 1963

	Q_R	Q_S	Q_L	Q_P	Q_I	A_C	A_M	$\overline{T}$
	Ly/Dy	Ly/Dy	Ly/Dy	Ly/Dy	Ly/Dy	cm	cm	°C
Mean	105.7	64.2	29.4	0.7	-23.2	2.2	2.2	4.0
Standard Dvn	38.1	55.1	31.8	2.0	7.5	1.1	1.7	1.5

Intercorrelation Coefficient Matrix

Q_R	1.000	-0.099	-0.162	-0.191	-0.338	0.273	-0.218	0.133
Q_S		1.000	0.924	-0.003	0.050	0.915	0.761	0.803
Q_L			1.000	0.110	0.104	0.873	0.803	0.757
Q_P				1.000	0.043	-0.011	0.365	-0.020
Q_I					1.000	0.007	0.339	-0.322
A_C						1.000	0.704	0.794
A_M							1.000	0.492
$\overline{T}$								1.000

Table 7.4: Summary Statistics of Energy Balance Study Carried Out at 300 m a.s.l. on Sverdrup Glacier, Devon Island, July 9 to August 10 1963. Sample Size = 33 Days. Correlation Coefficients Greater Than 0.292 are Significant at 5 % Level.

Situation	Y	Net Radiation A_o	Net Radiation B.L	Net Radiation U	Sensible Heat A_o	Sensible Heat B.L	Sensible Heat U	Air Temperature A_o	Air Temperature B	Air Temperature U
White Gla. 1960	A_C	1.6	1.36	1.8	1.5	1.68	0.5	-0.3	0.78	1.2
White Gla. 1960	A_M	1.9	0.88	1.6	1.7	1.28	0.8	0.4	0.57	1.2
White Gla. 1961	A_C	1.2	0.64	1.5	1.0	1.04	0.8	0.4	0.58*	0.9
White Gla. 1961	A_M	1.7	0.24	1.6	0.7	1.12	0.8	0.2	0.57	1.1
White Gla. 1962	A_C	-0.1	1.36	0.8	1.8	1.52	0.8	0.5	0.55	1.1
White Gla. 1962	A_M	3.2	0.32	1.9	2.0	1.84	0.9	-0.3	0.79	0.9
Sverdrup G 1963	A_C	1.4	0.64	1.1	[illegible]	1.44	0.4	-0.2	0.60	0.7
Sverdrup G 1963	A_M	3.3	-0.80	1.7	0.7	1.92	1.1	-0.1	0.58	1.5
Mean		1.8	0.58	1.5	1.3	1.48	0.8	0.1	0.63	1.1
Standard Dvn		1.1	0.70	0.4	0.5	0.32	0.2	0.3	0.10	0.2

Table 7.5: Comparison of Linear Regression Models of Ablation A_C or A_M in Terms of Net Radiation Q_R, Sensible Heat Q_S and Air Temperature T Respectively (L = Latent Heat of Fusion).

Model: $Y(t) = A_o + B.X(t) + U.e(t)$

Situation	Y	Sensible Heat A_o	Sensible Heat B.L	Sensible Heat U	Observed Abl. A_o	Observed Abl. B	Observed Abl. U	Air Temperature A_o	Air Temperature B	Air Temperature U
White Gla. 1960	A_C	1.5	1.68	0.5	0.5	1.02	0.9	-0.3	0.78	1.2
White Gla. 1961	A_C	1.0	1.04	0.8	0.5	0.87	0.7	0.4	0.58	0.9
White Gla. 1962	A_C	1.8	1.52	0.8	1.5	0.51	1.3	0.5	0.55	1.1
Sverdrup G 1963	A_C	1.0	1.44	0.4	1.2	0.45	0.8	-0.2	0.60	0.7
Mean		1.3	1.42	0.6	0.9	0.71	0.9	0.1	0.63	1.0
Standard Dvn		0.4	0.27	0.2	0.5	0.28	0.3	0.4	0.10	0.2

Table 7.6: Comparison of Linear Regression Models of A_C in Terms of Q_S, the Observed Ablation A_M and T Respectively.

A_M and temperature are given in Table 7.6

For the situations White Glacier 1960, 1961 and Sverdrup 1963 the sensible heat Q_S has the largest standard deviation and correlates best with A_C. In these three cases A_M is also well correlated with Q_S. For White Glacier 1962 it is the radiation Q_R which has highest standard deviation and correlates best with A_C. However, in this case the correlation between A_M and Q_R is poor whilst Q_S correlates highly with both A_C and A_M. In every case Q_R has the largest mean value. In qualitative terms, therefore, the expectations of the previous section are confirmed.

It is clear that the energy balance terms are not completely independent of each other. In two cases (White Glacier 1960 and Sverdrup 1963) there is strong interaction between Q_S and Q_L. In all cases Q_R is correlated with Q_I with quite consistent correlation coefficients in the range -0.330 to -0.396. In only one case (White Glacier 1962) is Q_R well correlated with Q_S. The various patterns of correlation between Q_R, Q_S and Q_L may be characteristic of different types of "weather". On the other hand, the relatively consistent pattern of correlation between Q_R and Q_I suggests that Q_I is more of a radiative process than a conductive process.

It is interesting that, in all cases except one, A_C and A_M are relatively well correlated with temperature T. Also interesting is the fact that in no case is Q_R significantly correlated with T so that temperature does not appear to be a simple "index" of net radiation.

With respect to the various regression coefficients, scrutiny of Table 7.5 is interesting. Under assumption (7.3) it was said that B.L = 1. On average this is not true: B.L is on average less than unity for Q_R and greater than unity for Q_S. This is due to the effects of interaction between the various energy balance terms, with generally destructive interactions for Q_R and constructive interactions for Q_S. On average U for Q_R is higher than for Q_S and T, corresponding to the lower correlation coefficients already noted. From the point of view of low U values it can be concluded that Q_S is a better predictor of ablation (A_C or A_M) than Q_R or T. Next best predictor is T with Q_R worst.

It is also interesting to examine the consistency of the regression models for the different situations. The coefficients of variation (standard deviation divided by mean value) of B in the various ablation models (A_C or A_M) are as follows:

Independent Variable	Q_R	Q_S	T
Coefficient of Variation of B	121 %	22 %	16 %

From the point of view of consistency of B values, the models using T are best (variability 16 %) with Q_S next best and 121 % variability of B for models with Q_R. The standard deviation of A_0 for temperature models is smaller than for Q_S and Q_R models.

In Table 7.6 models for A_C in terms of Q_S, A_M and T are compared. It can be seen that A_M is a worse predictor, from the point of view of having higher U values, of A_C than is Q_S. On the other hand, it is a little better than T as a predictor. Coefficients of variation for B in the various models for A_C are:

Independent Variable	Q_S	A_M	T
Coefficient of Variation of B	19 %	39 %	16 %

Once again the T models have the most consistent B values.

In summary then: Q_S is on average the best predictor of ablation in terms of having lowest Root Mean Square Error U. On the other hand, temperature seems to have more consistent model parameters although rather higher U values (which are, however, still lower than those for Q_R). For many practical modelling purposes the latter quality is more desirable so that ablation models using temperature are useful.

On the basis of results in Table 7.5 it is suggested that the ablation-temperature model for ice should be approximately

$$A(t) = 0.1 + 0.63\,T(t) \qquad (7.5)$$

This is based upon models for both A_C and A_M, but in fact models for A_C alone are very similar (Table 7.6). Actually, it is not clear that the intercept should be different from zero, with the value of 0.1 arising due to sampling effects. If the equation (7.5) is used to compute an estimate of mean ablation (A_P) for the four situations the difference between this estimate and A_C or A_M

can be regarded as an error. This error has mean value 0.0 cm and standard deviation 0.4 cm H_2O for the eight samples (4 situations and 2 ablation estimates).

v) A Further Comment About the Role of Net Radiation in Ablation

In Chapter 7(iv) the net radiation Q_R was shown to be generally a bad predictor of ablation. This is explainable in terms of the low variability of the net radiation according to the discussion in Chapter 7(ii) and by Equation (7.2) in particular. As Equation (7.2) is derived directly from the energy balance equation this finding is not inconsistent with the fact that net radiation is "most important". The ablation-energy balance system can be considered as a system that is mainly energized by net radiation, but the partition of energy within the system appears to be mainly controlled by sensible heat flux, with some help from latent heat flux. To fully explain this in physical terms a theory of interactions between the various sources and sinks would be needed. For development of such a theory, continuation of energy balance measurements on glaciers with improved instrumentation is required. However, the data sampling and presentation of results must be improved. For example, the usual method of expressing energy balance totals over periods of several days or weeks as percentages of the total energy is very misleading. As an illustrative example the 10-day means (9-day means for the last period) of the various terms for the White Glacier 1961 record are given in Table 7.7 (note that the first few days of discontinuous record have been discarded). In Table 7.8 the corresponding percentage values are given. The percentages in Table 7.8 obscure the fact that ablation during period 5 (31/07-09/08/61) was nearly double the average for the whole 59-day period. Furthermore, Q_R was actually a little above average during this period although it only constitutes 33 % of the total energy source for this period. From Table 7.7 it is clear that Q_R was not "most important" in period 5 and that the large ablation was due to large sensible heat flux Q_S supply during a period dominated by Foehn.

The low variability of net radiation in the four situations may have a physical cause, or it may be due to errors. With respect to the former, net radiation is a synthetic process compounded of several distinctly different processes which may tend to compensate each other. For example, the dependence of incoming short- and long-wave radiation upon cloud cover will be different. This is a

Period	N	$\bar{Q}_R$	$\bar{Q}_S$	$\bar{Q}_L$	$\bar{Q}_I$	$\bar{A}_C$
19/06-30/06	10 dy	159	30	-15	-47	1.6
01/07-10/07	10 dy	162	58	3	-53	2.1
11/07-20/07	10 dy	113	98	29	-34	2.6
21/07-30/07	10 dy	119	77	-8	-42	1.8
31/07-09/08	10 dy	135	256	19	-42	4.6
10/08-18/08	9 dy	84	99	-15	-36	1.6
Mean Values		128.7	103.0	2.2	-42.3	2.4

Table 7.7: 10-Day Mean Values of Energy Balance Components for White Glacier 1961. Units are Ly Dy^{-1} for Energy Balance Components $\bar{Q}_R$, $\bar{Q}_S$, $\bar{Q}_L$ and $\bar{Q}_I$ and cm H_2O dy^{-1} for $\bar{A}_C$.

Period	N	$\bar{Q}_R$	$\bar{Q}_S$	$\bar{Q}_L$	$\bar{Q}_I$	$\bar{A}_C$
19/06-30/06	10 dy	84 %	16 %	-8 %	-25 %	-67 %
01/07-10/07	10 dy	73 %	26 %	1 %	-24 %	-76 %
11/07-20/07	10 dy	47 %	41 %	12 %	-14 %	-86 %
21/07-30/07	10 dy	61 %	39 %	-4 %	-22 %	-76 %
31/07-09/08	10 dy	33 %	62 %	5 %	-10 %	-90 %
10/08-18/08	9 dy	46 %	54 %	-8 %	-20 %	-72 %

Table 7.8: 10-Day Mean Values of Energy Balance Components for White Glacier 1961. Data are Expressed as Percentages of Total Sources/Sinks.

very complex problem, see for example Ambach (1974). With respect to errors in the net radiation data, in all cases the long-wave radiation components were computed using empirical formulae. Ohmura (personal communication) suggests that this would tend to produce "smooth" results so that the variability of the net radiation is underestimated. A detailed study of radiation climate at Axel Heiberg Base Camp (Ohmura, personal communication) is in preparation. As the data originate from more modern instruments than were available in the early 1960s, further light may be thrown on the problem of the role of net radiation in ablation.

From the discussion in Appendix 3 it is clear that the correlation between ablation and net radiation will be frequency dependent if the variance of net radiation is itself frequency dependent. This might explain the fact that Lang (1968) found best correlation between net radiation and discharge from Aletschgletscher for hourly-sampled data and best correlation between air temperature and discharge for daily-sampled data. It is suggested that energy balance components should be analysed with respect to a sampling scale of one hour in accordance with standard practice in radiometry. Daily values could easily be computed from the hourly values and statistical and spectral analyses of the data could more easily be made.

For purposes of the present study it is chosen to make further discussion of ablation in terms of air temperature which, according to present findings, can be regarded mainly as a measure of the sensible heat flux.

CHAPTER 8

THE ROLE OF AIR TEMPERATURE IN ABLATION ON ARCTIC GLACIERS

1) Conceptual Model of Ablation in Terms of Temperature

On the basis of the statistical analysis an ablation-temperature model was advanced: Equation (7.5). It would be interesting to see if features of the model can be derived from physical considerations.

The intercept A_0 is best regarded as a kind of boundary condition: the ablation at $T = 0^{\circ}C$. It is difficult to test this statement directly as there are no cases with $T = 0^{\circ}C$ in the four energy balance series analysed. For the White Glacier 1961 series there are nine cases with temperature in the range 0 to $1^{\circ}C$. The mean temperature for these cases is $0.6^{\circ}C$, and the mean ablation is 1.1 cm H_2O dy^{-1}. There is actually no reason to expect that A_0 should be exactly zero at $T = 0^{\circ}C$ because melting could occur during part of a day although the mean temperature for the day might be less than $0^{\circ}C$ and because the glacier surface could be at $0^{\circ}C$, under conditions of strong insolation, whilst the air temperature was actually negative.

The problem of the slope B in Table (7.5) is easier to discuss. There are two main heat sources which depend directly upon air temperature: the sensible heat flux Q_S and the flux of incoming long-wave radiation Q_{LW}.

The sensible heat flux can be expressed approximately by the Thornthwaite-Holzmann equation:

$$Q_s = \rho c_p k^2 (T_2 - T_1)(u_2 - u_1) / \ln^2\left(\frac{z_2}{z_1}\right) \qquad (8.1)$$

where p is the density of air, k is von Karman's constant, c_p is the specific heat of air at constant pressure, T_2, u_2, T_1 and u_1 are temperature and wind respectively at each of two heights z_2 and z_1. All units are c.g.s. units. If z_1 is taken as equal z_0, the surface roughness length, then $u_1 = 0$ and $T_1 = 0$ for a melting glacier.

The flux of incoming long-wave radiation can be written:

$$Q_{LW} = \epsilon_a \sigma (T + 273)^4 \quad (8.2)$$

where σ is the Stefan-Boltzman constant, ϵ_a is the effective emissivity of the atmosphere and $(T + 273)$ is the air temperature in ^oK.

Assuming that Q_s and Q_{LW} are the only temperature dependent heat sources:

$$\frac{\partial \bar{A}}{\partial T} = \frac{86400}{L}\left(\frac{\partial Q_s}{\partial T} + \frac{\partial Q_{LW}}{\partial T}\right) \quad (8.3)$$

where L is the latent heat of fusion for ice. The units of $\bar{A}$ are cm H_2O dy^{-1}, and the factor 86 400 is the conversion factor between Ly s^{-1} and Ly dy^{-1}. The second right-hand term amounts to about 0.12 cm H_2O $^oC^{-1}dy^{-1}$ for average temperature conditions and an assumed constant effective emissivity of 0.95. The first right-hand term will be denoted by β. It can be estimated for the different situations using the average values of the various quantities.

β is compared to the corresponding B values from Table (7.6) below:

Situation	z_2	$\bar{z}_0$	$\bar{u}_2$	β	B
White Glacier 1960	170	?*	300	0.49	0.78
White Glacier 1961	150	?*	310	0.53	0.58
White Glacier 1962	150	0.7	240	0.44	0.55
Sverdrup Gla. 1963	220	0.4	200	0.26	0.60
Mean				0.43	0.63

* a value of 0.6 cm was assumed for $\bar{z}_0$.

The agreement between β and B is certainly better than order of magnitude. The average discrepancy is -32%. When account is taken of the effect of long-wave radiation also, the discrepancy is reduced to -13%. This is very good considering the shortcomings of the present analysis. Wind speed and temperature are actually coupled, especially during Foehn events, so that β could be further increased. For the White Glacier 1960 and Sverdrup Glacier 1963 cases the sensible heat flux Q_S is strongly coupled with the latent heat flux Q_L

Situation	Y	A_C	B	U
White Glacier 1960	A	1.8	0.101	0.8
White Glacier 1960	A_M	1.9	0.077	1.0
White Glacier 1961	A_C	1.3	0.093	0.9
White Glacier 1961	A_M	0.9	0.104	0.8
White Glacier 1962	A_C	2.1	0.092	1.3
White Glacier 1962	A_M	1.5	0.175	0.4
Sverdrup Gla. 1963	A_C	0.8	0.152	0.6
Sverdrup Gla. 1963	A_M	0.6	0.185	1.3
Mean		1.4	0.122	0.9
Standard Dvn		0.6	0.042	0.3

Table 8.1: Linear Regression Models of Ablation (A_C or A_M) in Terms of Air Temperature Multiplied by Wind Speed in $cm\ s^{-1}$.

Model:

$$Y(t) = A_o + B.(u(t).T(t)) + U.e(t)$$

which would further increase β for the two cases. From these results it would seem that there is little discordance between the computed statistics and "theory".

From Equation (8.1) it might be claimed that it would be more physically meaningful to correlate the ablation with the product of the wind and temperature rather than with the temperature alone. This was done, and the results are given in Table 8.1. It was found that the A_o and B coefficients for this model were less consistent than for the model with temperature alone although the R.M.S. errors were lower: only a little larger than for the model with Q_S. The mean slope of the models in Table 8.1 is 0.0012 compared to a mean value of 0.0017 for $\beta/\bar{u}_2$. Once again the agreement is better than order of magnitude.

ii) Ablation and Positive Degree-Day Total

It is claimed that daily ablation A(t) is related to daily mean temperature by a linear equation of the form:

$$A(t) = A_o + B\,T(t) + U\epsilon(t) \tag{8.4}$$

A_o is estimated to be about 0.1 cm H_2O dy^{-1}, B is about 0.63 cm H_2O $^oC^{-1}dy^{-1}$ and U is about $\mp$ 1.1 cm H_2O dy^{-1} (these values are actually the averages from Table 7.5). The equation is probably only valid for days with temperature above 0^oC and ablation is probably zero for negative temperatures. This cannot be directly confirmed as the data analysed in the previous chapter only involved positive temperatures. For practical purposes it is usually the cumulative or total ablation over a period of many days (n) that is of interest. The ablation at the end of n days is denoted by $a_c(n)$. If there are n* days during the n day period with $T(t) \geq 0^oC$ then:

$$a_c(n) = n^*A_o + B\sum_{t=1}^{t=N} H(t)T(t) + U\sum_{t=1}^{t=N}\epsilon(t) \tag{8.5}$$

where H(t) is a logical variable equal to zero for $T(t) < 0^oC$ and equal to unity for $T(t) \geq 0^oC$. The second term in the equation can be rewritten in terms of DDT(n) which is the positive degree-day total for the n day period:

$$DDT(n) = \sum_{t=1}^{t=N} H(t)T(t) \tag{8.6}$$

The third term, which involves summation of the residuals of the regression equation, will exhibit a kind of Random-Walk with range RA(n) between maximum and minimum values during the n day period which will be given approximately by Hurst's Law (Hurst, 1951):

$$RA(n) = U\left(\frac{n}{2}\right)^{0.73} \tag{8.7}$$

According to (8.7) RA(n) will have values of 3.6, 11.5 and 19.1 cm H_2O respectively for 10, 50 and 100 day periods (assuming U = 1.1 cm H_2O dy^{-1}).

From Table 7.5 it can be seen that the mean value of A_0 is not significantly different (5 % level) from 0.0, and this value will henceforth be assumed for convenience. B will be assumed to be 0.63 with a standard deviation of about ±0.10 (values from Table 7.5) so that the cumulative ablation versus positive degree day total model for ice will be:

$$a_c(n) = 0.63\ DDT(n) \tag{8.8}$$

The main source of error will be the uncertainty in the coefficient as the Random-Walk term will be small by comparison.

It might have been better to express Equation (7.5) in terms of the daily positive degree day total (computed from daily degree hour totals) rather than in terms of the daily mean temperature T. At high temperatures the discrepancy will be entirely negligible, but for T in the range of about -3 to +3°C a discrepancy can arise. This is because temperature can be above (below) 0°C for part of the day whilst the daily mean temperature is below (above) 0°C. It was chosen to express (7.5) in terms of T because, as already shown in earlier chapters, T itself can be calculated using temperatures at distant weather stations. Errors arising on this account should be small over a whole season.

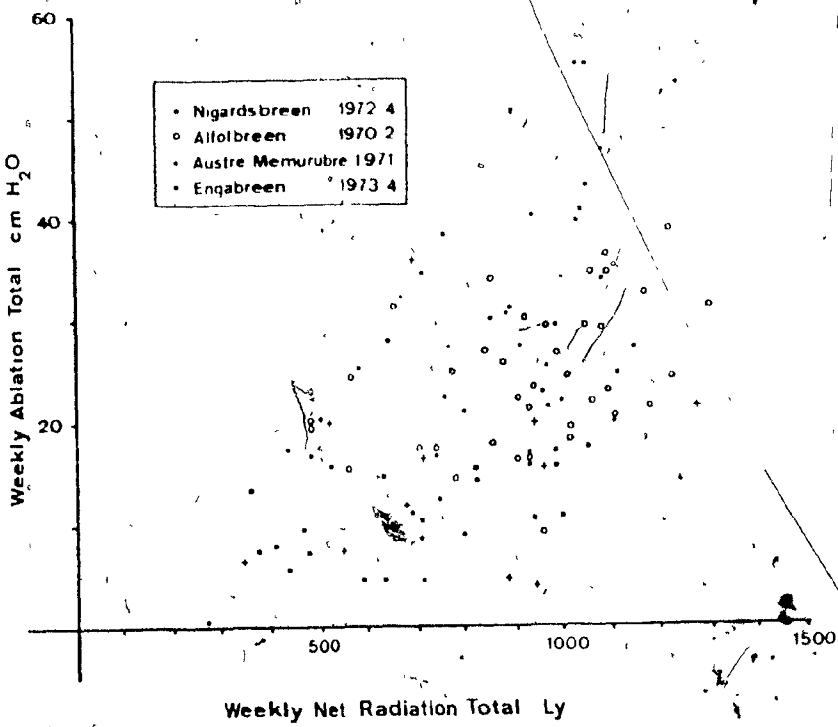


Fig 8.1: Weekly Totals of Specific Ablation Versus Net Radiation for Four Norwegian Glaciers in Period 1970-74

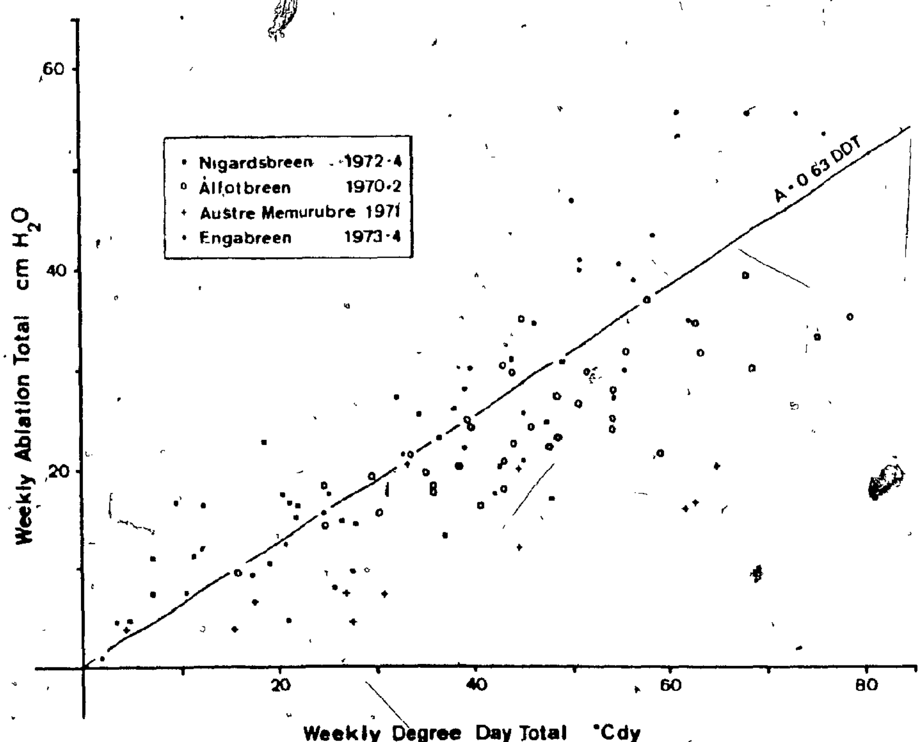


Fig 8.2: Weekly Totals of Specific Ablation Versus Degree-Days for Four Norwegian Glaciers in Period 1970-74. The Straight Line Represents the Model for White/Sverdrup Glacier

Kuz'min (1972, Chapter III) presents a number of different computational schemes for snowmelt and asserts that equations of the same form as (8.8) are: "simplest and most accurate of all the examined computational methods". However, he asserts "A shortcoming of these formulae is their local character", and he questions the linearity with respect to DDT.

iii) Comparison with Results from Norwegian Glaciers

For a number of years the Norges Vassdrags-og Elektrisitetsvesen (NVE) have carried out "special radiation studies" on glaciers in Norway. For these studies actinographs were installed on two glaciers for each summer, and from additional observations of cloudiness, air temperature, surface albedo etc estimations of net radiation were made, see Tvede (1971, p.70-73) for details. Data in the form of tabulations of weekly totals of net radiation together with observed weekly ablation are given by Tvede (1971, p.76-77) for Austre Memurubre and Ålfotbreen (1970), by Tvede (1973, p.79) for Austre Memurubre and Ålfotbreen (1971), by Tvede (1974, p.82-83) for Nigardsbreen and Ålfotbreen (1972), by Tvede (1975, p.54) for Nigardsbreen and Engabreen (1973) and by Tvede *et al* (1975, p.58) for Nigardsbreen and Engabreen (1974).

An analysis of these data was carried out for comparison with results already obtained (Chapter 7). Temperature data for the 10 situations analysed are not published and were kindly made available in computer print-out form by Dr Gunner Østrem of NVE. Weekly positive degree day totals were computed from the raw data for the same periods and locations for which the radiation and ablation data were available.

Scatter diagrams of ablation versus net radiation and degree-day total respectively are given in Figures 8.1 and 8.2 where the data refer to weekly totals. Data from Austre Memurubre for 1970 are excluded as, in this case, the only available ablation data refer to ablation averaged over the whole glacier (mean specific ablation). Both diagrams show considerable scatter, but it does appear that the relation between ablation and degree-day total is better than between ablation and net radiation. The straight line corresponding to Equation (8.8) is plotted in Figure 8.2 to allow comparison between the Norwegian results and the results from the Arctic.

[illegible] the data was carried out. Correlation coefficients, [illegible] between ablation (A), degree-day total (DDT) and net radiation (QR) for the 10 situations are given in Table 8.[illegible] together with the percentage importance of radiation for ablation, expressed by $\overline{QR}/\overline{A}$. In [illegible] out of the 10 cases the ablation is better correlated with degree-day total than with net radiation. The average correlation between ablation and DDT is 0.72 with coefficient of variation [illegible] whilst the corresponding figures for ablation and QR are 0.55 and 55 %. There are some cases where DDT and QR are themselves strongly correlated but on average their correlation is low (R = 0.39). It can be clearly seen that there is no relationship between the correlation of ablation with net radiation R(A,QR) and the percentage importance of radiation for ablation $\overline{QR}/\overline{A}$. For example, in the four cases where the percentage importance of radiation is less than 50 % the average of R(A,QR) is 0.54 whilst the average for the other six cases is 0.56.

According to Appendix 3 the correlation between net radiation and ablation should be governed by the ratio of their standard deviations, i.e. S_R/S_A. In Table 8.3 the correlation coefficients R(A,QR) are compared with S_R/S_A and the quantity $X = R(QR,DDT).(S_{DDT}/S_A)$ which expresses the interaction between net radiation and air temperature. If there were no interactions, R(A,QR) should be equal to S_R/S_A, i.e. on average equal to about 0.47, so that some reinforcement of the correlation must be occurring. The following regression model was computed:

$$R(A,QR) - S_R/S_A = -0.19 + 0.48\,X \quad R = 0.80 \tag{8.9}$$

Equation (8.9) demonstrates that R(A,QR) is primarily reinforced by the interaction of net radiation and air temperature with about 64 % explanation of variance. The intercept −0.19 could represent negative reinforcement effects of other, unspecified, energy sources whilst the slope 0.48 could represent the conversion factor between degree-day total and sensible heat flux total.

From these results it would seem that the generally low correlation between ablation and net radiation arises because of the generally low variability of net radiation compared to higher variability of degree-day total and, presumably, sensible and latent heat fluxes. This has been stated also by Tvede (1974, p.81).

Glacier	Year	N	R(A,DDT)	R(A,QR)	R(DDT,QR)	$(\overline{QR}/\overline{A})$
Alfotbreen	1970	15	0.70	0.58	0.62	46 %
	1971	14	0.88	0.13	-0.01	44 %
	1972	9	0.88	0.68	0.68	53 %
Nigardsbreen	1972	11	0.85	0.43	0.62	62 %
	1973	11	0.78	0.85	0.77	53 %
	1974	11	0.39	0.60	0.11	77 %
Engabreen*	1973	10	0.92	0.79	0.87	32 %
	1974	11	0.86	0.64	0.27	33 %
Austre Memurubre	1970*	9	(0.19)*	(0.84)*	(-0.15)*	(66 %)*
	1971	12	0.79	-0.06	0.07	76 %
Mean			0.72	0.55	0.39	
Standard Dvn			0.24	0.30	0.37	

*ablation refers to whole glacier

Table 8.2: Correlations Between Weekly Ablation A, Degree-Day Total DDT and Weekly Net Radiation QR for Four Norwegian Glaciers With Sample Size N (weeks).

Glacier	Year	N	R(A,QR)	S_R/S_A	X
Ålfotsbreen	1970	15	0.58	0.45	1.33
	1971	14	0.13	0.32	-0.02
	1972	9	0.68	0.26	1.29
Nigardsbreen	1972	11	0.43	0.24	1.38
	1973	11	0.85	0.40	1.24
	1974	11	0.60	0.50	0.17
Engabreen	1973	10	0.79	0.21	0.87
	1974	11	0.64	0.22	0.33
Austre Memurubre	1970*	9	0.84*	1.71*	-0.76*
	1971	12	-0.06	0.37	-0.20
Mean			0.55	0.47	0.56
Standard Dvn			0.30	0.45	0.76

*ablation refers to whole glacier.

$X = R(QR,DDT) . S_{DDT}/S_A$

Table 8.3: Comparison of the Correlation Between Ablation and Net Radiation R(A,QR), Relative Variability of Net Radiation S_R/S_A and Interaction Between Net Radiation and Degree-Day Total X for Four Norwegian Glaciers.

Glacier	Year	N	A_o	B	R(A,DDT)	$\overline{A}/\overline{DDT}$
Alfotbreen	1970	15	9.3	0.33	0.70	0.47
	1971	14	4.4	0.48	0.88	0.55
	1972	9	0.9	0.46	0.88	0.53
Nigardsbreen	1972	11	0.4	0.69	0.85	0.45
	1973	11	2.4	0.78	0.78	0.62
	1974	11	7.9	0.26	0.39	0.67
Engabreen	1973	10	-9.2	0.85	0.92	0.93
	1974	11	0.8	0.71	0.86	0.82
Austre Memurubre	1970*	9*	-*	0*	(0.19)*	-*
	1971	12	1.9	0.27	0.79*	0.34
Mean			2.1	0.54	0.72	0.60
Standard Dvn			5.3	0.23	0.24	0.19

*ablation refers to whole glacier

Table 8.4: Statistics of Linear Regression Model of Weekly Ablation Total Versus Weekly Degree-Day Total, Intercept A_o, Slope B, Mean Ablation $\overline{A}$ and Mean Degree-Day Total $\overline{DDT}$.

The correlation between ablation and net radiation is, however, a little reinforced by the interaction of net radiation and temperature.

Various statistics relating to the regression models of ablation in terms of degree-day total are given in Table 8.4 where A_o and B are the intercept and slope respectively of the regression model. Values for Austre Memurubre 1970 are omitted because the ablation refers to the whole glacier in that case (this is a pity as the situation was, apparently, an interesting one). Comparison between Table 8.4 and the corresponding Table 7.5 for Arctic situations is interesting. It should be borne in mind that Table 8.4 refers to weekly ablation totals so that A_o must be divided by seven to obtain the equivalent statistic for daily ablation. For the Norwegian situations the slope B is on average lower than for the Arctic situations, i.e. 0.54 compared to 0.63 cm $H_2O\,^oC^{-1}$, although its variability is greater, standard deviation of 0.23 compared to 0.10 cm $H_2O\,^oC^{-1}$. The difference in mean slopes of 0.54 (sample size 9) and 0.63 (sample size 8) is not significant at the 5 % level according to the test given by Kreyszig (1970, p.210). On the other hand, the ablation for the Norwegian situations does involve snow ablation as well as ice ablation so that the ablation per ^oC may actually be lower on account of this. The average intercept A_o of 2.1 cm H_2O week^{-1} or 0.3 cm H_2O dy^{-1} is not significantly different from zero.

In summary the results from the ten Norwegian situations do generally support the results from the four Arctic situations which were discussed in Chapter 7. However, this is not to say that the problem of ablation on Norwegian glaciers is completely solved by the degree-day approach any more than the problem was solved for Arctic glaciers. This point can be illustrated by recalling that, according to the results in Table 8.4, only about 58 % of the variance of ablation is, on average, explained by variations in degree-day totals. In addition there are admittedly individual situations where the degree-day model is especially bad, e.g. Austre Memurubre 1970 and Nigardsbreen 1974. In the latter case net radiation actually provided more than 100 % of ablation energy for a three-week period (Tvede *et al* 1975, Fig 47).

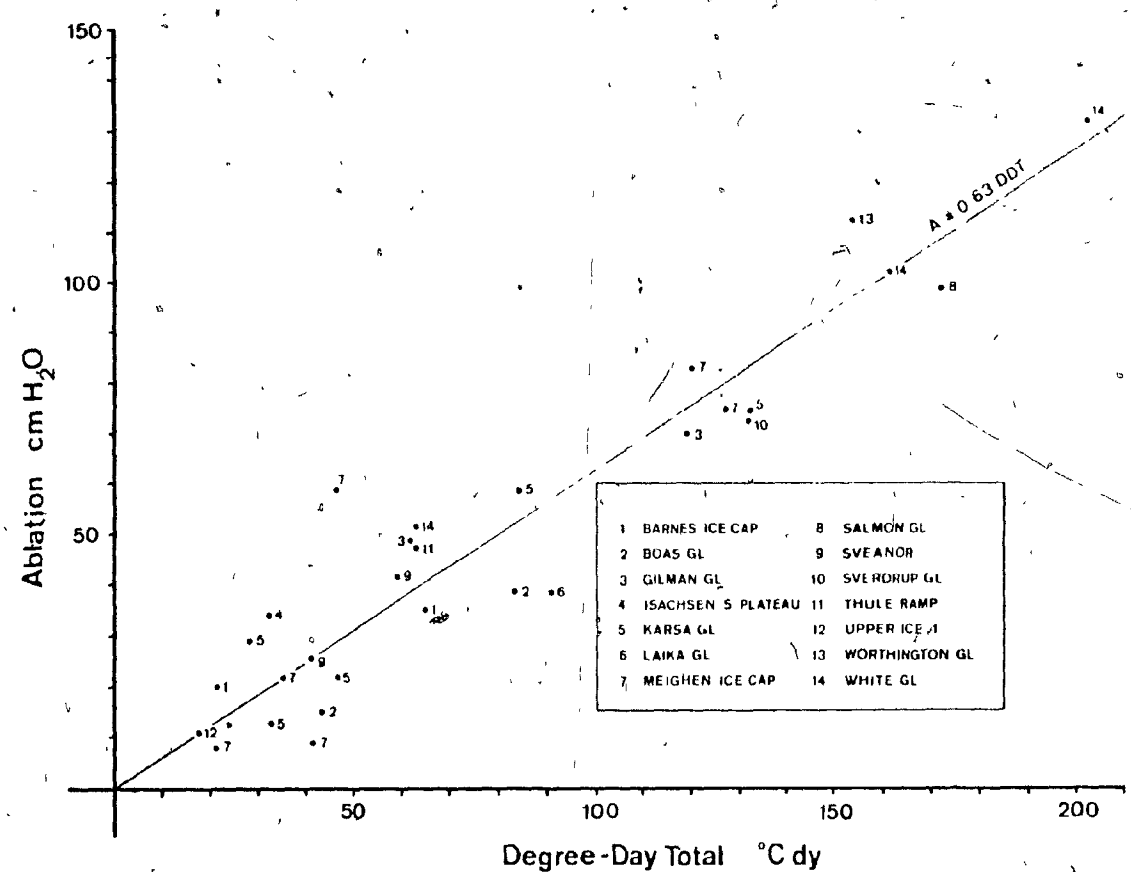


Fig 8.3: Specific Ablation Totals Versus Degree-Day Totals for 14 Different Glaciers (29 Situations in Total) for Arbitrary Sampling Periods. The Straight Line Represents the Model for White/Sverdrup Glacier

iv) Comparison with Results from Other Glaciers

For further comparison with the results already described, a literature search was carried out. Figure 8.3 is a scatter diagram of ablation totals versus degree-day total for arbitrary periods (few days to whole summer) on fourteen different glaciers. Results are either those quoted by the various authors or are computed from raw data tabulated by the various authors. The sources were: Barnes Ice Cap (Baffin Island) - Summers 1962 & 63 from Sagar (1966, p.64), Boas Glacier (Baffin Island) - Summers 1970 & 71 from Jacobs *et al* (1974), Gilman Glacier (Ellesmere Island) - Summers 1957 & 58 from Hattersley-Smith *et al* (1961) and Arnold (1968), Isachsen's Plateau - Summer periods in 1934 from Sverdrup (1935), Kårsa Glacier (Sweden) - Summer periods in 1942, 1943, 1946 and 1948 from Wallén (1948), Laika Ice Cap (Coburg Island) - Summer 1975 from Müller and Kappenberger (unpublished), Meighen Island Ice Cap - Summers 1960-62 & 1968-70 from Taylor-Alt (1975), Salmon Glacier (British Columbia) - Summer period in 1957 from Aidkin (1958), Sveanor (Northeast Land) - Summer periods in 1931 from Ahlmann (1933), Sverdrup Glacier (Devon Island) - Summer period 1963 from Keeler (1964), Thule Ramp (NW Greenland Ice Cap) - Summer periods in 1954 from Schytt (195 , Upper Ice 1 (Axel Heiberg Island) - Summer periods in 1960 from Havens (1964), Worthington Glacier (Alaska) - Summer period 1967 from Streten and Wendler (1968) and White Glacier (Axel Heiberg Island) - Summer periods in 1960-62 from Andrews (1964), Havens *et al* (1965) and Müller & Roskin-Sharlin (1967).

The data represent widely varied locations and conditions. In some cases ablation refers to ice and in others it refers to snow. The model (8.8) is plotted in Figure 8.3 for comparison. The fit of the results to the model is not too bad: differences between individual results and the model have an average of 1.6 cm H_2O with standard deviation 11.9 cm H_2O (sample size 29). If it is asserted that the results should fit the model the errors would be 3 % of the mean and 14 % of the variance. These errors are surprisingly low, and it would be difficult to claim that they falsify the model.

The question as to whether ice and snow have different ablation models is problematic. For example, ablation on Laika Ice Cap was 39 cm H_2O compared to 57 cm H_2O predicted by the model (8.8), and the discrepancy might be due to effects of snow ablation but, on the other hand, ablation at Upper Ice 1, also involving snow, is in good agreement with the model.

Gutersohn (1936) developed relations for daily ablation and specific discharge for eight Swiss glaciers in terms of temperature extrapolated from valley stations to the mean altitude of the glaciers (actually relatively close to the mean ELA). His temperatures do not take account of glacier "cooling effect", and his relationships are developed for selected "heitere Sommertage" in run-off records extending over many years. Despite these qualifications his models are in the same "ball park" as (8.8) with ice ablation in the range 0.4 to 0.7 cm H_2O $^oC^{-1}dy^{-1}$. Lliboutry (1964-65, p.452) reviews a number of works involving degree-day approaches to ablation and quotes coefficients in the range 0.36 to 0.69 cm H_2O $^oC^{-1}dy^{-1}$ whilst Orheim (1970) reports coefficients of 0.65 and 0.61 cm H_2O $^oC^{-1}dy^{-1}$ for Summers 1965-66 on Store Supphellbre (W. Norway). These latter results are particularly interesting in view of the large absolute amount of ablation (over 7 m H_2O !). A coefficient of 1.38 cm H_2O $^oC^{-1}dy^{-1}$ reported by Schytt (1964) should be mentioned as lying outside the "normal" range.

Degree-day approaches to snow melting are common for prediction of short-period run-off, see Linsley *et al* (1949, p.427-432), Becker (1969, p.26) and Kuzmin (1972, Chapter III). Widely differing coefficients are quoted. Church (1942, p.129) quotes 0.23 m H_2O $^oC^{-1}dy^{-1}$ for the Sierra Nevada and 0.49 cm H_2O $^oC^{-1}dy^{-1}$ for Finland. Gray (1970, p. 9.13) quotes different coefficients for "open site" and "forest site" snow ablation situations: 0.27 and 0.23 cm H_2O $^oC^{-1}dy^{-1}$ respectively. Zingg (1951) quotes a value of 0.45 cm H_2O $^oC^{-1}dy^{-1}$ for the Swiss Alps.

It seems *a priori* reasonable that there should be a difference in ice ablation and snow ablation coefficients. Snow has higher albedo and lower surface roughness than ice so that there should be less snow melt than ice melt under the same degree-day conditions. Furthermore, the hydraulic properties of snow are such that melt-water may be more easily retained within the snow pack as free water which may later refreeze within or under the snow pack. This can be summarized by saying that snow melt and snow ablation are not identical: identity of ice melt and ice ablation was certainly assumed in the discussion in Chapter 7.

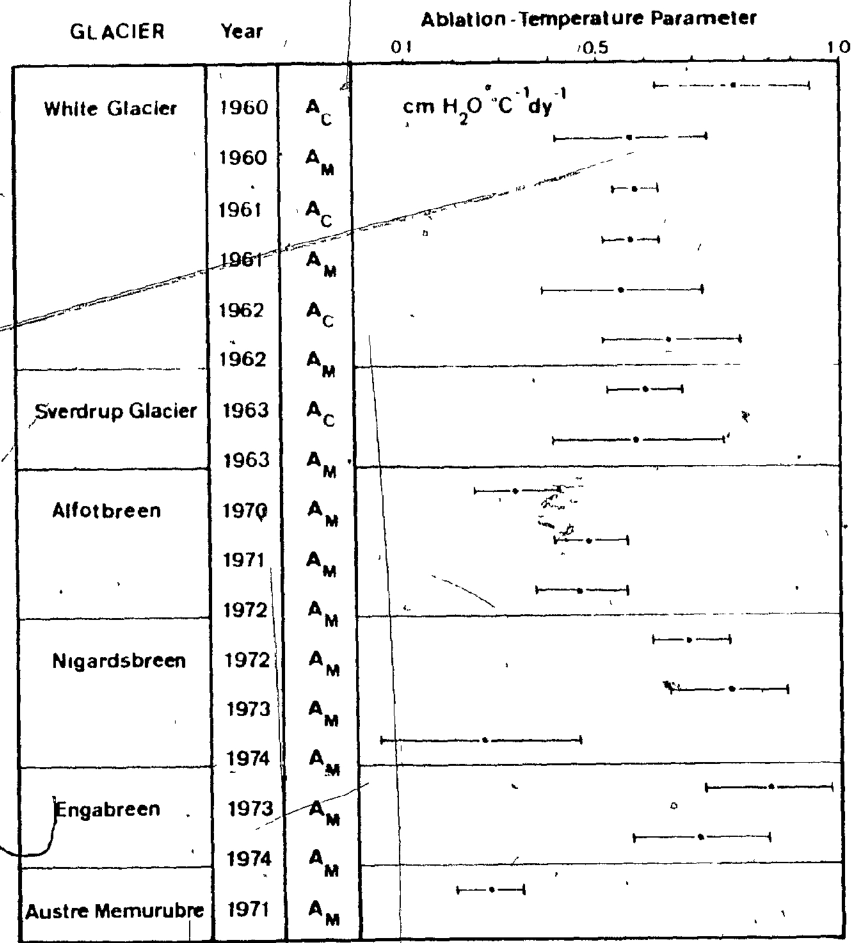


Fig 8.4: Comparison of Ablation-Temperature Parameter for Six Different Glaciers (13 Situations). Error Bars Denote Sampling Standard Deviations, A_C is Ablation Computed from Energy Balance Equation and A_M is Directly Measured Ablation

Ablation versus degree-day coefficients will be affected by the method of computation and definition of the degree-day total. For example, the computation of the coefficient as the ratio of ablation and degree-day totals over some relatively long period is relatively sensitive to errors. The method of computing coefficients by regression of daily values of ablation and temperature or degree-day total is preferable, see Chapter 8(ii). The method of computation by regression of cumulative ablation on cumulative degree-day totals is not advisable on account of the extreme non-stationarity in the cumulative data as stated by Andrews *et al* (1971) and Adam (1972). As pointed out by Arnold and Mackay (1964) it would be preferable to compute degree-day totals from degree hour totals rather than from daily mean temperatures as is done in Chapter 8(ii) of the present work.

v) Conclusions

The regression models of daily ablation versus temperature, developed in Chapter 7(iv), show reasonable mutual consistency (Table 7.5). A degree-day model is developed from these in Chapter 8(ii) which seems to be reasonable valid for other situations as an approximation, see Figures 8.2 to 8.4. However, problems do remain and the approximate nature of the model must be emphasised: it is certainly not a "law".

The model (8.8) will be applied to the problem of modelling glacier ablation using temperature data from distant weather stations in the following chapters.

A Note on the Autocorrelation of Residuals

The first-order autocorrelation coefficients for the residuals in the regression models linking ablation and temperature are, perhaps surprisingly, small in the range -0.3 to +0.3. Accordingly, the effect of autocorrelation of the residuals should be to increase the widths of the confidence intervals in Fig. 8.4 by a factor of about 10%.

CHAPTER 9

PARAMETRIC MODEL OF ABLATION ON WHITE GLACIER

i) Introduction

From the findings reported in Chapters 4 to 6 it is claimed that reasonably accurate series of local air temperature can be computed at points on White Glacier using temperature data interpolated from the distant weather stations Eureka (113 km) and Isachsen (280 km). From the findings of Chapters 7 and 8 it is claimed that ablation can be reasonably accurately computed from observed local air temperature. The logical next step is to attempt computation of summer ablation totals at sites on the glacier using temperature data from the distant weather stations together with the values already obtained for the relevant parameters. The usefulness/accuracy of the method should be assessed by comparison of the resulting computed ablation series with the observed ablation series. In practice this is not easy, and is discussed in the following section.

If the probability distribution of daily mean local temperature within a monthly sample is known or assumed, it is not necessary to compute a series of daily temperatures which must then be summed to give a degree-day total for the month. The procedure actually adopted was to interpolate monthly mean temperatures and the corresponding standard deviations from the distant weather stations. Next the local degree-day total was computed for the month by numerical integration of the normal probability curve corresponding to the given monthly mean and standard deviation. The ablation for each month was computed from the monthly degree-day total. This was done for the months of June-August, and the computed summer ablation a_c was taken as the sum of the ablation for each of the three months.

Details of the computation scheme are given in Appendix 4. Using the method, summer ablation was calculated for each year in the period 1960-72 at three levels on White Glacier: 210 m (Lower Ice), 370 m (Anniversary) and 870 m (Moraine).

ii) Use of Field Data for Checking the Model

Direct comparison of computed ablation with observed ablation on White Glacier is not possible for all years of record in the period 1960-72. This is because ablation itself is not observed during a routine mass balance programme involving visits to the glacier a few times, at most, during the summer season. Accordingly the computed summer ablation was compared to the observed annual net ablation (defined here as the negative of the annual net balance). Consequently some of the discrepancies between the two compared quantities will reflect the fact that they do represent different things: one source of discrepancy will be accumulation, winter and summer.

The model in Appendix 4 purports to compute ablation in a fixed date system (June-August) under the assumption that the ablating material is ice.

The observed data used in the present study comprise averages of the annual specific net ablation at a few stakes at or near each of the three altitudes: 210 m (Lower Ice), 370 m (Anniversary) and 870 m (Moraine). For logistic reasons (Müller, personal communication) it was not possible to visit White Glacier every spring in the 13-year period 1960-72. Hence separate series of winter and summer balances are not available. An additional problem arises if no spring visit to the glacier was made because it is not certain whether the last stake readings made in the previous autumn correspond to the end of the budget year (end of ablation season and of superimposed ice formation). On account of this problem it is possible that some mass changes are actually attributed to the wrong balance year. It would have been ideal if the series of observed net ablation, used in the present study, could have been based upon identical stakes for each year. This would have given homogeneous series. However, this was not possible because of loss of stakes, changes of stake location etc. Values for Lower Ice were generally based upon readings of *Lower Ice Diamond* for 1959/60-1963/64 and of stakes L100, L103 and L105-107 for 1965/66-1971/72 with the 1964/65 net ablation estimated from the L 71 cable. Values for Anniversary were based upon any available data from *Anniversary Profile* comprising stakes A1 - A11. Values for Moraine were based upon available data from *Moraine Profile* comprising stakes M2 - M8 and from *Moraine Diamond*. Accordingly the series of observed net ablation are not entirely homogeneous or uniform in quality. An objective estimate of the data accuracy would be difficult.

White Glacier net ablation data for 1959/60 to 1961/62 are plotted in Figure 1 of Muller (1963c) whilst 1969/70 to 1971/72 data are summarized in Braun (1976). The data are otherwise unpublished and the kind permission of Dr Fritz Müller to make use of the data is gratefully acknowledged.

The general problem of comparison of ablation, computed by some model, with observed net ablation can be illustrated by consideration of ~~the mass~~ balance equation in terms of net ablation a_n:

$$a_n(t) = a_w(t) + a_s(t) - c_w(t) - c_s(t) \qquad (9.1)$$

where $a_w(t)$, $c_w(t)$, $a_s(t)$ and $c_s(t)$ are winter ablation, winter accumulation, summer ablation and summer accumulation respectively for the t th balance year. Winter ablation $a_w(t)$ can, perhaps, be neglected. The corresponding computed ablation is $a_c(t)$, and the discrepancy between net ablation and computed summer ablation can be denoted by E(t):

$$E(t) = (a_s(t) - a_c(t)) - (c_w(t) + c_s(t)) \qquad (9.2)$$

where $a_s(t)-a_c(t)$ is the discrepancy between observed and computed summer ablation. For various reasons the right-hand terms of (9.2) need not be independent of each other.

On the lower parts of White Glacier the accumulation is small and the mean and standard deviation of E(t), $\bar{E}$ and S_E will be only weakly influenced by accumulation and its variability whilst higher on White Glacier the effect will be stronger. It is known that variations of accumulation on arctic glaciers from year-to-year are relatively small, see Müller (1966), Koerner (1970), Hatterley-Smith (1974, p.83) and Taylor-Alt (1975). From this point of view the comparison of $a_n(t)$ and $a_c(t)$ will still be a meaningful exercise.

A further problem is the fact that summer ablation and annual net balance are known to be spatially quite variable (1 to 100 m scale) in addition to being strongly dependent upon altitude, see Müller (1963c, p.38) and Young (1972). The computation of a_c does not take account of these small-scale, topographically related, variations and it can only be hoped that the averaging of data from a few stakes, to compute the a_n values used in the study, will partly smooth out this effect in the observed data. From this point of view a_n may be

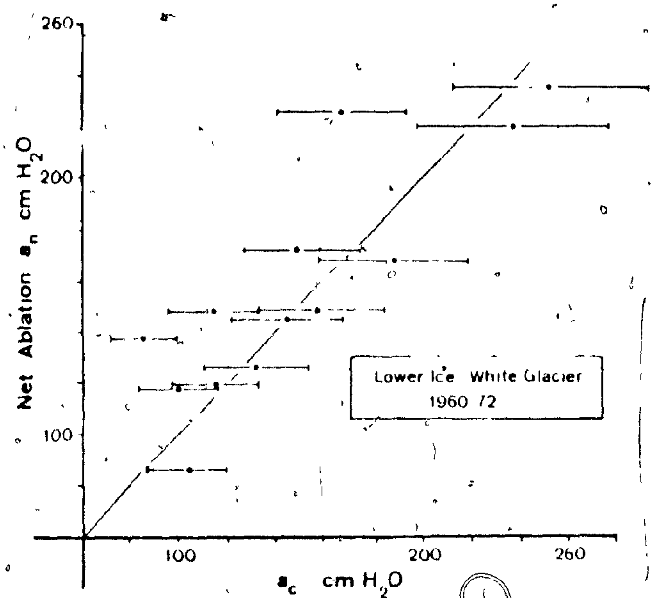


Fig 9.1: Comparison of Computed Summer Ablation a_c with Observed Net Ablation a_n at Lower Ice, White Glacier, for Period 1959/60 to 1971/72. Error Bars Denote Estimated Standard Deviation of Errors in Computing a_c

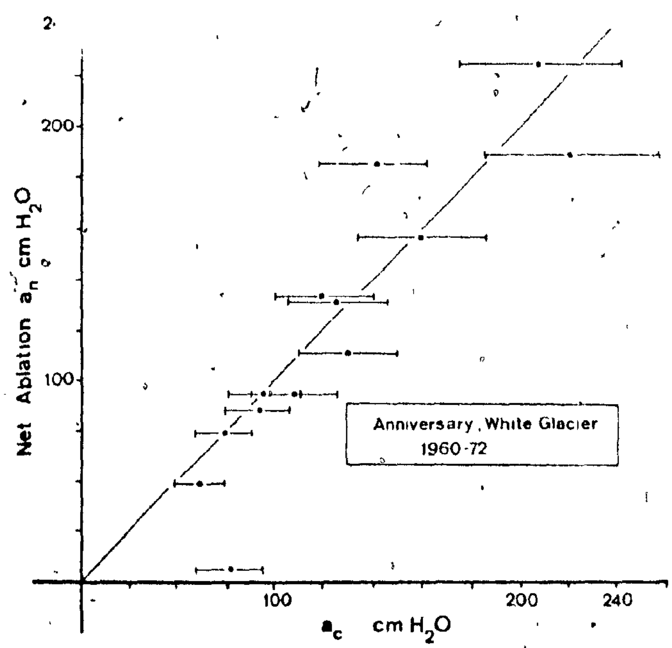


Fig 9.2: Comparison of Computed Summer Ablation a_c with Observed Net Ablation a_n at Anniversary Profile, White Glacier, for Period 1959/60 to 1971/72. Error Bars Denote Estimated Standard Deviation of Errors in Computing a_c

biased with respect to a "representative" value for the altitude concerned.

iii) Comparison of Observed Annual Net Ablation and Computed Summer Ablation on White Glacier 1959/60 to 1971/72

Observed net ablation at three altitudes on White Glacier for the balance years 1959/60 to 1971/72 are compared with the corresponding computed ablation a_c (see Appendix 5 for details of the computation scheme) in Figure 9.1 to Figure 9.3. The error bars given for a_c refer to the standard deviation of the error associated with uncertainty in the ablation-temperature parameter, see Chapter 5(ii). This will not be the only source of error in a_c nor will a_n be perfectly accurate.

Statistics for the discrepancies between a_n and a_c are as follows:

	$\overline{E}$	S_E	$(\overline{E}/\overline{a}_n)$	$(S_E/S_n)^2$
Lower Ice	+8.2 cm	±26.9 cm	+5.2 %	35.2 %
Anniversary	-4.5 cm	±24.3 cm	-3.8 %	18.5 %
Moraine	-27.4 cm	±17.7 cm	-75.5 %	16.1 %

where $\overline{E}$ and S_E are the 13-year mean and standard deviation of the discrepancy between a_n and a_c, $\overline{E}/\overline{a}_n$ is the mean discrepancy expressed as a percentage of the mean of a_n, and $(S_E/S_n)^2$ is the variance of the discrepancy expressed as a percentage of the variance of a_n.

From the Figures 9.1 to 9.3 and the above statistics it is clear that the performance of the model to compute a_c is fairly good from several points of view. Systematic discrepancies, represented by $\overline{E}$, appear to become increasingly negative with increasing altitude and accumulation. At Lower Ice and Anniversary the mean values of E, +8.2 cm and -4.5 cm respectively, are not significantly different from zero (at 5 % level). Average annual accumulation at Moraine is of the order of 10 to 12 cm H_2O (Weiss, personal communication) whereas $\overline{E}$ for Moraine is -27.4 cm H_2O. According to Appendix 5 an average accumulation of 10 cm could produce an $\overline{E}$ of -27 cm if the ablation-temperature parameter for melting snow were about 0.23 cm H_2O $^oC^{-1}$ dy^{-1} (cf. the corresponding value for melting ice of about 0.63 cm H_2O $^oC^{-1}$ dy^{-1}). Alternatively the large negative

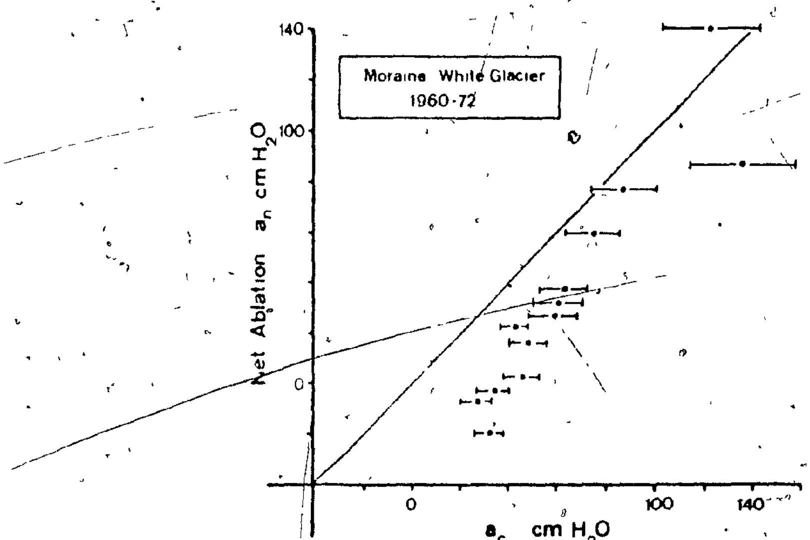


Fig 9.3: Comparison of Computed Summer Ablation a_c with Observed Net Ablation a_n at Moraine Profile, White Glacier, for Period 1959/60 to 1971/72. Error Bars Denote Estimated Standard Deviation of Errors in Computing a_c

values of F could be due to effects of superimposed ice, i.e. melt-water from melted snow refreezing *in situ* to form ice which must be remelted.

Random discrepancies, represented by S_E, appear to decrease with altitude. This is a little difficult to explain. It is certainly true that the errors in a_c due to uncertainty in the ablation-temperature parameter (represented by the error bars in Figures 9.1 to 9.3) become smaller with decrease of a_c (and increase in altitude), but it is doubtful if this effect is sufficient. It is possible that the computed local temperatures and degree-day totals become more accurate with increasing altitude but direct evidence, that this is in fact the case, is missing.

It is clear from Figures 9.1 to 9.3 that "anomalies" do occur. At Lower Ice there are three out of thirteen cases where the discrepancy between a_n and a_c is very large, i.e. the data points lie far from the 45° line in Figure 9.1. At Anniversary there are two such cases. At Moraine there appears to be a generally skewed relationship between a_n and a_c with one "anomaly". The 1963/64 budget year is notable in that a_n is substantially lower than a_c at all three altitudes, for example at Anniversary a_c is almost three times a_n, and at Moraine the net ablation a_n is substantially negative. Although the summer of 1964 is known to have been cool, with a low a_c value, the budget year 1963/64 must also have involved heavier accumulation than normal.

The 3 by 13 table of discrepancies between a_n and a_c was analysed by two-way analysis of variance (Kreyszig, 1970, p.277) to test for significant difference (5 % level) between budget years as well as the differences known to exist between the three different altitudes. The test failed to show significant differences. As a further check the discrepancies at each altitude were expressed as "standardized" data (i.e. deviations from the mean divided by the standard deviation) and the analysis of variance was repeated but without success (differences between altitudes being suppressed by the standardization). Although there are some years when there appears to be a systematic trend in the discrepancies between a_n and a_c at the different altitudes, it is best to regard the discrepancies as random with respect to years. Undoubtedly there should be significant differences between years, corresponding to different prevailing "weather" etc, but they are obscured by other effects, error etc.

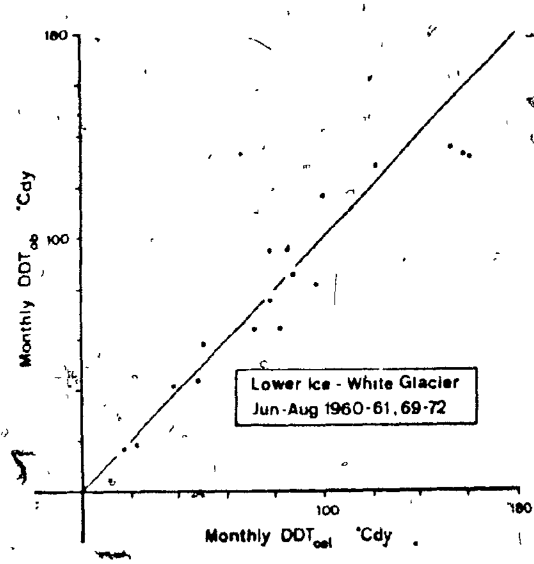


Fig 9.4: Comparison Between Observed and Calculated Monthly Degree-Day Totals at Lower Ice, White Glacier, for the Months June to August 1960-61 & 1969-72

Computation of regression equations between a_n and a_c for the different altitudes shows systematic bias, i.e. the regression coefficients are different from one (differences not significant at 5 % level). The correlation coefficients are 0.85, 0.91 and 0.93 at Lower Ice, Anniversary and Moraine respectively. The corresponding R.M.S. errors are $\mp$23.9, $\mp$23.9 and $\mp$16.5 cm respectively. These are only a little smaller than the corresponding values of S_I. From these considerations it is suggested that the apparent bias of the data points in Figures 9.1 to 9.3 with respect to the 45° line, corresponding to $\bar{a}_n = \bar{a}_c$, could be regarded as fortuitous.

In the following sections the situations at Lower Ice and Moraine will be examined in a little more detail.

iv) <u>The Performance of the a_c Model at Lower Ice for Six Budget Years</u>

There are six summers for which nearly complete temperature records are available at the Lower Ice station on White Glacier: 1960, 1961 & 1969-72. The observed degree-day totals DDT_{ob} for the months of June-August for these years are compared to the corresponding computed degree-day totals DDT_{cal} in Figure 9.4. On the whole, agreement is very good. The daily temperature data for the years 1960-61 and 1969-71 were used in the development of the cooling effect model (Chapters 4 & 5) so that the good agreement in Figure 9.4 cannot be taken as a further support of the cooling effect model. It can, however, be regarded as support for the validity of the computation of degree-day totals from mean temperature, see Appendix 4. The average monthly DDT_{ob} is 77.5°C dy with standard deviation $\mp$41.9°C dy with sample size 18. The average error between DDT_{cal} and DDT_{ob} is +3.5°C dy with standard deviation $\mp$13.5°C dy. Accordingly the errors between DDT_{cal} and DDT_{ob} amount to +4.5 % of the mean of DDT_{ob} and 10.4 % of the variance of DDT_{ob} for the sample of 18 months. The mean error is not actually significantly different (at 5 % level) from zero.

The mean discrepancy between a_n and a_c for the six years is +2.9 cm H_2O compared to +8.2 cm H_2O for the full thirteen years. The average error in degree day totals for the six summers is actually larger than for the 18 months, i.e. +10.6°C dy. This error does not even have the correct sign to explain the discrepancy between a_n and a_c. The main source of discrepancy between a_n and a_c

cannot, therefore, be related to errors in computing degree-day totals but should rather be related to errors in computing ablation from degree-day total. As a test of this, a_n values for the six years were regressed on the summer degree-day totals, observed and calculated. The regression equations were:

$$a_n = 52.2 + 0.45\ DDT_{ob} \quad R = 0.83\ N = 6 \tag{9.3}$$

and

$$a_n = 53.8 + 0.42\ DDT_{cal} \quad R = 0.92\ N = 6 \tag{9.4}$$

The striking agreement between the two equations is encouraging, particularly in view of the small sample size. The slopes of 0.45 and 0.42 cm H_2O $^oC\ dy^{-1}$ have 95 % confidence intervals of ∓0.42 and ∓0.25 cm H_2O $^oC^{-1}$ dy^{-1} respectively. They are, therefore, not significantly different (at 5 % level) from each other, but they are also not significantly different from the assumed ablation-temperature parameter with value 0.63 cm H_2O $^oC^{-1}$ dy^{-1}. However, the close agreement of the slopes of equations (9.3) and (9.4) with the parameter given by Zingg (1951) is noteworthy.

The values of a_c calculated for the 13 years at the three altitudes were adjusted according to Equation (9.4). By this means the 13-year mean discrepancy between a_n and a_c at Lower Ice was reduced to +3.9 cm H_2O, but results at Anniversary and Moraine were much worse than before so that Equation (9.4) cannot be valid for those locations. Actually the intercept in (9.4) should contain information about the length of ablation season, see Equation (8.5) and should, therefore, be smaller at Anniversary and Moraine. But this possibility cannot be tested.

In summary it appears that the discrepancies between observed net ablation at Lower Ice and the computed summer ablation are more likely to be due to errors in the ablation-temperature model than in the cooling effect and degree-day total versus temperature models. It should not be forgotten that the discrepancies are on average relatively small.

v) Effect of Precipitation on Model Performance at Moraine Camp

Precipitation and accumulation data are not available at Moraine Camp for the whole 13-year record. An attempt was made to compare net ablation at Moraine with precipitation records interpolated from Eureka and Isachsen.

However, in view of the findings of Chapter 2 with respect to interpolation of precipitation on the 100 km scale, there was no *a priori* expectation of success. Precipitation totals at Eureka and Isachsen were computed for the winter season, September to May, and for the summer season, June to August, using data given in "*Arctic Summary*". An interpolated series for Axel Heiberg Island was computed using weighting factors of 0.7 and 0.3 for Eureka and Isachsen respectively. Winter precipitation is denoted WP, and summer precipitation is denoted SP. The correlation coefficients for the various series were:

	a_n	a_c	WP	SP
a_n	1.00	0.93	0.07	-0.00
a_c		1.00	0.08	0.11
WP			1.00	-0.28
SP				1.00

With the exception of the correlation between a_n and a_c, none of the correlation coefficients are significant (at 5 % level) or usefully large. As the interpolation errors involved in computing WP and SP are about 94 % and 96 % of variance, see Chapter 2(iii), it may be suggested that the low correlations reflect excessive errors in SP and WP.

The lack of quantitative relationship between weather station precipitation and glacier accumulation can be illustrated by an example: the budget year 1963/64 was a year of notably positive mass balance for White Glacier (Müller, 1966), Meighen Ice Cap (Taylor-Alt, 1975, p.5 and Paterson, 1969a) and Devon Ice Cap (Koerner, 1970). The summer of 1964 was a cool one, with low a_c for White Glacier, but precipitation at Eureka and Isachsen was not excessively high for either of the periods September 1963 to May 1964 or June 1964 to August 1964. It seems clear that the 1963/64 budget year must have been a year of excessive accumulation on glaciers without, at the same time, being a "wet" year at surface weather stations.

The point may be illustrated by a further example. An investigation of accumulation at Upper Ice II at 1920 m a.s.l. on the McGill Ice Cap, Axel Heiberg Island, was carried out in a deep snow shaft during summer 1961 (Müller, 1963a).

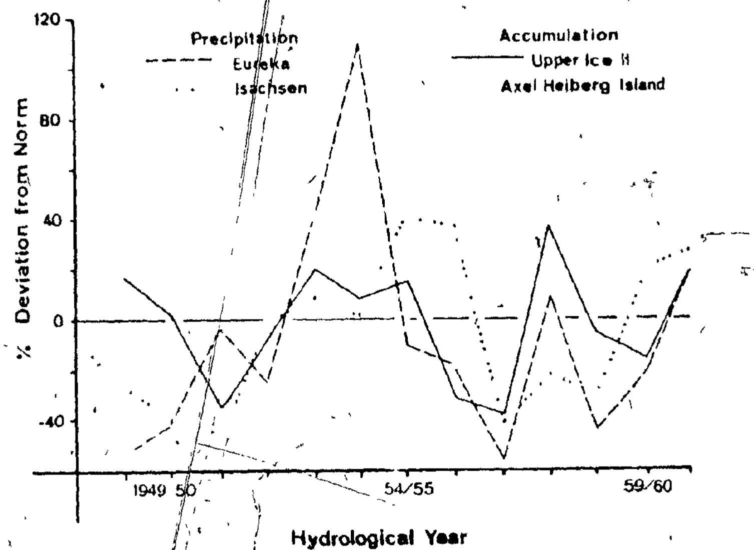


Fig 9.5: Comparison Between Annual Precipitation Totals (August to July) at Eureka and Isachsen and Annual Accumulation Measured in a Deep Snow Shaft at Upper Ice II on Axel Heiberg Island

Reasonably accurate data for annual accumulation back to the 1920s were obtained. Accumulation data are plotted together with Eureka and Isachsen annual precipitation in Figure 9.5 for the period of overlapping records, i.e. 1948/49 to 1960/61. The hydrological year assumed for computation of the annual precipitation totals was chosen to be August to July because August temperatures at 800mb are already generally below freezing. The data in Figure 9.5 are expressed as percentage deviations from the 10-year norms for 1951/52 to 1960/61 for the separate records; the norms used were 68 mm and 98 mm for Eureka and Isachsen annual precipitation and 295 mm for Upper Ice II annual accumulation. From Figure 9.5 there is clear little relation between the various series, for example Eureka precipitation was 110 % above norm for the year 1953/54 whilst Isachsen precipitation and Upper Ice II accumulation were very close to normal. A similar lack of relationship is apparent between the Gilman Glacier accumulation series (Hattersley-Smith, 1964) and Eureka and Alert precipitation series.

It is not suggested that there is no relationship between net ablation at Moraine and precipitation *per se*. In fact, there must be some kind of relationship. However, the technique of interpolating precipitation data from distant weather stations (10^2 km scale) is valueless in attempting to establish such a relationship.

vi) Summary

The model, Appendix 4, is quite successful in that it computes relatively accurate long-term averages of ablation at Lower Ice and Anniversary although discrepancies are quite substantial on a year-to-year basis. Average performance of the model at Moraine is very poor because the model does not take account of accumulation, but even here the computed ablation is a reasonably accurate "index" of the net ablation. The model, as described in Appendix 4, could undoubtedly be improved with a view to improving the year-to-year performance of the model. However, it is suggested that the major problem to be solved for the future is the problem of modelling precipitation and accumulation processes. The parametric approach, quite successful for modelling ablation, is probably invalid for such modelling.

Mean Temperature	Standard Deviation of Temperature 1.0	2.0	3.0	4.0	5.0	6.0
-9.0	0	0	0	0	0	2
-8.0	0	0	0	0	1	4
-7.0	0	0	0	1	3	7
-6.0	0	0	0	2	5	11
-5.0	0	0	1	4	9	16
-4.0	0	0	2	7	14	23
-3.0	0	1	6	13	21	31
-2.0	0	4	11	21	30	41
-1.0	2	10	21	31	42	53
0.0	11	22	33	44	56	67
1.0	32	41	51	61	72	83
2.0	61	64	71	81	91	102
3.0	91	91	96	103	112	122
4.0	121	120	122	127	135	144
5.0	153	150	151	154	159	168
6.0	182	181	180	182	186	193

Table 10.1: Computed Monthly Positive Degree-Day Total (31-day month) as Function of Monthly Mean Temperature and Monthly Standard Deviation of Daily Temperatures.

CHAPTER 10

MODELS OF WHITE GLACIER NET ABLATION VERSUS SUMMER TEMPERATURE

i) Introduction

In the previous chapter the computation of summer ablation on White Glacier, using a parametric model, is discussed. The approach is a new one and the results cannot, therefore, be compared simply to results obtained for other glaciers. The simpler approach of comparing glacier summer balance or net balance for a number of years to the corresponding summer mean temperature or some function of temperature is one that has been carried out for a number of glaciers, e.g. see Martin (1974) and Hoinkes & Steinacker (1975). The non-linear curves of accumulation at the equilibrium line or glaciation limit versus summer mean temperature given by Ahlmann (1924, p.264 and 1948, p.48); Loewe (1971, Fig 2) and Liestøl (personal communication, 1976) can be interpreted also as ablation versus summer temperature curves. Accordingly, for purposes of comparing White Glacier with other glaciers (results for twelve other glaciers are given in the following chapter) it is interesting to express both a_n and a_c in terms of summer mean temperature. This can be done for both $\overline{T}_P$ and $\overline{T}_{IN}$, but the latter will be more interesting as it allows comparison of results from other glaciers where it is not possible to take account of glacier cooling effect, i.e. $\overline{T}_P$ is not known.

It should be first noted that the relationship of monthly degree-day total, and hypothetical monthly ablation, with monthly mean temperature is non-linear. In Table 10.1 and Figure 10.1 this relationship is illustrated under the assumptions of Appendix 4, i.e. that daily temperatures within the monthly sample are stationary and normally distributed with standard deviation S. This means that summer degree-day totals, calculated as a sum of monthly degree-day totals, cannot be exactly expressed as a function, linear or non-linear, of the summer mean temperature. However, it should be noted that the curves in Figure 10.1 are "quasi-linear" or "locally linear" over narrow temperature ranges of several ^{o}C; monthly mean temperature at any particular location will not change from one year to another by more than a couple of ^{o}C.

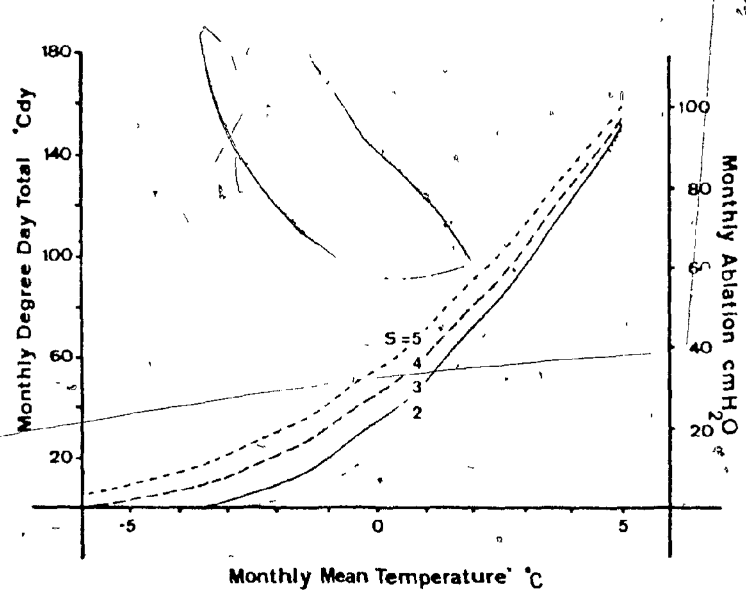


Fig 10.1: Monthly Degree-Day Total as a Function of Monthly Mean Temperature Under the Assumption of a Normal Distribution With Standard Deviation S

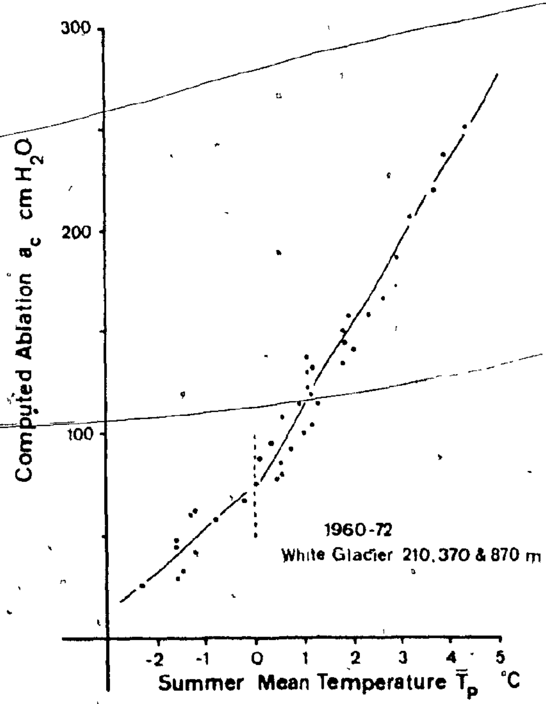


Fig 10.2: Computed Summer Ablation a_c Versus Computed Summer Mean Temperature $\overline{T}_p$ at 210, 370 & 870 m a.s.l. on White Glacier for Period 1960-72

ii) Observed Net Ablation and Computed Summer Ablation Compared to Computed Summer Temperature

In Figure 10.2 the computed summer ablation a_c for the three altitudes on White Glacier are compared to the corresponding computed summer (June-August) mean temperatures $\overline{T}_P$ for the 13-year period 1960-72. The non-linearity and lack of exact relationship is readily apparent; these effects are due to the non-linearity of relationship between the monthly quantities shown in Figure 10.1 and to the fact that the different months have different standard deviations. Separate linear regression equations were computed for positive temperatures (sample size 29) and for negative temperatures (sample size 10) to illustrate the linear approximation of a_c in terms of summer temperature. The regression equations are:

$$a_c = 75.4 + 20.9\,\overline{T}_P \quad R = 0.81 \quad N = 10 \quad \overline{T} < 0^oC \qquad (10.1)$$

and

$$a_c = 72.4 + 40.5\,\overline{T}_P \quad R = 0.98 \quad N = 29 \quad \overline{T} > 0^oC \qquad (10.2)$$

It is noteworthy that the intercepts, 75.4 and 72.4 cm H_2O $^oC^{-1}$ respectively, are almost "matched". According to Equations (10.1) and (10.2) the assumptions made in Appendix 4 lead to the hypothetical expectation that the sensitivity of summer ablation to changes in summer temperature should be between about 21 and 41 cm H_2O $^oC^{-1}$. This range of values reflects the values chosen for the various parameters in implementing the model described in Appendix 4 as well as the average temperature conditions.

As a further comparison of a_n and a_c, regression models between a_n or a_c and $\overline{T}_P$ or $\overline{T}_{IN}$ were computed for each of three altitudes separately, i.e. for Lower Ice, Anniversary and Moraine. The intercept A, slope B and their 95 % confidence intervals k_A and k_B together with the corresponding correlation coefficient R are given in Tables 10.2a and 10.2b. Comparison of Tables 10.2a and 10.2b illustrates the differences between models in terms of $\overline{T}_P$ and $\overline{T}_{IN}$ respectively: in the latter case both slopes and intercepts are lower than in the former case.

Correlation coefficients for the relations between a_c and $\overline{T}_P$ or $\overline{T}_{IN}$ are in all

Location	Y	X	A	k_A	B	k_B	R
Lower Ice	a_n	T_p	92.8	8.5	33.4	12.8	0.86
Lower Ice	a_c	T_p	65.7	3.3	43.1	4.3	0.99
Anniversary	a_n	T_p	65.4	7.1	42.8	14.1	0.89
Anniversary	a_c	T_p	74.4	2.7	39.4	5.3	0.98
Moraine	a_n	T_p	62.4	4.4	33.5	10.9	0.90
Moraine	a_c	T_p	85.1	1.8	27.6	4.5	0.97

Table 10.2a: Statistics for Linear Regression Model Between a_n or a_c and T_p where A is Intercept and B is Slope and k_A k_B are 95 % Confidence Intervals for A and B respectively. R is the Corresponding Correlation Coefficient with Sample Size 13.

Location	Y	X	A	k_A	B	k_B	R
Lower Ice	a_n	T_{IN}	69.7	10.8	27.8	15.1	0.86
Lower Ice	a_c	T_{IN}	36.2	3.9	35.7	5.4	0.99
Anniversary	a_n	T_{IN}	35.8	9.5	35.5	16.8	0.89
Anniversary	a_c	T_{IN}	47.1	3.5	32.7	6.1	0.98
Moraine	a_n	T_{IN}	39.1	3.6	28.0	12.8	0.90
Moraine	a_c	T_{IN}	66.0	1.4	23.0	5.1	0.97

Table 10.2b: Statistics for Linear Regression Model Between a_n or a_c and T_{IN} where A is Intercept and B is Slope and k_A and k_B are 95 % Confidence Intervals for A and B respectively. R is the Corresponding Correlation Coefficient With Sample Size 13.

cases very high. Linear approximation of a_c in terms of temperature is, therefore, quite accurate for the narrow ranges of temperature to be expected at each altitude: the standard deviation of T_p, for example, is only $\pm 1.2^{\circ}C$. The slope for the relationship between a_c and $\overline{T}_p$ or $\overline{T}_{IN}$ decreases with altitude whilst the intercept increases. This can be readily explained by scrutiny of Figure 10.1.

With respect to the comparison of models for a_c and a_n the situation is a little complicated. In no case is the slope for the a_n model significantly different (at 5 % level) from the slope of the a_c model. This is encouraging. However, the slopes for the a_n models at Lower Ice appear too low in comparison to those at Anniversary. Furthermore, the intercepts in a_n and a_c models appear quite inconsistent and are in all cases significantly different from each other (at 5 % level). The discrepancies for Moraine can certainly be explained in terms of accumulation which is included in a_n but not in a_c. Possibly a similar explanation would be adequate for the Anniversary models. However, there is, for several reasons, a problem with the a_n models for Lower Ice. This suggests that either the hypothesis works worst for Lower Ice or that there is something wrong with the a_n data from Lower Ice. No firm conclusions can be drawn. The former possibility would be a little surprising because Lower Ice situations are proportionally well represented in the data base which was used to develope the parametric models (Chapters 4-5 and 7-8). On the other hand, it must be mentioned that the ablation stake data for Lower Ice are in some years problematic because of loss of record due to stakes melting out before they could be redrilled. This problem is less common for the other two altitudes, Anniversary and Moraine. However, there is certainly something offensive about crying "bad data" too easily when faced with results that appear unfavourable to the hypothesis being tested.

Final figures for the mean specific net balance of the whole glacier are not yet available (Müller, personal communication) and will serve as a basis for a future glacier-climate study. Using the method outlined in Appendix 7 together with assumed average temperature conditions over the whole glacier and the model given in Table 10.1 (with an assumed standard deviation of $\pm 3^{\circ}C$ for monthly temperature samples) it can be estimated that the sensitivity of the mean specific net balance for the whole glacier to changes in summer mean temperature will be

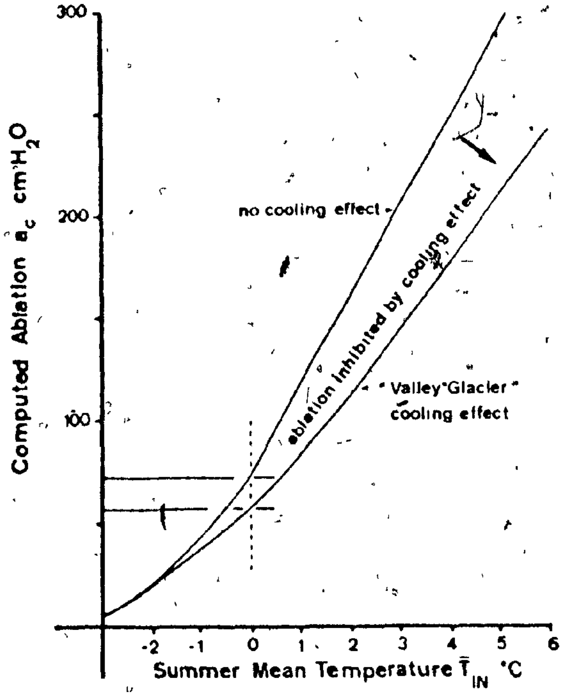


Fig 10.3: Illustration of Cooling Effect as an Inhibitor of Ablation The Two Curves Represent Computed Ablation a_c as a Function of Summer Mean Temperature T_{IN} Under the Assumptions of "No Cooling" and "Valley Glacier Cooling" Respectively.

of the order of -24 cm H_2O $^oC^{-1}$. This is a prediction which must await future testing.

In conclusion it can be said that there are certainly useful relationships between net ablation a_n at various altitudes and summer mean temperature. The errors involved in the linear models range from 26 % of variance at Lower Ice to 19 % of variance at Moraine. The sensitivity of net ablation to changes of temperature in the large scale atmosphere, represented by $\overline{T}_{IN}$, amounts to between about 36 and 28 cm H_2O $^oC^{-1}$ depending upon altitude (and thereby average temperature conditions). However, there remain various problems (especially at Lower Ice) which might be explainable in terms of either errors in the observed data or in the parametric model to compute a_c.

iii) Glacier Cooling Effect as an Inhibitor of Ablation

One of the reasons for studying the cooling effect of glaciers, e.g. in Chapters 4 and 5 of the present work, is to be able to take account of this effect in the computation of ablation using data from distant weather stations. The question naturally arises as to th quantitative influence of cooling effect on computed ablation. This question was studied by repeating the computation of a_c, using the method outlined in Appendix 4, but resetting the relevant parameters, so that the cooling effect is zero, i.e. $\overline{T}_P = \overline{T}_{IN}$. Comparison between the resulting computed ablation and that previously computed, assuming "Valley Glacier" cooling effect, is made in Figure 10.3 (NB the curves are subjectively smoothed through the corresponding point clusters). The difference between the two curves for any particular value of temperature $\overline{T}_{IN}$ represents the reduction or inhibition of ablation due to cooling effect.

A simple physical interpretation can probably be placed on the curves in Figure 10.3: the upper curve might represent ablation at the glacier edge whilst the lower curve represents ablation at points some distance from the glacier edge, e.g. at Lower Ice, Anniversary or at Moraine. This point can be illustrated by an example. Müller (1963c, p.42) gives 1961/62 net ablation figures for each stake (11 in total) in the Anniversary Profile. From the figures he gives, the average net ablation near the glacier centre (stakes A4 to A8) is calculated to be 204.6 with 95 % confidence interval $\pm$23.5 cm H_2O. The corresponding average ablation near the glacier edge (stakes A1, A2, A10 and A11) is 251.0 with 95 %

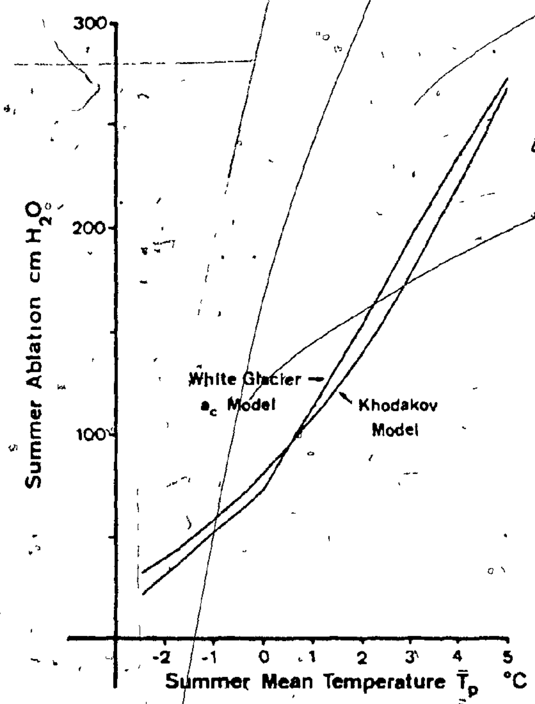


Fig 10.4 : Comparison Between the White Glacier a_c Model and the Khodakov (1975) Model for Summer Ablation as a Function of Summer Mean Temperature Over the Glacier. Summer is Defined as June to August

confidence interval ±29.6 cm H_2O. The computed summer mean temperature $\overline{T}_{IN}$ at Anniversary for summer 1962 was 4.7°C which, using Figure 10.3, gives an ablation of 200 cm H_2O according to the lower curve and 284 cm H_2O according to the upper curve. The former is in excellent agreement with the observed net ablation near the glacier centre whilst the latter is rather too high. One possibility is that some cooling effect already exists at the glacier edge (it should be recalled from Chapter 5 that the Outwash station shows cooling effect although it is not on the glacier). This cannot be directly tested because there were no weather stations at the glacier edge.

If it is accepted that the magnitude of the cooling effect increases with distance from the glacier edge it may be concluded, in qualitative terms at least, that the curves in Figure 10.3 represent an "explanation" of the fact that ablation along a contour line generally decreases with distance from the glacier edge. However, ablation patterns can be very complex in detail, see Adams (1966, Figures 27 and 28) for an example.

iv) Comparison of a_c with the Model of Khodakov

Khodakov (1975) quotes a model of summer ablation a_s versus summer (June-August) mean temperature at the same site:

$$a_s = 0.096(T + 10)^{2.93} \qquad (10.3)$$

The non-linear equation was established by regression analysis with a sample size of 93. Sources of data are not specified in the paper, but they are compiled from pre-IHD literature (Golubev, personal communication, 1977).

It is not the purpose of the present discussion to assess the validity of Khodakov's model, for which more information would be needed, but rather to point out the close similarity between his model and the a_c model. This is illustrated in Figure 10.4 where the Equations (10.1) and (10.2) are plotted together with Equation (10.3). The point can be further illustrated by comparison of the White Glacier a_c values for the 13 years and 3 altitudes with the values a_{kh} computed from the corresponding summer temperatures according to Equation (10.3). The discrepancy $(a_c - a_{kh})$ is denoted by D whose 13-year mean and standard deviation are denoted by $\overline{D}$ and S_D respectively. The results for the three altitudes on White Glacier are as follows:

	$\overline{D}$	$\overline{D}/\overline{a}_c$	S_D	$(S_D/S_c)^2$
Lower Ice	+ 8.5 cm	5.7 %	±12.0 cm	5.5 %
Anniversary	+ 6.5 cm	5.2 %	±11.6 cm	6.0 %
Moraine	− 1.8 cm	−2.8 %	±10.3 cm	9.2 %

where S_c is the standard deviation of a_c. Clearly, the discrepancies between a_c and a_{kh} are small percentages of the means and variances of a_c. This suggests that the assumptions made in Appendix 4, upon which the computation of a_c depends, are also reasonably valid for the situations modelled by Khodakov (1975).

CHAPTER 11

COMPARISON WITH OTHER GLACIERS

i) Introduction

In the following sections the relationship between mass balance and air temperature will be discussed for 12 other glaciers: 3 in the Canadian Arctic, 1 in northern Sweden, 2 in the French Alps, 2 in the Austrian Alps and 4 in the Swiss Alps. In most cases it will only be possible to discuss the relationship between mean specific annual net balance and summer mean temperature at a weather station some distance from the glacier (10^1 to 10^2 km scale). This might be termed a "lumped system" approach and is discussed in Appendix 7. In the eight Alpine cases discussed it is also possible to take account of the effect of precipitation on the mass balance, which was not possible for White Glacier because the nearest weather station was too distant.

Several points should be mentioned before proceeding to a discussion of results:

1) *The parameters relating specific ablation and mean specific ablation respectively to temperature will not, in general, be the same. They may be related in the way discussed in Appendix 7.*

2) *The parameters relating mean specific ablation and mean specific balance respectively to temperature should be the same if the accumulation and temperature are independent of each other. This is discussed in Appendix 8.*

3) *Because of the non-linear relation between summer mean temperature and summer degree-day total it may be expected that the relation between mean specific balance (or ablation) and temperature should be non-linear. However, as mentioned in Appendix 7, this non-linearity may be very weak if the changes in temperature from one summer to another are small, i.e. of the order of 1 to 2°C.*

4) *The amount of change in mass balance per degree of summer mean temperature should, all things being equal, be related to the length of the summer.*

	b_n	a_c	(a_c+b_n)
1960/61	-26	27	1
1961/62	-108	84	-24
1962/63	-24	45	21
1963/64	+25	18	43
1964/65	+6	22	28
1965/66	-7	41	34
1966/67	0	15	15
1967/68	+5	40	45
1968/69	+6	29	35
1969/70	-1	33	32
1970/71	-50	53	3
Mean	-15.8	37.0	21.2
Standard Dvn	36.7	19.5	20.9

Table 11.1: Comparison of Observed Net Balance b_n of Meighen Ice Cap from Taylor-Alt (1975, p.5) and Computed Summer Ablation a_c.

ii) Meighen Ice Cap, Canada

Glaciological and climatological investigations have been carried out on Meighen Ice Cap since 1959 by scientists from Polar Continental Shelf Project. Mean specific net balance data for the ice cap are quoted by Taylor-Alt (1975, p.5) for the period 1960/61 to 1970/71. More detailed (winter and summer balances) data for 1959/60 to 1965/66 are given by Paterson (1969a).

An attempt was made to compute summer ablation a_c on Meighen Ice Cap using the same method as for White Glacier (Appendix 4) with appropriate parameters. Temperatures were interpolated from Eureka and Isachsen using simple distance-weighting factors of 0.38 and 0.62 respectively. Using the June-August 1960-62 monthly mean temperatures given by Arnold (1965) the following relationship was found:

$$T_P = -2.9 + 0.71\ T_{IN} \tag{11.1}$$

where T_P is the monthly mean temperature at the "Main Ice" station on Meighen Ice Cap at 240 m a.s.l.. This relationship is in qualitative agreement with those for other ice cap situations (see Table 4.2 and 4.3). The strong cooling effect may be due to the relatively long travel times of air passing over the ice cap or might be related to the prevailing fogginess which is commented on by several authors.

Equation (11.1) was used in the computation of a_c. The computed summer ablation a_c and observed net balance b_n are compared in Figure 11.1 and Table 11.1. It can be seen that a_c tends to overestimate $-b_n$ by an average of 21 cm H_2O: the discrepancies between $-b_n$ and a_c amount to -134 % and 32 % of the mean and variance respectively of $-b_n$.

Discrepancies will be due to:

- errors in computing degree-day totals
- effects of accumulation

Taylor-Alt (1975, Tables 3:6a and 6b) quotes observed degree-day totals for the months June-August for the years 1960-62 and 1968-70. These are plotted against the corresponding computed monthly degree-day totals in Figure 11.2. At low temperatures there is a tendency for the computed total to overestimate

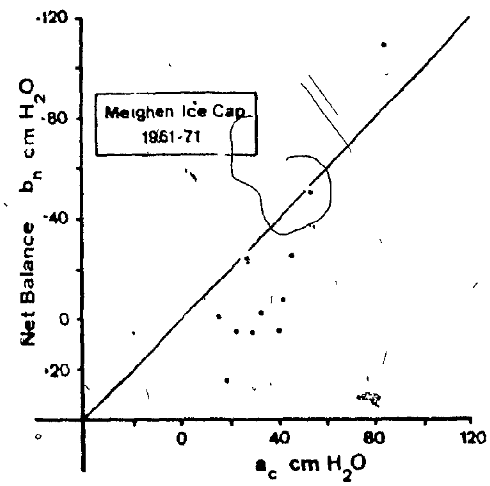


Fig 11.1: Comparison Between Computed Summer Ablation a_c and Observed Net Balance b_n for the Meighen Island Ice Cap for Period 1960/61 to 1970/71

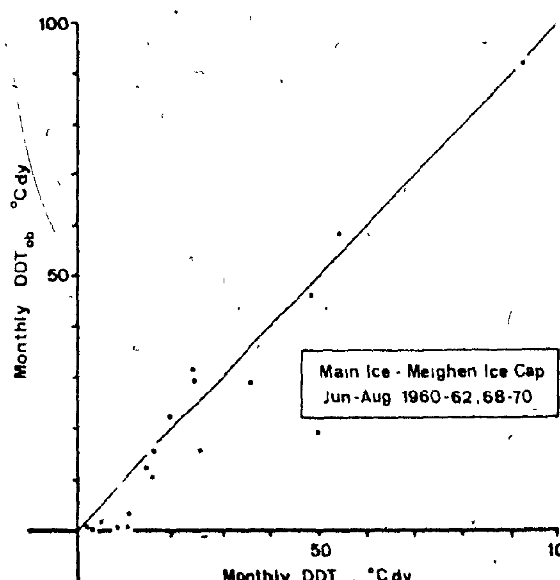


Fig 11.2: Comparison Between Observed and Calculated Monthly Degree-Day Totals at Main Ice, Meighen Island Ice Cap for the Months June-August 1960-62 & 1968-70

the observed quantity but, on the whole, agreement is fair. The mean error in computing summer degree-day totals is +11.8°C dy (mean of six) which should be equivalent to about 7.4 cm H_2O ablation.

From figures given by Paterson (1969a) the average annual accumulation can be taken as about 17.6 cm H_2O (actually this figure refers to the 1959/60 to 1965/66 average). According to Appendix 5 this could produce a discrepancy between a_c and $-b_n$ bigger than 17.6 cm H_2O.

The correlation between b_n and a_c is -0.90, corresponding to "explanation" of 81 % of the variance of b_n.

The equation for regression of b_n on $\overline{T}_P$, the computed local summer (June-August) mean temperature, is as follows:

$$b_n = -76.8 - 39.0\ \overline{T}_P \quad R = -0.82 \quad N = 11 \qquad (11.2)$$

The corresponding equation for regression of b_n on $\overline{T}_{IN}$ is:

$$b_n = +35.2 - 27.1\ \overline{T}_{IN} \quad R = -0.81 \quad N = 11 \qquad (11.3)$$

Comparison of the two equations illustrates the effect of taking account of cooling effect. The sensitivity of the mass balance to change in local temperature is -39 cm H_2O $^{o}C^{-1}$ and to changes of temperature in the large-scale atmosphere -27 cm H_2O $^{o}C^{-1}$. It might be noted that Equation (11.3) can be obtained almost exactly by algebraic manipulation of (11.1) and (11.2).

Regression of the 7-year summer balance series, computed from figures given by Paterson (1969a) for 1959/60 to 1965/66, in terms of $\overline{T}_P$ gives:

$$b_s = -86.1 - 37.7\ \overline{T}_P \quad R = -0.94 \quad N = 7 \qquad (11.4)$$

The agreement between the slopes of (11.2) and (11.4) is very close. The R.M.S. errors of (11.2) and (11.4) are $\mp$ 21 cm H_2O and $\mp$ 14 cm H_2O respectively. It seems clear that variations in summer air temperature, and of a_c, are a major

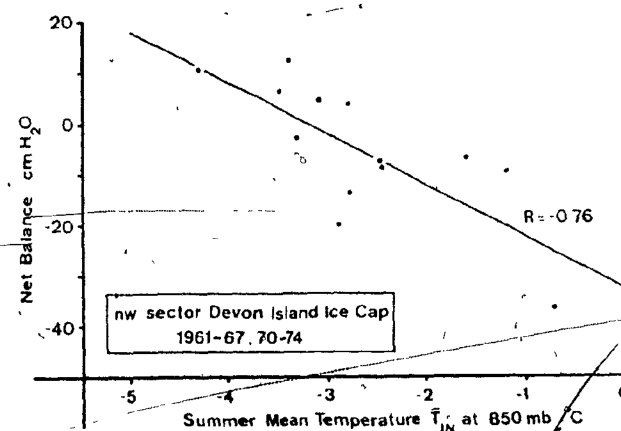


Fig 11.3: Observed Net Balance of the NW Sector Devon Island Ice Cap Versus Interpolated Summer (June-August) Mean Temperature at 850 mb for Periods 1961-67 & 1970-74

source of variations in the mass balance of Meighen Ice Cap. Taylor-Alt (1975) used a Synoptic-Energy Balance approach to study the same problem. She does not quote errors involved in her approach, but they will almost certainly be lower than those quoted in the present section. On the other hand, the Synoptic Energy Balance approach requires observations of local meteorological elements for each summer of study whilst the a_c approach described here only needs a few summers of local data to "calibrate" the model which can then be applied to summers for which there are no local observations.

iii) Devon Ice Cap, Canada

Mean specific net balance data for the north-west sector of the Devon Island Ice Cap from Dr R.M. Koerner of Polar Continental Shelf Project are reproduced by Kasser (1973, p.168) for 1960/61 to 1966/67 & 1969/70. Data for 1969/70 to 1973/74 are given by Koerner in *"Ice"* (various dates).

Summer mean (June-August) temperatures at 850 mb were computed for the location of the Devon Island I.C.S. by interpolation of 850 mb temperatures at Resolute and Eureka. Thule and Clyde data would have been useful for this purpose, but the Clyde Upper Air Station was closed in 1970, and Thule data are difficult to obtain. The observed mean specific net balance for the north-west sector of the ice cap is plotted against 850 mb summer mean temperature $\overline{T}_{IN}$ in Figure 11.3. The regression equation is:

$$b_n = -32.0 - 10.1\ \overline{T}_{IN} \qquad R = -0.76 \qquad N = 12 \qquad (11.5)$$

The corresponding R.M.S. error is $\pm$ 9.0 cm H_2O.

The slope of the regression equation seems very low, i.e. -10.1 cm H_2O $^oC^{-1}$. This is most likely due to the fact that the temperature over most of the ice cap is low and that, at low temperature, the degree-day total and, hence, the ablation are relatively insensitive to temperature changes compared to the situation at higher temperature (see Figure 10.1). The 12-year mean of the 850mb temperature is -2.7^{o}C which would correspond to a local temperature of about -4.9^{o}C according to Equation (4.5). A quantitative investigation of this problem using the model in Appendix 7 would be very difficult because of various uncertainties, e.g. the problem as to whether air temperatures over the whole ice cap are described by Equation (4.5) or whether a transition to weaker cooling effect occurs at lower altitudes.

The validity of the model in Equation (11.2) can be checked. For various reasons the net balance figures for 1967/68 and 1968/69 could not be separately determined: the combined total for the two years was -28.2 cm H_2O as determined by Koerner. Values of $\overline{T}_{IN}$ for the 1968 and 1969 summers were -2.4 and -1.7°C respectively. According to Equation (11.5) the corresponding net balance values would be -7.8 and -14.8 cm H_2O respectively with 67 % probable error of about $\mp$ 9 cm H_2O. The 2-year sum would be -22.6 cm H_2O which is in fair agreement with the observed 2-year total of -28.2 cm H_2O, an error of -20 %. By way of contrast, the 12-year mean net balance is -4.9 cm H_2O and the 2-year total predicted using this mean would be -9.8 cm H_2O, an error of -65 %.

It seems clear that summer temperatures play a major role in controlling the mass balance of the Devon Island Ice Cap. This is consistent with the finding by Bradley (1975, p.270) of a strong correlation between the annual equilibrium line altitude (ELA) of Devon Ice Cap and the altitude of July freezing levels.

iv) Decade Glacier, Canada

Mass balance data (winter and summer balance) for Decade Glacier on Baffin Island are published by Kasser (1973, p.175-176) for the period 1965/66 to 1969/70. Data were collected by Glaciology Division, Environment Canada, and a description of work on Decade Glacier is given by Østrem *et al* (1967).

An attempt was made to compute summer ablation on Decade Glacier using the model derived for White Glacier: Equations (10.1) and (10.2) were used to compute specific ablation quantities from upper air data at Clyde. The cooling effect of the glacier was assumed identical to White Glacier and the computation was made for each 100 m altitude band between 450 and 1450 m a.s.l. using 950, 900 and 850 mb data at Clyde. Total volumetric ablation was obtained by area-weighting of the computed specific ablation quantities for the individual altitude bands and finally the mean specific ablation a_c was obtained by dividing total ablation by the glacier area. The calculation could not be done for Summer 1970 because the Upper Air station at Clyde was closed during that summer.

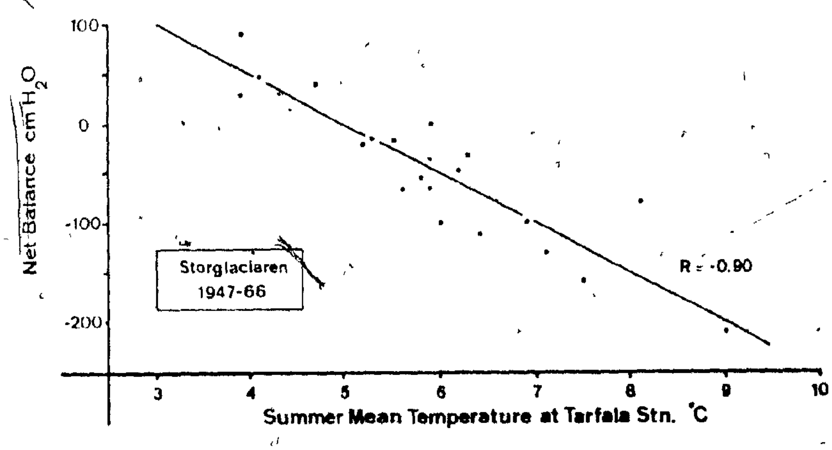


Fig 11.4: Net Balance of Storglaciären, Northern Sweden, Versus Summer (June-August) Mean Temperature at the Tarfala Station Near the Glacier Snout for Period 1946/47 to 1965/66

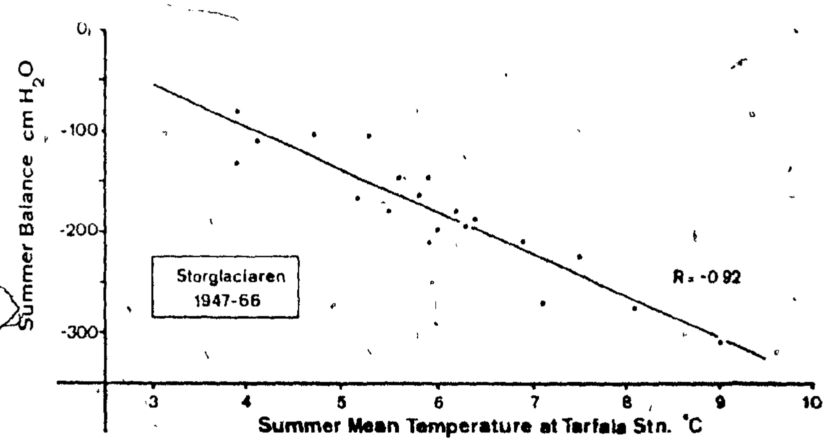


Fig 11.5: Summer Balance of Storglaciären, Northern Sweden, Versus Summer (June-August) Mean Temperature at the Tarfala Station Near the Glacier Snout for Period 1946/47 to 1965/66

computed mean specific ablation a_c can be compared to the observed mean specific summer balance b_s as follows:

	b_s	a_c
1965/66	−97 cm H_2O	75 cm H_2O
1966/67	−29	49
1967/68	− 9	38
1968/69	−102	93
4-year mean	−59	64

The average discrepancy given by $(a_c+b_s)/b_s$ is only − 8 %. This is not bad considering that no local information from Decade Glacier was used. Unless this agreement is merely fortuitous it can be concluded that Decade Glacier is very similar to White Glacier with respect to both cooling effect and the relation between ablation and temperature.

Regression of the mean specific net balance of Decade Glacier on summer (June-August) mean temperatures at 900 mb over Clyde gave a slope of −40 cm H_2O $^oC^{-1}$ (sample size of only 4 !).

v) Storglaciären, Northern Sweden

Programmes of glaciological and climatological observations on Storglaciären in Swedish Lapland have been carried out since 1946 by Stockholm University. Schytt (1967) showed that the 20-year 1946/47 to 1965/66 summer ablation series is well correlated (R = 0.92) with summer (June-August) mean temperature at the Tarfala station (1130 m a.s.l.) near the glacier snout. In an earlier paper Schytt (1962) states that present temperatures (since 1946) would have to fall by 1.2^{o}C to bring the glacier into equilibrium with its present accumulation and mass distribution.

In Figures 11.4 and 11.5 the mean specific net balance and summer balance respectively are plotted against summer (June-August) mean temperature at the Tarfala station for the balance years 1946/47 to 1965/66. The data are taken from Schytt (1967, Table 1). The corresponding regression equations for the net balance b_n and summer balance b_s are:

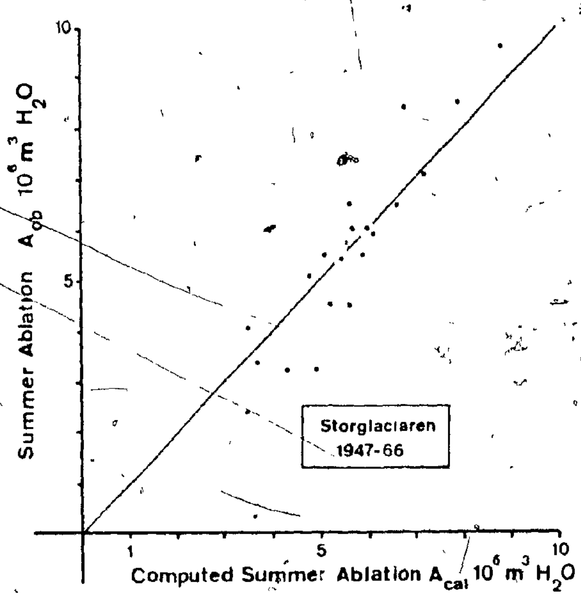


Fig 11.6: Comparison Between Computed Summer Ablation A_{ob} and Observed Summer Ablation A_{cal} for Storglaciären for Period 1946/47 to 1965/66. Units are 10^6 m^3 H_2O

$$b_n = 251.5 - 50.5\ T \qquad R = 0.90 \qquad N = 20 \tag{11.6}$$

and

$$b_s = 74.0 - 42.7\ T \qquad R = 0.92 \qquad N = 20 \tag{11.7}$$

with corresponding R.M.S. errors of ∓ 32 cm H_2O and ∓ 24 cm H_2O respectively. Comparison of the two equations is interesting for several reasons: the difference in intercept, the difference in slope and the close similarity of the correlation coefficients. If T is 0^oC the net balance b_n would be +252 cm H_2O and the summer balance would be +74 cm H_2O according to the regression equations. The discrepancy would be +177 cm H_2O compared to the known 20-year mean accumulation of 132 cm H_2O. However, the accumulation is also correlated with temperature. The effect of this can be assessed using Equation (A8.4) from Appendix 8:

$$R(b_n,T) = \frac{S_c}{S_b} R(c,T) - \frac{S_a}{S_b} R(a,T) \tag{A8.4}$$

Substitution of the various statistics relating to Storglaciären gives:

$$-0.90 = 0.54(-0.26) - 0.83(0.92) \tag{11.8}$$

If accumulation and temperature were independent, everything else remaining equal, the correlation between net balance and temperature would be reduced to $-0.83 \times 0.92 = -0.76$. Accordingly, the high correlation of net balance with temperature reflects not only the high correlation of ablation with temperature but also the weak dependence of accumulation on temperature. Presumably the latter will be related to a weak association of "cool" summers with "wet" summers and the influence of temperature in determining whether precipitation falls as snow or as rain.

The gradient of net balance with respect to temperature is -50.8 cm H_2O $^oC^{-1}$ compared to -42.7 cm H_2O $^oC^{-1}$ for summer balance. The gradient of accumulation with respect to temperature is, in this case, -7.8 cm H_2O $^oC^{-1}$ which accounts exactly for the discrepancy.

Model	Cooling Effect	G	$\overline{E}$	S_E
1	Class 1a	-0.005	3.20	∓0.75
2	Class 1	-0.005	2.31	∓0.73
3	Class 2	-0.003	0.87	∓0.81
4	Class 2	-0.005	0.07	∓0.81
5	Class 2	-0.007	-0.71	∓0.81
6	Class 3	-0.005	-1.71	∓0.79

Table 11.2: 20-year Mean and Standard Deviations $\overline{E}$ and S_E of Error Between Computed Total Ablation A_{cal} and Observed Total Ablation A_{ob} Computed by 6 Different Models for Storglaciären, Units are 10^6 m^3 H_2O.

An attempt was made to compute total (volumetric) summer ablation on Storglaciären using a similar method as for White Glacier. Details of the computing scheme are given in Appendix 6: temperature data from Tarfala station (Schytt, 1967, Table 2) were used as input data. The calculation was repeated several times assuming different temperature lapse rates and different cooling effects, .t in all cases the relationship between specific ablation and local temperature was assumed to be given by either Equation (10.1) or (10.2). Results for six runs of the computation are given in Table 11.2 where $\overline{E}$ and S_E are the 20-year means and standard deviations of the errors between computed ablation A_{cal} and observed ablation A_{ob}. *A priori* the basic assumptions of Model No 4 are most reasonable, i.e. Class 2 or Valley Glacier cooling effect and a vertical temperature gradient of -0.5°C per 100 m, and agreement between A_{cal} and A_{ob} is, in fact, best in this case. Observed and computed ablations A_{ob} and A_{cal} corresponding to Model 4 are plotted in Figure 11.6 from which it can be seen that agreement is quite good. The 20-year average error between A_{cal} and A_{ob} is 0.07×10^6 m^3 H_2O which corresponds to a mean error of +1,3 %. The variance of the error is 18.4 % of the variance of A_{ob}. However, from Figure 11.6 there appears to b some bias with under- (over-) estimation of A_{ob} by A_{cal} for warm (cool) summers.

From these results it can be concluded that not only is the mass balance of Storglaciären controlled by temperature, as already pointed out by Schytt (1967), but that the mechani of this control is quantitatively very similar to that of White Glacier.

vi) Glacier de Sarennes and Glacier de Saint-Sorlin, French Alps

Glacier de Sarennes and Glacier de Saint-Sorlin are two small glaciers (ca 1 km^2) in the Massif des Grandes Rousses, French Alps. Mass balance data from Glacier de Sarennes have been collected since 1948/49 by the Service des Eaux et Forêts (Kasser, 1967) whilst mass balance measurements on Glacier de Saint-Sorlin have been made since 1956/57 by the Laboratoire de Glaciologie in Grenoble (Lliboutry, 1974).

Martin (1974) has reported results of a multiple regression analysis of 16-year (1956/57 to 1971/72) mean specific net balance data from these two glaciers using summer (May-August) mean temperature and annual (September-August) total precipitation at nearby climatological stations, at Chazelet (1780 m a.s.l.) and Village de Saint-Sorlin (1550 m a.s.l.) respectively, as independent variables. It is not necessary to give full details here as Martin (1974) presents a full discussion. The regression equations he obtained were as follows:

Glacier de Sarennes

$$b_n' = 0.233P' - 56.7T' \quad R = 0.94 \quad N = 16 \qquad (11.9)$$

Glacier de Saint-Sorlin

$$b_n' = 0.227P' - 51.4T' \quad R = 0.86 \quad N = 16 \qquad (11.10)$$

where P is the annual precipitation in mm H_2O and the prime ' denotes deviation from the 16-year means. From data given by Martin (1974, Tables 5 & 6) the R.M.S. errors for the two equations were computed to be $\mp$ 31 and $\mp$ 21 cm H_2O.

The close similarity of the two equations is noteworthy. The gradient of the net balance with respect to temperature is -57 cm H_2O $^oC^{-1}$ for Glacier de Sarennes and -51 cm H_2O $^oC^{-1}$ for Glacier de Saint-Sorlin. These are larger than those found for the glaciers previously considered. Presumably because of the longer 4-month summer. The correlation between precipitation and temperature is negligible. Further statistical information of interest is given in Table 11.4.

From data given by Martin (1974, Tables 5 & 6) the following simple regression equations were computed:

Glacier de Sarennes

$$b_n' = -62.9T' \quad R = -0.62 \quad N = 16 \qquad (11.11)$$

Glacier de Saint-Sorlin

$$b_n' = -55.7T' \quad R = -0.54 \quad N = 16 \qquad (11.12)$$

Two things are worthy of note: firstly the relatively low correlation coefficients and secondly the increase in gradient of the net balance with respect to temperature in comparison to those for the multiple regression models. For

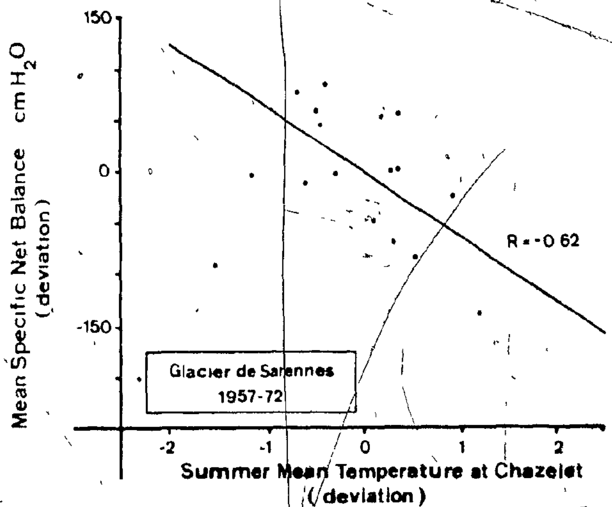


Fig 11.7: Net Balance of Glacier de Sarennes, French Alps, Versus Summer (May-August) Mean Temperature at Chazelet for Period 1956/57 to 1971/72. Data are Deviations from Means

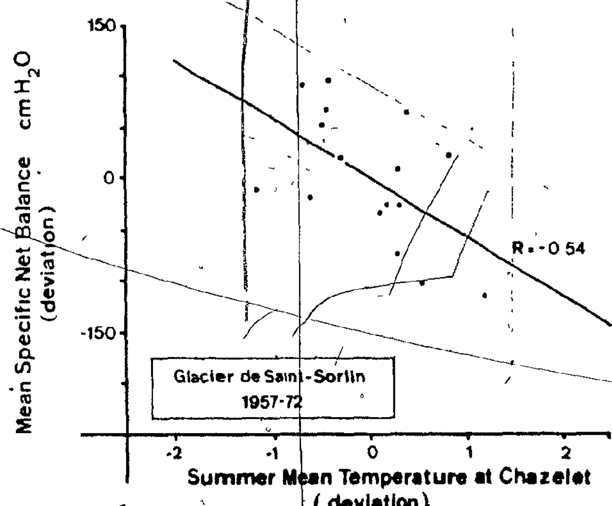


Fig 11.8: Net Balance of Glacier de Saint-Sorlin, French Alps, Versus Summer (May-August) Mean Temperature at Chazelet for Period 1956/57 to 1971/72. Data are Deviations from Means

example, in the case of Glacier de Sarennes: temperature alone only explains 38 % of the variance of b_n whilst temperature and precipitation together explain 88 %, in the first case the slope of b_n with respect to T is -57 cm H_2O $^oC^{-1}$ whilst in the second case it is -63 cm H_2O $^oC^{-1}$. Scatter diagrams are given in Figures 11.7 and 11.8.

vii) Hintereisferner and Kesselwandferner, Austrian Alps

Mass balance measurements have been made on Hintereisferner since 1952/53 and on Kesselwandferner since 1957/58 by the Institut für Meteorologie und Geophysik der Universität Innsbruck. The Hintereisferner mass balance programme is discussed by Hoinkes (1970). More detailed discussion of the relationships with climatic elements is given by Hoinkes and Rudolph (1962), Hoinkes *et al* (1968) and Hoinkes and Steinacker (1975).

The approach taken by Hoinkes and his co-workers to the problem of relating the mass balance of Hintereisferner to temperature and precipitation at a nearby weather station (station Vent at 1900 m a.s.l.) is rather complicated to describe, but Hoinkes and Steinacker (1975) can be quoted as follows:

"In an earlier contribution to simple glacio-meteorology (Hoinkes *et al* 1968) it was shown that daily observations of air temperature at Vent (1900 m) and of precipitation falling as snow on the glacier could be used to assess net ablation on Hintereisferner. Taking as average a lapse rate of $-0.6^oC/100$, melting conditions at the terminus of Hintereisferner (2400 m) commence when daily mean temperatures in Vent exceed 3^oC; they prevail over the whole drainage basin with temperatures in excess of 10^oC. Air temperature was reduced from Vent to the terminus of Hintereisferner by subtracting 3^oC, and cumulative positive degree-days were calculated for the potential ablation period May to September. Precipitation falling at Vent with air temperature below 3^oC was considered as fresh snow on the whole glacier. As a threshold for precipitation 3 mm was chosen, corresponding to about 5 mm of water equivalent on the glacier. To melt this amount of fresh snow two positive degree-days were considered necessary, which were subtracted for each 3 mm of precipitation. Thus numerically degree-days are equivalent to two-third of the below 3^oC precipitation (in mm) at Vent. In this way a cumulative curve of degree-days was obtained which is called the TS curve. In two recent papers it was shown that the TS(3^o) sum at

Vent could also be used to estimate the mean specific mass balance of Hintereisferner for the budget years following 1962/63. The connection was clearly better than with the sum of positive degree-days without considering the retardation of ice melt by falls of fresh snow (Hoinkes, 1970, 1971)."

Hoinkes and Steinacker (1975, p.145) proceed to apply the concept to the whole observation period 1952/53 to 1968/69. They state that the correlation of mean specific net balance with the sum of positive degree-days reduced to an elevation of 2400 m is -0.72. Taking account of the heat necessary to melt fresh snow, i.e. using $TS(3^o)$, improves the correlation to $R = -0.76$. They introduce further corrections for the effect of fresh snow only on the higher parts of the glacier ($R = -0.81$), for the amount of winter snowfall at Vent ($R = -0.85$) and for the variation in length of the ablation season with altitude ($R = -0.91$).

It would certainly seem that there is good evidence that the mass balance of Hintereisferner can be expressed in terms of precipitation and temperature at the nearby weather station Vent. The approach is in an interesting contrast to that of Martin (1974). There is a problem, however: the late Professor Hoinkes was a leading exponent of the dominant role of radiation in controlling glacier variations, and his great success with the approach described here would seem to deny this role. This problem is discussed in detail in Appendix 9.

From the data given by Hoinkes and Steinacker (1975, Table 1) the R.M.S. error corresponding to the final correlation of -0.91 is calculated to be ∓ 22 cm H_2O and the slope of the regression equation is 0.51 cm H_2O $^oC^{-1}$ dy^{-1}. This contrasts with the figure of 0.63 cm H_2O $^oC^{-1}$ dy^{-1} which was deduced for the White Glacier. It is noteworthy that Hoinkes assumes a figure of 0.25 cm H_2O $^oC^{-1}$ dy^{-1} for pure snowmelt.

For comparison of Hintereisferner with other situations a multiple regression analysis was carried out whereby the mean specific net balance was regressed on summer (May-September) mean temperatures at station Vent and on annual (October-September) precipitation (in mm H_2O) at Vent. A similar regression was carried out for the mass balance of the Kesselwandferner which is immediately adjacent to Hintereisferner. The 21-year (1952/53 to 1972/73) Hintereisferner and 16-

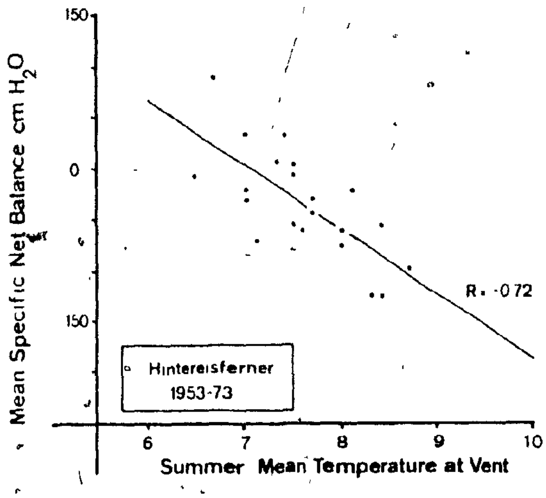


Fig 11.9: Net Balance of Hintereisferner, Austrian Alps, Versus Summer (May-September) Mean Temperature at Vent for Period 1952/53 to 1972/73

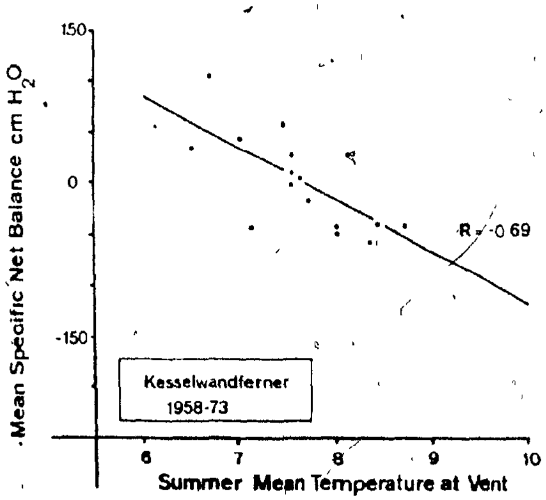


Fig 11.10: Net Balance of Kesselwandferner, Austrian Alps, Versus Summer (May-September) Mean Temperature at Vent for Period 1957/58 to 1972/73

year (1957/58 to 1972/73) Kesselwandferner series were taken from Kasser (1967 and 1973) and from Kuhn (1976). Climatological data from Vent up to 1967/68 are reported by Hoinkes (1970, p.81) and later data were kindly made available by Dr M. Kuhn of Innsbruck (personal communication, 1976). The regression equations were as follows:

Hintereisferner

$$b_n' = 0.292P' - 41.2T' \quad R = 0.83 \quad N = 21 \qquad (11.13)$$

Kesselwandferner

$$b_n' = 0.281P' - 29.5T' \quad R = 0.85 \quad N = 16 \qquad (11.14)$$

where the prime ' denotes deviations from the 21-year and 16-year means respectively.

Corresponding R.M.S. errors are $\mp$ 29 cm H_2O and $\mp$ 24 cm H_2O respectively.

Some further details of the statistics are given in Table 11.4, but it should be mentioned that there is a relatively strong correlation between T' and P'. The corresponding simple regression equations were:

Hintereisferner

$$b_n' = -62.1T' \quad R = -0.71 \quad N = 21 \qquad (11.15)$$

Kesselwandferner

$$b_n' = -48.9T' \quad R = -0.69 \quad N = 16 \qquad (11.16)$$

These relationships are illustrated in Figures 11.9 and 11.10. Noteworthy is the consistently lower gradient of Kesselwandferner mass balance with respect to temperature compared to that of Hintereisferner. A rough calculation based upon the model in Appendix 7 indicates that this may be due to the fact that the main mass of Kesselwandferner is higher lying, and subject to lower temperatures, than the main mass of Hintereisferner.

viii) Aletschgletscher, Silvrettagletscher, Limmerngletscher and Griesgletscher, Swiss Alps

Long-term mass balance measurements are carried out on four glaciers in

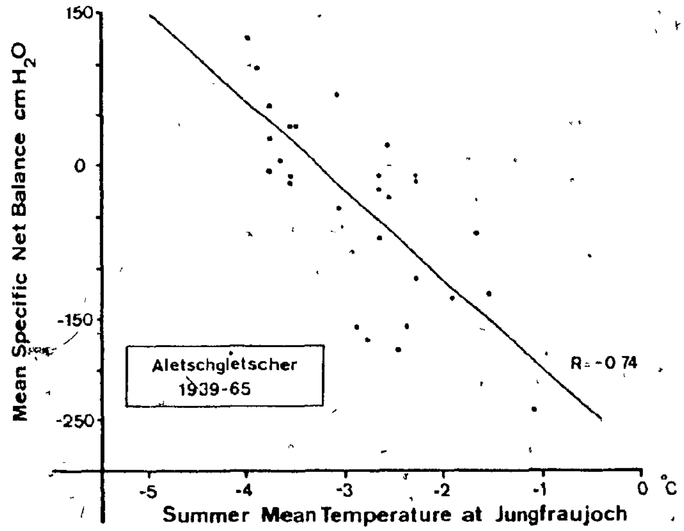


Fig 11.11: Net Balance of Aletschgletscher, Swiss Alps, Versus Summer (May-September) Mean Temperature at Jungfraujoch for Period 1938/39 to 1964/65

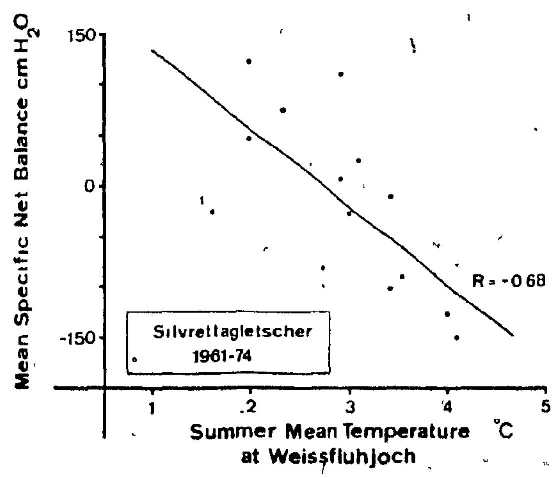


Fig 11.12: Net Balance of Silvrettagletscher, Swiss Alps, Versus Summer (May-September) Mean Temperature at Weissfluhjoch for Period 1960/61 to 1973/74

Switzerland by the Versuchsanstalt für Wasserbau, Hydrologie und Glaziologie (VAW) of the Eidgenössische Technische Hochschule (ETH) Zürich.

In the present study the following mass balance series were analysed: a 27-year series (1938/39 to 1964/65) from Aletschgletscher, a 14-year series (1960/61 to 1973/74) from Silvrettagletscher, a 20-year series (1954/55 to 1973/74) from Limmerngletscher and a 13-year series (1961/62 to 1973/74) from Griesgletscher. Sources of data were Kasser (1967 and 1973) and Kasser & Aellen (1973, 1974 and 1975). The Silvrettagletscher, Limmerngletscher and Griesgletscher series are based upon direct, i.e. ablation stake, measurement whilst the Aletschgletscher series is based upon hydrological data from the *Massaboden* limnograph station and is described (perhaps unfairly) as being of "uncertain reliability" by Sugden and John (1976, p.103).

Regression models were computed for the glacier mean specific balances in terms of summer (May-September) mean temperature and annual (October-September) total precipitation at nearby weather stations. The weather station records which were used were Jungfraujoch (Sphinx) at 3576 m a.s.l. for Aletschgletscher, Weissfluhjoch at 2667 m a.s.l. for Silvrettagletscher, and Gütsch ob Andermatt at 2284 m a.s.l. for Limmerngletscher and Griesgletscher. Monthly mean temperature and monthly total precipitation for the above stations were extracted from the station records published annually in *"Annalen der Schweizerischen Meteorologischen Zentralanstalt"* except for precipitation which is not observed at Jungfraujoch. In this case the corrected annual (1 October to 30 September) total precipitation from the *Aletschwald* totalizer (2040 m a.s.l.) was used. The records of mass balance analysed for Aletschgletscher and Limmerngletscher did not comprise the full available records as the initiation of mass balance measurements actually predated the establishment of the relevant weather stations (i.e. Jungfraujoch and Gütsch ob Andermatt). In addition, the method of computing the mass balance for Aletschgletscher was changed in 1965 by use of hydrological data from the newly established *Blatten bei Naters* limnograph station.

Multiple regression equations with summer mean temperature and annual precipitation as independent variables were as follows:

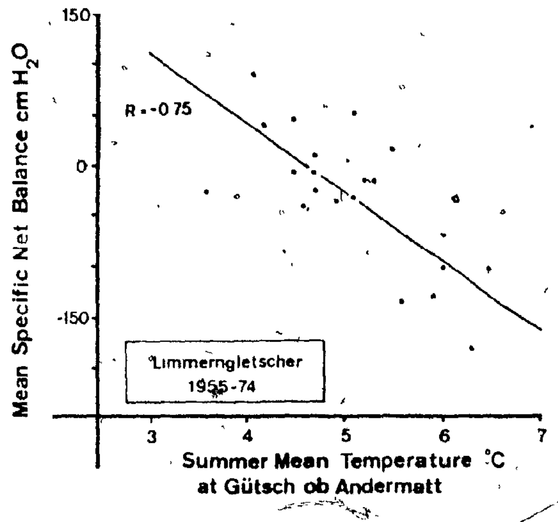


Fig 11.13: Net Balance of Limmerngletcher, Swiss Alps, Versus Summer (May-September) Mean Temperature at Gütsch ob Andermatt for Period 1954/55 to 1973/74.

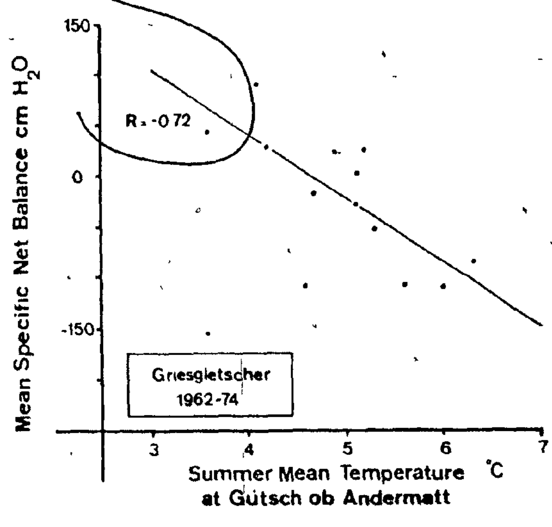


Fig 11.14: Net Balance of Griesgletscher, Swiss Alps, Versus Summer (May-September) Mean Temperature at Gütsch ob Andermatt for Period 1961/62 to 1973/74

Aletschgletscher

$$b_n' = 2.264P' - 65.7T' \qquad R = 0.89 \qquad N = 27 \qquad (11.17)$$

Silvrettagletscher

$$b_n' = 0.257P' - 57.7T' \qquad R = 0.91 \qquad N = 14 \qquad (11.18)$$

Limmerngletscher

$$b_n' = 0.127P' - 57.5T' \qquad R = 0.83 \qquad N = 20 \qquad (11.19)$$

Griesgletscher

$$b_n' = 0.068P' - 54.3T' \qquad R = 0.76 \qquad N = 13 \qquad (11.20)$$

where the prime ' denotes deviation from the long-term means.

Comparison of the different situations is interesting. In all cases more than 50 % of the variance is explained and the gradients of mass balance with respect to temperature are fairly consistent from one situation to another unlike the gradients of mass balance with respect to precipitation.

Simple regression equations involving temperature alone were as follows:

Aletschgletscher

$$b_n' = -87.3T' \qquad R = -0.74 \qquad N = 27 \qquad (11.21)$$

Silvrettagletscher

$$b_n' = -77.7T' \qquad R = -0.68 \qquad N = 14 \qquad (11.22)$$

Limmerngletscher

$$b_n' = -69.1T' \qquad R = -0.75 \qquad N = 20 \qquad (11.23)$$

Griesgletscher

$$b_n' = -63.4T' \qquad R = -0.72 \qquad N = 13 \qquad (11.24)$$

These relationships are illustrated in Figures 11.11 to 11.14.

The gradients of mass balance with respect to temperature in the simple regression equations (11.21) to (11.24) are steeper than in the multiple regression equations (11.17) to (11.20). This is similar to the other situat-

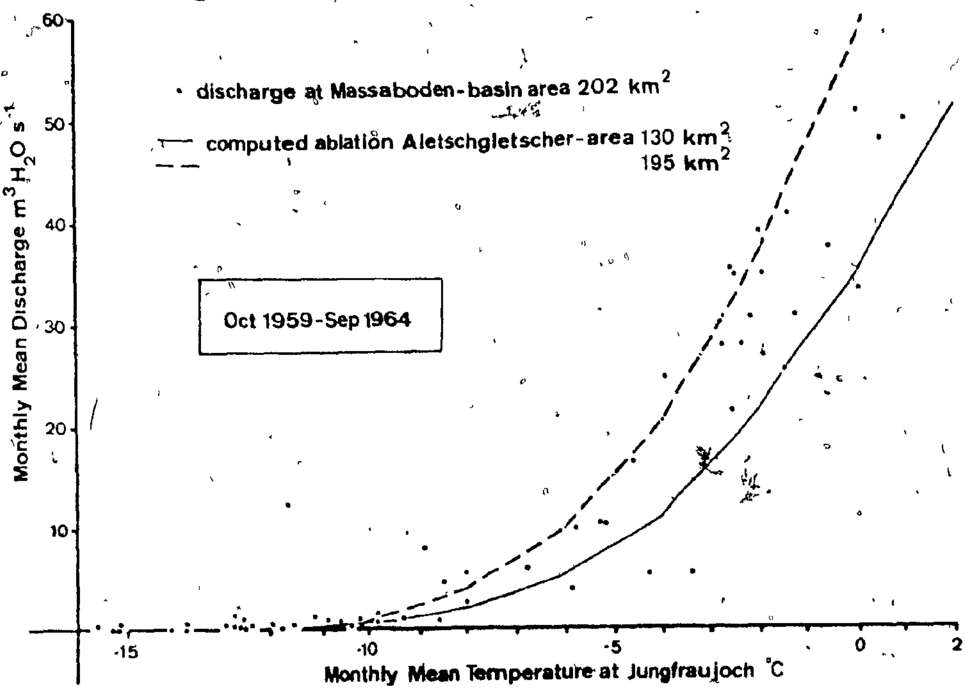
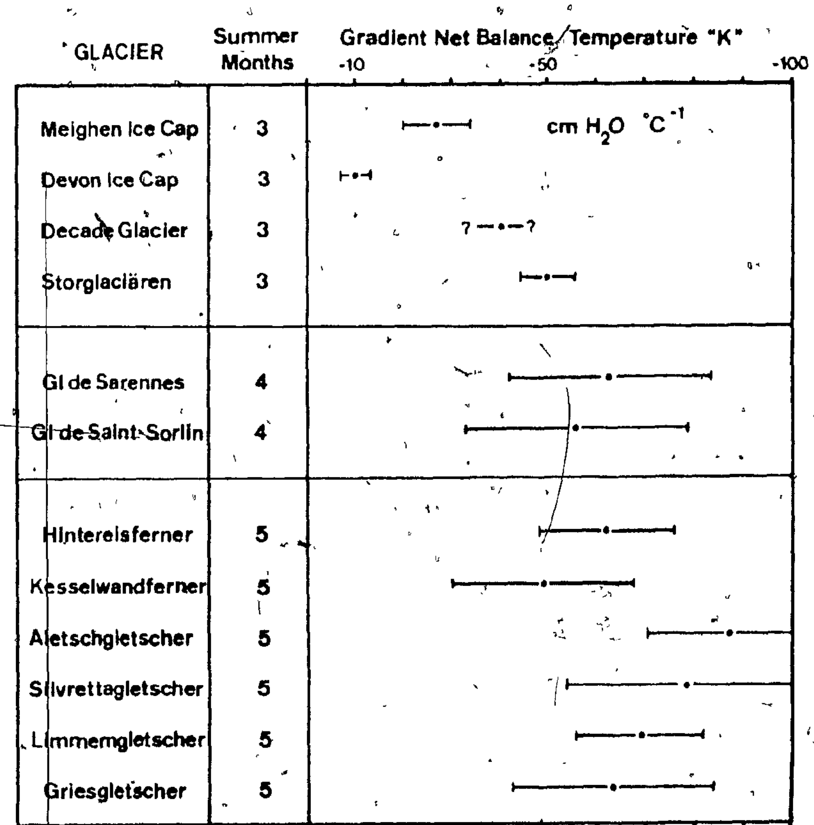


Fig 11.15: Observed Monthly Mean Discharge at *Massaboden* Versus Monthly Mean Temperature at Jungfraujoch. The Two Curves Represent the Discharge Computed as a Function of the Temperature at Jungfraujoch for the Area of Aletschgletscher and for the Area of the Basin Respectively.

ions and is, presumably, related to interactions between temperature and precipitation and between temperature and accumulation. Further statistical information is given in Table 11.4.

The slope of the equation for Aletschgletscher seems rather steep. This might be because the temperature at Jungfraujoch is already influenced by the cooling effect of Aletschgletscher whilst the other stations, i.e. Weissfluhjoch and Gütsch ob Andermatt, are uninfluenced by the cooling effect of their glaciers. If it is assumed that the cooling effect of Aletschgletscher at Jungfraujoch is the same as for White Glacier, the gradient of -87.3 cm H_2O $^oC^{-1}$ in (11.21) may be scaled with a factor of 0.83 for comparison with the other situations. This would give an adjusted gradient of -72.5 cm H_2O $^oC^{-1}$ compared to the average gradient for the other three situations of -70.1 cm H_2O $^oC^{-1}$ which is with respect to temperature variations in the large-scale atmosphere. This agreement is so close that the explanation might, perhaps, be considered as correct.

Kasser (1967, Table 23) presents monthly run-off data measured at the *Massaboden* limnograph station at 687 m a.s.l. below the Aletschgletscher for the 60-month period October 1959-September 1964. These data are plotted against the corresponding monthly mean temperatures at the Jungfraujoch station at 3580 m a.s.l.. As Aletschgletscher covers 64 % of the drainage basin for *Massaboden* an attempt was made to compute monthly total ablation expressed in units of m^3 H_2O s^{-1}, for the glacier using monthly mean temperature at Jungfraujoch. For the computation a temperature lapse rate of $-0.6^oC/100$ m was assumed so that monthly mean temperatures could be computed for each 100 m altitude inverval on the glacier. These were converted into monthly degree-days using the model given in Table 10.1 and assuming a monthly standard deviation of 3^oC. Specific ablation was computed for each 100 m altitude interval by assuming 0.63 cm H_2O $^oC^{-1}$ dy^{-1} and total ablation was computed by area-weighting the specific ablation. According to Kasser (1967, Table 15 parts 2 and 4) the glacier area is 130 km^2, and the basin area is 195 km^2. Total ablation was computed from specific ablation for both areas as glacier-free areas will contribute melt-water from snowmelt for some months. The two different estimates of total ablation are plotted in Figure 11.15: all things being equal, the correct value should lie between the two curves. Overall, the agreement of the observed discharge with the com-



EQUATION ; $b_n = A + K.T$

Fig. 11.16: The Gradient K of Mean Specific Net Balance b_n With Respect to Summer Mean Temperature T for 12 Different Glaciers as a Function of the Assumed Length of Summer. Error Bars Denote Computed Standard Deviations of K

puted discharge curves is quite fair, anomalously large discharges could be due to contributions from precipitation. Figure 11.15 does illustrate the non-linearity of the relationship between total ablation and air temperature at one site on a month-to-month basis.

ix) Summary

In all twelve cases examined there are significant (at 5 % level) correlations between mean specific net balance and air temperature at weather stations situated some distance from the glacier (10^1 to 10^2 km scale).

Summary statistics for the corresponding simple regression models are given in Table 11.3 & Figure 11.16 where K is the parameter linking net balance to summer mean temperature, S_K is its sampling standard deviation, R_{bT} is the coefficient of correlation between net balance and temperature, N is the number of years of record and M is the assumed length of summer in months. In all cases "summer temperature" refers to temperature in the large-scale atmosphere, i.e. not adjusted for glacier cooling effect. Aletschgletscher is possibly the exception.

There are several anomalies in Table 11.3 and Figure 11.16 which have already been discussed in the previous sections, e.g. the low K value for Devon Ice Cap, the high K value for Aletschgletscher and the discrepancy between K values for Hintereisferner and Kesselwandferner.

The increase of K with the length of summer M is not as marked as might have been expected. For Storglaciären with a 3-month summer K is -51 cm H_2O $^oC^{-1}$, the average K value for the two French glaciers is -60 cm H_2O $^oC^{-1}$ whilst the average of K for the six situations with 5-month summers is -68 cm H_2O $^oC^{-1}$. Although these differences could be fortuitous and more cases would be needed to draw definitive conclusions, it is suggested that these differences are real. It appears that the response of mass balance to temperature changes of arctic glaciers is smaller, i.e. lower K, than the response of Alpine glaciers on account of the shorter summer.

It is certainly not claimed that temperature is the only element responsible for variations in mass balance. In fact, in four out of the twelve cases temperature "explains" less than 50 % of the variance of the mass balance, i.e.

Glacier	Period	N	M	K	S_K	R_{bT}
Meighen Ice Cap	1961-71	11		-27	± 7	-0.81
Devon Ice Cap	1961-74	12*	3	-10	± 3	-0.76
Decade Glacier	1966-69	4	3	(-40)§	(± 6)§	(-0.98)§
Storglaciären	1947-66	20	3	-51	± 6	-0.90
Glacier de Sarennes	1957-72	16	4	-63	±21	-0.62
Glacier de Saint-Sorlin	1957-72	16	4	-56	±13	-0.54
Hintereisferner	1953-73	21	5	-62	±14	-0.71
Kesselwandferner	1958-73	16	5	-49	±19	-0.68
Aletschgletscher	1939-65	27	5	-87	±16	-0.74
Silvrettagletscher	1961-74	14	5	-78	±24	-0.68
Limmerngletscher	1955-74	20	5	-69	±13	-0.75
Griesgletscher	1962-74	13	5	-63	±21	-0.72

* data for balance years 1967/68 and 1968/69 missing

§ N.B. sample size is very small

Table 11.3: Summary Statistics for Linear Regression Model Relating Mean Specific Net Balance for 12 Glaciers to Summer Mean Temperature. N is Number of Years of Record, M is the Assumed Length of Summer in Months, K is the Gradient of Net Balance with Respect to Temperature in cm H_2O $^oC^{-1}$, S_K is the Sampling Standard Deviation of K, and R_{bT} is the Correlation Coefficient for Net Balance and Summer Temperature.

R_{bT} is less than -0.71. For the eight Alpine situations it is possible to study the additional effect of precipitation variations on the mass balance. In Table 11.4 information (B, C and R_m) relating to the multiple regression equations, involving temperature and precipitation, are repeated together with the correlation coefficients R_{bT}, R_{bP} and R_{TP}. It can be seen that in five out of the eight cases the correlation between mass balance and precipitation R_{bP} is slightly better than the correlation between mass balance and temperature R_{bT} although most of these differences are not statistically significant (5 % level). In most cases there are moderate correlations between precipitation and temperature R_{TP} so that; for example, the temperature carries some information about precipitation variations. This will be one reason why the B parameters in Table 11.4 are not identical to the K parameters in Table 11.3.

From comparison of Tables 11.3 and 11.4 it is clear that the introduction of annual precipitation as an extra independent variable leads to better "explanation" of the variances of the mass balances. For example, the mean-square of R_m in Table 11.4 is 0.74 compared to a mean-square of 0.47 for the corresponding R_{bT} values. On the other hand, the consistency of the C parameters from one situation to another is poor compared to the consistencies of B and K. The coefficients of variability for the eight situations are ∓ 156 %, ∓ 22 % and ± 18 % respectively for C, B and K. This means that the repeatability of the multiple regression models, involving temperature and precipitation together, is poorer than that of the simple regression models, involving temperature alone. This could be the result of topographic effects whereby the relationship between the precipitation at a weather station and the accumulation on a glacier is unique to each station-glacier pair.

x) Conclusions

The relationships found between glacier mean specific net balance and summer mean temperature at weather stations distant from the glacier and, in particular, the relatively good consistency of the K parameter can be explained in terms of the results of previous chapters as follows:

(a) Variations of summer specific ablation on glaciers are partly controlled by the local summer degree-day total which is in turn controlled by

Glacier	B	C	R_m	R_{bT}	R_{bP}	R_{TP}	N	M
Gla.de Sarennes	-56.7	0.233	0.94	-0.62	0.75	-0.08	16	4
Gla.de Saint-Sorlin	-51.4	0.227	0.86	-0.54	0.70	-0.06	16	4
Hintereisferner	-41.2	0.292	0.83	-0.71	0.72	-0.48	21	5
Kesselwandferner	-29.5	0.281	0.85	-0.68	0.77	-0.46	16	5
Aletschgletscher	-65.7	2.264	0.89	-0.74	0.72	-0.35	27	5
Silvrettagletscher	-57.7	0.257	0.91	-0.68	0.77	-0.27	14	5
Limmerngletscher	-57.5	0.127	0.83	-0.75	0.57	-0.27	20	5
Griesgletscher	-54.3	0.068	0.76	-0.72	0.50	-0.38	13	5

Table 11.4: Statistics for Multiple Regression Models Relating Deviations of Mean Specific Net Balance (b_n') to Deviations of Summer Temperature (T') and Annual Precipitation in millimetres (P') at Nearby Weather Stations. The Sample Size is N, the Length of Summer is M Months, R_m is the Multiple Correlation Coefficient, and R_{bT}, R_{bP} and R_{TP} are Correlation Coefficients Between the Single Variables.

Model: $b_n' = B.T' + C.P'$

the local summer mean temperature.

(b) Variations of local temperature over glaciers are related to the temperature variations in the large-scale atmosphere which are detected by weather stations around the glacier, i.e. uninfluenced by the glacier.

(c) Variations of specific ablation on glaciers control the mean specific net balance of the glacier to a greater or lesser extent depending upon the concurrent variations of accumulation.

In Table 11.5 statistics relating to the mass balances of the twelve glaciers under "present" climate are given: the mean and standard deviation of the net balance, $\bar{b}_n$ and S_b, over the N years of record together with the corresponding standard deviations of summer mean temperature S_T. The variability of mass balance of Canadian arctic glaciers (Meighen Ice Cap, Devon Ice Cap and Decade Glacier) is somewhat lower than the variability for Alpine glaciers although the variability of temperature is greater. This is consistent with the lower K values.

Using "present" $\bar{b}_n$ values together with the corresponding K values it is possible to estimate the hypothetical temperature change from "present" temperatures which would be necessary to bring each glacier into equilibrium with its present mass distribution. This change is denoted by (T_0-T_1) in Table 11.6. The necessary temperature changes are very small, i.e. -1°C or smaller (Kesselwandferner already has a +ive balance). This is interesting as some authors, e.g. Hoinkes (1955, p.501), have suggested that secular changes in temperature of "only" ∓ 1°C are insufficient to explain the alternations between "advance" and "retreat" of Alpine glaciers that have been observed since systematic observations were started in the late 19th century.

There is, as a matter of fact, some evidence of a new glacier advance. For example, in Switzerland the percentage of glaciers in advance has risen from about 25 % in 1964/65 to over 50 % in 1974/75 (Kasser and Aellen, 1976, Fig 7) which has not been exceeded since 1919/20. The relationship between advance/retreat and mass balance is, of course, a complex one so that one should not conclude that this advance is necessarily caused by lowering of summer temperatures, see Posamentier (1977). However, on the basis of the present analysis, the author suggests this as a possibility.

Glacier	Period	N	$\bar{b}_n$	S_b	S_T
Meighen Ice Cap	1961-71	11	-16 cm	± 37 cm	± 1.1 °C
Devon Ice Cap	1961-74	12*	-5	± 14	± 1.1
Decade Glacier	1966-69	4	-31	± 50	± 1.2
Storglaciaren	1947-66	20	-49	± 74	± 1.3
Glacier de Sarennes	1957-72	16	-41	± 63	± 0.6
Glacier de Saint-Sorlin	1957-72	16	- §	± 64	± 0.6
Hintereisferner	1953-73	21	-33	± 53	± 0.6
Kesselwandferner	1958-73	16	+3	± 45	± 0.6
Aletschgletscher	1939-65	27	-39	± 92	± 0.8
Silvrettagletscher	1961-74	14	-17	± 86	± 0.8
Limmerngletscher	1955-74	27	-30	± 69	± 0.8
Griesgletscher	1962-74	14	-22	± 67	± 0.8

* data for balance years 1967/68 and 1968/69 missing

§ mean value not published

Table 11.5: Mass Balance and Temperature Statistics Under "Present" Climate for 12 Glaciers. $\bar{b}_n$ and S_b are Mean and Standard Deviation of Mean Specific Net Balance for a Period of N Years. S_T is the Standard Deviation of Summer Mean Temperature Over the Same N Year Period.

Glacier	$\bar{b}_n$	K	(T_0-T_1)
Meighen Ice Cap	-16 cm	-27 cm $^oC^{-1}$	-0.6 ^oC
Devon Ice Cap	-5	-10	-0.5
Decade Glacier	-31	-40	-0.8
Storglaciären	-49	-51	-1.0
Glacier de Sarennes	-41	-63	-0.7
Glacier de Saint-Sorlin	- §	-56	-
Hintereisferner	-33	-62	-0.5
Kesselwandferner	+3	-49	+0.1
Aletschgletscher	-39	-87	-0.4
Silvrettagletscher	-17	-78	-0.2
Limmerngletscher	-30	-69	-0.4
Griesgletscher	-22	-63	-0.3

§ mean value not published

Table 11.6: Change of Temperature (T_0-T_1) from "Present" Temperature T_1 to Hypothetical Temperature T_0 Corresponding to Zero Mass Balance for 11 Glaciers. $\bar{b}_n$ is "Present" Mass Balance in cm H_2O and K is Gradient of Mass Balance with Respect to Temperature in cm H_2O $^oC^{-1}$ for "Present" Glacier.

CHAPTER 12

CONCLUDING REMARKS

i) Usefulness of the Present Study

Air temperature data from standard weather stations at some distance from the glacier - up to several hundred kilometres - can be used to give information about air temperatures over glaciers. In the context of the White Glacier this means that gaps in the observed records of local temperature - of lengths of a few days to whole summers - can be filled with reasonably accurate computed temperature. This has obvious utility because the collection of temperature data for even part of a summer involves heavy investment in terms of money, man-power and logistics.

It is concluded that variations of air temperature are generally a better measure of short-term ablation variations than are variations of net radiation. This is a useful result because air temperature is simply and cheaply measured in comparison to net radiation whose measurement requires great care.

Summer ablation and annual mass balance of many glaciers can be estimated using temperature data from the standard climatological network. This is useful because there are few meteorological records from glaciers whose mass balance, run-off or tongue activity are reasonably well known. As emphasis has been placed upon obtaining repeatable models, the year-to-year errors in such models will tend to compensate on the secular time-scale.

The problem of explaining or predicting glacier fluctuations should, according to the above, reduce to the problem of explaining or predicting climate as it is defined on the scale of resolution represented by the existing network. This formulation of the problem is useful as it allows one to make further study by analysis of the relatively long records from permanent stations rather than the short, and typically discontinuous, records from temporary weather stations on glaciers.

ii) Limitations of the Present Study

Some remarks should be made about several interesting problems which the present study has either failed to solve or not attempted.

a) No attempt has been made to "explain" the general climate or its fluctuations: it has simply been taken as defined by the data at the various permanent weather stations such as Eureka, Isachsen, etc. Such explanation is beyond the scope of the present study and can, in fact, be treated as a separate non-glaciological problem using accepted climatological techniques.

b) The sampling units for the statistical analyses are periods of record. It could be claimed that sampling should be with respect to prevailing weather type, i.e. that a synoptic climatological approach should be used. This hypothesis should be critically tested in the future although it might be problematic because one would either have to analyse smaller samples than are analysed in the present study or less homogeneous ones. In any case, a weather type approach could not avoid the use of statistics or the problems of inference.

c) Although least-squares methods have been used in the statistical analyses, more importance has been placed upon obtaining repeatable models rather than upon mere reduction of variance. From the various quoted results it is clear that all the models could, in principle, be improved with respect to further reduction of unexplained variance by the introduction of new independent variables. However, this should not be done at the cost of poorer repeatability of the models.

d) The results presented in Chapters 4 & 5 are a little unsatisfactory as the question of time and spatial variability of the parameters, for example the transition between unglacierized and glacierized environments or between winter and summer situations, is not solved; computed parameters are simply classified with respect to several simple classes. Furthermore, it is still not entirely clear how a particular location can be assigned to a particular class on an *a priori* basis. Numerical simulation combined with carefully planned field experiments would be necessary to study these problems.

e) From analyses of short-term energy-balance data in Chapters 7 & 8 it is suggested that ablation is poorly related to net radiation because the latter is, in most cases, not sufficiently variable. This conclusion should not be extended to the question of longer-term, e.g. annual, variations of ablation because the relevant statistics are not known. Furthermore, the short-term energy-balance data are themselves somewhat unreliable (particularly with respect to the long-wave radiation components which were estimated using empirical formulae) so that the conclusions could be affected by errors in the data. More field measurements with modern instruments are required for complete resolution of the problem.

f) The parametric approach implemented in the present study does not appear to be suitable for modelling processes involving precipitation, particularly accumulation. This is probably due to the relatively poor spatial autocorrelation of precipitation together with effects of topography. The problem of modelling glacier accumulation in terms of climate is still, therefore, a major problem awaiting solution. For glaciers in the Canadian Arctic, the first requirement would be a denser precipitation station network.

g) The statistical methods used in the study are admittedly primitive, e.g. the ordinary least-squares method does not take account of effects of possible autocorrelation in the various time-series so that computed confidence intervals must be interpreted cautiously.

APPENDIX 1

PHYSICAL MODELS OF THE THERMAL MODIFICATION OF AIR BY GLACIERS

i) Introduction

In principle the air temperature over a glacier (or anywhere else) can be described by the first law of thermodynamics in combination with other governing equations of the atmosphere (SMIC, 1971, p.105). Solution of the equations would represent the best and most "physical" approach to the problem. In practice, however, solution of the equation is not easy: if the equations are at all realistic they are too complicated for analytical solution, and there will seldom be enough detailed meteorological data from the glacier and its near surroundings to specify boundary conditions etc.

An empirical approach to the problem of air temperatures over glaciers is proposed in the present work. The approach involves the postulation of some simple equation relating the variables of interest. Unspecified coefficients in the equation are then evaluated by the statistical analysis of data. The assessment of the "validity" of the approach can, to some extent, be made by an examination of the computed statistics, i.e. on an *a posteriori* basis. However, it is also necessary to demonstrate, as far as it is possible, that the postulated equation and assumptions do bear some logical relationship to the problem at hand. This will be attempted in the present Appendix with the help of heuristic arguments.

The models which will be discussed refer to the following situation:

A parcel of air is transported over the glacier by the horizontal wind field from the warm unglacierized surroundings. The air parcel loses heat to the cool glacier surface by turbulent heat exchange whilst gaining some heat from overlying air by the same mechanism. However, there is a net heat loss, and cooling of the air parcel occurs.

ii) Simple Diffusion Model

For simplicity a flat semi-infinite glacier is considered with the horizontal wind velocity perpendicular to the glacier edge. The steady state assumption is made, i.e. the local derivative of temperature with respect to time is neglected in comparison to the advective derivative. For these simple, and unrealistic, assumptions the heat equation reduces to:

$$u(z)\frac{\partial T}{\partial x} = \frac{\partial}{\partial z}K(z)\frac{\partial T}{\partial z} \qquad (A1.1)$$

where u(z) and K(z) respectively are the horizontal wind speed and turbulent exchange coefficient at height z. Even for this simple equation analytical solution is hardly possible. A further drastic simplification can be made by assuming that u and K are constant with respect to height:

$$u\frac{\partial T}{\partial x} = K\frac{\partial^2 T}{\partial z^2} \qquad (A1.2)$$

This is clearly unrealistic for the Prandtl Layer over a glacier although it may not be too bad for the Ekman Layer. However, Ohmura (1972) and Müller *et al* (1973) show that thermal modification of air by a "small" glacier occurs mainly in the Prandtl Layer.

The boundary conditions chosen for solution of (A1.2) are appropriate for vertically well mixed air, with initial lapse rate c, advecting from a homogeneous region with surface temperature T' over a flat semi-infinite glacier with a fixed surface temperature of T". The origin of the x-axis is chosen so that the glacier edge is at x = 0. The boundary conditions are:

$$x \leq 0 \quad z = 0 \quad T = T'$$
$$x > 0 \quad z = 0 \quad T = T''$$
$$x \leq 0 \quad z > 0 \quad T = T' + cz$$

The solution of (A1.2) with the given boundary conditions is:

$$T(x,z) = (T'' - T')\,\mathrm{erfc}\,(\eta) + T' + cz \qquad (A1.3)$$

where $\eta = z/2\sqrt{\frac{Kx}{u}}$ and erfc is the Complementary Error Function (Carslaw and Jaeger, 1959, Appendix III).

A change of variable in (A1.3) can be made by putting $T_{IN}(z) = T' + cz$ and $T^{**}(z) = T'' + cz$ which gives:

$$T(x,z) = \mathrm{erf}(\eta)\, T_{IN}(z) + \mathrm{erfc}(\eta)\, T^{**}(z) \qquad (A1.4)$$

Clearly $T_{IN}(z)$ is the initial state of the air at height z whilst $T^{**}(z)$ is the final state of the air after complete "domestication" of the air by the cool glacier surface. The solution T(x,z) at any point (x,z) is a linear combination of the initial and final states. $erf(\eta)$ varies from unity outside of the thermal influence of the glacier, i.e. $x \leq 0$ or at large z, to zero at large x or small z whilst $erfc(\eta)T^{**}(z)$ varies from zero to T'' at the glacier surface.

Equation (A1.4) shows that the solution depends not only upon "location" as represented by (x,z) but also upon "weather" as represented by u and K. For the purposes of the present discussion (x,z) can be regarded as fixed, i.e. the coordinates of some weather station on the glacier. However, T and T_{IN} must be regarded as functions of time t, e.g. they could be regarded as daily mean temperatures for successive days. The wind speed and turbulent exchange coefficient will vary from day to day also. However, their variations will probably be reasonably well represented by a stationary random process. As the ratio of u and K appear in (A1.4) as part of the argument of the error and complementary error functions, it is suggested that their variations will ~~not~~ cause too big an error if neglected. This is based upon the fact that u and K tend to be positively correlated and, in any case, the error function is a relatively insensitive function of its argument. The final state T** will also be a function of time because the glacier surface temperature will be a function of time (never more than 0°C, however). The equation (A1.4) can be replaced by:

$$T(x,z,t) = \overline{erf(\eta)}\; T_{IN}(z,t) + \overline{erfc(\eta)}\; \overline{T}^{**}(z) + U\epsilon(t) \qquad (A1.5)$$

where the overbar ‾ denotes "average value over the period of record", and $U\epsilon(t)$ is an unknown stochastic process, assumed stationary, with mean value of zero and variance U^2. Fluctuations of u, K and T** from their mean values for the period of record are "absorbed" into the $U\epsilon(t)$ term. Equation (A1.5) is useful if the latter term is small. Regression analysis of "synthetic" data, where T was generated from T_{IN} according to (A1.4), supports (A1.5) as a reasonable approximation of (A1.4) with time-dependent parameters.

The purpose in discussing the present model is heuristic: the model is not realistic. In fact, Equation (A1.4) predicts stronger "domestication" than the more realistic model of Ohmura (1972) for similar values of u and K. This is because the model neglects the increase of u and K with height so that supply of sensible heat to the air at height z from overlying air is underestimated. Another unrealistic feature of (A1.4) is neglect of variations of glacier surface temperature in the downwind direction. This may be realistic for high summer conditions with initially very warm advecting air, but under cooler conditions it is possible that the glacier surface temperature can be $0^{o}C$ near the glacier edge but cooler further down wind because of the reduced supply of sensible heat flux. Once the surface temperature is below $0^{o}C$ it is free to adjust itself according to the energy balance equation and can no longer be considered as independent of the temperature of the overlying air.

iii) Prandtl Layer Model

Equation (A1.1) can be solved with more realistic assumptions concerning the height variations of u and K, e.g. a logarithmic increase of u and linear increase of K in the Prandtl layer. The equation must, however, be solved numerically, see for examples Ohmura (1972) and Müller *et al* (1973). This is done by representing the differentials as finite-differences between values of the various variables at points in a two-dimensional grid with origin at the edge of the glacier. A finite-difference approximation of the left-hand side of Equation (A1.1) may be written:

$$u\frac{\partial T}{\partial x} = \frac{u_{i+1,k}T_{i+1,k} - u_{i,k}T_{i,k}}{d} \qquad (A1.6)$$

whilst the right-hand side term may be written:

$$\frac{\partial}{\partial z}K\frac{\partial T}{\partial z} = \frac{1}{h^2}\left[K_{i,k+\frac{1}{2}}T_{i,k+1} - (K_{i,k+\frac{1}{2}} + K_{i,k-\frac{1}{2}})T_{i,k} + K_{i,k-\frac{1}{2}}T_{i,k-1}\right] \qquad (A1.7)$$

where i and k denote values on i th (horizontal) and k th (vertical) grid point and d & h are respectively the horizontal and vertical grid spacings. $K_{i,k+\frac{1}{2}}$ and $K_{i,k-\frac{1}{2}}$ denote representative K values for the air layers between grid points. It can be shown that (A1.6) and (A1.7) are represented by an equation of the form:

$$T_{i+1,k} = \alpha_{i,k}T_{i,k} + \beta_{i,k}T_{i,k+1} + \gamma_{i,k}T_{i,k-1} \qquad (A1.8)$$

where $\alpha_{i,k}$, $\beta_{i,k}$ and $\gamma_{i,k}$ are suitably defined in terms of grid point values of u and K.

Equation (A1.8) can be solved for the temperature at grid point (i+1,k) if values at grid points (i,k), (i,k+1) and (i,k-1) are already known. The solution can be carried out step by step if suitable boundary conditions are specified: the temperature of the advecting air upwind of the glacier, the surface temperature of the glacier and the temperature along the top boundary of the solution space which could be either the top of the Prandtl layer or bottom of the free atmosphere. In principle the finite-difference approach can be extended to the three dimensional and non-steady state problem and can take account of spatial variations of the glacier surface temperature and any chosen variations of u and K. If an additional equation is introduced to describe the energy balance of the glacier surface it should also be possible to compute the glacier surface temperature as part of the solution instead of prescribing it.

There are, however, some practical objections to this approach. For example, it is not possible to choose arbitrary values for the grid spacings d and h, they must be chosen according to a Stability Criterion. Ohmura (1972) chose values of 1 m for both d and h. A solution extending 1 km downwind and 40 m vertically would, in this case, require 41 000 grid points (of which 2 039 grid points would be used for the boundary conditions). The computation might have to be repeated every day for a whole summer. If time-dependence was introduced into Equation (A1.1) all grid-point values would have to be stored and the time-step for integration would also have to be chosen according to a stability criterion. It is suggested that the amount of computation involved makes the approach impractical for predictive purposes although it has very great value as a diagnostic tool. Furthermore, the specification of boundary conditions and suitable values of wind speed etc would require observational data from close (1 km scale) to the glacier which is seldom the case.

For the purposes of the present discussion, it can be pointed out that the Equation (A1.8) is linear as long the chosen formulations for u and K do not involve temperature or its spatial gradients. Successive elimination between equations for each value of i and k will ultimately lead to a linear equation relating the temperature at the (i,k) grid point to the boundary conditions. The coefficients in the linear combination will depend upon i and k with, presumably, proportionally greater weighting on the closer boundary. In particular, the precise location of the upper boundary will probably be irrelevant as long as it is chosen sufficiently high. As the solution proceeds in the downwind direction it should become decreasingly dependent upon the upwind boundary condition and increasingly dependent upon the surface boundary condition. From this point of view, the Prandtl layer and simple diffusion models are qualitatively similar.

iv) The Parametric Model

From the analyses of the previous sections, it is suggested that a plausible general model for T(x,z,t) is given by:

$$T(x,z,t) = A(x,z) + B(x,z)\, T_{IN}(z,t) + U(x,z)\, \epsilon(t) \qquad \text{(A1.9)}$$

where T(x,z,t) is the temperature at some point (x,z) on the glacier at time t, e.g. daily mean air temperature at a glacier weather station, $T_{IN}(z,t)$ is the equivalent-altitude temperature upwind of the glacier edge, A(x,z), B(x,z) and U(x,z) are parameters which must be evaluated empirically, e.g. by least-squares analysis of field data, and $\epsilon(t)$ is an unknown stochastic process with zero mean value and unit variance.

The logical connection between (A1.9) and the results of the previous sections is a weak one: the connection is based upon intuition and analogy. Accordingly, Equation (A1.9) is a hypothesis which must be tested rather than a logically deduced statement of fact whose testing is hardly necessary. The actual testing of the hypothesis is outlined in Chapters 3 to 6.

The parameter A expresses mainly the average effect of the glacier surface temperature whilst B parameterizes the average wind and turbulence conditions. Fluctuations from average conditions are then lumped into the $U\,\epsilon(t)$ term which can be regarded as an "error". The latter should be small for (A1.9) to be a useful parameterization. This is a characteristic of the parametric approach whereby certain quantities are known to be variable but are treated as if they were constant.

The main assumptions implicit in (A1.9) are:

(a) *Negligible heat storage in the air on the daily time-scale in comparison to the advective heat source and the cooling of the air by turbulent heat flux into the glacier surface.*

(b) The dynamics of the system, e.g. wind speed and exchange coefficient, as well as the glacier surface temperature play a relatively passive role in transforming the air temperature. This is equivalent to saying that time variation in T_{IN} is the main source of variation in T at some fixed location.

Finally, a methodological problem should be mentioned. The hypothesis, i.e. Equation (A1.9), might be tested and not found to be falsified. It might then be accepted as "true" and might be "explained" in terms of the arguments advanced in the present Appendix, i.e. in terms of factors such as advection and turbulence. There may, however, be other logically possible mechanisms whose parameterization would also lead to a hypothesis in the form of Equation (A1.9) so that the "explanation" would not be a unique one. This is an aspect of the "process-response" problem but is probably not a serious problem from the practical point of view.

APPENDIX 2

TWO-WAY ANALYSIS OF VARIANCE OF MONTHLY MEAN ERRORS FOR 1972 TEMPERATURES

The monthly mean temperatures at 10 Axel Heiberg stations were computed from the corresponding monthly means $\overline{T}_M$ using the various Class Models. This gives the estimate $\overline{T}_p$ for each month and station. The corresponding observed temperatures are $\overline{T}$, and the Monthly Mean Error $\overline{E}$ is given by $(\overline{T}_p - \overline{T})$. The values of $\overline{E}$ for each month and station are as follows:

Station	May	June	July	Aug	Mean	Standard Dvn
Fjord	-0.3	0.0	-1.1	-0.7	-0.5	0.5
Base Camp	0.3	0.4	0.1	-0.3	0.1	0.3
Valley	1.2	0.3	-0.7	-0.1	0.2	0.8
Baby	0.2	-0.1	0.3	0.8	0.3	0.4
Gordon's	[illegible]	-0.7	0.5	-0.4	-0.7	1.0
Ermine	-2.7	-1.2	0.2	-0.3	-1.0	1.3
Outwash	3.2	0.4	-1.0	-0.3	0.6	1.8
Lower Ice	1.3	0.9	0.1	-0.4	0.5	0.8
Anniversary	1.2	0.3	0.2	-0.3	0.4	0.6
Moraine*	(0.2)	(-0.6)	(-1.3)	-		
Mean	0.3	0.0	-0.2	-0.2	0.0	1.0
Standard Dvn	1.8	0.6	0.6	0.4		

* Moraine data not used in the computation because of missing August data.

The means and standard deviations of the rows may be different from each other on account of "station effect," and the means and standard deviations of the columns may be different on account of "time effect". The purpose of the analysis is to see if these differences are statistically significant. The method follows Kreyszig (1970, p.277). It is assumed that the 36 values in the table correspond to 36 random variables E_{st} which are independent and normal and have the same (unknown) variance although the means may be different. The hypothesis to be tested is that all the mean values are in fact the same, i.e. that the population means are all $0^{o}C$.

Source of Variations	DOF	SOS	Mean Square
Between Rows (Station Effect)	8	10.60	1.33
Between Columns (Time Effect)	3	1.53	0.51
Deviations	24	32.87	0.95
Total	35	35.00	

DOF = Degrees of Freedom   SOS = Sum of Squares

The ratio of mean square "station effect" to mean square deviation is v_1 which in this case has the value 1.40. The ratio of mean square "time effect" to mean square deviation is v_2 which in this case has the value 0.54.

The 5 % significance level is chosen so that $F(c_1)$ has the value 0.95 for (8,24) degrees of freedom at $c_1 = 3.12$. Now v_1 is less than c_1 so there is no significant (5 % level) difference between stations. $F(c_2)$ has the value 0.95 for (3,24) degrees of freedom at $c_2 = 8.64$. Now v_2 is less than c_2 so that there is no significant (5 % level) difference between months. In view of these two findings the hypothesis that the mean values are the same cannot be rejected and the 36 values in the table are best regarded as arising randomly from a single population. However, the most questionable assumption is probably that the variances are the same.

If it were asserted, the previous conclusion notwithstanding, that the mean error E_{st} for the s th station and t th month could be described by a linear model of the form

$$E_{st} = \alpha_s + \beta_t + e_{st}$$

the variance of the error e_{st} would be 0.82^2. This would correspond to an error of about 67 % of the variance of E_{st}.

Despite the statistical findings, the suspicion remains that the model performance is relatively poor for May. This need not be surprising as the data used for computing the various model parameters are heavily biased towards high-summer situations. For May the computed mean temperature $\overline{T}_p$ is, on average, 1.7°C too high for the group of four stations represented by Valley, Outwash, Lower Ice and Anniversary whilst it is 2.0 to 2.7°C too low for

Gordon's and Ermine. At the same time computed temperatures are quite accurate for Fjord, Base Camp, Moraine and Baby. This pattern may well be fortuitous or due to spatial autocorrelation, but it could be speculated that it reflects a real pattern of enhanced cooling and heating effect characteristic of the pre-melt season. According to this explanation the first four stations might be under the influence of a cold "pool" extending down-glacier from Moraine into the Expedition River valley, possibly constrained by topography, whilst Gordon's and Ermine stations lie within a "heat island" where the underlying tundra is already heating the atmosphere although still covered with snow. It should be pointed out that this particular scenario could not be described by the model assumed for the two-way analysis of variance computation and would not be detected by it.

APPENDIX 3

STATISTICAL PROPERTIES OF ENERGY/MASS BALANCE EQUATIONS

In principle energy or mass balance equations can be expressed in the form:

$$Y(t) = \sum_{i=1}^{i=N} X_i(t) \qquad \text{(A3.1)}$$

where $Y(t)$ is the value of the balance at time t and $X_1(t)$ to $X_N(t)$ are values of the N sources and sinks at time t. There is a convention that sources are +ive and sinks are -ive. If (A3.1) is regarded as representing the energy balance equation then $Y(t)$ would be the energy used to melt ice whilst $X_1(t)$ to $X_N(t)$ would represent net radiation, sensible heat flux, latent heat flux, heat conduction into ice and heat from precipitation etc ($N = 5$). If (A3.1) is regarded as representing the mass balance equation then $Y(t)$ would be the annual net balance and $X_1(t)$ to $X_N(t)$ would represent accumulation and ablation, or winter and summer balances etc ($N = 2$).

The mean values over a period T ($t = 1$ to $t = T$), denoted by the overbar, would be:

$$\overline{Y(t)} = \sum_{i=1}^{i=N} \overline{X_i(t)} \qquad \text{(A3.2)}$$

Deviations from the mean, denoted by the prime, would be:

$$Y(t)' = \sum_{i=1}^{i=N} X_i(t)' \qquad \text{(A3.3)}$$

For the assumption of stationarity the mean values will be independent of time t whilst for non-stationary processes the mean values would have to be specified as a function of time t.

Multiplication of (A3.3) by the deviation of the j th sink/source followed by averaging over the period of length T gives an expression involving covariances:

$$COV(Y,X_j) = \sum_{i=1}^{i=N} COV(X_i X_j) \qquad \text{(A3.4)}$$

where

$$COV(Y,X_j) = \sum_{t=1}^{t=T} \frac{(Y(t)' . X_j(t)')}{(T-1)} \qquad \text{(A3.5)}$$

Recall that the product moment correlation coefficient between Y and X_j, denoted by $R(Y,X_j)$ is simply the covariance $COV(Y,X_j)$ divided by the standard deviations of Y and X_j for the period of length T. It can be shown that:

$$R(Y,X_j) = \frac{1}{S} \sum_{i=1}^{N} S_i R(X_i,X_j) \qquad \text{(A3.6)}$$

where S is the standard deviation of Y and S_j is the standard deviation of X_j. The index j can take values from 1 to N so that (A3.6) represents the j th equation in a system of N equations. The correlation coefficient $R(X_i,X_j)$ represents the interaction or coupling between i th and j th sources/sinks. In the case of mutually independent sources/sinks (orthogonality assumption) the correlation coefficients take the values:

$$\begin{aligned} R(X_i,X_j) &= 1.0 \quad \text{for } i = j \\ &= 0.0 \quad \text{for } i \neq j \end{aligned} \qquad \text{(A3.7)}$$

In this particularly simple case (A3.6) gives:

$$R(Y,X_j) = \frac{S_j}{S} \qquad \text{(A3.8)}$$

Equations (A3.6) or (A3.8) illustrate the fact that the correlation between Y and any particular source/sink X_j does not depend upon the mean value $\overline{X}_j$ but depends upon the standard deviations of the various terms and the interactions between the terms. In general, if S_j is small compared to S then X_j will correlate poorly with Y. (A3.8) expresses the common-sense notion that, if X_j is only slightly variable during the period T, it cannot cause relatively large variations in Y and that these variations must be caused by some other more variable factor in the energy/mass balance equation.

APPENDIX 4

DETAILS OF COMPUTATION SCHEME FOR SUMMER ABLATION ON WHITE GLACIER

i) Symbols for Variables

$T_{IN}(z,t)$ = Interpolated monthly mean temperature for month t and altitude z

$S_{IN}(z,t)$ = Interpolated standard deviation associated with above

$T_p(z,t)$ = Monthly mean local temperature computed from $T_{IN}(z,t)$

$S(z,t)$ = Standard deviation associated with above computed from $S_{IN}(z,t)$

N = Number of days in t th month

$\overline{T}_{IN}(z)$ = Mean of $T_{IN}(z,t)$ for June-August

$\overline{T}_p(z)$ = Mean of $T_p(z,t)$ for June-August

MDT(z,z) = Computed positive degree-day total for month t

f(x) = Probability density function for daily mean temperature in month t

DDT(z) = Computed positive degree-day total for June-August

MAB(Z,t) = Computed ablation total for month t

$a_c(z)$ = Computed ablation total for June-August

t = Month (June=6, July=7, August=8)

x = Dummy variable

ii) The Equations

$$T(z,t) = \alpha_1 + \beta_1 T_{IN}(z,t) \quad (A4.1)$$

$$S(z,t) = \sqrt{\beta_1^2 S_{IN}(z,t)^2 + U_1^2} \quad (A4.2)$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-(x-T_p)^2/2S^2\right] \quad (A4.3)$$

$$MDT(z,t) = N\int_0^{2S} f(x)\,x\,dx \quad (A4.4)$$

$$MAB(z,t) = \alpha_2 + \beta_2 MDT(z,t) \quad (A4.5)$$

$$DDT(z) = MDT(z,6) + MDT(z,7) + MDT(z,8) \quad (A4.6)$$

$$a_c(z) = MAB(z,6) + MAB(z,7) + MAB(z,8) \quad (A4.7)$$

iii) The Parameters

For White Glacier the following values were adopted:

From Table 5.5: α_1 = -0.7 °C, β_1 = 0.834 and U_1 = 1.8 °C.

From Table 7.5: α_2 = 0.0 cm H_2O dy^{-1} and β_2 = 0.63 cm H_2O dy^{-1} $°C^{-1}$.

iv) Comments

The parameters α_1, β_1, α_2 and β_2 are assumed constant with respect to time and location. Equation (A4.3) is based upon the assumption that daily mean temperatures within a month are normally distributed about the monthly mean. Equation (A4.4) describes the computation of positive degree-day total for the month in terms of integration over the ensemble of probable daily mean temperatures with lower limit of integration set at 0°C. The upper limit of integration was actually taken as two standard deviations above the mean and the integral was computed as a sum using steps of 0.2°C. Positive degree-day totals are normally understood as time-summations, and replacement of a time-summation by an ensemble-summation is only valid for a stationary process.

$T_{IN}(z,t)$ and $S_{IN}(z,t)$ were actually calculated from 1000 mb and 850 mb data at Eureka and Isachsen using simple distance weighting with weights 0.71 (Eureka) and 0.29 (Isachsen). Monthly ablation values for May were actually computed but were found to be very small, and ablation for May is excluded. The calculation was made for every summer in the period 1960-72 so that 39 $a_c(z)$ values were obtained: values for 13 years at z = 210 m a.s.l., z = 370 m a.s.l. and z = 870 m a.s.l..

None of the various assumptions will be exactly fulfilled. In particular, the parameters α_2 and β_2 describe <u>ice melt</u> rather than <u>snow melt</u>. It can be expected that $a_c(z)$ will involve error. The testing of the hypothesis will concentrate on the question as to whether these errors are, in fact, large or not.

APPENDIX 5

EFFECT OF ACCUMULATION ON THE COMPARISON OF a_c AND a_n

The computed ablation for the t th summer is $a_c(t)$. It is computed from the formula:

$$a_c(t) = \beta_2 DDT(t) \quad (A5.1)$$

where DDT(t) is the computed degree-day total for the summer and β_2 is a parameter valid for melting ice. If $a_c(t)$ is compared to the observed net ablation $a_n(t)$ there is a discrepancy between them given by E(t):

$$E(t) = a_n(t) - a_c(t) \quad (A5.2)$$

It will be recalled that:

$$a_n(t) = a_s(t) - c_w(t) - c_s(t) \quad (A5.3)$$

where $a_s(t)$ is the true summer ablation and $c_w(t)$ and $c_s(t)$ are the winter and summer accumulation respectively, assumed to be in the form of snow.

The ablation period is assumed to be divided into two sub-periods: a period of snow melt with degree-day total $DDT_1(t)$ and a period of ice melt with degree-day total $DDT_2(t)$ where:

$$DDT(t) = DDT_1(t) + DDT_2(t) \quad (A5.4)$$

It is assumed that the parameter valid for melting snow is β_1 so that:

$$a_s(t) = \beta_1 DDT_1(t) + \beta_2 DDT_2(t) \quad (A5.5)$$

In the ablation area where all snow accumulation (c_w and c_s) is ablated during the summer, i.e. $DDT_1(t) \leqslant DDT(t)$, the following will hold:

$$c_w(t) + c_s(t) = \beta_1 DDT_1(t) \quad (A5.6)$$

From (A5.1 to A5.5) the following expression for E(t) can be deduced:

$$E(t) = \beta_1 DDT_1(t) - \beta_2 DDT_1(t) - c_w(t) - c_s(t) \quad (A5.7)$$

Elimination of $DDT_1(t)$ between (A5.6) and (A5.7) gives:

$$E(t) = -(\beta_2/\beta_1)(c_w(t) + c_s(t)) \quad \text{(A5.8)}$$

If β_1 and β_2 are identical then Equation (A5.8) expresses the triviality that the discrepancy between a_n and a_c is simply numerically equal to the accumulation. However, if $\beta_1 < \beta_2$, the discrepancy will be numerically larger than the accumulation.

From Equations (A5.4 to A5.6) it can be shown that:

$$a_s(t) = \frac{\beta_1 - \beta_2}{\beta_1}(c_w(t) + c_s(t)) + \beta_2 DDT(t) \quad \text{(A5.9)}$$

If β_1 and β_2 are identical then the summer ablation $a_s(t)$ will depend only upon the summer meteorological conditions, represented here by DDT(t). If $\beta_1 < \beta_2$ then the summer ablation will also be dependent upon the accumulation, i.e. "ablation is not independent of accumulation".

It can be simply shown that the net ablation a_n will be given by:

$$a_n(t) = \beta_2(DDT(t) - DDT_1(t)) = \beta_2 DDT_2(t) \quad \text{(A5.10)}$$

Because $DDT_1(t) \leqslant DDT(t)$ in the ablation area, $a_n(t)$ will be positive.

In the accumulation area where all snow accumulation (c_w and c_s) is not ablated, i.e. $DDT_1(t) = DDT(t)$ and $DDT_2(t) = 0$, the Equation (A5.6) must be replaced by the inequality:

$$c_w(t) + c_s(t) > \beta_1 DDT_1(t) \quad \text{(A5.11)}$$

The net ablation $a_n(t)$ will then be given by:

$$a_n(t) = \beta_1 DDT_1(t) - (c_w(t) + c_s(t)) \quad \text{(A5.12)}$$

Because of the condition expressed by (A5.11) $a_n(t)$ will be negative.

APPENDIX 6

COMPUTATION SCHEME FOR TOTAL ABLATION ON STORGLACIAEREN

i) Symbols for Variables

$\overline{T}_{Tarf}$ = Summer mean (June-August) temperature at Tarfala station at 1130 m a.s.l. near snout of Storglaciären

$\overline{T}_p(z_i)$ = Computed summer mean (June-August) temperature at altitude z_i on Storglaciären

$a_c(z_i)$ = Computed summer ablation at altitude z_i on Storglaciären in cm H_2O

$S(z)$ = Area in km^2 of the 50 m altitude band centred on altitude z_i

A_{cal} = Computed total ablation in 10^6 m^3 H_2O

ii) The Equations

$$\overline{T}_p(1130) = \alpha_1 + \beta_1 \overline{T}_{Tarf} \qquad (A6.1)$$

$$\overline{T}_p(z_i) = \overline{T}_p(1130) + G.(z_i - 1130) \qquad (A6.2)$$

$$a_c(z_i) = 72.4 + 40.5.\overline{T}_p(z_i) \qquad \overline{T}_p(z_i) \geq 0^oC \qquad (A6.3a)$$

$$a_c(z_i) = 75.4 + 20.9.\overline{T}_p(z_i) \qquad \overline{T}_p(z_i) < 0^oC \qquad (A6.3b)$$

$$z_i = 1125 + 50i \qquad (A6.4)$$

$$A_{cal} = \sum_{i=1}^{i=u} \frac{a_c(z_i)}{100} . S(z_i) \qquad (A6.5)$$

iii) The Parameters

The computation was made for several choices of the various parameters. For example, the α_1 and β_1 parameters in the cooling effect equation (A6.1) were chosen as follows:

Run Number	Class	α_1	β_1
1	Class 1a	1.2	0.933
2	Class 1	0.1	0.999
3	Class 2	-0.7	0.834
4	Class 3	-2.5	0.890

These are actually the parameters for the different Axel Heiberg classes.

The computation was made for several choices of vertical temperature gradient G = -0.003, -0.005 and -0.007 $^{o}C\ m^{-1}$.

iv) Comments

The purpose of the present exercise is to compute total ablation on Storglaciären under the assumption that it acts like an Axel Heiberg situation. For example, Equations (A6.3a) and (A6.3b) are based upon the Axel Heiberg/Devon Island ablation model, see Chapter 10 and Appendix 4 for details. *A priori* the choice of parameters for Run Number 3 should be the most likely, i.e. Storglaciären should behave like Class 2 (Lower Ice etc) rather than Class 1a (Ermine station), Class 1 (Base Camp etc) or Class 3 (Ice Cap situations).

The vertical temperature gradient of -0.005 $^{o}C\ m^{-1}$ is most likely and the other values were only used for completeness.

Equations (A6.3a) and (A6.3b would lead one to expect an average ablation gradient of 10-20cm H_2O/100m as opposed to the value of 55cm H_2O/100m quoted by Schytt (1967). This seems to indicate a serious discrepancy, but the model does perform reasonably well, see Fig. 11.6 and page 96.

APPENDIX 7

TOTAL ABLATION OF A GLACIER IN RELATION TO TEMPERATURE AT A POINT

Suppose that:

$$a(z,t) = \beta \, DDT(z,t) \qquad (A7.1)$$

where $a(z,t)$ and $DDT(z,t)$ are the summer specific ablation and degree-day total respectively for the t th balance year, and altitude z and β is a parameter.

The glacier is divided into N altitude bands of thickness Δz.

The total area of the glacier is S and the area of the i th altitude band, centred on altitude z_i, is S_i where i takes values from 1 to N. Clearly:

$$S = \sum_{i=1}^{i=N} S_i \qquad (A7.2)$$

The total ablation A(t) for the t th summer is the sum of contribution from the N altitude bands:

$$A(t) = \sum_{i=1}^{i=N} a(z_i,t) \; S_i \qquad (A7.3)$$

The mean specific ablation a(t) is given by A(t)/S (NB the overbar ‾ usually used to denote mean specific quantities is reserved in the present study for time-averages.) It can be shown that:

$$a(t) = \frac{\beta}{S} \sum_{i=1}^{i=N} DDT(z_i T) \; S_i \qquad (A7.4)$$

In general the relationship between the summer degree-day total $DDT(z_i,t)$ and the corresponding summer mean temperature $T(z_i,t)$ is non-linear. However, if summer temperatures at z_i are not too variable from summer to summer the relation can be linearly approximated:

$$DDT(z_i,t) = d_i + f_i \; T(z_i,t) \qquad (A7.8)$$

where d_i and f_i are constants for the altitude z_i and are dependent upon average temperature conditions at that altitude.

If (A7.8) is substituted into (A7,4) then:

$$a(t) = \frac{\beta}{S} \sum_{i=1}^{i=N} (d_i + f_i\, T(z_i,t))\, S_i \qquad (A7.9)$$

The temperature at a weather station at altitude z_o is $T(z_o,t)$. If temperature is a linear function of altitude with vertical gradient g:

$$T(z_i,t) = T(z_o,t) + ig\Delta z \qquad (A7.10)$$

Substitution of (A7.10) into (A7.9) gives:

$$a(t) = \frac{\beta}{S} \sum_{i=1}^{i=N} \left[d_i + f_i\, ig\Delta z + f_i\, T(z_o,t) \right] S_i \qquad (A7.11)$$

Equation (A7.11) is clearly linear with respect to $T(z_o,t)$. This result depends upon the validity of (A7.8) which rests in turn upon the assumption that variations of $T(z_i,t)$ from year to ~~year are~~ quite small, e.g. of the order of 1 to 2 °C at most.

Equation (A7.11) may be expressed as follows:

$$a(t) = A_o + B\ T(z_o,t) \qquad (A7.12)$$

where

$$A_o = \frac{\beta}{S} \sum_{i=1}^{i=N} (d_i + f_i\, ig\Delta z)\, S_i$$

and

$$B = \frac{\beta}{S} \sum_{i=1}^{i=N} f_i S_i$$

Several authors have regressed mean specific net balance or summer balance on summer mean temperature at some station near the glacier (10^1 km scale). Equation (A7.12) may be considered as constituting an "explanation" to such regression models. The intercept A_o contains information about the altitude differences between the weather station and various parts of the glacier, lapse rate, mass distribution of the glacier and relationship between temperature and degree-day total whilst the slope B contains information about the mass distribution of the glacier, the relationship between temperature and degree-day total and the relationship between degree-day total and specific ablation. The terms A_o and B should show secular variations because of the changes in areas S and S_i as the glacier advances or retreats. From this point of view the system described in Equation A7.12 is non-stationary.

APPENDIX 8

ABLATION, NET BALANCE AND TEMPERATURE

The specific net balance $b_n(t)$ for the t th balance year is given by:

$$b_n(t) = c(t) - a(t) \qquad (A8.1)$$

where c(t) and a(t) are the accumulation and ablation for the t th balance year. Deviations from the N-year average which is denoted by the overbar ¯ are denoted by the prime ', thus:

$$b_n(t)' = c(t)' - a(t)' \qquad (A8.2)$$

The deviation of summer mean temperature T(t) from its N-year average is denoted by T(t)'. Multiplication throughout (A8.2) by T(t)' followed by averaging over the N years gives:

$$\overline{b_n(t)'T(t)'} = \overline{c(t)'T(t)'} - \overline{a(t)'T(t)'} \qquad (A8.3)$$

The N-year standard deviations of $b_n(t)$, c(t), a(t) and T(t) are denoted by S_b, S_c, S_a and S_T. Division of (A8.3) by $S_b.S_T$ gives:

$$R(b,T) = \frac{S_c}{S_b} R(c,T) - \frac{S_a}{S_b} R(a,T) \qquad (A8.4)$$

where R(b,T), R(c,T) and R(a,T) are coefficients of linear correlation of temperature with net balance, accumulation and ablation respectively.

It is quite likely that c(t) and T(t) will be moderately negatively correlated because "wet" summers are often "cool" summers and because lower than average temperatures will favour snowfall rather than rainfall. However, the influence that this will have in increasing R(b,T) will depend upon the relative magnitudes of S_c, S_a and S_b.

Two interesting cases can be considered:

i) The accumulation is nearly constant, i.e. $S_c \ll S_b$, so that:

$$R(b,T) \approx R(a,T) \qquad (A8.5)$$

ii) a(t) and c(t) are orthogonal, R(c,T) is zero and $S_c = S_a$ so that $S_b = \sqrt{2}\, S_a$ then:

$$R(b,T) = -0.71\ R(a,T) \qquad (A8.6)$$

In the second case (ii) T(t) will "explain" at most 50 % of the variance of b(t), and a greater percentage of explanation would only be possible with a negative non-zero value of $R(c,\mathring{T})$.

Division of (A8.3) by $S_T^{\ 2}$ gives:

$$\frac{\overline{b_n(t)'T(t)'}}{S_T^{\ 2}} = \frac{\overline{c(t)'T(t)'}}{S_T^{\ 2}} - \frac{\overline{a(t)'T(t)'}}{S_T^{\ 2}} \qquad \text{(A8.7)}$$

where the left-hand term represents the rate of increase of net balance with respect to temperature whilst the right-hand terms represent the rates of increase of accumulation and ablation respectively with respect to temperature.

APPENDIX 9

THE NET BALANCE OF HINTEREISFERNER IN RELATION TO METEOROLOGICAL ELEMENTS

The late Professor Hoinkes was probably one of the leading exponents of the notion that glacier variations are controlled by variations in the supply of energy by radiation. He discussed results of short-period energy balance measurements made in the early 1950s on glaciers in the eastern Alps (Hoinkes,1955) and showed that radiation was the major heat source. This is in agreement with results from other areas. From this he concluded (Hoinkes, 1955, p.500) that: "The glaciers in the Alps will, therefore, react more strongly to oscillations in the duration and intensity of solar radiation or albedo than to variation of air temperature or to precipitation."

Further, Hoinkes (1955, p.500) said: "In recent years many authors, on the basis of careful studies, have come to the conclusion that the summer temperature is to be regarded as the most important factor influencing the behaviour of glaciers. This result is not in contradiction to the results of the measurements which are given here (according to which radiation is the main source of energy for the ablation of the Alpine glaciers) so long as it is not combined with the idea that the greater heat exchange from air to ice during a hot summer is sufficient to account for the greater ablation. In an Alpine climate in most cases a high summer temperature means weather with much radiation and with infrequent incursions of cold air and snowfalls on the glaciers. Higher air temperature naturally contributes to some extent to the greater ablation, but it appears that it is to be regarded mainly as an index of higher radiation and to the less power of reflection of the glacier surface."

Hoinkes (1955, p.500) discussed the relation between glacier/advance and retreat in Switzerland with summer (June-September) sunshine duration and days with snowfall at several high-altitude climatological stations for the period 1890 to 1954 and said: "The correlation of the curve, indicating the number of advancing glaciers (under observation) as a percent of the total for Switzerland, and the curves for sunshine duration and, in particular, for new snowfalls is definite and quantitatively adequate." He did not quote a correlat-

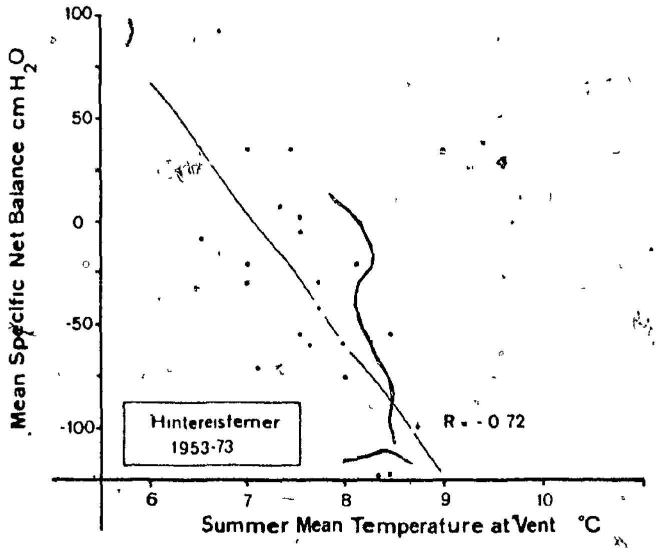


Fig A9.1: Net Balance of Hintereisferner, Austrian Alps, Versus Summer (May-September) Mean Temperature at Vent for Period 1952/53 to 1972/73

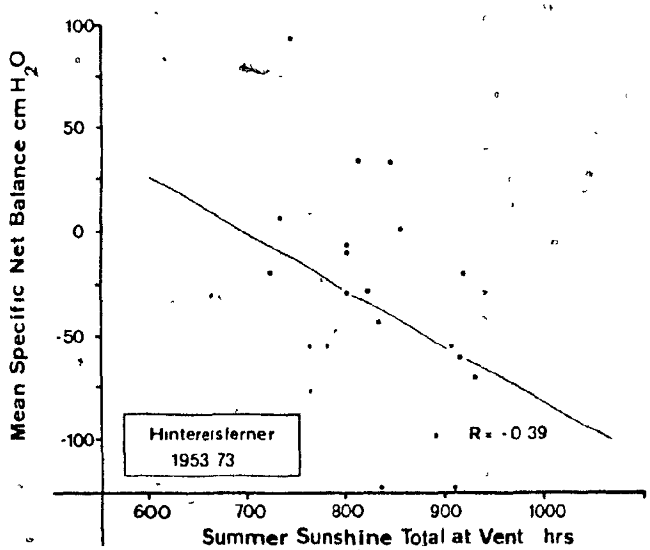


Fig A9.2: Net Balance of Hintereisferner, Austrian Alps, Versus Summer (May-September) Sunshine Total at Vent for Period 1952/53 to 1972/73 ion coefficient, however.

This emphasis on radiation may seem in contradiction to the success with which Hoinkes and Steinacker (1975) relate the 17-year Hintereisferner mass balance series to a quantity combining temperature and precipitation at Vent weather station. However, Hoinkes *et al* (1968, discussion) state: "At least in summer, temperature shows a positive correlation with radiation, therefore temperature here is simply a substitute for radiation, and summer snow a substitute for albedo."

A partial assessment of the validity of this point can be made by analysing data published by Hoinkes at various times. There are actually no long series of net radiation measurements from Hintereisferner although there are sunshine data from Vent weather station (Hoinkes, 1970, p.81). According to Hoinkes and Rudolph (1962, p.22-23) the short-wave radiation from sun and sky (global radiation) at the tongue of Hintereisferner can be accurately computed from sunshine duration at Vent. Furthermore, they assert that such computed global radiation is, under certain conditions, "approximately proportional to the radiation balance" (1962, p.23). Accordingly, one might reasonably expect a strong correlation between sunshine duration at Vent and net balance of Hintereisferner or Kesselwandferner. As a test of this, correlation coefficients were computed between the various series: b_N the mean specific net balance, ST the summer (May-Sept) mean temperature, WP the winter (October-April) precipitation total, SP the summer (May-Sept) precipitation total and SS the summer (May-Sept) sunshine duration. All the climatological observations relate to Vent. The 21-year (1952/53 to 1972/73) Hintereisferner and 16-year (1957/58 to 1972/73) Kesselwandferner mass balance series were taken from Kasser (1967 and 1973) and from Kuhn (1976) whilst the Vent climatological data up to 1968 are reported by Hoinkes (1970, p.81) with more recent data kindly made available by Dr M. Kuhn of Innsbruck. The correlation coefficients were as follows:

Hintereisferner 21 years (1952/53 to 1972/73)

	b_N	ST	WP	SP	SS
b_N	1.00	-0.71	(0.23)	0.61	-0.39
ST		1.00	(-0.01)	-0.50	0.37
WP			1.00	(-0.23)	(-0.34)
SP				1.00	-0.52
SS					1.00

Kesselwandferner 16 years (1957/58 to 1972/73)

	b_N	ST	WP	SP	SS
b_N	1.00	-0.69	(0.16)	0.74	(-0.32)
ST		1.00	(-0.05)	-0.49	(0.29)
WP			1.00	(-0.13)	(-0.32)
SP				1.00	-0.47
SS					1.00

() = Not significant at 5 % level.

The patterns of correlation are very similar for Hintereisferner and Kesselwandferner. Particularly noteworthy is the relatively weak correlation between mass balance and sunshine duration compared to much stronger correlations between mass balance and summer temperature and precipitation. The sunshine duration is also only weakly correlated with temperature.

Another interesting feature is the weak correlation between mass balance and winter precipitation compared to a stronger correlation with summer precipitation. This is consistent with the analysis of a 21-year accumulation series from Kesselwandferner reported by Ambach and Eisner (1967, p.27). The low correlation with winter precipitation may be partly due to the notorious difficulty of accurate snow gauging as well as depending upon topographic factors.

The fact that the mass balance of Hintereisferner and Kesselwandferner are more strongly controlled by temperature than by sunshine duration can be also demonstrated by partitioning the series into separate samples for "warm" and "cool" summers and for "sunny" and "dull" summers. In the former case differences between means of mass balance for the two sub-samples are significant (at 5 % level), and in the latter case they are not.

The coefficient of variation of the 21-year sunshine duration series is only 9.3 %. This suggests that variations of global radiation from year to year are small. The standard deviation of the summer mean temperature series is only $\mp$ 0.6 $^{\circ}$C, but from Equations (11.15) and (11.16) this is equivalent to variations of $\mp$ 37 and $\mp$ 29 cm H_2O in the mass balance of Hintereisferner and Kesselwandferner.

According to these results it is suggested that variations of mass balance of Hintereisferner and Kesselwandferner are probably controlled by variations of temperature rather than by variations of radiation. This is a reasonable inference based upon the available evidence. However, it should be repeated that there is no series of net radiation data so that no 100 % reliable conclusion can be drawn.

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Abbreviations:

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NVE = Norges Vassdrags- og Elektrisitetsvesen

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