TEMPERATURE - ENTROPY DIAGRAM AND ITS APPLICATION TO GAS - ENGINE CYCLES

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AND

ITS APPLICATION TO GAS-ENGINE CYCLES

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by

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AUTHORITIES

Clerk - The Gas, Petrol & Oil Engine

Wimperis - The Internal Combustion Engine

Boulvin - The Entropy Diagram (trans. Donkin)

Berry - The Temperature - Entropy Diagram

Golding - The Theta-Phi Diagram

Burstall - Reports to Gas-Engine Research Committee of the Institution of Mechanical Engineers.

Sankey - Appendix to Report of the Committee on the Efficiency of Internal Combustion Engines of the Institution of Civil Engineers.

Goudie - A Direct Graphical Method of Deriving the Temperature-Entropy Diagram of the Gas-Engine from the Indicator Diagram. (Proc. I. Mech. E., Feb. 1908).

SYMBOLS USED

a - constant

b - "

c - specific heat

c_p - " " at constant pressure

c_v - " " " " volume

γ - $\frac{c_p}{c_v}$

E - internal energy

η - efficiency

J - Joule's equivalent

p - pressure absolute

Q - quantity of heat

r - ratio of compression

R - constant

s - "

t - temperature on thermometer

T - " absolute

v - volume

φ - entropy

All logarithms are hyperbolic

ENTROPY

A substance having five characteristic properties, viz. specific pressure, specific volume, intrinsic energy, temperature, and entropy (p, v, E, T, ϕ), it is, in general, completely defined if any two of these are known, for the relations between any three of these properties may be expressed by the equation $x = f(y,z)$. In graphical solutions of problems any pair of properties may be used as co-ordinates; if the relation between one pair of properties is given by a curve to one pair of co-ordinates, it may, by means of the fundamental equation, be transferred to any other. The pv diagram is the one most commonly used, since it is the one most easily obtained, but for many investigations great advantages are offered by the temperature-entropy diagram.

Entropy may be defined as $\int \frac{dQ}{T}$ between any two points, taken along any reversible path between those points. It depends therefore on the two states only and not on the way of changing from one to the other. Entropy may be measured from any arbitrary zero. If, in Fig. 1, OT and Oϕ represent the axes of temperature and entropy respectively, we can at once see that any isothermal change will be represented by a line parallel to Oϕ, and any isentropic change by a line parallel to OT. Changes taking place under conditions of constant pressure, constant volume, or constant intrinsic energy will be represented by curves, varying with the conditions of the substance and of the change. An adiabatic change, if reversible, is also isentropic, but an adiabatic change is not necessarily isentropic: for example, if a gas expand through a non-conducting porous plug, no heat is added, but the entropy increases. In all ordinary cases of expansion, however, an adiabatic change may be assumed to be isentropic.

The Carnot cycle on the $T\phi$ diagram can be easily shown to be a rectangle, and to be a cycle of maximum theoretical efficiency between its temperature limits; but it can also be shown that there is an unlimited number of cycles between the same temperature limits of the same maximum efficiency, composed not of two isothermals and two adiabatics, but of two isothermals and two isodiabatics, along which the interchanges of heat are balanced.

Let AB in Fig. 1 represent any reversible process and c the specific heat during this change. Then to raise the temperature of unit weight by amount dt, heat $dQ = c\,dt$ must be expended. Also by definition $d\phi = \frac{dQ}{T}$ or $dQ = T\,d\phi$. Therefore $c\,dt = T\,d\phi = dQ$, whence

$$Q = \int_{t_1}^{t_2} c\,dt = \int_{\phi_1}^{\phi_2} T\,d\phi \quad ------- \quad (1)$$

This last integral is the area under the curve; hence the heat necessary to produce any change is represented by the area under the curve in the $T\phi$ diagram. This is the first important deduction from our definition of entropy. From these relations we can also get $c = \frac{T\,d\phi}{dT}$, or the subtangent at any point on the curve representing a reversible change represents the specific heat at that point. If the curve be a constant volume curve, the subtangent represents the specific heat at constant volume; if a constant pressure curve, the specific heat at constant pressure. This, however, unfortunately does not assist us in finding the specific heat of a substance, since we have no method of drawing $T\phi$ curves except through the knowledge of, or reliance on, predetermined specific heat.

Perfect Gases

A perfect gas may be defined as one whose properties follow the equation $pv = RT$, where R is a constant, and whose specific heat at constant pressure and at constant volume are constants. The law of the mechanical equivalent of heat applied to such a body gives the equation $dQ = dE + \frac{1}{J}\, pdv$ (2) where Q is the heat supplied and E the change in internal energy. If a change takes place at constant volume, the heat supplied, dQ, must be equal to $c_v\, dT$; hence since dv is zero, $dE = c_v dT$; in other words the internal energy depends solely on the temperature and an isothermal change is also isodynamic. If a change takes place in unit mass of gas from temperature T_1 to temperature T_2 at constant pressure, the heat supplied is $c_p(T_2 - T_1)$; if at constant volume, $c_v(T_2 - T_1)$. The change in internal energy is the same in the two cases, since the two end temperatures are the same; therefore the difference in the heat supplied is equivalent to the external work done. Hence

$c_p(T_2 - T_1) - c_v(T_2 - T_1) = \frac{1}{J} p(v_2 - v_1) = \frac{1}{J} R(T_2 - T_1)$, since $pv = RT$. Therefore $c_p - c_v = \frac{R}{J}$, or since $\frac{c_p}{c_v} = \gamma$, $c_v(\gamma - 1) = \frac{R}{J}$. Equation (2) may also be written $dQ = c_v dT + \frac{1}{J} p\, dv$. Therefore $d\phi$, which is $\frac{dQ}{T}$, is equal to $c_v \frac{dT}{T} + \frac{1}{J} p \frac{dv}{T} = c_v \frac{dT}{T} + \frac{R}{J} \frac{dv}{v}$; hence on integrating, $\phi = c_v \log T + \frac{R}{J} \log v + \text{const.} = c_v \log \frac{pv}{R} + (c_p - c_v) \log v + \text{const.} = c_v \log p + c_v \log v - c_v \log R + c_p \log v - c_v \log v + \text{const.} = c_v(\log p + \gamma \log v) + \text{const.} = c_v \log (pv^{\gamma}) + \text{const.}$ By means of the fundamental equation $pv = RT$, we can eliminate either p or v from this last expression for ϕ, and obtain the value of the entropy in terms of temperature and volume or in terms of temperature and pressure, giving $\phi = c_v \log (Tv^{\gamma-1}) + \text{const.}$ or $\phi = c_p \log (Tp^{\frac{1-\gamma}{\gamma}}) + \text{const.}$ These results may also be obtained more directly from equation (2) and the fundamental equation $pv = RT$. By combining these, equation (2) may be written $dQ = c_v dT + (c_p - c_v) T \frac{dv}{v}$. By substituting in this from the differentials of the logarithms of the fundamental gas equation, equation (2) may also be written in the forms $dQ = c_p dT - (c_p - c_v) T \frac{dp}{p}$ or $dQ = c_v T \frac{dp}{p} + c_p T \frac{dv}{v}$. These three forms on dividing through by T and integrating give three different expressions for the entropy in terms of any two of the three variables p, v, T, which can be written in a convenient form thus:-

$$\phi_2 - \phi_1 = c_v \log \frac{T_2}{T_1} + (c_p - c_v) \log \frac{v_2}{v_1} \qquad (3)$$

$$\phi_2 - \phi_1 = c_p \log \frac{T_2}{T_1} - (c_p - c_v) \log \frac{p_2}{p_1} \qquad (4)$$

$$\phi_2 - \phi_1 = c_v \log \frac{p_2}{p_1} + c_p \log \frac{v_2}{v_1} \qquad (5)$$

From these we may obtain at once the equations for constant volume lines as

$$\left.\begin{aligned} \phi_2 - \phi_1 &= c_v \log \frac{T_2}{T_1} \\ \text{or} \quad \phi_2 - \phi_1 &= c_v \log \frac{p_2}{p_1} \end{aligned}\right) \qquad (6)$$

and for constant pressure lines as

$$\left.\begin{aligned} \phi_2 - \phi_1 &= c_p \log \frac{T_2}{T_1} \\ \text{or} \quad \phi_2 - \phi_1 &= c_p \log \frac{v_2}{v_1} \end{aligned}\right) \qquad (7)$$

$T\ \phi$ Curves

We have now derived certain general equations that define the connection between the four variables p, v, T and ϕ; in actual problems we usually have a relation between the two characteristic properties p and v and require to find the corresponding relation between T and ϕ. The commonest relation in the p v plane is that shown on an indicator card, which in general may be expressed by the equation $p\ v^n$ = a constant. (8). Hence $p_1\ v_1^n = p_2\ v_2^n$ or $\frac{p_1}{p_2} = \frac{v_2^n}{v_1^n}$. From the fundamental gas equation we have $p_1\ v_1 = R\ T_1$ and $p_2\ v_2 = R\ T_2$, whence $\frac{p_1 v_1}{p_2 v_2} = \frac{T_1}{T_2}$.

On substituting this value of $\frac{p_1}{p_2}$ in the previous equation we get

$$\frac{v_2^{\,n}}{v_1^{\,n}} = \frac{T_1}{T_2}\cdot\frac{v_2}{v_1} \quad \text{or} \quad \frac{v_2^{\,n-1}}{v_1^{\,n-1}} = \frac{T_1}{T_2}$$

Hence from equation (3) we get $\phi_2 - \phi_1 = c_v \log \frac{T_2}{T_1} + (c_p - c_v) \log\left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}} = c_v \log \frac{T_2}{T_1} - \frac{c_p - c_v}{n-1} \log \frac{T_2}{T_1} =$

$$\left\{c_v - \frac{c_p - c_v}{n-1}\right\} \log \frac{T_2}{T_1} = \frac{c_v n - c_p}{n-1} \log \frac{T_2}{T_1} = c_v \frac{n-\gamma}{n-1} \log \frac{T_2}{T_1} \qquad (9)$$

Equations (8) and (9) are general expressions, covering practically all variations in perfect gases, and giving the relations in the p v and T ϕ planes respectively. For isothermal changes T is a constant; hence from the fundamental equation pv = a constant, and hence from equation (8) n = 1. Then from equation (9) $\phi_2 - \phi_1 = c_v \cdot \infty \cdot 0$, i. e., ϕ may have any value whatever and an isothermal change is represented in the T ϕ plane by the equation ϕ = a constant. For isentropic changes, or adiabatic in the case of reversible cycles, $\phi_2 = \phi_1$ and equation (9) gives $n = \gamma$, whence equation (8) becomes $p\,v^{\gamma}$ = a constant. If p is a constant, equation (4) gives $\phi_2 - \phi_1 = c_p \log \frac{T_2}{T_1}$; if v is a constant, equation (3) gives $\phi_2 - \phi_1 = c_v \log \frac{T_2}{T_1}$, as already found. From equation (8) in these cases n = 0 and ∞ respectively. All possible variations of equations (8) and (9) are indicated in Table 1 and Fig. 2.

GAS ENGINE CYCLES

In applying the principles and results given above to the various theoretical gas-engine cycles the classification given by Dugald Clerk will be followed. In all the figures capital letters are used to denote points on the temperature-entropy, small letters corresponding points on the pressure-volume diagram. The letters T and ϕ followed by subscripts denote the temperature and entropy at the points denoted by the subscript.

Type 1 (See Fig. 3). Heat received at constant volume without previous compression. The cycle is as follows:- From a to b the piston moves forward, drawing in the charge; at b the inlet valve closes and the mixture is fired, instantaneously in the ideal case, the pressure rising along bd; adiabatic expansion then takes place, as shown by de, to the initial pressure; ea is the return or exhaust stroke, expelling the products of combustion. The portion a b ofthe stroke does not appear on the $T\phi$ diagram, since during it no change of condition of the gas takes place; it represents only the mechanical moving of the charge into the cylinder and is no part of the heat cycle. The heat taken into the cylinder is represented by the area O B D P; that rejected by the area O B E P; the difference,or area B D E, represents the work done during the cycle. The efficiency of the cycle therefore is given by the fraction $\frac{O\,B\,D\,P - O\,B\,E\,P}{O\,B\,D\,P}$. Now B D is a constant volume line; if in equation (6) we take ϕ_1 as zero and T_B as the initial temperature, we may write the equation of the line as

$\phi = c_v(\log T - \log T_B)$. Differentiating $d\phi = c_v\frac{dt}{T}$.

The area under the curve or area O B D P = $\int_0^{\phi_2} T\,d\phi = \int_{T_B}^{T_D} T \cdot c_v \frac{dT}{T} = c_v (T_D - T_B)$. The line B E is a constant pressure line and in a similar way we can find the area under it, or area O B E P, to be $c_p (T_E - T_B)$. Hence the efficiency of the process is $\frac{c_v (T_D - T_B) - c_p (T_E - T_B)}{c_v (T_D - T_B)}$ or $1 - \gamma \frac{T_E - T_B}{T_D - T_B}$. If the expansion be incomplete, continued only to the point f say, the line G F is a constant volume line. The heat rejected may be divided into two portions, that represented by the area under F G, and that under G B, or $c_v (T_F - T_G)$ and $c_p (T_G - T_B)$ respectively. Hence the efficiency = $\frac{c_v (T_D - T_B) - C_v (T_F - T_G) - C_p (T_G - T_B)}{C_v (T_D - T_B)} = 1 - \frac{T_F - T_G + \gamma (T_G - T_B)}{T_D - T_B}$.

It is evident at a glance from the $T\phi$ diagram that the efficiency is decreased when expansion is less than that required to reach the initial pressure. Type II (See Fig.4). Heat received at constant pressure after previous compression. Two cylinders, a pump and a motor cylinder, are required in this type, but their diagrams may be superposed on one another. a b represents the suction stroke of the pump; b c the compression, assumed adiabatic, the gas passing into a receiver along the line cc'; it then enters the motor cylinder along the line c' d, receiving heat, at constant pressure. The supply of heat is cut off at d, from where a-diabatic expansion takes place to the initial pressure at e; e a is the return or exhaust stroke, at constant pressure. The lines a b and c c' again do not appear in the $T\phi$ diagram, for the same reason as before. The heat taken into the cylinder is represented by the area O C D P; that rejected by the area O B E P; the efficiency is therefore $\frac{O\,C\,D\,P - O\,B\,E\,P}{O\,C\,D\,P}$. Now C D and B E are both constant pressure lines; hence, as before, area under C D = $c_p\,(T_D - T_c)$ and area under B E = $c_p\,(T_E - T_B)$. Hence $\eta = \frac{c_p\,(T_D - T_c) - c_p\,(T_E - T_B)}{c_p\,(T_D - T_c)} = 1 - \frac{T_E - T_B}{T_D - T_c}$. Now for the line C D, from the equation of the curve, $c_p\,(\log T_D - \log T_c) = \phi_D - \phi_c$; similarly for the line B E $c_p\,(\log T_E - \log T_B) = \phi_E - \phi_B$. But $\phi_D - \phi_c = \phi_E - \phi_B$ since the lines B C and E D are adiabatics. Therefore $\log T_D - \log T_C = \log T_E - \log T_B$ or $\log \frac{T_D}{T_c} = \log \frac{T_E}{T_B}$

and therefore $\frac{T_D}{T_c} = \frac{T_E}{T_B}$. Hence $T_B = \frac{T_c}{T_D} \cdot T_E$ and $T_E = \frac{T_D}{T_c}\,T_B$.

Substituting in the equation above

$$\eta = 1 - \frac{T_E - T_E \frac{T_c}{T_D}}{T_D - T_c} = 1 - \frac{T_E}{T_D} \quad \text{or} \quad \eta = 1 - \frac{T_B \frac{T_D}{T_c} - T_B}{T_D - T_c} = 1 - \frac{T_B}{T_c}$$

This last form shows the result usually derived from the p v diagram, that the efficiency in this case depends only on the temperatures at the beginning and end of compression, or, since for an adiabatic line the volumes are proportional to the tempera-tures, only on the volumes at the beginning and end of compression. If the expansion be incomplete, say to a volume a f twice that at the beginning of compression, the heat taken in is the same as before, area O C D P, but the heat rejected is greater, being now represented by the area O B G F P. Hence the work is now represented by the area B C D F G, against B C D E for the full expansion. The work done and the efficiency are therefore both diminished by any cutting off of expansion.

If in this type it were desired to open to exhaust at the same volume as at the beginning of compression, the diagram shows that with the compression assumed this is impossible, unless we suppose that the highest temperature also is not reached, since the constant volume line through B requires a higher temperature for the same entropy than the constant pressure line through C. The only way of securing this economically is therefore to have a high compression, when the temperature at C is higher and the line CD is shifted in a direction parallel to the ϕ axis. This is what is actually done in the engines attempting to work on this cycle, which usually follow the Otto mechanical cycle and so open to exhaust at the same volume as that at the beginning of compression.

Type III (See Fig:5) Heat received at constant volume after previous compression. The action may be regarded as requiring two cylinders, but the pump and motor diagrams may conveniently be combined, as in Type II. The pump draws in a volume a b, which is compressed adiabatically along the line b c and into a receiver along the line $c\ c^1$. The compressed gases enter the motor cylinder along the line $c^1\ c$; heat is added instantaneously and the pressure and temperature rise along the line c d. When the supply of heat is cut off at d, adiabatic expansion along d e takes place down to the original pressure; an exhaust stroke at constant pressure completes the cycle. The heat taken in is represented as before by the area O C D P; the heat rejected by O B E P; and the work done by B C D E. The efficiency is therefore $\frac{OCDP - OBEP}{OCDP}$. C D is a constant volume line; hence the area under it is $c_v\ (T_D - T_c)$. B E is a constant pressure line and hence the area under it is $c_p\ (T_E - T_B)$. The efficiency is therefore

$$\frac{c_v\ (T_D - T_C) - c_p.\ (T_E - T_B)}{c_v\ (T_D - T_c)} = 1 - \gamma\ \frac{T_E - T_B}{T_D - T_c}.$$

When the expansion is continued only to the volume existing at the beginning of compression we have the ordinary four-stroke Otto cycle. H B then represents the closing line of the cycle, the drop of pressure and temperature at constant volume. B H is therefore a constant volume line, so that the area under it is $c_v\ (T_H - T_B)$. The heat taken in is the same as before; the efficiency is then $\frac{c_v\ (T_D-T_C)-C_v(T_H-T_B)}{c_v\ (T_D-T_C)}$ $= 1 - \frac{T_H - T_B}{T_D - T_C}$. Also the lines C D and B H are similar, and therefore $\frac{T_H}{T_D} = \frac{T_B}{T_C}$ and hence as in Type II we can reduce the efficiency to $1 - \frac{T_H}{T_D}$ or $1 - \frac{T_B}{T_C}$, or in this case also the efficiency depends only on the temperatures at the beginning and end of compression, or, since compression is supposed adiabatic, only on the ratio of compression. A third case is given by Clerk, where the expansion is not full, but is carried to a volume greater than that at the beginning of compression. F G shows this case when the volume at the end of expansion is double that at the beginning of compression. The efficiency, if desired, can easily be found in the same way as previously, the work done being the area B C D F G, and the heat rejected being composed of the two areas, O B G R, under the constant pressure line B G, and R G F P, under the constant volume line G F. A glance at the T ϕ diagram shows that the efficiency in this case is intermediate between that in the first two cases, and that the efficiency decreases as the expansion is lessened.

TYPE I A. Fig 6 This differs from Type I in that, while the expansion stroke is adiabatic, the exhaust or return stroke, which really becomes then a compression stroke, is isothermal. The expansion is continued until the temperature falls to that at the beginning of the cycle. The line of addition of heat B D is a constant volume line; that of rejection of heat E B is isothermal. The area under B D is therefore $c_v\ (T_D - T_B)$. The area under E B is the rectangle O B E P, the sides of which are T_B and $\phi_E - \phi_D$. Now from the equation of the line B D, $\phi_D = c_v\ (\log T_D - \log T_B) = c_v \log \frac{T_D}{T_B}$; hence the area under E B $= T_B \cdot c_v \log \frac{T_D}{T_B}$. The efficiency is therefore

$$\frac{c_v\ (T_D - T_B) - c_v \cdot T_D \log \frac{T_D}{T_B}}{c_v\ (T_D - T_B)} = 1 - \frac{T_D \log \frac{T_D}{T_B}}{T_D - T_B}.$$

It is evident at once from the form of the diagram that the theoretical efficiency of this type is very high, but the immense expansion required and the extremely slow working necessary to allow of even approximately isothermal compression render an engine of this type excessively cumbrous and uneconomical from the standpoint of output with respect to size and weight. Incomplete expansion has the effect of cutting off part of the area representing the work done, as shown by F G, and hence of decreasing the efficiency, as in all the other cases.

COMPARISON OF ENGINES.

The $T\phi$ diagram can be advantageously employed in comparing one type with another, or in comparing different cases of the same type under changing conditions. For this purpose it is superior to the pv diagram, especially in presenting at a glance the relative efficiency of different cycles, while if used in conjunction with the pv diagram, we have before us all the information which can be obtained regarding any theoretical cycle.

Figure 7 shows the relation between the Lenoir, the Otto, and the Atkinson cycles; the first is an example of Clerk's Type I; the other two of Type II, the Atkinson with complete expansion produced by the mechanism of the engine, the Otto with expansion to a volume equal to that at the beginning of compression. The area B E F shows the work done during a cycle in the Lenoir engine; B C D E that done in a cycle in the Otto; and B C D F that in a cycle in the Atkinson. Very evidently the first is much the least efficient and the last the most efficient of the three. If the compression in the two latter is increased, the work done becomes $B\,C'\,D'\,E'$ and $B\,C'\,D'\,F'$ respectively. The difference between these two areas (now $B\,E'\,F'$) is less than before (B E F); hence we see that the theoretical superiority of the Atkinson is less at higher compressions.

Further, the increased size, complexity and friction in the mechanism of the Atkinson more than counterbalance the theoretical superiority to the Otto, so that in practice the Otto is the most efficient, and the Atkinson has been little more than an ingenious attempt at improvement.

For engines employing adiabatic compression and expansion there are three symmetrical types of thermodynamic cycle, which are each cycles of maximum efficiency for the conditions assumed. The first of these is the constant temperature type, the well-known Carnot cycle. In this adiabatic compression raises the temperature from the lower to the upper limit; during isothermal expansion at the upper temperature the whole heat supply is received; when the supply is cut off, adiabatic expansion reduced the temperature to the lower limit; and then isothermal compression brings the working fluid back to its initial conditions. The second symmetrical cycle is the constant pressure type. In this we have first adiabatic compression as before from the lower to the upper pressure; heat is then supplied at the constant upper pressure, with consequent rise of temperature and increase of volume; when the heat supply is cut off, adiabatic expansion to the lower pressure takes place, followed by the rejection of heat at the constant lower pressure and consequently, diminishing temperature and volume. The third symmetrical cycle is the constant volume type. Again the first change is adiabatic compression, here from maximum to minimum volume; at the end of compression heat is added at constant volume, with consequent increase of both pressure and temperature; when the heat supply is cut off, adiabatic expansion reduces the pressure and temperature until the volume reaches that existing before compression; finally heat is rejected at constant volume and diminishing temperature and pressure. Figure 8 shows these three symmetrical cycles, for the same range of compression in all three; A B G H represents work done during a Carnot cycle; A B E F that during a constant pressure cycle; and A B C D that during a constant volume cycle. It can be easily shown, by the method employed before, that in all three the efficiency is the same and depends only on the ratio of the volume before compression to that after compression, being $1 - (\frac{1}{r})$. In the Carnot cycle the efficiency is the greatest ideally possible under any conditions, since the temperature at the end of compression is the maximum reached in the cycle; in the others the efficiency is not the greatest ideally possible for the temperature range, but is the greatest possible for the conditions assumed. The $T\phi$ diagram shows that the work done per cycle in the Carnot type is very much less than in either of the others; the pv diagram shows that the range of volume is considerable, being about twelve times;

hence a large unwieldy machine is required for a small output. The constant pressure type gives the greatest output per cycle, shown by area A B E F, but from the pv diagram the range of volume is about eighteem times; hence here too a large cylinder is required, though for a much larger output than in the Carnot type. The work per cycle in the constant volume type, represented by area A B C D, is not very much less than for the constant pressure type, while the pv diagram shows that the range of volume is very much less, being only five times. Hence, under these conditions, a much greater output per unit of weight or size would be given by an engine working on the constant volume type.

The Carnot cycle is outside the bounds of practical consideration for two reasons: the exceedingly small output per unit of size of cylinder, and the inability to secure even approximately isothermal compression except at excessively slow speeds. Figure 9 shows a comparison between the constant pressure and constant volume types under conditions more nearly approaching those met with in practice, i.e. where the extreme pressure and temperature range is the same and where in the constant pressure type expansion takes place only to a volume equal to that at the beginning of compression. This latter is approximately the cycle of the Diesel engine. A B C D shows the work done per cycle in the constant volume type; A E C D that in the constant pressure type. The latter shows evidently a considerable advantage in efficiency. Reference to the pv diagrams, however, shows that the range of volume is about 14 to 1 in the constant pressure cycle but only 5 to 1 in the constant volume type, thus necessitating a cylinder of about three times the size in the former case. This to some extent offsets the advantage of greater efficiency between the given temperature limits, and moreover this advantage in efficiency is less at lower temperatures and pressures than those shown, which hitherto have been more frequent in practice.

There is also great difficulty in regulating and controlling the combustion practically in engines of the constant pressure type. These various reasons have militated against engines of Type II taking the place in commercial practice to which their theoretical superiority entitles them.

VARIATIONS WITHIN ONE TYPE.

The T ϕ diagram lends itself admirably to comparisons of different conditions in any given cycle. A series of diagrams of Type II to which most gas-engines in use belong will show this. Figure 10 shows the effect of varying the quantity of heat supplied per cycle; that is of having rich or weak mixtures in the charge. We can easily find that the work done per cycle changes, but the efficiency is constant. Figure 11 shows the effect of varying the compression. Here not only does the work done per cycle increase with increased compression, but the efficiency also increases, efficiency, as we have already seen, being greater at greater compressions. This appears at once from the T ϕ diagram, where the areas under the three curves, representing the heat taken in per charge are equal, but the area representing the work done increases considerably. Figure 12 gives the effect of varying the suction temperature alone, and Figure 13 the effect of varying suction temperature and its natural corollary, the quantity of heat per cycle, since a smaller mass of gas is drawn in at the higher temperature. In both these cases the efficiency will remain the same, since it depends only on the ratio of compression, but the work done per cycle will decrease as the temperature rises.

VARIABLE SPECIFIC HEAT.

In the discussion so far the specific heat of the working substance has been treated as constant. It is, however, now generally admitted that the specific heat is not constant, but increases with increasing temperature, the rate of increase being different in different gases. Though all experimenters agree in finding this increase of specific heat, they are very discordant as to numerical results and no published figures have so far found general acceptance; hence it is impossible to make calculations of any value allowing for increasing specific heat. One instance is however taken in order to examine the effect of the increasing specific heat on the efficiency and on the entropy diagram. That selected is the Otto cycle shown in Figs: 8 and 9 and repeated in Figure 14, diagram A B C D. Starting with equation (2) in the form $dQ = c_v \, dt + (c_p - c_v) T \frac{dv}{v}$, let us examine the changes in the equations caused by variable specific heat. Assume Prof.Burstall's statement of the results of Mallard and Le Chatelier's experiments to be the law of variation, this is $c_v = a + s\,T$

$$c_p = b + s\,T$$

where a, b and s are constants to be determined by experiment. Substituting in the equation above gives $d\,Q = (a + s\,T)\,d\,T + (b - a)\,T \frac{dv}{v}$. Hence $d\phi = \frac{dQ}{T} = \left(\frac{a}{T} + s\right) d\,T + (b-a) \frac{dv}{v}$. Integrating $\phi_1 - \phi_2 = a \log$ (over)

$\frac{T}{T_1} + s\ (T-T) + (b-a) \log \frac{v}{v_1}$. For a constant volume line, taking $\phi_1 = 0$, we get $\phi = a \log \frac{T}{T_1} + s\ (T-T_1)$. In Figure 14, A B E F is plotted from this equation, assuming the working substance as nitrogen, for which Burstall gives $c_v = .170 + .0000\ 872\ t$, which equals $.1939 + .0000\ 872\ T$. It was assumed, for the sake of comparison, that the temperature reached was the same as before; the actual temperature reached does not affect the efficiency, though it does affect the work done. A simple expression for the efficiency with varying specific heat cannot be obtained, but the efficiency in any particular case may be obtained from the $T\phi$ diagram by the ratios of the areas, as is done below for this case. The equation of the curve is $\phi = a \log \frac{T}{T_1} + s\ (T-T_1)$. The area under the curve $= \int T\, d\phi$ or, substituting from the differential of the equation of the curve, the area $= \int T\ (a \frac{dT}{T} + s\ dT) = \int (a\ dT + s\ T.d\ T) = a\ T + s\ \frac{T^2}{2} + \text{const.}$

The temperature limits for B E are 559° and 1973° absolute, hence the area under it is 430.5 heat units; the limits for A F are 290° and 1250°, so that the area under it is 250.5 units. The efficiency is therefore $\frac{430.5 - 250.5}{430.5} = 42\%$. The efficiency of the process with specific heat assumed constant can be found from the compression ratio as $1 - \left(\frac{1}{5}\right)^{.408} = 48\%$. Hence in this case the efficiency has been reduced from 48% to 42% by allowing for an increasing specific heat. Though these figures are not reliable, yet they indicate that the effect of the specific heat increasing with rise of temperature will be to decrease the theoretical thermal efficiency of any cycle, or in other words, to increase the "efficiency ratio" of a cycle by lessening the difference between the efficiency actually attained and that theoretically possible for that cycle under ideally perfect conditions. It therefore helps to account for part of "the missing quantity".

CONVERSION OF THE ACTUAL INDICATOR CARD.

The actual conversion of a gas-engine indicator diagram into a $T\varphi$ diagram is rendered difficult from several causes. For a $T\varphi$ diagram the quantity of heat concerned per cycle must be known; this demands an accurate knowledge of the explosive mixture. In practically all gas-engines there is considerable variation in the explosive mixture under running conditions; in the hit-and-miss type of governing, for example, the mixture after a missed stroke is much richer than the average and a greater weight of gas will be contained in it on account of the greater density due to the cooling effect of the missed stroke. If governing is done by throttling the fuel, again the composition of mixture and weight of gas in it will vary; if both air and gas are controlled we still find that the proportions necessarily vary, in order to get good explosive mixtures at the varying compressions resulting. The best that can be done in any case is to use average indicator cards and an average mixture calculated from the observed volumes of gas and air used. A second serious difficulty in the conversion of a pv to a Tϕ diagram is the determination of c_p and c_v. No satisfactory definite values for these quantities at the higher temperatures have yet been published, and there is a further difficulty in applying what is known to the mixture in varying proportions of gas, air and products of combustion that takes part in every cycle. A third difficulty is presented by the decrease in volume after combustion due to the chemical rearrangements in the products produced by the combustion. It has been calculated by Dugald Clerk that the shrinkage on combustion varies from 4% with a mixture of 1 gas to 5 air, to 2.2% with a mixture of 1 gas to 10 air. Finally, it is necessary to knpw a definite temperature somewhere in the cycle in order to plot a correct diagram. Only with difficulty can the temperature of the gases be measured during the cycle, and then only approximately, because any temperature observed will be not the mean of the whole mass, but the temperature at the point of observation only, since it is now definitely established that the temperature throughout the mass of gas is not uniform, but varies from point to point. A common assumption is to take the temperature of the charge at the beginning of compression as equal to that of the issuing jacket water. Prof.Reeve stated that he assumes the round figure of 600° F. abs. If the above mentioned necessary quantities are known or assumed, the $T\phi$ diagram may be plotted from the indicator diagram by using the equations given above. In Captain Sankey's method a chart is prepared on which constant pressure and constant volume lines are laid out at convenient distances apart; then the diagram is plotted by finding points at the intersection of the proper constant volume and constant pressure lines. Graphic methods of obtaining the $T\phi$ diagram from the indicator card have been developed, notably by Prof.Boulvin and by W.J.Goudie (Proc.I. Mech.E.) These are both based on the use of constructional logarithmic curves, and the former involves a transferring first to the Tv plane and from that to the $T\phi$. Neither of these methods, however, has come into general use, but either would be advantageous in cases where a number of diagrams with the same values of c_v and γ were to be transferred. Goudie's method can be applied also to cases in which the specific heat is assumed variable, as is pointed out in the paper referred to. Figure 15, taken from this paper, shows Burstall's trial D_4 worked out with variable specific heat assumption (in full lines) and with constant specific heat (in dotted lines). Figures 16 and 17 are diagrams taken from Berry, transferred from the pv diagram by Boulvin's graphic method. In treating of the effects of ratio of compression, time of firing, quantity of heat per charge, and other variables, on the efficiency, a single standard engine using a standard working substance may be assumed. The standard recommended by the Committee of the Institution of Civil Engineers and now generally followed is that of "a perfect air-gas engine operated between the same maximum and minimum volumes as the actual engine, receiving the same total amount of heat per cycle, but without jacket or radiation loss, and starting from one atmosphere and the selected initial temperature of 139° F." If however it be desired to compare different working substances or fuels, account must be taken of the varying values of c_v and γ .

The study of compression and expansion lines, the latter especially, in the $T\varphi$ diagram of the actual engine, throws a great deal of light on what is happening in the cylinder during those strokes. In Fig:15, for instance, the expansion line is found at first to slope slightly to the right, then more steeply to the right; it then becomes nearly vertical for a time and finally slopes to the left. During the whole of this period the temperature has been dropping; hence while the curve slopes to the right, heat is being added to the working substance. The source of this added heat is not well established. Some authorities would put it down as due to "afterburning"; others, as due, in great part at least, to the increase of specific heat at higher temperature; yet others, to the dissociation at high temperatures of some of the chemical compounds formed by the combustion, heat being produced by their re-combination as the temperature falls; while some believe it can be largely explained as due to the interchange of heat between the cylinder walls and the gases inside. According to this last theory the hot gases, immediately after explosion, give up heat to the cooler walls; as the gases expand, their temperature decreases and there comes a time when it falls below that of the walls, which then give up heat to the gases. One would hardly expect, however, that the temperature of the gases would fall so rapidly in comparison with that of the walls, which are all the time transmitting and giving up heat to the jacket water, that the walls would be able to give up heat to the gases at or soon after the middle of the stroke, when the surface of the cylinder is increasing most rapidly. Quite probably several of these influences may combine in cases where the heat actually does increase during the stroke, especially in such a case as that of Fig:16, where the expansion line in the $T\phi$ diagram slopes continuously to the right. In Fig:15 during the latter part of the stroke the line slopes to the left, showing that the heat is decreasing. In this case the fall of temperature due to expansion is taking place faster than can be overcome by the supply of heat from any of the sources mentioned. This reasoning shows too that even though the expansion line might be vertical, this does not necessarily mean that the expansion is really adiabatic, but only that the various influences counteract each other to such a degree as to produce an apparently adiabatic expansion. As a matter of fact, the expansion line in practically all cases does slope to the right showing that heat has been received from some source during expansion.

The compression line also gives opportunity for interesting deductions. As a general thing it slopes first to the right, showing an increase of entropy and a reception of heat from the cylinder walls by the gases. As compression proceeds a point is reached where the temperature due to compression is equal to that of the gases; the curve then becomes vertical, and on further compression slopes to the left, showing that the gases, owing to the compression, have reached a higher temperature than the walls, and are then giving up heat to the walls. It sometimes happens that the loss actually exceeds the gain due to compression, in which case the temperature may decrease towards the end of compression; Fig:17, giving a case of late ignition, shows this. In Fig:15 the compression curve at the end of compression slopes again towards the right, showing that at higher compressions and speeds the heat due to compression may be greater than is taken away by the walls. This additional heat may also be due to early ignition. The time of firing of course affects the combustion line, also making it approach more or less nearly to a constant volume line. "The character of the exhaust line is of no significance, as it does not represent the history of a fixed quantity of substance. Its sole importance is to close the diagram and thus to make the area of the $T\varphi$ diagram the heat equivalent of the work recorded by the indicator card."

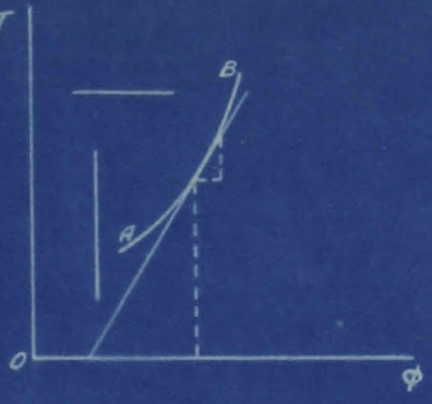
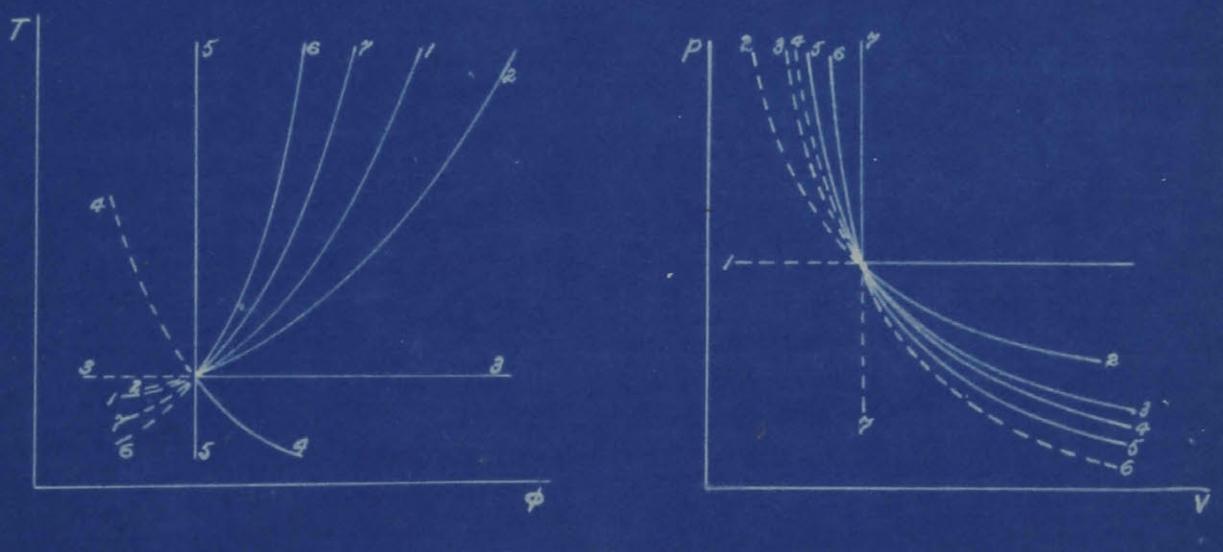


Fig. 1.

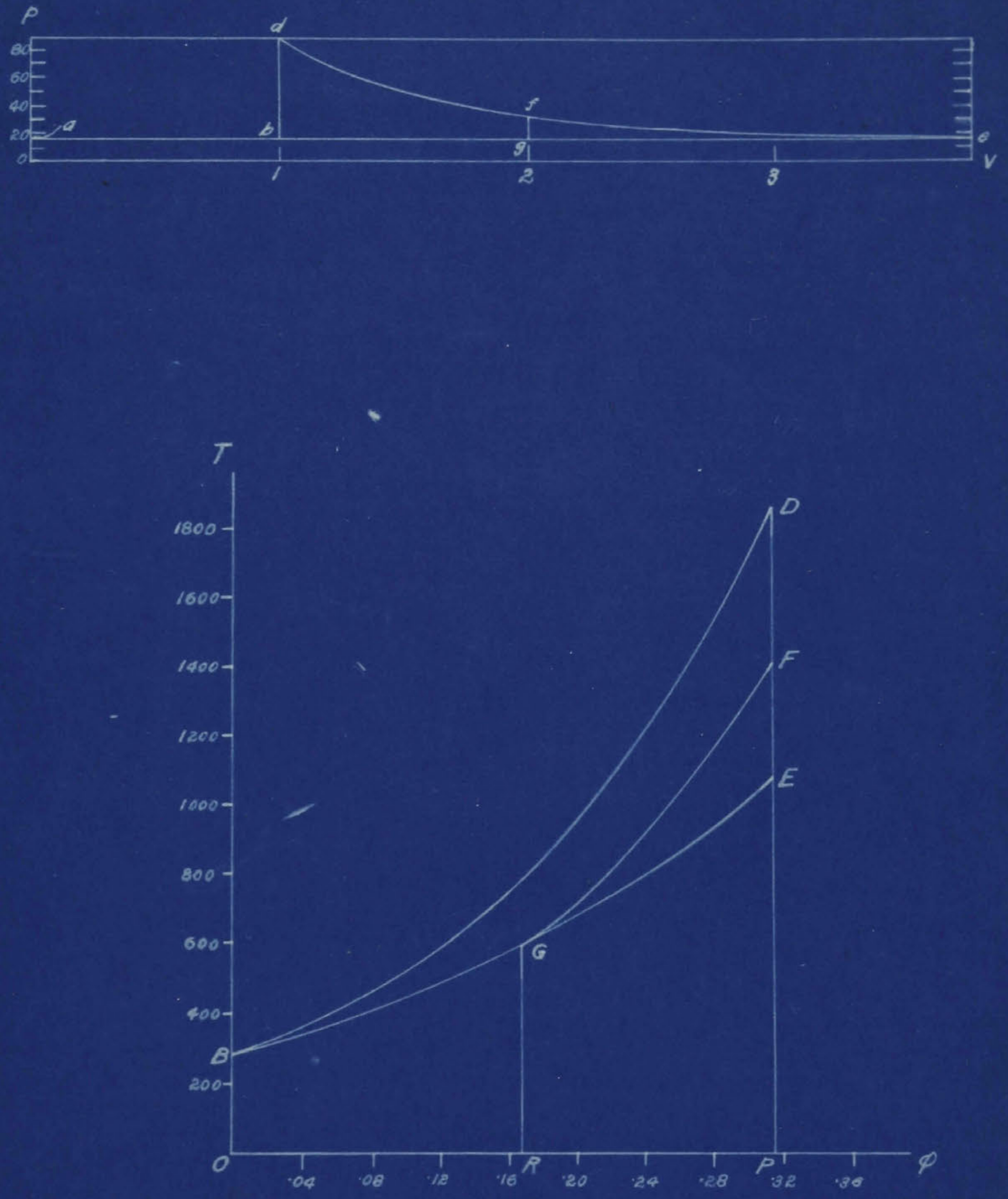
TABLE OF VARIATIONS.

Nº of Curve in Fig. 2	pv Co-ordinates		Tφ Co-ordinates	
	Value of n	Form of the Curve	Value of $c_v \frac{n-\gamma}{n-1}$	Form of the Curve
1	0	$p = \text{constant}$	c_p	φ and T increase
2	$0 < n < 1$	$pv^n = \text{constant}$	$> c_p$	or decrease together
3	1	$pv = \text{constant}$	∞	$T = \text{constant}$
4	$1 < n < \gamma$	$pv^n = \text{constant}$	negative	φ increasing, T decreasing, or vice-versa
5	γ	$pv^\gamma = \text{constant}$	0	$\phi = \text{constant}$
6	$\gamma < n < \infty$	$pv^n = \text{constant}$	$< c_v$	φ and T increase or
7	∞	$v = \text{constant}$	c_v	decrease together

TABLE I.



Figs. 2.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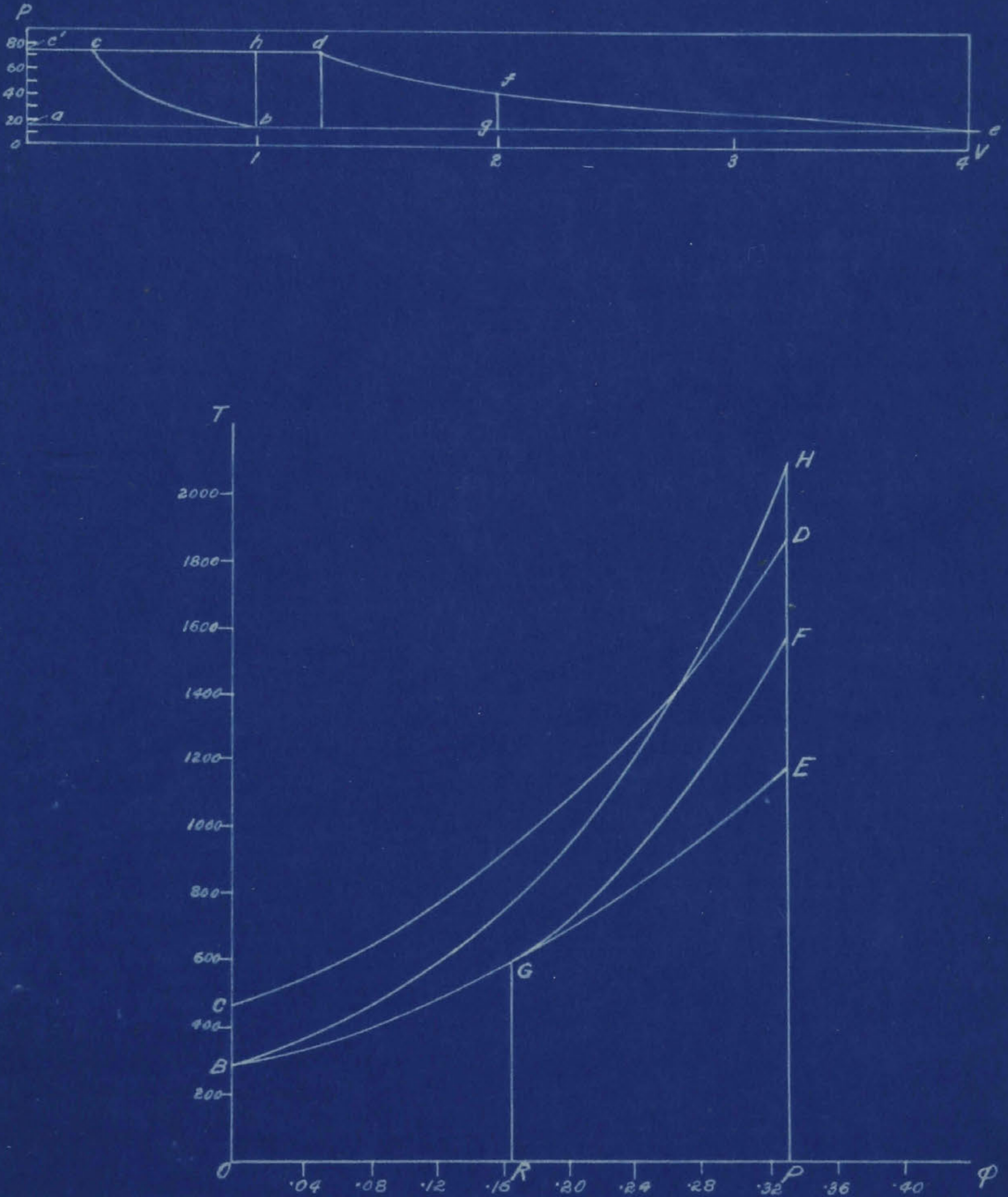
80
60
40
20
0
a
d
b
f
g
e
V
1
2
3
T
1800
1600
1400
1200
1000
800
600
400
200
D
F
E
G
B
O
R
P
φ
·04
·08
·12
·16
·20
·24
·28
·32
·36

Figs. 3.


P
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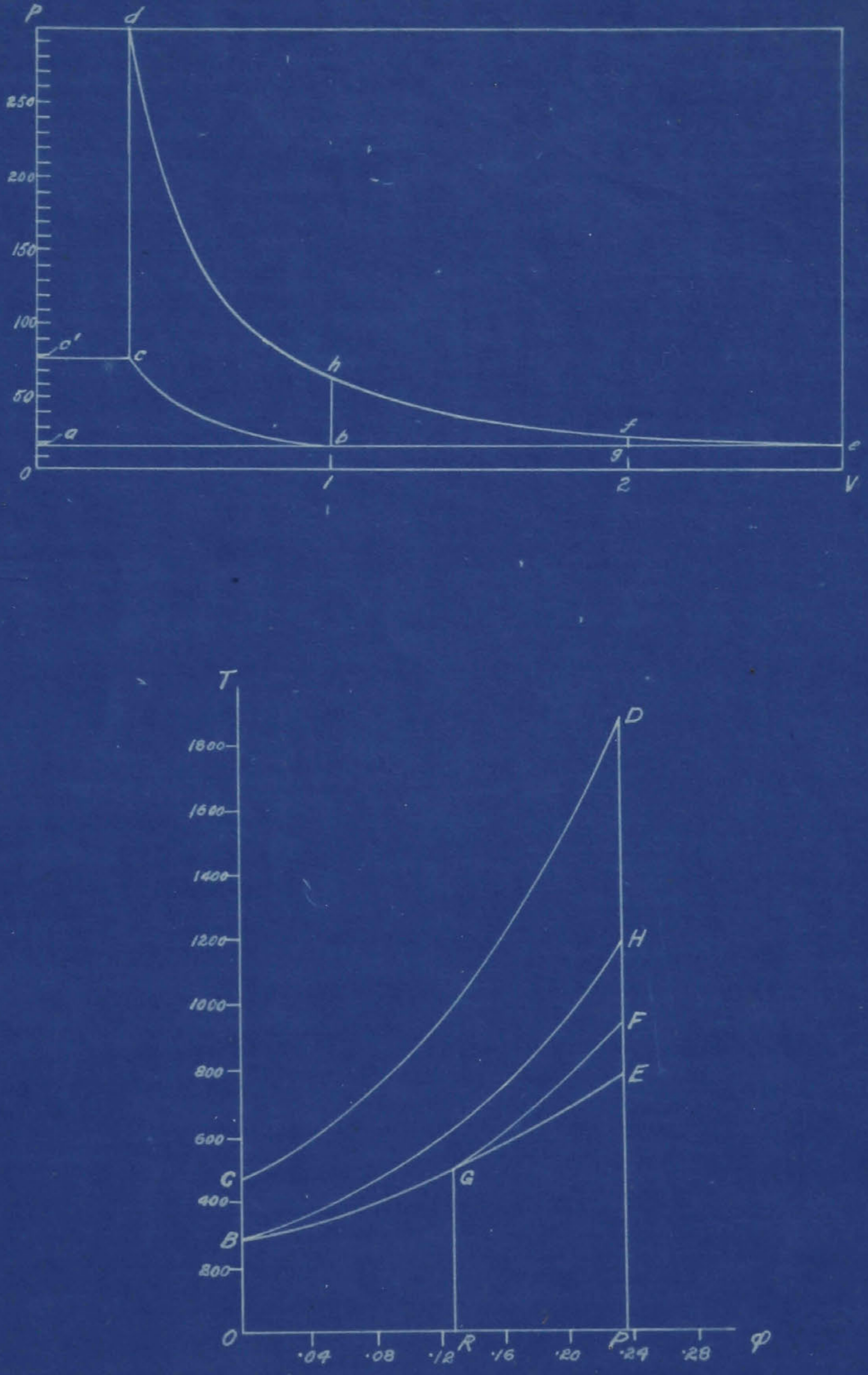
60
40
20
0
c'
c
h
d
f
a
b
g
e
1
2
3
4
V
T
2000
1800
1600
1400
1200
1000
800
600
400
200
H
D
F
E
G
C
B
O
R
P
·04
·08
·12
·16
·20
·24
·28
·32
·36
·40
φ

Figs. 4.


P
d
250
200
150
100
c'
c
h
50
a
b
f
g
e
0
1
2
V
T
D
1800
1600
1400
1200
H
1000
F
800
E
600
C
G
400
B
200
0
·04
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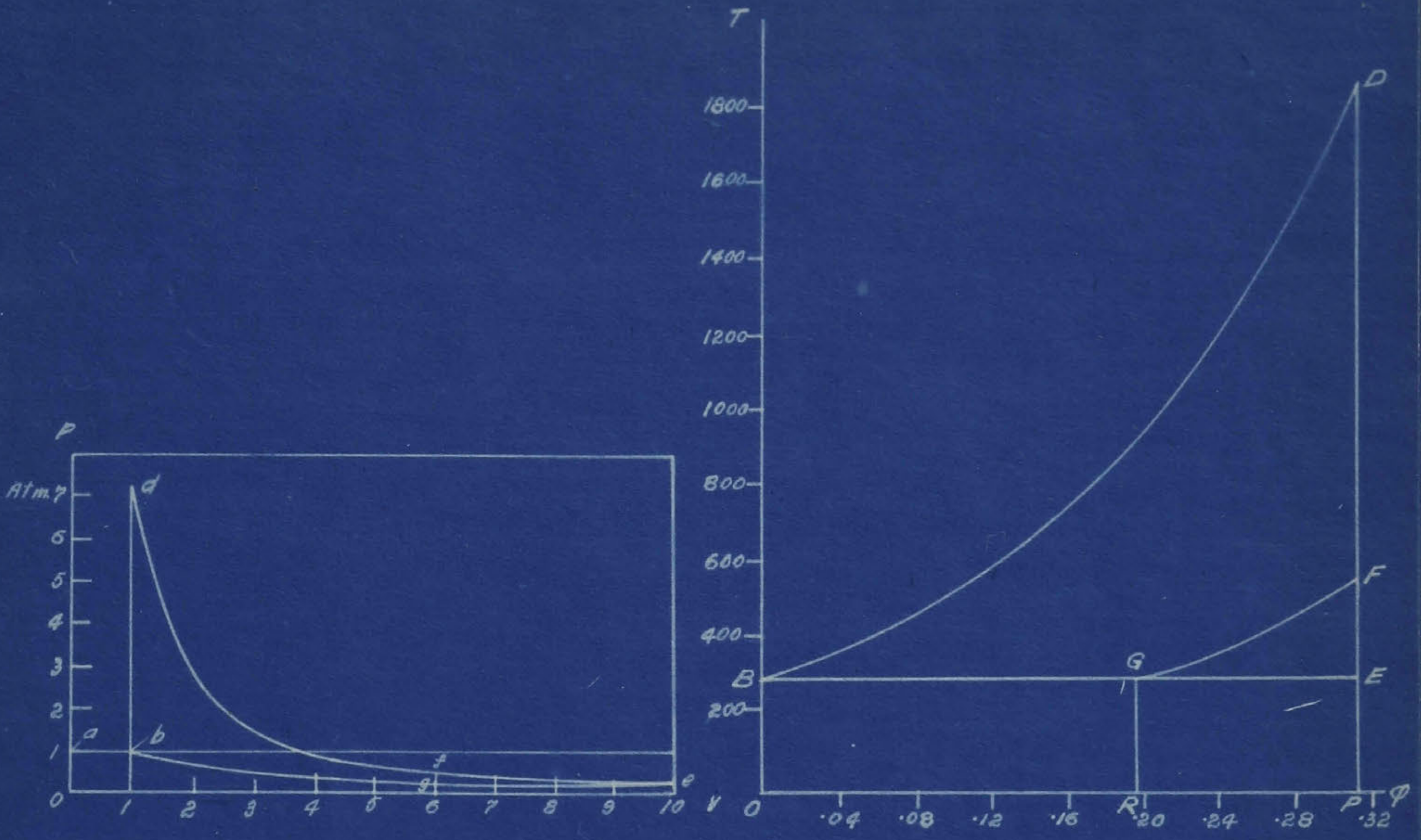
R
·16
·20
P
·24
·28
φ

Figs. 5.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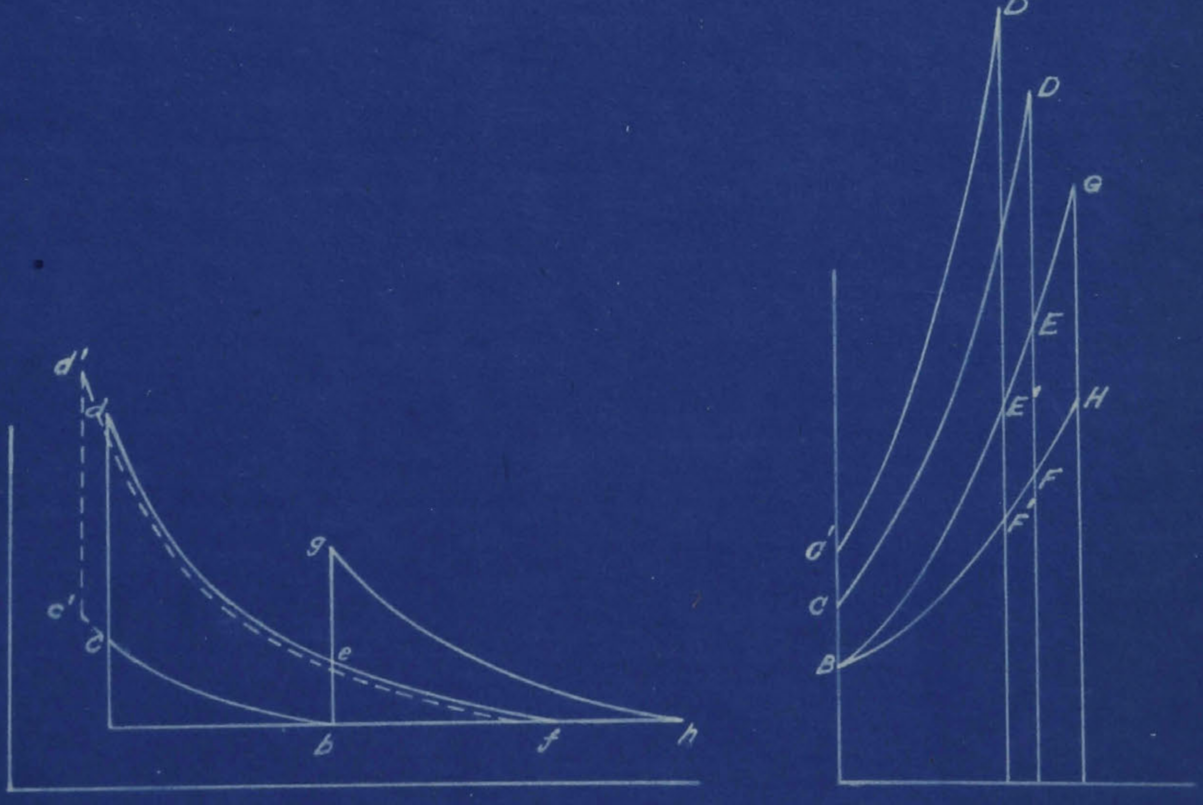
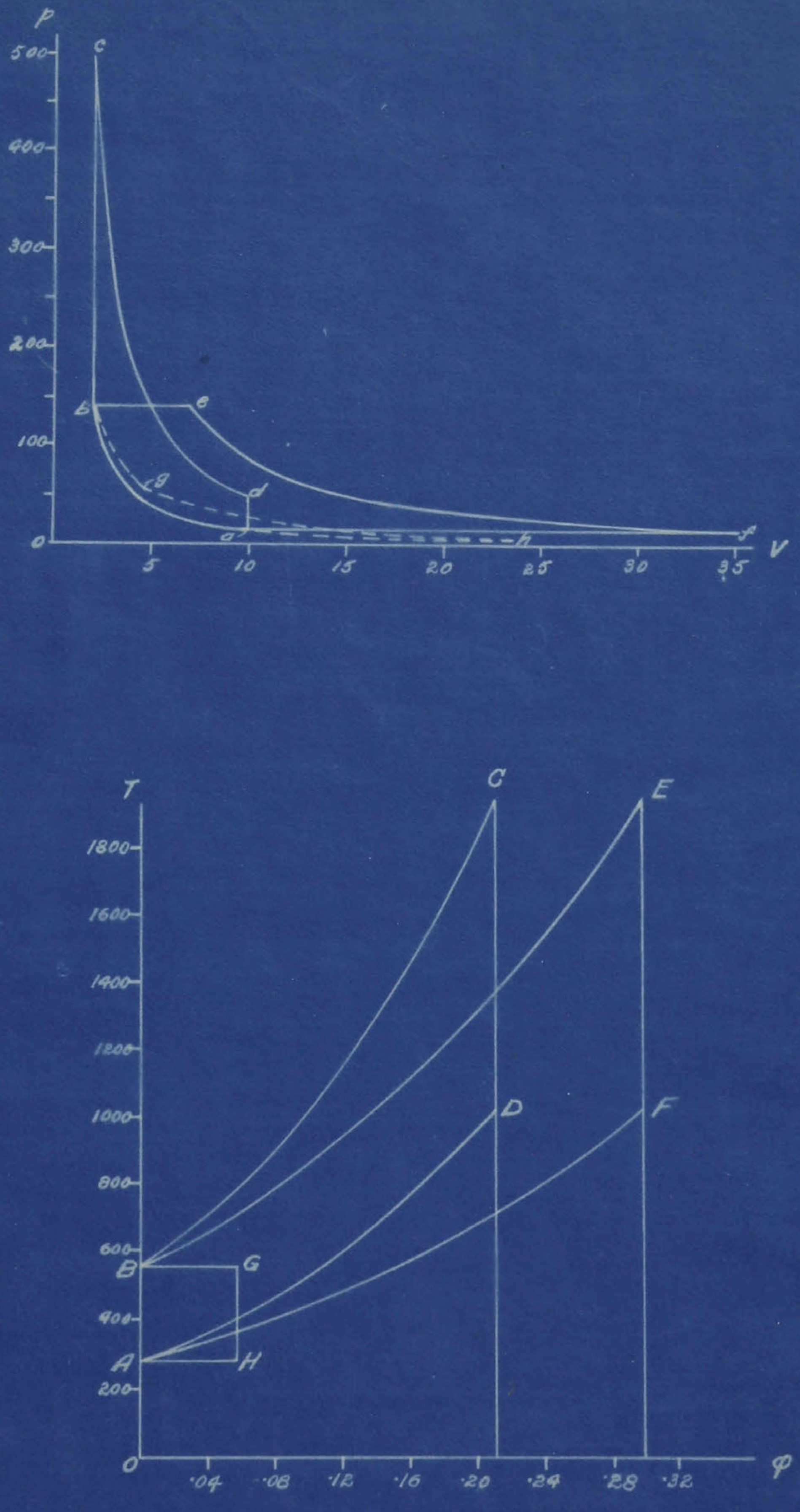
Atm.7
6
5
4
3
2
1
0
a
b
d
f
g
e
1
2
3
4
5
6
7
8
9
10
v
T
1800
1600
1400
1200
1000
800
600
400
200
D
F
G
B
E
0
.04
.08
.12
.16
R
.20
.24
.28
P
.32
φ

Figs. 6.


d'
d
c'
c
g
e
b
f
h
D'
D
G
E
E'
H
F
F'
C'
C
B

Figs. 7.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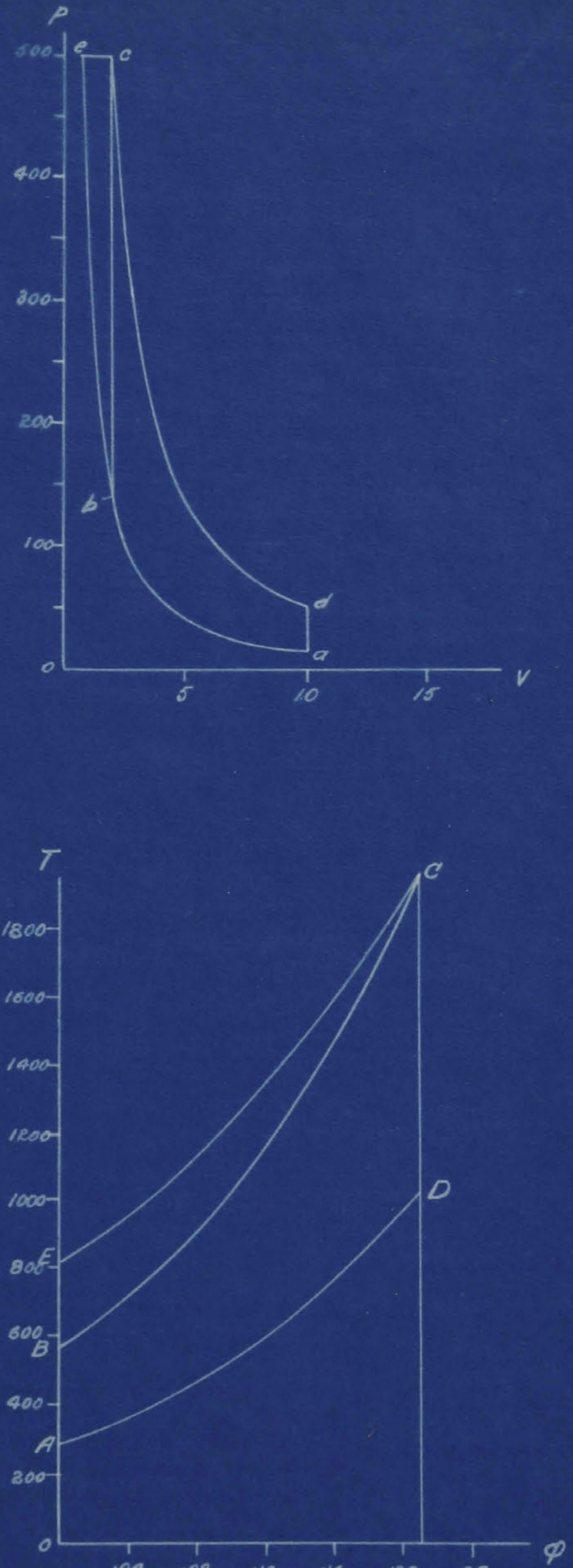
400
300
200
100
0
c
b
e
g
d
a
h
f
5
10
15
20
25
30
35
V
T
1800
1600
1400
1200
1000
800
600
400
200
0
C
E
D
F
B
G
A
H
·04
·08
·12
·16
·20
·24
·28
·32
φ

Figs. 8.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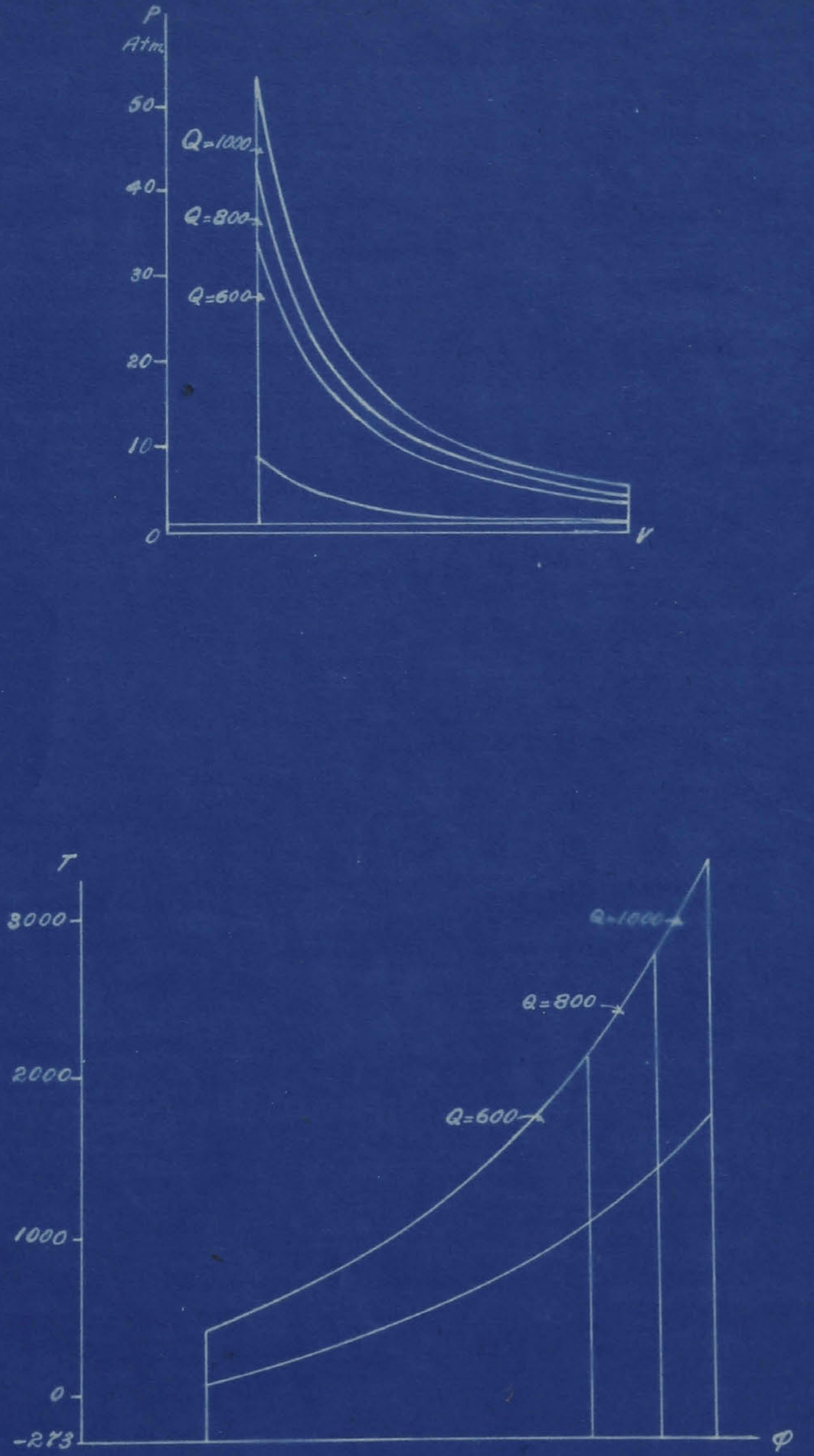
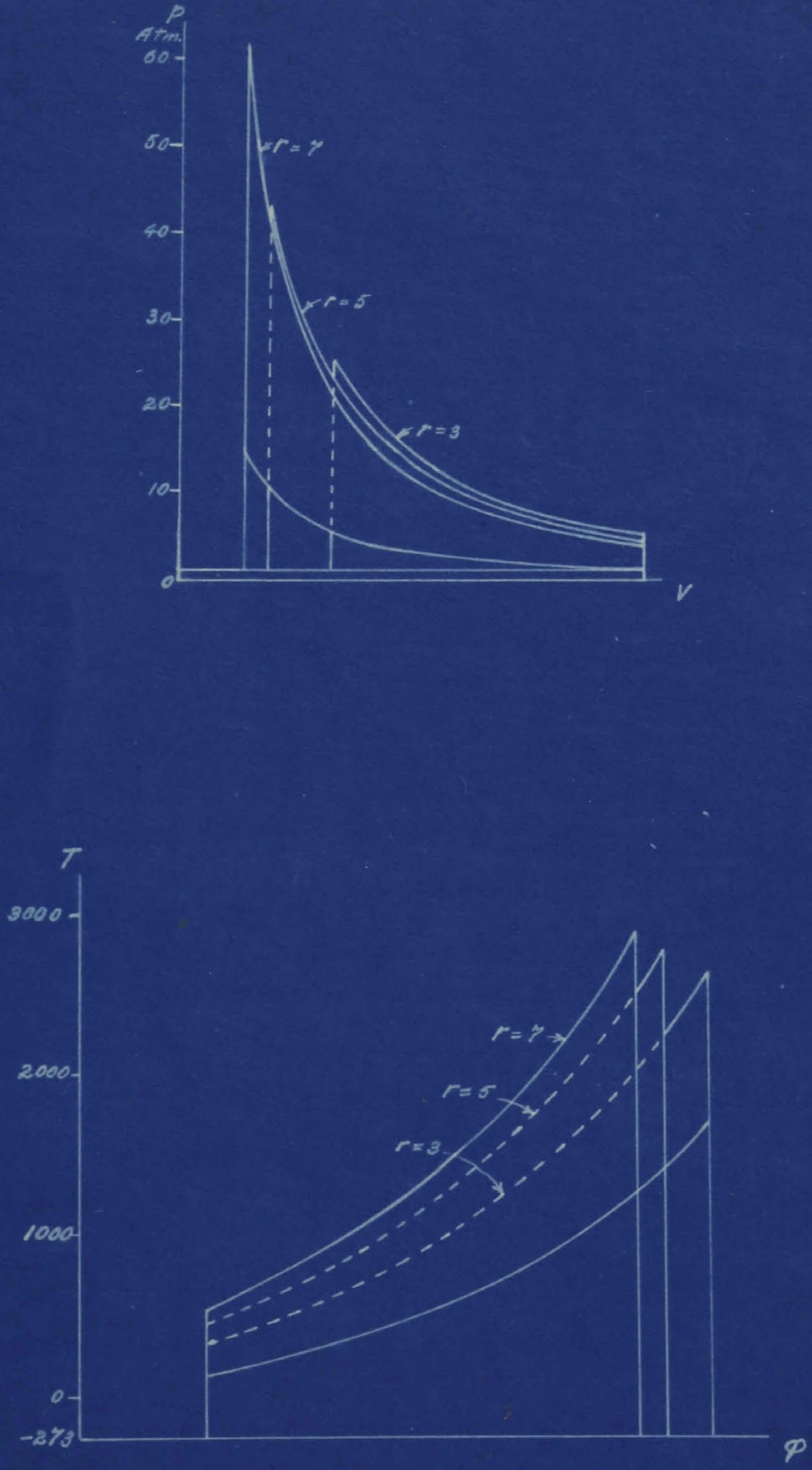
500
400
300
200
100
0
e
c
b
d
a
5
10
15
V
T
1800
1600
1400
1200
1000
800
600
400
200
0
C
D
E
B
A
·04
·08
·12
·16
·20
·24
φ

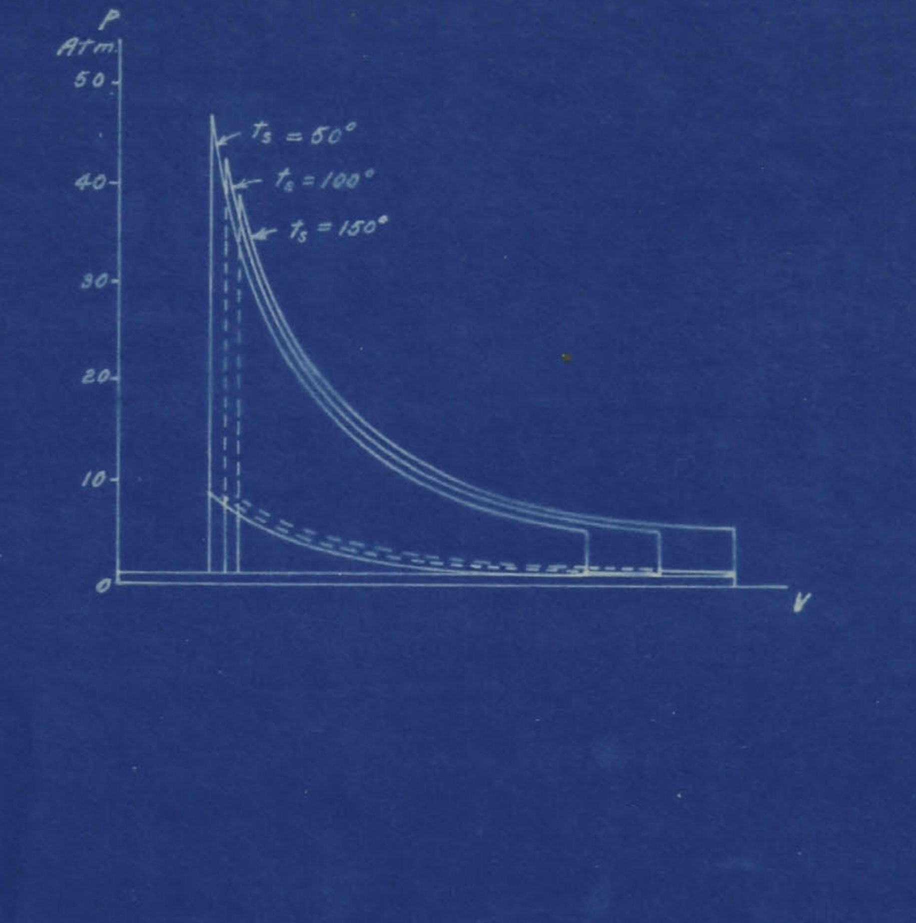
Figs. 9.


P
Atm
50
40
30
20
10
0
V
Q=1000
Q=800
Q=600
T
3000
2000
1000
0
-273
φ
Q=1000
Q=800
Q=600

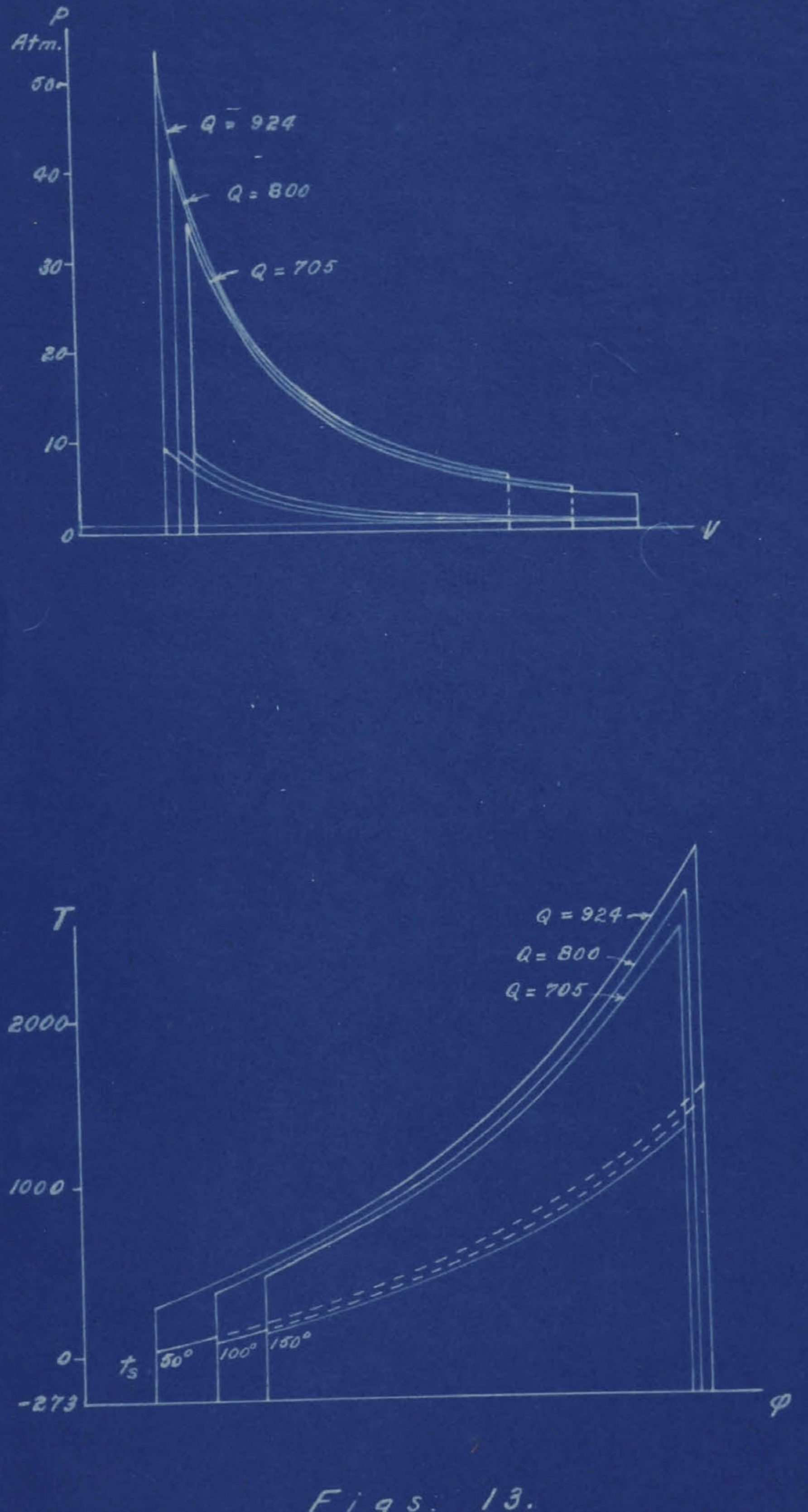
Figs. 10.



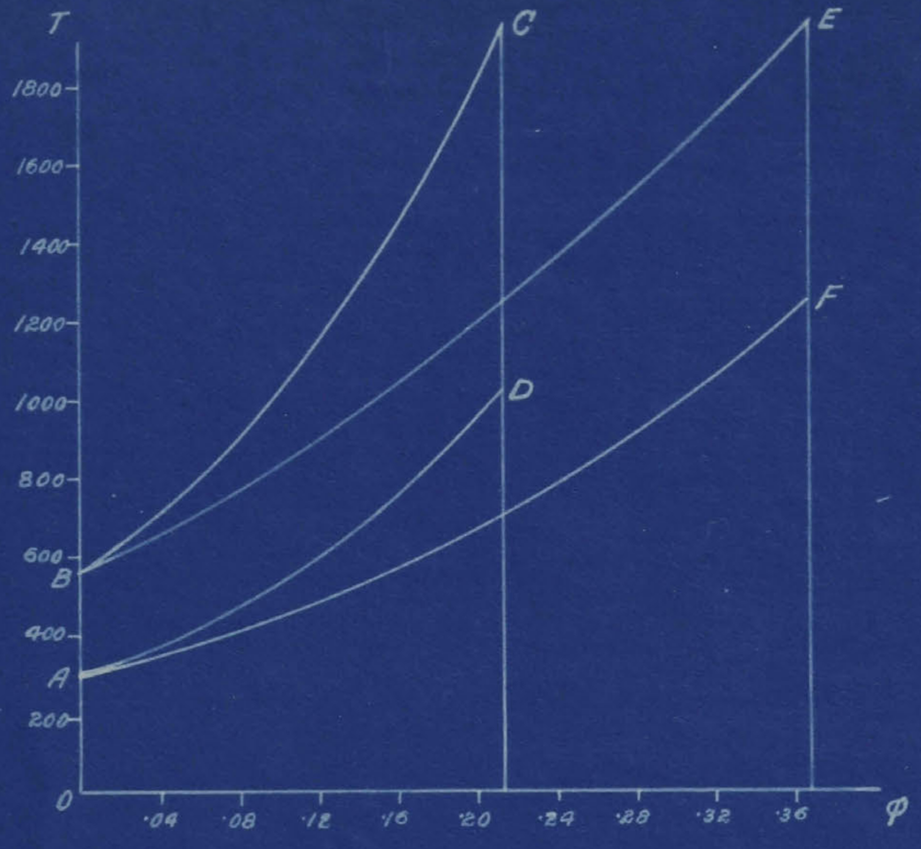
Figs. 11.


P
Atm.
50
40
30
20
10
0
V
$t_s = 50°$
$t_s = 100°$
$t_s = 150°$
T
2000
1000
0
-273
φ
$t_s = 50°$
$t_s = 100°$
$t_s = 150°$

Figs. 12.


p
Atm.
50
40
30
20
10
0
Q = 924
Q = 800
Q = 705
V
T
2000
1000
0
-273
Q = 924
Q = 800
Q = 705
t_s
50°
100°
150°
φ

Figs. 13.


T
1800
1600
1400
1200
1000
800
600
B
400
A
200
0
·04
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·12
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·20
·24
·28
·32
·36
φ
C
E
D
F

Fig. 14.

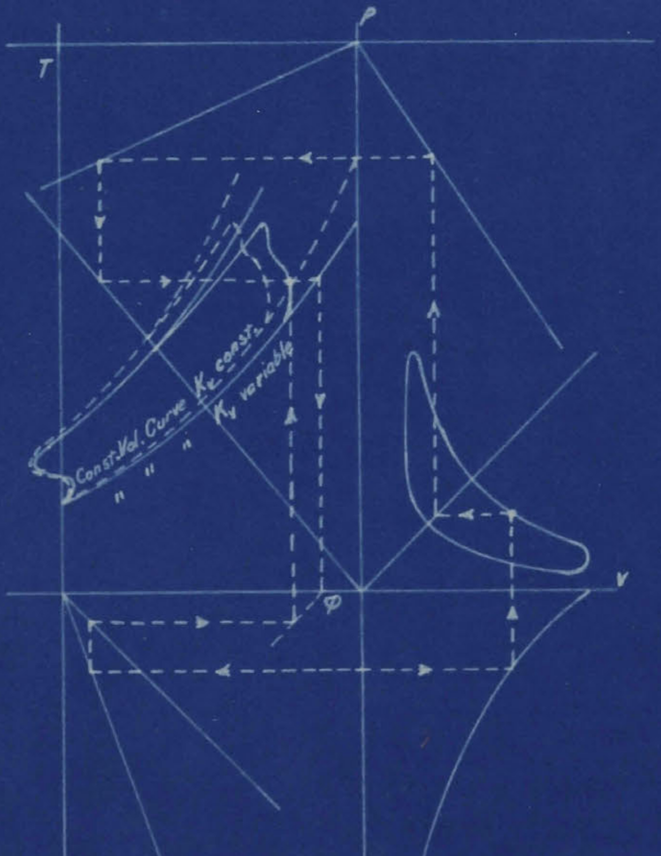
P
T
Const. Vol. Curve K_v const.
" " " K_v variable
φ
V

Fig. 15.

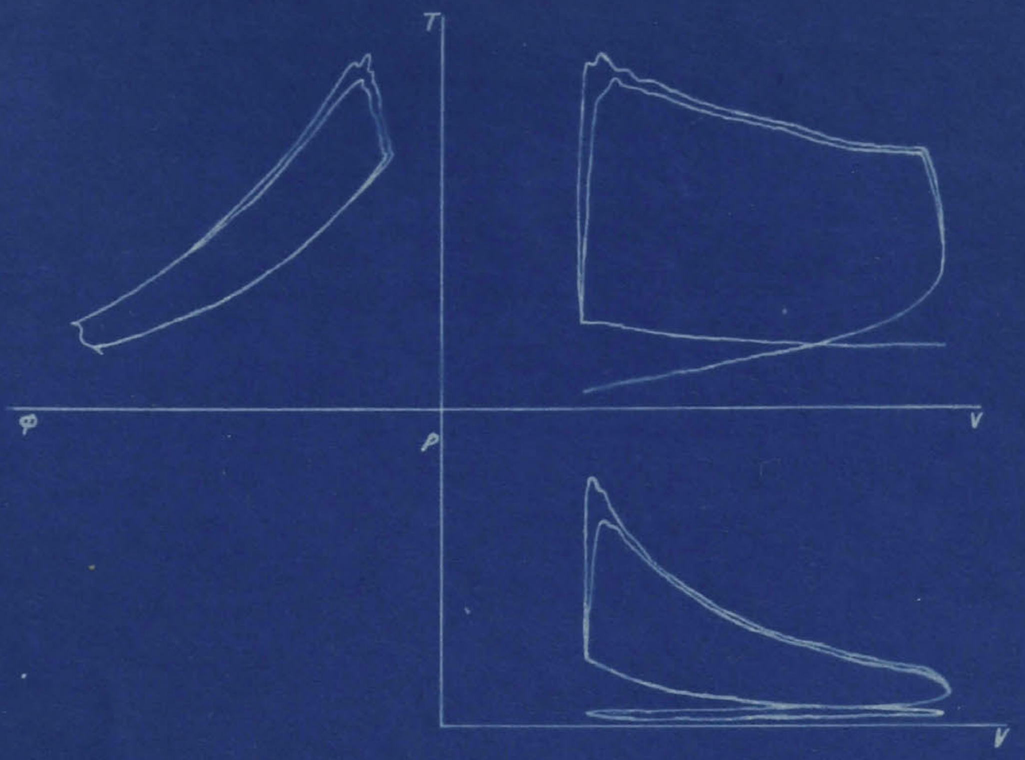

Fig. 16.

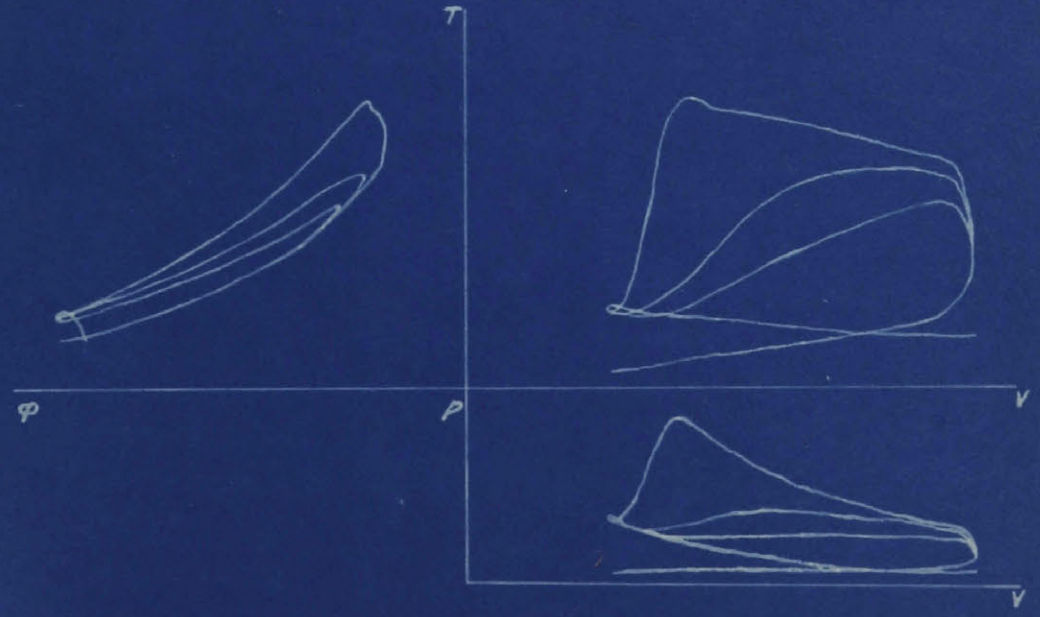

Fig. 17.