Oligopolistic Electricity Markets under Cap-and-trade and Carbon Tax

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To my father: I owe you everything I am and anything I will ever be...

To my mother: Your love makes me the man I am and the man I will always be...

To my brother: On your wedding I cried because it was the last time you and me were going to be...

To my older sister: I am truly proud of you and I hope that you always see...

To my baby sister: I love you so much and my baby girl you will always be...

To my angel: It was always going to be you and no one but you... forever and ever it's you and me...

Dedicated to...

My loving parents,

My Brother, his wife and their two adorable boys: Tarek and Jad

My two sisters,

My soul mate,

and

K. G. A. S. A.

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> ***"I could have never asked for better parents, but I hope my parents never ask for a better son;"***

يا رب كيف أشكرك ..... و شكرك نعمة تستحق الشكر

Abstract

Global warming is one of the most alarming phenomena facing our planet today. There is a general consensus among scientists that in order to slow down the heating of the Earth's oceans and atmosphere, human-induced greenhouse gases (GHG) should be regulated. Being a major GHG producer, the electricity industry's emissions should form part of any global emission regulation initiative.

The attainment of this goal is however subject to several complicating factors: (i) electricity is an essential commodity to the welfare of all modern societies, thus raising electricity prices is not an acceptable solution on its own, since such a step would have significant adverse effects on the demand-side economy; (ii) in parallel, any emission regulation scheme that significantly influences profits would have an adverse effect on the business of generating power; (iii) electricity cannot be economically stored in large quantities and, as a result, demand and generation must be continuously balanced; (iv) today, generation and demand are no longer balanced through a rate-regulated monopoly but through an hourly electricity market to which power producing companies (Gencos) and load-serving entities respectively submit supply offers and demand bids; (v) Since existing electricity markets are oligopolistic (few competing entities each being a price-maker), Gencos exercise market power by gaming through their offers. The addition of emission regulating schemes offers Gencos new opportunities to game on the basis of how polluting they are. This complicates the market-clearing process and renders its outcome more difficult to predict.

In this thesis, we design and test two emission regulation schemes in the context of electricity markets, one based on cap-and-trade (CAT) and the other on carbon tax (CTX). Furthermore, we compare the schemes' ability to meet emission regulation goals subject to the aforementioned complicating factors.

Under CAT, we first re-design the typical hourly oligopolistic electricity market into a joint electricity and emission permits trading market. This dual market includes Genco gaming that takes advantage of hourly production costs, Genco emission intensities, self-allocated Genco emission caps (permits), emission permits trading, and demand elasticity. In addition, under CTX, we re-design the hourly electricity market into one that includes a carbon tax penalty. Here, Genco gaming takes advantage of hourly production costs, Genco emission intensities, demand elasticity and the carbon tax penalty.

Furthermore, under CAT, we develop two novel approaches to allocate the commitment interval electricity sector cap among Gencos: (a) Gencos receive permits for free from a social planner (SP). This is done on the basis of maximizing social-welfare (SW) over the commitment interval while accounting for the effects that these permits have on the hourly operation of the electricity market; (b) Gencos receive permits based on an auction where, in addition to maximizing SW, the SP accepts bids from Gencos to influence the permit allocation, and where Gencos pay for their allocations at the auction clearing price.

In contrast, under the CTX scheme, there is no explicit emissions cap. Rather, the desired cap is attained implicitly through an hourly tax penalty, the parameters of which are computed to maximize SW over the commitment interval. This computation accounts for the effect of the resulting hourly tax rate on Genco gaming and on the ensuing outcome of the oligopolistic market-clearing process.

Finally, the thesis provides a thorough analytic and numerical comparison of both CAT and CTX under different scenarios. Groundwork results suggest that both schemes have significantly varying effects on market power and profits, effects that contest some preconceived ideas about both regulation schemes. These results suggest that:

Under CAT, the proposed SW auction to allocate the sector cap among the competing Gencos seems to be the preferred allocation scheme over free allocation schemes based on grand-fathering and SW maximization. This is so, because the SW auction eases some of the inherent drawbacks of a cap-and-trade system, in particular, by sending appropriate economic signals to invest in emission reduction. However, the auction does introduce new uncertainties into the prediction of the market equilibrium, not only due to the permits trading aspect of cap-and-trade and the hourly self-allocation but due to uncertainty in the permits auction bidding strategies.

In contrast, under CTX, the proposed carbon tax structure not only produces the desired economic signals to invest in cleaner technologies but is subject to fewer sources of uncertainty when predicting the market outcome. Finally, when compared to an equivalent cap-and-trade scheme under the proposed auction, a carbon tax leads to higher profits for producers as well as to a higher consumer surplus.

Thus, pending further studies, we conclude that the carbon tax structure proposed in this thesis is the recommended emission regulation scheme for an oligopolistic electricity market.

Resume

Le réchauffement planétaire est un des phénomènes les plus alarmants des nos jours. Il existe un consensus entre les scientifiques qu'afin de ralentir le réchauffement des océans et de l'atmosphère, les émissions de gaz à effet de serre anthropique (GES) devraient être réglementées. Étant le secteur de l'électricité un grand producteur de GES, ses émissions devraient faire partie de toute initiative globale de réglementation.

La réalisation de cet objectif est toutefois sujette à plusieurs facteurs de caractère compliquant : (i) l'électricité est un produit essentiel pour le bien-être de toutes les sociétés modernes ; augmentant ainsi les prix de l'électricité n'est pas par elle seule une solution acceptable, car une telle démarche aurait des effets négatifs importants sur l'économie des consommateurs ; (ii) en parallèle, tout régime de réglementation des émissions qui influence considérablement les bénéfices aurait un effet négatif sur les entreprises de génération d'énergie électrique; (iii) ne pouvant pas facilement être stockée en grandes quantités, l'électricité requiert que la demande et la génération soient équilibrés en tout temps; (iv) aujourd'hui, la génération et la demande ne sont plus équilibrées par un monopole avec des taux réglementés, mais plutôt par un marché de l'électricité horaire auquel les entreprises de génération (Gencos) et les entités représentant les consommateurs respectivement soumettent des offres d'approvisionnement et de soumissions de demande ; (v) étant donné que les marchés de l'électricité existants sont oligopolistiques (où quelques entités en concurrence partagent un monopole), les Gencos exercent un pouvoir de marché par le biais stratégique de leurs offres. L'ajout des régimes de réglementation des émissions offre de nouvelles possibilités aux Gencos de manipuler les prix sur la base de leurs niveaux d'émissions. Cela complique le processus de fermeture du marché et rend son équilibre plus difficile à prévoir.

Dans cette thèse, nous concevons et vérifions deux régimes de réglementation des émissions dans le contexte des marchés de l'électricité, un fondé sur le plafonnement et l'échange (CAT) et l'autre sur la taxe carbone (CTX). En outre, nous comparons la capacité des deux régimes pour atteindre les objectifs de règlementation d'émissions sujets aux facteurs de difficulté susmentionnés.

Sous CAT, tout d'abord nous remplaçons la conception d'un marché de l'électricité oligopolistique horaire typique par un marché conjoint d'électricité et d'échanges de permis d'émission. Ce double marché tient compte des manipulations du marché de la part des Gencos (Genco gaming) pour tirer parti des coûts de production horaires, des intensités d'émissions des générateurs, des allocations horaires de permis auto- réparties par les Gencos, du droit d'échange de permis avec le marché externe et de l'élasticité de la demande. En outre, sous CTX, nous remplaçons le marché de l'électricité horaire par une version qui inclut une pénalité basée sur une taxe carbone. Ici, la manipulation du marché par les Gencos tire parti des coûts de production horaires, des intensités d'émission des générateurs, de l'élasticité de la demande et de la pénalité imposée par la taxe carbone.

En outre, sous CAT, nous développons deux nouvelles approches pour partager parmi les Gencos le plafond sur les émissions imposé sur le secteur de l'électricité pendant l'intervalle d'engagement spécifié: (a) Les Gencos reçoivent gratuitement les permis d'un planificateur social (SP). Ceci est basé sur la maximisation du bien être sociale (SW) pour l'intervalle d'engagement entier, tout en comptabilisant les effets de ces permis sur le fonctionnement horaire du marché double de l'électricité et des permis ; (b) Les Gencos sont accordés de permis sur la base d'une vente aux enchères où, en plus d'optimisant le SW, le SP accepte des soumissions des Gencos à fin d'influencer l'attribution des permis, et où les Gencos payent pour leurs permis au prix de compensation de la vente aux enchères.

En revanche, au titre du régime CTX, il n'y a aucun plafond d'émissions explicite. Plutôt, le plafond souhaité est atteint implicitement à travers une pénalité fiscale horaire, dont les paramètres sont calculés en maximisant le SW sur l'intervalle d'engagement. Ce calcul représente l'effet du taux de carbon horaire qui en résulte de la maximisation sur la manipulation strategique du marché par les Gencos et sur l'équilibre correspondant du marché oligopolistique.

Enfin, la thèse offre une comparaison analytique et numérique approfondie de CAT et CTX sous différents scénarios. Les résultats de base suggèrent que les deux approches ont des sensiblement différents effets sur le marché et les profits, des effets qui défient certaines idées préconçues sur le règlement d'émissions. Ces résultats suggèrent que :

Sous CAT, la vente aux enchères SW proposé dans cette thèse pour partager le plafond du secteur parmi les Gencos semble être le régime préférée par rapport aux régimes de répartition gratuits basés sur la maximisation du SW ainsi que sur la clause d'antériorité (grand-fathering). Il en est ainsi parce que la vente aux enchères SW facilite certains des inconvénients inhérents d'un système de plafonnement et échange, en particulier, en envoyant des signaux économiques appropriées pour investir dans la réduction des émissions. Toutefois, la vente aux enchères introduit des nouvelles incertitudes dans la prédiction de l'équilibre du marché, non seulement en raison de l'échange de permis présent dans le régime plafonnement et échange et de l'auto-distribution horaire des permis accordés à l'intervalle d'engagement, mais en raison de l'incertitude dans les stratégies de soumission des Gencos dans la vente aux enchères.

En revanche, sous CTX, la structure de taxe carbone proposée non seulement produit les signaux économiques souhaités pour investir dans des technologies plus propres mais cette approche est susceptible à moins de sources d'incertitude lorsque qu’il s’agit de prédire l'issue du marché. Enfin, comparé à un régime de plafonnement et échange équivalent dans le cadre de la vente aux enchères proposée, une taxe carbone entraîne des bénéfices et un surplus plus élevés pour les producteurs et les consommateurs respectivement.

Ainsi, dans l'attente d'autres études, nous concluons que la structure de taxe carbone développée dans cette thèse est le régime de réglementation d'émissions recommandée pour un marché de l'électricité oligopolistique.

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Acronyms

Genco:	Power Generating Company
GHG:	Greenhouse gases
SP:	Social planner
ISO:	Independent system operator
IC:	Incremental cost
SW:	Social-welfare
MPEC:	Mathematical Problem with Equilibrium Constraints
SCED:	Security-Constrained Economic Dispatch
NE:	Nash equilibrium
SF:	Supply function
GF:	Grand-fathering
CAP:	Cap-and-trade
CTX:	Carbon tax

1 Chapter 1 Introduction

*"Life is a collection of me*morable moments we experience or inflict on others; unique people choose to struggle to create unique memories and live unique lives;"

1.1 Thesis Motivation

Global warming is an issue that has been heavily scrutinized, discussed and debated over the past several years. The major debate today is not over the consequences of global warming but over how to slow it down. There is a general consensus that global warming is a human induced phenomenon whose consequences range from adverse to catastrophic. However, researchers, experts and politicians are still a long way from uniting over the optimum way to decelerate this phenomenon.

But what is global warming?

Global warming of the Earth's surface and lower atmosphere is caused by excess levels of the sun's energy being trapped in the Earth's atmosphere.

There is a natural blanket around the earth that traps the sun's heat and keeps the planet approximately 30 degrees C warmer than it would be otherwise. This blanket is the result of the so-called Greenhouse gases (GHGs) [1], which include carbon dioxide (CO_2), Methane (CH_4), Nitrous oxide (N_2O), Sulphur Hexafluoride (SF_6), Hydrofluorocarbons (HFCs), and Perfluorocarbons (PFCs).

The usual levels of GHGs are supplemented by man-made emissions of carbon dioxide from the burning of coal, oil, and natural gas, from methane and nitrous oxide produced by farming activities and from deforestation.

Due to industrialization and the growth of economies across the globe the levels of GHGs in the atmosphere have been rising at alarming rates. As a result, the GHG blanket around the Earth is thickening and the rate at which the sun's heat escapes into outer space is decreasing, all of which contributes to accelerated global warming.

<u>What are the numbers?</u>

Carbon dioxide which is mainly associated with the burning of fossil fuel makes up almost 60 % of the levels of GHG in the atmosphere. Carbon dioxide levels have been rising by almost 10 % every 20 years. Moreover, the Earth's surface average temperature has risen by 0.74° C over the last century and is expected to increase by almost 4° C by the year 2100 under current emission trends [1].

<u>How to reduce global GHG emissions?</u>

Aside from government expenditure on public awareness of the relation between GHG emissions and global warming and on educating the public on means to reduce GHG emissions, there is an agreement that large-scale emission regulation schemes based on market instruments or economic incentives are the only viable alternatives. These instruments are of two main types: (i) financial; to stimulate emission reduction; and (ii) caps that limit national or international levels of GHG emissions.

<u>What is special about the electricity industry?</u>

In most countries, the electricity sector is the largest single producer of GHG emissions. Thus, an effective market-based emission regulation scheme should successfully target emissions produced from the electricity sector. This goal is however complicated by the fact that electricity is an essential social commodity whose consumption is relatively inflexible with respect to its price. In addition, electricity has the characteristic that it cannot be stored in a cost-effective manner. Thus, to avoid adverse effects on social-welfare, especially in fragile economies, emission regulation schemes should not have a disrupting influence on the economics of electricity.

What is the challenge?

In most parts of the world and over the past several years, the electricity sector has been restructured, a term that sometimes is referred to as deregulated or liberalized [2]. Basically, what this means is that, in contrast with a vertically-integrated monopoly that generates, transmits and distributes electricity at a regulated price, in a deregulated environment privately-owned power generating companies (Gencos) compete to sell electricity for a price set by demand and supply. The result is that electricity is now a commodity traded in so-called electricity markets [3]. These multi-faceted markets are complex in their structure and operation and are susceptible to price manipulation and market power by profit-driven power generating companies [4]. Because of this complexity, the impact of market-based emission regulation schemes is difficult to model and predict. This is the principal challenge that motivated this thesis.

1.2 Thesis Objective and Scope

This thesis models and investigates two market-based emission regulation schemes applied to the electricity sector, namely, cap-and-trade (CAT) and a carbon tax (CTX). We model these schemes in the context of deregulated electricity markets, and analyze their implications on the medium-to-short-term operation of such markets. The electricity markets we consider are oligopolistic, in other words, where Gencos can game by manipulating their supply offers to increase electricity prices along with their profits. Our analysis therefore also assesses how the two emission regulation schemes affect the Gencos' gaming strategies and market power.

This thesis makes two basic assumptions: (i) The transmission network has no congested lines, which results in one single electricity system marginal price as opposed to a locational marginal price at each substation; (ii) Investments in GHG-reduction technology do not affect generation costs and emission intensities within the relatively short time horizons of our studies, which range from 24 hours to one year.

The work in this thesis is conducted and presented from several perspectives: (i) The consumer; (ii) The producers or Gencos; (iii) Society as a whole as viewed by a social planner (SP). The SP can be a government entity such as a regulating body that has complete information of the system parameters and is able to model and analyze Genco gaming strategies [5].

1.3 Literature Review

1.3.1 Cap-and-trade

The first method we examine to regulate emissions in the electricity sector is cap-and-trade (CAT). Under this method, a cap is set on the level of emissions in the electricity sector over a commitment interval, typically a year. The cap is then allocated among the Gencos either for free, based on a benchmark such as historical emissions (grand-fathering), or sold by the government at an auction. An individual Genco cap can be viewed as the number of emission permits allocated to the Genco at the beginning of the commitment interval. Throughout this interval however, the Genco can buy additional permits or sell excess permits provided that at the end of the interval, the overall permits used are equal to the emissions produced.

One way to study the effects of CAT on the electricity sector, often seen in the literature, is to assume that the level of emission permits traded by the sector with the external permits market does not affect the permits price, γ_0, which is assumed to remain constant over the commitment interval. We refer to this approach as the *constant emission permits price methodology*.

Under this methodology, the hourly cost of each Genco is augmented by an amount equal to γ_0 multiplied by the Genco's hourly emissions, an extra cost that is present whether a Genco has to buy permits from the external market or receives these permits for free. The latter statement may not be too obvious but it can be explained by considering as an opportunity cost the revenue a Genco could make by not producing electricity and instead selling its freely obtained permits at the price γ_0. Thus, since the generation cost increases, so do the electricity price and the Gencos' profits coupled with a decrease in the load's surplus. Moreover, since the increased generation cost of a Genco depends on its emission intensity, high polluting power plants become more expensive than low polluting ones, which results in a shift in generation towards less polluting

Gencos and in a reduction in the total emissions produced by the sector. The degree of this shift is clearly dependent on the permits price γ_0.

In contrast to a constant emission permits price, the methodology followed in this thesis accounts for the elasticity of the permits price in terms of the permits traded by the electricity sector. A model that considers the electricity sector as an emission permits price-maker, as opposed to a price-taker, is more realistic since it can account for the Gencos' strategic positions in both the electricity and permits markets as well as the interaction between these two markets.

1.3.1.1 The effects of emission permits trading on the electricity sector

1.3.1.1.1 Perfect electricity markets

The following work is based on the constant emission permits price methodology:

a) Palmer, Burtraw and Shih in [6] study the implications of different federal proposals to set emission caps on the U.S. electricity sector in different regions over different commitment intervals. The paper simulates the operation of several U.S. regional electricity markets and considers interregional electricity trade. It also considers a detailed composition of technologies and fuel used to supply electricity at each hour over the commitment interval, for four different demand levels (super-peak, peak, shoulder, and base load). The paper computes how generation and the composition of fuel used, along with electricity prices, will change with each of the proposals considered. The paper concludes that the cost of setting emission caps on the electricity sector is lower than the benefits of reducing emissions;

b) In [7], the authors analyze the effects on the Finnish electricity sector of a cap-and-trade system applied to the Nordic region. The paper concludes that Gencos with low emission intensities will benefit significantly from the introduction of a cap-and-trade system, which will encourage investments in low polluting technologies. The authors argue that the increase in electricity price due to the pass-through of the permits price not only shifts production to low polluting Gencos but significantly increases their profit;

c) Bode [8] uses the constant emission permits price methodology to apply CAT to a hypothetical electricity market, obtaining similar results to [7]. Bode concludes that, because of the increase in electricity price under CAT, all Gencos, including high polluting ones, increase their profits significantly from business-as-usual (BAU);

d) Rosnes [9] studies the effect of the added cost of permits price due to CAT on the self-scheduling decision of a Genco with a thermal power plant. The paper considers the start-up costs and not just the variable generation cost. The paper concludes that the introduction of CAT reduces the effect of the fixed start-up cost on a Genco's self-scheduling decision because it increases the marginal variable Genco's generation cost.

Besides the constant emission permits price methodology, the following work is pertinent:

In [10], the authors study the effects of a cap-and-trade scheme applied to the provinces of Ontario, Quebec and New Brunswick where permits are grand-fathered to individual Gencos and there is free electricity trade with the states of New York and New England. In this work, the authors do not consider the opportunity cost of free permits given to Gencos, and thus the increase in each Genco's cost is equal to the fixed permits price multiplied by the amount of permits the Gencos buy at the external market. To do so, the paper assumes that the social planner will provide each Genco with 85% of the amount of permits it will need to cover its emissions. Thus, the marginal generation cost of each Genco will increase by 15% of the price of permits (this is equivalent to an increase in fuel prices). The price of permits is assumed fixed and known. Not surprisingly, the paper argues that such a scheme has no major effect on electricity generation and trade in the regions considered.

In [11], the authors study the effect of CAT on the electricity sectors in Nordic countries through the use of a partial equilibrium model. Aside from electricity production, the model includes other energy-consuming sectors, such as heating. The model is simulated with different emission caps over all the sectors included and permits-trading is permitted only within these sectors. Although the authors argue that their model considers detailed variations in electricity demand, they still assume a fixed constant emission permits price over the commitment interval equal to the marginal abatement cost of emission reduction in the capped regions. This is the dollar amount it costs all capped sectors to reduce emissions by an extra ton over the commitment interval at the specified emissions target (shadow price). By applying the Kyoto Protocol emission targets, in contrast to [10], the paper concludes that cap-and-trade will affect inter-regional electricity trading in a significant way.

Hindsberger et al [12] model an electricity market in the Baltic Sea region subject to CAT along with a green certificates policy that promotes the use of renewable sources of energy[1]. The authors do not consider an external permits market but a closed one consisting of the electricity sectors in the countries considered. Thus, to model permits trading, caps are set on the electricity sector, and emission permit trading is modeled among the countries' electricity sectors. The permits price is an endogenous variable depending on the total

[1] Green certificates are awarded to Gencos for power they generate through renewable sources and each Genco has a cap on the minimum number of certificates it owns at the end of a commitment interval. Excess certificates can be sold at a certificates market, which are bought by Gencos that did not meet their minimum cap.

emissions cap set on the different electricity sectors included[2]. This is the price that each Genco pays for the emissions it produces at the end of the commitment interval beyond the permits it receives for free. The authors conclude that because of the shift in generation from high to low polluting Gencos (as a result of emission caps), regardless of the amount of free permits allocated, low polluting Gencos will benefit the most from the introduction of the cap-and-trade system thus encouraging investments in such technologies.

In [13], the authors study the negative effects on energy intensive industries of a CAT system applied to the Benelux electricity sector, also proposing a novel pricing method to alleviate these effects. The CAT system is modeled by applying a cap on the total level of emissions produced by the electricity sectors in the countries considered over the commitment interval. The shadow price of the emissions constraint is the permits price in the model, and the authors use this price to calculate the total emission costs for each individual Genco, with no permits allocated for free.

1.3.1.1.2 Imperfect electricity markets

Sijm, Neuhoff and Chen [15] use the COMPETES model to study the effect of the generation mix on the pass-through cost of emission permits onto electricity prices under the constant emission permits price methodology in an oligopolistic electricity market. The COMPETES model computes the Cournot Nash equilibrium of the Gencos' supply offers after accounting for the price of permits γ_0 in each Genco's marginal generation cost as per the constant

[2] Note that this endogenous variable is the shadow price of the total emissions cap and is the marginal emission abatement cost discussed in [11]. The Balmorel project [14] models in details the electricity market in the Baltic Sea region and can be used to derive a similar aggregate emission abatement costs for the region. Note that a marginal abatement cost could be computed for each country, and if permits trading is feely permitted among countries, then the permits price will be equal to that of the country with the lowest marginal abatement cost.

emission permits price methodology. Results show that the increase in electricity price in markets with a large dependence on coal-fired power plants (high polluting), for instance in Germany, is higher than the increase in markets with a higher concentration of nuclear power plants (low polluting), such as France.

Chen, Sijm, Hobbs and Lise [16] use the COMPETES model to study the effect of the degree of competition (degree of oligopoly) in the electricity market on the pass-through of the cost of emission permits onto the price of electricity. Results indicate that the higher the degree of market competitiveness (less market power) the higher is this pass-through cost. They deduce that gaming reduces the effect of CAT on the electricity prices. Results also indicate that under free allocation of permits, all Gencos enjoy windfall profits due to the increase in electricity price. Finally, results show that under the constant emission permits price methodology, even if permits are auctioned to Gencos, that is, the Gencos are charged for their allocated permits at the permits price (more on that later), all Gencos will still make higher profits under CAT than under BAU. Lise, Sijm and Hobbs derive similar results in a more recent study in [17] by simulating the COMPETES model on 20 European countries.

Sijm summarizes the aforementioned results and explain them qualitatively in [18] and [19].

Bonacina and Gulli in [20] study the impact of CAT in an electricity market with one dominant gaming Genco (leader) and other Gencos that act as followers and are price takers. The authors analytically compute the strategic Cournot behavior of the dominant firm under constant emission permits price methodology. The authors derive similar results to [16] which indicate that the pass-through cost of the permits price under gaming is lower than under perfect market conditions.

Lise et al in [21] use the EMILIE model to study the effects of CAT systems on the dispatch of oligopolistic electricity markets. The model uses Cournot Nash to compute the Gencos' gaming strategies with total emissions cap set on the electricity sector. The permit price is then the shadow price of the emissions constraint which is then used to calculate the cost incurred by each Genco to purchase permits to account for its emissions. The effect of gaming in the electricity market is to increase the electricity price and reduce the permits price. The permits price decrease under such a model because the increase in marginal supply offers due to gaming results in a decrease in the elastic demand which in turns results in lower generation levels and thus lower emissions.

In [22] Kemfert also uses the EMILIE model to do a similar study to that in [21] and conclude that under Cournot gaming where the Gencos game in the electricity market only, low polluting but expensive Gencos benefit more than high polluting cheaper Gencos. This is because of the large shift of generation dispatch from high polluting to low polluting Gencos under gaming in an oligopolistic market under emission constraints.

In this thesis we use Cournot Nash equilibrium to study gaming strategies in oligopolistic electricity markets, an approach similar to that used in The COMPETES and EMILIE models. However, in our analysis, the electricity sector is a price maker in the external permits markets, thus we model Genco gaming in both electricity and permits markets.

Linares et al in [23] present a generation expansion model in the Spanish electricity sector under a cap-and-trade system. The electricity market considered is an oligopolistic one and the sector is a price maker in the external permits market. The authors study the hourly Gencos' gaming strategies through a Cournot Nash equilibrium based on a conjectural variation approach. In traditional Cournot-based models, when computing a Genco's optimum generation dispatch, the effect of its dispatch on the electricity price is defined through the elasticity of demand. Under a conjectural variation approach, a gaming Genco considers its residual demand to account for the effect of an incremental change in its output on the electricity price when computing its optimum MW output. The residual demand of the gaming Genco is the system demand minus the strategic supply offers of other Gencos, conjectured by the gaming Genco. Although the authors assume that the electricity sector is a price maker in the permits market (the assumption we make in this thesis), they do not consider the effect of permits trading on the hourly electricity supply offers of competing Gencos. This is because in [23] permits trading is assumed to occur at super periods (consisting of several hours) where the total permits allocated to each Genco is defined by the social planner. Results show that the electricity price increases significantly by the introduction of CAT and all Gencos (including low polluting ones) make windfall profits.

1.3.1.2 Allocation of the commitment interval electricity sector cap among Gencos

1.3.1.2.1 Free permit allocation among Gencos

Recall that under the constant emission permits price methodology, the effect of CAT on the Gencos' supply offers is seen only through the permits price γ_0. Thus, the free permits Gencos receive over a commitment interval affect neither the Gencos' supply offers nor the electricity market clearing. Furthermore this permit allocation does not affect the load surplus. The only effect of free permits is to increase a Genco's profit by reducing the number of permits the Genco has to buy to account for its emissions over the commitment interval. If a Genco's emissions are below the amount of free permits it receives, then these permits are sold at the external market at the permits price, with no effect on the Gencos' offered marginal generation costs. In contrast, in this thesis, by modeling gaming in both electricity and permits markets, Gencos benefit from emission permits they receive for free. Thus in our work, the hourly strategic supply offers of Gencos and the ensuing market-clearing are influenced by permits trading and by the allocation of free permits among Gencos.

Under the constant emission permits price methodology, the following authors studied the effect of permit allocation among Gencos:

a) In order to minimize the effect of CAT on electricity consumers in a perfect market, Paul et al propose in [24] allocating permits to load-serving entities. The authors argue that such entities can sell the free permits they receive at the external permits market, which will, at least partially, offset the increase in electricity price due to the cost of permits on the production side of electricity.

b) Neuhoff et al in [25] study free permit allocation methods based on generation levels in the previous commitment interval, instead of grand-fathering based on historical emissions. They find that such an allocation produces lower electricity prices than grand-fathering based on long-term historical emissions.

c) In [26] Martinez and Neuhoff theoretically highlight the importance of auctioning permits to reduce the Gencos' windfall profits and provide extra revenue for the government. In [27], the authors suggest that free allocation distorts investment in power plants and therefore advocate the use of an auction.

Additional work on free permits allocation includes that of Bohringer et al in [28] and Mackenzie et al in [29] that discuss optimal free allocation schemes in a general context and not specifically within the electricity sector. Bohringer et al argue that if the producers of a certain commodity are price makers of emission permits, then to optimize social-welfare, the commitment interval permits should be allocated on a periodic basis in a manner that depends on periodic emissions. Mackenzie et al suggests that by accounting for an external factor in the allocation of permits among Gencos, a better social-welfare can be achieved. We note that in this thesis, we examine a free permit allocation method among Gencos that maximizes social-welfare and considers hourly gaming in both electricity and permits markets, while accounting for a strategic or heuristic temporal self-allocation of these permits.

1.3.1.2.2 Auction-based allocation of permits among Gencos

In contrast to rewarding Gencos with free permits, current legislatures are suggesting allocating such permits via an auction at the beginning of the commitment interval. Under an auction-based allocation, Gencos bid to acquire permits and are charged for these permits at the auction price.

Researchers have studied the effects of an auction-based permit allocation among Gencos under constant emission permits price methodology. To do so, they assume that the auction price is equal to the constant permits price γ_0 of the external market, and the cost of buying permits at γ_0 is deducted from each Genco's profit [30]. Such a model characterizes a permits auction by simply charging the Gencos for emission permits at the constant price, thus reducing their profits. The point here is that instead of Gencos enjoying increased profits through the increase in electricity price as a result of the CAT system, the auction money will go to the government. An auction for emission permits over a commitment interval, however, introduces additional sources of market power and market uncertainty that cannot be accounted for under the aforementioned market model.

In this thesis, based on the methodology we proposed in the previous subsection to model cap-and-trade, we develop an auction model for emission permits that maximizes social-welfare while accounting for the effects of these permits on the hourly operation of an oligopolistic market. Moreover, the auction accepts bids from Gencos to acquire more permits which raise the price of permits. By receiving more permits from the auction the Gencos can use these permits to improve their hourly strategic positions in the electricity market and to increase their profits as long as the auction price is not excessively high.

Burtraw et al in [31] - [33] study the effects of the initial cap allocation among Gencos on the operation of perfect electricity markets in the United States. They base their study on the constant emission permits price methodology and compare free allocation based on grand-fathering (GF) to an auction-based allocation method. The authors argue that both GF and auctioning of permits have identical effects on the Gencos' supply offers and on prices; however auctioning is preferred because the government can then channel some of the total costs of CAT back to the consumers.

In [34], Burtraw et al, also argue in favor of an auction-based permit allocation as it reduces the Gencos' opportunity cost of free permits under CAT based on the constant emission permits price methodology. Thus, they find that auctioning permits leads to a price of electricity being close to the actual system generation cost.

Burtraw et al in [35] consider an auction-based allocation of emission permits among Gencos and calculates the actual cost that Gencos would incur paying for these permits based on the constant emission permits price methodology under perfect electricity market conditions. The authors then compute the proportion of emission permits that has to be allocated for free so as to compensate Gencos for the cost of auctioned permits.

In [36], Crampton et al argue in favor of auctioning permits as opposed to free permit allocation, mainly because of the revenue it raises which can be used to invest in emission reduction. The authors discuss different types of auctions in a general context not specific to the electricity sector and recommend an ascending clock auction. Under such an auction, the price of permits is fixed and participants submit the number of permits they wish to buy at that price. If the demand for permits exceeds the supply, then the price of the permits is increased. The process is repeated until equilibrium is reached.

Holt et al in [37] conduct a qualitative study of different auction designs and discuss general design aspects of an emission permits auction. In contrast to [36], Holt et al recommend a single-round, sealed bid, uniform price permits auction that they claim is ideal under current electricity markets. This is of the same format as the auction we propose in this thesis, however, in our auction, in addition to maximizing the Gencos' bids, permits are allocated based on maximizing SW taking into account the effect of permits on the hourly operation of electricity and permits markets.

1.3.2 Carbon tax

The second emission regulation scheme we study in this thesis is a carbon tax that penalizes Gencos for the emissions they produce at a rate set by the social planner.

1.3.2.1 Perfect electricity markets

Vorspools et al in [38] apply a fixed tax rate on the Gencos' emissions and compare the results to a fixed tax rate on the primary fuel used by Gencos. The authors use a bottom-up model, PROMIX, to simulate the effects of different levels of the aforementioned taxes on the generation dispatch of the Belgium power system. The paper concludes that an emissions tax is more effective than a tax on primary fuel, as the former induces a shift in generation from high polluting Gencos to low polluting ones. However, the authors find that beyond a specific emissions tax rate, there is no more shift in generation from high to low polluting Gencos.

In [39], Vorspools et al study the effects of a fixed carbon tax on generation dispatch and electricity trading between eight interconnected European countries. They conclude that such a tax leads to a net decrease in total emissions levels and also affect cross-country electricity trading as different countries have a varying generation mix.

The authors in [40] study the effects of a carbon tax with a fixed rate together with a white certificate policy[3] on the operation of electricity markets. The authors analyze how the electricity market dispatches under different tax rates and white certificate prices.

1.3.2.2 Imperfect electricity markets

The authors in [41] derive the Cournot Nash equilibrium gaming strategies of Gencos under a carbon tax with a fixed rate and under transmission constraints. The effect of the tax rate in this model is an increase in the incremental generation cost of each Genco based on its emission intensity. The authors also impose limits on total emissions produced by the sector but do not attempt to compute the optimum tax parameter that would achieve these limits. The model is implemented on a three Genco, 6 bus example, and results show that while the prices at all buses increase under the tax rate, the profit of each Genco decreases regardless of the Genco's emission intensity.

In this thesis we also use Cournot Nash equilibrium to study Genco gaming strategies under carbon tax, however in the carbon structure we examine, the ensuing tax rate is a function of the hourly emissions produced by the electricity sector. Moreover, the tax rate parameters affect the Genco hourly supply offers which allows low polluting Gencos to benefit from the carbon tax system. Moreover, we develop a method to set the tax parameters such that

[3] A white certificate policy sets energy saving or efficiency targets on producers or consumers of electricity. Fulfilling such targets is rewarded by white certificates that can be sold at a market. Entities that fail to reach their efficiency targets have to buy certificates to account for this failure.

social-welfare is maximized and the electricity sector's emission targets over a commitment interval are met.

In [42], the authors model the equilibrium outcome of what they refer to as the "electric power supply chain network" as a transportation network equilibrium model. The authors consider Gencos owning several units and load-serving entities (that they refer to as power suppliers) and demand at different locations. Under a fixed carbon tax rate, a Genco's decision over the amount of MW power to sell to the load-serving entity is optimized as well as the load-serving entity's decision to sell power to the demand. Thus, the paper optimizes physical bilateral contracts under a carbon tax, however it does not model gaming with supply offers to a power pool. In [43], the authors use the same model as in [42] to compute the tax rate that would meet emission targets under bilateral contracts that optimize each Genco's profit.

1.3.3 Comparing cap-and-trade against carbon tax

In [44], Shapiro qualitatively compares both emission regulation approaches, suggesting that a carbon tax can induce the required emission reductions without giving entities the opportunity to manipulate the flexible mechanisms of a cap-and-trade. Furthermore, the author argues that a carbon tax provides more stability in markets exposed to emission regulation, thus providing a safer environment for long-term investments. Thus, Shapiro recommends using a global carbon tax system over cap-and-trade to regulate emissions and defends the former as a more economic-efficient approach. However, the author concedes that applying a global tax-based emission regulation scheme is difficult to implement as different countries will find it hard to agree on certain aspects of a tax system, such as the tax rate. Cooper in [45] provides a similar qualitative comparison of both emission regulation approaches that supports Shapiro's suggestions and recommendations.

Kahn et al [46] recommend a global carbon tax initiative with different tax rates for different countries as a substitute for the global cap-and-trade system, the Kyoto protocol. The authors theoretically argue that investment in emission reduction under a price-based emission regulation approach such as CTX will result in lower long-term emissions than under similar investments under an equivalent quantity-based emission regulation approach such as CAT. Nonetheless, investment in emission reduction under a quantity-based emission regulation approach, such as cap-and-trade, will result in lower permits prices but with the same total level of emissions. Thus, a carbon tax will result in a continuous incentive to reduce emissions, in contrast to a cap-and-trade where such incentives mainly depend on the cap allocation among polluting entities.

In [47], Pizer and Burtraw and others study the effects of different emission regulation policies on a detailed model of the electricity sector. However, in contrast to the aforementioned papers, the authors treat carbon tax and cap-and-trade as equivalent market-based emission regulation approaches with equivalent effects on emissions and emission prices. This is because the authors use the constant emission permits price methodology to model the effects of CAT[4]. Recall that under this methodology, a CAT system increases the marginal generation cost of each Genco by an amount that depends on the permits' price. Thus, under CTX with a tax rate equivalent to the permits' price under CAT, the effect of the emission regulation scheme on the dispatch of the electricity market is such that it leads to the same increase in the marginal generation cost of each Genco. The authors compare these market-based approaches to other emission reduction policies that force a change in the

[4] Under this methodology the only difference between CAT and CTX is that in the former, if (some) permit are allocated to Gencos for free, then the Gencos can maintain (some of) the benefit of the emission regulation approach. However, if under CAT all permits are allocated based on an auction, then both schemes are analogous.

emission intensity or fuel efficiency of power plants. The authors conclude in favour of using market-based emissions regulation approaches as the cost of emission regulation under such approaches are lower than under stringent emission reduction policies.

This thesis presents a quantitative comparison between the effects of cap-and-trade and carbon tax on oligopolistic electricity markets, however, in contrast to [47], we account for the effect of each scheme on hourly market-clearing and on Gencos' gaming strategies. Simulations show that both schemes have varying effects on market power, market-clearing and profits; and support the recommendations in [44]-[46] derived based on qualitative analysis.

In [48], Green studies the relation between fuel and electricity prices under carbon tax and cap-and-trade and the ensuing effects on the Gencos' risk measured in terms of standard deviation of their profits. The paper uses supply function Nash equilibrium to model an electricity market with an underlying generation mix based on that of the United Kingdom. Green acknowledges the effect of the electricity sector on the price of permits, and based on Newberry's work in [49], he suggests that the electricity sector will set the price of permits such that the marginal generation cost of coal-fired power plants (high polluting but cheap Gencos) become higher than that of gas-fired power plants (low polluting but more expensive Gencos). Thus, a rise in the price of gas will raise the price of permits. Because of this coupling between the fuel prices and the price of permits, the standard deviation of the profit of Gencos with gas/coal-fired power plants is lower under CAT than under CTX. In contrast, Green finds that Gencos using nuclear power face more profit risk under CAT than under CTX because of the volatility of the price of permits under CAT. In his work, Green does not consider the allocation of permits among Gencos and thus Gencos, under CAT, have to buy all the permits they need from the external permits market.

In [50], the authors state that the volatility of the permits price under CAT, and the complexity of designing a cap-and-trade system are some of its major drawbacks. The paper suggests that the permit allocation among Gencos has to be done carefully in order to reduce the price of electricity under CAT, however it acknowledges the difficulty of this target. The paper qualitatively argues that market manipulation under CAT significantly reduces the effectiveness of this system to reduce emissions in the long-term. Thus, although the paper acknowledges the unpopularity of imposing new taxes among politicians, especially in Capitol Hill, it argues in favor of such a carbon tax as a superior emission regulation approach to CAT.

Metcalf in [51] proposes setting a carbon tax system with an optimum tax rate that accounts for the marginal social cost of emissions, however the author notes that the social cost of carbon ranges between 3 $/t to 95 $/t, according to different estimates of the IPCC [1]. Thus, the paper suggests imposing a tax rate that increases over time with the revenue used to fund reductions in income taxes. Although this work recommends the use of carbon tax, mainly due to the revenue it provides, the author suggests that a cap-and-trade system facilitates larger reductions in total emissions through tighter caps. However, the author argues that such caps result in significantly reduced profits of entities whose emissions are capped.

In [52] Grubb and Newberry make a theoretical comparison between carbon tax and cap-and-trade and argue in favor of CTX. However, they suggest that auctioning permits under CAT is equivalent to CTX, thus recommending the evolution of existing cap-and-trade systems into global ones. Moreover, they encourage the shift from free allocation to an auction-based permit allocation among Gencos. Finally, the authors state that the electricity industry is the single biggest source of carbon emissions globally and is “a prime driver of projected global emissions growth”. Thus, they conclude that, while designing a global emission regulation scheme is a challenge, applying such a scheme to the electricity sectors around the world would be “a huge step forward”.

1.3.4 Supply function Nash equilibrium with generation limits

Supply function Nash equilibrium (SFE) was first analyzed by Klemperer and Mayer [53] and has been recently used to study gaming strategies and price-setting influences in deregulated electricity markets [54]-[58]. Under supply function equilibrium, power generating companies compete by submitting price schedules for levels of power they are willing to produce over a range of demand levels. Solving for the SF Nash equilibrium is computationally demanding, in addition to yielding an infinite number of strategic choices out of which no specific one stands out.

To sidestep the dilemma of the continuum of SFE and the complexity of finding such equilibria, previous researchers have made simplifying assumptions on: the number of competing Gencos, generation costs, generation limits, form of permitted supply function offers, and finally on market rules.

For example, in [57] Green restricts supply functions to special linear forms with zero intercepts that render the solution to the Klemperer and Mayer equations (and therefore the corresponding SFE) unique. Baldick et al in [59], consider only affine functions (linear with non-zero intercepts) or piece-wise affine functions when lower generation limits are active. Holmberg uses a price cap and capacity constraints to single out a single equilibrium [60], [61]. In [62], Hogan et al find unique solutions by ruling out unstable equilibria and by considering price caps and capacity constraints. The effect of generation capacity constraints on SFE is also examined in [63]-[65].

The effect of network constraints on gaming and market power under a SFE was studied in [66]-[71]. In [72], SFE is used to optimize Gencos’ bidding strategies considering forward contracts.

In this thesis we develop a mixed integer-based formulation to solve for the supply function equilibrium under capacity constraints for Gencos with asymmetric generation costs and capacity constraints. We use this formulation to solve for the SFE when competing Gencos offer affine supply functions.

1.4 Claim of Originality

A gap in the previously reviewed literature that this thesis attempts to narrow is to examine the various aspects of oligopolistic electricity markets under emissions regulation in concert rather than separately, seeing that these aspects are strongly coupled. The various aspects that apply to cap-and-trade are: (i) The allocation of the commitment period sector cap among the Gencos be it with or without an auction; (ii) The self-allocation of a Genco cap into hourly fractions to be used in the hourly market; (iii) Genco gaming in the hourly market that takes advantage of both emission intensities, emission trading, elastic prices for both power and emission permits and cost. A similar set of set of strongly linked aspects apply in the case of a carbon tax as discussed below.

The original joint approach taken in this thesis to study the effect of cap-and-trade on oligopolistic electricity markets is illustrated in the figure below.

1) Commitment Interval Cap Allocation among Gencos (Social-Welfare Auction)

$$\max_{\mathbf{e}} \left\{ SW(\mathbf{e}) - \sum_i F_i(e_i) \right\}$$

Subject to:

1.1) $\sum_k e_k \le e^0$

1.2) $e_k^f \le e_k$

1.3) Individual Genco Commitment Interval Profit Maximization ; $\forall k$

$$\max_{\{e_{kt};\forall t\}} \left\{ pr_k(\mathbf{e}) \right\}$$

Subject to:

1.3.1) $\sum_t e_{kt} = e_k$; $\forall k$

1.3.2) Hourly Market-Clearing ; $\forall t$

$$\min_{g_{it}, d_t, \Delta e_t} \left\{ \sum_i C_{it}(g_{it}) - B_t(d_t) + CEP(\Delta e_t) \right\}$$

Subject to:

$\sum_i g_{it} = d_t \qquad (\lambda_t)$

$g_i^{\min} \le g_{it} \qquad (\mu_{it}^{\min})$

$g_{it} \le g_i^{\max} \qquad (\mu_i^{\max})$

$\Delta e_t = \sum_i \rho_i g_{it} - \sum_i e_{it} \qquad (\gamma_t)$

Figure 1-1: The three-level optimization problem used in this thesis to model cap-and-trade.

More specifically, the main original contributions of this thesis are:

1) The design of a cap-and-trade emission regulation scheme that, in addition to the standard electricity trading, includes emission permits trading under a joint hourly market. This joint market allows us to study the effect of permits trading on the hourly electricity dispatch and to compute the optimum hourly Genco supply offers and permits trades;

2) The analytical and numerical computation of the Cournot Nash Genco gaming strategies and of the resulting outcome of the joint electricity and emission permits hourly market. This outcome is computed in terms of a number of parameters, of which one set is the self-assigned hourly Genco emission caps;

3) The self allocation of a Genco's emissions cap over the commitment interval into hourly fractions and its effect on the short and long term. This temporal allocation is tested under a heuristic and under a Nash equilibrium approach;

4) Analyzing the effects of allocating the commitment interval sector cap among Gencos on the basis of grand-fathering in conjunction with its temporal allocation and with the joint electricity and emission permits trading hourly market;

5) Analyzing the effects of allocating the commitment interval sector emissions cap among Gencos on a basis other than Grand-fathering. Here, we developed a novel approach based on maximizing social-welfare that solves a tri-level mathematical problem with equilibrium constraints (MPEC). The innermost level is the joint hourly market, the second level is the temporal Genco cap self-allocation, and the third level is the allocation of the sector commitment interval emissions cap among the Gencos;

6) The allocation of the commitment interval sector emissions cap among Gencos on the basis of an auction is examined from a quantitative rather than a qualitative perspective. Such an auction is an extension of the MPEC problem solved in point (5) where Gencos influence their permits allocation by bidding and modifying the SW function;

7) Re-designing the electricity market to account for a carbon tax scheme based on a tax penalty on the hourly total emissions produced by the sector.

While current carbon tax systems implemented on the electricity sector or studied in relevant literature are based on a fixed rate, our scheme results in a variable hourly tax rate that depends on the tax parameters set by the SP and on the total hourly emissions produced by the sector;

8) The analytical and numerical computation of the Cournot Nash equilibrium solution that defines the strategic hourly supply offers of competing Gencos under the new electricity/carbon tax penalty model;

9) A scheme that computes the optimum carbon tax parameters to meet a specified emission target over the commitment interval. The scheme is based on an MPEC that maximizes social-welfare while accounting for the effect of the tax parameters on the hourly gaming strategies of the re-designed electricity market;

10) A quantitative comparison of the effects of cap-and-trade and carbon tax on the operation of electricity markets and on the Gencos' market power and profits. In this comparison, both schemes are designed to respect equivalent emission caps and maximize social-welfare;

11) A mixed-integer formulation to compute the SFE in cases where Gencos own multiple generating units with asymmetric generation costs and upper and lower generation limits.

1.5 Thesis Structure

Chapter one presents the motivation, objectives and scope of the thesis. It discusses previous relevant work and presents the thesis' claims of originality.

Chapter two gives an overview of the operation of deregulated electricity markets without emission regulation. In this chapter we introduce the notion of gaming and describe two Nash equilibrium modeling strategies to evaluate Gencos' gaming: Cournot and supply function. We end the chapter by presenting numerical simulations depicting the effects of gaming using both modeling strategies.

In chapter three, we begin discussing the cap-and-trade emission regulation scheme. We present an overview of the scheme, explain how it works and highlight some of its design concerns. We end the chapter with an overview of the challenges of applying the scheme in the electricity sector.

Chapter four is a comprehensive analysis of a cap-and-trade system applied to the electricity sector. In this chapter we re-model the electricity market and the Gencos' gaming strategies under a cap-and-trade system. We also investigate the challenges of designing an effective and efficient cap-and-trade system in a deregulated electricity market. We present quantitative examples throughout the chapter illustrating the quantitative analysis.

In chapter five we study the carbon tax emission regulation scheme and how it can be applied to the electricity sector. We first propose a carbon tax system based on an hourly tax penalty, and then re-model the electricity market and the Gencos' gaming strategies under such a system. Taking into consideration the modified electricity market model, we design a carbon tax system that minimizes the effects on social welfare while achieving emission regulation targets. We present quantitative examples throughout the chapter illustrating the quantitative analysis.

In chapter six, we conclude our study of the effects of emission regulation schemes on the electricity sector by providing quantitative and qualitative comparisons between cap-and-trade and equivalent carbon tax schemes.

In chapter seven we present a summary of the thesis, its major conclusions and some future prospects.

2 The Structure and Operation of Electricity Markets

"Real abundance has very little to do with being able to give ourselves whatever we want, but rather with being free of the part of us that's forever wanting something better and new in order to feel new and better;" Ahmed Shawqi.

In this chapter we present an overview of electricity markets. We explain how electricity is traded periodically to meet the predicted demand and how a market operator dispatches sufficient generation according to the offers to sell electricity submitted by the Gencos. Finally, we elucidate how Gencos define these offers (game) in order to increase their profits according to two Nash equilibrium models, namely, Cournot and supply function equilibrium (SFE).

2.1 Nomenclature

The following is a partial nomenclature for this chapter.

Parameters:

ng : Number of Gencos;

nt : Number of time periods in the commitment interval;

Δt : length of time period;

$\boldsymbol{\rho} = \{\rho_i; \forall i\}$: Vector of Genco emissions intensities (t/MWh);

$\mathbf{g}^{\max} = \{g_i^{\max}; \forall i\}$: Vector of Genco maximum generation levels (MW);

$\mathbf{a}_t^* = \{a_{it}^*; \forall i\}$; $\forall t$: Vector of first-order parameters of Genco actual cost functions at time t (\$/MWh);

$\mathbf{b}_t^* = \{b_{it}^*; \forall i\}$; $\forall t$: Vector of second-order parameters of Genco actual cost functions at time t (\$/MW^{2}h);

λ_t^0 : First-order parameter of demand benefit function at time t (\$/MWh);

α_t : Second-order parameter of demand benefit function at time t (\$/MW^{2}h);

Variables:

$\mathbf{a}_t = \{a_{it}; \forall i\}$; $\forall t$: Vector of first-order parameters of Genco offered cost functions at time t (\$/MWh);

$\mathbf{b}_t = \{b_{it}; \forall i\}$; $\forall t$: Vector of second-order parameters of Genco offered cost functions at time t (\$/MW^{2}h);

$\mathbf{g}_t = \{g_{it}; \forall i\}$; $\forall t$: Vector of Genco output levels at time t (MW)

d_t : Demand level at time t (MW);

<u>Lagrange Multipliers:</u>

λ_t : Price of electricity at time t ($/MW);

$\boldsymbol{\mu}_t^{\max} = \left\{\mu_{it}^{\max}; \forall i\right\}; \forall t$: Vector of Lagrange multipliers of maximum generation constraints at time t ;

$\boldsymbol{\mu}_t^{\min} = \left\{\mu_{it}^{\min}; \forall i\right\}; \forall t$: Vector of Lagrange multipliers of minimum generation constraints at time t ;

<u>Functions:</u>

$C_{it}\left(g_{it}\right)$: Generation cost function offered by Genco i at time t ($/h);

$IC_{it}\left(g_{it}\right)$: Incremental generation cost function offered by Genco i at time t ($/MWh);

$B_t\left(d_t\right)$: Demand benefit function at time t ($/h);

$IB_t\left(d_t\right)$: Incremental demand benefit function at time t ($/MWh);

2.2 Structure of Electricity Markets

Since 1997, the electricity sectors in most countries have been significantly restructured (the terms liberalized or deregulated are also used to describe this reform) [2]. What this means is that power generating companies (Gencos) now compete to sell electricity to load-serving entities[5] at a price set by the equilibrium conditions between the demand and supply of electricity according to the rules of the electricity market [4]. An electricity market is the mechanism that facilitates the trading of electric power to achieve this equilibrium while meeting technical and reliability constraints on electricity generation, transmission and distribution.

2.2.1 Pool trade

The power pool is an independent non-profit entity, hereafter referred to as the pool, which receives bids to buy and offers to sell electricity from the market participants (respectively load-serving entities and Gencos) [73]-[75]. The pool market-clearing is based on the maximization of a social welfare function (total consumer benefits minus total generation cost offers) subject to the power system's security and technical constraints. The result of this maximization defines the dispatch of the Gencos' outputs and the demand consumption levels, along with the marginal price of electricity which, under marginal pricing, is the system incremental generation cost or the "cost of the last MW of power delivered" [76]. An alternative pricing mechanism is the so-called pay-as-bid, where the rate at which a Genco gets paid for its electricity generation is equal to its supply offer [77]. For the purpose of this thesis we only consider marginal pricing, this being the most common pricing mechanism.

[5] Load-serving entities are retailers that buy electricity at the pool price or according to a bilateral contract and then sell this electricity to consumers at a fixed retail price. For the remainder of this thesis we neglect the difference between retail and wholesale demand and we refer to it as the load or the demand.

Each Genco's offer to the pool can be represented by a function $IC(g)$ relating its incremental cost offer, IC, to its generation output g. Equivalently, a Genco can offer to sell electricity through its supply function (SF), $g = S(\lambda)$, relating its MW output to the electricity price λ. The two offer forms are equivalent if the incremental cost $IC(g)$ is monotonically non-decreasing in g since, at market-clearing, one relation is the inverse of the other.

A Genco games by offering an $IC(g)$ function that differs from its true incremental cost curve, $IC^*(g)$ or, equivalently, by submitting a supply function offer $S(\lambda)$ that differs from its true supply function $S^*(\lambda)$.

Pool trade is usually settled in two steps, the day-ahead and the real-time markets, a design that is implemented by pools such as the Pennsylvania-New Jersey-Maryland (PJM), New England (NE) and New York (NY) power pools [3].

2.2.2 Day-ahead market

The day-ahead market is a voluntary forward market in which clearing prices are calculated for each hour of the next operating day based on the Gencos' offers and the demand bids submitted to the pool. The day-ahead energy market is cleared using a Security-Constrained Unit Commitment (SCUC) and Economic Dispatch (ED) for each hour in the next operating day [78]. A day-ahead market that clears on an hourly basis is the most common market design.

Electricity supply and demand can be unpredictable due to random outages and events such as rapidly changing weather conditions. The day-ahead market provides its participants with an opportunity to manage this unpredictability in the short-term. Market participants often use the day-ahead market to lock in energy prices as a way of managing risk against the sudden changes in energy prices that could occur in the real-time market. This hedging concept is also applied through long-term bilateral contracts as discussed later.

2.2.3 Real-time market

The real-time energy market is a balancing market in which the clearing prices are calculated several times per hour (typically five to fifteen minutes) based on a Security-Constrained Economic Dispatch performed on the actual system conditions at market-clearing [79]. This spot pool trading serves to ensure that supply and demand are economically balanced at all times within each hourly interval. Note that the balancing of demand and generation inside very short intervals of seconds to minutes is the responsibility of the primary and secondary regulation mechanisms, which are not economically based [80].

The real-time market carries the most risk for participants since unforeseen events, such as a sudden transmission line failures or unexpected weather conditions, can cause spot prices to rise or fall dramatically [79]. Nevertheless, the real-time market is essential to fill the gaps in supply and demand not covered by the day-ahead market. Typically, the day-ahead market balances the largest fraction of the total demand, leaving a relatively small unbalance for the real-time market. For this reason, in this thesis we focus on the day-ahead market.

2.2.4 Bilateral trade

The other form of trading in the current deregulated electricity markets is through bilateral contracts, namely contracts between two agents to purchase or sell, at set prices, one or more market products over a specified time period [81]. We note here that although the main electrical product sold is power in MW or energy in MWh, a Genco can also market different types of reserve as well as reactive power. These products are called ancillary services [82].

When applied to electricity markets a bilateral contract is an agreement between two market participants, generator “x” and load “y” under which generator “x” has a legal obligation to provide a certain MW amount to load “y” at a set price at the time of market-clearing. Bilateral contracts also include the start and end times, along with the duration of each contract, in addition to a number of legal obligations and rights. Such bilateral contracts are negotiated from two weeks to six months prior to market-clearing [83].

Bilateral contracts provide price stability because they are fixed and are not subject to price fluctuations in the day-ahead or real-time markets [83], [84]. Furthermore bilateral contracts provide market participants with a method to transfer some of their wholesale settlement obligations and reduce their operating reserve charges. On the other hand, bilateral contracts are subject to the risk that the real-time price may be more advantageous [85], [86].

Bilateral contracts can be divided into two categories, physical and financial. The parties involved in a physical bilateral contract assume the responsibility and obligation to physically transfer an agreed-to MW amount, at an agreed-to price, between specified injection and withdrawal points in the transmission grid. Conversely, financial bilateral contracts limit the legal responsibility of the parties involved to guaranteeing the price for the specified amount, but not necessarily to physically generate this power into the grid, which in this case is generated by the pool [87].

2.2.5 Mixed pool/bilateral trade

A common market design today referred to as mixed pool/bilateral is one that enables its participants to trade through bilateral contracts, through the pool, or via a combination of the two as depicted in Fig. 2-1 [88],[89] . There are two forms of such mixed markets: centralized electricity markets [90] where the principal form of trade is the pool trade; and de-centralized markets where the bilateral trade is more dominant [74], [91].

In such markets, the pool runs a SCED similar to the one performed when the trade is strictly through a pool. However the difference is that in a mixed market, the SCED in the day-ahead market has to include, within its constraints, the privately negotiated physical bilateral contracts. Note however that before any physical bilateral contract becomes legally binding market participants have to obtain the ISO's approval [90].

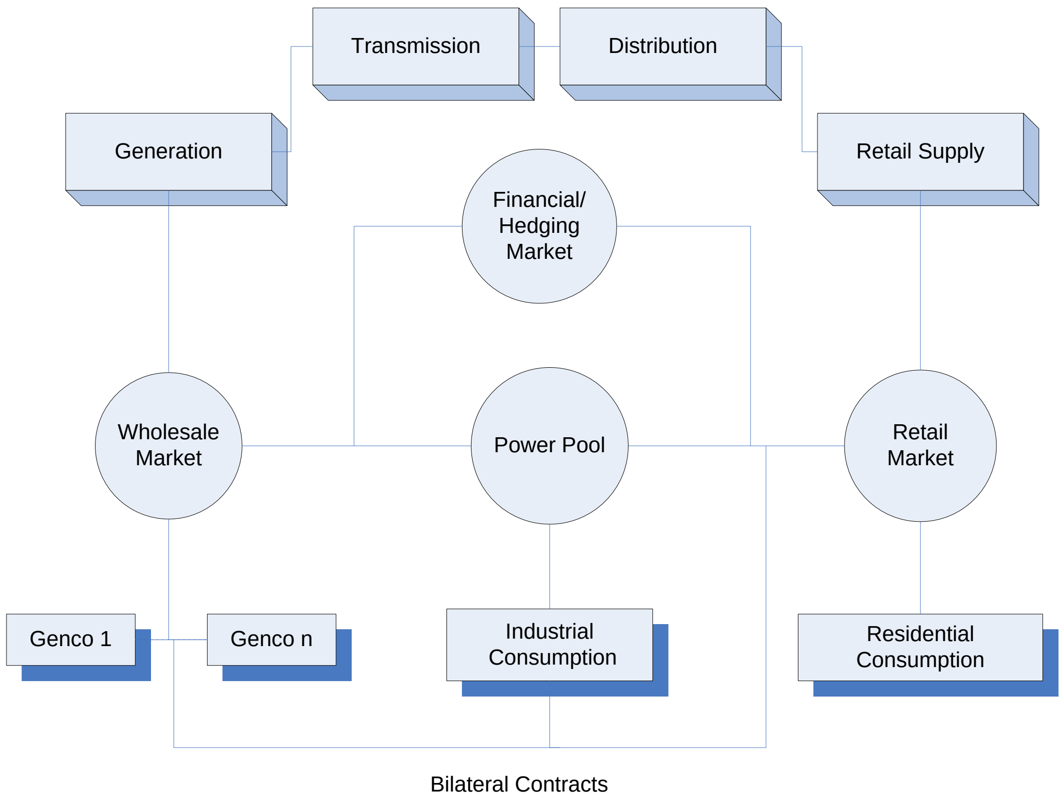

Figure 2-1: Structure of electricity markets.

2.2.6 Operation of electricity markets

The operation of electricity markets consists of short, medium and long-term decisions made by either the system operator or by the competing Gencos. These decisions and obligations are briefly discussed below and are summarized in Fig 2-2.

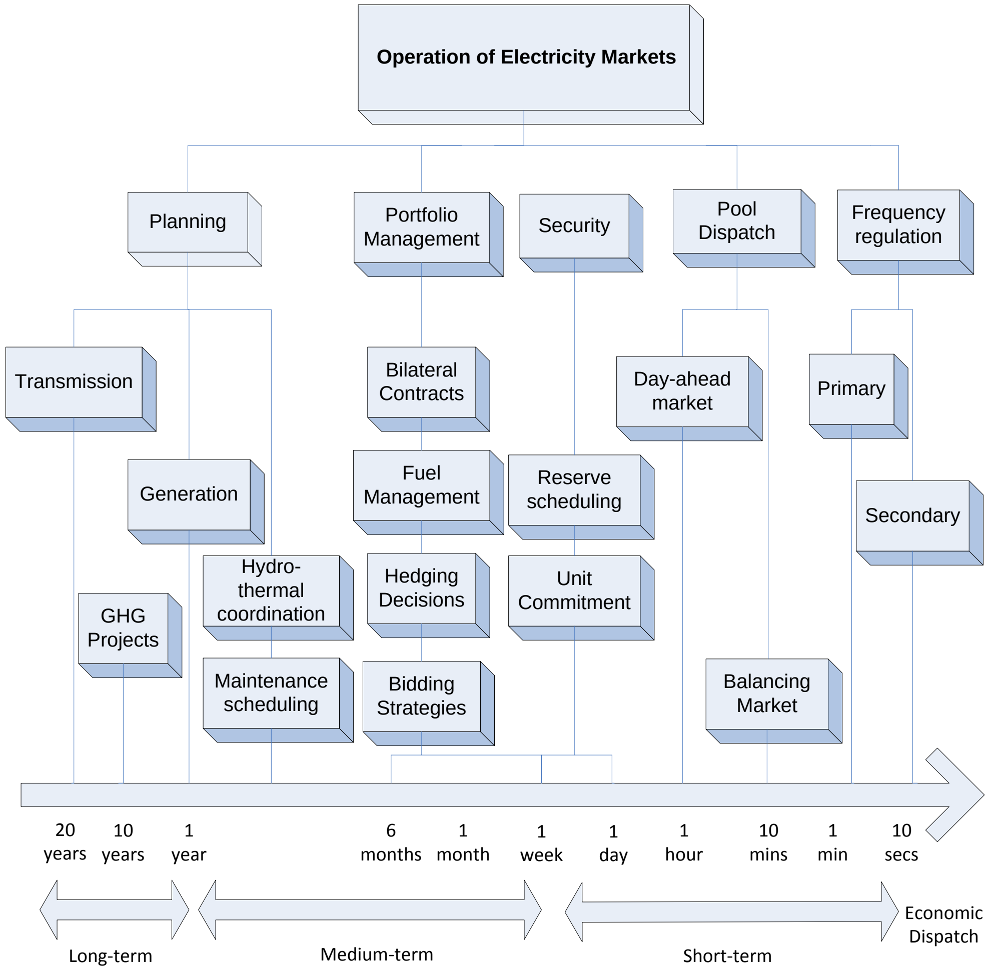

Figure 2-2: Time-line of the operation of electricity markets.

2.2.6.1 Long-term

Planning:

Long-term operating decisions are categorized as a planning problem. This includes transmission and generation planning, investing in greenhouse gas (GHG) emission reduction projects, hydro-thermal coordination and maintenance scheduling [92], [93].

In most current electricity markets, transmission lines are owned by independent companies (Transcos). Transcos apply to a regulating body (Transmission System Operator) to build new high voltage transmission lines whose location and number are decided upon on the basis of congestion relief, supply of predicted increasing loads, security considerations and environmental norms. Such high voltage lines are expensive and investing in such projects is usually planned 5 to 10 years ahead. Transcos are traditionally compensated for the use of their lines at a fixed rate [94].

Privately owned power generating companies (Gencos) plan on the type and number of new power plants they intend to add to their fleet over time horizons ranging up to 20 years. This decision is primarily driven by profit but it is also heavily influenced by emission regulation mechanisms as well as by the projected market rules and performance [95].

To meet emission standards, individual Gencos invest in long-term GHG reduction projects over horizons of 10 years or more. Such projects may include carbon capture mechanisms in existing power plants, carbon sinks and the development of renewable sources of electricity generation.

Hydro-thermal coordination and maintenance scheduling decisions are functions with time horizons of less than one year.

2.2.6.2 Medium-term

Portfolio management: Gencos control the risk and maximize the benefits of trading in electricity markets by setting their trading portfolios within approximately six months prior to the pool market-clearing [96], [97]. This includes: negotiating financial/bilateral contracts; purchasing hedging financial instruments such as financial transmission rights or other congestion management tools [98],[99]; fuel management which consists of locking fuel prices and exchange rates; and finally computing their strategic offers to sell electricity to the pool which is the central focus of this thesis.

Security: The ISO schedules enough reserve to meet security requirements at market-clearing [100], including dispatching units to be on as required. This analysis is performed up to seven days prior to market-clearing [101].

2.2.6.3 Short-term

Pool dispatch: This refers to the day-ahead and balancing market-clearings discussed earlier.

Frequency regulation: This is responsible for the balancing of demand and generation inside very short time intervals of seconds to minutes. It includes local primary regulation which is frequency based, as well as centralized secondary or load-following regulation [80].

2.3 Hourly Pool Dispatch

2.3.1 Piece-wise linear generation cost offers

The market rules can be such that hourly generation cost functions offered to the hourly pool dispatch are piece-wise linear functions of generation output as shown in Fig. 2.3. Equivalently, this implies that the incremental cost functions offered are piece-wise constant or in "block form" as shown in Fig. 2.4.

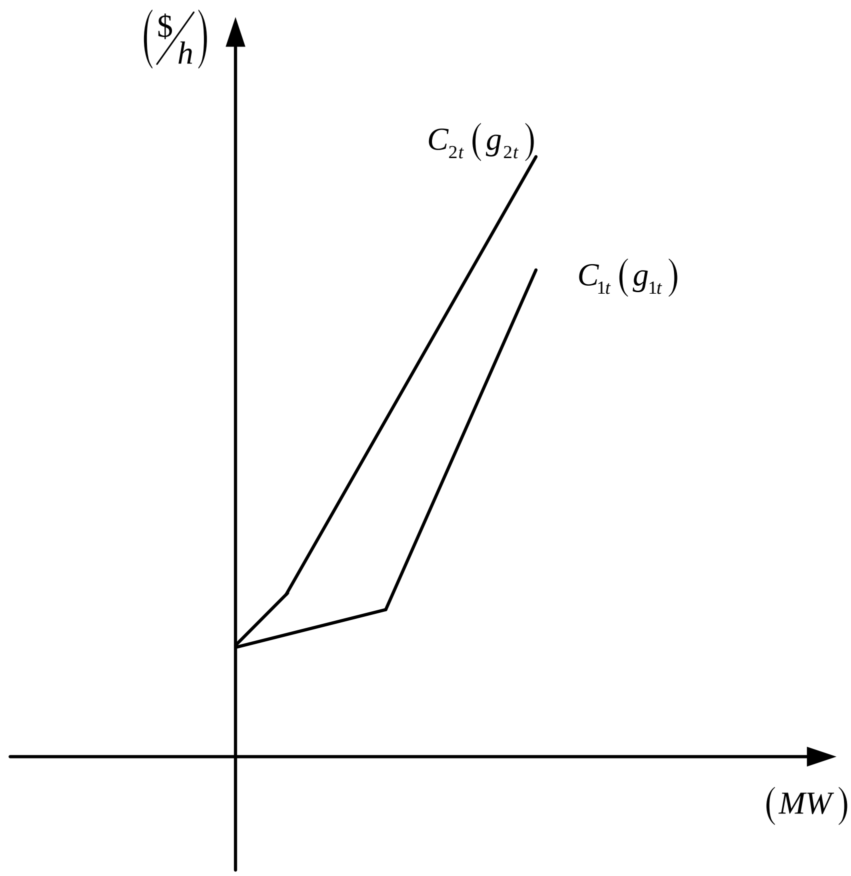

Figure 2-3: Piece-wise linear hourly generation cost functions.

Under such offers, the pool orders the offered hourly IC blocks in a monotonically increasing so-called merit-order and establishes a staircase relationship between system incremental cost (or price) and demand as illustrated in Fig. 2.4. The hourly market-clearing price λ_t is then defined by the point at which the hourly demand intersects the price versus demand curve, as seen in Fig. 2.4 [102].

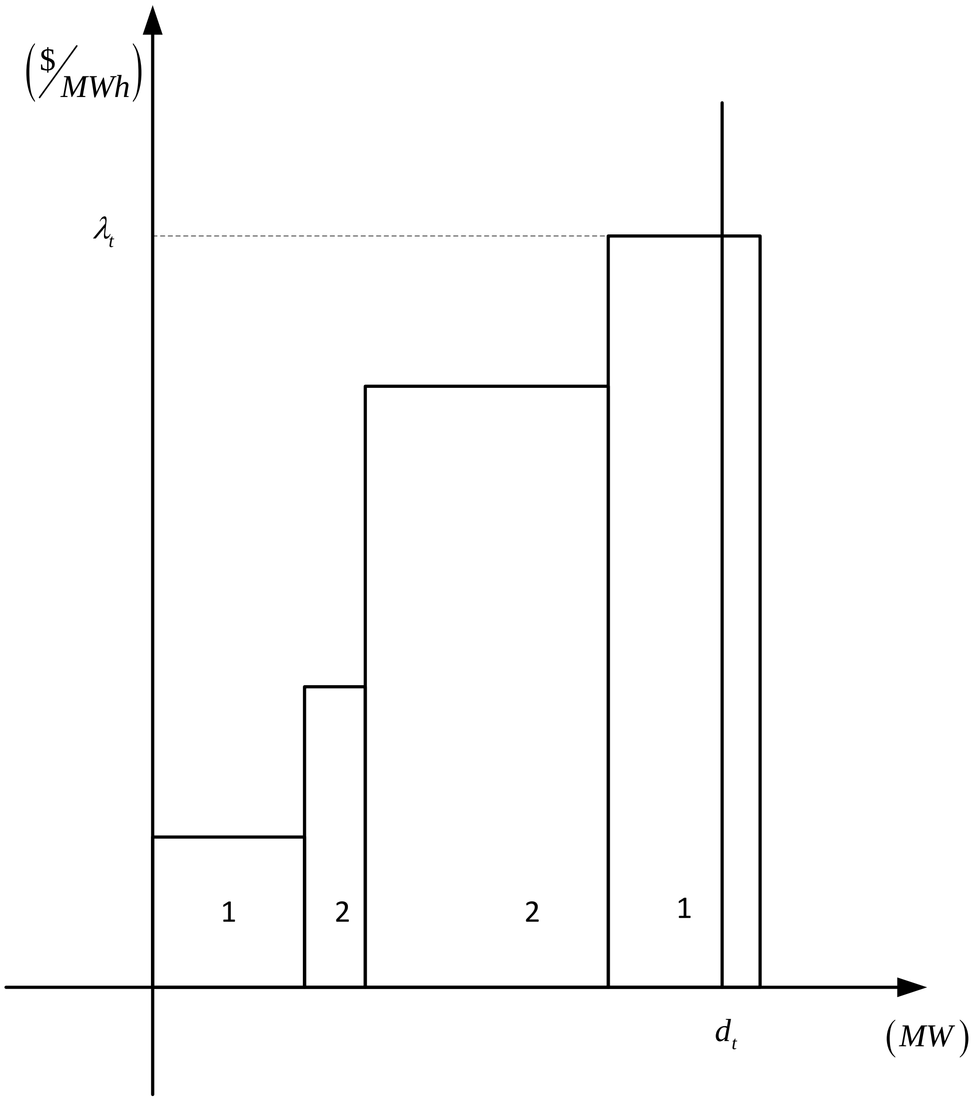

Figure 2-4: Hourly market-clearing under block generation incremental cost offers.

2.3.2 Non-linear generation cost offers

Instead of piece-wise linear cost functions, the market rules may require that the Gencos offer continuous cost functions as shown in Fig. 2.5.

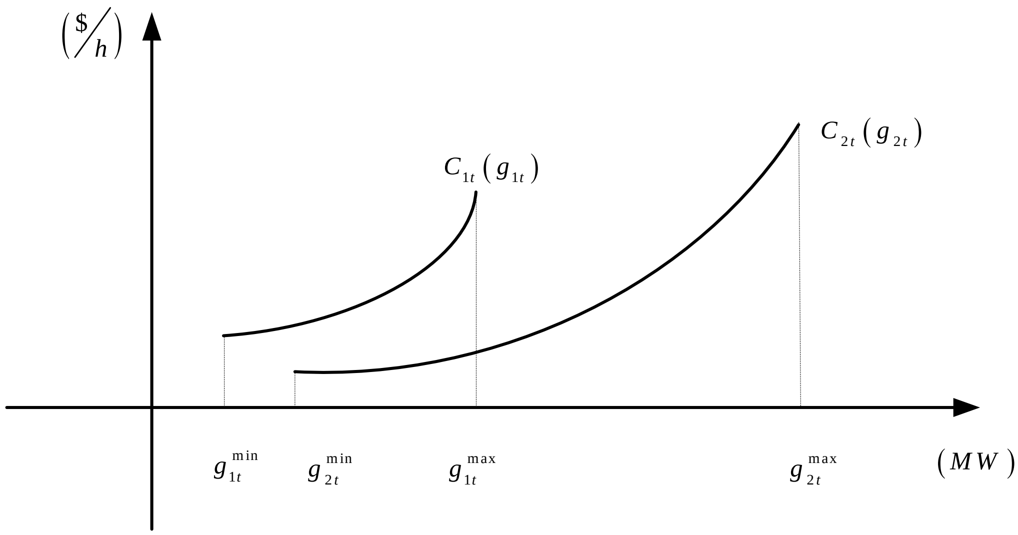

Figure 2-5: Nonlinear hourly generation cost functions.

In general, the hourly market-clearing process at each time period t is obtained through the following optimization problem,

$$\min_{\{g_{it};\forall i\}} \sum_i C_{it}(g_{it}) \tag{2.1}$$

subject to the hourly power balance,

$$\sum_i g_{it} = d_t \qquad (\lambda_t) \tag{2.2}$$

and to the hourly upper and lower generation limits,

$$g_{it} \le g_{it}^{max} \; ; \forall i \qquad (\mu_{it}^{max}) \tag{2.3}$$

$$g_{it}^{min} \le g_{it} \; ; \forall i \qquad (\mu_{it}^{min}) \tag{2.4}$$

The solution to this optimization problem represents the hourly market-clearing at time t and defines the following: 1) the vector of generation levels at time t $\{g_{it};\forall i\}$; 2) the electricity price λ_t at time t.

The market-clearing is such that the vector $\{g_{it};\forall i\}$, meets the generation limits for each Genco i , and balances the demand level d_t . Moreover the market-clearing is such that the incremental generation costs, $\{IC_{kt}(g_{kt});\forall k\}$ are equal to the electricity price λ_t for all free Gencos (operating strictly within their generation limits).

The hourly market-clearing is analytically derived in Appendix A and is illustrated in Fig. 2.6 for linear IC functions.

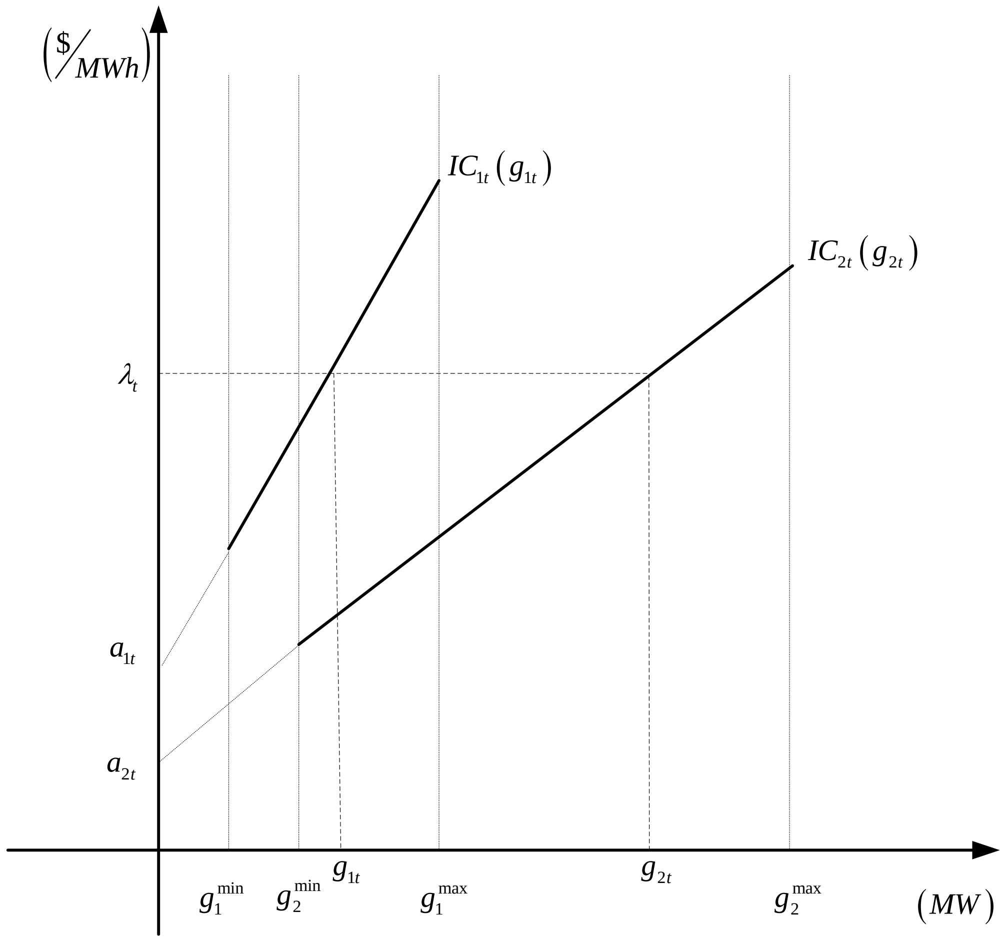

Figure 2-6: Market-clearing under linear incremental generation cost function offers.

2.3.3 Elastic hourly demand

Electricity demand might be elastic, that is, the demand has an associated hourly benefit function, denoted by $B_t(d_t)$, which represents the demand bid[6]

[6] In this work we treat the aggregate demand bid as a given and we do not consider demand-side strategic bidding in which load-serving entities manipulate their bids to increase the demand surplus.

for electricity and reflects how much the demand values its electricity consumption at time t .

The hourly demand benefit function for a load-serving entity, for instance, measures the revenue the load-serving entity makes by selling electricity at time *t* to consumers on the retail side at the agreed-upon price. The hourly demand benefit function for energy-intensive industries, such as a steel production factory, represents the benefit earned by the factory by selling steel manufactured by consuming electricity.

A typical demand benefit function at time t is,

$$B_t(d_t) = \lambda_t^0(d_t) - 0.5\alpha_t(d_t)^2 \tag{2.5}$$

As shown in Fig.2.7, the hourly demand benefit function, $B_t(d_t)$ is concave and is such that a higher consumption of electricity (an increase in the demand level d_t) leads to a decline in the rate at which the benefit increases, which is the opposite behavior to the convex generation cost function.

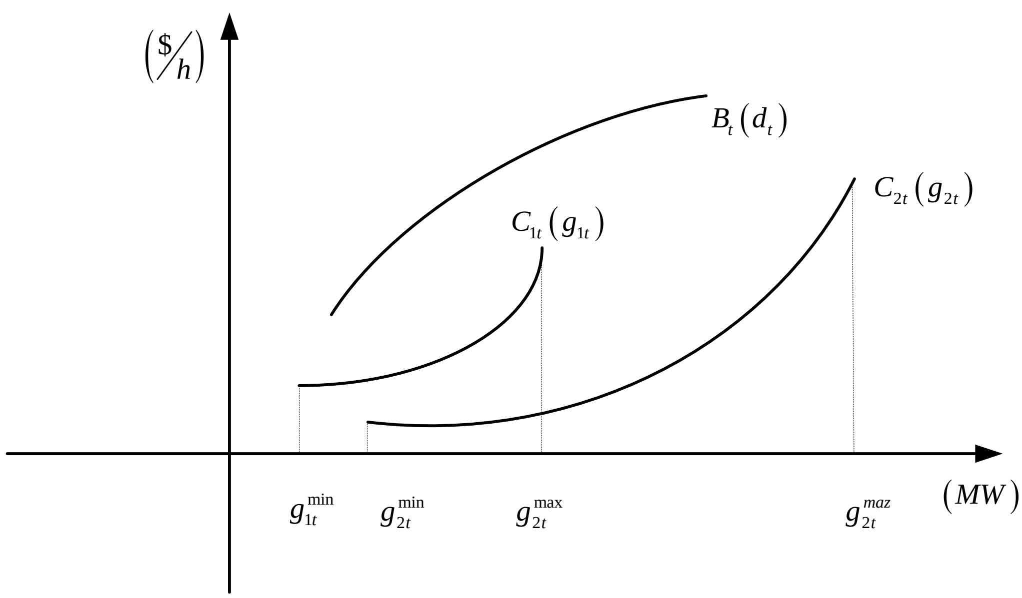

Figure 2-7: Hourly nonlinear generation cost and demand benefit functions.

To incorporate the demand benefit function into the hourly market-clearing, the objective function of the optimization problem defined by (2.1) to (2.4) is modified to maximize the hourly social-welfare (SW), namely, the difference between the demand benefit and the total generation cost,

$$\max_{\{g_{it};\forall i\},d_t} \sum_i B_t(d_t) - C_{it}(g_{it}) \tag{2.6}$$

The solution to this optimization problem represents the hourly market-clearing at time t under elastic demand and defines the following: 1) the vector $\{g_{it};\forall i\}$ of generation levels at time t; 2) The demand level d_t and the electricity price, λ_t, both at time t.

The market-clearing is such that the vector $\{g_{it};\forall i\}$, meets the generation limits for each Genco i and balances the demand level d_t. Moreover the market-clearing is such that the incremental demand benefit $IB_t(d_t)$, along with the incremental generation costs, $\{IC_{kt}(g_{kt});\forall k\}$ are equal to the electricity price, λ_t, for all free Gencos (operating strictly within their generation limits). The hourly market-clearing for linear incremental cost functions under elastic demand is analytically derived in Appendix A and is illustrated in Fig. 2.8.

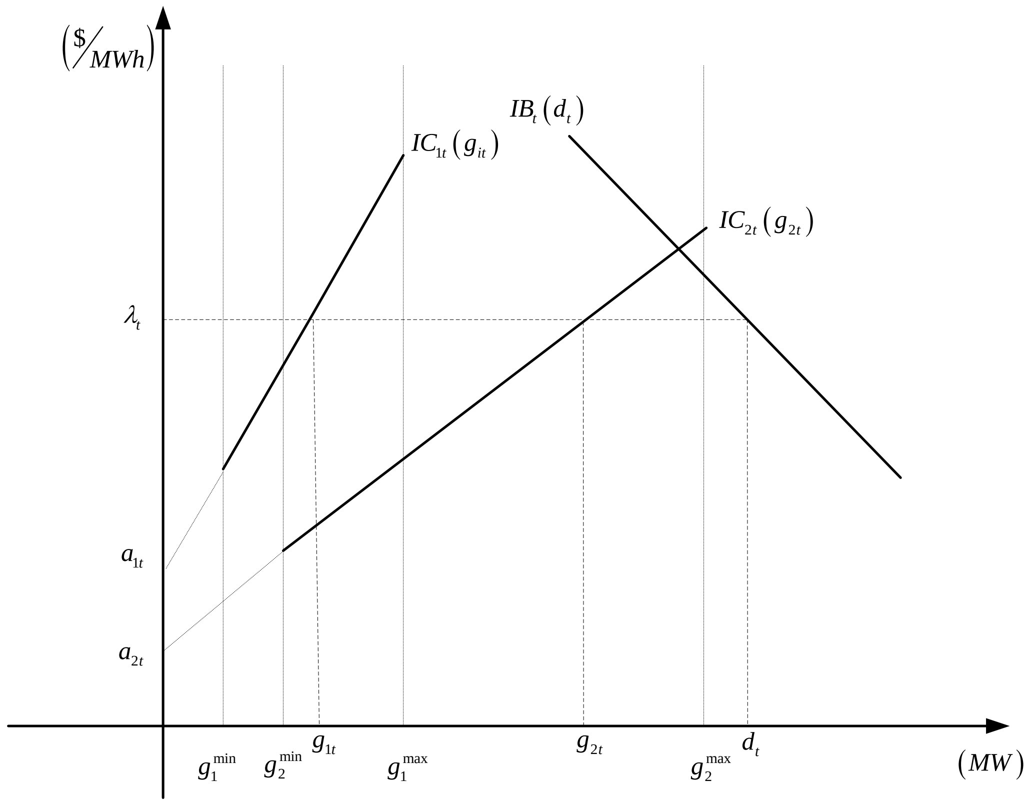

Figure 2-8: Hourly market-clearing that includes maximizing the hourly demand benefit function.

Note from Fig. 2.8, that the incremental demand benefit function characterizes the elasticity of demand to the electricity price. This elasticity is such that an increase in electricity price at time t requires a higher incremental benefit, which results in a decrease in the corresponding demand level.

2.4 Gaming in the Pool Dispatch

In the previous discussion of the power pool market-clearing we stated that Gencos offer hourly generation cost functions $C_{it}\left(g_{it}\right)$ (or supply offers) for each time period *t*. These offers are strategically computed by the Gencos with the aim of increasing their profits while recognizing the associated risk of being excessively greedy *[103]*. This strategy is known as *gaming* and the extent to which a Genco can increase its profit by gaming is defined as *market power.*

It is well known that under perfect market conditions, that is, when a large number of Gencos of similar size compete, gaming offers no advantage. As a result, the best strategy is for each Genco *i* to offer at true cost, $C_{it}^{*}\left(g_{it}\right)$ [104]. The price of electricity then reflects the actual system marginal cost of electricity generation.

Nonetheless, current electricity markets are far from being perfect [4], in general they are oligopolistic markets made up of a small number of competing Gencos each with its own market power. These competing Gencos therefore game by offering to the power pool hourly generation cost functions, $C_{it}\left(g_{it}\right)$ that inflate their true costs and increase their profit, that is,

$$C_{it}\left(g_{it}\right) \geq C_{it}^{*}\left(g_{it}\right); \forall i,t \qquad (2.7)$$

By offering above its true generation cost at time t, a Genco i aims to increase the hourly electricity price, λ_t thus increasing its hourly profit, pr_{it}, given by[7],

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[7] In this thesis, for simplicity, each Genco owns only one generating unit. However all of the results here derived can readily be extended to Gencos owning several generating units.

$$pr_{it} = \lambda_t g_{it} - C_{it}^*(g_{it}) \tag{2.8}$$

There is however a risk to a Genco that by offering an overly inflated cost function, $C_{it}(g_{it})$, its profit may diminish compared to offering its true generation cost, $C_{it}^*(g_{it})$. This is so since the increase in electricity price will at some point be offset by a decrease in generation level and in revenue due to the market-clearing solution seeking generation from cheaper sources. Thus, imperfect competition sets an inherent limit on how much any one Genco can increase its profit by offering to supply power above its true generation cost.

As an example of gaming strategies, consider the case of hourly quadratic generation cost function offers,

$$C_{it}(g_{it}) = a_{it}(g_{it}) + 0.5b_{it}(g_{it})^2 \; ; \forall i, t \tag{2.9}$$

where the demand benefit function is given by (2.5). With this type of offer at time t, Genco i can game through its cost parameters a_{it} and b_{it} such that,

$$\left.\begin{aligned} a_{it} &\geq a_{it}^* \\ b_{it} &\geq b_{it}^* \end{aligned}\right\} \tag{2.10}$$

Assuming for the moment that at market-clearing all generation limits are inactive, from Appendix A, the KKT necessary conditions that define the hourly market-clearing at time t are,

$$\sum_i g_{it} = d_t \tag{2.11}$$

and,

$$\lambda_t = IC_{it}(g_{it}) = a_{it} + b_{it} g_{it} \qquad ; \forall i \tag{2.12}$$

$$\lambda_t = IB_t(d_t) = \lambda_t^0 - \alpha_t d_t \tag{2.13}$$

From (2.12) and (2.13), the hourly generation and demand levels can be expressed, respectively, as,

$$g_{it} = \frac{\lambda_t - a_{it}}{b_{it}} \qquad ;\forall i \tag{2.14}$$

and,

$$d_t = IB_t^{-1}(\lambda_t) = \frac{\lambda_t^0 - \lambda_t}{\alpha_t} \tag{2.15}$$

The function $IB_t^{-1}(\lambda_t)$ is the inverse of the demand benefit function and characterizes the hourly demand as a function of the electricity price at time t. Note that a lower α_t represents higher demand elasticity.

Using equations (2.14) and (2.15) in the power balance(2.11), we can express the hourly electricity price at time t as a function of all the Gencos' offer parameters as follows,

$$\lambda_t = \frac{\left(\frac{\lambda_t^0}{\alpha_t}\right) + \sum_i \left(\frac{a_{it}}{b_{it}}\right)}{\sum_i \left(\frac{1}{b_{it}}\right) + \frac{1}{\alpha_t}} \tag{2.16}$$

From(2.16), we see that by increasing one or both of its offer parameters a_{it} and b_{it}, at time t, Genco i can increase the electricity price λ_t at that time period. The effect on the hourly market-clearing of gaming strictly with the slope b_{it} of the incremental generation cost function, $IC_{it}(g_{it})$, is seen in Fig. 2.9, while the effect of gaming strictly with the IC-intersect, a_{it}, is seen in Fig. 2.10. Both figures show that the hourly market clears at a higher electricity price, and that the dispatched generation level of the gaming Genco decreases.

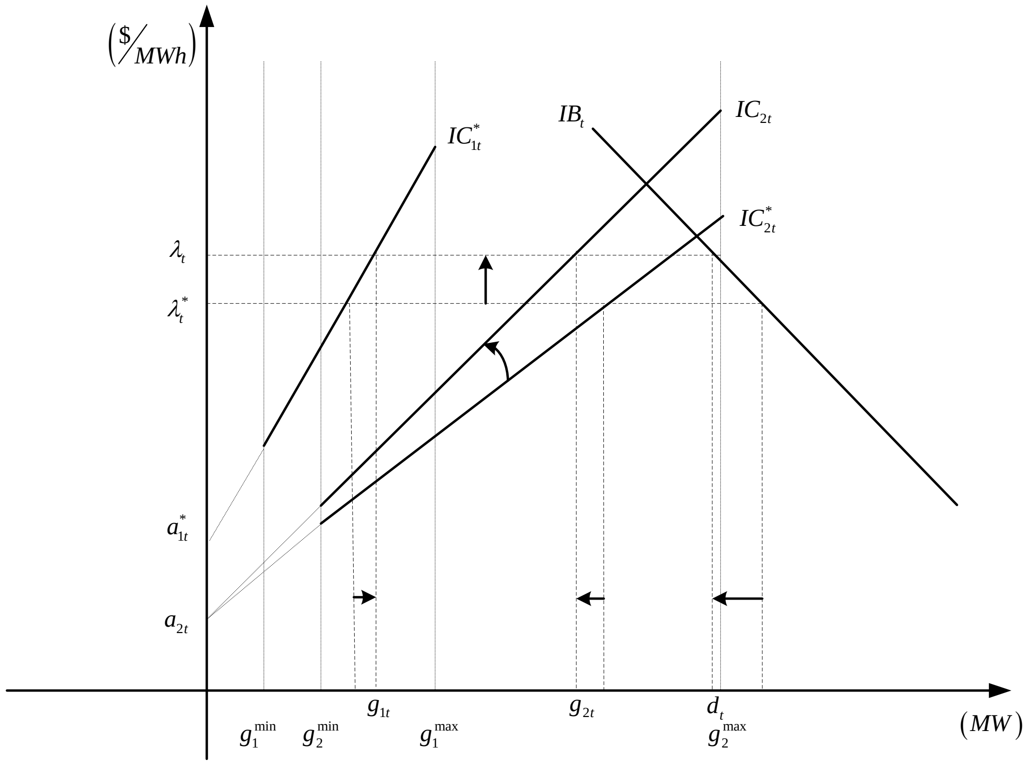

Figure 2-9: Hourly market-clearing when Genco 2 games with the slope b_{2t} of its hourly incremental generation cost function.

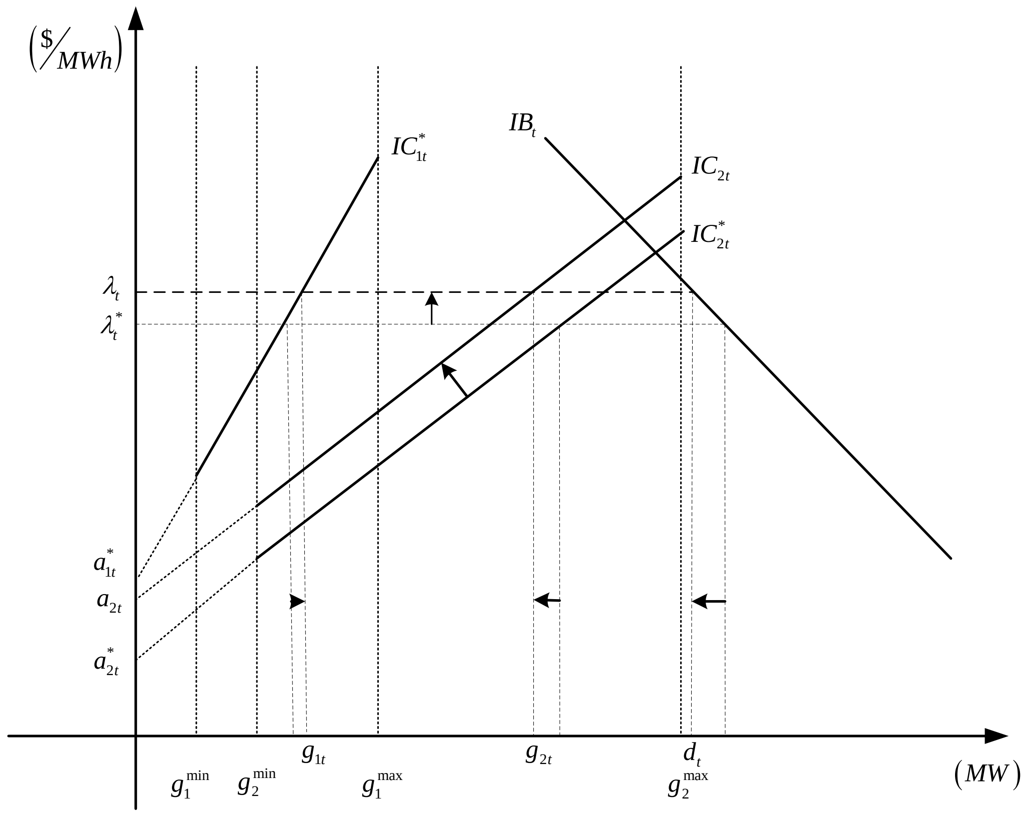

Figure 2-10: Hourly market-clearing when Genco 2 games with a_{2t} , the IC-intersect of its hourly incremental generation cost function.

The question now is how each Genco computes its gaming strategy recognizing that the hourly electricity market-clearing solution will be affected by this gaming.

2.4.1 Nash equilibrium

Nash equilibrium (NE) is one way to model gaming strategies in games where competing players have the ability to affect the outcome and increase their profits. At the Nash equilibrium, if it exists, no competing player (Genco) can increase its utility function[8] given that all the other players (Gencos) do not change their respective strategies. A Nash equilibrium gaming strategy is the strategy that meets the Nash equilibrium condition [105].

Thus, at the NE, the hourly profit of each Genco i given by (2.8) is maximized with respect to the Genco's offered cost function, given that all other Gencos $\{j; \forall j \neq i\}$ do not change their offered cost functions.

In this thesis, we rely on the common assumption that in imperfect electricity markets such as the oligopolistic markets we study, Gencos behave rationally and base their gaming strategies on the NE. However, we recognize that if Gencos game irrationally or if they illegally collude amongst themselves to increase their profits, then the NE solution cannot predict the outcome of the market.

The NE model allows the system operator or social planner to study policy implications on market-clearing in imperfect electricity markets. This model is ideal for comparing the performance of different market designs and market rules, a comparison that is one of the objectives of this thesis.

Finally, it is worthwhile to note that although Gencos might be tempted by the possibility of higher profits by deviating from the Nash equilibrium solution, they would then be susceptible to price wars and the associated risk of making less profit. Still, despite the Nash equilibrium solution ensuring a degree of market stability together with high profits, since we live in a world dictated by greed, rational behavior does not always prevail, as the deep financial crisis of 2008 demonstrated [106].

[8] In this thesis a Genco's utility function is its profit.

2.4.2 Cournot Nash equilibrium

In Cournot games, players (Gencos) compete with their outputs, and a Cournot Nash equilibrium occurs when the conditions of NE are satisfied with respect to the competing players outputs (generation levels) [107].

Thus, in the context of electricity markets, the profit of each Genco i at time t, is maximized with respect to its generation level, g_{it} given that the generation levels of all other competing Gencos $\{j; \forall j \neq i\}$, remain unchanged. That is, the decision variables when computing the Cournot NE strategies at time t are the generation levels $\{g_{it}; \forall i\}$.

Appendix B shows that the hourly market-clearing at each time t will correspond to the Cournot Nash equilibrium if the Genco hourly cost function offers are of the form,

$$C_{it}(g_{it}) = a_{it}^{*}(g_{it}) + 0.5b_{it}(g_{it})^{2}; \forall i \tag{2.17}$$

such that,

$$b_{it} = b_{it}^{*} + \alpha_{t}; \forall i \tag{2.18}$$

Note that the gaming strategies defined by (2.18) correspond to a uniform shift in the slope of the hourly incremental generation cost of each Genco i at each time period t. This shift is defined by the parameter α_t of the hourly demand benefit function $B_t(d_t)$. Recall that this parameter is the inverse of the elasticity of the hourly demand at time t with respect to the hourly electricity price. From (2.18) the lower the elasticity of the demand (higher α_t) the higher is the shift in each Genco's incremental cost offer and the more aggressive the Gencos are in their gaming strategies. This is rational because low demand elasticity means that the consumers are willing to pay almost any price for electricity, thus increasing the market power of the Gencos.

We note that theCournot strategy (2.18) is valid whether or not the Gencos' outputs are subject to upper and lower limits. However, despite the relative computational simplicity of finding the Cournot equilibrium, its implementation is problematic. Since the pool market rules require that the Gencos' offers be in the form of a cost function similar to that of equation (2.17) (as opposed to a constant output), as discussed in the next section, this tempts Gencos to game further by deviating from the Cournot strategy and earn them higher profits.

2.4.3 Supply function Nash equilibrium

Supply function Nash equilibrium (SFE) is an alternative to Cournot NE that can also be used to study gaming strategies and price-setting influences in deregulated electricity markets [54]-[58]. Under supply function equilibrium, power generating companies (Gencos) compete to supply electricity to the consumers through their supply function offers. They do so through price schedules for levels of power they are willing to produce over a range of demand levels[9]. The supply function Nash equilibrium is then satisfied with respect to variations in the Gencos' supply functions. By gaming in price and quantity, SFE offers a higher degree of flexibility over Cournot equilibrium where the gaming is only in quantity. In fact, we conjecture that SFE is closer to the real way in which Gencos game in a pool market. Nonetheless, as will be seen, finding the SFE is considerably more complex than finding the Cournot equilibrium [108], [109].

Gaming through a Nash equilibrium (NE) with respect to supply functions (SFE) was first investigated by Klemperer and Mayer (KM) who proved that, when the units have unlimited capacity and the offered supply functions are

[9] We note that if the SF of a Genco is monotonic in the price, the SF is the inverse function of the Genco IC function. Thus, under this assumption, offers based on SFs are equivalent to offers based on ICs.

differentiable in the electricity price, the SFE is governed by a set of non-linear differential equations [53]. Besides the computational difficulty of solving the KM equations, another drawback is that if the demand is deterministic, then these equations have a continuum of solutions, essentially an infinite number of strategic choices out of which no specific one stands out. In order to get around this non-uniqueness dilemma, Klemperer and Mayer added the assumption that the demand was stochastic with a known duration-curve. Despite this, the computation of the SFE remains a formidable task.

Other important comparisons are: (i) Cournot Nash equilibrium does not pose overwhelming computational difficulties or lack of uniqueness; (ii) SFE does not require that the demand be elastic, which is a more realistic assumption, especially in the short-term.

Finally, in comparing SFE and Cournot, we must also consider that electricity markets generally require that competing Gencos offer price schedules (i.e. supply functions or incremental costs) over a specified time horizon (or range of demand levels), not just a constant output. This is why in order to implement Cournot, the offer function of cost versus output shown in equation (2.18) is used. This cost versus output function is equivalent to Genco *i* offering the supply function,

$$g_{it}(\lambda_t) = \begin{cases} g_i^{\max} & ;\ a_{it}^* + (b_{it}^* + \alpha_t) g_i^{\max} \le \lambda_t \\ \dfrac{\lambda_t - a_{it}^*}{b_{it}^* + \alpha_t} & ;\ a_i^* + (b_{it}^* + \alpha_t) g_i^{\min} \le \lambda_t \le a_{it}^* + (b_{it}^* + \alpha_t) g_i^{\max} \\ g_i^{\min} & ;\ \lambda_t \le a_{it}^* + (b_{it}^* + \alpha_t) g_i^{\min} \end{cases} \qquad (2.19)$$

However, if all Gencos follow the Cournot strategy and offer according to equation (2.19), there is an incentive for any one Genco k to break from the joint Cournot strategy and offer a more profitable SF. To see this, recall that the hourly profit of Genco *k* at time t is,

$$pr_{kt} = \lambda_t g_{kt} - C_{kt}^*\left(g_{kt}\right) \tag{2.20}$$

Now, we show that if all Gencos but Genco *k* offer according to (2.19), Genco *k* can alter its supply function offer and improve its profit as shown next. Although the proof is valid for the general case with active generation limits, for simplicity, we assume that all Gencos operate within their limits.

First, if Genco *k* alters its SF by dg_{kt}, then from the power balance, it follows that,

$$dg_{kt} = -\left(\frac{d\lambda_t}{\alpha_t} + \sum_{i \neq k} \frac{d\lambda_t}{b_{it}^* + \alpha_t}\right) \tag{2.21}$$

Then, the increment in the profit of Genco *k* becomes,

$$dpr_{kt} = d\lambda_t g_{kt} - \lambda_t\left(\frac{d\lambda_t}{\alpha_t} + \sum_{i \neq k} \frac{d\lambda_t}{b_{it}^* + \alpha_t}\right) + IC_{kt}^*\left(g_{kt}\right)\left(\frac{d\lambda_t}{\alpha_t} + \sum_{i \neq k} \frac{d\lambda_t}{b_{it}^* + \alpha_t}\right) = 0 \tag{2.22}$$

which simplifies to,

$$\lambda_t = IC_{kt}^* + \left(\frac{1}{\dfrac{1}{\alpha_t} + \sum_{i \neq k} \dfrac{1}{b_i^* + \alpha_t}}\right) g_{kt} \tag{2.23}$$

Thus, if the non-complying Genco k offers according to,

$$C_{kt}\left(g_{kt}\right) = a_{kt}^*\left(g_{kt}\right) + 0.5 b_{kt}\left(g_{kt}\right)^2 \tag{2.24}$$

where,

$$b_{kt} = b_{kt}^* + \left(\frac{1}{\dfrac{1}{\alpha_t} + \sum_{i \neq k} \dfrac{1}{b_i^* + \alpha_t}}\right) \tag{2.25}$$

then the profit of the non-complying Genco k at time t will be higher than its profit under the Cournot Nash equilibrium as will be illustrated in section 2.5.2.

Thus, although Cournot is easier to calculate and, as will be seen, produces higher profits than SFE, its implementation in real electricity markets makes it susceptible to further gaming and price instability. It is therefore conjectured that SFE models more closely the gaming strategies of Gencos in actual electricity markets.

Despite these limitations, Cournot models are still extensively applied in numerous fields to study the behavior of gaming strategies [110]. In this thesis, we also employ Cournot to model gaming in electricity markets subject to emission regulation schemes. In Appendix C however we develop a novel SFE method which is applicable under generation constraints [111][10]. This Appendix also derives the SFE when Gencos game with the IC offer intersects (a - parameters) under inactive generation limits leading to cost function offers of the form,

$$C_{it}(g_{it}) = a_{it}^{*}(g_{it}) + 0.5b_{it}(g_{it})^{2}\ ;\forall i \tag{2.26}$$

where,

$$b_{it} = b_{it}^{*} + \frac{1}{\sum_{k \neq i} \frac{1}{b_{kt}^{*}}} \qquad ;\forall i \tag{2.27}$$

We point out the similarities among the strategic offers under Cournot in equation (2.18), for the non-compliant Genco in equation (2.25), and finally

[10] A variation of this method that derives SFE under emission constraints is a challenging problem and a timely research prospect.

under SFE when Gencos game with the IC intersect in equation (2.27). In the next section, these three strategies are compared numerically.

2.5 Quantitative Analysis of "Business-as-Usual" Market-Clearing

In this section, we present examples of hourly market-clearing under perfect market conditions and compare them to the imperfect oligopolistic market case when Gencos game. The perfect market and the imperfect market with gaming define what is called "business-as-usual" or BAU. Later in this thesis, we compare BAU with how markets clear when we impose emission regulation schemes and Gencos game not only on the basis of their true cost but on the basis of how polluting they are. This comparison is one of the major aims of this research.

To illustrate the BAU case, we make the following assumptions:

1) Demand level: we consider two demand levels, one with demand benefit parameter λ_t^0 set at a relatively low value of 400 \$/MWh (if the electricity price is above 400 \$/MWh, the demand is zero). This choice results in the market clearing at a relatively low demand level. Alternatively, we also consider the case where the demand benefits more by consuming electricity with $\lambda_t^0 = 700$ \$/MWh, which leads to relatively higher demand;

2) Gaming model: We model the Gencos' gaming strategies by computing the Nash equilibrium under Cournot and under supply function gaming, the latter being carried out on the IC intersect parameter, a . We also present the market-clearing outcome when one of the Gencos, Genco 2, deviates from the Cournot strategy and instead offers based on (2.25). This non-compliant strategy is referred to as "Cournot-2".

2.5.1 Simulation parameters

The parameters of the Gencos' cost functions and emission intensities for both demand levels are given in Table 2-1, while those of the hourly demand benefit function are given in Table 2-2. Note that the same example with the same parameters will be used throughout the thesis. A Genco's emission intensity is the tons of carbon a Genco emits per MWh output it produces. Emission intensity is a measure of how "clean" versus how "polluting" a Genco is.

In the illustrative examples used throughout this thesis, of the four Gencos, Genco 1, is the most polluting but the cheapest to operate while Genco 4 is the least polluting but the most expensive to operate. This illustrates the not-unusual case of a coal-fired power plant with high emission intensity and low fuel cost competing against, say, a gas-fired power plant with high incremental cost but low emission intensity. Beyond the BAU model, when we consider emission regulations, it will be shown that Gencos can then game according to how polluting they are in addition to how high their operational costs are.

	Genco 1	Genco 2	Genco 3	Genco 4
Linear true cost parameter: a_{it}^{*} $(\$/MWh)$	10	15	20	50
Quadratic true cost parameter: b_{it}^{*} $\left(\$/MW^{2}h\right)$	0.05	0.05	0.05	0.05
Emission intensity: ρ_i (t/MWh)	2	1	0.8	0.5

Table 2-1: Gencos' true generation cost parameters and emission intensities.

	Low demand level	High demand level
Linear demand benefit parameter: λ_t^0 $(\$/MWh)$	400	700
Quadratic demand benefit parameter: α_t $\left(\$/MW^{2}h\right)$	0.8	0.8

Table 2-2: Demand benefit parameters.

2.5.2 Simulations

Tables 2-3 and 2-4 compare four sets of market-clearing results when all Gencos offer: (1) At true cost (No Gaming); (2) According to Cournot (Cournot); (3) According to SFE when gaming with the offered IC intersect, a (SFE); (4) According to Cournot while Genco 2 is non-compliant and offers so as to maximize its own profit as shown in equation (2.25) (Cournot-2).

2.5.2.1 Results for low demand level

		No gaming	Cournot	Cournot -2	SFE
Offered linear cost parameters ($\$/MWh$)	a_{11}	10	10	10	15
	a_{21}	15	15	15	19
	a_{31}	20	20	20	22
	a_{41}	50	50	50	-
Offered non-linear cost parameters ($\$/MW^2h$)	b_{11}	0.05	0.85	0.85	0.05
	b_{21}	0.05	0.85	0.26	0.05
	b_{31}	0.05	0.85	0.85	0.05
	b_{41}	0.05	0.85	0.85	0.05
Demand level (MW)	d_1	471	372	406	467
Generation levels (MW)	g_{11}	257	109	77	223
	g_{21}	157	103	233	156
	g_{31}	57	97	65	88
	g_{41}	0	62	30	0
Electricity marginal price ($\$/MWh$)	λ_1	23	103	76	27
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	717	430	455	672
Genco profits ($\$/h$)	pr_{11}	1,653	9,816	4,897	2,448
	pr_{21}	617	8,786	12,753	1,192
	pr_{31}	82	7813	3,516	384
	pr_{41}	0	3,173	742	0
Load surplus ($\$/h$)	pr_{d1}	88,898	55,236	65,818	87,165
Social-welfare ($\$/h$)	SW_1	91,250	84,823	87,725	91,188

Table 2-3: Market-clearing at the low demand level period with and without gaming.

2.5.2.2 Analysis of Table 2-3

i) <u>No Gaming</u>

Here Gencos naively do not game and their hourly offers are their true generation costs. Genco 1 with the lowest true generation cost has the highest level of MW production, while the generation level of Genco 4, with the highest true generation cost, does not produce. This is reasonable as the market-clearing tries to find the optimum generation dispatch that will meet the demand at the least total cost.

Since the cheaper Gencos generate more power their revenues from the sale of electricity are higher. The hourly Genco profits also follow this pattern. The cheapest Genco 1 makes the highest hourly profit at 1,653 $/h, and the most expensive Genco 4 makes the lowest at zero.

Note that emission levels have no bearing on the market-clearing solution in this "business-as-usual" (BAU) case.

ii) Gaming

Under Cournot and SF equilibrium models, when Gencos game and increase their offers, the hourly electricity price increases. In addition, the most expensive Genco 4, previously dispatched at zero output under true cost offers, now, under Cournot gaming, produces non-zero output. In general, all Gencos' profits increase with gaming however the biggest profit increase occurs with Cournot gaming.

Under Cournot, the increase in electricity price is substantial as compared to the increase under SF equilibrium. This is because the Gencos' gaming strategies under Cournot are strictly dependent on the elasticity of the demand with respect to the price. Alternatively, under SFE a Genco's offer considers the supply function offers of all other Gencos. Thus, the only restriction to a Genco's market power under Cournot is the demand elasticity, while under SFE a Genco's market power is also restricted by the other Gencos' offers. As a result, the Gencos' profits under Cournot are higher than under SFE.

Moreover, by gaming, in general, cheaper Gencos still make higher profits than more expensive Gencos. That is cheaper Gencos have more market power than more expensive ones in oligopolistic markets with no emission regulation.

From column 5, we see that if Genco 2 becomes non-compliant with the Cournot strategy as discussed in the previous section, then it stands to increase its profit significantly even though the price drops. This suggests that the Cournot strategy is excessively aggressive and can entice Gencos to deviate from it to their benefit.

2.5.2.3 Results for high demand level

		No Gaming	Cournot	Cournot-2	SFE
Offered linear cost parameters $(\$/MWh)$	a_{12}	10	10	10	18
	a_{22}	15	15	15	22
	a_{32}	20	20	20	25
	a_{42}	50	50	50	-
Offered non-linear cost parameters $(\$/MW^2h)$	b_{12}	0.05	0.85	0.85	0.05
	b_{22}	0.05	0.85	0.26	0.05
	b_{32}	0.05	0.85	0.85	0.05
	b_{42}	0.05	0.85	0.85	0.05
Demand level (MW)	d_2	839	668	726	831
Generation levels (MW)	g_{12}	380	183	128	344
	g_{22}	280	177	401	277
	g_{32}	180	171	116	210
	g_{42}	0	136	81	0
Electricity marginal price $(\$/MWh)$	λ_2	29	166	119	36
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i2}$	1,182	749	791	1,012
Genco profits $(\$/h)$	pr_{12}	3,602	27,674	13,543	5,834
	pr_{22}	1,954	25,925	37,631	3,774
	pr_{32}	806	24,233	11,170	2,161
	pr_{42}	0	15,280	5,421	0
Load surplus $(\$/h)$	pr_{d2}	281,418	178,437	211,045	275,930
Social-welfare $(\$/h)$	SW_2	287,781	271,550	278,810	287,700

Table 2-4: Market-clearing at the high demand level period with and without gaming.

2.5.2.4 Analysis of Table 2-4

Under perfect market conditions or with gaming under Cournot or SF equilibrium, the electricity price and demand levels are higher for the higher demand benefit function. This is predictable since the benefit obtained by consuming electricity is higher and, in maximizing SW, the pool puts more emphasis on increasing the demand level. Consequently, the Gencos are dispatched at higher MW levels, thus increasing the net marginal cost of production and therefore the electricity price. Moreover, due to the higher generation levels and higher electricity prices, the profits of all Gencos are higher compared to the case with lower demand benefit.

Under a higher demand level, Gencos with low generation costs increase their profits by a higher percentage by gaming (under both Cournot and SF) than the corresponding increase with lower demand. On the other hand, Gencos with higher generation costs increase their profits by a lower percentage by gaming under higher demand. For example, with low demand, under Cournot, Genco 1 increases its profit by 494 %, while with higher demand, under Cournot, the same Genco increases its profit by 668 %. On the other hand, for low demand, the more expensive Genco 3 increases its profit by 9,470 % by gaming, while by gaming at high demand; it increases its profit by 2,905 %. Similar results occur under SFE.

This indicates that the market power of Gencos with low generation costs is enhanced during time periods with high demand levels, while the market power of Gencos with high generation costs is diminished during time periods with high demand levels. This is because at high demand levels, expensive Gencos cannot increase their generation levels by the same proportion as at lower demand levels; otherwise the market-clearing will not be economically optimal.

2.5.3 Major findings from simulations on the analysis of BAU market-clearing under gaming

1) Under market-clearing based on maximizing social-welfare, cheap Gencos are dispatched to produce more MW output than the more expensive Gencos and thus earn higher profits. This is true irrespective of whether they game or not;

2) The effect of demand level on the market-clearing and on the Gencos' profits and market power is significant, the higher the greater is market power;

3) The market-clearing and resulting Gencos' profits are independent of the Gencos' emission intensities under a market model that does not take into account emission regulation.

2.6 Emission Regulation in Electricity Markets

Under Business-as-Usual, the hourly market-clearing is based on economic dispatch and does not consider constraints on emission levels. Thus, the BAU Genco gaming strategies only take into account generation costs, generation limits and demand elasticity, excluding emission intensities. This leads to cheap but dirty Gencos making excessively higher profits compared to the cleaner but more expensive Gencos. Thus, under this market model there are no incentives for Gencos to invest in reducing their emission intensities, which can then lead to rising emission levels as the demand increases.

Therefore we propose that effective emission regulation in the electricity sector should:

1) Produce economic signals for investing in emission reduction technologies: The market-clearing structure should be modified to favor the dispatch of cleaner Gencos over more polluting ones, even though the latter might be more expensive. Low polluting Gencos should be able to increase their profits compared to their BAU profits and to high polluting Gencos. The effective generation incremental cost of low polluting Gencos should be reduced under the new market rules accounting for emission regulation, thus encouraging investment in low polluting generation alternatives;

2) Reduce long-term emissions through short-term implementation: Emission reduction normally applies to a specified long time-horizon, typically of a year or more. However, this long-term target can and should be implemented within the structure of the short-term electricity market based on hourly market-clearing by setting short-term emission targets that add up to the long-term target. We claim this since the hourly day-ahead market-clearing structure is well-established and will therefore continue to operate even if emissions are accounted for.

To achieve these two goals, this thesis therefore remodels the current electricity market structure in the short and medium term horizons to account for emission regulation. We do this by introducing into the electricity market structure two emission regulation approaches from those being actively considered by governments, cap-and-trade (CAT) and carbon tax (CTX). An alternative emission regulation scheme is an intensity-based one where the goal is to reduce the average emission intensity of a power system. In this thesis the reference point for comparing different emission regulation schemes is BAU.

2.7 Chapter summary

Electricity has become a commodity traded through multifaceted markets. In pool-based markets, the market operator receives hourly offers to sell from the competing Gencos and matches these offers against the hourly demand benefit function. The market operator then computes hourly generation dispatch and demand levels as well as electricity prices. In an oligopolistic market with relatively few competitors, Gencos game by submitting offers to sell above their true costs, thus raising the market-clearing prices and increasing their profits.

These Genco gaming strategies are typically modeled through Nash equilibrium models such as Cournot or supply function equilibrium. Cournot leads to more aggressive gaming and higher prices and profits; moreover, Cournot is considerably easier to compute than SFE, which is why Cournot gaming is more commonly used to study gaming behavior. The effect of the demand level on the hourly market-clearing and on market power is significant.

Under Business-as-usual Cournot NE, cheap Gencos have higher market power than more expensive ones, that is they are more competitive as they can offer less aggressively, be dispatched at higher generation levels and make higher profits. Under BAU, emission intensities have no bearing on the hourly market-clearing, on gaming strategies or on profits. Thus, if cheap Gencos have high emission intensities and less polluting Gencos are more expensive, under BAU, emission levels from the electricity sector will be high and will continue to rise as Genco profits will encourage investing in cheap but high polluting Gencos.

Since emissions have to be regulated in the electricity sector, next we modify the electricity market structure by implementing and comparing two common emission regulation schemes: cap-and-trade and carbon tax.

3 Cap-and-Trade Emission Regulation Scheme

"People become attached to their burdens sometimes more than their burdens become attached to them;" George Bernard Shaw.

This chapter starts with a general overview of cap-and-trade systems where we elucidate the main components of this emission regulation scheme. We then present in section 3.2 examples of cap-and-trade systems around the world. Section 3.3 discusses the general challenges of implementing a cap-and-trade emission regulation system in the electricity sector. Finally we conclude in section 3.4 with a brief summary of the chapter and some conclusions.

3.1 Cap-and-Trade Overview

Cap-and-trade (CAT) has become the most discussed emission regulation scheme around the world. It is now a "buzz word" that politicians use in their election campaigns [112]. In-fact, President Obama[11] is an advocate of the cap-and-trade emission regulation approach as is the European Union [114], [115].

The cap-and-trade approach sets a limit on the absolute level of emissions that a group of entities such as countries or regions or sectors produce over a specified time horizon. Under cap-and-trade, capped entities share emission abatement costs by trading emission caps amongst each other. This flexibility allows the relatively more polluting entities to alleviate the financial burden they might face trying to meet the emissions cap.

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[11] Although President Obama's plan to implement a GHG emissions cap-and-trade system in the U.S. is being challenged by his political adversaries, he remains an advocate of the system [113].

3.1.1 Terminology

We now explain the main terminology associated with cap-and-trade systems:

Cap: The limit in tons set on the total absolute level of emissions produced by a group of entities over a specific time horizon.

Entities: Emission caps are enforced on entities such as countries, sectors, industries, or individual installations.

Installations: Installations can be power generating plants, factories, or methods of transportation. The distinction between entities and installations will be made clearer when we discuss the Kyoto Protocol.

Permits: A permit is an allowance or a permission to produce one ton (t) of emissions. A cap is divided into a collection of permits allocated to the chosen entities.

Commitment interval: Is the time horizon over which an emission cap is imposed. In terms of permits, the commitment interval defines the time horizon during which the permits are applicable. At the end of the commitment interval each entity has to have in its emissions bank (registry) enough permits to account for all emissions it produced over the commitment interval. Note than in some cap-and-trade systems that are applied through successive phases or commitment intervals, excess permits at the end of one commitment interval (phase) can be banked for use in a future commitment interval. On the other hand, a permit deficit can result in a high penalty or in a lower cap for the following commitment interval.

Trade: Permits can be traded throughout the commitment interval at an exchange house or bilaterally (over-the-counter) for a price determined by the demand and supply of those permits. It is the trade aspect of this regulation scheme that lessens the financial obligation faced by a capped entity in reducing its emissions. Thus, if the cost of reducing emissions is high for an entity, it can opt to purchase permits at a relatively lower price. On the other hand, an entity whose cost of emission reduction is lower can sell excess permits for a profit. Thus, under perfect market conditions, at equilibrium, the price of emission permits is equal to the emission reduction cost of the entities being capped.

Business-as-usual (BAU): Refers to the conditions before a cap-and-trade emission regulation system is employed. BAU does not refer to the past but refers to the present without emission regulation. BAU operation and BAU emissions are the operation and emissions under these conditions.

3.1.2 Design concerns

A number of concerns arise when attempting to design a cap-and-trade system:

1) **What constitutes emissions:** Although there are many varieties of green-house gases that lead to global warming, all GHG emissions are expressed in equivalents tons of CO_2.

2) **Capped entities:** Which entities are to be capped? That is, should caps be set on country, sector, industry, or on individual installations-basis? The answer to this depends on the purpose of the cap-and-trade system. If it is to mitigate national emissions to meet international requirements, then setting caps on the country level is required. On the other hand, if the purpose of the cap-and-trade system is to send social and political signals on the problem of global warming resulting from emissions, then sector or industrial- based caps are used.

4) **Commitment interval:** How to choose the commitment time horizon? Shorter horizons are superior to relatively longer ones in that they grant the operators of the cap-and-trade system the flexibility to change the cap in subsequent horizons. On the other hand, longer commitment intervals

provide flexibility and allow capped entities to invest in emission-abatement measures to meet their caps.

5) **Intra versus inter-permit trading:** Should permits in one cap-and-trade system be traded in a closed market pertaining to that specific system, or should entities be given the flexibility to trade their permits with other cap-and-trade systems through a global market?

6) **How specific should the caps be?**: If caps are placed on countries then which emission-producing sectors should be included in the caps? For instance, should emissions from the transportation sector within a capped county be accounted for? Moreover should the caps be set on a company owning a fleet of emission-producing installations, or should the caps be set on each of the company's individual installations? Do you cap the production side or the demand side; for instance, in the electricity sector, should one place caps on power generating companies or on load-serving entities?

This thesis examines the aforementioned cap-and-trade concerns through comprehensive and detailed analysis under two themes:

a) **Cap:** How to set the total cap and how will an emissions cap effect market operation, profits and social-welfare.

A hard cap guarantees the desired reduction in absolute levels of emissions over the commitment interval. Nonetheless, the initial cap in a cap-and-trade system has a ripple effect on permit prices, prices of commodities, and ultimately on social-welfare.

A stringent cap will reduce the supply of permits thus making the permits more valuable and raising their price. Although higher permit prices will lead to higher commodity prices and a reduction in social-welfare, they are advantageous in that they signal the need to invest in emission reduction technologies. On the other hand, a more generous cap might be favorable in terms of prices and social-welfare, but will not result in effective emission reduction. Thus, the cap should be a compromise between reducing emissions, maintaining the integrity of the permit prices, and not disrupting the economy adversely.

Another concern is on what basis the cap should be set? For example, as a percentage of the emissions produced by an entity over a given time horizon? If so, how long should this time horizon be and how long into the past should it go? In this thesis, caps are based on BAU emission levels.

2) **Permit allocation:** The effect of a cap-and-trade system on prices and social-welfare is not solely dependent on the initial cap; this effect also depends in a significant way on how this cap is allocated among individual sub-entities.

As mentioned earlier, trading permits allows capped entities to share the costs of reducing emissions. Thus, at equilibrium, the permits price should reflect the true cost of reducing emission within the capped system, and should send a signal for investing in emission-reducing technologies. The price of traded permits throughout the commitment interval is based on demand and supply of these permits, and depends heavily on the initial allocation of these permits.

A situation where the permit allocation results in low demand for permits, and consequently lower prices, might ultimately result in the wrong price signals on the need to invest in reducing emissions. On the other hand, a permit allocation that leads to high permit prices might increase the cost of capped entities that have to buy permits and might result in a ripple-through effect on the prices of commodities produced by these entities.

Allocating the cap amongst the chosen sub-entities is a controversial and intricate design concern. We address this concern comprehensively in the next chapter when we model a cap-and-trade system applied to the electricity sector.

3.2 Global Cap-and-Trade systems

3.2.1 The U.S. Clean Air Act Amendment

The first use of a cap-and-trade system as a market-based emission regulation scheme was in the United States under Title IV of the 1990 Clean Air Act Amendments. The system was aimed at regulating SO_2 emissions from the electricity sector and was implemented in two phases. Phase I started in 1995 and phase 2 began in the year 2000. Phase II covered all the coal-fired power plants with a capacity greater than 25 MW totaling around 1,420 generating units [116].

Other state-based cap-and-trade systems were implemented in the United States to regulate NO_2 on the state level. Several such systems were adopted by some northeast states by the year 1999.

3.2.2 Kyoto Protocol

The first cap-and-trade system implemented on an international level is the Kyoto protocol that came into effect in 2005 and sets emission caps on 37 countries plus the European Union. Under this protocol, the cap is such that the total emissions of six GHGs from specified sectors are to be reduced to 95% of their corresponding 1990 levels during the commitment interval of 2008 until 2012 [117].

The Kyoto protocol provides countries with the opportunity to reduce financial and technical burdens of emission caps through three measures: 1) pre-set individual caps can be translated into group-based caps by forming pools of several countries; 2) emission permits can be traded internationally; 3) additional permits can be obtained by investing in emission-reducing projects in other industrialized countries that are part of the protocol (Joint Implementation) or countries not considered industrialized by the protocol (Clean Development Mechanism) [118].

Under the Kyoto protocol, the entities responsible to reduce their emissions to meet their caps are the countries that have ratified the protocol. Each of these countries can choose to reduce their emissions to meet their cap over the permitted time horizon using any method they find ideal. Some countries (or group of countries) chose to create their own cap-and-trade systems, where they allocate permits they received from the protocol to individual GHG producing installations within their borders in one of four sectors identified by the protocol. These installations can then trade their emission permits within these new regional or local cap-and-trade systems or with the international cap-and-trade system set by the Kyoto protocol. Furthermore, these installations can take advantage of the flexibility offered by the Kyoto protocol and acquire additional permits through the Joint Implementation or Clean Development Mechanism. Fig. 3-1 illustrates the hierarchy and design of the Protocol.

The European Union is one example of a group of countries joining together to form one pool that trade within its own cap-and-trade system under the European Union emissions trading scheme (EU ETS) [119].

In Dec. 2009, world leaders met in Copenhagen for the United Nations Climate Change Conference to negotiate new global and individual emission targets. Unfortunately, there was a lack of general consent on such targets, and the Kyoto Protocol remains the single, biggest international binding agreement for GHG emission targets [120].

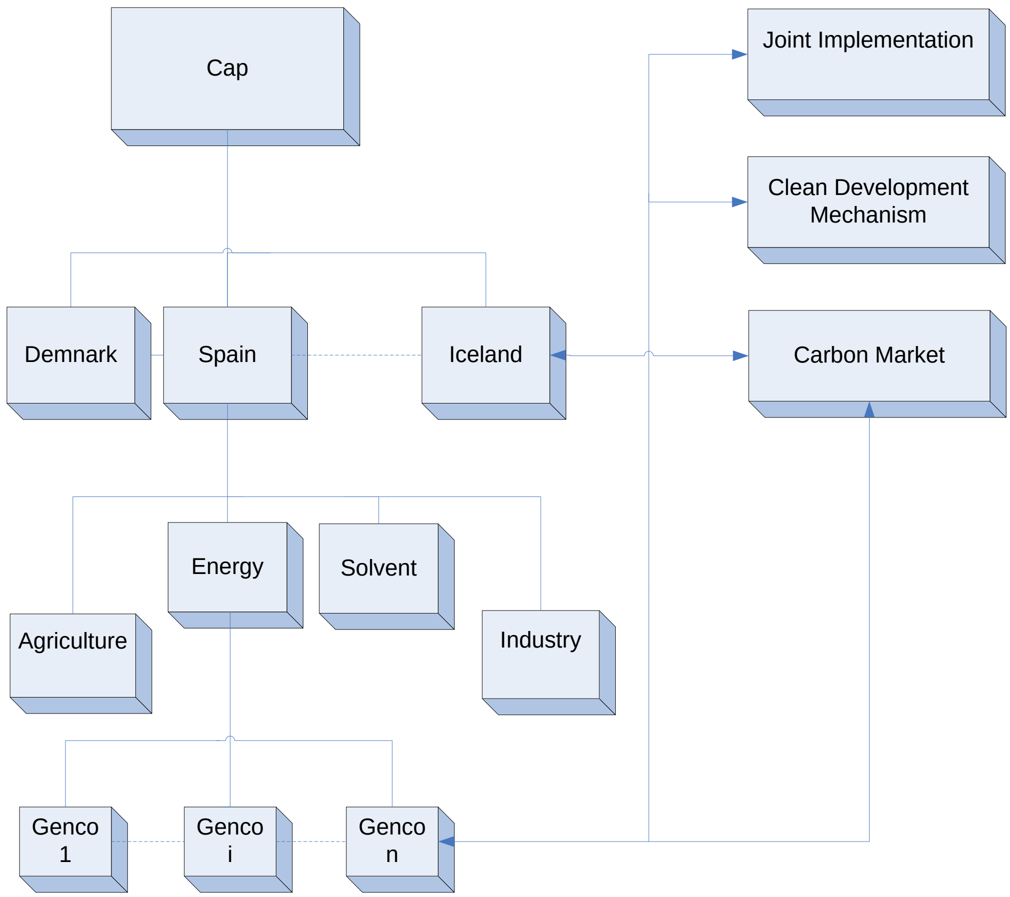

Figure 3-1: Cap-and-trade model under the Kyoto Protocol.

3.2.3 European Emission Trading Scheme (EU ETS)

The biggest cap-and-trade system implemented and currently operating is the European Union Emissions Trading Scheme (EU ETS), which was established in response to the Kyoto protocol [114]. As part of its obligations, the European Union has to reduce its emissions to 92% of its 1990 level during the commitment interval of 2008-2012 [121]-[123]. This total cap is distributed into smaller caps on each member state of the European Union.

Each member state has to present to the European Commission a National Allocation Plan that describes how each state will allocate its emission permits among local installations corresponding to the several phases of the EU ETS [124]. The first phase was the period of 2005-2007; the second phase coincides with the Kyoto protocol first commitment interval, 2008-2012; and finally the third phase will run from 2012 until 2020 [125], [126].

3.2.4 Regional Greenhouse Gas Initiative

In 2009, the first cap-and-trade system in the United States to regulate GHG emissions was launched under the Regional Greenhouse Gas Initiative (RGGI). The initial targets, set for the electricity sector only, include 10 states with the goal of maintaining emissions at levels equivalent to those of 2000, and by the year 2019, at levels equal to 90% of these initial levels [127].

3.2.5 Western Climate Initiative

The western climate initiative (WCI) includes seven U.S. states and four Canadian provinces and is a comprehensive regional effort to reduce GHG emissions by 15 % below 2005 levels by 2010. The system will cover around 90% of emissions from different sectors, including the electricity sector [128].

3.3 Cap-and-Trade Applied to the Electricity Sector

3.3.1 Importance of the electricity sector in capping emissions

The electricity sector is the single largest emission-producing sector in most countries. For instance, according to the 2010 U.S. Greenhouse Gas Inventory Report, by the U.S. Environmental protection Agency, the electricity sector in the U.S. accounted for 2,363 Mt (42%) of the 5,572 Mt of CO_2 equivalent emissions[12] produced in the U.S. in 2008 [129]. According to a similar report by Environment Canada, the electricity sector in Canada accounted for 119 Mt (16%) of the 734 Mt of CO_2 equivalent emissions produced in Canada in 2008 [130]. Finally, the European Environmental Agency states that the electricity sector accounted for 26% of the total GHG emissions produced by the European Union in 2007 [131].

Thus accounting for emissions produced by the electricity sector is essential to maintain the effectiveness of a cap-and-trade system in slowing down global warming.

In addition to being a major polluter, the electricity sector is unique in that it is already deregulated, restructured and liberalized. As discussed in chapter 2, the electricity sector trades electricity though a well-defined, multifaceted market. Thus, the infrastructure to apply a market-based emission regulation scheme such as the cap-and-trade system is readily available.

Moreover, the importance of electricity as a social commodity makes the electricity sector ideal to assess a cap-and-trade emission regulation scheme and

[12] Greenhouse gas emissions are typically expressed in carbon dioxide (CO_2) equivalents based on global warming potentials.

avoid adverse economic ripple results due to its implementation. Extreme negative effects on prices and load surpluses due to errors or misjudgments in the design of the cap-and-trade system are not tolerable [132]. Thus, “getting the cap-and-trade system right” in the electricity sector is vital.

3.3.2 Major design concerns

The main design concerns of cap-and-trade systems in general brought up in section 3.1 are now elaborated upon within the context of the electricity sector.

1) **Cap:** To study the effects of an emissions cap on the operation of electricity markets, the hourly market-clearing and Gencos’ gaming strategies are re-modeled to account for hourly permits trading and Gencos’ emission intensities. Moreover, emission caps set over commitment intervals are divided into hourly fractions considering Gencos’ desire to maximize their profits.

2) **Permit allocation among Gencos:** Under current cap-and-trade systems, the allocation of the sector cap among Gencos is performed using two methods, grand-fathering (GF) and auctioning:

a) Grand-fathering: Under this method, emission permits are allocated to Gencos for free based on a set benchmark that favors polluting Gencos over less polluting ones. This benchmark can be the Gencos’ emission intensities or historical emission levels. In this thesis, permits are grand-fathered based on the Gencos’ BAU emission levels. The aim here is to reduce emissions while avoiding drastic deviations from BAU market-clearing, particularly in what concerns electricity prices, profits and social-welfare.

b) Auctioning: Under this allocation method, an auction is held where Gencos bid to buy permits for the entire commitment interval. The price of permits is determined by the auction type and by the Gencos’ bids.

3.3.3 Grand-fathering versus Auctioning

We now present a qualitative comparison between both emission permits allocation schemes, while in the next chapter we present a more exhaustive quantitative analysis.

3.3.3.1 Grand-fathering

Without GF, Gencos with high BAU emission levels would need to buy considerable emission permits over the commitment interval to meet their cap. Buying permits is an added burden on Gencos that is reflected in their marginal generation costs, changing their hourly offers to sell electricity under imperfect markets, affecting hourly market-clearings, and ultimately varying profits and social-welfare.

Thus, under GF, additional permits are allocated to Gencos with higher BAU emissions, thus reducing the number that have to be purchased by these Gencos; ultimately reducing the effect of the cap on BAU market-clearing and social-welfare.

There are several disadvantages to grand-fathering permits:

i) Gencos are not necessarily interested in following BAU market-clearing if by deviating from BAU they can increase their profits. A cap-and-trade system provides Gencos with the potential to deviate from BAU by trading permits in an imperfect permits market. Thus, giving Gencos free permits, some argue, is in essence giving them a "free-ride" to abuse the trade aspect of the cap-and-trade system;

ii) Grand-fathering can also be viewed as rewarding polluting Gencos for their BAU emissions, especially if high polluting Gencos, with high BAU emissions, end up making relatively high profits under the cap-and-trade system. This does not give the proper signal for Gencos to invest in reducing their emissions in the long-term, which refutes the long-term purpose of an emission regulation scheme;

iii) Having the social planner decide on the allocation of GF permits leads to undue negotiation, often of a political nature, which is an added burden on the planner.

3.3.3.2 Auctioning

The main advantage of allocating permits at a price is to avoid giving Gencos the "free ride" discussed earlier. The expected decrease in Gencos' profits due to the auctioning of permits will provide the required incentive for Gencos to invest in long-term emission reduction technologies. Moreover, an auction obliges Gencos to decide on their own how many permits they require over the commitment interval, thus relieving the social planner from this burden.

Nonetheless, auctioning permits has some drawbacks:

i) An auction is similar to the market-clearing process discussed in chapter 2, but takes place only once at the beginning of a commitment interval (or several times during that interval) and the commodity sold is emission permits as opposed to electricity. Having an auction for permits creates new gaming opportunities for Gencos. For example, Gencos can bid to acquire a large proportion of the offered permits at the auction, thus controlling the supply and ultimately the price of the permits throughout the commitment interval. Moreover, due to the ripple effect of the allocation of permits amongst Gencos on the hourly electricity market-clearing, auctioning permits creates another channel through which Gencos can influence the hourly electricity prices;

ii) Designing and running the auction is yet another challenge for the social planner as are the allowed participants and what to do with the revenue from the auction.

Examples of permit auctions in current global cap-and-trade systems are:

(a) Under the RGGI, all initial permits will be auctioned to the power generating companies operating in the states under regulation;

(b) Under the EU ETS, in phase 1 most permits were allocated free of charge with only around 1 % of the initial permits auctioned [133]. In phase 2, 10 % of initial permits were auctioned, and in phase 3 the European commission stipulates that no permits will be given free of charge for power generating companies and all permits will have to be auctioned in the electricity sector [134].

3.3.3.3 Maximum social-welfare

One of the contributions of this thesis is the permit allocation scheme that maximizes social-welfare by taking into account the effect of commitment interval permits, as well as Gencos' emission intensities and generation costs, on hourly permits trading, gaming strategies, hourly market clearing and total profits. The scheme can be applied to an electricity market were permits are allocated to Gencos for free by the SP, or to one where permits are allocated through an auction. Under a free allocation, just like GF, the SP has the final say in the permit allocation; while under an auction Gencos influence the allocation through their bids. The mechanics of this scheme will be explained gradually in the next chapter, and the quantitative comparison between free allocation and auction-based allocation will be presented at the end of the chapter.

3.4 Chapter summary

The flexibility offered by a cap-and-trade system renders it an attractive instrument to reduce emissions in the electricity sector. Nonetheless, with this flexibility some drawbacks arise, principally: design concerns due to the complexity of cap-and-trade systems; and potential for market abuse which may lead to enhanced market power and undesired Genco profits. Thus an efficient and effective cap-and-trade system applied to the electricity sector is one that reduces emissions in the sector by the desired amount, with minimal upset to BAU market-clearing, and without rewarding high polluting Gencos with relatively high profits.

In the next chapter we study the effects of emission caps and permit allocation among Gencos, on the operation of an electricity market with an hourly market-clearing that is re-modeled to account for hourly emission caps and permits trading. We also present and discuss a novel permit allocation scheme that maximizes social-welfare and can accept Genco bids for additional permits.

4 Electricity Markets Operating under Cap-and-Trade

"Calling upon thought to collect or otherwise integrate your understanding about the conflict you feel is like asking the pieces of a jigsaw puzzle to assemble themselves;" Guy Finley

In this chapter, we expand on the two main concerns relating to electricity markets operating under cap-and-trade: The first is how does the allocation of the total permits granted to the electricity sector (cap) among the competing Gencos affect the hourly clearing of an imperfect electricity market operating in conjunction with an imperfect external emission permits market. The second is how do individual Gencos self-allocate their total permits at the hourly level in a strategic manner to benefit from gaming in both electricity and permits markets.

In addition, in this chapter we develop a permit allocation scheme that maximizes social-welfare by accounting for the effect of the permit allocation on hourly market-clearings, considering that each Genco self-allocates its permits strategically into hourly fractions. We also investigate the effects of how Gencos' bids influence this allocation scheme under a novel permit auction model run by the social planner.

4.1 Nomenclature

The following is a partial nomenclature for this chapter (some of the nomenclature from chapter 2 is repeated for easy access).

Parameters:

ng : Number of Gencos;

nt : Number of time periods in the time horizon under scrutiny;

Δt : Length of time period;

$\boldsymbol{\rho} = \{\rho_i; \forall i\}$: Vector of Genco emission intensities (t/MWh);

$\mathbf{g}^{\max} = \{g_i^{\max}; \forall i\}$: Vector of Genco maximum generation levels (MW);

$\mathbf{a}_t^* = \{a_{it}^*; \forall i\}; \forall t$: Vector of first-order parameters of Genco actual cost functions at time t (\$/MWh);

$\mathbf{b}_t^* = \{b_{it}^*; \forall i\}; \forall t$: Vector of second-order parameters of Genco actual cost functions at time t (\$/MW^{2}h);

λ_t^0 : First-order parameter of demand benefit function at time t (\$/MWh);

α_t : Second-order parameter of demand benefit function at time t (\$/MW^{2}h);

γ_t^0 : First-order parameter of cost of emission permits function at time t (\$/t);

β_t : Second-order parameter of cost of emission permits function at time t (\$h/t^2);

e^0 : Total emissions cap on the electricity sector over the commitment interval or specified time horizon (t);

$\mathbf{e} = \{e_i; \forall i\}$: Vector of total permits allocated to Gencos (t);

$\boldsymbol{\pi}^0 = \{\pi_i^0; \forall i\}$: Vector of first-order parameters of Genco bid functions at the permits auction ($/t);

$\boldsymbol{\pi}^1 = \{\pi_i^1; \forall i\}$: Vector of second-order parameters of Genco bid functions at the permits auction ($h/t^2);

$\mathbf{e}^f = \{e_i^f; \forall i\}$: Vector of free permits allocated to Gencos (t);

Variables:

$\mathbf{a}_t = \{a_{it}; \forall i\}; \forall t$: Vector of first-order parameters of Genco offered cost functions at time t ($/MWh);

$\mathbf{b}_t = \{b_{it}; \forall i\}; \forall t$: Vector of second-order parameters of Genco offered cost functions at time t ($/MW^{2}h);

$\mathbf{g}_t = \{g_{it}; \forall i\}; \forall t$: Vector of Genco output levels at time t (MW);

d_t : Demand level at time t (MW);

$\mathbf{e}_t = \{e_{it}; \forall i\}; \forall t$: Vector of hourly permits self-allocated by Gencos at time t (t/h);

Δe_t : Net hourly permits traded by the electricity sector at time t (t/h);

$\mathbf{e} = \{e_i; \forall i\}$: Vector of Genco permits over the commitment interval (t);

Lagrange multipliers:

λ_t : Price of electricity at time t ($/MW);

γ_t : Price of emission permits at time t ($/t);

$\boldsymbol{\mu}_t^{\max} = \{\mu_{it}^{\max}; \forall i\}; \forall t$: Vector of Lagrange multipliers of maximum generation constraints at time t ;

$\boldsymbol{\mu}_t^{\min} = \{\mu_{it}^{\min}; \forall i\}; \forall t$: Vector of Lagrange multipliers of minimum generation constraints at time t ;

σ : Auction price of the emission permits allocated for the entire commitment interval ($/t);

<u>Functions:</u>

$C_{it}(g_{it})$: Generation cost function offer by Genco i at time t ($/h);

$IC_{it}(g_{it})$: Incremental generation cost function offer by Genco i at time t ($/MWh);

$B_t(d_t)$: Demand benefit function at time t ($/h);

$IB_t(d_t)$: Incremental demand benefit at time t ($/MWh);

$CEP_t(\Delta e_t)$: Emission permits trading function at time t ($/h);

$ICEP_t(\Delta e_t)$: Incremental cost of emission permit trading function at time t ($/t);

4.2 Introduction

4.2.1 Solution outline

As explained in chapter 2, under BAU electricity markets, Gencos submit hourly offers to supply electricity to the pool, and based on these offers and on hourly demand benefit parameters, the market clears on an hourly basis. The market-clearing equilibrium defines generation levels, electricity prices, demand levels, as well as profits.

In cap-and-trade (CAT) systems, the standard electricity market model is extended to allow for Genco emission caps and emission permits trading. Since Genco caps are allocated over long-term commitment intervals, to integrate a cap-and-trade system into an hourly electricity market, each Genco self-allocates a fraction of its long-term emission cap to each hour interval during which the Genco can also buy or sell permits from an external emission permits market.

This gives rise to the following four challenges addressed in detail in this chapter:

(1) Remodeling the hourly market-clearing discussed in Chapter 2 to include individual Genco hourly caps and emission permits trading;

(2) Developing a Cournot-based gaming strategy at the hourly market-clearing step in which Gencos take advantage of: (a) Electricity demand elasticity and incremental cost, (b) Permits price elasticity, self-allocated hourly permits and emission intensity;

(3) Dividing into hourly amounts the total commitment interval permits allocated to each Genco. This is done either heuristically or strategically according to the Nash equilibrium;

(4) Allocating the electricity sector cap among Gencos. This is done either for free or through an auction.

4.2.2 Chapter outline

The model for the hourly market-clearing under cap-and-trade, including hourly permits trading and hourly Genco gaming is presented and derived in section 4.3. In section 4.4, the hourly Genco self-allocation of the commitment interval permits under a heuristic and strategic approach is discussed. Section 4.5 addresses how to allocate the commitment interval permits to Gencos either for free or through our proposed auction. Finally, section 4.6 contains a brief summary of the chapter and its main conclusions.

4.3 Hourly Market-Clearing with Emission Permits Trading

4.3.1 Emission permits trading cost function

We first define the hourly emission permits trading cost function, $CEP_t(\Delta e_t)$ which models the hourly cost at time *t* to the electricity sector of trading permits, Δe_t, with the external market[13]. This function is defined as follows,

$$CEP_t(\Delta e_t) = \gamma_t^0 \Delta e_t + 0.5\beta_t (\Delta e_t)^2 \tag{4.1}$$

where the positive parameters γ_t^0 and β_t are assumed known[14] for all t. Note that this cost is positive when buying permits ($\Delta e_t > 0$) and negative when selling ($\Delta e_t < 0$). In addition, as shown in Fig. 4-2, the incremental cost of permits traded with the external market at time *t*, $ICEP_t(\Delta e_t) = \gamma_t^0 + \beta_t \Delta e_t$, increases with Δe_t. Finally, although the permits traded, Δe_t, have no limits, we do assume that, $ICEP_t(\Delta e_t) = \gamma_t^0 + \beta_t \Delta e_t > 0$, which is equivalent to requiring that, $\Delta e_t > -\gamma_t^0 / \beta_t$, a selling lower bound that is unlikely to be violated.

[13] It is assumed that there exists a world market where emission permits are traded at a price that depends on the demand for such permits in a known manner. As one of the major participants in this market, the electricity sector therefore has an influence on the price of permits.

[14] γ_t^0 and β_t are estimated from historical emission permit prices and how these prices vary with the demand for such permits [135].

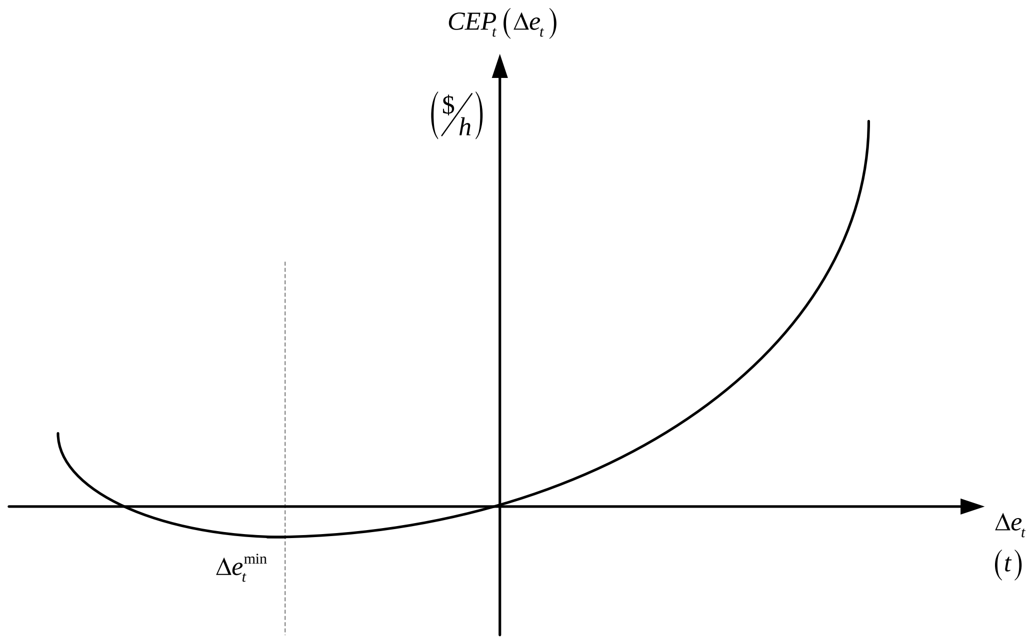

Figure 4-1: Hourly emission permits trading cost function.

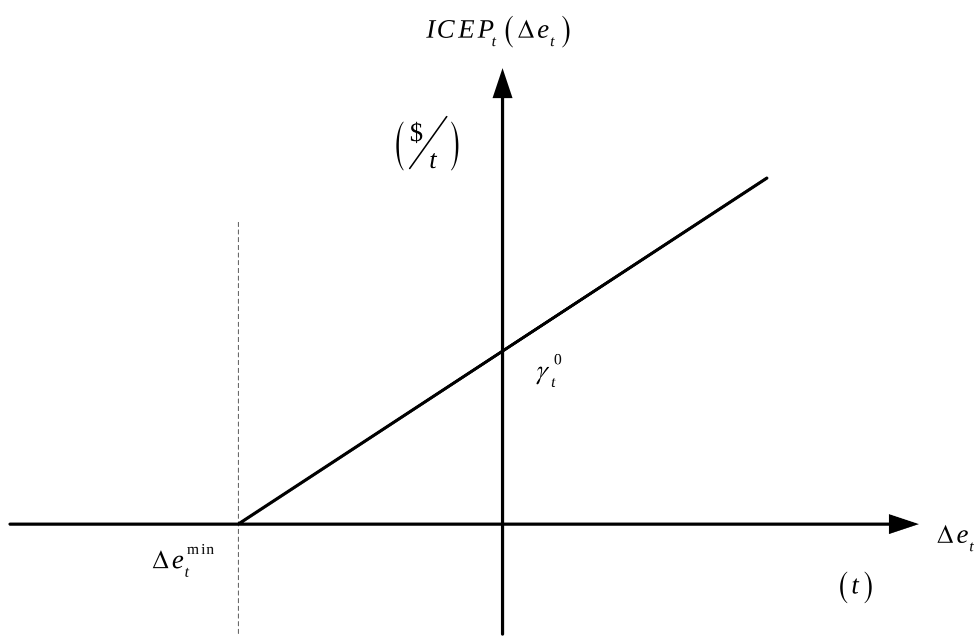

Figure 4-2: Hourly emissions permit trading incremental cost function.

4.3.2 Hourly pool dispatch including permits trading with external market

To implement the cap-and-trade system in the electricity sector, we re-model the market-clearing process at time *t* by modifying the defining optimization problem of (2.1) - (2.4) to include the cost to the electricity sector of buying external permits, $CEP_t(\Delta e_t)$. This problem takes the form,

$$\min_{\{g_{it};\forall i\},d_t,\Delta e_t} \sum_i C_{it}(g_{it}) - B_t(d_t) + CEP_t(\Delta e_t) \tag{4.2}$$

subject to the hourly power balance,

$$\sum_i g_{it} = d_t \qquad (\lambda_t) \tag{4.3}$$

to the hourly emission permits equation ,

$$\Delta e_t = \sum_i \left(\rho_i g_{it} - e_{it}\right) \qquad (\gamma_t) \tag{4.4}$$

and to the upper and lower generation limits, respectively,

$$g_{it} \le g_{it}^{\max} \,;\, \forall i \qquad (\mu_{it}^{\max}) \tag{4.5}$$

and,

$$g_{it}^{\min} \le g_{it} \,;\, \forall i \qquad (\mu_{it}^{\min}) \tag{4.6}$$

The minimization in (4.2) is now carried out over the hourly variables, $\left(\{g_{it};\forall i\}, d_t\right)$ as well as with respect to the new decision variable, Δe_t, the total permits bought by the electricity sector. The new objective function is subject to the power balance equation(4.3), to the generation limits(4.5) and (4.6), as well as to(4.4), a new equality between Δe_t , the permits bought at time t by the electricity sector for all Gencos, and the difference between the total emissions produced at time t and the sum of the self-assigned permits at that hour. This hourly difference is balanced by either buying from or selling to the external emission permits market[15].

We note that in the hourly market-clearing stage at time t the quantities $\{e_{it}; \forall i\}$ are treated by the market operator as parameters chosen by each Genco i. However, as shown later in this chapter, when the Gencos compute their commitment interval strategy, the hourly permits become variables self-allocated by each Genco i in a strategic or heuristic manner.

In this hourly stage, we also assume that there are no constraints on either the total emissions produced or on the level of permits traded. An alternative model would be one where the SP imposes a limit on the total emissions produced at each time *t*. Here, however, we chose not to let the SP intrude into the hourly market, thus restricting the SP's role to the more reasonable one of ensuring that the emission targets over the longer commitment interval are met.

Thus, in order to participate in this new hourly market accounting for CAT, each Genco i submits not only a cost offer function, $C_{it}(g_{it})$, but also a self-assigned hourly permit, e_{it}.

The Karush-Kuhn-Tucker (KKT) necessary conditions under which the joint hourly emissions and electricity market clears at time *t* require that the price of electricity be given by,

[15] If Δe_t is positive then the sector buys permits at time *t,* else if Δe_t is negative then the sector sells permits at time *t*. We exclude the possibility of buying more permits than required and banking them for later use.

$$\begin{aligned}\lambda_t &= IB_t(d_t) = \lambda_t^0 - \alpha_t d_t \\ &= \lambda_t^0 - \alpha_t \left(\sum_i g_{it} \right)\end{aligned} \tag{4.7}$$

and that the price of the total permits bought in the external market at time t be of the form,

$$\begin{aligned}\gamma_t &= ICEP_t(\Delta e_t) = \gamma_t^0 + \beta_t \Delta e_t \\ &= \gamma_t^0 + \beta_t \left(\sum_i \left(\rho_i g_{it} - e_{it} \right) \right)\end{aligned} \tag{4.8}$$

Finally, the KKT conditions require that the Gencos' incremental cost offers satisfy,

$$IC_{it}(g_{it}) = \lambda_t - \rho_i \gamma_t + \mu_{it}^{\min} - \mu_{it}^{\max} \qquad ; \forall i \tag{4.9}$$

$$\left.\begin{aligned}\mu_{it}^{\max} &\le 0 \\ \mu_{it}^{\min} &\le 0\end{aligned}\right\} \qquad ; \forall i \tag{4.10}$$

as well as the complementarity slackness conditions,

$$\left.\begin{aligned}\mu_{it}^{\min}\left(g_{it} - g_{it}^{\min} \right) &= 0 \\ \mu_{it}^{\max}\left(g_{it} - g_{it}^{\max} \right) &= 0\end{aligned}\right\} \qquad ; \forall i \tag{4.11}$$

4.3.3 Gaming in the hourly pool dispatch with permits trading

To find the Genco outputs, g_{it}, and thus solve the hourly market-clearing problem, it follows from (4.9) that it is necessary to understand how each Genco *i* will define its strategic cost function offer $C_{it}\left(g_{it} \right)$ at each time period *t.* This gaming strategy, as will be seen, is influenced by the self-allocated hourly Genco permits, e_{it} [136].

If Gencos do not game in the hourly market then in (4.9) we define,

$$IC_{it}(g_{it}) = IC^*_{it}(g_{it}) \qquad ;\forall i \tag{4.12}$$

where $IC^*_{it}\left(g_{it}\right) = a^*_{it} + b^*_{it} g_{it}$ is the true incremental cost of Genco i at time t. However, if Gencos game according to the Cournot Nash equilibrium, then we proceed as follows:

Consider the profit of Genco i during period t including the hourly permits trading payments, $\gamma_t(\rho_i g_{it} - e_{it})$, which is positive if buying permits, $(\rho_i g_{it} - e_{it}) > 0$, and negative if selling, $(\rho_i g_{it} - e_{it}) < 0$,

$$pr_{it} = \lambda_t g_{it} - C^*_{it}(g_{it}) - \gamma_t(\rho_i g_{it} - e_{it}) \tag{4.13}$$

In (4.13), the Genco's revenue from electricity sales is $\lambda_t g_{it}$ while the true cost of generation is $C^*_{it}(g_{it})$. In addition, permits are traded with the external market at the hourly market price of $\gamma_t = \gamma^0_t + \beta_t\left(\sum_i\left(\rho_i g_{it} - e_{it}\right)\right)$.

We now recall that, in the hourly market-clearing, the level of permits self-allocated by Genco i to hour t, e_{it}, is treated as a parameter. Then, if, as required by the conditions of Cournot Nash equilibrium, we maximize the Genco's hourly profit in (4.13) with respect to its output, g_{it}, it follows from Appendix D that the hourly offer by Genco i at time t is such that,

$$\begin{aligned} IC_{it}(g_{it}) &= IC^*_{it}(g_{it}) + \left(\alpha_t + \beta_t\rho_i^2\right) g_{it} - \beta_t\rho_i e_{it} \\ &= IC^*_{it}(g_{it}) + \alpha_t g_{it} + \beta_t\rho_i\left(\rho_i g_{it} - e_{it}\right) \quad ;\forall i \end{aligned} \tag{4.14}$$

The gaming strategy for Genco *i* with emissions trading can also be expressed in the form,

$$IC_{it}(g_{it}) = a_{it} + b_{it} g_{it} \tag{4.15}$$

Where,

$$\begin{aligned} a_{it} &= a_{it}^{*} - \beta_t \rho_i e_{it} \\ b_{it} &= b_{it}^{*} + \alpha_t + \beta_t \rho_i^2 \end{aligned} \tag{4.16}$$

Condition (4.14) defines the Cournot Nash gaming strategy of Genco *i* at time *t* under a cap-and-trade system modeled with an hourly permit trading function. In (4.14), the term $IC_{it}^{*}(g_{it}) + \alpha_t g_{it}$ is the standard Cournot Nash gaming strategy under BAU that takes advantage of the load elasticity as explained in section 2.4. The extra term in the gaming strategy, namely, $\beta_t \rho_i \left(\rho_i g_{it} - e_{it} \right)$, implies that if Genco *i* buys permits at time *t* from the external market, that is, if $\rho_i g_{it} - e_{it} > 0$, then the Genco becomes incrementally more expensive (and vice-versa). This is intuitively reasonable since an incrementally more expensive Genco will produce less power and will therefore buy fewer permits from the external market. Another observation from (4.14) is that Gencos with higher emission intensities become incrementally more expensive than cleaner Gencos.

Now, as shown in Appendix D, simultaneously solving relations, (4.7)-(4.11), plus the gaming strategy, (4.14), we can express the hourly outputs, $\{ g_{it}; \forall i \}$ at time t, as linear explicit functions of the hourly permits, $\{ e_{it}; \forall i \}$ and the Lagrange multipliers, $\{ \mu_{it}^{\min}, \mu_{it}^{\max}; \forall i \}$ at time t. We denote these explicit linear functions by,

$$g_{it} = g_{it}(\mathbf{e}_t, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) \qquad ; \forall i \tag{4.17}$$

where the aforementioned vectors are defined as,

$$\begin{aligned} \mathbf{e}_t &= \left\{ e_{it} ; \forall i \right\} \\ \boldsymbol{\mu}_t^{\min} &= \left\{ \mu_{it}^{\min} ; \forall i \right\} \\ \boldsymbol{\mu}_t^{\max} &= \left\{ \mu_{it}^{\max} ; \forall i \right\} \end{aligned} \tag{4.18}$$

In addition to (4.17), the hourly market-clearing solution includes the complementarity slackness conditions, (4.11), here denoted by,

$$\begin{aligned} & s_t(\mathbf{e}_t, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) = \\ & \sum_i \left\{ \mu_{it}^{\min} \left(g_{it}^{\min} - g_{it}(\mathbf{e}_t, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) \right) + \mu_{it}^{\max} \left(g_{it}^{\max} - g_{it}(\mathbf{e}_t, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) \right) \right\} = 0 \end{aligned} \tag{4.19}$$

and the inequalities,

$$\left. \begin{aligned} g_{it}^{\min} &\le g_{it}(\mathbf{e}_t, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) \\ g_{it}(\mathbf{e}_t, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) &\le g_{it}^{\max} \end{aligned} \right\} \quad ; \forall i \tag{4.20}$$

and

$$\left. \begin{aligned} \boldsymbol{\mu}_t^{\min} \\ \boldsymbol{\mu}_t^{\max} \end{aligned} \right\} \le \mathbf{0} \tag{4.21}$$

4.3.4 Quantitative analysis

In this section, we study the effects of the emission permits trading model on the hourly market-clearing when:

1) Permits are allocated for free amongst Gencos in two ways: (a) Grand-fathering, based on the Gencos' BAU emissions and (b) In equal amounts. The possibility of an auction to obtain emission permits as mentioned earlier is examined in section 4.5;

2) Total emission targets are set to 15 % and 30 % below BAU.

In the following simulations, we do not yet consider the temporal allocation of permits by the Gencos. In other words, we assume that the commitment interval is one time period only. In addition, we consider two demand levels, one high and one low.

Recall that BAU refers to market-clearing without cap-and-trade assuming that the Gencos game according to Cournot by taking into account demand elasticity (see equations (2.17) and (2.18) in section 2.4). Under cap-and-trade, Gencos game by taking into account demand elasticity, emission intensities and the permits allocated to each hour (as per equations (4.15) and (4.16)).

4.3.4.1 Simulation parameters

In this section we use the standard example and simulation parameters used throughout the thesis, that is:

	Genco 1	Genco 2	Genco 3	Genco 4
Linear true cost parameter: a_{it}^{*} $(\$/MWh)$	10	15	20	50
Quadratic true cost parameter: b_{it}^{*} $(\$/MW^2h)$	0.05	0.05	0.05	0.05
Emission intensity: ρ_i (t/MWh)	2	1	0.8	0.5

Table 4-1: Gencos' cost parameters and emission intensities.

	Low demand	High demand
Linear demand benefit parameter: λ_t^0 $(\$/MWh)$	400	700
Quadratic demand benefit parameter: α_t $(\$/MW^2h)$	0.8	0.8

Table 4-2: Demand benefit parameters.

The parameters of the hourly emission permits trading cost function for both demand levels are given in Table 4-3.

Linear permits trading cost parameter: γ_t^0 $(\$/t)$	50
Quadratic permits trading cost parameter: β_t $(\$h/t^2)$	0.1

Table 4-3: Parameters of the hourly emission permits trading cost function for both demand levels.

4.3.4.2 Results for low demand level

4.3.4.2.1 Genco permits allocated based on BAU emissions

					15% reduction	**30% reduction**
BAU Genco emissions (t/h)	Genco 1	218	**Allocated Genco permits** (t/h)	e_{11}	185	153
	Genco 2	103		e_{21}	88	72
	Genco 3	78		e_{31}	66	54
	Genco 4	31		e_{41}	26	22
Total	Σ	430	**Total permits (cap)**	Σ	365	301

Table 4-4: Permits grand-fathered to Gencos based on BAU emissions.

Based on the gaming strategy of (4.15) and (4.16), and the permit allocation of Table 4-4, the following market-clearing results are obtained.

		BAU	15% reduction in BAU emissions	30% reduction in BAU emissions
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	-27	-21
	a_{21}	15	6	8
	a_{31}	20	15	16
	a_{41}	50	49	49
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	0.85	1.25	1.25
	b_{21}	0.85	0.95	0.95
	b_{31}	0.85	0.91	0.91
	b_{41}	0.85	0.88	0.88
Demand level (MW)	d_1	372	325	319
Generation levels (MW)	g_{11}	109	60	51
	g_{21}	103	92	91
	g_{31}	97	96	97
	g_{41}	62	78	81
Electricity marginal price $(\$/MWh)$	λ_1	103	140	145
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	430	327	310
Total cap (t/h)	$\sum_{i=1}^{4} e_{i1}$	-	365	301
Permits traded with external market (t/h)	Δe_1	-	-66	-51
	Δe_2	-	4	18
	Δe_3	-	11	23
	Δe_4	-	12	19
Permits marginal price at external market $(\$/t)$	γ_1	-	46	51
Genco absolute and per unit profits $(\$/h),(\$/MWh)$	pr_{11}	9,816 (90)	10,693 (178)	9,389 (184)
	pr_{21}	8,786 (85)	11,051 (120)	10,632 (117)
	pr_{31}	7,813 (81)	10,792 (112)	10,684 (110)
	pr_{41}	3,173 (51)	6,234 (80)	6,533 (81)
Load surplus $(\$/h)$	pr_{d1}	55,236	42,362	40,689
Social-welfare $(\$/h)$	SW_1	84,823	81,133	77,928

Table 4-5: Market-clearing at low demand level, under an emissions cap equivalent to 15% and 30% reductions in total BAU emissions.

Analysis of Table 4-5

i) Cap set to 15% below BAU emissions

From Table 4-4, because grand-fathering has been used to allocate permits, the most polluting Genco 1 with BAU emissions at 218 t/h receives the highest number of permits at 185 t/h; while the least polluting Genco 4 receives the lowest number of permits at 26 t/h. .

Recall from section 3.3 that the main motivation for an allocation scheme based on BAU emissions is to meet the cap without deviating excessively from the BAU market-clearing conditions. Nonetheless, Table 4-5 shows that by applying the cap-and-trade system, the MW output levels of all Gencos decrease except that of the least polluting yet most expensive Genco 4, which increases from 62 MW to 78 MW. Meanwhile the output of the most polluting yet cheapest Genco 1 decreases significantly from 109 MW under BAU to 60 MW under CAT, despite the comparatively large number of permits the Genco receives. Note however that despite the very low emission intensity of Genco 4, because of the low number of permits it receives, Genco 4 must buy permits to the amount of 12 t/h from the external market

These new generation levels exemplify the effect of emission regulation on market-clearing which, as discussed in section 2.5, should favor low polluting Gencos over more polluting ones. This is also reflected in the Genco b-parameter IC offers defined by (4.16). Because of its low emission intensity, the b-parameter of the IC offer of Genco 4 is the smallest at 0.88 $/MW^{2}h, while that of the most polluting Genco 1 is the highest at 1.25 $/MW^{2}h. These changes imply that the least polluting Genco 4 now becomes incrementally less costly thus increasing its competitiveness. In contrast, the cheapest but most polluting Genco 1 now becomes incrementally more costly thus reducing its competitiveness.

However, recall from (4.14) that the Gencos' CAT gaming strategies also depend on the amount of permits traded by each Genco, which in turn depend on the amount of permits they receive. Thus receiving a low number of permits and having to buy permits from the external market, not only increases the cost of the expensive Genco 4 but also reduces the advantage it might enjoy from its low emission intensity. This is reflected in the a-parameter of the IC offers of the Gencos, defined by (4.15), which show that, by only receiving few permits, Genco 4 fails to reduce its offered a-parameter significantly from its true high value of 50 $/MWh, and offers at 49$/MWh. On the other hand, by receiving a large number of permits, the high polluting Genco 1 now offers at -27 $/MWh, a number significantly less than its true offer of 10 $/MWh.

We also note that the intent of the grand-fathering clause does not work very well since the large number of permits that Genco 1 receives (185 t/h) is not used for it to maintain an output close to BAU. Instead, the output of Genco 1 is reduced significantly and it ends up selling a large proportion of its allocated permits (66 t/h) at the external market rate of 46 $/t. Moreover, the shift in generation dispatch from high polluting but cheap Gencos to low polluting but more expensive Gencos result in an increase in the total generation cost of meeting the hourly demand level, which increases the electricity price from 103 $/h to 140 $/h.

With regard to Genco profits, we see that, under CAT, all increase from their BAU levels. For instance, the profit of Genco 1 increases from 9,816 $/h (or 90 $/MWh) to 10,693 $/h (or 178 $/MWh), while that of the less polluting Genco 4 increases from 3,173 $/h (or 51 $/MWh) to 6,234 $/h (or 80 $/MWh). Although the per unit profit of the least polluting Genco 4 increases from 51 to 81 $/MWh, it is still significantly less than that of the most polluting Genco 1 at 178 $/MWh. This is also true for the absolute profit levels.

The high sale of permits along with the sale of electricity at a higher price, despite a reduction in the level of electricity produced, results in an increase in the profit of Genco 1 under CAT as compared to BAU. Meanwhile the increased profit of the least-polluting most expensive Genco 4 is attributed to increased revenue from higher electricity sales at a higher price.

With regard to the demand, by having to rely on less polluting but more expensive Gencos, under CAT the market clears at a higher price, at a lower demand surplus and at a lower social-welfare.

Summarizing the simulation results of Table 4-5, we see that CAT with grand-fathering is only somewhat successful in reducing short-term emissions by clearing the market in favor of the less polluting Gencos despite their higher generation costs. *The results also suggest that, under CAT with GF, high polluting cheaper Gencos benefit disproportionately compared to the low polluting expensive Gencos by, for example, allowing the polluting Gencos to sell permits instead of using such permits for the intended use of not deviating excessively from BAU.* Such a profit trend does not provide incentives or signals to invest in long-term emission reduction.

Another interesting observation is that the total emissions produced by the sector is 327 t/h which, as seen from Table 4-5, is significantly lower than the total cap imposed, 365 t/h. This is because there is a total sale of permits from the sector to the external market which indicates that a cap of 15% reduction in BAU emissions is too generous. Thus we now analyze the results of imposing a stricter emissions cap.

ii) <u>Cap set to 30% below BAU emissions</u>

By applying a tighter emissions cap, Table 4-4 shows that all Gencos now receive lower permits, with the most polluting Genco 1 still allocated the highest number, while the least polluting Genco 4 allocated the fewest number of permits.

By allocating fewer total permits among Gencos, the generation costs increase and Gencos are forced into a more aggressive gaming strategy as shown by the increased a-parameters of the IC offers. The only Genco that does not change its offer significantly is Genco 4 because by being allocated fewer permits and having the least emission intensity, the number of permits the Genco has to buy from the external market increases only slightly from 12 t/h to 19 t/h.

The effect of the tighter cap is a larger shift towards low polluting but expensive Gencos and away from high polluting yet cheap Gencos. Nonetheless, although the emissions cap now represents a decrease of 30% instead of 15% from BAU, (i.e. doubled), the shift in generation is not as sharp as expected. For instance, the MW output of Genco 1 decreases from 60 MW to 51 MW, while that of Genco 4 increases only from 78 MW to 81 MW.

With tighter emission caps the profit of the least polluting but most expensive Genco 4 increases slightly from 6,234 $/h to 6,533 $/h; while the absolute profits of all other Gencos decrease. For instance, the profit of Genco 1 decreases from 10,693 $/h to 9,389 $/h, a value slightly less that its BAU profit.

The above trend in the Gencos' profit under tighter caps can be rationalized as follows: The profit of the most polluting Genco 1 decreases because, with less allocated permits, Genco 1 sells fewer permits. Moreover by applying tighter emission caps, Genco 1 is dispatched to produce less MW output. On the other hand, Genco 4 with the lowest emission intensity is dispatched to produce more MW under tighter caps and the amount of permits it buys increases only by a small amount. Thus, its net increase in revenue is higher than its net increase in cost and its profit increases.

However, it is interesting to note that, under a tighter emissions cap, the per unit profit of the most polluting cheapest Genco 1 increases from 178 $/MWh to 184 $/MWh and is still significantly higher than the per unit profit of the least polluting Genco 4 which only increases from 80 $/MWh to 81 $/MWh.

Finally, we note that the total emissions produced by the sector is now 310 t/h, which is higher than the total cap imposed, 301 $/t. This means that there is a total purchase of 9 permits by the sector from the external market, indicating the cap is well defined. Recall, that under a cap equivalent to 15% reduction in BAU emissions, there was a net sale of 38 permits from the sector. Thus in the next simulations in this section we consider only a cap equivalent to 30% reduction.

4.3.4.2.2 Genco permits allocated equally

To study the effects of the permit allocation among Gencos on the hourly market-clearing, including profits and social-welfare, we now consider allocating permits to Gencos in equal amounts, as opposed to allocating them based on the Gencos' BAU emissions as is the case with GF.

As shown in Table 4-6, the major difference between the two schemes is that by allocating permits equally, the most polluting cheapest Genco 1 now gets 75 permits as opposed to being allocated 153 permits under GF. Moreover, the least polluting most expensive Genco 4 now gets a bigger number of permits, 75, compared to the 22 permits it got under GF.

					GF	**Equal allocation**
BAU Genco emissions (t/h)	Genco 1	218	**Allocated Genco permits** (t/h)	e_{11}	153	75
	Genco 2	103		e_{21}	72	75
	Genco 3	78		e_{31}	54	75
	Genco 4	31		e_{41}	22	75
Total	Σ	430	**Total permits (cap)**	Σ	300	300

Table 4-6: Permits allocated to Gencos in equal amounts.

Based on the gaming strategy of (4.15) and (4.16), and the permit allocations of Table 4-6, the following market-clearing results are obtained.

		BAU	GF	Equal allocation
Offered linear cost parameters ($\$/MWh$)	a_{11}	10	-21	-5
	a_{21}	15	8	7
	a_{31}	20	16	14
	a_{41}	50	49	46
Offered non-linear cost parameters ($\$/MW^2h$)	b_{11}	0.85	1.25	1.25
	b_{21}	0.85	0.95	0.95
	b_{31}	0.85	0.91	0.91
	b_{41}	0.85	0.88	0.88
Demand level (MW)	d_1	372	319	319
Generation levels (MW)	g_{11}	109	51	41
	g_{21}	103	91	93
	g_{31}	97	97	100
	g_{41}	62	81	85
Electricity marginal price ($\$/MWh$)	λ_1	103	145	145
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	430	310	297
Total cap (t/h)	$\sum_{i=1}^{4} e_{i1}$	-	301	300
Permits traded with external market (t/h)	Δe_{11}	-	-51	6
	Δe_{21}	-	18	17
	Δe_{31}	-	23	5
	Δe_{41}	-	19	-33
Permits marginal price at external market ($\$/t$)	γ_1	-	51	50
Genco absolute and per unit profits ($\$/h$),($\$/MWh$)	pr_{11}	9,816 (90)	9,389 (184)	5,163 (126)
	pr_{21}	8,786 (85)	10,632 (117)	10,995 (118)
	pr_{31}	7,813 (81)	10,684 (110)	12,047 (120)
	pr_{41}	3,173 (51)	6,533 (81)	9,519 (112)
Load surplus ($\$/h$)	pr_{d1}	55,236	40,689	40,583
Social-welfare ($\$/h$)	SW_1	84,823	77,928	78,306

Table 4-7: Market-clearing at low demand level, under an emissions cap equivalent to 30% reduction in total BAU emissions when permits are grand-fathered based on BAU or distributed equally among Gencos.

Analysis of Table 4-7

Under equal distribution of permits, by receiving fewer permits, Genco 1 now buys 6 permits from the external market as opposed to selling 51 permits; which increases the cost of Genco 1, thus forcing it to bid more aggressively by increasing the a-parameter of its IC-offer given in (4.16). On the other hand, Genco 4 now sells 33 permits as opposed to buying 19 permits, which reduces its cost and increases the competitiveness of this expensive Genco, which is reflected in the lower a-parameter of its IC-offer.

Due to the more aggressive offer of Genco 1, its generation level goes down from 51 MW to 41 MW; on the other hand the generation level of Genco 4 goes up from 81 to 85 MW. Thus there is a shift in generation outputs from polluting Gencos to less polluting ones. Nonetheless, because the demand level goes down by applying a tighter emissions cap, the shift in generation outputs does not induce a visible change in electricity price.

Note also that the significant change in the pattern of permit trading does not result in a visible change in the permits price from the external market. This is because the total number of permits traded by the sector with the external market does not change significantly; a different allocation scheme primarily shifts the level of permits traded amongst Gencos within the electricity sector.

The aforementioned change in the Genco's strategic offers and subsequent shift in generation outputs illustrate the effect of permit allocation among Gencos and of permits trading on the hourly market-clearing.

By allocating permits equally among Gencos, the profit of all Gencos increase compared to their profits under GF based on BAU emissions, except that of the most polluting cheapest Genco 1. The profit of Genco 1 goes down from 9,389 $/h to 5,960 $/h, while the profit of the least polluting most expensive Genco 4 increases from 6,533 $/h to 9,519 $/h. Note that the profit of Genco 1 is now lower than the profit of Genco 4; moreover, the profit of the most polluting Genco 1 under cap-and-trade is even considerably lower than its BAU profit. The main reason for the aforementioned changes in Genco profits is the shift in permits trading resulting from the shift in allocated permits among Gencos. Moreover, the per unit profit of the most polluting Genco 1 at 126 $/MWh is now only slightly higher than that of Genco 4 at 112 $/MWh.

Interestingly, by allocating permits equally among Gencos, and shifting permits from high polluting Gencos to lower polluting ones, the profit of the load goes down only slightly from 40,689 $/h to 40,583 $/h, while social-welfare in fact increases from 77,928 $/h to 78,306 $/h.

This example illustrates the significant effect of allocated permits and subsequent permits trading on the Gencos' profits. When permits are allocated using a grand-fathering approach, high polluting Gencos make significantly higher profits than low polluting ones. However by allocating permits equally among Gencos, profits change significantly, favoring less polluting Gencos despite their high generation costs. This shift in profits produces more correct signals to invest in emission reduction. *It is interesting that this change in profit has no visible negative effect on prices, consumer surplus or social-welfare.*

4.3.4.3 Results for high demand level

Before considering the temporal allocation of permits among time periods, we now examine the effect of different demand levels, which in this work represents the central characteristic of a time-varying demand. We therefore now perform the same simulations as in the previous section but for a high demand.

4.3.4.3.1 Genco permits allocated based on BAU emissions

					15% reduction	**30% reduction**
BAU Genco emissions (t/h)	Genco 1	366	**Allocated Genco permits** (t/h)	e_{11}	311	256
	Genco 2	177		e_{21}	150	124
	Genco 3	137		e_{31}	117	96
	Genco 4	68		e_{41}	58	48
Total	Σ	748	**Total permits (cap)**	Σ	636	524

Table 4-8: Permits Grand-fathered to Gencos based on BAU emissions.

Based on the gaming strategy of (4.15) and (4.16), and the Genco permit allocations of Table 4-8, the following market-clearing results are obtained.

		BAU	15% reduction in BAU emissions	30% reduction in BAU emissions
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	-52	-41
	a_{21}	15	0	3
	a_{31}	20	11	12
	a_{41}	50	47	48
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	0.85	1.25	1.25
	b_{21}	0.85	0.95	0.95
	b_{31}	0.85	0.91	0.91
	b_{41}	0.85	0.88	0.88
Demand level (MW)	d_1	668	617	606
Generation levels (MW)	g_{11}	183	127	113
	g_{21}	177	165	163
	g_{31}	171	171	171
	g_{41}	136	154	159
Electricity marginal price $(\$/MWh)$	λ_1	166	206	215
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	749	633	605
Total cap (t/h)	$\sum_{i=1}^{4} e_{i1}$	-	636	524
Permits traded with external market (t/h)	Δe_1	-	-56	-31
	Δe_2	-	15	39
	Δe_3	-	20	41
	Δe_4	-	19	32
Permits marginal price at external market $(\$/t)$	γ_1	-	50	58
Genco absolute and per unit profits $(\$/h),(\$/MWh)$	pr_{11}	27,674 (151)	27,438 (216)	24,626 (218)
	pr_{21}	25,925 (146)	30,207 (183)	29,753 (183)
	pr_{31}	24,233 (142)	30,103 (176)	30,391 (178)
	pr_{41}	15,280 (112)	22,512 (146)	23,786 (150)
Load surplus $(\$/h)$	pr_{d1}	178,437	152,244	146,727
Social-welfare $(\$/h)$	SW_1	271,550	262,505	255,283

Table 4-9: Market-clearing at high demand level, under emissions caps equivalent to 15% and 30% reductions in total BAU emissions.

Analysis of Table 4-9

i) <u>With 15% reduction in BAU emissions</u>

The differences in Genco gaming strategies, generation levels, and levels of traded permits, among Gencos, are similar to those under low demand and can be rationalized by the same analysis. We note that, although under high demand Gencos are allocated more permits than under low demand, in order to meet the high demand level; more permits will have to be bought by the sector and less sold at the external market. This increases the permit price from 46 $/t to 50 $/t. Moreover, buying more permits, increases the cost of Gencos and forces them to game more aggressively, which is reflected in the a-parameter of the Gencos' IC-offers, given by equation(4.16). By offering more aggressively, and increasing the total cost of generation, along with the higher demand benefit parameters, the electricity price increases from 140 $/MWh at low demand, to 206 $/MWh at this higher demand.

At the higher demand level, the profit of each of the four Gencos is considerably higher than its corresponding profit at the lower demand level. For instance the profit of Genco 1 increases from 10,693 $/h to 27,438 $/h, and that of Genco 4 increases from 6,234 $/h to 22,512 $/h. The increase in the profits of all Gencos is attributed to the significantly higher revenue from the sale of electricity.

An important difference between the two demand levels, is that at high demand, the profit of Genco 1 under CAT, at 27,438 $/h, is slightly lower than its BAU profit of 27,674 $/h. Recall that at the low demand level, the profit of Genco 1 under CAT with 15% reduction was higher than its BAU profit. Moreover, the difference in the profits of the most polluting Genco 1 and least polluting Genco 4 is now lower than the corresponding difference at low demand. *This indicates that in order to meet the high demand, low polluting*

Gencos become more valuable despite their high generation cost, and high polluting Gencos lose some advantage.

Finally, we note that the sector now produces 633 t/h which is almost equal to the total cap imposed of 636 t/h. This indicates that, unlike the case at low demand, a cap equivalent to 15% reduction in BAU emissions is not too generous at high demand. Nonetheless, for the sake of comparison we apply a cap equivalent to a 30% reduction in BAU emissions and analyze the results.

ii) With 30% reduction in BAU emissions

Similar to low demand, by allocating less permits to Gencos, the cost of Gencos increase thus forcing them to be more aggressive in their gaming strategies, which is reflected in the a-parameter of the Gencos IC-offers. Moreover, there is a shift in generation from high polluting Gencos to low polluting ones.

While only the profit of the least polluting Genco increases by applying tighter caps at low demand level, the profits of the two least polluting Gencos 3 and 4 now increase by applying tighter caps at higher demand level. For instance the profit of the most polluting Genco 1, decreases from 27,438 $/h to 24,626 $/h, while that of the least polluting Genco 4, increases from 22,512 $/h to 23,786 $/h. Note that with a tighter emissions cap at high demand level, the profit of Genco 1 is still higher than that of Genco 4, the difference in profit is however much lower than the corresponding difference at low demand level.

4.3.4.3.2 Genco permits allocated equally

To compare the effect of permit allocation among Gencos at a high demand level, we simulate the case when permits are allocated equally. Note that, unlike the low demand case, since a cap equivalent to 15% reduction was not too generous, we apply the equal permit allocation for a cap equivalent to 15% reduction in BAU emissions.

					GF	Equal allocation
BAU Genco emissions (t/h)	Genco 1	366	**Allocated Genco permits** (t/h)	e_{11}	311	159
	Genco 2	177		e_{21}	150	159
	Genco 3	137		e_{31}	117	159
	Genco 4	68		e_{41}	58	159
Total	Σ	748	**Total permits (cap)**	Σ	636	636

Table 4-10: Permits allocated to Gencos using GF based on BAU emissions, and permits allocated equally.

Based on the gaming strategy of (4.15) and (4.16), and the Genco permit allocations in Table 4-10, the following market-clearing results are obtained.

		BAU	GF	Equal allocation
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	-52	-22
	a_{21}	15	0	-1
	a_{31}	20	11	7
	a_{41}	50	47	42
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	0.85	1.25	1.25
	b_{21}	0.85	0.95	0.95
	b_{31}	0.85	0.91	0.91
	b_{41}	0.85	0.88	0.88
Demand level (MW)	d_1	668	617	616
Generation levels (MW)	g_{11}	183	127	108
	g_{21}	177	165	169
	g_{31}	171	171	177
	g_{41}	136	154	162
Electricity marginal price $(\$/MWh)$	λ_1	166	206	207
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	749	633	608
Total cap (t/h)			636	636
Permits traded with external market (t/h)	Δe_1	-	-56	56
	Δe_2	-	15	10
	Δe_3	-	20	-17
	Δe_4	-	19	-78
Permits marginal price at external market $(\$/t)$	γ_1	-	50	47
Genco absolute and per unit profits $(\$/h),(\$/MWh)$	pr_{11}	27,674 (151)	27,438 (216)	18,287 (169)
	pr_{21}	25,925 (146)	30,207 (183)	31,329 (185)
	pr_{31}	24,233 (142)	30,103 (176)	33,207 (188)
	pr_{41}	15,280 (112)	22,512 (146)	28,432 (176)
Load surplus $(\$/h)$	pr_{d1}	178,437	152,244	151,855
Social-welfare $(\$/h)$	SW_1	271,550	262,505	263,111

Table 4-11: Market-clearing at high demand level, under an emission cap equivalent to 15% reduction in total BAU emissions, when Genco permits are grand-fathered based on BAU emissions, and when permits are allocated equally among Gencos.

Analysis of Table 4-11

Under this higher demand level, results agree with those at lower demand level that by allocating more permits to less polluting Gencos, the Genco profits provide proper signals for long-term investments in emission regulation without noticeable negative effects on market-clearing and consumer surplus.

4.3.4.4 *Major findings from simulations on the effect of hourly permits trading on market-clearing and Genco market power*

From simulations over one time period we make the following observations:

1) The cap-and-trade model is successful in meeting short-term emission targets by dispatching generation based on Gencos' generation costs as well as on their emission intensities. This leads to a shift in generation dispatch from high polluting cheap Gencos to less polluting more expensive ones;

2) The allocation of permits and the trading of these permits with the external market are key factors in Genco profits under CAT;

3) The allocation of permits among Gencos affects both the Gencos' gaming strategies and the ensuing hourly market-clearing;

4) The demand level is important, because at high demand level, low polluting Gencos become more valuable despite their high generation costs.

While observations 1) and 2) are consistent with findings of other studies based on the constant permits price methodology[16], observations 3) and 4) signify the importance of accounting for the effects of hourly permits trading on the hourly market-clearing when studying the allocation of commitment interval permits among Gencos.

[16] Recall from section 1.3 that under the constant permits price methodology permits trading does not affect the hourly market-clearing.

4.4 Self-Allocation of Commitment Interval Genco Caps into Hourly Fractions

The challenge of how to self-allocate the total commitment interval Genco cap into hourly fractions $\{e_{it}\}$ is now considered. This is examined through two approaches: a strategic one based on Nash equilibrium and a heuristic one.

4.4.1 Strategic method

Under this method, the SP assumes that the Gencos allocate the hourly permits in a way that maximizes their individual profits over the commitment interval. To model such an allocation, the SP finds the Nash equilibrium hourly permit allocation over the entire commitment interval. Under this strategic method, the SP takes into consideration the effect of the hourly permits on the hourly external market permit price as well as on the hourly Genco gaming strategies and market-clearing equilibria.

The profit of each Genco *i* over the commitment interval consists of: (i) its total revenue from selling electricity, $\sum_t \lambda_t g_{it}$; (ii) minus its true total generation cost, $\sum_t C_{it}^*(g_{it})$; (iii) minus its total cost of buying balancing emission permits from the external permits market, $\sum_t \gamma_t \left(\rho_i g_{it} - e_{it}\right)$. The total profit of Genco *i* can therefore be written as,

$$pr_i = \sum_t \left\{\lambda_t g_{it} - C_{it}^*(g_{it}) - \gamma_t \left(\rho_i g_{it} - e_{it}\right)\right\} \tag{4.22}$$

Note that if the total commitment interval permits were bought by each Genco from the SP at an auction, there would be an added cost to Genco *i* that is not included in (4.22). However, at this stage of the problem neither the cost of the total permits acquired by each Genco nor the manner in which these permits are acquired are relevant. These considerations are treated in section 4.5.

Thus, given its total commitment interval permits, e_i, each Genco *i* chooses the variables under its control, namely, the hourly permits, $\{e_{it};\forall t\}$, so as to maximize pr_i subject to the condition that $\sum_t e_{it} = e_i$. Since each Genco has the same objective, we assume that each Genco *i* seeks a Nash equilibrium with respect to its controllable variables $\{e_{it};\forall t\}$.

4.4.1.1 Solution

To formulate this Nash equilibrium problem, recall from the hourly market-clearing solution and Appendix D, that the hourly generation levels are known in terms of the hourly permits through the explicit forms $g_{it} = g_{it}(\mathbf{e}_t, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max});\forall i,t$. In addition, since the prices are given by $\lambda_t = \lambda_t^0 - \alpha_t\left(\sum_i g_{it}\right)$ and $\gamma_t = \gamma_t^0 + \beta_t \Delta e_t$, and since $\Delta e_t = \sum_i \left(\rho_i g_{it} - e_{it}\right)$, the total profit of Genco *i* defined by (4.22) becomes an explicit function of $\{\mathbf{e}, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}\}$, here denoted by $pr_i\{\mathbf{e}, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}\}$, where the additional vectors, $\{\mathbf{e}, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}\}$, are defined as,

$$\mathbf{e} = \{\mathbf{e}_t;\forall t\} = \{e_{it};\forall i,t\} \tag{4.23}$$

$$\boldsymbol{\mu}^{\min} = \{\boldsymbol{\mu}_t^{\min};\forall t\} = \{\mu_{it}^{\min};\forall i,t\} \tag{4.24}$$

and,

$$\boldsymbol{\mu}^{\max} = \{\boldsymbol{\mu}_t^{\max};\forall t\} = \{\mu_{it}^{\max};\forall i,t\} \tag{4.25}$$

Moreover, since at this profit-maximizing stage the SP has allocated all total commitment interval permits, $\{e_i;\forall i\}$, these quantities are treated as constants.

To find the Nash Equilibrium with respect to the hourly permits controlled by each Genco *i*, the profit of Genco i , $pr_i\left\{\mathbf{e},\boldsymbol{\mu}^{\min},\boldsymbol{\mu}^{\max}\right\}$, is maximized with respect to its controllable variables, $\{e_{it};\forall t\}$, assuming that all other Gencos keep their hourly permits unchanged. For every Genco *i*, this maximization takes the form,

$$\max_{\{e_{it};\forall t\}} pr_i\left\{\mathbf{e},\boldsymbol{\mu}^{\min},\boldsymbol{\mu}^{\max}\right\} \tag{4.26}$$

subject to the requirement that $\left\{e_{jt};\forall t\right\};\forall j\neq i$ remain constant and that the sum of the hourly permits of Genco *i* is equal to its total allocated permits, e_i ,

$$\sum_t e_{it} = e_i \qquad (\theta_i) \tag{4.27}$$

In addition, the following non-negativity condition must be imposed,

$$0\leq e_{it} \qquad \left(\theta_{it}^{\min}\right) \qquad ;\forall t \tag{4.28}$$

together with the complementarity slackness requirements and non-equality constraints from the hourly market-clearing stage, (4.19) - (4.21).

The KKT necessary conditions of the Genco profit maximizing problems result in *ng* sets of implicit equations relating the hourly Genco outputs and permits, $\left\{g_{it},e_{it};\forall i,t\right\}$, to the total Genco permits, $\{e_i;\forall i\}$, as well as the Lagrange multipliers associated with all inequalities, namely, $\left\{\boldsymbol{\mu}^{\min},\boldsymbol{\mu}^{\max},\boldsymbol{\theta},\boldsymbol{\theta}^{\min}\right\}$. These equations allow us to establish the following set of implicit linear relations among all pertinent variables,

$$\begin{aligned}
g_{it} &= g_{it}(\mathbf{e}^0, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}, \boldsymbol{\theta}, \boldsymbol{\theta}^{\min}) && ;\forall i,t \\
e_{it} &= e_{it}(\mathbf{e}^0, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}, \boldsymbol{\theta}, \boldsymbol{\theta}^{\min}) && ;\forall i,t \\
d_t &= d_t(\mathbf{e}^0, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}, \boldsymbol{\theta}, \boldsymbol{\theta}^{\min}) && ;\forall t \\
\lambda_t &= \lambda_t(\mathbf{e}^0, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}, \boldsymbol{\theta}, \boldsymbol{\theta}^{\min}) && ;\forall t \\
\gamma_t &= \gamma_t(\mathbf{e}^0, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}, \boldsymbol{\theta}, \boldsymbol{\theta}^{\min}) && ;\forall t
\end{aligned} \tag{4.29}$$

where the aforementioned vectors are defined as,

$$\mathbf{e}^0 = \{e_i; \forall i\} \tag{4.30}$$

and,

$$\boldsymbol{\theta} = \{\theta_i; \forall i\} \tag{4.31}$$

and finally,

$$\boldsymbol{\theta}^{\min} = \{\theta_{it}^{\min}; \forall i,t\} \tag{4.32}$$

The necessary conditions that define a Nash equilibrium temporal allocation along with the hourly market-clearing under Cournot Nash equilibrium are derived in Appendix D.

4.4.2 Heuristic method

We also consider a heuristic temporal allocation method under which all Gencos use the same temporal allocation. Since in our model the only variable amongst the time periods is the demand level (or the demand benefit function parameters), we assume that a heuristic temporal allocation of permits will be based on the demand level. Intuitively, one would think that to earn higher profits more permits would be self-allocated to high demand periods where individual Genco emissions are expected to be high, however under less than full competition, intuition does not always lead to expected results as verified by simulations. As a result, we consider three heuristic temporal allocation schemes, namely, (i) Gencos allocate more permits to time periods with higher demand levels; (ii) Gencos allocate permits equally amongst time periods regardless of the demand levels; (iii) Gencos allocate more permits to time periods with lower demand levels.

4.4.3 Quantitative analysis

In this section we study the effects of the temporal self-allocation of commitment interval Genco permits on market-clearing and profits. The basic assumptions of the simulations are:

1) The commitment interval permits are grand-fathered to Gencos based on their individual BAU emissions over that commitment interval;

2) Total permits allocated to the sector over the commitment interval are 15 % below total BAU emission levels;

3) The commitment interval consists of two time periods: period 1 with low demand level, and period 2 with high demand level.

4) Under both heuristic and strategic temporal allocation schemes, the Gencos game according to the Cournot NE based on their emission intensities, hourly permits and demand elasticity, as per equations (4.15) and (4.16). Note that the strategic temporal allocation of permits must take into consideration the aforementioned gaming strategy as an implicit relation.

Under the above assumptions, we now examine the effect of the following four temporal allocation methods on the hourly market-clearing, profits and social-welfare:

A) 70 % of total permits awarded to each Genco are allocated to the period with higher demand level, time period 2;

B) The total permits given to each Genco are allocated equally to each time period;

C) 70 % of total permits awarded to each Genco are allocated to the period with lower demand level, time period 1;

D) The temporal allocation meets the Nash equilibrium conditions presented and discussed in section 4.4.1.

4.4.3.1 Simulation parameters

In this section's demonstrations we use the standard simulation parameters used throughout the thesis and repeated below for convenience:

	Genco 1	Genco 2	Genco 3	Genco 4
Linear true cost parameter: a_{it}^{*} $(\$/MWh)$	10	15	20	50
Quadratic true cost parameter: b_{it}^{*} $(\$/MW^2h)$	0.05	0.05	0.05	0.05
Emission intensity: ρ_i (t/MWh)	2	1	0.8	0.5

Table 4-12: Gencos' cost parameters and emission intensities.

	Time period 1 (low demand)	Time period 2 (high demand)
Linear demand benefit parameter: λ_t^0 $(\$/MWh)$	400	700
Quadratic demand benefit parameter: α_t $(\$/MW^2h)$	0.8	0.8

Table 4-13: Demand benefit parameters.

	Time period 1 (low demand)	Time period 2 (high demand)
Linear permits trading cost parameter: γ_t^0 $(\$/t)$	50	50
Quadratic permits trading cost parameter: β_t $(\$h/t^2)$	0.1	0.1

Table 4-14: Parameters of the hourly emission permits trading cost function.

4.4.3.2 Results for heuristic temporal allocation method A

Since 70 % of permits are allocated to time period 2 with higher demand, the resulting temporal allocation is as depicted in Table 4-15.

Heuristic method A	Time period 1 (low demand)		Time period 2 (high demand)		Commitment interval permits	
Genco permits (t)	e_{11}	149	e_{12}	348	e_1	497
	e_{21}	72	e_{22}	167	e_2	238
	e_{31}	55	e_{32}	128	e_3	183
	e_{41}	25	e_{42}	59	e_4	84

Table 4-15: Temporal allocation of permits that favors the period with high demand level (period 2).

With the temporal allocation in Table 4-15, the hourly market-clearings for both time periods are shown in Table 4-16 below.

Heuristic method A	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Offered linear cost parameters $(\$/MWh)$	a_{11}	-20	a_{12}	-60		
	a_{21}	8	a_{22}	-2		
	a_{31}	16	a_{32}	10		
	a_{41}	49	a_{42}	47		
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	1.25	b_{12}	1.25		
	b_{21}	0.95	b_{22}	0.95		
	b_{31}	0.91	b_{32}	0.91		
	b_{41}	0.88	b_{42}	0.88		
Demand levels (MW)	d_1	319	d_2	624		
Generation levels (MW)	g_{11}	50	g_{12}	137		
	g_{21}	91	g_{22}	166		
	g_{31}	97	g_{31}	170		
	g_{41}	81	g_{42}	150		
Electricity marginal price $(\$/MWh)$	λ_1	145	λ_2	201		
Permits marginal price at external market $(\$/t)$	γ_1	51	γ_2	45		
Total emissions $(t/h),(t)$	$\sum_{i=1}^{4}\rho_i g_{i1}$	309	$\sum_{i=1}^{4}\rho_i g_{i2}$	651	$\sum_{t=1}^{2}\sum_{i=1}^{4}\rho_i g_{it}$	960
Total cap $(t/h),(t)$	$\sum_{i=1}^{4} e_{i1}$	301	$\sum_{i=1}^{4} e_{i2}$	702	$\sum_{i=1}^{4} e_i$	1002
Permits traded with external market $(t/h),(t)$	Δe_{11}	-48	Δe_{12}	-74	Δe_1	-123
	Δe_{21}	19	Δe_{22}	-1	Δe_2	19
	Δe_{31}	23	Δe_{32}	8	Δe_3	31
	Δe_{41}	15	Δe_{42}	16	Δe_4	31
Genco profit $(\$/h),(\$)$	pr_{11}	9,189	pr_{12}	29,009	pr_1	38,199
	pr_{21}	10,603	pr_{22}	30,292	pr_2	40,895
	pr_{31}	10,716	pr_{32}	29,730	pr_3	40,446
	pr_{41}	6,738	pr_{42}	21,453	pr_4	28,191
Load surplus $(\$/h),(\$)$	pr_{d1}	40,669	pr_{d2}	155,504	pr_d	196,173
Social-welfare $(\$/h),(\$)$	SW_1	77,915	SW_2	265,988	SW	343,904

Table 4-16: Market-clearing when 70% of permits are allocated to the high demand period 2

4.4.3.3 Analysis of Table 4-16

i) Market-clearing

Although allocating more permits to the period with higher demand level appears to be the most rational heuristic temporal allocation method, it yields a high permit price of 51 \$/t at time period 1 (low demand level) and a lower permit price of 45 $/t during period 2 (higher demand level). These results can be rationalized as follows: The total emissions produced at time period 1 exceed the low number of total permits allocated to that period, and there is a net hourly purchase of permits by the sector from the external market, which increases the price of the permits. On the other hand, by allocating a high number of permits to time period 2, there is a net hourly sale of permits by the sector which reduces the price of the permits.

Under this temporal allocation scheme the generation level of the most polluting Genco 1 for both time periods is lower than that of each of the other three more expensive yet less polluting Gencos. Producing a low MW output decreases the level of total emissions produced by Genco 1, which, along with the high number of permits it is allocated, renders it the only Genco that sells permits over the two time periods. Note also that since Genco 1 is selling a high number of permits over both time periods, it is the Genco that benefits the most from the high permit prices at both time periods.

ii) Profits and social-welfare

The tradeoff between loss of revenue from the diminished sale of electricity and gain in revenue from the rise in sale of excess permits, yields a total profit of 38,199 $ for Genco 1.

Genco 2 makes the highest total profit at 40,895 $, followed by Genco 3 at 40,446 $. Genco 4, which is the least polluting yet most expensive Genco, still makes the lowest profit at 28,191 $. The reason for the relatively low profit observed by Genco 4 is that by being allocated the least number of total permits, the Genco has to buy the most number of permits over both periods at high permit prices. It is a good sign that the profit of the most polluting but cheapest Genco 1 is lower than the profits of the two less polluting Gencos 2 and 3. This shows that the CAT system is giving correct signals to invest in emission reduction. However, the profit of the least polluting Genco 4 is still the lowest.

4.4.3.4 Results for heuristic temporal allocation method B

Under this method, permits are allocated equally to each time period regardless of the demand level. The temporal allocation is depicted in Table 4-17.

Heuristic method B	Time period 1 (low demand)		Time period 2 (high demand)		Commitment interval permits	
Genco permits (t)	e_{11}	248	e_{12}	248	e_1	497
	e_{21}	119	e_{22}	119	e_2	238
	e_{31}	91	e_{32}	91	e_3	183
	e_{41}	42	e_{42}	42	e_4	84

Table 4-17: Equal temporal allocation of permits among both time periods.

With the temporal allocation of permits in Table 4-17, the hourly market-clearings for both time periods are shown in Table 4-18 below.

Heuristic method B	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Offered linear cost parameters $(\$/MWh)$	a_{11}	-40	a_{12}	-40		
	a_{21}	3	a_{22}	3		
	a_{31}	13	a_{32}	13		
	a_{41}	48	a_{42}	48		
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	1.25	b_{12}	1.25		
	b_{21}	0.95	b_{22}	0.95		
	b_{31}	0.91	b_{32}	0.91		
	b_{41}	0.88	b_{42}	0.88		
Demand levels (MW)	d_1	339	d_2	603		
Generation levels (MW)	g_{11}	77	g_{12}	110		
	g_{21}	94	g_{22}	163		
	g_{31}	96	g_{31}	172		
	g_{41}	72	g_{42}	159		
Electricity marginal price $(\$/MWh)$	λ_1	129	λ_2	217		
Permits marginal price at external market $(\$/t)$	γ_1	36	γ_2	60		
Total emissions $(t/h),(t)$	$\sum_{i=1}^{4}\rho_i g_{i1}$	361	$\sum_{i=1}^{4}\rho_i g_{i2}$	599	$\sum_{t=1}^{2}\sum_{i=1}^{4}\rho_i g_{it}$	960
Total cap $(t/h),(t)$	$\sum_{i=1}^{4} e_{i1}$	500	$\sum_{i=1}^{4} e_{i2}$	500	$\sum_{i=1}^{4} e_i$	1002
Permits traded with external market $(t/h),(t)$	Δe_{11}	-94	Δe_{12}	-29	Δe_1	-123
	Δe_{21}	-25	Δe_{22}	43	Δe_2	19
	Δe_{31}	-15	Δe_{32}	46	Δe_3	31
	Δe_{41}	-6	Δe_{42}	38	Δe_4	31
Genco profit $(\$/h),(\$)$	pr_{11}	12,406	pr_{12}	24,190	pr_1	36,596
	pr_{21}	11,415	pr_{22}	29,630	pr_2	41,045
	pr_{31}	10,702	pr_{32}	30,363	pr_3	41,065
	pr_{41}	5,757	pr_{42}	23,789	pr_4	29,546
Load surplus $(\$/h),(\$)$	pr_{d1}	45,969	pr_{d2}	145,620	pr_d	191,589
Social-welfare $(\$/h),(\$)$	SW_1	86,248	SW_2	253,591	SW	339,839

Table 4-18: Market-clearing when total permits are allocated equally to each time period.

4.4.3.5 Analysis of Table 4-18

i) Market-clearing

By allocating permits equally among both time periods, the permits price at time period 1 is now lower, and the permits price at time period 2 (with higher demand level), is now higher than the corresponding prices under the temporal allocation that favors the period with higher demand level. This is because now more permits have been bought at time period 2 which increases the permit price, while more permits are sold at time period 1 which reduces the permit price at that period.

Moreover, the electricity price at time period 1 is lower, while the price at time period 2 is higher than the corresponding prices under the previous permit temporal allocation. This is because with more permits given to time period 1, Genco 1, the cheapest most polluting Genco, produces more MW output, while Genco 4, the most expensive least polluting Genco produces less MW output at that period. This reduces the marginal generation cost at time period 1 and reduces the electricity price. The opposite effect in time period 2 induces a higher electricity price.

ii) <u>Profits and social-welfare</u>

Table 4-19 depicts the Genco profits and load surplus, as well as the SW under heuristic temporal allocation methods A and B.

		Heuristic temporal allocation	
		A	**B**
Genco profit ($)	pr_1	38,199	36,596
	pr_2	40,895	41,045
	pr_3	40,446	41,065
	pr_4	28,191	29,546
Load surplus ($)	pr_d	196,173	191,589
Social-welfare ($)	SW	343,904	339,839

Table 4-19: Genco profits and load surplus as well as SW for two temporal allocation methods.

By allocating permits equally among both time periods (method B), as opposed to favoring the period with high demand level (method A), all Gencos make higher profits, except the most polluting yet cheapest Genco 1. As seen in Table 4-19, the profit of Genco 1 goes down from 38,199 $ to 36,596 $, while the profits of all other Gencos go up. For instance, the profit of Genco 4, the most expensive but least polluting Genco, increases from 28,191 $ to 29,546 $.

The profit of Genco 1 decreases under allocation method B, because it cannot take advantage of selling permits at the high permit price at time period

1. Moreover, by allocating fewer permits to period 2 with high demand level, Genco 1, being the most polluting, loses revenue by having to decrease its generation level at that period as now there are fewer free permits at that period to account for its emissions. *These results demonstrate the importance of hourly permits trading on the market-clearing and the ensuing Genco profits*.

On the other hand, Genco 4 now makes more profit because when all Gencos allocate fewer permits to the period with high demand level, the output of Genco 4 (being the least polluting) becomes more valuable and its generation level increases. Furthermore, by increasing the MW output of the highly expensive Genco, the system marginal generation cost increases, raising the electricity price, which emphasizes the benefit to Genco 4 at that time period.

However, to be able to meet the demand at optimum conditions, the dispatch decreases the demand level at that period which reduces the benefit of the demand. This decrease in the benefit of the load and increase in electricity price reduce the surplus of the load during period 2 significantly, which outweighs its increase in surplus during period 1. This results in a decrease in the load's surplus from 196,173 $ to 191,589 $, which induces a decrease in the SW from 343,904 $ to 339,839 $.

From results so far we conclude that:

1) Although the profit of the least polluting Genco 4 increases and the profit of the most polluting Genco 1 decreases by allocating more permits to a period with lower demand level, the profit of Genco 1 is still considerably higher;

2) By allocating more permits to the period with low demand level, the load surplus and social-welfare decrease.

Thus, by penalizing high polluting Gencos, both load surplus and social-welfare decline.

4.4.3.6 Results for heuristic temporal allocation method C

Here 70 % of permits are allocated to the lower demand time period 1. The corresponding temporal allocation is depicted in Table 4-20.

Heuristic method C	Time period 1 (low demand)		Time period 2 (high demand)		Commitment interval permits	
Genco permits (t)	e_{11}	348	e_{12}	149	e_1	497
	e_{21}	167	e_{22}	72	e_2	238
	e_{31}	128	e_{32}	0	e_3	183
	e_{41}	59	e_{42}	25	e_4	84

Table 4-20: Temporal allocation of permits that favor the low demand period 1.

With the temporal allocation of permits in Table 4-20, the hourly market-clearings for both time periods are shown in Table 4-21 below.

Heuristic method C	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Offered linear cost parameters $(\$/MWh)$	a_{11}	-60	a_{12}	-20		
	a_{21}	-2	a_{22}	8		
	a_{31}	10	a_{32}	16		
	a_{41}	47	a_{42}	49		
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	1.25	b_{12}	1.25		
	b_{21}	0.95	b_{22}	0.95		
	b_{31}	0.91	b_{32}	0.91		
	b_{41}	0.88	b_{42}	0.88		
Demand levels (MW)	d_1	359	d_2	583		
Generation levels (MW)	g_{11}	104	g_{12}	83		
	g_{21}	98	g_{22}	159		
	g_{31}	94	g_{31}	173		
	g_{41}	63	g_{42}	168		
Electricity marginal price $(\$/MWh)$	λ_1	113	λ_2	233		
Permits marginal price at external market $(\$/t)$	γ_1	21	γ_2	75		
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	413	$\sum_{i=1}^{4} \rho_i g_{i2}$	548	$\sum_{t=1}^{2} \sum_{i=1}^{4} \rho_i g_{it}$	960
Total cap (t/h)	$\sum_{i=1}^{4} e_{i1}$	702	$\sum_{i=1}^{4} e_{i2}$	246	$\sum_{i=1}^{4} e_i$	1002
Permits traded with external market (t/h)	Δe_{11}	-140	Δe_{12}	17	Δe_1	-123
	Δe_{21}	-69	Δe_{22}	87	Δe_2	19
	Δe_{31}	-53	Δe_{32}	84	Δe_3	31
	Δe_{41}	-27	Δe_{42}	59	Δe_4	31
Genco profit $(\$/h)$	pr_{11}	13,361	pr_{12}	17,111	pr_1	30,472
	pr_{21}	10,799	pr_{22}	27,539	pr_2	38,338
	pr_{31}	9,612	pr_{32}	29,920	pr_3	39,533
	pr_{41}	4,426	pr_{42}	25,776	pr_4	30,202
Load surplus $(\$/h)$	pr_{d1}	51,594	pr_{d2}	13,6059	pr_d	187,653
Social-welfare $(\$/h)$	SW_1	89,793	SW_2	236,405	SW	326,198

Table 4-21: Market-clearing when 70% of the total permits are allocated to the low demand period 1.

4.4.3.7 Analysis of Table 4-21

i) Market-clearing

Similar to the previous results, by allocating more permits to period 1 with low demand level, the electricity price decreases in this time period because cheaper but more polluting Gencos, such as Genco 1, can increase their generation levels and demand can be met at lower total generation cost without having to buy additional permits from the external market. On the other hand, the electricity price during period 2 increases (as compared to equal permit allocation) because scarcity of permits at that period leads to a shift in generation from high polluting cheap Gencos to less polluting but more expensive Gencos. Moreover, the increase in the demand of permits during period 2 leads to an increase in permit price, and the increase in permits sold by the sector during period 1, leads to a decrease in permit price at that period.

ii) Profits and social-welfare

Table 4-22 depicts the Genco profits and load surplus, as well as SW under heuristic temporal allocation methods A, B and C.

		Heuristic temporal allocation		
		A	B	C
Genco profit ($)	pr_1	38,199	36,596	30,472
	pr_2	40,895	41,045	38,338
	pr_3	40,446	41,065	39,533
	pr_4	28,191	29,546	30,202
Load surplus ($)	pr_d	196,173	191,589	187,653
Social-welfare ($)	SW	343,904	339,839	326,198

Table 4-22: Genco profits and load surplus as well as SW for the three heuristic temporal allocation methods.

By allocating more permits to the period with low demand level (method C), the profits of all Gencos (except the least polluting and most expensive Genco 4) decrease compared to the other allocation methods. The profit of Genco 1 decreases from 36,596 $ under method B to 30,472 $ under method C, while the profit of Genco 4 goes up from 29,546 $ to 30,202 $.

The profit of Genco 4 increases under allocation method C, because by allocating fewer permits to a period with high demand level, emission permits become scarce, and the generation level of the Genco with the lowest emission intensity becomes more valuable. The reason for this is that the amount of permits this clean Genco has to buy is low even if its generation level increases significantly to meet the high demand. Thus a Genco whose output would have been low because of its high generation cost, will be dispatched to generate more to avoid an excessively high permit price at a time period with few permits and high demand level.

Under the temporal allocation scheme C that favors the period with low demand level, the profit of the least polluting Genco 4 is now almost equal to that of the most polluting one. However, again, although allocating more permits to a period with low demand level produces better signals for investing in emission reduction, the profit of the load decreases from 191,589 $ to 187,653 $, which results in a reduction in SW from 339,839 $ to 326,198 $.

From Table 4-22, we see that by allocating permits differently amongst time periods with varying demand levels, the total profits of each Genco changes. While one temporal allocation increases the profit of one specific Genco, it might decrease the profit of a different Genco with different emission intensity and marginal cost.

For instance, while cheap yet highly polluting Gencos prefer a temporal allocation where most permits are allocated to the period with high demand level, expensive yet low polluting Gencos prefer an allocation that favors the period with low demand level. Moreover, Gencos with average generation costs and average emission intensities prefer permits to be equally allocated among the time periods.

The pervious simulations suggest that there is no one heuristic temporal allocation method that will satisfy all Gencos and thus can accurately predict the Gencos' allocation decisions. Consequently a strategic temporal allocation method that maximizes each Genco's profit according to Nash equilibrium is required to be able to analyze the effects of the cap-and-trade model.

4.4.3.8 Results for strategic temporal allocation based on NE

Recall from section 4.4.1 that, under this NE-based method, hourly permits are allocated to each Genco so that its profit is maximized given that the permit temporal allocations of all other Gencos are unchanged. The temporal allocation obtained by this NE scheme is depicted in Table 4-23.

Strategic method based on NE	Time period 1 (low demand)		Time period 2 (high demand)		Commitment interval permits	
Genco permits (t)	e_{11}	0	e_{12}	497	e_1	497
	e_{21}	0	e_{22}	238	e_2	238
	e_{31}	0	e_{32}	183	e_3	183
	e_{41}	81	e_{42}	3	e_4	84

Table 4-23: Strategic temporal allocation of permits based on NE.

Under this strategic temporal allocation scheme, all Gencos except the least polluting Genco 4, self-allocate all their permits to the time period with the higher demand level (period 2). Meanwhile, Genco 4 allocates 81 of its total permits (almost all its permits) to the time period with the lower demand level (period 1) and allocates only 3 permits to the period with higher demand level. This NE allocation agrees with the conclusions we drew from the results in Table 4-22 by testing several heuristic temporal allocation schemes. Recall that we deduced that the most polluting Genco 1 stands to benefit most by allocating fewer permits to the period with low demand level, while the least polluting most expensive Genco 4 benefits the most by allocating more permits to the period with low demand level.

With the NE temporal allocation of permits in Table 4-23, the hourly market-clearing is shown in Table 4-24 below.

Strategic allocation based on NE	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	a_{12}	-89		
	a_{21}	15	a_{22}	-9		
	a_{31}	20	a_{32}	5		
	a_{41}	46	a_{42}	50		
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	1.25	b_{12}	1.25		
	b_{21}	0.95	b_{22}	0.95		
	b_{31}	0.914	b_{32}	0.914		
	b_{41}	0.875	b_{42}	0.875		
Demand levels (MW)	d_1	296	d_2	646		
Generation levels (MW)	g_{11}	16	g_{12}	171		
	g_{21}	86	g_{22}	171		
	g_{31}	98	g_{31}	169		
	g_{41}	96	g_{42}	135		
Electricity marginal price $(\$/MWh)$	λ_1	163	λ_2	183		
Permits marginal price at external market $(\$/t)$	γ_1	66	γ_2	29		
Total emissions $(t/h),(t)$	$\sum_{i=1}^{4} \rho_i g_{i1}$	245	$\sum_{i=1}^{4} \rho_i g_{i2}$	715	$\sum_{t=1}^{2}\sum_{i=1}^{4} \rho_i g_{it}$	960
Total cap $(t/h),(t)$	$\sum_{i=1}^{4} e_{i1}$	81	$\sum_{i=1}^{4} e_{i2}$	921	$\sum_{i=1}^{4} e_i$	1002
Permits traded with external market $(t/h),(t)$	Δe_{11}	32	Δe_{12}	-155	Δe_1	-123
	Δe_{21}	86	Δe_{22}	-67	Δe_2	19
	Δe_{31}	79	Δe_{32}	-48	Δe_3	31
	Δe_{41}	-33	Δe_{42}	65	Δe_4	31
Genco profit $(\$/h),(\$)$	pr_{11}	322	pr_{12}	33,422	pr_1	33,744
	pr_{21}	6,827	pr_{22}	30,015	pr_2	36,842
	pr_{31}	8,597	pr_{32}	28,216	pr_3	36,814
	pr_{41}	12,817	pr_{42}	15,666	pr_4	28,483
Load surplus $(\$/h),(\$)$	pr_{d1}	35,116	pr_{d2}	166,964	pr_d	202,080
Social-welfare $(\$/h),(\$)$	SW_1	-65,016	SW_2	-276,390	SW	341,406

Table 4-24: Market-clearing when hourly permits are allocated strategically based on Nash equilibrium.

4.4.3.9 Analysis of Table 4-24

i) Market-clearing

Similar to the previous analysis of heuristic allocation methods A, B and C, the more polluting Gencos, such as Genco 1, stand to benefit most by allocating more permits to the period with high demand when the electricity price is high. At low demand levels, the price of permits is high but the output of the more polluting Gencos is low thus reducing the number of permits they need to buy.

On the other hand, the most expensive yet least polluting Genco 4 benefits the most by allocating more permits to the period with low demand because by reducing the number of permits allocated to the high demand period, the corresponding generation output of this Genco will be relatively high.

ii) Profits and social-welfare

Table 4-25 depicts the Genco profits and load surpluses, as well as SW under the heuristic temporal allocation methods A, B, C as well as under a strategic temporal allocation based on the NE.

		Heuristic temporal allocation			Strategic temporal allocation based on NE
		A	B	C	
Genco profit (\$)	pr_1	38,199	36,596	30,472	33,744
	pr_2	40,895	41,045	38,338	36,842
	pr_3	40,446	41,065	39,533	36,814
	pr_4	28,191	29,546	30,202	28,483
Load surplus (\$)	pr_d	196,173	191,589	187,653	202,080
Social-welfare (\$)	SW	343,904	339,839	326,198	337,963

Table 4-25: Genco and load surpluses as well as SW for the four different temporal allocation methods.

Under a strategic temporal allocation, the profit of the most polluting Genco 1 is 33,744 \$, this is higher than its profit under the heuristic method C where most permits are allocated to the period with low demand level (period 1). However it is lower than its profit under the two alternative temporal allocation methods A and B. Similarly the profit of the least polluting Genco 4 under the strategic allocation is 28,483 \$, which is higher than its profit under the heuristic method A where more permits are allocated to the high demand period 2, however it is lower than its profit under the other two allocation methods. The profits of Gencos 2 and 3 are lower under the strategic temporal allocation method compared to their profits under any of the three heuristic allocation methods.

It is very interesting to note that the load surplus under the strategic allocation method is 202,080 \$, which is higher than its profit under the three heuristic allocation methods. Moreover, the SW under the strategic method is

337,963 $, which is surpassed by heuristic methods A, and B where the SW are, respectively, 343,904 $, and 339,839 $.

4.4.3.10 Major findings from simulations on the Gencos' temporal self-allocation of permits

1) The temporal allocation of permits significantly affects the hourly market-clearing, Genco profits, as well as the load surplus and SW;

2) Allocating more permits to time periods with low demand levels increases the profit of very low polluting Gencos and decreases the profit of very high polluting Gencos, thus producing proper signals to invest in emission reduction. Unfortunately this temporal allocation reduces the load surplus and the SW;

3) There is no one heuristic temporal allocation method that will satisfy all Gencos;

4) Under the NE strategic temporal allocation, high polluting Gencos prefer to allocate more permits to high demand periods.

4.5 Allocation of Sector Cap among Gencos

4.5.1 Free allocation

Given e^0, the total emissions cap set by the SP on the electricity sector over a commitment interval, an important issue is how to divide this cap among the Gencos.

The SP can divide this cap among the Gencos for free based on some systematic approach such as the Gencos' historic emissions (grand-fathering) or, as suggested in the next section, on the basis of maximizing social welfare. The main characteristic of both approaches is that Gencos cannot influence the amount allocated which is received free of charge from the SP. In subsection 4.5.2, we consider an alternative in which Gencos can influence the amount of permits allocated by bidding into a permits auction and paying for the permits received.

4.5.1.1 Grand-fathering

As discussed in section 3.3, the benchmark used in this thesis under grand-fathering (GF) is the Gencos' BAU emission levels. Business-as-usual refers to the operation of the power system under the current market structure with no emission regulation scheme. Grand-fathering can also be based on average historical emissions.

4.5.1.2 Maximum social-welfare

While grand-fathering is the common method of free permits allocation among Gencos, its critics suggest that it rewards high polluting Gencos for their high emissions and give them an unfair advantage over less polluting ones. Thus in this thesis, we propose and develop a new benchmark for the free allocation of permits among Gencos in which the SP maximizes social-welfare over the specified commitment interval. The SP is assumed to have access to the true costs and benefits of generators and consumers.

Social-welfare is defined by,

a) The total demand benefit, $\sum_t B_t(d_t)$;

b) Minus the total true generation cost, $\sum_{i,t} C^*_{it}(g_{it})$;

c) Minus the sum of what the Gencos pay to the external emission permits market, $\sum_t \left(\gamma^0_t + \beta_t \Delta e_t\right) \Delta e_t$;

As discussed in section 4.3, the generation levels $\{g_{it}; \forall i\}$, demand level d_t, and total hourly permits traded by the sector Δe_t, can be explicitly expressed as a function of the hourly emission permits $\{e_{it}; \forall i, t\}$. Moreover, using the heuristic or strategic temporal allocation method presented and discussed in section 4.4, the hourly emission permits are implicitly defined as functions of the total permits e_i allocated to each Genco i. A detailed derivation of the equations that define this implicit relation is provided in Appendix D.

Thus, maximizing social-welfare over the Gencos' total emission permits $\{e_i; \forall i\}$ given the explicit and implicit relations between these permits and social-welfare, defines the permit allocation among Gencos and their ensuing temporal allocation, as well as the hourly market-clearing where Gencos use their hourly permits, emission intensities, as well as demand and external permits price elasticity to game according to the Cournot NE.

4.5.2 Auction-based allocation

An alternative method of dividing the sector cap, e^0, among the Gencos is to set up an auction in which each Genco *i* bids for its allocation, e_i. Gencos will now be charged for the allocated permits at a rate defined by the auction's permit incremental cost, σ. This approach may also allow for a fraction of the permits to be grand-fathered for free, while the rest are auctioned.

An auction for emission permits will be held once or few times over the commitment interval and is not to be confused with the hourly trading of permits in which Gencos can purchase or sell permits at the hourly permit price.

4.5.2.1 Social-welfare auction

Current cap-and-trade systems in Europe and North America are employing an auction method to allocate the electricity sector emissions cap over the commitment interval among individual Gencos. Thus, in this thesis, we design an auction, here called social-welfare auction (SW auction), whose objective is to maximize the sector's social-welfare which now includes the combined auction bids by the Gencos [137]. This is similar to the method of allocating permits for free by maximizing social-welfare, however now Gencos can influence this allocation through their submitted bids. The permit auction objective function (*PAOF*) to be maximized is therefore,

(a) The total demand benefit, $\sum_{t} B_t(d_t)$;

(b) Minus the total true generation cost, $\sum_{i,t} C_{it}^{*}(g_{it})$;

(c) Minus the total sum of what the Gencos pay to the external emission permits market, $\sum_{t} \left(\gamma_t^0 + \beta_t \Delta e_t\right) \Delta e_t$;

(d) Plus the bid functions, $F_i(e_i)$, submitted by the Gencos to influence how the SP distributes the commitment interval sector cap into Genco permits, $\mathbf{e} = \left\{e_i ; \forall i\right\}$. These bid functions represent the benefit each Genco i associates with acquiring its share e_i of the total commitment interval permits e^0 and take the standard form,

$$F_i(e_i) = \pi_i^0\left(e_i - e_i^f\right) - 0.5\pi_i^1\left(e_i - e_i^f\right)^2 \qquad ;\forall i \qquad (4.33)$$

with positive bid parameters, π_i^0, in \$/t, and π_i^1, in (\$/t^2) specified by each Genco i. The positive parameters, $\{e_i^f ; \forall i\}$, represent the commitment interval permits given to each Genco free-of-charge, the sum of which is a fraction[17] of the total sector cap e^0.

From the incremental bid, $IF_i(e_i) = \pi_i^0 - \pi_i^1(e_i - e_i^f)$, depicted in Figure 4.3, we see that Genco *i* is willing to pay incrementally less as the permits allocated, e_i, increases. Note that by increasing its bid (say through the parameter π_i^0), Genco i is allocated more permits and the more the Genco can produce without having to buy permits from the external market. Of course, as the bids are raised, the resulting increased amount paid to acquire these permits eventually cancels out any possible gain.

The permit auction objective function (*PAOF*) is then,

$$PAOF = \sum_t B_t(d_t) - \sum_{i,t} C_{it}^*(g_{it}) - \sum_t \left(\gamma_t^0 + \beta_t \Delta e_t\right)\Delta e_t + \sum_i F_i(e_i) \tag{4.34}$$

which is maximized by the SP with respect to the vector of commitment interval Genco permits, $\mathbf{e}$, subject to the sector's cap constraint,

$$\sum_i e_i \le e^0 \qquad (\sigma) \tag{4.35}$$

as well as to the lower bounds on each e_i, defined by the permits given to the Gencos for free,

$$e_i^f \le e_i \qquad ;\forall i \tag{4.36}$$

[17] In most countries, the free-of-charge fraction will in due course go to zero.

The Lagrange multiplier of the permits allocation constraint σ , is the marginal price charged to each Genco i for its allocated permits, e_i . Thus, Genco i is charged a total amount $\sigma(e_i - e_i^f)$ for total permits it receives to be used over the commitment interval. Appendix D shows the details of the auction model.

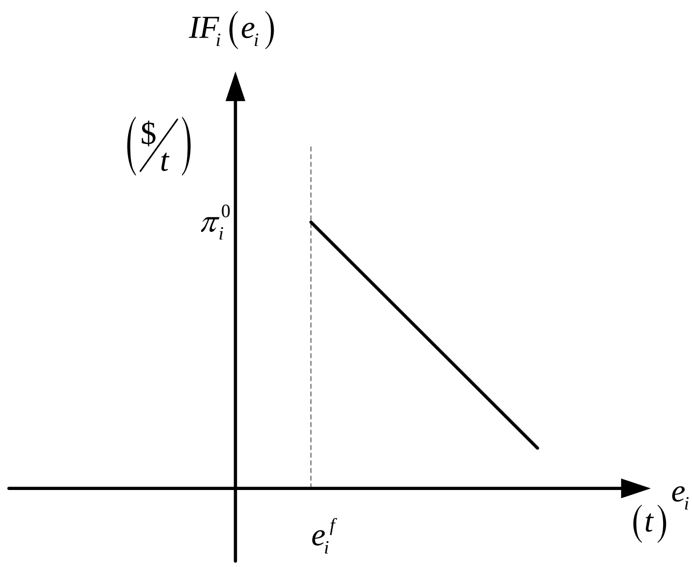

Figure 4-3: Incremental Genco bid function into the permits auction.

4.5.3 Quantitative analysis

In the following examples we study the effects of different total permit allocation methods among Gencos, on hourly market-clearing, profits and social-welfare. The allocation methods we consider are:

A) Grand-fathering (GF): Permits are allocated to Gencos for free based on their BAU emissions over the commitment interval;

B) Maximum social-welfare (SW): Permits are allocated to Gencos for free based on maximizing social-welfare;

C) Social-welfare Auction (SW auction): Permits are allocated to Gencos at the auction price, based on their bids and based on maximizing social-welfare.

We first compare the two free allocation methods; we then study the effect of the auction bids on the SW auction; and finally we compare all three allocation methods in terms of profits and SW.

For the examples in this section we assume that:

1) Total permits allocated to the sector over the commitment interval are 85 % of the corresponding BAU emissions;

2) Permits awarded to each Genco are strategically allocated among time periods based on the NE;

3) The commitment interval consists of two time periods: period 1 with low demand, and period 2 with high demand.

4.5.3.1 Simulation parameters

In this section we use the standard simulation parameters used throughout the thesis:

	Genco 1	Genco 2	Genco 3	Genco 4
Linear true cost parameter: a_{it}^{*} $(\$/MWh)$	10	15	20	50
Quadratic true cost parameter: b_{it}^{*} $(\$/MW^2h)$	0.05	0.05	0.05	0.05
Emission intensity: ρ_i (t/MWh)	2	1	0.8	0.5

Table 4-26: Gencos' cost parameters and emission intensities.

	Time period 1 (low demand)	Time period 2 (high demand)
Linear demand benefit parameter: λ_t^0 $(\$/MWh)$	400	700
Quadratic demand benefit parameter: α_t $(\$/MW^2h)$	0.8	0.8

Table 4-27: Demand benefit parameters.

	Time period 1 (low demand)	Time period 2 (high demand)
Linear permits cost parameter: γ_t^0 $(\$/t)$	50	50
Quadratic permits cost parameter: β_t $(\$h/t^2)$	0.1	0.1

Table 4-28: Parameters of the hourly emission permits trading cost function.

4.5.3.2 Results under grand-fathering of total permits

Grand-fathering (GF)	Time period 1 (low demand)		Time period 2 (high demand)		Commitment interval permits	
Genco permits (t)	e_{11}	0	e_{12}	497	e_1	497
	e_{21}	0	e_{22}	238	e_2	238
	e_{31}	0	e_{32}	183	e_3	183
	e_{41}	81	e_{42}	3	e_4	84

Table 4-29: Total permits allocated to each Genco under GF, and the resulting strategic temporal allocation of these permits.

With the permit allocation shown in Table 4-29, the hourly market-clearings for both time periods are shown in Table 4-30 below.

Grand-fathering (GF)	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	a_{12}	-89		
	a_{21}	15	a_{22}	-9		
	a_{31}	20	a_{32}	5		
	a_{41}	46	a_{42}	50		
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	1.25	b_{12}	1.25		
	b_{21}	0.95	b_{22}	0.95		
	b_{31}	0.914	b_{32}	0.914		
	b_{41}	0.875	b_{42}	0.875		
Demand levels (MW)	d_1	296	d_2	646		
Generation levels (MW)	g_{11}	16	g_{12}	171		
	g_{21}	86	g_{22}	171		
	g_{31}	98	g_{31}	169		
	g_{41}	96	g_{42}	135		
Electricity marginal price $(\$/MWh)$	λ_1	163	λ_2	183		
Permits marginal price at external market $(\$/t)$	γ_1	66	γ_2	29		
Total emissions $(t/h),(t)$	$\sum_{i=1}^{4} \rho_i g_{i1}$	245	$\sum_{i=1}^{4} \rho_i g_{i2}$	715	$\sum_{t=1}^{2} \sum_{i=1}^{4} \rho_i g_{it}$	960
Total cap $(t/h),(t)$	$\sum_{i=1}^{4} e_{i1}$	81	$\sum_{i=1}^{4} e_{i2}$	921	$\sum_{i=1}^{4} e_i$	1002
Permits traded with external market $(t/h),(t)$	Δe_{11}	32	Δe_{12}	-155	Δe_1	-123
	Δe_{21}	86	Δe_{22}	-67	Δe_2	19
	Δe_{31}	79	Δe_{32}	-48	Δe_3	31
	Δe_{41}	-33	Δe_{42}	65	Δe_4	31
Genco profit $(\$/h),(\$)$	pr_{11}	322	pr_{12}	33,422	pr_1	33,744
	pr_{21}	6,827	pr_{22}	30,015	pr_2	36,842
	pr_{31}	8,597	pr_{32}	28,216	pr_3	36,814
	pr_{41}	12,817	pr_{42}	15,666	pr_4	28,483
Load surplus $(\$/h),(\$)$	pr_{d1}	35,116	pr_{d2}	166,964	pr_d	202,080
Social-welfare $(\$/h),(\$)$	SW_1	-65,016	SW_2	-276,390	SW	337,963

Table 4-30: Market-clearing when total permits are allocated to Gencos using a grand-fathering (GF) approach based on their total BAU emissions.

Analysis of Table 4-30

The results in this table are the same as those in Table 4-24 in the previous section. Thus we refer the readers to the analysis following Table 4-24 in section 4.4.3.9.

4.5.3.3 Results when permits are allocated to Gencos for free based on maximizing social-welfare

Recall from section 4.5.1.2, that under this allocation method, permits are awarded for free by the SP to the Gencos according to maximum social-welfare. The method assumes that each Genco allocates its permits strategically to each time period based on the NE model described in section 4.4. Moreover, the method accounts for the effect of the Genco hourly permits on each Genco's Cournot NE strategy in the hourly joint electricity/permits market and its equilibrium. Table 4-31 shows the permit allocation under this method and the subsequent temporal allocation.

Maximum social-welfare	Time period 1 (Low demand)		Time period 2 (High demand)		Commitment interval permits	
Genco permits (t)	e_{11}	345	e_{12}	0	e_1	345
	e_{21}	0	e_{22}	610	e_2	610
	e_{31}	0	e_{32}	47	e_3	47
	e_{41}	0	e_{42}	0	e_4	0

Table 4-31: Total permits allocated to each Genco under maximum social-welfare and the resulting strategic temporal allocation of these permits.

Under maximum social-welfare allocation, Genco 2, the second most polluting and second cheapest Genco is awarded 610 permits which is the highest number of permits, while Genco 1 (the most polluting cheapest Genco) gets the second biggest number of permits with 345 permits. Genco 3 gets only 47 permits, while Genco 4, the least polluting but most expensive Genco gets zero permits. Thus, under maximum SW, all Gencos except Genco 2 get fewer permits than under grand-fathering.

The strategic NE temporal allocation is such that Genco 1 allocates all its permits to the low demand time period 1; and Genco 2 and 3 allocate all their permits to the high demand period 2. The justification of this temporal allocation is of the same nature as the one given for Table 4-23.

With the permit allocation shown in Table 4-31, the hourly market-clearings for both time periods are shown in Table 4-32 below.

Maximum social-welfare	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Offered linear cost parameters $(\$/MWh)$	a_{11}	-59	a_{12}	10		
	a_{21}	15	a_{22}	-46		
	a_{31}	20	a_{32}	16		
	a_{41}	50	a_{42}	50		
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	1	b_{12}	1		
	b_{21}	1	b_{22}	1		
	b_{31}	1	b_{32}	1		
	b_{41}	1	b_{42}	1		
Demand levels (MW)	d_1	324	d_2	620		
Generation levels (MW)	g_{11}	80	g_{12}	85		
	g_{21}	80	g_{22}	217		
	g_{31}	89	g_{31}	167		
	g_{41}	75	g_{42}	151		
Electricity marginal price $(\$/MWh)$	λ_1	141	λ_2	204		
Permits marginal price at external market $(\$/t)$	γ_1	50	γ_2	44		
Total emissions $(t/h),(t)$	$\sum_{i=1}^{4} \rho_i g_{i1}$	348	$\sum_{i=1}^{4} \rho_i g_{i2}$	596	$\sum_{t=1}^{2} \sum_{i=1}^{4} \rho_i g_{it}$	944
Total cap $(t/h),(t)$	$\sum_{i=1}^{4} e_{i1}$	345	$\sum_{i=1}^{4} e_{i2}$	657	$\sum_{i=1}^{4} e_i$	1002
Permits traded with external market (t/h)	Δe_{11}	-186	Δe_{12}	170	Δe_1	-16
	Δe_{21}	80	Δe_{22}	-393	Δe_2	-313
	Δe_{31}	71	Δe_{32}	87	Δe_3	158
	Δe_{41}	38	Δe_{42}	75	Δe_4	113
Genco profit $(\$/h),(\$)$	pr_{11}	19,629	pr_{12}	8,847	pr_1	28,476
	pr_{21}	5,901	pr_{22}	57,106	pr_2	63,007
	pr_{31}	6,970	pr_{32}	26,231	pr_3	33,201
	pr_{41}	4,838	pr_{42}	19,369	pr_4	24,207
Load surplus $(\$/h),(\$)$	pr_{d1}	41,876	pr_{d2}	153,738	pr_d	195,615
Social-welfare $(\$/h),(\$)$	SW_1	79,215	SW_2	265,476	SW	344,506

Table 4-32: Market-clearing when total permits are allocated to Gencos based on maximizing SW.

4.5.3.4 Comparing the two free allocation methods, GF and SW

Table 4-33 shows the allocation of total permits among Gencos under GF and under the maximum social-welfare method, while Table 4-34 depicts the Genco and load surpluses, as well as SW under both allocation methods.

		GF	Maximum social-welfare
Commitment interval permits	e_1	497	345
	e_2	238	610
	e_3	183	47
	e_4	84	0

Table 4-33: Permit allocation among Gencos under GF and maximum SW.

		GF	Maximum social-welfare
Genco profits ($)	pr_1	33,744	28,476
	pr_2	36,842	63,007
	pr_3	36,814	33,201
	pr_4	28,483	24,207
Load surplus ($)	pr_d	202,080	195,615
Social-welfare ($)	SW	337,963	344,506

Table 4-34: Comparing GF and SW permit allocation methods.

Analysis of Table 4-34

The profits of all Gencos, except that of Genco 2, are lower under a maximum-social welfare permit allocation than their corresponding profits under grand-fathering. The profit of Genco 2 under maximum SW is 63,007 $, while it is 36,842 $ under GF.

This increase in the profit of Genco 2 by allocating permits based on maximizing SW can be rationalized as follows: By receiving a large number of permits, Genco 2 ends up selling 393 permits during period 2 at the permit price of 44 $/t and generating the highest output of 217 MW during period 2 at the high price of 204 $/MWh. Moreover, Genco 2 generates 80 MW at the period with low demand level (period 1), a high level of output only exceeded by Genco 3 that produces 89 MW.

By allocating permits based on maximizing SW, the social-welfare increases from 337,963 $ to 344,506 $ while the surplus of the load decreases from 202,080 $ to 195,615 $, as compared to corresponding levels under GF. The profit of the load decreases because under maximum SW, fewer permits are allocated to the cheaper yet most polluting Genco 1, thus increasing the system generation cost of meeting the demand and the emission targets. Moreover, the strategic temporal allocation now is such that Genco 1 allocates all its permits to the period with low demand level (period 1), thus under maximum SW more permits are now rewarded to the period with low demand level as compared to under GF. As was discussed in the previous section's simulations, by reducing the permits allocated to the period with high demand level, the electricity price at that period increases, and the demand benefit decreases which leads to a decrease in the load's surplus.

By maximizing social-welfare, the second most polluting and second cheapest Genco 2 is allocated the largest number of free permits, which results in it making the highest profit. *We deduce that in order to maximize social-welfare and meet emission reduction objectives, cheap Gencos with relatively high emission intensities are rewarded with large amounts of free permits which results in them making significantly high profits.*

Moreover, under the SW allocation scheme, the most polluting Genco still makes higher profit than the least polluting Genco. *This suggests that a cap-and-trade emission regulation scheme that meets short-term emission objectives and maximizes social-welfare is not effective in producing market signals to reduce emissions in the long-term.*

4.5.3.5 Results when permits are allocated based on the proposed social-welfare auction

We now simulate and analyze results when Gencos are charged for permits they receive under the social-welfare auction model developed in section 4.5.2.1. Recall that in this auction model, as shown in equation 4.33, permits are allocated to Gencos based on maximizing SW including the Genco bids.

Table 4-35 illustrates the effect of Genco bids on the permit allocation among Gencos under the SW auction. The table also shows in brackets the corresponding allocation of these permits between the two periods under strategic temporal allocation based on the NE. We note that in these simulations Gencos do not receive any proportion of their allocated permits for free, that is $e_i^f = 0; \forall i$.

Social-welfare auction		**Auction bids** $\{\pi_1^0, \pi_2^0, \pi_3^0, \pi_4^0\}$					
		$\{0,0,0,0\}$	$\{1,0,0,0\}$	$\{2,0,0,0\}$	$\{2,1,0,0\}$	$\{2,1,3,0\}$	$\{2,1,3,6\}$
Genco permits (t)	e_1 (e_{11}, e_{12})	345 (345, 0)	927 (0, 927)	1002 (0, 1002)	340 (340, 0)	341 (341, 0)	339 (252,87)
	e_2 (e_{21}, e_{22})	610 (0, 610)	75 (0, 75)	0 (0, 0)	662 (0, 662)	0 (0, 0)	0 (0, 0)
	e_3 (e_{31}, e_{32})	47 (0, 47)	0 (0, 0)	0 (0, 0)	0 (0, 0)	661 (0, 661)	0 (0, 0)
	e_4 $(e_{41}.e_{42})$	0 (0, 0)	0 (0, 0)	0 (0, 0)	0 (0, 0)	0 (0, 0)	663 (0, 663)

Table 4-35: Permit allocation among Gencos under the SW auction for different Genco bids.

Table 4-36 shows the hourly market-clearings, profits and SW for the permit allocations shown in Table 4-35.

Social-welfare auction		Auction bids $\{\pi_1^0, \pi_2^0, \pi_3^0, \pi_4^0\}$					
		$\{0,0,0,0\}$	$\{1,0,0,0\}$	$\{2,0,0,0\}$	$\{2,1,0,0\}$	$\{2,1,3,0\}$	$\{2,1,3,6\}$
Demand level (MW)	d_1 d_2	324 620	289 654	289 654	323 621	323 619	314 624
Generation levels (MW)	g_{11} g_{12}	80 85	10 235	10 245	79 85	79 88	61 110
	g_{21} g_{22}	80 217	85 147	85 138	80 222	80 155	81 158
	g_{31} g_{32}	89 167	99 145	99 145	89 163	89 223	91 167
	g_{41} g_{42}	75 151	94 128	94 127	76 151	76 153	81 190
Electricity marginal price $(\$/MWh)$	λ_1 λ_2	141 204	169 176	169 176	142 204	142 205	149 201
Total emissions (t)	$\sum_{i=1}^{4} \rho_i g_{i1} + \sum_{i=1}^{4} \rho_i g_{i2}$	944	1028	1039	944	932	922
Total cap (t)	$\sum_{i=1}^{4} e_i$	1002	1002	1002	1002	1002	1002
Permits traded with external market (t) (t/h)	Δe_1 $(\Delta e_{11}, \Delta e_{12})$	-16 (-186, 170)	-438 (20, -458)	-492 (20, -513)	-13 (-183, 170)	-7 (-183, 176)	2 (-130, 132)
	Δe_2 $(\Delta e_{21}, \Delta e_{22})$	-313 (80, -393)	157 (85, 72)	223 (85, 138)	-359 (80, -439)	235 (80, 155)	239 (81, 158)
	Δe_3 $(\Delta e_{31}, \Delta e_{32})$	158 (71, 87)	196 (79, 116)	195 (79, 116)	201 (71, 130)	-412 (71, -483)	206 (73, 133)
	Δe_4 $(\Delta e_{41}, \Delta e_{42})$	113 (38, 75)	111 (47, 64)	111 (47, 64)	113 (38, 75)	114 (38, 76)	-528 (40, -568)
Permits marginal price at external market $(\$/t)$	γ_1 γ_2	50 44	73 29	73 30	51 44	51 42	56 36
Permits marginal price at auction $(\$/t)$	σ	50	25	27	51	51	52
Genco profits $(\$)$	pr_1	11,266	28,481	28,072	10,806	11,427	14,131
	pr_2	32,575	25,919	24,302	31,715	28,186	29,224
	pr_3	30,865	27,522	27,296	30,515	33,468	32,097
	pr_4	24,207	21,414	21,296	24,145	24,700	19,203
Load surplus $(\$)$	pr_d	195,615	204,656	204,621	195,782	194,980	195,291
Social-welfare $(\$)$	SW	294,528	307,992	305,587	292,963	292,761	289,946

Table 4-36: Market-clearing when permits are allocated using the SW auction for several Genco bids.

Analysis of Tables 4-35 and 4-36

When the vector of Genco auction bids is $\{0,0,0,0\}$, then the allocation of permits is the same as the case that maximizes social-welfare and shown in Table 4-31. One important difference however is that Gencos are not allocated permits free of charge. Instead, Gencos are now charged for their allocated permits at the permit allocation incremental rate of $\sigma = 50$ $/t , an extra charge that results in a decrease in the profits of those Gencos receiving permits. For instance, the profit of Genco 2 that receives the highest number of permits goes down considerably from 63,007 $ to 32,575 $. Note however that Genco 2 still makes the highest profit followed closely by the less polluting Genco 3 at 30,865 $, while the most polluting but cheapest Genco 1 now makes the least profit at 11,266 $. By being charged for the permits received, the profit of Genco 4 does not change since it does not receive any permits.

Thus, these results suggest that charging Gencos for their allocated permits as required by the SW auction provides the necessary long-term signals to invest in emission reduction provided that Gencos submit zero bids into the auction. This is reflected in the profit of the most polluting cheapest Genco being considerably less than that of the least polluting yet most expensive one.

However, as shown in Table 4-35, in our SW auction model, Gencos can influence the number of permits they receive and consequently their profits by bidding non-zero amounts into the auction. The corresponding effect on market-clearing is show in Table 4-36, and is now discussed for each bidding vector:

When the vector of bids is $\{1,0,0,0\}$, that is, if Genco 1 is the only Genco that increases its bid, it now gets 927 instead of 345 permits, while Genco 2 gets 75 permits instead of the 610 permits it got under zero bids. The profit of Genco 1 therefore increases significantly from 11,266 $ to 28,481 $, while that of Genco 2 goes down from 32,575 $ to 25,919 4 $. Both Gencos 3 and 4 do not get any permits now and both their profits decrease. For instance, the profit of Genco 4 goes from 24,207 \$ to 21,414 \$. Thus by bidding into the auction, the profit of the most polluting Genco 1 is again the highest, yet again sending a "bad" economic signal.

Two other interesting consequences are also observed: the auction price of the initial permits goes down from 50 \$/t to 25 \$/t; and the hourly permits price at the time period with low demand level (period 1) goes up from 50 \$/t to 73 \$/t, while it goes down from 44 \$/t to 29 \$/t at time period 2.

The auction permit price σ decreases from 50 to 25 \$/t because of the significant change in the permit allocation amongst the Gencos that results from Genco 1 being the only one to submit a non-zero bid. Thus, since the most polluting Genco 1 receives the largest number of permits, its need for permits is lowered. Thus, the effect of increasing the cap e^0 by one ton on the SW (that is, the auction permit price σ) will not be as significant as when the need for permits by Genco 1 remains strong. As will be seen below, this is not the case when Gencos other than Genco 1 bid non-zero amounts and Genco 1 receives fewer permits.

Clearly, being the only one to bid, the profit of Genco 1 increases because it then receives a high number of permits at a lower auction price. This allows Genco 1 to: i) make profit by selling a significant number of permits during period 2 to the external market at a price higher than that of the auction; ii) increase its generation level significantly during period 2 and benefit by selling electricity at a high price.

Finally, when Genco 1 receives considerably more permits, the load surplus increases from 195,615 \$ to 204,656 \$. This is reasonable because of the significantly higher generation level of the cheapest Genco 1 at the high demand period 2, which induces a lower electricity price and a higher demand level at that period.

When the vector of bids is $\{2,0,0,0\}$ **:** Genco 1 now receives all of the 1002 permits available for the electricity sector. However by doubling its bid, the auction permit price increases from 25 $/t to 27 $/t. More importantly, the total cost of buying all permits at this higher price is no longer justifiable by the additional revenue Genco 1 makes from these permits (through selling more at the external permits market, and generating more MW output). Consequently, the profit of Genco 1 decreases from 28,481 $ to 28,072 $.

We now consider the possibility of Genco 2 bidding so as to take back some of the permits allocated to Genco 1 when Genco 1 is the only one bidding.

When the vector of bids is $\{2,1,0,0\}$, Genco 2 gets 666 permits and Genco 1 only gets 336 permits, while the other Gencos do not get any permits. The profit of Genco 2 is now 31,914 $, which is higher than both previous instances when Genco 1 was the only bidding Genco, but lower than its profit (32,575 $) when all Gencos were bidding at zero $/t. With this new bidding vector, the permit allocation amongst Gencos is similar to that under zero bids (maximum social-welfare) with similar hourly market-clearing results.

By losing a significant amount of permits to Genco 2, Genco 1 cannot sell as much permits as before to the external market, and more importantly its generation level at the period with high electricity price (period 2) decreases significantly; as a result, the profit of the Genco 1 goes down from 28,072 $ to 10,806 $. The profit of the least polluting most expensive Genco 4 is now 24,145 $ which is significantly higher than that of the most polluting cheapest Genco 1.

It is interesting that the profits of Gencos 3 and 4 increase when Genco 2 bids to receive more permits. This is because, although both Gencos receive zero permits (whether Genco 2 bids or not), when fewer permits are allocated to the most polluting cheapest Genco 1 to offset the high level of emissions it produces, the low polluting Gencos 3 and 4 are dispatched to produce more MW power and their profits increase. However, Genco 3 can still bid to receive permits hoping to increase its revenue stream.

When the vector of bids is $\{2,1,3,0\}$, Genco 3 now gets all the permits from Genco 2 and its profit goes up from 30,515 $ to 33,468 $, while that of Genco 2 goes down from 31,715 $ to 28,186 $. By receiving the biggest share of the permits, and by allocating them all to the high demand level period 2, Genco 3 now has the highest MW output during period 2 which increases its profit.

Although the least polluting most expensive Genco 4 is yet to receive any permits, its profit also increases slightly from 24,145 $ to 24,700 $ when permits are taken away from Genco 2 to the less polluting Genco 3. This is because with fewer permits given to a rather polluting Genco, the generation level of the least polluting Genco 4 increases slightly. However, Genco 4 can bid to get some permits and try to increase its profit.

When the vector of bids is $\{2,1,3,6\}$, by bidding for permits, Genco 4 now receives 663 permits, while Genco 1 receives the remaining 339 permits. However by receiving the biggest number of permits at the high auction price of 52 $/t, the profit of Genco 4 decreases from 24,700 $ to 19,203 $. On the other hand, the profit of Genco 1 increases from 11,427 $ to 14,131 $. The profit of Genco 4 decreases because by being allocated a large number of permits, the increase in the MW output of Genco 4 is not as significant as that of a more polluting Genco such as Genco 1. This is because what limits the generation output of Genco 4 is its high marginal generation cost and not emission intensity. Thus by getting more permits Genco 4 does not benefit, as it has to pay a high premium for these permits, and it can only sell excess permits to the external market at a price lower than the auction price at both periods. We note that the increase in market power of Genco 4 by selling permits does not compensate for the added cost of paying a high auction price for them.

Although the amount of permits given to Genco 1 is almost the same, the profit of Genco 1 increases because by increasing the generation dispatch of the least polluting most expensive Genco 4, the output of Genco 1 can increase as the low emission intensity of Genco 4 allows the more polluting Genco 1 to generate more without the need for more emission permits.

To further illustrate the effect of the allocated permits on the hourly generation dispatch, consider once again the case when Genco 1 (the most polluting but cheapest Genco) is the only Genco that bids for permits, and the bidding vector changes from $\{0,0,0,0\}$ to $\{1,0,0,0\}$, thus increasing the amount of permits allocated to Genco 1 from 345 t to 927 t. In this case, the generation output of the most polluting cheapest Genco 1 during period 2 increases significantly from 85 MW to 235 MW which induces a significant increase in the profit Genco 1 from 11,266 $ to 28,481 $.

The above auction simulations over different Genco bids, suggest that by increasing their auction bids, up to a limit, Gencos can acquire higher permit allocations and increase their profits. The simulations also show that more polluting cheaper Gencos stand to benefit more through bidding than less polluting but more expensive Gencos. However, while high polluting Gencos need to bid into the auction to acquire permits and avoid low profits, low polluting Gencos can retaliate by bidding themselves and denying high polluting Gencos such high profits.

While further studies regarding the bidding strategies of the competing Gencos into the auction is beyond the scope of this thesis, simulations show that a NE defining these bids is likely to exist. Moreover, results show that under the proposed SW auction high polluting cheap Gencos are denied the high profits they enjoyed under free allocation. Finally, while the SW auction introduces new uncertainties in modeling the electricity market under CAT, the proposed auction provides low polluting expensive Gencos with a new form of market power, allowing them to earn higher profits than high polluting cheap Gencos. As a result, the cap-and-trade system under the SW auction provides better signals to invest in emission reduction, as compared to GF or maximum SW free allocation of permits.

4.5.3.6 Major findings from the simulations on the allocation of sector cap among Gencos

Table 4-37 summarizes the allocation of the sector cap among Gencos under: i) GF; ii) maximum social-welfare; iii) social-welfare auction. Table 4-38 depicts the Genco profits and load surplus as well as the SW under these three cap allocation methods.

		GF	Maximum social-welfare	Social-welfare auction					
				$\{0,0,0,0\}$	$\{1,0,0,0\}$	$\{2,0,0,0\}$	$\{2,1,0,0\}$	$\{2,1,3,0\}$	$\{2,1,3,6\}$
Commitment interval permits (t)	e_1	497	345	345	927	1002	341	341	339
	e_2	238	610	610	75	0	662	0	0
	e_3	183	47	47	0	0	0	661	0
	e_4	84	0	0	0	0	0	0	663

Table 4-37: Permit allocation among Gencos under GF, maximum social-welfare, and the SW auction.

		GF	Maximum social - welfare	Social-welfare auction					
				$\{0,0,0,0\}$	$\{1,0,0,0\}$	$\{2,0,0,0\}$	$\{2,1,0,0\}$	$\{2,1,3,0\}$	$\{2,1,3,6\}$
Genco profit $(\$/h)$	pr_1	33,744	28,476	11,266	28,481	28,072	10,806	11,427	14,131
	pr_2	36,842	63,007	32,575	25,919	24,302	31,715	28,186	29,224
	pr_3	36,814	33,201	30,865	27,522	27,296	30,515	33,468	32,097
	pr_4	28,483	24,207	24,207	21,414	21,296	24,145	24,700	19,203
Load surplus $(\$/h)$	pr_d	202,080	195,615	195,615	204,656	204,621	195,782	194,980	195,291
Social-welfare $(\$/h)$	SW	337,963	344,506	294,528	307,992	305,587	292,963	292,761	289,946

Table 4-38: Profits and SW for different permit allocation schemes.

From the simulations in this section we make the following observations:

1) Allocating permits based on maximizing SW but charging Gencos for these permits provides the necessary long-term signals to invest in emission reduction;

2) Under the SW auction, Gencos can bid to acquire more commitment interval permits and increase their total profits. Low polluting expensive Gencos can now deny high polluting cheap Gencos from making high profits, which result in better signals and incentives to invest in emission reduction as compared to GF or maximum SW free allocation of permits;

3) The least polluting most expensive Genco 4 has no incentive to bid into the SW auction since, being clean, it does not need permits, thus avoiding the expense that comes with them;

4) Initial results suggest the existence of a Nash equilibrium defining the Gencos' auction bids; however the computation of such equilibrium is beyond the scope of this thesis;

5) An interesting and important point to be researched further is the manner in which the money collected by the SP from the permit auction is used. For example, one can look at the possibility that this income be used to invest in new technologies that reduce the emission intensities and energy efficiencies of power plants.

4.6 Chapter Summary

(i) In this chapter we implemented a cap-and-trade emission regulation scheme in the electricity sector by re-modeling the customary hourly market-clearing to account for hourly emissions, self-assigned permit allocations and permits trading. Moreover, the hourly Cournot NE Genco gaming strategies were accordingly re-modeled to factor in hourly permits, price elasticity of permits at the external market, emission intensities, as well as the customary marginal generation costs and demand elasticity. The effect of all of these extensions on market-clearing, market power and profits were numerically tested and examined in detail;

(ii) Results show that the emissions cap and how this cap is allocated among Gencos, the Gencos' emission intensities and the hourly permits trading have a marked effect on the Gencos' market power, hourly market-clearing and profits. For example, high polluting but cheap Gencos lose market power compared to BAU without emission regulation. However, when the cap allocation strategies are based on grand-fathering, the aim of which is to avoid large deviations from BAU, high polluting but cheap Gencos tend to receive exceptionally high levels of permits. As a result, at market-clearing these Gencos end up with excess permits, which they then sell in the external market instead of using them to offset their own emissions. Thus, under GF, high emission but cheap Gencos are able to maintain their market power and high profits, while low polluting Gencos continue to earn significantly lower profits.

(iii) We also developed a strategic method based on the NE for Gencos to self allocate permits among the time periods of the commitment interval. Heuristic allocation methods based on the demand level were also tested. Results showed that Gencos with different marginal costs and emission intensities do not benefit uniformly from a given heuristic scheme and it is difficult to settle on one that would be acceptable to all Gencos. This suggested that a more stable temporal permit allocation would be one based on the NE. This is the approach taken in subsequent studies in this thesis.

(iv) This chapter also examined how to allocate the commitment interval sector cap among the various Gencos. Different such permit allocation methods were numerically tested and compared. A grand-fathering method where permits are allocated for free based on BAU emissions was the first one considered. A second novel free allocation method was also proposed and tested. Under this allocation method, permits are allocated to Gencos based on maximizing social-welfare while accounting for the effects of the Genco permit allocation on the hourly gaming strategies and market-clearing.

v) Since, rather than free allocation, current governments are proposing a cap distribution scheme based on an auction, we also examined this alternative. To do so we incorporated the maximum social-welfare allocation method into an auction, where Gencos bid to influence the commitment interval allocation outcome but are charged for these permits. The effects of the auction bids on permit allocation and on market-clearing were compared against results based on GF and on a free allocation that maximizes social-welfare.

(vi) Results suggest that under the socially-accepted and common permit allocation scheme based on grand-fathering, high polluting but cheap Gencos make disproportionately higher profits than less polluting more expensive Gencos, and that the scheme therefore fails to promote investment in emission reduction. Similar results occur under a free allocation of permits based on maximizing SW, in which case higher polluting but cheaper Gencos are still rewarded with high permits and profits. However, when Gencos are charged for permits under the maximum social-welfare allocation scheme (the same as a SW auction with zero bidding), the profits of high polluting Gencos decline, thus sending a more reasonable economic signal.

(vii) In the auction model proposed and studied here, results show that through their bids, Gencos can exercise another form of market power in which less polluting but expensive Gencos can now deny more polluting cheaper Gencos the high profits they enjoy under free permit allocation. Thus, while auctioning permits adds another complexity to an electricity market operating under CAT, the proposed SW auction provides better signals and incentives to invest in emission reduction, than free allocation of permits based on GF and maximum SW.

As a final word, advocates of cap-and-trade (CAT) claim that this scheme, especially its permits trading flexibility, will not only regulate emissions but will moderate the negative effect of an emission cap on prices and profits. This chapter however shows first and foremost that CAT is a complex system to design and implement, and, furthermore, that it opens the door to additional gaming opportunities. Our analysis suggests that permit trading will be used by Gencos to increase their market power and make windfall profits, actions that will not mitigate the effect of an emissions cap on the economy.

5 Electricity Markets Operating under Carbon Tax

"An invasion of armies can be resisted, but not an idea whose time has come;" Victor Hugo

In this chapter, we examine the carbon tax (CTX) approach to regulate emissions in the electricity industry. A number of governments have considered CTX as an alternative to cap-and-trade [138]. Under this approach, Gencos pay a premium or tax for the carbon emissions they emit while producing electricity.

Since most societies and governments are reluctant to accept the notion of new taxes, an emission regulation approach based on a carbon tax begins with a big handicap. Nonetheless, in this thesis we propose and design a novel carbon tax emission regulation model that can be efficiently integrated into an electricity market while meeting desired emission targets. In chapter 6, we offer qualitative and quantitative comparisons of the CTX and CAT emission regulation approaches and make a recommendation as to which approach is preferable.

This chapter is organized as follows: Section 5.1 defines the basic nomenclature used in this chapter; section 5.2 gives an overview of carbon tax and highlights the challenges of implementing a carbon tax scheme into an electricity market; section 5.3 models and analyses the hourly market-clearing under a carbon tax scheme; section 5.4 explains our approach to set the optimum carbon tax parameters; and finally in section 5.5 we present the chapter summary and highlight important conclusions.

5.1 Nomenclature

The following is a partial nomenclature for this chapter (some of the nomenclature from chapters 2 and 4 is repeated for easy access).

Parameters:

ng : Number of Gencos;

nt : Number of time periods in the commitment time interval;

Δt :length of time period;

$\boldsymbol{\rho} = \{\rho_i ; \forall i\}$: Vector of Genco emission intensities (t/MWh);

$\mathbf{g}^{\max} = \{g_i^{\max} ; \forall i\}$: Vector of maximum Genco output levels (MW);

$\mathbf{a}_t^* = \{a_{it}^* ; \forall i\}; \forall t$: Vector of first-order parameters of Genco actual cost functions for at time t (\$/MWh);

$\mathbf{b}_t^* = \{b_{it}^* ; \forall i\}; \forall t$: Vector of second-order parameters of Genco actual cost functions at time t (\$/MW^{2}h);

λ_t^0 : First-order parameter of demand benefit function at time t (\$/MWh);

α_t : Second-order parameter of demand benefit function at time t (\$/MW^{2}h);

e^0 : Emissions cap on the electricity sector within the commitment interval (t);

Variables:

$\mathbf{a}_t = \{a_{it} ; \forall i\}; \forall t$: Vector of first-order parameters of Genco offered cost functions at time t (\$/MWh);

$\mathbf{b}_t = \{b_{it} ; \forall i\}; \forall t$: Vector of second-order parameters of Genco offered cost functions at time t (\$/MW^{2}h);

$\mathbf{g}_t = \{ g_{it}; \forall i \}; \forall t$: Vector of Genco output levels at time t (MW)

d_t : Demand level at time t (MW);

γ^0 : First-order parameter of carbon tax cost function (\$/t);

β : Second-order parameter of carbon tax cost function (\$h/t^2);

$\mathbf{x}_t = \{ x_{it}; \forall i \}; \forall t$: Vector of Genco emissions at time t (t/h);

$\mathbf{x} = \{ x_t; \forall t \}$: Vector of total hourly emissions produced by all Gencos (t/h)

<u>Lagrange multipliers:</u>

λ_t : Price of electricity at time t (\$/MW);

$\boldsymbol{\mu}_t^{\max} = \{ \mu_{it}^{\max}; \forall i \}; \forall t$: Vector of Lagrange multipliers of maximum generation constraints at time t;

$\boldsymbol{\mu}_t^{\min} = \{ \mu_{it}^{\min}; \forall i \}; \forall t$: Vector of Lagrange multipliers of minimum generation constraints at time t;

$\boldsymbol{\gamma} = \{ \gamma_t; \forall t \}$: Vector of hourly carbon tax rate (\$/t);

$\delta^{\min}$: Lagrange multipliers of the minimum constraint on the first-order parameter of the carbon tax cost function;

$\delta^{\max}$: Lagrange multipliers of the maximum constraint on the first-order parameter of the carbon tax cost function;

<u>Functions:</u>

$C_{it}(g_{it})$: Generation cost function offered by Genco i at time t (\$/h);

$IC_{it}\left(g_{it}\right)$: Incremental generation cost function offer by Genco i at time t ($/MWh);

$B_t\left(d_t\right)$: Demand benefit function at time t ($/h);

$IB_t\left(d_t\right)$: Incremental demand function at time t ($/MWh);

$CTX_t(x_t)$: Carbon tax scheme at time t ($/h);

$ICTX_t(x_t)$: Incremental carbon tax scheme at time t ($/t);

5.2 Carbon Tax Overview

Taxing certain commodities from specific industries is a common practice that governments employ to raise funds or to earn extra revenue. In some cases, however, a tax is used to raise the price of a commodity, such as tobacco, thus discouraging its use. A carbon tax applied to the electricity industry is intended to decrease levels of emissions that generating companies emit while producing electricity.

Under a carbon-tax emission regulation approach, power generating companies (Gencos) pay a premium (tax) for all their carbon emissions in the form of $ per ton of carbon emitted. This rate depends on a tax scheme defined by the social planner (SP) and designed so that the emission target is met with minimal adverse impact on social welfare.

As discussed in chapter 2, electricity generation is dispatched through an electricity market that is based on hourly offers by Gencos to supply power to consumers. The resulting hourly market-clearing defines the hourly generation and demand levels as well as prices and profits.

Thus, initially, an appropriate tax scheme must be defined and the hourly electricity market must be modified to incorporate such a scheme, including how the Gencos' gaming strategies are affected by this modification. In a later section, we show how the tax scheme parameters are set optimally by the SP in order to meet the commitment interval sector emissions cap with minimal impact on social-welfare.

5.3 Hourly Market-Clearing with Carbon Tax

5.3.1 Carbon tax scheme

The hourly carbon tax scheme proposed in this thesis first requires the definition of a carbon tax penalty added to the hourly electricity market. This carbon tax penalty in \$/h is in the form of a function of the total sector emissions at time *t*, x_t,

$$CTX_t = \gamma_t^0 (x_t) + 0.5\beta_t (x_t)^2; \quad \forall t \tag{5.1}$$

The parameters defining the tax scheme, γ_t^0 and β_t, respectively in \$/t and \$h/t ^{2} are strictly positive and are determined by the SP in a manner described in section 5.4.

The total emissions in t/h produced by all Gencos at time t is given by,

$$x_t = \sum_i \rho_i g_{it} \quad ; \forall t \tag{5.2}$$

As shown below, the incremental carbon tax penalty, $ICTX_t(x_t) = \gamma_t^0 + \beta_t x_t$, defines the carbon tax rate at time t, denoted by γ_t, that is charged to each Genco *i* for its individual hourly emissions, $\rho_i g_{it}$. Thus at time t, the carbon tax paid by Genco i for its hourly emissions, is given by, $\gamma_t(\rho_i g_{it})$ and is measured in units of \$/h. As seen in Fig. 5.1, the tax rate is monotonically increasing in the total sector emissions; the more emissions the sector produces, the higher the carbon tax rate.

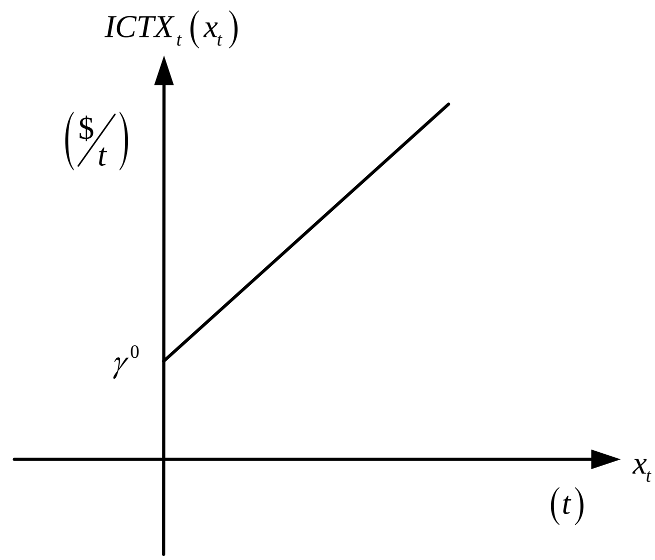

Figure 5-1: Hourly incremental tax rate.

5.3.2 Carbon tax parameters

In this thesis, although the carbon tax parameters can be time varying, we reasonably assume that these remain constant over at least the entire commitment interval. Thus,

$$\left.\begin{aligned} \gamma_t^0 &= \gamma^0 \\ \beta_t &= \beta \end{aligned}\right\} ;\forall t \tag{5.3}$$

Despite this assumption, the hourly carbon tax rate charged to Gencos is still time-dependent through its dependence on the hourly emissions, x_t,

$$\gamma_t = ICTX_t(x_t) = \gamma^0 + \beta x_t \tag{5.4}$$

The basic hourly rate at which Gencos are taxed for their emissions, is given by the parameter γ^0. This notion of a fixed rate agrees with current carbon tax proposals. However, as shown in Fig. 5-1, in this thesis we consider a more general hourly carbon tax rate that increases with the sector's hourly emissions, x_t, at the constant rate of β. Such a progressive tax rate will be seen to affect the Genco hourly offer strategies in the sense that the cleaner Gencos now have an exploitable advantage over the dirtier Gencos.

Finally, we observe from (5.4) that, since the hourly total emissions produced by the electricity sector may fluctuate considerably with the hourly demand, so will the hourly carbon tax rate γ_t. The rationale here is that a variable tax rate has more flexibility to meet the objectives of the carbon tax than a fixed rate, namely to meet the emissions target and to do so with minimal impact on social welfare.

5.3.3 Hourly pool dispatch with carbon tax penalty

To implement the carbon tax scheme in the electricity sector, we re-model the market-clearing process at time *t* by modifying the defining optimization problem of (2.1) - (2.4) to include the hourly carbon tax penalty, $CTX_t(x_t)$,

$$\min_{\{g_{it};\forall i\},x_t,d_t} \sum_k C_{it}(g_{it}) - B_t(d_t) + CTX_t(x_t) \tag{5.5}$$

subject to the power balance,

$$\sum_i g_{it} = d_t \qquad (\lambda_t) \tag{5.6}$$

the total emissions produced by all Gencos at time t,

$$x_t = \sum_i \rho_i g_{it} \qquad (\gamma_t) \tag{5.7}$$

and the generation upper and lower limits,

$$g_{it} \le g_{it}^{\max} \qquad ;\forall i \qquad (\mu_{it}^{\max}) \tag{5.8}$$

and,

$$g_{it}^{\min} \le g_{it} \qquad ;\forall i \qquad (\mu_{it}^{\min}) \tag{5.9}$$

The minimization in (5.5) is carried out over the hourly variables, $\left(\{g_{it};\forall i\},d_t,x_t\right)$, subject to the power balance equation (5.6), to the generation limits (5.8) and (5.9), as well as to (5.7) which is the equality that defines the total emissions produced by the electricity sector at time *t*, $x_t=\sum_i \rho_i g_{it}$.

Recall that $C_{it}\left(g_{it}\right)$ represents the strategic cost offer submitted to the pool by Genco *i* to meet the demand at time *t*. As shown below, these strategic Genco offers are obtained from the Cournot Nash equilibrium accounting for the tax scheme.

The Karush-Kuhn-Tucker (KKT) necessary conditions under which the electricity market clears at time *t* require that the price of electricity be given by,

$$\begin{aligned}\lambda_t &= IB_t(d_t)=\lambda_t^0-\alpha_t d_t\\ &=\lambda_t^0-\alpha_t\left(\sum_i g_{it}\right)\end{aligned} \qquad (5.10)$$

and that the Lagrange multiplier γ_t be of the form,

$$\begin{aligned}\gamma_t &= ICTX_t(x_t)=\gamma^0+\beta x_t\\ &=\gamma^0+\beta\left(\sum_i \rho_i g_{it}\right)\end{aligned} \qquad (5.11)$$

Observe that, as the Lagrange multiplier of the hourly emissions balance equation (5.7) γ_t is equal to the marginal cost of emissions, in other words the cost of the next ton of emissions produced by the sector. As such, under marginal pricing, γ_t becomes the carbon tax rate paid by the Gencos to the SP[18].

Finally, we observe that the KKT conditions require that the Gencos' incremental cost offers satisfy,

$$IC_{it}(g_{it}) = \lambda_t - \rho_i \gamma_t + \mu_{it}^{\min} - \mu_{it}^{\max} \qquad ; \forall i \tag{5.12}$$

$$\left. \begin{aligned} \mu_{it}^{\max} \geq 0 \\ \mu_{it}^{\min} \geq 0 \end{aligned} \right\} \qquad ; \forall i \tag{5.13}$$

as well as the complementarity slackness conditions,

$$\left. \begin{aligned} \mu_{it}^{\min}\left(g_{it} - g_{it}^{\min}\right) = 0 \\ \mu_{it}^{\max}\left(g_{it} - g_{it}^{\max}\right) = 0 \end{aligned} \right\} \qquad ; \forall i \tag{5.14}$$

5.3.4 Genco gaming with carbon tax

If Gencos do not game in the hourly market, for example, under perfect market conditions, then in (5.12) we define,

$$IC_{it}(g_{it}) = IC_{it}^{*}(g_{it}) \qquad ; \forall i \tag{5.15}$$

where $IC_{it}^{*}(g_{it}) = a_{it}^{*} + b_{it}^{*} g_{it}$ is the true incremental cost of Genco i at time t. However, if Gencos game according to Cournot Nash, then we proceed as follows:

Consider the profit of Genco i during period t,

[18] As was the case in the permits auction under CAT, the question of what the SP will do with the money collected from the carbon tax is an interesting and important one, but beyond the scope of this thesis.

$$pr_{it} = \lambda_t g_{it} - C_{it}^*(g_{it}) - \gamma_t(\rho_i g_{it}) \tag{5.16}$$

This hourly profit consists of the Genco's revenue, $\lambda_t g_{it}$, minus its true generation cost, $C_{it}^*(g_{it})$, minus the tax the Genco pays for its hourly emissions, $\gamma_t\left(\rho_i g_{it}\right)$. As required by Cournot Nash, we now maximize the Genco's hourly profit (5.16) with respect to its output, g_{it}, then, as detailed in Appendix E, the hourly offer by Genco *i* at time *t* is such that,

$$IC_{it}(g_{it}) = IC_{it}^*(g_{it}) + \left(\alpha_t + \beta\rho_i^2\right) g_{it} \qquad ;\forall i \tag{5.17}$$

Thus, the gaming strategy for Genco *i* under carbon tax is to offer an IC function of the form,

$$IC_{it}(g_{it}) = a_{it} + b_{it} g_{it} \tag{5.18}$$

where,

$$\begin{aligned} a_{it} &= a_{it}^* \\ b_{it} &= b_{it}^* + \alpha_t + \beta_t\rho_i^2 \end{aligned} \tag{5.19}$$

Condition (5.17)defines the Cournot Nash gaming strategy of Genco *i* at time *t*. In the term, $IC_{it}^*(g_{it}) + \alpha_t g_{it}$, we recognize the standard Cournot Nash gaming strategy (when there is no carbon tax) that takes advantage of the load elasticity. The extra term in the gaming strategy, $\beta\rho_i^2 g_{it}$, reflects the added marginal cost of the hourly carbon tax. It implies that Gencos with higher emission intensities become incrementally more expensive than cleaner Gencos. This is intuitively reasonable and favorable, since Gencos with higher emission intensities will now lose some of their BAU market power, producing less power and emitting less, thus meeting the purpose of the carbon tax.

Note that the extra term, $\beta\rho_i^2 g_{it}$, in the gaming strategy depends on the carbon tax parameter β which, from (5.4), defines how much the hourly tax rate, γ_t, changes with the hourly total emissions of the electricity sector, x_t. A higher β , means a higher sensitivity of the tax rate to the hourly sector's emissions and makes the polluting Gencos less competitive.

Now, as shown in Appendix E, by simultaneously solving relations (5.10) - (5.12), plus the gaming strategy, (5.17), we can express the hourly outputs, $\{g_{it}; \forall i\}$ at time *t*, as implicit functions of the hourly tax parameters γ^0 and β as well as the Lagrange multipliers, $\{\mu_{it}^{\min}, \mu_{it}^{\max}; \forall i\}$, at time *t*. We denote these functions by,

$$g_{it} = g_{it}(\gamma^0, \beta, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) \qquad ; \forall i \tag{5.20}$$

where the aforementioned vectors are defined as,

$$\begin{aligned} \boldsymbol{\mu}_t^{\min} &= \{\mu_{it}^{\min}; \forall i\} \\ \boldsymbol{\mu}_t^{\max} &= \{\mu_{it}^{\max}; \forall i\} \end{aligned} \tag{5.21}$$

In addition to (5.20), the hourly market-clearing solution includes the complementarity slackness conditions, (5.14),

$$\begin{aligned} & s_t(\gamma^0, \beta, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) = \\ & \sum_i \left\{ \mu_{it}^{\min} \left(g_{it}^{\min} - g_{it}(\gamma^0, \beta, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) \right) + \mu_{it}^{\max} \left(g_{it}^{\max} - g_{it}(\gamma^0, \beta, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) \right) \right\} = 0 \end{aligned} \tag{5.22}$$

as well as the inequalities,

$$\left. \begin{aligned} g_{it}^{\min} &\le g_{it}(\gamma^0, \beta, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) \\ g_{it}(\gamma^0, \beta, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}) &\le g_{it}^{\max} \end{aligned} \right\} \qquad ; \forall i \tag{5.23}$$

and

$$\left.\begin{matrix}\boldsymbol{\mu}_t^{\min} \\ \boldsymbol{\mu}_t^{\max}\end{matrix}\right\} \leq \mathbf{0} \tag{5.24}$$

5.3.5 Quantitative analysis

In this section we study the effects of the carbon tax scheme on the hourly market-clearing for two arbitrarily chosen levels[19] of the carbon tax parameter γ^0:

1) Low: characterized by a value of 30 \$/t;

2) High: characterized by a value of 50 \$/t.

In the following simulations, we assume that the second order carbon tax parameter β is equal to 0.01 (\$/t)/ (t/h), and that the commitment interval is one time period only. In addition, we consider two demand levels, one high and one low.

Recall that BAU refers to market-clearing without carbon tax, assuming that Gencos game according to Cournot NE by taking into account demand elasticity and generation costs only(see equations (2.17) and (2.18) in section 2.4). Under carbon tax, Gencos game by taking into account emission intensities and carbon tax elasticity (as per equations (5.18) and (5.19)).

[19] In the next section, we show how to set the tax parameters systematically in a way that maximizes social welfare and meets emission targets.

5.3.5.1 Simulation parameters

In this section we use the standard simulation parameters used throughout the thesis. As a reminder these parameters are:

	Genco 1	Genco 2	Genco 3	Genco 4
Linear true cost parameter: a_{it}^{*} $(\$/MWh)$	10	15	20	50
Quadratic true cost parameter: b_{it}^{*} $(\$/MW^2h)$	0.05	0.05	0.05	0.05
Emission intensity: ρ_i (t/MWh)	2	1	0.8	0.5

Table 5-1: Gencos' cost parameters and emission intensities.

	Low demand	High demand
Linear demand benefit parameter: λ_t^0 $(\$/MWh)$	400	700
Quadratic demand benefit parameter: α_t $(\$/MW^2h)$	0.8	0.8

Table 5-2: Demand benefit parameters.

5.3.5.2 Results for low demand level

		BAU	Tax parameter $\gamma^0 = 30\ \$/t$	Tax parameter $\gamma^0 = 50\ \$/t$
Emission tax 2nd order parameter ($/t)/ (t/h)	β	-	0.01	0.01
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	10	10
	a_{21}	15	15	15
	a_{31}	20	20	20
	a_{41}	50	50	50
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	0.85	0.89	0.89
	b_{21}	0.85	0.86	0.86
	b_{31}	0.85	0.856	0.856
	b_{41}	0.85	0.853	0.853
Demand level (MW)	d_1	372	335	315
Generation levels (MW)	g_{11}	109	62	37
	g_{21}	103	97	93
	g_{31}	97	99	100
	g_{41}	62	77	84
Individual emissions (t/h)	$\rho_1 g_{11}$	218	124	73
	$\rho_2 g_{21}$	103	97	93
	$\rho_3 g_{31}$	78	80	80
	$\rho_4 g_{41}$	31	38	42
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	430	339	289
Electricity marginal price $(\$/MWh)$	λ_1	103	132	148
Emission tax rate $(\$/t)$	γ_1	-	33	53
Genco absolute and per unit profits $(\$/h),(\$/MWh)$	pr_{11}	9,816 (90)	3,321 (54)	1,154 (31)
	pr_{21}	8,786 (85)	7,878 (81)	7,296 (78)
	pr_{31}	7,813 (81)	8,231 (83)	8,378 (84)
	pr_{41}	3,173 (51)	4,844 (63)	5,876 (70)
Load surplus $(\$/h)$	pr_{d1}	55,236	44,916	39,601
Social-welfare $(\$/h)$	SW_1	84,823	69,190	62,305

Table 5-3: Market-clearing under BAU and under CTX with different tax parameters.

Analysis of Table 5-3

i) <u>Tax parameter $\gamma^0 = 30\ \$/t$</u>

By applying a carbon tax, the gaming strategies of Gencos are affected as can be seen through the b-parameter of the IC-offer of each Genco, given by equation(5.19). Recall from equation (5.19) that the a-parameter of these IC-offers remain unchanged at their true values while Gencos with lower emission intensities offer lower b-parameters than those with higher intensities.

Comparing these new modified offers with BAU, we observe a shift in generation levels from high polluting cheap Gencos to less polluting more expensive ones, as well as a significant decrease in demand level. The decrease in generation level of the most polluting cheapest Genco 1 from 109 MW to 62 MW is paralleled by a much smaller increase in the generation level of the least polluting most expensive Genco 4 from 62 MW to 77 MW. In addition, because of the increased energy price, the demand level decreases from 372 MW to 335 MW. As a result of this shift in generation levels and decrease in demand level, total emissions produced by the sector decrease significantly from 430 t/h to 339 t/h, however the electricity price increases from 103 $/MWh to 132 $/MWh.

By applying the carbon tax, the profits of the two most polluting cheapest Gencos 1 and 2 decrease significantly, while the profits of the two least polluting more expensive Gencos 3 and 4 increase. For instance the profit of Genco 1 decreases significantly from 9,816 $/h to 3,321 $/h, while that of Genco 4 increases from 3,173 $/h to 4,844 $/h. Interestingly, the profit of the least polluting yet highly expensive Genco 4 is now higher than the profit of the cheapest yet most polluting Genco 1. Also, the profits of Gencos 1, 2 and 3 is such that Gencos with lower emission intensities make higher profits. Moreover, the per unit profit of Genco 1 decreases from 90 $/MWh to 54 $/MWh, while that of Genco 4 increases from 51 $/MWh to 63 $/MWh.

By applying a carbon tax, the surplus of the load decreases significantly from 55,236 $/h to 44,916 $/h which induces a decrease in the social-welfare, from 84,823 $/h to 69,190 $/h. This decrease in load surplus is expected due to the significant decrease in demand level and increase in electricity price.

We now study the effects of increasing the tax parameter γ^0.

ii) Tax parameter $\gamma^{0} = 50\ \$ / t$

By increasing the tax parameter γ^{0} from 30 to 50 $/t, the generation level of the dirty Genco 1 decreases significantly from 62 MW to 37 MW, while that of the least polluting Genco 4 increases slightly from 77 MW to 84 MW. The demand level decreases from 335 MW to 315 MW and the total emissions produced decrease from 339 t/h to 289 t/h. These results reinforce the conclusion that a carbon tax is a powerful tool in reducing the MW outputs of high polluting Gencos despite their low generation costs and in reducing emissions. Note that the strategic Genco offers do not change by increasing the tax parameter γ^{0}, because the Gencos' gaming strategies depend on the non-linear tax parameter, β which is constant in these simulations.

Moreover, by increasing the tax parameter γ^{0}, the profits of the two most polluting cheapest Gencos 1 and 2 decrease significantly, while the profits of the two least polluting more expensive Gencos 3 and 4 increase. For instance the profit of Genco 1 decreases significantly from 3,321 $/h to 1,154 $/h, while that of Genco 4 increases from 4,844 $/h to 5,876 $/h. The profit of the least polluting yet highly expensive Genco 4 is now significantly higher than the profit of the cheapest yet most polluting Genco 1. Moreover, the per unit profit of Genco 1 decreases from 54 $/MWh to 31 $/MWh, while that of Genco 4 increases from 63 $/MWh to 70 $/MWh.

Furthermore, the load surplus decreases significantly from 44,916 $/h to 39,601 $/h which induces a decrease in the social-welfare, from 69,190 $/h to 62,305 $/h.

Thus, this first case suggests that, under CTX, effective short-term emission reduction is possible as the generation dispatch favors the less polluting Gencos regardless of their higher generation costs. Moreover, the generator profit trend motivates investing in emission reduction in the long-term.

5.3.5.3 Results for high demand level

We now study the effect of demand level by simulating the hourly market-clearing at high demand. Table 5-4 shows the results.

		BAU	Tax parameter $\gamma^0 = 30\ \$/t$	Tax parameter $\gamma^0 = 50\ \$/t$
Emission tax 2nd order parameter (\$/t)/ (t/h)	β	-	0.01	0.01
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	10	10
	a_{21}	15	15	15
	a_{31}	20	20	20
	a_{41}	50	50	50
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	0.85	0.89	0.89
	b_{21}	0.85	0.86	0.86
	b_{31}	0.85	0.856	0.856
	b_{41}	0.85	0.853	0.853
Demand level (MW)	d_1	668	627	607
Generation levels (MW)	g_{11}	183	130	104
	g_{21}	177	171	167
	g_{31}	171	174	175
	g_{41}	136	153	160
Individual emissions (t/h)	$\rho_1 g_{11}$	366	259	208
	$\rho_2 g_{21}$	177	171	167
	$\rho_3 g_{31}$	137	139	140
	$\rho_4 g_{41}$	68	76	80
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	749	646	596
Electricity marginal price $(\$/MWh)$	λ_1	166	198	215
Emission tax rate $(\$/t)$	γ_1	-	36	56
Genco absolute and per unit profits $(\$/h),(\$/MWh)$	pr_{11}	27,674 (151)	14,540 (112)	9,394 (90)
	pr_{21}	25,925 (146)	24,345 (201)	23,315 (140)
	pr_{31}	24,233 (142)	25,213 (145)	25,471 (146)
	pr_{41}	15,280 (112)	19,264 (126)	21,272 (133)
Load surplus $(\$/h)$	pr_{d1}	178,437	157,313	147,221
Social-welfare $(\$/h)$	SW_1	271,550	240,676	226,672

Table 5-4: Market-clearing under BAU and under CTX with different tax parameters.

i) Tax parameter $\gamma^0 = 30\ \$/t$

The effects of applying a carbon tax on the market-clearing at a high demand level period are similar to those under lower demand, the major differences being:

1) The decrease in generation level of the most polluting Genco 1 and the increase in generation level of the least polluting Genco 4 are more pronounced. Moreover, the gap between the generation levels of the most and least polluting Gencos is now higher;

2) The gap between the absolute and per unit profits of the least polluting Genco 4 and the most polluting Genco 1 is higher than under a lower demand level.

These results indicate that at high demand levels, the carbon tax model is more effective in favoring low polluting Gencos than at low demand levels.

ii) Tax parameter $\gamma^0 = 50\ \$/t$

By increasing the tax parameter γ^0, from 30 $/t to 50 $/t at high demand level, the effects on market-clearing and profits are similar to that under lower demand level. The major difference is that, similar to the low tax parameter case, the differences in the profits and generation levels between the most and least polluting Gencos are more pronounced under high demand level than under lower demand level.

5.3.5.4 Major findings from simulations on the effects of the hourly carbon tax rate on market-clearing

From simulations over one time period we make the following observations:

1) Effective short-term emission reduction is met as the generation dispatch favors the less polluting Gencos regardless of their higher generation costs. Moreover, the profit trend motivates investing in emission reduction in the long-term;

2) Increasing tax parameters have the desired short-term effects of increasing the profit of the more polluting Gencos while decreasing that of the less polluting Gencos;

3) At high demand levels, the carbon tax model is more effective in favoring low polluting Gencos over more polluting ones in terms of dispatch and resulting profit.

5.4 Computing the Optimum Carbon Tax Parameters

We now compute the tax parameters γ^0 and β that will maximize the electricity sector's social-welfare while retaining the total emissions produced by the sector over a specified time horizon[20] below a required level. In this computation, the effect of the tax parameters on the hourly market-clearing modeled in section 5.3 is accounted for.

The social-welfare to be maximized with respect to the unknown tax parameters is given by,

$$SW = \sum_{t} B_t(d_t) - \sum_{i,t} C_{it}^*(g_{it}) - \sum_{t} \left(\gamma^0 + \beta x_t\right) x_t \tag{5.25}$$

where :

(a) $\sum_{t} B_t(d_t)$ is the total demand benefit;

(b) $\sum_{i,t} C_{it}^*(g_{it})$ is the total true generation cost;

(c) $\sum_{t} \left(\gamma^0 + \beta_t x_t\right) x_t$ is the total Genco tax paid over the commitment interval.

To formulate this maximization problem, recall from the hourly market-clearing solution that the hourly generation levels are known in terms of the hourly emission tax parameters through the forms in equation (5.20), $g_{it} = g_{it}(\gamma^0, \beta, \boldsymbol{\mu}_t^{\min}, \boldsymbol{\mu}_t^{\max}); \forall i, t$. Note that this relation accounts for the hourly gaming Cournot strategies, where Gencos use demand elasticity, generation costs and intensities as well as the tax penalty elasticity in their hourly offers.

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[20] The time horizon is synonymous to the commitment interval under CAT.

In addition, since the hourly demand level and emission tax rate are given by $d_t = \left(\sum_i g_{it} \right)$ and $\gamma_t = \gamma^0 + \beta_t x_t$, respectively, and since $x_t = \sum_i \rho_i g_{it}$, the social-welfare over the entire time horizon (or commitment interval) defined by (5.25) becomes an implicit function of $\{\gamma^0, \beta, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}\}$, here denoted by $SW\{\gamma^0, \beta, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}\}$.

The social-welfare, $SW\{\gamma^0, \beta, \boldsymbol{\mu}^{\min}, \boldsymbol{\mu}^{\max}\}$ is maximized with respect to the tax parameters γ^0 and β subject to the sector's permissible emissions constraint,

$$\sum_t x_t \le e^0 \qquad (\sigma) \tag{5.26}$$

and,

$$\gamma_t^0 \le \gamma_{\max}^0 \qquad ;\forall t \qquad \left(\delta_t^{\max} \right) \tag{5.27}$$

and,

$$\gamma_{\min}^0 \le \gamma_t^0 \qquad ;\forall t \qquad \left(\delta_t^{\min} \right) \tag{5.28}$$

And possible similar constraints on β .

Furthermore as detailed in Appendix E, the optimization is also subject to the complementarity slackness requirements and non-equality constraints from the hourly market-clearing stage, (5.22) - (5.24).

As will be shown in the simulations in this section, the social-welfare can be optimized over one or both of the tax parameters for different resulting tax rates, market-clearings and profits. Setting the second order parameter, β , to a relatively low level results in a more even tax rate over the commitment interval but might lead to a lower SW.

5.4.1 Quantitative analysis

In the following examples, we compute the hourly tax parameters while maintaining the total emissions produced by the sector over a commitment interval to a level equal to or below 15% BAU. We consider two cases:

1) The second order tax parameter β is set by the SP at 0.01 (\$/t)/ (t/h). and the SW is optimized over the first order parameter, γ^0, only;

2) The SW is optimized over both tax parameters, γ^0 and β.

We recall that both tax parameters, γ^0 and β, are assumed constant over the whole commitment interval.

5.4.1.1 Simulation parameters

In this section's demonstrations we use the standard simulation parameters used throughout the thesis,

	Genco 1	Genco 2	Genco 3	Genco 4
Linear true cost parameter: a_{it}^{*} $(\$/MWh)$	10	15	20	50
Quadratic true cost parameter: b_{it}^{*} $(\$/MW^2h)$	0.05	0.05	0.05	0.05
Emission intensity: ρ_i (t/MWh)	2	1	0.8	0.5

Table 5-5: Gencos' cost parameters and emission intensities.

	Low demand	High demand
Linear demand benefit parameter: λ_t^0 $(\$/MWh)$	400	700
Quadratic demand benefit parameter: α_t $(\$/MW^2h)$	0.8	0.8

Table 5-6: Demand benefit parameters.

5.4.1.2 Results when the second order tax parameter β is set by the SP and the maximum SW tax rate is found by optimizing over the first order parameter γ^0 only.

Setting the second order tax parameter β at 0.01 (\$/t)/(t/h) (that is, for every increase in the hourly emissions by 100 t/h, the carbon tax rate increases by 1 \$/t), the optimum tax parameter γ^0 that maximizes SW over the commitment interval and meets the total emissions target is 26 \$/t. The resulting market-clearing is shown in Table 5-7.

Analysis of Table 5-7 (next page)

The hourly market-clearing is consistent with the previous general observations from Tables 5-3 and 5-4. The generation dispatch under CTX in both time periods is now biased by the emission intensities of the Gencos and not only by their generation costs. For instance, Genco 1 with the lowest generation cost and highest emission intensity produces the least MW output during both time periods.

The electricity price and tax rate are higher at the time period with higher demand level (period 2), which is consistent with the higher generation and emission levels at that period.

The total profit of the most polluting Genco 1 over the time horizon of both time periods is the lowest at 19,376 \$, while the profit of the least polluting most expensive Genco 4 at 23,595 \$ is higher than that of Genco 1. Thus, the method is both successful in restricting emissions to the desired level and in encouraging investments in emission reduction.

	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Carbon tax 1st order parameter $(\$/t)$	γ_1^0	26	γ_2^0	26		
Carbon tax 2st order parameter $(\$h/t^2)$	β	0.01	β	0.01		
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	a_{12}	10		
	a_{21}	15	a_{22}	15		
	a_{31}	20	a_{32}	20		
	a_{41}	50	a_{42}	50		
Offered non-linear Cost parameters $(\$/MW^2h)$	b_{11}	0.89	b_{12}	0.89		
	b_{21}	0.86	b_{22}	0.86		
	b_{31}	0.8564	b_{32}	0.8564		
	b_{41}	0.8525	b_{42}	0.8525		
Demand level (MW)	d_1	339	d_2	631		
Generation Levels (MW)	g_{11}	66	g_{12}	134		
	g_{21}	98	g_{22}	171		
	g_{31}	99	g_{31}	174		
	g_{41}	75	g_{42}	151		
Individual emissions (t/h)	$\rho_1 g_{11}$	133	$\rho_1 g_{12}$	268	$\sum_1^2 \rho_1 g_{1t}$	401
	$\rho_2 g_{21}$	98	$\rho_2 g_{22}$	171	$\sum_1^2 \rho_2 g_{2t}$	269
	$\rho_3 g_{31}$	79	$\rho_3 g_{32}$	139	$\sum_1^2 \rho_3 g_{3t}$	219
	$\rho_4 g_{41}$	38	$\rho_4 g_{42}$	76	$\sum_1^2 \rho_4 g_{4t}$	113
Total emissions (t/h)	$\sum_{i=1}^4 \rho_i g_{i1}$	348	$\sum_{i=1}^4 \rho_i g_{i2}$	654	$\sum_{t=1}^2 \sum_{i=1}^4 \rho_i g_{it}$	1002
Electricity marginal price $(\$/MWh)$	λ_1	129	λ_2	195		0
Emission tax rate $(\$/t)$	γ_1	30	γ_2	33		0
Genco profits $(\$/h),(\$/MWh)$	pr_{11}	3,817	pr_{12}	15,559	pr_1	19,376
	pr_{21}	7,982	pr_{22}	24,528	pr_2	32,510
	pr_{31}	8,205	pr_{32}	25,168	pr_3	33,373
	pr_{41}	4,673	pr_{42}	18,922	pr_4	23,595
Load surplus $(\$/h)$	pr_{d1}	45,884	pr_{d2}	159,120	pr_d	205,005
Social-welfare $(\$/h)$	SW_1	70,562	SW_2	243,298	SW	313,860

Table 5-7: Market-clearing when SW is maximized over the first order tax parameter only.

5.4.1.3 Results when the maximum SW tax rate is found by optimizing over the first and second order parameters γ^0 and β

The optimum carbon tax parameters that maximize SW while meeting emission targets are γ^0 = 0 \$/t and β = 0.05 (\$/t)/ (t/h). The resulting market-clearing is shown in Table 5-8.

	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Emission tax 1st order parameter $(\$/t)$	γ_1^0	0	γ_2^0	0		
Emission tax 2st order parameter $(\$h/t^2)$	β	0.05	β	0.05		
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	a_{12}	10		
	a_{21}	15	a_{22}	15		
	a_{31}	20	a_{32}	20		
	a_{41}	50	a_{42}	50		
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	1.039	b_{12}	1.039		
	b_{21}	0.897	b_{22}	0.897		
	b_{31}	0.880	b_{32}	0.880		
	b_{41}	0.862	b_{42}	0.862		
Demand level (MW)	d_1	348	d_2	626		
Generation levels (MW)	g_{11}	74	g_{12}	124		
	g_{21}	100	g_{22}	171		
	g_{31}	100	g_{31}	176		
	g_{41}	73	g_{42}	155		
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	365	$\sum_{i=1}^{4} \rho_i g_{i2}$	637	$\sum_{t=1}^{2} \sum_{i=1}^{4} \rho_i g_{it}$	1002
Electricity marginal price $(\$/MWh)$	λ_1	122	λ_2	199		
Emission tax rate $(\$/t)$	γ_1	17	γ_2	30		
Genco profits $(\$/h),(\$)$	pr_{11}	5,602	pr_{12}	15,506	pr_1	21,108
	pr_{21}	8,696	pr_{22}	25,618	pr_2	34,314
	pr_{31}	8,560	pr_{32}	26,459	pr_3	35,018
	pr_{41}	4,510	pr_{42}	20,203	pr_4	24,713
Load surplus $(\$/h),(\$)$	pr_{d1}	48,323	pr_{d2}	156,853	pr_d	205,176
Social-welfare $(\$/h),(\$)$	SW_1	75,691	SW_2	244,639	SW	320,330

5-8: Market-clearing when SW is maximized over both carbon tax parameters.

Analysis of Table 5-8

i) <u>Market-clearing</u>

By maximizing SW over both tax parameters, the first order parameter, γ^0 turns out to be lower than when SW is maximized over a fixed β set to 0.01 (\$/t)/ (t/h) . However, the second order tax parameter is now higher at 0.05 (\$/t)/ (t/h).

A second result of this two tax parameter optimization is that the resulting hourly tax rates are lower. When the SW is maximized over the first order tax parameter γ^0 only and β is set to 0.01 \$/t (Table 5-7), the resulting hourly carbon tax rates are 30 \$/t at the low demand period 1 and 33 \$/t at the high demand period 2. On the other hand, when the SW is optimized over both tax parameters (Table 5-8), the resulting hourly carbon tax rates are 17 \$/t at the low demand period 1 and 30 \$/t at the high demand period 2.

Recall from equation (5.19) in section 5.3 that, under CTX, the Gencos' hourly gaming strategies (reflected in the b-parameter of the IC offers) become more aggressive with a higher second order tax parameter. This is observed in the b-parameters of the Gencos' IC offers in both Tables; in Table 5-8, with higher β , the b-parameters are higher than those in Table 5-7. Moreover, the increase in the b-parameter of high polluting Gencos, such as Genco 1, is more prominent as compared to low polluting Gencos such as Genco 4. Thus, as β increases, the more polluting Gencos lose market power. Despite the significantly lower value for γ^0 in Table 5-8, the generation level at 124 MW of the most polluting Genco 1 at the high demand period 2 is lower than that under Table 5-7 at 134 MW. Similarly, the corresponding generation level (155 MW) of the least polluting Genco 4 in Table 5-8, is higher than that under Table 5-7 (151 MW). Note however than, under low demand (period 1), since the tax rate is low, the market-clearing still dispatches a significant MW level from the more polluting Gencos.

As a result of the previous generation dispatch, the electricity price under Table 5-8 (with higher β) is lower than that under Table 5-7 during period 1 and is higher during period 2. It is lower during period 1 because at that period the market-clearing dispatches more of the cheapest most polluting Genco 1 and less of the most expensive least polluting Genco 4, while during period 2 the opposite result induces a higher price in Table 5-8.

ii) <u>Profits and social-welfare</u>

Table 5-9 shows the Genco profits, load surplus and SW under both methods of finding the tax parameters.

		Pre-set β	Optimum β
Genco profits ($\$/h$)	pr_1	19,376	21,108
	pr_2	32,510	34,314
	pr_3	33,373	35,018
	pr_4	23,595	24,713
Load surplus ($\$/h$)	pr_d	205,005	205,176
Social-welfare ($\$/h$)	SW	313,860	320,330

Table 5-9: Profits and SW under different carbon tax parameters.

We observe that compared to the pre-set β (Table 5-7), the Genco profits are higher under the optimum parameter β (Table 5-8) due to the lower carbon tax rate in this latter case. Despite the more polluting Gencos losing market power due to the higher β , the lower tax rate compensates for this loss and ensures the higher all-around profits. Finally, the load surplus and SW are also higher with the optimum β .

5.4.1.4 *Major findings from simulations on computing the optimum tax parameters over a commitment interval*

1) Computing tax parameters using the maximum social-welfare method is successful in restricting emissions over the commitment interval to the desired levels;

2) The resulting carbon tax model penalizes high polluting Gencos while rewarding low polluting ones, thus encouraging investments in emission reduction in the long-term;

3) When the SW is optimized over both tax parameters, the hourly tax rates are lower than when the second order parameter is fixed, thus inducing higher Genco profits and load surplus.

5.5 Chapter Summary

In this chapter, the hourly-market clearing along with hourly Gencos' gaming strategies were re-modeled to account for a carbon tax emission regulation scheme. The CTX reduces the market power of cheap but polluting Gencos and shifts generation from these Gencos to less polluting but more expensive ones. Moreover, under CTX, the profits of low polluting but expensive Gencos are significantly higher than the profits of the more polluting cheaper Gencos.

We also developed a model to compute the carbon tax parameters on the basis of maximizing social-welfare while accounting for the effects of the tax parameters on hourly gaming strategies and market-clearing, as well as respecting total emission targets over the specified commitment interval. The resulting carbon tax model favors low polluting Gencos over higher polluting but cheaper ones, a result that sends the right economic signals to invest in low-emission generation technologies.

6 A Comparison of CAT and CTX on Electricity Markets

"Cowardice asks the question - is it safe?
Expediency asks the question - is it politic?
Vanity asks the question - is it popular?
But conscience asks the question - is it right?
And there comes a time when one must take a position…that is neither safe, nor politic, nor popular; but one must take it because it is right." Martin Luther King Jr.

In the previous chapters, we designed and tested both a cap-and-trade and a carbon tax emission regulation model. Although practically and philosophically different, both models reduce the electricity sector's emissions by a desired amount over a specified time horizon or a commitment interval. In this chapter, we compare the implications of both models on oligopolistic electricity markets with hourly market-clearing.

Sections 6.1 and 6.2 of this chapter present qualitative and quantitative comparisons between these two emission regulation models.

6.1 Qualitative Comparison

6.1.1 Advantages of cap-and-trade

The major advantages of cap-and-trade (CAT) over carbon tax (CTX) are:

1) The notion of setting a cap on the level of emissions produced by the electricity sector is an easy concept to promote since it guarantees that emissions will be cut by a desired amount. In contrast, a carbon tax cannot guarantee that emissions will be curtailed by a given amount. Such a tax is instead a disincentive to emit that does not guarantee satisfying the desired emission target;

2) Under cap-and-trade, capped entities can relieve part of the financial burden imposed by the emission caps by trading permits among each other or with an external emission permits market. The trading of permits distributes the overall sector cap more efficiently among Gencos. Moreover it offers some Gencos an added opportunity to game and increase their market power. On the other hand, a carbon tax does not offer such added gaming opportunities, which may be viewed as a drawback by the producers and as an advantage by the consumers;

3) Cap-and-trade systems can be applied nationwide or even globally to include other sectors and nations. Since the issue of global warming is by definition global, having a scheme to tackle the issue where nations come together to share the burden is reasonable. On the other hand, a tax is a national policy that is not necessarily shared by other nations, both in principle and in the degree to which the tax is applied;

4) Carbon tax is a concept that is not generally favored by the general population, which may associate it with wasteful governments looking for additional sources of revenue. Thus, cap-and-trade may be more socially acceptable.

6.1.2 Advantages of carbon tax

The major advantages of a carbon tax emission regulation scheme over cap-and-trade are:

1) A carbon tax is simple to implement and design as compared to a cap-and-trade scheme. Designing a carbon tax requires setting the tax structure, its parameters and resulting tax rates, while designing a cap-and-trade system requires allocating permits individually amongst Gencos and setting up a permits trading mechanism. While under both schemes, Gencos will operate strategically, modeling the Gencos' strategic position is a much more challenging task under cap and trade, as we saw in chapter 4. Thus, from the point of view of the SP, creating an efficient cap-and-trade system is also complex and therefore subject to uncertainty, all of which makes the behaviour of a power system operating under CAT harder to predict than under CTX;

2) A carbon tax eliminates the additional gaming opportunities offered to Gencos under CAT by exploiting their ability to trade emission permits, as shown in Chapter 4. As seen, such gaming leads to higher prices and to a more complex, harder-to-predict market behaviour.

6.2 Quantitative Comparison

In this section we compare the market-clearing behaviour, both hourly and over the commitment interval, under cap-and-trade and carbon tax. We do this through two illustrative cases:

(1) Single time period case: Initially, both emission regulation schemes are applied to a commitment interval of one time period. Here, under the CAT model, the total cap, equivalent to a 30% reduction in BAU emissions, is grand-fathered to the Gencos based on BAU, while, under CTX, the carbon tax parameters are computed to reduce emissions to the same level as under CAT.

(2) Multiple time period case: We also compare both schemes when applied to a commitment interval of two time periods. In this comparison, CAT is subject to a sector cap equal to 15% reduction in emissions from BAU. Under CAT, permits are allocated to Gencos based on maximizing social-welfare with a strategic Genco temporal self-allocation based on the NE and are given away for free. Under CTX, the tax parameters are computed to maximize SW and to set the total emissions equal to (or below) the cap under CAT.

6.2.1 Simulation parameters

Here we use the standard simulation parameters used throughout the thesis:

	Genco 1	Genco 2	Genco 3	Genco 4
Linear true cost parameter: $a_{it}^{*}\ (\$/MWh)$	10	15	20	50
Quadratic true cost parameter: $b_{it}^{*}\ (\$/MW^2h)$	0.05	0.05	0.05	0.05
Emission intensity: $\rho_i\ (t/MWh)$	2	1	0.8	0.5

Table 6-1: Gencos' cost parameters and emission intensities.

	Time period 1 (low demand)	Time period 2 (high demand)
Linear demand benefit parameter: $\lambda_t^0\ (\$/MWh)$	400	700
Quadratic demand benefit parameter: $\alpha_t\ (\$/MW^2h)$	0.8	0.8

Table 6-2: Demand benefit parameters.

The parameters for both time periods of the hourly emission permits trading cost function under CAT are given in Table 6-3.

Linear permits cost parameter: γ_t^0 $(\$/t)$	50
Quadratic permits cost parameter: β_t $(\$h/t^2)$	0.1

Table 6-3: Parameters of the hourly emission permits trading cost function.

6.2.2 Single time period case

Table 6-5, shows the hourly market-clearing under: i) BAU; ii) cap-and-trade with a cap of 30 % reduction in BAU emissions. The cap is allocated to the Gencos according to a grand-father clause (Table 6-4); iii) a carbon tax where the tax parameters are calibrated so that the total emissions are equal to those found under CAT.

BAU Genco emissions (t/h)	Genco 1	218	**GF Allocated Genco permits** (t/h)	e_{11}	153
	Genco 2	103		e_{21}	72
	Genco 3	78		e_{31}	54
	Genco 4	31		e_{41}	22
Total	Σ	430	**Total permits (cap)**	Σ	301

Table 6-4: Permits grand-fathered to Gencos based on BAU emissions.

		BAU	CAT	CTX
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	-21	10
	a_{21}	15	8	15
	a_{31}	20	16	20
	a_{41}	50	49	50
Offered non-linear Cost parameters $(\$/MW^2h)$	b_{11}	0.85	1.25	0.89
	b_{21}	0.85	0.95	0.86
	b_{31}	0.85	0.91	0.8564
	b_{41}	0.85	0.88	0.8525
Demand level (MW)	d_1	372	319	324
Generation levels (MW)	g_{11}	109	51	47
	g_{21}	103	91	94
	g_{31}	97	97	100
	g_{41}	62	81	83
Electricity marginal price $(\$/MWh)$	λ_1	103	145	141
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	430	310	310
Total cap (t/h)	$\sum_{i=1}^{4} e_{i1}$	-	301	-
Permits traded with external market (t/h)	Δe_1	-	-51	-
	Δe_2	-	18	-
	Δe_3	-	23	-
	Δe_4	-	19	-
Permits marginal price/ Carbon tax $(\$/t)$	γ_1	-	51	36
Genco absolute and per unit profits $(\$/h),(\$/MWh)$	pr_{11}	9,816 (90)	9,389 (184)	2,689 (57)
	pr_{21}	8,786 (85)	10,632 (117)	8,217 (87)
	pr_{31}	7,813 (81)	10,684 (110)	8,955 (90)
	pr_{41}	3,173 (51)	6,533 (81)	5,840 (70)
Load surplus $(\$/h)$	pr_{d1}	55,236	40,689	41,998
Social-welfare $(\$/h)$	SW_1	84,823	77,928	67,699

Table 6-5: Market-clearing under BAU, CAT, and CTX, where BAU emissions are reduced by 30%.

Analysis of Table 6-5

The parameters of the CTX function in (5.1) that produce the same level of emissions as those under CAT are $\gamma^0 = 5\$/t$ and $\beta = 0.1\$h/t^2$. Here, only the linear tax parameter γ^0 was varied, while the sensitivity of the carbon tax rate ($\beta = 0.1\$h/t^2$) was kept at the same level as the sensitivity of the permits price in the external market under CAT. In this example, since the Genco permit allocation under CAT is grand-fathered, there is no SW maximization in either CAT or CTX.

Table 6-5 shows that the shift in generation from cheap high polluting Gencos to more expensive less polluting Gencos is more pronounced under CTX than under CAT. This is because under CAT, permits trading affects the Gencos' gaming strategies (allowing high polluting Gencos to maintain some of their market power); while under CTX, the offers are based on demand elasticity and emission intensities only.

For instance, the most polluting cheapest Genco 1 can use the high number of permits it receives under GF to sell permits to the external market, thus partially offsetting the negative effect of its high emission intensity. Consequently, under CAT, Genco 1 can game less aggressively than under CTX, maintaining some of its competitiveness. As a result, the generation level of Genco 1 goes down from 109 MW (BAU) to 51 MW under CAT, while it goes down to 47 MW under CTX.

Furthermore, the generation level of the least polluting most expensive Genco 4 goes up from 62 MW (BAU) to 81 MW under CAT, while it goes up to 83 MW under CTX. This is because, under CAT, GF allocates the least number of permits to Genco 4, which is then forced to buy permits, thus increasing its cost and forcing it to game more aggressively. This effect does not exist under CTX and the least polluting Genco 4 can then use its low emission intensity to become more competitive despite having the highest generation cost.

Moreover, under CAT, the demand level goes down from 372 MW (BAU) to 319 MW, while under CTX it goes down only to 324 MW. Consequently the electricity price goes up from 103 $/MWh (BAU) to 145 $/MWh under CAT, while it goes up only to 141 $/MWh under CTX. This is an interesting and significant result suggesting that, under CAT, Gencos in general are more aggressive thus raising the total generation cost, which forces the market to clear at a lower demand level and a higher electricity price.

By applying CAT, the profits of all Gencos increase from BAU except that of the most polluting cheapest Genco 1 whose profit decreases slightly from 9,816 $/h to 9,389 $/h. While this is a positive investment signal, CTX produces an even stronger signal, reducing the profit of Genco 1 to 2,689 $/h.

Although the profits of all Gencos are lower under CTX than under CAT, under CTX, the profit of the least polluting most expensive Genco 4 is higher than the profit of the most polluting cheapest Genco 1. In contrast, under CAT, the opposite occurs. In other words, under CTX there is an obvious and significant profit advantage for low polluting Gencos over high polluting ones despite steep differences in marginal generation costs.

6.2.3 Multiple time period case

In this section we compare both emission regulation schemes over a commitment interval of two time periods. CAT is subject to a sector cap equal to 15% reduction in emissions from BAU. Under CAT, permits are allocated to Gencos based on maximizing social-welfare with a strategic Genco temporal self-allocation based on the NE and are given away for free. Under CTX, the tax parameters are computed to maximize SW and to set the total emissions equal to (or below) the cap under CAT.

This comparison is reasonable as both emission regulation schemes are compared under the same maximum social-welfare conditions with approximately equivalent sector emissions. Recall that under CAT, permits trading with the external market means that the total emissions produced by the sector may end up above or below the set cap. On the other hand, under CTX, there is no permits trading market and the total emissions produced by the sector can be set to or below a desired level such as that of the CAT cap.

An alternative to set the tax parameters would be to require that the total emissions under CTX be set equal to the total emissions under CAT; however since the latter level of emissions is not known a priori when the tax parameters are defined, typically a year or more in advance, this approach is not realistic.

6.2.3.1 Business-as-usual for two-period case

Table 6-6 recalls the outcome of the market-clearing under BAU without emission regulation. Gencos are assumed to use demand elasticity and actual generation costs to game based on the Cournot NE.

BAU	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	a_{12}	10		
	a_{21}	15	a_{22}	15		
	a_{31}	20	a_{32}	20		
	a_{41}	50	a_{42}	50		
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	0.85	b_{12}	0.85		
	b_{21}	0.85	b_{22}	0.85		
	b_{31}	0.85	b_{32}	0.85		
	b_{41}	0.85	b_{42}	0.85		
Demand levels (MW)	d_1	372	d_2	668		
Generation levels (MW)	g_{11}	109	g_{12}	183		
	g_{21}	103	g_{22}	177		
	g_{31}	97	g_{31}	171		
	g_{41}	62	g_{42}	136		
Electricity marginal price $(\$/MWh)$	λ_1	103	λ_2	166		
Total emissions $(t/h),(t)$	$\sum_{i=1}^{4}\rho_i g_{i1}$	430	$\sum_{i=1}^{4}\rho_i g_{i2}$	749	$\sum_{t=1}^{2}\sum_{i=1}^{4}\rho_i g_{it}$	1,179
Genco profit $(\$/h),(\$)$	pr_{11}	9,816 (90)	pr_{12}	27,674 (151)	pr_1	37,490
	pr_{21}	8,786 (85)	pr_{22}	25,925 (146)	pr_2	34,711
	pr_{31}	7,813 (81)	pr_{32}	24,233 (142)	pr_3	32,046
	pr_{41}	3,173 (51)	pr_{42}	15,280 (112)	pr_4	18,453
Load surplus $(\$/h),(\$)$	pr_{d1}	55,236	pr_{d2}	178,437	pr_d	233,674
Social-welfare $(\$/h),(\$)$	SW_1	84,823	SW_2	271,550	SW	356,373

Table 6-6: Market-clearing under BAU.

6.2.3.2 Cap-and-Trade for two-period case

Table 6-7 recalls the commitment interval permits allocated to Gencos for free and the subsequent temporal self-allocation of these permits based on maximizing social-welfare.

Maximum social-welfare	Time period 1 (low demand)		Time period 2 (high demand)		Commitment interval permits	
Genco permits (t)	e_{11}	345	e_{12}	0	e_1	345
	e_{21}	0	e_{22}	610	e_2	610
	e_{31}	0	e_{32}	47	e_3	47
	e_{41}	0	e_{42}	0	e_4	0

Table 6-7: Permit allocation under CAT among Gencos based on maximizing SW, and the subsequent temporal strategic allocation based on NE.

Table 6-8 recalls the market-clearing under CAT resulting from the permit allocations in Table 6-7 when the Gencos use demand and permit price elasticity, emission intensities and generation costs as well as hourly permits to game according to Cournot. The contents of Table 6-8 are compared below to those of Tables 6-6 and 6-9.

	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Offered linear cost parameters $(\$/MWh)$	a_{11}	-59	a_{12}	10		
	a_{21}	15	a_{22}	-46		
	a_{31}	20	a_{32}	16		
	a_{41}	50	a_{42}	50		
Offered non-linear cost parameters $(\$/MW^2h)$	b_{11}	1	b_{12}	1		
	b_{21}	1	b_{22}	1		
	b_{31}	1	b_{32}	1		
	b_{41}	1	b_{42}	1		
Demand levels (MW)	d_1	324	d_2	620		
Generation levels (MW)	g_{11}	80	g_{12}	85		
	g_{21}	80	g_{22}	217		
	g_{31}	89	g_{31}	167		
	g_{41}	75	g_{42}	151		
Electricity marginal price $(\$/MWh)$	λ_1	141	λ_2	204		
Permits marginal price at external market $(\$/t)$	γ_1	50	γ_2	44		
Total emissions $(t/h),(t)$	$\sum_{i=1}^{4} \rho_i g_{i1}$	348	$\sum_{i=1}^{4} \rho_i g_{i2}$	596	$\sum_{t=1}^{2}\sum_{i=1}^{4} \rho_i g_{it}$	944
Total cap $(t/h),(t)$	$\sum_{i=1}^{4} e_{i1}$	345	$\sum_{i=1}^{4} e_{i2}$	657	$\sum_{i=1}^{4} e_i$	1002
Permits traded with external market $(t/h),(t)$	Δe_{11}	-186	Δe_{12}	170	Δe_1	-16
	Δe_{21}	80	Δe_{22}	-393	Δe_2	-313
	Δe_{31}	71	Δe_{32}	87	Δe_3	158
	Δe_{41}	38	Δe_{42}	75	Δe_4	113
Genco profit $(\$/h),(\$)$	pr_{11}	19,629	pr_{12}	8,847	pr_1	28,476
	pr_{21}	5,901	pr_{22}	57,106	pr_2	63,007
	pr_{31}	6,970	pr_{32}	26,231	pr_3	33,201
	pr_{41}	4,838	pr_{42}	19,369	pr_4	24,207
Load surplus $(\$/h),(\$)$	pr_{d1}	41,876	pr_{d2}	153,738	pr_d	195,615
Social-welfare $(\$/h),(\$)$	SW_1	79,215	SW_2	265,476	SW	344,506

Table 6-8: Market-clearing under CAT when total permits are allocated to Gencos for free based on maximizing social-welfare.

6.2.3.3 *Carbon Tax for two-period case*

Table 6-9 recalls the hourly market-clearing when a carbon tax system is used and the tax parameters are computed using the maximum social-welfare method. Moreover, the SW is maximized over both tax parameters, γ^0 and β , while keeping the resulting emissions equal to or below the cap imposed under CAT.

	Time period 1 (low demand)		Time period 2 (high demand)		Total	
Emission tax 1st order parameter $(\$/t)$	γ_1^0	0	γ_2^0	0		
Emission tax 2st order parameter $(\$h/t^2)$	β	0.05	β	0.05		
Offered linear cost parameters $(\$/MWh)$	a_{11}	10	a_{12}	10		
	a_{21}	15	a_{22}	15		
	a_{31}	20	a_{32}	20		
	a_{41}	50	a_{42}	50		
Offered non-linear Cost parameters $(\$/MW^2h)$	b_{11}	1.039	b_{12}	1.039		
	b_{21}	0.897	b_{22}	0.897		
	b_{31}	0.880	b_{32}	0.880		
	b_{41}	0.862	b_{42}	0.862		
Demand level (MW)	d_1	348	d_2	626		
Generation levels (MW)	g_{11}	74	g_{12}	124		
	g_{21}	100	g_{22}	171		
	g_{31}	100	g_{31}	176		
	g_{41}	73	g_{42}	155		
Total emissions (t/h)	$\sum_{i=1}^{4} \rho_i g_{i1}$	365	$\sum_{i=1}^{4} \rho_i g_{i2}$	637	$\sum_{t=1}^{2}\sum_{i=1}^{4} \rho_i g_{it}$	1002
Electricity marginal price $(\$/MWh)$	λ_1	122	λ_2	199		
Emission tax rate $(\$/t)$	γ_1	17	γ_2	30		
Genco profits $(\$/h),(\$)$	pr_{11}	5,602	pr_{12}	15,506	pr_1	21,108
	pr_{21}	8,696	pr_{22}	25,618	pr_2	34,314
	pr_{31}	8,560	pr_{32}	26,459	pr_3	35,018
	pr_{41}	4,510	pr_{42}	20,203	pr_4	24,713
Load surplus $(\$/h),(\$)$	pr_{d1}	48,323	pr_{d2}	156,853	pr_d	205,176
Social-welfare $(\$/h),(\$)$	SW_1	75,691	SW_2	244,639	SW	320,330

Table 6-9: Market-clearing under CTX when SW is optimized over both tax parameters.

6.2.3.4 Comparison of CAT and CTX market-clearing versus BAU

i) Generation and demand levels

Under CTX, the generation level of the most polluting cheapest Genco 1 goes down considerably from its BAU levels, at both time periods. At time period 1, the generation level of Genco 1 goes down from 109 MW to 74 MW, while at time period 2 it goes down from 183 MW to 124 MW.

Under CAT, the generation level of Genco 1 goes down only to 80 MW during period 1, while it goes down to 85 MW during period 2. This is due to the strategic temporal allocation of permits of Genco 1 that favors time period 1, thus allowing Genco 1 to be less aggressive (more competitive) in its gaming strategy during period 1, while forcing it to buy more permits during period 2 and offering more aggressively, thus losing some market share.

Under both emission regulating schemes, the generation level of the least polluting most expensive Genco 4 goes up at both time periods from its BAU levels. However, the total emissions produced by the sector under CAT are lower than the cap. This means that the arbitrarily set cap of 85% of BAU is too generous for the assumed external market price, given that under CAT there is a net sale of permits to the external market[21]. Under CTX, during period 1 the generation of Genco 4 goes up from 62 MW at BAU to 73 MW and from 136 MW

[21] As discussed in the conclusions to this thesis, the issue of how to set the sector cap e^0 is one that requires further investigation. However, preliminary thoughts suggest that an optimum sector cap e^0 could be computed by adding a 4th layer to the MPEC problem discussed in Chapter 4. This problem would maximize SW including the cost of buying external permits over e^0 , now treated as a variable, in such a way that the marginal SW with respect to e^0 is equal to the external market permit price. In this way, the electricity sector would have no incentive to trade permits with the external market.

to 155 MW during period 2. Under CAT, the generation level of Genco 4 goes up to 75 MW during period 1 (almost equal to that under CTX) and to 151 MW during period 2 (lower than that under CTX). These results occur despite the lower total emissions produced by the sector under CAT as compared to CTX. This is because under CAT, Genco 4 has to buy permits at both periods, which forces it to offer aggressively and reduces the market power it could have gained due to its low emission intensity.

Another interesting observation, which shows the effect of permits trading on the Gencos' market power, is the generation level of Genco 2. The MW output of Genco 2 during period 2 increases from 177 MW under BAU to 217 MW under CAT, despite its high emission intensity. By receiving the highest number of permits and by strategically allocating all these permits to period 2, Genco 2 sells 393 t/h of permits during period 2, thus allowing it to game competitively at that period and increase its market power. Moreover, by allocating zero permits to period 1 and instead buying permits during that period, the generation level of Genco 2 goes down from 103 MW under BAU to 80 MW under CAT. On the other hand under CTX, the generation level of Genco 2 goes down only to 100 MW during period 1 and goes down to 171 MW during period 2 (considerably less than its MW output under CAT).

Moreover, the demand levels at both periods are higher under CTX than under CAT. Even though total emissions are lower under CAT than under CTX, permits trading influences market-clearing by reducing the generation cost of polluting Gencos (that sell permits) and by increasing the cost of low polluting Gencos (that buy permits). Thus, in order to meet the demand at the least possible total generation cost, the market-clearing has to reduce demand.

The market power and consequently the generation levels of Gencos under CTX depend on emission intensities as well as on generation costs. Under CTX, the more polluting cheap Gencos lose market power, while less polluting more expensive Gencos increase their market power. However, under CAT, the generation levels also depend on the self-allocated hourly permits and on hourly permits trading. *This adds to the volatility of the market-clearing under CAT, and enhances the market power of cheap but high polluting Gencos*.

ii) <u>Electricity price</u>

As expected, under both emission regulation schemes, the hourly electricity price at both periods are higher than under BAU. However an interesting observation is that for both time periods, the electricity price under CTX is lower than under CAT. During period 1 with low demand level, the price under CAT is 141 $/MWh while under CTX it is 122 $/MWh. Similarly, during period 2 with high demand, the price under CAT is 204 $/MWh while under CTX it is 199 $/MWh.

This result agrees with the comparison in the previous section, and shows that even if permits were allocated to maximize social-welfare, permits trading enhances the overall market power of Gencos resulting in a more expensive system marginal generation cost under CAT than under CTX.

These results challenge the ascription of high prices to a carbon tax, and refute the claim that cap-and-trade softens the effect of an emissions cap on electricity prices.

iii) <u>Emission price</u>

Under cap-and-trade, the high demand of permits in period 1 due to the temporal permit allocation that favors the high demand period 2 induces a higher permit price in period 1 as compared to period 2. However under carbon tax, the lower emissions produced in period 1, due to the lower demand level, leads to an hourly tax rate during period 1 that is significantly lower than the rate during period 2. Unlike CTX, the trend of the price of emissions (permits price) under CAT is not so obviously related to the demand level and total emissions produced. Under CAT, this price depends on the allocated permits among Gencos and more importantly on how Gencos self-allocate these permits among the different time periods.

iv) Profits and SW

Table 6-10 shows the total profits and SW over the commitment interval under BAU and under both emission regulation schemes.

		BAU	CAT	CTX
Total emissions (t)	$\sum_{t=1}^{2}\sum_{i=1}^{4}\rho_i g_{it}$	1,179	944	1002
Genco profits $(\$)$	pr_1	37,490	28,476	21,108
	pr_2	34,711	63,007	34,314
	pr_3	32,046	33,201	35,018
	pr_4	18,453	24,207	24,713
Load surplus $(\$)$	pr_d	233,674	195,615	205,176
Social-welfare $(\$)$	SW	356,373	344,506	320,330

Table 6-10: Profits and SW under BAU, CAT and CTX.

As explained in chapter 2, the BAU profits of Gencos are based on generation costs, Gencos with lower generation costs making higher profits. Under CAT, the profits of all Gencos, except for the most polluting cheapest Genco 1, increase, while, by applying the carbon tax, the profits of all Gencos, except the two most polluting Gencos 1 and 2, increase. Decreasing the profits of high polluting Gencos and increasing those of low polluting ones from their BAU levels is a positive result for an emission regulation scheme since it rewards low polluting Gencos and penalizes high polluting ones.

Nonetheless, one result that stands out is the huge increase in the profit of the second most polluting and second cheapest Genco 2 under CAT. This increase is a result of the large number of permits the Genco receives for free under a maximum SW allocation. As detailed in section 4.5, all the permits the Genco receives increase its revenue from selling power and excess permits to the external permits market. As expected, under CTX, the profit of the polluting Genco 2 is considerably lower than under CAT.

Another interesting result is that the profit of the most polluting cheapest Genco 1 under CAT is significantly higher than its profit under CTX. This suggests that while a carbon tax is effective in penalizing high polluting Gencos and thus reducing their profits considerably from BAU, cap-and-trade allows such Gencos to reduce the burden of the emissions cap on their profits. This is because, under CAT, high polluting Gencos can sell permits and thus increase their revenue.

Another observation is that the profit of the least polluting most expensive Genco 4 under CTX is significantly higher than the profit of the most polluting cheapest Genco 1. On the other hand, under CAT, the profit of Genco 4 is lower than that of Genco 1. This result is important in view of the relatively high generation cost of Genco 4 compared to that of Genco 1. Recall, from Table 6-1, that the a-parameter of the generation cost of Genco 4 is 50 $/MWh, while that of Genco 1 is 10 $/MWh which, under BAU, leads to a much higher profit for Genco 1 compared to Genco 4. However CTX reduces the incremental cost of low polluting Gencos and increases that of high polluting ones so significantly that the profit of Genco 4 becomes much higher than that of Genco 1.

Under both emission regulation schemes, the load surplus is lower than under BAU, mainly due to the higher electricity price during both periods under each scheme. However, the load surplus is higher under CTX than under CAT. This is because of the lower demand level and higher electricity prices under CAT. This result implies that the effect of CTX on the consumer side is less severe than the effect of CAT, despite its touted higher flexibility due to its permits trading capability. On the other hand, while the SW under both schemes is lower than BAU, the SW under CAT is higher than that under CTX, which is attributed to the high windfall profits Gencos make under CAT.

6.2.4 Major findings from numerical comparisons of CAT and CTX

1) While permits trading under CAT allows high polluting but cheap Gencos to preserve their market power through selling permits, low polluting but expensive Gencos lose market power by having to buy permits. Moreover, this dependence on permits trading leads to a more volatile market-clearing under CAT which makes prices harder to predict. In addition, by requiring low polluting Gencos to buy permits, the total generation cost under CAT is higher than under CTX. This leads to higher electricity prices and lower demand levels under CAT and, ultimately, to lower load surpluses under CAT as compared to CTX;

2) A major disadvantage of a cap-and-trade system is that Gencos can significantly increase their profits by selling emission permits. These sales allow high polluting but cheap Gencos to reduce the negative effect of emissions caps on their profits. Under carbon tax, because of the absence of this extra profit-making channel, a polluting Genco has no way of compensating for its high emissions and its profit is significantly reduced from BAU;

3) Thus, while under CAT the profitability of a Genco depends on how the sector cap is allocated among Gencos, under CTX, regardless of differences in generation costs, low polluting Gencos are more profitable than high polluting ones. This makes low polluting power plants a more favorable investment. Moreover, CTX encourages Gencos to invest in emission reduction technologies in their high polluting power plants to avoid a significant loss of profit.

6.2.5 Comparison between CTX and CAT with social-welfare auction

In the CAT system considered in the previous section, permits were allocated free of charge by the SP among Gencos so as to maximize social-welfare. Under this approach, Gencos have no say in the permits they receive. We now compare CAT and CTX when the permits in the CAT system are allocated using the social-welfare auction proposed in section 4.5, according to which, instead of being allocated for free, the allocated permits are charged at the shadow price, σ .

Table 6-11 shows the Genco profits, load surplus and SW, as well as the sector emissions, over the commitment interval under CAT with permit allocation based on the SW auction under different auction bids, all with the same cap. The table also compares these results to: i) CAT when permits are allocated for free to maximize SW; ii) the equivalent[22] CTX system; iii) BAU.

		BAU	CTX	CAT						
				Maximum SW	SW Auction					
					$\{0,0,0,0\}$	$\{1,0,0,0\}$	$\{2,0,0,0\}$	$\{2,1,0,0\}$	$\{2,1,3,0\}$	$\{2,1,3,6\}$
Total emissions (t)	$\sum_{t=1}^{2}\sum_{i=1}^{4}\rho_i g_{it}$	1,179	1002	944	944	1028	1039	944	932	922
Genco profits ($)	pr_1	37,490	21,108	28,476	11,266	28,481	28,072	10,806	11,427	14,131
	pr_2	34,711	34,314	63,007	32,575	25,919	24,302	31,715	28,186	29,224
	pr_3	32,046	35,018	33,201	30,865	27,522	27,296	30,515	33,468	32,097
	pr_4	18,453	24,713	24,207	24,207	21,414	21,296	24,145	24,700	19,203
Load surplus ($)	pr_d	233,674	205,176	195,615	195,615	204,656	204,621	195,782	194,980	195,291
Social-welfare ($)	SW	356,373	320,330	344,506	294,528	307,992	305,587	292,963	292,761	289,946

Table 6-11: Profits and SW under BAU, CTX, and CAT with different permit allocations.

Analysis of Table 6-11

[22] By equivalent we mean that the carbon tax parameters are adjusted so that the emissions under CTX are equal to the cap under CAT.

When permits are allocated based on the SW auction with the auction bids equal to $\{0,0,0,0\}$ (which is the same as maximizing SW) and these permits are paid for at the shadow price, σ , Genco 2, which receives the most permits, no longer makes a windfall profit compared to the free allocation case. The profit of this Genco at 32,575 $ is now lower than its BAU profit at 34,711 $ and than its profit under CTX at 34,314 $. Moreover, the profit of Genco 1 under CAT is now lower than its profit under CTX, and even lower than the profit of the least polluting most expensive Genco 4 under CAT.

Thus, the CAT system with SW maximization (auction bids equal to $\{0,0,0,0\}$) and with payment for the allocated permits at σ , is comparable to CTX in terms of penalizing high polluting Gencos, and in terms of producing proper signals to invest in emission reduction.

Table 6-11 however also illustrates that by being the only Genco to bid into the auction to receive more permits, the most polluting cheapest Genco 1 benefits significantly by increasing its profit from 11,266 $ to 28,481 $, which is higher than its profit under CTX at 21,108 $ and higher than the profit of the least polluting most expensive Genco 4 at 21,414 $. On the other hand, the table also shows that if other less polluting Gencos bid, the profit of the polluting Genco 1 drops significantly.

Recall from section 4.5, a significant characteristic of our proposed SW auction, is that the profit of the most polluting yet cheapest Genco 1 is at the mercy of the bids of less polluting more expensive Gencos. As shown in Table 6-11, the only instance when Genco 1 can benefit from the auction (that is by increasing in profit) is if the other Gencos do not bid, which in practice would not happen since the other Gencos can always increase their profits by bidding.

Thus, comparing the Genco profits for any of the auction bids shown, we see that all Gencos do better in terms of profits under CTX (with the exception of the unrealistic bid where only the dirtiest Genco 1 bids). Moreover, the load surplus and social-welfare under CTX are higher than their corresponding values under CAT with the SW auction regardless of the Genco bids.

6.2.5.1 Major findings from comparing the SW auction simulations to the equivalent CTX system

1) Charging Gencos for their allocated initial permits under the SW auction developed in section 4.5 is similar to penalizing them under CTX, and both schemes produce similar profit results when Gencos do not bid into the SW auction;

2) Although the most polluting Genco has a strong incentive to bid into the SW auction, its potential profit increase by so doing is severely restricted by the bids from the other cleaner Gencos. This suggests that at a possible NE (where all Gencos bid and such that no one Genco can increase its profit by changing its bid), the permit allocation will not reward the high polluting Genco with a higher profit. In other words, the more polluting Gencos may receive more permits by submitting a high bid but at the expense of a higher permit price;

3) All Gencos seem to do better in terms of profits under CTX. The load surplus seems to be also higher under CTX than under the SW auction.

6.3 Chapter Summary

Under BAU, without emission regulation, the market-clearing dispatch favours the cheaper Gencos irrespective of their emission intensities. In contrast, under both emission regulation schemes: cap-and-trade and carbon tax, the hourly market-clearing dispatch shifts toward the low polluting Gencos.

Under BAU, Gencos game based on their generation costs and demand elasticity, thus, cheaper Gencos have higher market power irrespective of their emission intensities. In contrast, under CTX, Gencos use both use their generation costs and demand elasticity as well as emission intensities and tax elasticity to game in the hourly market. Thus, under CTX, low polluting Gencos, increase their competitiveness and market power even if they have higher generation costs.

Due to the hourly emissions tax under CTX, the profits of the high polluting Gencos decrease to levels that are significantly lower than those under BAU. Moreover, under CTX, the profits of the high polluting Gencos are lower than the profits of the less polluting but more expensive Gencos.

Thus, a carbon tax produces correct signals to invest in low polluting power plants as such investments will be justified by higher profits. Moreover, Gencos owning high polluting power plants will be compelled to invest in emission reduction technologies to avoid significant loss of profit.

On the other hand, under cap-and-trade, in addition to their emission intensities, generation costs and demand elasticity, Gencos use hourly emission trading as well as permits price elasticity in their gaming strategies. Thus, unlike CTX, high polluting cheap Gencos can retain their competiveness and market power provided they are allocated a relatively high number of emission permits (as under grand-fathering and maximum SW allocation). In addition, if low polluting expensive Gencos are allocated few permits, they will have to buy them, thus offsetting the positive effects of their low emission intensities on their market power. Consequently, permits trading under CAT results in an increase in the total generation cost of meeting the demand at the stipulated emissions cap. This leads to higher electricity prices and lower demand levels, as well as to lower load surpluses under CAT as compared to CTX. Moreover, the hourly market-clearing under CAT is less predictable than that under CTX because of the effects of the hourly permits trading under CAT.

Furthermore, under CAT with free permit allocation, high polluting cheap Gencos that are allocated a high share of these permits sell these permits and increase their profits. These Gencos end up making considerably higher profits than less polluting but more expensive Gencos, thus discouraging Gencos from investing in emission reduction.

Thus, while under CTX, low polluting Gencos make higher profits than higher polluting ones, regardless of differences in generation costs, under CAT, Genco profits depend significantly on how permits are allocated.

Preliminary results suggest that the proposed SW auction successfully penalizes high polluting Gencos and may prevent such Gencos from making windfall profits. Nonetheless, it seems that, under CTX, all Gencos earn higher profits than under the SW auction with competitive permit bids.

7 Précis, Conclusions and Future Prospects

"Don't look for your dreams to come true; look to become true to your dreams;"

7.1 Thesis Précis

In this thesis, we re-modeled current electricity markets to account for the application of the two eminent emission regulation schemes being applied or considered, cap-and-trade and carbon tax. We then applied each scheme to an imperfect electricity market (oligopoly) with hourly supply offers and demand bids and studied the effects of each scheme on hourly Genco gaming strategies and market-clearing over an extended time horizon.

7.1.1 Cap-and-trade

In a cap-and-trade system, the electricity sector emissions cap defined over a commitment interval, is assumed to be imposed by a regulating body or social planner (SP) on the basis of a national economic and environmental impact study. A major challenge is then how to allocate this sector emissions cap into individual Genco commitment interval caps that are then strategically used by each Genco in the hourly electricity market.

The most common cap allocation method, known as grand-fathering, rewards more polluting Gencos with a bigger share of the total sector cap, which furthermore is given away for free. Grand-fathering exists because of the political influence of high polluting and inexpensive generating companies that argue that without such a free and generous permit allocation, their profits together with social welfare would be negatively affected. In addition, consumers judge that grand-fathering prevents electricity prices from shooting up under emission caps compared to business-as-usual (BAU).

Nonetheless, a new sector cap allocation method is emerging and gaining popularity among decision makers, namely an emission permits auction where Gencos submit bids to acquire commitment interval permits, for which they are charged at the auction price. Advocates of this allocation method, argue that an auction shifts the allocation onus to the Gencos themselves through their bids for emission permits. Moreover, a permits auction also raises extra revenue that can be recycled by the SP back into the green economy. Recent directives by the European Union and the U.S. stipulate that the emissions cap of the electricity sector will be allocated among Gencos based on an auction. These are the major motivations for developing a new auction model for the electricity generation sector as well as for the quantitative study of its effects on market equilibrium, as done in this thesis.

We first developed a permits allocation scheme that maximizes social-welfare over the commitment interval. This SW maximization accounts for the realities of an oligopoly where Gencos compete by gaming in both the electricity and permits markets. We then developed a new permits allocation scheme based on an auction by taking the aforementioned maximum social-welfare allocation approach and adding the feature that Gencos can influence the permits allocation by submitting bids which essentially alter the SW definition and shift the allocation in their favour. Finally, we compared the common grand-fathering allocation scheme with the two new methods developed here.

For all three allocation methods, including grand-fathering, it is assumed that Gencos game in both electricity and permits market, as well as through their strategic temporal self-allocation of emission permits. The context of the application and comparison of the permit allocation methods consider several realistic features absent in current literature:

(i) Electricity and emission permits are traded in a joint market cleared on an hourly basis;

(ii) This joint electricity and emission permits market is not perfect (an oligopoly) in the sense that individual Gencos can influence the market prices of both electricity and emission permits. This influence is modeled and analyzed through the Cournot Nash equilibrium;

(iii) The analysis considers the temporal self-allocation of the commitment interval (typically one year) Genco emission caps into hourly fractions so as to improve the strategic position of competing Gencos.

7.1.2 Carbon tax

A major challenge of applying a carbon tax to an electricity market is to set up a tax structure, the effect of which is to reduce emissions by a desired amount over a commitment interval.

To do so, we re-modeled the hourly electricity market to clear by maximizing a modified objective function equal to the hourly social-welfare (based on the Genco cost offers) minus an hourly carbon penalty on the sector's hourly emissions. The resulting new market-clearing produces a variable hourly tax rate that is a linear function of two tax penalty parameters and of the sector's hourly emissions. We then re-formulated the Cournot Nash equilibrium hourly Genco gaming strategies to account for the tax scheme.

Finally, we developed a mathematical problem with equilibrium constraints (MPEC) solved by the SP (or carbon tax designer) over a commitment interval whose solution: (a) defines tax penalty parameters that result in the sector emissions being less than or equal to the desired target; (b) selects the tax penalty parameters that maximize the true social-welfare over the commitment interval including the tax penalty and its effects on hourly gaming.

7.1.3 The comparison

The effects of the two proposed emission regulation schemes on market-clearing results were compared against each other and against business-as-usual (BAU) through various case studies on the basis of such quantities like market power, Genco profits and consumer surplus.

7.2 Major Conclusions

7.2.1 Cap-and-trade

The analysis of an oligopolistic electricity market operating under cap-and-trade clearly suggests that hourly market-clearing, Genco profits and consumer surplus are significantly influenced by how the sector's emissions cap is allocated among Gencos (recall that this is so because, in addition to generation costs and emission intensities, the hourly Gencos' Cournot NE strategic offers under CAT are also strongly influenced by hourly permits trading). Thus, we conclude that:

<u>Under grand-fathering and maximum SW allocation with free permits:</u>

a) High polluting but cheap Gencos receive a high proportion of emission permits whose surplus they can then sell to the external permits market. This increases their revenue and allows them to offer more competitively, thus maintaining their market power;

b) On the other hand, low polluting but expensive Gencos have to buy permits from the hourly external market which increases their cost and forces them to offer more aggressively;

c) Moreover, high polluting cheap Gencos make considerably higher profits than low polluting more expensive ones.

Thus, the expectation that, under emission caps, low polluting Gencos would gain market power while high polluting ones would lose such power is practically nullified when caps are allocated according to grand-fathering or maximizing SW with free permits. Moreover, this profit outcome does not provide an incentive for polluting Gencos to invest in emission reduction technologies; neither does it produce signals to invest in low polluting power plants.

Under the proposed SW auction (with permit payments):

a) Under the proposed SW auction, the premium that Gencos pay for their emission permits limits profits out of selling excess permits in the external hourly market;

b) Since clean Gencos do not need permits, they have no incentive to bid into the auction. Thus, unlike the dirtier Gencos that need to acquire permits, the clean Genco profits are not affected by the permits auction price;

c) Although the most polluting Gencos have a strong incentive to bid, their maximum potential profit can never materialize since it can only be reached if the cleaner Gencos do not bid and sacrifice some profit. Although further study is still required, a more likely auction equilibrium is one where all Gencos bid, the clean ones very little but enough to prevent the dirtier Gencos from acquiring too many permits at the expense of the clean ones' profits.

Thus, the preferred permit allocation scheme (among grand-fathering, SW maximization and SW auction) seems to be the SW auction since low polluting Gencos then gain while high polluting ones lose market power. Moreover, the profit outcome provides an incentive for polluting Gencos to invest in emission reduction technologies while producing economic signals to invest in low polluting power plants.

7.2.2 Carbon tax

a) While the Cournot NE Genco gaming strategies under CAT are such that the Gencos' market power depends on generation costs, emission intensities and hourly permits trading, under CTX, market power depends on generation costs and emission intensities but not on permits trading. Consequently we observe the following:

i) Due to the lack of permits trading under CTX, compared to BAU, market power always shifts from high polluting cheap Gencos to less polluting more expensive ones. In contrast, under CAT, this shift in market power may not occur if the dirty Gencos can acquire a large number of permits either for free or by bidding into the permits auction;

ii) Due to the effect of permits trading on the Gencos' gaming strategies under CAT, the total cost of meeting the demand is higher under CAT compared to CTX. Thus, under CTX, electricity prices are lower and demand levels are higher, leading to higher load surpluses;

iii) The market-clearing outcome under CTX is easier to model than under CAT and therefore more predictable. This is because the hourly permits trading under CAT strongly depend on the intricate temporal self-allocation of each Genco's commitment interval permits into hourly fractions.

b) In addition to the influence of permits trading on hourly gaming strategies under CAT, the revenue from selling excess permits alleviates the effect of emission caps on the profits of high polluting cheap Gencos. In comparison, a carbon tax always penalizes such Gencos for their high emissions. As a result:

i) While the profits of high polluting cheap Gencos may increase under CAT compared to BAU, under CTX, such profits always decrease significantly from their BAU levels;

ii) While the profits of low polluting but expensive Gencos increase under both CAT and CTX compared to their BAU levels, the profits under CTX are higher than under CAT;

iii) Moreover, under CTX, low polluting expensive Gencos make considerably higher profits than high polluting ones, despite the high polluting Gencos being significantly cheaper to operate. In comparison, under CAT, the relative profits of clean and dirty Gencos depend strongly on the initial allocation of permits. For instance, under the common GF permit allocation scheme, or under our proposed maximum SW scheme, high polluting cheap Gencos make considerably higher profits than low polluting more expensive ones.

c) To avoid a significant loss of profit and a decrease in market power, under CTX, high polluting Gencos have strong incentives to invest in reducing their emission intensities. Moreover, CTX provides signals to invest in low polluting power plants despite their high marginal costs, in contrast to CAT with free permit allocation based on GF or maximizing SW, which send no such signals.

d) In comparing the CAT system where permits are allocated through the SW auction to CTX with total emissions equal to the CAT cap we conclude:

i) Both methods produce similar signals to invest in emission reduction in the sense that clean Gencos earn higher profits than dirty ones;

ii) All Gencos do better in terms of profits under CTX;

iii) The load surplus and SW are higher under CTX.

The main conclusions of our study can be summarized as follows:

The proposed SW auction to allocate the sector cap among the competing Gencos eases some of the inherent drawbacks of a cap-and-trade system, in particular, by sending appropriate economic signals to invest in emission reduction. However, we recognize that the auction does introduce new uncertainties into the prediction of the market equilibrium, not only due to the permits trading aspect of cap-and-trade and the hourly self-allocation but due to uncertainty in the permits auction bidding strategies.

In contrast, the proposed carbon tax structure not only produces the desired economic signals to invest in cleaner technologies but is subject to fewer sources of uncertainty when predicting the market outcome. Finally, when compared to an equivalent cap-and-trade scheme under the proposed auction, a carbon tax leads to higher profits for producers as well as to a higher consumer surplus.

Thus, pending further studies, we conclude that the carbon tax structure developed in this thesis is the recommended emission regulation scheme for an oligopolistic electricity market.

7.3 Future Prospects

Among the many issues that remain to be studied in more detail, the following stand out:

a) The ideas presented here should be tested on bigger systems with cost and intensity data as well as with emission permits prices derived from an actual system such as that of the EU;

b) The electricity sector cap could be elastic by allowing it to vary up or down according to a known incremental cost of such changes on the overall regional economy;

c) Further studies should be carried out on how Gencos will likely bid into the SW auction. The market outcome of this strategic Genco behavior should be compared against that of an equivalent carbon tax model. Does the carbon tax approach remain the preferred one?

d) The SW auction MPEC problem with its three nested optimizations could be simplified by approximating the coupling between the auction, the hourly market and the temporal Genco self-allocation of its commitment interval permits;

e) Other forms of Nash equilibrium to model the hourly Genco gaming strategies should be examined. For example, by replacing the Cournot NE strategy used in this thesis by the supply function equilibrium;

f) The long-term prospect of investing the money collected by the SP from the SW auction or from the carbon tax into research that could reduce emission intensities at the least cost possible.

7.4 Final Thought

Electric power systems, as is the case of other sectors of the economy, have been traditionally operated and planned under two misguided assumptions:

i) That there is an inexhaustible supply of hydro-carbon fuels, thus attaching no cost to the fact that such fuels will run out;

ii) That there is no environmental impact from the burning of hydro-carbons, again attaching no cost to increasing evidence that there is indeed a major negative impact.

These assumptions must be discarded to the garbage heap of disastrous historical decisions. Governments, industry and the public, instead of relying on rhetoric and delaying tactics, must be repeatedly pressured to act, and to do so based on scientific objective studies.

If what this thesis suggests, namely that a carbon tax, distasteful as it might appear to some, is a preferable emission regulation approach relative to cap-and-trade, then such a tax should be seriously considered, not just to limit emissions but to fund research to prepare for the eventual depletion of hydro carbons.

> ***“Verily never will Allah change the condition of a people until they change that which is in their hearts” The Holy Quran, Chapter 13, verse 11.***

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Appendices

Appendix A: Business-as-Usual Pool Dispatch

Non-linear generation cost offers

The hourly electricity market-clearing at time *t* is defined by the following optimization,

$$\min_{\{g_{it};\forall i\},d_t} \sum_k C_{it}(g_{it}) \tag{A.1}$$

subject to,

$$\sum_i g_{it} = d_t \qquad (\lambda_t) \tag{A.2}$$

$$g_{it} \leq g_{it}^{\max};\ \forall i \qquad (\mu_{it}^{\max}) \tag{A.3}$$

$$g_{it}^{\min} \leq g_{it};\ \forall i \qquad (\mu_{it}^{\min}) \tag{A.4}$$

Where $C_{it}(g_{it})$ is the hourly generation cost function offered by Genco i at time t. This function will be determined in the next section.

The Lagrangian function of the above optimization problem at each time t is,

$$\begin{aligned} L &= \sum_i C_i(g_i) - B(d) - \lambda\left(\sum_i g_i - d\right) \\ &-\sum_i \mu_i^{\min}\left(g_i^{\min} - g_i\right) - \sum_i \mu_i^{\max}\left(g_i - g_i^{\max}\right) \end{aligned} \tag{A.5}$$

The KKT necessary optimality conditions are,

$$\sum_i g_{it} = d_t \tag{A.6}$$

and,

$$IC_{it}(g_{it}) - \lambda_t + \mu_{it}^{\min} - \mu_{it}^{\max} = 0 \qquad ;\forall i \tag{A.7}$$

As well as the complimentarity conditions,

$$\left.\begin{matrix} \mu_{it}^{\max} \le 0 \\ \mu_{it}^{\min} \le 0 \end{matrix}\right\} \qquad ;\forall i \tag{A.8}$$

and the complementarity slackness conditions,

$$\left.\begin{matrix} \mu_{it}^{\min}\left(g_{it} - g_{it}^{\min}\right) = 0 \\ \mu_{it}^{\max}\left(g_{it} - g_{it}^{\max}\right) = 0 \end{matrix}\right\} \qquad ;\forall i \tag{A.9}$$

and finally the inequality constraints,

$$g_{it} \le g_{it}^{\max} \; ; \; \forall i \tag{A.10}$$

and,

$$g_{it}^{\min} \le g_{it} \; ; \; \forall i \tag{A.11}$$

Solving the set of non-linear equations (A.6) - (A.11) provides the solution to the market-clearing at time *t*. The aforementioned set of equations can be solved using a non-linear solver such as *MINOS* or through a binary search over λ.

If the generation limits are not active, the solution of the market-clearing at time t is such that,

$$IC_{it}(g_{it}) = \lambda_t \qquad ;\forall i \tag{A.12}$$

Elastic hourly demand

To incorporate the demand benefit function discussed in section 2.3.3 into the hourly market-clearing, the objective function of the optimization problem is modified to,

$$\min_{\{g_{it};\forall i\},d_t} \sum_k C_{it}(g_{it}) - B_t(d_t) \tag{A.13}$$

And the following equation is added to the list of KKT necessary conditions,

$$\lambda_t = IB_t\left(d_t\right) = \lambda_t^0 - \alpha_t d_t \tag{A.14}$$

Solving the set of non-linear equations (A.6) - (A.11), in addition to (A.14) provides the solution to the market-clearing at time t. Again, as an example, if the generation limits are not active, the solution of the market-clearing at time t is such that,

$$IC_{it}(g_{it}) = IB_t\left(d_t\right) = \lambda_t \qquad ;\forall i \tag{A.15}$$

Appendix B: Business-as-Usual Cournot Gaming in the Pool Dispatch

The profit of Genco i assuming it owns only one generating unit (the extension to Gencos owning more than one unit is relatively straightforward), is given by,

$$pr_{it} = \lambda_t g_{it} - C^*_{it}(g_{it}) \quad \text{(B.1)}$$

For the hourly market dispatch at time *t* to be at a Cournot Nash Equilibrium, Genco i should be generating at a MW level that maximizes its profit given by (B.1), provided all other Gencos do not change their generation levels. This is equivalent to solving the following optimization problem,

$$\max_{g_{it}} \left(pr_{it} \right) \quad \text{(B.2)}$$

subject to the following limits,

$$g_{it}^{\min} \leq g_{it} \qquad \left(\mu_{it}^{\min} \right) \quad \text{(B.3)}$$

$$g_{it} \leq g_{it}^{\max} \qquad \left(\mu_{k}^{\max} \right) \quad \text{(B.4)}$$

The Lagrangian function of the above optimization problem is,

$$L_{pr_{it}} = \lambda_t g_{it} - C^*_{it}\left(g_{it} \right) - \mu_{it}^{\min}\left(g_{it}^{\min} - g_{it} \right) - \mu_{it}^{\max}\left(g_{it} - g_{it}^{\max} \right) \quad \text{(B.5)}$$

One of the KKT optimality conditions requires that,

$$\frac{dL_{pr_{it}}}{dg_{it}} = \lambda_t + \frac{d\lambda_t}{dg_{it}} g_{it} - IC^*_{it} + \mu_{it}^{\min} - \mu_{it}^{\max} = 0 \quad \text{(B.6)}$$

where, from the power balance in (A.2) and from the hourly market-clearing optimality condition (A.14), we know that,

$$\lambda_t = \lambda_t^0 - \alpha_t \sum_k g_{kt} \tag{B.7}$$

and that,

$$\frac{d\lambda_t}{dg_{it}} = -\alpha_t \tag{B.8}$$

The Cournot Nash equilibrium hourly profit optimality condition (B.6) for Genco *i* can now be re-written as,

$$\lambda_t - \alpha_t g_{it} - IC_{it}^* + \mu_{it}^{\min} - \mu_{it}^{\max} = 0 \tag{B.9}$$

We conclude that for the hourly electricity market to clear at a Cournot Nash equilibrium, conditions (A.2) – (A.11), in addition to (A.14) have to be satisfied as well as condition (B.9) for all Gencos *i*. We recognize these conditions as the necessary optimality conditions arising from the hourly market-clearing problem when the offered generation cost function is given by,

$$C_{it}(g_{it}) = C_{it}^*(g_{it}) + 0.5\alpha_t g_{it}^2 \qquad ;\forall i \tag{B.10}$$

Appendix C: Business-as-Usual Supply Function Gaming in the Pool Dispatch

Mixed-Integer Formulation of supply function equilibrium with generation limits

Finding the SFE under generation limits is a combinatorial problem seeing as we do not know a priori at which of the three possible states a unit operates when satisfying the SFE (that is, at a minimum, at a maximum, or in between).

In this thesis, the definition of SFE first proposed by Klemperer and Mayer [53] for Gencos owning a single unit without generation limits is extended to include generation limits and Gencos owning multiple units. At this more general SFE, while some units may operate at either an upper or a lower bound, other units will operate within their limits.

At a SFE, these three possible outcomes must satisfy the following three necessary conditions:

(NC1) If a unit operates within its bounds, as shown below, the conditions defining the SFE are similar to those of Klemperer and Mayer;

(NC2) If a unit operates at its upper bound then it should not be possible for the owning Genco to increase its profit by increasing the unit's offered IC or, equivalently, by decreasing its SF offer to the point where the unit output is released from its upper bound and λ increases.

(NC3) If the unit operates at its lower bound then it should not be possible for the owning Genco to increase its profit by decreasing the unit's IC offer or, equivalently, by increasing its SF offer to the point where the unit output is released from its lower bound and λ decreases.

These three SFE necessary conditions are now expressed into an equivalent combinatorial problem in mixed-integer form:

Let L be a large positive number (e.g. the largest expected price or price cap). Then, for each generating unit i, we define three binary 0/1 variables, u_i, v_i, w_i, through the inequalities,

$$\frac{\left(\lambda - IC_i(g_i^{\max})\right)}{L} \le u_i \le 1 + \frac{\left(\lambda - IC_i(g_i^{\max})\right)}{L} \tag{C.1}$$

$$\frac{\left(IC_i(g_i^{\min}) - \lambda\right)}{L} \le v_i \le 1 + \frac{\left(IC_i(g_i^{\min}) - \lambda\right)}{L} \tag{C.2}$$

$$u_i + v_i + w_i = 1 \tag{C.3}$$

The above three relations are equivalent to the following conditional statements: (i) $u_i = 1$ if $\lambda \ge IC_i(g_i^{\max})$ (that is, if $g_i = g_i^{\max}$) and 0 otherwise; (ii) $v_i = 1$ if $\lambda \le IC_i(g_i^{\min})$ (that is, if $g_i = g_i^{\min}$) and 0 otherwise; and (iii) $w_i = 1$ if $IC_i(g_i^{\min}) \le \lambda \le IC_i(g_i^{\max})$ and therefore if $g_i^{\min} \le g_i \le g_i^{\max}$. Note that because of condition (C.3), for each unit i only one of the three binary variables can be equal to 1.

From (C.1) to (C.3) it follows that the output of unit *i* at market clearing can be written in the following explicit mixed-integer form,

$$g_i = u_i g_i^{\max} + v_i g_i^{\min} + w_i S_i(\lambda) \tag{C.4}$$

Moreover, the power balance relation between generation and demand requires that,

$$\sum_i \left(u_i g_i^{\max} + v_i g_i^{\min} + w_i S_i(\lambda)\right) = d = d_0 - \beta\lambda \tag{C.5}$$

Where, as seen in (C.5), the demand d can be price sensitive or not, depending on the value of the parameter $\beta \geq 0$.

To find the set of differential equations governing the SFE with generation limits, we allow Genco k to be the only Genco to vary the supply functions of the units under its ownership, one unit at a time.

Suppose then that unit $i \in G_k$ is the only one that varies its supply function incrementally. The profit of Genco k will then vary by,

$$\begin{aligned} d(pr_k)\big|_i &= d\lambda \sum_{j\in G_k} g_j + \lambda\left(dg_i + \sum_{\substack{j\in G_k \\ j\neq i}} w_j \frac{\partial S_j}{\partial \lambda} d\lambda \right) \\ &\quad - IC_i^*(g_i)dg_i - \sum_{\substack{j\in G_k \\ j\neq i}} w_j IC_j^*(g_j)\frac{\partial S_j}{\partial \lambda} d\lambda \end{aligned} \tag{C.6}$$

To compute the increment dg_i we have three situations:

(a) If unit i is within its bounds ($w_i = 1$), then from (C.5)[23],

$$w_i dg_i = w_i\left(-\beta(d\lambda) - \sum_{j\neq i} w_j \frac{\partial S_j}{\partial \lambda} d\lambda \right) \tag{C.7}$$

(b) If unit i is at its upper bound ($u_i = 1$), then we must test the variation of the profit of Genco k by decreasing the SF offer of unit i until $S_i(\lambda) = g_i^{\max}$ (or equivalently by increasing the IC offer until $IC_i(g_i^{\max}) = \lambda$) and unit i is just released from its upper bound, therefore giving,

[23] This assumes that no other unit $j \neq i$ is operating at the edge of being either released from or fixed to one of its limits by an infinitesimal change in λ. In such rare cases, dg_i would be defined by two different expressions, one for a positive and another for a negative $d\lambda$.

$$u_i dg_i = u_i \left(-\beta d\lambda - \sum_{j \neq i} w_j \frac{\partial S_j}{\partial \lambda} d\lambda \right) \tag{C.8}$$

(c) If unit i is at its lower bound ($v_i = 1$), then we must test the variation of the profit of Genco k by increasing the SF offer until $S_i(\lambda) = g_i^{\min}$ so that unit i i is just released from its lower bound (or equivalently by decreasing the IC offer until $IC_i(g_i^{\min}) = \lambda$) and unit i is just released from its lower bound, therefore giving,

$$v_i dg_i = v_i \left(-\beta(d\lambda) - \sum_{j \neq i} w_j \frac{\partial S_j}{\partial \lambda} d\lambda \right) \tag{C.9}$$

Since from (C.3) only one of the three binary variables u_i, v_i, w_i can be equal to 1, then, whether unit i is at a maximum, at a minimum or in between, it follows that,

$$\frac{\partial g_i}{\partial \lambda} = -\beta - \sum_{j \neq i} w_j \frac{\partial S_j}{\partial \lambda} \tag{C.10}$$

Then, from (C.6), the variation in the profit of Genco k when only the supply function of its unit i varies is given by,

$$\begin{aligned}
\left.\frac{\partial(pr_k)}{\partial\lambda}\right|_i &= \sum_{j\in G_k} g_j + \lambda\left(\left(-\beta-\sum_{j\neq i} w_j\frac{\partial S_j}{\partial\lambda}\right)+\sum_{\substack{j\in G_k\\ j\neq i}} w_j\frac{\partial S_j}{\partial\lambda}\right)\\
&\quad - IC_i^*(g_i)\left(-\beta-\sum_{j\neq i} w_j\frac{\partial S_j}{\partial\lambda}\right)-\sum_{\substack{j\in G_k\\ j\neq i}} w_j IC_j^*(g_j)\frac{\partial S_j}{\partial\lambda}\\
&= \sum_{j\in G_k} g_j + \left(IC_i^*(g_i)-\lambda\right)\beta-\lambda\sum_{j\notin G_k} w_j\frac{\partial S_j}{\partial\lambda}\\
&\quad+\left(\sum_{\substack{j\in G_k\\ j\neq i}} w_j\left(IC_i^*(g_i)-IC_j^*(g_j)\right)\frac{\partial S_j}{\partial\lambda}\right)\\
&\quad+\sum_{j\notin G_k} w_j IC_i^*(g_j)\frac{\partial S_j}{\partial\lambda}
\end{aligned} \tag{C.11}$$

The three possible conditions for a SFE with generation limits can now be expressed by the following set of inequalities,

$$-u_i L \le \left.\frac{\partial(pr_k)}{\partial\lambda}\right|_i \le v_i L \quad ;\{\forall i\in G_k;\forall k\} \tag{C.12}$$

From (C.12), we see that if $w_i=1$ then $\left.\frac{\partial(pr_k)}{\partial\lambda}\right|_i=0$ thus meeting NC1; if $u_i=1$ then $\left.\frac{\partial(pr_k)}{\partial\lambda}\right|_i\le 0$ thus meeting NC2; and finally if $v_i=1$ then $\left.\frac{\partial(pr_k)}{\partial\lambda}\right|_i\ge 0$ thus meeting NC3.

Inequality (C.12) together with (C.11) is a set of differential equations with inequalities in λ that characterizes the SFE with generation constraints.

Analytical solution to the affine supply function equilibrium: Illustrative case under inactive generation limits

Here we assume that each Genco k offers to supply power at time t from its unit i according to an affine IC offer of the form,

$$IC_{it}\left(g_{it}\right) = a_{it} + b_{it}g_{it} \tag{C.13}$$

This is equivalent to submitting a supply function of the form,

$$S_{it}\left(\lambda_t\right) = \left(\frac{\lambda_t - a_{it}}{b_{it}}\right) \tag{C.14}$$

For the remainder of this section we assume that at each time period t each Genco k games only with the IC -intersects or a -parameters under its ownership $\{a_{it}; i \in G_k\}$ and that the Genco always offers the true values of its b -parameters or IC slopes $\left\{b_{it} = b_{it}^*; i \in G_k\right\}$.

Moreover, unlike the Cournot Nash equilibrium, finding a solution for the Supply Function Equilibrium problem does not necessarily require an elastic demand. Thus, without loss of generality, in this derivation for simplicity of presentation we assume an inelastic demand. Finally, for the same reason, we assume that each Genco owns only one power generating unit.

Given an arbitrary set of offers ***a***, the objective function (A.1) in the optimization problem defining the hourly market-clearing is re-written as,

$$\min_{\{g_{it};\forall i\}}\left\{\sum_i C_{it}\left(g_{it}\right)\right\} \tag{C.15}$$

The Lagrangian function of the hourly market-clearing at each time t is then,

$$L_t = \sum_i C_{it}(a_{it}, g_{it}) - \lambda_t\left(\sum_i g_{it} - d_t\right) \tag{C.16}$$

In addition to the power balance (A.2), the KKT conditions include,

$$a_{it} + b^*_{it} g_{it} - \lambda_t = 0 \qquad ;\forall i \tag{C.17}$$

From which we can express the generation levels in terms of the Lagrange multipliers as,

$$g_{it} = \frac{\lambda_t - a_{it}}{b^*_{it}} \qquad ;\forall i \tag{C.18}$$

Replacing equation (C.18) in the power balance equation we get,

$$\sum_i \left(\frac{\lambda_t - a_{it}}{b^*_{it}} \right) = d_t \tag{C.19}$$

From (C.19) we can solve for the Lagrange multiplier,

$$\lambda_t = \frac{d_t + \sum_i \left(\frac{a_{it}}{b^*_{it}} \right)}{\sum_i \left(\frac{1}{b^*_{it}} \right)} \tag{C.20}$$

Recalling that the hourly profit of Genco k is,

$$pr_{kt} = \lambda_t g_{kt} - C^*_{kt}(g_{kt}) \qquad ;\forall k \tag{C.21}$$

and that from the power balance we can express g_{kt} in terms of the other Genco output levels, the profit of each Genco becomes,

$$pr_{kt} = \lambda_t \left(d_t - \sum_{i \neq k} g_{it} \right) - C^*_{kt}(d_t - \sum_{i \neq k} g_{it}) \qquad ;\forall k \tag{C.22}$$

According to the conditions of a Supply Function Nash Equilibrium, we vary the hourly offer of Genco k , by da_{kt} while the offers of the other Gencos are kept constant. This change in a_{kt} will lead to a change in the market equilibrium. In particular from (C.18) each Genco's output level will vary according to,

$$dg_{it} = \begin{cases} \dfrac{d\lambda_t}{b^*_{it}} & ;\forall i \neq k \\ \dfrac{d\lambda_t - da_{kt}}{b^*_{kt}} & ;i = k \end{cases} \tag{C.23}$$

In addition, from equation (C.22) the hourly profit of Genco k at time t will vary by,

$$dpr_{kt} = (d\lambda_t)\,g_{kt} + (\lambda_t)\left(-\sum_{i\neq k} dg_{it}\right) - IC^*_{kt}(g_{kt})\left(-\sum_{i\neq k} dg_{it}\right) \tag{C.24}$$

which, from (C.23), becomes,

$$\begin{aligned} dpr_{kt} &= (d\lambda_t)\,g_{kt} + \left(\lambda_t - IC^*_{kt}(g_{kt})\right)\left(-\sum_{i\neq k}\frac{d\lambda_t}{b^*_{it}}\right) \\ &= \left(g_{kt} - \left(\sum_{i\neq k}\frac{1}{b^*_{it}}\right)\left(\lambda_t - IC^*_{kt}(g_{kt})\right)\right)d\lambda_t \end{aligned} \tag{C.25}$$

Furthermore, from equation (C.20), a change in a_{kt} will lead to a change in the electricity marginal price equal to,

$$d\lambda_t = \frac{\dfrac{da_{kt}}{b^*_{kt}}}{\displaystyle\sum_i\left(\frac{1}{b^*_{it}}\right)} \tag{C.26}$$

Now, at the Nash Equilibrium, the following must be true,

$$\frac{dpr_{kt}}{da_{kt}} = \left(g_{kt} - \left(\sum_{i\neq k}\frac{1}{b^*_{it}}\right)\left(\lambda_t - IC^*_{kt}(g_{kt})\right)\right)\frac{d\lambda_t}{da_{kt}} = 0 \qquad ;\forall k \tag{C.27}$$

which when combined with equation (C.26) yields,

$$\left(g_{kt} - \left(\sum_{i \neq k} \frac{1}{b_{it}^*} \right) \left(\lambda_t - IC_{kt}^*(g_{kt}) \right) \right) \frac{\frac{1}{b_{kt}^*}}{\sum_i \left(\frac{1}{b_{it}^*} \right)} = 0 \qquad ; \forall k \tag{C.28}$$

Finally, equation (C.28) can be expressed as,

$$IC_{kt}^*(g_{kt}) + \left(\frac{1}{\sum_{i \neq k} \frac{1}{b_{it}^*}} \right) g_{kt} = \lambda_t \qquad ; \forall k \tag{C.29}$$

We conclude that for the hourly electricity market to clear at a supply function Nash equilibrium, conditions (A.2) – (A.11) have to be satisfied as well as condition (C.29) for all Gencos k. We recognize these conditions as the necessary optimality conditions arising from the hourly market-clearing problem when the offered generation cost function is given by,

$$C_{kt}(g_k) = C_k^*(g_k) + 0.5 \left(\frac{1}{\sum_{i \neq k} \frac{1}{b_{it}^*}} \right) g_k^2 \qquad ; \forall k \tag{C.30}$$

A similar result attributed to Green [57] can be derived if only the slope of the offered incremental cost function is allowed to vary.

Appendix D Electricity Markets Operating under Cap-and-Trade

Gaming in the hourly pool dispatch with permits trading

The hourly electricity market at time *t* under cap-and-trade dispatches according to the following optimization problem,

$$\min_{\{g_{it};\forall i\},d_t,\Delta e_t} \sum_k C_{it}(g_{it}) - B_t(d_t) + CEP_t(\Delta e_t) \qquad \text{(D.1)}$$

subject to the hourly power balance,

$$\sum_i g_{it} = d_t \qquad (\lambda_t) \qquad \text{(D.2)}$$

where Δe_t is the hourly emission permits bought by the electricity sector from the external permits market,

$$\Delta e_t = \sum_i \left(\rho_i g_{it} - e_{it}\right) \qquad (\gamma_t) \qquad \text{(D.3)}$$

The optimization is also subject to the upper and lower generation limits, respectively,

$$g_{it} \le g_{it}^{\max}\ ;\ \forall i \qquad (\mu_{it}^{\max}) \qquad \text{(D.4)}$$

and,

$$g_{it}^{\min} \le g_{it}\ ;\ \forall i \qquad (\mu_{it}^{\min}) \qquad \text{(D.5)}$$

The Lagrangian function of the above optimization problem is,

$$
\begin{aligned}
L_t &= \sum_i C_{it}(g_{it}) - B_t(d_t) + CEP_t(\Delta e_t) \\
&- \lambda_t \left(\sum_i g_{it} - d_t \right) - \gamma_t \left(\Delta e_t - \sum_i \left(\rho_i g_{it} - e_{it} \right) \right) \\
&- \sum_i \mu_{it}^{\min} \left(g_{it}^{\min} - g_{it} \right) - \sum_i \mu_{it}^{\max} \left(g_{it} - g_{it}^{\max} \right)
\end{aligned}
\tag{D.6}
$$

And the KKT necessary optimality conditions are,

$$IC_{it}(g_{it}) - \lambda_t + \gamma_t \rho_i + \mu_{it}^{\min} - \mu_{it}^{\max} = 0 \qquad ; \forall i \tag{D.7}$$

$$\lambda_t = IB_t(d_t) = \lambda_t^0 - \alpha_t d_t \tag{D.8}$$

$$\gamma_t = IC_t(\Delta e_t) = \gamma_t^0 + \beta_t \Delta e_t \tag{D.9}$$

$$\sum_i g_{it} = d_t \tag{D.10}$$

$$\Delta e_t = \sum_i \left(\rho_i g_{it} - e_{it} \right) \tag{D.11}$$

together with the complementarity slackness conditions,

$$\left. \begin{aligned} \mu_{it}^{\min} \left(g_{it} - g_{it}^{\min} \right) = 0 \\ \mu_{it}^{\max} \left(g_{it} - g_{it}^{\max} \right) = 0 \end{aligned} \right\} \qquad ; \forall i \tag{D.12}$$

$$\left. \begin{aligned} \mu_{it}^{\max} \geq 0 \\ \mu_{it}^{\min} \geq 0 \end{aligned} \right\} \qquad ; \forall i \tag{D.13}$$

and the inequality constraints,

$$g_{it} \leq g_{it}^{\max} \; ; \; \forall i \tag{D.14}$$

and,

$$g_{it}^{\min} \leq g_{it} \; ; \; \forall i \tag{D.15}$$

Now recall that the hourly profit of Genco *i* at time *t* is given by,

$$pr_{it} = \lambda_t g_{it} - C_{it}^*(g_{it}) - \gamma_t \left(\rho_i g_{it} - e_{it} \right) \tag{D.16}$$

For the hourly market dispatch at time *t* to be at a Cournot Nash Equilibrium, Genco *i* should be generating electricity at a level that maximizes its hourly profit given that all other Gencos do not change their output levels. This is equivalent to solving the optimization problem,

$$\max_{g_{it}} \left(pr_{it} \right) \tag{D.17}$$

subject to the following limits,

$$g_{it}^{\min} \le g_{it} \qquad \left(\mu_{it}^{\min} \right) \tag{D.18}$$

$$g_{it} \le g_{it}^{\max} \qquad \left(\mu_{k}^{\max} \right) \tag{D.19}$$

The Lagrangian function of the above optimization problem is,

$$L_{pr_{it}} = \lambda_t g_{it} - C_{it}^* \left(g_{it} \right) - \gamma_t \left(\rho_i g_{it} - e_{it} \right) - \mu_{it}^{\min} \left(g_{it}^{\min} - g_{it} \right) - \mu_{it}^{\max} \left(g_{it} - g_{it}^{\max} \right) \tag{D.20}$$

with the KKT optimality condition,

$$\frac{dL_{pr_{it}}}{dg_{it}} = \lambda_t + \frac{d\lambda_t}{dg_{it}} g_{it} - \gamma_t \rho_i - \frac{d\gamma_t}{dg_{it}} \left(\rho_i g_{it} - e_{it} \right) - IC_{it}^* + \mu_{it}^{\min} - \mu_{it}^{\max} = 0 \tag{D.21}$$

From the power balance (D.2) and from the hourly market-clearing optimality condition (D.8), we know that,

$$\lambda_t = \lambda_t^0 - \alpha_t \sum_k g_{kt} \tag{D.22}$$

thus,

$$\frac{d\lambda_t}{dg_{it}} = -\alpha_t \tag{D.23}$$

Similarly from the emissions balance (D.3) and the hourly market-clearing optimality condition (D.9) we know that,

$$\gamma_t = \gamma_t^0 + \beta_t \left(\sum_k \left(\rho_k g_{kt} - e_{kt} \right) \right) \qquad \text{(D.24)}$$

thus,

$$\frac{d\gamma_t}{dg_{it}} = \beta_t \rho_i \qquad \text{(D.25)}$$

The Cournot Nash equilibrium hourly profit optimality condition for Genco *i* (D.21) can now be re-written as,

$$\lambda_t - \alpha_t g_{it} + \beta_t \rho_i^2 g_{it} - \beta_t \rho_i e_{it} - IC_{it}^* + \mu_{it}^{\min} - \mu_{it}^{\max} = 0 \qquad \text{(D.26)}$$

We conclude that for the hourly electricity market to clear at a Cournot Nash equilibrium, conditions (D.7) - (D.15) have to be satisfied as well as condition (D.26) for all Gencos *i*. We recognize these conditions as the necessary optimality conditions arising from the hourly market-clearing problem, defined by (D.1) - (D.5), when the offered generation cost function are,

$$C_{it}\left(g_{it}\right) = C_{it}^*\left(g_{it}\right) + 0.5\left(\alpha_t + \beta_t \rho_i^2\right) g_{it}^2 - \beta_t \rho_i e_{it} g_{it} \qquad ;\forall i \qquad \text{(D.27)}$$

Market-clearing variables as functions of hourly emission permits

The KKT necessary conditions of the hourly market-clearing problem,(D.7) - (D.15) can be re-written in vector form as follows,

$$\mathbf{a}_t + \mathbf{B}_t \mathbf{g}_t - \lambda_t \mathbf{1} + \gamma_t \boldsymbol{\rho} - \boldsymbol{\mu}_t^{\max} + \boldsymbol{\mu}_t^{\min} = 0 \tag{D.28}$$

$$\lambda_t = IB_t = \lambda_t^0 - \alpha_t d_t \tag{D.29}$$

$$\gamma_t = ICEP_t = \gamma_t^0 + \beta_t \Delta e_t \tag{D.30}$$

$$\mathbf{1}^T \mathbf{g}_t = d_t \tag{D.31}$$

$$\Delta e_t = \boldsymbol{\rho}^T \mathbf{g}_t - \mathbf{1}^T \mathbf{e}_t \tag{D.32}$$

in addition to the complementarity conditions,

$$diag\left(\boldsymbol{\mu}_t^{\max}\right)\left(\mathbf{g}_t - \mathbf{g}_t^{\max}\right) = \mathbf{0} \qquad ;\forall t \tag{D.33}$$

$$diag\left(\boldsymbol{\mu}_t^{\min}\right)\left(\mathbf{g}_t - \mathbf{g}_t^{\min}\right) = \mathbf{0} \qquad ;\forall t \tag{D.34}$$

$$\boldsymbol{\mu}_t^{\max} \le 0 \tag{D.35}$$

$$\boldsymbol{\mu}_t^{\min} \le 0 \tag{D.36}$$

and the inequality constraints,

$$\mathbf{g}_t \le \mathbf{g}^{\max} \tag{D.37}$$

$$\mathbf{g}_t^{\min} \le \mathbf{g}_t \tag{D.38}$$

where the above vectors are defined as,

$$\begin{aligned} \mathbf{g}_t &= \{ g_{it}; \forall i \};\ \forall t \\ \mathbf{e}_t &= \{ e_{it}; \forall i \};\ \forall t \\ \mathbf{g}^{\max} &= \{ g_i^{\max}; \forall i \} \\ \boldsymbol{\mu}_t^{\max} &= \{ \mu_{it}^{\max}; \forall i \};\ \forall t \\ \boldsymbol{\mu}_t^{\min} &= \{ \mu_{it}^{\min}; \forall i \};\ \forall t \end{aligned} \quad \text{(D.39)}$$

Now, the Gencos' hourly offer strategies that define the Cournot Nash equilibrium are such that,

$$\begin{aligned} \mathbf{a}_t + \mathbf{B}_t\mathbf{g} &= \mathbf{a}_t^* - \beta_t diag(\boldsymbol{\rho})\mathbf{e}_t + diag\left(\mathbf{b}_t^* + \alpha_t\mathbf{1} + \beta_t diag(\boldsymbol{\rho})\boldsymbol{\rho}\right)\mathbf{g}_t \\ &= \mathbf{a}_t^* + diag\left(\mathbf{b}_t^*\right)\mathbf{g}_t + diag\left(\alpha_t\mathbf{1}\right)\mathbf{g}_t + \beta_t diag\left(\boldsymbol{\rho}\right)\left(diag(\boldsymbol{\rho})\mathbf{g}_t - \mathbf{e}_t\right) \end{aligned} \quad \text{(D.40)}$$

From the necessary conditions (D.28) to (D.32) along with condition (D.40), the generation levels at time *t*, $\mathbf{g}_t$ can be explicitly expressed as functions of the individual emission permits, $\mathbf{e}_t$ at time *t*, and of the Lagrange multipliers at time *t*, $\left(\boldsymbol{\mu}_t^{\max}, \boldsymbol{\mu}_t^{\min}\right)$ as follows:

$$\begin{aligned} \mathbf{a}_t^* - \beta_t diag(\boldsymbol{\rho})\mathbf{e}_t + diag\left(\mathbf{b}_t^* + \alpha_t\mathbf{1} + \beta_t diag(\boldsymbol{\rho})\boldsymbol{\rho}\right)\mathbf{g}_t &= \lambda_t\mathbf{1} - \gamma_t\boldsymbol{\rho} + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \\ &= \lambda_t^0\mathbf{1} - \alpha_t\left(\mathbf{1}^T\mathbf{g}_t\right)\mathbf{1} - \gamma_t^0\boldsymbol{\rho} \\ &\quad -\beta_t\left(\boldsymbol{\rho}^T\mathbf{g}_t - \mathbf{1}^T\mathbf{e}_t\right)\boldsymbol{\rho} + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \end{aligned}$$

(D.41)

The above relation can be re-written as,

$$\begin{aligned} diag\left(\mathbf{b}_t^* + \alpha_t\mathbf{1} + \beta_t diag(\boldsymbol{\rho})\boldsymbol{\rho}\right)\mathbf{g}_t + \alpha_t\left(\mathbf{1}^T\mathbf{g}_t\right)\mathbf{1} + \beta_t\left(\boldsymbol{\rho}^T\mathbf{g}_t\right)\boldsymbol{\rho} &= \beta_t diag(\boldsymbol{\rho})\mathbf{e}_t - \mathbf{a}_t^* + \lambda_t^0\mathbf{1} \\ &\quad -\gamma_t^0\boldsymbol{\rho} + \beta_t\left(\mathbf{1}^T\mathbf{e}_t\right)\boldsymbol{\rho} + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \end{aligned}$$

(D.42)

which simplifies to,

$$\left(diag\left(\mathbf{b}_t^* + \alpha_t \mathbf{1} + \beta_t diag(\boldsymbol{\rho})\boldsymbol{\rho}\right) + \alpha_t \mathbf{1}\mathbf{1}^T + \beta_t \boldsymbol{\rho}\boldsymbol{\rho}^T\right)\mathbf{g}_t = \left(\beta_t diag(\boldsymbol{\rho}) + \beta_t \boldsymbol{\rho}\mathbf{1}^T\right)\mathbf{e}_t - \mathbf{a}_t^* + \lambda_t^0 \mathbf{1} - \gamma_t^0 \boldsymbol{\rho} + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}$$

(D.43)

and finally to,

$$\begin{aligned}\mathbf{g}_t &= \left(diag\left(\mathbf{b}_t^* + \alpha_t \mathbf{1} + \beta_t diag(\boldsymbol{\rho})\boldsymbol{\rho}\right) + \alpha_t \mathbf{1}\mathbf{1}^T + \beta_t \boldsymbol{\rho}\boldsymbol{\rho}^T\right)^{-1} \begin{bmatrix} \left(\beta_t diag(\boldsymbol{\rho}) + \beta_t \boldsymbol{\rho}\mathbf{1}^T\right)\mathbf{e}_t - \mathbf{a}_t^* \\ +\lambda_t^0 \mathbf{1} - \gamma_t^0 \boldsymbol{\rho} + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \end{bmatrix} \\ &= \mathbf{g}_t^0 + \mathbf{H}_t \mathbf{e}_t + \mathbf{K}_t \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\right)\end{aligned}$$

(D.44)

where,

$$\begin{aligned}\mathbf{g}_t^0 &= \left(diag\left(\mathbf{b}_t^* + \alpha_t \mathbf{1} + \beta_t diag(\boldsymbol{\rho})\boldsymbol{\rho}\right) + \alpha_t \mathbf{1}\mathbf{1}^T + \beta_t \boldsymbol{\rho}\boldsymbol{\rho}^T\right)^{-1} \left[-\mathbf{a}_t^* + \lambda_t^0 \mathbf{1} - \gamma_t^0 \boldsymbol{\rho}\right] \\ \mathbf{H}_t &= \left(diag\left(\mathbf{b}_t^* + \alpha_t \mathbf{1} + \beta_t diag(\boldsymbol{\rho})\boldsymbol{\rho}\right) + \alpha_t \mathbf{1}\mathbf{1}^T + \beta_t \boldsymbol{\rho}\boldsymbol{\rho}^T\right)^{-1} \left[\left(\beta_t diag(\boldsymbol{\rho}) + \beta_t \boldsymbol{\rho}\mathbf{1}^T\right)\right] \\ \mathbf{K}_t &= \left(diag\left(\mathbf{b}_t^* + \alpha_t \mathbf{1} + \beta_t diag(\boldsymbol{\rho})\boldsymbol{\rho}\right) + \alpha_t \mathbf{1}\mathbf{1}^T + \beta_t \boldsymbol{\rho}\boldsymbol{\rho}^T\right)^{-1}\end{aligned}$$

(D.45)

Now, from equation (D.44) we can explicitly express all the variables in the hourly market-clearing optimization as functions of the individual emission permits, $\mathbf{e}_t$ at time *t*, and of the Lagrange multipliers, $\left(\boldsymbol{\mu}_t^{\max}, \boldsymbol{\mu}_t^{\min}\right)$ at time t, as follows:

1) <u>The demand level at time *t*:</u>

$$d_t = \mathbf{1}^T \mathbf{g}_t = \mathbf{1}^T \left(\mathbf{g}_t^0 + \mathbf{H}_t \mathbf{e}_t + \mathbf{K}_t \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\right)\right) = d_t^0 + \mathbf{h}_t^T \mathbf{e}_t + \mathbf{k}_t^T \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\right) \quad \text{(D.46)}$$

where,

$$\begin{aligned} d_t^0 &= \mathbf{1}^T \mathbf{g}_t^0 \\ \mathbf{h}_t &= \mathbf{H}_t^T \mathbf{1} \\ \mathbf{k}_t &= \mathbf{K}_t^T \mathbf{1} \end{aligned} \quad \text{(D.47)}$$

2) <u>The net permits traded by the electricity sector at time *t*:</u>

$$\begin{aligned}
\Delta e_t &= \boldsymbol{\rho}^T \mathbf{g}_t - \mathbf{1}^T \mathbf{e}_t \\
&= \boldsymbol{\rho}^T \left(\mathbf{g}_t^0 + \mathbf{H}_t \mathbf{e}_t + \mathbf{K}_t \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \right) \right) - \mathbf{1}^T \mathbf{e}_t \\
&= \boldsymbol{\rho}^T \mathbf{g}_t^0 + (\mathbf{H}_t^T \boldsymbol{\rho} - \mathbf{1})^T \mathbf{e}_t + \left(\mathbf{K}_t^T \boldsymbol{\rho} \right)^T \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \right) \\
&= \Delta e_t^0 + \left(\mathbf{m}_t^0 \right)^T \mathbf{e}_t + \mathbf{r}_t^T \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \right)
\end{aligned} \tag{D.48}$$

where,

$$\begin{aligned}
\Delta e_t^0 &= \boldsymbol{\rho}^T \mathbf{g}_t^0 \\
\mathbf{m}_t^0 &= (\mathbf{H}_t^T \boldsymbol{\rho} - \mathbf{1}) \\
\mathbf{r}_t &= \mathbf{K}_t^T \boldsymbol{\rho}
\end{aligned} \tag{D.49}$$

3) <u>The electricity price at time *t*:</u>

$$\begin{aligned}
\lambda_t &= \lambda_t^0 - \alpha_t d_t \\
&= \lambda_t^0 - \alpha_t \left(d_t^0 + \mathbf{h}_t^T \mathbf{e}_t + \mathbf{k}_t^T \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \right) \right) \\
&= \lambda_t^1 - \alpha_t \mathbf{h}_t^T \mathbf{e}_t - \alpha_t \mathbf{k}_t^T \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \right)
\end{aligned} \tag{D.50}$$

where,

$$\lambda_t^1 = \lambda_t^0 - \alpha_t d_t^0 \tag{D.51}$$

4) <u>The permits price at time *t*:</u>

$$\begin{aligned}
\gamma_t &= \gamma_t^0 + \beta_t \Delta e_t \\
&= \gamma_t^0 + \beta_t \left(\Delta e_t^0 + \left(\mathbf{m}_t^0 \right)^T \mathbf{e}_t + \mathbf{r}_t^T \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \right) \right) \\
&= \gamma_t^1 + \beta_t \left(\mathbf{m}_t^0 \right)^T \mathbf{e}_t + \beta_t \mathbf{r}_t^T \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \right)
\end{aligned} \tag{D.52}$$

where,

$$\gamma_t^1 = \gamma_t^0 + \beta_t \Delta e_t^0 \tag{D.53}$$

These relations are needed when solving the temporal self-allocation of the total permits allocated to each Genco for the commitment interval discussed next.

Strategic self-allocation of the commitment interval Genco cap into hourly fractions

a) Nash equilibrium condition

The profit of Genco *i* for the commitment interval is given by,

$$pr_i = \sum_t \left\{ \lambda_t g_{it} - C_{it}^*(g_{it}) - \gamma_t \left(\rho_i g_{it} - e_{it} \right) \right\} \tag{D.54}$$

Note that from the previous section in this Appendix, the profit of Genco *i* over the commitment interval can be expressed explicitly in terms of the hourly permits $\{e_{it}; \forall i, t\}$, and of the Lagrange multipliers $\{\mu_{it}^{\max}, \mu_{it}^{\min}; \forall i, t\}$. To find the Nash Equilibrium with respect to the hourly permits self-allocated by Genco *i*, its commitment interval profit, pr_i, is maximized with respect to its controllable variables $\{e_{it}; \forall t\}$ assuming that all other Gencos keep their temporal allocation unchanged.

However, we first have to be able to find the derivatives of the dependent variables, $\{\mu_{it}^{\max}, \mu_{it}^{\min}; \forall i, t\}$ with respect to the controllable variables $\{e_{it}; \forall t\}$.

b) Incremental variations

Recall from (D.33) and (D.34) that the KKT complementarity conditions of the hourly market-clearing at time t are:

$$diag\left(\boldsymbol{\mu}_t^{\max}\right)\left(\mathbf{g}_t - \mathbf{g}_t^{\max}\right) = \mathbf{0} \tag{D.55}$$

$$diag\left(\boldsymbol{\mu}_t^{\min}\right)\left(\mathbf{g}_t - \mathbf{g}_t^{\min}\right) = \mathbf{0} \tag{D.56}$$

Moreover, from (D.44) the generation levels at time t can be explicitly written in terms of $\mathbf{e}_t$ and the Lagrange multipliers $\left(\boldsymbol{\mu}_t^{\max}, \boldsymbol{\mu}_t^{\min}\right)$ at time t, as follows,

$$\mathbf{g}_t = \mathbf{g}_t^0 + \mathbf{H}_t \mathbf{e}_t + \mathbf{K}_t \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\right) \tag{D.57}$$

From the above three relations we can write the following,

$$diag\left(\boldsymbol{\mu}_t^{\max}\right)\left(\mathbf{g}_t^0 + \mathbf{H}_t \mathbf{e}_t + \mathbf{K}_t \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\right) - \mathbf{g}_t^{\max}\right) = \mathbf{0} \tag{D.58}$$

$$diag\left(\boldsymbol{\mu}_t^{\min}\right)\left(\mathbf{g}_t^0 + \mathbf{H}_t \mathbf{e}_t + \mathbf{K}_t \left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\right) - \mathbf{g}_t^{\min}\right) = \mathbf{0} \tag{D.59}$$

Considering infinitesimally small incremental variations, the above two equations become,

$$diag\left(\boldsymbol{\mu}_t^{\max}\right)\left(\mathbf{H}_t d\mathbf{e}_t + \mathbf{K}_t \left(d\boldsymbol{\mu}_t^{\max} - d\boldsymbol{\mu}_t^{\min}\right)\right) + diag\left(\mathbf{g}_t - \mathbf{g}_t^{\max}\right) d\boldsymbol{\mu}_t^{\max} = 0 \tag{D.60}$$

$$diag\left(\boldsymbol{\mu}_t^{\min}\right)\left(\mathbf{H}_t d\mathbf{e}_t + \mathbf{K}_t \left(d\boldsymbol{\mu}_t^{\max} - d\boldsymbol{\mu}_t^{\min}\right)\right) + diag\left(\mathbf{g}_t - \mathbf{g}_t^{\min}\right) d\boldsymbol{\mu}_t^{\min} = 0 \tag{D.61}$$

Collecting terms and re-writing the above two equations in matrix form we get,

$$\begin{bmatrix} \mathbf{C}_t & \mathbf{D}_t \\ \mathbf{E}_t & \mathbf{F}_t \end{bmatrix} \begin{bmatrix} d\boldsymbol{\mu}_t^{\max} \\ d\boldsymbol{\mu}_t^{\min} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_t \\ \mathbf{B}_t \end{bmatrix} d\mathbf{e}_t \tag{D.62}$$

Where,

$$\begin{aligned} \mathbf{A}_t &= -diag\left(\boldsymbol{\mu}_t^{\max}\right)\mathbf{H}_t \\ \mathbf{B}_t &= -diag\left(\boldsymbol{\mu}_t^{\min}\right)\mathbf{H}_t \\ \mathbf{C}_t &= diag\left(\boldsymbol{\mu}_t^{\max}\right)\mathbf{K}_t + diag\left(\mathbf{g}_t - \mathbf{g}_t^{\max}\right) \\ \mathbf{D}_t &= -diag\left(\boldsymbol{\mu}_t^{\max}\right)\mathbf{K}_t \\ \mathbf{E}_t &= diag\left(\boldsymbol{\mu}_t^{\min}\right)\mathbf{K}_t \\ \mathbf{F}_t &= -diag\left(\boldsymbol{\mu}_t^{\min}\right)\mathbf{K}_t + diag\left(\mathbf{g}_t - \mathbf{g}_t^{\min}\right) \end{aligned} \tag{D.63}$$

We can now find the required partial derivatives at time *t* by solving the following non-linear set of equations,

$$\begin{bmatrix} \mathbf{C}_t & \mathbf{D}_t \\ \mathbf{E}_t & \mathbf{F}_t \end{bmatrix} \begin{bmatrix} \dfrac{\partial \boldsymbol{\mu}_t^{\max}}{d\mathbf{e}_t} \\ \dfrac{\partial \boldsymbol{\mu}_t^{\min}}{d\mathbf{e}_t} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_t \\ \mathbf{B}_t \end{bmatrix} \tag{D.64}$$

From (D.64) and (D.57), we can now compute the following derivatives,

$$\frac{\partial \mathbf{g}_t}{\partial \mathbf{e}_t} = \mathbf{H}_t + \mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t} - \mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t} \tag{D.65}$$

c) <u>The NE optimization problem</u>

The maximization of the total profit of Genco *i* pr_i is subject to the single constraint requiring that the sum of the hourly self-allocated permits of Genco *i* be equal to its commitment interval permits, e_i. Recall that from (D.44) - (D.45) and (D.50)-(D.54), each Genco's generation level g_{it} at each time period t, as well as the electricity price λ_t and the permit price γ_t can be expressed in terms of the hourly permits $\{\mathbf{e}_t ; \forall t\}$ and the Lagrange multipliers $\left(\boldsymbol{\mu}_t^{\max}, \boldsymbol{\mu}_t^{\min}; \forall t\right)$. Thus the commitment interval profit maximization for each Genco *i* with respect to its temporal self-allocation can be written as,

$$\forall i \ldots \min_{\{e_{it};\forall t\}} \sum_t \left\{ C_{it}^*\left(g_{it}\right) + \gamma_t\left(\rho_i g_{it} - e_{it}\right) - \lambda_t g_{it} \right\} \tag{D.66}$$

Subject to,

$$\sum_t e_{it} = e_i \tag{D.67}$$

and to the hourly permits being non-negative,

$$0 \le e_{it} \qquad \left(\theta_{it}^{\min}\right) \qquad ;\forall t \tag{D.68}$$

This last inequality can be written in vector form as follows,

$$\mathbf{0} \leq \mathbf{e}_t \qquad \boldsymbol{\theta}_t^{\min} \qquad ;\forall t \tag{D.69}$$

The maximization of Genco *i*'s profit is also be subject to the complementarity conditions from the hourly market-clearing problem,

$$diag\left(\boldsymbol{\mu}_t^{\max}\right)\left(\mathbf{g}_t - \mathbf{g}^{\max}\right) = \mathbf{0} \qquad \left(\boldsymbol{\zeta}_t^{\max}\right) \qquad ;\forall t \tag{D.70}$$

$$diag\left(\boldsymbol{\mu}_t^{\min}\right)\left(\mathbf{g}_t - \mathbf{g}^{\min}\right) = \mathbf{0} \qquad \left(\boldsymbol{\zeta}_t^{\min}\right) \qquad ;\forall t \tag{D.71}$$

and to the inequality constraints,

$$\left.\begin{array}{ll} \boldsymbol{\mu}_t^{\max} \leq \mathbf{0} & \boldsymbol{\omega}_t^{A} \\ \boldsymbol{\mu}_t^{\min} \leq \mathbf{0} & \boldsymbol{\omega}_t^{B} \\ \mathbf{g}_t \leq \mathbf{g}^{\max} & \boldsymbol{\omega}_t^{C} \\ \mathbf{g}^{\min} \leq \mathbf{g}_t & \boldsymbol{\omega}_t^{D} \end{array}\right\} \forall t \tag{D.72}$$

where,

$$\begin{array}{l} \boldsymbol{\theta}_t^{\min} = \left\{\theta_{it}^{\min}; \forall i\right\};\ \forall t \\ \boldsymbol{\omega}_t^{A} = \left\{\omega_{it}^{A}; \forall i\right\};\ \forall t \\ \boldsymbol{\omega}_t^{B} = \left\{\omega_{it}^{B}; \forall i\right\}; \forall t \\ \boldsymbol{\omega}_t^{C} = \left\{\omega_{it}^{C}; \forall i\right\}; \forall t \\ \boldsymbol{\omega}_t^{D} = \left\{\omega_{it}^{D}; \forall i\right\}; \forall t \end{array} \tag{D.73}$$

d) <u>The necessary conditions</u>

The necessary conditions of the above optimization problem are:

1) The derivatives of the problem Lagrangian with respect to the hourly emissions permits, $\left\{e_{it};t\right\}$:

$$diag\left(diag\left(\frac{\partial \mathbf{g}_t}{\partial \mathbf{e}_t}\right)\right)\left(\mathbf{IC}_t^*\right)+diag\left(\frac{\partial \gamma_t}{\partial \mathbf{e}_t}\right)\left(diag\left(\boldsymbol{\rho}\right)\mathbf{g}_t\right)+\gamma_t diag\left(diag\left(\frac{\partial \mathbf{g}_t}{\partial \mathbf{e}_t}\right)\right)\boldsymbol{\rho}$$
$$-\gamma_t\mathbf{1}-diag\left(\mathbf{g}_t\right)\frac{\partial \lambda_t}{\partial \mathbf{e}_t}-diag\left(\frac{\partial \mathbf{g}_t}{\partial \mathbf{e}_t}\right)\lambda_t-\left(\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\max}\right)\left(\mathbf{g}_t-\mathbf{g}^{\max}\right)$$
$$-\left(\frac{\partial \mathbf{g}_t}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\max}\right)\left(\boldsymbol{\mu}_t^{\max}\right)-\left(\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\min}\right)\left(\mathbf{g}_t-\mathbf{g}^{\min}\right)-\left(\frac{\partial \mathbf{g}_t}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\min}\right)\left(\boldsymbol{\mu}_t^{\min}\right)$$
$$-\boldsymbol{\theta}+\boldsymbol{\theta}_t^{\min}-\left(\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)^T\boldsymbol{\omega}_t^A-\left(\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)^T\boldsymbol{\omega}_t^B-diag\left(diag\left(\frac{\partial \mathbf{g}_t}{\partial \mathbf{e}_t}\right)\right)\boldsymbol{\omega}_t^C+diag\left(diag\left(\frac{\partial \mathbf{g}_t}{\partial \mathbf{e}_t}\right)\right)\boldsymbol{\omega}_t^D=0\ ;\forall t$$

(D.74)

which simplifies to,

$$diag\left(diag\left(\mathbf{H}_t+\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}-\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\left(\mathbf{a}_t^*+\mathbf{B}_t^*\left(\mathbf{g}_t^0+\mathbf{H}_t\mathbf{e}_t+\mathbf{K}_t\left(\boldsymbol{\mu}_t^{\max}-\boldsymbol{\mu}_t^{\min}\right)\right)\right)$$
$$+diag\left(\beta_t\mathbf{m}_t^0\right)\left(diag\left(\boldsymbol{\rho}\right)\left(\mathbf{g}_t^0+\mathbf{H}_t\mathbf{e}_t+\mathbf{K}_t\left(\boldsymbol{\mu}_t^{\max}-\boldsymbol{\mu}_t^{\min}\right)\right)\right)$$
$$+diag\left(diag\left(\mathbf{H}_t+\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}-\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\left(\gamma_t^1+\beta_t\left(\mathbf{m}_t^0\right)^T\mathbf{e}_t+\beta_t\mathbf{r}_t^T\left(\boldsymbol{\mu}_t^{\max}-\boldsymbol{\mu}_t^{\min}\right)\right)\boldsymbol{\rho}$$
$$-\left(\gamma_t^1+\beta_t\left(\mathbf{m}_t^0\right)^T\mathbf{e}_t+\beta_t\mathbf{r}_t^T\left(\boldsymbol{\mu}_t^{\max}-\boldsymbol{\mu}_t^{\min}\right)\right)\mathbf{1}$$
$$-diag\left(\mathbf{g}_t^0+\mathbf{H}_t\mathbf{e}_t+\mathbf{K}_t\left(\boldsymbol{\mu}_t^{\max}-\boldsymbol{\mu}_t^{\min}\right)\right)\left(-\alpha_t\mathbf{h}_t\right)-diag\left(\mathbf{H}_t\right)\left(\lambda_t^1-\alpha_t\mathbf{h}_t^T\mathbf{e}_t-\alpha_t\mathbf{k}_t^T\left(\boldsymbol{\mu}_t^{\max}-\boldsymbol{\mu}_t^{\min}\right)\right)$$
$$-\left(\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\max}\right)\left(\mathbf{g}_t^0+\mathbf{H}_t\mathbf{e}_t+\mathbf{K}_t\left(\boldsymbol{\mu}_t^{\max}-\boldsymbol{\mu}_t^{\min}\right)-\mathbf{g}^{\max}\right)-\left(\mathbf{H}_t+\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}-\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\max}\right)\left(\boldsymbol{\mu}_t^{\max}\right)$$
$$-\left(\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\min}\right)\left(\mathbf{g}_t^0+\mathbf{H}_t\mathbf{e}_t+\mathbf{K}_t\left(\boldsymbol{\mu}_t^{\max}-\boldsymbol{\mu}_t^{\min}\right)-\mathbf{g}^{\min}\right)-\left(\mathbf{H}_t+\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}-\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\min}\right)\left(\boldsymbol{\mu}_t^{\min}\right)$$
$$-\boldsymbol{\theta}+\boldsymbol{\theta}_t^{\min}-\left(\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)^T\boldsymbol{\omega}_t^A-\left(\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)^T\boldsymbol{\omega}_t^B-diag\left(diag\left(\mathbf{H}_t+\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}-\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\boldsymbol{\omega}_t^C$$
$$+diag\left(diag\left(\mathbf{H}_t+\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}-\mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\boldsymbol{\omega}_t^D=0;\forall t$$

(D.75)

and then to,

$$diag\left(diag\left(\mathbf{H}_t\right)\right)\mathbf{a}_t^* + diag\left(diag\left(\mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t}\right)\right)\mathbf{a}_t^* - diag\left(diag\left(\mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)\right)\mathbf{a}_t^*$$
$$+diag\left(diag\left(\mathbf{H}_t\right)\right)\mathbf{B}_t^*\mathbf{g}_t^0 + diag\left(diag\left(\mathbf{H}_t\right)\right)\mathbf{B}_t^*\mathbf{H}_t\mathbf{e}_t + diag\left(diag\left(\mathbf{H}_t\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\max} - diag\left(diag\left(\mathbf{H}_t\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\min}$$
$$+diag\left(diag\left(\mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{g}_t^0 + diag\left(diag\left(\mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{H}_t\mathbf{e}_t + diag\left(diag\left(\mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\max}$$
$$-diag\left(diag\left(\mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\min}$$
$$-diag\left(diag\left(\mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{g}_t^0 - diag\left(diag\left(\mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{H}_t\mathbf{e}_t - diag\left(diag\left(\mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\max}$$
$$+diag\left(diag\left(\mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\min}$$

$$+diag\left(\beta_t\mathbf{m}_t^0\right)diag\left(\boldsymbol{\rho}\right)\mathbf{g}_t^0 + diag\left(\beta_t\mathbf{m}_t^0\right)diag\left(\boldsymbol{\rho}\right)\mathbf{H}_t\mathbf{e}_t + diag\left(\beta_t\mathbf{m}_t^0\right)diag\left(\boldsymbol{\rho}\right)\mathbf{K}_t\boldsymbol{\mu}_t^{\max} - diag\left(\beta_t\mathbf{m}_t^0\right)diag\left(\boldsymbol{\rho}\right)\mathbf{K}_t\boldsymbol{\mu}_t^{\min}$$

$$+diag\left(diag\left(\mathbf{H}_t + \mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t} - \mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)\right)\left(\gamma_t^1\boldsymbol{\rho} + \beta_t\boldsymbol{\rho}\left(\mathbf{m}_t^0\right)^T\mathbf{e}_t + \beta_t\boldsymbol{\rho}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\max} - \beta_t\boldsymbol{\rho}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\min}\right)$$
$$-\gamma_t^1\mathbf{1} - \beta_t\mathbf{1}\left(\mathbf{m}_t^0\right)^T\mathbf{e}_t - \beta_t\mathbf{1}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\max} + \beta_t\mathbf{1}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\min}$$

$$diag\left(\mathbf{g}_t^0\right)\alpha_t\mathbf{h}_t + diag\left(\mathbf{H}_t\mathbf{e}_t\right)\alpha_t\mathbf{h}_t + diag\left(\mathbf{K}_t\boldsymbol{\mu}_t^{\max}\right)\alpha_t\mathbf{h}_t - diag\left(\mathbf{K}_t\boldsymbol{\mu}_t^{\min}\right)\alpha_t\mathbf{h}_t - diag\left(\mathbf{H}_t\right)\lambda_t^1 + diag\left(\mathbf{H}_t\right)\alpha_t\mathbf{h}_t^T\mathbf{e}_t -$$
$$+diag\left(\mathbf{H}_t\right)\alpha_t\mathbf{k}_t^T\boldsymbol{\mu}_t^{\max} - diag\left(\mathbf{H}_t\right)\alpha_t\mathbf{k}_t^T\boldsymbol{\mu}_t^{\min}$$

$$-\left(\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\max}\right)\left(\mathbf{g}_t^0 + \mathbf{H}_t\mathbf{e}_t + \mathbf{K}_t\left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\right) - \mathbf{g}^{\max}\right) - \left(\mathbf{H}_t + \mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t} - \mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\max}\right)\left(\boldsymbol{\mu}_t^{\max}\right)$$
$$-\left(\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\min}\right)\left(\mathbf{g}_t^0 + \mathbf{H}_t\mathbf{e}_t + \mathbf{K}_t\left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\right) - \mathbf{g}^{\min}\right) - \left(\mathbf{H}_t + \mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t} - \mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\min}\right)\left(\boldsymbol{\mu}_t^{\min}\right)$$
$$-\boldsymbol{\theta} + \boldsymbol{\theta}_t^{\min} - \left(\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t}\right)^T\boldsymbol{\omega}_t^A - \left(\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)^T\boldsymbol{\omega}_t^B - diag\left(diag\left(\mathbf{H}_t + \mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t} - \mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)\right)\boldsymbol{\omega}_t^C$$
$$+diag\left(diag\left(\mathbf{H}_t + \mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\max}}{\partial\mathbf{e}_t} - \mathbf{K}_t\frac{\partial\boldsymbol{\mu}_t^{\min}}{\partial\mathbf{e}_t}\right)\right)\boldsymbol{\omega}_t^D = 0;\forall t$$

(D.76)

which finally simplifies to,

$$diag\left(diag\left(\mathbf{H}_t\right)\right)\mathbf{a}_t^* + diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)\right)\mathbf{a}_t^* - diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\mathbf{a}_t^*$$

$$+diag\left(diag\left(\mathbf{H}_t\right)\right)\mathbf{B}_t^*\mathbf{g}_t^0 + diag\left(diag\left(\mathbf{H}_t\right)\right)\mathbf{B}_t^*\mathbf{H}_t\mathbf{e}_t + diag\left(diag\left(\mathbf{H}_t\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\max} - diag\left(diag\left(\mathbf{H}_t\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\min}$$

$$+diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{g}_t^0 + diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{H}_t\mathbf{e}_t + diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\max}$$

$$-diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\min}$$

$$-diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{g}_t^0 - diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{H}_t\mathbf{e}_t - diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\max}$$

$$+diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\mathbf{B}_t^*\mathbf{K}_t\boldsymbol{\mu}_t^{\min}$$

$$+diag\left(\beta_t\mathbf{m}_t^0\right)diag\left(\boldsymbol{\rho}\right)\mathbf{g}_t^0 + diag\left(\beta_t\mathbf{m}_t^0\right)diag\left(\boldsymbol{\rho}\right)\mathbf{H}_t\mathbf{e}_t + diag\left(\beta_t\mathbf{m}_t^0\right)diag\left(\boldsymbol{\rho}\right)\mathbf{K}_t\boldsymbol{\mu}_t^{\max} - diag\left(\beta_t\mathbf{m}_t^0\right)diag\left(\boldsymbol{\rho}\right)\mathbf{K}_t\boldsymbol{\mu}_t^{\min}$$

$$+diag\left(diag\left(\mathbf{H}_t\right)\right)\gamma_t^1\boldsymbol{\rho} + diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)\right)\gamma_t^1\boldsymbol{\rho} - diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\gamma_t^1\boldsymbol{\rho}$$

$$+diag\left(diag\left(\mathbf{H}_t\right)\right)\beta_t\boldsymbol{\rho}\left(\mathbf{m}_t^0\right)^T\mathbf{e}_t + diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)\right)\beta_t\boldsymbol{\rho}\left(\mathbf{m}_t^0\right)^T\mathbf{e}_t - diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\beta_t\boldsymbol{\rho}\left(\mathbf{m}_t^0\right)^T\mathbf{e}_t$$

$$+diag\left(diag\left(\mathbf{H}_t\right)\right)\beta_t\boldsymbol{\rho}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\max} + diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)\right)\beta_t\boldsymbol{\rho}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\max} - diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\beta_t\boldsymbol{\rho}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\max}$$

$$-diag\left(diag\left(\mathbf{H}_t\right)\right)\beta_t\boldsymbol{\rho}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\min} - diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)\right)\beta_t\boldsymbol{\rho}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\min} + diag\left(diag\left(\mathbf{K}_t \frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\beta_t\boldsymbol{\rho}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\min}$$

$$-\gamma_t^1\mathbf{1} - \beta_t\mathbf{1}\left(\mathbf{m}_t^0\right)^T\mathbf{e}_t - \beta_t\mathbf{1}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\max} + \beta_t\mathbf{1}\mathbf{r}_t^T\boldsymbol{\mu}_t^{\min}$$

$$diag\left(\mathbf{g}_t^0\right)\alpha_t\mathbf{h}_t + diag\left(\mathbf{H}_t\mathbf{e}_t\right)\alpha_t\mathbf{h}_t + diag\left(\mathbf{K}_t\boldsymbol{\mu}_t^{\max}\right)\alpha_t\mathbf{h}_t - diag\left(\mathbf{K}_t\boldsymbol{\mu}_t^{\min}\right)\alpha_t\mathbf{h}_t - diag\left(\mathbf{H}_t\right)\lambda_t^1 + diag\left(\mathbf{H}_t\right)\alpha_t\mathbf{h}_t^T\mathbf{e}_t -$$

$$+diag\left(\mathbf{H}_t\right)\alpha_t\mathbf{k}_t^T\boldsymbol{\mu}_t^{\max} - diag\left(\mathbf{H}_t\right)\alpha_t\mathbf{k}_t^T\boldsymbol{\mu}_t^{\min}$$

$$-\left(\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\max}\right)\left(\mathbf{g}_t^0 + \mathbf{H}_t\mathbf{e}_t + \mathbf{K}_t\left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\right) - \mathbf{g}^{\max}\right) - \left(\mathbf{H}_t + \mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t} - \mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\max}\right)\left(\boldsymbol{\mu}_t^{\max}\right)$$

$$-\left(\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\min}\right)\left(\mathbf{g}_t^0 + \mathbf{H}_t\mathbf{e}_t + \mathbf{K}_t\left(\boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\right) - \mathbf{g}^{\min}\right) - \left(\mathbf{H}_t + \mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t} - \mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)^T diag\left(\boldsymbol{\zeta}_t^{\min}\right)\left(\boldsymbol{\mu}_t^{\min}\right)$$

$$-\boldsymbol{\theta} + \boldsymbol{\theta}_t^{\min} - \left(\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t}\right)^T\boldsymbol{\omega}_t^A - \left(\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)^T\boldsymbol{\omega}_t^B - diag\left(diag\left(\mathbf{H}_t + \mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t} - \mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\boldsymbol{\omega}_t^C$$

$$+diag\left(diag\left(\mathbf{H}_t + \mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\max}}{\partial \mathbf{e}_t} - \mathbf{K}_t\frac{\partial \boldsymbol{\mu}_t^{\min}}{\partial \mathbf{e}_t}\right)\right)\boldsymbol{\omega}_t^D = 0;\forall t$$

(D.77)

We refer to the above equation (D.77) as: $NC_{1t};\forall t$.

2) The derivatives of the problem Lagrangian with respect to the Lagrange multipliers $\{\theta_i;\forall i\}$:

$$\sum_t e_{it} = e_i \tag{D.78}$$

3) The derivatives of the problem Lagrangian with respect to the Lagrange multipliers $\{\zeta_{it}^{\max},\zeta_{it}^{\min};\forall i,t\}$:

$$diag\left(\boldsymbol{\mu}_t^{\max}\right)\left(\mathbf{g}_t-\mathbf{g}^{\max}\right)=\mathbf{0} \qquad ;\forall t \tag{D.79}$$

and

$$diag\left(\boldsymbol{\mu}_t^{\min}\right)\left(\mathbf{g}_t-\mathbf{g}^{\min}\right)=\mathbf{0} \qquad ;\forall t \tag{D.80}$$

4) The new complementarity conditions from the inequality constraints:

$$\left.\begin{aligned}
&\left(\boldsymbol{\theta}_t^{\min}\right)^T\left(\mathbf{0}-\mathbf{e}_t\right)=0\\
&\left(\boldsymbol{\omega}_t^{A}\right)^T\left(\boldsymbol{\mu}_t^{\max}-\mathbf{0}\right)=0\\
&\left(\boldsymbol{\omega}_t^{B}\right)^T\left(\boldsymbol{\mu}_i^{\min}-\mathbf{0}\right)=0\\
&\left(\boldsymbol{\omega}_t^{C}\right)^T\left(\mathbf{g}_t-\mathbf{g}^{\max}\right)=0\\
&\left(\boldsymbol{\omega}_t^{D}\right)^T\left(\mathbf{0}-\mathbf{g}_t\right)=0
\end{aligned}\right\}\forall t \tag{D.81}$$

5) The inequality conditions:

$$\left.\begin{array}{l} \mathbf{0} \le \mathbf{e}_t \\ \boldsymbol{\mu}_t^{\max} \le \mathbf{0} \\ \boldsymbol{\mu}_i^{\min} \le \mathbf{0} \\ \mathbf{g}_t \le \mathbf{g}^{\max} \\ \mathbf{0} \le \mathbf{g}_t \\ \boldsymbol{\theta}_t^{\min} \le \mathbf{0} \\ \boldsymbol{\omega}_t^{A} \le \mathbf{0} \\ \boldsymbol{\omega}_t^{B} \le \mathbf{0} \\ \boldsymbol{\omega}_t^{C} \le \mathbf{0} \\ \boldsymbol{\omega}_t^{D} \le \mathbf{0} \end{array}\right\} \forall t \qquad \text{(D.82)}$$

The above set of relations coupled with equations (D.64) and (D.65) yield an implicit relation between the hourly emission permits $\{e_{it}; \forall i, t\}$ and the total permits e_i allocated to each Genco i to be used over the commitment interval. This implicit relation defines our Nash equilibrium strategic temporal permit allocation.

Allocation of sector cap among Gencos using the social-welfare auction

a) Auction model

The auction model is defined by the following optimization,

$$\min_{\{g_{it};\forall i,t\},\{e_i;\forall i\},\{\Delta e_t,d_t;\forall t\}} \left\{ \sum_{i,t} C_{it}^{*}(g_{it}) - \sum_{t} B_t(d_t) + \sum_{t} CEP_t\left(\Delta e_t\right) - \sum_{i} F_i\left(e_i\right) \right\} \quad \text{(D.83)}$$

Subject to the total permits constraints,

$$\sum_{i} e_i \le e^0 \quad (\sigma) \quad \text{(D.84)}$$

and to the lower bound on the permits allocated to each Genco i which is set by the total permits grandfathered for free to the Genco,

$$e_i^f \le e_i \qquad \left(\delta_i^{\min}\right) \qquad ; \ \forall i \quad \text{(D.85)}$$

The above constraints in vector form are,

$$\mathbf{1}^T \mathbf{e} \le e^0 \quad (\sigma) \quad \text{(D.86)}$$

and,

$$\mathbf{e}^f \le \mathbf{e} \quad \left(\boldsymbol{\delta}^{\min}\right) \quad \text{(D.87)}$$

where,

$$\begin{aligned} \mathbf{e} &= \{e_i;\forall i\} \\ \boldsymbol{\rho} &= \{\rho_i;\forall i\} \\ \mathbf{e}^f &= \{e_i^f;\forall i\} \\ \boldsymbol{\delta}^{\min} &= \{\delta_i^{\min};\forall i\} \end{aligned} \quad \text{(D.88)}$$

b) The comprehensive social-welfare auction optimization:

Using the necessary conditions derived in the previous section that implicitly relate the total Genco permits $\mathbf{e}$ to the hourly permits $\{\mathbf{e}_t;\forall t\}$ the following problem is formulated,

$$\min_{\{e_i,\theta_i;\forall i\},\left\{e_{it},\mu_{it}^{\max},\mu_{it}^{\min},\frac{\partial\mu_{it}^{\max}}{\partial e_{it}},\frac{\partial\mu_{it}^{\min}}{\partial e_{it}},\theta_{it}^{\min},\zeta_{it}^{\max},\zeta_{it}^{\min},\omega_{it}^{A},\omega_{it}^{B},\omega_{it}^{C},\omega_{it}^{D},;\forall i,t\right\}}\left\{\sum_{i,t}C_{it}^{*}(g_{it})-\sum_{t}B_t(d_t)+\sum_{t}CEP_t\left(\Delta e_t\right)-\sum_{i}F_i\left(e_i\right)\right\}$$

(D.89)

where the hourly generation levels $\{g_{it};\forall i,t\}$, the hourly demand levels $\{d_t;\forall t\}$ and the hourly permits traded $\{\Delta e_t;\forall t\}$ can be explicit expressed in-terms of the hourly permits $\{\mathbf{e}_t;\forall t\}$ and the Lagrange multipliers $\left(\boldsymbol{\mu}_t^{\max},\boldsymbol{\mu}_t^{\min};\forall t\right)$. Recall that this explicit relation defines the hourly market-clearing under cap-and-trade and Cournot Nash equilibrium. The auction optimization function is subject to the following constraints,

$$\sum_t e_{it}=e_i \qquad \left(\theta_i^{*}\right) \tag{D.90}$$

$$\mathbf{1}^T\mathbf{e}\le e^0 \qquad (\sigma) \tag{D.91}$$

$$\mathbf{e}^f\le\mathbf{e} \qquad \left(\boldsymbol{\delta}^{\min}\right) \tag{D.92}$$

$$NC_{1t} \qquad ;\forall t \tag{D.93}$$

$$\begin{bmatrix}\mathbf{C}_t & \mathbf{D}_t\\ \mathbf{E}_t & \mathbf{F}_t\end{bmatrix}\begin{bmatrix}\dfrac{d\boldsymbol{\mu}_t^{\max}}{d\mathbf{e}_t}\\ \dfrac{d\boldsymbol{\mu}_t^{\min}}{d\mathbf{e}_t}\end{bmatrix}=\begin{bmatrix}\mathbf{A}_t\\ \mathbf{B}_t\end{bmatrix} \tag{D.94}$$

and finally,

$$\left.\begin{aligned}
& diag\left(\boldsymbol{\mu}_t^{\max}\right)\left(\mathbf{g}_t - \mathbf{g}^{\max}\right) = \mathbf{0} \\
& diag\left(\boldsymbol{\mu}_t^{\min}\right)\left(\mathbf{g}_t - \mathbf{g}^{\min}\right) = \mathbf{0} \\
& \left(\boldsymbol{\theta}_t^{\min}\right)^T \left(\mathbf{0} - \mathbf{e}_t\right) = \mathbf{0} \\
& \left(\boldsymbol{\omega}_t^{A}\right)^T \left(\boldsymbol{\mu}_t^{\max} - \mathbf{0}\right) = \mathbf{0} \\
& \left(\boldsymbol{\omega}_t^{B}\right)^T \left(\boldsymbol{\mu}_i^{\min} - \mathbf{0}\right) = \mathbf{0} \\
& \left(\boldsymbol{\omega}_t^{C}\right)^T \left(\mathbf{g}_t - \mathbf{g}^{\max}\right) = \mathbf{0} \\
& \left(\boldsymbol{\omega}_t^{D}\right)^T \left(\mathbf{0} - \mathbf{g}_t\right) = \mathbf{0}
\end{aligned}\right\} \forall t \tag{D.95}$$

such that,

$$\left.\begin{aligned}
& \mathbf{0} \le \mathbf{e}_t \\
& \boldsymbol{\mu}_t^{\max} \le \mathbf{0} \\
& \boldsymbol{\mu}_i^{\min} \le \mathbf{0} \\
& \mathbf{g}_t \le \mathbf{g}^{\max} \\
& \mathbf{0} \le \mathbf{g}_t \\
& \boldsymbol{\theta}_t^{\min} \le \mathbf{0} \\
& \boldsymbol{\omega}_t^{A} \le \mathbf{0} \\
& \boldsymbol{\omega}_t^{B} \le \mathbf{0} \\
& \boldsymbol{\omega}_t^{C} \le \mathbf{0} \\
& \boldsymbol{\omega}_t^{D} \le \mathbf{0}
\end{aligned}\right\} \forall t \tag{D.96}$$

Appendix E: Electricity Markets Operating under Carbon Tax

Gaming in the hourly pool dispatch with carbon tax

The hourly electricity market at time *t*, under cap-and-trade, dispatches according to the following optimization problem,

$$\min_{\{g_{it};\forall i\},d_t,x_t} \sum_k C_{it}(g_{it}) - B_t(d_t) + CTX_t(x_t) \qquad \text{(E.1)}$$

subject to the hourly power balance,

$$\sum_i g_{it} = d_t \qquad (\lambda_t) \qquad \text{(E.2)}$$

to the emissions to be taxed,

$$x_t = \sum_i \rho_i g_{it} \qquad (\gamma_t) \qquad \text{(E.3)}$$

and finally to the upper and lower generation limits , respectively,

$$g_{it} \le g_{it}^{\max} \; ; \; \forall i \qquad (\mu_{it}^{\max}) \qquad \text{(E.4)}$$

and,

$$g_{it}^{\min} \le g_{it} \; ; \; \forall i \qquad (\mu_{it}^{\min}) \qquad \text{(E.5)}$$

The Lagrangian function of the above optimization problem is,

$$\begin{aligned} L_t = & \sum_i C_{it}(g_{it}) - B_t(d_t) + CTX_t(x_t) - \lambda_t\left(\sum_i g_{it} - d_t\right) - \gamma_t\left(x_t - \sum_i \rho_i g_{it}\right) \\ & - \sum_i \mu_{it}^{\min}\left(g_{it}^{\min} - g_{it}\right) - \sum_i \mu_{it}^{\max}\left(g_{it} - g_{it}^{\max}\right) \end{aligned} \qquad \text{(E.6)}$$

And the KKT necessary optimality conditions are,

$$IC_{it}(g_{it}) - \lambda_t + \gamma_t \rho_i + \mu_{it}^{\min} - \mu_{it}^{\max} = 0 \qquad ; \forall i \qquad \text{(E.7)}$$

$$\lambda_t = IB_t\left(d_t\right) = \lambda_t^0 - \alpha_t d_t \tag{E.8}$$

$$\gamma_t = IC_t\left(\Delta e_t\right) = \gamma_t^0 + \beta_t x_t \tag{E.9}$$

$$\sum_i g_{it} = d_t \tag{E.10}$$

$$x_t = \sum_i \rho_i g_{it} \tag{E.11}$$

as well as the complementarity slackness conditions,

$$\left.\begin{aligned}\mu_{it}^{\min}\left(g_{it} - g_{it}^{\min}\right) = 0\\ \mu_{it}^{\max}\left(g_{it} - g_{it}^{\max}\right) = 0\end{aligned}\right\} \quad ;\forall i \tag{E.12}$$

$$\left.\begin{aligned}\mu_{it}^{\max} \geq 0\\ \mu_{it}^{\min} \geq 0\end{aligned}\right\} \quad ;\forall i \tag{E.13}$$

and finally the inequality constraints,

$$g_{it} \leq g_{it}^{\max}\ ;\ \forall i \tag{E.14}$$

and,

$$g_{it}^{\min} \leq g_{it}\ ;\ \forall i \tag{E.15}$$

Now recall that under a carbon tax system, the hourly profit of Genco *i* at time *t* is given by,

$$pr_{it} = \lambda_t g_{it} - C_{it}^*(g_{it}) - \gamma_t\left(\rho_i g_{it}\right) \tag{E.16}$$

For the hourly dispatch at time *t* to be at a Cournot Nash Equilibrium, Genco *i* should be generating electricity at a level that maximizes its hourly profit given that all other Gencos do not change their output levels. This is equivalent to solving the following optimization problem,

$$\max_{g_{it}} \left(pr_{it} \right) \tag{E.17}$$

subject to the following limits,

$$g_{it}^{\min} \leq g_{it} \qquad \left(\mu_{it}^{\min} \right) \tag{E.18}$$

$$g_{it} \leq g_{it}^{\max} \qquad \left(\mu_{k}^{\max} \right) \tag{E.19}$$

The Lagrangian function of the above optimization problem is,

$$L_{pr_{it}} = \lambda_t g_{it} - C_{it}^{*}\left(g_{it} \right) - \gamma_t \left(\rho_i g_{it} \right) - \mu_{it}^{\min}\left(g_{it}^{\min} - g_{it} \right) - \mu_{it}^{\max}\left(g_{it} - g_{it}^{\max} \right) \tag{E.20}$$

and the KKT optimality condition is,

$$\frac{dL_{pr_{it}}}{dg_{it}} = \lambda_t + \frac{d\lambda_t}{dg_{it}} g_{it} - \gamma_t \rho_i - \frac{d\gamma_t}{dg_{it}}\left(\rho_i g_{it} \right) - IC_{it}^{*} + \mu_{it}^{\min} - \mu_{it}^{\max} = 0 \tag{E.21}$$

From the power balance (E.2) and from the hourly market-clearing optimality condition (E.8), we know that,

$$\lambda_t = \lambda_t^0 - \alpha_t \sum_k g_{kt} \tag{E.22}$$

thus,

$$\frac{d\lambda_t}{dg_{it}} = -\alpha_t \tag{E.23}$$

Similarly from the emissions balance (E.3)and the hourly market-clearing optimality condition (E.9)we know that,

$$\gamma_t = \gamma_t^0 + \beta_t \left(\sum_k \rho_k g_{kt} \right) \tag{E.24}$$

thus,

$$\frac{d\gamma_t}{dg_{it}} = \beta_t \rho_i \tag{E.25}$$

From the above, the optimality condition (E.21) under Cournot Nash equilibrium with respect to the hourly profit of Genco *i* can now be re-written as,

$$\lambda_t - \alpha_t g_{it} + \beta_t \rho_i^2 g_{it} - IC_{it}^* + \mu_{it}^{\min} - \mu_{it}^{\max} = 0 \tag{E.26}$$

We conclude from (E.26) that for the hourly electricity market to clear at a Cournot Nash equilibrium, conditions (E.7) - (E.15) as well as condition (E.26) have to be satisfied for all Gencos. These conditions are recognized as those arising from the hourly market-clearing problem defined by (E.1) -(E.5) when the offered generation cost function are,

$$C_{it}(g_{it}) = C_{it}^*(g_{it}) + 0.5\left(\alpha + \beta_t \rho_i^2\right) g_{it}^2 \qquad ; \forall i \tag{E.27}$$

Market-clearing variables as functions of carbon tax parameters

Under a carbon tax system, the KKT necessary conditions of the hourly market-clearing problem, (E.7) - (E.15) can be re-written in vector form as follows,

$$\mathbf{a}_t + \mathbf{B}_t \mathbf{g}_t - \lambda_t \mathbf{1} + \boldsymbol{\rho} \gamma_t - \boldsymbol{\mu}_t^{\max} + \boldsymbol{\mu}_t^{\min} = 0 \tag{E.28}$$

$$\lambda_t = IB_t = \lambda_t^0 - \alpha_t d_t \tag{E.29}$$

$$\gamma_t = ICTX_t = \gamma^0 + \beta x_t \tag{E.30}$$

$$\mathbf{1}^T \mathbf{g}_t = d_t \tag{E.31}$$

$$x_t = \boldsymbol{\rho}^T \mathbf{g}_t \qquad (\gamma_t) \tag{E.32}$$

as well as the complementarity conditions,

$$diag\left(\boldsymbol{\mu}_t^{\max}\right)\left(\mathbf{g}_t - \mathbf{g}^{\max}\right) = 0 \tag{E.33}$$

$$diag\left(\boldsymbol{\mu}_t^{\min}\right)\left(\mathbf{g}_t^{\min} - \mathbf{g}_t\right) = 0 \tag{E.34}$$

$$\boldsymbol{\mu}_t^{\max} \leq 0 \tag{E.35}$$

$$\boldsymbol{\mu}_t^{\min} \leq 0 \tag{E.36}$$

and finally the inequality constraints,

$$\mathbf{g}_t \leq \mathbf{g}^{\max} \tag{E.37}$$

$$\mathbf{g}_t^{\min} \leq \mathbf{g}_t \tag{E.38}$$

Where the above vectors are defined as,

$$\begin{aligned}\mathbf{g}_t &= \{g_{it}; \forall i\}\\ \mathbf{g}^{\max} &= \{g_i^{\max}; \forall i\}\\ \boldsymbol{\mu}_t^{\max} &= \{\mu_{it}^{\max}; \forall i\}\\ \boldsymbol{\mu}_t^{\min} &= \{\mu_{it}^{\min}; \forall i\}\end{aligned} \tag{E.39}$$

Now, as derived in (E.27) the Gencos' hourly bidding strategies that define a Cournot Nash equilibrium are such that,

$$\begin{aligned}\mathbf{a}_t + \mathbf{B}_t\mathbf{g} &= \mathbf{a}_t^* + diag\left(\mathbf{b}_t^* + \alpha_t\mathbf{1} + \beta\, diag(\boldsymbol{\rho})\boldsymbol{\rho}\right)\mathbf{g}_t\\ &= \mathbf{a}_t^* + diag\left(\mathbf{b}_t^*\right)\mathbf{g}_t + diag\left(\alpha_t\mathbf{1}\right)\mathbf{g}_t + \beta\, diag\left(\boldsymbol{\rho}\right)\left(diag(\boldsymbol{\rho})\mathbf{g}_t\right)\end{aligned} \tag{E.40}$$

From the necessary conditions (E.28) - (E.32) along with equation (E.40), the generation levels at time t, $\mathbf{g}_t$ can be expressed as functions of the emissions tax parameters $\left(\gamma^0, \beta\right)$ and the Lagrange multipliers at time t, $\left(\boldsymbol{\mu}_t^{\max}, \boldsymbol{\mu}_t^{\min}\right)$ as follows,

$$\begin{aligned}\mathbf{a}_t^* + diag\left(\mathbf{b}_t^* + \alpha_t\mathbf{1} + \beta\, diag(\boldsymbol{\rho})\boldsymbol{\rho}\right)\mathbf{g}_t &= \lambda_t\mathbf{1} - \boldsymbol{\rho}\gamma_t + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\\ &= \lambda_t^0\mathbf{1} - \alpha_t\left(\mathbf{1}^T\mathbf{g}_t\right)\mathbf{1} - \boldsymbol{\rho}\left(\gamma^0 + \beta\boldsymbol{\rho}^T\mathbf{g}_t\right) + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}\end{aligned}$$

(E.41)

The above relation can be re-written as,

$$diag\left(\mathbf{b}_t^* + \alpha_t\mathbf{1} + \beta\, diag(\boldsymbol{\rho})\boldsymbol{\rho}\right)\mathbf{g}_t + \alpha_t\left(\mathbf{1}^T\mathbf{g}_t\right)\mathbf{1} + \beta\boldsymbol{\rho}\boldsymbol{\rho}^T\mathbf{g}_t = \lambda_t^0\mathbf{1} - \mathbf{a}_t^* - \gamma^0\boldsymbol{\rho} + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}$$

(E.42)

which simplifies to,

$$\left(diag\left(\mathbf{b}_t^* + \alpha_t\mathbf{1} + \beta\mathbf{R}\boldsymbol{\rho}\right) + \alpha_t\left(\mathbf{1}\mathbf{1}^T\right) + \beta\boldsymbol{\rho}\boldsymbol{\rho}^T\right)\mathbf{g}_t = \lambda_t^0\mathbf{1} - \mathbf{a}_t^* - \gamma^0\boldsymbol{\rho} + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \tag{E.43}$$

or,

$$\left(diag\left(\mathbf{b}_t^* + \alpha_t\mathbf{1}\right) + \alpha_t\left(\mathbf{1}\mathbf{1}^T\right)\right)\mathbf{g}_t + \beta\left(diag\left(\mathbf{R}\boldsymbol{\rho}\right) + \boldsymbol{\rho}\boldsymbol{\rho}^T\right)\mathbf{g}_t = \lambda_t^0\mathbf{1} - \mathbf{a}_t^* - \gamma^0\boldsymbol{\rho} + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min}$$

(E.44)

And finally can be written as,

$$\mathbf{H}_t\mathbf{g}_t + \beta\mathbf{K}\mathbf{g}_t = \mathbf{g}_t^0 - \boldsymbol{\rho}\gamma^0 + \boldsymbol{\mu}_t^{\max} - \boldsymbol{\mu}_t^{\min} \tag{E.45}$$

where,

$$\begin{aligned} \mathbf{g}_t^0 &= \lambda_t^0\mathbf{1} - \mathbf{a}_t^* \\ \mathbf{H}_t &= \left(diag\left(\mathbf{b}_t^* + \alpha_t\mathbf{1}\right) + \alpha_t\left(\mathbf{1}\mathbf{1}^T\right)\right) \\ \mathbf{K}_t &= \left(diag\left(\mathbf{R}\boldsymbol{\rho}\right) + \boldsymbol{\rho}\boldsymbol{\rho}^T\right) \end{aligned} \tag{E.46}$$

Computing the optimum carbon tax parameters

From the necessary conditions of the hourly market-clearing at each time period, the optimization variables in the global optimization problem defining the optimum tax parameters discussed in section 5.3, can all be written as implicit functions of the carbon tax parameters γ^0 and β as well as the Lagrange multipliers $\left(\boldsymbol{\mu}_t^{\max}, \boldsymbol{\mu}_t^{\min}\right)$ at time t. Maximizing the total social welfare over the commitment interval can be solved as one global optimization to find the optimum tax parameters that will meet a specific emissions constraint,

$$\min_{\gamma^0, \left\{\mu_{it}^{\max}, \mu_{it}^{\min}; \forall i,t\right\}} \left\{\sum_t \left[\sum_i C_{it}^*(g_{it}) - B_t(d_t) + \gamma_t \sum_i \rho_i g_{it}\right]\right\} \tag{E.47}$$

This SW optimization is subject to the emissions constraint on the electricity sector over the commitment interval under study,

$$\sum_t \sum_i \rho_i g_{it} \le e^0 \qquad (\sigma) \tag{E.48}$$

and to the lower and upper bounds on the linear parameter of the carbon tax,

$$\gamma^0 \le \gamma_{\max}^0 \qquad \left(\delta^{\max}\right) \tag{E.49}$$

and,

$$\gamma^0_{\min} \le \gamma^0 \qquad \left(\delta^{\min}\right) \tag{E.50}$$

as well as to the conditions,

$$\mathbf{H}_t\mathbf{g}_t + \beta\mathbf{K}\mathbf{g}_t = \mathbf{g}^0_t - \boldsymbol{\rho}\gamma^0 + \boldsymbol{\mu}^{\max}_t - \boldsymbol{\mu}^{\min}_t \tag{E.51}$$

$$d_t = \mathbf{1}^T\mathbf{g}_t \tag{E.52}$$

$$x_t = \boldsymbol{\rho}^T\mathbf{g}_t \tag{E.53}$$

And possible upper and lower bounds on the non-linear tax parameter,

$$\beta^{\min} \le \beta \le \beta^{\max} \tag{E.54}$$

The maximization is also subject to the complementarity conditions from the hourly market-clearing problem for each hour *t*,

$$diag\left(\boldsymbol{\mu}^{\max}_t\right)\left(\mathbf{g}_t - \mathbf{g}^{\max}\right) = \mathbf{0} \qquad \left(\boldsymbol{\zeta}^{\max}_t\right) \qquad ;\forall t \tag{E.55}$$

$$diag\left(\boldsymbol{\mu}^{\min}_t\right)\left(\mathbf{g}_t - \mathbf{g}^{\min}\right) = \mathbf{0} \qquad \left(\boldsymbol{\zeta}^{\min}_t\right) \qquad ;\forall t \tag{E.56}$$

and to the inequality constraints,

$$\left.\begin{array}{ll} \boldsymbol{\mu}^{\max}_t \le \mathbf{0} & \boldsymbol{\omega}^A_t \\ \boldsymbol{\mu}^{\min}_t \le \mathbf{0} & \boldsymbol{\omega}^B_t \\ \mathbf{g}_t \le \mathbf{g}^{\max} & \boldsymbol{\omega}^C_t \\ \mathbf{g}^{\min} \le \mathbf{g}_t & \boldsymbol{\omega}^D_t \end{array}\right\}\forall t \tag{E.57}$$