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Key Points:
- Radiative forcing scales logarithmically with gas concentration
- The log relationship holds for monochromatic radiance (besides broadband flux)
- The relationship results from radiative transfer (besides spectroscopy)

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Why logarithmic? A note on the dependence of radiative forcing on gas concentration

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Abstract Line-by-line radiative transfer computations show that the logarithmic dependence of radiative forcing on gas concentration not only applies to broadband irradiation fluxes such as in the well-known case of the CO_2 forcing, but also applies to the spectral radiance change due to both CO_2 and other gases, such as H_2O. The logarithmic relationship holds for monochromatic radiance requires an explanation beyond the conventional ideas based on the spectroscopic features of the gas absorption lines. We show that the phenomenon can be explained by an Emission Layer Displacement Model, which describes the radiance response to gas perturbation under normal atmospheric conditions such as temperature linearly varying with height and gas concentration exponentially decaying with height.

1. Introduction

It is interesting that the radiative forcing, i.e., the change in the radiation energy flux at the top of the atmosphere (TOA) or at the tropopause, caused by some greenhouse gases has a logarithmic dependency on the concentrations of these gases. For example, it is widely recognized that for every doubling of carbon dioxide (CO_2), the outgoing longwave radiation (OLR) is decreased by about a fixed amount (see Figure 1); logarithmic equations for calculating the radiative forcing of CO_2 are given by the Intergovernment Panel on Climate Change [e.g., *Shine et al.*, 1990]. Note that although "radiative forcing" sometimes includes the radiation flux changes due to rapid atmospheric adjustments, here we are concerned with the "instantaneous forcing" that is caused by a greenhouse gas change alone. The radiative forcing of water vapor (H_2O) can also be well approximated by scaling the effect in proportion to the change in the logarithm of its concentration. In analyzing the water vapor feedback, such scaling estimations are widely adopted [e.g., *Huang et al.*, 2007; *Soden et al.*, 2008; *Shell et al.*, 2008; *Vial et al.*, 2013; *Huang*, 2013; *Zhang and Huang*, 2014].

The logarithmic dependency is intriguing, considering that the dependency on the absorber concentration is exponential rather than logarithmic in the radiative transfer equation (RTE), which, when neglecting scattering, can be expressed as

$$R = \int_0^{\tau_s} B(T(\tau))e^{-\tau}d\tau \tag{1}$$

for optically thick atmosphere [*Goody and Yung*, 1989]. Here R is the monochromatic radiance, and B is the Planck function of temperature T at optical depth τ, which is measured from the TOA to an arbitrary altitude z and is computed by integrating the product of absorber number concentration ρ and its absorption cross-section k:

$$\tau = \int_z^{\infty} \rho(z)k(z)dz \tag{2}$$

where $e^{-\tau}$ gives the transmission function, T_r. Note that the absorber concentration ρ is imbedded in optical depth τ that only appears as an exponent in the above equation.

It needs to be pointed out that logarithmic behavior of mathematical functions is not unusual. However, as shown below, the logarithmic relationship between radiative forcing and gas concentration holds better, and more generally, than the accuracy that one would expect by approximating the exponential function $e^{-\tau}$ as logarithmic: $\log(\tau)$. What is of interest here is whether there is any physical reason that would lead to such a relationship.

The clues given in the textbooks usually point to the spectroscopic features of the absorption lines [e.g., *Goody and Yung*, 1989; *Pierrehumbert*, 2010]: both the absorption coefficient of an individual line from the

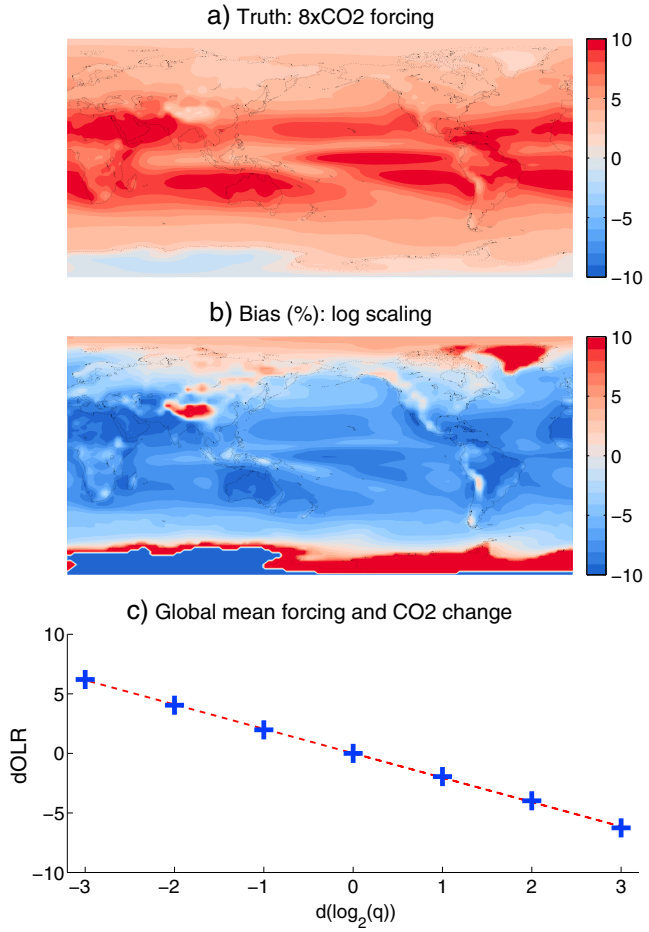

Figure 1. (a) The radiative forcing of 8 × CO_2 (unit: W m^{-2}). Here the all-sky forcing is measured by the change in the OLR, simulated by using a radiation model: MODTRAN. The simulation is based on a baseline CO_2 volume mixing ratio of 380 ppm and 3-hourly global 2.5°×2° resolution atmospheric profiles of 5 years (2000–2004) from a GCM: the Geophysical Fluid Dynamics Laboratory Coupled Model version 2.0. (b) Fractional bias (unit: 100%) in the 8 time CO_2 forcing analyzed by logarithmic scaling, i.e., scaling up the 2 time CO_2 forcing by 3 times. (c) Global mean forcing (unit: W m^{-2}, marked by the cross signs) versus CO_2 mixing ratio (q) changes. The results are based on a series of experiments, in which the CO_2 concentration is perturbed to 1/8, 1/4, 1/2, 2, 4, and 8 times the baseline value, respectively.

center to the wings of the line and the absorption strengths of all the lines from the center to the wings of the absorption band vary nonlinearly and as a result the *spectrally averaged* absorption saturates logarithmically with the absorber amount. Although such spectroscopic features certainly contribute to the logarithmic dependence of the broadband irradiance flux (the overall OLR), we find that the logarithmic relationship is also valid for spectrally revolved and even monochromatic radiance [*Bani Shahabadi and Huang*, 2014]. This means that the logarithmic dependence of radiative forcing requires not only a spectroscopic (spectral averaging) but also a radiative transfer explanation.

In the following, we will first show the logarithmic dependence of radiance based on accurate line-by-line (LBL) radiative transfer calculations. Then, we will show that a logarithmic dependence may arise from the radiative transfer under normal atmospheric conditions, which can be described by an Emission Layer Displacement Model. We will discuss the merits as well as the limitations of this explanation before concluding the paper.

2. Logarithmic Dependence of Radiance

It is known that the spectral mean absorption of an absorption line can be approximated as a logarithmic function of absorber amount given certain line shape functions (e.g., the "curve of growth" discussed by *Goody and Yung* [1989]). Moreover, the line strength of the absorption lines in the major absorption bands of such greenhouse gases as CO_2 and H_2O decays exponentially with the distance to band center [e.g., see of *Pierrehumbert*, 2010, Figure 4.12]. These spectroscopic features cause the spectrally averaged absorptivity to grow logarithmically with the absorber amount. This fact does contribute to the logarithmic dependence of spectrally integrated (broadband) radiation flux; one can easily verify this by computing the OLR in a one-layer atmosphere model (not shown). Because this explanation relies on spectral average, it would not be applicable to monochromatic radiance. However, LBL calculations show that the logarithmic dependence is also valid for monochromatic radiance.

Figure 2 shows that even when the atmospheric absorption is strong (or even saturated), such as in the H_2O rotational band in the far infrared (wave number less than 500 cm^{-1}) and in the CO_2 vibrational band (centered at 667 cm^{-1}), monochromatic radiance still varies in proportion with the

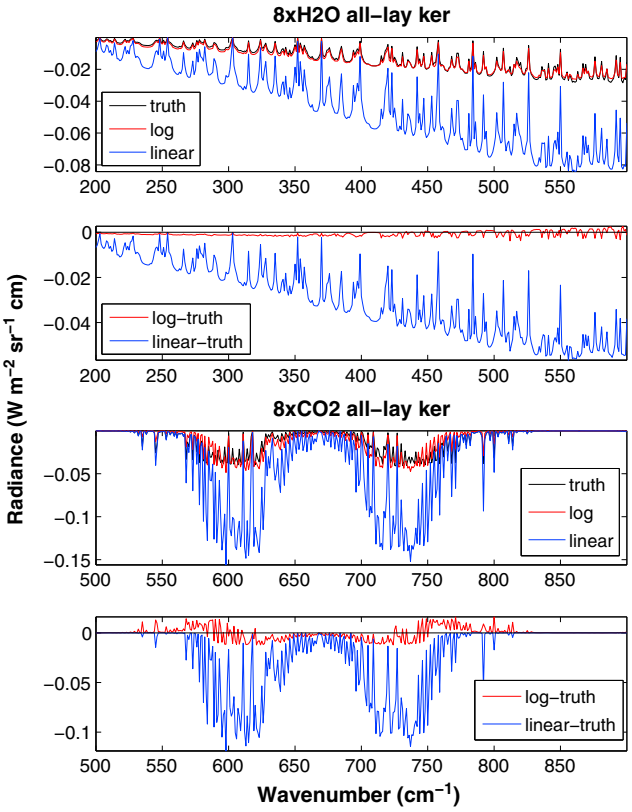

Figure 2. Changes in monochromatic radiance due to 8-time perturbation in (top) H_2O and (bottom) CO_2 of a standard atmospheric profile. The radiance is nadir, upwelling radiance at the tropopause, sampled one spectral point every 1 cm^{-1}. The result calculated by a line-by-line radiative transfer model ("truth") is compared to those logarithmically ("log") and linearly ("linear") scaled from the model-calculated radiance changes due to 1.1 time gas perturbation.

logarithm of the absorber concentration. The radiance change due to an 8 time change in absorber concentration can be accurately predicted from that due to a small 10% perturbation (80-fold amplification), by using a logarithmic scaling. For example, in the case of CO_2 forcing, first, the radiance change ΔR_0 due to 10% CO_2 concentration change, uniformly throughout the atmospheric profile, is calculated using the line-by-line radiative transfer model (LBLRTM) [*Clough et al.*, 1992] version 12.2. Then, the radiance change due to an 8 time CO_2 perturbation is predicated as $\Delta R_{\log} = \Delta R_0 * \frac{\log(8)}{\log(1.1)}$ in logarithmic scaling and $\Delta R_{\text{lin}} = \Delta R_0 * \frac{7}{0.1}$ in linear scaling. The logarithmic scaling well reproduces the true radiance change while linear scaling does not. As these changes are monochromatic radiance changes, the good logarithmic relationship demonstrated here cannot be explained by the line feature-based absorption saturation theory. Instead, such relationship must arise from the radiative transfer.

3. One-Layer Model

First, we examine whether the logarithmic relationship is possible for monochromatic radiance in a single-layer atmospheric model. Compared to the following section, a main simplification here is that the vertical inhomogeneity of the atmosphere is ignored. The outgoing radiance in this model can be calculated by summing the transmitted emission by the surface and the emission by the atmospheric layer:

$$R = B(T_1)(1-\varepsilon) + B(T_2)\varepsilon = B(T_2) + [B(T_1) - B(T_2)]e^{-\tau} \tag{3}$$

Here the surface is assumed to be a blackbody. T_1 and T_2 are the surface and atmospheric temperatures, respectively; ε is the atmospheric emissivity, which is equal to $(1-e^{-\tau})$, with τ being the optical depth of the layer. Except for some peculiar situations such as dimer-caused continuum absorption, τ is proportional to the absorber concentration ρ (see equation (2)).

From equation (3), R apparently is not a logarithmic function of τ (or ρ). Nevertheless, let us analyze its dependence on τ. When $\tau \gg 1$, i.e., the atmosphere is optically thick, R is equal to a constant $B(T_2)$, and there is no sensitivity to τ. When $\tau \ll 1$, i.e., the atmosphere is optically thin, R is linearly dependent on τ. An interesting case is when τ has a magnitude of 1. Given equation (3), the radiance change due to a perturbation in gas absorber, or equivalently in optical depth, around $\tau_0 = 1$, can be approximated as

$$\Delta R \simeq -Be^{-\tau_0}\left(\Delta\tau - \frac{1}{2}\Delta\tau^2 + \frac{1}{6}\Delta\tau^3 - \ldots\right) \tag{4}$$

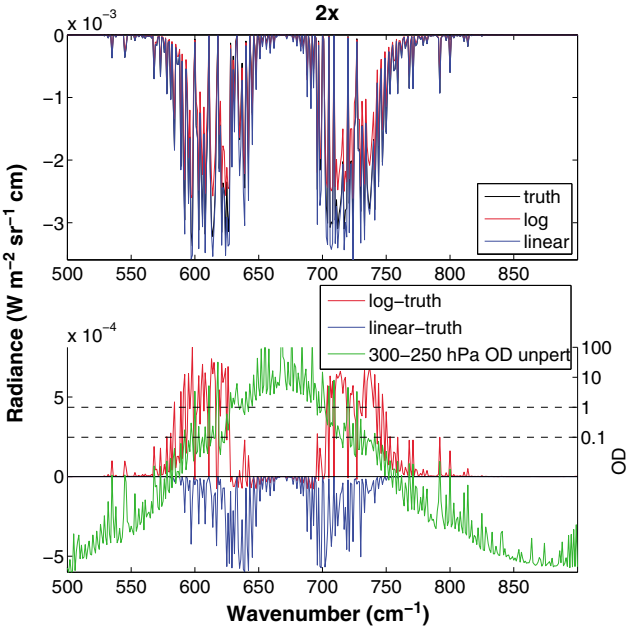

Figure 3. Radiance change due to CO_2 perturbation. Same as Figure 2 except for a doubling perturbation limited to the atmospheric layer between 250 and 300 hPa. A line of the optical depth of the layer (unperturbed) is drawn to provide a reference.

Consider a perturbation that multiplies the absorber amount by a factor of *a*. As the optical depth change can be expressed as $\Delta\tau = (a-1)\tau_0 = a\tau_0 - \tau_0$, ΔR can be estimated by a linear scaling as

$$\Delta R = K_{\text{lin}}\left[(a-1) + O\left((a-1)^2\right)\right] \quad (5)$$

where K_{lin} is the radiance sensitivity kernel $\frac{\partial R}{\partial \tau}$.

On the other hand, if taking a logarithmic scaling

$$\begin{aligned}\Delta(\ln(\tau)) &= \ln(\tau_0 + \Delta\tau) - \ln(\tau_0) \quad (6)\\ &= \ln(1+\Delta\tau) \simeq \Delta\tau - \frac{1}{2}\Delta\tau^2 \\ &\quad + \frac{1}{3}\Delta\tau^3 - \ldots\end{aligned}$$

Comparing the $\Delta\tau$—dependent parts of equations (4) and (6), ΔR can also be estimated as

$$\Delta R = K_{\log}\left[\ln(a) + O\left((\ln(a))^3\right)\right] \quad (7)$$

Here ln(…) denotes the natural logarithm. $K_{\log}$ is the radiance sensitivity kernel $\frac{\partial R}{\partial \ln(\tau)}$.

The higher-order residual term in equation (7) means that the logarithmic scaling can better approximate ΔR than the linear scaling. It can be shown that up to halving or doubling the absorber amount (and thus the optical depth τ), the bias in the logarithmic scaling is within 10% of the truth values, while the linear scaling can err by more than 100%.

Furthermore, we use the LBLRTM to examine the one-layer model here, in order to account for possible complications such as line broadening and overlapping. We consider a 2 time perturbation in CO_2 concentration in an atmospheric layer located between 250 and 300 hPa for a standard middle-latitude summer profile [*McClatchey et al.*, 1972]. This layer is selected because the outgoing radiance in the CO_2 absorption band (640–710 cm^{-1}) is very sensitive to the CO_2 perturbation in this layer. Figure 3 shows that when the layer optical depth is greater than one logarithmic scaling performs better, and when the optical depth is small, the linear scaling performs better. However, neither logarithmic nor linear scaling can reproduce the radiance change as well as in the previous, full-atmosphere case (Figure 2).

In summary, although the exponential form of transmission function $e^{-\tau}$ can be reasonably approximated by a logarithmic function of τ over a certain range of τ values around unity, the limited accuracy of this approximation cannot explain the good performance of the logarithmic scaling at large perturbation magnitudes: equation (7) suggests that the bias in the case of an 8 time perturbation would exceed 100%, which is not the case as shown in Figures 1 and 2. Moreover, this approximation cannot explain the logarithmic dependence at high optical depth values because when the atmospheric absorption is saturated, the one-layer model would predict zero radiative forcing. So the logarithmic relationship must also arise from the vertical variation of the atmospheric conditions. Interestingly, as shown below, the inhomogeneity of the atmosphere under normal conditions facilitates, rather than inhibits, the logarithmic relationship.

4. Emission Layer Displacement Model

In a nonscattering plane-parallel atmosphere, the outgoing radiance can be considered as contributed by the transmitted emission of a number of successive layers in the atmospheric column. The RTE (equation (1)) can be solved by summing the weighted source functions from all the discretized layers [*Goody and Yung*, 1989]:

$$R = \sum_i B_i W_i \quad (8)$$

Here B_i is the source function of a layer located at optical depth τ_i and is equal to the Planck function at the layer temperature T_i, $B_i = B(T(\tau_i))$, and W_i is the so-called "weighting function," which is the derivative of transmission function:

$$W_i = dT_r(z_i)/dz = e^{-\tau} d\tau/dz \tag{9}$$

where z denotes altitude. Note that for the sake of simplicity in the discussion, we neglect the sublayer inhomogeneity, which is small when a high-resolution vertical coordinate is adopted.

Due to the "$\tau = 1$ law," R in equation (8) is effectively contributed by the few layers around the altitude where the optical depth τ is about unity. Suppose the contributions from a total of m layers near $\tau = 1$ make the bulk of R and their weighting functions are $W(\tau_i)$, and their source functions are $B(\tau_i)$, i ranging from 1 to m:

$$R = \sum_{i=1}^{m} B(\tau_i) W_i \tag{10}$$

When the gas concentration in the atmospheric column is uniformly increased to a times its original value, so does the optical depth at any given level, i.e.,

$$\tau_i' = a \cdot \tau_i \tag{11}$$

We can still find m layers whose weighting functions W_i' are identical to those in the unperturbed atmosphere; i.e., $W_i' = W_i$. Because W is only a function of τ, these layers in the perturbed atmosphere correspond to where $\tau = \tau_i/a$ in the unperturbed atmosphere. Because B_i are also functions of τ, this new set of source functions become $B(\tau_i/a)$. So R becomes

$$R = \sum_{i=1}^{m} B(\tau_i/a) W_i \tag{12}$$

Now realizing that the Planck function in the longwave spectrum is a rather linear function of temperature at terrestrial temperatures (within 200–300 K, the deviation from the linear approximation is generally less than 10% for wave number less than 1000 cm^{-1}), B can be expressed as a linear function of altitude z given a constant lapse rate:

$$B = c_1 + c_2 z \tag{13}$$

Letter c here denotes constants, whose exact values are of no concern (it is the same in the following).

Meanwhile, if the volume mixing ratio of the gas absorber is constant (e.g., the case for CO_2) or exponentially decreasing (e.g., the case for H_2O) with altitude, the number density ρ in equation (2) can be expressed as an exponential function of z. Moreover, if the absorption cross-section k is also constant or varies exponentially with z (e.g., in the wings of a Lorenzian line), it follows that the optical depth can be expressed as an exponential function of z:

$$\tau = c_3 e^{z/c_4} \tag{14}$$

On a related note, the $\tau = 1$ law, i.e., the weighting function maximizing at the altitude where $\tau = 1$, follows from this equation. The monochromatic radiance in the absorption bands usually has a bell-shaped weighting function, which means that the photons arriving at the model top mostly emerge from the layers within a relatively short vertical range. So the conditions concerning gas absorber distribution (ρ) and optical property (k) used in the above derivation only need to be met within this vertical range.

Combining equations (13) and (14) yields

$$B = c_5 + c_6 \ln(\tau) \tag{15}$$

Then differencing equations (10) and (12), we obtain the change in radiance as

$$\Delta R = \sum_{i=1}^{m} [B(\tau_i/a) - B(\tau_i)] W_i = c \ln(a) \tag{16}$$

which has a logarithmic dependence on the gas concentration.

Note that the above result derived for the monochromatic radiance under nadir view can be reproduced for a slant path, simply by scaling the gas concentration in equation (2) by a constant $1/\cos(\theta)$, where θ is the zenith angle. Although trivial, this relationship under a slant path is verified by LBLRTM simulations. As the

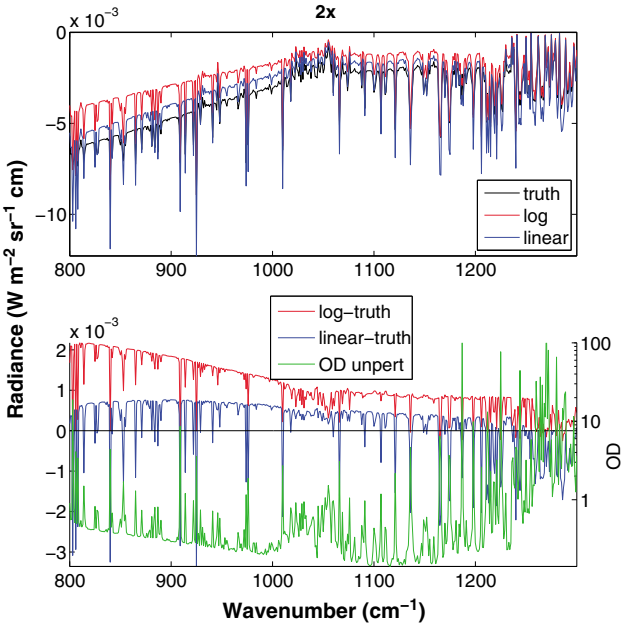

Figure 4. Radiance change due to H_2O perturbation. Same as Figure 2 but for a doubling perturbation and shown for the window region.

total radiation energy flux is simply the spectral and angular integration of radiance, the derived model would also explain the logarithmic dependence of the OLR (irradiance flux).

5. Discussions

The emission layer displacement (ELD) model derived above has a few advantages for explaining the logarithmic relationship. First, this model is derived based on monochromatic radiative transfer (requiring no spectral averaging) and thus explains the logarithmic scaling behavior of the monochromatic radiance as seen in Figure 2.

Second, this model is not based on approximation expansion series like equation (7). A perturbation of larger magnitude simply displaces the emission layer to a further distance (measured in τ). To the extent that the B-τ relationship in equation (15) holds, the effect of a large perturbation can be accurately predicted by scaling up the effect of a small perturbation. This explains the high accuracy in the logarithmic scaling results as seen in Figures 1 and 2.

A few key conditions that the derivation of the ELD model relies on can be used to predict when the logarithmic relationship may not hold. These situations include the following:

1. Unsaturated atmospheric absorption, such as in the window region. The ELD model essentially requires that when a perturbation occurs a replacement layer of similar weighting function value can be found for each emission layer. This condition is generally met for the radiance in those saturated gas absorption bands where the weighting function is bell-shaped (i.e., maximized at a certain level and decaying to small values within a vertical distance on both sides) and vertically moves, as a whole, within the atmosphere in response to a perturbation. Even though its shape may broaden or narrow slightly, this does not significantly change the result if only the B-τ relationship in equation (15) holds. However, in the window region (800–1250 cm^{-1}), the atmospheric absorption, mainly due to the H_2O continuum (and ozone, around 1042 cm^{-1}), is not saturated and the outgoing radiance is largely contributed by the surface emission. When H_2O is perturbed, the weighting function here does not maintain its shape, and the change in surface contribution cannot be represented by a displacement. So in the window region, the ELD model does not apply, and the logarithmic scaling can be expected to have a much worse performance (Figure 4 and see also *Bani Shahabadi and Huang* [2014]).
2. Continuum absorption. The optical depth scaling (equation (11)) may break down when τ has a nonlinear dependence on gas concentration. This effect does not significantly affect the logarithmic dependence in the absorption bands where τ is dominated by the line absorption, but it affects the window region where τ is mainly contributed by the H_2O continuum (which has a quadratic dependence on H_2O concentration).
3. Nonuniform perturbation. The exact displacement, and thus the radiance change, is determined by the relationship between the optical depths in the perturbed and unperturbed atmospheres. When the gas variations in different vertical layers are not uniform, equation (11) does not hold any more, and thus, the logarithmic relationship may break down. This is usually not an issue for the radiative forcing of well-mixed greenhouse gases such as CO_2 but calls into question the cases of variable gases such as H_2O and O_3. Nevertheless, in such cases, the analysis of the one-layer model suggests that it is still advisable to apply logarithmic scaling, as opposed to linear scaling, when the optical depth of the perturbation layer is greater than 1.

4. Overlapped absorption. This is another reason that may affect the logarithmic relationship. At a given wave number, if the absorption is dominated by one gas, equation (14) remains a good approximation; if not, however, additional z-dependent terms need to be added to the right-hand side of equation (14), which invalidates the simple relationship given by equation (15). This contributes to the less logarithmic behavior of the other greenhouse gases such as CH_4 and N_2O, whose absorption lines are significantly overlapped by H_2O lines.
5. Unusual temperature profile. This concerns equation (13). A simple case of an isothermal atmosphere that has a different temperature from the surface basically reduces to the one-layer model discussed in section 3. LBL simulations show that the logarithmic scaling indeed cannot reproduce the radiance change as well as before. For the outgoing radiance, whose weighting function peaks in the stratosphere (which has a negative temperature lapse rate), the logarithmic relationship still holds, but the radiative forcing is of an opposite sign. In this regard, the logarithmic relationship may be most impacted when the weighting function peaks around the tropopause.

6. Conclusions

In this paper, we show that the logarithmic dependence not only applies to broadband irradiance flux as in the well-known case of doubling CO_2 but is also manifested by spectrally resolved radiance changes due to both CO_2 and other gases such as H_2O.

The conventional ideas based on the spectroscopic features of the absorption lines thus would have difficulty in explaining the logarithmic relationship. In comparison, an Emission Layer Displacement Model best suits the facts concerning the logarithmic dependency of the radiance. This model means that the logarithmic relationship is generally valid for outgoing radiance that emerges from within the atmosphere. In addition, this model also predicts a few main situations when the relationship breaks down. Among them, most noteworthy is the case of unsaturated absorption (in the atmospheric window region).

Understanding the logarithmic dependence of radiative forcing on atmospheric gas absorber concentration is not only academically interesting by itself but also shed light to understanding climate feedback and sensitivity. The logarithmic nature of the forcing dependence allows us to efficiently estimate the radiative forcing and feedback using analytical methods. The analysis here suggests that such analytic methods can be applied to not only broadband but also spectrally resolved radiation.

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