

AVICENNA (D. 1037), LOGICAL THEORY, AND THE ARISTOTELIAN TRADITION

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ABSTRACT

In this study I examine Avicenna's (d. 1037) theory of conditional propositions, (or "if, then" sentences, *qaḍāyā šarṭiyya muttaṣila*), and his system of repetitive and conjunctive syllogisms (*qiyāsāt istiṭnā'iyya*, *qiyāsāt iqtirāniyya*). I show that Avicenna's theory of conditional propositions is conceived as a rejection of Alfarabi's "context theory"-based system of conditional propositions and conditional syllogisms (*qaḍāyā šarṭiyya*). I also show that Avicenna's "if, then" connectors operate as propositional connectives in the modern, technical sense of that term. However, the theoretical bases of Avicenna's conjunctive syllogistic belong to the *Prior Analytics*. The system of conjunctive syllogisms and quantified conditionals, which is one of Avicenna's most important contributions to the history of formal logic, is explicable in terms of Aristotle's syllogistic theory. Stoic logic, on the other hand, plays a minor role.

RÉSUMÉ

Dans cette étude, j'examine la théorie des propositions conditionnelles (*qaḍāyā šarṭiyya muttaṣila*) d'Avicenne (m. 1037) ainsi que son système des syllogismes répétitifs et conjonctifs (*qiyāsāt istiṭnā'iyya et qiyāsāt iqtirāniyya*). J'établis que Avicenne a formulé sa théorie des propositions conditionnelles afin de rejeter le système des propositions conditionnelles et syllogismes hypothétiques (*qiyāsāt šarṭiyya*) d'Alfarabi (m. 950), qui s'est fondé sur une théorie de langue dans laquelle le contexte dialectique demeure au centre de l'analyse des propositions et des syllogismes (appelée "context theory"). Ainsi je démontre que le connecteur conditionnel "si, alors" dans la logique hypothétique d'Avicenne fonctionne comme l'opérateur logique au sens technique du terme. Pourtant, les bases théorétiques du syllogisme conjonctif sont tirées des *Premiers Analytiques* d'Aristote. Le système du syllogisme conjonctif et la théorie des conditionnelles quantifiées, que je considère ici parmi les apports les plus importants à l'histoire de la logique formelle, sont explicables à la lumière de la théorie syllogistique d'Aristote. Cependant, la logique stoïcienne ne joue pas un rôle essentiel.

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INTRODUCTION

This study has two main objectives. The first is to examine Avicenna's theory of conditionals. According to the convention Avicenna inherited from earlier logicians—especially Alfarabi—what I call conditionals, or “if, then” sentences, Avicenna calls “connective conditional propositions (*qaḍāyā šarṭiyya muttaṣila*)”.¹ The second objective is to examine how Avicenna deploys conditionals in syllogistic forms of reasoning.

These objectives are closely related, but if there is any deeper philosophical conclusion to be drawn from this study, it is that for Avicenna the second project is prior to the first project. In Avicenna's systematic development of his theory of conditionals and conditional syllogisms in ŠQ V-IX, Avicenna presents his theory of restricted (*‘alā t-taḥqīq*) and simpliciter (*muṭlaq*) conditionals,² and his theory of quantified conditionals, *before* his presentation of the conjunctive syllogistic (*qiyāsāt iqtirāniyya*) in ŠQ VI,³ and before his exposition of repetitive syllogisms (*qiyāsāt istiṭnā'iyya*) in ŠQ VIII.⁴ Nevertheless, despite the appearance that Avicenna develops his theory of conditionals prior to his theory of the syllogism, in fact, the converse is closer to the mark. It is Avicenna's intuitions about what makes an argument with conditionals good that dictate the final form of his theory of conditional propositions, not the reverse.

¹ These stand in contrast with “disjunctive conditional propositions (*qaḍāyā šarṭiyya munfaṣila*)”, which will not be dealt with in this study.

² See N. Rescher, “Avicenna on the Logic of the ‘Conditional’ Proposition,” in *Studies in the History of Arabic Logic* (Pittsburgh: Pittsburgh University Press, 1963), 76-86.

³ Other manuscripts of this portion of the *Šifā'* make what is called Book VII in the edition follow Book V in the edition. The order in the printed edition is: V (conditional propositions) → VI (conjunctive syllogisms) → VII (conditional proposition equipollence). Other manuscripts (e.g. Ayasofia 2442 and Nuruosmaniye 2710) have the following order: (conditional propositions) → (conditional proposition equipollence) → (conjunctive syllogisms). This latter must be the correct one, since Avicenna uses the equipollence relations from ŠQ VII in the reduction of imperfect conjunctive syllogisms to perfect ones in ŠQ VI.

⁴ On the translation of *istiṭnā'* as “repetitive”, see K. Gyekye, “The Term ‘istithnā’ in Arabic Logic”, *Journal of the American Oriental Society* 92 (1972): 88-92).

The truth that intuitions about what makes an argument good dictate the account of what makes a conditional true is evidently on display in Avicenna. Without a doubt, the theoretical work in ŠQ V and VII is undertaken in order to make possible an account of valid inferences with conditional premises and conclusions that closely parallels Aristotle's syllogistic theory set out in *An. Pr.* A1-7. In other words, it is because Avicenna feels that Aristotle's account of logical validity is *the* correct account that he feels the need to develop a theory of quantified conditionals, and a doctrine of genuine versus absolute following (*luzūm*) and concomitance (*muwāfaqa*, *ma'iyya*). Yet, this belief also impels him to reject earlier theories of the conditional such as those found in Alfarabi (and likely originating in the work of Galen⁵) that are based on complete (*tāmm*) and incomplete (*ġayr tāmm*, *nāqis*) connection (*ittiṣāl*, *ittibā'*) and incompatibility (*inād*, *ta'ānud*) because they are incompatible with Aristotle's views of syllogistic validity.

To varying degrees, secondary literature has not appreciated the fact that Avicenna's theory of conditionals was dictated by his belief that Aristotle's account of what it is for a conclusion to follow necessarily from the premises on account of their formal properties. For example, in Nicholas Rescher's work on Avicenna's quantified conditionals, he takes Avicenna to have a Stoic doctrine of propositionality in which the truth of a proposition can vary over a period of time. Thus, a universal affirmative conditional (A-conditional), on Rescher's account of Avicenna, is true when there is no time in which the antecedent is true and the consequent is false. An E-conditional is true if it is always the case that the antecedent is true and the consequent is false; an I-conditional is true if there is at least one instance in which the antecedent and consequent are true; the O-conditional is true when there is an instance when

⁵ N. Shehaby, "Introduction" to Avicenna, *The Propositional Logic of Avicenna*, trans. N. Shehaby (Dordrecht and Boston: D. Reidel, 1973), 5f; M. Maróth, *Ibn Sīnā und die periptatetische "Aussagenlogik"* (Leiden: Brill, 1998), Chapter 4, §1.

the antecedent is true and the consequent is false. As we will see in Chapter 2, such a view of the conditional is actually closer to Alfarabi than Avicenna. Rescher's interpretation fails as an interpretation of Avicenna as soon as we consult ŠQ (something that Rescher could not have done since he did not have access to the printed edition of the *Šifā'*). Avicenna explicitly denies that he quantifies over times or instances (*mirār*), as Rescher would have it.⁶ Rather, as I discuss in Chapters 3 and 5, Avicenna quantifies over mentally supposed states (*aḥwāl*) or conditions (*šurūṭ*) that may or may not ever be realized at any time *t*, and, in fact, may not even be realizable (e.g. "if three is even, then it is divisible by two" is true on one reading Avicenna gives to conditionals). More important for our present purposes is the fact that Avicenna did not develop his theory of quantified conditionals in order to give an account of the five Stoic indemonstrables. Rather, Avicenna developed this theory in order (1) to allow opposition relations between conditionals to be isomorphic with the opposition relations in categorical propositions, (2) to extend the Aristotelian distinction between perfect and imperfect syllogisms to syllogisms with conditional premises and conclusions, (3) to allow the reduction of imperfect syllogisms by means of perfect ones using direct and indirect reduction, and (4) to make room for a doctrine of per impossible proofs of a conditional or categorical conclusion, something that is impossible on the traditional Aristotelian conception of per impossibile syllogisms.⁷

⁶ E.g. Avicenna, *Šifā': Maṭīq: al-Qiyās*, vol. 4, ed. I. Madkūr, S. Zāyed (Cairo: al-Hay'a al-ʿĀmma li-šūʿn al-Maṭābiʿ al-Amīriyya, 1964), 272.15. Hereafter, this work will be cited as follows: "Avicenna, ŠQ", followed by the chapter number in capitalized Roman numerals (e.g. "Avicenna, ŠQ, V" refers to *Šifā', al-Qiyās*, Book 5).

⁷ See Alfarabi, *Al-Fārābī's Short Commentary on Aristotle's Prior Analytics*, trans. N. Rescher (Pittsburgh: University of Pittsburgh Press, 1963), 82, n.3. Many of the concepts that Avicenna draws on in his exposition in ŠQ V-IX have precedents going as far back as Theophrastus. For example, Theophrastus used quantifiers in conditional forms of sentences, introduced middle terms, and organized valid forms into moods and figures when dealing with what Lejewski has called "prosleptic syllogisms" and also what Bobzien calls "wholly hypothetical syllogisms"; on the latter see C. Lejewski, "On Prosleptic Syllogism", *Notre Dame Journal of Formal Logic* 2 (1961): 148-76; on the former see S. Bobzien, "Wholly Hypothetical Syllogisms", *Phronesis* 45 (2000): 87-154. As will become clear by the end of

In this respect, Maróth's and Shehaby's studies, which emphasize the Aristotelian foundations of Avicenna's theory of conditionals and syllogisms with conditional premises and conclusions, were most welcome. Yet Maróth and Shehaby both claim that Avicenna's conditional propositions can be meaningfully interpreted as truth-functional conditionals. The claim that Avicenna's conditionals are truth-functional has two parts. It claims first that Avicenna's conditionals are *propositional*; it also claims that Avicenna's conditionals are *truth-functional*. In Chapter 4, I show that Avicenna does make the required logical distinctions for his "if, then" connectives to be meaningfully called "propositional" connectives. To that extent, Maróth and Shehaby are correct (though for the wrong reasons) to call Avicenna's syllogistic theory with conditional premises and conclusions a "propositional logic". They are wrong, however, to say that Avicenna's conditionals are "truth-functional". Not only does such a claim fail to find any basis in ŠQ, but implicit in this is a wide-ranging claim about Avicenna's ideas about logical validity, since it is from a particular view of logical validity that a truth-functional theory of conditionals arises in the first place.

It is perhaps worth dwelling on this point at length.⁸ First, let us assume that Avicenna holds what is called the "Classical" view of validity. An argument composed of premises and a conclusion is Classically valid if and only if it is impossible for the conclusion to be false while the premises are true. Classical validity authorizes the inference of any conclusion *Q* from an inconsistent premises set, e.g. $P \& \sim P$ (the principle is commonly shorthanded as "Ex Falso Quodlibet", or simply EFQ), since the premises cannot ever be true, regardless of the truth or falsity of the conclusion. Classical validity also allows us to validly infer a necessary truth from

the thesis, Avicenna's real contribution comes in systematizing a great deal of disparate material into a remarkably consistent theory of conditional propositions and syllogisms.

⁸ The definitions and names of the logical principles are adopted from S. Read, *Relevant Logic: A Philosophical Examination of Inference* (Oxford and New York: B. Blackwell, 1988), Chapter 2, §1-3.

any premise, even a false one (let us refer to this principle with the acronym for “necessary truth”, NT). Let us further assume that Avicenna holds what is called the principle of Deduction Equivalence. In its most basic form, this principle states that Q follows from P and background assumptions $P_1, P_2, \dots, P_n$ if and only if the conditional “if P , then Q ” follows from $P_1, P_2, \dots, P_n$. With these principles in mind, consider the principle EFQ, which says that Q follows from $P \& \sim P$. Then, by Deduction Equivalence, “if P , then Q ” follows from “ $\sim P$ ” (“ $\sim P$ ” being treated here as an assumption assimilable to $P_1, P_2, \dots, P_n$). Thus, Classical validity dictates that “if P , then Q ” must be true whenever $\sim P$ is true, i.e. when P is false.

On the other hand, the principle of Classical validity says that anything can imply a necessary truth (principle NT); or, in other words, Q follows from P and Q (“ Q ” being treated here as an assumption assimilable to $P_1, P_2, \dots, P_n$). Then, by Deduction Equivalence, “if P , then Q ” follows from Q , in which case “if P , then Q ” must be true whenever Q is true. In this case, “if P , then Q ” is true solely on account of the truth of the consequent Q ’s being true. The truth-value of the antecedent is, thus, irrelevant.

From all this we can understand the following. “If P , then Q ” is true not only when (1) P and Q are both true, but also (2) when P is false and Q is true, and (3) when P and Q are both false. “If P , then Q ” is (4) false only when P is true and Q is false. But this simply means that “if P , then Q ” is truth-functional (i.e., the truth-value of the conditional is entirely determined from the truth-values of its antecedent and consequent). In other words, only if we impute to Avicenna Classical validity, with its principles EFQ and NT, and we also assume that he held Deduction Equivalence, would it then be safe to claim that his conditionals are “truth-functional”. The converse holds too. It only makes sense to claim that Avicenna understands conditionals as truth-functional if we are also willing to say that his notions about what makes

an argument good are substantially in line with Classical validity and the principle of Deduction Equivalence.

The results of Chapter 5 can leave no doubt. Avicenna's notion of logical following (*luzūm*) is strictly Aristotelian in its fundamentals, and is thus incompatible with Classical validity. Chapter 5 also leaves no doubt that Avicenna would never accept the principle of Deduction Equivalence. In essence, this principle states that the truth-conditions of "if, then" sentences should be able to give full expression to our intuitions about the necessary and sufficient conditions for a conclusion to logically follow from premises. Avicenna is willing to say that a syllogism in Barbara can be expressed as a true, universal affirmative conditional with the premises as the antecedent and the conclusion as the consequent. Yet, Avicenna's true universal affirmative conditionals give formal expression to notions of following that are not valid as syllogisms, e.g. "always: if some Greeks are hairy, then some hairy things are Greek" is certainly a true conditional, but it does not, properly speaking, express a type of following between a premise and a conclusion because it is not a syllogism (it only has one premise, there is no middle term, both of the terms in the conclusion are in the premises, etc.).

In the following chapters, the basic challenge is to understand Avicenna as offering a formal analysis of the following (*luzūm*) of a conclusion from premises that is rooted in the foundational concepts of Aristotle's syllogistic. On the other hand, in ŠQ V-IX Avicenna also offers an analysis of the following (*luzūm*) of a consequent from an antecedent that is more general than the following of a conclusion from premises, but at the same time allows for their use as premises and conclusions in syllogistic arguments. What Maróth and Shehaby did not fully appreciate was the extent to which Avicenna's commitment to the idea that Aristotle's syllogistic is the only correct formal account of what makes a good argument guided his

thinking about conditionals. By the end of this dissertation, I hope to have offered an interpretation of Avicenna's text that grasps more fully Avicenna's commitment to the Aristotle and the Aristotelian tradition.

What might be called Avicenna's "logical conservatism" should not make us think that Avicenna's conjunctive syllogisms represent a quaint or naïve extension of Aristotle's logical theory of the syllogism to conditional propositions. Chapter 2 will show that Alfarabi's conditional syllogistic is very limited: (1) it cannot yield conditionals as conclusions, (2) it cannot handle cases where both the major and minor premise are conditionals, (3) only the major premise can be a conditional, the antecedent and consequent of which must be categorical propositions, and (4) the minor premise *must* repeat either the antecedent, the consequent, or the contradictory opposites of either, i.e. if the minor premise is not a repetition, then the syllogism is inconcludent. Chapter 2 suggests some reasons for why Alfarabi might have had such a restrictive view of conditional reasoning, but here I want to point to an interesting feature of Avicenna's conjunctive syllogistic that takes Avicenna's syllogistic far beyond the expressive power of Alfarabi's conditional syllogistic. Consider the following scenario. John is a businessman from Las Vegas who travels to Berlin for work very frequently. Based on your knowledge of John, you think the following two conditional premises are true: (A) if John is in Nashville, then he is not in Las Vegas; and (B) if John is not in Las Vegas, then he is in Berlin for business. By transitivity, the following conclusion *must* be true (C): if John is in Nashville, then he is in Berlin for business.⁹ But (C) is clearly false. Yet,

⁹ This argument is a variant of the following argument using subjective conditionals, which according to Dorothy Edgington (D. Edgington, "On Conditionals", in *Handbook of Philosophical Logic*, vol. 14, eds. D. Gabbay, F. Guenther (Dordrecht: Springer, 2007), 127-221, 144) was introduced by Ernest Adams (E. Adams, "The Logic of Conditionals", *Inquiry* 8/2 (1965): 166-97). This argument is used to show how transitivity (If A, then B; if B, then C entails that if A, then C) can fail (i.e. we accept the premises as true, but the conclusion is false) in instances in which the premises are subjunctive (also called "counterfactual") conditionals. Edgington points out that if we take the

formally expressed, this argument is (A*) “if P , then Q , and if Q , then R , then if P , then R ”, which is also obviously a valid argument form that relies on the (intuitively obvious) transitivity of implication. How do we account for the clear invalidity of the concrete argument on the one hand with the intuitive validity of the argument form on the other? Unfortunately, Alfarabi’s account of the syllogistic as being divided into a categorical (*ḥamlī*, *ḡazmī*) and a conditional (*ṣarṭī*) is of no help because the argument is not even formally expressible in Alfarabi’s syllogistic. The categorical syllogistic is no help here since the premises and the conclusion are all conditionals. But the conditional syllogistic is no help either. In the conditional syllogistic that Alfarabi inherited from late antique Greek authors, the minor premise must be a repetition of the antecedent, the consequent, or their contradictory opposite. Thus, in A* the only propositions allowed for assertion as minor premises in Alfarabi’s conditional syllogistic are either “ P ”, “ Q ”, “ $\sim P$ ”, or “ $\sim Q$ ”. Obviously, “if Q , then R ” is excluded from consideration because it is not a repetition of the antecedent, the consequent, or their contradictory opposites. In other words, it is not so much that Alfarabi’s syllogistic goes against our intuitive feeling that A* is valid by declaring that it is not, but at least offers an explanation about the underlying reasons for A’s invalidity. The case is, in fact, much more desperate: A cannot even

premises (1) “If Brown had been appointed, Jones would have resigned immediately afterwards”, and (2) “If Jones had died before the appointment was made, Brown would have been appointed” as counterfactually true (Edgington is using Lewis’ possible worlds semantics for counterfactuals (D. Lewis, *Counterfactuals* (Malden, MA: Blackwell, 2001) in her discussion), we will not say that (3) “If Jones had died before the appointment was made, Jones would have resigned immediately afterwards” must be (counterfactually) true as well. Those who used possible world semantics for counterfactual conditionals point out that we will be willing to concede (1) is counterfactually true in a possible world in which Jones has died before the appointment. Avicenna, like Aristotle, would probably have found possible worlds semantics implausible; nor is Avicenna interested in counterfactuals in his theory of conditionals. Nevertheless, his theory of quantified conditionals over states (*aḥwāl*) gets at a basic insight that underlies Lewis and Stalnaker’s quantification over possible worlds, though Avicenna enlists the aid of Aristotelian quantifiers. Avicenna’s and Lewis’ diagnoses for why the above arguments are invalid are closely analogous, since they both require us to examine the conditions under which we assert the consequent given the antecedent is true. In the above example about John, the basic intuition is that we would given assent to the consequent in (B) “if John is not in Las Vegas, then he’s in Berlin” as a characterization of John’s work schedule, but we would not assert that John is in Berlin knowing that John is in Nashville. Lewis would say that the possible worlds expressed by the propositions “John is not in Las Vegas” and “John is in Nashville” must be the same in order to guarantee transitivity in this case.

be formalized in Alfarabi's syllogistic, in the sense that it is not a categorical syllogism, and it is not a conditional syllogism of the modus ponens/modus tollens variety.

Avicenna is quite conscious of heavy restrictions imposed on the syllogistic's ability to formalize arguments with conditional premises if we abide by the Farabian view of conditional syllogisms. It is for this very reason that he develops his conjunctive syllogistic, which, *inter alia*, allows us to give reasons why argument A is clearly invalid, even though its formal expression suggests that it should be valid. Avicenna applies the notion of the middle term to purely conditional syllogisms such as A*, consciously paralleling the formal analysis of A* with the transitivity of first-figure syllogisms Barbara, Celarent, Darii, and Ferio.¹⁰ Avicenna's first achievement is thus to extend the transitivity of the first-figure categorical syllogisms to purely conditional syllogisms such as A*. Yet, Avicenna's theory of the conjunctive syllogisms with quantified conditionals also gives an ability to diagnose why A is invalid despite the fact that A* seems to be unproblematic. Following Avicenna's theory of conjunctive syllogisms, we might say that A* does not give adequate expression to the formal properties of the following in the antecedents and consequents of the premises and conclusion. As I mentioned above, in Chapter 3, we will discuss how Avicenna introduces "genuine" and "absolute" varieties of implication between antecedents and consequents, whereas in Chapter 5 I will discuss how they are used in syllogisms. For the moment, it is sufficient to note the following. Avicenna would say that A is a first-figure conjunctive conditional syllogism, meaning thereby that the middle part (Q, "John is not in Las Vegas") is the consequent in the first premise, and antecedent in the second premise. Of the first-figure conjunctive conditional syllogisms, only

¹⁰ On transitivity, see G. Patzig, *Aristotle's Theory of the Syllogism: a Logico-Philosophical Study of Book A of the Prior Analytics*, trans. J. Barnes (Dordrecht: D. Reidel, 1968), Chapter 3, especially 50-7.

four are concludent: conditional Barbara,¹¹ conditional Celarent,¹² conditional Darii,¹³ and conditional Ferio.¹⁴ The quantifiers here are the crucial factor, but they are masked by the overly simplistic formalization of A with A*. If the premise set of A were to be formalized as an Avicennian first-figure purely conditional conjunctive syllogism, then we would have: (A**) “always: if A is B, then H is not Z, and (at least) once: if H is not Z, then J is D”. According to Avicenna’s semantics of conditionals, the first premise “if John is in Nashville, then he’s not in Las Vegas” is true as an A-conditional because under no conditions, real or imagined, is it consistent to hold that John is in Nashville and that he is in Las Vegas. On the other hand, the second premise (the major premise) is true as an I-conditional, because it is not true to say that under all real or imagined conditions John’s absence from Las Vegas is inconsistent with his being in Berlin on business. But according to Avicenna’s theory of the conjunctive syllogism, A** is not a concludent mood of the first figure, since its major premise is an I-conditional rather than an A- or an E-conditional.

Yet, saying that A** is “not a concludent mood of the first-figure” is only a satisfactory answer to the question about A’s invalidity if we have already accepted the viability of Avicenna’s figure- and mood-based theory of conjunctive syllogisms. But what is really going on here? Or, to put the question another way, what intuitions about the notion of following (*luzūm*) in conditional reasoning is Avicenna’s theory of quantified conditionals giving formal expression to? Clearly the trouble with A is the failure of transitivity. On Avicenna’s reading of the second premise in A (as an I-conditional), “if John is not in Las Vegas,

¹¹ Conditional Barbara: “always: if A is B, then H is Z, and always: if H is Z, then J is D. Therefore, always: if A is B, then J is D”.

¹² Conditional Celarent: “always: if A is B, then H is Z, and never: if H is Z, then J is D. Therefore, never: if A is B, then J is D”.

¹³ Conditional Darii: “(at least) once: if A is B, then H is Z, and always: if H is Z, then J is D. Therefore, (at least) once: if A is B, then J is D”.

¹⁴ Conditional Ferio: “(at least) once: if A is B, then H is Z, and never: if H is Z, then J is D. Therefore, not always: if A is B, then J is D”.

then he's in Berlin on business", will be true because the consequent is consistent with the supposition of the antecedent as true: John's being in Berlin on business is entirely consistent with John's not being in Las Vegas. Avicenna might say that the second premise is a true *I*-conditional due to the fact that the concomitance (*ma'iyya*) of John's not being in Las Vegas and John's being in Berlin is a purely contingent state of affairs. On the other hand, John's not being in Las Vegas is under all conditions consistent with his being in Nashville. The problem is that in the conclusion John's being in Nashville is under all conditions *inconsistent* with his being in Berlin on business. Consider A again:

Premise 1: "if John is in Nashville, then he is not in Las Vegas";
 Premise 2: "if John is not in Las Vegas, then he is in Berlin on business";
 *Conclusion: "if John is in Nashville, then he is in Berlin on business".

Now, consider A* again:

Premise 1: if *P*, then *Q*;
 Premise 2: if *Q*, then *R*;
 Conclusion: if *P*, then *R*.

With the Avicennian reading of Premise 1 as an *A*-conditional, *Q* will be consistent with *P* no matter what conditions are added to *P*, so long as the conditions are consistent with *P* itself. This means that John's not being in Las Vegas will be consistent with his being in Nashville under all conditions, so long as the conditions are consistent with John's being in Nashville. If that is the case, then let $\sim R$ be one of those conditions that is added to the antecedent *P*. On Avicenna's account, $\sim R$ ("John is not in Berlin on business") can be added to *P*, since it is a condition that is consistent with *P*. Thus, our new argument is the following:

Premise 1*: "if John is in Nashville and he is not in Berlin on business, then John is not in Las Vegas";
 Premise 2: "if John is not in Las Vegas, then he is in Berlin on business";
 Conclusion: "if John is in Nashville and he is not in Berlin on business, then he is in Berlin on business".

This new argument can now be formalized anew as:

Premise 1*: if $P \& \sim R$, then Q ;
Premise 2: if Q , then R ;
Conclusion*: if $P \& \sim R$, then R .

But Conclusion* on this reading of “if, then” sentences cannot be true.

Thus, by using Avicenna’s notion of quantification over possible states of affairs consistent with the supposition of the truth of the antecedent, Avicenna’s theory of quantified conditionals gives us a way of shedding light on the ambiguities of our use of “if, then” sentences. His system of figures and perfect and imperfect moods on the other hand makes sure that such failures of transitivity as in the above example do not occur. Avicenna’s saying that A is invalid because it is “not a concludent first-figure syllogism” is not an uncritical generalization of Aristotle’s theory of quantified categorical propositions to conditional ones. His move to consider antecedents and consequents as states (*aḥwāl*) that are true or false under certain conditions (*ṣurūṭ*) is a remarkable insight, even if it is seriously flawed in other ways.¹⁵ Nevertheless, it must be considered quite an extraordinary achievement as a semantics for conditionals for scientific and natural language discourse. Post-classical logicians seem to have recognized it as such, and made it the basis of their own accounts of conditional reasoning.¹⁶

¹⁵ See the critical approach to Avicenna’s theory of conditionals and conjunctive syllogisms adopted by post-classical Arabic logicians, see K. El-Rouayheb, “Impossible Antecedents and their Consequences: Some Thirteenth-Century Arabic Discussions”, *History and Philosophy of Logic* 30 (2009): 209-11.

¹⁶ *Ibid.*, 209.

CHAPTER 1: THE STUDY OF AVICENNA’S THEORY OF CONDITIONAL¹⁷ PROPOSITIONS AND REPETITIVE AND CONJUNCTIVE SYLLOGISMS

§1.0: CHAPTER OVERVIEW

This chapter has two goals. One is to examine studies of Avicenna’s (d. 1037) hypothetical syllogistic by Nicholas Rescher, Nabil Shehaby, Helmut Gätje and Miklós Maróth and to identify shortcomings in their work. The second is to review scholarly literature on the history of ancient and medieval logic in order to see how we might learn from that literature how best to address shortcomings in the treatment of Avicenna’s hypothetical syllogisms.¹⁸

¹⁷ In this dissertation, I will be mainly concerned with what are today called “conditionals” or “if, then” sentences. This is not, however, how the ancient logicians referred to them. Alfarabi and Avicenna referred to them as “connective conditional propositions” or “*qaḍāyā šarṭiyya muttaṣila*”. The expression “connective” or “*muttaṣila*” is the Arabic translation of the Stoic “*sunēmmenos*”. “Connection” was originally a Stoic word (early Peripatetics such as Theophrastus used a different word), but later Aristotelians such as Alexander of Aphrodisias adopted the word when speaking about hypothetical syllogisms. It likely then that this originally Stoic notion found its place in the strongly Aristotelian Arabic logical lexicon.

¹⁸ In this chapter only, I will refer to Avicenna’s system of conjunctive and repetitive syllogisms with conditional premises and conclusions as a “hypothetical syllogistic” or “hypothetical syllogisms”. The term is even somewhat inaccurate when applied to Alfarabi’s division of syllogisms into “categorical [*ḥamlī, ǧazmī*]” and “conditional syllogism [*qiyās šarṭī*]” because Alfarabi distinguishes between these latter and “hypothetical syllogisms [*qiyās waḍ’ī*]”. By hypothetical syllogism, Alfarabi means any type of argument from a hypothesis. Understood in this way, Alfarabi’s “hypothetical syllogism” could be said to correspond to Aristotle’s “syllogism from a hypothesis”, which he discusses in the *An. Pr.* A23 and B8. On the other hand, “conditional syllogism” refers specifically to modus ponens and modus tollens-like argument schemata, which have no genuine correlate in Aristotle’s logic. I follow Shehaby and Rescher in translating “*qiyās iqtirānī*” as “conjunctive syllogism”, though Asad Ahmed (Avicenna, *Avicenna’s Deliverance: Logic*, trans. A. Ahmed (Karachi: Oxford University Press, 2011), see glossary) in his recent translation of Avicenna’s *Kitāb an-Naǧāt* uses the word “connective syllogism” to render this phrase. Ahmed adopts “repetitive syllogism” to render the Arabic “*qiyās istiṭnā’ī*” whereas Shehaby calls them “exceptive syllogisms”. Kwame Gyekye argued convincingly (K. Gyekye, “The Term *istiṭnā’* in Arabic Logic”, *Journal of the American Oriental Society* 92/1 (1972): 88-92) that the former translation should be adopted instead of the latter on lexical and conceptual grounds. Gyekye notes that in the Arabic Organon the term *istiṭnā’* is used to render the Greek work *prosthesis* (ibid, 88f). The Stoics seems to have used a derivative of this word, viz. *prolepsis*, to refer to the “repeated” minor premise in modus ponens and modus tollens argument forms. Gyekye notes that *prolepsis* was used by Peripatetics such as Theophrastus to refer to a type of syllogism, and not just to refer to the function of the minor premise in modus ponens for example (cf. C. Lejewski, “On Prosleptic Syllogisms”, *Notre Dame Journal of Formal Logic* 2 (1961): 158-176). As Gyekye notes, the reason why Shehaby opted for “exclusive syllogism” as a translation of “*qiyās istiṭnā’ī*” is because Alfarabi says that these syllogisms are so named because of the exceptive particle “*ḥarf istiṭnā’*” that attaches to the minor premise (see Alfarabi, *Al-Mantiq ‘inda l-Fārābī*, ed. R. ‘Aǧam, vol. 1 (Beirut: Dār al-Mašriq, 1985), 31). But for all the evidence that Gyekye has collected, “repetitive syllogism” conveys better the sense of the Stoic idea of *prolepsis* and the way Alfarabi and Avicenna use *istiṭnā’* and its cognates in their logical works.

Thus, in thematic scope, this chapter encompasses the entire dissertation. However, the primary focus will be on secondary literature and, as such, the use of primary sources here will be incidental to the exposition of the general problematic.

Nicholas Rescher's work on Avicenna's hypothetical propositions remains the most influential interpretation to date, and in §1.1 we will see that Rescher's interpretation of Avicenna's theory of hypothetical propositions has many remarkable virtues. However, §1.2 will show that there are strong historical and conceptual grounds for doubting the plausibility of Rescher's importing of aspects of Stoic logical theory into his interpretation of Avicenna's hypothetical propositions. Unlike Rescher, Nabil Shehaby and Miklós Maróth emphasize the Aristotelian background to Avicenna's thinking about hypothetical syllogism. However, in §1.3 I will discuss how and why, despite Shehaby's and Maróth's recognition of the importance of Peripatetic logicians such as Alfarabi on Avicenna, Alfarabi's role as an important precursor to Avicenna's theories about hypothetical propositions and syllogisms remains misunderstood. §1.4 takes up another criticism of Shehaby and Maróth, namely, their contention that Avicenna's hypothetical particles are can be understood as truth-functional operators and, further, that Avicenna's hypothetical syllogistic is interpretable as a propositional calculus. The work of §1.4 is to show that this reading of Avicenna's hypothetical syllogistic is conceptually unfeasible and historically implausible. Finally, in §1.5 I begin the task of looking for an alternative interpretation of Avicenna's hypothetical syllogistic. Taking a cue from the insightful work of Helmut Gätje, this last section reviews studies of Aristotle's formal theory of the. I suggest that Avicenna's hypothetical syllogistic is best understood in light of the increased importance of demonstration in post-Farabian Arabic logic, and, further, as a

conscious effort on Avicenna's part to work out tensions in Aristotle's theory of the syllogism: understood both as a theory of formal reasoning.

§1.1: VIRTUES OF RESCHER'S INTERPRETATION OF AVICENNA'S THEORY OF HYPOTHETICAL ("IF...THEN..." AND "EITHER...OR...") PROPOSITIONS

In many respects, Nicholas Rescher's discussions of implication in the works of Avicenna, Nağmaddīn al-Kātibī (d. 1276) and Muḥammad ibn Fayḍallāh aš-Šīrwānī (fl. 15th c.) remain the sustained treatment of the nature of implication in Avicenna and among later Arabic logicians.¹⁹ As such, it must serve as a starting point for our study of Avicenna's hypothetical syllogisms.²⁰ Consistent with his conviction that Stoic logical doctrines were operative in the writings of Arabic logicians like Avicenna Rescher sees Avicenna's quantification of conditional (*muttaṣil*, lit. "connective") and disjunctive (*munfaṣil*) propositions with the quantifiers 'always' (*kullamā* or *dā'iman*), 'sometimes' (*qad yakūn*), 'never' (*laysa al-batta*) and 'not always' (*laysa kullamā*) or equivalently 'sometimes not' (*qad lā yakūn*) as a variation on the temporally quantified form of implication first formulated by Diodorus

¹⁹ The latter author's name, as it appears on the MS of his *Šarḥ at-Takmil fī l-Mantiq* is: Muḥammad ibn Fayḍallāh ibn Muḥammad Amīn aš-Šīrwānī. Rescher notes that his name is not to be found in Brockelmann's *Geschichte der Arabischen Litteratur* or in its supplementary volumes. What is more, the title of Šīrwānī's book suggests that it is a commentary on a text entitled *At-Takmil fī l-Mantiq*. This title does not appear in *GAL* or its supplements either. Rescher observes (N. Rescher, A. vander Nat, "The Theory of Modal Syllogistic in Medieval Arabic Philosophy", in *Studies in Modality*, ed. N. Rescher (Oxford: Blackwell, 1974), 20) that the the codex (British Museum OR12405), which includes the *Šarḥ at-Takmil* (fol. 72-104), also "includes a commentary on the well-known tract *Al-Ḥāshiyyah* (or *Al-Risālah*) fī l-Mantiq" by the same author on the well-known as-Sayyid aš-Šarīf 'Alī ibn Muḥammad al-Ġurġānī (d. 1413). I am not sure if Rescher's description is correct. The "tract" referred to by Rescher is Ġurġānī's famous gloss on a commentary on Kātibī's *Ar-Risāla aš-Šamsiyya fī Qawā'id al-Manṭiqiyya* by Muḥammad ibn Quṭbaddīn Muḥammad ar-Rāzī at-Taḥṭānī (d. 1364), entitled *Tahrīr al-qawā'id al-manṭiqiyya*. Šīrwānī's "commentary" must, in fact, be classified as a "super-gloss" on the *Šamsiyya*. Unfortunately, no glossator by the name of Šīrwānī is listed by Brockelmann; cf. *GAL* I¹, 466f; *GAL Suppl.* I, 845-8. It has been suggested that the author of this super-gloss might be Kamāladdīn Mas'ūd ibn Ḥusayn ash-Shīrwānī ar-Rūmī (*GAL Suppl.* II, 326) (d. 1499), who authored commentaries on *kalām* and dialectic theory. Unfortunately, I have no way of verifying whether this suggestion is correct.

²⁰ N. Rescher, "Avicenna on the Logic of the 'Conditional' Proposition," in *Studies in the History of Arabic Logic* (Pittsburgh: Pittsburgh University Press, 1963), 76-86. See also idem. *Temporal Modalities in Arabic Logic* (Dordrecht: D. Reidel, 1967) and N. Rescher and A. Vander Nat, "The Theory of Temporal Modalities in Arabic Logic and Philosophy," in *Studies in Modality* (London: Blackwell, 1974), 17-56.

Cronus (ca. 284 B.C.).²¹ The conditions for a true “Diodorean implication” are the following: a proposition q is implied by a proposition p if and only if for all times t , if p is true at t , q is true at t .²² At the same time, Rescher also notices that Avicenna treats the propositional quantifiers ‘always’, ‘sometimes’, ‘never’ and ‘not always’ as analogous to the subject-term quantifiers ‘every’, ‘some’, ‘none’ and ‘not every’ of typical Aristotelian categorical propositions. Rescher applies both of these ideas to Avicenna’s quantified hypothetical propositions, yielding the following truth-conditions for Avicenna’s canonical forms of conditional propositions:

Universal affirmative conditional: ‘always: if a is b , then j is d ’ is true iff for every time t , ‘ a is not b ’ holds at t or ‘ j is d ’ holds at t ; or equivalently for every time t , it is not the case that both ‘ a is b ’ and ‘ j is not d ’ hold together at time t .²³

Universal negative conditional: ‘never: if a is b , then j is d ’ is true iff for every time t , it is not the case that both ‘ a is b ’ and ‘ j is d ’ hold together at t .²⁴

Particular affirmative conditional: ‘sometimes: if a is b , then j is d ’ is true iff there is a time t such that both ‘ a is b ’ and ‘ j is d ’ hold together at t .²⁵

Particular negative conditional: ‘not always: if a is b , then j is d ’ is true iff there is a time t such that ‘ a is b ’ holds at t and ‘ j is not d ’ holds at t .²⁶

Truth-conditions for Avicenna’s disjunctive propositions are as follows:

Universal affirmative disjunctive: ‘always: either a is b or j is d ’ is true iff for every time t , ‘ a is b ’ holds at t or ‘ j is d ’ holds at t , but not both simultaneously.²⁷

Universal negative disjunctive: ‘never: either a is b or j is d ’ is true iff for every time t , both ‘ a is b ’ and ‘ j is d ’ hold at t or both ‘ a is not b ’ and ‘ j is not d ’ hold at t .²⁸

²¹ Rescher, “Avicenna on Conditionals”, 79. Cf. B. Mates, *Stoic Logic* (Berkeley: University of California Press, 1952); idem., “Diodorean Implication,” *The Philosophical Review* 58/3 (1949): 239-242; A. Prior, “Diodoran Modalities”, *Philosophical Quarterly* 4/1 (1955): 205-213.

²² Rescher, “Avicenna on Conditionals”, 79.

²³ In symbols: $(t) (A/t \rightarrow C/t)$ or equivalently $(t) \sim(A/t \ \& \ \sim C/t)$; Rescher, “Avicenna on Conditionals”, 81. Here, ‘ $\rightarrow$ ’ and ‘ $\&$ ’ symbolize material implication and conjunction respectively. The variables A/t and C/t are propositional variables that are themselves functions of time t whose truth-values (‘true’ and ‘false’) vary according to time t .

²⁴ In symbols: $(t) \sim(A/t \ \& \ C/t)$; *ibid.*

²⁵ In symbols: $\exists(t) (A/t \ \& \ C/t)$; *ibid.*

²⁶ In symbols: $\exists(t) (A/t \ \& \ \sim C/t)$; *ibid.*

²⁷ In symbols: $(t) (A/t \vee C/t)$, where ‘ $\vee$ ’ is exclusive disjunction. This formula is equivalent to $(t) [(A/t \vee C/t) \ \& \ \sim(A/t \ \& \ C/t)]$, where ‘ $\vee$ ’ is inclusive disjunction. Rescher, “Avicenna on Conditionals,” 82.

Particular affirmative disjunctive: ‘sometimes: either a is b or j is d’ is true iff there is a time t such that ‘a is b’ or ‘j is d’ holds at t but not both simultaneously.²⁹

Particular Negative Disjunctive: ‘not always: either a is b or j is d’ is true iff there is a time t such that both ‘a is b’ and ‘j is d’ hold at t or both ‘a is not b’ and ‘j is not d’ hold at t .³⁰

In fact, this semantics for Avicenna’s quantified hypothetical propositions, both conditionals and disjunctives, has several virtues. First, Rescher is able to provide a convincing account of Avicenna’s analogy between categorical and hypothetical propositions by giving us a square of opposition for hypothetical propositions that is isomorphic with the categorical propositions. In other words, just as the universal affirmative and the particular negative categorical propositions divide truth and falsehood between them, as do the universal negative and particular affirmative, Avicenna holds that the same is true for hypotheticals. It is clear that Rescher sought to make his semantics reflect this fact by holding that universal affirmative conditionals and disjunctives are true if and only if the negation of their particular negative counterparts are true; and similarly, holding that universal negative conditionals and disjunctives are true if and only if the negation of their particular affirmative conditionals and disjunctives are true.

At the level of propositions, Rescher’s semantics is able to accommodate Avicenna’s vision of the analogy between hypothetical and categorical propositions, which Avicenna states explicitly in ŠQ VII.

[Text 1] If you know the affirmative, negative, universal and particular [hypothetical proposition, viz. *qaḍiyya šarṭiyya*], then you already know contradiction, contrariety, subcontrariety and subalternation [for hypothetical propositions]. There is, then, no need to return to teaching you about them

²⁸ In symbols: $(t) \sim(A/t \vee C/t)$; *ibid.* This formula is equivalent to $(t) [(\sim A/t \ \& \ \sim C/t) \vee (A/t \ \& \ C/t)]$.

²⁹ In symbols: $\exists(t) (A/t \vee C/t)$; *ibid.*

³⁰ In symbols: $\exists(t) \sim(A/t \vee C/t)$; *ibid.*

here for they are formed in the same way as they are in categorical propositions.³¹

In his article, Rescher considers only contradiction and conversion at length but, as he notes, his model for Avicenna's hypothetical propositions, with one important assumption, reflects Avicenna's doctrines for contrariety, subcontrariety and subalternation also.³² Avicenna claims that conversions for conditional and disjunctive propositions are exactly analogous to their categorical counterparts, namely, that the conversion of antecedent and consequent in particular affirmative and universal negative propositions is a valid deduction and the conversions of the universal affirmative and particular negative are invalid. Particular affirmative conditionals (and thus universal negative conditionals) in Rescher's semantics convert because of the commutative property of conjunction;³³ universal affirmative conditionals (and thus particular negative conditions) do not convert because the conversion of the antecedent and consequent is not a truth-preserving transformation for material implication. According to Rescher's model, universal affirmative and particular negative disjunctive propositions should also convert unlike their categorical counterparts because of the commutativity of exclusive disjunction. Yet, as Rescher notes, there are good reasons to believe that perhaps Avicenna's statement in this regard was "incautious" and that he would

³¹ Avicenna, ŠQ VII, 362.4-8; cf. Avicenna, *The Propositional Logic of Avicenna*, trans. N. Shahaby (Dordrecht: D. Reidel, 1973), 163.

³² "With regard to other kinds of immediate inference, it is clear that subalternation (*A* to *I*, *E* to *O*), contrariety (of *A* and *E*) and subcontrariety (of *I* and *O*) also hold with respect to 'conjunctive conditional' and to 'disjunctive conditional' propositions;" Rescher, "Avicenna," 83f. In order to bring all of these immediate inferences into harmony, we must assume that, in essence, no sentence may serve as an antecedent in a connective proposition that is eternally unrealized. This condition insures the contrariety of the universal affirmative and negative conditionals and thereby the subcontrariety of the particular affirmative and negative conditionals. This condition also assures the validity of subalternation deductions. The immediate inferences for disjunctive propositions are valid without any additional assumptions.

³³ Commutation: ($P \& Q$) iff ($Q \& P$) or, in other words, the order of the conjuncts is irrelevant to the truth of the conjunction. The same holds for disjunction: ($P \vee Q$) iff ($Q \vee P$).

accept the conversion of these disjunctives as well. In fact, he says as much in his discussion of the properties of disjunctive propositions in the ŠQ V.³⁴

Third, though he did not have access to the text in the *Šifā'* where Avicenna makes this claim, Rescher is able to account for the mutual implications (*talāzum*) that Avicenna claims holds between affirmative and negative universal conditional propositions on the one hand and between affirmative and negative particular propositions on the other. About the universal conditional propositions, Avicenna says:

[Text 2] The universal connective propositions that [1] agree in quantity [2] differ in quality and [3] whose consequents contradict each other [*al-mutanāqiḍa at-tawālī*] are mutually implicative [*mutalāzima*].³⁵

Avicenna says later at ŠQ VII, 371 that the same rule holds for particular connective propositions: particular connective propositions agreeing in quantity, differing in quality, and with contradictory consequents mutually imply each other. The above rules stipulate the following four equivalences: (1) 'always: if A is B, then J is D' is equivalent to 'never: if A is B, then J is not D' and (2) 'never: if A is B, then J is D' is equivalent to 'always: if A is B, then J is not D' and (3) 'sometimes: if A is B, then J is D' is equivalent to 'sometimes not: if A is B, then J is not D' and (4) 'sometimes not: if A is B, then J is D' is equivalent to 'sometimes: if A is B, then J is not D'. Consulting Rescher's symbolic renditions of Avicenna's conditional propositions, with the help of double negation and De Morgan's laws we see that Rescher's interpretation does, in fact, reflect all the above equivalences.³⁶

Fourth, Rescher's semantics reflects implications that Avicenna stipulates for connective and disjunctive propositions. Avicenna says:

³⁴ ŠQ V, 246; cf. Avicenna, *Propositional Logic*, 47.

³⁵ ŠQ VII, 368; translation is from Avicenna, *Propositional Logic*, 167.

³⁶ Rescher, "Avicenna on Conditionals", 80f.

[Text 3] Let us speak then [...] about mutual implication between disjunctive and connective propositions. We say: genuine affirmative disjunctive propositions [*al-munfaṣilāt al-ḥaqīqiyya al-mūǧiba*] with affirmative disjuncts imply connective propositions in which the contradiction of one of the disjuncts serves as antecedent and a propositions identical to the second disjunct serves as consequent.³⁷ Either disjunct may serve as antecedent as long as [both disjuncts] agree in quantity and quality. For example: if we say: ‘always: either every A is B or every J is D’ then this implies ‘whenever [*kullamā*] not every A is B, then every J is D’ and ‘whenever not every J is D, then every A is B’.³⁸

Avicenna illustrates the rule in the above passage using a universal genuine disjunctive propositions with affirmative disjuncts, though Avicenna says at ŠQ VII, 377 that an analogous rule applies to particular genuine disjunctive propositions with affirmative disjuncts. On the other hand, at ŠQ VII, 377, Avicenna makes the interesting claim that this implication is not reversible.³⁹ That is to say, the connective propositions mentioned in the passage above do *not* imply one and the same disjunctive proposition. In fact, Rescher’s semantics reflects each aspect of the above rule, including the non-convertibility of the implication relation. It is worth dwelling on this point further since it demonstrates the richness and elegance of Rescher’s semantics. Avicenna says that the disjunctive proposition ‘always: either every A is B or every J is D’ implies the connective propositions ‘always: if not every A is B, then every J is D’ and ‘always: if not every J is D, then every A is B’. Rescher’s symbolic rendition of these formulae has Avicenna claim that (1) (t) (A/t $\vee$ C/t) implies both (2) (t) ($\sim$ A/t $\rightarrow$ C/t) and (3) (t) ($\sim$ C/t $\rightarrow$ A/t) where ‘ $\vee$ ’ symbolizes exclusive disjunction.⁴⁰ Applying De Morgan’s laws to (2) and (3) yields (2’) (t) (A/t $\vee$ C/t) and (3’) (t) (C/t $\vee$ A/t), where ‘ $\vee$ ’ symbolizes inclusive

³⁷ I read “‘aynu tālīhi tāliyan” for “‘aynu tālīhi tāliyan”, though the editor of the Arabic text records no manuscript variations attesting to this reading. Shehaby translates this sentence without comment. His translation, which hardly reproduces the Arabic syntax, cannot be what Avicenna intended as it makes no sense in light of the syntactical rules Avicenna is sets down nor in respect of the examples that follow.

³⁸ Avicenna, ŠQ VII, 376; translation is from Avicenna, *Propositional Logic*, 173.

³⁹ Avicenna, *Propositional Logic*, 174.

⁴⁰ Rescher, “Avicenna on Conditionals”, 82.

disjunction. Transforming exclusive disjunction into a logically equivalent proposition expressed with inclusive disjunction (1') (t) $[(A/t \vee C/t) \& \sim(A/t \& C/t)]$ shows Avicenna's reasoning to be perfectly valid. (1') (t) $[(A/t \vee C/t) \& \sim(A/t \& C/t)]$ obviously implies (2') (t) $(A/t \vee C/t)$ due to simplification and (1') also implies (3') due to simplification and commutativity of disjunction. Rescher's semantics also reveals the validity of Avicenna's claim that neither (2') nor (3') implies (1') since $p \rightarrow p \& q$ is obviously not a theorem of the propositional calculus.

Fifth, Rescher's proposed semantics reflects implications that Avicenna says exist between disjunctive propositions. In Book VII of the *Šifā'*, Avicenna says:

[Text 4] As for mutual implication (*talāzum*) between disjunctive propositions (*munfaṣilāt*), we say: affirmative disjunctive propositions with affirmative disjuncts imply disjunctive propositions that agree with them in quantity, differ in quality and whose antecedents are contradictory [i.e. the antecedent of the second is the contradictory opposite of the original]. An example for universally quantified propositions (*kulliyyāt*) is: 'always: either every A is B or every J is D' implies 'never: either not every A is B or every J is D' or similarly 'never: either not every J is D or every A is B'.⁴¹

And at ŠQ VII, 381 Avicenna says the same rule holds, *mutatis mutandis*, for particular propositions: 'sometimes: either every A is B or every J is D' implies 'sometimes not: either not every A is B or every J is D' or 'sometimes not: either not every J is D or every A is B'. Rescher's symbolic rendition of the claim about the implication between universals has Avicenna claiming that the proposition (1) (t) $(A/t \vee C/t)$ implies (2) (t) $\sim(\sim A/t \vee C/t)$ where, as above, 'V' symbolizes exclusive disjunction. Rewriting (1) and (2) in terms of inclusive disjunction yields (1') (t) $[(A/t \vee C/t) \& \sim(A/t \& C/t)]$ and (2') (t) $\sim[(\sim A/t \vee C/t) \& \sim(\sim A/t \& C/t)]$. In fact, with the help of distribution laws, it is quite easy to derive (2') from (1'). Rescher's interpretation once again can be taken to vindicate Avicenna's doctrines of hypothetical propositions.

⁴¹ Avicenna, ŠQ VII, 379.17-80.4; cf. Avicenna, *Propositional Logic*, 176.

Sixth, and perhaps most importantly, this semantics offers a historically and conceptually convincing account of what Rescher sees to be Avicenna's use of notions of implication and propositionality from Stoic logic in combination with Aristotelian notions of subject-term quantification. Rather than present Avicenna as just another late antique logician who tried to force the square peg of a logic of terms into the round hole of a logic of sentences, Rescher's semantics for Avicenna's hypothetical propositions shows Avicenna's doctrines to be both conceptually defensible and logically sound.

§1.2.0: INADEQUACIES OF RESCHER'S INTERPRETATION OF AVICENNA'S THEORY OF HYPOTHETICAL ("IF...THEN..." AND "EITHER...OR...") SYLLOGISMS

The above discussion should suffice as an indication of the merit of Rescher's interpretation of Avicenna's doctrines of hypothetical propositions. At the very least, I hope it is clear that Rescher's interpretation of Avicenna's hypothetical syllogisms as a truth-functional theory of temporally quantified conditional and disjunctive propositions should not have been ignored in its details as thoroughly as Shehaby and Maróth do. Yet, despite these virtues, Rescher's account may be shown to be untenable on historical and conceptual grounds.

§1.2.1: HISTORIOLOGICAL CRITICISMS OF RESCHER'S INTERPRETATION

Encouraged by formal analyses of Stoic logic by Arthur Prior, Benson Mates and Jan Łukasiewicz, Rescher was able to draw on a large body of historical evidence in secondary literature that pointed to the historical importance of the Stoics in the history of formal logic.⁴² It is a credit to Nicholas Rescher's perspicacity that he saw the history of Arabic logic as

⁴² J. Łukasiewicz, "Zur Geschichte der Aussagenlogik," *Erkenntnis* 5 (1935): 111-131.

unfolding in the context of late Antiquity and thus that the history of Arabic logic shares essential features with the history of formal logic that focuses on Greek and Latin authors. The trouble with Rescher's historical interpretation is that there is no solid evidence of Stoic influence on Arabic logic or in any other field of Arabic philosophy. As a result, facts that Rescher takes for granted such as his belief that Avicenna held a Stoic view of propositionality, are not supported by concrete historical evidence.⁴³ On the other hand, Dimitri Gutas and Robert Wisnovsky have demonstrated the centrality of Aristotelian thought in many important realms of Avicenna's philosophy.⁴⁴ In fact, in several articles and books Rescher himself has detailed the thoroughgoing Aristotelianism of the Arabic logical curriculum from its inception says:

Arabic logic, like the rest of medieval Arabic science and philosophy, is entirely western and has nothing to do with "Oriental philosophy." It developed wholly in the wake of the classical Greek tradition as preserved in, and transmitted through Hellenistic Aristotelianism.⁴⁵

Additionally, Karl Dürr, the author of *The Propositional Logic of Boethius*, has observed that there is no evidence of Stoic influence in Boethius' works on hypothetical syllogisms and for all of Christopher Martin's criticism of Dürr's study, this is a rare instance in which Martin agrees with Dürr's assessment.⁴⁶ In his book on Avicenna's hypothetical syllogistic, Miklós Maróth believes that Dürr's verdict for Boethius is equally valid for Avicenna:

⁴³ There is indirect evidence of Stoic influence on early ethical philosophy in al-Kindī for example; see T. Druart, Al-Kindī's Ethics, *The Review of Metaphysics* 47/2 (1993): 329-57. However, textual evidence in the form of translation of texts is non-existent. Fehmi Jadaane has collected indirect evidence of Stoic influence on the development of Arabic philosophy, mainly in logic, physics, and ethics; F. Jadaane, *L'influence du stoicism sur la pensée musulmane* (Beirut: Dar el-Meshreq Éditeurs, 1968), though see D. Gutas "Pre-Plotinian philosophy in Arabic (other than Platonism and Aristotelianism): a review of the sources", *Aufstieg und Niedergang der römischen Welt* II.36.7 (Berlin, Walter de Gruyter, 1994): 4939-4973 for a critical review of Jadaane's work.

⁴⁴ D. Gutas, *Avicenna and the Aristotelian Tradition* (Leiden: Brill, 1988); R. Wisnovsky, *Avicenna's Metaphysics in Context* (Ithica: Cornell University Press, 2002).

⁴⁵ N. Rescher, "Arabic Logic: A Brief Account," in *Studies in the History of Arabic Logic*, 13.

⁴⁶ K. Dürr, *The Propositional Logic of Boethius* (Amsterdam: North-Holland, 1951), 11; C. Martin, "The Logic of Negation in Boethius," *Phronesis* 36/3 (1991): 280. In reference to her work on Boethius' commentary on Cicero's

In [Dürr's] forward, the author tries to identify Boethius' sources. He compares Boethius' system to all that we know about the ancient Peripatetics, the Stoics and the views of the late commentators. This comparison led him to conclude that Boethius wrote using the works of the ancient Peripatetics such as Theophrastus and Eudemus. These were his sources, whereas the Stoics had no part in his writings.

Avicenna's terminology agrees with the Peripatetic terminology [...] and obviously would agree with Boethius' who used the same sources.⁴⁷

My intention is not to completely rule out Stoic influence on Arabic logic. Rather, I agree with the implicit judgment of Tony Street and Paul Thom, both of whom have written extensively on Avicenna's logic and the history of Arabic logic.⁴⁸ These authors have not explicitly rejected Rescher's thesis that Avicenna and post-Avicennian logicians made use of Stoic doctrines in constructing their syllogistic theories. Instead, based on a large amount of historical evidence pointing to the centrality of Aristotle's philosophy in the Arabic philosophical tradition, they have come to the conclusion, even if they have not explicitly stated, that Avicenna's syllogistic, even his hypothetical syllogistic, is best understood in the context of Aristotelian rather than Stoic logical theory.

§1.2.2: CONCEPTUAL CRITICISMS OF RESCHER'S INTERPRETATION

From a conceptual standpoint, Rescher's suggested semantics cannot work when we take into account the totality of the material on hypothetical propositions in ŠQ V-VII and aspects of Avicenna's theory of assertoric or, more properly, 'absolute (*muṭlaq*)' categorical propositions discussed in secondary literature. From the conceptual perspective, my

Topics (E. Stump, Boethius's In Ciceronis Topica and Stoic Logic", in *Studies in Medieval Philosophy*, ed. J. Wippel (Washington DC: Catholic University Press of America, 1987), 1-22) Martin chides Eleanor Stump for using Boethius as a purported source of Stoic logic. Martin does not elaborate on his criticisms of Stump's thesis, but Maróth's work suggests that Martin is correct.

⁴⁷ M. Maróth, *Ibn Sīnā und die peripatetische "Aussagenlogik"* (Leiden: Brill, 1989), 35.

⁴⁸ T. Street, "An Outline of Avicenna's Syllogistic," *Archiv für Geschichte der Philosophie* 84 (2002): 129-160; P. Thom, *Medieval Modal Systems: Problems and Concepts* (Aldershot, Hants/Burlington, Vermont: Ashgate, 2003), 65-80.

substantial criticisms of Rescher's semantics are of two types. The first relies on recent work by Tony Street about temporality in Avicenna's categorical propositions.⁴⁹ Street's findings show that it is unlikely that Avicenna held a view of propositionality in which propositions can have different truth-values at different times. The second is not only a criticism of Rescher but those of others, whose interpretations of Avicenna's hypothetical syllogisms take an extensional approach rather than seeing Avicenna's hypothetical syllogistic as being based on the primary and complementary concepts of connection (*ittiṣāl*) and incompatibility (*'inād*) between antecedent and consequent.⁵⁰

Simple hypothetical propositions in Avicenna's hypothetical syllogistic are composites of two categorical propositions—be they quantified or indefinite, affirmative or negative and modified or unmodified. The pair of categorical propositions are related to each other by a conditional particle, such as 'if...then...' or 'either...or...', which signifies (*yataḍamman*) the mode of the connection that exists between the two categorical propositions, e.g. complete connection (*ittiṣāl tāmm*), incomplete connection (*ittiṣāl ġayr tāmm*), complete incompatibility (*'inād tāmm*) or incomplete incompatibility (*'inād nāqīṣ*). As was mentioned above, the mode of signification, viz. the nature of the connection between the categorical propositions, is itself subject to universal and particular quantification and affirmative and negative qualification as was discussed above. Rescher's semantics tries to mirror Avicenna's quantification and qualification of the connection between the two categorical propositions by making the categorical propositions functions of time *t* whose truth-values will differ at different

⁴⁹ Street, "Outline of Avicenna's Syllogistic", 132f.

⁵⁰ An 'extensional approach' to truth-conditions for conditional sentences means that the truth-value of the compound proposition is deducible exclusively from the truth of the atomic propositions out of which it is composed. For now, this schematization is sufficient for our purposes. However, we will see in §3 that Avicenna completely reconceptualises the concept of "connection" from being undergirded by *ittibā'* to being undergirded by "concomitance (*ma'iyya*, *muwāfaqa*, *murāfaqa*)".

moments in time. The canonical examples, which Rescher takes from Avicenna's *Iṣārāt* and *Daneshnāme*, are indeed sentences whose truth will vary according to the time at which the sentence is uttered, e.g. 'the sun is up', 'it is day' and 'it is night'.⁵¹ Sentences of this type straightforwardly lend themselves to variability in truth-values over a period of time.

But these are not the most common type of categorical proposition in Avicenna's syllogistic system in general nor are they even the most common type of proposition to appear in Avicenna's treatment of hypothetical syllogisms in the *Šifā'*.⁵² Rather, the most common form of categorical absolute propositions (*muṭlaq*) is the well-known quantifier-subject-predicate form, e.g. 'every J is D' or 'no J is D'. Common examples of this type of proposition, such as 'every man is risible' or 'no man is a stone', evidently do not vary in their truth-values in the same way as 'it is day'. Of course temporality does play some role in Avicenna's theory of categorical propositions. Tony Street and Paul Thom have both observed that Avicenna reads temporal modalities in both the subject and the predicate of his assertoric categorical propositions. This is to say, however, that quantifying categorical propositions over time as Rescher does will directly impact whether truth-conditions for Avicenna's categorical assertoric propositions.

Let us take, for example, the universal affirmative assertoric 'every j is d', which Avicenna's reads as "every object picked out by j in the mind or outside the mind, whether always or sometimes, is a d without further adding that it is described as d at such and such a

⁵¹ Rescher, "Avicenna on Conditionals", 81f.

⁵² For example, see Š.Q VII where Avicenna sets out the canonical forms of conditional and disjunctive propositions. Formal variations in the conditional and disjunction are prescribed according to the quantity and quality of the categorical propositions that serve as antecedent or consequent in addition to the variations in the quality and quantity of the conditional and disjunction themselves.

time, or in such and such circumstances, or always”.⁵³ The implication of Avicenna’s reading of the universal affirmative assertoric is that as long as an object in mental or extra-mental existence, in past, present or future, is picked out at least once by the term ‘j’ (e.g. ‘man’) and the term ‘d’ (e.g. ‘risible’), then the proposition is true. Further, there is never any question of the proposition’s truth changing its value depending on the moment in which the utterance occurs; the sentence would simply be always and at every time true. Keeping the above distinctions in mind, consider again Rescher’s truth-conditions for the particular affirmative conditional ‘sometimes: if every a is b, then every j is d’. Rescher takes Avicenna to be saying ‘there is a time t such that both ‘every a is b’ and ‘every j is d’ are true at t’. However, if Avicenna really does read categorical propositions as Street and Thom claim, then if there is a t such that ‘every a is b’ is true at t and ‘every j is d’ is true at t, these categorical propositions will be true not only at time t but presumably at all times. But if “every a is b” and “every j is d” are true at all times, then the universal affirmative “always: if every a is b, then every j is d” is true, viz. the truth of the particular affirmative conditional on Rescher’s reading entails the truth of the universal affirmative conditional! In a similar way, the particular negative conditional ‘sometimes not: if every A is B, then every J is D’ is taken by Rescher to mean ‘there is a time t such that ‘every A is B’ is true at t and ‘not every J is D’ is true at t’. Again, if Thom and Street are correct, then ‘every A is B’ and ‘not every J is D’ will be true not only at t but at all times, which means that the universal negative “never: every A is B, and every J is D” is true. Rescher’s interpretation of conditions combined with Avicenna’s doctrine of subject

⁵³ In the *Iṣārāt wa-Tanbīhāt*, Avicenna says the following about how the subject-term of a categorical proposition should be taken: “1. [...] we mean that every single thing described as J, be it in mental supposition or extramental existence, be described as J always, or sometimes or whatever; 2. that thing is described as B without further adding that it is so described at such and such a time, or in such and such circumstances, or always”; T. Street, “An Outline of Avicenna’s Syllogistic,” *Archiv für Geschichte der Philosophie* 84/2 (2002): 134; cf. Thom, *Medieval Modal Systems*, 65f.

term ampliation entails that the truth of the particular negative conditional implies the truth of the universal negative conditional! Technically speaking, this eliminates any (semantic) distinction between universal and particular propositions, since the particular propositions under the new reading come to imply their universal counterparts. The square of opposition is undone, the direction of subalternation is reversed and contrariety is no more. This reading of Avicenna's conditionals reduces Avicenna's logic of conditional propositions to simply a way of talking about either (1) eternally coincidental sentences, in the sense that true affirmative (universal or particular, it doesn't matter) conditional propositions say no more than that the antecedent is coincidental with the consequent at all times; or (2) eternally non-coincidental propositions, in the sense that true negative conditional propositions say no more than that the antecedent and consequent are never affirmed together. But a logic of sentential coincidence is clearly not what Avicenna had in mind when he was thinking about hypothetical syllogisms.

In order to present my second objection to Rescher's semantics, it seems best to begin with an example that will illustrate my point. In the fifth point above—about universal affirmative disjunctive propositions with affirmative disjuncts implying universal negative disjunctive propositions with identical consequents and contradictory antecedents—Avicenna denies that the converse implication holds:

[Text 5] But it is not necessary that the implication convert such that if the expression 'never: either not every A is B or every J is D' [were true], it would also be true that 'always: either every A is B or every J is D'.⁵⁴

According to Rescher's interpretation however, the implication is convertible because of the commutative property of disjunction. At first sight, this is clearly not a shortcoming that, on

⁵⁴ Avicenna, ŠQ VII, 380.9-15; cf. Avicenna, *Propositional Logic*, 176.

its own, should be a reason for serious reservations. Consider, however, Avicenna's whole argument:

[Text 6] But it is not necessary that the implication convert such that if the expression 'never: either not every A is B or every J is D' were true, it would also be true that 'always: either every A is B or every J is D'. [This is so] because a [universal negative] proposition with an impossible disjunct that is not incompatible with the other disjunct can be true [*li-annahu qad yaşduqu mā fīhi 'l-muḥālu 'l-ğayru 'l-mu'ānid*]. For example, the expression [S1] 'never: either not every human is an animal or void exists (or does not exist)' does not imply that [S2] ['always:] either every human is an animal or void exists (or does not exist)'.⁵⁵

A universal negative disjunctive proposition like S1 above is true because, stated simply, there is no possibility of incompatibility of any kind existing between either of the disjuncts. In this instance, the truth and falsehood of the disjuncts is entirely irrelevant to the truth or falsehood of the whole proposition. What is relevant is only the existence or non-existence of incompatibility (*inād*) between them. S1 would be true because

[Text 7] neither disjunct is incompatible with the other nor is either the contradictory of the other. Even if the contradictory of one of the disjuncts, viz. [the contradictory of] the impossible disjunct, were always true with the other disjunct, [propositions like S1 would still be true] as long as the truth [of the contradictory of the impossible disjunct] does not imply the other disjunct in such a way that if [the other disjunct] were false, would require removal [*raf*] of the [negative particle] from the [impossible] disjunct.⁵⁶

Thus, a sentence like S1 is true because (1) the disjuncts from which it is composed are not incompatible with each other, and (2) neither disjunct implies the contradictory of the other. As Avicenna notes, even the contradictory of the patently false 'not every human is an animal', which is the obviously true 'every human is an animal', might be true in every instance but its truth or falsehood is never material to the truth or falsehood of 'the void exists' or 'the void

⁵⁵ Ibid.

⁵⁶ Avicenna, ŠQ VII, 284.5-10; cf. Avicenna, *Propositional Logic*, 80. In this last part, Avicenna rules out the truth of universal negative disjunctive propositions like "never: void exists or void doesn't exist".

does not exist'. Thus, S1 is true because there is never a situation in which 'not every human is an animal' is incompatible with 'void exists' or 'void does not exist'. On the other hand S2 is clearly false for precisely the same reasons that S1 is true. A true universal affirmative disjunctive proposition signifies that a type of incompatibility exists, whereas in S2 there is clearly no relation of incompatibility between 'every human is an animal' and 'void exists' or 'void doesn't exist'. Once again, the truth or falsehood of the categorical disjuncts is of no consequence. Only the fact of some type of incompatibility is of consequence. Thus, since S1 can be true while S2 is false, there can be no relation of mutual necessary implication (*talāzum*) between them.

There are far-reaching ramifications in Avicenna's use of notions of incompatibility and connection (*ittiṣāl*) for laying down truth-conditions for disjunctive and conditional propositions as far as our valuation of Rescher's interpretation is concerned. Rescher reads Avicenna's hypothetical sentences extensionally. By this I mean that Rescher claims that the truth or falsity of Avicenna's conditional and disjunctive propositions can be deduced exclusively from the truth and falsity of the antecedent, consequent or pair of disjuncts. In Rescher's defense, Shehaby and Maróth have gathered textual evidence that appears to suggest that this is how Avicenna thought about the truth-conditions for hypothetical propositions.⁵⁷ In addition, the many virtues of Rescher's interpretation discussed above stand as convincing evidence that this is a sound interpretation of Avicenna's hypothetical propositions.

Yet the discussion about the non-convertibility of implication relations shows decisively that this cannot, in fact, be the whole story, for such theories leave us ill-equipped

⁵⁷ See Avicenna, *Propositional Logic*, 222-234; Maróth, *Aussagenlogik*, 52f, 63-68.

to understand other logical decisions that Avicenna makes when constructing his system of hypothetical syllogisms. Rescher's study—and this criticism applies with equal force to Shehaby and Maróth as we will see—has overlooked the intensional character of Avicenna's hypothetical propositions in the sense that, as in the example mentioned above, the truth or falsity of the antecedent, consequent or the pair of disjuncts is not enough or even important for determining whether the conditional proposition is true or not. Rather, a universal or particular affirmative conditional proposition according to Avicenna is true if and only if there is some sort of relation of connection between the antecedent and the consequent, the determination of which can be entirely independent of the question of whether the antecedent and the consequent are true. Similarly, a universal or particular affirmative disjunctive proposition is true if and only if there is a relation of complete or partial incompatibility (*'inād tāmm* or *'inād nāqīṣ*) between the two disjuncts, the determination of which may be independent of the question of whether the disjuncts themselves are, in fact, true. In order to achieve a fuller understanding of Avicenna's hypothetical syllogisms, the manner in which the notions of connection and incompatibility operate in Avicenna's theory of hypothetical propositions will be a major focus in the coming chapters.

§1.3: AFTER RESCHER 1: SHEHABY, GÄTJE AND MARÓTH ON ALFARABI AND AVICENNA'S THEORY OF HYPOTHETICAL (“IF...THEN...” AND “EITHER...OR...”) SYLLOGISMS

Let us pause to take stock of the argument so far. Despite the many merits of Rescher's interpretation of Avicenna's theory of conditional and disjunctive propositions, the aim of the last section was to show that there are good conceptual and historical reasons for believing that it does not reflect Avicenna's thinking about propositionality or Avicenna's truth-conditions for hypothetical propositions and, thus, that it would serve as a poor basis on which

to build an interpretation of Avicenna's hypothetical syllogistic. Rescher believed Avicenna had appropriated these doctrines, viz., propositionality and the extensional nature of the truth-conditions for hypothetical propositions, from the Stoics, ostensibly by means of the syncretic, late-antique Aristotelian commentaries.⁵⁸ Yet the first aspect of my criticism of Rescher's work on this topic is that there is no historical evidence of direct, textual transmission of Stoic philosophy into Arabic. The second aspect of my criticism of Rescher's interpretation is that given what we now know about how Avicenna reads categorical propositions and what we know about his hypothetical syllogisms as presented in the *Šifā'*, Rescher's model does not reflect Avicenna's thinking about hypothetical propositions. The historical and conceptual aspects of Rescher's work serve to justify each other. After all, it would be historically plausible to impute Stoic views of propositionality and extensional truth-conditions to Avicenna *if* it could be shown that to have Avicenna hold such views is conceptually defensible. Conversely, such an interpretation of Avicenna's doctrine of hypothetical propositions would only be defensible *if* the imputation to Avicenna of Stoic logical doctrines was historically grounded. The conclusion of the last section is that neither position is viable.

Subsequent studies by Nabil Shehaby, Helmut Gätje and Miklós Maróth benefitted enormously from the publication of ŠQ V-IX.⁵⁹ All three may be viewed as continuing and extending the Nicholas Rescher's work in 1963 in the sense that they correct four major shortcomings in Rescher's article: (1) they move beyond Avicenna's theory of hypothetical propositions to treat his whole theory of hypothetical syllogisms; (2) they collect a large

⁵⁸ A. Specia, *Hypothetical Syllogistic and Stoic Logic* (Leiden and Boston: Brill, 2001).

⁵⁹ Shehaby's translation of ŠQ V-IX in Avicenna, *op. cit.*; Maróth, *op. cit.*; Helmut Gätje, "Zur Lehre von den Voraussetzungsschlüssen bei Avicenna", *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 2 (1985): 140-204.

amount of Greek, Latin and Arabic textual sources that were absent from Rescher's work; (3) they use this material to trace possible historical precedents for Avicenna's thought to Stoic and Aristotelian sources; (4) and they use material collected from the *Šifā'* to emphasize Avicenna's indebtedness to the Aristotelian tradition more than Rescher had been able to do using the material available to him in 1963.

Nevertheless, I believe that these studies perpetuate and often amplify some of the shortcomings in Rescher's work. Much of Rescher's work on Arabic logic is concerned to enlist the "unexploited potential" of Arabic logical works as a "source of insight into the later stages of the history of Greek logic."⁶⁰ Though a praiseworthy aim in itself, in the case of hypothetical syllogisms an absence of straightforwardly Aristotelian sources for Avicenna's doctrines in ŠQ V-IX in the Greek literature leads Rescher to look to remote Stoic sources for precedents for Avicenna's logical doctrines, while potential Arabic Aristotelian sources much closer at hand remain unexplored. Maróth shares Rescher's interest in Avicenna as a later representative of late antique philosophy but, unlike Rescher, he tries to use Avicenna's *Šifā'* as a source for reconstructing the lost "propositional logic" of Theophrastus and Eudemus:

In our search for the Peripatetic propositional logic, we are confronted by a basic difficulty: the incompleteness of our sources. This is the chief reason that, even today, scholars consider the theory of hypothetical syllogisms as something of an irrelevant digression. [...] However, after examining these volumes [of Avicenna's *Šifā'*], it is clear that Ibn Sīnā worked with sources that contained a great deal of material about the Peripatetic propositional logic that is lost to us.⁶¹

A significant consequence of this historiographical attitude is that in no work on hypothetical syllogisms is the possibility that Alfarabi (d. 950) served as an important source of Avicenna's

⁶⁰ N. Rescher, *Galen and the Syllogism* (Pittsburgh: University of Pittsburgh Press, 1966), 21.

⁶¹ Maróth, *Aussagenlogik*, 6; translated from original German.

thinking about hypothetical propositions or syllogisms pursued at length. And the little that has been said is often conflicting and always cursory.

Tony Street has noted that a nuanced approach to Alfarabi's influence on Avicenna's logical doctrines is lacking in scholarly literature. Street points out, following Fritz Zimmermann, that Nabil Shehaby hardly discusses Alfarabi at all in his introduction to and commentary on his translation of ŠQ V-IX and, when Alfarabi is mentioned, Shehaby points to fundamental differences between the two authors.⁶² On the other hand, some authors have exaggerated the similarities between Avicenna's and Alfarabi's treatment of hypothetical syllogisms as well as the former's indebtedness to the latter. In his 1976 review of Nabil Shehaby's *The Propositional Logic of Avicenna*, Fritz Zimmermann notes that Shehaby "underestimated Alfarabi's importance."⁶³ However, Zimmermann then goes on to draw a conclusion for which there is no evidence and which, in any event, cannot be correct. About Avicenna's hypothetical syllogisms, Zimmermann says that

So far as it is possible to tell, Avicenna follows Alfarabi so closely in logic that one suspects that the direct basis for Avicenna's exposition [in ŠQ] is Alfarabi's *Great Commentary* [on the *Prior Analytics*].⁶⁴

And in his translation of Alfarabi's *Great Commentary* on Aristotle's *De Interpretatione*, Zimmermann extends this claim to include all of Avicenna's logic saying, simply, that in his view, "there is little in the logic of Avicenna that is not foreshadowed in that of al-Farabi."⁶⁵

There are of course, certain obvious instances where Avicenna's hypothetical syllogisms

⁶² Shehaby does point out in the form of schematic diagrams the important differences between Alfarabi's and Avicenna's division of syllogisms. It should be noted that Helmut Gatje has correctly pointed out that Shehaby's diagram misrepresents Avicenna's division of conjunctive syllogisms; see Avicenna, *Propositional Logic*, 22. and Gatje, "Voraussetzungsschlüssen", 202.

⁶³ F. Zimmermann, review of *The Propositional Logic*, by Avicenna, trans. N. Shehaby, *Der Islam* 53 (1976): 307.

⁶⁴ Ibid.

⁶⁵ F. Zimmermann, introduction to *Al-Farabi's Commentary and Short Treatise on Aristotle's De Interpretatione* (London: Oxford University Press, 1981), lxxxiv.

reflect Alfarabi's, both of which have been noted by Helmut Gätje: their treatment of deductive forms like *modus ponens* and *modus tollens*, and how conditional and disjunctive propositions signify connection (*ittiṣāl*) and incompatibility (*inād*) between the elements of the proposition.⁶⁶ Yet, Tony Street has noted several "irreconcilable differences" between Avicenna's and Alfarabi's categorical syllogistics and, in regard to his hypothetical syllogistic, Avicenna says explicitly that the system of hypothetical syllogism set forth in the *Šifā'* and elsewhere is his own innovation.⁶⁷ To say that Alfarabi's *Great Commentary* is the basis of Avicenna's exposition of his hypothetical syllogistic suggests that Avicenna was blithely unaware of his debt to Alfarabi. In any event, Helmut Gätje has noted that according to the evidence, the system of hypothetical syllogisms propounded by Avicenna, with its basic division of syllogisms into conjunctive and repetitive types, is not found in any of Alfarabi's extant works or in any other author before Avicenna.⁶⁸

Other scholars, however, have been more circumspect. In his 1963 article about Avicenna's hypothetical propositions, Rescher notes in passing weak similarities between Avicenna's and Alfarabi's hypothetical syllogisms.⁶⁹ However, based on his translation of what he calls Alfarabi's "Short Commentary" on Aristotle's *Prior Analytics*, it is clear that Rescher was also aware of the fundamental, even irreconcilable, differences between Alfarabi and Avicenna and thus does not overstate certain points of agreement. Like Rescher, Helmut Gätje hesitates to draw strong conclusions about the effect of Alfarabi's logic, if any, on Avicenna's based on a small number of parallels between the two thinkers. Gätje's position is noncommittal,

⁶⁶ Gätje, "Voraussetzungsschlüssen," 161f. In fact, Alfarabi and Avicenna do not in share the view that conditionals signify connection. This is a result of careless readings of ŠQ V 1 by Maróth, Shehaby and Gätje. In this chapter, Avicenna begins by a summary of what has been said on the topic of conditionals to date. He then rejects this notion of connection, and the notion never appears again in any substantial way in ŠQ V-IX (see chapter 3 for details).

⁶⁷ Ibid., 158.

⁶⁸ Ibid.

⁶⁹ Rescher, "Avicenna on Conditionals", 83. Gätje, "Voraussetzungsschlüssen", 167, 173.

claiming only that “the extent to which Alfarabi was important for particular doctrines Avicenna held is clearly something that is judged differently” by different people. It is clear however from the rest of Gätje’s article that he believes, but does not explicitly argue, that both the similarities and the differences between Avicenna and Alfarabi are not merely fortuitous and that the complex of ideas from which Avicenna’s hypothetical syllogistic is formed originates in Avicenna’s ancient Greek and Arabic sources.⁷⁰

Miklós Maróth is the only author to argue at length that Avicenna drew extensively on Alfarabi’s logical works, particularly from Alfarabi’s *Great Commentary on Aristotle’s Prior Analytics*, only a small part of which has survived.⁷¹ The chief aim of Maróth’s book is to reconstruct a genuinely Peripatetic doctrine of hypothetical syllogisms that he simply assumes to be (1) historically and conceptually determinate, (2) ascribable to Theophrastus and Eudemus but which finds some inspiration in Aristotle, (3) continuously debated until at least the time of Averroes by thinkers as far apart in time and thought as Galen, Boethius and Avicenna and (4) conceptually and terminologically distinct from but bearing some resemblance to the Stoic logical doctrines. In his discussion of Alfarabi, Maróth’s particular claim is that Avicenna dealt with hypothetical syllogisms “in the same spirit (Geist)” as Alfarabi and that their sources were “identical” and thus that Alfarabi’s treatment of hypothetical syllogisms is a representative of this same Peripatetic tradition.⁷²

In order to show the similarity between Avicenna and Alfarabi’s accounts of hypothetical syllogisms, Maróth relies primarily on Nicholas Rescher’s translation of a brief chapter on hypothetical syllogisms (*qiyāsāt šartiyya*) in the short treatise entitled *al-Qiyās aṣ-Ṣaḡīr*. It must be said that the evidence Maróth cites as evidence for his claim is at times

⁷⁰ Gätje, “Voraussetzungsschlüssen”, 203.

⁷¹ Maróth, *Aussagenlogik*, 210.

⁷² *Ibid.*, 212ff.

dubious. The following points constitute Maróth strongest.⁷³ First, he notes that like Avicenna in ŠQ VIII, Alfarabi divides hypothetical syllogisms into connective and disjunctive syllogisms and uses the same terms ‘connective conditional (*šarṭī muttaṣil*)’ and ‘disjunctive conditional (*šarṭī munfaṣil*)’ in his division of the types of hypothetical syllogisms as Avicenna does. Second, according to Maróth, Alfarabi understands conditional and disjunctive propositions to be composed of two categorical propositions joined by a logical connective, and Maróth believes that this view of propositional form is consistent with Peripatetic usage. Avicenna says as much at ŠQ V and this is certainly the standard form that Avicenna adopts when he uses letters to represent terms in hypothetical propositions composed of categorical propositions in ŠQ VII. Third, Maróth notes that Alfarabi divides conditional propositions into complete (*tāmm*) and incomplete (*ğayr tāmm*) and disjunctive propositions into complete (*tāmm*) and deficient (*nāqis*) propositions. Maróth notes that this division of the hypothetical propositions is “entirely in accordance” with Galen’s division and also, though Maróth does not say this, with Avicenna’s in ŠQ V. Fourth, Maróth notes that Alfarabi and Avicenna recognize as valid the reduction of a conditional proposition of the form ‘if it rains, then the earth becomes wet’ to a categorical proposition in which the subject is ‘rain’ and the predicate is ‘such a thing that when it falls, the earth becomes wet’.⁷⁴ And finally, Maróth claims that in his *Rhetoric*, Alfarabi “indirectly” recognizes the reduction of a disjunctive proposition like ‘either it is not day or the sun is out’ to a conditional proposition such as ‘if it is day, then the

⁷³ For his entire argument, see *ibid.*, 209-214.

⁷⁴ *Ibid.*, 213; Maróth must be referring to Š.Q V, 256 but Avicenna’s brief remarks there do not warrant Maróth’s conclusion at all. Avicenna says that he will discuss this particular point at a later time but I cannot find the place in the text of Š.Q where he does. Shehaby refers us to a section of Š.Q VII that is completely unrelated to this particular question; see Avicenna, *Propositional Logic*, 55.

sun is out', in a way that recalls Avicenna's lengthy discussion of this transformation in ŠQ VII.⁷⁵

Where does this leave us? It seems that the Alfarabi's influence on how Avicenna eventually formulated his hypothetical syllogistic in ŠQ is as disputed as it is poorly understood. Finding a solution to this problem is clearly a task that requires attention. I believe that a brief glance at Avicenna's and Alfarabi's treatment of hypothetical syllogisms reveals many incongruities between the two thinkers' hypothetical syllogistics. But this is not to deny that Alfarabi's thinking about hypothetical propositions and syllogisms was crucial for Avicenna's formulation of his hypothetical syllogistic. In a much-celebrated, highly disputed, and under-appreciated passage at the end of ŠQ VI, Avicenna allows himself a moment of personal reflection, after having set forth his account of conjunctive hypothetical syllogisms (*qiyās šarṭī iqtirānī*):

[Text 8] In our native land we came to know of a long annotated book on this topic which we have not seen since we left our country and travelled around to look for a means of living. However, it might still be there. About eighteen years after we figured out this part of the philosophical sciences, we came across a book on hypothetical propositions and syllogisms attributed to the eminent later scholar (*fāḍil al-mut'aḥḥirīn*). It seems to be wrongly imputed to him. It is neither clear nor reliable.⁷⁶

Avicenna then presents an extraordinarily detailed list of particular doctrines that are incompatible with those of the long, annotated work that Avicenna believes to have been falsely attributed to Alfarabi. Tony Street has gathered together logical doctrinal evidence that suggests that, at least in this passage, "the eminent later scholar" might be Alfarabi, not

⁷⁵ Maróth, *Aussagenlogik*, 213. I assume Maróth is referring to Š.Q VII, 376-384; cf. Avicenna, *Propositional Logic*, 173-179.

⁷⁶ Translated in T. Street, "The Eminent Later Scholar' in Avicenna's *Book of the Syllogism*", *Arabic Sciences and Philosophy* 11 (2001): 217; cf. Š.Q VI, 356; cf. Maróth, *Aussagenlogik*, 7; Shehaby, *Propositional Logic*, 159; Gutas, *Avicenna and the Aristotelian Tradition*, 106.

Alexander of Aphrodisias as suggested by Maróth, Gätje and Shehaby. This passage suggests a new approach to the question of the force of Alfarabi's thought on Avicenna's hypothetical syllogistic. Thus, the following chapters will try to gauge the precise extent to which Avicenna hypothetical syllogistic departs from Alfarabi's. With what remains of Alfarabi's logical works, I will explore not only the similarities between Avicenna's and Alfarabi's hypothetical syllogistics, but in what way Avicenna contrasted his views about hypothetical propositions and syllogisms with those of Alfarabi.

§1.4: AFTER RESCHER 2: SHEHABY AND MARÓTH ON AVICENNA'S "PROPOSITIONAL LOGIC"

Whether as a positive influence or as a foil against whom Avicenna formulated his own views, Alfarabi's impact on Avicenna has remained unclear. The reasons for this are many. One is simply the absence of sources of Alfarabi's hypothetical syllogistic. Others, as discussed in §1.3, are a result of historical reductionism: either to interpret Avicenna's hypothetical syllogistic, like Rescher, Maróth and Shehaby have, exclusively in terms of sometimes very remote Greek sources for which we have no concrete evidence of textual transmission, while ignoring immediate Arabic precursors; or overstating, as Zimmermann does, Alfarabi's importance without appreciating the conceptual divide separating Alfarabi's and Avicenna's hypothetical syllogistics.

I believe there are, however, interpretive reasons that have precluded a more nuanced approach to considering Alfarabi's impact on Avicenna's hypothetical syllogistic. As we saw in §1.2.2, Nicholas Rescher's interpretation of Avicenna's theory of hypothetical propositions assumes that Avicenna's conditional and disjunctive particles 'if...then...' and 'either...or...' behave like truth-functional operators on propositional variables. As a consequence, truth-

conditions at any particular time t for conditional and disjunctive propositions are understood extensionally. I mean by this that the truth of any conditional or disjunctive proposition at time t is a function of the truth of its propositional variables at time t . Yet, we also saw in §1.2.2 that this cannot be what Avicenna was thinking when he was formulating his views on hypothetical propositions: the truth of a disjunctive proposition, for example, is entirely independent of the question of the truth of its disjuncts. What matters is the existence of an incompatibility or *‘inād* between the disjuncts that matches the incompatibility signified by the disjunctive particle. An analogous situation holds for conditionals. What is crucial is that while Avicenna and Alfarabi disagree fundamentally about hypothetical syllogisms, they do seem to agree on at some deeper level. Yet, Maróth and Shehaby have each followed Rescher in interpreting truth-conditions for Avicenna’s hypothetical propositions extensionally, with the result that this very important aspect of agreement between these two great thinkers has been largely masked from view.

As early as his 1976 review of Shehaby’s *The Propositional Logic of Avicenna*, Fritz Zimmermann observed that:

Stoic logic has received so much attention in the last decade or so mainly because it was thought that they conceived of propositions composed of logical connectives that work as truth-functional operators and, as such, were a precursor to modern (formal) propositional logic. Recently however [the book by Michael] Frede, *Die stoische Logik* has put a damper on this enthusiasm. Yet, with the title of his book and the construction of truth-tables, Shehaby suggests that there is a connection between the Stoic-oriented part of Avicenna’s logic and modern propositional logic. [...] In any case, according to Avicenna the meaning of logical connectives is determined by the contentual (inhaltlichen) relation existing between the parts of the [hypothetical] proposition—obviously, this is

contrary to the formal definition of operators as truth-functional in modern logic.⁷⁷

Zimmermann's comments are interesting for several reasons, but for now they, along with the argument from §1.2.2, suffice to show that there are good reasons for being sceptical about whether Avicenna's conditional and disjunctive particles really work like truth-functional logical operators. Writing in a similar vein in his review of Miklós Maróth's *Ibn Sīnā und die peripatetische "Aussagenlogik"*, Tony Street notes that Maróth also provides truth-tables for what he calls Avicenna's conditional, biconditional, inclusive disjunctive and exclusive disjunctive propositions, the evidence for which Street feels is often inadequate.⁷⁸ Street cites with approval the judgment of Lenn Goodman who was the first to suggest that "Avicenna's propositional connectives are not truth-functional."⁷⁹

Zimmermann's and Street's criticisms are surely correct—at least in part—but they also appear to be based on the principle of historical prudence (or incredulity). In other words, Street and Zimmermann do not show that negative consequences result from interpreting Avicenna's conditional and disjunctive particles 'if...then...' and 'either...or...' as truth-functional operators as in classical propositional logic. After all, what is wrong with interpreting grammatical particles as logical operators in ancient logical texts if such interpretations yield interesting results? Indeed, Shehaby and Maróth have ample precedent for their interpretations in works of the likes of Łukasiewicz, Mates, Dürr and others. Above all, passages from Avicenna's ŠQ—like the following cited by Maróth and Shehaby in their

⁷⁷ Zimmermann, review of Avicenna, *The Propositional Logic of Avicenna*, 308. Zimmermann refers to M. Frede, *Die stoische Logik* (Göttingen: Vandenhoeck und Ruprecht, 1974). For more on this issue, see chapter 4.

⁷⁸ Maróth, *Aussagenlogik*, 52f, 63.

⁷⁹ T. Street, review of Miklós Maróth, *Ibn Sīnā und die peripatetische "Aussagenlogik"*, in *Philosophy East and West* 45/2 (1995): 285f, n. 9.

treatments of what they call Avicenna's material conditional—lend themselves to such an interpretation if read out of context:

[Text 9] You have known the truth-conditions of the restricted connective proposition [*haqīqiyya*], also [called] the implicative [*luzūmiyya*]. <The restricted conditional propositions is true> when the antecedent alone is false and when the consequent and the antecedent are together false. [On the other hand, if the restricted conditional proposition is true, then] it is not possible [*lā yaǧūzu*] for the antecedent to be true and the consequent false because false statements cannot be implied by true statements.⁸⁰

Maróth and Shehaby use this quote as evidence for interpreting Avicenna's restricted conditional as a classical material conditional. But this cannot be correct. In clear violation of the usual truth-table semantics for material implication, Avicenna allows the restricted conditional proposition with false antecedent and consequent 'if man is a creature that caws, then the raven is a creature that talks' to be false.⁸¹

I shall dwell on this point further. Paul Thom has noted that despite the well-known interpretations of Stoic logic by Łukasiewicz and Mates, the examples that the Stoics use when discussing the indemonstrables suggest that the disjunctive propositions serving as the major premise in deductions like modus tollens were not intended to have been read truth-functionally. "Rather," says Thom, the disjunctive major "was meant to be read intensionally, as stating that in some sense it is impossible for the first and the second [disjuncts] to hold together."⁸² Galen says that, in such cases, the disjunction states a conflict,⁸³ e.g. "Dion is not both in at Athens and on the Isthmus". Thom's discussion suggests that a disjunctive proposition such as (1) "either Dion is in Athens or Dion is on the Isthmus" is true (to the extent that it states a conflict "not both: Dion is in Athens and Dion is on the Isthmus") if and

⁸⁰ Avicenna, ŠQ V, 240.17-1.4; cf. Avicenna, *Propositional Logic*, 42f; Maróth, *Aussagenlogik*, 52.

⁸¹ ŠQ V, 239; cf. Avicenna, *Propositional Logic*, 41.

⁸² P. Thom, "Three Conceptions of Formal Logic," *Vivarium* 48 (2010): 239.

⁸³ *Ibid.*

only if it is impossible that both disjuncts be true together.⁸⁴ The sentence would be false just in case there were such a possibility. Sentence (1) above is true because it is obviously impossible that both ‘Dion is in Athens’ and ‘Dion is on the Isthmus’ be true at once. However according to this intensional way of reading disjunctions, the proposition (2) ‘either Dion is walking or Theon is talking’ is false if read as expressing conflict because it is certainly possible that both ‘Dion walks’ and ‘Theon talks’ be true together.⁸⁵ What is crucial to note is whether or not Dion is walking and Theon is talking at the moment of the utterance appears to be irrelevant to deciding whether the disjunction as a whole is true. Said differently, the truth of sentences like ‘Dion walks or Theon talks’ read truth-functionally depends on whether or not Dion walks and Theon talks, whereas ‘it is incompatible that Dion walk and Theon talk’ does not. In fact, determining the truth of such a sentence seems to require investigation of a completely different set of principles.

But this is only part of the story. In the passage quoted above from Zimmermann’s review above and similarly in Street’s review of Maróth, there is an implicit assumption by both scholars that the trouble with Shehaby’s interpretation is the decision to interpret Avicenna’s hypothetical syllogistic as a *truth-functional* propositional calculus. Yet, there are plenty of propositional calculi that are not truth-functional. The following question suggests itself: Is it possible to find a propositional calculus, even if it is not truth-functional, that would

⁸⁴ I believe I have understood Thom’s text properly. Nonetheless, Stephen Menn has alerted me to the fact that there are different strengths of conflict (see S. Bobzien, “Peripatetic Hypothetical Syllogistic in Galen”, *Rhizai* 2 (2004): 62), which Galen called “complete” and “incomplete” conflict (a similar distinction is found in Alfarabi; see chapter 2 below). A disjunction “either A or B” stating complete conflict is true iff one and only one of A or B can be true. The typical example is “either x is odd or it is even”, there being no possibility that x is neither even nor odd, nor is there any possibility that x is both even and odd. A disjunction “either A or B” is incomplete iff it is impossible that A and B are true together. Incomplete conflict obtains in the case that A and B both fail to be true. This appears to be the type of conflict mentioned in Thom’s example “Dion is not both in Athens and on the Isthmus”, since it is perfectly possible for Dion to be neither in Athens nor on the Isthmus.

⁸⁵ Thom notes (ibid., 239) that Galen called this type of sentence one that concerns “ ‘things that sometimes occur together and sometimes do not’ and these latter statements are ‘absolutely useless for demonstration’ ”.

in fact correctly mirror Avicenna's intuitions about hypothetical syllogisms? After all, Thom's discussion of incompatibility suggests that the semantics for hypothetical syllogistics whose disjunctions and conditionals rely on notions of connection and incompatibility can be interpreted in terms of alethic modalities—to say that the propositions 'P and Q are incompatible' would then be precisely to say that 'it is necessary that not both P and Q and not neither A nor B—seems to suggest that a modal propositional logic that cannot be give a truth-functional semantics like Lewis' S5 just might provide the basis for that elusive semantics for Avicenna's hypothetical syllogisms.

In 1991, Chris Martin said the following about Karl Dürr's *The Propositional Logic of Boethius*, a book that was important for Shehaby and Maróth's work on Avicenna's hypothetical syllogisms:

Although [Dürr's book] has been the standard authority for even the most curious and critical of recent historians, Dürr's work turns out to be quite useless as a guide to Boethian logic. While he might be said to offer a logic for compound propositions Boethius in no way provides a propositional logic of the sort which Łukasiewicz proposed to write the history. Dürr's method is quite extraordinarily crude. He simply symbolises Boethius' schemata for hypothetical syllogisms with the conditional construed first as material and then as strict implication. The test of Boethius' worth as a logician is whether or not the translation is a theorem of *Principia Mathematica* or of Lewis' S5.⁸⁶

Thankfully, Shehaby's and Maróth's studies of Avicenna's hypothetical syllogisms possess none of the "crudeness" of Dürr's study of Boethius. Nevertheless, Martin's remarks point to a crucial issue that lies at the heart of the question of how we interpret Avicenna's hypothetical syllogisms. Based on historical evidence gathered by Michael Frede, Zimmermann has suggested what Street has briefly though convincingly shown, namely, that Avicenna's

⁸⁶ Martin, "Negation in Boethius", 279.

hypothetical syllogistic cannot be interpreted as a classical propositional calculus.⁸⁷ Yet, as Martin says in connection with his work on Boethius' hypothetical syllogisms, the question we should be asking is not whether Avicenna's logic is a *classical* propositional logic but whether it is not *propositional* logic.⁸⁸ To claim that a logic is of the propositional variety is to claim, at least in part, that the logic operates with an adequate notion of propositionality. Martin's claim in connection with Boethius is not that the latter did not have any conception of propositionality but, rather, Martin claims that Boethius did not have a notion of propositionality that would warrant interpretation as a modern propositional logic.

In an article on negation in Boethius, Christopher Martin lays down a minimum requirement for a "logic of compound sentences" to be considered a genuine propositional calculus. The distinction, avers Martin, reduces to the question of whether a logician makes use of, if not explicitly formulates, a distinction between propositional content and propositional force—the so called "Frege point". The chief intuition of the distinction between propositional content (or the meaning of the proposition) and force (or the different speech acts in which the identical proposition may appear) is described by Peter Geach in the following way: "a proposition may occur in discourse now asserted, now unasserted, and yet be recognizably the same proposition."⁸⁹ What Geach and others mean by drawing this distinction is that a proposition such as *p* (e.g. 'The door is closed') may appear in many different speech contexts such as commands ('Make it such that the door be closed.'), prohibitions ('It is prohibited that the door be closed.'), assertions ('I assert that the door is

⁸⁷ Street, review of Maròth's *Aussagenlogik*, 283f.

⁸⁸ Martin, "Negation in Boethius", 279.

⁸⁹ P. Geach, *Logical Matters*, 254f; cf. M. Dummett, *Frege: Philosophy of Language*, 2nd ed. (London: Duckworth, 1981), 295-363.

closed'), questions ('Is the door closed?'), and suppositions ('If the door were closed...') and regardless of the different uses to which *p* is put, the sense of *p* remains constant throughout.⁹⁰

The notion that there is a third element of a proposition that is wholly distinct from its meaning and its reference and which is called its force, is part of a larger theory of speech acts.⁹¹ At present, however, we are only interested in the Frege point insofar as it has ramifications for propositional logic. According to Christopher Martin, an implicit or explicit recognition of the so-called Frege point is a necessary condition for a propositional logic. In other words, if a logician formulates a theory of deduction of compound sentences that does not, consciously or unconsciously, allow for the meaning of a proposition to remain constant irrespective of whether it is asserted (e.g. when it appears as a premise) or unasserted (e.g. when it appears as the antecedent of a conditional sentence), then it cannot be the case that this logic be called a propositional logic in the ordinary sense. Thus, to claim as Dürr does but also as Maróth and Shehaby do, that a particular logician's hypothetical syllogistic is interpretable as a propositional logic is to make strong claims about that thinker's (i) notion of propositionality, (ii) the syntactic behaviour of the conditional and disjunctive particles from which hypothetical proposition are constructed, (iii) certain requirements on inferential validity that are, at least partially, a consequence of (i) and (ii). All three of these claims must arise from recognition of the Frege point. Thus, an important task that must be undertaken before it can be decided whether Avicenna's hypothetical syllogistic is a propositional calculus is determining whether Avicenna's thinking about hypothetical propositions and syllogisms accords with propositional logic on all three counts. Each of these three conditions, which will require thorough examination in later chapters, demands brief consideration with respect to

⁹⁰ See also, Dummett, *Frege: Philosophy of Language*, 302f.

⁹¹ See, for example, J. Austin, *How to Do Things with Words*, 2nd ed. (Cambridge: Harvard University Press, 1975).

Avicenna. For all the parallels that Miklós Maróth likes to draw between Avicenna and Boethius, Avicenna is *not* Boethius. In fact, in Chapter 4 I will argue that based on Martin’s own conditions, Avicenna’s “if, then” connector operates as a proposition connective in the technical meaning of that term. The justification of this claim is that in fact Avicenna (1) recognizes and deploys the Frege point in elaborating his theory of conditional propositions, and (2) that he uses the atomic nature of propositions treated in this way to allow conditionals with indefinitely complex propositional content.

§1.5: AFTER RESCHER 3: AVICENNA AND ARISTOTLE’S LOGICAL THEORY

In §1.2.1, I noted that Nicholas Rescher’s article on Avicenna’s hypothetical proposition emphasizes the importance of Stoic logic in Avicenna’s hypothetical syllogistic at the expense of Aristotle’s. In §1.2.2 I suggested that this decision leads to an interpretation of Avicenna’s hypothetical propositions that leaves us ill-equipped to grasp important aspects of Avicenna’s theory from a conceptual standpoint and from a historical standpoint. In §1.3, I claimed that from a historical perspective, Rescher, Shehaby and Maróth appear not to see Alfarabi’s thinking about entailment (*luzūm*), connection (*ittiṣāl*) and incompatibility (*‘inād*) as formative of Avicenna’s thinking about hypothetical propositions, largely due to their concern with remote Stoic sources which, until recently, were thought to present a truth-functional theory of propositions.⁹² A full appreciation of Aristotle’s importance in Avicenna’s and Alfarabi’s logical thinking reveals how much at odds they are with the common interpretation of Stoic logic as a propositional logic. Thus, in §1.4 I reviewed evidence from a study of Boethius’ hypothetical syllogisms which rejects interpretations of Peripatetic hypothetical syllogistics as

⁹² See R. O’Toole, and R. Jennings, “The Megarians and the Stoics”, in *Handbook of the History of Logic*, vol. 1, eds. D. Gabbay and J. Woods (Amsterdam: Springer, 2004), 397-522.

propositional calculi. Avicenna's logical thinking is far subtler than Boethius' however: despite Maróth's claims to the contrary, Avicenna and Boethius have little in common.

It is not sufficient, however, merely to reject Shehaby and Maróth's interpretation of Avicenna's hypothetical syllogistic without offering some alternative account that both makes sense of Avicenna's relationship to Alfarabi and allows us to appreciate more fully Avicenna's true achievement in ŠQ V-IX. Reactions to Avicenna's system of hypothetical syllogisms, especially those in ŠQ VI have been largely negative. Shehaby concludes his overview of Avicenna's hypothetical syllogisms on a sour note:

Though there is much to say against Avicenna's ideas on the subject of conditional propositions and syllogism, there is no doubt as to their historical significance. The vivid picture which the text reveals of the Peripatetic doctrines in addition to many of the Galenic views will be of much interest to the historian of late Greek logic. The most important aspect of this picture is perhaps the role which the Peripatetics played in diverting the attention of philosophers from the worthy step which Stoic thinkers had taken. The Peripatetic influence is clear in Avicenna's case.⁹³

It seems that Shehaby's disappointment originates with the expectation—mercifully unrealized—that we find in Avicenna's hypothetical syllogistic Stoic-like “antecedents” to modern propositional logic as “discovered” by Łukasiewicz and Mates a generation before. Avicenna's Peripateticism appears, in Shehaby's eyes to be nothing but a sterile diversion.

Fritz Zimmermann seconds Shehaby's verdict:

In general, [Shehaby] has admirably avoided the temptation to exaggerate Avicenna's importance for ancient or modern logic. The result of his work is primarily negative: in many respects, Avicenna's contribution is disappointing. If we want to deal justly with Arabic logic, then we must swallow this bitter pill (lit. “bite the sour apple”) and work out Arabic logic's particular features, however slight they might actually be.⁹⁴

⁹³ Shehaby, Introduction to Avicenna, *Propositional Logic*, 21f.

⁹⁴ Zimmermann, review of Shehaby, *The Propositional Logic of Avicenna*, 308.

Such attitudes bring us no closer to understanding what Avicenna was trying to achieve in the ŠQ V-IX. Avicenna's status as being influenced by the Peripatetics should not have the pejorative sense that Shehaby seems to intend it. Rather, it is precisely Avicenna's relation to Aristotle and Aristotle's logical theory that requires attention.

ŠQ VIII, the book in which Avicenna treats repetitive syllogisms (*qiyāsāt istiṭnā'iyya*), is relatively brief but fascinating nevertheless. While the book begins with an excursus on the difference between repetitive and conjunctive syllogisms, the remainder is devoted to enumerating moods of valid and invalid repetitive syllogisms with conditional and disjunctive premises. Yet, Avicenna's attitude toward repetitive syllogisms as presented in ŠQ VIII is dismissive, as Nabil Shehaby has rightly emphasized.⁹⁵ Avicenna calls the figures of this type of syllogism "well-known (*mašhūr*)" and seems to want to keep his entire presentation of this type of syllogism at arm's length. As he so often does in the *Šifā'*, Avicenna discusses at length viewpoints that are not his own before he takes up any systematic criticism.⁹⁶ What is more, Avicenna sees the division of these "well-known" figures into distinct moods as "excessiveness (*takalluf*)" in the sense that he sees many of them as formally indistinguishable from others.⁹⁷ On the other hand, there is no indication that he feels that repetitive syllogisms are invalid in the sense that it is possible that the premises all be true and the conclusion false (assuming for the moment that Avicenna accepts Classical validity. He does not; see Chapter 5). Following a terse summary of the conclusions of chapter one of ŠQ VIII, Avicenna begins a sharp invective

⁹⁵ Shehaby, Introduction to Avicenna, *Propositional Logic*, 5.

⁹⁶ Shehaby's translation of *mašhūr* as 'indemonstrable' is without philological basis and, even worse, extremely misleading in the sense that it causes us to miss Avicenna's deep suspicion regarding this type of syllogism; see Shehaby, *Propositional Logic*, 274f. About the circumstances and the mode of composition of the *Šifā'*, see L. Goodman, *Avicenna*, 2nd ed. (London and New York: Routledge, 1992), 28-32.

⁹⁷ ŠQ VIII, 397.4-9; Shehaby, *Propositional Logic*, 190.

against some of Aristotle's detractors. Though tirade-like in tone, this is certainly one of the most telling texts in ŠQ:

[Text 10] [You ought to know] that this excessiveness of theirs (i.e. setting out eight moods with conditional propositions as major premises, some of which are formally indistinguishable from one another) arises from one reason only, namely, their loss of the first teacher's meticulous treatment of hypothetical syllogisms (*tafṣīl al-qiyāsāt aš-šarṭiyya*) and their resultant need to delve into the subject on their own. What's more, they have only made matters worse by their ignorance of conjunctive syllogisms (*qiyāsāt iqtirāniyya*) among [hypothetical syllogisms], by their adoption of these repetitive syllogisms, by assuming the task of enumerating the number [of valid moods] that seemed most seemly to them and by their disapprobation of this affair (i.e. the treatment of hypothetical syllogisms) being analogous to the one the first teacher undertook with respect to categorical syllogisms. Thus have they exacerbated this travesty by contradicting [Aristotle].⁹⁸

Clearly, Avicenna is critical of repetitive syllogisms because of his belief that this way of dealing with deductions from a hypothesis is antithetical to the way Avicenna believed Aristotle dealt with them in his "lost book" on hypothetical syllogisms.⁹⁹ It is also clear from this passage and others that Avicenna sees something fundamentally, even irreducibly, different about the logical properties of repetitive syllogisms (*qiyāsāt istiṭnā'iyya*) and conjunctive syllogisms (*qiyāsāt iqtirāniyya*) and that this difference lies at the heart of Aristotle's logical theory.

Helmut Gätje has recognized more clearly than others that Avicenna's novel division of syllogisms into conjunctive and repetitive types arises from Avicenna's conscious recognition of a fundamental incompatibility between conjunctive and repetitive syllogisms:

Above all, it is important to observe that Avicenna groups categorical syllogisms and all pure and mixed hypothetical syllogisms into a single superset, the latter of which are based more or less directly on the model of Aristotelian syllogistic.

⁹⁸ Cf. Shehaby, *Propositional Logic*, 190.

⁹⁹ Shehaby, Introduction to *Propositional Logic*, 5.

[Syllogisms belonging to this superset] stand opposed to syllogisms of the Chrysippian type, and thus [opposed to] those syllogisms in which the second premise is got by way of taking out (Herausnahme, excepting, *istiṭnā*) the antecedent or the consequent from the hypothetical proposition that serves as the first premise.¹⁰⁰

Citing Avicenna's discussion at the beginning of ŠQ VIII, Gätje notes that the *sine qua non* of a valid conjunctive syllogism, whether of the categorical or the hypothetical variety, is the existence of the "middle-part" (*ğuz' muštarak*), which can be a term or an entire proposition, that is shared by the major and minor premises.¹⁰¹ On the other hand, the repetitive syllogism is characterized by the peculiar relationship between the first and second premise, viz. that the second premise is acquired by the taking-out from or the repetition of the antecedent or consequent or the contradictory of the antecedent or the contradictory of the consequent of the major premise and then designating it as the minor premise.

In the secondary literature about Avicenna's hypothetical syllogistic, this is more or less the end of the story. The inquiry terminates with the conclusion that Avicenna's division of syllogisms into conjunctive and repetitive varieties arises primarily out of his antithetical attitude to Stoic ways of doing logic, his preference for Aristotle's type of logic, and also from a desire to vindicate Aristotle's theory of the middle term against critics who charged that the middle-term requirement on syllogistic validity was improperly extended to syllogisms with hypothetical premises. Yet, while this is surely part of the story, it cannot be all of it. It is certainly not the most interesting. Avicenna's chauvinism sharpens his criticism of those who have been unfaithful to Aristotle. This criticism was a frequently repeated by Aristotelians such as Alexander, who criticized the Stoics for a syllogistic theory that did not accord with

¹⁰⁰ Gätje, "Voraussetzungsschlüssen", 164f.

¹⁰¹ Ibid., 166.

Aristotle's logical principles.¹⁰² Nevertheless, I claim that Avicenna's logical doctrines in ŠQ V-IX are based on logical considerations that are much more fundamental than Avicenna's desire to vindicate Aristotle. In order to discover why Avicenna's hypothetical syllogistic takes the shape that it does in ŠQ V-IX and to evaluate Avicenna's relationship with Alfarabi, we must begin, unsurprisingly, by looking again at Avicenna's syllogistic but now through the lens of Aristotle's views of syllogistic validity, fallacy, and demonstrative science. As such, the rest of this chapter will be devoted to reviewing studies of Aristotle's syllogistic, which I believe offer us a way into understanding Avicenna's hypothetical syllogisms anew. Unfortunately, no work of this sort has been undertaken in Arabic philosophy, and thus, this and later chapters dealing with this particular aspect of Avicenna's hypothetical syllogisms (especially Chapter 5) will possess all the inadequacies of a line of inquiry in its infancy.

Alfarabi divides syllogisms into categorical (*hamlī* or *ğazmī*) syllogisms and hypothetical syllogisms (*šarṭī*) syllogisms.¹⁰³ As far as I can tell, this division appears to be based on the nature of the premises in the syllogism, or more specifically, a formal property of the major premise: if the major premise is categorical, then the syllogism is a categorical syllogism; if the major premises is a hypothetical, viz. a conditional or a disjunctive proposition, then the syllogism is a hypothetical syllogism. Avicenna's division of syllogisms into conjunctive and repetitive types is different in the sense that it relies on an altogether different criterion: if the syllogism's validity depends on the presence of a middle part or shared part (*ğuz' muštarak*), then the syllogism is conjunctive, subdivisions of which are syllogisms having only categorical premises and conclusions and syllogisms that can have hypothetical and categorical premises and conclusions. On the other hand, if the validity of the syllogism depends on repetition

¹⁰² I. Mueller, "Stoic and Peripatetic Logic", *Archiv für Geschichte der Philosophie* 51 (1969): 173-187; M. Frede, "Stoic vs. Peripatetic Syllogistic", *Archiv für Geschichte der Philosophie* 56/1 (1974): 1-32.

¹⁰³ J. Lameer, *Al-Fārābī and Aristotelian Syllogistic Theory*, 42-47.

(*istiṭnā'*) of an antecedent or consequent or one of a pair of disjuncts or their contradictory opposites, then the syllogism is a repetitive syllogism. Taking this and some of Gätje's findings mentioned above into consideration, it seems that behind Avicenna's Aristotelian chauvinism lies an Aristotelian notion of syllogistic validity. It is not by chance that hypothetical and categorical syllogisms whose validity rests on, among other things as we will see, the presence of the middle term, are precisely those syllogisms that Avicenna regarded as being in line with Aristotle's own views. Yet, it is certainly false that Avicenna regarded repetitive syllogisms, which rely on the repetition of one of the parts of the hypothetical proposition or its contradiction, as *invalid* in the sense that it is possible for the premises to be true and the conclusion false. Insofar as Avicenna's reformulation of *per impossibile* syllogisms is a composite of one conjunctive hypothetical syllogism and one repetitive syllogism, the validity of *per impossibile* syllogisms requires that repetitive syllogisms be valid also.¹⁰⁴

The reasons, then, for Avicenna's highly critical view of repetitive syllogisms are more complicated than that he feels them to be invalid deductive schema: his use of them in *per impossibile* syllogisms entirely precludes our entertaining such a conclusion. Two closely related but nevertheless distinct questions arise. The first: is why did Avicenna feel it necessary to move away from earlier divisions of syllogisms into categorical and hypothetical and adopt an entirely different division, which Avicenna believed was without historical

¹⁰⁴ In fact, I have once again begged the question that Avicenna holds Classical validity for his repetitive and conjunctive syllogisms. In fact, he does not. Chapter 5 will show that his conjunctive syllogisms are modelled on Aristotle's categorical syllogisms in which logical consequence must be characterized by what Paul Thom calls "strong relevance" (this statement is adequate because Avicenna does not consider syllogisms with more than two premises). He also holds that formally concludent syllogisms such as "if every man is risible, and every risible thing is an animal, then every man is an animal" should not be called a syllogism because the conclusion is not better known than either of the premises. I call Avicenna's requirement that syllogisms be ampliative the "Productivity Principle". It has wide ramifications in his conjunctive syllogistic; see chapter 5.

precedent?¹⁰⁵ The second question is: if Avicenna's argument about repetitive syllogisms is not simply that they are invalid, then what is the basis of Avicenna's criticism? The beginnings of an answer to both of these questions may be found in Avicenna's discussion of repetitive syllogisms and conjunctive syllogisms in ŠQ IX. In this closely-argued chapter, Avicenna undertakes the remarkable task of showing that in the same way that any argument in syllogistic form (*qawl qiyāsi*) with categorical premises and conclusion may be shown to be valid by means of formal reduction (lit. "completion", *yutimmu*) to a syllogism in a mood of the three canonical figures of categorical syllogisms, arguments in syllogistic form that make use of hypothetical premises may similarly be shown to be valid by some type of formal reduction to one of the moods of the three figures of conjunctive syllogisms Avicenna sets out in detail in ŠQ VI.¹⁰⁶ But this claim is accompanied by an important qualification, namely, that arguments put into repetitive syllogistic form may be completed by conjunctive syllogisms *if* the aim in putting an argument with hypothetical premises in syllogistic form is that the syllogism be "productive".¹⁰⁷ On the one hand, then, repetitive syllogisms are, in Avicenna's mind, valid: it is impossible that an argument in the form of a repetitive syllogism with true premises yield a false conclusion. Yet, based on the qualification I have pointed to above, Avicenna's argument in ŠQ IX that repetitive syllogisms are completed by conjunctive syllogisms is not so much a question of syllogistic validity as of productivity. Tied up with formal considerations of syllogistic validity, in ŠQ IX there is an additional concern, which appears to originate in Aristotle's treatment of demonstration (*apodeiktikos*, *burhān*) *Posterior Analytics*, that syllogisms generate new knowledge, or, in Avicenna's parlance, "reveal hidden knowledge (*yubayyinu* 'l-

¹⁰⁵ Rescher, "Avicenna on Conditionals", 82f; T. Street, "Arabic Logic", in *The Handbook of the History of Logic*, vol. 1, ed. D. Gabbay and J. Woods (Amsterdam: Elsevier, 2004), 546; Gätje, "Voraussetzungsschlüssen", 158.

¹⁰⁶ ŠQ IX, 415; cf. Avicenna, *Propositional Logic*, 203.

¹⁰⁷ "*idā urīda an yakūna al-qiyāsu mufīdan*"; *ibid.*

ḥafi)” and, in Avicenna’s view, there is an important sense in which repetitive syllogisms do not.¹⁰⁸

The notion that a syllogism ought somehow to generate new knowledge lies at the heart of Aristotle’s ideas about the role of the syllogism in his vision of the demonstrative sciences set out in the *Posterior Analytics*. Jonathan Barnes and Myles Burnyeat agree that in the *Posterior Analytics* Aristotle intends to set out a method for instructing students in philosophy. Barnes claims that the demonstrative method is intended to convey philosophical theses to students in a classroom atmosphere. As a consequence of this pedagogical orientation, the *Posterior Analytics* should not, in Barnes view, be understood as stipulating a method of philosophical research into genuine unknowns. Barnes uses this conclusion that the *Posterior Analytics* is for instructing students to explain why we do not find any demonstrative syllogisms in Aristotle’s extant works. The reason, says Barnes, is that Aristotle’s preserved works are lecture notes on his current research and are, to that extent, do not fit the mould of demonstrative pedagogy. Burnyeat disagrees with Barnes however. He says that the *Posterior Analytics* should not be understood as prescribing a method of conveying new knowledge to students but as a method for deepening the knowledge students already possess by revealing the causes and justifications for why a particular fact is what it is and, additionally, cannot be other than what it is.¹⁰⁹ With respect to Aristotle’s syllogism, “a demonstration”, says Jonathan Barnes, summarizing Aristotle’s view, “is a sort of syllogism; that is it has the form of one of the fourteen syllogistic moods which Aristotle acknowledges as

¹⁰⁸ Š.Q IX, 416; cf. Shehaby, *Propositional Logic*, 203.

¹⁰⁹ J. Barnes, Introduction to Aristotle, *Aristotle’s Posterior Analytics*, trans. J. Barnes (Oxford: Clarendon Press, 1975); idem., “Proof and the Syllogism”, in *Aristotle on Science: the Posterior Analytics*, ed. Enrico Berti (Padova: Edritice Antenore, 1981), 17-59; M. Burnyeat, “Aristotle on Understanding Knowledge”, in *Aristotle on Science: the Posterior Analytics*, ed. Enrico Berti (Padova: Edritice Antenore, 1981), 97-139.

valid”.¹¹⁰ However, it is differentiated from the other species of syllogisms by the fact that it depends

on things which are true and primitive and immediate and more familiar than and prior to and explanatory of the conclusion (for in this way the principles will also be appropriate to what is being proved). For there will be deduction even without these, but there will not be demonstration; for it will not produce understanding.¹¹¹

The upshot of Barnes and Burnyeat’s interpretations for Aristotle’s syllogistic is to make them nothing more than pedagogical tool, a classroom instrument, rather than a necessary part of the scientific method. Whatever the case may be, Barnes’ and Burnyeat’s interpretations are not how Alfarabi and Avicenna interpreted Aristotle. Aristotle’s *Posterior Analytics* as interpreted in late antiquity, particularly by Alfarabi and his remote and proximate successors, was understood to present a theory of demonstration that was universally applicable to scientific inquiry. In fact, Alfarabi’s *Kitāb al-Ġadal* leaves no doubt that the *Topics* and not the *Posterior Analytics* was the most suitable method for instructing students in philosophy, for it is the *Topics* that “is preparatory for engaging in the certain sciences”.¹¹² Drawing on programmatic statements in Alfarabi’s *Ihsā’ al-‘ulūm*, Deborah Black has keenly observed that in Alfarabi’s thought and afterward, demonstration occupies the highest order of a hierarchically conceived notion of syllogistic sciences. Alfarabi

considers the purpose of the syllogism to be fulfilled primarily by the method of demonstration, as articulated by Aristotle in the *Posterior Analytics*. As he states in *Ehṣā’ al-‘olūm*, “logic seeks its principal intention only in this part, and the remainder of its parts have been invented only for its sake”.¹¹³

¹¹⁰ J. Barnes, “Aristotle’s Theory of Demonstration”, *Phronesis* 14/2 (1969): 123-152, quote at 123f.

¹¹¹ C. Hamblin, *Fallacies* (London: Methuen, 1970), 77.

¹¹² Alfarabi, *Al-Manṭiq ‘inda l-Fārābī*, vol. 3, ed. R. Aḡam (Beirtu: Dār al-Mašriq, 1986), 31: [al-ḡadal] yuwaṭṭī’u li-l-‘ulūmi l-yaqīniyya.

¹¹³ D. Black, “al-Fārābī ii. Logic”, *Encyclopaedia Iranica*, vol. 1, 350.

In the quote from the *Posterior Analytics* cited previously, demonstration seems to be merely a proper subspecies of syllogism, the latter of which is more general in the sense that there may be valid arguments constructed syllogistically that do not count as demonstrations because they do not “produce understanding”.¹¹⁴ Yet, as Avicenna received Aristotle’s theory of demonstrative knowledge from Aristotle’s commentators, with Alfarabi as one of their chief representatives, demonstration takes on a larger role in his discussion of repetitive syllogisms in ŠQ VIII and IX role than we might expect. As an active participant in this tradition, Avicenna is compelled to balance three distinct exegetical commitments. As for the first commitment, the generic view of syllogisms as formally valid deductions as set out at the beginning of the *Prior Analytics* seems to allow room for the validity of repetitive syllogisms like modus ponens and modus tollens, though they admittedly do not conform to the special Aristotelian middle-term condition. As a consequence, by the end of late antiquity Aristotelians of different stripes, as different as Avicenna and Alfarabi, were able to accept repetitive syllogisms as valid. However, as a partisan of Aristotle, Avicenna’s was also committed to showing that the specific Aristotelian view of logical validity which relies on the presence of the middle term to be the most fundamental and also productive of new knowledge. This second commitment encourages Avicenna’s argument that productive repetitive syllogisms must be completed by conjunctive syllogisms and to that extent conjunctive syllogisms are more fundamental. The last commitment is a consequence of the increased importance of demonstration in the post-Farabian era of Arabic logic. Avicenna’s argues in ŠQ IX that, since we can show that in most cases the formal characteristics of repetitive syllogisms, though valid, do not guarantee the production of new knowledge, they

¹¹⁴ *An. Post.* I ii 71b 25; *epistesthai* is translated by Barnes as “understanding”. Note too that the word ‘deduction’ is a translation of the Greek ‘*sullogismoi*’, which is normally translated simply as ‘syllogism’.

cannot be used in the demonstrative sciences. To this extent, repetitive syllogisms are inferior to conjunctive syllogisms since they fail to fulfill the most fundamental purpose of syllogistic method in demonstrative inquiry as understood at the end of late antiquity.

The motivations behind Avicenna's idiosyncratic formulation of hypothetical syllogisms arise from a set of strictly Aristotelian doctrines related to formal logic in the *Prior Analytics* and to demonstrative science in the *Posterior Analytics*. Yet what caused Avicenna to diverge so far afield from the relatively straightforward hypothetical syllogistic of Alfarabi is, paradoxically enough, a result of Alfarabi's own efforts as an advocate of Aristotelian demonstrative science. Yet the demands of demonstrative science—a theory of generating necessary knowledge—and formal logic—an inquiry into the nature of deductive validity—should not be seen as necessarily opposed to one another, though as we will see in coming chapters, the balancing act Avicenna is compelled to perform is palpable in Avicenna's proofs. A unique feature of Aristotle's theory of the syllogism is that to a large extent it seeks to place formal restrictions on syllogisms in such a way that valid syllogisms are at least minimally productive—to the extent that these restrictions “can be expressed formally”.¹¹⁵ Paul Thom identifies three notions of validity that are at the back of Aristotle's mind when Aristotle discusses the theory of the syllogism, particularly in the second book of the *Prior Analytics*. The first notion is already familiar to us, viz. that a syllogism is valid if and only if there is no possibility of deriving a false conclusion from true premises (this is called Classical validity).¹¹⁶ The second notion, which Thom calls B-validity, is that in every valid syllogism the truth of any subsequence of premises does not necessitate the truth of the conclusion. As Thom observes, a syllogism valid in this way requires that any proposition that occurs as the

¹¹⁵ Thom, “Conceptions of Formal Logic”, 236.

¹¹⁶ Ibid., 233.

conclusion must have false instances—excluded are conclusions like “every man is a man”—and any proposition that occurs as a premise must have true instances—excluded are premises like “some man is not a man”. Syllogisms that are B-valid can have no redundant premises. The third peculiar property of Aristotle’s notion of syllogistic validity is what Thom calls A-validity.¹¹⁷ A-valid syllogisms are those in which the conclusion can be true while the premises are all false. A consequence of holding syllogisms to this type of validity is that no premise may be implied by the conclusion, which clearly guarantees the invalidity of circular syllogisms. After discussing these two types of validity, Thom says:

Given that Aristotelian syllogisms are both B-valid and A-valid, and that these properties reflect Aristotle’s requirements that good deductive reasoning be neither redundant nor (in the strict sense) circular (to the extent that these requirements can be expressed formally), it is safe to conclude that Aristotle’s formal syllogistic was guided primarily by a conception of logic as a theory of reasoning[.]¹¹⁸

where, by a “theory of reasoning”, Thom means a theory about the “logical relations that connect propositions to one another” but in the context of human activities such as debate and argumentation.¹¹⁹ As far as possible, Aristotle wants “good” reasoning, that is, reasoning free of circularity and irrelevant premises, to be coextensive with syllogistic reasoning. If not in Aristotle’s syllogistic then certainly in Avicenna’s and Alfarabi’s, the aims of formal logic and demonstrative science dovetail to the extent that in late antiquity good reasoning *par excellence* was demonstrative reasoning.

Aristotle’s treatment of logical fallacies also shows how formal concerns about logical validity naturally merge with concerns about demonstration. Unlike the treatment of fallacies as deliberate sophistries in the *Sophistical Refutations*, Aristotle’s treatment of formal fallacies in

¹¹⁷ Ibid., 235.

¹¹⁸ Ibid., 236.

¹¹⁹ Ibid., 233.

Book II, chapters 16-21 of the *Prior Analytics* has more to do with fallacy as a “failure in demonstration.”¹²⁰

A man may not reason syllogistically at all, or he may argue from premises which are less known or equally known, or he may establish the antecedent by means of its consequents; for demonstration proceeds from what is more certain and prior.¹²¹

Fallacies such as begging the question, misconception of refutation, consequent and non-cause as cause all impugn the demonstrative nature of a syllogism. Though this particular passage is from the *Prior Analytics*, it clearly echoes the passage from the *Posterior Analytics* cited above in the sense that Aristotle envisions demonstration here too as a passage from premises that are “certain and prior” to conclusions that are “less known”. Even if someone reasons, says Aristotle, in a formally syllogistic way, the person’s argument may still fail and not necessarily due to straightforward Classical invalidity (as discussed by Thom above). There are also epistemic properties that premises and conclusions must possess such as “priority”, “certainty”, “equally known” and “less known” because the syllogism is being used in the context of demonstration. It seems, then, late antique philosophers like Avicenna and Alfarabi would also say that syllogisms that commit such fallacies are invalid, though only to the extent that these fallacies have a formal aspect. For example, according to Charles Hamblin, William D. Ross claimed that Aristotle’s view about a question-begging inference was that it “can be represented formally as a syllogism in which the conclusion follows from one of the premises alone, independently of the other [premise]”.¹²² Thus, if we say that certain fallacies that are amenable to formal representation are invalid, then we are then committed to rejecting

¹²⁰ Hamblin, *Fallacies*, 67: “To beg and assume the original question is a species of failure to demonstrate the problem proposed (64b 28)”.

¹²¹ *Ibid.*, 192.

¹²² *Ibid.*, 213.

inference schema such as ‘p therefore p’ and ‘p; q; therefore q’.¹²³ In fact, the rejection of both of these inferences is a consequence of A- and B- type validity Thom ascribes to Aristotle.

Logical fallacies in the Aristotelian tradition have two equally important aspects. From a formal aspect, Aristotle’s devotees in the Arabic tradition such as Avicenna and Alfarabi appear to have considered syllogisms in their formal aspects and in view of their efficacy in demonstrative science. These two aspects are recognized as objects of distinct investigation, yet as we saw in Thom, they are not pursued wholly independently of one another. Formal syllogistic validity in Aristotle was pursued in light of the fact that syllogisms were to be used in argument and thus the needs of demonstration as a practical tool for scientific discovery appear to have influenced the formal aspects of Aristotle’s syllogistic. I propose to understand Avicenna’s hypothetical syllogism as a conscious effort by Avicenna to work out a tension that is palpable in Aristotle’s treatment of the syllogism in the *Prior* and *Posterior Analytics* and also in his treatment of fallacies. It is clear from Avicenna’s discussion of hypothetical syllogisms in ŠQ VIII and IX that they clearly present a problem for Avicenna from the side of syllogistic validity and from the side of syllogistic productivity. Formally, Avicenna appears to have divided syllogisms into conjunctive and repetitive syllogisms, motivated by the realization that repetitive and conjunctive syllogisms rely on different principles of validity. From the side of demonstration, Avicenna’s division into repetitive and conjunctive syllogisms is motivated by his belief that repetitive syllogisms are to a certain extent not good inference schema because, in Avicenna’s eyes, they do not lead to new understanding in the way that syllogisms with middle terms do. The final chapter of this study will show how Avicenna’s novel system of hypothetical syllogisms was the result of his conscious effort to work out both the formal and

¹²³ Ibid., 192.

practical challenges of extending a basically Aristotelian logical theory to deductive schema that Aristotle never dealt with in any surviving work. Unlike Shehaby and Zimmermann, I hope to conclude that Avicenna's hypothetical syllogistic is a remarkable success.

CHAPTER 2: ALFARABI ON CONDITIONALS

§1 INTRODUCTION

Unlike Avicenna (d. 1037), at no point in his extant logical works does Alfarabi (d. 950) give a systematic account of ‘if...then’ sentences. Alfarabi discusses conditional syllogisms (*qiyāsāt šartīyya*) in some of his epitomes of the books of the *Organon*.¹²⁴ However, these brief discussions are not accompanied by an exposition of the syntactic or semantic properties of conditional sentences. Strictly speaking Alfarabi cannot be said to propound a proper logical doctrine of conditional propositions. Yet, conditionals appear frequently, and at crucial junctures, in many of Alfarabi’s treatises on logic. Despite the importance of conditional syllogisms in Alfarabi’s overall theory of the syllogism, as well as the importance of Alfarabi’s doctrines of conditional syllogisms to Avicenna’s own syllogistic theory, scholarly literature has not accorded much importance to this aspect of Alfarabi’s logical thought.¹²⁵ As such, this chapter sets out to discuss important aspects of Alfarabi’s development of conditional reasoning by examining several key texts from Alfarabi’s logical canon that are relevant to his thinking about conditional propositions and conditional inferences. Unlike many historical accounts of syllogistics, whether of Aristotle or Avicenna, in this chapter I want to highlight the powerful influence exerted on the development of Alfarabi’s conditional propositions and syllogisms by the context theory of logic, which the classical Islamic philosophers inherited from Greek late antiquity. In §2, I discuss the importance of the different grades of assent (*taṣdīq, inqiyād aḍ-ḍihn*) that a reasoner gives to a proposition according to the argumentative

¹²⁴ Alfarabi, *Al-Manṭiq ‘inda l-Fārābī*, vol. 2, ed. ‘R. Aḡam, (Beirut: Dār al-Mašriq, 1985), 11-64. Hereafter, I refer to this work as follows: Alfarabi, *Madḥal*; idem., *Al-Manṭiq ‘inda l-Fārābī*, vol. 2, ed. R. ‘Aḡam, (Beirut: Dār al-Mašriq, 1985), 65-93. Hereafter, I refer to this work as follows: Alfarabi, *Qiyās*.

¹²⁵ In Joep Lameer’s superb work on Alfarabi’s syllogistic, there are chapter length treatments of Alfarabi’s categorical syllogisms, induction, example (*taṣṭīl*), which Lameer translates as ‘paradigm’, analogy from the present to the absent (*istidlāl bi-š-šāhid ‘alā l-ḡā’ib*), and legal deduction (*qiyās fiqhī*). Not four pages are given conditional syllogisms; cf. J. Lameer, *Al-Fārābī and Aristotelian Syllogistics*, 44-7.

context in which the proposition appears. The truth of conditionals, as well as the type of assent the reasoner gives to them, is based on the notion of connection (*ittiṣāl*) that does (or does not) exist between the antecedent and consequent. Regardless of whether or not the antecedent and consequent are true in themselves, the fact there is a connection between them such that the consequent is true given the antecedent is true determines whether the conditional is true or false. The strength of this connection is also central to eliciting assent to the conditional from the audience. §3 discusses features of Alfarabi's conditional propositions that arise out of their use in demonstrative, dialectical and rhetorical argument, and, in particular, how this argumentative background affects Alfarabi's thinking about conditional propositions and conditional inferences. This section provides further evidence that Alfarabi's conditionals must be understood in terms of connection, rather than the truth of the antecedent and consequent. In addition, due to the strong influence exercised by the argumentative context that shaped Alfarabi's thinking about conditionals, the atomic sentences that constitute conditional propositions must be categorical propositions. In other words, he does not develop a conditional syllogistic of nested conditionals, nor one that yields conditionals as conclusions, despite the fact that such doctrines were developed by late antique Peripatetics. Nor do not find Alfarabi explicitly developing a doctrine of conditional contradiction, which would be required for a ramified theory of conditional syllogisms that allowed nested conditionals. §4 tries to provide a reasonable conjecture about what Alfarabi thought makes a conditional sentence true in a context, and how this context affects the level of assent the reasoner gives to the conditional. This is done by examining Alfarabi's treatment of implication (*luzūm*) in his paraphrase of the *Categories* of Aristotle (APCA). This conjecture attempts to take into account one of Alfarabi's basic logical insights, namely, that there is no

single, monolithic reading of conditionals that captures their use in all the argumentative contexts in which they can be meaningfully deployed. Rather, the conditions under which a conditional elicits the reasoner's assent vary according to the pragmatic assumptions and expectations of the interlocutors. In the language of the context theory of logic, this means that the mind gives different strengths of assent to conditionals according to whether the conditional is being deployed in demonstrative, dialectical, or rhetorical contexts. Finally, in §5 I will show that the variable strength approach to the strength of the implicative relation between antecedent and consequent developed in §4 requires a notion of syllogistic validity for conditional syllogisms that is context-sensitive also. Again, unlike many well-known, contemporary accounts of inferential validity, Alfarabi holds that, depending on the argumentative context, interlocutor's will require assent to the conclusions to follow in various strengths from assent to the premises. In the language of context theory, this means that interlocutors tend to give their assent to the conclusion in a non-demonstrative context according to less rigorous standards than in demonstrative contexts. Once again, I try to provide a reasonable conjecture about what Alfarabi might have thought about the variation in the way the conclusion follows from the premises in different argumentative contexts. I conclude with some brief comments about how Alfarabi's conditionals compare to some contemporary accounts of the semantics of conditionals, followed by brief remarks about how Alfarabi's conditional syllogisms relate to Aristotle's brief comments about arguments from a hypothesis in *Prior Analytics* A44.

§2 THE ‘CONTEXT THEORY’ OF LOGIC: TRUTH, ASSENT, AND CONDITIONALS

Alfarabi’s use of conditional sentences is shaped in crucial ways by the rich legacy of the ‘context theory’ of logic, which he inherited from late antique, Greek commentary tradition on Aristotle’s *Organon*.¹²⁶ Alfarabi, like other classical Islamic philosophers, developed many concepts lying at the foundations of his logical doctrines as solutions to, or extensions of, a constellation of problems surrounding the question of how each of the books of the *Organon* deal with a unitary subject called ‘logic’,¹²⁷ as well as the question of how each of the books of the *Organon* can be distinguished from each other despite this underlying unity. At the risk of schematizing a delicate textual history,¹²⁸ we could say that the classical Islamic philosophers accounted for the ultimate unity of the *Organon* by claiming that the formal theory of the syllogism developed by Aristotle in the *Prior Analytics* provided the formal principles for demonstrative (*Posterior Analytics*), dialectical (*Topics*), rhetorical (*Rhetoric*), poetical (*Poetics*), and sophistical (*Sophistici Elenchi*) modes of argumentation. For Alfarabi, as well as other classical Arabic philosophers, logic was syllogistic.

Yet, by Alfarabi’s day, though perhaps as early as Alexander of Aphrodisias (fl. 200),¹²⁹ it was recognized that conditional (also called ‘hypothetical (*wadʿī*)’)¹³⁰ syllogisms were an important subset of the set of the valid deduction schema available in philosophical debate.

¹²⁶ D. Black, *Logic and Aristotle’s Rhetoric and Poetics*, 17–51.

¹²⁷ *Ibid.*, 52.

¹²⁸ *Ibid.*, 79: ‘The development by the Islamic philosophers of an alternative solution to the problem of how to divide and classify the logical disciplines seems to be closely linked to their resolution of another key issue in the Alexandrian versions of the context theory, that of the degree to which all the logical arts, including rhetoric and poetics, are syllogistic in their structures. In this regard, there is general agreement among the Islamic philosophers that both rhetoric and poetics are syllogistic in some way, although there remains considerable diversity in the syllogistic interpretation provided for them’.

¹²⁹ Citing Boethius’ authority, Miklós Maróth reports that a hypothetical syllogistic of some sort was developed by Aristotle’s students Theophrastus and Eudemos; Maróth, *Aussagenlogik*, 33ff. Cf. J. Barnes, *Terms and Sentences: Theophrastus and Hypoetical Syllogisms*, (London: British Academy, 1984).

¹³⁰ Lameer, *Al-Fārābī and Aristotelian Syllogistics*, 45.

Thus, in his epitomes of the books of the *Organon*, we find Alfarabi making extensive use of conditional syllogisms in the rhetoric, dialectic, and demonstration. That being said, the formal properties of conditional premises and syllogisms developed by Alfarabi had to be elastic enough to accommodate their use in a wide variety of argumentative contexts in which the interlocutors engage in debate while harbouring different goals. In a rhetorical exchange between a speaker and an audience, the aim is persuading (*qanā'a*, *iqnā'*) an audience;¹³¹ in a dialectical exchange, the aim is to discover and then refute (*tabkīt*, *ibtāl*) the position of an opponent if one is the questioner or to defend a position from refutation if one is the respondent;¹³² in poetics it is stimulating the listener's imagination (*taḥyīl*);¹³³ in demonstration, the aim is eliciting in one's self or in another certainty of the objective truth (*burhān*) of a proposition. Deborah Black has noted that the classical Islamic philosophers moved away from using premises' modality, truth-values, or the part of the soul from which premises originate as a way of distinguishing between these syllogistic arts. Black sees Avicenna as the culmination in a trend, in which the distinction between the five syllogistic arts was made to rest on the strength of the audience's assent (*taṣdīq*) or, more generically, acquiescence (*id'ān*), to the proposition rather than the proposition's content corresponding to states of affairs or not corresponding to them.¹³⁴ However, as Black also notes, there is evidence from *Kitāb al-Alfāz* that Alfarabi also formulated a doctrine that distinguishes between the syllogistic arts according to the different grades of the 'mind's compliance (*inqiyād aḡ-*

¹³¹ Black, *Rhetoric and Poetics*, 103.

¹³² Alfarabi, *Al-Manṭiq 'inda l-Fārābī*, vol. 3, ed. R. 'Aḡam, (Beirut: Dār al-Mašriq, 1985), 13-96. Hereafter, I refer to this book as follows: Alfarabi, *ḡadal*. There are two dissertation-format translations of Alfarabi's *Kitāb al-ḡadal*. The first is by Dominique Mallet, "La dialectique dans la philosophie d'Abū Naṣr al-Fārābī", (PhD diss., Université de Lille III, 1992). The second is by Michael DiPasquale, "Alfarabi and the Starting Point of Islamic Philosophy: a Study of the *Kitāb al-Jadal* (Book of Dialectic)", (PhD diss., Harvard University, Harvard University, 2002).

¹³³ Black, *Rhetoric and Poetics*, 181-92.

¹³⁴ Black, *Rhetoric and Poetics*, 76. For a thematically related treatment of *taṣdīq* (and *id'ān*), see also W.C. Smith, 'Faith as *Taṣdīq*', in *Islamic Philosophical Theology*, ed. P. Morewedge (Albany: State University of New York Press, 1979), 96-119.

dihn)’ with the propositions in a syllogistic process of reasoning.¹³⁵ Thus, it also seems to be Alfarabi’s view that syllogism, especially as developed by Aristotle in *Prior Analytics*, is a genus for the different species of the syllogism developed in the other five syllogistic books of the *Organon*. This should be seen as Alfarabi’s attempt at providing a partial solution to the problem of the unity of the syllogistic arts, the terms of which Alfarabi largely inherited from the late antique Greek logical commentary tradition.¹³⁶ The “horizontal” distinction between each of the syllogistic arts (e.g. what distinguishes demonstration from dialectic), as well as the “vertical” distinction between the species of the syllogism and its genus (e.g. in what sense is the syllogism of the *Prior Analytics* different from the syllogism described in the *Topics*) rest on Alfarabi’s analysis of the mental act of compliance or assent that attaches to propositions in a syllogism-formatted argument. All of the syllogistic arts share in the fact that a mental act of assent, which varies in strength according to the context in which the proposition is expressed, attaches to the propositions from which the premises and conclusions of the syllogism are composed.¹³⁷ These sentences are, thus, composed of two parts. One part of the sentence is the *mental act* of assent; the other part is the proposition (e.g. ‘X is Y’) that is the object to which the mind gives its assent. As a consequence of this distinction, it is possible to say that it is ultimately the variation in the ways in which we give assent to propositions, and not necessarily the propositions themselves, that allows us to differentiate among the syllogistic arts. For example, Alfarabi says that there is a generic notion of the syllogism, alluding to the syllogism as outlined in the *Prior Analytics*, just to the extent that it leads to an

¹³⁵ Black, *Rhetoric and Poetics*, 75f.

¹³⁶ Black, *Rhetoric and Poetics*, 36-51.

¹³⁷ Alfarabi, *Kitāb al-Alfāz al-Musta’mala fī al-Manṭiq*, ed. M. Mahdi (Beirut: Dār al-Mašriq, 1968), 96.2-3. It is important to note that it is not until §55 that Alfarabi finally explicitly identifies as ‘syllogisms’ those things he refers to prior to §55 as ‘the ways and things’ that lead the mind to give its assent to something. Alfarabi’s words suggest that, in his view, logic and syllogism are coextensive. This weakens the claim that Alfarabi was not a clear predecessor of the doctrine of the division of logic in *taṣawwur* and *taṣdīq*, and strengthens Avicenna’s claim to being the originator of the doctrine.

unqualified (*mutlaq*) act of mental assent. On the other hand, there is a poetic syllogism, which is distinguished from all the other kinds of syllogism, just to the extent that it leads the mind to what Alfarabi calls ‘poetical assent (*al-inqiyād aš-ši’rī*)’. It is reasonable to take Alfarabi to be claiming that the proposition ‘X is Y’ can be common to both the poetic and demonstrative modes of assent, but what distinguishes the conclusion of a poetic syllogism from a demonstrative syllogism is the modality, or strength, of the mental act itself, not necessarily the propositional content of the judgment.

[Text 1] The generic and unqualified things that lead the mind to give unqualified assent are called ‘syllogisms’. The subclasses of these generic items, wherein each subclass leads the mind to a subclass of mental assent, are called ‘subclasses and species of syllogisms’. Thus, those subclasses [of syllogism] that lead the mind to poetical assent are ‘poetical syllogisms’. Those that lead the mind to rhetorical assent are ‘rhetorical syllogisms’, and supplementary considerations are added by which these syllogisms are brought to conclusion. Those that lead the mind to sophistical acts of assent that come across [the mind] are ‘sophistical syllogisms’, and supplementary considerations are added by which these syllogisms are brought to conclusion—e.g. ruses that are used [by the questioner] to trick the respondent in such a way that the location of the sophistry is obscured from him, and what the respondent must use to learn the sophistries that will refute him, and guarding his views from suspicion of their falsehood or from being misled by a sophistry. Those that lead the mind to dialectical assent are dialectical syllogisms, and supplementary considerations are added by which these syllogisms are brought to conclusion—e.g. ruses that trick the respondent in such a way that the opinion [that the questioner seeks to] oppose is obscured so the respondent does not take precautions [against the questioner’s attack], and ruses that the respondent uses to learn from the questioner what opinion of his is being refuted so he can take precaution and prevent the questioner from employing his syllogisms [against him]. Those that lead the mind to give its assent to that which is certain truth are called ‘demonstrations’ and ‘certain syllogisms’. Supplementary considerations are added by which demonstrations are brought to conclusion, and ways that make it easier for the mind to investigate demonstrations, as well as those non-logical considerations a person relies on in order to arrive at the truth. Yet, the foremost aim of logic is the study of demonstrations. As for the other kinds of syllogism, when one becomes acquainted with them and comes to distinguish

them from demonstration, one learns by virtue of [studying the non-demonstrative syllogisms] what must be used when one's aim is true belief, and what must be avoided.¹³⁸

We can find, therefore, in Alfarabi too a tendency to move away from “the objective truth of the proposition which is known, towards the knowing act of evaluating and accepting it as true”.¹³⁹ In other words, in the context of an argument the modality of the mind's assent to the proposition ‘X is Y’ is just as important to the logical analysis in the thinking of the classical Arabic philosophers as the proposition ‘X is Y’ itself. This is by no means to deny the centrality of objective truth values in Alfarabi's logic, nor should it be taken as denying that all propositions are either true or false (bivalence). Indeed, by insisting, along with the majority of late antique Greek Peripatetics that demonstration represents the *telos* of logical inquiry despite the existence of other species of syllogism,¹⁴⁰ there clearly remains in Alfarabi's mind an ineluctable relation between the act of assent and the assignment of truth-value to a proposition. Indeed, as Black notes in her discussion of Avicenna's ‘imaginative syllogism’, the above passage shows that in Alfarabi's thinking ‘the primary focus of assent remains the determination of what is true. To give one's assent to any proposition necessarily presupposes the prior consideration of whether the proposition is true or false’.¹⁴¹ Rather, in Alfarabi as well as Avicenna there is a ‘shift of emphasis from the veracity of the cognitive act as a representation of some object to the way in which the cognition itself is accepted by the knower’.¹⁴² A reasoner will say that P is true, yet he entertains different criteria in different argumentative contexts. After considering what relation the proposition P bears to the state of affairs it represents within the particular speech-context in which the reason places

¹³⁸ Alfarabi, *Alfāz*, 98.11-100.2.

¹³⁹ Black, *Rhetoric and Poetics*, 76f.

¹⁴⁰ *Ibid.*, 34.

¹⁴¹ *Ibid.* 181.

¹⁴² *Ibid.*, 76.

himself, the speaker will then tailor the degree of his assent, or the force with which he says “P is true”, according to the demands of that speech-context. Of course, this makes the reasoner’s saying ‘P is true’ ambiguous. In Alfarabi’s way of thinking, we are able to disambiguate the speaker’s context-sensitive assignment of truth to propositions by examining the nature and strength of his assent to proposition in question.¹⁴³

In order to illustrate the complex relationship between assent or the mind’s compliance to a proposition (*taṣdīq*, *inqiyād aḍ-ḍihn*) on the one hand and the propositions objective truth on the other, it would be helpful to examine what Jonathan Lear has called the ‘argumentative role’ of each of the propositions in a generic conditional syllogism. Jonathan Lear pointed to the importance of appreciating the argumentative role each premise plays for understanding Aristotle’s brief treatment of hypothetical syllogisms in the *Prior Analytics*. However, since context theory puts perhaps even greater emphasis on the argumentative context, an analysis of the argumentative role of propositions in conditional syllogisms is indispensable.¹⁴⁴ Lear says that Aristotle’s categorical syllogistic does not recognize the argumentative role of sentences in a deduction. For example, Aristotle’s syllogistic does not distinguish between a sentence in a deduction that is merely supposed to be true by two opponents for the sake of argument, or for the sake of probing the logical implications of accepting the sentence as true, and a sentence that is true as such, as a necessary first principle of a science. Aristotle’s categorical syllogistic identifies which arguments are valid according to purely formal characteristics of the premises (quantify, quality, etc.). Aristotle does not aim to analyze deductions according to the argumentative role played by each sentence in the deduction. Thus, in *Prior Analytics* A23, when Aristotle claims that hypothetical

¹⁴³ I will return to this important point with respect to the truth of conditionals in particular in §4.

¹⁴⁴ J. Lear, *Aristotle and Logical Theory* (Cambridge: Cambridge University Press, 1980), 36.

sylogisms “are brought about through syllogistic figures”, his claim is that this is so only to the extent that such a hypothetical syllogism contains a *categorical* syllogism as a proper part of the deduction.¹⁴⁵ This may be illustrated by considering the following schematized hypothetical argument borrowed from Lear:

(H) ‘You agree to accept *Q* if *P*; but... so *P*; but you agreed to accept *Q* if *P*; therefore, you must accept *Q*.’¹⁴⁶

In the above hypothetical argument, the part of the argument “but...so *P*” is a direct syllogism that is formally reducible to one of the valid figures of Aristotle’s categorical syllogistic, where “...” represents a series of premises and *P* represents a categorical proposition that serves as a conclusion of a categorical syllogism of the A-, I-, E- or O- type. Lear believes that Aristotle does not intend to formalize, however, into his syllogistic the act of agreement between the opponents that they accept *Q* as a necessary result of *P*’s being deduced from a categorical syllogism.

In contrast to Aristotle, there is good reason to believe that Alfarabi developed his theory of conditional as a way of formalizing the argumentative role that the prior agreement between the opponents plays in a dialectical exchange. Indeed, Alfarabi formalizes both the act of prior agreement between two opponents and the mental act of supposition as a conditional proposition, in the sense that the propositions from which a conditional are composed are given or conceded (*waḍʿ*) by one or more of the interlocutors. The first sense of *waḍʿ* relates primarily to a prior act of agreement between *two opponents*. This sense of *waḍʿ*, which might be translated as positing or laying down, does not necessarily connote a particular type of mental activity that accompanies the act of positing or laying down of premises. However, Alfarabi assimilates *waḍʿ* to the notion of *farḍ* or *iftirāḍ*, viz. supposition,

¹⁴⁵ Ibid., 34.

¹⁴⁶ Ibid.

which does connote a mental process. Thus, the specific meaning of *waḍʿ* as a prior act of agreement between two opponents to entertain the existence of a connection between an antecedent and consequent can also take on the meaning of mental supposition. In *Ġadal* Alfarabi says:

[Text 2] As for the connective conditional [syllogism],¹⁴⁷ the connection in it may be clear in itself [*bayyin bi-naḥsihi*], or it may not be clear in itself, and thus require demonstration of the truth¹⁴⁸ [*ṣiḥḥa*] of the connection in it, for it is a fact that the chief consideration [*malāk al-amr*] in the connective conditional [syllogism] is the truth of the connection [*ṣiḥḥatu l-ittiṣāl*] and the truth of the repeated proposition (*ṣiḥḥatu mā yustaṭnā*). As for the truth [*ṣiḥḥa*] of the antecedent and the consequent, no conditional expression [*qawl ṣarṭī*] signifies [their truth, sc. *ṣiḥḥa*] and it may happen that neither of them is true [*ṣaḥīḥan*]. Rather, a conditional expression only signifies [*yataḍammanu*] the soundness of the connection [*ṣiḥḥata l-ittiṣāl*].¹⁴⁹ Even if neither the antecedent nor the consequent is true [*ṣaḥīḥan*], the expression's being a conditional is not undermined¹⁵⁰. The proof of this is that the truth-value [lit. 'the matter' or 'the actual state of affairs', *al-amr*] with regard to the antecedent and consequent rests on the asserted proposition [i.e. the minor premise, *al-mustaṭnā*]. Thus, the contradiction of the consequent can be asserted (*yustaṭnā*) due to the fact that it is true [*ṣaḥīḥ*], yielding thereby the contradiction of the antecedent. If, however, [the antecedent and consequent] were true because of what was posited about them, then it would be impossible to except the contradictory of the consequent by virtue of the fact that it is true and yields thereby the contradictory of the antecedent, since the two contradictories cannot be true simultaneously [*id kānā an-naqīḍāni lam yumkin an yaṣḍuqā ma'an*]. Rather, the antecedent and the consequent are supposed [*yafrūdu*] to have the quality

¹⁴⁷ Sc. *al-qiyās aṣ-ṣarṭī al-muttaṣil*

¹⁴⁸ Reading "*tatabayyanu*" for "*yatabayyanu*".

¹⁴⁹ Some authors (S. Afnan, *Avicenna: His Life and Works*, (London: George Allen & Unwin, 1958), 93; Avicenna, *Remarks and Admonitions, Part One: Logic*, trans. S. Inati, (Toronto: Pontifical Institute of Medieval Studies, 1984), 13) have translated 'taḍammun' as 'implication', which, if adopted, would be a source of great confusion. 'Taḍammun' is used to talk about the way in which terms signify meanings; in the way that, for example, the term 'human' signifies animal. It is for this reason that Ahmed (Avicenna, *Avicenna's Deliverance: Logic*, trans. A. Ahmed, (Karachi: Oxford University Press, 2011), 10f, 174) translates 'taḍammun' as 'inclusion', viz. a concept such as human includes the concept of animal in it because of the genus-species relation between them. Similarly, Goichon (Avicenna, *Livre des directives et remarques*, trans. A.-M. Goichon, (Paris: Vrin, 1951), 82f) takes 'taḍammun' to mean the way in which a term (nom, *lafẓ*) such as 'triangle' refers (se refere, *yadullu 'alā*) indirectly to a concept such as 'figure', which is a constitutive part of the concept to which the term properly belongs, viz. 'three-sided figure'. Obviously, none of these is quite the sense that Alfarabi intends to convey here.

¹⁵⁰ Reading "*lam yubṭal bihimā*" for "*lam tubṭal bihimā*".

[*kayfiyyatihimā*] that they have only in so far as they are taken to be so by hypothesis [*bi-l-waḍ'*], not in so far as they are inescapably true in themselves [*lā 'alā annahumā ṣaḥīḥāni fī anfusihiṁā lā maḥāla*]. It is for this reason that every conditional syllogism [*qiyās šarṭī*] is also a syllogism from a hypothesis [*qiyās bi-l-waḍ'*], since the two components of the conditional particle—the antecedent and the consequent—are hypothesized in such a way that neither one of them has to be true according to the one who hypothesized them.¹⁵¹

According to Alfarabi, the syllogism from a hypothesis described by Aristotle in A23 is a genus for conditional syllogisms generally. The feature that relates them is the fact that in both kinds of syllogism, the antecedent (*muqaddam*) (or the hypothesis (*waḍ'*) in a syllogism from a hypothesis) and the consequent do not have a definite truth values insofar as they are parts of the conditional proposition. Of course, the antecedent and consequent may be objectively true in themselves (*fī anfusihiṁā*) when considered on their own. But if the reasoner gives his assent to the conditional (or the hypothesis) as a whole, this does not entail that he gives his assent to the parts of the conditional (or to the hypothesis and what follows from it). In Alfarabi's analysis, the reasoner's assent is given to, or there is compliance of the reasoner's mind with, the propositions that are objectively true (*ṣaḥīḥ*), which in the case of a conditional syllogism are the conditional itself as a major premise and the minor premise. As a premise, assent can be given to the conditional as a whole, based on the fact that the connection expressed by conditional sentence corresponds to the actual state of affairs. The minor premise as far as its propositional content is concerned is identical to antecedent of the conditional or it is the contradictory opposite of consequent. In the case of modus ponens, what distinguishes the antecedent of the conditional and the minor premise is not their propositional content. Rather, the reasoner attaches his assent to the proposition expressed by the minor premise, but does not give any assent to the proposition expressed by the

¹⁵¹ Alfarabi, *Ġadal*, 103.

antecedent qua member of the conditional (the same proposition is expressed in both instances). In other words, giving assent to a conditional involves the reasoner simultaneously adopting a definite attitude toward the truth of the connection signified by the conditional proposition the act of supposition (*fard*, *iftirād*), and the suspension of his mind's compliance with the antecedent or consequent of the conditional. To justify his view, Alfarabi asks us to consider a situation in which this is not the case, viz. assume for the sake of argument that giving assent to the conditional as a whole is also to give assent to the antecedent and the consequent. Say it is night time and reasoner wants to reason about it being day or night. The reasoner executes a syllogism in modus tollens, with a conditional major premise 'if the sun is up, then it is day'. Since it is obviously night out, we then want to assert 'but it is not day' in order to conclude 'Therefore, the sun is not up'. However, our assumption has blocked our ability to assert the minor premise, since, having given our assent to the conditional, we have also thereby given our assent to the fact that the sun is up. This leaves us in the undesirable position of having given our assent to a proposition and its contradictory opposite. Though Alfarabi does not mention it, a similar argument can be made for modus ponens. If the conditional 'if the sun is out, then it is day' means nothing more than 'the sun is out, and it is day', then modus ponens is, technically speaking, nothing more than a *petitio principii*. Thus, in order not to make nonsense of valid deduction schema, Alfarabi shows that a sharp distinction must be made between the proposition when it is a member of a conditional and when it is not. When it is a member of a conditional that a reasoner has given his assent to, the reasoner has not, in fact, given his assent to the proposition itself, but to the connection between it and the other member of the conditional. However, when the proposition

expressed by the antecedent or the consequent appears as a minor premise or a conclusion, then the reasoner's assent does attach to the proposition itself.

On Alfarabi's view assent does not attach to the antecedent and consequent qua constituent parts of a conditional sentence. The reasoner will give his assent to a conditional proposition P saying "P is true" once he has verified that the nature of the connection between the antecedent and consequent is of a strength required by the speech-context. Considerations involving the reasoner's assent are distinct from the question of P's truth-value. The conditional is true just in case there is a certain type of connection (*ittiṣāl*) between the antecedent and the consequent. As we will see, in Alfarabi's view (drawing on Galen, but not exclusively)¹⁵², this connection may be *per accidens* or *per se*. If it is *per accidens*, then the connection between antecedent and consequent is completely coincidental. (P1) "if Dion is walking, then Theon is leaving" is a true conditional *per accidens*. If the connection is *per se*, then the connection between the antecedent and consequent is either for-the-most-part. Read *per se* with for-the-most-part P1 is false, but (P2) "if the sky is clear in winter, then it will be colder" is true. There is also *per se* necessary connection. P1 and P2 would both be false on this reading of the conditionals, but (P3) "if there is a man, then there is an animal" is true. As I will discuss in greater detail below, it seems that Alfarabi has in mind that the degree or strength of assent given to a conditional proposition "if A, then B" must be in line with the strength of the connection between antecedent and consequent. If the speaker considers P1, he will perhaps notice that the connection between antecedent and consequent is *per accidens*, and also that P1 is false when read as having *per se* connection. With these truth-values in hand, depending on the requirements of the speech-context, the reasoner will give assent to

¹⁵² See S. Bobzien, "Peripatetic Hypothetical Syllogistic in Galen", *op. cit.*

P1 “it is true that if Dion is walking, then Theon is leaving”. Yet, his assent will not have the strength of certitude as required by a demonstrative speech-context, but perhaps he would give his assent to P with strength appropriate to a rhetorical speech-context. There is, then, a parity in emphasis in mental representation (truth as correspondence) and mental act (assent-giving) in Alfarabi’s account of conditional proposition, which reflects the argumentative backdrop against which the context theory of syllogism was developed by classical Islamic philosophers. Awareness of this backdrop is particularly important when we consider Alfarabi’s theory of conditionals and conditional syllogisms. If logic is to give shared, objective criteria for the validity of, for example, demonstrative inferences as well as poetic and rhetorical inferences—viz. in argumentative contexts where the speakers very often lie, tell half-truths, and generally dissimulate, then naturally the aim of our analysis of propositions and syllogisms in such contexts will be aimed less toward the objective truth of the propositions expressed in the argument and more toward what propositions will the audience give its assent to. In non-demonstrative contexts, on the other hand, a wedge is opened up between the proposition’s objective truth and the audience’s compliance with it. In such contexts, there will be propositions to which assent attaches but whose correspondence to contingent states of affairs is not exact.

In none of these types, however, is the truth of the conditional determined by the straightforward determination of the antecedent and consequent’s correspondence to current or possible states of affairs, or the antecedent’s and consequent’s straightforward assertibility or deniability.¹⁵³ There is no question that the objective truth of the antecedent and the

¹⁵³ Grice’s view (P. Grice, ‘Indicative Conditionals’, in *Studies in the Way of Words* (Cambridge, Massachusetts: Harvard University Press, 1989), 58-87) that material conditionals as the logical interpretation of how conditionals are used in natural language has been shown to be indefensible; see E. Adams, *The Logic of Conditionals: An Application of Probability to Deductive Logic* (Dordrecht and Boston: D. Reidel, 1975); for psychological studies

consequent is determined by the correlation (or non-correlation) of what it expresses to the state of affairs. However, in the context theory of logic developed by the classical Islamic philosophers, the correlation of the proposition to the state of affairs is not the exclusive unit of analysis. Rather, along side the question of the antecedent's and consequent's being true (or false) sits the question of whether, how, and to what end the listener actively *gives his assent* to the them. In the non-demonstrative syllogistic arts, what you can *get your audience to give its assent to* is more important than whether the antecedent and consequent are true or not. The aim of the five syllogistic arts is as much the production of *attitudes* of different strengths toward a particular proposition as it is the production of a proposition as a conclusion. Nor is there only one attitude that a reasoner attaches to propositions amenable to truth and falsity. Rather, the strength of the reasoner's assent to a proposition varies according to the argumentative context in which the proposition is used. The attaching of different grades of assent to propositions in dialectical and rhetorical contexts is due to the listener's recognition of the contingency of the matter of these propositions. It is often the case that propositions expressed by the conditionals can be said to correspond to the state of affairs, but to a greater or lesser extent, and as a consequence, the mind's compliance with them will be similarly graduated.¹⁵⁴ Thus, instead of looking for conditions under which a conditional is true, it seems more suitable to look for conditions under which a reasoner gives his assent to a conditional. With respect to conditionals, determining the conditions under which a listener

showing empirically that indicative conditionals are not normally understood as material conditionals, see J. Evans, D. Over, *If* (New York: Oxford University Press, 2004), 38. See also J. Bennett, *A Philosophical Guide to Conditionals* (Oxford and New York: Oxford University Press, 2003), 20-33. For possible worlds semantics of counterfactual conditionals, see R. Stalnaker, 'A Theory of Conditionals', in *IFS, Conditionals, Belief, Decision, Chance, and Time*, ed. R. Stalnaker, W. Harper, G. Pearce (Dordrecht and Boston: D. Reidel, 1981), 41-56; D. Lewis, *Counterfactuals* (Malden, Massachusetts: Blackwell Publishers, 2001). For conditionals as conditional assertions, see G. Von Wright, *Logical Studies* (London: Routledge and Kegan Paul, 1957), 127-65; C. Gauker, *Conditionals in Context*, (Cambridge Massachusetts: MIT Press, 2005).

¹⁵⁴ Cf. Evans, and Over, *If*, 38f.

will give assent to a conditional, it must be determined under what conditions a listener will give his assent to the consequent *given* that the listener gives his assent to the antecedent.

The importance of assent in determining in what senses a conditional is said to be true by an audience has implications in how we think about the syllogistic validity of arguments with conditional premises. For example, an argument constructed according to the schema *modus ponens* is valid in the truth-valuation sense, if there is no value assignment to the sentences of the premises and the conclusion that makes the former all true but the latter false. If there is such a value assignment, then the argument is invalid. Analogously, an argument in the scheme of *modus ponens* is valid in the sense of assertibility, if there is no value assignment to the premises and the conclusion that makes all the premises assertible, but the conclusion is either not assertible or is deniable. And, once again, the argument is invalid if there is such a value assignment.¹⁵⁵ The difficulty with applying this criterion for validity to Alfarabi's conditional syllogisms is that, depending on the context in which the argument takes place, a listener will give his assent, will be induced to say *that* a conclusion is true, viz. to say that it corresponds to the state of affairs, given his assent to the premises, but only to a greater or lesser degree. This is especially true in non-demonstrative arts such as dialectic and rhetoric. In dialectical and rhetorical discourses the conclusion may not necessarily follow *necessarily* from the premises, but it follows in enough cases that a listener

¹⁵⁵ Aristotle's notion of validity (i.e. what conditions must be fulfilled to be a syllogism) are formulated for categorical syllogisms only. However, George Boger's work on Aristotle (G. Boger, 'Aristotle's Underlying Logic', in *The Handbook of the History of Logic*, vol. 1, ed. J. Woods, D. Gabbay (Amsterdam and Boston: Elsevier, 2004), 234) can be taken to show that, loosely speaking, Aristotle's notion of validity is close enough to contemporary ones that it can be used here without too much violence being done to the text and context. John Woods and Andrew Irvine (J. Woods, A. Irvine, 'Aristotle's Early Logic', in *The Handbook of the History of Logic*, vol. 1, ed. J. Woods, D. Gabbay, (Amsterdam and Boston: Elsevier, 2004), 38) make what I think is a helpful distinction between Aristotle's notion of categorical syllogistic validity and syllogistic simpliciter. While the former is very different from contemporary ideas about deductive validity, the latter is rather closer. For accounts of Aristotle's notion of syllogistic validity, viz. what it means to be a syllogism, see T. Smiley, 'What is a Syllogism?', *Journal of Philosophical Logic* 2/1 (1973): 136-154; J. Corcoran, 'A Mathematical Model of Aristotle's Syllogistic', *Archiv für Geschichte der Philosophie* 55/2 (1973): 191-219.

will be convinced to give his assent to the conclusion, given that he gives his assent to the premises. In arguments from premises about contingent events or, for example, about ethical analyses of the good and the just that are known to have many exceptions, a listener may be induced to give his assent to propositions talking about what the good and the just are, in spite of his knowledge of exceptions that might otherwise be considered as falsifying the premise.¹⁵⁶ Analogously, a listener might consider a syllogistic argument good despite his knowledge that in certain instances the conclusion sometimes does not follow from the premises.

§3 CONDITIONALS IN ARGUMENTATIVE CONTEXTS

Two of the richest sources for Alfarabi's thoughts about conditionals are two epitomes of Aristotle's *Topics*, *Ġadal* and *Taḥlīl*.¹⁵⁷ In these two works, Alfarabi gives a very precise account of the structure of a dialectical exchange between a questioner (*as-sā'il*) and a respondent (*al-muḡīb*). As we will see, these pragmatic conditions determine to a large extent some of the syntactic and semantic properties of Alfarabi's conditional propositions.

Unlike a rhetorical argument, which has the structure of a single individual addressing a passive audience in order to convince them to act in a certain manner,¹⁵⁸ a dialectical exchange is one involving a single questioner (Q) and a single respondent (R).¹⁵⁹ The exchange between Q and R is closer to a competition with a winner and a loser than a straightforward deduction. Q's aim in the argument is to discover the view that R has been tasked with defending, and then showing that the view is logically inconsistent.

¹⁵⁶ Alfarabi, *Ġadal*, 20.

¹⁵⁷ Alfarabi, *Al-Mantiq 'inda l-Fārābī*, ed. R. 'Aḡam, vol. 3 (Beirut: Dār al-Mašriq, 1985), 95-129. Hereafter, I cite this work as follows: Alfarabi, *Taḥlīl*. Roughly speaking, *Ġadal* seems to be a summary of books I and VIII of the *Topics*, whereas Ta. seems to be related to *Topics* II to VII but also to *Prior Analytics* 27-32; see Dominique Mallet, "Le kitāb al-Taḥlīl d'Alfarabi", *Arabic Sciences and Philosophy* 4/2 (1994): 317-335.

¹⁵⁸ Black, *Rhetoric and Poetics*, 103f.

¹⁵⁹ Alfarabi, *Ġadal*, 14.

[Text 3] The activity of this art [*ṣināʿa*] is debate [*al-muǧādala wa-l-ǧadal*]. It is addressing another [*muḥāṭaba*] with widely-accepted statements [*aqāqwīl mašhūra*] by which the person, if the questioner [*sāʿil*], seeks to show the falsity [*ibṭāl*] of whichever of the disjuncts of a contradictory pair [*an-naqīd*] the questioner obtains from a respondent [*muǧīb*] who is tasked with defending it. If <the person> is the respondent, then he seeks to defend by means of [these statements, sc. *aqāwīl mašhūra*] whichever of the disjuncts from a pair of contradictories the questioner aims to show to be false. Thus, the aim of the questioner is to show the falsity of <the statement> of the respondent who has been tasked with defending it. The aim of the respondent is to defend the statement against the questioner who has been tasked with showing its falsity. Aristotle was of the opinion that dialectic was primarily meant for showing that statements are false, according to which showing a statement to be false is to produce [*yuntiǧ*] the opposite of the statement we seek falsity of [as a conclusion of a syllogism]. However, [dialectic] is suited primarily for showing the falsity [*ibṭāl*, of an opponent's statement]. [Dialectic] is suited for substantiating statements [*iṭbāt*] in a secondary way.¹⁶⁰

A dialectical exchange may be characterized as a dialectical competition, the aim of which is primarily refutation. The quaesitum (*maṭlūb*)—viz. a question in the form of a disjunction of a pair of contradictory or contrary statements whose ‘subject terms are universal’—organizes the exchange.¹⁶¹ For example, the quaesitum may be in the form of ‘*Aab or Oab*’, e.g. ‘Is all killing injustice or is some killing is not injustice?’, or ‘*Aab or Eab*’, e.g. ‘Is all killing injustice or is no killing injustice?’.¹⁶² Once R concedes to Q one of the disjuncts of the quaesitum, say *Aab*,¹⁶³ by means of question and answer, Q’s objective for the rest of the exchange is to elicit further concessions from R. Only these concessions and no others may be used by Q as his set

¹⁶⁰ Alfarabi, *ǧadal*, 14.2-9.

¹⁶¹ See Alfarabi, *ǧadal*, 13.6.

¹⁶² For this form of the quaesitum (*maṭlūb*), see Alfarabi, *Tahḥīl*, 96.

¹⁶³ The argument format is simplified here in order to focus on the formal logical aspects of the debate. In reality, Q does not know the thesis R is trying to defend. As a consequence, Q uses devices to try to get R to reveal the thesis to be overthrown. On the other hand, R tries to prevent Q’s discovering the thesis he has been tasked with defending by dissimulation, ambiguity, and misdirection. In fact, perhaps the majority of the debate is given to this sort of jockeying for position. In the post-classical period, the analysis and formalization of these methods became a scientific discipline in their own right called ‘*adab al-baḥṭ wa-l-munāẓara*’. For now and in the rest of the chapter, I systematically suppress these combative prolegomena. See also §2, Text 1 above.

S of premises by means of which he refutes R. Q successfully refutes R's claim 'Aab' when a subset of the premises conceded by R S* can be combined into a syllogism, the conclusion of which is identical to the other disjunct of the quaesitum. In this particular case, Q refutes R by constructing a syllogism from S* whose conclusion is 'Oab'. On the other hand, R successfully defends 'Aab' by not conceding premises that, when taken together, Q can use to construct just such a syllogism. Thus, R's objective is to allow Q a set of premises S, no subset of which can be combined to form syllogisms whose conclusion is 'Oab'. In other words, Q shows R's position is 'logically inconsistent' in the sense that he is able to derive from R's set of conceded premises according the rules of categorical or hypothetical syllogisms the contradictory or the contrary of the thesis R is tasked with defending.

In order to illustrate, consider the following highly implausible exchange—in fact, all the examples in this chapter are highly implausible—between R and Q. R concedes “All killing is injustice” (Aab). It thus falls upon Q to have R concede enough premises that allow him to construct a syllogism that concludes, “So, not all killing is injustice” (Oab). The argument, whatever its plausibility or implausibility, might go as follows:

Q: All killing is injustice or some is not? What do you say (Quaesitum: 'Aab or Oab')?

R: I will allow you that all killing is injustice (R concedes: Aab).

Q: Will you allow that everything is by God's decree?

R: Certainly.

Q: Then, will you allow that *this* act of killing was by God's decree?

R: I must (Universal Instantiation, (UI)).

Q: Do you concede, then, that some act of killing is by God's decree.

R: Certainly (Existential Generalization (EG), R concedes: Iaf).

Q: I think you will also concede that all that God decrees is justice; will you not?

R: I do (R concedes: *Afb*).

Q: Then you must concede also that nothing that God decrees is injustice. Is that right?

R: It is (R concedes: *Efb* by rule of obversion, $Axy \text{ } Ex\bar{y}$, for any x and y).

Q: But, if you concede that some act of killings is God's decree (*Iaf*) and you also concede that nothing of God's decree is injustice (*Efb*), then it seems you must concede that some act of killing is not injustice (*Oab*, by Ferio); is this not so?

R: You are right.

In the above scenario, R concedes the following set of statements $S1=\{Aab, Iaf, Afb, Efb\}$ as well as the concessions that $S2=\{\text{for all } x, x \text{ is by God's decree}\}$, from which immediately follows $S3=\{\text{this act of killing is by God's decree}\}$ by UI, from which *Iaf* in $S1$ follows immediately by EG. Thus, in the above exchange, R concedes all of $S=\{S1, S2, S3\}$, but of these Q only needs $\{Iaf, Efb\}$ to yield the other disjunct of the original quaesitum *Oab* by Ferio, $\{Iab, Ebc\} \text{ } Oac$ for any a, b , and c .

Let us now examine how the above considerations affect Alfarabi's thinking about conditional propositions and conditional syllogisms. This will be accomplished primarily by examining Alfarabi's recommendations for how to construct premises and syllogisms, a task which Alfarabi assigns to the topoi (*al-mawāḍi'*).

The topoi (*mawāḍi'*, sing. *mawḍi'*) occupy an important place in Alfarabi's logical theory.¹⁶⁴ Alfarabi does not restrict their use to dialectical investigation. In fact, they are, in

¹⁶⁴ There is not a great deal of secondary literature on dialectic and the topoi in the Arabic philosophical tradition. Nevertheless, see: M. Maróth, 'Die Rolle der Topik Avicennas in den arabischen Wissenschaften', *Acta Antiqua Academiae Scientiarum Hungariae* 29 (1981): 33-41; idem., *Aussagenlogik*, 88-99. See also N. Rescher, *The Development of Arabic Logic* (Pittsburgh: (University of Pittsburgh Press, 1964), 15032; idem. 'Al-Kindī's Sketch of Aristotle's Organon', in *Studies in the History of Arabic Logic* (Pittsburgh: University of Pittsburgh Press, 1963), 32-7; idem. 'The Logic Chapter of Muḥammad ibn Aḥmad al-Khwārizmī's Encyclopaedia, Keys to the Sciences (c. A.D. 980)', *Studies*, 74f; Black, *Rhetoric and Poetics*, 156-7; Lameer, *Al-Fārābī and Aristotelian Syllogistics*, 149; H. Hugonnard-Roche, A. Elamrani-

his view, central to all of the intellectual disciplines (*aṣ-ṣanā'i' al-fikriyya*). Alfarabi opens *Taḥlīl* with the following words:

[Text 4a] It is incumbent on us now to say how we find a syllogism for every quæstio that we hypothesize in any discipline, from where the syllogism is obtained, from which things we generate the premises of each syllogism that is sought for quæsitum, and the way [to generate them]. Above all, this is accomplished by familiarity with the topoi, viz. the universal premises whose particulars are used as major premises in each and every syllogism and in each and every discipline. [This is so] since each one of the universal topoi includes many particular premises, some of which are used in dialectic, some in rhetoric, some in the demonstrative sciences, and some in other intellectual disciplines.¹⁶⁵

Thus, despite their application in practically all of the philosophical disciplines, in Alfarabi's view the topoi still provide no more than rules of thumb for constructing premises that are easily adopted into categorical or conditional syllogisms. In this passage, the topoi are called “universal premises (*muqaddimāt kulliyya*)”, but this does not mean that the terms from which they are composed pick out individual objects subsumed under the categories, in the way that “animal” and “men” in the universal premise “all men are animals” pick out individual men and animals. Rather, the topoi are universal in the sense that they are intended as rules for constructing any number of premises, which are likely to gain the assent of the opponent. In making use of the topoi, the speaker is not necessarily interested in constructing true premises, as much as he is interested in constructing premises that are probable, or instill enough conviction in the mind to gain the opponent's assent. In order to gain the opponent's assent, these rules rely on the different ways that a predicate attaches to a subject. Often the topical rule is stated in terms the five predicables, which for Alfarabi (though not necessarily

Jamal, ‘Les topiques’, *Dictionnaire des Philosophes Antiques*, vol. no., ed. R. Goulet (Paris: Editions du Centre nationale de la recherche scientifique, 1989-2003, 524-526.

¹⁶⁵ Alfarabi, *Al-Manṭiq 'inda l-Fārābī*, vol. 2, ed. R. 'Ağam (Beirtu: Dār al-Mašriq, 1986), 95-129. Hereafter, I will refer to this work as follows: Alfarabi, *Taḥlīl*. The quote cited above is Alfarabi, *Taḥlīl*, 95. See Alfarabi's characterization of the topoi in Alfarabi, *Ġadal*, 68.

for Aristotle)¹⁶⁶ are five ways a universal predicate (*maḥmūl kullī*) Y attaches to a subject (*mawḍūʿ*) in response to the question “What is X?” (where X is some individual such as Zayd whom we see from afar and ask “What is that?”; Y is a universal predicate if two or more things share in the fact that Y is predicated of them).¹⁶⁷ If, to the question “What is X?”, we respond “X is Y”, then Y is predicated of X in five different ways. The predicate Y is called a “genus [*ǧins*]” when Y constitutes the substance (*ǧawhar*) of two (or more) things X₁ and X₂, in the most generic sense of what it is to be X₁ and what it is to be X₂. The predicate Y is called a “species [*nawʿ*]” when Y constitutes the substance of two (or more) things X₁ and X₂ in the most specific sense of what it is to be X₁ and what it is to be X₂. The predicate “animal” is, thus, called the “genus” of both Zayd and ‘Amr, because both share in the fact that “animal” is said of both of them in the most generic sense of the questions “What is Zayd?” and “What is ‘Amr?”. The predicate “human” is called the “species” of both Zayd and ‘Amr because both share in the fact that “human” is said of both of them in the most specific sense of the question “What is Zayd?” and “What is ‘Amr?”. On the other hand, if X₁ and X₂ share in the fact that both are called Y but not in way that we are speaking about the substance of X₁ or X₂, then predicate Y is called an “accident [*ʿaraḍ*]”.¹⁶⁸ If the predicate Y is used to respond to a question about what X₁ is, i.e. a question about X₁’s substance, in such a way that the substance of X₁ is distinguished from the substance of X₂ by the fact that the substance of the former has Y and the latter does not, then Y is called a “differentia [*faṣl*]”. Thus, “rational” is called a “differentia” because it is a predicate that distinguishes what Plato is from what Bucephalus is, though both are animals. If the predicate Y is used to respond to a question about what

¹⁶⁶ Cf. S. Abed, *Aristotelian Logic and the Arabic Language in Alḥārābī* (Albany, NY: SUNY Press, 1991), 2f.

¹⁶⁷ Alfarabi, *Al-Manṭiq ʿinda l-Fārābī*, vol. 1, ed. R. Aḡam (Beirut: Dār al-Mašriq, 1985), 55-62, 60. Hereafter, I cite this work as follows: Alfarabi, *Tawḥīʿa*.

¹⁶⁸ *Ibid.*, 61.

distinguishes X_1 from X_2 , but not in a way that is connected to the substance of X_1 or X_2 , then Y is called a “proprium [*ḥāssa*]”. The predicate “risible” is a proprium because it is a predicate that is used to distinguish Plato from Bucephalus, though the difference is not at the level of the substance of the individual.

Alfarabi uses topoi to generate categorical as well as conditional premises for a syllogism in a dialectical exchange. First, consider the following prescription for generating universal affirmative or negative categorical premises from “topoi from definition” in *Tahḥīl*.

[Text 4b] Among [the topoi] are those that are derived in the manner of definition. First, we find the subject [of the quaestium, sc. *maṭlūb*] and then see if the predicate of the quaesitum is in its [the subject’s] definition [*ḥadd*]. If it is, then it necessarily follows [*lazima bi-ḍ-ḍarūra*] that the predicate is in all of the subject. It is plain, then, that a first-figure syllogism is formed. Alternatively, if we find that [the predicate in the quaesitum] is absent from [*maslūban ‘an*, viz. the subject’s definition], it necessarily follows that the predicate is negated of all of the subject [of the quaesitum] and a syllogism in the first figure is formed.¹⁶⁹

We can schematize the scenario described by Alfarabi in the following way. The reasoner is debating a quaestium such as “either bats are hairy or bats are scaly”. The reasoner is charged with getting his opponent to give his assent to “some bats are hairy”. In order to do so, he looks at the definition of bat, which is “winged mammal”. Finding “mammal” in the definition of bat, the reasoner knows that, in fact, the predicate “mammal” applies not only to some bats but to everything referred to by the term “bat”. This yields the universal affirmative categorical premise “(all) bats are mammals”. This premise is then used as the minor premise in the following first-figure syllogism Barbara (following the premise-order in the Arabic tradition where the minor premise generally appears first) with “all bats are hairy” as the conclusion, the speaker reasons as follows “if (all) bats are mammals and (all) mammals are hairy, then (all) bats are hairy”. If the reasoner is charged with refuting the quaestium’s other

¹⁶⁹ Alfarabi, *Tahḥīl*, 101.

disjunct, he needs a syllogism that yields the contradiction or the contrary of the other disjunct in the quaestium, viz. the conclusion can be “some bats are not scaly” or “no bats are scaly works well”. Following Alfarabi’s prescriptions, the reasoner proceeds as follows. He persues the definition of “bat” and finds that the term “scaly” is not to be found in “winged mammal”. From this, he is able to construct a universal negative “no mammal is scaly”, which is used as the major premise in the following first-figure syllogism, which yields the contrary of the quaesitum’s second disjunct “no bats are scaly” as a conclusion: (Celarent, with minor premise appearing first): “if (all) bats are mammals and (no) mammal is scaly, then no bat is scaly”. The important point to notice in Text 4b is that the topical rules are set out based on relations of inclusion and exclusion that hold between terms falling under the five predicables. The reason why the above “topoi from definition” work is because the definition states the genus (*ğins*) “mammal” and differentia (*faşl*) “winged”, which allows us to generate a universal affirmative premise based on the fact that genus terms are those that are shared univocally among the species belonging to the genus. Analogously, the fact that “scaly” is not an element in the definition of “bat” entails that “scaly” is neither an element of the substance (*ğawhar*) of “bat” that it has in common with other objects (viz. it is not the genus of bat), nor is it an element of the substance of “bat” that distinguishes it from other members of the genus (viz. it is not the differentia). From the facts generated by the relations between genus and differentia, we can conclude that no bat is scaly, with the implicit assumption being that not bat is *essentially* scaly, though it may be scaly accidentally (*bi-l-‘arad*).¹⁷⁰

¹⁷⁰ I am thankful to Stephen Menn for encouraging me to rethink my analysis of how Alfarabi uses the topoi. I had originally claimed that the variables used to state the topoi are simply “linguistic entities” such as “terms” that we can attach universal quantifiers to. I now realize that what makes the topoi work, so to speak, are the relations of inclusion and exclusion (partial and complete) that the topical rules assume to hold between the terms that the topical rules take as objects. These relations of inclusion and exclusion are the basis for the theory of the five predicables as Alfarabi seems to have understood it.

Consider now a topos that generates a conditional premise, an example of which appears in *Ġadal*: “if X is in Y, then the contrary of X is in the contrary of Y”, possible instantiations of this rule include any number of premises such as the following: ‘if pain is evil, then pleasure is good’, ‘if God is perfect, then creation is deficient’, and ‘if men incline to injustice, then women incline to justice’.¹⁷¹ In this case, the use of this topos has generated a conditional sentences, which, despite the fact that the antecedent and consequent are indefinite categorical propositions, might reasonably be interpreted as universally quantified; thus, e.g. (P1) ‘if all pain is evil, then all pleasure is good’. Consider a scenario in which Q and R agree to debate the quaesitum ‘is all pain evil or not?’ In this, R decides to defend ‘all pain is evil, in which case Q takes on the task of constructing a syllogism or series of syllogisms that conclude (C) ‘some pain is not evil’ only from premises obtained from R. Using this topos, Q might proceed in the following way. He will try to convince R to concede P1, and then to concede further premises that allow Q to construct a (probably) categorical syllogism with (P2) ‘some pleasure is not good’ as the conclusion. With P1, P2 and modus tollens in hand, Q can then directly refute R by forcing him to concede C. It is this use of the topoi that Alfarabi seems to have in mind when he says near the beginning of *Tahḥīl*:

[Text 5] Once we have thoroughly familiarized ourselves with the topoi, then we analyze the quaesitum into a pair of contradictories, and we place each of them on its own as a thesis [*waḍʿ*] for which we seek to substantiate it [*īḥbātahu*] by producing it as a conclusion [of a syllogism], or show its falsity [*ibṭālahu*] by producing its opposite [*muqābil*] as a conclusion [of a syllogism].

Then we analyze the thesis [*al-waḍʿ*] into its predicate and subject, and we consider each of them in turn on its own. Then we sort carefully through [*nastaqriʿu, istiqrāʿ*] the topoi until we have gone through all of them. If we then

¹⁷¹ in *kāna* ‘š-šayʿu mawḡūdun fī amrin mā fa-diddu dālika ‘š-šayʿu mawḡūdun fī diddi dālika l-amri; literally, ‘if the thing is in something else, then the contrary of that thing is in the contrary of that something else’; Alfarabi, *Ġadal*, 68.2-3. It is convenient, though perhaps not entirely accurate, to think of topoi as premises whose quantifiers range over terms or sentences rather than individuals. The examples listed above are not Alfarabi’s.

find in the thesis we hypothesized or among its parts [sc. the predicate or subject] something that is characteristic of a topos familiar to us, then we have found the syllogism by which we show the thesis is true or show that it is false.¹⁷²

As in the imaginary exchange between Q and R in this section, Q analyzes the thesis defended by R ‘All pain is evil’ into its subject and predicate parts. The subject and predicate are found to be suited to have the topos ‘if X is in Y, then the contrary of X is in the contrary of Y’ applied to them. Applying this topos to the subject ‘pain’ and predicate ‘evil’ generates a premise ‘if all pain is evil, then all pleasure is good’, which is the key step in constructing the conditional syllogism that refutes R’s conceded statement. As alluded to above, the antecedent and the consequent of the conditional are *theses* obtained from the opponent in a dialectical exchange, and neither R nor Q needs to be committed to their being true or false. Said differently, since they are either hypotheses or derived from hypotheses, Q and R do not take the antecedent and the consequent, as a result of their use in a dialectical context, as being subject to assent or denial, *taṣdīq* and *takdīb*. In particular, Q’s interest in the conditional is chiefly as providing a way of constructing a conditional syllogism (in this case, modus tollens) that allows him to falsify R’s concession. The conditional does this by providing a connection between the hypothesis (antecedent) and another sentence (consequent). Q rests assured that R will accept the conditional ‘if all pain is evil, then all pleasure is good’ because it is authorized by the topos that ‘if X belongs to Y, then the opposite of X belongs to the opposite of Y’. Nor does it concern Q that this topos generates quite a large number of clearly sophistical conditionals. Yet, in dialectical and rhetorical exchanges it is not of chief importance that the conditional or its parts be true; they only need to be convincing to R so that he will concede them.¹⁷³

¹⁷² Alfarabi, *Tahṭīl*, 95–6.

¹⁷³ Black, *Rhetoric and Poetics*, 105.

However, it is not the case that the topoi are applied exclusively to the thesis' subject and predicate. In *Tahlīl*, which is Alfarabi's most exhaustive treatment of the topoi, the 'topos from implications' does not analyze the thesis into its subject and predicate elements. Instead, it operates directly on the thesis, generating a variety of forms of conditional proposition according to what purpose the conditional premise and syllogism will serve in the dialectical exchange.

[Text 6] And among the topoi are those derived from implications, viz. the topoi of existence and elimination [*wuḡūd wa-l-irtifā'*]. This is when we look into each of pair of theses making up the quaesitum and consider: 'by virtue of what P [lit. 'thing (*aš-šay'*)'] is the thesis; or 'what P is by virtue of the thesis' being'? So whatever kind of P we come across, we select it. Then, [1] if the topos that we selected is of the kind that the thesis is by virtue of P, we make P the antecedent and the thesis the consequent. We then assert [*nastaṭnī*] the antecedent to produce the thesis as it is, whether it is affirmative or negative. [And the deduction] will be in the first-figure of the connective conditional syllogisms [viz. *modus ponens*].

[2] If, on the other hand, what we find is that P is by virtue of the thesis, then we make the thesis the antecedent and P, i.e. the thing that we have come across, the consequent. Then we assert the opposite [*muqābil*] of the consequent, viz. that is the opposite of the thing we come across [viz. P], producing thereby the opposite of the thesis. [The conclusion of this syllogism] is the other part that is disjoined to [the thesis] in the quaesitum.

Or, [alternatively], we consider: by virtue of the elimination [*irtifā'*]¹⁷⁴ of what P is the thesis eliminated; or what P is eliminated by the elimination of the thesis?

¹⁷⁴ The adjective 'negative (*salbī, sālib*)' describes the quality of the sentence, indicating that the sentence is such that it possesses a negative particle in its logical structure. Contradiction (*tanāquḍ*) and contrariness (*taḍādd*), and opposition (*taqābul*) generally, are best understood as characterizing the quality of the logical relation *between two sentences* and in relation to each other. Thus, a single sentence might be described as 'negative', but only a pair of sentences can be contradictory or contrary, each with respect to the other. On the other hand, *irtifā'*, literally 'elimination' and similar in its import to the phrase 'negated of (*maslūban 'an*)', is a cognitive or linguistic act carried out by the reasoner on a sentence of a given quality that converts the quality of the sentence to its opposite. To this extent, 'negation' would be an appropriate translation of *irtifā'*, which conveys Alfarabi's intended meaning, if we keep the following point in mind. Though *irtifā'* behaves in some ways like propositional negation, it should not be understood a purely linguistic unary function that takes a sentence as an operator and generates a sentence as a value. It often has, though not necessarily in this particular passage, a metaphysical counterpart. Sometimes, when we say about something that it has been eliminated or negated (e.g. 'elimination of the thing (*irtifā' aš-šay'*)), we do not mean exclusively that a *sentence* has been negated, but sometimes, the

Then, if we come across P, by virtue of whose elimination the thesis is eliminated, we make P's elimination the antecedent and we attach it [*ardafnāhu*] to the thesis. We then assert the antecedent, yielding the elimination of the thesis, in such a way that, if the thesis were affirmative [*mūǧib*], it would become negative and if the [thesis] were negative [*sāliban*], it would become affirmative. In general, the conclusion will be the opposite [quality] of P. Thus, [by virtue of the deduction] P is shown to be false. This latter topos is used to show every thesis that one hypothesizes is false.

If we come across a P that is eliminated by virtue of the elimination of the proposition [*qaḍiyya*, i.e. sc. the thesis, *al-waḍ'*] we hypothesized, then we make the elimination of the proposition the antecedent, and the elimination of P the consequent. Then we assert [*nastaṭni*] the opposite of the consequent to produce the existence of the thesis. So the previous topos was for showing a thesis is false [*ibṭālihi*], and this latter one is for showing that a thesis is true [*itbātihi*].¹⁷⁵

In this topos, the subject and predicate play no role at all. Instead of the predicate 'belonging to' a subject as in a categorical thesis, this topos describes the thesis as 'being by virtue' of something else, or being absent or eliminated by virtue of the presence or absence of something else. Whereas the former approach relates terms from the categories to terms from the categories, the latter is a way of relating facts, conditions or events to each other, in the way that (the fact, condition or event of) its being day is by virtue of the (fact, condition or event of the) sun's being out. 'Implications', between antecedent and consequent for Alfarabi are best expressed using conditionals because they are a linguistic form that speaker conventionally use to express the way in which the presence or absence of a fact, condition, or event is somehow *by virtue of* (i.e. connected to, responsible for, related to, causally associated with, prior or posterior to) the presence or absence of another fact, condition, or event. Indeed, Alfarabi mentions that this topos is used most often to argue—often speciously in

absence of a condition outside the soul that is in line with what the sentence expresses about it. In a complementary way, the *presence* of the thesis (*wuǧūd al-waḍ'*), say, means the presence of a condition outside the soul that is in line with what the thesis expresses about it.

¹⁷⁵ Alfarabi, *Tahṭīl*, 102.

Alfarabi's view--about causal relations between events (e.g. the cutting of this or that nerve being the cause of paralysis) or between substances (e.g. the sense in which the being of animal causes the being of human). Thus, it seems that the chief virtue of the 'topoi from implications' in relation to arguments like this is that they allow us to formalize arguments about causes as conditionals that are then used to construct conditional syllogisms. Thus, Alfarabi's presentation of the 'the topoi from implications' might be used in a dialectical context in the following way.

Q and R are out for a walk at noon. What they can only make out as a small, black object flies past them quickly some distance off. They agree to clarify the kind of which the object might be a member according to these argumentative parameters: *quaesitum*: 'it is either a bat or not'; R opts to defend 'it is a bat'. Q then peruses the zoological data related to bat behaviour available to him, and proposes that R concede (P1) 'if it is a bat, then night has fallen'. R, rashly, concedes, but before he realizes his error (he conceded P1 without considering his current circumstances, viz. that it is noon), Q quickly asserts (P2) 'but it is noon so night cannot have fallen'. Too late to retract his concession, R grudgingly concedes P2. Q triumphantly produces the contradiction of R's thesis ('it is not a bat' from P1 and P2 by *modus tollens*).

As usual, in the dialectical context described above, this *topos* from implication is being used negatively, i.e. in order to show that the thesis defended by R cannot be correct. Clearly, R and Q are hardly any closer to discovering what the object is, though they are that much closer to knowing what the object is not. The 'implications' that Q peruses in this example are data about phenomena that bear some sort of connection to the phenomenon expressed in the thesis. Though Alfarabi says that this *topos* is usually used to talk about causes,¹⁷⁶ *nightfall causes* bats to appear in the sky only in a secondary or derivative sense. If the fact that bats come out at night is due to features of their physiology or their nocturnal feeding patterns, then perhaps these are more accurately called 'causes' of bats taking wing at nightfall. If this is so,

¹⁷⁶ Alfarabi, *Tahḥīl*, 104ff.

then nightfall might be called an inseparable yet accidental concomitant of bats flying at night rather than a cause as such. The point here, however, is that in both cases R intuitively accepts the conditional ‘if bats are out, then night has fallen’ because he recognizes (without necessarily knowing why) that the phenomenon expressed in the antecedent somehow or other *is by virtue of* the existence of the phenomenon expressed in the consequent. This topos is thus not concerned with investigating the nature of connection between the phenomena (causal, purely accidental, inseparable accidental, relational, mathematical, etc.), but simply takes advantage of conventional intuitions to generate conditional propositions. Throughout, the aim of the opponents in the schematic argument described above is investigating the phenomena expressed in the antecedent and consequent, not with the connection expressed by the conditional sentence as a whole. R gives his assent to the conditional because it expresses a real connection that exists between the phenomena expressed in the antecedent and consequent. This connection, however, is taken at face value, and does not itself become the object of investigation in the debate.

With respect to the foregoing discussion of dialectic and the topoi with respect to conditional sentences, the argumentative context in which conditional sentences are used endows Alfarabian conditionals with peculiar, but deeply-ramified formal properties. As we observed in §2, the reasoner does not give his assent to the antecedent and consequent *qua* parts of a conditional when he gives assent to the conditional as whole. However, after examining the structure of dialectical and rhetorical argument, we can see the reason for this seeming peculiarity. As we have observed in this section, conditionals arise in argumentative contexts in which the interlocutors debate a particular thesis that is subject to dispute. The interlocutors may harbour motives for engaging in argument that are geared more toward

getting an audience, or a single listener to give its assent to a desired opinion rather than showing that the opinion is true. In the arguments with conditional syllogisms described above, it is usually the case that the disputed thesis or the thesis we want the listener to give his assent to is the antecedent, the consequent, or their contradictory opposites. Thus, as we saw in the case of conditional topoi, assent to the antecedent, the consequent, or their contradictory opposites is usually the very point under dispute. To that extent, it would be circular to expect the parties to the debate to give their assent to the conditional based on their prior knowledge of the truth of either the antecedent or the consequent. Rather, the parties to the debate will be induced to give their assent to the conditional based on their recognition that the *connection between* the antecedent and the consequent, the nature of which is usually dictated by the topoi, corresponds to the states of affairs. Consider the two conditionals generated from the two topoi discussed above: (a) ‘if all pain is evil, then all pleasure is good’, and (b) ‘if bats are out, then night has fallen’. The advertised purpose of the topoi is to generate premises that are, if not true, then at least engender some sort of compliance in the listener’s mind; that is, they do not at first glance strike the listener as so implausible as to be rejected outright. As for (b), the reason why R might be tempted to accept it is due to his observations of bat behaviour in the past. In particular, he has no doubt observed that, without exception, bats never fly while the sun is up. In Alfarabi’s technical language, this could be expressed by saying that the bats’ being (*wuǧūd*) implies the sun’s elimination (*irtifā*). In the examples explicitly discussed by Alfarabi, the reasons that justify R’s belief about why, say, bats only come out at night, are not important in the course of the argument. The point is that it is R’s recognition of a connection between the phenomena that leads R to concede the conditional sentence expressing this connection. In fact, since Q and R

are trying to settle what exactly the black shape flying in front of them is, then their assent to the conditional cannot be based on their assent to the antecedent or consequent, or their knowledge that they are true or false. In (a), the topoi ‘if Y belongs in X, then the opposite of Y belongs in the opposite of X’ describes a certain connection that a reasoner might *reasonably* accept as existing between two things, events, states, or conditions which are expressible in a subject-predicate format. The point of the argument between Q and R with (a) as a major premise is to settle, even if negatively, whether all pain is evil. R is thus not likely to give assent to the conditional ‘if all pain is evil, then all pleasure is good’ based on his conviction that the consequent is true without any consideration given to the truth of the antecedent. In this case, R might feel justified in conceding ‘if all pain is evil, then all pleasure is good’ is due to a certain, obviously specious but no less convincing, connection we recognize to exist between opposites such as pain/pleasure and evil/good. Thus, in Alfarabi’s way of thinking, the use of conditionals in a dialectic, and presumably a fortiori, in a rhetorical, context is not interpretable as the speaker asserting, or adopting a definite cognitive attitude toward, either the antecedent or the consequent. Rather, he gives his assent to the conditional to the extent that he recognizes that the connection between the antecedent and consequent that the conditional signifies really does exist, viz. that the antecedent and consequent are connected in such a way that the listener will give his assent to the consequent given his assent to the antecedent. Clearly, then, Alfarabi’s view of conditionals is not amenable to an interpretation in which the truth or falsity of the conditional is in any way dependent on the truth or falsity of its constitutive parts. This is not merely to say, however, that Alfarabi’s conditionals do not appear to be amenable to a truth-functional interpretation of ‘if...then...’ sentences as material implication. Material conditionals are true in all cases in which the antecedent is false. By

contrast, in the context of a dialectical or rhetorical exchange, it does not make sense for two interlocutors to make a hypothesis (*wadʿ*) about something they already know to be false or impossible. It seems that in Alfarabi's way of thinking about the use of conditionals in dialectical exchanges, a conditional with a false or impossible antecedent would violate a pragmatic assumption that the conditional be useful in a debate. Nor are Stalnaker-Lewis interpretations appropriate either. In order to evaluate whether or not an Alfarabi conditional is true, we are not required to look to see whether the consequent is true according to a 'stock of beliefs' that are hypothetically altered to make the antecedent true.¹⁷⁷ Rather, the focus is on what the listener will give his assent to given that he gives his assent to the antecedent.¹⁷⁸ In non-demonstrative contexts, for example, what is objectively true and what the audience gives its assent to are not coextensive.

Finally, it is also important to realize that the most common form of atomic sentences that will serve in the antecedent and consequent position in a conditional are categorical propositions of the form *Aab*, *Eab*, *Iab* or *Oab* for any *a* and *b* that can be plausibly located in one of the ten categories. Nor is there any reason to believe that Alfarabi would disallow singular terms as subjects: '*s* is *b*' or '*s* is not *b*', where *s* is a *this* and *b* is plausibly located in one of the ten categories. What is more, we find in *APCA* that Alfarabi allows fact-like or event-like propositional contents to be expressed by the antecedent and consequent of a conditional. For example, the following 'event-like' propositions are used as antecedents and consequents: 'Zayd comes', 'Amr departs', 'Sirius rises in the morning', 'the day will be hot and the rain will

¹⁷⁷ R. Stalnaker, 'A Theory of Conditionals', in *IFS*, 44.

¹⁷⁸ Stalnaker, 'A Theory of Conditionals', 43 says: 'According to this line of thought, a conditional is to be understood as a statement which affirms that some sort of logical or causal connection holds between the antecedent and consequent. [In order to determine whether a conditional understood in this way is true], you should look, not at the truth values of the two clauses, but at the relation between the propositions expressed by them'. As we will see, the connective scheme to conditional evaluation is what informs Alfarabi's thoughts about conditionals.

cease', 'it is day', and 'the sun is out'.¹⁷⁹ While none of these propositions is obviously expressible in the form of a universal or particular categorical proposition, it is easy to see how they could serve as one part of a dialectical quaesitum, e.g. 'Either Zayd comes or he doesn't', or 'the rain will cease or it won't'. Thus, despite Alfarabi's claim in *Ġadal* that 'the only thing sought from a quaesitum is whether the predicate belongs to the subject or not', which, if taken strictly, would restrict the types of quaesitum to disjunctions of subject-predicate categorical propositions, practically speaking, Alfarabi does not seem strictly committed to such a view.¹⁸⁰ What such a view does exclude, however, is the very possibility that we can have a debate or argue about the truth and falsity of a conditional, or that we can try to elicit assent to a conditional *as the main objective of an argument*. This is true to the extent that Alfarabi does not seem to consider the case in which the quaesitum itself is composed of conditionals, i.e. that the quaesitum can be of the form 'either if P, then Q, or not-(if P, then Q)'. In order to develop such an argumentative scheme, Alfarabi would have to develop, one, a conditional syllogistic that allows nested conditional; two, a conditional syllogistic that can yield conditional conclusions; and three, a doctrine of conditional contradiction. In the logical structure of the dialectical exchange, the disjuncts of the quaesitum circumscribe the choice of opinions R can defend and Q can refute. Say, for instance, that R decides to defend 'if P, then Q'. Then Q's task is to get R to concede premises that allow Q to construct a *conditional* syllogism that yields the *contradictory* of 'if P, then Q' as a *conclusion*. This single requirement requires that Q know what the contradictory of a conditional is, that he be able to construct a conditional syllogism in the moods of modus tollens or modus ponens whose antecedent and consequent are the conditional 'if P, then Q' or its contradiction, and that he be able to

¹⁷⁹ Alfarabi, 'Al-Fārābī's Paraphrase of the Categories of Aristotle', trans. D. Dunlop, *The Islamic Quarterly* 5 (1959): 34. Hereafter, I will cite this work as follows: Alfarabi, *APCA*.

¹⁸⁰ Alfarabi, *Ġadal*, 82.

generate the contradictory opposite of ‘if P, then Q’ as a conclusion. But Alfarabi does not theorize at any length about any of these three items in his extant logical works.

This is not to say that Alfarabi did not formulate or discuss such doctrines in other works that we no longer possess. Indeed, it is highly unlikely that this could be the case. Alfarabi must have been aware that there were late antique logicians, such as Galen and Boethius,¹⁸¹ who espoused the view that the conditionals ‘if A, then B’ and ‘if A, then not-B’ are logically inconsistent, in the sense that a speaker will never assert both simultaneously.¹⁸² Indeed, Avicenna explicitly mentions the existence of such a view (and goes to great lengths to refute it) in the book 5 of *Qiyās of Šifā’*. Similarly, it is well-documented that late antique Peripatetics like Boethius developed hypothetical syllogisms that allowed conditionals and nested conditionals as premises and conclusions. The truly perplexing question in my view is this: why was not Alfarabi sufficiently impressed by the importance of these issues to include them in his epitomes, even if only to mention them in passing? The above discussions in this section and the last perhaps provide at least a partial explanation. In Alfarabi’s use of conditionals, it seems to be the case that the nature and purpose of the quaesitum determines what formal features of conditionals are developed and what formal features are not. Not only in *Ġadal*, but also in *Taḥlīl*, which presents practical guidelines for constructing arguments in *all*

¹⁸¹ C. Martin, ‘The Logic of Negation in Boethius’, *Phronesis* 36/3 (1991): 279; idem. ‘Denying Conditionals: Abaelard and the Failure of Boethius’ Account of the Hypothetical Syllogism’, *Vivarium* 45 (2007): 153-68; cf. R. Stalnaker, ‘A Defense of Conditional Excluded Middle’, in *IFS*, 87-104.

¹⁸² This is how the contradiction of indicative conditionals is sometimes interpreted for natural language, e.g. E. Adams, ‘The Logic of Conditionals’, *Inquiry* 8 (1965): 184. In this article Adams is interested in giving an analysis of indicative conditionals as they are used in natural language. In Adams’ view, a conditional expresses the probability that a reasoner will assert the consequent given a certain probability that the antecedent. One pragmatic assumption in this theory is that a speaker will never be justified in asserting a consequent when he knows that the antecedent is false. Adams says (ibid., 178) “a pair of conditional statements of the form ‘if p then q’ and ‘if p then not q’ are seldom if ever justifiably asserted on the same occasion. When such a pair of statements is made on the same occasion, it is usually the case that one is asserted in contradiction to the other, and this carries the implication that the contradicted statement is false or at least that it may be justifiably denied (and non-vacuously)”.

of the sciences, arguments tend to be limited to disputes about predicable-type relationships (species/genus, definition, differentia, property accidental or otherwise, individual substance (i.e. *this-es*), and existence/non-existence). As a consequence, Alfarabi consistently presents the quaesitum, a disjunction of two opposite sentences with interrogative force, in the form of a categorical proposition such as ‘is *Aab* or *Oab*?’ Of course, the purpose of the quaesitum is to provide the basic standard by which a thesis is overthrown in an argument, and to organize the initial conditions of the inquiry (what Q aims to derive, what R thesis defends). Yet, the formal property of the disjuncts as exclusively categorical propositions circumscribes the scope of the types of question that can be asked in such exchanges. In other words, it is difficult to ask questions about causes, for example, which are, as we saw in the discussion of the ‘topoi from implications’ best formalized in conditional sentences. Yet, the quaesitum-based dialectical exchange which focuses on arguments organized around predicable-type questions makes it so that conditionals do not appear as elements of the quaesitum. But the placement of conditionals in the quaesitum demands the explicit formulation of both a rule about the formal contradiction of a conditional, and a doctrine of conditional syllogism that, at the very least, could countenance conditionals as conclusions of conditional syllogisms. It is one thing for R to concede ‘if A, then B’ as a thesis, but Q must know, at least with regard to form, what is required to contradict such a conditional in order to refute R’s concession. Also, the quaesitum-based argument format requires that Q refute R by producing the contrary or contradictory of R’s thesis as a *conclusion* of a conditional syllogism. Thus, Alfarabi would also be required to develop a conditional syllogistic that can not only take conditionals (and nested conditionals) as premises, but also produce conditionals (and nested conditionals) as conclusions. Alfarabi’s silence on these important points is, at first glance, surprising. It

seems, however, that when we consider the dialectical backdrop in which conditionals are used, we see that the quaesitum-based argumentative structure that focuses on predicable-type topoi and arguments makes the development or discussion of such doctrines unimportant if not unnecessary.

§4 CLASSIFICATION OF IMPLICATION (*LUZŪM*) IN APCA: CONDITIONS OF ASSENT, CONTRADICTION

In APCA §58 Alfarabi directly links the notion of necessary implication (*luzūm bi-ḡ-ḡarūra*) to the formation of connective conditional propositions.

[Text 7] Things between which there is necessary following [*mutalāzimāt*] are the things from which connective conditionals are composed. Things between which there is opposition are the things from which disjunctive conditionals are composed. It is an additional feature of [the propositions] that are characterized by complete following that if either the antecedent or the consequent is asserted [*yustaṭnā*], then the other follows from it [*lazima ‘anhu l-āḡar*, sc. as conclusion], and if the opposite of one of them is repeated, then necessarily the opposite of the other follows from it. As for [the propositions] whose following is incomplete, it is only appropriate to except the antecedent or the opposite of the consequent in order for it [sc. the set of premises, i.e. the conditional major and repeated minor] to become a syllogism.¹⁸³

It is difficult to overemphasize the importance of this move since it allows (1) all implication relations to be formalized in the language of conditional propositions; and (2) it allows us to construct valid syllogisms based on the standard schema, primarily modus tollens (MT) and modus ponens (MP), with conditional major premises; finally, (3) it allows us to reason demonstratively about sentences between which relations of necessary implication exist. Nevertheless, this should not make us lose sight of the fact that Alfarabi also recognizes types of implication that are not necessary. In other words, Alfarabi says that an antecedent implies a consequent even if the connection between them is not necessary. Thus, in this section, out

¹⁸³ Alfarabi, APCA, §58, 35.10-14.

task will be to explore how Alfarabi speaks about the different grades of implication that exist according to the different strengths of connection between antecedent and consequent. In Alfarabi's treatment of conditionals, the conditional sentences (1) 'if Paul comes, Peter goes', (2) 'if it is windy, then it is cool', and (3) 'if this shape is square, then it has four sides' are formalized indifferently as 'if P, then Q'. However, Alfarabi would also say that (1) signifies accidental implication (*luzūm bi-l-'araḍ*), (2) implication for the most part (*'alā l-akṭar*), and (3) necessary implication. Thus, the notion of connection underlies each of these types of conditional, but the nature of the connection in each is different.

In fact, we are already quite familiar with Alfarabi's discussion of implication from his development of the 'topoi from implication' from *Tahḥīl*. The 'topoi from implications (*mawāḍi' mina l-lawāzim*)' are rules for forming conditional sentences that rely on common intuitions about *by-virtue-of* relations—these are, as Alfarabi notes, often causal relations¹⁸⁴—that hold between pairs of events, states, substances, accidental or essential properties, or phenomena, taken broadly. These topoi rely, in particular, on the reasoner's intuitions about how some Y (be it a state, event, substance, property or phenomenon) is somehow or other 'by virtue of (bi...)' X (be it some other state, event, substance, property or phenomenon). With this intuition in hand, the reasoner is then justified in constructing a conditional sentence where the sentence expressing Y is the consequent and the sentence expressing X is the antecedent.

¹⁸⁴ The important relation between the notion of causality as a basis for our use of conditionals in everyday speech is recognized in the philosophical literature. For example, speaking about counterfactual conditionals, Dorothy Edgington says: 'it is worth adding that subjunctive conditionals are supposed to do a lot of work for us within philosophy, as well as in ordinary life. They have been used to 'analyse' causation, dispositions, laws, and play a large part in some accounts of perception and knowledge. On the first, causation, I think we need to appeal to causal notions to get subjunctive conditionals right, and the order of explanation goes that way round'; D. Edgington, 'On Conditionals', in *Handbook of Philosophical Logic*, vol. 14, ed. D. Gabbay, F. Guenther (Dordrecht: Springer, 2007), 216. See also J. Collins, 'Counterfactuals, Causation, and Preemption', in *Philosophy of Logic*, ed. D. Jacquette (Dordrecht: Elsevier B.V., 2007), 1127-43; J. Williamson, 'Causality', in *Handbook of Philosophical Logic*, vol. 14, ed. D. Gabbay, F. Guenther (Dordrecht: Springer, 2007), 95-126.

These topoi allow us to say ‘X implies Y’, to the extent that it is possible to say that Y is (or is not) by virtue of the being (or non-being) of X.

Alfarabi does not take the notion of implication as the object of investigation in *Tahḥlīl*, but he does, and in quite some detail, in APCA.¹⁸⁵ As we have seen in previous sections, Alfarabi’s classification of the types of implication in APCA is sensitive to the argumentative contexts in which conditionals expressing implicative relations appear as premises. Alfarabi’s tendency to give pride of place to what he calls ‘necessary implication (*luzūm bi-ḍ-ḍarūra*)’ is characteristic of the well-known, if not somewhat problematic, Peripatetic lionization of demonstration and demonstrative premises, while still attempting to maintain a place for dialectical, rhetorical and even poetical reasoning.¹⁸⁶ As such, Alfarabi’s classification of implication also includes subdivisions such as ‘per accidens implication (*luzūm bi-l-‘araḍ*)’ and implication for the most part (*‘alā l-aḳṭar*), which, based on the examples Alfarabi provides, seem to correspond to types of implication in rhetorical and dialectical conditional syllogisms respectively. Thus, despite Alfarabi’s giving pride of place to necessary implication, it is

¹⁸⁵ The motivation for his discussion of implication appears to arise out of questions surrounding the meaning of the Greek expression *hē tou einai akalouthēsis*, which is translated in the Arabic *Categories* as ‘*luzūm al-wuḡūd*’ (K. Georr, *Les Catégories d’Aristote dans leurs versions syro-arabes* (Beirut: Institut français de Damas, 1948), 243) and in English as ‘implication of existence’ (e.g. Aristotle, *Aristotle’s Categories and De Interpretatione*, trans. T. Ackrill (Oxford: Clarendon Press, 1963). This expression, which is found at *Categories* 14a30, 35, 14b15, 30, 15a9, is used in the chapters on priority, posteriority, and simultaneity. In this context, ‘implication of existence’ is often said to be ‘reciprocal (*pros antistrephonta*, *bi-t-takāfu*)’ or not (Georr, *Les Catégories d’Aristote*, 241), and ‘of necessity (*bi-ḍ-ḍarūra*, *ex anagkēs*)’ (ibid., 230) or not, i.e. ‘accidentally (*bi-l-‘araḍ*, *kata sumbebēkos*)’, (ibid., 233). Alfarabi’s wording in his epitome of the *Categories* closely matches the Sergius of Rašaina’s (d. 536) Arabic translation of Aristotle, often word for word; on Sergius of Rašaina, see ibid., 17–24. As we will see, Alfarabi moves substantially beyond Aristotle’s text just when he explicitly connects the discussion about the being of something following from something else with the construction of conditional and disjunctive premises and syllogisms in a way that strongly recalls his discussion of the ‘topoi of implications’ in *Tahḥlīl*. Though al-Ḥasan b. Suwār’s (born in 942) marginal notes on the *Categories* make no mention of this constellation of issues, such an obvious concern with showing the intertextual consistency in between the *Categories*, *Topics*, and the late antique discussion of hypothetical syllogisms suggests that Alfarabi’s ideas in APCA grew out of a late antique commentary tradition that seems to have existed no later than Proclus (d. 485). As noted by Fritz Zimmermann (Alfarabi, *Long Commentary*, 128, n. 3), in his long commentary on *De Interpretatione*, Alfarabi’s condemnation of Proclus’ incomprehension of Aristotle’s doctrine of metathetic sentences adopts Proclus’ use of reciprocal and nonreciprocal implication (ibid., 123–31) in order to clarify Aristotle’s meaning (at *De Interpretatione* 20a20–3).

¹⁸⁶ Black, *Rhetoric and Poetics*, 35.

important to keep in mind that, for Alfarabi, there is no single “correct” reading of conditionals of the form ‘if P, then Q’. Rather, the sort of implication expressed by conditional sentences, divided according to the weakness or strength of the connection between the antecedent and consequent, depends crucially on the argumentative context in which the conditional is deployed.

As in *Tahḥīl*, in *APCA* two things are called implicates (*mutalāzimān*), or to stand in a relation of implication, when ‘if one of them is, then the other is by virtue of the being [of the first] (*idā wuġida aḥaduhumā wuġida al-āḥaru bi-wuġūdihi*)’.¹⁸⁷ As we saw in the previous two sections, Alfarabi does not strictly delimit the types of sentential content that can be expressed by implicates, in the sense that he allows content that is amenable to expression in subject-predicate sentences, and content that is not, e.g. sentences talking about facts and states of affairs. Nevertheless, as we discussed in the last section, Alfarabi does not here seem to consider it important to consider in what sense an *implication* might be by virtue of the being of *another implication*, i.e. a relation of implication between pairs of implications. In general then, the types of thing that are said to imply something else are, broadly speaking, states of affairs on the one hand and predicable-type relations on the other.

Alfarabi’s analysis of implication begins with a division of the types of implication according to whether the antecedent implies the consequent *per se* (*bi-d-ḡāt*) or *per accidens* (*bi-l-‘arḍ*). This analysis of implication, in the final analysis, reduces to an examination of the strength of the connection between the antecedent and the consequent. The antecedent’s implying the consequent *accidentally* signifies the weakest form connection between antecedent and consequent (if there is any connection at all), whereas the antecedent’s

¹⁸⁷ Alfarabi, *APCA*, 34.

implying the consequent necessarily signifies the strongest connection between antecedent and consequent. As an example of *per accidens* implication, Alfarabi gives us the conditional proposition ‘if Zayd comes, then ‘Amr departs’. In this case, there is not any clear principle that gives insight into why ‘Amr’s departure, as expressed in the consequent, should be by virtue of, or responsible for Zayd’s arrival, as expressed in the antecedent. Alfarabi’s words suggest that it simply happens to be the case that ‘Amr’s departure and Zayd’s arrival coincide. Thus, it seems that Zayd’s arrival implies ‘Amr’s departure in a purely accidental way, in the sense that there is no underlying principle requiring the inseparability or perpetual concomitance of ‘Amr’s departure and Zayd’s arrival. In this weakest sense of implication, it would be unintuitive, but no less correct, to say that, for example, ‘Socrates is white’ *implies* ‘the sun is round’.

[Text 8] The consequent may follow *per accidens* [*qad yakūnu lāziman bi-l-‘araḍ*], as when we say ‘if Zayd arrives, then ‘Amr departs’—in the case that this happened to occur at some moment. For ‘Amr’s departure is a consequent of Zayd’s arrival, but [a consequent] *per accidens*.¹⁸⁸

On the other hand, an antecedent implies the consequent *per se* when the following of the latter from the former is not accidental, in the sense that there is presumably some underlying principle, or set of them, that determines that when the antecedent is, the consequent is by virtue of the antecedent.

[Text 9] A consequent [that is implied by the antecedent] *per se* may be implied [1] for the most part [*‘alā l-akṭar*]. For example, in the statement ‘if Sirius reaches the zenith in the morning, then the heat will intensify and the rains will cease’, this [viz. the intensity of the day’s heat and the cessation of rain] is a consequence of Sirius appearing on the horizon [*lāzimun li-ṭulū‘i ‘š-šī‘rā*], but one that happens for the most part. Or, [2] a consequent [that is implied by the antecedent] *per se* may follow of necessity [*bi-ḍ-ḍarūra*]. This [type of consequent] is implied perpetually [*ad-dā‘im al-luzūm*] and it is inseparable from

¹⁸⁸ Alfarabi, *APCA*, §56, 34.9-11.

the thing by virtue of whose existence it exists [*lā yumkinu an yufāriqa ‘š-šay’a l-laḍī bi-wuğūdihi wuğida*]. Whenever the thing is, the consequent is, and it is never at any moment unaccompanied by <the consequent>.¹⁸⁹

It is clear that the two types of *per se* implication presented here, namely, implication for-the-most-part and implication of necessity, do not hold between antecedent and consequent out of chance. The consequent’s (the intensification of heat and cessation of rain) being implied by the antecedent (Sirius’ passing the zenith in the morning) for-the-most-part seems to be due to the fact that astronomical phenomena have some sort of regular, law-like, though not entirely determinative effect on the weather and other natural processes in the world. In other words, implication *per se*, but for the most part, seems to be due to physical laws that are, nevertheless, not entirely deterministic. For example, as a matter of fact, cloudless winter days in Montréal are generally colder than days where there are clouds. Of course, it happens that sometimes a winter day is clear and unusually warm. Yet, in spite of knowledge of these exceptions, Alfarabi would not say that someone who says “it is a clear, winter day in Montréal” implies ‘it is a cold’ has spoken wrongly. Rather, Alfarabi wants to include conditional premises, likely in dialectical arguments, that while not true eternally and unchangingly, are true often enough to be acceptable in non-demonstrative contexts. The connection between the antecedent and consequent in this type of implication is strong enough to be ‘widely-accepted’, but not so strong that it is falsified by an instance or instances of the antecedent not coinciding with the consequent.

On the other hand, necessary implication, which is also classified under the *per se* division, represents the strongest type of implication between the antecedent and consequent envisioned by Alfarabi. Furthermore, just as demonstrative *categorical* syllogisms must have

¹⁸⁹ Alfarabi, *APCA*, §55, 34.11-16. Note Alfarabi’s explicit identify of alethic and statistical necessity.

premises in which the predicates being in the subject is necessary, Alfarabi likely envisioned conditionals with a necessary connection between antecedent and consequent as being primarily suited to demonstrative *conditional* syllogisms. In a general sense, necessary implication represents a type for which it is impossible that the antecedent ever be without being accompanied (or not accompanied) by the consequent. However, Alfarabi subdivides *per se* necessary implication into complete implication (*luzūm tāmm*) and incomplete implication (*luzūm ġayr tāmm*). Alfarabi describes the former in the following words:

[Text 10] Two things between which is complete following are such that if either of them is, then the other necessarily is by virtue of it, viz. if the first of them is, then necessarily the second is, and if the second is, then necessarily the first is.¹⁹⁰

It is important to note that the order of the antecedent and consequent in this type of implication relation is unimportant. Whichever of X or Y happens to be (*ittafaqa*), then the other is by virtue of it. This is not the case with incomplete following, in the sense that the which proposition is treated as the given is important. Alfarabi says:

[Text 11] Two things between which is incomplete following are such that if the first is, then the second necessarily is, but if the second is, it does not follow of necessity that the first is [*lam yalzam ġarūratan wuġūdu l-awwal*]. These are two things for which the being of one does not follow from the being of the other reciprocally [*humā allaḡānī lā yatakāfānī fī luzūm al-wuġūd*]. This is like man and animal. For if man is, then animal is of necessity, but if animal is, it does not follow of necessity that man is.¹⁹¹

With respect to incomplete following, the order in which the antecedent and consequent are spoken is important, since if X is given, then it is necessary that Y is, though the converse does not follow. Consider, for example, two of Alfarabi's favourite examples: (a) 'if the sun is up, then it is day', and (b) 'if human is, then animal is'. In (a), the antecedent and the consequent

¹⁹⁰ Alfarabi, APCA, §57, 34.17-19.

¹⁹¹ Alfarabi, APCA, §57, 34.20-35.4.

stand in a complete relation of implication. This means that the connection between the antecedent and consequent is such that it is impossible for the consequent to not be when it is given that the antecedent is. But since the implication is complete, it does not matter which of ‘it is day’ or ‘the sun is up’ is given to be. This means that in complete implication the antecedent and consequent are convertible, meaning that switching the order of the antecedent and consequent—whichever of the pair of propositions is hypothesized as the given—will not affect the truth of the conditional. In (b), on the other hand, the ordering of the antecedent and consequent is important insofar as it is important which of ‘human is’ and ‘animal is’ is assumed to exist. If human is taken to be, then it is certainly impossible that human is but there is no animal. However, on the assumption that animal is, it is certainly quite possible for an animal to be and it not be a human. Thus, for (b) to be true, the antecedent must be ‘man is’. For (b) to be false is just for the assumption (in this case, that ‘animal is’) to leave it open to a situation in which, given that the antecedent is, the consequent still is not.

Although conversion does not hold for incomplete implication, contraposition does. As in *Tahḥīl*, in APCA Alfarabi also enlists the aid of the notion of the removal (*irtifāʾ*) as an alternative way of talking about necessary types of implication.¹⁹²

[Text 12] [Of a pair of things between which is incomplete following,] they are such that the being of one implying the being of the other is not reciprocal. If the consequent is removed, then of necessity the thing implying it is removed.

¹⁹² Stephen Menn and Robert Wisnovsky note (S. Menn, R. Wisnovsky, “Yahyā ibn ‘Adī *On the Four Scientific Questions Concerning the Three Kinds of Existence*”, *Mélanges de l’Institut Dominicain d’Études Orientales du Caire* 29 (2012): 95, n. 114) that in Yahyā ibn ‘Adī’s treatise “On the Four Scientific Questions Concerning the Three Kinds of Existence” uses the term *irtifāʾ*, which Menn and Wisnovsky translate as “removal”, as a translation of *anairesis*. Just as in Alfarabi, this term is “commonplace in many parallel accounts of prior by nature” in classical works such as Aristotle’s *Protrepticus*, Porphyry’s *Commentary on the Categories*, Elias’ *Commentary on Porphyry’s Isagoge*, Alexander’s *Questions*, and Alexander’s essay against Xenocrates. I suspect that a close study of these texts, along with commentaries on the *Topics* and Galen’s *Intitutio Logica* would reveal the origins of Alfarabi’s doctrine of conditionals propositions and conditional syllogisms.

For example, consider man and animal. If animal is removed, then it follows of necessity that man is removed. For if animal were removed and man were not removed, but, rather, remained in existence [*baqiya fī l-wuḡūd*] while [still accepting the premise that] by virtue of man's being, animal is, then it follows necessarily that if animal is removed, then at the time that it is not, it is. Then there is something that is and is not at the same time, and in exactly the same manner. But that is absurd [*muḥāl*]. Based on this example, with regard to two things, the being of one of which follows reciprocally from the being of the other, it follows that the removal of either one of them entails the removal of the other.¹⁹³

By an *ad absurdum* argument about man and animal, Alfarabi suggests, without formally proving it, that contraposition holds for incomplete implication, viz. X's incompletely implying Y entails that Y's not being incompletely implies that X is not ('if X is, then Y is' 'if Y is not, then X is not'). It is perhaps worth dwelling on Alfarabi's proof of contraposition of incomplete implication at length. The proof runs as follows. Assume X incompletely implies Y, i.e. assume 'if X is, then necessarily Y is'. Then, in order to generate a contradiction later, assume further that *it is not the case that* Y's not being incompletely implies X's not being, i.e. assume 'not-(if Y is not, then necessarily X is not)'. This step forces Alfarabi to give voice to his intuitions about what the contradictory of conditional is. According to Alfarabi, the conditional sentence 'if animal is not, then human is not' signifies a connection of incomplete necessity between the antecedent and the consequent, or, said differently, signifies that the antecedent incompletely implies the consequent. Thus, the meaning of this conditional is that it is impossible that human be and animal not be, or, informally, it is impossible for there to be something outside the soul called 'human' but not 'animal'.¹⁹⁴ To contradict this statement is to say that it is possible at one and the same moment that animal is not but, somehow, human

¹⁹³ Alfarabi, *APCA*, §57, 34.23–35.4.

¹⁹⁴ S. Menn, 'Al-Fārābī's *Kitāb al-Ḥurūf* and His Analysis of the Senses of Being', *Arabic Sciences and Philosophy* 18 (2008): 59–97.

is. Informally, this would be to say that it is possible for there to be an object outside the soul that is simultaneously a human but not an animal. Thus, it seems that Alfarabi's intuition about the contradictory of a conditional is that it is in the form of a conjunction in which the leading conjunct is identical to the original antecedent and the final conjunct is the contradictory of the original consequent. And the conjunction itself signifies, as Alfarabi says, temporal coincidence. This conclusion, viz. that the contradiction of a conditional sentence in Alfarabi is formally a conjunction rather than another conditional as in contradiction of indicative conditionals, is borne out by the rest of Alfarabi's "proof" of the validity of the rule of contraposition. The ad absurdum assumption allows Alfarabi to construct a conditional syllogism with (P1) the original assumption 'if human is, then animal is', and (P2) 'human is' which he obtains from the ad absurdum assumption that there is in fact something that is human.¹⁹⁵ P1 and P2 with modus ponens yields 'animal is', whereas the other conjunct 'animal is not' was assumed ad absurdum, yielding the desired contradiction. Alfarabi could not have reasoned in the way he does here in order to generate this ad absurdum argument if the contradiction of a conditional were another conditional. On the other hand, it is perfectly consistent with Alfarabi's way of talking here to say that, in general, the contradiction of a conditional 'if P, then Q' is the conjunction 'P and not-Q'. Yet, the formal similarity of Alfarabi's rule of conditional contradiction with the negation of the material conditional should not make us lose sight of the fact that they could not be more different with respect to their semantics. The material conditional 'if P, then Q' is false just when its contradictory is true, viz. when P is true and Q is false. Alfarabi's conditional 'if P, then Q' is also false just in case its contradiction is true. But the contradiction of 'if P, then Q' will be true when it is

¹⁹⁵ Perhaps, strictly speaking, in two steps of this "proof" Alfarabi would have had to rely on conjunction elimination, viz. 'P and Q' $\Rightarrow$ 'P' and 'P and Q' $\Rightarrow$ 'Q'. It seems likely, however, that took the elimination steps to be obvious.

possible that the state of affairs expressed by P coincide with absence of the state of affairs expressed by Q. In this case, the modality makes the current truth-value of P and Q irrelevant. It is the consistency of the state of affairs expressed by P with the absence of the state of affairs expressed by Q that is signified by the contradiction of Alfarabi's conditionals.

Having considered Alfarabi's comments in *APCA*, *Ġadal*, and *Taḥlīl*, we are now in a position to provide a reasonable conjecture about the truth-conditions for conditionals. As I discussed in §2, to say that a conditional is true is to say only that a reasoner says that the conditional is true. As we have noted throughout this paper, however, a reasoner harbours different criteria for calling a conditional true in different contexts. Thus, onus for making suitable distinctions between the notions of truth in context falls on the strength with which the mind's assent attaches to the conditional in question. Broadly speaking, the context-sensitivity of the modality of the reasoner's mental assent to a conditional makes room for a variety of different attitudes that the reasoner can adopt towards it. For a theory of conditionals, this means that the ambiguity of the reasoner's assessment that a conditional is true is resolved by looking to the different strengths of the implication relation between antecedent and consequent. The weaker the connection between the antecedent and consequent, the weaker the mental assent to the conditional composed of them. In determining whether or not to give assent to a proposition, the reasoner looks to see if, and to what extent, the connection between antecedent and consequent correspond to what is the case. However, the *current* truth-values of the antecedent and consequent are not necessarily relevant to the reasoner's decision to call a conditional true or false. Rather, it is his observation of the variation in the *frequency* with which the connection between the antecedent and the consequent mirrors what is the case that accounts both for the variation of

the strengths of implication in Alfarabi's conditionals, and for the variation in the grades of assent to those conditionals. As we saw at the beginning of this section Alfarabi does not feel the need to consistently use an overt linguistic marker that indicates the modality of implication between antecedent and consequent signified by the conditional expression. On the face of it, there is no overt linguistic indicator that when the reasoner says that 'if human is, then animal is' and 'if it is December in Montreal, then it is cold out' are true, he holds each conditional to different criteria.¹⁹⁶ Yet, for the reasoner to say that a conditional is true means different things in different argumentative contexts. When the reasoner says that a conditional is true in a demonstrative argument, this means that he has observed that the consequent is always true given the truth of the antecedent. This means that there is necessary implication between the antecedent and consequent, and further, that the reasoner gives the strongest form of assent to the conditional in question. On the other hand, the reasoner also says that a conditional is true in a dialectical argument. But in this context, this should be taken to mean that he has observed that the consequent is almost always true given the truth of the antecedent. There is, thus, for-the-most-part implication between the antecedent and the consequent, and further, that the reasoner gives fairly strong assent to the conditional. And when he says a conditional is true in a rhetorical argument, this means only that he has observed that the consequent was true at least once when the antecedent was true.

¹⁹⁶ See Text 8 and Text 9 for Alfarabi's statistical reading of the modalities. In his so-called Short Treatise on Aristotle's *De Interpretatione*, Alfarabi is more explicit about the interdefinability of the 'primary modes' of necessity and possibility and statistical/temporal modalities: 'Necessary is what exists permanently, not having ceased nor going to cease, and cannot not exist at any time. Possible is what does not exist now but is apt to exist and apt not to exist at any time in the future. The absolute is of the nature of possibility, but has come to exist now after having had the possibility of existing and the possibility of not existing, though it has the possibility of not existing again in the future'; Alfarabi, *Alfarabi's Commentary and Short Treatise on Aristotle's De Interpretatione*, trans. F. Zimmermann, (London: Oxford University Press, 1981), 242. On the notion of primary or basic modalities, and its role in the development of Avicenna's division of existence into necessary and possible, see R. Wisnovsky, *Avicenna's Metaphysics in Context*, (Ithica, NY: Cornell University Press, 2003), 219-25.

This also means there is a *per accidens* type of implication between the antecedent and consequent, and further, the reasoner gives the weakest form of assent to the conditional.

*Truth-Conditions for Alfarabi's necessary, for-the-most-part, and per accidens conditionals*¹⁹⁷

For any sentence *A* and *C* expressing some state of affairs, the conditional sentence 'if *A*, then *C*' is true...

[1] with *necessary implication* if and only if there is no instance in which *A* is true and *C* is not true. Thus, 'if *A*, then *C*' is false when there is such an instance;

[2] with *for-the-most-part implication* if and only if it is more often the case that *C* coincides with *A* than not-*C* coincides with *A*. Thus, 'if *A*, then *C*' is false when *A* coincides with *C* and not-*C* with equal frequency, or *A* coincides with not-*C* with greater frequency;

and [3] with *per accidens implication* if and only if there is an instance in which *A*'s being true and *C*'s being true coincide. Thus, 'if *A*, then *C*' is false when there is never such an instance, in which case 'if *A*, then not-*C*' will be true in the sense of [1].

It might seem amiss to claim that the sentence 'if it is December in Montreal, then it is cold out' is true with the explicit admission that the antecedent may very well be true and the consequent come out false. Yet, it is not amiss when we realize that the conditional is not true according to the conditions required for necessary conditionals, but is according to the conditions for 'for-the-most-part' conditionals. This fact is corroborated by the observation that 'if it is December in Montreal, then it is cold out' will likely draw the interlocutor's assent in a dialectical or rhetorical debate because it is *rarely* the case that the antecedent is true and the consequent false. On the other hand, since the conditional signifies a statistical rather than a sempiternal connection between two states of affairs, the conditional is false when read

¹⁹⁷ Technically speaking, the locution 'necessary conditional' means that it is a conditional sentence with necessary implication, 'for-the-most-part conditional' means that it is a conditional sentence possessing for-the-most-part implication, and 'per accidens conditional' means that it is a conditional sentence possessing per accidens implication.

as a necessary conditional, and the connection indicated by the conditional is not likely to elicit assent from the respondent in the case that the interlocutors are engaged in a demonstrative argument. In short, a true conditional is a conditional that a reasoner calls true *in a context*. The reasoner calls a conditional true according to the observed frequency with which the consequent is true given the antecedent. Without inconsistency, he may call one and the same conditional true in one context and false in another. This ostensive inconsistency is resolved by looking to the degree of assent the reasoner gives to the consequent given the truth of the antecedent. He may call a conditional such as ‘if human is, then animal is’ and ‘if it is December in Montreal, then it is cold out’ true, but his assent to the former is strong, whereas his assent to the latter is significantly weaker. As we saw above, Alfarabi prefers to speak about the strength and weakness that inheres in proposition generally and conditional in particular in terms of the notion of the ‘site of opposition (*mawḍi‘ al-‘inād*)’ to the proposition, which may be characterized by the doubt or reservations that the reasoner has about the proposition in question. In the case of conditionals, the basis of this site of opposition will be the frequency with which the reasoner observes the consequent coming out false while the antecedent is true. In the case of necessary conditionals, this never happens so there is no site of opposition to such a conditional. In the case of rhetorical conditionals, the site of opposition is potentially much greater, since in a rhetorical argument the audience may be induced to say that a conditional is true based on a single instance in which the antecedent and consequent coincide.

§5 CONDITIONAL PROPOSITIONS AND INFERENTIAL VALIDITY

Consistent with Alfarabi’s lionization of demonstrative methods, Alfarabi is less concerned with per accidens implication and per se implication for-the-most-part than with

necessary implication, whether complete or incomplete. The reason for this is that Alfarabi seems to believe that these types of following are not suited to carrying out deductions in which a true conclusion follows *of necessity* from a set of true premises. Yet, it is only natural for Alfarabi to entertain such a view of syllogisms given Aristotle's generic definition of the syllogism at the opening of the *Prior Analytics*, which states that a discourse is a syllogism if, inter alia, it is *impossible* that the conclusion be false given that the admission that the premises are true. But if this is so, then there is a problem. Consider an example of per se implication of the for-the-most-part variety in the following conditional: (P1) 'if Sirius passes the zenith in the morning, then it will be hot'. For Alfarabi, this means that there is 'for-the-most-part' implication relation between the sentences, 'Sirius reached its zenith in the morning' and 'it will be hot'. An antecedent implies a consequent for-the-most-part if the connection between the state of affairs expressed by the antecedent and the consequent holds with law-like regularity. Observing this fact, a reasoner gives assent to the conditional when he sees that the consequent is true in most cases in which the antecedent is true. Say that in the course of an argument a respondent gives his assent to P1. This means that the respondent, seeing that it is hardly ever the case that Sirius reaches its zenith in the morning but the temperature remains mild, gives his assent to the conditional. Then, as it turns out the questioner and respondent observe that today is a day that Sirius reached its zenith in the morning, and thus the respondent also feels obliged to assert (P2) 'but Sirius passes the zenith in the morning'. Now, Alfarabi classes modus ponens, along with modus tollens, as a conditional syllogistic figures in *Qiyās* and *Madḥal*,¹⁹⁸ and, thus, given that respondent has conceded that both P1 and P2 are true, the canonical notion of syllogistic validity tells us that it should be impossible that

¹⁹⁸ Alfarabi, *Qiyās*, 82f; cf. Alfarabi, *Al-Fārābī's Short Commentary on Aristotle's Prior Analytics*, trans. N. Rescher (Pittsburgh: University of Pittsburgh Press, 1963), 74-7. Alfarabi, *Madḥal*, 31ff.

the conclusion be false. If we take impossibility statistically, as Alfarabi normally does, the inferential validity of modus ponens requires that there never be a case in which P1 and P2 are true and the conclusion is false, i.e. there is never a situation in which Sirius passes the zenith in the morning and yet the day is rainy and mild. Yet, consider the truth conditions for for-the-most-part conditionals like P1 above. P1 is true if and only if it is statistically more frequent that Sirius reaches its zenith in the morning and it is a hot than it is that Sirius reaches its zenith and it mild or cold. So P1 will still be true and an interlocutor will still give his assent to P1 knowing that there are cases in which Sirius reaches its zenith in the morning and the weather remains cold, rainy or mild. Such a notion of implication makes room for the following scenario: Sirius reaches its zenith in the morning and the weather remains cold. Thus, P1 is true and the respondent will give his assent to it because its truth is based on statistical frequency and he will also be willing to give his assent to P2 since he observed Sirius reach its zenith in the morning. Yet the respondent will still not give his assent to the conclusion that it is a hot day, for the simple reason that it is not. So modus ponens with conditional premises read ‘for-the-most part’ (and, a fortiori, per accidens) is classically invalid.

What are we to make of this result? Deborah Black has observed that Alfarabi, like other classical Islamic philosophers, countenanced the use of fallacious inferences schemes such as ‘denying the antecedent’ (DA) and ‘affirming the consequent’ (AC) in rhetorical, and especially, enthymemic forms of reasoning.¹⁹⁹ Thus, one approach might be simply bite the bullet, so to speak, and admit that in arguments in rhetorical or dialectical contexts in which for-the-most-part conditionals are admissible, arguments in modus ponens are just as invalid

¹⁹⁹ Black, *Rhetoric and Poetics*, 170f.

as arguments in which we deny the antecedent or affirm the consequent. This approach would be consistent with the general tendency in Alfarabi to hold up demonstration as the genuine form of syllogistic, and to hold up the other species of syllogism as inferior. On this reading, rhetoric and dialectical are inferior to demonstration because they produce ‘variable opinions’ rather than certainty,²⁰⁰ admit premises of contingent rather than necessary matter,²⁰¹ false premises rather than eternally true premises,²⁰² premises derived authority and sense perception rather than from first principles,²⁰³ and now also because they admit the use of fallacies in syllogistic argumentation.²⁰⁴ Yet, this would ignore important distinctions that Alfarabi explicitly makes in *Kitāb al-Ḥaṭāba* between syllogisms that are genuinely productive such as modus ponens and modus tollens, and those that are only ostensibly (*fī ‘ẓ-zāhir*) productive such as DA and AC. In reference to conditional enthymemic reasoning, Alfarabi notes that valid inferences and ostensibly valid inferences are rhetorically persuasive only if we suppress the asserted minor premise (*muṣṭaṭnā*) in each.²⁰⁵ Yet, the reason for withholding the asserted minor in each case is different. In the case of modus ponens and modus tollens, syllogisms which will be recognized as genuinely valid (*ṣaḥīḥ*) by the audience, the asserted minor is withheld. For example, the minor is suppressed in the case of modus ponens so the

²⁰⁰ Ibid., 108.

²⁰¹ Ibid., 86ff.

²⁰² Ibid., 87.

²⁰³ Ibid., 98.

²⁰⁴ Ibid., 170.

²⁰⁵ Alfarabi, *Deux ouvrages inédits sur la Rétorique*, ed. J. Langhade, M. Grignaschi (Beirut: Dār al-Mašriq, 1986), 95.6-13: ‘Connective conditional syllogisms are only persuasive (*muqni’a*) when the conditional proposition is stated explicitly, the asserted [minor] premise is withheld, and one simply sets forth the conclusion. In this art (i.e. in rhetoric) the conclusion of a connective conditional syllogism may be the opposite of the consequent, or the opposite of the antecedent. [Whatever conclusion the speaker decides to work with] will depend on what the speaker feels will be most beneficial to him. By withholding the asserted [minor] premise, the locus of the sophistry in each of these conclusions will be obscured, for at first glance (*fī bādi’ r-ra’y*) most people (*ḡumhūr*) can hardly tell what must be asserted, or which assertion will produce the conclusion. For all of this is obscure to the majority of people’. Hereafter, I will cite this work as follows: Alfarabi, *Rétorique*. Cf. Alfarabi, *Kitāb fī l-Mantiq: al-Ḥaṭāba*, ed. Muḥammad Salīm Sālim (Cairo: al-Hay’a al-Miṣriyya al-‘Āmma li-l-kitāb, 1976), 47.10-5. Hereafter, I will cite this work as follows: Alfarabi, *Ḥaṭāba*.

speaker avoids further questioning about the propriety of asserting the antecedent in the minor premise.²⁰⁶ The minor is withheld in the case of *modus tollens*, the validity of which will also be recognized by the audience, so that the locus of opposition (*mawḍi‘ al-‘inād*) to the view being argued over, the persistence of which is necessary to the process of persuading one’s audience, is not revealed.²⁰⁷ For if the locus of opposition to the viewpoint stands exposed, the audience will turn to view the viewpoint with a more critical eye, changing the modality of their assent to the opinion from persuasion to refutation.²⁰⁸ By contrast, Alfarabi says that the assertion of the minor premise must be suppressed in the case of fallacious arguments such as DA and AC because the audience will realize that the arguments are, in fact, formal fallacies—in Alfarabi’s words, the particular combination of premises is ‘unsound’ or ‘corrupt’, sc. *fāsid*—and, as a consequence, the arguments will lose their persuasive power.

[Text 13] If the conclusion [*natīqa*] is the opposite of the consequent, then the asserted minor premise is the opposite of the antecedent. This combination [of premises] is only ostensibly [*fī ‘ẓ-ẓāhir*], not genuinely [*fī l-ḥaqīqa*], productive. If the asserted [minor premise] is explicitly mentioned, then there is no guarantee that the audience [*sāmi‘*] will not perceive [the locus of the sophism sc. *mawḍi‘ al-muḡālata*], and as a consequence, his conviction will vanish. For this reason, it is necessary that [the speaker] withhold [the asserted minor premise]. If the conclusion is the antecedent [of the conditional], then it is only presumed to produce [this conclusion, i.e. the antecedent (*muqaddam*)] by asserting the consequent as it was set down. This is also not productive in reality. This combination [of premises] is rarely used except when the speaker deems it

²⁰⁶ Alfarabi, *Rétorique*, 97.6-9; idem., *Ḥaṭāba*, 48.6-9.

²⁰⁷ Alfarabi, *Rétorique*, 97.4-6; idem., *Ḥaṭāba*, 48.4-6.

²⁰⁸ Black, *Rhetoric and Poetics*, 112: ‘Since rhetorical assent is a form of decisive adherence to one contrary, in the face of an equally strong objective probability that the rejected contrary is the true one, the logician is left without any explanation of why the mind does indeed incline one way, rather than the other. The production of rhetorical assent cannot, therefore, be due solely, or even primarily, to the truth and modality of rhetorical propositions. The very nature of rhetorical acceptance is that it is primary and unhesitating, and thus able to subsist despite the awareness of the possibility that it is false, or that not everyone accepts it as true. As soon as doubts regarding these rhetorical premises reach the point that they make the believer feel the need for investigation, his assent has lost its innocence, the very innocence that made it rhetorical belief. Thus, as soon as the opposite of which the holder of a rhetorical belief is aware becomes an active force, the believer is thrust into the realm of dialectical investigation’. For more details on the ‘locus of opposition (*mawḍi‘ al-‘inād*)’, see *ibid.*, 111-13.

likely that it will elicit conviction [in the audience]. In this case, it is also necessary to withhold the asserted minor premise so that [the audience] does not perceive the unsoundness of this combination of premises [*fasād ta'līfīhi*], and, the argument become thereby unpersuasive.²⁰⁹

Thus, even in argumentative contexts in which the interlocutors are willing to admit weaker forms of conditionals such as *per accidens* and *for-the-most-part*, Alfarabi still holds that the audience distinguishes between formally valid syllogisms such as MP and MT, and invalid premise combinations such as AC and DA. And the audience makes this distinction despite the fact that the conclusions do not follow necessarily from the premises in any of these schema, whether in classically valid schema such as MP and MT, or classically invalid schema such as AC and DA, when weaker forms of conditional are used as major premises. If the primary aim of employing syllogisms in demonstrative contexts is the generation of a conclusion from the premise set necessarily, the same cannot be said for syllogisms in non-demonstrative contexts. In employing syllogisms in these latter type of context, the primary aim is eliciting assent or compliance in the audience's mind with the conclusion rather than generating a true conclusion of necessity. The notion of validity aptly characterizes demonstrative arguments to the extent that the concern in a valid argument is with the *truth* of the conclusion given the *truth* of the premises. In dialectical and rhetorical arguments on the other hand the speaker's is concerned with eliciting assent to the conclusion given that they audience gives its assent to the premises. It seems more appropriate, then, to speak about the 'success' and 'failure' or the 'felicity' or 'infelicity' of a rhetorical or dialectical argument, to the extent that calling an argument 'successful' or a 'failure', 'felicitous' or 'infelicitous' makes it clear that the aim of these types of argument is with inculcating mental compliance in the audience rather than generating a true conclusion of necessity from a pair of premises.

²⁰⁹ Alfarabi, *Rétorique*, 95.14-6.3; idem., *Ḥaṭāba*, 47.16-8.3.

Consider the inference with the for-the-most-part condition major premise (P1) ‘if it is December in Montreal, then it is cold out’ discussed above. This conditional is true because it is more often the case that it is December in Montreal and it is cold than it is December in Montreal and it is warm or mild. This is so, despite the explicit admission that there exists at least one case in which it is December in Montreal and it is not cold. As we observed above, however, if we demand that an argument in MP, which uses P1 as a major premise, satisfy the requirement of classical validity, then this argument will be invalid because given a warm December day in Montreal, the speaker may get his audience to concede P1 and P2 ‘but it is now December in Montreal’, and the conclusion ‘it is cold’ may still be false. Yet, consider: there is a sense in which a mother who argues with her son about dressing for the cold during his visit to Montreal in December is making a good argument when she gets him to concede that, on the one hand, it is hardly ever the case that December in Montreal is warm, and, on the other hand, that he will soon be in Montreal in December. The fact that we feel, even if just intuitively, that this is a good argument is registered by our genuine surprise upon hearing that her son returned to inform her that he did not, in fact, need his jacket at all because the weather during his visit was unusually warm. Our reaction is not so much to say that her inference was invalid, but to remark at the exceptionality of Montreal’s weather. Moreover, if her son were to be advised in the same fashion on another occasion, he would still likely be convinced to give his assent to the premises once more, since the connection between cold weather and Montreal in December still holds for the most part.

Alfarabi’s discussion from *Kitāb al-Ḥaṭāba* suggests that we hold MP to be a good (or successful, or felicitous) argument, even if the argument is classically invalid because a for-the-most-part conditional is used as a major premise. It seems that at the heart of this issue is

the requirement, as stipulated in Aristotle's generic definition of the syllogism in the *Prior Analytics*, that the connection between the premises and the conclusion is too stringent in all but the demonstrative syllogistic arts. This requirement brings schema like MP and MT to the same level as DA and AC, when it is intuitive in Alfarabi's eyes that arguments in the former are good and those in the latter are not. Speaking in reference to using formal fallacies such as AC in rhetorical debate, Deborah Black notes that despite the fact that rhetorical discourse often makes use of argument schema that 'do not formally entail their conclusions,

there is some plausibility involved in the acceptance of those sorts of arguments. Given a sufficiently strong connection between the antecedent and the consequent, a context in which connections between the consequent and other conditions are few, it is not improbable that the presence of the consequent does suggest the truth of the conditional, even if the entailment is not formally conclusive'.²¹⁰

It seems that an analogous situation should be entertained with respect to MP and MT. That is to say, despite the fact that the truth of premises in a syllogism with a for-the-most-part conditional as a major premise do not *necessitate* the truth of the conclusion, there is still a sense in which the assent to the for-the-most-part conditional follow by the assertion of the antecedent as a minor premise will elicit, but in a weaker way, assent to the consequent as a conclusion of the syllogism, even if the entailment is not of the strength required by Aristotle in his generic definition of the syllogism.

In short, the context theory forces on the logician the analysis of conditionals in which the connection between the antecedent and consequent varies according to the dialectical context in which the conditional proposition is used. In a demonstrative context, the connection between antecedent and consequent is necessary; in a dialectical it is for-the-most

²¹⁰ Black, *Rhetoric and Poetics*, 170f. Cf. Evans and Over, *If*, 32.

part; in a rhetorical it is per accidens. Each of these conditionals is true *in suitable contexts* in the sense that the connection between the antecedent and consequent expressed by the conditional proposition corresponds to the actual state of affairs in the way *required by dialectical context*. Thus, a true for-the-most-part conditional in a demonstrative context will be false because the connection between the antecedent and consequent in such a context must be sempiternal. On the other hand, a for-the-most-part conditional is also true in a dialectical or rhetorical exchange, since all that is required in such contexts is that the antecedent and consequent be connected accidentally.²¹¹ Yet, our discussion above shows that not only does the context demand different grades of assent in relation to the actual state of affairs, but also a sensitivity to the dialectical context demands variable degrees in the strength of the connection between the *premises* and *conclusion* of the conditional syllogism. In other words, it requires the admission that dialectical contexts require different grades of syllogistic validity. MP is syllogistically (and classically) invalid with a for-the-most-part conditional because it may happen that it is admitted that the premises are all true and the conclusion still happens to come out false. However, it seems inappropriate to require a single, monolithic criterion of syllogistic validity in all dialectical contexts, as required by many contemporary formal accounts of logical validity. Ancient philosophers--and the classical Islamic philosophers are not exceptional in this regard--proved to be extremely adept at adapting the notion of the syllogism as developed in the *Prior Analytics* to the demands of the different species of syllogism that they took Aristotle to be developing in the five syllogistic arts of the *Organon*. I suggest, rather, that a conditional syllogism in MP executed in a dialectical or rhetorical context with a for-the-most-part conditional major entails its

²¹¹ Jonathan Evans and David Over (Evans and Over, *If*, 38) note that when conditionals are used in natural language environments, people “do not expect a ‘true’ conditional to apply universally” which gives further evidence in the authors’ view that people tend to “interpret ‘all’ fuzzily or vaguely to mean ‘nearly all’”.

conclusion, but in a weaker sense than in a demonstrative argumentative context, for example. Alfarabi considers just such a possibility in *Kitāb al-Ḥaṭāba*. Once again returning to the question of the problem of how to distinguish between the five syllogistic arts in the face of their underlying syllogistic unity, Alfarabi moves to distinguish between unqualified (or “simpliciter”, *muṭlaq*) and qualified senses of the term ‘syllogism’ based this time on how strongly (or weakly) the premises entail the conclusion. In the face of claims that a wide variety of clearly distinct argument forms, such as enthymemes, induction, and analogy, can somehow all still meaningfully be grouped under the name ‘syllogism’,

[Text 14] logicians [*aṣḥāb al-mantiq*] maintain that this name [viz. syllogism, *qiyās*] refers to the combinations of premises that produce necessarily, whether [these premise combinations] are categorical, conditional, or per impossibile [*‘alā ṭarīqi l-ḥulf*]. Furthermore, they have designated [the premise combination that is necessarily productive] as a ‘syllogism’, and not the inductive [syllogism, *istiqrā’*] or [the syllogism from] analogy [*tamṭīl*]. According to them, enthymemes are more deserving of the name ‘syllogism’ than the syllogism from analogy, though this is the opposite of how the majority of logicians [*ḡumhūr*] have understood the matter, and it is also the opposite of how many of the scholastic theologians [*kaṭīrun mina l-mutakallimīn*] have understood it [i.e. the Mu‘tazilites].²¹² Similarly, sophistical discourses [*al-aqāwīl al-sūfiṣṭā’iyya*] they at times call ‘syllogisms [*qiyāsāt*’], but not simpliciter [*lā ‘alā ṭarīqi l-iṭlāq*]. Rather, they call sophistical discourses ‘sophistical syllogisms’, and [they call] the enthymeme a ‘rhetorical syllogism [*qiyāsan ḥuṭabiyyan*]’. As for the simpliciter sense of ‘syllogism’ [*wa-amma l-qiyāsu bi-l-iṭlāq*], they designate thereby the discourse [*qawl*] from which the conclusion follows necessarily. But the enthymeme includes [premise combinations] that are genuine syllogisms as well as those that are only ostensibly syllogisms.²¹³

Alfarabi observes that ‘syllogism’, taken in its most generic sense, means a set of premises that necessarily entail the conclusion. In terms of conditional syllogisms, this means that

²¹² For Mu‘tazilite ideas about the syllogism, see J. van Ess, *Die Erkenntnislehre des ‘Aḡuaddīn al-Īcī* (Wiesbaden: Franz Steiner Verlag, 1966), 382-94. For Mu‘tazilite influence on the classical Islamic philosophers, see P. Adamson, ‘Al-Kindī and the Mu‘tazila: Divine Attributes, Creation and Freedom’, *Arabic Sciences and Philosophy* 13 (2003): 45-77.

²¹³ Alfarabi, *Rétorique*, 85.4-11; idem., *Ḥaṭāba*, 41.14-2.3.

‘conditional syllogism’ in an unqualified sense denotes a premise set composed of a conditional major premise and another, non-conditional minor premise, which, if asserted, necessitate the assertion of the conclusion. Consistent with the late antique tendency to make demonstration the end of syllogistic methods, this generic sense is made to align exactly with the sense in which the premises in a demonstrative syllogism must entail their conclusion, namely, of necessity. In the case of a demonstrative syllogism, in order to ensure that the premises necessitate the conclusion, the connection between the antecedent and the consequent of the conditional major premise must be necessary too, whether of the complete or the incomplete variety. Thus, in the case of demonstrative syllogisms, *the strength of the connection between the antecedent and consequent indexes the strength of the premises’ entailment of the conclusion*. Said differently, the strength of the following of the conclusion from the premises is limited by the strength of the connection between the antecedent and consequent of the conditional major premise. This observation may be generalized to the two other types of conditional discussed by Alfarabi in APCA. In a conditional syllogism with a for-the-most-part conditional major premise, the frequency with which the connection between the antecedent and consequent expressed by the conditional proposition corresponds to the actual state of affairs is the frequency with which the premises and the conclusion will be true together. In a conditional syllogism with a per accidens conditional major premise, the frequency with which the conclusion will be true given that the premises are true is just the frequency with which the consequent of the per accidens conditional is true given that the antecedent of the per accidens conditional is true.

In sum, for each argumentative context, viz. in each argument in which the interlocutors agree to use syllogisms that will elicit only a certain degree of assent, the context

theory of logic assigns an appropriate syllogistic art. Alfarabi distinguishes between these syllogistic arts in various ways. At times he recognizes that the distinction between them is at the level of the premises' truth values; at other times he distinguishes between them at the level of the modality of their premises; at other times, he seems inclined to distinguish them, like Avicenna would after him, with the rank of assent that the premises and conclusion elicit from the listener. The result of the above discussion is to realize that he also distinguishes between the different syllogistic arts—regardless of whether the syllogism deployed in the argument is categorical, conditional, or per impossibile—according to the way in which the premises entail the conclusion. Depending on the pragmatic conditions under which the argument takes place, not only will the interlocutors come to an implicit or explicit agreement about the level of assent they require from their opponent's premises, but they will also expect or even explicitly stipulate the suitable manner in which the conclusion follows from the premises vis-à-vis the context; in other words, they will implicitly expect or explicitly stipulate a suitable notion of validity vis-à-vis the context. Indeed, this result should strike us as intuitive: of course, we do not expect the premises of arguments in historical disciplines to entail their conclusions in the same way that premises in, say, mathematics.

Conditional syllogisms represent a special case of this general rule. In the case of conditional syllogisms such as MP and MT, the variability in the strength of the premises' entailment of the conclusion is entirely determined by the strength of the implication relation between the antecedent and the consequent. The stronger the connection between antecedent and consequent, the more circumscribed the locus of opposition to assent to the conclusion given truth of the premises. In a demonstrative syllogism, there is no locus of opposition to the connection between the antecedent and consequent since the connection is

sempiternal, and as a consequence, there is strong assent to the conclusion given the premises. In rhetorical arguments, the locus of opposition to the connection between the antecedent and consequent is greater due to the weakness of the *per accidens* type of implication. And, as a consequence, the strength of assent to the conclusion given the truth of the antecedent is greatly diminished as well. Our degree of assent to a conclusion ‘so it is not a triangle’ is very strong given that we give our assent to ‘if this figure is a triangle, then it has only three sides’ and ‘but it does not have only three sides’. But this is not chiefly due to the strength of our assent to the minor premise. Rather, we are willing to give a high rank of assent to the conclusion because we recognize that it is quite impossible for the consequent of the *conditional* to be false and the antecedent of the *conditional* to be true. On the other hand, if we are arguing about weather patterns, then we are strongly inclined to give our assent to the conclusion ‘so it is not December in Montreal’ given that we give our assent to the conditional major premise ‘if it is December in Montreal, then it is cold’ and the minor premise ‘but it is not cold’. The locus of opposition to the conclusion, viz. the knowledge that the conclusion may, in fact be false, is magnified in this argument by our knowledge that there are instances when Montreal is warm in December, though rarely. Yet, in spite of this locus of opposition, the conclusion elicits our assent. A context theory-based concept of validity must take into account the fact that, unconsciously or consciously, two interlocutors will adopt different validity criteria based on the pragmatic conditions under which their argument takes place.

With these considerations in mind, call a syllogistic argument in a context in which the interlocutors agree, explicitly or implicitly, to require that the conclusion follows necessarily from the premises, an argument in a *demonstrative context*. Call a syllogistic argument in a context in which the interlocutors agree, explicitly or implicitly, to require that the conclusion

follow from the premises for the most part, an argument in a *dialectical context*. Finally, call a syllogistic argument in a context in which the interlocutors agree, explicitly or implicitly, to require that the conclusion follow from the premises per accidens, an argument in a *rhetorical context*. Since the aim of arguments executed in demonstrative contexts is generating a true conclusion from true premises, say that a conditional syllogism in a demonstrative context is valid when it is never the case that the conclusion is false and the premises are true; it is invalid if there is such a case. On the other hand, call a conditional syllogism in a dialectical context valid (in the sense of ‘successful’ or ‘felicitous’ discussed above) when it is hardly ever the case that the audience fails to give assent to the conclusion though it does give its assent to the premises; it is invalid (in the sense of ‘infelicitous’, ‘fails’) if it is often the case that the audience fails to give its assent to the conclusion but does give its assent to the premises. For similar reasons, call a conditional syllogism in a rhetorical context valid when there is at least one instance in which the audience gives its assent to the conclusion and the premises together; it is invalid in the case that there is never an instance in which it gives its assent to both together.²¹⁴

The crucial test for this view of syllogistic validity for conditional syllogisms is whether it can make meaningful distinctions between valid inference schema such as MT and MP and fallacious ones such as DA and AC. Maintaining this distinction becomes particularly important in the case of arguments with for-the-most-part and per accidens conditional major premises, since in Text 13 Alfarabi is so emphatic in holding that, for example, MT is, as he says, ‘genuinely productive (*fī l-ḥaqīqa*)’, whereas DA is only ostensibly productive (*fī ‘ẓ-ẓāhir*).

²¹⁴ This does not entail, however, that the argument with the same premise set and the contradiction of the conclusion would be valid in a demonstrative context, since there is clearly a difference between refusing to give assent to *P* on the one hand, and affirming not-*P* on the other. For a similar distinction between assertion and rejection, see T. Smiley, ‘Rejection’, *Analysis* 56/1 (1996): 1-9.

In order for a conditional syllogism with for-the-most-part conditionals to be valid in a dialectic context, it must be hardly ever the case that the conclusion is false while the premises are true. Take the usual example of a true for-the-most-part conditional as the major premise 'if it is December in Montreal, then it is cold out' and let the minor premise be 'but, it is not cold out'. This premise pair yields the conclusion by MT 'then it is not December in Montreal'. This inference is valid in a dialectic context because it is hardly ever the case that we will deny the conclusion, viz. we affirm that it is December in Montreal, while also giving assent to the premises, viz. we give assent to the fact that it is warm or mild out (minor premise) and that it is hardly ever the case that it is warm or mild out and December in Montreal (conditional major premise). Now consider the fallacy DA with the conditional major 'if it is December in Montreal, then it is cold out' and the minor premise 'but it is not December in Montreal'. The purported conclusion is 'then it is not cold out'. In order for this inference to be valid it would have to be hardly ever the case that we do not give assent to the conclusion in spite of our giving assent to the premises. Assent to the conditional premise is assent to the fact that it rarely happens that it is warm out and it is December, as well as assent to the fact that there are, nevertheless, instances in which it is warm out and it is December in Montreal. What sort of assent can the assertion of the minor premise 'but it is not December in Montreal' in combination with the conditional major elicit? In fact, the minor premise is irrelevant to the information provided by the conditional: the conditional tells us only about the connection between the weather in Montreal in December, and nothing about the weather in any other time of the year. Thus, it may very often happen that the conclusion 'then it is not cold out' is false while the premises are true. DA is invalid. Consider AC, the other common fallacy, with the same conditional major 'if it is December in Montreal, then it is cold out' and the minor

premise ‘but it is cold out’. The purported conclusion is ‘then it is December in Montreal’. Once again, in order to be valid, it must be hardly ever the case that we deny the conclusion, viz. we deny that it is December in Montreal, but we nevertheless give assent to the pair of premises, viz. we give assent to the fact that it is, in fact, cold out, and that it sometimes happens, though rarely, that it is warm or mild and it is December in Montreal. What sort of assent does the minor premise in combination with the conditional major elicit vis-à-vis the conclusion ‘it is December in Montreal’? The conditional tells us that it is cold given that it is December in Montreal. In other words, it points out only a single condition, from all of the countless conditions, under which the statement ‘it is cold out’ is true. Thus, even if the conclusion ‘then it is December in Montreal’ is certainly not ruled out by the combination of premises, more often than not the conclusion will be false due to the large number of other conditions under which ‘it is cold out’ is true. AC is invalid.

§6 CONCLUSION

Of contemporary accounts of conditionals, our conjectures about Alfarabi’s truth conditions and validity for conditional syllogisms suggests some similarities with Ernest Adams’ account of the semantics of conditional sentences.²¹⁵ With respect to truth conditions, both Alfarabi and Adams’ account of indicative (natural-language conditionals) are non-truth functional. For example, according to Adams’ account of indicative conditionals, indicative conditionals with false antecedents are simply indeterminate rather than being true or false.²¹⁶ This indeterminacy is a technical result of Adams’ use of probability theory to give an

²¹⁵ E. Adams, *The Logic of Conditionals: An Application of Probability to Deductive Logic* (Dordrecht and Boston: D. Reidel, 1975).

²¹⁶ Evans, and Over, *If*, 25.

interpretation of indicative conditionals,²¹⁷ but the interpretation is lent greater plausibility by the fact that we do not normally use indicative conditionals in the cases in which we already believe that the antecedent is false prior to uttering the sentence. In Alfarabi's case, his use of conditionals originates from a dialectical context in which the antecedent is a hypothesis (*wad'*) that the respondent and questioner give their assent to prior to carrying out a deduction in the format of a conditional syllogism. Since the antecedent is a hypothesis the truth or falsity of which the questioner and respondent have set out to determine, it makes no sense for questioner or respondent to *hypothesize* a proposition they already know to be false. Another important similarity between Adams' account of conditionals and some of Alfarabi's intuitions is the idea of indexing the strength of the following of the conclusion from the premises with the strength of the implication between the antecedent and consequent. For Adams, this means that the probability of the conclusion can be no greater than the sum of the probabilities of each of the individual premises. For Alfarabi, this simply means that if the implicative relationship between the antecedent and consequent is necessary, the premises entailing the conclusion will be necessary. On the other hand, if the implicative relation between the antecedent and consequent is only for the most part, then the conclusion will follow from the premise-pair for the most part. But perhaps most important of all, in both Adams' account and the account of Alfarabi's use of conditionals here there is a basic intuition that a theory of conditionals for natural language contexts must recognize that conditionals are used often, even perhaps primarily, in circumstances in which the degree to which we believe the consequent to be true given the truth of the antecedent admits of gradations, and

²¹⁷ According to Adams, the probability of our belief in the indicative conditional $P(A \rightarrow B) = P(A \& B)/P(A)$. Obviously, if we believe that A is false, then we believe that A has 0% chance of coming about. $P(A)$ thus equals 0, and the probability of ' $A \rightarrow B$ ' is indeterminate (since a fraction with a denominator of numerical value 0 is undefined).

that truth-conditions and notions of inferential validity must be devised in order to compensate for this fact. This intuition led Adams to speak of conditionals in terms of gradations in the probability that our subjective belief that the state of affairs represented in the consequent will come about given our belief that the state of affairs represented in the antecedent comes about. It led Alfarabi to speak about conditionals in terms of gradations of assent to a conditional.

Nevertheless, though they share some basic intuitions, Adams' account of conditionals and Alfarabi's ideas about conditionals presented here are very different. Adams' account of conditionals is motivated by the way conditionals are used in decision making. Thus, in his writings, the pragmatic conditions under which his theory develops are sensitive to these uses of conditionals only. As we saw, Alfarabi does not share Adams' interest in using conditionals for decision-making at all. Alfarabi does say that, in general, dialectical arguments normally have premises from ethics, and to that extent Alfarabi might use conditionals in an argument about what is good or just, and, thus, what ought to be done. However, the conditions that motivate his use of conditionals are the conditions that obtain in dialectical, rhetorical, and demonstrative argumentation. Whereas the pragmatics of decision-making stand at the center of Adams' account of conditionals, making decisions about what is good do not motivate Alfarabi's use of conditionals as such. Finally, because Adams' account of conditionals focuses on their use for decision-making, conditionals come to be interpreted as means to speaking about the probability that our beliefs are true or false and then acting according to what is most probable. For Alfarabi, conditionals are a means to engaging in a syllogistically formatted argument the aim of which is to elicit in the listener or the audience at large some sort of conviction (*iqḍān*) about the conclusion. The argumentative nature of Alfarabi's use of

conditionals is essential to understanding them properly. For Adams, conditionals appear to be primarily geared to individual decision making process rather than eliciting any sort of assent in a listener. Thus, Adams would use a conditional inference only if the conclusion is probable, whereas what determines Alfarabi's use of conditionals is whether or not the speaker has confidence the conditional inference will have the desired effect on his audience. Its objective or quantitative probability is of secondary importance.

Given that Alfarabi's use of conditionals arises in his epitomes of the *Topics*, the *Rhetoric* and the *Categories*, it seems that the dialectical scenario described in §2 and §3 would be familiar to Aristotle. Yet, it appears also that the underlying logical mechanics of the Farabian conditional are a far cry from anything in the *Prior Analytics* or the *Topics*. Let us comparing the results of our analysis with Jonathan Lear's comments about Aristotle's brief treatment of hypothetical syllogisms in *Prior Analytics* A44. First, Aristotle:

Further, we must not try to reduce hypothetical syllogisms; for with the given premises it is not possible to reduce them. For they have not been proved by syllogism, but all are assented to by agreement. For instance if a man should suppose that unless there is one [potentiality] of contraries, there cannot be one science and should then argue that not every [potentiality] is of contraries, e.g. of what is healthy and what is sickly: for the same thing will then be at the same time healthy and sickly. He has shown that there is not one [potentiality] of contraries, but he has not proved that there is not a science. And yet one must agree. But the agreement does not come from a syllogism but from an hypothesis. This cannot be reduced: but the argument that there is not a single potentiality can. The argument perhaps was a syllogism: but the other was an hypothesis.²¹⁸

In this passage from *An. Pr.* A44 Aristotle presents the reader with a hypothetical scenario in which two opponents debate whether or not, for any given pair of contrary objects or states,

²¹⁸ *Prior Analytics* A44 50a16-28; quoted in Lear, *Aristotle and Logical Theory*, 40. The translation is Lear's.

there is a single science that has as its object both members of the contrary states.²¹⁹ The disputants agree to accept the thesis that there is not a single science for any given pair of contrary states *on condition that* it be proved that it is not the case that, for any pair of contrary states, there is a single potentiality underlying them. One disputant then proceeds to construct a *reductio ad absurdum* proof that there is at least one pair of contrary states that is not underlied by a single potentiality (or power or, maybe better, faculty (*dunamis*)).²²⁰ If we assume that for every pair of contrary states there is a single potentiality, then this entails that, in particular, the contrary states of health and sickness return to a single potentiality. But holding this latter view forces us to conclude that we can make the following contrary predications of the same individual X at the same time, viz. “X is healthy” and “X is sick”, where the predicate here expresses the inherence of the capacity in the subject. Our hypothetical opponents hold that this last entailment is absurd and so the contradictory of their assumption is proved true, viz. not every potentiality is of contraries (or, in other words, there is a pair of contraries (health and sickness) that do not return to a single potentiality). With this proposition proved, the disputants are bound by their earlier agreement to accept that there is not a single science for every pair of contrary states. The important observation to make at this point is that Aristotle says that the conclusion of the *reductio*, viz. that there is not a single potentiality for every pair of contrary states, has been definitively shown or proved. He explicitly denies that the thesis that there is not a single science for every pair of

²¹⁹ R. Smith, Notes to Aristotle, *Prior Analytics*, trans. R. Smith (Indianapolis, Cambridge: Hackett, 1989), 175. Smith also notes (ibid.) that the context of the passage is dialectical. I am not sure that I agree with Smith’s interpretation of this passage in important respects. Like earlier interpreters, Smith appears to take the agreed-upon proposition to be a conditional “if there is not a single potentiality [...] for a pair of contraries, then there is not a single science of them either”. I have indicated in many places above why I believe Lear is right to say that this approach to Aristotle’s text is wrong.

²²⁰ Robin Smith notes (ibid.) that this second porition of the argument is in the form of a *reductio*.

contrary states is proved, because this latter thesis only comes about from an agreement to accept it *if* it is proved that there is not a single potentiality for contraries.

Alfarabi's theory as outlined above bears little resemblance to Aristotle's brief comments about hypothetical syllogisms. In the above passage from A44 Aristotle holds any attempt to formalize hypothetical reasoning to the categorical syllogistic developed in the *Prior Analytics* as bound to fail. The reason for this appears to be how he understands the prior act of agreement between the speaker and his opponent. By this I mean that Aristotle appears to see the agreement between the speaker and his opponent as a *promise* to give his assent to the speaker's thesis given that certain conditions obtain. In Aristotle's example,²²¹ this promise obligates the speaker's opponent to concede the thesis—a thesis that Aristotle believes is in fact false—that there is not a single science for contrary objects or states on the condition that the speaker can prove that there is not one potentiality of contraries. The next step in the scenario is that the speaker then proves syllogistically that there is not one potentiality of contraries. The *proof* that there is not one potentiality of contraries, not the mere supposition that there is not one potentiality of contraries fulfills the condition, obligating the speaker's opponent to concede the speaker's thesis that there is not a science. Concede the opponent must, but Aristotle is explicit that this whole process does not constitute a proof of the speaker's thesis; indeed, Aristotle would vociferously deny that the thesis is true. For Aristotle, a proof must come from a syllogism, which is for Aristotle nothing but a categorical syllogism of the kind outlined in *Prior Analytics* 1-7. A conclusion from mere agreement does not constitute a proof of the speaker's thesis. In fact, according to Aristotle, the only thing that

²²¹ In a previous version of this dissertation, I misunderstood Aristotle's argument. I would like to express my gratitude to Professor Stephen Menn for bringing this error to my attention.

has been proven in this whole process is the statement following the initial agreement, viz. that there is not one potentiality of opposites.

Thus, when I claim that Alfarabi has formalized this act of agreement into the syllogistic, I mean that what was treated by Aristotle as a commissive speech act, viz. a promise between a speaker and opponent, Alfarabi gives a formal logic counterpart as the antecedent and consequent in a conditional proposition. The conditional *promise* in Aristotle becomes a conditional *proposition* with the conjectural ‘assent-conditions’ given above. As a commissive there is no sense in which the prior act of agreement between speaker and opponent in Aristotle’s example above can be said to be true, since amenability to truth and falsity is the province of assertives;²²² a promise might be described as felicitous or infelicitous, but never as apophantic.

Finally, unlike Aristotle, Alfarabi considers these conditional syllogisms as genuine proofs of the conclusion and not, as in Aristotle, conclusions that are a result of mere agreement. Aristotle rejects entirely the idea that the kind of hypothetical reasoning outlined in the passage above is a proof because it is not in the form of one of the canonical moods of his syllogistic. The deductive steps Aristotle describes above do not, when taken as a whole, constitute a syllogism, i.e. it is not ‘a discourse in which, certain things having been supposed, something different from the things supposed results of necessity because these things are so’. Rather, it is a discourse in which certain things having been supposed, something different from the things supposed results *because of our prior agreement*. But this is not a proof. For Alfarabi, however, conditionals syllogisms really do qualify as syllogisms (1) because the conclusion follow from the premises due to a implicative relation between antecedent and

²²² Black does note (Black, *Rhetoric and Poetics*, 54f) that, at times, Alfarabi does seem to entertain that there is a sense in which non-assertives, viz. non-apophantic discourses, might be said to be true or false. However, Alfarabi’s position is clear: only apophantic statements are true and false in a genuine sense.

consequent and (2) because the “things supposed” in the premises are, in Alfarabi’s view, different from the things that result from them. “Different,” not in the sense of different in quality (e.g. ‘Socrates is a bear’ and ‘Socrates is not a bear’ are different in quality), nor merely because the major premise is a conditional rather than a categorical proposition like the conclusion. The antecedent of the conditional, the consequent of the conditional, or their contradictory opposites *qua* conclusion or repeated minor premise differ from the antecedent, consequent or their contradictory opposites *qua* members of the conditional because of their illocutionary force, where the former are assertives and the latter suppositions. Alfarabi explicitly recognizes that the difference between the antecedent in the conditional and the antecedent in the repeated minor when he says that the former is simply hypothesized (*wuḍīʿa*) or supposed (*yufraḍu*), while the latter is asserted (*yustaṭnā*). With these logical distinctions in tow, Alfarabi is able to preserve the argumentative structure of Aristotelian dialectic but to bring hypothetical reasoning to level of demonstration that Aristotle reserved exclusively for his categorical syllogisms.

CHAPTER 3: AVICENNA AND ALFARABI ON THE SUBJECT MATTER OF LOGIC, AND AVICENNA'S REJECTION OF COMPLETE (TĀMM) AND INCOMPLETE (ĠAYR TĀMM) CONNECTION

§3.0 INTRODUCTION

As is well attested in the secondary literature, Avicenna was the first to hold that the subject matter (*mawḍūʿ*) of logic is secondary intelligibles (*al-maʿqūlāt at-tāniya*).²²³ It is less well-known to what extent, if any, this view ramified into other aspects of his logic. This chapter will examine how Avicenna's doctrine concerning the subject matter of logic makes possible one substantial departure from the late antique theories of conditional propositions (*qaḍāyā šartīyya muttašila*). Section §3.1.2 will consider Avicenna's views on the subject matter of logic (*mawḍūʿ al-manṭiq*). Instead of rehashing what has already been written, I will try to contextualize Avicenna's doctrine more than has been done in the past by examining Alfarabi's understanding of the subject matter of logic.

The current literature suggests that Avicenna developed his doctrine of the subject matter of logic in opposition to doctrines like the one outlined by Alfarabi.²²⁴ Yet, Fritz Zimmermann has noted that Alfarabi was the first to use the idea of secondary intelligibles in the first book of *Kitāb al-Ḥurūf* where Alfarabi discusses the subject matter of logic in some

²²³ A.I. Sabra, *op. cit.*; W. Kneale, M. Kneale, *The Development of Logic* (Oxford: Clarendon Press, 1962), 230; and K. El-Rouayheb, "Post-Avicennian Logicians on the Subject Matter of Logic: Some Thirteenth- and Fourteenth-Century Discussions", *Arabic Science and Philosophy* 22 (2012): 69-90.

²²⁴ Thus, Sabra says ("Avicenna on the Subject Matter of Logic", 762): "Avicenna seems to have had some such statements [as Alfarabi's] in mind when he wrote in chapter 3 of the *Introduction [Madḥal]* that "there is no value in the doctrine of those who say that the subject of logic is to investigate utterances in so far as they indicate notions".

detail.²²⁵ And Stephen Menn has more recently noted that Alfarabi appears to be the “inventor” of the expression,²²⁶ saying that for Alfarabi a “second intention is a concept applying to concepts, so something that is predicated of thoughts or “intelligibles” in the soul rather than directly of external things”, a typical example of which is *being-a-predicate*.²²⁷ This characterization of Alfarabi’s understanding of secondary intelligibles is quite close to Avicenna’s. Nevertheless, there are important differences between how the two thinkers think about how second intelligibles come to be in the mind. The main difference, as I will show in §3.1.1 is that Alfarabi says that primary intelligibles and not secondary intelligibles are the subject matter of logic. In §3.1.2 I will show how Avicenna is, in fact, deeply indebted to Alfarabi in how he conceives of secondary intelligibles, but he insists that secondary intelligibles are the subject matter of logic, not primary intelligibles. Thus, despite Avicenna’s appropriation of Alfarabi’s idea, he remains highly critical Alfarabi’s use of it in developing his doctrine of the subject matter of logic in *Kitāb al-Ḥurūf*. In §3.1.2 I claim that Avicenna is motivated to depart from Alfarabi for two reasons. One, Avicenna is interested in establishing logic as a branch of the philosophical sciences. In particular, Avicenna wants to fit logic into the Alexandrian division of the sciences as part of theoretical philosophical (*falsafa nazariyya*), rather than insisting with most Peripatetics and Aristotle’s Neoplatonist commentators that logic is a tool of philosophy and not a sub-discipline of philosophy.²²⁸ And two, stemming partially from his belief that logic is a science, Avicenna entertains a notion of the formality

²²⁵ I am thankful to Stephen Menn for reminding me of Zimmermann’s brief but insightful discussion of this important text from the *Ḥurūf* in Zimmermann’s introduction to his translation of Alfarabi’s lemmatic commentary and short treatise on Aristotle’s *De Int*.

²²⁶ S. Menn, “Al-Fārābī’s *Kitāb al-Ḥurūf* and His Analysis of the Senses of Being”, *Arabic Sciences and Philosophy* 18 (2008): 81.

²²⁷ Ibid.

²²⁸ See D. Gutas, *Medical Theory and Scientific Method in the Age of Avicenna*, in *Islamic Medical and Scientific Tradition*, ed. P. Pormann, vol. 1 (London, New York: Routledge, 2011), 33-47; idem., “Paul the Persian on the Classification of the A Milestone between Alexandria and Baghdad”, *Der Islam* 60/2 (1983): 231-67.

proper to logical inquiry that sets him apart from many Aristotle's Neoplatonist commentators, but which finds important precedent in Alexander of Aphrodisias.²²⁹

With Avicenna's ideas about the formality proper to logical inquiry in mind, in §3.2 I will discuss Avicenna's dismissal of the well-established division of "if...then..." propositions into complete (*tāmm*) and incomplete (*ġayr tāmm*) in ŠQ V, a division that is clearly on display in Alfarabi and in earlier authors such as Galen. Avicenna's rejection of this classification is based on two major criticisms, which are rooted in Avicenna's ideas about the formality of logical inquiry. One criticism is that the traditional classification is based on the propositional matter (*mādda*) of "if...then..." sentences rather than on their propositional form (*ṣūra*). Avicenna argues that since secondary intelligibles are the proper subject matter of logic, an investigation into the nature of the logical connection (*ittiṣāl*) between the antecedent and consequent in an "if...then..." statement carried out in terms of primary intelligibles will be unsuited to logical inquiry. Yet, claims Avicenna, this is precisely the error that the classification of connection into complete and incomplete commits. The upshot of the first criticism is that the classification of connection into complete and incomplete makes the utterance of connective conditionals useless (lit. "without benefit [*bi-lā fā'ida*]"). Avicenna's claim, which is borne out by the conclusions of chapter 2, is that in order to construct a connective conditional proposition according to what might be called the "Galenic" method, the speaker must already know and be in a position to assert the antecedent and the consequent.²³⁰ Avicenna argues that if the speaker already knows that the consequent is true, there will be no point in putting it into a connective conditional expression.²³¹ Avicenna proposes an alternative division of connection, which he believes will avoid both of these

²²⁹ J. Barnes, *Truth, Etc.: Six Lectures on Ancient Logic* (Oxford: Oxford University Press, 2007), chapter 4.

²³⁰ Susanne Bobzien, "Wholly Hypothetical Syllogisms", *Phronesis* 45/2 (2000): 87-137.

²³¹ M. Frede, "Stoic vs. Peripatetic Syllogistic", *Archiv für Geschichte der Philosophie* 56/1 (1974): 1-32.

shortcomings in the conventional view of connection. Avicenna divides connection into coincidental connection (*ittifāqī*), which he also calls connection *simpliciter* (*‘alā l-iṭlāq*), and genuine connection (*ḥaqīqī*), which he also calls implicative (*luzūmī*) or restricted (*‘alā t-taḥqīq*). The notion of concomitance in truth (Avicenna uses all of these as if they were synonyms *murāfaqa*, *muwāfaqa*, and most commonly *ma‘īyya*) underlies both of these kinds of connection. By recasting the analysis of connection in terms of concomitance in truth rather than in terms of following (*ittibā‘*), Avicenna thinks he is able to circumvent the shortcomings of the traditional classification of connection. Nevertheless, the results of chapter 2 suggest that, at least as a criticism of Alfarabi’s doctrines about conditionals, Avicenna’s criticisms sometimes miss the mark.

The final section of this chapter will turn to Avicenna’s discussion of repetitive syllogisms in ŠQ VIII. In that chapter, Avicenna’s ambivalent attitude toward repetitive syllogisms is motivated by his criticism of complete and incomplete connection that he took up in ŠQ V 1. Avicenna is not merely a parroting of Alexander’s oft-repeated gripe against repetitive syllogisms that they are not syllogisms according to Aristotle’s conditions.²³² In ŠQ VIII, Avicenna presents a summary of a view of repetitive syllogisms similar to what we find in Alfarabi’s epitomes of the *Prior Analytics* and in Galen’s *Insitutio Logica*, in which the division of inference schemes is carried out according to the division of connection into complete and incomplete types. The chief aim of §3.3 is to discuss why Avicenna believes that the Galenic account of repetitive syllogisms with connective conditional propositions is incoherent. Avicenna’s evidence for his position lies in his argument that moods, which are in reality formally identical, are claimed to be formally distinct in the Galenic doctrine of conditionals.

²³² M. Frede, “Stoic vs. Peripatetic Syllogistic”, 2.

Once again, Avicenna's focus on moods recalls a notion of the formality of logical inquiry that is reasonable to impute to Alexander of Aphrodisias.²³³ In general, this chapter shows how, relying on Peripatetic precedent, Avicenna argues that the complete/incomplete connection dichotomy, which can be found in Alfarabi, and in Greek logicians of late antiquity such as Galen, is not defensible as a logical doctrine of conditional reasoning, neither as a logical theory of conditional propositions nor as a logical theory of conditional syllogisms.

§3.1.1 ALFARABI ON THE SUBJECT MATTER OF LOGIC

Citing Alfarabi's "enigmatic" use of the term "secondary intelligibles" in his lemmatic commentary on *De Int.*, Sabra notes that while both Avicenna and Alfarabi seem to have used the concept of secondary intelligibles, Sabra was unable to see how Alfarabi thought this concept was relevant to the question of the subject matter of logic.²³⁴ Alfarabi deals with this the question of the subject matter of logic, in three different texts.²³⁵ One is in chapter two of *I'U*, in which his treatment of logic (*ṣinā'atu l-manṭiq*) follows his treatment of the linguistic sciences. This order is not accidental since the familiar analogy between the aims and utility of logic and grammar figures prominently in Alfarabi's thinking about the subject matter of logic as well. The second discussion, which is relevant the question of logic's subject matter, is the text from *APCA*, §41 treated earlier in §2, in which Alfarabi concludes his discussion of

²³³ J. Barnes, *Truth, Etc.*, 282.

²³⁴ Sabra, "Avicenna on the Subject Matter of Logic", 756.

²³⁵ In fact, there are four. The fourth is in the first section of *'Uyūn al-Masā'il*; see Alfarabi, *Al-Fārābī's philosophische Abhandlungen*, ed. F. Dieterici, (Frankfurt am Main: Institute for the History of Arabic-Islamic Science, 1999), 56. However, this text is probably not authored by Alfarabi. See M. Cruz Hernandez, Introduction to "El 'Fontes Quaestionum' (*'Uyūn al-Masā'il*) de Abū Naṣr al-Fārābī," ed. M. Cruz Hernandez, in *Abū Naṣr Muḥammad ibn Muḥammad al-Fārābī (d. 339/950): Texts and Studies*, ed. F. Sezgin, (Frankfurt am Main: Institute for the History of Arabic-Islamic Science, 1999), iv. 150f; F. Rahman, *Prophecy in Islam: Philosophy and Orthodoxy*, (London: George Allen & Unwin, 1958), 21f, n. 2; T. Druart, "Al-Fārābī, Emanation, and Metaphysics", *Neoplatonism and Islamic Thought*, ed. P. Morewedge, (Albany: SUNY Press, 1992), 127; D. Black, *Logic and Aristotle's Rhetoric and Poetics in Medieval Arabic Philosophy*, (Leiden; New York: Brill, 1990), 71, n.53. Nevertheless, see J. Lameer, *Al-Fārābī and Aristotelian Syllogistics: Greek Theory and Islamic Practice*, (Leiden; New York: Brill, 1994), 24.

Aristotle's *Categories* with an exposition of what he counts as logical intelligibles (*ma'qūlāt mantiqiyya*) as opposed to non-logical intelligibles. This APCA text complements Alfarabi's discussion of logic's subject matter, in the sense that the text sets out the types of intelligibles (and terms) that serve as the objects of logical operations. The third text is the longest and the most important. It appears in the opening chapters of Alfarabi's *Kitāb al-Ḥurūf*,²³⁶ in which Alfarabi discusses, *inter alia*, Aristotle's doctrine of the categories (*maqūlāt*) and the subject matter of logic and the philosophical sciences, and in what sense they are distinct from each other. In this text, Alfarabi develops a doctrine of primary (*ma'qūlāt uwal*) and secondary intelligibles (*ma'qūlāt tāniya* or *ma'qūlāt tawānin*), which he eventually brings to bear on the problem of the subject matter of logic. However, he concludes that primary intelligibles (*al-ma'qūlāt al-uwal*) are the subject matter of logic (and the other sciences) and not secondary intelligibles as Avicenna would later claim.

In *I'U*, Alfarabi says on several occasions that the art of logic is the complete set of all the rules (*ḡumlatu l-qawānīn*) that “rectify the intellect (lit. “straighten it out,” *tuqawwimu l-‘aqla*) and set it on the path to correct judgment (*aṣ-ṣawāb*) and truth.”²³⁷ According to Alfarabi, logic aims to guard the intellect from error in passing a judgment related to truth and falsehood. In addition, logic is primarily a defensive, even prophylactic, discipline that serves as an instrument (*ālā*) that holds scientific and philosophical investigation to the norms of right reasoning. Alfarabi illustrates logic's normative character by three analogies. Alfarabi says that logic is analogous to grammar (*‘ilm an-naḥw*) and prosody (*‘ilm al-‘arūd*), in the sense that the way in which grammar furnishes rules governing the sound and unsound

²³⁶ Zimmermann, Introduction to Alfarabi, *Al-Farabi's Commentary and Short Treatise on Aristotle's De Interpretatione*, trans. F. Zimmermann (London: Oxford University Press, 1981), xxxiv; S. Menn, “Al-Fārābī's Kitāb al-Ḥurūf and His Analysis of the Senses of Being”, 81, n. 32.

²³⁷ Alfarabi, *Catalogo de las ciencias*, ed. A. G. Palencia, (Madrid: Consejo Superior de Investigaciones Científicas, Patronato Menéndez y Pelayo, Instituto Miguel Asín, 1953), 21f.

arrangement of verbal expressions (*alfāz*) and prosody furnishes rules governing the proper and improper arrangement of poetic meter (*awzān aš-šī'r*), is similar to the way logic furnishes rules that determine correct and incorrect arrangement of intelligibles (*ma'qūlāt*) in philosophical discourse. The third comparison, comparing logic to weights and measures (*al-mawāzīn wa-l-makāyīl*), adds a further element to the analogy:

[Text 1] The logical rules [*al-qawānīn al-manṭiqiyya*] are an instrument by which one appraises [*yamtaḥīnu*] the intelligibles that one is apt to err in or to fall short in perceiving their true nature. <In this way, logical rules> resemble weights and measures. For the latter are things by which one appraises bodies [*yamtaḥīnu bihā fī kaṭīrin mina 'l-aḡsām*] whose determination one may have erred in or fallen short in evaluating the true measure of. Such is, for example, the straightedge [*al-masāṭir*], by which one appraises [*yamtaḥīnu*] the lines whose straightness sense perception is likely to be uncertain or mistaken in perceiving.²³⁸

Logic supplies us with a set of norms which we hold up our philosophical arguments up to, and which we determine the correctness or erroneousness of our arguments by. But this third analogy indicates that logic acts as a necessary supplement to our otherwise intuitive grasp of argumentative validity. We can eyeball a straight line or a circle but, on closer inspection, this method always proves to be insufficient, requiring the aid of a straightedge or a compass. Similarly, each person has an intuitive grasp of the validity of arguments that he hears, but Alfarabi seems suspicious of the suitability of mere intuition as a guide in philosophical discourse. For Alfarabi, logic provides a set of rules that enhance our already intuitive grasp of an argument's validity, protecting us from falling into error on our own or being led astray by others.

Indeed, when we consider the question of logic's utility (*fā'ida*, *manfa'a*), the scope of logic's application—to arguments we make to ourselves, we make to others, and that others

²³⁸ Alfarabi, *Catalogo de las ciencias*, 29. *imtaḥana/yamtaḥīnu/imtiḥān* here means to “appraise” or “measure up” according to a standard. See N. Rescher, “Yahyā ibn ‘Adī’s Treatise ‘On the Four Scientific Questions Regarding the Art of Logic’”, in *Studies in Arabic Logic*, ed. N. Rescher (Pittsburgh: University of Pittsburgh Press, 1967/8), 38-47.

present to us—is crucial to Alfarabi’s thinking about logic. Logic’s dialectical backdrop, against which its rules are formulated as norms universally applicable to all manner of philosophical discourse and to each individual’s arguments, is an indispensable element in Alfarabi’s thinking. In other words, logic’s main function is as a set of normative rules that regulate philosophical discourse which is first and foremost carried out by means of *argument*. Perhaps the ideal of philosophical debate is demonstration, but in reality, most philosophical argumentation happens at the level of dialectical exchanges between parties to a disagreement. It is at logic’s universal applicability that the analogy between logic and (Arabic) grammar terminates, for “grammar only furnishes rules that govern the speech of a single nation (*alfāz ummatin-mā*), whereas logic furnishes common rules that apply universally to the speech of all nations (*ta‘ummu alfāza l-umami kullihā*).”²³⁹ This universality arises from the imperative to construct justifications (or proofs, sc. *ḥuǧǧa/ḥuǧaǧ*) of our opinions, both to ourselves and to others, for, in either case, it is the proof that necessitates the soundness of the opinion (*al-ḥuǧǧa tūǧibu ṣiḥḥata r-ra’y*).²⁴⁰ Since logic is therefore a set of universally recognized discursive norms that circumscribe the bounds of permissible deductive steps, it is only indirectly involved in the deductive process of knowledge acquisition (by a “deductive process” of acquiring knowledge I mean only the syllogistically mediated process of moving from known premises to unknown conclusions).²⁴¹ Logic does not so much generate knowledge as much as it generates certainty (*yaqīn*) that our deductive steps are not awry.

²³⁹ Alfarabi, *Catalogo de las ciencias*, 33.

²⁴⁰ *Ibid.*, 27.

²⁴¹ Burnyeat and Barnes (see §1) both agree that the theory of demonstration that Aristotle sets out in the *Posterior Analytics* cannot possibly be a process that generates new knowledge. In other words, they rule out the possibility that Aristotle conceived of demonstration as an ampliative process, where the logical combination of premises yields a conclusion that was not previously part of the set of all our objective knowledge. There has, however, been more recent work on Aristotle’s *Posterior Analytics* that moves substantially beyond Burnyeat and Barnes; e.g. J. Lennox, “Divide and Explain: the *Posterior Analytics* in Practice”, in *Philosophical Issues in Aristotle’s Biology*, ed. A. Gotthelf, J. Lennox (Cambridge: Cambridge University Press, 1987), 90-119.

Logic also generates certainty (which obviously different from knowledge) by allowing us to justify our knowledge to ourselves and other by dictating rules for constructing proofs.

The justification of our views to ourselves turns the mode of the philosophical discourse “inward (*dāḥil*),” to the level of intelligibles (*ma‘qūlāt*) rather than words (*alfāz*). In the inner discourse, the normative rules supplied by logic eliminate the subjective aspect of our opinions by providing a set of universally recognized norms according to which our arguments are evaluated. When this set of norms, which provides objective, universally acknowledged criteria for the construction of valid arguments, is applied to the inner discourse of the soul, the person become secure that his beliefs qua conclusions of a process of reasoning are not based on a mere presentiment of their soundness.

[Text 2] If we have these rules and we try to derive a conclusion and justify it to ourselves [*wa-taṣṭihahu ‘inda anfusinā*], we will not let our minds wander about unguided [lit. “swim about”, *tasbahu*], searching for what we want to prove among a countless number of things, allowing our minds to come and go as chance has it, and approaching the question from points of view that might lead us to err such that we imagine that something is true when, in fact, it is not, while being unaware of <our mistake>. Rather, we must know which path to follow to [the conclusion], which things will convey us [to the desired conclusion], where we will begin the journey, how we will stop when once our minds are certain, or how our minds will come to know things little by little until we have necessarily attained our aim. All the while, we must be aware of the things that would cause us to err and obscure our judgment, so that we might avoid them on our journey. When all this happens, we will be certain that we have found the truth in what we have derived, and that we have not erred. What is more, if we see anything that we derived that causes us to suspect that we have passed over something, we can immediately test it [against the rules of logic] so that if there is a mistake, we can easily remedy the mistake.²⁴²

What is interesting about this passage is that logic does not just provide a criterion for distinguishing the true opinion from the false opinion. By deploying a journey metaphor, Alfarabi hints that logic dictates the discursive rules of argumentation, i.e. which type of

²⁴² Ibid., 24f.

argument will be used (e.g. categorical or hypothetical), the types of premise we must start from, the order of the premises, the deductive steps that will be taken, and the rules for avoiding false reasoning and sophistry. What is more, the rules of logic are not subjective in the sense that they reflect the nature of an individual mind's inner workings. As Alfarabi says, exactly the same rules that apply to the inner dialog of intelligibles are used when philosophical discourse turns outward (*hāriġ bi-ṣ-ṣawt*). Even with this shift in the mode of the discourse from inward intelligibles to outward expressions (*alfāz*), logic is necessary for all the same reasons, viz. its dictating the discursive rules of dialectic, argumentative validity, proof, and its supplementing our instinctive grasp of erroneous and sophistical argumentation. Alfarabi's vision of logic turns on an exact one-to-one correspondence between intelligibles and the expressions that signify them: the same rules apply to both with exactly the same consequences. In other words, according to Alfarabi, the constant shift in the mode of discourse in a single dialectical exchange, the move from intelligibles to speech and from speech to intelligibles, has no impact on the nature and content of logic as a discipline.

We learned earlier that Alfarabi believed that the goal of logic was protecting the reasoner by prescribing a set of discursive norms which, if followed, will lead to certainty in the reasoner's opinions, by rendering invulnerable to error the deductive process by which the reasoner arrived at these opinions. If certainty in one's opinions is the goal of logic, then its utility lies in the process of justification (*taṣḥīḥ*), i.e. proof. In philosophical discourse, proof furnishes the best means to achieve this certainty. The proof will exhibit the truth of one's views to one's self, if the discourse is internal, and to others, if the discourse is external. In fact, Alfarabi justifies the dichotomous nature of the logic's subject matter (intelligibles qua significands of expressions *and* expressions qua signifiers of intelligibles) by explicit reference

to the need to externally justify one's opinions to others through the medium of words, and to internally justify one's opinions to one's self through the medium of intelligibles.

[Text 3] The objects of logic, viz. those things about which the rules [of logic] are set down, are intelligibles insofar as they are signified by expressions and expressions insofar as they signify intelligibles. This is so because we justify an opinion to ourselves by thinking [*nufakkiru*], then giving an account [to ourselves, (*narwī*)], and then setting up in our minds [*nuqīmu fī anfusinā*] <mental> objects and intelligibles [*umūran wa-ma'qūlātin*] that are suitable for justifying this opinion. We justify <the opinion> to someone else by presenting him with arguments [*nuḥāṭibuhum bi-l-aqāwīl*] through which he will understand the various concepts that are suitable for justifying this opinion. Nor is it possible to justify any opinion with just any sort of intelligible, in any number and under any condition, composition or order. Rather, each opinion we want to prove needs definite <mental> objects and intelligibles [*umūrin wa-ma'qūlātin maḥdūda*] that are of a certain number, standing in certain conditions, and compounded and ordered in a certain fashion. And this is also how the verbal expressions by which we express these intelligibles must be when we want to justify our opinion to someone else.²⁴³

According to Alfarabi, the reason why the subject matter of logic is characterized by what we might justly call “the intelligible/expression dichotomy” is (1) due to the need for proofs or justifications of opinions that we come to hold by means of deductions, and (2) that the arena for these proofs may be internal through the medium of concepts or external through the medium of speech. In both cases, Alfarabi assumes that the discursive norms that the arguments are held to, i.e. the rules of logic, are exactly analogous in both arenas.

Alfarabi's discussion of the aims and utility of logic is part of an ancient debate among Peripatetics, Stoics, and Platonists about the instrumentality of logic vis-à-vis the rest of philosophy.²⁴⁴ Results from studies by Jonathan Barnes and Hans Gottschalk suggest that the outlines of this debate originate perhaps as early as Andronicus of Rhodes (d. 60 B.C.), who in his critical edition of Aristotle's corpus gathered Aristotle's works on logic into a single unit he

²⁴³ Ibid., 31f.

²⁴⁴ See Jonathan Barnes' discussion of logical utilitarianism among Stoics, Peripatetics, and Galen in chapter 2 of J. Barnes, *Logic and the Imperial Stoa* (Leiden, New York, Köln: Brill, 1997), especially 19-23. For the Platonists, see C. Evangelou, *Aristotle's Categories and Porphyry* (Leiden, New York: Brill, 1988), 7f, and chapter 4 for Plotinus.

called the *Organon*. Andronicus was also perhaps the first Peripatetic to argue explicitly for logic's utility vis-à-vis the rest of philosophy.²⁴⁵ What is more, “logical utilitarianism” was an attitude toward logic held by such august figures as Alexander and Galen, both of whom exercised a profound influence on the development of Arabic philosophy. For example, Barnes notes that, citing Aristotle's opening comments of the *Prior Analytics*, Alexander insisted that the aim of syllogistic is the production of demonstrative proofs, and that “consequently a philosopher should study only those logical forms which are of use—of conceivable use—for the formalization of scientific arguments”.²⁴⁶ Similarly, according to Barnes, Galen believed that “the scientist needs logic in order to make certain discoveries and in order to organize and confirm what has been discovered, so that logic is an indispensable instrument of research and exposition. But beyond that it has no function [...]”.²⁴⁷ Rather than arising from a particular text or aporia in the Aristotle's text, these debates were carried out in the interest of establishing the authorship of an entire text (e.g. *De Int.*) or with general methodological principles (e.g. logical utilitarianism).

Alfarabi's discussion of the subject matter of logic is different from his discussion of logic's aim and utility. It arises out of a particular textual tradition that became popular in Neoplatonic commentaries on Aristotle's *Categories*,²⁴⁸ in which commentators would go through the motions of settling the question, among others,²⁴⁹ of what the purpose (*skopos*) of Aristotle's *Categories* is before beginning their lemmatic commentary. The question that

²⁴⁵ H. Gottschalk, “The Earlier Aristotelian Commentators”, in *Aristotle Transformed: The Ancient Commentators and Their Influence*, ed. R. Sorabji (Ithaca: Cornell University Press, 1990), 55-82.

²⁴⁶ J. Barnes, *Logic and the Imperial Stoa*, 20.

²⁴⁷ *Ibid.*

²⁴⁸ On this textual tradition, see P. Hoffmann, “Catégories et langage selon Simplicius—la question du ‘skopos’ du traité aristotélicien des ‘Catégories’ ”, in *Simplicius: sa vie, son oeuvre, sa survie*, ed. I. Hadot (Berlin: de Gruyter, 1987), 61-90. See also Evangeliou, *Aristotle's Categories and Porphyry*, chapter 1.

²⁴⁹ Evangeliou, *Aristotle's Categories and Porphyry*, 18; for an example in the Arabic tradition, see, e.g., R. Wisnivosky, “Yahyā ibn ‘Adī's Discussion of the Prolegomena to the Study of a Philosophical Text”, in *Law and Tradition in Classical Islamic Thought*, eds. M. Cook, et al. (Basingstoke, Hampshire: Palgrave Macmillan, 2013), 187-202.

should draw our attention here is this. How does a discussion in Aristotle's late antique Neoplatonic commentators about the *skopos* of the *Categories*, which is variously translated as purpose, goal, or aim, get taken up by Alfarabi and transformed into a discussion about the subject matter of logic?

In order to move toward an answer to this question, let us take Simplicius' commentary on the *Categories*.²⁵⁰ Continuing a practice that became institutionalized in Greek and Arabic late antiquity, Simplicius first considers a number of questions about the *Categories* before moving to lemmatic commentary. One of the more fundamental questions Simplicius addresses is this: what is the purpose (*skopos*) of the *Categories*? In surveying the different answers given to this question until his day, Simplicius presents Porphyry's solution to this question as the correct one, though he also imputes versions of Porphyry's solution to earlier Peripatetic thinkers such as Boethus of Sidon (d. ca. 10 B.C.) and Alexander (fl. ca. 200 A.D.).²⁵¹ He presents Porphyry's solution both as a culmination of three earlier views about the purpose of the *Categories* and as a synthesis of all three of them. To the question "What is the *Categories* about?" all parties were in agreement, according to Simplicius, that the *Categories* is "about ten simple things, which, since they are most universal, they are called 'genera' ".²⁵² Yet, beyond this preliminary agreement, the parties stood at odds. To the question, "What is the *Categories* about?", one party responded that it is about words or expressions (*phônai*), another that it is about things (*ta onta*, or *pragmata*), and the last that the *Categories* is about notions (*noemata*).

[Text 4] Now some say that they are about words [*phônai*], and that the goal is about simple words, and that it is the first part of logic. Just as the first part of the book on propositions [i.e. *De Int.*] is about composite words, but not about realities [*pragmata*], so this [book], being about the parts of the proposition, would be about words [...]

²⁵⁰ Simplicius, *On Aristotle's Categories*, 1-4, trans. M. Chase (Ithaca: Cornell University Press, 2003).

²⁵¹ Simplicius, *On Aristotle's Categories*, 1-4, 10.10-19

²⁵² Ibid. 9.6-8.

Others, however, do not accept this goal. It does not, they say, pertain to the philosopher to theorize about words, but rather to the grammarians, who investigate their modifications, configurations, and changes in word-endings, as well as their proper usages and their types. They say the goal is about the very beings which are signified by words; and that these are what is said [*to legomenon*] [...]

In opposition to these considerations too, however, is [the fact that] the present book is a part of the study of logic, whereas to occupy oneself with beings *qua* beings is to engage in that philosophy which is metaphysical, and in general primary [...]

Others say that the goal is neither about significant words nor about signified realities, but rather about simple notions [*noêmata*]. For if, they say, the discussion in the [*Categories*] is about the ten genera, and the latter are posterior and conceptual, then the discussion is about notions [...]. These people, however, should have considered that to speak about notions *qua* notions does not pertain to the study of logic, but rather to that of the soul. Of these people, each one had an imperfect grasp of the goal [...]²⁵³

And Simplicius, with most of Aristotle's later Neoplatonist commentators,²⁵⁴ adds a fourth view (the correct one in his eyes) saying that there is a sense in which the *Categories* is about all of them: the *Categories*, Simplicius concludes, "is about simple, primary words which signify the primary and most generic beings by means of simple, primary notions".²⁵⁵

There are two important conclusions that we might draw with respect to how this debate about the purpose of the *Categories* might have been adapted by Alfarabi's for a discussion of the subject matter of logic in general. The first is that despite Simplicius' insistence that intelligibles (*noêmata*) play a role in this story about the aim of the *Categories*, it is not clear what he means when he says that the categories *qua* expressions (*phônai*) signify things (*ta pragmata*, *ta onta*) by means of intelligibles (*noêmata*). Earlier in his commentary,

²⁵³ Ibid., 9.4-10.7.

²⁵⁴ The "inclusive interpretation" as Evangeliou calls it, is perhaps most forcefully (if not disingenuously) stated by Iamblichus (in Evangelios, *Aristotle's Categories and Porphyry*, 32): "O men! You fight each other without really fighting each other. Though you tell the truth, none of you is wholly right. And none of you is entirely wrong, though you tell lies".

²⁵⁵ Simplicius, *On Aristotle's Categories*, 1-4, 13.18-21.

Simplicius appears to be wrestling with how to fit intelligibles into the mechanics of how words signify things.²⁵⁶ In order to do so, he appeals to a Platonic-Aristotelian theory of cognition, which stipulates that at the moment of cognition the knower, act of knowing, and object of knowledge are one, and, thus, share the same nature.

[Text 5] For neither are significant expressions wholly separate from the nature of beings, nor are beings detached from the names which are naturally suited to signify them. Nor, finally, are notions extraneous to the nature of the other two; for these three things were previously one and became differentiated later. For Intellect, being identical with realities and with intellection, possesses as one both beings and the notions of them by virtue of its undifferentiated unity; and there [sc. in the intelligible word] there is no need for language.²⁵⁷

Simplicius seems to be trying to justify or at least give a clear sense to the role he claims intelligibles play in what we might for convenience call this “economy of signification”. Yet, his reasoning is less than satisfactory for several reasons. Firstly, it falls prey to the charge of psychologism that he leveled against the party that contended that the categories are intelligibles. Second, while this justification may explain why notions share a nature with the beings that are signified by categories, it does so at the expense of clarifying the relation between terms and intelligibles, despite the fact that Simplicius claims that categories signify things *by means of* intelligibles. Thirdly and most importantly, Simplicius’ reasoning does not address the question of signification vis-à-vis intelligibles at all. All he has argued is that intelligibles and beings share a nature at some point in their history. Thus, in spite of his efforts, Simplicius’ commentary leaves the precise details about the place of notions in words’ signification of things an open question for later commentators to work out.

²⁵⁶ This portion of the Porphyrian solution to the purpose of the *Categories* is not found in Porphyry’s surviving commentary on the *Categories*. It seems to be a later development of his theory, likely due to the influence of Iamblichus; see R. Sorabji, *The Philosophy of the Commentators*, vol. 3 (Logic and Metaphysics) (Ithaca: Cornell University Press, 2005), 74: “Iamblichus, according to Simplicius in Cat. 2,13-14, gave an intellectual (*noera*) interpretation not only for substance, but for nearly every topic in his (now lost) commentary on the *Categories*, and Simplicius quotes many of them [...]”.

²⁵⁷ Simplicius, *On Aristotle’s Categories*, 1-4, 12.13-20.

Alexander also introduced intelligibles into his discussion of the *skopos* of the *Categories*, and whose solution Simplicius calls among the “more complete”.²⁵⁸ Simplicius quotes from Alexander’s (lost) commentary on Aristotle’s *Categories* at some length.²⁵⁹

[Text 6] This book is the beginning [*arkhê*] of the study of logic since “the proposition [*ho lógos*] is significant because its primary elements are significant. Now, Aristotle wants to show what the notions notions [*noêmata*] that are signified by the primary and simple elements are. In order to do this, he carries out a division of being [*tò ón*] not into particular individuals (for these are uncircumscribable [*aperilêpta*]²⁶⁰ and unknowable, owing to their multiplicity and the fact that they undergo various changes). Rather, he divides [being] into the highest genera, the ten genera presented here, which Aristotle calls “categories”, since they are the most generic, and since they do not themselves serve as the substrate for any other thing but are predicated of others. Thus, the aim is to examine the simple and most generic elements of a proposition that signify simple notions about simple realities.”²⁶¹

Like the solution Simplicius prefers, the categories are verbal expressions, but unlike the Porphyrian solution, Alexander says that verbal expressions signify notions (*noêmata*), not things directly. First, these verbal expressions do not signify just any notion that comes to the mind, but those that are about things outside the mind. Second, since these verbal expressions are predicates of the highest genera, the notions they signify are not of concrete objects, which only naturally stand in the subject term position. Instead, these verbal expressions signify intelligibles about things outside the mind about which scientific knowledge is possible (in the strong way required by Aristotle at beginning of the *Posterior Analytics*²⁶²). Thus, according to Alexander these categories cannot, *inter alia*, signify notions about concrete, individual objects due to the fact that they are subject to change, can only ever serve as the substrate for properties, and are so numerous as to lack a unifying principle. In order to

²⁵⁸ *Ibid.*, 10.9.

²⁵⁹ Hoffmann, “Catégories et langage selon Simplicius”, 71f.

²⁶⁰ For *aperilêpta* I have adopted Chase’s translation for Hoffmann’s “insaisissables”.

²⁶¹ P. Hoffmann, 71 (from French); cf. Michael Chase’s translation at Simplicius, 10.9-20.

²⁶² See P. Adamson, “Knowledge of Universals and Particulars in the Baghdad School”, in *Islamic Medical and Scientific Tradition*, ed. P. Pormann, vol. 1 (London, New York: Routledge, 2011), 69.

determine what kinds of thing these intelligibles can be about, it might be useful to consider an example. The verbal expressions “human” and “man” signify intelligibles in the mind that are about realities outside it. In this case, the realities happen to be quiddities (viz. the things you speak about when some asks “What is X?”, where X is a particular individual). As a consequence, the verbal expressions “human” and “man” belong in the category substance. The verbal expressions “three”, “red”, “slave” are similar but they do not so obviously accord with Alexander’s scheme. They are each predicate expression that signify intelligibles in the mind that correspond to some state of affairs outside it. In this instance, the predicate “three” serves as a response to the question “How many are X?”, the predicate “red” to the question “How is X?”, and the predicate “slave” to the question “What relation does X have (with respect to Y)?”. All of these predicates may be said about individual Xs, which generally are subject to change, are contingent, and are too multifarious, *qua* individual beings, to be organized according to a principle. There cannot, therefore, be any scientific knowledge about X as an individual. However, the important point to notice is that in each of the example questions mentioned above, we are asking about (1) a real state of affairs, which hold of an X that may or may not be eternal, unchanging and necessary. Nevertheless, regardless of whether X itself can be the object of scientific knowledge, (2) there are intelligibles *red*, *three*, and *slave* with a unitary and constant meaning that do circumscribe states of affairs outside the mind. Thus, (3) the predicate terms “red”, “three”, and “slave” belong to the categories quality (or “How?”), quantity (or “How many?”), and relation insofar as they signify notions that circumscribe such states of affairs. In conclusion, we may reasonably construe Alexander’s claim that the aim of the *Categories* is to examine those terms that fulfill conditions (1) – (3).

Whether or not the above is a convincing interpretation of Alexander, I think it is clear that Simplicius' claim that his and Porphyry's solution to the question of the purpose of the *Categories* is shared and even anticipated by Alexander is disingenuous. Notions play no role at all in Porphyry's discussion of this issue in his catechism commentary on Aristotle's *Categories*, since according to this account categories are terms directly signifying things. Notions occupy an important place in Alexander's discussion on the other hand, since according to his account category terms signify notions. Yet, despite these differences, there is an important similarity that must be highlighted. I would like to draw the reader's attention to the fact that *all* parties to this debate reduce the question of the purpose of the *Categories* to a question of determining what exactly categories are. So the first party argues that since categories are words, the purpose of the *Categories* is the study of words; the second that since categories are things, the purpose of the *Categories* is the study of things; the third that since in the categories are notions, the purpose of the *Categories* is the study of notions; the fourth argues that since the categories are words significant of things outside the soul, the purpose of the *Categories* is the study of the words significant of things outside the soul; the Alexandrian account argues that since the categories are terms signifying of notions circumscribing states of affairs outside the mind, the purpose of the *Categories* is the study of expressions answering to that description. Indeed, Porphyry makes this way of addressing the problem of the *skopos* of the *Categories* explicit in his catechism commentary, saying that the question of the *Categories* *skopos* can only be determined when the subject matter of the *Categories* has been properly determined.²⁶³ I believe, then, that the most important outcome of this debate, at least for the history of Arabic philosophy and logic, is *not* what Aristotle's commentators had primarily intended this debate

²⁶³ Porphyry, *On Aristotle's Categories*, trans. S. Strange (Ithaca: Cornell University Press, 1992), 33.

to settle, i.e. the purpose of the *Categories*. Rather, I suggest that it is plausible to conclude that what Arabic logicians such as Alfarabi took away from this debate are ideas about what the nature of Aristotle's categories is, and that any adequate answer to this question must involve giving an account of the relation between words, things, and notions.

I have just presented evidence from the Greek texts that all parties to the ancient and late antique debate about the *skopos* of Aristotle's *Categories* seem to have reduced this debate to one about the precise nature of the categories. I will now discuss evidence for the claim that the Greek discussions of the *skopos* in the prolegomena of the commentaries on the *Categories* were understood by Alfarabi as, *inter alia*, an exposition on the nature of the most basic elements of *logic* and not merely the *Categories*. The evidence for this claim comes in two parts. First, Alfarabi's discussion of the subject matter of logic in his summary of Aristotle's *Categories* is closely related to Alexander's discussion of the nature of the categories in his exposition on purpose of the *Categories* recorded in Simplicius' commentary. Second, Alfarabi's text about the subject matter of logic in *Kitāb al-Ḥurūf* takes a discussion about the nature of the categories as a starting point. There are similarities to Alexander in these passages from the *Ḥurūf* too. However, in the passages from the *Ḥurūf* about the subject matter of logic, Alfarabi approaches the question from a different angle, namely, from a Porphyrian angle. In these passages, Alfarabi introduces the distinction between primary and secondary intelligibles. But, in a way analogous to Porphyry, Alfarabi uses this distinction in order to show that the subject matter of logic is primary intelligibles. I will show that Alfarabi's distinction is indebted to Porphy's earlier distinction between words of first and second imposition, which Porphyry introduced in his own discussion of the *skopos* and subject matter of the *Categories*. Once more,

ancient discussions of the purpose of the *Categories* inform Alfarabi's thinking about the subject matter of the whole logical discipline.

In *APCA*, §41 Alfarabi discusses the particular types of intelligible and verbal expression that serve as the subject matter of logic. There, he identifies both genus and species terms and genus and species concepts as the paradigmatic logical objects, i.e. as the paradigmatic objects subject to the rules of logic. Having just completed his discussion of all of Aristotle's categories and before moving on to his epitome of the *Postpraedicamenta*, Alfarabi concludes his summary with the following words:

[Text 7] These then are the summa genera, under which fall all sensible things [*al-ašyā' al-mahsūsa*], and which are the intelligibles that are most inclusive of these sensible things. These genera and the species subordinate to each of them may be regarded *qua* intelligibles of existent sensible objects and representations [*miṭālāt*] of existent things in the soul. If [these representations in the soul] are taken in this way, then [the summa genera and their subordinate species] will be intelligibles existing in the intellect, rather than logical intelligibles [*fa-idā uḥidat hākaḍā kānat hiya al-mawǧūda al-ma'qūla wa-lam takun manṭiqiyya*]. However, if they are taken *qua* universal intelligibles that define sensible things, and insofar as terms signify them, then they are logical intelligibles and [*ma'qūlāt manṭiqiyya*] are called categories [*maqūlāt*]. In this latter case, [these intelligibles] have two relations, a relationa to concrete individuals and a relation to verbal expressions, and it is by virtue of the fact that they possess these two relations that they become logical [properly speaking]. Similarly, when they are considered insofar as some are superordinate to others and some are subordinate to others, or *qua* predicates and subjects, or insofar as some define others according to the ways of definition mentioned earlier, viz. the definition "What is X [*mā huwa š-šay'*]?" and "Which kind of thing is X [*ayyu šay'in huwa*]?"—in all these cases, they are logical intelligibles. On the other hand, if they are taken as divorced from all definitions and taken instead as simply intelligibles of existent things, whether as intelligibles of the disciplines of physics, or geometry or any other theoretical discipline, then [these intelligibles] will not be called categories.²⁶⁴

²⁶⁴ Alfarabi, *APCA*, 26.12-23. Cf. al-Ḥasan b. Suwār's notes on the *Categories*: "[Ibn Suwār] states first that Aristotle's aim in the *Categories* is to discuss those 'single utterances in first position (*fī al-waḍ' al-awwal*) which signify the highest genera of things (*al-umūr*) by means of the affections (*āṭār*) [produced] by them in the soul, and [to discuss] things insofar as they are signified by the utterances;" cited and translated by Sabra, "Avicenna on the Subject Matter of Logic," 754. Thus, Alfarabi's thoughts about the subject matter of logic are closely related to the question of the *Categories*' place in the *Organon*. There seems to be a sense in which the *Categories* furnishes the

Alfarabi's discussion differs from earlier Greek discussions in which the nature of the categories is discussed. As I have mentioned before, the question of the nature of the categories was subsumed within the question of the *skopos* of the *Categories*. In this text from Alfarabi's *APCA* the question of the nature of the categories is no longer carried out with this end in mind. Instead, it is subsumed under the larger project of identifying what the basic elements of logic are. Second, Alfarabi's thinking here seems motivated by those of his contemporaries—exemplified dramatically by the famous debate between Alfarabi's teacher Abū Bišr Mattā ibn Yūnus (d. 940) and the grammarian Abū Sa'īd aṣ-Ṣirāfī (d. 979), the latter of whom was critical of logic and laid the charge that it was redundant since it performed the same role as grammar. Earlier authors such as Simplicius and Alexander, both of whom as we have seen insist that the categories are verbal expressions—significant verbal expressions but verbal expressions nevertheless, would easily fall prey to the charge by grammarians that logic had nothing to offer to the sciences since it was made redundant by grammar. Simplicius and Porphyry insisted that categories are verbal expressions and both proved to be at a loss to say what exactly the role of notions is vis-à-vis words and things. To the extent that Alexander's text says that words signify intelligibles rather than the terms verbal expressions signifying them, Alfarabi would have found Alexander's solution to the *skopos* problem more appropriate to a defense of logic against critics. Indeed, it would be Alexander's claim that the category predicates signify intelligibles that would make it easier for Alfarabi to vindicate Abū Bishr Mattā ibn Yūnus' claim against Ṣirāfī and the grammarians that logic concerns meaning (*ma'nan*) not utterances (*alfāz*), and thus that logic is not a universal grammar, but a norm of

subject matter of logic insofar as the *Categories* is read, as Alfarabi and his contemporaries did, as providing a theory of terms from which all propositions and syllogisms are constructed.

right thinking.²⁶⁵ Nevertheless, Alfarabi's text places much more emphasis than Alexander's text on the fact that the categories are intelligibles. As a result, Alfarabi distinguishes the kind of intelligible that serves as the subject matter of logic from intelligibles that are merely mental representations of things (*miṭālāt*) outside. This idea seems to derive from Alfarabi's understanding of Alexander's exclusion of verbal expressions that refer to notions of changing, contingent individuals. Yet, Alfarabi goes further than the Alexander text by explicitly linking the categories with the task of constructing genus-species type definitions.

Despite these differences, the nature of the most basic logical elements in this text from APCA is similar to Alexander's, in the sense that the categories taken as verbal expressions signify notions or intelligibles rather than things. Further, in both Alfarabi and Alexander, the relation between intelligibles and the terms signifying them on the one hand and intelligibles and things on the other, is different. Terms and notions stand in a relation of signification, whereas intelligibles circumscribe concrete individuals to the extent that they can be used to define sensible objects. Further, in both Alexander and Alfarabi there is an emphasis on the categories as a classification of predicates. In both authors, this classification is carried out by considering which kinds of predicates are used to answer definition-type questions about a concrete individual X. Predicates answering to the question "What is X?" belong to the category of substance; predicates answering to the question "When is X?" belong to the category of when; those answering to the question "What kind is X?" belong to the category of quality; those answering to the question "How many are X?" belong to the category of quantity, etc. Finally, in Alexander there is a concern to distinguish the sub-scientific knowledge that we have of particular, changing, and contingent concrete individuals from the

²⁶⁵ P. Adamson, "Knowledge of Universals and Particulars", 75.

scientific knowledge that attends knowing quiddities or states of affairs circumscribed by the intelligibles signified by the category terms. Alfarabi not only wants to distinguish between logical intelligibles, viz. the categories, and mere representations of concrete individuals. He also wants to distinguish logical intelligibles from intelligibles that constitute the subject matter of the other sciences. Alfarabi seems to mean by this that what distinguishes the categories from the intelligibles in the other sciences is not that they possess some wholly different nature. Rather, logical intelligibles are distinguished from the intelligibles used in physics or politics by the goal of the inquiry in which they are being used. The goal of logical inquiry is definition whereas the goal of physics is an inquiry into things that have the principle of motion in themselves. Logical intelligibles are then distinct from the intelligibles of physics to the extent the former serve as genus and species terms in constructing definitions, whereas the latter are not.

In *Kitāb al-Ḥurūf* Alfarabi takes up the question of the subject matter of logic once again. Once more we find that his thoughts on the subject matter of logic start out from questions relating to the nature of categories. In the midst of this discussion Alfarabi's distinction between primary and secondary intelligibles arises. Once again it is clear that Alfarabi's distinction is best understood against the backdrop of the Greek discussions of the purpose of the Aristotle's *Categories*. We have already seen how Alfarabi's emphasis on intelligibles seems to stem both from his understanding of Alexander's commentary on the purpose of logic and from debates about the usefulness of logic among his contemporaries. As we have seen, it appears that Alfarabi's emphasis on the idea that the categories *qua* verbal expressions signify intelligibles rather than things is taken from Alexander. However, Alfarabi's division of *intelligibles* into primary and secondary seems to come from Porphyry's commentary on

Aristotle's *Categories*, in which Porphyry draws a distinction between *terms* of first and second imposition. The evidence presented from *Kitāb al-Ḥurūf* thus constitutes the second body of evidence for my claim that Arabic logicians understood Peripatetic and Neoplatonic prolegomena about the purpose of the *Categories* as texts relevant to the question of the subject matter of logic more generally. I further claim that in *Kitāb al-Ḥurūf* Alfarabi melds Porphyry's distinction between terms of first and second imposition and Alexander's emphasis on terms signifying intelligibles rather than things to arrive at a novel thesis that the subject matter of logic is first intelligibles, which are none other than the categories themselves. In this discussion in *Kitāb al-Ḥurūf*, secondary intelligibles are useful to Alfarabi as way of distinguishing what he believes to constitute the subject matter of logic from what does not. It is Avicenna's contribution to the philosophy of logic to reject Alfarabi's thesis and claim instead that *secondary* intelligibles constitute the subject matter of logic. This latter claim will be dealt with in greater detail in §3.1.2.

The texts from *Kitāb al-Ḥurūf* under examination, which run from §3 to §13 of Mahdi's edition, read in a way to suggest that they form a unified whole. Alfarabi begins his discussion, by explaining why the *Categories* is organized around interrogative particles such as when (*matā*), where (*ayna*), how much (*kam*), how (*kayf*), and so on. As Alexander had alluded before, Alfarabi observes that the *Categories* presents a classification of predicates in which each predicate is assigned to an interrogative particle, which in Alfarabi's view serves as a name for the whole set of predicates falling beneath it. For example, distinct predicates $P_1, P_2, \dots, P_n$ will fall under the particle "when [*matā*]" whenever speakers, and more specifically, philosophers, use $P_1, \dots, P_n$ to respond to questions in which the interrogative particle "when" is used. On the other hand, distinct predicates $Q_1, \dots, Q_m$ will fall under the particle "what [*mā*]" or its cognate

“what-is-it-ness [*māhiyya*]” (or just “quiddity”) when philosophers use $Q_1, \dots, Q_m$ to respond to questions in which the interrogative particle “what” is used. In Alfarabi’s words:

[Text 8] It should be kept in mind that the philosophers give the name of these interrogative particles [*hurūf*] or their cognates [*muštaqqun minhā*] to [1] most of the things [*ašyā*] that we enquire about by them, as well as [2] to what is appropriate as a response to them. Every way of responding to [a question] using the interrogative particle “When [is X]?” they designate with the word [*lafza*] “when”. Every way of responding to the question “Where [is X]?” they designate with the word “where”. Every way of responding to the question “How [is X]?” they designate with the word “how” or “quality [*kayfiyya*]”. Every way of responding to the question “How much [*kam*] [is X]?” they designate with the word “how much” or “quantity [*kammiyya*]”. They designate every way of responding to the question “Which [*ayyu*] [thing is X?]” with the word “which”. Every way of responding to the question with “what [*mā*] [is X]” they designate with the word “what” or “quiddity [*māhiyya*]”, though they do not designate the way of responding to the question with the interrogative particle “is there [*hal*] [an X]”²⁶⁶ with the word “is there” but [with the word] “there is [*inna š-šay*]”.²⁶⁷

In this passage, Alfarabi is tackling the basic ambiguity in the technical vocabulary of the philosophers, especially when we are speaking about Aristotle’s categories. This returns of course to the famous question about the purpose of the Categories: is it about words or things? As we have discussed above, however, in the *Hurūf* as in APCA §41 Alfarabi also has extracted the question from its original context. Further, with the diagnosis that much of the problem is just a matter of confusion over what words designate, Alfarabi removes much of the force that originally inspired this controversy. According to Alfarabi, the names of the categories “when”, “what”, “how”, “how much” are used to designate both (1) a set of things that we use the interrogative particles to enquire about, and (2) the set of predicates that that we use in our responses to such questions using these particles. Thus, human would fall under “what” in two senses. As a linguistic object (as a predicate) “human” falls under “what” because it is one

²⁶⁶ I.e. Does X exist? or simply Is there an X? The response would require a one-place predicate “X is”, which is not a predicate analyzed in the *Categories*.

²⁶⁷ Alfarabi, *Kitāb al-Hurūf*, ed. M. Mahdi, 3rd ed. (Beirut: Dar El-Mashreq, 2004), 62.12-20. Hereafter I refer to this work as follows: Alfarabi, *Hurūf*.

way of responding to the question “What is X?” where X refers to some concrete individual (say, for example, Socrates). But human as a being outside the soul is also classified under “what” because it is part of the being of a concrete X outside the soul.

Thus, the categories of substance, quality, quantity, etc. are interrogative particles that are used, according to the convention of the philosophers, to organize our thinking about certain relations between predicates, intelligibles and things. At this initial formulization, Alfarabi appears to be saying that there are at least two important elements that form the basis of a philosophical discourse. There are concrete things about which we enquire, and there are the elements of language which are used to give answers to these questions. For Alfarabi, the crucial aspects of language that constitute the basic elements (or at least the most philosophically interesting elements) of a philosophical discourse are the predicates that philosophers invoke as answers to questions posed with these interrogative particles about concrete individuals. Thus, in the *Categories* Aristotle identifies the philosophically interesting predicates, and he does this by determining what the philosophically interesting forms of question there are. Namely, they are the types of question that ask about beings, but not beings qua concrete individual, for in Alfarabi’s (and Alexander’s) view when philosophers say that a concrete X is said to be in different senses.

[Text 9] Convention has it that [the expressions] “substance in an unqualified sense”, and similarly “being in an unqualified sense”, designate [1] this sensible X [lit. “this sensible that is pointed to”], which is not used to characterize other things except in an accidental and unnatural way, as well as [2] what is used to apprise of what X is. Due to the fact that the meaning of X’s substance is its being, its quiddity, and part of its quiddity, that which is being in itself and is not being for anything else at all is substance in an unqualified sense, just as it is being in an unqualified sense, so that no other thing is attached to it, nor is it qualified by anything else. Similarly, that which apprises of what this X is is the substance of this X. And also because it is only predicated of something else in such a way that [apprises of] what it is, it too is substance in an unqualified sense, not conditioned by anything else, for it is in all respects the substance for

everything that is predicated of it. As for everything else that is predicated of this X, none of them serves as the substance for [this X], even if it is the substance for something else. It is thus substance in relation [to something else], in a qualified sense, and a property of this X [*‘araḍun fī hādā l-mušāri ilayhi*].²⁶⁸

Alfarabi here appears to be recalling Alexander’s claim in Text 6 that the *Categories* presents not only a division of predicates but also a division of beings. As Alexander insisted, this is not a division of beings in the sense of a division of individuals. A concrete X, according to Alfarabi, is said to be *substance* in an unqualified sense, or to have being in an unqualified sense, meaning that “it is being on its own and is not the being for anything else”. Yet, those things, namely predicates such as “man” and “animal”, which apprise us of what X is are also called “substance [*ḡawhar*]” or “being [*dāt*]” in an unqualified sense (*ḡawharun* or *dātun ‘alā l-ittlāq*) by the philosophers. But by this it is meant that “man” and “animal” are the substance of X, or that they constitute the being of X, *only* to the extent that they are invoked by a speaker to say what X is. According to Alfarabi, the other predicates that are used to characterize X do not constitute the substance or being of X because they are never invoked by the philosophers to say what X is. Instead, they are invoked in order to characterize other aspects of X. Thus, the division of being in the *Categories*, as Alexander had alluded to before, is not carried out on substance or being in the sense of classifying *these* concrete items. Rather, the division of being in the *Categories* is, first, carried out on substance or being, which Alfarabi says the philosophers also call substance or being in “an unqualified sense” and constitutes what this concrete X is. Properties that subsist in the concrete individual X that do not apprise us of what X is belong to one of the other nine categories. These elements might be said to constitute the being of X, but only in a secondary or derivative sense.

²⁶⁸ Ibid., 63.6-17.

Thus, in Texts 8 Alfarabi has justified the principle underlying the classification of predicates into ten in Aristotle's *Categories*. And in Text 9, he has shown that this division is based on prior division of being into two: that which is called being in an unqualified sense and that is called being in a qualified sense. The predicates that belong to the class of "what" fall under substance because they are used to answer question about what this X is. All the other predicates fall under the interrogative particles other than "what" because they are not used to inform us of what X is. To this extent, Alfarabi's account so far has given us an account of the relation between words and being in the *Categories*. But what about intelligibles, which we saw play such an important role in Alfarabi's thinking in *APCA*? In fact it is to this question that Alfarabi moves to next in the *Ḥurūf*. In general, intelligibles play a crucial mediating role between the words that signify them and the being or properties that the intelligibles characterize. As we saw earlier in this section, the idea that notions mediate the relation of signification between words and things was alluded to by Simplicius in his commentary on Aristotle's *Categories*. It may be, then, that Alfarabi is trying to give an adequate interpretation of Simplicius' scattered comments on this topic in his *Categories* commentary in the light of the more rigorously stated view of Alexander of Aphrodisias. Nevertheless, Alfarabi's views are quite different from those mentioned by Simplicius and later commentators.

[Text 10] Every meaning grasped by the intellect [*kullu ma'nā ma'qūl*], which [1] is signified by an utterance [*lafẓatun mā*], and [2] by means of which this X [lit. this thing that is pointed to] is characterized [*yūṣafu bihi*] we call a "category [lit. an (unit of) expression, *maqūla*]". Some categories apprise us of what this X is; some of how much; some of them apprise us of how it is; some of them apprise us of where it is; some of them apprise us of when it is, was or will be; some of them apprise us of what it is related to; some that it is in a position [*waḍ'un wa-annahū mawḍū'*]; some that there is something covering its surface; some of them that it is passively affected; and some of them that it is active.²⁶⁹

²⁶⁹ Ibid., 62.21-63.5.

According to Alfarabi, the predicates falling under the different categories signify intelligibles rather than things. In particular, they signify some intelligible aspect, element or characteristic of a concrete individual X in the mind. The intelligible, then, serves two functions. First, it serves as the object signified by the predicate term. Second, since they “characterize [*yaṣifu, waṣf*]” things, intelligibles allow us to close the gap between the things that we speak about and the predicates we use to speak about them. By characterizing, or in Alexander’s terminology “circumscribing”,²⁷⁰ not this X itself (the first sense of “being in an unqualified sense” in Text 9) but those aspects of concrete X that constitute what X is (the second sense of “being in an unqualified sense” in Text 9), the intelligible allows us to fix the reference of the predicate term to the appropriate aspects of concrete X’s being; being in an unqualified sense if speaking about those aspects of X that constitute what X is, or being in a qualified sense, if we are not speaking about those aspects of concrete X that constitute what it is.

As is clear from Text 10 and consistent with the Text 7 from APCA discussed earlier, Alfarabi identifies the categories [*maqūlāt*] with intelligibles rather than words (or with things). Alfarabi’s claim must face one rather steep philological barrier. It is difficult to argue that the categories, or *maqūlāt* in Arabic, which Aristotle discusses in the *Categories*, are intelligibles (*ma’qūlāt*) when the Arabic word for category (*maqūla*, pl. *maqūlāt*) is obviously a derivative of the word *maqūla* (lit. that which is spoken) rather than *ma’qūl* (lit. that which is grasped by the intellects) or *ma’qūla*. *Maqūla* and *maqūla* are both obvious paronyms forms of the word *qawl* (statement, verbal expression), so if Alfarabi is to defend his claim that the

²⁷⁰ Alfarabi uses “circumscribed [*munḥāz*]” in the *Ḥurūf* also; see S. Menn, “Al-Fārābī’s *Kitāb al-Ḥurūf* and His Analysis of the Senses of Being”, 78.

maqūlāt are actually *ma'qūlat*, he must find some way of surmounting this linguistic obstacle.²⁷¹

Once again, he does this by taking advantage of the fact that philosophers tend to not distinguish carefully between the things outside the soul, the representations of them in the soul, and the words we use to speak about them. Alfarabi claims that there is a sense in which each is something “spoken [*maqūl*]”:

[Text 11] “That which is spoken [*maqūl*]” sometimes means that by means of which something is uttered [*mā kāna malfūzan bihi*], regardless of whether [the thing uttered] is significant or not. For sometimes “expression” means most generically every utterance [*kullu lafẓin*], regardless of whether it is significant or not. Or, [“that which is spoken [*maqūl*]” sometimes means that by means of which something significant is uttered [*wa-qad yu'nā bihi malfūzan bihi dāllan*], for sometimes “expression” means most specifically every significant utterance [*kullu lafẓin dāllin*], be it a noun [*ism*], a verb [*kalima*], or particle [*adā*]. Or [“that which is spoken [*maqūl*]” sometimes means that which is signified by means of some utterance [*wa-qad yu'nā bihi madlūlan 'alayhi bi-lafẓin mā*]. Sometimes it means that which is predicated of something [*yu'nā bihi maḥmūlan 'alā šay'in mā*]. Sometimes it means that which is grasped by the intellect [*yu'nā bihi ma'qūlan mā*], for sometimes “expression” means an expression located in the soul [*fa-inna al-qawla qad yadullu 'alā al-qawli l-markūzi fī n-nafsi*]. Sometimes it means that which is defined [*yu'nā bihi šay'in maḥdūdin*], for a definition is a kind of expression. Sometimes it means that which is afforded a descriptive definition [*marsūman*], for a descriptive definition is also a kind of expression [*qawlun mā*]. For all these reasons have the categories been called “those which are said” [*wa-bi-hādihi summiyati l-maqūlātu maqūlātin*], because it is common among all these senses [of “what is said [*maqūl*]”] that [1] it is signified by an utterance [*lafẓ*], and that [2] it is predicated of a sensible *this* [*li-anna kulla wāḥidin minhā ḡtama'a fīhi an kāna madlūlan 'alayhi lafẓun wa-kāna maḥmūlan 'alā šay'in mā mušārun ilayhi maḥsūsin*].²⁷²

Any intelligible meeting the minimum conditions set out by Alfarabi could, in this technical sense Alfarabi has just outlined, count as “something spoken [*maqūla*]”, and could then be called a category (*maqūla*), but intelligibles must have been foremost in Alfarabi’s mind. First, the intelligible must be the object signified by some utterance (*lafẓ*); and second, the intelligible must be predicated of a concrete X. The intelligible must be predicated, not in the

²⁷¹ Given the importance of paronyms to Alfarabi’s logic and metaphysics, Alfarabi would have taken this linguistic argument against his position seriously; see *ibid.*, 65.

²⁷² Alfarabi, *Ḥurūf*, 63.18–64.4.

strict linguistic sense; but in the sense that the intelligible, as Alfarabi says in Text 10, characterizes or circumscribes aspects of a concrete X, whether those aspects constitute the concrete X's substance or some non-substantial aspect of concrete X.

So far I have shown that Alfarabi's understanding of the categories as intelligibles is probably a development of an idea originally found in Alexander's lost commentary on the *Categories*. Even if Alfarabi could not have had Alexander's commentary in front of him, Simplicius' discussion of the *skopos* of the *Categories* in his commentary on the *Categories* preserves this important text. What is more, Dexippus' commentary on Aristotle's *Categories* is clearly beholden to Porphyry's commentaries on the *Categories*, Dexippus' commentary is emphatic that the categories qua predicates signify notions and not things.²⁷³ Thus, Dexippus' text constitutes evidence of a later tradition among Neoplatonists that melded Alexander's view that the predicates signify notions with overall Porphyrian approach to the *Categories*. Alfarabi's discussion is an adaptation of Alexander's text, and it differs from Simplicius' text in that it says that words, and especially predicates, signify intelligibles rather than things. Nevertheless, Alfarabi also seems to be trying to make sense of Simplicius' oblique claim that the *Categories* is about words signifying things *by means of* notions.

Alfarabi takes it that he has established that the categories are intelligibles. They are intelligibles not qua representations (*miṭālāt*) of individuals but intelligibles that characterize individuals and that are signified by predicate expressions. At this point in the text Alfarabi now moves to investigate more closely what this special sort of intelligible is. It is at this point in the text that Alfarabi introduces the distinction between primary and secondary intelligibles. In making this distinction Alfarabi is building on an earlier distinction made by

²⁷³ Dexippus, *On Aristotle's Categories*, trans. J. Dillon (Ithaca: Cornell University Press, 1990), 24f.

Porphyry applying to words, not intelligibles. Porphyry introduces this distinction in the midst of his opening comments on the title, purpose and subject matter of the *Categories*. Porphyry, like many of Aristotle's Peripatetic and Neoplatonist commentators, believed that the *Categories* is about words. (Porphyry's view about the *Categories* would have been subject to the same criticism laid against logic by Şirāfī and the grammarians many generations later). Porphyry is perhaps the first to directly link the question of the purpose of the *Categories* with the question of its subject matter; and, in particular, to make an adequate response to the former question to depend on giving an adequate analysis of the latter. As I alluded to above, the challenge for someone holding the view that the *Categories* is about words becomes to draw a suitable distinction between the task of the *Categories* and grammar. Before Porphyry, Alexander also held that the *skopos* of the *Categories* was to examine words, but a certain class of words to the exclusion of others; namely, those words that signified notions that circumscribed those aspects of a concrete X's being that the questions "What is X?", "How is X?", "Where is X?", etc. enquire about. Porphyry utilizes a different set of principles in order to distinguish the *Categories* from grammar. Porphyry does not follow Alexander in holding that the words investigated in the *Categories* signify notions. Since Porphyry holds that this class of words signifies things (*pragmata*), Porphyry classifies words according to their level of abstraction from the *things* they signify. In particular, Porphyry draws a distinction between words of first imposition (*prôte thesis*) and words of second imposition (*deutera thesis*). Words, such as "man" and "animal" for example, are words of first imposition since they signify parts of the substance of a concrete individual. Words "verb" and "noun" are words of second imposition since they do not signify anything that has to do with any aspect of a concrete X. We might say that they are words about properties of words.

In his so-called “catechism” commentary the *Categories*, Porphyry directly links together solutions to the questions of (1) why the *Categories* has the title it has, (2) what its purpose (*skopos*) is, and (3) what its subject matter is. Porphyry says that in order to understand why the *Catagories* is called the “*Categories*”, we must understand its purpose (*skopos*), and that we cannot understand its purpose until we have understood what its subject matter is.²⁷⁴ Porphyry introduces this discussion in the context of a development history of the origins of human language, which many later Neoplatonic commentators on the *Categories* follow closely.²⁷⁵ This history traces the development of universal terms from singular terms. In order to speak about the things around them and to communicate with others about them, humans first attached particular utterances such as “chair” and “tree”, and “white” to particular objects, viz. *this* chair, *this* tree, and *this white*, and in this way connections are drawn between words and things. At a second step in this development the expressions “chair”, “tree”, and “white” ceased to signify exclusively concrete objects, and began to signify the substance, quantity, quality, etc. of individual objects. These latter expressions were a result of the recognition that though *this* chair differs from *that* chair in number, there is an underlying unity in this objects and the expression “chair” as a genus arises as a result of this recognition. The words that signify things in this way, viz. not as a *this* or a *that* but as an underlying unity within members of the genus, are what Porphyry calls words of first imposition. Words of second imposition are of a different nature, though Porphyry does not explore words of second imposition at great length since they are not the subject matter of the *Categories*.²⁷⁶ What is clear only is that unlike words of first imposition, which are a result of

²⁷⁴ Porphyry, *On Aristotle's Categories*, 33.

²⁷⁵ Hoffmann, “Catégories et langage selon Simplicius”, 79ff.

²⁷⁶ Though, interestingly, Porphyry says at *in Cat.* 58.30-35 that words of second imposition are the subject matter of *De Int.*, since, on his view, *De Int.* divides words into nouns and verbs, which are the paradigms of words second

our speaking about things and communicating about things with other people, words of second imposition are a result of the mind's reflection about expressions themselves. Words of second imposition are words that signify classifications of different classes of expression rather than classes of being.

[Text 12] Thus calling this sort of thing 'gold' and that material that shines so brightly 'the sun' belongs to the primary imposition (*prôte thesis*) of words,²⁷⁷ while saying that the expression 'gold' is a noun belongs to their secondary imposition (*deutera thesis*), which signifies the qualitatively different types of expressions. The subject of this book is primary impositions of expressions, which is used for communicating about things. For it concerns simple significant words insofar as they signify things—not however as they differ from one another in number, but as differing in genus. For things and expressions are both practically infinite in number. But his [Aristotle's] intention is not to list words one by one [...] but since things that are many in number are one in species or in genus, the infinity of beings and of the expressions that signify them is found to be included under a list of ten genera. Since beings are comprehended by ten generic differentiae, the words that indicate them have also come to be ten in genus, and are themselves also so classified. Thus predications (*katêgoriai*) are said to be ten in genus, just as beings themselves are ten in genus. So since the subject of the book is significant expressions differing in genus, insofar as they signify, and people used to call speaking of things according to a certain signification, and in general the utterance of a significant expression about something, as 'predication' (*katêgorein*), it was quite reasonable for him to give the title *Categories* to this elementary discussion of simple

imposition. According to Porphyry, in *De Int.* Aristotle "is discussing their [words'] secondary imposition, which is no longer concerned with expressions that signify things qua signifying them, but rather with expressions that signify types of words, qua being of such types. For being a noun or a verb is a type of word, and whether an expression has its proper use or is metaphorical or is in some way figuratively used also belongs to the second sort of inquiry about words, not to the first". If Porphyry's claim is accepted at face-value, then this would allow us to raise some fairly serious charges against this view, for it entails at least two problems. First, the claim that *De Int.* is about words of second imposition such as noun and verb allows us to make a convincing argument that even if the *Categories* is not about grammar, it would seem that *De Int.* is. Second, the claim that *De Int.* is about words of second imposition allows us to argue against the internal unity of the *Organon*, since how can the books have different subject matters and still constitute parts of unified science? In any case, these are two problems finding solutions to which do not seem to have concerned Alfarabi much. He seems perfectly comfortable saying that logic shares its subject matter with the other philosophical sciences, viz. primary intelligibles. And this latter claim is consistent with the claim that logic is not a science but an instrument of philosophy since it does not have an subject matter distinct from any of the other philosophical sciences. On the other hand, the question of the whether logic is science or an instrument of philosophy was a major concern for Yaḥyā ibn 'Adī and Avicenna.

²⁷⁷ This seems somewhat inconsistent since there is only one sun; but since it is *sui generis* and we are talking about certain attributes it has, it might still be a universal in that sense.

expressions, which considers them according to the genus insofar as they primarily signify beings.²⁷⁸

Although Porphyry does not delve into the question of what the nature of words of second imposition are and how they come about, we are able to glean some of their properties from his comments. First, words of first and second imposition differ in the objects that they signify. Words of first imposition signify aspects of concrete X. In the case of words from the category substance, these words signify the being of concrete X. Words falling under the other interrogative particles signify properties and accidents that concrete X possesses. On the contrary, words of second imposition do not bear any relation to concrete X. They signify properties of words of first imposition, such as being tensed for verb, or untensed for nouns, or taking a definite particle for nouns, or not taking a definite article for verbs.²⁷⁹ In addition, I believe it is safe to say that no words of second imposition can fall under any of the ten categories, for in no instance do terms such as “noun” or “verb” or “particle” serve as an answers to questions such as “What is X?”, or “How is X?”, or “How much is X?”, or “Where is X?”, or “When is X?”, etc., where X is understood as a *this*, unless we are to include individual verbal expressions in the class of primary substance discussed above.

Despite the differences in *what* they signify, words of first and second imposition arise from the same type of mental process applied to concrete individuals on the one hand, and individual expressions on the other. Both types of word arise from the need to speak with others in a general way about shared aspects of concrete X, Y and Z for words of first imposition, and individual instances of speech in the case of words of second imposition. In addition, the *relation* between the words and what they signify is the same in both. Two words of first imposition, e.g. “human” and “animal”, do not pick out numerically different concrete

²⁷⁸ Porphyry, *On Aristotle's Categories*, 57.19-58.21.

²⁷⁹ E.g. see Porphyry, *On Aristotle's Categories*, 34.

objects X, the concrete object *this* human and the concrete object *this* animal. These words are applied equally well to the same concrete X. Rather in the case of “human”, it signifies a part of the substance that is shared by this human X, this Y and this Z. In addition to signifying part of the substance of this human X, Y, and Z, “animal” also signifies part of the substance that is shared by, for example, Socrates, and Bucephalus. Similarly with the words “verb” and “noun”, they do not signify this or that verbal expression. They signify certain shared properties (having a particle, being tensed) shared by individual instances of verbal expressions or nominal expressions that, when taken together, form a unity.

Alfarabi’s notion primary and secondary intelligibles thus differs from Porphyry’s in at least one fundamental way. Porphyry talks about *words* of first and *words* of second imposition. Words of first imposition *signify* either parts of the substance or (accidental) properties belonging to concrete individuals. Words of second imposition are used to speak about properties shared by individual linguistic entities. Words of second imposition then signify these properties to a community of speakers (grammarians, philosophers, grammar school students, etc.). Porphyry explicitly identifies the categories with words of first imposition, and the subject of the *De Int.* with words of second imposition. Alfarabi on the other hand, following the interpretation of the nature of the categories adopted by Alexander and some later Neoplatonists such as Dexippus, talks about primary intelligibles that characterize parts of the substance or (accidental) properties belonging to concrete individuals. Yet, Alfarabi goes further than any earlier thinker by then identifying the categories with primary intelligibles. Unlike Porphyry’s discussion of words of second imposition, Alfarabi also goes into much greater detail about the nature of secondary intelligibles. Yet, Alfarabi like Porphyry does not appear analyze them for their intrinsic philosophical interest. Rather, he

uses them in order to demonstrate his main thesis in this portion of the *Ḥurūf*, viz. that primary intelligibles constitute the subject matter of logic and serve as the atomic elements of complex logical statements.²⁸⁰ First intelligibles come about as representations (*miṭālāt*) of sensible objects. On the other hand, secondary intelligibles, on Alfarabi's view, appear to represent a later event in the development of human thought, since secondary intelligibles are "intelligibles of intelligibles that do not arise from sensible objects, this [i.e. the existence of such intelligibles] would not have been obvious to us at the beginning [*laysa bayyinan lanā munḍu awwali l-amri*]"²⁸¹ Alfarabi suggests that secondary intelligibles might have existed all along in the mind, but it was, he implies, the philosophers who first discovered them. In the following passage, Alfarabi describes how second intelligibles arise in the mind from first intelligibles. First intelligibles arise from sensible objects obtaining in the soul. Meanings (*ma'ānin*) attach to these zero-order intelligibles on account of their presence in the soul. These meanings, such as *being-a-genus* and *being-a-species*, and *being-a-definiens* for a definiendum attach to the intelligibles of sensibles. Insofar as these meanings attach to representation of sensible things in the soul, they are called "primary intelligibles" since it is qua something predicated of many or qua genus that the primary intelligibles are predicated of things outside the soul. Since these meanings are also mental objects, meanings come to attach to these meanings as well. Insofar as these meanings attach to these meanings, they are called "secondary intelligibles", since they are predicated of intelligibles rather than things outside the soul.

[Text 13] When those intelligibles arising in the soul from sensible objects [*maḥsūsāt*] obtain in the soul, then, insofar as [these sensible objects] are present in the soul, concomitants [concomitant properties, *lawāḥiq*] attach to them [viz. representations of sensibles], on account of which some [of these

²⁸⁰ Cf. Steven Menn, "Al-Fārābī's Kitāb al-Ḥurūf and His Analysis of the Senses of Being", 81, n. 32.

²⁸¹ Alfarabi, *Ḥurūf*, 64.5-7.

representations of sensibles] come to be genera, some species, and some come to define others. For the meaning [*al-ma'nā*] on account of which [sensible in the soul] is a genus or species, which means *to be predicated of many*, is a meaning that attaches to the intelligible insofar as it is in the soul in the soul. The case is similar with respect to relations that attach to [intelligibles of sensibles], which dictate that some are more specific than others, or some are more general than others. These, too, are meanings that attach to [first intelligibles] insofar as they are in the soul. Similarly, that some [intelligibles] define others is a result of states [*aḥwāl wa-umūr*] attaching to them insofar as they are in the soul. Likewise our saying that they [first intelligibles] are “known [*ma'lūma*]” and that they are “grasped by the intellect [*ma'qūla*]” are also things that attach to them insofar as they are in the soul. But these very [meanings] that attach to them [intelligibles of sensibles] after they obtain in the soul are themselves intelligibles [*umūrun ma'qūlatun*], but they do not obtain in the soul because they are representations [*miṭālātu maḥsūsātīn*] of sensible objects, or because they depend on sensible objects, or because they are intelligibles of things outside the soul. These are second intelligibles [*al-ma'qūlāt at-tawānī*].²⁸²

In this passage Alfarabi gives a succinct account of how primary and secondary intelligibles arise in the soul. In analogy with the levels of abstraction in Porphyry's distinction between words of first and second imposition, intelligibles start out as representation of sensible objects obtaining in the soul (call them “zero-order” intelligibles”). Porphyry makes the move from “zero-order” words signifying concrete individuals to universal words of first imposition part of the (contingent) history of the development of human language. He is driven to this view in part because of the fact that concepts play no mediating role in his theory of signification. Alfarabi on the other hand says that primary intelligibles are generated by the natural processes of the mind. Thus, primary intelligibles are generated from mere representations of individual objects on account of the fact that concomitant meanings or properties (*lawāḥiq*, Alfarabi is not consistent in his usage, which further suggests that the doctrine he is setting out here is a novelty; he also uses *ma'ānin*, *aḥwāl*, and *'awāriḍ* to speak about the same phenomenon) attach to the intelligibles of sensible objects. Examples of these properties, which accrue to zero-order intelligibles of individuals merely on account of the fact

²⁸² Ibid., 64.8-19.

that they obtain in the mind, are *being-a-genus*, *being-a-species*, and *being-predicated-of-many*. When these properties accrue to zero-order intelligibles, the latter then are called “primary intelligibles”, and are what Alfarabi identifies with the categories. Alfarabi seems to mean that an intelligible of *this animal* becomes the primary intelligible *animal* predicable of many individuals outside the soul when the mind recognizes that *animal* may be predicated of numerically different primary substances (e.g. Socrates, Plato, and Bucephalus). What is more, at least with respect to the terms from the category of substance, this shift from a zero-order intelligible (*this animal*, e.g. Bucephalus) to a primary intelligible (*animal*) would also involve a shift in the types of being the intelligibles characterize. Zero-order intelligibles circumscribe being in its most basic sense, viz. the sense of what is in itself and when it does not serve as the being for any other thing (see Text 9 above). Primary intelligibles of substance characterize being in the sense that they circumscribe those aspects of concrete individuals that constitute what it is. Due to the fact that primary intelligibles are in the mind, they too are subject to the same kinds of concomitant property that accrue to zero-order intelligibles of concrete individuals. When such properties accrue to primary intelligibles, secondary intelligibles are the result. At this point in his exposition, Alfarabi’s characterization of secondary intelligibles is negative. The main difference between primary and zero-order intelligibles on the one hand and secondary intelligibles on the other is that the former are, representations of things outside the soul (zero-order intelligibles) or they depend on things outside the soul (*tastanidu ilā maḥsūsāt*, viz. primary intelligibles). Secondary intelligibles, on the other hand, are not like this, since they are a result of concomitant properties such as *being-predicated-of-many* attaching to these very meanings attaching to primary intelligibles. Alfarabi sees this process of meanings attaching to intelligibles of increasingly higher order as repeatable

indefinitely (*ilā ġayri nihāyatin*, so properties of properties, and properties of properties of properties, *ad infinitum*).²⁸³ In the passage following the one translated above (Text 13), Alfarabi describes how second intelligibles are subject to these same states (*aḥwāl*) that first intelligibles are. As a consequence, second intelligibles are intelligibles of intelligibles. These intelligibles would pick out properties and be predicated of the intelligibles *animal*. They would not pick out properties or be predicated of any being outside the soul. It is noteworthy that Alfarabi explicitly draws an analogy between his idea of primary and secondary *intelligibles* and Porphyry's division of words into first and second imposition (*waḍ' awwal*, *waḍ' tānin*). However, Alfarabi treats Porphyry's distinction as one proper to Arabic grammar, rather than one proper to the *Categories* or to logic, since Arabic grammatical inflections attach to the terms of second imposition, (e.g. the word "raf" itself when appearing in a sentence, can be nominative, genitive or accusative, the word "naṣb" can be nominative, genitive or accusative, etc.) when they are used in speech.

[Text 14] Also, since they are intelligibles, these meanings may be subject to those states [*aḥwāl*] that attach to the first intelligibles. As a result, [the meanings] that attach to the first intelligibles attach to them [meanings] also,

²⁸³ This may be one of the texts that Alfarabi is referring to in his lemmatic commentary on *De Int.* that seems to have so baffled Sabra (A. Sabra, "Avicenna on the Subject Matter of Logic", 756). Alfarabi calls the copula "is" qua connector "one of the secondary concepts". Alfarabi continues, saying "that it is neither absurd nor impossible that the secondary concepts go on to infinity, as you have heard me say many times and as I have set down in writing" (ibid., Sabra's translation). Stephen Menn (S. Menn, "Al-Fārābī's *Kitāb al-Ḥurūf* and His Analysis of the Senses of Being", *Arabic Sciences and Philosophy* 18 (2008): 59-97) has discussed in what sense "is [*mawǧūd*]" in the statement "X is" is a secondary intelligible. The situation that Alfarabi is discussing in the context of the *Categories* and in portions that Menn has studied are not quite analogous, since existence is not one of the classes of predicates discussed in the *Categories*. As Alfarabi says, the article "Is there an X [*hal huwa šay'*]" does not pick out a particular class of predicates in the way that "What is X?" does, for example. As Menn notes the question "Is there is an animal?" has two senses in Alfarabi's way of thinking. In the first sense, we may be asking whether there is something outside the soul that we would predicate "animal" of, and if we respond to question saying "Animal is", then we have predicated "animal" of that substance outside the soul. Menn says that "is" is being used in this sense as a "first-order" concept since it is being predicated of "the external instance of the concept, and not the concept itself (ibid., 82)". In the second sense, "Is there an animal?" asks not about some substance outside the soul that we might predicate animal of. Rather, we are asking about the *concept* animal; in particular, whether "there is some thing of which the concept holds (ibid., 81)", and if we respond by saying that "Animal is", we are saying that, in fact, there is such a thing that the concept animal characterizes. We are *not* saying anything about some thing outside the soul that animal is its part of its quiddity.

such that they too come to be species, genera, and serve to define each other. Even the act of knowing itself, which attends an object when it obtains in the soul, becomes an object of knowledge [...] ²⁸⁴ And knowledge, meaning the act of knowing, becomes an object of knowledge by means of another act of knowing; and so on indefinitely. Even a genus is in a genus, and that too, may be repeated indefinitely. This is analogous to how terms [*alfāz*] that are set down in second imposition are. For the same grammatical inflections [*i'rāb*] attach to terms of first imposition [*al-waḍ' al-awwal*] as attach to [terms of second imposition] also. Thus, the term “raf” may itself be nominative [*yakūnu marfū'an bi-raf*], and the term “naṣb” may be accusative [*maṣṣūban bi-naṣb*], and so on. ²⁸⁵

It seems that Alfarabi believes that any object grasped by the intellect will be subject to a variety of meanings (or concomitant properties) attaching to it merely on account of its mental status. Alfarabi does not give a comprehensive list of the types of concomitant accidents. He has already mentioned meanings such as *being-predicated-of-many*, *being-a-genus*, *being-a-species*, *being-a-definiens* for a definiendum, to which we can add *being-an-object-for-an-act-of-knowing*. Alfarabi says that the unlimited repetition of these mental states attaching to lower-order intelligibles, which leads to the generation of nth-order levels of abstraction, does not make our logical theory incoherent. ²⁸⁶ In fact, asserts Alfarabi, the meaning of *being-a-genus*, *being-a-species*, and *being-a-definiens* for a definiendum attaching to an intelligible of any order remains the same regardless of how far abstracted from the zero-order intelligible it is. These states, insists Alfarabi, retain a constant meaning in the face of repeated abstraction from zero-order intelligibles. Nevertheless, Alfarabi, drawing on Alexander and the Neoplatonist commentators on Aristotle's *Categories*, is interested in characterizing the nature of the predicates intelligibles themselves, not with the states or concomitant properties attaching to the intelligibles objects. Be that as it may, drawing on Stephen Menn's recent work on one-place existential predicates (“X is”), we may speculate about how secondary

²⁸⁴ There appears to be a problem in the text (or the edition).

²⁸⁵ Alfarabi, *Ḥurūf*, 64.20–65.8.

²⁸⁶ See §10 of Alfarabi, *Ḥurūf*, 65.22–66.16.

intelligibles function. Secondary intelligibles are concepts applying to concepts, and so they may be thought of as predicates or properties attaching to concepts rather than as predicates or properties of things outside the soul. When, for example, we ask “What is X?”, we are asking our interlocutor to indicate to us the quiddity outside the soul constitutes X’s being. To this end, the respondent furnishes us with predicates that are predicable of numerically different individuals (e.g. concrete X, Y and Z), which afford us knowledge of what X is. In this case the intelligibles furnished to answer the question “What is X?” are being used in their primary sense, since they are being predicated of an individual substance outside the soul. Since secondary intelligibles are concepts of concepts, the question “What is P?” is going to have a sense that has nothing to do with the category of substance, one, because “P” will not refer to a concrete outside the soul, but to an intelligible inside it; two, we are not asking the speaker to furnish us with predicates that constitute what P is outside the soul, since P is an intelligible. In fact, none of the interrogative particles from the *Categories* really applies to P since the *Categories* is a classification of predicates about different aspects of the substance and properties of concrete X. The best we might come up with in response to the question “What is P?”, where P is the intelligible *animal* for example, is “*Animal* is a universal”, “*Animal* is predicated of many”, “*Animal* is a species”, “*Animal* is a genus”, and “*Animal* is a definiens” for the definiendum *human*. And, for example, by “*Animal* is predicated of many”, we would mean that the concept *animal* has the property that it may be predicated of numerically distinct individuals; and by “*Animal* is a genus”, we would mean that the concept *animal* has the property that it may be used to characterize the substance of numerically distinct individuals. In each of these answers, we are predicating properties of concepts and not of individuals. All of these are, of course, the states that Alfarabi refers to in his discussion of primary and

secondary intelligibles, but he does not pursue the sorts of questions that are appropriate to ask about secondary intelligibles in any great detail.

I suspect that the reason why Alfarabi is not interested in investigating these states, meanings, concomitants properties, or accidents is that they are tangential to his major concern in this series of texts. These passages from the *Ḥurūf* may be read as serving as preparation for Alfarabi's major claim in this portion of the *Ḥurūf*, namely that the primary intelligibles (*al-ma'qūlāt al-uwal*) are the subject matter of logic and the other philosophical sciences. We have seen how Alfarabi follows a Porphyrian view of reducing the question of the *skopos* of the *Categories* to determining its subject matter, though, as we have seen, this reduction was present in much earlier treatments of the question of the purpose of the *Categories*. However, what is new in Alfarabi is to drastically expand the scope of this ancient debate to the question of the subject matter of *logic*. I can find no evidence that other authors did what Alfarabi does here. Indeed, Porphyry's claim in his catechism commentary on the *Categories* that the *Categories* is about words of first imposition and *De Int.* is about words of second imposition, a claim which he does not seem to take to be controversial, suggests that Alfarabi is striking quite an original note in these passages from the *Ḥurūf*. That being said, Alfarabi's viewpoint is not entirely without precedent. Alfarabi claims that logic has the same subject matter, viz. primary intelligibles, as all the other philosophical sciences. This view is consistent with the instrumentality of logic, for in order for logic to be an independent philosophical discipline on par with, say, physics or ethics, it must at the least have its own subject matter. Alfarabi claims however that it does not, and so the claim of logic's instrumentality is justified further.

[Text 15] These intelligibles [primary intelligibles] are first with respect to all of these secondary [intelligibles, "secondary" here meaning nth order intelligibles]

with $n > 1$, *wa-hāḍihi l-ma'qūlātu hiya l-uwalu bi-l-idāfati ilā hāḍihi <l-ma'qūlati> t-tāniyati kullihā*. The first terms are set down as first on account of their signifying these [primary intelligibles] and signifying combinations of them. And these [primary intelligibles] are the primary subject matters of the art of logic, and the science of physics, the science of politics and ethics [*'ilmi l-madani wa-t-ta'limi*], and the science of metaphysics.

For it is on account of their [primary intelligibles'] being signified by terms [*alfāz*], their being universals [*kullīya*, viz. their being “predicated of many”], their being predicates and subjects, their serving to define each other, their being the objects of enquiry, and their being used to furnish answers in an enquiry about them—[it is for all these reasons that] they are *logical* intelligibles [*manṭiqiyya*]. Then one examines them and investigates the types of composites they form with each other insofar as the states mentioned earlier attach to them. [One also investigates] the states of the composites after they have been composed. When the component intelligibles from which [the composite intelligibles] are composed are examined in light of these states, [then under these conditions] the composites [*murakkabāt*, composites of primary intelligibles] come to serve as instruments directing the intellect to correct judgments about intelligibles, and to guard [the intellect] from making mistakes in those things one is liable to make errors in.²⁸⁷

The major distinction Alfarabi wants to draw in the first paragraphs of this passage is between primary intelligibles on the one hand, and all higher order intelligibles that arise from the repeated attachment of concomitant properties to lower order intelligibles. For it is only the former and none of the latter which serve as the subject matter of logic. Alfarabi then justifies his reason for identifying the categories, viz. primary intelligibles, as the subject matter of logic. The basic reason, says Alfarabi, that primary intelligibles should be called “logical intelligibles [*ma'qūlāt manṭiqiyya*]” is that they are the intelligible objects that serve as the basic elements manipulated (by predication, definition, signification, enquiry about, etc.) by philosophers in their philosophical discourse. The task of logical investigation, then, is to examine these primary intelligibles in light of the meanings that accrue to primary intelligibles and composites of primary intelligibles, such as *being-predicated-of-many*, *being-in-a-genus*, and *being-in-a-species*. And it is the concepts that result from this task of logical

²⁸⁷ Ibid., 66.17-67.8.

inquiry are precisely the concepts of logic that “serve as instruments [*alāt*] directing the intellect to correct judgments about intelligibles, and to guard it from making mistakes in those things one is liable to make errors in”. These concepts would be secondary (and presumably possibly higher order) intelligibles. These are not the subject matter of logical inquiry; they are, rather, its fruit.

Alfarabi’s move to identify the subject matter of the categories with all of logic because the categories constitute the basic building blocks of philosophical discourse puts him in a difficult position. His account affords the means to providing detailed answers to some perennially troubling questions among Aristotelians of all stripes, e.g. What is the purpose of the *Categories*? What is the subject matter of the *Categories*? What are the categories, viz. are they words, notions, or things? How does the *Categories* relate to the rest of the *Organon*? How do the categories relate to more complex logical statements? In what sense is logic an instrument of philosophy? What is the relationship of logic to philosophy? What are the basic elements of logical discourse? What are the basic building blocks of the philosophical sciences? Yet, Alfarabi’s account suffers from three serious drawbacks that he does not adequately anticipate. First, Alfarabi does not explore what the nature of the states (*aḥwāl*), meanings (*maʿānin*), or properties (*lawāḥiq*, *ʿawāriḍ*) that attach to primary intelligibles are. These concepts are bridled with a great deal of explanatory work in Alfarabi’s theory, but he does not feel the need to explore what they are in any depth. More seriously, the claim that logic and the other philosophical sciences share the same subject matter, viz. primary intelligibles, has two troubling consequences. The first difficulty is that if the subject matter of physics and logic are the same, how are we supposed to distinguish logic and physics? Second, Alfarabi’s claim is inconsistent with his insistence on the instrumentality of logic and

therefore its non-scientific status. According to Alfarabi's own account in Text 9, the intelligibles falling under the category of substance such as *human* and *animal* are called, according to the convention of the philosophers, "substance [ḡawhar]" or "being [dāt]" in an "unqualified sense [alā al-iṭlāq]". There is, then, a real sense in which logic, on Alfarabi's account of logic's subject matter, is (1) about beings, and, thus, (2) it should qualify as a proper science of philosophy. And if logic is about beings in some sense of that term, then what about its status as a discipline concerned with form (ṣūra) rather than matter (mādda)? Alfarabi seems to be grappling with a few of these questions at the end of his account of the subject matter of logic in the *Ḥurūf*. On others, however, Alfarabi is largely silent.

[Text 16] As for the rest of the philosophical disciplines, [primary intelligibles] are examined in them insofar as they are intelligibles of things outside the soul, and without regard for the terms signifying them, nor for the concomitant accidents [ʿawāriḍ] discussed above that come to attach to them. Yet, man is compelled to examine them [primary intelligibles] with these states [bi-tilka l-aḥwāl] in order that they [primary intelligibles] come to be known by means of them [concomitant states], but when they [primary intelligibles] come to be known, they are examined without regard to them [the concomitant states].²⁸⁸

Indeed, one is compelled to examine them with these states. Then, by means of these, one is able to infer conclusions to the questions one's philosophical discipline investigates, until when one has finished studying them, these states vanish. Or, one's aim regarding them is not that they be examined with a view to their having these states, even if they [states] are not, in fact, detachable from them [primary intelligibles] [aw yaḡʿalu l-maqṣada minhā an tuḥada lā min ḡihati mā lahā tilka l-aḥwālu wa-in kānat lā tanfakku minhā].

Alfarabi admits that we cannot distinguish the subject matter of logic from that of the other philosophical disciplines. They are distinguished according to the manner in which they are examined by the philosopher. Primary intelligibles are the subject matter of logic, but only to the extent that they are examined in light of the properties that accrue to them on account of their presence in the soul. Yet, logical deductions were understood to play an important role

²⁸⁸ There seems to be a problem with the text at this point.

in the development of philosophical disciplines such as physics and metaphysics. Alfarabi is right to note that even in the above philosophical sciences, it is inescapable that some mind be given to these states that attach to primary intelligibles on account of their presence in the soul, for these states come about as part and parcel of the deductive process of moving from premises to conclusions in any science. In Alfarabi's mind, the difference between the philosophical sciences on the one hand and logic on the other comes in the way in which the primary intelligibles are regarded in light of the concomitant properties that attach to them. In logic primary intelligibles are examined in light of the accidental states that accrue to them from their presence in the mind. This was, as was mentioned earlier, the main business of logical inquiry. In the other philosophical sciences the accidental states are present because in the move from premises to conclusions is an inescapably mental activity. These accidental states thus play an instrumental role, facilitating the move from premises to conclusion. Nevertheless in the non-logical disciplines of philosophy, primary intelligibles are not examined in light of these accidental properties, but in light of the properties that accrue to the objects the primary intelligibles characterize in concrete individuals outside the soul. These states are thus present, and inevitably so, but they do not determine the nature of the inquiry. Thus, logic examines the intelligibles animal and man insofar as they share in the property of being predicated of many things; and the concept of universal is the fruit of this investigation. Physics also examines the intelligibles animal and human, but only insofar as they are the type of thing whose principle of motion lies within or without it. To put the matter more simply, it is the presence of these states such as *being-predicated-of-many* in our investigation that distinguishes logical investigation from physics or metaphysics.

These concomitant properties thus play a crucial role not only in explaining the formation of primary and secondary intelligibles from zero-order representations of concrete individuals. They are also important for distinguishing logic from the philosophical disciplines. What is more, the fact that these states attach to the primary intelligibles does not appear to be an entirely adequate way of distinguishing logic from the other philosophical sciences, since it is clear that the division of sciences in the Aristotelian scheme is carried out on beings. Primary intelligibles may be qualified in a real sense that Alfarabi himself recognizes. Accidental states attaching to primary intelligibles do not seem to do the trick. But what would a discourse about secondary intelligibles (or states of states) look like? It would investigate properties of intelligibles that are predicable of many things. It would ask whether such intelligibles may be grouped into further categories (e.g. predicable of many *per se*, predicable of many *per accidens*). It would ask whether there are different classifications of composite intelligibles (e.g. composites of intelligibles that one subject and one predicate, one subject and two predicates, etc.), and it might ask whether there are different classifications of composites according to the nature of the connector (e.g. composites that are connected by a copula (*rābiṭa*) or connected by a conditional particle (*šarṭ*)). This is not Alfarabi's idea of what logic is about. But it is Avicenna's.

§3.1.2 AVICENNA ON THE AIMS, UTILITY AND SUBJECT MATTER OF LOGIC

Avicenna approaches the question of the subject matter of logic from a different direction than Alfarabi. Like Alfarabi, he is concerned to establish against the grammarian that logic is indispensable to the development of the philosophical sciences. Like Alfarabi, Avicenna continues a line of inquiry that had been heavily trod by Aristotle's successors. As

we saw, Alfarabi's views about the subject matter of logic seem to stem from a commentary tradition on the *Categories* that was concerned with the purpose of the *Categories*. Avicenna approaches the question of the subject matter of logic from a different angle. Greek antecedents to his concerns about the subject matter of logic have, as we discussed in §3.1.2, more to do with attempting to establish logic as a proper science among the philosophical sciences. Yet, the debate between the Baghdad philosophers and the grammarians seems to have been a very strong factor in Avicenna's thinking. And in particular, the work of Yaḥyā ibn 'Adī seems to have been an important, if unacknowledged, source of inspiration for Avicenna's thinking about the subject of logic. Though Avicenna's comes to very different conclusions than Ibn 'Adī does about the subject matter of logic, he follows him in a few important respects. According to Peter Adamson, Yaḥyā ibn 'Adī was as concerned as Alfarabi was to vindicate the position claimed by philosophers such as Alfarabi and Abū Bišr, that the value in logic lies in the fact that it concerns meanings rather than the expressions. To this end, Yaḥyā appears to have devoted several short treatises to this question. In one of these treatises, Ibn 'Adī deals explicitly with the question of whether logic is a science that is distinct from the other philosophical sciences. According to Ibn 'Adī, a discipline will be an independent discipline of philosophy if it fulfills any one of two conditions. A science is set apart from other by virtue of the fact that it has a (1) subject matter (*mawḍū'*) that is proper to it and no other, or it is set apart by virtue of the fact that its (2) purpose (*ḡāya*) is proper to it and no other discipline.²⁸⁹ Based on this principle, Ibn 'Adī then claims that logic is distinct from grammar on both counts. Logic and grammar are distinguished by their subject matter and they are distinguished by their purpose. Ibn 'Adī claims that grammar has expressions

²⁸⁹ For this discussion I rely on P. Adamson, "Knowledge of Universals and Particulars", at 75f.

(*alfāz*) as its subject matter and that the purpose of grammar is setting down norms for the correct usage of these expressions. Following in part the view of early Greek authorities such as Alexander and Porphyry, Yaḥyā ibn ‘Adī identifies a special subgroup of expressions as the subject matter of logic, viz. those “expressions insofar as they refer to universal things [*huwa al-alfāzu d-dāllatu ‘alā l-umūri l-kulliyya*]”. Ibn ‘Adī’s statement seems to be formulated in an intentionally vague way so that it the “universal things” being referred to by expressions could be intelligibles (as Alfarabi had held) or quiddities and properties predicated of sensible objects (as Porphyry appears to have held). Nevertheless, Ibn ‘Adī has clearly departed from Alfarabi in claiming that some sort of expression is the subject matter of logic, since as we have seen Alfarabi says quite explicitly that it is intelligibles. In justifying his claim that the subject matter of logic is “expressions insofar as they refer to universal things”, Ibn ‘Adī’s makes use of the fact that the end of logic is demonstration:

[Text 17] That the subject-matter of [the art of logic] is expressions insofar as they refer to universal things is clear from the fact that—since demonstration obviously only of referring expressions, and every referring expression must refer either to a particular or universal meaning [*ma‘nan*], and demonstration is a syllogism that possesses certainty [*qiyās yaqīnī*], and every syllogism that possesses certainty is free of ambiguity and unmixed with doubt, and anything unmixed with ambiguity is distinct and separated from it [*sc. ambiguity*], and what is like this is defined [*maḥdūd*]*—therefore whatever is known by demonstration is defined. But the defined is the certain, and none of the particulars [ḡuz’iyyāt] is certain, so therefore none of the particulars is demonstrated. (By “demonstrated”, I mean here whatever is such as to receive the form of demonstration, even if it has not received such a form). Every subject-matter is a subject-matter for the art of logic. Therefore the subject-matter for the art of logic is expressions that refer to universal things”.*²⁹⁰

Ibn ‘Adī’s reasoning and conclusions are drastically different from his “master” Alfarabi. I have already mentioned the fact that Ibn ‘Adī says that the subject matter of logic is expressions rather than intelligibles as Alfarabi claimed. However, the main difference

²⁹⁰ Yaḥyā ibn ‘Adī, *The Philosophical Treatises*, ed. S. Khalifat (Amman: University of Jordan, 1988), 422.1-9; cited and translated in P. Adamson, “Knowledge of Universals and Particulars in the Baghdad School”, 76.

between them is how Ibn ‘Adī uses the end of logic, viz. demonstration (*burhān*), as a way of circumscribing exactly which kind of expression is the subject matter of logic. Alfarabi concluded that the subject matter of logic is primary intelligibles largely based on his reading of the commentators’ debates about the purpose of the *Categories*. Alfarabi’s also reasoned that primary intelligibles are the subject matter of logic based on the fact that they are the basic building blocks for complex logical statements. On the other hand, Ibn ‘Adī’s thinking about the subject matter of logic on the other hand is clearly dictated by his thinking about the nature of demonstration, and in particular about the nature of demonstrative syllogisms. He is not guided as much by a concern about the nature of the categories or with identifying the most basic logical element as he is by the question of which logical units are suitable to carrying out demonstrative syllogisms. Ibn ‘Adī says that since the end of logic is demonstration, then the objects that logic deals with must be able to serve in demonstrative types of reasoning. Thus, these objects must be those about which it is possible to have certainty (recalling Text 6 above from Alexander and Aristotle’s opening chapters from *An. Post.*) and to construct syllogisms that yield certain conclusions from certain premises, and they must be the types of thing from which we can construct definitions. But, concludes Ibn ‘Adī, since particular things do not serve in demonstrative syllogisms, since they are not objects of certain knowledge, and, thus, are not the objects of definition, particulars are no good as a subject matter of logic. Particulars cannot be the subject matter of logic since they fail to serve logic’s end, namely the ends of demonstration. On the force of this argument, Ibn ‘Adī concludes the only other possibility is that “universal things” are the subject matter of logic.

One major difference between Ibn ‘Adī and Avicenna is that the former agrees with Alfarabi and many other ancient thinkers that the subject matter of logic is also the subject matter of all the other sciences, except obviously, grammar. This statement is indicative of Ibn ‘Adī’s particular motives for writing the treatise cited above. His main concern, as Adamson rightly notes, is with opposing a group of his contemporary grammarians in Baghdad who claimed that logic was irrelevant or useless. His clearly was not to carve out a place for logic among the philosophical sciences. This is a major difference between Ibn ‘Adī and Avicenna, the latter of whom was naturally less concerned with the immediate debates carried out by the Baghdad circle than Ibn ‘Adī, who became intimately involved in them. Still, despite the differences in their motives and the fact that Avicenna identifies Alfarabi’s secondary intelligibles as the subject matter of logic, Avicenna’s *reasoning* is much closer to Yaḥyā ibn ‘Adī’s than it is to Alfarabi’s. First, like Ibn ‘Adī Avicenna is interested in examining the ways in which we might distinguish sciences from each other; second, Avicenna’s reasoning like Ibn ‘Adī’s is end-oriented in the sense that he begins from an intuition about the purpose of logic and from there moves to to identify the most basic logical elements. On the other hand, Avicenna’s thinking on the topic shows little of the concern that Alfarabi’s does with the commentary tradition on the *Categories*. Rather, begins by reviewing the Alexandrian division of philosophy into practical (*‘amalī*) and theoretical (*naẓarī*). After considering this division, Avicenna moves to consider in what sense logic could be classified among the theoretical sciences. It is at this point in Avicenna’s argument that Ibn ‘Adī’s influence appears.

Following Alexandrian divisions of the sciences, Avicenna divides the sciences into the theoretical and the practical, and theoretical philosophy is divided into metaphysics, physics, and mathematics.

[Text 18] The aim of philosophy is that the true nature of all things be investigated to the extent of man's ability. Existent things are either things whose existence is not by our choosing or action, or they are things whose existence is by our choosing and action. Knowledge about the objects belonging in the first category is called "theoretical philosophy [*falsafa nazariyya*]". Knowledge about the objects belong in the second category is called "practical philosophy [*falsafa 'amaliyya*]". The aim of theoretical philosophy is perfecting the soul so that it knows; the aim of practical philosophy is perfecting [*takmil*] the soul not so much that it just know, but that it knows what must be known in order to act [...]

The divisions of the sciences consider the sense of beings insofar as [beings] are [1] in motion both in conception and in substance, such that they are connected with species of matter; or they consider the sense of beings insofar as they are [2] separable from motion in conception but not in substance; or they consider the sense of beings insofar as they are [3] separable from motion in conception and in substance. The first division of the sciences [1] is physics, the second [2] is pure mathematics, and arithmetic as it is normally understood (the analysis of number qua number is not part of this science); the third [3] division of the sciences is the divine science [*al-ilm al-ilāhī*, i.e. metaphysics]. Beings in nature fall into these three classes, and, thus, the theoretical philosophical sciences are these.

Practical philosophy is connected to the teaching of opinions by means of which the public human collectivity is ordered. It is known as "government of the city [*tadbīr al-madīna*]", and is called "the science of government [*ilm al-siyasa*]". Or <practical philosophy> is connected to the ordering of the private human collectivity. It is known as "government of the household [*tadbīr al-manzil*]". Or <practical philosophy> is connected to the ordering of the single individual's purification of his soul. This is called "the science of ethics [*ilm al-aḥlāq*]"" [...]

The aim of theoretical philosophy is knowledge of the truth [*al-haqq*]; the aim of practical philosophy is knowledge of the Good [*al-khayr*].²⁹¹

Logic is noticeably absent from Avicenna's summary division of the sciences. Thus, in order to argue for the view that logic is a philosophical science, indeed, a member of theoretical philosophy, Avicenna begins by considering the different senses in which we may ask "What is X?".

[Text 19] Quiddities of things [*māhiyyāt al-aṣyā'*] are in concrete things [*fī l-a'yān*] or they are in conception [*fī t-taṣawwur*]. Thus, they [quiddities] have three senses [*i'tibārāt*]. There is the sense of quiddity for what is to be this quiddity

²⁹¹ Avicenna, *Madḥal*, 12.3-14.18.

without relation to either of the modes of being [in concrete reality, or in conception] and what attaches to it on account of its being in that way. Then there is its sense insofar as it is in concrete things [*fī l-a'yān*], on account of which concomitant properties [*a'rād*] attach to it that are particular to its being in this way. Then there is its sense insofar as it is in conception, on account of which concomitant properties attach to it that are particular to its being in this way, e.g. hypothesis [*wad'*], predication [*ḥaml*], and universality [*kulliyya*] and particularity [*ḡuz'iyya*] in relation to predication, and essentiality [*dātiyya*] and accidentality [*'araḍiyya*] in relation to predication, and other things you will come to learn. For there is no essentiality or accidentality in predication in external beings [*fī l-mawḡūdāti l-ḥāriḡati*], nor a thing's being a [grammatical] subject or predicate, nor a [thing's being] a premise or syllogism, nor anything else of that sort.²⁹²

Based on Avicenna's distinction between essence and existence, the question "What is X?" can be asked in such a way as to have three distinct senses. Say, for example, that X is human. In the first sense, "What is human?" asks for those properties that are *essential* to human. In particular, we are asking for the genus (animal) and differentia (rational) of human that constitute its definition. On Avicenna's view to predicate "rational animal" of human is not to say to predicate "rational animal" of anything outside the soul, or to say that there is a quiddity "rational animal" the is just the being of human. Nor, however, when we ask "What is human?" in this sense are we asking about the properties human has *qua* intelligible in the mind. In the second sense, "What is human?" asks for properties that human possess as a being with a quiddity outside the soul. In this case, "What is human?" corresponds to the typical question from the category of substance, where we ask for we are asking about is a quiddity for a sensible object outside the soul. In our response "Human is animal", animal is a predicate that we predicate of a thing outside, and the being of human is this quiddity. In addition, the other predicates of the categories when applied to human "human is white", or "human is sitting" are properties that accrue to human on account of its being in concrete reality.

²⁹² Ibid., 15.1-8.

The third sense is, for Avicenna, the most important because it relates to his immediate concern with determining what the place of logic is among the philosophical sciences. In the third sense when we ask “What is X?”, we are asking about properties in X that it has insofar as they accrue to X *qua* mental object. Thus, the X we are asking about is an intelligible, and any properties that we predicate of X will be *qua* its being in the mind. Then when we ask “What is human?” (or more clearly “What is (the intelligible) human?”, the properties that we supply as an answer will be properties that attach to intelligibles and not beings or the essence of human. Interestingly, Avicenna draws on examples of properties that attach to X *qua* intelligible from logic and grammar. From logic he supplies examples such as “predicate [*ḥaml*]” and “syllogism [*qiyās*]”; from grammar he mentions “[grammatical] subject [*mubtada*]” and “[grammatical] predicate [*ḥabar*]”. Avicenna seems to be saying that when we ask “What is human?” and mean thereby to ask about it *qua* intelligible, the predicates we supply may be drawn from grammar or logic. This statement rings a bit oddly however, since it makes most sense (and is consistent with Alfarabi and Ibn ‘Adī’s thought on this matter) that the response “[grammatical] predicate” or “[grammatical] subject” is given to the question “What is human?” when we intend to ask what human is *qua* part of *speech*. Nevertheless, Avicenna makes his reasoning clear enough. What human is *qua* intelligible are quiddities such as universality [*kullīyya*] (with respect to predication) or grammatical subject. As Avicenna says, what is common to these quiddities (and what distinguishes them from the second sense of quiddity) is that neither of these quiddities are not predicated of any object outside the soul.

By this argument, Avicenna has thus far established that the objects that he will identify as the subject matter of logic, i.e. the secondary intelligibles such as “*being-a-universal*”, “*being-a-syllogism*”, “*being-a-hypothesis*”, etc. are intelligibles that attach to

intelligibles. In this way his thinking is perfectly in line with Alfarabi's as detailed above. What he has not done however is clearly distinguished the subject matter of logic from that of grammar. In fact, it seems that Avicenna believes that grammar and logic share the same kind of subject matter to the extent that it is common to both disciplines that their subject matters are quiddities that do not have any extramental instantiations. Following the conditions laid down by Ibn 'Adī, Avicenna next moves to distinguish the subject matters of grammar and logic according to their end.

[Text 20] And if we want to think about things and know them [*wa-idā aradnā an nufakkira fī l-ašyā'i wa-na'lamuhā*], then, of necessity, we need to enter them [things] into conception [*fī t-taṣawwur*]. Then the states [*aḥwāl*] that they have in conception necessarily accrue to them [due to their being in conception]. Then, of necessity, we need to examine the states that they [things] have in conception, particularly when we wish to inquire into the unknowns [*maḡhūlāt*] and [when we wish] them to be knowns [*ma'lūmāt*]. Yet, it is unavoidable that things are only unknown in relation to the mind; likewise, things are only known in relation to it. The state [*ḥāl*] and property [*ʿarīḍ*] that accrue to them [things in conception] so that we move from the knowns among them [things in conception] to the unknowns among them is a state and property that accrues to them in conception, even if what [properties] it has in itself also exist with these [states and properties in conception]. We must, therefore, have a knowledge of these states [*ilmun bi-hāḍihi l-aḥwāl*], and [knowledge of] how many [*kam hiya*] they are, and what qualities they have [*wa-kayfa hiya*], and how they are to be consider in this fashion. This inquiry [*hāḍā n-naẓar*] is not an inquiry into things insofar as they exist in one or other of the two modes of existence mentioned earlier [in concrete reality and in conception]; it is, rather, [an inquiry] insofar as it is useful to perceiving the states [*aḥwāl*] of these two [modes] of existence. [This being so], whoever believes that philosophy is an investigation of things insofar as they exist and are divided up according to the two [modes] of being mentioned earlier [concretely and in conception], then according to such a person this science will not be a part of philosophy. Instead, according to this person [this science] will be an instrument of philosophy insofar as this it is useful to [the investigation into things insofar as they exist]. On the other hand, whoever believes that philosophy deals with every theoretical inquiry [*baḥṭ nazarī*], and [deals with it] from every perspective, then [this science] will be a part of philosophy [*yakūnu ǧuz'an mina l-falsafati*], and an instrument to all the other parts of philosophy.²⁹³

²⁹³ Avicenna, *Madḥal*, 15.9-16.5.

In the earlier passage, Avicenna identified the kinds state or property that accrue to quiddities (or composites of them) qua mental objects, e.g. universality, grammatical subject, and syllogism, which are, as we said, what Avicenna calls in the *Metaphysics* of the *Šifā'* secondary intelligibles. Like Alfarabi, Avicenna says that these intelligibles will accrue to quiddity X on account X's mere presence in the conception. However, Avicenna appears to believe that based on this criterion alone without any further qualification states such as *being-a-grammatical-subject* in a nominal sentence will be on par with *being-an-antecedent* in a conditional proposition. This is a problem for Avicenna, for if he wants to hold like Alfarabi that secondary intelligibles include both grammatical concepts such as noun and verb and logical concepts such as being-a-syllogisms. In order to distinguish the secondary intelligibles that are proper to logic from those proper to grammar, Avicenna (following Yaḥyā ibn 'Adī) appeals to the end which logical concepts serve. To be sure, Avicenna does not follow Ibn 'Adī in claiming that they end of logic is demonstration (*burhān*), but he does follow him in saying that we can distinguish the subject matter of logic from grammar because these two disciplines have different ends. Those secondary intelligibles that Avicenna claims constitute the proper subject matter of logic are distinct from the secondary intelligibles constituting the subject matter of grammar because only the former and not the latter are part of the process of deductively acquiring knowledge by moving from known premises to unknown conclusions. Avicenna is even more explicit on this point in a short passage from the *Metaphysics* of the *Šifā'*:

[Text 21] As you have known the object of logic was the secondary intelligible concepts (*al-ma'ānī al-ma'qūla al-tāniya*)—those that depend on (*tastanidu ilā*) the primary intelligible concepts—insofar as they may be of use in arriving at the unknown from the known, and not insofar as they are thoughts (*ma'qūla*) having

an intellectual existence that is not attached to matter at all or is attached to non-corporeal matter.²⁹⁴

This is only part of Avicenna's argument however. The more important facet of Avicenna's argument is identifying what sorts of questions the science of logic asks. Alfarabi seems to have had some difficulty extracting himself from the idea that the class of scientific questions we could ask about a quiddity X were limited to those in the *Categories*. Nevertheless, his work on one-place existential predicates and his claim that "is" in at least one of its senses is a secondary intelligible seems to have been at least a starting point for Avicenna's discussion here. Alfarabi had claimed that when we say that "X is [*mawǧūd*]", where we mean that X *qua* concept is instantiated outside the mind, we are taking "is" as a secondary intelligible (i.e. we are predicating something of a concept, not of a thing outside the soul).²⁹⁵ As we saw earlier, Alfarabi also said that the properties "being-predicated-of-many" and "being-a-genus" for example were also properties predicated of intelligibles, viz. primary intelligibles such as *animal* in response to the question "What is animal (qua intelligible)?" . Avicenna wants to take Alfarabi's intuition that we can ask about an object's quiddity, and about its quality and quantity but the object about which we are asking is an intelligible only that has no relation to any quiddity outside the mind. Since the subject matter of logic is primary intelligibles, Alfarabi had asked "What is X?" where X was a primary intelligible (*animal*), and the response given was a secondary intelligible (e.g. something predicated of many). Since Avicenna holds that the subject matter of logic is secondary intelligibles, Avicenna wants to ask "What is X?", where X is a *secondary intelligible*, e.g. "What is an antecedent?", and the response (we might call it a tertiary intelligible) is some property of the quiddity "antecedent" possesses on

²⁹⁴ Sabra, "Avicenna on the Subject Matter of Logic", 753, trans. Sabra, from Avicenna, *Aṣ-Šifā'*, *al-Ilāhiyyāt*, eds. G. Anawati, and Sa'īd Zāyid (Cairo: al-Hay'a al-Āmma li-Šu'ūn al-Maṭābi' al-Amīriya, 1960), 10f.

²⁹⁵ Menn, "Al-Fārābī's *Kitāb al-Ḥurūf* and His Analysis of the Sense of Being", 81.

account of its participation in the deductive process of acquiring knowledge. The basic point of Text 20, then, is to argue that the purely mental objects such as “antecedent”, “consequent” and “connective conditional proposition” for example are quiddities to which certain properties accrue. These properties accrue to them not merely due to their being in mind, but because they arise necessarily from the process of deductively moving from known premises to unknown conclusions. These quiddities are not purely subjective states of mental representations, are objective and stable concepts that can be the object of a rigorous scientific investigation whose concepts are not contingent and subject to change.

Avicenna’s main criticisms, then, is aimed at the Alexandrian division of the sciences, which Avicenna summarizes at the opening of the *Šifā’*, for failing to make room for an objective philosophical discourse whose objects of inquiry have no extramental status. On the other hand, Avicenna adopts Alfarabi’s idea of secondary intelligibles, leaving it essentially intact. He rejects Alfarabi’s the claim that primary intelligibles are the subject matter of logic in the *Ḥurūf*, which was inspired by Pophryry’s discussion of the purpose and subject matter of the *Categories* in his commentaries on that work. He also rejects Alfarabi’s claim in Text 3 that the subject matter of logic is “intelligibles insofar as they are signified by expressions, and expressions insofar as they signify intelligibles”, which was also inspired by Alfarabi’s reading of the lengthy debates about the purpose of the *Categories* among Aristotle’s Peripatetic and Neoplatonist commentators. Comparing the two thinkers shows clearly both Alfarabi’s debt to these late antique commentary traditions; it also puts in stark contrast simultaneously Avicenna’s radical departure from that tradition, but his great debt to Alfarabi.

Thus, while Avicenna may disagree with Alfarabi on important issues, Avicenna is quite clearly beholden to Alfarabi’s thought on several crucial points: the notion of secondary

intelligibles, and the senses of being laid in the *Ḥurūf* make possible both Avicenna's claim that the subject matter of logic is second intelligibles and that logic is in a real sense part of philosophy and not simply an instrument. It is not clear to me, then, who Avicenna has in mind when he launches into a sharp invective against those who "hold that the subject matter of logic is "expressions insofar as they signify meanings, and the logicians art is to talk about expressions insofar as they signify meanings [*al-mantiqū mawḍū'uhu n-naẓaru fī l-alfāẓi min ḥayṭu tadullu 'alā l-ma'ānī wa-l-maṭiqiyyu innamā šinā'atuhu an yatkallama 'alā l-alfāẓi min ḥayṭu tadullu 'alā l-ma'ānī*]"

In Text 3 we see Alfarabi saying something similar, namely, that the subject matter of logic is "intelligibles insofar as they are signified by expressions, and expressions insofar as they signify intelligibles". Avicenna agrees (with substantial qualifications) with the first part of the conjunct that the subject matter of logic is intelligibles, but he adamantly disagrees with the second. In Text 6 we saw that Alexander might have held something like this view in his discussion of the purpose of the *Categories*. However, it is more likely that the individual Avicenna has in mind is his contemporary Yaḥyā ibn 'Adī, who claimed that the subject matter of logic is expressions [*alfāẓ*] insofar as they signify universals, without adding the other part of the claim, viz. that the subject matter of logic is also universals insofar as they are signified by expressions.

[Text 22] There is no good in the one who says that the subject matter of logic is expressions insofar as they signify meanings, and the logicians art is to talk about expressions insofar as they signify meanings. Instead, the matter is I have said. They have been thus foolish and stupid because they failed to gather what the reality of the subject matter of logic is, and the class of being that is distinct to it. For they have found being [*mawḡūd*] to have two modes: the being things have outside [the soul], and their being in the mind. They then assigned the investigation of being outside to one or other of the philosophical disciplines, and the investigation of the being in the mind to a discipline or part of a discipline [i.e. not to a philosophical discipline]. They thus did not make any further distinctions so that they could recognized that things [*umūr*] in the mind are either [1] things that are conceived of in the mind but derived from

outside, or they are [2] things that accrue to objects in the mind on account of [object's] being in the mind, but nothing outside [the soul] correlates to [*lā yuḥādī*] them. One of these [classes of mental being] might then have been assigned to the subject matter of logic. As to the question of which one of [the classes of mental being that is the subject matter of logic], it is the second [i.e. 2]. As to the question of which properties accrue to them, it is the property that becomes a means to bringing about in the soul another intellectual form that had not been before, or that is instrumental to bringing this [new intellectual form] about, or that hinders it [i.e. hinders the coming about of a new intellectual form].²⁹⁶

Thus in Avicenna's mind the claim that the subject matter of logic is secondary intelligibles is closely tied to the more important question of logic's place among the philosophical disciplines. The way that Avicenna saw to make logic a member of the philosophy and not merely an instrument of it was, first, to identify logic's subject matter with the class of objects that have no extramental correlates. Further, in order to distinguish these the objects of logic from those of, say, grammar, Avicenna says that the properties that we predicate of the subject matter of logic are those that are instrumental in generating new knowledge in soul. The X's in the questions "What is X?", "How many are X?", and "What quality does X have?" are concepts such as antecedent, predicate, and syllogism. And the properties that we generate in investigating these questions, such as *concludent* and *inconcludent* with respect to *syllogism*, or *impossible* and *possible* with respect to *antecedent*, and *per se* and *per accidens* with respect to predicate, are the fruit of logical inquiry.

§3.2.0 COMPLETE/INCOMPLETE VS. COINCIDENTAL/RESTRICTED FOLLOWING: INTRODUCTION

In §3.1, we saw that Avicenna holds that the subject matter of logic is secondary intelligibles such as subject, predicate, quantifier, universal, and particular. We also saw that Avicenna believes that the proper business of logic is investigating the nature of and the

²⁹⁶ Avicenna, *Madḥal*, 23.5-24.2.

relations between these concepts. In §3.2, I will explore one important ramification of this doctrine, namely, Avicenna's rejection of the complete (*tāmm*)/incomplete (*ġayr tāmm*) division of the connection between antecedent and consequent, a division that was held by several esteemed late-antique logicians, Alfarabi being only the most recent of Avicenna's predecessors.²⁹⁷ We will see that Avicenna's dispassionate epitome of complete/incomplete connection, as well as the notion of following (*ittibā'*) that Avicenna says underlies it closely parallel Alfarabi's discussion in *APCA*. Nevertheless, in ŠQV Avicenna *rejects* this venerable doctrine as unsuited to a division of conditional expressions that looks at the formal, logical properties of the antecedent and consequent and the connection between them.

The reasons for Avicenna's rejection of this division appear to be rooted in two distinct but closely related notions of logical form. The first notion is rooted in Avicenna's thinking about logic's subject matter and logic's status as a science that examines the formal aspects of arguments. As we will see in §3.2.1, Avicenna believes that logic examines second-order conceptual objects such as *predicate*, and *syllogism*, since they are logic's subject matter. Remember that for Alfarabi, the subject matter of logic is primary intelligibles such as *human* and *animal*. As we saw in §3.1.1, logic examines animal and human as intelligibles and seeks to discover properties about them such as that *animal* is predicated of many and human is a species of animal. The trouble with this view, according to Avicenna, is this. If we hold Alfarabi's view of the subject matter of logic, properties such as *being-predicated-of-many* and *being-a-species*, which accrue to primary intelligibles such as *animal* and *human*, accrue to them due to properties that hold of and between their instantiations outside the soul. For example

²⁹⁷ For the complete/incomplete division of following, see Maróth, *Aussagenlogik*, 38–55. See also Susanne Bobzien's recent article on Galen's theory of hypothetical syllogisms in his *Introduction to Logic*; S. Bobzien, "Peripatetic Hypothetical Syllogistic in Galen-Propositional Logic off the Rails?", *Rhizai: Journal for Ancient Philosophy and Science* 2 (2004): 57–102.

we say that the concept animal is predicated of many. The question is: why do we say that? We say that *animal* is predicated of many not because of properties it has *qua* mental object but because it is predicated of sensible objects outside the mind, and for which it serves as part of their being. We say that that the concept human is subordinate to the concept animal? Why do we say that? We do not say this because of their properties as mental objects; we say it because animal is the material cause of human, or because of every object outside the soul that we predicate human of we also predicate animal, but not the reverse, or because animal is the quiddity of human, etc. Thus, based on the Farabian view of the subject matter of logic, the discipline of logic is formal in the sense that its subject matter is intelligibles. However, Avicenna will argue that it is not formal in the sense that the properties (e.g. *being-a-predicate*) that we attribute to primary intelligibles (e.g. *animal*) arise *not* on account of primary intelligibles *qua* intelligibles, but on account of properties and relations possessed by instantiations of primary intelligibles outside the soul.

Logic as a science of form has quite a different meaning in Avicenna's way of thinking. Like Alfarabi logic is formal in the sense that its subject matter, viz. secondary intelligibles such as syllogism and predicate, are concepts. More importantly in Avicenna's mind however is the fact that the properties that the science discovers about these objects such as *valid* and *invalid* in relation to syllogism, and *per se* and *per accidens* in relation to predicate are properties that belong to these objects on account of their presence in the mind, but more importantly, on account of their being instrumental in the process of deductively acquiring knowledge. These properties do *not* come about on account of relations that hold between quiddities outside the soul such as *human* and *animal*. This is, in my view, the main reason for Avicenna's insistence in the above passages from the *Madḥal* that the subject matter of logic and the

properties that they possess have no extramental correlates. In §3.2, Avicenna will apply these notions to his critique of the division of conditionals into complete and incomplete.

The above notion of logical form has to do with the most *basic elements* of logic taken as a science of form. The second intuition about logical form that informs Avicenna's critique of the classification of conditionals into complete and incomplete has to do with what the form of an *argument* is. Here, Avicenna seems to be relying on an intuition that informs a discussion by Alexander of Aphrodisias in the *Topics*. As Jonathan Barnes has observed, in this passage we find Alexander giving voice to the idea that the opposite of an argument's matter is an argument's *mood*.

[Text 23] Aristotle and his followers ... lay it down that dialectic is a certain syllogistic method; and they think that syllogisms do not in the least differ one from another insofar as they are syllogisms—their differences are, some of them, according to the form of the propositions, some according to the moods and the figures, and some according to the matter with which they are concerned. The first of these differences make some syllogisms probative—or predicative—, as we call them—and others hypothetical. The second makes some perfect and other imperfect, and some in a first figure, some in a second, and some in a third ... And the difference according to matter—makes some demonstrative and some dialectical and some eristical.²⁹⁸

As I discussed in greater length in §2, Alexander is here concerned to balance the claim that the syllogism is a unitary genus, and the various criteria for dividing its members into species. What Alexander is claiming in this passage is that it is possible to group syllogisms according to whether they are direct or indirect, or according to their mood, or their figure, or according to their matter. However, Barnes cites this text because it gives us an intuition into what Alexander thinks what logic form is by his contrast between an argument's matter on the one hand and the following three elements: (1) the argument's being direct or indirect, (2) its

²⁹⁸ Alexander of Aphrodisias, in *Top.* 1.19-2.16; cited and translated in J. Barnes, *Truth, Etc.*, 282. I have modified Barnes' translation slightly to reflect the conventional translation of *schêma* as "figure". So where I have put "figure" or its plural Barnes puts "shape" or its plural.

mood, and (3) its figure. All of these, according to Barnes, are in some sense part of an argument's form in the sense that they are used as a contrast with an argument's matter. For Alexander (1) an argument's being direct or indirect is equivalent to saying that the argument is composed of either categorical or hypothetical premises. An argument's (3) figure is determined by the placement of the middle figure in categorical syllogisms and the prosleptic minor premise (and its quality) in hypothetical syllogisms. And (2) an argument's mood is determined by the quantity and quality of subject and predicate terms of the premises and conclusion within each figure. Thus, Alexander identifies several elements all of which constitute aspects of an argument's form: the species of proposition of the premises, the placement of the middle term vis-à-vis the major and minor in the premises in categorical syllogisms, the repeated minor premise (and its quality) in hypothetical syllogisms, and the quality and quantity of the predicate and subject terms (in categorical syllogisms). The matter of an argument would be everything that is not one of the above elements, viz. the concrete terms that the argument itself is about. The main point to take from this passage from Alexander on the *Topics* is that a formal analysis of the syllogism (whether of the categorical or hypothetical variety) must provide formal criteria for distinguishing between different species of syllogisms. If an analysis does not do this, then it has failed one of its main tasks.

In §3.3.1 we will see that Avicenna's claims that the Galenic classification of conditional propositions into complete and incomplete fails to appropriately distinguish between argument moods (*ḍarb*, pl. *ḍurūb*) among repetitive syllogisms with conditional major premises. From the elements of argument form identified in the above passage from Alexander, only two are relevant to the analysis of repetitive syllogisms, viz. the type of proposition, and the mood (perfect, imperfect). The notion of figure is irrelevant to the

analysis of Galenic types hypothetical syllogisms because there is no middle term. Moreover, mood is determined not by the quantity and quality of premises by which of the antecedent and consequent is repeated or their contradictory opposites is repeated in the minor premise. The Galenic account of these types of hypothetical syllogisms says that two syllogisms with the following forms are belong to different moods, the first being complete and the second incomplete: (1) if P, then Q; P. Therefore, Q; and (2) if A, then B, where the connection (*ittiṣāl*) between P and Q in the major premise of 1 is complete (*tāmm*), and the connection between A and B in in the major premise of 2 is incomplete. Avicenna shows that with such a classification of conditional major premises no *formal* distinction between 1 and 2 can be drawn. Appeal must be made to the matter of 1 and 2 in order to maintain their division into perfect and imperfect moods.

Avicenna proposes that we make a fresh start. According to Avicenna's new classification, we should divide connective conditional expressions according to whether the connection between the antecedent and the consequent signifies a simple coincidence of truth (*yaṣḍuqu ma'a*) or signifies that the antecedent implies the consequent (*al-muqaddamu yalzimuhu t-tālī*). Avicenna calls the former "connection *simpliciter* (*'alā l-iṭlāq*)" or "coincidental (*ittifāqī*).” Avicenna calls the latter “*restricted* (*'alā 't-taḥqīq*)” or “*real* (*ḥaqīqī*).” In §3.2.1 I will show how ŠQ V 1 is devoted to clarifying what Avicenna believes a connective conditional proposition is. For Avicenna, this involves clarifying, first, in what sense it is a proposition; and second, what the nature of the connection (*ittiṣāl*) between the antecedent and consequent is. Of these issues, it is evident that Avicenna moves away from Alfarabi's views regarding the second, viz. classifying connection in a connective conditional expression as complete or incomplete. The idea of complete/incomplete connection disappears from Avicenna's

development of the hypothetical syllogistic, only to resurface once again in ŠQVIII where Avicenna discusses repetitive syllogism at length.

§3.2.1 THE NATURE OF CONNECTIVE CONDITIONAL PROPOSITIONS AND THE COMPLETE/INCOMPLETE DIVISION OF CONNECTION (ITTIŞĀL)

ŠQ V 1 constitutes the opening volley in Avicenna's lengthy and often critical treatment of hypothetical syllogisms in ŠQ V-IX. Avicenna justifies delving into the subject with the remark that since many of the statements needing proof in philosophical disciplines such as physics and metaphysics are conditional in form, there is a need for a syllogistic machinery that can take conditional sentences as premises and yield conditional sentences as conclusions. Avicenna points to formal similarities between categorical and conditional propositions, but concludes that, despite these similarities, it is not possible to deduce a conditional-sentence conclusion from only categorical sentences as premises. It is clear from his brief comparison of conditional and categorical sentences that Avicenna feels the differences between categorical and conditional propositions to be more fundamental than the similarities.²⁹⁹ Generally, Avicenna agrees with Alfarabi that conditional sentences are propositions, by which both mean that they are indicative sentences with propositional content referring to objects outside of the soul. Avicenna says that the conditional proposition (*qāḍiyya šarṭiyya*) and the categorical proposition (*qāḍiyya ḥamliyya*) share several properties. They both fall under the category of apophantic discourse (*qawl ḡāzim*) in the sense that, by the utterance of a categorical or conditional sentence, the sentence is subject to affirmation or denial. They are also both said to be propositions that possess a conceptual content (*taṣawwur*

²⁹⁹ Avicenna, ŠQV 231.6-232.10.

li-ma'nā) that is conceptually distinct from and prior to the conceptual relation (*taṣawwur li-nisba*) it may or may not have with some extramental object (*ḥārīḡ*), the existence or nonexistence of which determines the truth and falsity of the conditional. In short, Avicenna shares with Alfarabi a correspondence theory of truth for conditional propositions in analogy with categorical propositions. Yet, despite the agreement of categorical and conditional propositions at the level of semantics (i.e. at the level of truth), Avicenna maintains syntactic differences that appear to be, in his eyes, more fundamental. Despite the fact that conditional and categorical propositions both relate “parts to parts (*anna hunāka ḥukman bi-nisbati ḡuz'in ilā ḡuz'*),” Avicenna says that they differ with regard to the formal properties (*hay'a*) of the relation. The relata of conditional propositions are indicative statements (*ta'līf ḥabarī*) rather than subject and predicate terms as in categorical sentences. An affirmative categorical judgment says that “the first is the second (*awwaluhā huwa tānīhā*), e.g. man is a writer (*al-insānu kātib*),” whereas an affirmative conditional judgment says that the sentential content of the indicative sentences are related by following (*'alā sabīli l-mutāba'a*) or by incompatibility (*'alā sabīli l-mu'ānada*). Such is the case in Avicenna's example, wherein he says that the indicative statements “it is day” and “the sun is up” are connected by a relation of following (*'alā sabīli l-ittibā'*).

Avicenna agrees with Alfarabi that conditional sentences are propositions in the sense mentioned above. Yet, following these brief remarks, Avicenna's account of how the connection between the antecedent and consequent should be analyzed in light of its logical properties diverges sharply from Alfarabi. Avicenna clearly says that he will begin his discussion by “summarizing what has been said about the matter of connection and

incompatibility so far (*abda'u bi-qtiṣāṣi mā qīla fī amri l-ittiṣāli wa-l-ʿinād*),”³⁰⁰ alluding to the fact that he is presenting conventional views with which he does not necessarily agree. Reporting what others have said on the subject, Avicenna quickly summarizes the two different types of connection formalized in connective conditional expressions:

[Text 24] They say: connection is either complete or incomplete [...] As for complete connection, it is in those [conditional expressions] in which the antecedent implies the consequent and the consequent implies the antecedent. They say, for example: “whenever the sun is up, then it is day” and “whenever it is day, then the sun is up.” As for incomplete connection, it is in <the conditional expression> in which the antecedent implies the consequent but not the converse. For example, they say: “whenever this is a man, then it is an animal.” This does not convert, for it is not the case that if this is an animal, then it is human also.³⁰¹

According to the traditional account, the logical connection between antecedent and consequent, whether incomplete or complete, is underwritten by different types of following (*ittibāʿ*). According to traditional view, the most general type of conditional expressions gives formal expression to many different types of following, of which Avicenna alludes to three explicitly. The first sense, however, is the one that concerns us here. In this sense of following, the hypothesizing or “setting down” (*wadʿ*) of the antecedent is said to require, in itself (*yaqtaḍī li-dātihi*), the following of the consequent in a self-evident way.³⁰² To explain

³⁰⁰ Ibid., 232.11. Shehaby gives a misleading translation of this important sentence, which obscures the point of Avicenna’s project in ŠQ V: “Let us start with an *accurate* account of what has been said on connection and conflict (*italics added*)”; see Avicenna, *Propositional Logic*, 36.

³⁰¹ Avicenna, ŠQ, 232.11-17. The parallels with Alfarabi are obvious enough, but the idea of the necessity of the connection is conspicuously absent; see §2 above. I am only able to account for their absence in two ways. One, is that for Avicenna (and I think for Alfarabi too) the verb *lazima/yalzamu/luzūm* means to follow and connotes that this following is necessary. This is certainly how our authors use it in a slightly non-technical sense when they are arguing a particular point. After setting out premises from which they wish to draw a conclusion for the reader, they will use *lazima/yalzamu* to mark that the conclusion follows (logically or materially, but in any case necessarily). The second way of accounting for the absence of necessity in Avicenna’s summary is that Alfarabi felt the need to emphasize the necessity of *luzūm* because he was discussing different types of following, some of which were purely accidental, others that were law-like but not necessary in a statistical sense. If this interpretation is correct, his attaching modalities would be redundant strictly speaking, but appropriate in order to avoid confusion with other, weaker types of following.

³⁰² Ibid., 233.15.

what he means, Avicenna enlists the aid of the well-known example of the sun's being up requiring that it be day:

[Text 25a] For example, they say that the hypothesizing of the sun being up implies in existence and in the intellect [*yalzimuhu fī l-wuġūd wa-l-'aql*] that it is day. This antecedent condition [*malzūm*, lit. "that from which the consequent follows"] may be a cause [*'illa*] of the existence of the second as is the case of the above example, or it might be an inseparable effect [*ma'lūl ġayru mufāriq*] [of some cause in the consequent position], e.g. "if it is day, then the sun is up," or it might be relative [*muḍāyif*], or each [of the antecedent and consequent] might be a cause of the other, or they might be the effect of a single cause simultaneously, e.g. thunder and lightning in relation to the movement of the wind in the clouds, or [the antecedent might follow from the consequent] in some other way, which does not concern us here.³⁰³

Avicenna's remarks here recall an important discussion of the senses of prior (*at-taqaddum*) and posterior (*at-ta'ahhur*) in Alfarabi's paraphrase of the *Categories*.³⁰⁴ Alfarabi says that something is said to be prior (or posterior) to something else in five distinct senses: prior in time (*bi-z-zamān*), in nature (*bi-ṭ-ṭab'*), in rank (*bi-l-martaba*), in perfection (*bi-l-kamāl*), and causal priority (*at-taqaddum bi-annah sababu wuġūdi š-šay'i*). Only priority in nature and causal priority are relevant to understanding Avicenna's criticism of complete and incomplete following. Recall from chapter 2 that Alfarabi, following an ancient model for the semantics of conditionals, divides following into complete and incomplete types according to the way in which the antecedent implies the consequent. From APCA we learn that the notions of priority and causal priority underwrite these two categories of following. What is more, the type of priority that characterizes the connection between the antecedent and consequent dictates the order of the antecedent and consequent so that the conditional comes out true. Alfarabi says that a relation of natural priority holds between X and Y when there is a non-equipollent relation of existence between them (*lā yatakāfa'āni fī luzūmi l-wuġūdi*). This happens in the case

³⁰³ Ibid., 233.15-234.4.

³⁰⁴ Alfarabi, APCA, (paragraph 59) 35.25-36.27.

that the antecedent X entails the existence of the consequent Y, but the consequent Y does not entail the existence of the antecedent X. Causal priority holds between an X and Y in which such a relation *does* exist, viz. that X is entails that Y is, and that Y is entails that X is. Yet, in this latter case, it is nevertheless clear that one of X or Y is the cause of the other. In order to illustrate this point, Alfarabi uses the example of the relation of consequence between animal (Y, prior) and human (X, posterior), and the numbers one (Y, prior) and two (X, posterior).

[Text 25b] For the consequent [*al-lāzim*, Y] is said to be prior to that which entails it [X] when the thing that entails [Y] is not part of the being of the consequent [*laysa šay'an li-wuğūdi l-lāzimi*]³⁰⁵. The thing that entails the consequent is posterior in nature [*huwa l-muta'ahhiru bi-t-ṭab'*]. For example, in the case of human and animal and two and one, in each case the one that is prior [viz. animal and one] is the thing whose elimination entails the elimination of the other [viz. human and two], but [the prior thing's] existence [animal, one] does not entail the existence of the other [human, two].

This is how the consequent [*al-lāzim*] is in non-equipollent relations, for Y [prior consequent] is entailed by X [posterior antecedent], but Y is not complementary in entailing the existence of X. Also, X is eliminated when Y is, but Y is not eliminated when X is. Animal is entailed by human, but animal is not complementary in entailing human. Human is eliminated when animal is eliminated, but eliminating human does not eliminate animal. Animal, then, is the prior in nature, and human is posterior. The same is true in the case of two, which is posterior, and one is the prior [...]

Of a pair of things [X and Y] that *are* in an equipollent relation of existence, Y is prior on account of its being the cause [*as-sabab*]. For example, the sun rising and day are like this, for they are in an equipollent relation of existence, despite which the sun rising is the cause of day, and so it is prior to it because it alone is the cause [of day]. In general, the cause insofar as it is a cause, and regardless of what type it may be, is prior to the thing that comes to be from it. For this reason, it is possible that some cause is prior in time to the thing that comes to be from it [*mina š-šay'i l-kā'ini 'anhu*], e.g. the house and the wall [*al-binā' wa-l-ḥā'it*].³⁰⁶ In this case [the house] is prior in two respects; because it is prior as cause, and prior in time.³⁰⁷

³⁰⁵ 'Ağam reads "*sababan*" for Dunlop's "*šay'an*"; Alfarabi, *Al-Mantiq 'inda l-Fārābī*, ed. R. 'Ağam, vol. 1 (Beirut: Dār al-Mašriq, 1985), 129 [Hereafter, I will cite 'Ağam's edition as follows: Alfarabi, *Maqūlāt*]. In this instance, I believe that Dunlop's reading is the right one.

³⁰⁶ I take Alfarabi to be saying that the house is prior to the wall as a final cause, and thus, it possesses explanatory priority. Alfarabi's claim that the house is prior in time to the wall is more difficult to account for. He might be

Thus the priority between antecedent and consequent determines the kind of following that must hold in between an antecedent and consequent in order for the conditional composed from them to be true. The following between human and animal on the one hand and the following between its being day and the sun's being up on the other are different sorts of following. Alfarabi calls the former type of following between antecedent and consequent incomplete because human implies animal but animal does not imply human. The reason why animal must stand in the consequent place and human in the antecedent place for a conditional having incomplete following to be true is because animal (consequent) is prior in nature to human (antecedent); for this reason human entails animal, but the converse is false. Alfarabi explains natural priority in terms of substance. Animal is prior in nature to human because animal is part of the substance of human (to the question "What is a human?", we respond "An animal"). On the other hand, human is not prior in nature to animal because we do not reply to the question "What is an animal?" with "A human." This type of following stands in sharp contrast to complete following. In this latter type, we pick out the sun's rising as the cause of its being day, but the causal priority of the former does not require a certain order in antecedent or the consequent. The placement of "the sun is up" in the antecedent position because it is cause or in the consequent position because it is an inseparable effect has no impact on the truth of the conditional. It is this fact that distinguishes complete from incomplete following in Alfarabi.

Yet, Avicenna sees these two types of following as one in essence, in the sense that both are concerned with seeing the antecedent and the consequent in respect of their status as

thinking of Aristotle's discussion in *Metaphysics* VII, 7 where the builders thinking of the house's *form* is prior to the process of building the house. If this is so, then it makes sense to say that the house as a form in the thought of the builder is prior to the wall, where the wall is part of the process of building the house.

³⁰⁷ Alfarabi, *APCA*, 36.3-23; idem, *Maqūlāt*, 129-30.

causes, effects, relatives, inseparable accidents, etc. Two things seem to worry Avicenna here. One is that the following of the consequent from the antecedent is due to the nature of the antecedent qua its propositional content, not qua antecedent. Thus, in the example of human and animal, animal follows from human not due to the formal logical properties of animal qua consequent or human qua antecedent, but qua their respective ranks in the genera-species hierarchy. To understand the connection between antecedent and consequent in a connective conditional proposition in terms of its being complete or incomplete is to look at the antecedent and consequent in respect of their status as causes or effects, wholes or parts, universals or particulars or as relatives. Having determined in what kind of causal/effect relation day and the sun stand, we can then determine whether the connection between them is complete or incomplete. The nature of the connection is determined by the antecedent and consequent qua their content, not by the formal or logical properties of the antecedent, consequent or the connection between them. According to Avicenna, however, to speak about the antecedent and consequent in these terms is not suitable to a discussion in the context of logical investigation. Remember that according to Avicenna, the subject matter of logic is secondary intelligibles, which are concepts such as antecedent, consequent, and, presumably, connection (*ittiṣāl*). If connection too is to constitute part of the subject matter of logic, then to speak about the connection between antecedent and consequent in terms of the link (*‘alāqa*) in extramental reality between cause and effect, whole and part, and the like is to investigate the nature of secondary intelligibles in terms of primary intelligibles. For Avicenna however, a basic principle of his logical theory is that the investigation of the nature of the connection between antecedent and consequent *as a concept of logic* cannot be carried out with concepts that circumscribe objects subsisting outside the soul. By definition, logical concepts do not

have extramental correlates. Since the subject matter of logic for Alfarabi is primary intelligibles, to speak about the connection between antecedent and consequent qua connection between cause and effect and the like poses no problem. With an eye toward Alfarabi's rule-based notion of logic (see chapter 2), we can construct, as an example, a Farabian rule for conditional conversion for the logic of conditionals:

Farabian Rule of conditional conversion: the antecedent P and the consequent Q in a true conditional proposition “if P, then Q” may be converted while preserving truth, yielding the true “if Q, then P” if P (e.g. sun) is a cause of Q (e.g. day) and Q is an inseparable accident of P, or if P (e.g. master) and Q (e.g. slave) are relatives in a binary relation (e.g. the relation called indifferently “masterhood” or “slavehood”). Otherwise, conversion is a non-truth-preserving operation. If the conversion is truth-preserving, the connection between P and Q is called a “complete connection”; if “if P, then Q” is true and a connection between P and Q exists but conversion is not truth-preserving, then the connection between P and Q is incomplete.³⁰⁸

Based on the results of section 1.1 and 1.2, I believe that Avicenna's view is that concepts like cause and inseparable accident have no place in an investigation into the logical properties characterizing the connection between an antecedent like “the sun is up” and a consequent such as “it is day.” This problem is one factor motivating Avicenna's departure from the traditional division of connection into complete and incomplete.

The first factor is inspired by Avicenna's concern with the formality of logical investigation. The second is motivated by a concern with its potential to generate knowledge. Avicenna worries that, since the consequent's following from the antecedent is a self-evident fact returning to the nature of the antecedent, we come to have knowledge of the consequent's following from the antecedent before it comes to be expressed as a consequent of hypothesis in a conditional expression. If we already know that the consequent follows from the

³⁰⁸ Bobzien notes (S. Bobzien, “Peripatetic Hypothetical Syllogistic in Galen—Propositional Logic off the Rails?”, *Rhizai: Journal for Ancient Philosophy and Science* 2 (2004): 57-102) that a consequence of Galen's way of thinking about the semantics of conditionals is that, technically speaking, they do not have conditions that we can stipulate beforehand under which the conditional propositions is be.

antecedent, then there is no point, according to Avicenna, in setting down the antecedent as a hypothesis so that the mind moves from the antecedent to the consequent. The mind is, so to speak, already there. It is clear that Avicenna's worries about the conventional division of the connection between antecedent and consequent serve as the negative backdrop against which Avicenna's puts forward his alternative division of connection (*ittiṣāl*). Avicenna begins by rehashing what he said at the beginning of ŠQ V 1. At the most basic level, says Avicenna, a connective conditional expression is one that signifies that the sentence *P* is true along with (*ma'a*) the sentence *Q*, and, additionally, that whenever *P* is true, then *Q* is true. A connective conditional expression, in which the antecedent and consequent are related in such a way, is said to have following (*ittibā'*) between the antecedent and consequent. As we saw above, a consequent follows from an antecedent properly speaking when (1) the antecedent implies the consequent in extramental existence due to a link (*'alāqa*), and (2) when the antecedent implies the consequent in the mind (*fī l-'aql*) due to a relation (*nisba*) that the mind entertains between them. Avicenna says that this relation can be of various types. The relation between antecedent and consequent can be a relation of predicate to subject (e.g. "if man is, then animal is"), or a relation of cause to effect (e.g. "if the sun is up, then it is day"), or a relation of possession to thing possessed, or a relation of inseparable but accidental possession (e.g. "if man is, then something risible is"), or a relative (e.g. "if slave is, then master is"). The difficulty that Avicenna explicitly alludes to is this. In the Arabic philosophical tradition of hypothetical reasoning of which Avicenna and Alfarabi are members, connective conditional expressions signify the existence of some sort of connection between the antecedent and consequent expressions from which the conditional is composed. According to Avicenna's presentation of the conventional view, the connection signified is one of following (*ittibā'*).

According to this view, connective conditional expressions such as “if Socrates is human, then Socrates is an animal,” or “if the sun is up, then it is day” signify that there is incomplete connection between the antecedent and consequent of the first expression and complete connection between the antecedent and consequent of the second expression. This means that in the first expression, the antecedent implies the consequent and following is due to the subject-predicate relation that holds between human and animal. In the second expression, this means both that the antecedent implies the consequent and the following is due to the fact that the antecedent causes the consequent; and that the consequent implies the antecedent and this following is due to the effect’s being an inseparable byproduct of its cause. Thus, Avicenna says that connective conditional expressions like the ones above are *true* because a relation in extramental existence holds between the referents of the antecedent and consequent. Avicenna also says that a true antecedent *implies* a true consequent when there is this relation in extramental reality and the intellect recognizes this link and places it in one of the analytical categories mentioned above (whole, part, universal, particular, cause, effect, possession, possessed, relatum). However, according to Avicenna, the purpose of setting down a hypothesis as an antecedent in a connective conditional expression is so that, by our entertaining the hypothesis in our mind, the mind moves from the antecedent hypothesis to the consequent or consequents that follow from it. For Avicenna, forming a conditional in the mind is an ordered process that involves the mind’s moving from the first thought to the second. First, one hypothesizes the antecedent and then, by means of this act of hypothesizing, the mind moves to the consequent. The trouble with the way connective conditionals are discussed under conventional assumptions is that, in Avicenna’s view, there is no order and there is no movement. In Avicenna’s words:

[Text 26] However, if the mind's knowledge of the consequent's extramental being has already preceded <the act of hypothesizing> so that the mind does not arrive at <the consequent> by means of the hypothesizing of the first, whether it arrives at it as a first principle of a science [*awwaliyyan*] or by investigation [*bi-n-naẓar*], then there is no use [*lā fā'idata fi*] in hypothesizing the antecedent so that the mind moves from it to the consequent.³⁰⁹

According to Avicenna, there is no point in uttering a connective conditional proposition if we already know the consequent by some other means. In this case, the setting down of the antecedent does not lead the mind to any place it has not already been. Take for example the connective conditional “if Socrates is human, then Socrates is an animal.” The expression is *true* since there is a causal connection between human and animal in extramental reality, which manifests itself in speech in the form of a true categorical “man is an animal.” However, in conceiving of the antecedent “Socrates is human” as a hypothesis, our conception of human is inseparable from our conception of animal, such that when we hypothesize that Socrates is human, we are also, by this very act, hypothesizing that Socrates is an animal because of animal's status as material cause of human. Similarly, in the expression, “if master is, then slave is,” our entertaining the concept master as a hypothesis is inseparable from our conceiving of the concept slave. The hypothesis of the concept does not lead us to conceive of slave as a byproduct of our initial hypothesis. They are a single, unitary mental act. According to Avicenna, the conventional account of connection in connective conditionals is that (1) the antecedent implies the consequent in extramental reality and (2) the antecedent stands in a relationship to the consequent which the mind recognizes as one of cause to effect, whole to part, universal to particular, or substance to inseparable accident. However, according to Avicenna, this is to make connective conditionals useless, in the sense that they do not fulfill the role that Avicenna assigns to them in his logical system. The hypothesizing of the

³⁰⁹ Ibid., 237.

antecedent must not lead to just any consequent, but a consequent that was not already known by the mind. Yet, conceiving of the connection in conventional terms makes it so that the antecedents and consequents of true conditional propositions turn out to be related in such a way that precludes our knowing the antecedent and not the consequent.

A theory of the connection in connective conditionals that (1) is not formal enough to be suited for logical investigation and (2) that makes them useless—in the specific way Avicenna means it—is, in Avicenna’s mind, a sufficient reason to abandon this line of thinking entirely. Let us begin to think about conditionals anew. He says:

[Text 27] [A] It is necessary that the expression signifying that something is true with something else, and that whenever the first is true, the other is true—such an expression must be a proposition [i.e. subject to truth and falsehood], but <an expression with such a meaning> cannot be counted among the propositions that are categorical in form [*ḥamliyyāt*]. Thus, <such expressions> must be counted among the propositions that are conditional in form [*ṣartīyyāt*], and <in particular> among the *connective* <conditional expressions> [...] ³¹⁰ [B] Let the connective <expression> be either *simpliciter* [*‘alā l-iṭlāq*], such that what is claimed in it is only that the consequent is true with [*ma‘a*] the antecedent; or *restricted* [*‘alā t-taḥqīq*], such that what is claimed is that the truth of the consequent is from [the hypothesis of] the antecedent [*‘ani l-muqaddam*]. The first of these is more general than the second, since they are divided according to whether the concomitance of the antecedent and the consequent is by an implication [*bi-luzūm*] or by a coincidence [*bi-t-tifāq*]. ³¹¹

In the division of connective conditional propositions into *simpliciter* and *restricted*, both types are now said to express concomitance (*ma‘iyya*) rather than following (*ittibā’*). In this text, Avicenna appears to be hearkening back to an Aristotelian division of propositions into those that are categorical and those that are not categorical, all of the latter kind of which Aristotle generically calls “hypothetical”. ³¹² A single propositions that signifies that one (or more) of its elements are true “with [*ma‘a*]” other elements in the proposition cannot, in Avicenna’s eyes,

³¹⁰ See also chapter 4, Text 6 below.

³¹¹ Ibid., 237.1-16.

³¹² See chapter 5 below.

be classified among the categorical propositions, since the elements of categorical propositions are terms, which, as concepts (*taṣawwurāt*), are not subject to truth valuation in Avicenna's logical theory.³¹³ If Aristotle's thoughts about types of proposition that should be classified among hypotheticals was somewhat nebulous, Avicenna's was not. Following the precedent of late antique logicians, Avicenna divided what Aristotle generically classified as "hypothetical" propositions into "connective conditional" propositions, viz. "if...then..." propositions, and "disjunctive conditional" propositions, viz. "either...or..." propositions. Of these types, the only reasonable candidate for expressing that some proposition is true with another another is, in Avicenna's view, the connective conditional proposition, since the disjunctive type is used to signify different grades of incompatibility. Thus, in Text 27 part A Avicenna is drawing the reader's attention to the fact that there is a need for a propositional form by means of which we can express the fact that two (or more) propositions are true together. And Avicenna concludes that the most appropriate form for expressing concomitance in truth is what had come to be called in the Arabic logical tradition a "connective conditional proposition [*qaḍiyya šarṭiyya muttaṣila*]", which usually took the form of an "if...then...[*idā...fa...*]" sentence.

In Text 27 part B, Avicenna identifies what he takes to be the two most important, or at least the logically most relevant, senses of "concomitance in truth [*ma'iyya*]", both of which are expressed with the identical sentential form of "if...then...". First, a sentence of the form "if A, B" expresses the fact that A is truth with B in an "unqualified [*muṭlaq*, '*alā l-iṭlāq*]" sense. Second, a sentence of the form "if A, B" expresses the fact that A is true with B in a qualified or "restricted [*alā t-taḥqīq*]" sense. A restricted reading of "if A, B" expresses not only the fact

³¹³ H. Wolfson, "The terms *Tasawwur* and *Tasdiq* in Arabic Philosophy and their Greek, Latin and Hebrew Equivalents", *The Moslem World* 33 (1943): 114-128.

that *A* is true with *B*, but additionally signifies that the hypothesis of *A* as true is responsible or correlated with the truth of *B*. An unqualified or *simpliciter* reading of “if *A*, *B*” does not carry this extra meaning. It only signifies that *A* and *B* are true together and does not indicate that the truth of *B* relies on or tied up with the truth of *A*.

When Avicenna says that a *simpliciter* connective conditional expression signifies that the antecedent and consequent are true together, the “together” suggests a modalized reading of the definition. Indeed, Avicenna says that “the true [connective conditional] without implication is one in which the antecedent’s being true does not rule out [*lā yamna‘u*] the truth of the consequent accompanying it [*wa-innamā yakūnu ṣ-ṣādiq bi-lā luzūmi mā yakūnu l-muqaddamu fīhi ṣādiqan lā yamna‘u an yuqārinahu ṣidqu t-tālī*]”.³¹⁴ First, a true *simpliciter* conditional can only be true when there is a true antecedent. Given the true antecedent, we then consider whether it is possible for the consequent to be true. If the consequent can be true with the true antecedent, the *simpliciter* conditional will be true. But, if the true antecedent precludes such a possibility, the *simpliciter* conditional will be false. Furthermore, if we recall that Avicenna holds a correspondence theory of truth (viz. a sentence is true if the intelligible quiddity signified by the expression circumscribes something in extramental reality), the manner of assessing the truth of a *simpliciter* conditional can be reformulated in the following terms. Given that the antecedent expression signifies a concept with an extramental correlate, we must then verify whether or not it is possible for the consequent expression to express a concept that has an extramental correlate. If it is possible, then the conditional expression as a whole is true; otherwise it is false.

³¹⁴ Avicenna, ŠQ V, 238.15-6.

Thus, according to the truth-conditions Avicenna sets out, a *simpliciter* connective conditional is false when the antecedent is false and the consequent is true, when the consequent is true and the antecedent is false and when both the antecedent and consequent are false. Does this mean that we can say Avicenna's *simpliciter* connective conditional, "if P, then Q," is true if and only if the truth-functional conjunction of the antecedent and the consequent, "P&Q," is true? Not quite. There is no doubt that Avicenna's "if P, then Q" read *simpliciter* will be true whenever P and Q are both true, with the further stipulation that the subject matter of P and Q are entirely irrelevant, e.g. "if Germany is in the EU, then two is even" is true *simpliciter* because the processes for evaluating the truth of the antecedent and the consequent are irrelevant to each other. The modality located in the semantics of a *simpliciter* conditional becomes relevant in cases in which the processes for evaluating the truth of the antecedent and consequent *are* relevant to one another. Consider the connective conditional that have the same subject:

(S1) "if Theon sits, then Theon walks."

On this reading, if we read S1 as a *simpliciter* conditional with truth conditions of a truth-functional conjunction, then S1 is true if and only if "Theon sits and Theon walks" is true. Whether or not we take the conjunction to be true depends on how we allow temporality and modality to affect our judgment. If we say that S1 should be read in such a way that the antecedent and consequent are evaluated at some time *t*, then S1 is false since Theon cannot sit and walk at once. However, by saying that Theon sits and walks we can also mean to indicate that Theon has the capacity for sitting and walking (perhaps he was paralyzed in the past, but to indicate that he has now recovered or is in the process of recovery we say that he sits and he walks, i.e once more Theon has the ability to sit and the ability to walk). In this

case, the temporal considerations are not relevant since we are not talking about the mutual exclusivity of certain bodily postures at a time t . S1 is then true since Theon's ability to sit at one time does not inhibit his ability to walk at some other time and so it has no effect on our consideration of the truth of the whole conjunction. Consider if by uttering S1, we are talking in the context of a particular time t . The process of determining whether S1 is true is as follows. Take the antecedent as true at t , viz. take "Theon sits" to be true at t . This would mean that Theon is circumscribed by the concept signified by the term "sits" at time t . Given this, then it is impossible for the consequent to be true at t since Theon's walking at t precludes his being seated at t . Equivalently, Theon cannot at the same time be an object circumscribed by the concept sits while also being circumscribed by the concept walks. However, if we understand S1 as making a statement about Theon's *ability* to sit and his *ability* to walk, then it seems that S1 could be true despite its intuitive falsity. Another possibility is that the *simpliciter* conditional expresses the fact that the antecedent and consequent are true, but at times t_1 and t_2 , where t_1 is not equal to t_2 . Indeed, in Avicenna's thinking about the meaning of absolute categorical propositions, the proposition "A is B" is true if at some point in the past, present or future, the thing picked out by A is a B.³¹⁵ Thus, for an A that is contingently described as B, viz. A is sometimes B and sometimes not, it would be perfectly reasonable, according to this reading of Avicenna's *simpliciter* conditionals, to say "if A is B, then A is not B", by which we mean that "A is B" is true, but at time t_1 , and "A is not B" is true, but at time t_2 . Thus, the modal language in Avicenna's characterization of a true *simpliciter* conditional plays an important role in discovering the truth-conditions for *simpliciter*

³¹⁵ T. Street, "An Outline of Avicenna's Syllogistic", 134f.

conditionals. In many (if not most) cases *simpliciter* conditionals will be semantically equivalent to (material) conjunctions,³¹⁶ but not in all.³¹⁷

The difference between *simpliciter* and restricted conditional lies in the kind of concomitance that holds between their antecedents and consequents. In Text 27, Avicenna said that true *simpliciter* conditionals signified a relation of concomitance in truth between antecedent and consequent. Our analysis above cannot offer a decisive interpretation, but the evidence suggests that a *simpliciter* conditional has the same truth conditions as a material conjunction: it is only true when both the antecedent and consequent are true; in all other cases it is false. In contrast, the concomitance of antecedent and consequent in restricted conditionals is not related to whether or not the antecedent or consequent are true. Said differently, in order to determine the truth-value of a restricted conditional we cannot look to

³¹⁶ A forceful objection to my understanding Avicenna's *simpliciter* conditionals as being basically akin to conjunctions can be raised, since such an understanding makes nonsense of modus ponens and modus tollens with a *simpliciter* conditional as a major premise. This is, of course, true. Avicenna is aware of such an objection, and he explicitly says (see chapter 3.2.1 below) that only *restricted* conditionals can be used in repetitive syllogisms or *qiyāsāt istiṭnā'iyya*, which is the class of syllogism into which modus ponens and modus tollens fall. A second and a third objection may be outlined as follows. The claim that a *simpliciter* conditional proposition is basically a conjunction flies in the face of our intuition that whatever else a conditional might be, its truth-conditions (if it has any at all) should be weaker than those of a conjunction. What is more, it is disingenuous to say that Avicenna is formulating a theory of "if...then..." propositions, one category of which he takes to be semantically nothing more than a conjunction. These latter two objections lose part of their force when we remember that, unlike in Stoic formalism, which took the form as primary, what was primary for Peripatetics, later Aristotelian commentators, and Galen, was the meaning that the conditional sentence is signifying. For Avicenna and his Aristotelian forebears (see M. Frede, "Stoic vs. Peripatetic Syllogistic", *op. cit.*; S. Bobzien, "Peripatetic Hypothetical Syllogistic in Galen—Propositional Logic off the Rails?", *op. cit.*) the form that the speaker chose to express the meaning was of secondary importance. In the case of connective conditionals, the underlying meaning they are intended to convey is concomitance in truth. Avicenna feels that the only suitable form for expressing this meaning is the "if...then..." sentence. Yet, the idea of concomitance in truth has two senses, both of which can receive logical analysis. Avicenna analyses unqualified concomitance in truth, or concomitance *simpliciter*, as akin to conjunction. The second, restricted concomitance, is the sense of concomitance in truth that is relevant to use in his theory of repetitive and conjunctive syllogisms. This latter type is Avicenna's contribution to the logical theory of entailment. Avicenna's sentiments are contrary to our notion of logic as being a science about form of arguments. As we saw earlier, Avicenna believed that logic was about second intelligibles and that an analysis of their properties is what logic is about. Avicenna's doctrine of logical form is not ours, but it is a rigorous, intriguing, and defensible doctrine nevertheless.

³¹⁷ The truth-functionality of conjunction strikes most as intuitive, but it was not necessarily so for ancient logicians. For a statement of some of the challenges posed by ancient theories of conjunction see C. Normore, "Medieval Connectives and Hellenistic Connections", in *Atoms, Pneuma, and Tranquility*, ed. M. Osler (Cambridge: Cambridge University Press, 1991), 25-39.

whether the antecedent and consequent are true in correspondence with the current states-of-affairs. Rather, the truth of a restricted conditional returns to whether the antecedent and consequent are concomitants in implication. Avicenna's informal way of marking out a conditional as restricted is by attaching the phrase "implies" to the consequent of a restricted conditional. This is also his informal way of testing whether a conditional in question is true *simpliciter* or restricted. For example, his informal method of showing that the true *simpliciter* "if the earth is round, then man is rational" is false restricted is to alter the consequent so the conditional reads "if the earth is round, then this *implies that* man is rational". If it strikes as unintuitive to say that the earth's roundness implies man's rationality, then we have informally shown that the above conditional is false as a restricted conditional. Beneath this informal meaning of implication, lies an elaborate semantics of suppositional states (*aḥwāl*) and conditions (*šurūt*).³¹⁸ Formally speaking, when Avicenna (informally) says that "if A is B, then this implies J is D", he means the following technical definition in mind. A *restricted* connective conditional expression "if A is B, then J is D" is true when there is not condition "H is Z", real or hypothetical, which when attached to the antecedent, is consistent with "A is B" being true, but is inconsistent with "J is D" being true.³¹⁹ Thus a true *restricted* conditional such as "if A is B, then J is D" is true because no matter what condition "H is Z" is added to the antecedent to generate a new compound antecedent "A is B and H is Z", the truth of "J is D" remains with the new antecedent "A is B and H is Z". On the other hand, the *restricted* "if A is B, then J is D" will be false in case there is some "H is Z", which can be added to the antecedent

³¹⁸ These basic ideas are developed into an entire theory of quantified conditionals in ŠQ V 4. See §5 for technical details and its Aristotelian inspiration.

³¹⁹ E.g. Avicenna, ŠQ V, 265.1-10. This is how Avicenna formulates the truth-conditions for the unquantified conditional. This suggests that for Avicenna the most basic of primary understanding of a *restricted* conditional "if A is B, then J is D" is what later in ŠQ V he calls a universal affirmative conditional (what I will call an "A-conditional"). This suggests that for Avicenna the default reading of *restricted* conditionals is with the universal quantifier suppressed. Thus, Avicenna implicitly takes "if A is B, then J is D" read as a restricted conditional is implicitly taken as "always: if A is B, then J is D".

“A is B”, such that the compound antecedent “A is B and H is Z” is inconsistent with the truth of “J is D”. In formulating the truth-conditions of *restricted* conditionals in this way, Avicenna makes room for a theory of conditionals that can deal meaningfully with impossible antecedent. Avicenna considers two such conditionals in ŠQ V: (S2) “if man is not an animal, then he does not possess sense perception”; and (S3) “if man caws, then crows are rational (*idā kāna l-insān nā‘iqan*³²⁰ *fa-l-ğurābu nāṭiqan*).” According to Avicenna, S2 is true when read as a restricted conditional but false when read *simpliciter*. On the other hand, S3 is false on both readings because they have impossible antecedents. If *simpliciter* conditionals are semantically equivalent to material conjunctions, then neither S2 nor S3 can be true. In order to account for the truth of S2, Avicenna must make a distinction between the antecedent’s (and consequent’s) being true or false *in itself* (*fī nafsi l-amr*) (i.e. not on hypothesis), and being true or false *on a hypothesis* (*bi-l-waḍ‘*). In S2 and S3, everyone already knows that the antecedent and consequent are false in themselves. Avicenna avers, however, that the truth-in-itself status of the antecedent and consequent in a *restricted* conditional are not what concerns us. What is at stake, rather, is whether or not the concomitance between the antecedent and the truth of the consequent is consistent with the hypothetically true antecedent *under all hypothetical conditions*. Consider S2. For S2 to be true, we must first suppose that man is not an animal. Then we must see if there is any condition that can be consistently attached to the antecedent but is inconsistent with the consequent. In the case of S2, we must see if there is some condition “H is Z” that can be attached to the antecedent “man is not an animal and H is Z” such that H’s being Z is consistent with man’s not being an animal but *is* consistent with man’s having sense perception. If no such “H is Z” can be found, then S2 is true and our

³²⁰ Reading “*nā‘iqan*” with other manuscripts; Avicenna, ŠQV, 239.1.

informal method of attaching “implies” to the consequent of S2 “if man is not an animal, then this implies that he does not have sense perception” is vindicated. In this case at least, our intuition about the truth of S2 accords with Avicenna’s technical definition. Now, consider S3. First let us test S3 with our informal method. Does the following conditional seem true or false: “if man caws, then this implies that crows are rational”? Intuitively this S3 seems false, even though S2 and S3 share in the feature that their antecedents and consequents are unquestionably false. If S3 is false as a restricted conditional, then we should be able to find some condition *C* that is consistent with man’s crowing but is inconsistent with crows’ being rational. In fact, the condition “crows are not rational” is perfect consistent with the antecedent but inconsistent with the consequent. Therefore, we have found such a *C*, and S3 must be false as a *restricted conditional*. Again, the informal test of restricted implication parallels the underlying technical machinery Avicenna sets out. And thus, S3 “need not be true on either reading, neither because in itself <the consequent [lit. “this (*hāḍā*)”]> is true with <the antecedent, [lit. “that [*dālika*]”]>—in fact, both antecedent and consequent are false—nor because one of them implies the other.”³²¹

Nevertheless, in Avicenna’s view, a *restricted* connective conditional constructed of a true antecedent and consequent might still be false. Consider (S4): “if Theon laughs, then Dion leaves.” Say that S4 is true on a *simpliciter* reading, i.e. that Theon laughs and that Dion leaves are true together. Is S4 a true conditional when read as a restricted conditional? For S4 to be true under this reading, Dion must leave in every case that we hypothesize Theon laughs. Yet S4 clearly fails to fulfill this requirement, since on the hypothesis that Theon belongs to the extension of the term “laughs,” there is no underlying principle that requires that Dion belong

³²¹ Ibid., 239.1-3.

to the extension of the term “leaves.” We are free to hypothesize a scenario in which Theon laughs and Dion fails to leave. Thus, S4 read as a restricted is false, despite the assumption that it is true *simpliciter*. This is what Avicenna means when he says that restricted conditionals are more specific (*aḥaṣṣ*) than *simpliciter* conditionals. A true *simpliciter* connective conditional expression is one formed from any pair of currently true sentences. On the other hand, a restricted conditional does not necessarily require the truth of the antecedent and consequent at all, since Avicenna obviously envisions this reading of connective conditionals as handling conditionals with impossible antecedents like those mentioned above. What is required for the truth of restricted conditionals is for the truth of the consequent to be true with the antecedent on the supposition that the antecedent is true. However, our exegesis has revealed another aspect that Avicenna does not quite explicitly allude to in ŠQ V 1. It is not enough that the supposition of the antecedent’s truth be consistent with the truth of the consequent. Or said differently, it is not enough that it be possible, or sometimes the case, that the consequent be true supposing the truth of the antecedent. It does not make much sense for us to claim that “man is not an animal” *implies* “man does not have sensory perception” if the supposition of the antecedent’s truth leads to the consequent in only some instances or only in some cases. If this were allowed, then we could claim on the one hand that “man is not an animal” implies “man does not have sensory perception” while still entertaining the existence of another scenario in which “man is not an animal” is true and “man does not have sensory perception” is false. Avicenna certainly does not want to develop a notion of implication in which we can simultaneously claim that the antecedent *implies* the consequent and at the same time be forced to concede that it is possible that the antecedent be true and the consequent false. Under no reading of “if...then...” is this a desirable outcome. Thus, in

order to block this possibility, Avicenna says that not only must the consequent be true with a hypothetically true antecedent but that this concomitance in truth must hold under all conditions under which the antecedent is hypothesized. Thus, S2 is true because not only will the consequent be true on the hypothesis of the antecedent's truth but the consequent will hold under all conditions, actual or hypothetical, under which the antecedent is hypothesized. For this same reason, S4 is false. "Theon laughs" and "Dion leaves" are both currently true, but if the conditional is taken as a restricted conditional then "Dion leaves" will have to be true not just for what is presently the case, but for all cases and under all conditions, real or hypothetical, in which Theon laughs. But, as I mentioned above, S4 does not fulfill this requirement, and so it is false.

If the above interpretation is correct, then there is an inconsistency in Avicenna's discussion that deserves to be discussed briefly. S4 was found to be true *simpliciter* but false as *restricted conditional*. The example was straightforward because Theon's laughing and Dion's leaving are purely contingent states-of-affairs. Consider, however, the following example that Avicenna considers at length in ŠQ V 4: (S5) "if man is rational, then donkeys bray". Avicenna says that S5 is true *simpliciter* conditional and a true *restricted* conditional (in fact, it is a true as a universal affirmative "always: if man is rational, then donkeys bray").³²² Yet our intuition and the technical machinery outlined above both indicate that S5 is false. First, our intuition tells us that S5 is false as a restricted conditional, for when we attach "implies" to the consequent we get the intuitively false "if man is rational, then this implies that donkeys bray". Second, we must ask ourselves: is there a condition, real or hypothetical, that can be attached to the antecedent of S5 "man is rational" that is consistent the antecedent but

³²² Avicenna, ŠQ V, 265.10-5.

inconsistent with the consequent “donkeys bray”? There is; namely, “donkeys do not bray”. Remember, it does not matter if the condition attached to the antecedent is true or false (Avicenna says this explicitly as we will see in Chapter 5). It need only be entertainable by the mind. What matters is its consistency with the antecedent. Thus, the compound antecedent is consistent “man is rational and donkeys do not bray”, but it is inconsistent with the consequent “donkeys bray”. Avicenna’s argument is that the truth of the consequent is sufficient for the truth of the conditional as a whole. This justification, however, seems to be beside the point, and, in any case, contrary to the principles that Avicenna has set down so carefully. I am unable to account for why Avicenna holds this view, but it may be that he was uncomfortable using a false condition to show that S5 is false as *restricted conditional*. We should not imagine that this problem is not trivial. If all true *simpliciter* conditions in which the antecedent and consequent are “eternal truths” are also true as a restricted conditional, then maintaining any sharp distinction between concomitance in truth on the one hand and implicative concomitance on the other becomes very difficult to maintain. (This problem becomes especially clear in relation to I-conditionals “once: if A is B, then J is D”, which are logically useless because they are always true, no matter what the I-conditional’s antecedent and consequent might be, and thus can be validly deduced from any set of premises.³²³)

Despite the problems with Avicenna’s theory, let us turn to the question of how Avicenna thinks his scheme is different from the division of following into complete and incomplete that he has just summarized in the early chapter of ŠQ V. I will approach this problem by examining how Avicenna’s *simpliciter*/restricted division differs from the account of conditionals that we find in Alfarabi. Avicenna faults this account of conditional

³²³ See K. El-Rouayheb, “Impossible Antecedents and their Consequences: Some Thirteenth-Century Discussions”, *History and Philosophy of Logic* 30 (2009): 209-25, especially 215f.

propositions on two counts. First, starting from his belief that the subject matter of logic is secondary intelligibles, Avicenna accuses the opposing account of conditionals of turning the investigation into the nature of the *logical* connection between antecedent and consequent into an investigation into the nature of the conceptual relations (*nisba*) between cause and effect, relatives, or the like. Avicenna is unhappy with this aspect of the account he summarizes in ŠQ V because it investigates the connection between antecedent and consequent insofar as the link (*‘alāqa*) between them has an extramental correlate. Sections 1.1 and 1.2 of this chapter have shown that a principle of Avicenna’s logic is that the concepts of logic should not circumscribe extramental objects. On the other hand, Avicenna seems to believe that his alternative division of conditionals into *simpliciter* and restricted is consistent with the subject matter of logic as secondary intelligibles. In this case, I think it is possible to present Avicenna’s and Alfarabi’s viewpoints as genuinely distinct. In section 3.1.1 I argued that Alfarabi holds that the subject matter of logic is primary intelligibles. What this means for Alfarabi is that intelligibles such as *human* and *animal* are the objects that logic is about, whereas to discover the properties such as *being-a-predicate* and *being-a-subject*, which accrue to primary intelligibles due to their mental status, is the aim of logic. Alfarabi is perfectly justified and consistent in holding that following or *ittibā’* is just one of these properties possessed by primary intelligibles such as *man* and *animal*. The Farabian logician examines the objects of logic, the primary intelligibles, viz. those terms falling under any one of the ten categories, and notices that certain properties hold of them. In the case of following, these properties are binary relations between two primary intelligibles, e.g. *one* and *two*, or *sun* and *day*. And, as it happens, the ideal linguistic medium for expressing binary relations of following between primary intelligibles is a conditional proposition. Take two pairs of primary

intelligibles, e.g. *master* and *slave*; *human* and *animal*. Upon examination, you discover that among their properties is the fact they are *relata*. Therefore, you conclude that a relation of perfect connection exists between *master* and *slave*; in the language of APCA there is an “equipollent relation of existence [*yatakāfa’āni fī luzūmi l-wuğūd*]” between them. The logical properties of *master* and *slave* can be expressed in the form of the following conditional propositions: (1) “if there is a master, there is a slave”, (2) “if there is a slave, there is a master”, (3) “if there is no master, there is no slave”, and (4) “if there is no slave, there is no master”. Upon examination of *animal* and *human*, you discover that among their conceptual properties is the fact that animal is naturally prior to human at the level of substance. As a consequence of this relation of priority at the level of substance, their concepts have the property of having a “non-equipollent relation of existence”. The logical properties of animal and human can be expressed in the form of the following conditional propositions: (5) “if there is a human, there is an animal”, and (6) “if there is not an animal, there is not a human”. Propositions (1-6) are theses in a Farabian art of logic, in the sense that they give expression to properties possessed by primary intelligibles due to their presence in the mind. In this case, the property, or secondary intelligible, that accrues to *master* and *slave* as well as *animal* and *human* is *following-from*. The secondary intelligible *following-from* has two subspecies: complete and incomplete, which are inferential properties of different strengths possessed by different primary intelligible pairs.

As illustrated by the above example of *animal* and *human*, the relation of following that holds between a pair of primary intelligibles on the Farabian theory of logic arises from the observation that certain relations (e.g. priority and posteriority) hold between animal and human at level of substance. In other words, we are able to state logical theses (5-6) about the

primary intelligibles *animal* and *human* because of the natural priority of animal to human at the level of substance. But for Alfarabi, this approach is perfectly consistent with his views on the subject matter of logic, in which the predications we make about primary intelligibles are founded on the properties possessed by the being circumscribed by these concepts outside the mind. To this extent, Avicenna's criticism of Alfarabi's unbefitting logical inquiry is unjust, in the sense that Avicenna formulates his criticism without taking into consideration Alfarabi's fundamental views on the subject matter of logic. As we know from section 3.1.2, Avicenna held that the subject matter of logic is secondary intelligibles such as *being-an-antecedent*, and *being-a-consequent*. The aim of logic is to examine these kinds of secondary intelligible in light of the properties that accrue to them both on account of their presence in the mind, and on account of their role in the process of deductively moving from known premises to unknown conclusions. In Avicenna's view it seems, the binary relation property that accrues to the secondary intelligibles *being-an-antecedent* and *being-a-consequent* due to its participation in the mental process of deductive knowledge acquisition is the (tertiary intelligible) *being-a-concomitant-in-truth-with* ("P *ma'a* Q", or "P *murāfiqun li-Q*", or "P *muttafiqun ma'a* Q"). What is more, this binary relation has two subspecies: unqualified (or *simpliciter*) and restricted. The binary relation residing between *being-an-antecedent* and *being-a-consequent* has the property *concomitance-in-truth* in an unqualified way whenever the truth of the consequent does not rule out the truth of the antecedent. They have the property *concomitance-in-truth* in a restricted sense if the consequent is true under any conditions, real or hypothesized, in which the antecedent is true. Interestingly, these latter statements about when the secondary intelligibles *being-an-antecedent* and *being-a-consequent* have the property *being-concomitant-in-truth-with* are the truth-conditions for *simpliciter* and restricted conditional propositions. They

are also theses of Avicennian logical science. These truth conditions are consistent with Avicenna's view of the subject matter of logic, in the sense that the truth conditions make no reference to primary intelligibles such as *animal* and *human*. They are, rather, stated in terms of real or hypothetical conditions or states (*aḥwāl*) under which the antecedent and consequent are evaluated. Truth conditions in the Farabian scheme cannot be given with the same generality; though, of course, the "generality" Alfarabi's logic aims at is not the same as Avicenna's. The theses (1-6) of the Farabian logic express complete and incomplete relations of entailment that hold between primary intelligibles insofar as the intelligibles are present in the mind. They are not—indeed, they cannot serve as—a general statement of truth conditions for complete and incomplete conditionals. Rules of Avicennian generality do not exist in Alfarabi's way of thinking about logic.

The second shortcoming, in Avicenna's view, is that thinking about the connection in terms of the relation between antecedent and consequent insofar as they are related as cause to effect, inseparable accidents, pair of effects arising from a single cause, whole to part, universal to particular, relatives and so on, makes the utterance of connective conditional expressions pointless (*bi-lā fā'ida*) in a syllogistic argument. According to Avicenna it is useless to introduce such conditionals into (repetitive) syllogisms such as modus ponens and modus tollens in order to draw as a conclusion the antecedent, the consequent, or their contradictory opposites. As we saw above, sentences (1-6) are logical theses expressing properties holding between pairs of primary intelligibles. We state these theses in the form of (true) conditionals because we know that the primary intelligibles are related with complete connection in (1-4) and incomplete connection in (5-6). We know the nature of this connection between *master* and *slave* because of the knowledge we have about those properties of substance circumscribed

by these primary intelligibles. Similarly, we know the nature of the connection between *human* and *animal* because of the knowledge we have about the natural priority of animal to human at the level of substance. In other words, sentences (1-6) arise from our knowledge of facts in the world. According to Avicenna's understanding of this scheme, constructing a syllogism in *modus ponens*, e.g. "if there is a master, there is a slave; but there is a master. Therefore, there is a slave" is pointless since we already knew going into the deduction that the primary intelligibles *slave* and *master* circumscribe properties inhering in substance outside the soul. If this is, in fact, Avicenna's criticism, it is inaccurate and unjust. The Farabian complete or incomplete conditional is an expression that states an equipollent or non-equipollent relation of existence holds between primary intelligibles. It states, in other words, that a relation exists between two primary intelligibles. It does not allow us to say anything about whether the primary intelligibles themselves circumscribe anything outside the mind. Alfarabi's theory of conditional propositions is perfectly capable of allowing for scenarios in which it is doubtful whether the primary intelligible in the antecedent position circumscribes an object outside the soul, and yet still be in relation of incomplete or complete following. Avicenna's criticism is unjust too, for even if we did already know for a fact that the conclusion of the (repetitive) syllogism "there is a slave" is true prior to the deduction, this does not take away from the pragmatic utility of using such conditionals in a debate such as the ones I have described in chapter 2. At no point in his lengthy development of the logic of conditionals in ŠQ V-IX does Avicenna evince any interest in the context sensitivity of conditional truth-conditions or syllogistic validity. To his credit, Alfarabi on the other hand displays great interest in incorporating sensitivity to context in his theory of conditional reasoning.

According to Avicenna, the reason why we make conditional expressions is that the initial act of hypothesizing the antecedent leads us (*intaqala*, v.n. *intiqāl*) to consider consequents that are consistent with this hypothetical antecedent, but which we otherwise admit are objectively false (e.g. “if three is even, then it is divisible by two” is typical example of a true restricted connective conditional proposition). What is more, when we introduce conditional expressions into syllogistic arguments, we intend thereby to draw conclusions that follow necessarily from premises that have been supposit as true. For Avicenna, the logical analysis of such arguments must harmonize with how these arguments are being used. The point of logical analysis is not to say that, for example, “if two is, one is; but two is. Therefore one is” is valid, but the argument “if two is, one is; but one is. Therefore two is” is invalid because the primary intelligibles are characterized by a non-equipollent relation of existence, which authorizes the conclusion in the first argument, but does not in the second, since the aim of such an analysis is stating properties of primary intelligibles. Rather, the task of logical analysis is to take the secondary intelligible *being-a-syllogism*, and in particular the subspecies of it *being-a-repetitive-syllogism* and investigate the properties (tertiary intelligibles) that accrue to this secondary intelligible when such a concept is used in the mental activity of deducing new knowledge from known premises. For example, in Avicenna’s mind two properties will accrue to *being-a-repetitive-syllogism* insofar as it is a concept utilized for this end, viz. *being-concludent* and *being-inconcludent*. Logical investigation reveals that the tertiary intelligible *being-concludent* is a property of the secondary intelligible *repetitive-syllogism* under certain conditions, and *being-inconcludent* will hold of the *repetitive syllogism* whenever some or all of those conditions fail to hold. Avicenna’s notion of concomitance in truth is important for his analysis of syllogisms, because it preserves the level of generality that is not possible in a

Farabian theory of the subject matter of logic. According to Avicenna,³²⁴ we can predicate the property “concludent” of the concept *repetitive-syllogism* only on condition that (1a) the minor premise asserts the antecedent of the major premise, or (1b) the contradictory opposite of consequent; and (2a) that the antecedent and the consequent of the conditional major are characterized by restricted concomitance in truth, viz. the consequent is true under all conditions under which the antecedent is hypothesized as true; or, said differently but a direct consequence of (2a): they cannot be characterized by unqualified (or *simpliciter*) concomitance in truth, viz. the consequent’s truth merely compatible with the truth of the antecedent.³²⁵ Requiring that the conditional major be characterized by restricted concomitance once again allows Avicenna’s analysis to have the generality required by his doctrine of the subject matter of logic. This doctrine bars Avicenna from replacing (2a) and (2b) with the simple requirement that the conditional major must be characterized by complete connection. Why? The answer to this question is interesting, and points up how distinct Avicenna’s and Alfarabi’s views of logic are from each other. If my understanding of Alfarabi’s view on the subject matter of logic is correct, incomplete (and complete) connection is a secondary intelligible, a property possessed by a pair of primary intelligibles such as *being-an-animal* and *being-a-human*. If my understanding of Avicenna’s view on the subject matter is correct, *concomitance-in-truth* is a tertiary intelligible, a property possessed by a pair of secondary intelligibles *being-an-antecedent* and *being-a-consequent*. To be consistent with his views on the subject matter of logic, Avicenna must state the concludency rules for repetitive syllogisms in terms of properties possessed by secondary intelligibles, whereas to say “the conditional must have

³²⁴ See chapter 4, section 3, Text 7 for more details.

³²⁵ The view Avicenna is criticizing ŠQ V and VIII allows “repetitive syllogisms” that affirm the consequent and deny the antecedent in cases that the connection between antecedent and consequent is complete. The unnamed writer whom Avicenna is criticizing calls these separate moods (*ḍurūb*). Avicenna convincingly argues that they are bogus moods; see section 3.3.1 below.

incomplete connection” is to state a rule about properties (complete connection) possessed by primary intelligibles such as *being-human* and *being-animal*, not secondary intelligibles such as *being-an-antecedent*, *being-a-consequent*, and *being-a-premise*, or *being-a-syllogism*. When we talk about connection for Avicenna and Alfarabi, we are ultimately talking about properties of primary intelligibles. To state concludency conditions for repetitive syllogisms in terms of properties possessed by primary intelligibles is to confuse the levels of analysis.

§3.3.0 AVICENNA ON REPETITIVE SYLLOGISMS IN ŠQ VIII: INTRODUCTION

In the previous section, I discussed two shortcomings that Avicenna attributes to the division of connective conditionals into those expressing complete and incomplete connection. Given Avicenna’s opposition to this classification, it is not surprising that these concepts appear nowhere in the rest of Avicenna’s discussion of connective and disjunctive conditional expressions in ŠQV, conjunctive syllogisms (*qiyās iqtirānī*) in ŠQVI, or formally valid but non-syllogistic inferences in ŠQVII. They do, however, reemerge to play an important role in Avicenna’s treatment of repetitive syllogisms (*qiyās istiṭnā’ī*) in ŠQ VIII. Once again, the complete/incomplete division of connection is the object of Avicenna’s intense and protracted criticism. But Avicenna now pursues this criticism of Farabian-type theories of the conditional further by considering how it ramifies into the theory of repetitive syllogisms. In particular, Avicenna extends this criticism by connecting it to the question of syllogistic productivity and formalism. Avicenna’s implicit claim in ŠQ V that the complete/incomplete dichotomy fails to treat conditional propositions as formal (*ṣūra*) in the logical sense is explicitly stated in ŠQ VIII

in the discussion of syllogistic form (*ṣūra*) versus syllogistic matter (*mādda*).³²⁶ However, Avicenna also introduces a new element into his criticism of the complete/incomplete dichotomy. He claims that taking connective conditionals (and disjunctives) as either complete or incomplete and assigning to each a separate syntactical mood (*ḍarb*) multiplies the number of moods in an unnecessary way. The number of valid deduction schemes for repetitive syllogisms that are genuinely distinct from one another is small; According to Avicenna there are, in fact, only two: *modus ponens* and *modus tollens*. However, Avicenna alleges that the classification of connection as complete (*tāmm*) and the doctrine of conditional conversion (*ʿaks*) this classification entails compels us to make distinctions between moods that are, in fact, formally identical.

§3.3.1 AVICENNA’S *SIMPLICITER* AND *RESTRICTED* CONDITIONALS IN REPETITIVE SYLLOGISMS IN ŠQ VIII

Avicenna’s presentation of repetitive syllogisms with connective conditional premises is puzzling for several reasons. In it, he outlines eight valid moods using the language of complete and incomplete connection. However, interspersed with it is criticism of the foundations of the very account of repetitive syllogisms he is outlining. Another reason is that Avicenna does not systematically develop his critique. Instead, Avicenna’s remarks unfold over the course of the chapter without alluding explicitly to his primary aims or the principles that inform his particular objections. As far as can be told, Avicenna holds syllogisms to three necessary criteria: one is that (Pr1) all syllogisms must be productive in the sense that they must generate new knowledge; two, (Pr2) syllogisms must be formal in the sense that an

³²⁶ For a discussion of matter and form with respect to modality see A. Ahmed, “The Jiha/Tropos-Mādda/Hūlē Distinction in Arabic Logic and its Significance for Avicenna’s Modals”, in *The Unity of Science in the Arabic Tradition*, ed. S. Rahman, T. Street, H. Tahiri (Amsterdam: Springer, 2008), 229-53.

analysis of their validity must not rest on primary intelligible concepts; the third relates to the deduction schema (lit. mood [*ḍarb*, pl. *ḍurūb*]). This third criterion requires that (Pr3) a purportedly imperfect hypothetical syllogistic mood with a connective conditional premise must be formally distinguishable from the perfect mood to which it is reducible. Pr1 and Pr2 seem to motivate Avicenna's development of the *simpliciter/restricted* division of conditional propositions treated in the last section. What is different in ŠQ VIII is, of course, that the questions of productivity (Pr1), formality (Pr2), and formal distinctions between perfect and imperfect moods (Pr3) are extended to strings of propositions, viz. deductions instead of propositions.

The main focus of ŠQ VIII is determining the nature of the relationship between connective conditionals and repetitive syllogisms. There are two types of connective conditional expressions in Avicenna's logic: *simpliciter* and *restricted*. The former type, according to Avicenna, is unsuited to serve as a premise in a repetitive syllogism because the use of a *simpliciter* conditional as a premise violates Pr1. In other words, the use of a *simpliciter* conditional in a repetitive syllogism will not induce in the reasoner knowledge he did not already know. In his discussion of what he calls the first mood (*al-ḍarbu l-awwal*) of repetitive syllogisms with a connective conditional premise, Avicenna speaks about the *simpliciter* connection between antecedent and consequent as one of coincidence (*ittifāq*). As I discussed in the last section, this name alludes to the fact that in a connective conditional with a *simpliciter* connection, there need not be any link between the antecedent and consequent. Remember, the *simpliciter* connective conditional proposition is true when the antecedent and consequent are true. A consequence of these truth conditions is if the reasoner knows that a *simpliciter* conditional is true, then he knows that the antecedent and consequent are both

true prior to and independent of his utterance of the conditional expression. In this fact lies the trouble. In the following quote, Avicenna considers a form of modus ponens: “if P, then Q. But P. Therefore Q.” In this schema, the second premise, which is tantamount to the assertion of Q, is embodied in the expression “but Q.” This assertion Avicenna calls the “exception (*istiṭnā*).” What exactly the exception of Q yields (*afāda*, *yufīdu*) depends crucially on how we take the connection between the antecedent P and consequent Q.

[Text 28] Let us now look into the connective conditional expression [*al-muttaṣil*] that produces this type of exception [*istiṭnā*] and see what the nature of the exception [*ḥāl al-istiṭnā*] is when the connection [in the connective conditional expression] is coincidental [*‘alā sabīli l-muwāfaqa*] and when the connection is by implication [*‘alā sabīli l-luzūm*]. We say, in fact, that the exception [of the antecedent] yields nothing when the connection is coincidental. This is so because the consequent is not something that is implied by the setting down of the antecedent [*li-anna t-tāliya lā yakūnu šay’an yalzimu min waḍ’i l-muqaddam*]. Rather, the consequent is something that is made contiguous with the antecedent [by the mind] [*ḡu’ila muwāṣilan li-l-muqaddam*] because of [the consequent’s] being known to be true in itself along with [*ma’a*] the antecedent. However, the thing whose existence is known [viz. Q, the consequent of the conditional, which is identical to the conclusion of the syllogism] has no need for a syllogism [from which it is deduced]. Thus, the consequent must be something that is unknown in itself [*maḡhūlan bi-naḡsihi*], but its following from and contiguity with the antecedent must be known. In this case, if the existence of the antecedent is known, then the existence of <the consequent> will be known [as a consequence]. Say, for example, we say: “if A is B, then J is D.” If we then except by saying “but A is B,” but we already knew that J is D, then the syllogism does not produce new knowledge [*lā yufīdu ‘ilman ḡadīdan*] that J is D. On the other hand, if J’s being D were unknown, but its following from [*mutāba’atuhu*] A’s being B were known, then if we come to know that A is B is true [*iḍā ṣaḡḡa lanā*], we will come to know as a result that J is D.³²⁷

In order for a *simpliciter* conditional to be true, the truth of the antecedent and consequent must be known prior to the construction of the conditional expression. Yet this requirement makes it such that the conclusion we seek from constructing a deduction in modus ponens is already known. Avicenna’s argument is similar to the nearly circular deduction schema “P and

³²⁷ Avicenna, ŠQ VIII, 390.7-16.

Q; but P; therefore Q.” Whether or not we agree that the assertion of a conjunction is logically equivalent to the individual assertion of its conjuncts, we may at least agree with Avicenna that our “conclusion” Q is certainly a piece of knowledge we would already possess before we actually carry out the deduction. Modus ponens with a *simpliciter* conditional is a thought process that adds nothing to our store of knowledge and thus violates Pr1. On the basis of this fact, Avicenna concludes that “the types of connection used in repetitive syllogisms must be connections in which there is a connection of implication” between the antecedent and consequent.³²⁸ By this, Avicenna excludes the case in which the conclusion of the syllogism, which is identical to the consequent of the connective conditional, is known before the syllogism is carried out. Rather, the extent of what should be known prior to the deduction is that the consequent follows from and is contiguous with the antecedent such that *if* the antecedent were supposed true, the truth of the consequent would follow. In this latter scenario, when we conclude the deduction “Therefore Q,” Q is added to our store of knowledge, whereas before, it had been genuinely unknown (*mağhūl*).

With this argument, Avicenna dispenses with the entire class of conditional propositions in which the truth of the antecedent and consequent are not directly relevant to each other. By limiting the types of conditional proposition to those in which there is a connection of implication, Avicenna wants to put forward *restricted* conditionals as the only feasible form of conditional premise in a repetitive syllogism. Avicenna would like to claim that only *restricted* conditionals avoid making the hypothetical syllogistic nothing more than question-begging. Avicenna’s opponents would object that *restricted* conditionals are not the only types of proposition in which there is an implicative relation between the antecedent and

³²⁸ Ibid., 390.16-17.

consequent. Indeed, proponents of the complete/incomplete dichotomy of connection could claim that connective conditionals signifying complete and incomplete connection between the antecedent and consequent certainly qualify as propositions in which there is an implicative connection. In fact, Avicenna anticipates this claim and stridently rejects it. Avicenna says that repetitive “syllogisms” in which the connective conditional premise is taken to signify complete or incomplete connection, fail to satisfy conditions Pr2 and Pr3, and therefore they should not, in fact, be counted as syllogisms. Avicenna’s main discussion comes in his presentation of the “third mood” of the repetitive syllogism in which one of the premises is a connective conditional. This mood may be schematized in the following way: (M3) “if M, then N. But N. Therefore M.” To this, the traditional account adds the important caveat that the type of connection between the antecedent and consequent is “complete implication,” by which is meant that the connection in the connective conditional is such that conversion of the conditional premise preserves truth.³²⁹ In his argument, Avicenna begins by demonstrating how this stipulation makes M3 fail Pr2, then Pr3. First, consider Avicenna’s presentation of the view he is attacking:

[Text 29] The third of the well-known moods is that the consequent of a [connective conditional proposition] signifying a complete implicative connection [*tāmm al-luzūm*] is repeated, yielding the antecedent. They say, “but its productivity is not self-evident [*laysa intāğuhu bayyinan li-nafsihi*]; in fact, it is an imperfect syllogism that can be perfected along the following lines. To say that the implication is complete is to say that the implication is convertible. In that case, we may make the consequent an antecedent and the antecedent a consequent. Then, we except what is now the antecedent, which was previously the consequent. This produces what is now the consequent, which was previously the antecedent.”³³⁰

In the Aristotelian tradition, to perfect (*akmala, yukmilu*) a mood that is not self-evident, i.e. imperfect (*ğayr kāmīl*), unlike the first-figure in the categorical syllogisms, is typically

³²⁹ Ibid., 391.8.

³³⁰ Ibid., 391.8-12.

accomplished by converting one of the convertible a-, i-, or e-premises so that the mood is formally identical to one of the moods of the first-figure (or to a non-first-figure mood that has already been shown to be reducible to a first-figure mood). For example, the reduction of the second-figure mood Cesare (No A is B and every C is B; therefore no C is A) to the first-figure Celarent (Every N is M and no M is P; therefore no N is P) requires the conversion of the universal negative major premise “No A is B” to “No B is A.” Assuming that the conversion of the universal negative preserves truth, viz. that if “No A is B” is true, then of necessity “No B is A” is true, this “reduction” of Cesare to Celarent is a proof that Cesare is a *concludent* syllogism, in the sense that it shows that if the premises in an argument constructed according the Cesare are true, then it is impossible for the conclusion to be false. In the quote from ŠQ VIII above, the same idea of reduction to a more primitive mood lies behind the traditional view’s claim that this third mood is not perfect, and, as a consequence, its being counted a syllogism must be demonstrated. On the assumption that conversion in this case preserves truth, Cesare is revealed as a *concludent* syllogism by showing that it can be made formally identical to Celarent by converting the universal negative major premise. Analogously, M3 can be revealed as a *hypothetical* syllogism by showing that it can be made to be formally identical to the more primitive, self-evident first mood (M1) “if P, then Q; but P. Therefore Q,” with the proviso that the connection between the antecedent and consequent is complete. The demonstration of this formal identity relies on the assumption that the conversion of the connective conditional premise is valid, meaning that if “if P, then Q” is true, then of necessity “if Q, then P” is true. In fact, on the traditional account, the convertibility of the connective conditional is guaranteed by the assumption that the connection signified by the connective conditional is complete implication (*tāmm al-luzūm*), which means precisely that conversion of

the conditional premise preserves truth. Thus, M3 can be shown to be formally identical to M1 merely by the conversion of the conditional premise “if M, then N” to “if N, then M.” This conversion, while keeping the repeated premise and the conclusion as they stand, shows that M3 and M1 are formally identical, revealing thereby that M3 is a concludent syllogism, albeit an imperfect one.

After outlining what Avicenna calls the “third mood”, Avicenna presents two serious objections to this entire way of thinking about repetitive syllogisms. The first objection is general in the sense that it applies with equal force to all moods of the conventional division of repetitive syllogisms in which moods are differentiated according to whether the connection in the conditional premise is called complete or incomplete. The second objection is specific to moods in which the connection in the conditional premise is called complete. As I alluded to above, Avicenna’s first objection is based on the fact that the conventional analysis of M3 is not formal in the sense required by logic. Avicenna believes this violation of Pr2 is a direct consequence of talking about connection in terms of completeness and incompleteness. Avicenna’s second objection is based on his claim that M3 is not in reality distinct from M1 and thus violates Pr3. Avicenna begins with the question of the formal nature of our analysis of repetitive syllogisms. He says:

[Text 30] What the just person must believe with regard to this is that the investigation into the form of the syllogism [*anna n-naẓara fī ṣūratī l-qiyās*] is an investigation that is restricted to the thought elicited by the premise [*al-muqtaṣaru ‘alā mūǧibi mafhūmi l-muqaddama*] insofar as it is the premise that is hypothesized [*al-muqaddima al-mafrūda*]. As for [an investigation into] whether it has this or that matter [*mādda*] or this or that special characteristic [*ḥuṣūṣiyya*], this is not an investigation of the form [of the syllogism] at all. If we say, “if A is B, then J is D” and we make it a premise upon which we build a syllogism, then one must consider [*yaltafitu*] what one grasps of this premise with regard to its form and then judge according to what is demanded by its formal elements [*al-ḥāṣṣ min ṣūratihā*]. As for whether the consequent converts

with the antecedent, it is an investigation into the non-formal aspects of the premise; in fact, it is an investigation into the matter of the premise.³³¹

Avicenna is here expanding on an idea that was only implicit in his discussion of complete and incomplete connection in ŠQ V 1. In §3.2, we saw that Avicenna criticized the conventional analysis of the logical connection between antecedent and consequent in terms of concepts that have extramental correlates, e.g. cause and effect. In ŠQ V 1, it was clear that Avicenna objected to basing a classification of connection as one-way or two-way implication on, for example, what kind of causal relationship the antecedent stood in with respect to the consequent. The Farabian account of logical connection dictated that in constructing a connective conditional sentence, the naturally prior thing, e.g. animal, must stand in the consequent place, whereas the naturally posterior thing must stand in antecedent place, e.g. human. In this case, an antecedent and consequent whose connection in extramental reality is that of priority and posteriority are not convertible just as animal and human are not convertible when they appear in connective conditional expressions such as “if Socrates is human, then Socrates is an animal.” However, if the effect, e.g. its being day, is an inseparable effect of a cause, e.g. the sun, then a connective conditional with this antecedent–consequent pair is convertible because the effect is inseparable from the cause. Avicenna’s claim is that this is not an investigation that is proper to logic because logic examines the nature and the relationships between secondary intelligibles, viz. those concepts that have no extramental correlates. In ŠQ VIII, Avicenna extends this analysis to syllogisms. Our demonstration that M3 is a syllogism requires that we reduce M3 to M1 by conversion of the conditional expression. Yet, according to Avicenna the requirement that the antecedent and consequent be convertible violates the formal nature of our analysis of M3. For Avicenna, the

³³¹ Avicenna, ŠQ VIII, 391.13–392.2.

convertibility of the antecedent and consequent is not a thought elicited by the setting down of the conditional premise qua hypothetical expression, nor is it elicited by considering the antecedent qua antecedent or consequent qua consequent. Rather, the convertibility of the antecedent and consequent is something that results from our knowledge of the matter (*mādda*) of the conditional premise.³³² But logic, avers Avicenna, only investigates the formal properties of the syllogism, not its matter, and any theory that requires the consideration of the matter of the propositions cannot, properly speaking, be a logical theory. Rather, in the analysis of repetitive syllogisms,

[Text 31] one must turn to consider the condition [*šarṭ*], its antecedent, and its consequent insofar as the consequent is a consequent, the antecedent an antecedent, and what is implied by all of these in a general way, regardless of their matter.³³³

Avicenna's second objection is that the conventional analysis of M3 violates Pr3. According to Avicenna, the reduction of M3 to M1 as prescribed by the traditional account relies on the assumption that the antecedent and consequent of the conditional premise are convertible. Avicenna's claim is that this very assumption, in reality, makes M3 no more than a substitution instance of M1 rather than a mood that is genuinely distinct from M1. In order to understand Avicenna's argument regarding repetitive syllogisms, we must first understand how Avicenna sees conversion operating in the reduction of imperfect to perfect categorical syllogisms. As an example, consider the typical reduction of the second-figure Cesare (predicate <quantifier> subject) (BeC, BaA $\Rightarrow$ CeA) to the first-figure Celarent (NeM, MaL $\Rightarrow$ NeL). In order to show that BeC, BaA $\Rightarrow$ CeA is a concludent syllogism, we must show that it can be made formally identical to a first-figure mood using truth-preserving operations on the

³³² Avicenna, ŠQ VIII, 393.5-6.

³³³ Ibid., 392.16-17. Cf. Shehaby, Propositional Logic, 186.

premises. $BeC, BaA \Rightarrow CeA$ can be made *formally* identical to $NeM, MaL \Rightarrow NeL$ by converting the universal negative major premise BeC to CeB . This operation yields $CeB, BaA \Rightarrow CeA$, which is identical to the first-figure Celarent in its logical form (obviously, the difference in dummy variables is irrelevant); and thus, Cesare is revealed or demonstrated (*istabāna, yastabīnu*) to be a syllogism. The fact that perfecting Cesare requires the conversion of the minor premise is a clear sign that the two moods are formally distinct. However, Avicenna also introduces the notion of the use of second- and first-figure moods as a further way of distinguishing between one mood and another. In particular, the formal distinction between a first-figure mood like Celarent and a second-figure mood like Cesare should be mirrored in some way by the manner in which the moods are utilized in a particular reasoner's formulation of an actual argument. Avicenna considers the following scenario. Say that in constructing an argument, the thought that no A is B (for some appropriate A and B) comes to one's mind at the outset, before any logical inferences have taken place. Add to the first premise, AeB , a second premise, CaB ; then two ways of proceeding with the deduction present themselves. Either we can convert the universal negative premise and, with it, put together a first-figure syllogism with premises BeA, CaB to generate the conclusion, CeA , or we can rely on the syllogism Celarent and directly conclude CeA from the premises AeB and CaB without having to convert the universal negative premise AeB . Formally speaking, the choice is unimportant since both moods have been accepted as equally conclusive. However, as Avicenna points out, in the heat of an argument, the imperfect Cesare plays an important role in the case that the conversion of the universal negative happens to be unavailable to the reasoner at the outset of the reasoning process.

[Text 32] As you know, it often happens that what occurs to the mind at the outset [*as-sābiq ilā ḡ-dih̄n*, i.e. a pre-deductive thought] is that no A is B. Then, after that, it occurs to the mind [*yaḥṭuru bi-l-bāl*] that it is not necessary that no B be A; or even this [latter thought] does not occur to the mind. <In either case>

then, no A is B is what occurs to the mind at the outset. Then, if this thought, which occurred to your mind at the outset remains as it is [i.e. unconverted] and it can produce a conclusion [e.g. by Cesare rather than Celarent] by attaching some other premise to it [e.g. “every C is B”], then there is no need for the conversion [i.e. of “no A is B” to “no B is A”]. The case of the particular affirmative is similar. This, then, is a sense in which we can put the other two figures to use. In spite of this, the convertend [i.e. “no A is B”] nevertheless implies the converse (i.e. “no B is A”) [*wa-ma’a dālīka fa-inna al-‘aksa lāzimun li-l-ma’kūs*].³³⁴

In the above scenario, Avicenna is not trying to reduce a syllogism like the imperfect Cesare to the perfect Celarent. Rather, an idealized reasoner is trying to derive a conclusion from a given set of premises. In this scenario, Avicenna takes it for granted that the reasoner knows that both forms of argument are concludent syllogisms. The reasoner has two premises before him: AeB (e.g. “nothing sleeping is human”) and, say, CaB (e.g. “everything rational is human”). One way of reasoning would be to convert AeB (“nothing sleeping is human”) to BeA (“no human sleeps”) and to produce the desired conclusion (“nothing rational is sleeping”) by a first-figure syllogism. However, in this case, this option is not open to the reasoner because he does not think to convert the universal negative premise at all, or there is some ambiguity about the universal negative premise that makes the reasoner unsure that the conversion is materially valid. Since the ideal way of proving the conclusion is blocked by the reasoner’s skepticism, he must find some other way of producing the conclusion, namely, by using Cesare directly, i.e. generating the conclusion “nothing rational is sleeping” directly, without making recourse to the conversion of the universal negative premise. The result of this thought-exercise is to show that there are occasions in which the use of second- and third- figure moods is useful or even necessary, even if the use of a first-figure mood would make the truth of one’s conclusion self-evident to your opponent. Far from being motivated by mere curiosity

³³⁴ Ibid. 393.13-394.1.

about the properties of a logical calculus, the above scenario shows that there are practical benefits in deriving syllogistic figures other than the first.

However, perhaps the most important conclusion to be drawn from Avicenna's argument is to see how the formal validity of conversion modifies the relationship between imperfect and perfect syllogisms. Before Cesare is reduced to Celarent, its status as a concludent syllogism remains undemonstrated: it is just an argument form. The reduction of Cesare to Celarent shows decisively that Cesare is valid syllogistically. And the formal validity of the conversion of universal negative categorical propositions is what makes this *formal* reduction possible. Avicenna's detractors want to maintain that the relationship between M3 and M1 is exactly analogous to that between, for example, Cesare and Celarent. Remember that M3 is "if A is B, then J is D. But J is D. Therefore, A is B" and M1 is "if A is B, then J is D. But A is B. Therefore, J is D," with the crucial qualification that the conditional premises in both argument forms are stipulated as signifying perfect implication (*luzūm tāmm*). Thus, M1 and M3 are formally distinct, at least ostensibly, in that in the former we assert the antecedent to produce the consequent of the conditional premise, whereas in the latter we assert the consequent to produce the antecedent of the conditional premise. The stipulation that the implication between the antecedent and consequent is perfect (*luzūm tāmm*) amounts to holding that the conversion of the antecedent and consequent preserves truth. Unlike in the case of e-conversion, however, in which the validity of the conversion was due to the formal properties of the proposition, viz. the quantity and quality of the categorical proposition, the adherents of the traditional view must admit that "if A is B, then J is D" $\Rightarrow$ "if J is D, then A is B" is not a formally valid inference, meaning that if we assume that "if A is B, then J is D" is true, then the form of "if A is B, then J is D" on its own does not guarantee the truth of "if J is D, then

A is B.” Thus, in order for the adherents to hold simultaneously that conditional conversion is formally invalid but that the antecedent and consequent in both of the above syllogisms are convertible, they must maintain that we know pre-deductively that both “if A is B, then J is D” and “if J is D, then A is B” are true. In this way, it is possible for Avicenna’s opponents to maintain the analogy between the use of Cesare or Celarent and the use of M3 and M1. Let “A is B” signify “the sun is up” and “J is D” signify “it is day”. When it happens that (1) “if A is B, then J is D” occurs to the mind at the outset of an argument, two ways of concluding that “A is B” are open to us. One way is that our knowledge that if J is D, then A is B, allows us to add (1)’s converse, (3) “if J is D, then A is B,” to the steps of the deduction. Since we were also given (2) “J is D” as a premise at the outset of the argument, the combination of (2) and (3) yields the desired conclusion that A is B by M1. Yet, the advocate of this view maintains a more efficient way of proceeding is open to us. Instead of tasking ourselves with the conversion of (1) and generating the conclusion by M1, we can produce the desired conclusion directly from M3, saving the step of having to make (3) a step in the deduction. This is the traditional view as conveyed in Avicenna’s words in the following passage.³³⁵

[Text 33] When this [i.e. “if A is B, then J is D”] occurs to the mind at the outset, it implies something [*lazimahu šay’un*, i.e. it implies “if J is D, then A is B”], which implies, in turn, a third [*yalzamuhu tālītun*, i.e. the conclusion “A is B” of the premise set {“if J is B, then A is B”, “J is B”}]. Since what is implied by the first implication [*lāzimu l-lāzim*, i.e. the “first implication” is “if A is B, then J is D,” *lāzim* here being used in the first sense; “what is implied by” the first implication is “if J is D, then A is B”; *lāzim* being used again in the second sense] is something that implies the first antecedent [*lāzimun li-l-malzūmi l-awwal*, i.e.

³³⁵ Before considering Avicenna’s words, it is important to note that Avicenna uses the notion of implication (*lazima*, *al-lāzim*, *lāzimun*, *lāzimun li-*, *luzūm*, *yalzamu*/*yalzum*) to speak about (1) the way an antecedent implies a consequent and (2) the way a set of premises imply a conclusion. What unites these different notions is Avicenna’s view that, in both, the mind must move (*intaqala*, *yantaqilu*) from the antecedent/premise set to the consequent/conclusion. In this passage, sense 2 is assumed, unless I indicate otherwise. Lastly, this is an important text, but Avicenna’s language is extremely vague. Thus, although the reader will no doubt be annoyed by the number of bracketed and parenthetical comments in the translation, I believe they are necessary. I considered removing them and explaining Avicenna’s intention in the paragraphs following the quotation, but the passage would be incomprehensible in their absence.

“if J is D, then A is B” implies “A is B”; *lazima* being used now in the first sense], there is no need to task my mind with moving from this [i.e. “if A is B, then J is D”] to what was first implied [i.e. “if J is D, then A is B”], and then from what was first implied to the third implication [i.e. “the third” being “A is B,” the conclusion of the premise set {“if J is D, then A is B”, “J is D”}], which is the consequent of the second implication [*lāzimu tānin*]. Rather, I will let my mind move immediately [*dufatan*] to the second implication [*al-lāzm at-tānī*, i.e. “if J is D, then A is B,” “implication (*lāzim*)” having sense one] as an initial implication [i.e. as the initial premise, *lāzim* being in the first sense]. As such, I have no need to consider the first syllogism [i.e. M1] when I use this syllogism [*inda l-isti‘māl*, i.e. M3], even if I must use it [i.e. M1] at the time that I want to demonstrate [*istibāna*] that the syllogism [i.e. M3] is productive [*muntiğ*]. If you did all this, then you would have generated knowledge [*istafadta šay’an*] and abridged the inquiry. In this way is it [i.e. the use and form of M3] analogous to the moods of the second and third-figure.³³⁶

Avicenna’s opponents are trying to make two basic points. First, just as using Cesare instead of Celarent saves a step, viz. the conversion step, in the deduction of a conclusion from a pair of premises, the reasoner can use M3 instead of M1 to deduce his desired conclusion directly rather than having to convert the conditional premise. But, secondly, they also want to hold that M1 is still necessary for the formal reduction of M3 to M1. It is only by means of M1 that M3 can be shown to be a syllogism in the first place. For our purposes, Avicenna’s final sentence is the crucial one, because by it he attacks this second claim. Just as in the case of Cesare and Celarent, adherents of the traditional view make a distinction between the use of Cesare in constructing an argument and its *formal reduction* to a perfect mood on the other. Avicenna’s opponents will claim that when we have a choice between using Cesare or Celarent, using Cesare is more efficient, so to speak, than using Celarent, because using the latter requires the additional conversion step. A similar situation holds, says the traditional view, with regard to the use of M1 and M3. There is no point, goes the argument, in the reasoner obliging himself to take with (*yukallifu nafsahu*) the conversion step, and then using M1 to

³³⁶ Ibid., 394.9-14.

reach the conclusion when he can use M3 to reach the same conclusion in a single step. However, merely because Celarent and M1 are dispensable at the level of use does not mean that they are eliminable at the level of syllogistic form. As Avicenna notes, his opponents hold that just like we need Celarent to reveal (*istabāna*, *yastabīnu*) that Cesare is a concludent syllogism, we need M1 to reveal that M3 is concludent syllogism. Just like Celarent and Cesare are formally distinguished by the position of the terms in the major premises, M1 and M3 are formally distinguished by which of the antecedent and consequent is asserted and which of them serves as the conclusion. And just like Cesare is perfected, viz. revealed to be a concludent syllogism, by conversion of the universal negative major premise, M3 is perfected by switching the antecedent and consequent of the conditional premise.

Avicenna denies all this, but his claim is stronger than just that there is no point in using M3 instead of M1. He claims that, in fact, M1 and M3 are not even formally distinct. In other words, M1 and M3 might differ in the dummy variables used to represent them, but are otherwise formally identical. Avicenna erases the use-form dichotomy with respect to M3 and M1 in a single blow.

[Text 34] But the matter is not like this at all. It is only of any use to bring this conversion to mind when you³³⁷ need to know that the implication is complete, which is no more than your needing to know and bringing to mind that this, which is the consequent [i.e. “J is D”] has a connection of priority [*nisbatu t-taqdīm*] to what is now the antecedent [i.e. “A is B”]. But when you need to bring this to mind (i.e. bring to mind the consequent’s priority to the antecedent) at the outset in order to put together your syllogism [i.e. by converting the antecedent and the consequent of the “if A is B, then J is D”], then you have brought to mind “if J is D, then A is B.” Then, when you except, saying “but J is D,” then you have done nothing more than except the antecedent of the premise that you have actually brought to mind [*aḥṭarta bi-l-bāl bi-l-fi’l*, i.e. “if J is D, then A is B”]. So there is not really any use for the first premise [i.e. “if A is B, then J is D”] as an element of a syllogism [*ḡuz’u qiyāsin*]. The most consideration

³³⁷ Reading “taḥtāḡ” for “yaḥtāḡ”.

given to it is that because of it, you remember something [i.e. “if J is D, then A is B”] which does not imply it but occurs with it incidentally [*ya’riḍu ma’ahu*].³³⁸

Whereas the conversion of the universal negative was formally valid, in the sense that $A \in B$ implies $B \in A$ regardless of the content of the terms A and B, it cannot be said that “if A is B, then J is D” implies “if J is D, then A is B” regardless of the content of “A is B” and “J is D”. In order to know that “A is B” and “J is D” convert, one must know that the propositional content of “J is D” is such that it can also serve in the antecedent place in a true connective conditional with “A is B”. The conversion, in other words, is not available at the level of the form. Because the conversion of the antecedent and consequent is not a formally valid operation, knowledge that the connective conditional is complete amounts to knowing both “if A is B, then J is D”, “if J is D, then A is B” separately at the outside of the argument, viz. as premises, before any steps of reasoning have been taken. With this in mind, consider variations of M1 and M3 in which the converse of the antecedent and consequent are added as a third premise: (M1') {“if M is N, then O is P”, “if O is P, then M is N”, “M is N”} $\Rightarrow$ “O is P”; and (M3') {“if W is X, then Y is Z”, “if Y is Z, then W is X”, “Y is Z”} $\Rightarrow$ “W is X”. Ostensibly, there still appears to be a formal difference between M1' and M3', viz. in M1' the antecedent of the first premise is asserted yielding the consequent of the first premise as a conclusion; whereas in M3', the consequent of the first premise is asserted yielding the antecedent of the first premise as a conclusion. What is more, M3' at least appears to reduce to M1', in the sense that our ability to produce “W is X” as the conclusion in M3' from the assertion of “Y is Z” as a premise seems to depend on or require the validity of M1'. But these formal distinctions are shown to come to nothing. If we just change the order of the conditional premises in M3', we get (M3''): {“if Y is Z, then W is X”, “if W is X, then Y is Z”, “Y is Z”} $\Rightarrow$ “W is X”, which is identical to M1', ignoring the differences

³³⁸ Ibid., 394.16-395.7.

in dummy variables as obviously irrelevant. If we remember that changing the order of premises is of no logical significance, then the upshot of Avicenna's argument is the evaporation of any real formal distinction between the moods M1 and M3 and any other moods in which the conditional premises are stipulated as complete. In other words, take some account of hypothetical syllogisms in which the conditional premise is stipulated as a *complete* connective conditional, meaning that the antecedent and consequent are convertible, but not as a result of a formally valid inference. Then, this account's formally distinguishing between two moods based on which of the antecedent and consequent are asserted and which of them serves as the conclusion is illusory. In fact, these moods, which are supposedly formally distinct, collapse into one.

By eliminating *simpliciter* following from the types of connection that can be used in repetitive syllogisms, Avicenna showed that some sort of logical connection between antecedent and consequent has to be in place in order for repetitive syllogisms to be useful. Avicenna wanted to hold up *restricted* connection as the only alternative that made sense in the context of repetitive syllogisms. However, since complete and incomplete connection were well-known to signify a connection between antecedent and consequent, Avicenna had to argue to eliminate this option as well. Avicenna does this by showing, first, that the analysis of connection as complete and incomplete is not appropriate to a formal analysis of syllogisms. Then Avicenna argues that the purported convertibility or non-convertibility of complete and incomplete connective conditionals relies on non-logical considerations. Finally, Avicenna shows that distinguishing moods, which are stipulated to have complete conditional premises, according to which of the antecedent and consequent is repeated as premise and as conclusion, leaves such moods as no more than mere substitution instances of each other.

This assault on the complete/incomplete dichotomy leaves Avicenna free to propose his own simplified rules for connective conditional syllogisms, leaving Avicenna's *restricted* connection as the only suitable interpretation of the connection between antecedent and consequent:

[Text 35] The eighth mood: excepting the contradiction of the consequent of a conditional having complete implication. This is not, in reality, a mood that is distinct from the one preceding it [i.e. the seventh mood is one in which the exception of the contradiction of the antecedent yields the contradiction of the consequent]. Rather, you should know that [1] the exception of the contradiction of the antecedent is inconcludent (*lā yuntiġ*), [2] that the exception of the antecedent yields the consequent, [3] that the exception of the consequent is inconcludent, and that [4] the exception of the contradiction of the consequent yields the contradiction of the antecedent.³³⁹

Thus, the only productive repetitive syllogisms with connective conditional major premises are modus ponens and modus tollens. What is more, the assertion of the consequent and of the contradiction of the antecedent are both explicitly rejected as invalid. Yet, it is obvious that the following argument is at least materially valid: “if the sun is up, then it is day. But it is day. Therefore, the sun is up.” What does Avicenna mean when he says that the argument forms (1) and (3) are inconcludent (lit. “sterile”, *‘aqīm*)”? The relevant condition that forms (1) and (3) must fulfill with regard to this question is that, on the assumption that the premises are true, it is necessary that the conclusion is true. As we discussed in the previous section, for Avicenna a *restricted* conditional is true if and only if the consequent is true under all conditions, whether real or hypothetical, in which the antecedent is true. Notice that this definition allows a *restricted* conditional to be true in the case in which the consequent is true and the antecedent is false where truth and falsity are defined according to the correspondence theory of truth discussed above. With this observation in mind, consider a formal representation of (1) with a *restricted* conditional (P1) “if P is Q, then R is S” as a first

³³⁹ Ibid., 397.1-4.

premise, (P2) “R is S” as a second premise and “P is Q” as a conclusion. In order to show that an argument form like (1) is inconcludent, Avicenna must show that it is *possible* for “P is Q” to be false, given that P1 and P2 are true. Avicenna can show this if we remember that he holds a correspondence theory of sentential truth, meaning that a sentence “P is Q” is true if and only if the thought grasped from the sentence is in line with what is actually the case outside the soul; “P is Q” is false in case the thought grasped from the sentence is not in line with what is actually the case outside the soul. Consider “P is Q”. Say it is contingently true, viz. the thought grasped by “P is Q” is not in line with what is actually the case, but there are certain hypothetical conditions under which “P is Q” is true. Make “P is Q” the antecedent in a counterfactual *restricted* conditional “if P is Q, then R is S,” where “R is S” is always true, under the hypothetical conditions in which “P is Q” is true and also under those conditions in which “P is Q” is false. Thus: P1 is true in the sense that the consequent is true whenever and under all conditions in which the antecedent “P is Q” is true. P2 is true because it’s always true. But the conclusion “P is Q” is false because, as stipulated, what we understand of “P is Q” is not in line with what is actually the case outside the soul. Therefore, (1) is sterile. A similar argument may be constructed for argument form (3).

In conclusion, in ŠQ V and VIII, Avicenna goes to extraordinary lengths to show that the complete/incomplete connection has no place in logical theory. Avicenna attacks this doctrine of conditionals on several different levels. Based on Avicenna’s theory of second intelligibles, Avicenna shows that the complete/incomplete dichotomy analyzes logical connection in terms of primary intelligibles. Secondly, Avicenna shows that taking connection as complete or incomplete makes the utterance of the conditional statement pointless because the speaker already knows that the antecedent and consequent are both true. For Avicenna,

uttering a connective conditional should lead the mind from entertaining an antecedent as hypothetically true to a consequent that one does not *know* to be true except under the conditions stated in the antecedent. Complete and incomplete connection does not let us do this and so connective conditionals, taken in that sense do, not fulfill their primary objective. Finally, Avicenna shows that the division of eight productive and sterile hypothetical moods with connective conditional premises rests on the claim that there is a formal distinction between a “if P, then Q. But P. Therefore, Q”, and “if P, then Q. But Q. Therefore P.”, where the connection between the sentences P and Q in the conditional premise is stipulated as complete. Avicenna shows decisively that this claim is false. Thus, not only does using complete and incomplete conditionals in syllogisms lead the reasoner to pointlessly “deduce” knowledge he already knows, but it leads to a situation in which moods that are conventionally claimed to be distinct are, in fact, formally identical. Having thoroughly discredited the traditional account of connection, Avicenna proposes an entirely different division of connection. In Avicenna’s new scheme, connection is based on the idea of concomitance in truth. *Simpliciter* connection is similar, though as we discussed not identical to, truth-functional conjunction. A *simpliciter* conditional, or, equivalently a coincidental conditional, is one in which the truth of the antecedent and the truth of the consequent are consistent. A *restricted* or genuine conditional is one in which the consequent is true under all conditions, whether actual or hypothetical, in which the antecedent is true. Talking about connection in terms of concomitance in truth (*ma’iyya*) also avoids the serious defects in the traditional doctrine of connection. It is sufficiently formal in the sense that the terms under which the logical nature of the connection between antecedent and consequent are investigated in terms like hypothesis, conditional, truth and quantifiers. And to utter *restricted*

conditionals is not a pointless exercise, since the act of hypothesizing the antecedent as true leads the mind to consider what propositions can be true under all the possible conditions in which the antecedent is true. According to Avicenna, only this latter type of conditional can be used in repetitive syllogisms. As such, Avicenna's genuine conditionals have been shown to provide the basis for a much more coherent doctrine of deductions with "if...then..." statements as premises. What remains to be seen, in Chapter 5, is (A) how Avicenna develops the main insights in ŠQ V 1 into a theory of connective conditions that quantify over the hypothetical states; and (B) how *simpliciter* and *restricted* conditionals work in Avicenna's novel system of conjunctive hypothetical syllogisms (*qiyās šarṭī iqtirānī*).

CHAPTER 4: INTERPRETING AVICENNA'S CONDITIONAL SYLLOGISTIC AS A PROPOSITIONAL LOGIC

§4.0 INTRODUCTION

In this chapter, I make two claims. One is that Avicenna has a propositional logic. The other is that Alfarabi does not. Nabil Shehaby and Miklós Maróth have both written books about what they call Avicenna's "propositional logic", but these authors do not provide sufficient justification that satisfies the criteria demanded of a propositional logic. In one sense, then, I agree with Maróth and Shehaby: their judgment that Avicenna's conditional syllogistic is a propositional logic appears to me to be, on the whole, correct. These authors may be criticized, however, for merely assuming rather than arguing this point as I try to do in what follows. As I mentioned in §1, I rely primarily on Christopher Martin's penetrating analyses of Boethius's hypothetical syllogistic and Abelard's theory of conditionals and entailment. According to Martin, for any sentential logic to be properly called a "propositional logic", it must contain a notion of propositional connectives. What this means will be stated more explicitly below, but for now, it is sufficient to understand the difference between a grammatical (or sentential) connective such as "if, then" on the one hand, and a propositional connective on the other. A sentential connective is a syntactical marker that takes well-formed verbal utterances (of any syntactical complexity) and forms them into a new, well-formed sentence according to a language's syntax. A propositional connective works in a similar fashion, but instead of forming more complex sentences from simpler sentences, it must form more complex propositional content from simpler propositional content. Thus, the notion of propositional connective, which is the *sine qua non* of a propositional logic, rests on

an adequate appreciation of propositional content. As far as I can tell, propositional content, viz. a form of words in which something is put forward for consideration, is claimed by contemporary philosophers of logic to be a unitary property of sentences that remains constant in the face of the sentence's deployment in a variety of speech contexts. According to Martin, this notion lies at the heart of Frege's distinction between a proposition's content and its force, between what it propounds and how what is propounded is used by a speaker. Peter Geach called this distinction the Frege point, and Martin claims that any logic of compound sentences must understand this distinction in order for it to qualify as a propositional logic in the technical sense accorded to the term by philosophers.

There are thus two major questions to grapple with in this chapter. One is explicating further Martin's discussion of the basic conditions that qualify a sentential logic as a propositional logic; the second is seeing whether Avicenna's theory of conditional propositions and conditional syllogisms fulfills these conditions. The former is the task of §4.1, the latter the task of §4.2 and §4.3. §4.1 concludes by identifying two criteria against which Avicenna's conditional syllogistic will be measured in order to determine whether Avicenna's conditional syllogistic is a propositional logic. The first condition is that Avicenna must recognize the difference between a proposition's force and its content; that is, he must recognize what is, in effect, the Frege point. The second condition is whether or not Avicenna's sentential connective "if, then" qualifies as a propositional connective. I claim that Avicenna's conditional syllogistic is a propositional logic only to the extent that it can be shown to respect these two basic principles. By the same criteria, Alfarabi's conditional syllogistic recognizes the first condition, but fails to recognize the second. Alfarabi makes a distinction akin to the Frege point, but does not treat grammatical connectives as proposition-forming operators on

propositions. §4.2 provides evidence that Avicenna recognizes the Frege point in a way that is similar to Alfarabi. However, whereas Avicenna's formulation of what is today called the "Frege point" arises from his analysis of the nature of propositions, Alfarabi's arises from his working within the context theory of logic described in §2. In §4.3 I focus on Avicenna's treatment of propositional connectives, and, in sum, the evidence is clear that Avicenna's conditional syllogistic succeeds in qualifying as a propositional logic.

Martin claims Abelard to be the greatest philosophical logician between Aristotle and the Stoics at one end and Buridan on the other, for his work on propositional theory. Abelard died in 1142, Avicenna in 1037. Avicenna's "discovery" of propositional logic preceded Abelard's by 97 years. Should not this be taken as evidence of Avicenna's superiority as a logician to Abelard? The question is, in my view, meaningless. Avicenna and Abelard worked under incommensurable intellectual conditions. In Abelard's day, few of Aristotle's works had been translated into Latin, and Abelard did not have access to the work of any of the Greek commentators. As is well known, Avicenna's situation was quite the opposite. Avicenna benefitted from an abundance of excellent translations of all of the *Organon*, and most of Aristotle's Greek (and Syriac) commentators. As a consequence, Avicenna's solutions must be understood as responses to particular logical problems that arose from the Greek and Syriac commentators in late antiquity. The continuity of the philosophical tradition within which Avicenna worked imposed certain limitations or restrictions on the kinds of solution that could be offered. In other words, Avicenna's logical theory could not but propose solutions that made sense within the logical commentary tradition. Such exegetical strictures were much weaker in Abelard's case. Whatever the case, Avicenna's achievement may be

recognized in its own right, and he certainly deserves a place among the greatest logicians in history.

§4.1 PROPOSITIONAL LOGIC IN ISLAMIC LATE ANTIQUITY? ENGAGING IN ANACHRONISM

In §1 I pointed out that Miklós Maróth and Nabil Shehaby both call Avicenna's conditional syllogistic a "propositional logic". In making this claim, these authors are not merely saying that in ŠQ V-IX Avicenna formulates a logic of compound sentences. In fact, their claim is much stronger. The truth tables that Maróth and Shehaby deploy in order to illustrate the truth-conditions for Avicenna's connective and disjunctive conditional propositions show that they claim that Avicenna's conditional syllogistic develops a theory of the proposition in which the truth of the conditional or disjunctive sentence may be determined solely from a consideration of the truth of the parts of the conditional. In other words, Shehaby and Maróth claim that "if, then" and "either, or" in Avicenna's logic behave as truth-functional logical operators. In §1, however, I also mentioned that scholars such as Tony Street, Fritz Zimmermann, and indirectly Lenn Goodman and Khaled El-Rouayheb, have expressed strong reservations about the claim that Avicenna's conditional syllogistic is interpretable as a *truth-functional* propositional logic. However, these authors do not provide much evidence to justify their reservations. This is, however, a shortcoming that is easily rectified. Consider Avicenna's conclusion of ŠQ V, 3:

[Text 1] In general, what puts one into error with regard to the truth of the connective and disjunctive [conditional expression] is the mistaken belief about the [conditional] proposition that the main objective with regard to its truth-value is the consequent or antecedent such that its truth-value is considered [ie. the truth-value of the antecedent or the consequent].³⁴⁰

³⁴⁰ Avicenna, ŠQ V, 261.8-10: *wa-ğummā'u mā yūqī'u l-ğalaṭa fī amri l-muttaṣili wa-l-munfaṣili subūqu l-wahmi fī l-qaḍiyyati ilā anna l-ğaraḍa fihā tālin aw muqaddamun fa-tu'tabaru ḥāluhu.*

As far as I can tell, this text and many others like it speak decisively against the claim that Avicenna's conditional propositions can be treated as truth-functional.

In the present chapter, I want to extend Street's and Zimmermann's criticism of Maróth and Shehaby: in what sense is Avicenna's conditional syllogistic even a *propositional* logic? As Christopher Martin says,

Though generally not made explicit by modern historians, the concepts of propositional content and propositional operation are nevertheless presupposed by the symbolic apparatus which they typically use to represent the claims of ancient and medieval logics.³⁴¹

In other words, to use truth-tables and modern logical symbols to interpret Avicenna's conditional syllogisms is not a neutral hermeneutical device, but creates the expectation that Avicenna should conform to certain ideas about the nature of the proposition that he may not in fact agree with.

What does it mean, then, to claim that a sentential logic such as Avicenna's conditional syllogistic is "propositional logic" in the technical sense give to the term by contemporary logicians? Christopher Martin's discussion of Boethius' "propositional logic" allows us to give this question quite a precise meaning. According to Martin, any philosophical account of language that begins from the assumption that the meaning of a compound sentence can be constructed from its parts must do two things. Obviously, it must provide some account of how the meaning of a compound sentence can be constructed from its atomic components. But any such account also has to account for the different ways a single sentence can be employed by speakers in various speech contexts.³⁴² In his philosophy of language, Frege attempted to accomplish these two tasks, viz. to provide an account for the constructive nature of sentential meaning, and to provide an account of how meaning is utilized in different

³⁴¹ C. Martin, "The Logic of Negation in Boethius", *Phronesis* 36 (1991): 277-301, quoted at 277.

³⁴² *Ibid.*, 281.

speech contexts such that sentential meaning remains constant in the face of different employments by speakers, by introducing a distinction between the propositional content of a sentence and its force. The ramifications for logic are that Frege's distinction between propositional force and content allows us to treat sentential meaning as a unitary entity that remains constant in the face of its employment in different speech acts. As a direct consequence, using Frege's distinction allows us to treat these propositional units as atomic propositional contents from which more complex propositions can be built using propositional connectives (e.g. "if, then", "and", "or", "but", "only if", etc.). Martin says that insisting on the distinction between propositional force and propositional content has been called by Peter Geach the "Frege point" the recognition of which is needed in order to properly grasp the "doctrine of truth-functional connectives".³⁴³

While it is true that Geach coined the label "Frege point", his chief concern was investigating how to properly understand the relation between a sentence's propositional content, defined as "a form of words in which something is propounded, put forward for consideration", and the speaker's assertion of that proposition. For Geach, it is especially important to recognize (even if, once stated, it should strike us as a sort of platitude) that the content of the proposition is a self-contained unit, in the sense that it is not altered by its appearing in different speech acts, e.g. assertion, question, command, or supposition. Geach asks the reader to consider the proposition "P or Q" where "or" is taken as a truth-functional operator. The assertion of "P or Q" does not mean that "P" is being asserted in the context, or that "Q" is being asserted. When we say, then, that the truth of "P or Q" is determined by the truth of P and Q, this admission contains the implicit admission that P and Q have truth values

³⁴³ P. Geach, *Logical Matters* (Berkeley and Los Angeles: University of California Press, 1972), 258.

“independently of being actually asserted”.³⁴⁴ In this example, insistence on the Frege point allows us to, one, treat “P” and “Q” as individual semantic units from which the compound “P or Q” is constructed; and two, to distinguish between the assertion of “P” and the assertion of “Q”, on the one hand, from the assertion of the compound “P or Q” on the other.

Thus, if Avicenna’s conditional syllogistic is to be interpreted as a propositional logic in any real sense, then, according to Martin, it would have to put forward a doctrine about propositions that recognizes, in effect, the Frege point. In other words, there must be evidence in Avicenna’s text, direct or indirect, that he believes that it is of the logical nature of propositions that what we grasp of what they express remains unaltered in the face of their deployment in a variety of different argumentative contexts. Yet, cognizance of the Frege point is not a necessary and sufficient condition for a logic of compound sentences to qualify as a propositional logic in a technical sense. Martin says:

By *propositional logic* I mean any account of compound propositions and the arguments based on them which, cognizant in effect of the Frege point, treats at least some sentential connectives as propositional content forming operations on propositional contents. The operations need not be defined truth-functionally [...] but they must take propositional contents as arguments and yield propositional contents as values.³⁴⁵

In Martin’s view, the grammatical particle “if, then”, for example, operates as a sentential connective when it connects two independent sentences “the sun is up” and “it is day” to form the grammatically sound sentence “if it is up, then it is day”. “If, then” acts as a *sentential* connective because it connects the verbal expressions uttered by a speaker according to the conventions of English usage to form a new verbal expression that is not identical to either of the original utterances. On the other hand, “if, then” will only be a *propositional* connective when it joins together propositional content, to form a new propositional content that is

³⁴⁴ Ibid.

³⁴⁵ Martin, “The Logic of Negation in Boethius”, 281 (italics in the original).

not identical to either of the original propositions taken individually. According to Martin's requirements, "the sun is up" and "it is day" are each propositions because they are forms of words in which something is put forward for consideration. In order for "if, then" to operate like a genuine logical connective and not merely a sentential one, it must connect "the sun is up" and "it is day" into "if the sun is up, then it is day" in such a way that the new compound proposition is "a form of words in which something is propounded" in the same way as "the sun is up" is.³⁴⁶ It is important to note that in order for "if, then" to be a properly propositional connective, it must join the propositional content that is expressed in the antecedent and consequent regardless of their complexity. In other words, "if, then", as a propositional connective, must accept any sort of propositional content, whether a simple categorical or an extremely complex proposition formed from a series of nested conditionals, and yield a new propositional content. There must, in Martin's words, be a "notion of the propositional compounding of propositional contexts to form new contents of *arbitrary complexity*".³⁴⁷ And it is recognizing the so-called "Frege point" that allows this sort of compounding of propositional contents of arbitrary complexity into new content, since this doctrine stipulates that propositional content, i.e. the form of words in which something is put forward for consideration, may be treated as unitary objects that remain unchanged in the face of their deployments as antecedents, consequents, disjuncts, or conjuncts.

This is a lot to ask from Avicenna, especially since he entertained views about the nature and scope of logic and logical theory that are otherwise opposed to those espoused by Frege and most modern logicians. Indeed, even asking whether Avicenna's conditional syllogistic is a propositional logic is to engage in anachronism. That being said, I believe that *if*

³⁴⁶ Geach, *Logical Matters*, 255.

³⁴⁷ C. Martin, "Logic", in *The Cambridge Companion to Abelard* (Cambridge/New York: Cambridge University Press, 2004), 163 (*italics added*).

it can be shown that Avicenna's conditional syllogisms fulfill the following two conditions, then it can be said in a meaningful way that Avicenna conditional syllogistic is a propositional logic.³⁴⁸

Condition FP ("Frege point"): In his conditional syllogistic, Avicenna must show himself cognizant of, in effect, the Frege point. In other words, he must develop his theory of conditional propositions in such a way that he makes a distinction between the truth of the compound propositional unit and the truth of the propositional elements from which it is compounded.

Condition PropCon ('Propositional Connective'): Avicenna must recognize nested conditionals of *arbitrary complexity* as syntactically well-formed, and he must also develop a *calculus*, viz. a set of sentential transformation rules and rules of deduction, that makes room for the construction of syllogisms with arbitrarily complex compound propositional content as premises and arbitrarily complex compound propositional content as conclusions.

It is the task of the rest of this chapter is to show that Avicenna's conditional syllogistic fulfills both of these conditions.

To begin, however, I shall discuss at greater length why I believe Alfarabi's logic fulfills only FP. To facilitate exposition, I reproduce the relevant passage from *Ġadal* (Text 2, translated in chapter 2).

[Text 2] As for the connective conditional [syllogism], the connection in it may be clear in itself (*bayyin bi-naḥsihi*), or it may not be clear in itself [*ḡayr bayyin bi-naḥsihi*], and thus require demonstration of the truth [*ṣiḥḥa*] of the connection in it, for it is a fact that the chief consideration [*malāk al-amr*] in the connective conditional [syllogism] is the truth of the connection [*ṣiḥḥatu 'l-ittiṣāl*] and the

³⁴⁸ Whatever the case may be, the proof of Avicenna's worth as a logician does not and should not hinge on whether or not his conditional syllogistic qualifies as a propositional logic; personally, I find the question largely irrelevant. Nevertheless, there are three reasons why I have decided to deal with this question in detail. First, Shehaby and Maróth claim that Avicenna has a propositional logic, and so their claims deserve scrutiny. Second, it is common on scholarly literature to show ancient logicians dealing with similar sets of problems as modern logicians wrestle with today. For Avicenna, it would be helpful, I think, to make his logical thought comprehensible to non-Arabists and Arabists alike. Finally, if it can be shown that Avicenna has a propositional logic, then the use of symbolic logic is suitable and will not cover over distinctions that Avicenna is trying to make. We certainly should not be despondent, as Nabil Shehaby shows himself to be (N. Shehaby, Introduction to Avicenna, *The Propositional Logic of Avicenna*, trans. N. Shehaby, Dordrecht/Boston: (D. Reidel, 1973), 22f) when we find Avicenna propounding doctrines that expose their Aristotelianism rather than logical doctrines that ostensibly antedate the *Principia Mathematica* by this or that many centuries. Indeed, it is in an examination of Avicenna's development of the Aristotelian tradition that a true estimation of his greatness lies.

truth of the repeated proposition [*ṣiḥḥatu mā yustaṭnā*]. As for the truth [*ṣiḥḥa*] of the antecedent and the consequent, no conditional expression [*qawl ṣarṭī*] signifies [their truth, sc. *ṣiḥḥa*] and it may happen that neither of them is true [*ṣaḥīḥan*]. Rather, a conditional expression only signifies [*yataḍammanu*] the soundness of the connection [*ṣiḥḥata l-ittiṣāl*]. Even if neither the antecedent nor the consequent is true (*ṣaḥīḥan*), the expression's being a conditional is not undermined. The proof of this is that the truth-value [lit. "the matter" or "the actual states-of-affairs", *al-amr*] with regard to the antecedent and consequent rests on the asserted proposition [i.e. the minor premise, *al-mustaṭnā*]. Thus, the contradiction of the consequent can be asserted (*yustaṭnā*) due to the fact that it is true [*ṣaḥīḥ*], yielding thereby the contradiction of the antecedent. If, however, <the antecedent and consequent> were true because of what was posited about them, then it would be impossible to except the contradictory of the consequent by virtue of the fact that it is true and yields thereby the contradictory of the antecedent, since the two contradictories cannot be true simultaneously [*id kânā an-naqīḍāni lam yumkin an yaṣḍuqā ma'an*]. Rather, the antecedent and the consequent are assumed [*yafrūdu*] to have the quality [*kayfiyyatihimā*] that they have only in so far as they are taken to be so by hypothesis [*bi-l-waḍ'*], not in so far as they are inescapably true in themselves [*lā 'alā annahumā ṣaḥīḥāni fī anfusihiḥimā ḥāla*]. It is for this reason that every conditional syllogism (*qiyās ṣarṭī*) is also a syllogism from a hypothesis [*qiyās bi-l-waḍ'*], since the two components of the conditional particle—the antecedent and the consequent—are hypothesized in such a way that neither one of them has to be true according to the one who hypothesized them.³⁴⁹

In this important passage, Alfarabi is trying to give a precise answer to what it means to say that a conditional proposition is true. The essence of his claim is that saying that a conditional such as "if the sun is up, then it is day" is true does not entail that either "the sun is up" or "it is day" is true. In Alfarabi's view, to say that a conditional is true should only be taken to mean that there is a certain connection (*ittiṣāl*) between the states-of-affairs expressed by the two propositions, *not* that the antecedent on its own, or the consequent on its own, is true. For our purposes, this intuition by Alfarabi is crucial since, at least for Martin, the ability to distinguish the truth or falsity of the component propositions from the truth or falsity of the conditional as a whole is a *sine qua non* of recognition of the Frege point.³⁵⁰ Nor is this merely an off-hand

³⁴⁹ Alfarabi, *Ġadal*, 103.

³⁵⁰ Thus, Martin claims (Martin, "Logic", 166f) that, unlike Boethius, Abelard clearly and consciously makes the required distinction.

remark from Alfarabi. The *ad impossibile* argument that Alfarabi presents in this passage is presented in order to show that making a clear analytical distinction between, on the one hand, the speaker's assertion of the conditional, and the assertion of either of its parts in the minor premise (i.e. the premise to which "but" attaches) and conclusion, on the other, is necessary to preserving the validity of *modus tollens*. If we assume such a distinction is not available, then to assert a conditional such as "if the sun is out, then it is day" is also to assert either or both of "the sun is out" and "it is day". On this assumption, asserting the contradictory of the consequent "but it is not day" as required by *modus tollens* would not yield "then the sun is not out" as a conclusion, but would rather be tantamount to the speaker's first asserting that it is day and then asserting that it is not day immediately after. Alfarabi's conclusion to this *reductio* argument is telling. While the conditional proposition as a compound unit can have a definite truth value, the antecedent and consequent qua parts of the conditional do not. If the antecedent or consequent qua member of the conditional is formally affirmative (or negative) in quality, this does not authorize the conclusion that the affirmatively stated antecedent, for example, is true or that the negatively stated antecedent is false. Rather, "the antecedent and the consequent are assumed (*yafrūdu*) to have the quality (*kayfiyyatihimā*) that they have only in so far as they are taken to be so by hypothesis (*bi-l-waḍ'*), not in so far as they are necessarily true in themselves". Thus, at the very least, Alfarabi seems to put forward a theory about the proposition that allows him, one, to identify some linguistic entity or something grasped by the mind of the listener, which is expressed by the proposition the speaker puts forward for consideration by the listener hypothetically or *non-assertorically*; two, to hold that a proposition has objective truth value when considered on its own (*fī nafsīhi*), particularly in assertoric speech contexts; and three, to maintain the truth-value of the

compound conditional with the explicit denial of truth-value to the composite parts of the conditional qua antecedent and consequent of a conditional.

The above three consequences of Alfarabi's treatment of conditional propositions allow Alfarabi to treat the constituent parts of a conditional as unitary carriers of meaning that remain unchanged when they are introduced into a hypothetical speech context, and when they are introduced into an assertive speech context. In the above passage, Alfarabi shows himself clearly cognizant of the fact that in *modus ponens*, for example, what the antecedent means is identical to what the asserted minor premise means. The difference between them is only noticeable at the level of usage: qua antecedent, the propositional content is hypothesized; qua minor premise, it is asserted (*yustaṭnā*).³⁵¹ For this reason, I think it is not inaccurate to say that Alfarabi, unconsciously or consciously, has an understanding of what is now called propositional content in the technical sense described by Peter Geach and Christopher Martin. Nevertheless, I do *not* think it would be correct to say that Alfarabi's treatment of this particular view of propositionality evolved from his thinking about the nature of conditional propositions and conditionality, as it evolved in Avicenna's ŠQ V and VIII. Rather, I claim that Alfarabi's discussion in which he formulates what is, in effect, the Frege point, evolves from his working under conceptual conditions dictated by the context theory of logic (see chapter 2). Of course, Avicenna was working within the confines of context theory too. However, in developing his thoughts about conditionals, Avicenna puts forward the idea that, from a logical point of view, *any sentence* expresses a unitary content that is importable to many different speech acts as a property of propositions. The distinction made by Alfarabi seems to arise from his analysis of what I called in chapter 2 the dialectical

³⁵¹ Shehaby usually translates "*yustaṭnā*", and then follows with the parenthetical clarification "(asserted)". In order to avoid unnecessary redundancy, I see no reason why we cannot simply translate "*yustaṭnā*" as "asserted" in the appropriate contexts.

context of a debate. In chapter 2, I pointed out that Alfarabi's logical theory allows us to distinguish the objective truth values of the sentences that eventually come to serve as antecedent or consequent in a conditional from their assertion by parties to the dialectical exchange. This distinction between the proposition's objective truth value on the one hand and the reasoner's giving his assent to the proposition does not arise from Alfarabi's analysis of what it means for a sentence to be conditional. Rather, in a dialectical exchange between two opponents, when one uses a topos to build an argument that enlists the aid of modus tollens or modus ponens, the antecedent and consequent turn out to be unasserted, but only because what they assert is precisely what the questioner and the respondent are trying to establish. Thus, the distinction between propositional force and content for arises in Alfarabi's thinking from the dialectical conditions under which a dialectical exchange takes place. With this in mind, we can perhaps account further for a curious feature of Alfarabi's conditional syllogistic alluded to in chapter 2. As far as the extant texts reveal his complete system, Alfarabi does not allow or even consider the possibility of nested conditionals as premises or conclusions in his conditional syllogistic. (Nested conditionals are conditional sentences whose antecedent or consequent (or both) are themselves conditionals. He does, however, allow complex but unnested antecedents and consequents, e.g. "if P and Q and R and S..., then V", or "if P, then Q and R and S and T...".³⁵²) In §2, I partially accounted for this peculiarity of Alfarabi's treatment by saying that the quaesitum-responsum approach to the use of conditionals makes it so that Alfarabi uses conditionals chiefly as a means for arguing about the truth or falsity of category-like predications. I said that the consequence of this tendency is that conditionals themselves

³⁵² Consult *al-Qiyās al-Ṣaḡīr*: Alfarabi, *Al-Mantiq 'inda l-Fārābī*, vol. 2, ed. R. 'Aḡam, (Beirut: Dār al-Mašriq, 1985), 84; idem., *Al-Fārābī's Short Commentary on Aristotle's Prior Analytics*, trans. N. Rescher (Pittsburgh: University of Pittsburgh Press, 1963), 76; and *Al-Madḥal ilā l-Qiyās* mentions speaks in a similar fashion about disjunctive conditionals: idem., *Al-Mantiq 'inda l-Fārābī*, vol. 2, ed. R. 'Aḡam, (Beirut: Dār al-Mašriq, 1985), 32f.

do not tend to appear as antecedent and consequents. Only *quaestia* of disjunctions of *categorical* propositions appear in Alfarabi's logical treatises. As such, the question of whether or not the "if, then" grammatical particle can act like a propositional connective by joining together *arbitrarily complex* propositional contents is simply not at stake in Alfarabi's way of thinking. To put it more simply, I claim that the distinction between a proposition's force and its content arises in Alfarabi's discussion from the logical requirement that a dialectical exchange be a non-question-begging activity, and also that Alfarabi's chief syntactic concerns are motivated by debates about category-like predicates. Thus, Alfarabi tends to think of conditionals as composites of simple, categorical antecedents and consequents, rather than composites of arbitrary complexity. This being the case, Alfarabi does not develop a calculus that allows for the manipulation of such arbitrarily complex conditionals. Thus, the evidence suggests that while Alfarabi upholds condition FP, he formulates it for different reasons than Avicenna, which lead Alfarabi to not extend this insight into a theory of logical connectives as required by condition PropCon.

§4.2 CONDITION FP: AVICENNA ON DOUBT AND CONDITIONALITY

In §3, I dealt extensively with Avicenna's discussion in ŠQ V, 1 about the sense in which a connective conditional proposition is *connective*. By doing so, I bracketed off Avicenna's important discussion of the sense in which a connective conditional proposition is *conditional*. As I noted in §3, Avicenna's decisive rejection of the dichotomy of perfect and imperfect connection as unsuited to the formal analysis of conditional propositions is extraordinary not only for its subtlety, but also for its being a quite radical departure from a long and respected logical tradition in late antiquity of classifying the connection between the antecedent and consequent in conditional propositions. A similar observation can be made with respect to

Avicenna's discussion of conditionality. In ŠQ V 1, Avicenna reports the view of an unnamed authority who is said to have held that—at this point in the text, Avicenna does not say explicitly whether he feels the view to be sound or not³⁵³—“the connective conditional proposition is *conditional* if its antecedent is like the statement about which there is uncertainty”.³⁵⁴ Maróth makes two important observations about this passage. The first is historical. The opinion that the antecedent in a conditional proposition must be uncertain is, according to Maróth, well-attested in the works of ancient and late-antique Peripatetics. Maróth reports that a similar criterion for the conditionality of a conditional was held by Theophrastus, Alexander, Ammonius, and Philoponus, and that Boethius also discusses the role of uncertainty in conditional propositions. The second observation, which is for our purposes more important, is that the idea of the uncertain premise, in opposition to the certain premise, strongly suggests that the late-antique thinkers before Avicenna formulated their doctrines about conditional propositions in the context of their discussion of non-demonstrative syllogisms such as dialectical and rhetorical syllogisms. I believe Maróth is quite right to take the term “*al-maškūk fīhi*” as a technical term for premises in rhetorical and dialectical arguments.³⁵⁵ Maróth's remarks suggest that by the phrase “an uncertain statement”, Avicenna is referring to the antecedent's status in a conditional proposition as a

³⁵³ Nevertheless, Avicenna uses the verb *ẓanna/yazunnu* which is often used to advert to the fact that he thinks the opinion being reported is unsound or simply wrong; Avicenna, ŠQ V, 233.10. In his discussion (Maróth, *Aussagenlogik*, 120-32) of the element of doubt or uncertainty (Zweifel) that characterizes conditionals, Miklós Maróth does not alert the reader to the fact that, as we will see, Avicenna is *staunchly opposed* to characterizing conditionals in this way.

³⁵⁴ Avicenna, ŠQ V, 233.10-1: *wa-ẓanna ba'dhum anna [l-qadīyyata] š-šarṭiyyata l-muttaṣilata innamā takūnu šarṭiyyatan bi-an yakūna muqaddimuhā ka-[l-qawli]-l-maškūki fīhi*. Cf. Maróth, *Aussagenlogik*, 126. Maróth's translation reads the passage as making a slightly stronger statement: “some believe that the hypothetical conditional proposition is hypothetical because the proposition is uncertain (Zweifelhaft)”.

³⁵⁵ Thus, Maróth sometimes translates the term “*maškūk fīhi*” as *dubium* as in Boethius; see Maróth, *Aussagenlogik*, 129.

thesis (*thesis*, *wadʿ*, *assumptio*³⁵⁶) that is set down by two interlocutors in order to verify its truth or falsehood by means of a dialectical exchange. This view that the antecedent must be uncertain or disputed in order for a proposition to be considered a conditional that is suited to dialectical exchange appears to have been held by several well-known Peripatetic philosophers. Maróth cites Ammonius, who says that the antecedent is called the *metalambanomenon* “because of [its] uncertain and disputed status”.³⁵⁷ In his commentary on the *Prior Analytics*, Alexander notes that Theophrastus considered it unnecessary to demonstrate a conditional proposition whose antecedent is self-evident,³⁵⁸ and Alexander himself holds that “when everything is obvious and, as a consequence, the proof of the assumption (sc. *assumptio*) is rendered unnecessary, there is no syllogism either”.³⁵⁹ As I mention in §2 with regard to Alfarabi, the late-antique Greek antique logicians also appear to have the following thought in mind. Every *quaesitum* is a disjunction with interrogative force, e.g. “Is all justice good or not?” From this, a pair of contradictory or contrary theses—in this case, the interlocutors may decide to prove (or refute) either of the pair of theses “all justice is good”, or “some justice is not good”—may be adopted by the interlocutors for the sake of argument. If, for example, the interlocutors decide to investigate the thesis that all justice is good, then “all justice is good” will serve as an antecedent whose truth and falsehood are in question. The Peripatetics mentioned above, then, appear to hold that if the point (the hypothesis or *wadʿ*, which is one of the pair of disjuncts) under dispute is in fact self-evidently true or false, then no one is served by setting it out as a disputed thesis in the first place, nor is

³⁵⁶ See Maróth, *Aussagenlogik*, 132.

³⁵⁷ Maróth, *Aussagenlogik*, 122.

³⁵⁸ Ibid.

³⁵⁹ Ibid.

there any point in placing it as an antecedent in a conditional proposition, or in constructing a syllogism proving or disproving it.

In his thinking about conditionals in ŠQ V, Avicenna attempts to move away from a formal logical analysis of conditional propositions, rooted in a theory of dialectic, that *requires* that the antecedent be uncertain in order for the conditional to be included in a deduction. Rather, Avicenna wants to distinguish between, on the one hand, what the reasoner knows independently (lit. “from outside (*min ḥāriġ*)”) about the truth-status of the antecedent and consequent (or the grade of certainty or uncertainty about their truth status) and, on the other, their formal status as antecedent and consequent. Thus, in line with Alfarabi’s statement in *Ġadal* discussed above and in chapter 2, Avicenna also claims that a sentence qua antecedent in a connective conditional proposition “only indicates the hypothesis (*al-waḍʿ*) without indicating that the hypothesized antecedent is or is not”.³⁶⁰ Against ancient Peripatetic authorities, then, when considering a sentence in the antecedent position of a conditional, Avicenna wants to eliminate from consideration any question of the modality of the reasoner’s knowledge (certain, uncertain, non-committal) about whether the sentence is in line with the actual states-of-affairs. But he also wants to eliminate from consideration the question of whether or not the sentence itself is in line with the actual state of affairs. With respect to the analysis of the antecedent of a conditional proposition, Avicenna wants to eliminate from the logical analysis both a consideration of our cognitive attitude toward the truth of the antecedent as well as a consideration of its truth as such, leaving the reasoner to consider the antecedent qua hypothesis and nothing more.

[Text 3] It is not the case that if we say, “if such-and-such is, then such-and-such is”, that we mean that such-and-such actually is, such that the meaning of this

³⁶⁰ Avicenna, ŠQ V, 235.12-3: *wa-l-muqaddamu fi š-šarṭi l-muttaṣili yadullu ‘alā l-waḍʿi faqaṭ laysa fihi anna l-muqaddama al-mawḍū‘a mawḡūdun aw laysa bi-mawḡūdin.*

[sc. “if such-and-such is, then such-and-such is] would [according to this interpretation] be that such-and-such actually is and, along with it, such-and-such actually is, with the result that the antecedent would be a true proposition in itself, and the consequent would be true in itself.³⁶¹

For Avicenna, a necessary condition for a sentence to qualify as a proposition (*qaḍiyya*) is that it be receptive of true and false judgment. Thus, there is a sense in which Avicenna denies that the sentences serving as antecedent and consequent are propositions, strictly speaking, because when they are considered *qua antecedent* or *qua consequent*, they are denied truth values. Of course, such a sentence considered in itself will be the object of assent (or denial, *takḍīb*) insofar as we can assess whether or not the thought expressed in the sentence stands in line with the actual states-of-affairs. Similarly, considered in itself, the reasoner will be able to adopt a cognitive attitude (certainty, doubt, uncertainty, ambivalence, indifference) toward the truth or falsity of the thought contained in the sentence.

[Text 4] If we look at the antecedent and consequent from outside [*min ḥāriğ*, viz. not *qua antecedent* and *consequent*] then the consequent might become uncertain, if the aim of [introducing it into an argument] is to produce it as a conclusion. Or the antecedent [might become uncertain] if the aim of [introducing it into an argument] is to show it is false.³⁶²

Looking “at the antecedent and consequent from outside” means to evaluate what the antecedent and consequent express against the actual states of affairs. By doing so, we commit ourselves both to their truth or falsehood and a conviction about their truth and falsehood. However, the mind may regard these sentences *qua antecedent* and *consequent* (regardless of what we think about them objectively) as mere thoughts

³⁶¹ Avicenna, ŠQ V, 235.13-6: *fa-laysa idā qulnā in kāna kaḍā kāna kaḍā huwa anna kaḍā yurīdu an yakūna hattā yakūna ma'nā hāḍā anna kaḍā yurīdu an yakūna wa-ma'ahu kaḍā yurīdu an yakūna fa-yakūna l-muqaddamu fī nafsīhi qaḍiyyatan šādiqatan wa-t-tālī šādiqan.*

³⁶² Avicenna, ŠQ V, 236.19-20: *wa-ammā idā naẓara ilayhimā min ḥāriğin fa-rubbamā šāra t-tālī huwa l-maškūku fīhi idā kāna l-qaṣḍu muttağihan naḥwa intāğīhi awi l-muqaddamu idā kāna l-qaṣḍu muttağihan naḥwa ibṭālīhi.*

expressed by the antecedent and consequent to which we may merely “turn our attention (*al-iltifātu ilayhi*)” in a way that neither implies our commitment to their truth or falsehood in an argument, or our subjective feeling (certainty, doubt, etc.) that they are true or false. Consider Avicenna’s analysis of the nature of the consequent and then the antecedent of a connective conditional. About the consequent, Avicenna says

[Text 5] it has neither truth nor falsity, until the hypothesis [*al-waḍ‘ al-mawḍū‘*, viz. the antecedent] is known, even if <the consequent> is true or false taken on its own. Nor is the antecedent qua antecedent subject to doubt or assent, but only to our attending to it insofar as it implies or does not imply the consequent. Thus, <considered in itself>, it may happen that there is no doubt about [the antecedent’s] falsity, such as in the expression “if ten were odd, then it would not be divisible in two”. Or yet, [an antecedent] may be hypothesized as established and true in itself as a means of verifying the consequent. However, considered as elements of a conditional expression, neither the antecedent nor the consequent require <that they be subject to assent>. Thus, neither of them is such that they are subject to assent when they are considered as antecedent or consequent. And what is not <subject to assent> is not subject to uncertainty.³⁶³

Against previous Peripatetic logicians, Avicenna allows sentences to serve as antecedents whose truth values are not, in themselves, genuinely subject to dispute. Avicenna allows the sentence “ten is odd” to serve as the clearly false antecedent in the (true) restricted conditional “if ten is odd, then it is not divisible by two”. In this example there is obviously no sense in which the antecedent’s truth-value is uncertain, but Avicenna remains adamant that “ten is odd” can, in spite of the sentence’s clear falsehood, play the role of an antecedent in a (true) genuine connective conditional. Avicenna, then, clearly rejects the widely held view that a conditional is a genuine

³⁶³ Avicenna, ŠQ V, 236.10-8: *wa-hādā lā šidqa fihi wa-lā kaḍiba ḥattā yu‘lama l-waḍ‘u l-mawḍū‘u wa-in kāna nafsu qawlinā yakūnu šādiqan aw kāḍiban waḥdahu wa-laysa l-muqaddamu ayḍan ma‘riḍan min ḥaytu huwa muqaddamun li-š-šakki fihi aw li-t-taṣḍīqi lahu bal innamā l-iltifātu ilayhi min ḥaytu huwa muqaddamun anna t-tālī yalzamuhu aw lā yalzamuhu fa-rubbamā kāna ḡayra maškūkin fi buṭlānihi ka-qawlinā in kānati l-‘aṣratu fardan fa-lā niṣfa lahu bal rubbamā kāna waḍ‘uhu ‘alā annahu ṭābitun ḥaqququn fi nafsīhi li-yaṣiḥḥa bihi t-tālī wa-ammā min ḥaytu huwa šartīyyun fa-laysa l-muqaddamu minhu wa-lā t-tālī taqtaḍīhi fa-laysa aḥaduhumā bi-ḥaytu yaṣduqu bihi wa-huwa muqaddimun aw tālin wa-mā lam yakun kaḍālīka fa-laysa maškūkan fihi.*

conditional only if the antecedent is in doubt. Instead, Avicenna claims that what qualifies a connective conditional proposition as *conditional* is the recognition that the consequent follows from the antecedent, on the assumption that the antecedent holds. In order to make this argument, Avicenna resorts to several important distinctions about the nature of propositions. One is Avicenna's distinction between an expression considered in itself, and an expression considered qua its role as an antecedent or consequent in a connective conditional proposition. When a sentence such as "ten is odd" is considered in itself, it becomes a proposition (*qadiyya*) properly speaking because it becomes an object that is subject to assent and denial according to whether or not ten's being an odd number is in line with what is actually the case. But certainly it is not, so the proposition "ten is odd", is false and subject to denial or the assent of its contradictory, and also subject to our certainty about its being false. On the other hand, according to Avicenna, expressions are not genuine propositions when they fail to be subject to truth and falsehood. This happens, *inter alia*, when they serve as antecedents and consequents in conditional propositions. When a sentence appears as antecedent or consequent in a conditional, the mind merely "attends to" whatever thought is expressed by the antecedent or consequent expression, but the reasoner in effect suspends adopting a definite cognitive attitude toward it.

Thus, like Alfarabi, Avicenna holds a view of the proposition that allows us to make a sharp distinction between, on the one hand, what is expressed in the antecedent and consequent of a conditional, and the truth of what the antecedent and consequent express. As I said in §3, Avicenna like Alfarabi believes that a conditional ("if A is B, then J is D") as a single propositional unit is true and false, but its being true or false does not

depend on the truth-value of either the antecedent or the consequent: as we have seen, for Avicenna, the antecedent and consequent qua components of a conditional do not have truth-values at all. Avicenna says:

[Text 6] It is necessary that the expression signifying that something is true with something else, and that whenever the first is true, the other is true—such an expression must be a proposition [i.e. subject to truth and falsehood], but <an expression with such a meaning> cannot be counted among the propositions that are categorical in form [*ḥamliyyāt*]. Thus, <such expressions> must be counted among the propositions that are conditional in form [*ṣartīyyāt*], and <in particular> among the *connective* <conditional expressions>.³⁶⁴

According to Avicenna, when a speaker asserts a sentence that has the form of a conditional, we do not take him to be asserting the antecedent or the consequent. Rather, we understand the speaker to be asserting that a relation of strong concomitance holds (see §3 for details) between what the antecedent expresses and what the consequent expresses, such that the consequent is true under all conditions in which the antecedent is assumed to be true. The assertion is that a relation holds between two hypothetical states of affairs; it is not about whether or not the states of affairs as such themselves hold. Indeed, the example of the antecedent sentence “ten is odd” and the consequent “ten is not divisible by two” illustrates the point that conditionals are expressions about relations between states of affairs, not about states of affairs.³⁶⁵ Avicenna calls the conditional “if ten is odd, then it is not divisible by two” true, but makes no recourse to the truth or falsity of “ten is odd” or “ten is not divisible by two” to establish its truth. Rather, “ten is not divisible by two” is true whenever we

³⁶⁴ Avicenna, ŠQ V, 237.1-4.

³⁶⁵ Here, I intend “states-of-affairs” to have the same broad sense that Avicenna uses the word “states (*aḥwāl*)” and “conditions (*ṣurūt*)” in ŠQ V 4, in which states-of-affairs should not be understood as implying that what the antecedent and consequent express has any correlate in extramental existence. In ŠQ V 4, in which Avicenna sets out his theory of quantified conditionals (see §5), the universal and particular quantifiers quantify over precisely these real or hypothetical “states” and “conditions”. These can be states like the sun’s being up, or purely hypothetical “states” such as two’s being odd.

hypothesize that ten is an odd number, for on this hypothesis, and based on the definition of an odd number, the consequent clearly follows.

Thus, and again similar to Alfarabi, Avicenna formulates a doctrine in which what a sentence expresses—in Avicenna’s words, that which the mind “attends to”—maintains its unitary meaning such that it is unaffected by its hypothetical deployment as an antecedent or consequent, and then its deployment as an asserted minor premise, or its assertion as a conclusion. The main evidence for this claim is that Avicenna explicitly denies that the assertion of the conditional as a single unit can be taken as entailing the assertion of the antecedent or the consequent. In the deduction of “it is day” from the premise set “if the sun is up, then it is day; but, the sun is up”, the expression “the sun is up” appears twice. In the antecedent, Avicenna claims that it is deployed non-assertorically, viz. qua antecedent in a conditional that asserts no more than that the consequent is true *on the admission that* the antecedent is true. In its second occurrence, however, “the sun is up” is considered “in itself (*fī nafsīhi*)”, that is separate from its conditional context, and hence, assertorically. The failure to make the distinction between “the sun is up” when it is used to make a law-like statement about a relation between contingent states of affairs and “the sun is up” as used to make a statement about what is currently the case is a failure to recognize the distinction between a proposition’s content and its force. Where Boethius failed, Alfarabi and Avicenna succeeded.

Let us take stock. In §4.1 I noted Christopher Martin’s observation that any theory of sentential meaning must carry out two tasks. First, it must account for the production of compound meaning from simpler semantic units. Second, it must account for the stability of

simple and compound semantic units in the various speech acts that sentences are used in. Martin notes that Frege was able to develop a theory of language that performed these tasks by urging that a distinction be made between a proposition's content ("a form of words in which something is propounded") and its force. In the field of logic, recognition of what Geach calls the "Frege point" allowed logicians to develop a theory of propositional connectives, and hence a propositional logic in the technical sense of the term. In §4.1 and §4.2, I have suggested that Alfarabi and Avicenna understood the difference between a proposition's content and its force, to the extent that they both see that it is possible to entertain in the mind—in Avicenna's words, to "attend to"—the meaning of a sentence in a non-assertoric speech context (e.g. the supposed antecedent in a conditional), and to assert a sentence expressing the same meaning in an assertoric speech context (e.g. second premise in a deduction in *modus ponens*). To this extent it can be said that Alfarabi and Avicenna are aware of what is, in effect, the Frege point.

Nevertheless, in §4.2 I have tried to show too that Avicenna's motivations for formulating this doctrine differ substantially from Alfarabi's. Alfarabi shares with ancient (e.g. Theophrastus) and late-antique (e.g. Boethius, Alexander) thinkers the need to hold that the reason for the denial that antecedents and consequents *qua* members of a conditional have truth-values arises from their theory of the *topoi* and dialectic, in which conditional propositions are used to argue about and establish the truth or falsity of the conditional's antecedent and consequent. According to this way of thinking, since the antecedent and consequent of a conditional (or their contradictory opposites) are precisely what is at stake in the exchange between the questioner and respondent, the antecedent must be somehow in question or subject to doubt (*maškūk fīhi*). If not, i.e. if its truth-value has already been

established, or is self-evident, or more evident than the conclusion, for example, then there is no point in debating its truth or falsity in the first place.³⁶⁶ The view is not without merit, since, as Avicenna describes the justification for this view given by late-antique logicians, “if the antecedent were evident and plain, then its consequent, i.e. that which follows from it, would be evident and plain as well. [In this case,] it would be absurd to establish [the consequent] by means of a syllogism since syllogisms reveal the unknown, since what is [already] evident has no need to be deduced by means of a syllogism”.³⁶⁷ Yet, as we have seen, Avicenna shows himself diametrically opposed to this view of the nature of conditionality. The most obvious sign of his disagreement is his admission of self-evidently and even necessarily false antecedents in (true) conditional propositions. Ignoring for the moment the important question of why Avicenna proposes another such radical departure from late antique tradition,³⁶⁸ the relevant question here is: if the antecedent’s being subject or not subject to doubt is not a suitable criterion for the conditionality of a connective conditional proposition, then what is? As I explained in §4.2, the sole criterion for a compound sentence to be a connective conditional—a connective conditional is *conditional*—is that there be a connection of concomitance between the propositional content of the antecedent and the propositional content of the consequent, regardless of the state of affairs in extramental reality. The truth-value of a connective conditional is established by determining the nature of this concomitant relation between the antecedent and consequent. Note that the truth of the conditional does not depend on whether or not there are dialectical or rhetorical topoi

³⁶⁶ Notice the importance of Aristotle’s definition of demonstration at *An. Post.* A2 71b20. See J. Barnes, “Aristotle’s Theory of Demonstration”, *Phronesis* 14/2 (1969): 123-152, especially 124.

³⁶⁷ Avicenna, ŠQ IX, 416.4-6: *innahu law kāna l-muqaddamu bayyinan wāḍiḥan la-kāna lāzimuhu wa-tābi’uhu bayyinan wāḍiḥan wa-kāna fī itbātihi bi-l-qiyāsi muḥālan idī l-qiyāsu yubayyinu l-ḥafīyya fa-ammā l-bayyinu fa-lā ḥāḡata bihi ilā an yuqāsa ‘alayhi.*

³⁶⁸ See §5 below.

that generate true conditionals or not.³⁶⁹ Nor, as I pointed out in §3, does it matter what the nature of the connection between the antecedent and consequent is. As Avicenna says (ŠQ V, 237.10), “the criterion for the truth [of a conditional] is simply the existence of some sort of connection (*fa-inna l-mu‘ayyinata fī ṣ-ṣidqi fī l-wuğūdi ‘alāqatun mā*)”, without our knowing the exact nature of the connection between the antecedent and the consequent, “whether it be a relation of predication, or a relation of genuine possession [e.g. rationality for Socrates], or a consequence of possession [e.g. risibility for Socrates], or because one is the cause and the other the effect, or because one is a whole and the other is part, or because one is a universal and the other is a particular, or similar relations”.³⁷⁰ For Avicenna a connective conditional is conditional because it is a compound sentence that signifies a connection of concomitance between the meaning signified by the antecedent and the consequent. It is a *true* connective conditional proposition if this concomitance holds under all conditions that we add to the antecedent. What is more, the assertability of this conditional must not be taken to depend on the assertability of its antecedent or consequent, but only on the speaker’s recognition that the connection of concomitance holds. The objective truth-values of the antecedent and consequent are irrelevant to the valuation of the compound whose constituents they are.

Avicenna’s discussion of condition FP arises, broadly speaking, from a wider concern about the nature of conditional propositions’ conditionality and what it means to say that a *logical* connection exists between the antecedent and consequent. In other words, it does not arise primarily from a concern about the dialectical uses to which conditionals are put, as in Alfarabi and other ancient and late-antique Greek and Latin authors. In sum, what distinguishes Alfarabi’s and Avicenna’s treatment of conditional FP is the context in which the

³⁶⁹ Unlike Boethius; see Martin, ‘Logic’, 174f; see also Martin, ‘The Logic of Negation in Boethius’, 290.

³⁷⁰ Avicenna, ŠQ V, 237.5–7.

two thinkers develop their thoughts on this issue. For Alfarabi the question is relevant to a proper description of the practice of the syllogistic arts; for Avicenna his discussion of condition FP arises from his thinking about the nature of conditional propositions as such. The logical ramifications of this difference in their motivations will become clearer in the following section.

§4.3 CONDITION WFF: AVICENNA ON NESTED CONDITIONALS AND REPETITIVE SYLLOGISMS (*QIYĀSĀT ISTIṬNĀ' IYYA*)

Despite the fact that Alfarabi and Avicenna appear to have understood the difference between a proposition's force and its content, a distinction which lies at the heart of condition FP, I still claim that Avicenna has a propositional logic and Alfarabi does not. As I explained in §4.1, for a logic of compound sentences to be called a propositional logic in the technical sense given to it by contemporary philosophers, it must have a notion of propositional connectives. In contrast to sentential connectives (they could also be called “grammatical connectives”) such as “if, then”, “or”, and “and” that join together verbal utterances to form compound utterances of arbitrary complexity, propositional connectives must join together propositional content to form new, compound propositional content of arbitrary complexity. It is not sufficient that, e.g. a conditional composed of categorical antecedent and consequent, such as “if A is B, then C is D”, be considered well-formed formulae. Nested conditionals, e.g. “if if A is B, then C is D, then if E is F, then G is H”, also must be allowed to serve as premises and conclusions as well. In §2 and in greater detail in §4.1, I suggested that Alfarabi's interest in conditionals arises primarily from his concern with the *topoi*, which are used to settle questions about whether or not a certain predicate from one of the ten categories belongs or does not belong to a certain subject. As a consequence, this format for argumentative

exchange tends to discourage questions that ask about concomitance, e.g. causality, which are most naturally treated, from the standpoint of a formal language, in the language of conditionals. Thus, the motive to develop a conditional syllogistic with conditionals as antecedent, consequent, or conclusion does not seem to have been a pressing concern in Alfarabi's eyes. The fact that there is no evidence that Alfarabi recognized nested conditionals of any sort suggests that he did not treat grammatical connectives as genuine propositional connectives because, in his view, these connectives only join together atomic, categorical propositions rather than propositions of arbitrary complexity.

Yet, recognition of the well-formedness of nested conditionals is a necessary, but not a sufficient condition for “if, then” as a sentential connective to operate as a genuine propositional connective. Boethius recognized the well-formedness of a large number of nested conditionals as premises, and even developed a rudimentary calculus for generating nested conditionals as conclusions.³⁷¹ In spite of this, Christopher Martin has insisted that Boethius' account of hypothetical syllogisms shows no cognizance of “the propositional compounding of propositional contents to form new contents of arbitrary complexity”.³⁷² If the claim that neither Alfarabi nor Boethius had a propositional logic is unsustainable, is the same true of Avicenna?

In other words, is there evidence that Avicenna recognizes that at least one sentential connective such as “if, then” functions as propositional connective because it joins together propositional content to form a new, compound propositional content of arbitrary complexity? In order to answer this question, we first need to make precise what the qualification “of arbitrary complexity” means. For Martin, this means that Avicenna would

³⁷¹ Such a calculus was available even as early as Theophrastus (d. 287 BC); see J. Barnes, *Terms and Sentences: Theophrastus on Hypothetical Syllogisms* (London: British Academy, 1984).

³⁷² Martin, “Logic”, 163.

have to have a principle of substitution. Stated in the language of logical functions, Martin says that propositional logics are characterized by a “principle of substitution: If ‘p’ and ‘F(p)’ are propositional contents, then so is ‘F(q)’ where ‘q’ is any propositional content”.³⁷³ For Avicenna’s “if, then” propositions to fulfill this condition with respect to “if, then” connectives, they must behave according to the following principle: the sentential connective “if, then” will be a genuine propositional connective if whenever “P”, “Q”, and “if P, then Q” are propositional contents, then so is “if R, then S”, where “R” and “S” are any propositional content. Translating Martin’s formulation into terms that are suitable to Avicenna’s conditional syllogistic, the principle of substitution states any proposition must be able to serve in the antecedent and consequent place of an “if, then” expression, and that the new proposition arising from the combination must be reintroducible into the antecedent or consequent place of another conditional, and that this substitution be indefinitely repeatable. Furthermore, however complex the compound proposition might be, the final product of these indefinitely repeated substitution steps must still be a well-formed formula, and, therefore subject to the rules of contradiction and conversion, and introducible as a premise or conclusion of a syllogism.

In fact, Avicenna is explicit in urging us to treat “if, then” in just such a fashion. However, this is not immediately apparent from Avicenna’s initial comments in ŠQ V, where he only notes that the main connective in connective conditional expression (*qawl muttaṣil*) can join together categorical, connective conditional, and disjunctive conditional sentences (*munfaṣil*).³⁷⁴ As an example of a nested conditional, Avicenna provides the example “if whenever it is day, the sun is up, then whenever it is night, then the sun has set [*in kāna*

³⁷³ Martin, ‘The Logic of Negation in Boethius’, 281f.

³⁷⁴ Avicenna, ŠQ V, 253.4-8.

*kullamā kāna nahārun kānati š-šamsu ṭālī‘atan, fa-kullamā kāna laylun kānati š-šamsu ġāribatan]”.*³⁷⁵

Unlike Alfarabi, Avicenna explicitly recognizes the well-formedness of nested conditional propositions. Nevertheless, on their own, Avicenna’s words do not show that he sees the sentential connective “if, then” as capable of operating on propositional content of arbitrary complexity. Avicenna is more explicit, however, in his discussion of repetitive syllogisms (*qiyās istiṭnā’i*) in ŠQ VIII. There, Avicenna distinguishes between the different deductive schema according to the quality, viz. affirmative or negative, of the asserted minor premise (*al-mustaṭnā*, i.e. the non-conditional “repeated” premise that normally takes second place in the premise list). Thus, modus ponens is simply described as the conditional syllogism in which the minor premise (*mustaṭnā*) is “identical to the antecedent [*ayn al-muqaddam*]”; modus tollens is described as the syllogism in which the minor premise is “the contradiction of the consequent [*naqīḍ t-tālī*]”; and the inconcludent (*lā yuntiġu*) deductive form, which is today called “Affirming the Consequent”, is labeled as the argument scheme in which the minor premise asserts the “contradiction of the antecedent [*naqīḍ l-muqaddam*]”. In a terse summary of all the valid and invalid argument schema for repetitive syllogisms with connective conditional majors, Avicenna says:

[Text 7] Rather, you ought to know that [1] the assertion of the contradiction of the antecedent is inconcludent [*istiṭnā’u naqīḍi l-muqaddami lā yuntiġu*]; [2] the assertion of the antecedent yields the consequent [*istiṭnā’u l-muqaddami yuntiġu t-tālī*]; [3] the assertion of the consequent is inconcludent; [4] and the assertion of the contradictory of the consequent yields the contradiction of the antecedent.³⁷⁶

Based on the most common examples that Avicenna provides in ŠQ V, by “antecedent” and “consequent”, we expect that he is referring to categorical propositions that Avicenna formalizes as, for example, “A is B”, or in other places as “every A is B”; and by “contradiction”

³⁷⁵ Ibid., 7-8.

³⁷⁶ Avicenna, ŠQ VIII, 397.2-4; see also §3.

of the antecedent or consequent, he appears to be referring to sentences having the logical form, for example, “A is not B” or “not every A is B”. If we were to understand Avicenna’s words this way, however, then would be in error:

[Text 8] You should not give any heed to the view according to which the assertion [of the minor premise] can only be a categorical [*<al-muqaddimatu> l-istiṭnā’iyyatu lā takūnu illā ḥamliyyatan*]. Know, rather, that the asserted premise will be derived from the antecedent or the consequent [of the conditional itself] or its contradictory. After all, if the antecedent and consequent can be any type of premise, then so must the asserted [minor premise be able to take any of these sentential forms]. Thus, if a speaker says “if if the sun is up, then it is day, then the day is a consequence of the sun [*in kāna in ṭala’ati š-šamsu kāna nahāran*]”, and then one would like to assert the antecedent, then one would be asserting a conditional expression [viz. “if the sun is up, then it is day”].³⁷⁷

Thus, in Text 7 when Avicenna says that a syllogism in which “the assertion (*istiṭnā*) of the antecedent yields the consequent”, Avicenna means *any* proposition of whatever form, whether it be categorical, connective conditional, disjunctive conditional, or a complex compound of any of them; if its antecedent propositional content is asserted in the minor premise, then the content of the consequent, whatever its complexity, can be asserted as a conclusion. This is an important finding; in Avicenna’s description of *modus ponens* in Text 7 as the syllogism in which “the assertion of the antecedent yields the consequent”, the names “antecedent” and “consequent” have the generality of propositional variables such as “P” and “Q”, in the sense that “antecedent” and “consequent” in Avicenna’s language, like the propositional variables “P” and “Q”, are syntactic markers for sentences expressing propositional content of arbitrary complexity. It is thus Avicenna’s view that a sentence expressing propositional content of any sort can appear in a non-assertive speech context, namely as an antecedent in a conditional. Moreover, this same content can then be asserted (*yustaṭnā*) in the minor premise with assertive propositional force (whose presence is signaled

³⁷⁷ Ibid., 10-5.

by the repetitive particle “but...”) to generate the conclusion. In this case, the conclusion is a sentence with propositional content identical to the consequent, but differing only in the fact that it is being asserted in the conclusion, while it is unasserted qua member of the conditional.

With the above in mind, let us return to our discussion of Christopher Martin’s claim that for a logic of compound sentences to qualify as a propositional logic, it must be characterized by a principle of substitution. Martin claims that it is this principle of substitution that allows a propositional connective such as “if, then” to fill its antecedent and consequent positions with propositions of arbitrary complexity to generate a new, well-formed proposition that can serve as a conditional premise in a syllogism and be subject to the same rules of contradiction, conversion, and validity as “if, then” sentences composed of simple, categorical antecedents and consequents. Stating again the principle of substitution for “if, then” sentences, the grammatical connective “if, then” will be a genuine propositional connective if whenever “P”, “Q”, and “if P, then Q” are propositional contents, then so is “if R, then S”, where “R” and “S” are any propositional content. With respect to Avicenna, we should be clear that, at least in this context, it makes sense to use propositional variables such as “P” and “Q” as standing for “antecedent” and “consequent”, and as a way of referring to the antecedent and consequent places of Avicenna’s connective conditionals as he discusses them in Texts 7 and 8. When Avicenna describes *modus ponens* as the connective conditional syllogism in which “what is identical to the antecedent is asserted [*istiṭnā’u ‘ayni l-muqaddam*]”, he implicitly admits two things. One, he admits that the conditional “if P, then Q” can be an indifferently complex compound proposition composed of an antecedent “P” and a consequent “Q”, where “P” and “Q” are arbitrarily complex. This is tantamount to accepting

the rule of substitution. Yet, Avicenna also admits that the content expressed in “P” can appear in the antecedent place of the conditional unasserted, and asserted in the minor premise, and yet remain the same, objectively identical content throughout.

In sum, Avicenna’s terse statement of the valid connective conditional syllogism is formulated with a generality that allows all valid and invalid moods to be stated in four rules rather than dozens and dozens as in Boethius.³⁷⁸ This brevity is made possible by Avicenna’s theory of propositionality. Avicenna recognizes that the sentential connective “if, then”, when considered from the standpoint of a logic of compound sentences, connects together sentences in its antecedent and consequent position, viz. the sentences referred to by the terms “antecedent”, “consequent”, “contradiction of the antecedent”, and “contradiction of the consequent” in Text 7. Moreover, “if, then” in Avicenna’s thinking connects them together in such a way that the new compound itself can be (1) of arbitrary complexity, (2) well-formed, and (3) that what the antecedent and consequent express, regardless of their composition, is unasserted when it occupies the antecedent or consequent position in an “if, then” sentence, but asserted when it occupies the minor premise and conclusion position in a syllogism. In other words, Avicenna’s ability to treat antecedents and consequents with the generality required by the principle of substitution is made possible by his use of the difference between propositional content and force. Avicenna does not have a truth-functional propositional logic as Maróth and Shehaby have claimed; but he does have a propositional logic.

³⁷⁸ See Martin, “The Logic of Negation in Boethius”, 285: “Lacking propositional operations and substitution, Boethius must account for all of the various kinds of compound proposition in turn and he must do so for all combinations of quality among their components. Though he is certainly not clearly aware of these constraints, his treatment of compound propositions does to some extent conform to them. Hence the ‘striking and tedious feature’ of the exposition noted by the Kneales. In giving his account of compound propositions Boethius rings all possible changes on their components”. See also K. Dürr, *The Propositional Logic of Boethius* (Amsterdam: North-Holland Publishing, 1951), 32-58.

CHAPTER 5: AVICENNA ON *PRIOR ANALYTICS* A23: CONJUNCTIVE SYLLOGISMS, REPETITIVE SYLLOGISMS, AND REDUCTION

§5.1 INTRODUCTION

Tony Street recently noted that Avicenna's division of the syllogism into conjunctive and repetitive types is a distinctive feature of Avicenna's syllogistic.³⁷⁹ Khaled El-Rouayheb has pointed to the uniqueness of Avicenna's notion of quantified conditionals (a crucial ingredient in Avicenna's broader theory of the conjunctive syllogisms), observing that although post-classical Arabic logicians accepted Avicenna's conjunctive syllogistic theory in its broad outlines, and they adopted a critical attitude toward particular aspects of the theory.³⁸⁰ Beyond the general observation that Avicenna was trying to extend Aristotle's theory of categorical syllogisms to conditional syllogisms, no systematic attempt has yet been made to explain why Avicenna felt the need to introduce a new division of the syllogism.

If Avicenna's new division of the syllogism into conjunctive and repetitive types is new, many of the principles and arguments he invokes in order arrive at this novel division were present in Aristotle's commentators, particularly in Alexander of Aphrodisias' commentary on the *Prior Analytics*. Avicenna's relationship with Alexander is a complex one. He is critical of many of Alexander's theses about hypothetical proposition and syllogisms, even if he does not mention Alexander by name. He criticizes Alexander's understanding of conditionals as failing to distinguish between a conditional proposition "if P, then Q" and a conditional promise to accept Q on the conditional that P is demonstrated. He criticizes Alexander for saying that hypothetical syllogisms are not really syllogisms because they do not show anything. And he

³⁷⁹ T. Street, "Introduction" to Avicenna, *The Deliverance*, trans. A. Ahmed (Karachi: Oxford University Press, 2010), xxvi.

³⁸⁰ K. El-Rouayheb, "Impossible Antecedents and Their Consequence: Some Fourteenth-Century Discussions",

criticizes Alexander for failing to understand that conditionals as a premise and conclusion in a hypothetical syllogism may be asserted in the same way that categorical premises and conclusions can. Yet, Avicenna accepts much of Alexander's thought about hypothetical syllogisms. One of the major concerns for Avicenna is to show that repetitive syllogisms are useful or can generate new knowledge. Avicenna tries to do this by various means, especially by introducing the division of conditionals into *simpliciter* and restricted types. However, in this chapter we will see that Avicenna invokes an even strong principle that an argument is *invalid* (if not inconcludent) if no new knowledge is produced in the conclusion. This is readily on display in Alexander who argued against valid Stoic logical schema such as "if P, then P; but P. Therefore, P", are not syllogisms because they were useless, in the sense that they do not show anything.³⁸¹ As we will see, Avicenna takes Alexander's insistence that syllogisms must show something very seriously. Avicenna seems to accept this principle from Alexander by incorporating it as a principle of conclendency into his conjunctive syllogistic. Avicenna says, therefore, that the syllogism "if every human is risible, and everything risible is an animal, then every man is an animal" is not really a syllogism because there is a sense in which knowledge of the premises already presupposes knowledge of the conclusion.³⁸² Often, however, Avicenna argues against Alexander's stronger claims in order to make room for repetitive syllogisms in his vision of the syllogistic. Thus, in ŠQ IX Avicenna insists conjunctive syllogisms with all conditional premises and conclusions (what Bobzien calls "wholly hypothetical syllogisms") can *establish* conclusions that were not previously known in such a way that we can assert them with the same force as we assert the conclusion of a categorical syllogism, but that what we are asserting is a conditional proposition. This is

³⁸¹ M. Frede, "Stoic vs. Aristotelian Syllogistic", 24.

³⁸² Alexander argued this position too; *ibid.*, 26, 29.

against Alexander who had held that “strictly speaking, arguments with hypothetical conclusions, and especially the so-called totally hypothetical syllogisms, are not really syllogisms at all”.³⁸³

Under the heading of the conjunctive syllogism, Avicenna includes not only Aristotle’s categorical syllogisms, but also syllogisms with connective and disjunctive conditional premises and conclusions. Avicenna developed his general theory of conditional propositions for a specific reason. He wanted to incorporate them in a syllogistic theory committed to extending the foundational concepts in Aristotle’s categorical syllogistic, as developed in *An. Pr.* A1-7, to cover syllogisms with conditional premises and conclusions. These foundational concepts are, first, a theory of quantification and qualification that, while incompletely developed, nevertheless generated a square of opposition for conjunctive syllogisms that is analogous to Aristotle’s theory of A-, I-, E-, and O-categorical propositions. The second foundational concept Avicenna adopts is a general theory of the middle part, which generates conjunctive syllogistic figures and, in coordination with the theory of quantified conditionals, conjunctive syllogistic moods. The conclusency of all conjunctive syllogisms, whether composed of purely categorical premises and conclusions, or purely conditional premises and conclusions, or a mix of both, is a generalization of Aristotle’s idea of the middle term (*ḥadd*, *ḥudūd*), which is shared between premises of a categorical syllogism. Avicenna’s extension of this notion to syllogisms with conditional premises and conclusions allows him to set down the three syllogistic figures for all conjunctive syllogisms without qualification. As Paul Thom has shown, the concept of the middle part or shared term also extends the peculiar nature of logical validity in Aristotle’s categorical syllogistic to Avicenna’s own conjunctive syllogisms.

³⁸³ Ibid., 25.

Finally, Avicenna adopts the same methods of proving imperfect conjunctive syllogisms by means of direct or indirect reduction to perfect conjunctive syllogisms. The idea that the self-evident nature and non-perfectibility of the first-figure moods to moods that are more self-evident, the reducibility of imperfect moods to first-figure moods, the two-premise and three-term format of syllogistic arguments, and the kind of formal logical necessity that holds between the conclusion of a syllogistic argument and its premises, are generalized to all forms of conjunctive syllogisms, regardless of the formal differences of the premises and conclusions. Discussing these three components of Avicenna's conjunctive syllogisms will be the task of §5.2.

In §5.3 I will attempt to answer the obvious question: why does Avicenna do all this? There are two reasons. First, the peculiar way in which Avicenna read Aristotle's *An. Pr.* A23; and second, the widespread sentiment shared by many late-antique logicians including Avicenna, that the only correct account of logical following, viz. what conditions an argument form must fulfill in order to *be* a syllogism, is Aristotle's. The following thesis about Avicenna's conjunctive and repetitive syllogisms and what Avicenna was trying to accomplish in ŠQ IX 1 suggests itself. As we will see in §5.3, in the course of ŠQ V-IX, Avicenna is consciously trying to pry apart three distinct ways of thinking about logical following (*luzūm*), which on his account earlier authors conflated. (Type 1) The following of a consequent from an antecedent (e.g. on some reading of the proposition "if *P*, then *Q*", the consequent *Q* is said to *follow from* the antecedent *P*). (Type 2) The following of a conclusion *c* from its premises *P* in an Aristotelian syllogism (e.g. the conclusion *Aac* is said to follow from the premises *Aab* and *Abc* under certain precisely delineable conditions). (Type 3) The following of a conclusion *c* from its premises *P* in a conditional syllogism such as modus ponens or modus tollens (e.g. the

conclusion “not-*P*” is said to follow from the premises “if *P*, then *Q*” and “not-*Q*” on account of a “repetition”). For Avicenna’s part, in ŠQ V and VII he presents a theory of Type 1 following that applies to antecedent and consequents, but that can also be used to formalize Type 2 and Type 3 following (see chapters 3 and 4). Then in ŠQ VI he presents his theory of conjunctive syllogisms, which should be read as a general theory of Type 2 following that allows the reasoner to use conditionals with type 1 following in scientific arguments (see §5.2 below). In ŠQ VIII, he gives a somewhat dismissive account of Type 3 following, while showing that previous attempts that try to make Type 3 following fit the mold of Type 2 following are misguided (see chapter 3).

With these results in hand, Avicenna returns in ŠQ IX 1 to a basic dilemma in Aristotle’s *Prior Analytics*. In *An. Pr.* A7 Aristotle shows that all imperfect syllogisms can be made perfect by reduction to the two universal moods of the first-figure (Barbara and Celarent).³⁸⁴ In *An. Pr.* A23, which modern commentators agree should be read as following A7,³⁸⁵ Aristotle says that he will now show that “every syllogism without qualification can be so treated (40b20)”. By “every syllogism without qualification” Aristotle means not only categorical syllogisms with more than three premises, but per impossibile syllogisms and syllogisms from a hypothesis (40b25-31). Thus, 40b25-31 can be reasonably read as a claim that per impossibile syllogisms and hypothetical syllogisms in general can be reduced to the universal syllogisms of the first-figure. Yet, at *An. Pr.* A44 50a29-31 Aristotle seems to forcefully assert just the opposite: “Further we must not try to reduce hypothetical syllogisms; for with the given premises it is not possible to reduce them. For they have not been proved by deduction, but assented to by

³⁸⁴ R. Wisnovsky, “Arabic logicians on perfect and imperfect syllogisms: A supplement to Patzig’s ‘Historical Excursus’”, in *Avicenna and his Legacy: A Golden Age of Science and Philosophy*, ed. T. Langermann (Turnhout: Brepols, 2010), 257-273.

³⁸⁵ E.g. G. Striker, *Commentary on Aristotle, Prior Analytics Book A*, trans. G. Striker (Oxford: Clarendon Press, 2009), 190.

agreement". Thus, 50a29-31 can be plausibly read as an attempt to show that per impossibile and hypothetical syllogisms cannot be reduced to the universal syllogisms of the first-figure. Which view is the correct view? For his part, Avicenna seems to have adopted preferred to adopts An. Pr. A23 and ignore A44. The opening salvo of Avicenna's ŠQ IX 1 reads as follows:

[Text 1] Every syllogistic argument that produces a categorical <conclusion> is perfected by means of one of the three figures of the categorical syllogism. In general, repetitive syllogisms are perfected by means of conjunctive syllogisms when the aim is for the <repetitive> syllogism to be productive. We say: it has been proved that the per impossibile syllogism is perfected by the conjunctive syllogism and the repetitive syllogism [ŠQ VIII 4]. Further, it has been shown that the conditional syllogism's productivity is suited to being perfected by conjunctive syllogisms [ŠQ VI]. And since the discussion in the [*Prior Analytics*] is only about productive categorical syllogisms, the intended meaning of "conjunctive <syllogism>" and "categorical <syllogism>" is the same. We say: it has also been shown that the disjunctive repetitive syllogism is on account of a similarity between it and the connective [ŠQ VII and VIII]. And further, that the connective <conditional syllogism> in which the contradiction of the consequent is asserted is proved by means of <the syllogism> in which the asserted premise is the antecedent [ŠQ VIII]. Thus, if it be proved that the assertion of the antecedent in <the repetitive syllogism> only produces through a conjunctive syllogism, then this is shown for all conditional and categorical syllogisms.³⁸⁶

First, Avicenna must believe that Aristotle's proof of the reducibility of syllogisms from a hypothesis by means of perfect syllogisms is inadequate. We should therefore approach Avicenna's division of syllogisms into repetitive and conjunctive, his theory of quantified conditionals, and his theory of conjunctive syllogisms with an eye toward completing Aristotle's claim in A23. In ŠQ IX Avicenna claims that if it can be shown that modus ponens is perfectible by means of a conjunctive syllogism, then Aristotle's claim in A23 can be vindicated. In order to vindicate Aristotle, Avicenna is forced to reorganize and supplement considerably Aristotle's original theory of the syllogism with insights that would shape the contours of Arabic logic for centuries.

³⁸⁶ Avicenna, ŠQ IX, 415.6-6.3.

§5.2.1 INTRODUCTORY REMARKS ON AVICENNA'S CONJUNCTIVE SYLLOGISMS (QIYĀSĀT IQTIRĀNIYYA):

Avicenna opens ŠQ V with a declaration about his main objective for this chapter and the three chapters to follow:

[Text 2] Among premises there are those that are categorical or conditional. Similarly among quaestia there are those that are categorical and those that are conditional. Among the categorical <propositions> are those that are affirmed without a syllogism and those that need a syllogism <to be affirmed>. The case is similar among conditional <propositions>, for many theses in mathematics, physics, and metaphysics are connective and disjunctive conditional [in sentential form]. However, whereas categorical <propositions> can be proved by categorical and conditional syllogisms alike, conditional <propositions> are not produced from the categorical <sylogism> as you know. There are, therefore, conditional syllogisms that produce conditional <conclusions>, whether <the premises> are pure conditionals or a mix of <of categorical and conditional premises> as we shall demonstrate.³⁸⁷

As modern commentators have noted, the *Topics* served as an important source for ancient logicians such as Boethius who developed systems of hypothetical syllogistics based on the ideas contained in it.³⁸⁸ As we saw in §2, a similar claim could be made with respect to Alfarabi, since some of the essential features of his doctrines of conditional propositions and conditional syllogisms are located in and informed by doctrines drawn from his work on Aristotle's *Topics*. As we saw in §3 and §4, Avicenna believed that a “*Topics*-based” approach to developing a theory of conditionals is bad logic. If, as Avicenna thought, the *Topics* is not a sufficient source for developing a logic of conditionals, then the *Prior Analytics*, which presents no theory of conditional syllogisms at all, must be considered deficient in this respect. For it is obvious that there are scientific arguments in disciplines such as mathematics, physics, and metaphysics in which conditional sentences serve as both premises and conclusions.

³⁸⁷ Avicenna, ŠQ V 231.6-12.

³⁸⁸ E. Stump, “Topics: Their Development and Absorption in Consequences”, in *The Cambridge History of Late Medieval Philosophy* (Cambridge: Cambridge University Press, 1982), 277-99, especially 275-81.

Even though Avicenna believes that the *Prior Analytics* lacks the explicit formulation of a conditional syllogistic, he clearly thinks that it provides sufficient theoretical foundations for developing a conditional syllogistic. From Text 1, it is obvious that Avicenna intends to adopt Aristotle's assertoric syllogistic, which Aristotle outlines in *An. Pr.* A1-7, as the basis for his conditional syllogistic. Indeed, Avicenna adopts for his conjunctive syllogistic the most fundamental notions in Aristotle's treatment of assertoric syllogisms: (1) the "common term (or "middle part", *ḥadd muštarak*, *ǧuz' muštarak*)" in respect of whose position in the premises the syllogistic figures are determined, (2) the quantity and quality (A, I, E O) of the premises and conclusion, and finally (3) rules of direct and indirect reduction of imperfect to perfect syllogistic forms.

In the *Prior Analytics* Aristotle aims to give an adequate formalization of arguments with categorical premises and conclusions, Avicenna, on the other hand, is interested in supplementing what he takes to be Aristotle's incomplete account in the *Prior Analytics* by giving an adequate formalization of arguments with conditional premises and conclusions. (This is Avicenna's task in ŠQ V and VII.) As a logician, Avicenna was not interested exclusively in providing an adequate syntax and truth-conditions for connective and disjunctive conditional propositions. Avicenna's chief aim in ŠQ VI is to give a full account of the conditions under which a conclusion of an argument involving conditional premises and conclusions follows from the argument's premises on account of the formal properties of both. This Avicenna does by (A) identifying a set of perfect and imperfect two-premise argument forms with conditional premises and conclusions in which the conclusion follows from the premises. For the perfect syllogisms, this following is self-evident. Thus, Avicenna's second task in ŠQ VI is (B) showing that the conclusions of imperfect argument forms also follow from

their premises entirely on account of their formal properties by means of direct or indirect reduction of the imperfect to the perfect argument forms. Avicenna's final task (carried out in ŠQ IX) is (C) to show that every conditional syllogism without qualification is reducible by means of the perfect conditional syllogisms.

§5.2.2 AVICENNA'S CONJUNCTIVE SYLLOGISMS: ON THE "MIDDLE PART", SYLLOGISTIC FIGURES, AND "STRONG RELEVANCE"

Avicenna opens ŠQ VI with a treatment of the so-called "totally hypothetical" syllogisms,³⁸⁹ recalling an ancient extension of Aristotle's use of the word by his student Theophrastus.³⁹⁰ Thus "Barbara" for conjunctive conditional syllogisms composed of purely connective conditional premises and conclusion looks like the following: "always: if A is B, then J is D and always: if J is D, then H is Z. Therefore, always: if A is B, then H is Z". Avicenna calls all syllogisms that rely on the "middle term" principle, regardless of whether they are purely categorical, purely conditional or a mix of both, "conjunctive syllogisms (*qiyāsāt iqtirāniyya*)". The analogy with the figures of the categorical syllogism is obvious, and, as we will see ramifies widely.

[Text 3] The syllogisms formed from [purely] connective <conditional premises> are those formed from two connective <conditional expressions> that share a part, viz. an antecedent and a consequent. <This sharing> is arranged on the model of the three categorical figures: either the middle extreme [*ḥadd*]³⁹¹ is the consequent in one and the antecedent in the other (this is called the "first-figure"), or the extreme is the consequent in both (this is called the "second-figure"), or the extreme is the antecedent in both (this is called the "third-

³⁸⁹ In §5.2, my discussion is based on Avicenna's discussion of these types of syllogism. Of course, there are others, but Avicenna devotes most of his attention to these.

³⁹⁰ J. Barnes, "Terms and Sentences: Theophrastus on Hypothetical Syllogisms", *Proceedings of the British Academy* 69 (1983): 279-326.

³⁹¹ Since the shared parts are whole propositions rather than predicate or subject terms, I translate "*ḥadd*" as "extreme" rather than the usual "term".

figure”). There is no syllogism from two particulars, from two negatives, nor from a negative minor whose major is particular.³⁹²

The requirement that the middle part be present in all kinds of conjunctive syllogism determines not only the form that a conjunctive syllogism may take, but also the kind of following or “syllogistic consequence” that can stand between premises and conclusions. As Günther Patzig has pointed out, in Aristotelian syllogistic, being-in-a-figure “names both a [formal] *property* of the syllogism itself and the *class* to which the different syllogisms belong by virtue of this property”.³⁹³ Indeed, for a syllogism to be in a figure in the Aristotelian sense is tantamount to holding that a certain type of logical following holds between conclusions and premises in an argument that has that syllogistic form. Thus, in the two-premise assertoric syllogistic of the type outlined by Aristotle in *An. Pr.* A1-7, the following of the conclusion from the premises is characterized by what Paul Thom calls “strong relevance”, and “all and only” two-premised syllogisms characterized by strong relevance are in an Aristotelian figure.³⁹⁴ Thom describes strong relevance in the following terms: a relation between conclusion *c* and premise-pair *P* is strongly relevant if and only if (1) if a term is in the conclusion, then it is in some premise; and (2) if a term is in only one of the premises, then it is in the conclusion.³⁹⁵ Yet, Aristotle’s definitions of the figures are often incomplete, and are not formulated with a conscious interest in how the conclusion is related to the premises by means of the middle term.³⁹⁶ In the *Iṣārāt wa-Tanbīhāt*, Avicenna also characterizes the relationship between the middle extreme such that the syllogistic following in all his conjunctive syllogisms (which, of course, includes pure, conditional, pure categorical, and mixed

³⁹² Avicenna, ŠQ VI, 295.6-11.

³⁹³ G. Patzig, *Aristotle’s Theory of the Syllogism: A Logico-Philological Study of Book A of the Prior Analytics*, trans. J. Barnes (Dordrecht: D. Reidel, 1968), 101 (italics in original).

³⁹⁴ P. Thom, *The Syllogism* (Munich: Philosophia, 1981), 27-8.

³⁹⁵ Thom, *The Syllogism*, 28.

³⁹⁶ There is, thus, a sense in which Aristotle’s definition of the figures does not match his practice; see Patzig, *Aristotle’s Theory of the Syllogism*, Chapter 4.

sylogisms) is similarly characterized by strong relevance. But his treatment also displays a concern with *how* the conclusions follow from conditional premises. The examples appearing in the following quote from the *Iṣārāt* clearly stand for terms in a categorical syllogism in Barbara “every J is B and every B is A, so every J is A”. However Avicenna might just as well be talking about conjunctive syllogisms of conditional Barbara: “always: if J is D, then H is Z and always: if H is Z, then A is B, then always: if J is D, then A is B”.

[Text 4] In the conjunctive syllogism there is something that is shared and repeated called the “middle term”, such as B in the previous example. For each of the premises, there is something that is unique to it, such as J and A in the example. The conclusion only comes about from the coming together of this pair of extreme parts, for which we say “every J is A”. Of the two [extreme parts], the one that becomes the subject term or antecedent in the conclusion—such as J in the example—is called the “minor”. The one that becomes the predicate or consequent in the conclusion is called the “major”. The premise having the minor <part> is called the “minor <premise>”, and the one having the major <part> is called the “major premise”. Their [i.e. the premises’] combination is called a “conjunction”. The configuration [*hay’a*] of the premise combination and how the middle term is positioned with respect to the extreme terms is called a “figure”. Among the class of conjunctions, only what is productive is called a “syllogism”.³⁹⁷

In all two-premise, conjunctive syllogisms there is (i) a part, whether formally a categorical term, categorical sentence, or a compound of conditionals of any complexity, that is shared among the premises; (ii) there is a part that is unique to each premise; and (iii) the conclusion “is such that it only comes about through the coming together” of the extreme parts. In a two-premise conjunctive syllogism, there are six possible positions—call them a_1 , a_2 , a_3 , a_4 , a_5 , and a_6 —that the parts of a conjunctive syllogism can occupy. If we adopt Paul Thom’s notation for representing syllogisms and if we ignore quantity and quality, as Aristotle and Avicenna do when defining the figures, then the possible subject-predicate/antecedent-consequent positions in Avicenna’s conjunctive syllogism may be represented as follows:

³⁹⁷ Avicenna, *Iṣārāt*, 428–9.

$$\left[\begin{array}{c} \frac{a_1 a_2 \quad a_3 a_4}{a_5 a_6} \end{array} \right]$$

Property (i) eliminates the possibility of any conclusion following from more than three or more parts, though, without further stipulations, syllogisms fewer would be allowed:

$$\left[\begin{array}{c} \frac{a_1 a_1 \quad a_1 a_1}{a_1 a_1} \end{array} \right] \left[\begin{array}{c} \frac{a_1 a_1 \quad a_1 a_1}{a_2 a_2} \end{array} \right] \left[\begin{array}{c} \frac{a_1 a_1 \quad a_1 a_2}{a_1 a_2} \end{array} \right]$$

The above three (inconcludent) “syllogisms” all have middle extremes, technically speaking. Moreover, the conclusions in each “follow” validly from the premises according to the requirements of what is called “Classical validity” (an argument is Classically valid if it is impossible for its antecedents to be true and its conclusion false). However, property (ii) eliminates the above argument forms from being syllogisms since it requires that there cannot be just one or two identical parts in the premises. Properties (i) and (ii) together are sufficient to limit the number of parts in a conjunctive syllogism to exactly three distinct parts. If we do not take into account the position of the parts in the conclusion, then syllogisms with properties (i) and (ii) taken together are also a sufficient basis from which derive the three figures in the way Aristotle does in *An. Pr.* A4-6, viz. only according to the position of the middle part. Thus, we can call (i) and (ii) together the “Middle Part” principle. Yet, what about (iii)? Stipulating that syllogisms must have property (iii) eliminates any two-premise conjunctive syllogism in which there are only three unique parts occupying four positions in the premise pair, viz. there is one and only one middle term (shared part). However, without stipulating (iii), it might be thought that a formally concludent syllogism might allow its conclusion to follow from only one of its premises or neither, as in the following examples:

$$\left[\begin{array}{c} \frac{a_1 a_2 \quad a_2 a_3}{a_1 a_2} \end{array} \right] \qquad \left[\begin{array}{c} \frac{a_1 a_2 \quad a_2 a_3}{a_4 a_4} \end{array} \right]$$

Eliminating “syllogisms” that follow from only one premise or because they are tautologous is a formal requirement that tries to be consistent with Aristotle’s description of the syllogism as argument in which, *inter alia*, “something other than what is laid down results (24b19)”. For Avicenna this “Productivity Principle” of conjunctive syllogisms gives formal expression to the requirement that a conjunctive syllogism be useful in the sense of producing new knowledge. What is clear in Avicenna (though not clear in Aristotle³⁹⁸) is that a syllogism must obey the Productivity Principle in order to qualify as a syllogism *in the formal sense*. Avicenna is willing to disqualify syllogisms (“syllogism” now used in the sense of concrete arguments) such as “if every human is risible and if every risible thing is animal, then every human is an animal” on the grounds that “not everything that takes the form of a syllogism produces like a syllogism”.³⁹⁹ Avicenna admits that this is in Barbara, so the conclusion follows from the premises solely on account of the form of each. Formally speaking the syllogism is sound, but the outcome of this process of reasoning is not something that the reasoner could have been ignorant of prior to the setting down of the premises. Prior to setting down premises, Avicenna claims the reasoner must already have known the conclusion, since in Avicenna’s mind it makes little sense to use a proprium of a species of animal (risibility) as a means for “discovering” that the species (man) belongs to the genus (animal). Nevertheless, this Productivity Principle is sound as a *formal* guarantee that there be no formal redundancy in the

³⁹⁸ E.g. see G. Striker’s commentary on 24b18–20, Aristotle, *Prior Analytics Book I*, 80. In practice however, Aristotle assumes these conditions; see Patzig, *Aristotle’s Theory of the Syllogism*, chapter 4.

³⁹⁹ Avicenna, ŠQ 422.10; translation is Shehaby’s (Avicenna, *Propositional Logic*, 208) with slight modifications.

premises and the conclusion, in the sense that the conclusion does not follow from the premises on account of a single premise only. It is also a guarantee that the conclusion not be irrelevant to the premises, in the sense that the truth of the conclusion (e.g. “all humans are human”) does not depend in any way on the truth or falsity of the premises. Thus, Avicenna’s making clear that the Productivity Principle is a necessary condition for being a syllogism is important because it reveals that in developing the conjunctive syllogism he was concerned with the manner in which the conclusions followed from the premises and not merely with the placement of the middle term.⁴⁰⁰ Just like the relation between the conclusion and premises of Aristotle’s categorical syllogisms, which display what Thom calls “strong relevance”, Avicenna’s definition of the properties of the middle part vis-à-vis the premises and conclusion guarantees strong relevance between all conclusions and two-premise syllogisms in the three figures of conjunctive syllogisms, regardless of whether the syllogism is composed of categorical terms, sentences or conditionals.

§5.2.3 AVICENNA’S CONJUNCTIVE SYLLOGISMS: A-, I, E, O-CONDITIONALS

Continuing the analogy between the fundamental concepts of Aristotle’s categorical syllogisms and his conjunctive syllogisms, Avicenna also quantifies and qualifies his conditional propositions in the universal affirmative (A-conditionals), universal negative (E-conditionals), particular affirmative (I-conditionals), and particular negative (O-conditionals). In the truth-conditions for Avicenna’s restricted conditionals (*ḥaqīqī*, *luzūmī*, *‘alā t-taḥqīq*) discussed in §3, I mentioned that a restricted conditional is true if and only if the consequent is

⁴⁰⁰ It is not clear to what extent this is a doctrine originates with Avicenna. This three-part definition of the middle term appears in the *Iṣārāt* entitled “Special Properties of Conjunctive Syllogisms”, which suggests that he arrived at these formulae as a consequence of his thinking about conjunctive syllogisms. Alfarabi defines the figures in the same way that Aristotle does. For example, see Alfarabi, *Al-Fārābī’s Short Commentary on Aristotle’s Prior Analytics*, trans. N. Rescher (Pittsburgh: University of Pittsburgh Press, 1963), 60.

true under all conditions in which the antecedent is posited as true. When Avicenna first formulates these truth-conditions, he does not mention quantifiers in his discussion, though it is clear from his early exposition in ŠQ V 1 that the qualification “under all conditions” lends itself to universal quantification; and, if suitably modified, particular quantification. In ŠQ V 4, Avicenna makes explicit the quantification over conditions (or “states [aḥwāl]”, but main he uses “conditions [šurūṭ]”; the terms are used interchangeably).

[Text 5] Let us then speak of the universal affirmative connective conditional. We say that in the expression “whenever J is B, then H is Z”, the meaning of the expression “always” is not just a generalization over all instances⁴⁰¹ so that it is as if one has said “every instance in which J is B, H is Z”. Rather, it is also a generalization over every state that attaches to the expression “J is B”, so that there is no state or condition that attaches to it so that the condition makes “J is B” exist save that “H is Z” exists. For it may be that the antecedent is a state of affairs that does not repeat and is not periodic, but remains stable continuously.⁴⁰²

In this chapter, Avicenna discusses many different types of examples of conditionals. Let us take three. The first example is (a) “always: if this is a pair, then this is even”; a second example that Avicenna provides is (b) “always: if the void exists, then it has dimension”; and the third (c) “always: if the sun is hot, then the earth is round”. Avicenna’s idea is that in true conditional statements, it is not merely the case that the consequent is true in every instance that the antecedent is true. “Always”, when applied to conditionals, behaves as a quantifier quantifying over the conditions under which the consequent follows from the antecedent. Moreover, Avicenna denies that “always”, when attached to conditionals, works to quantify

⁴⁰¹ I read “*mirār*” for the edition’s “*murād*”. This is obviously a mistake by edition typesetter as no alternative readings are noted in the footnotes to the edition. It is not, however, a typo since this particular error is repeated throughout this chapter. Professor Shehaby does not consistently adopt the correct reading. Thus, he correctly reads “*mirār*” for “*murād*” in the translation of this passage (Avicenna, *Propositional Logic*, 63), but in a different passage a short while later, in which Avicenna is rehashing the ideas in from the above passage (Avicenna, *Propositional Logic*, 69f), Shehaby choses to retain “*murād*” without comment. This is unfortunate, given the importance of this latter passage to understanding the finer points of Avicenna’s theory of conditional qualification.

⁴⁰² Avicenna, ŠQ V 265.1-7.

over every instance (*mirār*) in which the consequent follows from the supposition of the antecedent. For Avicenna, it is not the case that an A-conditional is true if and only if the consequent is true in every instance in which the antecedent is true. The reason for this is that the use of instances in setting down the truth-conditions of a conditional lends itself to a simpliciter reading of conditionals. In other words, the instances over which “always” quantifies would, on this reading, apply only to instances that occur as a matter of fact. Thus, example 1 and 3 would be true as A-conditionals. Example 2 on the other hand would come out false because the consequent is never true; in fact, it is impossible. These truth conditions are consistent with those Avicenna gives for simpliciter conditionals. However, Avicenna has to have something else in mind for his theory of quantified conditionals, since he says that quantified conditionals are not called true or false on account of the truth or falsity of the antecedent or consequent, but on account of the following between them. Indeed, he says that in an argument with conditionals, the antecedent qua antecedent is not used with the expectation that it is true as a matter of fact. In fact, the class of sentences that can stand in the antecedent position in a connective conditional is not determined by their being subject to truth or falsity by correspondence to actual states of affairs. Rather, antecedents express meanings that are put forward in mental supposition (*farḍ*, *iftirāḍ*).

[Text 6] It is evident, then, that the antecedent qua antecedent is not expected to exist as a matter of fact. It is, rather, only a supposition, which then is subclassified as sometimes a true supposition, at <other> times true with respect to some <other> supposition, or one is not concerned with its truth, but has suspended judgment. The meaning of the supposition <of the antecedent> is not that you suppose it in the present or you suppose it in the future. It is, rather, that if the supposition <of the antecedent> is sound, then what follows from it is sound. On the other hand, if the impossible is supposed as an antecedent, then its content is just the supposition.⁴⁰³

⁴⁰³ Avicenna, ŠQ V, 271.3-7.

As I noted in §3, Avicenna is motivated by, *inter alia*, a desire to use conditionals in per impossibile syllogisms in which conditionals with impossible antecedents are used to make valid syllogistic arguments. Thus, Avicenna's theory of quantified conditionals must be able to set down truth-conditions for conditionals in which it is possible for an A-conditional to be true, but both the antecedent and consequent are false as a matter of fact. It is for this reason that Avicenna strongly identifies the content and function of a conditional's antecedent with supposition. But just as the content of the antecedent itself is a result of mental supposition, so too are the states or conditions (*aḥwāl*, *šurūṭ*) over which "always" quantifies. There appears to be, then, an important sense in which, "always", in Avicenna's theory of universally quantified conditionals, operates like "every" in Avicenna's theory of universally quantified terms. In his categorical syllogistic, Avicenna holds that the universal quantifier "every" quantifies a subject term J that picks out a wider set of objects than just those that currently exist, have existed, or will exist in the future. Rather, "every" quantifies over "every single thing described as J, be it in mental supposition or extramental existence be it described as J always, or sometimes, or whatever".⁴⁰⁴ In a somewhat analogous way, "always" when attached to conditionals quantifies over possible states; "possible" in the sense that the set of states that "always" quantifies over is large than the set of states that could be, have been, or will be realized in matter of fact. Rather, "always" also quantifies over states that are known with certainty to be impossible in matter of fact (*wa-in kāna muḥālan fī nafsīhi*),⁴⁰⁵ but the reasoner is capable of making suppositions about them in such a way that the realizes that these suppositions sustain genuine implications.

⁴⁰⁴ Quoted and translated in T. Street, "An Outline of Avicenna's Syllogistic", *Archiv für Geschichte der Philosophie* 84/2 (2002): 129-160, at 134.

⁴⁰⁵ Avicenna, ŠQ V, 273.1.

[Text 7] Let us then make a definitive statement about the universal connective conditional. We return <to do this> now. We say: the universal conditional proposition is only universal if the consequent follows every setting down of the antecedent, not only in every instance [the antecedent is set down], but in every state [*aḥwāl*] as well. But which states are these? They are those states that follow from the supposition of the antecedent, or that can be supposed to belong to it, follow from it, and be consistent with it, either [1] because they are predicates of the subject term of the antecedent, if <the antecedent> is a categorical proposition; or [2] because other antecedents are attached to it if it is not a categorical proposition (I mean antecedents that can be true with <the supposition of> its truth, not those that are impossible with it, even if <the antecedent> is impossible in matter of fact); or [3] because of some admission <by an interlocutor> that demands <the antecedent> and makes it admissible, even if <the antecedent> is impossible in matter of fact. Indeed, <these states can be supposed> not only if the antecedent is true, but also if <the antecedent> is false, but is supposed hypothetically. For <the antecedent, when true> has concomitant and accidental properties [*lawāzim wa-ʿawāriḍ*] that obtain in it, or <when false> that would obtain in it if it were supposed true. In a similar way [the antecedent sustains necessary and accidental properties] if it is taken according to the admission of an interlocutor if the conditional is used in dialectical exchange.⁴⁰⁶

Consider example 3. Is the conditional “always: if the sun is hot, then the earth is round” true as a universal affirmative connective conditional? In order to be true, the consequent “the earth is round” must be true under any condition that attaches to the antecedent and that is consistent with it—“consistent” in the sense of “conceivable”. In this case, one of the conceivable conditions that is consistent with the fact that the sun is hot is that the earth not be round. There is, then, a condition, viz. “the earth is not round” or “the earth is a cube”, that, while obviously false as a matter of fact, can be consistently supposed to be true with the antecedent “the sun is hot” and that contradicts the following of the consequent. The universal affirmative connective in example 3 is thus false.

How do we test examples 1 and 2? They are, after all, conditionals that Avicenna wants to hold up as true as universal conditionals. Example 2 is especially important since it can serve as a true conditional premise in a per impossible syllogism that has as its ultimate

⁴⁰⁶ Avicenna, ŠQ V, 272.13-273.6.

objective the conclusion that the void does not exist. On the other hand, example 1 is an example of a conditional that is analytically true, so it seems it should be true when universally quantified as well. It was easy to establish that example 3 was false because all that was needed was a single condition consistent with the truth of the antecedent but inconsistent with the truth of the consequent. However, in order to establish the truth of a conditional, which conditions are relevant to establishing that examples 1 and 2 are true? Avicenna presents a possible objection to his theory of universally quantified conditionals. The test of the truth of an A-conditional has become a question of whether or not the consequent follows from the antecedent under all conditions that attach to the antecedent, regardless of whether the antecedent or attached conditions are true as a matter of fact or hypothetically true in supposition only, but false otherwise. This may be so, but then there does not seem to be any real difference between an impossible antecedent like “the void exists” in example 2, and “this is a pair and it is not divisible by two” in the *true* conditional “if this is a pair and it is not divisible into two, then it is odd”.⁴⁰⁷ The imagined objector tries to force Avicenna to concede that he must accept this conditional as true according to his own principles,

[Text 8] for it cannot be said that this conditional is false because of a transformation of the antecedent, because [according to your own principles] the truth of conditionals is not determined by the truth of the antecedent and the consequent, but rather by <the conditionals’> truth when there is an implication. Indeed, most of the conditionals used in the sciences when the per impossible syllogisms are employed are of this description, for their antecedents are impossible, and yet it is not said that they are false for their having impossible antecedents and consequents. The case is similar if someone says “if this is a pair and it is not divisible into two, then this pair is odd”, for this is true, even if the antecedent is impossible. Thus, <you must hold> that there are states that are not impossible in supposition which, even if they are impossible as a matter of fact, if the antecedent is supposed under them, then the consequent will not follow. For example then, it is not the case that whenever it is supposed that this is a pair, then it is even; rather, it will only be

⁴⁰⁷ Avicenna, ŠQ V, 273.16. This conditional is called true by the objector (who is Avicenna himself), and it seems it is true on Avicenna’s reading of implication.

the case if nothing is supposed along with it [i.e. the antecedent] that contradicts <the consequent>. However, if something is supposed along with it [i.e. the antecedent] that contradicts <the consequent>, then <the implication> is nullified. For even if <the addition of the condition> is impossible in matter of fact, its being impossible in matter of fact does not preclude its possibility in supposition. Thus, it is not the case that from everything supposed to be a pair it follows that it is even, for [it has been now shown] that there are impossible suppositions that preclude this.⁴⁰⁸

Thus, if we take this objection seriously not even example 1 “always: if this is a pair, then it is even” can be true as an A-conditional, for we have found a condition “it is not divisible into two” which, when attached to the antecedent “this is a pair”, implies something contradictory to the consequent “it is even”, in which case we should despair of ever finding a true A-conditional. For even the empty tautology “if *P*, then *P*” can technically speaking, have a condition attached to it in supposition, namely “not-*P*” such that in the true conditional “if *P* & not-*P*, then *P*”, the consequent *P* cannot be true with the attached condition not-*P*. In response to the claim that the truth-conditions on A-conditionals are so strong that there are no A-conditionals, Avicenna responds that true A-conditionals can be had

[Text 9] by attaching to the antecedent a condition, which, in meaning, blocks any condition that occasions the following of a consequent whose following is not necessary solely by virtue of <the antecedent>. Such as if you were to say “whenever this is a pair in the manner in which it is possible to be a pair, then it is even”, and “whenever this is void in such a way that if void were supposed to have the existence that it was supposed to have, or it is admitted for the sake of argument [*ilzāman*] to have the existence it is supposed to have, or is a concomitant of its supposition if it is possible and there is no condition that contradicts the meaning that is grasped from the notion of the void [*mafḥūm al-ḥalā'iyya*], then it is a dimension”.⁴⁰⁹ Therefore it is imperative in the connective conditional proposition to consider along with it [i.e. the antecedent] additional meanings similar to these. Otherwise, there will be no universal at all.⁴¹⁰

⁴⁰⁸ Avicenna, ŠQ V, 273.12-4.5.

⁴⁰⁹ For Avicenna, we “grasp the meaning” of a notion when there is a nominal definition; for a discussion, see S. Menn, “Avicenna’s Metaphysics”, in *Interpreting Avicenna: Critical Essays*, ed. P. Adamson (New York, Cambridge: Cambridge University Press, 2012), 143-69, 164f. For the doctrine of nominal definition in *An. Post.* II 8-10, see D. Demoss, D. Demereux, “Essence, Existence, and Nominal Definition in Aristotle’s *Posterior Analytics* II 8-10”, *Phronesis* 33/2 (1988): 133-54.

⁴¹⁰ Avicenna, ŠQ V, 274.13-5.4

Thus, in order to produce true A-conditionals, the reasoner must add an implicit understanding of the antecedent, which eliminates any consequent that is not a genuine consequence of the setting down of the antecedent. This (admittedly ad hoc) “limiting condition” seems to work to circumscribe the acceptable conditions that “always” must quantify over in order for the conditional to be true. According to this “limiting condition” in example 1 “always: if this is a pair, then it is even” “always” does not have to quantify over conditions such as “it is not divisible into two” because the “limiting condition” added to the antecedent has eliminated it for its scope. In general, we could say that this “limiting condition” is intended to pick out attributes that are in or entailed by the quiddity of the subject term of the antecedent, and to eliminate them as possible states over which the “always” quantifier must quantify. Example 3 still fails to be true on this reading because even after eliminating all the conditions that are inconsistent with the quiddity of the concept “sun”, or are inconsistent with the entailments of the quiddity of the concept “sun”, it is still possible to set down as a condition in the antecedent “the earth is a cube”. Example 2 is retained as a true A-conditional as long as the “limiting condition” eliminates from the scope of “always” any condition that is incompatible with concomitant properties that arise from the supposition of void’s existing.

Thus the A-conditional “always: if J is B, then H is Z” is true if and only if the consequent “H is Z” is consistent with “J is B” and its limiting conditions L_c under all conditions $C_1...C_n$, where $C_1...C_n$ are consistent with “J is B” and L_c . L_c is a set of conditions $L_1L_2L_3...L_m$ that are consistent with the quiddity of J, the propria of J, the differentia of J, its genus, species, and with any other concomitant properties of J. Thus, if a condition C_i is found to be inconsistent with “J is B” or any of $L_1...L_m$, it is not a condition falling within the scope of

the universal quantifier over propositional states “always”. It is now a simple matter of deriving truth-conditions for all the other quantifiers. *O*-conditional “not always: if J is B, then H is Z” is true if and only if there is at least one condition C_i that is consistent with “J is B”, its limiting conditions L_C , and with “H is not Z”. The *I*-conditional “once: if J is B, then H is Z” is true if and only if there is at least one condition C_i that is consistent with “J is B”, its limiting conditions L_C , and with “H is Z”. Finally, the *E*-conditional “never: if J is B, then H is Z” is true if and only if “H is not Z” is consistent with “J is B” and L_C under all conditions $C_1...C_n$ consistent with “J is B” and L_C .

§5.2.4 AVICENNA’S CONJUNCTIVE SYLLOGISMS: DIRECT AND INDIRECT REDUCTION IN AVICENNA AND ARISTOTLE

In analogy to Aristotle’s categorical syllogistic, Avicenna’s syllogistic composed of purely conditional premises and conclusions singles out four moods of the first-figure as perfect (*kāmil*): conditional Barbara, conditional Celarent, conditional Darii, and conditional Ferio. In the Aristotelian manner of speaking, to say that a syllogism is perfect is to say two things. First, it is to say that the conclusion of the syllogism follows from the premises of logical necessity, viz. the conclusion follows from the premises only on account of the formal properties of the premises and conclusion. The following is called logically *necessary* because to say that an argument form is a syllogism is to say that there is no correct substitution of concrete terms such that the premises would be made true but the conclusion made false.⁴¹¹ These formal properties are the placement of the middle term vis-à-vis the extreme terms (i.e. the argument form’s figure), and the quantity and quality of the premises, viz. whether they are *A*-, *E*-, *I*-, or *O*-premises. For Aristotle, an argument form such as the following is in the

⁴¹¹ G. Patzig, *Aristotle’s Theory of the Syllogism*, 26f. For the idea of correct substitution, see *ibid.*, 7.

first-figure: “if A belongs to no B and B belongs to no C, then A belongs to no C”. It is not, however, a syllogism because it is inconcludent, meaning that a set of concrete terms can be uniformly substituted for A, B, and C that make the premises true with a universal affirmative conclusion (e.g. for A substitute man, for B stone, and for C animal), and another set can be found that makes the premises true and the conclusion true as a universal negative (e.g. for A substitute human, for B stone, for C horse). Thus, being a first-figure *argument form* is a necessary but not a sufficient condition for being a perfect *syllogism*. In addition to being in the first-figure, a perfect syllogism must also of course be concludent. And a concludent syllogism in the first-figure (and in the other two as well) is called a mood.

There is a second implicit claim that is tied up with saying that an argument form is a perfect syllogism. An important feature of concludent first-figure syllogisms is that their conclendency is self-evident.⁴¹² Thus, not only does the conclusion of a concrete syllogism in Barbara, Celarent, Darii or Ferio follow from its premises with logical necessity, this following is taken by Aristotle as a self-evident fact that not only has no need of proof, but cannot be proven in the first place. Having developed a general theory of conjunctive syllogistic figures and quantified conditionals, Avicenna is confident that all of these basic notions are transferrable to his treatment of purely conditional conjunctive syllogisms. For example:

[Text 10] There is no syllogism from two particular premises, from two negative premises, nor from a negative minor premise whose major is particular. The first-figure among the connective <conditionals>: its <conclendency> conditions are like those of the first-figure assertoric syllogisms. The expressions “A is B” and “J is D” symbolize categorical expressions [...]

The first mood is composed of two universal affirmatives: “whenever A is B, then J is D, and whenever J is D, then H is Z. Therefore, whenever A is B, H is Z”. It is perfect.

⁴¹² Ibid., 135.

The second mood from two universals, the major of which is negative: “whenever A is B, then J is D, and never: if J is D, then H is Z. Therefore, never: if A is B, then H is Z”. It is perfect.

The third mood is two affirmatives, the minor of which is particular: once: if A is B, then J is D, and whenever J is D, then H is Z. Therefore, once: if A is B, then H is Z”. It is perfect.

The fourth mood is from a particular affirmative minor and a universal negative major: once: if A is B, then J is D, and never: if J is D, then H is Z. Therefore, not always: if A is B, then H is Z”. It is perfect.⁴¹³

To say that there are perfect syllogisms is to imply that there are also imperfect syllogisms. The imperfect syllogisms are syllogisms both in the sense that they are (two-premise) argument forms in a figure, and in the sense that the conclusions of arguments follow necessarily from the premises. Just as in perfect syllogisms, the quantity and placement of the middle terms of the imperfect moods are enough to guarantee that it not possible to find a concrete substitution that satisfies the premises but falsifies the conclusion. What imperfect syllogisms lack, then, is the self-evidentiary nature of the necessity of the conclusion’s following from the premises on account of their formal properties. Thus, the process of perfecting imperfect syllogisms must reveal that, in fact, imperfect argument forms are *concludent* (in the sense defined above). However, only *inconcludency* in Aristotle’s syllogistic is, strictly speaking, demonstrable in a way that is independent of perfect syllogisms. Concludency can be demonstrated only insofar as the imperfect argument form can be shown to have the properties of a perfect syllogism. So, in order to prove that imperfect syllogisms are in fact syllogisms, in *perfecting* imperfect syllogisms, a set of procedures must be developed that somehow reveals that the conclusion in an imperfect argument form is related with the same sort of formal necessity to its premises as a first-figure syllogism’s conclusion is related to its premises. The normal procedure in a systematic treatment of the syllogism is that every

⁴¹³ Avicenna, ŠQ VI, 295.10-6.13.

sylogism identified as imperfect is accompanied by a proof of its conclusency. Given the strong analogy Avicenna has adopted between his conjunctive syllogistic and Aristotle's categorical syllogistic, Avicenna's treatment of purely connective conjunctive syllogisms can be no different. Here is Avicenna's description and reduction of Cesare (II):

[Text 11] The first mood, which is from two universal <premises> the major of which is negative, is "whenever A is B, then J is D and never: if H is Z, then J is D", which yields "never: if A is B, then H is Z". It is proven by the conversion of the major premise and reducing it to the second <mood> of the first <figure>. <Its proof> ad impossibile is: "if the conclusion is false, then its contradiction "once: if A is B, then H is Z" is true, to which is attached "never: if H is Z, then J is D", yielding "not always: if if A is B, then J is D". [But this contradicts the minor premise.]⁴¹⁴

"Proof" says Avicenna shortly before the above passage, "is by conversion ('aks), [and] *reductio ad absurdum* (ḥalf)..."⁴¹⁵ Conversion and *reductio* are not processes that are carried out on concrete arguments, and thus they are not required to work like syllogisms in the way that Aristotle and Avicenna requires of concrete arguments. They are, rather, processes that are carried out on the logical variables only.⁴¹⁶ Avicenna provides two proofs of the Cesare (II), one by conversion, which is called "direct reduction"; the other by *reductio*, which is called "indirect reduction". By conversion of the major premise "never: if H is Z, then J is D" to

⁴¹⁴ Avicenna, ŠQ VI, 300.14-1.2.

⁴¹⁵ "...and ecthesis (*iftirād*); ibid., 300.13. Like many scholars of Aristotle's logic, I will ignore ecthesis in Avicenna, both because it is not well understood in the Arabic tradition (see T. Street, "An Outline of Avicenna's Syllogistic", *Archiv für Geschichte der Philosophie* 84 (2002): 129-60, especially 139-42) or the Greek tradition for that matter (R. Smith, "What is Aristotelian Ecthesis?", *History and Philosophy of Logic* 3(1982): 113-127, consult 113), and also because it is tangential to my main argument. Street concludes a long discussion of ecthesis with the following words: "I have tried to include everything the texts have which may be used in attempting to understand Avicenna's move (or moves) indicated by "suppose that possible to be actual" – it has defeated me. Working out what is going on here is the major problem to be faced by any attempt to make sense of the Avicennian system (op. cit., 142)". Given the importance of ecthesis and the particular challenges it poses to interpreters, it seems wiser to leave it for a later study.

⁴¹⁶ Patzig's (*Aristotle's Theory of the Syllogism*, 133f) makes heavy weather of Aristotle's purported claim that every proof (i.e. argument which argues that a certain conclusion must be true) must be a syllogism and also that a proof (i.e. the proof that a certain syllogism is conclusive) also must be in a syllogistic form. Patzig's criticism is based on an ambiguity in the use of the word "proof", so I believe his words are unjustified. Otherwise Patzig's chapter 5 is lucid, and without doubt one of the best treatments of reduction in Aristotle. In general, Patzig's book is indispensable particularly when trying to understand the historical developments in late antiquity. Corcoran's, Smiley's, Johnson's, and Thom's work, while of course extremely valuable in themselves, are not always helpful in trying to understand how later logicians were thinking.

“never: if J is D, then H is Z”, conditional Cesare is reduced to the perfect conditional mood Celarent (I) “whenever A is B, then J is D, and never: if J is D, then H is Z. Therefore, never: if A is B, then H is Z”. According to Avicenna (and Aristotle) this procedure on its own is sufficient as a demonstration that in any argument having the formal properties of conditional Cesare (II), the conclusion in such an argument follows necessarily from the premises on account of the formal properties of the premises and conclusion alone. Though indirect reduction is not necessary for proving Cesare (II), since the latter may be proved by direct reduction, indirect reduction is indispensable for proving the conclusivity of conditional Baroco (II) and conditional Bocardo (III). In the *reductio* proof of Cesare (II), from “once: if A is B, then H is Z”, which is the contradictory of the conclusion, and the major premise “never: if H is Z, then J is D”, is produced the contradictory of the minor premise “not always: if A is B, then J is D”. Once again, Avicenna (and Aristotle) is of the view that forming a perfect first-figure syllogism (Ferio) from the contradiction of the conclusion and the major premise and replacing the conclusion with the contradiction of the minor premise proves that the conclusion of an argument in Cesare (II) follows logically from its premises.⁴¹⁷

Given the complexity of Avicenna’s theory of qualified and simpliciter/restricted conditionals, it should come as no surprise that there are conditions that must be placed on the conclusivity of pure conjunctive conditional syllogisms, even on first-figure syllogisms. Avicenna’s direct and indirect reductions do not alert the reader to the fact that there are certain conditions placed on the kind of major and minor conditional premises in order for a pure conjunctive conditional Cesare (II), for example, to be conclusive:

[Text 12] As for the case in which they [i.e. the premises] are both universal and the major is negative, such as “whenever H is Z, then J is D, and never: if A

⁴¹⁷ This procedure is called “transposition” and will be discussed in greater detail in §5.3.2. For Avicenna’s use of transposition (*half*) in his categorical syllogistic, see Street, “An Outline of Avicenna’s Syllogistic”, 139.

is B, then J is D”, then its status depends on whether the premises are [a] both simpliciter [*wifāqiyyatayn*], or [b] both restricted [*luzūmiyyatayn*], or [c] a mix of both. If both are simpliciter, then this will not be a proof of something unknown, and it will be as you have learned in relation to the first-figure [i.e. inconcludent]. As for the affirmative [minor premise], what follows from its being simpliciter or not simpliciter follows the first-figure: if the [negative premise] negates the implication only, but does not exclude <its being true> simpliciter, and the affirmative [minor premise] is simpliciter, then this syllogism is never concludent. For example: “whenever human is rational, then the donkey brays”, and never: if a pair is even, then it follows that the donkey brays. From this it is true that “Therefore, never: if human is rational, then it follows that a pair is even”. However, if a pair’s being dual is replaced by human’s being animal, then the conclusion is true, viz. “whenever human is rational, then it follows that human is animal”.⁴¹⁸

Clearly there is a tension between Avicenna’s desire to preserve the analogy between conditional conjunctive syllogisms and categorical syllogisms, and his theory of quantified simpliciter and restricted conditionals. In Aristotle’s assertoric syllogisms, the two proofs of Cesare’s (II) concludency, the two reductions to Celarent and Ferio, are as straightforward as Avicenna’s reductions of conditional Cesare in Text 11. In the “naïve” proofs of conditional Cesare, to show that an imperfect syllogism is concludent merely required the direct reduction of Cesare to Celarent (I), or the indirect reduction of Cesare to Ferio (I). Similarly in the simple proofs of conditional Cesare, it is, at first blush at least, sufficient for Avicenna to prove conditional Cesare to have (1) developed a generalized theory of figures and moods; (2) to have a conditional square of opposition in analogy with the categorical quantifiers; (3) to use conversion and transposition⁴¹⁹ along with (1) and (2) to show that the relation between premises and conclusions in imperfect syllogism is of the same logical necessity as in the first-figure syllogisms. However, other logical factors such as quantified conditionals, simpliciter versus restricted conditionals, and the Productivity Principle all serve to complicate this picture. For example, the proof given in Text 11 cannot apply to pure conjunctive conditional

⁴¹⁸ Avicenna, ŠQ V, 299.12-300.11.

⁴¹⁹ See §5.3.2 below.

sylogisms in which the conditional premises are both simpliciter, because syllogisms in which conditional premises are characterized by simpliciter following are only syllogisms homonymously, but not in reality, because they do not prove something that was previously unknown (*mağhūl*).

After dismissing in Text 12 purely connective conditional conjunctive syllogisms constructed from a pair of simpliciter premises as inconcludent, Avicenna moves on to consider other variations of conditional Cesare (II), but with different combinations of simpliciter and restricted premises. Avicenna says that conditional Cesare (II) with a simpliciter universal affirmative minor and a universal negative major premise, which “negates the implication only, but does not exclude <its being true> simpliciter”, is inconcludent (*lā yuntiğ*). And as a concrete substitution instance of such a sterile argument form, Avicenna provides the following example “whenever human is rational, then the donkey brays, and never: if a pair is even, then it follows that the donkey brays. Therefore, never: if human is rational, then it follows that a pair is even”. Two questions arise. First, what does Avicenna mean when he says that the premise, which can be formally represented as “never: if A is B, then *it follows that* J is D”, is a “negation of implication”? Second, what does he mean when he says that a universal negative conditional is a negation of implication, but is of the kind that does *not* exclude the possibility of the conditional being true as a simpliciter conditional? In §5.2.3 I suggested that for Avicenna, a consequent *genuinely* follows from an antecedent in a (universal affirmative) restricted conditional when the concomitance of the antecedent and consequent in mental supposition is not merely an accident. Following is genuine when the existence of the consequent in supposition and, *a fortiori* as a matter of fact, is consistent with the supposition of the antecedent under all conditions consistent with

the supposition of the antecedent. Thus, when we say that “J is D” follows from “A is B”, we mean that of all the states consistent with “A is B”, “J is D” is a condition that is so essential to our conceptualization of A’s being B that under no conditions is it conceivable to suppose “A is B” and not suppose “J is D”. It is in order to make manifest that this type of following stands between “A is B” and “J is D” that sometimes for “whenever A is B, then J is D”, Avicenna adopts an (optional) alternative formulation “whenever A is B, then it follows that J is D”. If this is the *A*-conditional, then the *E*-conditional is the contrary of this type of following. Avicenna believes that *E*-conditionals may be said to be “contrary” to *A*-conditions in two distinct ways. In the stronger sense, “never: if A is B, then J is D” means that under no conditions is an antecedent set down as a hypothesis and the consequent follows. In this sense, then, “never: if A is B, then J is D” is true when there is no condition “H is Z” such that “H is Z” can be attached to the antecedent “A is B” and the attachment of this condition authorizes the following of “J is D”. Take as an example “never: if man is a stone, then man is animal”. Is there some condition “H is Z” that can be (consistently) added to “man is a stone” that would authorize “man is an animal” as a consequent? If there is such an “H is Z”, then “never: if man is a stone, then man is an animal” is false. If no such “H is Z” can be found, then it is true. Thus, this strong reading of *E*-conditionals, excludes any possibility of the antecedent and consequent being even accidental concomitants. In this sense, a true strong *E*-conditional cannot be true when read as simpliciter conditional. In a weaker sense, “never: if A is B, then it follows that J is D” may be taken to mean that it is never the case that “A is B” *implies* “J is D”. What is being denied here is not that “A is B” and “J is D” are compatible under all conditions real or supposed, but that the fact that the *responsibility* for “J is D” being true is under no conditions linked to the truth of “A is B”. In this case the truth of “never: A is B, then

J is B” is consistent with the truth of a simpliciter reading of “if A is B, then J is D”. Avicenna adduces “never: if man is an animal, then two is even” as an example of a true *E*-conditional, but only in the sense that it is never said that *responsibility* for the truth of “two is even” depends on the fact that man is an animal. But since both antecedent and consequent are true in matter of fact, “if man is an animal, then two is even” is a true simpliciter conditional.

Let us now return to our text on the inconcludency of certain readings of Cesare (II). Cesare (II)—in fact, any purely connective conditional conjunctive syllogism—is inconcludent if the connective conditional premises are each simpliciter conditionals. Avicenna next considers Cesare (II) with mixes of simpliciter and restricted connective conditional premises. Avicenna says about the Cesare in which the affirmative minor premise is a simpliciter conditional and the negative premise “negates the implication only, but does not exclude <its being true> simpliciter”, such a syllogism is inconcludent. The earlier discussion of weak and strong kinds of *E*-conditionals allows us to conclude the following. If conditional Cesare (II) is composed of a simpliciter A-conditional minor and a *strong* *E*-conditional major premise, then the conditional Cesare (II) can be shown to be concludent either by direct reduction to Celarent by conversion of the major premise, or by indirect reduction to Ferio by transposing the contradiction of conditional Cesare’s conclusion with the contradiction of the A-conditional minor premise. However, if the minor premise in conditional Cesare (II) is taken as a simpliciter and the major is taken as a *weak* *E*-conditional, in which there is the negation of implication and which is, as a result, consistent with the truth of the antecedent and consequent in a simpliciter conditional, then conditional Cesare (II) is inconcludent.

In order to show that conditional Cesare (II) with a simpliciter A-conditional minor and a weak restricted *E*-conditional major, Avicenna appeals both to a stronger version of the

Productivity Principle mentioned earlier, and to Aristotle's method of providing term triples that show that no conclusion follows of logical necessity from the given premises. Remember from §5.2.2 that the weaker form of the Productivity Principle of syllogisms only guarantees that none of the middle term variables in the conclusion of a figure are in the conclusion. This condition made sure that no conclusion could follow logically from the premises on account of one of the premises, or neither of them. In passing I also mentioned that Avicenna regarded the categorical syllogism in Barbara "if every human is risible, and every risible thing is an animal, then every human is an animal" as formally valid, but that "not everything that is a syllogism *produces* like one".⁴²⁰ With these words, Avicenna drives a wedge between the ideal of syllogistic demonstrative productivity that leads the reasoner from knowns to unknowns, and the reality that no *formal* treatment of argumentative validity can guarantee this property for all arguments. Avicenna is able to make a similar point here with respect to conditional Cesare (II) with a simpliciter A-conditional minor and a weak E-conditional major. If the weak E-conditional is consistent with the simpliciter truth of the same conditional, then in reality conditional Cesare (II) so understood is in the end Cesare (II) with an (inconcludent) pair of simpliciter conditionals, as Avicenna had already claimed with respect to perfect pure connective conditional conjunctive moods.

[Text 13] If the major premise is [true] with the middle in the manner of accompaniment rather than in the manner of implication, and the middle and the minor are similarly <related>, let us then investigate whether something follows in this way [of laying down the premises] or not. In fact, it is more suitable to say that this is not a syllogism because it has not produced in us knowledge of something unknown. If the middle part is not a necessary concomitant [*multazim*] of the major, but merely aids in [acquiring knowledge of the major], then we already knew prior to deploying the syllogism that the

⁴²⁰ Avicenna, ŠQ IX, 422.8 (italics added).

major is in the minor, and with every existent thing and every supposition, regardless of whether or not we attended to the middle term beforehand.⁴²¹

Remember from §3 that for Avicenna, simpliciter connective conditionals are conditional sentences in which the truth of the consequent is consistent with the truth of the antecedent. Restricted conditionals as we have seen in §3 and §5.2.3 are conditional sentences in which the supposition of the consequent is consistent with the supposition of the antecedent. Unlike in the latter kind of conditional, in any true simpliciter conditional it is necessary that the antecedent and consequent are *known by the reasoner* employing them to have been true at some point in the past, present, or future.⁴²² Then conditional Cesare with a simpliciter A-conditional minor and a weak E-conditional major is inconcludent, because the weak E-conditional major is consistent with a simpliciter conditional with the same antecedent and consequent. It is inconcludent because it violates the stronger version of the Productivity Principle for syllogisms.

Text 13 implicitly relies on the fact that conditional Cesare (II) with a simpliciter A-conditional minor and a weak E-conditional major violates the Productivity Principle. Avicenna makes explicit use of Aristotle's practice in his categorical syllogistic of showing that the formal properties of certain premise pairs are always inconcludent by furnishing a pair of triples, one of which has three terms in which the premises come out true and the conclusion is true as a universal affirmative, while the other triple makes the premises true and the conclusion true as a universal negative.⁴²³ For conditional Cesare (II) with a simpliciter A-

⁴²¹ Avicenna, ŠQ VI, 297.8-14.

⁴²² Notice the similarity to Avicenna's ampliation of the "*muṭlaq*" categorical premise (see Street, "Outline of Avicenna's Syllogistic", 134). The similarity between the *muṭlaq* categorical and the conditional '*alā l-iṭlāq*' is not by chance, but points to the close analogy in Avicenna's mind between predication in a categorical proposition and following in the conditional proposition.

⁴²³ We should not say that the premises "yield" a universal affirmative, and other premises "yield" a universal negative. Their ability to yield is exactly what is being disproven. If it is necessary for there to be some conclusion, then it is necessary that it be either an A-, E-, I-, or O-proposition. However if it can shown to "yield"

conditional minor and a weak *E*-conditional major, Avicenna furnishes the following pair of triples. In order to produce a true weak *E*-conditional conclusion, the triples are: antecedent (minor term) “human is rational”, (middle term) “donkey brays”, and (major term) “pair is even”; in order to produce a true *A*-conditional conclusion: antecedent “human is rational”, (middle term) “donkey brays”, and (major term) “human is animal”. From these triples we can construct the following pair of arguments:

(Conclusion weak *E*-conditional): “whenever human is rational, then the donkey brays, and never: if a pair is even, then it follows that the donkey brays. Therefore, never: if human is rational, then it follows that a pair is even”.

Conclusion *A*-conditional): “whenever human is rational, then donkey brays, and never: if a pair is even, then it follows that human is an animal. Therefore, whenever human is rational, then human is an animal”.

As Günther Patzig has clearly and forcefully argued with respect to Aristotle assertoric syllogistic,⁴²⁴ it is sufficient to show that it is not possible for the premise set of conditional Cesare (II) to yield any conclusion (whether an *A*-conditional, *E*-conditional, *I*-conditional, or *O*-conditional) *of necessity* if we can find one set of triples that is consistent with an *E*-conditional conclusion and another set of triples that is consistent with an *A*-conditional conclusion. Since Avicenna has just provided such triples, conditional Cesare (II) with a simpliciter *A*-conditional minor and weak *E*-conditional major is inconcludent.

As we have seen, Avicenna’s theory of simpliciter/restricted conditionals with universal and particular quantifiers introduces much greater complexity into the task of deciding which purely connective conjunctive syllogisms are concludent and which are inconcludent. Nevertheless, this should not make us lose sight of the important parallels that remain between Avicenna’s conjunctive syllogisms and Aristotle’s categorical syllogisms.

to conclusions that are contrary to each other, then the necessity of *any* conclusion has been refuted. For a lucid discussion of this point, see Patzig, *Aristotle’s Theory of the Syllogism*, 172.

⁴²⁴ G. Patzig, *Aristotle’s Theory of the Syllogism*, 168–83, especially 174–6.

Avicenna still insists that direct and indirect reduction are sufficient to show that the conclusions of imperfect syllogisms follow with logical necessity from their premises. Further, he also uses the method of finding pairs of triples in order to reject moods in which the premise set is sterile.

§5.3.1 INTRODUCTORY REMARKS ON THE PROBLEM OF REPETITIVE SYLLOGISMS, AND THEIR REDUCTION TO CONJUNCTIVE SYLLOGISMS

For Aristotle, to say that an argument is good solely on account of the formal properties of its premises and conclusion is to say that the argument is in a syllogistic mood. In each of the moods of the three figures in *An. Pr.* A4-6, any conclusion (with the stipulated quantity and quality) follows of necessity from the two premises (with the stipulated quantity and quality) such that it is impossible for us to substitute appropriate concrete terms in the premises that make the premises true but make the conclusion come out false. Of the hundreds of possible combinations of quantity, quality, and position of the middle term (figure), in only four moods in the first-figure does the conclusion of the syllogism follow with logical necessity from the premises in such a way that the necessity of the following is *self-evident*. These four moods are called perfect. For all the other imperfect moods that yield conclusions on account of the formal properties of the premises and conclusion, this necessity must be *proved* because it is not self-evident, whereas the necessity of perfect moods is on principle indemonstrable because of its self-evidence. In order to prove the necessity of the following in imperfect moods, Aristotle uses two main methods, direct and indirect reduction. These two methods along with his rejection procedure, are sufficient as a proof method for showing that any particular two-premise/single conclusion, three term categorical argument form either entails

a conclusion with strong relevance (i.e. it is concludent), or no conclusion follows from premise pair of necessity (i.e. it is inconcludent).

As we have seen, Avicenna makes these fundamental logical concepts the foundation of his conjunctive syllogistic. Avicenna's main motivation for generalizing these foundational concepts in Aristotle to arguments with conditional premises and conclusions seems to have been his realization that Aristotle's syllogistic as it is developed in the *Prior Analytics* is quite inadequate to providing an account of logical following between conditional premises and conditional conclusions. This is so despite the fact that there are many arguments in mathematics, physics and metaphysics that have conditional premises and conclusions, and whose validity must be measured against some logical criteria. Aristotle's categorical syllogistic in the *Prior Analytics* (much less his theory of material consequences in the *Topics*)⁴²⁵ does not provide these criteria. Thus, it would appear that Avicenna felt the need to take the principles expounded in the *Prior Analytics* and develop his conjunctive syllogistic in analogy with Aristotle's categorical syllogistic.

Yet, much of Avicenna's work in ŠQ VI seems unnecessary. It is patently false that in Avicenna's day, there were no systems of logic which provided criteria for determining whether an argument with conditional premises and conclusions was valid or not. As we saw in §4, Avicenna was well aware of the fact that repetitive syllogisms (*qiyāsāt istiṭnā'iyya*) could take conditional propositions in the antecedent and consequent position and yield conditional propositions as a conclusion. Even if Avicenna was unhappy with the semantics of conditionals that relied on notions such as connection (*ittiṣāl*) and incompatibility (*'inād*) rather than implication (*luzūm*) and concomitance (*ma'iyya*, *muwāfaqa*) in truth (*fī l-wuḡūd*, i.e.

⁴²⁵ S. Read, "The Medieval Theory of Consequence", *Synthese* 187 (2012): 899-912, especially 904f.

what the antecedent and consequent express *corresponds* to some object in extramental existence) or supposition (*fī l-farḍ*), this does not seem to justify all of the work required to develop his conjunctive syllogistic. Yet Avicenna perceives that concludency conditions for conjunctive syllogisms are based on the presence or absence of shared part (*al-ḡuz' al-muštarak*), whereas concludency for repetitive syllogisms is based on “repetition (*istiṭnā*)” of propositional content with assertive force. These differences are fundamental.

[Text 14] Since we have spoken about conjunctive syllogisms, both categorical and conditional, we ought now to speak about repetitive syllogisms. We say that the repetitive syllogism is distinguished from the conjunctive in that one of the extremes of the quaesitum [i.e. one of the pair of disjuncts in a quaesitum of the form ‘*P* or not-*P*’] is actually present in the repetitive syllogism, whereas it is only potentially present in the conjunctive syllogism. For example, “every human is an animal and every animal is a body, so every human is a body”. Thus, neither the conclusion nor its contradiction is actually present in the conjunctive syllogism. However, if we say “if human is an animal, then human is a body” or “if human is not a body, then <human> is not an animal”, and then we say with respect to the first “but human is an animal”, this yields “human is a body”; and if we say <“but human is an animal”> with respect to the second, this yields <“human is a body”>, then we find that one of the extremes of the conclusion, namely the affirmative <conclusion>, is actually present in the first of the syllogisms as a consequent. Similarly, we find the second extreme (of the disjunctive quaesitum) actually present in the second syllogism as an antecedent. We say, then, that every repetitive syllogism is composed from a conditional premise and an repetitive premise that is either identical to one of the parts of <the conditional> or its contradictory opposite, and yields thereby the other member <of the conditional> or its contradictory opposite.⁴²⁶

It seems that undergirding Avicenna’s logical theory and practice lie two basic assumptions. Many ancient logicians—and in this Avicenna is not exceptional—seem to take it for granted that (1) the only correct account of the formal properties that an argument form must have in order for its conclusion to follow necessarily from its premises is the account that Aristotle gives in the *Prior Analytics*; and (2) that the only correct methods of proof, whereby any particular argument form can be proved to be a genuine syllogism or inconcludent, are direct

⁴²⁶ Avicenna, ŠQ VIII 389.6–390.3.

and indirect reduction, and rejection from pairs of concrete triples. Yet, any logician that accepts (1) and (2) must realize that the argument forms that Avicenna includes under the heading of repetitive syllogisms are incompatible with a view of logic that takes (1) and (2) as basic. This is why Avicenna introduces the division of syllogisms into repetitive and conjunctive: he realizes that they operate according to different notions of what makes a valid argument, and how an argument can be proved valid. This insight lies behind Avicenna's harsh ridicule of philosophers who divided what Avicenna calls repetitive syllogisms into "moods", calling some "perfect" and others "imperfect", and using "reduction" of imperfect moods to perfect moods to demonstrate the conclusivity of the former by means of the latter. As we saw in §3, Avicenna's biting critique is justified.

[Text 15] They felt compelled to expound all of these forced and ill-conceived notions [*takkalluf*] for one reason, namely their having lost what the First Teacher [Aristotle] dealt with at great length about conditional syllogisms. Thus, they were obliged to delve into the matter on their own. Add to this their obliviousness to conjunctive [hypothetical] syllogisms, so that when they came upon these repetitive <sylogisms>, they deemed the number that appeared to them paltry, and they were appalled by the notion that these syllogisms should not be equal what the first teacher [Aristotle] expounded in the categorical syllogistic. Thus did they resort to exacerbating this outrage by contradicting [Aristotle].⁴²⁷

Let us return to the three fundamental principles on which Avicenna built his conjunctive syllogistic, namely, middle terms determining figure, quantification determining mood, and proof as direct and indirect reduction. Repetitive syllogisms cannot stand in figures because they do not have middle terms. Certainly, the antecedent is repeated in both premises of modus ponens, and the contradictory opposite of the consequent is "repeated" as a premise in

⁴²⁷ Avicenna, ŠQ VIII, 397.5-10. I am grateful to Stephen Menn for helping to clarify Avicenna's meaning in this passage.

modus tollens. But this analogy is deceptive. As Aristotle says in *An. Pr.* A23,⁴²⁸ the importance of the middle term, “C”, for example, lies in that its presence is necessary in order to conclude, for example, that “A belongs to B” so that our argument is not circular and not incoherent. We cannot *conclude* “A belongs to B” from the *assumption* that “A belongs to B” because then “the proposition originally in question will have been assumed (*An. Pr.* A23 40b30)”. On the other hand, if we assume “A belongs to C” but nothing more, or we assume “A belongs to C” and “C belongs to D” and “D belongs to E” and so on, but never predicate anything of B, then while a syllogism concluding “A belongs to E” is possible, no syllogism will be possible in relation to B, since there is no “connexion however being made with B (40b40)”. In an repetitive syllogism however, the “repeated” part, whether it is the “repetition” of the antecedent itself, or the “repetition” of the contradiction of the consequent does not connect anything to anything. In both cases, these “repetitions” only authorize the conclusion in the sense that they rely on logical distinctions that can be intuitively made between the occurrence of a proposition in an argument unasserted in one part of the argument, and asserted in the other part of the argument. In other words, the “repetition” that authorizes the conclusion in exclusive syllogisms ultimately relies on intuitions about conventions relating to the speech acts of supposition (as a speech act, not as a mental supposition about possibilities) and assertion. On the other hand, Avicenna’s system of conjunctive syllogisms, like Aristotle’s system of categorical syllogism does not rely on even an intuitive distinction between assertion and unasserted propositional contents.

Consider quantification. In Avicenna’s conjunctive syllogistic and Aristotle’s categorical syllogistic, syllogisms in a single figure are distinguished from one another by the

⁴²⁸ For details on the role of the middle term in *An. Pr.* A23, see T. Smiley, “Aristotle’s Completeness Proof”, *Ancient Philosophy* 14 (1994): 25-38.

combination of quantifiers in the premises. Thus, based on the few details that can be gleaned from Avicenna's negative account of previous logicians' treatment of what Avicenna calls "repetitive syllogisms", it seems to have been thought that syllogisms with one conditional major premise and an asserted minor premise counted as a "figure". Modus ponens could then be called a perfect mood (*ḍarb*) that was, unlike other moods, perfect in the sense that the logical necessity of the conclusion's following from the premises was self-evident and, thus, indemonstrable. Other "imperfect" moods such as modus tollens could then be "perfected" by "reduction" to this "first-figure", all of this in analogy with Aristotle's categorical syllogistic. On the other hand Avicenna developed a theory of quantified conditionals for his conjunctive syllogistic.⁴²⁹ Can these quantified conditionals be used in repetitive syllogisms? They cannot. In fact, simpliciter conditionals cannot be used at all (as we saw in §3) because they violate the Productivity Principle. On the other hand, one of the major characteristics of restricted, and one of the most important in Avicenna's mind, is that it allows impossible antecedents to appear in the antecedent and consequent position. Thus, in general, a true A-conditional with an impossible antecedent such as "if two is not a pair, then two is odd" is not only invalid but the antecedent cannot even be asserted in the first place. Similarly modus tollens is invalid when the antecedent is impossible but the consequent is possible, e.g. "always: if man is a stone, then man is corporeal". As for the rest of the types of quantified conditionals, I-, and O-conditionals seem to be assimilable to simpliciter conditionals and thus violate the

⁴²⁹ Avicenna was not the first to try to introduce the formal elements of Aristotle's logic (quantification, mood, and middle term) to non-categorical types of syllogism. Indeed, this tradition goes as far back as Theophrastus' analysis of Lejewski calls "proseptic syllogisms"; see C. Lejewski, "On Prosleptic Syllogisms", *Notre Dame Journal of Formal Logic* 2/ (1961): 158-76). These types of syllogism were also analyzed by Galen, and a scholium that has been attributed to Ammonius. See also S. Bobzien, "Wholly Hypothetical Syllogisms", *Phronesis* 45/2 (2000): 87-137.

Productivity Principle;⁴³⁰ weak E-conditionals work like simpliciter conditionals and so are invalid for the same reason; a strong E-conditional “never: if A is B, then J is D” is, on Avicenna’s calculation, equipollent with the following A-conditional “always: if A is B, then J is not D” and so it is vulnerable to the same incoherencies as A-conditionals used in repetitive syllogisms.⁴³¹

Finally, reduction. The basic question is this: are the tools of conversion and *reductio ad absurdum* sufficient to prove that all the repetitive syllogistic forms are concludent? The short answer is “no”. As we have seen, the methods of perfecting imperfect syllogisms by reduction to perfect syllogisms only make sense in the context of a logic for which terms such as “perfect”, “figure”, “mood” and “imperfect” have meaning, and as I have just showed, none of these terms can be suitably extended to repetitive syllogisms. Nevertheless, such a response is not adequate because at the opening of ŠQ IX 1 (see Text 1 above), Avicenna states his intention to show that repetitive syllogisms can be reduced to conjunctive syllogisms. The three questions to answer for this section are, first, what does Aristotle mean when he says that imperfect categorical syllogisms are reducible or perfected “by means of” perfect ones? Second, at *An. Pr.* A23 40b21, Aristotle says that he will show that not only are all categorical arguments with more than two premises reducible to Barbara and Celarent, but that “all syllogisms without qualification” can be reduced by means of perfect universal moods of the first-figure. Shortly thereafter (40b25), Aristotle says that, in fact, per impossibile syllogisms and hypothetical syllogisms are both perfected by means of the perfect, first-figure syllogisms (the latter kind of syllogism being a genus of the former). How does the claim that hypothetical syllogisms are perfected by the categorical syllogisms relate to the claim that

⁴³⁰ Avicenna, *Propositional Logic*, 272-4.

⁴³¹ Avicenna, ŠQ VII, 366.1.

categorical syllogisms with more than two premises are perfected by two-premise perfect figure syllogisms? Thirdly, as we can see from Text 1, Avicenna makes a similar claim, save only that he has substituted “conjunctive syllogism” for categorical syllogism. Does the introduction of conjunctive syllogisms allow Avicenna to make more headway in executing the claim Aristotle makes in *An. Pr.* A23?

One final note deserves attention. In his classic article on Stoic and Aristotelian logic, Michael Frede disputes Ian Mueller’s earlier claim that the main point of contention between Stoic and Peripatetic logicians was over the “priority” of their respective syllogistic systems, each party claiming that their syllogistic was the most basic.⁴³² In his concluding section Frede notes that the claim that the ancient logicians were arguing about whose syllogistic was prior “does not seem very plausible”.⁴³³ In his article Frede shows that much of the debate was given over to trying to establish some clear criteria for what exactly should count as a syllogism and what should not. According to Frede, the debate about whose syllogistic was prior was unlikely to have occurred since none of the parties seems to have agreed that their opponents syllogistic was a syllogistic at all. I do not dispute Frede’s argument or his evidence from the Stoics or from Alexander and some of Aristotle’s later commentators. However, I believe that Avicenna was engaged in something like a dispute over the priority of Aristotle’s syllogistic over other kinds of syllogistic that were common in his day. Frede says that one

of the causes of the controversy may have been that those engaged in the discussion did not have a clear notion of the kind of priority they wanted to argue for. After all, there are various kinds of priority which could be thought to hold between categorical and hypothetical syllogisms.

⁴³² M. Frede, “Stoic vs. Aristotelian Syllogistic”, 2; see I. Mueller, “Stoic and Peripatetic Logic”, *Archiv für Geschichte der Philosophie* 51 (1969): 173f.

⁴³³ M. Frede, 30.

I do not believe that Frede's conclusions hold for the late antique period. Susanne Bobzien has shown in her important article on totally hypothetical syllogisms that while there was clearly a dispute in Alexander's day about whether or not hypothetical syllogisms were really syllogisms, by the time that Philoponus wrote his commentary on the *Prior Analytics* "[n]either Philoponus nor any of the later sources voices any doubt that WHs are proper syllogisms. From Philoponus we can infer that this is a conscious change from Alexander".⁴³⁴ It is also clear in Alfarabi's and Avicenna's work that few save Averroes perhaps, who seems to have devoted an entire treatise to the topic and a good portion of his *Talḥiṣ al-Qiyās*,⁴³⁵ seem to have seriously entertained the idea that hypothetical syllogisms are not syllogisms *tout court*. Rather, it seems that by Avicenna's day, there was a more or less clear idea about what the valid and invalid hypothetical syllogisms were and what the valid and invalid categorical syllogisms were. Thus, it seems plausible to see Avicenna's argument that conditional syllogisms must be reduced to conjunctive syllogisms in order for the former's conclusency to be made manifest as being an argument about the priority of the conjunctive syllogisms to the hypothetical syllogisms. Finally, to the extent that Avicenna's conjunctive syllogistic is based on some of the main principles of Aristotle's categorical syllogistic, Avicenna's ultimate claim might be not that categorical syllogisms are prior to hypothetical syllogisms, but that the logical principle set out by Aristotle are prior to those principles upon which the hypothetical syllogistic rests.

⁴³⁴ Bobzien, "Wholly Hypothetical Syllogisms", 134.

⁴³⁵ See Averroes, *Talḥiṣ al-Qiyās*, ed. Butterworth (Cairo: al-Maṭbaʿa Dār al-Kutub al-Miṣrīya, 1948), see especially his comments on *An. Pr.* A44.

§5.3.2 ARISTOTLE ON DIRECT AND INDIRECT REDUCTION, AND THE REDUCTION OF PER IMPOSSIBILE SYLLOGISMS AND “SYLLOGISMS FROM A HYPOTHESIS”

As I have said before, direct and indirect reduction are methods of proof. They do not transform an inconcludent argument form into a syllogism. They only reveal that the argument form in question is concludent. To be more precise: direct and indirect reduction are operations that are performed on the variables of a syllogistic form (they are *not* operations on concrete substitution instances) that *reveal* that the conclusion of the argument form follows from the premises entirely on account of the formal properties of the premises and conclusion. Günther Patzig is right when he says that Aristotle “did not mean reduction to be understood as a proof [of a conclusion from premises] but as a procedure for *transforming* imperfect syllogisms into perfect”.⁴³⁶ Thus, reduction⁴³⁷ of an imperfect figure to a perfect one should be understood as a transformation, the primary means of effecting this transformation being conversion and reduction ad absurdum. Further, this transformation should have two characteristics, each of which is crucial in Aristotle’s mind. First, this transformation should make it obvious to the person carrying out the reduction of the imperfect to the perfect mood that a conclusion *c*’s following from the imperfect premise set *P* is necessary even if this necessity is not evident in the imperfect moods at first sight. Second, the extent to which a conclusion *c* can be shown to follow logically from its premises *P* is ultimately the extent to

⁴³⁶ Patzig, *Aristotle’s Theory of the Syllogism*, 134 (italics in original).

⁴³⁷ Gisela Striker argues persuasively (G. Striker, “Perfection and Reduction in Aristotle’s *Prior Analytics*”, *Rationality in Greek Thought*, eds. M. Frede, G. Striker (Oxford: Clarendon Press, 1996), 203-20) that a distinction ought to be made between the “perfection” of an imperfect syllogism and the “reduction” of a syllogism. In the latter, the reduction of a syllogism may include perfect first-figure syllogisms that can be reduced to other types of syllogism, even if they the latter are imperfect. Thus, in *An. Pr.* A7, Aristotle *reduces* but does not, strictly speaking, *perfects* Darii and Ferio by means of the concludent, second-figure syllogisms (Cesare and Camestres). He does this in order to carry out the program of *An. Pr.* A7 that all concludent syllogisms can be reduced to first-figure Barbara and Celarent. Nevertheless, in this chapter I will not always adhere strictly to this distinction. Since the logical machinery of reduction and perfection are essentially identical, I do not feel compelled to adhere strictly to this terminological distinction.

which it can be shown that c can be shown to be related to P in the manner of a perfect syllogism. This is not to say that imperfect syllogisms are inconcludent. This is to say that Cesare, for example, is only concludent to the extent that its conclusion c and its premises P are shown to have the formal properties of a perfect syllogism after certain authorized transformations are carried out on the premises and conclusion. Thus, the methods of direct and indirect reduction must reveal that the necessity of c 's following from an imperfect premise set P is somehow derived from, or in some sense flows from, c 's following from a perfect, first-figure premise set.

Thus, consider Aristotle's reduction of (third-figure) Darapti (*An. Pr.* A6 28a 18-21).

If they (sc. the premises) are universal, then, and when both P and R belong to every S , I say that P will belong to some R of necessity. For since the positive premiss converts, S will belong to some R , so that, since P belongs to all S and R to some S , it is necessary for P to belong to some R , for a syllogism in the first-figure comes about.

Let P be the premise set of Darapti $\{Aps, Ars\}$, let c be the conclusion set $\{Ipr\}$, where ' Aps ' is the major premise and ' Ars ' is the minor premise, and let Pc be the set whose members are the premises and conclusion of Darapti $\{Aps, Ars, Ipr\}$ in that order.⁴³⁸ In what follows, when I refer to a "concrete argument in Darapti" or simply an "argument in Darapti", e.g. "if human belongs to every rational and risible belongs to every rational, then human belongs to some risible" and so on, I am referring to any argument in which concrete terms such as 'human' and 'rational', and 'risible' substitute uniformly for p , r , and s in, for example, Darapti's Pc -set. When I speak about 'Darapti' simpliciter, I am referring to the Pc -set, which is a way of representing the formal structure of all concrete arguments in Darapti. With this terminology in mind, recall what Aristotle wants to show in the above passage. Aristotle appears to have in

⁴³⁸ Technically speaking, the order in which members of a set are set out in language does not distinguish them. If $A=\{1,2,3\}$, $B=\{2,1,3\}$, and $C=\{3,2,1\}$, $A=B=C$, by definition. In this discussion, I use the term "set" in a non-technical sense. I do not believe this obscures understanding of the present argument.

mind the following scenario: a reasoner is confronted with an argument in which it is claimed that if you admit that every rational being is human, and also that every rational being is risible, then that some risible entity is human follows necessarily. Since the concrete argument is not in the first-figure, it cannot be taken as evident that the conclusion does in fact necessarily follow from the premises. How, then, can we substantiate the claim that the logical form of the premises is a sufficient grounds for the necessary following of the conclusion? Aristotle provides a response in the above passage, first by considering the above concrete argument vis-à-vis its formal properties, viz. Darapti, and then uses conversion of the minor premise to reveal Darapti's special relationship to first-figure Darii.

<u>Darapti</u>	<u>Darii</u>
'if human belongs to every rational, and risible belongs to every rational, then human belongs to some risible.'	'if human belongs to every rational, and rational belongs to some risible, then human belongs to some risible.'
If P belongs to every S, and R belongs to every S , then P belongs to some R.	If P belongs to every S, and S belongs to some R , then P belongs to some R.
{Aps, Ar s, Ipr}	{Aps, I sr, Ipr}

It is clear from Figure 1 that any concrete argument in Darii will be nearly identical to the corresponding concrete argument in Darapti, with the same major premise, middle terms, and conclusion. The only difference between the two argument forms lies in the quantity of their minor premises and the placement of the middle term vis-à-vis the middle term of the major premise. With regard to quantity, the minor premise of Darii is a particular affirmative whereas the minor premise of Darapti is a universal affirmative. With regard to the placement of the middle term, the middle term in Darii's minor premise stands in the predicate position;

the middle term of Darapti's minor premise stands in the subject position. But these formal differences between Darapti and Darii can be eliminated when we realize that the minor premise in Darapti can be converted by the rule of imperfect conversion (or limitation): if Aab , then necessarily Iba , or simply Aab entails Iba , for any substitutions of a and b . Thus, an imperfect syllogism is called directly reducible to a perfect, first-figure syllogism when the imperfect syllogism differs formally from the perfect syllogism by a premise that formally entails a premise in one of the perfect moods by any one of the rules of conversion (i-conversion, e-conversion, or conversion by limitation). Darapti is, then, said to be 'reduced to' or 'perfected by' Darii when the conversion of the premise has made this reducibility apparent to the reasoner. This constitutes the proof of Darapti. Thus, Darapti is *reducible* to Darii because for any values of p , r , and s the major premise and conclusion of Darapti and Darii are identical, and Darapti's minor premise Ars entails the minor premise in Darii Isr by limitation. Darapti is *reduced* to Darii in the passage from *An. Pr.* A6 28a 18-21 because this relationship between Darii and Darapti is revealed by this transformation of the formal structure of Darapti into Darii. Finally, it seems that, according to Aristotle, this process of formal reduction of imperfect syllogisms to perfect by conversion is sufficient to reveal the necessity of Ipr 's following from the premises Aps and Ars , for any p , r , and s . More generally, as Thom has noted, the reduction by conversion implicitly relies on what is called the rule of "Cut", which for our purposes here simply means that if the conversion of one of the premises of an imperfect syllogism generates a perfect syllogism, then the conclusion c of the imperfect syllogism logically follows from its premises P .⁴³⁹ Thus, the conclusion of Darapti $\{Ipr\}$ follows

⁴³⁹ Thom, *The Syllogism*, 36-9.

logically from its premises $\{Aps, Ars\}$ because the minor premise *Ars* can be converted by limitation to *Isr* to form a syllogism that is identical to Darii $\{Aps, Isr, Ipr\}$.

As a further illustration, consider Aristotle's reduction of second-figure Cesare (*An. Pr.* A5 27a 5-9), which is also carried out by conversion of the universal negative major premise:

For let *M* be predicated of no *N* and of all *X*. Now since the privative premiss converts, *N* will belong to no *M*; but it was assumed that *M* belongs to all *X*, so that *N* will belong to no *X*—this was proved before.

Just as in the case of Darapti and Darii above, Aristotle is trying to show that when confronted by any concrete argument that has the formal properties of Cesare, we can conclude that the conclusion of that concrete argument follows from the premises with logical necessity, i.e., without any consideration of the matter of the premises or the conclusion, but solely by virtue of the formal properties of the premises. As with third-figure Darapti, Aristotle believes that in order to show this necessity of following it is sufficient to reduce the imperfect, second-figure syllogism Cesare to the perfect first-figure Celarent. In this case however, the reduction is carried out by the formal conversion of Cesare's major premise.

<u>Cesare</u>	<u>Celarent</u>
'if human belongs to no stone, and human belongs to every risible, then stone belongs to no risible.'	'if stone belongs to no human, and human belongs to every risible, then stone belongs to no risible.'
If M belongs to no N , and M nbelongs to every X, then N belongs to no X.	If N belongs to no M , and M belongs to every X, then N belongs to no X.
$\{Emn, Amx, Enx\}$	$\{Enm, Amx, Enx\}$

By means of Aristotle's above demonstration at 27a 5-9, we know that for any value of *m*, *n* and *x*, Cesare is *reducible by conversion* to Celarent because the member of Cesare's *Pc*-set that is not identical to Celarent's *Pc* set (viz. Cesare's major premise, *Emn*) entails a member of Celarent's

Pc-set (viz. Celarent's major premise *Enm*) by e-conversion. Cesare is *reduced by conversion* or *directly reduced* to Celarent when this relationship has been clarified to the reasoner. Finally, we can now claim that not only is 'no stone is risible' follows logically from the assumption that no stone is human, and that every risible entity is human, but for any *m*, *n*, and *x*, the conclusion *c* {*Enx*} follows logically from the premise set *P* {*Emn*, *Amx*} since if there is a member of Cesare's *Pc* set, call it *u*, that is not identical to any member of Celarent's *Pc* set, then there is a member of Celarent's *Pc* set, call it *v*, such that *u* entails *v* (by e-conversion). Thus, by means of the process of conversion Aristotle is able to account for the *necessity* of the conclusion's following from the premise set for any concrete argument in Cesare by locating it in the self-evident necessity of Celarent's conclusion's following from its premise set. By this same process of reduction by conversion, Aristotle is also able to make this necessity *evident* by transforming Cesare's major premise, revealing thereby that the following of Cesare's conclusion from its premises is no weaker than the necessity of the following of Celarent's conclusion from its premises.

Reduction by conversion is only one of Aristotle's methods of showing that a conclusion is a logical consequence of its premises; the other is, of course, *reductio ad absurdum*. Direct reduction is not sufficient to show that the conclusions of concrete arguments in Baroco (II) and Bocardo (III) follow necessarily from their premises. Consider a concrete argument in Baroco and then its formal representation (see Figure 3):

<u>Baroco</u>
'if white belongs to every stone, and white does not belong to some risible, then stone does not belong to some risible.'
If M belongs to every N,

and M does not belong to some X, then N does not belong to some X.
{ <i>Amn</i> , <i>Omx</i> , <i>Onx</i> }

If only direct reduction were available to us, then Baroco would appear to be inconcludent, though we would not be able to find triples to prove its inconcludency definitively. Suppose that Baroco were considered reducible by conversion to a first-figure syllogism, either Barbara, Celarent, Darii or Ferio. On this assumption, the conclusion *c* of any concrete argument in Baroco, which we might represent generally as {*Onx*}, follows logically from the premise set *P*, which we might represent as {*Amn*, *Omx*}, if the conclusion and one of the premises of Baroco are formally identical to a first-figure mood and the remaining premise entails the other premise in the first-figure mood by a rule of conversion. But in the case of Baroco, this is clearly impossible. The only convertible premise in Baroco is the major premise which converts to *Inm*, but the first-figure *Pc*-set {*Inm*, *Omx*, *Onx*} is sterile because, as Aristotle says (26a20-5), no conclusion follows of necessity in the first-figure (or in any figure for that matter) “if both intervals (i.e. premises) are particular, whether positive or privative, or if one is stated positively, the other privatively, or one indeterminate, the other determinate, or both indeterminate”. A similar problem presents itself with respect to third-figure Bocardo. If we represent Bocardo’s *Pc* set as {*Oab*, *Acb*, *Oac*}, the only convertible premise in Bocardo’s *Pc* set is universal affirmative minor premise. Converting it to a particular affirmative yields a first-figure *Pc* set {*Oab*, *Ibc*, *Oac*}, which, again, corresponds to no perfect first-figure mood.⁴⁴⁰

⁴⁴⁰ Aristotle defines the figures only in terms of the position of the middle term in relation to the extremes in premise set. The quantity and quality of the premises is not considered. Thus, by ‘first-figure’ syllogism, I mean a syllogism in which the middle term is subject for the major extreme and predicate for the minor extreme. I do not mean it in the sense that it is a concludent syllogism or that it is perfect. The inconcludent first-figure *Pc* set {*Oab*, *Obc*, *Oac*} clearly displays the property of transitivity Patzig says belongs to concludent, first-figure moods, so its reason for being imperfect must lie in the fact that it is self-evidently (or not) *inconcludent*; see G. Patzig, *Aristotle’s Theory of the Syllogism*, chapter 3.

Indirect reduction is Aristotle's way out of this dilemma. Consider his reductions of Baroco (*An. Pr.* A5 27a35-27b1) and Bocardo (*An. Pr.* A6 28a17-20):

(Baroco) Again, if M belongs to all N but does not belong to some X, it is necessary for N not to belong to some X. For if it belongs to all X and M is predicated of every N, it is necessary for M to belong to every X. But it was assumed that it did not belong to some.

(Bocardo) For if R belongs to all S but P does not belong to some S, it is necessary for P not to belong to some R. For if it belongs to all R and R belongs to all S, the P will also belong to all S; but it did not belong to all.

The practical question Aristotle is trying to answer when he sets out these reductions is the same as when he reduces imperfect syllogisms to perfect by conversion, namely, does the conclusion that some risible thing is not a stone follow with logical necessity when it is granted that every stone is white and some risible thing is not white? And, further, can this result be generalized to all concrete arguments having the formal properties of Baroco and Bocardo? Aristotle believes that both questions can be answered affirmatively, and his reduction of Baroco in A5 and Bocardo in A6 are intended as just such demonstrations. Moreover, Aristotle is again of the view that indirect reduction allows us, one, to ultimately *ground the necessity* of the conclusion's following from the premises in the necessity of a first-figure mood's conclusion's following from its premises; and, two, that indirect reduction is a process that makes the following of Baroco's conclusion from its premises evident.

In their respective discussions of Aristotle's doctrine of indirect reduction, Paul Thom and Günther Patzig have pointed to the importance of Aristotle's doctrine of conclusion-premise transposition (*metathesis*) in *An. Pr.* B8 for a better understanding of Aristotle's indirect reductions of imperfect moods.⁴⁴¹ Aristotle says (*An. Pr.* B8 59b1-6) that transposition is

⁴⁴¹ See Thom, *The Syllogism*, 39-41; see Patzig, *Aristotle's Theory of the Syllogism*, 151f. 'Conversion' is, of course, different from Aristotle's subject-predicate conversion (*antistrophe*). 'Transposition' is Thom's translation of *metathesis*, though R. Smith (*Aristotle, Prior Analytics*, trans. R. Smith (Indianapolis and Cambridge: Hackett, 1989),

to alter the conclusion and make a deduction (i.e. syllogism) that either the extreme does not belong to the middle or the middle to the last term. For it is necessary, if the conclusion has been converted and one of the propositions stands, that the other should be destroyed. For if it should stand, the conclusion also must stand.

As Aristotle says at *An. Pr.* B11 61a20-25, transposition is a transformation of a syllogism that “has already come about”, meaning that transposition is a process that begins from a syllogism whose conclusion is already known to follow necessarily from its premise set. With respect to the indirect reduction of Baroco and Bocardo, the relevant syllogism that has already been taken is Barbara. For the sake of facilitating exposition, let us represent Barbara with the premise set $P \{Aab, Abc\}$ and conclusion $c \ Aac$, and let its Pc -set be $\{Aab, Abc, Aac\}$.⁴⁴² Transposition, when applied to Barbara, allows us to make two distinct syllogisms that are each composed of two premises and a conclusion that follows necessarily from them. Transposition of Barbara’s conclusion with Barbara’s minor premise gives us a syllogism in the second-figure whose major premise is also Barbara’s major premise, whose minor premise is the contradictory opposite of Barbara’s conclusion, and whose conclusion is the contradictory opposite of Barbara’s minor premise. In other words transposition of Barbara generates the syllogism Baroco $\{Aab, Oac, Obc\}$. For, just as Aristotle says, by transposing Barbara’s conclusion with its minor premise, we generate a syllogism in which the fact that the middle term b does not belong to all of the minor term c follows of necessity from the premises Aab , the premise retained from Barbara, and Oac , the contradictory of Barbara’s conclusion. Analogously, transposition of Barbara’s conclusion with its major premise yields a third-figure syllogism whose major premise is the contradictory opposite of Barbara’s conclusion, whose

79) and the A. Jenkinson (Aristotle, *The Complete Works of Aristotle*, ed. J. Barnes, vol. 1 (Princeton: Princeton University Press, 1984), 39) translate the term as ‘conversion’. Given the mechanics of the operation, Thom’s translation seems suitable, since it also avoids confusion with subject-predicate conversion.

⁴⁴² Aristotle’s construction of Baroco and Bocardo by transposition appears in *An. Pr.* B8 59b25-35.

minor premise is retained, and whose conclusion is the contradictory opposite of Barbara's major premise, i.e. Bocardo $\{Oac, Abc, Oab\}$. Again, as Aristotle states, transposing Barbara's conclusion with its major premise yields a syllogism in which the fact that the middle term b does not belong to all of the major term the m follows from its premises, Oac , the contradiction of Barbara's conclusion, and Abc , the premise retained from Barbara. This process that I have just described is outlined by Aristotle in the following way (*An. Pr.* B8 59b 28-31). If we 'let A have been proved of C by means of middle term B', viz. we assume a syllogism in Barbara with Pc set $\{Aab, Abc, Aac\}$, then if we let Barbara be converted in this way,

then if A does not belong to every C but belongs to every B, then B will not belong to every C (Baroco, $\{Aab, Oac, Obc\}$, but stating the minor premise first); and if A does not belong to every C but B does, then A will not belong to every B (Bocardo, $\{Oac, Abc, Oab\}$).⁴⁴³

Aristotle here outlines a mechanical process of reducing imperfect syllogisms to perfect syllogisms that does not rely on major or minor premise conversion. Take any imperfect syllogistic argument form, and transpose the contradiction (or contrary) of the conclusion with the contradiction of the major or minor premise. Any transposition that generates a perfect syllogism is immediately known to be concludent. Thus, Baroco $\{Aab, Oac, Obc\}$ can be perfected by indirect reduction by transposing the contradiction of its conclusion and the contradiction of its minor premise, generating the perfect syllogism Barbara $\{Aab, Abc, Aac\}$, which is, in Aristotle's words, a syllogism that "has already come about". Similarly, Bocardo $\{Oac, Abc, Oab\}$ is perfect by indirect reduction by transposing the contradiction of the conclusion and the contradiction of the major premise, generating Barbara $\{Aab, Abc, Aac\}$, which is again a syllogism that has already come about.

⁴⁴³ Aristotle, *Prior Analytics*, trans. R. Smith, 79.

Thus, direct and indirect reduction of an imperfect to a perfect mood is a single proof that accomplishes two tasks. It proves that the conclusion c of the imperfect mood logically follows from the premises P , but it does so by a transformation, either of a single premise, or transposing a premise and conclusion, in such a way that the relationship between the perfect and imperfect syllogism stands revealed. There is an important observation that deserves mentioning before we move to Aristotle's reduction of syllogisms from a hypothesis in *An. Pr.* A23. Reduction is a proof that an imperfect syllogism's conclusion follows logically from its premises. It is not the proof of the truth or falsity of any proposition. This distinction becomes especially important in the distinction between per impossibile proofs of a *conclusion*, and *reductio ad impossibile* proofs of a *syllogism*, where indirect reduction performs only the latter, not the former. *Reductio* proofs of a syllogism have nothing to do with truth. They are proofs that a relation of logical following exists between premises and conclusions of certain argument forms. On the other hand, the aim of a per impossibile proof of a conclusion c takes it for granted that a relation of logical necessity exists between, on the one hand, premises not- c and premise p , the latter of which is agreed upon as true between the speakers, and an impossible conclusion c^* . The generation of the conclusion c^* , which authorizes the final conclusion c , is produced from a syllogism whose conclusency has already been established. Thus, direct and indirect reduction are about proving conclusency of an argument form, not about proving the truth or falsity of a proposition qua conclusion of a syllogistic deduction process.

In *An. Pr.* A23 Aristotle makes the ambitious claim that every concrete argument from which the conclusion follows necessarily from the premises can be formalized in one or other

of the syllogisms in *An. Pr.* A4-7 and reduced to a perfect, universal syllogism. Thus, Aristotle opens *An. Pr.* A23 (40b 18-23) with the following programmatic statement:

It is clear from what has been said (viz. in *An. Pr.* A7 29b 1-25) that the syllogisms in these figures are perfected through the universal syllogisms in the first-figure and are reduced to these. But that this will be so for any syllogism without qualification will become evident now, when we have proved that every syllogism comes about in one of the those figures.

It is necessary that every demonstration as well as every syllogism should prove that something belongs or does not belong, either universally or particularly, and further either in the ostensive way or from a hypothesis. (Arguments through the impossible are part of the syllogisms from a hypothesis.) First, then, let us speak about ostensive syllogisms, for once the proof has been given for these, it will be evident also what holds for arguments that lead to the impossible, and in general for syllogisms from a hypothesis.

Thus, at first sight, Aristotle's claim is more than that all valid syllogistic arguments whose premises and conclusions take the form of categorical propositions, viz. propositions that state that something does or does not belong to something universally or in part, can be reduced to Barbara or Celarent. Within the ambit of his claim Aristotle includes per impossibile syllogisms and syllogisms from a hypothesis in general, the former being designated by Aristotle as merely a subspecies of the latter. At first sight, Aristotle's claim seems to be that it is possible to use direct and indirect reduction to prove that the conclusion in a syllogism from a hypothesis follows from premises however they might be defined. Yet, as Jonathan Lear has noted, when Aristotle concludes *An. Pr.* A23

by saying that the arguments of the chapter [viz. A23] have shown that all proof and all syllogism necessarily come about through the three syllogistic figures, he is actually making a remarkably weak claim. He has not claimed to show that every syllogism in the broad sense [i.e. as stated at *An. Pr.* A1 24b20-2] is formalizable. He has only claimed to show that every direct syllogism is formalizable and that the hypothetical syllogism contains a direct syllogism as a proper part.⁴⁴⁴

⁴⁴⁴ Lear, *Aristotle and Logical Theory*, 35.

Repeating somewhat a discussion that I have already alluded to in §2, a hypothetical argument involves two interlocutors agreeing to accept Q if P . Aristotle's assumption is that the reasoners will then go through and prove P directly using an argument in the form of one of the syllogism in *An. Pr.* A4-6. In Lear's schematization of a hypothetical argument

'You agree to accept Q , if P ; but ... so P ; but you agreed to accept Q , if P ; therefore you must accept Q ',

the portion of the hypothetical argument that Lear represents with '...' is a direct syllogism in one of the syllogistic moods with P as its conclusion, for as Lear says, '[t]he part of the argument '...', so P ' is a direct syllogism and is thus assumed to be syllogistically formalizable'.⁴⁴⁵ Indeed, '...', so P ' is the *only* part of the argument that Aristotle believes is syllogistically formalizable in the sense that this portion of the hypothetical argument's formal properties correlate with a syllogism in one of the moods of *An. Pr.* A4-6, and are thus reducible to Barbara or Celarent. A similar situation holds with respect to per impossibile syllogisms, which Aristotle takes to operate in the following way. Two interlocutors want to demonstrate that Q . In order to do so they take one premise P that both agree is true and assume the contradictory opposite of Q , not- Q as a second premise. From these two premises, the reasoners syllogistically arrive at the conclusion R which both agree is impossible. Thus, they are forced to concede that not- Q is false and thus Q must be true. Let us adopt Lear's schematization of a per impossibile syllogism, which is now clearly revealed as a species of syllogism from a hypothesis:

'P, suppose not- Q , then R ; but that is impossible; therefore Q '.⁴⁴⁶

Again, as Lear notes, the portion of the per impossibile syllogism represented by 'P, suppose not- Q , then R ', is a direct syllogism and can thus be expressed in syllogistic form. According to

⁴⁴⁵ Ibid., 34.

⁴⁴⁶ Ibid., 35.

Lear—and this is the crucial point—“Aristotle does not think that the entire per impossible syllogism can be formalized as a chain of syllogistic inferences’.⁴⁴⁷ Aristotle claims only that syllogisms from a hypothesis can be reduced to one or other of the moods of *An. Pr.* A4-6 only insofar as the *argument to the hypothesis* contains a direct syllogistic argument that has the form of one of the moods in *An. Pr.* A4-6 that can be reduced to a perfect, first-figure mood. On the other hand, Aristotle explicitly denies the possibility that syllogisms from a hypothesis can be reduced to a categorical syllogistic mood. The crucial point, then, is this. Aristotle denies the very possibility that the conclusion of a syllogism from a hypothesis can be shown to follow necessarily from the premises by transforming one of its premises or a premise and a conclusion in a way that reveals the relationship it bears to a perfect, first-figure mood. The reason, strictly speaking, is that the conclusion Q is not deduced from a premise set, but Q is asserted by the interlocutors because of a prior agreement between them to accept Q on condition the P is proved.

Furthermore, one should not try to reduce the syllogisms from a hypothesis, since they cannot be reduced from what has been laid down. For they have not been proved by a syllogism, but are all accepted on the basis on an agreement. For example, if one had assumed the hypothesis that if there is not a single power of contraries, there is also not a single knowledge, and then one went on to argue that not every power is a power of contraries: not, for example, of the healthy and the unhealthy, for then the same thing would be healthy and unhealthy at the same time. Now that there is not a single power for all contraries has been proved, but that there is not a single knowledge has not been shown. And yet it is necessary to agree to this—though not on the basis of a syllogism, but on the basis of a hypothesis. This part, then, cannot be reduced, but the argument that there is not a single power can be, for this was perhaps a syllogism after all, while that was a hypothesis (*An. Pr.* A44 50a 15-30).⁴⁴⁸

In this passage, Aristotle divides the hypothetical argument into two parts. Employing the schematization of Aristotle’s hypothetical syllogisms reproduced from Lear, there is the

⁴⁴⁷ Ibid.

⁴⁴⁸ Translation by G. Striker (Aristotle, *Prior Analytics Book A*, 58f); cf. Lear, *Aristotle and Logical Theory*, 40; Aristotle, *Prior Analytics*, trans. R. Smith, 58.

portion of the hypothetical argument that is represented by (a) ‘but... so *P*’, and then there is the remainder of the argument (b) ‘You agree to accept *Q*, if *P*; [...] therefore you must accept *Q*’. According to Aristotle (a) concludes of necessity, whereas (b) only concludes because of a prior agreement between the interlocutors to accept *Q* if *P* is proved. In the example Aristotle provides, the conclusion of the direct syllogism *P* ‘there is not one single power for contraries’ is the only portion of the hypothetical argument that Aristotle says has been proved because it can be reduced to one or other of the moods in *An. Pr.* A4-6. But *P* is not the main objective in the minds of the interlocutors who carry out a hypothetical argument in this form—*Q* is. Yet, Aristotle expressly denies that the conclusion of the hypothetical argument *Q* has been proved because *Q* does not follow from its premises out of the necessity that is required from a perfect or imperfect syllogism. Rather, *Q* only can be said to ‘follow’ in the sense that by our prior agreement we have obliged ourselves to admit *Q* if *P*; but we do not admit *Q* on account that it has been shown to follow necessarily from a pair of premises on account of their logical formal properties alone. The case is similar with respect to per impossible syllogisms. Aristotle also divides per impossibile syllogisms into (a) the part of the argument that is reducible to a perfect mood (represented in Lear’s schematization by ‘*P*, suppose *not-Q*, then *R*’), and on account of this fact the conclusion *R* follows necessarily from the premises *P* and *not-Q*. The second conclusion *Q* is not, strictly speaking, the result of a syllogism, in the sense that it does not logically follow from any premises. Nevertheless, it does “follow” in the sense that we find ourselves accepting it because we obligated ourselves to accepting it on condition that *R* turned out to be (self-evidently) impossible. Thus, hypothetical syllogisms do not genuinely prove their conclusions, in the sense that their conclusions taken together with their premises need not belong to any perfect or imperfect syllogistic figure. To this extent, the conclusions

of hypothetical syllogisms do not follow from their premises of necessity in the way required by the theory of the assertoric syllogism Aristotle presents in *An. Pr.* A4-6. The only part of the hypothetical syllogistic that is formalizable as a perfect or imperfect syllogism is the proposition *P*, on account of whose acceptance we are obligated to accept the conclusion *Q*.

Thus, Aristotle's claim in A23 is only that hypothetical syllogisms are reducible to imperfect or perfect syllogism to the extent that the portion of the hypothetical argument that contains a direct syllogism can be reduced to a perfect or imperfect syllogism. Thus, the claim that we can reduce imperfect syllogisms "by means of" perfect ones and the claim that we can reduce syllogisms from a hypothesis "by means of" perfect syllogisms appears to be based on an ambiguity in the meaning of "by means of" in *An. Pr.* A4-7 and *An. Pr.* A23. In A4-7, the reduction to a perfect syllogism means that a syllogistic form is proved to be concludent based on its direct or indirect reduction to a perfect syllogism. The proof then is that the conclusion of any concrete argument having the form of the imperfect syllogism will follow logically from the premises. In A23 the reduction of a syllogism from a hypothesis "by means of" a perfect syllogism means that there is some part of the part of the syllogism that can be reduced by direct or indirect reduction (namely "but...*P*" in the syllogism from a hypothesis). Showing that the conclusion *Q* logically follows from the hypothesis *P* is not even considered, because *Q*'s only is said to "follow" from the hypothesis *P* homonymously. *Q* does not follow from *P* on account of their logical form, but on account of a promise. Consider the following example. Say that there is a hypothetical argument that has the following form: '*Oab* if *Ipr*; but *Apq*, *Iqr* so *Ipr*; therefore *Oab*'. Now, as Aristotle says only the argument in the form {*Apq*, *Iqr*, *Ipr*} is proven in the sense that it is in the mood *Darii*, and it is only this 'secondary' conclusion that follows necessarily from its premises {*Apq*, *Iqr*}. Now, there is no sense in which *Ipr* acts as

premise from which *Oab* follows necessarily, for in order to be a syllogism the *Pc*-set {*Ipr*, *Oab*} must at the very least be in a figure. Yet {*Ipr*, *Oab*} clearly belongs to no figure. Therefore, it is not formalizable as a syllogism in *An. Pr.* A4-6, and therefore has no possibility of being reducible to a perfect or imperfect mood. Despite the fact that we are obliged by some prior agreement based on dialectical conventions to accept *Oab* if *Ipr* is proved, Aristotle denies that *Oab* genuinely follows necessarily from *Ipr* in the way required by Aristotle's syllogistic theory.

§5.3.3 AVICENNA ON THE REDUCTION OF CONDITIONAL SYLLOGISMS

One of the main reasons why Aristotle felt no obligation to say that hypothetical arguments are formalizable or reducible to perfect or imperfect categorical moods in a strong sense, is that his syllogistic has no way of formalizing a hypothetical act of prior agreement as a conditional proposition. Agreement, like assertion, or rejection, are not the content of propositions, but they attach to propositions with a propositional force. It is only natural that Aristotle would say that conclusions concluded hypothetically from premises do not follow logically from their premises. Aristotle was not interested, at least in the *Prior Analytics*, in developing a calculus of conditional arguments in the first place.

In ŠQ IX, Avicenna clearly feels that he is carrying out a part of a project that was only adumbrated by Aristotle in *An. Pr.* A23, but never fully, or at least adequately, executed. For Aristotle, that project was to reduce syllogisms from a hypothesis by means of perfect syllogisms. Avicenna takes Aristotle's project to have been the perfection of repetitive syllogisms (e.g., schema such as *modus ponens* and *modus tollens*) by means of conjunctive syllogisms. The question is this. Why does Avicenna feel that conjunctive syllogisms hold the key to what he feels to be the proper understanding of that part of Aristotle's A23 that deals with the reduction of syllogisms from a hypothesis? In fact there are two closely related

reasons, one related to the transmission history of the *Prior Analytics* into Arabic;⁴⁴⁹ the other to logical theory of conditionals. The version of the *Prior Analytics* that Avicenna likely used was a product of a late antique tradition of logical inquiry centered around commentaries on the *Organon*.⁴⁵⁰ Other than Aristotle, Galen was likely the second most important source of logical theory in Arabic late antiquity. As is well known,⁴⁵¹ in his *Institutio Logica*, Galen provides a conditional syllogistic based on connection (*ittiṣāl*) and incompatibility (*inād*) that was the source of Alfarabi's thinking about conditional syllogisms.⁴⁵² Galen, to whom Avicenna referred to as being “strong in medicine but weak in logic”, drew the ire of Avicenna in ŠQ V for what the latter considered to be a theory of conditionals that was unsuited to logical theory (see §3).⁴⁵³ Yet, such was the stature of Galen's logic in the philosophical milieu in Ḥunayn ibn Isḥāq's (d. 873) and Isḥāq ibn Ḥunayn's (d. 910/11) (the translator of the Arabic *Organon* in Badawi's edition) Baghdad that in the Arabic translations of the *Prior Analytics*, the syllogisms called by Aristotle “ex hypotheseōs syllogismoi” and translated by Gisela Striker as ‘syllogisms from a hypothesis’ are translated in the Arabic *Prior Analytics* simply as ‘al-maqāyīs aš-šarṭiyya’ or “conditional syllogisms”. As Joep Lammeer observes about Alfarabi's writing on conditional syllogisms, ‘the conditional syllogism [(*qiyās šarṭī* or *maqāyīs šarṭiyya*)] only includes Stoic

⁴⁴⁹ A. Ahmed, “On Avicenna's Reception of Aristotelian Modal Syllogistics: A Study Based on the Conversion Rules and the BARBARA Problematic”, in *Before and After Avicenna: Proceedings of the First Conference of the Avicenna Study Group*, ed. D. Reisman, A. Al-Rahim (Leiden: Brill, 2003), 3-24; see A. Badawi, “Introduction” to Aristotle, *Maṭīq Aristū*, trans. Ḥunayn ibn Isḥāq, et al. (Cairo: Maktab Dar al-Kutub al-Miṣriyya, 1947), 7-32.

⁴⁵⁰ For example, on the profound importance of commentary traditions to translation in relation to Avicenna's metaphysics of causality, see R. Wisnovsky, *Avicenna's Metaphysics in Context* (Ithica: Cornell University Press, 2003).

⁴⁵¹ Shehaby, Introduction to Avicenna, *Propositional Logic*, page no.; M. Maróth, *Aussagenlogik*, 38-55; N. Rescher, “New Light from Arabic Sources on Galen and the Fourth Figure of the Syllogism”, *Journal of the History of Philosophy* 3/1 (1965): 27-41; idem. *Galen and the Syllogism* (Pittsburgh: University of Pittsburgh Press, 1966); S. Bobzien, “Pre-Stoic Hypothetical Syllogistic in Galen's *Institutio Logica*”, in *The Unknown Galen*, ed. V. Nutton (London: Institute of Classical Studies, 2002), 57-72; A. Specia, *Hypothetical Syllogistic and Stoic Logic* (Leiden: Brill, 2001).

⁴⁵² F. Zimmermann, Introduction to *Al-Farabi's Commentary and Short Treatise on Aristotle's De Interpretatione*, trans. F. Zimmerman (London: British Academy, 1981), lxxxiii.

⁴⁵³ Avicenna, ŠQ VIII, 398; translation is Shehaby's (Introduction to Avicenna, *Propositional Logic*, 25).

deductions [...] In this respect [Alfarabi's] use of terminology appears to be more in line with Galen who, in his *Institutio Logica* [...] applies the concept 'hypothetical syllogism' (*hypothetikos syllogismos*) in a similar, narrow sense". The same can be said of the Ishāq ibn Ḥunayn, who takes Aristotle to be talking about *conditional syllogisms* (*maqāyīs šarṭiyya*) in the sense it acquired in late antiquity, viz. as a two-premise deduction scheme with a conditional first premise and a categorical second premise.⁴⁵⁴ A hypothetical syllogism for Aristotle was an argument in which the conclusion followed from the hypothetical agreement between the interlocutors to accept *Q* if *P*. Strictly speaking a syllogism from a hypothesis has no premises, just a hypothesis that must be proved in order to move to the conclusion *Q*. In a conditional syllogism, the conclusion logically follows from a pair of premises. The first premise is a conditional *proposition* with an antecedent and a conclusion. The second premise is a "repetition" of the antecedent, or the "repetition" of the contradiction of the consequent. The conclusion logically follows from the premises if the "repeated" premise asserts the antecedent of the conditional or the contradictory opposite of its consequent. Obviously, conditional syllogisms are different in structure and conclude according to completely different validity conditions than syllogisms from a hypothesis, but Ishāq ibn Ḥunayn, Alfarabi, Avicenna, and even Averroes all took the former for the latter in their readings of *An. Pr.* A23.⁴⁵⁵ For our purposes, the most important consequence of this "conflation" is that the

⁴⁵⁴ Aristotle, *Manṭiq Aristū*, ed. A. Badawi (Cairo: Maṭba' Dār al-Kutub al-Miṣriyya, 1948), 216.17-8: *wa-ayḍan laysa yanbaḡi an yata'āṭā ḥalla l-maqāyisi š-šarṭiyyati li-annah laysa yumkinu an yaḥulla min ḍālika l-makāna l-mawḍū'a=eti dè tous ex hupotheseōs sullogismoū ou peiratéon anagein ou gār estin ek ton keiménōn anágein*. According to Lameer, Alexander of Aphrodisias is perhaps more faithful to Aristotle's intention; see Lameer, *Al-Fārābī and Aristotelian Syllogistics*, 45.

⁴⁵⁵ E.g. Averroes, *Talḥiṣ al-Qiyās*, ed. C. Butterworth, A. Harīdī, Q. Maḥmūd (Cairo: al-Hay'a al-Miṣriyya al-ʿĀmma li-l-Kitāb, 1983), 194.11. Averroes was obviously a keen reader of Aristotle and I believe he realized that the Galenic conditional syllogism was incommensurable with the way Aristotle talks about syllogisms from a hypothesis. Thus, while Averroes does use the expression "*qiyās šarṭī*" in his commentary on A23 and uses material he seems to have taken (copied?) directly from Alfarabi's *Qiyās aṣ-Ṣaḡīr*, he also says (ibid., 189.9) that there are syllogisms that prove from predication, and other syllogisms that prove "from a hypothesis (*min ḡihati l-ḥaml*)".

Arabic commentators on Aristotle such as Avicenna all seem to have felt the need to clothe the agreement to accept Q on condition that P in Aristotle's syllogisms from a hypothesis in the garb of a conditional proposition.⁴⁵⁶ Thus, Avicenna's solution to the problem of the reduction of conditional syllogisms to conjunctive syllogism in ŠQ IX 1, a solution that he calls Aristotle's own, is motivated by an attempt to find an adequate interpretation of (1) how we should understand the following (*luzūm*) between antecedent and consequent in the conditional premise, (2) how to understand the following (*luzūm*) between the premises and the conclusion, and finally (3) what part the conditional premise plays in the reduction of repetitive to conjunctive syllogisms.

As we have seen, Aristotle limited the reducibility of the syllogism from a hypothesis to the proof of the hypothesis ('but... so P ' in Lear's schematization), which authorizes the move from the hypothesis ' P ' to the conclusion ' Q '. This move is based on an agreement between interlocutors to accept Q on condition that P is proved. The agreement works like a commissive speech act in the sense that by uttering the very words " Q if P is proved", the speakers place on themselves a conditional obligation to accept Q . Aristotle's logic had no capacity to formalize a "logic of promises", so the only formalizable part of the syllogism from a hypothesis in Aristotle's mind is the sub-deduction "but...so P ". Indeed, Alexander of Aphrodisias, whose thought on hypothetical syllogistic was so important to Avicenna, seems to have conflated the proposition "if P , then Q " with the agreement to accept Q if P is accepted. As Michael Frede aptly observes, Alexander "treats hypothetical propositions like "if p then q "

⁴⁵⁶ "Commentator" is normally a predicate that only very weakly attaches to "Avicenna", though I believe it is meaningful in this particular instance given some that Avicenna clearly intended ŠQ IX as a supplement to or even an extension or correction of *An. Pr.* A23.

not as if they state a connection between “p” and “q”, but as if they either stated “q” with the proviso that p, or stated one’s willingness to accept a proof for “p” as a proof for “q”.”⁴⁵⁷

By Avicenna’s day, things had changed. Due to (1) the importance of Galen’s *Institutio Logica* to the development of conditional syllogistic in the Arabic logical tradition, and (2) the translation of the Greek “syllogism from a hypothesis” as “conditional syllogism” in the Arabic *Prior Analytics*, there was heavy pressure on logicians to turn what was in Aristotle a conditional promise to accept Q into a syntactically conditional sentence with some sort of propositional content. If the syntactical shift was perhaps straightforward, working out the semantics of this change seems to have been more difficult. If what used to be a conditional commissive speech act is now understood as a conditional premise in a two-premise, premise-conclusion syllogism such as modus ponens and modus tollens, what content does this conditional sentence signify, and how is the conditional supposed to operate in the syllogism?

In ŠQ IX 1, Avicenna considers three different options in answer to this pair of questions, only one of which, he finds, works with a satisfactory account of the reduction of repetitive syllogisms to conjunctive syllogism. First, let us schematize the conditional syllogisms in the following way. I will focus on modus ponens because Avicenna does, and because, as Avicenna says, modus tollens is derivable from modus ponens.

‘if P, then Q; but P. Therefore, Q’.

One option was to say that the conditional signified a self-evident connection between the antecedent P and a consequent Q. Since the connection is self-evident, the conditional itself is not subject to proof, since it is impossible to prove something that is known in an immediate way. Thus, in order for modus ponens to be of any use at all, it was maintained that P qua

⁴⁵⁷ M. Frede, “Stoic vs. Aristotelian Syllogistic”, 27.

antecedent had to be in doubt (*šakk*). However, after *P* had been proved by a categorical syllogism, the doubt in *P* no longer remains and we are then able to assert *P* in the second premise. Finally, this assertion of the content of the antecedent is what authorizes the assertion of *Q* as the conclusion. As we saw in §3, this view had several supporters among the Greek commentators. However, we also saw in §3 that Avicenna looked on this view with extreme hostility. In ŠQ IX, Avicenna returns once more to the question of the requirement that the antecedent be doubtful. Though he still feels the view is erroneous, the tone in this passage is more conciliatory.

[Text 16] The best effort that has been made to prove this [viz. to prove that the repetitive syllogism reduces to the conjunctive] is if we say what one of the scholars [*ba'du l-muḥaṣṣilīn*] has said <on this topic>: if the antecedent were evident, then what follows from it would be evident also, and it would be impossible to arrive at it by means of a syllogism. The syllogism makes the obscure evident, whereas the evident has no need to be reasoned to by a syllogism. He said: “if so is so, then so is so” entails that there is doubt in the antecedent. If <this doubt> were eliminated, then the consequent would follow in the correct way. If the antecedent were self-evident, then what point is there in attaching the conditional particle [‘if’] to it?⁴⁵⁸

Avicenna sees that the alleged need for a doubtful antecedent in order for the syllogism to have any point is founded on the assumption that anything that follows from a self-evident antecedent must itself be self-evident too. Yet, taken as the basis for an account of how a conditional works as a premise in, say, *modus ponens*, this principle has two serious drawbacks. The first is that it makes the following between the antecedent and consequent into something indemonstrable. By saying that something follows from a self-evident antecedent or an antecedent that has been proven by a categorical syllogism, the following itself has been completely excluded from our account of the conditional premise. The following itself, then, becomes something indemonstrable in the sense that it is merely an

⁴⁵⁸ Avicenna, ŠQ IX, 416.4-9.

immediate corollary of our knowledge of the antecedent. As an account of Aristotle's agreement, this is suitable in the sense that it does allow us to clothe the conditional commissive in the garb of a conditional sentence. However, it is not useful as an account of a premise in a syllogism. On this reading of the conditional, conditional premises in the philosophical sciences could only ever function as expressing indemonstrable first principles for each of the sciences. They would not themselves ever be subject to demonstration. They would thus not be formalizable as conjunctive syllogisms, nor could any type of argument with them be subject to direct or indirect reduction. Avicenna seems to believe that the consequences of holding this view are unacceptable, since as I noted in Text 2 there are theses of metaphysics, physics and mathematics that are conditional in form, and since they are theses and not axioms there must be some way of proving them. Finally, the whole point of ŠQ VI was to develop a calculus for proving conditional theses based on the foundations of Aristotle's categorical syllogistic. To understand conditionals as signifying indemonstrable, immediately intuited connections between an antecedent and consequent makes nonsense of those theses of the philosophical sciences that are conditional in form, and moots the large number of logical doctrines and philosophical insights contained in ŠQ V-VII.

There is another objection to this way of taking the conditional premise in modus ponens-like syllogisms, which is peculiar to Avicenna's view of what a conditional proposition should be. Taking conditionals as sentences that can only express a type of following that is immediately intuited and indemonstrable greatly restricts the use of conditionals. As can be seen in §5.2.3, conditionals for Avicenna have an extremely wide application, expressing all types of relations between real or mentally supposed states of affairs. Thus, in the argument he deploys to weaken the above view of conditional premises in repetitive syllogisms,

Avicenna insists that conditionals be allowed to express the following that holds between things that joined (*muta'alliq, ta'alluq*) in a generic and unqualified way. If we are willing to grant conditionals much wider currency while at the same time insisting that what follows from a self-evident antecedent is itself self-evident, then it is quite simple to show that Barbara violates the Productivity Principle. Avicenna puts this argument this way:

[Text 17] We do not concede the claim that the thing [call it *Q*] joined in a self-evident way to something else [call it *P*] whose truth is evident is itself evidently true. For it may be that something is self-evident [protasis *P*, for example], but what follows from it is not self-evident [apodosis *Q*, for example]. [Indeed, *P* may be self-evident], and the implication belonging to the self-evident thing may be self-evident, but what follows from it [apodosis *Q*] is known by some intermediary step [of reasoning]. After all, it is not the same thing to say that something is self-evident, and to say that something's following from something self-evident is self-evident. For all unknowns are arrived at by steps [of reasoning] by virtue of the fact that they follow from things that are self-evident or have been proved [by some other means]. If their following is not evident on account of some intermediary step of reasoning, but terminates at something that follows in an evident way, then if the terminal step is self-evident, and what implied it was self-evidently <known> without any intermediary step because it follows from something that is self-evident and whose implication is self-evident, then everything will become self-evident. From this it would follow that the first mood of the first-figure is inconcludent [*lā yuntiġu šay'an*]. That is because its premises [*qiyāsāt*]⁴⁵⁹ reduce to two self-evident premises, and then the conclusion self-evidently follows from them as you know. Then the relation of the premises to the conclusions is made analogous to the relation of the antecedent to the consequent. Then if the antecedent is something evident, (for example, if every *J* is *B* and every *B* is *A*"), and the consequent evidently follows from it (for example, "every *J* is *A*"), then it is necessary that "every *J* is *A*" is evident.⁴⁶⁰

Imagine a two-step deduction in Barbara (with quantifiers suppressed) of *AE* from premises *AC* and *CE*. *AC* is itself the result of a deduction from premises *AB* and *BC*. Similarly, *CE* is a result of a deduction from premises *CD* and *DE*. The final premises in the chain *AB, BC, CD, DE* are all self-evident, being, perhaps, indemonstrable axioms an Aristotelian science. Since each of them is self-evident and their implying their consequents is self-evident (by Barbara), then, by

⁴⁵⁹ Avicenna often refers to the premises alone as "syllogisms".

⁴⁶⁰ Avicenna, ŠQ IX, 416.13-7.10.

the principle Avicenna is attacking (what follows from something self-evident is self-evident), the premises *AC* and *CE* of the final conclusion *AE* are self-evident. Finally, since *AE* follows self-evidently (by Barbara) from two self-evident premises it too is self-evident. But this leads to the absurd conclusion that all the steps of reasoning were to no point, and the final conclusion *AE* was already known self-evidently before it had been deduced from first principles.

Avicenna feels that this argument provides sufficient grounds to reject the principle that “what follows from something self-evident is self-evident”. More to the point, it allows Avicenna to reject the view that the conditional premise in a repetitive syllogism simply expresses an intuitively obvious connection between antecedent and consequent, and that in order for a repetitive syllogism to be useful, antecedents such as “all humans are animals”, or “two is odd” would be inadmissible as antecedents in conditional premises because they are not subject to doubt (the first being self-evidently true, the second self-evidently false). Yet, Avicenna’s argument against this principle raises an important point: is it correct, as his argument suggests, to take the conditional not as a sentence that expresses a self-evident, indemonstrable connection between a pair of phenomena, but as an inference where the antecedent *P* (e.g. “every *A* is *B* and every *B* is *C*”) is understood as a premise whose assertion justifies the assertion of *Q* (e.g. “every *A* is *C*”) ? Several factors recommend this way of understanding the conditional. One is that instead of the units of analysis being only the antecedent and the consequent, as in the first proposal just discussed, this proposal brings the nature of the following between the antecedent and consequent into the investigation as a third unit of analysis. Second, in this second proposal, it is possible for the premises to be self-evident and the following itself to be self-evident—in the sense of the self-evidentiary nature

of logical following in perfect syllogisms, for example—but for the conclusion not be self-evident. The reason for this is

[Text 18] that the form of this [premise] combination is not fixed in the mind. For it may happen that it occurs to the mind to give assent to one of the premises but this is not accompanied by assent to the second <premise> because [assent to the second] never occurred to the mind, not because [the second premise's] truth is not evident. (The difference between these two things has been dealt with before.) In fact, both [premises] may be present [in the mind] together, but they are not arranged in a way that leads to the conclusion since the middle term is not perceived.⁴⁶¹

There are several obvious problems with this proposal. If conditionals have, as Avicenna calls it, a “syllogistic form (*ṣūra qiyāsiyya*)” then it is not clear how or in what sense the predicates “true” and “false” will attach to conditionals, since inferences are not called “true” or “false”, only propositions are. Second, even if we were to resolve this problem, the assumption in this second proposal is that the notion of following that conditionals express is identical to the type of following that holds between the conclusion and the premises of an imperfect or perfect syllogism. As attested by his theory of quantified conditionals, Avicenna would clearly want to allow for a much broader theory of conditional propositions than one that only allows strong relevance to hold between antecedent and consequent, and that only admits antecedents that are in a figure and conclusions that fulfill the conclusivity conditions on syllogisms. Although this suggestion would make it easy to reduce repetitive syllogisms to conjunctive syllogisms, the accomplishment would only be vacuous: repetitive syllogisms are simply conjunctive syllogisms according to this second proposal.

Avicenna does not focus on these problems, however. Instead, he points out that accepting this second proposal (that the conditional premise is a syllogism in the form of a conditional with the premises serving as the antecedent and the conclusion as the consequent)

⁴⁶¹ Avicenna, ŠQ IX, 418.17-9.3.

makes the second premise, in which we assert what is expressed in the antecedent, redundant (*faḍl*).

[Text 19] What we should say about this—and we will try to justify it as well as we can—is <the following>. With regard to anything [call it *Y*] that is joined in an evident way to a single evident thing [call it *X*], the coming to mind of the thing [i.e. *Y*] joined <to the single, evident thing [*X*]> aids the mind in grasping the soundness of the consequent's [*Y*] following [from *X*]. Thus, if “every *A* is *B*” is evident and the following of “*J* is *D*” is evident, then when “*A* is *B*” comes to mind there is absolutely no reason at all to use a syllogism to bring about the following of the consequent. For just as you brought the state “*A* is *B*” to mind when you say “if *A* is *B*...”, then it is as if you have said to yourself “if it is true that *A* is *B*, then *J* is *D*”. In that case, there is then no point to returning and positing “but *A* is *B*” because this is included in “*A* is *B*” being set out as the antecedent. [This is so,] because you only take it [“*A* is *B*”] as an antecedent—or you take it as something coming to the mind that would only come to mind hypothetically. In this case, hypothesizing it as an antecedent aids in knowing the truth of the consequent, even if in reality you had repeated the hypothesis of the antecedent—when the assertion [of “*A* is *B*”] is embedded in the setting out [of “*A* is *B*” as antecedent], and is thus an already completed action. But in that case, there is no need to single it out for being repeated [as an assertion in the second premise] as if it were prior to something that is only perceived after it [i.e. as if the consequent only became known after the assertion]. On the other hand, if the following [of the consequent from the antecedent] is not evident, then there is need to single it out as a repeated second premise.

The case is similar with regard to conjunctive syllogisms if they [i.e. their premises] are made antecedents, for the proof of their premises eliminates the need for the recurrence of the assertion [of the antecedent] so that the consequent, i.e. the conclusion, follows from the syllogism, which you regard as the antecedent of connective conditional <proposition>. Based on this, if the following of perfect [i.e. following of a perfect syllogism] the soul has no need to return to assert [the antecedent, which is a premise pair of a perfect syllogism]—as if we were to say “but every *J* is *B*, and every *B* is *A*”—since this [assertion] is embedded in the mind's setting out of the antecedent.⁴⁶²

If the conditional in a repetitive syllogism is understood to work in the same way as a syllogism, viz. if the premises of the syllogism are the antecedent and the conclusion is the consequent, then the assertion of the antecedent in the minor premise becomes redundant. If we are to understand the conditional “if *X*, then *Y*” in a manner analogous to a syllogism “if every *A* is *B*, and every *B* is *C*, then every *A* is *C*”, in which the mind has grasped the placement

⁴⁶² Avicenna, ŠQ IX, 417.15-8.15.

of the middle term of an evidently concludent syllogism, then when “X” comes to the mind of the reasoner, Y will immediately come to the mind of the reasoner. Avicenna notes that this is the reason why enthymemes often come in the form of conditionals, because they are merely syllogistic inferences (with a major or minor premise suppressed) in the form of a single conditional proposition.⁴⁶³

Avicenna’s view is a hybrid of each of these views. Unlike the second proposal, he wants to treat the conditionals used in repetitive syllogisms like genuine propositions, as in the first proposal. Unlike the first proposal however, he wants to make them amenable to demonstration by means of conjunctive syllogisms. He wants, in other words, for the following itself to be the object of a syllogistic demonstration. Like the second proposal, he wants to be able to say that the antecedent can be self-evident or made evident through a syllogism, but that this not entail that the consequent immediately become self-evident. Above all, he wants to make sure that repetitive syllogisms do not violate the strong version of the Productivity Principle that I spoke of before. The problem with modus ponens is that there are only two parts involved in inferring the conclusion from the premises--the antecedent and the consequent—because the very condition for validity for modus ponens is that the second premise be *identical* (in content) to the antecedent. Thus, the threat of redundancy—for an Aristotelian logician where redundancy is a type of invalidity—always presents a problem. With only two distinct proposition contents in the premises, how do we guarantee that the Productivity Principle is not violated? In other words, how do we guarantee that the conclusion that we eventually arrive at has a different epistemic status than the premises? In conjunctive syllogisms, this problem does not arise: the Middle Part Principle with the

⁴⁶³ Avicenna, ŠQ 419.12–421.8.

Productivity Principle together guarantee that the conclusion is different from either of the premises, but is not a tautology (“every stork is a stork”) or incoherent (i.e. none of the terms in the conclusion are in the premises). Earlier philosophers accounted for the different epistemic status of the conclusion in modus ponens-like arguments by stipulating that the antecedent in the conditional premise be doubtful. As we have seen, however, Avicenna is uncomfortable with making recourse to this stipulation since it makes conditionals like “if man is an animal, then man is a body” inadmissible as conditional premises.⁴⁶⁴ Avicenna’s solution is to focus on the unknown or undemonstrated epistemic status of the implication in the conditional. If the implication itself is not evident, then

[Text 20] let the implication be proven. Then, if the implication should be made evident by a proof [*ḥuǧǧa*], and the antecedent is then asserted at that time, then the consequent is produced. Then this is a productive syllogism [*fa-kāna qiyāsan mufīdan*]. Thus, it is possible for the antecedent to be evident, but the implication not evident, but is then proved. If it is proved, then the assertion of a self-evident antecedent will produce something that was unknown beforehand.⁴⁶⁵

Thus, Avicenna retains the notion of doubt, but he applies it to the implication itself, not to the antecedent. In the first proposal we discussed, it was inconceivable that the implication itself be subject to doubt because it had been *defined* as something that was immediately intuited. Thus, an repetitive syllogism will be consistent with the Productivity Principle, not when the antecedent is doubtful (though if it is, this is indifferent), but when the implication itself is doubtful, in the sense that it needs demonstration, ultimately by a conjunctive syllogism. Once the relation of following between the antecedent and consequent has been proved, then the assertion of the antecedent, whatever its epistemic status, authorizes the following of the consequent.

⁴⁶⁴ Avicenna, ŠQ IX, 422.14-5.

⁴⁶⁵ Avicenna, ŠQ IX, 417.11-15.

In this way, Avicenna is able to distinguish the following in a conditional from the following in a valid syllogism as well as from the type of immediate following that is characteristic of immediate concomitants (e.g. day/sun being up, two/dual etc.), and relatives (e.g. slave, master/father, son, etc.). His conditionals have truth conditions that are distinct from both of these ways of taking conditionals. Moreover, the calculus of conjunctive syllogisms along with the truth-conditions, a theory of quantification, a generalized theory of figures, and a generalized theory of rejection, allow him to demonstrate these with conditional conclusions. As for reduction, his results are still more or less the same as Aristotle's. The ambiguity in the claim that syllogisms from a hypothesis are reducible "by means of" perfect syllogisms allows Aristotle to prove his claim by showing that the hypothesis in the syllogism from a hypothesis can be proved by a categorical syllogism formally identical or reducible to a perfect syllogism. He does not say that it be proved that the agreed upon conclusion can be shown to be logically necessary. Avicenna shares this result with Aristotle, since it makes no sense to claim that modus ponens can be reduced to a perfect conjunctive syllogism. If such a claim were to be made, there would have to be a distinction among repetitive syllogisms into perfect and imperfect moods. But as I noted above, figures, moods, quantifiers, and reduction by conversion and transposition sit uneasily when extended to modus ponens-like argument schemes. Can the premises and conclusion of modus ponens somehow be manipulated to reveal conditional Barbara, Celarent, Darii or Ferio? Not at all. Thus, Avicenna is content to follow Aristotle in claiming only that each of the *premises* in a repetitive syllogism can be a conclusion that logically follows from a pair of premises according to the concludency conditions for conjunctive syllogisms. Yet, Avicenna's claim is necessarily different from Aristotle's because the Arabic *Prior Analytics* takes Aristotle to be talking about conditional

sylogisms rather than syllogisms from a hypothesis. Thus, Avicenna feels compelled to show that both premises of modus ponens-like syllogisms can be reduced to syllogisms that obey both the Middle Part Principle and the Productivity Principle. By using his conjunctive syllogisms, Avicenna is able to extend Aristotle's results by saying that both the conditional premise and the asserted minor premise (however complex its propositional structure) can be proved by syllogisms that obey both principles. This is certainly not Aristotle's claim; it is, though, an achievement.

CONCLUSION

This has been a study of Avicenna's theory of conditional propositions, and repetitive and conjunctive syllogisms. Despite this and the work of other scholars, Avicenna's logic of conditionals requires much careful study. Even after the work undertaken by Rescher, Shehaby, Gätje, and Maróth, Tony Street still states, in his introduction to Asad Ahmed's translation of the logic of Avicenna's *Kitāb an-Nağāt*, that:

Avicenna introduced an entirely new way of dividing the syllogistic, into *iqtirānī* and *istiṭnā'ī* [...] Everyone save obdurate Aristotelians like Averroes followed Avicenna in this innovation. One consequence it had was for what might be called metalogical analysis, and the proof by reduction to an impossibility was analysed by making use of this division [...] This analysis generated discussion in the later tradition, though none of this material has been properly assessed.⁴⁶⁶

In this study I have tried to go into greater logical and philosophical depth than previous studies. In particular, I have tried to add greater depth to our understanding of the nature of Avicenna's debt to Alfarabi and earlier logicians, particularly Avicenna's complicated relationship to Aristotle. Given the evidence presented in Chapters 2 and 3, Avicenna may be justly said to owe little to Alfarabi's conditional syllogistic, except perhaps as a foil against which he developed some of his doctrines. However, Avicenna clearly owes a great deal to Alfarabi in developing his thoughts on the subject matter of logic and in establishing logic as a philosophical science (even if Alfarabi rejects both theses). In ŠQ V Avicenna took as his starting point a pre-existing tradition of conditional propositions and syllogisms, which is loosely traceable to Alfarabi's epitomes of the *Prior Analytics*—in particular *al-Qiyās aṣ-Ṣaḡīr* and *al-Madḥal ilā l-Qiyās*. Two points about Alfarabi are worth noting. One is that developing a logic of conditionals, or extending the work done by earlier commentators, does not seem to have

⁴⁶⁶ T. Street, Introduction to Avicenna, *Avicenna's Deliverance: Logic*, trans. A. Ahmed (Karachi: Oxford University Press, 2011), xxvi.

been a pressing concern for Alfarabi. To be sure, Alfarabi does discuss conditional syllogisms (*qiyāsāt šarṭiyya*) in his epitomes of the *Prior Analytics*, and tangentially addresses questions related to a theory of conditional propositions in his epitome of the *Categories* (what Dunlop calls “Alfarabi’s Paraphrase of the *Categories* of Aristotle”), and in his epitomes of the *Topics* (*Kitāb al-Ġadal* and *Kitāb at-Taḥlīl*⁴⁶⁷). He does not, however, try to meld these disparate sources into a more unified, coherent account of syllogistic reasoning as Avicenna does. In Alfarabi’s logic there is a vague sense that conditional syllogisms complement categorical syllogisms, but this complementarity is not give full expression. We are left, then, doing the investigative guess-work of chapter 2.

This brings me to the second point. Alfarabi’s discussions of conditional propositions and conditional syllogisms come from a variety of often disparate sources, though there was an effort in some late antique commentaries to refine and reconcile these strands into a unified theory. The non-Aristotelian material clearly seems to have been provided by Galen’s (d. 199/216?) *Institutio Logica*. As Tony Street notes, the main translators of the *Organon* Ḥunayn ibn Ishāq (d. 873) and his son Ishāq ibn Ḥunayn (d. 910/1)

drew on earlier Syriac translations when available, and on the Greek commentaries of late antiquity. Ḥunayn’s primary interests were medical, and he held Galen in high regard; in consequence, he translated many of Galen’s logical works along with the medical works, including the *Institutio Logica*, a treatise on the number of syllogisms, fragments dealing with *On Interpretation*, and fragments of *On Demonstration*. Galen may well have dominated logical studies in Baghdad for one or two generations after Hunayn, but a reaction ultimately set in, and neither Alfarabi and Avicenna acknowledged any debt to Galen’s logical works; this does not mean that they did not share some of his ideas.⁴⁶⁸

⁴⁶⁷ *Kitāb at-Taḥlīl* contains some content that could be traced back to *An. Pr.* A27-30. Nevertheless, *Kitāb at-Taḥlīl* makes frequent use of topoi, whereas explicit mention of topoi is absent from *An. Pr.* A27-30.

⁴⁶⁸ T. Street, “Arabic Logic”, in *Handbook of the History of Logic*, vol.1, ed. D. Gabbay and J. Woods (Amsterdam and Boston: Elsevier, 2003), 531f.

In addition to Galen's theory of connection (*ittiṣāl*) and disjunction (*infiṣāl*), Alfarabi's conditional syllogistic developed out of material related to Aristotle's theory of dialectical topoi in the *Topics*, and from a theory of equipollence (*talāzum*) and incompatibility between terms that developed out of Aristotle's treatment of relations in the *Categories*.

Many of these terms appear in ŠQ V and ŠQ VII, but Avicenna discards most of them or uses them with different meanings. For example, "equipollence", which in Alfarabi's paraphrase of the *Categories* is a relation between terms, Avicenna takes to mean logical equivalence between conditional propositions. Most of the other concepts Avicenna simply discards as unsuited to the proper business of logic. Thus, a search for the distant origins of Avicenna's theory of conditional propositions and conjunctive syllogisms will prove unfruitful. The same cannot be said for Alfarabi however. A fascinating line of future research is to investigate how Greek and Syriac commentators were able to take these concepts from Galen's *Institutio*, material relating to the analysis of term relations in the *Categories*, and the analysis of topoi in the *Topics* and form them into an account of conditional propositions and syllogisms.

In his history of Arabic logic, Tony Street notes that by eliminating the *Categories* from his later logical texts while at the same time retaining Aristotle's analysis of simple types of terms and the types of signification (*dalāla*), "Avicenna was cutting out of his logic the two things to which the Syriac Christians [i.e. the commentators on the *Organon*] devoted most of their efforts".⁴⁶⁹

The student of logic, after learning what we have told him about regarding the simple terms, and learning the noun and the verb, can go on to learn propositions and their parts, and syllogisms, and definitions and their kinds, and the matters of syllogisms and the demonstrative and non-demonstrative

⁴⁶⁹ Ibid., 541.

terms and their genera and species, even if it does not occur to him that there are ten categories.⁴⁷⁰

Street rightly says that in excising most of the material in the *Categories* from logical investigation, Avicenna is motivated by his vision of what the subject matter of logic is. The categories of substance, quantity, quality, position, etc. are not second-order concepts or “second intentions”, as predicate, subject, antecedent and consequent are. The results of this dissertation suggest that the same reasoning is operating behind Avicenna’s rejection of a *Topics*-based approach to the analysis of logical consequence (*luzūm*). *Luzūm* for Avicenna seems to be what later Arabic logicians called a third-order concept or a “third intention”, which is of a higher order than “predicate”, “subject”, “antecedent”, and “consequent”.⁴⁷¹ Second intentions are concepts that obtain in the mind when the reasoner considers first-order concepts insofar as they can be arranged to deduce new knowledge.⁴⁷² Like the concepts “figure (*ṣakl*)” and “mood (*ḍarb*)”, “following (*luzūm*)” is a concept that obtains in the mind when the reasoner considers second order concepts such as “antecedent” and “consequent” as well as second-order concepts such as “premises” and “conclusion” insofar as their arrangement will yield formally concludent (*muntig̃*) or formally inconcludent (*‘aqīm*) arguments. The analysis of consequence in the *Topics*, which is based on relations of following between wholes and parts, genus and species, pairs of relata, definiendum and definiens, etc., has nothing to do with Avicenna’s analysis of logical consequence. In essence, Avicenna turns his back on a long tradition that pre-dated the Syriac commentators and had a long shelf-life in the Latin tradition, which saw the *Topics* as the natural venue for theories of logical

⁴⁷⁰ Avicenna, *Kitāb aṣ-Ṣifā’*: *Maqūlāt*, ed. M. Khodeiri, G. Anawati, S. Zāyed, A. Ahwānī (Cairo: Wizārat al-Taḳāfah wa-al-Irṣād al-Qawmī, 1959), 5.1-4; quoted and translated in Street, “Arabic Logic”, 541.

⁴⁷¹ See K. El-Rouayheb, “Post-Avicennian Logicians on the Subject Matter of Logic: Some Thirteenth- and Fourteenth-Century Discussions”, *Arabic Sciences and Philosophy* 22 (2012): 69-90, especially 80ff.

⁴⁷² See Street, “Arabic Logic”, 540.

consequence.⁴⁷³ This is one important factor in explaining why no comparable tradition of “consequentia” existed in Arabic logical texts.

Avicenna’s marginalizing the *Categories* and the *Topics* in his discussion of logic proper explains a profound difference between Alfarabi’s theory of conditional syllogisms and Avicenna’s theory of conjunctive and repetitive syllogisms, viz. the importance of context theory to the former and its near irrelevance to the latter.⁴⁷⁴ Deborah Black has suggested that the context theory of logic developed out of Greek and Arabic attempts to somehow preserve the conventional account the *Organon*’s underlying unity.⁴⁷⁵ As a consequence, Alfarabi’s theory of conditional propositions and syllogisms makes little sense unless it is seen as a participant in this exegetical tradition. In particular, the statistical sensitivity of conditional truth-values and argumentative validity discussed in Chapter 2 must strike us as unusual (if not just plain weird) if we do not take into consideration the variation in the degrees of assent which interlocutors give to premises and conclusions in the midst of a dialectical exchange. Alfarabi’s development of conditional propositions and conditional syllogisms reflects this basic truth very clearly. Avicenna’s belief that logic is about second intentions was the end of context theory of logic in the Arabic tradition, and ultimately destroyed any pretense to maintaining the unity of the *Organon* in the later Arabic tradition. Ibn Ḥaldūn’s keenly observed about the fate of the text of the *Organon*:

Its sections came to be nine; and all were translated in the Islamic community, and the philosophers dealt with [these books] by commentary and exposition. Alfarabi did [this], and Avicenna, and Averroes among the Andalusian philosophers. Avicenna wrote *The Cure*, in which he took in all seven

⁴⁷³ E.g. E. Stump, “Topics: the Development and Absorption into Consequences”, in *The Cambridge History of Later Medieval Philosophy*, ed. N. Kretzmann, J. Pinborg, A. Kenny, E. Stump (Cambridge: Cambridge University Press, 1982), 273–99.

⁴⁷⁴ See Street, “Arabic Logic”, 538f.

⁴⁷⁵ D. Black, *Logic and Aristotle’s Rhetoric and Poetics in Medieval Arabic Philosophy* (Leiden and New York: Brill, 1990), chapter 2.

philosophical disciplines. Then the later scholars came and changed the technical terms of logic; and they appended to the investigation of the five universals its fruit, which is to say the discussion of definitions and descriptions which they moved from the *Posterior Analytics*; and they dropped the *Categories* because a logician is only accidentally and not essentially interested in that book; and they appended to *On Interpretation* the treatment of conversion (even if it had been in the *Topics* in the texts of the ancients, it is none the less in some respects among the things which follow on from the treatment of propositions). Moreover, they treated the syllogistic with respect to its productivity generally, not with respect to its matter. They dropped the investigation of [the syllogistic] with respect to matter, which is to say, these five books: *Posterior Analytics*, *Topics*, *Rhetoric*, *Poetics*, and *Sophistical Fallacies* (though sometimes some of them give a brief outline of them). They have ignored [these five books] as though they had never been, even though they are important and relied upon in the discipline. Moreover, that part of [the discipline] they have set down they have treated in a penetrating way; they look into it in so far as it is a discipline in its own right, not in so far as it is an instrument for the sciences. Treatment of [the subject as newly conceived] has become lengthy and wide-ranging--the first to do that was Faḥraddīn ar-Rāzī and, after him, Ḥunaḡī⁴⁷⁶ (on whose books Eastern scholars rely even now). On this art, Ḥunaḡī has written *The Disclosure of Secrets*, which is long, and an abridgement, *The Short Epitome*, which is good for teaching, and another abridgement, *The Digest*, which in four folios takes up the cruxes and principles of the discipline--students use it frequently to this day and benefit from it.

The books and ways of the ancients have been abandoned, as though they had never been...⁴⁷⁷

I think it is somewhat misleading for Ibn Ḥaldūn to hold up Avicenna's *Šifā'* as evidence that he should be classed with Alfarabi and Averroes as a participant in the "commentary and exposition" tradition of the *Organon*. Certainly Ibn Ḥaldūn is right to point out that in the *Šifā'* Avicenna maintains the traditional ordering of the *Organon*. Yet, in this work he sets out doctrines such as the theory of second intentions and logical consequence that are major factors in the slow disintegration and modulation of the *Organon* into distinct fields of inquiry. The doctrines of the *Šifā'* make possible the change in Avicenna's treatment of logic in the

⁴⁷⁶ He is Afḍaladdīn al-Ḥunaḡī (d. 1248). For a long introduction to his life and works, see K. El-Rouayheb, Introduction to Afḍaladdīn al-Ḥunaḡī, *Kašf al-Asrār 'an Ġawāmiḍ al-Afkār*, ed. K. El-Rouayheb (Tehran: Institute of Iranian Philosophy and Institute of Islamic Studies Free University of Berlin, 2010), iii-l.

⁴⁷⁷ Ibn Ḥaldūn, *Prolégomènes d'Ebn Khaldoun: texte arabe (troisième partie)*, ed. M. Quatremère (Paris: B. Duprat, 1858), 112.8-3.12; quoted and translated in Street, "Arabic Logic", 580.

Iṣārāt and the *Nağāt*, where the syllogistic arts receive much less attention. Thus, Ibn Ḥaldūn's history of the text of the *Organon* in the Arabic logical tradition is correct, though Avicenna's place in it is more complicated than Ibn Ḥaldūn's narrative suggests. Like that of Alfarabi and Averroes, Avicenna's logic makes sense as a hermeneutic project connected to an *Organon*-centered problematic; and understood this way, it makes sense in a way that post-classical Arabic logic does not.⁴⁷⁸ Nevertheless, Avicenna's later works are a logical extension of doctrines he sets out in the *Šifā'* that will ultimately lead to the eclipse of the five syllogistic arts in his later works, and the fragmentation of the *Organon* in the post-classical period.

To a degree, Avicenna was able to develop a propositional logic (in the restricted sense I give this term in chapter 4) because he consciously and systematically excluded the *Topics* as a source of formal theory about logical consequence (*luzūm*). As I have just discussed, his rejection is based on his vision about what the subject matter of logic is and what it is not. (In particular, it is not what we find in the *Topics*.) For Avicenna, an analysis of the formal properties of logical consequence involves an analysis of the sense in which the concept of concomitance (*ma'iyya*, *muwāfaqa*) is deployed when speaking about things following from other things. It does *not* involved the analysis of paired complementaries such as cause/effect, genus/species, paired relata (e.g. slave/master, father/son), wholes/parts, etc., which are the basis for the theory of dialectical topoi in the *Topics*, and also the source of the “maximal propositions” in the Latin tradition of consequentia.⁴⁷⁹ Avicenna's rejection, then, marks a historical caesura, and a parting of ways between the Latin and Arabic traditions' theorizing about the nature of how a sentence or sentences are said to follow logically from other sentences. ŠQ V deserves to be recognized for one of the most historically momentous texts in

⁴⁷⁸ Street, Introduction to Avicenna, *Avicenna's Deliverance: Logic*, xxvi.

⁴⁷⁹ Stump, “Topics: their Development and Absorption into Consequences”, 276f.

the history of formal logic, for it is the results of these few pages that led post-classical Arabic logicians to formulate their thoughts about arguments with conditionals in a way that is far more “Aristotelian” than in the Latin tradition. There is an irony here. Avicenna’s conjunctive syllogistic is consciously modeled as an extension of Aristotle’s theory of the categorical syllogism to forms of conditional reasoning. This Aristotelian conservatism is clearly not a mark of an unoriginal thinker, but reveals Avicenna’s somewhat unorthodox if not idiosyncratic approach to the text of the *Prior Analytics*. As Street has noted, Avicenna was not any less “Aristotelian” than Alfarabi, but the former differed from the latter in the “implementation of his Aristotelianism”.⁴⁸⁰ Avicenna’s adopted a certain posture toward Aristotle’s text such that when he

collided with a crux in the text, he did not have to resort to exegetical strategies to find his way out. In fact, throughout the *Cure*, it is clear that he believed he had worked out the unified vision that motivates Aristotle’s presentation, and this allowed him to elide, transform and augment the system of the *Prior Analytics*.⁴⁸¹

Indeed, our findings in this study confirm the aptness of Street’s characterization of Avicenna’s hermeneutic outlook. Avicenna’s generalization of Aristotle’s logical theory to the conjunctive syllogistic is a prime example of Avicenna’s belief in his having “worked out” Aristotle’s “unified vision” of the syllogisms.

Street has noted that Avicenna’s division of the syllogism into conjunctive and repetitive was accepted by all logicians that followed him “save obdurate Aristotelians like Averroes”.⁴⁸² Additionally, Avicenna’s novel doctrine of quantified conditionals, his application of perfect and imperfect moods to conjunctive syllogisms, a system of syllogistic figures based on shared parts between the premises of conjunctive syllogisms, his application

⁴⁸⁰ Street, “Arabic Logic”, 536.

⁴⁸¹ Ibid.

⁴⁸² Street, Introduction to Avicenna, *Avicenna’s Deliverance: Logic*, xxvi.

of direct and indirect reduction to conjunctive syllogisms, and his adoption of rejection of inconcludent moods by substitution of concrete triples were all accepted by post-classical logicians. That being said, Avicenna's novelties were not accepted uncritically. As Khaled El-Rouayheb has shown in recent articles, Avicenna's views relating to the subject matter of logic, his doctrine of equipollence between A- and E-conditionals, his allowing impossible antecedents, and the vacuity of conditional Darapti (III) were the subject of spirited debate among post-classical logicians. That being said, it is not clear to what extent Avicenna himself saw his conjunctive and repetitive syllogisms as having as wide a currency and scope of application as his categorical syllogistic with its theory of sentential modalities. For example, Street has noted that in Avicenna's two major summa written after the *Šifā'*, viz. the *Išārāt* and the *Nağāt*,⁴⁸³ Avicenna limits his discussion of conjunctive and repetitive syllogisms to just those aspects that are "actually used to explain his formulation of the per impossibile syllogism (*qiyās al-ḥulf*)".⁴⁸⁴ In other words, despite Avicenna's titanic efforts in ŠQ V-IX, by the time of his writing the *Nağāt* and the *Išārāt*, the scope of application for conjunctive and repetitive syllogisms seems to have diminished somewhat in Avicenna's eyes. I can offer no reason that might account for this. In any case, most post-classical logicians such as Afḍaladdīn al-Ḥunaḡi and 'Alī ibn 'Umar al-Kātibī al-Qazwīnī (d. circa 1276) were as careful readers of Avicenna's *Šifā'* as they were of his other works, and they developed Avicenna's

⁴⁸³ "Summa" in the sense given to it by Dimitri Gutas (in D. Gutas, "The Logic of Theology (*Kalām*) in Avicenna", in *Logik und Theologie: Das Organon im Arabischen und im Lateinischen Mittelalter*, ed. D. Perler, U. Rudolph (Leiden and Boston: Brill, 2005), 59-72, especially 59: "It is becoming increasingly clear, as it is successively documented in a number of recent studies, that Avicenna's is a philosophical system, and indeed an Aristotelian system incorporating some significant Neoplatonic elements [...] which is amazingly rationalistic and self-consistent. This is something after which Avicenna strove explicitly and in earnest, and it constitutes his great historical achievement [...] His formulations of the system he set down in a new literary genre which he initiated, the philosophical *summa* within the covers of a single book—whether that book be the extremely concise essay *The Elements of Philosophy* (*Uyūn al-Ḥikma*) or the multi-volume magnum opus *The Cure* (*aš-Šifā'*)" (italics in the original).

⁴⁸⁴ Street, "Arabic Logic", 546.

conjunctive syllogistic extensively. The same cannot be said, however, for other post-classical logicians. Consider the somewhat scathing remarks made by the early Avicennian logician ‘Umar ibn Sahlān as-Sāwī (d. 1058). His remarks are interesting because his characterization of some figures of the conjunctive syllogistic as “unnatural” or “remote from nature (*ba’īdun ‘ani ṭ-ṭab’*)” strongly recall the charges Avicenna himself lays against the fourth figure of the categorical syllogistic in the *Iṣārāt*.⁴⁸⁵

[Text 1] The conjunction [*al-iqtirān*] obtains between two connective [conditional premises], two disjunctives, or between a categorical and a connective (in which case the sharing [*ṣarika*] is in the antecedent or the consequent), or between a categorical and a disjunctive, or between a connective and a disjunctive. We do not feel obliged, however, to exhaustively discuss all of the conjunctive syllogisms [*al-iqtirānāt*], for among them are those that are unnatural [*ba’īdun ‘ani ṭ-ṭab’i*], and whose concludency [*intāğ*] is only proved in an extremely laborious manner [*lā yastabīnu intāğuhu illā bi-kulfatin ṣadīdatin*]. Nor is it appropriate to delve into such tangential matters [*umūr waḥṣiyya*] in an abridged work <such as this>. Let us then limit ourselves to <sylogisms> whose concludency is closer to the sound nature. Whoever desires to investigate all of the conjunctive syllogisms further, the concludent and the inconcludent alike, then he should consult the books of the eminent later scholar [*afḍal al-muta’aḥḥirīn*], in which one may find more derivation of rules, and distinguishing the concludent <sylogisms> from the inconcludent than in other books that preceded him [i.e. Avicenna]. And if God should prolong my life [*wa-in aḥḥara llāhu fī l-ağali*, lit. delay the appointed time of death], then I intend to devote an entire book to <this topic>, which will contain the familiar [*ma’lūf*] and the unfamiliar [*ğarīb*, lit. “strange”] alike.⁴⁸⁶

In general, Ibn Sahlān’s critical evaluation of the conjunctive syllogistic developed by the “eminent later scholar” of the post-classical tradition seems have been a minority view. Ibn Sahlān’s remarks also reveal how even in the post-classical period, Avicenna’s syllogistic—not

⁴⁸⁵ N. Rescher, “New Light from Arabic Sources on Galen and the Fourth Figure of the Syllogism”, *Journal of the History of Philosophy* 3/1 (1965): 27-41, especially 33.

⁴⁸⁶ ‘Umar ibn Sahlān as-Sāwī, *Kitāb al-Baṣā’ir an-Naṣīriyya fī ‘ilm al-Manṭiq*, ed. M. Abduh (Cairo: Maktaba at-Taḳāfa ad-Dīniyya, 2005, [reprint: Cairo, ed. 1898]), 98.8-14.

that of Alfarabi or Aristotle—would become paradigmatic for Avicenna’s opponents and backers alike.⁴⁸⁷

⁴⁸⁷ For an interesting discussion of the identity of the “eminent later scholar” in Avicenna’s text, see T. Street, “‘The Eminent Later Scholar’ in Avicenna’s Book of the Syllogism”, *Arabic Sciences and Philosophy* 11/2 (2001): 205-18.

BIBLIOGRAPHY

- Adams, Ernest. "The Logic of Conditionals", *Inquiry* 8/2 (1965): 166-197.
- Adams, Ernest. *The Logic of Conditionals: An Application of Probability to Deductive Logic*. Dordrecht and Boston: D. Reidel, 1975.
- Adamson, Peter, ed. *Interpreting Avicenna: Critical Essays*. New York, Cambridge: Cambridge University Press, 2012.
- Afnan, Soheil. *Avicenna: His Life and Works*. London: George Allen & Unwin, 1958.
- Ahmed, Asad. "On Avicenna's Reception of Aristotelian Modal Syllogistics: a Study Based on the Conversion Rules and the BARBARA Problematic", in *Before and After Avicenna: Proceedings of the First Conference of the Avicenna Study Group*, ed. David Reisman and Ahmad Al-Rahim. Leiden: Brill, 2003.
- Ahmed, Asad. "The Jiha/Tropos-Mādda/Hūlē Distinction in Arabic Logic and its Significance for Avicenna's Modals", in *The Unity of Science in the Arabic Tradition*, ed. S. Rahman, T. Street, H. Tahiri. Amsterdam: Springer, 2008, 229-53.
- Alfarabi. *Catalogo de las ciencias*, ed. A. G. Palencia. Madrid: Consejo Superior de Investigaciones Científicas, Patronato Menéndez y Pelayo, Instituto Miguel Asín, 1953.
- Alfarabi. *Al-Fārābī's Short Commentary on Aristotle's Prior Analytics*, trans. Nicholas Rescher. Pittsburgh: University of Pittsburgh Press, 1966.
- Alfarabi. *Kitāb fī 'l-Mantiq: al-Ḥaṭāba*, ed. Muḥammad Salīm Sālim. Cairo: al-Hay'a al-Miṣriyya al-Āmma li-l-Kitāb, 1976.
- Alfarabi. *Al-Fārābī's Commentary and Short Treatise on Aristotle's De Interpretatione*, trans. Fritz Zimmermann. London: British Academy, 1981.
- Alfarabi. *Al-Mantiq 'inda l-Fārābī*, 4 vols., eds. Rafīq al-ʿAḡam, Majid Fakhry. Beirut: Dār al-Mašriq, 1985-1986.
- Alfarabi. *Deux ouvrages inédits sur la Rétorique*, ed. Jacques Langhade, M. Grignaschi. Beirut: Dār al-Mašriq, 1986.
- Alfarabi. "El 'Fontes Quaestionum' ('Uyūn al-Masā'il) de Abū Naṣr al-Fārābī," ed. M. Cruz Hernandez, in *Abū Naṣr Muḥammad ibn Muḥammad al-Fārābī (d. 339/950): Texts and Studies*, ed. Fuat Sezgin. Frankfurt am Main: Institute for the History of Arabic-Islamic Science, 1999.
- Alfarabi, *Al-Fārābī's philosophische Abhandlungen*, ed. Freidrich Dieterici. Frankfurt am Main: Institute for the History of Arabic-Islamic Science, 1999.
- Alfarabi. *Kitāb al-Ḥurūf* [*Al-Fārābī's Book of Letters*], 4th ed. Beirut: Dār al-Mašriq, 2004.

- Aristotle, *Manṭiq Aristū*, trans. Ḥunayn ibn Ishāq (d. 873) et al., ed. ‘Abdarrahmān Badawī. Cairo: Maṭba‘at Dār al-Kutub al-Miṣriyya, 1948-1952.
- Aristotle. *Aristotle’s Posterior Analytics*, trans. Jonathan Barnes. Oxford: Clarendon Press, 1975.
- Aristotle. *Prior Analytics*, trans. Robin Smith. Indianapolis and Cambridge: Hackett, 1989.
- Aristotle. *Topics: Books I and VIII*, trans. Robin Smith. Oxford: Clarendon Press, 1997.
- Aristotle. *Prior Analytics Book A*, trans. Gisela Striker. Oxford: Clarendon Press, 2009.
- Austin, John. *How to Do Things with Words*, 2nd ed. Cambridge, Massachusetts: Harvard University Press, 1975.
- Averroes. *Talḥīṣ al-Qiyās*. ed. C. Butterworth. Cairo: al-Maṭba‘a Dār al-Kutub al-Miṣriyya, 1948.
- Averroes. *Talḥīṣ al-Qiyās*, eds. Charles Butterworth, Qāsim Maḥmūd, Aḥmad Harīdī. Cairo: al-Hay’a al-Miṣriyya al-‘Āmma li-l-Kitāb, 1983.
- Avicenna. *Livre des directives et remarques*, trans. Anne-Marie Goichon. Paris: Vrin, 1951.
- Avicenna, *Aṣ-Ṣifā’, al-Madḥal*, ed. George Anawati, Muḥammad Al-Ḥuḍayrī, Fu‘ād Al-Ahwānī. Cairo: al-Maṭba‘a al-Amīriyya, 1952.
- Avicenna. *Al-Iṣārāt wa-Tanbīhāt*, ed. Sulaymān Dunyā. Cairo: Dār al-Ma‘ārif, 1957-1960.
- Avicenna. *Avicenna’s Propositional Logic*, trans. Nabil Shehaby. Dordrecht and Boston: D. Reidel, 1973.
- Avicenna. *Remarks and Admonitions*, trans. Shams Inati. Toronto: Pontifical Institute of Medieval Studies, 1984.
- Avicenna. *Avicenna’s Deliverance: Logic*, trans. Asad Ahmed. Karachi: Oxford University Press, 2011.
- Barnes, Jonathan. “Aristotle’s Theory of Demonstration”, *Phronesis* 14/2 (1969): 123-152.
- Barnes, Jonathan. “Terms and Sentences: Theophrastus on Hypothetical Syllogisms”, *Proceedings of the British Academy* 69 (1983): 279-326.
- Barnes, Jonathan. *Logic and the Imperial Stoa*. Leiden, New York, Köln: Brill, 1997.
- Barnes, Jonathan. *Truth, Etc.* Oxford: Oxford University Press, 2007.
- Bennett, Jonathan. *A Philosophical Guide to Conditionals*. Oxford and New York: Oxford University Press, 2003.
- Berti, Enrico, ed. *Aristotle on Science: the Posterior Analytics*. Padova: Edritice Antenore, 1981.
- Black, Deborah. *Logic and Aristotle’s Rhetoric and Poetics in Medieval Arabic Philosophy*. Leiden, New York: Brill, 1990.

- Bobzien, Susanne. "Wholly Hypothetical Syllogisms", *Phronesis* 45/2 (2000): 87-137.
- Bobzien, Susanne. "Pre-Stoic Hypothetical Syllogistic in Galen's *Institutio Logica*", in *The Unknown Galen*, ed. Vivian Nutton. London: Institute of Classical Studies, 2002, 57-72.
- Bobzien, Susanne. "Peripatetic Hypothetical Syllogistic in Galen—Propositional Logic off the Rails?" *Rhizai: Journal for Ancient Philosophy and Science* 2 (2004): 57-102.
- Boger, George. "Aristotle's Underlying Logic", in *Handbook of the History of Logic*, vol. 1, eds. John Woods, Dov Gabbay. Amsterdam and Boston: Elsevier, 2004, 101-246.
- Collins, John. "Counterfactuals, Causation, and Preemption", in *Philosophy of Logic*, ed. Dale Jacquette. Dordrecht: Elsevier, 2007, 1127-43.
- Corcoran, John. "A Mathematical Model of Aristotle's Syllogistic", *Archiv für Geschichte der Philosophie* 55/2 (1973): 191-219.
- Demoss, David, Devereux, Daniel. "Essence, Existence and Nominal Definition in Aristotle's *Posterior Analytics* II 8-10", *Phronesis* 33/2 (1988): 133-54.
- Dexippus. *On Aristotle's Categories*, trans. John Dillon. Ithaca: Cornell University Press, 1990.
- DiPasquale, Michael. "Alfarabi and the Starting Point of Islamic Philosophy: a Study of the *Kitāb al-Jadal (Book of Dialectic)*". PhD diss., Harvard University, Harvard University, 2002.
- Druart, Thérèse. "Al-Fārābī, Emenation, and Metaphysics", in *Neoplatonism and Islamic Thought*, ed. Parviz Morewedge. Albany: SUNY Press, 1992, 127-148.
- Dummett, Michael. *Frege: Philosophy of Language*, 2nd ed. London: Duckworth, 1981.
- Dürr, Karl. *The Propositional Logic of Boethius*. Amsterdam: North-Holland Publishing, 1951.
- Edgington, Dorothy. "On Conditionals", in *Handbook of Philosophical Logic*, vol. 14, eds. Dov Gabbay, Franz Guenther. Dordrecht: Springer, 2007, 127-221.
- El-Rouayheb, Khaled. "Impossible Antecedents and their Consequences: Some Thirteenth-Century Discussions", *History and Philosophy of Logic* 30 (2009): 209-225.
- El-Rouayheb, Khaled. "Post-Avicennian Logicians on the Subject Matter of Logic: Some Thirteenth- and Fourteenth-Century Discussions", *Arabic Sciences and Philosophy* 22 (2012): 69-90.
- van Ess, Josef. *Die Erkenntnislehre des 'Aḡuaddīn al-Īcī*. Wiesbaden: Franz Steiner Verlag, 1966.
- Evangeliou, Christos. *Aristotle's Categories and Porphyry*. Leiden, New York: Brill, 1988.
- Evans, Jonathan, Over, David. *If*. Oxford and New York: Oxford University Press, 2004.

- Frede, Michael. "Stoic vs. Peripatetic Syllogistic", *Archiv für Geschichte der Philosophie* 56/1 (1974): 1-32.
- Frege, Gottlieb. *The Frege Reader*, ed. Michael Beaney. Oxford: Blackwell Publishing, 1997.
- Galston, Miriam. "Al-Farabi on Aristotle's Theory of Demonstration", in *Islamic Philosophy and Mysticism*, ed. Parviz Morewedge. Delmar, NY: Caravan Books, 1981, 23-34.
- Gätje, Helmut. 'Zur Lehre von den Voraussetzungsschlüssen bei Avicenna', *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 2 (1985): 140-204.
- Gauker, Christopher. *Conditionals in Context*. Cambridge, Massachusetts: MIT Press, 2005.
- Geach, Peter. *Logic Matters*. Berkeley and Los Angeles: University of California Press, 1972.
- Georr, Khalil. *Les Catégories d'Aristote dans leurs versions syro-arabes*. Beirut: Institut français de Damas, 1948.
- Goodman, Lenn. *Avicenna*, 2nd ed. London and New York: Routledge, 1992.
- Grice, H. Paul. *Studies in the Way of Words*. Cambridge, Massachusetts: Harvard University Press, 1989.
- Gutas, Dimitri. "Paul the Persian on the Classification of the A Milestone between Alexandria and Baghdad", *Der Islam* 60/2 (1983): 231-67.
- Gutas, Dimitri. *Avicenna and the Aristotelian Tradition*. Boston and Leiden: Brill, 1988.
- Gutas, Dimitri. "Pre-Plotinian philosophy in Arabic (other than Platonism and Aristotelianism): a review of the sources", *Aufstieg und Niedergang der römischen Welt* II.36.7. Berlin, Walter de Gruyter, 1994, 4939-4973.
- Gutas, Dimitri. "Intuition and Thinking: the Evolving Structure of Avicenna's Epistemology", in *Aspects of Avicenna*, ed. Robert Wisnovsky. Princeton, New Jersey: Markus Wiener, 2001, 1-39.
- Gutas, Dimitri. "The Logic of Theology (Kalām) in Avicenna", in *Logik und Theologie: Das Organon im Arabischen und im Lateinischen Mittelalter*, eds. Ulrich Rudolph and Dominik Perler. Leiden and Boston: Brill, 2005, 59-72.
- Gyekye, Kwame. "The Term 'istithnā' in Arabic Logic", *Journal of the American Oriental Society* 92 (1972): 88-92.
- Hadot, Ilsetraut, ed. *Simplicius: sa vie, son oeuvre, sa survie*. Berlin: de Gruyter, 1987.
- Hamblin, Charles. *Fallacies*. London: Methuen, 1979.

- al-Ḥunaḡī, Afdaladdīn. *Kaṣf al-Asrār ‘an Ġawāmiḡ al-Afkār*, ed. Khaled El-Rouayheb. Tehran: Iranian Institute of Philosophy and the Institute of Islamic Studies of Free University of Berlin, 2010.
- Jadaane, Fehmi. *L'influence du stoicisme sur la pensée musulmane*. Beirut: Dar el-Machreq, 1969.
- Kneale, William, Kneal, Martha. *The Development of Logic*. Oxford: Clarendon Press, 1962.
- Lameer, Joep. *Al-Fārābī and Aristotelian Syllogistics: Greek Theory and Islamic Practice*. Leiden and New York: Brill, 1994.
- Lear, Jonathan. *Aristotle and Logical Theory*. Cambridge: Cambridge University Press, 1980.
- Lejewski, Czesław. "On Prosleptic Syllogism", *Notre Dame Journal of Formal Logic* 2 (1961): 158-176.
- Lewis, David. *Counterfactuals*. Malden, Massachusetts: Blackwell Publishers, 2001.
- Łukasiewicz, Jan. *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, 2nd ed. Oxford: Clarendon Press, 1957.
- Mallet, Dominique. "La dialectique dans la philosophie d'Abū Naṣr al-Fārābī". PhD diss., Université de Lille III, 1992.
- Mallet, Dominique. "Le kitāb al-Taḥlīl d'Alfarabi", *Arabic Sciences and Philosophy* 4/2 (1994): 317-335.
- Maróth, Miklós. "Die Rolle der Topik Avicennas in den arabischen Wissenschaften", *Acta Antiqua Academiae Scientiarum Hungariae* 29 (1981): 33-41.
- Maróth, Miklós. *Ibn Sīnā und die peripatetische "Aussagenlogik"*, trans. Johanna Till. Leiden: Brill, 1989.
- Martin, Christopher. 'The Logic of Negation in Boethius', *Phronesis* 36 (1991): 277-301.
- Martin, Christopher. "Logic", in *The Cambridge Companion to Abelard*. Cambridge and New York: Cambridge University Press, 2004, 158-199.
- Martin, Christopher. "Denying Conditionals: Abaelard and the Failure of Boethius' Account of the Hypothetical Syllogism", *Vivarium* 45 (2007): 153-68.
- Mates, Benson. "Diodorean Implication", *The Philosophical Review* 58/3 (1949): 239-242.
- Mates, Benson. *Stoic Logic*. Berkeley: University of California Press, 1952.
- Menn, Stephen. "Al-Fārābī's *Kitāb al-Ḥurūf* and His Analysis of the Senses of Being", *Arabic Sciences and Philosophy* 18 (2008): 59-97.
- Menn, Stephen, Wisnovsky, Robert. "Yaḡyā ibn ‘Adī On the Four Scientific Questions Concerning the Three Kinds of Existence", *Mélanges de l'Institut Dominicain d'Études Orientales du Caire* 29 (2012): 73-96.

- Mueller, Ian. "Stoic and Peripatetic Logic", *Archive für Geschichte der Philosophie* 51 (1969): 173-187.
- O'Toole, Robert, and Jennings, Raymond. "The Megarians and the Stoics", in *Handbook of the History of Philosophy*, vol. 1, eds. Dov Gabbay and John Woods. Amsterdam and Boston: Elsevier, 2004, 397-522.
- Patzig, Günther. *Aristotle's Theory of the Syllogism: A Logico-Philosophical Study of Book A of the Prior Analytics*, trans. Jonathan Barnes. Dordrecht: D. Reidel, 1968.
- Pormann, Peter. *Islamic Medical and Scientific Tradition*, 3 vols. London, New York: Routledge, 2011.
- Porphyry. *On Aristotle's Categories*, trans. Steven Strange. Ithaca: Cornell University Press, 1992.
- Prior, Arthur. "Diodoran Modalities," *Philosophical Quarterly* 4/1 (1955): 205-213.
- Rahman, Fazlur. *Prophecy in Islam: Philosophy and Orthodoxy*. London: George Allen & Unwin, 1958.
- Read, Stephen. *Relevant Logic: a Philosophical Examination of Inference*. Oxford: Basil Blackwell, 1988.
- Read, Stephen. "The Medieval Theory of Consequences", *Synthese* 187 (2002): 899-912.
- Rescher, Nicholas. *Studies in the History of Arabic Logic*. Pittsburgh: University of Pittsburgh Press, 1963.
- Rescher, Nicholas. *The Development of Arabic Logic*. Pittsburgh: University of Pittsburgh Press, 1964.
- Rescher, Nicholas, et al. *Studies in Modality*. Oxford: Blackwell, 1974.
- Rescher, Nicholas. *Galen and the Syllogism*. Pittsburgh: University of Pittsburgh Press, 1966.
- Rescher, Nicholas. "New Light from Arabic Sources on Galen and the Fourth Figure of the Syllogism", *Journal of the History of Philosophy* 3/1 (1965): 27-41.
- Sabra, Abdelhamid. "Avicenna on the Subject Matter of Logic", *Journal of Philosophy* 77/11 (1980): 746-64.
- Sanford, David. *If P, then Q: Conditionals and the Foundations of Reasoning*, 2nd ed. London and New York: Routledge, 2003.
- al-Sāwī, 'Umar ibn Sahlān. *Kitāb al-Baṣā'ir fī 'Ilm al-Manṭiq*, ed. Muḥammad Abduh. Cairo: Maktabat Muḥammad Ṣibāh, 1898.
- Simplicius. *On Aristotle's Categories, 1-4*, trans. Michael Chase. Ithaca: Cornell University Press, 2003.

- Smiley, Timothy. "What is a Syllogism?", *Journal of Philosophical Logic* 2/1 (1973): 136-154.
- Smiley, Timothy. "Aristotle's Completeness Proof", *Ancient Philosophy* 14 (1994): 27-38.
- Smiley, Timothy. "Rejection", *Analysis* 56/1 (1996): 1-9.
- Smith, Robin. "What is Aristotelian Ecthesis?" *History and Philosophy of Logic* 3 (1982): 113-127.
- Smith, Wilfred Cantwell. "Faith as Taṣdīq", in *Islamic Philosophical Theology*, ed. Parviz Morewedge. Albany: State University of New York Press, 1979, 96-119.
- Sorabji, Richard, ed. *Aristotle Transformed: The Ancient Commentators and Their Influence*. Ithaca: Cornell University Press, 1990.
- Sorabji, Richard, ed. *The Philosophy of the Commentators, 200-600 AD*, vol. 3 [Logic and Metaphysics]. Ithaca: Cornell University Press, 2005.
- Specia, Anthony. *Hypothetical Syllogistic and Stoic Logic*. Leiden: Brill, 2001.
- Stalnaker, Robert. "A Theory of Conditionals", in *IFS: Conditionals, Belief, Decision, Chance, and Time*, ed. Robert Stalnaker, William Harper, Glenn Pearce. Dordrecht and Boston: D. Reidel, 1981.
- Stalnaker, Robert. "A Defense of Conditional Excluded Middle", in *IFS*, eds. William Harper, Robert Stalnaker, Glenn Pearce. Dordrecht and Boston: D. Reidel, 1980, 87-104.
- Street, Tony. "An Outline of Avicenna's Syllogistic", *Archiv für Geschichte der Philosophie* 84/2 (2002): 129-160.
- Street, Tony. "Arabic Logic", in *Handbook of the History of Logic*, vol.1, eds., Dov Gabbay, John Woods. Amsterdam and Boston: Elsevier, 2004, 523-596.
- Street, Tony. "'The Eminent Later Scholar' in Avicenna's *Book of the Syllogism*", *Arabic Sciences and Philosophy* 11/2 (2001): 205-218.
- Striker, Gisela. "Perfection and Reduction in Aristotle's Prior Analytics", in *Rationality in Greek Thought*, eds. Michael Frede, Gisela Striker. Oxford: Clarendon Press, 1996, 203-20.
- Stump, Eleanor. "Topics: their Development and Absorption into Consequences", in *The Cambridge History of Later Medieval Philosophy*, eds. Norman Kretzmann, Jan Pinborg, Anthony Kenny, Eleanor Stump. Cambridge: Cambridge University Press, 1982.
- Thom, Paul. *The Syllogism*. Munich: Philosophia, 1981.
- Thom, Paul. *Medieval Modal Systems: Problems and Concepts*. Aldershot, Hants and Burlington, Vermont: Ashgate, 2003.
- Thom, Paul. "Three Conceptions of Formal Logic", *Vivarium* 48 (2010): 239.

- Williamson, Jon. "Causality", in *Handbook of Philosophical Logic*, vol. 14, eds. Dov Gabbay, Franz Guenther. Dordrecht: Springer, 2007, 95-126.
- Wisnovsky, Robert. *Avicenna's Metaphysics in Context*. Ithica: Cornell University Press, 2003.
- Wisnovsky, Robert. "Arabic Logicians on Perfect and Imperfect Syllogisms: a Supplement to Patzig's 'Historical Excursus'", in *Avicenna and his Legacy: a Golden Age of Science and Philosophy*, ed. Y. Tzvi Langermann. Turnhout, Belgium: Brepols, 2010, 257-273.
- Wisnovsky, Robert. "Yahyā b. 'Adī's Discussion of the Prolegomena to the Study of a Philosophical Text", in *Law and Tradition in Classical Islamic Thought: Studies in the Honor of Professor Hossein Modarressi*, eds. Michael Cook, Najam Haider, Intisab Rabb, and Asma Sayeed. New York: Palgrave, 2013, 187-202.
- Wolfson, Harry. "The terms Tasawwur and Tasdiq in Arabic Philosophy and their Greek, Latin and Hebrew Equivalents", *The Moslem World* 33 (1943): 114-128.
- Woods, John, Walton, Douglas. "Petitio Principii", *Synthese* 31/1 (1975): 107-127.
- Woods, John, Walton, Douglas. "The Petitio: Aristotle's Five Ways", *Canadian Journal of Philosophy* 12/1 (1982): 77-100.
- Woods, John, Irvine, Andrew. "Aristotle's Early Logic", in *Handbook of the History of Logic*, vol. 1, eds. John Woods, Dov Gabbay. Amsterdam and Boston: Elsevier, 2004, 27-99.
- von Wright, Georg. *Logical Studies*. London: Routledge and Kegan Paul, 1957.