DESIGN INVESTIGATION OF A POWER TRANSMISSION SHAFT

by

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INTRODUCTION

Sponsored by the American Society of Mechanical Engineers a Code for the Design of Transmission Shafting was issued by the American Standards Association in 1929. In this Code all commonly used theories are discussed and rational procedure for the design of the shafts under all conditions of loading is developed with recommendations as to the values of allowable stresses and factors to be used in the design formula. Apart from this code, formulae for design of transmission shaftings have been established by the classification societies like Lloyd's Register of shipping and Westinghouse Co. Ltd.

By making use of one or the other of these formulae given by the above mentioned authorities a shaft can be designed for satisfactory operation when the operating speeds are kept away from the critical speeds of the transmission system. However, experience has revealed that failures of transmission systems designed for safe running in a normal state of operation have been caused by excessive vibrations because the system was running at or nearly at a dangerous critical speed. If the transmission shaft system is one which couples an electrical set to an internal combustion engine, critical or disturbing amplitude occurring near the operating speed may cause the light to flicker to a prohibitive degree; excessively large vibration torques occurring at the generator armatures may cause loosening of core laminations and other forms of armature failure. A design of a transmission shaft system without vibration analysis is, therefore, hardly considered complete nowadays. The classification societies set definite rules to have the transmission systems vibrationally analyzed. Lloyd's Register of shipping judged, in 1943, that shipbuilders should submit critical speed calculations for both main and auxiliary heavy oil installations in order to obtain Lloyd Machinery certificate. This requirement was introduced into the society's rules in January 1944.

Since this work may deal to a great extent with the vibration problems of the designed transmission system it may be proper to give a short history of both torsional and lateral vibration in this introduction.

TORSIONAL VIBRATION

Interesting notes on the history of torsional vibration problems in land, marine, and aero installations are given by Ker Wilson[(2)*] in the introduction to the third edition of his book, "Practical Solution of Torsional Vibration Problems". The following is a short account on the history of the problems mainly in marine and land installations, to which the thesis is closely related.

Problems of vibration have been known and widely discussed since the last decade of the last century. Early work on torsional vibration, however, were mostly concerned with fundamental or one-node mode of vibration. In those days, the systems, in which the problem of torsional vibration arose, comprised mostly of heavy reciprocating steam engine masses on the one end and heavy driven machine on the other. The system was then reasonably reduced

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* Numbers refer to the references given in the Bibliography at the end of this thesis.

to approximately equivalent two-mass arrangement. Methods for calculating the critical speeds of systems containing several masses were published by Chree, Sankey and Millington in 1905 and Holzer in 1907. Not much progress in analytical and experimental work on the subject was, however, accomplished until the beginning of the first world war.

In the years following this 1914-1918 war, rapid development of internal combustion engines came into the scene; and during this period it was brought to the notice of the investigators that failure of transmission systems occurred not only at the transmission shafting as was the case in the earlier years, but also at the crankshaft, revealing the requirement of analysis for higher modes of vibration. Considerable progress was made in the subject during these years and first text books were published by Holzer and Wydler in 1921 and 1922 respectively; and a paper by Lewis came out in 1925. The first text books in English were published by Tuplin, Ker Wilson, and Den Hartog all in 1934. In the U.S.A. a large volume of contribution to the subject was made by F.P. Porter.

Harmonic analysis of the engine torque curve of single cylinder electric ignition engine was made and published by Muir and Terry in 1930. The most complete analyses were made by F.P. Porter in a paper entitled "Harmonic coefficients of Engine Torque Curves", published in Transactions of the American Society of Mechanical Engineers in 1943.[20]

Analysis on stiffness of crankshafts in torsion as well as bending was published by Timoshenko, in 1922 and 1923, in

Transactions of the A.S.M.E. An empirical formula for crankshaft stiffness in torsion was introduced by Carter in 1928; and this was followed by an alternative formula by Ker Wilson published in the 2nd Edition of his text book in 1940(1).

At the present time, there is a considerable amount of literature on torsional vibration problems. Various methods, analytical as well as graphical, have been published by various investigators, alongside with tremendous improvement in the techniques of measurements of vibration. However, although such a large volume of literature is available, and some problems are comparatively straightforward requiring little more than determination of the fundamental natural frequency of the system to make sure that there are no significant resonant conditions in the operating speed ranges, there are still cases where both analytical and experimental investigations are required.

LATERAL VIBRATION

In the same way as Holzer, Lewis, Porter etc. are noted in the field of torsional vibration there are some well known contributors, whose names come up frequently in the lateral vibration field, namely: Rayleigh, Dunkerley, Stodola, Myklestad, Prohl etc.

The problems of lateral vibration have been known since almost the same time as were torsional vibration problems, viz, the later years of the 19th century. A paper in this field on stability of a shaft between bearings was presented by Greenhill to I.M.E. in 1883. Dunkerley gave both theory and experimental results in his paper published in Phil. Transactions in 1894. Chree published his paper in Phil. Magazine in 1904. There have been numerous subsequent writers in the later years, but they mostly followed Dunkerley and Chree.

The first problem of the whirling and vibration of an overhung shaft carrying a symmetrical load of appreciable inertia was investigated and published by Lees in Phil. Magazine in 1923.

Stodola's "iteration" method of obtaining natural frequencies in lateral vibration chiefly for use in determining the whirling speeds of a turbine rotor, is given in his book, which was translated into English in 1927[(10)]. He gave the method both in analytical and graphical forms.

The usual method for determining the lateral vibration frequencies or critical whirling speeds of shafts is the method of Stodola. Recently, in 1944 and 1945 respectively, Myklestad and Prohl published another method of arriving at the result. This new method is referred to by some authors as an extension of the Holzer Tabulation Method. A notable feature of this method is the accuracy with which frequencies for all modes of vibration can be estimated. This method was used by Rankin in his paper entitled "Calculation of the multispan critical speed of flexible shafts by means of punched card machine[11], published in Transactions of the American Society of Mechanical Engineers, in 1946.

PREFACE

The present trend in designing a power transmission shaft between a diesel engine and an electrical generator is to keep the length of the shaft as short as possible. By so coupling the generator as rigidly as possible to the engine, arrangements are possible to raise the harmful critical speeds of the engine-generator system well above the operating speeds. But there may, however, be cases where the generator and engine cannot be operated close to each other or in the same room. In such cases it may be possible to keep the generator and engine in separate halls with a wall in between and to transmit power by using a long floating shaft through the wall.

This thesis is on theoretical design investigation of a long power transmission shaft between a medium size diesel engine and an alternator. It is assumed that the engine and generator need be located on opposite sides of a wall in different rooms; and to cope with some possibility that misalignment may occur, a floating shaft is used through the wall to drive the generator. The distance between the engine and generator is taken as 20 feet; and possibility of operation of the system in parallel with other sets of the same nature is presumed.

The engine used in this investigation is 1120 BHP, 300 rpm, 7 - cylinder in line, vertical, two-stroke diesel engine manufactured by the British Polar Engines Limited of Glasgow, U.K. The drawings of some component parts of the engine used are given in Appendix V at the end of this thesis. Additional data on the engine are given in Appendix I.

The generator used in this investigation is G.E., 1045 KVA, 300 rpm generator, Model No. API-24. A considerable amount of difficulties was experienced in obtaining data for the generator due to the company's strict observance of trade secrecy. However, most of the data which are useful for this investigation were obtained. But unfortunately, no drawings on the generator could be included and assumptions have had to be made as to some data which could not be obtained from the company.

The thesis is prepared in sections, each of which deals with a particular aspect. A section itself is divided into two parts, the first part dealing with the theoretical background and second part giving numerical applications of the theory to the system being investigated.

Procedures for designing the shaft and the flywheel necessary for the system are given in the first three sections. Investigation on the operating characteristics of the system is then made in the later sections. Torsional vibration characteristics and stresses occurring in the system are analyzed from sections IV to IX; the degree of light flicker is determined in section X; an estimation of the critical whirling speed of the system is made in section XI. Conclusion to the investigations is drawn in section XII, which shows that the system possesses satisfactory operating properties when checked up against the standards laid down by various authorities.

As it is difficult to cover the torstional vibration investigations in a single section, a span over six sections has been allowed. It may seem that some sections in the earlier part of this group are put irrelevantly to each other. However, each of the earlier sections of the group needs be introduced to determine particular data of the system that is useful in the later sections. Final summation to get the complete picture of the torsional vibration critical speeds and stresses occurring in the system is made in sections VIII and IX.

Determination of harmonic coefficients of engine torque curve, which seems to be suitable to describe in the later part to avoid abrupt change of topic is also given in section II, as the value of 7th harmonic coefficient is required in design of flywheel in section III.

CONTENTS

I. SHAFT DESIGN

1-1-0 THEORY

1.1.1 Shafts are generally circular in cross-section and made of ductile materials, and the maximum shear theory is generally taken as design criteria. However, some designers prefer to design by the maximum-shear theory as well as by maximum-normal-stress theory, and use the larger size of the two results.

The maximum shear and normal stresses in a shaft of outside diameter d_o in. and inside diameter d_i, carrying a torsional load of T lb.-in., a bending load of M lb.-in. and an axial load of F lb. simultaneously, can be expressed as follows:

$$S_{s_{max}} = \frac{16}{\pi d_o^3} \times \frac{1}{(1 - K^4)} \sqrt{\left(M + \frac{F d_o (1 + K^2)}{8}\right)^2 + T^2} \quad \ldots\ldots (1\text{-}1)$$

$$S_{t_{max}} = \frac{16}{\pi d_o^3} \times \frac{1}{(1 - K^4)} \left[M + \frac{F d_o (1 + K^2)}{8} + \sqrt{\left(M + \frac{F d_o (1 + K^2)}{8}\right)^2 + T^2} \right] \quad \ldots\ldots (1\text{-}2)$$

where K = ratio of inside to outside diameter of the shaft.

1-1-2 A.S.M.E. Code for Design of Transmission Shafting

Equations (1-1) and (1-2) are true when the shaft is subjected to steady torsional and bending loads, and when the axial load does not produce column action. In practice, however, the nature of the loads which the shaft carries is hardly subject to the above conditions. A rotating shaft is subjected to completely reversed stresses; and at the same time, the loads it carries may be subject to variation in intensity or to shocks; the axial load may produce column action.

The American Society of Mechanical Engineers issued a code for designs of transmitting shafting in 1929. The code is based upon the condition that the shaft is made of ductile material and it uses the maximum-shear theory as criterion.

The A.S.M.E. Code equation for a hallow shaft subjected to torsion, bending and axial load can be obtained from equation (1-1), when shock-and-fatigue factors and α factor are introduced, as follows:

$$d_o = \frac{1}{\sqrt[3]{1 - K^4}} \times \sqrt[3]{\frac{16}{\pi S_s} \sqrt{\left[K_m M + \frac{\alpha F d_o (1 + K^2)}{8}\right]^2 + (K_t T)^2}} \qquad \text{........... (1-3)}$$

where S_s = maximum permissible shear stress, psi.

K_m = combined shock and fatigue factor to be applied to the computed bending moment.

K_t = combined shock and fatigue factor to be applied to the computed torsional moment.

The recommended values of S_s, K_m and K_t are given in the above Code and in most of design reference books.

α = ratio of maximum to average intensity of stress, resulting from axial loading only.

$$= \frac{1}{1 - 0.0044\,(L/k)} \qquad \text{for } L/k \leqslant 115 \qquad \text{... (1-4)}$$

$$= \frac{S_y}{\pi n E} \left(\frac{L}{k}\right)^2 \qquad \text{for } L/k \geqslant 115 \qquad \text{... (1-5)}$$

L	=	length between supporting bearings, in.
k	=	radius of gyration of transverse cross-section of the shaft, in.
S_y	=	yield stress in compression, psi.
E	=	modulus of elasticity, psi.
n	=	constant for type of column end support
	=	1 for hinged ends
	=	1.6 for both ends pinned, guided and partially strained (as in bearings).
	=	2.25 for fixed ends.

1-1-3 Practical Design Procedure

Equation (1-3) is for the general case and can readily be used to design when the operational conditions and loading values are known. In practice, however, especially when a shaft is designed for a completely new system, the exact magnitude of bending moment is seldom known. It is, therefore, customary in practice to design the shaft by considering it to be loaded with the torsional moment alone, with introduction of some design factor.

Equation (1-3) with torsional moment alone becomes:

$$d_o = \frac{1}{\sqrt[3]{1 - K^4}} \sqrt[3]{\frac{16}{\pi\ S_s} (K_t\ T)} \quad \text{.........} \ (1\text{-}6)$$

If because of lack of accurate data, the shaft is designed by this simplified method, the torque transmitting the required horse power is multiplied by a factor K_1 called "the load factor". The load factor, K_1, is the ratio of the maximum torque to the average or normal torque and it depends on the type of the primemover and the driven machine. The values relevant to the system we are treating are mentioned below[13]:

Driver	Driven Machine	K_1
Electric Motor	Turbine blower, metal working machinery	1.25
	Centrifugal pump, wood working machinery	1.50
	Line shaft, ship propeller, double acting pump	1.75
	Triplex single-acting pump, elevator, crane	1.75
	Compressor, air or ammonia	1.75
	Rolling mill, rubber mill	2.50
Gas and Oil Engine	Values for electric-motor drive x 1.3 to 1.6, the factor depending on the coefficient of steadiness of flywheel.	

The value T can be expressed with Horse Power as:

$$T = \frac{396,000\ (H.P.)}{2\pi N} \qquad \text{............ (1-7)}$$

where H.P. = horse power

N = revolution per minute.

Substituting value of T from equation (1-7), and putting K_1 into equation (1-6), we get:

$$d_o = \frac{1}{\sqrt[3]{1 - K^4}} \times \sqrt[3]{\frac{3,168,000\ K_t\ K_1\ (H.P.)}{\pi^2\ S_s\ N}} \qquad \text{.... (1-8)}$$

1-1-4 Maximum Linear Deflection

For transmission shafting, it is considered good practice to limit the linear deflection to a maximum of 0.01 inches per foot of length. Taking the weight of bare shafting in pounds as $W = 2.6\ d^2\ L'$ and the vertical pull of belt as 40 lbs. per in. of width, as is usual practice, which gives a load of $W = 1.3\ d^2\ L'$ we get the following equations(11);

$$L' = \sqrt[3]{873\ d^2} \quad \text{for bare shafting} \quad \ldots\ldots \ (1\text{-}9)$$

$$L' = \sqrt[3]{175\ d^2} \quad \text{for shafting carrying pulleys etc.} \quad \ldots\ldots \ (1\text{-}10)$$

where L' = maximum distance between bearings, ft.

d = diameter of shafting, in.

The limitations in linear deflection of machinery shafting necessarily have to be more exacting. The limitations depend upon the service for which the shaft is intended. The deflection of any machine shaft supported on plain bearings is limited to a maximum of 0.0015 L, where L is the distance from the load point to the centre of bearing, in inches. The deflection of shafts carrying gears are limited to a maximum of 0.005/f inches at the gear, where f is the width of the gear face in inches. For very smooth-running gears, the deflection has to be much smaller than this value.

1-1-5 Maximum Angular Deflection

The angular deflection in transmission shafts is usually limited to a maximum of 1 degree in 20 diameters.

The angular deflection of machinery shaftings is usually limited to 6 min. per ft. for ordinary service; to 4-1/2 min. per ft.

for variable loads and 3 min. per ft. for suddenly reversed loads and long feed shafts. The angular movement of machine-tool spindles is limited to a maximum of 1/64 in. at the circumference of the face plate. Milling cutters are designed to have an angular twist of less than 1 deg. at the edge of the cutter.

The torsional deflection of circular shafting is given as:

$$\theta = \frac{583.61\ L\ T}{(d_o^4 - d_i^4)\ G} \qquad \ldots\ldots\ (1\text{-}11)$$

where L = length of shaft, in.

T = torque, lb-in.

d_o = outside diameter of shaft, in.

d_i = inside diameter of shaft, in.

G = modulus of rigidity of shaft material

θ = angular deflection, degree

1-2-0 APPLICATION

1-2-1 Shaft Design

From the above theory, we choose the following factors

S_s = 6000 psi.

K_t = 1.5

K_1 = 1.5 x 1.45 = 2.175 2.2.

K = 0.5

Now, because the design is to be done completely anew, the value of bending moment is not known. The size of shaft may be designed approximately by using equation (1-8). Engine data are given in Appendix I. Hence,

$$d_o = \sqrt[3]{\frac{3,168,000\ K_t\ K_1\ (H.P.)}{\pi^2 \cdot S_s\ N}} \times \frac{1}{\sqrt[3]{1 - K^4}}$$

$$= \sqrt[3]{\frac{3,168,000 \times 1.5 \times 2.2 \times 1120}{\pi^2 \times 6000 \times 300}} \times 0.02$$

$$= 8.75 \times 1.02 = 8.93 \quad \text{say } 9 \text{ in.}$$

This value of shaft diameter is only 0.25" less than the size of the crankshaft pins and journals. To simplify the design of flanges, adapter shaft between the engine and coupling, and the couplings, the size of the shaft may be stepped up by this amount to 9.25 inches, without causing much difference in the cost. Doing this we get

$$d_o = 9.25 \text{ in.}$$

$$d_i = 4.625 \text{ in.}$$

1-2-2 <u>Linear Deflection</u>

Since the shaft will be almost bare in the largest span, the allowable distance between the bearings may be estimated by using equation (1-9). Here we get:

$$L' = \sqrt[3]{873\ (d_o^2 - d_i^2)} = \sqrt[3]{873\ (9.25^2 - 4.625^2)} = 28.7 \text{ ft.}$$

Two bearings will be enough for a total distance of 20 ft. between the engine and the generator.

1-2-3 Angular Deflection in a Length of 20 d_o

Using equations (1-11) and (1-7), we get

$$\theta = \frac{583.61\ L\ T}{(d_o^4 - d_i^4)\ G}$$

$$= \frac{583.61 \times 20 \times 9.25 \times 396{,}000 \times 1120}{(9.25^4 - 4 \times 625^4) \times 12 \times 10^6 \times 2 \times \pi \times 300}$$

$$= 0.25 \text{ degrees.}$$

1-2-4 Couplings for the Transmission System

A drawing of the Ajax, rubber-bronze bushed flexible coupling is given in drawing No. (1) in Appendix V. The dimensions are the approximate values of the coupling manufactured by the Ajax Flexible Coupling Co. Inc. of Westfield, N.Y.

II HARMONIC COEFFICIENTS OF ENGINE TORQUE

II-1-0 THEORY

II-1-1 Total Output Torque

For designing the flywheel for an engine-generator system by the method shown in section III, and for calculation of the steady-state torsional vibration amplitudes of the designed engine-generator system, we need the values of resultant harmonic coefficients of the engine torque curve occurring at each crank of the engine. In this section we shall try to deal with determination of these resultant harmonic coefficients.

Torque output at one crank of an engine running at a uniform rotation can be represented as

Torque output = (Torque due to the gas pressure) + (Inertia torque correction for reciprocating weight) + (Dead weight correction for reciprocating part) + (Dead weight correction for unbalanced rotating part) + (connecting rod couple correction).

Using dimension of engine torque in lb. per sq.in. of piston area, so that the torque output per cylinder can be obtained by multiplying the determined harmonic coefficient by the piston area and crank throw, we can express above expression as follows[23]:

$$\frac{\text{Torque}}{AR} = \frac{\text{Torque}}{AR} \text{ due to gas pressure} + W_c F_a + \frac{W_{rec}}{A} F_b \operatorname{Cos} \psi + \frac{W_u R_u}{AR} \operatorname{Sin} (\alpha + \alpha_u + \psi) + 0.0000284034 \frac{Wd}{AR} (h_1 h_2 - k^2) N^2 F_c \quad \text{....... (II-1)}$$

where positive torque tends to move the crank in the direction of positive rotation.

A = piston area, sq.in.

R = crank radius, in.

g = acceleration due to gravity = 386.088 in. per sec.^{2}

W_{rec} = reciprocating weight, lb.

$$P = \frac{\pi^2}{900\ g} W_{rec}\ R\ N^2 = 0.0000284034\ W_{rec}\ R\ N^2$$

= centrifugal force of reciprocating weight, if it were at crank radius.

$$W_c = \frac{P}{A} = 0.0000284034\ \frac{W_{rec}}{A}\ R\ N^2 \qquad \ldots\ldots \text{(II-2)}$$

= centrifugal force per square in. of piston area of reciprocating weight if it were rotating at crank radius, psi.

N = rpm.

h_1 = distance of centre of mass of connecting rod from centre of crank pin, in.

h_2 = distance of centre of mass of connecting rod from centre of wrist pin, in.

k = radius of gyration of connecting rod about its centre of mass, in.

W_d = weight of connecting rod, lb.

α = crank angle from firing dead-centre position or from position of piston furthest from crankshaft, rad.

ψ = inclination of cylinder centre line with vertical in direction of rotation, rad.

W_u = weight of unbalanced rotating parts at crank, lb.

R_u = radius from shaft pin to centre of mass of W_u, in.

α_u = angle from crank arm to R_u in direction of rotation

K = crank-to-connecting-rod ratio

$$F_a = -\frac{1}{2}\frac{d}{d\alpha}\left(\text{Sin }\alpha + \frac{K \text{ Sin } 2\alpha}{2\sqrt{1 - K^2 \text{ Sin } \alpha}}\right)^2 \quad .... \text{(II-3)}$$

F_a can be expressed in a series form by expanding equation (II-3) and collecting like terms. Each term of expansion of F_a can be expressed in product form with coefficient in terms of K and variable parts as function of α. The coefficient parts are called "harmonic coefficients" of F_a. These harmonic coefficients of F_a are denoted by H_a.

$$F_b = \text{Sin }\alpha + \frac{K \text{ Sin } 2\alpha}{2\sqrt{1 - K^2 \text{ Sin}^2 \alpha}} \quad ...... \text{(II-4)}$$

H_b = harmonic coefficients of F_b

$$F_c = -\frac{K^2 (1 - K^2)}{2} \frac{\text{Sin } 2\alpha}{(1 - K^2 \text{ Sin}^2 \alpha)^2} \quad ...... \text{(II-5)}$$

H_c = harmonic coefficients of F_c.

Values of H_a, H_b and H_c for different $^1/K$ values are given in references (1-a) and (23). If, however, their values be desired and above references are not available, they can be calculated by using expressions given in tables (1) to (3)*.

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* Numbers refer to the tables given in Appendix II.

II-1-2 Gas Pressure Torque

In equation (II-1) the terms on the right-hand side other than the gas pressure torque can be calculated by using expressions described above.

If an indicator card of the engine under consideration is obtained, the harmonic coefficients due to gas pressure can be determined by using analytical methods or by sending the curve through a harmonic analyzer. F.P. Porter, in his paper published in 1943[23], gave harmonic coefficients of engine gas torque curve for eight widely different types of engine. For most practical purposes the data given for one of eight prototype engines of his paper are always sufficiently close to those of the engine being considered.

We shall use the values given in above paper for our purpose also.

II-1-3 Note on Summation of Harmonic Components

If "f" is a periodic curve of period 2π radians it can be represented in a Fourier series, like that in equation (II-1), as follows:

$$f = T_o + T_1 \text{ Sin } (\alpha + \phi_1) + T_2 \text{ Sin } (2\alpha + \phi_2) + T_3 \text{ Sin } (3\alpha + \phi_3) + \cdots \cdots \cdots + T_n \text{ Sin } (n\alpha + \phi_n) \qquad \ldots\ldots (II\text{-}6)$$

In this expansion T's are the amplitudes of the Sine waves and called "resultant harmonic coefficients"; ϕ's are the phase angles.

If engine torque value is expressed in above form, since the period is 2π, α may be taken as the angle of rotation of crank for two-stroke-cycle engine and 2α the angle of rotation for four-stroke-cycle engine.

Now, since we have

$$\text{Sin}\,(n\alpha + \phi_n) = \text{Sin}\,n\alpha\ \text{Cos}\,\phi_n + \text{Cos}\,n\alpha\ \text{Sin}\,\phi_n$$

we may write equation (II-6) in the following form

$$f = a_1\,\text{Sin}\,\alpha + a_2\,\text{Sin}\,2\alpha + a_3\,\text{Sin}\,3\alpha + \ \ldots\ + b_o +$$

$$+ b_1\,\text{Cos}\,\alpha + b_2\,\text{Cos}\,2\alpha + b_3\,\text{Cos}\,3\alpha + \qquad \ldots \quad \text{(II-7)}$$

where a's are called coefficients of Sine harmonics.

b's (excepting b_o) are called coefficients of Cosine harmonics

b_o gives the average height of the torque curve.

The relations between T_n, a_n, b_n are given by

$$T_n = \sqrt{a_n^2 + b_n^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(II-8)}$$

If two or more of harmonic series are in phase and of the same period as those on the right-hand-side of equation (II-1) and they are required to be summed up, the resultant harmonic coefficient may be obtained by adding algebraically all the a's and b's of like harmonic terms and combining the resulting coefficeints by means of equation (II-8).

II-2-0 APPLICATION

II-2-1 Procedure

To get the resultant harmonic coefficients of $\frac{\text{Torque}}{\text{AR}}$ value per cylinder of engine output curve, we shall use equation (II-1). The values on the right hand side of equation (II-1) can be found by using various equations included in the previous theory portion. Getting these values, Torque/AR may, if desired, be expressed in a series form like that shown in equation (II-6). However, we are interested only in the resultant harmonic coefficients, which may be useful in the sections to follow. Therefore, since we can save considerable work by concentrating on T_n values, we shall try to get what we need only.

In the following calculations, the coefficients of sine and cosine harmonics for different harmonic orders of the terms on the right hand side of equation (II-1) are found in tabular forms. The resultant harmonic coefficients are then calculated by summing these values by equation (II-8).

II-2-2 Correction for Inertia Torque of Reciprocating Weight

Using equation (II-2), we get,

$$W_c = 0.0000284 \times \frac{W_{rec}}{A} R N^2$$

Data for the right hand side are given in Appendix I, so that

$$W_c = 0.0000284 \times \frac{905}{141} \times 11.2 \times 300^2 = 143.5 \text{ psi.}$$

The calculated values of harmonic coefficients are given in table (4). In this table H_a values are obtained for $^1/K = 4.74$ from reference (23). The values show that the inertia torque has no cosine harmonics.

II-2-3 Dead Weight Correction for Reciprocating Part

This correction is given by the 3rd term on the right hand side of equation (3-1).

Since our engine is of the vertical-inline type we have $\psi = 0$. Hence,

Dead weight correction for reciprocating part = $\frac{W_{rec}}{A} F_b$

From which we see that:

Harmonic coefficients of dead weight correction for reciprocating part ...... $= \frac{W_{rec}}{A} H_b$

Therefore, with values given in Appendix I,

Harmonic coefficients of dead weight correction for reciprocating part ...... $= \frac{705}{141} H_b = 5 H_b$

Calculated values of coefficients are given in table (5) in which H_b values are obtained for $^1/K = 4.74$ from reference (23). These values show that, the correction torque has no cosine term.

II-2-4 Dead Weight Correction for Unbalanced Rotating Part

To get this correction, the unbalanced rotating weight has to be determined. A crank element is illustrated in figure (II-1). The unbalanced weight is first divided into various portions of regular shapes, which in the figure (II-1) are marked, I, II, III, IV. We shall try to calculate these unbalanced weights step by step as follows:

(i) Weight of pin

$$W_p = \frac{\pi}{4}\left[9.25^2 \times \frac{190}{25.4} - \frac{24^2}{(25.4)^2} \cdot \frac{110}{25.4}\right] \times 0.283 = 141.5 \text{ lb.}$$

(ii) Weight of portion I.

Now, in figure (II-1) we see that

$$o\,a = 410 - (130-42) = 410 - 88 = 322 \text{ mm.}$$

$$a\,b = \sqrt{322^2 - 117.15^2} = 300 \text{ mm.}$$

$$c\,a = 300 - 162 = 138 \text{ mm.}$$

Weight of I without oil hole $= \dfrac{138 \times 234.3 \times 130}{(25.4)^3} \times 0.283 = 72.61$ lb.

Weight of I with oil hole $= \left[256.5 - \dfrac{\pi}{4}\left(\dfrac{24}{254}\right)^2 \times \dfrac{130}{25.4}\right] \times 0.283$

$$= (256.5 - 3.589) \times 0.283 = 71.57 \text{ lb.}$$

Total weight $W_I = 72.61 + 71.57 = 144.18$ lb.

Centre of gravity of W_I from axis of shaft $= 162 + \dfrac{138}{2} = 231$ mm. $= 9.1"$.

(iii) Weight of Portion II

In figure (II-1), we have

$$o\,d = 410 \text{ mm}$$

$$b\,d = \sqrt{410^2 - 117.15^2} = 393 \text{ mm.}$$

$$a\,d = 393 - 300 = 93 \text{ mm.}$$

Weight of portion II $= \dfrac{93 \times 42 \times 234.3 \times 0.283}{(25.4)^3} = 15.81$ lb.

Hence for both sides, $W_{II} = 31.62$ lb.

distance of centre of gravity of W_{II} from shaft axis

$$= 300 + \frac{93}{2} = 346.5 \text{ mm} = 13.46 \text{ in.}$$

(iv) Weight of Portion III

Tables to calculate area of segments of circle are given in the hand books. The values of M in the following calculations are obtained from such tables[15].

We have:

Height of segment $H = 410 - 393 = 17$ mm.

Ratio of height to circle dia., $\frac{H}{D} = \frac{17}{820} = 0.0208$

Hence the area coefficient M is obtained from table as:

$M = 0.00398$

So that

$$\text{Area} = D^2 M = \frac{820 \times 820}{25.4 \times 25.4} \times 0.00398 = 4.148 \text{ in}^2.$$

$$\text{Weight of portion III} = 4.148 \times \frac{42}{25.4} \times 0.283 = 1.941 \text{ lb.}$$

$$\text{Weight for both sides} = W_{III} = 3.882 \text{ lb.}$$

Distance of the centre of gravity from shaft axis $= \frac{C^3}{12\ a}$

Where C is the length of the base of the sector

a is the area of sector

$$= \frac{(234.3)^3}{(25.4)^3 \times 12 \times 4.15} = 15.77 \text{ in.}$$

(v) Weight of portion IV

To get the weight of this portion, the following approximate method is used.

The portion IV is represented in (C), (D) and (E) of figure (II-1) as $a_1\ d_1\ e\ f\ a_2$. It is assumed that the curved face

d_1 e d_2 is replaced by a rectangular section d_1 g_1 g_2 d_2 of the same area as illustrated in (D) of the figure and sector a_1 f a_2 is replaced by a_1 h_1 h_2 a_2 in the same manner. We have from above:

$$\text{Area of } d_1 \text{ e } d_2 = 4.148 \text{ in}^2 \text{ sq.in.}$$

$$\text{So that } d_1 g_1 = \frac{4.148 \times 25.4}{234.3} = 0.45 \text{ in.}$$

Now, height of segment a_1 f a_2

$$H_1 = \text{o a} - \text{a b} = 322 - 300 = 22 \text{ mm.}$$

Hence

$$\frac{H_1}{D_1} = \frac{22}{644} = 0.03416$$

$$M = 0.008394$$

So that

$$a_2 h_1 = \frac{MD^2}{a_1 a_2} = \frac{0.008394 \times (644)^2 \times 25.4}{(25.4)^2 \times 234.3} = 0.585 \text{ in.}$$

The resulting figure of portion IV is illustrated in (F) of figure (II-1).

From above results, we get

$$a_1 g_1 = a_1 d_1 + d_1 g_1 = 4.11 \text{ in.}$$

$$h_3 g_1 = 4.11 - 0585 = 3.525 \text{ in.}$$

We denote h_3 g_1 h_1 as portion IV' and a_1 h_3 h_1 a_2 as portion IV".

Weight of IV' $= \frac{3.525}{2} \times \frac{88}{25.4} \times \frac{234.3}{25.4} \times 0.283 = 15.94$ lb.

and therefore, weight of both sides $W_{IV'} = 31.88$ lb.

Distance of C.G. from shaft axis $= a\ b + a_2\ h_1 + \frac{1}{3} \cdot h_3\ g_1 = 1.76"$

Again we get, weight of IV"

$$= \frac{88}{25.4} \times 0.585 \times \frac{234.3}{25.4} \times 0.283 = 5.29 \text{ lb.}$$

Wt. for both sides $W_{IV"} = 10.58$ lb.

Distance of C.G. from the shaft axis $= a\ b + \frac{0.585}{2} = 12.104"$

Getting the weights and distances of centres of gravity from the shaft axis we can get the total value of $W_u R_u$ for use in the expression for unbalanced rotating weight. Calculated results are given in table (6).

Now, from equation (II-1),

Unbalanced rotating weight correction $= \frac{W_u\ R_u}{AR}$ Sin $(\alpha + \alpha_u + \psi)$

For the type of engine being considered, $\psi = 0$.

And since the unbalanced weight is on the crank-pin side we get

$\alpha_u = 0$. So, we get:

Unbalanced rotating weight correction $= \frac{W_u\ R_u}{AR}$ Sin α

$$= \frac{6962.82}{141 \times 11.2} \text{ Sin } \alpha = 4.409 \text{ Sin } \alpha$$

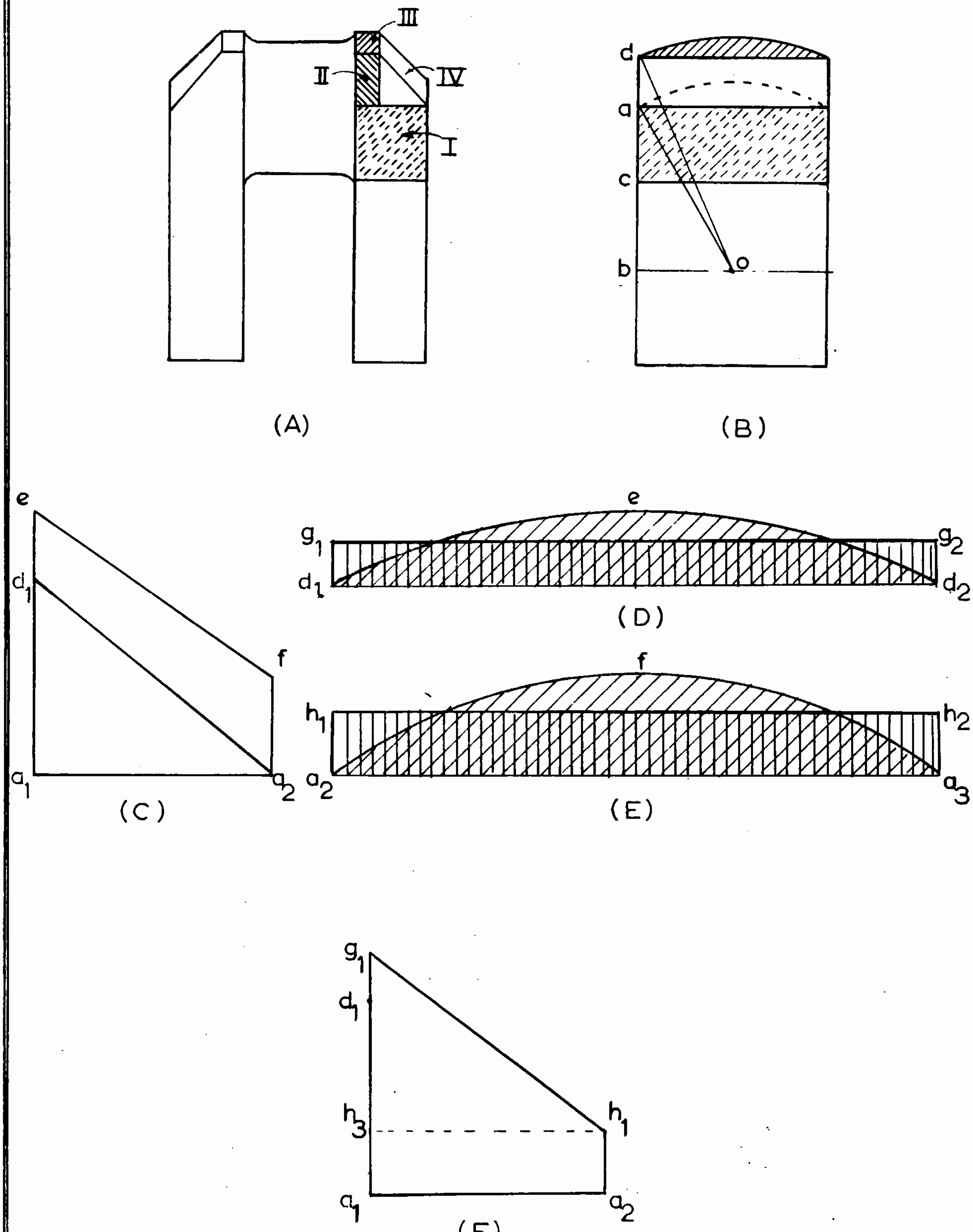

III
II
IV
I
(A)
d
a
c
b
o
(B)
e
d1
f
a1
a2
(C)
e
g1
g2
d1
d2
(D)
f
h1
h2
a2
a3
(E)
g1
d1
h3
h1
a1
a2
(F)

Figure (II - 1)

II-2-5 <u>Connecting Rod Couple Correction</u>

This is given by the last term in equation (II-1). For connecting rods of common engines $h_1\ h_2$ value does not differ much from k^2 that the value of $(h_1\ h_2 - k^2)$ is usually very small. The values of coefficients H_c at the same time is usually very small for the crank-to-connecting-rod ratio of commonly used engine. The effect of connecting rod couple is, therefore, neglected without appreciable error.

II-2-6 <u>Gas Pressure Torque</u>

The harmonic coefficients of gas pressure torque for the type closely equivalent to the engine being considered are obtained from reference (23). The engine is of 160 BHP per cylinder with approximate compression ratio of 1:14. Hence, with assumption of a mechanical efficiency of about 85% the group with mean harmonic coefficient of 25.298 for J - type engine is chosen.

II-2-7 <u>Corrected Harmonic Coefficients</u>

The values of correction coefficient values calculated in above sections show that, only first six harmonic coefficients need be corrected. Further, since all the correction values compose of sine coefficients only, we need to add all sine coefficients algebraically and the resulting sine coefficients compounded to the corresponding cosine coefficients of gas torque curve by means of equation (II-8) to get the resultant harmonic coefficients.

Algebraic summation of sine coefficients of the first sine orders are given in table (7). Table (8) gives the corrected resultant harmonic coefficients.

III DETERMINATION OF FLYWHEEL EFFECT

III-1-0 THEORY

III-1-1 For successful operation of alternators, flywheel effect necessary for the system is calculated with due regard to the standards laid down for this purpose.

The design requirements for successful operation of alternators become more exacting when a number of generating sets are run in parallel. Usually, the flywheel of engine-generator system for parallel operation is designed to fulfil three aspects as follows:

i) The coefficient of cyclic irregularity of the system is kept below certain limit so that the light flickering characteristics of the system confirm to the recoganized standard.

ii) If the alternator pole deviates to a great extent from its position under perfectly uniform rotation, the losses caused by the synchronizing current will become excessive so that the machine falls out of step. To avoid this kind of failure the angular deviation is kept low.

(iii) The system is so designed that the natural frequency of oscillation of electro-mechanical system does not coincide with any harmonic of engine impulse frequency.

III-1-2 Flywheel Effect for Required Cyclic Irregularity

The cyclic irregularity is defined as the ratio of the maximum variation in angular velocity, at the point under consideration, during one engine cycle to the mean velocity, when engine is running at any load up to and including rated load and speed. This can be expressed as

$$C_c = \frac{\text{Maximum Speed - Minimum Speed}}{\text{Mean Speed}} \qquad \text{...... (III-1)}$$

For a direct-coupled engine and generator, the cyclic irregularity refers to the angular velocity at the generator rotor. For systems with engines of more than two cylinders, the British Standards Institution[24] has laid down the following standards:

Engine Impulses per sec.		Cyclic irregularity not worse than
Less than 20	.....	1/50
10 to 20	.....	(Engine impulses per sec.)/1500
Above 20	.....	1/75

Number of engine impulses per second for a 2-S.C., S.A. engine is given by:

$$p = \frac{m \ N}{60} \qquad \text{.................. (III-2)}$$

where p = number of engine impulses per sec.

m = number of cylinders

N = revolution, rpm.

The flywheel effect, making due allowance for dynamic magnification due to electrical resonance should be sufficient to limit the C_c to the value determined for the engine type.

Denoting WK^2 = flywheel effect to limit the C_c to the desired value, with allowance for dynamic magnification due to electrical resonance, ton-ft.^{2}

WK_a^2 = flywheel effect to give a natural frequency equal to the lowest harmonic of engine impulses, ton-ft.^{2}

WK_b^2 = flywheel effect to limit the C_c to the desired value, neglecting dynamic magnifier due to electrical resonance, ton-ft.^{2}

It can be shown that

$$WK^2 = WK_a^2 + WK_b^2 \quad \text{.........} \quad \text{(III-3)}.$$

Ker Wilson[2] gives WK_a^2 for average alternators coupled to 2-S.C., S.A. engine as

$$WK_a^2 = \frac{1375000\ f}{N^4} \quad \text{ton-ft.}^2\text{/KVA} \quad \text{...} \quad \text{(III-4)}$$

$$= \frac{1375000 \times f \times E_a}{N^4 \times (PF) \times 1.34} \quad \text{ton-ft.}^2\text{/BHP} \quad \text{..} \quad \text{(III-5)}$$

where E_a = efficiency of alternator

PF = power factor

f = electrical frequency, cycles per second

N = revolution, rpm.

The approximate value of WK_b^2 is given as:

$$W\,K_b^2 = \frac{U\ D^2\ L}{C_c\ N^2} \quad \text{........} \quad \text{(III-6)}$$

where U = a factor, which is constant for a particular type of engine.

$$= \frac{T_n \times m}{11.65\,n} \quad \text{........} \quad \text{(III-7)}$$

T_n = maximum value of n^{th} order harmonic component of engine torque per cylinder, pound per square inch of piston area.

m = number of cylinders

n = engine impulses per revolution

D = cylinder diameter, in.

L = stroke, in.

III-1-3 <u>Flywheel Effect Necessary to Limit Angular Deviation</u>

The maximum angular deviation of ± 2-1/2 electrical degree has been adopted as a standard by British Standard Institution[24].

The relationship between the cyclic irregularity and angular deviation is given by:

$$C_c = 2 \cdot \theta \cdot n \qquad \text{(III-8)}$$

where θ = maximum angular deviation, rad. (mechanical)

n = number of oscillations in one revolution (Assuming that only the principal unbalanced harmonic order is important, the value of n is the number of working cylinders in the case of 2 S.C., S.A. engine).

If θ is expressed in electrical degree, equation (III-9) becomes:

$$C_c = \frac{\theta \cdot n}{28.6\, P} \qquad \text{(III-9)}$$

θ = maximum angular deviation, electrical deg.

P = number of pole pairs = $\frac{\text{No. of poles}}{2}$

$$= \frac{60\, f}{N} \qquad \text{(III-10)}$$

If the British Standard is taken, θ value in (III-10) must not be greater than $\pm$ 2-1/2 electrical deg. Putting this value, equation (III-10) becomes:

$$C_c = \frac{n}{12\ P} \quad \text{very nearly} \qquad \text{........ (III-11)}$$

From (III-11) and (III-12), we get,

$$C_c = \frac{N\ n}{720\ f} \qquad \text{........ (III-12)}$$

or $$= \frac{N\ m}{720\ f} \quad \text{since for 2-S.C., S.A. engine } n = m \qquad \text{..... (III-13)}$$

III-1-4 <u>Flywheel Effect Necessary to Avoid Electrical Resonance</u>

When the alternators are run in parallel, if a machine takes a momentary lead it is subjected to a retarding force due to increasing load thereby imposed upon it; at the same time an accelerating force acts on the lagging machine. While this effect is to restore synchronism, it cannot prevent them from passing beyond the synchronized position, and hence hunting results. At certain speeds, depending on the characteristics of the mains and prime movers, resonance occurs causing the amplitude of oscillation to build up greater and greater until finally one or other of the machines falls out of step.

It is possible for electrical resonance to occur when the frequency of any one of the harmonic impulses of engine torque coincides with the natural frequency of any one mode of oscillation of alternator. For satisfactory parallel operation, it is, therefore, necessary that the electro-mechanical system be tuned so that the natural frequency of the generator does not coincide with any harmonic of engine impulse frequency.

The natural frequency of an alternator on a relatively large network, or of two similar alternators running in parallel, is given by[2]:

$$F_a = \frac{742}{N} \sqrt{\frac{f \cdot KVA_k}{WK^2}} \qquad \text{.......... (III-14)}$$

where F_a = natural frequency of alternator, oscillations per min.

KVA_k = short circuit capacity of alternator

N = rpm.

f = electrical frequency, cyl./sec.

It is customary that the resonance is avoided by tuning the electro-mechanical system so that its natural frequency, given by equation (III-14) is lower than the lowest harmonic impulse of engine torque curve, with sufficient margin between the two.

For a 2 S.C., S.A., single-cylinder engine the lowest harmonic impulse frequency is the same as the rpm of the engine. The same value applies to multi-cylinder engines, since the lowest harmonic impulse frequency is that due to one cylinder, and it is also the lowest impulse frequency in cases when there is uneven firing in the cylinders of a multi-cylinder engine.

Equation (III-14) reveals that the tuning of the system can be done either by choosing suitable flywheel effect or by changing the short-circuit capacity of alternator. The former method is usually preferred.

The short-circuit capacity is smallest at no-load and greatest at full-load excitation. Thus for a constant flywheel effect, the value of F_a is smallest at no load and greatest at full-load. The short-circuit capacity at full-load should, therefore, be used to have assurance that satisfactory operation is obtained over the whole range of operation.

If a 30% margin is allowed between the natural freouency of alternator and the lowest impulse frequency, we get

$$F_i = 1.3\ F_a$$

where F_i = lowest impulse frequency, oscillation/min.

= N for 2-S.C., S.A. engine.

Hence,

$$N = 1.3\ F_a \quad \text{.......... (III-15)}$$

Putting the value of F_a from (III-15) into equation (III-16):

$$W\ K^2 = \frac{2,330,000\ f}{N^4}\ \text{ton-ft.}^2/\text{KVA} \quad \text{....... (III-16)}$$

$$= \frac{2,330,000 \times f \times E_a}{N^4\ (PF)\ (1.34)}\ \text{ton-ft.}^2/\text{BHP} \quad \text{... (III-17)}$$

III-1-5 Procedure to Use Design Theory

The following gives the appropriate steps to follow in the course of applying the design theory.

Step (1) Using equations (III-4) and (III-5) the value of $W\ K_a^2$ can be determined. Using equation (III-2) the value of p for the engine can be calculated. With this value of p, enter the table given on page (21) to get the appropriate value of Cc. With this value of C_c and T_n value given in table (8), $W K_b^2$ can be obtained by using equation (III-6). Summing up the values of $W K_a^2$ and $W K_b^2$ by equation (III-3) the value of $W K^2$ necessary to limit the C_c, determined above, can be obtained.

Step (2) The maximum value of C_c allowable for the angular deviation of $\pm$ 2-1/2 elect. degree can be determined by using equation (III-12) or (III-13). If the value of C_c determined here is higher than the value which was used in step (1), then $W K^2$ determined in step (1) will fulfil both the requirements described in sections (III-1-1) and (III-1-2). If the value of C_c of this step is lower than that used in step (1), then the flywheel effect obtained in step (1) will fulfil the requirement of section (III-1-1) but not that of section (III-1-2). To get appropriate value of $W K^2$ to fulfil both requirements the lower value of C_c obtained in here should be used in equation (III-6), and $W K^2$ recalculated by repeating step (1).

Step (3) The flywheel effect necessary to avoid electric resonance can be determined by using equations (III-16) and (III-17).

Step (4) We have obtained two values of $W K^2$ through above steps. To get the flywheel effect sufficient to cope with all the three requirements, the higher of the two $W K^2$ values must be used in our flywheel design.

III-2-0 APPLICATION

The following shows the calculation steps when above theory is applied to our engine-generator system. Data of engine and generator are given in Appendix I.

III-2-1 <u>Determination of Flywheel Effect to Avoid Flicker</u>

Using Equation (III-5)

$$W K_a^2 = \frac{1375000 \times f \times E_a \times BHP}{N^4 . (PF) 1.34}$$

$$= \frac{1375000 \times 50 \times 0.95 \times 1120}{(300)^4 \times 0.8 \times 1.034} = 8.41 \text{ ton-ft.}^2$$

The value of T_m = 11.365 from table (8). Hence, from equation (III-8)

$$U = \frac{11.365 \times 7}{11.65 \times 7} = 0.976$$

Using equation (III-2),

$$p = \frac{m N}{60} = \frac{7 \times 300}{60} = 35 > 20.$$

So that appropriate value of C_c = 1/75. Thus, from equation (III-6), with engine data from Appendix I.

$$W K_b^2 = \frac{U D^2 L}{C_c N^2} = \frac{0.976 \times (13.4)^2 \times 22.4 \times 75}{300^2} = 3.27 \text{ ton-ft}^2.$$

Hence, from equation (III-3) we have

$$W K^2 = 8.41 + 3.27 = 11.68 \text{ ton-ft}^2.$$

III-2-2 <u>Coefficient of Cyclic Irregularity to Limit Angular Derivation</u>

Using equation (III-13),

$$C_c = \frac{N m}{720 f} = \frac{300 \times 7}{720 \times 50} = \frac{1}{17.14} > \frac{1}{75}$$

So, the flywheel effect determined in III-2-1 above is sufficient to fulfil both requirements.

III-2-3 Flywheel Effect to Avoid Electrical Resonance

Using equation (III-17)

$$W K^2 = \frac{2{,}330{,}000 \times f \times E_a}{N^4 \, (P.F.)(1.34)}$$

$$= \frac{2{,}330{,}000 \times 50 \times 0.95}{(300)^4 \times 0.2 \times 1.34} = 14.3 \text{ ton-ft.}^3$$

III-2-4 Extra Flywheel for the System

Comparing the values of $W K^2$ obtained in above calculations we see that the maximum flywheel effect necessary for parallel operation with similar generating sets is 14.3 ton-ft.^{2}

Now, the flywheel effect of the generator rotor alone is:

$$= \frac{37{,}000}{2{,}240} = 16.5 \text{ ton-ft.}^2$$

This shows that no separate flywheel is necessary in the system, unless otherwise required for correcting some unsatisfactory vibration behaviour of the system.

IV EQUIVALENT LENGTHS

IV-1-0 THEORY

IV-1-1 To make the calculations of torsional vibration possible, the complicated engine system has to be reduced to a system with flywheels situated on a shaft of uniform diameter.

In this section we shall deal with the procedure for finding the equivalent lengths and torsional stiffness of the shaft sections of the system.

The shaft of uniform diameter to which the actual shafts are reduced is called the "equivalent shaft". Its diameter may be chosen arbitrarily; but its length between each two masses must be such that it is torsionally equivalent to the actual shaft between corresponding masses of the actual engine system. A torsionally equivalent shaft is one which twists through exactly the same angle as the actual shaft when equal and opposite torques of given amount are applied to the two ends.

IV-1-2 Torsional Stiffness

Torsional stiffness of a circular shaft is given by:

$$C = \frac{\pi d^4 G}{32 L} \quad \text{for solid shaft} \quad \ldots\ldots\ldots \text{(IV-1)}$$

$$C = \frac{\pi (d_o^4 - d_i^4) G}{32 L} \quad \text{for hallow shaft} \quad \ldots\ldots \text{(IV-2)}$$

where C = torsional stiffness, lb.-in/rad.

d = diameter of solid shaft, in.

d_i = inside diameter of hallow shaft, in.

d_o = outside diameter of hallow shaft, in.

G = modulus of rigidity of shaft material, psi.

L = length of shaft section, in.

IV-1-3 Overall Stiffness of Shafts in Series

The overall stiffness is given as:

$$\frac{1}{\sum C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \quad \ldots\ldots \frac{1}{C_n} \qquad \ldots \text{(IV-3)}$$

where C_1, C_2, C_3 .... C_n are stiffnesses of individual shafts

$\sum C$ = overall stiffness of the shaft built up with "n" shafts of stiffnesses C_1, C_2 .... C_n in series.

IV-1-4 Equivalent Lengths of Shaft Sections

(a) Uniform circular shaft.

Equivalent length is given as:

$$L_e = \frac{L \cdot d_e^4 \cdot G_e}{d^4 \cdot G}, \text{ when actual shaft is solid} \quad \ldots \text{(IV-4)}$$

$$L_e = \frac{L \cdot d_e^4 \cdot G_e}{(d_o^4 - d_i^4)\, G}, \text{ when actual shaft is hollow} \quad .. \text{(IV-5)}$$

where L_e = length of equivalent solid shaft, in.

L = length of actual shaft, in.

d_e = diameter of equivalent shaft, in.

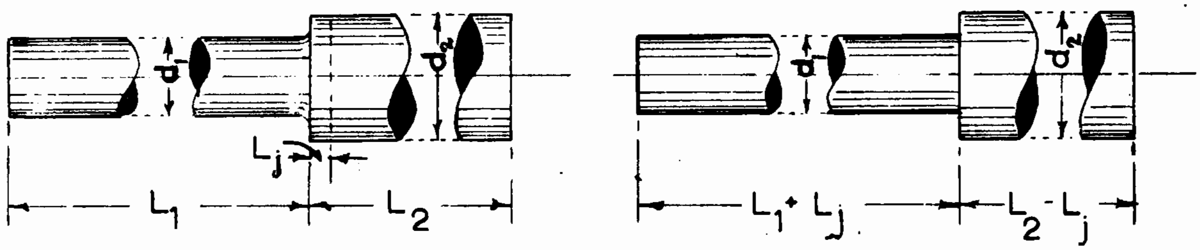
d, d_i, d_o, G are as described above.

G_e = modulus of rigidity of material of equivalent shaft.

(b) Stepped Shaft

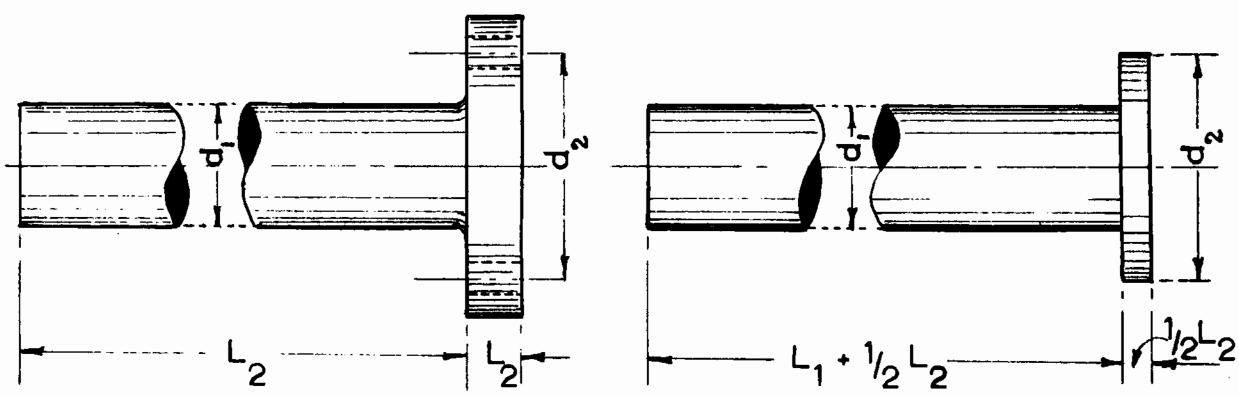
When a shaft of smaller diameter is joined to a shaft of larger diameter, the effective length of smaller shaft is greater than its actual length due to local deformation at the juncture.

Actual shaft sections	Representative sections for calculation of equivalent length

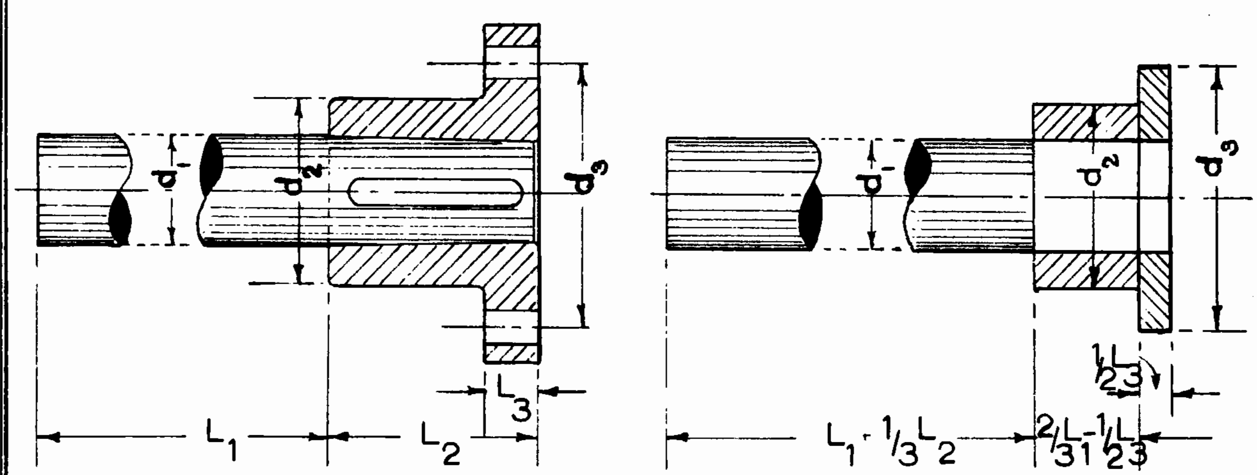


(a)

d_2/d_1	1·00	1·25	1·50	2·00	3·00	Infy
L_j/d_1	0	0·055	0·085	0·100	0·107	0·125



(b)



(c)

Figure (IV-1)

To make allowance for this effect a small length L_j; called "penetration length at the juncture", is added to the length of smaller shaft and the same amount is subtracted from the length of larger portion. Doing this the shaft may be represented by a shaft of the dimensions shown on the right hand side sketch in figure (IV-1-a), for calculation of equivalent length. From this we get

$$L_e = \left[\frac{(L_1 + L_j)}{d_1^4} + \frac{(L_2 - L_j)}{d_2^4} \right] \frac{d_e^4 G_e}{G} \quad \text{...... (IV-6)}$$

The recommended values of L_j are given in the table of the figure.

(c) Forged Coupling

The thickness of the flange of couplings currently used is about one-quarter the shaft diameter. For such type of coupling, if the penetration length is taken as 0.125 d_1, the shaft for calculation of equivalent length may be like that shown on the right hand side sketch in figure (IV-1-b). From this we get:

$$L_e = \left[\frac{L_1 + 1/2\, L_2}{d_1^4} + \frac{1/2\, L_2}{d_2^4} \right] \frac{d_e^4 G_e}{G} \quad \text{....... (IV-7)}$$

(d) Keyed Coupling

It is recommended to take the shaft to be unstrained by the coupling hub for a distance of one-third the length of the coupling, and to assume for the remaining length that the torque is carried by the boss and flange only. Taking the bolt circle diameter as the effective diameter of the flange we may represent the coupling by the shaft sections as that shown in sketch on the right hand side in figure (IV-1-3). We get the following formula

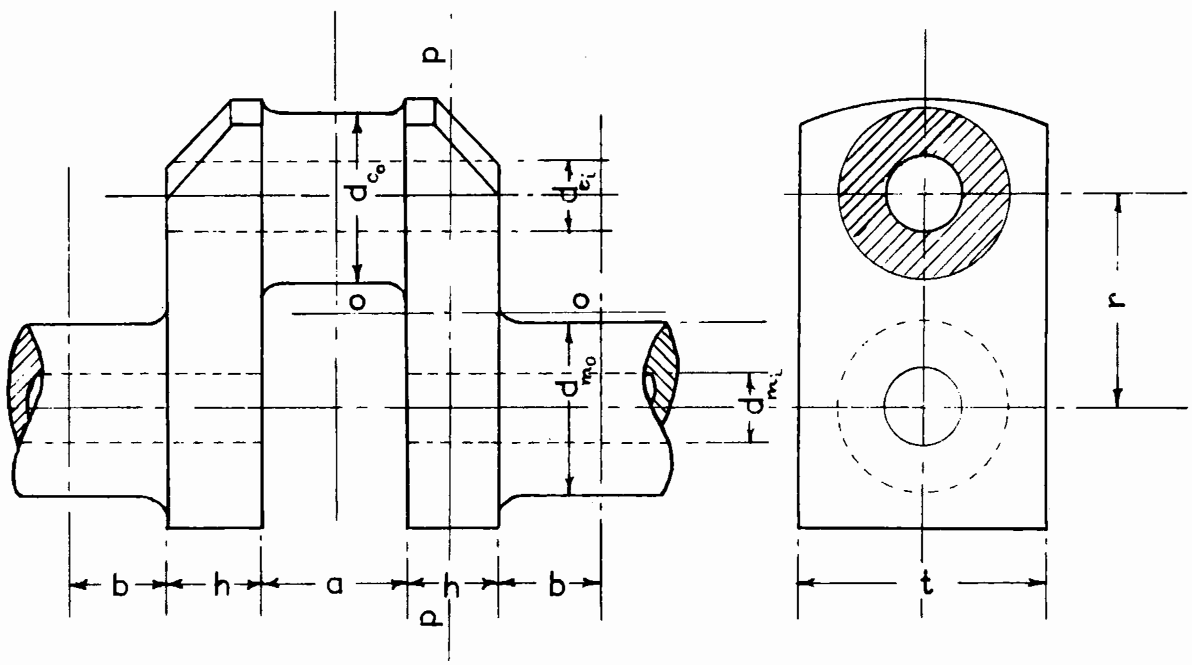


Figure (IV-2)

for equivalent length.

$$L_e = \left[\frac{(L_1 + 1/3\ L_2)}{d_1^4\ G_s} + \frac{(2/3\ L_2 - 1/2\ L_3)}{(d_2^4 - d_1^4)\ G_h} + \frac{1/2\ L_3}{(d_3^4 - d_1^4)\ G_h} \right] d_e^4\ G_e \qquad \text{......... (IV-8)}$$

where G_s = modulus of rigidity of shaft material

G_h = mofulus of rigidity of coupling material

(e) Tapered Shaft

Equation for equivalent length of a solid tapered shaft is given as

$$L_e = \frac{L \cdot d_e^4}{3\ (d_2 - d_1)} \left[\frac{1}{d_1^3} - \frac{1}{d_2^3} \right] \qquad \text{................ (IV-9)}$$

where d_1 = diameter at small end, in.

d_2 = diameter at large end, in.

L = axial length, in.

IV-1-5 <u>Equivalent Length of Crankshaft</u>

An element of crankshaft is illustrated in figure (IV-2). With assumption that

(a) deflection of the element is mainly due to the twist of the journals, twist of the crankpin, and bending of the webs,

(b) no local deformation exists at the junctions of the webs and pin or journals,

(c) the lever arm of the couple acting on crankwebs is equal to the crank throw,

(d) the bearing clearance is sufficient and the displacements of the journals are possible (no bearing contraint), the equation of equivalent length of the crank element is given by

$$L_e = c_e \left[\frac{b}{c_1} + \frac{a}{c_2} + \frac{2r}{B} \right] \qquad \text{.......... (IV-10)}$$

where c_e = torsional rigidity of equivalent shaft

$= \frac{\pi}{32} d_e^4 G_e$ for circular shaft ... (IV-11)

c_1 = torsional rigidity of the journal

$= G I_{p_1} = \frac{\pi}{32} (d_{mo}^4 - d_{mi}^4) G$ .... (VI-12)

c_2 = torsional rigidity of the crankpin

$= G I_{p_2} = \frac{\pi}{32} (d_{co}^4 - d_{ci}^4) G$ ..... (IV-13)

B = flexural rigidity of the web against bending in the plane p-p perpendicular to the plane of the drawing of figure (IV-2)

$= \frac{1}{12} h t^3 E$

I_p = polar moment of inertia of the section, in.^{4}

E = modulus of elasticity, psi.

In practice the above assumptions do not strictly hold.

(a) There are local deformations at the junctures.

(b) The effective lever arm of the couple is not exactly equal to the crank throw, due partly to attachment of the pins and partly to the bearing constraint.

(c) The equivalent length may be different from that obtained by using equation (IV-10) due to stiffening effect of the bearing constraint. Based on some mathematical assumptions and working on element of dimensions a = 2b for three cases, viz. no constraint, partial constraint, and complete constraint, Timoshenko(22-a) analytically showed that reductions in equivalent length of crank element under complete and partial constraints compared to that of no constraint case are about 17% and 13%.

(d) The degree of stiffening effect on the equivalent length per crank element depends not only on the degree of bearing constraint as mentioned in (c) above but also on the relative location on the crankshaft of the element under consideration. Analyzing a three-throw crank shaft of dimensions a = 2b for partial constraint case, Timoshenko(22-b) also showed that reduction from no-constraint equivalent length of the first and third elements is about 3% only where as the reduction for the second crank is about 7.8%.

No exact mathematical treatment, however, is possible. The following empirical formulae of modified forms of equation (IV-10), are used currently to estimate crankshaft equivalent length. Wherever possible it is recommended that there should be a comparison of calculation and experiment of a number of previous crankshafts of similar characteristics. The empirical formulae are(2):

B.C. Carter's Formula

$$L_e = d_e^4 \left[\left(\frac{b + 0.8h}{d_{mo}^4 - d_{mi}^4} \right) + \left(\frac{0.75\,a}{d_{co}^4 - d_{ci}^4} \right) + \left(\frac{1.5\,r}{h\,t^3} \right) \right] \qquad \ldots \text{(IV-11)}$$

W. Ker Wilson's formula

$$L_e = d_e^4 \left[\left(\frac{b + 0.4\,d_{mo}}{d_{mo}^4 - d_{mi}^4} \right) + \left(\frac{a + 0.4\,d_{co}}{d_{co}^4 - d_{ci}^4} \right) + \left(\frac{r - 0.2\,(d_{mo} + d_{co})}{h\,t^3} \right) \right]$$

.... (IV-12)

The above formulae are given for hollow pin and journal. If they are solid, internal diameters d_{ci} and d_{mi} may be made zero.

Wilson[2] recommends obtaining equivalent length from both equations (IV-11) and (IV-12) and taking the average value of the two results if the difference between them is not large. If the difference is large, value from equation (IV-11) is recommended if the webs are thin and narrow, and there is no overlapping of crankpin and journals. If, on the other hand, the crankwebs are thick and wide, and if there is overlapping of crankpin and journals the result from (IV-12) is recommended.

Timoshenko's[7] modification of equation (IV-10) is different from above two. The formulae for equivalent lengths for cases with no bearing constraint and with complete constraint are given as:

with no bearing constraint

$$L_e = d_e^4 \left[\left(\frac{b + 0.9\,h}{d_{mo}^4 - d_{mi}^4} \right) + \left(\frac{a + 0.9\,h}{d_{co}^4 - d_{ci}^4} \right) + \left(\frac{0.942\,r}{h\,t^3} \right) \right] \quad \text{........ (IV-13)}$$

with complete constraint

$$L_e = d_e^4 \left[\left(\frac{b + 0.9\,h}{d_{mo}^4 - d_{mi}^4} \right) + \left(\frac{a + 0.9\,h}{d_{co}^4 - d_{ci}^4} \right) \left(1 - \frac{r}{k}\right) + \left(\frac{0.942\,r}{h\,t^3} \right) \left(1 - \frac{r}{k}\right) \right] \quad \text{........ (IV-14)}$$

in which,

$$k = \frac{\dfrac{r\,(a+h)^2}{4\,c_3} + \dfrac{a\,r^2}{2\,c_2} + \dfrac{a^3}{24\,B_1} + \dfrac{r^3}{3\,B} + \dfrac{1.2}{G}\left(\dfrac{a}{2\,A} + \dfrac{r}{A_1}\right)}{\dfrac{ar}{2\,c_2} + \dfrac{r^2}{2\,B}}$$

where k = effective lever arm, of the couple

c_3 = torsional rigidity of the web in respect to twist around p-p = $\frac{t^3 h^3 G}{3.6 (t^2 + h^2)}$

B_1 = the flexural rigidity of journal

$= \frac{\pi}{64} (d_{mo}^4 - d_{mi}^4)$

A = area of the cross section of the web taken on 0-0 = $h\,t$

A_1 = area of the cross-section of the web taken on p-p = $t\,r$

Equations (IV-13) and (IV-14) give equivalent lengths for two extreme cases. The equivalent length in practice lies somewhere between the two values.

In equations (IV-11) through (IV-14), the three terms on the right hand side give respectively the equivalent lengths of one journal, one crankpin, and two webs. Knowing this, equipment lengths of more complicated designs of crankshaft can be obtained by breaking them into simple portions to which the terms just mentioned can individually be applied. Full accounts on such cases may be seen in reference (2).

IV-2-0 APPLICATION

We have determined the tentative size of shaft in section II. It has also been shown in section III that no extra flywheel effect is necessary other than that contributed by the generator rotor mass.

Knowing these properties the required transmission shaft system is planned out and illustrated in figure (IV-3).

The next step is to determine the equivalent lengths and torsional stiffnesses of the shaft sections preparatory to torsional vibration analysis in section (VI). To determine these properties we shall assume the diameter of equivalent shaft equal to 9.25 in. and its material is steel.

IV-2-1 Equivalent Length Between Cylinder Masses

The portion of crankshaft between two cylinder masses comprises two crankwebs, a length equal to that of crankpin, and a journal, total equivalent length of which will be equal to the equivalent length per crank element as discussed in the theory part.

The equivalent length of an element of crankshaft may be determined by using the formulae given. However, in our case the torsional stiffnesses between the cylinder masses, between the scavenge pump and No. 1 cyl. and between the damper and the scavenge pump are given by the engine manufacturing company and it is not necessary to find these values. The present task is to find the equivalent lengths of these sections and to find the equivalent length as well as stiffness of the shaft between the No. 7 cylinder mass and the generator rotor.

To get the equivalent length between cylinder masses, we use equation (IV-12). The stiffness is given as 316 x 10^6 lb.-in./rad. so that

$$L_e = \frac{\pi d_e^4 G}{32 C} = \frac{\pi (9.25)^4 \times 12 \times 10^6}{32 \times 316 \times 10^6} = 27.294 \text{ in.}$$

IV-2-2 Equivalent Length Between No. 1 Cylinder and Scavenge Pump Masses

The stiffness of this section is given as 139×10^6 lb.in./rad. Hence,

$$L_{e_2} = \frac{\pi \cdot d_e^4 \cdot G_e}{32\ C} = \frac{\pi \times 732 \times 12 \times 10^6}{32 \times 139 \times 10^6} = 62.05 \text{ in.}$$

IV-2-3 Equivalent Length Between Scavenge Pump and Damper Masses

The stiffness of this section is given as 90×10^6 lb.-in./rad. Hence,

$$L_{e_3} = \frac{\pi \cdot d_e^4 \cdot G_e}{32 \cdot C} = \frac{\pi \times 7321 \times 12 \times 10^6}{32 \times 90 \times 10^6} = 95.83 \text{ in.}$$

IV-2-4 Equivalent Length and Stiffness of the Shaft Between No. 7 Cylinder Mass and Generator

This shaft is subdivided into small sections for our calculation as illustrated in figure (IV-3).

Section (a) Section between point A of No. 8 main and end of crank shaft flange.

Here we use equation (IV-7). The dimensions are obtained from drawing No. (6). We get:

$$L_{e_a} = (6.69 + 1.181) + \frac{1.181 \times 7321}{32107} = 8.1433 \text{ in.}$$

Section (b) Section between the end of crankshaft and first coupling.

From figure (IV-3) we see that the equivalent length of the portion b_1 is the same as that already calculated above for subsection (a), i.e.

$$L_{e_{b_1}} = 8.1433 \text{ in.}$$

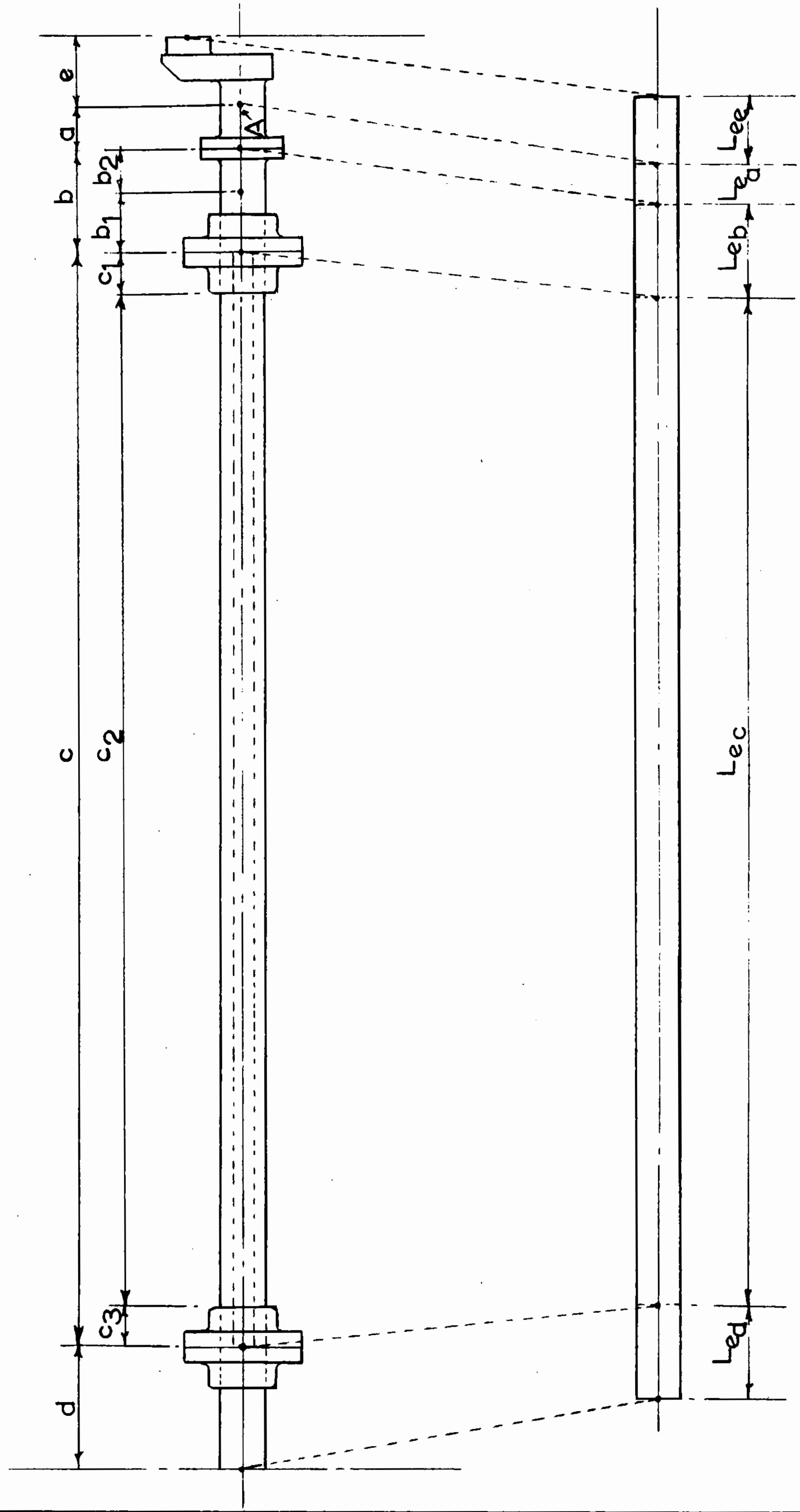


Figure (IV-3)

For section b_2 we use equation (IV-8) with dimensions given in drawing Nos. (6) and (1). This gives:

$$L_{e_{b_2}} = 6.695 + 2.75 + \frac{3.75 \times 7321}{36884} + \frac{1.75 \times 7321}{169679}$$

$$= 6.695 + 2.75 + 0.744 + 0.076 = 10.265 \text{ in.}$$

Hence equivalent length of section (b),

$$L_{e_b} = L_{e_{b_1}} + L_{e_{b_2}} = 8.143 + 10.265 = 18.408 \text{ in.}$$

<u>Section (c)</u> <u>Equivalent length of the main shaft</u>

This section is marked as "c" in figure (IV-3).

Equivalent lengths of the two couplings, i.e. equivalent lengths of sections c_1 and c_3, can be expressed, when we put $L_1 = 0$ and introduce d_{1o} and d_{1i} for outside and inside diameter of the shaft in equation (IV-8), as:

$$L_e = \left[\frac{1/3\ L_2}{(d_{1o}^4 - d_1^4)\ G_s} + \frac{(2/3\ L_2 - 1/3\ L_3)}{(d_2^4 - d_{1o}^4)\ G_h} + \frac{1/2\ L_3}{(d_3^4 - d_{1o}^4)\ G_h} \right] d_e^4 \cdot G_e$$

The values of second and third terms can be obtained directly from calculations done for section (b). Hence,

$$L_{e_{c_1}} = L_{e_{c_3}} = \frac{2.75 \times 7321}{6863} + 0.744 + 0.076 = 3.754 \text{ in.}$$

Equivalent length of section C_2, from equation (IV-1), is:

$$L_{e_{c_2}} = \frac{L_{c_2} \times d_e^4 \times G_e}{(d_{1o}^4 - d_{1i}^4) G_s} = \frac{199.5 \times 7321}{6863} = 212.814 \text{ in.}$$

Hence total equivalent length of section (c) is given by:

$$L_{e_c} = L_{e_{c_1}} + L_{e_{c_2}} + L_{e_{c_3}} = 2 \times 3.754 + 212.814 = 220.322 \text{ in.}$$

Section (d) The section between the generator mass and its coupling

Attempts were made to get accurate dimensions of the generator rotor. But it was not successful due to the Company's strict observance of trade accuracy. It is, therefore, assumed here that the total length of shaft between the rotor mass and coupling end is 24 in. and the shaft is of 9.25 in. diameter. In actual case, however, the shaft may be stepped to carry the bearings. Torsional stiffness of the shaft may then be obtained by using the equations described before. For our engine system, the assumption should not differ very much from actual value. Furthermore, the generator shaft contributing to the stiffness calculation occupies only a small fraction of the total length of the transmission shaft between the No. 7 cylinder and the generator masses so that the overall torsional stiffness calculated on this assumption should not differ appreciably from the value with actual rotor shaft.

To use equation (IV-8) we have $L_1 = 15.75$ in. All the other values are the same as those calculated for section (b). Hence,

$$L_{e_d} = 15.75 + 2.75 + 0.744 + 0.076 = 19.32 \text{ in.}$$

Section (e) The section between No. 7 cylinder mass and point A on No. 8 journal.

This is equal to one half the equivalent length of one crank element. Hence, we get from V-2-2:

$$L_{e_e} = 1/2\, L_{e_1} = 1/2 \times 27.294 = 13.647.$$

Summing up above values of equivalent length, we get the equivalent length between No. 7 cylinder and the generator masses as:

$$L_{e_4} = L_{e_a} + L_{e_b} + L_{e_c} + L_{e_d} + L_{e_e}$$

$$= 8.1433 + 18.408 + 220.322 + 19.32 + 13.648 = 279.841 \text{ in.}$$

IV-2-5 <u>Torsional Stiffness of Shafting Between No. 7 Cylinder and Generator Masses</u>

This is obtained by using equation (IV-1) with equivalent length just calculated as

$$C = \frac{\pi \cdot d_e^4 \cdot G_e}{32\, L_e}$$

$$= \frac{\pi \times 7321 \times 12 \times 10^6}{32 \times 279.841} = 30.8 \times 10^6 \text{ lb.in./rad.}$$

V EQUIVALENT MOMENTS OF INERTIA

V-1-0 THEORY

V-1-1 The equivalent moments of inertia of various masses for our system have been supplied by the engine and alternator manufacturers and we can even exclude this section. However, since the treatment may not be complete without literature on the method of determination of mass moments of inertia, the theory of determination of these values are included below.

V-1-2 Equivalent Mass Moment of Inertia of the Damper

The damper used on the engine being investigated is a viscous Lanchester damper. The equations for such a damper are reproduced below(3).

The equivalent mass moment of inertia due to vibration of the inertia ring, experienced by the housing can be shown as

$$J_e = \frac{J_d}{1 + \left(\frac{J_d\, w}{f}\right)^2} \qquad \text{............ (V-1)}$$

where J_e = equivalent mass moment of inertia due to the inertia ring, felt by the housing

J_d = mass moment of inertia of the ring

w* = phase velocity of vibration

f = viscous damping torque per unit velocity.

It can be shown that the work dissipated through the damper is maximum when $f = J_d\, w$. This value of f is called the optimum damping. With this optimum damping condition, we get from equation (V-1) that

---

* Since ω does not exist on the typewriter, w will be used to denote the phase velocity.

$$J_e = J_d/2 \qquad \ldots\ldots\ldots\ldots\ldots \text{(V-2)}$$

Hence, the total equivalent moment of inertia of the damper with optimum damping is given by:

$$J_{total} = J_h + J_d/2 \qquad \ldots\ldots\ldots \text{(V-3)}$$

where J_h = mass moment of inertia of the housing.

V-1-2 <u>Mass Moment of Inertia per Cylinder</u>

The total mass moment of inertia per cylinder can be obtained by summing up the following components.

i) Inertia of one crankpin

ii) Inertia of one journal

iii) Inertia of two crankwebs (and rotating weights if any are used)

iv) Inertia of one rotating part of connecting rod

v) Inertia due to reciprocating part.

To get the representative mass moment of inertia, the above may be determined. Determination of the first two components follow simply the fundamental formulae of mechanics. The third component can be obtained by graphical or analytical methods(2). The analytical method may be used for our purpose.

iii) <u>Analytical method of determining inertia of crankweb</u>

a) If the crankweb is not bevelled the moment of inertia can be found without difficulty by dividing the web into regular portions to which the fundamental formulae can be applied.

b) If the web is bevelled the procedure is to find the inertia of unbevelled web as described above and to make correction for bevelling.

Inertia of the bevelled portion can be found by using the formulae for hallow cone sections with cylindrical bore as follows:

$$J_{oo} = \frac{W\,K_{oo}^2}{g} \qquad \text{.............. (V-4)}$$

where J = inertia of the cone section, lb-in-sec.^{2}

W = weight of the cone section, lb.

$$= \frac{\pi\,\rho\,L}{3}\,(r_1 + 2\,r_2)\,(r_1 - r_2) \qquad \text{.. (V-5)}$$

K_{oo} = radius of gyration about the cone axis, in.

$$= \frac{3}{10}\left[\frac{r_1^{\,3} + 2\,r_1^{\,2}\,r_2 + 3\,r_1\,r_2^{\,2} + 4\,r_2^{\,3}}{r_1 + 2\,r_2}\right] \qquad \text{.. (V-6)}$$

r_1 = outside radius at the large end, in.

r_2 = radius at the small end = radius of the bore, in.

L = axial length, in.

ρ = specific weight of material, lb/in.^{3}

(iv) <u>Moment of inertia of rotating part of connecting-rod Weight</u>

The usual practice, in finding moment of inertia, is to replace the connecting rod by two concentrated masses W_{rot}' and W_{rec}' , one at the crankpin and the other at the wristpin, by using the following equations:

$$W_{rot}' + W_{rec}' = W_{con} \qquad \text{................. (V-13)}$$

$$W_{rot}' \cdot h_1 = W_{rec}' \cdot h_2 \qquad \text{.......... (V-14)}$$

where W_{con} = total weight of connecting rod

h_1 = distance of C.G. of connecting rod to the centre of crankpin

h_2 = distance of C.G. of connecting rod to the centre of wristpin.

This method disregards the difference between the moment of inertia of the replacing system and that of the original connecting rod. But this difference in moments of inertia is usually small and the result obtained by the above method is sufficiently correct to an acceptable degree of approximation.

Having the value of W'_{rot} the problem of finding its mass moment of inertia gives no difficulty.

(v) <u>Moment of inertia of reciprocating part</u>

The total reciprocating part per cylinder consists of

a) weight of one piston, complete with its component parts;

b) weight of one wristpin;

c) weight of cross-head, if any is used;

d) a reciprocating weight of connecting rod, W'_{rec} as determined above.

Now, disregarding the obliquity of the connecting rod, the approximate value of moment of inertia about the crankshaft axis due to the reciprocating part can be shown as:

$$J = W_{rec} \times r^2 \, (1 - \cos 2\alpha) \qquad \text{........ (V-15)}$$

where W_{rec} = total reciprocating weight, lb.

r = crank radius, in.

α = angle of rotation of crank from the t.d.c. position.

J = equivalent mass moment of inertia about crankshaft axis due to the reciprocating weight, lb-in-sec^2.

This has an average value of

$$J_{av} = \frac{W_{rec}}{2 \cdot g} r^2 \qquad \text{......... (V-16)}$$

Equation (V-16) shows that average mass moment of inertia of reciprocating weight about the crankshaft axis can be obtained by putting a concentrated weight equal to one-half the total reciprocating weight at the crank radius.

In common practice in determining the total mass moment of inertia per cylinder the average value of the inertia of reciprocating part is taken.

V-2-0 APPLICATION

V-2-1 The equivalent moments of inertia of various masses needed in the system for torsional vibration analysis are given by the manufacturers and, in fact, it is not necessary to do calculations for these values. However, calculations for the inertia of the damper and one cylinder mass may be given for illustrative purposes.

V-2-2 Moment of Inertia of the Damper

The moments of inertia of the hub and flywheel are given on the drawing No. (2).

Hence using equation (V-3) with an assumption for optimum damping:

$$J = J_h + \frac{J_d}{2}$$

$$= 329.5 + \frac{575}{2} = 617 \text{ lb-in-sec.}^2$$

This is the value given by the engine manufacturer also.

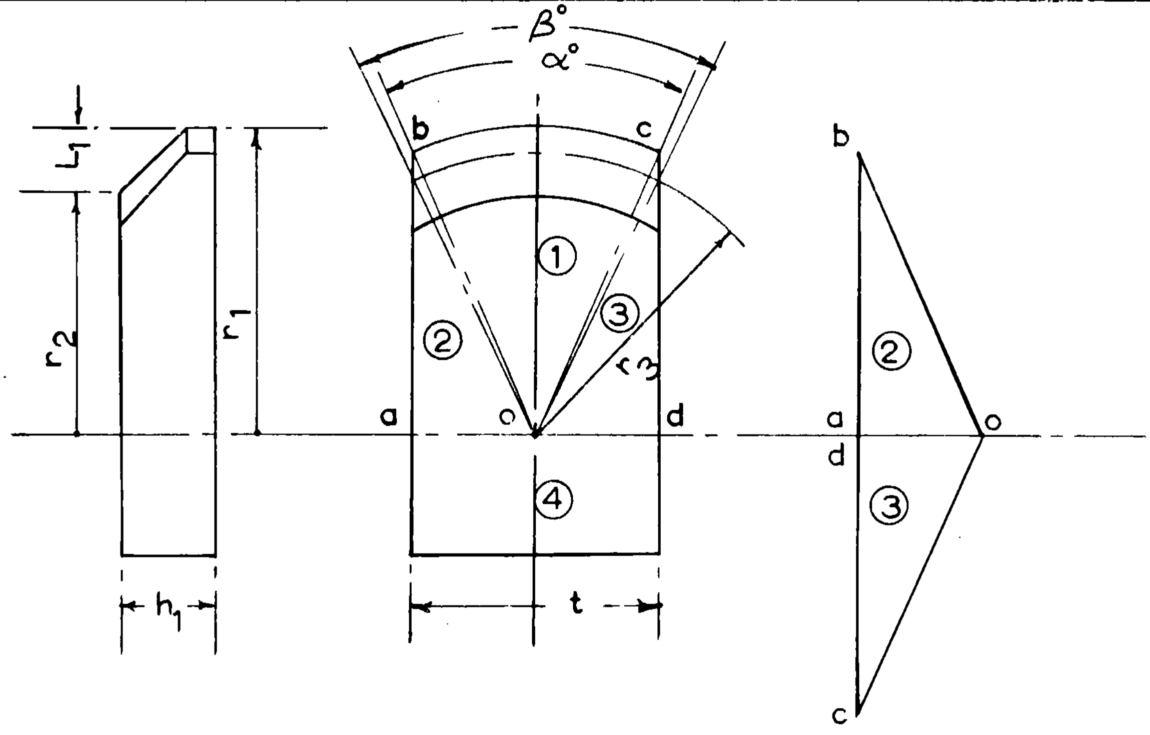


Figure (V - 1)

V-2-3 Moment of Inertia of the Engine Masses

All the dimensions in the following calculations are obtained from drawing No. (6)

(i) Inertia of one journal

This is given by:

$$J = \frac{\pi \cdot d^2}{4} \times L \times \frac{\rho}{g} \times \frac{d^2}{8} ,$$

where d = diameter of journal, in.

L = length of journal, in.

Hence,

$$J_j = \frac{\pi \times (9.25)^4 \times 10.23 \times 0.283}{32 \times 386} = 5.4 \text{ lb.in.sec.}^2$$

(ii) Moment of inertia of a crankpin

This is given by:

$$J_p = \frac{W_p}{g} (K_p^2 + r^2)$$

where W_p = weight of pin = $\frac{\pi d^2 L}{4}$ lb.

K_p = radius of gyration about pin axis = $\frac{d^2}{8}$

r = crank throw, in.

Hence,

$$K_p^2 = \frac{d^2}{8} = \frac{9.25^2}{8} = 10.7$$

$$r^2 = (11.22)^2 = 126.$$

Hence

$$J_p = \frac{\pi \times d^2 \times \rho \times L \times (K_p^2 + r^2)}{4 \times g} = \frac{\pi\,(9.25)^2\ 0.283 \times 7.49 \times 136.7}{4 \times 386}$$

$$= 50.5 \text{ lb.in.sec}^2$$

(iii) Moment of inertia of crankweb

A crankweb from drawing No. (6) is reproduced in figure (V-1) for clarity and convenience of reference. The crankweb may be divided into sections marked (1), (2), (3) and (4) for application of analytical formulae.

Inertia of section (1)

This is a circular sector, radius of gyration of which about the axis 0-0 is given by

$$K_1^{\,2} = \frac{r_1^{\,2}}{2}\,,$$ disregarding the bevel for the time being.

Hence,

$$K_1^{\,2} = \frac{16.15^2}{2} = 130.5 \text{ in.}^2$$

We have:

$$\text{Sin}\,\frac{\alpha}{2} = \frac{t}{2 \times r_1} = \frac{12.78}{2 \times 16.15} = 0.396$$

$$\frac{\alpha}{2} = 23.33^\circ$$

$$\alpha = 46.66^\circ$$

The mass of the sector is given by

$$m_1 = \frac{\pi \cdot \rho \cdot r_1^{\,2} \cdot h}{g} \times \frac{\alpha^\circ}{360}$$

$$= \frac{\pi \times 0.283 \times 16.15^2 \times 5.12 \times 46.66}{386 \times 360} = 0.4$$

Hence,

$$J_1 = m_1 K_1^2 = 0.4 \times 130.5 = 52.2 \text{ lb.in.sec.}^2$$

Inertia of sections (2) and (3)

For this calculation, section (3) is imagined to be rotated to take the position as shown on the extreme right diagram in figure (V-1) and the formula for a triangular lamina is applied to this figure.

We get, radius of gyration about 0 - 0 as:

$$K_2^2 = \frac{(2\,L)^2 + 12\,(\frac{t}{2})^2}{24}$$

$$= \frac{4 \times (14.83)^2 + 3 \times (12.878)^2}{24} = 57.2 \text{ in.}^2$$

Mass of this lamina is:

$$m_2 = \frac{\rho \times 2\,L \times \frac{t}{2} \times h}{2\,g} = \frac{0.283 \times 14.83 \times 12.78 \times 5.12}{2 \times 386}$$

$$= 0.355 \text{ lb.sec}^2 \text{ in}^{-1}$$

Hence

$$J_{2+3} = 0.355 \times 57.2 = 20.4 \text{ lb.in.sec.}^2$$

Inertia of section (4)

This is a simple rectangular lamina. We do the calculations as follows:-

$$m = \frac{\rho\, a\, h\, t}{g} = \frac{0.283 \times 6.38 \times 5.12 \times 12.78}{386} = 0.307 \text{ lb.sec.}^2 \text{ in.}^{-1}$$

$$K_{oo}^{2} = \frac{4\,a^2 + t^2}{12} = \frac{4 \times 6.38^2 + 12.78}{12} = 27.2 \text{ in}^2$$

Hence,

$$J_4 = 0.307 \times 27.2 = 8.35 \text{ lb.in.sec.}^2$$

Inertia of bevel

The bevel is a part of complete hollow cone section only. To get the weight of the bevel, the weight in equation (V-5) must be multiplied by the ratio of the angle subtended by the bevel at the centre O, to the complete circle. In determining the angle subtended by the sector, the radius to the locus of the C.G. of the bevel must be used. This is given by:

$$r_3 = r_1 - \frac{L_1}{3} = 10.5 - 1.15 = 15 \text{ in.}$$

Hence,

$$\sin\frac{\beta}{2} = \frac{12.78}{2 \times 15} = 0.425$$

$$\frac{\beta}{2} = 25.15^\circ$$

$$\beta = 50.3^\circ$$

Hence, from equation (V-5), the mass of the bevel

$$m_b = \frac{W_b}{g} = \frac{\pi\,\rho\,h_2}{3\,g}\,(r_1 + 2\,r_2)\,(r_1 - r_2) \times \frac{\beta^\circ}{360}$$

$$= \frac{\pi \times 0.283 \times 3.46\,(16.15 + 2 \times 2.69)(3.46) \times 50.3}{3 \times 386 \times 360} = 0.534 \ \text{lb.sec.}^2\,\text{in.}^{-1}$$

From equation (V-6), we get

$$K_{oo} = \frac{3}{10}\left[\frac{16.15^3 + (2 \times 16.15^2 \times 12.69) + (3 \times 16.15 \times 12.69^2) + (4 \times 12.69^3)}{16.15 + 2 \times 12.69}\right]$$

$$= 194.4 \text{ in.}^2$$

Hence

$$J_b = m_b K_{oo}^2 = 0.0534 \times 194.4 = 10.4 \text{ lb.in.sec.}^2$$

Moment of inertia of the web is obtained by summing the above results as:

$$\text{Inertia of a crankweb} = 52.2 + 20.4 + 8.35 - 10.4 = 70.55 \text{ lb.in.}^2\text{sec.}^2$$

(iv) Moment of inertia of rotating weight of connecting rod

Using the value of W'_{rot} given on drawing No. (3) we get:

$$J_{rot} = \frac{W'_{rot}}{g} \cdot r^2 = \frac{269}{386} \times 126 = 97.8 \text{ lb.in.sec.}^2$$

(v) Moment of inertia of reciprocating weight

Total reciprocating weight has been calculated and used in section II. This value was obtained as 705 lb.

Hence, using equation (V-16), we get

$$J_{rec} = \frac{W_{rec}}{2g} \cdot r^2 = \frac{705}{2 \times 386} \times 126 = 115.2 \text{ lb.in.sec.}^2$$

Moment of inertia per cylinder is obtained by summing the items (i) to (v) calculated above as:-

$$\text{Moment of inertia per cylinder} = 5.4 + 50.5 + (2 \times 70.55) + 87.8 + 115.2$$

$$= 400 \text{ lb.in.sec.}^2$$

This is exactly the value supplied by the engine manufacturer.

VI NATURAL FREQUENCIES

VI-1-0 THEORY

VI-1-1 The problem of finding out the natural frequencies of torsional vibration of a system with many degrees of freedom is not at all new. A number of methods have been established for the last fifty years. The most practical methods may be grouped in four, viz.

a. Holzer's Tabulation Method

b. F.M. Lewis' Distributed Mass Method

c. F.P. Porter's Equivalent Inertia Method

d. Graphical Methods.

These methods are described in detail in the literature on torsional vibration and it may be superficial to reproduce all of them in this section.

However, for our purpose, two methods may be used: one to get detailed results for further analysis and the other to serve as a check on the correctness of the natural frequencies obtained by the first method. We shall use the methods mentioned in (a) and (b) above for our purpose. However, since method (a) is very well established the procedure of working out a Holzer Table will not be treated in detail.

To work out a Holzer Table, the natural frequency value of the system is estimated by transforming the given multirotor system into an equivalent two- or three-rotor systems. The two-rotor system gives approximate value of the lowest natural frequency, the three-rotor system gives one-node and two-node frequencies.

Two-rotor and three-rotor systems are illustrated in figure (VI-1). Their natural frequency equations are given below:-

Natural frequency of two-rotor system:

$$w_n = \sqrt{\frac{C\ (J_1 + J_2)}{J_1\ J_2}} \qquad \text{.............} \ (VI\text{-}1)$$

Natural frequency equation for three-rotor system:

$$w_n^2 = \frac{1}{2}\left[\frac{C_1}{J_1} + \frac{C_2}{J_3} + \frac{(C_1 + C_2)}{J_2}\right] \pm$$

$$\pm\ \frac{1}{2}\sqrt{\left[\frac{C_1}{J_1} + \frac{C_2}{J_3} + \frac{(C_1 + C_2)}{J_2}\right]^2 - 4\,\frac{C_1\ C_2}{J_1\ J_2\ J_3}\,(J_1 + J_2 + J_3)}$$

.... (VI-2)

VI-1-2 F.M. Lewis' Distributed Mass Method

In this method, the inertia and stiffness of engine crank are uniformly distributed along the entire length of the engine. Then this portion is treated as a shaft of uniform stiffness and moment of inertia.

So that Lewis' method can be used, the analysis of torsional vibration of uniform shaft is given below:-

Torsional vibration of uniform shaft

A uniform shaft is shown in figure (VI-2)

Let I_p = polar moment of inertia of shaft section

G = modulus of rigidity

c = torsional rigidity = $G\ I_p$

j = mass moment of inertia of shaft per unit length

L = length of shaft

J_o = total mass moment of inertia of the whole shaft

$= j\ L$ ... (VI-3)

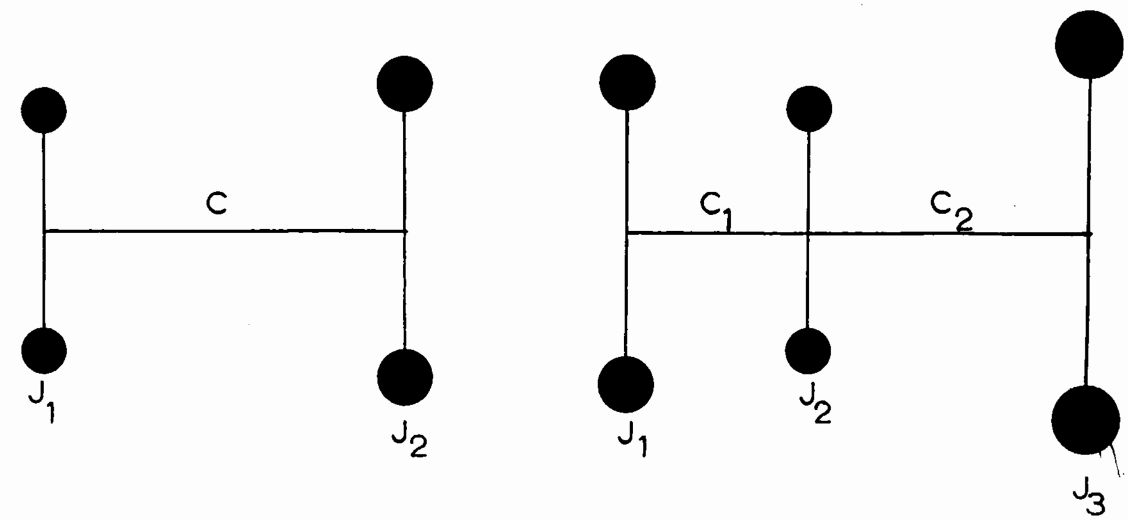


Figure (VI- 1)

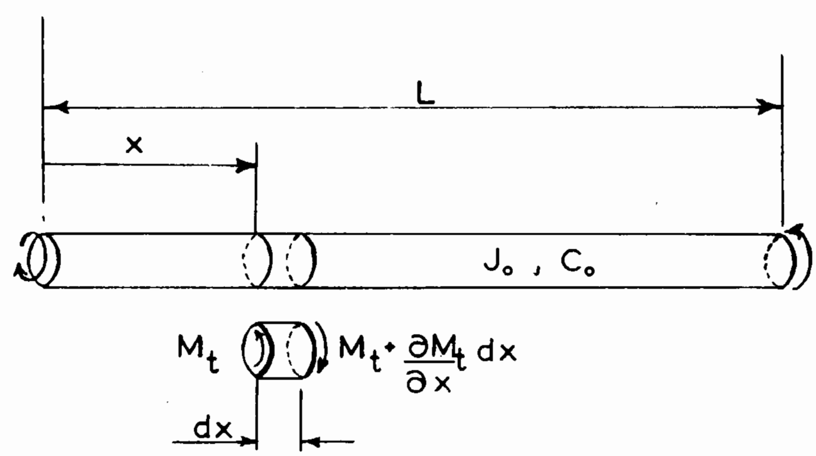


Figure (VI- 2)

C_o = overall stiffness of the shaft.

= c/L .... (VI-4)

w = angular velocity of vibration

θ = deflection, which is function of x and time "t".

a_x = amplitude of vibration at a distance x.

Equating the net torque acting on an element of length of dx and its inertia torque, the equation of motion can be obtained as:

$$j \frac{\partial^2 \theta}{\partial t^2} = c \frac{\partial^2 \theta}{\partial x^2} \qquad \text{..... (VI-5)}$$

Solving equation (VI-5), the amplitude of vibration at a distance x can be obtained as:

$$a_x = K_1 \text{ Sin } x\sqrt{\frac{j\ w^2}{c}} + K_2 \text{ Cos } x\sqrt{\frac{j\ w^2}{c}}$$

where K_1 and K_2 are constants. This can be written as:

$$a_x = A \text{ Cos } \left(x\sqrt{\frac{j\ w^2}{c}} + \alpha\right) \qquad \text{..... (VI-6)}$$

where A = an amplitude constant.

α = phase-angle of cosine wave.

Using equations (VI-3) and (VI-4) in equation (VI-6) we get:

$$a_x = A \text{ Cos } \left(\frac{x}{L} \cdot w\sqrt{\frac{J_o}{C_o}} + \alpha\right) \qquad \text{........ (VI-7)}$$

In equation (VI-7), $w\sqrt{\frac{J_o}{C_o}}$ is called "frequency coefficient" and we may denote this by

$$\beta = w\sqrt{\frac{J_o}{C_o}} \qquad \text{........ (VI-8)}$$

Using equation (VI-6) in (VI-7), we get

$$a_x = A \cos \left(\beta \cdot \frac{x}{L} + \alpha\right) \qquad \ldots\ldots\ldots \text{(VI-9)}$$

The maximum value of torque at any section along the shaft is given by:

$$M_{t_x} = c \cdot \frac{\partial a_x}{\partial x},$$

which with value of a_x, c and β from equations (VI-9), (VI-4)and (VI-8), becomes

$$M_{t_x} = A \; w \sqrt{J_o \; C_o} \; \sin \left(\beta \frac{x}{L} + \alpha\right) \qquad \ldots\ldots \text{(VI-10)}$$

VI-2-0 <u>APPLICATION</u>

VI-2-1 We have obtained the dimensions of equivalent shaft of various shaft sections in Section (IV) and equivalent masses in Section (V). The loading diagram with these values, ready for torsional vibration analysis, is illustrated in figure (VI-3). At this stage we are in a position to find the natural frequencies of the system.

The calculations of natural frequencies of the system are made by using Holzer tabulation method and the results are checked by Lewis' method.

VI-2-2 <u>Natural Frequencies by Holzer Method</u>

To work out the Holzer tables, the approximate values of natural frequency have to be obtained. To get these estimations the system is reduced to a three-mass system as shown in figure (VI-1-b). In this we lump all the engine masses to the centre

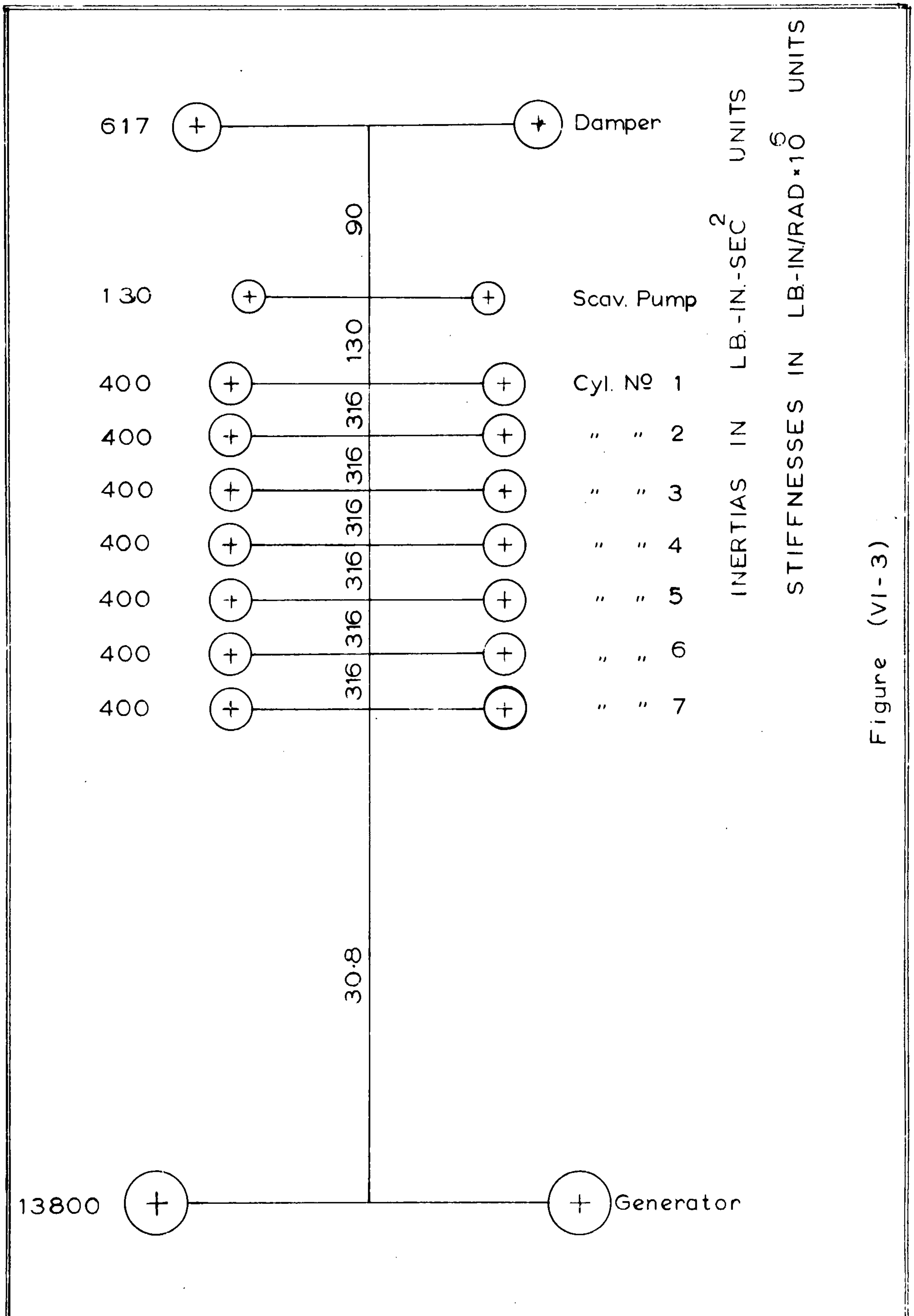

617
Damper
90
130
Scav. Pump
130
400
Cyl. Nº 1
316
400
" " 2
316
400
" " 3
316
400
" " 4
316
400
" " 5
316
400
" " 6
316
400
" " 7
30.8
13800
Generator
INERTIAS IN LB.-IN.-SEC^2 UNITS
STIFFNESSES IN LB.-IN/RAD×10^6 UNITS

Figure (VI-3)

cylinder mass, i.e. to No. 4 cylinder mass. The scavenge-pump mass is lumped to the damper mass. The stiffnesses of the shaft sections between the masses are obtained by using equation (IV-3) of Section (IV) as follows:-

$$\frac{1}{C_1} = \left[\frac{3}{316} + \frac{1}{30.8} \right] \frac{1}{10^6} = \frac{1}{23.85 \times 10^6}$$

which gives

$$C_1 = 23.85 \times 10^6 \text{ lb.in./rad.};$$

and also

$$\frac{1}{C_2} = \left[\frac{3}{316} + \frac{1}{139} + \frac{1}{90} \right] \frac{1}{10^6} = \frac{1}{36 \times 10^6}$$

which gives $C_2 = 36 \times 10^6$.

Getting all these values, we apply equation (VI-2). The following gives a few steps of calculations:

a)

$$\frac{C_1}{J_1} = \frac{23.85}{13800} = 1729$$

$$\frac{C_2}{J_3} = \frac{36 \times 10^6}{747} = 48193$$

$$\frac{C_1 + C_2}{J_2} = \frac{59.85 \times 10^6}{2800} = \underline{21390}$$

Sum of above three = 71312

1/2 sum = 35656

$(\text{Sum})^2 = 5.087 \times 10^9.$

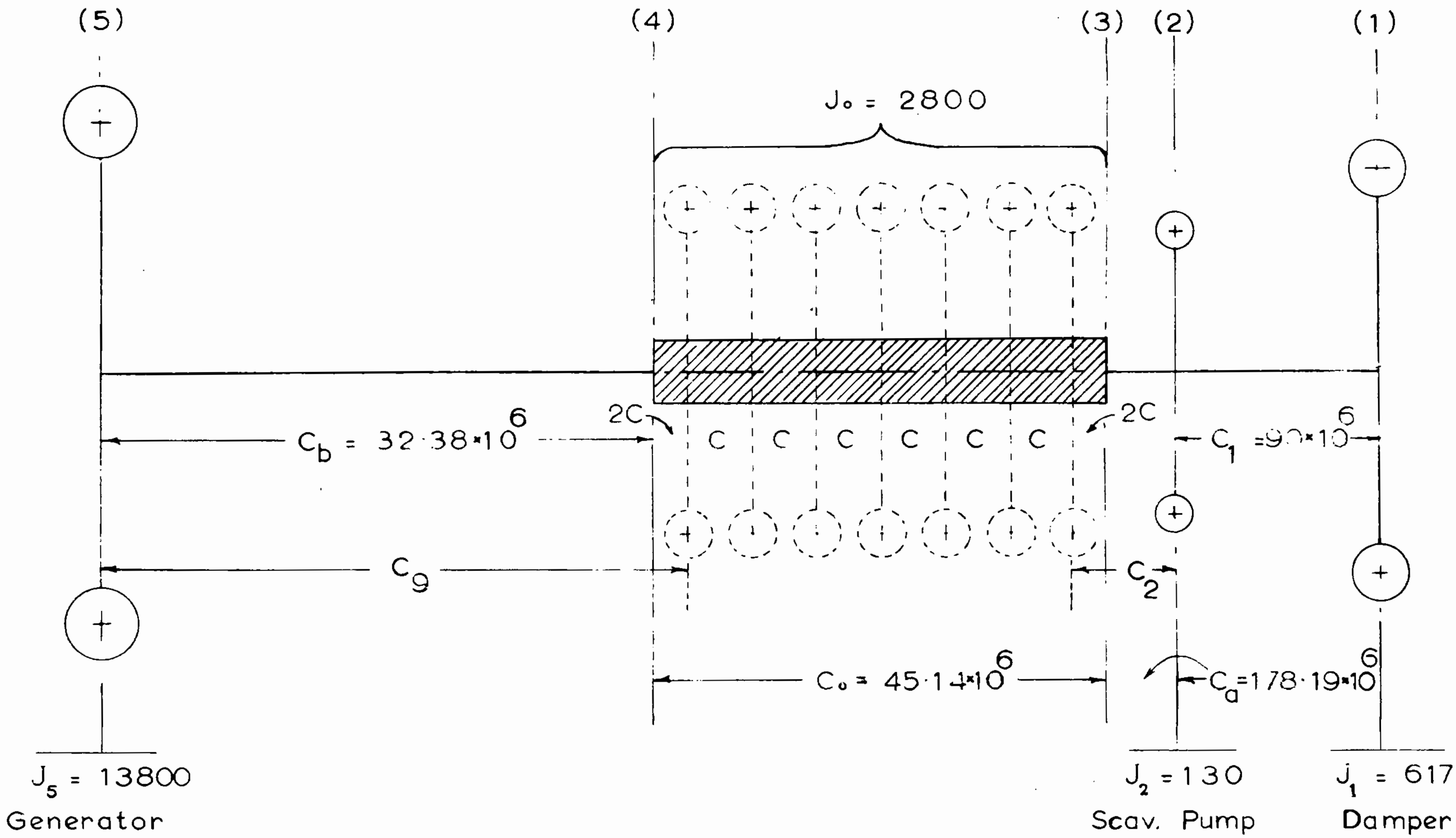

(5)
(4)
(3)
(2)
(1)
Jo = 2800
2C
C
C
C
C
C
C
2C
Cb = 32·38×10 6
C1 =90×10 6
Cg
C2
Co = 45·14×10 6
Ca=178·19×10 6
J5 = 13800
Generator
J2 = 130
Scav. Pump
J1 = 617
Damper

Figure (VI-4)

Again,

$$4\ \frac{C_1\ C_2}{J_1\ J_2\ J_3}\ (J_1 + J_2 + J_3) = \frac{4 \times 23.85 \times 36 \times 10^{12} \times 17347}{13800 \times 2800 \times 747} = 2.062 \times 10^9$$

Value of the discreminant term $= \pm \frac{1}{2}\sqrt{30.23 \times 10^8} = \pm 27585$

$$w^2_{n_1} = 35656 - 2758 = 8072$$

$$w^2_{n_2} = 35656 + 2758 = 63241.$$

Using these estimations we set up Holzer tables, with the known values of the system which have been obtained in Sections (IV) and (V). The Holzer tables for one-node vibration are given as tables (9-a), (9-b), and (9) in Appendix II. Tables (9-a) and (9-b) give the first two approximations and table (9) gives the third and final try for the frequency of this mode of vibration. Getting the closest approximation of natural frequency, the stress column, column K, of table (9) is filled in. The description given at the top of this column explains the meaning fully.

Holzer tables for two-node vibrations are given as tables (10-a), (10-b), (10-c), and (10) in Appendix II.

VI-2-3 <u>Application of Lewis' Method</u>

In this method the stiffnesses and moments of inertia of the engine crank shaft are distributed uniformity along the entire length of the engine. The system under investigation with engine distributed mass is illustrated in figure (VI-4). Using the notations in this figure the values of J_o, C_o, C_a, C_b are obtained as follows:

Total distributed engine inertia = $J_o = n\, J_e = 7 \times 400 = 2800$ lb.in.sec.^{2}

Stiffness of entire length of engine is obtained by using equation (IV-3) as:

$$\frac{1}{C_o} = \frac{n}{C} = \frac{7}{316 \times 10^6} = \frac{1}{45.14 \times 10^6},$$

Whence,

$$C_o = 45.14 \times 10^6 \text{ lb.in./rad.}$$

Again, from the same equation,

$$\frac{1}{C_a} = \frac{1}{C_2} - \frac{1}{2\,C}$$

$$= \frac{1}{139 \times 10^6} - \frac{1}{2 \times 316 \times 10^6} = \frac{1}{178.19 \times 10^6},$$

which gives

$$C_a = 178.19 \times 10^6 \text{ lb.in./rad.}$$

Similarly,

$$\frac{1}{C_b} = \frac{1}{C_9} - \frac{1}{2\,C}$$

$$= \frac{1}{30.8 \times 10^6} - \frac{1}{2 \times 316 \times 10^6} = \frac{1}{32.38 \times 10^6}$$

whence,

$$C_b = 32.38 \times 10^6 \text{ lb.in./rad.}$$

From above J_o and C_o values, we have

$$\sqrt{C_o\, J_o} = \sqrt{2800 \times 45.14 \times 10^6} = 3.555 \times 10^5$$

$$\sqrt{\frac{J_o}{C_o}} = \sqrt{\frac{2800}{45.14 \times 10^6}} = 7.875 \times 10^{-3}$$

All these values are given in figure (VI-4). The following is application of Lewis' Method to show that the natural frequencies obtained for our system by Holzer tabulation method are correct.

a. <u>One-node vibration</u>

Let $w^2 = 8530$ and damper deflection = 1 rad.

Then inertia torque of the damper,

$$M_{t_1} = J_1 \cdot w^2 \cdot a_1 = 617 \times 8530 \times 1 = 5.2630 \times 10^6 \text{ lb.-in.}$$

This also is the torque transmitted through the shaft 1-2, hence it is also $M_{t_{1-2}}$. Therefore, deflection between stations (1) and (2),

$$a_{1-2} = \frac{M_{t_{1-2}}}{C_1} = \frac{5.263 \times 10^6}{90 \times 10^6} = 0.0585 \text{ rad.}$$

$$a_2 = a_1 - a_{1-2} = 1 - 0.0585 = 0.9415 \text{ rad.}$$

Hence,

$$M_{t_2} = J_2 \cdot w^2 \cdot a_2 = 130 \times 8530 \times 0.9415 = 1.044 \times 10^6 \text{ lb.-in.}$$

$$M_{t_{2-3}} = (M_{t_{1-2}})_2 + M_{t_2} = (M_{t_{3-4}})_3$$

$$= (1.044 + 5.263) \times 10^6 = 6.307 \times 10^6 \text{ lb.-in.}$$

Therefore,

$$a_{2-3} = \frac{M_{t_{2-3}}}{C_a} = \frac{6.307 \times 10^6}{478.19 \times 10^6} = 0.0354 \text{ rad.}$$

$$a_3 = a_2 - a_{2-3} = 0.9415 - 0.0354 = 0.9016 \text{ rad.}$$

Hence, using equation (VI-9), we get

$$(a_{3-4})_3 = A \text{ Cos } \alpha = 0.9061 \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

And also from equation (VI-10),

$$(M_{t_{3-4}})_3 = A \cdot w \sqrt{J_o \cdot C_o} \text{ Sin } \alpha = 6.307 \times 10^6 \quad \ldots \text{(ii)}$$

Dividing equation (ii) by (i) and using the values of J_o and C_o obtained above,

$$\tan \alpha = \frac{6.307 \times 10^6}{0.9061 \times 92.36 \times 3.555 \times 10^5} = 0.2119$$

$$\alpha = 11.96 \text{ deg.}$$

Now, the frequency coefficient, by equation (VI-8) is:

$$\beta = w \sqrt{\frac{J_o}{C_o}} = 9.36 \times 0.007875 = 0.7273 \text{ rad.}$$

$$= 41.67 \text{ deg.}$$

Hence,

$$\beta + \alpha = 11.96 + 41.67 = 53.63 \text{ deg.}$$

From (i),

$$A = \frac{0.9061}{\text{Cos } \alpha} = \frac{0.9061}{0.9782} = 0.9263$$

Hence, using equation (VI-9), deflection at Station (4) is:

$$a_4 = A \ \text{Cos} \ (\beta + \alpha) = 0.9263 \ \text{Cos} \ 53.63 = 0.5493 \ \text{rad.}$$

Again from equation (VI-9),

$$(M_{t_{3-4}})_4 = A \cdot w \sqrt{J_o \, C_o} \quad \text{Sin} \ (\beta + \alpha) = M_{t_{4-5}}$$

$$= 0.9263 \times 92.36 \times 3.555 \times 0.8052 \times 10^5 = 24.489 \times 10^6 \ \text{lb.-in.}$$

$$a_{4-5} = \frac{M_{t_{4-5}}}{C_b} = \frac{24.489 \times 10^6}{32.38 \times 10^6} = 0.7563 \ \text{rad.}$$

$$a_5 = a_4 - a_{4-5} = 0.5493 - 0.7563 = -0.207 \ \text{rad.}$$

and

$$M_{t_5} = J_5 \cdot w^2 \cdot a_5 =$$

$$= -13800 \times 8530 \times 0.207 = -24.368 \times 10^6 \ \text{lb.-in.}$$

Hence

$$M_{t_{remainder}} = M_{t_{4-5}} + M_{t_5}$$

$$= (24.489 - 24.368) \, 10^6 = 0.121 \times 10^6 \ \text{small.}$$

This shows that the assumed frequency is a natural frequency of the system.

(b) Two-node vibration

Let $w^2 = 78000$, and deflection at the damper = 1 rad. Then,

$$M_{t_1} = 617 \times 78000 \times 1 = 48.126 \times 10^6 = M_{t_{1-2}}.$$

$$a_{1-2} = \frac{M_{t_{1-2}}}{C_1} = \frac{48.126 \times 10^6}{90 \times 10^6} = 0.5347 \text{ rad.}$$

$$\therefore a_2 = a_1 - a_{1-2} = 1 - 0.5347 = 0.4653 \text{ rad.}$$

Hence,

$$M_{t_2} = J_2 \cdot w^2 \cdot a_2 = 130 \times 78000 \times 0.4653 = 4.7181 \times 10^6 \text{ lb.-in.}$$

$$M_{t_{2-3}} = (M_{t_{1-2}})_2 + M_{t_2} =$$

$$= (48.126 + 4.7181) \times 10^6 = 52.8441 \times 10^6 \text{ lb.-in.}$$

$$a_{2-3} = \frac{M_{t_{2-3}}}{C_a} = \frac{52.8441 \times 10^6}{178.19 \times 10^6} = 0.2966 \text{ rad.}$$

$$\therefore a_3 = a_2 - a_{2-3} = 0.4653 - 0.2966 = 0.1687 \text{ rad.}$$

Hence, from equation (VI-9),

$$(a_{3-4})_3 = A \text{ Cos } \alpha = 0.1687 \text{ rad.} \qquad \text{......... (iii)}$$

And also from equation (VI-10),

$$(M_{t_{3-4}})_3 = A \cdot w \cdot \sqrt{J_o \cdot C_o} \text{ Sin } \alpha = 52.8441 \times 10^6 \text{ .. (iv)}$$

Dividing equation (iv) by (iii) and using the values of C_o and J_o, we get

$$\tan \alpha = \frac{52.8441 \times 10^6}{279.4 \times 3.555 \times 10.1687 \times 10^5} = 3.1537$$

$$\alpha = 72.39 \text{ deg.}$$

From equation (VI-8),

$$\beta = w\sqrt{\frac{J_o}{C_o}} = \frac{279.4 \times 0.007875 \times 180}{\pi} = 126.07 \text{ deg.}$$

$$\beta + \alpha = 126.07 + 72.39 = 198.46 \text{ deg.}$$

From (iii),

$$A = \frac{0.1687}{\text{Cos } \alpha} = \frac{0.1687}{0.3025} = 0.5577$$

Hence,

$$a_4 = 0.5577 \text{ Cos } (198.46) = -0.5577 \times 0.9486 = -0.529 \text{ rad.}$$

and

$$(M_{t_{3-4}})_4 = M_{t_{4-5}} = A \cdot w \sqrt{J_o \cdot C_o} \quad \text{Sin } (\beta + \alpha)$$

$$= -0.5577 \times 279.4 \times 3.555 \times 0.3166 \times 10^5 = -17.538 \times 10^6 \text{ lb.-in.}$$

Therefore,

$$a_{4-5} = \frac{M_{t_{4-5}}}{C_b} = \frac{-17.538 \times 10^6}{32.38 \times 10^6} = -0.5416 \text{ rad.}$$

$$a_5 = a_4 - a_{4-5} = -0.529 + 0.5416 = 0.0126 \text{ rad.}$$

$$M_{t_5} = J_5 \cdot w^2 \cdot a_5 = 13800 \times 78000 \times 0.0126 = 13.563 \times 10^6 \text{ lb.-in.}$$

$$M_{t_{remainder}} = -17.538 + 13.563 = -3.975 \times 10^6 \text{ small.}$$

VI-2-4 <u>Critical Speeds of the Engine System</u>

Having natural frequencies of the system above, the critical speeds are obtained as follows:-

(a) For one-node vibration

For this mode of vibration we have, from table (9), $w = 92.36$ rad./sec.

Hence, natural frequency of vibration, F, is given by:

$$F = \frac{60\,w}{2\pi} = \frac{60 \times 92.36}{2\pi} = 882.3 \text{ vpm.}$$

(b) For two-node vibration

The value of w is given in table (10), and

$$F = \frac{60\,w}{2\pi} = \frac{60 \times 278.92}{2\pi} = 2664 \text{ vpm.}$$

The critical speeds can be obtained from frequency values by the following relation:-

$$N_c = \frac{F}{n}$$

where N_c = critical speed, rpm.

n = harmonic order numbers.

The critical speeds obtained by using above calculated frequencies for one-node and two-node vibrations in this relation are given in Table (11).

VII PHASE DIAGRAMS

VII-1-0 THEORY

VII-1-1 Work done by one Cylinder per Cycle of Vibration

The work done by the n^{th} harmonic torque of a cylinder per cycle of vibration per unit deflection at the damper mass is given by:-

$$W_e = \pi \cdot T_n \cdot A \cdot R \cdot a_e \cdot \sin \phi \quad \ldots\ldots\ldots \text{(VII-1)}$$

where T_n = n^{th} harmonic of engine torque curve, lb. per sq.in. of cylinder area.

A = cylinder area, sq.in.

R = crank radius, in.

a_e = torsional vibration amplitude at the cylinder when deflection at the damper is unity.

ϕ = phase angle between the torque and the amplitude vectors.

To get the work input per cylinder for the actual vibration cycle the value of W_e in above equation may be multiplied by the vibration amplitude at the damper. But we do not need it at this moment. The work may be obtained as given in equation (VII-1) and multiplied by the actual damper deflection only after the summation has been done for all cylinders, in later sections.

Now, in an engine system, the torque vectors at various cylinders have the same magnitude but their phase angles are different. The vibration amplitudes, however, vary in magnitude but they are in phase. The work done by the cylinders, therefore, have to be added vectorially to get representative total work input.

In doing this it is customary to consider the torque vectors in phase and the amplitude vectors out of phase, as the value of work input given in equation (VII-1) is not changed if the direction of the torque and displacement vectors are interchanged. This manipulation renders the summation very convenient since the equal engine torque magnitude can be taken out as a common factor from summation terms.

The present section deals with the procedures of determining vector sum of engine amplitudes leading to vector summation of the work at all cylinder given by equation (VII-1), in later sections.

VII-1-2 Phase Diagrams

Summation of displacement vectors is achieved by drawing phase diagrams. This is best explained by considering a case of our engine system.

Assume that 360° in a phase diagram represents one vibration. Now with a two-stroke-cycle single acting engine, where working cycle occupies one revolution, there are no half order harmonics.

For an engine of m cylinders firing at equal intervals, the firing interval is $\frac{360°}{m}$ degrees of crankshaft rotation for 2-stroke engine.

Now at the critical speed of order 1, while the crankshaft makes a full revolution, each vector in the phase diagram also executes one cycle of vibration, so that the phase diagram rotates at the same speed as the crankshaft. Hence when the crankshaft rotates $\frac{360}{m}$ degrees between two consecutive firings the vectors in the phase diagram also turn $\frac{360}{m}$ degrees. The phase diagram for order 1 is, therefore, an exact reproduction of crank sequence diagram.

For vibration of order 2, where 2 vibrations occur during one engine revolution, the phase diagram turns at twice the speed of the crankshaft. So, while the engine turns $\frac{360}{m}$ degrees between consecutive firings, the phase diagram turns $\frac{360 \times 2}{m}$ degrees between the corresponding displacement vectors. So in this case the angle between the two successive vectors is twice the firing intervals of the crankshaft.

In general, for vibration of order n, the phase diagram rotates at n times the speed of the crankshaft. Thus 1° of crankshaft rotation is equivalent to n degree of vector rotation. For our engine type, the angle between consecutive vectors in phase diagrams, corresponding to $\frac{360}{m}$ degrees of crankshaft rotation between two consecutive engine firings, is given by $\frac{360\ n}{m}$, where n = 1, 2, 3, ...

Summing the above explanations, we can adopt the following procedure for getting the phase diagram for our engine.

Assume that No. 1 crank is at zero angle (vertical position) and that the angles of all other cranks are measured from No. 1 crank. Then,

(a) Since phase diagram for order 1, as explained above, is exact reproduction of crank sequence diagram it is easily obtained.

(b) The phase diagrams of higher orders are then obtained from diagram of order 1 by going around it according to firing sequence and increasing the angle between successive vectors by factors equal to the order numbers.

(c) The vectors in the phase diagram are all in phase for major orders, when $\frac{n}{m} = 1$, and vectors can be added algebraically.

VII-2-0 APPLICATION

VII-2-1 Phase Diagram of the Engine under Investigation

By applying the theory described above phase diagrams are drawn for the engine under consideration. These diagrams are shown in figure (VI-1). The engine crank sequence (same as firing order because engine is two-stroke engine) is 1-6-3-4-5-2-7. The phase diagram of orders 1, 8, 15, etc. is the exact replica of the crank sequence diagram. The diagram for the next higher orders, viz. orders 2, 9, 16, etc. is obtained by increasing the angle between successive cylinders to twice that in the first diagram. The angles between the cylinders in the third diagram are made three times that in the first diagram and so on until the diagram for major orders is reached.

VII-2-3 Vector Summation of Engine Amplitudes

Amplitudes of vibration of engine cylinder masses have been obtained in Holzer tables (9) and (10) of part (V) for one-and two-node vibrations respectively. These amplitudes are summarized in tables (12) and (13).

Vector summation of these amplitudes may be done by drawing vector diagrams with amplitude of each cylinder drawn in the direction of its own cylinder, or they may also be added analytically.

The analytical method is used here. For using this method we see that angles between number 1 and 6 cranks in the crank sequence diagram at the top of figure (VII-1) is 51.43°. Angle between number 3 crank and the horizontal is 12.86°, and angle between number 4 crank and the vertical is 25.215°. We have

Cos 51.43° = 0.6235	Sin 51.43° = 0.7818
Cos 12.86° = 0.9750	Sin 12.86° = 0.2220
Cos 25.215° = 0.9047	Sin 25.215° = 0.4260

Hence, the following relations can be written for the phase diagrams shown in the figure.

(i) Harmonic Orders: 1, 8, 15, etc.

$$\sum \vec{a_e} = \left[\left\{ a_1 + (a_6 + a_7) \times 0.6235 - (a_3 + a_2) \times 0.222 - (a_4 + a_5)\ 0.904 \right\}^2 + \left\{ (a_6 - a_7) \times 0.7818 + (a_3 - a_2) \times 0.975 + (a_4 - a_5) \times 0.426 \right\}^2 \right]^{1/2} \quad \ldots\ldots\ (VII-1)$$

(ii) Harmonic Orders: 2, 9, 16, etc.

$$\sum \vec{a_e} = \left[\left\{ a_1 + (a_5 + a_4) \times 0.6235 - (a_6 - a_7) \times 0.222 - (a_2 + a_3) \times 0.9047 \right\}^2 + \left\{ (a_5 - a_4) \times 0.7818 + (a_6 + a_7) \times 0.975 + (a_2 - a_3) \times 0.426 \right\}^2 \right]^{1/2} \quad \ldots\ldots\ (VII-2)$$

Similar equations can be written for other harmonic orders conveniently from the phase diagrams of figure (VII-1).

CALCULATIONS

Calculations for vector sum of engine amplitudes for harmonic orders 1, 8, 15, etc. of one-node mode of vibration is given below for illustrative purposes. Detailed calculation

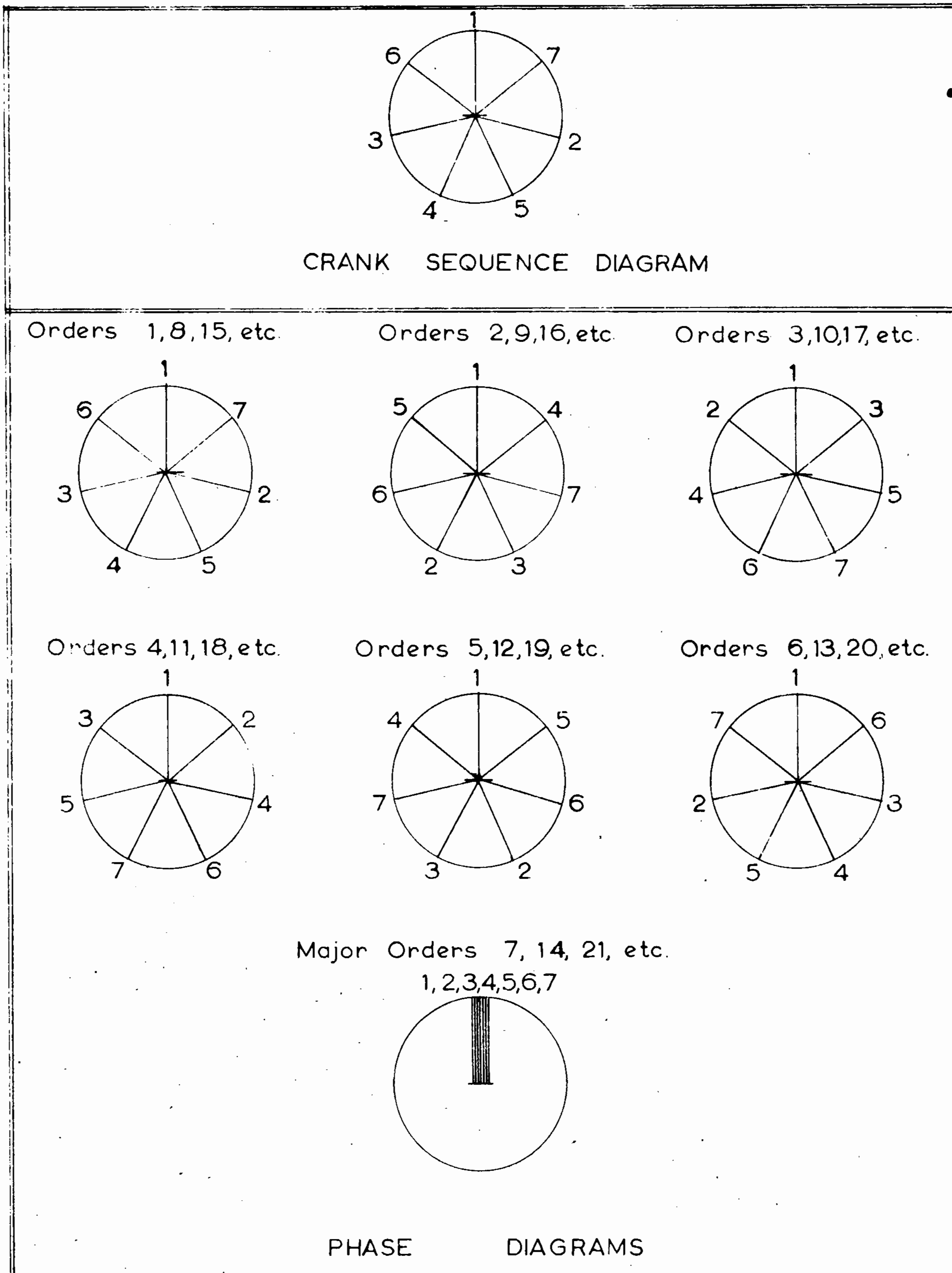

1
6
7
3
2
4
5
CRANK SEQUENCE DIAGRAM
Orders 1,8,15, etc.
Orders 2,9,16, etc.
Orders 3,10,17, etc.
Orders 4,11,18, etc.
Orders 5,12,19, etc.
Orders 6,13,20, etc.
Major Orders 7, 14, 21, etc.
1, 2,3,4,5,6,7
PHASE DIAGRAMS

Figure (VII-1)

steps for others may not be included here.

For harmonic orders 1, 8, 15, etc. we use equation (VII-1). The values of engine amplitudes in one-node vibration are given in table (12), which was prepared for these calculations. we have

$$a_1 = 0.9014 \qquad (a_3+a_2) \times 0.222 = 1.7092 \times 0.222 = 0.379$$

$$(a_6+a_7) \times 0.6235 = \underline{0.7993} \qquad (a_4+a_5) \times 0.9047 = 1.5268 \times 0.9047 = \underline{1.381}$$

$$\text{Sum} = 1.701 \qquad \text{Sum} = 1.760$$

$$\underline{1.701}$$

$$\text{Total vertical component} = \underline{0.059}$$

Again,

$$(a_6 - a_7) \times 0.7818 = 0.0683 \times 0.7818 = 0.0534$$

$$(a_4 - a_5) \times 0.426 = 0.0538 \times 0.426 = \underline{0.0229}$$

$$\text{Sum} = 0.0763$$

$$(a_3 - a_2) \times 0.975 - 0.0372 \times 0.975 = -\underline{0.0363}$$

$$\text{Total horizontal component} = \underline{0.04}$$

Hence,

$$\sum \vec{a_e} = \sqrt{(0.059)^2 + (0.04)^2} = 0.0713$$

Values of $\sum \vec{a_e}$ for other orders and mode of vibration were found in the same manner using the corresponding equations. The values of $\sum \vec{a_e}$ so determined are given in table (14). It may be seen that $\sum \vec{a_e}$ values for 1, 8, 15, etc. orders are the same as those for orders 6, 13, 20, etc. Equality of $\sum \vec{a_e}$ values for other orders can also be seen in table (14).

VIII VIBRATION STRESSES AT NON-RESONANT SPEEDS

VIII-1-0 THEORY

VIII-1-1 In studying the resonance diagrams of forced vibration problems with damping, in which the ratio of dynamic to static amplitudes (some time termed as dynamic magnifier) is plotted against the ratio of the system's natural frequency to the forcing frequency, some interesting aspects can be noted. The resonance diagram itself may not be illustrated here. On such diagram we can see the following interesting points:

(a) Damping has very little effect on the dynamic magnifier except close to resonance.

(b) The phase angle between the force and displacement changes from 0 degree when the frequency ratio is zero, to 90 degrees when the latter is 1, and to 180° when the frequency ratio is infinite.

(c) The maximum values of dynamic magnifier for the curves with various damping ratios do not occur at the resonant frequency but at somewhat lower frequencies.

(d) There are three different frequencies to be distinguished, viz. free undamped frequency, damped natural frequency, and frequency of maximum amplitude. These are grouped very closed together for small damping values.

Based on above study we could get the following approximations for practical use:-

(i) Since, from (a) above, damping has little effect except close to resonance, the difficulty in calculating the dynamic magnifier at the flanks of the resonance curve for damped vibration may be avoided by getting the curve on an assumption that the system is undamped. The curve for the region close to the resonance may be approximated by drawing a smooth curve when the dynamic magnifier at the resonance speed is obtained.

(ii) In practical problems of systems which undergo forced damped vibrations, the damping ratio is usually small so that, by (d) above, the damped frequency and forcing frequency for maximum amplitude may with little error be assumed to be identical with the free undamped frequency. Under this assumption, the value of dynamic magnifier is slightly different from the value at the resonance with forcing frequency. But this difference is very small and may be neglected.

Using the approximate procedures outlined in (i) and (ii) above the resonance curves under the action of various harmonic components of disturbing forces for our engine system may be obtained. The stresses at non-resonance speeds with the assumption that the system is undamped will be obtained in this section, the dynamic magnifiersand stresses at resonant speeds will be obtained in section (IX). By combining the two, the complete picture of the maximum stresses in the system will be obtained by drawing smooth curves as outlined in (i) above.

VIII-1-2 Vibration Stresses at Non-Resonant Speeds of Undamped System

Neglecting damping in the system, the maximum vibration stress in the engine system is given as:

$$S_s = S_{so} \times M \qquad \text{.......... (VIII-1).}$$

where S_s = vibration stress, neglecting damping, psi.

S_{so} = equilibrium stress, psi.

$= S_{s_h} \times \theta_o$ .... (VIII-2)

S_{s_h} = stress for unit deflection at the damper, from Holzer tables, psi.

θ_o = equilibrium amplitude as defined below

M = dynamic magnifier for undamped vibration

$$= \frac{1}{1 - \left(\frac{N}{N_c}\right)^2} \qquad \text{........... (VIII-3)}$$

N = revolution per minute of the engine

N_c = critical speed of the system, rpm.

VIII-1-3 Equilibrium Amplitude

The equilibrium amplitude of a multimass system usually refers to the No. 1 mass (Damper in our case). It is defined as the amplitude at mass No. 1, when the engine is rolling very slowly, tending to a stop, without any magnification due to resonance with an external pulsating couple. The equilibrium amplitude of any mode of vibration is obtained by equating the work done by the external couples in deflecting the shaft from its mean position to one extreme of its angular displacement corresponding to the equilibrium amplitude at mass No. 1, to the maximum potential energy of vibration. This is given as[(1-a)]

$$\theta_o = \frac{T_n \cdot A \cdot R \sum \overrightarrow{a_e}}{w_c^2 \sum J a^2} \text{ radians} \qquad \text{........ (VIII-4)}$$

where, T_n = maximum value of n^{th} order harmonic component of engine torque curve for one cylinder, lb. per sq. in. of cylinder area.

A = area of cylinder, sq.in.

R = crank radius, in.

$\sum \vec{a}_e$ = vector sum of engine amplitudes, obtained on assumption of unit amplitude at the damper.

$\sum J\, a^2$ = effective moment of inertia of system referred to the damper, i.e. the arithmetic sum of the products of the moments of inertia of the respective masses and squares of the amplitude at each mass, assuming unit amplitude at the damper.

w_c = natural frequency of the system.

VIII-1-4 Equilibrium Stresses

The stresses in the shaft system corresponding to the equilibrium amplitude at mass No. 1 are referred to as the equilibrium stresses.

Equilibrium stresses at various sections may be obtained by multiplying the stress values in columns K of tables (9) and (10) by equilibrium amplitudes. But, since the maximum stresses only are important, the calculations are usually made for this maximum value by using the highest stress values in columns K of tables (9) and (10).

Doing this, equation (VIII-2) may be written as

$$S_{s_o} = (S_{s_h})_{max.} \quad \theta_o \qquad \text{..........} \quad \text{(VIII-5)}$$

If, however, the stresses at the section other than the section of maximum stress is needed it can easily be obtained by multiplying the maximum stresses calculated by the above procedure by the ratio of stresses in column K of table (9) or (10) at the required section to the corresponding maximum value used in the above calculations.

VIII-2-0 APPLICATION

VIII-2-1 Equilibrium Amplitude at the Damper and Equilibrium Stresses

Since the stresses in columns K of tables (9) and (10) are given for one degree deflection at the damper, the equilibrium amplitude should be expressed in degrees. Equation (VIII-4) may, therefore, be written as:

$$\theta_o = \frac{180}{\pi} \cdot \frac{T_n \cdot A \cdot R \sum \vec{a_e}}{w_c^2 \cdot \sum J\, a^2} \text{ deg.}$$

The values of the factors are given as follows:

T_n values are given in table (8).

$\sum \vec{a_e}$ are given in table (13).

$A = \frac{\pi}{4} (13.4)^2 = 141.$ sq.in.

$R = 11.2$ in.

$w_c^2 = 8530$ for one-node vibration, from table (9).

$= 77800$ for two-node vibration, from table (10).

$\sum J\, a^2$ are calculated by using the inertia values of the masses and amplitudes at various points, when deflection at the damper is one radian, from tables (9) and (10). These values are given in tables (15) and (16) for one-node and two-node vibration, respectively.

Using these values in above equation, we get

<u>For one-node vibration</u>

$$\theta_o = \frac{141 \times 11.2 \times 180}{8530 \times \pi \times 2991.9} T_n \sum \vec{a}_e$$

$$= 0.00354 \; T_n \sum \vec{a}_e \qquad \ldots\ldots\ldots\ldots\ldots\ldots (VIII-6)$$

<u>For two-node vibration</u>

$$\theta_o = \frac{141 \times 11.2 \times 180}{77800 \times \pi \times 1087.5} T_n \sum \vec{a}_e$$

$$= 0.001068 \; T_n \sum \vec{a}_e \qquad \ldots\ldots\ldots\ldots (VIII-7)$$

The values of equilibrium amplitudes at the damper calculated by using equations (VIII-6) and (VIII-7), and equilibrium stresses by using equation (VIII-5) are given in tables (17) and (18).

VIII-2-2 <u>Torsional Vibration Stresses at Non-Resonant Speeds</u>

The stresses at non-resonant speeds are given by equation (VIII-1). The values of M are given by equation (VIII-3); and the equilibrium stresses, S_{s_o}, are given in table (17) and (18). The values of stresses at non-resonant speeds calculated by using above relations are given in tables (19) and (20), respectively, for one-node and two-node vibrations. In tables (19) and (20), only the stresses for harmonic orders that give rise to important stress magnitudes are included; stresses for all other orders, which are negligibly small, have been disregarded.

IX VIBRATION STRESSES AT RESONANT SPEEDS

IX-1-0 THEORY

IX-1-1 Energy Dissipation in the Damper

Let J_d = mass moment of inertia of the damper inertia ring.

f = viscous damping torque per unit velocity

θ_h = amplitude of oscillation of housing

w = angular frequency of oscillation

Then, the work dissipated in the damper, per cycle of oscillation can be proved as[3]:

$$W_d = \frac{\pi}{2} J_d w^2 \theta_h^2 \frac{\frac{2f}{J_d w}}{1 + \left(\frac{f}{J_d w}\right)^2} \qquad \text{........ (IX-1)}$$

If $\dfrac{W_d}{\frac{\pi}{2} J_d w^2 \theta_h}$ is plotted against $\dfrac{f}{J_d w}$, we may obtain a curve on which $\dfrac{\frac{2f}{J_d w}}{1 + \left(\frac{f}{J_d w}\right)^2}$ has a maximum value when $\dfrac{f}{J_d w} = 1$. The value of f at this point is called the optimum damping and the work as the optimum work. Hence,

$$f_{opt.} = J_d w \qquad \text{........... (IX-2)}$$

$$(W_d)_{opt.} = \frac{\pi}{2} \cdot J_d \cdot w^2 \cdot \theta_h^2 \qquad \text{......... (IX-3)}$$

IX-1-2 Energy Dissipation in the Engine

As can be seen from above, it is fairly easy to find energy dissipation in the damper. Energy dissipation through other components like pumps, propellers, etc. also follow the viscous characteristics.

But energy dissipation other than these forms is due to a number of causes like elastic hysteresis in the material of the shafting and the material of the running gears, movements of bearings, vibration of engine frame, viscous friction in the engine, etc., which are elusive and complicated to determine, and different from installation to installation. The problem of evaluating engine damping effect, therefore, is exceedingly difficult.

Owing to these difficulties, it is customary in practice, to use empirical formulae derived from analysis of torsiograph measurements on different types of engine for determination of overall engine damping. The formulae in current use are based on two types of assumption, viz.,

i) that the overall engine damping is mainly viscous in character, i.e. overall damping loss is proportional to the square of the vibratory amplitude.

ii) that the overall engine damping is mainly hysteresis in character, i.e. overall damping loss is proportional to the cube of the vibratory amplitude.

The resulting stress values calculated from equation derived on the first assumption have been found reasonably close to the actual values at the strong and fairly strong criticals; but it is learnt that, at the weak criticals, the results given by the formula based on this assumption are found to be low.

At the same time, the values given by the formula based on the second assumption are, in some cases, found to be too high.

In the absence of accurate method, however, we shall use the formula derived by the first assumption, for estimation of engine damping.

Based on the first assumption, work dissipated per cycle in the engine damping can be shown as:(1-b)

$$W_e \quad = \quad \pi \cdot f_e \cdot w \cdot \sum a_e^2 \qquad \ldots\ldots\ldots\ldots\ldots \text{(IX-4)}$$

where W_e = work dissipated at engine damping points per unit deflection at the damper mass, per cycle of oscillation.

f_e = viscous damping torque per unit vibrational velocity.

$\sum a_e^2$ = arithmetical sum of squares of amplitudes on the normal elastic curve at the engine cylinder, for unit deflection at the damper.

The relation in (IX-4) is true if f_e is the same at each damping points, i.e. at each cylinder mass. If f_e is not the same throughout, then $\sum f_e\ a_e^2$ should be used.

A large number of tests on different engines ranging from large slow-speed marine oil engines to small high-speed aero- and automobile engines are known to have been made, giving the rotation:

$$f_e \quad = \quad V \cdot J_e^{0.8} \ \text{ft.-lb./rad.} \qquad \ldots\ldots\ldots \text{(IX-5)}$$

where J_e = mass moment of inertia of crank masses per cylinder, lb.-in.-sec.^{2}

V = a coefficient.

The value of V at strong and fairly strong criticals, from these experiments, was found to vary from 12 to 40. These are extreme values, the value in most normal installations being from 12 to 25. Ker Wilson(1-b) recommends taking the average value of 21. Putting this value of f_e, equation (IX-4) becomes

$$W_e = 21 \pi J_e^{0.8} w \sum a_e^2 \qquad \text{.......... (IX-6)}$$

IX-1-3 Work Done by the Exciting Torques

It can be shown that energy input per cycle of vibration by the n^{th} order harmonic component of exciting torque per unit deflection at the damper is:(1-b),(3)

$$W_i = \pi T_n A R \sum \vec{a_e} \qquad \text{...... (IX-7)}$$

The components in this equation have been described before.

IX-1-4 Dynamic Magnifier at Resonant Speeds

Since the work input per cycle of vibration at resonant speed must be equal to the work dissipated through damping, we have from equations (IX-3), (IX-6), and (IX-7) for deflection at the damper of θ_h radians:

$$W_i = W_d + W_e$$

$$\pi T_n A R \sum \vec{a_e} \; \theta_h = \frac{\pi}{2} J_d \cdot w^2 \cdot \theta_h^2 + 21 \pi J_e^{0.8} w \sum a_e^2 \cdot \theta_h^2$$

whence,

$$\theta_h = \frac{T_n \cdot A \cdot R \cdot \sum \vec{a_e}}{\frac{J_d}{2} \; w^2 + 21 \; J_e^{0.8} \cdot w \cdot \sum a_e^2} \qquad \text{....... (IX-8)}$$

The equilibrium amplitude, θ_o, has been found in section (VIII) and given as:

$$\theta_o = \frac{T_n \cdot A \cdot R \cdot \sum \vec{a_e}}{w^2 \cdot \sum J \; a^2} \qquad \text{...... (VIII-4)}$$

Hence, dividing (IX-8) by (VIII-4), we get the dynamic magnifier at resonance as:

$$M_c = \frac{\theta_h}{\theta_o} = \frac{w \cdot \sum J \; a^2}{\frac{J_d}{2} \, w + 21 \; J_e^{0.8} \cdot \sum a_e^2} \qquad \text{..... (IX-9)}$$

<u>Note:</u> The above derivation was made with the assumption that the damper has "optimum damping" $f = J_d \; w$. If the damping constant, f, has the values other than the optimum, the work dissipated in the damper will be lower, and we can write

$$M_c = \frac{w \cdot \sum J \; a^2}{X \cdot \frac{J_d}{2} \cdot w + 21 \; J_e^{0.8} \sum a_e^2} \qquad \text{...... (IX-10)}$$

where

$$X = \frac{W_d}{(W_d)_{opt.}} = \frac{\frac{2f}{J_d \; w}}{1 + (\frac{f}{J_d \; w})^2} \qquad \text{......... (IX-11)}$$

IX-2-0 APPLICATION

IX-2-1 Dynamic Magnifier at Resonant Speeds for One-Node Vibration

We have seen that the equilibrium stresses for one-node vibration are much higher than those for two-node vibration. The damper, therefore, will have to be tuned to have optimum damping for one-node vibration. If the damper is so tuned, then the dynamic magnifier is given by equation (IX-9). We have the following values for one-node vibration.

$\sum J \cdot a^2 = 2991.9$ from table (15)

$w = 92.36$ from table (9)

$J_e = 400$

$\frac{J_d}{2} = \frac{575}{2} = 287.5$, drawing No. (2)

and from table (15),

$$\sum a_e^2 = 0.803 + 0.7508 + 0.6848 + 0.6078 + 0.5232 + 0.4344 + 0.3455$$

$$= 4.1495.$$

Hence, from equation (IX-9),

$$M_c = \frac{w \sum J \cdot a^2}{\frac{J_d}{2} \cdot w + 21\, J_e^{0.8} \cdot \sum a_e^2}$$

$$= \frac{92.36 \times 2991.9}{29.36 \times 287.5 + 21 \times 120.7 \times 4.1495} = 7.514$$

IX-2-2 Dynamic Magnifier at Resonant Speeds for Two-Node Vibration

For this mode of vibration, we have

$$\sum J \cdot a^2 = 1087.5, \quad \text{from table (16)}$$

$$w = 278.92, \quad \text{from table (10)}$$

$$\sum a_e^2 = 1.0966, \quad \text{from table (16).}$$

We have tuned the damper to its optimum damping value for one-node vibration. This value is

$$f = J_d \, w_1 = 575 \times 92.36 \ \text{lb.-in./rad./sec.}$$

Therefore, from equation (IX-11),

$$X = \frac{\dfrac{2 \times 575 \times 92.36}{575 \times 278.92}}{1 + \left(\dfrac{575 \times 92.36}{575 \times 278.92}\right)^2} \doteqdot 0.6$$

Hence, from equation (IX-10),

$$M_c = \frac{w \cdot \sum J a^2}{X \cdot \frac{J_d}{2} \cdot w + 21 J_e^{0.8} \cdot \sum a_e^2}$$

$$= \frac{278.92 \times 1087.5}{0.6 \times 287.5 \times 278.92 + 21 \times 120.7 \times 1.0966} = 5.96$$

IX-2-3 Maximum Resonant Stresses

The maximum stresses at resonant speeds are calculated by multiplying the equilibrium stresses obtained in section (VIII) by the dynamic magnifiers calculated above. The values obtained by this procedure are given in tables (21) and (22).

IX-2-4 Stress Diagrams

Having obtained the values of vibration stresses at non-resonant and resonant speeds, the stress diagrams for the system can be drawn. The stress diagrams for one- and two-node vibrations are given in Appendix III. These diagrams were drawn through the steps summarized below.

Diagram (A. III-1a) This diagram was obtained by plotting the stress values given in tables (19) and (21). Continuous curves were obtained by plotting the values in table (19). The peak stresses at resonant speeds were obtained from table (20). The approximate representative stress curves were then obtained by drawing smooth curves through these peak-stress points. The curves are shown dotted in the figure.

Diagram (A. III-1b) The mean transmission stress in the system can be obtained by the following steps.

Mean transmission torque is given as:

$$T_M = T_m \cdot A \cdot R. \text{ No. of cylinders.}$$

where T_m = mean value of engine torque,

= 25.298 psi. for our engine, from table (8).

A = area of cylinder, psi.

R = crank radius, in.

$$\therefore T_M = 25.298 \times 141 \times 11.21 \times 7 = 286000 \text{ lb.-in.}$$

Hence, mean transmission stress is:

$$S_{s_{mean}} = \frac{16\, T_M}{\pi\, d_e^3} = \frac{286000 \times 16}{\pi \times 9.25^3} = 1839 \text{ psi.}$$

Diagram (A. III-1) This was drawn by using the values obtained in diagram (A. III-1b) to show the complete picture of resultant stresses occurring in the system under one-node vibration.

Diagrams (A. III-2a) to (A. III-2) These diagrams give the stresses in the system under two-node vibration. These diagrams were drawn through the steps similar to those described for one-node stress diagrams above, by using the values given in tables (20) and (22).

IX-2-5 <u>Notes on the Stress Diagrams</u>

a. It may be noted that the mean transmission stress was taken as constant at all speeds in drawing the above diagrams. It is, therefore, apparent that the mean stress values in the diagrams at lower speeds, when the system is not transmitting its full power, are higher than the actual values. The exact values of mean stress can be calculated by using torque-speed curve of the alternator. Here, however, since the purpose of analysis is to see whether the system can withstand the maximum stresses, slight exaggeration on stresses at lower speeds can be allowed. No attempts will, therefore, be made to get the exact values, and the above diagrams will be used for further investigations.

b. The stress diagrams give stresses occurring in the solid equivalent shaft of 9.25" dia. For one-node vibration the maximum value of stress in column K of table (9), occurs in hallow shaft. Hence to get actual maximum operating stresses, the values obtained from diagrams (A. III-la) through (A. III-l) must be multiplied by a factor:-

$$\frac{d_e^3 \, d_o}{(d_o^4 - d_i^4)} = \frac{7325}{7325 - 458} = 1.07$$

X AMPLITUDE OF FORCED TORSIONAL VIBRATION AND CYCLIC IRREGULARITY AT THE GENERATOR, LIGHT FLICKERING CHARACTERISTICS

X-1-0 THEORY

X-1-1 A number of methods to determine the forced vibration amplitude at a mass of a multimass system have been published in the technical literature. For our purpose we shall use two methods as follows:-

(a) Holzer Tabulation Method

(b) F. P. Porter's Method

X-1-2 Holzer Tabulation Method

This method follows the same procedures as those in finding out the natural frequencies of the system. Here, however, the amplitude at the No. 1 mass is assumed at some arbitrary value and the table worked out as before; when the torque column is reached, the total torque acting on the mass being considered is taken into account. Hence, in forced vibration, total torque comprises the inertia torque by virtue of vibration and the forcing torque acting on the mass. When the final mass in the system is reached the torque value in the torque-summation column is equated to the external torque at that point to determine the value of amplitude at No. 1 mass, assumed previously.

For a system with input torque acting at one point only and a multimass system with forcing torques acting at various points but all in phase, the procedure is fairly simple. In the former case, no problem of torque summation arises. In the latter case, the forcing torques are taken into account by simply adding algebraically total torque values at the respective masses.

The method needs a great volume of work if used for a multimass system with forcing torques acting at various points and all out of phase. In this case amplitude at a required point may be obtained when the forcing torque at a mass is considered at a time, disregarding all other torque components at the other cylinders. Separate tabulation may be done for each torque component and the amplitude at the mass being considered may be obtained by vector summation of the amplitudes obtained in all tables.

Fortunately, however, in most of multi-cylinder engine systems, the major orders of vibration predominate so that approximation of cyclic irregularity can quite sufficiently be done by determining the forced vibration amplitude under such harmonic orders only.

X-1-3 F.P. Porter's Method

Strictly speaking, this method is an extension of Holzer tabulation for free vibration.

Consider a multimass system. Number the masses starting from a particular end, calling the mass at this end as mass No. 1. The mass at the other end is numbered L, to denote the last mass in the system. A Holzer table for free vibration, like those in Section (VI), is worked out starting with an assumption of unit deflection at the first mass. Tabulation is repeated starting with assumption of unit deflection at the last mass, working out backward to the first mass. Since the system is the same, with the only difference being the direction of working, and since they are both for free vibration, the remainder torque values at the last lines in the torque summation columns in the two tables must be the same. Amplitudes at a particular mass in the two tables may be different, since unit deflection is assumed at the first mass in the first table, and at the last mass in the second table, respectively. Using the values in these tables and external forcing torque magnitudes, forced vibration amplitude at a particular mass may be found by the formulae given below. Detail derivation of these formulae are not given here. It may be seen in F. P. Porter's paper, published in 1953[6].

Now, in above multimass system, let

i = denote forcing torque input points

M_i = forcing torque acting at the point i

a_n' = amplitude at the n^{th} point obtained in the Holzer table, which is worked through starting from the first mass.

a_n'' = amplitude at the n^{th} point obtained in the Holzer table, which is worked through starting from the last mass.

Then, forced vibration amplitude at the r^{th} mass is given as:

$$\theta_r \; (r \leqslant i) = - \frac{\overrightarrow{\sum_{n=i}} M_i \; a_i''}{\frac{a_r''}{a_r'} \sum_{n=1}^{r-1} J_n \, w^2 \, a_n' + \sum_{n=1}^{r} J_n \, w^2 \, a_n''} \qquad \text{........ (X-1)}$$

$$\theta_r\ (r \geqslant i) = - \frac{\overrightarrow{\sum\limits_{n=i}} M_i\ a_i^{'}}{\sum\limits_{n=1}^{r} J_n\ w^2\ a_n^{'} + \frac{a_r^{'}}{a_r^{''}} \sum\limits_{n=L}^{r+1} J_n\ w^2\ a_n^{'}} \qquad \text{.........} \ (X\text{-}2)$$

Now, at the first and last masses, we have

$$\sum_{n=1}^{0} J_n\ w^2\ a_n^{'} = 0$$

and

$$\sum_{n=L}^{L+1} J_n\ w^2\ a_n^{''} = 0$$

so that we get, from (X-1) and (X-2), respectively,

$$\theta_1 = - \frac{\overrightarrow{\sum\limits_{n=i}} M_i\ a_i^{''}}{\sum\limits_{n=L}^{1} J_n\ w^2\ a_n^{''}} \qquad \text{........} \ (X\text{-}3)$$

$$\theta_L = - \frac{\overrightarrow{\sum\limits_{n=i}} M_i\ a_i^{'}}{\sum\limits_{n=1}^{L} J_n\ w^2\ a_n^{'}} \qquad \text{..........} \ (X\text{-}4)$$

<u>Notes:</u> In the above formulae,

$$\sum_{L}^{1} J_n\ w^2\ a_n^{''} = \sum_{n=1}^{L} J_n\ w^2\ a_n^{'}$$

are the residual torque at the last lines in the torque-summation columns of Holzer tables. Equality has been explained previously.

Porter's formulae given above have the advantage over the Holzer Tabulation Method described in X-1-1, in that summation of the out-of-phase torque components at various points can be done analytically without going through a large number of tables.

X-2-0 APPLICATION

X-2-1 From the earlier Sections, we have seen that the 7th order harmonic, which also is strongest amongst the majors, predominates in our engine-gerator set. Hence, here, it should be sufficient to examine the coefficient of cyclic irregularity at the service speed under the 7th order harmonic engine torque component.

For illustrative purposes, we shall determine the value of coefficient of cyclic irregularity by using both methods described in the theory portion.

Now, from table (8), T_n for the 7th order harmonic = 11.368.

$$M_n = T_n A R = 11.368 \times 141 \times 11.2 = 17{,}950 \text{ lb.-in.}$$

At the service speed of 300 rpm, the frequency of vibration of 7th order harmonic is:

$$F = 7 \times 300 = 2100 \text{ vpm.}$$

Hence, the angular frequency is:

$$w = \frac{2 \pi F}{60} = \frac{2 \pi \times 2100}{60} = 220 \text{ rad./sec.}$$

$$w^2 = 48{,}400 \text{ rad.}^2/\text{sec.}^2$$

X-2-2 Holzer Forced-Vibration Tabulation Method

With above w^2 and M_n a Holzer table is worked out. This is shown as table (23). The columns of this table are almost exactly the same as those used for natural frequency calculations. The difference here is that the torque columns are made to include both the vibration and engine torques. The 7th order harmonic being a major, the engine torques at all cylinders are in phase; and so in working out this table, each engine torque component M_n is simply added at its own point in the torque column.

The table is first entered with an assumption that the forced vibration amplitude at the damper is x radians and worked out step by step, until the value of the remainder torque is obtained at the generator mass. Since at this point there is no external torque acting, we equate the torque value to zero which gives the value of x. Here,

$$M_{t_{remainder}} = -95448148\ x - 2136659.95$$

Equating this to zero, we get the value of x as:

$$x = -\frac{2136659.95}{954481480} = -0.002238556.$$

If we substitute this value of x in column D of the table, the forced vibration amplitudes at the various masses may be obtained. However, what we are looking for here is the amplitude of vibration at the generator, which is given as:

$$\theta_{generator} = -1.4761 \text{ x } - 0.0033069.$$

Substituting the value of x, we get:

$$\theta_{generator} = 1.4761 \times 0.002238556 - 0.0033069$$

$$= -0.000002507 \text{ radians.}$$

Note: In working table (23) above, correctness to a large number of digits was maintained throughout the table. Ordinarily, it should be quite sufficient to get correctness to a few digits only to get nearest value of vibration amplitude at the generator, preparatory to determination of the coefficient of cyclic irregularity. Here, however, because attempt is made not only to get the value but also to show the conformity of the results obtained by the two methods for the amplitude at the generator, which is excessively small in this case, correctness to a large number of digits is required to be maintained.

X-2-3 F. P. Porter's Method

To get the forced vibration amplitude at the generator, we use equation (X-4), which is:

$$\theta_{generator} = - \frac{\overrightarrow{\sum_{n=1}} \quad M_i \; a_i'}{\sum_{n=1}^{L} J_n \, w^2 \, a_n'}$$

The values needed for this equation are the free vibration amplitudes at various masses and the value of remainder torque in the torque-summation column, when a Holzer table for the required frequency is worked out starting from the damper, with assumption that damper deflection is 1 rad.

Here, it is not required to prepare a new table. The required values are given in table (23) when x is made equal to 1 rad. and the second parts in the amplitude and torque columns are dropped out. By doing this the following a_i' values are obtained.

Mass	Damper	Scav. Pump	Cyl. 1	Cyl. 2	Cyl. 3
a_i'	1	0.6682	0.4231	0.2894	0.1379
Mass	Cyl. 4	Cyl. 5	Cyl. 6	Cyl. 7	Generator
a_i'	-0.022	-0.1806	-0.3281	-0.4555	-1.472

From above,

$$\overrightarrow{\sum} a_e = 0.4231 + 0.2894 + 0.1379 - 0.022 - 0.186 - 0.3281 - 0.4555$$

$$= 0.1358$$

Hence,

$$\overrightarrow{\sum} M_i \, a_i' = 17950 \times 0.1358,$$

since 7th harmonic is a major order and the torques are acting in phase.

Now, as stated in the theory portion, the denominator on the right hand side of the above equation, $\sum_{1}^{L} J_i \, w^2 \, a_i'$ is the residual torque in the last column of the Holzer table for free vibration. This value is obtained from the last line in column G of table (23) when $x = 1$ and the second forcing torque term is dropped, viz.,

$$\sum_{1}^{L} J_n \; w^2 \; a_n' = - 95448148$$

Hence, from the above equation,

$$\theta_{generator} = - \frac{17950 \times 0.1358}{95448148} = - 0.00000255$$

which agrees very well with the value obtained in X-2-2.

X-2-4 <u>Coefficient of Cyclic Irregularity at the Generator</u>

This is given by equation (III-8) as:

$$C_c = 2 \cdot \theta \cdot n.$$

Hence, coefficient of cyclic irregularity at the service speed due to the 7th order harmonic engine torque is:

$$C_c = 2 \times 0.00000257 \times 7 = 0.000036.$$

This shows that the coefficient of speed fluctuation at the service speed is very much less than 1/75 as specified by the British Standards Institution(24).

XI CRITICAL WHIRLING SPEEDS

XI-1-0 THEORY

XI-1-1 As has been mentioned before, there are a number of different methods of arriving at the critical speeds in whirling of a transmission system. They are:

(1) Rayleigh's method

(2) Dunkerley's method

(3) Stodola's method

and (4) Extension of Holzer's tabulation method due to Mykelstad and Prohl.

Stodola's method is given both in graphical and numerical forms. This method in graphical form is used for estimation of the lowest critical speed in whirling of our transmission system.

XI-1-2 Stodola's Method in Graphical Form

In applying the Stodola Method to a particular system, a start is made by assuming a reasonable deflection curve and a frequency of vibration. The steps recommended in applying the method are summarized below:

Step 1. Assume a deflection curve for the system that is reasonable both as regards the shape and the scale of deflection. Generally, the static deflection curve is taken as a good assumption.

Step 2 Assume a frequency of vibration. Generally, $w^2 = 386$ is assumed so that $\frac{w^2}{g} = 1$ and the inertia loads $\frac{W_1}{g} y_1 w^2$, $\frac{W_2}{g} y_2 w^2$, etc. are given simply by the products $W_1 y_1$, $W_2 y_2$, etc., when w's are the weights and y's are the deflections at the masses.

Step 3. Assume that the system is loaded with the inertia loads $W_1 y_1$, $W_2 y_2$, etc. of step 2 and find the corresponding new deflection curve. Let the deflections measured from the new deflection curve are denoted by y_1', y_2', etc. Then, since the inertia loads which cause the deflections y_1', y_2', etc. must be equal to the inertia loads of step 2, it follows that

$$w' = w_{ass.} \sqrt{\frac{y_1}{y_1'}} = w_{ass} \sqrt{\frac{y_2}{y_2'}} \text{, etc.}$$

This can only be true if the ratio of the two deflection values is constant at all points, i.e. if the derived curve and the assumed curve are geometrically similar.

If the ratio is constant and equal to y/y', then the natural cyclic frequency F_n in vpm. is given as:

$$F_n = \frac{60\ w'}{2\ \pi} = \frac{60 \cdot \sqrt{386}}{2\ \pi} \sqrt{\frac{y}{y'}} = 187.5 \sqrt{\frac{y}{y'}} \quad \text{.. (XI-1)}$$

Step 4. If the ratio is not constant throughout the system, the derived curve of step 3 may be used as the next assumption and the above steps repeated. However, the process converges so rapidly when applied for finding the lowest frequency that the result obtained from the first assumption is generally sufficiently close.

XI-1-3 <u>Beam Deflection Formulae</u>

A simply-supported beam with overhang on one end, a simply-supported beam with overhangs on both ends, and a cantilever beam are illustrated in figure (XI-1). Some formulae for deflections of these systems, that will be useful for our analysis are given below, without derivation details.

a) Simply-supported beam with overhang on one end

This is shown in figure (XI-1-a). Deflection at the end C due to a concentrated load X at that point is:

$$y_c = \frac{a_1^2 (a_1 + L_1) X}{3 E I_z} \qquad \text{....... (XI-2)}$$

where EI_z = flexural rigidity of the beam.

b) Simply-supported beam with overhangs on both ends

This is illustrated in figure (XI-1-b). The deflections at the ends D and G due to the concentrated loads X and y acting as shown in the figure are:

$$y_d = \frac{a_2^2 (a_2 + L_2) X}{3 E I_z} + \frac{a_2^2 L_2 Y}{6 E I_z} \qquad \text{.... (XI-3)}$$

$$y_g = \frac{a_2^2 (a_2 + L_2) Y}{3 E I_z} + \frac{a_2^2 L_2 X}{6 E I_z} \qquad \text{.... (XI-4)}$$

c) Cantilever

This is shown in figure (XI-1-c). The deflection at the end due to a concentrated load acting at that point is:

$$y_h = \frac{L^3 . Y}{3 E I_z} \qquad \text{........ (XI-5)}$$

XI-2-0 APPLICATION

XI-2-1 To estimate the lowest critical whirling speed of our transmission system Stodola's graphical method is used, with the following assumptions:

a) Since the engine crankshaft portion has very short spans comparative to the other sections in the system, it is assumed to be rigidly fixed for the purpose of determining shaft deflections.

b) Since the Ajax couplings are of rubber-bronze bushed type, they are assumed to behave like ball-joints in lateral deflection of the shafting.

c) In the absence of exact dimensions, the generator rotor is assumed to be of 9-1/4 in. solid shaft. The length of the overhang portion is taken as 15 inches.

With these assumptions, the transmission system is portrayed in figure (XI-2). The two bearings on the main shaft are placed 144 in. apart with a 36 in. overhang on each end in this figure. The values of shaft spans, flexural rigidities, weight per unit run, and weights of couplings are given.

XI-2-2 Deflection Curves for Lowest Critical Speed Calculations

By making use of the values given in figure (XI-2) the deflection curves for the lowest critical speed estimation were drawn.

The curves so drawn are given in Appendix IV. Each of the diagrams gives the deflection curves for a section of shaft. The shaft sections in these diagrams are noted with the same letters as those in figure (XI-2), namely ABC for the generator rotor, DEFG for the main shaft and HI for the crankshaft overhang. The following is the summary of procedures followed to get these curves.

i) Static loading diagrams

Each shaft is divided into small sections, at the centre of each of which is assumed to act its weight. The load

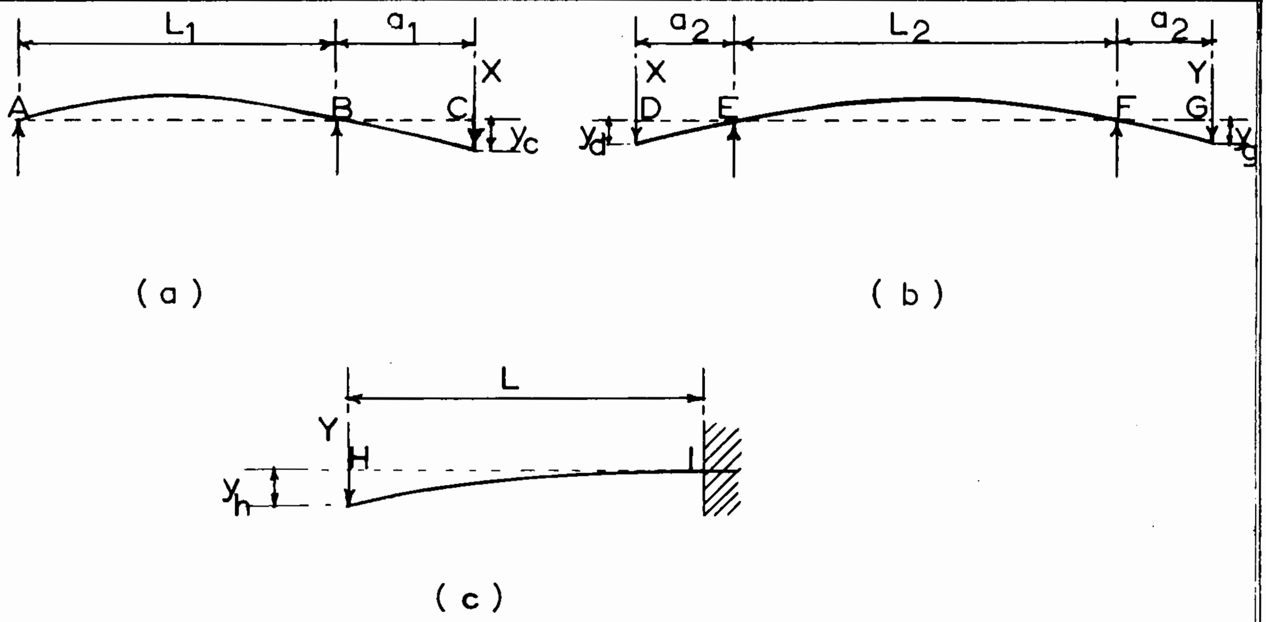


Figure (XI-1)

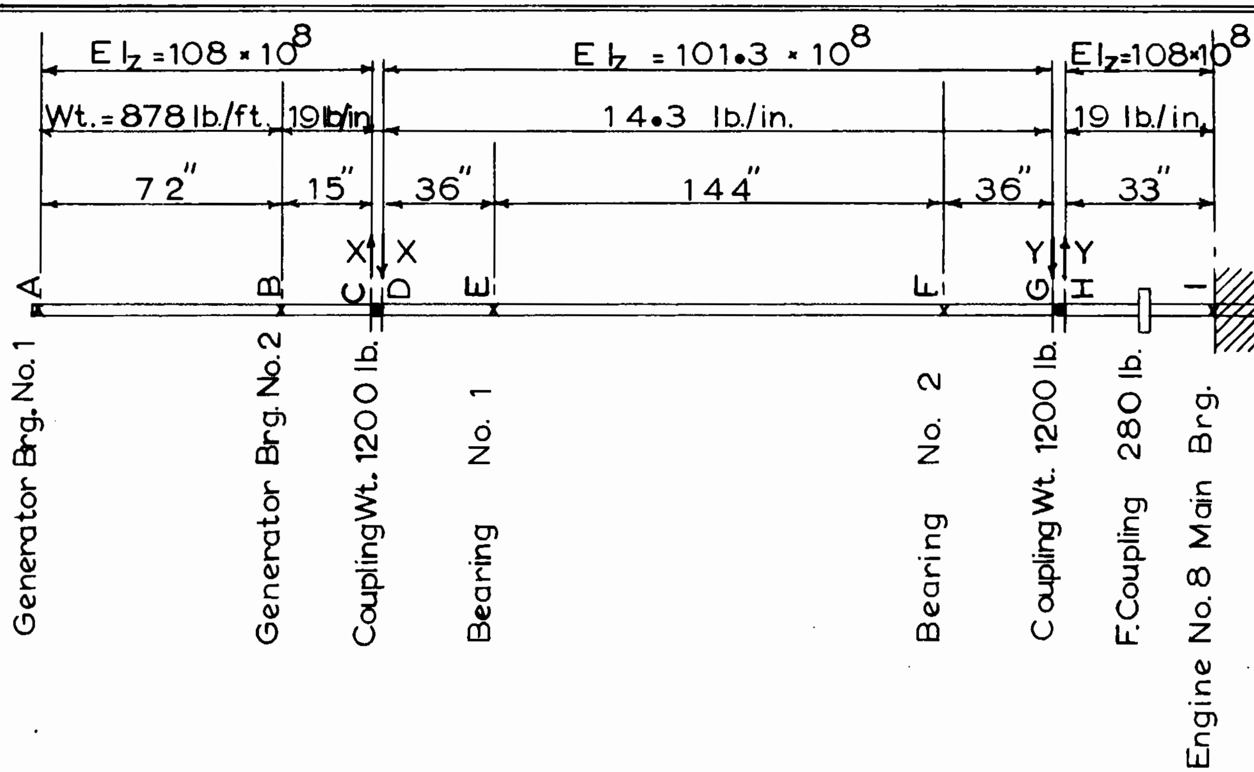


Figure (XI-2)

acting at each section is given in the diagrams. It may be noted here that for the lowest critical whirling speed, the loads of alternate span act in opposite directions. The directions of the loads also are illustrated in the diagrams.

ii) Coupled static deflection curves

The curves were obtained by using the static loading values and going through the steps similar to those described for the dynamic deflection curves below. The detailed drawings for obtaining these curves have been omitted.

iii) Inertia loadings

If a frequency, w, of 386 is assumed, the centrifugal force acting at each point is given by Wy, where W is the weight and y is the assumed deflection at that point. The static deflection curve described in (ii) above is taken as the assumed deflection curve so that the inertia loads can be obtained as the product of the section weight given in the static loading diagram and the corresponding coupled static deflection.

Note: It should be noted that, to save space, the curves for both uncoupled and coupled systems are given on the same sheet. The continuous lines and curves give the values for uncoupled system and dotted lines and curves give the values for coupled system, respectively.

iv) Dynamic bending moment diagrams (uncoupled)

These were obtained by using the dynamic loading values for uncoupled system and going through the steps of graphical statics. The corresponding load vector diagrams are given on the right of the B.M. diagrams. The diagrams are shown with

Bow's notation and no detailed explanations are necessary here. The lower B.M. diagrams are simply the upper ones actuated for integration to get slope diagrams.

v) Dynamic slope and deflection diagrams (uncoupled)

Having obtained the B.M. diagrams, drawing the slope and deflection curves should give no difficulty; detailed steps are given in the reference books.(4),(8),(10),21).

vi) Coupled inertia loadings

Since we have assumed ball-joints at the couplings, deflections at the points C and H must, when coupled, be respectively equal to the deflections at the points D and G. The deflections at these points in the diagrams so far obtained are not equal. There must, therefore, be forces X and Y acting at C, D and G, H as shown in figure (XI-2), when the shaft sections are coupled. The values of X and Y could be obtained as follows:-

(a) Deflection at C

Deflection due to X, from equation (XI-2),

$$y_{c_x} = -\frac{a_1^2 (a_1 + L_1) X}{3 E I_z} = -\frac{15^2 \times 87 X}{3 \times 108 \times 10^8} = -0.0000006 X$$

So that, with the uncoupled deflection value from Diagram (A. IV-1):

$$\text{Total deflection at C} = -0.00000051 - 0.0000006 X \quad \ldots. (1)$$

(b) Deflection at D

Deflection due to X and Y, from equation (XI-3), is:

$$y_{d_{x-y}} = \frac{a_2^{\,2}\,(a_2 + L_2)\,X}{3\,E\,I_z} + \frac{a_2^{\,2}\,L_2\,Y}{6\,E\,I_z}$$

$$= \frac{36^2 \times 180}{3 \times 101.3 \times 10^8}\,X + \frac{36^2 \times 144}{6 \times 101.3 \times 10^8}\,Y$$

$$= 0.00000768\,X + 0.00000307\,Y.$$

Using the value of uncoupled deflection from Diagram (A. IV-2):

Total deflection at D = 0.00000768 X + 0.00000307 Y - 0.0000466 .... (2)

(c) Deflection at G

Through the steps similar to those in (b), we get:

Total deflection at G = 0.00000307 X + 0.00000768 Y - 0.0000448 .... (3)

(d) Deflection at H

Deflection due to Y, from equation (XI-4), is:

$$y_{h_y} = -\frac{L^3\,Y}{3\,E\,I_z} = -\frac{33^3}{3 \times 108 \times 10^8}\,Y = -0.0000011\,Y$$

Hence, with the uncoupled deflection from Diagram (A. IV-3):

Total deflection at H = - 0.0000011 Y - 0.00000197 ... (4)

Equating (1) to (2), (3) to (4) and solving the resulting equations, we get:

X = 3.68 lb. and Y = 3.59 lb.

vii) Coupled dynamic deflection curve

With the values of X and Y obtained above, dynamic loading diagrams were corrected accordingly. The steps described in previous paragraphs were then repeated until the coupled dynamic deflection curves were obtained.

XI-2-3 Lowest Critical Whirling Speed

The ratios of the deflections on the assumed curve to the dynamic deflections, at the points of maximum deflection, are obtained below from the above diagrams. The ratios at maximum deflection points only are considered here.

At point of max. deflection of span AB, $\frac{y}{y'} = \frac{31 \times 10^{-4}}{8 \times 10^{-6}} = 388.$

" " " " " " " BE, $\frac{y}{y'} = \frac{27 \times 10^{-4}}{7 \times 10^{-6}} = 368$

" " " " " " " EF, $\frac{y}{y'} = \frac{52 \times 10^{-4}}{14 \times 10^{-6}} = 371$

" " " " " " " FI, $\frac{y}{y'} = \frac{23.5 \times 10^{-4}}{6 \times 10^{-6}} = 392$

which gives an average value of $\frac{y}{y'} = 384.$

If this average value is taken, the approximate value of lowest critical speed of whirling is from equation (XI-1):

$$N_c = 187.5 \sqrt{\frac{y}{y'}} = 187.5 \sqrt{384} = 3675 \text{ rpm.}$$

XII CONCLUSION

Some of official limitations laid down for a power transmission system were mentioned in the introduction at the beginning of this thesis. The following discusses in detail, the compatibility of the results obtained in the foregoing analysis for our transmission system with these standards.

1. Maximum Allowable Vibration Stresses

The Lloyd's Register of shipping mentions in its rule(25) that within speed limits of $N_s/1.075$ and $1.075\ N_s$, N_s being the full load rpm., the vibration stresses in the crankshaft and transmission shafting should not exceed the values given by

$$S'_{s_c} = \pm (4400 - 70\ d) \qquad \text{............... (XII-1)}$$

where S'_{s_c} = maximum value of vibration stress for continuous operation within the speed range specified above, psi.

d = shaft diameter, inches.
(In the case of crankshafts, d is the diameter of crankpin or journal whichever is smaller).

The same rule gives maximum allowable value of vibration stresses, in psi., due to transient critical speeds which have to be passed through in starting and stopping by:-

$$S'_{s_t} = \pm 3.75\ S'_{s_c} \qquad \text{............... (XII-2)}$$

Applying these equations to our system, where $d = 9.25$ in., we get the following allowable stress values:-

$$S'_{s_c} = \pm(4400 - 70 \times 9.25) = \pm 3752.5 \text{ psi.}$$

for the speed range of 289 rpm to 322 rpm.

$$S'_{s_t} = \pm 3752.5 \times 3.75 = \pm 14071 \text{ psi.}$$

The maximum stresses occurring in the system are given by figures (A. III-1b) and (A. III-2b) for one- and two-node vibration, respectively, in Appendix III. Since these occur at different points and the stresses in the two-node vibration are small, it is customary not to add the two, except in some extreme cases. Now, from figure (A. III-1b), maximum stress occurring in the hallow transmission shaft, within the speed range found above is:

$$S_{s_c} = \pm 1300 \times 1.07 = \pm 1391 \text{ psi.}$$

Again from figure (A. III-1b), maximum vibration stress at transient critical speed of 126 rpm is:

$$S_{s_t} = \pm 4950 \times 1.07 = \pm 5297 \text{ psi.}$$

Both of these values are below the limitations.

Note: In our case, the stresses in the hallow shaft at the point considered above of the system will be even smaller, if the stresses due to two-node criticals are taken into account. The total maximum stress occurring at the nodal point in the crankshaft between the cylinders 1 and 2, which has the second highest stresses, is much lower than the above values.

2. Light Flickering Level

The maximum values of cyclic irregularity C_c for allowable light flickering level, set down by the British Standards Institution[24], have been discussed in Section III and calculated for our engine as equal to $^1/75$ in (III-2-2).

The approximate value of cyclic irregularity C_c, actually occurring in our system was found to be equal to 0.000036, which is very much less than the above maximum limitation.

3. Maximum Vibration Torque at the Generator Armature

The Lloyd's Register of shipping recommends reducing the vibration torques applied at the generator armature to the lowest value and sets the following limitations for ordinary armatures:-

(a) Not more than twice full load engine torque over the speed range ± 7-1/2 per cent on each side of the full load revolutions.

(b) Not more than six times full load engine torque in passing through transient criticals.

Full load engine torque in our case is, from Section (IX-2-4), 286000 lb.-in.

Now, if we take the maximum stress obtained in (1) above = ± 1391 psi., which is the same as ± 1300 psi. in a solid shaft of diameter d_e,

Maximum vibration torque at the armature

$$= \pm 1300 \times \frac{\pi}{16} \times 9.25^3 = \pm 202{,}000 \text{ lb.-in.}$$

which is only about 70 per cent. of engine full-load torque.

Again, at the transient critical speed of 126 rpm., we have:

Maximum torque at the armature

$$= \quad 4950 \times \frac{\pi}{16} \times 9.25^3 \quad = \quad 769000 \text{ lb.-in.}$$

which is about 2.7 times the full load engine torque.

4. Whirling Speed of Engine-Generator System

It is customary to design so that the operating speed of the system is removed at least 20 per cent from any critical speed. The lowest critical speed of whirling of the system under consideration has been found in Section (XI) approximately. The value was found to be relatively far away up from the operating speed of the system so that there may not be any trouble due to whirling.

It is thus concluded that the system possess satisfactory operating properties in respect to the limitations laid down by the authorities.

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APPENDIX I

ENGINE AND GENERATOR DATA

1. The following is a list of data on the engine supplied by the manufacturer, in addition to the drawings given in Appendix V.

ENGINE

B.H.P. per cylinder.	160	
Compression ratio. . . . . (Approximately)	1:14	
Stroke	22.4	in.
r.p.m.	300	
Diameter of piston	13.4	in.
Area of cylinder bore.	140.6	$in.^2$
Weight of piston complete.	536	lb.
Total weight of a connecting rod complete (including bottom end bolts)	438	lb.
Length of a connecting rod (center to center of bearings)	135	cm.
Distance of C.G. of connecting rod from center of crank pin.	52.1	cm.

SCAVENGE PUMP

(a) Top piston and center rod, etc.

Stroke	24	cm.
Total reciprocating weight	442	lb.
Total rotating weight. Scavenge shaft rot. parts	+92	lb.
Net area of top piston	918	$in.^2$
Length of the connecting rods (1 center and 2 side) between centers of the bearings . . .	82	cm.

(b) Lower piston and side rods, etc.

Total reciprocating weight. . . . . 40 lb.

Total rotating weight . Scavenge shaft rot.parts .+170 lb.

2. The following is a list of data on the generator.

GENERATOR

Model	API -24
r.p.m.	300
Power Factor.	0.8
K.W.	836
K.V.A.	1045
Moment of inertia of the rotor.	13800 lb.in.sec^2.
Weight of stator.	4750 lb.
Weight of rotor	3900 lb.
Span between the centers of bearings.	72 in.

APPENDIX II

TABLES OF RESULTS

Table (1)

Harmonic Order		Expression for H_a						
1	Sin α		$+ K/4$		$+ K^3/16$		$+ 15K^5/512$	
2	Sin 2α	$- 1/2$				$- K^4/32$		$- K^6/32$
3	Sin 3α		$- 3K/4$		$- 9K^3/32$		$- 81K^5/512$	
4	Sin 4α			$- K^2/4$		$- K^4/8$		$- K^6/16$
5	Sin 5α				$+ 5K^3/32$		$+ 75K^5/512$	
6	Sin 6α					$+ 3K^4/32$		$+ 3K^6/32$

Table (2)

Harmonic Order		Expression for H_b			
1	Sin α	1.0			
2	Sin 2α		$+ K/2$	$+ K^3/8$	$+ 15K^5/256$
4	Sin 4α			$+ K^3/16$	$+ 3K^5/64$
6	Sin 6α				$+ K^5/85.3$

Table (3)

Harmonic Order		Expression for H_c		
2	Sin 2α	$- K^2/2$		$+ K^6/32$ +
4	Sin 4α		$+ K^4/4$	$+ K^6/8$ +
6	Sin 6α			$- 3K^6/32$ -

Table (4)	Inertia Torque Correction						
Harmonic Order		H_a	$a_n = W_c H_a$ psi	Harmonic Order		H_a	$a_n = W_c H_a$ psi
1	Sin α	0.053348	7.6554	4	Sin 4α	-0.0113842	-1.6336
2	Sin 2α	-0.50000648	-71.7509	5	Sin 5α	0.0015318	0.2198
3	Sin 3α	-0. 160955	-23.0970	6	Sin 6α	0.0001945	0.0279

Table (5)	Dead Weight Correction for Reciprocating Part						
Harmonic Order		H_b	$a_n = \frac{W_r H_b}{A}$ psi	Harmonic Order		H_b	$a_n = \frac{W_r H_b}{A}$ psi
1	Sin α	1	5	4	Sin 4α	-0.0006073	-0.0030365
2	Sin 2α	0.106696	0.5335	6	Sin 6α	0.0000052	0.00002595

Table (6)

Moment of Unbalanced Rotating Weight

Mass	Weight lb.	Moment Arm in.	Moment lb.-in.
Connecting rod. rot. mass	269	11.2	3012.80
Crank pin	141.5	11.2	1584.80
W_I	144.18	9.1	1312.04
W_{II}	31.62	13.64	431.29
W_{III}	3.882	15.77	61.22
W'_{IV}	31.88	13.57	432.61
W''_{IV}	10.58	12.104	128.06
Total	632.642		6962.82

Table (7)

Corrected a_n Values of First Six Harmonics

Back Reference	Correction	Sin α	Sin 2α	Sin 3α	Sin 4α	Sin 5α	Sin 6α
Table (4)	Inertia Torque of reciprocating weight	7.6554	-71.7509	-23.0970	-1.6336	0.2198	0.0279
Table (5)	Dead Weight correction for reciprocating part	5.0000	0.5335	-	-0.0030	-	0.00003
Page 19	Dead Weight correction for unbalanced rotating part	4.4091	-	-	-	-	-
	Resultant correction	17.0645	-71.2174	-23.0970	-1.6366	0.2198	0.02793
	Gas pressure a_n	61.9930	66.8010	47.0300	22.2400	13.8040	9.5660
	Corrected values of a_n	79.0575	-4.4164	23.9330	20.6034	14.0238	9.59393

Table (8)

Resultant Harmonic Coefficients of Engine Torque

Harmonic Order n	Sine Coefficient a_n	Cosine Coefficient b_n	$T_n = \sqrt{a_n^2 + b_n^2}$
0	-	25.298	25.298
1	79.0575	29.315	84.319
2	- 4.4164	0.882	4.505
3	23.9330	- 4.911	24.310
4	20.6034	- 6.667	21.655
5	14.0238	- 8.850	16.585
6	9.5939	- 6.566	11.626
7	9.5660	- 6.141	11.368
8	5.0450	- 5.594	7.533
9	2.8880	- 3.685	4.682
10	1.7770	- 3.503	3.928
11	0.2450	- 2.703	2.714
12	0.0850	- 1.715	1.717
13	- 0.1800	- 1.700	1.710
14	- 0.5290	- 1.120	1.283
15	- 0.2670	- 0.765	0.810
16	- 0.2700	- 0.768	0.814
17	- 0.1800	- 0.560	0.588
18		- 0.248	0.248

Table (9 - a)										
One-Node Vibration First Try, $w = 90$ rad./sec.; $w^2 = 8100$ rad.^{2}/sec.^{2}										
A	B	C	D	E	F	G	H	I	J	K
Mass	Equiv't dia.	Equiv't length	Moment of inertia	Torque per unit deflection	Deflection in plane of mass	Torque in mass	Total torque	Shaft stiffness	Change in deflection	Stress for 1 deg. defln. at Damper
	d_e in.	L_e in.	J lb.in.sec.^{2}	$J \cdot w^2/10^6$ lb.in./rad.	a radians	$J \cdot w^2 \cdot a/10^6$ lb.in.	$\Sigma\ J \cdot w^2 a/10^6$ lb.in.	C lb.in./rad.	Col.H/Col.I radians	Col.H/11.25d^3 psi.
Damper	9.25	95.9	617	4.9977	1.0000	4.9977	4.9977	90×10^6	0.0555	
Sc.Pump	9.25	62.1	130	1.0530	0.9445	0.9946	5.9923	139×10^6	0.0431	
Cyl.No.1	9.25	27.3	400	3.2400	0.9014	2.9205	8.9128	316×10^6	0.0282	
" "2	9.25	27.3	400	3.2400	0.8732	2.8292	11.7420	316×10^6	0.0372	
" "3	9.25	27.3	400	3.2400	0.8360	2.7086	14.4506	316×10^6	0.0457	
" "4	9.25	27.3	400	3.2400	0.7903	2.5606	17.0112	316×10^6	0.0538	
" "5	9.25	27.3	400	3.2400	0.7365	2.3863	19.3975	316×10^6	0.0614	
" "6	9.25	27.3	400	3.2400	0.6751	2.1873	21.5848	316×10^6	0.0683	
" "7	9.25	280.17	400	3.2400	0.6068	1.9660	23.5508	30.8×10^6	0.7646	
Generator	-	-	13800	111.7800	-0.1578	-17.6389	5.9119	-	-	-

Table (9 - b)										
One-Node Vibration Second Try, w = 94.3 rad./sec.; w^2 = 8900 rad.^{2}/sec.^{2}										
A	B	C	D	E	F	G	H	I	J	K
Mass	Equiv't dia.	Equiv't length	Moment of inertia	Torque per unit deflection	Deflection in plane of mass	Torque in plane of mass	Total torque	Shaft stiffness	Change in deflection	Stress for 1 deg.defln. at Damper
	d_e in.	L_e in.	J lb.in.sec.^{2}	$J \cdot w^2/10^6$ lb.in./rad.	a radians	$J \cdot w^2 \cdot a/10^6$ lb.in.	$\Sigma\ J \cdot w^2 a/10^6$ lb.in.	C lb.in./rad.	Col.H/Col.I radians	Col.H/11.25d^3 psi.
Damper	9.25	95.9	617	5.4913	1.0000	5.4913	5.4913	90×10^6	0.0610	
Sc.Pump	9.25	62.1	130	1.1570	0.9390	1.0864	6.5777	139×10^6	0.0473	
Cyl.No.1	9.25	27.3	400	3.5600	0.8917	3.1744	9.7521	316×10^6	0.0309	
" " 2	9.25	27.3	400	3.5600	0.8608	3.0644	12.8165	316×10^6	0.0406	
" " 3	9.25	27.3	400	3.5600	0.8202	2.9199	15.7364	316×10^6	0.0498	
" " 4	9.25	27.3	400	3.5600	0.7704	2.7426	18.4790	316×10^6	0.0585	
" " 5	9.25	27.3	400	3.5600	0.7119	2.5344	21.0134	316×10^6	0.0665	
" " 6	9.25	27.3	400	3.5600	0.6454	2.2976	23.3110	316×10^6	0.0738	
" " 7	9.25	280.17	400	3.5600	0.5716	2.0349	25.3459	30.8×10^6	0.8223	
Generator	-	-	13800	122.8200	-0.2507	-30.7910	-5.4451	-	-	-

Table (9)

One-Node Vibration

Third and Final Try, $w = 92.36$ rad./sec.; $w^2 = 8530$ rad.^{2}/sec.^{2}

A	B	C	D	E	F	G	H	I	J	K
Mass	Equiv't dia.	Equiv't length	Moment of inertia	Torque per unit deflection	Deflection in plane of mass	Torque in plane of mass	Total torque	Shaft stiffness	Change in deflection	Stress for 1 deg. defln. at Damper
	d_e in.	L_e in.	J lb.-in.-sec.^{2}	$J \cdot w^2/10^6$ lb.-ins./rad.	a radians	$J \cdot w^2 \cdot a/10^6$ lb.-in.	$\Sigma J \cdot w^2 a/10^6$ lb.-in.	C lb.-in./rad.	Col.H/Col.I radians	Col.H/11.25d^3 psi.
Damper	9.25	95.9	617	5.2630	1.0000	5.2630	5.2630	90×10^6	0.0585	591.1
Sc.Pump	9.25	62.1	130	1.1089	0.9415	1.0440	6.3070	139×10^6	0.0454	708.3
Cyl.No.1	9.25	27.3	400	3.4120	0.8961	3.0575	9.3645	316×10^6	0.0296	1051.7
" " 2	9.25	27.3	400	3.4120	0.8665	2.9565	12.3210	316×10^6	0.0390	1383.8
" " 3	9.25	27.3	400	3.4120	0.8275	2.8234	15.1444	316×10^6	0.0479	1700.9
" " 4	9.25	27.3	400	3.4120	0.7796	2.6600	17.8044	316×10^6	0.0563	2000.0
" " 5	9.25	27.3	400	3.4120	0.7233	2.4679	20.2723	316×10^6	0.0642	2276.8
" " 6	9.25	27.3	400	3.4120	0.6591	2.2489	22.5212	316×10^6	0.0713	2529.3
" " 7	9.25	280.17	400	3.4120	0.5878	2.0056	24.5268	30.8×10^6	0.7963	2754.6
Generator	-	-	13800	117.7140	-0.2085	-24.5434	-0.0166	-	-	-

Table (10 - a)

Two-Node Vibration

First Try, w = 252 rad./sec.; w^2 = 63500 $rad.^2/sec.^2$

A	B	C	D	E	F	G	H	I	J	K
Mass	Equiv't dia.	Equiv't length	Moment of inertia	Torque per unit deflection	Deflection in plane of mass	Torque in plane of mass	Total torque	Shaft stiffness	Change in deflection	Stress for 1 deg. defln. at Damper
	d_e in.	L_e in.	J $lb.in.sec.^2$	$J.w^2.a/10^6$ lb.in./rad.	a radians	$J.w^2.a/10^6$ lb.in.	$\Sigma J.w^2 a/10^6$ lb.in.	C lb.in./rad.	Col.H/Col.I radians	Col.H/1.25 d^3 psi
Damper	9.25	95.9	617	39.1795	1.0000	39.1795	39.1795	90×10^6	0.4353	
Sc.Pump	9.25	62.1	130	8.2550	0.5647	4.6616	43.8411	139×10^6	0.3154	
Cyl.No.1	9.25	27.3	400	25.4000	0.2493	6.3322	50.1733	316×10^6	0.1588	
" " 2	9.25	27.3	400	25.4000	0.0905	2.2987	52.4720	316×10^6	0.1661	
" " 3	9.25	27.3	400	25.4000	-0.0756	- 1.9202	50.5518	316×10^6	0.1500	
" " 4	9.25	27.3	400	25.4000	-0.2356	- 5.9842	44.5676	316×10^6	0.1410	
" " 5	9.25	27.3	400	25.4000	-0.3766	- 9.5656	35.0020	316×10^6	0.1108	
" " 6	9.25	27.3	400	25.4000	-0.4874	-12.3800	23.6220	316×10^6	0.0716	
" " 7	9.25	280.17	400	25.4000	-0.5590	-14.1986	8.4234	30.8×10^6	0.2735	
Generator	-	-	13800	876.3000	-0.8325	-729.5198	-721.0964	-	-	-

Table (10 - b)

Two-Node Vibration

Second Try, w = 270 rad./sec.; w^2 = 72900 rad.^{2}/sec.^{2}

A	B	C	D	E	F	G	H	I	J	K
Mass	Equiv't dia.	Equiv't length	Moment of inertia	Torque per unit deflection	Deflection in plane of mass	Torque in plane of mass	Total torque	Shaft stiffness	Change in deflection	Stress for 1 deg.defln. at Damper
	d_e in.	L_e in.	J lb.-in.-sec.^{2}	$J \cdot w^2/10^6$ lb.-in./rad.	a radians	$J \cdot w^2 \cdot a/10^6$ lb.-in.	$\Sigma J w^2 \cdot a/10^6$ lb.-in.	C lb.-in./rad	Col.H/Col.I radians	Col.H/11.25d^3 psi
Damper	9.25	95.9	617	44.9793	1.0000	44.9793	44.9793	90×10^6	0.4998	
Sc.Pump	9.25	62.1	130	9.4770	0.5002	4.7404	49.7197	139×10^6	0.3577	
Cyl.No.1	9.25	27.3	400	29.1600	0.1425	4.1553	53.8750	316×10^6	0.1705	
" " 2	9.25	27.3	400	29.1600	-0.0280	- 0.8165	53.0585	316×10^6	0.1679	
" " 3	9.25	27.3	400	29.1600	-0.1959	- 5.7124	47.3461	316×10^6	0.1499	
" " 4	9.25	27.3	400	29.1600	-0.3458	-10.0835	37.2626	316×10^6	0.1179	
" " 5	9.25	27.3	400	29.1600	-0.4637	-13.5215	23.7411	316×10^6	0.0751	
" " 6	9.25	27.3	400	29.1600	-0.5388	-15.7114	8.0297	316×10^6	0.0254	
" " 7	9.25	280.17	400	29.1600	-0.5642	-16.4521	- 8.4224	30.8×10^6	-0.2735	
Generator	-	-	13800	1006.0200	-0.2907	-292.4500	-300.8724	-	-	-

Table (10 - c)

Two-Node Vibration

Third Try, $w = 278.8$ rad./sec.; $w^2 = 77720$ rad.^{2}/sec.^{2}

A	B	C	D	E	F	G	H	I	J	K
Mass	Equiv't dia.	Equiv't length	Moment of inertia	Torque per unit deflection	Deflection in plane of mass	Torque in plane of mass	Total torque	Shaft stiffness	Change in deflection	Stress for 1 deg.defln. at Damper
	d_e in.	L_e in.	J lb.-in.-sec.^{2}	$J \cdot w^2/10^6$ lb.-in./rad.	a radians	$J \cdot w^2 \cdot a/10^6$ lb.-in.	$\Sigma J \cdot w^2 \cdot a/10^6$ lb.-in.	C lb.-in./rad.	Col.H/Col.I radians	Col.H/11.25d^3 psi
Damper	9.25	95.9	617	47.9532	1.0000	47.9532	47.9532	90×10^6	0.5328	
Sc.Pump	9.25	95.9	130	10.1036	0.4672	4.7204	52.6736	139×10^6	0.3790	
Cyl.No.1	9.25	27.3	400	31.0880	0.0882	2.7420	55.4156	316×10^6	0.1754	
" " 2	9.25	27.3	400	31.0880	-0.0872	- 2.7109	52.7047	316×10^6	0.1668	
" " 3	9.25	27.3	400	31.0880	-0.2540	- 7.8964	44.8083	316×10^6	0.1418	
" " 4	9.25	27.3	400	31.0880	-0.3958	-12.3046	32.5037	316×10^6	0.1029	
" " 5	9.25	27.3	400	31.0880	-0.4987	-15.5036	17.0001	316×10^6	0.0538	
" " 6	9.25	27.3	400	31.0880	-0.5525	-17.1761	- 0.1760	316×10^6	-0.0006	
" " 7	9.25	280.17	400	31.0880	-0.5519	-17.1575	- 17.3335	30.8×10^6	-0.5628	
Generator	-	-	13800	1072.5360	-0.0109	-11.6906	- 29.0241	-	-	-

x

Table (10)

Two-Node Vibration

Fourth and Final Try, $w = 278.92$ rad./sec.; $w^2 = 77800$ rad.^{2}/sec.^{2}

A	B	C	D	E	F	G	H	I	J	K
Mass	Equiv't dia.	Equiv't length	Moment of inertia	Torque per unit deflection	Deflection in plane of mass	Torque in plane of mass	Total torque	Shaft stiffness	Change in deflection	Stress for 1 deg. defln. at Damper
	d_e in.	L_e in.	J lb.-in.-sec.^{2}	$J \cdot w^2/10^6$ lb.-in./rad.	a radians	$J \cdot w^2 \cdot a/10^6$ lb.-in.	$\Sigma\, J \cdot w^2 \cdot a/10^6$ lb.-in.	C lb.-in./rad.	Col.H/Col.I radians	Col.H/1.125d^3 psi
Damper	9.25	95.9	617	48.0026	1.0000	48.0026	48.0026	90×10^6	0.5334	5391.1
Sc.Pump	9.25	62.1	130	10.1140	0.4666	4.7192	52.7218	139×10^6	0.3793	5921.1
Cyl.No.1	9.25	27.3	400	31.1200	0.0873	2.7168	55.4386	316×10^6	0.1754	6226.3
" " 2	9.25	27.3	400	31.1200	-0.0881	- 2.7417	52.6969	316×10^6	0.1668	5918.3
" " 3	9.25	27.3	400	31.1200	-0.2549	- 7.9325	44.7644	316×10^6	0.1417	5027.4
" " 4	9.25	27.3	400	31.1200	-0.3966	-12.3422	32.4222	316×10^6	0.1026	3641.3
" " 5	9.25	27.3	400	31.1200	-0.4992	-15.5351	16.8871	316×10^6	0.0534	1896.6
" " 6	9.25	27.3	400	31.1200	-0.5526	-17.1969	- 0.3098	316×10^6	-0.0010	- 34.8
" " 7	9.25	280.17	400	31.1200	-0.5516	-17.1658	-17.4756	30.8×10^6	-0.5674	-1962.7
Generator	-	-	13800	1073.6400	0.0158	16.9640	0.5116	-	-	-

Table (11)			
Critical Speeds $N_c = F/n$			
One-Node		Two-Node	
Harmonic Orders	Critical Speeds N_c	Harmonic Orders	Critical Speeds N_c
1	882	1	2664
2	441	2	1332
3	294	3	888
4	221	4	666
5	176	5	533
6	147	6	444
7	126	7	381
8	110	8	333
9	98	9	296
10	88	10	266
11	80	11	242
12	74	12	222
13	68	13	205
14	63	14	190
15	59	15	178
16	55	16	167
17	52	17	157
18	49	18	148
19	46	19	140
20	44	20	133
21	42	21	127

Table (12) Engine Amplitudes for One-Node Vibration

a_1 = 0.9014	a_2 = 0.8732	a_4 = 0.7903	a_6 = 0.6751
	a_3 = 0.8360	a_5 = 0.7365	a_7 = 0.6068
	a_2+a_3 = 1.7092	a_4+a_5 = 1.5268	a_6+a_7 = 1.2819
	a_2-a_3 = 0.0372	a_4-a_5 = 0.0538	a_6-a_7 = 0.0683
	a_3-a_2 = -0.0372	a_5-a_4 = -0.0538	a_7-a_6 = -0.0683

Table (13) Engine Amplitudes for Two-Node Vibration

a_1 = 0.0873	a_2 = -0.0881	a_4 = -0.3966	a_6 = 0.5526
	a_3 = - 0.2549	a_5 = -0.4992	a_7 = -0.5526
	a_2+a_3 = -0.3430	a_4+a_5 = -0.8958	a_6+a_7 = -1.1042
	a_2-a_3 = 0.1668	a_4-a_5 = 0.1026	a_6-a_7 = -0.0010
	a_3-a_2 = -0.1668	a_5-a_4 = -0.1026	a_7-a_6 = 0.0010

Table (14) Summary of $\Sigma \vec{a}_e$ Values

One-Node		Two-Node	
Harmonic Orders	$\Sigma \vec{a}_e$	Harmonic Orders	$\Sigma \vec{a}_e$
1, 8, 15, etc. and 6, 13, 20, etc.	0.0713	1, 8, 15, etc. and 6, 13, 20, etc.	0.3094
2, 9, 16, etc. and 5, 12, 19, etc.	0.0475	2, 9, 16, etc. and 5, 12, 19, etc.	0.0848
3, 10, 17, etc. and 4, 11, 18, etc.	0.4808	3, 10, 17, etc. and 4, 11, 18, etc.	1.0950
7, 14, 21, etc.	5.4193	7, 14, 21, etc.	2.4303

Table (15) $\Sigma J a^2$ for One-Node Vibration

Mass	J	a	a^2	$J \cdot a^2$
	Table 9	Table 9		Lb-Ins-Sec.^{2}
Damper	617	1.0000	1.0000	617
Sc. Pump	130	0.9415	0.8864	115.2
Cyl. No. 1	400	0.8961	0.8030	321.2
" " 2	400	0.8665	0.7508	300.3
" " 3	400	0.8275	0.6848	273.9
" " 4	400	0.7796	0.6078	243.1
" " 5	400	0.7233	0.5232	209.3
" " 6	400	0.6591	0.4344	173.4
" " 7	400	0.5878	0.3455	138.2
Generator	13800	-0.2085	0.0435	600.3
			$\Sigma J \cdot a^2$ =	2991.9

Table (16) $\Sigma J a^2$ for Two-Node Vibration

Mass	J	a	a^2	$J \cdot a^2$
	Table 10	Table 10		Lb-Ins-Sec.^{2}
Damper	617	1.0000	1.0000	617
Sc. Pump	130	0.4666	0.2178	28.3
Cyl. No. 1	400	0.0873	0.0076	3.1
" " 2	400	0.0881	0.0078	3.1
" " 3	400	0.2549	0.0650	26.0
" " 4	400	0.3966	0.1573	62.9
" " 5	400	0.4992	0.2492	99.7
" " 6	400	0.5526	0.3054	122.2
" " 7	400	0.5516	0.3043	121.7
Generator	13800	0.0158	0.00025	3.5
			$\Sigma J \cdot a^2$ =	1087.5

Table (17)

Equilibrium Stresses

One-Node Vibration

Harmonic Order n	Critical Speed N_c R.P.M.	Reslt.Harc. Component per cycle T_n	Vector Sum of engine Amplitudes $\Sigma \vec{a}_e$	Reslt.Harc. Component all cycles $T_n \cdot \Sigma \vec{a}_e$	Equilibrium Amplitude θ_o Degrees	Equilibrium Stress (Max.) S_{so}, psi.
	F/n Table (11)	Table (8)	Table (14)		Equation VIII - 6	Highest Value from Col. K of Table 9 x θ_o
1	882	84.319	0.0713	6.012	0.02130	58.67
2	441	4.505	0.0475	0.214	0.00076	2.09
3	294	24.310	0.4808	11.693	0.04139	114.03
4	221	21.655	0.4808	10.416	0.03687	101.56
5	176	16.583	0.0475	0.788	0.00279	76.85
6	147	11.626	0.0713	0.829	0.00294	7.70
7	126	11.368	5.4193	61.603	0.21808	600.72
8	110	7.533	0.0713	0.517	0.00183	5.04
9	98	4.682	0.0475	0.222	0.00079	2.15
10	88	3.928	0.4808	1.889	0.00687	18.92
11	80	2.714	0.4808	1.305	0.00462	12.73
12	74	1.717	0.0475	0.082	0.00029	0.80
13	68	1.710	0.0713	0.122	0.00043	1.19
14	63	1.238	5.4193	6.709	0.02375	65.22

Table (18)

Equilibrium Stresses

Two-Node Vibration

Harmonic Order n	Critical Speed N_c R.P.M.	Reslt.Harmc. Component per cycle T_n	Vector Sum of Engine Amplitudes $\Sigma \overrightarrow{a_e}$	Reslt.Harmc. Component all cycles $T_n \cdot \Sigma \overrightarrow{a_e}$	Equilibrium Amplitude θ_o Degrees	Equilibrium Stress (Max.) S_{so}, psi.
	F/n Table (11)	Table (8)	Table (14)		Equation VIII - 7	Highest Value from Col. K of Table 10 x θ_o
1	2664	84.319	0.309	26.055	0.0278	173.09
2	1332	4.505	0.085	0.383	0.0004	2.49
3	888	24.310	1.095	26.620	0.0284	176.83
4	666	21.655	1.095	23.712	0.0253	157.53
5	533	16.585	0.085	1.410	0.0015	9.34
6	444	11.626	0.309	3.592	0.0038	23.66
7	381	11.368	2.430	27.624	0.0295	183.68
8	333	7.533	0.309	2.328	0.0025	15.52
9	296	4.682	0.085	0.398	0.0004	2.49
10	266	3.928	1.095	4.301	0.0046	28.64
11	242	2.714	1.095	2.972	0.0032	19.92
12	222	1.717	0.085	0.146	0.0002	1.25
13	205	1.710	0.309	0.528	0.0006	3.74
14	190	1.238	2.430	3.008	0.0032	19.92

Table (19)

Undamped torsional vibration stresses at non-resonant speeds
One-Node Vibration

Harmonic Order n = 1
N_c = 882

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = M S_{so}$
88	0.1	1.01	59.26
176	0.2	1.04	61.02
265	0.3	1.10	64.54
353	0.4	1.19	69.82
529	0.6	1.56	91.53
706	0.8	2.78	163.10

Harmonic Order n = 3
N_c = 294

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = M S_{so}$
29	0.1	1.01	115.17
59	0.2	1.04	118.59
88	0.3	1.10	125.43
118	0.4	1.19	135.70
147	0.5	1.34	153.80
176	0.6	1.56	177.89
206	0.7	1.96	223.50
235	0.8	2.78	317.00
265	0.9	5.26	599.80
324	1.1	4.76	542.78
353	1.2	2.27	258.85
382	1.3	1.45	165.34
412	1.4	1.04	118.59

Harmonic Order n = 4
N_c = 221

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = M S_{so}$
66	0.3	1.10	111.72
88	0.4	1.19	120.86
111	0.5	1.34	136.09
133	0.6	1.56	158.43
155	0.7	1.96	199.06
177	0.8	2.78	282.34
199	0.9	5.26	534.21
243	1.1	4.76	483.43
265	1.2	2.27	230.54
288	1.3	1.45	147.26
310	1.4	1.04	105.62
332	1.5	0.80	81.25
354	1.6	0.64	65.00
376	1.7	0.53	53.83
398	1.8	0.45	45.70
420	1.9	0.38	38.59
440	2.0	0.34	34.53

Continued

Table (19) contd.

Undamped torsional vibration stresses at non-resonant speeds
One-Node Vibration

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration Stress $S_s = MS_{so}$
Harmonic Order $n = 5$, $N_c = 176$			
88	0.5	1.34	102.98
106	0.6	1.56	119.89
123	0.7	1.96	150.63
141	0.8	2.78	213.64
158	0.9	5.26	404.23
194	1.1	4.76	365.81
211	1.2	2.27	174.45
229	1.3	1.45	111.43
246	1.4	1.04	79.92
264	1.5	0.80	61.48
282	1.6	0.64	49.18
299	1.7	0.53	40.73
317	1.8	0.45	34.58
Harmonic Order $n = 6$, $N_c = 147$			
103	0.7	1.96	15.09
118	0.8	2.78	21.41
132	0.9	5.26	40.50
162	1.1	4.76	35.55
177	1.2	2.27	17.48

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration Stress $S_s = MS_{so}$
Harmonic Order $n = 7$, $N_c = 126$			
50	0.4	1.19	714.86
63	0.5	1.34	804.97
76	0.6	1.56	937.12
88	0.7	1.96	1173.41
101	0.8	2.78	1670.00
113	0.9	5.26	3159.79
139	1.1	4.76	2859.43
151	1.2	2.27	1363.63
164	1.3	1.45	871.04
176	1.4	1.04	624.75
189	1.5	0.80	480.58
202	1.6	0.64	384.40
214	1.7	0.53	318.38
227	1.8	0.45	270.32
240	1.9	0.38	228.27
252	2.0	0.34	204.25
315	2.5	0.19	114.14
378	3.0	0.13	78.09
504	4.0	0.07	42.05

Continued

Table (19) contd.

Undamped torsional vibration stresses at non-resonant speeds
One-Node Vibration

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = MS_{so}$
Harmonic Order n = 10, N_c = 88			
44	0.5	1.34	25.35
53	0.6	1.56	29.51
62	0.7	1.96	37.08
70	0.8	2.78	52.60
79	0.9	5.26	99.52
97	1.1	4.76	90.01
106	1.2	2.27	42.95
114	1.3	1.45	27.43
123	1.4	1.04	19.68
132	1.5	0.80	15.14
Harmonic Order n = 11, N_c = 80			
48	0.6	1.56	19.86
56	0.7	1.96	24.95
64	0.8	2.78	35.39
72	0.9	5.26	66.96
88	1.1	4.76	60.60
96	1.2	2.27	28.90
104	1.3	1.45	18.46
112	1.4	1.04	13.24

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = MS_{so}$
Harmonic Order n = 14, N_c = 63			
32	0.5	1.34	87.39
38	0.6	1.56	101.74
44	0.7	1.96	127.83
50	0.8	2.78	181.31
57	0.9	5.26	343.06
69	1.1	4.76	310.45
76	1.2	2.27	148.05
82	1.3	1.45	94.57
88	1.4	1.04	67.83
95	1.5	0.80	52.18
101	1.6	0.64	41.74
107	1.7	0.53	34.57
113	1.8	0.45	29.35
120	1.9	0.38	24.78

Table (20)

Undamped torsional vibration stresses at non-resonant speeds
Two-Node Vibration

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = M S_{so}$
Harmonic Order n = 1, N_c = 2664			
0	0	1	170.39
267	0.1	1.01	174.82
533	0.2	1.04	180.01
Harmonic Order n = 3, N_c = 888			
89	0.1	1.01	178.60
178	0.2	1.04	183.90
266	0.3	1.10	194.51
355	0.4	1.19	210.43
444	0.5	1.34	236.95
Harmonic Order n = 4, N_c = 666			
67	0.1	1.01	159.11
133	0.2	1.04	163.83
200	0.3	1.10	173.28
266	0.4	1.19	187.46
333	0.5	1.34	211.09
400	0.6	1.56	245.75
466	0.7	1.96	266.80

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = M S_{so}$
Harmonic Order n = 5, N_c = 533			
107	0.2	1.04	9.71
160	0.3	1.10	10.27
213	0.4	1.19	11.12
267	0.5	1.34	12.52
320	0.6	1.56	14.57
373	0.7	1.96	18.31
426	0.8	2.78	25.97
479	0.9	5.26	49.13
Harmonic Order n = 6, N_c = 444			
89	0.2	1.04	24.61
133	0.3	1.10	26.03
178	0.4	1.19	28.16
222	0.5	1.34	31.70
266	0.6	1.56	36.91
311	0.7	1.96	46.37
355	0.8	2.78	65.78
400	0.9	5.26	124.45

Continued

Table (20) contd.

Undamped torsional vibration stresses at non-resonant speeds
Two-Node Vibration

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = MS_{so}$
Harmonic Order n = 7, N_c = 381			
114	0.3	1.10	202.05
152	0.4	1.19	218.58
191	0.5	1.34	246.13
229	0.6	1.56	286.54
267	0.7	1.96	360.01
305	0.8	2.78	510.00
343	0.9	5.26	966.16
419	1.1	4.76	874.32
457	1.2	2.27	416.95
Harmonic Order n = 8, N_c = 333			
100	0.3	1.10	17.13
133	0.4	1.19	18.53
167	0.5	1.34	20.86
200	0.6	1.56	24.29
266	0.8	2.78	43.29
300	0.9	5.26	81.90
366	1.1	4.76	74.11
400	1.2	2.27	35.34
433	1.3	1.45	22.58

R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = MS_{so}$
Harmonic Order n = 10, N_c = 266			
133	0.5	1.34	38.38
160	0.6	1.56	44.68
186	0.7	1.96	56.13
213	0.8	2.78	79.62
239	0.9	5.26	150.65
293	1.1	4.76	136.33
319	1.2	2.27	65.01
346	1.3	1.45	41.53
372	1.4	1.04	29.79
399	1.5	0.80	22.91

Continued

Table (20) contd.							
Undamped torsional vibration stresses at non-resonant speeds Two-Node Vibration							
R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = MS_{so}$	R.P.M. N	$\frac{N}{N_c}$	Dynamic magnifier M	Vibration stress $S_s = MS_{so}$
Harmonic Order n = 11 N_c = 242				Harmonic Order n = 14 N_c = 190			
121	0.5	1.34	26.69	95	0.5	1.34	26.69
145	0.6	1.56	31.08	114	0.6	1.56	31.08
170	0.7	1.96	39.04	133	0.7	1.96	39.04
194	0.8	2.78	55.38	152	0.8	2.78	55.38
218	0.9	5.26	104.78	171	0.9	5.26	104.78
266	1.1	4.76	94.82	209	1.1	4.76	94.82
290	1.2	2.27	45.22	228	1.2	2.27	45.22
315	1.3	1.45	28.88	247	1.3	1.45	28.88
339	1.4	1.04	20.72	266	1.4	1.04	20.72
363	1.5	0.80	15.94	285	1.5	0.80	15.94
387	1.6	0.64	12.75	304	1.6	0.64	12.75
411	1.7	0.53	10.56	323	1.7	0.53	10.56

Table (21)				
Torsional vibration stresses at resonant speeds				
One-Node Vibration				
Harmonic order	Critical speed	Equilibrium stress	Dynamic magnifier	Maximum vibration stress
n	N_c Table (11)	S_{so} Table (19)	M_c	$S_{s_{max}} = M_c\ S_{so}$ psi
1	882	58.67	7.514	440.85
2	441	2.09	7.514	15.70
3	294	114.03	7.514	856.82
4	221	101.56	7.514	763.12
5	176	76.85	7.514	577.45
6	147	7.70	7.514	57.86
7	126	600.72	7.514	4513.81
8	110	5.04	7.514	37.87
9	98	2.18	7.514	16.38
10	88	18.92	7.514	142.17
11	80	12.73	7.514	95.65
12	74	0.80	7.514	6.01
13	68	1.19	7.514	8.94
14	63	65.22	7.514	490.06

Table (22)

Torsional vibration stresses at resonant speeds

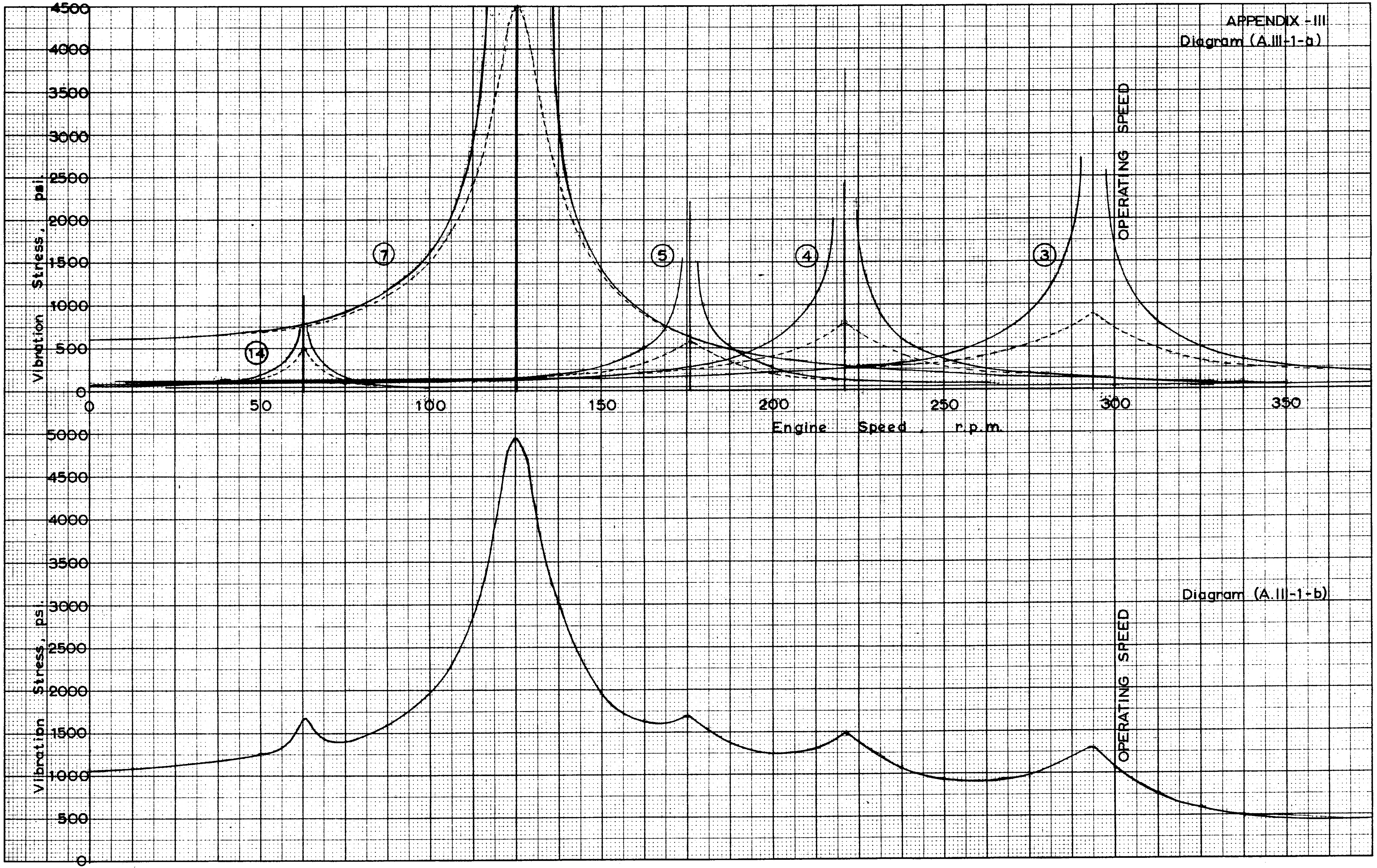
Two-Node Vibration

Harmonic order	Critical speeds	Equilibrium stress	Dynamic magnifier	Maximum vibration stress
n	N_c Table (11)	S_{so} Table (20)	M_c	$S_s = M_c\ S_{so}$ psi
1	2664	173.09	5.96	1031.60
2	1332	2.49	5.96	14.84
3	888	176.83	5.96	1053.90
4	666	157.53	5.96	938.88
5	533	9.34	5.96	55.67
6	444	23.66	5.96	141.01
7	381	183.68	5.96	1094.73
8	333	15.52	5.96	92.50
9	296	2.49	5.96	14.84
10	266	28.64	5.96	170.69
11	242	19.92	5.96	118.72
12	222	1.25	5.96	7.45
13	205	3.74	5.96	22.29
14	190	19.92	5.96	118.72

Table (23)

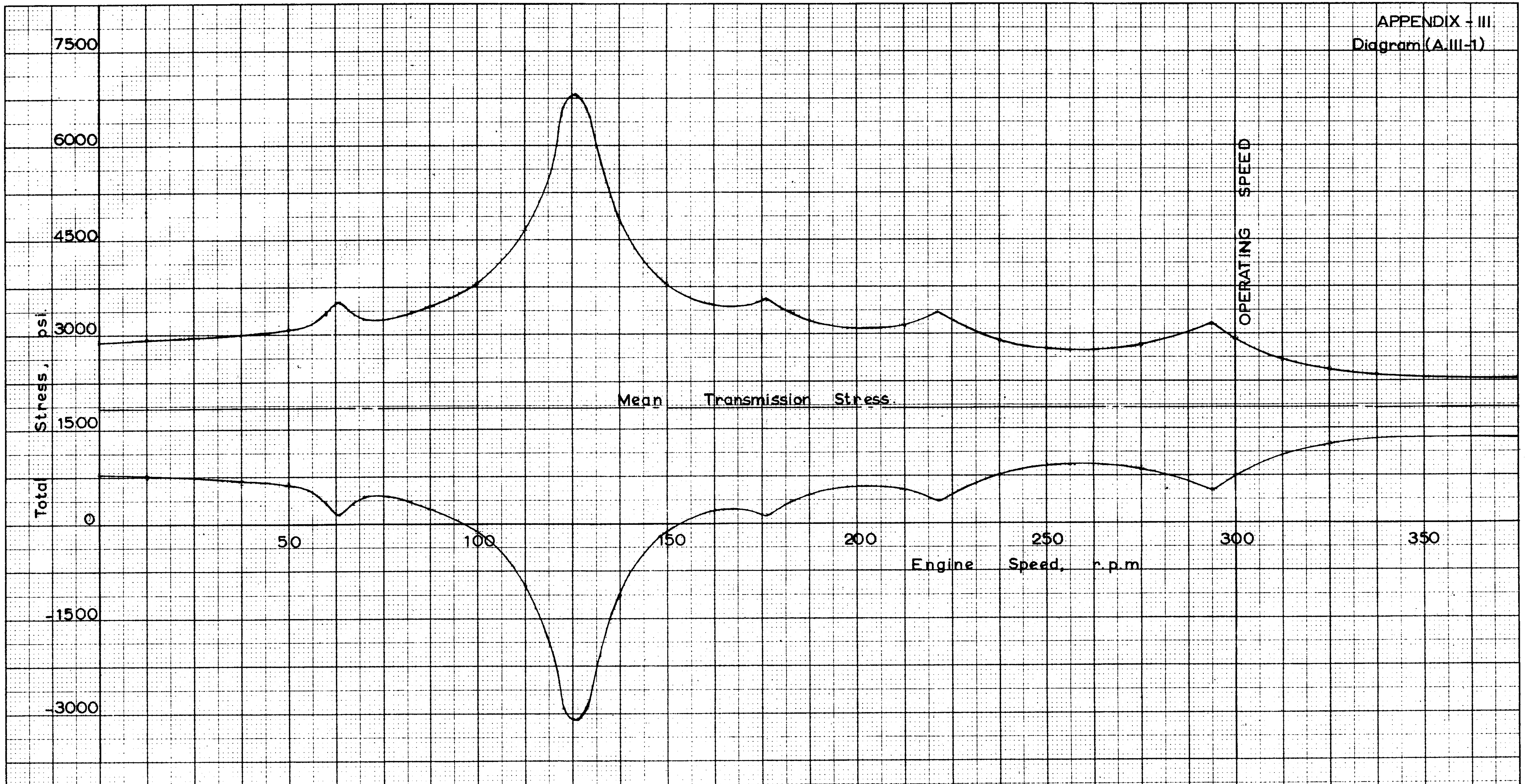
Forced Vibration Amplitudes Due to 7th: Harmonic Engine Torque at 300 RPM

A	B	C	D	E	F	G	H	I
Mass	Moment of Inertia	Accn. Torque per unit amplitude	Deflection in plane of mass	Torque in plane of mass: Acceleration torque	Torque in plane of mass: Engine harmn'c torque	Total torque	Shaft stiffness	Change of deflection
	J lb. in. sec.^{2}	$J \cdot w^2$ lb. in./rad.	θ radians	$J \cdot w^2 \cdot \theta$ lb. in.	M_n lb. in.	$\Sigma(J \cdot w^2 \cdot \theta + M_n)$ lb. in.	C lb. in./rad.	θ radians
Damper	617	29862800	x	$29862800x$	-	$29862800x$	90×10^6	$0.3318x$
Sc. Pump	130	6292000	$0.6682x$	$4201720x$	-	$34064520x$	139×10^6	$0.2451x$
Cyl. No. 1	400	19360000	$0.4231x$	$8191216x$	17950	$42255736x+17950$	316×10^6	$0.1337x+0.0000568$
" " 2	400	19360000	$0.2894x-0.0000568$	$5602784x-1099.65$	17950	$47858520x+348004$	316×10^6	$0.1515x+0.00011$
" " 3	400	19360000	$0.1379x-0.0001668$	$2669744x-3229.25$	17950	$50528264x+49521.1$	316×10^6	$0.1599x+0.0001567$
" " 4	400	19360000	$-0.0220x-0.0003235$	$-425920x-6263$	17950	$50102344x+61208.1$	316×10^6	$0.1586x+0.0001937$
" " 5	400	19360000	$-0.1806x-0.0005172$	$-3496416x-10012.99$	17950	$46605928x+69145.1$	316×10^6	$0.1475x+0.0002188$
" " 6	400	19360000	$-0.3281x-0.0007360$	$-6352216x-1424896$	17950	$40253712x+728462$	316×10^6	$0.1274x+0.0002305$
" " 7	400	19360000	$-0.4555x-0.0009665$	$-8818480x-18711.44$	17950	$31435232x+720847$	30.8×10^6	$1.0206x+0.0023404$
Generator	13800	667920000	$-1.4761x-0.0033069$	$-985916712x-2208744.65$	-	$-954481480x-2136659.95$		



HUGHES OWENS 314F 20x20

APPENDIX - III
Diagram (A.III-1)



Maximum Stresses due to One-Node Vibration occuring in the Main Transmission Shaft.

ACTUAL STRESS = 1.07 (CURVE VALUE)

APPENDIX - III

DIAGRAM (A.III-2-a)

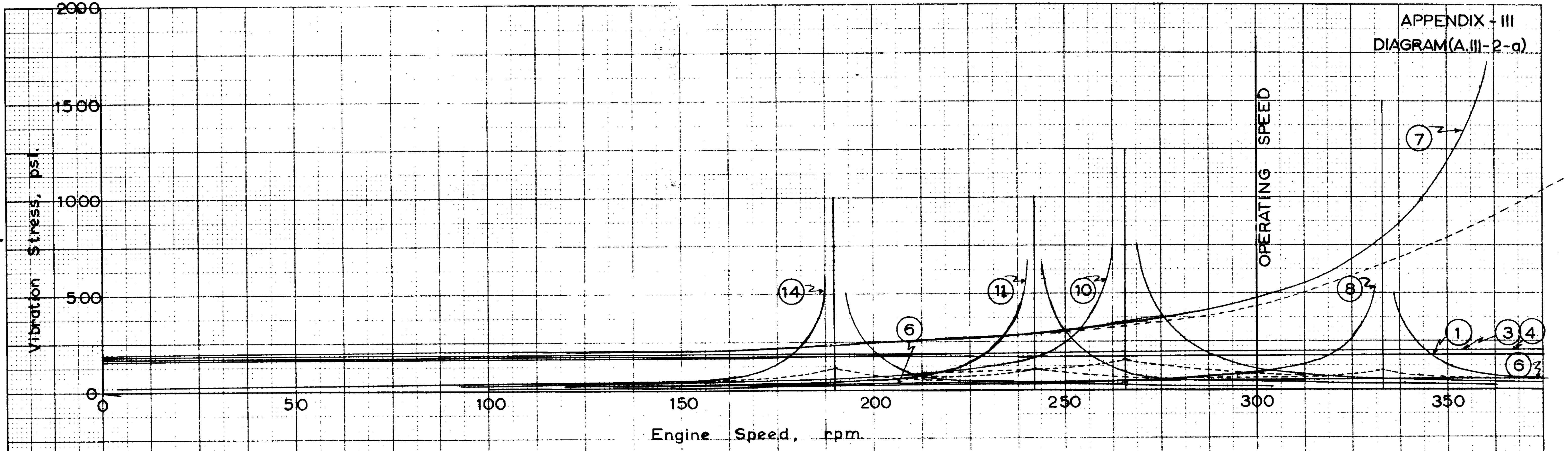
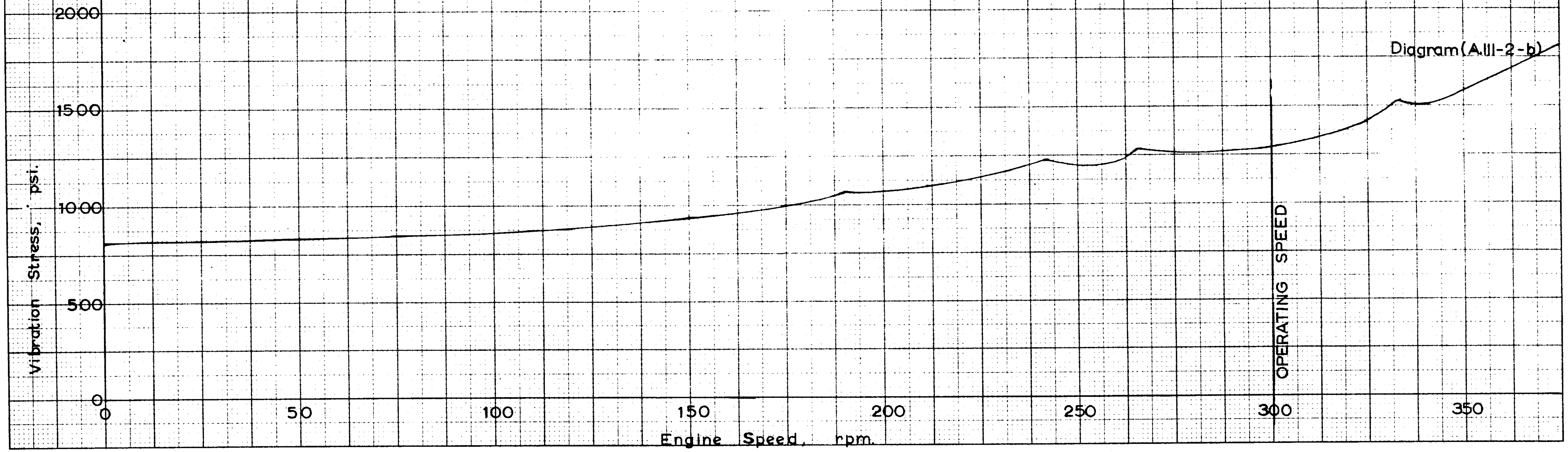
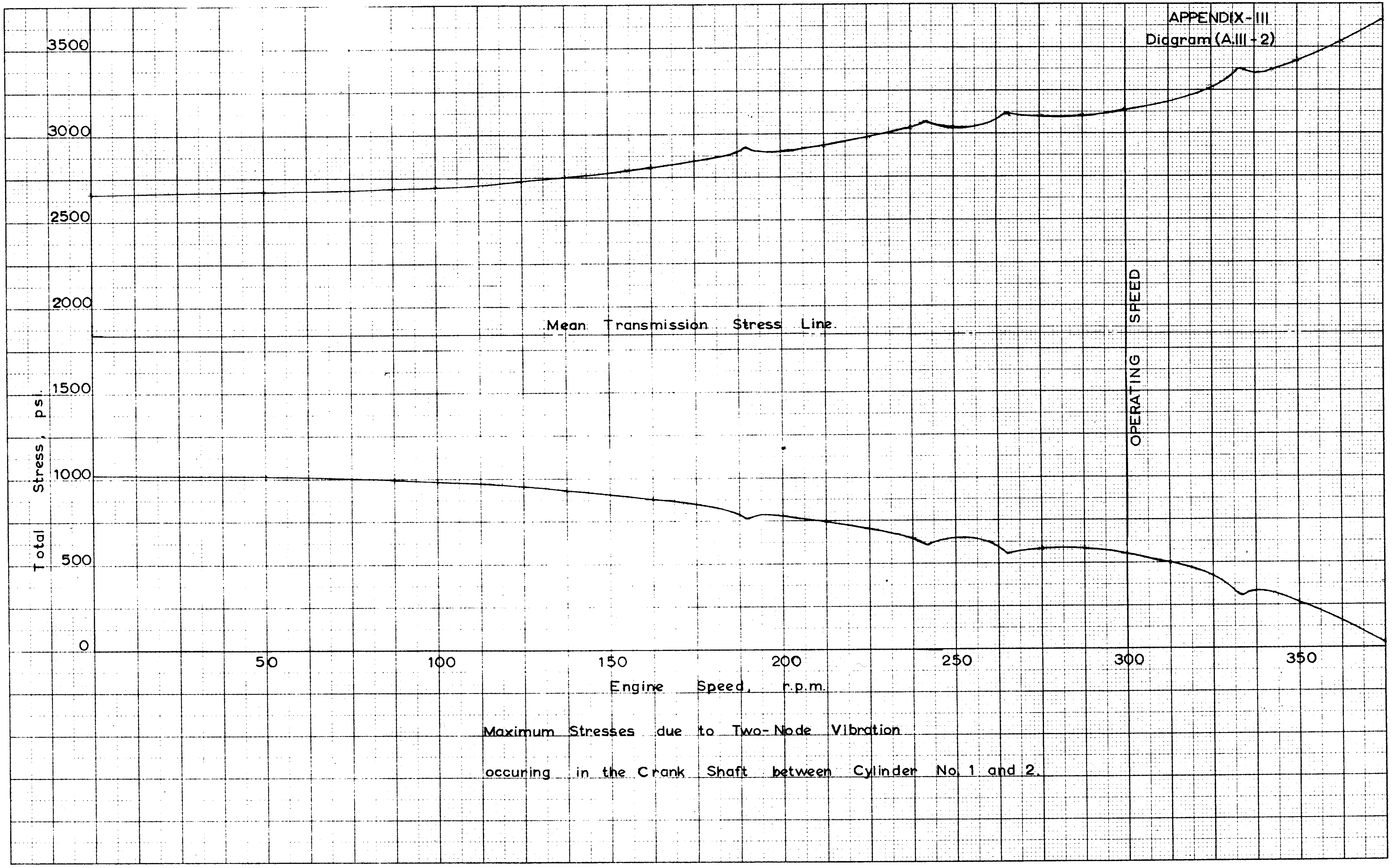


Diagram (A.III-2-b)



HUGHES OWENS 314F 20x20

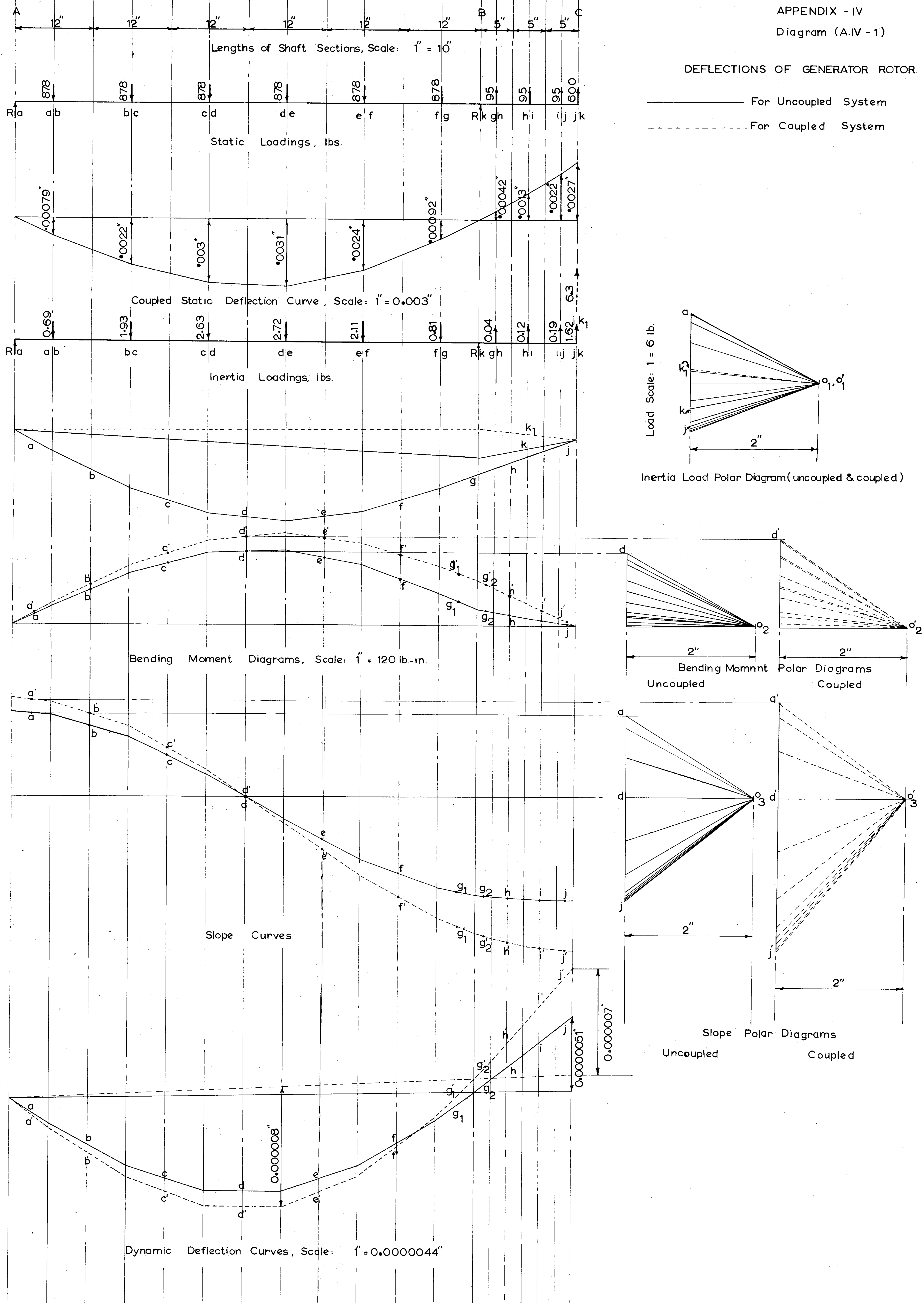


APPENDIX-III
Diagram (A.III-2)
3500
3000
2500
2000
1500
1000
500
0
Total Stress, psi.
Mean Transmission Stress Line.
OPERATING SPEED
50
100
150
200
250
300
350
Engine Speed, r.p.m.
Maximum Stresses due to Two-Node Vibration
occuring in the Crank Shaft between Cylinder No. 1 and 2.

APPENDIX - IV

Diagram (A.IV - 1)

DEFLECTIONS OF GENERATOR ROTOR.

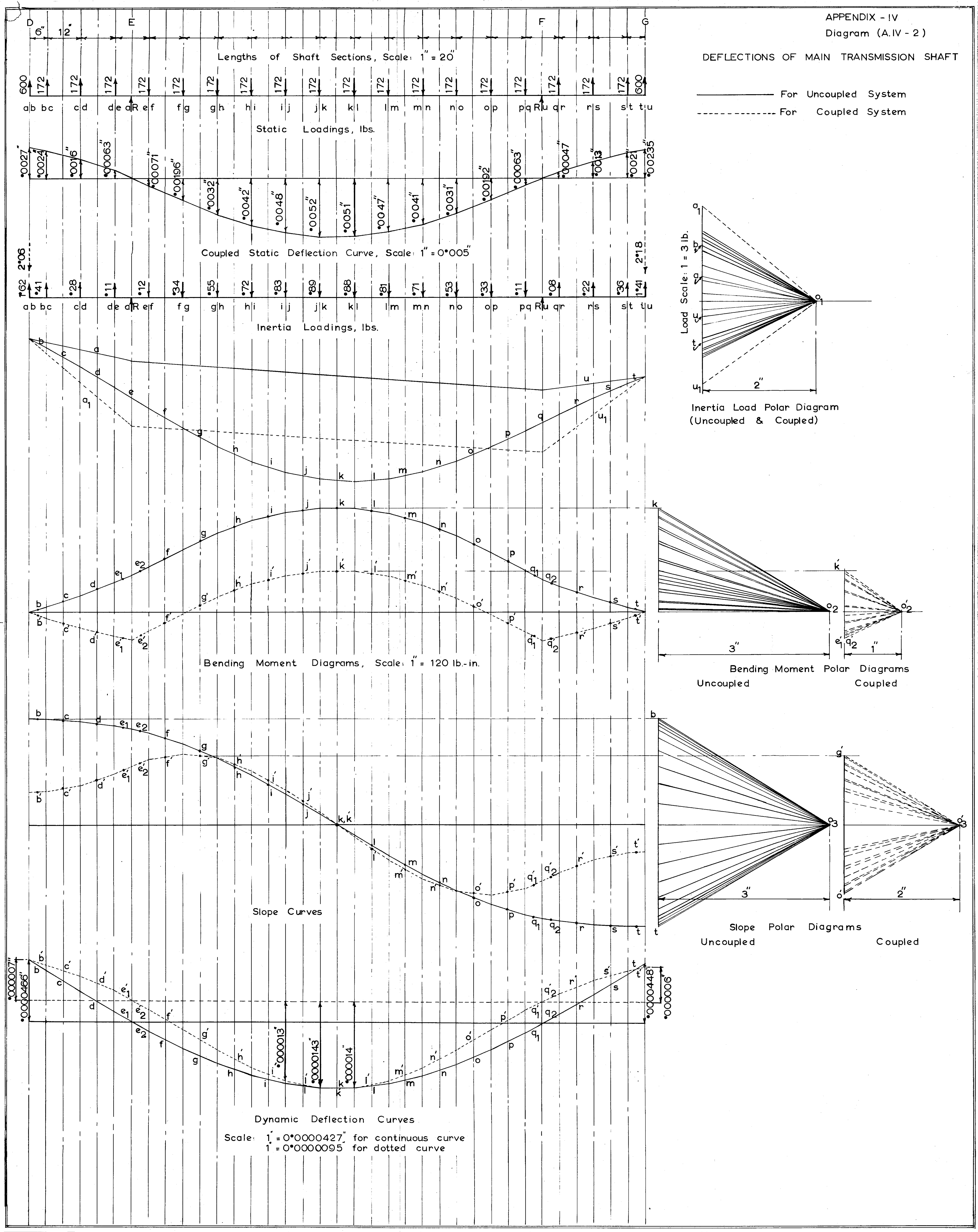


NOTE: Only few points are lettered on the polar diagrams to avoid confusion

APPENDIX - IV

Diagram (A.IV - 2)

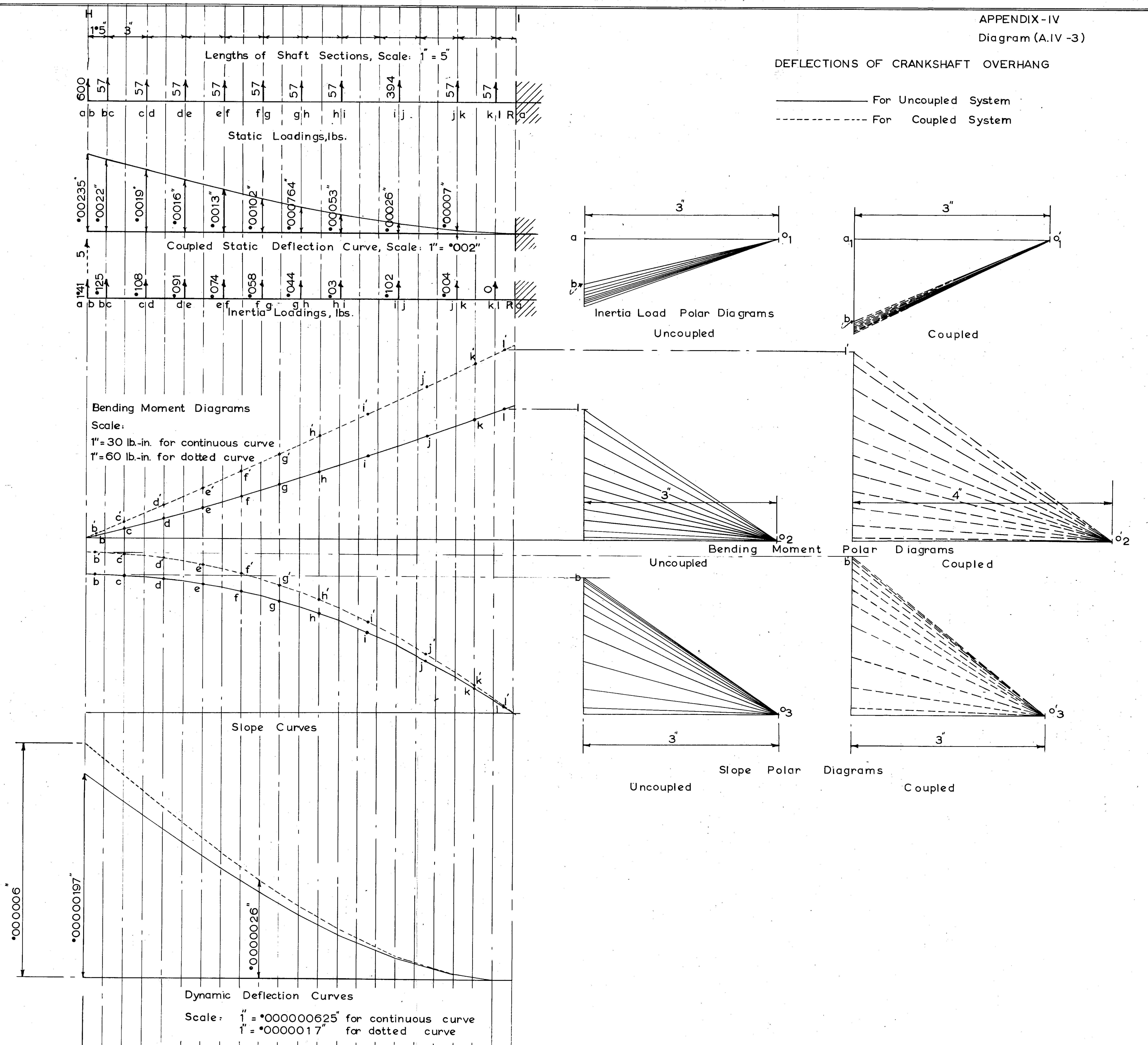
DEFLECTIONS OF MAIN TRANSMISSION SHAFT

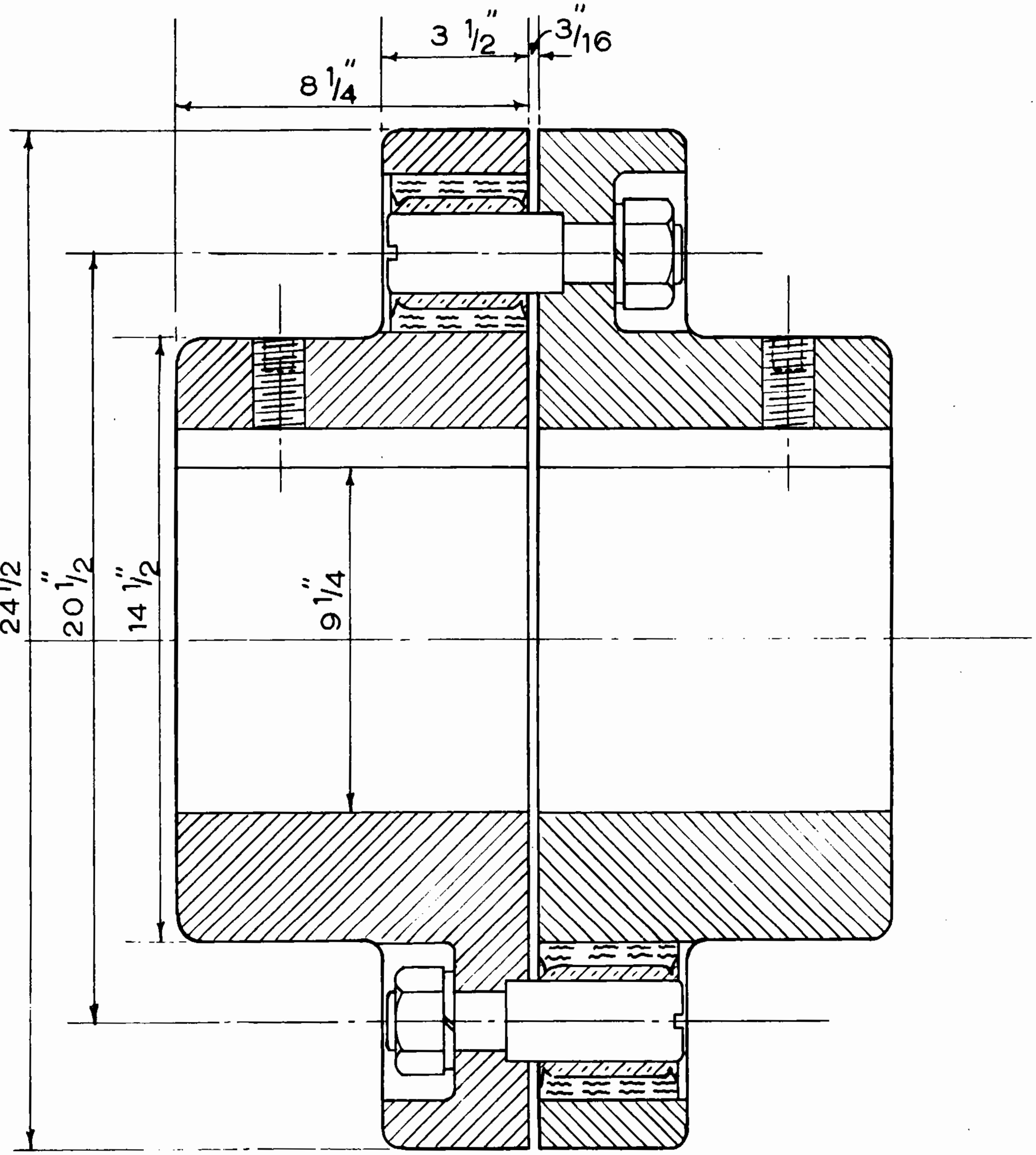


APPENDIX-IV

Diagram (A.IV-3)

DEFLECTIONS OF CRANKSHAFT OVERHANG





AJAX COUPLING

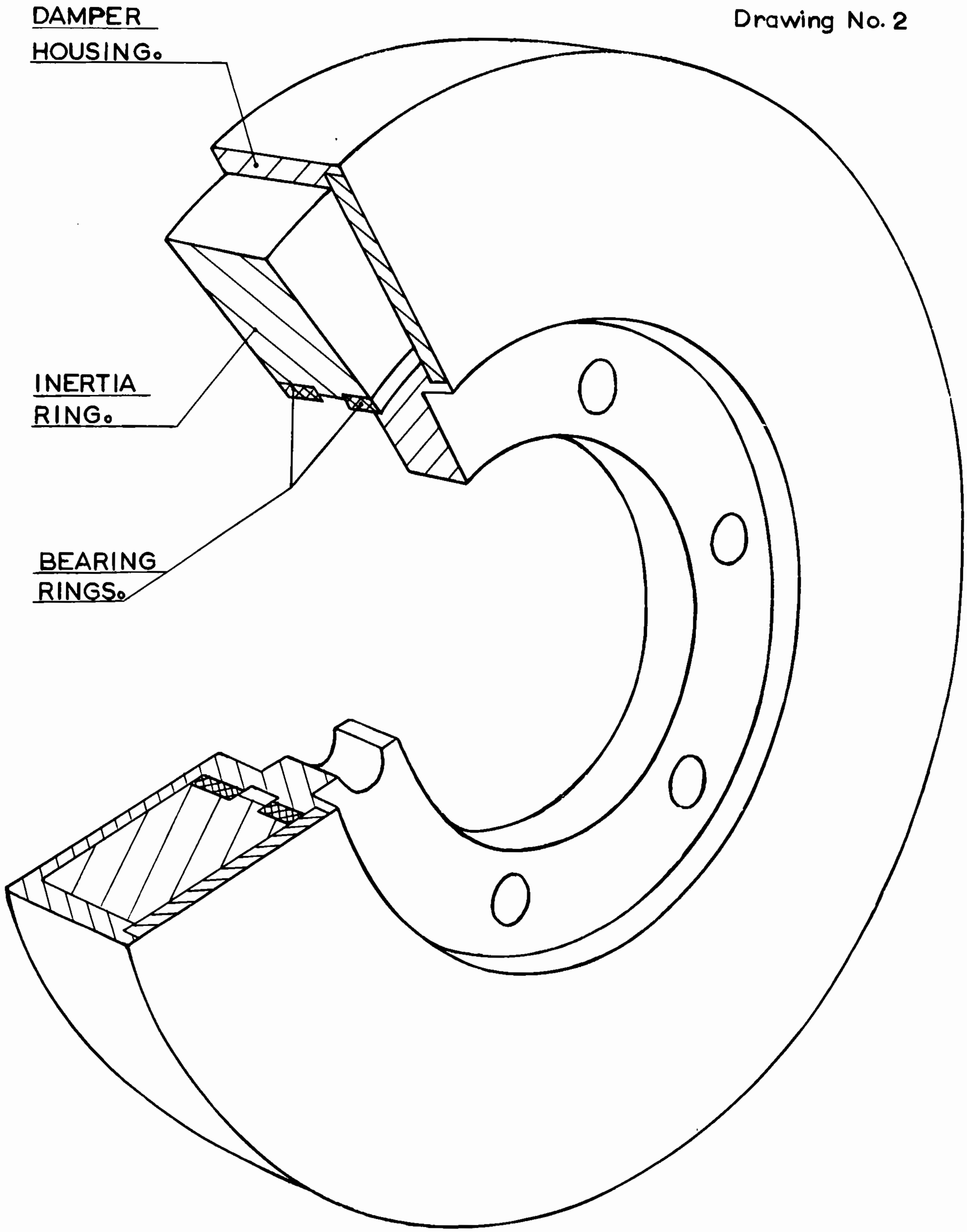
Approximate dimensions : Courtesy Alexander Fleck Ltd.

Approx. Weight = 1200 lb.

Number of studs = 16

Size of studs = 1 1/2"

APPENDIX - V
Drawing No. 2



INERTIA OF THE HOUSING = 329·5 lb.-in. sec.^{2}
INERTIA RING INERTIA = 575 lb.- in. sec.^{2}
EFFECTIVE INERTIA OF THE DAMPER = 617 lb.- in. sec.^{2}

Courtesy British Polar Engines Ltd.

FIG 1180

APPENDIX - V

Drawing No. 3

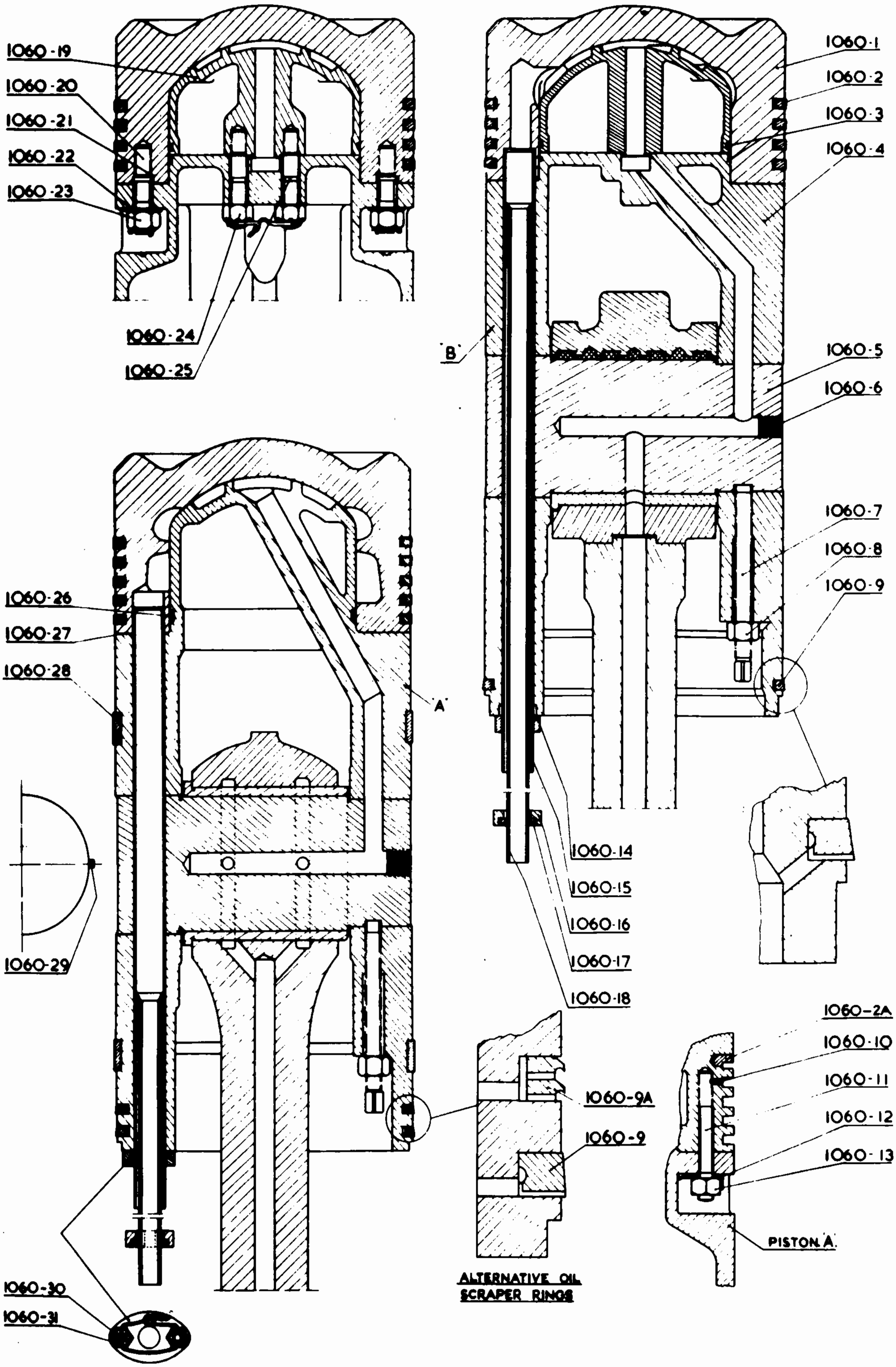


FIG.1060

Piston Details

Courtesy British Polar Engines Ltd.

APPENDIX - V

Drawing No. 4

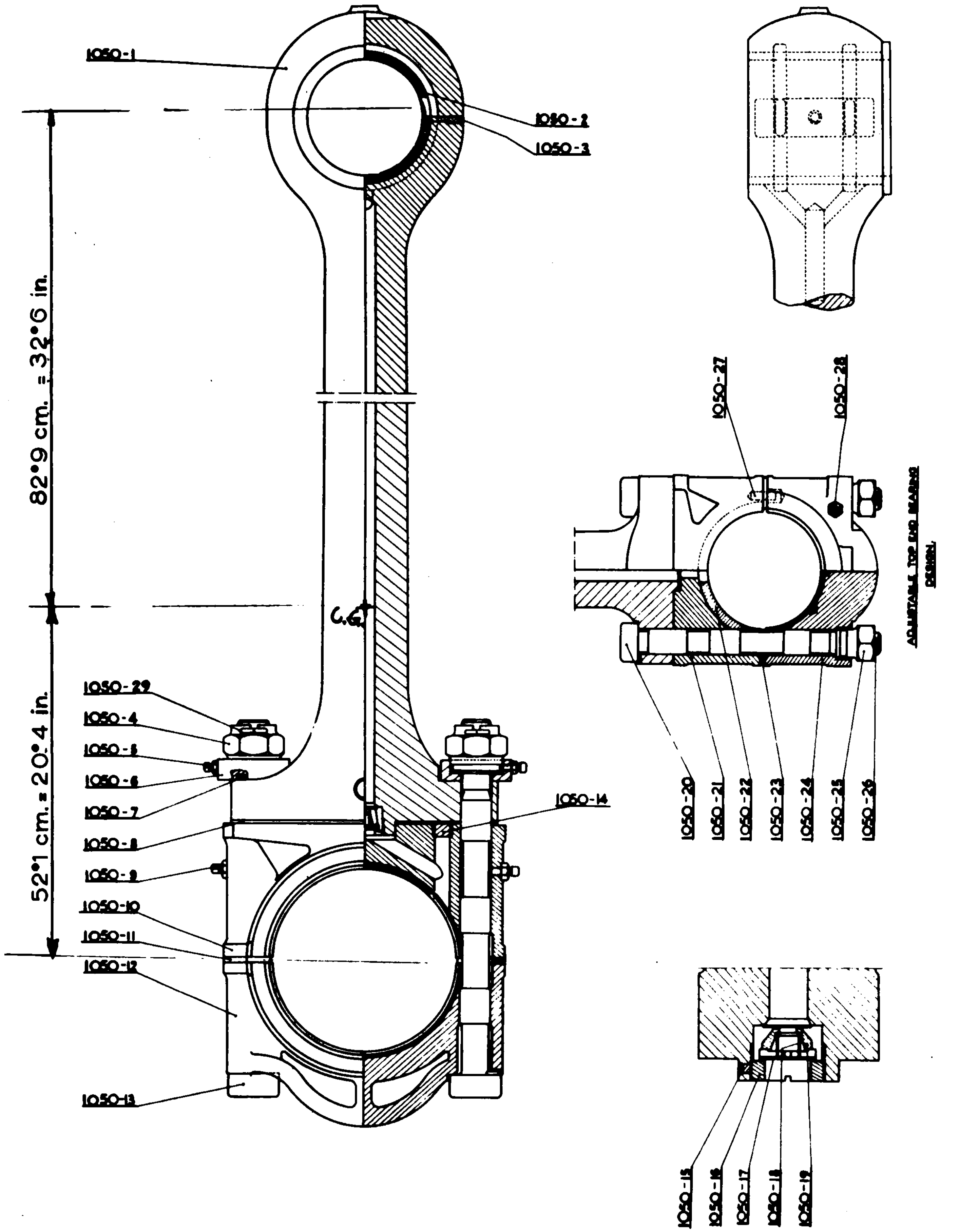
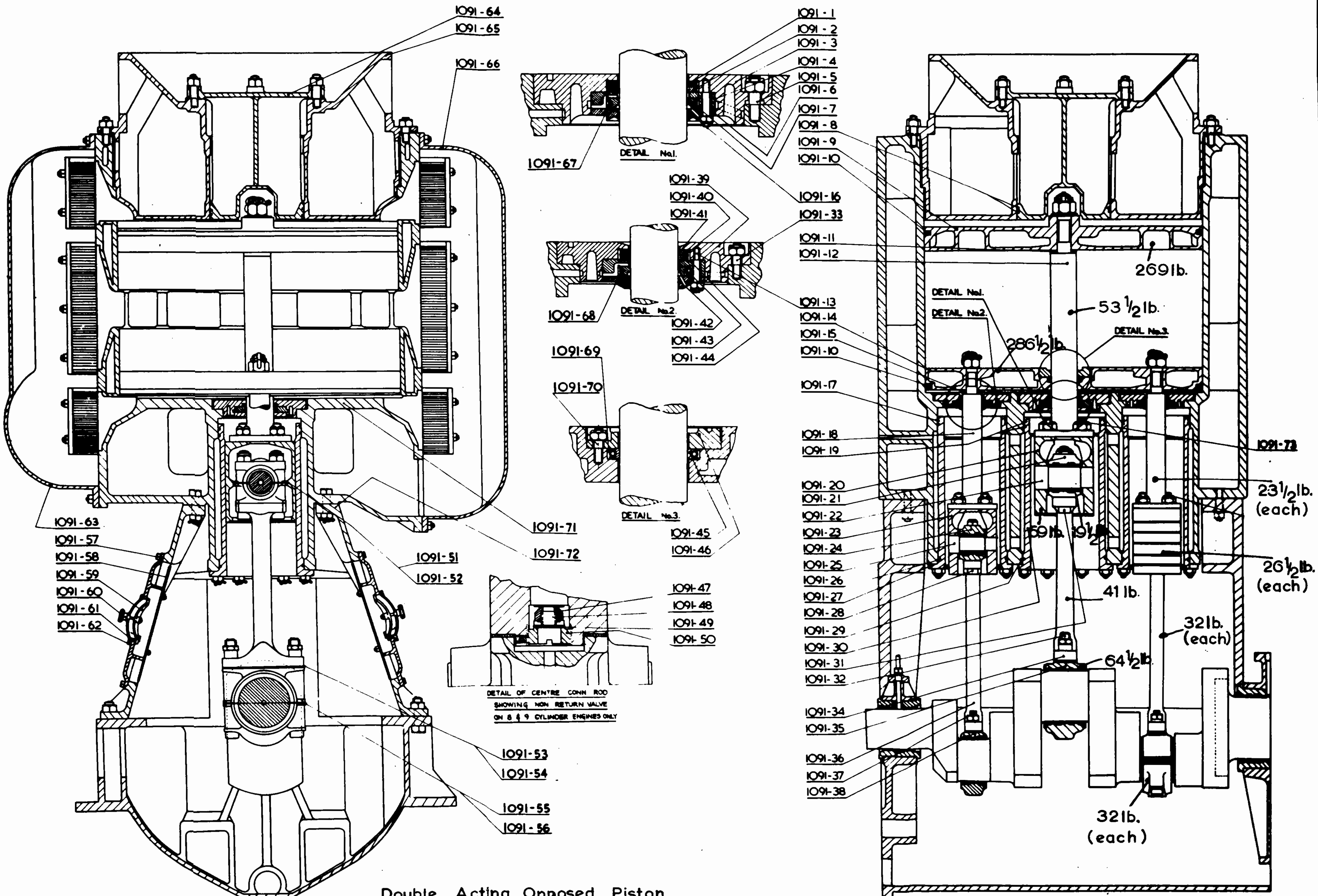


FIG 1050.

Connecting Rod Details

Courtesy British Polar Engines Ltd.



Double Acting Opposed Piston Scavenge Pump

Courtesy the British Polar Engines Ltd.

FIG. 1091.

APPROVED BY	DATE	B.H.P.	R.P.M.

APPENDIX - V
Drawing No. 6

TEST PIECE No 1 TAKEN FROM EACH END OF SHAFT
ULT. TENSILE STRENGTH 32/36 TONS/□"
YIELD POINT NOT LESS THAN 18 TONS/□"
ELONGATION PLUS U.T.S. TO BE NOT LESS THAN 57.
COLD BEND TEST 180° ON 1/4" RADIUS.

ITEM I FOR SPECIAL
ORDERS ONLY

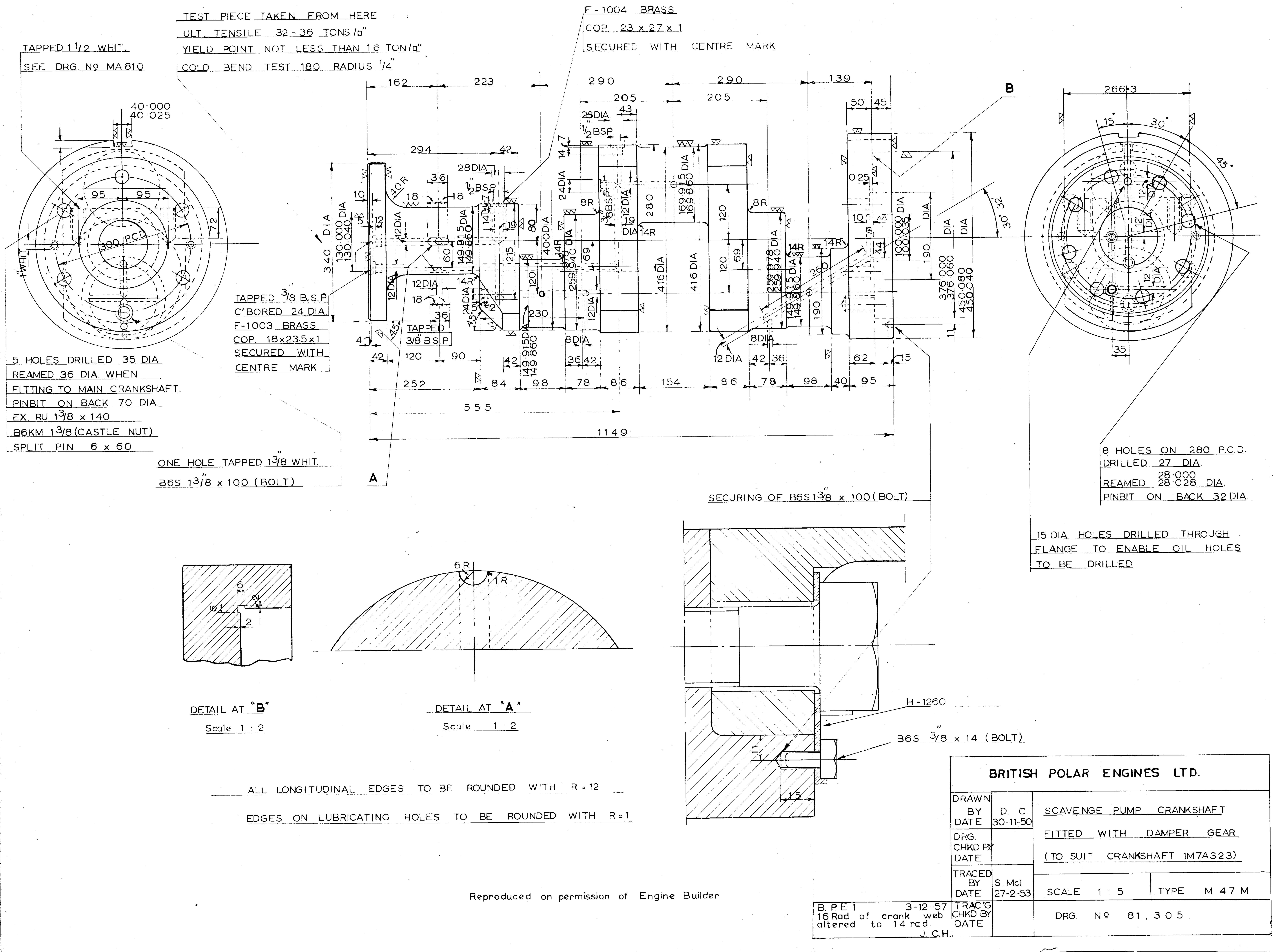
CRANKSHAFT

TYPE M57M, ITEM No M7A323 SCALE:- 1:10 & 1:6

DR'G No 1M7A323

B.P.E.4 1-9-54	B.P.E.3 5-10-50	B.P.E.2 29-8-49	B.P.E.1 4-12-48
Holes in coupling flange to be drilled 53 & reamed 55.000/55.030 J.C.H.	Drilling at section E-E altered R.I.C.	Dia. of hole altered from 36 dia. to 39 dia. J.M.D.	Flange modified to suit scave pump engine.

APPENDIX -V
Drawing No. 7



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