Merger Performance under Uncertain Efficiency Gains*

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Abstract

In view of the uncertainty over the ability of merging firms to achieve efficiency gains, we model the post-merger situation as a Cournot oligopoly wherein the outsiders face uncertainty about the merged entity's final cost. At the Bayesian equilibrium, a bilateral merger is profitable provided the non-merged firms sufficiently believe that the merger will generate large enough efficiency gains, *even if ex post none actually materialize*. The effects of the merger on market performance are shown to follow similar threshold rules. The findings are broadly consistent with stylized facts. An extensive welfare analysis is conducted, bringing out the key role of efficiency gains and the different implications of consumer and social welfare standards.

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1 Introduction

Mergers and acquisitions constitute a major feature of the economic landscape of most industrialized countries. Historically, mergers have displayed a clear tendency to occur in waves, stretching from the end of the nineteenth century to the present. To provide an idea of the numbers and resources involved, over the period 1981-1998, there were nearly 70,000 merger announcements worldwide, with each deal worth at least 1 million U.S. dollars, of which nearly 45,000 were actually implemented. The average deal was valued at 220 million U.S. dollars (base year 1995). Of these, 42% were horizontal mergers, defined as those involving two companies with sales in the same 4-digit industry, 54% were conglomerate mergers, and 4% were vertical mergers (Gugler, Mueller, Yurtoglu and Zulehner, 2003; henceforth GMYZ).

Mergers have been an important source of increase in market concentration, particularly outside the U.S. (Schmalensee, 1989). Antitrust policy on mergers has undergone extensive revisions over the last decade, both in the U.S. and in Europe. Not surprisingly, the topic has received considerable scholarly attention from industrial, business and financial economists over the last two decades. An extensive empirical and theoretical literature has explored the motives of mergers and their consequences on business activity. While both approaches have yielded useful insights, allowing industrial economists to reach a consensus on various aspects of merger performance, major points of controversy remain. In particular, important discrepancies exist between key theoretical findings and stylized facts based on empirical and event studies.

By their very nature, mergers pose a complex conceptual challenge, wherein structure and conduct are inextricably intertwined. The theoretical literature on horizontal mergers relies largely on the standard Cournot model. Since the pioneering work of Salant, Switzer and Reynolds (1983), henceforth SSR, a central postulate is that the pre-merger and the post-merger situations are represented as Cournot equilibrium points involving different market structures, with the merged entity being treated as a single player in the post-merger game. SSR showed that in the context of an n-firm symmetric Cournot oligopoly with linear demand and costs, for a merger to be profitable, it should comprise a pre-merger market share of at least 80%. Allowing the merging firms to exploit production synergies in some way, thereby lowering their post-merger costs, leads to a wider scope for profitable mergers (Perry and Porter, 1985, Farrell and Shapiro, 1990, and McAfee and Williams, 1992). A similar result holds under sufficiently concave demand (Fauli-Oller, 1996). By contrast, postulating Bertrand competition with differentiated products, Deneckere and Davidson (1985) establish that

every merger would be profitable.[1]

While some degree of controversy, mostly of a quantitative sort, persists, the empirical literature has delineated some important stylized facts, despite the diversity of data sets, countries, time periods, methodologies, and comparison standards adopted. Rather than reviewing the entire literature, we concentrate on the important conclusions that have direct bearing on the model proposed in the present paper. On the key issue of profitability, in the largest cross-national study to date (based on the aforementioned data set), GMYZ reports that nearly 60% of all horizontal mergers were profitable, with this proportion being higher in services than in manufacturing. As for sales (or revenues), it is essentially the other way around, with nearly 60% of merged firms experiencing a drop in sales. A similar negative effect is also reported for the post-merger market shares of the firms involved in the merger (Mueller, 1985). On the other hand, two other broad-based studies concluded that the profitability of acquired firms declined after the merger for U.K. firms (Meeks, 1977) and for U.S. firms (Ravenscraft and Scherer, 1987).[2] The overall conclusion one can draw from this rather mixed picture is that while horizontal mergers have some limited negative effects on sales, they do not appear to have, on average, a clear-cut impact on profitability.[3]

Many other consequences of horizontal mergers were investigated in the empirical literature. It is widely held that mergers typically lead to a price increase[4] (see, e.g., Kim and Singal, 1993 and Borenstein, 1990 for airline mergers)[5]. Regarding the effects on share prices, near-unanimity has emerged around the fact that initially the target firm's shareholders earn a substantial premium of about 30% on the merger while those of the acquirer tend to have more variable fortunes, with an average on the low side (see, e.g., Mueller, 1985). Furthermore, the acquirer's shares tend to experience a subsequent fall in value, a few years after the merger. For firms outside a merger, the evidence does not seem conclusive for recent time periods, but Banerjee and Eckard (1998) report significant losses of about 10% for the merger wave at the turn of the 19th century in the U.S.

As to the issue of whether mergers generate efficiency gains, of paramount impor-

[1] Few studies have adopted the Bertrand paradigm: See also Werden and Froeb (1994).

[2] There are many other studies on this central point, and the results are quite mixed. In particular, specific studies involving OECD countries have produced divergent results (Mueller, 1980).

[3] The comparison standard adopted by GMYZ and many other empirical studies is to compare the performance of the merged firms against a control group composed of (an average of) non-merged peers in the type of industry, over a period of the first five years or so following the merger.

[4] Interestingly, while the Cournot and Bertrand models yield strongly divergent conclusions on the profitability of mergers, they nonetheless do agree in their prediction that mergers increase prices.

[5] There are surprisingly few studies to this effect, particularly in view of the prominence of consumer surplus as a key criterion in the antitrust review process for mergers in the U.S. and elsewhere

tance for all aspects of the economics of mergers, the evidence is not direct as such gains are difficult to estimate, but rather deductive, and the findings are controversial[6]. While many studies, including Ravenscraft and Scherer (1987), report little support for a positive relationship, GMYZ concludes that 29% of all mergers engendered efficiency gains, as suggested by the observation of an increase in both profits and sales. Naturally, it is very difficult to disentangle the efficiency gain and the market power effects due to a merger. On the other hand, there appears to be a consensus reached on the basis of case studies and casual observation that while some mergers were successful in securing substantial efficiency gains[7], there is great variability on this issue.

In view of the lack of congruence between theoretical and empirical findings[8], the primary challenge of theoretical work on mergers is to come up with alternative models of merger behavior that would close this gap, while at the same time preserving the equilibrium nature of both the pre and post-merger situations, a consensual feature of the theoretical literature –at least since Farrell and Shapiro (1990). This paper constitutes an attempt in this direction within the framework of static analysis. The key novel ingredient is that the non-merging firms face uncertainty as to the efficiency gains, in terms of variable costs, that the merged firm could achieve. More precisely, they believe that with some fixed probability, the merged firm will end up with a lower unit cost than before and with the complementary probability, it will retain its original unit cost. The lower cost may correspond, for example, to the claim made by the merging firms to the antitrust agency, possibly appropriately discounted by the rival firms, or to a past average achieved by comparable mergers in related industries. Pre-merger competition is modelled as a standard Cournot oligopoly with identical firms while short-run post-merger competition involves a Bayesian Cournot equilibrium, with the merged firm alone being informed about its true cost. Demand and costs are assumed linear, both for simplicity and for ease of comparison with much of the literature.

This simple formulation seems natural and appropriate, in view of the stylized facts and other common observations following merger announcements. Indeed, for the merger to obtain the approval of antitrust authorities in most countries, the candidate firms have to convincingly document scope for significant efficiency gains, via the

[6] That mergers often result in savings on fixed costs, including such components as inefficient plant shut-downs, personnel consolidation and R&D expenses, is a well-accepted proposition. Likewise, mergers also arguably require substantial one-time transaction costs to actually be implemented. Except for some brief mention below, we follow the literature in ignoring these effects.

[7] For instance, a case study in Scherer et. al. (1975) reports a 40% increase in output per worker upon post-merger reorganization. Other success stories may be found in Fisher and Lande (1983).

[8] Observe that with the above stylized fact on profitability, the conclusions reached under the Cournot and the Bertrand approaches to mergers are equally far off the mark, in opposite directions.

exploitation of organizational and production synergies. In most cases, the approval of a merger presumes that the antitrust authority has been swayed by the firms' claims of lurking efficiency gains. Likewise, the initial positive reaction of the financial markets provides some support for the presumption that the merger is likely to lead to strong efficiency gains. In this respect, the magnitude of the upward shift in share prices suggests that an increase in market power alone is unlikely to yield the concomitant increase in expected profits. Another point is that the firms in the industry frequently react with apprehension to a merger announcement by two of their rivals. These typical facts lend credence to the postulate that all concerned parties generally hold beliefs about the prospect of efficiency gains that are naturally captured by a Bayesian model. Indeed, the revised Section 4 of the Horizontal Merger Guidelines issued by the U.S. Department of Justice and the Federal Trade Commission in 1997 states that "efficiencies are difficult to verify and quantify, in part because much of the information relating to efficiencies is uniquely in the possession of the merging firms. Moreover, efficiencies projected reasonably and in good faith by the merging firms may not be realized".[9] Further discussion in support of our Bayesian setting is given in Section 5.2.

One of our main results states that if the non-merged firms believe with a sufficiently high probability that the merged firm will experience a high enough efficiency gain, the merger will be profitable, even if one takes the worst-case scenario for the merged firm, wherein it ends up not experiencing any efficiency gain at all. Given that the answer to this central question rests on two threshold values, the belief and the efficiency gain, a natural question is whether these would typically be reasonable. We illustrate that these thresholds are indeed quite plausible. Similar threshold rules are shown to govern the effects of a merger on the merged firm's and outsiders' outputs as well as on industry price, using worst case, best case and ex ante (or expected) benchmarks.

In all theoretical models with complete information and no efficiency gains, whether based on Cournot or on Bertrand competition, mergers always exert a positive externality on non-merged firms. In a Bayesian formulation, the nature of this externality also follows a threshold rule depending on the same pair of parameter values, so that it may well be negative. Similar remarks may be made about market shares and sales. The set of possible outcomes following a merger is substantially expanded, with one or

[9] Fisher and Lande (1983) assert that "efficiencies still are enormously difficult to predict on a case-by-case basis...". Likewise, according to FTC chairman Robert Pitofsky, the efficiencies defense is "easy to assert and sometimes difficult to disprove" (Quoted in J. Kattan (1994), Efficiencies and merger analysis, *Antitrsut Law Journal*, 62, 513). One is tempted to add that if all the federal agencies empowered to ascertain the prospects of efficiency gains admit to the complexity of the task, the rival firms and outside analysts of the industry will typically find it beyond hope. As a consequence, these outsiders have no option after the merger other than to engage in Bayesian behavior.

both the merged firm and the outsiders, or neither of them, being possible beneficiaries.

In analyzing the welfare effects of mergers, we proceed along the three possible evaluation criteria: worst-case, ex ante (or expected) and best-case scenarios, from the merged firm's standpoint. For both consumer surplus and social welfare, the former benchmark yields a negative effect of mergers while the latter two lead to thresholds depending again on the belief and the efficiency gain levels. The threshold rule associated with the ex ante and worst case benchmarks confirms the central role played by expected efficiency gains in gauging the welfare effects of mergers, as in common antitrust practice in many countries. Another main conclusion of the paper is that an ex-ante profitable merger is necessarily social-welfare, but not always consumer-welfare, improving. This result provides support for a laisser-faire policy if the decisive criterion rests on social welfare, but not if it rests on consumer welfare. This underscores the importance of the selection of a decisive criterion for antitrust approval of a merger.

The present set-up also demonstrates that the merging firms have a strong incentive to overstate the extent of their potential efficiency gains ex ante, not only to secure approval of the merger by antitrust authorities, but also to twist the terms of Bayesian Cournot competition in their favor, in a short-run perspective.

All in all, our results form a major departure from the complete information equilibrium analysis of the literature starting with SSR. Particularly noteworthy is the fact that the novel features of the present paper hold even in the worst-case ex post outcome of no efficiency gains. In such a case, the *only difference* between the post-merger markets in this paper and in SSR is the *informational market power* of the merged firm. An important consequence of this difference is that, unlike most previous theoretical results, our conclusions are quite consistent with many empirical findings and stylized facts on the effects of mergers on profitability, sales and market shares, both for the merged firm and for the outsiders to the merger.

Informational market power thus emerges as a natural candidate for the fundamental asymmetry that mergers seem to trigger in favor of the merged firm, which previous models have not attempted to capture. By its very nature, this new type of asymmetry is transitory, as are most investigated effects of mergers. In this sense, the present theory constitutes a short-run analysis, but the short run is where most of the interest in mergers actually lies[10]. In addition, we can add a plausible dynamic extrapolation of our model to capture the resolution of uncertainty over the merged firm's cost. Our

[10]Indeed, a long run analysis would have to disentangle various other potential contributing factors, including industry-specific or economy-wide shocks, entry into or exit from the industry, other mergers within the same industry for which the original merged firm would now be an outsider, etc. Recall in this respect that most empirical studies consider horizons extending only from three to five years.

results are consistent with GMYZ's finding that over their five-year data window, from one year to the next, realized profits increased for profitable mergers but decreased for unprofitable mergers (see Section 5 where a dynamic extension of the model is discussed). A similar mechanism may be invoked to account for the initial substantial rise in share values that typically accompanies merger announcements, which often ends up spiraling downwards after one to three years.

This paper is organized as follows. After a model description in Section 2, the effects of mergers on market performance are presented in Section 3, followed by a detailed welfare analysis in Section 4. Section 5 is devoted to some dynamic extensions. All computations, proofs and quantitative illustrations are gathered in an Appendix.

2 The Model

In the pre-merger situation, consider an industry composed of $n+1$ identical firms choosing quantity levels of a homogenous product in a market with inverse demand $P = a - bQ$, with $a > 0$ and $b > 0$. Each firm has constant unit cost c, with $a > c > 0$. Each firm's pre-merger Cournot equilibrium output and profits are:

$$q = \frac{a-c}{b(2+n)} \qquad \text{and} \qquad \pi = \frac{(a-c)^2}{b(2+n)^2}.$$

We consider a bilateral (two-firm) merger only. In the post-merger situation, we postulate that the outsiders (non-merged firms) are uncertain over the resulting unit cost of the merged firm. In particular, they believe that with probability p the merged firm will end up with marginal cost $c_l < c$, thus having experienced efficiency gains equal to $\Delta c \equiv c - c_l$, while with probability $(1-p)$ its cost will remain c. Here, c_l may for instance be an average value attained by comparable mergers, or the value reflected in post-merger simulations accepted by the merger authorities, or the actual value claimed by the merging firms. The value of p reflects the subjective perception rivals have formed about the merged firm's ability to achieve the posited efficiency gain, given the information available to them about the case.

Let q_m^h and q_m^l be the merged firm's quantities conditional on low-cost type (i.e. with unit cost c_l) and high-cost type (i.e. with unit cost c) respectively, and Eq_m be its expected quantity. Each *outsider's* quantity is denoted by q_o. The Bayesian Nash equilibrium quantities are as follows[11] (recall that *after* the merger, the industry has n

[11] The computational details are in appendix 8.1.

firms, and $\Delta c = c - c_l > 0$ is the efficiency gain):

$$\begin{aligned} q_o &= \frac{a - c - p\Delta c}{b\,(n+1)}, \qquad q_m^l = \frac{2(a-c) + \Delta c(1 + n + p(n-1))}{2b\,(n+1)} \\ q_m^h &= \frac{2(a-c) + p\Delta c(n-1)}{2b\,(n+1)} \quad \text{and} \quad Eq_m = \frac{a - c + np\Delta c}{b\,(n+1)}. \end{aligned}$$

All these quantities are strictly positive if one assumes[12] $p < \frac{a-c}{\Delta c}$. The corresponding market prices are

$$P^l = \frac{2\,(a + nc_l) + \Delta c(n-1)(p+1)}{2\,(n+1)}$$

$$P^h = \frac{2\,(a + nc) + p\Delta c\,(n-1)}{2\,(n+1)} \quad \text{and} \quad EP = \frac{a + cn - p\Delta c}{n+1}.$$

The expected equilibrium profits of each outsider firm and the profits of *the merged* firm, conditional on its cost type, c or c_l, are respectively

$$E\pi_o = \frac{(a - c - p\Delta c)^2}{b\,(n+1)^2}, \qquad \pi_m^h = \frac{(2\,(a-c) + p\Delta c\,(n-1))^2}{4b\,(n+1)^2}$$

$$\text{and } \pi_m^l = \frac{(2\,(a-c) + \Delta c\,(1 + n + p\,(n-1)))^2}{4b\,(n+1)^2}$$

Note that it is always the case that $E\pi_o \leq \pi_m^h \leq \pi_m^l$, with equality throughout if and only if $p = 0$. In other words, the informational asymmetry created by the merger always works in favor of the merged firm, which now outperforms its rivals even in the worst case situation wherein their costs are all equal. Whether this informational rent is sufficient to compensate for the fact that the merged firm must now divide its profit between its two pre-merger partners is investigated in the following section.

3 Effects on Market Performance

This section provides a detailed account of the consequences of the merger on profits and outputs for both the merged firm and the outsider firms, as well as on industry price. In dealing with these effects, several options are possible. One is obviously to use expected profits and outputs at the Bayesian Cournot equilibrium. This profit measure is arguably the relevant indicator that determines the movement and magnitude of the merged firm's share price. We also study the worst-case scenario, wherein the merged

[12] Observe that certainty-equivalence holds, due to the linear specification, in that the merged firm's expected output is the p-weighted average of its outputs in the corresponding full information oligopolies. This property suggests that our conclusions extend to more general formulations, instead of our stylized binomial version, of the Bayesian feature of this model.

firm fails in achieving any ex-post efficiency gains at all, so that its post-merger realized profits are given by π_m^h, as well as the best-case scenario with realized profits π_m^l.

In the worst case scenario, the merger is profitable if $\pi_m^h > 2\pi$, the solution of which leads to one of our main results (note that as π_m^h is clearly the lowest possible realized profit, the threshold values below are the most conservative ones).

Proposition 1 *If the non-merged firms believe sufficiently, i.e. with*

$$p > p^{h*} = \frac{2(a-c)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{\Delta c\,(n+2)(n-1)}$$

that the merged firm will experience large enough efficiency gains

$$\Delta c > \frac{2(a-c)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{(n+2)(n-1)}$$

then the merger will be profitable, even in the worst case scenario. These gains can occur only if the original cost is high enough, i.e.,

$$c > \frac{2a\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{2\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)+(n+2)(n-1)}.$$

To provide some illustrative idea of the plausibility of the two threshold values given in Proposition 1, the following graphs depict the regions of (p, c_l) space for which a bilateral merger is profitable when the merged firm experiences no actual efficiency gains ex-post, for $a = 10$, $n = 5, 10$, or 15, and $c = 3$ or 7. In each case, the merger is profitable below the given curve and unprofitable above it.

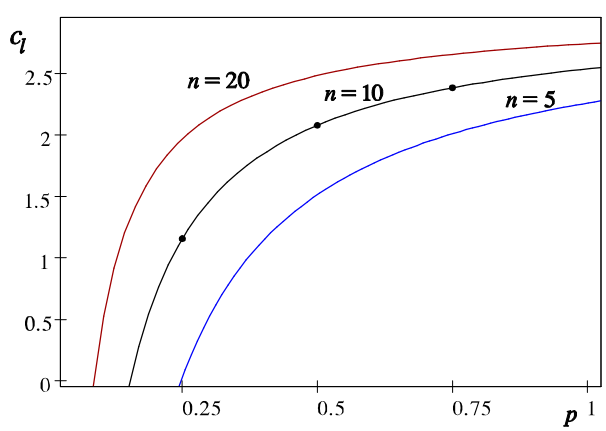


Figure 1. Profitability thresholds in the worst case scenario when $c = 3$.

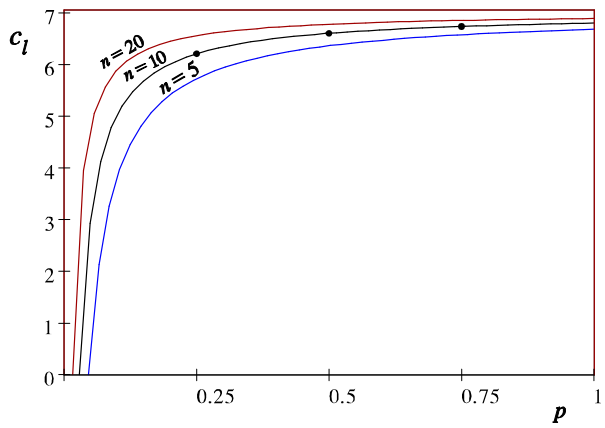


Figure 2. Profitability thresholds in the worst-case scenario when $c = 7$.

For the given parameters, it thus appears that the scope for mergers to be profitable in our setting is quite broad[13]. This is consistent with the empirical facts on profitability, whether one goes by the more optimistic picture presented by GMYZ or by the more pessimistic numbers of Ravenscraft and Scherer (1987) and others.

In expected rather than worst-case terms, mergers are even more likely to be profitable for the merging firms. It is easy to see that $E\pi_m = (1-p)\pi_m^h + p\pi_m^l \geq \pi_m^h$, so that the threshold for merger profitability, p^* (the exact formula is rather long and given in the appendix) is less demanding, thus providing wider scope for mergers. The following example offers a direct comparison between the profitability cost threshold (for c_l) ex-post in the worst case and ex-ante.

$a = 10$ $n = 10$ $c = 7$	$p = 0.25$	$p = 0.5$	$p = 0.75$
$\pi_m^h > 2\pi$	6.2097	6.6049	6.7366
$E\pi_m > 2\pi$	6.6754	6.8280	6.8828

Lastly, we can perform the same kind of comparison between the profits of the merged firm in the best case scenario, i.e., in the event the merged firm experiences efficiency gains, π_m^l and the pre-merger profits. Naturally, since $\pi_m^l \leq E\pi_m \leq \pi_m^h$, the probability threshold, p^{l*}, is even lower than p^*, where p^{l*} is given by[14]

$$p^{l*} = \frac{2(a-c)\left[n(\sqrt{2}-1) - (2-\sqrt{2})\right] - (n+1)(n+2)\Delta c}{(n-1)(n+2)\Delta c}.$$

We now investigate the effects of the merger on outputs and outsiders' profits.

[13] In the way of comparative statics, it can be shown that the profitability of a two-firm merger is enhanced by having a lower level of demand (a), a higher initial unit cost (c), or a higher initial number of firms (n). This last point is of particular interest as it constitutes a departure from the complete information case analyzed by SSR.

[14] As in Proposition 1 we need conditions on both Δc and c.

Proposition 2 *The merged firm expands output in the worst case scenario, or $q_m^h \geq 2q$ (resp., in the best case scenario, or $q_m^l \geq 2q$) if and only if*

$$p \geq p_m^h = \frac{2n(a-c)}{\Delta c(n-1)(n+2)} \left(\textit{resp., } p \geq p_m^l = \frac{2n(a-c)-(n+1)(n+2)\Delta c}{(n+1)(n+2)\Delta c} \right).$$

As for outsiders, the same threshold on the value of p governs the direction of change of their output as well as their profit, at the Bayesian Cournot equilibrium.

Proposition 3 *The merger increases an outsider firm's expected profit and output (i.e., $E\pi_o \geq \pi$ and $q_o \geq q$) if and only if*

$$p \leq p_o = \frac{a-c}{\Delta c(n+2)} .$$

The merged firm expands output in expected terms (i.e., $Eq_m \geq 2q$) if and only if $p \geq p_o$.

Figure 3 below summarizes all the possibilities in expected terms. It also includes the threshold belief, p^*, above which the merged firm will be profitable in expected terms. In Appendix 8.2 (Figure 3 explanation), it is shown that $p^* < p_o$. A more detailed exposition of all the possible ex-post realizations (high and low costs) is given in Figure 7 in Appendix 8.2. Depending on the belief and efficiency gain levels, a much richer picture emerges, relative to the full information model of SSR. All possible combinations on the contraction or expansion of output by the merged firm and the outsiders can emerge. Due to the informational asymmetry, the usual business stealing argument does not quite hold in the worse case scenario, where both the merged firm and the outsiders actually contract output when $p_o \leq p \leq p_m^h$. Likewise, the worst-case profits of the merged firm and the expected profits of the outsiders can both increase (for $p^* \leq p \leq p_o$). More interestingly, in contrast to all theoretical work on mergers, one novel feature that emerges here is that outsider firms may actually be harmed by a merger. While this issue has not received much attention in the empirical literature dealing with recent merger waves, it does provide theoretical support for the conclusion by Banerjee and Eckard (1998) that outsider firms experienced profit losses during the 19th century merger wave in the U.S. It also lends credence to a prevalent belief amongst business strategists that firms are often apprehensive of the prospect of a merger between two of their rivals.

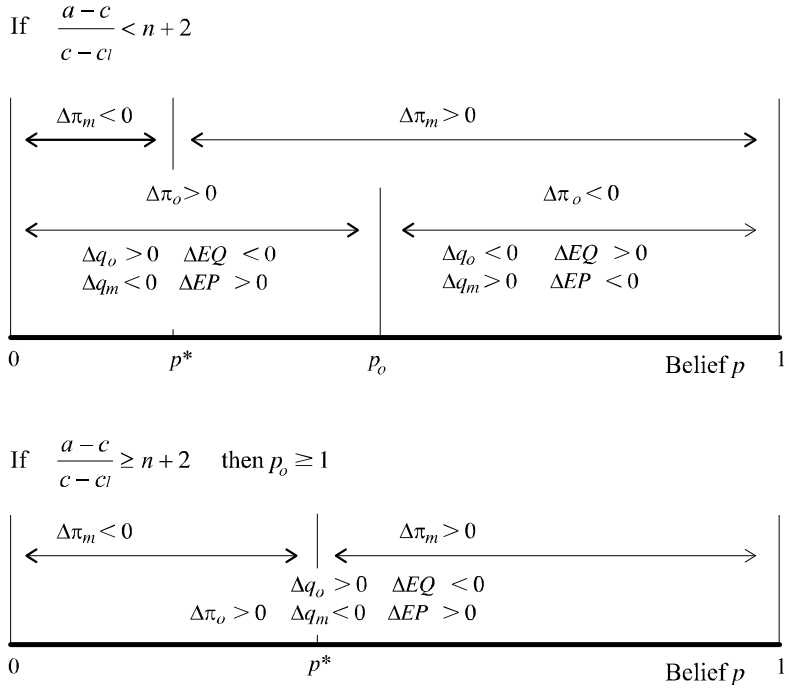


Figure 3. Market performace in expected terms.

All in all, the emerging picture squares well with the stylized facts, even from a (suggestive) quantitative standpoint. The theory at hand predicts a narrower scope for output expansion ($p \geq p_m^h$) than for profitability of the merger ($p \geq p^{h*}$), since $p^{h*} < p_o$ as shown in the explanation of Figure 3 in the Appendix 8.2. The corresponding empirical averages reported by GMYZ are about 40% and 60%, respectively[15]! Thus a merger may be profitable in our setting in case the merged firm contracts output, even if this means a loss in market share (recall that output and market share reductions for the merged firms take place in a majority of cases, Mueller, 1985).

These results suggest that the underlying informational asymmetry endows the merged firm with a new form of market power, of an informational nature, relative to the outsider firms. Whether this power is sufficient to overcome the usual mechanism that makes mergers more favorable to outsiders than to insiders depends on the levels of the potential efficiency gains and of the associated belief.

As to the effect on industry price, it is unambiguously upwards only when no efficiency gain is realized ex post.

Proposition 4 *(a)In the worst case scenario, a merger raises industry price ($P < P^h$). (b)A merger raises the expected price ($P < EP$) if and only if $p < p_o$.*

[15]If one were to include savings on fixed costs in efficiency gains, the scope for profitability would widen while the merged firm's output would remain unaltered. This would further reinforce our results.

(c)In the best case scenario, a merger raises industry price ($P < P^l$) if and only if

$$p > p^l = \frac{(n+1)(n+2)\Delta c - 2(a-c)}{\Delta c(n+1)(n-1)}.$$

This finding is consistent with empirical results reported in the few studies that have dealt directly with the price effects of mergers. These studies typically report modest to significant price increases, ranging from none (Borenstein, 1990 for the 1986 airline merger between TWA and Ozark) to around 10% (Kim and Singal, 1993).

We now provide an intuitive explanation of our results so far. Recall that in the full information model of SSR, each partner of a merged firm wishes to reduce output in the post-merger situation as it now takes into account the business-stealing externality it inflicts on its merging partner (this is the only way to hope for a price increase, a necessary condition for profits to increase for the merged firm). Non-merging firms react to this contraction by expanding output, due to the same externality. The resulting price increase is then not sufficient to imply higher profits for the merged firm.[16]

By contrast, in the present Bayesian setting, the merged firm exploits its informational market power that lies in the inability of the outsiders to adapt their outputs to its true, but unknown unit cost. Depending on the belief held by the outsiders, this new market power may well lead to the merged firm producing more than before the merger, despite the fact that the aforementioned externality effect is still present here. While a tendency for the outsiders' output to move in the opposite direction is still there, there is a range of values of p (between p_0 and p_m^h) for which all firms decrease their output after the merger, even in the worst case scenario. Similarly, there is a range of p (between p^{h*} and p_o) such that all firms' profits increase. This new diversity of strategic behavioral patterns is due to the interaction between the informational market power of the merged firm with the well-known effects of mergers in the standard SSR model (see also Gaudet and Salant, 1991). As a result, real-life merger behavior emerges as being compatible with static equilibrium theory, at least in the short run.

4 Welfare Analysis

Producer surplus, PS, consumer surplus, CS, and total (social) welfare, TW, before the merger are easily found to be:

$$PS = \frac{(n+1)(a-c)^2}{b(2+n)^2}, \quad CS = \frac{(c-a)^2(n+1)^2}{2(n+2)^2 b} \text{ and } TW = \frac{(c-a)^2(n+1)(n+3)}{2(n+2)^2 b}.$$

[16] By contrast, in the Bertrand case with substitute products, the merging partners raise their prices while outsider firms react by raising prices as well, due to the strategic complementarity of the price game. So prices rise even more, and the merger ends up being profitable for both insiders and outsiders.

In evaluating the welfare effects of a merger, we consider all the three possible evaluation benchmarks: worst-case scenario, best-case scenario, and expected terms.

We start our analysis with producer surplus. Ex-post producer surplus, conditional on the realized cost being high and low are $PS^h = \pi_m^h + (n-1)\pi_o^h$ and $PS^l = \pi_m^l + (n-1)\pi_o^l$ respectively. Expected producer surplus is $EPS = pPS^l + (1-p)PS^h$. All these expressions are in Appendix 8.3 (with the proofs in Appendix 8.5).

Proposition 5 *(a)In the worst-case scenario as well as in expected terms, a merger increases producer surplus, i.e.,* $\Delta PS^h = PS^h - PS > 0$ *and* $\Delta EPS = EPS - PS > 0$.
(b)In the best-case scenario $\Delta PS^l = PS^l - PS$ *may be positive or negative.*

In the worst-case scenario as well as in expected terms, a merger is beneficial to the industry as a whole as in the standard model of SSR. How these gains are divided between the insiders and the outsiders to the merger in our Bayesian setting depends of course on the usual pair of key parameters $(p, \Delta c)$. Interestingly, it is only when efficiency gains are realized ex-post that the industry as a whole may be adversely affected by a merger. In this case, the conjunction of informational market power and efficiency superiority of the merged firm may lead to a reduction in outsiders' profits exceeding the merger's profit expansion. This suggests one plausible explanation for the frequently observed apprehensive reaction of outsiders to a merger.

In the U.S., antitrust authorities generally take consumer welfare as the key indicator in merger cases. So a separate analysis of consumer surplus is highly desirable. Ex-post consumer surpluses, conditional on the actual realized cost, are $CS^h = \frac{1}{2}(a - P^h)Q^h$ and $CS^l = \frac{1}{2}(a - P^l)Q^l$. Expected consumer surplus is clearly the weighted average of the two conditional consumer surpluses: $ECS = pCS^l + (1-p)CS^h$. The expressions are provided in the Appendix. Recall that in case of no efficiency gains ex post, price always increases, and so ex post consumer welfare always decreases.

Proposition 6 *(a)In the worst-case scenario, a merger always lowers consumer surplus, i.e.,* $\Delta CS^h = CS^h - CS < 0$.
(b)In expected terms and in the best-case scenario, a merger may increase or decrease consumer surplus.

Thus, according to this result, if antitrust authorities adopted an absolutely conservative standard requiring that consumer surplus increase in the worst case scenario, then no merger would ever be permitted. On the other hand, going by the ex-ante or best-case standards, the conclusion depends on the levels of p and Δc once more.

We now prove Proposition 6 (b) via an example showing the change in ECS can be positive or negative, depending on the parameters of the model.[17]

Example 7 *Consider the parameter values* $a = 10$, $b = 1$, $n = 10$ *and* $c = 3$. *The dark shaded surface is the* 0 *plane whereas the light shaded area denotes the change in consumer surplus. Clearly, the higher the belief and the lower the cost, the more likely it is for consumer surplus to increase.*

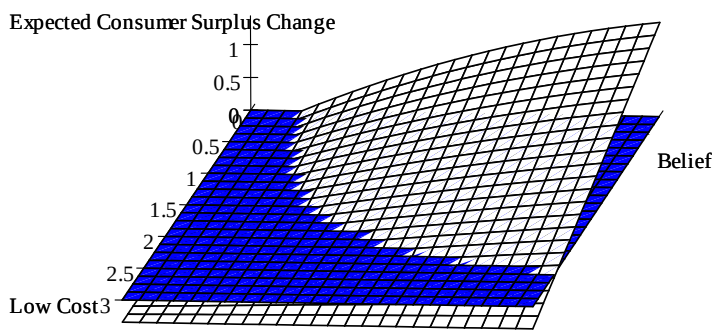


Figure 4.

In view of our Bayesian setting and of the prospect for an efficiency gain, the relationship between consumer and producer surpluses is more nuanced than in the standard SSR model. It makes sense here to ask whether an ex-ante profitable merger will necessarily improve consumer welfare. Example 7 settles this question in the negative, as depicted in the graph below, comparing the expected profitability threshold (dotted line) with the threshold for an increase in expected consumer surplus (solid line). Here, if expected consumer surplus increases so does the merger's expected profit, but not the other way around. Only if the merger is expected to be sufficiently profitable will it become beneficial to consumers as well. Hence, antitrust authorities acting on the basis of consumer surplus should not adopt an a priori laisser-faire merger policy.

[17] Consumer surplus in the best-case scenario follows a similar pattern. For example with parameters $a = 10, b = 1$, $n = 10$, $c = 7$, $p = 0.5$, and $c_l = 6.95$, the change in consumer surplus is $\Delta CS^l = -0.021843$. But when $c_l = 5.62$, the change becomes $\Delta CS^l = 1.1329$.

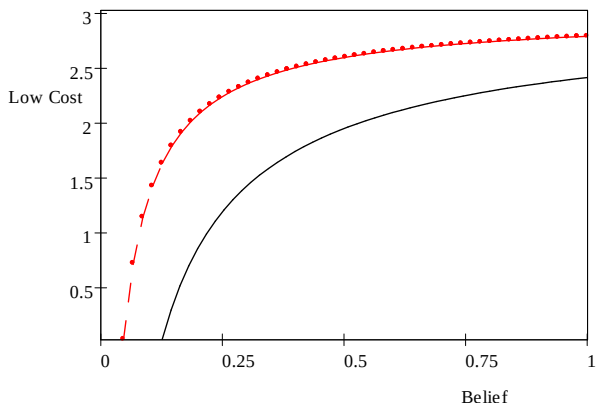


Figure 5.

We now consider the effects of a merger on social welfare.

Proposition 8 *(a)In the worst-case scenario, a merger always lowers social welfare, i.e.,* $\Delta TW^h = TW^h - TW < 0$.
(b)In expected terms, a merger may increase or decrease social welfare.

In addition, one may fully characterize the expected welfare change with a threshold belief (shown in Appendix 8.3, due to the length of the expression), along with an example indicating that a merger will be beneficial to society as a whole, unless p and Δc are both very small.[18] Interestingly, this may be viewed as a Bayesian-Cournot analog of Williamson's (1968) classical efficiency defense, arguing that a small efficiency gain is sufficient to make a merger welfare-improving in a competitive economy.

While Propositions 6 and 8 reveal similar effects of mergers on consumer and social welfare from a qualitative standpoint, an important divergence for merger policy is brought out in the next result, relating the private and social incentives for a merger.

Proposition 9 *In the best-case scenario as well as in expected terms, a merger will increase social welfare whenever the merger is expected to be profitable, i.e.,*

$$\Delta E\pi_m > 0 \Longrightarrow \Delta ETW = ETW - TW > 0 \text{ and } \Delta TW^l = TW^l - TW > 0.$$

This is consistent with Farrell and Shapiro's (1990, Proposition 5) finding that, under some conditions on demand and costs that are satisfied by our linear setting, if a merger with sure efficiency gains (i.e. with $p = 1$ here) is profitable to the merging firms, it will also be welfare improving.

The implications of this result are significant in that they suggest that if two firms' wish to merge were fully based on an expected profit calculation, then, on the basis

[18] This observation can be confirmed to be robust via other simulations.

of a social welfare criterion, antitrust authorities should adopt a laissez-faire policy. However, if efficiency gains do not materialize ex post, then the merger will always be detrimental to society, irrespective of the ex-ante private incentives (Proposition 8). To the extent that it is widely believed that in some cases, mergers are, at least partly, motivated by managerial hubris or empire-building, the implications of Proposition 9 should be viewed with due care. The result also raises in a clear-cut manner the issue of risk-bearing in merger policy.

A continuing controversy in merger policy is whether the key decisive criterion for approving mergers should be consumer or social welfare. Our results neatly bring out the commonalities and the divergences in appropriate public policy responses depending on which standard is adopted. With the somewhat intermediate standard given by the sum of expected consumer surplus and outsiders' profits, proposed by Farrell and Shapiro (1990), the present model would not prescribe a laissez-faire policy.

5 Further Results and Observations

This section presents two extensions of interest, not considered so far. The first deals with how our Bayesian setting would affect m-firm mergers, and the second with how uncertainty over efficiency gains might resolve over time. We then discuss a possible dynamic extension of the present analysis.

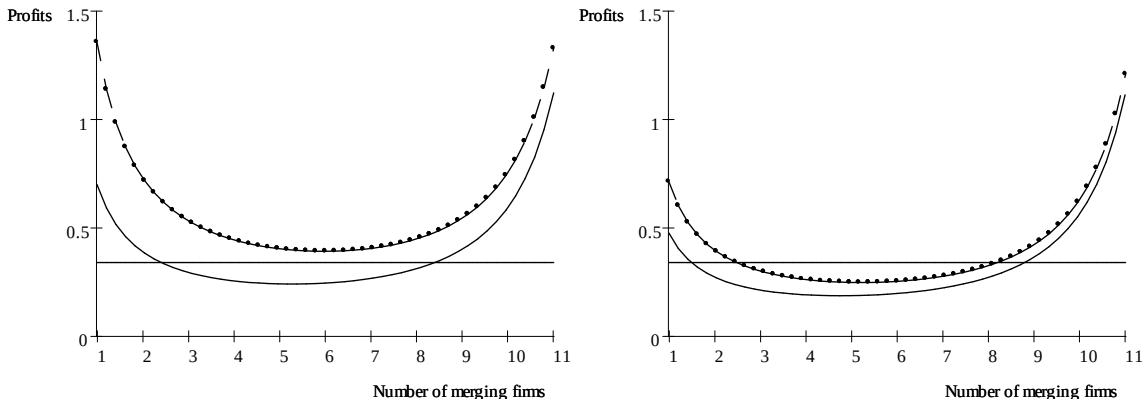
5.1 The Profitability of Multilateral Mergers

Extending consideration to multilateral mergers here, we provide plausible illustrations showing that (i) larger mergers do not necessarily fare better than bilateral mergers and (ii) larger mergers may not even be profitable when bilateral mergers are.

With s merging firms, each firm's profit in the worst-case and ex-ante (expected) benchmarks are respectively

$$\frac{\pi_m^h}{s} = \frac{\left(2a - 2c + p\left(c - c_l\right)\left(n - s + 1\right)\right)^2}{4sb\left(n - s + 3\right)^2} \text{ and } \frac{E\pi_m}{s} = p\frac{\pi_m^l}{s} + (1-p)\frac{\pi_m^h}{s}.$$

Consider $n = 10$, $a = 10$, $c = 3$, $c_l = 2$, $b = 1$ and either $p = 0.25$ or $p = 0.6$. Each of the following two graphs is a plot of π_m^h/s (solid lines) and $E\pi_m/s$ (dotted lines) as functions of s in the two cases.



Profits when $p = 0.6$ Profits when $p = 0.25$

Figure 6. Per partner profits.

In the case where $p = 0.6$, in expected terms, all mergers are profitable, and bilateral mergers are preferred by the merged firms to multilateral ones, at least up to 9 partners. On the other hand, in the worst case scenario the only profitable mergers are those involving $2, 10$ or all 11 firms. By contrast, recall that for this example, only mergers with $9, 10$ or 11 firms are profitable in the complete information Cournot model of SSR. Analogously, when $p = 0.25$, in the worst case scenario, no merger involving less than 9 firms is profitable, while in expected terms a bilateral merger is profitable!

These findings are obviously consistent with the observed reality that virtually all mergers involve two firms, another noteworthy divergence from previous models.

5.2 On the "Dynamics" of Profitability

This subsection discusses some possible dynamic extensions of the static Bayesian approach to mergers, paying close attention again to the stylized facts. A finer empirical finding of GMYZ is that, over their five-year data window, from one year to the next, realized profits increased for profitable mergers but decreased for unprofitable mergers. This seemingly strange finding turns out to be quite consistent with our results if one adds a plausible dynamic extrapolation capturing the resolution of uncertainty over time in our model. Assuming that profitable mergers tend to be those that indeed generate efficiency gains, the outsiders progressively learn about these gains, realize with more and more certainty that they face a lower-cost merged rival and react accordingly. Likewise, if unprofitable mergers are identified with those that failed to generate efficiency gains, rivals will progressively find out over time that they face a high-cost merged rival, and the latter's profits will move accordingly lower. In both cases, the

process eventually settles at the full information Cournot equilibrium that reflects the true efficiency gains actually achieved by the merger. This argument conveys clearly the sense in which the present analysis is of a short-run nature.

In this scenario, it is important to note that the learning process envisioned is not related to signaling; it is rather nonstrategic and is based on information about p gathered via firm reports, leaks in the investigative press, etc. Alternatively, it is reasonable to postulate that due to exogenous noise, for instance in demand or in macroeconomic variables, the outside firms cannot fully learn the true efficiency gains of the merged firm in one, or even a few, periods.[19] There is quite a bit of anectodal evidence in support of this slow learning of the extent of efficiency gains in mergers. For instance, in a report to the Federal Trade Commission advocating a two-stage process to review efficiency claims, one ex ante and one ex post, Brodley (1996) argues that "the ex post proceeding should normally be held between three and five years after the ex ante determination. Efficiency realization generally will require a longer time period than that used in competitive analysis of mergers".[20]

In the way of illustrative insight, consider the following example where the merger decision is based on expected merger profits.

Example 10 *Let* $a = 10, b = 1, n = 10, c = 7, p = 0.5,$ *and* $c_l = 6.8 \in (6.6049, 6.8280)$. *That is,* c_l *is between the 2 critical values of* c_l *in the tables following Proposition 1. As a result,* $\pi_m^h = 0.0984 < 2\pi = 0.125$ *and* $\pi_m^l = 0.1711 > 2\pi = 0.125$. *If the merger decision is based on ex ante expected profits, then, ex post, with probability* p *the cost is* c_l *and the profits are* $\pi_m^l = 0.1711 > 2\pi = 0.125$ *while with probability* $(1-p)$ *the cost is* c *and profits are* $\pi_m^h = 0.0984 < 2\pi$. *The subsequent (complete information) Cournot game is either one with the merged firm having cost* c_l *or one with the merged firm having cost* c. *In the former case, the merged firm's profits are* $0.2066 > \pi_m^l = 0.1711$ *and in the latter case, its profits are* $0.074\,38 < \pi_m^h = 0.0984$.

An alternative plausible dynamic extension would be to a multi-period framework allowing for signalling by the merged firm and learning on the part of outsider firms. Such an extension would produce an even more diverse set of possible equilibrium outcomes, including separating and pooling (perfect Bayesian) equilibria. In the following

[19] This assumption is common in Bayesian oligopoly models (see e.g. Mirman, Samuelson and Schlee, 1994)

[20] Two remarks are worth adding here. The first is that his recommended window of three to five years coincides with the time frame of the data used by nearly all empirical studies on mergers. The second is that the complexity of the post-merger process of reorganization and consolidation is such that one cannot reasonably expect it to be any faster. For instance, to some observers, the Daimler-Chrysler merger is not fully out of this phase yet.

example, the separating equilibrium yields an outcome that is fully consistent with the data on profit dynamics described above (though many other outcomes can emerge in other plausible examples).

Consider the industry in Example 10 operating over two periods with discount factor $\delta = 0.7$. It can be verified[21] that the following strategies and system of beliefs constitute a separating (perfect Bayesian) equilibrium:

First period choices:	$q_m^l = 0.5398, q_m^h = 0.3503, q_o = 0.2550$
Second period choices:	$q_m^l = 0.4546, q_m^h = 0.2727, q_o^h = 0.2727, q_o^l = 0.2546$
Beliefs:	$\Pr(\text{low cost} \mid q < 0.5398) = 0, \Pr(\text{low cost} \mid q \geq 0.5398) = 1$

In this equilibrium, the low-cost merged firm's profits would be 0.1947 in the first period and rise to 0.2066 in the second, while a high-cost merged firm's profits would be 0.1227 in the first period and go down to 0.0744 in the second. Prior to the merger, the total profits of the two firms are 0.1250 per period. Thus, comparing period by period, the merger is profitable for the low-cost merged firm but unprofitable for the high-cost one. Furthermore, in ex-ante terms, using discounted expected profits, the merger is profitable (0.2125 and 0.2992 before and after the merger, respectively). Note that in a separating equilibrium, all the information is fully revealed after the first period.

This two-period example also admits a pooling equilibrium, albeit with beliefs that do not survive the "intuitive criterion". Indeed, there is a pooling equlibrium in which the merged firm produces 0.3636 in the first period regardless of costs and each outsider produces 0.26364; however, to sustain such behavior in equilibrium, extreme beliefs such as $\Pr(\text{high cost} \mid q \neq 0.3636) = 1$ are necessary.

While such strategic dynamic models are appealing, going beyond two periods in this context is unfortunately intractable. At the same time, dynamic models with incomplete information typically give rise to too many equilibrium outcomes, and the present model is no exception.

6 Conclusion

This paper argues that many of the circumstances surrounding mergers call for a theoretical model wherein the firms outside the merger face a new type of rival, characterized by unknown unit costs, reflecting their natural initial uncertainty about the ability of the merged firm to achieve any (of the claimed) efficiency gains. This pervasive uncertainty also affects the approval decision of antitrust authorities, and triggers the

[21] The associated computations are tedious, and are available from the authors upon request.

favorable response by financial markets. Within the obvious confines of a static model, the proposed Bayesian Cournot equilibrium leads to an outcome that is broadly consistent with much of the empirical evidence on the industry effects of mergers, on profits, price and market shares for the merged firm as well as for outsiders, at least in the short run. All in all, the model at hand reflects a simple and natural modification of the standard Cournot approach, based on an informational advantage of the merged firm over outsiders, which confers additional market power to the merged firm, bringing about a surprising level of congruence with stylized facts.

In terms of welfare, mergers lower consumer and social welfare for sure only in the worst-case scenario. In the other two scenarios, welfare depends on the levels of belief and the efficiency gain. An ex-ante profitable merger is necessarily beneficial for expected social welfare but not necessarily for expected consumer welfare. Overall, these results vindicate the central role assigned to efficiency gains in merger policy.

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8 Appendix

This Appendix gathers the computational details, some extra figures, the quantitative illustrations that complement the results, and the proofs of the results in the text.

8.1 Finding the Bayesian Cournot Equilibrium

This part provides the computational details of Section 2. Each (outsider) firm's expected payoffs are:

$$\begin{aligned} E\pi_i &= p\pi_i^h + (1-p)\pi_i^l \\ &= p\left((a - bQ_{-i}^h - bq_i)q_i - cq_i\right) + (1-p)\left((a - bQ_{-i}^l - bq_i)q_i - cq_i\right) \\ &= aq_i - bEQ_{-i}q_i - bq_i^2 - cq_i. \end{aligned}$$

The 1st order condition yields the best response function: $q_i = \frac{a-bEQ_{-i}-c}{2b}$. As everybody knows the cost of $n-1$ firms but not the cost of the nth firm, the best response function of the *each outsider* becomes $q_o = \frac{a-c-bEq_m}{bn}$.

The *merged* firm's best response functions are:

$$q_m^h = \frac{a-c-b(n-1)q_o}{2b} \text{ and } q_m^l = \frac{a-c_l-b(n-1)q_o}{2b}.$$

Each *outsider's* and the merged firm's outputs are presented in the text.

The three versions of total output are:

$$Q^l = \frac{2n\,(a-c) + \Delta c\,(n+1-p\,(n-1))}{2b\,(n+1)},\ Q^h = \frac{2n\,(a-c) - p\Delta c\,(n-1)}{2b\,(n+1)}$$

$$\text{and } EQ = \frac{n\,(a-c) + p\Delta c}{b(n+1)}.$$

The corresponding prices are presented in the text.

The conditional expected profits of each outsider firm are:

$$\pi_o^h = E\pi_o + \frac{p\Delta c\,[(a-c) - p\Delta c]}{2b(n+1)} \text{ and } \pi_o^l = E\pi_o + \frac{\Delta c(1-p)\,[p\Delta c - (a-c)]}{2b(n+1)}$$

The expected profits of *each outsider* firm and conditional profits of the merged firm are again presented in Section 2. The expected profits of the merged firm is $E\pi_m = p\pi_m^l + (1-p)\pi_m^h$, or

$$E\pi_m = \frac{p^2\Delta c^2 (3n+1)(n-1) + p\Delta c\left(8n(a-c) + \Delta c(n+1)^2\right) + 4(c-a)^2}{4b(n+1)^2}$$

The change in expected profits is $\Delta E\pi_m = \pi_m - 2\pi$, or

$$\frac{\left(\begin{array}{c} p^2(3n+1)(n-1)(n+2)^2\Delta c^2 \\ +p(n+2)^2\Delta c\left(8n(a-c) + \Delta c(n+1)^2\right) - 4(c-a)^2(n^2-2) \end{array}\right)}{4b(n+1)^2(2+n)^2}$$

Threshold belief above which expected profits are higher than pre-merger profits:

$$p^* = \frac{\left(\begin{array}{c} -(n+2)\left[8n(a-c) + \Delta c(n+1)^2\right] \\ +\sqrt{(n+2)^2\left[8n(a-c) + \Delta c(n+1)^2\right] + 16(3n+1)(n-1)(a-c)^2(n^2-2)} \end{array}\right)}{2(3n+1)(n-1)(n+2)\Delta c}$$

8.2 Extra Figures and Explanations

This part contains supplementary figures and their explanations.

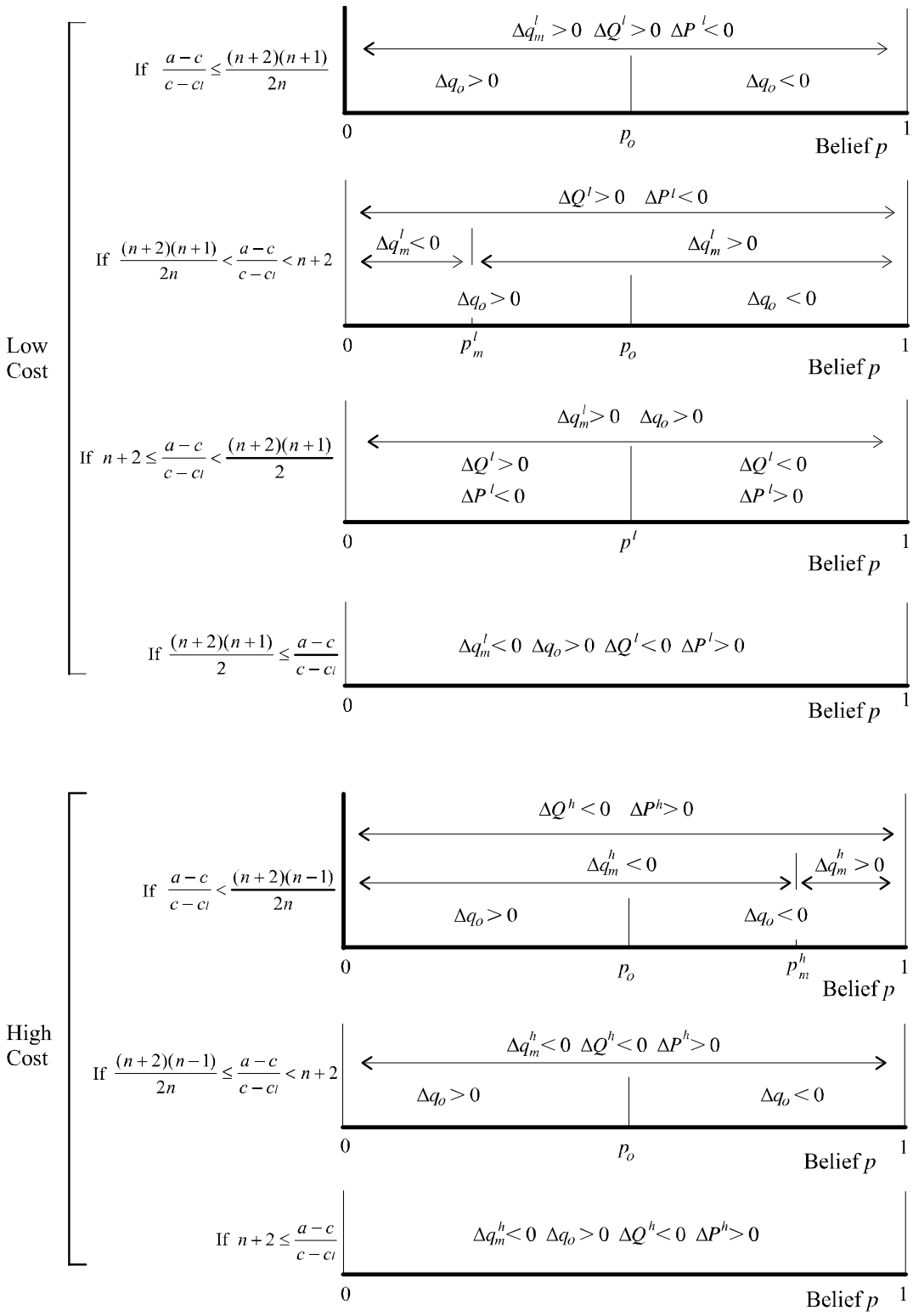

Figure 7. Market performance in low and high cost realizations.

Figure 3 explanation. Note that if $\frac{a-c}{c-c_l} \geq n+2$ then $p_o \geq 1$ hence, $E\pi_o \geq \pi$ and $q_o \geq q$ for all $p \in [0,1]$. If the merger is profitable in the worst case scenario, then $p > p^{h*}$ and it easy to show that $p_o > p^{h*}$ for all $(a-c) > 0$ and for all $\Delta c > 0$ since the inequality reduces to $n > -1$ which is always true. If the merger is profitable in the worst case scenario then it is profitable in expected terms as well, i.e., $p^* < p^{h*}$, hence, $p^* < p_o$. It follows immediately from the expressions presented in the previous section that $EQ > Q$ if and only if $p > p_o$. ■

Figure 7 explanation. For information regarding p_o see explanation of Figure 3.

Low Cost:

Further note that $p_m^l \geq 0$ if and only if $\frac{a-c}{c-c_l} \geq \frac{(n+2)(n+1)}{2n}$ and $p_m^l \leq 1$ if and only if $\frac{a-c}{c-c_l} \leq n+2$. It is easy to see that $\frac{(n+2)(n+1)}{2n} < n+2$.Observe that $p_m^l < p_o$ if and only if $\frac{a-c}{c-c_l} < n+2$ but otherwise both p_o and p_m^l are greater than 1 hence their relationship is insignificant. Similarly, notice that $p^l \leq 1$ if and only if $\frac{a-c}{c-c_l} \geq n+2$ and $p^l \geq 0$ if and only if $\frac{a-c}{c-c_l} \leq \frac{(n+2)(n+1)}{2}$. Again it is easy to see that $n+2 \leq \frac{(n+2)(n+1)}{2}$ is always true. Finally, the inequality $Q^l > Q$ reduces to $p < p^l$. Observe that if p^l is effective (i.e., $p^l < 1$) p_o and p_m^l are ineffective (i.e., greater than 1).

High Cost:

Note that it is always the case that $p_m^h \geq 0$ and $p_m^h \leq 1$ if and only if $\frac{a-c}{c-c_l} \leq \frac{(n+2)(n-1)}{2n}$. Similarly, it is easy to show that $p_m^h > p_o$ for all $(a-c) > 0$ and for all $(c-c_l) > 0$ since the inequality reduces to $n > -1$, which is always true. Finally the inequality $Q^h > Q$ reduces to $2(a-c) + p(c-c_l)(n-1)(n+2) > 0$, which is always true. ■

8.3 Welfare analysis details

This part provides the computational details of Section 4. We obtain:

$$PS^h = \frac{(2(a-c) + p\Delta c(n-1))^2}{4b(n+1)^2} + (n-1)\left(\frac{a-c-p\Delta c}{b(n+1)}\right)\left(\frac{2(a-c)+p\Delta c(n-1)}{2(n+1)}\right)$$

$$\begin{aligned} PS^l &= \frac{(2(a-c) + \Delta c(1+n+p(n-1)))^2}{4b(n+1)^2} \\ &\quad +(n-1)\left(\frac{a-c-p\Delta c}{b(n+1)}\right)\left(\frac{2(a+nc_l)+\Delta c(n-1)(p+1)}{2(n+1)} - c\right) \end{aligned}$$

Expected producer surplus, EPS, is:

$$\begin{aligned} EPS &= p\frac{(2(a-c) + \Delta c(1+n+p(n-1)))^2}{4b(n+1)^2} \\ &\quad +(1-p)\frac{(2(a-c)+p\Delta c(n-1))^2}{4b(n+1)^2} + (n-1)\frac{(a-c-p\Delta c)^2}{b(n+1)^2} \end{aligned}$$

The change in expected producer surplus, $\Delta EPS = EPS - PS$, is:

$$\begin{aligned}\Delta EPS &= p\frac{\left(2\left(a-c\right)+\Delta c\left(1+n+p\left(n-1\right)\right)\right)^2}{4b\left(n+1\right)^2}+\left(1-p\right)\frac{\left(2\left(a-c\right)+p\Delta c\left(n-1\right)\right)^2}{4b\left(n+1\right)^2}\\ &\quad +(n-1)\frac{\left(a-c-p\Delta c\right)^2}{b\left(n+1\right)^2}-\left(n+1\right)\frac{\left(a-c\right)^2}{b\left(2+n\right)^2}.\end{aligned}$$

The consumer surpluses are:

$$\begin{aligned}ECS &= p\left(\frac{2n\left(a-c_l\right)-\Delta c(n-1)(p+1)}{8b\left(n+1\right)^2}\right)\left(2n(a-c)+\Delta c(1+n-p(n-1)\right)\\ &\quad +(1-p)\frac{1}{2b}\left(\frac{2n\left(a-c\right)-p\Delta c\left(n-1\right)}{2\left(n+1\right)}\right)^2\end{aligned}$$

$$CS^h = \frac{1}{2b}\left(\frac{2n\left(a-c\right)-p\Delta c\left(n-1\right)}{2\left(n+1\right)}\right)^2$$

$$CS^l = \left(\frac{2n\left(a-c_l\right)-\Delta c(n-1)(p+1)}{8b\left(n+1\right)^2}\right)\left(2n(a-c)-\Delta c(n-1)(p+1)\right)$$

The expected total welfare is given by:

$$\begin{aligned}ETW &= p\frac{\left(2\left(a-c\right)+\Delta c\left(1+n+p\left(n-1\right)\right)\right)^2}{4b\left(n+1\right)^2}\\ &\quad +(1-p)\frac{\left(2\left(a-c\right)+p\Delta c\left(n-1\right)\right)^2}{4b\left(n+1\right)^2}+\left(n-1\right)\frac{\left(a-c-p\Delta c\right)^2}{b\left(n+1\right)^2}\\ &\quad +p\left(a-\frac{2\left(a+nc_l\right)+\Delta c(n-1)(p+1)}{2\left(n+1\right)}\right)\frac{\left(2n(a-c)+\Delta c(1+n-p(n-1)\right)}{4b\left(n+1\right)}\\ &\quad +\frac{(1-p)}{4b\left(n+1\right)}\left(a-\frac{2\left(a+nc\right)+p\Delta c\left(n-1\right)}{2\left(n+1\right)}\right)\left(2n(a-c)-p(n-1)\Delta c\right)\end{aligned}$$

which reduces to:

$$\begin{aligned}ETW &= p^2\Delta c^2\left(5n+7\right)\left(n-1\right)+p\Delta c\left[8\left(a-c\right)\left(n+2\right)+3\Delta c\left(n^2+2n+1\right)\right]\\ &\quad +4n\left(c-a\right)^2\left(n+2\right)\end{aligned}$$

The change in expected total welfare is then:

$$\Delta ETW = \frac{\left(\begin{array}{c} p^2\left(5n+7\right)\left(n-1\right)\left(n+2\right)^2\Delta c^2\\ +p\left(n+2\right)^2\Delta c\left(8\left(a-c\right)\left(2+n\right)+3\Delta c\left(n+1\right)^2\right)\\ -4\left(c-a\right)^2\left(2n+3\right)\end{array}\right)}{8\left(n+1\right)^2 b\left(n+2\right)^2}$$

The threshold belief above which post-merger expected total welfare is higher is

$$p' = \frac{\left(\begin{array}{c} -(n+2)\left[8(a-c)(n+2)+3\Delta c\left(n+1\right)^2\right]+\\ \sqrt{\begin{array}{c}(n+2)^2\left[8(a-c)(n+2)+3\Delta c\left(n+1\right)^2\right]\\ +16(n-1)(5n+7)\left(a-c\right)^2\left(2n+3\right)\end{array}}\end{array}\right)}{2(5n+7)(n-1)(n+2)\Delta c}$$

8.4 Illustrations

This part of the appendix provides some insight of a quantitative nature into some of the results of Section 4.

(a) *The values of* $(p, \Delta c)$ *and expected social welfare.*

Consider the parameter values $a = 10$, $b = 1$, $n = 10$, and $c = 3$.

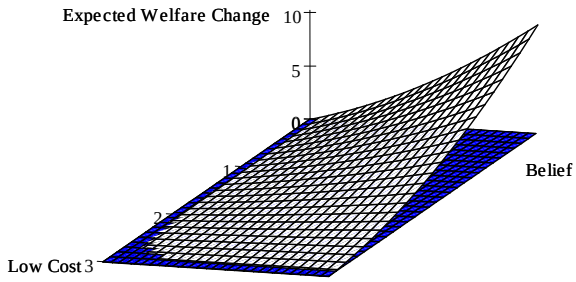

Figure 8.

It is clear that unless the belief p and the efficiency gain are extremely low, a merger will be advantageous to society, i.e., expected total welfare will increase.

Example 11 *Consider the parameter values* $a = 10$, $b = 1$, $n = 10$ *and* $c = 7$.

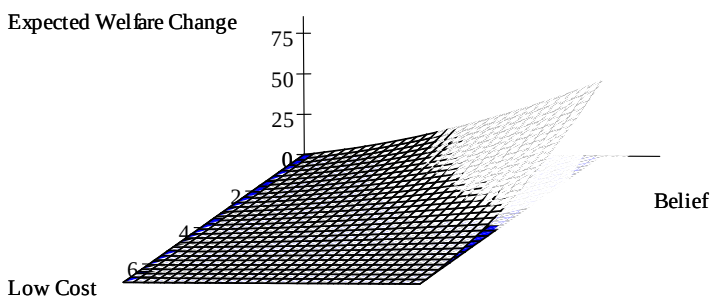

Figure 9.

The observations made in the previous example concerning the impact of a merger to society are reinforced. In fact, as can be shown to hold for consumer surplus as well, the opportunities for a socially beneficial merger are increased when the starting cost is higher. Naturally, higher starting costs provide more opportunities for efficiency gains that are advantageous to consumers as well.

8.5 Proofs

Proof of Proposition 1. The merged firm is profitable if $\pi_m^h > 2\pi$ which reduces to

$$p > \frac{2\left(a-c\right)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{\left(c-c_l\right)\left(n+2\right)\left(n-1\right)} = p^{h*}$$

It is easy to see that $p^{h*} > 0$ if $n \geq 2$. Moreover, $p^{h*} < 1$ if the cost gains are sufficient:

$$\frac{2\left(a-c\right)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{\left(n+2\right)\left(n-1\right)} < c - c_l.$$

For the lower bound on cost gains to be feasible ($c - c_l < c$) it must be that the pre-merger cost is at least:

$$\frac{2a\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{2\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)+\left(n+2\right)\left(n-1\right)} < c.$$

Note that the latter lower bound on c is always below a. ■

Proof of Proposition 2. In the worst case scenario, the merged firm expands if and only if $q_m^h \geq 2q$, or $p \geq p_m^h = \frac{2n(a-c)}{\Delta c(n-1)(2+n)}$. In the best case scenario, it expands if and only if $q_m^l \geq 2q$, or $p \geq p_m^l = \frac{2n(a-c)-(c-c_l)(n+1)(n+2)}{\Delta c(2+n)(n-1)}$. ■

Proof of Proposition 3. Outsider firms benefit from the merger and expand their output if $E\pi_o \geq \pi$ and $q_o \geq q$. Both inequalities reduce to $p \leq p_o = \frac{a-c}{\Delta c(n+2)}$. Similarly, the merged firm expands its output if $Eq_m \geq 2q$, which reduces to $p \geq p_o$. ■

Proof of Proposition 4. (a) Setting $P^h > P$ reduces to $2(a-c) + p\Delta c(n-1)(n+2) > 0$, which is always true. (b) Setting $EP > P$ reduces to $p < \frac{a-c}{\Delta c(n+2)} = p_o$. (c) Setting $P^l > P$ reduces to $p > p^l = \frac{(n+1)(n+2)\Delta c - 2(a-c)}{\Delta c(n+1)(n-1)}$. ■

Proof of Proposition 5. The change in Producer Surplus in the worst case scenario is given below:

$$\Delta PS^h = \frac{\left(2n\left(a-c\right)-p(n-1)\Delta c\right)\left(2(a-c)+p(n-1)\Delta c\right)}{4b\left(n+1\right)^2} - (n+1)\frac{\left(a-c\right)^2}{b\left(2+n\right)^2}.$$

Recall that we require that $p < \frac{a-c}{\Delta c}$ for an interior solution. Now observe that ΔPS^h is increasing in p since

$$\frac{d\Delta PS^h}{dp} = \frac{2(n-1)^2\Delta c^2\left(\frac{a-c}{\Delta c}-p\right)}{4b\left(n+1\right)^2} > 0.$$

Hence, it suffices to show that $\Delta PS^h|_{p=0} > 0$. Indeed

$$\Delta PS^h|_{p=0} = \frac{\left(a-c\right)^2}{b}\left(\frac{n}{\left(n+1\right)^2} - \frac{n+1}{\left(2+n\right)^2}\right) > 0.$$

To show $\Delta EPS > 0$ we take its derivative w.r.t. p and see that is it positive:

$$\frac{d\Delta EPS}{dp} = \frac{2p\left(3n+5\right)\left(n-1\right)\left(n+2\right)^2\Delta c^2 + \left(n+2\right)^2\Delta c\left(8\left(a-c\right)+\left(1+n\right)^2\Delta c\right)}{4b\left(n+1\right)^2\left(2+n\right)^2} > 0$$

So $\Delta EPS > 0$ if $\Delta EPS|_{p=0} > 0$. Then observe that $\Delta EPS|_{p=0} = \frac{(a-c)^2\left(n^2+n-1\right)}{b(n+1)^2(n+2)^2} > 0$.

It is easy to see that when the parameters take the following values: $a = 10, b = 1$, $n = 10$, $c = 7$, $p = 0.5$, $c_l = 6.6200$ the change in PS^l becomes $\Delta PS^l = -2.650\,9\times10^{-2}$. Similarly, when the parameters are $a = 10, b = 1$, $n = 10$, $c = 7$, $p = 0.5$, $c_l = 5.6200$ the change in PS^l becomes $\Delta PS^l = 0.607\,38$. ■

Proof of Proposition 6. The change in consumer surplus in the worst case scenario is given below:

$$\Delta CS^h = \frac{1}{2}\frac{\left(2n(a-c)-p\left(n-1\right)\Delta c\right)^2}{4b\left(n+1\right)^2} - \frac{1}{2}\frac{\left(a-c\right)^2\left(n+1\right)^2}{\left(n+2\right)^2 b} < 0$$

Observe that ΔCS^h is a decreasing function of p as $2n(a-c)-p\left(n-1\right)\Delta c > 0$. It is easy to see that at $p = 0$ where ΔCS^h takes its highest value it is already negative. ■

Proof of Proposition 8. The change in Total Welfare in the worst case scenario is given by the formula below:

$$\begin{aligned}\Delta TW^h &= \frac{\left(2n\left(a-c\right)-p(n-1)\Delta c\right)\left(2(n+2)(a-c)+p(n-1)\Delta c\right)}{8b\left(n+1\right)^2} \\ &\quad -(n+1)\frac{\left(a-c\right)^2}{b\left(2+n\right)^2} - \frac{1}{2}\frac{\left(a-c\right)^2\left(n+1\right)^2}{\left(n+2\right)^2 b}.\end{aligned}$$

It is easy to see that ΔTW^h is a decreasing function of p, thus, it suffices to show that $\Delta TW^h < 0$ at $p = 0$ where ΔTW^h takes its highest value. Indeed

$$\Delta TW^h|_{p=0} = \frac{(a-c)^2}{b}\left(\frac{-2n-3}{2\left(n+1\right)^2\left(2+n\right)^2}\right) < 0.$$

As Figures 8 and 9 illustrate the expected total welfare can increase or decrease depending on the parameters. ■

Proof of Proposition 9. Observe that for $\Delta E\pi_m > 0$ it suffices to show that

$$\begin{aligned}&p^2\left(3n+1\right)\left(n-1\right)\left(n+2\right)^2\Delta c^2 \ + \\ &p\left(n+2\right)^2\Delta c\left(8n\left(a-c\right)+\Delta c\left(n+1\right)^2\right) - \\ &4\left(c-a\right)^2\left(n^2-2\right) > 0.\end{aligned}$$

Let the coefficient of p^2 be denoted by

$$\alpha_m = \left(3n+1\right)\left(n-1\right)\left(n+2\right)^2\Delta c^2,$$

the coefficient of p be denoted by

$$\beta_m = (n+2)^2 \Delta c \left(8n(a-c) + \Delta c (n+1)^2\right)$$

and the constant be denoted by

$$\gamma_m = -4(a-c)^2 \left(n^2 - 2\right).$$

Note that $\alpha_m, \beta_m > 0$, hence $\Delta E\pi_m$ is increasing in p.

Now observe that for $\Delta ETW > 0$ it suffices to show that

$$\begin{aligned}
&p^2 (5n+7)(n-1)(n+2)^2 \Delta c^2 + \\
&p(n+2)^2 \Delta c \left(8(a-c)(2+n) + 3\Delta c (n+1)^2\right) - \\
&4(c-a)^2 (2n+3) > 0.
\end{aligned}$$

Similarly, let the coefficient of p^2 be denoted by

$$\alpha_w = (5n+7)(n-1)(n+2)^2 \Delta c^2,$$

the coefficient of p be denoted by

$$\beta_w = (n+2)^2 \Delta c \left(8(a-c)(2+n) + 3\Delta c (n+1)^2\right)$$

and the constant term by $\gamma_w = -4(a-c)^2 (2n+3)$. Again, note that $\alpha_w, \beta_w > 0$, hence ΔETW is increasing in p.

Next note that $\alpha_w > \alpha_m$, $\beta_w > \beta_m$ and $\gamma_w > \gamma_m$. When $\Delta E\pi_m(p^*) = 0$ we have $p^{*2}\alpha_m + p^{*2}\beta_m = -\gamma_m$. since $\alpha_w > \alpha_m$ and $\beta_w > \beta_m$ we have $p^{*2}\alpha_w + p^{*2}\beta_w > p^{*2}\alpha_m + p^{*2}\beta_m$ hence $p^{*2}\alpha_w + p^{*2}\beta_w > -\gamma_m$ but $\gamma_m < \gamma_w \Leftrightarrow -\gamma_m > -\gamma_w$. Thus, $p^{*2}\alpha_w + p^{*2}\beta_w > -\gamma_w$ which implies that $p^{*2}\alpha_w + p^{*2}\beta_w + \gamma_w > 0 \Leftrightarrow \Delta ETW(p^*) > 0$.

To conclude, when $p = p^*$ we have $\Delta E\pi_m(p^*) = 0$ and $\Delta ETW(p^*) > 0$ and for all $p > p^*$ we have $\Delta E\pi_m(p^*) > 0$ and $\Delta ETW(p^*) > 0$ since both $\Delta E\pi_m(p)$ and $\Delta ETW(p)$ are increasing functions of p as argued earlier.

>From $\Delta TW^h < 0$ and $\Delta ETW > 0$ it follows that $\Delta TW^l > 0$. ■