Price and Resource-Related Risk of the Wind Power Business in Electricity Markets

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December 2008

A thesis submitted to McGill University in partial fulfillment of the requirements of the degree of Master in Engineering

A mis padres, que han estado siempre apoyándome, por todo lo que han hecho por mí.

A mi hermano y a mis abuelos con mucho cariño.

A Hugo Alejandro, que ha estado a mi lado cada día de esta etapa, haciendo de ella una experiencia inolvidable.

Acknowledgements

I would like to thank Prof. Francisco Galiana for giving me the opportunity to pursue my Master of Engineering degree under his supervision, for his consideration and friendship. His optimism and wisdom are always source of inspiration.

My deepest gratitude to Hugo Alejandro, for his devoted support and for everything I have learned thanks to his dedication and infinite patience.

Thanks to the Power Engineering Laboratory and the Department of Electrical and Computer Engineering of McGill University for excellent facilities provided and to Sameh, José, Jonathan and Carlos for their friendship and collaboration.

Finally, I would like to thank the TALENTIA Scholarship Program, from the Government of Andalusia, Spain for the financial support.

Abstract

A generalized mathematical model is developed that allows the quantification of the economic risk of the investment in a wind farm under the two most common revenue options defined by how the output of the wind farm is paid to the owner: a) at a pre-established fixed-rate or, b) at the hourly electricity market prices. Sensitivities are also characterized in order to determine the influence of various market and resource-related parameters on the variability of the annual revenue and the corresponding economic risk. Particular to the fixed-rate revenue option, the developed model allows the owner of the wind farm to determine the minimum necessary rate to meet a pre-established risk-return requirement. The model is validated using actual wind power output and electricity market datasets. Understanding the sources of risk in any investment is crucial for the development of proper hedging strategies.

Resumé

Dans ce travail, un modèle mathématique généralisée est développé qui permet la quantification du risque économique de l'investissement dans un parc éolien dans le cadre des deux options de rémunération les plus communes, définies par la manière dont la production du parc éolien est rémunérée au propriétaire : a) à un tarif fixe pré-établi ou, b) au prix horaire du marché d'électricité. Des sensibilités sont aussi caractérisées afin de déterminer l'influence de divers paramètres reliés au marché d'électricité et à la ressource éolienne sur la variabilité des recettes annuelles et ses risques économiques. Dans l'option de recette à tarif fixe, le modèle développé permet au propriétaire du parc éolien de déterminer le tarif minimal nécessaire pour répondre à une exigence de risque pré-établie. Le modèle est validé en utilisant des données réelles de production d'énergie éolienne et du marché d'électricité. La connaissance des sources de risque dans un investissement est cruciale pour la conception des stratégies de couverture propres.

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Chapter 1: Introduction

During the last two decades, many countries and jurisdictions around the world have embarked in a restructuring process of their electricity industry with the objective of introducing competition in the generation segment and offering more choice to consumers. This new environment has been embodied into what the industry proudly calls an ‘electricity market’. Every restructuring process has been unique and experience shows that there is no recipe for success. In fact, a considerable number of electricity markets have failed to reach the initial objectives of regulators and governments and have been completely redesigned.

Regarding the expansion in the generation segment over the last years, wind power has emerged as the fastest-growing renewable energy generation technology due to a variety of reasons, namely: a) wind is a zero carbon emissions generation technology; b) unlike fossil-fuelled power plants, wind power as resource is free and available almost anywhere; c) wind turbine costs have dropped dramatically during the last years to the point that this technology in now competitive with conventional generation in terms of investment ($/MW) and operation ($/MWh). In many cases, and due to advances in wind resource forecasting, wind is now treated more and more as a conventional power sources in terms of scheduling, power imbalance charges, ramping control, etc.

There are basically two options for the trading of wind power around the world, regardless of the structure of the electricity industry.

Fixed-Rate Option: In this option (the most common), the owner of a wind farm is paid a fixed rate for the output of the wind farm over a predetermined period of time (usually 10 to 25 years). The rate paid to the owner is either defined by the regulating entity or established through a competitive RFP process. This rate usually recognizes the typical levelized cost of the investment (or Cost of Electricity, COE, in $/MWh) plus an overhead that includes minimum profits and, possibly, the environmental benefits brought about by wind power to the specific power system or region. For instance, in the province of Quebec, Canada, the average fixed rate accepted for the winning bidders in the 2,004 MW wind RFP released in 2008 was 8.7 CAD¢/kWh [1]. In Spain, the rate paid for wind power in the fixed rate scheme is 7.32 EUR¢/kWh for the first 20 years of operation [2].

Market-Price Option: In this option, large-scale wind farms are paid the electricity market clearing price (usually defined at the hourly timeframe). Since wind power is not dispatchable, it must be integrated into the system whenever available. Therefore, wind power is always assigned zero cost in the generation scheduling optimization process. Some markets pay wind farm owners the day-ahead market clearing prices while others pay the real-time prices. Other markets charge wind power real-time imbalance fees in order to encourage forecasting performance improvement [3]. Some other jurisdictions [2] give wind farm owners the option to receive either a pre-established fixed rate or the hourly electricity market price plus a premium, according to the owner's convenience. Unlike the case of the 'fixed rate' option for wind, under real-time rates the owner of the wind farm is faced with a variable revenue, which depends not only on the natural fluctuations of the wind resource but also on the fluctuations of the electricity market prices. Besides the market revenue, wind power may also face imbalance charges in some jurisdictions [4] much like conventional power sources. These imbalances are either priced at fixed pre-established rates or at real-time balancing prices.

The previous two alternatives (Fixed-Rate or Market-Price option) can be deemed as the fundamental revenue alternatives for a wind farm. These alternatives are adopted in one way or another in every jurisdiction around the world. There may be however some additional sources of revenue for a wind farm, which usually represent smaller portions of the total wind farm's revenue and are subject to specific conditions established by each particular regulating entity. Moreover, there are jurisdictions that implement none, all or some of them and under different locally customized versions. These sources are usually: a) *environmental revenue and carbon-related credits*, which recognize the fact that every MWh of wind power would displace, in general but not always, one MWh of power from conventional fossil-fuelled generators and their corresponding carbon emissions; b) *real-time imbalance settlement*s, whereby generators are charged (or paid) according to the difference between the day-ahead (or hours-ahead) wind power forecast with respect to the actual output in real-time operations, c) *capacity contribution*, by which, despite the intermittency of the wind resource, wind farms are recognized as contributors to the reliability of the system, especially to the supply mix during peak load hours.

The above three source of additional revenue usually represent only a percentage of the total revenue of a wind farm. In addition, there are several markets where none of these revenue options have been implemented; other markets have implemented only one or two or even all of them, usually under different set of rules and procedures, customized by the local power industry regulator or market operator. This work, therefore, focuses on the two fundamental widespread revenue options (fixed-rate option and electricity market price option for the trading of the electricity produced by the wind farm). The generalized nature of the developed model, however, allows for the introduction of additional revenue factors once their specific characteristics are clearly identified.

With this in mind, the objective of this work is to determine the most relevant market-related and resource-related factors that create economic risk in the form of uncertainty in the annual economic revenues for the two basic wind farm revenue options. In addition, the mathematical and statistical models developed in this work allow the estimation of sensitivities of risk with respect to the different resource and market-related risk factors. One of the main applications of this work is therefore to assist wind farm investors in identifying how both, market price and resource variability combined, affect the uncertainty in the economic return for their investment given by the sale of their electricity production.

Some sites that are considered adequate from the wind resource point of view (high capacity factors and low variability) may be located in jurisdictions dominated by volatile electricity markets. According to the risk aversion characteristic of a particular investor, such a setting may not be optimal. The sensitivities developed in this work provide information on how any strategy to reduce the effect of each risk factor (market and resource variability, wind farm capacity factor, etc) may help reduce the economic uncertainty for the investment. Since the costs of hedging against economic uncertainty are not negligible (for instance in the fuel markets [5]), considering all risk factors in an integrated way may facilitate the design of hedging instruments and improve the financial performance of the investment.

In fact, the consensus among investors in the power business is that the most decisive risk factor for their investment besides the regulatory uncertainty is the volatility in the electricity market prices [6]. If it is a fact that regulators have stressed the importance in developing more stable markets with substantially lower long-term regulatory uncertainties [7], the same cannot be said about the volatility in the electricity prices. The reason is that the process of price formation depends on factors that lie outside the market itself, the most important of which is the cost of fuel, especially natural gas. For instance, in the early 90's, large natural gas reserves were made available to the power industry in North and South America, which motivated an escalation in the construction of gas-fuelled power plants across the continent. The result is that the growing demand for natural gas put considerable pressure on the supply side, resulting in a corresponding price volatility. As a consequence, a number of electricity markets dominated by gas-fuelled plants started to experience volatility in the market clearing prices. This ongoing phenomenon has been aggravated by increasingly stringent environmental requirements for other types of fossil-fuelled plants, especially coal, which results in higher operating costs and/or penalties with the corresponding effect on electricity prices.

A substantial amount of research has been devoted to the issue of electricity market price volatility given the implications that it has for investors and consumers in terms of economic risks. For instance, in [6] the issue of electricity market price volatility is explored for the case of Ontario, Canada. A similar study is carried out in [8] where the price volatility in electricity markets in the United States is quantified based on time-series analysis.

Nonetheless, the volatility in the electricity prices is just one piece in the puzzle of understanding the economic risks faced by a generator for the sale of its electricity. For the owner of a wind farm, the production of electricity is subject to the availability of the wind resource, which depends on variable atmospheric conditions. For instance, in [4] a risk-minimization method is developed to determine the optimal short-run, hour-ahead energy contract level for wind generators. However, this method is only applicable to the specific trading rules of the electricity market in the UK and, therefore, cannot be generalized to other settings. In any case, the main conclusion here is that the intermittency of winds adds a resource-related risk to the inherent market-related risk in the operating revenue of a wind farm.

There are different methodologies for the quantification of the risks inherent to a given economic or financial activity. Among those methodologies, the so-called "Value at Risk" (VaR) criterion stands out as the most widespread [9]. This is the methodology that will be used in this work, of which a general overview will also be provided.

The objective of this work is, therefore, to quantify the extent to which both, the market and resource-related risk, influence the overall economic risks of a wind farm operating under the two most common revenue options (fixed-rate or electricity market-price).

Chapter 2: Models for Wind Resource and Electricity Market Prices

This section describes the statistical characterization of the different parameters and variables that influence the hourly and annual revenue of a wind farm operating under the two different market options previously described namely a **fixed-rate** scheme and an **hourly electricity market** scheme. Statistical models for the following variables are examined in this section:

- Wind Power Output
    - Through wind speed records and a wind farm power curve
    - Through actual wind power output records
- Electricity market prices
    - Through actual records
    - Through the market price versus demand relationship

Each of these models is described next. These models will be used in the subsequent variability analyses for the hourly and annual wind farm revenues, which is the main objective of this work.

2.1 Modeling Wind Power Output

In this section two approaches for modeling the output of a wind farms will be examined. The first one is based on the wind farm's wind power curve to convert wind speed data into power output (in MW). The second one is to use actual wind power output records from the wind farm of interest. Each of these alternatives is used depending on the dataset available (wind speeds or actual power outputs) and are described next.

2.1.1 Wind Speed – Power Curve Approach

This approach consists of modeling wind speeds in conjunction with a model of the wind farm's power curve. This procedure is the only way of characterizing the output of the wind farm in the project-planning phase, since actual power output records are obviously not available.

2.1.1.1 Modeling Wind

Wind is a weather related phenomenon, which depends on a series of complex atmospheric interactions mainly driven by the temperature differences that arise from the uneven warming of the atmosphere by the sun. Winds are usually represented by their speed and direction at a particular site and have been characterized for decades through physical measurement using anemometers. With the advent of high-performance computers supported by satellite data, the numerical simulation of the atmosphere (particularly winds) is becoming a more attractive option over direct physical measurement, especially for long-term wind resource assessment.

Wind speed U at a particular location can be modeled through a Weibull Probability Density Function (PDF) [10]. The Weibull PDF is completely determined by two parameters usually named the shape factor, k, and the scale factor, c. Both parameters are a function of the mean wind speed, $\overline{U}$, and the corresponding standard deviation σ_U. The Weibull PDF is given by:

$$p(U) = \left(\frac{k}{c}\right)\left(\frac{U}{c}\right)^{k-1} \exp\left[-\left(\frac{U}{c}\right)^{k}\right] \tag{1}$$

There are different models for estimating the parameters k and c of the Weibull distribution from a particular wind dataset. In this work, these parameters have been calculated using the Justus and Lysen methods respectively [10]. By the Justus method, the shape factor, k, is given by:

$$k = \left(\frac{\sigma_U}{\overline{U}} \right)^{-1.086} \tag{2}$$

This method provides accurate results only for the range $1 \leq k < 10$. Typical values for this parameter range between 1.5 and 3.5. On the other hand, the scale factor, c, can be estimated through the Lysen method as:

$$c = \overline{U}\left(0.568 + 0.433/k\right)^{-\frac{1}{k}} \tag{3}$$

Fig. 1 shows different examples of Weibull PDFs for different k and c factors. For an equal c factor, larger values for k indicate that the Weibull PDF has a sharper peak and, therefore, less variability (winds are more concentrated around the mean). The parameter c is generally related to the mean of the distribution.

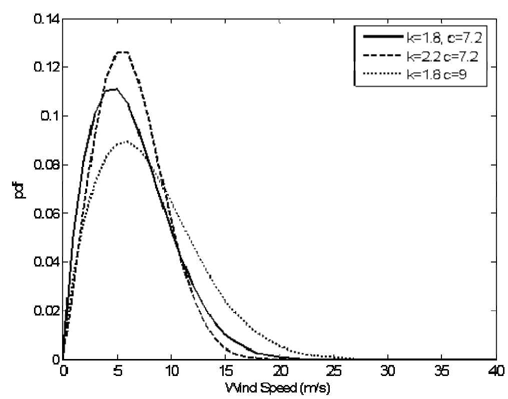


Fig. 1: Examples of typical wind speed Weibull PDFs for different parameters *k* and *c*.

Although the wind speeds at a specific location are determined by a Weibull PDF, they display typical patterns that vary according to different time-frames: hourly, monthly, seasonally, annual and even multi-decadal [11]. It is well-known that overnight winds are stronger than afternoon winds in some regions. Also, winds are generally weaker in summer when compared to colder seasons (for off-tropical regions). All these factors must be accounted for when assessing the wind resource for a particular region or site of interest.

In order to characterize the time-dependence of the wind resource, actual wind speed data for a specific location in the province of Quebec for twelve months period (2000-2001) have been used. These data are used to estimate typical wind power patterns and to understand and confirm their variability over different timeframes. Fig. 2 shows the hourly average wind speed for each month for the specific wind dataset. The wind resource for other locations around the world shows a similar season-dependent oscillation [12].

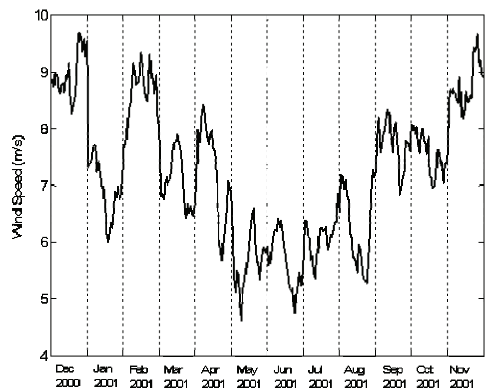


Fig. 2: Hourly average wind speed for each month for a specific location in Quebec 2000/01

From Fig. 2 it can be clearly seen how, for the particular location of the dataset, average wind speeds are higher during the fall and winter seasons (around 8 m/s), while lower average wind speeds are seen during the summer season (around 6m/s). Fig. 3 shows the k parameter as defined by (2) for each hour of each month from the wind dataset. These parameters, together with the parameters c (estimated from the averages in Fig. 2 and from (3)), are used to fully determine a Weibull wind speed distribution for each hour of the day and month. Such characterization of wind speeds by hour and month is necessary in order to simulate the correlation of wind speeds, system demand and electricity prices as described later.

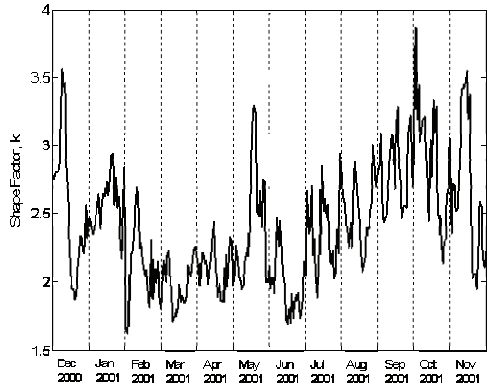


Fig. 3: Hourly shape factor *k* for each month for a specific location in Quebec 2000/01

From Fig. 3 it can be observed how larger values for the shape parameter k are seen during the fall, meaning that winds are more persistent (less variable) during this season, as opposed to spring, where the k parameter shows the lowest values. The scale factors, c, (one for each hour and month), show a behaviour similar to that of the shape factors k. As can be seen from (3), the scale factor c depends directly on the shape factor. Fig. 4 shows different actual wind speed histograms for different hours and months from the wind dataset. On top of each histogram, the corresponding Weibull PDF with parameters estimated using (2) and (3) is shown.

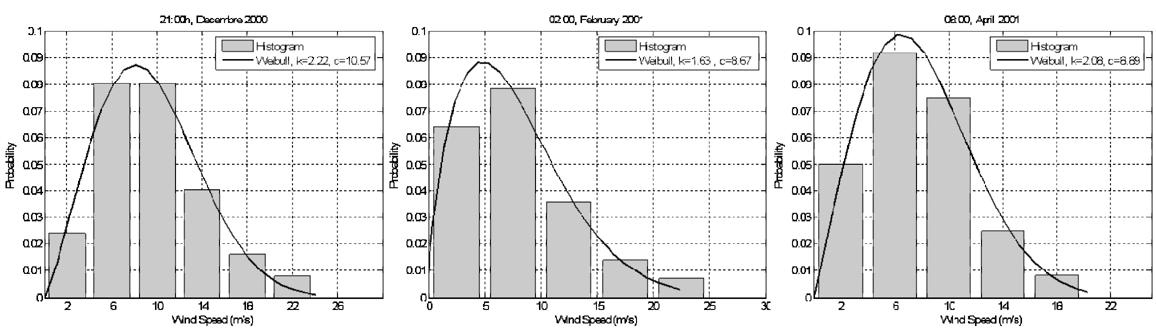


Fig. 4: Probability distributions for different hours and months for the wind data set.

The optimal bin width, h_n, for each histogram has been optimally computed from [13] as:

$$h_n = 3.49\sigma n^{-1/3} \tag{4}$$

where, for a given dataset, n is the number of data points, and σ is the sample standard deviation.

2.1.1.2 Modeling the Wind Power Curve and Wind Farm Output

A turbine's wind power curve gives the electric power output with respect to the wind speed at the turbine's location. This curve is usually given by the manufacturer, and depends on the technical characteristics of the turbine. Fig. 5 presents the power curve of a commercial GAMESA G80-2.0 MW turbine [14].

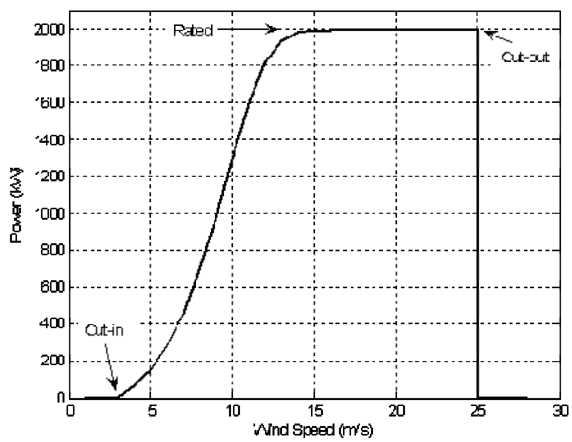


Fig. 5: Gamesa G80-2.0 MW power curve

As shown in Fig. 5, the wind power curve is composed of four regions:

1. Region below the 'cut-in' wind speed: the turbine stands still and produces no power. A minimum air momentum is necessary to get the mass of the blades under rotation subject to mechanical friction. The cut-in wind speed usually falls in the 4 m/s to 5 m/s range.

2. Region between the cut-in wind speed and rated speed: the power output increases with wind speed in a non-linear fashion, usually under a cubic relationship until the generator's maximum output is reached.

3. Region of maximum power output. In this region, the turbine's rotational speed is kept constant for wind speeds above the rated speed so the generator output does not exceed its rating.

4. Region above the ‘cut-out’ speed: For very high wind speeds (usually above the 25 m/s threshold) the turbine is shut down in order to avoid structural damage to the turbine.

The total output power of a wind farm is the sum of the power produced by each individual turbine that makes up the farm. However, wind speeds are not the same for all turbines at the same time; therefore, the turbines do not produce the same output simultaneously unless they are all in the region of maximum output. For this reason, the maximum power of a wind farm is generally lower than the sum of all turbine power ratings. In [15], an approach to estimate a regional power curve from wind speed measurements at a single site and actual wind power outputs from wind farms across the region has been proposed. In [16] the advantages and limitations of this approach have been examined. Appendix 1 describes how the regional wind power curve methodology developed in [15] can be adapted to simulate a wind farm’s power output from wind speed measurements. This methodology is particularly suitable for planning purposes where different wind farm sites and sizes must be examined at the feasibility stage.

2.1.2 Using Actual Wind Farm Output Data

An alternative to the use of wind speeds combined with the wind farm power curve is the use of actual wind power output records from the wind farm of interest. This can be done when the wind farm has already been in operation for a given period of time. Since these data are usually confidential or only available for short periods of time, in this work, simulated, publicly available wind power output for different projected wind farms across the province of Ontario are used [17].

Although the wind speeds can be statistically characterized by a Weibull distribution, the non-linear nature of the wind power curve, as described in the previous section, makes the characterization of the wind power output through a mathematical Probability Density Function (PDF) impossible. For instance, Fig. 6 shows a typical histogram of the wind power output for the test wind farm at a given hour of the day.

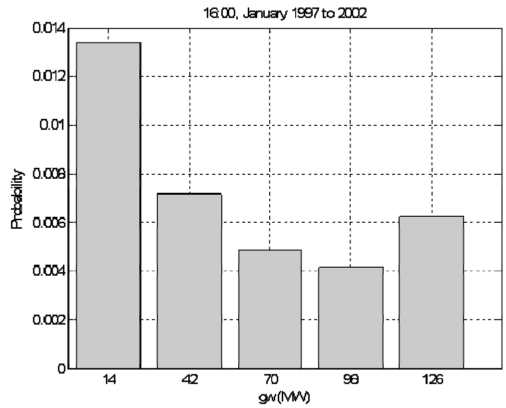


Fig. 6: Typical PDF for the hour 16:00 in January for the test wind farm

The histogram shown in Fig. 6 was calculated based on wind power output measurements recorded at 16:00 (4:00 pm) for all days in January and for years 1997 – 2001 for the test wind farm. If one estimates a similar histogram for each hour of the day and month in a year, 288 hourly wind power output distributions are obtained (24 hours times 12 months). It is important to mention that the characterization of the wind power distribution of Fig. 6 through a mathematical PDF is not required in this work. Only the first two moments (mean and variance) of the hourly wind power output as random variable are needed since the mathematical model will be expressed as a function of these two fundamental parameters.

Following the previously described grouping, there are, therefore, 288 means and 288 different variances, corresponding to the 288 unknown hourly distributions of wind power output. This grouping is essential for the statistical models developed in this work, as will be explained in the subsequent sections.

Fig. 7 shows, for illustrative purposes, the mean and variance of the wind power output for each hour and month for the available record (1997 – 2001) for the test wind farm. As described before, the mean wind power output and variance for hour 16:00 in January is estimated based on the wind power output seen at 16:00 h for all days in January for all the years in the dataset.

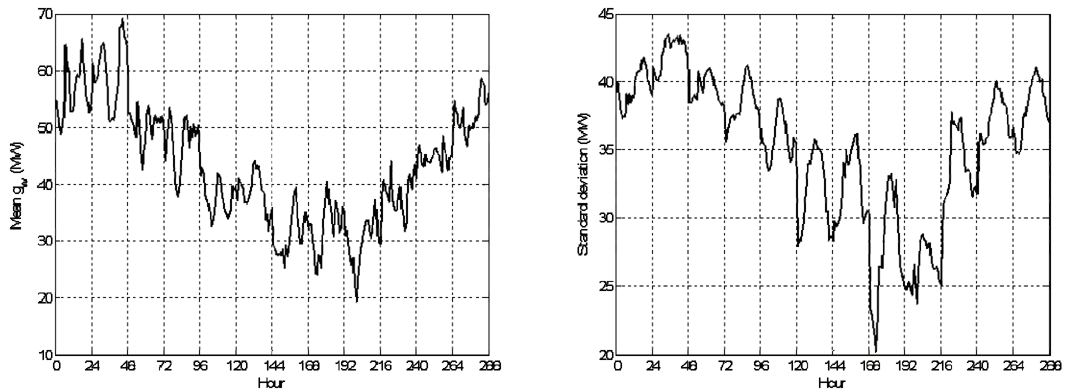


Fig. 7: Mean and variance for power output for each hour of each month for test wind farm for the years 1997 – 2001

2.2 Modeling Electricity Market Prices

An adequate model for electricity market prices is critical for the statistical characterization of the hourly revenues of a wind farm. The main objective of this work is to determine how the variability of the wind resource and the electricity prices influences the economic risk of a wind farm. As mentioned in the previous section, the mathematical model developed does not depend on the particular shape of the hourly PDF for the wind power output or the electricity prices (only the first two moments of both random variables). It is worth mentioning that modeling electricity prices is required only for the case of wind farms participating in the hourly electricity market price revenue option.

Regarding the electricity prices, there are two options available for their statistical characterization, which are described next.

2.2.1 Using Actual Records

Although production cost bids from generators are considered confidential by market operators, electricity market prices are usually made available to all generators and the public in general in order to improve the transparency of the market. In this work, we use a dataset of hourly prices in the electricity market of the province of Ontario, Canada, publicly available since the market inception (May 1st, 2002) [18].

In this work, the market price as a random variable is characterized in the same way as the wind power output, that is, the first two moments are assumed as the fundamental variables. They are calculated based on actual records for each of the 24 hours of the day and 12 months of the year (there will be, therefore, 288 means and 288 variances for the electricity price as random variable). The mathematical models developed in this work depend only on these two variables.

2.2.2 Using the Price–Demand Relationship

The electricity market price is a random variable closely correlated with the system demand. The reason is that generating resources are usually dispatched minimizing total operating costs, while ensuring that the system demand is completely met. As the system demand increases, more and more generating units with increasing operating costs must be called upon to produce electricity. Since the electricity market price is determined by the cost of the last generator being dispatched, the result is that higher electricity market prices are *usually* seen at higher system demands. This is not always the case, however, since generator unavailability, gaming and other market imperfections may also lead to high electricity prices for relatively low demand levels. After long periods of uninterrupted winter operation, large generating units are usually taken off-line for maintenance during the spring months when demand is at its lowest seasonal level.

Therefore, in this work, a price-demand relationship is considered for each season instead of a single price-demand relationship for the whole year. Actual price-demand data can be used to characterize their mutual relationship. In Appendix 2 it is shown through correlation analysis how the electricity market price as a random variable can be shown to be dependent on the system demand by a linear *Regression Function*, $r(d_h)$, of the form:

$$r\left(d_h\right) = \mathrm{E}\left[\lambda_s \mid d_s = d_h\right] = \alpha_s + \beta_s \cdot d_h \tag{5}$$

Where α_s and β_s are assumed for each season (winter, spring, summer and fall) and h and s stand for the hour of the day and season, respectively. In this work, it has been assumed that winter is composed of the months of December, January and February; spring, the months of March, April and May; and so on. This classification is due to the similarities in system demand behaviour within each season. In cool or warm seasons, electricity demand levels are usually high due to the widespread use of electric heaters or air conditioners. In Fig. A- 2 (Appendix 2) hourly averages of system demand by season can be seen for the test dataset (Ontario).

In Fig. A- 7 in Appendix 2 it can also be seen that the variance of the market prices grows in a quadratic fashion with demand level. This is a fundamental fact for the estimation of the dependency of market prices on system demands and reveals the notion that electricity markets become more volatile at higher demand levels. This phenomenon can be explained by using the notion of 'Supply-Function Equilibrium' under generation scarcity [19]. In this way, the *Conditional Variance Function*, $v(d_h)$, for the electricity prices with respect to the system demand is defined by:

$$v\left(d_h\right) = \mathrm{var}\left[\lambda_s \mid d_s = d_h\right] = \sigma_s + \rho_s \cdot d_h + \gamma_s \cdot d_h^2 \tag{6}$$

Summarizing, in this section the price-demand relationship has been determined by two functions of random variables, namely a *Regression Function* (5) and a *Conditional Variance Function* (6). In Fig. A- 5 of Appendix 2 curves with standard deviation (square-root of the variance) of the electricity prices for the Ontario dataset can be observed for each hour and season.

Chapter 3: Characterization of the Hourly Revenue of a Wind Farm

The annual revenue of a wind farm is made up of the revenues obtained at each hour during the year. This section starts the formal statistical characterization of the hourly revenue of a wind farm as the building-block of the annual revenue. This characterization is done for the two fundamental revenue options, the fixed-rate scheme and the hourly electricity market scheme.

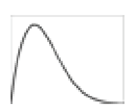
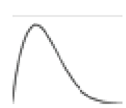
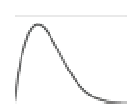
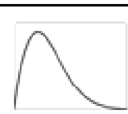
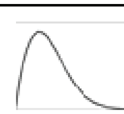
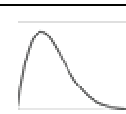
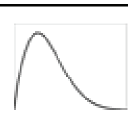
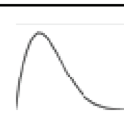
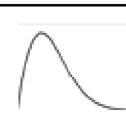
The hourly revenue of a wind farm depends on two random variables, namely the hourly wind farm output and the hourly electricity price (according to the hourly electricity market scheme). As mentioned in the previous section, the wind resource and the electricity prices are determined by a clear daily and seasonal pattern. Therefore, one cannot assume that the revenue at each hour of the year is represented by the same random variable. However, similarly to the electricity prices, winds are usually the result of marked daily cycles (temperatures, humidity, etc.). It is therefore usually assumed that the wind speed at say, 3:00 pm on a January day, behaves similarly to the wind speed at 3:00 pm the next day. Both wind speeds may well be substantially different but they are determined approximately by the same probability distribution. In fact, commercial wind resource assessment studies provide the client with wind speed PDFs for each hour of the day and month [19]. Again, since there are 24 hours in a day and 12 months in a year, there are, therefore, $24 \times 12 = 288$ different probability densities that characterize the annual wind resource for each hour of the year.

Summarizing, there are 8,760 different random variables (hourly revenues) that make up the annual revenue of a wind farm, one associated with each hour of the year. However, there are only 288 random variables that represent all of them. For instance, all the revenues corresponding to all 3:00 pm hours in January are represented by the same random variable, with a given mean and variance.

Table 1 clarifies these notions by showing the hypothetical PDFs associated with the 288 different random variables. The rows represent the 24 hours of the day whereas the columns represent the 12 months in the years. The number between parentheses below each month indicates the number of days in each month (which is equal to the number of different random variables (PDFs) that fully represent the hourly revenue for the corresponding month.

In this way, 31 hours in the year are represented by the random variable (PDF) corresponding to hour 1 in January (each 1:00 am in the 31 days in January), other 31 hours are represented by the random variable (PDF) of hour 2 in January. Similarly, 28 hours in the year are represented by the random variable (PDF) corresponding to hour 1 (1:00 am) in February, etc.

Table 1: Hourly PDF characterization

Months / Hours	January (31)	February (28)		December (31)
1			...	
2			...	
. . .	. . .	. . .	...	. . .
24				

The next sub-sections will characterize the hourly revenue of a wind farm for both revenue schemes based on the statistical representation described above.

3.1 Hourly Fixed-Rate Revenue

The revenue of a wind farm at hour *h* under the fixed-rate scheme is given by the product of the fixed-rate, π, and the wind farm power output for that specific hour, g_{w_h}.

$$R_{F\_h} = \pi \cdot g_{w\_h} \tag{7}$$

Where subscript *h* represents each hour in the year.

3.1.1 Expected Value of the Hourly Fixed-Rate Revenue

Since it is assumed that there is a random variable g_{w_h} representing the wind farm power output for each hour of the day, *h*, the expected value of the hourly fixed-rate revenue for that hour is simply given by:

$$\mathrm{E}\left[R_{F\_h}\right] = \mathrm{E}\left[\pi \cdot g_{w\_h}\right] = \pi \cdot \mathrm{E}\left[g_{w\_h}\right] \tag{8}$$

Where E[·] represents the expected value (or mean). Since the hourly revenue for each of the 8,760 hours in the year are represented by 288 random variables (and their corresponding PDFs), the subscript *g* will represent each of the 288 random variables or 'groups' ($g = 1,\ldots,288$). Therefore, hours $h = 1, 25, 49, \ldots, 721$ (hours 1:00 am for each of the 31 days in January) will correspond to group 1, hours $h = 2, 26, 50, \ldots, 722$ will correspond to group 2, and so on. In this way, groups 1 to 24 correspond to the 24 hours of all January days, groups 25 to 48 correspond to the 24 hours of all February days, and so on.

So, the random variable R_{F_g}, with $g = 1$, represents the hourly fixed-rate revenue for 1:00 am of January 1st and also for 1:00 am of January 2nd and 1:00 am of January 3rd, and so on for all days in January. Similarly, R_{F_184} represents the hourly fixed-rate revenue for 4:00 pm of August 1st and also for 4:00 pm of August 2nd and 4:00 pm of August 3rd, and so on for all days in August.

Therefore, the expected value of the hourly revenue corresponding to each group *g*, $\mathrm{E}[R_{F\_g}]$, will be given by:

$$\mathrm{E}\left[R_{F\_g}\right]=\mathrm{E}\left[\pi\cdot g_{w\_g}\right]=\pi\cdot\mathrm{E}\left[g_{w\_g}\right] \tag{9}$$

However, the expected value of g_{w_g} can be expressed as a function of the wind farm's capacity factor for group g, η_g. In this way,

$$\mathrm{E}\left[R_{F\_g}\right]=\pi\cdot\eta_g\cdot g_w^{\max} \tag{10}$$

where g_w^{max} is the wind farm's rated output.

For the particular wind power dataset, Fig. 8 shows these expected values for all groups in a year with the previously described grouping for a given π.

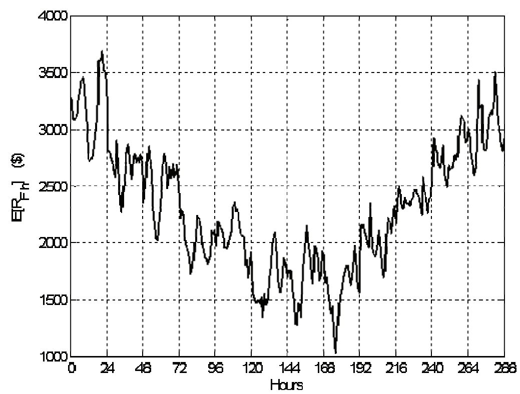


Fig. 8: Expected values of hourly revenues for all the hours and months in a year, for a given π.

From Fig. 8 it can be confirmed how the higher wind power outputs are seen during the winter months with a clear decline during the summer. Such a pattern is observed in most regions around the world.

3.1.2 Variance of Hourly Fixed-Rate Revenue

The variance of the hourly fixed-rate revenue for any hour h corresponding to group g is given by:

$$\mathrm{var}\left[R_{F\_g}\right]=\mathrm{var}\left[\pi\cdot g_{w\_g}\right]=\pi^2\cdot\mathrm{var}\left[g_{w\_g}\right] \tag{11}$$

Where var[·] represents the variance operator.

As an example, Fig. 9 shows the variance of the hourly revenues for each group for the same actual wind farm of Fig. 8:

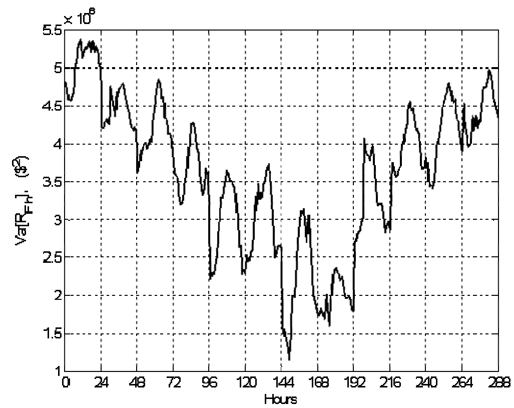


Fig. 9: Variances of hourly revenues for all the hours and months in a year, for a given π.

From Fig. 9 it can be seen that the lower revenue variances are seen during the summer months.

3.2 Hourly Market Revenue

Unlike the fixed-rate revenue option, where only one random variable is involved, the electricity market revenue option for hour h is given by the product of two random variables, the electricity price, λ_h and the corresponding wind power output g_{w_h}.

$$R_{M\_h} = \lambda_h \cdot g_{w\_h} \tag{12}$$

Again, the PDF for λ_h and g_{w_h} is not necessary in the subsequent analyses. Only their expected values and variances are needed. In fact, tests performed using actual data show that these two random variables cannot be characterized by a mathematical PDF.

Similarly to the fixed-rate revenue scheme, the following subsections will characterize the expected value and variance for the market revenue option.

3.2.1 Expected Value of Hourly Market Revenue

The expected value of R_{M_g} can be calculated by:

$$\mathrm{E}\left[R_{M\_g}\right]=\mathrm{E}\left[\lambda_h \cdot g_{w\_g}\right] \tag{13}$$

Therefore,

$$\mathrm{E}\left[R_{M\_g}\right]=\eta_g \cdot g_w^{\max} \cdot \mathrm{E}\left[\lambda_g\right]+\operatorname{cov}\left[\lambda_g \cdot g_{w\_g}\right] \tag{14}$$

Notice from (14) that the expected value of the hourly market revenue depends on the covariance the product of the market price and the wind power output. However, it is natural to assume that electricity market prices are uncorrelated with wind speeds (at least for small wind power penetration levels). This assumption was confirmed by using the actual dataset of wind power outputs and electricity prices for Ontario, where the corresponding correlation factor for 2007 for different wind farms is below 0.03. Therefore, the covariance between the market price and the wind power output can be assumed as zero. In this way,

$$\mathrm{E}\left[R_{M\_g}\right]=\eta_g \cdot g_w^{\max} \cdot \mathrm{E}\left[\lambda_g\right] \tag{15}$$

In other words, the expected value of the hourly market revenue is given by the product of the expected values of market price and wind power output, respectively.

In the case that a market price versus demand model is used (Section 2.2.2) with the form:

$$r\left(d_h\right)=\alpha_s+\beta_s \cdot d_h \tag{16}$$

The expected value of the hourly revenue can be calculated by the *Conditional Mean Theorem* [21] as:

$$\mathrm{E}\left[\lambda_h\right]=\mathrm{E}[r\left(d_h\right)] \tag{17}$$

Therefore,

$$\mathrm{E}\left[\lambda_h\right]=\alpha_s+\beta_s \cdot \mathrm{E}\left[d_h\right] \tag{18}$$

By grouping the hourly demands the same way as the other random variables:

$$\mathrm{E}\left[\lambda_g\right] = \alpha_s + \beta_s \cdot \mathrm{E}\left[d_g\right] \tag{19}$$

Substituting $E[\lambda_g]$ from (19) into (15) yields:

$$\mathrm{E}\left[R_{M\_g}\right] = \eta_g \cdot g_w^{\max} \cdot \left(\alpha_s + \beta_s \cdot \mathrm{E}\left[d_g\right]\right) \tag{20}$$

This way, the expected value of the hourly market revenue is now expressed as a function of the system demand through the regression model (5).

In order to validate the previous equations, Fig. 10 shows the expected values of the hourly market revenues (for each group) using the actual dataset for each hour of the year using the direct model of (15) and the price versus demand model developed in (20). These two values are compared with the expected value of the actual hourly revenues using actual price and wind power output datasets.

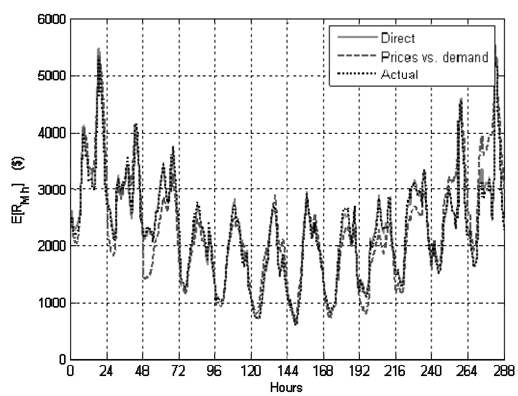


Fig. 10: Expected value of the 288 distributions of hourly market revenues, computed by different methods

In Fig. 10 it can be seen that both, the direct model (15) and the price versus demand model (20) represent closely the actual values. It can be seen that the difference between the direct model (15) and the actual values is unnoticeable which confirms the assumption of no correlation between hourly prices and wind power outputs.

3.2.2 Variance of the Hourly Market Revenue

The variance of the hourly market revenue of an hour h within a group g, R_{M_g}, is given by:

$$\mathrm{var}\left[R_{M\_g}\right]=\mathrm{var}\left[\lambda_g\cdot g_{w\_g}\right] \tag{21}$$

By the definition of variance,

$$\mathrm{var}\left[R_{M\_g}\right]=\mathrm{E}\left[\left(\lambda_g\cdot g_{w\_g}\right)^2\right]-\left(\mathrm{E}\left[\lambda_g\cdot g_{w\_g}\right]\right)^2 \tag{22}$$

In the first term of the right hand side, and from the hypothesis of independence between the electricity prices and the wind power outputs,

$$\mathrm{E}\left[\left(\lambda_g\cdot g_{w\_g}\right)^2\right]=\mathrm{E}\left[\lambda_g{}^2\right]\cdot\mathrm{E}\left[g_{w\_g}{}^2\right] \tag{23}$$

and, in the second term,

$$\left(\mathrm{E}\left[\lambda_g\cdot g_{w\_g}\right]\right)^2=\left(\mathrm{E}\left[\lambda_g\right]\right)^2\cdot\left(\mathrm{E}\left[g_{w\_g}\right]\right)^2=\left(\mathrm{E}\left[\lambda_g\right]\right)^2\cdot\left(\eta_g\cdot g_w^{\max}\right)^2 \tag{24}$$

Therefore,

$$\mathrm{var}\left[R_{M\_g}\right]=\mathrm{E}\left[\lambda_g{}^2\right]\cdot\mathrm{E}\left[g_{w\_g}{}^2\right]-\left(\mathrm{E}\left[\lambda_g\right]\right)^2\cdot\left(\eta_g\cdot g_w^{\max}\right)^2 \tag{25}$$

However, by the same definition of variance:

$$\mathrm{var}\left[\lambda_g\right]=\mathrm{E}\left[\lambda_g{}^2\right]-\left(\mathrm{E}\left[\lambda_g\right]\right)^2 \tag{26}$$

and

$$\mathrm{var}\left[g_{w\_g}\right]=\mathrm{E}\left[g_{w\_g}{}^2\right]-\left(\mathrm{E}\left[g_{w\_g}\right]\right)^2 \tag{27}$$

Therefore,

$$\mathrm{E}\left[\lambda_g{}^2\right]=\mathrm{var}\left[\lambda_g\right]+\left(\mathrm{E}\left[\lambda_g\right]\right)^2 \tag{28}$$

$$\mathrm{E}\left[g_{w\_g}{}^2\right]=\mathrm{var}\left[g_{w\_g}\right]+\left(\eta_g\cdot g_w^{\max}\right)^2 \tag{29}$$

Substituting (28) and (29) into (25) yields:

$$\mathrm{var}\left[R_{M\_g}\right]=\left(\mathrm{var}\left[\lambda_g\right]+\left(\mathrm{E}\left[\lambda_g\right]\right)^2\right)\cdot\left(\mathrm{var}\left[g_{w\_g}\right]+\left(\eta_g\cdot g_w^{\max}\right)^2\right)$$
$$-\left(\mathrm{E}\left[\lambda_g\right]\right)^2\cdot\left(\eta_g\cdot g_w^{\max}\right)^2 \tag{30}$$

Therefore,

$$\mathrm{var}\left[R_{M\_g}\right]=\mathrm{var}\left[\lambda_g\right]\cdot\left(\eta_g{}^2\cdot\left(g_w^{\max}\right)^2+\mathrm{var}\left[g_{w\_g}\right]\right)+\left(\mathrm{E}\left[\lambda_g\right]\right)^2\cdot\mathrm{var}\left[g_{w\_g}\right] \tag{31}$$

It can be seen from (31) that the variance of the hourly market revenue represented by its corresponding group *g* has been expressed as a function of the expected value and the variance of both, the electricity market price and the wind power output. The importance of this result is that once the wind farm's capacity factor for hour *h* is known (which can be obtained based on historical records), the variance of the revenue for that hour can be estimated based on the individual variances of both, the electricity prices and the wind farm outputs for that hour. Moreover, knowledge of the particular PDF for either random variable is not necessary.

Continuing with the analysis, if the model representing the relationship between the electricity market prices versus the system demand is used, applying the *Conditional Variance Theorem* [21] yields:

$$\mathrm{var}\left[\lambda_h\right]=\mathrm{E}[v\left(d_h\right)]+\mathrm{var}\left[r\left(d_h\right)\right] \tag{32}$$

Therefore

$$\mathrm{var}\left[\lambda_h\right]=\sigma_s+\rho_s\cdot\mathrm{E}\left[d_h\right]+\gamma_s\cdot\mathrm{E}\left[d_h^2\right]+\beta_s{}^2\cdot\mathrm{var}\left[d_h\right] \tag{33}$$

Or, for a group *g* in particular,

$$\mathrm{var}\left[\lambda_g\right]=\sigma_s+\rho_s\cdot\mathrm{E}\left[d_g\right]+\gamma_s\cdot\mathrm{E}\left[d_g^2\right]+\beta_s{}^2\cdot\mathrm{var}\left[d_g\right] \tag{34}$$

However, according to the expression for the variance,

$$\mathrm{var}\left[d_g\right]=\mathrm{E}\left[d_g{}^2\right]-\left(\mathrm{E}\left[d_g\right]\right)^2 \tag{35}$$

therefore,

$$\mathrm{E}\left[d_g{}^2\right] = \mathrm{var}\left[d_g\right] + \left(\mathrm{E}\left[d_g\right]\right)^2 \tag{36}$$

Substituting (36) into (33) yields,

$$\mathrm{var}\left[\lambda_g\right] = \sigma_s + \rho_s \cdot \mathrm{E}\left[d_g\right] + \gamma_s \cdot \left(\mathrm{var}\left[d_g\right] + \left(\mathrm{E}\left[d_g\right]\right)^2\right) + \beta_s{}^2 \cdot \mathrm{var}\left[d_g\right] \tag{37}$$

or, regrouping terms,

$$\mathrm{var}\left[\lambda_g\right] = \sigma_s + \rho_s \cdot \mathrm{E}\left[d_g\right] + \gamma_s \cdot \left(\mathrm{E}\left[d_g\right]\right)^2 + \left(\gamma_s + \beta_s{}^2\right) \cdot \mathrm{var}\left[d_g\right] \tag{38}$$

Putting (38) and (19) in (31) results in:

$$\begin{aligned} \mathrm{var}\left[R_{M\_g}\right] &= \left(\alpha_s + \beta_s \cdot \mathrm{E}\left[d_g\right]\right)^2 \cdot \mathrm{var}\left[g_{w\_g}\right] \\ &+ \left(\sigma_s + \rho_s \cdot \mathrm{E}\left[d_g\right] + \gamma_s \cdot \left(\mathrm{E}\left[d_g\right]\right)^2 + \left(\gamma_s + \beta_s{}^2\right) \cdot \mathrm{var}\left[d_g\right]\right) \cdot \left(\eta_g{}^2 \cdot \left(g_w^{\max}\right)^2 + \mathrm{var}\left[g_{w\_g}\right]\right) \end{aligned} \tag{39}$$

Regrouping terms:

$$\begin{aligned} \mathrm{var}\left[R_{M\_g}\right] &= \eta_g{}^2 \cdot \left(g_w^{\max}\right)^2 \cdot \left(\sigma_s + \rho_s \cdot \mathrm{E}\left[d_g\right] + \gamma_s \cdot \left(\mathrm{E}\left[d_g\right]\right)^2 + \left(\gamma_s + \beta_s{}^2\right) \cdot \mathrm{var}\left[d_g\right]\right) \\ &+ \mathrm{var}\left[g_{w\_g}\right] \cdot \left(\sigma_s + \alpha_s{}^2 + \left(\rho_s + 2\alpha_s\beta_s\right) \cdot \mathrm{E}\left[d_g\right] + \left(\gamma_s + \beta_s{}^2\right) \cdot \left(\mathrm{E}\left[d_g\right]\right)^2 + \left(\gamma_s + \beta_s{}^2\right) \cdot \mathrm{var}\left[d_g\right]\right) \end{aligned} \tag{40}$$

The importance of the previous result lies on the fact that the variance of the hourly market revenue has been decoupled, that is, it can now be computed as a function of the expected value and variance of the system demand and wind power output.

The results given by (31) and (40) have been compared with the values from the actual dataset, and are shown in Fig. 11.

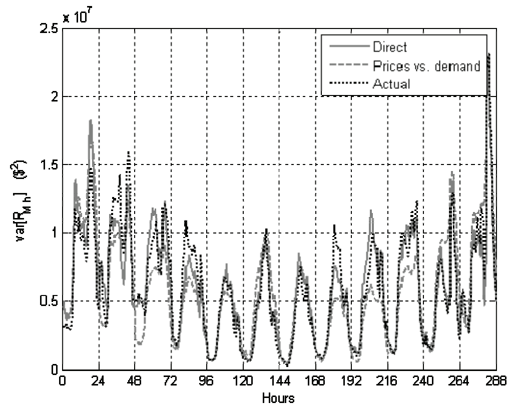


Fig. 11: Variances of the hourly market revenues, computed with the different methods

In Fig. 11, it can be seen that both models (31) and (40) follow well the actual values. Again, the small difference between the direct model (31) and the actual values support the assumption of zero covariance between the hourly market price and wind power output. The difference between the price versus demand model (40) and the actual values comes from the assumption of the linear regression and quadratic conditional variance in the market price versus demand model. However, the performance of this price versus demand model can be considered satisfactory for practical purposes.

Summarizing, Fig. 11 and Fig. 10 have confirmed the validity of the different models assumed for the mathematical estimation of the expected values and variances of the hourly fixed-rate and hourly market revenues for a wind farm.

Now that the mathematical models for the calculation of the expected values and variances of the hourly fixed-rate revenues and hourly market revenues have been developed and validated, the next chapter will explore the statistical characterization of the annual wind farm revenue for both options, fixed-rate and market based revenues.

Chapter 4: Characterization of the Annual Revenue of a Wind Farm

In the previous chapter, a statistical characterization of the wind farm's hourly revenue was carried out for both, fixed-rate and market revenue options. The reason why only the first two moments (expected value and variance) of the hourly revenue as random variable have been examined so far is that knowing these two quantities is enough for the complete stochastic characterization of the wind farm's annual revenue.

The wind farm's total annual revenue for both revenue options is given by the sum of a stream of hourly revenues (8,760 of them). Since each of the hourly revenues is a random variable, the annual revenue is also a random variable.

The annual revenue for the wind farm, R_A, is given by:

$$R_A = \sum_{h=1}^{8760} R_h \tag{41}$$

where R_A represents the annual revenue, in general, for both hourly revenue options.

In the following subsections mathematical expressions for the expected value and variance of the wind farm's annual revenue will be developed.

4.1 Expected Value of the Annual Revenue

The expected value of the annual revenue can be computed by adding up the expected values of all the hourly revenues over one year:

$$\mathrm{E}\left[R_A\right] = \sum_{h=1}^{8760} \mathrm{E}\left[R_h\right] \tag{42}$$

Again, R_A represents the annual revenue, and R_h represents the hourly revenue for either revenue option. As explained before, the 8,760 summands in (42) are represented by 288 different random variables (each one corresponding to an hour of the day and month).

4.2 Variance of the Annual Revenue

From (41), the variance of the annual revenue is therefore given by:

$$\mathrm{var}\left[R_A\right] = \mathrm{var}\left[\sum_{h=1}^{8760} R_h\right] \tag{43}$$

In general, the variance of a sum of n random variables X_i for i =1,…,n can be expressed as:

$$\mathrm{var}\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{cov}\left[X_i, X_j\right] \tag{44}$$

Note that if the random variables X_i were independent, the variance of the sum in (44) would simply be the sum of the variances of each summand. However, as will be described in this section, the hourly revenues for both revenue options are not independent and so the covariance between them must be included.

From (44),

$$\mathrm{var}\left[R_A\right] = \sum_{i=1}^{8760}\sum_{j=1}^{8760} \mathrm{cov}\left[R_i, R_j\right] \tag{45}$$

Moreover, the covariance between two random variables (X_i, X_j) can be expressed in terms of their correlation coefficient $\rho_{i,j}$ as follows:

$$\mathrm{cov}\left[X_i, X_j\right] = \rho_{i,j} \cdot \sqrt{\mathrm{var}\left[X_i\right]} \cdot \sqrt{\mathrm{var}\left[X_j\right]} \tag{46}$$

where, by definition:

$$\sqrt{\mathrm{var}\left[X_i\right]} = \mathrm{std}\left[X_i\right] \tag{47}$$

where std[X_i] is the standard deviation of the variable X_i. Therefore, by putting (46) and (47) into (45):

$$\mathrm{var}\left[R_A\right] = \sum_{i=1}^{8760}\sum_{j=1}^{8760} \rho_{i,j} \cdot \mathrm{std}\left[R_i\right] \cdot \mathrm{std}\left[R_j\right] \tag{48}$$

Equation (48) can be written in matrix form as:

$$\mathrm{var}\left[R_A\right] = \boldsymbol{\sigma}'\mathbf{P}\boldsymbol{\sigma} \tag{49}$$

Note that the vector **σ** is already known. It is the vector of 8,760 standard deviations of each hourly revenue option. The variances (standard deviations squared) were characterized in the previous section. On the other hand, **P**, the 8,760×8,760 matrix of correlation coefficients between hourly revenues, will be given by:

$$\mathbf{P} = \begin{bmatrix} \rho_{1,1} & \rho_{1,2} & \cdots & \rho_{1,8760} \\ \rho_{2,1} & \rho_{2,2} & \cdots & \rho_{2,8760} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{8760,1} & \rho_{8760,2} & \cdots & \rho_{8760,8760} \end{bmatrix} \tag{50}$$

Therefore, according to (48), if the standard deviations for the individual hourly revenues are known, the characterization of the correlation coefficients allows the complete characterization of the variance of the sum of the hourly revenues (48) (the wind farm's annual revenue).

By definition, the correlation coefficient, $\rho_{i,j}$, measures the strength and direction of the linear relationship between two variables (X_i, X_j). The correlation coefficient falls always within the range [-1.0, 1.0]. For a value of -1, there is a perfectly opposite correlation between both random variables, whereas for a value of +1, there is a perfect direct correlation. In general, the correlation coefficient between two random variables (X,Y), is given by:

$$\rho_{X,Y} = \frac{\mathrm{E}\left[\left(X-\mu_X\right)\cdot\left(Y-\mu_Y\right)\right]}{\sigma_X \cdot \sigma_Y} \tag{51}$$

where μ_X and σ_X and μ_Y and σ_Y represent the expected value and the standard deviation of both random variables X and Y, respectively. With this in mind, the random variables X and Y in (51) can be adapted to represent any of the 8,760 hourly revenues. Also since each hourly revenue takes place in consecutive order in time (each hour, obviously), a collection of these hourly revenues over a year constitute a time-series of length 8,760. Also, the correlation coefficient between any two elements of a time-series is usually called *auto-correlation* [22].

By definition, the auto-correlation between two realizations of the random variable that defines the time-series at different points in time, t and $t+k$, (with k being the time interval between the realizations) is given by:

$$\rho_{t,t+k} = \frac{\mathrm{E}\left[\left(X_t-\mu_t\right)\cdot\left(X_{t+k}-\mu_{t+k}\right)\right]}{\sigma_t \cdot \sigma_{t+k}} \tag{52}$$

In the case of a second-order stationary time-series, (52) could be significantly simplified because the random variable X_t would have a constant expected value and the variance and the auto-correlation would only depend on the time span k. Fig. 12 shows an example of a typical auto-correlation curve for a second-order stationary time-series.

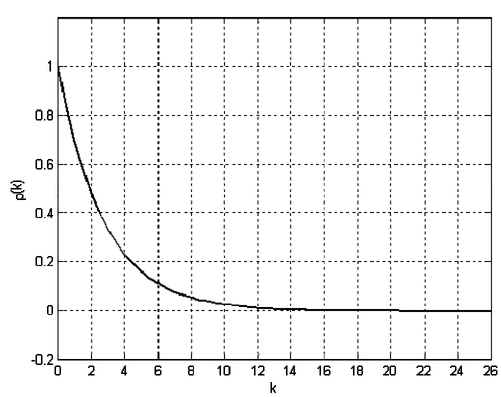


Fig. 12: Typical auto-correlation curve for a time series

In Fig. 12, the x-axis represents the time span k in (52). Note that this curve does not depend on t for a second-order stationary random process (or time-series). The y-axis represents the autocorrelation coefficient between the random variables X_t and X_{t+k}. Note that for k = 0 hours, the auto-correlation coefficient is one (it represents the auto-correlation of the random variable with itself). From Fig. 12 it can also be seen how the auto-correlation declines as k increases to finally tend to zero. This is the typical behaviour for a second-order stationary time-series. In this specific example, for k = 2, the autocorrelation is less than 0.5, and it is near zero for k greater than 12.

Notice that in (48) the auto-correlation coefficients could be thought of as a weighting factors in the summation. Also, given that these weighting factors are given by an auto-correlation function (as in Fig. 12), the summands (i, j) in (48) corresponding to hours that are close to each other (i similar to j) would be greater than summands corresponding to time intervals far apart from each other, vanishing to zero as the time interval goes to infinity.

The previous notions have been brought up in order to clarify the basic notions of the auto-correlation of a stationary time-series and how they influence the different summands in the annual revenue stream of hourly revenues.

However, given the seasonal nature of the wind resource and the electricity markets, it has been found experimentally that the time-series given by the stream of hourly revenues is not a stationary process (first or second-order). In fact, as was seen in Fig. 10 and Fig. 11, the hourly expected values and variances are not constant in time. For instance, the auto-correlation of revenues between hours 1 and 2 will be different from the auto-correlation between hours 2 and 3, even if the time interval between the involved revenues is one for both cases. Therefore, in the most general case, and according to (48) the auto-correlation between all possible combinations of hourly revenues should be computed in order to fully determine the matrix **P** in (50). According to the notation in (52), the auto-correlation matrix in (50) can be expressed as:

$$\mathbf{P} = \begin{bmatrix} \rho_{1,1} & \rho_{1,2} & \cdots & \rho_{1,8760} \\ \rho_{2,1} & \rho_{2,2} & \cdots & \rho_{2,8760} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{8760,1} & \rho_{8760,2} & \cdots & \rho_{8760,8760} \end{bmatrix} \tag{53}$$

Again, the diagonal terms of matrix rho are all equal to one. The entries in the first row ($\rho_{1,j}$) correspond to the auto-correlations of the revenue of hour 1 of the time-series (January 1st, 0:00 am, for instance) with respect to all the other 8,759 hourly revenues.

However, as it was seen in Fig. 10, the hourly revenues follow a strong intra-day pattern. According to the wind speed dataset for the province of Ontario, winds are stronger in the overnight hours and decline every morning at about 5:00 am. Although in some other regions winds may be stronger in the late afternoon, a daily wind speed pattern is observed almost anywhere. A similar behaviour occurs for the electricity prices, which follow closely the predictable system demand fluctuations.

With this in mind, we could assume that the correlation between the revenue at hour 1:00 am and 2:00 am (every day) will be the same for all the days of the year. In other words, it can be considered that the information that the revenue at 1:00 am provides about the revenue at 2:00 am is the same for all hours 1:00 am and 2:00 am throughout the year. From this point of view, the autocorrelation $\rho_{1,2}$ can be computed from (52) with vector X_1 containing, for instance, the hourly revenues at all hours 1:00 am of the year (hours 1, 25, 49, …, 8737) and vector X_2 containing the revenues at all hours 2:00 am of the year (hours 2, 26, 50,…8738). Therefore,

$$X_1 = \begin{bmatrix} R_1 & R_{25} & \ldots & R_{8737} \end{bmatrix} \tag{54}$$

$$X_2 = \begin{bmatrix} R_2 & R_{26} & \ldots & R_{8738} \end{bmatrix} \tag{55}$$

With the previous arrangement, all entries in the auto-correlation matrix **P** (53) can be calculated. It should be noticed that **P** is symmetric since $\rho_{t,\, t+k}$ is equal to $\rho_{t+k,\, t}$.

Now, recall the statistical similarity between the hourly revenues for the same hour of the day and month, which led to the grouping described in Chapter 3 ($X_t \approx X_{t+24d}$ and $X_{t+k} \approx X_{t+24d+k}$ for the same month). Therefore, the $\rho_{t,\ t+k}$ for $t \in [1, 24]$, (with t representing hours in the year) will be equal to the $\rho_{t+24\cdot d,\ t+24\cdot d+k}$, for $t \in [1, 24]$, and $d \in [1, 364]$, (with d representing the day ahead of the first day of the series). It follows that, since an auto-correlation is defined for each hour of the day, only the first 24 rows of the auto-correlation matrix **P** have to be computed.

For instance, in order to calculate the auto-correlation between the revenues at 1:00 am y 2:00 am for the second day of the year ($d = 1$, or entry $\rho_{25,26}$ in matrix **P**), the vectors to be compared are:

$$X_{1+24} = \begin{bmatrix} R_{25} & R_{49} & \ldots & R_{8737} \end{bmatrix} \tag{56}$$

$$X_{2+24} = \begin{bmatrix} R_{26} & R_{50} & \ldots & R_{8738} \end{bmatrix} \tag{57}$$

Comparing the vectors X_1 and X_{25} in (54) and (56), it can be observed that the only difference between both vectors is the first element. The same occurs if the vectors X_2 (55) y X_{26} (57) are compared. Summarizing, only the first 24 rows in matrix **P** have to be calculated. There are, therefore, 24 different auto-correlation functions similar to the ones shown in Fig. 12, one for each hour of the day.

Now that the correlation coefficients have been defined, the summation of the hourly revenues in (48) can be fully determined. Since the auto-correlation matrix **P** is symmetric, with its diagonal terms equal to one, (48) can be divided into two terms: the summation involving elements $i=j$, and the summation involving elements $i \neq j$, as follows:

$$\operatorname{var}\left[R_A\right] = \sum_{h=1}^{8760} \operatorname{var}[R_h] + 2\sum_{i=1}^{8760} \sum_{j>i}^{8760} \operatorname{cov}[R_i, R_j] \tag{58}$$

Or

$$\mathrm{var}\left[R_A\right]=\sum_{i=1}^{8760}\mathrm{var}[R_i]+2\sum_{i=1}^{8760}\sum_{j>i}^{8760}\rho_{i,j}\cdot\mathrm{std}[R_i]\cdot\mathrm{std}[R_j] \tag{59}$$

If the hourly revenues were uncorrelated from one another, the covariance terms would be all zero in (59) (or the auto-correlations for $i \neq j$ in (59) would be zero). In this case, the variance of the annual revenue would be just the summation of the variances of the hourly revenues (the first term of the right hand side of (59)).

In order to clarify the subsequent analyses, the term that corresponds to the summation of the variances of the hourly revenues in (59):

$$\sum_{i=1}^{8760}\mathrm{var}[R_i] \tag{60}$$

will be called simply the *individual-variance* term, whereas the term

$$2\sum_{i=1}^{8760}\sum_{j>i}^{8760}\rho_{i,j}\cdot\mathrm{std}[R_i]\cdot\mathrm{std}[R_j] \tag{61}$$

will be called the *cross-variance* term. The application of these definitions in Section 7.2 will give an idea of the relative importance of considering the auto-correlation between hourly revenues and how it affects the variance of the annual revenue as opposed to simply (and wrongly) considering uncorrelated hourly revenues.

Given the previous notions, the following subsections will develop mathematical expressions for the expected value and variance of the annual revenue for both revenue options.

4.3 Annual Fixed-Rate Revenue

The expression for the annual fixed-rate revenue, R_{F_A}, is given by:

$$R_{F\_A}=\sum_{h=1}^{8760}R_{F\_h}=\sum_{h=1}^{8760}\pi\cdot g_{w\_h} \tag{62}$$

The expected value of the annual fixed-rate revenue can therefore be computed as:

$$\mathrm{E}\left[R_{F\_A}\right]=\sum_{h=1}^{8760}\mathrm{E}\left[\pi\cdot g_{w\_h}\right] \tag{63}$$

Since the fixed-rate π is a constant, from (63):

$$\mathrm{E}\left[R_{F\_A}\right]=\pi\cdot\sum_{h=1}^{8760}\mathrm{E}\left[g_{w\_h}\right] \tag{64}$$

Since the expectation of the sum is the sum of the expectations,

$$\mathrm{E}\left[R_{F\_A}\right]=\pi\cdot\mathrm{E}\left[\sum_{h=1}^{8760}g_{w\_h}\right] \tag{65}$$

Therefore, from (62)

$$\mathrm{E}\left[R_{F\_A}\right]=\pi\cdot\mathrm{E}\left[g_{w\_A}\right] \tag{66}$$

Where $\mathrm{E}[g_{w\_A}]$ is the expected value of the annual energy output of the wind farm which can be also expressed as a function of the annual capacity factor η_A and the wind farm's installed capacity g_w^{max} as:

$$\mathrm{E}\left[R_{F\_A}\right]=\pi\cdot\eta_A\cdot g_w^{\max} \tag{67}$$

From (48), the variance of the annual fixed-rate revenue can be computed as:

$$\mathrm{var}\left[R_{F\_A}\right]=\sum_{i=1}^{8760}\sum_{j=1}^{8760}\rho_{F\_i,j}\cdot\mathrm{std}\left[R_{F\_i}\right]\cdot\mathrm{std}\left[R_{F\_j}\right] \tag{68}$$

Where $\mathrm{std}[R_{F\_i}]$ is the standard deviation of the hourly revenue R_{F_i} (these are the square roots of the corresponding hourly variances characterized in Section 4.2), Also, $\rho_{F\_i,j}$ is the auto-correlation between the hourly fixed-rate revenues R_{F_i} and R_{F_j}.

Since the fixed-rate paid for the output of the wind farm is constant, (68) can be written in terms of the variance of the hourly energy output, $\mathrm{std}[g_{w\_i}]$:

$$\mathrm{std}\left[R_{F\_i}\right]=\pi\cdot\mathrm{std}\left[g_{w\_i}\right] \tag{69}$$

Therefore,

$$\operatorname{var}\left[R_{F\_A}\right] = \pi^2 \sum_{i=1}^{8760} \sum_{j=1}^{8760} \rho_{gw\_i,j} \cdot \operatorname{std}\left[g_{w\_i}\right] \cdot \operatorname{std}\left[g_{w\_j}\right] \tag{70}$$

where $\rho_{gw\_i,j}$ is the auto-correlation between the hourly power outputs. These auto-correlations are the same as the $\rho_{F\_i,j}$ of (68) since the correlation coefficient between two random variables does not change if either variable is multiplied by a constant (in this case, by π). Note also that, by definition, the variance of the annual power output is given by:

$$\operatorname{var}\left[g_{w\_A}\right] = \sum_{i=1}^{8760} \sum_{j=1}^{8760} \rho_{gw\_i,j} \cdot \operatorname{std}\left[g_{w\_i}\right] \cdot \operatorname{std}\left[g_{w\_j}\right] \tag{71}$$

Therefore the variance of the annual fixed-rate revenue (70) can be expressed as a simple explicit function of the variance of annual wind power output.

$$\operatorname{var}\left[R_{F\_A}\right] = \pi^2 \cdot \operatorname{var}\left[g_{w\_A}\right] \tag{72}$$

Summarizing, the variance of the annual fixed-rate revenue has been expressed in terms of the variances (or standard deviations) of the hourly wind power outputs in (70) or in terms of the variance of the annual wind power output (72). Therefore, at this point, the expected value and the variance of the annual fixed-rate revenue have been completely determined. Although the result given by (72) is intuitive, the use of (70) may be necessary when the seasonal revenues are to be modeled separately.

4.4 Annual Market Revenue

The expression for the annual market revenue is given by:

$$R_{M\_A} = \sum_{h=1}^{8760} R_{M\_h} = \sum_{h=1}^{8760} \lambda_h \cdot g_{w\_h} \tag{73}$$

By applying the model of Section 4.1, the expected value of the annual fixed-rate revenue can be computed as:

$$\mathrm{E}\left[R_{M\_A}\right]=\sum_{h=1}^{8760}\mathrm{E}\left[\lambda_h\cdot g_{w\_h}\right] \tag{74}$$

From the hypothesis of independence between the hourly market price and output energy used in Section 3.2.2:

$$\mathrm{E}\left[R_{M\_A}\right]=\sum_{h=1}^{8760}\mathrm{E}\left[\lambda_h\right]\cdot\mathrm{E}\left[g_{w\_h}\right] \tag{75}$$

The expected value of the annual market revenue can also be expressed as a function of the hourly demand by substituting (18) into (75).

Also, from (59), the variance of the annual fixed-rate revenue can be computed as:

$$\mathrm{var}\left[R_{M\_A}\right]=\sum_{i=1}^{8760}\mathrm{var}[R_{M\_i}]+2\sum_{i=1}^{8760}\sum_{j>i}^{8760}\rho_{M\_i,j}\cdot\mathrm{std}[R_{M\_i}]\cdot\mathrm{std}[R_{M\_j}] \tag{76}$$

Where std[R_{M_i}] is the standard deviation of the hourly revenue R_{M_i}, and $\rho_{M\_i,j}$ the autocorrelation between the hourly market revenues R_{M_i} and R_{M_j}. The variance of the annual market revenue can be also expressed as a function of the hourly demand by using the expression for the variances of the hourly market revenues as given by (33).

Note that in the case of the market revenue option, the first two moments of the annual revenue (expected value and variance) cannot be expressed as an explicit function of E[g_{w_A}] and var[g_{w_A}], as in (66) and (72) for the case of the fixed-rate revenue.

4.4.1 Density Distribution of Annual Revenue

At this point, explicit expressions for the first two moments of the annual revenue based on the first two moments of the hourly revenues have been found. In turn, the first two moments of the hourly revenue have been estimated as a function of the first two moments of the wind power output and the electricity markets, separately. This 'statistical decoupling' is the main objective of this work. As was observed, in order to arrive to these results, no assumption about the shape of the PDF of any random variable has been made. Only their moments and their auto-correlations are needed. However, knowledge of the distribution of the annual revenue as a random variable is important since most of the risk-assessment theory is based on the assumption of normality, which simplifies considerably the calculations.

The *Central Limit Theorem* (CLT), in its most general form, indicates that the PDF of the sum of a large number of independent, identically distributed variables tends to a Gaussian distribution [23]. The importance (and the beauty) of the CLT is that the summand distributions may be of any shape.

As has been seen through this work, the wind farm's annual revenue is nothing more than the summation of 8,760 hourly revenues, each represented by an unknown random variable whose expected value and variance have been characterized. The issue here is that the CLT in its most general form cannot be applied directly in this work since, as it was seen, the hourly revenues are not independent and are not identically distributed (there are, in fact, 288 different random variables involved in the annual revenue).

Other versions of the CLT have been developed in the case where the random variables are not independent. However, these versions are based on complex and limited mathematical assumptions for considerably specific cases [24]. However, the fact that there is no explicit formulation of the CLT applicable to correlated (dependent) time-series does not mean that the annual revenue given by the sum of the 8,760 hourly revenues does not converge to a normal distribution. Since the mathematical proof of a CLT in this case is extremely difficult, if not impossible, one possible way of estimating the PDF of the annual revenue as random variable is through simulation.

One of way of estimating the PDF for the annual revenue is to carry out a time-series analysis of the Auto-Regressive, Moving-Average (ARMA) type [22], based on actual records of wind power and electricity price (and therefore, hourly revenues). An optimal ARMA(p, q) model can be fit to an actual long-enough (multi-year) time-series of hourly revenues. This model is defined by auto-regressive and moving-average polynomials of order p and q, respectively as given by:

$$R_h = c + \varepsilon_h + \sum_{i=1}^{p} \varphi_i R_{h-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{h-i} \tag{77}$$

where c is a constant, and ε_h is the error of the model at time h. Once the coefficients of the polynomials, φ_i and θ_i, have been found for the p and q, the statistical nature of the time-series, including its auto-covariance function has been determined. The next step is then to create synthetic hourly revenue time-series based on the optimal ARMA(p, q) model. For each synthetic series s (of length 8,760 hours), its corresponding annual revenue R_{A_s} is found and stored. If this process is repeated a sufficiently large number of times, it is possible to determine the actual PDF of the synthetically generated R_{A_s}. One way of determining whether an unknown random variable is normally distributed is to perform a 'Quantile-to-Quantile' or 'Q-Q test' [25]. This test would be applied to the syntethically generated R_{A_s}.

The synthetic time-series approach would not only tell whether the annual revenue is normally distributed but also would validate the complete statistical model developed in this work. The variance of the synthetically generated annual revenue should be close to the mathematical estimates previously found.

The results of the application of the previous notions will be shown in Section 7.2. It will be seen that the annual revenue is actually normally-distributed. Since a normal distribution is completely characterized by its first two moments, it can be assumed then at this point that the annual revenue of a wind farm has been completely characterized by the statistical models developed here.

The following chapter will explore in detail the economic risks of the annual revenue of a wind farm as a normally-distributed random variable with expected value and variance characterized as described in the previous sections.

Chapter 5: Economic Risk of the Annual Revenue of a Wind Farm

As explained in previous sections, one of the main objectives of this work is to identify how the market and resource-related parameters influence the economic risks in the operation of a wind farm. Particularly to the annual revenue, risk is a measure of its volatility and is closely related with the notion of loss. Risk would be zero if there were no chance to lose.

There are different types of risks and different ways to measure them. In this work, risk will be measured through the concept of Value at Risk (VaR), which is an amount (in monetary terms) that indicates that the investment will lose less than the specified monetary amount with a specified probability. This probability defines the *confidence interval* for the distribution of returns of investment (annual revenues, in our case), set usually between 90% to 95%. In this work, since the economic performance variable is annual revenue (in M$) the VaR will be defined by setting a minimum accepted return instead of considering the possibility of a loss. However, the fact of being below a certain minimum revenue could be considered as a loss: if a different investment could provide higher revenues, the investor would be worse off with respect to the best possible scenario [26].

Sometimes a complementary probability is issued, that is, one minus the confidence interval [27]. For instance, a VaR(10%) of 20 M$ for the annual revenue indicates that there is a 10% chance that the revenue will be less than 20 M$. This is the definition used in this work.

Fig. 13 shows an example of VaR for a confidence level of 90% applied to a normal distribution of annual revenues.

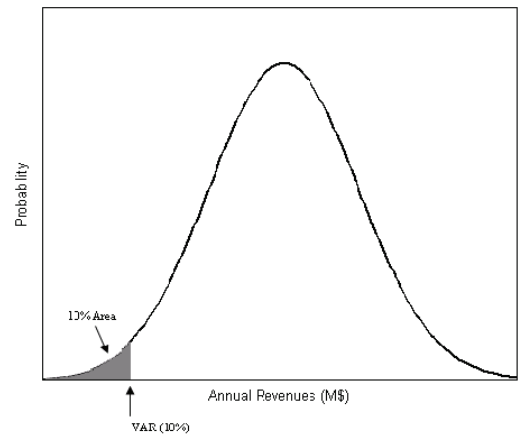


Fig. 13: Graphical representation of the VaR(10%) for a Gaussian PDF

In this example, and from the point of view of the distribution of annual revenues, the VaR is the quantile of the PDF which defines an area of 10% to its left.

One of the advantages of the VaR is that it is a normalized indicator, that is, it allows the direct comparison between different types of investments. This makes the VaR a popular criterion for the study of portfolio of investments. According to [9], the VaR is the most widespread approach to measure risk by power companies.

5.1 VaR Applied to the Distribution of Annual Revenues

As discussed in Section 4.4.1 and as will be confirmed later, it is assumed that the annual revenue is normally-distributed. This makes possible the mathematical representation of the VaR since the PDF (and cumulative density function, CDF) for a normal distribution can be expressed mathematically. Again, a normal distribution is completely defined by its first two moments (expected value and variance). The mathematical characterization of these two values has been one of the main objectives of this work.

In this way, the PDF of a normally-distributed (Gaussian) random variable $X \sim N(\mu, \sigma)$ is given by:

$$\mathrm{Prob}(X=x)=\mathrm{PDF}(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{78}$$

where X is the variable of interest (the annual revenue in our case), x is a specific value for the variable, μ is the expected value and σ the standard deviation. The area below the PDF curve to the left of a specific value, x, can be measured by means of the cumulative density function, CDF(x). For a normal distribution, the CDF is given by:

$$\mathrm{CDF}(x)=\mathrm{Prob}(X\leq x)=\frac{1}{2}\left(1+\mathrm{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right) \tag{79}$$

where *erf*(·) is the *Error Function* also called *Gauss Error Function*, and is defined by:

$$\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt \tag{80}$$

If we specify a given probability, P, of having $X \leq x$, the value of x satisfying this constraint is the value at risk associated to that probability, x=VaR(P). Equation (79), for a specific probability P, in per unit, can be written as:

$$P{=}\mathrm{Prob}(X\leq x)=\frac{1}{2}\left(1+\mathrm{erf}\left(\frac{\mathrm{VaR}(P)-\mu}{\sigma\sqrt{2}}\right)\right) \tag{81}$$

where VaR(P) is the only unknown. We can solve:

$$2P\text{-}1=\mathrm{erf}\left(\frac{\mathrm{VaR}(P)-\mu}{\sigma\sqrt{2}}\right) \tag{82}$$

and, therefore

$$\mathrm{erf}^{-1}\left(2\cdot P\text{-}1\right)=\frac{\mathrm{VaR}(P)-\mu}{\sigma\sqrt{2}} \tag{83}$$

where erf^{-1} is the inverse of the error function. Note the left hand side of (83) is a constant for a specified P. Fig. 14 shows the plot of the inverse of the error function, whose domain is defined in the interval $x \in [-1,1]$.

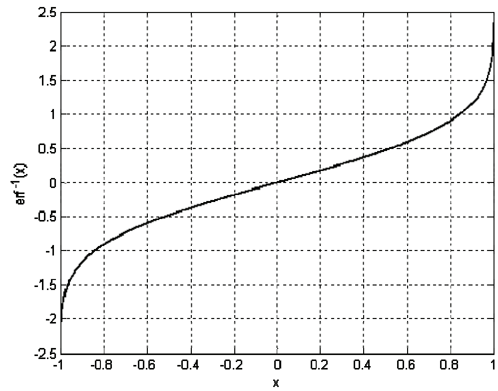


Fig. 14: Inverse error function *erf*$^{-1}$*(.)*

Fig. 14 shows how the function $erf^{-1}(x)$ goes from -∞ to ∞ as x goes from -1 to 1 and takes on a zero value when $x = 0$.

Continuing with the analysis, (83) can be solved for VaR(P) as:

$$\mathrm{VaR}(P) = \sigma \cdot \sqrt{2} \cdot \mathrm{erf}^{-1}\left(2P\text{-}1\right) + \mu \tag{84}$$

If we define R_A as the annual, normally-distributed revenue (either fixed-rate or market), then, from (84):

$$\mathrm{VaR}(P) = \sqrt{\mathrm{var}\left[R_A\right]} \cdot \sqrt{2} \cdot \mathrm{erf}^{-1}\left(2P\text{-}1\right) + \mathrm{E}\left[R_A\right] \tag{85}$$

This is the final mathematical representation of the VaR(P) in terms of the expected value and variance of the annual revenue, which have been characterized in Chapter 4. In Chapter 6 a sensitivity analysis will estimate how the VaR changes according to different parameters.

5.2 VaR Applied to the Return of the Investment

As explained before, the definition of VaR can be also applied to the distribution of the returns of the investment (in %) instead of the revenue (in M$). The *Internal Rate of Return*, IRR, is the discount rate that equalizes the amount of the investment with the present value of future revenues.

The IRR is a capital budgeting tool that allows the direct comparison of the financial performance of two different projects. Generally, the preferred investment is the one with the highest IRR subject to a *Minimum Accepted Rate of Return* (MARR) specified by the investors according to the type of investment. Generally, if the project's IRR does not exceed the specified MARR, it is rejected.

Mathematically speaking, the IRR is defined as the value (in %) which satisfies the following equality:

$$\mathrm{NPV} = \sum_{t=0}^{n} \frac{C_t}{\left(1+\mathrm{IRR}\right)^t} = 0 \tag{86}$$

where NPV is the *Net Present Value* of the stream of cash flows C_t at time t, and n is the number of periods of analysis (measured in years in our case).

Usually, the cash flow at time zero, C_0, is the negative of the upfront investment I. In this work, and in order to simplify the analysis, it will be assumed that the cash flow stream remains constant during the period of analysis (25 years in our case). This avoids the introduction of additional stochasticity into the problem (inflation, interest rate fluctuations, etc). Therefore, for a constant annual revenue, R_A (either R_M or R_F) (86) can be written as:

$$\mathrm{NPV} = -I + \sum_{t=1}^{n} \frac{R_A}{\left(1+\mathrm{IRR}\right)^t} = 0 \tag{87}$$

Note that the IRR is a random variable since R_A is a random variable itself.

A company can therefore set a MARR together with a specific probability P^* that the IRR will be lower than the especified MARR. Therefore,

$$\mathrm{VaR}(P^*) = \mathrm{MARR} \tag{88}$$

If this constraint is binding, the company knows that the probability of obtaining an IRR below the MARR is the acceptable P^*. Whether (88) can be satisfied depends of the distribution of the IRRs. If the return of the investment given by VaR(P^*) is greater than the MARR, the project would be accepted.

For a given MARR, equation (87) can be solved for the corresponding needed R_A^*:

$$R_A^* = \frac{I}{\sum_{t=1}^{n} \frac{1}{(1+\text{MARR})^t}} \tag{89}$$

It can be proven that (89) is a bijective function. This implies that, for every MARR, there is only one R_A^*. For this reason, VaR(P^*) can be directly applied to the distribution of annual revenues. Therefore,

$$\text{VaR}(P^*) = R_A^* \tag{90}$$

Since (90) is bijective, this expression implies that the probabilities of satisfying $R_A < R_A^*$ and IRR < MARR are equal. Therefore, the probability of being below the specified P^* is the same for both distributions (IRR and R_A). The advantage of applying directly the definition of VaR to the distribution of the annual revenues is that their PDF is known to be normal. Therefore, from (85) and (90) we have:

$$R_A^* = \sqrt{\text{var}\left[R_A\right]} \cdot \sqrt{2} \cdot \text{erf}^{-1}\left(2P^* - 1\right) + \text{E}\left[R_A\right] \tag{91}$$

In terms of the standard deviation, (91) can be written as:

$$R_A^* = \text{std}\left[R_A\right] \cdot \sqrt{2} \cdot \text{erf}^{-1}\left(2P^* - 1\right) + \text{E}\left[R_A\right] \tag{92}$$

Since both P^* and R_A^* are known, (92) specifies a linear relationship between E[R_A] and std[R_A]. This linear relationship must be satisfied if the company is to meet the limit revenue constraint (88). Also, (92) can also be expressed as a function of the expected value of the annual revenue, E[R_A]:

$$\mathrm{E}\left[R_A\right] = R_A^* - \mathrm{std}\left[R_A\right] \cdot \sqrt{2} \cdot \mathrm{erf}^{-1}\left(2P^*\text{-}1\right) \tag{93}$$

The importance of (93) is that only the distribution of annual revenue satisfying this linear relationship can meet constraint (88). Fig. 15 shows a plot of function (93) for values of P^* in the definition of VaR less than 0.5. Notice that the slope of this straight line is a function of P^* and takes on positive values for the assumed range of P^*.

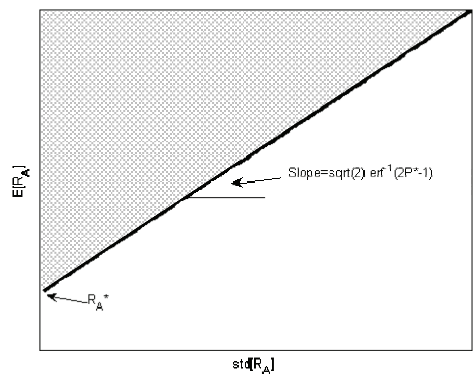


Fig. 15: Representation of the linear relation E[R_A]-std[R_A].

The usefulness of (93) (or Fig. 15) can be interpreted as follows. For a given wind farm selling its output at a specific electricity market prices (or fixed-rate), the corresponding expected value and standard deviation of the annual revenue, E[R_{M_A}] and std[R_{M_A}], respectively, can be estimated as described in Chapter 4. If constraint (88) is binding for this pair of values, they will lie on the solid line in Fig. 15. If the pair (std[R_{M_A}], E[R_{M_A}]) for the particular wind farm and electricity market lies in the region above the straight line (the shaded area), the associated return IRR for that particular pair will be greater than MARR (the project outperforms the minimum acceptable requirements). On the other hand, if the pair (std[R_{M_A}], E[R_{M_A}]) lies below the line (the white area), the investment is not acceptable to the investor since the probability of being below the MARR will be greater than P^*.

The previous analysis can be useful when comparing different wind resource and electricity market options for a wind farm, since the wind resource at different locations together with the characteristics of the different electricity markets will define different points throughout Fig. 15.

5.3 Estimation of the Optimal Fixed-Rate

In the specific case of the fixed-rate revenue, from (66) and (72) we can write the VaR as a function of the expected value and variance of the annual generation output of the wind farm, g_{w_A}, and the fixed-rate, π, as follows:

$$\mathrm{VaR}(P) = \sqrt{\pi^2 \cdot \mathrm{var}\left[g_{w\_A}\right]} \cdot \sqrt{2} \cdot \mathrm{erf}^{-1}\left(2P\text{-}1\right) + \pi \cdot \mathrm{E}\left[g_{w\_A}\right] \tag{94}$$

or

$$\mathrm{VaR}(P) = \pi \cdot \mathrm{std}\left[g_{w\_A}\right] \cdot \sqrt{2} \cdot \mathrm{erf}^{-1}\left(2P\text{-}1\right) + \pi \cdot \mathrm{E}\left[g_{w\_A}\right] \tag{95}$$

This expression can be useful for the owner of a wind farm for the negotiation of a fixed-rate with the local utility or power broker. Since the values of E[g_{w_A}] and std[g_{w_A}] are known or can be estimated using wind speed records (as will be explained in Appendix 1), the only unknown in (95) is the fixed-rate π. Then, for a specific R_A^* and P^* according to the owner's VaR strategy, (95) can be solved for a π^* as:

$$\pi^* = \frac{R_A^*}{\left(\mathrm{std}\left[g_{w\_A}\right] \cdot \sqrt{2} \cdot \mathrm{erf}^{-1}\left(2P^*\text{-}1\right) + \mathrm{E}\left[g_{w\_A}\right]\right)} \tag{96}$$

where π^* is the value of π which satisfies constraint (88). The importance of (96) is that π relates the fixed-rate with the statistical parameters of the wind resource and the owner's VaR strategy.

Another useful application for (96), is that the owner of the wind farm, when faced with the option of participating in an electricity market as opposed to the fixed-rate option, would be able to estimate the optimal value of the fixed-rate π that provides the same VaR of the market option.

The previous notions will be better clarified in Chapter 7 with the application of the methodology using actual data. The following Chapter will develop a sensitivity analysis based on the mathematical models for the expected value and variance of the wind farm's annual revenue for both revenue options (fixed-rate or market).

Chapter 6: Sensitivities

The objective of a sensitivity analysis is to measure how a variation in one parameter may affect any variable that depends on it.

Mathematically speaking, the sensitivity of a variable X with respect to a parameter p, S^X_p, is defined as the partial derivative of the X with respect to p [28]:

$$S^X_{p,d} = \frac{\partial X}{\partial p} \tag{97}$$

The sub-index d indicates that the sensitivity is a dimensioned measure, with the units of X/p. Sometimes, however, local sensitivity measures are normalized by some reference or central value. If we have the initial values X^0 and p^0, the normalized sensitivity can be expressed as:

$$S^X_p = \frac{\partial X}{\partial p}\frac{p^0}{X^0} \tag{98}$$

The advantage of the normalized sensitivities is that, since they are dimensionless, sensitivities with respect to different parameters can be compared. From this point on, the sensitivities estimated in this work are meant to be dimensionless (normalized).

In this work, normalized sensitivities for the expected value and variance of the annual revenue with respect to the following market and resource related parameters will be developed:

- The expected value of the hourly energy output of the wind farm, E[g_{w_h}]. Note hat since E[g_{w_h}] is equal to $\eta_h \cdot g_w^{max}$, (where η_h is the hourly capacity factor, and

g_w^{max} is the installed power in the wind farm), the dimensionless sensitivity with respect of any of these three variables (expected value of the output, the hourly capacity factor or the wind farm's installed capacity) will yield the same result.

- The variance of the hourly energy output of the wind farm, var[g_{w_h}].
- The expected value of the hourly market price, E[λ_h] (for the market revenue option).
- The variance of the hourly market price, var[λ_h] (for the market revenue option).
- The expected value of the hourly demand, E[$d_{\_h}$] (for the market revenue option).
- The variance of the hourly demand, var[$d_{\_h}$] (for the market revenue option).

This Chapter summarizes the mathematical expressions of the sensitivities of the hourly and annual revenues with respect to the above listed parameters. Expressions for the sensitivities of the VaR with respect to the moments of the annual revenue will also be estimated.

The detailed development of the mathematical expressions for the sensitivities shown here can be found in Appendix 3.

6.1 Sensitivities of the Expected Value and Variance of the Hourly Revenues

The expressions for the sensitivities of the hourly revenues with respect to the different parameters are shown in the following subsections.

6.1.1 Fixed-Rate Revenue

The sensitivities of the hourly revenues for the fixed-rate option are given in Table 2.

Table 2: Sensitivity of the hourly fixed-rate revenues

Sensitivity	*Expression*
$S_{\mathrm{E}\left[g_{w\_h}\right]}^{\mathrm{E}\left[R_{F\_h}\right]} = S_{\eta_h}^{\mathrm{E}\left[R_{F\_h}\right]} = S_{g_w^{\max}}^{\mathrm{E}\left[R_{F\_h}\right]}$	$\pi \cdot \frac{\mathrm{E}\left[g_{w\_h}\right]}{\mathrm{E}\left[R_{F\_h}\right]}$
$S_{\mathrm{var}\left[g_{w\_h}\right]}^{\mathrm{var}\left[R_{F\_h}\right]}$	$\pi^2 \cdot \frac{\mathrm{var}\left[g_{w\_h}\right]}{\mathrm{var}\left[R_{F\_h}\right]}$

The sensitivities of E[R_{F_h}] and var[R_{F_h}] with respect to the first two moments of the electricity market related parameters, d_h and λ_h, are zero (since the fixed-rate revenue does not depend on market-related parameters).

6.1.2 Market Revenue

The market revenue is affected by both, market and resource-related parameters. The corresponding sensitivities are shown in Table 3:

Table 3: Sensitivity of the hourly market revenues

Sensitivity	*Expression*
$S_{\mathrm{E}[\lambda_h]}^{\mathrm{E}[R_{M\_h}]} = S_{\mathrm{E}[g_{w\_h}]}^{\mathrm{E}[R_{M\_h}]} = S_{\eta_h}^{\mathrm{E}[R_{M\_h}]} = S_{g_w^{\max}}^{\mathrm{E}[R_{M\_h}]}$	$\pi \cdot g_w^{\max} \cdot \dfrac{\mathrm{E}[\lambda_h]}{\mathrm{E}[R_{M\_h}]}$
$S_{\mathrm{E}[d_h]}^{\mathrm{E}[R_{M\_h}]}$	$S_{\mathrm{E}[\lambda_h]}^{\mathrm{E}[R_{M\_h}]} \cdot \beta \cdot \dfrac{\mathrm{E}[d_h]}{\mathrm{E}[\lambda_h]}$
$S_{\mathrm{E}[g_{w\_h}]}^{\mathrm{var}[R_{M\_h}]} = S_{\eta_h}^{\mathrm{var}[R_{M\_h}]} = S_{g_w^{\max}}^{\mathrm{var}[R_{M\_h}]}$	$\dfrac{2 \cdot \eta_h^2 \cdot \left(g_w^{\max}\right)^2 \cdot \mathrm{var}[\lambda_h]}{\mathrm{var}[R_{M\_h}]}$
$S_{\mathrm{E}[\lambda_h]}^{\mathrm{var}[R_{M\_h}]}$	$\dfrac{2 \cdot \mathrm{E}[\lambda_h]^2 \cdot \mathrm{var}[g_{w\_h}]}{\mathrm{var}[R_{M\_h}]}$
$S_{\mathrm{E}[d_h]}^{\mathrm{var}[R_{M\_h}]}$	$S_{\mathrm{E}[\lambda_h]}^{\mathrm{var}[R_{M\_h}]} \cdot \beta \cdot \dfrac{\mathrm{E}[d_h]}{\mathrm{E}[\lambda_h]}$
$S_{\mathrm{var}[g_{w\_h}]}^{\mathrm{var}[R_{M\_h}]}$	$\left(\mathrm{var}[\lambda_h] + \left(\mathrm{E}[\lambda_h]\right)^2\right) \cdot \dfrac{\mathrm{var}[g_{w\_h}]}{\mathrm{var}[R_{M\_h}]}$
$S_{\mathrm{var}[\lambda_h]}^{\mathrm{var}[R_{M\_h}]}$	$\left(\eta_h{}^2 \cdot \left(g_w^{\max}\right)^2 + \mathrm{var}[g_{w\_h}]\right) \cdot \dfrac{\mathrm{var}[\lambda_h]}{\mathrm{var}[R_{M\_h}]}$
$S_{\mathrm{var}[d_h]}^{\mathrm{var}[R_{M\_h}]}$	$S_{\mathrm{var}[\lambda_h]}^{\mathrm{var}[R_{M\_h}]} \cdot \left(\gamma + \beta^2\right) \cdot \dfrac{\mathrm{var}[d_h]}{\mathrm{var}[\lambda_h]}$

Up to this point, the sensitivities for the hourly revenues have been characterized. The following subsection shows the sensitivities for the annual revenue.

6.2 Sensitivities of the Expected Value and Variance of the Annual Revenue

The sensitivities of the annual revenues will be expressed as a function of the sensitivities of the hourly revenues. These sensitivities will be shown for a generic annual revenue, R_A, (which may represent a fixed-rate or market revenue).

The expressions will be given as a function of a general parameter p_k, which represents any of the hourly parameters considered for the hourly sensitivities. For the specific hour k, p_k represents any of the following: E[g_{w_k}] (or η_k or g_k^{max}, since they yield the same result), var[g_{w_k}], E[λ_k], var[λ_k], E[d_k] or var[d_k]. The expressions for the sensitivities of the annual revenue with respect to a variation in the parameter p_k, are shown in Table 4:

Table 4: Normalized sensitivities of the annual revenue with respect to a change in the parameter p_k

Sensitivity	*Expression*
$S_{p_k}^{\mathrm{E}[R_A]}$	$S_{p_k}^{\mathrm{E}[R_k]} \cdot \dfrac{\mathrm{E}[R_k]}{\mathrm{E}[R_A]}$
$S_{p_k}^{\mathrm{var}[R_A]}$	$\dfrac{1}{\sqrt{\mathrm{var}[R_k]}} \cdot S_{p_k}^{\mathrm{var}[R_k]} \cdot \dfrac{\mathrm{var}[R_k]}{\mathrm{var}[R_A]} \cdot \sum_{h=1}^{8760} \rho_{k,h} \cdot \sqrt{\mathrm{var}[R_h]}$

Again, the complete deduction of these expressions is shown in Appendix 3.

In order to determine the sensitivity of the annual revenue with respect to a particular hourly parameter, it is just necessary to substitute the corresponding expressions in Table 2 and Table 3 in the corresponding hourly sensitivities with respect to parameter p_k in Table 4.

The sensitivities given in Table 4 measure how the annual revenue R_A is affected by a change in the parameter p for a specific hour k. In this work, in order to measure generalized, sustained annual changes in the market or resource-related parameters, it will be assumed that the same percentual change in the parameter takes place during all hours of the year (it would not make much practical sense to estimate, for instance, the sensitivity of the annual revenue to the change in the expected value of the wind power output at 4:00 am in March 27th).

In this way, in order to measure for instance how the E[R_A] is affected by a change in E[g_{w_h}], it will be assumed that at all the hours of the year (h=1,…,8,760), the corresponding E[g_{w_h}] will simultaneously increase by the same percentage amount. In this way, the sensitivities with respect to a change in parameter p_k for all hours of the year (represented as a sensitivity with respect to p_A) are given by Table 5:

Table 5: Sensitivities of the annual revenue with respect to a variation in the parameter p_k at all the hours of the year

Sensitivity	*Expression*
$S_{p_A}^{\mathrm{E}[R_A]}$	$\sum_{h=1}^{8760} S_{p_h}^{\mathrm{E}[R_A]}$
$S_{p_A}^{\mathrm{var}[R_A]}$	$\sum_{h=1}^{8760} S_{p_h}^{\mathrm{var}[R_A]}$

Again, the p_A represents a change in parameter p_k at all the hours of the year. Once the sensitivities of the first two moments of the annual revenue have been characterized, a similar sensitivity analysis can be carried out for the corresponding VaR, as shown next.

6.3 Sensitivity of the VaR

The sensitivity of the VaR with respect to a change in parameter p_A can be expressed as a function of the sensitivities of E[R_A] and var[R_A] with respect to the same change. Therefore, it can be proven that:

$$S_{p_A}^{\mathrm{VaR}(P)} = \frac{1}{\mathrm{VaR}(P)}\left(\frac{\mathrm{erf}^{-1}\left(2\cdot \mathrm{P}\text{-}1\right)}{\sqrt{2}}\cdot S_{p_A}^{\mathrm{var}[\mathrm{R}_A]}\cdot\sqrt{\mathrm{var}\left[\mathrm{R}_A\right]}+S_{p_A}^{\mathrm{E}[\mathrm{R}_A]}\cdot \mathrm{E}\left[\mathrm{R}_A\right]\right) \tag{99}$$

The proof can be found in Appendix 3.

This point wraps up the complete characterization of the first two moments of the annual revenue of a wind farm and the corresponding VaR attributes based on a characterization of the hourly revenues, for both revenue options (fixed-rate or market). The generalized mathematical model can be applied to any particular case once the market and resource-related parameters have been estimated using the particular actual data from the case under study. The following section will apply the developed model to an actual wind farm and electricity market.

Chapter 7: Application and Results

This section will show the results of the application of the developed methodology to a case of a wind farm operating under the two revenue options.

Two real datasets are used:

- Hourly electricity market clearing prices and system demand from Ontario's electricity market for the period between January 1st, 2002 through December 31st, 2006 (five years) [18].

- Hourly simulated wind power output dataset from planned wind farms in Ontario. This dataset has been released by Ontario Power Authority to provide future developers with information about the province's wind power resource. The wind power dataset covers the years 1997-2001 [17].

These datasets are publicly available and comprise, each 8,760 × 5 = 43,800 hourly values. The selected test wind farm has an installed capacity of 143 MW.

7.1 Hourly Revenues

From Ontario's dataset, the expected value and variance of the hourly revenues (the 288 distributions for each case) were already shown in Fig. 8 to Fig. 11 for the two revenue options (fixed-rate and market). In order to equalize the economic conditions given by both revenue options (so the involved risks could be compared directly), a fictitious fixed-rate, π, was used so that the annual expected values for both revenue options are equal. Fig. 8 and Fig. 9 show a comparison of both options.

The value of the fixed-rate π that equalizes both expected values is 53.32 \$/MWh. This value will be used whenver the risks (variances) associated to either revenue option are compared.

7.2 Annual Revenue

7.2.1 Expected Value

Once the expected values and variances for the hourly revenues are calculated from the actual dataset, the annual expected value can be easily calculated by using (42). Due to the fixed-rate used (π = 53.32 \$/MWh), the expected value for both cases is the same, therefore, $E[R_M] = E[R_F] = 20.1841$ M\$.

For the calculation of the annual variance, it is necessary to calculate the auto-correlation coefficients as indicated by (48). The way these coefficients were estimated is shown next.

7.2.2 Auto-Correlation Coefficients

The calculation of the auto-correlation coefficients between hourly revenues is necessary (48) in order to estimate the annual variance as a function of the variances of the hourly revenues. In Section 4.2 it was shown that the hourly revenue time-series are not stationary, so an auto-correlation relationship was proposed for each hour of the day. Fig. 16 shows these 24 auto-correlation relationships (curves), for the hourly fixed-rate revenues using the actual dataset for a time span of 48 hours ahead. Fig. 17 shows similar information for the whole year (8,760 hours ahead). Again, each of these curves represents the auto-correlation coefficientes for each of the 24 hours with respect to each subsequent hour (from the hour ahead up to 8,760 hours ahead). This process was explained in Section 4.2. The x-axis represents the time difference between the compared hourly revenues.

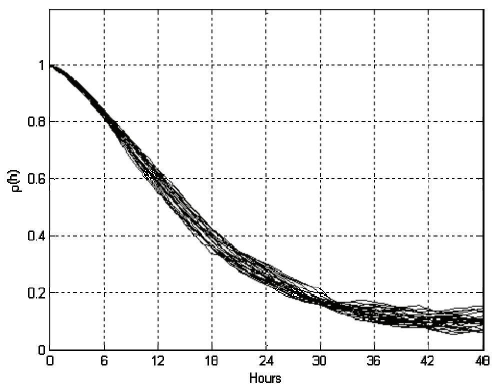


Fig. 16: First 48 hours of auto-correlation coefficient of the fixed-rate hourly revenues

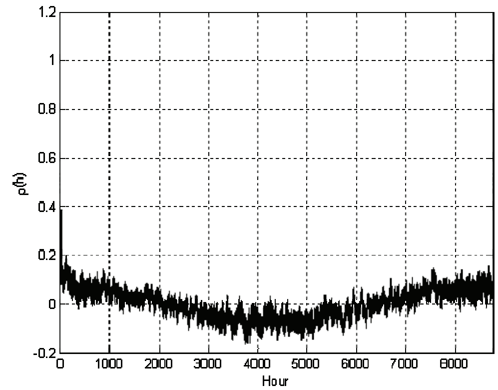


Fig. 17: Auto-correlation coefficient for all the hours of the year, for fixed-rate hourly revenues

In Fig. 16 it can be seen how, for a time difference of zero hours, the auto-correlation coefficient is obviously one (an hourly revenue at hour *h* compared with itself). A time interval of one hour is related to the auto-correlation between the revenue at a specific hour and the revenue one hour ahead. A similar logic applies to a time interval of two hours, and so on. In Fig. 16 it can be seen how the auto-correlation decreases as the time difference between the compared revenues increases. However, from Fig. 17, it can be seen how, contrary to the typical behaviour of stationary time-series, the auto-correlation coefficients do not become strictly zero but oscillate around the narrow interval [-0.1, 0.1]. This behaviour is caused by the seasonal characteristic of the hourly revenues. In the case of the fixed-rate revenue, this is caused exclusively by the seasonality in the winds.

This seasonal behaviour can be better seen by calculating a one-month moving average of the hourly revenue time-series. In fact, Fig. 18 shows the time-series for the fixed-rate hourly revenue from the dataset, together with a monthly moving average (730 hours).

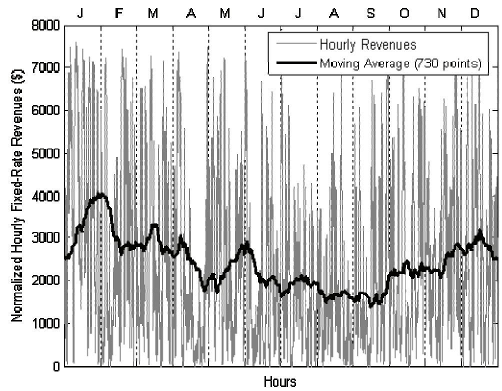


Fig. 18: One-year hourly fixed-rate revenue time-series and monthly moving average

The top of Fig. 18 indicates the month of the year. It can be seen how there is a clear declining trend in the hourly revenues from the end of January up to mid-August and a growing trend from then on. The period of this moving-average oscillation is around six months (about 4,000 hours). Therefore, for all hours belonging to a declining period, the auto-correlation coefficient with other hours on the same declining period should be slightly positive (both hours are on the same trend). Similarly, for all hours belonging to a revenue-growing period, the auto-correlation coefficient with other hours on the same growing period should be slightly positive as well (both hours are on the same trend). On the other hand, the auto-correlation coefficients should be slightly negative for hours lying on opposite trends. For instance, the hourly revenue in May lies on a declining trend whereas it lies on a growing trend in October. The auto-correlation between them is slightly negative. The time difference between opposite-trend hours is around 3,500 to 4,500 hours (around six months apart). This explains why, in Fig. 17, slightly negative auto-correlation coefficients are seen around this time span and why positive auto-correlation coefficients are seen anywhere else. A similar result was obtained for the auto-correlation coefficients for the hourly market revenues.

Due to the particular nature of the auto-correlation coefficients, several approximation attempts were made in order to simplify the estimation process.

a) Those auto-correlation coefficients lying between ±0.1 have been assumed to be zero. The error introduced in the calculation of the annual variance in the case of the fixed-rate revenue (70) is so large (approximately 400%) that this approximation was rejected with no further consideration.

b) The 24 curves of auto-correlation coefficients have been averaged out into a unique aggregated curve. The errors in the estimate of the annual variances are 1% in the case of the fixed-rate revenue and 2% in the case of the market revenue option. This approximation can, therefore, be considered adequate. Fig. 19 shows the avarege fo the 24 auto-correlation curves for both revenue options (fixed-rate and market) for a time span of 12 hours. As expected, the auto-correlation coefficients drop faster in the case of the market revenue option since there are two random variables involved in this case (wind power output and electricity prices) and therefore inter-hour volatility is higher.

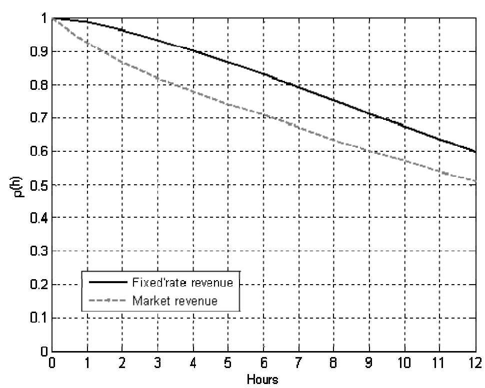


Fig. 19: First 12 hours of mean autocorrelations for both type of hourly revenues

c) Finally, the auto-correlation coefficients have been calculated as if the process were stationary. A single auto-correlation coefficient curve is obtained but the calculation process does not assume differences in the auto-correlation coefficients between the hours of the day. In this case, the induced errors in the estimation of the annual variance are 1% and 8% for the fixed-rate and market revenue options, respectively.

7.2.3 Annual Variance and PDF

Once the auto-correlation coefficients have been calculated, the variance of the annual revenue ('annual variance') has been completely characterized. As defined by (59), the variance of the annual revenue can be expressed as the sum of an *individual-variance* term (the variances of each hour's individual revenue) plus the *cross-variance* term (the covariance between each hour's revenues):

$$\operatorname{var}\left[R_A\right] = \sum_{i=1}^{8760} \operatorname{var}[R_i] + 2\sum_{i=1}^{8760}\sum_{j>i}^{8760} \rho_{i,j} \cdot \operatorname{std}[R_i] \cdot \operatorname{std}[R_j] \tag{100}$$

Note that (100) is the same as (59) (Section 4.2). One interesting exercise is to determine the relative weight of each of the summands in (100): the *individual-variance* and the *cross-variance* terms. The results are shown in Table 7 and Table 9 for the fixed-rate and market revenue options respectively.

Table 6: Annual variance for the fixed-rate revenue option

var[R_F] *(M\$)^2*	*'Individual-variance' in var[R_F]* *(M\$)^2*	*'Cross-variance' in var [R_F]* *(M\$)^2*
1.2246	0.0318	1.1928

From Table 6, it can be observed how the '*individual-variance'* tem is considerably lower that the *cross-variance* term (only 2.46% of the total variance). Coming back to Fig. 16, it can be seen that for the first 15 hours the auto-correlation coefficients are greater than 0.5. This means that in (100), for each *'individual-variance'* summand, there are 15 additional *'cross-variance'* summands which are multiplied by coefficients greater than 0.5, which, in turn, are multiplied by a factor of 2. This confirms the importance of the inter-hour covariance in the hourly revenues and how it must be estimated accurately (options b) and c) in the previous section).

Table 7 shows the results for the market annual revenue using the different models. The row 'Actual' shows the results of applying (76) directly to the 8,760 hourly revenues (obtained by multiplying hour by hour the electricity prices and wind power outputs from the used datasets). The row 'Direct Model' indicates the results of applying (76) with the hourly variances' given by (31). The row 'Indirect Model' refers to the application of the price versus demand relationship developed in Section 2.2.2, which leads to the hourly variances' shown in (40). This model used *Regression* and *Conditional Variance* functions ((5) and (6), respectively) to represent the electricitity prices as a function of system demand.

Table 7: Annual variance for the market revenue option under different methods

	$var[R_M]$ $(M\$)^2$	*'Individual-variance' in $var[R_M]$ $(M\$)^2$*	*'Cross-variance' in $var[R_M]$ $(M\$)^2$*
Actual	**9.8500**	0.0531	9.7968
'Direct' Model	**9.8671**	0.0531	9.8141
'Indirect' Model	**9.2259**	0.0531	9.1748

From Table 7 the excellent annual variance estimate provided by the developed model ('Direct') can be seen (second column, bold numbers). The relative error between the estimates provided by the 'Direct' Model with respect to the 'Actual' model is 0.2% (or 0.09% for the standard deviation). The error of the 'Indirect Model' in estimating the 'Actual' annual variance is only 6.29% (or 3.18% for the standard deviation).

From Table 7 it can also be seen that the the *individual-variance* term is considerably lower than the *cross-variance* term (the *individual-variance* term represents only a 0.5% of the total variance).

Following with the analysis, and once the first two moments of the annual revenue for both revenue options have been calculated, the corresponding PDFs can be laid out, assuming the hypothesis of normality (which will be confirmed later). Fig. 20 shows the normal distributions for both annual revenue options, using the values of Table 6 and Table 7 (the *'Actual'* row) and the expected values previously shown in Section 7.2.1.

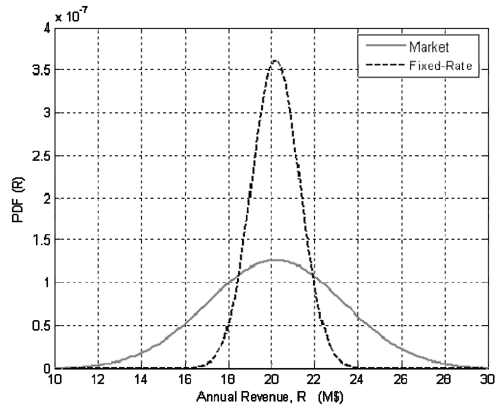


Fig. 20: PDF for the annual revenues.

Estimating these two PDFs through a mathematical model that decoupled the isolated effects of the market and wind resource on the variance of the annual revenue has been the main objective of this work.

From Fig. 20 it can be seen graphically that the larger spread in the market revenue option when compared to the fixed-rate revenue option (again, the fixed-rate, π, has been fixed to 53.32 \$/MWh in order for both PDFs to have the same expected value and thus facilitate the variance comparison). The fixed-rate revenue option may obviously have a different expected value from the market revenue option. The decision on the best return strategy for an investor faced with these distributions is determined by the compromise that exists between risk and return (variance against expected value). This was explained in detail in Chapter 5.

Since the assumption of normality is critical in the subsequent analyses, the following section will validate the model through a time-series analysis approach as explained in Section 4.4.1 using a generic ARMA model. The results will serve not only to confirm the hypothesis of normality but also to validate the numerical results found so far for the case of fixed-rate revenue.

7.2.4 Model Validation through an ARMA model

Using the actual wind power and market price dataset, an optimal ARMA(p,q) model has been fitted for the time-series of hourly market revenues in the case of the fixed-rate option. In the market revenue option, hourly revenues depend not only on the wind power output but also on the electricity prices. It was seen that the electricity maket prices usually depend on the system demand. It was also seen that the *variance* of the electricity prices usually depends on the demand level through a quadratic relationship (Fig. A- 7). Since the demand is a random variable, the variance of the electricity prices is therefore stochastic. The simulation of time-series with stochastic variance requires the use of the so-called GARCH time-series models [29]. The implementation of this model for the validation of the market revenue option is outside the scope of this work. The validation, therefore, will be limited to the fixed-rate revenue option, which will be shown to be well represented by an ARMA model.

Having decided on the time-series to be represented through an auto-regressive model, it is important to mention that this series had to be normalized to remove the seasonalities (trends), a necessary condition for the use of an ARMA model. The normalization is done by subtracting, at each hour of the actual time-series, the mean of the corresponding group ((9) and (15)) divided by the result by the corresponding group's standard deviation ((11) and (31)).

Different values of p (the order of the auto-regressive polynomial) and q (the order of the moving average polynomial) have been tested using the Matlab® System Identification Toolbox. From the analyses, it was found that the optimal parameters that minimize the Loss Function are $p = 6$ and $q = 0$. This means that this process is actually best represented by a pure Auto-Regressive model of order six (AR(6)). In the case of the fixed-rate revenue, the AR polynomial found was:

$$\begin{aligned} R_h = {} & 1 - 1.255\, R_{h-1} + 0.2057\, R_{h-2} + 0.01272\, R_{h-3} \\ & - 0.004263\, R_{h-4} - 0.01044\, R_{h-5} + 0.07767\, R_{h-6} \end{aligned} \tag{101}$$

Once the AR model has been identified, 8,760 hour-long artificial time-series can be gegenerated. Fig. 21 Shows, the first 1,000 hours of a normalized generated time-series compared with the actual normalized time-series.

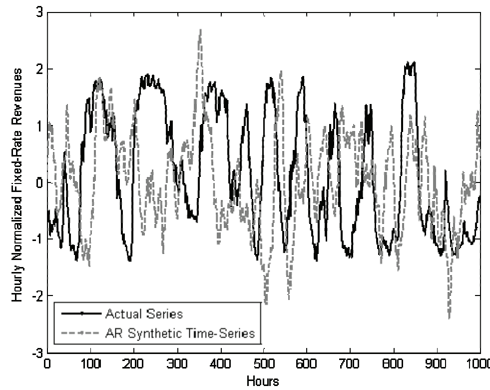


Fig. 21: A thousand hours of actual and synthetic time series for the fixed-rate hourly revenues

A total of 10,000 synthetic hourly revenue time-series have been generated. For each time-series, an annual revenue has been calculated by adding up all the terms in the series. Fig. 22 shows a histogram with 100,000 revenues thus obtained. Overlapped with the histogram, a normal PDF with the same expected value and variance of the 100,000 artificial revenues is shown. Fig. 23 shows the correspondent 'Q-Q' plot for those 100,000 revenues with respect to a normal distribution.

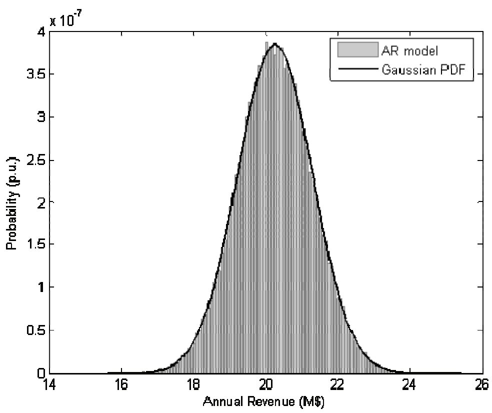


Fig. 22: Annual fixed-rate revenues' PDF computed by means of the AR-6 model

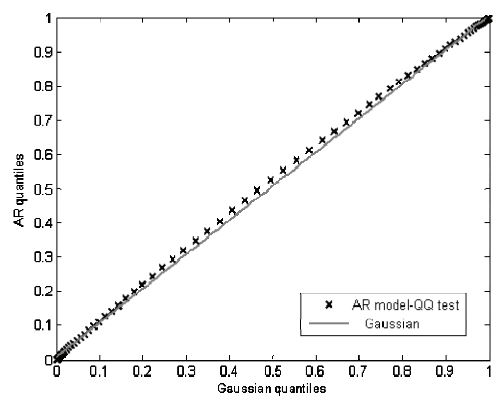


Fig. 23: QQ-test for the fixed-rate annual revenues' PDF

Fig. 22 shows at first glance the excellent fit of a normal distribution to the annual revenues. The 'Q-Q' test in Fig. 23 confirms the result. The 'Q-Q' line for the annual revenue is considerably close to that of a normal distribution (a straigth line with unitary slope). This result finally confirms that the annual revenue is normally-distributed.

The results from the AR(6) analysis can be used to estimate the overall numerical results of the proposed mathematical model as shown in Table 8:

Table 8: Results of the AR(6) for the annual fixed-rate revenue

	$E[R_F]$ $(M\$)$	$var[R_F]$ $(M\$)^2$	$std[R_F]$ $(M\$)$
AR(6)	20.2583	1.0807	1.0396
Mathematical model	20.1841	1.2246	1.1066

Comparing the results in Table 8 it can be seen that the expected value provided by the AR(6) model is only 0.37% higher than the expected value calculated through the mathematical model. In the case of the variance, the relative difference between both values is only 6.1%. These results are satisfactory and it can be now concluded that the mathematical model developed in this work has been validated.

7.3 Application of the Concept of Value at Risk

This section will show the results of applying the concept of VaR to both revenue options (fixed-rate and market) for all possible values of the probability P in (85). Although actual sensitivities were developed in Section 6.3, this section shows how the general behaviour of the VaR according to changes in some important parameters.

Fig. 24 and Fig. 25 show the values for the VaR obtained for the intervals $P \leq 0.5$ and $P \geq 0.5$ respectively, applying the definition of VaR to the annual revenue distributions shown in Fig. 20 (recall that $\pi = 53.32$ \$/MWh for the base-case for the fixed-rate revenue option).

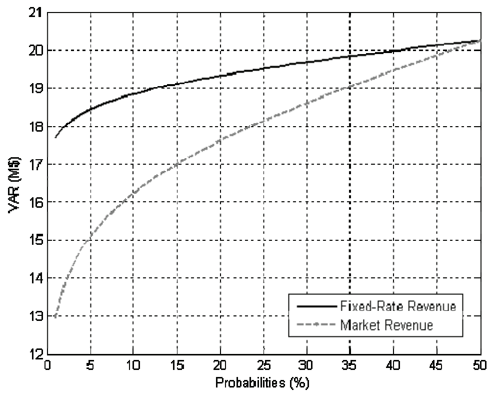


Fig. 24: VaR curves for the annual revenues with $P \leq 0.5$

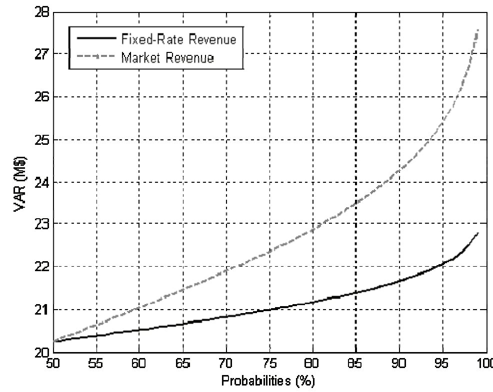


Fig. 25: VaR curves for the annual revenues with $P > 0.5$

In the case shown in Fig. 24 y Fig. 25, both curves cross each other at P = 50% (0.5) since both revenue distributions have the same expected value. It can also be observed that for values of P less than 50%, the VaR in the fixed-rate revenue option lies above the VaR for the market revenue option. The VaR is usually defined for low values of P (5% to 10%). This means that the fixed-rate revenue option is the preferred one for this specific case as given by its higher VaR on the typical range of P. In particular, at P=10%, the VaR for the fixed-rate option is 18.9 M$, 2.5 M$ more than the market revenue option.

In Fig. 26 it can be seen how the VaR for the fixed-rate revenue option changes with respect to the expected value or variance of the annual revenue distribution.

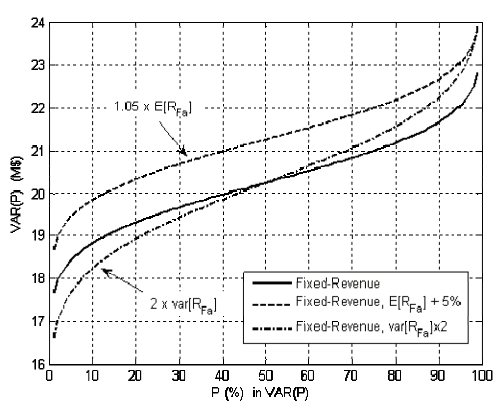


Fig. 26: Fixed-rate revenue VaR for different E[R_A] and var[R_A]

In Fig. 26, the expected value has been increased by 5% with respect to the base-case. It can be seen how the curve for the VaR shifts upwards. The contrary occurs when the expected value is reduced. The effect of a change in the variance is of a ‘rotation’ around the mid-point of the curve (P = 50%).

It is important to mention that the influence of a percentual change in the variance on the VaR is considerably less significant than the same change in the expected value. Therefore, in order to observe a significant effect, the variance had to be doubled with respect to the base-case. Similar results are obtained for the market revenue option. Actual sensitivities of the VaR with respect to different parameters have been developed in Chapter 6, and will be confirmed numerically in Section 7.4.2.

Fig. 27 shows how a change in the fixed-rate, π, of ±5% influences the VaR with respect to the base-case.

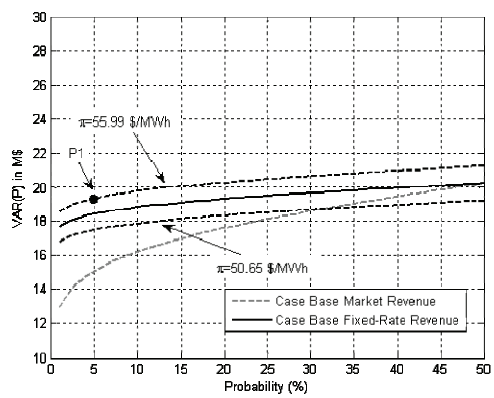


Fig. 27: Fixed-rate revenue VaR for different fixed-rates, π

The curves shown in Fig. 27 may help the owner of a wind farm to determine the best revenue option (a fixed-rate or the electricity market). These curves allow a comparison of the different VaR obtained in each case under a given probability P in VaR(P).

The curves also allow the owner of the wind farm or investor to estimate the necessary fixed-rate to achieve a given VaR(P). For instance, from Fig. 27, if the owner requires a VaR(5%) of 19.3 M$, he/she would negotiate a fixed-rate with the local utility or the power broker of around 55.99 $/MWh. This point is indicated as ‘P1’ in Fig. 27.

The analysis continues with an application of the expressions developed in Section 5.2.

Now, a predefined MARR can also be used to estimate the optimal fixed-rate to be negotiated with the local utility or power broker. For instance, if a MARR is set at 10%, then, for an investment in a 143MW wind farm, with an initial unitary investment cost of 1,000 $/kW [30] and a project life-time of 25 years, from (102), the corresponding minimum acceptable return, R_A^*, is 15.75 M$. Suppose that the minimum acceptable VaR is then defined as VaR(5%) = 15.75 M$. Then, from (96), the minimum aceptable fixed-rate can be estimated based on the expected value and variance of the wind power output, E[g_{w_A}], var[g_{w_A}], respectively (estimated at the project's planning stage). Since from the dataset E[g_{w_A}] = 378,515 MWh and var[g_{w_A}] = 427,847,139 MWh^2, the minimum accepted fixed-rate would be π^*= 45.73$/MWh. This situation is indicated as '*S1*' in Fig. 29. Fig. 28 shows the minimum acceptable π^* for different values of P^*, which satisifes VaR(P^*) = MARR = 10%.

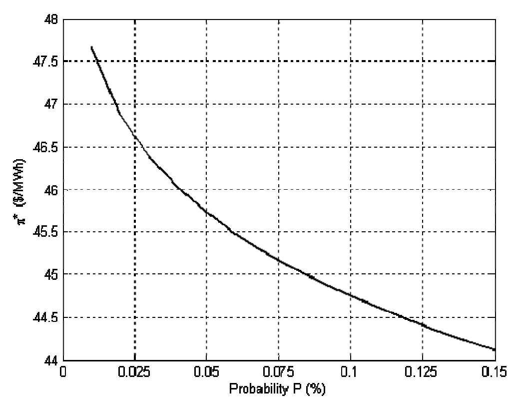


Fig. 28: Values of limit fixed-rate, π^*, for different values of P in Var(P) = MARR = 10%

As expected, if the value of P^* in VaR(P^*) = MARR is increased (that is, the possibility of obtaining IRRs lower than the MARR is increased), the owner of the wind farm or investor can settle for lower fixed-rates.

Now, the concept of the 'limit line' defined by the expected value and standard deviation of the annual revenue as described in Section 5.2 can be used to determine the financial feasibility of the project. Following the case where VaR(5%) = MARR = 10% was defined (which was equivalent to saying VaR(5%) = 15.75 M$) and considering that from the dataset E[g_{w_A}]= 378,515 MWh and var[g_{w_A}]= 427,847,139 MWh2 the 'limit line' given by (93) is:

$$\mathrm{E}\left[R_A\right] = 15.75 + 1.65 \cdot \mathrm{std}\left[R_A\right] \tag{103}$$

Fig. 29 shows the limit curve for the specific case under analysis with the situations 'S2' and 'S3' indicating the expected revenue given by the dataset (E[R_F] = E[R_M] = 20.1841 M$.) for the fixed-rate revenue option and the market revenue option, respectively.

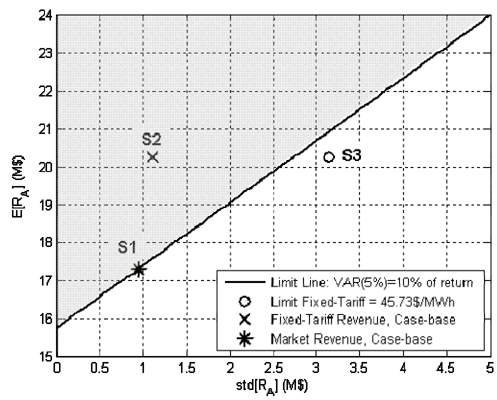


Fig. 29: Limit-line E[R_A] versus std[R_A], and different revenue-fixed rate situations

As expected, the situation 'S1' (the minimum fixed-rate necessary to achieve a pre-established MARR) falls on the 'limit-line. It can be seen that situation 'S2' would be acceptable for the investor while situation 'S3' would not for the assumed VaR(5%) = 15.75 M$ as defined by the 'limit-line'. In other words, for the same expected revenue of 20.1841 M$, and according to the definition of risk, the fixed-rate option would be preferrable over the market revenue option.

Summarizing, this section has shown how the VaR is affected by different major parameters and how the risk acceptance as given by the VaR defined by the investor can determine the optimal revenue option (whether to participate in the fixed-rate or the market option).

The next section will show numerical results for the sensitivities developed in Chapter 6 related to the expected value and variance of the annual revenue and the parameters for the VaR criterion.

7.4 Sensitivities

This section will show numerical results for the sensitivities obtained in Chapter 6. All sensitivities have been validated throught an incremental analysis by changing a parameter by a small amount and calculating the change on the corresponding variable. In all cases the difference between the exact incremental analysis and the mathematical sensititives have been less than 0.5%. This small difference validates the proposed model.

7.4.1 Annual Sensitivities

Table 9 shows the sensitivities of the expected values for both annual revenue options (fixed-rate or market). As explained in Chapter 6, the sensitivities of E[R_M] with respect to E[λ] give the same value as with respect to E[g_w]. Also, all the sensitivities with respect to E[g_w] give the same result as with respect to E[η] and E[g_w^{max}]. As explained in Section 6.2, annual changes in the parameter will be considered.

Table 9: Sensitivities of the expected value of the annual revenue

Sensitivities of the annual revenue's expected value	
$S_{\mathrm{E}[\lambda_A]}^{\mathrm{E}[R_{M\_A}]} = S_{\mathrm{E}[g_{w\_A}]}^{\mathrm{E}[R_{M\_A}]}$	1
$S_{\mathrm{E}[d_A]}^{\mathrm{E}[R_{M\_A}]}$	2.48
$S_{\mathrm{E}[g_{w\_A}]}^{\mathrm{E}[R_{F\_A}]}$	1

From Table 9 it can be concluded that the parameter that affects the most the annual market revenue is a change in the system demand. This is due to the dependency of the prices on the system demand. A sustained one percent change on the system demand throught the year leads to an increase of 2.48% in the expected market revenue. This important result allows wind farm investors to measure the effect of future load growth on the economic performance of their investment if the same price-demand relationship is mantained. On the other hand, any percentual change on the expected value (capacity factor) of the wind power output leads to an equivalent percentual change in the expected annual revenue.

Table 10 shows the sensitivities of the variances of the annual market revenue with respect to the expected values of some parameters. Note that from (72) the variance of the fixed-rate revenue does not depend on the expected value of any parameter.

Table 10: Sensitivities of the variance of the annual revenue in the market option with respect to expected values

Sensitivities of the annual revenue's variance	
$S_{\mathrm{E}[\lambda_A]}^{\mathrm{var}[R_{M\_A}]}$	1.32
$S_{\mathrm{E}[d_A]}^{\mathrm{var}[R_{M\_A}]}$	3.23
$S_{\mathrm{E}[g_{w\_A}]}^{\mathrm{var}[R_{M\_A}]}$	0.41

Again, the parameter that influences the most the variance of the annual market revenue is the expected value of the demand (a one-percent change in the average demand translates into a change of 3.23% in the annual revenue variance). However, the key result is that the most important risk factor for the annual market revenue is the expected value of the prices when compared to the expected value of wind power output (capacity factor of the wind farm). The change caused by a one-percent change in the average electricity prices is 3.2 times (1.32/0.41) an equivalent change in the average wind power output.

Table 11 shows the sensitivities of the variances of the annual revenue for both revenue options with respect to the variance of different parameters.

Table 11: Sensitivities of the variance of the annual revenue with respect to variances

Sensitivities of the annual revenue's variance	
$S_{\mathrm{var}\left[\lambda_A\right]}^{\mathrm{var}\left[R_{M\_A}\right]}$	0.34
$S_{\mathrm{var}\left[d_A\right]}^{\mathrm{var}\left[R_{M\_A}\right]}$	0.07
$S_{\mathrm{var}\left[g_{w\_A}\right]}^{\mathrm{var}\left[R_{M\_A}\right]}$	0.79
$S_{\mathrm{var}\left[g_{w\_A}\right]}^{\mathrm{var}\left[R_{F\_A}\right]}$	1

The important result in Table 11 is that, in the market option, the change in the variability of the wind power resource (variance) is what most affects the variance of the annual revenue. In the case of the fixed-rate revenue option (last row), a unitary increase in the variability of the winds translates into an equivalent increase in the variability of the annual revenue, since the wind power output is the only source of variability in this case.

Summarizing the previous results, with respect to the expected value of the different parameters, it is a change in the average demand the factor that affects the most the risks (variance) in the market revenue option. Any change in the average wind power output (or the capacity factor of the wind farm) or average electricity prices affect equally the risk in the annual revenue.

Now, regarding variance of the different parameters, it is a change in the variability of the wind resource the factor that affects the most the risks in the annual revenue, followed by a change in the variability of the electricity prices.

7.4.2 Sensitivities of the VaR

Sensitivities of the VaR(P) have also been calculated for both revenue options with respect to the parameters E[λ_A], E[g_{w_A}], E[d_A], var[λ_A], var[g_{w_A}], var[d_A]. Again, these sensitivities are assumed for a sustained change in the corresponding parameter, that is, a percentual change is assumed to take place at all hours in the year. Expressions for the sensitivities of the VaR(P) were given by (99). Values of sensitivities have been calculated for values of P in the range [0.05, 0.2].

Fig. 30 shows the sensitivities with respect to change in the expected values of the parameters (E[λ_A], E[g_{w_A}] and E[d_A]). In Fig. 30, 'S_M' refers to the sensitivities of the VaR applied to the market revenue whereas 'S_F' refers to the fixed-rate revenue.

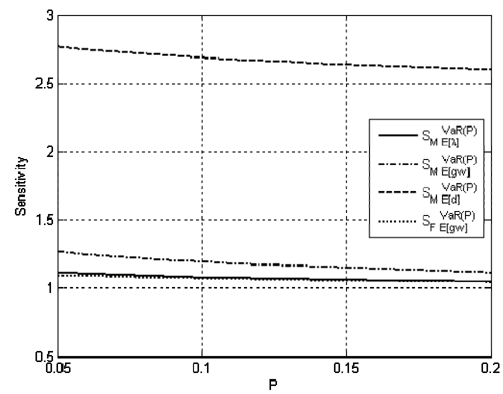


Fig. 30: Sensitivities of the VaR with respect the expected value of the parameters, for *P*є[0.05, 0.2]

In Fig. 30 It can be observed how in all cases, the sensitivities with respect to the expected values are not only positive (an expected result) but also greater or equal to unity for the examined range of *P*. Also, the sensitivities go down as *P* increases. Similar to the previous sensitivities, the sensitivity of the VaR with respect to the expected value of the demand is about 2.5 times the sensitivities with respect to other expected values.

Fig. 31 shows the results of the sensitivities of the VaR(P) with respect to the variances of the variables λ_A, g_{w_A} and d_A:

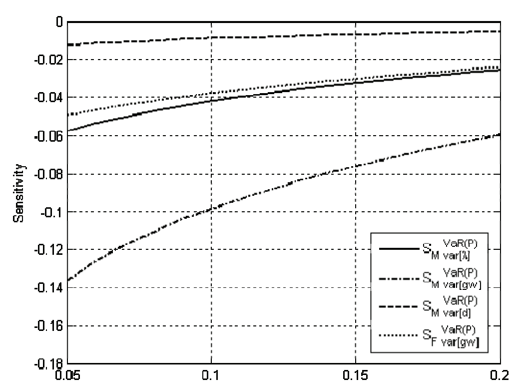


Fig. 31: Sensitivities of the VaR with respect the variances of the parameters, for *P*є[0.05, 0.2]

From Fig. 31 it can be seen how all sensitivities with respect to the variance of the different parameters are negative. This result is expected since any increase in variability in any of the parameters increase the risks in the annual revenue and, therefore, reduce the VaR(*P*) for any *P* less than 50% (*P* is usually defined at 5% or 10%).

It can be seen that the sensitivities with respect to the variances of the parameters are lower than the sensitivities with respect to the expected values. It can also be seen how the parameter that affects the VaR the most is a change in the variability (variance) of the wind power output. On the other hand, a change in the variability in the demand is the factor that affects the least the VaR.

Summarizing, the previous results indicate that the most effective risk-minimizing strategy for investors is to hedge, if possible, against the variability in the wind power output. Hedging against the volatility in the electricity market prices would have an effect of less than half on the VaR when compared to a hedging against the variability in the wind power output. These strategies depend, however, on the availability of different hedging instruments, which seem to be more common for electricity prices than for the power output. These results can therefore contribute to the development of those hedging instruments now that their effects on the risks to investors have been quantified.

This analysis concludes the ideas developed in this work, whose objective has been to understand how the market and wind resource variability affects the risks of the investment in a wind farm.

Chapter 8: Conclusions and Future Work

This work has developed a generalized mathematical model that allows the quantification of the risks of the investment in a wind farm under two different revenue options: a) the owner receives a pre-definied (sometimes negotiated) fixed-rate for the output of the wind farm, typically over the life-time of the project; b) the owner sells the output of the wind farm at the hourly clearing price of an electricity market. This work has also developed mathematical expressions for sensitivities with respect to different parameters, which allows the identification of the influence of the variability of the market prices and the wind resource on the total economic risk of the investment, under both revenue options.

The mathematical model is based on the statistical characterization of the hourly revenue as the building-block for the total annual revenue of a wind farm. One of the advantages of the model is that only the first two moments (expected value and variance) of the hourly revenue of the wind farm are required. No assumption about the particular PDF for the hourly revenue is needed. In fact, tests show that the PDF of the hourly revenues cannot be represented mathematically. Based on the statistical characterization of the hourly revenue, the first two moments of the annual revenue have also been mathematically characterized. It was shown that the annual revenue is normally distributed, through a synthetic time-series analysis based on an Auto-Regressive AR-6 model. Since a normal distribution is completely determined by its first two moments, therefore the annual revenue has been completely mathematically characterized.

Regarding the electricity market revenue option, the model allows the representation of the electricity prices through a price-demand relationship which provides the analyst with information about how the variability of the demand may affect the risks of the investment.

Although the mathematical model developed in this work is applicable to any wind farm regardless of its size and particular location, the different mathematical expressions have been verified through simulation using actual price, demand and wind power output datasets. In case wind power output datasets are not available (for instance, during the planning phase of the project), this work also proposes a methodology to convert wind speed records obtained during resource exploration into wind power output records using the notion of intra-wind farm wind diversity.

It was seen how the variance of the annual revenue of the wind farm is made up of two components, namely an 'individual-variance' term (related to the variance of each independent hourly revenue) and a 'cross-variance' term (related to the auto-correlation that exists between the hourly revenues at different hours). It was seen that the 'cross-variance' term makes up almost 98% of the variance of the annual revenue. This result indicates the importance of simulating accurately the auto-correlation between the hourly revenues at different hours. This work has shown how different assumptions of auto-correlation lead to different accuracies in the estimation of the annual revenue.

The fact that the annual revenue of the wind farm has been completely characterized as normally-distributed leads to a considerabl simplification of the Value at Risk analysis since the quantiles of a normal distribution can be represented mathematically. This opens a series of possibilities, such as determining the relationship that the expected value and variance of the annual revenue must meet in order to satisfy a particular VaR requirement. Moreover, it was shown that this relationship is linear, which facilitates the analyses even in graphical form.

In the case of the fixed-rate annual revenue option, a mathematical expression has also been developed to estimate the optimal fixed-rate to be negociated by the owner of the wind farm in order to satisfy his/her own VaR requirement. For instance, it was shown that in the case of Ontario, for a 143MW wind farm, for a unitary investment cost of 1,000 $/kW [30], a project life-time of 25 years, the minumum fixed-rate that would satisfy a VaR(5%) = MARR = 10% is 45.73$/MWh. A similar analysis could also allow the owner to estimate the fixed-rate that would yield the same VaR(*P*) as the one provided by the market revenue option.

Finally, sensitivities have been developed that allow the quantification of the influence of different market and wind-resource parameters on the expected value, variance and associated VaR related to the investment in a wind farm.

From the actual dataset used, it was found that percentual changes in the *expected value* of the different market and wind-resource parameters affect the revenues more than percentual changes in the *variance* of the parameters. It was found that a one-percent change in the expected value of the demand increases the expected value of the annual market revenue by 2.48% and its variance by 3.23%. This results shows the importance of the demand growth on the attributes of the annual revenue of the wind farm, supposing that the market behaves as described by the price-demand relationship developed.

However, considering the variance of the parameters alone, a change in the variability of the wind resource is the issue that affects the most the variability of the annual revenue. This is particularly true for the fixed-rate revenue option, since this option obvioulsy does not depend on the conditions of an electricity market. It was shown in this case how a percentual change in the expected value or variance of the wind resource leads to an equivalent change in the expected value and variance of the annual fixed-rate revenue.

Regarding the VaR(*P*), it was seen how all sensitivities with respect to the expected value of the market and wind resource-related parameters are positive and decrease as *P* increases. This confirms how any average increase (market conditions or wind resource) improves the performance of the project in terms of risk. As in the case of the variance of the annual revenue, using the actual dataset indicates that the parameter that affects the VaR the most is a change in the average annual demand (any percentual change is ‘amplified’ by 2.5 times). On the other hand, it was found how all sensitivities with respect to variances were negative. This is expected since any increase in the variability in any market or resource-related parameter increases the variability of the annual revenue and, therefore, increases the risks (or, equally, reduces the VaR(*P*) for the same *P*). The largest sensitivity found in the case of the variances was related to the wind resource. For the particular actual dataset a one-percent change in the variance of the wind power output reduces the VaR(at 5%) by 0.14%. The same sensitivity with respect to the variance of the electricity prices was -0.06%. This indicates rather surprisingly that the best risk-hedging strategy is the use of resource-related hedging instruments as opposed to the more-common market-price hedging instruments.

In general, the developed sensitivities allow investors to determine how future conditions may affect the risks involved in their invesments. Issues include, for instance, a) the inception of new generation technologies and market rules and how they may affect the electricity prices (both, in average and variability); b) the growth in system demand; or c) the inter-annual wind resource variability (for instance, the wide-reaching 'El-Niño Southern Oscillation').

Regarding future work related to the ideas presented here, the model can be upgraded to incorporate other sources of revenue (or costs) for the wind farm, including real-time imbalance settlements, forecasting penalties or environmental markets. The corresponding model however could only be applied to the specific case study since these revenue options vary considerably among jurisdictions. There are even several cases where none of these options have implemented simultaneously.

The time-series analysis used in the validation of the model assumed an optimized Auto-Regressive AR(6) model for the fixed-rate renevue option (which only includes the wind power output as random variable). However, the market revenue option includes electricity prices. Research indicates that the best auto-regressive representations for price time-series (including commodities, stocks, etc) are defined by stochastic-variance time-series models such as the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. The implementation of this model would be the best option for validating the results for the annual wind farm market-revenue option.

Finally, the VaR(*P*) analysis developed here could be used for the study of portfolio diversification. Instead of considering only one wind farm in the analysis operating under either revenue option (fixed-rate or market), the case of a portfolio investment in different wind farms under different revenue options (in different jurisdictions) could be examined. One of the results is to determine which proportions of the overall investment fund should be allocated to each of the wind farms of the portfolio.

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Appendix 1: Intra-Wind Farm Power Output Diversity

The ideal way of determining a wind farm's power output curve is to obtain a wind speed record at the exact location of each wind turbine and use the manufacturer's individual wind power curve to estimate the power curve for the whole wind farm. Usually, however, limited wind speed records are available, which are meant to cover large areas during the wind resource surveying stages.

In [15], a multi-turbine power curve approach has been developed based on the statistics that characterize geographically dispersed winds within a specific region. In [16] the details of a realistic multiple-turbine power curve have been estimated and the results have been compared to the findings of [15].

As an example, Fig. A- 1 shows a normalized power output curve for an individual Gamesa G80-2.0MW, [14] (80m in diameter with a swept area of 5027m^2). This normalized curve is compared to the hypothetical wind power output curve of a large number of wind turbines evenly scattered over a 300km × 300km region as detailed in [16]. The power output curve of any wind farm could be considered to lie between the limits given by the single-turbine power curve and the wide-area power curve. For instance, Fig. A- 1 shows the hypothetical wind power output curve for a wind farm composed by 200 G80-2.0MW evenly spaced Gamesa turbines, which, according to typical wind farm layouts (with wind turbines separated by a distance equal to five times its diameter), would cover an area of 19.8 km^2.

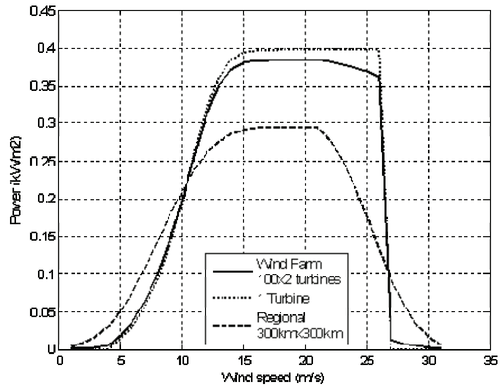

Fig. A- 1: Normalized power curves for a single wind turbine, a regional 300kmx300km wind farm and a wind farm of 100x2 Gamesa G80-2.0 MW turbines

From Fig. A- 1 it can be seen how a multi-turbine normalized power curve is 'smoother' than the single-turbine curve. The reason is the diversity in the winds since, as the area covered by the wind farm increases, it is less likely that all wind turbines will 'see' the same wind speeds simultaneously. The results are two-fold: a) it is unlikely that the wind farm will produce no output or, at the same time, b) it is unlikely that it will produce the maximum possible output given by the sum of the individual ratings of all turbines.

Although this model is approximate, it can be the only alternative to the simulation of the output of a wind farm when limited wind speed records for the area covered by the wind farm are available.

Appendix 2: Models for Demand and Electricity Prices

This section characterizes the inherent relationship between the electricity prices and the system demand.

It is a widely recognized fact that the total system demand displays a marked hourly and seasonal pattern, mainly determined by the ambient temperatures and work-business patterns. In fact, Fig. A- 2 shows the hourly average system demand for each hour and season for the Ontario electricity market for 2007.

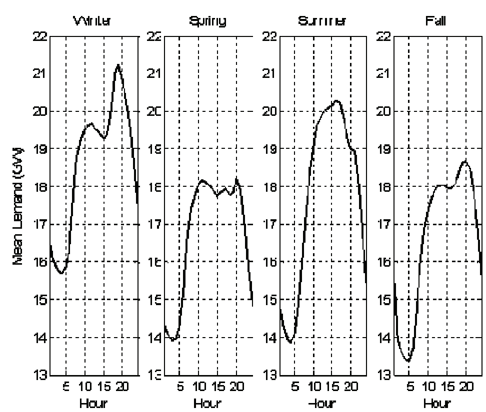

Fig. A- 2: Hourly average demand for Ontario, 2007

In Fig. A- 2 it can be confirmed how higher average demands are seen during the winter (heating loads) and summer (cooling loads) seasons. Regarding the hourly timeframe it can be observed how the peak demand hours are usually seen between 10:00 a.m. and 8:00 p.m. for all seasons, with the overnight hours showing the lowest demand levels.

The patterns are generally seen in all jurisdictions with the difference that some of them are 'summer-peaking' as opposed to 'winter-peaking'.

Fig. A- 3 shows the standard deviation of the system demand for each hour and season. It can be seen how the higher standard deviations are seen during the summer. This can be explained by the more variable temperature cycles and work-business profiles in this season when compared to the winter.

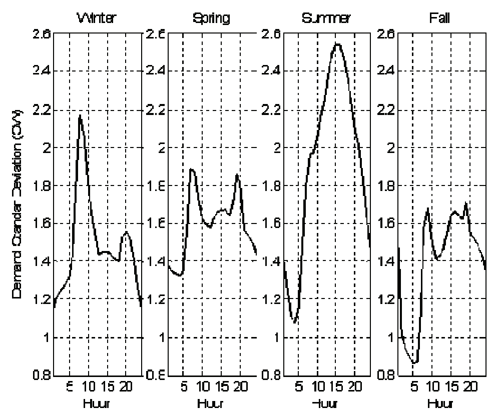

Fig. A- 3: Hourly standard deviation for the Ontario demand, 2007

In order to have an idea of the typical variability of the electricity market prices, a database for the Ontario electricity market in 2007 has been used to compute the hourly and seasonal expected values, which are shown in Fig. A- 4.

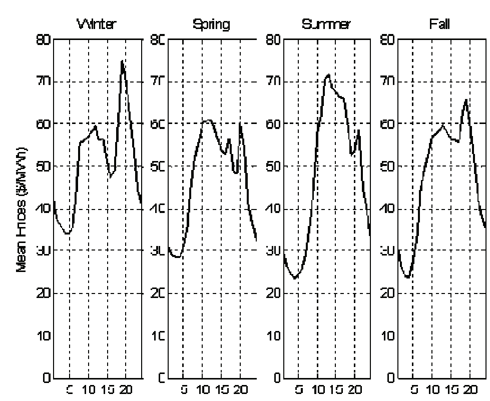

Fig. A- 4: Hourly average price for Ontario, 2007

Fig. A- 5 shows the hourly and seasonal standard deviations for the price dataset, showing again that larger price spreads are seen during the summer, as it was the case with the demands.

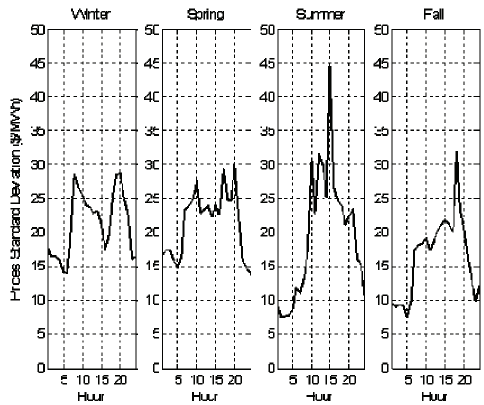

Fig. A- 5: Hourly standard deviation for the prices in Ontario, 2007

The similarities between the hourly-seasonal pattern of the demand and the electricity prices (shown in Fig. A- 2 and Fig. A- 4) indicate that there is a strong correlation between them. In fact, the correlation coefficients (Section 4.2, (51)) between the hourly electricity price and hourly demands for the actual dataset has been estimated for each season, and are shown in Table A- 1:

Table A- 1: Correlation coefficients for the seasonal prices w.r.t the seasonal demand

Season	*Winter*	*Spring*	*Summer*	*Fall*
Correlation coefficient	0.733	0.746	0.768	0.723

From Table A- 1, it can be seen how the correlation remains fairly constant throughout the year, with a small increase during the summer.

As a result of these correlation levels it can be assumed that, for each season, the distributions for the market electricity prices are conditioned to the corresponding demand's distributions. As explained in [21], for two jointly distributed random variables (X,Y), a regression function, $r(x)$, and a conditional variance function, $v(x)$, can be defined as follow:

$$r(x) = \mathrm{E}[Y \mid X = x] \tag{104}$$

$$v(x) = \mathrm{var}[Y \mid X = x] \tag{105}$$

In the case of study, each seasonal demand has been considered to be an independent variable, X, while the corresponding seasonal electricity market prices have been considered to be the conditioned variable, Y. Several functions have been evaluated in order to find the best fitting functions for each season: exponential and linear functions in the case of $r(x)$; linear and quadratic functions in the case of $v(x)$.

Fig. A- 6 shows a scatter of the electricity prices versus the corresponding demand for the spring season, for the years 2002 to 2006, as well as the corresponding regression function. By computing the residual electricity market prices around each type of regression, it has been found that the linear regression fits better than the exponential regression.

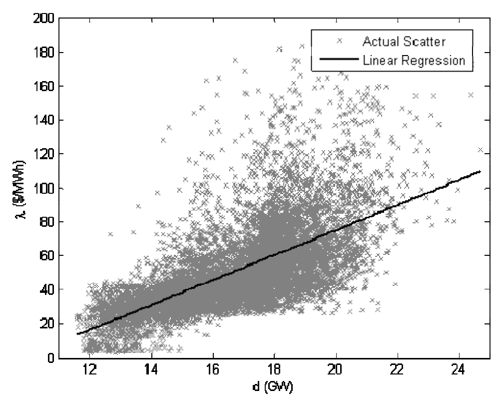

Fig. A- 6: Scatter of the electricity prices versus the demand, spring seasons, years 2003 to 2007

Similar results have been found for the other seasons, which leads to the following definition of seasonal regression function, r(d_h):

$$r(d_h) = \mathrm{E}[\lambda \mid d = d_h] = \alpha_s + \beta_s d_h \tag{106}$$

where the parameters α_s and β_s have to be estimated for each season, and the variables d_h and λ_h correspond to hourly samples of demand and electricity market prices within the corresponding season.

The parameters α_s and β_s have been estimated by simply applying a least squares regression to the actual pairs (d_h, λ_h).

In order to estimate the conditional variance function, $v(d_h)$, it was necessary to have a distribution of electricity-price variances versus demand. To do that, each seasonal scattered plot (d_h, λ_h) has been split into demand bins by applying the optimal bin width equation (4). The electricity-price variances have been computed for each bin, leading to a distribution of price-variances versus demands, where the demand array entries are the bins' middle points. For this scatter, the least square method has been applied in order to compute the parameters σ_s, ρ_s and γ_s of the conditional variance function.

Fig. A- 7 shows a scatter of the electricity prices-variance versus the corresponding demand array for the Spring season for the years 2003 to 2007, as well as the corresponding conditional variance function. From the analyses it has been found that, in this case, the quadratic regression fits better than the linear regression for all the seasons.

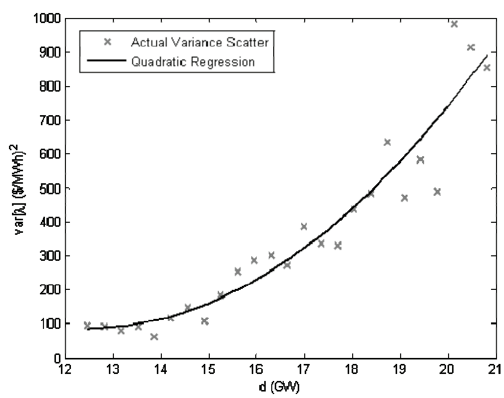

Fig. A- 7: Scatter of the electricity prices' variance versus de demand. Spring season, years 2003 to 2007

From these results, the seasonal conditional variance function, $v(d_h)$, can be defined as:

$$v(d_h) = \mathrm{var}\left[\lambda \mid d = d_h\right] = \sigma_s + \rho_s d_h + \gamma_s d_h^2 \tag{107}$$

where the parameters σ_s, ρ_s and γ_s have to be estimated for each season.

Table A- 2 shows the value of the parameters for each season corresponding to the Ontario market data set (demand and electricity prices) 2007:

Table A- 2: Seasonal parameters for the electricity market prices-demand relationship expression

	α_s ($/MWh)	β_s ($/(MW^2h))	σ_s ($/MWh)^2	ρ_s ($^2/(MW^3h^2))	γ_s ($^2/(MW^4h^2))
Winter	-92.15	0.0079	-1,035	0.065	7.84 e-7
Spring	-71.88	0.0074	1,877	-0.288	1.15 e-5
Summer	-76.79	0.0073	-880	0.075	-4.12 e-7
Fall	-72.04	0.0074	2,666	-0.414	1.65 e-5

Appendix 3: Calculation of Sensitivities

This section develops the mathematical expressions for the sensitivities of the expected value and variance of the annual revenue for both revenue options with respect to different market and resource-related parameters.

A-3.1 Sensitivities of the Hourly Revenues

A-3.1.1 Fixed-Rate Revenue

In Section 3.1.2 the following expressions for the expected value and the variance of the hourly fixed-rate revenue were found:

$$\mathrm{E}\left[R_{F\_h}\right] = \pi \cdot \mathrm{E}\left[g_{w\_h}\right] \tag{108}$$

or

$$\mathrm{E}\left[R_{F\_h}\right] = \pi \cdot \eta_h \cdot g_w^{\max} \tag{109}$$

and

$$\mathrm{var}\left[R_{F\_h}\right] = \pi^2 \cdot \mathrm{var}\left[g_{w\_h}\right] \tag{110}$$

Note that expressions (108) to (110) are the same as expressions (9) to (11)

It can be seen how the expected value of the hourly revenue only depends on the expected value of the hourly generation output of the wind farm, or on the hourly capacity factor for a given wind farm (a given g_w^{max}).

The different normalized sensitivities of the hourly revenues' expected values can be expressed as:

$$S_{\mathrm{E}[g_{w\_h}]}^{\mathrm{E}[R_{F\_h}]} = \frac{\partial\left(\mathrm{E}\left[R_{F\_h}\right]\right)}{\partial\left(\mathrm{E}\left[g_{w\_h}\right]\right)} \cdot \frac{\mathrm{E}\left[g_{w\_h}\right]}{\mathrm{E}\left[R_{F\_h}\right]} = \pi \cdot \frac{\mathrm{E}\left[g_{w\_h}\right]}{\mathrm{E}\left[R_{F\_h}\right]} \tag{111}$$

$$S_{\eta_h}^{\mathrm{E}[R_{F\_h}]} = \frac{\partial\left(\mathrm{E}\left[R_{F\_h}\right]\right)}{\partial\left(\eta_h\right)} \cdot \frac{\eta_h}{\mathrm{E}\left[R_{F\_h}\right]} = \pi \cdot g_w^{\max} \cdot \frac{\eta_h}{\mathrm{E}\left[R_{F\_h}\right]} \tag{112}$$

for given initial values $\mathrm{E}[R_{F\_h}]$, $\mathrm{E}[g_{w\_h}]$, and η_h. Notice that, since:

$$\mathrm{E}\left[g_{w\_h}\right] = \eta_h \cdot g_w^{\max} \tag{113}$$

the sensitivities (111) and (112) will give the same result. It also happens if we compute the sensitivity with respect to g_w^{max}, $S_{g_w^{\max}}^{\mathrm{E}[R_{F\_h}]}$.

On the other hand, in (110) can be seen how the variance of the hourly fixed-rate revenue only depends on the variance of the hourly wind farm output energy. Therefore, the following sensitivity can be computed:

$$S_{\mathrm{var}[g_{w\_h}]}^{\mathrm{var}[R_{F\_h}]} = \frac{\partial\left(\mathrm{var}\left[R_{F\_h}\right]\right)}{\partial\left(\mathrm{var}\left[g_{w\_h}\right]\right)} \cdot \frac{\mathrm{var}\left[g_{w\_h}\right]}{\mathrm{var}\left[R_{F\_h}\right]} = \pi^2 \cdot \frac{\mathrm{var}\left[g_{w\_h}\right]}{\mathrm{var}\left[R_{F\_h}\right]} \tag{114}$$

A-3.1.2 Market Revenue

For the market revenue there were two options:

1. The hourly expected value and variance could be expressed as a function of the hourly price λ_h, and the hourly output energy g_{w_h} through the equations:

$$\mathrm{E}\left[R_{M\_h}\right]=\eta_h\cdot g_w^{\max}\cdot\mathrm{E}\left[\lambda_h\right] \tag{115}$$

$$\mathrm{var}\left[R_{M\_h}\right]=\mathrm{var}\left[\lambda_h\right]\cdot\left(\eta_h{}^2\cdot\left(g_w^{\max}\right)^2+\mathrm{var}\left[g_{w\_h}\right]\right)+\left(\mathrm{E}\left[\lambda_h\right]\right)^2\cdot\mathrm{var}\left[g_{w\_h}\right] \tag{116}$$

Note that expressions (115) and (116) are the same as expressions (15) and (31).

2. Or they could be expressed as a function of the hourly demand d_h, through the equations:

$$\mathrm{E}\left[R_{M\_h}\right]=\eta_h\cdot g_w^{\max}\cdot\left(\alpha_s+\beta_s\cdot\mathrm{E}\left[d_h\right]\right) \tag{117}$$

$$\begin{aligned}\mathrm{var}\left[R_{M\_h}\right]&=\eta_h{}^2\cdot\left(g_w^{\max}\right)^2\cdot\left(\sigma_s+\rho_s\cdot\mathrm{E}\left[d_h\right]+\gamma_s\cdot\left(\mathrm{E}\left[d_h\right]\right)^2+\left(\gamma_s+\beta_s{}^2\right)\cdot\mathrm{var}\left[d_h\right]\right)\\&+\mathrm{var}\left[g_{w\_h}\right]\cdot\left(\sigma_s+\alpha_s{}^2+\left(\rho_s+2\alpha_s\beta_s\right)\cdot\mathrm{E}\left[d_h\right]+\left(\gamma_s+\beta_s{}^2\right)\cdot\left(\mathrm{E}\left[d_h\right]\right)^2+\left(\gamma_s+\beta_s{}^2\right)\cdot\mathrm{var}\left[d_h\right]\right)\end{aligned} \tag{118}$$

Note that expressions (117) and (118) are the same as expressions (20) and (40)

In the first case, the following sensitivities can be computed:

$$S_{\mathrm{E}\left[\lambda_h\right]}^{\mathrm{E}\left[R_{M\_h}\right]}=\frac{\partial\left(\mathrm{E}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{E}\left[\lambda_h\right]\right)}\cdot\frac{\mathrm{E}\left[\lambda_h\right]}{\mathrm{E}\left[R_{M\_h}\right]}=\pi\cdot g_w^{\max}\cdot\frac{\mathrm{E}\left[\lambda_h\right]}{\mathrm{E}\left[R_{M\_h}\right]} \tag{119}$$

(In this case, the normalized sensitivity with respect to the hourly capacity factor, η_h, or with respect to the installed power $g_w{}^{max}$ gives again the same result)

$$S_{\mathrm{E}\left[g_{w\_h}\right]}^{\mathrm{var}\left[R_{M\_h}\right]}=\frac{\partial\left(\mathrm{var}\left[R_{M\_h}\right]\right)}{\partial\left(\eta_h\right)}\cdot\frac{\eta_h}{\mathrm{var}\left[R_{M\_h}\right]}=\frac{2\cdot\eta_h^2\cdot\left(g_w^{\max}\right)^2\cdot\mathrm{var}\left[\lambda_h\right]}{\mathrm{var}\left[R_{M\_h}\right]} \tag{120}$$

$$S_{\eta_h}^{\mathrm{var}\left[R_{M\_h}\right]}=S_{g_w^{\max}}^{\mathrm{var}\left[R_{M\_h}\right]}=S_{\mathrm{E}\left[g_{w\_h}\right]}^{\mathrm{var}\left[R_{M\_h}\right]} \tag{121}$$

$$S_{\mathrm{var}\left[g_{w\_h}\right]}^{\mathrm{var}\left[R_{M\_h}\right]}=\frac{\partial\left(\mathrm{var}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{var}\left[g_{w\_h}\right]\right)}\cdot\frac{\mathrm{var}\left[g_{w\_h}\right]}{\mathrm{var}\left[R_{M\_h}\right]}=\left(\mathrm{var}\left[\lambda_h\right]+\left(\mathrm{E}\left[\lambda_h\right]\right)^2\right)\cdot\frac{\mathrm{var}\left[g_{w\_h}\right]}{\mathrm{var}\left[R_{M\_h}\right]} \tag{122}$$

$$S_{\mathrm{E}[\lambda_h]}^{\mathrm{var}\left[R_{M\_h}\right]} = \frac{\partial\left(\mathrm{var}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{E}\left[\lambda_h\right]\right)} \cdot \frac{\mathrm{E}\left[\lambda_h\right]}{\mathrm{var}\left[R_{M\_h}\right]} = \frac{2 \cdot \mathrm{E}\left[\lambda_h\right]^2 \cdot \mathrm{var}\left[g_{w\_h}\right]}{\mathrm{var}\left[R_{M\_h}\right]} \tag{123}$$

$$S_{\mathrm{var}[\lambda_h]}^{\mathrm{var}\left[R_{M\_h}\right]} = \frac{\partial\left(\mathrm{var}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{var}\left[\lambda_h\right]\right)} \cdot \frac{\mathrm{var}\left[\lambda_h\right]}{\mathrm{var}\left[R_{M\_h}\right]} = \left(\eta_h^{\,2} \cdot \left(g_w^{\max}\right)^2 + \mathrm{var}\left[g_{w\_h}\right]\right) \cdot \frac{\mathrm{var}\left[\lambda_h\right]}{\mathrm{var}\left[R_{M\_h}\right]} \tag{124}$$

All these sensitivities can be written in terms of the d_h instead of λ_h by substituting the expressions of E[λ_h] and var[λ_h] given in (19) and (32), as a function of d_h.

The different sensitivities with respect to E[d_h] and var[d_h] are:

$$S_{\mathrm{E}[d_h]}^{\mathrm{E}\left[R_{M\_h}\right]} = \frac{\partial\left(\mathrm{E}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{E}\left[d_h\right]\right)} \cdot \frac{\mathrm{E}\left[d_h\right]}{\mathrm{E}\left[R_{M\_h}\right]} \tag{125}$$

$$S_{\mathrm{var}[d_h]}^{\mathrm{E}\left[R_{M\_h}\right]} = \frac{\partial\left(\mathrm{E}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{var}\left[d_h\right]\right)} \cdot \frac{\mathrm{var}\left[d_h\right]}{\mathrm{E}\left[R_{M\_h}\right]} \tag{126}$$

$$S_{\mathrm{E}[d_h]}^{\mathrm{var}\left[R_{M\_h}\right]} = \frac{\partial\left(\mathrm{var}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{E}\left[d_h\right]\right)} \cdot \frac{\mathrm{E}\left[d_h\right]}{\mathrm{var}\left[R_{M\_h}\right]} \tag{127}$$

$$S_{\mathrm{var}[d_h]}^{\mathrm{var}\left[R_{M\_h}\right]} = \frac{\partial\left(\mathrm{var}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{var}\left[d_h\right]\right)} \cdot \frac{\mathrm{var}\left[d_h\right]}{\mathrm{var}\left[R_{M\_h}\right]} \tag{128}$$

The derivative terms can be computed in two different ways. The first way, by directly deriving from equations (117) and (118). The second way, by applying the chain rule as follows:

$$\frac{\partial\left(\mathrm{E}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{E}\left[d_h\right]\right)} = \frac{\partial\left(\mathrm{E}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{E}\left[\lambda_h\right]\right)} \cdot \frac{\partial\left(\mathrm{E}\left[\lambda_h\right]\right)}{\partial\left(\mathrm{E}\left[d_h\right]\right)} = S_{\mathrm{E}[\lambda_h]}^{\mathrm{E}\left[R_{M\_h}\right]} \cdot \frac{\mathrm{E}\left[R_{M\_h}\right]}{\mathrm{E}\left[\lambda_h\right]} \cdot \frac{\partial\left(\mathrm{E}\left[\lambda_h\right]\right)}{\partial\left(\mathrm{E}\left[d_h\right]\right)} \tag{129}$$

$$\frac{\partial\left(\mathrm{E}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{var}\left[d_h\right]\right)} = \frac{\partial\left(\mathrm{E}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{var}\left[\lambda_h\right]\right)} \cdot \frac{\partial\left(\mathrm{var}\left[\lambda_h\right]\right)}{\partial\left(\mathrm{var}\left[d_h\right]\right)} = S_{\mathrm{var}[\lambda_h]}^{\mathrm{E}\left[R_{M\_h}\right]} \cdot \frac{\mathrm{E}\left[R_{M\_h}\right]}{\mathrm{var}\left[\lambda_h\right]} \cdot \frac{\partial\left(\mathrm{var}\left[\lambda_h\right]\right)}{\partial\left(\mathrm{var}\left[d_h\right]\right)} = 0 \tag{130}$$

$$\frac{\partial\left(\mathrm{var}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{E}\left[d_h\right]\right)}=\frac{\partial\left(\mathrm{var}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{E}\left[\lambda_h\right]\right)}\cdot\frac{\partial\left(\mathrm{E}\left[\lambda_h\right]\right)}{\partial\left(\mathrm{E}\left[d_h\right]\right)}=S_{\mathrm{E}[\lambda_h]}^{\mathrm{var}\left[R_{M\_h}\right]}\cdot\frac{\mathrm{var}\left[R_{M\_h}\right]}{\mathrm{E}\left[\lambda_h\right]}\cdot\frac{\partial\left(\mathrm{E}\left[\lambda_h\right]\right)}{\partial\left(\mathrm{E}\left[d_h\right]\right)} \tag{131}$$

$$\frac{\partial\left(\mathrm{var}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{var}\left[d_h\right]\right)}=\frac{\partial\left(\mathrm{var}\left[R_{M\_h}\right]\right)}{\partial\left(\mathrm{var}\left[\lambda_h\right]\right)}\cdot\frac{\partial\left(\mathrm{var}\left[\lambda_h\right]\right)}{\partial\left(\mathrm{var}\left[d_h\right]\right)}=S_{\mathrm{var}[\lambda_h]}^{\mathrm{var}\left[R_{M\_h}\right]}\cdot\frac{\mathrm{var}\left[R_{M\_h}\right]}{\mathrm{var}\left[\lambda_h\right]}\cdot\frac{\partial\left(\mathrm{var}\left[\lambda_h\right]\right)}{\partial\left(\mathrm{var}\left[d_h\right]\right)} \tag{132}$$

Note that equation (130) equals zero since E[R_{M_h}] does not depends on var[λ_h]. In (129) to (132), the sensitivity terms have already been defined, and the other derivative terms can be computed from equations (19) and (32) as follows:

$$\frac{\partial\left(\mathrm{E}\left[\lambda_h\right]\right)}{\partial\left(\mathrm{E}\left[d_h\right]\right)}=\beta_s \tag{133}$$

$$\frac{\partial\left(\mathrm{var}\left[\lambda_h\right]\right)}{\partial\left(\mathrm{var}\left[d_h\right]\right)}=\gamma_s+\beta_s^{\,2} \tag{134}$$

Therefore, equations (125) to (128) can be written as:

$$S_{\mathrm{E}[d_h]}^{\mathrm{E}\left[R_{M\_h}\right]}=S_{\mathrm{E}[\lambda_h]}^{\mathrm{E}\left[R_{M\_h}\right]}\cdot\beta_s\cdot\frac{\mathrm{E}\left[d_h\right]}{\mathrm{E}\left[\lambda_h\right]} \tag{135}$$

$$S_{\mathrm{var}[d_h]}^{\mathrm{E}\left[R_{M\_h}\right]}=0 \tag{136}$$

$$S_{\mathrm{E}[d_h]}^{\mathrm{var}\left[R_{M\_h}\right]}=S_{\mathrm{E}[\lambda_h]}^{\mathrm{var}\left[R_{M\_h}\right]}\cdot\beta_s\cdot\frac{\mathrm{E}\left[d_h\right]}{\mathrm{E}\left[\lambda_h\right]} \tag{137}$$

$$S_{\mathrm{var}[d_h]}^{\mathrm{var}\left[R_{M\_h}\right]}=S_{\mathrm{var}[\lambda_h]}^{\mathrm{var}\left[R_{M\_h}\right]}\cdot\left(\gamma_s+\beta_s^{\,2}\right)\cdot\frac{\mathrm{var}\left[d_h\right]}{\mathrm{var}\left[\lambda_h\right]} \tag{138}$$

A-3.2 Sensitivities of the Annual Revenue

As explained in Chapter 4, the annual revenue can be computed as the sum of the 8,760 hourly revenues for the specific year, and its expected value and variance can be computed, in general, as:

$$\mathrm{E}\left[R_A\right]=\sum_{h=1}^{8760}\mathrm{E}\left[R_h\right] \tag{139}$$

$$\operatorname{var}\left[R_A\right]=\sum_{i=1}^{8760}\sum_{j=1}^{8760}\rho_{i,j}\cdot\operatorname{std}\left[R_i\right]\cdot\operatorname{std}\left[R_j\right] \tag{140}$$

where, again, R_A and R_h represent here the annual and the hourly revenue for any of the two type of revenues explained in this work.

Note that expressions (139) and (140) are the same as expressions (42) and (48).

A-3.2.1 Expected Value

The dimensionless sensitivity of the annual revenue's expected value with respect to a change in the hourly parameter p_h at hour k, p_k, can be computed as:

$$S_{p_k}^{\mathrm{E}[R_A]}=\frac{\partial\mathrm{E}\left[R_A\right]}{\partial p_k}\cdot\frac{p_k}{\mathrm{E}\left[R_A\right]}=\frac{\partial\left(\sum_{h=1}^{8760}\mathrm{E}\left[R_h\right]\right)}{\partial p_k}\cdot\frac{p_k}{\mathrm{E}\left[R_A\right]} \tag{141}$$

where the parameter p_k represents any of the hourly parameters with respect to which the sensitivities of E[R_h] and var[R_h] where defined, for the specific hour $h = k$: E[g_{w_k}] (or η_k or g_k^{max}, which gave the same results), var[g_{w_k}], E[λ_k], var[λ_k], E[d_k] and var[d_k].

Since the derivative of the sum is equal to the sum of the derivatives, we have:

$$S_{p_k}^{E[R_A]}=\sum_{h=1}^{8760}\frac{\partial\mathrm{E}\left[R_h\right]}{\partial p_k}\cdot\frac{p_k}{\mathrm{E}\left[R_A\right]} \tag{142}$$

However, only the hourly revenue at hour k is affected by changes in parameter p_k. Therefore, the derivatives of hourly revenues' expected values corresponding to hours $h \neq k$ will be zero. Then, only the term k of the sum will be kept:

$$S_{p_k}^{E[R_A]}=\frac{\partial\mathrm{E}\left[R_k\right]}{\partial p_k}\cdot\frac{p_k}{\mathrm{E}\left[R_A\right]} \tag{143}$$

which can be expressed as a function of the sensitivities of the hourly revenues' expected values as:

$$S_{p_k}^{\mathrm{E}[R_A]} = S_{p_k,d}^{\mathrm{E}[R_k]} \cdot \frac{p_k}{\mathrm{E}\left[R_A\right]} = S_{p_k}^{\mathrm{E}[R_k]} \cdot \frac{\mathrm{E}\left[R_k\right]}{\mathrm{E}\left[R_A\right]} \tag{144}$$

Recall that subscript d refers to the absolute sensitivity (with the original dimensions) as explained by (97) in Chapter 6.

Regarding the annual sensitivities, it is assumed that a given parameter changes by the same amount during all hours of the year, as explained in.

If a change in a given parameter p takes place for all hours during the year, the corresponding change in the annual revenue (the first two moments) is given by the sum of the hourly changes. This can be expressed as:

$$S_{p_A}^{\mathrm{E}[R_A]} = \sum_{h=1}^{8760} S_{p_h}^{\mathrm{E}[R_A]} = \sum_{h=1}^{8760} S_{p_h,d}^{\mathrm{E}[R_h]} \cdot \frac{p_h}{\mathrm{E}\left[R_A\right]} \tag{145}$$

where p_A represents a simultaneous change in the parameter p at all the hours.

Equation (145) can be also written as:

$$S_{p_A}^{\mathrm{E}[R_A]} = \frac{1}{\mathrm{E}\left[R_A\right]} \cdot \sum_{h=1}^{8760} S_{p_h,d}^{\mathrm{E}[R_h]} \cdot p_h \tag{146}$$

Notice that in (146), each term h in the summation is the change, in \$, produced in the hourly revenue R_h by the unitary change in the parameter p_h.

Equation (146) can also be expressed in terms of the dimensionless hourly sensitivities:

$$S_{p_A}^{\mathrm{E}[R_A]} = \frac{1}{\mathrm{E}\left[R_A\right]} \cdot \sum_{h=1}^{8760} S_{p_h}^{\mathrm{E}[R_h]} \cdot \mathrm{E}\left[R_h\right] \tag{147}$$

A-3.2.2 Variance

In the same way, the sensitivity of the variance of the annual revenue with respect of a change in the parameter p_h at hour $h = k$, p_k, can be computed as:

$$S_{p_k}^{\mathrm{var}[R_A]} = \frac{\partial\,\mathrm{var}\left[R_A\right]}{\partial p_k} \cdot \frac{p_k}{\mathrm{var}\left[R_A\right]} = \left(\sum_{i=1}^{8760} \sum_{j=1}^{8760} \rho_{i,j} \frac{\partial\left(\mathrm{std}\left[R_i\right] \cdot \mathrm{std}\left[R_j\right]\right)}{\partial p_k} \right) \cdot \frac{p_k}{\mathrm{var}\left[R_A\right]} \tag{148}$$

In (148), the correlation coefficient $\rho_{i,j}$ is out of the derivative since it has a constant value for a given (i, j). The derivative of the product of the standard deviations, σ_i and σ_j, can be developed, leading to the equation:

$$\begin{aligned} S_{p_k}^{\mathrm{var}[R_A]} &= \frac{\partial\,\mathrm{var}\left[R_A\right]}{\partial p_k} \cdot \frac{p_k}{\mathrm{var}\left[R_A\right]} = \\ &\left(\sum_{i=1}^{8760} \sum_{j=1}^{8760} \rho_{i,j} \left[\frac{\partial \mathrm{std}\left[R_i\right]}{\partial p_k} \cdot \mathrm{std}\left[R_j\right] + \frac{\partial \mathrm{std}\left[R_j\right]}{\partial p_k} \cdot \mathrm{std}\left[R_i\right] \right] \right) \cdot \frac{p_k}{\mathrm{var}\left[R_A\right]} \end{aligned} \tag{149}$$

which is the same as:

$$S_{p_k}^{\mathrm{var}[R_A]} = \frac{p_k}{\mathrm{var}\left[R_A\right]} \cdot \left(\sum_{i=1}^{8760} \sum_{j=1}^{8760} \rho_{i,j} \frac{\partial \mathrm{std}\left[R_i\right]}{\partial p_k} \cdot \mathrm{std}\left[R_j\right] + \sum_{i=1}^{8760} \sum_{j=1}^{8760} \rho_{i,j} \frac{\partial \mathrm{std}\left[R_j\right]}{\partial p_k} \cdot \mathrm{std}\left[R_i\right] \right) \tag{150}$$

In (150), the derivative terms are different from zero only when $i=k$ and $j=k$ respectively. Therefore, we can write:

$$S_{p_k}^{\mathrm{var}[R_A]} = \frac{p_k}{\mathrm{var}\left[R_A\right]} \cdot \left(\sum_{j=1}^{8760} \rho_{k,j} \frac{\partial \mathrm{std}\left[R_k\right]}{\partial p_k} \cdot \mathrm{std}\left[R_j\right] + \sum_{i=1}^{8760} \rho_{i,k} \frac{\partial \mathrm{std}\left[R_k\right]}{\partial p_k} \cdot \mathrm{std}\left[R_i\right] \right) \tag{151}$$

which is the same as:

$$S_{p_k}^{\mathrm{var}[R_A]} = \frac{p_k}{\mathrm{var}\left[R_A\right]} \cdot \frac{\partial \mathrm{std}\left[R_k\right]}{\partial p_k} \cdot 2 \cdot \sum_{h=1}^{8760} \rho_{k,h} \cdot \mathrm{std}\left[R_h\right] \tag{152}$$

Since the standard deviation is the square root of the variance, equation (152) can be also written as:

$$S_{p_k}^{\mathrm{var}[R_A]} = \frac{p_k}{\mathrm{var}\left[R_A\right]} \cdot \frac{\partial \sqrt{\mathrm{var}\left[R_k\right]}}{\partial p_k} \cdot 2 \cdot \sum_{h=1}^{8760} \rho_{k,h} \cdot \sqrt{\mathrm{var}\left[R_h\right]} \tag{153}$$

And from the derivative of a square root, we have:

$$S_{p_k}^{\mathrm{var}[R_A]} = \frac{p_k}{\mathrm{var}\left[R_A\right]} \cdot \frac{1}{2 \cdot \sqrt{\mathrm{var}\left[R_k\right]}} \frac{\partial\, \mathrm{var}\left[R_k\right]}{\partial p_k} \cdot 2 \cdot \sum_{h=1}^{8760} \rho_{k,h} \cdot \sqrt{\mathrm{var}\left[R_h\right]} \tag{154}$$

or, in other words,

$$S_{p_k}^{\mathrm{var}[R_A]} = \frac{p_k}{\mathrm{var}\left[R_A\right]} \cdot \frac{1}{\sqrt{\mathrm{var}\left[R_k\right]}} \cdot S_{p_k,d}^{\mathrm{var}[R_k]} \cdot \sum_{h=1}^{8760} \rho_{k,h} \cdot \sqrt{\mathrm{var}\left[R_h\right]} \tag{155}$$

Equation (155) is a function of the dimensional sensitivity of var[R_h] with respect to p_k, and all the other variables are already defined. This equation can be also written in terms of the dimensionless sensitivity of var[R_h] as:

$$S_{p_k}^{\mathrm{var}[R_A]} = \frac{1}{\sqrt{\mathrm{var}\left[R_k\right]}} \cdot S_{p_k}^{\mathrm{var}[R_k]} \cdot \frac{\mathrm{var}\left[R_k\right]}{\mathrm{var}\left[R_A\right]} \cdot \sum_{h=1}^{8760} \rho_{k,h} \cdot \sqrt{\mathrm{var}\left[R_h\right]} \tag{156}$$

All the dimensionless sensitivities where defined in Chapter 6.

At this point, the sensitivity of the hourly variance with respect to a variation in an hourly parameter p_h has been defined. However, as explained in Section 6.2, a simultaneous change in all the hourly parameters is going to be considered, so that the effect of all the hourly changes will be added. Therefore, the sensitivity of the annual revenue's variance with respect to a simultaneous change at all the hours in the parameter p is given by:

$$S_{p_A}^{\mathrm{var}[R_A]} = \sum_{h=1}^{8760} S_{p_h}^{\mathrm{var}[R_A]} \tag{157}$$

A-3.3 Sensitivities of the VaR Applied to the Annual Revenue

As seen before, the Value at Risk applied to the annual revenue, for a given probability P, depends on the expected value and on the standard deviation of the annual revenue ($\mathrm{E}[R_A]$ and $\mathrm{std}[R_A]$), with the equation:

$$\mathrm{VaR}(P) = \mathrm{std}[R_A] \cdot \sqrt{2} \cdot \mathrm{erf}^{-1}\left(2P\text{-}1\right) + \mathrm{E}[R_A] \tag{158}$$

Note that expression (158) is the same as expression (85).

Notice that the VaR is measured in \$, since it is being applied to the annual revenue. The sensitivity of the VaR with respect to a change in any parameter p at hour k, p_k, is given by:

$$S_{p_k}^{\mathrm{VaR}(P)} = \frac{\partial\left(\mathrm{VaR}(P)\right)}{\partial p_k} \cdot \frac{p_k}{\mathrm{VaR}(P)} = \left(\sqrt{2} \cdot \mathrm{erf}^{-1}\left(2P\text{-}1\right)\frac{\partial \mathrm{std}[R_A]}{\partial p_k} + \frac{\partial \mathrm{E}[R_A]}{\partial p_k}\right) \cdot \frac{p_k}{\mathrm{VaR}(P)} \tag{159}$$

which can be written, again, in terms of the annual variance, $\mathrm{var}[R_A]$, instead of the annual standard deviation, as:

$$S_{p_k}^{\mathrm{VaR}(P)} = \left(\sqrt{2} \cdot \mathrm{erf}^{-1}\left(2P\text{-}1\right) \cdot \frac{\partial\left(\sqrt{\mathrm{var}\left[R_A\right]}\right)}{\partial p_k} + \frac{\partial\left(\mathrm{E}\left[R_A\right]\right)}{\partial p_k}\right) \cdot \frac{p_k}{\mathrm{VaR}(P)} \tag{160}$$

Developing the derivative of the integral of $\mathrm{var}[R_A]$:

$$S_{p_k}^{\mathrm{VaR}(P)} = \left(\frac{\mathrm{erf}^{-1}\left(2P\text{-}1\right)}{\sqrt{2 \cdot \mathrm{var}\left[R_A\right]}} \cdot \frac{\partial\left(\mathrm{var}\left[R_A\right]\right)}{\partial p_k} + \frac{\partial\left(\mathrm{E}\left[R_A\right]\right)}{\partial p_k}\right) \cdot \frac{p_k}{\mathrm{VaR}(P)} \tag{161}$$

Which, in terms of the sensitivities of $\mathrm{var}[R_A]$ and $\mathrm{E}[R_A]$ can be written as:

$$S_{p_k}^{\mathrm{VaR}(P)} = \frac{1}{\mathrm{VaR}(P)}\left(\frac{\mathrm{erf}^{-1}\left(2P\text{-}1\right)}{\sqrt{2 \cdot \mathrm{var}\left[\mathrm{R}_A\right]}} \cdot S_{p_k}^{\mathrm{var}[\mathrm{R}_A]} \cdot \mathrm{var}\left[\mathrm{R}_A\right] + S_{p_k}^{\mathrm{E}[\mathrm{R}_a]} \cdot \mathrm{E}\left[\mathrm{R}_A\right]\right) \tag{162}$$

If, again, all the parameters p_k change at the same time at all the hours, the total change in the VaR can be measured by:

$$S_{p_A}^{\mathrm{VaR}(P)} = \sum_{h=1}^{8760} S_{p_h}^{\mathrm{VaR}(P)} \tag{163}$$

which can be expressed as a function of the sensitivities of var[R_A] and E[R_A] with respect to the same change in all the parameters p_k as:

$$S_{p_A}^{\mathrm{VaR}(P)} = \frac{1}{\mathrm{VaR}(P)}\left(\frac{\mathrm{erf}^{-1}\left(2P\text{-}1\right)}{\sqrt{2}} \cdot S_{p_A}^{\mathrm{var}[\mathrm{R}_A]} \cdot \sqrt{\mathrm{var}\left[\mathrm{R}_A\right]} + S_{p_A}^{\mathrm{E}[\mathrm{R}_A]} \cdot \mathrm{E}\left[\mathrm{R}_A\right]\right) \tag{164}$$