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Circular Cylinders in Cross-Flow

by

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January 1999

**A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements of the degree of
Doctor of Philosophy in Mechanical Engineering.**

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0-612-50265-1

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Abstract

Small groups of circular cylinders of equal diameter, in tandem, side-by-side and staggered configurations, were investigated in steady mean cross-flow, and under conditions of an impulsive start, for Reynolds numbers from 500 to 6000. Experiments were conducted in water in three facilities, using flow visualization, hot-film anemometry and particle image velocimetry (PIV). The emphasis was on acquiring an improved physical understanding of the fluid behaviour, and the dynamics of the vortical structures in the flow, as the number of cylinders, the spacing between the cylinders, and the angle of incidence with respect to the oncoming flow, were varied.

For tandem cylinders in impulsively started flow, the development showed constrained streamwise growth and lateral expansion of the gap recirculation zones at small and intermediate pitch ratios. Under steady cross-flow conditions, reattachment of the free shear layers from the upstream cylinder, onto the surface of the downstream cylinder, was found to occur in an alternating, non-continuous, and non-simultaneous fashion, in a process synchronized with Kármán vortex shedding from the downstream cylinder. Within the gap between the cylinders, bounded on either side by the reattaching shear layers, was found weakly rotating or stagnant fluid, with vorticity concentrations only near the points of reattachment.

The temporal development of the side-by-side configuration was dominated by strong flow through the gaps between the cylinders, rapid breakup of the recirculation zones behind the outer cylinders, gap vortex shedding, and the formation of a counter-rotating vortex pair in the combined wake. Under steady mean cross-flow conditions, the PIV vorticity data showed instantaneous variation in the base-bleed flows, the gap flow deflection angles and the vortex formation lengths. In the three-cylinder side-by-side configuration, vortex shedding frequency measurements were sensitive to the measurement location.

For the two-cylinder staggered configuration in steady cross-flow, nine flow patterns were identified, and processes of vortex pairing, enveloping, impingement and induced separation were observed. Some new insight was gained into previously published force coefficient and Strouhal number data, by considering the flow patterns responsible. The study revealed that vortex shedding frequencies are more properly associated with the individual shear layers, rather than with the individual cylinders.

Sommaire

Des petits groupes de cylindres circulaires de diamètre identique, en configuration en ligne, en rangée et générale, ont été étudiés en écoulement permanent, ainsi qu'en écoulement avec impulsion initiale, pour des nombres de Reynolds compris entre 500 et 6000. Les expériences ont été menées dans l'eau, dans trois installations, en utilisant la visualisation de l'écoulement, l'anémométrie à température constante, et la vélocimétrie des images des particules (PIV). Le but de l'investigation a été l'amélioration de la compréhension physique de l'écoulement, ainsi que la dynamique des tourbillons, lorsque le nombre de cylindres, leurs espacements et l'angle d'incidence par rapport à l'écoulement varient.

Concernant les cylindres en ligne en écoulement avec impulsion, l'étude a montré un élargissement limité dans la direction de l'écoulement, et une expansion transversale des zones de recirculation, pour les petits et moyens espacements entre cylindres. En écoulement permanent, le rattachement des deux couches de cisaillement du cylindre en amont sur le cylindre en aval, se produit de manière alternée, discontinue, et non-simultanée, et en synchronisation avec la formation de tourbillons de type Kármán au voisinage du sillage du cylindre en aval. Dans les espacements entre les cylindres, limités de chaque côté par les deux couches de cisaillement rattachées, on trouve un fluide faiblement recirculant ou stagnant, avec accumulation de vorticit  uniquement   proximit  des point de rattachement.

Le développement en temps, en configuration en rangée, est dominé par un écoulement fort à travers les espacements entre cylindres adjacents, une rupture rapide des zones de recirculation au voisinage des sillages des cylindres externes, le détachement tourbillonnaire à côté des espacements entre cylindres, et la formation d'une paire de tourbillons contre-tournant dans le sillage combiné des cylindres. Sous un écoulement permanent, les données de la vorticité obtenues par PIV ont mis en évidence la variation instantanée de l'écoulement suintant pour les petits espacements, l'angle de déviation de l'écoulement dans l'espace entre les cylindres, et la longueur de la région de formation des tourbillons. Pour les configurations en rangée de trois cylindres, les fréquences de détachement des tourbillons ont montré une sensibilité à l'égard du positionnement des mesures.

Pour les deux cylindres en configuration générale (en quinconce) immergés dans un écoulement permanent, neuf comportements types d'écoulement ont été identifiés. Les processus d'accouplement, d'enveloppement, et d'empiètement des tourbillons et du détachement induit, ont été observés. Un nouvel éclairage a été apporté en ce qui concerne les coefficients de forces ainsi que les nombres de Strouhal publiés, en considérant les comportements types d'écoulement impliqué. L'étude a déterminé que les fréquences de détachement des tourbillons sont associées aux couches de cisaillement individuelles plutôt qu'aux cylindres individuels.

Statement of Contribution to Original Knowledge

The subject of this study was the fluid dynamics of groups of circular cylinders in steady and unsteady upstream cross-flow. To this author's knowledge, the contributions of the thesis to original knowledge include: (i) the first investigation of tandem and side-by-side cylinder configurations in impulsively started flow, in which the spatial and temporal development of the flow fields were described and quantified; (ii) the first in-depth investigation of three side-by-side circular cylinders in steady cross-flow, to combine visualization and hot-film measurements; and (iii) the first systematic investigation of two staggered cylinders in steady cross-flow, from a strictly fluid dynamics point of view, from which nine flow patterns were identified. The study is also one of the first known applications of particle image velocimetry (PIV) to impulsively started flow, and the second known application of PIV to multiple cylinder configurations. The contributions of the work are listed as follows:

1. The fluid dynamics of tandem, side-by-side and staggered configurations of circular cylinders of equal diameter were studied experimentally under conditions of steady cross-flow and impulsively started cross-flow, in the low subcritical Reynolds number regime for a single circular cylinder. A flow pattern identification approach was adopted for each configuration, and comparison was made to previously published results where available. The flow patterns were determined from flow visualization video images and from instantaneous, full-field velocity and vorticity data obtained by PIV. Good correspondence between the techniques was demonstrated.
2. The three basic configurations of two circular cylinders were studied in common test facilities using three investigative techniques: flow visualization, hot-film anemometry and PIV. Two of the test facilities, a small vertical water towing tank

and a main water towing tank, were purposely built for the research and with the PIV technique in mind.

3. A number of previously observed flow patterns for multiple cylinders in steady cross-flow were clarified, by considering the flow visualization video images and the instantaneous PIV data, including the shear layer reattachment dynamics for two tandem circular cylinders, the base-bleed flow patterns for two and three side-by-side circular cylinders, the biased flow pattern for two side-by-side circular cylinders, and anti-phase vortex shedding synchronization.
4. Vortex shedding frequency measurements showed that up to three frequencies can be found for some multiple cylinder configurations, and that the number of frequencies and their strengths are sensitive to the measurement location. The experiments showed that vortex shedding from a two-cylinder staggered arrangement in steady cross-flow is more properly shear-layer-dependent rather than cylinder-dependent.

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Acknowledgments

None of this work would have been possible without God's providence and grace, nor could it have been completed without the assistance of other people. I need to acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada (NSERC), Les Fonds pour la Formation de Chercheurs et l'Aide à la Recherche (FCAR) of the Province of Québec and the J. W. McConnell Foundation. I am also indebted to my two graduate supervisors, Prof. S. J. Price and Prof. M. P. Païdoussis, who together have provided additional financial support, guidance and direction to my research, equipment and resources for the work's completion, and much patience. They have always been very thorough in reviewing my work, and in making valuable comments, suggestions and criticisms, and these were always appreciated. I have learned a great deal from them both.

Portions of this study could not have been completed without the assistance of a number of students, including Maruxa Campo, Léo Grépin, Olivier Lefebvre, Katie Leong, Jennifer Smith, and Stephen S. T. Wong. Stephen, in particular, undertook the flow visualization experiments and hot-film anemometry measurements that I needed, which greatly benefited the interpretation of the PIV results. I have also valued discussions with my colleagues and friends at McGill University and elsewhere, including M. H. Akbari, Hekmat Alighanbari, Judd Kaiser, Tim Lee, Christian Semler, and David S. K. Ting. The assistance of Lal Alakatussery, Jamie Anderson of MIT, John Boisvert, Ewart Brundrett, George Dedic, Mary Fiorilli, Jean-Louis Lopes, Brad Luckhart, Bill Mark, Scott Manktelow, Spiro Mitsialis, Gary Savard, Lois and Robert Sumner, Scott Sumner and George Tewfik, all need to be acknowledged.

Lastly, I must thank my wife, Jennifer, for her love, prayers, friendship and understanding. Her wonderful faith and trust in God is always an encouragement to me.

Nomenclature

A	vortex area [mm ²]
A_p	acceleration parameter [nondimensional]
AR	cylinder aspect ratio [nondimensional]
a	streamwise position of eddy from the base of the cylinder [mm]
b	cross-stream or transverse separation of eddies [mm]
C_D, C_D'	mean and r.m.s. drag coefficients [nondimensional]
C_L, C_L'	mean and r.m.s. lift coefficients [nondimensional]
D	cylinder diameter [mm]
d_p	particle diameter [μ m]
F_D	drag force [N]
F_L	lift force [N]
f	frequency [Hz]
f_s	vortex shedding frequency [Hz]
G	gap between two neighbouring cylinders [mm]
g	gap between the free end of the cylinder and the wall of the towing tank [mm]; acceleration due to gravity [mm/s ²]
h	transverse vortex spacing [mm]
h/l	vortex spacing ratio [nondimensional]
K	constant defined in (A.2) [s ⁻¹]
L	centre-to-centre longitudinal pitch [mm]
L/D	centre-to-centre longitudinal pitch ratio [nondimensional]
L_f	vortex formation length [mm]
L_R	length of recirculation zone from the base of the cylinder [mm]
L_u	longitudinal turbulence integral length scale [mm]
l	path coordinate, cylinder length or span, longitudinal vortex spacing between two vortices in a single row [mm]

M_x, M_y	horizontal and vertical image magnification factors [mm/pixel]
N_s	Stokes number [nondimensional]
P	centre-to-centre pitch [mm]
P/D	centre-to-centre pitch ratio [nondimensional]
Re	Reynolds number, based on cylinder diameter D [nondimensional]
r, r_1	radial coordinate, Oseen vortex core radius[mm]
St	Strouhal number [nondimensional]
$s.g.$	specific gravity
sq	squareness ratio [nondimensional]
T	centre-to-centre transverse pitch [mm]
T/D	centre-to-centre transverse pitch ratio [nondimensional]
T_s	vortex shedding period [s]
$t, \Delta t$	time, time difference between images comprising a pair [ms]
$t^*, \Delta t^*$	nondimensional time, nondimensional time difference [nondimensional]
TI_u	longitudinal freestream turbulence intensity [nondimensional]
U	steady freestream velocity, steady towing velocity, terminal or asymptotic constant velocity attained after an impulsive start [mm/s]
U_c	vortex convection velocity [mm/s]
u	streamwise velocity component, local cylinder velocity [mm/s]
u_f	fluid velocity [mm/s]
u_p	particle velocity [mm/s]
u_θ	tangential velocity component [mm/s]
v	transverse velocity component [mm/s]
w	out-of-plane velocity component [mm/s]
w_s	sedimentation rate [mm/s]
x	streamwise coordinate [mm]
x_c	streamwise location of the vortex centroid [mm]
y	transverse coordinate [mm]
y_c	transverse location of the vortex centroid [mm]

z spanwise coordinate [mm]

Greek Symbols

α angle of incidence or incoming flow angle [°]
 Γ, Γ_o circulation, Oseen vortex strength [mm²/s]
 Γ/UD nondimensional circulation, or vortex strength [nondimensional]
 δ_x, δ_y horizontal and vertical grid spacing [pixels, mm]
 λ_u longitudinal turbulence length microscale [mm]
 ν_f kinematic viscosity [m²/s]
 ρ_f fluid density [kg/m³]
 ω_z in-plane vorticity [s⁻¹]
 ω_{zp} peak vorticity [s⁻¹]

Acronyms

ABF asymmetric biased flow pattern
BB base-bleed flow pattern
BF biased flow pattern
CTA constant temperature anemometry
FV flow visualization
IS induced separation flow pattern
KVS Kármán vortex shedding flow pattern
LDV laser Doppler velocimetry
NVS no vortex shedding synchronization flow pattern
PIV particle image velocimetry
r.m.s. root mean square
SBB single bluff-body flow pattern
SBB1, 2 single bluff-body flow patterns types 1 and 2
SBF symmetric biased flow pattern
SLR shear layer reattachment flow pattern

SLR1, 2, 3	shear layer reattachment flow patterns types 1, 2 and 3
SVS	synchronized vortex shedding flow pattern
VI	vortex impingement flow pattern
VPE	vortex pairing and enveloping flow pattern
VPSE	vortex pairing, splitting, and enveloping flow pattern

Preface



Fig. 115 — Carpi, La Sagra.
Pittore della fine del secolo XIV: *San Cristoforo*.

Painting of Saint Christopher carrying the Christ child across a river, attributed to Giovanni da Modena, 14th Century, and found in Bologna, Italy (from Venturi 1911). Theodor von Kármán purportedly saw this painting (or another like it) when visiting Italy, and noticed an alternating pattern of vortices in the river downstream of the Saint's legs (Tokaty 1971). The actual existence of the vortices in the painting is left to the imagination of the reader.

Chapter 1

Introduction

1.1 Circular Cylinders in Cross-Flow

Artists from the time of Leonardo da Vinci, and earlier, have portrayed the often complex motion of a fluid in a variety of ways through their observations of the created world. In the painting of Saint Christopher dating from the 14th Century, found in the Preface, a complicated pattern of waves and vortices may be seen in the flow around the Saint's legs as he crosses the river carrying the Christ child. From an artistic or religious point of view, the river current may be of secondary importance to the subjects of the painting, but from a more scientific and analytical point of view, the river's depicted behaviour is, in effect, a very reasonable representation of the interaction between a moving fluid and a group of obstacles immersed in its path, that is commonly observed in nature and technology. This was apparent to Theodor von Kármán when he purportedly saw a painting much like this one during a visit to Bologna, Italy, in which he recognized the familiar alternating pattern of vortices in the wake of the Saint's legs on the surface of the river (Tokaty 1971). This pattern of vortices, "discovered" by an artist who lived nearly six hundred years before him, would eventually bear von Kármán's name.

In practice, each of Saint Christopher's legs may be simply thought of as a structure of slender shape and nearly circular cross-section, not unlike a circular cylinder in cross-flow. By virtue of its common occurrence in many forms and in different applications, the flow of fluid around a circular cylinder has been well studied, and is one of the classical problems of fluid dynamics. The flow field is typically marked by the

periodic, alternate formation and shedding of vortices from opposite sides of the cylinder, and a regular pattern of vortices in its wake known as the Kármán vortex street. Cylinder-like structures can be found, both alone and in groups, in the designs for heat exchangers, cooling systems for nuclear power plants, offshore structures, buildings, chimneys, power lines, struts, grids, screens, and cables, in both air- and water-flow. In many of these engineering applications, the strong periodic shedding of Kármán vortices from a cylinder is responsible for problems with flow-induced vibration and noise, the production of highly turbulent and three-dimensional fluid motion in its wake, and structures that experience considerable drag forces. A complete understanding of the fluid dynamics for the flow around a circular cylinder encompasses such fundamental subjects as the boundary layer, the free shear layer, the wake and the dynamics of vortices.

Less well studied and understood are the modifications to the fluid dynamics which may occur when two or more circular cylinders are placed in close proximity to one another. The flow field of a multiple cylinder configuration may involve complex interactions between the shear layers, vortices and Kármán vortex streets, and the structures themselves. And although slender structures of circular or other bluff cross-section are commonly found in groups in engineering applications, in many instances an understanding of the fluid dynamics is limited to “only” being able to predict the occurrence of flow-induced vibration and noise. Otherwise, an accurate understanding of the flow field for many arrangements of cylinders remains unresolved, as would be the case of the flow around the legs of Saint Christopher.

This present study of “Circular Cylinders in Cross-Flow” has thus been motivated by both fundamental and practical considerations, ranging from acquiring improved physical insight into multiple vortex formation and shedding processes, to a need to identify the fluid dynamic mechanisms in fluid-structure interaction and the potential for flow-induced vibration and noise. The objectives of the study are outlined in the following section.

1.2 Objectives of the Study

The primary objective of this study was to acquire, through experiments, an improved physical understanding of the fluid dynamics of small groups of circular cylinders in steady* and impulsively started cross-flow, and in particular the dynamics of the vortical structures in the flow. Three basic configurations of tandem, side-by-side and staggered arrangements of cylinders were examined, as a starting point for future studies of more complicated multiple cylinder groups, such as a cylinder array, for example. The understanding of the fluid behaviour was not based solely on measured quantities, of vortex shedding frequencies or the fluid forces acting on the structures, for example, as in many previously published studies of multiple cylinders in cross-flow. Rather, the emphasis was on the identification of flow patterns through the application of two complementary investigative techniques, dye injection flow visualization and particle image velocimetry (PIV). The PIV technique has been applied previously only in a limited manner to the flow about groups of bluff-bodies, but is a highly effective quantitative flow visualization method. PIV readily provides the researcher with a time history of the instantaneous, in-plane vorticity field, in a nonintrusive fashion, which is arguably the best means of studying the flow about bodies prone to vortex shedding and complex vortex interactions.

The fluid behaviour in each multiple cylinder group was then gauged through changes in the flow patterns, identified from flow visualization images and the PIV vorticity data, as the number of cylinders, the spacing between the cylinders, the orientation of the cylinders with respect to the oncoming flow, and to some degree the Reynolds number, were varied. Measurements of various parameters were also acquired

* Throughout the thesis, the words “steady flow” or “unsteady flow” refer to the nature of the oncoming mean flow, or freestream, in which the cylinders are situated. Thus, it may be said that under steady mean flow conditions, the cylinder flow field is fundamentally unsteady, or periodic, in the subcritical Reynolds number regime.

in the current investigation, but the emphasis on the data was of a secondary nature to the flow patterns.

Apart from acquiring an overall assessment of the fluid behaviour fundamentals for three basic configurations of cylinders in cross-flow, a second objective of the study was the fluid dynamics of the most general form of two equal-diameter cylinders in close proximity, that being a staggered configuration at arbitrary spacing and arbitrary angle of incidence to the oncoming flow. Such a configuration is perhaps the most commonly found in engineering applications, but its fluid dynamics, apart from measurements of the fluid forces acting upon the cylinders and the frequencies of vortex shedding, has been the subject of rather limited study. In this thesis, the staggered arrangement of cylinders was considered strictly from a flow pattern perspective for the first time.

A third objective of the research was to gain an understanding of how the steady flow patterns develop, and to this end the fluid behaviour for the two most simple cases, tandem and side-by-side configurations, was studied under conditions of an impulsive start from rest. There are presently very few known investigations of groups of bodies in unsteady mean flow, and this study is perhaps the first. The cylinders were tested under similar conditions of geometry and Reynolds number that were employed in the steady flow experiments, with all tests undertaken in the low subcritical Reynolds number regime for a single circular cylinder. Although the complete flow development could not be studied from the start of motion to the attainment of steady mean flow conditions, insight was nevertheless gained into the initial stages of vortex formation and vortex shedding.

The work undertaken in this thesis also aimed to address a number of unanswered questions in the literature regarding the flow around specific multiple cylinder configurations in cross-flow. A survey of the relevant literature showed that there are still some misunderstandings concerning the fluid behaviour of multiple cylinder systems, some of which may have been caused by poor experimental conditions, measurements acquired on a time-averaged basis, and poor or nonexistent flow visualization, for example. Thus, some of areas which needed to be clarified were: the origins of two or

more vortex shedding frequencies and maximum and minimum values in the measured lift and drag forces on the cylinders, the structure of enclosed regions of recirculating fluid, and the degree of flow organization or synchronization which may exist.

1.3 Outline of the Thesis

The thesis begins with a review and a brief survey of the literature pertaining to the fluid dynamics of a single circular cylinder in cross-flow, under steady mean flow and impulsively started conditions, in Chapter 2. This review is not meant to be exhaustive, but is included for completeness, since hundreds of studies have been published concerning numerous aspects of the fluid dynamics of a circular cylinder. A similar review and survey of the literature, for small groups of circular cylinders in steady cross-flow and impulsively started cross-flow, is found in Chapter 3. Here, the subject matter is more vast, but there have been considerably fewer investigations than in the case of the single cylinder.

In Chapters 4 and 5 of the thesis, the details of the experimental facilities and apparatus chosen for the present study, followed by the measurement instrumentation and means of data analysis and processing, are discussed, respectively. Each of the three major facilities used for the experiments, two of which were purposely designed for this work, is described separately in Chapter 4, along with some details pertaining to the cylinder models used in each of the facilities. Of the three investigative techniques adopted in the present research, the particle image velocimetry (PIV) technique receives the most treatment in Chapter 5, since it is the newest of the three employed and the most widely used in the study.

The experimental results are then presented in Chapters 6 through 9, including discussion and analysis with respect to the different test facilities, investigative techniques, flow conditions, multiple cylinder configurations and previously published results where found. A preliminary study of the single circular cylinder, in steady cross-flow and after an impulsive start, is first reported in Chapter 6. These experiments were conducted as a reference for the remaining multiple cylinder experiments, but also to

demonstrate the suitability of the experimental apparatus and instrumentation selected. Results for tandem, side-by-side and staggered configurations of two and three circular cylinders are then presented in Chapters 7, 8 and 9, respectively, beginning first with the steady flow experiments and followed by the impulsively started flow experiments in each case (where applicable).

The thesis ends with a summary of the major conclusions and contributions of the research, and some recommendations for future study, in Chapter 10. This is followed by an extensive list of references, a single Appendix pertaining to the fluid dynamics and performance of seeding particles (used with the PIV technique), and together nearly 150 Tables and Figures.

Chapter 2

The Circular Cylinder in Cross-Flow

2.1 Introduction

The flow of fluid around a bluff-body experiences considerable resistance, and typically involves separation of the boundary layers from the surfaces of the body and the formation of a significant wake downstream. The circular cylinder, shown in Figure 2.1, is the classical example of a two-dimensional bluff-body and it has received considerable attention from the research community for perhaps hundreds of years, partly because it is a structure that is commonly observed in nature, and employed in engineering and other applications (see reviews by Gerrard (1978), Coutanceau & Defaye (1991), Williamson (1996b), and Zdravkovich (1997), among others); in Zdravkovich's (1997) book alone, some 1000 papers from 100 journals in five languages were reviewed. The geometrical simplicity of the circular cylinder belies its complex fluid behaviour under steady mean flow conditions, which is marked by the formation of two separated free shear layers, the alternate formation of spanwise vortices in its near-wake region, and a wake which is both highly turbulent and three-dimensional in nature. The vortices are shed in a periodic fashion at a known frequency, and two rows of regularly spaced vortices of opposite sign in each row form in the wake, in a pattern known as a Kármán vortex street. Each of these features is shown in Figure 2.2.

Slender circular cylinders of uniform cross-section, with spanwise length, l , constant diameter, D , and aspect ratio $AR = l/D$, are considered, as shown in Figure 2.1. When the cylinder is immersed in a cross-flow of uniform freestream velocity, U , the x -

axis is oriented with the mean flow direction, and the y - and z -axes are in the mutually perpendicular transverse and spanwise directions, respectively. The Reynolds number, Re , is based on the cylinder diameter and the freestream velocity, as $Re = UD/\nu_f$, for fluid kinematic viscosity ν_f . The flow features described above, and shown in Figure 2.2, are intrinsic to the subcritical regime of Reynolds number, from $Re = 350$ to 3×10^5 (see Section 2.2.3), in which the experiments described in this thesis were conducted. Vortex shedding from a circular cylinder, at frequency f_s or period $T_s = f_s^{-1}$, is typically expressed in nondimensional form as the Strouhal number, $St = f_s D/U$.

In the following sections, a brief review of the fluid dynamics of a single, isolated circular cylinder is presented, for cylinders of smooth surface finish, high aspect ratio (with $AR > 10$), which are rigidly mounted (since the fluid dynamics may become highly modified, otherwise); for the case of incompressible, uniform, low turbulence cross-flow, under steady mean flow conditions (Section 2.2) or after an impulsive start (Section 2.3); and in the absence of significant extraneous effects caused by the finite length of the cylinder and its end conditions, meaning the flow may be considered predominantly two-dimensional about the central portion of its span. The Chapter also includes a brief review of experimental considerations (Section 2.4), and their influence on the single-cylinder fluid dynamics, since the present study of multiple cylinders in cross-flow followed solely an experimental approach.

2.2 Steady Mean Flow

Under steady mean cross-flow conditions, the flow field of the circular cylinder undergoes a complex evolution with increasing Reynolds number, as outlined in Figure 2.3, and many aspects of its fluid dynamics have been studied extensively both experimentally and numerically. In Figure 2.3, the flow regimes are grouped into seven different classifications, although the number of regimes and their definitions typically differ from study to study, as do the Reynolds number boundaries (compare the classification schemes and Reynolds number limits found in Roshko (1961), Achenbach (1968), Chen (1987), Coutanceau & Defaye (1991), Williamson (1996b) and

Zdravkovich (1997), for example). The seven flow regimes shown here range from the creeping flow regime in highly viscous flow at extremely low Reynolds numbers (Figure 2.3(a)), to the transcritical or post-critical regime at extremely high Reynolds numbers (Figure 2.3(g)). Periodic, alternate vortex shedding is observed in most regimes, as shown. A more succinct classification has recently been proposed by Zdravkovich (1997), in which the fluid behaviour under separated flow conditions is predominantly governed by three types of transition from laminar to turbulent flow: (i) within the cylinder wake, encompassing the low, transitional, and high Reynolds number regimes in Figure 2.3(c); (ii) within the separated free shear layers, corresponding to the subcritical regime in Figure 2.3(d); and (iii) within the cylinder boundary layers, covering the critical to post-critical regimes in Figures 2.3(e-g).

Current research efforts being conducted on a more fundamental level are focused on the three-dimensional structure of the circular cylinder flow field, and particularly its interplay with transition from laminar to turbulent flow at low and intermediate Reynolds numbers (see Williamson 1996a, for example), neither of which have received as much consideration as the cylinder flow's dominant two-dimensional features (see Figure 2.4, for example). The circular cylinder has also received more practical attention in the fields of flow-induced vibration and fluid-structure interaction (Chen 1987, Price 1995), for which the dominant Reynolds numbers are often in the high subcritical to post-critical regimes, shown in Figures 2.3(d-f), often found in industrial applications. In the following subsections, some additional details of flow regimes for a single circular cylinder are outlined. The Reynolds number boundaries indicated in the text are those found in the review article of Countanceau & Defaye (1991), and from other references.

2.2.1 Very Low Reynolds Numbers

For very low Reynolds numbers of $Re < 5$, the cylinder flow field is symmetrical and stable, and remains attached to the cylinder surface. The flow is commonly referred to as constituting the creeping flow regime, as shown in Figure 2.3(a). From $Re = 5$ to 40, the flow separates from the surface of the cylinder, and two separated shear layers enclose a

recirculation zone in the near-wake region containing a pair of standing eddies of equal strength and opposite rotation, as sketched in Figure 2.3(b) (Coutanceau & Bouard 1977a). Separation initially occurs near the downstream stagnation point, but slowly moves forward on the surface of the cylinder with increasing Reynolds number, as the recirculation zone increases in size. The flow field looks similar to that for a circular cylinder impulsively set into motion, as described in Section 2.3, with the absence of secondary concentrations of vorticity.

2.2.2 Laminar Vortex Shedding and Transition to Turbulence

The breakdown of the recirculation zone and its pair of standing eddies, with increasing Reynolds number, leads to periodic vortex formation and shedding in the cylinder wake, and the formation of a Kármán vortex street, as shown in Figure 2.3(c). Primary, or Kármán, vortices are shed in an alternating fashion from the two separated free shear layers. Kármán vortex shedding now becomes characteristic of the circular cylinder in steady cross-flow for Reynolds numbers spanning several orders of magnitude, as shown in Figures 2.3(d-g).

The transition to vortex shedding begins with a waviness along the wake centreline, a decrease in the length of the recirculation zone, and the appearance of “gathers,” which are narrow regions of spanwise vorticity extending from the rear of the recirculation zone into the intermediate wake (Gerrard 1978). Instability within the recirculation zone may then cause one of the standing eddies to become detached from the cylinder. The separated shear layers, which at lower Reynolds numbers used to enclose the recirculation zone, instead now become free and roll up, in an alternating fashion from one side of the cylinder and then the other, into vortices of opposite sign. Each vortex is then shed, in the same alternating manner, into the cylinder wake, to form a regular, staggered pattern of two rows of vortices of opposite sign known as a Kármán vortex street (Coutanceau & Defaye 1991). This vortex street remains intact for more than one hundred diameters from the cylinder, although turbulence, three-dimensional fluid motion, diffusion of vorticity, and vorticity cancellation across the street eventually

lead to its disappearance. These effects are more pronounced at higher Reynolds numbers, particularly in the subcritical regime, as described in Section 2.2.3.

Laminar vortex shedding is observed from $Re = 40$ to 180, and possibly higher depending on experimental conditions. The cylinder end conditions are important in determining the shedding frequency, and whether the vortices are shed at an oblique angle to the cylinder axis or parallel to it. Cells of lower frequency vortex shedding near the cylinder ends may also be found (Williamson 1989).

As the Reynolds number is increased from $Re = 150$ to 180, the appearance of organized three-dimensional structures within the vortex street results in two distinct transitions in the Strouhal number-Reynolds number relationship, near $Re = 180$ and $Re = 260$, associated with two different modes A and B of three-dimensional instability, respectively (Williamson 1996a; Wu *et al.* 1996), shown in Figure 2.4. These structures appear as vortex loops oriented in the streamwise direction superimposed on the spanwise Kármán vortices in the near-wake region, with modes A and B having spanwise wavelengths of 3 to $4D$ and $1D$, respectively. Randomly occurring three-dimensional structures, known as vortex dislocations, also appear in the cylinder wake, as a precursor to the transition to a fully turbulent wake and to fully turbulent vortices at higher Reynolds numbers (Williamson 1992).

2.2.3 Subcritical Flow

As previously mentioned, the experiments conducted in the present study fell entirely within the low subcritical flow regime. This fluid behaviour, illustrated in Figure 2.3(d), begins near $Re = 350$ as the Kármán vortices become increasingly turbulent, boundary layer separation exhibits spanwise nonuniformities (Yokoi & Kamemoto 1992, 1993), and the free shear layers become increasingly unstable with the formation of transition waves and shear layer instability (or Kelvin-Helmholtz) vortices, as investigated by Bloor (1964), Wei & Smith (1986), Unal & Rockwell (1988), Prasad & Williamson (1997), and others. The Kármán vortices comprising the vortex street are linked to the vortex formation region at the base of the cylinder through complex, streamwise vortical

structures (Lin *et al.* 1995b, 1996). Separation of the laminar boundary layers occurs near $\pm 82^\circ$ from the forward stagnation point throughout the subcritical regime, and Kármán vortex shedding occurs at a nearly constant value of the Strouhal number, close to $St = 0.2$ (Zdravkovich 1997).

Despite the constancy of the Strouhal number, both the base pressure coefficient and the vortex formation length vary considerably in subcritical flow; the vortex formation length, L_f , marks the point in the near-wake region where the free shear layers roll up into fully formed Kármán vortices (Noca *et al.* 1998). Furthermore, based on instantaneous flow patterns, a number of additional modes of fluid behaviour can be identified in the subcritical regime, apart from the dominant alternate vortex shedding activity. Some of these modes show symmetrical or irregular forms of vortex formation and shedding, as identified by Lin *et al.* (1995b).

2.2.4 Critical, Supercritical and Post-Critical Flow

With a further increase in Reynolds number to near $Re = 2 \times 10^5$, the entire flow field of the circular cylinder may be considered turbulent, with the exception of the boundary layers on its surface. Further changes to the flow field are now influenced by changes in the cylinder boundary layer, as the cylinder enters the critical, supercritical, and post-critical regimes (Figures 2.3(e-g)). Although the definitions of these flow regimes often differ, depending on the researcher, the following events typically occur (see Coutanceau & Defaye (1991), for example).

Within the critical flow regime (Figure 2.3(e)), the boundary layer undergoes laminar separation, which is followed by turbulent reattachment. Transition to turbulence occurs within a separation bubble on the cylinder surface. The final separation of the boundary layer is now delayed, and the cylinder wake becomes increasingly narrow, with the minimum wake width attained at the critical Reynolds number, around $Re = 3 \times 10^5$. Asymmetric formation of the pair of separation bubbles may lead to a deflected wake and vortex street within the critical regime, as shown in Figure 2.3(e).

At Reynolds numbers slightly greater than the critical value, laminar separation and turbulent reattachment of the boundary layer become increasingly irregular in time and position on the cylinder surface (about the circumference and along the span). Premature transition, or the formation of a separation bubble on one side of the cylinder only, leads to the absence of regular vortex shedding. This irregular behaviour categorizes the supercritical flow regime, beginning near $Re = 7 \times 10^5$ (see Figure 2.3(f)). In the transcritical or post-critical regime which follows, shown in Figure 2.3(g), typically for $Re > 3.5 \times 10^6$, the cylinder boundary layer is fully turbulent from near the upstream stagnation point, and organized vortex shedding resumes. The critical and post-critical regimes are highly sensitive to freestream turbulence and surface roughness (Roshko 1961; Achenbach 1968), and consequently there is considerable scatter in measurements of the vortex shedding frequency and the fluid forces acting on the cylinder (Zdravkovich 1997).

2.3 Impulsively Started Flow

The time evolution of the flow field for a circular cylinder impulsively set into motion is one of the classic examples of unsteady fluid dynamics. Its study has provided physical insight into a number of time-dependent fluid dynamic processes, including two-dimensional unsteady boundary layer formation and separation, vortex formation and vortex shedding. The flow has often been used as a test-case for computational and numerical studies (Akbari & Price 1997). The evolution of the flow is typically examined at different values of the nondimensional elapsed time t^* , where $t^* = tU/D$ for elapsed time t , cylinder diameter D , and steady flow velocity U (on which Re is based; see also Section 4.4, and Figure 4.6) achieved after the impulsive start.

The development of the flow is characterized by the diffusive and convective transport of vorticity from the surface of the cylinder. A symmetrical recirculation zone forms in the cylinder near-wake, containing a pair of stationary eddies of equal strength and opposite rotation, as shown in Figure 2.5(a). Small regions of secondary vorticity also form near the point of boundary layer separation; these regions are not observed

under steady cross-flow conditions. The recirculation zone remains stable and symmetric up to $t^* = 4.0$ to 6.0 , depending on the Reynolds number, after which asymmetry develops, followed by a transition to the more familiar alternate vortex shedding seen under steady mean flow conditions. Because of the absence of regular, periodic vortex shedding in the early stages of its development, however, a shedding frequency or Strouhal number cannot be defined for the flow around an impulsively started circular cylinder, until the transition to periodic flow is complete between $t^* = 50$ and 100 .

Experimental work by Honji & Taneda (1969), Nagata *et al.* (1975, 1979, 1985a,b & 1989), Coutanceau & Bouard (1977b), Bouard & Coutanceau (1980), Wu & Chu (1989), and Chu & Liao (1992) have provided a detailed quantitative description of the recirculation zone development for Reynolds numbers up to and greater than $10,000$. Their measurements have included the length of the recirculation zone from the base of the cylinder, L_R , and the streamwise and transverse locations of the primary eddies, a and b , as defined in Figure 2.5(b). Sarpkaya (1991) and others have measured the unsteady lift and drag forces. The flow field has also been studied numerically, with some success for low Reynolds numbers and small elapsed times, by Ta Phuoc Loc & Bouard (1985) and others.

The question of whether it is physically possible to achieve impulsively started flow was investigated by Sarpkaya (1991), who reiterated that for any experimental apparatus, there will always be an initial acceleration that will have some effect on the flow development. Based on Sarpkaya's experiments, a dimensionless acceleration parameter, A_p , where $A_p = DU^{-2}(du/dt)$ for acceleration du/dt , can be used to assess whether or not a flow is nearly impulsively started. According to Sarpkaya (1991), a flow may be considered to be impulsively started, provided $A_p > 0.23$.

2.4 Experimental Considerations

The flow about a circular cylinder is sensitive to experimental conditions, and in particular the end conditions, which may impart considerable changes to the flow's perceived strongly two-dimensional character. Flow conditions at the ends of the cylinder

may cause unwanted but real three-dimensional effects that influence Kármán vortex shedding and other parameters over significant portions of its span. Other factors influencing the flow field include the cylinder aspect ratio, AR, its surface finish, wall interference effects imposed by small test facilities (when compared to the size of the model), as well as the quality and uniformity of the mean flow.

Since many of the disagreements in published experimental data for the circular cylinder in cross-flow can be attributed to differences in experimental conditions, some minimum requirements were suggested by Cantwell & Coles (1983), in their often-cited study of a circular cylinder in the subcritical regime. The requirements are applicable to experiments designed to simulate uniform, two-dimensional, incompressible flow about an unconfined circular cylinder in a low-turbulence freestream (as in the present study), and included: using high aspect ratio cylinders of $AR > 10$, ensuring the testing environment is of low solid blockage ratio less than 10%, maintaining a low freestream turbulence intensity of less than 1%, and keeping the Mach number less than 0.25. The influence of experimental conditions on the impulsively started flow field has not been extensively investigated, however, but many of the same recommendations would seem to apply.

2.4.1 End Conditions and Cylinder Aspect Ratio

The effects of cylinder end geometry and aspect ratio on vortex shedding are well-documented in the literature, and include such phenomena as oblique shedding (with respect to the cylinder axis) and cellular regions of lower shedding frequency and higher base pressure (Gerich & Eckelmann 1982; Williamson 1989). The effects of the end conditions may extend over the entire span of the cylinder at low Reynolds numbers (even for cylinders of high aspect ratio), while at subcritical Reynolds numbers the effects are more often restricted to cell regions of several diameters in length near the ends of the cylinder. End conditions are of importance in both wind tunnel and water tunnel testing, and in such facilities as a towing tank.

To ensure a region of nominally two-dimensional vortex shedding about the central portion of the cylinder, in which to acquire measurements and visualize the flow, the cylinder aspect ratio must be sufficiently high. Moreover, it is often necessary to mount end plates onto the cylinder near its spanwise extremities, typically to remove the unwanted influence of the test section wall boundary layers, and to help maintain the two-dimensional flow conditions over greater portions of the span. Fox & West (1990) noted that, in the absence of end plates fitted to a circular cylinder in a wind tunnel, a spanwise region of $15D$ may be affected by the test-section wall boundary layers, whereas with the use of end plates the region of affected flow was reduced to $3.5D$. A number of studies of the effects of end plates have been undertaken, with application to the subcritical regime (Stansby 1974; Fox & West 1990; Szepessy & Bearman 1992), and these have been summarized in a design of optimized end-plate geometry by Szepessy (1993). This end-plate design takes into account the desired improvement of spanwise conditions, the removal of the test-section wall boundary layers, and also the undesirable formation of a horseshoe vortex structure at the cylinder-end plate junction. Some evidence suggests, however, that end plates may not be needed in the subcritical regime provided the cylinder aspect ratio is sufficiently high, i.e. $AR > 20$ to 30 (Fox & West 1990; Szepessy 1993).

Different types of cylinder end conditions are found when conducting tests in a towing tank, as compared to a wind tunnel or water tunnel. First, there is an absence of test-section wall boundary layers in a towing tank, which is an advantage, since end plates will usually not have to be used. Second, since cylinders are typically mounted in the vertical direction in a towing tank, the cylinder will often penetrate the free surface of the water. This end condition may be undesirable, since complex interaction between the spanwise, vertical Kármán vortices and the free surface may occur. Third, there exists in a towing tank experiment a small gap between the free end of the cylinder and the bottom wall (or floor) of the test facility. This end condition is probably the most important of the three, and Slaouti & Gerrard (1981) recommended a maximum gap of $0.15D$ to $0.2D$ to ensure minimum spanwise effect on the vortex shedding, in experiments at $Re = 75$ to

100. Some insight into the influence of the free end may also be found in studies of a cantilevered, finite-length circular cylinder with a free end. These studies have noticed (i) a region of disturbed flow that may extend up to seven diameters from the free end for a cylinder of aspect ratio $AR > 13$ (Farivar 1981; Okamoto & Sunabashiri 1992; Fox & West 1993a,b); (ii) lower vortex shedding frequencies within this affected region; (iii) the transition to the lower shedding frequency in the region of disturbed flow may be step-wise at discrete frequencies (Farivar 1981) or may decrease in a continuous manner (Okamoto & Sunabashiri 1992); (iv) the existence of a second region of disturbed flow near the tip of the cylinder where there is no regular vortex shedding (Farivar 1981; Ayoub & Karamcheti 1982); (v) spanwise flow around the free end of the cylinder, and a complex trailing vortex structure formed there (Kawamura *et al.* 1984), both of which interfere with the interaction of the free shear layers, increase the length of the vortex formation region, promote weaker and more symmetric vortex shedding (Okamoto & Sunabashiri 1992), increase the base pressure, and lower the local drag coefficient (Luo *et al.* 1996).

Each of these flow phenomena generated by different, physical end conditions means that measured data for a single cylinder will differ from one test facility to another for apparently identical conditions of Reynolds number and other parameters. This has been particularly the case within the low and intermediate ranges of Reynolds number, in determining the Strouhal number-Reynolds number relationship (Williamson 1996a), for which there has been much disagreement in the past. Computational studies of the circular cylinder using two-dimensional codes ideally represent a flow field without end conditions, but generally show disagreement with published experimental data for $Re > 180$, since they fail to account for other, intrinsic three-dimensional structures unrelated to the end conditions (see Figure 2.4, for example).

2.4.2 Wall Interference

Wall interference effects on bluff-bodies are well-documented in the literature, and may include contributions from (i) solid blockage, due to the size of the model in proportion to

size of the test section; (ii) wake blockage, due to the size of the wake in proportion to the dimensions of the test facility; and (iii) pressure gradients in the mean flow direction caused by the growth of test section wall boundary layers. Semi-empirical methods for blockage correction, however, are typically available only for small solid blockage ratios, and they are useful for adjustments to simple measured quantities, such as the drag coefficient (Sumner & Brundrett 1995). In many cases, an increase in blockage is analagous to an increase in the Reynolds number, and results in a higher value for the Strouhal number, but often the influence is unknown and may be more complex or nonlinear. At high subcritical Reynolds numbers, wall interference effects have been found to be insignificant for blockage ratios less than 6% (West & Apelt 1982), although at lower Reynolds numbers, blockage levels of even 2% have noticeable effects on measured quantities (Coutanceau & Bouard 1977a,b).

2.4.3 Mean Flow Quality

Provided the mean flow is uniform across the span of the cylinder, and also in the transverse direction, the quality of the mean flow will most influence the cylinder flow field through the intensity and scale of the freestream turbulence. At low subcritical Reynolds numbers, increased turbulence intensity primarily acts upon the separated shear layers and the vortex formation region (Norberg 1986). Elevated levels of freestream turbulence typically affect the flow field in a similar manner to an increase in the Reynolds number, meaning that major changes in the flow pattern could result, particularly in the critical and post-critical regimes (Figures 2.3(e-g)). The turbulence length scale, which corresponds to average sizes of the turbulent eddies within the flow, will have an effect on flow structures of similar geometric scale or size (Nakamura 1993). For example, small scale turbulence structure on the order of the shear layer thickness may disturb the natural growth of the free shear layer, its interaction with the opposite shear layer, and the level of instabilities within it. In a towing tank, effects similar to freestream turbulence may arise from vibration associated with the towing system, as well as the settling of water between experiments (Slaouti & Gerrard 1981).

Chapter 3

Circular Cylinders in Cross-Flow

3.1 Introduction

When two or more circular cylinders are situated in close proximity to one another in steady upstream cross-flow, the fluid behaviour is more complex than the case of the single, isolated circular cylinder, as reviewed in Chapter 2. In many engineering applications, a circular cylinder is often grouped with similar structures, such that the flow field of each body is modified by the presence of its neighbours. The effects may extend to the separation of the boundary layers from the cylinders, the behaviour and interaction of the free shear layers, the formation and shedding of Kármán vortices, and the development of the wakes of the cylinders and the patterns of vortices found there.

Compared to the body of knowledge for a single circular cylinder in cross-flow, the understanding of the fluid dynamics of groups of cylinders is much less, although many studies may be found in the literature. Often, these studies have been prompted by practical concerns, although the study of multiple cylinders in cross-flow has also greatly aided the understanding of wake interference and the dynamics and interactions of vortices, for example. In this Chapter, a brief review is presented of the fluid dynamics of simple multiple cylinder groups in steady mean cross-flow, beginning with an overview of the standard configurations in the subsections to follow.

3.1.1 Multiple Cylinder Configurations

In simple multiple cylinder configurations in cross-flow, shown in Figure 3.1, where each circular cylinder is of the same diameter D , the three most important variables governing the fluid behaviour are (i) the spacing between the cylinders, (ii) the orientation of the cylinders in the x - y plane relative to the oncoming flow, and (iii) the Reynolds number. The two idealized arrangements of the cylinders which have received the most study are the tandem and side-by-side configurations, shown in Figures 3.1(a,b), where the geometry is described by the centre-to-centre longitudinal and transverse spacings, or pitches, L and T , respectively. In each case, the spacing between the cylinders is usually expressed in nondimensional form, as the longitudinal or transverse pitch ratio, L/D or T/D , as the case may be.

The most general arrangement of two cylinders, however, is the staggered configuration, which is likely the arrangement most encountered in practice, since in very few cases will an oncoming flow be perfectly aligned with the cylinders, or precisely perpendicular to them. The geometry of the staggered pair of cylinders, shown in Figure 3.1(c), is set by the centre-to-centre pitch P between the cylinders and the angle of incidence α ; as in the case of the tandem and side-by-side configurations, the spacing is typically expressed as a dimensionless centre-to-centre pitch ratio, P/D . Alternatively, it may be defined in a similar manner to the tandem and side-by-side configurations, by the longitudinal and transverse spacings L and T , as shown in Figure 3.1(d). Perhaps because of the added variable used to define the geometry, however, the staggered configuration has received far less research attention, despite its predominance in engineering applications.

Other simple configurations of cylinders include three cylinders in an equilateral triangular arrangement, studied by Sayers (1987, 1990) and Lam & Cheung (1988), and four cylinders in a square arrangement, studied by Sayers (1988, 1990) and Lam & Lo (1992), shown in Figures 3.1(e,f). These configurations are of interest since they often appear as subsets of much larger in-line and staggered multiple cylinder arrays. The four basic kinds of regular multiple cylinder arrays are shown in Figures 3.1(g-j), and include

the (i) in-line rectangular configuration, (ii) the staggered rotated-square configuration, (iii) the staggered normal triangular configuration, and the (iv) staggered parallel triangular configuration. None of these larger groupings of cylinders have been addressed in this thesis, however, but many of the smaller two-cylinder and three-cylinder tandem, side-by-side and staggered geometries which were examined, may be found repeated throughout these arrays.

3.1.2 Approaches to Understanding the Fluid Behaviour

The fluid dynamics of multiple cylinder configurations immersed in steady cross-flow have been examined by many researchers, and more than 120 such studies were collected and reviewed as part of the current research, with about 36 studies summarized in Tables 3.1 to 3.4. Furthermore, the behaviour of cylinders in cross-flow has been reviewed in articles by Zdravkovich (1977, 1987, 1993), Nishimura (1986), Ohya *et al.* (1989), and a new review will appear in a future volume of the recent publication of Zdravkovich (1997), on the fluid dynamics of a single circular cylinder. In these studies, a number of approaches have been used in an attempt to classify the fluid behaviour of multiple circular cylinders in steady cross-flow, based on a combination of theory, measurement and observation of the flow. Concerning small groups of two and three cylinders, in tandem, side-by-side and staggered configurations (Figures 3.1(a-d)), on which the present study is focused, none of the approaches has been universally successful.

For two-cylinder configurations in steady mean cross-flow, the simplest of the approaches is a more theoretical treatment by Zdravkovich (1987), who classified the fluid behaviour into two basic types of interference, based on the location of the downstream cylinder with respect to the upstream cylinder. The two types of interference, and the conditions under which they occur, are shown in Figure 3.2; these include: (i) wake interference, when one of the cylinders is partially or completely submerged within the wake of the other, and (ii) proximity interference, when the two cylinders are located close to one another, but neither is submerged in the wake of the other. Wake interference is the more readily apparent phenomenon occurring for two cylinders when

the wakes intersect. An example of wake interference would be the widening of the two separated free shear layers from one cylinder located immediately upstream of a second cylinder, in a tandem configuration (Figure 3.1(a)), such that vortex formation from the first cylinder is suppressed (seen, for instance, in Figures 3.3(a,b)). Proximity interference, on the other hand, is often difficult to predict since often there is no obvious or visible displacement of a wake, for example, but it typically has an effect on the vortex shedding behaviour. An example of proximity interference for a two-cylinder arrangement is a specific frequency or phase synchronization between two Kármán vortex streets, typically seen for the side-by-side configuration of two cylinders (seen, for example, in Figures 3.4(c,d)). Although of some usefulness from a practical point of view, the results of this study show that a rather elaborate interference classification system of Zdravkovich (1987) often fails to account for the complex behaviour of experimental data and the wide range of flow patterns observed.

A second approach to understanding the fluid behaviour of multiple cylinders in steady cross-flow has been through the interpretation of experimental data, and more specifically, measurements of vortex shedding frequencies, static pressure coefficients on the surfaces of the cylinders, and lift and drag force coefficients. These measurements are acquired for different arrangements of two or more cylinders, as the pitch ratio, incidence angle and Reynolds number are varied, and the changes to the flow field and other features are then inferred from the behaviour of the experimental data. As examples: (i) vortex shedding frequency data have been used to infer the existence of two Kármán vortex shedding processes for staggered cylinder configurations, by Kiya *et al.* 1980; (ii) static pressure coefficient data have been offered as evidence of a stable pair of eddies within the gap between two tandem cylinders, and continuous reattachment of the two shear layers, by Igarashi (1981, 1984), Hiwada *et al.* (1982) and Ljungkrona (1992); and (iii) peak lift force measurements for two staggered cylinders, and rapid changes in the lift force for small adjustments to the geometry, have been attributed to the strong influence of the high speed flow deflected through the gap between cylinders, as described by Zdravkovich & Pridden (1977). Such an approach is useful from an engineering design

point of view, and has led to many recommendations for avoiding the occurrence of flow-induced vibration, for example. However, determining the fluid behaviour from measured quantities is prone to misinterpretation, particularly when done without the benefit of accompanying flow visualization (the studies of Zdravkovich & Pridden (1977), Kiya *et al.* (1980), and Eastop & Turner (1982) are specific examples of the measurement-only approach), or from time-averaged data acquired in flows which are fundamentally periodic (such as the single cylinder and multiple cylinders).

A third approach to understanding the fluid behaviour of multiple cylinders in cross-flow, and arguably the most useful, is to identify from flow visualization (and other means) the various flow patterns which arise for different choices of the geometry. The understanding of tandem cylinders in cross-flow has greatly benefited from this approach, for example, as demonstrated by Igarashi (1981, 1984), where up to eight different flow patterns were identified from a *combination* of flow visualization and measured data for various combinations of the longitudinal pitch ratio and the Reynolds number. The side-by-side configuration has also been addressed in a similar manner, by Williamson (1985) and Kumada *et al.* (1984). The equally spaced triangular and square clusters of three and four cylinders (Figures 3.1(e,f)) have also been investigated with remarkable effectiveness through the flow pattern approach, by Lam & Cheung (1988) and Lam & Lo (1992). Their interpretation of Strouhal number data, for example, based on their own extensive flow visualization survey, was by far superior to the measurement-only approach of Sayers (1987, 1988, 1990) for the same configurations.

The flow pattern approach has not been followed, to date, for the staggered configuration, which is both the most general and most commonly occurring arrangement of two circular cylinders. Furthermore, no one approach has been successfully or uniformly applied to all three of the basic two-cylinder configurations shown in Figures 3.1(a-d). Consequently, one of the prime objectives of the present study has been to consider a complete range of two-cylinder configurations in cross-flow from a common, flow pattern approach.

3.2 Tandem Configuration

When the cylinders are arranged in-line and parallel to the mean flow in a tandem configuration (Figure 3.1(a)), the downstream cylinder is shielded from the oncoming flow by the cylinder immediately upstream. This configuration falls within the wake interference region of Zdravkovich (1987) shown in Figure 3.2, and at small spacings between the cylinders Kármán vortex shedding from the lead cylinder is suppressed. Many studies of tandem cylinders in steady cross-flow have been undertaken by other researchers, from low Reynolds numbers to the critical and post-critical regimes for a single cylinder, and for columns of two, three, four and five cylinders, spaced from one to more than twenty diameters apart in some cases. A number of investigations of two-cylinder tandem configurations, relevant to the two-cylinder experiments conducted in the present study, are summarized in Table 3.1. Other studies have focused on the heat transfer aspects (Kostic & Oka 1972; Oka *et al.* 1972, Aiba & Yamazaki 1976; Aiba *et al.* 1980, 1981; Hiwada *et al.* 1982), mass transfer (Nishimura *et al.* 1986), interference with a ground plane (Taniguchi *et al.* 1982; Luo *et al.* 1996), and the effects of forced oscillation (Gowda & Prabhu 1987; Mahir & Rockwell 1996a).

The tandem configuration is sensitive to both the longitudinal pitch ratio, L/D , and the Reynolds number, and up to eight different flow patterns were identified by Igarashi (1981, 1984) for two tandem circular cylinders of equal diameter in steady cross-flow. Other studies, as well as the present research, have shown that they can be grouped into three basic types of wake interference behaviour. The three basic flow patterns are: (i) single bluff-body behaviour at small L/D , in Figure 3.3(a), where the two cylinders are sufficiently close to act as a single structure; (ii) shear layer reattachment behaviour at intermediate L/D , in Figure 3.3(b), where the separated free shear layers from the upstream cylinder reattach onto the surface of the downstream cylinder; and (iii) Kármán vortex shedding from each cylinder at larger L/D , shown in Figure 3.3(c). Researchers have noticed that the transition from shear layer reattachment to Kármán shedding from the upstream cylinder is sudden, and the third type of behaviour has been called the

“jumped” flow pattern because of the abrupt changeover. The transition is also known to be bistable, with intermittent appearance of the two flow patterns on either side of the “critical” (or transitional, not to be confused with the behaviour of the cylinder boundary layers) pitch ratio (and investigated by Kiya *et al.* 1992).

Still of issue in the understanding of the fluid behaviour of the tandem configuration is precisely what occurs in the gap between the two cylinders, under reattachment conditions, as well the exact nature of the reattachment process. Various researchers (including Igarashi 1981, 1984; Lee & Panagakos 1997) have suggested that a stable pair of eddies forms within the gap, as shown in Figure 3.3(b), and others have indicated periodic shedding of the gap eddies as another vortex formation mechanism (Igarashi 1981, 1984; Hetz *et al.* 1991). Igarashi (1981, 1984) also showed that shear layer reattachment could be synchronized with Kármán vortex formation from the downstream cylinder, or could be a continuous process, depending on the pitch ratio and the Reynolds number.

3.3 Side-by-Side Configuration

In the side-by-side configuration of circular cylinders, the structures are aligned in a row oriented perpendicular to the oncoming flow and the flow pattern is dependent primarily on the centre-to-centre transverse pitch ratio, T/D , separating the cylinders.* In the side-by-side geometry, the dominant form of interference based on the analysis of Zdravkovich (1987) is of the proximity type (see Figure 3.2). Investigations of two side-by-side circular cylinders of equal diameter in steady cross-flow, summarized in Table 3.2, have shown the existence of four flow patterns caused by the influence of the neighbouring cylinder, which are mostly insensitive to the Reynolds number. The flow patterns are illustrated in Figure 3.4, and they are: (i) single bluff-body behaviour when the two cylinders are in contact, in Figure 3.4(a); (ii) weak flow through the narrow gap

* Some studies, such as Williamson (1985), have used the width of the gap between the cylinders, G (shown in Figure 3.1(b)), to denote the spacing, rather than the more commonly used centre-to-centre pitch, T . The conversion is simply $T = G + D$, or $T/D = G/D + 1$.

between the cylinders, at small T/D , shown in Figure 3.4(b); (iii) a biased flow pattern with synchronized vortex shedding at intermediate T/D , shown in Figure 3.4(c); and (iv) a symmetrical flow pattern of synchronized vortex shedding at higher T/D , shown in Figure 3.4(d). The configuration has been studied primarily under steady mean flow conditions, but has also been examined with the influence of a ground plane (Taniguchi *et al.* 1982; Boyle & Langston 1989), and with forced oscillation (Gowda & Deshkulkarni 1988; Mahir & Rockwell 1996b).

Of particular note in the two-cylinder case is the biased flow pattern, variously observed for $T/D \sim 1.2$ to 2.2 by other researchers, in which the flow through the gap is deflected in a continuous manner towards one of the cylinders. The flow pattern then shows a narrow near-wake region behind the cylinder towards which the flow is deflected, a wider near-wake behind the other cylinder, and two dominant vortex shedding frequencies (or Strouhal numbers) are measured. Some researchers have shown the biased flow pattern to be bistable, as it switches intermittently from being directed towards one cylinder to the other (Bearman & Wadcock 1973; Kim & Durbin 1988).

Interaction between the two simultaneously occurring vortex shedding processes is also of interest for two side-by-side cylinders, and at large T/D the two vortex streets typically synchronize in an anti-phase fashion, as shown in Figure 3.5(a). Here, vortices are shed simultaneously on both sides of the gap between the cylinders (Williamson 1985), and the resulting pattern of vortices in the combined wake of the two cylinders is stable. In some cases, in-phase synchronization of the two vortex streets is observed (shown in Figure 3.5(b)), but this has been shown to be an unstable pattern, and is not often observed.

When the biased flow pattern is observed, similar anti-phase synchronization may be seen, although Williamson (1985) identified two other “modes” of vortex interaction, for $Re \leq 200$. In the “first harmonic mode,” shown in Figure 3.5(c), vortices which form on both sides of the biased gap flow are squeezed and enveloped into the stronger vortices from the outer shear layer of the configuration, on the side of the wake to which the gap flow is deflected. This process eventually produces a single vortex street in the combined

wake of the cylinders. In the “second harmonic mode,” shown in Figure 3.5(d), which was said to be dominant for $T/D = 1.6$ to 1.9 , the combined wake of the two cylinders is marked by pairs of vortices on one side, behind the cylinder with the narrower near-wake region, and single vortices of larger size on the opposite side. In this case, the gap vortices are amalgamated into the side of the wake where the pairs of vortices are seen.

Studies of three-cylinder side-by-side configurations in steady cross-flow (summarized in Table 3.3), where the same pitch ratio T/D separates each cylinder from the others, have also suggested the existence of four basic flow patterns. The flow patterns were formally identified by Kumada *et al.* (1984) as: (i) single bluff-body behaviour when the cylinders are in contact, in Figure 3.6(a); (ii) an asymmetric biased flow field at small T/D shown in Figure 3.6(b), which may be bistable (as suggested by Eastop & Turner (1982)), and in which there are three different sizes of near-wake regions; (iii) a symmetric biased flow pattern for intermediate T/D , shown in Figure 3.6(c), where the two gap flows are deflected towards the outer cylinders; and (iv) synchronized, unbiased vortex shedding at large T/D , for which anti-phase vortex shedding synchronization is likely. When more cylinders are added side-by-side to form a row, the fluid behaviour becomes even more complex, and bistable behaviour at small pitch ratios gives way to “metastability,” as any number of near-wake regions and gap flows may intermittently interact and coalesce (Ishigai & Nishikawa 1975; Auger & Coutanceau 1978, 1979; Moretti & Cheng 1987; Cheng & Moretti 1988; Zdravkovich & Stonebanks 1990; Le Gal *et al.* 1996).

3.4 Staggered Configuration

The staggered configuration of two cylinders (shown in Figures 3.1(c,d)) is more commonly encountered in engineering applications, since the idealized tandem and side-by-side arrangements do not always materialize in practice. Unfortunately, it has often received little attention from the research community, at least when considering the numbers of studies for each of the three basic configurations. Based on Zdravkovich's (1987) more theoretical approach, shown in Figure 3.2, both wake interference and

proximity interference contribute to the fluid dynamics of the staggered cylinder pair, and extensive sets of experimental data have revealed considerable complexity to the flow as P/D and α are varied. The complexity lies in the interaction of four separated free shear layers, two Kármán vortex formation and shedding processes, and vortex interactions within the two parallel Kármán vortex streets (see Figure 3.7). Unfortunately, few extensive flow visualization studies have been undertaken for two staggered cylinders, from which the true complexity of the flow can be ascertained, although a few photographs may be found in the work of Suzuki *et al.* (1971), Ishigai *et al.* (1972, 1973), Price (1976) and Moriya & Sakamoto (1985). Furthermore, the effects of Reynolds number have yet to be examined in any study of the staggered geometry to date, and such an investigation would likely reveal further variation in the flow patterns.

Most studies of two staggered cylinders, summarized in Table 3.4, have followed the measurement approach, and have acquired information on the vortex shedding frequency in the combined wake of the cylinders, and the lift and drag forces acting upon the downstream cylinder. Of particular interest have been specific staggered geometries that give rise to extreme values of the lift force or drag force (Price 1976; Zdravkovich & Pridden 1977; Moriya & Sakamoto 1985), and a number of different explanations have been provided in the literature (which were recently summarized by Ting *et al.* 1998). The configuration has been mostly studied at high subcritical Reynolds numbers, since these are more commonly found in industrial applications. It has also been the focus of combined theoretical and experimental examinations of fluid-structure interactions (Ting *et al.* 1998).

3.5 Impulsively Started Flow

Small groups of circular cylinders have not been investigated extensively under unsteady cross-flow conditions, including the specific case of impulsively started flow where the cylinders are initially at rest, and at a moment later are set into motion at a constant velocity. Only in unpublished work, by Williamson (1997) for the case of two side-by-side cylinders, has any apparent emphasis been placed on how the steady mean flow

patterns develop. Experiments conducted as part of this thesis have, perhaps for the first time, described in detail the starting flow dynamics for some simple, multiple cylinder configurations (with results published by Sumner *et al.* 1997a,b,c).

3.6 Experimental Considerations

Much as in the case of the single cylinder, experimental conditions also have an influence on the behaviour of multiple cylinders in cross-flow. Experimental effects associated with small cylinder aspect ratio, high solid blockage ratio, improper end conditions, or high freestream turbulence, have likely influenced many published experimental results (see data in Tables 3.1 to 3.4), and have contributed to some misunderstandings and disagreements in published results. As an example, the flow pattern boundaries for two tandem cylinders in steady cross-flow were first proposed by Igarashi (1981, 1984), based on the longitudinal pitch ratio, L/D , and the Reynolds number. The more recent work by Ljungkrona & Sundén (1993), however, identified some discrepancies in the boundaries under similar conditions, and it seems that differences in aspect ratio ($AR = 3$ versus $AR = 16$), blockage ratio (16% versus 6%), and freestream turbulence intensity (0.6% versus 0.1%) may be the reasons for the different boundaries. Furthermore, few studies have examined all three basic configurations in a single facility, with a consistent set of experimental conditions, with the exception of Kiya *et al.* (1980).^{*} Among the few who have addressed experimental conditions for cylinder groups, Ljungkrona (1992) examined the effects of increased longitudinal freestream turbulence intensity, TI_u , on two tandem circular cylinders; in this case, higher freestream turbulence was found to be analagous to an increase in the Reynolds number (see also Section 2.4.3).

The choice of measurement or investigative technique has also influenced the understanding of multiple cylinders in cross-flow. Studies that follow a time-averaged measurement approach, to study the turbulent flow field [e.g. Kumada *et al.* 1984], or a time-averaged visualization approach, such as the oil film technique [e.g. Igarashi 1984],

^{*} This work, however, focused solely on measurements of the vortex shedding frequency, at a single Reynolds number.

often mean that the strong periodic components intrinsic to multiple cylinder flow fields are missed. Instantaneous measurements (through the use of PIV, as in this study, for example) or phase averaging [e.g. Kolar *et al.* 1997], have proven more useful in many respects, but care must be taken in interpreting the full field data acquired on an instantaneous basis.

Chapter 4

Experimental Apparatus

4.1 Introduction

In this investigation of the fluid dynamics of multiple circular cylinders in steady and impulsively started cross-flow, experiments were conducted in three different testing facilities in the Fluid-Structure Interactions Laboratory 2 at McGill University. These facilities included the existing Kempf & Remmers water tunnel, and two new water towing tank facilities purposely designed for this work. The new experimental apparatus was designed for use with particle image velocimetry (PIV): a large towing tank was used for experiments under steady flow conditions, and a smaller facility was used for studying impulsively started flow.

4.2 Water Tunnel

Flow visualization experiments and vortex shedding frequency measurements, under steady mean flow conditions, were acquired in a closed, recirculating Kempf & Remmers water tunnel (Figure 4.1), with a test section of 260 mm $\times$ 260 mm in cross-section (additional details are provided by Price & Serdula 1995). Freestream velocities from $U = 20$ to 200 mm/s were used in the experiments, although velocities up to 15 m/s can be attained in the facility. In the lower velocity range of the present experiments, the flow in the test section had a longitudinal turbulence intensity of $TI_u = 0.5\%$ and a velocity profile uniformity of better than 95%.

4.3 Main Water Towing Tank Facility

Most of the experiments were conducted in a newly designed and constructed water towing tank facility (Figure 4.2). A towing tank was chosen in preference to a wind tunnel or water tunnel since it is better suited for particle image velocimetry (PIV) measurements (Sections 5.4 to 5.7). Other advantages of a towing tank include (i) reduced power requirement compared to a water tunnel or wind tunnel (since less power is needed to move a model through stationary fluid, compared to moving fluid past a stationary model); (ii) better suited to the study of unsteady flows, since the fluid remains stationary; (iii) better suited to the study of flows at low Reynolds numbers, and particularly at low velocities; and, (iv) in principle, its environment of zero “freestream” turbulence intensity, when compared to a wind tunnel or water tunnel (Gad-el-Hak 1987). Some of the disadvantages of a water towing tank facility include (i) sensitivity to vibration from the motion system and external sources, which may be transmitted to the model being tested; (ii) a limited period of time in which to acquire data; (iii) the researcher must wait for the water to become stationary before conducting a subsequent experiment; and (iv) residual transient effects from the startup of motion must be accounted for (Gad-el-Hak 1987).

4.3.1 Description of the Towing Tank

The size of the new towing tank (Figure 4.2) was based on a review of experimental facilities summarized by Coutanceau & Defaye (1991), and on a similar facility at the Ocean Engineering Laboratory of the Massachusetts Institute of Technology (Anderson 1996; Anderson *et al.* 1998). The tank measures $0.76 \text{ m} \times 0.76 \text{ m} \times 3.05 \text{ m}$, with a volume of nearly 1.77 m^3 of water. The tank was constructed of untempered glass of 19.1 mm thickness, to minimize image distortion, permit access with a laser, and for scratch-resistance. The glass tank was supported by a steel frame, which also prevented the glass sides from bending under the pressure of the water. Styrofoam insulation and rubber gaskets were located between the glass panels and the steel frame. The total weight of the

towing tank (including the water) exceeded 2300 kg, including some 1400 kg of glass and 340 kg of steel framework. The tank is supported by ten vibration-resistant leveling legs.

The mobile platform (or carriage) from which the cylinder models were mounted was supported by four linear bearings and two rails running the length of the tank. The two rails were precision-aligned and leveled with surveying equipment. The platform was driven by a linear motion system, comprising a stepping motor, a microstep controller and computer software. The towing velocity ranges from 10 to 500 mm/s. Components for the PIV imaging system (discussed in Section 5.4.4) are mounted on a positioning rail beneath the tank.

4.3.2 Fluid Medium

The fluid medium was unfiltered, industrial (tap) water, which was initially treated with household chlorine bleach (see Willert 1992). Some pure, laboratory-grade sodium chloride was also added to the water, to improve the buoyancy characteristics of the seeding particles used for the PIV technique (see Section 5.4.3). While the presence of the chlorine bleach served to minimize the growth of algae, it was eventually removed from the water since it interfered with the fluorescence of the rhodamine-impregnated seeding particles being used at the time, and an adverse chemical reaction occurred upon addition of significant amounts of sodium chloride; a similar interference, with fluorescein dye, was noted by Willert (1992). In the end, the chosen fluid medium for the experiments was unfiltered, industrial (tap) water, without chlorine bleach and without sodium chloride.

Free surface contamination has some effect on lower Reynolds number vortex shedding from a circular cylinder (Slaouti & Gerrard 1981), where density and temperature gradients are also important (Anagnostopoulos & Gerrard 1976). The influence of free surface contamination was not specifically addressed in this work, undertaken in the lower subcritical Reynolds number regime, however recent studies suggests that its influence may extend from one to three cylinder diameters below the surface (Vlachos *et al.* 1998; Warncke Lang & Gharib 1998). The measurements

reported here, however, were taken well below the free surface (some 13 diameters, typically), and so free surface contamination is not likely to be a problem in the present study.

4.3.3 Background Turbulence Signature

In theory, the initially quiescent water in the towing tank means an environment of nominally zero freestream turbulence intensity into which the model is towed. In practice, it is difficult to eliminate the smallest levels of residual, background fluid motion. Of further concern are vibrations induced by the towing system, which are particularly evident at low towing velocities, that may superimpose small-scale oscillatory behaviour on the model being tested. In the frame of reference moving with the model, this vibration translates into an apparent turbulence structure or fluid motion, from which conventional, statistical turbulence quantities can be computed.

To assess the effects of vibration and residual motion in the water, a calibrated single-component hot-film sensor (see Section 5.3) was mounted on the platform on the test section centreline (with no model fitted) and towed over a range of velocities. A number of dominant frequencies were seen in the velocity time signals and the power spectra (Figure 4.3), some of which were independent of the towing velocity. The frequencies were believed to be caused by various components of the motion system, including the tooth pitch of the timing belt and pulleys, the resolution (in steps/revolution) of the stepping motor, and the wandering “crab-like” motion of the carriage at low towing speeds as it is guided by the two parallel rails.

Calculations of a longitudinal turbulence intensity, integral length scale, and microscale (Figure 4.4) showed an apparent background turbulence on the order of $TI_u = 2\%$ at higher towing velocities, although its major contribution is at a single frequency nearly one order of magnitude greater than a typical vortex shedding frequency, and at a power level some two orders of magnitude lower. This frequency also leads to an apparent streamwise integral length scale of $L_u = 3.5$ mm and a microscale of $\lambda_u = 2$ mm.

Background motion was also assessed using the PIV system (see Sections 5.4 to 5.7), to determine the residual levels of vorticity in the water at the beginning of an experiment. A typical mean background vorticity magnitude for the towing tank (averaged over a 57×57 grid of vorticity data) was approximately 0.25 s^{-1} , with a maximum of 1.7 s^{-1} . In contrast, the minimum meaningful levels of vorticity magnitude which were used to distinguish the shear layers and vortices in the cylinder flow fields, and to compute the vortex strengths, for example, ranged from 1.5 to 4.5 s^{-1} (see Section 5.7.2 and the vorticity plots in subsequent Chapters).

4.4 Vertical Water Towing Tank

A small, vertical water towing tank (Figure 4.5) was adopted for investigating cylinders under nearly impulsively started flow, with dimensions of $305 \text{ mm} \times 610 \text{ mm} \times 1200 \text{ mm}$. Cylinder models were mounted in a cantilevered fashion from a flat plate, which was towed upwards guided by two rails located on the rear wall of the tank. The towing velocity and the impulsive start were controlled by selecting appropriate weights to be released. Velocity-time data for a nearly impulsive start are shown in Figure 4.6, for a range of acceleration parameters, A_p . The steady velocity U attained by the cylinder models after the impulsive start, and used to compute the Reynolds number for impulsively started conditions, was defined by the terminal or asymptotic value of the cylinder velocity u seen in Figure 4.6. As noted in Section 2.3, the acceleration parameter is simply the acceleration of the cylinder(s) expressed in nondimensional form, with $A_p > 0.23$ indicating nearly impulsively started conditions. In most cases, the vertical towing tank yielded nearly impulsive starts of $A_p = 0.2$ to 0.8 , although for a small number of experiments its value was too low, and these results were usually rejected. The acceleration of the cylinders seemed to be sensitive to the number of cylinders mounted on the platform, and the arrangement of the cylinders, with configurations of greater total drag (such as the side-by-side arrangement) resulting in lower values of A_p .

4.5 Cylinder Design

4.5.1 Cylinders Used in the Water Tunnel

The circular cylinder models used in the water tunnel were constructed of plexiglas. Each cylinder was fitted with two dye injection ports of 0.8 mm diameter, at the mid-span position near the points of laminar boundary layer separation (at approximately $\pm 82^\circ$ from the forward stagnation point). For the multiple cylinder experiments, the cylinders were of $D = 16$ mm, $AR = 16$, and solid blockage ratio of 6.3% (per cylinder). No end-plates were fitted.

4.5.2 Cylinders Used in the Main Water Towing Tank

For the experiments in the main water towing tank, the circular cylinder models were made of stainless steel, machined and polished smooth to $D = 25.4$ mm. Each cylinder had a nominal aspect ratio of $AR = 27$ and a nominal solid blockage ratio of 3.5%, depending on the depth of water in the tank. These parameters were ideal for multiple cylinder experiments, particularly compared to some experiments of other researchers that were often conducted with low aspect ratio cylinders and in high blockage conditions (see Tables 3.1 to 3.4, and tabulated data in Cantwell & Coles (1983)).

When illuminated by a light sheet (for flow visualization and PIV), a stainless steel cylinder creates a shadow and a solid plexiglas cylinder behaves as a cylindrical lens (which was noticed in the water tunnel experiments, as an intensified region of light “behind” the cylinder away from the light source). To improve the lighting conditions, Brücker (1995) used a thin-walled plexiglas tube, filled with water, as the cylinder model. A similar water-filled cylinder ($D = 19.1$ mm., 3.2 mm wall thickness) was tested in the towing tank, but this option was rejected since the cylinder noticeably deflected when being towed.

4.5.3 Cylinder End Conditions in the Main Water Towing Tank

In the main water towing tank, the effects of different cylinder end boundary conditions were investigated. This was done using the same towed hot-film probe discussed in Section 4.3.3, that could be positioned in the cylinder wake and along its span. Four different end geometries and their combinations were examined: (i) the upper end of the cylinder protruding directly through the free surface of the water; (ii) the upper end of the cylinder fitted with an end plate; (iii) the lower end of the cylinder near the bottom of the tank left as a simple free end, with two gap distances tested ($g/D = 0.2$ and 0.8 , where g corresponds to the distance between the end of the cylinder and the bottom of the tank); and (iv) the lower end of the cylinder fitted with an end plate. The end plates were designed based on the optimized design of Szepessy (1993), with a streamwise length of $8D$, a cross-stream width of $7D$, and a distance from the cylinder axis to the leading edge of $3.5D$. The end plates were manufactured from plexiglas of 6.4 mm thickness, and machined to a sharp leading edge. The end plate mounted on the lower end of the cylinder was fixed in place, while that for the upper end could easily be positioned at any spanwise location. Under various combinations of the four end configurations, the length of the cylinder immersed in the water ranged from $l = 680$ to 700 mm, the aspect ratio ranged from $AR = 27$ to 28 , and the solid blockage ratio (excluding the end plates) ranged from 3.4% to 3.5% .

The effects of the end conditions were investigated by acquiring spanwise measurements of the vortex shedding frequency f_s (the measurement uncertainty in St was $\pm 10\%$), at $x/D = 5.0$, $y/D = 0.8$, and from the centre span ($z/D = 0$) to the upper or lower ends ($z/D = \pm 13.5$). Towing velocities ranged from $U = 50$ to 200 mm/s, corresponding to $Re = 1300$ to 5000 . For each set of boundary conditions, f_s and St at $z/D = 0$ were the same, within the range of experimental uncertainty (Figure 4.7).

For a large gap, $g/D = 0.8$ (Figure 4.7(a)), there is an affected region of flow that extends between $5.5D$ to $7.5D$ from the free end. This result is consistent with an affected region of $7D$ noted by Fox & West (1993a,b) for a finite cylinder of $30D$ at $Re =$

44,000. This affected region includes a span of $2D$ to $4D$ with a measurable, lower shedding frequency corresponding to $St = 0.15$ that is independent of the Reynolds number, which is consistent with Farivar (1981) who obtained $St = 0.16$ and Fox & West (1993a,b) who reported $St = 0.15$ to 0.18 for the affected region. There is also a remaining spanwise portion of $2.5D$ to $3.5D$ for which there is no measurable shedding frequency, that is supported by the results of Luo *et al.* (1996). The results (Figure 4.7(a)) show that the region of affected flow is Re-dependent, extending further from the end at lower Reynolds numbers (with some 50% of the span below the centreline experiencing lower-frequency or nonexistent vortex shedding).

The results for the small gap, $g/D = 0.2$, show a reduced span for the region of affected flow, but with a similar Re-dependence (Figure 4.7(b)). The affected region extends $3.5D$ to $4.5D$ from the free end, corresponding to some 30% of the span below the centreline. A region of lower shedding frequency, with $St = 0.17$, ranged from $2D$ to $3D$, and was independent of the Reynolds number. A region with no measurable vortex shedding extended only $1.5D$ from the free end. The shorter affected region is caused by a reduction in the downwash about the free end of the cylinder at the smaller g/D .

By fitting an end plate to the cylinder, the affected region was effectively eliminated, within the limits of experimental uncertainty (Figure 4.7(c)). A constant Strouhal number could be detected to within $0.25D$ of the end plate. A similar result was noticed for the free surface end condition (Figure 4.7(d)), although an affected region of $1D$ to $3D$ could have been expected based on experiments of Vlachos *et al.* (1998) and Warncke Lang & Gharib (1998).

The results showed that the fluid behaviour at the centre of the span was independent of any choice of end boundary condition. The simplest combination of end conditions, with the cylinder directly penetrating the free surface and a free end of $g/D = 0.2$, seemed to ensure spanwise independence of the shedding frequency over nearly 80% of the cylinder span, within the limits of experimental uncertainty. Since spanwise uniformity of the flow was the desired result, it was this combination of end conditions which was used for the single circular cylinder experiments. It is noted, however, that the

constancy of the vortex shedding frequency along the span does not indicate that the flow is strictly two-dimensional, since oblique shedding conditions could still exist (Williamson 1989). Rather, the uniformity ensured that the PIV measurements would not be acquired within one of the affected flow regions, where more complex three-dimensional fluid motion exists (see the “knots” in the flow visualization results of Slaouti & Gerrard (1981), for example).

In the multiple cylinder experiments, circular end plates of diameter $3.5D + P + 3.5D$, for centre-to-centre pitch P , were fitted to the configurations near the bottom of the towing tank. The diameter was based on the optimized design of Szepessy (1993) for rectangular end plates, noted above. The end plates were used primarily to ensure proper alignment of the cylinders along their spans, rather than to improve the end conditions.

4.5.4 Cylinders Used in the Vertical Water Towing Tank

The circular cylinders used in the vertical towing tank, which examined conditions of nearly impulsively started motion, were constructed of plexiglas. Each cylinder had a diameter of $D = 25.4$ mm, an aspect ratio of $AR = 9$ and a solid blockage ratio of 4.2%.* The cylinders were mounted in a cantilevered fashion to the mobile platform on the rear wall of the tank. No end plates were fitted to the cylinders, and the gap between the cylinder and the front wall of the tank was $0.2D$.

* Although the aspect ratio of the plexiglas cylinders was low, it is thought that aspect ratio effects may be less significant under impulsively started conditions. Previously published experiments on an impulsively started circular cylinder have used cylinders of $AR = 8$ to 42, and good agreement between the experimental results, and with two-dimensional numerical simulations, suggests that three-dimensional effects are unimportant as the flow starts from rest. Only Polidori *et al.* (1992) have examined, briefly, the influence of the cylinder end conditions in impulsively started flow, without any significant conclusions. In terms of the solid blockage ratio, Nagata *et al.* (1975) suggest minimum wall interference effects for blockage ratios of less than 10%.

Chapter 5

Instrumentation and Data Analysis

5.1 Introduction

In investigating the fluid dynamics of the multiple cylinder configurations, a three-fold experimental approach was followed, using flow visualization, hot-film anemometry and particle image velocimetry (PIV). Flow visualization was used first, in the water tunnel and the vertical water towing tank, to gain an initial qualitative understanding of the fluid behaviour of nearly every multiple cylinder configuration examined, under steady upstream cross-flow and impulsively started conditions. Flow visualization images from the water tunnel were also used to estimate the vortex shedding frequencies in the flow fields. Hot-film anemometry was used next, to provide supplementary information on the range of dominant frequencies in the flow fields. This technique was chosen since vortex shedding frequencies could not be measured with the PIV system, and because low-frequency events were difficult to measure with flow visualization. By conducting the flow visualization experiments first, the hot-film sensor could be positioned more effectively in the water tunnel, and the fluid dynamic origins of the frequencies could be better assessed. The PIV technique was the third and last technique employed, in both the vertical towing tank and the main water towing tank. Interpretation of the instantaneous velocity and vorticity data provided by the PIV technique was greatly aided by the flow visualization experiments conducted beforehand.

5.2 Flow Visualization

Conventional dye injection flow visualization was adopted for the water tunnel experiments, using fluorescein and rhodamine dyes, similar to the experiments of Price & Serdula (1995), Grépin (1996), Wong (1997) and Smith (1998). The experimental setup in the water tunnel is shown in Figure 5.1. The injected dye forms a streakline within the flow field, which by definition is a line joining all fluid elements that have passed through a given point, namely the dye injection point (Lugt 1983; Yang 1989). In a periodic or unsteady flow, streaklines differ from other common representations of a flow, namely pathlines and streamlines, meaning that care must be taken in interpreting the fluid motion from injection techniques, particularly far downstream from the injection point. In this study, the dye injection method was effective at showing the vortex formation and shedding processes in the near-wake regions of the cylinders, close to the points of dye injection, for example, but rapid diffusion of the dye within the turbulent combined wake of the cylinders rendered the method less useful thereafter.

In the water tunnel experiments, the two dyes were injected into the flow through ports on the cylinder near the points of laminar boundary layer separation, as noted in the description of the cylinders in Section 4.5.1. A thin section of light, generated by one or two xenon arc lamps, illuminated the flow, and images of the dye streaklines were acquired by a JVC KY-25U Super-VHS video camera (shown in Figure 5.1). Digital images were produced from individual video frames using a video printer and a scanner, and the images were enhanced using Adobe Photoshop 3.0 or Corel PhotoPaint 7.0 software.

A second flow visualization method, employing seeding particles, was used in the vertical towing tank when investigating multiple cylinder configurations in impulsively started flow. Nearly neutrally buoyant particles (described in Section 5.4.3) were suspended in the water, and some of the heavier particles were allowed to settle on the upper surfaces of the cylinder models. As the cylinders were set into motion, the particle motions were recorded using the laser light sheet and the digital camera of the PIV system

(described in Sections 5.4.2 and 5.4.4), using longer exposure times than used for PIV such that the moving particles appeared as short, bright streaks. This method proved to be effective, but like other visualization techniques employing tracer particles, its accuracy is sensitive to differences in density between the fluid and the particles, and the sizes and shapes of the particles (Yang 1989).

5.3 Hot-Film Anemometry

A single-component hot-film anemometer probe was used to measure vortex shedding frequencies in the water tunnel and in the main water towing tank, to assess the background turbulence signature in the towing tank (as discussed previously in Section 4.3.3), and to determine the effects of the cylinder end conditions in the towing tank (discussed previously in Section 4.5.3). The system consisted of a TSI FlowPoint Model 1500 anemometer and data acquisition software, and either a TSI Model 1231 conical sensor or a TSI Model 1269 wedge-type sensor. Each probe was regularly calibrated over the speed ranges of the water tunnel and the towing tank.

The hot-film probe was used to obtain time histories of the streamwise velocity component, from which the power spectrum was computed to determine the dominant (vortex shedding) frequencies in the flow. In the water tunnel, the sampling rate was typically 20 Hz, and the sampling time ranged from 5 to 10 minutes (300 to 600 s). For vortex shedding frequencies of $f_s = 0.5$ to 2 Hz, the sampling time covered 150 to 1200 vortex shedding periods, with a measurement uncertainty in the Strouhal number of $\pm 1\%$. The probe was located at the centre span of the cylinder models, corresponding to $z/D = 0$, and at various streamwise and cross-stream positions in the combined wake of the cylinders (shown in Figure 5.1), from $x/D = 2.0$ to 5.0, and from $y/D = -4.0$ to 4.0. In the towing tank, the sampling time was limited to 5 to 20 vortex shedding periods, using a sampling rate of 50 to 200 Hz. The measurement uncertainty in the Strouhal number was ± 10 to 20%.

5.4 Particle Image Velocimetry (PIV)

Particle image velocimetry (PIV) is a non-intrusive, full-field measurement technique, suitable for nominally two-dimensional flow fields, by which a velocity field is measured (velocimetry) based on sequentially acquired images of seeding particles introduced into the fluid medium, as these particles follow the fluid motion. The particles are illuminated by a thin sheet of light produced by a laser and a cylindrical lens, and successive images of the illuminated seeding particles are recorded with a camera, from which average in-plane particle displacements are found. Velocity, vorticity and streamline fields are then readily computed from the particle displacement information, and it is the ease of determining an in-plane vorticity field that makes PIV a valuable tool. The PIV technique has been recently reviewed by Raffel *et al.* (1998), where the development of specialized, three-dimensional PIV systems, such as stereo PIV, holographic PIV, and dual-plane PIV, is described.

The data obtained from PIV are typically instantaneous in nature, rather than phase-averaged or time-averaged. PIV has been applied by a number of researchers to the flow past a single circular cylinder (Wu *et al.* 1994a, 1996; Lin *et al.* 1995a,b; Brede *et al.* 1996; Chyu & Rockwell 1996a,b; Lin *et al.* 1996; Soria 1996; Noca *et al.* 1998), and also to multiple circular cylinders but in a limited fashion (Kwon *et al.* 1996). Constant temperature anemometry (CTA) and laser Doppler velocimetry (LDV), can also be used to find the vorticity field, but since they are point-measurement techniques, the process becomes time-intensive, as a phase-averaging approach is needed to ensure a meaningful description of the flow field over a vortex shedding period (since time-averaged measurements of vorticity are of little value in a fundamentally periodic flow). The time-intensive nature of this technique usually means that the resolution of the vorticity field is lower than that obtained with PIV. Phase-averaging techniques have been applied to cylinder flow fields by Cantwell & Coles (1983), Hayakawa & Hussain (1987, 1989), and Lyn *et al.* (1995); and to a multiple cylinder flow field by Kolar *et al.* (1997). The advantages of CTA and LDV are that three velocity components are measured rather than

just two in conventional PIV, and the entire vorticity vector field can be obtained rather than an in-plane, two-dimensional field.

5.4.1 Digital Cross-Correlation PIV

In this study, the digital counterpart of conventional PIV was adopted, based on the method of Willert & Gharib (1991) and Willert (1992). An overview of the system is shown in Figure 5.2. Particle images were recorded digitally rather than on film or photographic transparencies, and all analysis was performed computationally without the need for time-intensive photographic processing and optical interrogation. With digital PIV (or DPIV, as some refer to it) there is only one exposure of the particle images on a single frame, allowing information in two successive digitized images to be spatially cross-correlated to yield the particle displacement information. The spatial cross-correlation process is undertaken in small corresponding subsections of the pair of images, each known as an interrogation window. The location of the cross-correlation peak within the interrogation window then corresponds to the average particle shift or displacement of all the individual particles within the given interrogation window. By employing a local spatial cross-correlation between two sequential single-exposed particle images, the directional ambiguity associated with conventional PIV is removed (the latter technique employs doubly exposed images and uses an autocorrelation process). The result is a rectangular grid of displacement vectors, from which the velocity field is obtained by dividing by the time between successive images. A thorough treatment of the spatial cross-correlation process, and the extraction of the displacement vector field, can be found in Willert (1992) and Raffel *et al.* (1998).

More specifically, a Dantec FlowGrabber digital PIV system was used in this study, with additional software from the Graduate Aeronautical Laboratories of the California Institute of Technology. Other software was programmed by the author. Images of the seeding particles were captured by a digital camera, which contains a photo-sensitive CCD array, and sequences of images were stored in the memory of a frame grabber card on a personal computer.

5.4.2 Illumination System

PIV images are recorded from a section of the flow illuminated by a sheet of light parallel to the sensor array of the digital camera. For experiments in the main water towing tank, where the cylinders were suspended vertically from the platform (or carriage), the light sheet was in the horizontal plane, as shown in Figure 5.2. For experiments in the vertical towing tank, where the cylinders were horizontal, the light sheet was instead in the vertical plane. The light sheet (green, 514.5 nm wavelength) was 3 to 5 mm in thickness, and was generated by a 5 W Spectraphysics Model 165-09 argon-ion laser, a fibreoptic manipulator, a multimode fibreoptic cable, and a light sheet probe with a cylindrical lens, each shown in Figure 5.2.* Other components of the illumination system included a mechanical shutter and a pulse timing generator, also shown in Figure 5.2, to pulse the laser light for proper exposure of the particle images, and to ensure its synchronization with the digital camera. For most experiments, the cylinders were illuminated from the downstream direction and from the side, to minimize the effect of the shadow created by the opaque stainless steel cylinders in the main towing tank, or to minimize a cylindrical lens effect created by the transparent plexiglas cylinders in the vertical towing tank, and to ensure complete illumination of the wake.

5.4.3 Seeding Particles

The seeding particles used for PIV must be small enough to accurately follow the fluid motion, while at the same time be large enough to scatter or emit sufficient light to enable them to be properly imaged (Willert 1992). Parameters of interest include the tracking error, which is the difference between the local velocity of the fluid and the velocity of the particle; the shape of the particles; the mean particle diameter and the size range of the particles; and the relative density between the particle and the fluid (Willert 1992). The ideal particle is (i) small and spherical in shape, to minimize aerodynamic effects in

* Use of the fibreoptic cable meant that the light sheet probe could be easily positioned without the need for complicated optics. The drawback to its use is attenuation of light intensity from the laser. The cylindrical lens produced a fan-shaped light sheet with a reduction of intensity at its edges.

regions of high shear and to accurately follow the fluid, and (ii) neutrally buoyant, to minimize the tracking error and the sedimentation (or settling) velocity. Spherical particles, however, are poor at scattering light at 90° to the light source, as required by the PIV technique. The particle seeding density is also important for PIV, and like the other parameters, must be matched to the imaging system, magnification, and other experimental conditions.

In the course of this study, a number of different seeding particles were tested. In the main water towing tank, the initial choice was fluorescent polymer spheres of diameter $d_p = 80$ to $180\ \mu\text{m}$, manufactured by Prof. J. Katz at The Johns Hopkins University. These particles were impregnated with a combination of rhodamine and fluorescein dyes, and under illumination by the argon-ion laser light sheet, the particles fluoresced bright orange-yellow, considerably improving the signal-to-noise ratio of the imaging system. Although these particles proved to be remarkably neutrally buoyant, the mean particle diameter proved to be too small for the intended application. Spherical amberlite resin particles of $d_p = 150$ to $250\ \mu\text{m}$ were then selected, with a specific gravity of $s.g. = 1.02$. The amberlite particles were immersed in a solution of methanol and rhodamine dye in order to make them fluorescent. The methanol served to break the surface tension and release trapped air from the porous surface area of the particles, enabling the rhodamine dye to impregnate the surface. The particles were then thoroughly rinsed in tap water to remove residual rhodamine dye. Although the manufacture of these particles proved relatively simple, the process of dye-impregnation rendered the particles heavier, with $s.g. = 1.07$ to 1.10 , and the sedimentation rate was found to be unacceptable. In order to use the dye-impregnated amberlite-resin particles in the towing tank, the density of the fluid medium would have to be changed to render the particles nearly neutrally buoyant. As discussed in Section 4.3.2, this process was problematic, and the amberlite-resin particles were not used. Some irregularly shaped, white, non-fluorescent, and nearly neutrally buoyant ($s.g. = 1.03$) granular particles were found to be satisfactory, however, with a size range of $d_p = 100$ to $500\ \mu\text{m}$. These particles were chosen to seed both the vertical towing tank and the main water towing

tank for all of the experiments reported in the thesis, and a discussion of their performance and suitability is found in Appendix A. Large-diameter seeding particles, of comparable size, were also used by Anderson *et al.* (1998) and Leweke & Williamson (1998), both who used a similar digital PIV technique.

Because the seeding particles were not strictly neutrally buoyant, the particles were redistributed throughout the water in each of the towing tanks before beginning a series of new experiments. Heavier particles that settled more quickly to the bottom of the tank interfered with the imaging system, and these, in particular, were redistributed between tests. The background motion was then allowed to dissipate for several minutes before another experiment was conducted.*

5.4.4 Imaging System

The imaging system for the PIV system included a Dantec FlowGrabber Double Image 700 digital camera, shown in Figure 5.2, that was interfaced to a microcomputer with an EPIX 4MEG video frame grabber using standard RS-170 video as an input. The frame grabber enabled 16 images (of 250 kBytes each) to be acquired with a resolution of 512×480 pixels (Figure 5.3). The camera was mounted in a fixed position (laboratory reference frame), on the positioning rail beneath the main towing tank (with a first-surface mirror), or on a tripod next to the small vertical towing tank. A proximity sensor and switch on the main towing tank was used to initiate image capture. The pulse timing generator/video trigger generator, shown in Figure 5.2, was used to synchronize the image acquisition process and the exposure time (length of light pulse duration).

Asynchronous image capture enabled two images to be acquired in quick succession, at a time difference less than the standard video frame rate of 30 Hz. Timing

* Dong *et al.* (1997), in their towing tank, waited only several minutes for the background motion to subside once the seeding particles were injected, before commencing an experiment. In contrast to this study, their particles were injected locally to avoid seeding the entire towing tank. Leweke & Williamson (1998) waited about 30 minutes for the background fluid motion to dissipate, after the seeding particles were uniformly distributed.

was set to control the duration of the light sheet pulse, typically 5 ms, and to control the time difference between pulses, typically 5 ms also, yielding a time difference of about 10 ms between successive images in an image pair. The time duration of the laser pulse is a compromise between particle streaking and particle intensity, whereas the time difference between image pairs is tailored to the maximum expected velocity in the flow and the maximum particle displacement which can be tolerated. The frame grabber enabled pairs of images to be acquired at intervals of 0.2 s, or at a rate of 5 Hz.

5.5 PIV Implementation

Use of PIV requires some previous knowledge of the flow, such as the range of velocities and the sizes of various structures or regions of interest. This knowledge is used to set such parameters as the image magnification and size, and the particle size and seeding density, for example.

5.5.1 Image Magnification and Dimensions

The field of view is determined by the focal distance between the camera and the light sheet, and is specified by the horizontal and vertical image magnification factors M_x and M_y , respectively. The magnification factors are also equal to the horizontal and vertical dimensions of a single pixel. For CCD cameras with rectangular pixels, $M_x \neq M_y$, and their difference is defined by the squareness ratio, $sq = M_y/M_x$, with $sq = 1.167$ for the given camera. The magnification is found by imaging an object of known length (typically the cylinder diameter), and ranged from $M_x = 0.2$ to 0.3 mm/pixel in these experiments, in order for the cylinders and their wake regions to be imaged. Since a mean particle image diameter slightly larger than the mean pixel “diameter” is recommended (mean particle diameters of 2 to 3 pixels were used by Willert & Gharib (1991)), seeding particles on the order of $d_p \sim 200 \mu\text{m}$ were needed in the towing tank (hence the range of particle diameters used, as mentioned in Section 5.4.3).

In this study, the 512×480 pixel CCD array imaged flow field regions from about 100×100 mm ($\sim 4D \times 4D$) to 150×150 mm ($\sim 6D \times 6D$). The local spatial cross-

correlation between the two images is performed within an interrogation window that is a subset of the entire image (Figure 5.4), and the interrogation window size was set at 32×32 pixels in this study. Each interrogation window represents one local spatial cross-correlation operation, yielding a single displacement vector. Subsequent interrogation windows are usually overlapped onto the immediately preceding interrogation window, as a form of oversampling. In these experiments, by moving the interrogation window about the image in increments of 8 pixels (75% overlap) in both directions, a rectangular grid of about 57×57 vectors was generated, at spacings from 1.5 to 2.5 mm (0.05 to $0.1D$).

5.5.2 Measurement Resolution

For an accurate spatial cross-correlation, each interrogation window should contain between five and twenty particle images (Willert & Gharib 1991). The seeding density of the towing tank can then be estimated, but in practice it is determined by trial-and-error. The maximum particle displacement should be limited to $1/3$ or $1/4$ the size of the interrogation window, to minimize the number of particles leaving or entering the window during the time Δt between the two images comprising a pair. In this study, the maximum displacement was 8 pixels (~ 1.5 to 2.5 mm, or 0.05 to $0.1D$). A maximum expected velocity of 200 mm/s gives a maximum $\Delta t = 10$ ms.

The velocity dynamic range, which is the ratio of the largest to smallest measurable displacements, is primarily affected by the interrogation window size (Willert & Gharib 1991; Willert 1992). A small interrogation window will mean more particles leaving and entering the interrogation window between the two images comprising the pair, and consequently fewer particles will be matched (for same Δt , seeding density and displacement). Willert (1992) estimated the velocity dynamic range for digital PIV at 200:1 to 1000:1 for linear displacements.

The spatial resolution, which represents the range of sizes of flow structure which can be detected, is primarily limited by the particle seeding density (Willert & Gharib 1991; Willert 1992). Lower seeding densities mean that less of the finer-scale structure

of the flow can be resolved. A smaller interrogation window and step size will also improve the resolution, particularly for smaller flow structures and in regions of higher velocity gradients, yet a reduction in window size simultaneously reduces the velocity dynamic range, which is a more pronounced effect (Willert & Gharib 1991).

5.5.3 Measurement Uncertainty

The measurement uncertainty of PIV is influenced by a number of factors, such as particle image size, interrogation window size, particle seeding density*, the number of particles in the interrogation window, computational and instrumentation effects, and flow properties such as velocity gradients, curvature of the particle trajectories, and three-dimensional (out-of-plane) motion, for example. According to Willert & Gharib (1991), the displacement error may range from 1 to 10% depending on the seeding density and the maximum displacement being measured. In this study, the displacement error was estimated by applying, digitally, a known uniform horizontal or vertical displacement to an image. The original image and the shifted image were then cross-correlated, and the computed displacement vector field was compared with the known, applied uniform displacement. The results in Figure 5.5, which are expressed as a standard error (r.m.s.) in the computed displacement vector magnitude, show an error of up to 5% for prescribed 1 to 10 pixel displacements, which yields an uncertainty of 10% in the vorticity field. In other studies using the digital PIV technique, Willert (1992) reported a displacement error from 1 to 10% and a vorticity error of 5 to 10%, Anderson *et al.* (1998) reported a displacement error of 5% and an uncertainty of 3 to 15% in circulation (or vortex

* A lower seeding density will produce a higher error (broader cross-correlation peak) if there is a velocity gradient. The size and shape of the cross-correlation peak can itself be used as a gauge of measurement accuracy, where a higher and more defined peak represents a better cross-correlation, and a more accurate measure of displacement. The FlowGrabber software expressed the peak size and shape as a confidence value between 0.5 and 1.0. Confidence statistics typically averaged around 0.95 to 0.98 in this work.

strength), and Leweke & Williamson (1998) estimated a 6% uncertainty in circulation (or vortex strength).

5.5.4 Three-Dimensional Effects

As mentioned, the PIV technique is suited to measurements in predominantly two-dimensional flow fields, such as that of a circular cylinder in cross-flow, where the u - v velocity components commonly of interest lie in the x - y plane (which is also the plane of the light sheet). It is now well-established, however, that the flow field of the circular cylinder has an intrinsic three-dimensional structure for most Reynolds numbers (Williamson 1996b). This means that in addition to the in-plane velocity components, there may also be a substantial out-of-plane (z -direction) velocity component field, w , normal to the plane of the light sheet. Out-of-plane particle motion means an increased likelihood for the loss of particle-pair matches in the spatial cross-correlation, as particles enter and leave the interrogation window in the z -direction. This leads to an increase in the measurement uncertainty for the in-plane particle displacements (or the velocity components of interest, u and v). Recognizing this, care must be taken in the implementation of the PIV technique, to ensure that the motion of the seeding particles in the z -direction, normal to the plane of the light sheet, is small compared to the light sheet thickness. The effects can be minimized with an appropriate choice of Δt (Raffel *et al.* 1998). In strongly three-dimensional flows, additional uncertainty in the in-plane velocity components may also be caused by perspective projection errors, particularly for particles nearest to edges of the images (Raffel *et al.* 1998).

5.6 PIV Data Processing

Before computing the velocity and vorticity fields, a number of corrections are typically made to the raw displacement vector field obtained from the cross-correlation process (Figure 5.6). In this study, corrections were made to account for the presence of the cylinders in the images and to remove spurious vectors. Particle offset bias correction (shown in Figure 5.6) was not employed, since the correction was extremely minor in

most cases. This correction accounts for uneven distributions or intensities of the particle images, which may bias the location of the cross-correlation peak and the centre for the averaged displacement.

5.6.1 Accounting for the Presence of the Cylinder(s) in the Images

In images which include the cylinder model(s), the cross-correlation process computes erroneous displacement vectors in their vicinity, as shown in Figure 5.7(a). Additional software was written to remove the displacement vectors at the location(s) of the cylinder(s), as shown in Figure 5.7(b), to a distance of one-half the size of the interrogation window greater than the cylinder radius (for the case of 75% overlap), in order to account for interrogation windows overlapping both the fluid and a cylinder. Consequently, the velocity and vorticity fields cannot be resolved close to the surface(s) of the cylinder(s), to a distance of 16 pixels (~ 3 to 5 mm, or 0.1 to 0.2 D).

5.6.2 Removal of Spurious Vectors

Regions of low particle seeding density, high velocity gradient, and significant rotation, and differences in particle image intensity, may cause the cross-correlation process to compute erroneous (or “outlier”) displacement vectors (Willert 1992). These vectors are also computed in the location of the cylinder model and the shadow (shown in Figures 5.2 and 5.3), where there are no particle images (also seen in Figures 5.7(a,b)). “Outlier” vectors are identified by searching for a vector that is significantly different in magnitude and direction from its neighbours. If the difference exceeds a certain threshold, the vector is removed and reinterpolated. In this work, a 4 pixel threshold was used for the 32×32 pixel interrogation window with 75% overlap (as used by Willert 1992), leading to some 100 to 500 vectors being removed and reinterpolated depending on whether the cylinder model and/or shadow is present, and representing between 3% to 15% of the entire field. A reinterpolated field is shown in Figure 5.7(c). Based on simulations, Høst-Madsen & McCluskey (1994) report that a raw displacement field containing up to 25% randomly

distributed spurious vectors can still yield an accurate result once the “outliers” are removed.

5.6.3 Smoothing of the Displacement Field

Some form of data smoothing is commonly employed in the presentation of PIV velocity data, with its primary purpose to give a more pleasing appearance to isovorticity contour lines.^{*} According to Willert (1992), because the particle image field is oversampled by overlapping the interrogation windows, the displacement vector field can be neighbourhood smoothed with a smoothing “kernel” smaller than the interrogation window. In this study, a 3×3 grid point spatial filter was used, with weighting based on the distance from the central grid point (Willert 1992), permissible for a 32×32 pixel interrogation window with 75% overlap, a vector grid spacing of 8 pixels, and a grid of 4×4 vectors within each window (see results in Figure 5.7(d)).

5.7 Vorticity and the Identification of Vortices

The velocity field (Figure 5.7(e)) is found from the smoothed displacement vector field, by multiplying by the magnifications M_x and M_y (defined in Section 5.5.1), and dividing through by the time difference between images in a pair, Δt . The in-plane vorticity field (Figure 5.7(f)), of instantaneous nature in this study, is then found, which allows flow structures such as shear layers and vortices to be easily identified. The in-plane vorticity ω_z is the curl of the in-plane velocity field, denoted by streamwise and transverse components u and v , respectively (with positive vorticity in the CCW direction):

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \quad (5.1)$$

^{*} Using a standard error estimate based on the maximum vorticity within a given field, smoothing altered the velocity and vorticity fields by about 10%, which is within the limits of measurement uncertainty. Data smoothing was also programmed to account for the blanked-off region representing the cylinder model(s).

Because the calculation of the in-plane vorticity ω_z involves velocity differences, it is invariant with the reference frame (i.e. moving with the flow, with the cylinder, or at the convection speed of the vortices), unlike the more popular streamline representations in which vortex identification can be misleading (Lugt 1983; Gursul *et al.* 1990).

5.7.1 Computing the Vorticity Field

Vorticity is typically determined by a finite difference method, since it represents a derivative quantity. Since difference methods tend to amplify existing measurement uncertainties (Willert 1992), ω_z is often computed from the local circulation $\Gamma_{i,j}$ about a given grid point (i,j) , computed along the path of the eight nearest neighbours using the trapezoidal rule, and grid spacings δx , δy , as shown in Figure 5.8. This approach uses integration rather than differentiation, and the circulation is computed as

$$\begin{aligned} \Gamma_{i,j} = & \left(u_{i-l,j-l} + u_{i,j-l} \right) \frac{\delta x}{2} + \left(u_{i,j-l} + u_{i-l,j-l} \right) \frac{\delta x}{2} \\ & + \left(v_{i-l,j-l} + v_{i-l,j} \right) \frac{\delta y}{2} + \left(v_{i-l,j} + v_{i-l,j-l} \right) \frac{\delta y}{2} \\ & - \left(u_{i-l,j-l} + u_{i,j-l} \right) \frac{\delta x}{2} - \left(u_{i,j-l} + u_{i-l,j-l} \right) \frac{\delta x}{2} \\ & - \left(v_{i-l,j-l} + v_{i-l,j} \right) \frac{\delta y}{2} - \left(v_{i-l,j} + v_{i-l,j-l} \right) \frac{\delta y}{2}. \end{aligned} \quad (5.2)$$

The vorticity at grid point (i,j) is then computed from the local circulation as

$$(\omega_z)_{i,j} = \frac{\Gamma_{i,j}}{4\delta x\delta y}. \quad (5.3)$$

In either method, vorticity cannot be reliably computed near a solid boundary (such as the surface of the cylinder), since some of the nine grid points used may themselves lie beyond the boundary (or on the cylinder). Although in principle these grid points could assume the towing velocity U of the cylinder, the accuracy of the computed vorticity was felt to be questionable, and this approach was not followed. Rather, vorticity near the

cylinder was simply not computed. As mentioned in Section 5.6.1, this usually affects a zone next to the cylinder of 3 to 5 mm (0.1 to 0.2D).

5.7.2 Vortex Identification and Description

The definition of a vortex is varied, but has been defined as “the rotating motion of a multitude of material particles around a common centre” (Lugt 1983). Here, it is more convenient to consider a vortex as a concentration of like-sign vorticity within a specified closed boundary, in which a peak value of vorticity ω_{zp} may be found. Ideal models for a vortex consider this concentration to be circular in shape, with ω_{zp} at the centre. The extent of appreciable vorticity is contained within a core bounded by an irrotational outer region in which the tangential velocity slowly decays. The Oseen model for a vortex has been used successfully to model vortices in the turbulent cylinder wake (Zhou & Antonia 1993): the tangential velocity u_θ at radius r , for strength Γ_o and core radius r_1 , is given by

$$u_\theta = \frac{\Gamma_o}{2\pi r} \left[1 - \exp\left(-1.26(r/r_1)^2\right) \right]. \quad (5.4)$$

In a real flow field, a vortex may exist in the presence of other vortical structures, and consequently lie in the presence of a strain field. Under such conditions, the vortex boundary may no longer be circular in shape. While ω_{zp} may still be used to identify the existence of a vortex within a vorticity field, the noncircular vortex shape may mean that the peak value within the boundary no longer properly represents a vortex centre. In such cases, the vorticity centroid (x_c, y_c) may be more appropriate to define the vortex location (Cantwell & Coles 1983):

$$x_c = \frac{\sum_{i,j} x_{i,j} (\omega_z)_{i,j} \delta x \delta y}{\sum_{i,j} (\omega_z)_{i,j} \delta x \delta y}, \quad (5.5)$$

$$y_c = \frac{\sum_{i,j} y_{i,j} (\omega_z)_{i,j} \delta x \delta y}{\sum_{i,j} (\omega_z)_{i,j} \delta x \delta y}.$$

Whether circular or irregular in shape, the boundary of a vortex may be a contour of constant ω_z that (i) encloses the concentration of vorticity being considered (along with the identifying peak), and (ii) does not render the vorticity field messy. The local in-plane velocity components along the bounding contour are then used to compute the vortex strength or circulation Γ , for local velocity vector $\vec{u}$ and closed path element $d\vec{l}$:

$$\Gamma = \oint_C \vec{u} \cdot d\vec{l} . \quad (5.6)$$

The circulation may be also be found from Stokes' theorem, for vortex area A :

$$\Gamma = \oint_C \vec{u} \cdot d\vec{l} = \iint_A \omega_z dA . \quad (5.7)$$

Determining the value of the bounding isovorticity contour, or the lowest level of meaningful vorticity used to define the vortex, must be done carefully since it will have an influence on Γ , x_c , y_c and A . Its value may be based on (i) the measurement uncertainty; (ii) the smallest amount of vorticity which may be accurately resolved by the measurement system; (iii) the highest level of the residual, background, or freestream vorticity found in the "irrotational" mean flow (see Section 4.3.3); or (iv) a percentage of the maximum vorticity in the flow field. The latter criterion was used by Cantwell & Coles (1983) to find the vortex area for a single circular cylinder, where a lower limit of 10% of the peak vorticity in the vortex was specified.

In this study, two methods were used to compute the circulation. The first method computed Γ using (5.6), along a path of constant ω_z corresponding to 10% of the maximum vorticity in the flow field, and was included as part of the FlowGrabber software. This method was used to compute the vortex strengths for impulsively started single and side-by-side circular cylinders. The second method computed Γ using (5.7), where like-sign vorticity greater than the specified lower limit was summed over the region defining the vortex. This method was applied to the vortex strengths of cylinders in steady upstream flow, and to tandem cylinders under impulsively started conditions. Although both methods give equivalent results, the second method was preferred, since the contour selection process of the first method could only be done on an interactive basis, and was less precise.

The velocity, streamline and vorticity fields may be used to identify vortices; however, the vorticity field is the most appropriate representation of the flow field to use since it is invariant with the choice of reference frame. Vortices which do not remain attached to the cylinder, or convect at a velocity U_c different than the freestream velocity U , will be very difficult to identify using the velocity or streamline fields, since these are reference-frame dependent. Unless the vortex convection velocity U_c is known beforehand, vortex identification using velocity and streamline fields may lead to erroneous interpretations of the fluid behaviour, particularly for unsteady and/or periodic flows (Gursul *et al.* 1990). An example of vortex identification in the wake of a circular cylinder, from this study, is shown in Figure 5.9, using three different representations. In the laboratory reference frame (Figure 5.9(a)), with the cylinder fixed and freestream velocity U , the vortex street cannot be easily identified. In a reference frame which moves with the vortices (Figure 5.9(b)), the vortices are apparent; however, U_c was known beforehand, which may not be the case for a flow not previously investigated. The vortex in question is most clearly identified in the vorticity data (Figure 5.9(c)).

Chapter 6

Single Circular Cylinder in Cross-Flow

6.1 Introduction

Initial experiments were conducted in the water tunnel and the two towing tanks using a single, isolated circular cylinder, in order to assess the reliability of the experimental apparatus and instrumentation. The cylinder was tested in the low subcritical Reynolds number regime, and under conditions of steady upstream cross-flow and impulsively started cross-flow, which were the same conditions to be used in the multiple cylinder experiments to follow. In both imposed mean flow environments, favourable comparison was made with published experimental data.

6.2 Circular Cylinder in Steady Mean Flow

The fluid dynamics of a single circular cylinder under steady upstream mean flow conditions was first investigated using flow visualization and hot-film anemometry in the Kempf & Remmers water tunnel. Dye injection flow visualization images of the cylinder near-wake region clearly illustrated the laminar separation of the cylinder boundary layer, the two free shear layers extending from the separation points, shear layer roll-up, alternate formation and shedding of Kármán vortices, and the beginnings of the Kármán vortex street (Figure 6.1). Hot-film anemometry, used in both the water tunnel and the main water towing tank, provided measurements of the vortex shedding frequency for a range of subcritical Reynolds numbers; a sample time history of the streamwise velocity signal, and a corresponding power spectrum of the streamwise velocity fluctuations, are

shown in Figures 6.2(a,b). Strouhal number data plotted in Figure 6.2(c), from a number of experiments conducted in both the water tunnel and the towing tank, show a mean $St = 0.2$ for $Re = 250$ to $25,000$. This mean Strouhal number, and its nearly constant value for the range of Reynolds number tested, compare favourably with published data (Zdravkovich 1997). Strouhal number data obtained in the towing tank were higher than those obtained in the water tunnel, at equivalent Reynolds numbers, but remained within the range of Strouhal numbers measured by other researchers (Zdravkovich 1997). The differences in the Strouhal number data between the water tunnel and the towing tank are likely caused by different experimental conditions, such as blockage, cylinder aspect ratio, cylinder end conditions, turbulence levels, and by a higher degree of measurement uncertainty for the hot-film probe measurements in the towing tank (see Section 5.3).

The single circular cylinder was also examined in the main towing tank facility using the PIV system, to obtain time histories of the instantaneous, in-plane velocity and vorticity fields in the near-wake region. For these experiments, the image magnification ranged from $M_x = 0.2$ to 0.3 mm/pixel, and the time difference between image pairs was $\Delta t = 10$ ms. The imaged region of the flow field covered an area between 100×100 mm ($\sim 4D \times 4D$) and 150×150 mm ($\sim 6D \times 6D$), and yielded a spatial resolution for the velocity vector and vorticity fields of 1.5 to 2.5 mm (0.05 to $0.1D$). In the single cylinder experiments, and in the multiple cylinder experiments which followed (described in Chapters 7, 8 and 9), PIV images were acquired near the end of a towing run of about 2.5 m, corresponding to a nondimensional elapsed time from the start of towing of $t^* = tU/D = 98$ at $Re = 1900$ (for elapsed time t from the start of motion).

The velocity and vorticity fields obtained with the PIV system (Figure 6.3) compared favourably with other instantaneous PIV vorticity data for a single circular cylinder (Lin *et al.* 1995a), and also with the streakline patterns from the dye injection flow visualization experiments (Figure 6.1). The velocity and vorticity data (Figure 6.3) clearly show the cylinder near-wake region, the two free shear layers, alternate vortex formation and shedding, and the Kármán vortex street. When compared to streakline representations, however, the instantaneous representation of the vorticity field appears

much more disordered, and the shed vortices in particular appear to have much less coherence or structure. The disorderliness may be removed by averaging a number of instantaneous vorticity fields, as shown by Lin *et al.* (1995a); however, such a technique requires reference to the phase of the vortex shedding period for proper averaging, something that was not available in this study.

In addition to showing the flow structure of the cylinder near-wake region, the PIV velocity and vorticity data were also used to provide quantitative information on the Kármán vortices, up to $x/D = 8.0$. Experiments conducted at $Re = 1900$ yielded measurements for the location of the vorticity centroid (or vortex centre), x_c and y_c , the peak vorticity within the Kármán vortex, ω_{zp} , the vortex circulation or strength, Γ , and the vortex area, A . The vortex convection velocity U_c was also determined, along with the streamwise spacing of the Kármán vortices in one row of the vortex street, l , the vortex street spacing ratio h/l , and vortex formation length, L_f . These data are compared in the subsections that follow, primarily to results from Cantwell & Coles (1983) at $Re = 140,000$ ($AR = 25$, blockage ratio = 4%) for $x/D = 1.0$ to 7.4. Their study is perhaps the most frequently cited by other researchers, and so it is used here, although it was conducted at a considerably higher Reynolds number. Their measurements were obtained by a flying hot-wire technique and a phase-averaging approach. The results of Cantwell & Coles (1983) are not the only published Kármán vortex data available, and other data have been obtained by Kiya & Matsumura (1985) at $Re = 16,000$ ($AR = 15$, blockage ratio = 6.7%) for $x/D = 8.0$; Tanaka & Murata (1986) at $Re = 37,000$ ($AR = 13$, blockage ratio = 4.8%) for $x/D = 0.5$ to 15; Hussain & Hayakawa (1987) at $Re = 13,000$ ($AR = 32$, blockage ratio = 6%) for $x/D = 10$ to 40; and Zhou & Antonia (1993) at $Re = 5600$ ($AR = 28$, blockage ratio = 3.6%) for $x/D = 10$ to 60. Each of these studies used some form of conditional averaging or sampling technique based on hot-wire data. In contrast to these studies, the PIV experiments in this work relied solely on instantaneous measurements of velocity and vorticity, and no phase-averaging approach was followed (as in the work by Lin *et al.* 1995a).

6.2.1 Kármán Vortex Location, Spacing and Size

The centre of a Kármán vortex was defined as the location of the vorticity centroid, at x_c and y_c , subject to a lower vorticity magnitude threshold of $\omega_z D/U = 0.5$. The location of the peak vorticity was only used to identify the existence of a Kármán vortex, but was not used to define its centre, since it rarely coincided with the vorticity centroid. The mismatch between the location of the peak vorticity and the vorticity centroid is attributed to (i) the instantaneous nature of the vorticity data, and (ii) the noncircular shape of the Kármán vortices, which is clearly seen in the vorticity data in Figure 6.3.

As Kármán vortices are shed from a circular cylinder, the vortex centres slowly move away from the wake centreline as the vortices increase in size and reduce in strength. The movement of the vortices away from the centreline with increasing distance from the cylinder is illustrated by the measurements of the transverse location of the vortex centroid, y_c , shown in Figure 6.4(a). Here, the data of Cantwell & Coles (1983) show a mean $y_c/D = 0.35$ for $x/D = 2.6$ to 6.8, and the data of Tanaka & Murata (1986) show that the vortex centre follows a line inclined at approximately 5° to the wake centreline, over $x/D = 3.0$ to 12.0. Instantaneous measurements of y_c obtained by PIV in the present study exhibit a wide range of scatter, however, and yielded a mean $y_c/D = 0.29$ for $x/D = 2.7$ to 6.7.

Within the cylinder wake, both the streamwise and transverse spacings of the vortices in each of the rows of the Kármán vortex street are known to vary with both the Reynolds number and the distance from the cylinder. Published data compiled by Sallet (1969) give a streamwise spacing of $l/D = 4.0$ to 4.3 in the intermediate cylinder wake, for $Re = 1000$ to 5000, although the vortices are spaced closer together nearer to the cylinder. A similar result was found by Cantwell & Coles (1983), with a mean streamwise spacing of $l/D = 4.3$ for $x/D = 2.6$ to 6.8. From the present PIV experiments, a comparable mean streamwise spacing of $l/D = 3.8$ was obtained, for $x/D = 3.5$ to 8.0, although this result was based on the very few vorticity fields which showed more than one Kármán vortex (because of the limited field of view of the camera).

The vortex spacing ratio, of transverse to streamwise spacing of the vortices within an ideal, stable Kármán vortex street, should theoretically have a value of $h/l = 0.28$. Published experimental data for a single cylinder have shown that within the near-wake and intermediate-wake, the spacing ratio is usually smaller than the theoretical value, and it only approaches the theoretical value with increasing distance from the cylinder (Tanaka & Murata 1986; Hussain & Hayakawa 1987). For example, the experiments of Tanaka & Murata (1986) yielded a spacing ratio of $h/l = 0.1$ to 0.3 for $x/D = 3.0$ to 12.0 , while the results of Hussain & Hayakawa (1987) gave $h/l = 0.20$ to 0.43 for $x/D = 10$ to 40 . The present PIV experiments revealed a mean spacing ratio of $h/l = 0.20$ for $x/D = 3.5$ to 8.0 , although as shown in Figure 6.4(a), there was considerable variation in this value.

As the vorticity contained in the Kármán vortex is diffused with increasing distance from the cylinder, the vortex size or area increases. Cantwell & Coles (1983) showed that the vortex area, A , increases by about 14% per diameter between $x/D = 3.6$ and 6.2 (Figure 6.4(b)). The vortex areas computed from the present PIV data were considerably smaller than those of Cantwell & Coles (1983), and a straight line fit to the data indicates an increase of only 4% per diameter between $x/D = 2.7$ and 6.7 . This difference in vortex area may be attributed to (i) the difference in Reynolds number between the two studies, and to (ii) the choice of vorticity magnitude threshold used to define the vortex area.

6.2.2 Peak Vorticity and Vortex Strength Measurements

The peak vorticity ω_p within a shed Kármán vortex decays with streamwise distance from the cylinder, yet measurements of the magnitude of the peak vorticity, and its rate of decay, vary widely in the literature (Figure 6.5). This variation is likely caused by (i) different amounts of averaging and interpolation incorporated in the data-reduction processes; (ii) different spatial resolutions (and velocity vector grid densities) of the measurement systems; (iii) the effective number of velocity vectors or vorticity values used to compute the vorticity field (Soria 1996); and (iv) differences in the Reynolds

number ($Re = 1900$ for this study, compared with $Re = 140,000$ for the study of Cantwell & Coles (1983)). Cantwell & Coles (1983) estimated a peak-vorticity decay rate of 22% per cylinder diameter of vortex travel, near $x/D = 5.0$, whereas the present PIV data indicated a decay rate of about 17% per cylinder diameter near the same streamwise location. The scatter of the PIV peak vorticity data shown in Figure 6.5 reflects the use of instantaneous vorticity data for their computation, as opposed to the ensemble averaging techniques of the other researchers.

The circulation or strength of a shed vortex was computed through integration of like-sign vorticity about the vortex area, that was greater than the lower specified limit on meaningful vorticity (and set as $\omega_z D/U = 0.5$, as previously mentioned). Like the peak vorticity data, Kármán vortex circulation also decays with increasing distance from the cylinder (Figure 6.6). Cantwell & Coles (1983) measured a vortex strength decay rate of 8% per cylinder diameter of vortex travel near $x_c/D = 5$. From the present PIV data, a straight-line fit indicates a decay rate of about 6% per diameter between $x_c/D = 2.7$ and 6.7. The near-wake data of Cantwell & Coles (1983) and Tanaka & Murata (1986) also suggest that the circulation attains a peak value a short distance from the cylinder, near $x_c/D = 3.0$ to 4.0, but this peak was not readily apparent in the PIV data.

6.2.3 Vortex Convection Velocity

Researchers have shown that the Kármán vortex convection velocity, U_c , slowly increases with distance from the cylinder before assuming a nearly constant value (Cantwell & Coles 1983; Tanaka & Murata 1986; Hussain & Hayakawa 1987; Zhou & Antonia 1992). This convection velocity behaviour in the range $x/D = 10$ to 60 is only weakly dependent on the Reynolds number, for $Re = 80$ to 13,700 (Zhou & Antonia 1992). Published results, shown in Figure 6.7, have indicated a constant mean value for the convection speed ranging from $U_c = 0.82U$ (Tanaka & Murata 1986) to $U_c = 0.87U$, $0.875U$, $0.885U$, and $0.89U$ at $x/D = 10$, 20, 30, and 40 respectively (Hussain & Hayakawa 1987). The present PIV experiments yielded a mean convection velocity of $U_c = 0.82U$, and an

apparent increase in speed with distance from the cylinder, although there was considerable scatter in the measurements (Figure 6.7).

6.2.4 Vortex Formation Length

The vortex formation length L_f is a quantitative means of determining the location of shear layer roll-up, vortex formation, and vortex shedding, in the near-wake of a circular cylinder, and it exhibits a remarkable variation with the Reynolds number (Noca *et al.* 1998). This variation may be illustrated by comparing the cylinder flow fields shown in Figures 6.1 and 6.3. Here, the vortex formation length is noticeably longer in the flow visualization experiment, at $Re = 500$ to 1000 (Figure 6.1), when compared to the PIV vorticity results, at $Re = 1900$ (Figure 6.3). This Reynolds number dependence of L_f has been important in understanding the behaviour of the tandem configuration (Section 7.2.), and whether or not the second cylinder is completely immersed inside the vortex formation region of the cylinder upstream (Ljungkrona *et al.* 1991).

The vortex formation length is also strongly correlated with the vortex shedding frequency, and a short vortex formation length (or smaller near-wake region) usually means a higher vortex shedding frequency, and, conversely, a long vortex formation region (or larger near-wake region) corresponds to a lower vortex shedding frequency. This interrelationship between the shedding frequency and the near-wake region size is important in understanding the origins of two or more vortex shedding frequencies in side-by-side configurations (Section 8.2). The vortex formation length also undergoes changes when a cylinder is forced to oscillate, and is a function of the oscillation frequency and amplitude (Chyu & Rockwell 1996c).

Although there are many definitions of the vortex formation length for a circular cylinder (Noca *et al.* 1998), the definition adopted here was based on the vorticity field. Specifically, the vortex formation length was defined as the furthest streamwise extent of the vorticity comprising a fully formed Kármán vortex, when the vortex is still attached to the cylinder (in one field) but just about to be shed (in the subsequent field). The data

obtained from the present PIV experiments show good agreement with published data, in Figure 6.8.

6.3 Circular Cylinder in Impulsively Started Flow

The circular cylinder was investigated under nearly impulsively started conditions in the small vertical water towing facility, using flow visualization and PIV, for $Re = 1500$ to 1900 , $A_p = 0.4$ to 0.6 , and $t^* < 6.0$ (the measurement uncertainty in t^* was estimated at 5%). The acceleration parameter A_p was sufficiently high that the flow could be considered nearly impulsively started, according to the study by Sarpkaya (1991). The flow visualization experiments were undertaken with the seeding particle technique described in Section 5.2, and some results are shown in Figure 6.9 (Sumner *et al.* 1997c). Since the plexiglas cylinders behave as cylindrical lenses in the presence of the laser light sheet, a shadow and an intensified region of light can be seen to lower left of the cylinder models in each of the images. Some results from the PIV experiments are shown in Figure 6.10.

6.3.1 Flow Field Development

The flow visualization images in Figure 6.9 clearly illustrate the temporal development of the flow field as the cylinder is impulsively set into motion, and the growth of the recirculation zone and the formation of the pair of eddies of equal strength and opposite rotation are both seen. Secondary vorticity downstream of the point of separation can also be distinguished, from $t^* = 3.0$ to 4.1 , and some asymmetry within the recirculation zone is noticed at $t^* = 4.1$, with the lower eddy (in the picture) extending further from the cylinder in the streamwise direction.

Similar features of the starting flow field were also found in the PIV experiments (Sumner *et al.* 1997a,b), as shown in the nondimensional velocity and vorticity data in Figure 6.10. Here, the data are rendered nondimensional with the terminal cylinder velocity U , which was determined from the digital images and the PIV data, with an uncertainty of about 3%. In Figure 6.10(a), the velocity field is shown in the experiment

reference frame, with the cylinder being towed and the fluid initially stationary; in Figure 6.10(b), the local cylinder velocity $u < U$ has been superimposed on the velocity field, to create an inertial reference frame. In both reference frames, the recirculation zone and the pair of eddies can be distinguished, however each of the features is best represented by the vorticity data in Figure 6.10(c), shown as isovorticity contour lines.* The only features that could not be distinguished from the PIV data, but could be seen in the flow visualization images, were the secondary vorticity concentrations found downstream of the separation points. Other PIV experiments at higher magnification (not shown) did reveal the regions of secondary vorticity, although their detection was limited by the inability of the PIV technique to resolve the flow field close to the cylinder surface (as discussed in Section 5.6.1).

6.3.2 Temporal Growth of the Recirculation Zone

The growth of the recirculation zone behind the circular cylinder, as it is impulsively set into motion, may be quantified by examining the behaviour of the streamwise position of the primary eddy centres, a , the transverse spacing of the primary eddy centres, b , the length of the recirculation, L_R , and the primary eddy circulation, Γ . Each of the geometrical parameters was defined in Figure 2.5(b), and results from a number of PIV experiments from the present study are plotted alongside previously published data in Figure 6.11. Here, the primary eddy centres were determined graphically from the vorticity contour data, and correspond to the geometric centres of the primary eddies, rather than the locations of the peak vorticity or the vorticity centroid (although under impulsively started conditions, the three locations are nearly equivalent). The length of the recirculation zone was found in a similar manner as the vortex formation length under steady mean flow conditions (Section 6.2.4), by determining the downstream extent of the

* The vorticity contour data in Figure 6.10(c) may be computed from either of the two velocity fields shown, since it is independent of the reference frame. The smallest magnitude vorticity contour plotted was approximately 10 percent of the maximum vorticity in the flow field.

minimum magnitude vorticity that sensibly defined the region. The measurement uncertainty associated with a/D , b/D , and L_R/D was estimated at no more than 5%.

For Γ , the closed path of integration corresponded to the contour line of minimum vorticity magnitude sensibly enclosing the eddy, bounded on the upstream side by the y -axis at the centre of the cylinder. The measurement uncertainty associated with Γ/UD was estimated to be 10%.

As shown in Figure 6.11, the PIV recirculation zone data for a , b , L_R , and Γ compare favourably with the previously published data of Bouard & Coutanceau (1980), Nagata *et al.* (1989), Wu & Chu (1989), and Chu & Liao (1992) for a similar range of Reynolds number. No significant influence of Reynolds number was noticed in the range tested, $Re = 1500$ to 1900 , within the limits of experimental uncertainty; this result is also in agreement with other studies of the impulsively started circular cylinder.

Following the impulsive start, the primary eddies grow in size and their centres are displaced further from the cylinder base, as shown in Figure 6.11(a), although their transverse spacing remains relatively constant for $t^* = 1.0$ to 4.0 , as shown in Figure 6.11(b). The length of the recirculation zone increases in a like manner as the streamwise position of the primary eddy centres increases, as shown in Figure 6.11(c). The recirculation zone length seems to attain a value of one cylinder diameter by $t^* = 4.0$; this may be the maximum length for which the recirculation zone remains symmetric, since for $t^* > 4.0$ to 6.0 the zone is known to develop asymmetry and begin to transition to alternate vortex shedding (although this transitory behaviour was not investigated in this study).

Measurements of the primary eddy circulation, in Figure 6.11(d), show an increase in strength as the recirculation zone develops in time, and the strength seems to approach a maximum value of $\Gamma/UD = 3.0$ by $t^* = 4.0$. This maximum value of the *attached* primary eddy circulation, at the end of the impulsive start, is close to the maximum value of the *shed* Kármán vortex circulation, under steady mean flow conditions (as shown in Figure 6.6 for $x/D < 4.0$). This agreement suggests that a significant portion of the primary eddy circulation may be fed into the first Kármán

vortex that is formed, a level that may be maintained as regular, alternate vortex shedding is established; further study of the transition between the impulsive start and steady cross-flow conditions is needed to determine if this indeed is the case.* Small differences between the published eddy circulation data (Nagata *et al.* 1989) and those obtained from the present PIV data reflect (i) the extreme sensitivity and difficulty of computing such quantities (Nagata *et al.* 1985a,b); (ii) the use of slightly different definitions for the bounding contours of the recirculation zone, particularly on the upstream side; and (iii) the use of different lower limits for the meaningful vorticity considered in computing the circulation.

6.4 Summary

This investigation of the single circular cylinder, under steady mean cross-flow and nearly impulsively started conditions, showed that good agreement existed between the three investigative techniques of flow visualization, hot-film anemometry and PIV. In particular, there was a basic correspondence between the dye streaklines, which are influenced by the “integrated” history of the dye and by diffusion, and the in-plane vorticity field, which is instantaneous in nature, as both methods illustrated the same features of the flow. This correspondence is significant, since it reduces the possibility of misinterpreting the fluid behaviour by considering instantaneous velocity, streamline, or vorticity data on their own. This would be especially important for the study of the multiple cylinder configurations following, since a number of them have not been extensively tested (such as two staggered circular cylinders, presented in Chapter 9).

* This would be an ideal investigation to use CFD, since numerical and computational techniques have proved much more reliable in reproducing the starting fluid dynamics of a circular cylinder (even for low subcritical Reynolds numbers, for example), for small elapsed times, when compared to the same flow under steady conditions. Two-dimensional codes may even be effective in such a study, although it is not yet precisely known how the three-dimensional character of the cylinder flow field develops from an impulsive start.

The two techniques were also complementary, in that each revealed unique aspects of the flow that the other technique could not. For example, the visualization experiments enabled the flow to be studied for long periods of time, on the order of 150 to 1200 vortex shedding periods. This enabled large-time-period variations in the vortex shedding behaviour to be detected, such as intermittent appearance of symmetric or irregular modes of vortex shedding, that interrupted the dominant periodic, alternate Kármán vortex shedding. On the other hand, only a single shedding period could typically be captured with the PIV technique, because of the moving cylinder and the fixed camera, and the limited number of images which could be stored in the memory of the frame grabber. The PIV technique did highlight, however, the natural variability in the instantaneous velocity and vorticity representations of the flow field, that could not be detected with flow visualization. For example, the instantaneous PIV vorticity data revealed noticeable differences, between tests, in the size and shape of the Kármán vortices, the length of the vortex formation region, the thickness of the shear layers, and the extent of the vorticity in the base region. In contrast, these structures appeared to be mostly of the same size and shape in the flow visualization video images.

The investigation of the single circular cylinder also demonstrated the reliability of the experimental apparatus and instrumentation, and that acceptable experimental conditions (with minimum influence of external effects) were established, since there was good qualitative and quantitative agreement with established results in the literature. This result was especially significant for the case of the impulsively started circular cylinder, since the good correspondence with published data meant that the small vertical towing tank could reliably reproduce conditions of a nearly impulsive start.

Chapter 7

Tandem Configuration

7.1 Introduction

The tandem configuration of circular cylinders results when the cylinders are arranged in-line and parallel to the oncoming flow, such that the cylinders downstream are completely shielded by the leading cylinder (shown schematically in Figure 3.1(a)). In this study, two-cylinder and three-cylinder tandem configurations were tested under conditions of steady mean cross-flow and an impulsive start, in the same limited range of low subcritical Reynolds number that was used in the initial single cylinder experiments (reported in Chapter 6).

Under steady upstream flow conditions, the emphasis was on improving the physical understanding of the three basic, known flow patterns for two tandem cylinders, of single bluff-body behaviour, shear layer reattachment, and Kármán vortex shedding from the upstream cylinder, which were shown schematically in Figure 3.3. Under impulsively started conditions, little is presently known of the behaviour of tandem cylinders, and how the three steady flow patterns originate once the cylinders are set into motion. Here, the emphasis was on qualifying and quantifying the starting flow in much the same manner that was done for the single cylinder (Section 6.3), and to illustrate the effects of varying the longitudinal pitch ratio and the number of cylinders.

7.2 Steady Mean Flow

Under steady upstream cross-flow conditions, two cylinders in tandem were investigated in the water tunnel, using flow visualization, and in the main water towing tank, using PIV and hot-film anemometry. Tandem configurations of $L/D = 1.0, 1.5, 2.0, 2.5, 3.0, 4.0, 4.5$ and 5.0 were tested in the water tunnel for $Re = 850$ to 1350 , and configurations of $L/D = 1.0, 1.125, 1.25, 1.5, 2.0, 2.5, 3.0, 3.5$ and 4.0 were tested in the towing tank, at a single Reynolds number of $Re = 1900$.

7.2.1 Vortex Shedding Frequencies

The hot-film anemometry experiments conducted in the towing tank included additional tandem configurations of $L/D = 1.75, 2.25, 2.75, 3.25, 3.75, 4.25$ and 4.75 , and covered a wider range of low subcritical Reynolds number than did the PIV experiments, from $Re = 600$ to 6000 . Vortex shedding frequency measurements for the *downstream* cylinder are presented in Figure 7.1, and the data compare favourably to published results in a similar range of low subcritical Reynolds number. The published data show a large range of scatter over the entire range of L/D , but this is likely caused by differences in the Reynolds number (investigated by Ljungkrona (1992)) and experimental conditions, such as blockage, aspect ratio and freestream turbulence (studied by Ljungkrona *et al.* (1991)).

The results in Figure 7.1 show that higher frequency vortex shedding occurs at small pitch ratios, to $L/D \sim 1.5$, which is a property of single bluff-body behaviour (discussed further in Section 7.2.2). Here, the vortex formation length and the near-wake region are both shorter than a single cylinder, corresponding to the higher shedding frequency, as mentioned in Section 6.2.4.

The same Strouhal number data (Figure 7.1) then show a vortex shedding frequency lower than that of a single cylinder, in a range of pitch ratio that corresponds to the shear layer reattachment flow pattern (described in Sections 7.2.3 to 7.2.5). In this manner, the second cylinder acts as a flow stabilization device through the lowering of the vortex shedding frequency, although this may have some implications in the

occurrence of flow-induced vibrations. This flow pattern is marked by a steady decrease in Strouhal number for $L/D \sim 1.5$ to 2.5 , and a relatively constant value near $St = 0.15$ for $L/D \sim 2.5$ to 3.5 , suggesting that there may be more than one type of reattachment behaviour. In fact, three different types of reattachment behaviour were identified in this study, as discussed further in Section 7.2.4.

For large pitch ratios, of $L/D > 4.0$, the Strouhal number becomes very close to that of a single circular cylinder, at $St = 0.18$ to 0.20 . (see Figure 6.2(c)). This result indicates that vortex shedding behind the second cylinder is less influenced by the presence of the cylinder upstream, and coincides with the steady flow pattern of Kármán vortex shedding in the gap between the cylinders.

The transition to the third flow pattern is shown by the jump in the vortex shedding frequency in Figure 7.1, which occurs over a range of pitch ratio, from $L/D = 3.25$ to 3.75 in the present results at $Re = 1900$. In this transitional range, two vortex shedding frequencies may be measured, one corresponding to the shear layer reattachment flow pattern, and the other corresponding to the second flow pattern involving Kármán vortex shedding from the upstream cylinder. In general, the power spectra from the towing tank experiments showed only one dominant frequency, low in one test and high in another test, for example, rather than two frequencies simultaneously as would be measured in longer-duration wind tunnel or water tunnel experiments.

This transition is highly sensitive to the Reynolds number, mostly attributed to variations in the vortex formation length for a single cylinder (and here the upstream cylinder, specifically), as discussed by Ljungkrona *et al.* (1991). The transition from shear layer reattachment to Kármán vortex shedding from the upstream cylinder has also been found to be bistable (Igarashi 1981) and hysteretic, depending on whether L/D (or Re) is increased or decreased (Zdravkovich 1987). Neither property was observed in this study, since the hot-film anemometry measurements were acquired only in the towing tank, and the experiments were of insufficient duration to discern bistability.

7.2.2 Tandem Configurations with Small L/D

From $L/D = 1.0$ to approximately 1.5 in this study, the two tandem cylinders behaved like a single bluff-body, as shown in Figures 7.2 and 7.3, and this range of pitch ratio is consistent with published results. Other researchers have shown that the combined drag force on the two cylinders is less than the sum of the drag forces for two isolated circular cylinders (Zdravkovich & Pridden 1977), and in this sense the second cylinder acts as a drag reduction device. At $L/D = 1.0$, with the cylinders in contact, and at $L/D = 1.25$ (Figure 7.3(b)), shear layers from the upstream cylinder do not reattach onto the downstream cylinder but instead wrap around both cylinders, rolling-up into Kármán vortices in the near-wake of the downstream cylinder. The shorter vortex formation region for these tandem configurations, compared to a single circular cylinder, may be seen by comparing the flow visualization results in Figures 6.1 and 7.2(a). In the cavity region between the two cylinders, small concentrations of vorticity may be seen, that are contiguous to the adjacent wrapped shear layers (Figures 7.3(a,b)). The existence of distinct cavity vortices, however, as described by Hetz *et al.* (1991) for a group of five in-line cylinders, could not be distinguished from the vorticity data.

In the wake of the cylinders, a single Kármán vortex street forms (Figures 7.2(a) and 7.3(a,b)), much like that observed for a single cylinder although at a higher frequency of vortex shedding. As in the case of a single circular cylinder (see discussion in Section 6.2.4), the shorter vortex formation length correlates well with the higher shedding frequency. In this sense, the second cylinder intensifies the vortex shedding compared to a single cylinder, which may be an advantage from a mixing or flow-induced vibration point of view. The flow visualization experiments also showed that symmetric vortex shedding could occasionally interrupt the alternate vortex shedding behaviour (Figure 7.2(b)), and shear layer (Kelvin-Helmholtz) vortices could also be observed. These instability vortices formed in a symmetric fashion adjacent to the downstream cylinder, and coalesced into the Kármán vortices in the near-wake region. The appearance of the shear layer vortices is likely caused by the streamwise lengthening of the two shear layers

from the upstream cylinder, as they must extend further downstream (compared to a single cylinder) in order to enclose (or wrap around) the second cylinder.

7.2.3 Tandem Configurations with Intermediate L/D

Perhaps the least understood of the three basic flow patterns for two tandem circular cylinders is the shear layer reattachment flow pattern at intermediate L/D . Previous studies have variously suggested (i) simultaneous and continuous reattachment of the two shear layers, and (ii) the formation of a pair of eddies of equal strength and opposite rotation in the gap between the cylinders, although little supporting flow visualization evidence has been shown in the literature.

In this study, the investigation of the shear layer reattachment flow pattern (Figures 7.4 and 7.5) instead showed that (i) the shear layer reattachment process was an alternating, antisymmetric process, rather than continuous; (ii) the alternating behaviour was synchronized with Kármán vortex formation in the near-wake of the downstream cylinder; (iii) the simultaneous reattachment of both shear layers was not observed; and (iv) a stable pair of vortices of equal strength and opposite rotation was rarely, if ever, observed in the gap between the two cylinders. The alternating reattachment behaviour occurred together with the wrapped shear layer behaviour seen at small L/D (Section 7.2.2), such that when there was a reattaching shear layer on one side of the configuration, a wrapped shear layer would be seen on the opposite side. Igarashi (1981) also suggested the existence of a synchronized, alternating shear layer reattachment flow pattern, but only for $L/D < 1.6$, and for $Re > 25,000$.

In this process, the wrapped shear layer extends from the separation point on the upstream cylinder, and rolls up into a Kármán vortex in the near-wake of the downstream cylinder (seen in Figures 7.4(b,c), and the middle column of data in Figure 7.5(c)). Once the Kármán vortex is shed, the wrapped shear layer reattaches onto the front face of the downstream cylinder (Figure 7.5(a)). A small region of recirculating fluid, seen as a concentration of vorticity in the PIV results in Figure 7.5, is found locally near the reattachment point, but this does not extend appreciably into the gap between the

cylinders. Similar features, in fact, may be seen in the flow visualization results of Wu *et al.* (1994b) at $L/D = 3.0$ and 4.0 , $Re = 1000$.

7.2.4 Three Types of Shear Layer Reattachment Behaviour

In the present study of the shear layer reattachment flow pattern for two tandem cylinders, three different types of behaviour were identified depending on the pitch ratio, with each type of behaviour shown schematically in Figure 7.6. A distinction between the first two types of behaviour was already made based on the Strouhal number data, in Figure 7.1 (Section 7.2.1).

The first type of behaviour, sketched in Figure 7.6(a), was seen at $L/D = 1.5$ and 2.0 (Figures 7.4(a,b) and 7.5(a,b)). Here, recirculating fluid is highly localized within the shear layer, near the point of reattachment. As the shear layer transitions from a reattached state to a wrapped state (as described in Section 7.2.3), the recirculating fluid is convected about the downstream cylinder, and becomes entrained into the newly forming Kármán vortex.

In the second type of behaviour, sketched in Figure 7.6(b), and seen at $L/D = 2.5$ and 3.0 (Figures 7.4(c,d) and 7.5(c,d)), the recirculating fluid accumulating at the reattachment point divides into two concentrations of vorticity; this type of behaviour was seen only in the flow visualization videos. One concentration enters the gap between the two cylinders, and the other enters the base region of the downstream cylinder. The latter concentration of vorticity crosses the wake and becomes entrained into the Kármán vortex forming on the opposite side of the cylinder. With increasing L/D , the division of vorticity near the reattachment point seems to occur further back on the surface of the downstream cylinder. Similar behaviour may in fact be identified in the flow visualization results of Wu *et al.* (1994b) for $L/D = 4.0$, $Re = 1000$. Unfortunately, the division of vorticity could not be found in the present PIV vorticity data, although future experiments at a higher magnification, that focuses more specifically on the near-wake region of the downstream cylinder, might prove successful.

The third type of behaviour, sketched in Figure 7.6(c), was first seen at $L/D = 3.0$ (Figures 7.4(d) and 7.5(d)). The behaviour is marked by the appearance of shear layer instabilities, in the form of waviness or undulations. At $L/D = 3.5$ (Figure 7.4(e)), the waviness becomes increasingly replaced by the symmetric formation of shear layer (or Kelvin-Helmholtz) vortices near the surface of the second cylinder. A similar flow pattern may be seen in the results of Kühn & Klatt (1992) at $L/D = 3.5$, $Re = 1000$. More discussion of the third type of behaviour is found in Section 7.2.6.

Other researchers have also suggested different modes of shear layer reattachment. Igarashi (1981) distinguished three types of behaviour: (i) synchronized, alternating shear layer reattachment, noted earlier, for $L/D < 1.6$; (ii) simultaneous and continuous shear layer reattachment, with a stable pair of eddies in the gap between the cylinders, for $L/D = 1.6$ to 2.4 ; and (iii) the same behaviour but with intermittent shedding of the gap eddies, for $L/D > 2.4$. Observation of the three patterns was mostly restricted to much higher Reynolds numbers than the present study, for $Re > 25,000$, however the second two patterns were said to persist to $Re = 10,000$ and perhaps lower. Unfortunately, as this study showed (see Section 7.2.5), stable gap eddies were never found. The instantaneous PIV velocity data of Kwon *et al.* (1996), at $Re = 17,800$, also suggest different types of the shear layer reattachment behaviour, although different than the ones identified here. Their data seem to show *one* shear layer undergoing reattachment at $L/D = 1.3$ and *two* shear layers undergoing reattachment at $L/D = 2.1$. The sparseness of the velocity vectors in their results, however, and the absence of derived vorticity data, prevent a more detailed description from being achieved.

7.2.5 Fluid Motion in the Gap Between the Cylinders

In the gap between the two cylinders, bounded by the shear layers extending from the upstream cylinder, formation of a stationary pair of gap vortices has been suggested by a number of researchers, including Kostic & Oka (1972), Igarashi (1981, 1984), and Lee & Panagakos (1997), in experiments at higher Reynolds numbers. The existence of the gap eddy pair has been inferred from time-averaged measurements of surface static pressure,

the output of multiple hot-film sensors, and from long-duration surface oil flow patterns, even though the time-averaged gap velocity profile measurements of Zdravkovich & Pridden (1977) show no appreciable gap streamwise flow velocity, at $L/D = 2.5$. Previously published flow visualization results, however, have not clearly shown the existence of the eddy pair, except for the photographs of Huhe-Aode *et al.* (1985) at $L/D = 3.0$, $Re = 100$. The eddy pair would then seem to be a low Reynolds number flow phenomenon, similar to that observed in cylinder arrays by Price *et al.* (1995).*

In this study, organized recirculating fluid motion (in the form of a pair of stationary eddies) was never seen in a steady or stable manner in the gap between the cylinders (Figures 7.4 and 7.5).† The PIV results showed that three instantaneous gap flow patterns, illustrated in Figure 7.7, could arise at any L/D : (a) stagnant fluid within the gap; (b) a vorticity concentration near the reattachment point, which was the dominant behaviour; or (c) weakly recirculating fluid. The weakly recirculating fluid, however, shown in Figure 7.7(c), was never seen on a continuous basis and no obvious symmetrical pattern of vorticity was ever seen in the gap between the cylinders (with supporting evidence in Figures 7.4 and 7.5).

7.2.6 Transitional Flow Patterns at Large L/D

In the Strouhal number data from the towing tank at $Re = 1900$, shown in Figure 7.1, transitional behaviour was found for $L/D = 3.25$ to 3.75 . In this range of pitch ratio,

* The existence of a pair of eddies in the gap also implies some form of continuous and simultaneous shear layer reattachment on the downstream cylinder, which itself was not observed in this study. Smoke flow visualization photographs of Ljungkrona (1992) and Lee & Panagakos (1997) show what may be interpreted as continuous reattachment, but the visualization technique does not highlight fluid motion in the gap or just inside the shear layers.

† Since a stable pair of eddies was not observed in the gap between the cylinders, the regular (or irregular) shedding of gap vortices, a mechanism that was suggested by Hetz *et al.* (1991) for a tandem grouping of five cylinders, was not observed either.

either the shear layer reattachment flow pattern, or Kármán vortex shedding from the upstream cylinder, could be observed. PIV data which illustrate the occurrence of both flow patterns at the same pitch ratio are shown in Figure 7.8, for $L/D = 3.5$: shear layer reattachment is shown in Figure 7.8(a), and Kármán vortex shedding behind the upstream cylinder is shown in Figure 7.8(b). The two flow patterns are perhaps better distinguished from the instantaneous streamline patterns in Figure 7.9.

These results imply that the transitional or “critical”^{*} pitch ratio for the tandem configuration, at $Re = 1900$, is $L/D = 3.5$. The data from the water tunnel, however, continued to show the shear layer reattachment flow pattern, at $L/D = 3.5$ (Figure 7.4(e)), 4.0, and 4.5 (Figures 7.10(a,b)). Only at $L/D = 5.0$ were the two distinct flow patterns observed in the water tunnel, as shown in Figures 7.10(c,d), meaning the transitional or “critical” pitch ratio was $L/D = 5.0$ in that facility. Furthermore, Kármán vortex shedding from the upstream cylinder at $L/D = 5.0$ was only seen for the higher of the two Reynolds numbers tested in the water tunnel, at $Re = 1320$ (Figure 7.10(d)). This result is also supported by the flow visualization results of Ishigai *et al.* (1972) at $Re = 2060$ to 4000, which show shear layer reattachment up to $L/D = 4.0$, and Kármán shedding beginning at $L/D = 5.0$.

The shear layer reattachment activity which continued to be observed in the water tunnel experiments was the third type of behaviour (Figure 7.6(c)), with shear layer instability waves at $L/D = 3.5$ (Figure 7.4(e)), and nearly symmetric formation of shear layer instability vortices[†] at $L/D = 4.0$, 4.5 and 5.0 (Figures 7.10(a-c)). At $L/D = 4.5$ and

^{*} The use of the word “critical” is historical in nature, and is not meant to imply transition in the cylinder boundary layers.

[†] Although the formation of the instability vortices is observed to be nearly symmetric, the departure from symmetry could be caused by circumferential or azimuthal misalignment of the upstream cylinder and its dye injection ports. Small angular differences in the source of the dye injection (as measured from the actual upstream stagnation point) between one side of the cylinder and the other would mean that different streaklines were being observed, lending an appearance of asymmetry in the formation of the shear layer vortices.

5.0, two cycles of these vortices form before the vortices impact the downstream cylinder (Figures 7.10(b,c)). The shear layer vortices form at $y/D > 0.5$ (i.e. outside the cross-stream extent of the cylinder radius), and wrap around the downstream cylinder as they are convected along the shear layer. Similar behaviour can be discerned from the flow visualization results of Ishigai *et al.* (1972) at $L/D = 3.0$ and 4.0 , $Re = 3400$ to 3900 .*

7.2.7 Kármán Vortex Shedding from the Upstream Cylinder

Beyond the transitional or “critical” pitch ratios of $L/D = 3.5$ and 5.0 , at $Re = 1900$ in the towing tank and $Re = 1320$ in the water tunnel, respectively, Kármán vortex shedding behind the upstream cylinder is observed in a continuous fashion. The PIV results in Figure 7.11 clearly show complete formation and shedding of Kármán vortices in front of the downstream cylinder. The shed vortices impinge upon the downstream cylinder, and are then convected around it and into its near-wake region. This impinging flow is highly turbulent and three-dimensional, and is periodically sheared as vortices are alternately shed (Igarashi 1981). Here, the upstream cylinder behaves as a turbulence generator.

As a consequence of the impinging flow, Kármán vortex shedding from the *downstream* cylinder becomes highly disturbed or irregular, and in many instances it appears to be absent in the flow visualization videos and the PIV data. This is similar to results presented later for two staggered circular cylinders, for the vortex impingement (VI) flow pattern (Section 9.8), where the absence of a well-defined vortex street in the wake of the cylinder pair is due to turbulence, vorticity and three-dimensional flow generated by the Kármán vortices from the upstream cylinder. Although other researchers have suggested that regular Kármán vortex shedding occurs from the downstream

* The nearly symmetrically formed shear layer vortices convect vorticity and turbulent fluid into the near-field of the downstream cylinder, but they do not appear to prevent alternate, antisymmetric Kármán vortex shedding in the near-wake of the downstream cylinder. It is difficult to determine from the flow visualization videos, however, precisely how the alternating vortex pattern arises in the near-wake of the downstream cylinder.

cylinder, at the same frequency and phase as from the upstream cylinder (Igarashi 1981), it could not clearly distinguished in the present results.

In the water tunnel, however, vortex shedding from the upstream cylinder at $L/D = 5.0$ and $Re = 1320$ (Figure 7.10(d)) was still not the dominant behaviour. Instead, the vortex shedding appeared weak, exhibited streamwise variation in the vortex formation length, and could be seen to slowly transition to shear layer reattachment through a streamwise lengthening of the vortex formation region and a slow change in the phase and mechanism of vortex formation (from anti-symmetric Kármán vortex formation to nearly symmetric Kelvin-Helmholtz vortex formation).

7.2.8 Reynolds Number Effects

The two different values of the transitional or “critical” L/D , at which Kármán vortex shedding is first observed behind the upstream cylinder, highlight the Reynolds number sensitivity of the tandem configuration (Igarashi 1981, 1984; Ljungkrona 1992). As these researchers have found, the transitional L/D increases as the Reynolds number is lowered, although there are insufficient published data in the lower subcritical regime to accurately determine the L/D - Re relationship. Both transitional “critical” pitch ratios from the present study are consistent with the published data, although the value of $L/D = 3.5$ from the towing tank is somewhat low, and the value of $L/D = 5.0$ from the water tunnel is slightly high. Nonetheless, a lower-than-anticipated transitional L/D might be expected from the towing tank, since small levels of background or residual motion might provide enough disturbance to the shear layer reattachment process to initiate Kármán shedding from the upstream cylinder. Furthermore, a higher-than-anticipated “critical” L/D from the water tunnel is consistent with the wider range of T/D for the biased flow pattern that was detected for two side-by-side circular cylinders, in the same facility, that was attributed to experimental effects (see Section 8.2).

7.3 Impulsively Started Flow

The impulsively started flow field for a tandem configuration of circular cylinders has not been previously investigated, but it was expected to be quite similar to a single circular cylinder impulsively set into motion (described in Section 6.3). The important new feature of the flow field, shown schematically in Figure 7.12, is the gap recirculation zone, that contains a pair of primary eddies of equal strength and opposite rotation as seen behind the downstream cylinder. It is the temporal development of this new recirculation zone that best illustrates the interference effects of the added cylinder, and provides insight into the transition from impulsively started flow to steady mean flow, and the development of the three basic steady mean flow patterns of (i) single bluff-body behaviour, (ii) shear layer reattachment, and (iii) Kármán vortex shedding from the upstream cylinder, each described in Section 7.2.

In this study, two-cylinder and three-cylinder tandem configurations were tested in the small, vertical water towing tank, using the seeding particle flow visualization technique* and PIV. The Reynolds number ranged from $Re = 1200$ to 3800 , in the same range of low subcritical Reynolds numbers examined for the impulsively started single cylinder (in Section 6.3), and the flow was investigated for $t^* < 6.0$ and $A_p = 0.2$ to 0.8 . Higher accelerations were attained in the vertical towing tank for the tandem cylinders, when compared to the single cylinder and, in particular, to the side-by-side cylinder (Section 8.3) experiments, but this was attributed to the lower resistance or drag of the

* The seeding particle flow visualization technique, described in Section 5.2, was successfully applied to an impulsively started single cylinder (Section 6.3) and impulsively started side-by-side cylinders (Section 8.3), in the small vertical water towing tank, described in Section 4.4. The technique was not as successful with the tandem configuration, however, since the shielding effect of the “upper” (upstream) cylinders prevented particles from collecting on the “upper” surfaces of the cylinders “below” (downstream). This meant that the recirculation zones were more difficult to visualize, since less particles could be entrained into the near-wake regions of the “lower” cylinders.

tandem configuration.* Two-cylinder configurations were tested at pitch ratios of $L/D = 1.0, 1.5, 2.0, 2.5$ and 3.0 , and three-cylinder configurations were tested at $L/D = 1.0, 1.5$ and 2.0 .

Based on the flow visualization and PIV experiments, the temporal development of the flow field after an impulsive start, for two-cylinder and three-cylinder configurations, exhibited three types of behaviour. Each type was analogous to one of three steady flow patterns (from Section 7.2, or Figure 3.3), and the flow patterns are grouped and discussed in the following subsections according to (i) tandem cylinders in contact, $L/D = 1.0$, which approximates a single bluff-body flow pattern (Section 7.3.1); (ii) tandem cylinders at intermediate L/D , which would transition to a shear layer reattachment flow pattern (Sections 7.3.2 and 7.3.4); and (iii) tandem cylinders at large L/D , which behave increasingly like single, isolated impulsively started cylinders (Section 7.3.3). In the limited range of Reynolds number of the experiments, no apparent Reynolds number dependence was found, and similar flow structures were identified at the same t^* values; this result is consistent with the behaviour of a single cylinder, under impulsively started conditions, in the same part of the low subcritical regime.

7.3.1 Tandem Configurations with $L/D = 1.0$

When the tandem cylinders are in contact, at $L/D = 1.0$, both two-cylinder and three-cylinder configurations behave as a single bluff-body, in much the same manner as an impulsively started single circular cylinder (Section 6.3) and impulsively started side-by-side cylinder configurations (Sumner *et al.* 1997a; and Section 8.3.1 of the thesis). Both the flow visualization results (Figure 7.13) and the PIV vorticity data (Figure 7.14) show

* Although little is known of the temporal evolution of the fluid force coefficients for groups of cylinders in impulsively started cross-flow, extensive measurements are available in the literature for steady flow conditions. Force measurements for the tandem configuration show that the upstream cylinder's drag coefficient is usually less than that of a single cylinder, and the downstream cylinder's drag coefficient is often negative (indicating a thrust force), within the single bluff-body and shear layer reattachment flow patterns (Igarashi 1981, Hiwada *et al.* 1982). This result would seem to account for the greater accelerations attained for the tandem cylinders, as they were freely towed in the vertical tank.

the growth of a single quasi-attached recirculation zone in the near-wake region of the most downstream cylinder, containing a pair of eddies of equal strength and opposite rotation. The recirculation zone and the eddies remain symmetrical until $t^* \sim 4.5$, for both the two-cylinder and three-cylinder configurations, which is also similar to a single impulsively started cylinder. The recirculation zone then becomes asymmetric (Figure 7.13(b), $t^* = 4.7$; Figure 7.14(a), $t^* = 4.6$), beginning the transition to alternate, Kármán vortex shedding (seen in Figure 7.14(a), $t^* = 5.4$, for example). The initial stages of this transformation are marked by the appearance of “gathers,”^{*} at $t^* \sim 5.5$, which are narrow regions of in-plane vorticity extending from the recirculation zone into the wake (seen in Figure 7.13(a), $t^* = 5.3$; Figure 7.13(b), $t^* = 5.5$; Figure 7.14(a), $t^* = 5.4$; Figure 7.14(b), $t^* = 5.2$).

As in the case of the single impulsively started cylinder, the temporal growth of the recirculation zone can be quantified by measurements of the streamwise location of the primary eddy centres from the base of the downstream cylinder, a , the cross-stream spacing of the primary eddy centres, b , and the length of the recirculation zone from the base of the downstream cylinder, L_R , all of which are defined in Figure 7.12(b). These measurements, as well as the area of a primary eddy, A , the peak vorticity within a primary eddy, ω_{zp} , and the primary eddy circulation or strength, Γ , were obtained from the PIV vorticity data, and are shown plotted alongside PIV data for a single cylinder in Figure 7.15.

Essentially, there is little difference in the size, shape and development of the recirculation zone for a single cylinder, two tandem cylinders in contact, and three tandem cylinders in contact, as borne by the measurements of a , b , L_R , and A , in Figures 7.15(a-d) respectively. Measurements of peak vorticity and eddy strength, in Figures 7.15(e,f) respectively, also show no apparent difference between the two-cylinder and three-cylinder groups, however the single cylinder data show a marked departure for both

^{*} “Gathers” are observed under steady mean flow conditions at low Reynolds numbers, before the onset of Kármán vortex shedding (Coutanceau & Defaye 1991), as mentioned in Section 2.2.2.

quantities. The higher peak vorticity measurements for the single cylinder, in Figure 7.15(e), may be attributed to the higher image magnification (or finer grid resolution) for the single cylinder experiments ($M_x = 0.15$ to 0.20 mm/pixel for the single cylinder, compared with $M_x = 0.28$ mm/pixel for the tandem configurations), which enables more accurate (and consequently higher) estimates of peak vorticity to be measured. The higher primary eddy circulation measurements for the single cylinder, in Figure 7.15(f), when compared to the tandem configurations, may be caused by (i) the same differences in magnification; (ii) higher values of A_p for the single cylinder experiments; (iii) some presently unknown aspect of the fluid behaviour, perhaps connected to the lower drag force of the tandem configuration, that would manifest itself in weaker circulation measurements; or (iv) a combination of these effects.

The PIV vorticity data (Figure 7.14) also show the development of shear layers alongside the cylinders (Figure 7.14(a), $t^* = 3.1$ to 4.6), which are contiguous with the recirculation zone in the near-wake region; these structures are not readily apparent in the flow visualization images (Figure 7.13), however. The shear layers greatly resemble the wrapped shear layer behaviour observed in the steady flow single bluff-body flow pattern, for $L/D < 1.5$ (Section 7.2.2). In some of the flow visualization and PIV results, small concentrations of vorticity are seen in the cavity regions between neighbouring cylinders (Figure 7.13(a), $t^* = 3.2$; Figure 7.14(a), $t^* = 3.1$ and 3.8); cavity vortices were also noticed by Hetz *et al.* (1991), in a study of a five-cylinder tandem configuration at small L/D . Here, the vorticity concentrations are contiguous to the shear layers enveloping the cylinders, and are similar to features seen under steady flow conditions at small L/D (Section 7.2.2), although the exact structure of the cavity region could not be determined precisely. Flow visualization images for the three-cylinder groups suggested that lateral growth or expansion of the cavity vorticity concentrations behind the upstream cylinder, for $t^* < 2.5$, may interrupt shear layer development alongside the cylinders, and eventually lead to the breakup of the recirculation zone (the growth is seen in Figure 7.13(b), $t^* = 1.6$). The expansion behaviour is not found in the PIV vorticity results, however.

7.3.2 Two-Cylinder Configurations at Intermediate L/D

When the tandem cylinders are no longer in contact, but rather there is a gap between them, a recirculation zone can form behind each of the cylinders, with each zone containing a pair of eddies of equal strength and opposite rotation. Similar features are noticed for both two-cylinder and three-cylinder configurations, although the discussion of the three-cylinder configurations is left to Section 7.3.4.

At intermediate pitch ratios, of $L/D = 1.5$ and 2.0 , the temporal development of the gap recirculation zone for the two-cylinder tandem arrangement is affected by the presence of the cylinder immediately downstream, as shown in the flow visualization images in Figure 7.16, and the PIV vorticity data in Figures 7.17 and 7.18, for two-cylinder configurations of $L/D = 1.5$ and 2.0 , respectively. In each case, the flow field seems to indicate that a shear layer reattachment flow pattern would result once steady mean flow conditions are established; similar conditions were observed at each Reynolds number tested, and at each t^* (see the results for $L/D = 1.5$, in Figures 7.17(a,b,c), at $Re = 1300, 1900$ and 2800 , for example). Recirculation zone parameters (defined in Figure 7.12(b)) computed for the upstream and downstream cylinders are presented in Figures 7.19 and 7.20, respectively.

The presence of the second cylinder constrains the streamwise growth of each gap recirculation zone, as it reaches the leading surface of the next cylinder (seen, for example, in Figure 7.16(a), $t^* = 1.6$; Figure 7.16(b), $t^* = 3.0$; Figure 7.17(b), $t^* = 1.8$; Figure 7.18(a), $t^* = 3.0$). This constrained growth is reflected in the streamwise position of the gap eddy centres (Figure 7.19(a)), which show $a/D < 0.5$ for $L/D = 1.5$ and 2.0 . The constrained growth is also shown in data for the length of the recirculation zone (Figure 7.19(d)), with maximum values of $L_R/D = 0.5$ and 1.0 for configurations of $L/D = 1.5$ and 2.0 , respectively.

As the gap recirculation zone grows, it eventually completely fills the gap between the cylinders (seen, for example, in Figure 7.16(a), $t^* = 2.0$; Figure 7.16(b), $t^* = 3.0$; Figure 7.17(b), $t^* = 1.8$; Figure 7.18(a), $t^* = 3.0$). At $L/D = 1.5$, the recirculation zone completely occupies the entire gap by $t^* \sim 1.8$. At $L/D = 2.0$, the recirculation zone takes

longer to span the gap between the cylinders, until $t^* \sim 3.0$. The existence of a stable pair of eddies between the cylinders is similar to a *steady* flow pattern, suggested by Igarashi (1981, 1984) and other researchers, that was said to occur under shear layer reattachment conditions, although the results of this study (Sections 7.2.3 to 7.2.5) showed that this was not the case.

As the flow develops further in time, and the gap eddies can no longer extend in the streamwise direction, the gap recirculation zone then expands symmetrically and laterally into the outer flow (shown, for example, in Figure 7.16(a), $t^* = 2.4$; Figure 7.16(b), $t^* = 3.5$; Figure 7.17(a), $t^* = 2.0$; Figure 7.18(a), $t^* = 3.5$). This lateral expansion is shown by the greater transverse spacing of the primary eddies (Figure 7.19(b)), which for a tandem configuration of $L/D = 1.5$ increases to nearly twice the spacing for a single cylinder, at $b/D \sim 1.25$ by $t^* = 4.5$; the expansion is less pronounced at $L/D = 2.0$, since the gap primary eddies have greater room to expand in the streamwise direction. Despite the lateral expansion of the gap primary eddies, however, data for the primary eddy area (Figure 7.19(c)) show no appreciable change from a single cylinder.

Continued development of the flow sees the gap eddies being symmetrically convected about the downstream cylinder (Figure 7.16(a), $t^* = 2.8$; Figure 7.18(b), $t^* = 4.9$ to 5.7). The shed gap eddies are subsequently entrained into the shear layers, and then into the near-wake region of the downstream cylinder, precipitating asymmetry in its recirculation zone (Figure 7.17(c), $t^* = 5.3$ and 6.2) and the formation of “gathers” (Figure 7.16(b), $t^* = 4.1$; Figure 7.17(c), $t^* = 5.3$). Despite the entrainment of the gap vortices, and the presence of the cylinder upstream, the temporal development of the recirculation zone behind the downstream cylinder behaves in a nearly identical fashion to a single circular cylinder, as shown in Figures 7.20(a-f).

7.3.3 Two-Cylinder Configurations at Large L/D

As the pitch ratio is increased, the second cylinder in the tandem configuration has increasingly less influence on the development of the gap recirculation zone, and its growth more approximates that of a single impulsively started circular cylinder. As

shown in the flow visualization images, in Figure 7.21, and in the PIV vorticity data for $L/D = 2.5$ and 3.0 , in Figures 7.22 and 7.23, respectively, symmetric recirculation zones again form in the near-wake regions of each cylinder, each containing a pair of eddies of equal strength and opposite rotation. Lateral expansion of the gap recirculation zone becomes less evident, as shown by the transverse spacing of the gap eddies (Figure 7.19(b)), which slowly approaches the single cylinder data with increasing pitch ratio. The primary eddy area, however, now becomes markedly larger than that of a single impulsively started cylinder (Figure 7.19(c)).

In time, the recirculation zones each become asymmetric, and the appearance of “gathers” may be seen from the *two* zones before a primary eddy is shed (Figure 7.21(a), $t^* = 4.3$; Figure 7.21(b), $t^* = 5.2$; Figure 7.22, $t^* = 4.5$; Figure 7.23, $t^* = 4.4$); this behaviour is in contrast to tandem cylinders at intermediate L/D (Section 7.3.2) where the “gathers” are only seen behind the downstream recirculation zone. As in the previous case (Section 7.3.2), temporal development of the downstream recirculation zone still behaves in a nearly identical fashion to a single cylinder, as shown in Figures 7.20(a-f).

At large L/D , the recirculation zone from the upstream cylinder will still completely fill the gap between the cylinders, although it now happens nearer to the point in time where the flow field begins to become asymmetric. At $L/D = 2.5$, the primary eddies fill the gap between the cylinders by $t^* \sim 4.4$ (Figure 7.21(a), $t^* = 4.3$; Figure 7.22, $t^* = 4.5$), and data for the length of the recirculation zone show a terminal value of $L_R/D = 1.5$ (Figure 7.19(d)). At $L/D = 3.0$, a terminal value of $L_R/D = 2.0$ appears by $t^* \sim 5.5$ (Figure 7.19(d)). Based on these data, the point in time at which the upstream recirculation zone completely fills the gap between the two tandem cylinders, can be found from Figure 7.24. The fact that the primary eddies extend between the cylinders, up to $L/D = 3.0$ (and perhaps beyond), suggests that a shear layer reattachment flow pattern would still evolve at larger L/D once steady upstream flow conditions were reached, but perhaps of the second type of behaviour (Section 7.2.4, and Figure 7.6(b)).

Considering all four tandem configurations together, the streamwise location of the primary eddy centre, the primary eddy area, the length of the recirculation zone, the

peak vorticity, and the primary eddy circulation, all increase with L/D (Figures 7.19(a,c-f) and 7.20(a,c-f)), although the effect is much more pronounced for the upstream cylinder.

7.3.4 Three-Cylinder Configurations with $L/D > 1.0$

The fluid behaviour of a three-cylinder tandem configuration with $L/D > 1.0$ was expected to be similar to the two-cylinder tandem configurations, discussed in Sections 7.3.2 and 7.3.3, since strong similarity between the two geometries is seen under steady upstream flow conditions (Aiba & Yamazaki 1976; Aiba *et al.* 1980; Igarashi & Suzuki 1984). In fact, even four-cylinder and five-cylinder tandem arrangements (Aiba *et al.* 1981; Igarashi 1986; Hetz *et al.* 1991) have similar steady flow patterns of (i) single bluff-body behaviour, (ii) shear layer reattachment, and (iii) Kármán vortex shedding from the upstream cylinders, although the pitch ratio boundaries are different, and combinations of two of the flow patterns may be seen in different gaps at a single L/D .

In the present study, the three-cylinder groups were tested only at $L/D = 1.5$ and 2.0, corresponding to intermediate pitch ratios. The flow visualization results, in Figure 7.25, and the PIV vorticity fields, in Figures 7.26 and 7.27 for $L/D = 1.5$ and 2.0, respectively, show that recirculation zones form behind each of the three cylinders, each containing a pair of primary eddies of equal strength and opposite rotation. During the course of their development, the recirculation zones of the upstream and middle cylinders eventually expand to completely fill the gaps between the cylinders. At $L/D = 1.5$, the temporal growth of the recirculation zones behind the upstream and middle cylinders is particularly constrained by the cylinders immediately downstream, and a lateral expansion into the outer flow is again seen (Figure 7.25(a), $t^* = 3.5$; Figure 7.26(a), $t^* = 2.6$), as in the case of the two-cylinder configuration (Section 7.3.2). At $L/D = 2.0$, the lateral expansion of the recirculation zones is less pronounced, and occurs later in the time history (Figure 7.25(b), $t^* = 3.7$; Figure 7.27, $t^* = 3.6$), since there is greater space between the cylinders for the recirculation zones to expand in the streamwise direction without interference. As the lateral expansion occurs, the primary eddies from the upstream and middle cylinders leave their recirculation zones and become entrained into

the shear layer structures bounding the three cylinders, shown more clearly at $L/D = 1.5$ (Figure 7.25(a), $t^* = 4.0$; Figure 7.26(a), $t^* = 3.6$). The release of the gap primary eddies is more likely to occur from the upstream cylinder, however, rather than from the middle cylinder. The release coincides with the appearance of asymmetry in the flow pattern, the formation of “gathers” from the recirculation zone behind the downstream cylinder (Figure 7.25(a), $t^* = 4.0$), and a transition to alternate vortex shedding from the downstream cylinder (Figure 7.28). As in the case of two impulsively started tandem cylinders, the resulting steady flow pattern will exhibit shear layer reattachment, which may be discerned even in the starting flow (Figure 7.26(b), $t^* = 2.4$ and 2.8).

One feature of the impulsively started flow field for a single cylinder that was not typically seen in any of the experiments, for single, tandem and side-by-side cylinders, was the formation of secondary eddies or vorticity, downstream of the points of boundary layer separation and near the surface of the cylinder (as shown schematically in Figure 7.12(a)). The secondary vorticity was not detected because of the low magnification of the images (which were intended to view the entire flow and show all the cylinders, rather than focus on specific regions), and the inability to resolve the velocity and vorticity fields close to the surfaces of the cylinders. Nonetheless, other secondary vorticity concentrations were detected for the tandem configuration, in regions where they were easier to resolve, as shown in Figure 7.29. The secondary vorticity took the form of a small gap vortex pair, with each vortex of apparently equal strength and size but opposite in rotation, that could develop before or after the middle cylinder, or in front of the downstream cylinder, for $t^* > 4.0$. The signs of the secondary vortices are oriented in an opposite manner to the primary eddies. The secondary vorticity was not seen in all of the PIV data, however, nor was it seen immediately behind the upstream cylinder, although the failure to detect it in this location was likely caused by the inability to view the upstream cylinder, with the PIV system, at high t^* .

Measurements of the streamwise location of the primary eddies, a , the transverse spacing of the eddies, b , the length of the recirculation zone, L_R (all defined in Figure 7.12(b)), the primary eddy area, A , and the peak vorticity and primary eddy strength, were

obtained for the upstream, middle, and downstream cylinders, for $L/D = 1.5$ and 2.0 . The data are presented in Figures 7.30 and 7.31, respectively, and comparison is made with a single impulsively started circular cylinder.

The data in Figures 7.30 and 7.31 show that the temporal growth of the three recirculation zones is different for each cylinder, and that there is a marked streamwise evolution in the growth, to a behaviour more like a single cylinder, with each successive cylinder downstream. In other words, the recirculation zone behind the downstream cylinder most closely approximates that of a single cylinder, while that of the upstream cylinder is the most different. The effect is most evident in the data for $L/D = 1.5$, where the streamwise positions of primary eddies (Figure 7.30(a)), the primary eddy areas (Figure 7.30(c)), the lengths of the recirculation zones (Figure 7.30(d)), and the peak vorticity and circulation (Figures 7.30(e,f)), for the upstream and middle cylinders, are all consistently lower than a single cylinder. This is in contrast to the data for the downstream cylinder, at $L/D = 1.5$, which are generally in good agreement with the single cylinder. Data for the transverse spacing of the primary eddies (Figure 7.30(b)) also show a similar tendency, with greater lateral expansion of the recirculation zone behind the upstream cylinder, and virtually no expansion for the downstream cylinder. Similar effects may be seen in the data for $L/D = 2.0$, particularly for the transverse spacing of the primary eddies (Figure 7.31(b)) and the lengths of the recirculation zones (Figure 7.31(d)).

It is not known whether this trend continues for four-cylinder and five-cylinder impulsively started tandem configurations, since these larger multiple cylinder groups could not be accommodated (at the same magnification) within the field of view of the camera. Published steady upstream cross-flow results, however, indicate that the addition of a fourth and fifth cylinder in tandem does not contribute to a further streamwise evolution to the flow. Rather, “fully developed” conditions seem to develop for the third and fourth cylinders, with the exception of the last cylinder, based on measurements of the drag coefficient, for example (Ishigai & Nishikawa 1975; Aiba *et al.* 1981; Igarashi 1986).

Chapter 8

Side-by-Side Configuration

8.1 Introduction

The side-by-side configuration results when the cylinders are arranged parallel to one another in a row, oriented transverse to the oncoming flow (shown schematically in Figure 3.1(b)). In the present study, two-cylinder and three-cylinder side-by-side configurations were examined under the conditions of steady mean cross-flow and impulsively started cross-flow, in the same range of low subcritical Reynolds number that was used in the initial single cylinder experiments (outlined in Chapter 6), and in the investigation of the tandem configuration (discussed in Chapter 7). All three investigative techniques of flow visualization, hot-film anemometry and PIV were used.

As in the case of the tandem arrangement, the study of the side-by-side configuration was focused on improving the physical understanding of the fluid behaviour, for the known steady flow patterns for two and three side-by-side cylinders (which were shown in Figures 3.4 and 3.6). Under impulsively started conditions, however, little is presently known of the behaviour of side-by-side cylinders, and how the steady mean flow patterns arise once the transient effects from the startup have disappeared. For this flow condition, the emphasis was on qualifying and quantifying the starting flow field in a similar fashion to the single cylinder and tandem cylinders, as detailed in Sections 6.3 and 7.3, to illustrate the influence of varying the transverse pitch ratio and the number of cylinders.

8.2 Steady Mean Flow

Under steady mean flow conditions, side-by-side configurations were first tested in the Kempf & Remmers water tunnel, using flow visualization and hot-film anemometry. The results from these experiments were published by Sumner *et al.* (1997d, 1999) and by Wong (1997). Experiments were then conducted in the main water towing tank, using PIV, and these results have also been published by Sumner *et al.* (1998, 1999).

In the water tunnel, two-cylinder configurations with $T/D = 1.0$ to 6.0 and three-cylinder configurations with $T/D = 1.0$ to 3.0 were examined, with the spacing between the cylinders incremented by $0.5D$. In these experiments, the mean flow velocity ranged from $U = 30$ to 190 mm/s, yielding Reynolds numbers of $Re = 500$ to 1500 for the flow visualization studies, and a higher range of Reynolds number of $Re = 500$ to 3000 for the measurements of vortex shedding frequency using hot-film anemometry. A higher range of Reynolds number was not possible in the flow visualization experiments, since the dye concentrations rapidly diffused under the more turbulent and three-dimensional conditions in the combined wake of the cylinders, rendering the streaklines incoherent. In the PIV experiments in the towing tank, the towing speed ranged from $U = 50$ to 120 mm/s, corresponding to Reynolds numbers of $Re = 1300$ to 2900 . In these experiments, the spacing between the cylinders was freely variable, and two-cylinder configurations of $T/D = 1.0, 1.125, 1.5, 1.75, 2.0, 2.5$ and 3.0 , and three-cylinder configurations of $T/D = 1.0, 1.125, 1.25$ and 1.5 , were tested.

8.2.1 Vortex Shedding Frequencies

Strouhal number data obtained using hot-film anemometry, for the two-cylinder and three-cylinder side-by-side arrangements in the water tunnel, agree well with previously published data, as shown in Figure 8.1. Most of the published experiments were conducted at higher Reynolds numbers than the present study, but the Strouhal number data as a whole seem to be remarkably insensitive to Reynolds number throughout the subcritical regime, from $Re = 500$ to $111,000$ (almost three orders of magnitude in Reynolds number), for both two-cylinder and three-cylinder arrangements. Some

representative power spectra for two-cylinder side-by-side geometries are shown in Figure 8.2; power spectra for the three-cylinder arrangements are presented later, in Section 8.2.8.

The Strouhal number data for two-cylinder side-by-side arrangement, in Figure 8.1(a), show that a single frequency of vortex shedding is measured when the two cylinders are in contact, at $T/D = 1.0$, and for small pitch ratios of $T/D < 1.25$. Here, the cylinders behave as a single bluff-body (see Section 8.2.2), with a single vortex street in the combined wake of the cylinders. The single Strouhal number, of $St \sim 0.1$, is the dominant frequency in the power spectrum, in Figure 8.2(a). Its value corresponds to half the value measured for a single circular cylinder at comparable Reynolds numbers, and would seem to reflect the increased spacing of two diameters between the two free shear layers. In Figure 8.2(a), a second Strouhal number of $St = 0.18$ is also found in the power spectrum, however this corresponds not to vortex shedding from a single cylinder (the similar vortex shedding Strouhal numbers are just a coincidence), but to the detection of Kármán vortices shed from both of the free shear layers.

As shown in Figure 8.1(b), a similar result is found for three side-by-side cylinders in contact, at $T/D = 1.0$, and for very small pitch ratios, where $St \sim 0.05$ to 0.06 is obtained. This nearly corresponds to one-third the shedding frequency for a single circular cylinder, and seems to reflect the increased spacing of three diameters between the two separated shear layers (although this would be correct only if boundary layer separation occurred at $\pm 90^\circ$ from the forward stagnation point). Single bluff-body behaviour is also seen for three side-by-side cylinders in contact, as will be discussed further in Section 8.2.2.

From $T/D \sim 1.25$ to 2.5 , the Strouhal number data for two side-by-side cylinders (Figure 8.1(a)) show that two vortex shedding frequencies may be measured in the combined wake of the two cylinders, where a second high frequency of $St \sim 0.3$ to 0.4

suddenly appears.* In the power spectra for the biased flow range, in Figures 8.2(b-d), two frequency peaks are detected, and the energy content seems to be equally divided between the two frequencies (at this particular measurement location). Flow visualization and other experiments have shown that the detection of two frequencies corresponds to the biased flow pattern, where the flow through the gap between the two cylinders becomes deflected towards one of the two cylinders (as mentioned in Section 3.3, and described in more detail in Section 8.2.4). In the biased flow pattern, a wider near-wake region forms behind one of the cylinders, away from the deflected gap flow, which sheds Kármán vortices at the lower of two shedding frequencies. A narrower near-wake region forms behind the other cylinder, which sheds vortices at the higher frequency. As the pitch ratio increases, the difference between the two frequencies decreases, and the two slowly approach the Strouhal number of a single cylinder, by $T/D \sim 2.5$, of $St \sim 0.2$ (seen in the Strouhal data in Figure 8.1(a), and in the spacing between the peaks in the power spectra, in Figures 8.2(b-d)). For $T/D > 2.5$, each cylinder sheds vortices at the same frequency (as shown by the power spectrum in Figure 8.2(e)), although there remains some interference between the two cylinders, namely anti-phase synchronization to the two parallel vortex streets (as discussed in more detail in Section 8.2.5), up to $T/D \sim 4.5$.

Similar behaviour is seen for the three-cylinder arrangements, in Figure 8.1(b), where two frequencies are measured from $T/D \sim 1.5$ to 2.75 or 3.0. Flow visualization and other experiments (see Sections 8.2.6 to 8.2.8) demonstrate that the two frequencies correspond to a symmetric biased flow pattern (shown schematically in Figure 3.6(c)), where the two gap flows are symmetrically deflected away from the flow axis towards the outer cylinders. In this flow pattern, a wider near-wake region forms behind the middle cylinder from which Kármán vortices are shed at the lower of the two frequencies, and

* Although Figures 8.1(a,b) show a range of Strouhal numbers at some pitch ratios, in fact no more than two frequencies were detected in a given experiment (as shown in the power spectra, in Figure 8.2). The scatter of the data simply shows that there is some variation in the magnitudes of the two frequencies at a given pitch ratio, possibly influenced by experimental effects, and variations in the actual measured mean flow velocity.

narrower near-wakes form behind the two outer cylinders, each of which sheds vortices at the same, higher frequency. Vortex shedding similar to a single cylinder is found at higher pitch ratios, of $T/D > 2.5$ to 3.0, where a Strouhal number of $St \sim 0.2$ is measured. There still remains some proximity interference at the higher pitch ratios, however, in the form of anti-phase vortex street synchronization (see Section 8.2.7).

The Strouhal number data in Figure 8.1(b) also show that three vortex shedding frequencies may be found for some three-cylinder arrangements. First, the published data show that intermediate frequencies of $St = 0.2$ to 0.25 can be found from $T/D \sim 1.125$ to 1.5, which corresponds to the asymmetric biased flow pattern sketched in Figure 3.6(b). In this case, a narrow near-wake region forms behind one of the outer cylinders, from which Kármán vortices are shed at the highest of the three frequencies. Progressively wider near-wake regions are then found behind the other two cylinders, which shed vortices at the two lower frequencies. Further discussion is found in Section 8.2.6.

Second, three frequencies were measured in the present study at $T/D = 1.5$, with a single low Strouhal number and a pair of high Strouhal numbers being detected. Since the flow visualization experiments showed a mostly symmetric biased flow pattern (shown in Figure 3.6(c), and discussed in Section 8.2.8), the two high frequencies were first thought to be attributed to small differences in the sizes of the near-wake regions behind the outer cylinders, i.e. if one of the cylinders had a slightly wider near-wake region than the other, perhaps due to misalignment of the cylinders, this would account for the second high frequency measurement. The pair of high frequencies at $T/D = 1.5$, however, is now believed to be caused by a pairing and enveloping process involving the low frequency and high frequency shed Kármán vortices, as discussed in more detail in Sections 8.2.7 and 8.2.8.

At the uppermost pitch ratio for the biased flow pattern for two cylinders, and the symmetric biased flow pattern for three cylinders, there is some disagreement between the present study and the previously published data for the number of frequencies measured. The water tunnel data show that two frequencies are measured at $T/D = 2.5$, for both cylinder arrangements, which extends the previously reported range of the biased flow

patterns for both. The extensions may be caused by experimental effects in the water tunnel, such as high blockage and lower aspect ratio, and are mentioned further in Sections 8.2.4 and 8.2.7 for two cylinders and three cylinders, respectively.

8.2.2 Side-by-Side Cylinder Configurations with $T/D = 1.0$

When the cylinders are in contact with one another, at $T/D = 1.0$, the flow pattern resembles a single bluff-body, with alternate Kármán vortex formation and shedding from the two free shear layers, and a single Kármán vortex street in the combined wake of the two cylinders. This behaviour is shown in the flow visualization images for two cylinders (no good-quality flow visualization images were available for the three-cylinder arrangements) in Figure 8.3(a), and in the instantaneous, in-plane vorticity data in Figure 8.4. Similar single bluff-body behaviour was seen for tandem cylinders in contact, as described in Section 7.2.2.

Kármán vortex shedding from the cylinders in contact showed some variability, in both the flow visualization video and the instantaneous PIV vorticity data. In the water tunnel, regular alternate vortex shedding from the two-cylinder configuration was often supplanted for periods of time by irregular or symmetric vortex shedding, as shown in Figure 8.3(b). Additionally, the vorticity fields tended to be of poorer quality and the Kármán vortices were less well defined at $T/D = 1.0$, when compared to other T/D values, which suggested a weaker vortex formation process. The less intense vortex shedding may be attributed to the different base geometry when compared to a larger circular cylinder of twice or three times the diameter. The experiments also revealed a range of vortex shedding behaviour, including (i) variation in the vortex formation length, with vortices forming near to the cylinders (shown in Figure 8.4(a)) or, alternatively, further into the near-wake; and (ii) variation in the cross-stream extent of the formed vortex, which could occasionally extend across the wake centreline. These variations could reflect different “modes” of fluid behaviour or different stages in the shedding cycle (a complete cycle could not be captured in one towing run with the current PIV system), but

are likely attributable to the instantaneous nature of the PIV data (as noted by Sheridan *et al.* 1998).

The PIV vorticity data also showed that the separated shear layers often broke up into small concentrations of vorticity during the vortex formation process; they can be discerned from the vorticity fields in Figure 8.4(b). These shear layer vortices are similar to those studied by Wei & Smith (1986) and Chyu & Rockwell (1996b), for example, and are related to Kelvin-Helmholtz instabilities in the shear layers. The shear layer vortices are seen, at times, to combine into larger vortices during the process of formation and shedding (Figure 8.4(b)), in a process known as amalgamation (Freymuth 1985).

8.2.3 Two-Cylinder Configurations with Small T/D

When there is a small gap between two side-by-side cylinders, a portion of the oncoming flow is directed into the base region of the cylinders, and the single bluff-body flow pattern at $T/D = 1.0$ becomes slightly modified. As shown in the PIV vorticity data in Figure 8.5 for two cylinders at $T/D = 1.125$ (no flow visualization images of side-by-side cylinders at small pitch ratios were acquired, since the apparatus in the water tunnel was restricted to spacing intervals of $0.5D$), the shear layers which form on either side of the gap flow do not roll-up into Kármán vortices, and remain completely within the near-wake region. Instead, the dominant shear layers remain those forming from the outside surfaces of the two cylinders, adjacent to the mean flow, which alternately roll-up into Kármán vortices to form a single vortex street in the combined wake of the two cylinders (seen forming in Figures 8.5(b,c), for example).

The flow through the gap between the cylinders behaves in a similar fashion to base-bleed, as studied by Bearman (1967) and noted by Bearman & Wadcock (1973). The higher-momentum base flow noticeably increases the vortex formation length in comparison to two cylinders in contact, as shown in Figure 8.5(a) (in comparison with Figure 8.4(a)). Despite this effect, there was no noticeable change in the Strouhal number, which remains at $St \sim 0.1$ similar to the two cylinders in contact (see Figure 8.1(a)).

On an instantaneous basis, the PIV vorticity data revealed three basic types of base-bleed behaviour at small pitch ratios, shown in both Figures 8.5 and 8.6; different behaviours were observed each time a new experiment was conducted. In the first type of behaviour, the gap flow remained parallel to the flow axis and the near-wake region was highly extended (Figures 8.5(a) and 8.6(a)). Here, the parallel gap flow shear layers persist until about $x/D = 2.0$, where the two shear layers end without vortex formation.

In the second type of behaviour, which was the most often observed, the base flow was deflected towards one of the cylinders (Figures 8.5(b) and 8.6(b)), and the near-wake region of the two-cylinder pair was then of intermediate length. In the towing tank, the deflection always seem to favour the “upper” cylinder,* however this was probably caused by minor misalignment of the two cylinders. The gap flow likely could freely deflect to either cylinder, since it seemed to deflect away from the next-forming Kármán vortex (Figure 8.5(b)). This second type of behaviour may also represent a transitional behaviour between the single bluff-body flow pattern at $T/D = 1.0$, discussed in Section 8.2.2, and the biased flow pattern at intermediate T/D , discussed in the Section following. Alternatively, the biased base-bleed flow may represent one stage of a larger shedding period that could not be captured by the PIV system in this study (typically up to 40% of the period could be captured in a single test, for side-by-side configurations with low frequency vortex shedding).

In the third type of behaviour, there was no appreciable gap flow, and a very short near-wake region resulted (Figures 8.5(c) and 8.6(c)). Three identical variations in behaviour were also noticed for staggered cylinder configurations with a base-bleed flow pattern, as described in Section 9.4.

8.2.4 Biased Flow Pattern for Two Cylinders at Intermediate T/D

From $T/D = 1.25$ to 2.5 (corresponding to intermediate T/D), the hot-film anemometry experiments for two side-by-side cylinders revealed two dominant frequencies in the

* In other words, the base flow was always deflected towards one side of the towing tank. There is no “upper” cylinder, for the cylinders were mounted vertically, and this refers to the Figures only.

combined wake of the cylinders (Figure 8.1(a), Figure 8.2(b-d)), which indicated the presence of the biased flow pattern. The flow visualization videos confirmed the existence of the biased flow pattern, with the gap flow deflected towards one of the two cylinders, as shown in Figure 8.7 for $T/D = 1.5$ and 2.0. Kármán vortex shedding was seen from each cylinder, at different frequencies, and two parallel vortex streets could be observed in the combined wake (Figure 8.7(b)).

The PIV vorticity data for $T/D = 1.25, 1.5, 1.75$ and 2.0, shown in Figure 8.8, also clearly show the biased flow pattern, with a narrower near-wake region behind one of the cylinders, and a wider near-wake region behind the other cylinder away from direction of gap flow deflection. At $T/D = 2.0$ (Figure 8.8(d)), the biased flow pattern is more apparent in the relative lengths of the near-wake regions of the two cylinders, rather than in the deflection of the gap flow; in the Figure, the “upper” cylinder has a slightly shorter vortex formation length than the “lower” cylinder. The two parallel vortex streets were more clear at the higher pitch ratios, at $T/D = 1.75$ and 2.0 (Figures 8.8(c,d)).

Concerning the synchronization of the two Kármán vortex streets, the two types of complex interaction observed by Williamson (1985) at low Reynolds numbers, discussed in Section 3.3 and shown schematically in Figures 3.5(c,d), were not clearly observed in the present study at $Re = 500$ to 3000. The complex interaction was perhaps not detected for two reasons: (i) in the context of the instantaneous vorticity field from PIV, the vortices become rapidly distorted and lose coherence (or definition) once they are shed from the cylinders, masking any amalgamation process (shown in Figures 3.5(c,d)); and (ii) the amalgamation process may occur further downstream from the cylinders, outside the range of detection of the PIV system used in this study. The sole form of flow organization observed was anti-phase synchronization of the two vortex shedding processes, shown schematically in Figure 3.5(a), whereby vortices were formed and shed in a symmetric manner on opposite sides of the deflected gap flow. The synchronization was seen only in the immediate vicinity of the two cylinders (as in the VPE, VPSE, and SVS flow patterns identified for two staggered cylinders, discussed in Sections 9.6 and 9.7), however, and no established regular pattern of rows of vortices was seen in the

combined wake of the two cylinders (as noted for large T/D , Section 8.2.5). Such behaviour, under biased flow conditions, would seem to broadly correspond to the “quasiperiodic regime” noted by Peschard & Le Gal (1996).

Both the flow visualization and PIV results showed that the gap flow deflection angle δ , defined in Figure 8.9, varied with the transverse pitch ratio and tended to decrease as the cylinders were located further apart. This trend is similar to the difference between the high and low vortex shedding frequencies for the biased flow pattern (Figure 8.1(a)), which also decreases as the pitch ratio is increased. Although the deflection angles from the flow visualization experiments are higher than those from the PIV experiments, the difference may be caused by the same experimental effects which caused the two shedding frequencies to be extended to higher T/D (mentioned in Section 8.2.1, and discussed further below). The full-field PIV data (in Figure 8.8) also show that the *instantaneous* deflection angle at a given T/D may itself vary over a wide range, from vorticity field to vorticity field, and from experiment to experiment. This variation is indicated by the error bars in Figure 8.9(a). Also shown in Figure 8.9(a) are two values for δ at $T/D = 1.125$, which reflect the two extreme cases for the base-bleed flow mentioned in Section 8.2.2, and shown in Figures 8.6(a,c).

As mentioned in Section 8.2.1, two vortex shedding frequencies were measured in the water tunnel at $T/D = 2.5$ (Figures 8.1(a) and 8.2(d)), which extended the usual range of the biased flow pattern from $T/D = 2.2$. A small degree of gap flow deflection was also detected at $T/D = 2.5$, as shown in Figure 8.9(a), although it can barely be discerned from the flow visualization picture shown in Figure 8.10(a). No apparent deflection is seen in the vorticity data for $T/D = 2.5$ (Figure 8.11), however, and this difference suggests that the detection of multiple frequencies in the water tunnel at $T/D = 2.5$ might be caused by external influences (i.e. from the experimental setup in the water tunnel), or it may possibly be the result of a transitional behaviour between a predominantly biased flow pattern and a predominantly symmetrical flow pattern (shown schematically in Figures 3.3(b,c)).

The biased flow pattern is also known to be bistable, with the deflection of the gap flow “flip-flopping” from being directed towards one cylinder and then to the other (Bearman & Wadcock 1973; Kim & Durbin 1988). The bistable characteristic is not believed to be caused by misalignment of the cylinders or other extraneous influences, but is thought to be an intrinsic property of the flow (Bearman & Wadcock 1973; Kim & Durbin 1988). In the same manner, the transverse configuration may also be considered a “critical” or transitional arrangement between two staggered configurations of greater and lesser incidence to the oncoming flow, which would account for the bistable nature; this is explored further in Section 9.10.2. Peschard & Le Gal (1996), however, who did not observe bistable behaviour in their low Reynolds number water tunnel experiments, suggest that it may be caused by turbulent perturbations in the incoming flow. In this study, and specifically in the water tunnel experiments, the bistable nature was not observed either, and the flow remained deflected consistently towards the same cylinder for long-duration flow visualization experiments. Only the experiments conducted in the towing tank showed that the near-wakes could be freely deflected towards either of the two cylinders, but in different tests, with no changes in the alignment of the cylinders or in the Reynolds number (although specific examples are not shown in Figure 8.8). In the present case, it is suspected that residual motion of the water in the towing tank (i.e. settling of the water after completion of an experiment, or after redistribution of the seeding particles) may be responsible for the selection of the biased flow direction in a given experiment.

8.2.5 Two-Cylinder Configurations with Large T/D

As the pitch ratio is increased, the flow through the gap is no longer biased towards one of the cylinders, and the two near-wake regions become similar in size. Kármán vortices are now shed at the same frequency from both cylinders, and the Strouhal number is identical to that of a single, isolated circular cylinder, as shown in Figures 8.1(a) and 8.2(e). As shown in the flow visualization results in Figure 8.10, and in the PIV vorticity

data in Figures 8.11 and 8.12, the combined wake of the two cylinders contains two parallel Kármán vortex streets.

Despite the vortex shedding frequency reverting to the value for a single cylinder, some interference remains between the two cylinders in the form of anti-phase synchronization to the two side-by-side vortex streets and formation processes.* Anti-phase synchronization was illustrated schematically in Figure 3.5(a), and is marked by symmetrical formation and shedding of vortices on either side of the gap between the two cylinders, and side-by-side correspondence of Kármán vortices of opposite sign across the two vortex streets. The synchronization or interaction of the vortex streets was observed in the water tunnel experiments up to $T/D = 4.5$ (Figure 8.10(b)), but was no longer evident at higher pitch ratios. The synchronization is clearly shown in Figures 8.10(a,b), 8.11(a) and 8.12, where in the latter two examples the corresponding vortices in the two vortex streets are labeled as A, B and C. The matching of vortices is found both in the vortex formation regions, as the vortices are shed, and also in the vortex streets.

Occasionally, an in-phase synchronization to the two vortex streets was noticed (defined in Figure 3.5(b)), as shown in Figure 8.11(b), however the predominant flow pattern was anti-phase vortex shedding, as indicated by Williamson (1985); its predominance broadly corresponds to the “Locking Π Phase Opposition” regime of Peschard & Le Gal (1996). Although Williamson (1985) indicated that in-phase shedding leads to the formation of a single, binary vortex street in the combined wake of the two cylinders, such a development was not observed in the present PIV experiments. A possible reason for this is that the combined wake of the two cylinders could not be visualized for a sufficient distance downstream with the given PIV system.

Since the Kármán vortices were well-defined in the PIV vorticity data, for the large T/D side-by-side configurations, reliable measurements of the Kármán vortex strength or circulation could be obtained, for $T/D = 2.5$ and 3.0; the results for $T/D = 3.0$

* Anti-phase vortex shedding should be expected, based on the symmetry of the starting flow field for side-by-side cylinder configurations (Sumner *et al.* 1997a,b), which is strongly symmetric about the gap flow. See also Section 8.3.

only are shown in Figure 8.13. The circulation measurements were computed using equation (5.7), with a lower vorticity magnitude of 0.75, in the same manner done for the single cylinder experiments in steady cross-flow (reported in Section 6.2).

Similar to the single cylinder results (Section 6.2.2), the circulation measurements in Figure 8.13 show a decay with streamwise distance from the cylinder. No difference in the decay rates for the inner and outer vortices was noticed, within the limits of experimental uncertainty (estimated at 10 to 20%). This result contrasts with the phase-averaged vorticity measurements of Kolar *et al.* (1997) for two side-by-side square cylinders at $T/D = 3.0$, where the strengths of the vortices on either side of the gap were found to decay at a greater rate than those adjacent to the outer flow. The convection velocities of the vortices, obtained in the same manner as the single cylinder (Section 6.2.3), were measured as $U_c = 0.84U$ in the streamwise range of $x/D = 2.0$ to 6.0 . This result compares favourably with $0.83U$ for a single circular cylinder as measured by Cantwell & Coles (1983), $0.80U$ for two side-by-side circular cylinders as measured by Kim & Durbin (1988), and somewhat less well with $0.75U$ for two side-by-side square cylinders as measured by Kolar *et al.* (1997). No difference in U_c was noticed for the vortices shed alongside the gap, or vortices shed alongside the outer flow, within the limits of experimental uncertainty, similar to what was observed by Kolar *et al.* (1997).

8.2.6 Three-Cylinder Configurations with Small T/D

Kumada *et al.* (1984) identified four different flow patterns for the three-cylinder side-by-side configuration in steady cross-flow, and these were shown schematically in Figure 3.6. At very small pitch ratios from $T/D = 1.0$ to 1.125 , the single bluff body flow pattern is observed (Figure 3.6(a)), as discussed in Section 8.2.1, and a single vortex street forms in the combined wake of the three cylinders. From $T/D = 1.125$ to 1.35 , an asymmetric biased flow pattern develops (Figure 3.6(b)), in which one of the outer cylinders has a wider near-wake region, and the remaining two cylinders have narrower near-wake regions. As some of the published Strouhal number data showed (Figure 8.1(b)), up to three vortex shedding frequencies may be identified with the asymmetric biased flow

pattern, and these likely correlate with three near-wake regions of three different sizes (as discussed in Section 8.2.1).

In the water tunnel, the spacing between the cylinders was limited to $0.5D$ increments, and the asymmetric biased flow pattern could not be observed. The PIV experiments, however, verified the flow pattern's existence, as shown in Figure 8.14. The pattern is very similar to the base-bleed flow pattern seen for two side-by-side cylinders, described in Section 8.2.3, with a single vortex street in the combined wake of the three cylinders; the base-bleed effect, and the single vortex street, were also noticed by Eastop & Turner (1982). The vorticity data in Figure 8.14(a) show a large Kármán vortex forming from the outer shear layer of the "lower" cylinder, which comprises one side of the single Kármán vortex street.

These results show that the wider near-wake region could freely develop behind either of the two outer cylinders, in different experiments (compare Figures 8.14(a,b)). The different sizes of the three near-wake regions, as well as the streamwise extents of the deflected gap flows, are perhaps shown more clearly in Figure 8.15, where deflection to either side of the towing tank is again illustrated for different tests. As in the case of the biased flow pattern for two side-by-side cylinders, described in Section 8.2.4, it is very possible that the selection of the gap flow direction, and the location of the widest near-wake region, (i) would exhibit bistable properties in wind tunnel experiments;* (ii) could be influenced by minor misalignment of the three cylinders; and (iii) could be influenced by transient effects and residual motion in the towing tank. The velocity data also illustrate the variability of the gap flow deflection angle, on an instantaneous basis,

* In the experiments of Eastop & Turner (1982), bistable flow was found only at $T/D = 1.375$, which would seem to correspond to a transitional flow pattern alternating between the asymmetric and symmetric biased flow patterns (these two types of behaviour were also seen to alternate in the present study, in different experiments, at $T/D = 1.25$, as noted in Section 8.2.7, and shown in Figure 8.16). Eastop & Turner (1982) did not detect any bistable behaviour for $T/D > 1.375$, and this is consistent with a stable, symmetric biased flow pattern for which the gap flow deflection angles do not change (as noted by Ishigai *et al.* (1972)).

with highly angled gap flow in Figure 8.15(a), and a largely undeflected base flow emanating from the gap between the two “lower” cylinders in Figure 8.15(b).

8.2.7 Three-Cylinder Configurations with Intermediate T/D

With an increase in the transverse pitch ratio to $T/D = 1.25$, the asymmetric biased flow pattern continues to be observed for the three-cylinder side-by-side group, but its appearance alternates with a more symmetric biased flow pattern, in different experiments in the towing tank. Several sets of vorticity data are shown in Figure 8.16, where Figures 8.16(a,c) show the asymmetric biased flow pattern described above, and Figures 8.16(b,d) show a pattern which is more symmetric about the central cylinder. In the symmetric biased flow pattern, shown schematically in Figure 3.6(c)), the two gap flows are deflected at nearly the same angle from the centreline towards the outer cylinders, and a wide near-wake region is found behind the central cylinder. Low frequency Kármán vortices are shed from the central cylinder, while higher frequency Kármán shedding occurs from the two outer cylinders, each of which has a narrow near-wake region and sheds vortices at nearly the same frequency (see discussion in Section 8.2.1, and Figure 8.1(b)). In the gap flow deflection angle data, plotted in Figure 8.9(b) for the three-cylinder arrangement, the deflection ranges for both the asymmetric and symmetric biased flow patterns are shown.

With a further increase in the pitch ratio to $T/D = 1.5$, the symmetric biased flow pattern is observed consistently, as shown in the flow visualization results in Figure 8.17(a) and in the vorticity data in Figure 8.18. The pattern was also observed in the water tunnel at higher transverse pitch ratios of $T/D = 2.0$ and 2.5 (Figures 8.17(b,c)), but the flow field reverted to an anti-phase synchronized vortex shedding behaviour by $T/D = 3.0$ (Figure 8.17(d)). Unfortunately, none of these larger side-by-side configurations could be accommodated within the field of view of the PIV system, without changing the magnification (and reducing the resolution of the vorticity field). The deflection angle of the gap flow decreases as the cylinders are spaced further apart, as shown in Figure 8.9(b); these data show that by $T/D = 3.0$, there is no further deflection of the gap flow.

The detection of the symmetric biased flow pattern for pitch ratios up to $T/D = 2.5$ in the water tunnel, as suggested by the Strouhal number data in Figure 8.1(b) where two frequencies are still detected at this pitch ratio, means the observed range is extended beyond the reported value of $T/D = 2.2$ (Kumada *et al.* 1984). A similar increase in the T/D range was found for the biased flow pattern for two side-by-side cylinders in the water tunnel, as mentioned in Sections 8.2.1 and 8.2.4, and the reasons for the occurrence are likely the same.

In the symmetric biased flow pattern, large-scale Kármán vortices are shed from the wide near-wake region behind the central cylinder, as shown in Figures 8.17(a,b) and in the last three vorticity fields in Figure 8.18(b). Three parallel vortex streets are then nominally found in the wake of the cylinder group, as shown in Figure 8.17(c). The flow visualization pictures also show some complex interaction between the larger vortices shed from the central cylinder, and the smaller Kármán vortices shed from the outer cylinders. Specifically, vortices forming on either side of the gap seem to be synchronized in an anti-phase manner, despite the difference in shedding frequency across the gap. This is shown by the pairing of vortices across the gap in Figure 8.17(b), and in the last two vorticity fields in Figure 8.18(a). Furthermore, once this pair of vortices is formed, it is either completely enveloped (Figure 8.18(a)) or partially enveloped (Figure 8.18(b)) by a Kármán vortex shed from the outer shear layer of the outer cylinder. Although dominant at $T/D = 1.5$, the process is no longer observed at $T/D = 2.5$ (Figure 8.17(c)). This enveloping process is not too clear in the PIV vorticity data at $T/D = 1.5$, in Figure 8.18, however it may be discerned in the last vorticity field in Figure 8.16(b), for example, at $T/D = 1.25$. This pairing and enveloping behaviour, of the gap vortices, was also found for two staggered circular cylinders in steady cross-flow, as described later in Section 9.6.

As mentioned, once the biased flow pattern disappears, the cylinders each shed vortices at the same frequency which corresponds to that of a single, isolated circular cylinder (see Figure 8.1(b)). Vortex shedding now occurs in the predominantly anti-phase mode of behaviour, as in the case of two side-by-side cylinders at large T/D (Section 8.2.5).

8.2.8 Vortex Shedding for the Symmetric Biased Flow Pattern

In the process of obtaining the Strouhal number data for three side-by-side cylinders (Figure 8.1(b)), the power spectra were computed at each pitch ratio. In each case, frequency measurements were also obtained from the hot-film sensor at a number of locations in the combined wake of the three cylinders, rather than at a single position. These measurements indicated that the number of frequencies detected, and their strengths, were often position-dependent. The Strouhal number data in Figure 8.1(b), for example, showed that up to three values could be found at $T/D = 1.5$, with one low frequency and two high frequencies. In Figure 8.19, some power spectra for the $T/D = 1.5$ configuration of three cylinders are plotted for a range of streamwise, x/D , and cross-stream, y/D , measurement locations.

The spectra show that the low Strouhal number, of $St = 0.05$ to 0.06 , is detected throughout the wake. The low frequency corresponds to vortex shedding from the wider near-wake region of the central cylinder (verified by counting the vortices from the video frames), and is particularly strong at $x/D = 3.0$ (Figures 8.19(e,f)). This streamwise location appears to be well correlated with the point of vortex formation for the central cylinder, where the Kármán vortices are at their strongest and are just about to be shed (see Figure 8.17(a) and the second and third vorticity fields in Figure 8.18(b)).

The highest frequency of vortex shedding, at $St = 0.31$ to 0.33 , is also found throughout the wake. It is associated with vortex shedding from the two outer cylinders (and confirmed from the flow visualization video), and is strongest nearer to the cylinders, at $x/D = 2.0$, and at the outside of the wake, for $y/D = 2.5$ and 3.0 (Figures 8.19(b,c)). By inspection of the vorticity data in Figure 8.18, these locations correspond to the end of the vortex formation regions for the outer cylinders, where the Kármán vortices are fully formed and are about to be shed.

In Figure 8.19(e,f), however, the second high frequency peak, of $St = 0.27$ to 0.28 , is also found. By inspection, this frequency appears to be the difference between the first two shedding frequencies. Since this third frequency is found further from the cylinders, where the low frequency shedding signal is the strongest, at $x/D = 3.0$, it seems to be

associated with the pairing and enveloping of the high frequency and low frequency Kármán vortices (the process was described in Section 8.2.7 and shown in Figure 8.17(a), for example). Further evidence of the association between the second high frequency and the vortex pairing and enveloping process is also found in the Strouhal data, as the two high frequencies are not detected at $T/D = 2.0$ and 2.5 , which is consistent with the gradual disappearance of the process as the pitch ratio is increased (see Section 8.2.7, and Figures 8.17(a-c)).*

8.3 Impulsively Started Flow

The fluid behaviour of the side-by-side configuration of circular cylinders has not been previously studied under the conditions of an impulsive start, however it was expected to be more complex than the tandem configuration previously described in Section 7.3, in particular because of the existence of the biased flow pattern for two cylinders under steady mean flow conditions. Williamson (1997) had acquired some flow visualization photographs of the starting flow, related to his study of side-by-side cylinders in steady cross-flow (Williamson 1985), however the photographs were left unpublished. Many of the key features of the flow field are similar to those identified for the single cylinder, as shown schematically in Figure 8.20(a), although the gap flow will be seen to be the dominant flow structure (in Sections 8.3.2 and 8.3.3).

In this study, two-cylinder and three-cylinder side-by-side configurations were investigated in the small, vertical water towing tank, using the seeding particle flow visualization technique (described in Section 5.2) and PIV. Many of these results were published during the course of the present study, by Sumner *et al.* (1997a,b,c). In these experiments, the Reynolds number varied from $Re = 1500$ to 2800 , in the same range of

* Eastop & Turner (1982) did not detect a third frequency in the range of $T/D = 1.375$ to 2.25 , but the furthest downstream their probe was positioned was $x/D = 1.2$, and this may be the reason it was missed. The two researchers varied the cross-stream position of the probe, but not the streamwise position. This result illustrates the sensitivity of the measurement location when it comes to detecting multiple frequencies in the flow fields of multiple cylinders, and the need to make measurements at more than one location.

the low subcritical regime used for the single cylinder and tandem cylinders under impulsively started flow, reported in Sections 6.3 and 7.3, respectively. The two techniques enabled the flow field development to be captured for $t^* < 6.0$, under nearly impulsively started conditions of $A_p = 0.2$ to 0.9 . Two-cylinder configurations were tested at transverse pitch ratios of $T/D = 1.0, 1.5, 2.0, 2.5$ and 3.0 , and three-cylinder configurations were tested at $T/D = 1.0, 1.5$ and 2.0 .

Based on the flow visualization and PIV experiments, the present results were consistent with the unpublished work of Williamson (1997), although they revealed considerably more of the fluid behaviour. In contrast to the impulsively started tandem cylinders, where three basic types of behaviour were observed (each corresponding to one of the steady flow patterns), only two basic types of behaviour were found here, for impulsively started two-cylinder and three-cylinder side-by-side configurations. These two types of behaviour are grouped simply by (i) the absence of a gap between the cylinders, at $T/D = 1.0$, when the cylinders are in contact (Section 8.3.1, Figure 8.20(b)); and (ii) the presence of a gap between the cylinders, for $T/D > 1.0$ (Sections 8.3.2 and 8.3.3, Figure 8.20(c)). In both basic types of behaviour, the starting flow field remained symmetric over the complete range of transverse pitch ratio tested, and for the full range t^* , and no evidence of a biased flow pattern or deflection of the gap flow was observed. This result suggests that the transition to a biased flow pattern, for both the two-cylinder and three-cylinder groups, occurs much later in the temporal development of the flow field before steady mean flow conditions are attained. The results also showed, that within the limited range of Reynolds number tested, no significant influence of the Reynolds number existed, and similar flow structures were identified at the same t^* values. This Reynolds number insensitivity was expected, based on the behaviour of (i) a single cylinder under impulsively started conditions (Section 6.3), (ii) tandem cylinders under similar conditions (Section 7.3), and (iii) side-by-side cylinders under steady cross-flow conditions (Section 8.2.1).

8.3.1 Cylinder Configurations with $T/D = 1.0$

When the side-by-side cylinders are in contact, at $T/D = 1.0$, the temporal development of the flow field is similar to that for a single cylinder under impulsively started conditions (Section 6.3), and to tandem cylinders in contact (Section 7.3.1), with the flow pattern resembling that of a single bluff-body. The flow visualization images and the PIV vorticity data in Figures 8.21 and 8.22, respectively, both illustrate the formation of a single, large, quasi-attached recirculation zone in the combined wake of the cylinders, containing a symmetric pair of eddies of equal strength and opposite rotation. In the PIV vorticity data, small concentrations of secondary vorticity may be seen, in opposite rotation to the primary eddies, for two cylinders from $t^* = 2.5$ to 3.8 (Figure 8.22(a)), and for three cylinders at $t^* = 2.9$ and 3.4 (Figure 8.22(b)). The secondary eddies arise from induced fluid motion between the primary eddy and the cylinder wall, and are also a feature of the single cylinder flow field after an impulsive start (see Section 2.3).

The similarity to a single cylinder extends to some of the parameters used to quantify the size of the recirculation zone, defined in a similar manner to a single cylinder, as shown in Figure 8.20(b). These parameters were computed interactively from the velocity data, using the first method described in Section 5.7.2, from which the circulation was obtained by the line integral in equation (5.6). A lower limit of 1.0 was specified for the dimensionless vorticity magnitude in these calculations.

Plotted alongside data for the single cylinder, in Figure 8.23, the temporal growths of the streamwise position of the primary eddies from the base of the cylinders, a , the length of the recirculation zone from base of the cylinders, L_R , and the primary eddy strength or circulation, Γ , are mostly independent of the number of cylinders (shown in Figures 8.23(a,c,d), respectively). Some diversion from the common behaviour may be noticed for the single cylinder data at $t^* = 4.0$, however, since this coincides with the initial signs of asymmetry in the recirculation zone, and its breakup and transition to alternate, Kármán shedding for $t^* = 4.0$ to 6.0. The continued, nearly linear temporal growth of a , L_R , and Γ for the two-cylinder and three-cylinder groups, for $t^* = 4.0$ to 6.0

(as shown in Figures 8.23(a,c,d), respectively), suggests that the development of asymmetry in the recirculation zone, and the transition to alternate vortex shedding, occurs later in the time history as additional cylinders are added in contact. This is supported by the flow visualization images in Figure 8.21, which show symmetric recirculation zones up to $t^* = 5.0$, and no appearance of “gathers” extending from the rear of the zones that precede the development of asymmetry (see the results for tandem cylinders, in Section 7.3, for example).

In Figure 8.23, the data for two and three cylinders, and the elapsed time, are made nondimensional with the single cylinder diameter D , rather than using $2D$ or $3D$ as the case may be. In any case, use of the latter geometrical scales to render the data dimensionless did not yield a closer collapse of the data over the entire range of t^* , and even failed to collapse the data for the transverse spacing of the primary eddies, b , in Figure 8.23(b). That the recirculation zone parameters for single and multiple side-by-side circular cylinders did not collapse onto common curves reflects the obvious differences in the shape of the base region between two or three side-by-side cylinders and a single cylinder of two or three times the diameter. Measurements of the length of the recirculation zone, L_R , were also compared to similar measurements for an impulsively started flat plate (Taneda & Honji 1971), but no apparent convergence of the multiple cylinder data towards the flat plate data was observed for three adjacent cylinders.

8.3.2 Two-Cylinder Configurations with $T/D > 1.0$

When there is a gap between the two cylinders (shown in Figure 8.20(a)), with $T/D > 1.0$, the temporal development of the flow becomes more complex. The flow visualization pictures in Figure 8.24, and the vorticity data in Figure 8.25, for two-cylinder groups of $T/D = 1.5$ to 3.0 , show that symmetric and stable recirculation zones are not observed in the near-wake regions of the individual cylinders. Rather, the combined wake region is dominated by the strong gap flow between the cylinders, and the rapid formation, shedding, and interaction of eddies (or vortices) alongside the gap flow. The gap flow

initiates the breakup of the recirculation zones behind the cylinders, and vortex shedding, much sooner in the time evolution than for a single cylinder. This breakup eventually evolves into alternate, Kármán vortex shedding at higher t^* , although it is not shown in Figures 8.24 and 8.25.

Shedding of the initial gap vortices was estimated to occur at $t^* = 2.5$ to 3.0 , $t^* = 3.0$ to 3.5 , and $t^* = 3.5$ to 4.0 for $T/D = 1.5$, 2.0 and 2.5 , respectively. Shedding of the initial gap vortices is not observed, however, at the largest pitch ratio of $T/D = 3.0$, at least up to $t^* = 5.5$, although it will occur later in the time history (but outside the range of capture of the flow visualization and PIV systems in the present study). For those transverse pitch ratios where shedding is observed, new gap vortices form once the initial gap vortices are shed (see Figure 8.25(b), $t^* = 3.5$, for example), and the near-wake region once again contains a pair of asymmetrical, quasi-attached recirculation zones.

Interestingly, for $T/D = 1.5$ to 2.5 (Figures 8.24(a-c) and 8.25(a-c)), the combined wake of the cylinder pair is marked by the formation of a single counter-rotating vortex pair, that is symmetrical in both alignment with the flow axis and in the strengths of the two vortices. The gap vortices which comprise the pair mutually propel themselves away from the cylinders, and convect more rapidly into the wake than the outer vortices. No counter-rotating vortex pair was observed at $T/D = 3.0$, however. Here, the pair may form further from the cylinders outside the range of the camera, and at t^* outside the range of capture for the imaging system, but otherwise may not form at all at $T/D = 3.0$. If this is the case, the disappearance of the vortex pair may mark a transition from one type of behaviour, at intermediate T/D , to another type of behaviour at large T/D , which would broadly correspond to the biased flow and the synchronized vortex street flow patterns under steady upstream flow conditions (see Figures 3.6(c,d)).

Circulation measurements of the initially formed gap vortices and outer vortices, while they remain attached to the cylinders, show a steady increase in strength with time (Figure 8.26). The peak strength attained by the attached vortices increases with the pitch ratio, since they remain attached for longer periods of time before they are shed. Once the vortices are shed from the cylinders, the strength decays with time (and with distance

from the cylinder), in much the same manner seen for Kármán vortices shed from a single cylinder (in Section 6.2.2 and Figure 6.6) or from two side-by-side cylinders (in Section 8.2.5 and Figure 8.13), both in steady cross-flow. In Figures 8.26(a-c), the time at which eddies are shed from the cylinders can be easily discerned from the point at which the increase in strength stops, and this matches well with the flow visualization and vorticity data. A peak circulation value and a decay are not observed in the circulation data at $T/D = 3.0$ (Figure 8.26(d)), however, since shedding of the gap vortices occurs later in the time history.

Despite the rapid distortion of the gap vortices, the data in Figure 8.26 show that the strengths of the gap vortices and the outer vortices are of approximately equal magnitude while they remain attached to the cylinders. Additionally, by comparing data in Figures 8.26(a-d), the initial strengths of the attached vortices are shown to be basically independent of the pitch ratio.*

Data for the streamwise location of the vortices from the cylinder base, a , and cross-stream spacing of like vortices, b , are shown in Figure 8.27; the parameters a and b are defined in Figure 8.20(c).† These data (for the streamwise eddy location, a) show the rapid acceleration of the initial gap vortices for $T/D = 1.5$ to 2.5, as vortices are shed from the cylinder pair, convected into the combined wake of the cylinders, and as the counter-

* The vortex strength data for $T/D = 2.5$ (Figure 8.26(c)) are somewhat higher than those of the other three pitch ratios, but this is attributed to higher values of A_p in these experiments, with $A_p = 0.7$ to 0.9 compared to $A_p = 0.3$ to 0.4 for the others. Although the flow is nearly impulsively started in both cases, it more closely approximates the impulsive start at the higher acceleration parameters, and this results in proportionally higher values for the eddy strengths. The higher acceleration parameters for the $T/D = 2.5$ experiments also mean that the shedding of the outer vortices, which was not observed for $T/D = 1.5$, 2.0 and 3.0, will occur earlier in the time history. This is seen in the PIV vorticity data in Figure 8.25(c), at $t^* = 3.9$, for example, and in a peak value of outer vortex strength in Figure 8.26(c), followed by a slow decay.

† The length of the recirculation zone, L_R , defined in Figure 8.20(b) for side-by-side cylinders in contact, is not used when there is a gap between the cylinders, as here, since the two recirculation zones rapidly become asymmetric as the primary vortices alongside the gap flow are shed.

rotating vortex pair is quickly formed. Similar but less accentuated behaviour is noticed for the outer vortex centres, since these vortices remain attached to the cylinders for $t^* < 6.0$, and the movement of the centres is more similar to a single impulsively started circular cylinder (see data for a/D presented earlier in Figure 6.11(a)). The acceleration of the gap vortices varies markedly with T/D , and decreases as the spacing between the two cylinders is increased (and the interaction of the vortices is reduced). By contrast, the streamwise location of the outer vortices in time is generally independent of the pitch ratio. At $T/D = 3.0$ (Figure 8.27(d)), the streamwise motion of the gap vortex centres and the outer vortex centres are no longer distinct, further indicating a different type of behaviour at higher pitch ratios.

Data for the cross-stream spacing of the vortices, denoted by b , show how the gap vortices slowly approach each other to form the counter-rotating vortex pair; this trend is most apparent at $T/D = 2.5$ (Figure 8.27(c)), where the spacing decreases by one diameter from $t^* = 1.0$ to 6.0 . The spacing of the outer vortices remains relatively constant after the impulsive start, similar to a single impulsively started cylinder (see data in Figure 6.11(b)).

8.3.3 Three-Cylinder Configurations with $T/D > 1.0$

The impulsively started flow field for three side-by-side cylinders at $T/D = 1.5$ and 2.0 is much the same as the two-cylinder case (discussed in Section 8.3.2), with the flow through the gaps between adjacent cylinders influencing its development. The flow visualization results in Figure 8.28, and the PIV vorticity data in Figure 8.29, show that the flow field remains symmetric about the central cylinder, the gap flows remain unbiased (in contrast to the steady flow pattern, which shows a symmetric biased flow pattern at $T/D = 1.5$ and 2.0 , see Section 8.2.7), and the gap primary eddies which form behind the outer cylinders (see designations in Figure 8.20(a)) are quickly shed from the recirculation zone. Once the initial gap vortices are shed, others quickly form in their places to maintain the semblance of quasi-attached recirculation zones behind the two outer cylinders. As in the case of two side-by-side cylinders, the initially formed gap

eddies convect into the combined wake of the three cylinders and interact strongly (and particularly at the smaller of the two pitch ratios tested, $T/D = 1.5$), to form a counter-rotating vortex pair. The pair is shown most clearly in the results for $T/D = 1.5$ (Figure 8.28(a), $t^* = 3.9$, and Figure 8.29(a), $t^* = 3.7$), since it occurs further from the cylinders at $T/D = 2.0$, outside the view of the fixed digital camera. Asymmetry in the flow pattern, in terms of the deflection of the gap flows, was not observed in any of the flow visualization images or vorticity data and is presumed to occur for $t^* > 5.0$ to 6.0, as the transient effects subside and steady upstream flow conditions are reached.

Behind the central cylinder, a more stable and symmetric recirculation zone forms, containing a pair of quasi-attached eddies of equal strength and opposite rotation. These two inner eddies, which lie adjacent to the gap flows, are not shed from the central cylinder during the development of the flow. Rather, at $T/D = 1.5$, the growth of the central recirculation zone and its inner gap vortices is inhibited by the shedding of the outer gap vortices from the outer cylinders, as shown in Figure 8.29(a), $t^* = 3.7$. At $T/D = 2.0$, the growth of the recirculation zone behind the central cylinder is less influenced by the shed gap vortices from the outer cylinders, as seen in Figures 8.28(b) and 8.29(b).

Circulation measurements for the vortices on either side of the gap, in Figure 8.30, illustrate both the shedding of the outer gap vortices, and the impeded growth of the central recirculation zone. During the initial stages of temporal development, the inner and outer gap vortices each have the same strength, but peak values of the outer gap vortex circulation are reached just before they are shed; the peak circulation is higher at $T/D = 2.0$ (see Figure 8.30(b)), since the vortices remain attached longer to the outer cylinders under reduced interference from the central cylinder and the gap flow. A decay in vortex strength then follows vortex shedding as the vortices are convected away from the cylinders, and the beginning of the decay in strength can be used to pinpoint the time at which shedding of the outer gap eddies occurs; this may be estimated at $t^* = 2.8$ and 3.8, for $T/D = 1.5$ and 2.0, respectively. Concerning the strengths of the inner vortices, these continue to increase once the outer gap vortices are shed, however the growth in each case is no longer linear, but seems to settle to $\Gamma/UD = 3.5$ and 4.0, for $T/D = 1.5$ and

2.0, respectively. The higher circulation at $T/D = 2.0$ seems to indicate that the growth of the central recirculation zone is less impeded by the shed outer gap vortices.

The rapid convection of the gap vortices from the outer cylinders, as they form a counter-rotating vortex pair in the combined wake, is shown in the vortex position measurements in Figure 8.31. The gap vortices convect more quickly in the streamwise direction for the $T/D = 1.5$ configuration, as shown in Figure 8.31(a), although the rate of cross-stream movement appears to be the same for both $T/D = 1.5$ and 2.0, with a one diameter decrease in spacing (b/D) between $r^* = 1.5$ and 5.0. In contrast, the centres of the inner gap vortices remain at a relatively fixed position from the base of the cylinder, of $a/D = 0.5$ from $r^* = 2.5$ to 5.0. The cross-stream spacing of these eddies, which remain attached to the central cylinder, also remains relatively constant at $b/D = 0.6$ to 0.7, which is similar to a single impulsively started circular cylinder.

8.3.4 Further Discussion

For both the two-cylinder and three-cylinder configurations, the strengths of the attached vortices remain remarkably the same following an impulsive start, despite different rates of growth and distortion following the impulsive start. Furthermore, the data as a whole indicate that much of this initial temporal behaviour, which precedes the shedding of the gap vortices, is insensitive to the pitch ratio and, possibly, to the number of cylinders. A similar constancy is found in the analysis of the counter-rotating vortex pair: not only is the vortex pair found for *both* the two-cylinder and three-cylinder flow fields following an impulsive start, but when the strength of one of the vortices in the pair is made nondimensional with the pitch, T , rather than the cylinder diameter, D , there is a collapse of the circulation data, as shown in Figure 8.32. This result indicates that the strength of the counter-rotating vortex pair is proportional to the proximity of the cylinders, and by consequence to the transverse spacing, b/D , of the initially formed gap vortices. This result is consistent with a counter-rotating vortex pair not being observed, within the permissible range of r^* , for two cylinders at $T/D = 3.0$, since interaction and mutual attraction of the gap vortices is considerably weakened.

Chapter 9

Staggered Configuration

9.1 Introduction

The staggered configuration represents the most general arrangement of two circular cylinders, and the one most likely to be encountered in real, engineering applications, either on its own as a two-cylinder group, or as a component of a much larger multiple cylinder system. In many respects, however, it is perhaps the least thoroughly studied. A survey of the literature shows that little emphasis has been placed on obtaining a thorough understanding of the flow field for two staggered cylinders, as the centre-to-centre pitch ratio, P/D , and the angle of incidence, α (as defined in Figure 3.1(c)), are varied. Although extensive measurements of the fluid forces acting on the cylinders, and the vortex shedding frequencies, are available, there are few flow visualization (or other) studies from which to interpret the data from a purely fluid-dynamic viewpoint, and to show, for example, whether the measured frequencies are in fact attributable to vortex shedding. Also, the vast majority of these studies have been undertaken in the high subcritical regime (see Table 3.4), which may be useful when considering real, engineering flows (such as marine risers, twin chimneys or transmission lines), but there is still very little understanding of the influence of the Reynolds number.

9.1.1 Overview of Experiments

In this investigation of the staggered configuration, the emphasis is on the identification of the steady mean flow patterns for various combinations of P/D and α , at a single

Reynolds number in the low subcritical regime. Modifications to the flow patterns attributed to changes in Reynolds number are left to a future study. The identification of flow patterns done by other researchers for the tandem and side-by-side configurations has never before been attempted for the staggered configuration.

In this study, the two-cylinder staggered arrangement was examined only under steady mean cross-flow conditions, using flow visualization and PIV. The flow visualization experiments were conducted in the water tunnel, for configurations corresponding to various combinations of nine different pitch ratios, $P/D = 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5$ and 5.0 , and eight different angles of incidence (for most P/D settings), $\alpha = 10^\circ$ to 80° in steps of 10° . The flow was visualized only for a limited range of Reynolds number, $Re = 850$ to 1350 . Although hot-film anemometry was not used with the staggered configuration, vortex shedding frequencies were estimated (where they could be) from the individual video frames; discussion of the vortex shedding frequency data is presented in Section 9.9. Once the flow visualization experiments were completed, staggered arrangements were tested in the main water towing tank at a single Reynolds number, of $Re = 1900$, at combinations of nine different pitch ratios, $P/D = 1.0, 1.125, 1.25, 1.5, 2.0, 2.5, 3.0, 3.5$ and 4.0 , and nine different angles of incidence, $\alpha = 5^\circ, 10^\circ, 15^\circ, 20^\circ, 30^\circ, 45^\circ, 60^\circ, 70^\circ$ and 75° . The complete range of staggered configurations tested in the two facilities is shown in Figure 9.1, for the P/D versus α and L/D versus T/D descriptions (defined in Figures 3.1(c,d)).

9.1.2 Overview of Results

Traditionally, it is accepted that for the flow about a pair of circular cylinders in the staggered configuration, a smaller near-wake region forms behind the upstream cylinder, where vortices are shed at higher frequency, and a wider near-wake forms behind the downstream cylinder, where vortices are shed at lower frequency. The widths of the near-wake regions are then governed by the amount of incoming flow deflected or forced through the gap between the cylinders.

Based on the new results presented here, however, this traditional viewpoint is found to be a severe oversimplification of the actual behaviour for two staggered circular cylinders. In this study, nine different flow patterns were identified from the flow visualization video images and the instantaneous PIV vorticity data, depending on the combination of the two geometrical parameters P/D and α . In the sections to follow in this chapter, each of the nine flow patterns is examined in detail; to simplify the discussion, each of the flow patterns was assigned an acronym. The nine flow patterns include: (i) type 1 single bluff-body behaviour, SBB1; (ii) type 2 single bluff-body behaviour, SBB2; (iii) base-bleed, BB; (iv) shear layer reattachment, SLR; (v) induced separation, IS; (vi) gap vortex pairing and enveloping, VPE; (vii) gap vortex pairing, splitting and enveloping, VPSE; (viii) synchronized vortex shedding, SVS; and (ix) vortex impingement, VI. A number of these flow patterns can actually be identified from the rather limited flow visualization results of Ishigai *et al.* (1972), but no systematic identification of flow patterns was then undertaken, as done here. As mentioned in Section 3.1.2, the flow pattern identification approach is similar to Igarashi (1981, 1984) for two tandem cylinders, Lam & Cheung (1988) for three cylinders in an equilateral triangle, and Lam & Lo (1993) for four cylinders in a square configuration.

9.2 Single Bluff-Body Flow Patterns (SBB1, SBB2)

When the two staggered circular cylinders are in contact, at $P/D = 1.0$, the flow pattern resembles a single bluff-body over the entire range of incidence angle, from the two-cylinder tandem configuration at $\alpha = 0^\circ$, described in Section 7.2.2, to the two-cylinder side-by-side configuration at $\alpha = 90^\circ$, described in Section 8.2.2. Flow visualization images and PIV vorticity data, presented in Figures 9.2 and 9.3 respectively, show that the near-wake region contains two free shear layers that alternately shed Kármán vortices. With a small increase in incidence from $\alpha = 0^\circ$ (see Figures 7.2 and 7.3(a)) the wrapped shear layer behaviour of the two-cylinder tandem configuration is no longer observed. Instead, the free shear layers seem to form by boundary layer separation from both the

upstream and downstream cylinders, rather than solely from the upstream cylinder as in the tandem case. In the combined wake of the two cylinders, a single Kármán vortex street is found.

The flow visualization results, shown in Figure 9.2, suggest that the vortex formation region is shortened as the angle of incidence increases, with shear layer roll-up occurring closer to the cylinders. This decrease in the vortex formation length, with increasing angle of incidence, was not apparent in the instantaneous PIV data, and the results in Figure 9.3 actually seem to show the opposite tendency. As with many of the other multiple cylinder arrangements that were examined in this study, the instantaneous PIV data showed considerable variability in the flow patterns, including the point of shear layer roll-up, for example. In Figures 9.3(a,b,d), the CW vortex forms downstream of the CCW vortex (shown by solid contour lines, and corresponding to positive vorticity), while in Figures 9.3(c,e) the CW vortex forms upstream of the CCW vortex. This may suggest that more than one flow pattern is possible, but this could not be verified in the present study, with the limited amount of PIV data from the towing tank.

Although similar single bluff-body behaviour is seen over the entire range of incidence angle, two flow patterns could be distinguished from the flow visualization videos. The two types of single bluff-body behaviour, designated SBB1 and SBB2, were seen for $\alpha = 0^\circ$ to 45° and $\alpha = 45^\circ$ to 90° , respectively, but were not readily apparent in the PIV data.

9.2.1 Single Bluff-Body Flow Pattern Type 1 (SBB1)

The type 1 single bluff-body flow pattern (SBB1) is observed at small angles of incidence, where the “lower” shear layer (“lower” in the figures) from the upstream cylinder is significantly extended in the streamwise direction. Although not clearly shown in the individual flow visualization images in Figures 9.2(a-e), nor in the PIV vorticity data of Figure 9.4, the flow visualization videos showed that the extended upstream (or “lower”) shear layer is prone to the development of instabilities, first in the form of waviness and second in the form of shear layer (Kelvin-Helmholtz) vortices.

Kármán vortices from the “lower” shear layer seem to form from the coalescence of a number of the shear layer instability vortices, a behaviour which may be discerned in Figure 9.2(e). By contrast, the Kelvin-Helmholtz vortices were rarely seen in the “upper” shear layer, formed behind the downstream cylinder, and consequently the Kármán vortices from there form in a more regular fashion. Despite the different mechanisms for primary vortex formation from the two shear layers, a regular Kármán vortex street is still seen in the combined wake of the cylinders, and vortices are still believed to be shed at the same frequency from each shear layer. The presence of the instability vortices, however, does have an influence on the “stability” of the alternate vortex shedding pattern, in the sense that (i) there appears to be reduced interaction between the two shear layers, and (ii) Kármán shedding is often interrupted by irregular shedding patterns (discussed separately in Section 9.2.3).

9.2.2 Single Bluff-Body Flow Pattern Type 2 (SBB2)

The type 2 single bluff-body flow pattern (SBB2) was observed in the flow visualization images at higher angles of incidence, as shown in Figures 9.2(f-h) and in the time sequence of vorticity data shown in Figure 9.5. In this flow pattern, the streamwise lengths of the two separated shear layers, and their degrees of instability, are more similar. Alternate vortex shedding often appeared weaker, or less intense, when compared to that from a single cylinder, in much the same manner seen for side-by-side circular cylinders in contact (see Section 8.2.2). Again, the difference in the base geometries between the cylinder pair and a larger circular cylinder of almost twice the diameter is most likely responsible.

The instantaneous vorticity data show variation in the size and shape of the Kármán vortices, and in the streamwise position where they form. In Figure 9.5, large-scale Kármán vortices spanning nearly two diameters in the y -direction are shown forming immediately behind the cylinders, in contrast to the SBB1 PIV data of Figure 9.4, where vortex formation occurs further from the cylinders, and the vortices are smaller in size. In Figures 9.3(d,e), however, which show the SBB2 flow pattern, the vortices are of

smaller size and form further from the cylinder. These results, and those from two and three side-by-side cylinders in contact (Section 8.2.2), suggest that shorter vortex formation lengths correspond to the formation of larger-size Kármán vortices.

9.2.3 Variable Behaviour

The flow visualization videos show that vortex shedding for two cylinders in contact is often irregular, and more so it seems than in the case of a single cylinder, or other staggered configurations with $P/D > 1.0$. Similar variability in the shedding behaviour was noticed for side-by-side configurations, as mentioned in Section 8.2.2 (and shown in Figure 8.3(b)). Apart from variability in the vortex formation region, that has already been noted, irregular and symmetric forms of vortex shedding were seen in the flow visualization videos (some examples are found in Figure 9.6). In a number of cases, periodic, alternate vortex shedding was interrupted as two smaller vortices were shed in succession from a single shear layer, as shown in Figures 9.6(a,b,e,f,g), before the regular shedding cycle resumed. Again, the greater inclination to irregular behaviour, for cylinders in contact, may be caused by (i) a different base geometry when compared to a single cylinder of (nearly) twice the diameter; (ii) asymmetry, in the x -direction, in the points of separation of the shear layers; and (iii) different lengths to the two shear layers.

9.3 Base-Bleed Flow Pattern (BB)

The SBB2 flow pattern becomes slightly modified when the pitch is increased by a small amount, of about 0.125 to 0.25 diameters, permitting the oncoming flow to enter the base region of the two cylinders through the narrow gap between them. The staggered arrangement now approximates the side-by-side configuration at small transverse pitch ratio, discussed in Section 8.2.3, that shows the effects of base-bleed. The base-bleed flow pattern (BB) for the two-cylinder staggered configuration was studied only in the towing tank, with PIV, and was seen for combinations of $P/D = 1.125$ to 1.25 and $\alpha = 45^\circ$ to 90° ; examples for $\alpha = 60^\circ$ and 75° are shown in the PIV vorticity results of Figure 9.7.

As in the case of two side-by-side cylinders (Section 8.2.3), base-bleed increases the vortex formation length (compare Figures 9.3 and 9.7, for example), and would likely reduce the drag force acting on the cylinders (if force measurements had been acquired in the present study). The instantaneous PIV data revealed a wide variation in the vortex formation length, in addition to variation in the base-bleed flow deflection angle and its streamwise extent and “strength,” illustrated by the velocity data shown in Figure 9.8. Here, weak (low magnitude) gap flow and highly deflected gap flow, away from the downstream cylinder, correlates with a shorter vortex formation region (Figures 9.8(a,b)), as does the remarkable case of reverse gap flow deflection (i.e. towards the *downstream* cylinder, Figure 9.8(f)). The fact that the gap flow could deflect instantaneously in such a manner means that, contrary to what was observed for all other staggered cylinder configurations with $P/D > 1.0$, the wider near-wake region is now behind the *upstream* cylinder. Such a result would support a contention by Zdravkovich (1987), discussed further in Section 9.10.2, that a biased flow pattern can be observed deflected towards either cylinder (and hence is bistable) for nearly side-by-side configurations (such as in the BB flow pattern), with a streamwise stagger of up to $\pm 0.15 L/D$. The BB flow patterns in Figures 9.7 and 9.8 have $L/D = 0.3$ to 0.6 , which is much higher than the range suggested by Zdravkovich (1987).

Regardless of the deflection of the base flow, the BB flow pattern results in a single Kármán vortex street in the combined wake of the two cylinders, formed by vortices shed from the two outer shear layers in much the same fashion as the SBB2 flow pattern. The initial stages of the single Kármán vortex street can be seen in Figure 9.7(a).

9.4 Shear Layer Reattachment Flow Pattern (SLR)

The SBB1 flow pattern becomes slightly modified when the pitch ratio is increased and a gap forms between the two cylinders. At small angles of incidence, a separated shear layer now forms from the inside of the upstream cylinder, which reattaches onto the outer surface of the cylinder immediately downstream. In contrast to the BB flow pattern, the oncoming mean flow is not permitted to flow through the gap between the cylinders. The

structure of the combined wake of the two cylinders remains essentially the same as the SBB1 flow pattern, with two free shear layers and two rows of Kármán vortices. The outer shear layer of the upstream cylinder, which participates in the Kármán vortex formation process, remains highly extended in the streamwise direction, and may contain instabilities in the form of shear layer (Kelvin-Helmholtz) vortices. The increased spacing between the shear layers seems to cause them to shed vortices on an independent basis, perhaps at different frequencies, and without a definitive phase relationship.

Each of these features marks the shear layer reattachment flow pattern (SLR), and is shown in the flow visualization results in Figure 9.9, and in the PIV vorticity data in Figure 9.10, for a variety of configurations ($P/D = 1.125$ to 3.0) of low incidence ($\alpha = 5^\circ$ to 15°). The shear layer vortices in the upstream, extended shear layer may partially be seen in Figure 9.9(b), as they coalesce into the newly forming Kármán vortex. They may also be seen as concentrations of vorticity in the “lower” shear layer in Figures 9.10(e,g). The same SLR flow pattern may be found in the flow visualization results of Ishigai *et al.* (1972), at $L/D = 2.0$, $T/D = 0.5$ ($P/D = 2.06$, $\alpha = 14^\circ$), and $Re = 1690$, although no systematic identification of flow patterns was undertaken in this work. Shear layer reattachment is also found in triangular and square clusters of three and four cylinders, respectively, particularly under conditions of “fully-shielded” flow (Lam & Cheung 1988; Lam & Lo 1993).

Of particular interest in the SLR flow pattern, as in the case of two tandem cylinders in steady mean cross-flow at intermediate pitch ratios (Sections 7.2.3 to 7.2.5), is (i) the behaviour of the shear layer reattachment process, and whether or not it is synchronized with Kármán vortex shedding elsewhere in the group of cylinders; and (ii) the fluid dynamics within the interstitial region, bounded by the reattached shear layer on one side, and by the free shear layer from the upstream cylinder on the opposite side.

Concerning the reattachment process, the flow visualization videos showed that it was not highly synchronized with vortex formation elsewhere in the flow field. Rather, the reattachment appeared steady, similar to the behaviour of triangular and square clusters of circular cylinders at incidence, investigated by Lam & Cheung (1988) and Lam

& Lo (1993). Based on this result, synchronization of the reattachment process with Kármán vortex shedding appears to be unique to the tandem configuration only.

Concerning the interstitial region, the flow visualization images in Figure 9.9 show mostly stagnant fluid in the gap between the cylinders, which is supported by the PIV data in Figure 9.10. These instantaneous data show very low levels of vorticity between the cylinders, even for SLR flow patterns at larger pitch ratios, of $P/D = 2.0$, 2.5 and 3.0 (Figures 9.10(e,f,g), respectively). In contrast, the temporal behaviour of the vorticity field, illustrated in Figure 9.11, shows that after the reattachment of the shear layer, some flow is deflected in front of the downstream cylinder, where it forms a localized region of recirculating fluid (Figure 9.11(a)). Neither flow separation, nor the production of significant opposite-sign vorticity, are induced about the downstream cylinder because of the deflected flow, in contrast to the induced separation flow pattern (IS) described in Section 9.5 to follow (which occurs at larger angles of incidence). The flow deflected in front of the downstream cylinder is shown in the instantaneous velocity data in Figure 9.12, from which an instantaneous reattachment point may be discerned (from images acquired from the reattachment side of the arrangement, in Figure 9.12(b)), although in practice reattachment varies over a small region of the surface of the cylinder rather than at a point (Lee & Panagakos 1997).

9.5 Induced Separation Flow Pattern (IS)

At larger angles of incidence, shear layer reattachment (the SLR flow pattern) can no longer be maintained, and the oncoming mean flow is deflected through the gap between the two cylinders. The inner shear layer, which used to reattach onto the downstream cylinder, is now part of a small near-wake region behind the upstream cylinder, that is biased away from the downstream cylinder and towards the outer flow. The two shear layers from the upstream cylinder each roll-up into small-diameter Kármán vortices at a frequency higher than that of a single circular cylinder (see the discussion of the Strouhal number data in Section 9.9). The Kármán vortices from the inner shear layer, however, form very close to the surface of the downstream cylinder, and appear to induce a

separation of the flow from this cylinder. Production of vorticity about the second cylinder accompanies the separated flow, and these two features mark the induced separation flow pattern (IS), shown in both the flow visualization images and PIV vorticity data presented in Figures 9.13 and 9.14, respectively. The IS flow pattern may also be discerned from the results of Ishigai *et al.* (1972), for a staggered configuration of $L/D = 1.0$, $T/D = 0.5$ ($P/D = 1.12$, $\alpha = 27^\circ$), at $Re = 2560$.

In Figure 9.14, the IS flow pattern is shown for a range of pitch ratios and angles of incidence, each marked by flow being deflected through the gap between the cylinders, Kármán vortex formation from the inner shear layer of the upstream cylinder (Figures 9.14(c,e)), and production of significant amounts of opposite-sign vorticity on the “lower” side of the downstream cylinder (Figures 9.14(c,d)). At small pitch ratios, the flow appears to be very similar to the BB flow pattern (Section 9.3), which is particularly the case for the large instantaneous gap flow deflection angle in Figure 9.14(b). Figures 9.14(a,b) both show the same staggered arrangement of $P/D = 1.125$, $\alpha = 30^\circ$, but with very different gap flow deflection angles and vortex formation lengths.

As shown in the flow visualization videos, the induced, separated flow (or generated vorticity) on the downstream cylinder is not a steady phenomenon, but rather occurs in a periodic fashion, at the same frequency as the Kármán vortex shedding from the upstream cylinder. A pairing of vorticity of opposite sign is noticed across the gap, but the paired vorticity is enveloped by an outer shear layer vortex shed from the upstream cylinder, to form a single larger composite vortex. The pairing and enveloping action is shown in Figures 9.13(b) and 9.14(d), and in the time sequences of vorticity shown in Figure 9.15. Within the pair, the dominant vorticity is that induced on the surface of the downstream cylinder (Figures 9.14(c,d), and the third vorticity field in Figure 9.15). At times, the vortex formed from the inner shear layer of the upstream cylinder is weak and may be destroyed.

The induced separation and vorticity production on the “lower” surface of the downstream cylinder is distinct from Kármán vortex formation, in that it does not involve the formation of a free shear layer and roll-up of a shear layer. It is similar to the vorticity

generation seen when a vortex impinges upon another body in its path, as in the case of the vortex impingement (VI) flow pattern identified for two staggered cylinders, and discussed later in Section 9.8. Similar induced separation and vorticity formation is also seen about the second row of cylinders in a normal triangular array (Oengören & Ziada 1995), particularly for arrays with small pitch ratios. In the array, the induced vortex formation from cylinders in the second row is distinct from Kármán vortex shedding, in that no free shear layers are involved. Rather, the induced formation is caused by the Kármán vortices shed from the cylinders in the first row, as they are squeezed around cylinders in the second row. The induced vortex formation occurs at the same frequency as the Kármán shedding from cylinders upstream, and pairing of vorticity of opposite sign is seen just beyond the second row.

The combined wake of the cylinder group remains similar to that in the SBB1 and SLR flow patterns, and includes two rows of vortices: one row of regular, Kármán-type vortices formed from the outer shear layer of the downstream cylinder, being shed at low frequency; and one row of the composite vortices spaced and formed at different, higher frequency from the inner shear layers (seen in Figures 9.14(a,b) and 9.15).

9.6 Gap Vortex Pairing Flow Patterns (VPE, VPSE)

Most flow patterns for the staggered pair are dominated by the oncoming flow being deflected through the gap between the cylinders, and high frequency alternate vortex shedding from the upstream cylinder, similar to what was observed in the IS flow pattern (Section 9.5). At higher pitch ratios and angles of incidence, however, the gap is now appreciably wider, and the induced separation and production of vorticity on the inside face of the downstream cylinder transitions to a separated, free shear layer, which periodically rolls-up into Kármán vortices. These Kármán vortices are shed at the same frequency as those from the upstream cylinder, and pair up in a symmetrical (or anti-phase) fashion with vortices shed from the opposite side of the gap. A counter-rotating vortex pair is then formed at the exit of the gap, which is enveloped by a Kármán vortex from the outer shear layer of the upstream cylinder. This rather complicated interaction

of vortices is sketched in Figure 9.16. Similar behaviour was observed in the present study for three side-by-side cylinders, at $T/D = 1.5$ (see Section 8.2.7), and in the first harmonic mode of vortex shedding for two side-by-side cylinders, as reported by Williamson (1985), and shown earlier in Figure 3.5(c).

From the flow visualization videos, two flow patterns involving this type of pairing phenomenon could be distinguished: (i) one pattern with complete enveloping of the gap vortex pair, and designated the “vortex pairing and enveloping” flow pattern (VPE), Figure 9.16(a); and, (ii) a second pattern with incomplete enveloping, and instead splitting of the vortex pair, designated the “vortex pairing, splitting and enveloping” flow pattern (VPSE), Figure 9.16(b). The VPSE pattern was observed for greater ranges of pitch ratio and angle of incidence than the VPE pattern. A VPSE flow pattern can be identified from the flow visualization results of Ishigai *et al.* (1972), for a staggered geometry of $L/D = 0.68$, $T/D = 1.0$ ($P/D = 1.21$, $\alpha = 34^\circ$), at $Re = 1600$.

In both the VPE and VPSE flow patterns (and the IS and SVS flow patterns, discussed in Sections 9.5 and 9.7, respectively), *three* of the four shear layers and their associated vortex formation and shedding process have the same, high frequency of vortex shedding (or induced separation, as the case may be). This contrasts with the traditional understanding of the staggered configuration, mentioned in Section 9.1.1, which says that high frequency and low frequency vortex shedding are identified with individual cylinders rather than individual shear layers. The two different frequencies of vortex shedding mean that the wake of the cylinders contains an irregular street of two rows of vortices.

9.6.1 Vortex Pairing and Enveloping Flow Pattern (VPE)

Complete enveloping of the gap vortex pair, by a Kármán vortex from the outer shear layer of the upstream cylinder, is the main feature of the VPE pattern (Figure 9.16(a)). The flow visualization and PIV vorticity data, presented in Figures 9.17 and 9.18, respectively, illustrate the flow being deflected through the gap between the cylinders, the pairing of the gap vortices, the enveloping process at the gap exit, and the formation of

two rows of Kármán vortices (a single street) in the combined wake of the two cylinders. A composite Kármán vortex (with net positive vorticity or CCW rotation) from the enveloped pair is shown in Figure 9.18(a), at $x/D = 4.5$. As the angle of incidence is increased within the VPE flow pattern, the gap flow becomes less deflected, and enveloping occurs further downstream (compare Figures 9.18(a) and 9.18(b)).

The temporal development of the VPE flow is shown in two examples in Figure 9.19, with the composite CCW (positive vorticity) Kármán vortex being formed in Figure 9.19(a). Shedding of a Kármán vortex from the outer shear layer of the downstream cylinder may be seen in Figure 9.19(b); it occurs at a lower frequency than vortex shedding from the other three shear layers. In many cases, the flow visualization videos showed the VPE pattern to be intermittent, appearing for only one or two cycles at a time. The low frequency shedding itself was often weak and intermittent, with the outer shear layer of the downstream cylinder occasionally being deflected away from the cylinders into the outer flow, as the stronger high-frequency vortex shedding interfered with and modulated this process. Interruption of the low frequency shedding often meant that two subsequent vortices combined and rolled around each other, as if the convection velocity of the first vortex was lowered, or that of next vortex was increased.

9.6.2 Vortex Pairing, Splitting and Enveloping Flow Pattern (VPSE)

The more prevalent of the two gap vortex pairing and enveloping flow patterns was the VPSE pattern, in which the enveloping process is incomplete. As shown in the flow visualization results of Figure 9.20, and in the vorticity data of Figure 9.21, the vortex pairing phenomenon about the exiting gap flow is weaker, and each inner gap vortex is split into two concentrations of vorticity during the enveloping process (see Figure 9.16(b)). The flow visualization results of Ishigai *et al.* (1972), for staggered configurations of $L/D = 1.0$, $T/D = 1.0$ ($P/D = 1.4$, $\alpha = 45^\circ$), at $Re = 2140$, and $L/D = 2.0$, $T/D = 1.0$ ($P/D = 2.24$, $\alpha = 27^\circ$), at $Re = 3500$, correspond to the VPSE flow pattern, but the pairing, splitting, or enveloping processes cannot be clearly distinguished.

Gap vortex pairing in the present study is seen in Figures 9.20(a-d), where there is now a greater distance between the two vortices across the gap flow when compared to the IS and VPE patterns. In the sequences of vorticity fields shown in Figure 9.22, pairs of opposite-sign vortices arranged in a row can be seen behind the upstream cylinder, extending into the wake, as shown in the last vorticity field of Figure 9.22(a), and the second-to-last vorticity field of Figure 9.22(b).

Splitting of the vorticity is seen in Figures 9.20(a,c) and 9.21(c,d). During the splitting process, vorticity may be forced across the near-wake of the downstream cylinder and towards the outer shear layer of this cylinder (seen in the third vorticity field of Figure 9.22(a), for example). Here, the outer shear layer of the downstream cylinder continues to shed Kármán vortices at low frequency, but with continued modulation from the higher frequency gap vortex formation; these large, low frequency vortices are seen in Figures 9.20(d), 9.21(a,b,c), and 9.22(a).

Alternatively, the split vorticity may form a third row of smaller vortices (or concentrations of vorticity) in the combined wake of the cylinders, seen as a third row of vortices in Figure 9.20(d), and as a concentration of vorticity parallel to the flow axis in Figures 9.21(d) and 9.22(b). The combined wake of the two cylinders then contains three rows of vortices, with two rows of like-sign vortices behind the upstream cylinder. The two parallel rows of like-sign vortices represent an unstable arrangement, however, as some of the flow visualization video results showed.

As with many of the other multiple cylinder configurations tested, the instantaneous vorticity data revealed temporal variation in the VPSE flow pattern as well. An example provided in Figure 9.23 shows variation in the gap flow deflection angle at a given geometry, with the VPSE process maintained under both conditions (as would be seen in the vorticity field, not shown).

9.7 Synchronized Vortex Shedding Flow Pattern (SVS)

The most commonly observed flow pattern for two staggered circular cylinders in steady mean cross-flow is similar to that for two side-by-side circular cylinders at intermediate

and large pitch ratios, discussed in Sections 8.2.4 and 8.2.5, where the flow field may be biased towards one of the cylinders, and where the two vortex shedding processes become synchronized. Typically, it is observed for staggered configurations of intermediate and larger transverse pitch ratio, $T/D > 1.5$, and for the complete range of longitudinal pitch ratio, $L/D = 0$ to 5.0. The flow pattern is marked by: (i) deflected flow through the gap between the cylinders; (ii) two different widths of near-wake regions; (iii) two dominant frequencies of vortex shedding; (iv) two vortex streets in the combined wake of the cylinders; (v) synchronization of the gap vortex shedding process; and (vi) synchronization of the two vortex streets, that occurs in an anti-phase fashion.

This behaviour is an evolution of the gap vortex pairing phenomenon, seen in the IS, VPE and VPSE flow patterns, but formation of a well-defined counter-rotating vortex pair *structure* is not seen, nor is the enveloping process, because of the still greater distance between the gap shear layers. Rather, the synchronized vortex shedding flow pattern (SVS), as it is called, is marked by two Kármán vortex streets in the combined wake of the two cylinders, yielding four rows of vortices. In each of the flow visualization pictures, shown in Figure 9.24, and in the PIV vorticity data shown in Figure 9.25, strong interaction occurs between the two shear layers on either side of the gap, along with simultaneous shedding of Kármán vortices from the gap at the same frequency. Pairing of the vortices across the gap, during their formation, is seen at smaller pitch ratios in Figures 9.24(a,e) and 9.25(b,c). Pairing of gap vortices further downstream, just after they have been shed, is seen in Figures 9.24(b,c,d) and 9.25(d,f) at intermediate pitch ratios. Pairing of gap vortices at larger pitch ratios often occurs far downstream from the upstream cylinder, as shown in Figures 9.24(h,i) and 9.25(g,h,j,k). In each case, the pairing of vortices occurs across the gap at some distance apart, and a counter-rotating vortex pair is not formed for any significant period of time. The two side-by-side vortex streets may be seen in Figures 9.24(a,d,e) and 9.25(e,h,i,j,k), each synchronized in an anti-phase manner. Only in a very few instances was in-phase (or no apparent phase) synchronization of the vortex streets detected, and only in the instantaneous PIV data from the towing tank.

As in many of the other staggered flow patterns, vortex shedding frequencies are mostly unique for each individual shear layer rather than for each cylinder. In most, but not all, SVS configurations, the two shear layers from the upstream cylinder, and the inner shear layer from the downstream cylinder, shed vortices at the same frequency. Lower frequency Kármán shedding then occurs only from the fourth shear layer, from the outer side of the downstream cylinder; these larger vortices may be seen forming in Figures 9.24(b,e) and 9.25(a,e). The difference in shedding frequency across the near-wake region of the downstream cylinder may be discerned from the symmetric vortex formation shown in Figures 9.24(a,b,d) and 9.25(b), and from the irregular vortex street behind this cylinder. High frequency modulation of the low frequency vortex formation, denoted by interruptions to its regular shedding behaviour, may be seen by the pairs of outer vortices in Figures 9.24(a,d), and the irregular pattern behind the downstream cylinder in Figure 9.24(f).

For SVS configurations of larger pitch ratio and larger incidence, however, both shear layers from the downstream cylinder seem more inclined to shed vortices at the same, lower frequency, with reduced high frequency modulation. This may mean that a subdivision of the SVS flow pattern is needed, but extensive vortex shedding frequency measurements with a hot-film anemometer, positioned in each of the shear layers, would be required to verify the frequency relationship. Low frequency events were difficult to measure precisely from the flow visualization videos, because of high frequency modulation from vortex shedding elsewhere in the configuration, and by limited video recording time for sufficient numbers of vortices to be counted.

The most reliable and extensive measurements of vortex strength were obtained for the SVS flow pattern, since vortices from each of the four different shear layers could easily be identified. In other flow patterns, such as VPE and VPSE, vortices from the upstream cylinder could not always be clearly identified as coherent vortical structures from the PIV vorticity data. Vortices were designated as belonging to the upstream or downstream cylinder, and as being shed from the inside (adjacent to the gap) or the outside (adjacent to the mean flow) of the staggered configuration; the shear layers were

identified in Figure 3.7. Measurements of vortex strength for the downstream cylinder were acquired in limited numbers, because the field of view of the digital camera was not always sufficiently large to image beyond the vortex formation region of the downstream cylinder, and also because these vortices formed at much lower frequencies than those from the upstream cylinder.

Some results for the vortex strength are shown in Figures 9.26 and 9.27, plotted against the streamwise distance from the centre of each cylinder, rather than from the centre of the staggered configuration. Here, the circulation was computed using the interactive contour selection method described in Section 5.7.2, using equation (5.6), subject to a lower vorticity contour magnitude of 1.5. Although this cut-off value was high, the circulation values were not meant to be treated as absolute; the purpose was to illustrate basic trends in the vortex strength for the SVS flow pattern (similar to the approach of Leweke & Williamson 1998). Hence, the circulation data are lower than they would be had they been computed with the second method described in Section 5.7.2, using equation (5.7) and employing a lower cut-off value. The vortex strength data presented in Figures 9.26 and 9.27 were chosen for specific staggered configurations that best illustrated the basic trends seen consistently throughout the complete SVS data set, which would be too large to present here.

The results in both figures show that for all the vortices, regardless of pitch ratio, incidence, upstream or downstream cylinder, inside or outside shear layers, there is a power law decay in strength with streamwise distance from the centres of the cylinders. In Figure 9.26(a), the effects of pitch ratio at constant incidence are illustrated for Kármán vortices shed from the inside shear layer from the upstream cylinder. Results are given for staggered configurations of $\alpha = 30^\circ$, and show an increase in strength as the spacing between the cylinders is increased from $P/D = 2.0$ to 4.0. This increase in strength is proportional to the width and size of the near-wake region behind the upstream cylinder, which is allowed to expand as the downstream cylinder is situated further away (compare

the two extreme cases, in Figures 9.21(f) and 9.25(j)).^{*} In Figure 9.26(b) the effects of varying incidence angle at constant pitch ratio are illustrated, again for vortices shed from inner shear layer of the upstream cylinder, for staggered configurations of $P/D = 2.5$. In this case, an increase in incidence has the same effect as an increase in pitch ratio, producing an increase in the vortex strength. In both cases, the increase in vortex strength is caused by reduced interference from the second cylinder, and less distortion of the near-wake region behind the upstream cylinder (compare two of the cases, in Figures 9.25(d) and 9.25(f), for $\alpha = 45^\circ$ and 75° , respectively).

In Figure 9.27(a), the strengths of the Kármán vortices from the inner and outer shear layers of both cylinders are compared, for a single SVS flow pattern corresponding to $P/D = 2.0$ and $\alpha = 75^\circ$; the vorticity field is shown in Figure 9.25(c). These circulation measurements illustrate two common trends. First, Kármán vortices shed from the downstream cylinder are usually higher in strength than those shed from the upstream cylinder. The reason for this may simply be that the vortices shed from the downstream cylinder are larger, and hence more vorticity is integrated when computing the circulation using equation (5.7). The larger scale of the Kármán vortices shed from the downstream cylinder is not obvious in Figures 9.24 and 9.25, however. Second, Figure 9.27(a) shows that the Kármán vortices shed from the outer shear layers are more often of greater strength than those shed from the inner shear layers. Since the vortices are shed in a symmetric fashion across the gap, and pair up in an anti-phase manner between the two parallel vortex streets, it might be expected that the inner Kármán vortices are weaker than those on the outside, due to cancellation of opposite-sign vorticity. This trend was not consistently observed for all the SVS configurations, however. For example, vortex strength data from small incidence SVS configurations typically showed no difference in circulation between inner and outer vortices from the upstream cylinder, as shown by the data in Figure 9.27(b). The latter figure also shows that the difference in strength between the Kármán vortices shed from the upstream and downstream cylinders

^{*} The configuration of $P/D = 2.0$, $\alpha = 30^\circ$, actually corresponds to the VPSE flow pattern.

decreases as the angle of incidence is increased (or for that matter, as the pitch ratio is increased, at constant incidence).

9.8 Vortex Impingement Flow Pattern (VI)

At large centre-to-centre pitch ratios and small angles of incidence, the two staggered cylinders resemble the tandem configuration at large L/D , described in Sections 7.2.7 and 7.2.8. For these staggered geometries, unbiased Kármán vortex shedding occurs from the upstream cylinder at a frequency close to that of a single circular cylinder.^{*} The Kármán vortices then impinge upon the downstream cylinder, hence the vortex impingement flow pattern (VI), shown in schematic form in Figure 9.28, where the main features of the flow pattern are described. Each of these features, including the incident Kármán vortices shed from the upstream cylinder (labeled A and B in Figure 9.28), induced vorticity (labeled C), and split vorticity (labeled D), is shown in the flow visualization results in Figure 9.29,[†] and in the PIV results in Figure 9.30. Kármán vortex shedding from the downstream cylinder (labeled E in Figures 9.28 and 9.30), while still observed in some form in an intermittent fashion, is highly disturbed by the impingement of the vortex street from the upstream cylinder. Nearly identical Strouhal number data measured in the wakes of the upstream and downstream cylinders[‡] show that synchronization between the

^{*} Verified by the Strouhal number data, discussed in Section 9.9, and shown in Figure 9.32(g).

[†] The VI flow pattern is less clear in the flow visualization images (Figure 9.29), where it more closely approximates the IS flow pattern. This is caused by an influence of the Reynolds number ($Re = 860$ to 910 for the flow visualization experiments, compared with $Re = 1900$ for the PIV experiments), in the same manner as for two cylinders in tandem (Sections 7.2.6 and 7.2.8). Just as the “critical” L/D for two tandem cylinders, marking the transition from shear layer reattachment to Kármán vortex shedding from the upstream cylinder, is Re -dependent, so it would be expected that “critical” combinations of P/D and α for two staggered cylinders, marking the transition from SLR and IS to VI, would be similarly influenced by the Reynolds number.

[‡] Again, discussed in Section 9.9, and shown in Figure 9.32(g).

impingement flow and Kármán vortex shedding from the downstream cylinder apparently exists, with a lock-in condition established (mentioned in a review of vortex-body interaction by Rockwell (1998)). A VI flow pattern may also be distinguished in the flow visualization results of Ishigai *et al.* (1972), for a staggered geometry of $L/D = 4.0$, $T/D = 1.0$ ($P/D = 4.12$, $\alpha = 14^\circ$), at $Re = 2790$.

The interaction between a vortex street and the body in its path is, according to Gursul & Rockwell (1990), primarily influenced by two parameters: (i) the relative scales of the vortex street and the body, denoted by the streamwise spacing of the Kármán vortices in one row of the vortex street, l , and (in this case) the cylinder diameter, D , expressed as the dimensionless ratio l/D ; and (ii) the transverse offset of the vortex street centreline from the centre of the body, which in the case of two staggered cylinders corresponds to the value of the transverse pitch ratio, T/D . In the present case, the ratio $l/D = 4.0$ is taken from the vortex shedding experiments for a single cylinder, in Section 6.2.1, while the three staggered configurations shown in Figure 9.30, of $P/D = 3.5$, $\alpha = 10^\circ$ (Figure 9.30(a)), $P/D = 4.0$, $\alpha = 5^\circ$ (Figure 9.30(b)), and $P/D = 4.0$, $\alpha = 15^\circ$ (Figure 9.30(c,d)), correspond to $T/D = 0.61$, 0.35 , and 1.04 , respectively. Based on the study of Gursul & Rockwell (1990), for $l/D = 4.0$, the staggered cylinder VI flow pattern corresponds to a “large-scale” vortex street, and the three staggered configurations in Figure 9.30 correspond to the second and third types of interaction mechanisms identified by Gursul & Rockwell (1990), where either one or two rows of Kármán vortices are disturbed.

In the tandem case, the alternately shed Kármán vortices from the upstream cylinder are aligned with the centre of the cylinder downstream, with $T/D = 0$, and both rows of vortices impinge on the downstream cylinder. These vortices become highly strained and distorted, and induce vorticity of opposite sign in the near-wake of the downstream cylinder (see Sections 7.2.7 and 7.2.8). Considering the staggered case, a VI flow pattern with impingement of two rows of vortices is shown in Figures 9.30(a,b), and with one row of vortices in Figures 9.29(a,b) and 9.30(c,d). In both cases, the PIV vorticity data show induced vorticity generated about the second cylinder (labeled C in

Figure 9.30), caused by an impinging vortex, and rapid deformation of the impinging vortices.

Where two rows of vortices impinge on the downstream cylinder, vortices from the inner row (labeled B in Figures 9.30(a,b)) are split into two concentrations of vorticity during the impingement process (labeled B and D in Figures 9.30(a,b)). Each portion is convected around the downstream cylinder and induces vorticity in its near-wake region (labeled C). In Figure 9.30(b), the split vorticity (labeled D) that is convected about the outside of the downstream cylinder is entrained into the newly forming Kármán vortex (labeled E).

At small T/D , where only one row of vortices impinges on the downstream cylinder, vortices from this row may still be split into two concentrations of vorticity (Figure 9.30(c)), but otherwise may remain intact and, instead, simply be deformed or distorted at larger T/D . Opposite-sign vorticity is induced about the second cylinder in either case (labeled C). Vortices from the outer row (labeled A), which do not directly impinge on the downstream cylinder, shown in Figure 9.30(d), may still be strained and distorted as they pass close to the downstream cylinder, although opposite-sign vorticity is not induced.

The significance of the VI flow pattern is that the vortex-body interaction will cause large changes in the time-dependent pressure forces and loading on the downstream cylinder (Gursul & Rockwell 1990; Rockwell 1998), and hence will significantly affect the r.m.s. values of the fluid force coefficients acting on the downstream cylinder. Such a phenomenon is important in the study of flow-induced vibrations of multiple cylinder systems (Rockwell 1998). Discussion of the r.m.s force coefficient data for two staggered cylinders is left to Section 9.11.

9.9 Vortex Shedding Frequencies

The flow field of two staggered circular cylinders is often characterized by the occurrence of two vortex shedding frequencies, a higher frequency from the upstream cylinder, and a lower frequency from the downstream cylinder (Zdravkovich 1987). This study has

shown, however, that the vortex shedding frequencies are more properly associated with the individual *shear layers* rather than the cylinders. More specifically, vortex shedding from the inner shear layer of the downstream cylinder was often synchronized (at the same shedding frequency) as alternate Kármán vortex shedding from the two free shear layers of the upstream cylinder. This was particularly evident in the VPE, VPSE and SVS flow patterns (as discussed in Sections 9.6 and 9.7).

Furthermore, the present investigation has shown that vortex shedding from the upstream cylinder is nearly always present, particularly when there is flow through the gap between the cylinders. In past studies, detection of only one Strouhal number, typically of lower value, implied the suppression of vortex shedding from the upstream cylinder (Zdravkovich 1987), but this study showed that this is not always the case. The high-frequency shedding from the upstream cylinder was only absent for the SBB1, SBB2, BB and SLR flow patterns. Because of the nature of the flow field, the higher frequency may not be detectable in the combined wake of the cylinder pair, due to a weaker amplitude of the signal and the smaller scale of the formation and shedding process, but it does not mean that such a process is absent (some of the results of Kiya *et al.* (1980), for example, would have us believe that it is absent). Furthermore, this higher frequency may involve vortex formation and shedding from three of the four shear layers, as in the case of the IS, VPE, VPSE and SVS flow patterns, where vorticity generation from the inner side of the downstream cylinder is highly synchronized with vortex shedding from the upstream cylinder.

These two key physical properties of the vortex shedding processes in the staggered configuration, identified in the present study, effectively confirm what Ishigai *et al.* (1972) noted: that the Strouhal number data for two staggered cylinders are strongly dependent on the measurement location.* According to Ishigai *et al.* (1972), to properly determine the number of fundamental frequencies associated with a given staggered

* The sensitivity of the measurement location was also realized by Kiya *et al.* (1980), but it does not seem to have been fully factored into their results and their subsequent interpretation of the flow (which was made without the benefit of flow visualization).

geometry, and to assess their fluid dynamic origins, measurements need to be made in the near-wake regions of each individual cylinder, in addition to the combined wake of the configuration. This was to some extent realized in this study, since vortex shedding frequencies were obtained on a frame-by-frame basis from the flow visualization videos, by counting and timing the individual vortices, rather than from a measuring probe at a single location (as for the side-by-side configuration, Section 8.2.1). The accuracy of the frame-by-frame analysis approach has been demonstrated previously by comparison with hot-film results (Sumner *et al.* 1997d), however it could not be used to reliably count vortices from the outer shear layer of the downstream cylinder, since these vortices were usually shed at a lower frequency than the other three shear layers, and insufficient numbers of vortices were recorded on the video for accurate measurements. Also, vortex shedding from this shear layer had significant higher frequency modulation from the synchronized vortex formation of the other three shear layers. Furthermore, the method failed to uncover any other high frequency components that may be present in the flow, that could be attributed to vortex pairing and enveloping for example, as discovered for three side-by-side cylinders at $T/D = 1.5$ (see Sections 8.2.1, 8.2.7 and 8.2.8).

Strouhal number data from this study, together with data from previously published work, are plotted against the centre-to-centre pitch ratio and the incidence angle in Figures 9.31 and 9.32, respectively. Since the experiments of Ishigai *et al.* (1972) and Moriya & Sakamoto (1985) were based on the L/D versus T/D notation, only some of their Strouhal number data could be readily converted into a P/D versus α plot (for analysis at constant incidence or pitch ratio) without using bilinear interpolation. Overall, the present results are consistent with those of other studies, although the lower Reynolds number range seems to yield greater values for the higher of the two Strouhal numbers detected. Kiya *et al.* (1980), however, failed to measure the very high frequencies of vortex formation which were found in the present study for the IS, VPE, and VPSE flow patterns, for example, since their measuring probe was located mostly in the combined wake of the two cylinders, and not in the interstitial region or behind the upstream cylinder (and hence some of the scatter in St for $\alpha = 0^\circ$ to 30° in Figures 9.32(a-d)).

9.9.1 Strouhal Number Data at Constant Incidence

For staggered configurations of constant incidence angle, α , an increase in the pitch ratio, P/D , corresponds to a lessening of interference, and for each cylinder vortex shedding becomes more like that of single, isolated circular cylinder. Here, two different sets of Strouhal number data are presented for staggered configurations of $\alpha = 25^\circ$ to 35° (Figure 9.31(a)) and $\alpha = 55^\circ$ to 65° (Figure 9.31(b)). Each set of data represents a small range of staggered geometries of nearly constant incidence, of $\alpha = 30^\circ$ and 60° , respectively. In each case, the variation of the Strouhal number with P/D is similar to that of two side-by-side cylinders, presented in Section 8.2.1.

For the first set of data, corresponding to staggered configurations of smaller incidence, $\alpha = 25^\circ$ to 35° , five flow patterns are included (SBB1, IS, VPE, VPSE, SVS) as the pitch ratio is increased, as shown in Figure 9.31(a). Here, the SBB1 flow pattern corresponds to a single, low frequency vortex shedding, of $St \sim 0.12$, demonstrating single bluff-body behaviour with one vortex shedding process and one Kármán vortex street, since the cylinders are in contact (or very nearly in contact). As the pitch ratio is increased, and the flow transitions away from the SBB1 and IS flow patterns, a second, high frequency of vortex shedding is suddenly detected.* In the VPE, VPSE and SVS flow patterns which follow at higher pitch ratios, two frequencies of vortex shedding are found, with the difference between them becoming smaller as the cylinders are displaced further apart, and the wake interference is lessened. As this study has shown, the higher Strouhal number is associated with vortex shedding from the upstream cylinder, and from the inner shear layer of the downstream cylinder, whereas the lower Strouhal number is due to vortex shedding from the outer shear layer of the downstream cylinder. The smaller difference between the frequencies, with increasing pitch ratio, is also seen in the Strouhal number data for two side-by-side cylinders (Section 8.2.1), and correlates well with the relative sizes of the near-wake regions behind each of the cylinders. As the pitch

* A second, high frequency may also be found for the IS flow pattern, see Figures 9.32(b-e), but it is not shown in the selected (published) St data at constant pitch ratio in Figure 9.31(a).

ratio is increased further, the two frequencies once again revert to a single frequency, at $P/D \sim 4.5$, where each cylinder now sheds vortices in a similar manner to a single, isolated circular cylinder, at $St \sim 0.2$. This result does not mean, however, that the interference of the second cylinder has disappeared, for there may still be anti-phase synchronization of the vortex streets (as noticed in the SVS flow pattern, Section 9.7).

In the present study, two Strouhal numbers may be found at higher pitch ratios than in other studies, similar to what was noticed for side-by-side cylinders (Section 8.2.1). Again, this difference may be attributed to differences in Reynolds number, the influence of blockage and aspect ratio, or a combination of these effects. It is suspected that a similar difference occurs for $\alpha = 60^\circ$, but insufficient data were acquired at larger pitch ratios to demonstrate this.

The second set of Strouhal data, in Figure 9.31(b), corresponds to staggered configurations of greater incidence, $\alpha = 55^\circ$ to 65° . Four flow patterns are observed in this range of incidence (SBB2, BB, VPSE, SVS) as the pitch ratio is increased. Similar characteristics to the smaller incidence case, in Figure 9.31(a), are seen, with a single low frequency for the SBB2 and BB flow patterns,^{*} and two frequencies for the VPSE and SVS patterns. The major difference is that the larger-incidence staggered configuration reverts to a single vortex shedding frequency similar to that for a single cylinder, at $St \sim 0.2$: at $P/D \sim 3.5$ rather than at $P/D \sim 4.5$ as in the previous case. Again, there may still be some interference for $P/D > 3.5$, in the form of anti-phase synchronization of the two side-by-side vortex streets. Interestingly, the two upper pitch ratio limits for the measurement of two shedding frequencies ($P/D \sim 4.5$ for $\alpha = 30^\circ$, in Figure 9.31(a), and $P/D \sim 3.5$ for $\alpha = 60^\circ$, in Figure 9.31(b)), may be linearly extended to $P/D = 2.5$ for $\alpha = 90^\circ$ (or more properly, $T/D = 2.5$, since $\alpha = 90^\circ$ corresponds to the side-by-side

^{*} Some high frequencies found by the other researchers, in the range of P/D corresponding to the SBB2 and BB flow patterns, are also shown in Figure 9.31(b). Kiya *et al.* (1980) indicate that they may be associated with weaker and more broad-banded spectral peaks, however (see Section 9.9.2). The higher frequencies were not detected in the present study

configuration), which agrees exactly with the upper limit of the biased flow pattern, found in the water tunnel, for two side-by-side cylinders (see Section 8.2.4).

9.9.2 Strouhal Number Data at Constant Pitch Ratio

For staggered configurations of constant centre-to-centre pitch ratio, P/D , between the cylinders, an increase in the angle of incidence, α , changes the geometry from the tandem configuration, at $\alpha = 0^\circ$, to the side-by-side configuration, at $\alpha = 90^\circ$. For smaller P/D , the increase in incidence angle often means moving from a flow field with one Strouhal number, in nearly tandem configurations, to a flow field with two Strouhal numbers, in nearly side-by-side configurations, as shown in Figure 9.32.

When the cylinders are in contact, at $P/D = 1.0$ (Figure 9.32(a)), single bluff-body behaviour with a single vortex shedding frequency is found for the entire range of incidence. At small angles of incidence, for $\alpha = 0^\circ$ to 10° , the single Strouhal number is higher than for a single, isolated circular cylinder, much as was observed for the tandem configuration at small L/D (see Section 7.2.1). Here, the higher frequency is caused by a shortening of the near-wake region, and corresponds to a portion of the SBB1 flow pattern (see Section 9.2.1). From $\alpha = 10^\circ$ to 45° , still within the SBB1 flow pattern, the Strouhal number steadily decreases to a value of $St \sim 0.1$, which corresponds to one half of the vortex shedding frequency for a single, isolated circular cylinder. The lowering of the Strouhal number occurs as the near-wake region lengthens and widens with increasing incidence. For $\alpha > 45^\circ$, the SBB2 flow pattern is observed (see Section 9.2.2), and in this range of incidence angle the Strouhal number remains relatively constant at $St \sim 0.1$, providing a good quantitative distinction between the SBB1 and SBB2 flow patterns. This value is consistent with $St = 0.1$ measured for two side-by-side cylinders in contact, at $T/D = 1.0$ (see Section 8.2.1). Although the data from Kiya *et al.* (1980) include some higher frequencies at $\alpha = 60^\circ$ and 75° (in Figure 9.32(a)), these were associated with weaker and more broad-banded spectral peaks, possibly unrelated to periodic vortex shedding.

For staggered configurations of $P/D = 1.4$ to 1.6 (Figure 9.32(b)), a single low vortex-shedding frequency is found for $\alpha = 0^\circ$ to 18° , corresponding to the SLR flow pattern. Low frequency shedding occurs from the cylinder group as a whole, by the interaction of the outer shear layers of the upstream and downstream cylinders (see Section 9.4), and as in the case of the SBB1 flow pattern, shedding occurs at a frequency lower than that of a single cylinder. The low Strouhal number of $St \sim 0.1$ persists at higher angles of incidence, up to and including the side-by-side configuration at $\alpha = 90^\circ$, but a second Strouhal number four to five times higher appears suddenly at $\alpha = 18^\circ$, marking the abrupt transition from the SLR flow pattern to the IS flow pattern, as flow is deflected through the gap for the first time (see the discussion of the IS flow pattern, Section 9.5). Two vortex shedding frequencies are then found for the remaining angles of incidence, as the flow pattern transitions to the VPE and VPSE patterns, although the difference between the high and low frequencies becomes smaller.

Similar behaviour in the Strouhal number variation is also found at nominal constant pitch ratios of $P/D = 2.0$ (actually, $P/D = 1.8$ to 2.2) and 2.5 (actually, $P/D = 2.5$ to 2.7), in Figures 9.32(c,d), although there is an added transition to the SVS flow pattern. At these higher pitch ratios, however, the difference between the high and low Strouhal numbers becomes progressively smaller, and vortex shedding from the upstream cylinder, and the inner shear layer of the downstream cylinder, is now between three and four times greater than the lower frequency shedding from the downstream cylinder. In both cases, a single vortex shedding frequency is found only for the SLR pattern. In Figure 9.32(d), a single shedding frequency of $St = 0.2$, the same as that of a single cylinder, is also found at $\alpha = 90^\circ$, but this corresponds to the side-by-side configuration at the end of the biased flow pattern.

For the range of $P/D = 2.8$ to 3.2 (Figure 9.32(e)), the same flow patterns as in Figures 9.32(c,d) are observed as the incidence angle is varied; however the lower of the two Strouhal numbers has increased towards the value for a single cylinder. In the range of incidence corresponding to the SVS flow pattern, two frequencies are no longer

consistently observed; rather, from $\alpha = 70^\circ$ to 90° only a single frequency is observed, meaning there is anti-phase synchronization of the two vortex streets, but each cylinder is now shedding at the same frequency.

The VI flow pattern first becomes evident at $P/D = 3.5$, for small angles of incidence, and is characterized by a single Strouhal number since both cylinders are believed to shed vortices at the same frequency (see Section 9.8). The one Strouhal number is shown in the Strouhal number data for $P/D = 3.4$ to 3.6 , and $P/D = 3.8$ to 6.2 , or in Figures 9.32(f) and 9.32(g), respectively. For these staggered configurations of high pitch ratio, two vortex shedding frequencies are rarely detected, except within the VPSE flow pattern at $P/D = 3.5$ (Figure 9.32(f)), and for small ranges of incidence within the SVS flow pattern (Figures 9.32(f,g)). Furthermore, the vortex shedding frequencies, whether low or high, now closely approximate vortex shedding from a single circular cylinder, near $St = 0.2$. This result does not indicate that no-interference conditions are established, however, since in most cases either the VI or SVS flow pattern exists.

9.10 Résumé of Flow Patterns

A résumé of the flow patterns for two staggered circular cylinders in steady cross-flow, identified in this study, is shown in Figure 9.33, both in a P/D versus α diagram and a L/D versus T/D diagram. Based on this information, approximate boundaries for the nine distinct flow patterns were drawn, shown in Figure 9.34 for the two different types of notation. For the tandem configuration, similar flow pattern diagrams were produced by Igarashi (1981, 1984) and Ljungkrona (1992), based on L/D and the Reynolds number.

In regions where limited experiments were undertaken (see Figures 9.1 and 9.33), the extents of the flow patterns were difficult to assess, and the suggested boundaries are marked with dashed lines in Figure 9.34. The boundaries do not mean that a sudden transition always occurs from one flow pattern to another. An abrupt transition may be noticed between the SLR and IS flow patterns, for example, where the oncoming flow is suddenly deflected through the gap between the cylinders. However, in most cases the change is gradual, and two flow patterns will alternate in an irregular or intermittent

fashion for configurations close to a boundary, which was seen in the case of two tandem cylinders, in the changeover from shear layer reattachment to Kármán vortex shedding from the upstream cylinder (Section 7.2.6). In addition, there may be an actual transitional flow pattern not seen in the present study because of the limited number of experiments. The boundaries may also exhibit hysteretic qualities, depending on whether the pitch ratios and/or the incidence angle is raised or lowered, but this could not be investigated in the present study since there was no means to continuously vary the staggered geometry under flow conditions in a given test.

9.10.1 Reynolds Number Effects

Since the experiments on which Figure 9.34 is based were conducted for a limited range of Reynolds number, from $Re = 850$ to 1900 , the boundaries should be considered applicable only for the lower subcritical Reynolds number regime (for a single circular cylinder). As the results in Chapters 7 and 8 demonstrated, when considered together with previously published work, significant Reynolds number effects are mostly seen for the tandem configuration of two cylinders (see Sections 7.2.1 and 7.2.8). The side-by-side configuration, on the other hand, was shown in this study to be remarkably insensitive to the Reynolds number, particularly in terms of the Strouhal number data and in a qualitative assessment of the flow patterns and their T/D boundaries (see Section 8.2.1). Thus, the flow patterns and their boundaries identified for the staggered configuration, shown in Figure 9.34, should be more sensitive to changes in the Reynolds number when the tandem configuration is approached, at lower angles of incidence, i.e. from $\alpha = 0^\circ$ to 30° . Conversely, there should be less sensitivity to changes in the Reynolds number when the staggered geometry is close to the side-by-side configuration, at high angles of incidence, i.e. from $\alpha = 60^\circ$ to 90° .

9.10.2 Bistable Behaviour

The biased flow pattern observed for two side-by-side cylinders at intermediate T/D , described in Section 8.2.4, has been observed by some researchers to be bistable, with the

narrower and wider near-wake regions, and the direction of the gap flow, switching from one cylinder to the other. When observed, the switching phenomenon occurs for periods much greater than the vortex shedding period, and it happens irregularly (Kim & Durbin 1988). Bistable flow was not seen in the present side-by-side cylinder experiments, however, as discussed in Section 8.2.4.

A number of explanations for the switching phenomenon have been proposed, including that they are caused by turbulent perturbations in the mean flow (Peschard & Le Gal 1996). Zdravkovich (1987) explained it based on the two-cylinder staggered configuration. In this sense, bistability arises when the staggered configuration is at the “critical” angle of incidence of $\alpha = 90^\circ$, between forward-stagger and rearward-stagger, at a geometry equivalent to the side-by-side configuration. The cylinder with the narrower near-wake region then acts as an “upstream” cylinder, and the cylinder with the wider near-wake region acts as a “downstream” cylinder. Based on the flow patterns identified in the current investigation, the forward- and rearward-stagger would involve the VPSE and SVS flow patterns, respectively.

Within the staggered configuration, Zdravkovich (1987) contends that the bistable phenomenon extends to nearly side-by-side configurations with $\pm 0.15 L/D$ (although without any apparent supporting evidence), while Price & Païdoussis (1984) showed that for a small stagger of $\pm 0.10 L/D$ bistability was no longer evident. In any case, the possibility of bistable behaviour in a staggered configuration means that, under certain conditions, a narrower near-wake could form behind the *downstream* cylinder. Although bistable flow was not observed in any of the experiments reported in this thesis, an example was already cited for the BB flow pattern (in Section 9.3) of a narrower near-wake region behind the downstream cylinder, but this was observed only instantaneously, i.e. it was not a steady pattern as would be the case for true bistable behaviour.

Bistable properties have also been found for mean lift coefficients, by Zdravkovich & Pridden (1977), for a staggered configuration of $L/D = 3.0$, $T/D = 0.25$ ($P/D = 3.01$, $\alpha = 4.8^\circ$), at $Re = 60,000$. For this geometry, the mean lift changed irregularly in time between higher and lower values. Although these researchers

attributed the phenomenon to bistable gap flow, the findings of the current study suggest that bistability is caused by changes, back-and-forth, from the SLR flow pattern to the IS or VI flow patterns, i.e. alternating between gap-flow and no gap-flow. This conclusion is supported by the mean static surface-pressure data of Zdravkovich & Pridden (1977), where one profile shows a (time-averaged) pressure minimum on the inner front face of the downstream cylinder, consistent with induced separation and vortex formation (IS) or vortex impingement (VI).

9.11 Interpretation of Fluid Force Measurements

Of particular interest for two staggered circular cylinders in steady mean cross-flow, in addition to the vortex shedding frequencies, are measurements of the fluid forces acting on the cylinders, and, more specifically, how the forces change with the pitch ratio and the angle of incidence. Force measurements, in conjunction with measurements of the vortex shedding frequencies, are also important in a practical sense in the prediction of flow-induced vibrations in two-cylinder systems, and in a more theoretical manner in determining the fluidelastic forces (Ting *et al.* 1998), for example.

Previous studies of the staggered cylinder pair have mostly considered the forces acting upon the downstream cylinder, and specifically the lift and drag forces, F_L and F_D respectively, expressed in nondimensional form (rendered such through division by $0.5\rho U^2 DI$) as mean time-averaged lift and drag coefficients, C_L and C_D , respectively. Also of interest are the r.m.s values of the fluid forces, expressed in dimensionless form as the r.m.s. lift and drag coefficients, C_L' and C_D' , respectively. As shown much earlier, in Figure 3.1(d), a positive lift coefficient is defined for an inward-directed lift force, i.e. the downstream cylinder experiences a force directed towards the x -axis of the staggered cylinder geometry.

Now that flow patterns have been identified for the staggered arrangement of two circular cylinders in steady upstream cross-flow, the behaviour of previously published data for the fluid forces acting upon the cylinders can now be interpreted from a fluid dynamics point of view. In the following two subsections, experimental data for the

mean and r.m.s. lift and drag force coefficients from two studies are examined in some detail. The experiments, by Zdravkovich & Pridden (1977) at $Re = 60,000$, and by Moriya & Sakamoto (1985) at $Re = 65,300$, were undertaken at much higher Reynolds numbers than the current study (which was conducted at $Re = 850$ to 1900), but these are perhaps the most extensive sets of force data available for two staggered cylinders, in this range of P/D , from the rather limited number of published studies (compiled in Table 3.4).

9.11.1 Analysis and Interpretation of Lift Force Measurements

In Figure 9.35, previously published data for the mean and r.m.s. lift force coefficients, measured on the downstream cylinder, are superimposed on the newly identified flow pattern boundaries. These data are plotted as contour lines of constant mean or r.m.s. lift coefficient with respect to the longitudinal and transverse pitch ratios, L/D and T/D , respectively. From the contour lines can be discerned regions of maximum or minimum C_L and C_L' , and, by the closer spacing of the contour lines, geometries for which small changes in L/D or T/D could mean rapid and/or abrupt changes in the lift force. Again, it is important to note that there is an order of magnitude difference in the Reynolds number between the previously published force coefficient data and that of the flow patterns (and their boundaries) determined from this study.

The lift force contours for the downstream cylinder in the staggered pair show a number of key features, that include: (i) a region of high inward-directed mean lift force known as the “inner” lift peak, for staggered configurations of very small transverse pitch, seen in Figure 9.35(a) for $L/D \sim 1.1$ to 3.5 and $T/D \sim 0.2$; (ii) a second region of high inward-directed mean lift force known as the “outer” lift peak, for staggered configurations of large longitudinal pitch and small transverse pitch, seen in Figure 9.35(a) as a contour for $L/D > 3.0$ and $T/D \sim 1.0$ to 1.5 and extending to large L/D ; (iii) a region of nearly zero mean lift in the no-interference region of Zdravkovich (1987), which was shown earlier in Figure 3.2; (iv) a region of outward-directed (or repulsive) mean lift, for nearly side-by-side configurations; and (v) similar “inner” and “outer”

r.m.s. lift peaks, seen in Figure 9.35(b). The highest mean lift forces are those experienced at the “inner” lift peak.

The “inner” mean lift force maximum has been attributed to high speed flow being forced through the gap between the two cylinders, when the cylinders are beginning to approximate a tandem configuration (Zdravkovich 1987). The strong gap flow creates a significantly lower pressure on the front and inner faces of the downstream cylinder, causing the high “inner” lift to be experienced. The abrupt disappearance of the “inner” mean lift peak, as T/D is lowered further, has been attributed to the sudden disappearance of the gap flow with the increased wake interference. The flow patterns identified in the present study essentially confirm this interpretation of the fluid behaviour, as shown by the superimposed boundaries in Figure 9.35(a), meaning the “inner” mean lift peak (denoted by the chain-dotted mean lift coefficient contour line in Figure 9.35(a), labeled “ C_L max (inner)”) is caused by the IS flow pattern, which is marked by strong gap flow forced in front of the downstream cylinder (see Section 9.5).^{*} Based on this interpretation, the sudden disappearance of the “inner” lift force peak would be caused by an abrupt changeover from the IS to the SLR flow pattern (see Section 9.4, and also the comments to follow). Here, the strong gap flow can no longer be maintained, and the inner shear layer from the upstream cylinder reattaches onto the downstream cylinder, effectively preventing high speed flow through the gap between the cylinders. A sketch is shown in Figure 9.36(a). A similar “inner” r.m.s. lift force maximum for the same combinations of L/D and T/D , shown in Figure 9.35(b), would also presumably be caused by the same mechanism.

Since, in Figure 9.35(a), the “inner” lift contours do not exactly fall within the boundaries of the IS flow pattern, the above interpretation has taken into account an order

^{*} Flow is also deflected in front of the downstream cylinder in the SLR flow pattern (see Section 9.4, and Figure 9.12), but this is believed to be more of an instantaneous or intermittent occurrence, rather than the steady phenomenon (i.e. consistently seen) in the IS flow pattern. Some additional discussion on the deflected flow is found at the end of Section 9.11.2, concerning the origins of the minimum mean drag coefficient.

of magnitude difference in the Reynolds numbers between the data of Zdravkovich & Pridden (1977) and the present study, which would presumably see the boundary between the SLR and IS flow patterns shift to lower values of T/D at higher Reynolds numbers. This assumption is reasonable, since (i) shear layer instabilities also increase with the Reynolds number, meaning that shear layer reattachment would be increasingly unlikely to occur for higher T/D ; and (ii) the tandem configuration itself, and nearly tandem staggered configurations by extension, have already been shown to be sensitive to Reynolds number (Sections 7.21 and 7.2.8), and the flow pattern boundaries of Igarashi (1981, 1984) and Ljungkrona (1992) already show a contraction of the shear layer reattachment flow patterns with an increase in Reynolds number to high subcritical values.*

The “outer” mean inward-directed lift force maximum is located near the boundary of the wake interference and no-interference regions, shown in Figure 3.2, and has been attributed to a number of sources, summarized by Price (1976), which include: (i) a portion of the oncoming flow deflected through the gap between the cylinders, although the flow is not nearly as strong as that which caused the “inner” lift peak, and this accounts for the maximum “outer” mean lift coefficient being lower than the maximum “inner” mean lift coefficients; (ii) complete displacement of the wake of the upstream cylinder (Zdravkovich 1987); (iii) a shift in the stagnation point on the downstream cylinder away from the flow axis, such that a component of the drag force now acts in the inward-directed lift direction; and (iv) the production of additional circulation along the inside of the downstream cylinder. Based on the superposition of the flow pattern boundaries in Figure 9.35(a), each of these interpretations is supported by

* In other results, Sun *et al.* (1992) noticed large vibrations of the downstream cylinder in a staggered arrangement of $P/D = 2.2$, as the incidence was varied from $\alpha = 8^\circ$ to 18° (a change from $L/D = 2.2$, $T/D = 0.3$ to $L/D = 2.1$, $T/D = 0.7$), at $Re = 3.25 \times 10^5$. According to Figure 9.35(a), this would correspond to the “inner” lift force and transition from the SLR to the IS flow pattern. As further evidence of the Reynolds number sensitivity of the “inner” lift peak, Sun *et al.* (1992) did not notice these vibrations in the supercritical regime, $Re = 6.5 \times 10^5$.

the findings of the present study, and the source of the “outer” lift peak can be attributed to two different mechanisms: first, flow through the gap at smaller values of L/D , corresponding to the VPE and VPSE flow patterns; and second, vortex impingement at larger values of L/D , corresponding to the VI flow pattern. Each of the mechanisms is shown in Figure 9.36(b), and is discussed in more detail in the following paragraphs.

In the first mechanism, associated with the VPE and VPSE flow patterns (Section 9.6), the oncoming flow is deflected through the gap between the two cylinders, causing a static pressure distribution on the front and inner surfaces of the downstream cylinder that favours a high, inward-directed lift force. The gap flow in the VPE and VPSE flow patterns does not pass as close to the surface of the downstream cylinder as in the IS flow pattern (Section 9.5), and this difference may account for the “outer” mean lift maximum being lower than that of the “inner” mean lift maximum. Here as well, the wake of the upstream cylinder may be thought of as “completely displaced,” as suggested by Zdravkovich (1987), which can be seen in the PIV vorticity results in Figure 9.21, for example. A shift in the stagnation point on the leading face of the downstream cylinder is a less accurate interpretation of the VPE and VPSE flow patterns, except perhaps at small angles of incidence, as further inspection of Figure 9.21 reveals. The role of the VPE and VPSE flow patterns would seem to account for the closed contour of high inward-directed mean lift in Figure 9.35(a), of $C_L = 0.6$,^{*} once small shifts are made in the flow pattern boundaries to account for the influence of the Reynolds number.

In the second mechanism, associated with the VI flow pattern, a favourable surface static pressure distribution on the downstream cylinder that creates the high lift conditions is caused not by the deflected gap flow, although some does occur in the VI flow pattern (see Figure 9.29), but rather from vortex impingement and the circulation (or vorticity) induced thereby. As Price (1976) suggested, the production of circulation along the inside surface of the downstream cylinder may be partly responsible for the “outer” lift peak, and this is certainly a feature of the VI flow pattern (see Figure 9.30, and

^{*} This is believed to be the correct value for the closed contour, based on neighbouring mean lift coefficient contour lines, that is shown incorrectly in Zdravkovich & Pridden (1977) as $C_L = 0.4$.

vorticity concentrations labeled C). The induced vorticity is particularly strong for staggered configurations of larger T/D , as shown in Figure 9.30(d), when only one row of vortices impinges on the downstream cylinder; this would seem to account for the chain-dotted mean lift coefficient contour line in Figure 9.35(a), labeled “ C_L max (outer),” found near the upper boundary of the VI flow pattern. This “outer” lift peak contour extends to very high values of L/D (Price 1975), and the results of the present study showed that vortex impingement could be certainly be found for staggered configurations of $L/D = 4.0$ and beyond.

The decrease in the magnitude of the “outer” lift coefficient as T/D is decreased, shown in Figure 9.35(a), is also consistent with the VI flow pattern: at smaller T/D , two rows of vortices impinge on the downstream cylinder (see Figures 9.30(a,b)), and the induced vorticity about the second cylinder is weakened, since the impinging vortices are split into two concentrations of vorticity (labeled B and D in Figure 9.30).^{*} With an increase in T/D , the VI pattern transitions to the SVS pattern in the no-interference region[†] of Zdravkovich (1987), and the lift force lowers.

In the r.m.s. lift coefficient data of Moriya & Sakamoto (1985), shown in Figure 9.35(b), there also appears an “outer” r.m.s. lift coefficient maximum. Since it appears in the vicinity of the “outer” mean lift peak discussed above, it likely has the same fluid dynamic origins. It is also found near the confluence of the flow pattern boundaries for the IS, VPE, VPSE, VI, and SVS flow patterns, and thus it may also be caused by intermittent and transitional behaviour between the various patterns.

^{*} A better understanding of the role of the induced vorticity in determining the fluid forces on the cylinder might be found by adopting, at a future date, the methods of Lin & Rockwell (1996) and Unal *et al.* (1997), which enable the fluid forces to be directly estimated from the vorticity field.

[†] The no-interference region of Zdravkovich (1987), shown in Figure 3.2, is a misnomer since the SVS flow pattern is observed there. Although there may be minimal distortion or displacement of the near-wake of the upstream cylinder, there is still interference in the form of anti-phase synchronization of the vortex shedding processes.

9.11.2 Analysis and Interpretation of Drag Force Measurements

In Figure 9.37, previously published data are presented for the mean and r.m.s. drag coefficients measured on the downstream cylinder, with the flow pattern boundaries from the current study superimposed. As in the case of the lift coefficient measurements, the data are plotted as contour lines of constant mean or r.m.s. drag coefficient with respect to L/D and T/D . Once again, the order of magnitude difference in the Reynolds number, between the previously published force coefficient data and that of the flow patterns (and their boundaries) determined from this study, is emphasized.

Analysis of the drag force coefficient data for the downstream cylinder in the staggered pair shows a number of key features: (i) a region of minimum mean drag force in the vicinity of the “inner” lift peak discussed in Section 9.11.1, shown in Figure 9.37(a) for staggered configurations of intermediate L/D and very small T/D ; (ii) a region of negative drag force, or thrust, also found for the nearly the same staggered configurations of the “inner” lift peak, shown in Figure 9.37(a); (iii) a maximum r.m.s. drag coefficient that coincides with the beginning of the “outer” mean and r.m.s. lift peaks, shown in Figure 9.37(b). Zdravkovich (1987) contends that the “outer” mean lift force maximum has no corresponding effect on the mean drag coefficient, since an “outer” mean drag force coefficient minimum is not found in Figure 9.37(a), nor in the mean drag coefficient data of Moriya & Sakamoto (1985), not shown here. The location of the “outer” lift force maximum does correspond, however, to a region of maximum *r.m.s.* drag coefficient, as shown in Figure 9.37(b).

The origin of the minimum drag force and the thrust force has been attributed by Zdravkovich (1987) as “a by-product of the gap flow.” Based on the flow patterns identified in the present study, the minimum and negative values of the mean drag coefficient fall entirely within the SLR boundaries, however, where flow through the gap is suppressed (see Section 9.4). Suppression of the gap flow, through reattachment of the inner shear layer from the upstream cylinder, would seem to be a logical origin of the minimum drag coefficient for two staggered cylinders, since (i) the upstream cylinder completely shields the downstream cylinder from the oncoming flow, and (ii) the

minimum drag for two tandem cylinders is also noticed under shear layer reattachment conditions (Ljungkrona 1992), where a thrust force is often experienced by the second cylinder (and seen in Figure 9.37(a) for slightly staggered arrangements). Interestingly, Price & Païdoussis (1984) remarked that when the downstream cylinder is in a location of minimum drag, the lead cylinder is correspondingly in a position of maximum drag force.

As remarked by Ohya *et al.* (1989), the minimum drag configuration for two cylinders in steady cross-flow does not occur surprisingly in the tandem arrangement, but rather for a slightly staggered, nearly tandem geometry. This may now be thought of as a by-product of shear layer reattachment (in the SLR flow pattern), which may cause more surface area on the front face of the downstream cylinder to experience negative static pressures, compared to the tandem configuration, yielding the lower resultant drag force. Ohya *et al.* (1989) suggest that an outward shift of the stagnation point on the downstream cylinder may be responsible for the lower drag coefficient, caused by a small amount of flow deflected through the gap in front of the downstream cylinder. This deflected flow was discerned from the present PIV velocity data, on an instantaneous basis, immediately in front of the downstream cylinder in the SLR flow pattern, as shown earlier in Figure 9.12. This flow is believed to manifest itself as a vorticity concentration in front of the downstream cylinder, within the bounded interstitial region, as shown in Figure 9.11(a); although this flow structure could also be responsible for lower pressures in front of the downstream cylinder, which would effectively lower the drag force, it was not consistently observed (as shown in by other PIV vorticity data in Figure 9.10).

Chapter 10

Conclusions and Recommendations

10.1 Conclusions and Contributions

In this study, small groups of two or three circular cylinders arranged in tandem, side-by-side and staggered configurations were investigated in steady cross-flow, and under conditions of an impulsive start, within the low subcritical Reynolds number regime for a single circular cylinder. The emphasis of the study was on acquiring an improved physical understanding of the flow around simple multiple cylinder systems, and the dynamics of the vortical structures in the flow, as the number of cylinders, the spacing between the cylinders, and the angle of incidence with respect to the oncoming flow, were varied. This was accomplished through extensive experiments in water, and undertaken in three experimental facilities: a closed test section, recirculating water tunnel; a small, vertical water towing tank; and a main water towing tank facility. The latter two facilities were specially designed and constructed for this research.

The improved understanding of the fluid dynamics for the multiple cylinder configurations was acquired by two means: first, on a qualitative basis through the identification of flow patterns; and, second, on a quantitative basis through measurements of vortex shedding frequency, velocity, vorticity, vortex strength, and other quantities. A three-fold experimental approach was adopted using conventional dye-injection flow visualization, single-component hot-film anemometry, and particle image velocimetry (PIV). The PIV technique enabled the flow to be studied through the temporal

development of the instantaneous, in-plane vorticity field, and the current study represents one of the first extensive applications of PIV to groups of bluff-bodies in cross-flow. Furthermore, the investigation is one of the first to use the PIV technique in the study of impulsively started flow. In each of these cases, the interpretation of the instantaneous vorticity data was greatly aided by the flow visualization video images acquired in this study, as well as by published results where available, and good correspondence was noticed between the two means of observing the flow field, and with existing published experimental data and observations. Importantly, the effects of increasing or decreasing the centre-to-centre pitch ratio, and/or the angle of incidence, were clearly seen. In the small range of Reynolds number tested, however, from approximately $Re = 500$ to 6000 (with most experiments conducted at $Re = 1900$), few significant Reynolds number effects were observed (which in some ways is an important conclusion in itself), but a more in-depth investigation of its effects is still needed for some multiple cylinder configurations. This work is left to a future study.

The full-field approach in particular yielded physical insight into the fluid dynamics that cannot often be found through extensive measurements of vortex shedding frequency, surface static pressure, and the lift and drag forces acting upon the cylinders. For example, as a thorough survey of past studies of the tandem, side-by-side and staggered cylinder arrangements revealed. Citing the study herein of two staggered circular cylinders in steady cross-flow, for example (for which more specific conclusions will follow), the results of the present study offered some new insight into the fluid dynamic origins of maximum and minimum values of the mean lift and drag force coefficients, found by other researchers, for specific geometries.

As a basis for the multiple cylinder experiments, each of the investigative techniques was first applied to the flow about a single, isolated circular cylinder, for which the flow field has been well-studied by many researchers under both steady and impulsively started mean cross-flow conditions. Although little new information was gained in the study of the single circular cylinder, the experiments validated the experimental approach and demonstrated the usefulness and reliability of the apparatus

and instrumentation. The multiple cylinder experiments which followed dealt with three basic configurations that covered practically the full range of possible two-cylinder and three-cylinder geometries: the tandem, side-by-side and staggered arrangements already mentioned. This research was one of very few that has attempted to examine all three in a single test facility, and under the same mean flow conditions.

10.1.1 The Tandem Configuration

This study included the first known investigation of tandem circular cylinders in impulsively started cross-flow, and three basic flow patterns were identified for two-cylinder and three-cylinder arrangements using a combination of flow visualization and PIV, and supported by measured quantities from the PIV velocity and vorticity data. The flow patterns are shown in Figure 10.1, and include (a) single bluff-body behaviour when the cylinders are in contact, (b) constrained streamwise growth and lateral expansion of the gap recirculation zones at small and intermediate longitudinal pitch ratios, and (c) independent formation of recirculation zones similar to a single impulsively started circular cylinder at larger pitch ratios. It was suggested that the latter two flow patterns, for which there is a gap between the cylinders, both transition to a shear layer reattachment flow pattern under steady mean flow conditions, despite the apparent and increasing independence of the cylinders at the larger pitch ratios.

Under steady mean cross-flow conditions, the flow visualization, hot-film anemometry and PIV results of the current study confirmed the existence of the three basic flow patterns previously reported by other researchers. These and other patterns identified in the present study are shown in Figure 10.2, comprising (a) single bluff-body behaviour, (b) type 1 shear layer reattachment, (c) type 2 shear layer reattachment, (d) type 3 shear layer reattachment, and (e) Kármán vortex shedding from the upstream cylinder. Here, a similar nomenclature is adopted to aid in the classification of the flow patterns, which was first initiated in the study of the staggered configuration presented in Chapter 9. Hence, the fluid dynamics of two tandem cylinders in steady cross-flow may now be classified by the SBB, SLR1, SLR2, SLR3 and KVS flow patterns (this is the first

use of the SLR1, SLR2, SLR3 and KVS designations, however SLR1, SLR2 and SLR3 are subgroups of the SLR designation introduced in Chapter 9, and KVS includes the vortex impingement, VI, flow pattern). A further summary of the flow patterns, and specific features, are found in Table 10.1.

Apart from verifying the three basic flow patterns, the study clarified the fluid dynamics of the shear layer reattachment flow pattern, and showed for the first time that (i) reattachment of the free shear layers from the upstream cylinder, onto the surface of the cylinder downstream, is not a continuous process; (ii) only one of the two shear layers undergoes reattachment at one particular time in the shedding cycle, and both shear layers are never observed to reattach simultaneously onto the downstream cylinder; (iii) the shear layer reattachment is highly synchronized with Kármán vortex formation from the cylinder downstream; and (iv) up to three different shear layer reattachment behaviours could be identified, depending on the longitudinal pitch ratio (and shown at two times, t_1 and t_2 , in the shedding cycle). This study also clarified, for the first time, the flow within the gap between the two cylinders, bounded on either side by the shear layers from the upstream cylinder. Specifically, the flow visualization and PIV vorticity data revealed that (i) a stable, symmetric pair of vortices was never consistently seen in the gap between the cylinders, as inferred by a number of researchers; (ii) concentrations of vorticity in the gap were more likely to be found in the vicinity of the reattachment point, and synchronized in a like manner with Kármán vortex formation from the cylinder downstream; and (iii) the dominant motion in the gap was weakly rotating or stagnant fluid. Further evidence of the Reynolds number sensitivity of the tandem configuration was also found, and specifically its influence on the transition from the shear layer reattachment flow pattern to Kármán vortex formation from the upstream cylinder. In this matter, good correspondence was maintained between some limited vortex shedding frequency measurements and the PIV vorticity data in determining the value of the “critical” longitudinal pitch ratio, and favourable comparison was also made with previously published Strouhal number data. The study also revealed considerable

temporal variation to all of the flow patterns, when viewed on an instantaneous basis from the PIV vorticity data.

10.1.2 The Side-by-Side Configuration

For the side-by-side configuration of circular cylinders, this study included the first known investigation of this arrangement under conditions of an impulsive start from rest. Two basic flow patterns were identified from a combination of flow visualization images and PIV vorticity data, for two-cylinder and three-cylinder side-by-side configurations, as shown in Figure 10.3. The temporal development of each of the patterns was also quantified using parameters obtained from the PIV velocity and vorticity data, and the flow field may be broadly classified into (a) single bluff-body behaviour when the cylinders are in contact with one another, and (b) vortex shedding and interaction when there are gaps between the cylinders. In the former flow pattern, a single recirculation zone forms in the combined wake of the cylinders, and its development and those of the primary eddies are markedly similar to a single impulsively started circular cylinder. In the latter pattern, strong flow through the gap(s) between the cylinders precipitates (i) the rapid breakup of the recirculation zones behind the outer cylinders, (ii) the shedding of vortices alongside the gap flows, and (iii) uniquely, the formation of a counter-rotating vortex pair in the combined wake. An increase in pitch ratio, under impulsively started conditions, typically translated into delayed breakup of the recirculation zones, delayed shedding of the initially formed gap vortices, and resulted in the counter-rotating vortex pair being first observed further downstream from the cylinders. The increase in pitch ratio had little influence on the strengths of the attached primary eddies before they were shed, as did the number of cylinders. The impulsively started flow field failed to illustrate, however, precisely how the biased steady cross-flow patterns develop for two and three side-by-side cylinders, since the starting flow remained symmetric, and the gap flows remained undeflected, throughout the limited range of time in which it was studied.

Under steady mean cross-flow conditions, the present research essentially confirmed the existence of known flow patterns identified in previous studies, as was the

case for the tandem configuration in steady mean cross-flow, through flow visualization experiments and the use of PIV. Few studies of three side-by-side cylinders were found in the review of the literature, and so this work was of some importance in further understanding the fluid dynamics unique to this grouping of bluff-bodies. The flow patterns are summarized for two-cylinder and three-cylinder side-by-side geometries in Figure 10.4, and in a more detailed fashion in Table 10.2, and include (a) single bluff-body behaviour when the cylinders are in contact, (b) base-bleed and asymmetric biased gap flow at small transverse pitch ratio, (c) biased gap flow at intermediate pitch ratio, and (d) anti-phase synchronization to the vortex shedding processes at both intermediate and larger pitch ratios. These patterns have been assigned designations of SBB, BB, BF, SVS, ABF and SBF (the first use of these designations in the thesis, with the exception of the BB and SVS patterns), similar to those first adopted for the staggered configuration in Chapter 9 and now applied also to the tandem configuration (see Section 10.1.1). When examined on the basis of the instantaneous velocity and vorticity data, the steady flow patterns showed remarkable variation, particularly in the direction of the base-bleed flow, the deflection angle of the biased flow at intermediate pitch ratios, and the length of the vortex formation region, for example.

Vortex shedding frequency measurements were also undertaken for the side-by-side cylinders, using hot-film anemometry, and these confirmed the existence of two Strouhal numbers for the biased flow patterns, and up to three Strouhal numbers for some three-cylinder flow patterns. Through a combination of flow visualization, PIV vorticity data, and frequency measurements at a number of positions in the combined wake of the cylinders, the origin for a third frequency at $T/D = 1.5$, for three cylinders, was traced to a complex vortex pairing and enveloping process that was also found in the staggered configuration of two cylinders.

10.1.3 The Staggered Configuration

The staggered configuration of two cylinders was investigated only under steady mean cross-flow conditions, but was one of the first known studies to approach its fluid

dynamics from the basis of flow pattern identification, rather than from point-measurements or theory. The flow visualization and PIV experiments revealed considerable complexity to the fluid behaviour, at a fixed Reynolds number, as the centre-to-centre spacing of the cylinders and the angle of incidence (within the plane of the cylinders) were each separately varied over wide ranges (from approximately one to four diameters in spacing, and from 0° to 90° in incidence, i.e. for intermediate angles between the tandem and side-by-side configurations for two cylinders). As in the case of the tandem and side-by-side configurations, good agreement was found between the flow visualization images and the instantaneous vorticity data provided by the PIV system. The flow visualization images were of greater importance for the staggered configuration, however, in helping to properly interpret the vorticity data, since there is an absence of flow visualization studies of two staggered cylinders in the literature. As for the tandem and side-by-side configurations, the variable nature of the instantaneous flow patterns was also seen for two staggered cylinders, with differences in the gap flow deflection angle and the vortex formation length, for example, for a given geometry in different experiments.

This study was the first to demonstrate that up to nine different flow patterns may arise for the two-cylinder staggered arrangement, and these are summarized, along with their designations of SBB1, SBB2, BB, SLR, IS, VPE, VPSE, SVS and VI, in Figure 10.5. The pitch ratio and incidence angle boundaries for the flow patterns were also proposed in the study, and were presented earlier in Figure 9.34. Further details are included in Table 10.3, where specific features of the individual flow patterns are summarized. These boundaries now make it possible to more accurately predict the nature of the flow for a given geometry of two cylinders, although it must be remembered that the present experiments were conducted for a limited range of Reynolds number in the low subcritical regime, and that the boundaries for nearly tandem geometries especially will be the most sensitive to changes in the Reynolds number.

The flow patterns and their boundaries were also shown to help identify the fluid dynamic origins for various phenomena noted by other researchers in their measurements

of the lift and drag force coefficients. A difficulty here is that there exists an order of magnitude difference in the Reynolds number between the present study, in which the flow patterns are identified ($Re = 850$ to 1900), and those studies which report the force measurements ($Re = 60,000$ to $65,300$). Nevertheless, the origin of the inner, inward-directed lift force coefficient maximum, and its sudden change in magnitude, are likely caused by a changeover between the SLR and IS flow patterns.

The study of the staggered pair of cylinders showed that for the majority of the flow patterns, fluid is permitted to flow through the gap between the cylinders, and when it does, vortex shedding from the upstream cylinder is nearly always observed in some form and at some scale. This observation contradicts a contention by Zdravkovich (1985) that strong gap flow can occur in the absence of vortex shedding from the upstream cylinder. Nevertheless, Zdravkovich's conclusion is essentially correct, if the only information available is a velocity-time signal at a *single* point in the combined wake of the two cylinders, as was shown in a critical assessment of previously published Strouhal number data and those obtained from the flow visualization videos in the present study (see Section 9.9).

The flow visualization videos also provided new insight into the vortex shedding behaviour, concerning the origins of the two vortex shedding frequencies, one low and the other high, that are commonly detected. The results revealed the frequencies are more properly associated with the individual free shear layers, rather than with the individual cylinders. In this manner, the shear layers from the rearmost cylinder, in many cases, do not usually shed vortices at the same, lower frequency. Instead, the two shear layers seem to act independently, with only the outer shear layer shedding Kármán vortices of low frequency. Vortex shedding from the inner shear layer of the downstream cylinder is instead synchronized with higher-frequency Kármán vortex shedding from the upstream cylinder.

Due to the large number of experiments which would be needed to cover a complete range of geometries, the staggered cylinder pair was not tested under impulsively started conditions. Based on the results for the tandem and side-by-side

arrangements under an impulsive start, however, a combination of the two types of behaviour would be expected, resembling more the tandem configuration at smaller incidence, and the side-by-side configuration at larger incidence.

10.1.4 Implications and Applications of the Research

The conclusions and contributions of this study are important from a physical or fundamental standpoint, in improving the understanding of the flow around groups of bluff-bodies in cross-flow. For example, new insight into proximity and wake interference effects, and the synchronization and interaction of multiple vortex shedding processes and vortex streets, was obtained. Another reason for studying the flow around small groups of cylinders in cross-flow was to better understand the fluid-dynamic origins of flow-induced vibrations noted in many multiple cylinder systems. Many of the flow features for the two-cylinder and three-cylinder configurations may also be found in some form in larger multiple cylinder arrays, for example, that are particularly prone to vibration problems. The direct application of these results to real industrial problems, however, must be tempered by recognition of the vast difference in the Reynolds number between this study and those in most practical conditions, as well as differences in mean flow properties such as turbulence and large-scale unsteadiness, for instance, that are more often encountered outside the laboratory. It is also well known, although not often cited in studies of small groups of cylinders, that the properties of the flow field for small groups of cylinders may not correspond well to the behaviour of larger multiple cylinder groups, in which the smaller geometries form a subset of the whole.

For tandem cylinders in steady cross-flow, the synchronization of shear layer reattachment with vortex shedding from the second cylinder is one of the more important findings, when considering interaction between the fluid motion and potential structural motion, since there is now a more direct link between the dynamics of the two cylinders. Thus, even without formal vortex shedding from the upstream cylinder, periodic behaviour will be strongly communicated upstream (as also shown in the surface-pressure and surface-hot-film measurements of Lee & Panagakos (1997)). The Reynolds number

sensitivity of the transition of the reattachment flow pattern to Kármán vortex shedding is also of importance in knowing whether or not vortex shedding will occur from the upstream cylinder.

For side-by-side cylinders in steady cross-flow, it was seen that frequency detection of periodicities in the flow may be highly location-dependent, as in the case of three cylinders, and may arise from other sources apart from Kármán vortex shedding. This too has implications in flow-induced vibrations, particularly for design data that have been solely based on frequency measurements at a single location, for example.

For staggered cylinders, a wide variety of flow patterns has now been identified, some occurring with very small changes in the incidence or direction of the oncoming flow. This suggests that flow-induced vibrations may be caused by a combination of fluid dynamic effects, which now include synchronized or alternating shear layer reattachment, or vortex impingement, in addition to turbulence and Kármán vortex shedding. With some new understanding of the fluid dynamic origins of extreme values of the lift and drag forces, designs for multiple-cylinder structures in cross-flow can now be optimized or modified to suppress or promote a particular flow pattern (for greater ranges of the angle of the incident flow, for instance). As a case in point, reduced drag force may now be achieved through imposing the SLR flow pattern. The study has also revealed that alternate vortex shedding occurs in the majority of staggered configurations, even from the upstream cylinder in geometries of small incidence and pitch ratio, which means that the potential for vortex-induced vibrations may still exist at multiple frequencies even for cylinders in close proximity. It has also been shown that vortex shedding frequencies are shear-layer-dependent, rather than cylinder-dependent, which may promote a rethinking of how Strouhal number data should be interpreted. The concept of the universal Strouhal number (Adachi *et al.* 1996; Nakamura 1996) might also be examined further, to assess its applicability to vortex shedding from multiple cylinder configurations. Much of the information needed to compute the universal Strouhal number can be readily provided for vortex shedding from multiple cylinders by the PIV technique (apart from the vortex shedding frequency, however), including the

distance between a pair of free shear layers and the local flow velocity in the neighbourhood of the shear layer formation region.

For all configurations, instantaneous measurements revealed considerable variability to the flow patterns, that is not reflected in time-averaged measurements. How this result affects design data based on long time-averaged series of pressure or velocity data remains to be understood, but it likely means that peak values of lift and drag, that could be attributed to temporal variations in vortex formation length or other aspects of the flow pattern, for example, have been ignored in time-averaged measurements. Similar debate has been underway recently in the wind engineering community, concerning the relative importance of averaged pressure measurements versus peak measurements, for example (Melbourne 1997).

10.2 Recommendations for Future Work

At the outset of this research, much still remained to be understood or clarified concerning the fluid dynamics of small groups of structures immersed in a moving fluid, as identified by a survey of the literature. At the conclusion of this study, there is still much more which remains to be investigated, but in many respects it is hoped that the physical understanding of cylinders in cross-flow has been enhanced by the contributions of this research. The goals which were initially established at the start of the research were ambitious, but a study of this nature, which tries to be all-encompassing, most often falls short in the interests of the time needed to perform the measurements and in the space needed to present the results. A number of interests and unanswered questions concerning multiple cylinders had to be set aside for future studies, including: (i) the influence of the Reynolds number for each configuration; (ii) the behaviour of the full range of staggered geometries under impulsively started conditions; (iii) the fluid dynamics of the triangular and square clusters of cylinders, using PIV; and (iv) more thorough measurements of vortex shedding frequency. A number of these goals were also set aside due to limitations of the experimental techniques; in the subsections to follow, some recommendations for various improvements to existing apparatus and

instrumentation are presented (in Sections 10.2.1 and 10.2.2), and some areas of future study are outlined (in Section 10.2.3).

Before presenting the recommendations, however, it must be realized that the vast majority of the work investigating the flow around multiple cylinders has been experimental in nature. Recent numerical studies by Chang & Song (1990), Morris *et al.* (1994, 1995), Braun & Kudriavtsev (1995), Blazik-Borowa & Flaga (1996), Tsutsui *et al.* (1997) and Edamoto & Kawahara (1998), for example, have begun to examine the behaviour of multiple bluff structures, but these studies and those of a more theoretical nature (such as by Peschard & Le Gal (1996), for the side-by-side configuration), are still primarily restricted to low Reynolds numbers, two-dimensional flows, and small elapsed time periods from the startup of motion. Thus, the study of multiple cylinders remains a challenge to both the computational fluid dynamicist and the experimentalist.

10.2.1 Improvements to the Experimental Apparatus

The use of three different test facilities proved to be valuable in the current study of multiple cylinders in water cross-flow, particularly when the same ranges of Reynolds number, pitch ratio, incidence angle and number of cylinders, could be accommodated in each apparatus. Nonetheless, differences in the levels of wall interference, freestream turbulence, and in the cylinder aspect ratio, for example, remained. In future work, improvements could be incorporated to each of the three experimental facilities to both expand and improve the results.

The Kempf & Remmers water tunnel has become an established apparatus, and improvements would simply be made to the cylinder models mounted in its test section. Specifically, the mounting hardware should be changed to permit $0.125D$ and $0.25D$ increments in the pitch, which would be needed to complete the flow visualization survey of all the staggered cylinder configurations that were eventually tested in the towing tank using PIV (compare the different symbols in Figure 9.1(a), on the basis of the centre-to-centre pitch ratio, P/D). This would be especially important in further study of the two-cylinder and three-cylinder side-by-side configurations, for which the base-bleed flow

pattern and the asymmetric biased flow pattern were only studied by PIV. The cylinder end conditions in the water tunnel may also deserve some attention, since higher “critical” or transitional pitch ratios, or boundaries between flow patterns (for tandem and side-by-side configurations, specifically) were found in the water tunnel when compared to the towing tank results, and the differences were attributed in part to aspect ratio effects. Improvement might be made in this regard by the use of end plates, or through a study of vortex shedding along the cylinder span, using dye injection and hot-film anemometry (i.e. with dye injection ports along the cylinder span, and the probe mounted through the test section roof or floor).

Concerning the two towing tanks, their usefulness was clearly demonstrated, but modifications to the motion systems ought to be considered in any future work. These would serve to (i) improve the repeatability of impulsive-starts in the vertical tank, for more control over the acceleration parameter, A_p , and to trigger the start of image capture from the moment the cylinders are released; (ii) alleviate the vibrations of the cylinder and platform at lower towing velocities in the main towing tank, particularly for any future experiments incorporating an oscillating cylinder; and (iii) permit unsteady cross-flow conditions to be more easily investigated in the main towing tank (presently, two computers would be needed to accomplish this, one to control the motion of the platform, and another to control image acquisition). Concerning the vibration problem in the main towing facility, mentioned briefly in Section 4.3.3, this may be reduced through (i) use of a rubber V-belt or steel cable in place of the timing belt, perhaps located along the centre of the tank rather than on the side; (ii) removal of one of the guide rails, and replacement of the linear bearings on that side of the tank with a simple pair of wheels, to alleviate the “crab-like” wandering motion of the carriage at low towing velocities; and (iii) refinement of the startup or acceleration portion of the cylinder motion, as the cylinder is towed from rest to a steady velocity, in order to quickly attenuate any residual transient effects.

As in the case of the water tunnel, a more thorough investigation of cylinder end conditions in both towing tanks, using PIV, should also be considered. Only a cursory

study was undertaken as part of this thesis, in the main towing tank using hot-film anemometry, with the results presented in Section 4.5.3. Such experiments may be conducted as part of a more thorough examination of three-dimensional flow around a circular cylinder or groups of cylinders [e.g. Wu *et al.* 1994b] in cross-flow.

10.2.2 Improvements to the Instrumentation

Although each of the three investigative tools of dye injection flow visualization, single-component hot-film anemometry, and PIV, was successfully applied to the multiple cylinder configurations, improvements could be made to each of the techniques. The purpose of these improvements would be to enhance the quality of the information and to extend the capabilities of the current experimental apparatus.

Concerning the flow visualization setup, improvements could be made to the video images in terms of lighting, by using the thinner and more intense argon-ion laser light sheet rather than the xenon arc lamp in the present setup (see schematic in Figure 5.1). This would also make the dye fluorescence more intense, improving the contrast of the streaklines against the background flow, and reducing the need to do extensive digital image enhancement. The shadows and intensified regions of light, generated by the plexiglas cylinders behaving as cylindrical lenses, would still remain; however, a mirror might be mounted on the far wall of the test section to reflect some of incident light sheet back into the flow.

Use of Super-VHS video was extremely valuable in studying the fluid motion, particularly in a periodic flow such as that about the circular cylinder; however, the quality of the images is much poorer than photographs from a SLR camera. A two-fold flow visualization approach might therefore be considered in the future, using video to identify various modes of synchronization, and still photographs to provide higher-quality pictures for publication. The use of a high-speed, high-resolution digital camera might also be considered, since this technology is rapidly improving.

A new dye injection technique might also be an option for future flow visualization experiments in the water tunnel, in particular one which was developed by

Hoyt & Sellin (1995) and successfully applied to the flow around a circular cylinder and the flow through cylinder arrays (Hoyt & Sellin 1997), in water, up to $Re = 30,000$. This technique used a special dye fluid which resisted breakup and diffusion under highly turbulent conditions. Adoption of this technique would considerably extend the range of Reynolds number for the flow visualization experiments. The use of a dye injection probe or rake might also be explored.

In terms of measuring the vortex shedding frequencies in the flow field, new experiments should consider a *combination* of hot-film anemometry and the frame-by-frame timing and analysis of individual vortices. Hot-film anemometry permits more reliable measurements of low-frequency vortex shedding to be obtained, and can be used to identify other dominant frequencies not directly related to Kármán vortex shedding, as noticed in the present study for three side-by-side cylinders in steady cross-flow, at $T/D = 1.5$ (in Section 8.2.8). The frame-by-frame analysis is valuable, however, for more accurately identifying which vortex formation processes are specifically responsible for the measured frequencies. Future hot-film measurements should also be conducted throughout the combined-wake of the cylinders, perhaps up to five diameters downstream in the case of the side-by-side and staggered configurations, and across the entire wake on both sides of the centreline, in case there is a biased flow pattern. This has been done to some extent in the experiments reported in this thesis. (Failure to measure at a number of locations would have meant that a second high frequency peak would have been missed in the study of three side-by-side cylinders at $T/D = 1.5$, and high-frequency Kármán shedding from the upstream cylinder would have been left undetected in many staggered cylinder flow patterns.)

Concerning the present PIV system, the major limitations are (i) the low resolution of the digital images at 512×480 pixels; (ii) the limited number of images which could be acquired in a given test (which was 16 images, or 8 pairs); and (iii) the large size and irregular shape of the seeding particles. With a higher-resolution digital camera, more of the finer flow structure could be uncovered and more reliable and repeatable measurements of vortex strength, for example, could be obtained. If more

images could be acquired, by using a frame grabber with a larger memory and a faster transfer rate (the current time between successive vorticity fields is 0.2 s, or a rate of 5 Hz, which is less than the theoretical rate of 15 Hz for asynchronous image capture), more of the vortex shedding period could be captured, with the camera still remaining in the fixed position beneath the main towing tank. Also, a greater range of t^* could then be captured under impulsively started conditions, and the time-dependent nature of the flow could be more finely resolved. The possibility of incorporating a reference signal, which would synchronize image capture with a specific point in the vortex formation or shedding cycle, would also be advantageous, since it could be used for phase averaging of the vorticity data (as done by Lin *et al.* (1995a)). A reference signal of this sort would be especially needed for studies of an oscillating cylinder, but would more likely be referred to the position of the cylinder.

Concerning the seeding particles, smaller fluorescent particles would be more desirable, in terms of the accuracy in following the fluid motion (as discussed in Appendix A, for example), but the often conflicting issues of neutral buoyancy, reflectivity and cost would remain. Different seeding particles could also mean an overall reduction in the undesirable background motion in the towing tank, caused by the present need to periodically redistribute the large, heavy particles during a series of experiments.

10.2.3 Further Areas of Study

This thesis began in the Preface with a painting of Saint Christopher, seen in a familiar setting carrying the Christ child across a river. Although left to the imagination of the reader for the most part, on the surface of the water in the vicinity of the Saint's legs may be seen complex patterns of fluid vortices, as if the two limbs were behaving as a pair of staggered circular cylinders immersed in cross-flow, as investigated in the present study.

In reality, however, the flow around the Saint's legs differs considerably from the idealized experiments conducted in this thesis, as do most real flows of interest to engineers and others. In fact, even a cursory examination of the painting shows that in carrying the Christ child across the river, Saint Christopher is immersed in a flow which

is unsteady, nonuniform, highly turbulent, and perhaps of a two-phase nature if the river is transporting significant amounts of sediment. Periodic, wave-like motion may also be superimposed. The river channel itself is a much different environment than any laboratory facility, with the confining or blocking effects of the two irregularly shaped river banks, the shallow height of the water which gives rise to very low aspect ratio bluff-structures, the presence of the roughened surface of the river bed and its boundary layer which will interact with the Saint's legs and feet, and the influence of a free surface that is highly distorted by the vortical structures. Furthermore, Saint Christopher's legs are far different from the smooth, rigid circular cylinders tested in the current investigation. Rather, his legs are of nonuniform cross-section, perhaps of different sizes and shape depending how deep they are immersed in the river, and far from smooth or rigid. And, in carrying the Christ child across the river, the bluff-structures which are his legs are each in large scale motion normal to the river flow, and may even oscillate under the weight of the shifting baby in his arms.

As this example has illustrated, much more remains to be understood of the fluid dynamics of multiple circular cylinders in cross-flow, and particularly when it comes to real engineering flows. In many applications, the Reynolds numbers are orders of magnitude higher than those studied in the laboratory, and there is always some danger when extrapolating laboratory results to real flows (Zdravkovich 1991). Although a number of laboratory studies have already been conducted which examine the low Reynolds number regime (Williamson 1985; Peschard & Le Gal 1996), and the very important critical and post-critical regimes (Suzuki *et al.* 1971; Okajima 1979; Sun *et al.* 1992), further work is still needed (and particularly for the staggered configuration). Published multiple cylinder experiments in real flows are few. Although some studies of real flows around twin towers, chimneys, and transmission lines may be found (Price 1975), extensive and systematic comparisons to laboratory studies, and explanations of any discrepancies, have yet to be made.

Another variable which could be examined in the context of two and three cylinders in cross-flow is the effect of different cylinder diameters in the same

configuration. Some limited work in this regard has been undertaken for the tandem (Hiwada *et al.* 1979; Igarashi 1982), side-by-side (Lam *et al.* 1993) and staggered (Bokaian & Geoola 1984; Baxendale *et al.* 1985; Tsutsui *et al.* 1997) arrangements. Multiple cylinders of noncircular cross-section also deserve more study, since these too are found in engineering applications. Investigations of tandem elliptical cylinders (Ota & Nishiyama 1986) and square cylinders (Takano *et al.* 1981; Ohya *et al.* 1989; Kolar *et al.* 1997) have shown basic similarity to the circular cylinder systems, with the differences mostly attributed to the angle of orientation of the bodies to the oncoming flow, and the fixed separation points in the case of the square cylinders.

The impulsive start is just one of many time-dependent scenarios for a circular cylinder which, prior to this study, have not been investigated (and published) using the PIV technique, but would perhaps warrant new quantitative treatment. Taneda (1977), for example, has presented extensive flow visualization results for a number of time-dependent separated flows involving a circular cylinder (and other bluff-bodies), undergoing uniform acceleration or deceleration, step changes in velocity, oscillation, and rotation; many of these basic time-dependent flows have received little attention elsewhere in the literature.

Studies of multiple cylinders in which one or both are undergoing forced or free oscillation, would also be of some benefit. Some initial investigations have already been reported (Mahir & Rockwell 1996a,b), and further work would assist in understanding how flow patterns change depending on the amplitude, frequency, and direction of the cylinder motion, providing fluid dynamic bases for various theories of fluid-structure interaction (Price 1995). This work would further benefit the understanding of flow-induced vibrations, and the influence of the cylinder configuration thereon. The possibility of feedback control of vortex shedding from multiple cylinders could also be examined, as has been the case for a single circular cylinder (Warui & Fujisawa 1996).

In terms of more applied research concerned with multiple cylinders in cross-flow, efforts could be focused on specific geometrical arrangements, and modifications to the cylinders, with the aim of drag reduction, lift reduction, turbulence generation and

suppression, vortex shedding enhancement and suppression, and separation and flow directional control, for example. The base-bleed effect, noted by Bearman & Wadcock (1973) for two side-by-side cylinders, and identified in this study for different arrangements of two staggered cylinders (in Section 9.4), could further be explored in its drag suppression role (and particularly for the staggered cylinder arrangement). Zdravkovich (1981) examined a number of devices aimed at vortex shedding control or suppression, for a single circular cylinder, for example, but their effectiveness in multiple cylinder systems has not yet been studied.

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Appendix A

Seeding Particle Dynamics

A.1 Introduction

The accuracy of the measurements obtained from particle image velocimetry (PIV) is greatly dependent on the choice of the seeding particles, and whether or not the motion of the seeding particles correctly corresponds to the actual motion of the fluid. When choosing the seeding particles, it is important to consider the size, shape and specific gravity of the particles, as well as the particle's light reflection properties. In this study, some white, granular, irregularly shaped seeding particles were chosen, with an estimated size range of $d_p = 100$ to $500\text{ }\mu\text{m}$, and a specific gravity of $s.g. = 1.03$. This specific gravity is close to unity, meaning that the particles are nearly neutrally buoyant when immersed in the water of the towing tank, which has a density of $\rho_f = 1000\text{ kg/m}^3$ and a kinematic viscosity of $\nu_f = 1.01 \times 10^{-6}\text{ m}^2/\text{s}$.

The choice of the granular seeding particles was made primarily based on their high degree of light reflection when compared to the other seeding particles tested (further discussion on the different seeding particles tested can be found in Section 5.4.3). The greater light reflection of the chosen particles can be attributed to their large size, white colour and irregular shape. The irregular shape of the particle means there are more surfaces for the incident light to reflect 90° to the camera, when compared to a spherical particle.

In the brief analysis which follows, the dynamics of the chosen seeding particles are examined. Flow velocities in the experiments typically ranged from $u_f = 0$ to 150 mm/s, and if the greatest acceleration which would be experienced is that of a nearly impulsive start, then for $A_p = 0.4$, $U = 100$ mm/s, and $D = 25.4$ mm, the maximum acceleration in the flow would be $du_f/dt = 150$ mm/s². Frequencies in the flow include vortex shedding at $f = 0.5$ to 1.0 Hz and shear layer frequencies perhaps an order of magnitude higher. The particles will have the most difficulty accurately responding to the highest frequencies in the flow, and thus the Stokes number of interest is $N_s = 0.4$, where $N_s = [v_f / (2 \pi f d_p^2)]^{1/2}$. The flow also has streamwise velocity gradients in the region behind the towed cylinder, estimated at $du_f/dx = 2$ s⁻¹, and cross-stream velocity gradients across the shear layer, estimated at $du_f/dy = 10$ to 20 s⁻¹.

A.2 Particle Dynamics and Performance

To simplify the particle dynamics, the following assumptions are made: (i) the particle may be considered spherical; (ii) Stokes Law is applicable, based on the relative velocity between the particle and the fluid; (iii) the only force acting on the particle is the drag force; (iv) the effects of fluid acceleration, pressure gradients, and streamwise velocity gradients may be neglected; (v) the effects of cross-stream velocity gradients and lift forces may be neglected; (vi) interactions between particles, and between particles and the cylinder surface, may be neglected; and, (vii) the particle may be assumed to be nearly neutrally buoyant. Obviously, many of these assumptions are strictly invalid, and the effects of nonsphericity are likely significant (Emrich 1981); however, these assumptions allow an order-of-magnitude estimate of the particle dynamics to be obtained in a simple manner.

Based on the above assumptions, Stokes' law for particle acceleration du_p/dt and particle specific gravity $s.g.$ (Merzkirch 1987) can be written as

$$\frac{du_p}{dt} = \frac{18\nu_f}{d_p^2} \left(\frac{1}{s.g.} \right) (u_f - u_p). \quad (A.1)$$

Using (A.1), estimates of the velocity tracking error can be obtained, which is the difference between the fluid velocity and the particle velocity, $(u_f - u_p)$. Considering the case of a seeding particle which is initially at rest, and then subjected to a particle acceleration of $du_p/dt = 150 \text{ mm/s}^2$, to reach a final particle velocity of $u_p = 100 \text{ mm/s}$, for $100 \text{ }\mu\text{m}$ and $300 \text{ }\mu\text{m}$ particles the tracking errors will be 0.1 mm/s (0.1%) and 0.8 mm/s (0.8%), respectively. These errors are both less than 1%, meaning that the seeding particles accurately follow the fluid motion.

Another estimate of the performance of the seeding particles may be found from the solution to (A.1), to estimate the particle response time (Merzkirch 1987). The solution is of the form

$$u_p = u_f (1 - \exp(-Kt)), \quad (\text{A.2})$$

where

$$K = \frac{18\nu_f}{d_p^2} \left(\frac{1}{s.g.} \right). \quad (\text{A.3})$$

Using (A.2) and (A.3), seeding particles of $100 \text{ }\mu\text{m}$ and $300 \text{ }\mu\text{m}$, that are initially at rest, should attain 95% of the fluid velocity within 2 ms and 15 ms, respectively. The response time for the $100 \text{ }\mu\text{m}$ particles is acceptable, as it is comparable to the laser pulse duration time and the time between pulses, for the PIV system, each on the order of 5 ms (see Sections 5.4.2 and 5.4.4 for further information on the PIV illumination and imaging systems). The response time for the larger particles, however, is slower than desired.

A third estimate of the seeding particle performance is found from the sedimentation rate, which is simply defined as the settling velocity of the particles under the action of gravity. Since the seeding particles are not strictly neutrally buoyant, but heavier than water, the sedimentation velocity (Merzkirch 1987) is found to be

$$w_s = \left(\frac{gd_p^2}{18\nu_f} \right) (s.g. - 1). \quad (\text{A.4})$$

For a $300 \text{ }\mu\text{m}$ seeding particle, the sedimentation rate is found from (A.4) as $w_s = 1.5 \text{ mm/s}$. This corresponds to a drop of 0.015 mm in the 10 ms time difference between

images comprising a pair. Since this distance is considerably less than the laser light sheet thickness of 3 to 5 mm, the effects of out-of-plane particle motion, due to sedimentation (as opposed to velocities normal to the light sheet, in three-dimensional flows), are minimal.

Based on this analysis, the seeding particles chosen for the experiments are acceptable in terms of the tracking error and the sedimentation rate, although the response time for the larger particles is high. In effect, the choice of particles was a reasonable compromise between accurate representation of the fluid motion and the need for high reflectivity of the incident light.

Tables and Figures

Researchers	Re	L/D	Facility	Technique	AR	Blockage	TI _u	Measurements
Arie <i>et al.</i> (1983)	1.57×10^5	2.0-10.0	Wind tunnel	Pressure	11	9%	0.3%	a,b,c,g,h,n
Hiwada <i>et al.</i> (1982)	15,000-80,000	1.0-6.0	Wind tunnel	FV, CTA, pressure	8, 20	7, 13%	0.2%	a,f,g,i,p,q
Hori (1959)	200-12,000	1.2-3.0	Wind tunnel	CTA, pressure	120	1%	0.03%	a,b,d,k
Huhe-Aode <i>et al.</i> (1985)	100, 300, 1000	1.5-10.0	Towing tank	FV, CTA	30	2%	not given	a
Igarashi (1981)	8700-52,000	1.0-5.0	Wind tunnel	FV, CTA, pressure	4	6%	0.6%	a,b,c,f
Igarashi (1984)	$11,500-1.03 \times 10^5$	1.0-1.5	Wind tunnel	FV, CTA, pressure	3	13%	0.6%	a,b,c,f
Ishigai <i>et al.</i> (1972, 1973)	1500-15,000	1.0-5.0	Wind tunnel	FV, CTA, pressure	11	9%	not given	a,b
Jendrzeczyk & Chen (1986)	$15,000-1.5 \times 10^5$	1.4-10.0	Water tunnel	Force balance	12	9%	1.0-11.0%	a,g,h
Kiya <i>et al.</i> (1980)	15,800	1.0-5.5	Wind tunnel	CTA	21	2%	0.8%	a
Kiya <i>et al.</i> (1992)	20,000-37,000	2.5-4.0	Wind tunnel	CTA, pressure	11	10%	0.1-10.0%	o
Kostic & Oka (1972)	13,000-40,000	1.6-9.0	Wind tunnel	Pressure, therm.	5	20%	2.8%	b,f,j,q
Kwon <i>et al.</i> (1996)	17,800	1.0-5.0	Water channel	PIV	11	2%	2.5%	l
Lee & Panagakos (1997)	24,000-51,000	2.0, 3.2	Wind tunnel	FV, CTA, pressure	20	5%	0.35%	a,b,c,i,j,n
Ljungkrona & Sundén (1993)	3300-40,000	1.25-4.0	Wind tunnel	FV, pressure	16	6%	0.1%	a,b,c
Ljungkrona <i>et al.</i> (1991)	20,000	1.25-5.0	Wind tunnel	FV, pressure	16	6%	0.1-3.2%	a,b,c,f
Nishimura <i>et al.</i> (1986)	800-10,000	1.2-7.2	Water tunnel	FV	5	9%	0.6%	p
Oka <i>et al.</i> (1972)	2950-40,000	1.0-40.0	Wind tunnel	CTA, pressure, therm.	5-34	3-20%	0.7-26.0%	a,b,f,q
Okajima (1979)	$40,000-6.2 \times 10^5$	1.1-6.3	Wind tunnel	FV	7	8%	0.1%	a,f,i,j
Sun <i>et al.</i> (1992)	$3.25 \times 10^5, 6.5 \times 10^5$	2.2-4.0	Wind tunnel	Pressure	15	5%	0.12%	b,c
Wu <i>et al.</i> (1994b)	1000	3.0-7.0	Water tunnel	FV, pressure	24	4%	0.1%	-
Wu <i>et al.</i> (1994b)	17,000-40,000	3.0-7.0	Wind tunnel	CTA, pressure	20	5%	0.15%	a,n
Zdravkovich & Pridden (1977)	60,000	2.5-7.0	Wind tunnel	CTA, pressure	33	2%	0.1%	b,d,e,f,j
Zhang & Melbourne (1992)	1.11×10^5	2.0-10.0	Wind tunnel	Pressure, force balance	8	5%	0.4-11.5%	b,c,f,g,h,o
Current Study	850, 1350	1.0-5.0	Water tunnel	FV, CTA	16	6%	0.5%	a
Current Study	1900	1.0-4.0	Towing tank	PIV, CTA	27	4%	-	a,l,m

Table 3.1. Experimental investigations of two tandem circular cylinders of equal diameter in steady upstream cross-flow. Measured quantities: (a) Strouhal number, (b) mean surface pressure distribution, (c) r.m.s. surface pressure distribution, (d) base pressure coefficient, (e) gap pressure coefficient, (f) mean drag coefficient, (g) r.m.s. drag coefficient, (h) r.m.s. lift coefficient, (i) separation angle, (j) reattachment angle, (k) time-averaged velocity profiles, (l) velocity field, (m) vorticity field, (n) spanwise coherence or correlation measurements, (o) time intervals for bistable flow, (p) Sherwood number, (q) Nusselt number. Abbreviations: FV = flow visualization; CTA = constant temperature anemometry; PIV = particle image velocimetry; therm. = thermocouple.

Researchers	Re	T/D	Facility	Technique	AR	Blockage	TI _u	Measurements
Bearman & Wadcock (1973)	25,000	1.0-7.0	Wind tunnel	FV	31	4%	0.2%	a,b,c,d,e,k
Hori (1959)	200-12,000	1.2-3.0	Wind tunnel	CTA, pressure	120	1%	0.03%	a,b,c,h
Ishigai <i>et al.</i> (1972)	1500-15,000	1.25-3.0	Wind tunnel	FV, CTA, pressure	11	18%	not given	a
Jendrzejczyk & Chen (1986)	15,000-1.5×10 ⁵	1.35, 2.7	Wind tunnel	Force balance	12	17%	1.0-11.0%	a,f,g
Kamemoto (1976)	662	1.5-3.0	Towing tank	FV, CTA	75	3%	-	a
Kamemoto (1976)	30,000	1.0-3.0	Wind tunnel	CTA	5	13%	not given	a
Kim & Durbin (1988)	3300	1.0-3.0	Wind tunnel	FV, CTA, pressure	27	3%	0.2%	a,c,i
Kiya <i>et al.</i> (1980)	15,800	1.0-5.5	Wind tunnel	CTA	21	2%	0.8%	a
Kwon <i>et al.</i> (1996)	17,800	1.1-3.0	Water channel	PIV	11	2%	2.5%	i
Le Gal <i>et al.</i> (1990)	110	1.0-7.5	Water tunnel	FV	50	4%	1.0%	-
Peschard & Le Gal (1996)	90-150	1.0-6.0	Water tunnel	FV	7	not given	1.0%	-
Spivack (1946)	5000-93,000	1.0-6.0	Wind tunnel	CTA	39	5%	0.02%	a,h
Sun <i>et al.</i> (1992)	3.25×10 ³ -6.5×10 ⁵	2.2	Wind tunnel	Pressure	15	10%	0.12-10.0%	b,c
Williamson (1985)	50-150	1.0-6.0	Wind tunnel	FV	279	1%	not given	a
Williamson (1985)	200	1.0-6.0	Water tunnel	FV	13	8%	not given	a
Zdravkovich & Pridden (1977)	60,000	1.0-3.0	Wind tunnel	Force balance	33	5%	0.1%	d,e
Current study	500-3000	1.0-6.0	Water tunnel	FV, CTA	16	13%	0.5%	a
Current study	1200-3000	1.0-3.0	Towing tank	PIV	27	7%	-	i,j

Table 3.2. Experimental investigations of two side-by-side circular cylinders of equal diameter in steady upstream cross-flow. Measured quantities: (a) Strouhal number, (b) mean surface pressure distribution, (c) base pressure, (d) mean drag coefficient, (e) mean lift coefficient, (f) r.m.s. drag coefficient, (g) r.m.s. lift coefficient, (h) time-averaged velocity profiles, (i) velocity field, (j) vorticity field, (k) spanwise coherence or correlation measurements. Abbreviations: FV = flow visualization; CTA = constant temperature anemometry; PIV = particle image velocimetry. Blockage ratio based on total blockage for two cylinders.

Researchers	Re	T/D	Facility	Technique	AR	Blockage	TI_{η}	Measurements
Eastop & Turner (1982)	45,000-111,000	1.2-2.6	Wind tunnel	CTA, pressure	14	21%	0.7%	a,c,d,f
Kumada <i>et al.</i> (1984)	10,000-32,000	1.0-3.75	Wind tunnel	FV, CTA, pressure	15	20%	0.2%	a,b,c,d,e,g
Current study	500-3000	1.0-3.0	Water tunnel	FV, CTA	16	19%	0.5%	a
Current study	1200-3000	1.0-1.5	Towing tank	PIV	27	11%	-	h,i

Table 3.3. Experimental investigations of three side-by-side circular cylinders of equal diameter in steady upstream cross-flow. Measured quantities: (a) Strouhal number (b) mean surface pressure distribution, (c) base pressure, (d) mean drag coefficient, (e) mean lift coefficient, (f) separation angle, (g) time averaged velocity profiles, (h) velocity field, (i) vorticity field. Abbreviations: FV = flow visualization; CTA = constant temperature anemometry; PIV = particle image velocimetry. Blockage ratio based on total blockage for three cylinders.

Researchers	Re	Geometry	Facility	Technique	AR	Blockage	TI _h	Measurements
Bokaian & Geoola (1984)	1500-24,000	$L/D = 1.5-4.0$ $T/D = 0.0-6.0$	Water channel	Force balance	18	10%	6.5%	d,e
Ishigai <i>et al.</i> (1972, 1973)	1500-15,000	$L/D = 0.68-4.0$ $T/D = 0.5-3.0$	Wind tunnel	FV, CTA, pressure	11	9%	not given	a,b
Kiya <i>et al.</i> (1980)	20,000- 37,000	$P/D = 0.0-5.5$ $\alpha = 0^\circ-90^\circ$	Wind tunnel	CTA	11	19%	0.8%	a
Moriya & Sakamoto (1985)	65,300	$L/D = 2.0-6.0$ $T/D = 0.0-1.5$	Wind tunnel	Force balance, FV, pressure	10	20%	0.4%	a,b,c,d,e,f,g
Price (1976)	17,000- 80,000	$L/D = 6.0-18.0$ $T/D = 0.0-2.42$	Wind tunnel	Force balance, FV, pressure	37, 42	5-12%	1.0-11.0%	b,d,e
Price & Paidoussis (1984)	17,000- 86,000	$L/D = 1.5-5.0$ $T/D = 0.75-2.0$	Wind tunnel	Force balance	24	6%	0.5%	d,e
Sun <i>et al.</i> (1992)	3.25×10^5 - 6.5×10^5	$P/D = 2.2$ $\alpha = 12.5^\circ$	Wind tunnel	Pressure	15	10%	0.12-10.0%	b,c
Suzuki <i>et al.</i> (1971)	1300	$P/D = 2.0$ $\alpha = 0^\circ-15^\circ$	Water tunnel	FV	not given	not given	not given	-
Suzuki <i>et al.</i> (1971)	1×10^5 - 6.3×10^5	$P/D = 1.1-3.9$ $\alpha = 0^\circ-15^\circ$	Wind tunnel	Pressure	6-18	12-24%	not given	b,d
Zdravkovich & Pridden (1977)	60,000	$L/D = 0.0-5.0$ $T/D = 0.0-3.0$	Wind tunnel	Force balance, pressure	33	5%	0.1%	b,d,e
Current Study	850, 1350	$P/D = 1.0-5.0$ $\alpha = 0^\circ-90^\circ$	Water tunnel	FV, CTA	16	13%	0.5%	a
Current Study	1900	$P/D = 1.0-4.0$ $\alpha = 0^\circ-90^\circ$	Towing tank	PIV	27	7%	-	h,i

Table 3.4. Experimental investigations of two staggered circular cylinders of equal diameter in steady upstream cross-flow. Measured quantities: (a) Strouhal number, (b) mean surface pressure distribution, (c) r.m.s. surface pressure distribution, (d) mean drag coefficient, (e) mean lift coefficient, (f) r.m.s. drag coefficient, (g) r.m.s. lift coefficient, (h) velocity field, (i) vorticity field. Abbreviations: FV = flow visualization; CTA = constant temperature anemometry; PIV = particle image velocimetry. Blockage ratio based on total blockage for two cylinders.

Designation	Full Name	Occurrence	Key Features
SBB	Single bluff-body	$L/D = 1.0$ to 1.5	Wrapped shear layers
SLR1	Shear layer reattachment, type 1	$L/D = 1.5$ to 2.5	Shear layer reattachment synchronized with Kármán vortex shedding
SLR2	Shear layer reattachment, type 2	$L/D = 2.5$ to 3.0	As above, with interaction across the base region of the downstream cylinder
SLR3	Shear layer reattachment, type 3	$L/D = 3.0$ to 3.5	As above, with shear layer instability vortices preceding reattachment
KVS	Kármán vortex shedding	$L/D > 3.5$	Kármán vortex shedding from the upstream cylinder

Table 10.1. Résumé of flow patterns identified in the present study for two tandem circular cylinders of equal diameter in steady upstream cross-flow, $Re = 600$ to 6000 .

Designation	Full Name	Occurrence	Key Features
SBB	Single bluff-body	$T/D = 1.0$ to 1.125	Single, low frequency Kármán vortex shedding process
BB	Base bleed	$T/D = 1.125$ to 1.25	Weak gap flow of variable deflection angle and a single vortex street
BF	Biased flow	$T/D = 1.25$ to 2.5	Gap flow deflected towards one of the cylinders
SVS	Synchronized vortex shedding	$T/D = 2.5$ to 4.5	Anti-phase synchronized Kármán vortex shedding

(a)

Designation	Full Name	Occurrence	Key Features
SBB	Single bluff-body	$T/D = 1.0$ to 1.125	Single, low frequency Kármán vortex shedding process
ABF	Asymmetric biased flow	$T/D = 1.125$ to 1.25	Three sizes of near-wake regions
SBF	Symmetric biased flow	$T/D = 1.25$ to 2.5	Gap flows symmetrically deflected outwards towards the outer cylinders
SVS	Synchronized vortex shedding	$T/D > 2.5$	Anti-phase synchronized Kármán vortex shedding

(b)

Table 10.2. Résumé of flow patterns identified in the present study for side-by-side cylinders of equal diameter in steady upstream cross-flow, $Re = 500$ to 3000 : (a) two cylinders; (b) three cylinders.

Designation	Full Name	Occurrence	Key Features
SBB1	Single bluff-body, type 1	$P/D = 1.0$ to 1.125 $\alpha = 0^\circ$ to 45°	Instabilities in the shear layer from the upstream cylinder
SBB2	Single bluff-body, type 2	$P/D = 1.0$ to 1.125 $\alpha = 45^\circ$ to 90°	Single, low frequency Kármán vortex shedding process
BB	Base bleed	$P/D = 1.125$ to 1.25 $\alpha = 45^\circ$ to 90°	Weak gap flow of variable deflection angle and a single vortex street
SLR	Shear layer reattachment	$P/D = 1.125$ to 4.0 $\alpha = 0^\circ$ to 20°	Reattachment of the inner shear layer from the upstream cylinder
IS	Induced separation	$P/D = 1.125$ to 3.0 $\alpha = 10^\circ$ to 30°	Narrow gap flow induces separation and vorticity generation on the inner surface of the downstream cylinder
VPE	Vortex pairing and enveloping	$P/D = 1.25$ to 3.5 $\alpha = 20^\circ$ to 45°	Pairing of gap vortices and enveloping of the pair by a Kármán vortex from the outer shear layer of the upstream cylinder
VPSE	Vortex pairing, splitting and enveloping	$P/D = 1.25$ to 4.0 $\alpha = 20^\circ$ to 90°	As above, but with incomplete enveloping of the gap vortex pair
SVS	Synchronized vortex shedding	$P/D = 1.5$ to 5.0 $\alpha = 15^\circ$ to 90°	Anti-phase synchronization of gap Kármán vortex shedding
VI	Vortex impingement	$P/D = 3.0$ to 5.0 $\alpha = 0^\circ$ to 20°	Impingement of Kármán vortices from the upstream cylinder onto the downstream cylinder

Table 10.3. Résumé of flow patterns identified in the present study for two staggered circular cylinders of equal diameter in steady upstream cross-flow, $Re = 850$ to 1900 . The observed ranges of pitch ratio and incidence represent the upper and lower limits for which the flow pattern is observed (see Figure 9.34(a)), and do not necessarily mean that the flow pattern will be observed for all combinations of P/D and α specified.

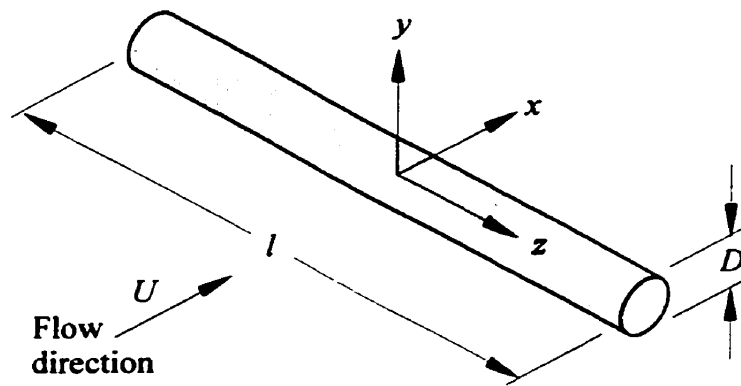


Figure 2.1. Circular cylinder in cross-flow.

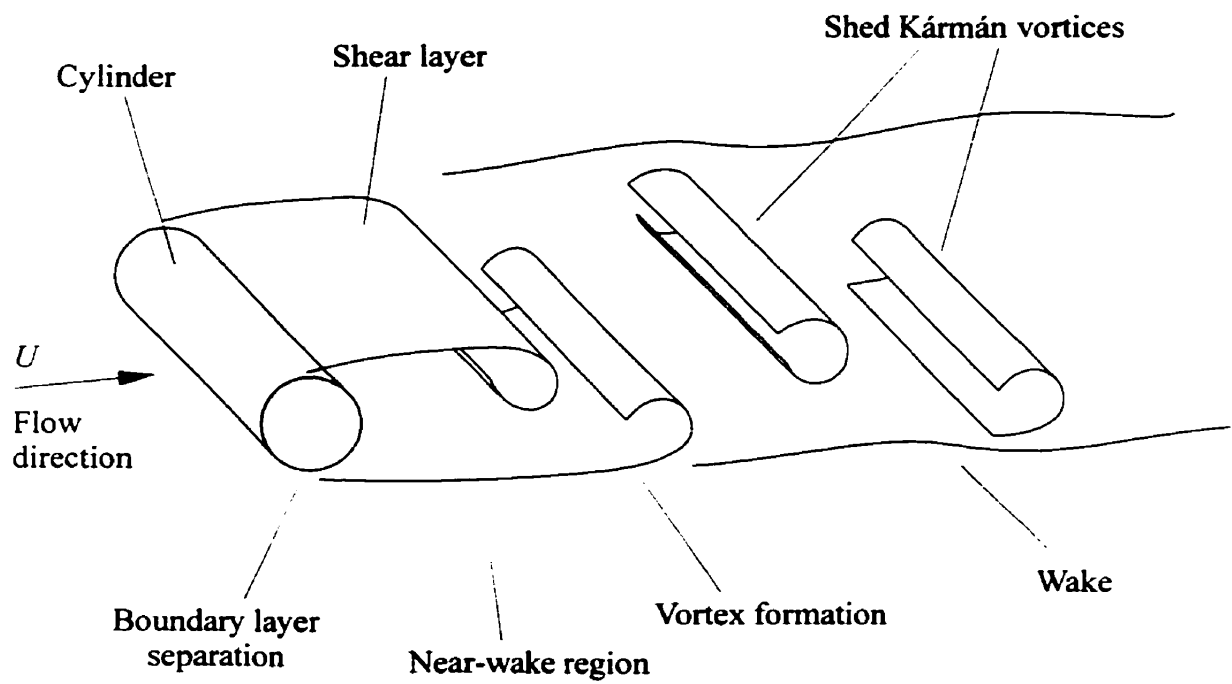


Figure 2.2. Periodic, alternate vortex shedding from a circular cylinder in steady cross-flow, at subcritical Reynolds numbers.

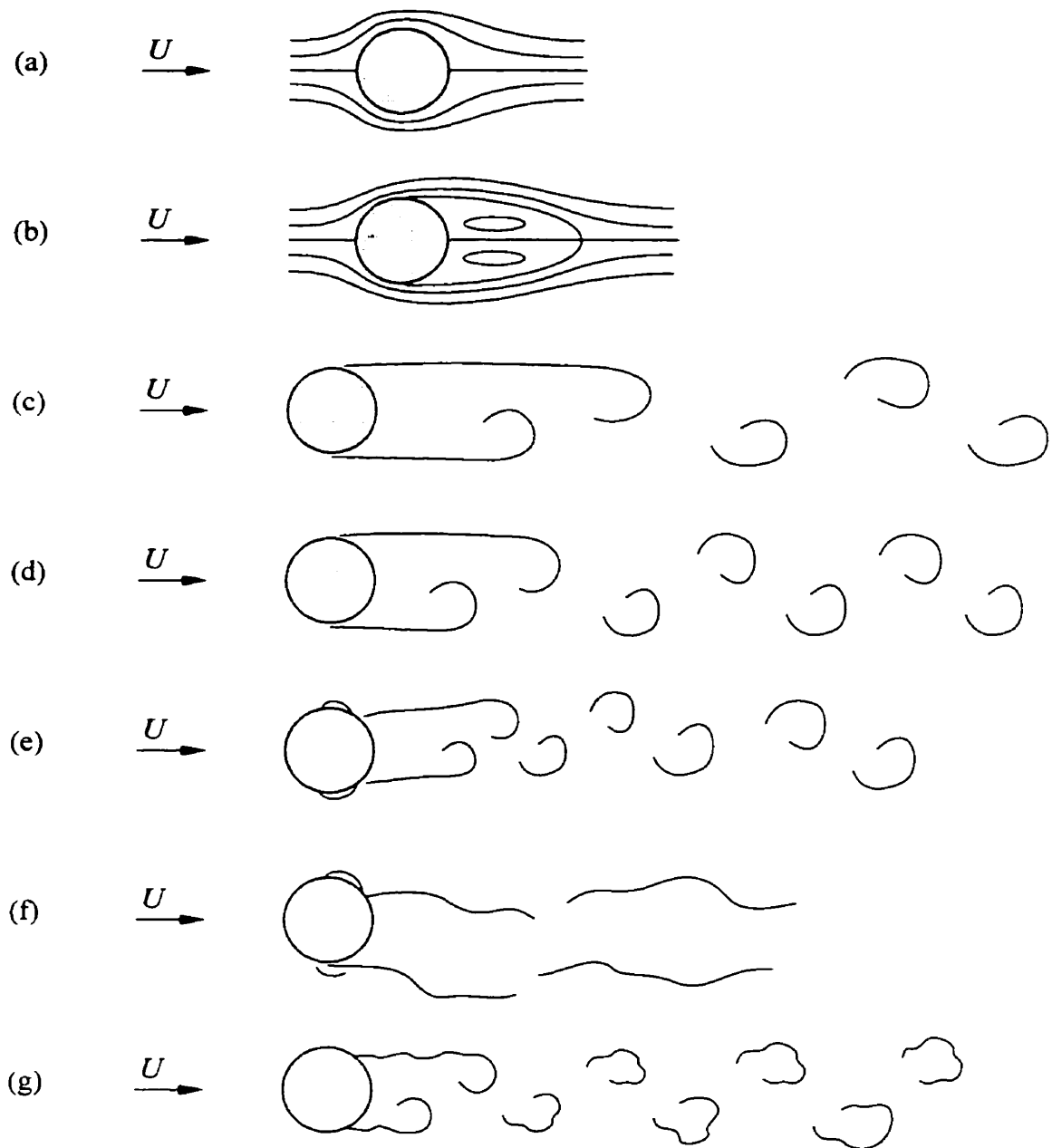
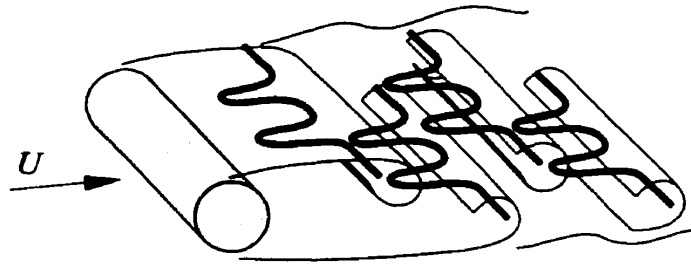


Figure 2.3. Reynolds number evolution for a circular cylinder in steady upstream cross-flow: (a) creeping flow, $Re < 5$; (b) attached recirculation zone, $Re = 5$ to 40 ; (c) low, transitional, and high Reynolds numbers, laminar and transitional vortex shedding, $Re = 40$ to 350 ; (d) subcritical regime, $Re = 350$ to 2×10^5 ; (e) critical regime, $Re = 2 \times 10^5$ to 7×10^5 ; (f) supercritical regime, $Re = 7 \times 10^5$ to 3.5×10^6 ; (g) transcritical or post-critical regime, $Re > 3.5 \times 10^6$. The Reynolds number ranges specified are approximate, since the upper and lower limits are sensitive to experimental conditions, and differ from study to study. The ranges are based on the review by Coutanceau & Defaye (1991), and other references.

Mode A
 $Re = 180$ to 260



Mode B
 $Re > 230$

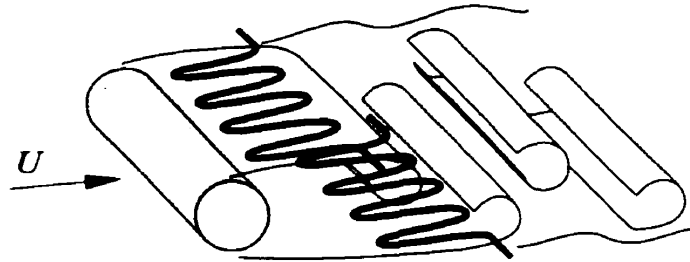
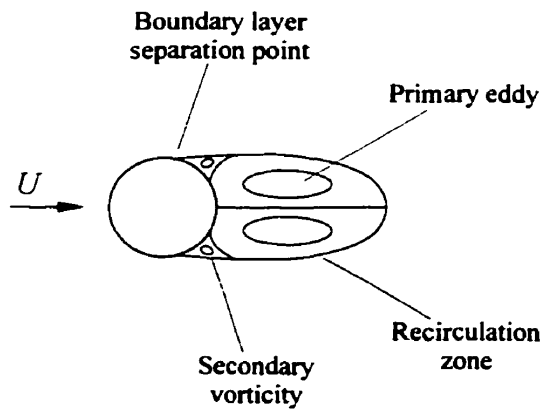
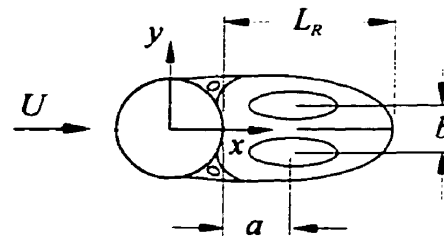


Figure 2.4. Three-dimensional vortical structures in the near-wake of a circular cylinder at transitional Reynolds numbers, with mode A vortex loops from the primary vortex cores, and mode B streamwise vortex pairs.



(a)



(b)

Figure 2.5. Circular cylinder impulsively set into motion: (a) features of the flow field; (b) notation used in the quantitative description.

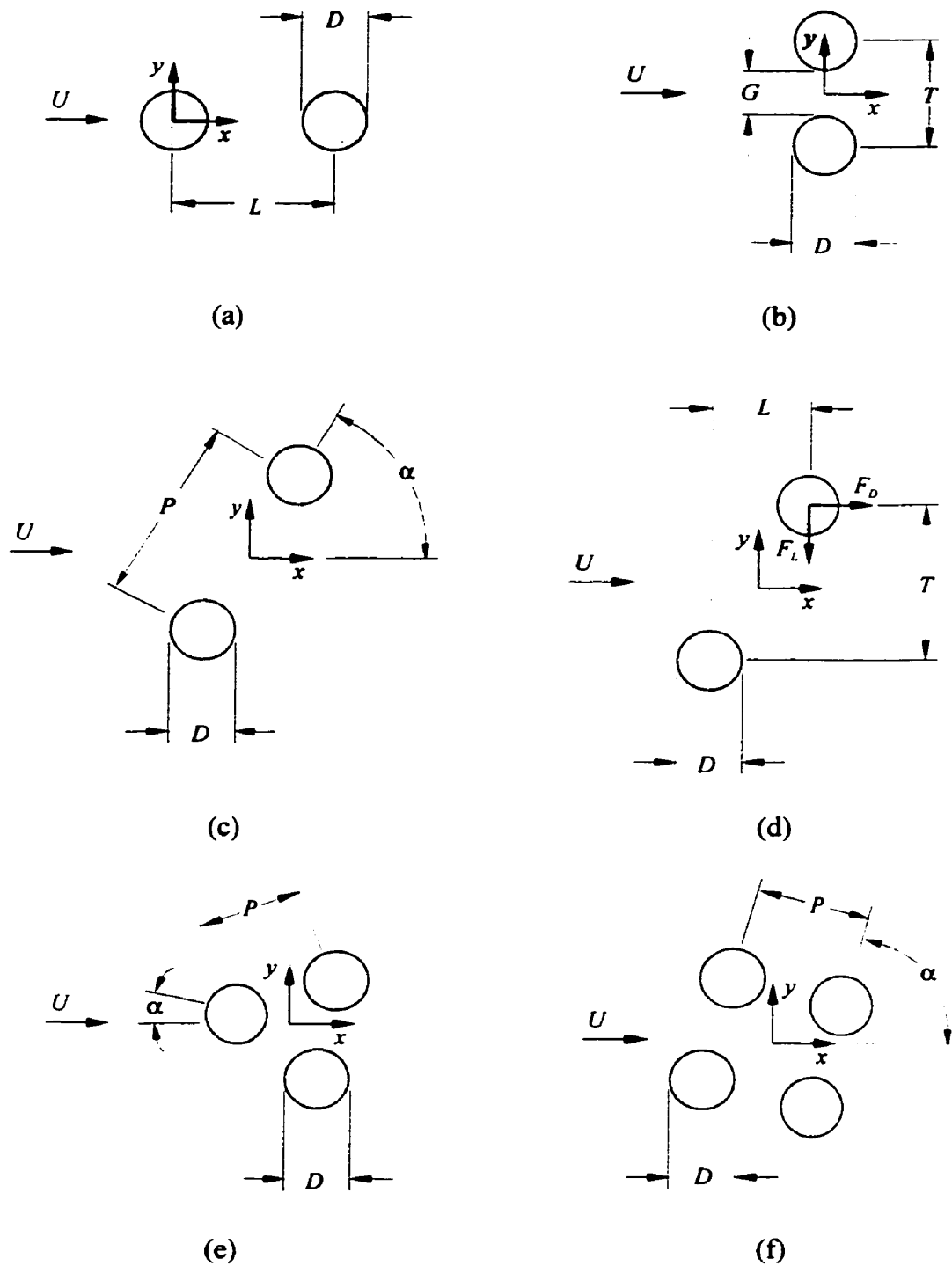
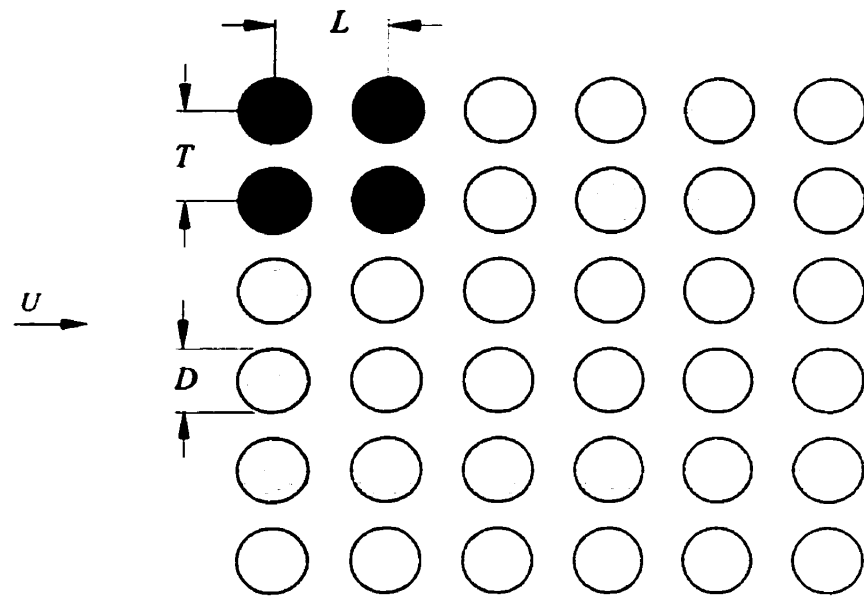
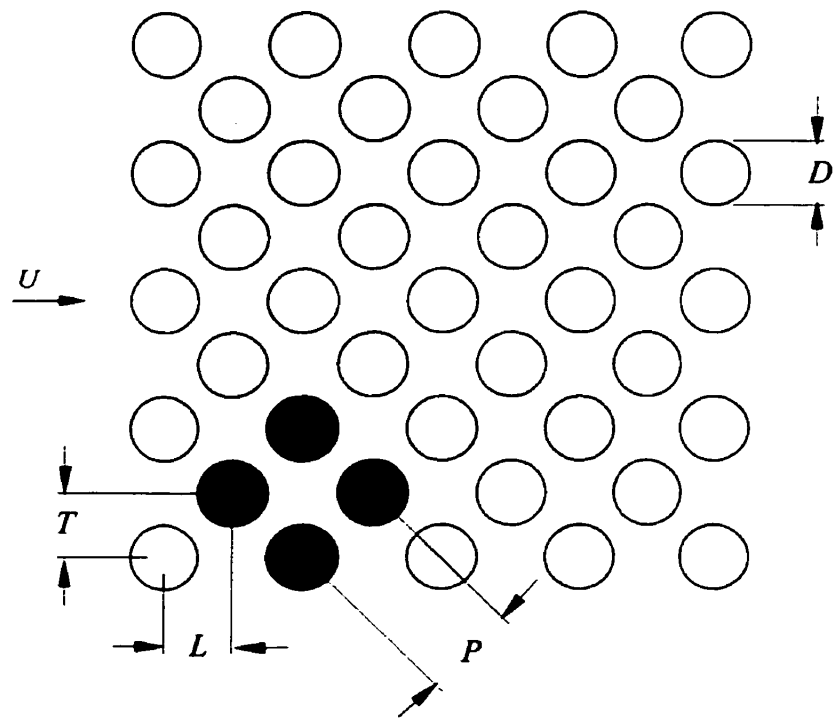


Figure 3.1. Circular cylinders of equal diameter in cross-flow: (a) tandem configuration; (b) side-by-side configuration; (c) staggered configuration; (d) staggered configuration with alternative nomenclature, showing the lift (directed inwards, towards the flow axis) and drag forces experienced by the downstream cylinder; (e) triangular cluster; (f) square cluster; (g) square array; (h) rotated square array; (i) normal triangular array; (j) parallel triangular array.



(g)



(h)

Figure 3.1 (continued).

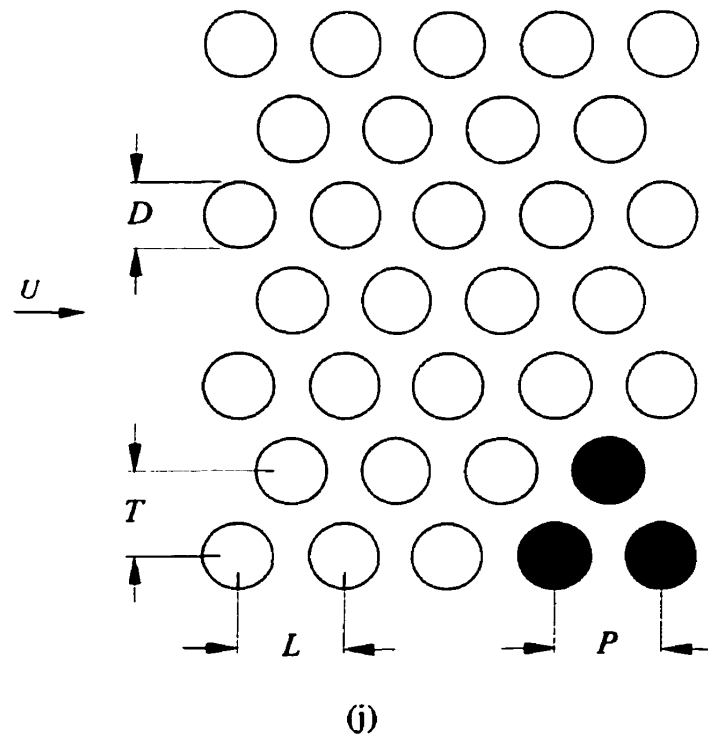
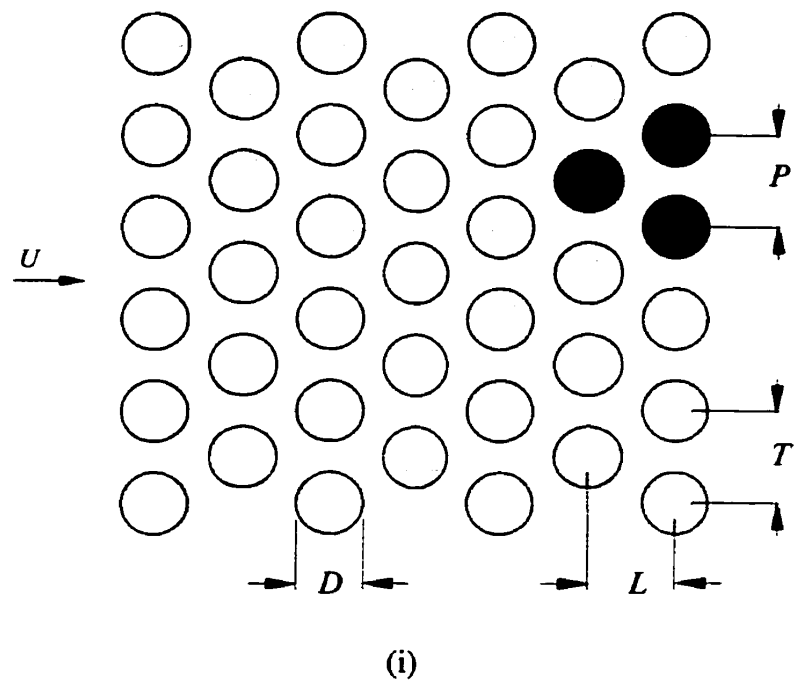


Figure 3.1 (continued).

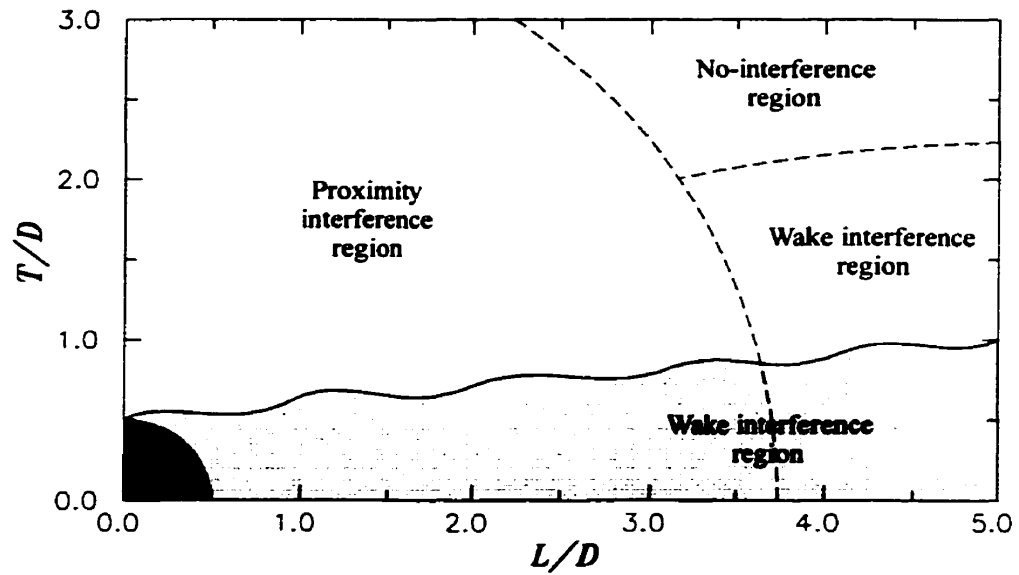


Figure 3.2. Wake interference and proximity interference boundaries for two staggered circular cylinders of equal diameter, in steady upstream cross-flow, based on Zdravkovich (1987). The proximity interference region also extends to negative values of L/D (not shown here).

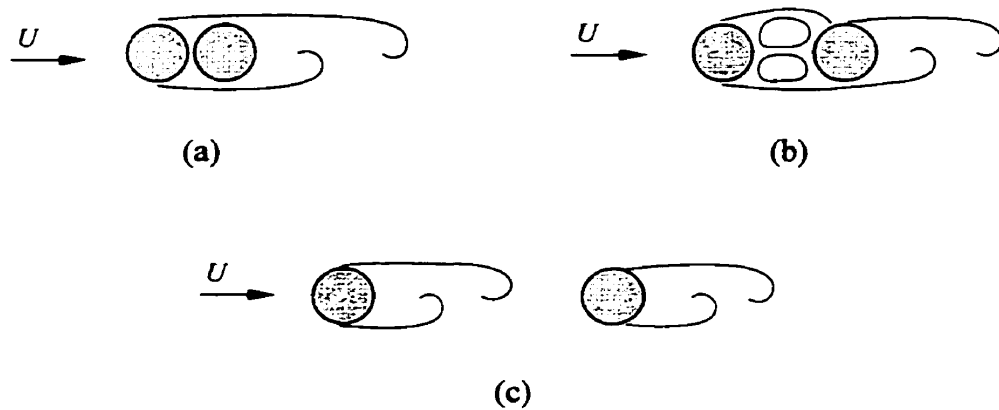


Figure 3.3. Two tandem circular cylinders in steady cross-flow: (a) single bluff-body behaviour at small L/D ; (b) shear layer reattachment at intermediate L/D , and (c) Kármán vortex shedding from each cylinder at large L/D .

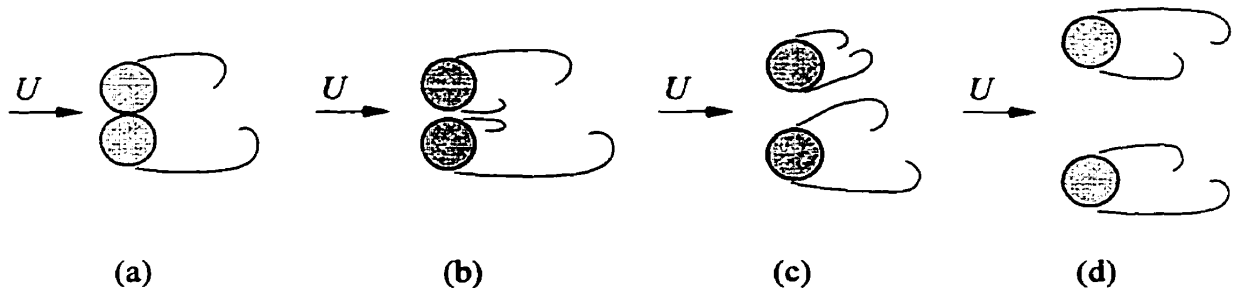


Figure 3.4. Two side-by-side circular cylinders in steady cross-flow: (a) single bluff-body behaviour when the cylinders are in contact; (b) base-bleed flow pattern at small T/D ; (c) biased flow pattern with synchronized vortex shedding at intermediate T/D ; (d) synchronized vortex shedding at large T/D .

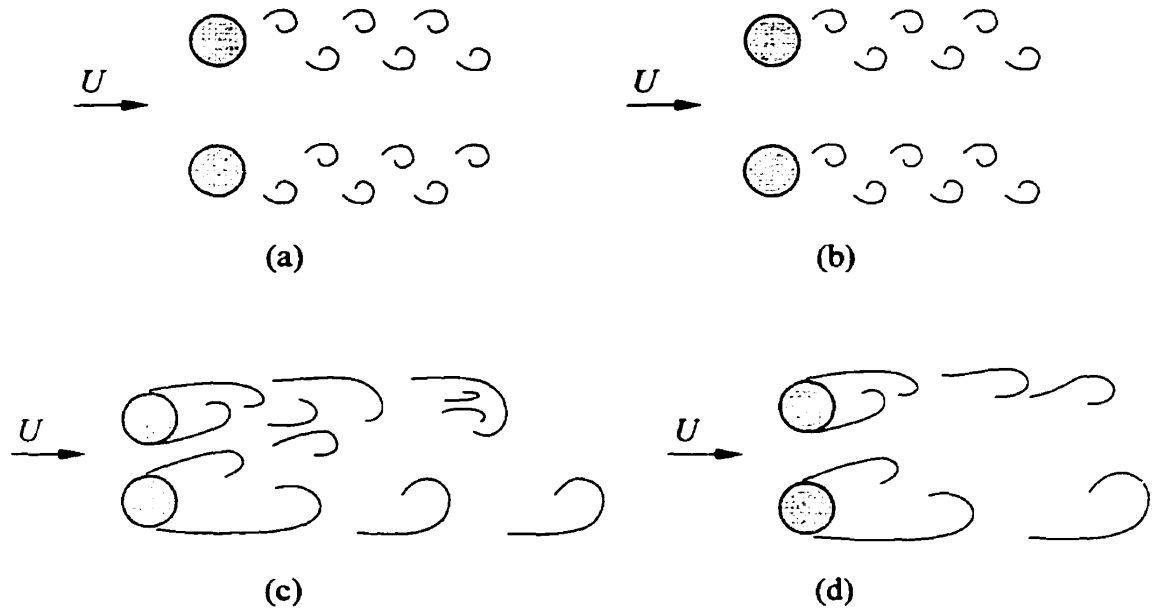


Figure 3.5. Vortex dynamics and interaction in the combined wake of two side-by-side cylinders in steady upstream cross-flow: (a) anti-phase synchronization of the Kármán vortex streets; (b) in-phase synchronization of the Kármán vortex streets; (c) first harmonic mode of vortex shedding; (d) second harmonic mode of vortex shedding.

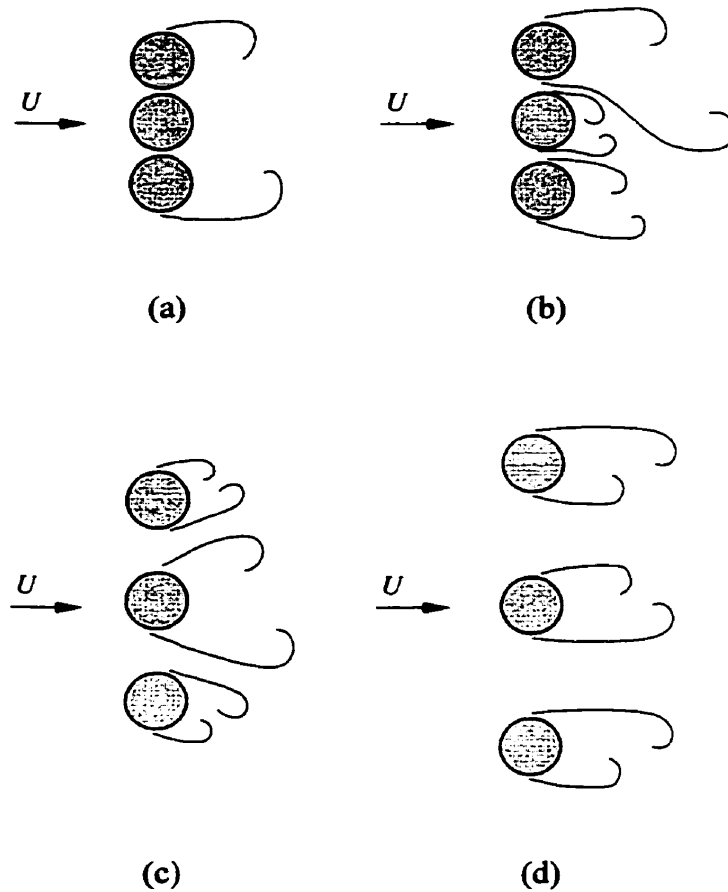


Figure 3.6. Three side-by-side circular cylinders in steady cross-flow: (a) single bluff-body behaviour at very small T/D ; (b) asymmetric biased flow pattern at small T/D , (c) symmetric biased flow pattern (likely with synchronized vortex shedding) at intermediate T/D , and (d) synchronized vortex shedding at large T/D .

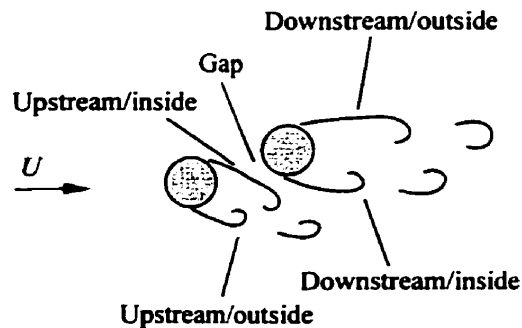


Figure 3.7. Shear layer and vortex designations for two staggered circular cylinders in steady upstream cross-flow.

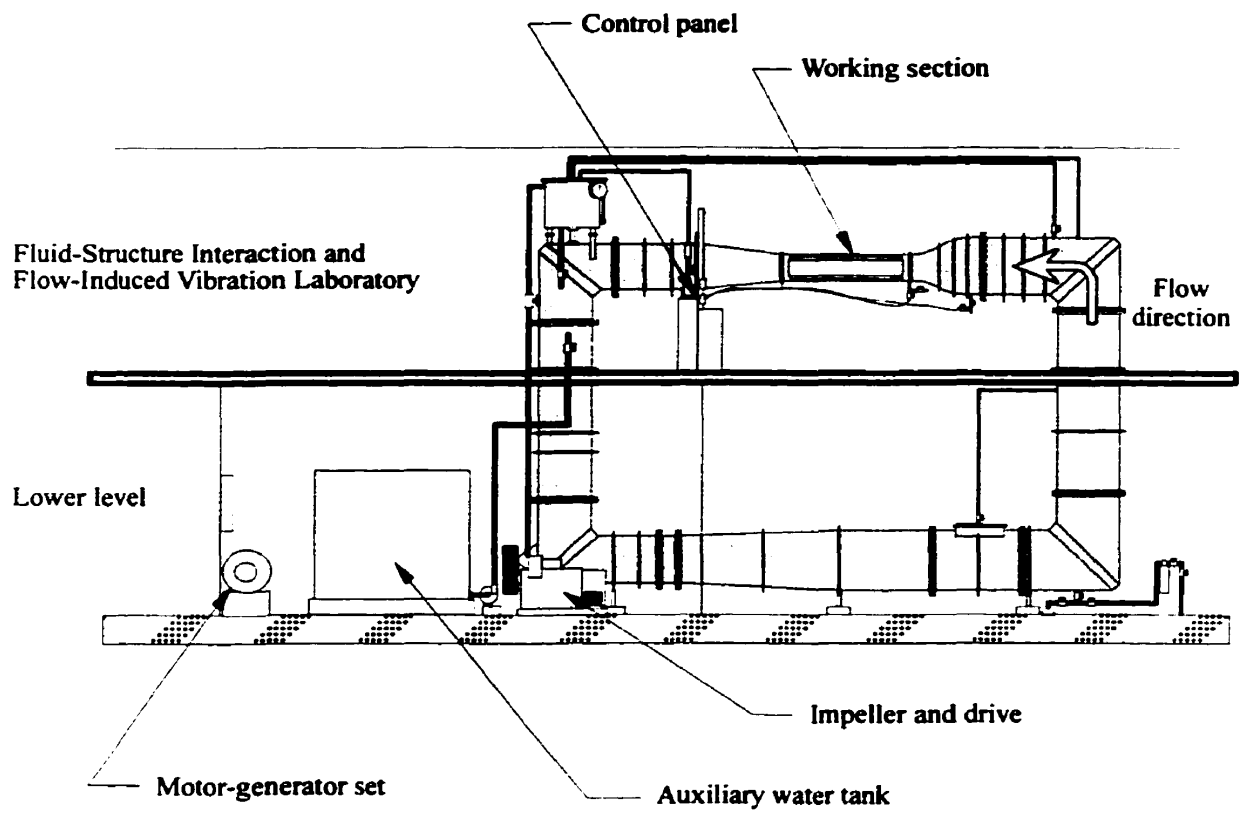


Figure 4.1. The Kempf & Remmers water tunnel.

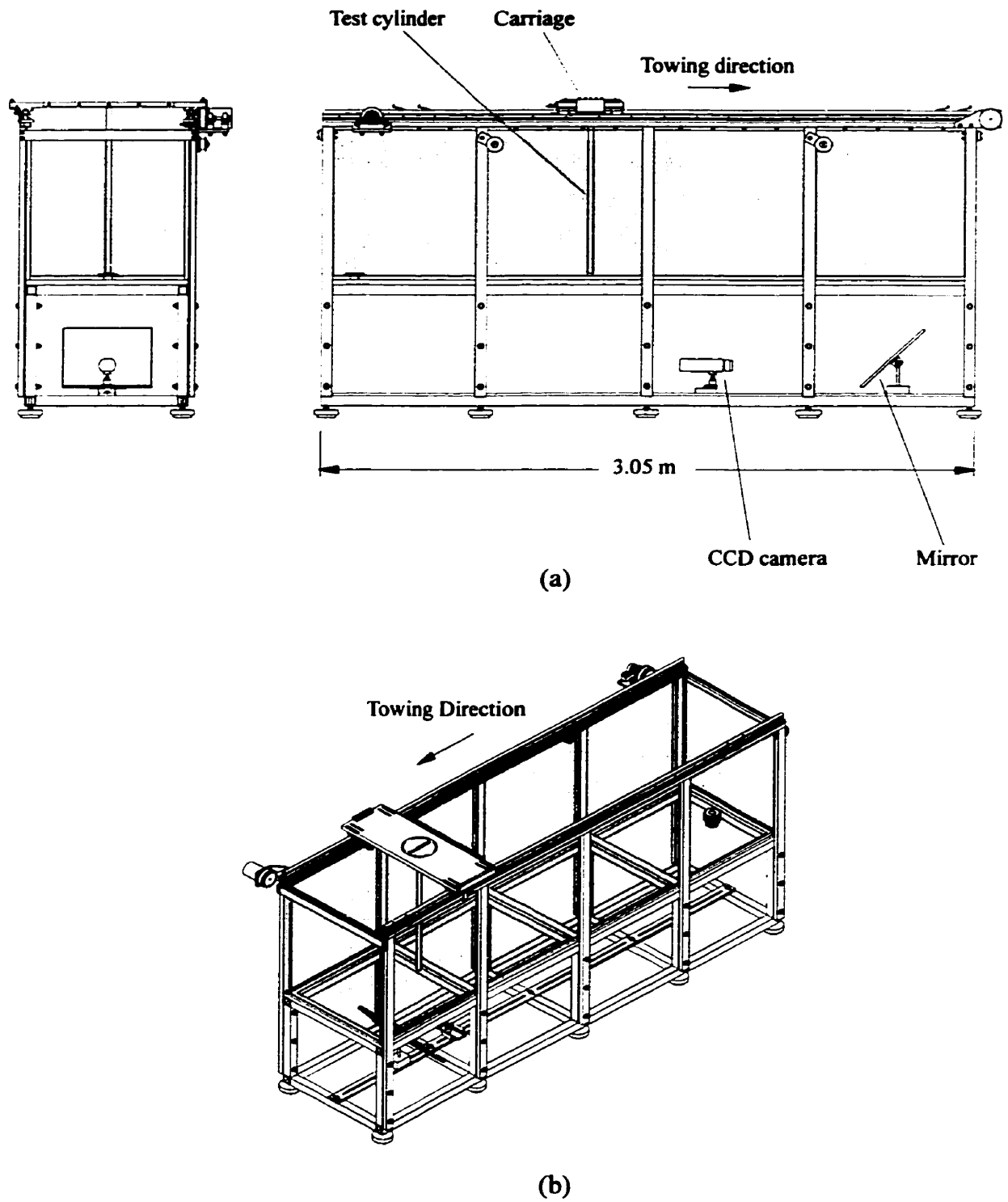


Figure 4.2. Main water towing tank facility: (a) end and side views shown with a single circular cylinder (without an end plate fitted) rigidly suspended vertically from the carriage; (b) isometric view.

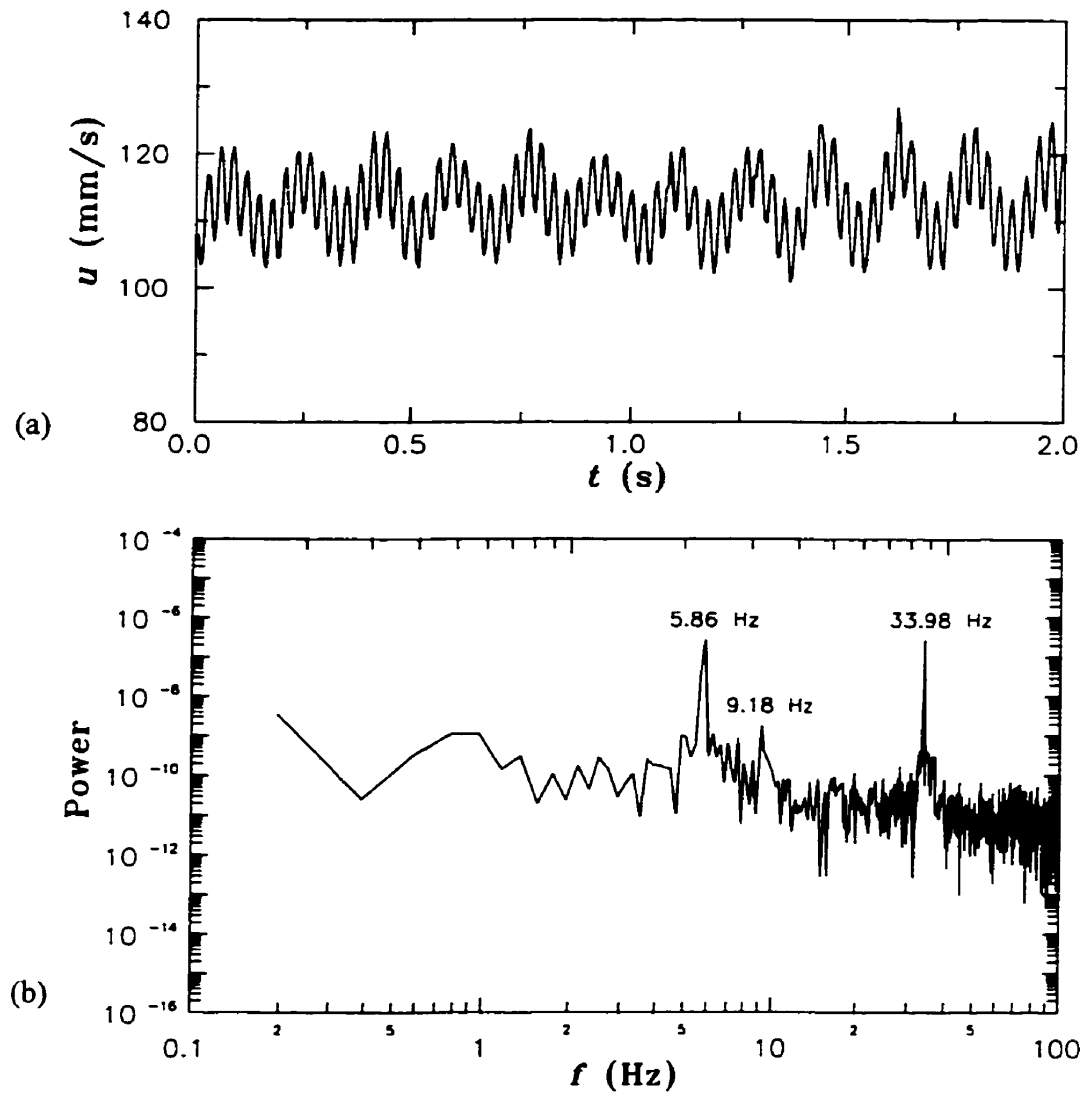


Figure 4.3. Apparent turbulence structure in the towing tank from a towed hot-film probe: (a) time history of streamwise (longitudinal) velocity component; (b) corresponding power spectrum. Sampling rate of 200 Hz, 1024 samples, mean towing velocity of 112 mm/s, $TI_u = 5.0\%$, $L_u = 1.10$ mm, $\lambda_u = 0.8$ mm.

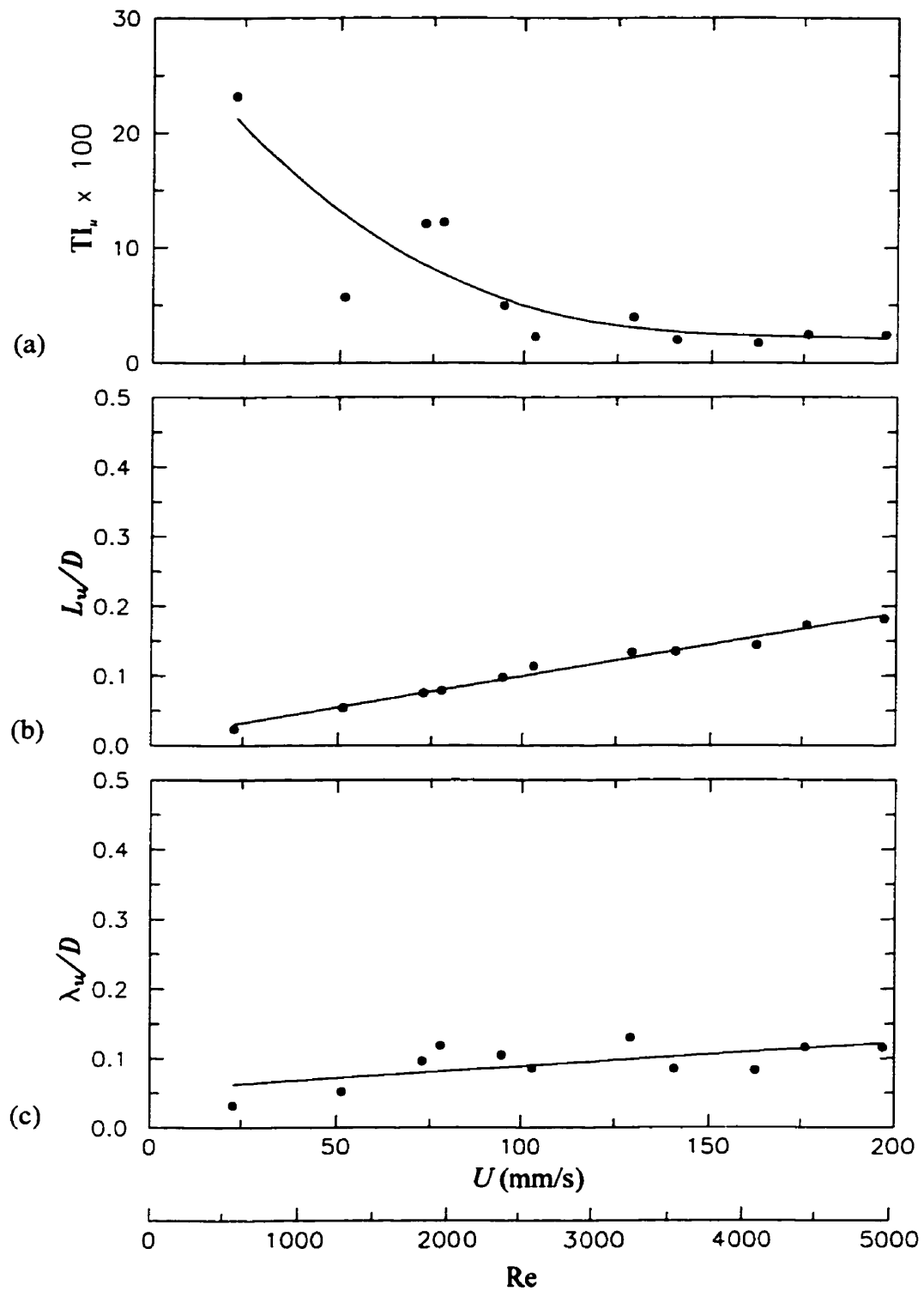


Figure 4.4. Structure of the apparent turbulence in the main towing tank (based on diameter D of the towed circular cylinder): (a) longitudinal turbulence intensity, (b) streamwise integral length scale, (c) streamwise length microscale. Lines added for illustration purposes only.

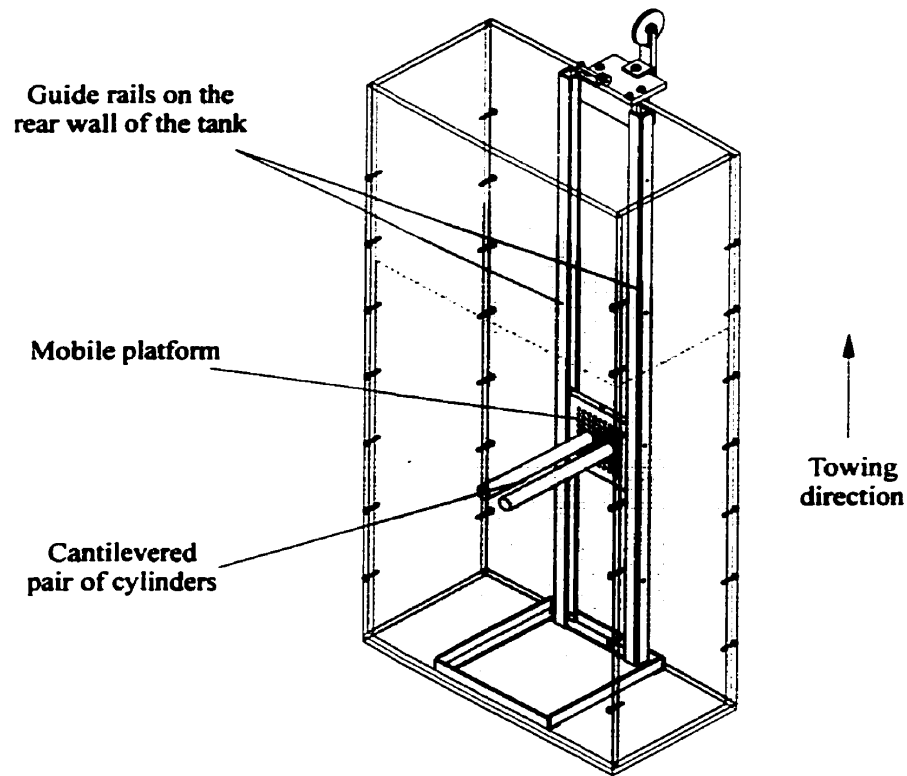


Figure 4.5. Vertical water towing tank, shown with two side-by-side circular cylinders mounted on the platform.

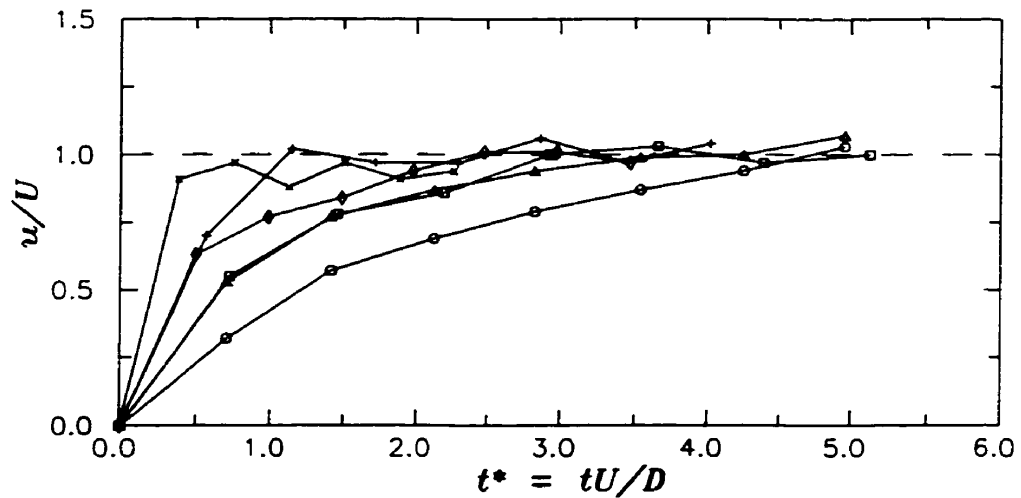


Figure 4.6. Some representative velocity-time profiles for a nearly impulsive start in the vertical towing tank: $\circ$, $A_p = 0.24$; $\triangle$, $A_p = 0.28$; $\square$, $A_p = 0.34$; $\diamond$, $A_p = 0.40$; $+$, $A_p = 0.87$; $\times$, $A_p = 1.32$. Lines added for illustration purposes only.

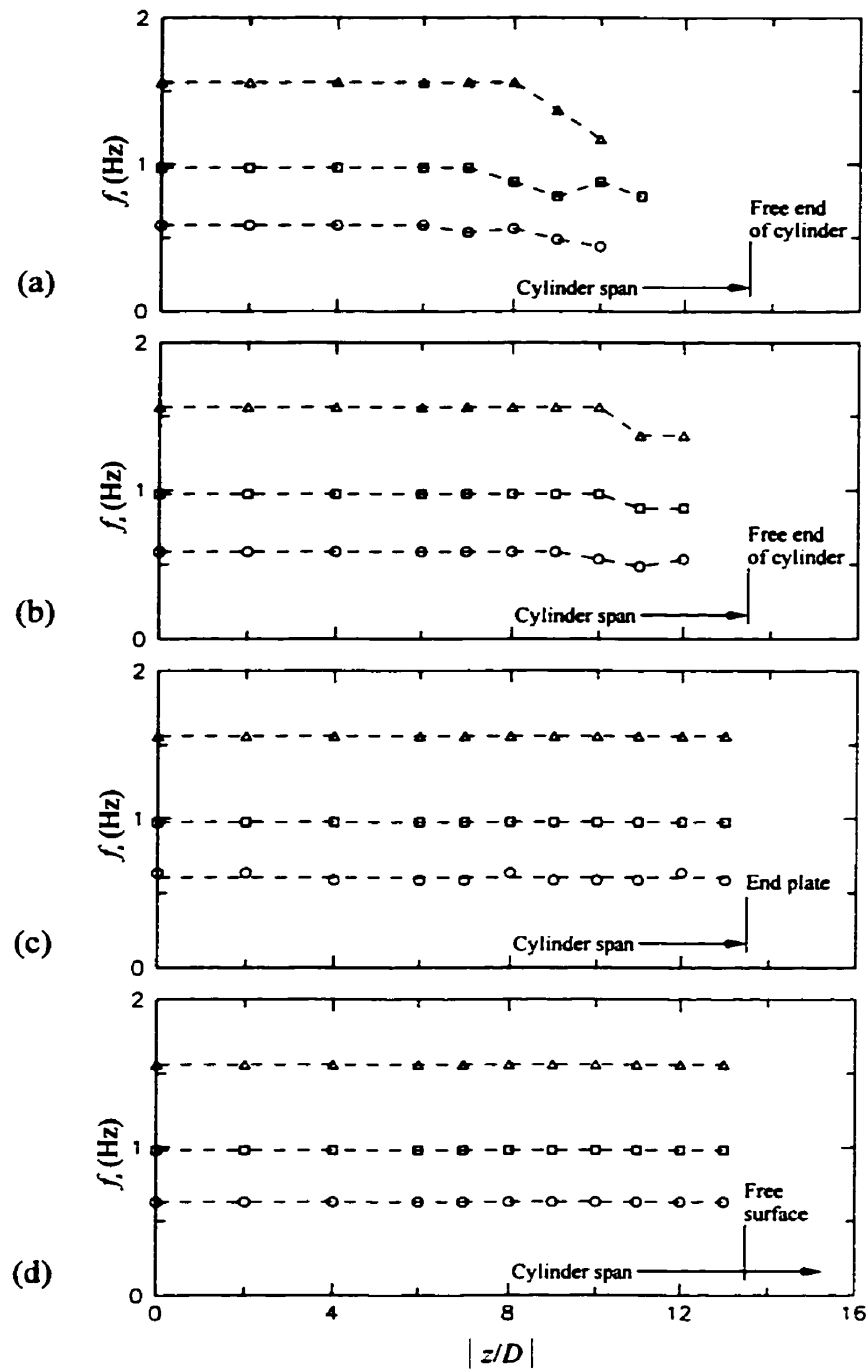


Figure 4.7. Spanwise measurements of the vortex shedding frequency, f_s , for a towed cantilevered circular cylinder, $AR = 27$ (centre span at $z/D = 0$): z measured (a) towards the free end with $g/D = 0.8$; (b) towards the free end with $g/D = 0.2$; (c) towards an end plate; (d) towards the free surface. $\circ$, $U = 75$ mm/s ($Re = 1900$); $\square$, $U = 125$ mm/s ($Re = 3100$); $\triangle$, $U = 200$ mm/s ($Re = 5000$). Measurement uncertainty in vortex shedding frequency estimated to be ± 10 to 20% ; measurement uncertainty in towing velocity estimated at $\pm 1\%$.

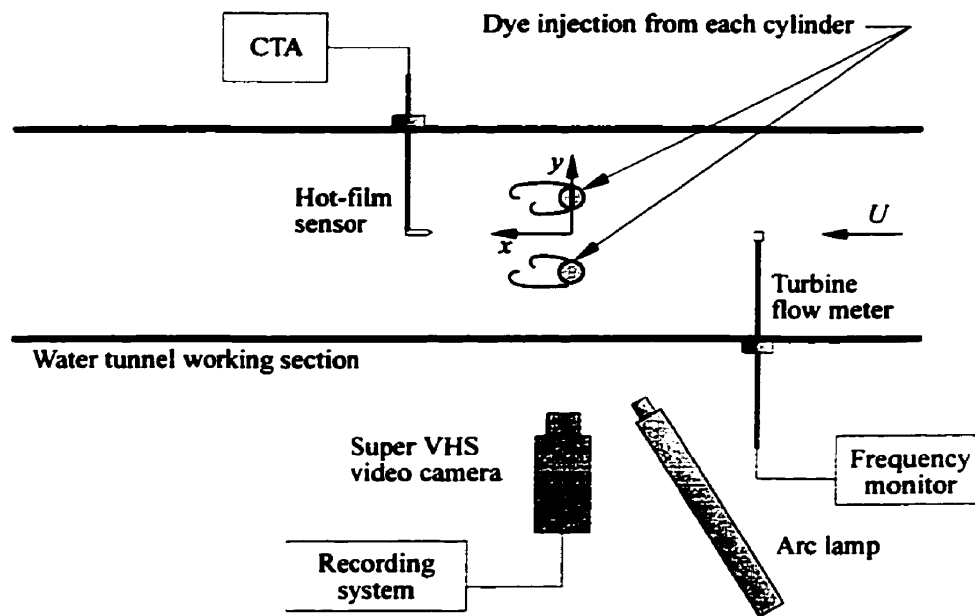


Figure 5.1. Water tunnel setup for flow visualization and hot-film anemometry. The turbine flow meter is used to measure the freestream velocity, and is retractable. A first surface mirror (not shown) is used with the video camera to visualize the flow.

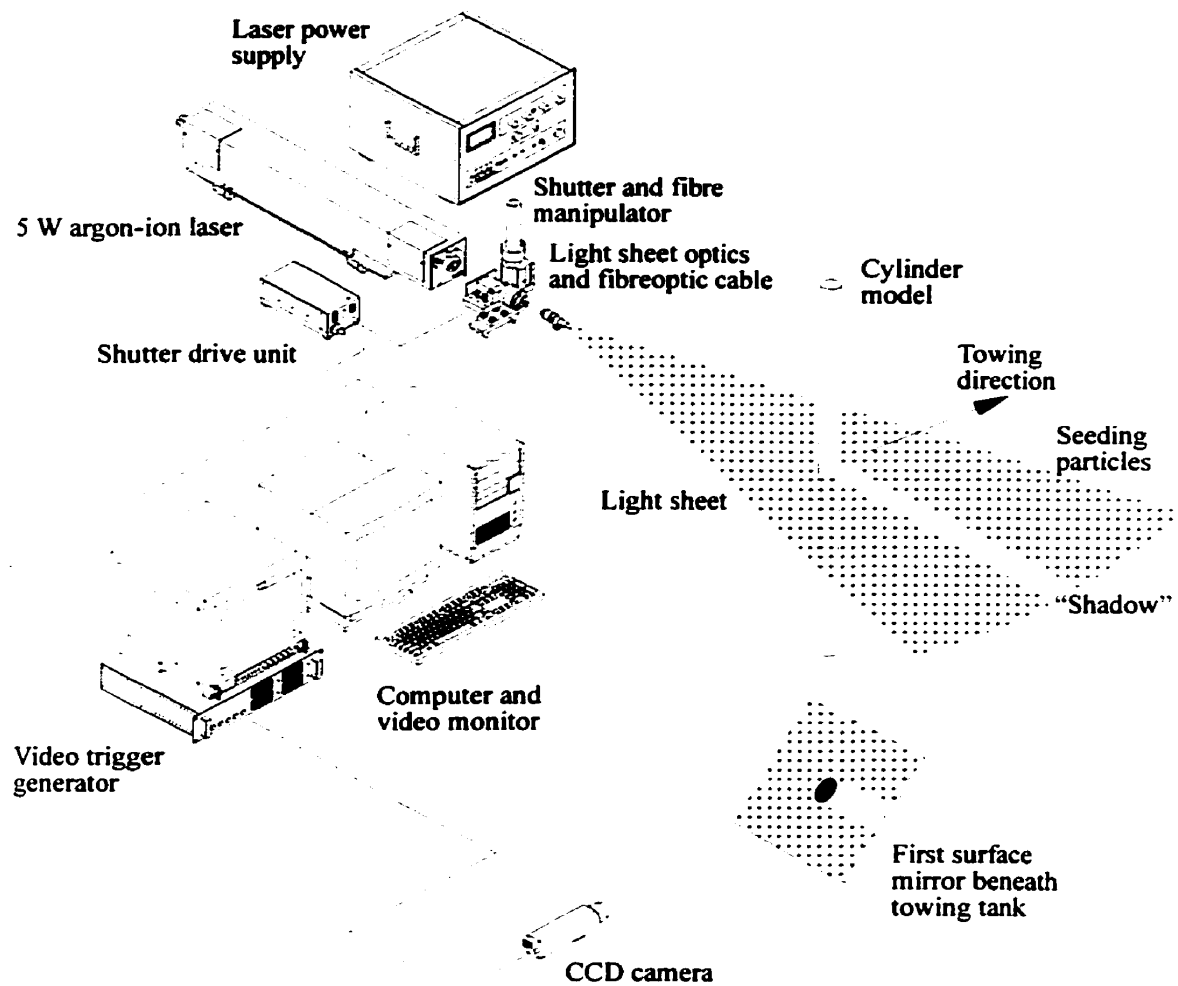


Figure 5.2. Components of the particle image velocimetry (PIV) system.

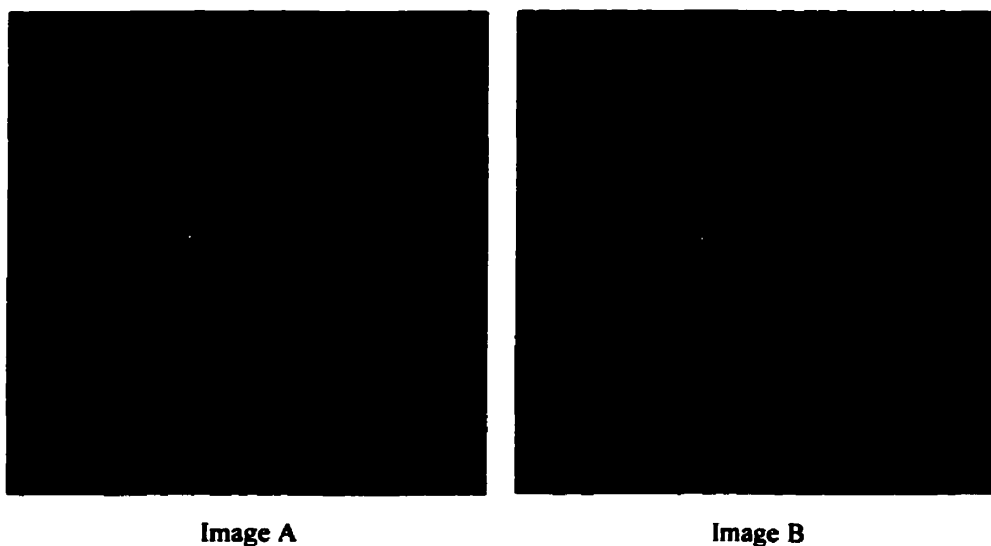


Figure 5.3. An example of a pair of digitized images acquired with the CCD camera (512×480 pixels, $M_x = 0.26$ mm/pixel, $\Delta t = 11$ ms, $sq = 1.167$) showing seeding particles and two impulsively started circular cylinders arranged side-by-side, $T/D = 1.5$, $Re = 2200$, $A_p = 0.3$, $t^* = 2.7$. In these images, because of the orientation of the camera, the cylinders are seen as being towed from top to bottom; the cylinders were actually towed upwards in the vertical water towing tank. Since the two plexiglas cylinders behave as cylindrical lenses in the presence of the light sheet, an intensified region of light and a shadow may be distinguished to the right hand side of the cylinders in each of the images.

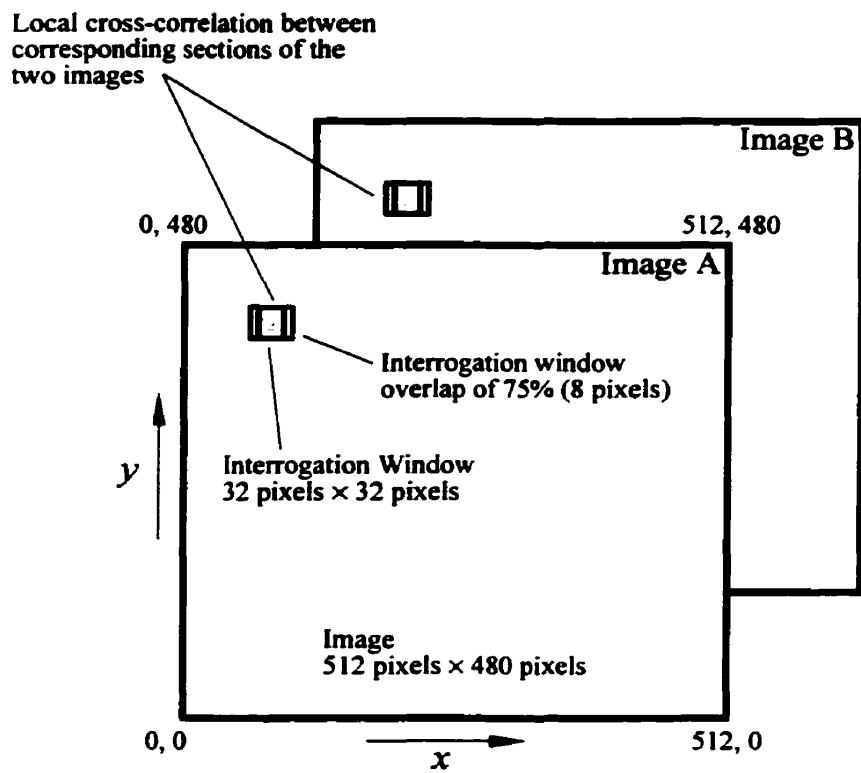


Figure 5.4. Illustration of the local spatial cross-correlation process between corresponding interrogation windows in the two images comprising a pair.

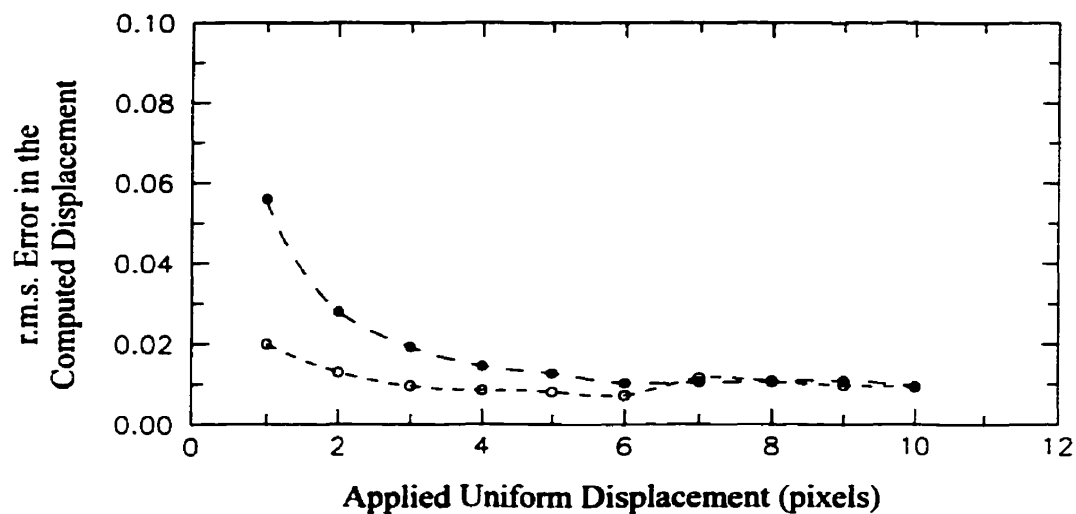


Figure 5.5. Measurement uncertainty in the displacement vector magnitude computed by the PIV system, after “outlier” removal (4 pixel threshold) and data smoothing, for applied uniform displacements: ○, horizontal direction; ●, vertical direction. Dashed lines added for illustration purposes only.

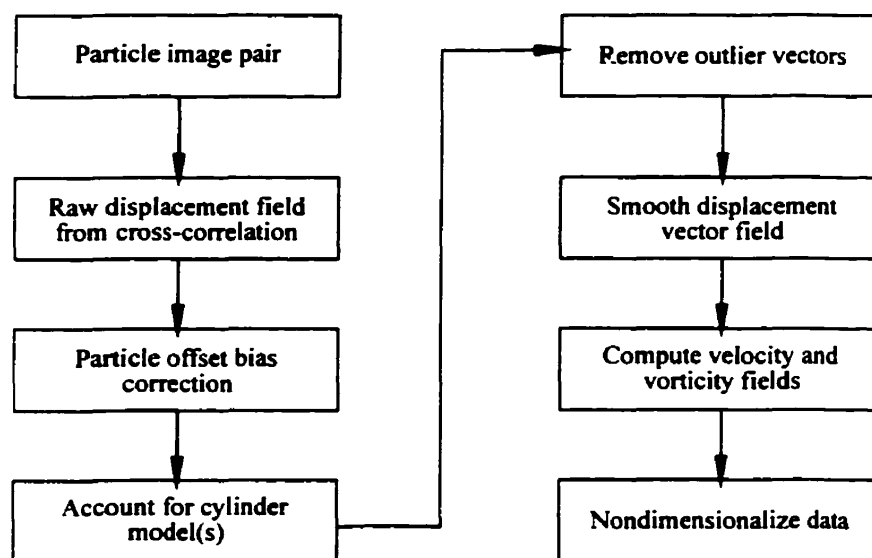


Figure 5.6. PIV data processing steps.

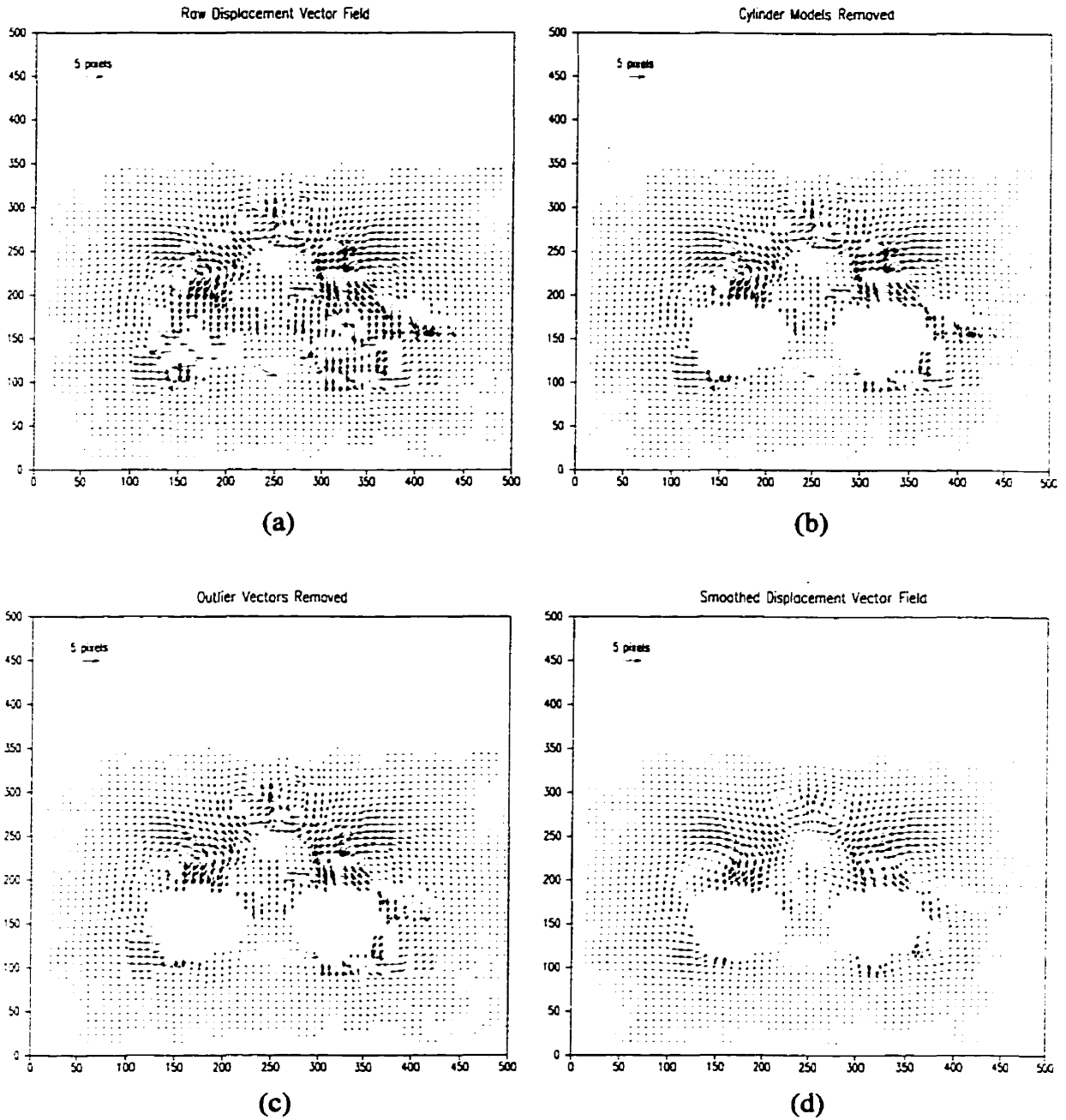


Figure 5.7. Processing of PIV data, from the image pair in Figure 5.3 for two side-by-side circular cylinders impulsively set into motion (towing direction from top to bottom), $T/D = 1.5$, $Re = 2200$, $A_p = 0.3$, $t^* = 2.7$: (a) raw displacement vector field, 32×32 pixel interrogation window, 75% overlap, 8 pixel spacing, 61×59 vectors; (b) displacement vector field with cylinders removed; (c) outlier vectors removed and reinterpolated (4 pixel threshold); (d) smoothed displacement vector field; (e) velocity field (mm/s), $M_t = 0.26$ mm/pixel, $sq = 1.167$, $\Delta t = 11$ ms; (f) vorticity field (s^{-1}), 59×55 points.

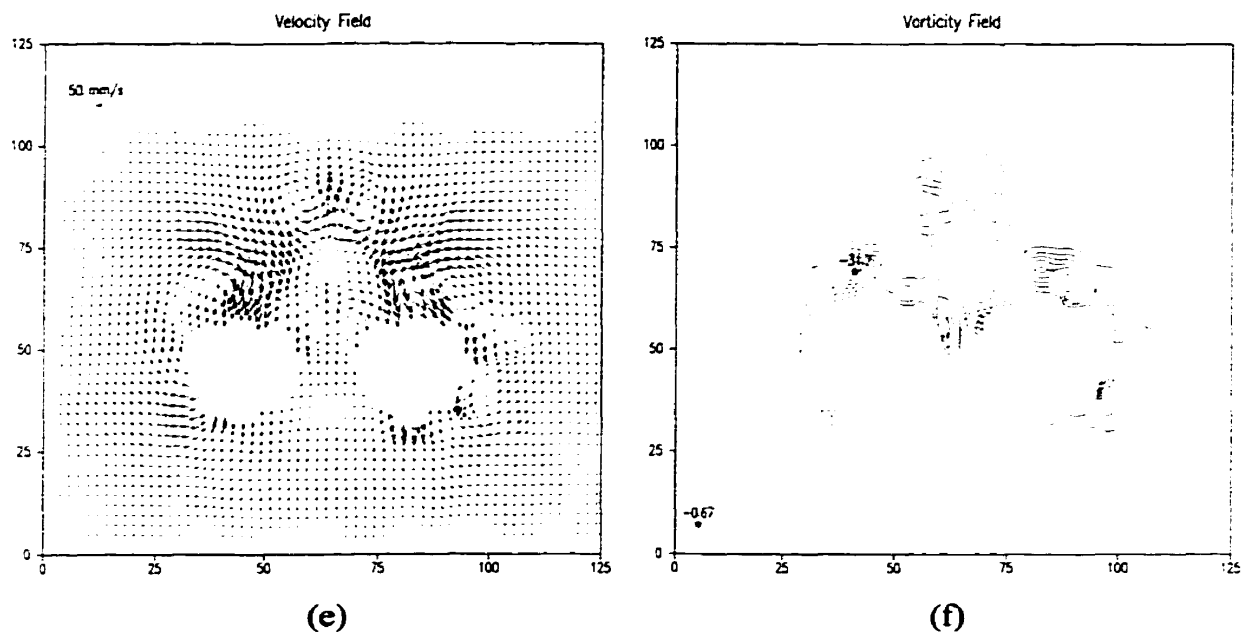


Figure 5.7 (continued).

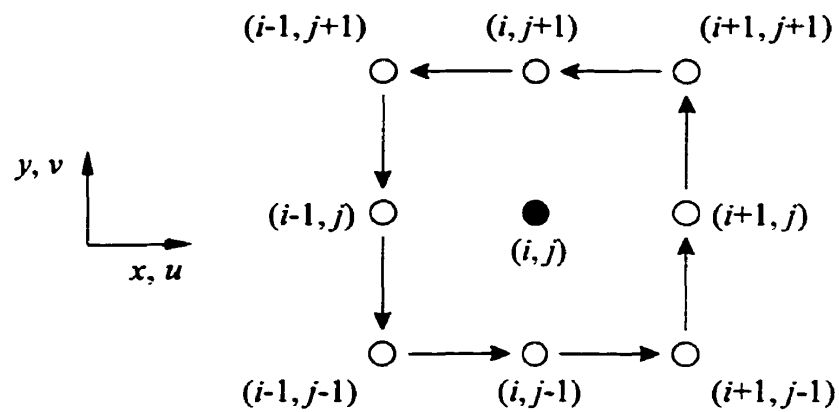


Figure 5.8. Path used to compute the local circulation in (5.2), to obtain the local in-plane vorticity at grid point (i, j) .

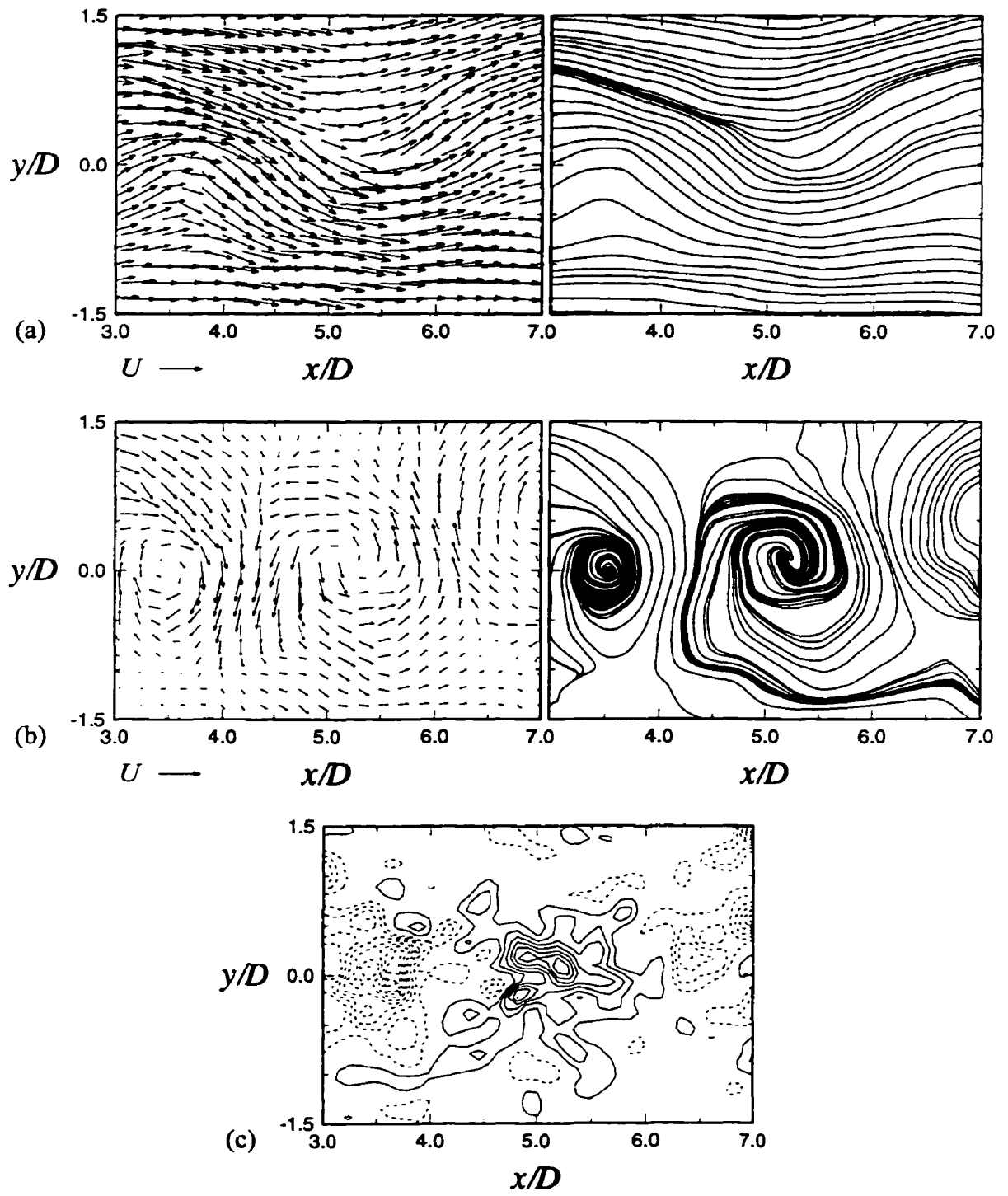


Figure 5.9. Kármán vortex shed from a circular cylinder at $Re = 1900$, with $x_c = 5.1, y_c = -0.1$, $\omega_{zp}D/U = 4.0, \Gamma/UD = 2.45, A/D^2 = 2.1$: nondimensional, instantaneous, in-plane (a) velocity and streamline fields (every second vector omitted for clarity, streamlines arbitrary); (b) $U_c = 0.82U$ removed; (c) vorticity field (minimum vorticity contour magnitude 0.5, contour increment 0.5, dashed lines represent negative (CW) vorticity).



Figure 6.1. Dye-injection flow visualization of a circular cylinder in steady upstream cross-flow, $Re = 500$ to 1000 (Wong 1997). The intensified region of light above the cylinder is caused by the cylinder's plexiglas construction, which behaves as a cylindrical lens.

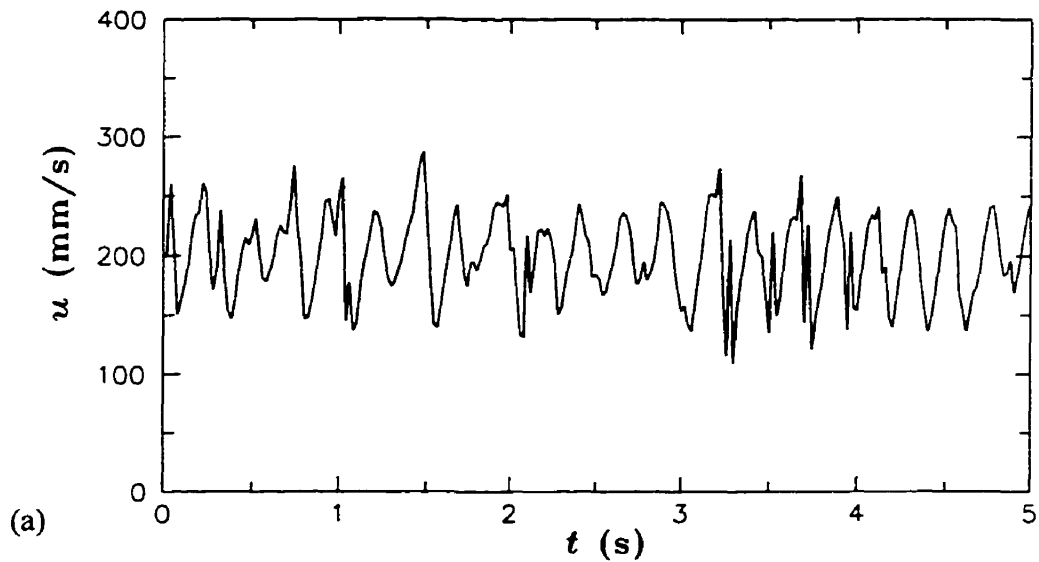


Figure 6.2. Vortex shedding from a single circular cylinder: (a) velocity signal from the hot-film sensor located at $x/D = 5.0$, $y/D = 0.8$ in the water tunnel, with $U = 220$ mm/s, $Re = 2200$, 20 Hz sampling frequency; (b) corresponding power spectrum (vertical scale arbitrary); (c) Strouhal number data: $\circ$, water tunnel, Grépin (1996), Wong (1997), Smith (1998); $\bullet$, towing tank.

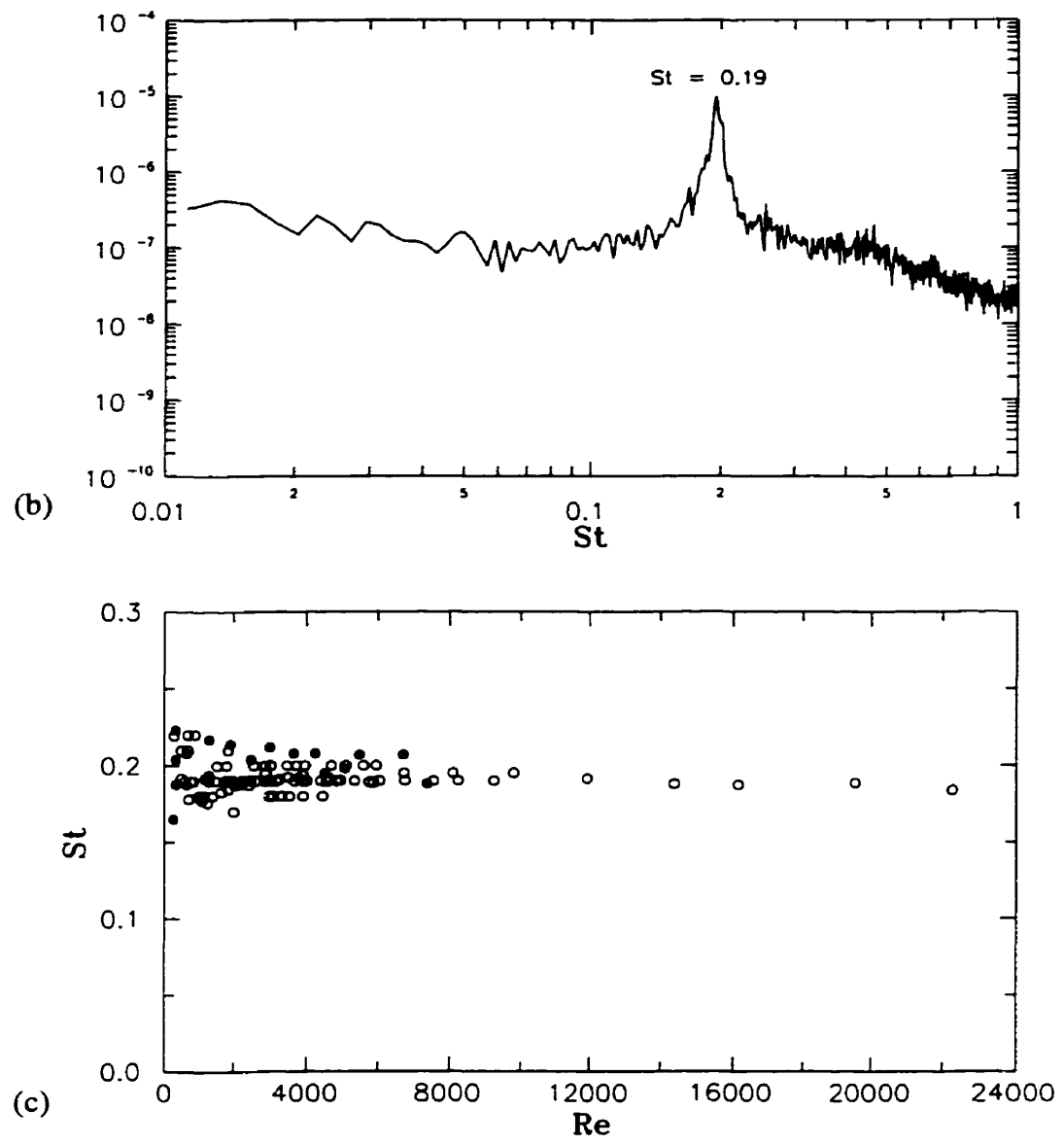


Figure 6.2 (continued).

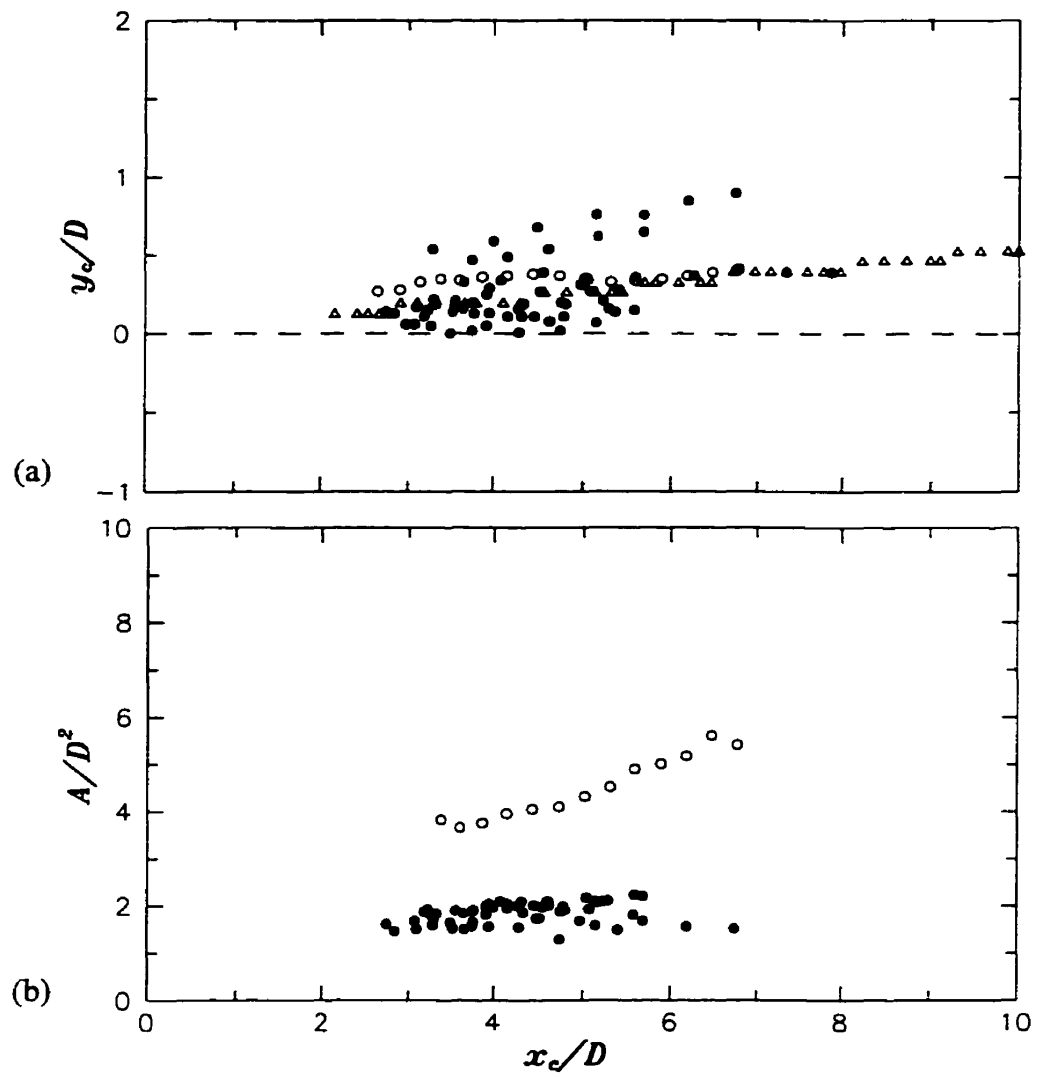


Figure 6.4. Kármán vortex data for a circular cylinder in steady cross-flow: (a) transverse location of the vortex centroid; (b) vortex area. $\circ$, Cantwell & Coles (1983), $Re = 140,000$; $\triangle$, Tanaka & Murata (1986), $Re = 37,000$; $\bullet$, PIV data, $Re = 1900$.

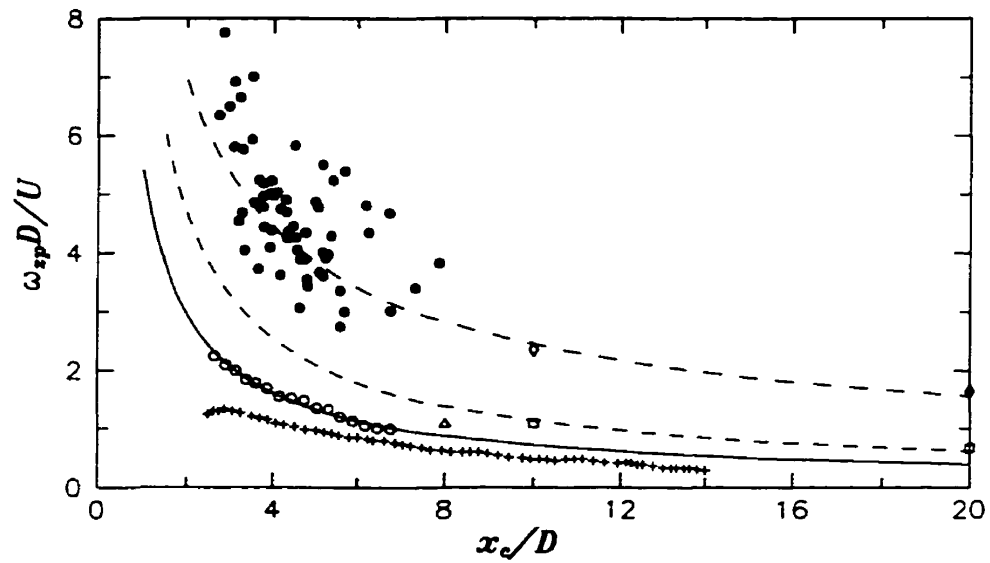


Figure 6.5. Peak vorticity data for a circular cylinder in steady cross-flow: $\circ$, Cantwell & Coles (1983), $Re = 140,000$; $\triangle$, Kiya & Matsumura (1985), $Re = 16,000$; $\square$, Hussain & Hayakawa (1987), $Re = 13,000$; $+$, Tanaka & Murata (1986), $Re = 37,000$; $\diamond$, Zhou & Antonia (1993), $Re = 5600$; $\bullet$, PIV data, $Re = 1900$. Curves added for illustration purposes only.

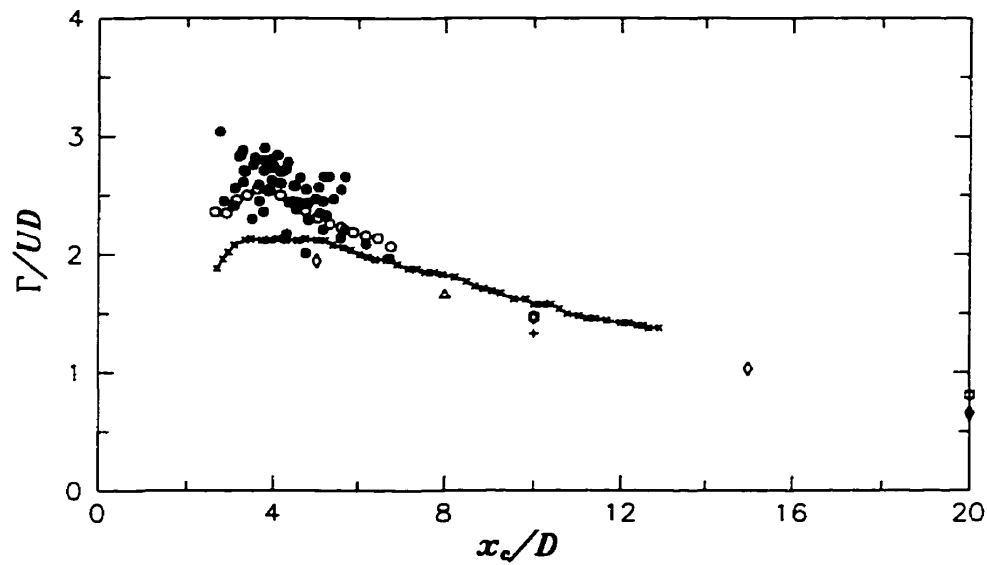


Figure 6.6. Kármán vortex circulation (or strength) for a circular cylinder in steady cross-flow: $+$, Bloor & Gerrard (1966), $Re = 2000$; $\diamond$, Schmidt & Tilmann (1972), $Re = 5100$; $\circ$, Cantwell & Coles (1983), $Re = 140,000$; $\triangle$, Kiya & Matsumura (1985), $Re = 16,000$; $\times$, Tanaka & Murata (1986), $Re = 37,000$; $\square$, Zhou & Antonia (1993), $Re = 5600$; $\bullet$, PIV data, $Re = 1900$.

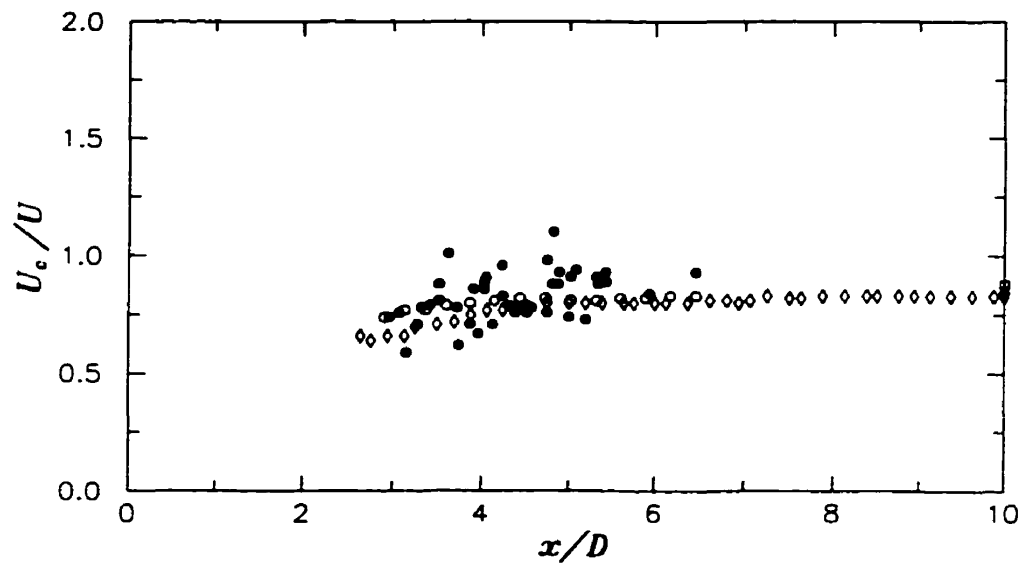


Figure 6.7. Kármán vortex convection velocity for a circular cylinder in steady cross-flow: $\circ$, Cantwell & Coles (1983), $Re = 140,000$; $\square$, Hussain & Hayakawa (1987), $Re = 13,000$; $\diamond$, Tanaka & Murata (1986), $Re = 37,000$; $\triangle$, Zhou & Antonia (1992), turbulent wake; $\bullet$, PIV data, $Re = 1900$.

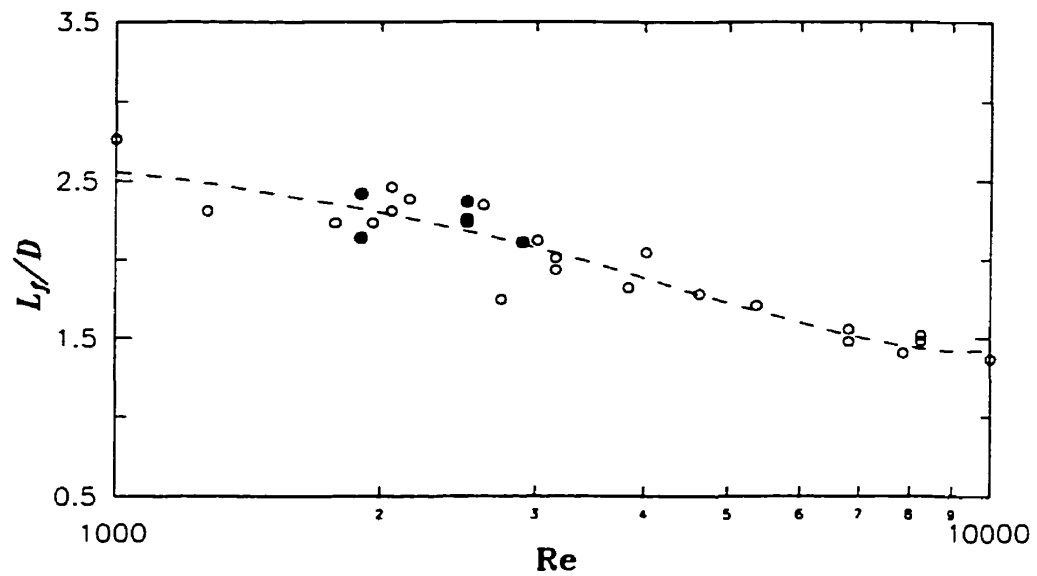


Figure 6.8. Vortex formation length for a circular cylinder in steady cross-flow, as a function of Reynolds number: $\circ$, data compiled by Norberg (1987); $\bullet$, PIV data.

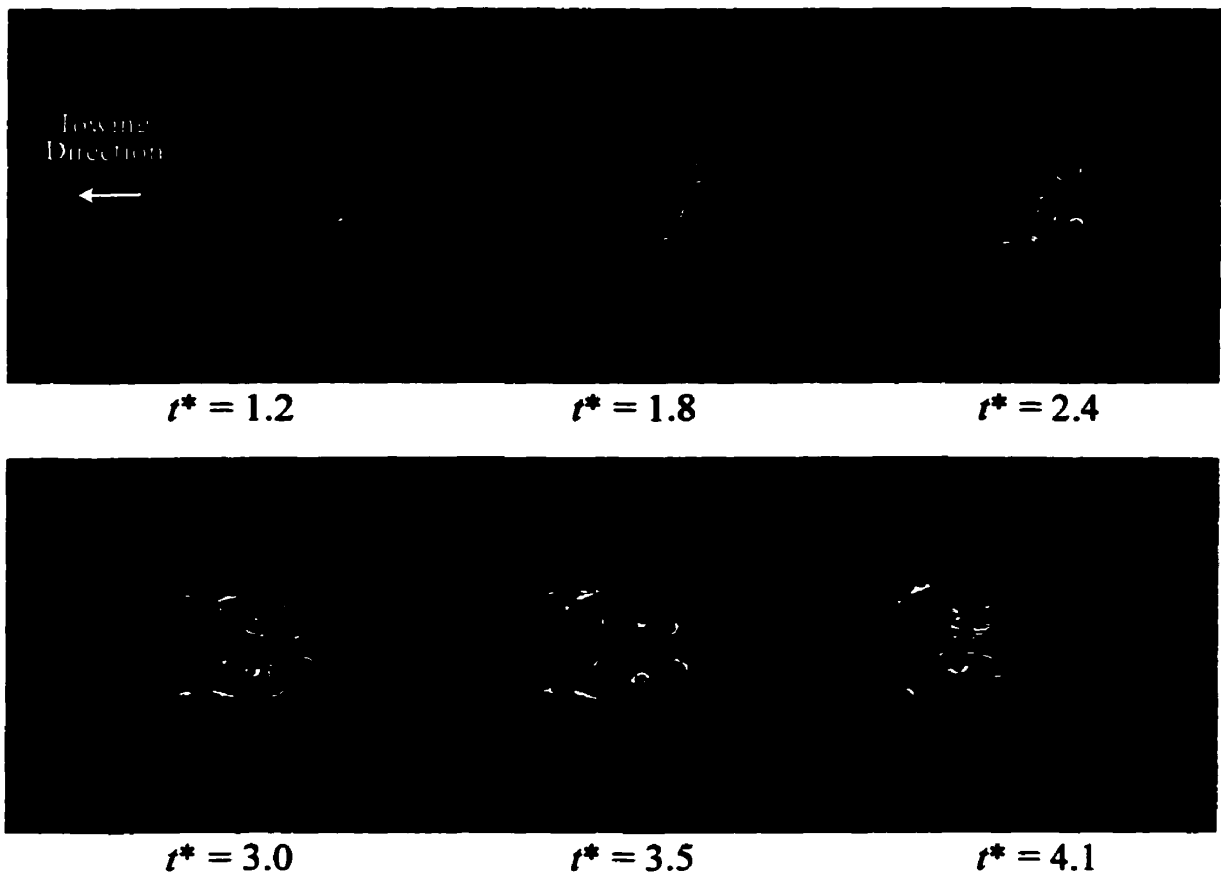


Figure 6.9. Flow visualization of an impulsively started circular cylinder, $Re = 1900$, $A_p = 0.4$.

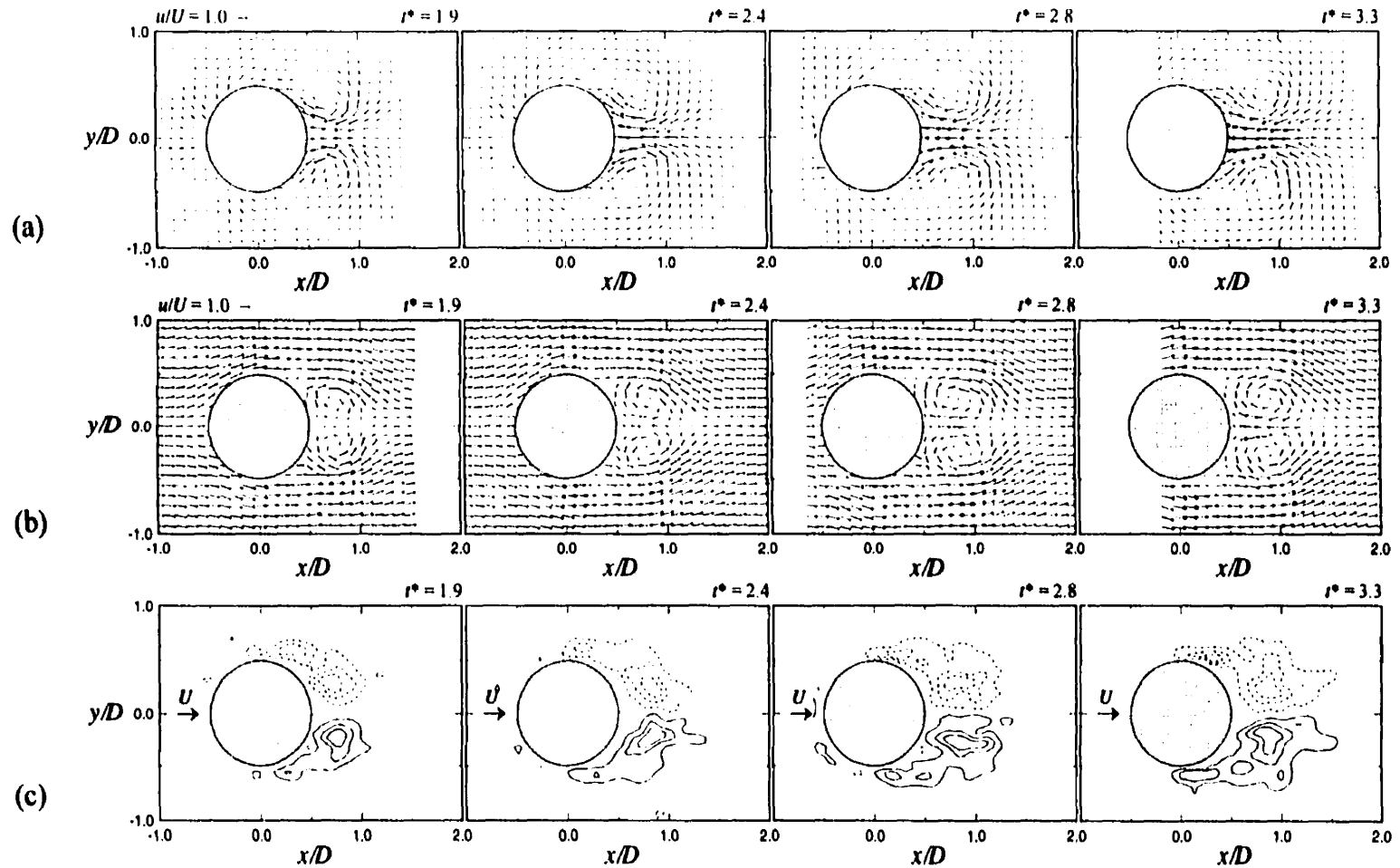


Figure 6.10. Circular cylinder impulsively set into motion, $Re = 1500$, $A_p = 0.5$, $M_r = 0.25$ mm/pixel: nondimensional, instantaneous in-plane (a) velocity field, in the experiment reference frame, with the cylinder being towed from right to left in stationary fluid (every second vector omitted for clarity); (b) inertial reference frame, with the instantaneous towing velocity superimposed; (c) vorticity field (minimum vorticity contour magnitude 2.0, contour increment 4.0, dashed lines represent negative (CW) vorticity).

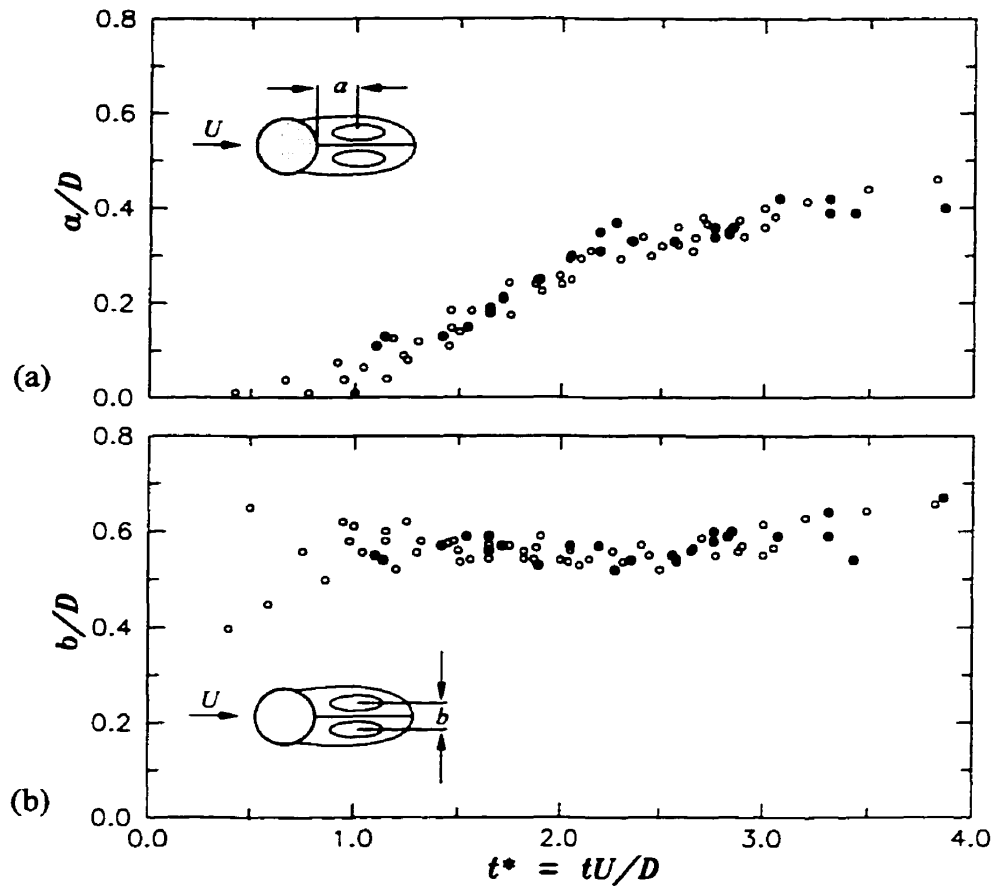


Figure 6.11. Recirculation zone data for an impulsively started circular cylinder: (a) streamwise position of the primary eddies from the base of the cylinder; (b) transverse spacing of the primary eddies; (c) streamwise extent of the recirculation zone from the base of the cylinder; (d) circulation or strength of a primary eddy. $\circ$, published data for $Re = 1000$ to 3000 (Bouard & Coutanceau 1980; Nagata *et al.* 1989; Wu & Chu 1989; Chu & Liao 1992); $\bullet$, present PIV experiments.

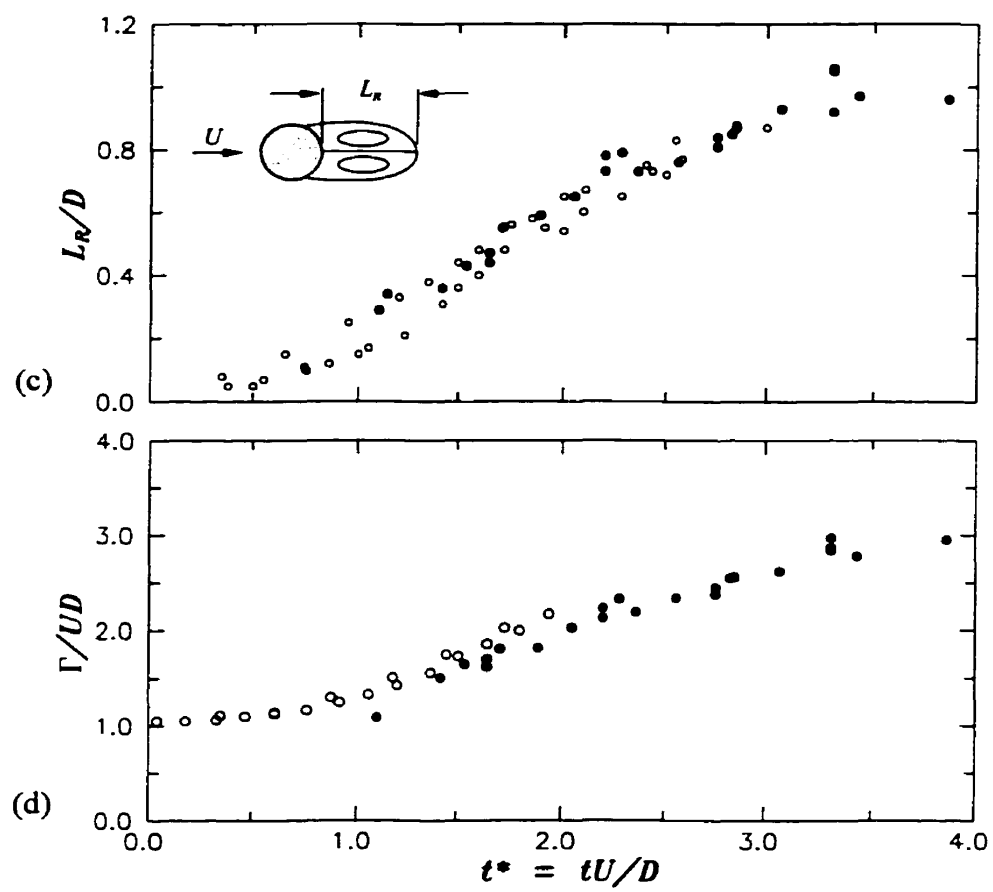


Figure 6.11 (continued).

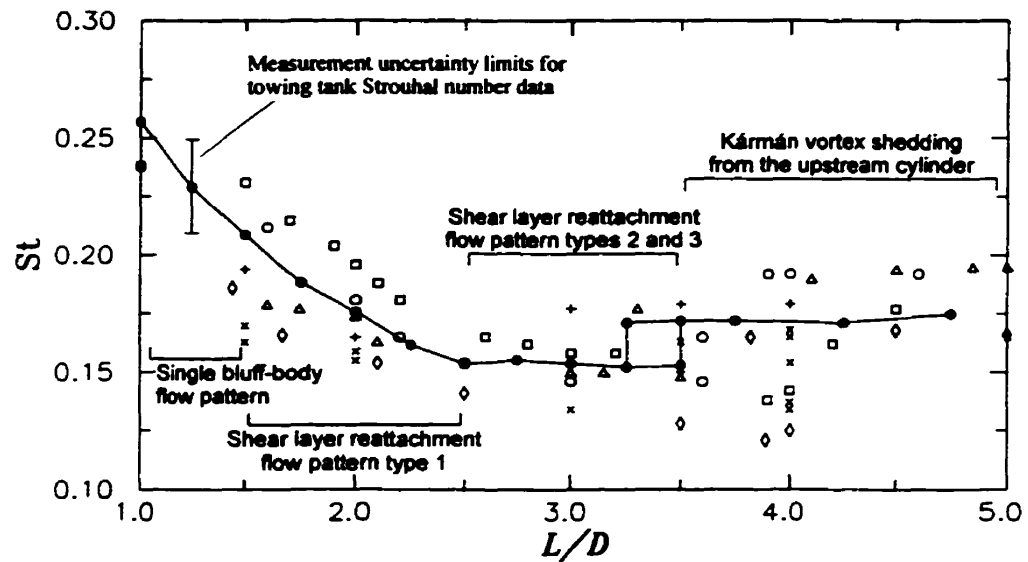


Figure 7.1. Strouhal number data, for the downstream cylinder, of two cylinders in tandem in steady cross-flow at low subcritical Reynolds numbers: $\times$, Huhe-Aode *et al.* (1985), $Re = 300$; $+$, Huhe-Aode *et al.* (1985), $Re = 1000$; Δ , Igarashi (1981), $Re = 8800$; $\diamond$, Ishigai *et al.* (1972), $Re = 8000$; $\square$, Oka *et al.* (1972), $Re = 5100$; $\circ$, Oka *et al.* (1972), $Re = 8800$; $\bullet$, present towing tank experiments, $Re = 600$ to 6000 . Lines added for visual purposes only, measurement uncertainty estimated at 20% for the towing tank data.

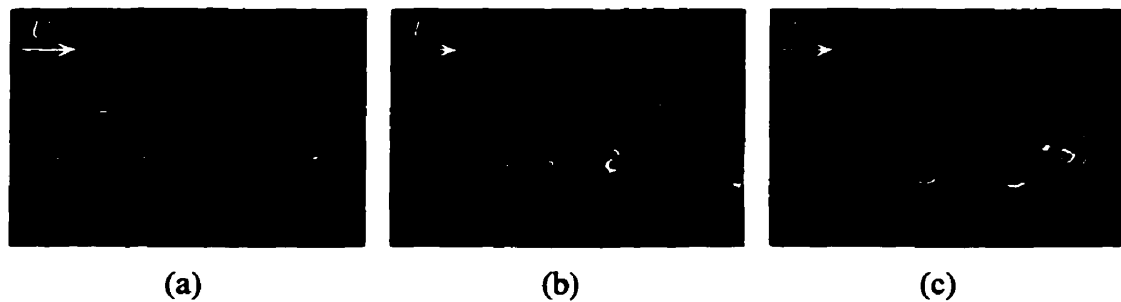


Figure 7.2. Flow visualization of two tandem cylinders at $L/D = 1.0$, $Re = 1270$, showing single bluff-body behaviour with (a) alternate vortex shedding, (b) irregular vortex shedding, and (c) shear layer instability vortices.

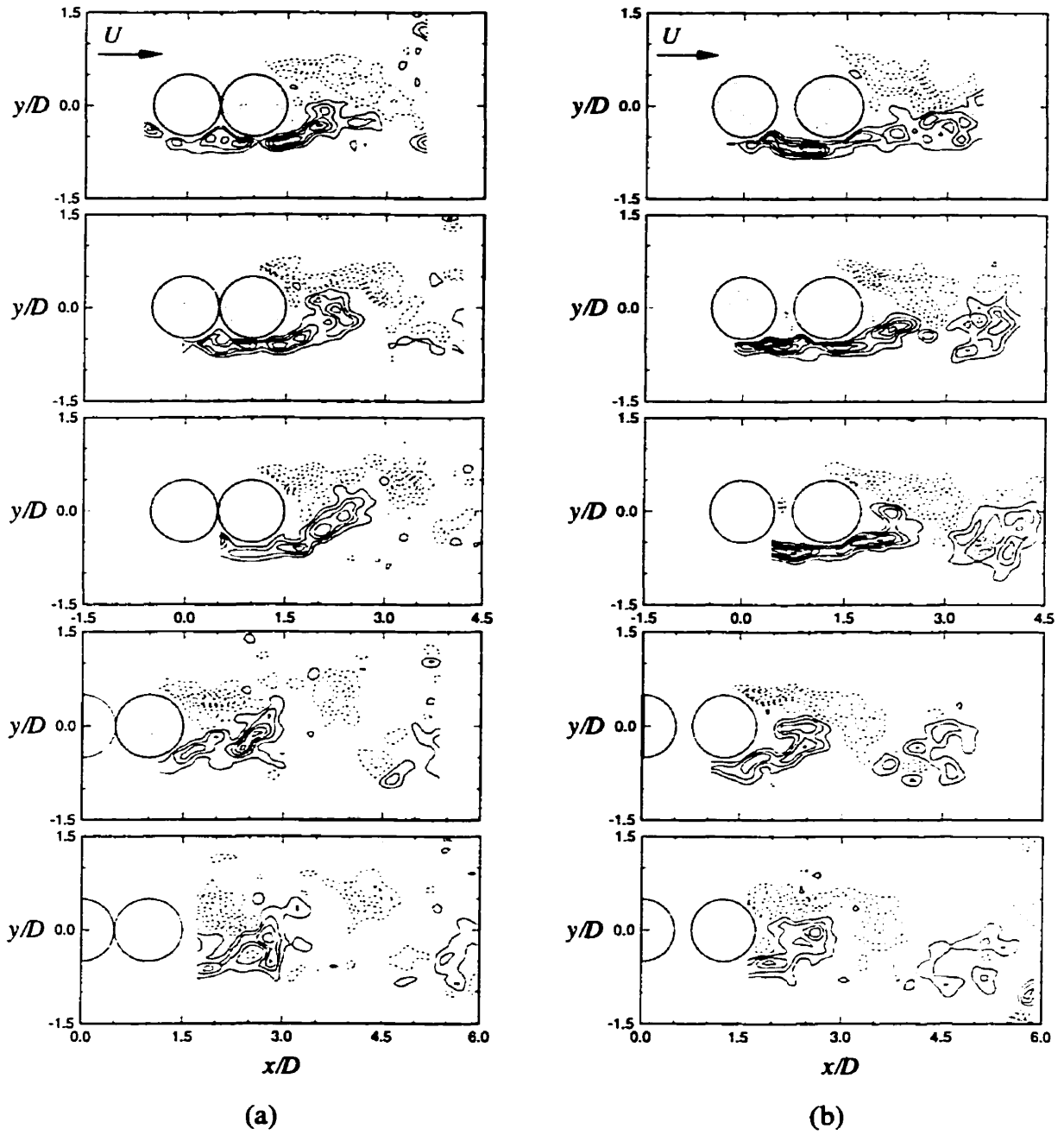


Figure 7.3. Vorticity data for two cylinders in tandem, $Re = 1900$, $M_r = 0.21$ mm/pixel, $\Delta t^* = 0.6$: (a) $L/D = 1.0$, $\Delta t/T = 0.15$, (b) $L/D = 1.25$, $\Delta t/T = 0.14$. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.



(a)



(b)



(c)



(d)



(e)

Figure 7.4. Flow visualization of two tandem cylinders in steady cross-flow, showing shear layer reattachment: (a) $L/D = 1.5$, $Re = 900$; (b) $L/D = 2.0$, $Re = 850$; (c) $L/D = 2.5$, $Re = 850$; (d) $L/D = 3.0$, $Re = 880$; (e) $L/D = 3.5$, $Re = 1320$.

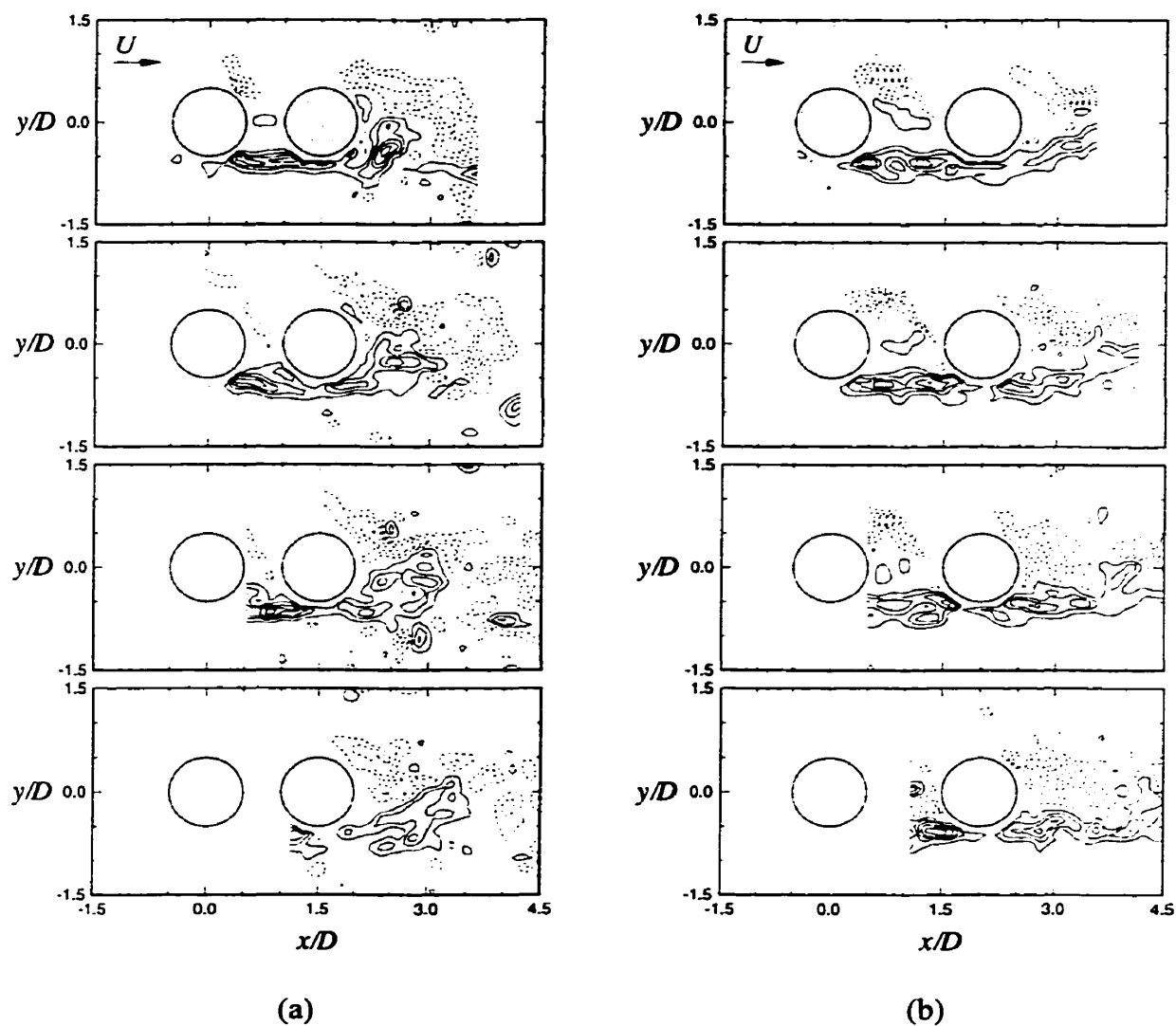
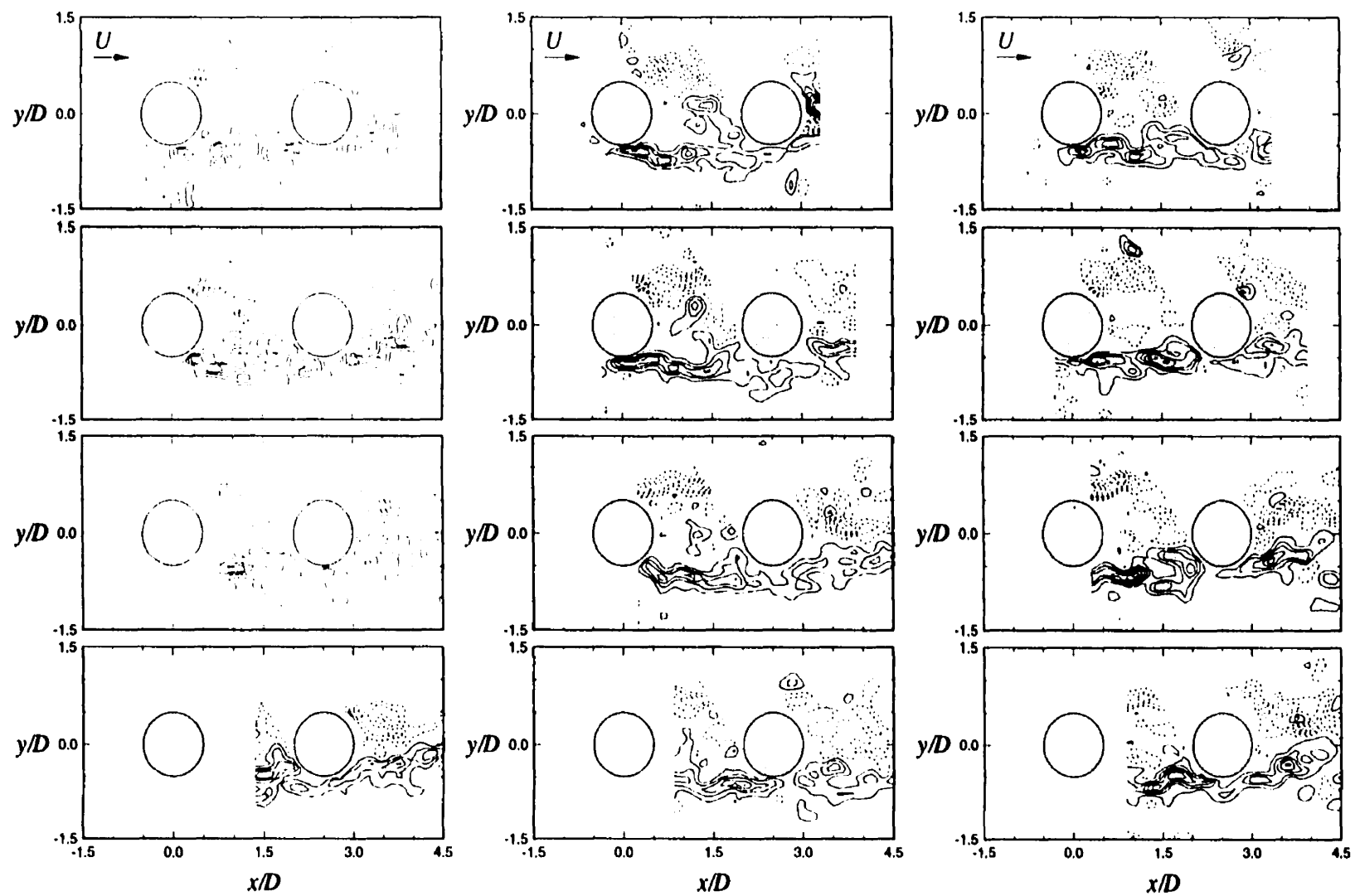
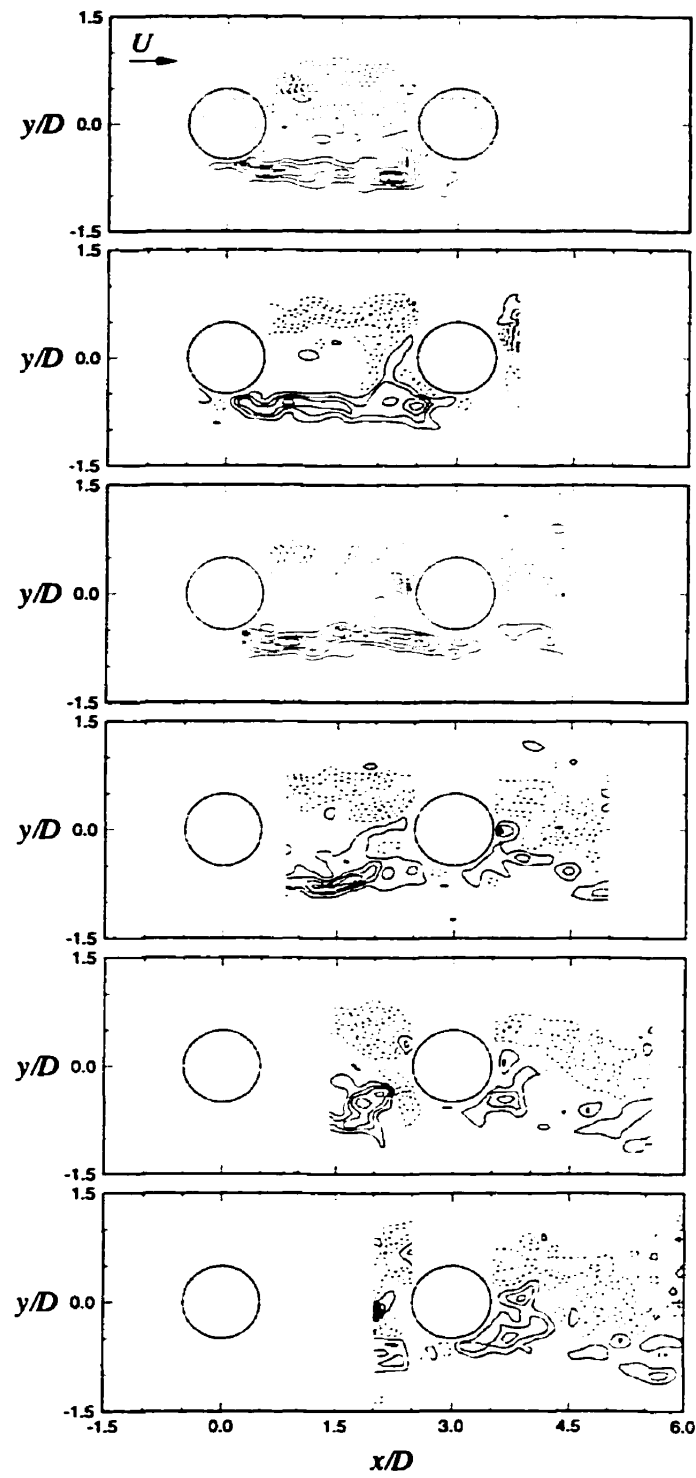


Figure 7.5. Vorticity field for two tandem cylinders with shear layer reattachment, in steady cross-flow, $Re = 1900$, $M_r = 0.21$ mm/pixel, $\Delta t^* = 0.6$: (a) $L/D = 1.5$, $\Delta t/T_r = 0.12$; (b) $L/D = 2.0$, $\Delta t/T_r = 0.11$; (c) $L/D = 2.5$, $\Delta t/T_r = 0.09$, three different examples shown; (d) $L/D = 3.0$, $\Delta t/T_r = 0.09$. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.



(c)

Figure 7.5 (continued).



(d)

Figure 7.5 (continued).

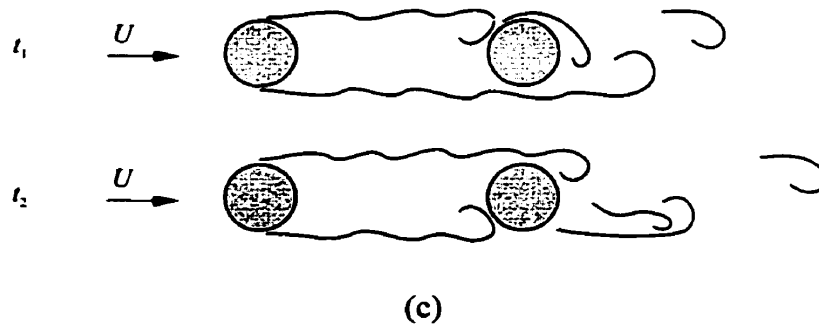
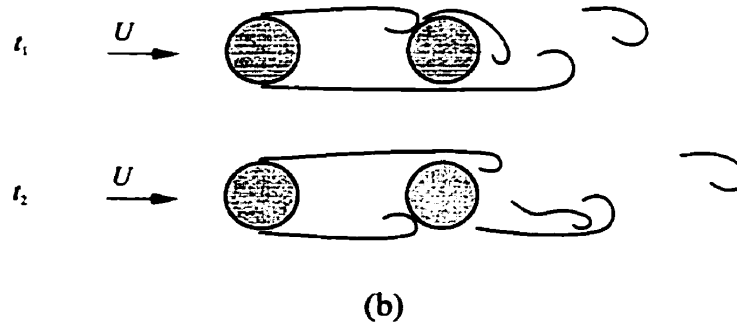
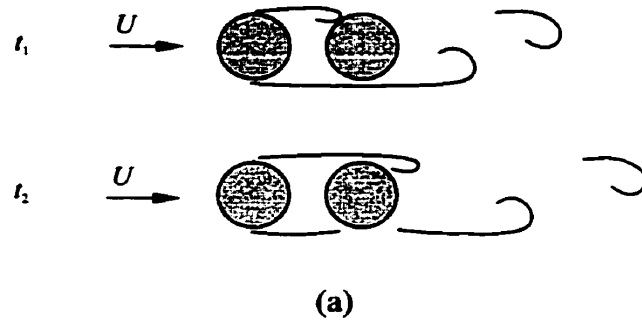


Figure 7.6. Three types of shear layer reattachment behaviour for two tandem cylinders in steady cross-flow, each shown at two times t_1 and t_2 in the shedding cycle: (a) first type, $L/D = 1.5$ and 2.0 ; (b) second type, $L/D = 2.5$ and 3.0 ; (c) third type, $L/D = 3.5$ and 4.0 .

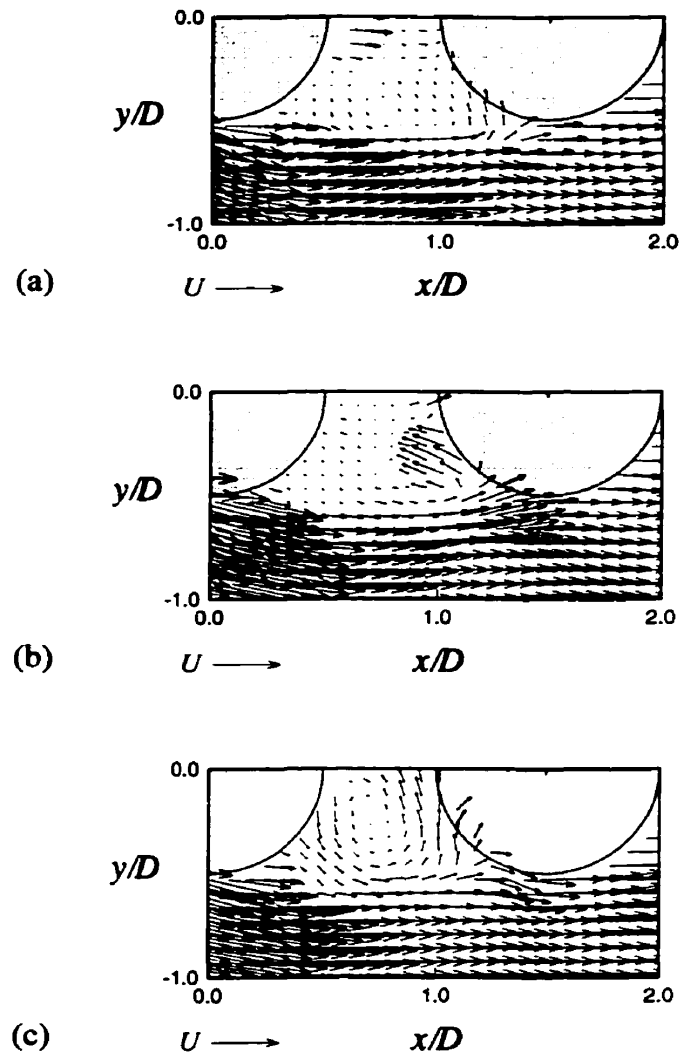


Figure 7.7. Instantaneous gap flow patterns for two tandem cylinders, $L/D = 1.5$, $Re = 1900$, $M_i = 0.21$ mm/pixel: (a) shear layer reattachment with stagnant fluid in the gap; (b) vorticity concentration near the reattachment point; (c) shear layer reattachment with recirculating fluid in the gap.

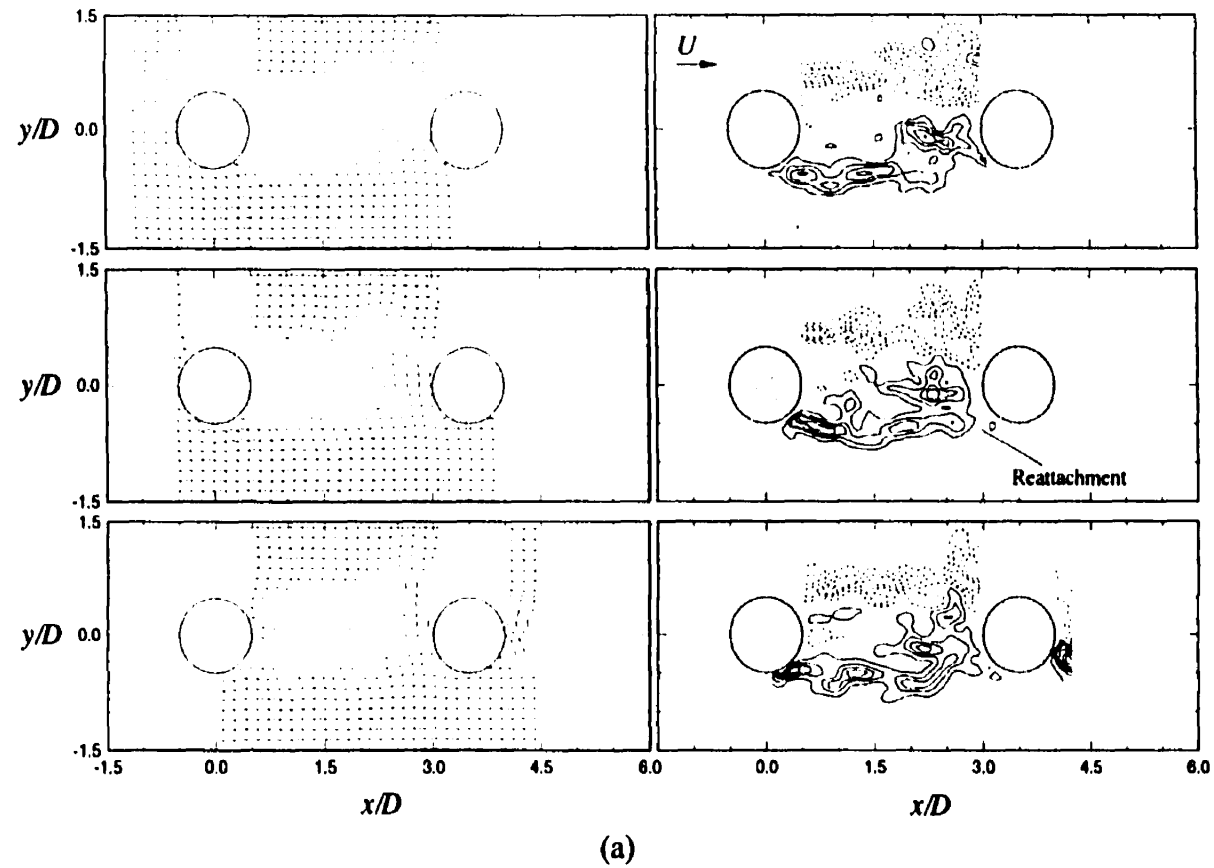


Figure 7.8. Two tandem cylinders in steady cross-flow, showing transitional behaviour at $L/D = 3.5$, $Re = 1900$, $M_t = 0.21$ mm/pixel, $\Delta t^* = 0.6$: (a) shear layer reattachment flow pattern, $\Delta t/T_c = 0.09$; (b) flow pattern showing Kármán vortex shedding from the upstream cylinder, $\Delta t/T_c = 0.10$. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity, every second velocity vector omitted for clarity.

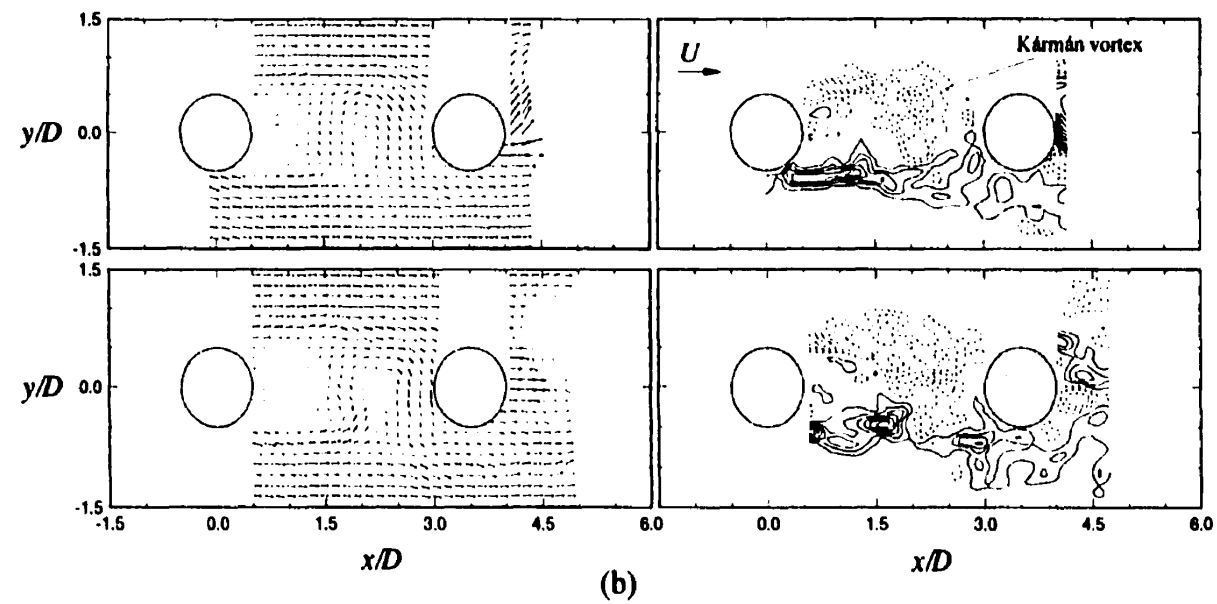
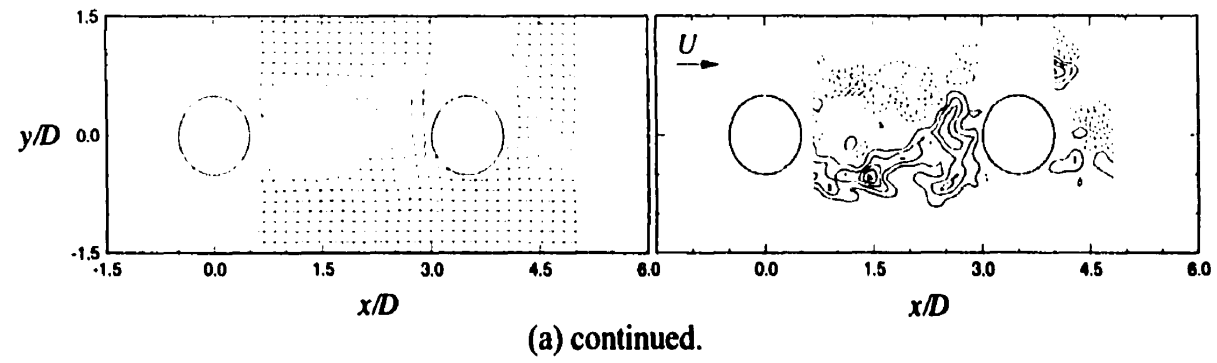


Figure 7.8 (continued).

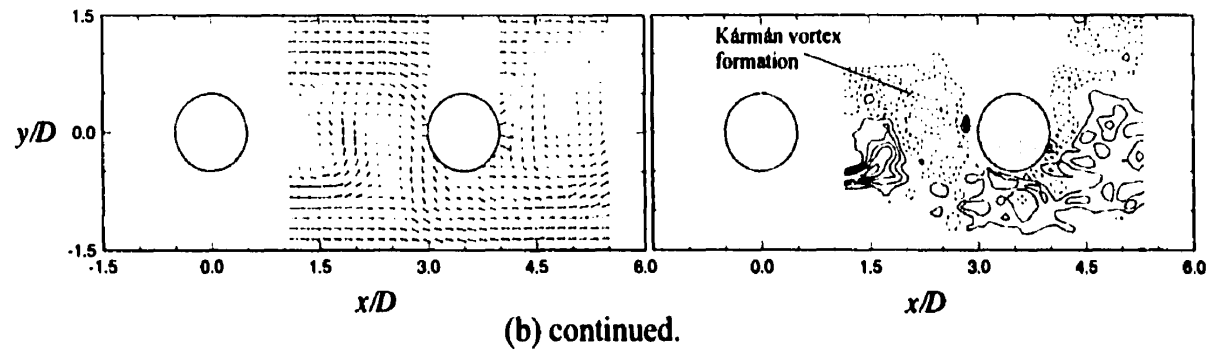


Figure 7.8 (continued).

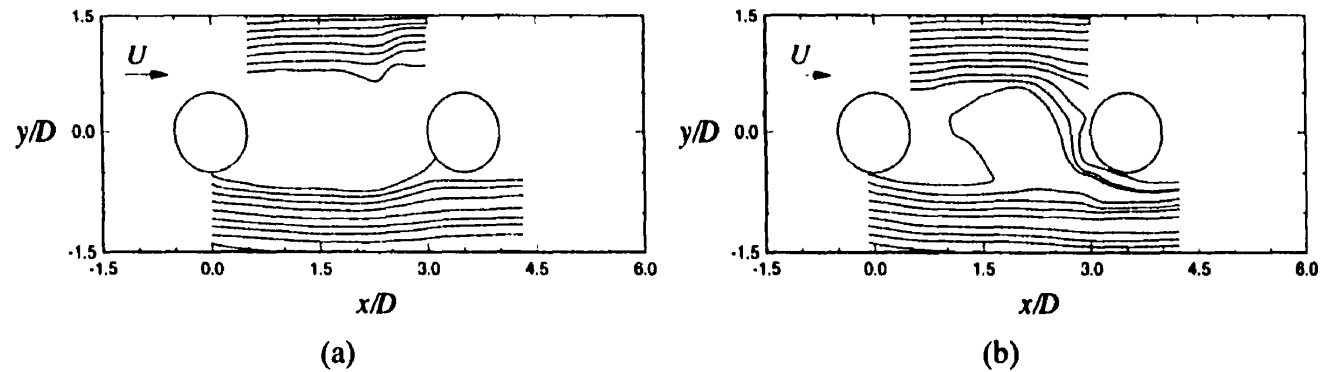
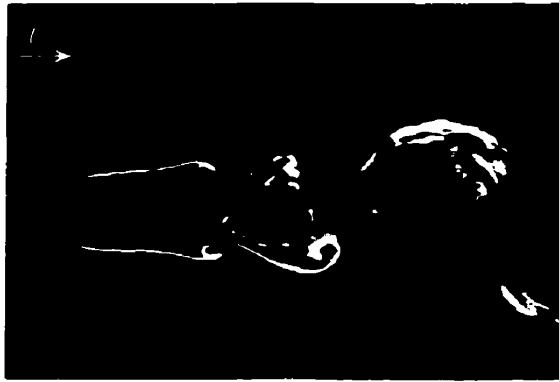
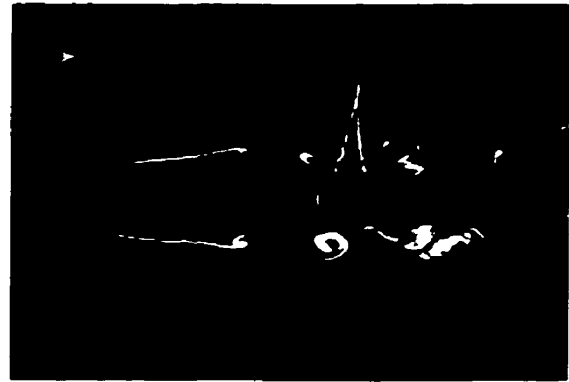


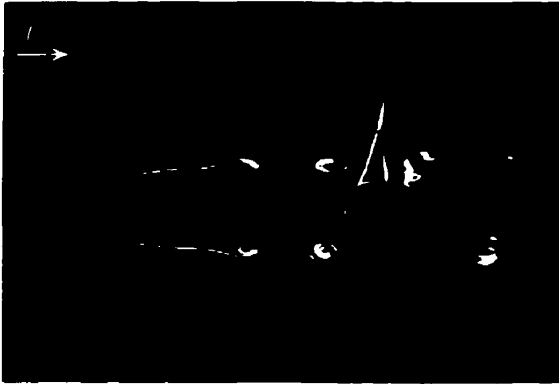
Figure 7.9. Transitional behaviour for two tandem cylinders in steady cross-flow, $L/D = 3.5$, $Re = 1900$, $M_t = 0.21$ mm/pixel: (a) shear layer reattachment flow pattern; (b) flow pattern showing Kármán vortex shedding from the upstream cylinder. Streamlines arbitrarily chosen.



(a)



(b)



(c)



(d)

Figure 7.10. Flow visualization of tandem cylinders in steady cross-flow, at large pitch ratio: (a) $L/D = 4.0$, $Re = 900$; (b) $L/D = 4.5$, $Re = 930$; (c) $L/D = 5.0$, $Re = 930$, showing shear layer reattachment and instability vortices; (d) $L/D = 5.0$, $Re = 1320$, showing Kármán vortex shedding from the upstream cylinder.

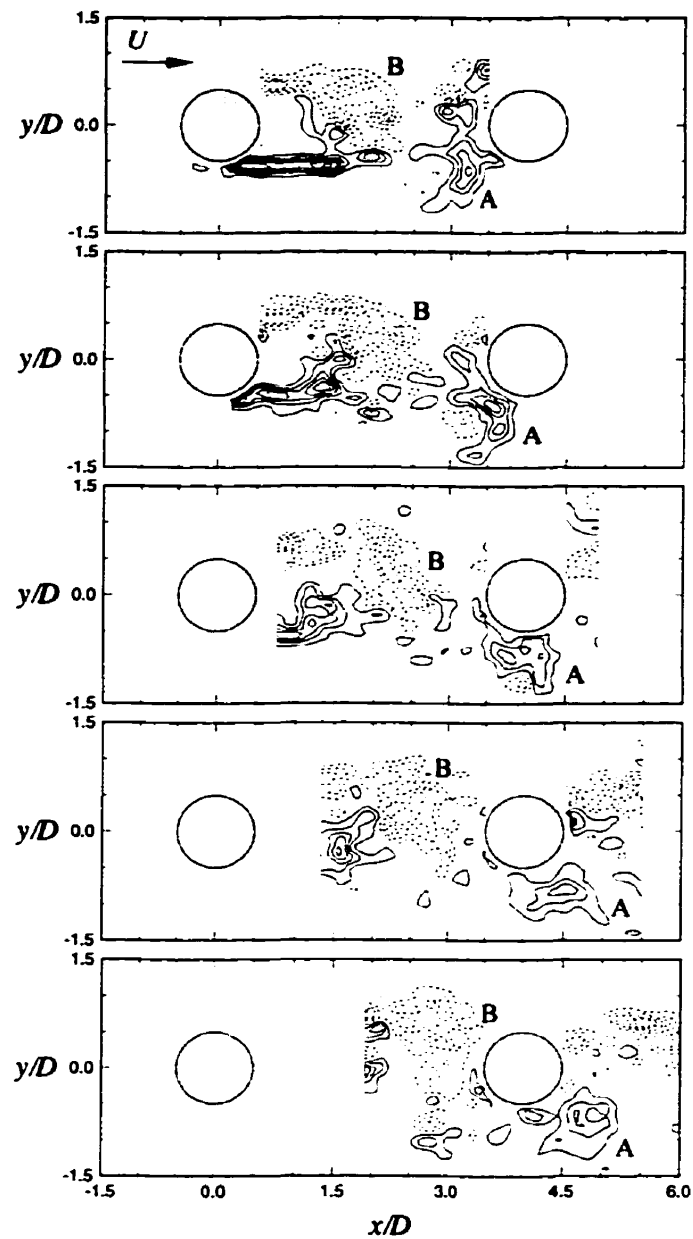
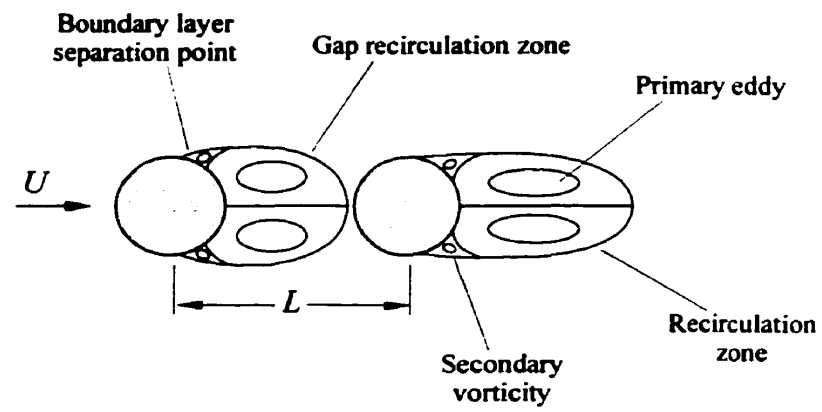
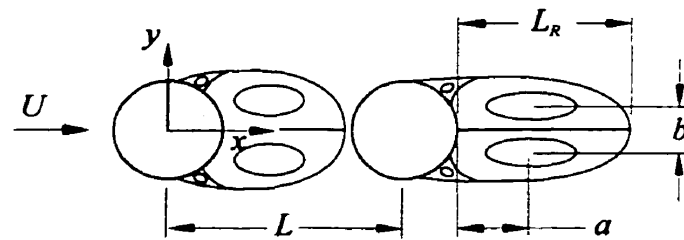


Figure 7.11. Vorticity data for two tandem cylinders in steady cross-flow, showing Kármán vortex shedding from the upstream cylinder, $L/D = 4.0$, $Re = 1900$, $M_r = 0.21$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_r = 0.10$. A shed CCW Kármán vortex (labeled A) is shown impinging on the downstream cylinder, while a CW Kármán vortex (labeled B) is shown being formed. Minimum vorticity contour magnitude 1.0, contour increment 1.5, solid lines represent positive (CCW) vorticity, dashed lines represent negative (CW) vorticity.

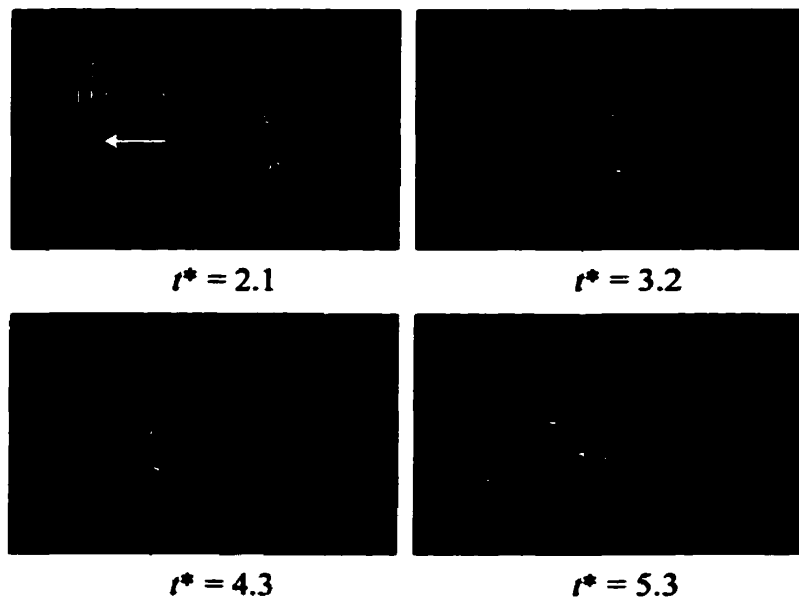


(a)

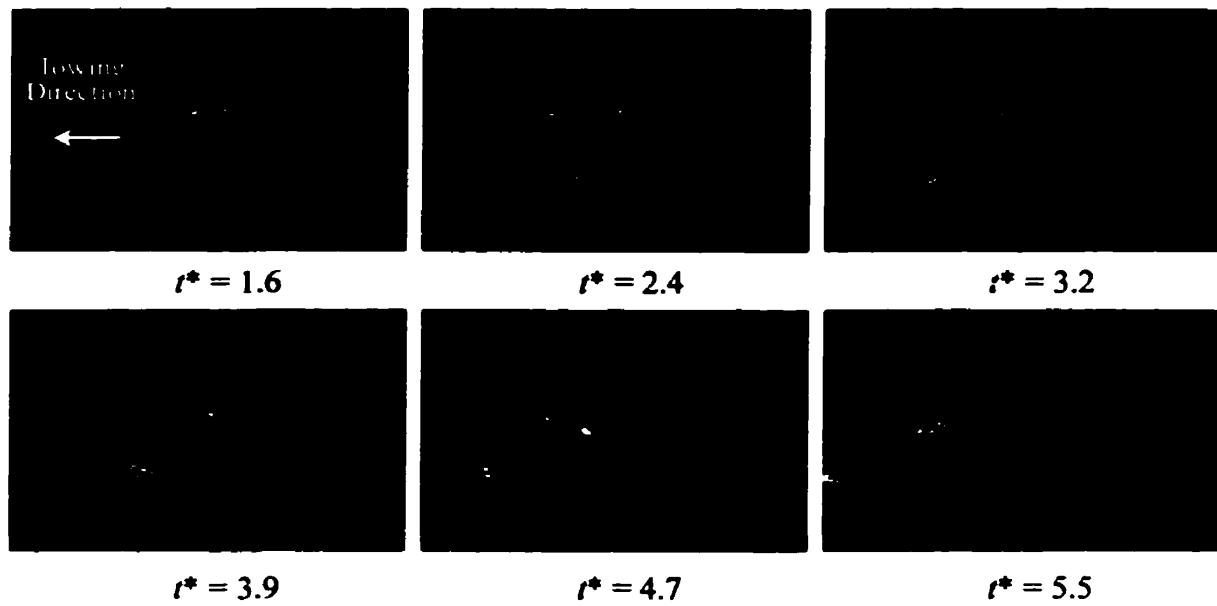


(b)

Figure 7.12. Tandem circular cylinders impulsively set into motion: (a) features of the flow field; (b) notation used in the quantitative description.

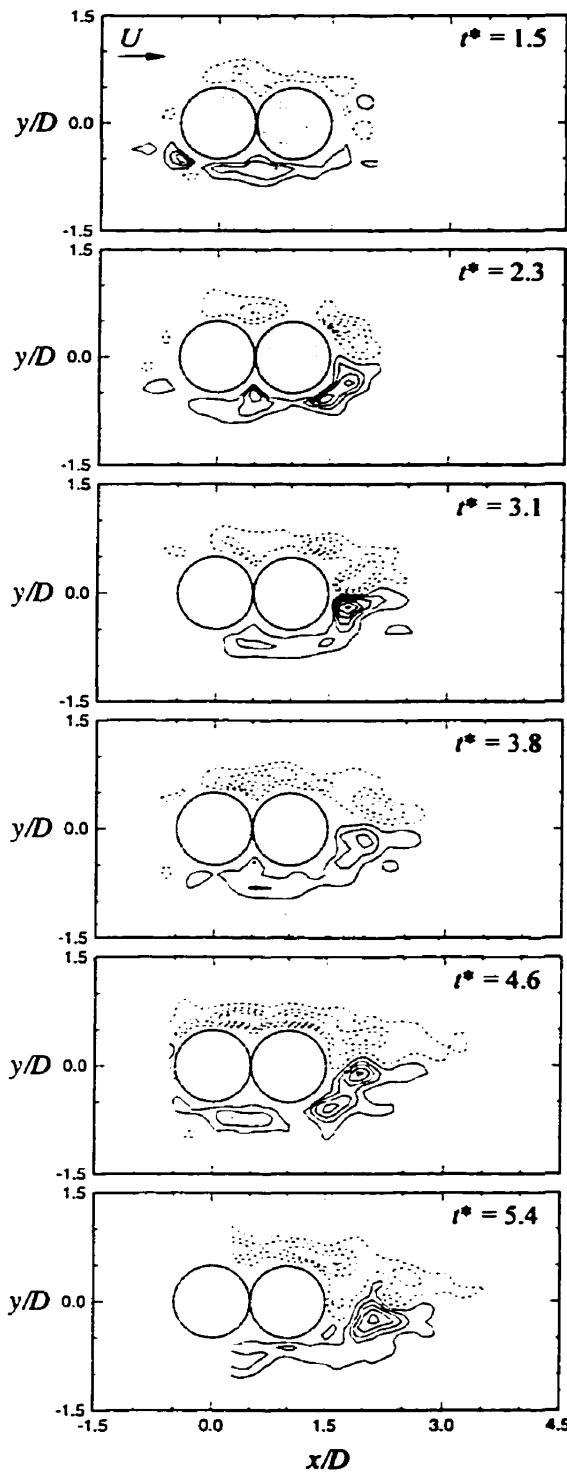


(a)

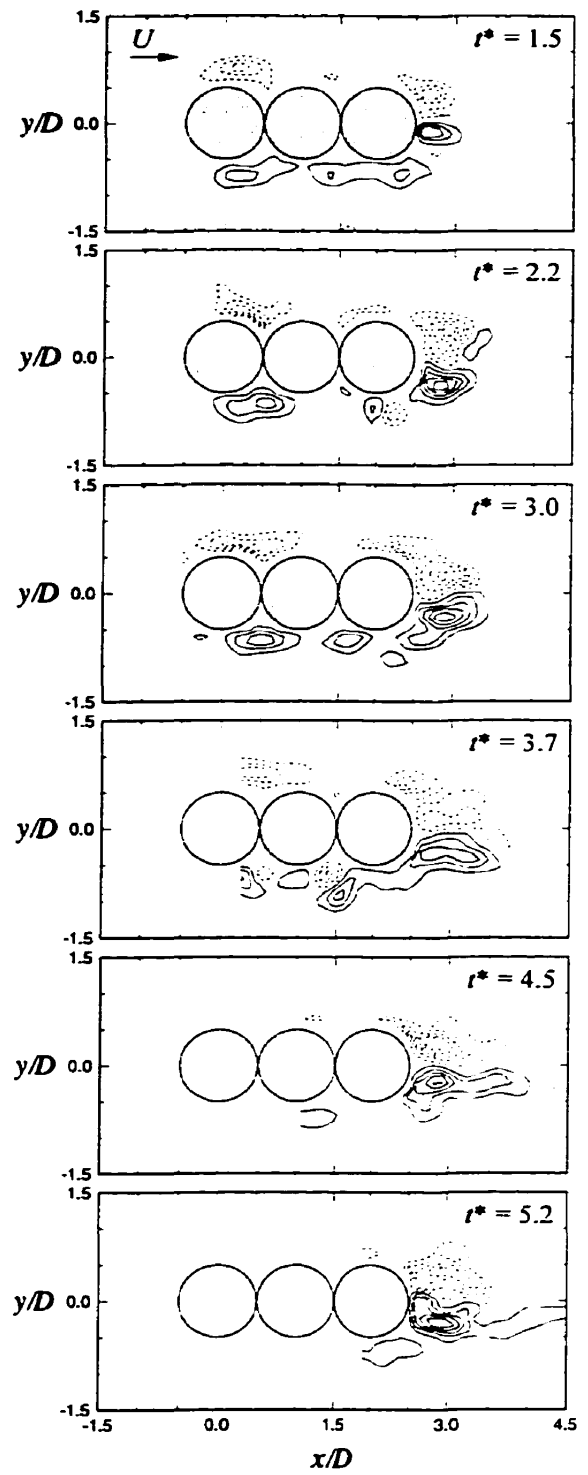


(b)

Figure 7.13. Flow visualization of two impulsively started tandem cylinders, $L/D = 1.0$: (a) two cylinders, $Re = 3400$, $A_p = 0.2$; (b) three cylinders, $Re = 2500$, $A_p = 0.3$.



(a)



(b)

Figure 7.14. Impulsively started tandem cylinders, $L/D = 1.0$, $Re = 2400$, $A_p = 0.3$, $M_x = 0.28$ mm/pixel: (a) two cylinders; (b) three cylinders. Minimum vorticity contour magnitude 0.75, contour increment 1.5, dashed lines represent negative (CW) vorticity.

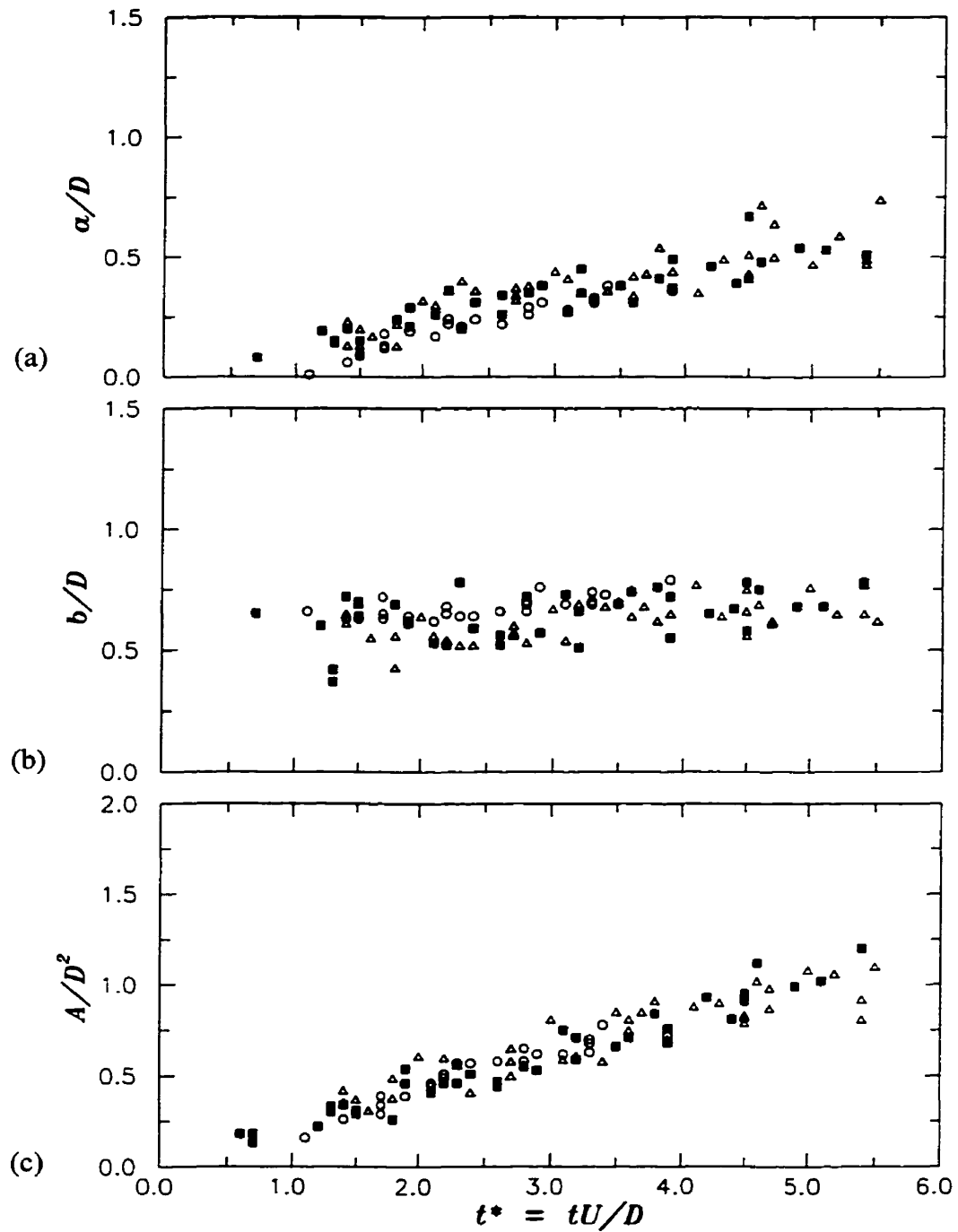


Figure 7.15. Recirculation zone data for impulsively started tandem cylinders in contact, $L/D = 1.0$: ○, one cylinder, $Re = 1500$ to 1800 , $A_p = 0.4$ to 0.6 , $M_r = 0.15$ to 0.20 mm/pixel, vorticity cutoff magnitude 1.0 ; ■, two cylinders, $Re = 1900$ to 2400 , $A_p = 0.3$ to 0.4 , $M_r = 0.28$ mm/pixel, vorticity cutoff magnitude 0.5 ; △, three cylinders, $Re = 2200$ to 2900 , $A_p = 0.3$ to 0.5 , $M_r = 0.28$ mm/pixel, vorticity cutoff magnitude 0.5 . (a) Streamwise position of the primary eddies from the base of the downstream cylinder; (b) transverse spacing of the primary eddies; (c) area of a primary eddy; (d) streamwise extent of the recirculation zone from the base of the downstream cylinder; (e) peak vorticity; (f) primary eddy circulation.

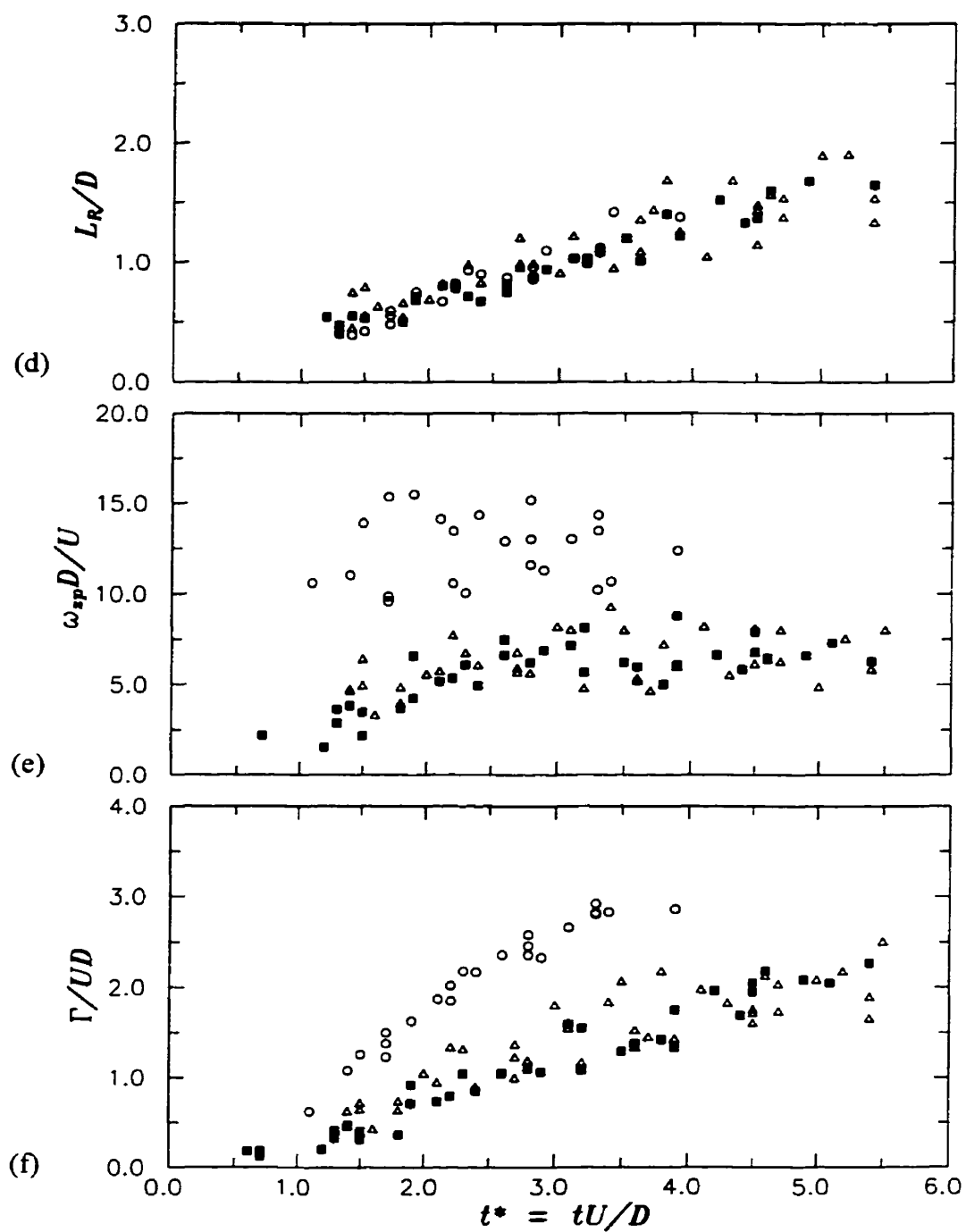


Figure 7.15 (continued).

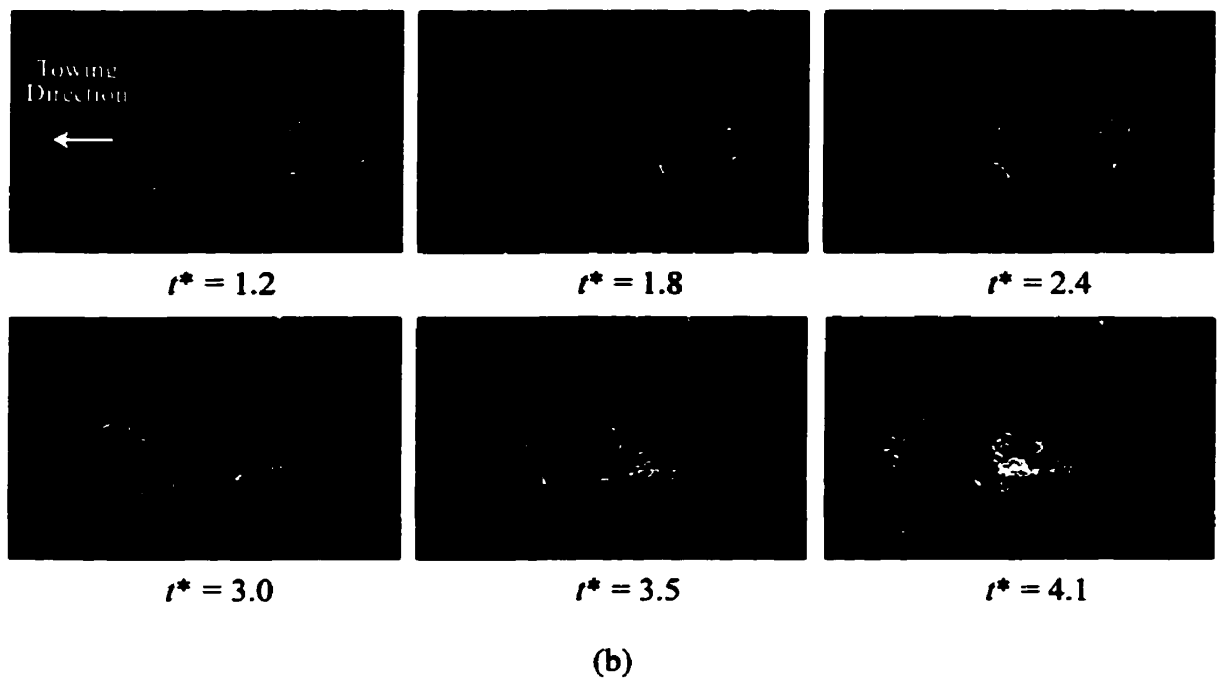
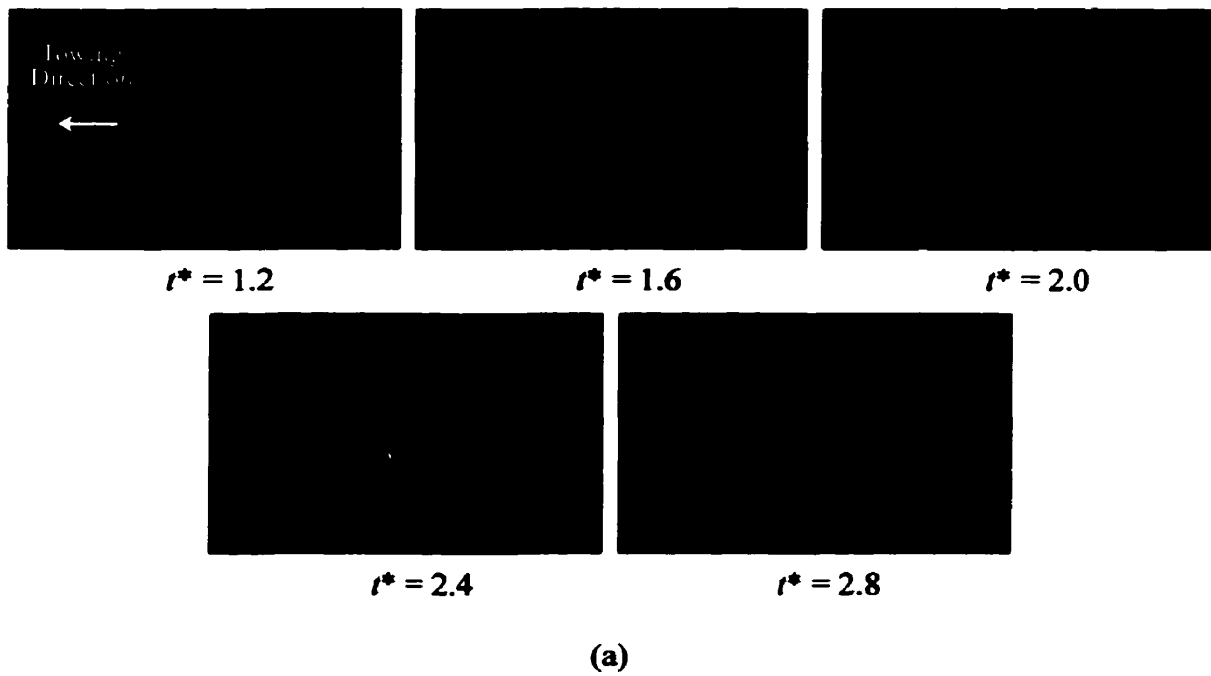


Figure 7.16. Flow visualization of two impulsively started tandem cylinders: (a) $L/D = 1.5$, $Re = 1300$, $A_p = 0.8$; (b) $L/D = 2.0$, $Re = 1900$, $A_p = 0.3$.

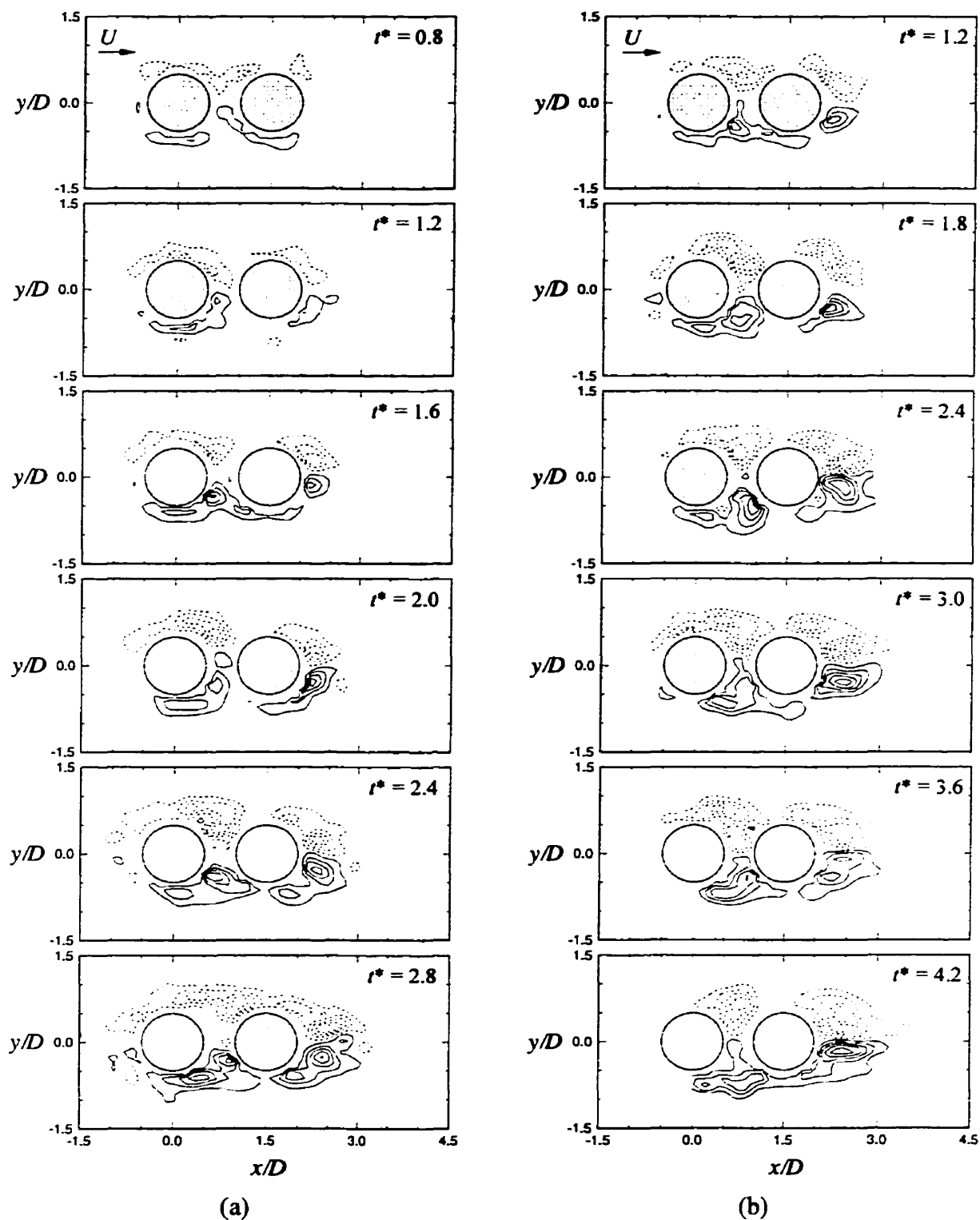
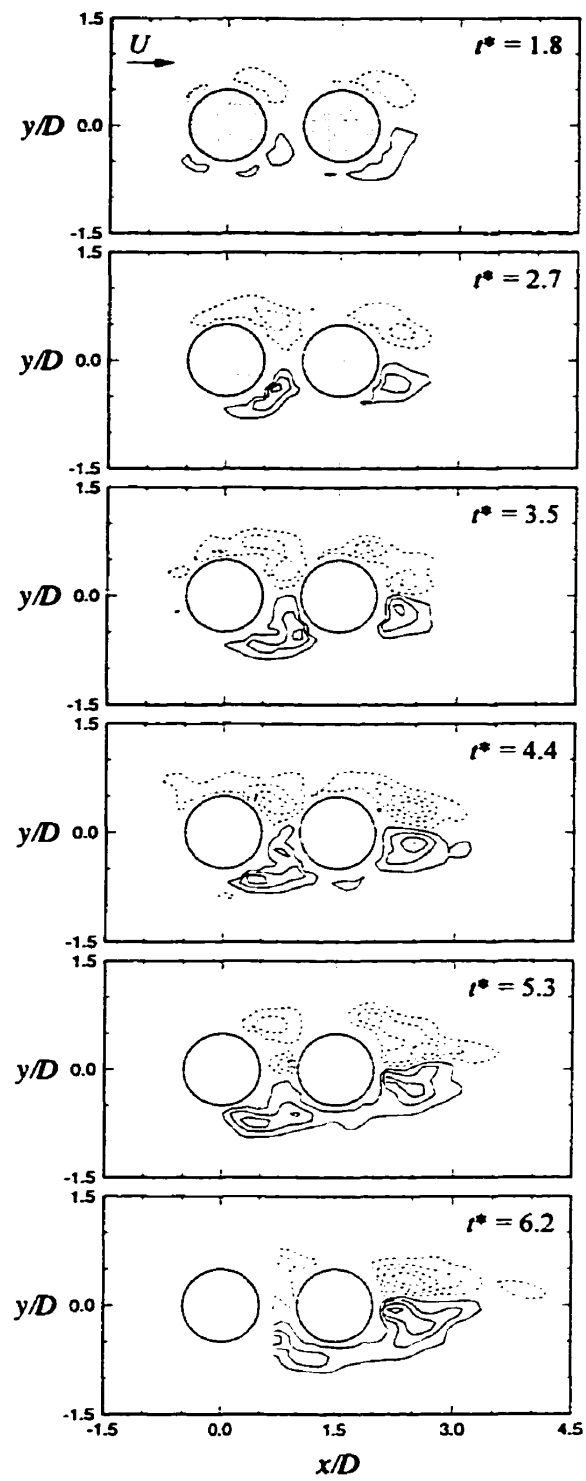


Figure 7.17. Two impulsively started tandem cylinders, $L/D = 1.5$, $M_r = 0.29$ mm/pixel: (a) $Re = 1300$, $A_p = 0.8$; (b) $Re = 1900$, $A_p = 0.3$; (c) $Re = 2800$, $A_p = 0.2$. Minimum vorticity contour magnitude 0.75, contour increment 1.5, dashed lines represent negative (CW) vorticity.



(c)

Figure 7.17 (continued).

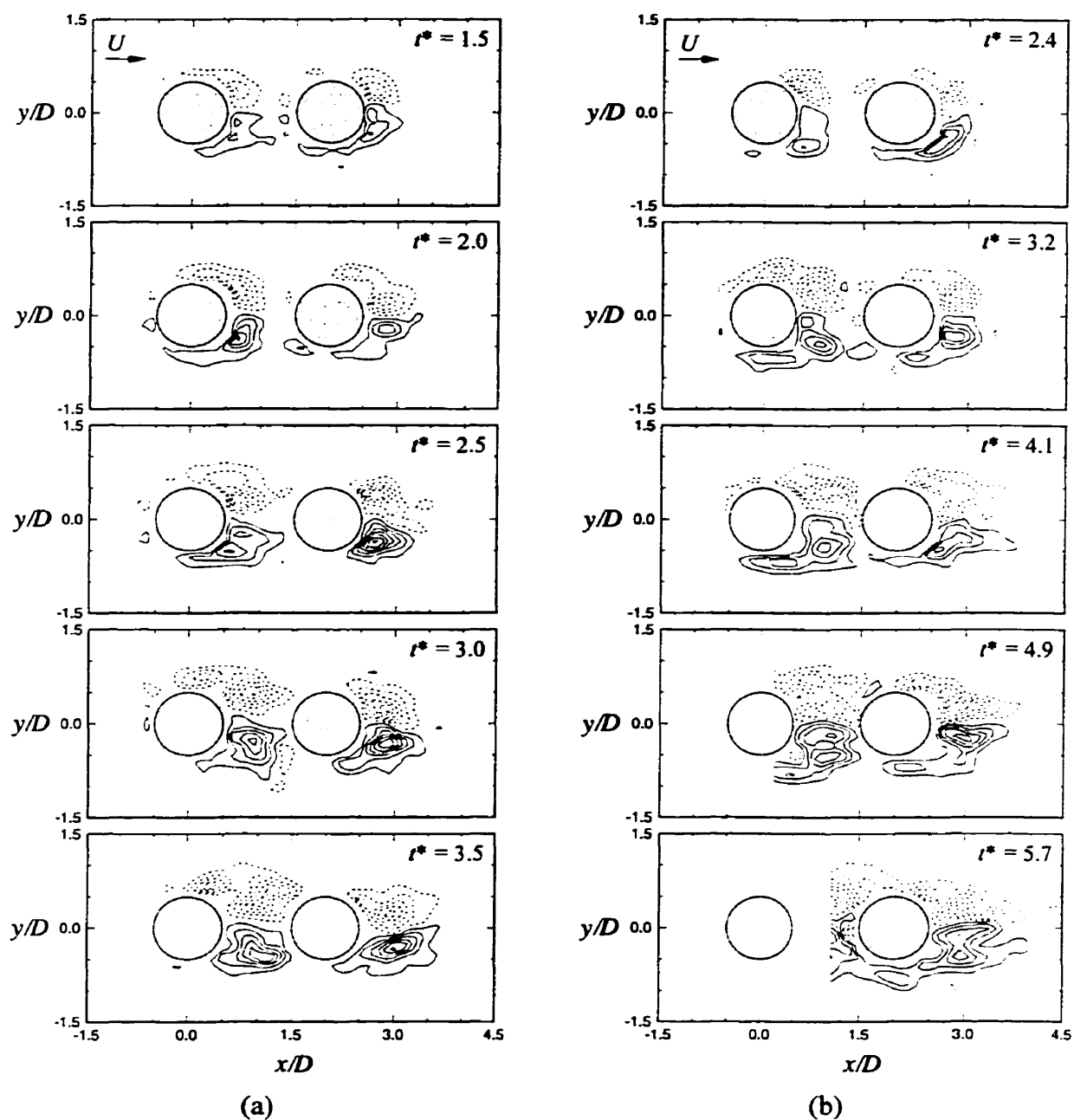


Figure 7.18. Two impulsively started tandem cylinders, $L/D = 2.0$, $M_t = 0.28$ mm/pixel: (a) $Re = 1600$, $A_p = 0.7$; (b) $Re = 2600$, $A_p = 0.3$. Minimum vorticity contour magnitude 0.75, contour increment 1.5, dashed lines represent negative (CW) vorticity.

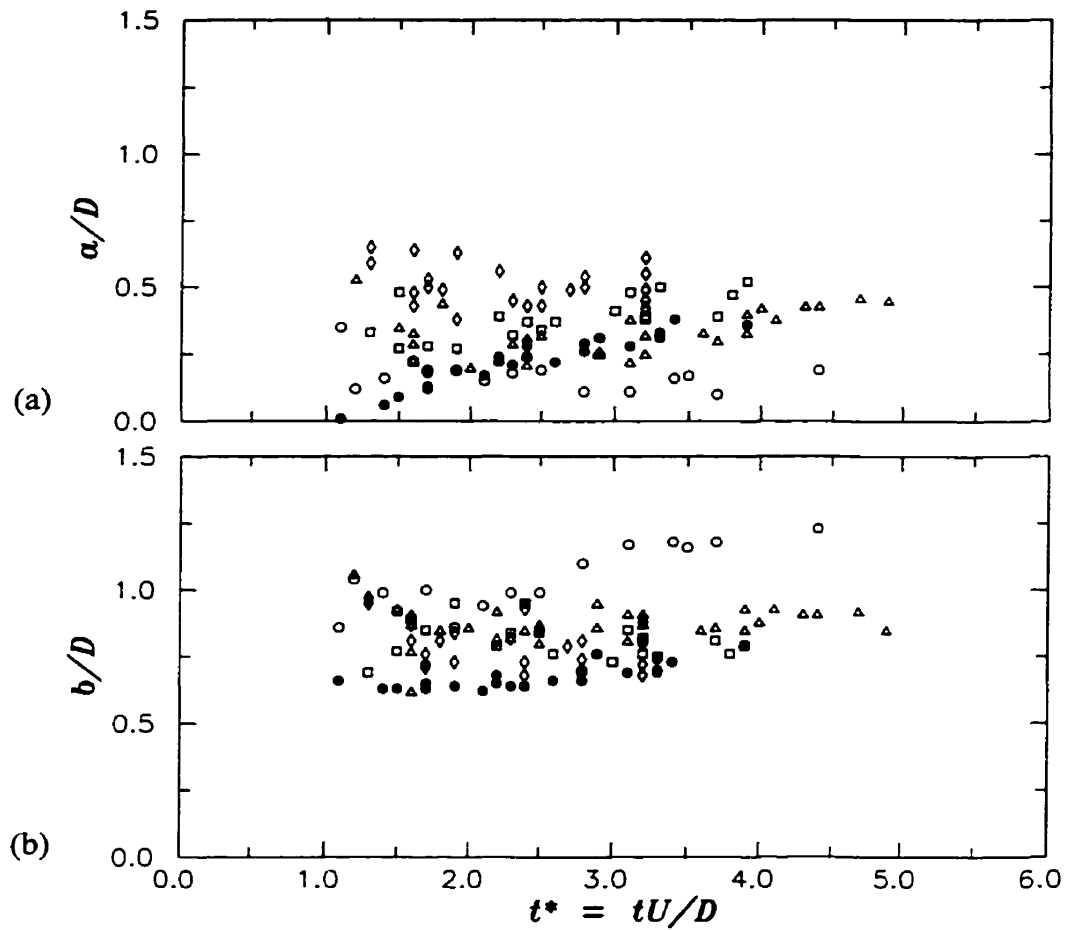


Figure 7.19. Recirculation zone data for two impulsively started tandem cylinders, upstream cylinder: $\circ$, $L/D = 1.5$, $Re = 1800$ to 2200 , $A_p = 0.3$ to 0.4 , $M_x = 0.29$ mm/pixel, vorticity cutoff magnitude 0.5 ; $\triangle$, $L/D = 2.0$, $Re = 2000$ to 2500 , $A_p = 0.3$ to 0.4 , $M_x = 0.28$ mm/pixel, vorticity cutoff magnitude 0.5 ; $\square$, $L/D = 2.5$, $Re = 2100$ to 2600 , $A_p = 0.3$ to 0.4 , $M_x = 0.28$ mm/pixel, vorticity cutoff magnitude 0.5 ; $\diamond$, $L/D = 3.0$, $Re = 1800$ to 2900 , $A_p = 0.3$ to 0.6 , $M_x = 0.28$ mm/pixel, vorticity cutoff magnitude 0.5 ; $\bullet$, single cylinder, $Re = 1500$ to 1800 , $A_p = 0.4$ to 0.6 , $M_x = 0.15$ to 0.20 mm/pixel, vorticity cutoff magnitude 1.0 . (a) Streamwise position of the primary eddies from the base of the cylinder; (b) transverse spacing of the primary eddies; (c) area of a primary eddy; (d) streamwise extent of the recirculation zone from the base of the cylinder; (e) peak vorticity; (f) primary eddy circulation.

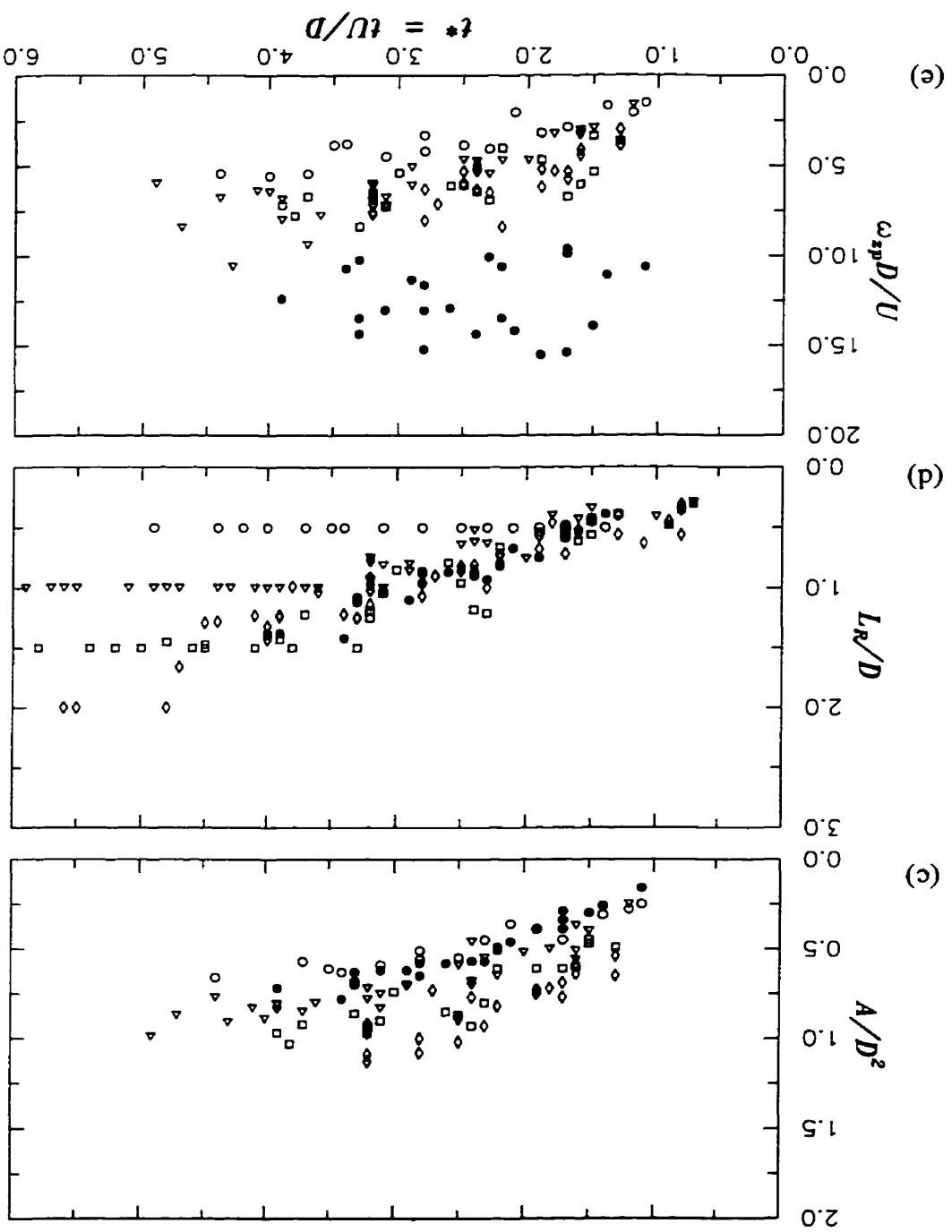


Figure 7.19 (continued).

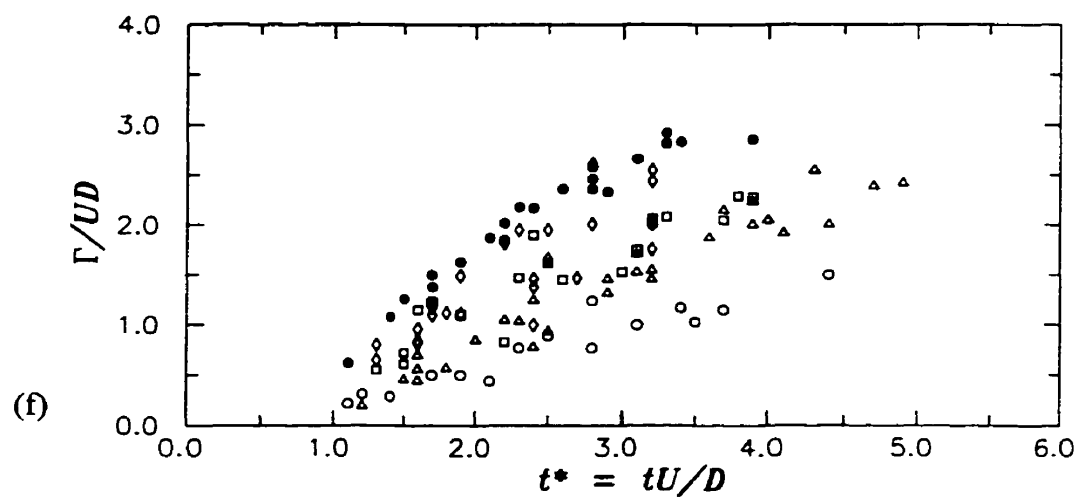


Figure 7.19 (continued).

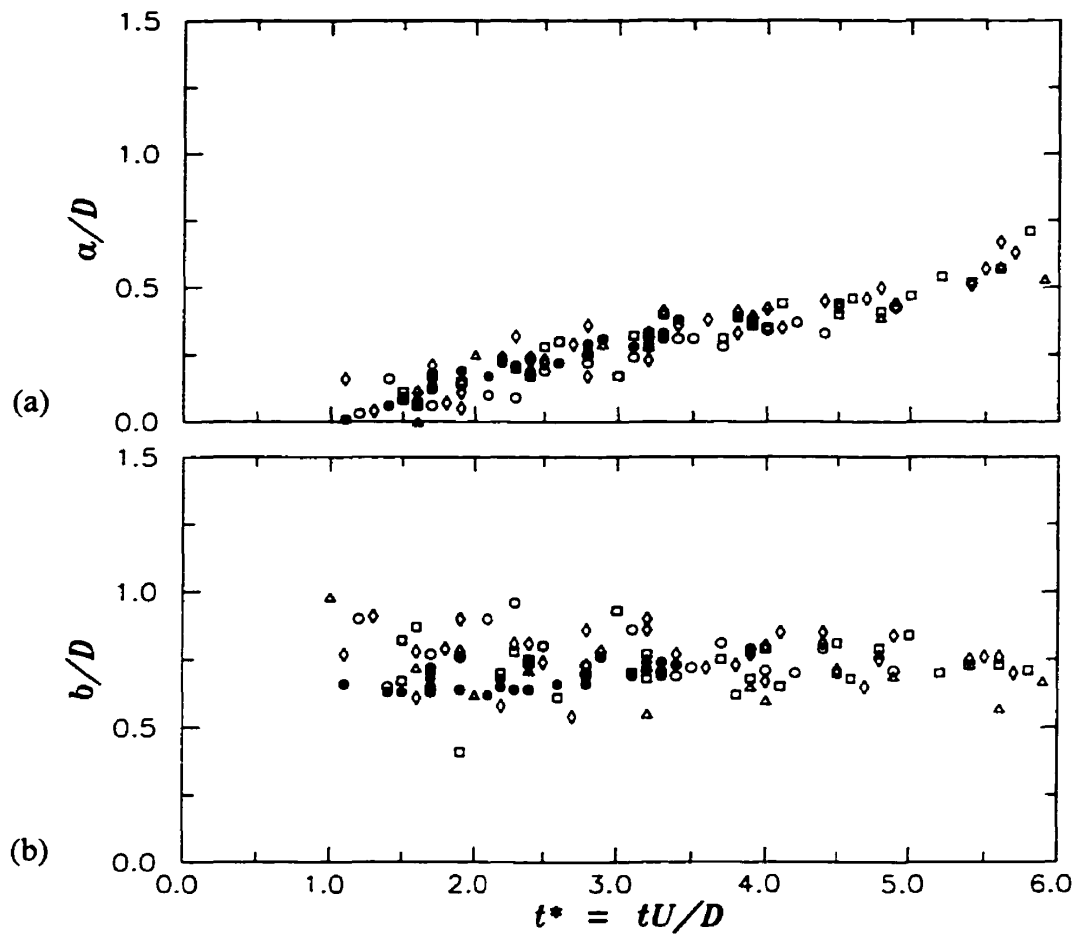


Figure 7.20. Recirculation zone data for two impulsively started tandem cylinders, downstream cylinder: $\circ$, $L/D = 1.5$, $Re = 1800$ to 2200 , $A_p = 0.3$ to 0.4 , $M_r = 0.29$ mm/pixel, vorticity cutoff magnitude 0.5 ; $\triangle$, $L/D = 2.0$, $Re = 2000$ to 2500 , $A_p = 0.3$ to 0.4 , $M_r = 0.28$ mm/pixel, vorticity cutoff magnitude 0.5 ; $\square$, $L/D = 2.5$, $Re = 2100$ to 2600 , $A_p = 0.3$ to 0.4 , $M_r = 0.28$ mm/pixel, vorticity cutoff magnitude 0.5 ; $\diamond$, $L/D = 3.0$, $Re = 1800$ to 2900 , $A_p = 0.3$ to 0.6 , $M_r = 0.28$ mm/pixel, vorticity cutoff magnitude 0.5 ; $\bullet$, single cylinder, $Re = 1500$ to 1800 , $A_p = 0.4$ to 0.6 , $M_r = 0.15$ to 0.20 mm/pixel, vorticity cutoff magnitude 1.0 . (a) Streamwise position of the primary eddies from the base of the cylinder; (b) transverse spacing of the primary eddies; (c) area of a primary eddy; (d) streamwise extent of the recirculation zone from the base of the cylinder; (e) peak vorticity; (f) primary eddy circulation.

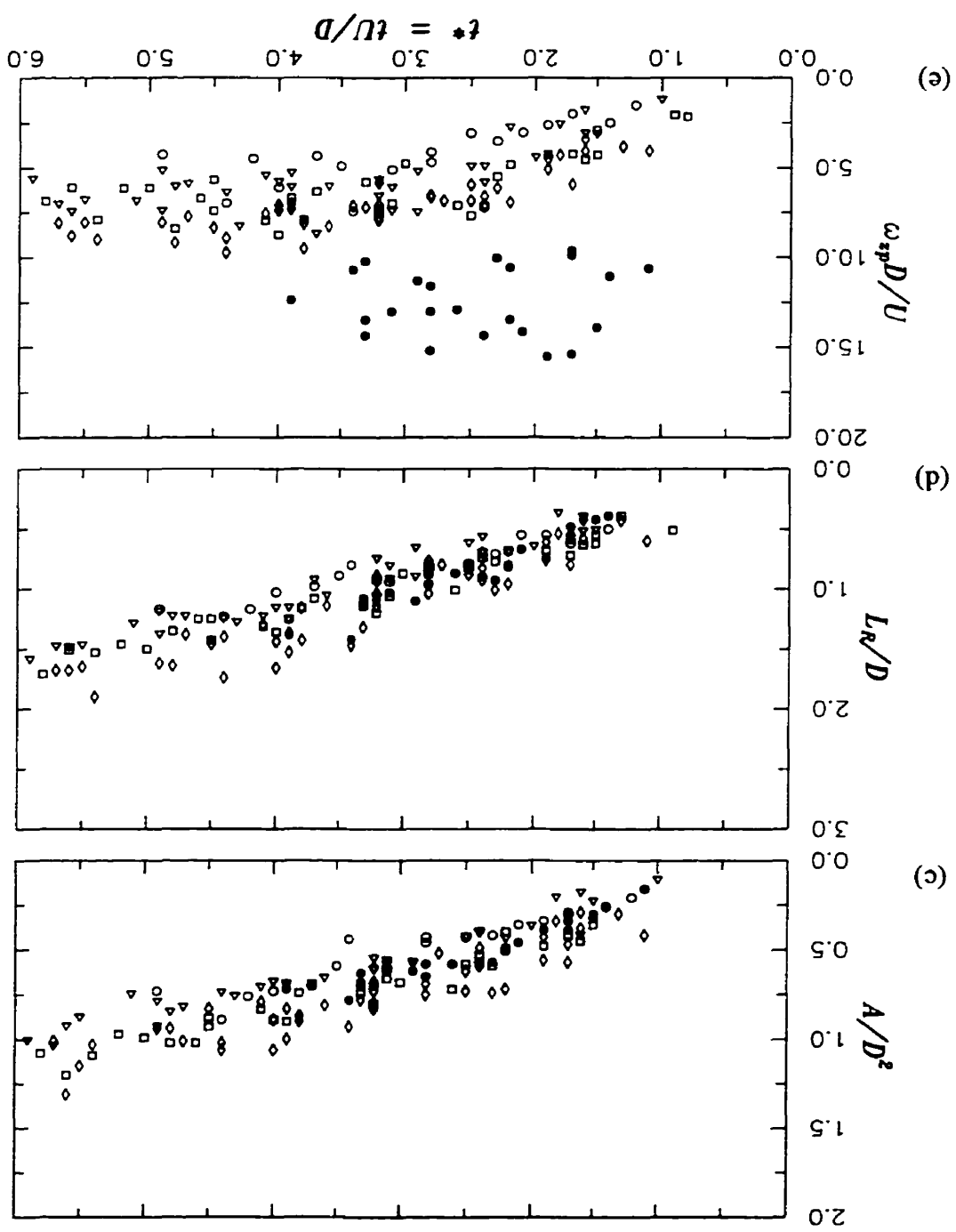


Figure 7.20 (continued).

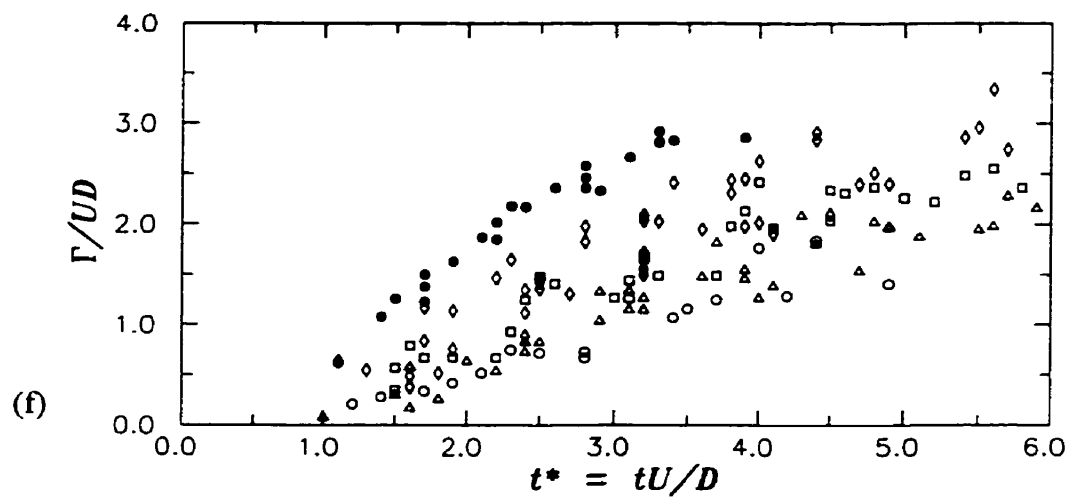
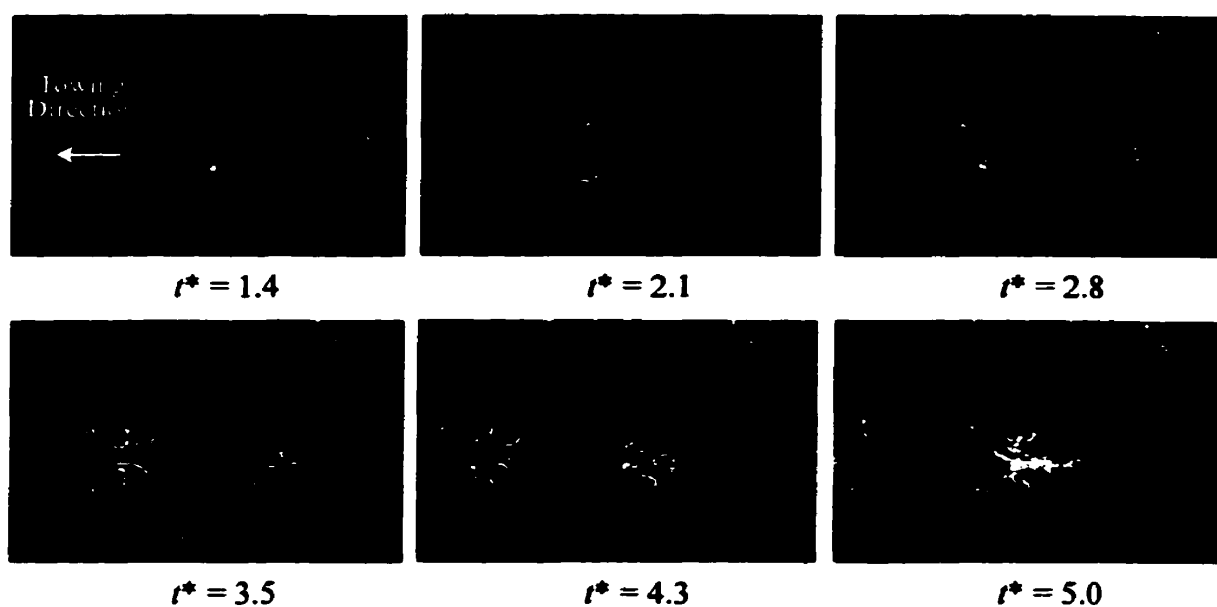
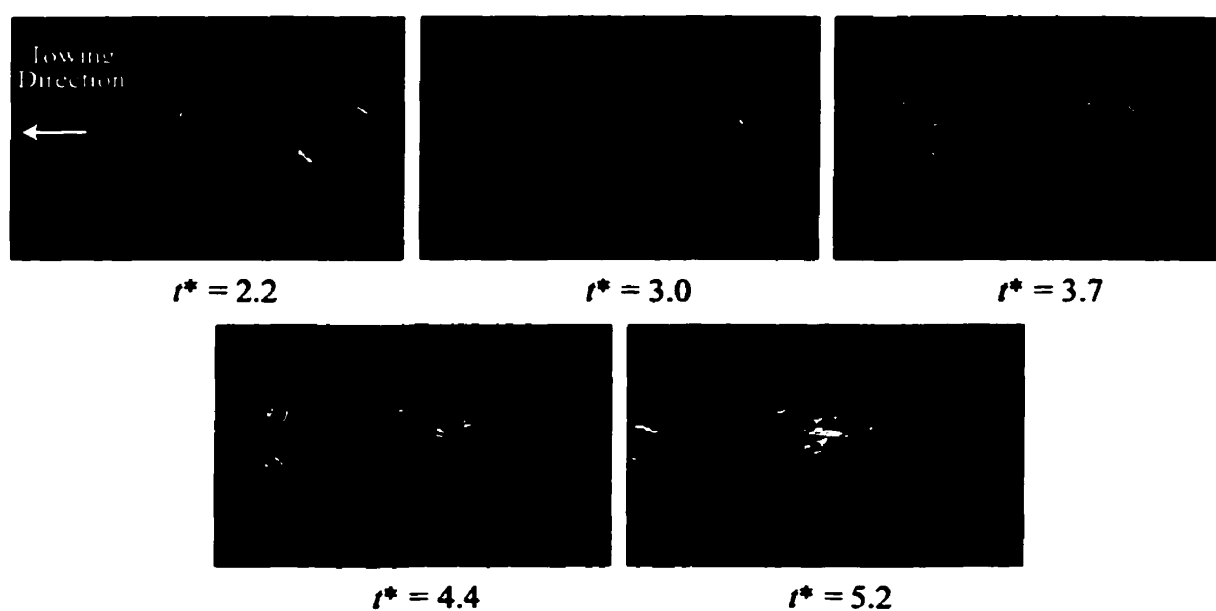


Figure 7.20 (continued).



(a)



(b)

Figure 7.21. Flow visualization of two impulsively started tandem cylinders: (a) $L/D = 2.5$, $Re = 2300$, $A_p = 0.2$; (b) $L/D = 3.0$, $Re = 2400$, $A_p = 0.3$.

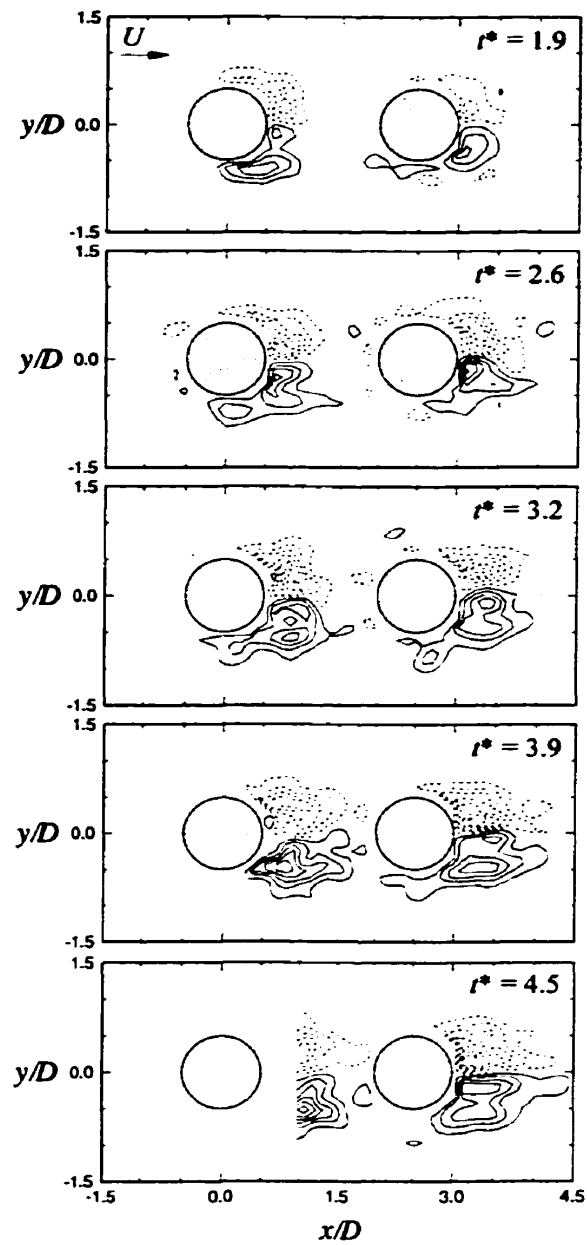


Figure 7.22. Two impulsively started tandem cylinders, $L/D = 2.5$, $Re = 2100$, $A_p = 0.3$, $M_x = 0.28$ mm/pixel. Minimum vorticity contour magnitude 0.75, contour increment 1.5, dashed lines represent negative (CW) vorticity.

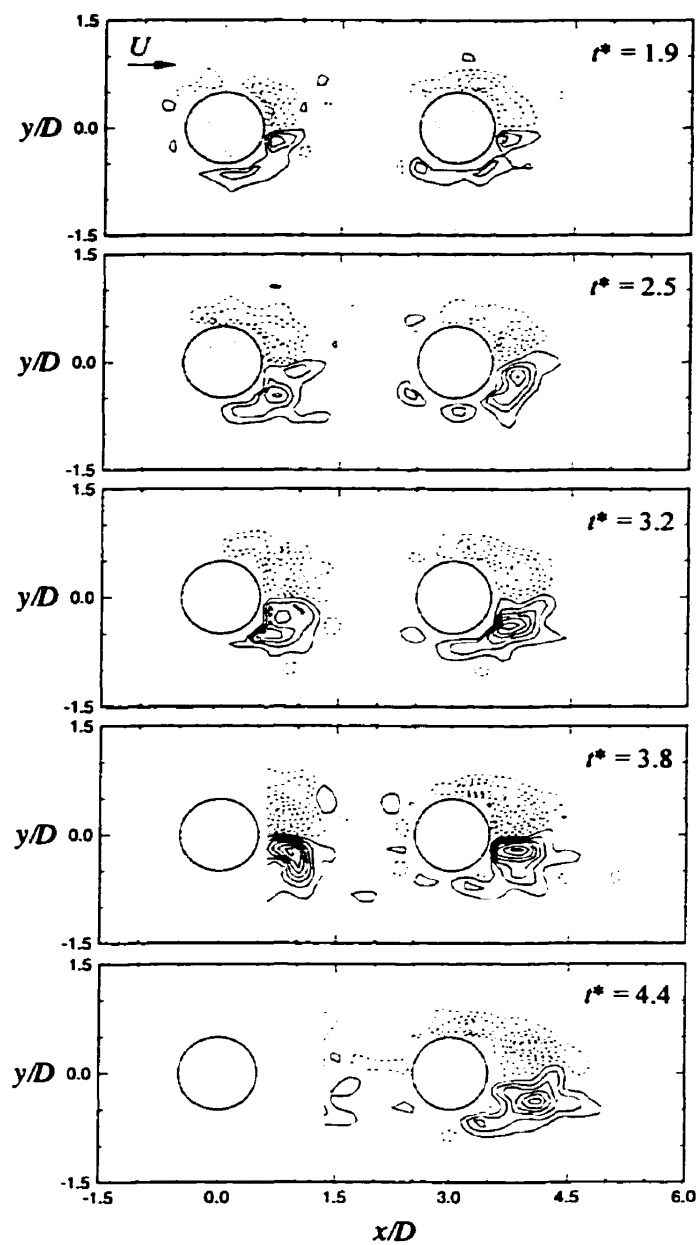


Figure 7.23. Two impulsively started tandem cylinders, $L/D = 3.0$, $Re = 2000$, $A_p = 0.4$, $M_x = 0.28$ mm/pixel. Minimum vorticity contour magnitude 0.75, contour increment 1.5, dashed lines represent negative (CW) vorticity.

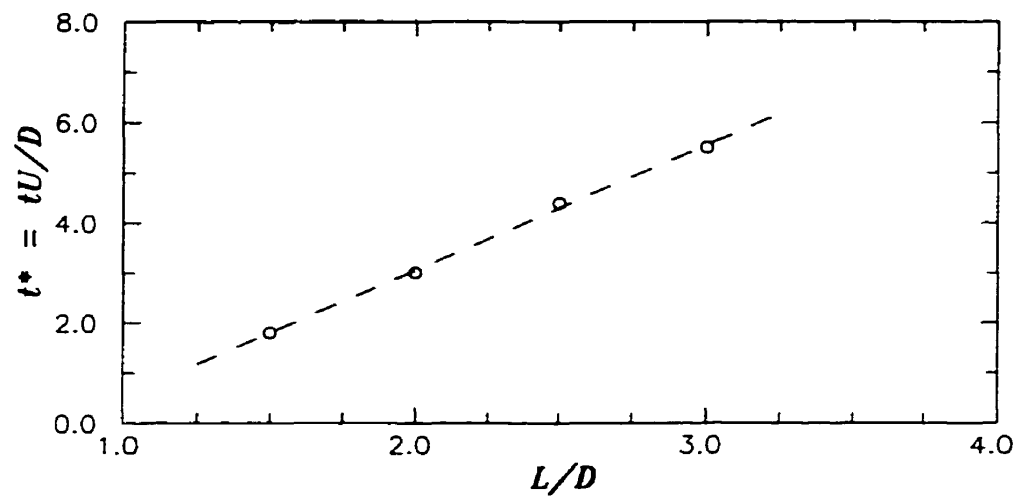


Figure 7.24. Time at which the primary eddies completely fill the gap between two impulsively started tandem circular cylinders.

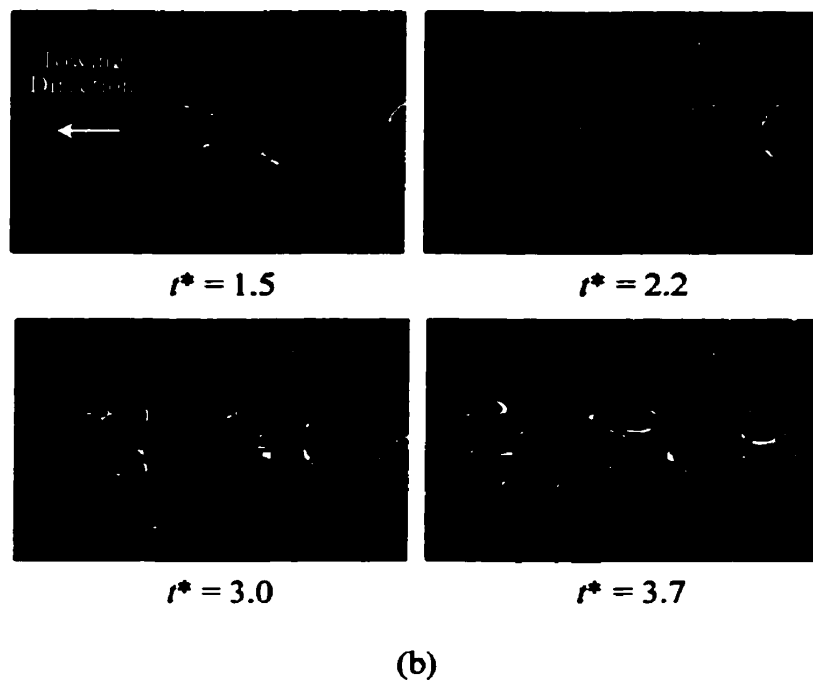
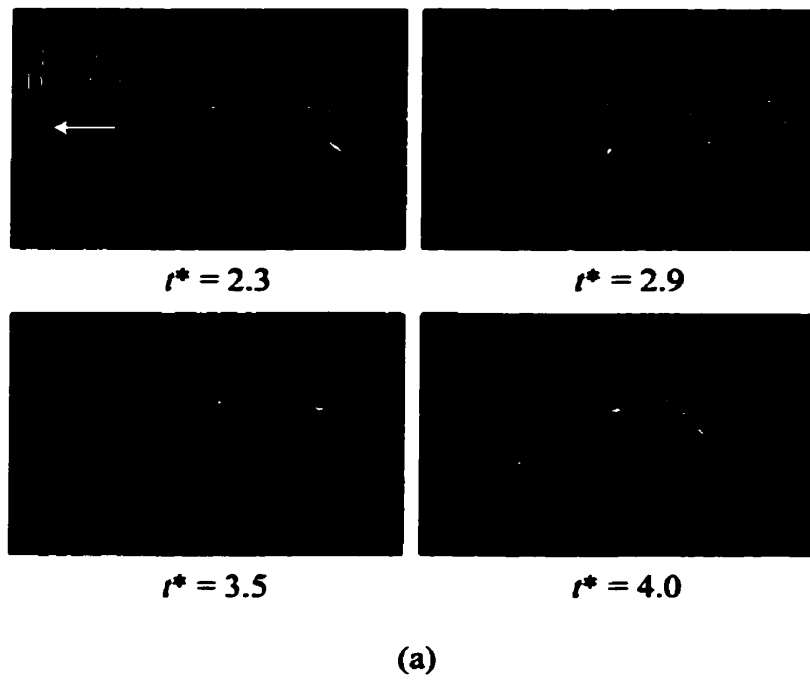


Figure 7.25. Flow visualization of three impulsively started tandem cylinders: (a) $L/D = 1.5$, $Re = 1800$, $A_p = 0.3$; (b) $L/D = 2.0$, $Re = 2400$, $A_p = 0.3$.

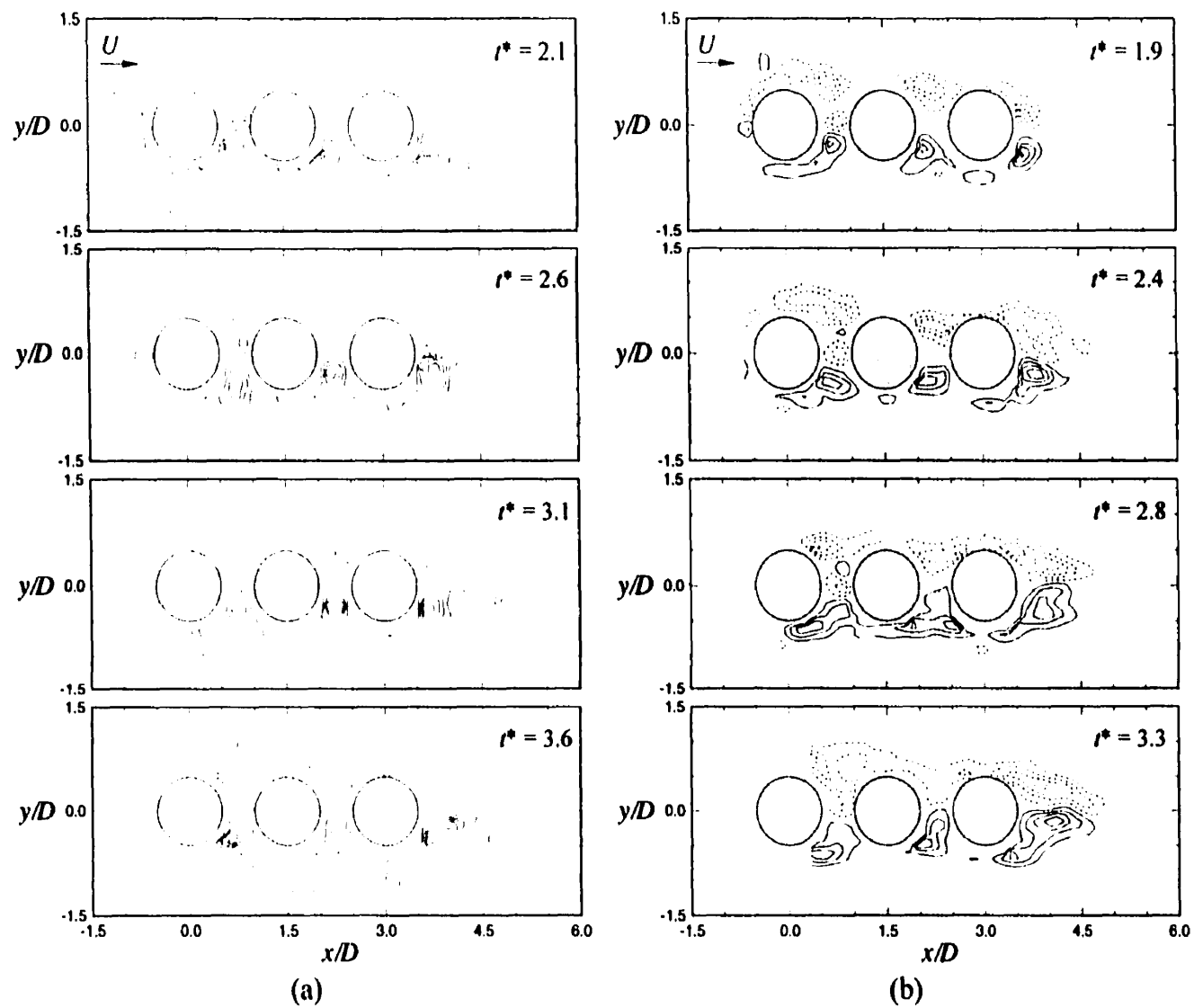


Figure 7.26. Three impulsively started tandem cylinders, $L/D = 1.5$: (a) $Re = 1600$, $A_p = 0.7$, $M_t = 0.29$ mm/pixel; (b) $Re = 1500$, $A_p = 0.4$, $M_t = 0.28$ mm/pixel. Minimum vorticity contour magnitude 0.75, contour increment 1.5, dashed lines represent negative (CW) vorticity.

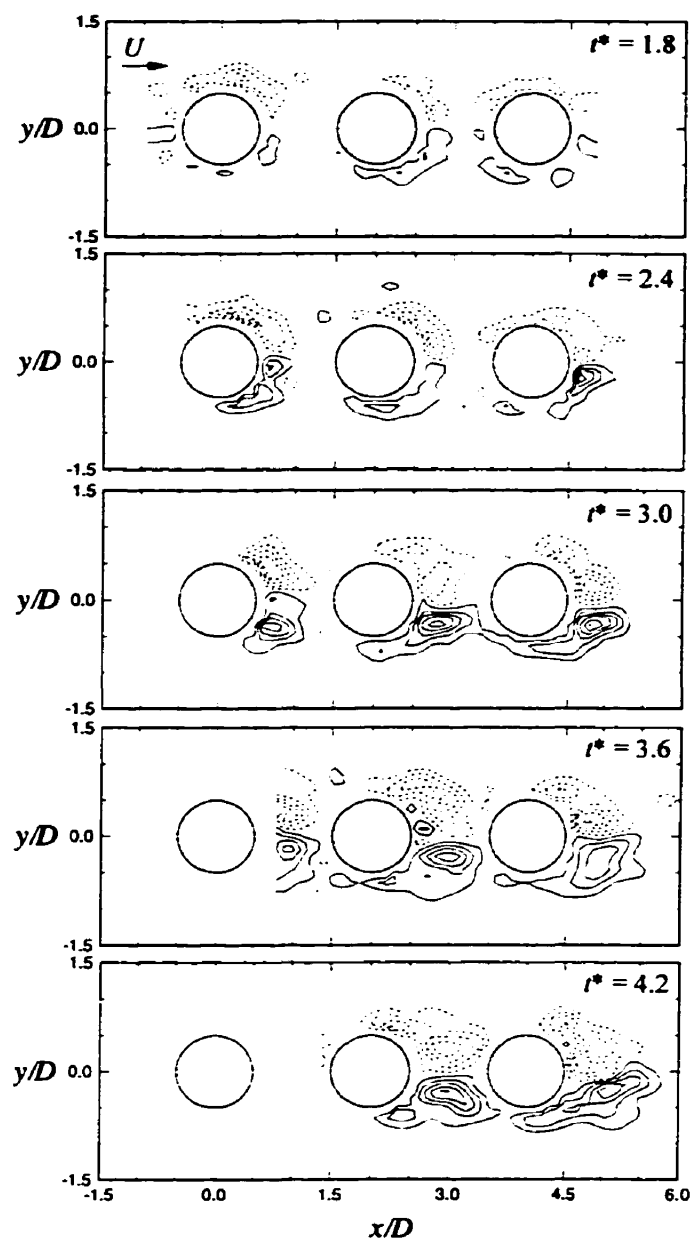


Figure 7.27. Three impulsively started tandem cylinders, $L/D=2.0$, $Re = 1900$, $A_p = 0.4$, $M_x = 0.29$ mm/pixel. Minimum vorticity contour magnitude 0.75, contour increment 1.5, dashed lines represent negative (CW) vorticity.

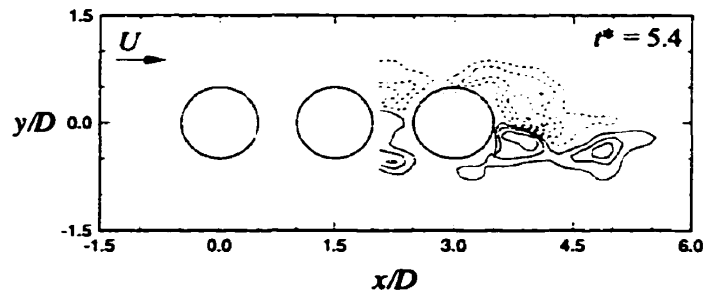


Figure 7.28. Breakup of the recirculation zone and the onset of Kármán vortex shedding for three impulsively started tandem cylinders, $L/D = 1.5$, $Re = 2900$, $A_p = 0.3$, $M_r = 0.28$ mm/pixel. Minimum vorticity contour magnitude 0.75, contour increment 1.5, dashed lines represent negative (CW) vorticity.

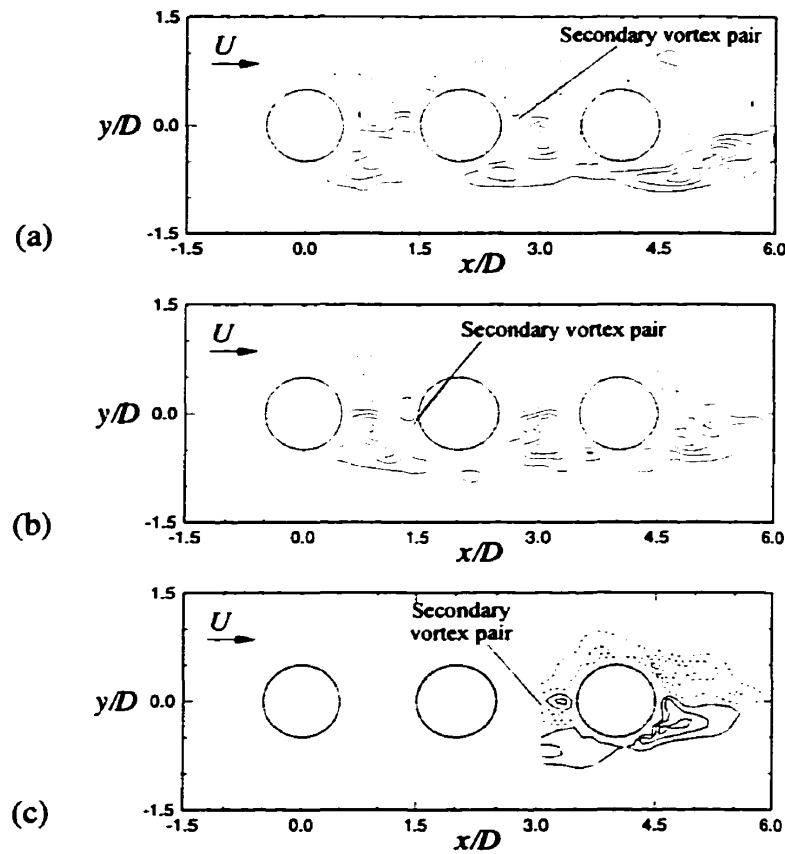


Figure 7.29. Secondary vorticity for three impulsively started tandem cylinders, $L/D = 2.0$, $M_r = 0.29$ mm/pixel: (a) behind second cylinder, $Re = 2200$, $A_p = 0.4$, $t^* = 4.1$; (b) in front of second cylinder, $Re = 3200$, $A_p = 0.3$, $t^* = 5.1$; (c) in front of third cylinder, with asymmetric recirculation zone, $Re = 3100$, $A_p = 0.3$, $t^* = 6.7$. Minimum vorticity contour magnitude 0.75, contour increment 1.5, dashed lines represent negative (CW) vorticity.

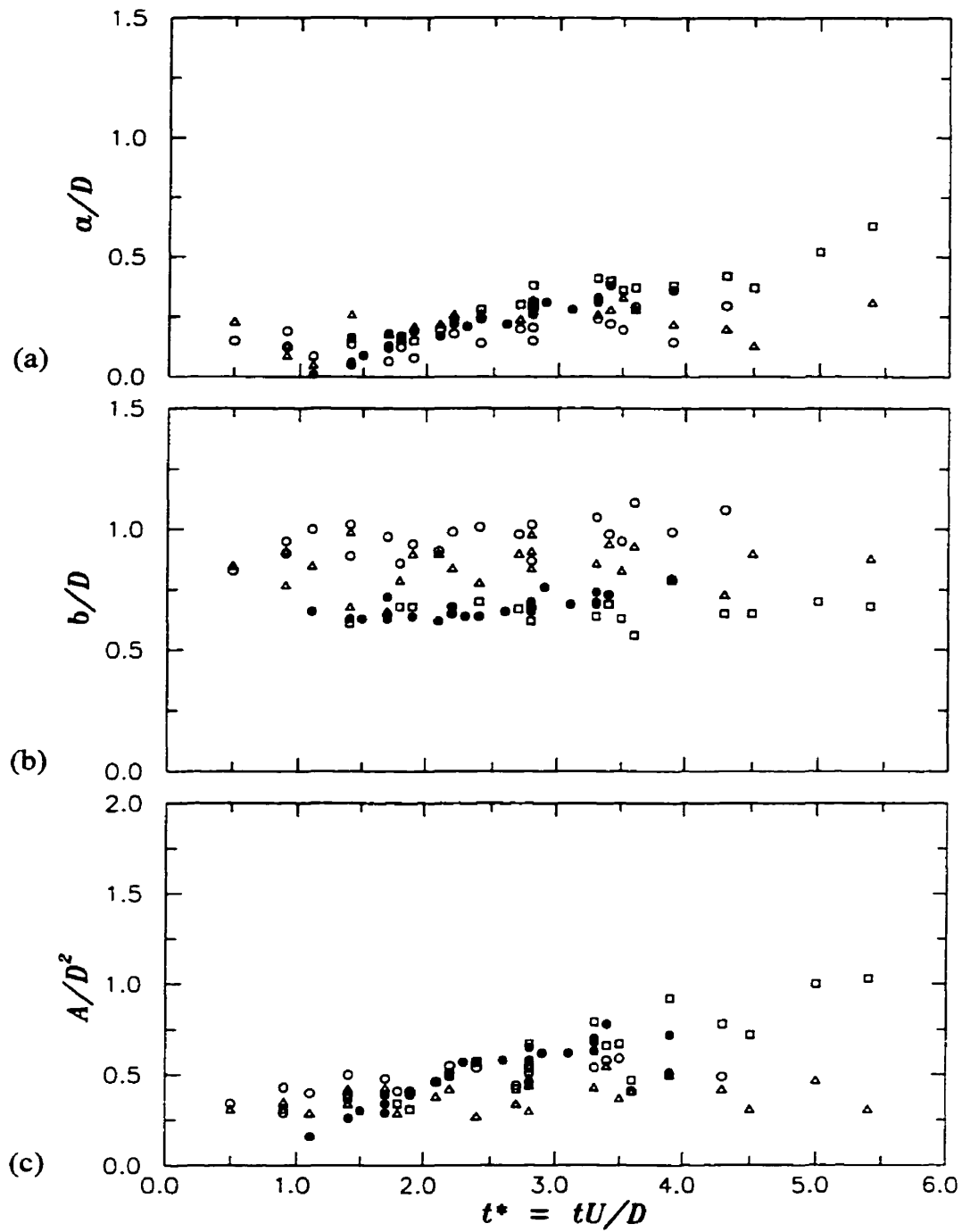


Figure 7.30. Recirculation zone data for three impulsively started tandem cylinders, $L/D = 1.5$, $Re = 1500$ to 2900 , $A_p = 0.3$ to 0.6 , $M_x = 0.28$ to 0.29 mm/pixel, vorticity cutoff magnitude 0.5 : $\circ$, upstream cylinder; $\triangle$, middle cylinder; $\square$, downstream cylinder; $\bullet$, single cylinder, $Re = 1500$ to 1800 , $A_p = 0.4$ to 0.6 , $M_x = 0.15$ to 0.20 mm/pixel, vorticity cutoff magnitude 1.0 . (a) Streamwise position of the primary eddies from the base of the cylinder; (b) transverse spacing of the primary eddies; (c) area of a primary eddy; (d) streamwise extent of the recirculation zone from the base of the cylinder; (e) peak vorticity; (f) primary eddy circulation.

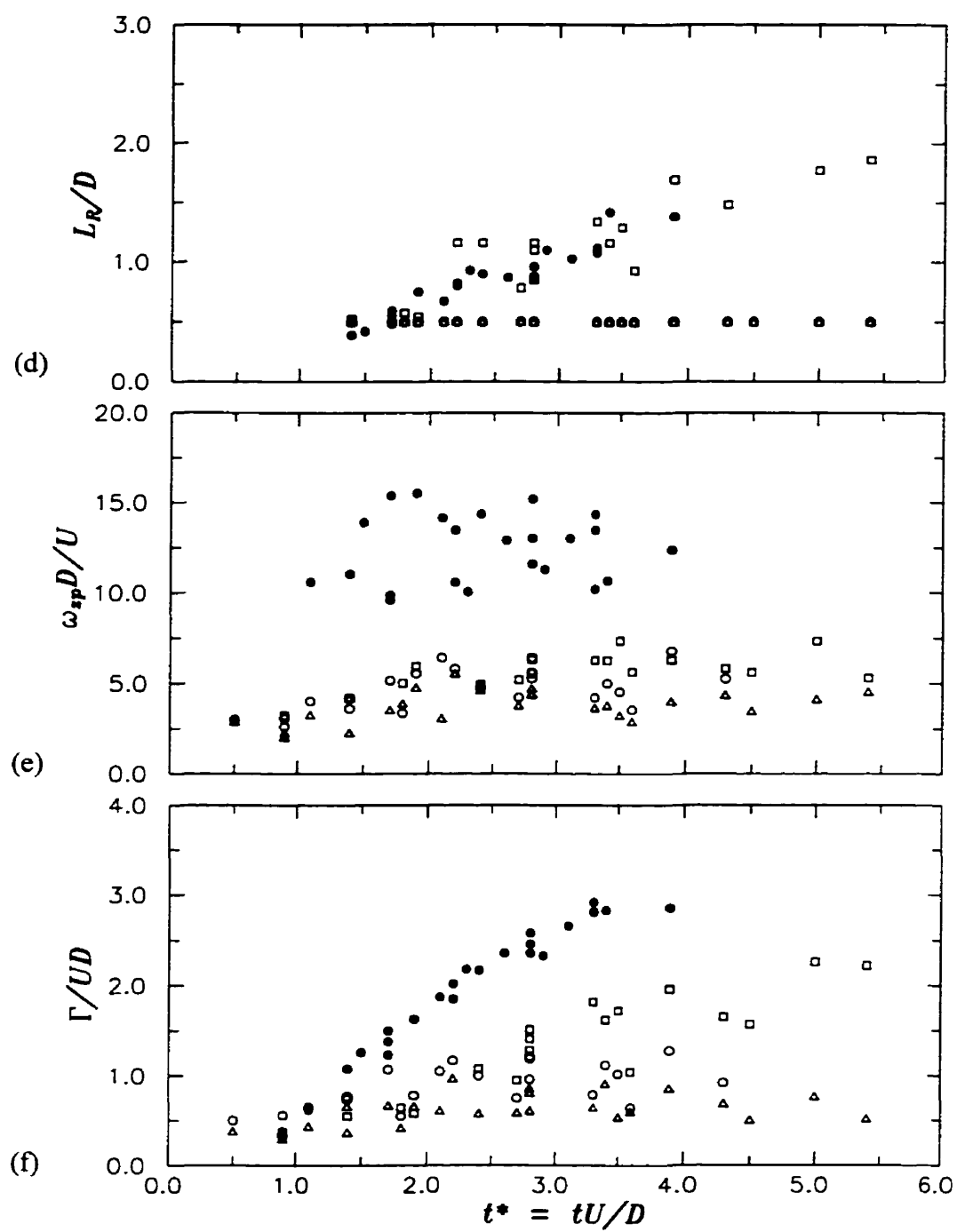


Figure 7.30 (continued).

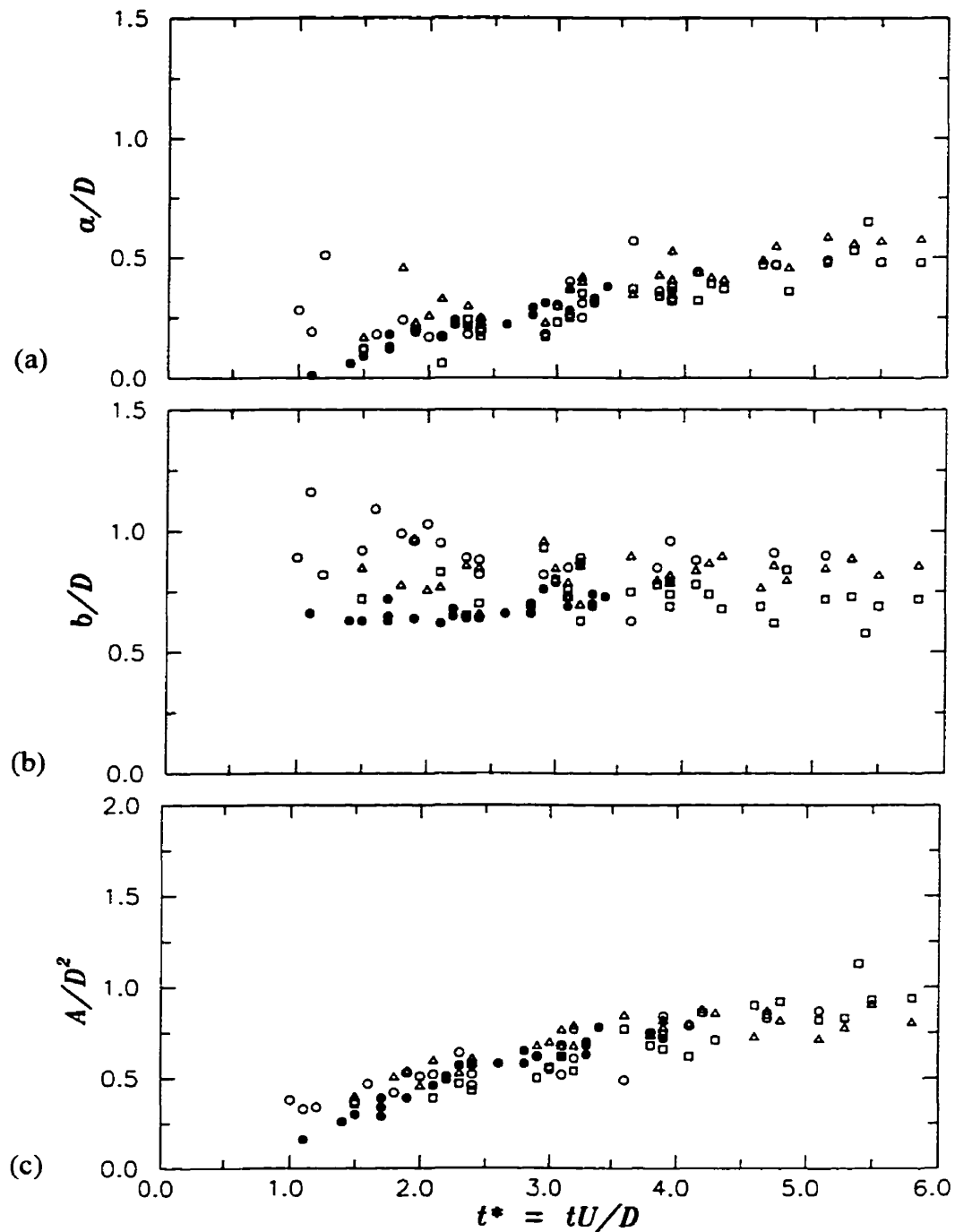


Figure 7.31. Recirculation zone data for three impulsively started tandem cylinders, $L/D = 2.0$, $Re = 1900$ to 3400 , $A_p = 0.3$ to 0.4 , $M_r = 0.28$ mm/pixel, vorticity cutoff magnitude 0.5 : $\circ$, upstream cylinder; $\triangle$, middle cylinder; $\square$, downstream cylinder; $\bullet$, single cylinder, $Re = 1500$ to 1800 , $A_p = 0.4$ to 0.6 , $M_r = 0.15$ to 0.20 mm/pixel, vorticity cutoff magnitude 1.0 . (a) Streamwise position of the primary eddies from the base of the cylinder; (b) transverse spacing of the primary eddies; (c) area of a primary eddy; (d) streamwise extent of the recirculation zone from the base of the cylinder; (e) peak vorticity; (f) primary eddy circulation.

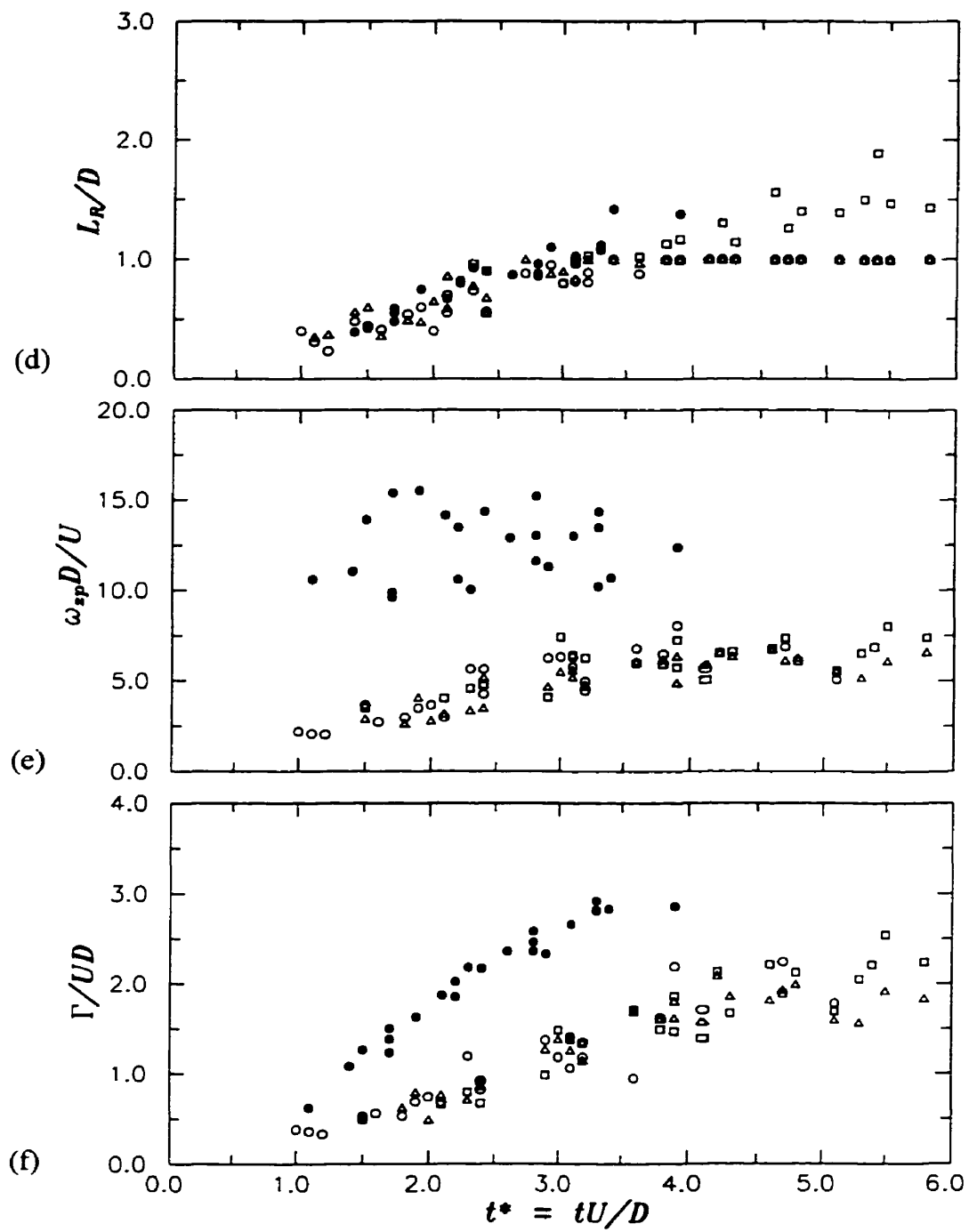


Figure 7.31 (continued).

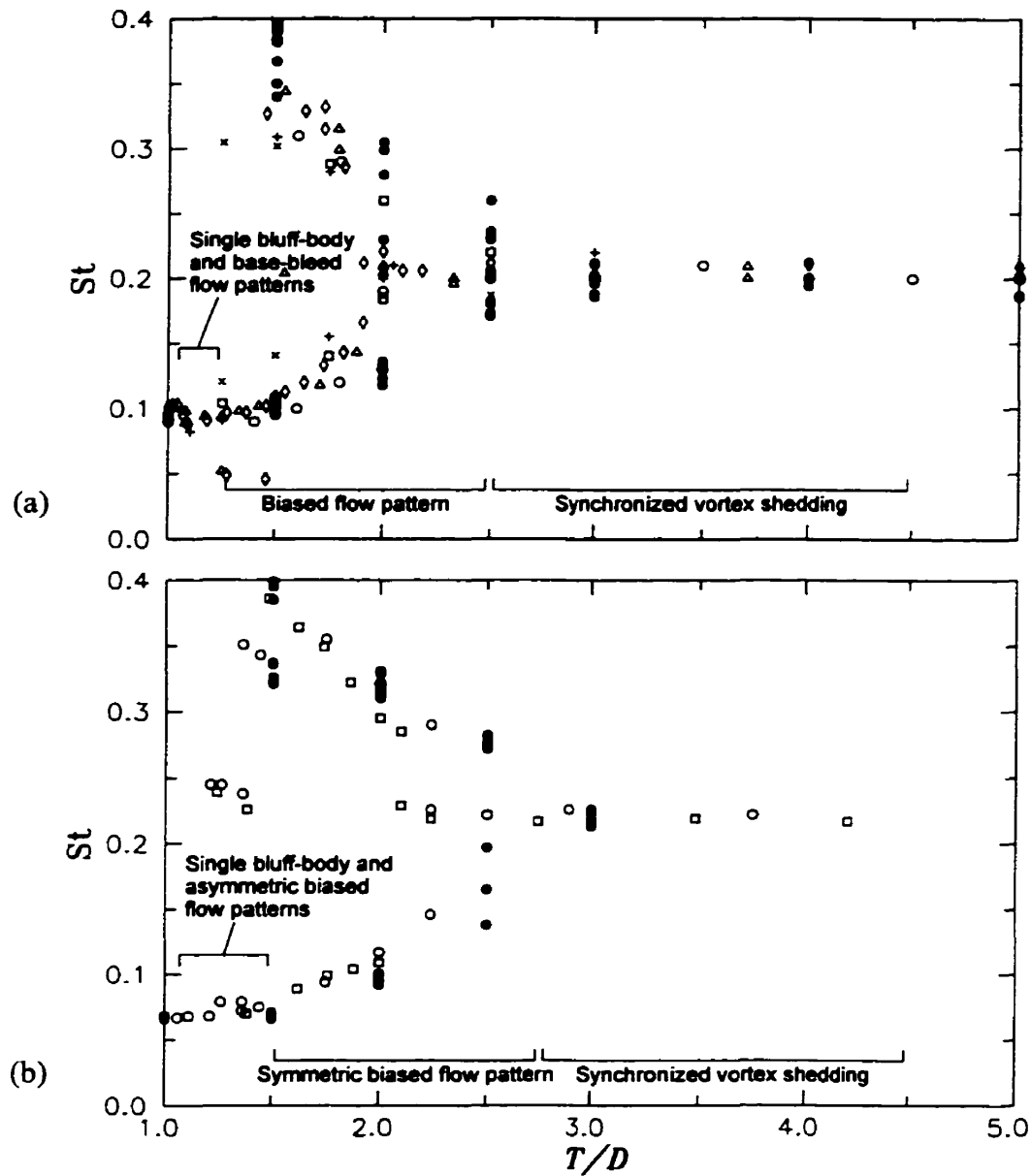


Figure 8.1. Strouhal number data for side-by-side cylinder configurations in steady cross-flow. (a) Two cylinders: $\circ$, Kiya *et al.* (1980), $Re = 15,800$; $\square$, Kim & Durbin (1988), $Re = 3300$; $\triangle$, Bearman & Wadcock (1973), $Re = 25,000$; $\diamond$, Spivack (1946), $Re = 5000$ to $93,000$; $\times$, Ishigai *et al.* (1972), $Re = 1500$ to $15,000$; $+$, Kamemoto (1976), $Re = 662$ and $30,000$; $\bullet$, present study, $Re = 500$ to 3000 . (b) Three cylinders: $\circ$, Kumada *et al.* (1984), $Re = 10,000$ to $32,000$; $\square$, Eastop & Turner (1982), $Re = 45,000$ to $111,000$; $\bullet$, present study, $Re = 500$ to 3000 .

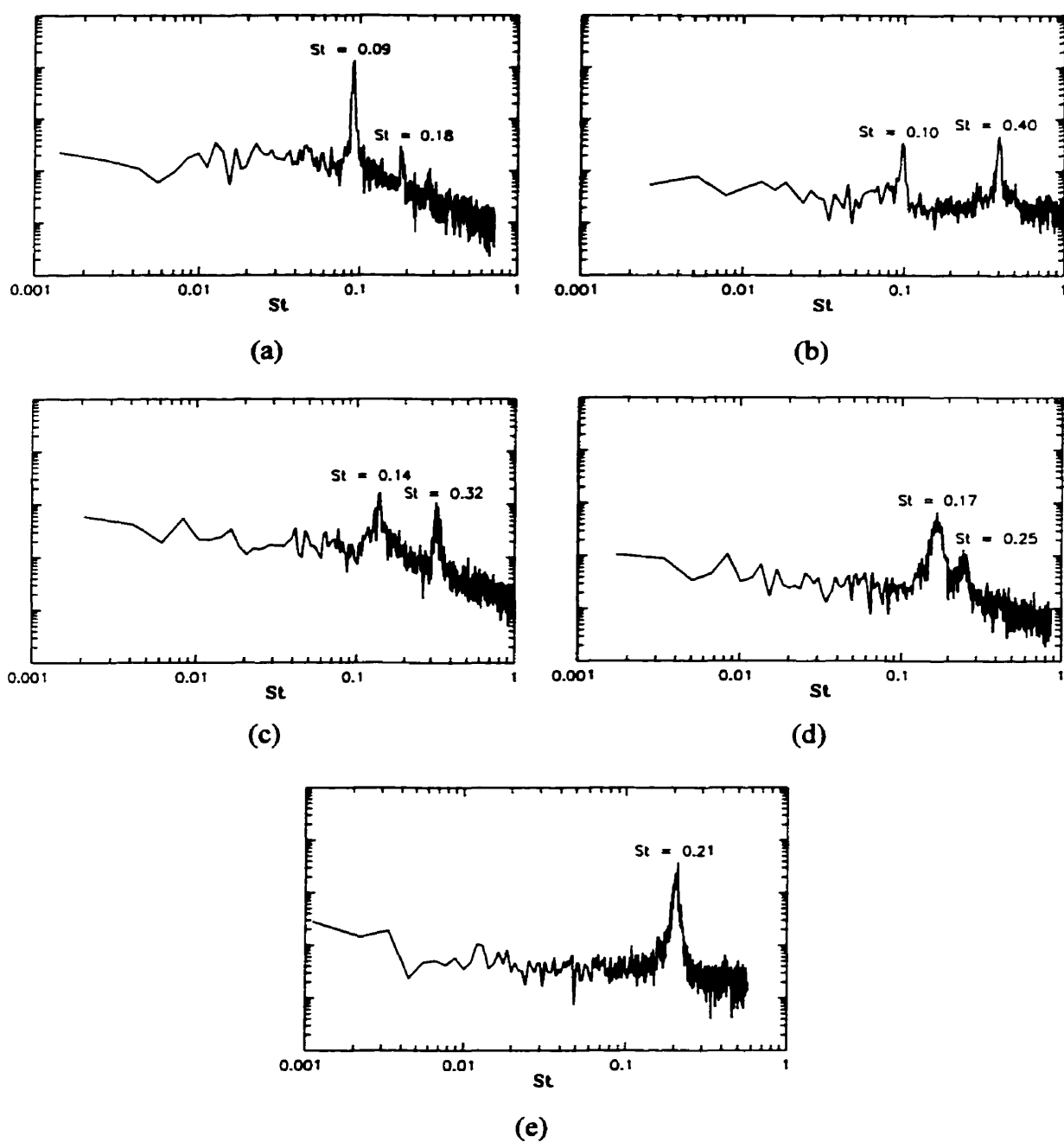


Figure 8.2. Representative power spectra for two side-by-side cylinders in steady cross-flow: (a) $T/D = 1.0$, $Re = 1700$; (b) $T/D = 1.5$, $Re = 1850$; (c) $T/D = 2.0$, $Re = 1200$; (d) $T/D = 2.5$, $Re = 1450$; (e) $T/D = 3.0$, $Re = 2200$. Vertical scale is arbitrary, but the same for each spectrum. Measurement locations at $x/D = 2.0$ to 3.0 , $y/D = 1.0$ to 2.0 .

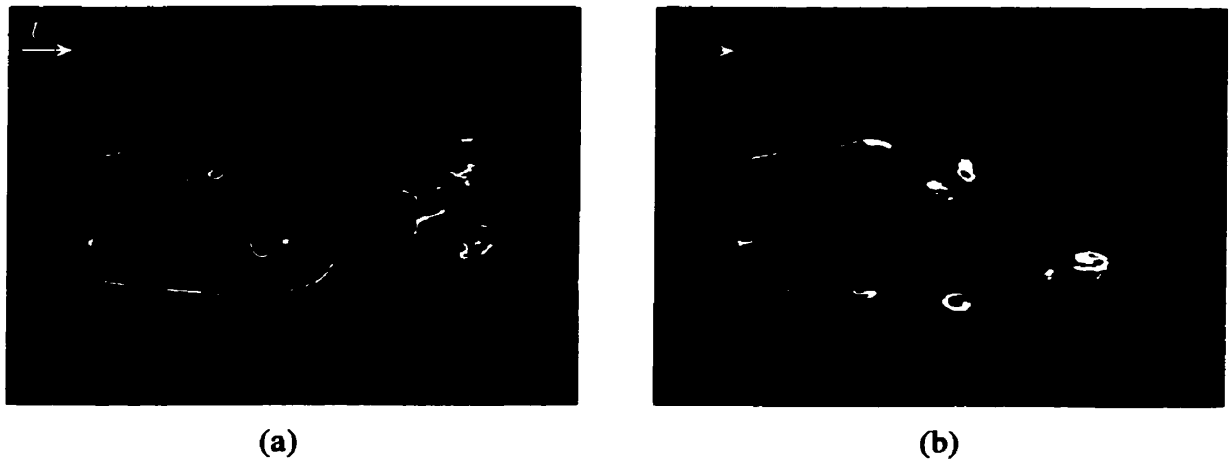


Figure 8.3. Flow visualization of two side-by-side cylinders in steady cross-flow, $T/D = 1.0$: (a) single bluff-body flow pattern with alternate vortex shedding, $Re = 1320$; (b) irregular vortex shedding, $Re = 820$.

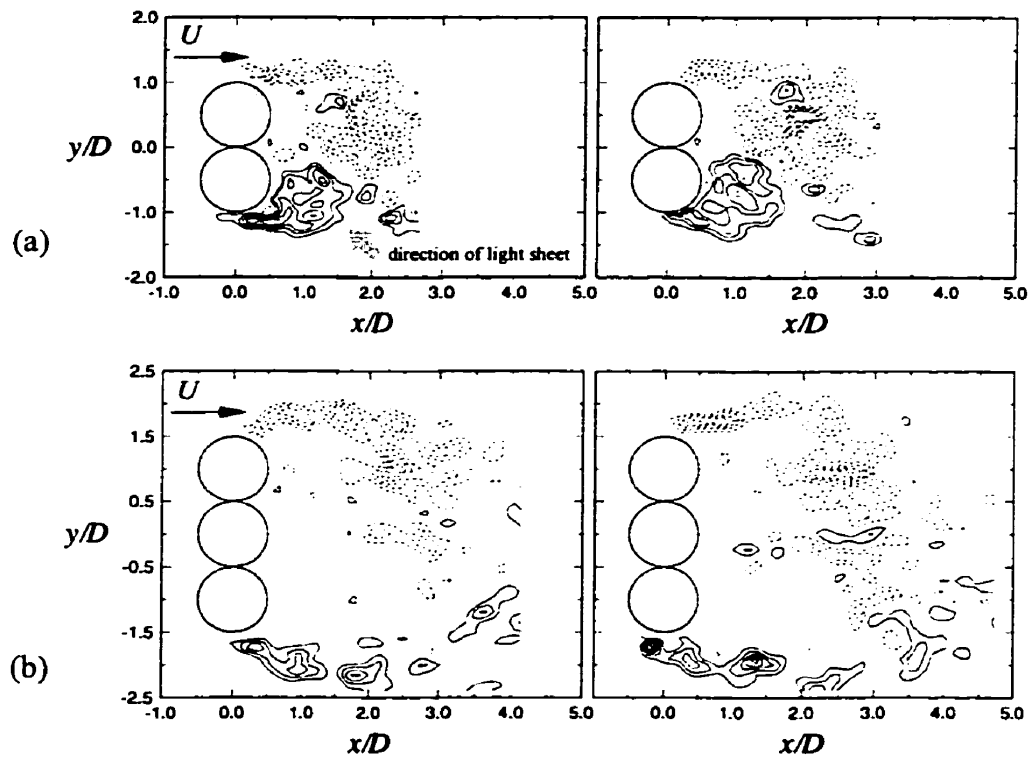
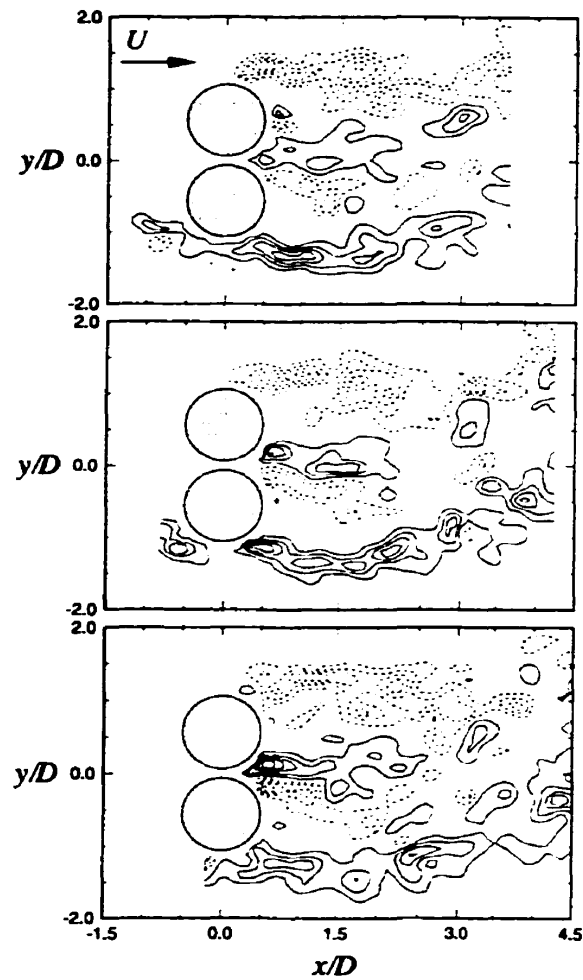
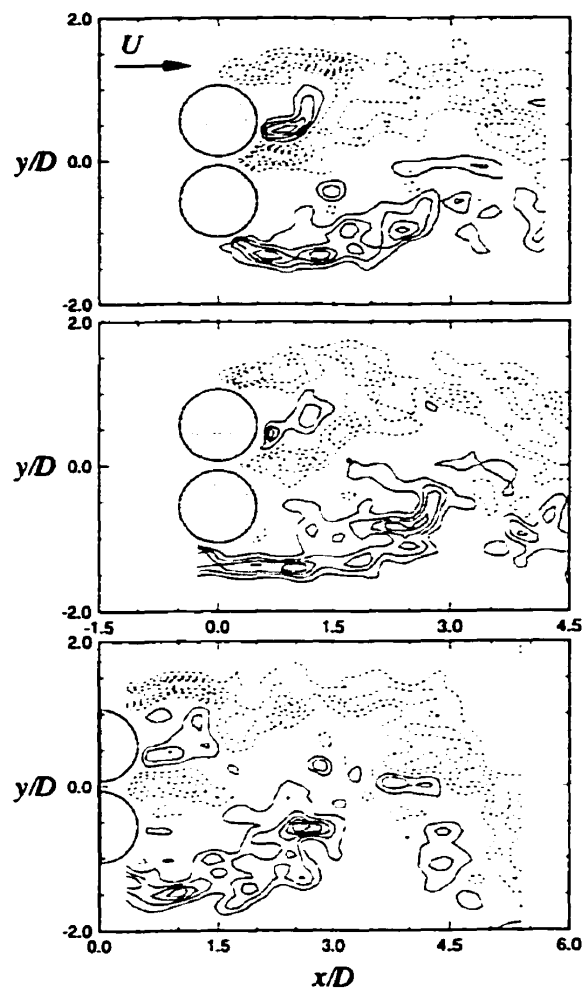


Figure 8.4. Side-by-side cylinders in steady cross-flow, $T/D = 1.0$, $M_t = 0.25$ mm/pixel: non-dimensional, instantaneous, in-plane vorticity field for (a) two cylinders, $Re = 1300$, $\Delta t^* = 0.4$, $\Delta t/T_c = 0.04$; (b) three cylinders, $Re = 1900$, $\Delta t^* = 0.6$, $\Delta t/T_c = 0.04$. Minimum vorticity contour magnitude 1.5, contour increment 1.5, dashed lines represent negative (CW) vorticity. The direction of the laser light sheet is shown in the first vorticity field in (a).

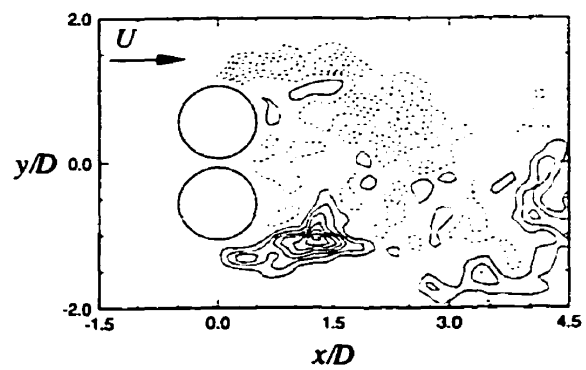


(a)

Figure 8.5. Two side-by-side cylinders in steady cross-flow, showing the base-bleed flow pattern and instantaneous variation in the near-wake region, at $T/D = 1.125$, $Re = 1900$, $M_r = 0.25$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_r = 0.05$: (a) symmetric gap flow, showing lengthened near-wake region; (b) biased gap flow, with intermediate length near-wake region; (c) negligible gap flow, showing shortened near-wake region. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.



(b)



(c)

Figure 8.5 (continued).

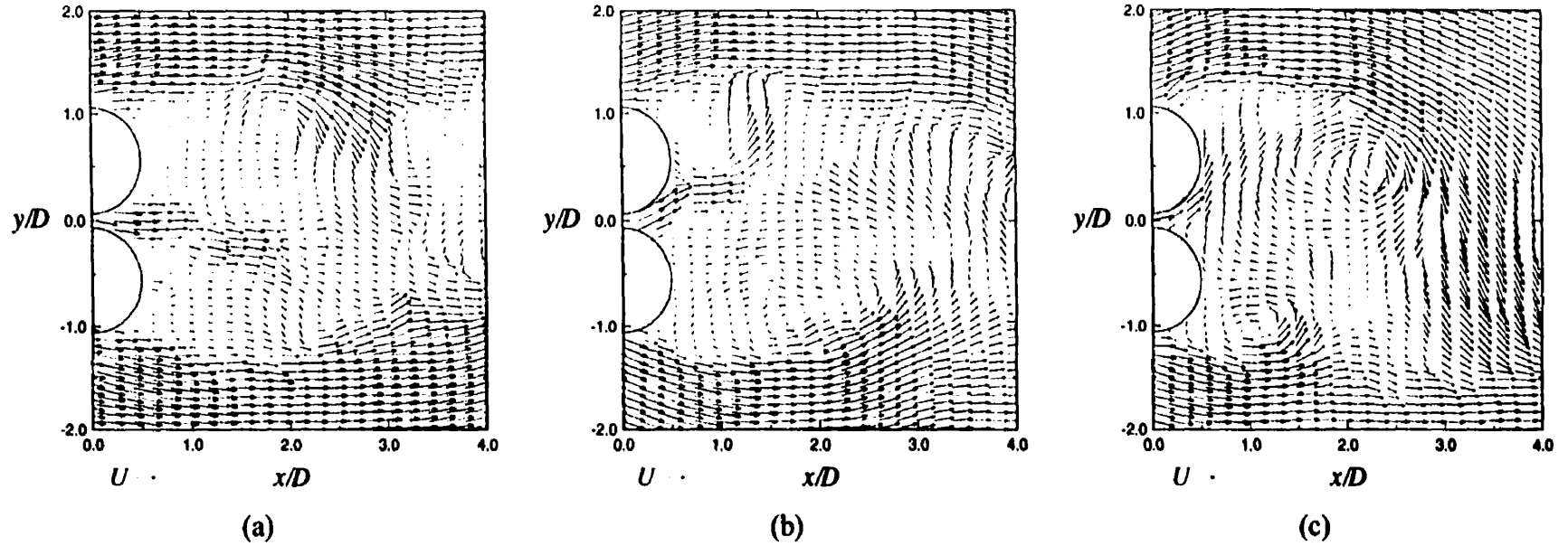


Figure 8.6. Two side-by-side cylinders in steady cross-flow, showing instantaneous variation in the base-bleed flow pattern, $T/D = 1.125$, $Re = 1900$, $M_r = 0.25$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_r = 0.05$: (a) symmetric gap flow, with lengthened vortex formation region; (b) biased gap flow, with intermediate vortex formation region; (c) negligible gap flow, with shortened vortex formation region. Every second vector in the x -direction omitted for clarity.

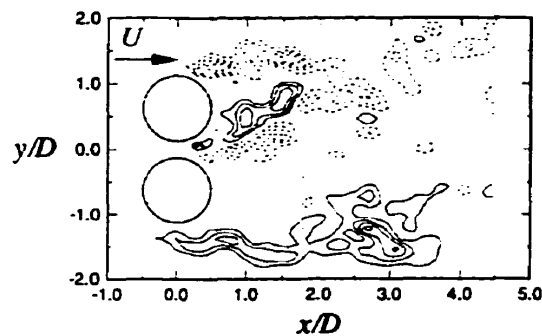


(a)

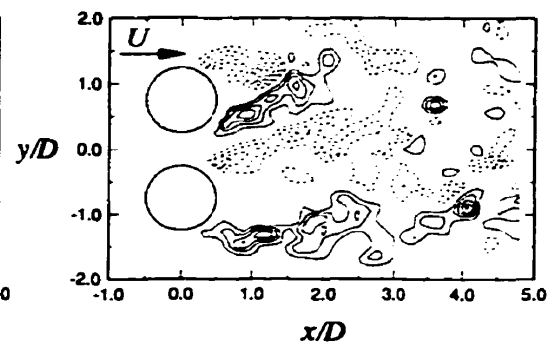


(b)

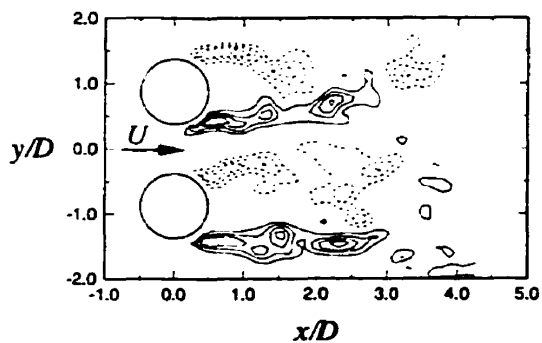
Figure 8.7. Flow visualization of two side-by-side cylinders in steady cross-flow, with a biased flow pattern, for $Re = 1000$ to 3000 : (a) $T/D = 1.5$; (b) $T/D = 2.0$.



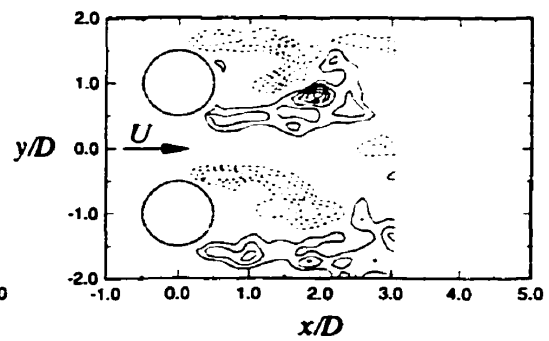
(a)



(b)



(c)



(d)

Figure 8.8. Side-by-side cylinders in steady cross-flow, with a biased flow pattern, $M_r = 0.25$ mm/pixel: (a) $T/D = 1.25$, $Re = 2500$; (b) $T/D = 1.5$, $Re = 1900$; (c) $T/D = 1.75$, $Re = 1900$; (d) $T/D = 2.0$, $Re = 1900$. Minimum vorticity contour magnitude 1.5, contour increment 1.5, dashed lines represent negative (CW) vorticity.

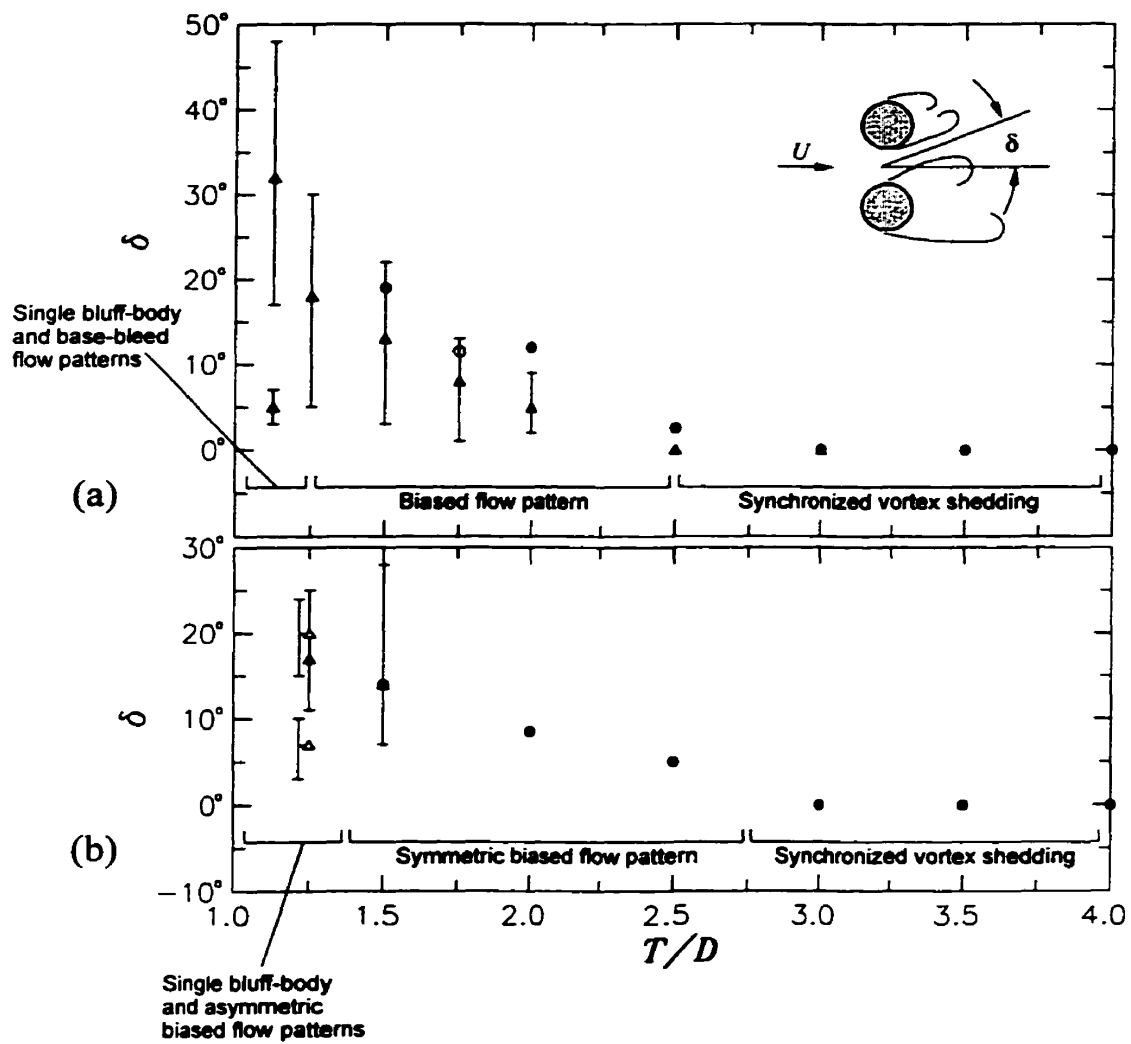


Figure 8.9. Gap flow deflection angle, δ , for side-by-side cylinders in steady cross-flow: (a) two cylinder; (b) three cylinders. ●, flow visualization; ▲, PIV; △, asymmetric biased flow pattern, PIV; ○, estimated from a flow visualization picture of Kim & Durbin (1988). The error bars show the range of instantaneous deflection angle from the PIV measurements. Despite the large time-variability or “error” in δ , the uncertainty is estimated at 2° .



(a)



(b)

Figure 8.10. Flow visualization of two side-by-side cylinders in steady cross-flow at large T/D , showing anti-phase vortex shedding, for $Re = 1000$ to 3000 : (a) $T/D = 2.5$; (b) $T/D = 4.5$.

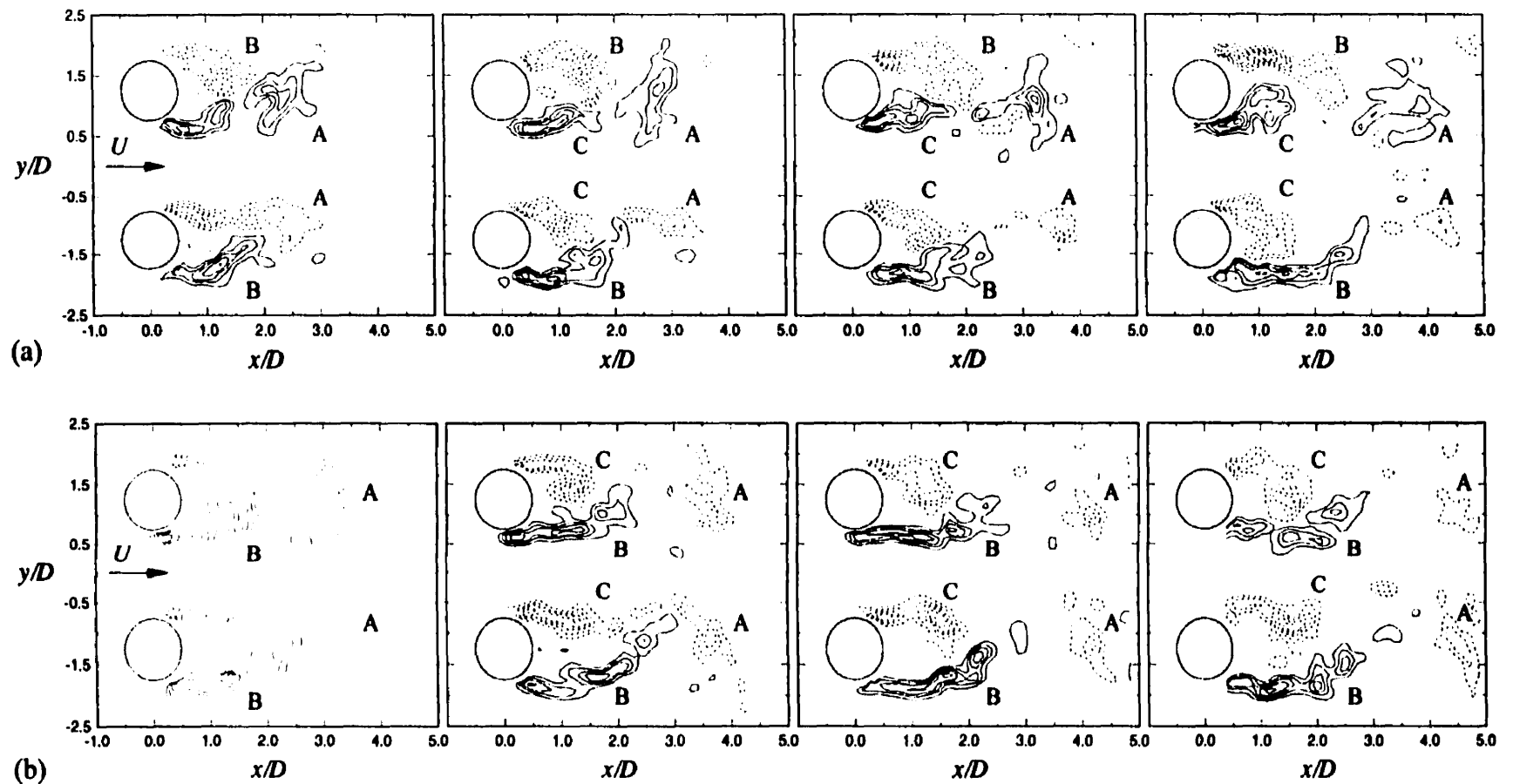


Figure 8.11. Two side-by-side cylinders in steady cross-flow, $T/D = 2.5$, $Re = 1900$, $M_t = 0.25$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_t = 0.13$: (a) anti-phase vortex shedding; (b) in-phase vortex shedding. Corresponding vortices labeled as A, B and C. Minimum vorticity contour magnitude 1.5, contour increment 1.5, dashed lines represent negative (CW) vorticity.

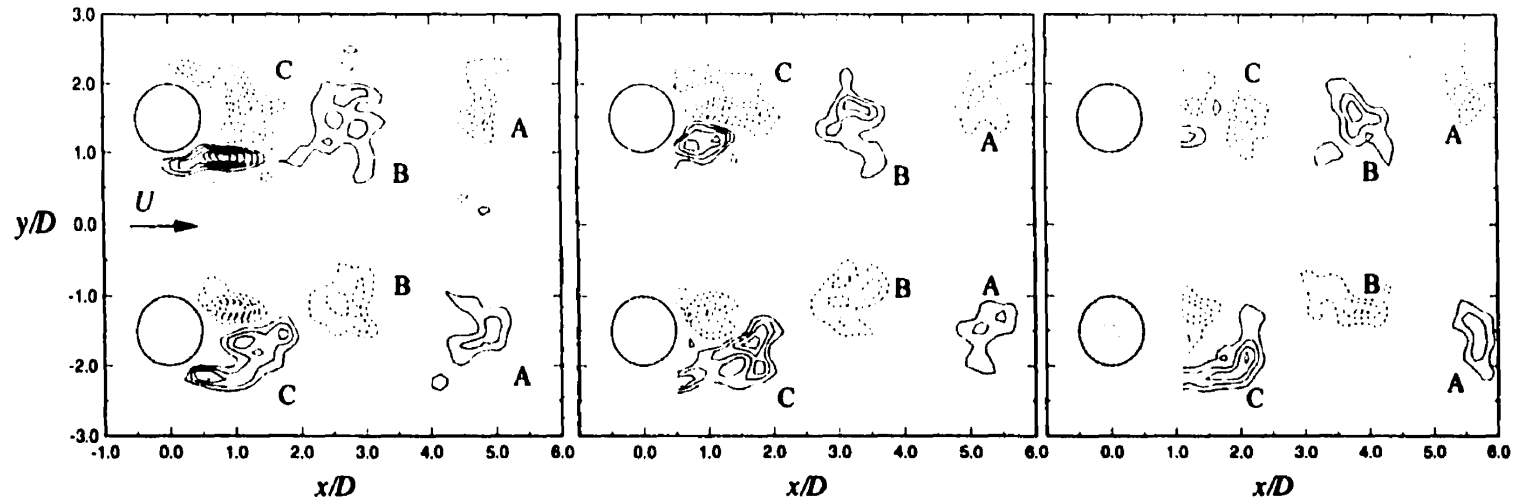


Figure 8.12. Anti-phase vortex shedding synchronization for two side-by-side cylinders in steady cross-flow, $T/D = 3.0$, $Re = 1900$, $M_t = 0.25$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_t = 0.12$. Paired vortices labeled as A, B and C. Minimum vorticity contour magnitude 1.5, contour increment 1.5, dashed lines represent negative (CW) vorticity.

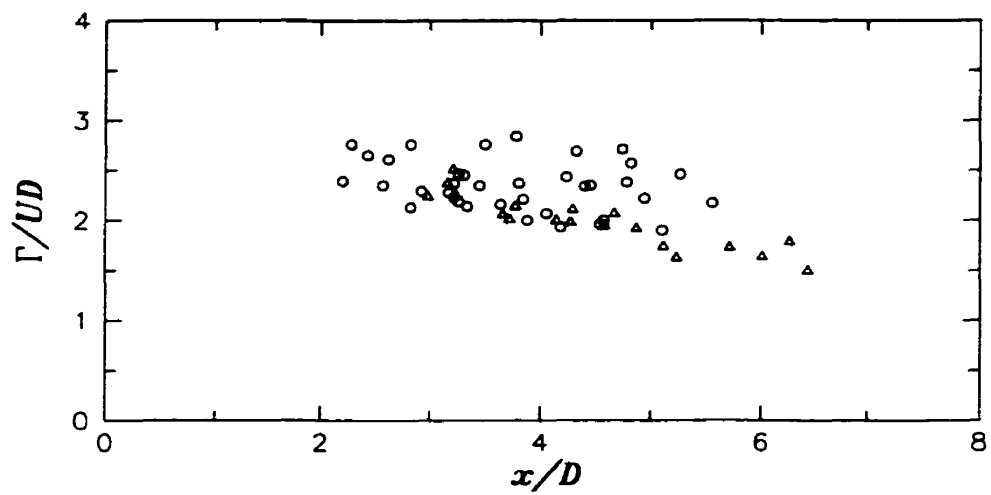


Figure 8.13. Vortex strength as a function of streamwise distance from the cylinder centre for two side-by-side cylinders in steady cross-flow, $T/D = 3.0$, $Re = 1900$ to 2500 : ○, vortices shed from the inner (gap) shear layers; △, vortices shed from the outer shear layers. The measurement uncertainty is estimated to be 10 to 20%.

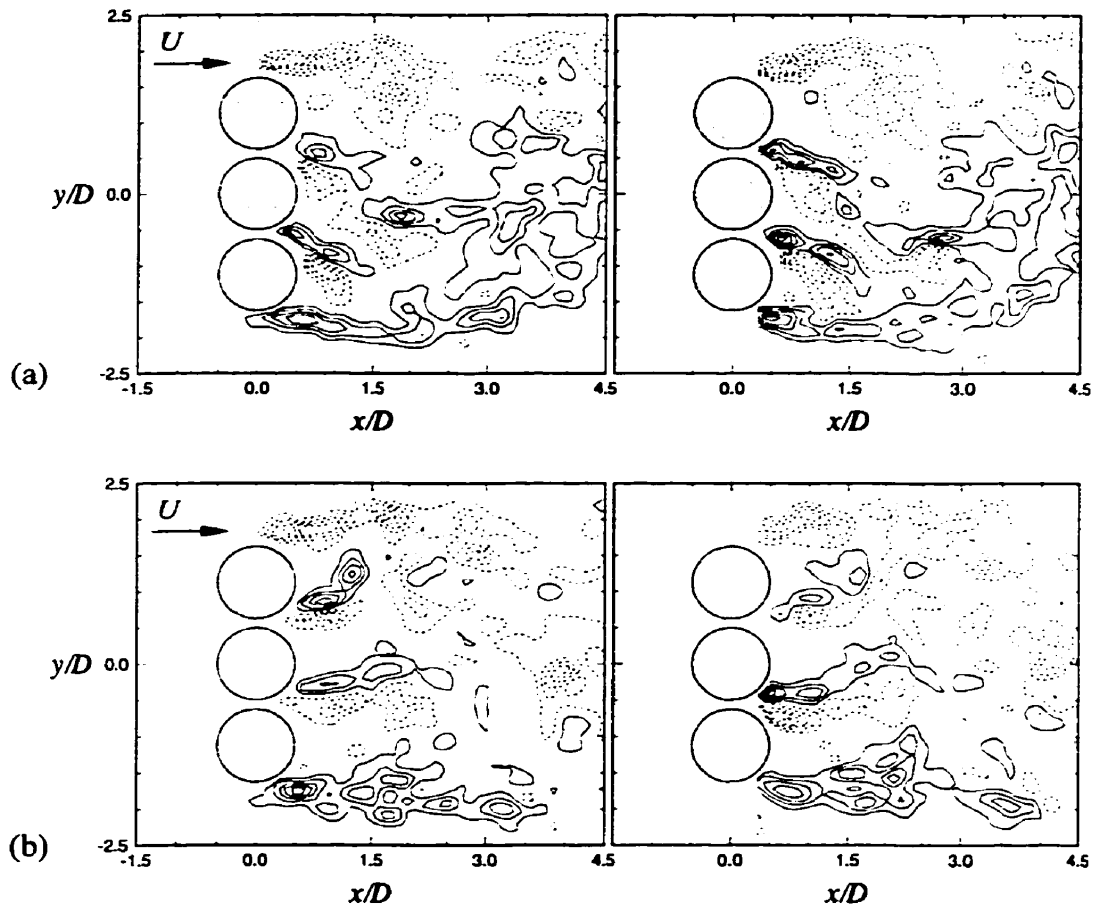


Figure 8.14. Three side-by-side cylinders in steady cross-flow, $T/D = 1.125$, $Re = 1900$, $M_r = 0.25$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_s = 0.04$: (a) wider near-wake behind the "upper" cylinder (in the figure); (b) wider near-wake region behind the "lower cylinder". Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.

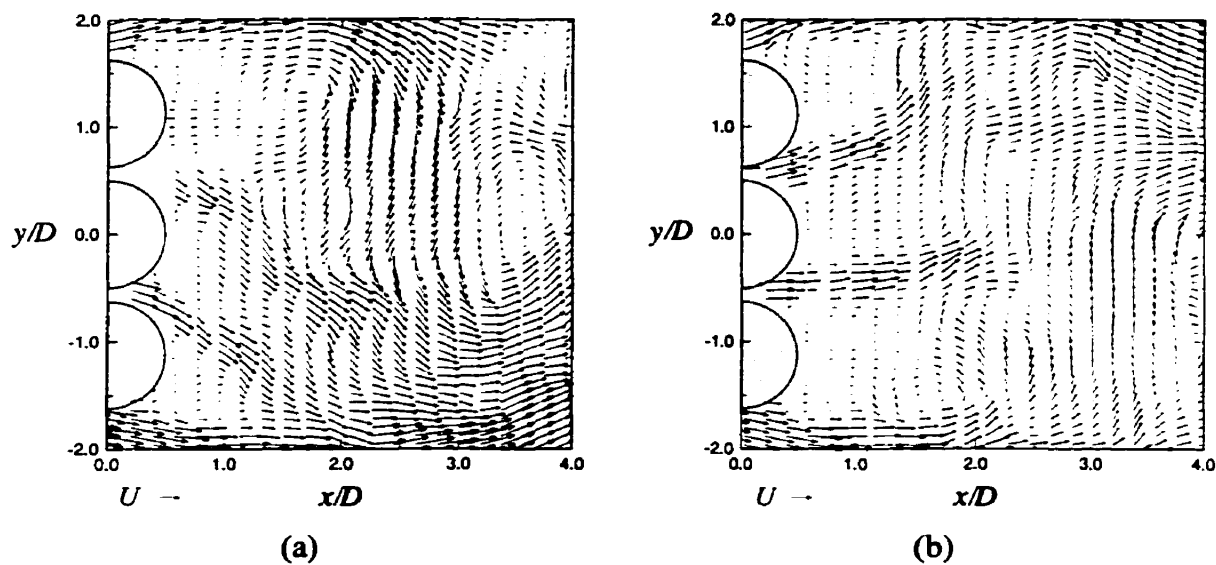


Figure 8.15. Velocity field for three side-by-side cylinders in steady cross-flow, $T/D = 1.125$, $Re = 1900$, $M_x = 0.25$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_s = 0.04$: (a) wider near-wake behind the “upper” cylinder (in the figure); (b) wider near-wake region behind the “lower” cylinder. Every second vector in the x -direction omitted for clarity.

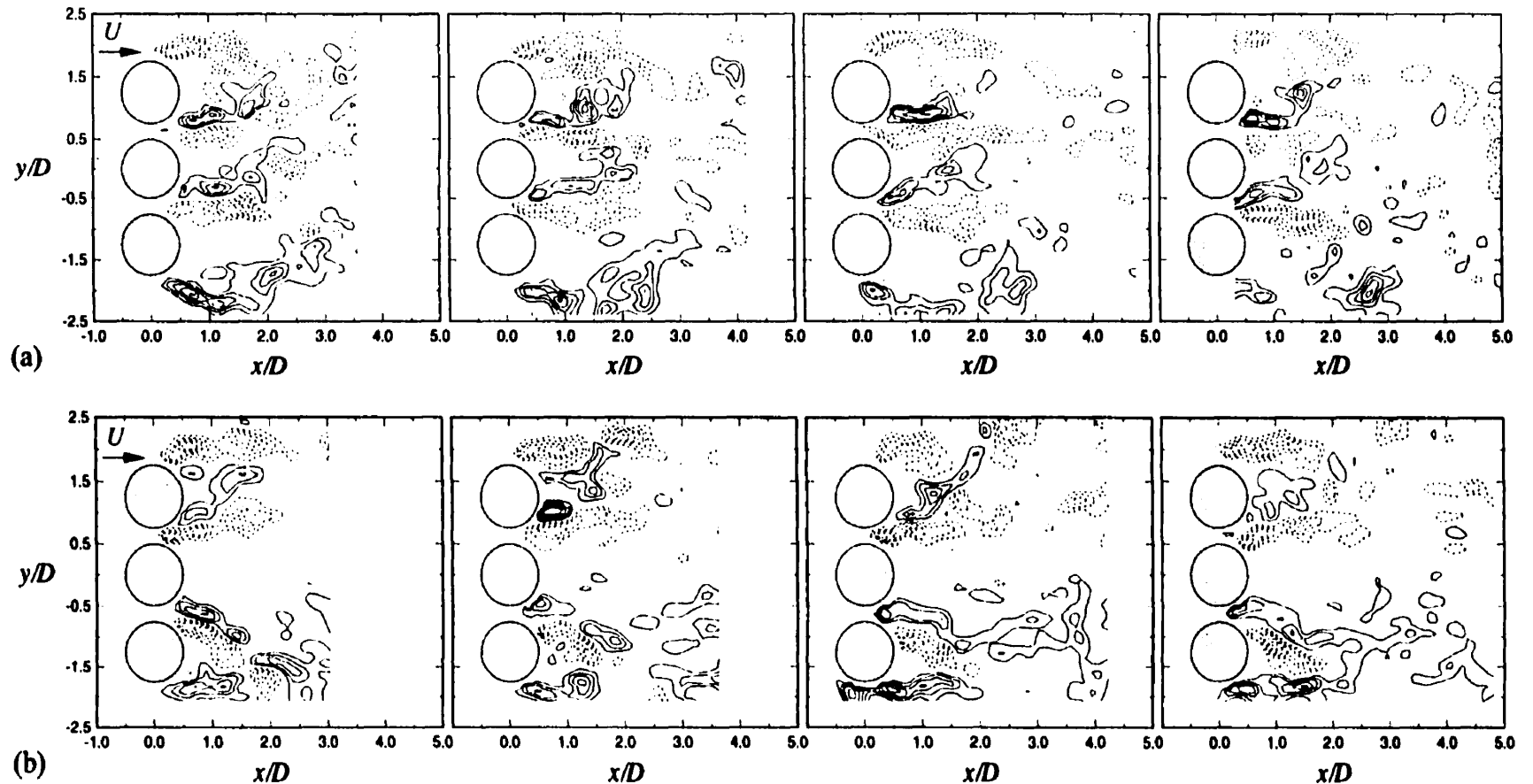


Figure 8.16. Three side-by-side cylinders in steady cross-flow, $T/D = 1.25$, $M_t = 0.25$ mm/pixel: (a) asymmetric biased flow pattern, with the wider near-wake region behind the “lower” cylinder (in the figure), $Re = 1900$, $\Delta t^* = 0.6$, $\Delta t/T_c = 0.05$ or 0.14 based on the low or high vortex shedding frequency, respectively; (b) symmetric biased flow pattern, $Re = 1900$, $\Delta t^* = 0.6$, $\Delta t/T_c = 0.05$ or 0.14 ; (c) asymmetric biased flow pattern, with the wider near-wake region behind the “lower” cylinder, $Re = 2500$, $\Delta t^* = 0.8$, $\Delta t/T_c = 0.06$ or 0.19 ; (d) nearly-symmetric biased flow pattern, $Re = 2900$, $\Delta t^* = 0.8$, $\Delta t/T_c = 0.06$ or 0.19 . Minimum vorticity contour magnitude 1.5, contour increment 1.5, dashed lines represent negative (CW) vorticity.

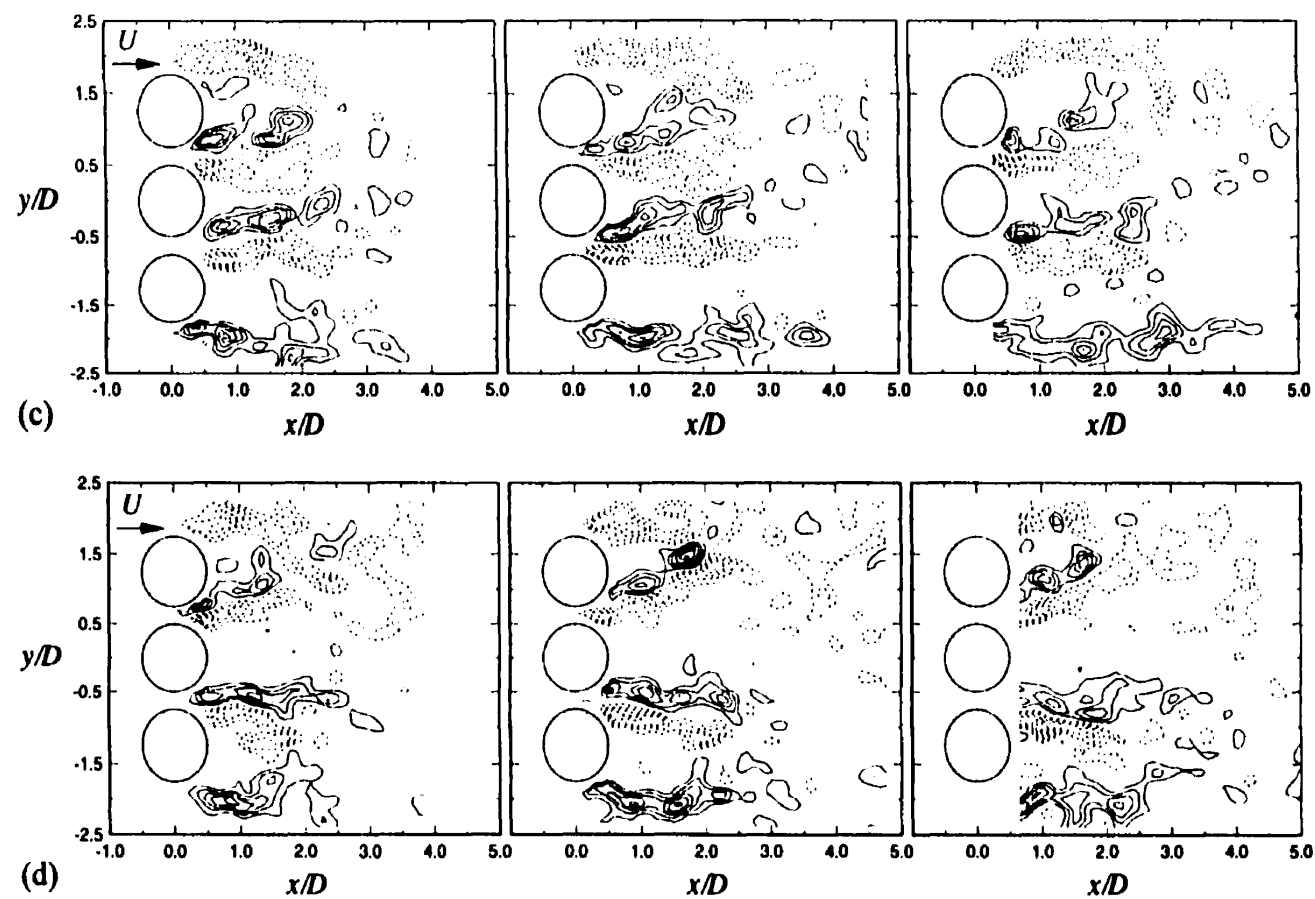


Figure 8.16 (continued).



(a)



(b)



(c)



(d)

Figure 8.17. Flow visualization of three side-by-side cylinders in steady cross-flow, $Re = 1000$ to 3000 : (a) $T/D = 1.5$, symmetric biased flow pattern; (b) $T/D = 2.0$, symmetric biased flow pattern; (c) $T/D = 2.5$, symmetric biased flow pattern; (d) $T/D = 3.0$, anti-phase vortex shedding.

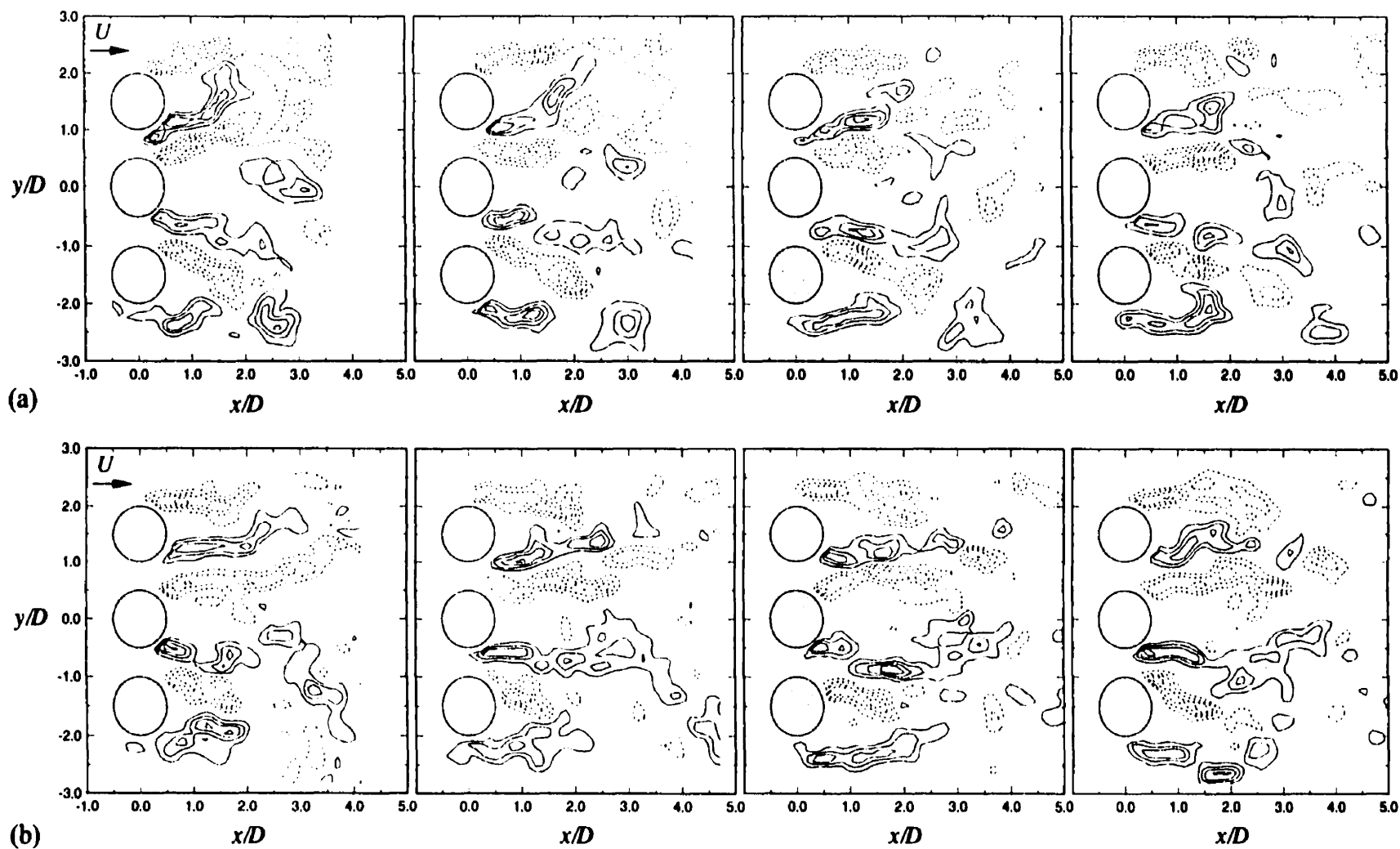


Figure 8.18. (a,b) Symmetric biased flow pattern for three side-by-side cylinders in steady cross-flow, $T/D = 1.5$, $Re = 1900$, $M_t = 0.25$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_s = 0.04$ or 0.22 based on the low or high vortex shedding frequency, respectively: Minimum vorticity contour magnitude 1.5, contour increment 1.5, dashed lines represent negative (CW) vorticity.

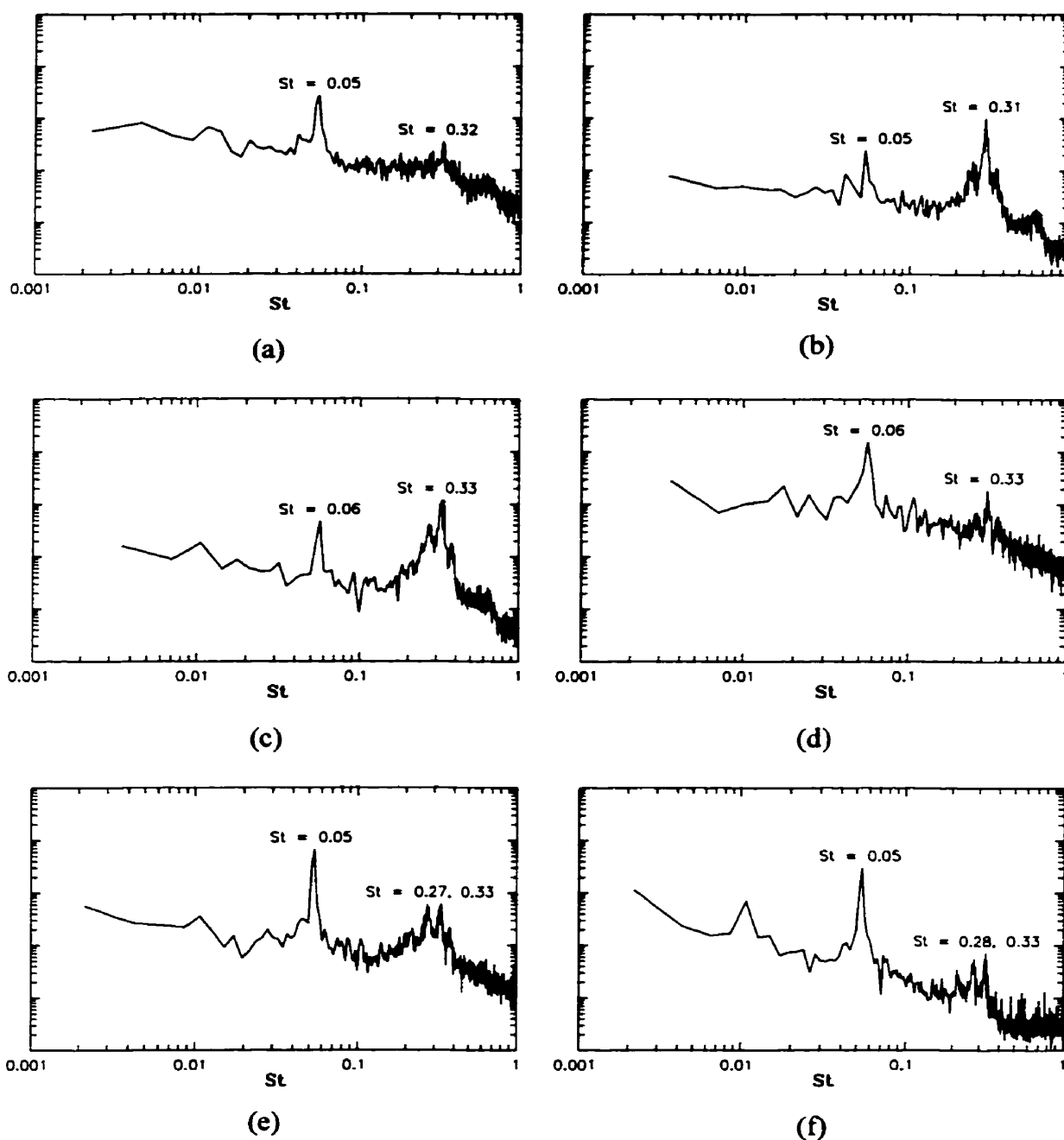


Figure 8.19. Representative power spectra for three side-by-side cylinders in steady cross-flow, $T/D = 1.5$, at the following measurement locations: (a) $x/D = 2.0$, $y/D = 2.0$, $Re = 1750$; (b) $x/D = 2.0$, $y/D = 2.5$, $Re = 1200$; (c) $x/D = 2.0$, $y/D = 3.0$, $Re = 1100$; (d) $x/D = 3.0$, $y/D = 1.5$, $Re = 1100$; (e) $x/D = 3.0$, $y/D = 2.5$, $Re = 1800$; (f) $x/D = 3.0$, $y/D = 3.5$, $Re = 1800$. Vertical scale is arbitrary, but the same for each spectrum.

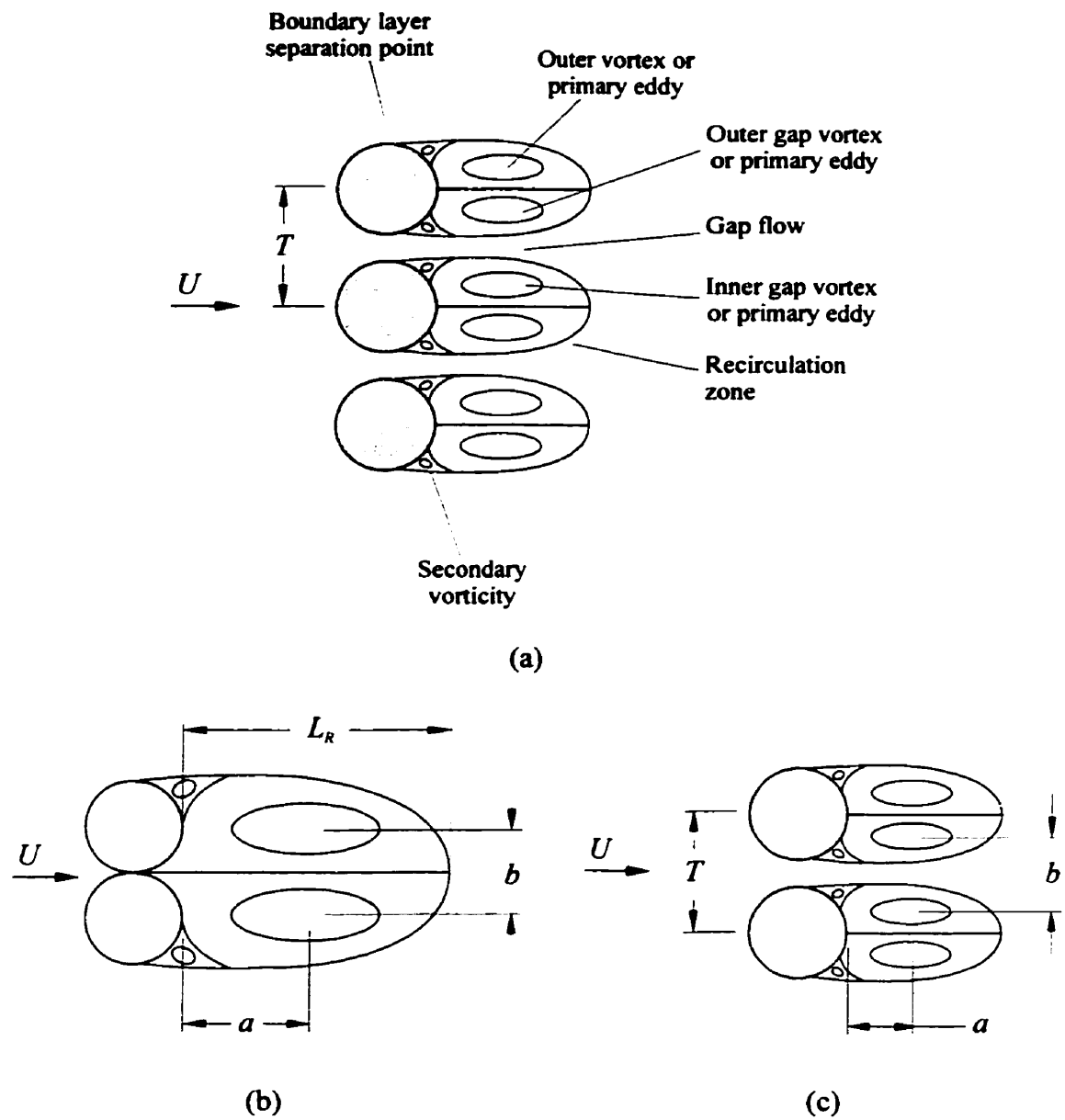


Figure 8.20. Side-by-side cylinders impulsively set into motion: (a) features of the flow field; (b) nomenclature for cylinders in contact; (c) nomenclature for cylinders with a gap between them.

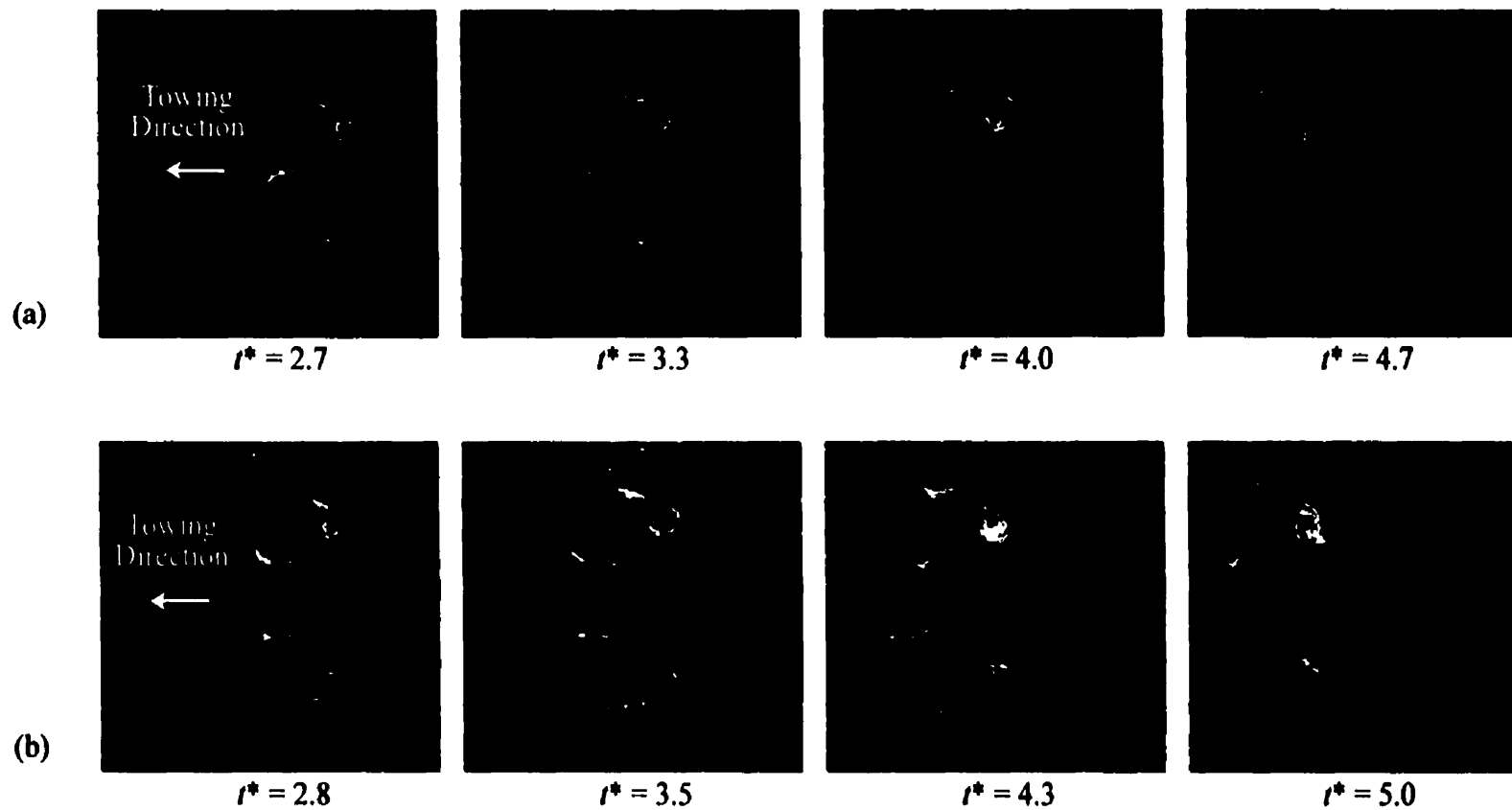


Figure 8.21. Flow visualization of impulsively started side-by-side cylinders, $T/D = 1.0$: (a) two cylinders, $Re = 2100$; (b) three cylinders, $Re = 2300$.

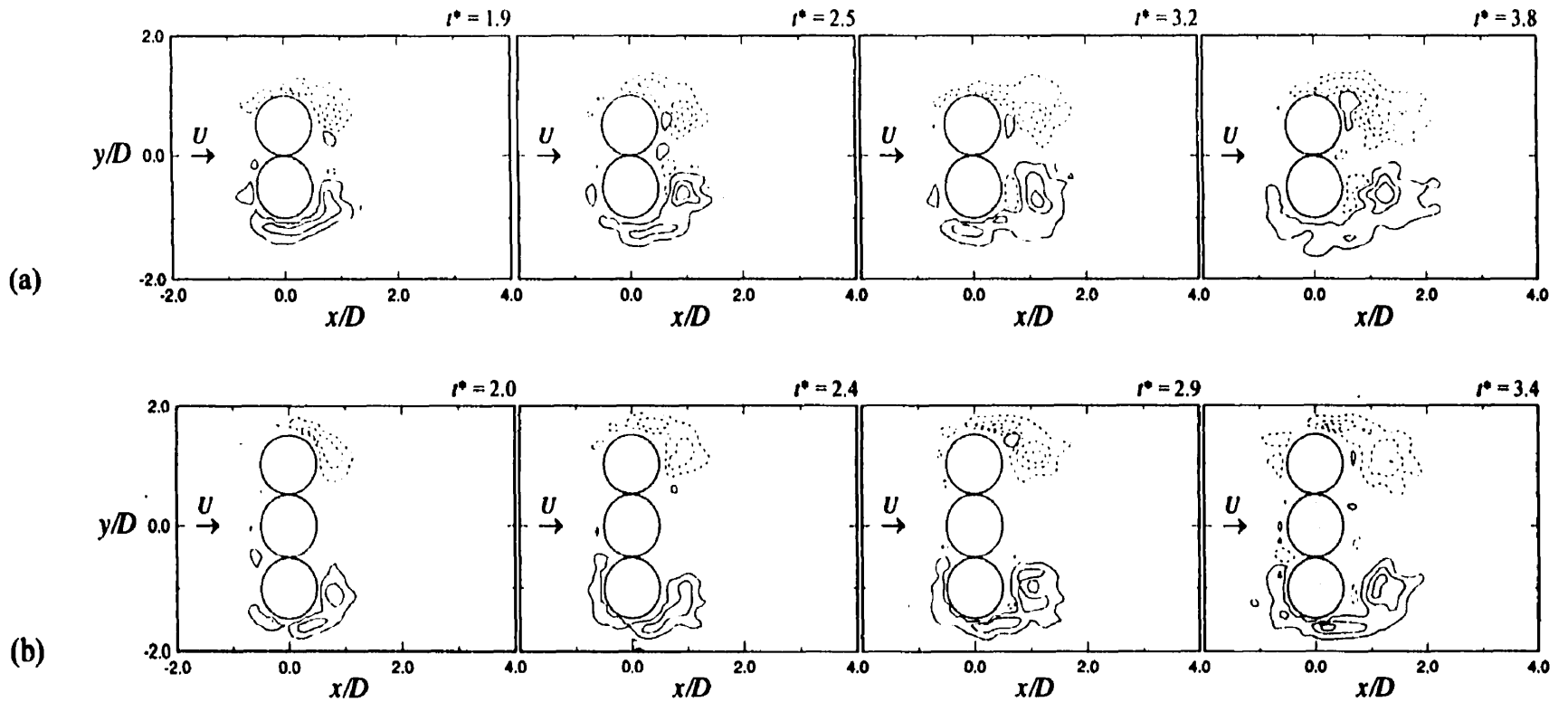


Figure 8.22. Impulsively started side-by-side cylinders in contact, $T/D = 1.0$, $A_p = 0.3$, $M_r = 0.25$ mm/pixel: (a) two cylinders, $Re = 2000$; (b) three cylinders, $Re = 1600$. Minimum vorticity contour magnitude 1.0, contour increment 4.0, dashed lines represent negative (CW) vorticity.

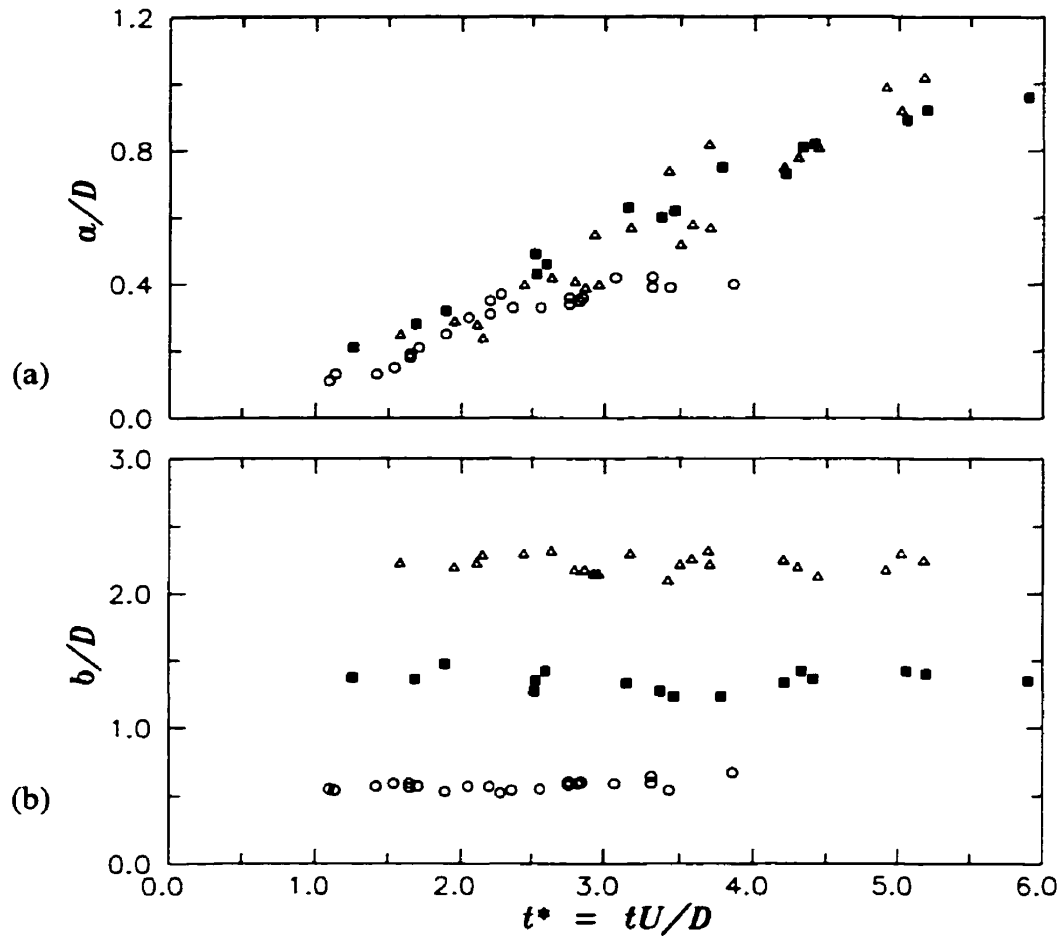


Figure 8.23. Recirculation zone data for impulsively started side-by-side cylinders in contact, $T/D = 1.0$: $\circ$, one cylinder, $Re = 1500$ to 1800 , $A_p = 0.4$ to 0.6 , $M_r = 0.15$ to 0.20 mm/pixel; $\blacksquare$, two cylinders, $Re = 2000$ to 2800 , $A_p = 0.2$ to 0.4 , $M_r = 0.25$ mm/pixel; $\triangle$, three cylinders, $Re = 1600$ to 2400 , $A_p = 0.3$ to 0.5 , $M_r = 0.25$ mm/pixel. (a) Streamwise position of the primary eddies from the base of the cylinders; (b) transverse spacing of the primary eddies; (c) streamwise extent of the recirculation zone from the base of the cylinders; (d) primary eddy circulation or strength. Vorticity cutoff magnitude 1.0.

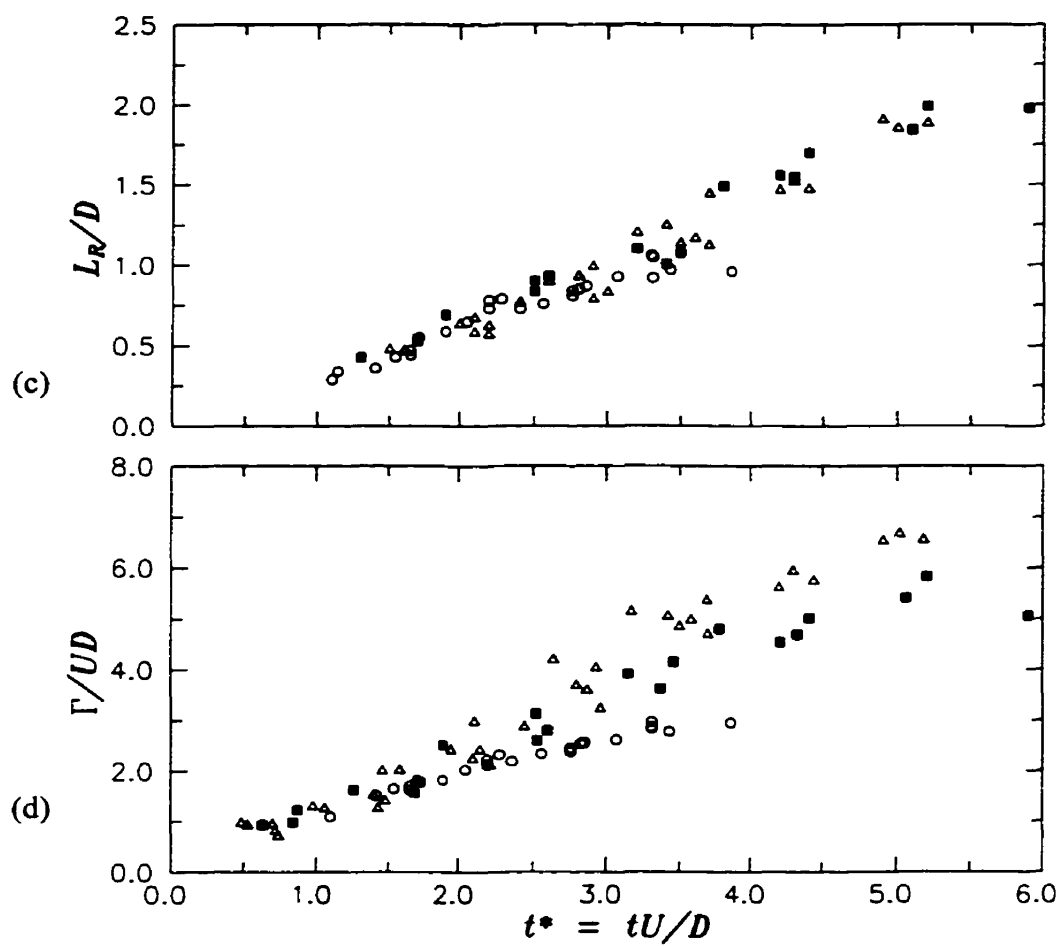


Figure 8.23 (continued).

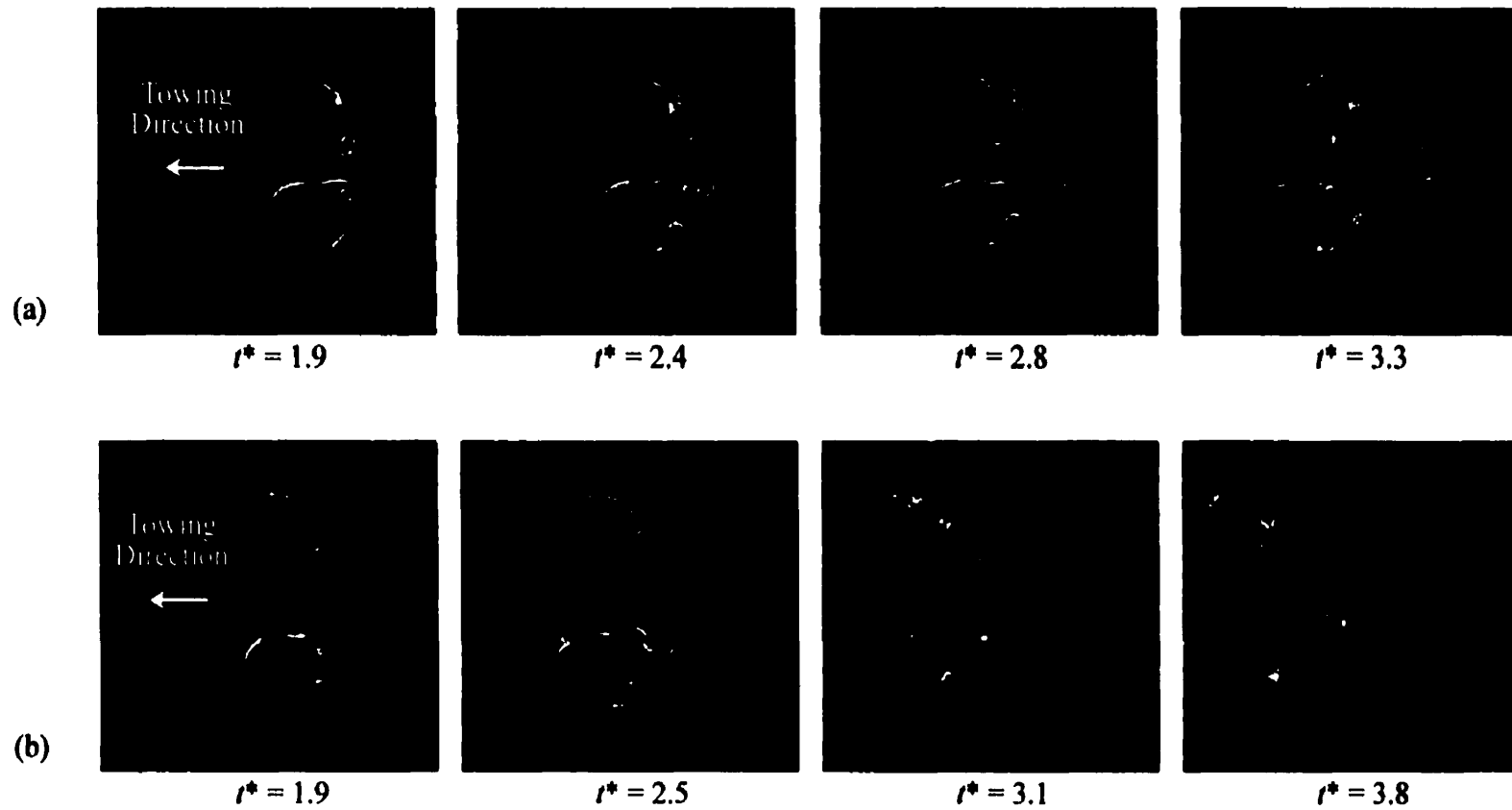
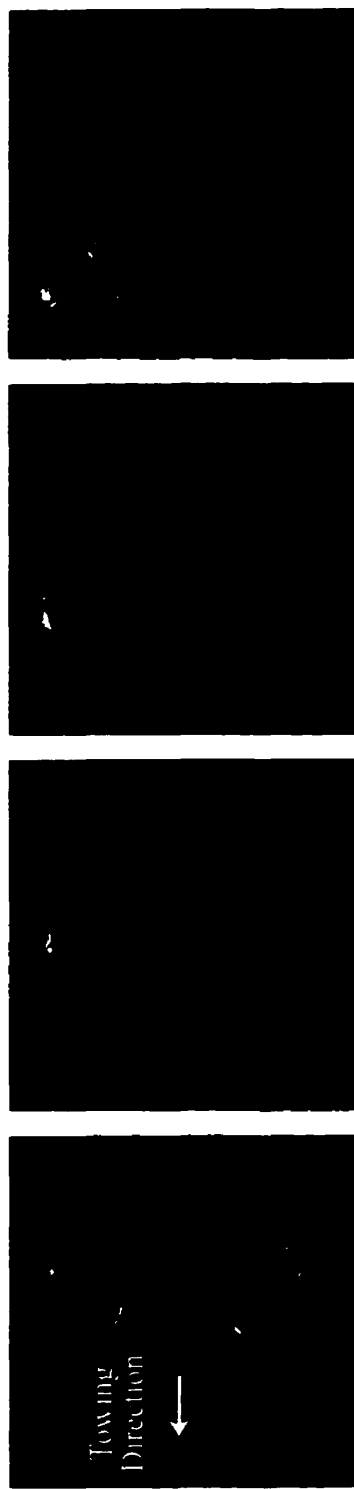
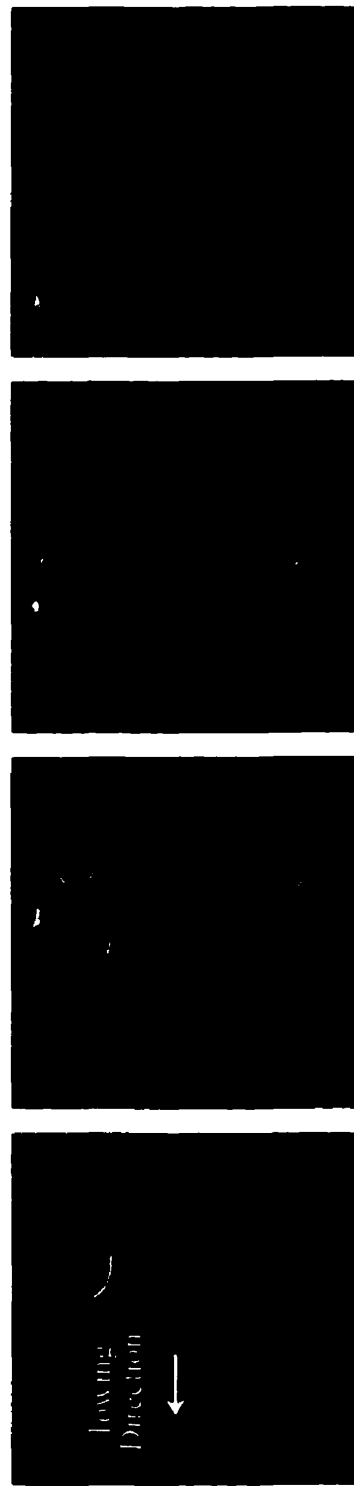


Figure 8.24. Flow visualization of two impulsively started side-by-side cylinders: (a) $T/D = 1.5$, $Re = 1500$; (b) $T/D = 2.0$, $Re = 2000$; (c) $T/D = 2.5$, $Re = 2000$; (d) $T/D = 3.0$, $Re = 2600$.



(c)



(d)

Figure 8.24 (continued).

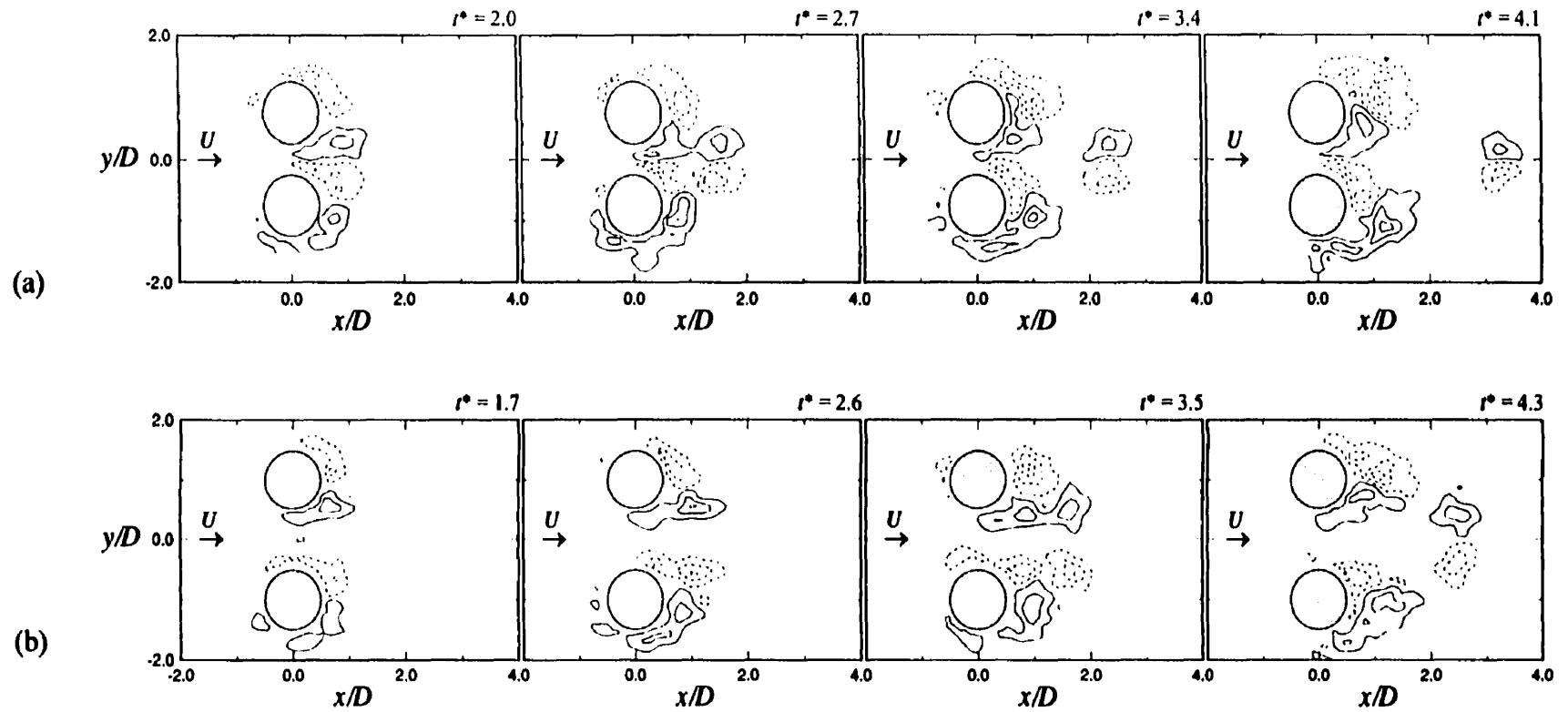


Figure 8.25. Two impulsively started side-by-side cylinders, $M_t = 0.25$ mm/pixel: (a) $T/D = 1.5$, $Re = 2200$, $A_p = 0.3$; (b) $T/D = 2.0$, $Re = 2800$, $A_p = 0.3$; (c) $T/D = 2.5$, $Re = 2500$, $A_p = 0.7$; (d) $T/D = 3.0$, $Re = 2300$, $A_p = 0.3$; Minimum vorticity contour magnitude 1.5, contour increment 4.0, dashed lines represent negative (CW) vorticity.

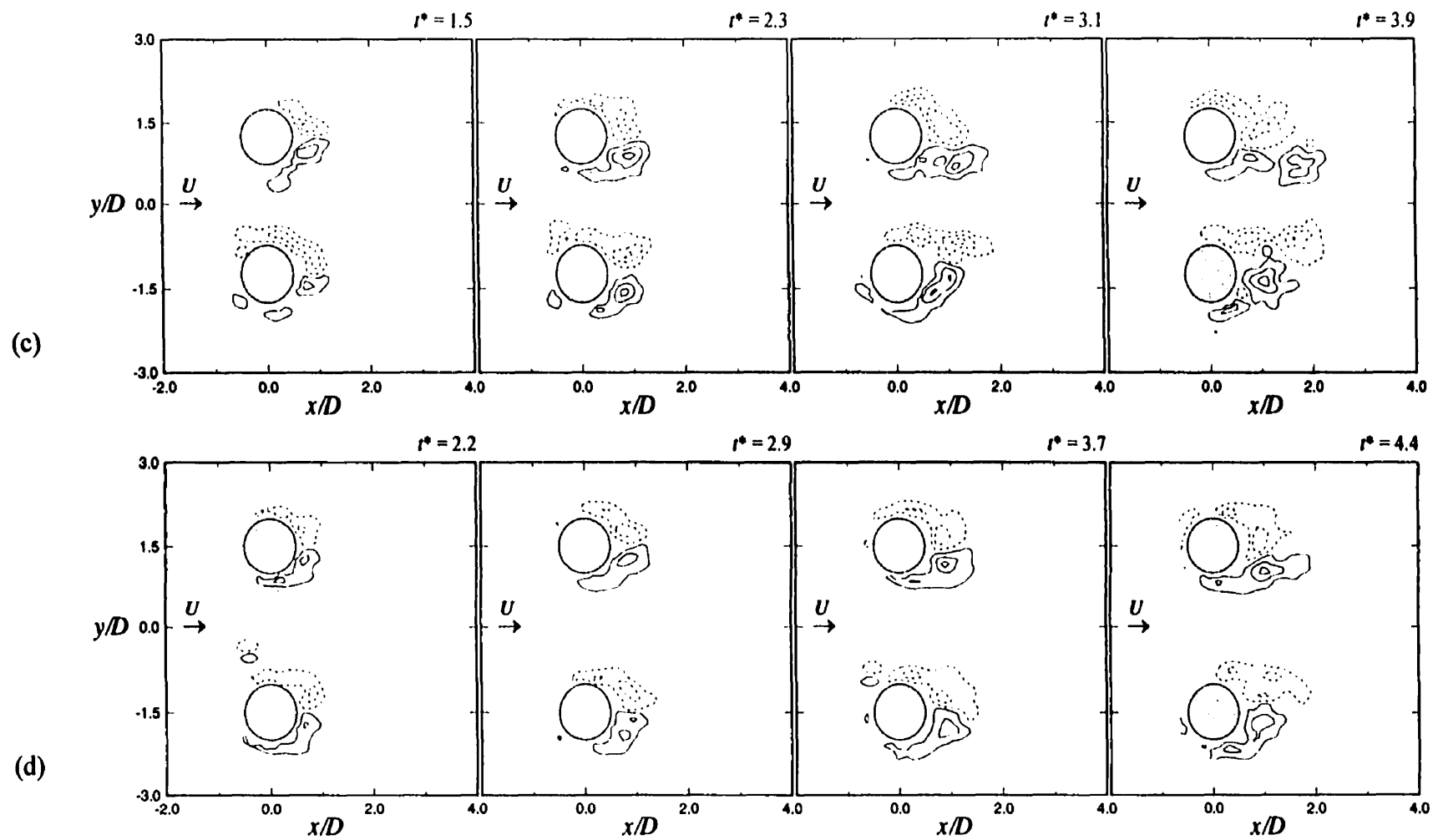


Figure 8.25 (continued).

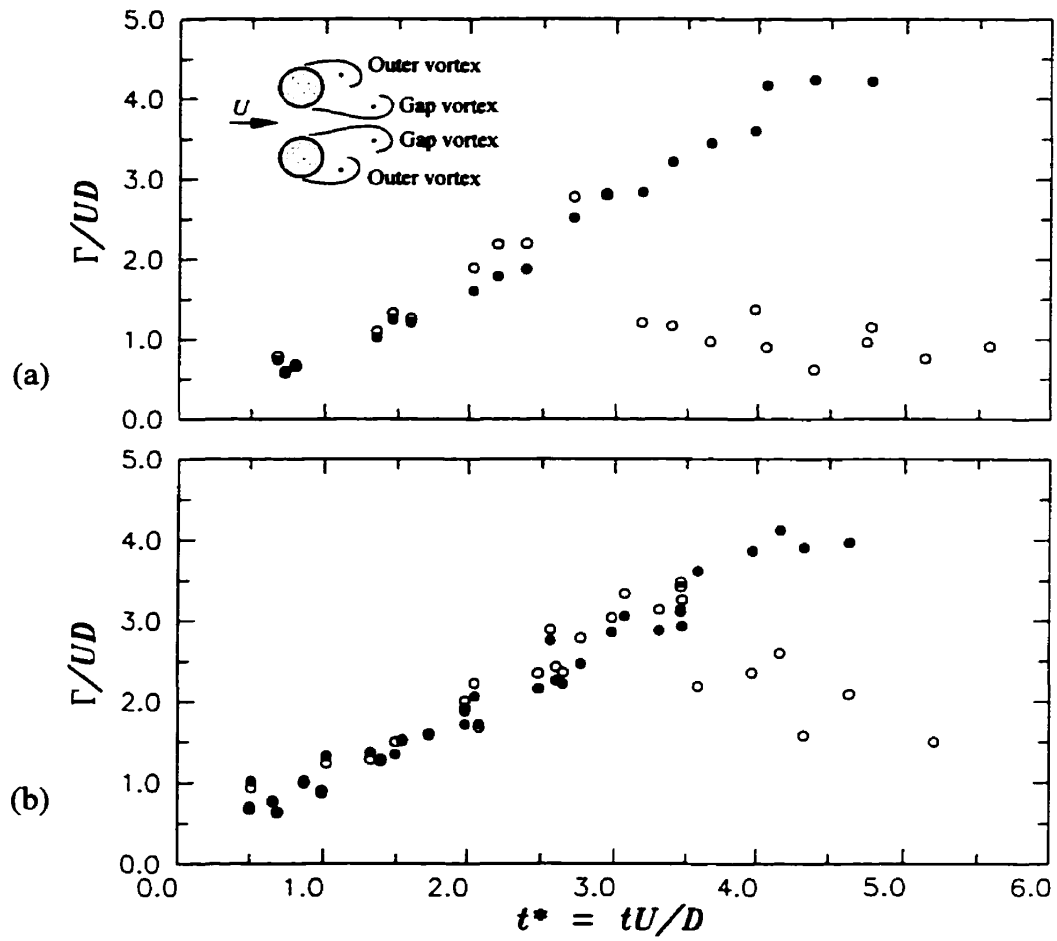


Figure 8.26. Vortex strength data for two impulsively started side-by-side cylinders, $M_\infty = 0.25$ mm/pixel: (a) $T/D = 1.5$, $Re = 2200$ to 2500 , $A_p = 0.3$; (b) $T/D = 2.0$, $Re = 1600$ to 2800 , $A_p = 0.3$ to 0.4 ; (c) $T/D = 2.5$, $Re = 1800$ to 2500 , $A_p = 0.7$ to 0.9 ; (d) $T/D = 3.0$, $Re = 1900$ to 2400 , $A_p = 0.3$. $\circ$, gap vortices; $\bullet$, outer vortices. Vorticity cutoff magnitude 1.0.

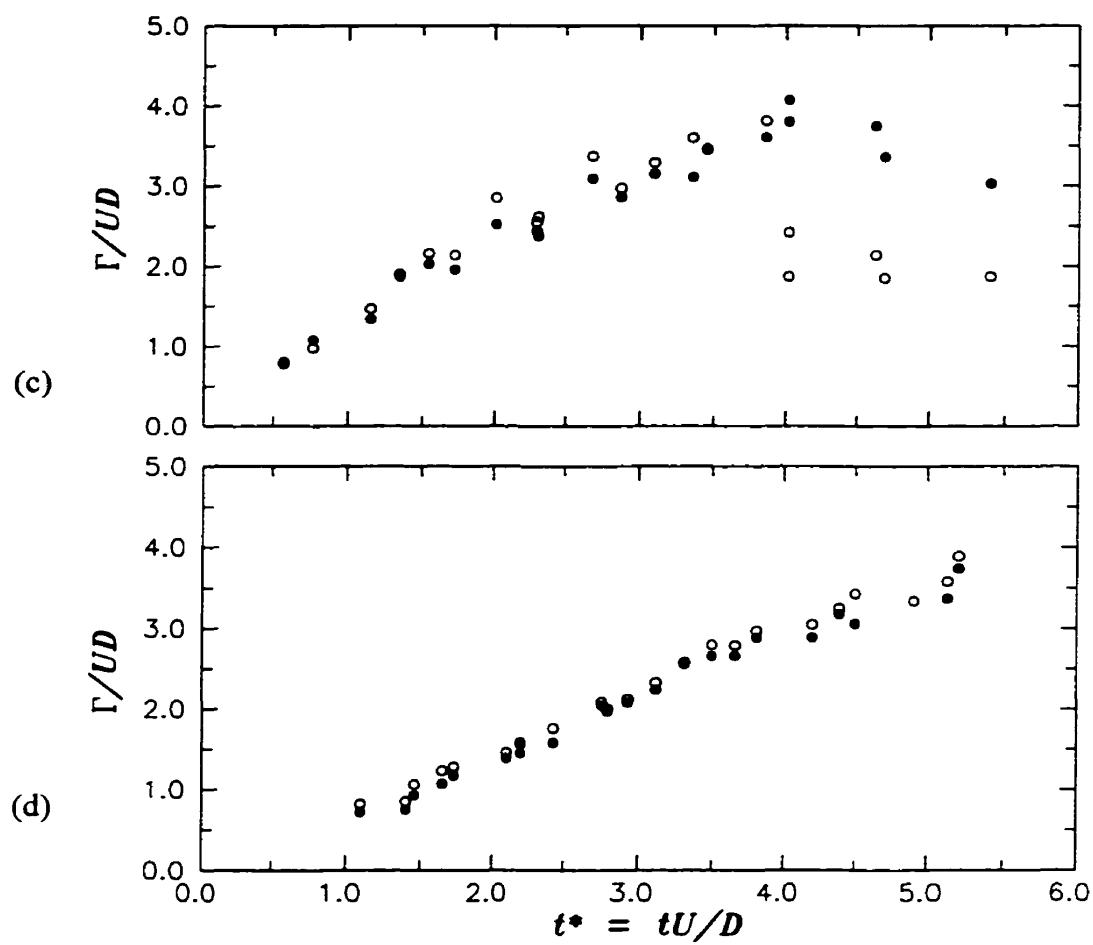
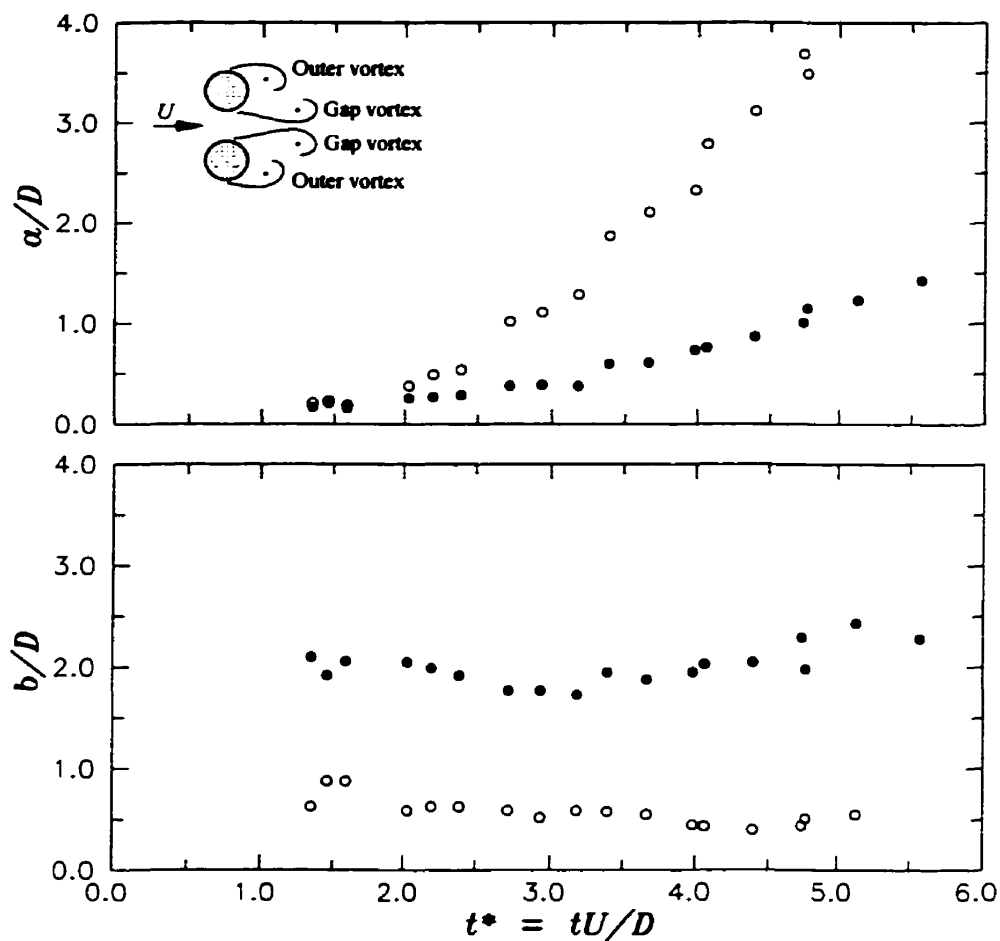
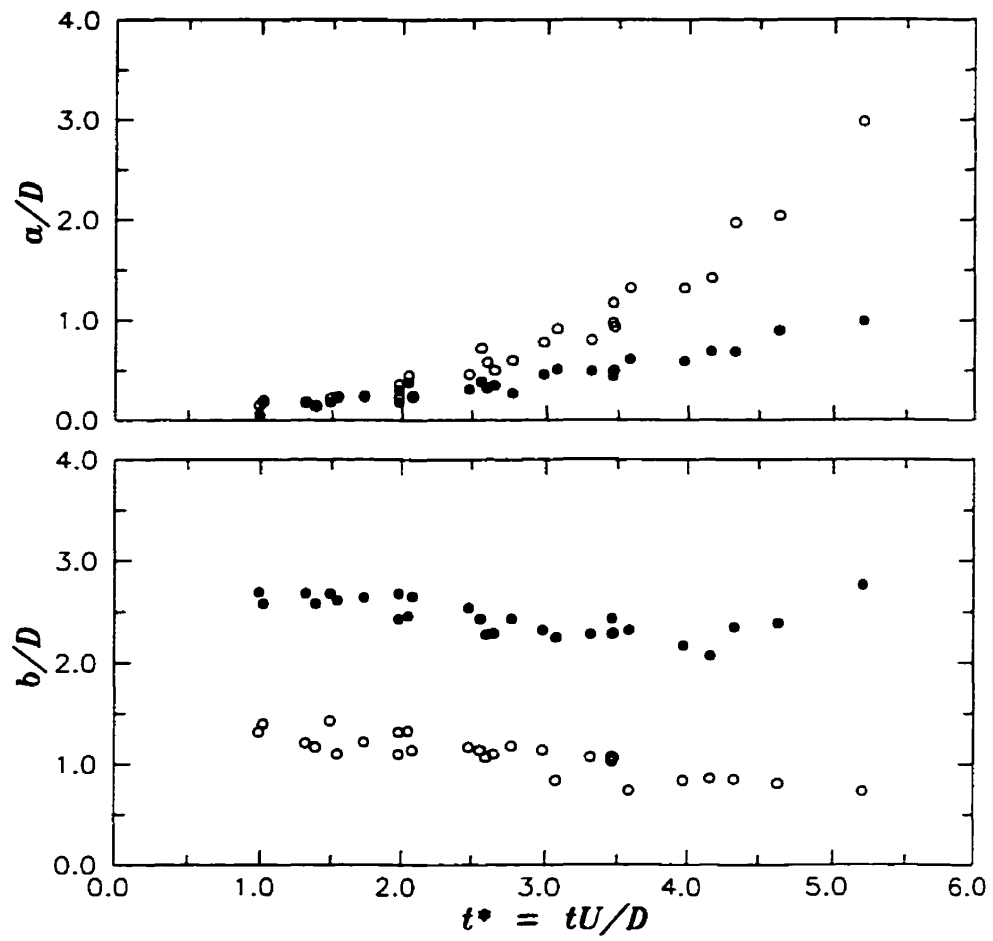


Figure 8.26 (continued).



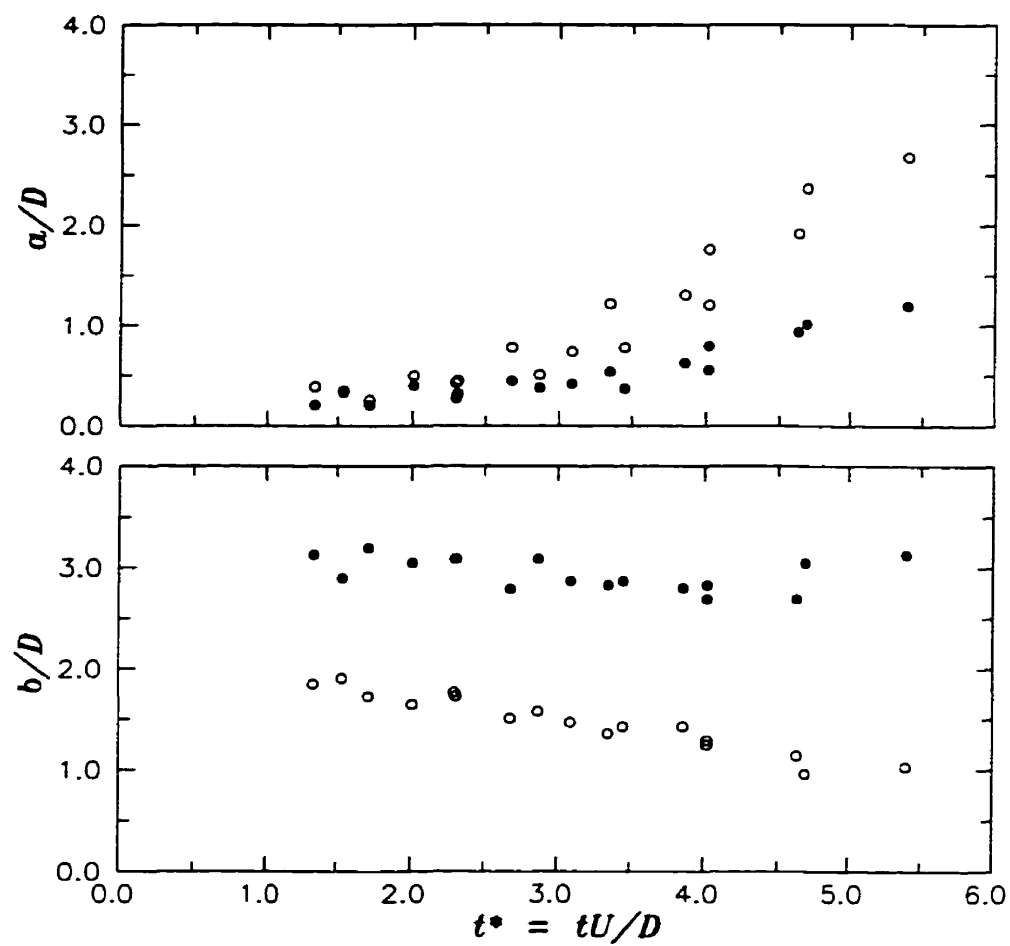
(a)

Figure 8.27. Streamwise position and transverse spacing of the vortices, for two impulsively started side-by-side cylinders, $M_r = 0.25$ mm/pixel: (a) $T/D = 1.5$, $Re = 2200$ to 2500 , $A_p = 0.3$; (b) $T/D = 2.0$, $Re = 1600$ to 2800 , $A_p = 0.3$ to 0.4 ; (c) $T/D = 2.5$, $Re = 1800$ to 2500 , $A_p = 0.7$ to 0.9 ; (d) $T/D = 3.0$, $Re = 1900$ to 2400 , $A_p = 0.3$. $\circ$, gap vortices; $\bullet$, outer vortices. Vorticity cutoff magnitude 1.0.



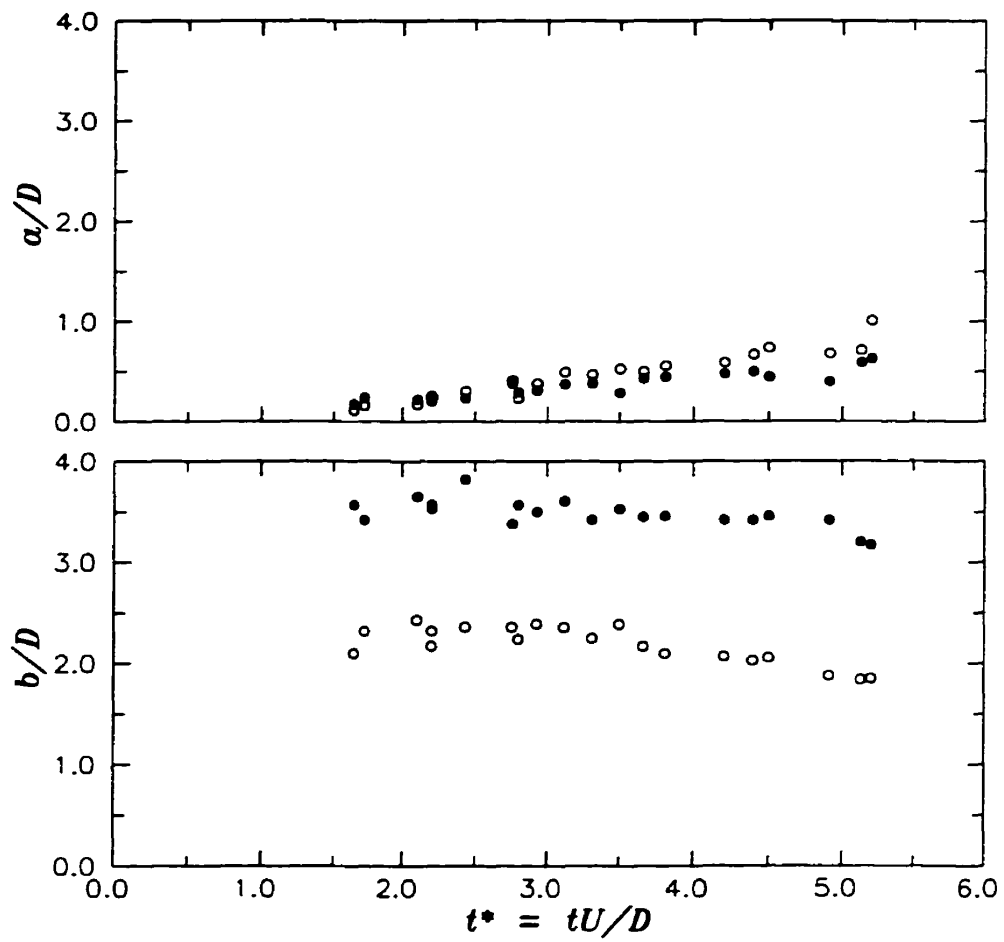
(b)

Figure 8.27 (continued).



(c)

Figure 8.27 (continued).



(d)

Figure 8.27 (continued).

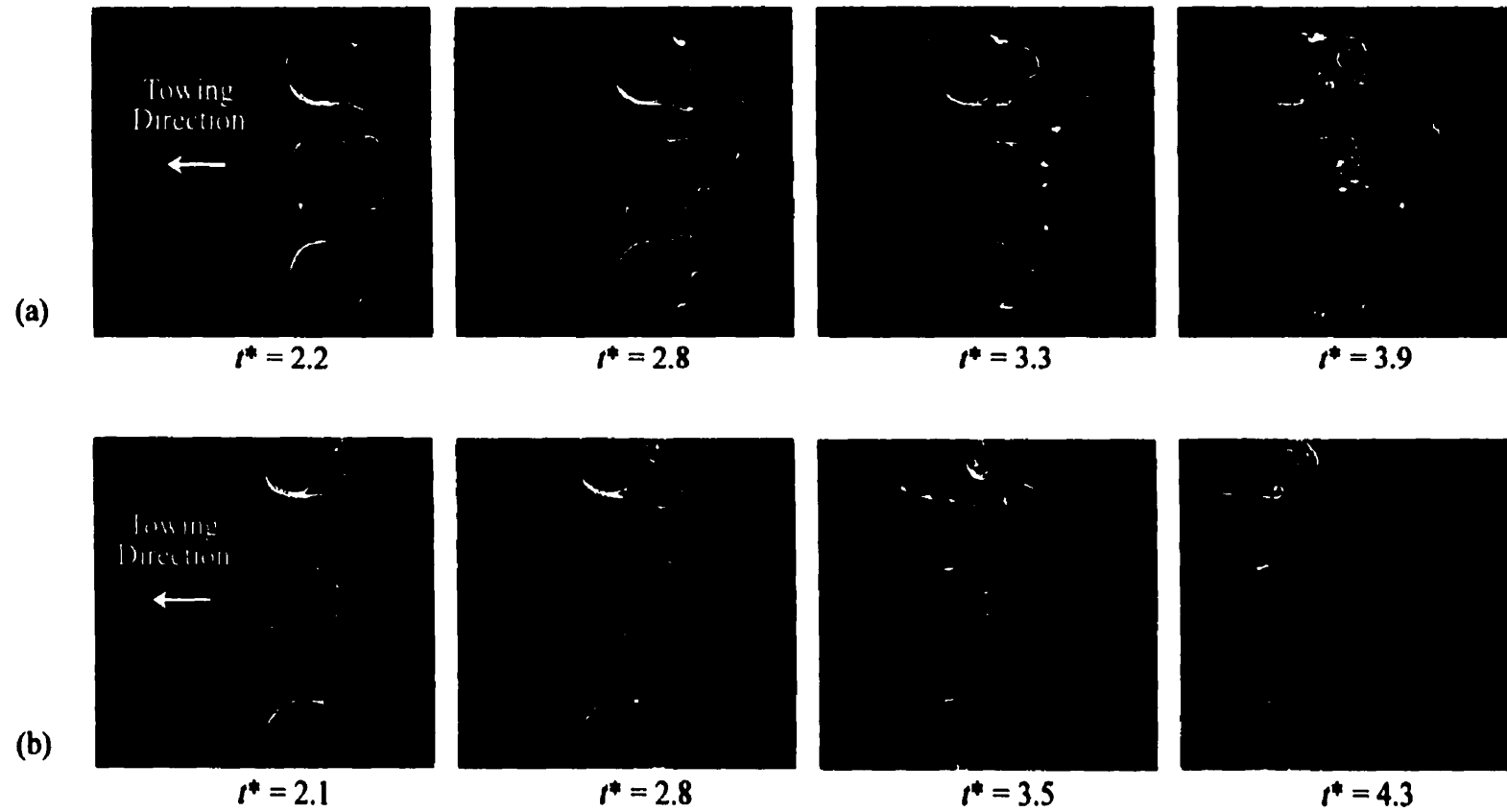


Figure 8.28. Flow visualization of three impulsively started side-by-side cylinders: (a) $T/D = 1.5$, $Re = 1800$; (b) $T/D = 2.0$, $Re = 2300$.

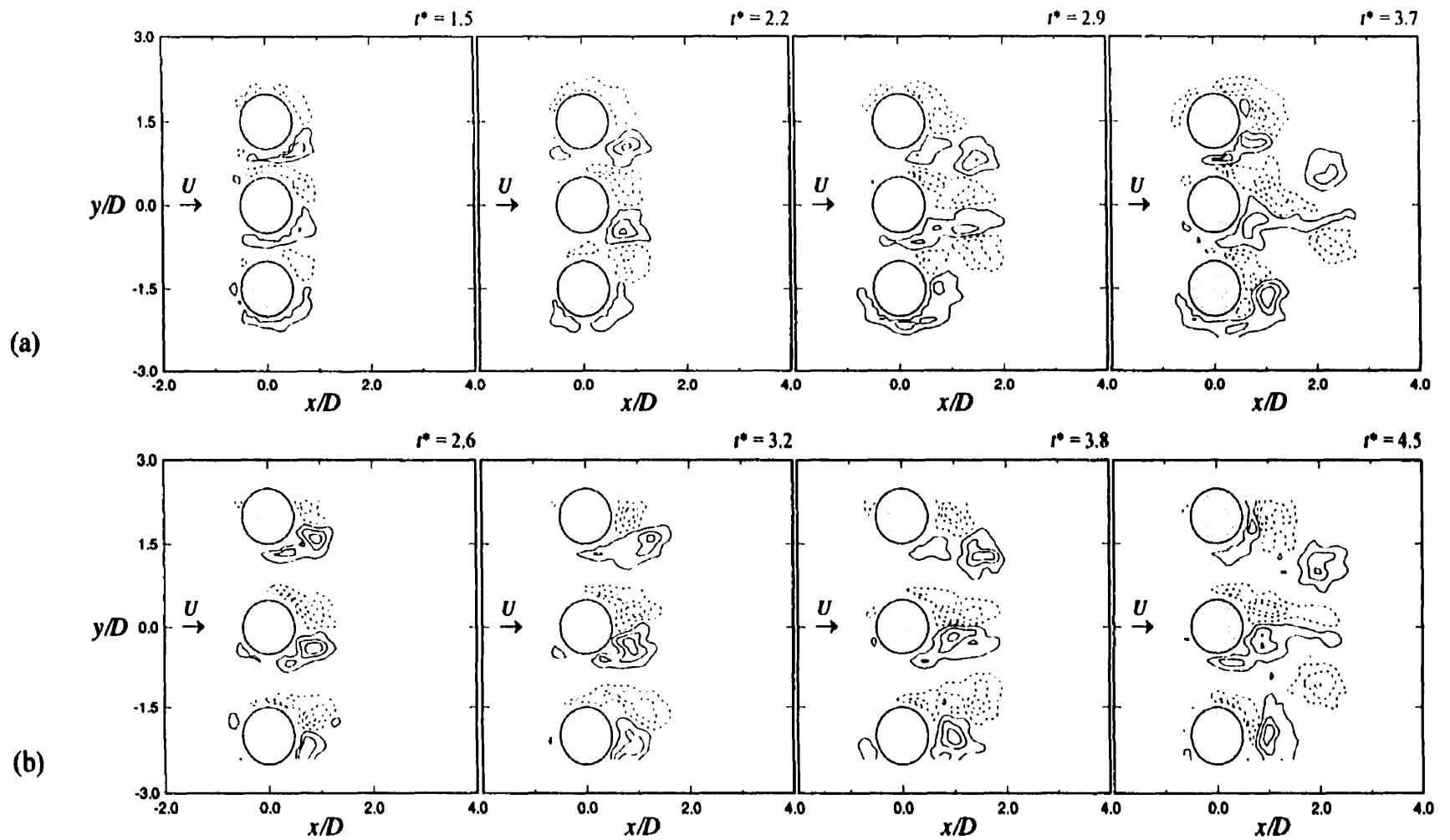


Figure 8.29. Three impulsively started side-by-side cylinders, $M_t = 0.25$ mm/pixel: (a) $T/D = 1.5$, $Re = 2300$, $A_p = 0.3$; (b) $T/D = 2.0$, $Re = 2000$, $A_p = 0.4$. Minimum vorticity contour magnitude 1.5, contour increment 4.0, dashed lines represent negative (CW) vorticity.

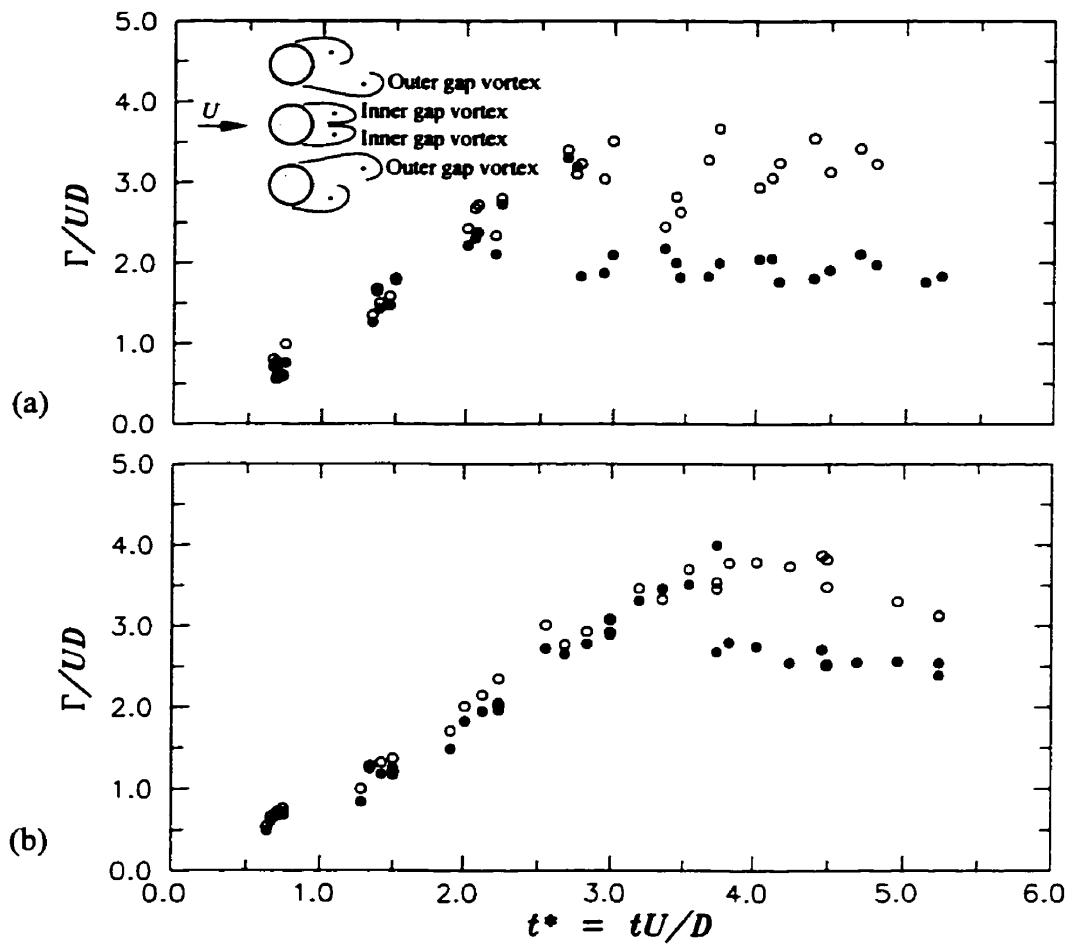


Figure 8.30. Vortex strength data for three impulsively started side-by-side cylinders, $A_p = 0.3$ to 0.4 , $M_r = 0.25$ mm/pixel: (a) $T/D = 1.5$, $Re = 2000$ to 2400 ; (b) $T/D = 2.0$, $Re = 2100$ to 2400 . $\circ$, inner gap vortices (behind central cylinder); $\bullet$, outer gap vortices (behind outer cylinders). Vorticity cutoff magnitude 1.0.

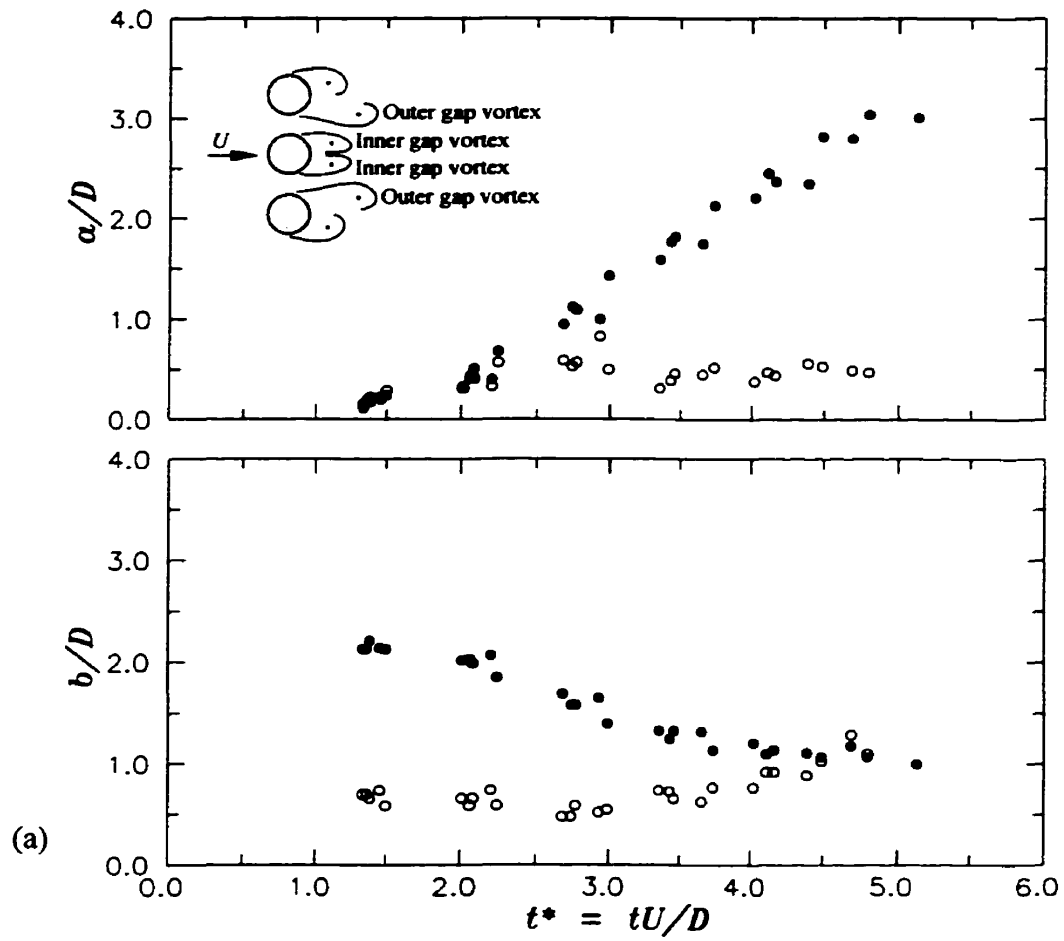


Figure 8.31. Streamwise position and transverse spacing of the vortices for three impulsively started side-by-side cylinders, $A_p = 0.3$ to 0.4 , $M_r = 0.25$ mm/pixel: (a) $T/D = 1.5$, $Re = 2000$ to 2400 ; (b) $T/D = 2.0$, $Re = 2100$ to 2400 . ○, inner gap vortices (behind central cylinder); ●, outer gap vortices (behind outer cylinders). Vorticity cutoff magnitude 1.0.

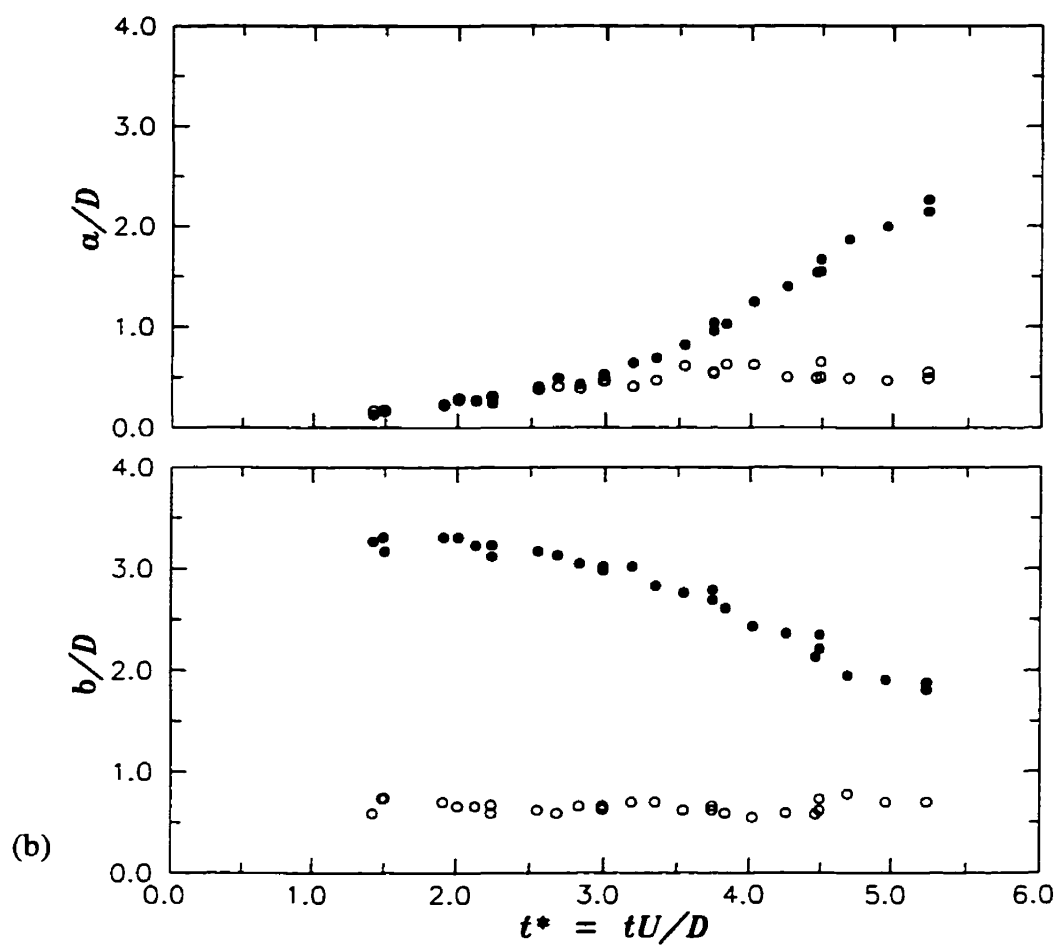


Figure 8.31 (continued).

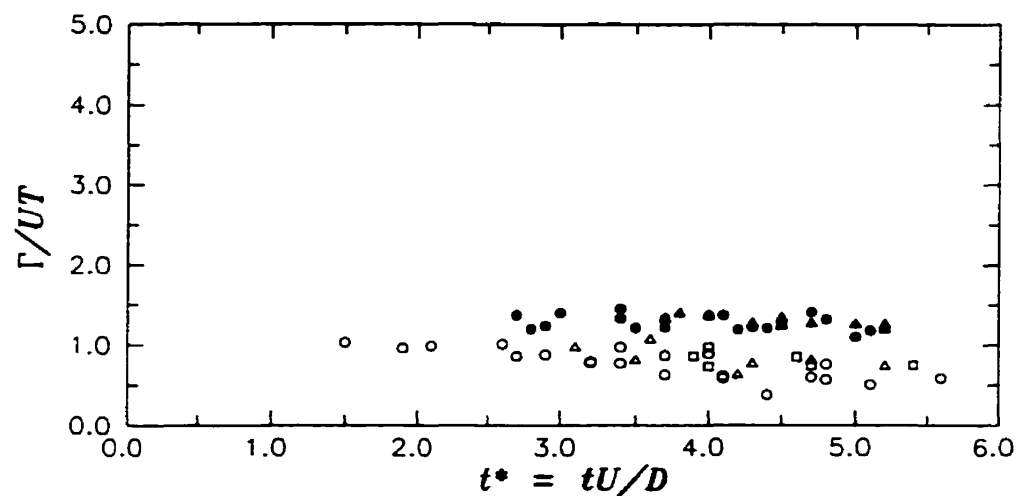


Figure 8.32. Strength of a vortex within the far-wake counter-rotating vortex pair, for impulsively started side-by-side cylinders, $M_r = 0.25$ mm/pixel. Two cylinders: ○, $T/D = 1.5$, $Re = 2200$ to 2500 , $A_p = 0.3$; △, $T/D = 2.0$, $Re = 1600$ to 2800 , $A_p = 0.3$ to 0.4 ; □, $T/D = 2.5$, $Re = 1800$ to 2500 , $A_p = 0.7$ to 0.9 . Three cylinders, $A_p = 0.3$ to 0.4 : ●, $T/D = 1.5$, $Re = 2000$ to 2400 ; ▲, $T/D = 2.0$, $Re = 2100$ to 2400 . Vorticity cutoff magnitude 1.0.

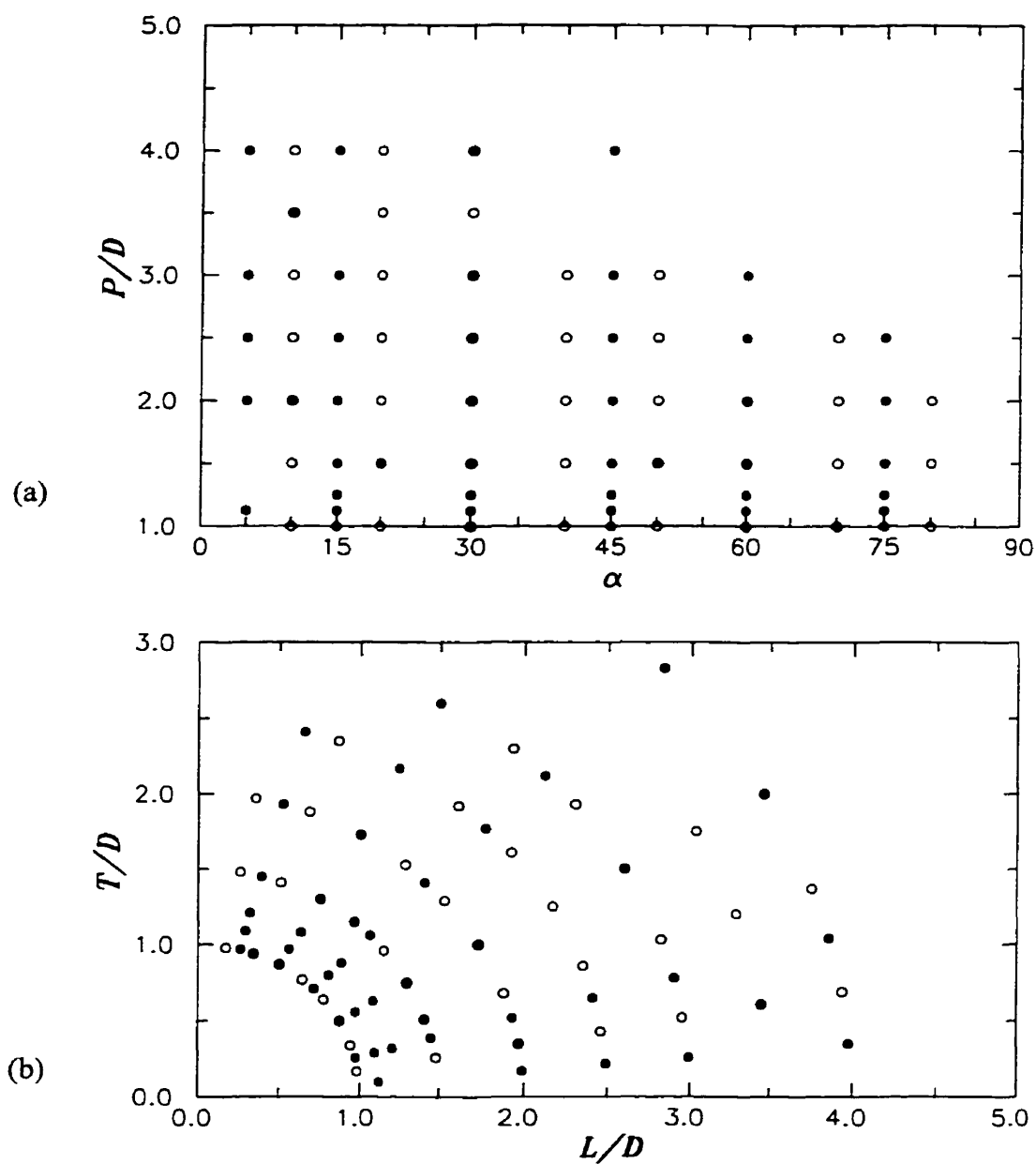
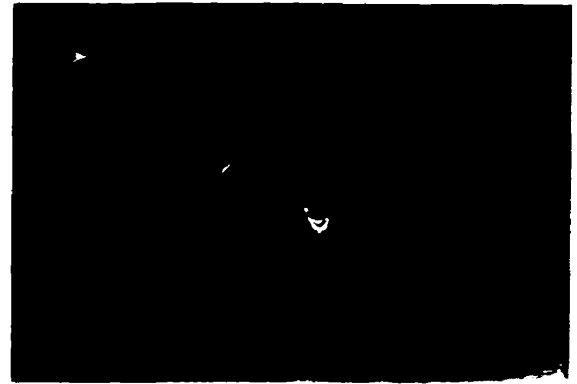


Figure 9.1. Staggered configurations tested in the present study: (a) P/D - α notation; (b) L/D - T/D notation. $\circ$, flow visualization experiments; $\bullet$, PIV experiments.



(a)



(b)



(c)



(d)

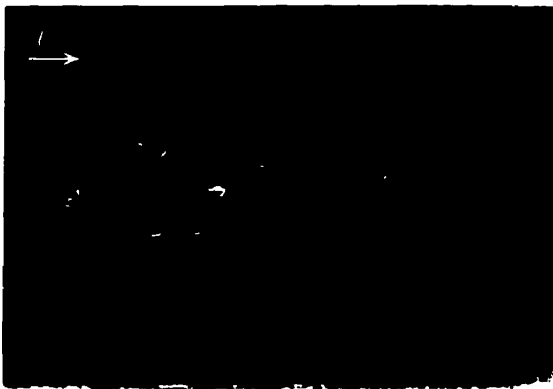
Figure 9.2. Flow visualization of two staggered circular cylinders in contact, $P/D = 1.0$, in steady cross-flow: (a) $\alpha = 10^\circ$, $Re = 900$; (b) $\alpha = 20^\circ$, $Re = 1270$; (c) $\alpha = 30^\circ$, $Re = 1320$; (d) $\alpha = 40^\circ$, $Re = 930$; (e) $\alpha = 50^\circ$, $Re = 1270$; (f) $\alpha = 60^\circ$, $Re = 1320$; (g) $\alpha = 70^\circ$, $Re = 860$; (h) $\alpha = 80^\circ$, $Re = 1320$.



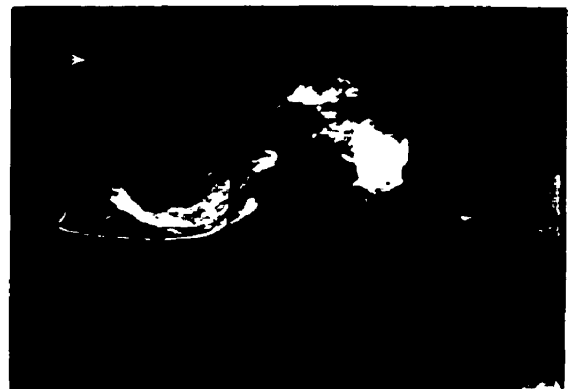
(e)



(f)



(g)



(h)

Figure 9.2 (continued).

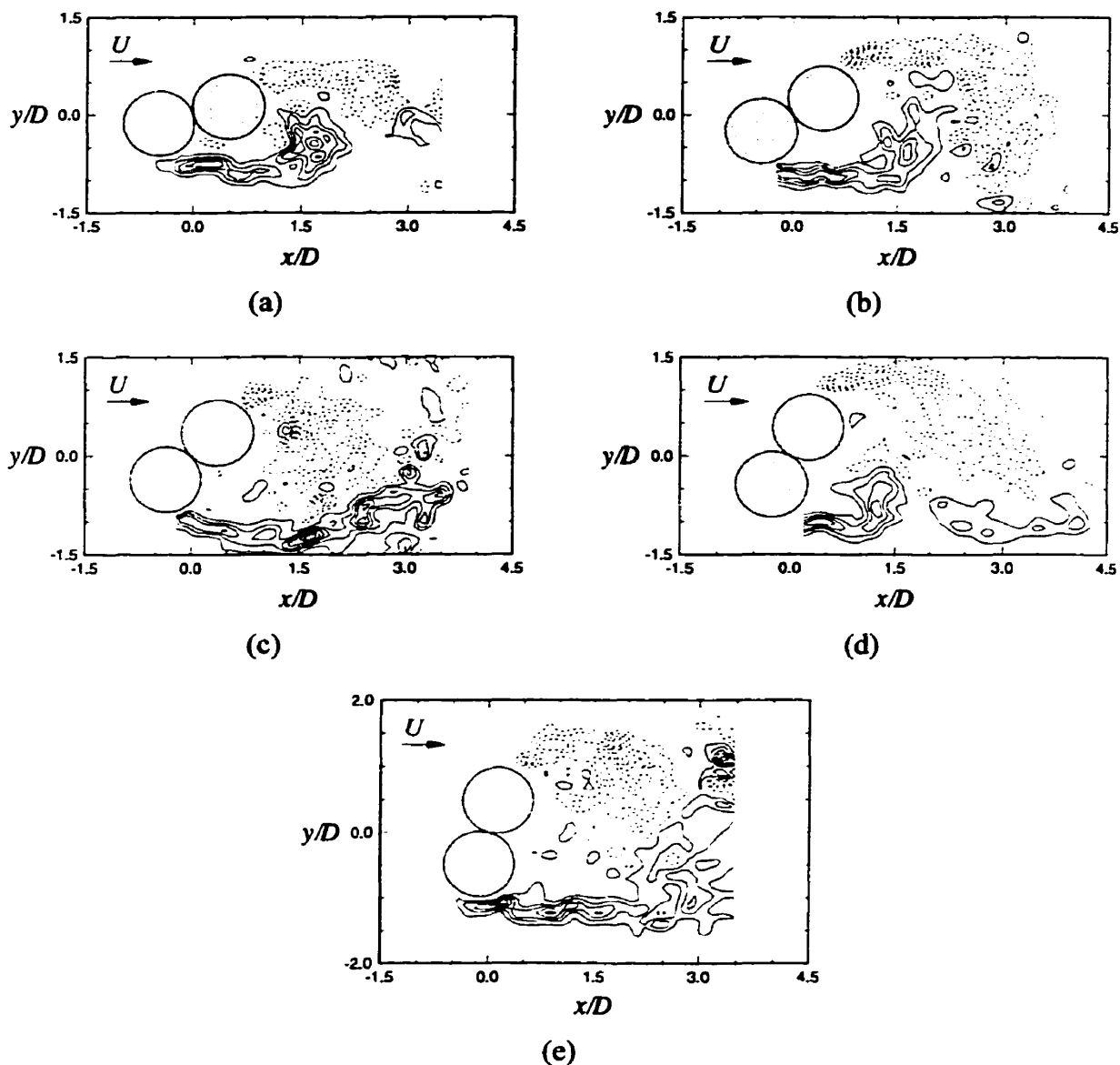


Figure 9.3. Vorticity field for staggered cylinders in steady cross-flow, with $P/D = 1.0$, $Re = 1300$, $M_r = 0.25$ mm/pixel: (a) $\alpha = 15^\circ$; (b) $\alpha = 30^\circ$; (c) $\alpha = 45^\circ$; (d) $\alpha = 60^\circ$; (e) $\alpha = 75^\circ$. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.

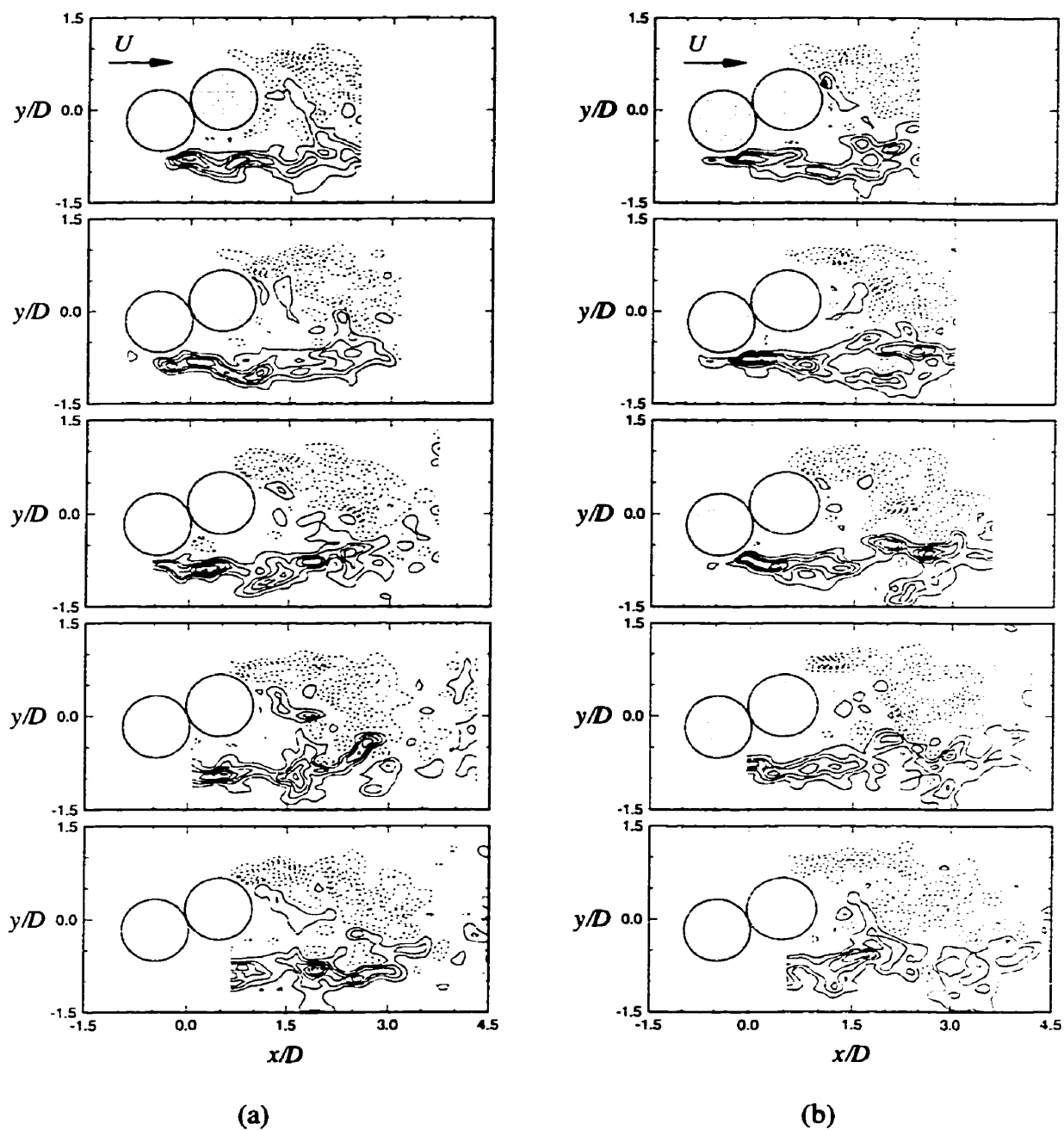


Figure 9.4. SBB1 flow pattern, $P/D = 1.0$, $\alpha = 20^\circ$, $Re = 1900$, $M_r = 0.21$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_r = 0.06$: (a) vortex forming from the downstream cylinder shear layer; (b) vortex being shed from the upstream cylinder shear layer. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.

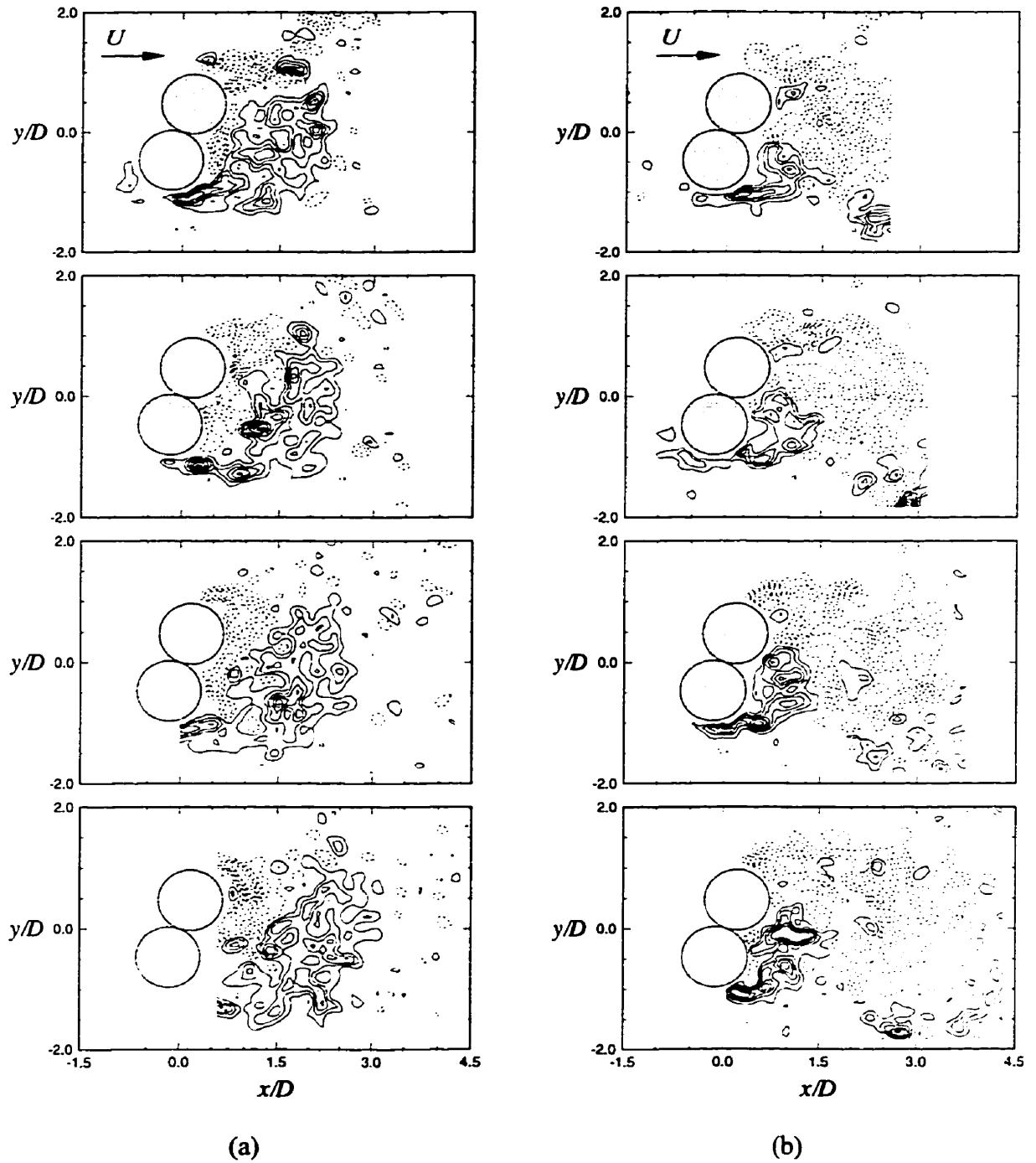


Figure 9.5. SBB2 flow pattern, $P/D = 1.0$, $\alpha = 70^\circ$, $Re = 1900$, $M_r = 0.21$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_r = 0.06$: (a) vortex forming from the upstream cylinder shear layer; (b) vortex forming from the downstream cylinder shear layer. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.



(a)



(b)



(c)



(d)

Figure 9.6. Irregular vortex shedding behaviour for two staggered cylinders in steady cross-flow, with $P/D = 1.0$: (a) $\alpha = 10^\circ$, $Re = 900$; (b) $\alpha = 20^\circ$, $Re = 880$; (c) $\alpha = 30^\circ$, $Re = 900$; (d) $\alpha = 40^\circ$, $Re = 930$; (e) $\alpha = 50^\circ$, $Re = 850$; (f) $\alpha = 60^\circ$, $Re = 910$; (g) $\alpha = 70^\circ$, $Re = 860$; (h) $\alpha = 80^\circ$, $Re = 1320$.



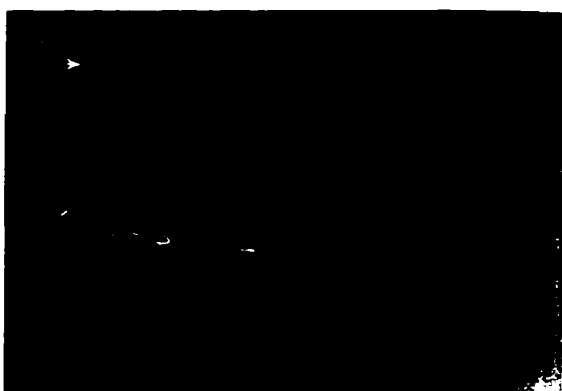
(e)



(f)



(g)



(h)

Figure 9.6 (continued).

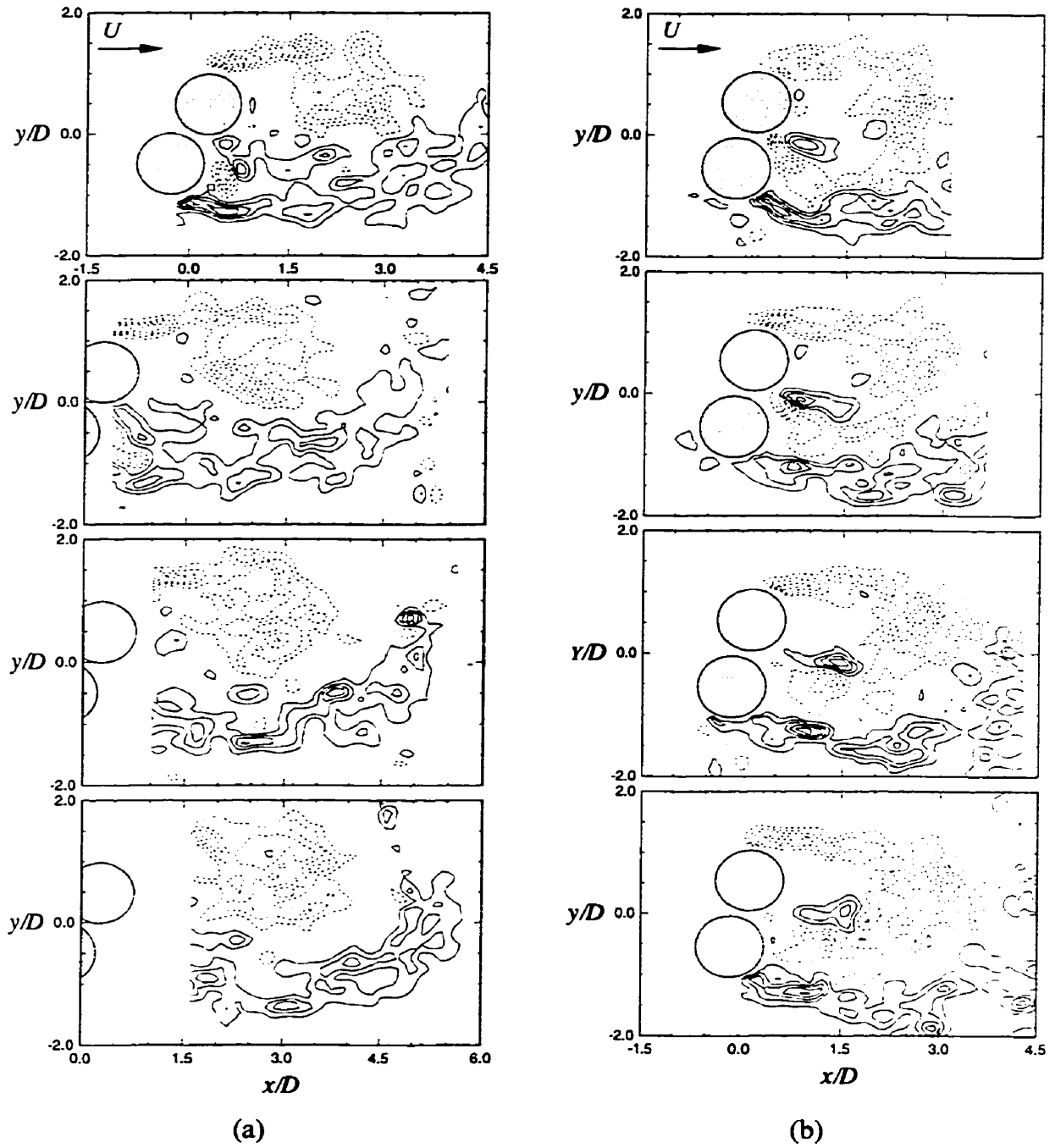


Figure 9.7. BB flow pattern, $P/D = 1.125$, $Re = 1900$, $M_r = 0.25$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_c = 0.06$: (a) $\alpha = 60^\circ$, showing Kármán vortex formation; (b) $\alpha = 75^\circ$, showing a lengthened near-wake region. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.

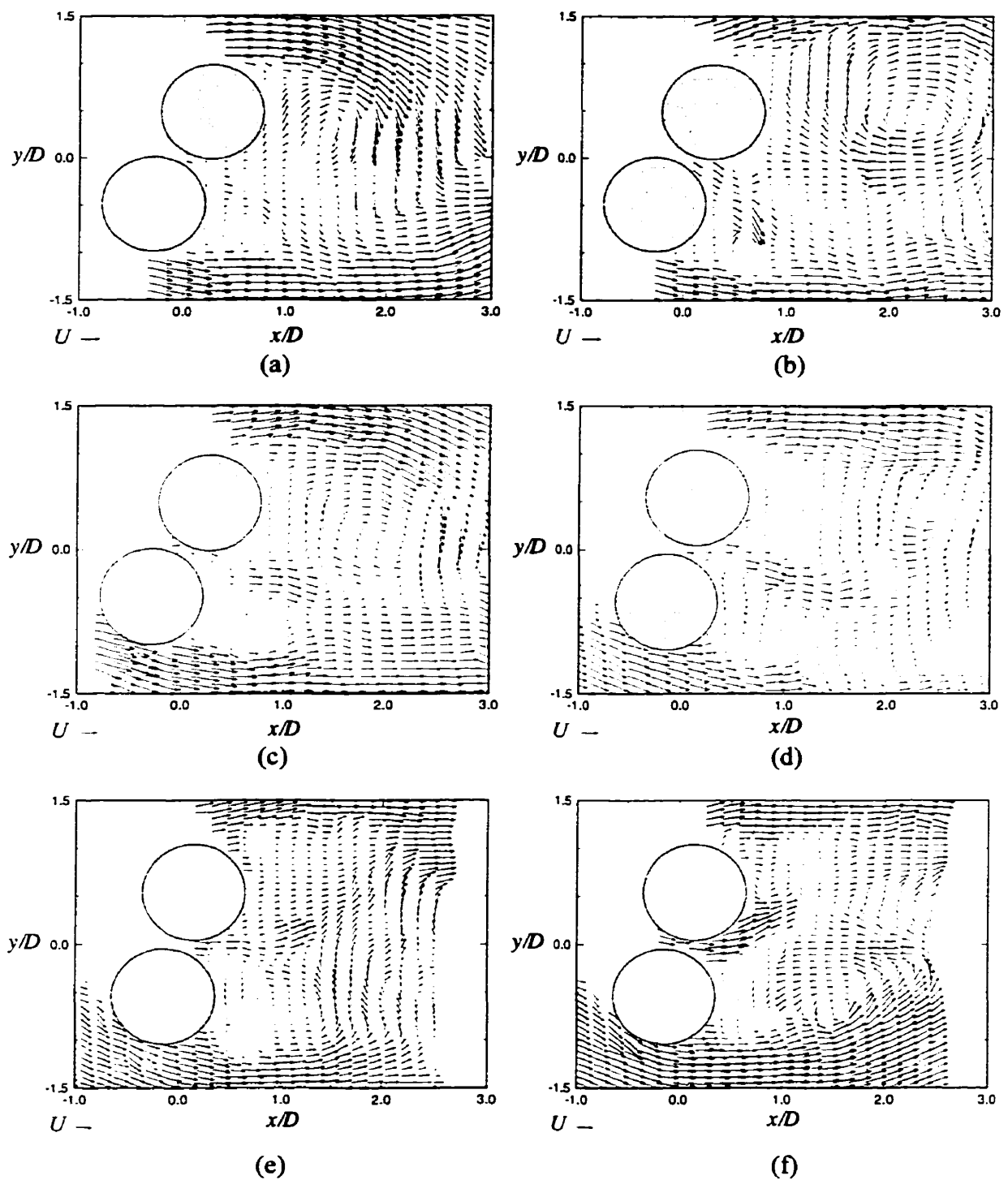


Figure 9.8. BB flow pattern, $P/D = 1.125$, $Re = 1900$: (a) $\alpha = 60^\circ$, $M_r = 0.25$ mm/pixel, no gap flow; (b) $\alpha = 60^\circ$, $M_r = 0.25$ mm/pixel, downward low magnitude gap flow; (c) $\alpha = 60^\circ$, $M_r = 0.25$ mm/pixel, downward high magnitude gap flow; (d) $\alpha = 75^\circ$, $M_r = 0.25$ mm/pixel, downward gap flow; (e) $\alpha = 75^\circ$, $M_r = 0.21$ mm/pixel, undeflected low magnitude gap flow; (f) $\alpha = 75^\circ$, $M_r = 0.21$ mm/pixel, upward deflected gap flow. Every second vector in the x -direction omitted for clarity.



(a)



(b)

Figure 9.9. Flow visualization of the shear layer reattachment (SLR) flow pattern: (a) $P/D = 1.5$, $\alpha = 10^\circ$, $Re = 1300$; (b) $P/D = 2.0$, $\alpha = 10^\circ$, $Re = 880$.

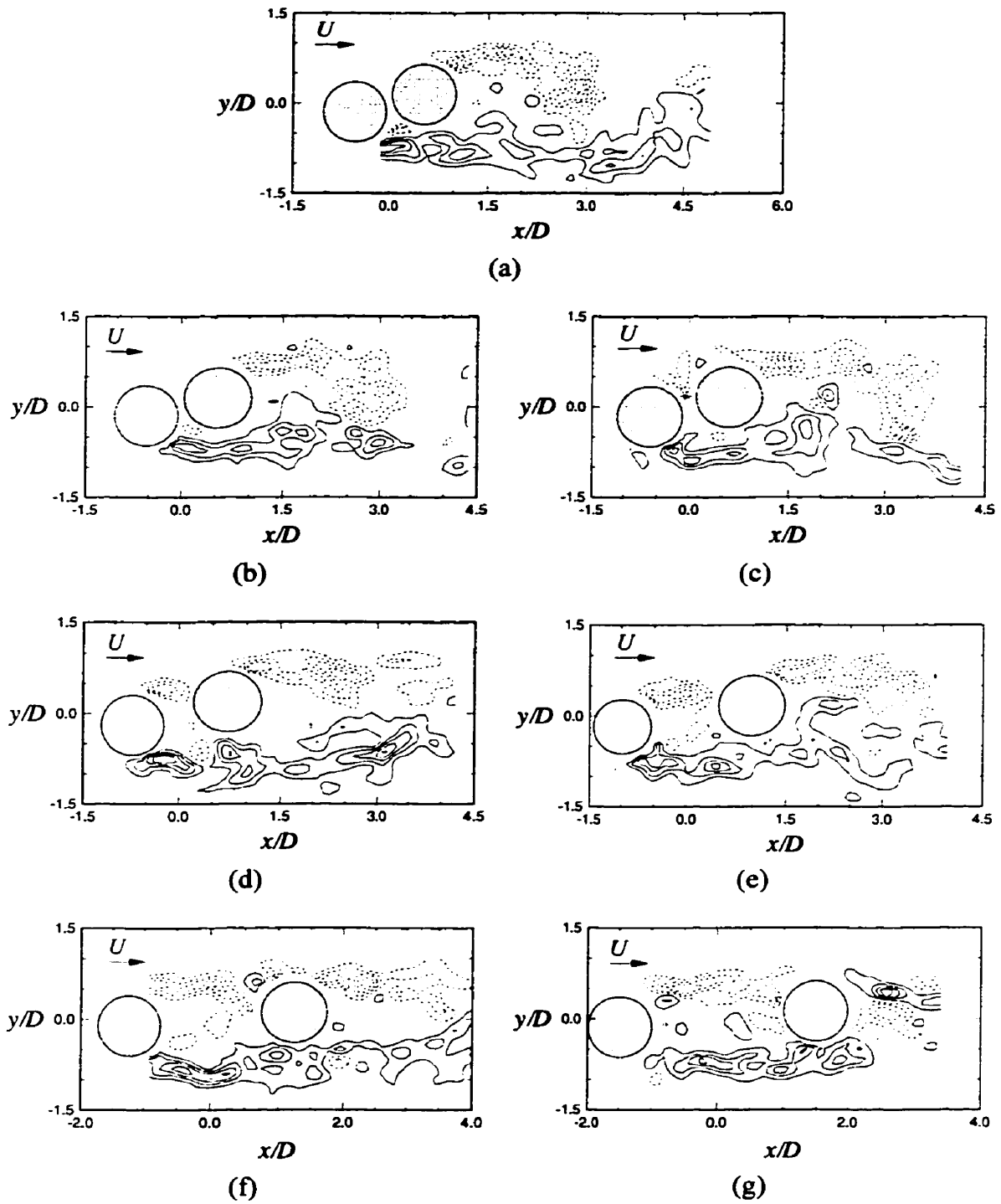


Figure 9.10. SLR flow pattern, $Re = 1900$, $M_r = 0.25$ mm/pixel: (a) $P/D = 1.125$, $\alpha = 15^\circ$, long vortex formation region; (b) $P/D = 1.125$, $\alpha = 15^\circ$, short vortex formation region; (c) $P/D = 1.25$, $\alpha = 15^\circ$; (d) $P/D = 1.5$, $\alpha = 15^\circ$; (e) $P/D = 2.0$, $\alpha = 10^\circ$; (f) $P/D = 2.5$, $\alpha = 5^\circ$; (g) $P/D = 3.0$, $\alpha = 5^\circ$. Minimum vorticity contour magnitude 1.0, contour increment 1.0, dashed lines represent negative (CW) vorticity.

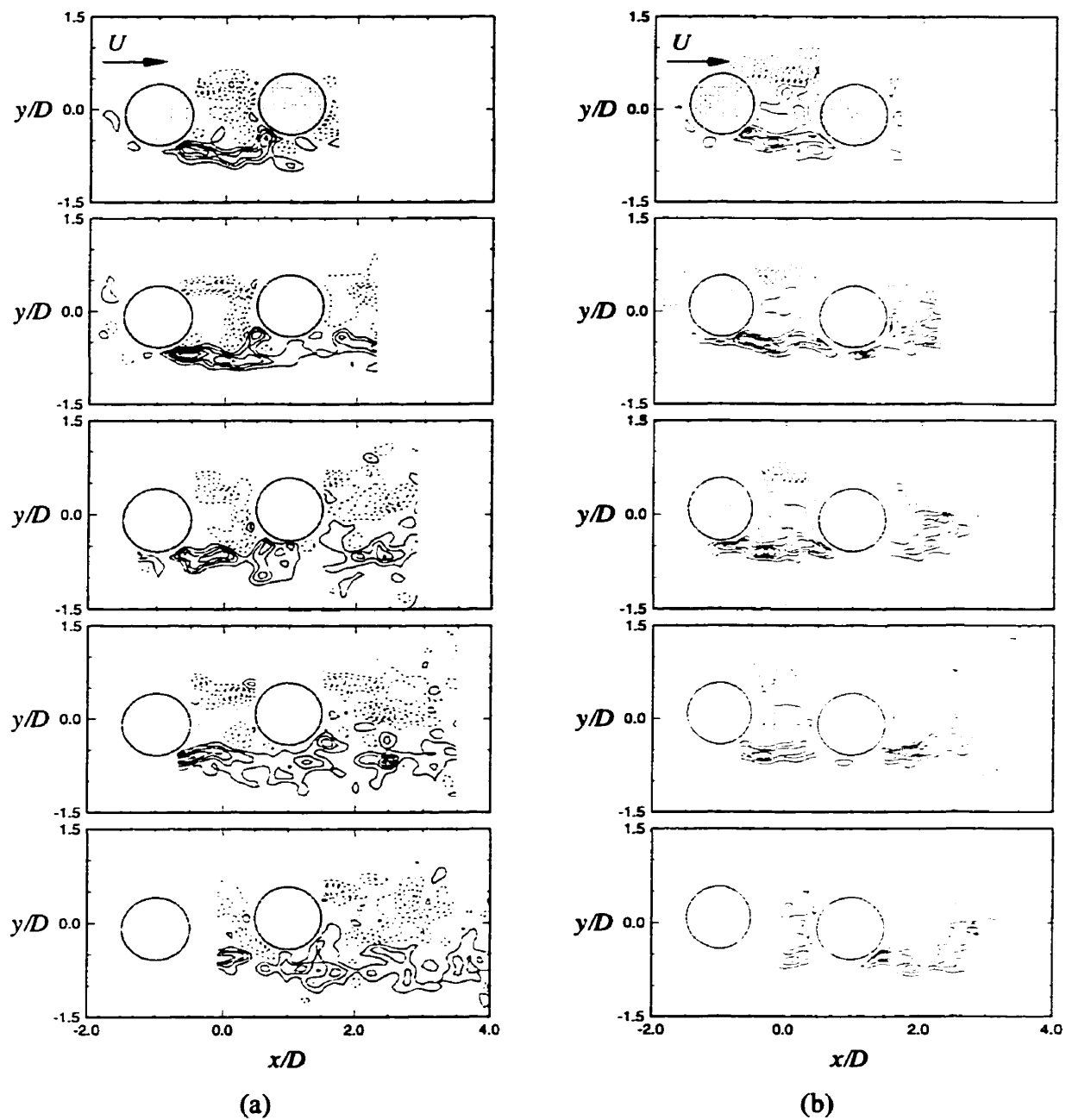


Figure 9.11. SLR flow pattern, $P/D = 2.0$, $\alpha = 5^\circ$, $Re = 1900$, $M_r = 0.21$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_r = 0.10$: (a) from the free shear layer side (i.e. with $\alpha = +5^\circ$); (b) from the reattachment side (i.e. with $\alpha = -5^\circ$). Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.

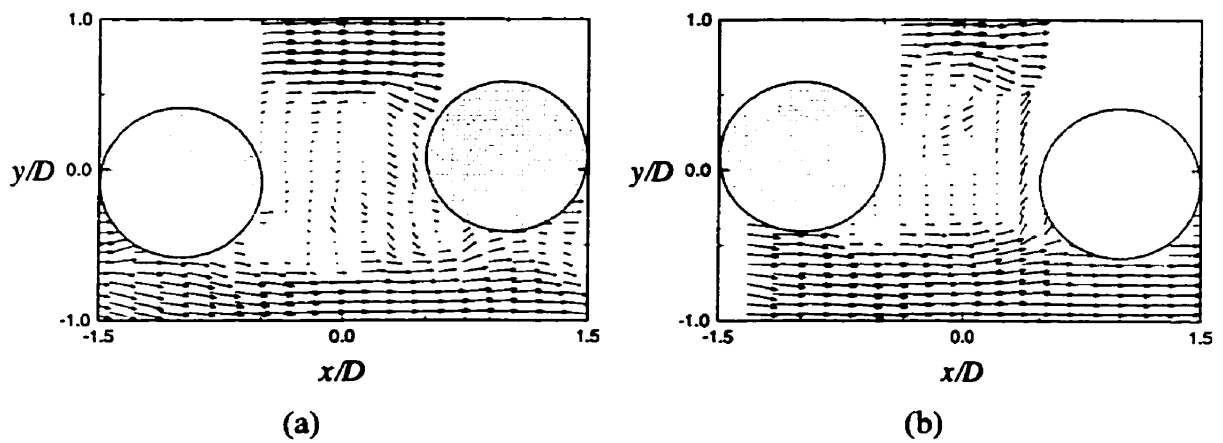


Figure 9.12. SLR gap flow patterns, $P/D = 2.0$, $\alpha = 5^\circ$, $Re = 1900$, $M_r = 0.21$ mm/pixel: (a) from the free shear layer side (i.e. with $\alpha = +5^\circ$), showing stagnant fluid in the gap, and flow deflected in front of the downstream cylinder; (b) from the reattachment side (i.e. with $\alpha = -5^\circ$), showing stagnant fluid in the gap, the reattachment point, and flow deflected in front of the downstream cylinder. Every second velocity vector in the x -direction omitted for clarity.

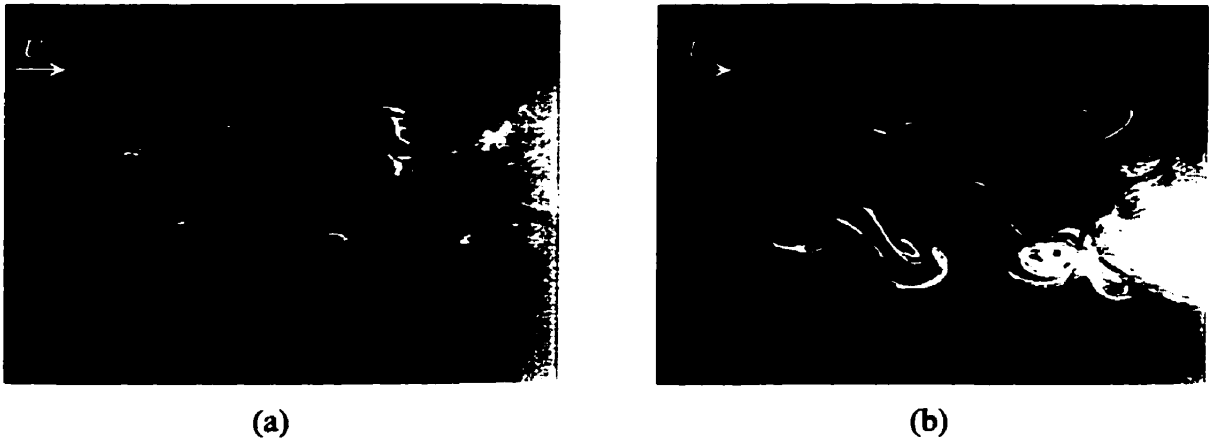


Figure 9.13. Flow visualization of the induced separation (IS) flow pattern: (a) $P/D = 1.5$, $\alpha = 20^\circ$, $Re = 1320$; (b) $P/D = 2.0$, $\alpha = 20^\circ$, $Re = 850$.

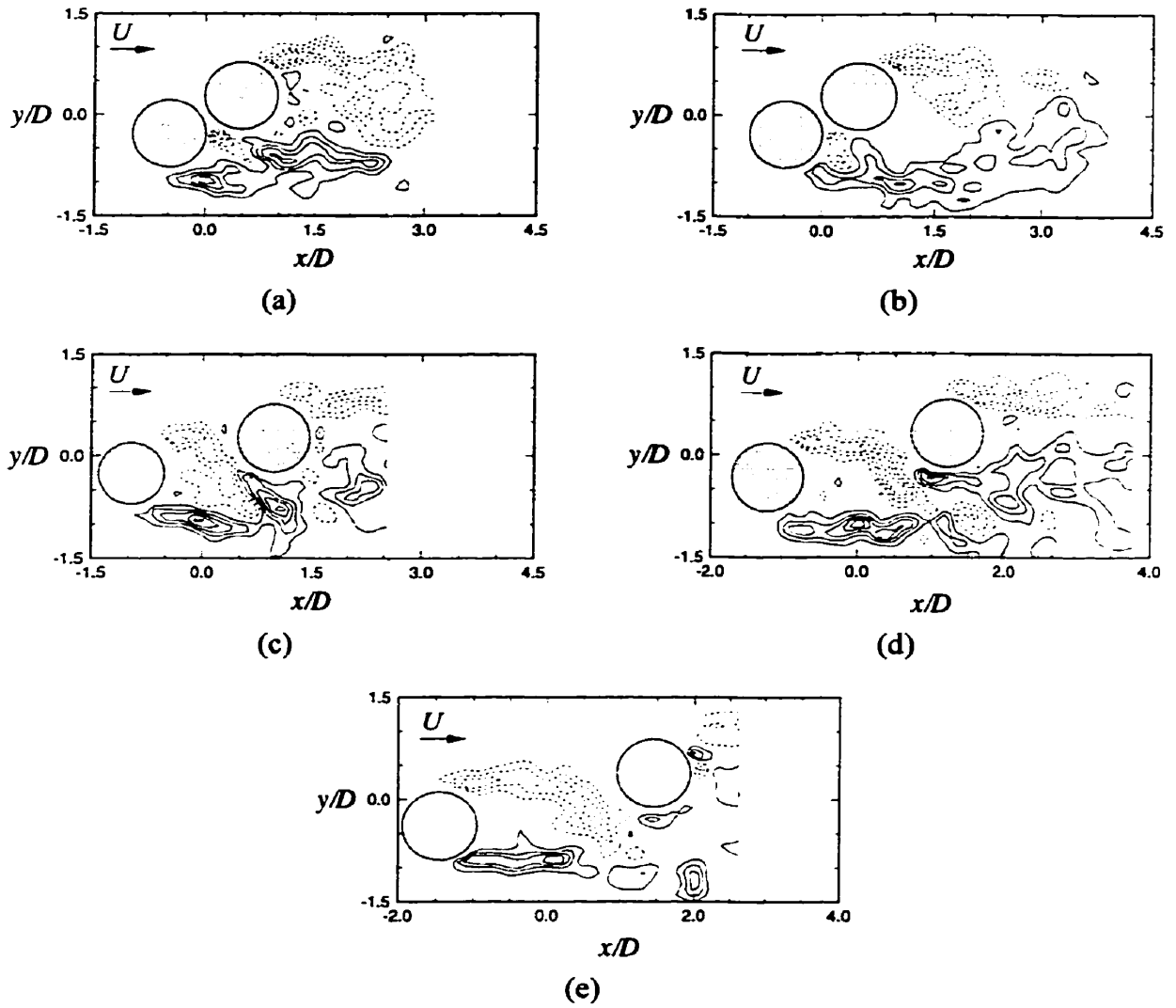


Figure 9.14. IS flow pattern, $Re = 1900$, $M_r = 0.25$ mm/pixel: (a) $P/D = 1.125$, $\alpha = 30^\circ$, small gap deflection; (b) $P/D = 1.125$, $\alpha = 30^\circ$, large gap deflection; (c) $P/D = 2.0$, $\alpha = 15^\circ$; (d) $P/D = 2.5$, $\alpha = 15^\circ$; (e) $P/D = 3.0$, $\alpha = 15^\circ$. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.

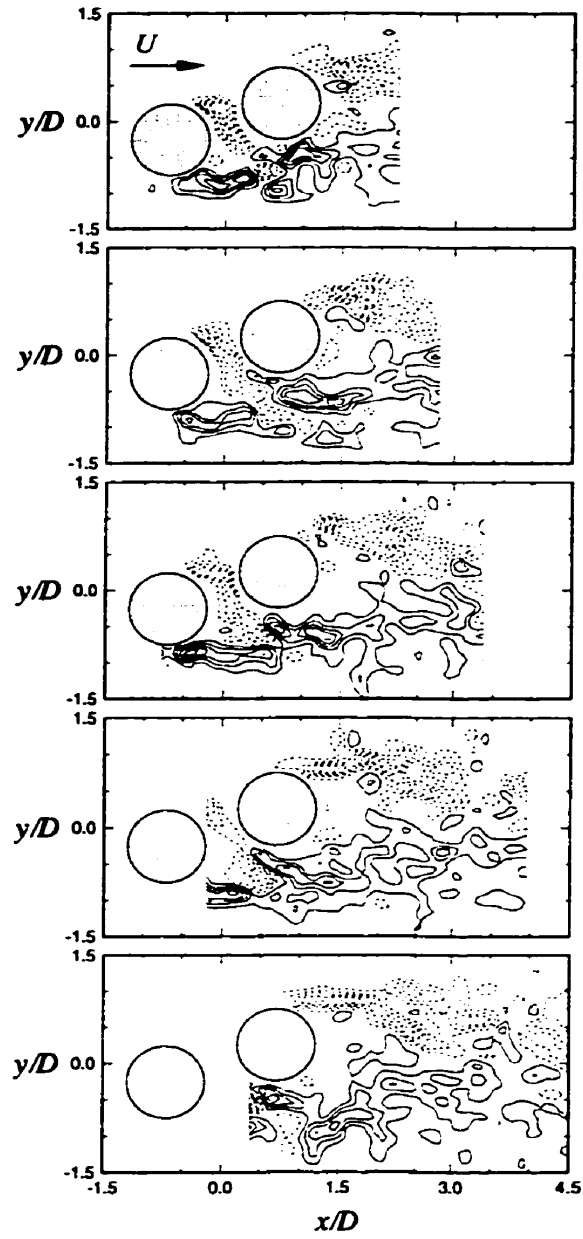


Figure 9.15. IS flow pattern, $P/D = 1.5$, $\alpha = 20^\circ$, $Re = 1900$, $M_t = 0.21$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_s = 0.28$. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.

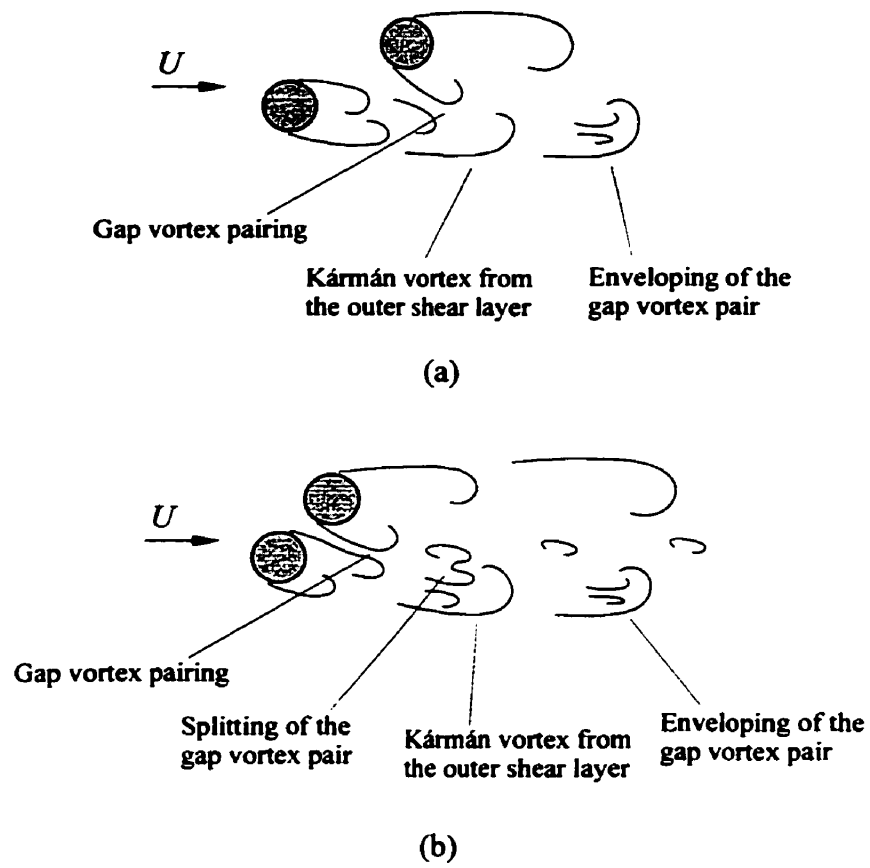


Figure 9.16. Staggered cylinder flow patterns with gap vortex pairing: (a) complete enveloping of the gap vortex pair by an outer vortex from the upstream cylinder; (b) incomplete enveloping and splitting of the gap vortex pair, during the enveloping process.



Figure 9.17. Flow visualization of the gap vortex pairing and enveloping (VPE) flow pattern, $P/D=1.5$, $\alpha=30^\circ$, $Re=850$.

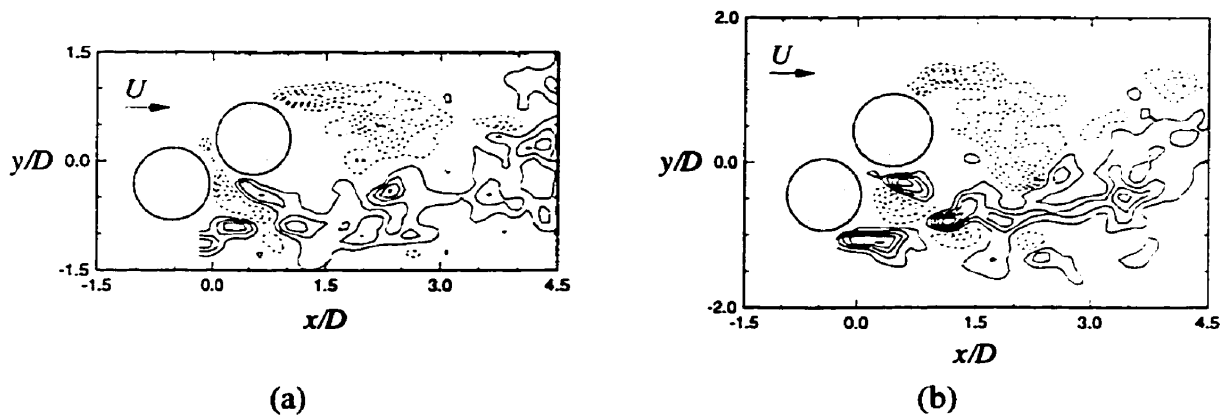


Figure 9.18. VPE flow patterns, $P/D=1.25$, $Re=1900$, $M_t=0.25$ mm/pixel: (a) $\alpha=30^\circ$; (b) $\alpha=45^\circ$. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.

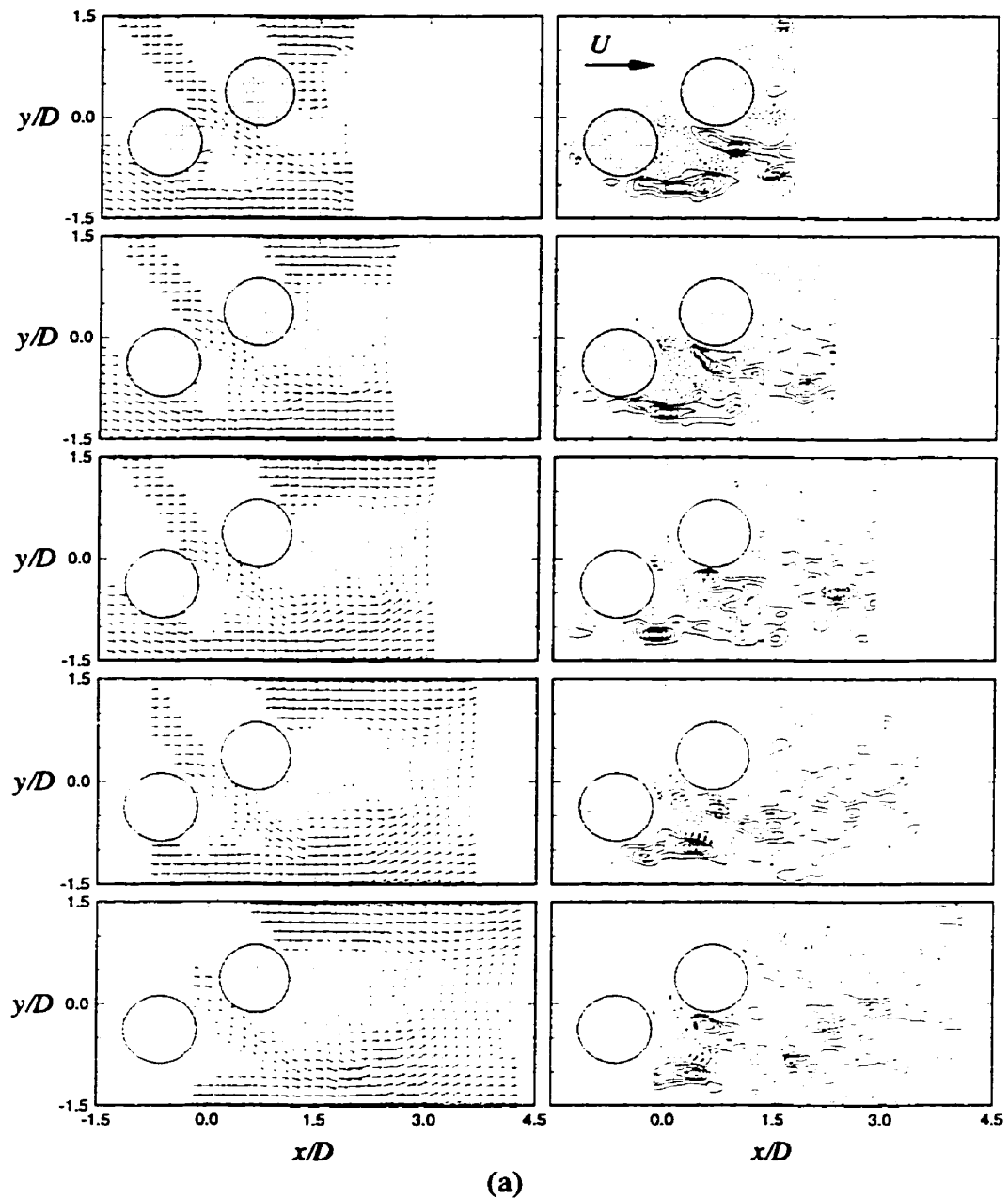


Figure 9.19. VPE flow pattern, $P/D = 1.5$, $\alpha = 30^\circ$, $Re = 1900$, $M_r = 0.21$ mm/pixel, $\Delta r^* = 0.6$, $\Delta t/T_r = 0.07$ or 0.52 : (a) showing composite vortex formation from the gap vorticity; (b) showing Kármán vortex formation from the outer shear layer of the downstream cylinder. Every second velocity vector in each direction omitted for clarity. Minimum vorticity contour magnitude 1.0, contour increment 1.0, dashed lines represent negative (CW) vorticity.

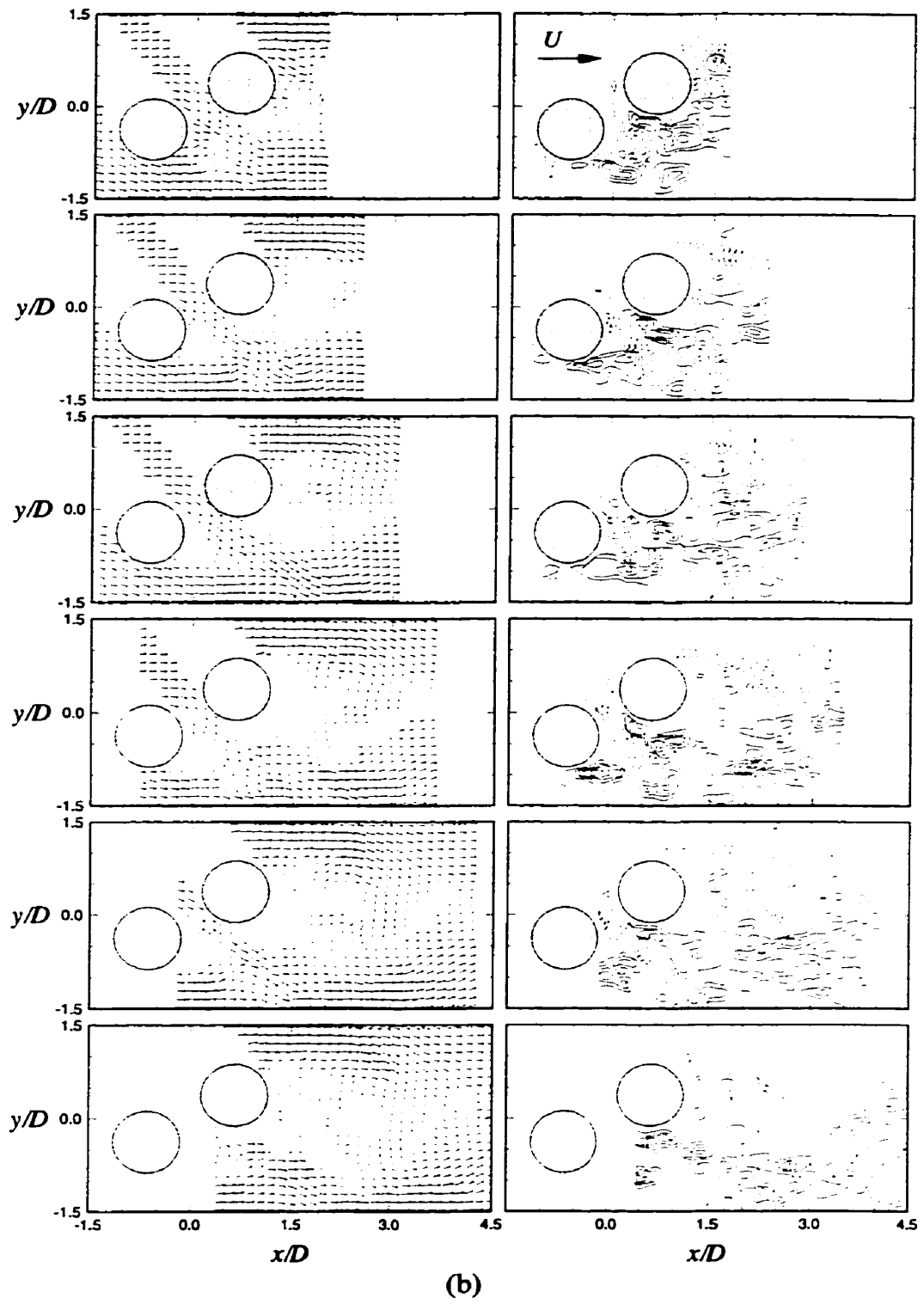
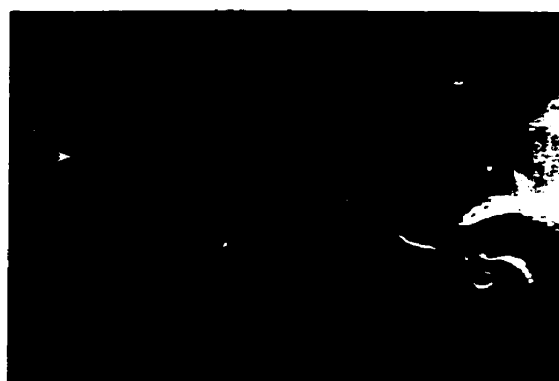


Figure 9.19 (continued).



(a)



(b)



(c)



(d)

Figure 9.20. Flow visualization of the vortex pairing, splitting and enveloping (VPSE) flow pattern: (a) $P/D = 1.5$, $\alpha = 50^\circ$, $Re = 910$; (b) $P/D = 1.5$, $\alpha = 60^\circ$, $Re = 900$; (c) $P/D = 1.5$, $\alpha = 70^\circ$, $Re = 910$; (d) $P/D = 2.0$, $\alpha = 40^\circ$, $Re = 880$.

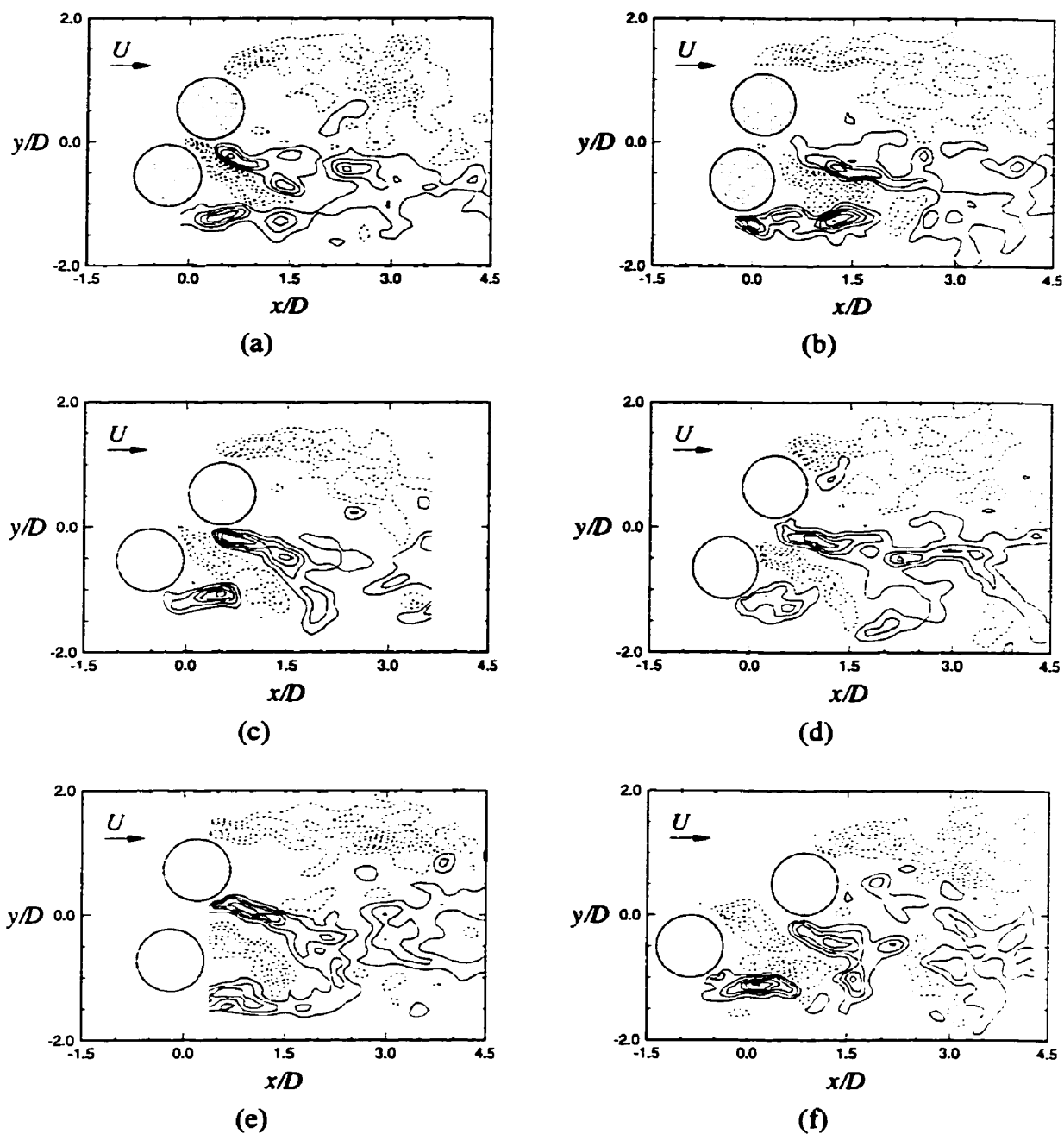
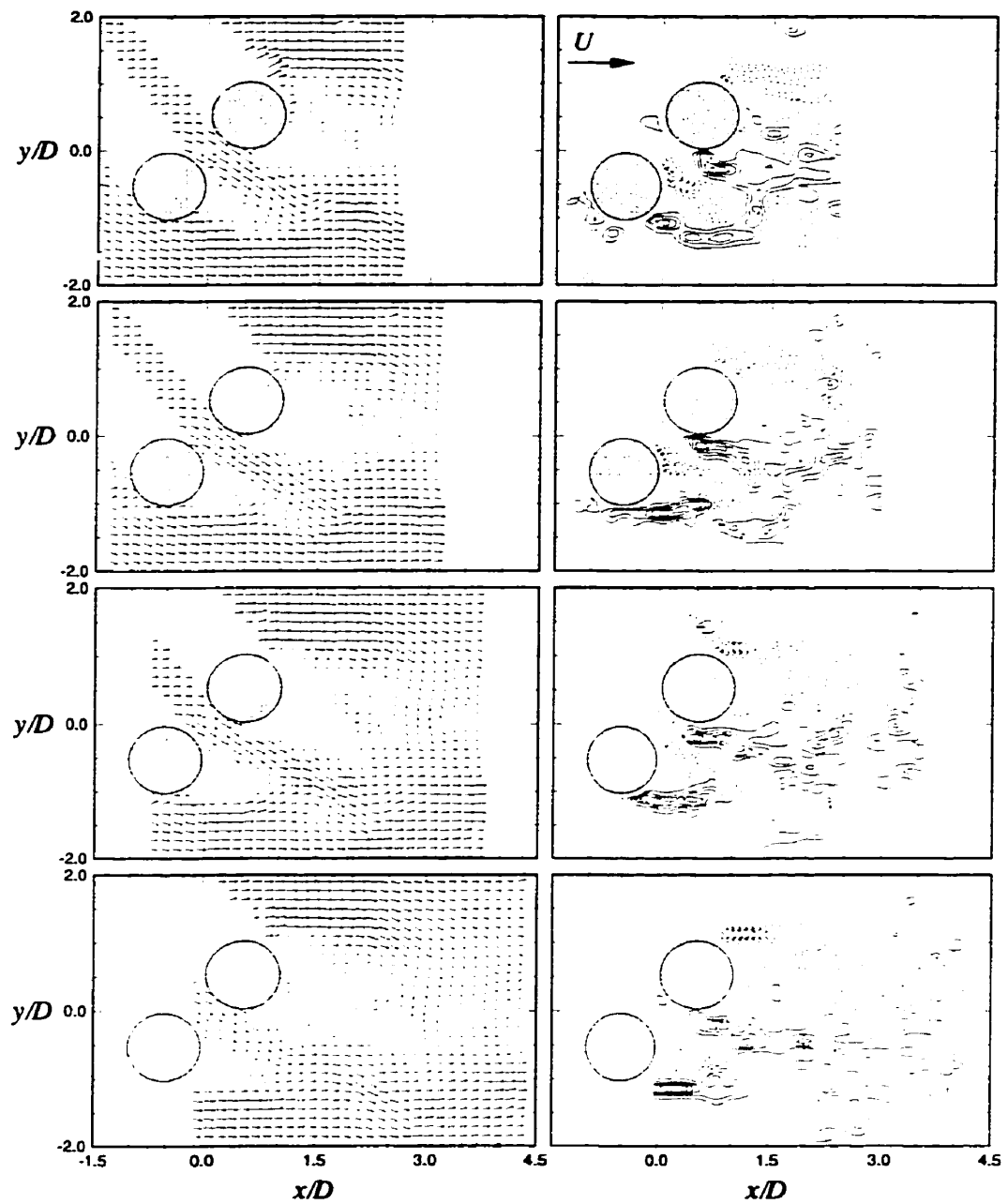
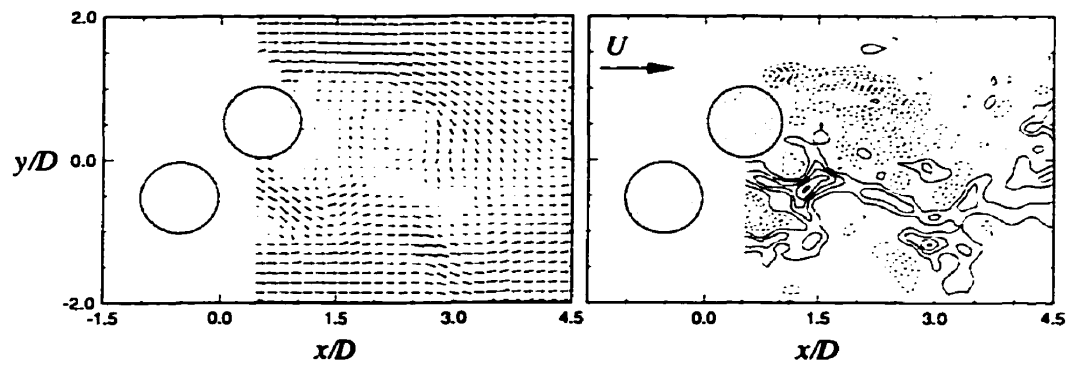


Figure 9.21. VPSE flow patterns, $Re = 1900$, $M_r = 0.25$ mm/pixel: (a) $P/D = 1.25$, $\alpha = 60^\circ$; (b) $P/D = 1.25$, $\alpha = 75^\circ$; (c) $P/D = 1.5$, $\alpha = 45^\circ$; (d) $P/D = 1.5$, $\alpha = 60^\circ$; (e) $P/D = 1.5$, $\alpha = 75^\circ$; (f) $P/D = 2.0$, $\alpha = 30^\circ$. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.

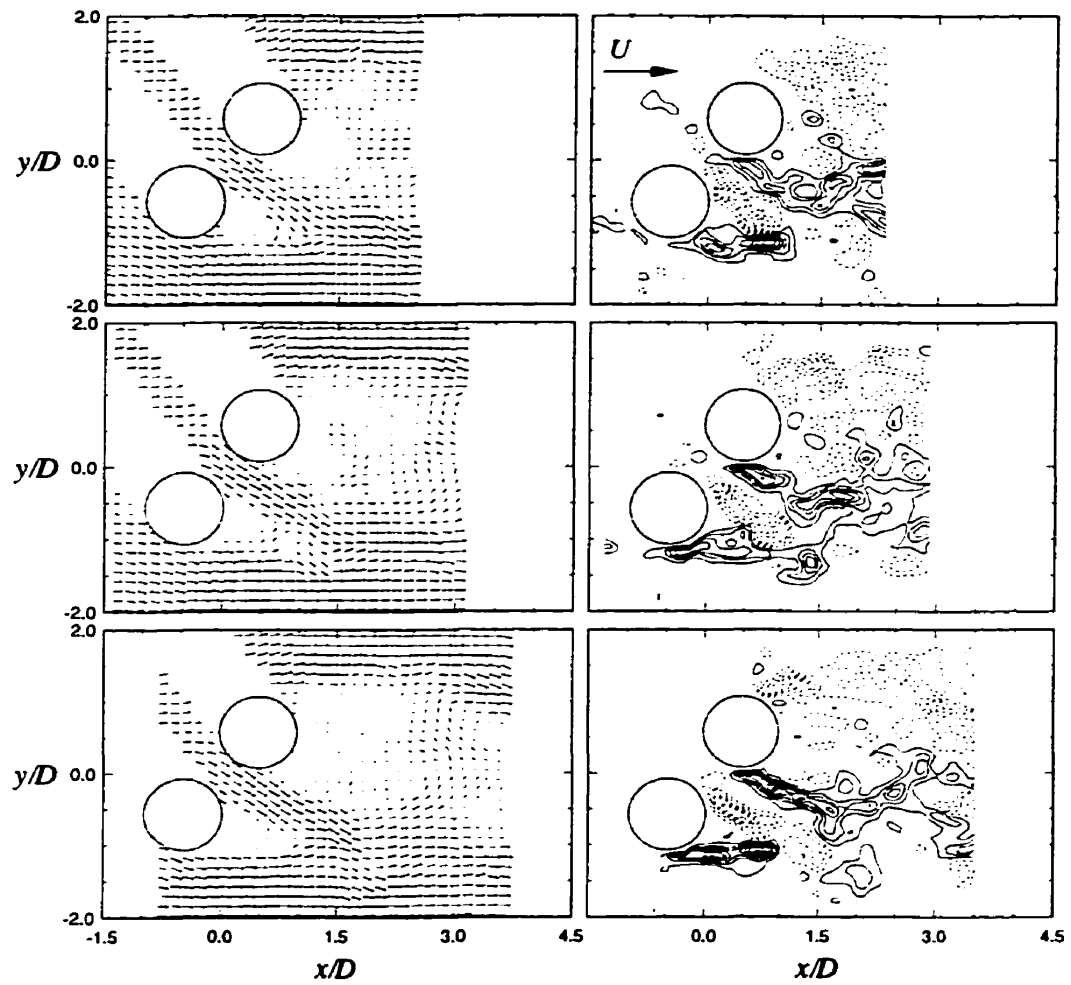


(a)

Figure 9.22. VPSE flow pattern, $P/D = 1.5$, $Re = 1900$, $M_r = 0.21$ mm/pixel, $\Delta t^* = 0.6$, $\Delta t/T_i = 0.06$ or 0.31 : (a) $\alpha = 45^\circ$; (b) $\alpha = 50^\circ$. Every second velocity vector in each direction omitted for clarity. Minimum vorticity contour magnitude 1.0, contour increment 1.0, dashed lines represent negative (CW) vorticity.

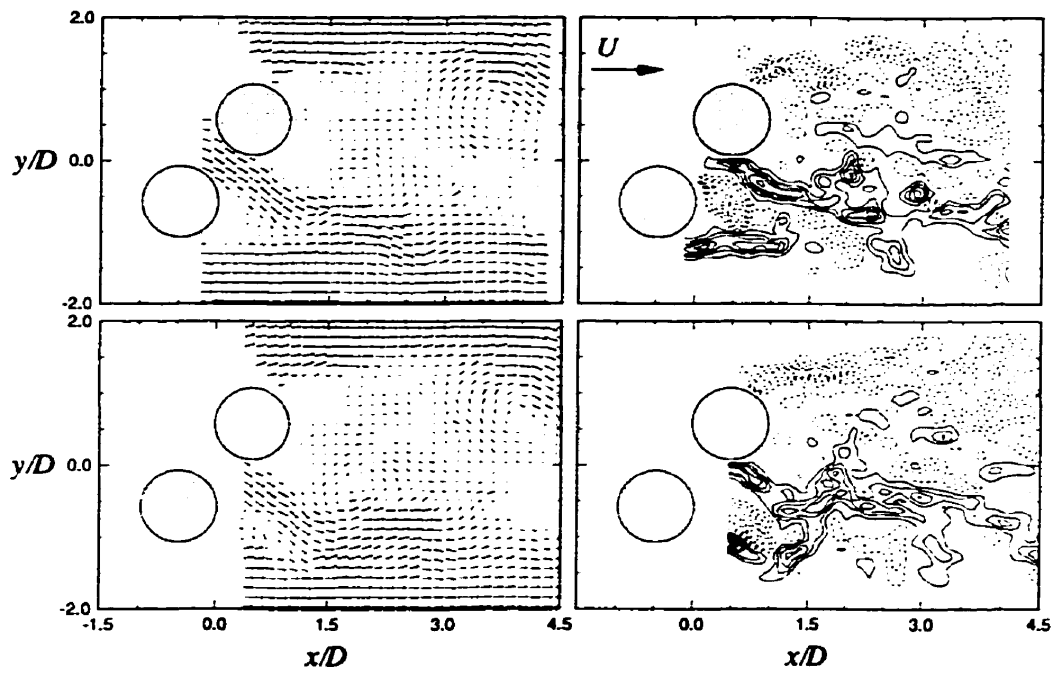


(a) continued.



(b)

Figure 9.22 (continued).



(b) continued.

Figure 9.22 (continued).

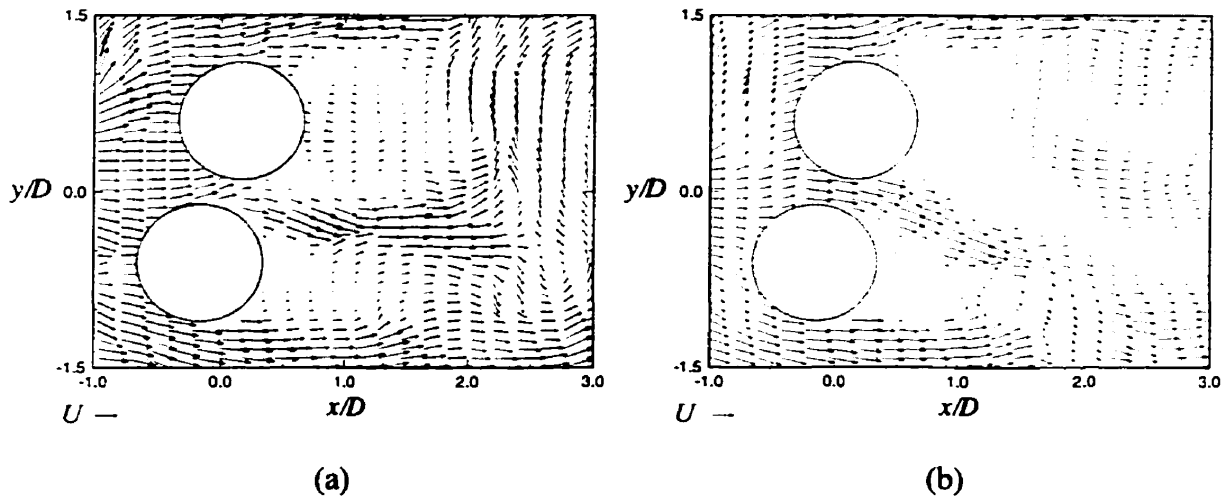


Figure 9.23. Variation in gap flow deflection angle, $P/D = 1.25$, $\alpha = 75^\circ$, $Re = 1900$, $M_r = 0.25$ mm/pixel: (a) large deflection angle; (b) small deflection angle. Every second velocity vector in the x -direction omitted for clarity.



(a)



(b)



(c)



(d)



(e)



(f)

Figure 9.24. Flow visualization of the synchronized gap vortex shedding (SVS) flow pattern: (a) $P/D=2.0$, $\alpha=60^\circ$, $Re=880$; (b) $P/D=2.0$, $\alpha=70^\circ$, $Re=860$; (c) $P/D=2.0$, $\alpha=80^\circ$, $Re=1290$; (d) $P/D=2.5$, $\alpha=50^\circ$, $Re=900$; (e) $P/D=2.5$, $\alpha=70^\circ$, $Re=900$; (f) $P/D=3.0$, $\alpha=40^\circ$, $Re=910$; (g) $P/D=3.0$, $\alpha=50^\circ$, $Re=930$; (h) $P/D=3.5$, $\alpha=30^\circ$, $Re=830$; (i) $P/D=4.0$, $\alpha=30^\circ$, $Re=930$.



(g)



(h)



(i)

Figure 9.24 (continued).

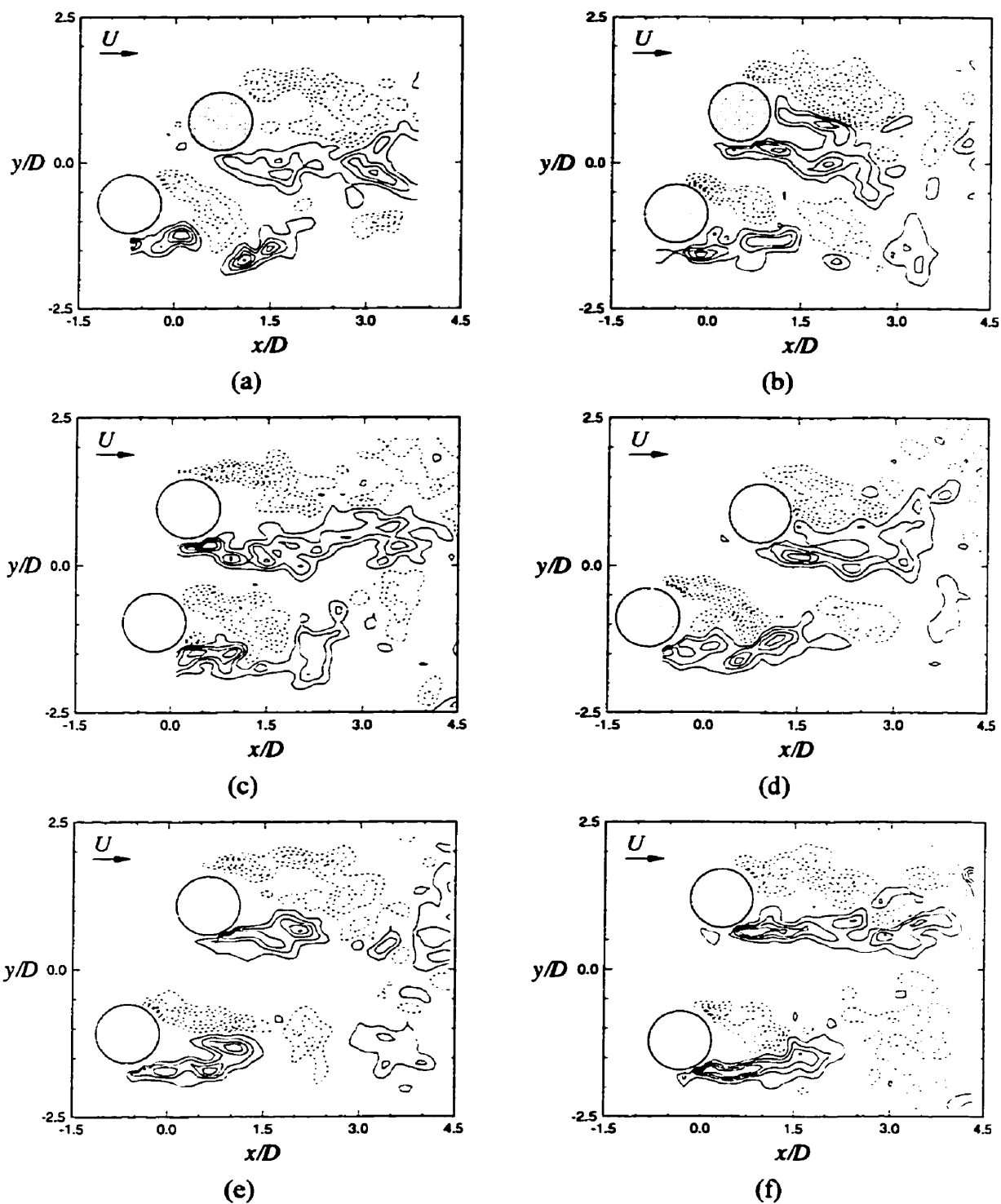
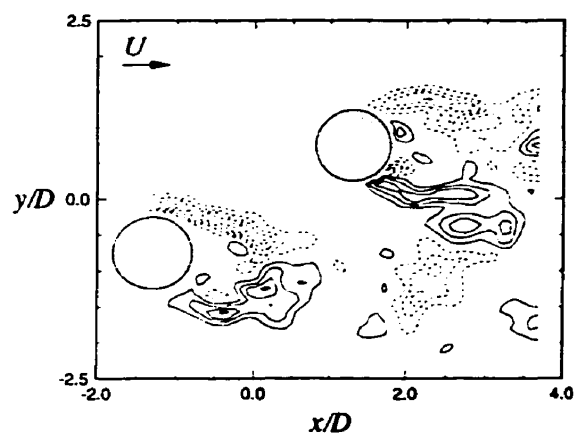
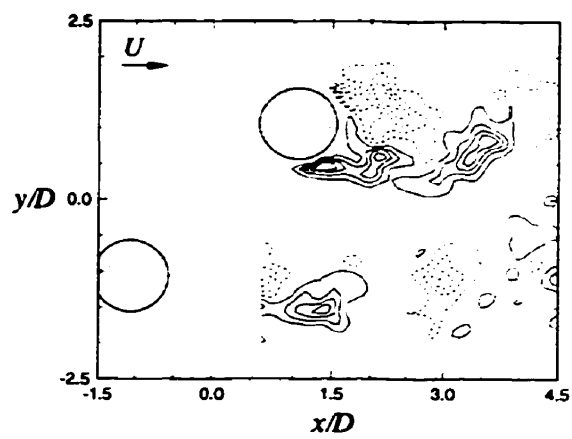


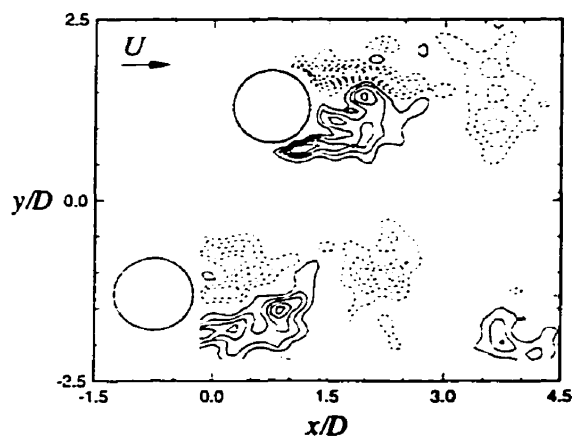
Figure 9.25. SVS flow pattern, $Re = 1900$, $M_r = 0.25$ mm/pixel: (a) $P/D = 2.0$, $\alpha = 45^\circ$; (b) $P/D = 2.0$, $\alpha = 60^\circ$; (c) $P/D = 2.0$, $\alpha = 75^\circ$; (d) $P/D = 2.5$, $\alpha = 45^\circ$; (e) $P/D = 2.5$, $\alpha = 60^\circ$; (f) $P/D = 2.5$, $\alpha = 75^\circ$; (g) $P/D = 3.0$, $\alpha = 30^\circ$; (h) $P/D = 3.0$, $\alpha = 45^\circ$; (i) $P/D = 3.0$, $\alpha = 60^\circ$; (j) $P/D = 4.0$, $\alpha = 30^\circ$; (k) $P/D = 4.0$, $\alpha = 45^\circ$. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.



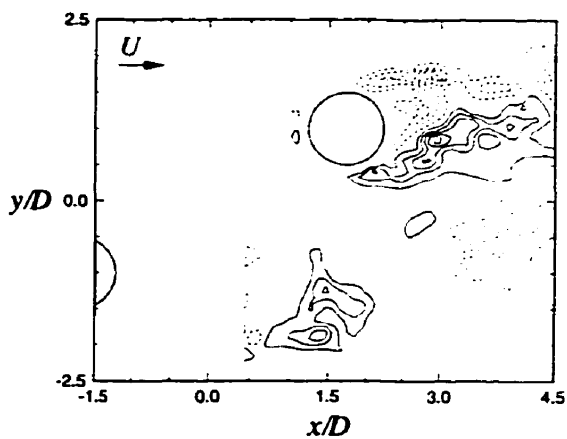
(g)



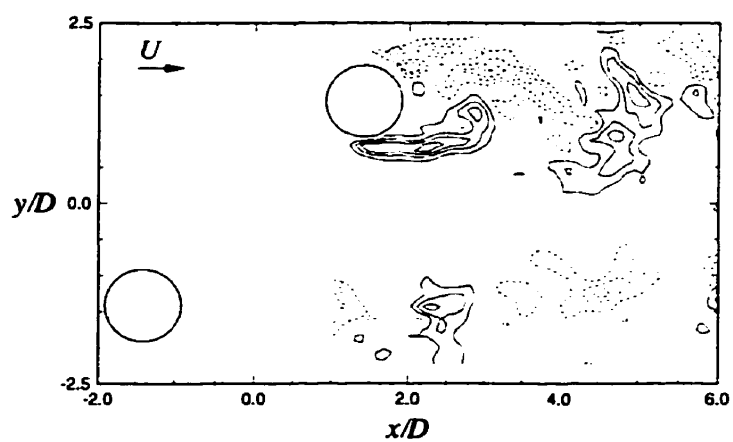
(h)



(i)



(j)



(k)

Figure 9.25 (continued).

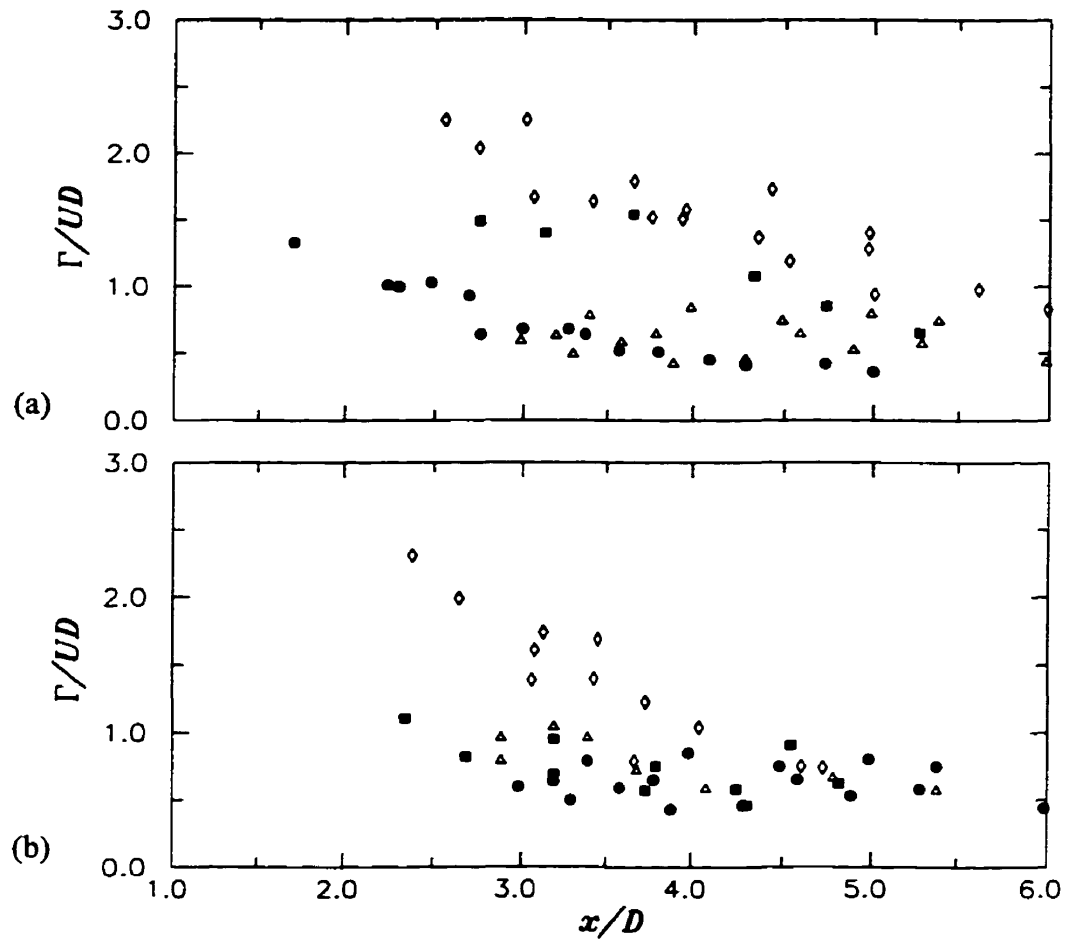


Figure 9.26. Strength of Kármán vortices shed from the inner shear layer of the upstream cylinder, for a two-cylinder staggered configuration in steady cross-flow, SVS flow pattern, $Re = 1900$, vorticity cutoff magnitude 1.5. (a) Constant incidence of $\alpha = 30^\circ$: $\bullet$, $P/D = 2.0$; $\triangle$, $P/D = 2.5$; $\blacksquare$, $P/D = 3.0$; $\diamond$, $P/D = 4.0$. (b) Constant pitch ratio of $P/D = 2.5$: $\bullet$, $\alpha = 30^\circ$; $\triangle$, $\alpha = 45^\circ$; $\blacksquare$, $\alpha = 60^\circ$; $\diamond$, $\alpha = 75^\circ$. Streamwise coordinate x/D measured from the different cylinder centres, and not from the centre of the configuration.

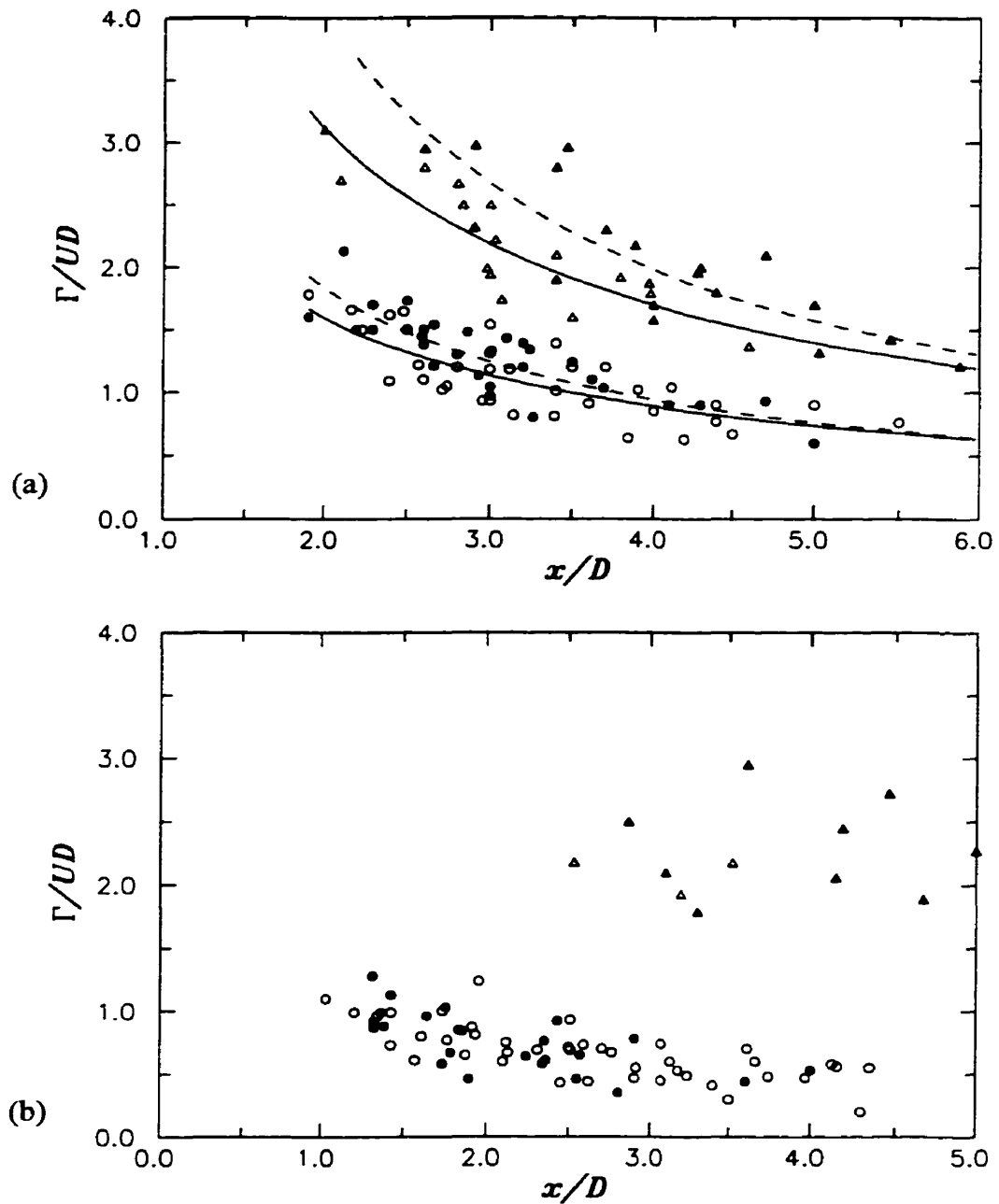


Figure 9.27. Kármán vortex strength measurements for two staggered cylinders in steady cross-flow, SVS flow pattern, $P/D = 2.0$, $Re = 1900$, vorticity cutoff magnitude 1.5: (a) $\alpha = 75^\circ$; (b) $\alpha = 45^\circ$. Upstream cylinder: $\circ$, inner vortices; $\bullet$, outer vortices. Downstream cylinder: $\triangle$, inner vortices; $\blacktriangle$, outer vortices. Power law curve fit lines shown for illustration purposes only: solid lines = inner vortices; dashed lines = outer vortices. Streamwise coordinate x/D measured from the different cylinder centres, and not from the centre of the configuration.

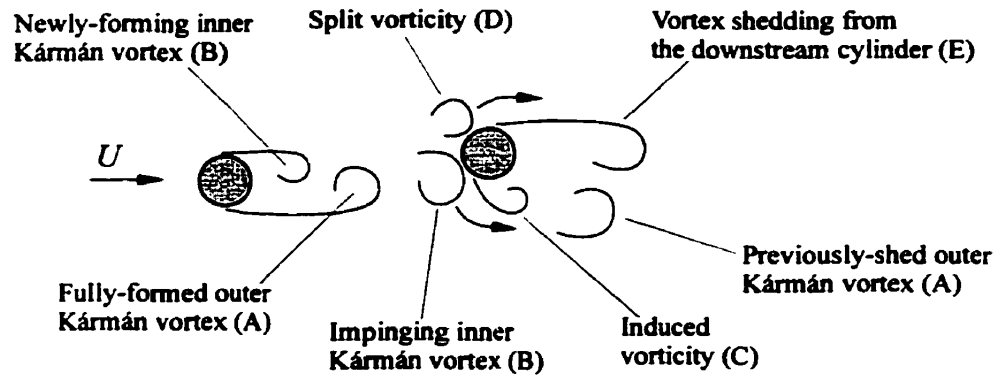


Figure 9.28. The vortex impingement (VI) flow pattern for two staggered circular cylinders in cross-flow.

- A - Kármán vortex from the outer shear layer of the upstream cylinder
- B - Kármán vortex from the inner shear layer of the upstream cylinder
- C - induced vorticity on the downstream cylinder
- D - vorticity concentration split upon vortex impingement
- E - Kármán vortex from the outer shear layer of the downstream cylinder

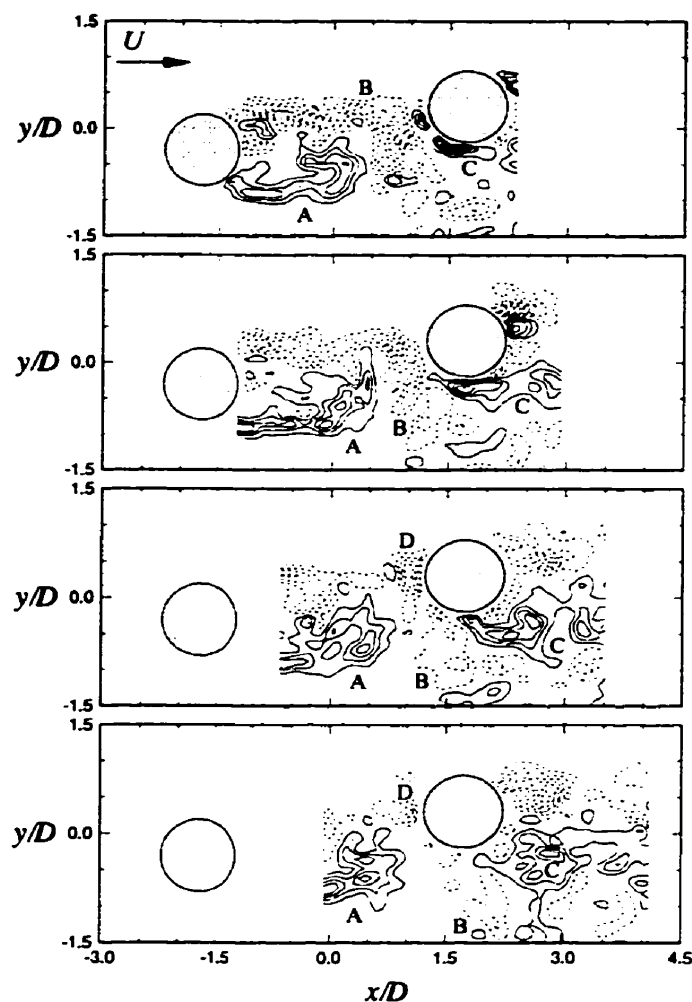


(a)



(b)

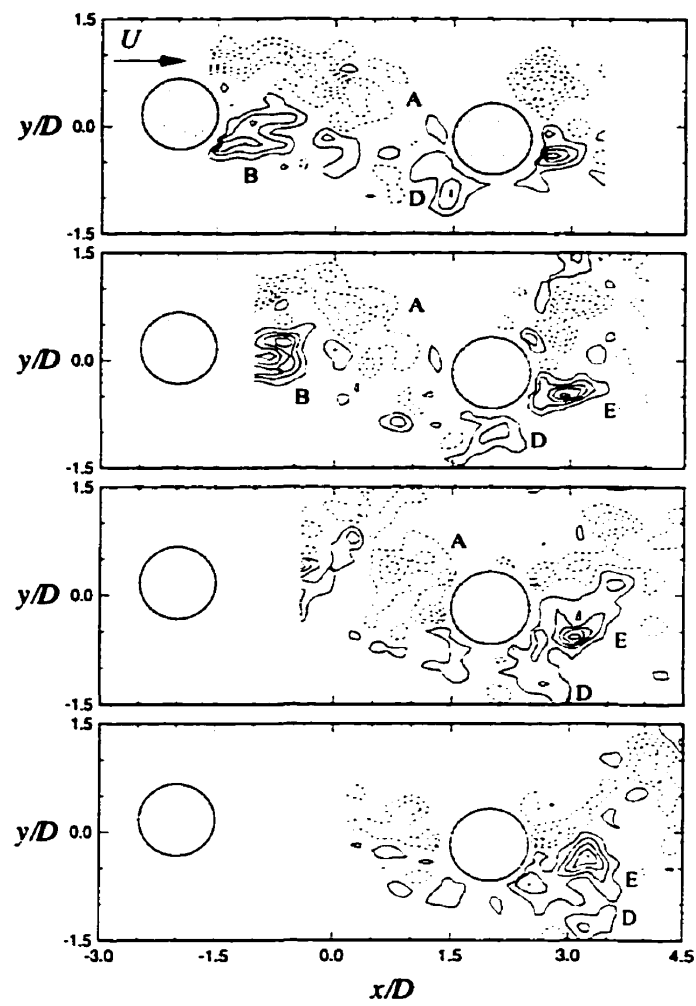
Figure 9.29. Flow visualization of the vortex impingement (VI) flow pattern: (a) $P/D = 3.5$, $\alpha = 10^\circ$, $Re = 910$; (b) $P/D = 4.0$, $\alpha = 10^\circ$, $Re = 860$.



(a)

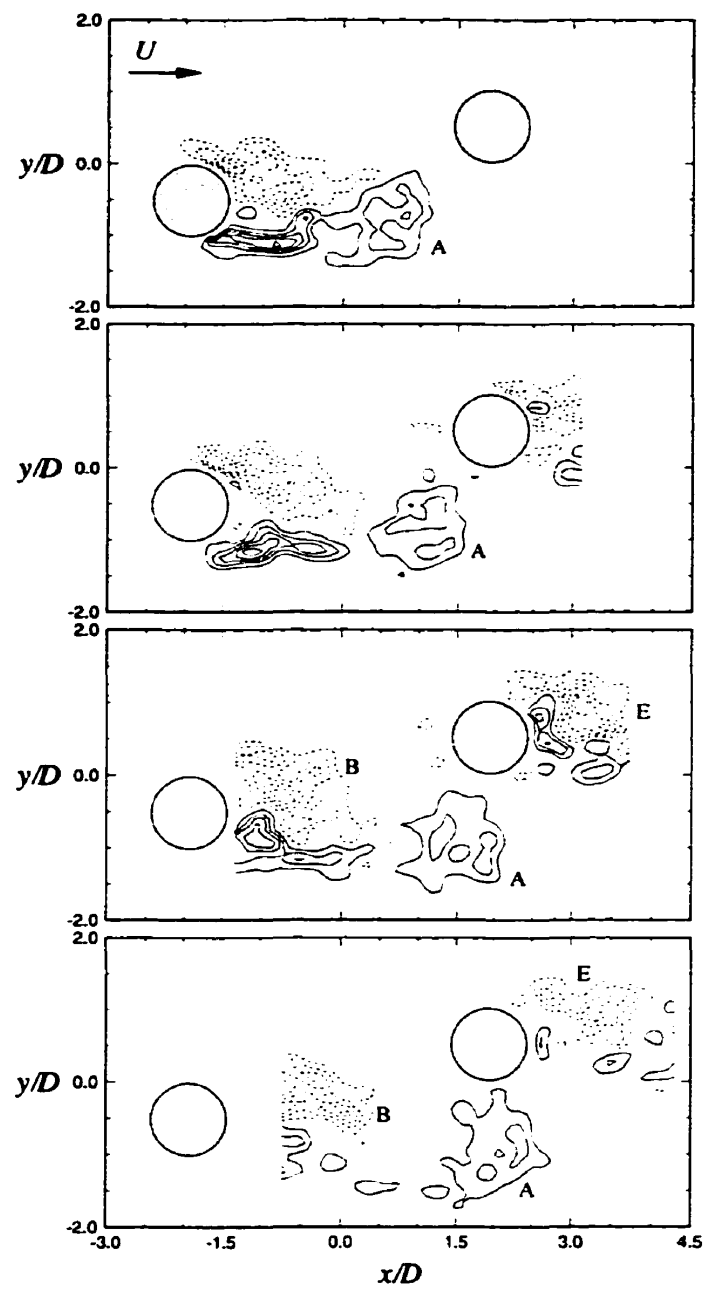
Figure 9.30. VI flow pattern, $Re = 1900$, $\Delta t^* = 0.6$: (a) $P/D = 3.5$, $\alpha = 10^\circ$, $M_r = 0.22$ mm/pixel, $\Delta t/T_r = 0.11$; (b) $P/D = 4.0$, $\alpha = 5^\circ$, $M_r = 0.25$ mm/pixel, $\Delta t/T_r = 0.10$; (c) $P/D = 4.0$, $\alpha = 15^\circ$, $M_r = 0.25$ mm/pixel, $\Delta t/T_r = 0.11$; (d) $P/D = 4.0$, $\alpha = 15^\circ$, $M_r = 0.25$ mm/pixel, $\Delta t/T_r = 0.11$. Minimum vorticity contour magnitude 1.0, contour increment 1.5, dashed lines represent negative (CW) vorticity.

- A - Kármán vortex from the outer shear layer of the upstream cylinder
- B - Kármán vortex from the inner shear layer of the upstream cylinder
- C - induced vorticity on the downstream cylinder
- D - vorticity concentration split upon vortex impingement
- E - Kármán vortex from the outer shear layer of the downstream cylinder



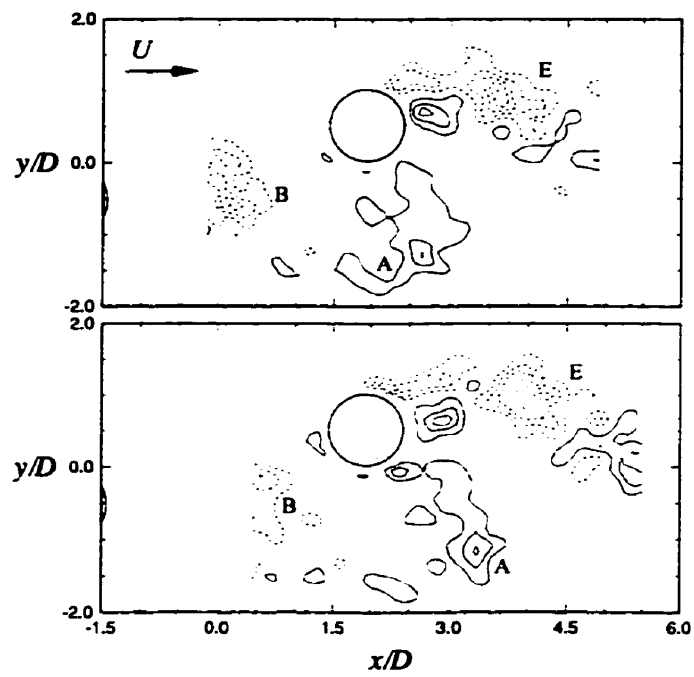
(b)

Figure 9.30 (continued).

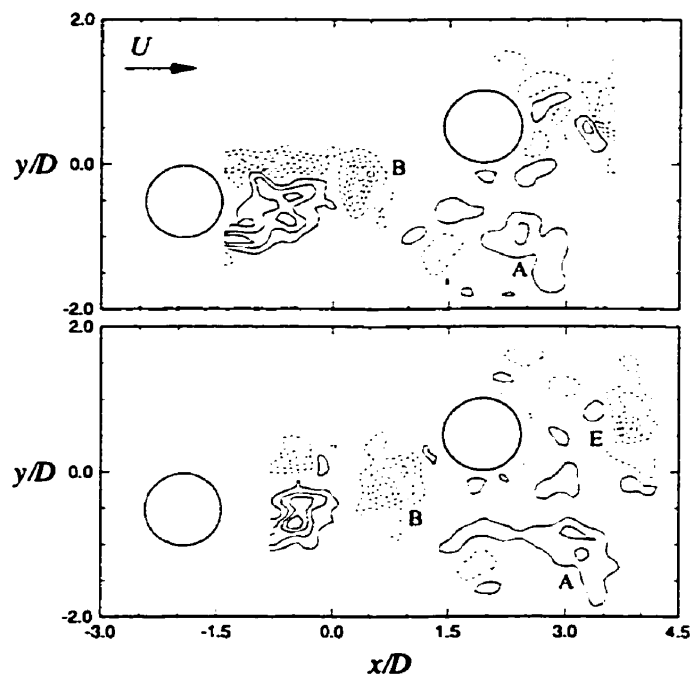


(c)

Figure 9.30 (continued).

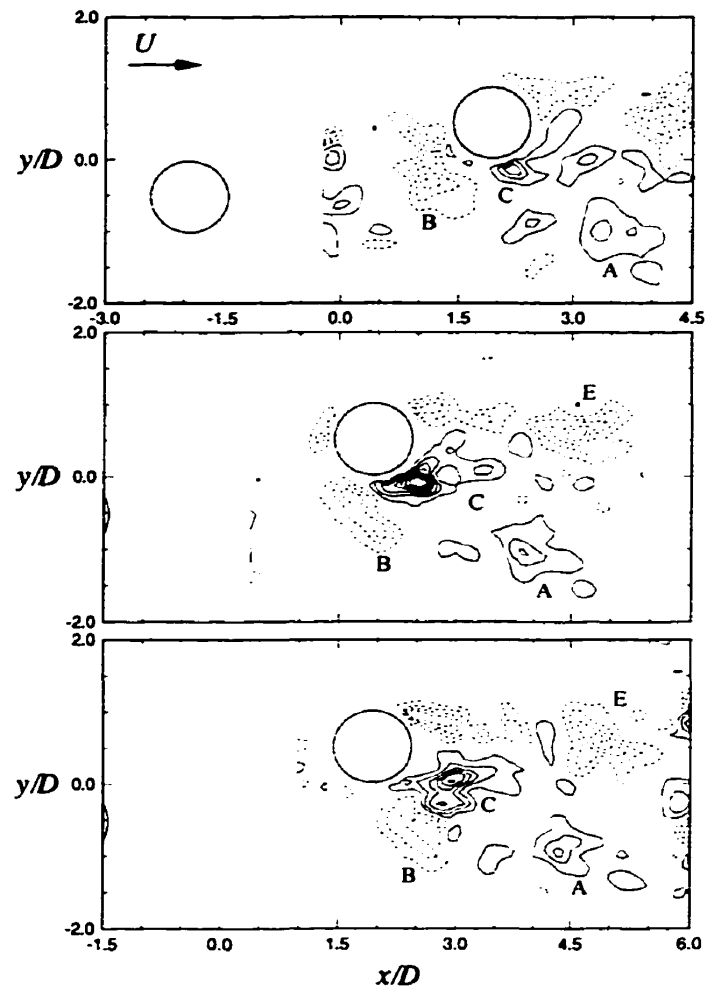


(c) continued.



(d)

Figure 9.30 (continued).



(d) continued.

Figure 9.30 (continued).

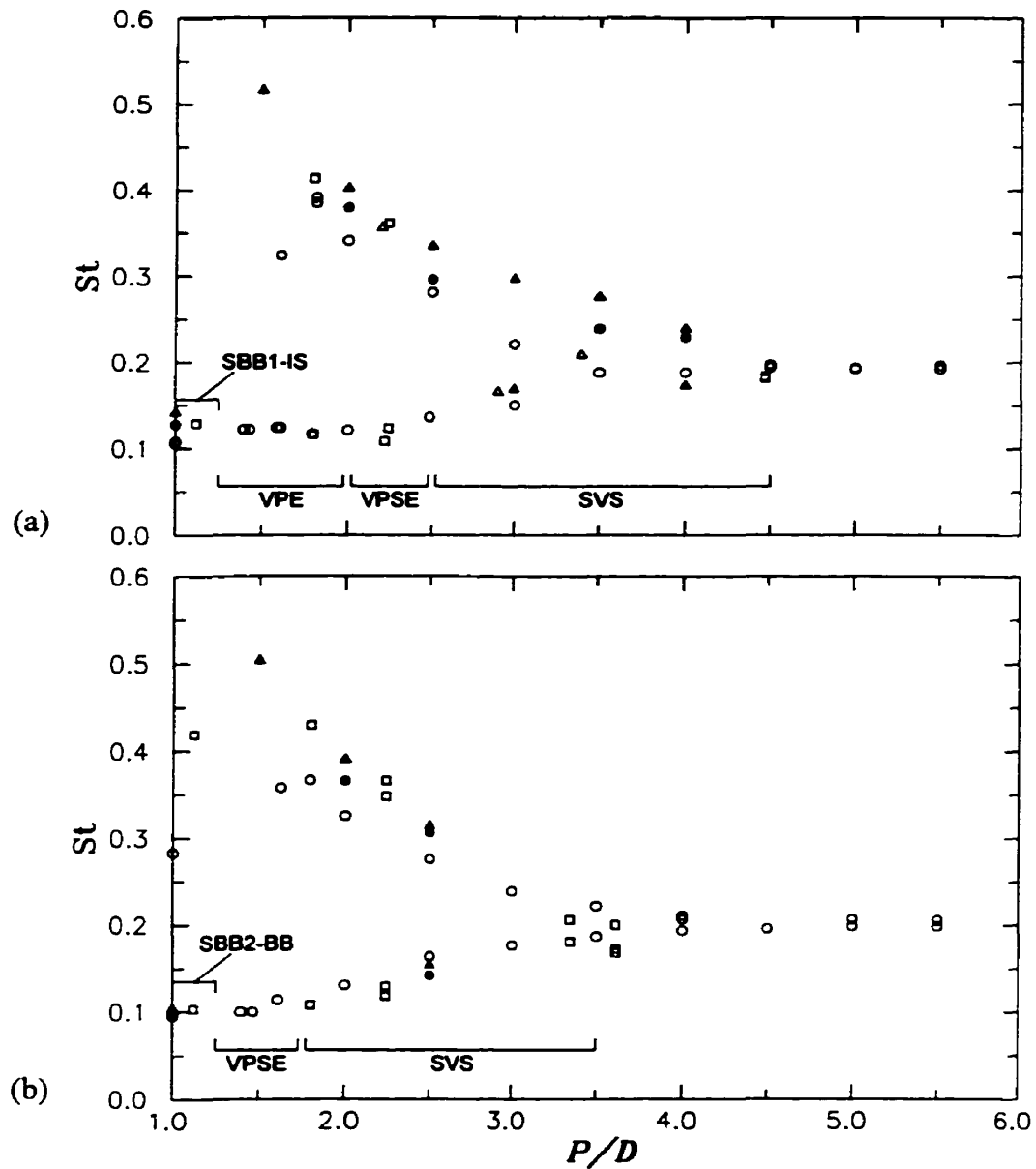


Figure 9.31. Strouhal number data for two staggered cylinders in steady cross-flow: (a) $\alpha = 25$ to 35° ; (b) $\alpha = 55$ to 65° . $\circ$, Kiyama *et al.* (1980), $Re = 17,000$; $\square$, Ishigai *et al.* (1972), $Re = 1500$ to $15,000$; $\triangle$, Moriya & Sakamoto (1985), $Re = 65,300$; $\bullet$, present study, $Re = 1290$ to 1350 ; $\blacktriangle$, present study, $Re = 830$ to 930 . BB, base-bleed; IS, induced separation; SBB1, single bluff-body type 1; SBB2, single bluff-body type 2; SVS, synchronized vortex shedding; VPE, gap vortex pairing and enveloping; VPSE, gap vortex pairing, splitting and enveloping.

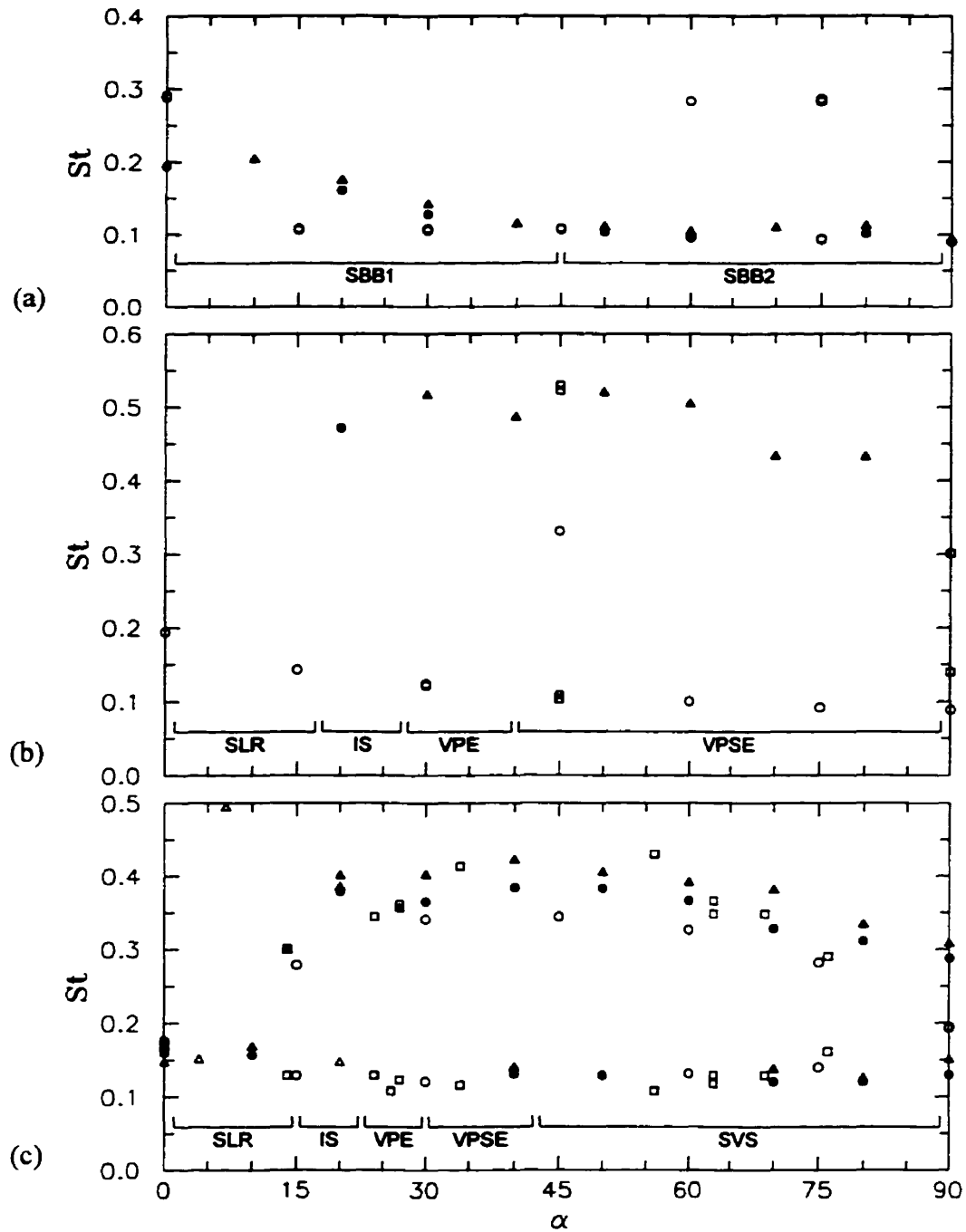


Figure 9.32. Strouhal number data for two staggered cylinders in steady cross-flow: (a) $P/D = 1.0$; (b) $P/D = 1.4$ to 1.6 ; (c) $P/D = 1.8$ to 2.2 ; (d) $P/D = 2.5$ to 2.7 ; (e) $P/D = 2.8$ to 3.2 ; (f) $P/D = 3.4$ to 3.6 ; (g) $P/D = 3.8$ to 6.2 . $\circ$, Kiya *et al.* (1980), $Re = 17,000$; $\square$, Ishigai *et al.* (1972), $Re = 1500$ to $15,000$; $\triangle$, Moriya & Sakamoto (1985), $Re = 65,300$; $\bullet$, present study, $Re = 1260$ to 1350 ; $\blacktriangle$, present study, $Re = 820$ to 930 . BB, base-bleed; IS, induced separation; SBB1, single bluff-body type 1; SBB2, single bluff-body type 2; SLR, shear layer reattachment; SVS, synchronized vortex shedding; VI, vortex impingement; VPE, gap vortex pairing and enveloping; VPSE, gap vortex pairing, splitting and enveloping.

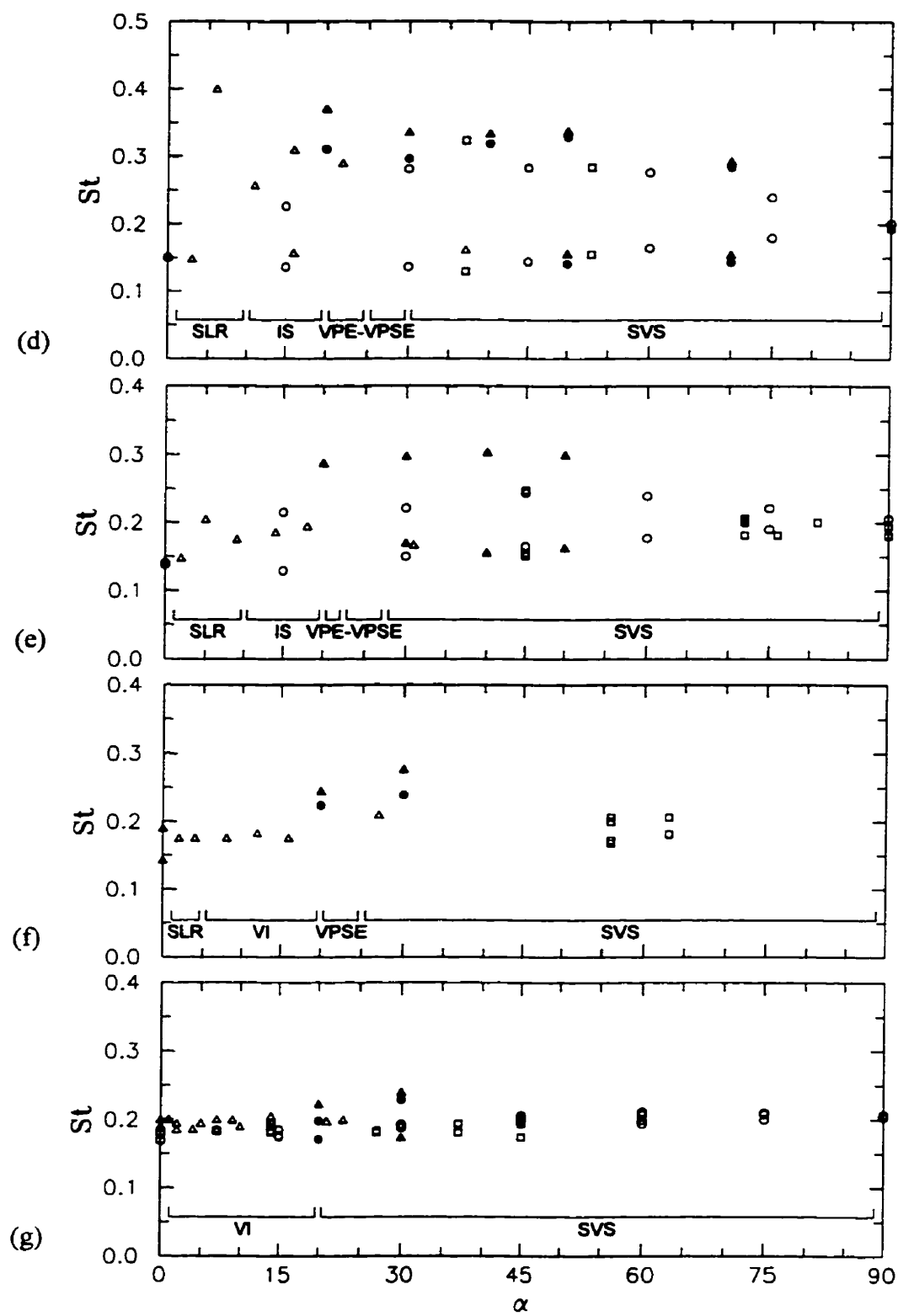


Figure 9.32 (continued).

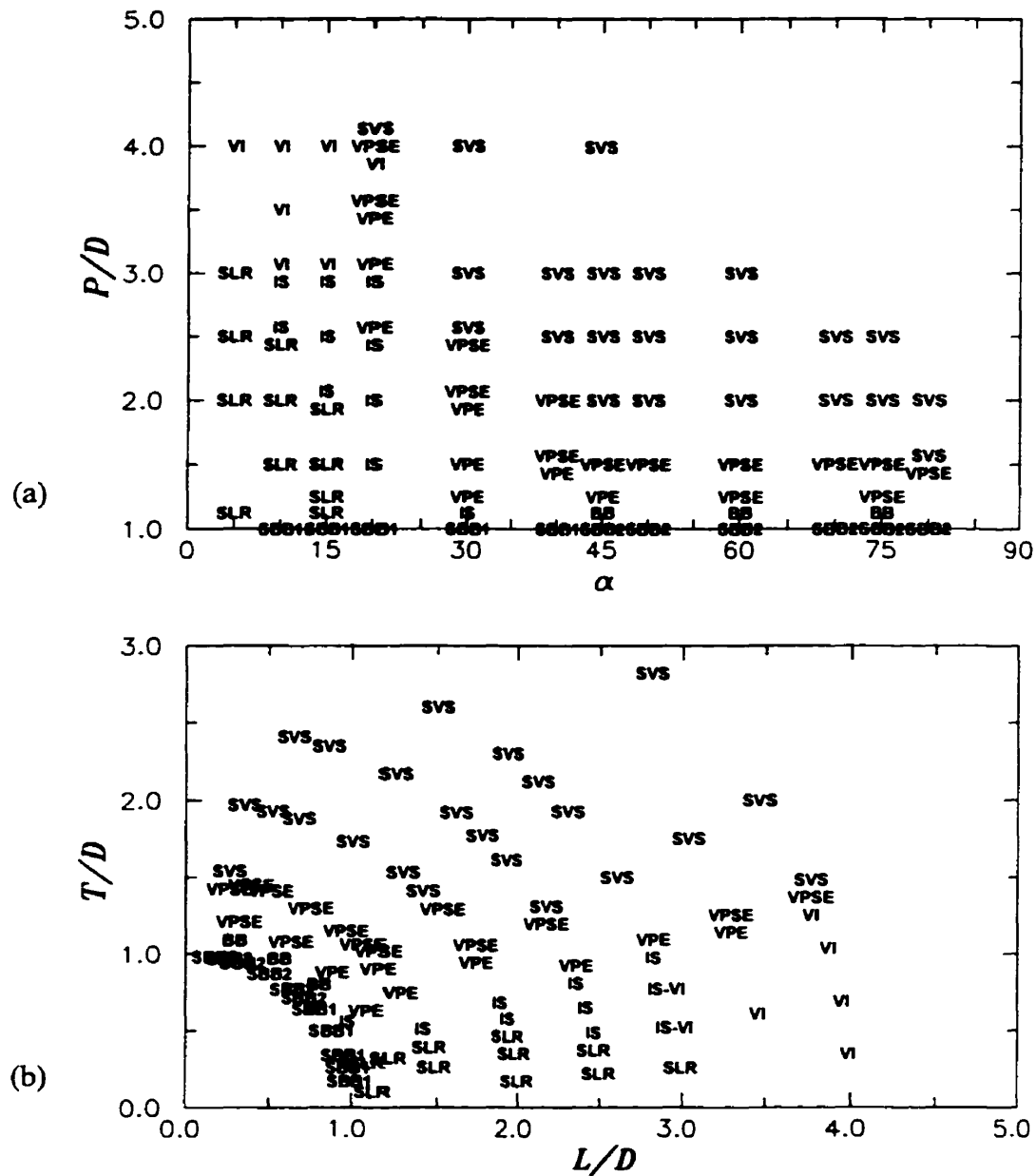


Figure 9.33. Flow patterns identified for two staggered cylinders in steady cross-flow, $Re = 850$ to 1900 : (a) P/D - α notation; (b) L/D - T/D notation. BB, base-bleed; IS, induced separation; SBB1, single bluff-body type 1; SBB2, single bluff-body type 2; SLR, shear layer reattachment; SVS, synchronized vortex shedding; VI, vortex impingement; VPE, gap vortex pairing and enveloping; VPSE, gap vortex pairing, splitting and enveloping.

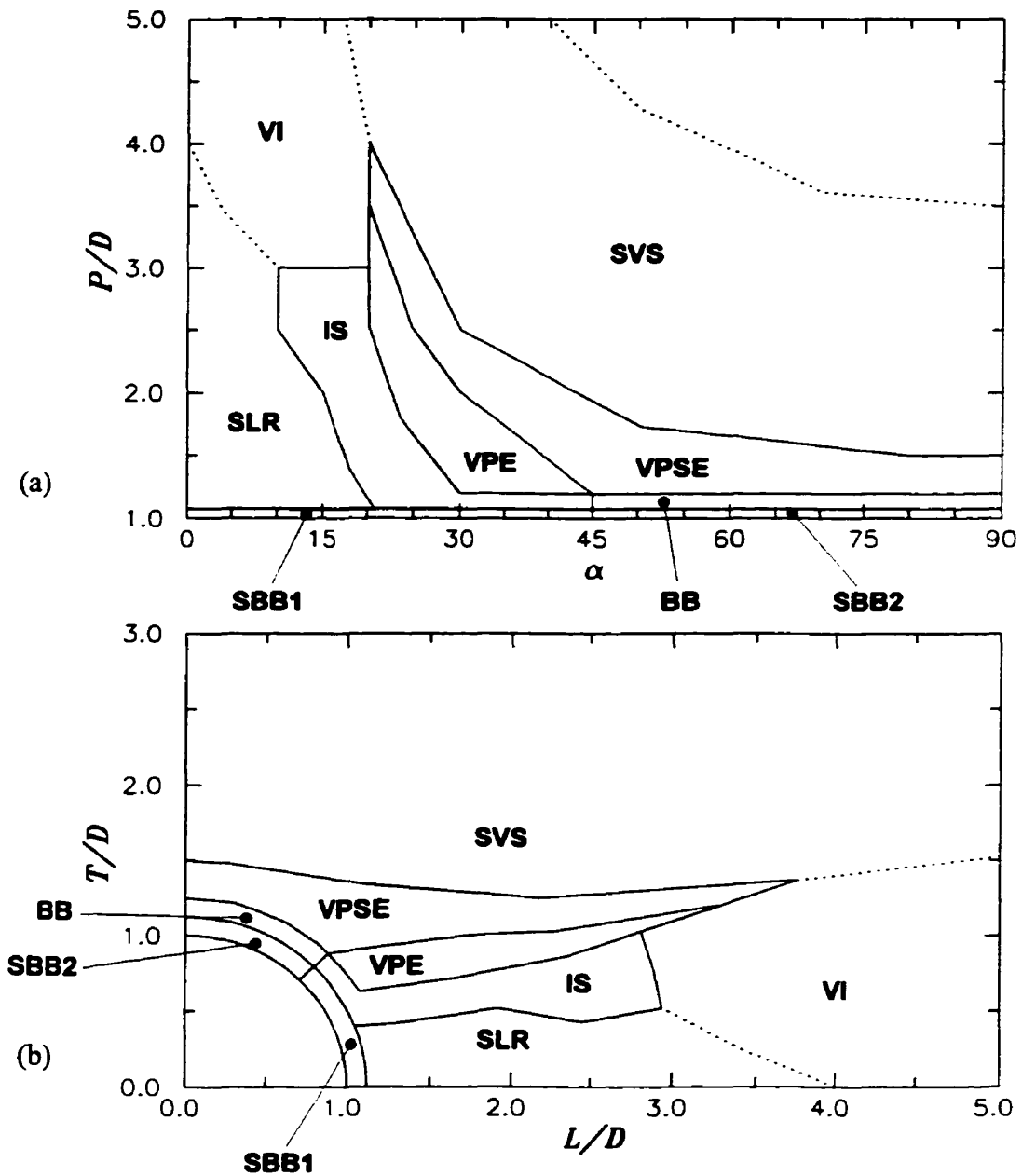


Figure 9.34. Flow pattern boundaries for two staggered cylinders in steady cross-flow, $Re = 850$ to 1900 : (a) P/D - α notation; (b) L/D - T/D notation. BB, base-bleed; IS, induced separation; SBB1, single bluff-body type 1; SBB2, single bluff-body type 2; SLR, shear layer reattachment; SVS, synchronized vortex shedding; VI, vortex impingement; VPE, gap vortex pairing and enveloping; VPSE, gap vortex pairing, splitting and enveloping. Dashed lines represent proposed boundaries.

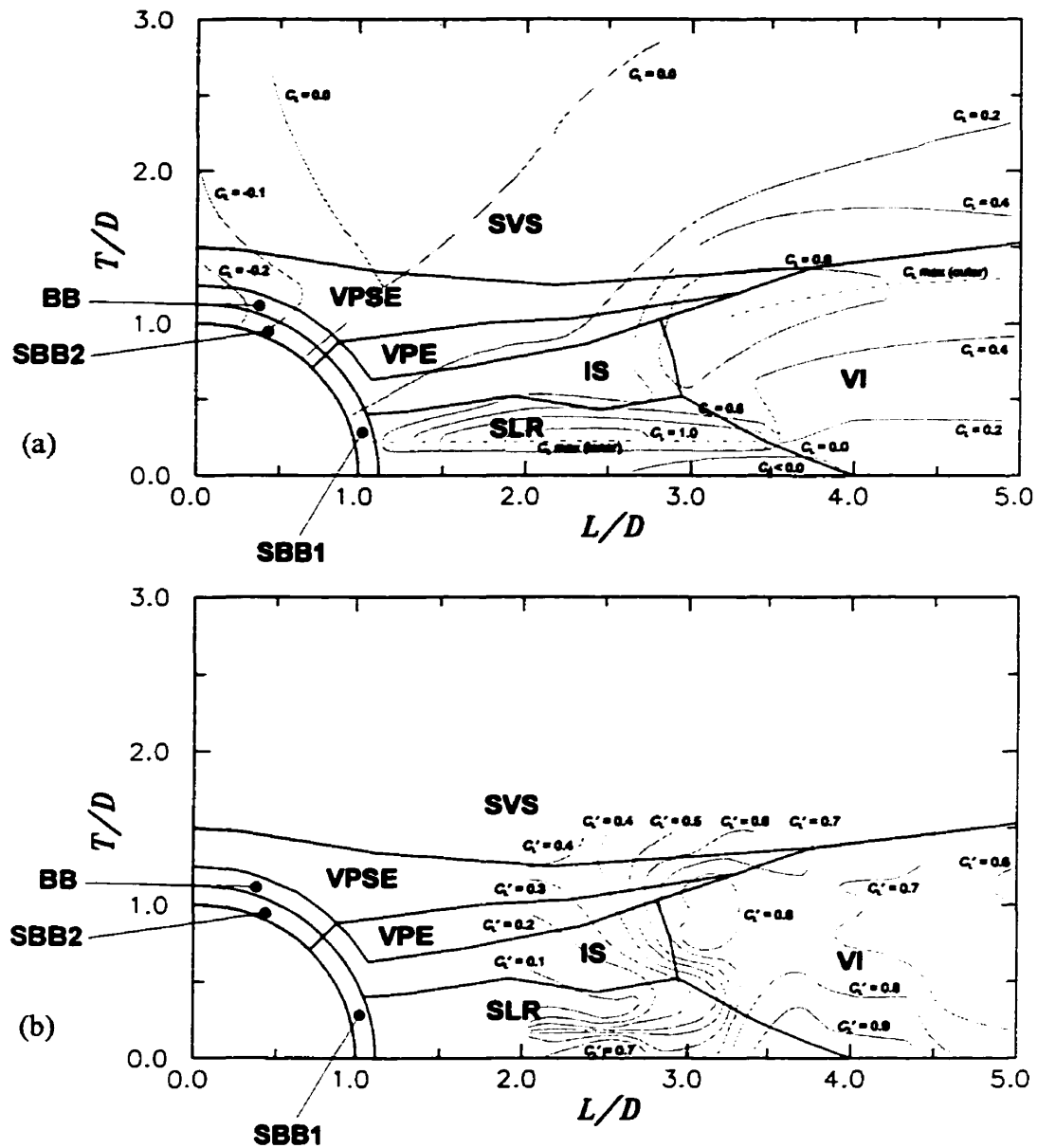


Figure 9.35. Lift force coefficient data, for the downstream cylinder, for two staggered cylinders in steady cross-flow, plotted as contours of constant lift coefficient, with flow pattern boundaries from the present study superimposed: (a) mean lift force coefficient, Zdravkovich & Pridden (1977), $Re = 60,000$; (b) r.m.s. lift force coefficient, Moriya & Sakamoto (1985), $Re = 65,300$. BB, base-bleed; IS, induced separation; SBB1, single bluff-body type 1; SBB2, single bluff-body type 2; SLR, shear layer reattachment; SVS, synchronized vortex shedding; VI, vortex impingement; VPE, gap vortex pairing and enveloping; VPSE, gap vortex pairing, splitting and enveloping.

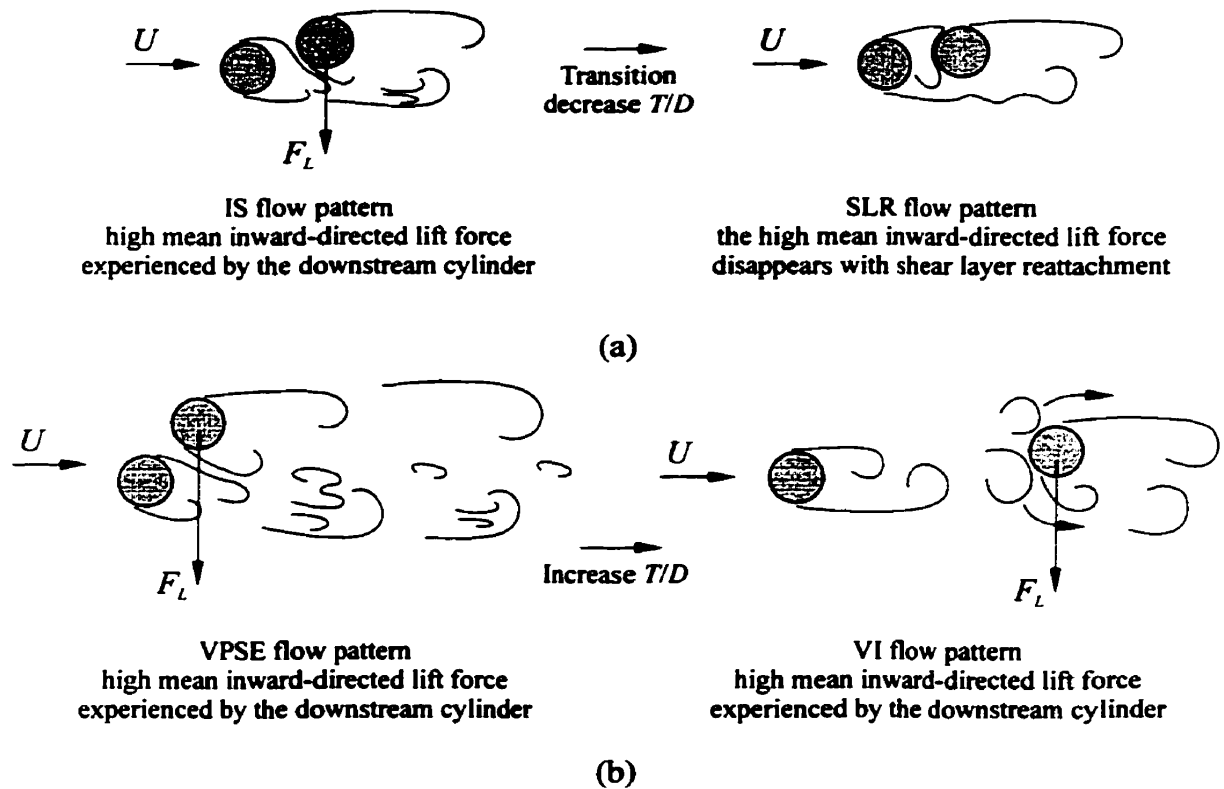


Figure 9.36. Fluid dynamic origins of the “inner” and “outer” mean lift coefficient peaks, for the downstream cylinder in a two-cylinder staggered pair, in steady cross-flow: (a) the “inner” lift peak is attributed mostly to the IS flow pattern, shown on the left, and its disappearance may be attributed to a sudden transition to the SLR flow pattern, shown on the right; (b) the “outer” lift peak is attributed to the VPSE flow pattern at lower values of L/D , shown on the left, and to the VI flow pattern at higher values of L/D , shown on the right. The forces vectors are shown for illustration purposes only, and the lengths of the vectors do not correspond to their relative magnitudes.

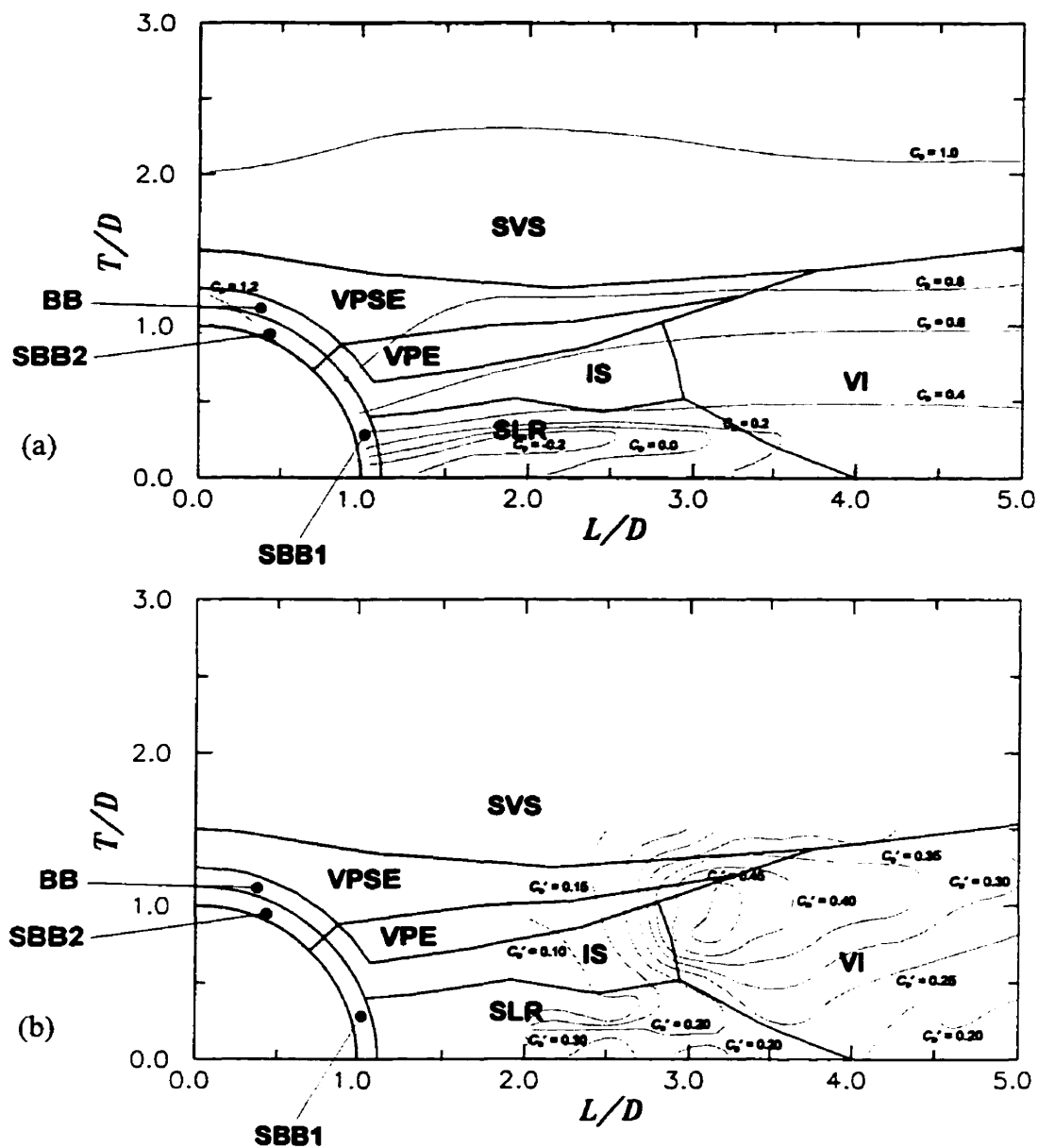


Figure 9.37. Drag force coefficient data, for the downstream cylinder, for two staggered cylinders in steady cross-flow, plotted as contours of constant drag coefficient, with flow pattern boundaries from the present study superimposed: (a) mean drag force coefficient, Zdravkovich & Pridden (1977), $Re = 60,000$; (b) r.m.s. drag force coefficient, Moriya & Sakamoto (1985), $Re = 65,300$. BB, base-bleed; IS, induced separation; SBB1, single bluff-body type 1; SBB2, single bluff-body type 2; SLR, shear layer reattachment; SVS, synchronized vortex shedding; VI, vortex impingement; VPE, gap vortex pairing and enveloping; VPSE, gap vortex pairing, splitting and enveloping.

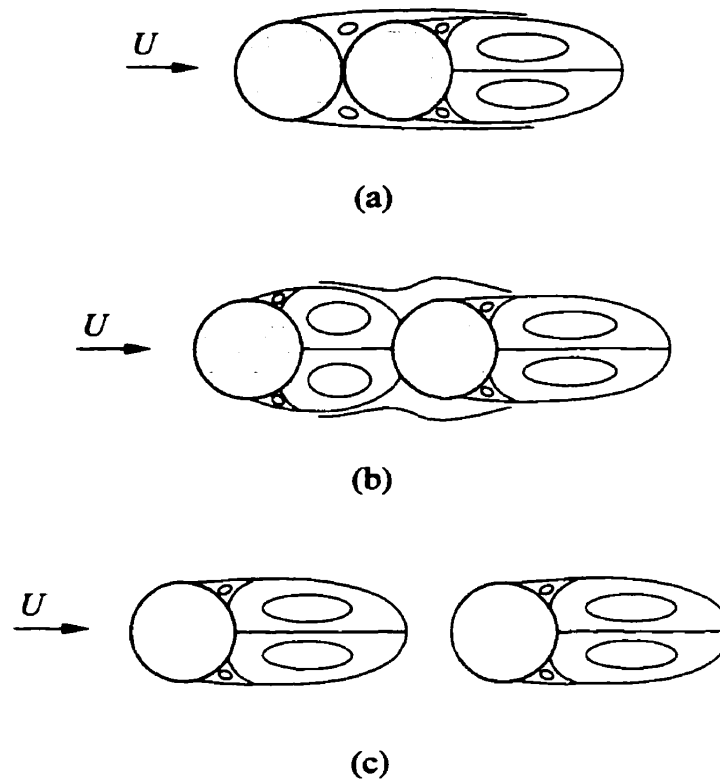


Figure 10.1. Flow development identified in the present study for two tandem circular cylinders of equal diameter impulsively set into motion, $Re = 1200$ to 3800 : (a) tandem cylinders in contact; (b) tandem cylinders with a small gap; (c) tandem cylinders with a large gap. Similar patterns are seen for three-cylinder configurations.

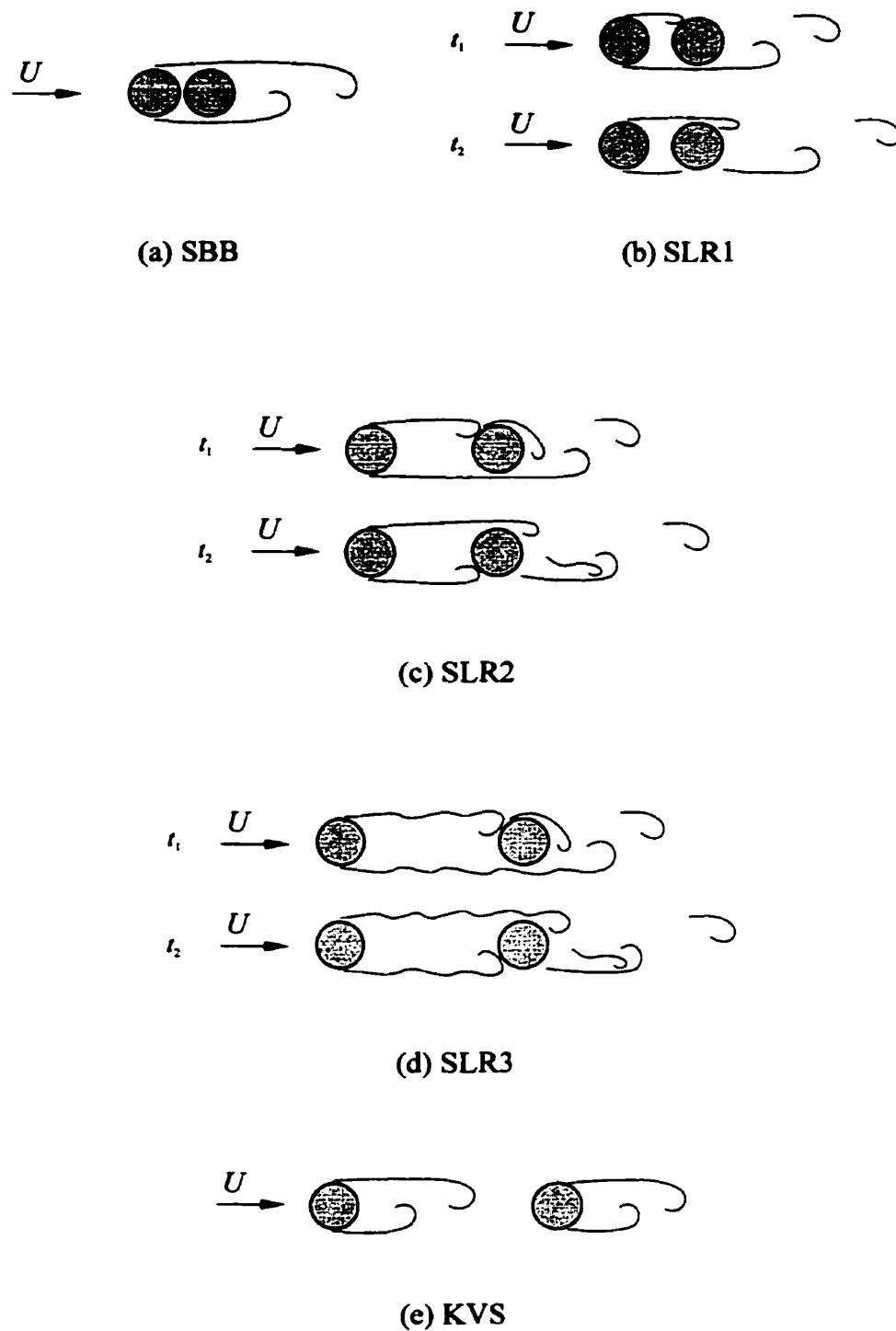


Figure 10.2. Low subcritical regime flow patterns identified in this study for two tandem circular cylinders of equal diameter, in steady cross-flow, $Re = 500$ to 3000 : (a) single bluff-body flow pattern, SBB; (b) type 1 shear layer reattachment flow pattern, SLR1; (c) type 2 shear layer reattachment flow pattern, SLR2; (d) type 3 shear layer reattachment flow pattern, SLR3; (e) Kármán vortex shedding flow pattern, KVS.

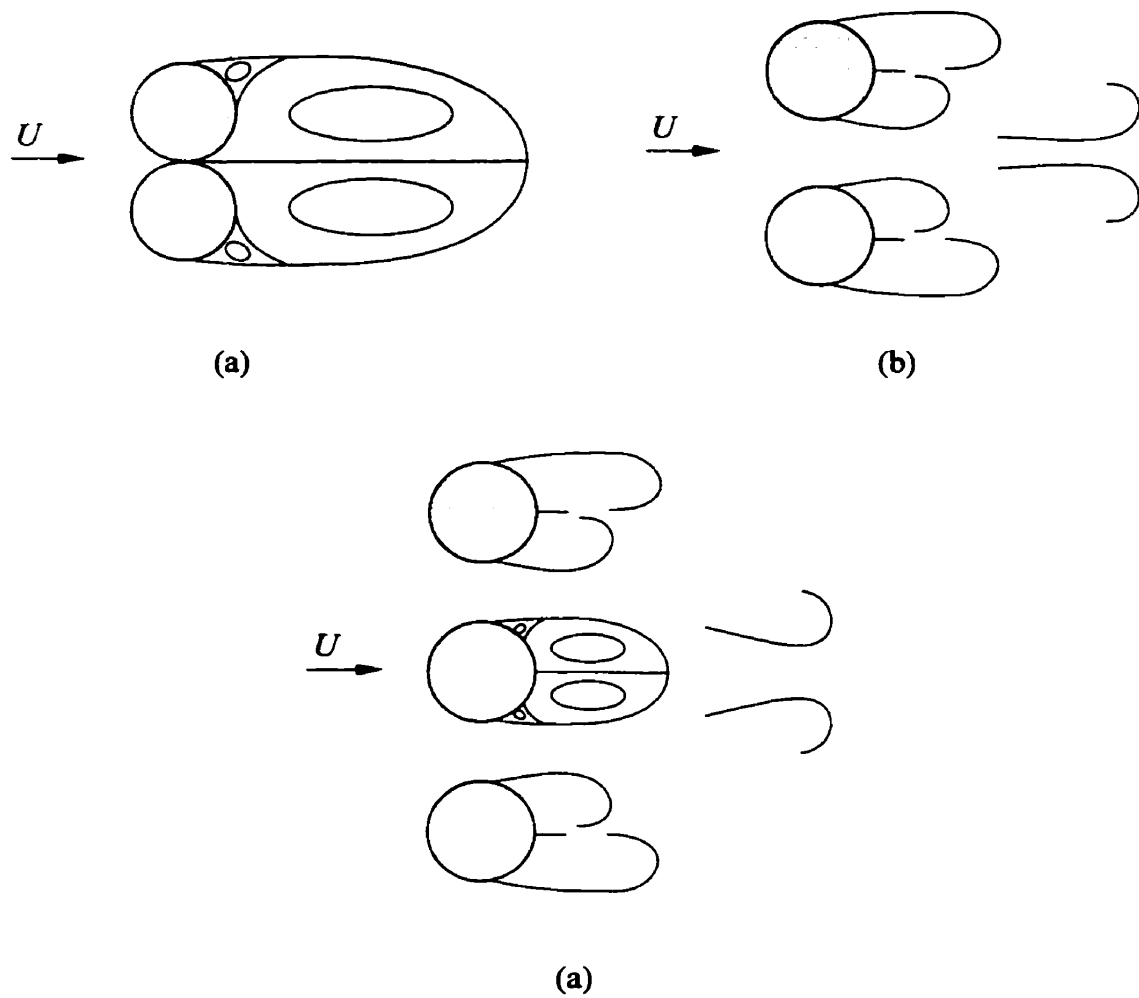


Figure 10.3. Flow development identified in the present study for side-by-side circular cylinders of equal diameter impulsively set into motion, $Re = 1500$ to 2800 : (a) side-by-side cylinders in contact; (b) two side-by-side cylinders with a gap between them; (c) three side-by-side cylinders with a gap between them.

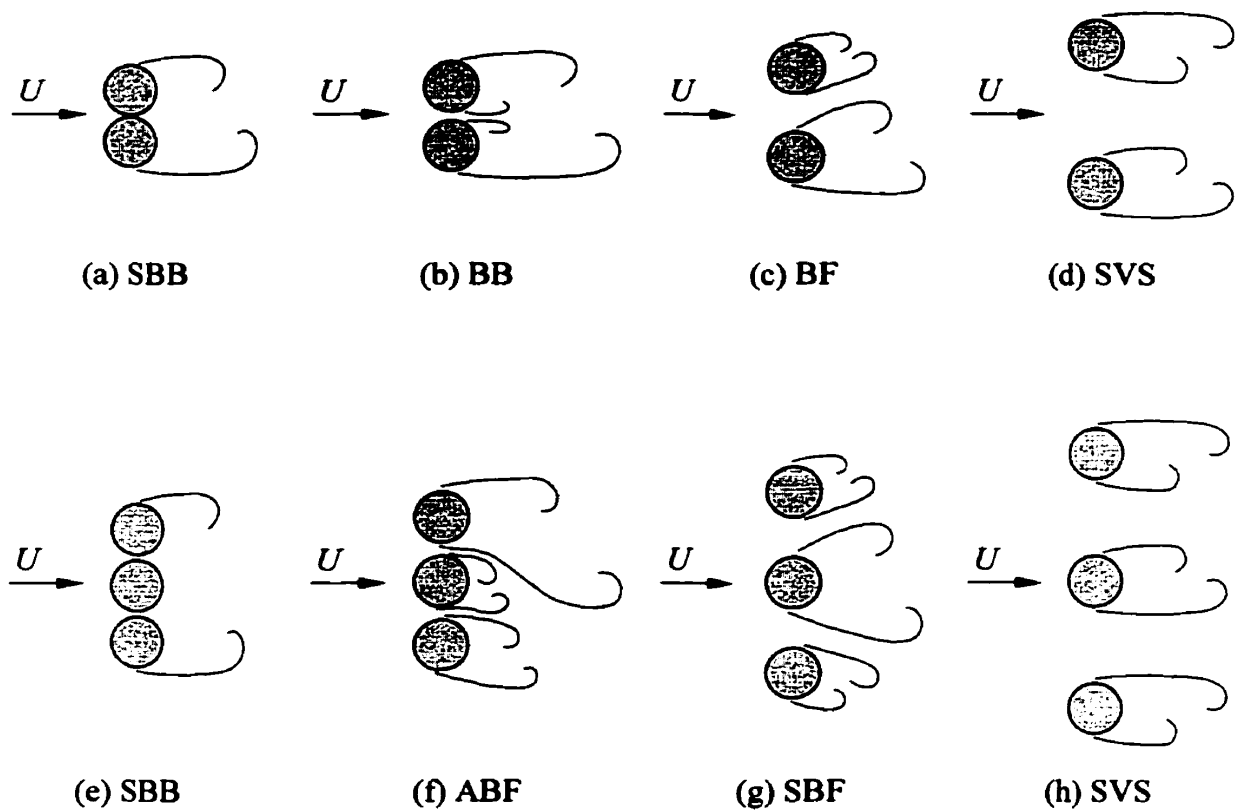


Figure 10.4. Low subcritical regime flow patterns identified in this study for two and three side-by-side circular cylinders of equal diameter, in steady cross-flow, $Re = 500$ to 3000 : (a) single bluff-body flow pattern, SBB; (b) base-bleed flow pattern, BB; (c) biased flow pattern, BF; (d) synchronized gap vortex shedding flow pattern, SVS; (e) single bluff-body flow pattern, SBB; (f) asymmetric biased flow pattern, ABF; (g) symmetric biased flow pattern, SBF; (h) synchronized vortex shedding flow pattern, SVS.

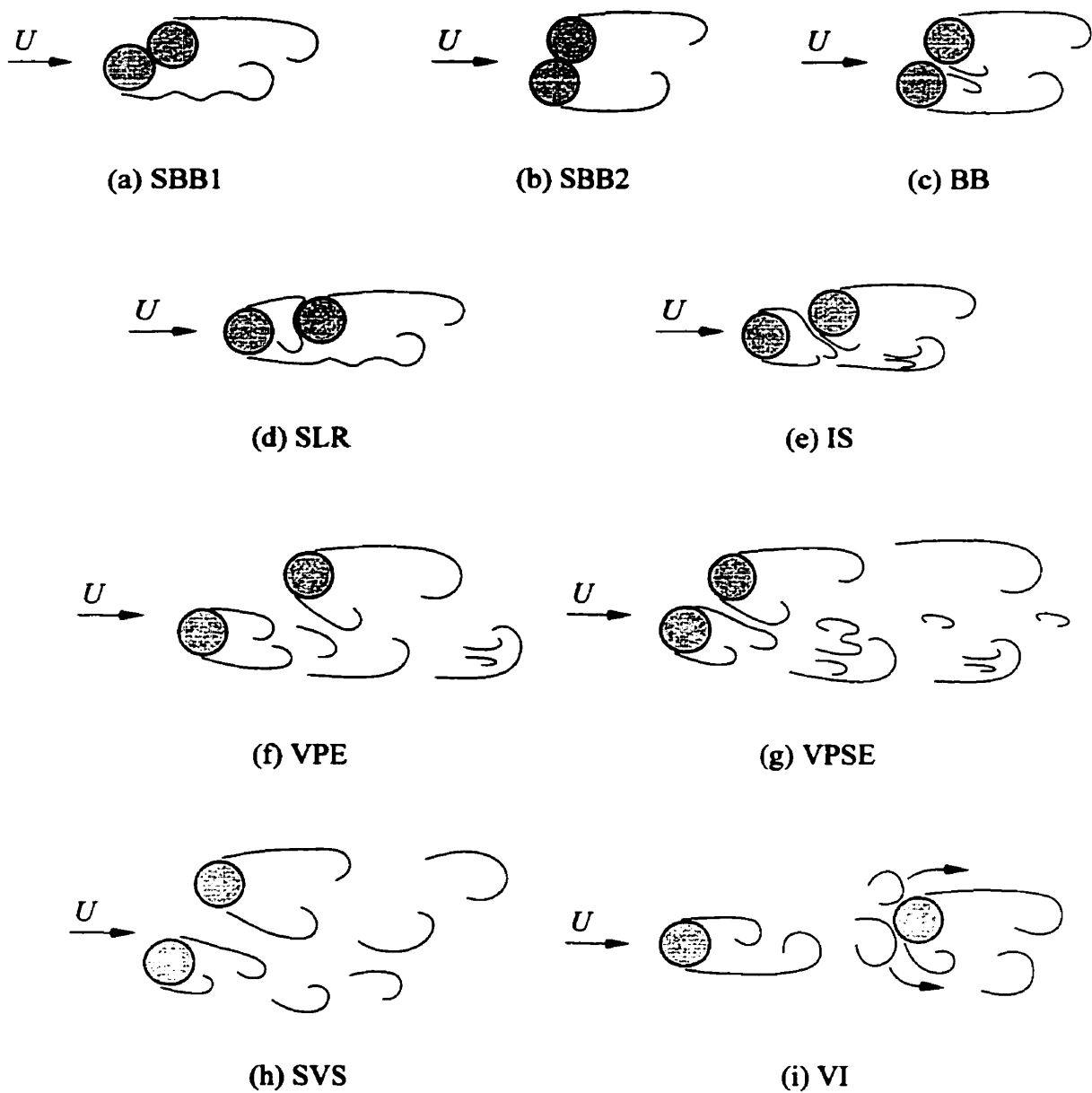


Figure 10.5. Low subcritical regime flow patterns identified in this study for two staggered circular cylinders of equal diameter, in steady cross-flow, $Re = 850$ to 1900 : (a) type 1 single bluff-body flow pattern, SBB1; (b) type 2 single bluff-body flow pattern, SBB2; (c) base-bleed flow pattern, BB; (d) shear layer reattachment flow pattern, SLR; (e) induced separation flow pattern, IS; (f) gap vortex pairing and enveloping flow pattern, VPE; (g) gap vortex pairing, splitting and enveloping flow pattern, VPSE; (h) synchronized gap vortex shedding flow pattern, SVS; (i) vortex impingement flow pattern, VI.