ESSAYS ON INTERNATIONAL INVESTMENT

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Abstract

The proposed thesis comprises two essays on international investment. Each essay proposes a new theory and provides empirical evidence. The first essay develops a three-moment international asset-pricing model (TM-IAPM) under full integration and deviations from purchasing power parity (PPP) that prices coskewness. The model also embeds the standard IAPMs as special cases when explicit restrictions are imposed. We further apply the model to investigate the time-series behavior of market, size, value, and momentum premiums in the United States, Japan, and the United Kingdom equity markets. We find that the model explains most of the variation of these premiums during the 1980s and 1990s and that the coskewness risk is more important than covariance risk. We also find that the model performs well out-of-sample. The direct implication of our result is that linear IAPMs are misspecified and that investors should use nonlinear models to price international assets. The second essay proposes a way to disentangle the test of market efficiency from the test of the postulated equilibrium model. Indeed, the efficient market hypothesis, which stipulates that prices fully reflect available information, is one of the most important building blocks in finance and economics. Unfortunately, there is no consensus on this important issue since the methodologies used to test market efficiency are subject to the well-established joint-hypothesis problem. We derive three propositions that build on the well-know Sharpe ratios with the specific aim to split the test from the joint-hypothesis into two separate tests. We apply the new approach to examine the efficiency of the United States, Japan, and the United Kingdom markets over the period 1981-2000. Our results suggest that the rejection of the efficient market hypothesis may be premature. We thoroughly discuss the bias underlying the traditional approaches and propose a way to solve the problems.

Résumé

Cette présente thèse comporte deux essais sur l'investissement international. Chaque essai propose une nouvelle théorie et fournit un test empirique. Le premier essai développe un modèle d'évaluation des actifs internationaux à trois moments (le TM-IAPM). Le modèle tient compte de l'intégration des marchés et de possibles déviations sur la parité du pouvoir d'achat. Le modèle ainsi dérivé rémunère le coskewness (le co-moment juste après la covariance) et tient les modèles d'évaluation des actifs internationaux (IAPMs) standards comme des cas particuliers lorsque des restrictions supplémentaires sont imposées. Nous appliquons le modèle pour étudier le comportement des primes de risque de marché, de taille, de valeur, et de momentum des États-Unis, du Japon, et du Royaume-Uni. Nous trouvons que non seulement le modèle explique la plupart des variations de ces primes durant les années 80 et 90, mais surtout que le risque associé au coskewness est plus important que celui relié à la covariance. Nous constatons également que le modèle performe bien hors échantillon. L'implication directe de ce résultat est que les IAPMs classiques sont mal spécifiés et que les investisseurs devraient employer les modèles non linéaires pour évaluer les actifs dans un contexte international. Quant au deuxième essai, il propose une manière de démêler le test d'efficience des marchés du test du modèle d'équilibre postulé. En effet, l'hypothèse d'efficience des marchés, qui stipule que les prix reflètent pleinement l'information disponible, n'est pas testable car les tests existants sont sujets au problème de l'hypothèse jointe. Se basant sur les ratios de Sharpe, nous dérivons une nouvelle approche qui permet de séparer le test joint d'efficience des marchés en deux tests distincts. Nous appliquons cette nouvelle approche afin d'examiner l'efficience des marchés boursiers américains, japonais, et britanniques sur la période 1981-2000. Nos résultats suggèrent que le rejet de l'hypothèse d'efficience des marchés reporté dans la littérature existante est prématuré, car les modèles d'évaluation des actifs utilisés sont plus souvent qu'autrement incorrects. Nous discutons aussi du biais inhérent dans les méthodes traditionnelles et proposons des façons de résoudre ces problèmes.

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To the memory of my twin sister Tabara Sy

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INTRODUCTION

The pricing of securities in an open international context and the efficiency of the international markets have been among the most active areas of research in finance. This thesis contributes to this area of research.

On the one hand, the extant literature on international asset pricing is incomplete and deficient in that there is no model that is able to explicitly price the higher moments of returns. Indeed, all existing international asset pricing models (IAPMs) consider only the first two moments of returns and overlook the higher-order moments such as skewness. However, there is mounting evidence that higher-order moments are not only significant in several international markets but they are indeed priced in a segmented-market economy. On the other hand, the efficient market hypothesis (EMH)—which stipulates that prices fully reflects the available information—is perhaps the most important building block in the both finance and economics. However, the actual state of the literature is not satisfactory because there is no specific test of market efficiency. This is so because all tests are simultaneously a test of the market efficiency and of the equilibrium model used. This well known phenomenon is usually referred to as the *joint-hypothesis* problem.

The first essay of this thesis, chapter II, attempts to fill this gap in the international asset pricing literature by deriving an IAPM that takes into account coskewness risk. More specifically, we develop an explicit three-moment IAPM (TM-IAPM) from the first-order condition of a consumer-investor's problem by allowing the intertemporal marginal rate of substitution (IMRS) to be nonlinearly influenced by the aggregate wealth portfolio. We expand the IMRS as a second-order Taylor series and aggregate the resulting equilibrium in an open international context characterized by relative price uncertainty and heterogeneous consumption tastes. We show that the expected returns are not solely influenced by their conditional covariance with the market and inflation factors (as predicted by the standard IAPM) but also by their conditional coskewness with the factors. The derived TM-IAPM hold as particular cases the standard IAPMs and can easily be extended to price further higher-order comoments such as cokurtosis.

Next, we investigate whether the TM-IAPM helps explain the behavior of the market, size, value, and momentum premiums observed in the world's three foremost international markets (the United States, Japan, and the United Kingdom) during the 1980s and 1990s. We modify the instruments-based approach to explicitly model conditional covariance and coskewness and to test the TM-IAPM and the restrictions implied by the standard IAPMs using specifications that allow for time-varying risk and prices of risk. We find that the TM-IAPM helps explain most of the time-series variation of the market, size, value, and momentum premiums across the United States, Japan, and the United Kingdom. We also find that inflation covariance is reliably priced beyond market covariance. More importantly, we find that conditional coskewness is persuasively priced independently of the dynamic specification of price of risk, and that the conditional coskewness is more significant than conditional covariance in explaining the variation of the market, size, value, and momentum premiums. Collectively, these results suggest that a nonlinear IAPM must be used to price securities.

The second essay, chapter III, proposes a new approach to solve the joint-hypothesis problem. To attain this goal, we rely on the scale-independence property underlying Sharpe ratios to demonstrate the Sharpe ratio of any portfolio can be reproduced by a precise combination of the Sharpe ratios of the factors when the postulated equilibrium model holds. Since this theoretical restriction is independent of the portfolio locus on the efficient frontier if the equilibrium model holds, the framework can be used to disentangle the test of market efficiency from the test of the postulated equilibrium model. In other words, separate tests for market efficiency and for the validity of equilibrium model can be devised within this new framework.

We apply the approach and next examine the efficiency of the United States, Japan, and the United Kingdom over the period 1981-2000. We test the efficiency of these markets vis-à-vis the most important anomalies reported in the literature; that is, the size, value, and momentum effects. We find that the three anomalies investigated in this study are potentially driven by the failure of the asset pricing models postulated, suggesting that the rejection of the EMH may be premature. We also investigate the implications of our results for event studies and performance measurement.

Chapter IV summarizes the main findings of this thesis and suggests some research avenues related to the tenor of the thesis.

CHAPTER I:

LITERATURE REVIEW

The literatures on asset pricing and market efficiency are among the most active areas of research in finance and economics. As discussed by Fama (1970, 1976), the two topics are interrelated because we need an asset pricing model to test for market efficiency. The purpose of this chapter is to briefly review the allied works. We will first discuss the literature on asset pricing and then discuss that of market efficiency.

I. THE LITERATURE ON ASSET-PRICING

The concept of diversification—the art of distributing one's assets on different securities—has always been well understood by international investors. For example, as early as 1994, one could read in the working paper of the largest British investment firm, The Investment Registry, some passages clearly referring to the concept of diversification (e.g., Lofthouse (1997)): "*Every investor, large or small, instinctively feels that his capital should be split up into sections, and not invested in any one particular security, so that there shall not be too many eggs in one basket (1904, p.28-29). [...] It is, in our opinion, impossible to construct a really safe investment list consisting of British stocks only, for the simple reason that these stocks are bound all to move together. The best practical substitute which we can suggest for world-wide distribution of investment capital is the careful choice of varied enterprises to make up an investment combination: great attention being paid to the point that no two companies selected shall be identical in their objects or in their trade interest (p.75).*"

None-the-less, the modern literature on asset pricing really started with the first scientific treatment of the concept of diversification by Markowitz (1952). One of Markowitz's main contributions is to show that it is not the level of risk of an asset in isolation (the variance) that matters but the contribution to each asset to the portfolio total risk (the covariance). Sharpe (1964) builds on the concept of diversification to derive the first formal asset-pricing model usually known as the capital asset pricing model (CAPM). The key idea of Sharpe's derivation is the simplifying assumption that risk can equivalently be captured from the covariation between each asset with the market portfolio. Sharpe's CAPM was further refined by the works of Lintner (1965) and Mossin (1966).

Given the rather restrictive hypotheses assumed while deriving the CAPM, several interesting papers have been published that improve or relax some of these hypotheses. Most of these researches show that the structure of the CAPM does not change even when some hypotheses are relaxed. For example, both Brennan (1970) and Mayers (1972) show that the model remain the same even when taxes are taken into account or the market portfolio includes non-traded assets, respectively. Black (1972) derives a zero-beta model that is basically the same as the classic CAPM when there is no riskless asset. Williams (1977) shows the robustness of the structure of the CAPM when the hypothesis of homogenous return expectations is relaxed. Further, Breeden (1979) relaxes the static nature of the CAPM to derive the consumption-based CAPM (CCAPM). The CCAPM is similar to classical CAPM; the only difference is that systematic risk is measured with regard to aggregate consumption rather than aggregate wealth.

The most outstanding extensions of the CAPM are probably the intertemporal CAPM (ICAPM) of Merton (1973) and Ross's (1976) arbitrage pricing theory (APT). Merton's model is primarily aimed to relax the static nature of the CAPM by considering an intertemporal portfolio choice problem while assuming continuous trading and Markovian stochastic processes. This framework produced a multifactor model that held the CAPM as a special case when the investment opportunity set is constant. Ross's APT yields the same functional form as the ICAPM; the only differences is that the model is derived to verify the absence of arbitrage paradigm, and that the factors are not necessarily state variables.

A common feature of all the previous theoretical works is that they assume a segmented-market economy. However, thanks primarily to the relatively low historical correlations between international markets, the international dimension of investment has always been an important in finance. The structure of the classical CAPM has been proven to hold even in an integrated international setting in which investors consume the same goods (Grauer, Litzenberger, and Stehle (1976)). However, a more important question is to know what happens if the setting of the traditional CAPM is extended to allow the existence of country-specific investment and consumption particularities.

This interesting question has been investigated by Solnik (1974), who show that in an integrated context characterized by purchasing power parity (PPP) deviations and heterogeneous national-investors tastes, additional risk premia related to exchange rate risk should be considered. In other words, expected returns can be different from the riskless asset even when the portfolios are constructed to be uncorrelated to the market factor. Solnik's (1974) international asset pricing model has been subsequently refined by Sercu (1980), Stulz (1981), and Adler and Dumas (1983).

The main insight behind the IAPMs is the recognition that the gain from international investing cannot be directly internalized since the exchange rate risk faced cannot be diversified away. Empirically, most of the early researches, which were mainly based on the unconditional version of the IAPM, were unfavorable (e.g., Solnik (1974), Stehle (1977), and Korajczyk and Viallet (1989)). However, the importance of exchange risk as a driving force of expected return has been confirmed by recent studies that estimate the conditional version of the IAPM (e.g., Dumas and Solnik (1995), De Santis and Gerard (1998), Carrieri (2001), and Carrieri, Errunza, and Majerbi (2005)). Consistent to the IAPM, these studies confirm the statistical and economic significance of the exchange risk for both developed and emerging markets.

Nonetheless, several recent researches highlight that higher moments of returns are not trivial in the international markets (see Harvey and Siddique (1999) and the references therein) and that nonlinear asset-pricing models work better than linear models (e.g., Ghysels (1998)). The problem is that the studies that take into account the higher-order comoments of returns in the pricing of securities are either based on a segmented-market economy (e.g., Rubinstein (1973), Friend and Westerfield (1980), Sears and Wei (1985), Lim (1989), Chapman (1997), Fang and Lai (1997), Harvey and Siddique (2000), and Dittmar (2002)) or *ad hoc* when they consider the international dimension of investing (e.g., Bansal, Hsieh, and Viswanathan (1993) and Nummelin (1997)). The main goal in the first part of this thesis, chapter II, is to shed some light on this issue by deriving and testing a *formal* IAPM that can handle the higher moments of returns.

II. THE LITERATURE ON MARKET EFFICIENCY

While, traditionally, financial economists were always aware of the concept of market efficiency,[1] the literature "*was little more than a collection of anecdotes, rules of thumbs, and shuffling of accounting data*" (Merton (1994, p.452)) before the path-breaking works of Samuelson (1965). Samuelson was the first to point out the close link between market efficiency and martingales. Indeed, assuming that economic agents are risk-neutral with a common constant time-preference, the author shows that prices will actually behave as martingales. The direct implications of this result is that prices will equate their fundamental values (the expected present value of future dividends) and investors will always get the fair price, independently of their comparative advantage in gathering information.

The efficient market hypothesis (EMH) later received a dramatic impulse with Fama's (1970) survey of the literature. In that paper, Fama presented the classical definition of market efficiency that continues to be the industry standard today: the statement that security prices fully reflect all available information. The limiting conditions implied by the costs to getting prices to reflect information and by economic viability have been examined by Grossman and Stiglitz (1980) and Jensen (1978), respectively. Fama (1970) also refined the concept market efficiency by introducing the sub-martingale model, which implies that the expect value of future

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[1] See Leroy (1989) for the discussion of the analogy between the concept of market efficiency and the Ricardian theory of competitive equilibrium. See also Cootner (1964) for the collection of the most important early works on market efficiency.

price is always greater than the present price (or expected returns are positive). The implication of the sub-martingale model of market efficiency is that no trading rule based on available information can beat the simple buy-and-hold strategy. Fama, crediting Roberts (1967), also identified the classical taxonomy of market efficiency: the weak form of market efficiency (the information set includes only the history of prices); the semi-strong form (the information set includes publicly available information); and the strong form (the information set includes private information).

On empirical grounds, the early empirical studies (mostly in the 1960s and 1970s) supported market efficiency. This literature is summarized by the empirical work of Jensen (1968), who showed that even finance professionals such as mutual fund managers are apparently unable to select portfolios that systematically beat the market. Nonetheless, the latter researches (mostly in the 1980s and 1990s) were less favorable. Indeed, the recent empirical literature has uncovered numerous *anomalies* that seem to contradict the EMH. The most significant anomalies documented by the empirical literature include the January seasonal effect (Rozeff and Kinney (1976)); the closed-end funds effect (Malkiel (1977)); the weekend effect (French (1980)); the size effect (Banz (1981)); the earnings-to-price effect (Basu (1983)); the book-to-market effect (Rosenberg, Reid, and Lanstein (1985)); the contrarian effect (De Bondt and Thaler (1985)); the Value-line effect (Stickel (1985)); the turn of the month effect (Ariel (1987)); the holiday effect (Lakonishok and Smidt (1988)); the leverage effect (Bhandari (1988)); the weather effect (Saunders (1993)); and the momentum effect (Jegadeesh and Titman (1993)).

Fama and French (1992, 1993, 1995, 1996) reviewed thoroughly the literature on anomalies to contend that the anomalies are rather explained by the failure of the CAPM. They also find that most of these anomalies vanish when a three-factor pricing model (TFPM) that consists of the market and two other mimicking factors based on size and boo-to-market is used. However, the issue is not resolved because there are several other explanations that have been advanced to explain the anomalies.

For instance, Black (1993) argues that the anomalies are mainly driven by data mining rather than mispricing. He contends that "*most of the so-called anomalies that have plagued the literature on investments seem likely to be the result of data-mining. We have literally thousands of researchers looking for profit opportunities in securities. They are all looking at roughly the same data. Once in a while, just by chance, a strategy will seem to have worked in the past. The researcher who finds it writes it up, and we have a new anomaly*" (p.9). However, this explanation has been put aside mainly because the out-of-sample evidence on some of anomalies is strong (e.g., Fama and French (1998)). Also related to the robustness issue are the Loughran (1997) and Knez and Ready's (1997) critiques. Loughran finds that the size and value effects are driven by the January seasonal in the value effect and the behavior of the small-growth firms while Knez and Ready report that the size premium vanishes when one percent of the most extreme observations are trimmed each month, suggesting that the anomalies may be driven by few observations.

Another explanation advanced in the literature is that the anomalies appear merely because markets are inefficient. Among others, Daniel and Titman (1997) and Daniel, Titman, and Wei (2001) lean towards this idea. Using a methodology that controls for firm characteristics across portfolios of different levels of risk, they find that once the effects of firm characteristics are controlled for, expected returns are not positively related to risk. They consequently dismiss the role of risk to explain market anomalies and attribute the results to mispricing. Proponents of this explanation also include La Porta, Lakonishok, Shleifer, and Vishny (1997), who argue that the value effect is inconsistent with a risk-based explanation, but is rather a manifestation of expectational errors made by investors.

Whether the one or the other of the above explanations hold cannot be determined from the existing tests because, as Fama (1991, p.1593) puts it: "*In truth, though, the existing tests can't tell whether the anomalies result from a deficient asset-pricing model or persistent mispricing of securities.*" The problem underlying the existing tests is correctly identified by Fama (1991, p.1575-76): "*Ambiguity about information and trading costs is not, however, the main obstacle to inferences about market efficiency. The joint-hypothesis problem is more serious. Thus, market efficiency per se is not testable. It must be tested jointly with some model of equilibrium, an asset-pricing model.*"

The main challenge in the second part of the present thesis, chapter III, is to develop a framework that can effectively be used to split the traditional joint-test of market efficiency into two distinct tests of market efficiency and the postulated equilibrium model, thus resolving the long-standing joint-hypothesis drawback.

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CHAPTER II:

A THREE-MOMENT INTERNATIONAL ASSET-PRICING MODEL: THEORY AND EVIDENCE

I. INTRODUCTION

There is mounting evidence that the cross sectional variation in expected returns can not be explained by the single-factor capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965).[2] In response, a number of authors have relaxed the assumptions that underlie the linear risk return relationship, dating back to the pioneering work of Kraus and Litzenberger (1976) who study unconditional skewness. More recently, Bansal and Viswanathan (1993) and Leland (1997) develop nonlinear factor models and Harvey and Siddique (2002a) derive a conditional version of the three-moment CAPM.[3]

International asset-pricing models (IAPMs) developed under full capital market integration and deviations from purchasing power parity (PPP) follow from the aggregation of the single country CAPM and deliver a linear risk-return relationship.[4] Although there is evidence against these IAPMs and that conditional skewness is important for international returns (Harvey and Siddique (1999)), the derivation of

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[2] See for example, Fama and French (1992, 1995) on the importance of size and book to market ratio.

[3] Dittmar (2002) provides a detailed discussion of nonlinear pricing kernels and their performance relative to linear and multifactor kernels for the cross section of U.S. equity returns. See also Chapman (1997), Fang and Lai (1997), and Chung, Johnson, and Schill (2004).

[4] The well known IAPMs include Solnik (1974), Stulz (1981), and Adler and Dumas (1983). See Karolyi and Stulz (2003) for an excellent discussion.

an explicit IAPM that takes this moment into account has not yet been done. This study attempts to fill this gap in the literature by extending the extant IAPMs to incorporate the effect of skewness on valuation. More specifically, we seek to answer the following interrelated questions: What type of risk-return relation should we expect in an integrated global capital market characterized by PPP deviations? In other words, do systematic terms beyond market and inflation covariance enter the pricing process? Are these new components of expected return systematically priced in the international markets?

We develop an explicit three-moment IAPM (TM-IAPM) from the first-order condition of a consumer-investor's problem by allowing the intertemporal marginal rate of substitution (IMRS) to be nonlinearly influenced by the aggregate wealth portfolio. We expand the IMRS as a second-order Taylor series and aggregate the resulting equilibrium in an international context characterized by relative price uncertainty and heterogeneous consumption tastes. We show that the expected returns are not solely influenced by their conditional covariance with the market and inflation factors (as predicted by the standard IAPM) but also by their conditional coskewness with the factors. The derived TM-IAPM is reduced to the standard IAPMs when coskewness is not priced.

Next, we investigate whether the TM-IAPM helps explain the behavior of the market, size, value, and momentum premiums observed in the world's three foremost international markets (the United States, Japan, and the United Kingdom) during the 1980s and 1990s. Given the importance of the size, value, and momentum effects, the litmus test for any candidate asset-pricing model is its ability to explain these premiums.[5] We use the TM-IAPM to decompose the variance of the market, size, value, and momentum premiums and assess the relative importance of the sources of systematic risk.

Because all models tested here are conditional, their estimation requires modeling conditional second and third comoments of returns. We build on Campbell (1987), Harvey (1989, 1981), and Dumas and Solnik (1995), and modify the instruments-based approach to explicitly model conditional covariance and coskewness, as well as to test the TM-IAPM and the restrictions implied by the standard IAPMs using specifications that allow for time-varying risk and prices of risk.

Our main empirical findings can be summarized as follows. The TM-IAPM helps explain most of the time-series variation of the market, size, value, and momentum premiums across the United States, Japan, and the United Kingdom. As expected, the world price of market covariance is positive and significant. The price of inflation covariance risk is significantly negative except for the UK inflation factor when only UK assets are considered.[6] Overall, the prices of market and inflation covariance are

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[5] The size effect is the propensity of small firms to out-perform large firms (Banz (1981)), the value effect is the tendency of value stocks (stocks with a high book-to-market ratio) to out-perform growth stocks (Rosenberg, Reid, and Lanstein (1985)), while the momentum effect is the tendency of winners (stocks that performed well in the past) to out-perform losers over the medium-term (Jegadeesh and Titman (1993)).

[6] In our model, the price of inflation risk will be negative if the risk-aversion of the national investors is sufficiently high. This feature of our model is similar to that of Adler and Dumas (1983).

consistent with the predictions of the IAPM and confirm Dumas and Solnik (1995) and De Santis and Gerard's (1997) results.

More importantly, conditional coskewness is persuasively priced independently of the dynamic specification of price of risk. The prices of market and inflation coskewness appear significant most of the time. Accordingly, we always reject at any standard level of statistical significance the restrictions implied by the nested IAPMs. Furthermore, as a confirmation of the nonlinear nature of the international risk-return relationship, we find that the conditional coskewness is more significant than conditional covariance in explaining the variation of the market, size, value, and momentum premiums.

The remainder of the study is structured as follows. The next section (Section II) presents the model. Section III describes the data, presents some descriptive statistics, and outlines the methodology. Section IV discusses the empirical results. Some concluding remarks are offered in Section V.

II. THE MODEL

Throughout the study, all consumption units (c) and returns are measured in terms of US dollar, the reference currency in a world of L countries ($\ell = 1 \ldots L$). Rational investors are concerned with real (deflated) consumption units and returns. The first order condition describing the investor's consumption and investment optimal decision (e.g., Hansen and Jagannathan (1991)) virtually offers a unique framework that encompasses all asset-pricing models (e.g., Cochrane (2001)). The basic equation of asset pricing can be written as follows:

$$E_t[m_{t+1}(1+\rho_{pt+1})]=1, \tag{1}$$

where E_t is the conditional expectations operator conditioned on time t information, $m_{t+1}=u'(c_{t+1})/u'(c_t)$ represents the investor's IMRS, and ρ_p is the real return on portfolio p ($p=1...P$).[7] Equation (1) states that in equilibrium, the marginal cost of postponing consumption at time t to buy an extra unit of portfolio p, $u'(c_t)$, must equal the expected marginal utility of selling the holdings and consuming the proceeds at $t+1$, $E_t[u'(c_{t+1})(1+\rho_{pt+1})]$.

To derive an explicit model from equation (1), the approach favored by financial economists is to *take the IMRS as given*; that is to specify the IMRS as a function of observable variables and parameters. Expanding the IMRS and assuming the existence of a conditionally risk-free asset (with real return ρ_f), the expected real return on the portfolio can be written as:

$$E_t[\rho_{pt+1}]=E_t[\rho_{ft+1}]-(1+E_t[\rho_{ft+1}])Cov_t[\rho_{pt+1},m_{t+1}], \tag{2}$$

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[7] The IMRS is also known as a stochastic discount factor, a pricing kernel, a Radon-Nicodym derivative, or an equivalent martingale measure.

where Cov_t stands for the conditional covariance operator. In equation (2), systematic risk is measured by conditional covariation with the IMRS; viz., nothing is rewarded over the riskless rate of return if the conditional covariance with the IMRS is zero. A given portfolio earns a positive risk premium if it is negatively correlated with the IMRS; since a negative correlation means that the portfolio is likely to earn less than average when it is needed the most. Beyond its simplicity, generality, and flexibility, the main advantage of this approach (over the general equilibrium approach) is that it "*allows to conveniently separate the step of specifying economic assumptions of the model from the step of deciding which kind of empirical representation to pursue*" (Cochrane (2001, p. *xv*)).

The assumption of the existence of a representative agent (e.g., Lucas (1978) and Breeden (1979)) for country ℓ allows the IMRS to be expressed as a function of aggregate consumption, c_ℓ. As noted by Cochrane (2001, p.160) the point of CAPMs is to "*avoid the use of consumption data, and to use wealth or the rate of return on wealth instead.*" Without loss of generality, writing the aggregate consumption as a function of aggregate wealth, $c_{\ell t} = g(w_{\ell t})$, and expanding the IMRS as Taylor's series about the point $w_{\ell t+1} = w_{\ell t}$ gives:[8]

$$m_{t+1} = 1 + \sum_{n=1}^{N} \{g'(w_{\ell t})(w_{\ell t+1} - w_{\ell t})\}^n \frac{u^{(n+1)}(c_{\ell t+1})}{n!u'(c_{\ell t})} + o_N \,, \tag{3a}$$

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[8] The function $g(.)$ is assumed differentiable at the point $w_{\ell t+1} = w_{\ell t}$ with a partial derivative that is independent of aggregate wealth.

where $u^{(n)}$ denotes the n^{th} partial derivative of u with respect to c_ℓ and o_N is the Lagrange remainder of the approximation. Using the definition of the return on aggregate wealth, $\rho_{\ell t+1} = (w_{\ell t+1} - w_{\ell t}) / w_{\ell t}$, equation (3a) simplifies as:

$$m_{t+1} = 1 + \sum_{n=1}^{N} \{g'(w_{\ell t}) w_{\ell t}\}^n \frac{u^{(n+1)}(c_{\ell t+1})}{n!\, u'(c_{\ell t})} (\rho_{\ell t+1})^n + o_N . \tag{3b}$$

Note that both equations (3a) and (3b) are exact representations of the IMRS. However, since the true functional form of the IMRS is unobservable, only an approximation can be used; that is, one must determine an order N at which the expansion of the IMRS is truncated. Linear factor models can be seen as first order approximations of the IMRS. When the investor has preference ordering beyond the second moment of returns (e.g., Scott and Horvath (1980)), then at least an additional order should be considered. While restricting the IMRS to be linear ($N = 1$) seems unreasonable given our previous discussion, an outsized truncation of the approximation could however lead to pricing models with too many parameters; which increase the chance of overfitting the data and diminish the power to statistically reject false null hypotheses.

Given the parsimony concerns as well as the need to meaningfully interpret the various components of the approximation, a second-order Taylor approximation ($N = 2$) has been preferred in the literature:[9]

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[9] See for example Harvey and Siddique (2000a) and the numerous references therein. In this study, we will not explicitly analyze the terms of order higher than $N = 2$. The inclusion of

$$m_{t+1} = \alpha_{\ell 0t} - \alpha_{\ell 1t}\rho_{\ell t+1} - \alpha_{\ell 2t}(\rho_{\ell t+1})^2, \qquad (3c)$$

where by identification with (3b) if follows $\alpha_{\ell 0t} = 1 + o_N$, $\alpha_{\ell 1t} = -g'(w_{\ell t})w_{\ell t}u''(c_{\ell t})/u'(c_{\ell t})$, and $\alpha_{\ell 2t} = -(1/2)\{g'(w_{\ell t})w_{\ell t}\}^2 u'''(c_{\ell t})/u'(c_{\ell t})$. Since $g'(w_{\ell t}) = \partial c_{\ell t}/\partial w_{\ell t}$ can be seen as the marginal propensity to consume with respect to aggregate wealth, it follows that $\alpha_{\ell 1t}$ can be interpreted as the relative risk aversion of the national investor.[10]

If the marginal utility of consumption is positive (i.e., nonsatiety with respect to consumption) then $\alpha_{\ell 1t}$ will be positive if the investor is risk-averse ($u''(c_{\ell t}) < 0$). Pratt (1964) argues that a good utility function must exhibit decreasing absolute risk-aversion; this is a sufficient condition for $u'''(c_{\ell t}) > 0$ (or $\alpha_{\ell 2t} < 0$ for a risk-averse investor). Furthermore, since $u'''/u' = (-u'''/u'')(-u''/u')$, $\alpha_{\ell 2}$ will also be a function of the investor's level of absolute prudence, $-u'''/u''$. The latter term aims to measure the strength of the precautionary savings motive under uncertainty (e.g., Kimball (1990)).

The standard IAPM and its various special cases are based on the a priori hypothesis that the IMRS is linear in the market return. Yet, several studies have

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such terms would considerably complicate our empirical task without affecting the conclusion about the presence of nonlinearities. The extension of our model to include the next higher-order moments is though straightforward.

[10] In the particular case of a two-period economy, because the investor consumes everything in the second period, we have $g'(.) = 1$ and $\alpha_{\ell 1t} = -c_{\ell t}u''(c_{\ell t})/u'(c_{\ell t})$.

stressed the importance of taking into account nonlinear risk-return relationships. For example, Bansal and Viswanathan (1993) and Bansal, Hsieh, and Viswanathan (1993) emphasize the fact that so long as there exist some securities whose payoffs are nonlinear in the factors, the linear factor-pricing model does not hold.[11] In the same vein, Ghysels (1998) provides evidence that nonlinear models work better empirically than linear factor models.

Plugging (3c) into equation (2) yields the three-moment CAPM (TM-CAPM), which states that, in equilibrium, the expected excess return on portfolio p is not only a function of conditional market covariance but also of conditional market coskewness:

$$E_t[\rho_{pt+1}] = E_t[\rho_{ft+1}] + \theta_{\ell 1t} Cov_t[\rho_{pt+1}, \rho_{\ell t+1}] + \theta_{\ell 2t} Cov_t[\rho_{pt+1}, (\rho_{\ell t+1})^2], \qquad (4)$$

where $\theta_{\ell 1t} = (1 + E_t[\rho_{ft+1}])\alpha_{\ell 1t}$ is the equilibrium local price of conditional market covariance and $\theta_{\ell 2t} = (1 + E_t[\rho_{ft+1}])\alpha_{\ell 2t}$ is the local price of conditional market coskewness (in real terms). Equation (4) states how investors assess the relationship between the IMRS and the local market factor. In equilibrium, the price of conditional market covariance should be positive when investors are risk-averse. If

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[11] This is of course the case for stocks because their payoffs are bounded from below (limited liability) (e.g., Black (1976)). The presence of bubbles (e.g., Blanchard and Watson (1982)), volatility feedback effects (e.g., Pindyck (1984)), agency problems (e.g., Brennan (1993)), overreactions to news (e.g., Veronesi (1999)), transaction costs (e.g., Cao, Coval, and Hirshleifer (2002)), and short-sale restrictions (e.g., Hong and Stein (2003)) may also introduce nonlinearities in payoffs.

such is the case, then they will also require a negative price of coskewness when they are mainly motivated by precautionary savings motive.

To derive the nominal TM-CAPM with stochastic inflation, first observe that the portfolio real rate of return is given by:

$$\rho_{pt+1} = \frac{1 + \mathrm{R}_{pt+1}}{1 + \pi_{\ell t+1}} - 1, \tag{5}$$

where R_p is the portfolio nominal rate of return and π_ℓ stands for the country's rate of inflation. Next expand (5) as a second-order Taylor series (around $\pi_{\ell t+1} = 0$) and obtain $\rho_{pt+1} = -1 + \varpi_{\ell t+1}(1 + \mathrm{R}_{pt+1})$, where $\varpi_{\ell t+1} = (1 + \pi_{\ell t+1})^{-1} \approx 1 - \pi_{\ell t+1} + \pi_{\ell t+1}^2$.[12] Finally, substitute this latter expression into (4), neglect the cokurtosis terms, rearrange, and obtain the nominal TM-CAPM:

$$\begin{aligned} E_t[\mathrm{R}_{pt+1}] = \xi_{\ell t} + \Delta_{pt} + \theta^*_{\ell 1t} Cov_t[\mathrm{R}_{pt+1}, \mathrm{R}_{\ell t+1}] + (1^* - \theta^*_{\ell 1t}) Cov_t[\mathrm{R}_{pt+1}, \pi_{\ell t+1}] \\ + \theta^*_{\ell 2t} Cov_t[\mathrm{R}_{pt+1}, (\mathrm{R}_{\ell t+1})^2] + \{\theta^*_{\ell 2t} - (1^* - \theta^*_{\ell 1t})\} Cov_t[\mathrm{R}_{pt+1}, (\pi_{\ell t+1})^2]' \end{aligned} \tag{6}$$

where $\theta^*_{\ell 1t} = \theta_{\ell 1t} / E_t[\varpi_{\ell t+1}]$ and $\theta^*_{\ell 2t} = \theta_{\ell 2t} / E_t[\varpi_{\ell t+1}]$ can be seen as *nominalized* local prices of market covariance and coskewness risks, $1^* = 1 / E_t[\varpi_{\ell t+1}]$,

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[12] Note that $\varpi_{\ell t+1} \approx \frac{3}{4} + (\pi_{\ell t+1} - \frac{1}{2})^2$ is always positive. As the inverse to the gross rate of inflation, $\varpi_{\ell t+1}$ can be seen as a multiplier that transforms nominal returns into real ones.

$$\begin{aligned}\xi_{\ell t} = E_t[\mathrm{R}_{ft+1}] - \{&\theta^*_{\ell 1t}Cov_t[\pi_{\ell t+1},\mathrm{R}_{\ell t+1}] + \theta^*_{\ell 1t}Var_t[\pi_{\ell t+1}] + (2\theta^*_{\ell 1t} + \theta^*_{\ell 2t})Skew_t[\pi_{\ell t+1}] \\ &+\theta^*_{\ell 2t}Cov_t[\pi_{\ell t+1},(\mathrm{R}_{\ell t+1})^2] - (\theta^*_{\ell 1t} + 2\theta^*_{\ell 2t})Cov_t[\pi_{\ell t+1},\mathrm{R}_{\ell t+1}\pi_{\ell t+1}] - \theta^*_{\ell 1t}Cov_t[\mathrm{R}_{\ell t+1},(\pi_{\ell t+1})^2]\}\end{aligned}, \tag{7a}$$

combines the terms that are not correlated with the market and inflation factors and equals the nominally local riskless rate of return (if one is available), and

$$\Delta_{pt} = \theta^*_{\ell 1t}Cov_t[\mathrm{R}_{pt+1}\pi_{\ell t+1},\pi_{\ell t+1}] - \theta^*_{\ell 1t}Cov_t[\mathrm{R}_{pt+1}\pi_{\ell t+1},\mathrm{R}_{\ell t+1}] - (\theta^*_{\ell 1t} + 2\theta^*_{\ell 2t})Cov_t[\mathrm{R}_{pt+1},\mathrm{R}_{\ell t+1}\pi_{\ell t+1}], \tag{7b}$$

regroups the cross terms; because these non-diagonal terms can always be mitigated through orthogonalization, we shall combine them into a single component. Equation (6), which illustrates well the intuition that inflation stochasticity requires an additional premium in nominal returns, collapses to the classic TM-CAPM of Kraus and Litzenberger (1976) when inflation is not stochastic. The model is also reduced to the usual Sharpe-Lintner CAPM when inflation is static and market coskewness is not priced.

The domestic TM-CAPM assumes that investors intertemporally substitute consumption units in a single-country context. The question as to what happens when the investor's opportunity set is extended to include the local and international markets has been addressed by several researchers in the case of the CAPM (see for example, Adler and Dumas (1983) and Stulz (1995)) but not for the TM-CAPM. The usual approach consists in aggregating the first-order condition over all national investor groups.[13]

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[13] Adler and Dumas (1983) discuss some of the caveats related to aggregation.

To obtain the TM-IAPM, multiply (6) by $x_{\ell t} / \theta^*_{\ell 1t}$, where $x_{\ell t}$ is the country ℓ's relative wealth in the world, and sum over all countries:

$$\begin{aligned} E_t[R_{pt+1}] = \varsigma_{ft} + \psi_{pt} + \beta_{Wt} Cov_t[R_{pt+1}, R_{Wt+1}] + \sum_{\ell=1}^{L} \beta^{\pi}_{\ell 1t} Cov_t[R_{pt+1}, \pi_{\ell t+1}] \\ + \sum_{\ell=1}^{L} \beta_{\ell 2t} Cov_t[R_{pt+1}, (R_{\ell t+1})^2] + \sum_{\ell=1}^{L} \beta^{\pi}_{\ell 2t} Cov_t[R_{pt+1}, (\pi_{\ell t+1})^2] \end{aligned} , \qquad (8)$$

where $R_{Wt+1} = \sum_{\ell} x_{\ell t} R_{\ell t+1}$ is the nominal return of the world market factor; $\beta_{Wt} = \Theta_t = 1/(\sum_{\ell} x_{\ell t} / \theta^*_{\ell 1t})$ is the world price of conditional market covariance; $\varsigma_{ft} = \Theta_t \sum_{\ell} x_{\ell t} \xi_{\ell t} / \theta^*_{\ell 1t}$ is the nominally global riskless rate of return; $\psi_{pt} = \Theta_t \sum_{\ell} x_{\ell t} \Delta_{pt} / \theta^*_{\ell 1t}$ aggregates the cross terms; $\beta^{\pi}_{\ell 1t} = \Theta_t x_{\ell t} (\frac{1^*}{\theta^*_{\ell 1t}} - 1)$ is the world price of conditional inflation covariance; $\beta_{\ell 2t} = \Theta_t x_{\ell t} \theta^*_{\ell 2t} / \theta^*_{\ell 1t}$ is the world price of conditional market coskewness; and $\beta^{\pi}_{\ell 2t} = \Theta_t x_{\ell t} \{ \frac{\theta^*_{\ell 2t}}{\theta^*_{\ell 1t}} - (\frac{1^*}{\theta^*_{\ell 1t}} - 1) \}$ is the world price of conditional inflation coskewness.

Note that β_W will be positive if the national investors are risk-averse ($\theta_{\ell 1} > 0 \, \forall \ell$) and $\beta_{\ell 2}$ will be positive if (in addition to risk-aversion) their behavior is not triggered by precautionary savings motive ($\theta_{\ell 2} > 0 \, \forall \ell$). The world prices of inflation covariance and coskewness are more difficult to interpret. Depending of the signs of $\frac{1^*}{\theta^*_{\ell 1t}} - 1$ and $\frac{\theta^*_{\ell 2t}}{\theta^*_{\ell 1t}} - (\frac{1^*}{\theta^*_{\ell 1t}} - 1)$, both signs can be observed on the price of inflation covariance and coskewness. A sufficient condition for the price of inflation risk to be negative is that the local price of conditional market covariance is higher than one ($\theta_{\ell 1} > 1$). Similarly, a sufficient condition for the price of inflation coskewness to be positive is that the risk-aversion of the national investors is sufficiently high ($\theta_{\ell 1} > 1$) whilst their absolute prudence is low ($\theta_{\ell 2} > 0$).

The TM-IAPM contains as many local market and inflation coskewness premiums as there are national investor groups. These premiums originate from the relaxation of the linearity hypothesis underlying the CAPM. If conditional coskewness is not priced, then the risk-return relationship will be linear and the TM-IAPM will be reduced to the standard IAPM of Adler and Dumas (1983). Solnik's (1974) version of the IAPM is obtained when it is further assumed that the inflation rate in the local currency is non-stochastic; that is, the price of inflation covariance is reduced to a pure reward-to-exchange rate risk. When PPP holds, then a global TM-CAPM with a world market covariance term and as many local market coskewness terms as there are national-investor groups will hold. The latter market coskewness terms will not necessarily combine into a single world-market-coskewness component as posited in Nummelin (1997). Nummelin's specification will only hold in the special case where the prices of coskewness are proportional to the countries' relative weights, $\beta_{\ell 2t} = \beta_{W 2t} x_{\ell t}$ (i.e., when $\theta_{\ell 1t} = \theta_{\ell 2t}\ \forall \ell$).[14] The world CAPM (W-CAPM) (e.g., Grauer, Litzenberger, and Stehle (1976)) is a particular case of Nummelin's specification where market coskewness is not priced.

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[14] This is however quite a restrictive condition; see the discussion in footnote 52 of Adler and Dumas (1983).

III. DATA AND METHODOLOGY

A. Data and Summary Statistics

We use monthly data from three countries, namely the United States, Japan, and the United Kingdom. Our sample runs from January 1981 to December 2000 (240 observations). Three different classes of data are used: factors, instruments, and premiums. As factors, we use: the world market return, which is the Morgan Stanley Capital International (MSCI) world equity index; inflation rates, which are the rates of change of the local consumer price indexes; and local market returns, which are the capitalization-weighted returns of all the securities considered within each country.

The instruments used to forecast returns are motivated by prior research (e.g., Fama and Schwert (1977), Chen, Roll, and Ross (1986), Harvey (1989, 1991), and Ferson and Harvey (1993)). The set of instruments includes: a constant; JAN: a January dummy variable; FED: the US Federal funds rate; DIV: the dividend yield of the world market index; DEF: the spread between Moody's Baa and Aaa yields; and TERM: the spread between the US Treasury bonds with maturity over ten years and the three-month US Treasury bill yields. All instruments are lagged variables.

We conduct the empirical test by examining whether the TM-IAPM explains the market, size, value, and momentum premiums. The market premium is computed as the difference between the market portfolio and the risk-free rate of interest. As risk-free rates, we use one-month Treasury bill rates for the US and UK markets. Since there are no Treasury bill rates in Japan, we use the Central Bank of Japan discount rate.

The size, value, and momentum premiums are computed as follows. For each month t from July of year $y-1$ to June of year y, we rank the stocks within each country based on their size in June $y-1$, their book-to-market (BEME) ratio in June $y-1$, and their return between $t-12$ and $t-2$. We then use these three rankings to calculate the quintile breakpoints (20, 40, 60, and 80 percent) for size, BEME, and prior return. The stocks are subsequently sorted into five size groups, five BEME groups, and five prior return groups based on these breakpoints. The portfolio returns are computed by value-weighting the stock returns within the portfolios. The size premium is the spread between the small (below the 20 percent size breakpoint) and large size (above the 80 percent size breakpoint) quintile portfolios. The value premium is the spread between the value (above the 80 percent BEME breakpoint) and growth (below the 20 percent BEME breakpoint) quintile portfolios. Finally, the momentum premium is the spread between the winner (above the 80 percent prior return breakpoint) and loser (below the 20 percent prior return breakpoint) quintile portfolios.[15]

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[15] The data used to compute the local market return and the premiums originate from DATASTREAM. Our sample includes 7,079 firms (4,620 firms for the United States, 1,567 firms for Japan, and 892 firms for the United Kingdom). We also use exchange rates to translate all the local series in US dollar (the reference currency). The data on the consumer price indexes, Treasury bill rates, and exchange rates are obtained from the International Financial Statistics (IFS) database. Finally, the data for FED, DEF, and TERM are obtained from the Federal Reserve Statistical Release (H15) while DIV is obtained from MSCI.

Table I provides summary descriptive statistics for the world and inflations factors as well as the various premiums. All the variables are expressed as continuously compounded monthly returns. Panel A focuses on the unconditional moments of the variables. Most of the stylized facts reported in the literature (e.g., Fama and French (1998) and Liew and Vassalou (2000)) were robust during the 1981-2000 period. Indeed, the market premium is significant in the United States and the United Kingdom. The average return of the value stocks is significantly higher than the average return of growth stocks for both Japan and the United Kingdom. The losing firms realize lower average returns (than the winning firms) in the United States and the United Kingdom. Even though the size premium is economically important at about 5 percent per year across the countries, the effect is however statistically insignificant at the 10 percent level.

Another notable result is the very low volatility of the US inflation factor, which is at least ten folds lower than the closest factor-volatility. This result is not surprising given that the Federal Reserve traditionally cares about price stability in setting the US monetary policy (e.g., Greenspan (1997)) and that the US inflation factor does not embed an exchange rate component, since the US dollar is the reference currency. To mitigate this shortcoming, we eliminate the US inflation factor in our empirical tests.

For several variables, the unconditional skewness, which should be equal to zero under the null of a normal distribution, is significant at the 1 percent level. In addition, without exception, the distribution of the variables is leptokurtic, to the extent that the null hypothesis of conformity of the distribution with normality, based on the values of skewness and kurtosis (using Bera-Jarque's (1982) statistic), is always rejected at the 1 percent level.

TABLE I ABOUT HERE

Several authors have documented that returns are partially predictable given information known in advance. In light of this evidence, non-exploitation of the predictability of returns could bias the tests. Therefore, we consider whether commonly used instruments could help predict the first three moments of the market, size, value, and momentum premiums. Following Harvey and Siddique (2000b), we estimate the following system for each premium, r_p:

$$\begin{aligned} E_t[r_{pt+1}] &= Z_t\gamma_{1p} + \phi_{1p} r_{pt} \\ E_t[(r_{pt+1})^2] &= Z_t\gamma_{2p} + \sum_{j=1}^{2} \phi_{2pj} r^2_{pt+1-j} \ , \\ E_t[(r_{pt+1})^3] &= Z_t\gamma_{3p} + \sum_{j=1}^{2} \phi_{3pj} r^3_{pt+1-j} \end{aligned} \qquad (9)$$

where Z are the instruments, the γ's are vectors of parameters, and the ϕ's are scalars.[16] We estimate system (9) for each premium using *generalized least squares* (GLS) and present the Wald tests of their time-variation. The results are shown in Panel B of Table I. Overall, we can conclude that the first three moments of international

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[16] Note that we modify the mean equation relative to Harvey and Siddique (2000b) and introduce the lagged return to account for the short-term contrarian effect (e.g., Jegadeesh (1990)).

returns are time-varying. This justifies the use of the conditional method that we describe below.

B. Methodology

Dumas and Solnik (1995) were the first to test a conditional version of the IAPM in which market and currency risks, as well as their prices, are allowed to vary over time.[17] They parameterized the IMRS and used the *generalized method of moments* (GMM) of Hansen (1982) to estimate the ensuing system. This has the main advantage of limiting the number of estimated parameters, because the IMRS is the same in all the equations of the system.

Unfortunately, this method also entails some limitations. As summarized by De Santis and Gerard (1997, p.383), "*their approach does not provide any measure of the potential deviation of each asset from the model restrictions. This is because the pricing errors in the model depend on conditional second moments that are not explicitly parameterized. Second, for the same reason, their approach cannot be used to measure, at each point in time, the importance of currency risk premiums relative to the market premium.*"

Since the comparison of the systematic sources of variation of the market, size, value, and momentum premiums is of particular interest in this study, we modify the approach used by Dumas and Solnik (1995) to address the questions raised above.

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[17] For unconditional versions of the tests of IAPMs, see Solnik (1974), Stehle (1977), and Korajczyk and Viallet (1989).

Following Harvey (1989), we proceed by explicitly modeling the first moment of returns and then use forecasting errors to construct the conditional covariance and coskewness terms without assuming a particular functional form.[18] Doing so, though, dramatically increases the number of orthogonality conditions, which renders the GMM extremely difficult if not impossible to implement. As discussed below, we solve this problem by using GLS. The price we pay in terms of efficiency for precluding GMM is, however, small compared to the gains in terms of robustness of the estimates, and that we are able to assess the relative importance of the systematic sources of variation of international returns.

To derive the econometric system for the TM-IAPM, we first express conditional covariance and coskewness as a function of the various forecasting errors. We begin by modeling the first moments of returns using the following linear filters:[19]

$$v_{pt+1} = r_{pt+1} - (Z_t \delta_p + \phi_p r_{pt}), \tag{10}$$

where the v's are the (zero-mean) forecasting errors for the various premiums, and the δ's and ϕ's are time-invariant weighting vectors used by investors to derive the expected excess returns.

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[18] One of the disadvantages of the GARCH approach, which is used by De Santis and Gerard (1997), is "*the strong assumption made about the functional form of the second moments*" (Harvey (1989, p.290)). In our framework, all the covariance and coskewness terms are obtained without assuming any a priori dynamic.

[19] We explored several functional forms for predicting returns including a quadratic model and find similar results.

With the forecasting errors defined in equation (10), the second and third comoment terms in the TM-IAPM ensue naturally:[20]

$$Cov_t[R_{pt+1}, R_{Wt+1}] = E_t[v_{pt+1} R_{Wt+1}], \tag{11a}$$

$$Cov_t[R_{pt+1}, \pi_{\ell t+1}] = E_t[v_{pt+1} \pi_{\ell t+1}], \tag{11b}$$

$$Cov_t[R_{pt+1}, (R_{\ell t+1})^2] = E_t[v_{pt+1} (R_{\ell t+1})^2], \tag{11c}$$

$$Cov_t[R_{pt+1}, (\pi_{\ell t+1})^2] = E_t[v_{pt+1} (\pi_{\ell t+1})^2]. \tag{11d}$$

Substituting (11a) through (11d) into equation (8) and using the linearity property of the expectations operator yields the following expressions for the prediction errors of the TM-IAPM on the premiums:[21]

$$e_{pt+1} = r_{pt+1} - \psi_{pt} - v_{pt+1} \left\{ \beta_{Wt} R_{Wt+1} + \sum_{\ell=1}^{L} \left(\beta^{\pi}_{\ell 1t} \pi_{\ell t+1} + \beta_{\ell 2t} (R_{\ell t+1})^2 + \beta^{\pi}_{\ell 2t} (\pi_{\ell t+1})^2 \right) \right\}, \tag{12}$$

where $E_t[e_{pt+1}] = 0$. With this econometric structure, it is easy to test the TM-IAPM (to gauge how well the TM-IAPM explains market, size, value, and momentum premiums) and the nested IAPMs (to examine the validity of the restrictions that they imply). The following system of equations:

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[20] With equations (11a) and (11b), we do not need to model the first moment of the factors (R_{Wt+1} and $\pi_{\ell t+1}$). Further, with the definition of conditional coskewness in (11c) and (11d), we are able to explicitly incorporate conditional coskewness in the instruments-based framework.

[21] Note that the nominally global riskless rate of return vanishes because the premiums are excess returns.

$$\varepsilon_{t+1} = \begin{pmatrix} \left[r_{pt+1} - (Z_t \delta_p + \phi_p r_{pt}) \right]' \\ \left[r_{pt+1} - \psi_{pt} - \upsilon_{pt+1} \left\{ \beta_{Wt} R_{Wt+1} + \sum_{\ell=1}^{L} \left(\beta_{\ell 1t}^{\pi} \pi_{\ell t+1} + \beta_{\ell 2t} (R_{\ell t+1})^2 + \beta_{\ell 2t}^{\pi} (\pi_{\ell t+1})^2 \right) \right\} \right]' \end{pmatrix}', \tag{13}$$

which stacks together the forecasting and prediction errors, is used to estimate the various parameters of the TM-IAPM.

As noted by Chapman (1997) and Cochrane (2001), the use of an optimal weighting matrix may improve efficiency at the expense of high pricing errors. In the GMM system, the parameterization of the first moment of returns dramatically increases the number of orthogonality conditions, because we need at least as many instruments as the maximum number of parameters in any equation. Since the objective function to be minimized not only contains the squared pricing errors of each equation but also the many squared cross products between the pricing errors and the (non-constant) instruments, we indeed find that the GMM system yields highly volatile pricing errors for system (13).

One way to solve this problem is to restrict the GMM objective function so as minimize solely the squared pricing errors. The resulting restricted GMM specification will be independent to the weighting matrix and is in fact equivalent to estimating an *ordinary least squares* (OLS) (e.g., Ogaki (1993)). Because of the potential contemporaneous correlations between the errors of the different equations, we further improve the efficiency of the estimation by making GLS corrections. Formally, we estimated the parameters by minimizing the following quadratic form:

$J_{GLS} = T^{-1} \sum_{t=0}^{T-1} [\varepsilon_{t+1}]' \hat{\Omega}^{-1} [\varepsilon_{t+1}]$, where the cross-equation covariance matrix Ω is estimated with Zellner's (1962) SUR effects from the first stage OLS residuals.[22]

Note that in system (13) we have not specified the functional form of the prices of risk. Because several authors have found that the change in reward-to-risk has a greater impact on returns than the change in risk (e.g., Ferson and Harvey (1991)), we consequently allow for time-varying prices of risk. We have two choices for the estimation of the prices of market covariance and coskewness: either constrain them to be concordant with theory or estimate them without restriction. We estimate (13) with and without restrictions on the prices of risk. In the unrestricted estimation, we assume that price of risk varies linearly with the default and term risk premiums: $\psi_{pt} = \mathcal{Q}_t \varphi_p$, $\beta_{Wt} = \mathcal{Q}_t \varphi_W$, $\beta^{\pi}_{\ell 1t} = \mathcal{Q}_t \varphi^{\pi}_{\ell 1}$, $\beta_{\ell 2t} = \mathcal{Q}_t \varphi_{\ell 2}$, and $\beta^{\pi}_{\ell 2t} = \mathcal{Q}_t \varphi^{\pi}_{\ell 2}$; where the φ's are weighting vectors and $\mathcal{Q}_t$ is the subset of Z_t that contains a constant, the default risk premium, and the term risk premium.[23]

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[22] Our GLS estimation can be seen as a tradeoff between the robustness of the OLS and the efficiency of the GMM; see Cochrane (2001, sections 10.2 and 11.5) for the discussion of this issue. See also Pindyck and Rubinfeld (1981, p.331-333) for a detailed discussion of SUR effects and Green (2000, p.615-616) for the efficiency gain accruing to GLS relative to OLS.

[23] Note that our specification of ψ_{pt} as a linear function of $\mathcal{Q}_t$ imposes, similar to the GMM, the restrictions that the prediction errors of the model are unpredictable from the default and term instruments. Similar results are obtained when we assume that price of risk varies also with the lagged short-term interest rate and dividend yield. See Jagannathan and Wang (1996, p.11) for an excellent discussion of the pertinence of choosing these two business cycle variables for capturing time-variation in price of risk.

We also restrict the parameters. Since restrictions generally make it more difficult to reach the global optimum, for the sake of parsimony, we further impose the constancy of the parameter in order to mitigate the potential convergence problems. We assume the following expressions for the prices of risk: $\psi_{pt} = \kappa_p$, $\beta_{Wt} = | \kappa_W |$, $\beta^{\pi}_{\ell 1t} = \kappa^{\pi}_{\ell 1}$, $\beta_{\ell 2t} = -| \kappa_{\ell 2} |$, and $\beta^{\pi}_{\ell 2t} = \kappa^{\pi}_{\ell 2}$; where the κ's are scalars.

IV. EMPIRICAL RESULTS

A. The Three-Moment IAPM with Constant Prices of Risk

Table II presents the results from the GLS estimation of the TM-IAPM with constant prices of risk. We use two approaches to estimate system (13). First, we estimate the model jointly for all three countries to end up with a system of 24 equations (a forecasting equation and a pricing equation for each of the market, size, value, and momentum premiums) and 104 parameters to be estimated. In the second approach, we estimate the system country-by-country; this entails a system of 8 equations (two equations for each premium) and 40 parameters to be estimated.[24]

For each country and across the three countries, the point estimate of the price of world market covariance is significant at the 1 percent level of statistical significance. The prices of market covariance range from 0.46 ($t = 2.28$) for the UK to 1.26 ($t = 8.39$) in the joint system. The estimated price of inflation covariance is

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[24] The country-by-country approach has both advantages and disadvantages over the joint-approach: we estimate fewer parameters and have distinct prices of risk for each country but we can not constrain the world market and currency risks to be identical across countries.

always negative and is significant in six of the eight cases. Given these results, we always reject at the 1 percent level the CAPM relative to the IAPM (the results from the formal tests are not reported in the Table). At first sight, this result seems to contrast with Dumas and Solnik (1995) and De Santis and Gerard (1997) who find that the CAPM and the IAPM can be distinguished only when prices of risk are allowed to vary over time. The difference is because our estimation accounts for conditional coskewness, which explains many of the episodes of negative premiums (e.g., Harvey and Siddique (2000b)).[25]

TABLE II ABOUT HERE

In the joint estimation, the world price of conditional market coskewness is negative and significant for the Japanese ($t =$ -5.74) and the UK ($t =$ -2.74) market factors and marginally significant for the US market factor ($t =$ -1.79). However, the estimated prices of market coskewness are significant in only 2 out of 6 cases in the country-by-country systems. As we will see later in the exposition, this may be driven by the restrictions on the prices of risk. In contrast, the point estimates of the prices of inflation coskewness are always significant at the conventional levels of statistical significance, suggesting a nonlinear international risk-return relationship. This is formally confirmed by the results presented in Panel B, which significantly rejects the marginal restrictions of the standard IAPM relative to the TM-IAPM.

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[25] Note that we obtain similar results as Dumas and Solnik (1995) and De Santis and Gerard (1997) when coskewness is not included (in the constant-price-of-risk IAPM system). These results are available from the authors upon request.

B. The Three-Moment IAPM with Time-Varying Prices of Risk

Dumas and Solnik (1995) and De Santis and Gerard (1997) emphasize the importance of time-varying prices of risk to test IAPMs. We specify the prices of risk as a function of the past realizations of the term and default risk premiums. The number of estimated parameters is 64 and 144 in the country-by-country and joint systems, respectively.

The results are reported in Table III. As expected, the null hypothesis of constant price of risk is often rejected. In 22 out of the 32 cases, the null of time-invariant prices of risk is rejected at the 5 percent level. Panels A to D of Figure 1 plot the evolution of the prices of risk estimated in the joint system. A visual look of the plots confirms the dynamic nature of prices of risk.

TABLE III & FIGURE 1 ABOUT HERE

In the joint estimation, the average price of world market covariance is and is positive (2.24; t = 22.84). Similar results obtain in the country-by-country estimations. This evidence is important since the world price of risk is not constrained to be positive. Further, as depicted in Panel A of Figure 1, the point estimates of the price of world market covariance is higher than 1 in most of the cases in the joint estimation, consistent with investor risk aversion. The average price of Japanese inflation is always negative and highly significant. in all our specifications. The price of UK inflation covariance does, however, fluctuate between positive and negative values albeit it is significantly negative on average. Without exception, the unrestricted prices of market coskewness are significantly positive.

Coupled with their high level of risk-aversion, this evidence implies that national investors are not primarily motivated by precautionary savings motive. This result is somewhat consistent with Harvey and Siddique's (2000b) finding that the price of world market skewness is often positive (in 90.50 and 45.09 percent of cases in the GMM and ML frameworks, respectively).

Panel B of the Table III supports the main contention of the paper: that conditional coskewness is important and that the IAPM, which overlooks this term, is misspecified. In other words, the functional relationship postulated by the standard IAPMs appears to be inconsistent with the data since the (omitted) coskewness terms matter.

C. Variance Decomposition

The approach developed in this study has two main advantages. First, it allows us to unambiguously test the restrictions implied by the standard IAPMs. Second, it allows us to assess the systematic sources of variation of the market, size, value, and momentum premiums.

Using (12) and the dynamic expressions of prices of risk, the variance of the observed premium can be decomposed into a cross effect, $\frac{Var[\mathcal{Q}_t\varphi_p]}{Var[r_{pt+1}]}$, and four systematic components: market covariance, $\frac{Var[\mathcal{Q}_t\varphi_W v_{pt+1}\mathrm{R}_{Wt+1}]}{Var[r_{pt+1}]}$; inflation covariance, $\frac{Var[\sum_{\ell=2}^{L}\mathcal{Q}_t\varphi_{\ell 1}^{\pi} v_{pt+1}\pi_{\ell t+1}]}{Var[r_{pt+1}]}$; market coskewness, $\frac{Var[\sum_{\ell=1}^{L}\mathcal{Q}_t\varphi_{\ell 2} v_{pt+1}(\mathrm{R}_{\ell t+1})^2]}{Var[r_{pt+1}]}$; and inflation coskewness, $\frac{Var[\sum_{\ell=2}^{L} \mathcal{Q}_t \varphi_{\ell 2}^{\pi} v_{pt+1} (\pi_{\ell t+1})^2]}{Var[r_{pt+1}]}$. The comparison of these different ratios will allow us to determine the importance of the various sources of systematic variation of international returns.

Panel A of Table IV presents the results from the decomposition of variance for the joint-system with time-varying risk and prices of risk. With only 1.43 percent of the variation of international risk premia, the cross effect, which combines the substitution effects between the various factors, has the weakest impact. Covariation with the world market factor, which is expected to be the dominant source of systematic variation of international returns, accounts on average for only 2.12 percent of the variation. The relative importance of the world market covariance risk is with the range of the R^2s estimated by De Santis and Gerard (1997) in their Table 4.[26] A more remarkable result, though, is that the world market covariance is dominated by inflation covariance during 1981-2000. This again confirms the importance of accounting for inflation risk for international assets.

The most intriguing results are observed on the coskewness components. By far, these are the most significant sources of systematic variation of returns. Market coskewness explains nearly 18 percent of the variation of the market, size, value, and

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[26] De Santis and Gerard (1997) report R^2s that varies between 1.57 percent for the German and 3.93 percent for the US markets when only the world market covariance is taken into account. The results on the world market covariance are also consistent with L'Her, Sy, and Tnani (2002), who find that the market and three other global factor betas account for about 4 percent of a large cross-section of international returns during the 1990s.

momentum premiums across the countries. Its contribution is quite large for market premiums, accounting for about 47 percent of the total variance in the case of the United States. To visualize the behavior of the risk premiums, we plot in Panels A to F of Figure 2 the various systematic components of the US market premium estimated in the joint system. As it can be seen from the comparison of the plots in Panels A and C, the coskewness terms account for most of the variation of the US market premium, especially during the episodes of market crash.

TABLE IV & FIGURE 2 ABOUT HERE

The contribution of the coskewness terms is very important. Indeed, combining both the coskewness terms, we see that these nonlinear components account on average for about 28 percent of the premiums' variations, which is more than four times larger than the 6.54 percent obtained when combining the two covariance terms. This evidence, again, demonstrates the nonlinear nature of the international risk-return relationship. Panel B of Table IV reports summary statistics and diagnostics for the residuals (the *pseudo*-R^2s and the root mean squared errors (RMSE)) obtained from the IAPM and TM-IAPM.[27] In all cases, the TM-IAPM explains more than the half of the total variation and on average yields about one-third lower pricing errors than the IAPM.

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[27] Following De Santis and Gerard (1997), we use the label "*pseudo*" to emphasize the fact that the R^2s are obtained by estimating the joint system. Further, following Ghysels's (1998) suggestion to use RMSE for comparing models, we report the RMSEs for both the IAPM and the TM-IAPM.

D. Robustness of the results

Since the expansion of the IMRS to price coskewness risk appreciably increases the dimension and flexibility of the model, there is a risk of overfitting the data in sample. If such is the case, then the model could work well in sample but ends up to perform quite poorly in real situations. Here, we investigate this issue and examine the performance of the TM-IAPM and the IAPM out-of-sample.

We begin with the division of our full sample (1981:1-2000:12) into two equal subperiods of ten years each (1981:1-1990:12 and 1991:1-2000:12). We use the first subperiod as an initial estimation window to estimate the various parameters of the TM-IAPM and IAPM systems. We then use these estimates to construct the forecasts of the various premiums in January 1991. We subsequently roll it one-month forward and update using ten-year moving windows, thus creating a time-series of forecasting errors for the various models.[28] The resulting sample covers 120 monthly observations (from 1991:1 to 2000:12).

Table 5 shows the results. As expected, the obtained pricing errors increase relative to the results obtained in sample. Still, the TM-IAPM continues to beat the IAPM even in the out-of-sample tests. For all the premiums across the countries (except the RMSE for the UK value premium in country-by-country estimation), the TM-IAPM yields lower out-of-sample pricing errors than the IAPM. The difference

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[28] We also considered a five-year rolling windows and find similar results. See Swanson and White (1997) for a discussion of the rolling-window method in econometric modeling.

in the squared forecast errors between the two models is always significant at the 5 percent level.[29] Overall, the out-of-sample evidence confirms the superiority of the TM-IAPM over the classical IAPM. We therefore conclude that our results are not driven by overfitting.

V. CONCLUSION

We expand the intertemporal marginal rate of substitution as a nonlinear function of the local market factor and aggregate the resulting model in an open international context to obtain a three-moment international asset-pricing model (TM-IAPM). The model suggests that the returns are influenced by their conditional covariance with the world market and inflation factors, as well as by their conditional coskewness with the factors. Most of the standard IAPMs are special cases of the TM-IAPM.

Using data on the world's foremost international markets, namely the United States, Japan, and the United Kingdom, we test the predictions of the TM-IAPM and check the restrictions implied by the IAPM. We also investigate whether the model can explain the time-series behavior of the size, value, and momentum premiums. The TM-IAPM seems to explain a large part of the variation of these

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[29] To test the difference of distribution between the squared forecast errors of the TM-IAPM and the IAPM, we relied on Wilcoxon's Signed Rank test (the *p*-values from this non-parametric test are obtained via Monte Carlo simulation). We also investigated the difference of distribution between the squared errors using Kruskal-Wallis's χ^2-statistic and find similar results.

premiums during 1981-2000. More importantly, we find evidence that conditional market covariance and inflation premia are not sufficient to describe expected returns because the conditional coskewness terms, which capture the nonlinearity of the risk-return relationship, are always significantly priced.

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Table I
Descriptive Statistics, 1981–2000

This table reports summary descriptive statistics for the world market and inflation factors as well as the market, size, value, and momentum premiums. The statistics presented in Panel A are the first four unconditional moments as well as the Bera-Jarque's (B-J) $\chi 2$ statistic for the test of normality. In Panel B, we present the results from the GLS-based Wald tests for time-variation in conditional mean, variance, and skewness of the various market, size, value, and momentum premiums. The test is based on the estimation of the following system:

$$\begin{aligned} E_t[r_{pt+1}] &= Z_t\gamma_{1p} + \phi_{1p} r_{pt} \\ E_t[r^2_{pt+1}] &= Z_t\gamma_{2p} + \sum_{j=1}^{2}\phi_{2pj} r^2_{pt+1-j} , \\ E_t[r^3_{pt+1}] &= Z_t\gamma_{3p} + \sum_{j=1}^{2}\phi_{3pj} r^3_{pt+1-j} \end{aligned} \quad (9)$$

where r_p is the market, size, value, or momentum premium and Z are the six instruments, which include: a constant, JAN: a January dummy variable, FED: the lagged US Federal funds rate, DIV: the lagged dividend yield of the world market index, DEF: the spread between Moody's Baa and Aaa yields, TERM: the spread between the US Treasury bonds with maturity over ten years and the and three-month US Treasury bill yields. The statistics reported are the χ^2-values along with the associated p-value [in brackets] and the Pseudo-R^2 from the mean equation. The system is estimated with GLS. Three, two, and one asterisks denote statistical significance at the 1, 5, and 10 percent levels, respectively. All the variables are expressed in US dollar. The mean and standard deviation are reported in percent per month. The sample covers 240 monthly observations (from January 1981 to December 2000).

Variable	Country	Panel A. Unconditional moments				
		Mean	Std. Deviation	Skewness	Excess Kurtosis	B-J
World		1.16***	4.14	-0.48***	1.63***	35.55***
Inflation	United States	0.29***	0.25	0.71***	1.80***	52.70***
	Japan	0.41*	3.54	0.71***	1.70***	48.84***
	United Kingdom	0.23	3.31	0.24	2.17***	49.61***

Table I–*Continued*

Variable	Country	Panel A. Unconditional moments				
		Mean	Std. Deviation	Skewness	Excess Kurtosis	B-J
Market	United States	0.78***	4.33	-0.65***	3.38***	131.28***
	Japan	0.45	5.48	0.04	2.27***	51.62***
	United Kingdom	0.73**	5.50	-0.84***	2.90***	112.43***
Size	United States	0.41	5.55	0.98***	4.21***	215.96***
	Japan	0.43	6.11	0.20	1.61***	27.69***
	United Kingdom	0.41	4.87	0.78***	2.96***	111.68***
Value	United States	0.38	3.66	0.46***	0.74**	13.84***
	Japan	1.06***	5.32	0.72***	5.40***	312.07***
	United Kingdom	0.74**	5.11	-0.14	3.21***	103.50***
Momentum	United States	0.73*	5.81	-0.29*	0.97***	12.73***
	Japan	0.28	7.61	-0.70***	4.38***	211.58***
	United Kingdom	1.09***	5.92	-0.68***	2.70***	91.77***

Variable	Country	Panel B. Conditional moments						
		Mean			Variance		Skewness	
		R^2 (%)	χ^2	p-value	χ^2	p-value	χ^2	p-value
Market	United States	4.00	9.74	[0.136]	16.01	[0.025]	13.79	[0.055]
	Japan	3.54	13.03	[0.043]	45.29	[0.000]	38.67	[0.000]
	United Kingdom	4.79	11.49	[0.074]	14.96	[0.037]	14.81	[0.039]
Size	United States	22.59	67.49	[0.000]	80.24	[0.000]	49.40	[0.000]
	Japan	5.24	13.13	[0.041]	16.74	[0.019]	11.59	[0.115]
	United Kingdom	8.78	19.31	[0.004]	19.30	[0.007]	18.96	[0.008]
Value	United States	6.52	14.12	[0.028]	12.39	[0.088]	10.31	[0.172]
	Japan	3.72	8.85	[0.182]	22.94	[0.002]	21.24	[0.003]
	United Kingdom	2.82	8.81	[0.184]	37.08	[0.000]	10.35	[0.170]
Momentum	United States	2.81	4.48	[0.613]	29.42	[0.000]	2.58	[0.921]
	Japan	2.75	9.28	[0.159]	28.92	[0.000]	17.57	[0.014]
	United Kingdom	6.46	16.01	[0.014]	10.39	[0.167]	10.83	[0.146]

Table II
Restricted GLS Estimation of the TM-IAPM, 1981–2000

The following restricted version of the TM-IAPM is estimated:

$$\varepsilon_{t+1} = (v_{pt+1} \quad e_{pt+1}) = \left(\begin{array}{c} \left[r_{pt+1} - (Z_t \delta_p + \phi_p r_{pt}) \right]' \\ \left[r_{pt+1} - \kappa_p - v_{pt+1} \left\{ |\kappa_W| \mathrm{R}_{Wt+1} + \sum_{\ell=2}^{L} \left(\kappa_{\ell 1}^{\pi} \pi_{\ell t+1} + |\kappa_{\ell 2}| (\mathrm{R}_{\ell t+1})^2 + \kappa_{\ell 2}^{\pi} (\pi_{\ell t+1})^2 \right) \right\} \right]' \end{array} \right)', \tag{13}$$

where r_p is the market, size, value, or momentum premium, R_W is the world market return, R_ℓ is the local market return, π_ℓ is the local inflation rate, and Z are our six instruments (see Table I). The system is estimated simultaneously across all the three countries (the United States, Japan, and the United Kingdom) on a joint-estimation basis or is estimated on a country-by-country basis. In Panel A, we report the GLS estimates of the parameters $|\kappa_W|$, $\kappa_{\ell 1}^{\pi}$, $|\kappa_{\ell 2}|$, and $\kappa_{\ell 2}^{\pi}$, which respectively represent the prices of market and inflation covariance and coskewness, along with the respective *t*-statistics (in parenthesis). In Panel B, we test the restrictions of the IAPM that coskewness does not matter using a Wald test. We report the χ^2-statistic along with the robust *p*-value [in brackets]. All the variables are expressed in US dollar. The sample covers 240 monthly observations (from January 1981 to December 2000).

Price of risk		Country-by-country estimations						Joint estimation	
		US		Japan		UK			
		Panel A. Prices of risk							
		Estimate	*t*-statistic	Estimate	*t*-statistic	Estimate	*t*-statistic	Estimate	*t*-statistic
Market covariance	World	1.006	(3.508)	1.189	(4.475)	0.464	(2.284)	1.257	(8.392)
Inflation covariance	Japan	-2.710	(-5.027)	-1.143	(-2.572)	-1.026	(-2.714)	-1.714	(-6.790)
	UK	-0.522	(-0.909)	-1.433	(-2.915)	-0.216	(-0.673)	-1.071	(-4.379)
Market coskewness	US	-5.464	(-1.919)	-0.275	(-0.099)	-2.635	(-1.341)	-2.697	(-1.791)
	Japan	-5.968	(-4.280)	-0.006	(-0.007)	-3.185	(-2.882)	-3.097	(-5.741)
	UK	-0.300	(-0.095)	-0.042	(-0.019)	-2.751	(-1.468)	-3.667	(-2.740)
Inflation coskewness	Japan	22.078	(4.034)	20.495	(3.100)	10.637	(2.605)	20.317	(6.333)
	UK	47.797	(4.761)	36.775	(4.314)	9.334	(2.429)	42.726	(12.167)
		Panel B. Test of the IAPM							
		χ^2 statistic	*p*-value	χ^2 statistic	*p*-value	χ^2 statistic	*p*-value	χ^2 statistic	*p*-value
Wald test		58.10	[0.000]	46.58	[0.000]	32.08	[0.000]	235.25	[0.000]

Table III
Unrestricted GLS Estimation of the TM-IAPM, 1981–2000

The hypothesis that conditional covariance and coskewness explain the market, size, value, and momentum premiums observed in international markets is tested by estimating the following unrestricted version of the TM-IAPM:

$$\varepsilon_{t+1} = (v_{pt+1} \quad e_{pt+1}) = \begin{pmatrix} \left[r_{pt+1} - (Z_t\delta_p + \phi_p r_{pt})\right]' \\ \left[r_{pt+1} - \mathcal{Q}_t\varphi_p - v_{pt+1}\left\{\mathcal{Q}_t\varphi_W \mathrm{R}_{Wt+1} + \sum_{\ell=2}^{L}\left(\mathcal{Q}_t\varphi_{\ell 1}^{\pi}\pi_{\ell t+1} + \mathcal{Q}_t\varphi_{\ell 2}(\mathrm{R}_{\ell t+1})^2 + \mathcal{Q}_t\varphi_{\ell 2}^{\pi}(\pi_{\ell t+1})^2\right)\right\}\right]' \end{pmatrix}', \tag{13}$$

using GLS, where r_p is the market, size, value, or momentum premium, R_W is the world market return, R_ℓ is the local market return, π_ℓ is the local inflation rate, and Z are our six instruments (see Table I), of which the subset $\mathcal{Q}$ contains a constant, the default and term risk premiums. The system is estimated simultaneously across all three countries (the United States, Japan, and the United Kingdom) on a joint-estimation basis or is estimated on a country-by-country basis. For each price of risk, we report the mean value with the *t*-statistic related to the null hypothesis that the mean value is equal to zero (in parentheses below), the percentage of negative values of the prices of risk, and the χ^2-statistic from the Wald test of the time variation in the prices of risk with the robust *p*-value [in brackets below]. In Panel B, we test the restrictions of the IAPM that coskewness does not matter using a Wald test. We report the χ^2-statistic along with the robust *p*-value. All the variables are expressed in US dollar. The sample covers 240 monthly observations (from January 1981 to December 2000).

Price of risk	Country-by-country estimations									Joint estimation		
	US			Japan			UK					
	Mean	Negative %	χ^2	Mean	Negative %	χ^2	Mean	Negative %	χ^2	Mean	Negative %	χ^2
	Panel A. Price of risk											
	Price of covariance											
World	1.989	13.75	19.180	2.719	3.75	7.940	1.871	4.58	9.350	2.244	5.00	33.870
	(13.038)		[0.000]	(25.781)		[0.019]	(21.147)		[0.009]	(22.841)		[0.000]
Japanese inflation	-5.101	100.00	0.070	-4.089	100.00	4.270	-4.948	100.00	7.100	-4.491	100.00	7.930
	(-411.593)		[0.968]	(-43.014)		[0.118]	(-40.642)		[0.029]	(-66.177)		[0.019]
UK inflation	-2.652	93.75	8.920	-2.734	100.00	5.290	0.553	30.83	3.300	-1.804	92.92	14.860
	(-20.253)		[0.012]	(-19.951)		[0.071]	(6.613)		[0.192]	(-19.769)		[0.001]

Table III–*Continued*

	Country-by-country estimations									Joint estimation		
Price of risk	US			Japan			UK					
	Mean	Negative %	χ^2	Mean	Negative %	χ^2	Mean	Negative %	χ^2	Mean	Negative %	χ^2
	Price of coskewness											
US market	32.699 (19.536)	9.58	20.800 [0.000]	36.878 (18.688)	10.00	31.910 [0.000]	23.979 (20.025)	8.75	10.650 [0.005]	29.659 (22.286)	5.00	58.120 [0.000]
Japanese market	12.590 (11.653)	17.08	23.670 [0.000]	9.888 (12.256)	16.25	26.030 [0.000]	11.703 (35.056)	3.33	2.540 [0.281]	12.373 (16.418)	13.33	62.920 [0.000]
UK market	18.973 (43.920)	1.67	1.710 [0.426]	17.594 (52.092)	0.00	1.700 [0.428]	18.028 (60.539)	0.00	0.780 [0.677]	18.625 (119.208)	0.00	1.660 [0.436]
Japanese inflation	82.411 (34.033)	0.00	12.240 [0.002]	82.690 (28.569)	0.42	16.210 [0.000]	89.772 (33.811)	1.67	18.670 [0.000]	84.373 (30.047)	0.00	72.830 [0.000]
UK inflation	102.308 (60.141)	0.00	4.340 [0.114]	85.830 (39.129)	0.83	9.180 [0.010]	77.780 (37.481)	0.83	12.780 [0.002]	85.161 (40.927)	0.00	47.160 [0.000]
	Panel B. Test of the IAPM											
IAPM			890.91 [0.000]			856.70 [0.000]			774.45 [0.000]			2951.90 [0.000]

Table IV
Variance Decomposition of the Market, Size, Value, and Momentum Premiums, 1981–2000

This table uses the TM-IAPM system to decompose the variance of market, size, value, and momentum premiums into a cross effect and four systematic components (market covariance, inflation covariance, market coskewness, and inflation coskewness) and reports (in Panel A) their relative variance (in proportion of the premium's total variance). The last column reports the mean values across the 12 premiums while the last row of Panel A reports the sum of the five components. Panel B reports summary statistics and diagnostics for the residuals (the pseudo-R^2s and the root mean squared errors (RMSE)) obtained from the IAPM and TM-IAPM for each premium equation. All the numbers are reported in percent. The method used to decompose the variance is described in Section III.C. All estimates are obtained from the joint system with time-varying risk and prices of risk. The sample covers 240 monthly observations (from January 1981 to December 2000).

Component	Market premium			Size premium			Value premium			Momentum premium			*Mean*
	US	Japan	UK	US	Japan	UK	US	Japan	UK	US	Japan	UK	
							Panel A. Variance decomposition						
Cross effect	0.09	2.39	0.24	2.97	1.70	1.84	3.53	1.52	0.04	1.60	0.29	0.92	*1.43*
World covariance	6.00	1.97	5.19	1.18	0.88	1.48	1.04	2.10	1.90	1.58	1.00	1.19	*2.12*
Inflation covariance	6.00	4.26	7.70	3.22	3.01	5.13	3.83	4.16	5.07	3.35	4.06	3.25	*4.42*
Market coskewness	47.41	31.25	36.18	7.88	7.33	13.99	10.06	15.52	11.58	12.84	10.82	9.83	*17.89*
Inflation coskewness	10.81	8.81	21.56	6.24	6.64	14.34	11.38	9.39	8.66	6.80	8.48	7.69	*10.07*
Sum	*70.31*	*48.68*	*70.87*	*21.49*	*19.56*	*36.78*	*29.84*	*32.69*	*27.26*	*26.17*	*24.65*	*22.88*	*35.93*
							Panel B. Goodness of fit						
R^2 (IAPM)	5.09	5.95	4.54	3.22	6.33	6.26	5.87	8.54	6.80	5.66	3.83	6.38	*5.71*
R^2 (TM-IAPM)	69.41	66.36	69.52	51.32	51.91	56.86	51.83	55.28	52.49	53.86	60.89	50.56	*57.52*
RMSE (IAPM)	4.27	5.39	5.45	5.53	5.99	4.78	3.59	5.16	5.00	5.72	7.57	5.81	5.36
RMSE (TM-IAPM)	2.43	3.23	3.09	3.93	4.31	3.25	2.58	3.62	3.58	4.01	4.84	4.23	3.59

Table V
Out-of-Sample Evidence, 1991–2000

This table provides the out-of-sample evidence on the performance of the IAPM and TM-IAPM. We first estimate system (13) and its IAPM version in the January 1981-December 1990 window to obtain the various parameters. We use these estimated parameters in January 1991 to obtain the forecast of the various premiums in that month. We subsequently roll it one month forward and repeat, thus creating a time series of predicted values of the premiums as well as the forecasting errors. The sample covers 120 monthly observations (from January 1991 to December 2000). For each premium, we report the root mean squared forecasting errors (RMSE), the root median squared forecasting errors (RMeSE), and the (two-sided) *p*-values associated with Wilcoxon's test of difference of the squared forecasting errors obtained from the IAPM and the TM-IAPM. Panel A reports the results obtained from the joint estimation while Panel B reports those obtained from the country-by-country estimations. With the exception of the *p*-values, all the figures are reported in percent per month.

Goodness-of-fit	Market premium			Size premium			Value premium			Momentum premium			Mean
	US	Japan	UK	US	Japan	UK	US	Japan	UK	US	Japan	UK	
							Panel A. Joint estimation						
RMSE (IAPM)	4.11	5.80	5.29	7.08	6.31	5.14	4.16	4.94	5.10	7.02	8.09	6.52	5.80
RMSE (TM-IAPM)	3.63	4.39	4.33	6.13	5.26	4.40	3.53	4.03	4.72	5.69	7.20	4.95	4.85
RMeSE (IAPM)	2.35	3.39	2.60	3.47	3.41	2.57	2.14	2.20	2.98	3.47	3.19	3.30	2.92
RMSeE (TM-IAPM)	1.49	2.18	1.83	2.57	2.54	1.70	1.43	1.44	1.73	2.28	1.96	2.27	1.95
Wilcoxon's *p*-value	0.000	0.018	0.002	0.020	0.020	0.002	0.002	0.002	0.002	0.010	0.002	0.012	NA
							Panel B. Country-by-country estimations						
RMSE (IAPM)	4.01	5.84	5.22	7.08	6.45	5.13	4.15	5.06	5.14	7.22	8.31	6.50	5.84
RMSE (TM-IAPM)	4.81	4.77	5.20	6.21	5.42	4.84	3.79	4.10	5.29	5.95	7.17	5.53	5.26
RMeSE (IAPM)	2.52	3.56	2.52	3.38	3.43	2.52	2.20	2.12	2.81	3.42	3.25	3.37	2.93
RMeSE (TM-IAPM)	1.56	2.33	1.90	2.34	2.78	1.65	1.37	1.58	2.20	2.22	2.08	2.34	2.03
Wilcoxon's *p*-value	0.000	0.018	0.000	0.020	0.056	0.000	0.002	0.012	0.018	0.014	0.024	0.038	NA

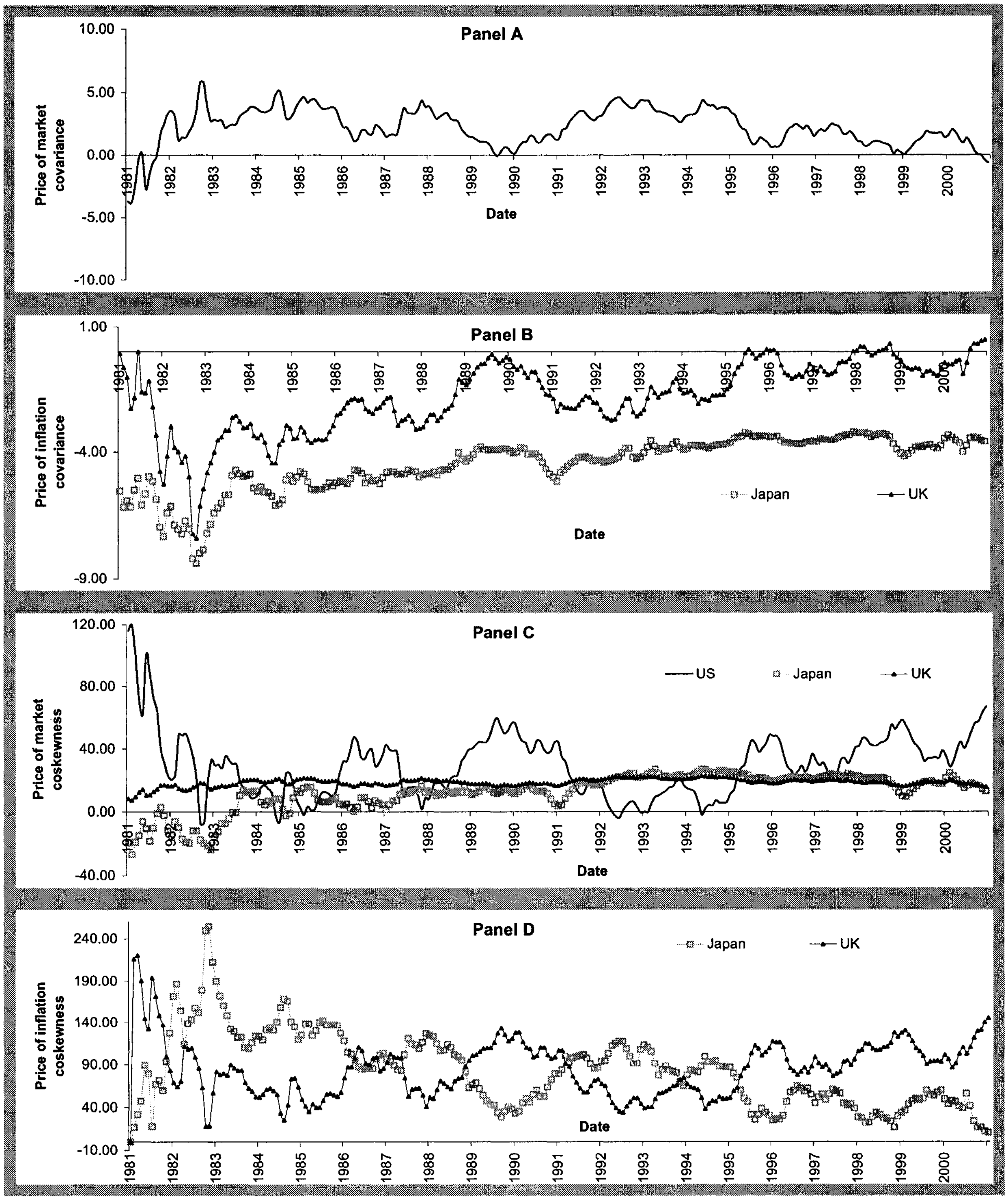
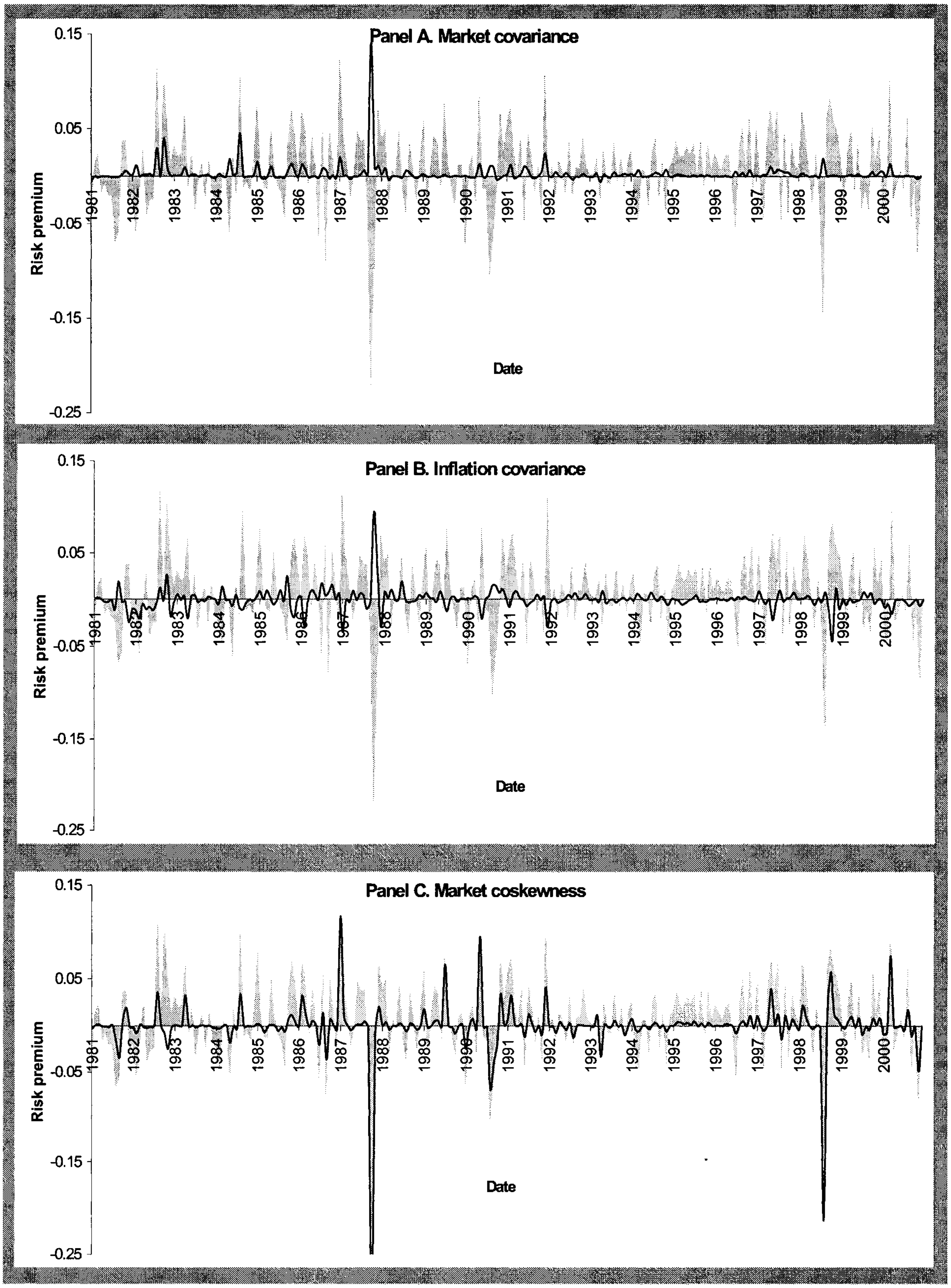


Figure 1. Time variation of the prices of risk, 1981–2000.

Panels A to D plot the time series of the estimated prices of market covariance, inflation covariance, market coskewness, and inflation coskewness, respectively. These are estimated as linear functions of the lagged default and term premiums. The estimates are obtained from the joint-system. The sample covers 240 monthly observations (from January 1981 to December 2000).


Panel A. Market covariance
Risk premium
0.15
0.05
-0.05
-0.15
-0.25
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
Date
Panel B. Inflation covariance
Risk premium
0.15
0.05
-0.05
-0.15
-0.25
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
Date
Panel C. Market coskewness
Risk premium
0.15
0.05
-0.05
-0.15
-0.25
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
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1999
2000
Date

Figure 2–*Continued*

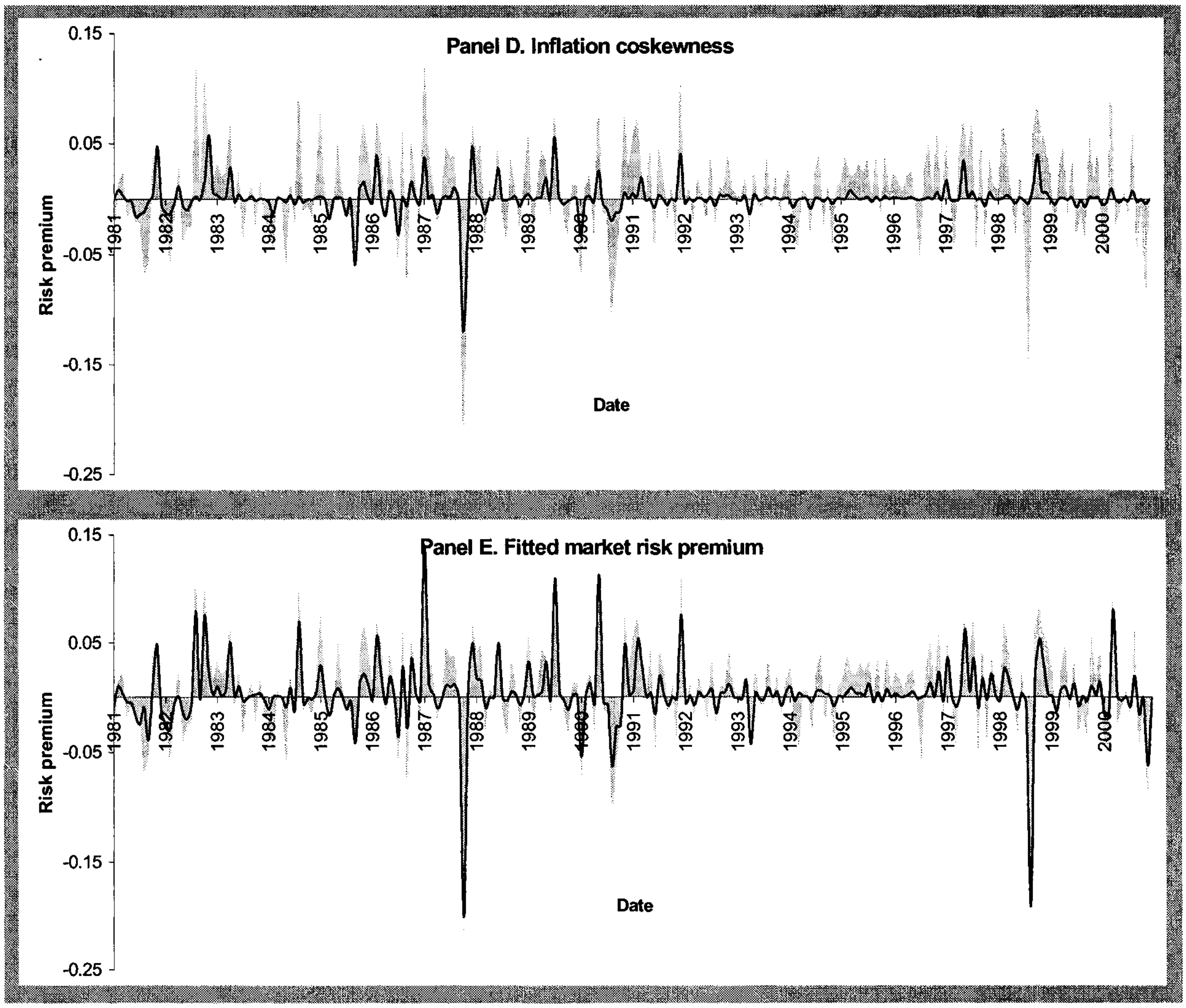

Figure 2. Decomposition of the US market premium, 1981–2000.

Panels A to E plot the time series of the systematic components of the US market premium. The systematic components plotted are respectively: market covariance, $Q_t\varphi_{W}v_{pt+1}R_{Wt+1}$; inflation covariance, $\sum_{\ell=2}^{3} Q_t\varphi^{\pi}_{\ell 1}v_{pt+1}\pi_{\ell t+1}$; market coskewness, $\sum_{\ell=1}^{3} Q_t\varphi_{\ell 2}v_{pt+1}(R_{\ell t+1})^2$; and inflation coskewness, $\sum_{\ell=2}^{3} Q_t\varphi^{\pi}_{\ell 2}v_{pt+1}(\pi_{\ell t+1})^2$. The estimates are obtained from the joint-system. The sample covers 240 monthly observations (from January 1981 to December 2000). The observed market risk premium is plotted in shadowed area.

CHAPTER III:

TEST OF MARKET EFFICIENCY: A NEW APPROACH

> *"An efficient capital market is a market that is efficient in processing information. The prices of securities observed at any time are based on 'correct' evaluation of all information available at that time. In an efficient market, prices 'fully reflect' available information."* (Fama (1976, p.133))

I. INTRODUCTION

Efficient capital markets are essential for a well-functioning economic system. In an efficient market, prices reflect intrinsic value; economic agents with informational advantage can not systematically make abnormal profits; and the existing resources will be efficiently allocated. The efficient market hypothesis (EMH henceforth) is one of the most important issues in finance and economics.[1] Much of the theory on the EMH can be traced back to Bachelier (1900), but the modern approach really started with Samuelson (1965) and received a dramatic impetus with Fama (1965). Unfortunately, there is no consensus on the empirical validity of the EMH despite a very large number of studies. While the early studies (mostly in the 1970s) supported market efficiency, the recent evidence (mostly in the 1980s and 1990s) has uncovered numerous anomalies that seem to contradict the EMH.[2]

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[1] See Fama (1970, 1991), Leroy (1989), and Dimson and Mussavian (1998) for excellent reviews of the literature.

[2] An anomaly refers to empirical patterns in the data that are not predicted by extant theories. Several financial economists attribute the term anomaly to Kuhn (1970). The most

Explanations for the anomalies can be broadly classified into two groups.[3] First, the anomalies emerge because the equilibrium models used to compute abnormal returns are inappropriate. As Fama and French (FF, 1993, 1996) argue, most of the anomalies can be explained by exposure to risk missed by capital asset-pricing model (CAPM) of Sharpe (1964) and Lintner (1965).[4] The second explanation suggests that the anomalies appear merely because markets are inefficient. Using a methodology that controls for firm characteristics across portfolios of different risk levels, Daniel and Titman (1997) find that once the effects of firm characteristics are controlled, expected returns are not positively related to risk exposure. They consequently dismiss the role of risk to explain market anomalies and attribute the results to mispricing. Proponents of this explanation also include La Porta, Lakonishok, Shleifer, and Vishny (1997),

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significant anomalies documented by the empirical literature include the January effect (Rozeff and Kinney (1976)); the closed-end funds effect (Malkiel (1977)); the weekend effect (French (1980)); the size effect (Banz (1981)); the earnings-to-price effect (Basu (1983)); the book-to-market effect (Rosenberg, Reid, and Lanstein (1985)); the contrarian effect (De Bondt and Thaler (1985)); the Value-line effect (Stickel (1985)); the turn of the month effect (Ariel (1987)); the holiday effect (Lakonishok and Smidt (1988)); the leverage effect (Bhandari (1988)); the weather effect (Saunders (1993)); and the momentum effect (Jegadeesh and Titman (1993)).

[3] See Fama (1998) for a discussion of the explanations based on behavioral finance and Davis, Fama, and French (2000) for discussions of the issues related to data and sample selection.

[4] The Fama and French's stance is consistent with Ball's (1978) hypothesis that some anomalies are due to omitted variables or other specification errors in implementing the CAPM. Also consistent with this view is Berk's (1995) assertion that some firm characteristics are related to the portion of the cross-section of expected returns left unexplained by an incorrectly specified asset-pricing model. Other recent examples are given by Jagannathan and Wang (1996) and Lettau and Ludvigson (2001), who argue that taking into account the effects of time-varying investment opportunities in the calculation of risk could account for the anomalies.

who argue that the value effect is inconsistent with a risk-based explanation, but is rather a manifestation of expectational errors made by investors.[5]

All existing tests are affected by the joint hypothesis problem. Specifically, the tests can fail either because the EMH or the equilibrium model is false or because both are false.[6] Hence, the main objective of this study is to disentangle the test of the EMH from the test of the equilibrium model. To attain this goal, we rely on Sharpe ratios (Sharpe (1994)) to derive theoretical restrictions on equilibrium prices that are independent of securities' characteristics when the postulated equilibrium model holds. Capitalizing on this property, we derive a test of market efficiency that does not critically depend on the validity of the equilibrium model.

In Section II, we illustrate the shortcomings of the traditional test of market efficiency. In particular, we show that depending on its level of variability and correlation with the market factor, a portfolio can spuriously yield high positive abnormal returns when it actually under-performs the market. The main purpose is to

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[5] Ferson, Sarkissian, and Simin (1999) show that attribute-sorted portfolios can appear to be useful risk factors even when the attributes are constructed to be completely unrelated to risk. For other examples of mispricing, see Heston, Rouwenhorst, and Wessels (1999) for the size characteristic for European countries, Brennan, Chordia, and Subrahmanyam (1998) for the evidence on security returns, and Daniel, Titman, and Wei (2001) for the evidence in Japan. See also Lakonishok, Shleifer, Vishny (1994), Haugen (1995), and Daniel, Hirshleifer, and Subrahmanyam (1998) for examples of the role of biases in investor decision-making.

[6] According to Fama (1976, p.137): "*This is the rub in tests of market efficiency. Any test is simultaneously a test of efficiency and of assumptions about the characteristics of market equilibrium. [...] If the tests are unsuccessful, we face the problem of deciding whether this reflects a true violation of market efficiency (the simple proposition that prices fully reflect available information) or poor assumptions about the nature of market equilibrium.*"

demonstrate that the traditional test of market efficiency can produce incoherent results when the equilibrium model used is inadequate. This phenomenon is illustrated with size-ranked portfolios for 1927-2000.

In Section III, we introduce the theoretical setting to resolve the joint hypothesis problem. We derive the restrictions implied by asset-pricing models on Sharpe ratios, which are critical to disentangle the test of the EMH from that of the equilibrium model. We demonstrate that the Sharpe ratio of any portfolio can be reproduced by a precise combination of the Sharpe ratios of the factors when the postulated equilibrium model holds. We exploit this unique framework to construct separate tests for the EMH and the equilibrium model in Section IV.

To illustrate the validity of our approach for all current and future anomalies, we next examine the efficiency of the United States, Japan, and the United Kingdom over the period 1981-2000. In Section V, we present the data and examine the robustness of the reported anomalies for the United States, Japan, and the United Kingdom. Most of the stylized facts reported in the literature are robust in the three countries during our sample period.

Next, we implement our new test to examine the EMH for the United States, Japan, and the United Kingdom. We test the efficiency of these markets vis-à-vis the most important anomalies reported in the literature; that is, the size, book-to-market

(BEME), and momentum effects.[7] We use two asset-pricing models: the CAPM and the four-factor pricing model (FFPM), which is the Fama-French three-factor pricing model (TFPM) augmented with the momentum factor (e.g., Carhart (1997)).[8] In Section VI, we present the results from the formal tests of the EMH and the equilibrium model. We find that the three anomalies investigated in this study are potentially driven by the failure of the equilibrium models postulated in the empirical tests, which suggests that the rejection of the EMH may be premature.

In Section VII, we investigate the implications of our results for performing event studies and measuring performance. In particular, we derive a corrected measure of cumulative abnormal returns. This measure is designed to provide a more accurate picture of the true situation, once the postulated equilibrium model fails. We illustrate the misspecification of the traditional approach using investment strategies based on size, value, and momentum. We finally conclude in Section VIII by summarizing the evidence.

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[7] The size effect is the propensity of small firms to out-perform large firms, the value effect is the tendency of stocks with a larger BEME ratio (value stocks) to out-perform stocks with a low BEME ratio (growth stocks), while the momentum effect is the tendency of winners (stocks that performed well in the past) to out-perform losers (stocks that performed poorly in the past) over the medium-term.

[8] Note that the approach proposed is not restricted to the use of the CAPM or the FFPM. These models are chosen because of their standing in the current literature (e.g., Graham and Harvey (2001)).

II. THE FALLACY OF THE TRADITIONAL TEST

This section illustrates the problem of the traditional tests of market efficiency. Most of the existing tests of the EMH are based on the following model:

$$r_{pt} = \alpha_p + \beta_{pm} r_{mt} + \varepsilon_{pt}, \tag{1}$$

where r_{pt} ($p = 1 \ldots N$) and r_{mt} are portfolio and market excess returns, β_{pm} is the portfolio beta, α_p is the abnormal return (usually known as Jensen's (1968) alpha), and ε_{pt} is the error term of the model. Because the CAPM does not predict an abnormal return, researchers have attempted to test market efficiency by *directly* examining whether the intercept from equation (1) is zero (e.g., Black, Jensen, and Scholes (1972) and Gibbons, Ross, and Shanken (GRS, 1989)).

This traditional test is subject to a serious drawback. To highlight the problem, divide both sides of equation (1) by the portfolio standard deviation (σ_p) and obtain,

$$s_{pt} = \tau_p + \rho_{pm} s_{mt} + u_{pt}, \tag{2}$$

where $s_{pt} = r_{pt} / \sigma_p$ and $s_{mt} = r_{mt} / \sigma_m$ are respectively the standardized excess returns of the portfolio and the market, $\rho_{pm} = \beta_p \sigma_m / \sigma_p$ is the correlation coefficient between the portfolio and the market, and $\tau_p = \alpha_p / \sigma_p$ is the portfolio abnormal performance. Since equation (2) is simply a direct standardization of (1), it follows that testing market efficiency using $\alpha_p = 0$ from (1) is equivalent to testing $\tau_p = 0$ from (2).

Combining (1) and (2), we obtain the following expression for the portfolio abnormal return:

$$\alpha_p = \sigma_p \left(E[s_{pt}] - \rho_{pm} E[s_{mt}] \right). \tag{3}$$

From equation (3), it is apparent that the traditional test can spuriously deliver an abnormal return even when the portfolio under-performs the market; i.e., $E[s_{pt}] - \rho_{pm} E[s_{mt}] > 0$ whilst $E[s_{pt}] < E[s_{mt}]$. That is, when the following inequalities hold: $\rho_{pm} E[s_{mt}] < E[s_{pt}] < E[s_{mt}]$.

We illustrate the problem by considering ten size-ranked decile-portfolios for 1927-2000.[9] Table I shows some descriptive statistics on the ten portfolios. The first column shows the mean excess returns of each portfolio. The average return is higher for the small portfolio and monotonously decreases with portfolio size, to the extent that the big portfolio exhibits the lowest average return.

TABLE I ABOUT HERE

The next column shows the standard deviations of the various portfolios. The standard deviation systematically decreases with size-decile. Since the higher volatility of small firms could be the explanation of the size effect, the issue is not resolved and further analysis is thus needed. Because the CAPM requires that all portfolios in the

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[9] The data, for the United States, are graciously provided by Ken French.

mean-variance efficient frontier yield the same Sharpe ratio (that of the market), we show in the third column of Table I the Sharpe ratios of the different size-ranked decile portfolios. The smaller-size portfolios produced lower Sharpe ratios during the 74-year period. Although the low Sharpe ratio observed on the small portfolio suggests an inferior performance relative to the market, it does not necessarily imply that this portfolio delivers a negative abnormal return based on the traditional test.

To illustrate the potential contradiction, we report in the last two columns of Table I the abnormal return of each portfolio along with the average relative portfolio performance, which is the difference of Sharpe ratios between the portfolio and the market. Interestingly, the small portfolio manages a positive abnormal return while it actually under-performs the market. Consistent with equation (3), this result is the consequence of the low amplitude of ρ_{pm} and the high level of σ_p for the small portfolio. As we show below, low amplitude of ρ_{pm} is an indication of the failure of the CAPM, so that the benchmark used to compute abnormal returns is incorrect. The parameter σ_p works as an amplifier for the bias when raw returns (rather than standardized returns) are used.[10]

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[10] Appendix D shows that the bias resulting from the misspecification of the traditional approach when the CAPM fails is about $(1-\rho_{pm})(\sigma_p/\sigma_m)E[s_m]$.

III. THEORETICAL FOUNDATION

To establish the framework for EMH test, we first present the restrictions implied by asset-pricing models on Sharpe ratios. The following multifactor model embeds most of the standard models: [11]

$$E[r_{pt} \mid \Phi] = \sum_{k=1}^{K} \beta_{pk} E[r_{kt} \mid \Phi], \tag{4}$$

where $E[. \mid \Phi]$ is the expectations operator conditional on the available information set Φ, r_{kt} is the excess return on factor k ($k = 1...K$) at time t, and β_{pk} is the loading on factor k. For convenience, and without loss of generality, we assume that the factors are uncorrelated with each other.

Rational expectations imply that realized returns depart from the expected returns only by a disturbance term, so that from (4) we obtain,

$$\varepsilon_{pt} = r_{pt} - \sum_{k=1}^{K} \beta_{pk} r_{kt}, \tag{5}$$

with the regularity conditions $E[\varepsilon_{pt} \mid \Phi] = E[\varepsilon_{pt} r_{kt} \mid \Phi] = 0$. With this structure, the portfolio variance can be written as,

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[11] The model could be seen as either the intertemporal CAPM (ICAPM) of Merton (1973) or the asset-pricing theory (APT) of Ross (1976).

$$\sigma_p^2 = \sum_{k=1}^{K} \beta_{pk}^2 \sigma_k^2 + \sigma_\varepsilon^2, \qquad (6)$$

where $\sigma_p^2 = Var[r_{pt} \mid \Phi]$, $\sigma_k^2 = Var[r_{kt} \mid \Phi]$, and $\sigma_\varepsilon^2 = Var[\varepsilon_{pt} \mid \Phi]$; and $Var[. \mid \Phi]$ stands for the conditional variance operator. Equation (6) decomposes the portfolio conditional variance into two terms: the variation explained by the model—often referred to as the systematic risk—and the unexplained variance, σ_ε^2, usually known as the unsystematic risk.[12] With this setting, we can explicitly derive the restrictions implied by the multifactor model on the Sharpe ratio of portfolio p.

PROPOSITION 1: *If the equilibrium model holds, then the Sharpe ratio of any portfolio* p *is equal to a weighted sum of the Sharpe ratios of the factors, where the weights are equal to the correlation coefficients between the portfolio and the factors and are located in a K-hypersphere of center zero and radius* $\sqrt{\delta_p}$.

Proof: Because the factors are uncorrelated with each other, the betas can be written as: $\beta_{pk} = \sigma_{pk} / \sigma_k^2$; where $\sigma_{pk} = Cov[r_{pt}, r_{kt} \mid \Phi]$ stands for the conditional

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[12] The unsystematic risk can be driven by firm-specific events that can be eliminated through diversification. More importantly, the unsystematic risk can be driven by missing factors that affect all firms, and hence will not vanish as the number of securities in the portfolio gets reasonably large. Appendix E illustrates the importance of using diversified portfolios while testing for market efficiency.

covariance between the portfolio and factor k. Dividing both sides of equation (4) by σ_p yields the following restriction on the Sharpe ratio of the portfolio:

$$E[s_{pt} \mid \Phi] = \sum_{k=1}^{K} \rho_{pk} E[s_{kt} \mid \Phi], \tag{7}$$

where $s_{kt} = r_{kt} / \sigma_k$ is the standardized excess return of the factor k and $\rho_{pk} = \sigma_{pk} / \sigma_p \sigma_k$ is the correlation coefficient between portfolio p and factor k.

To complete the proof, we need to identify the restrictions implied by the equilibrium model on the correlation coefficients (the weights). Using (6), the square of ρ_{pk} can be written as, [13]

$$\rho_{pk}^2 = \delta_p \varpi_{pk}. \tag{8}$$

Thus, the squared correlation coefficient comprises of a product of two important terms. The first term, which measures the contribution of the factors to the total variation of the portfolio (the goodness-of-fit of the model), is defined as, [14]

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[13] Since by definition we have $\rho_{pk} = \beta_{pk}\sigma_k / \sigma_p$, it follows that $\rho_{pk}^2 = \beta_{pk}^2 \sigma_k^2 / \sigma_p^2$. Replacing in the latter expression σ_p^2 by its value in equation (6) and solving yields equation (8).

[14] The term δ_p could be interpreted as the coefficient of determination of the regression of the portfolio excess return on the factors (equation (2)). As a particular case, for the CAPM, this term reduces to the squared correlation coefficient between the portfolio and the market factor. See Fama (1976, p.67) for an alternative demonstration based on return distribution.

$$\delta_p = 1 - \frac{\sigma_\varepsilon^2}{\sigma_p^2}. \tag{9}$$

while the second term on the right side of equation (8), ϖ_{pk}, can be seen as a measure of the contribution of each factor to the systematic variation of the portfolio, [15]

$$\varpi_{pk} = \frac{\beta_{pk}^2 \sigma_k^2}{\sum_{k=1}^{K} \beta_{pk}^2 \sigma_k^2} = \frac{\beta_{pk}^2 \sigma_k^2}{\sigma_p^2 - \sigma_\varepsilon^2}. \tag{10}$$

Using equations (8) and (10), we obtain the restrictions implied by the equilibrium model on the correlation coefficients between the portfolio and the factors:

$$\sum_{k=1}^{K} \rho_{pk}^2 = \delta_p . \tag{11}$$

This is the equation of a *K*-hypersphere of center zero and radius $\sqrt{\delta_p}$, and the proof is complete. Q.E.D.■

Proposition 1 demonstrates that the performance of any portfolio p can be *fully* reproduced by a combination of the factors. Interestingly, the way in which the factors are combined (equation (11)) is independent of the portfolio locus on the efficient

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[15] It is important to note that when factor k does not contribute to the portfolio systematic risk, then its contribution to the portfolio performance is trivial $(\varpi_{pk} = 0)$. At the other extreme, $\varpi_{pk} = 1$ when factor k is the only source of systematic variation of the portfolio.

frontier if the equilibrium model holds. Indeed, from equation (9), we observe that the goodness-of-fit parameter will be different from one ($\delta_p \neq 1$) if the factors do not *fully* explain the (well-diversified) portfolio variation ($\sigma_\varepsilon^2 \neq 0$), *and the equilibrium model is consequently rejected.* In contrast, if the factors are the driving forces behind the portfolio volatility, then the goodness-of-fit parameter will converge to one ($\delta_p = 1$) and the postulated equilibrium model will hold. This important observation is formalized by the following proposition.

PROPOSITION 2*: When portfolio* p *is well diversified, then the postulated equilibrium model is rejected when the null hypothesis* $H_0^{APM} : \sum_{k=1}^{K} \rho_{pk}^2 = 1$ *is rejected in favor of the alternative hypothesis* H_A^{APM} : $\sum_{k=1}^{K} \rho_{pk}^2 < 1$.

Proof: The proof follows immediately from the combination of equations (9) and (11), and the hypothesis that the portfolio is well diversified. Q.E.D.■

Proposition 2 is the basis of the solution we propose in this study to solve the joint-hypothesis problem. Furthermore, the vast majority of existing empirical studies on the EMH are performed while assuming the validity of the CAPM. Therefore, it is important to understand the boundaries implied by the CAPM on Sharpe ratios.

COROLARRY 1*: If the CAPM holds, then it will be impossible to beat the market.*

Proof: Under CAPM, only the market factor matters ($\beta_{pk} = \varpi_{pk} = 0 \; \forall k \neq m$). Replacing into (10) yields $\varpi_{pm} = 1$ so that, from (8) and (9), we have $\rho_{pm}^2 = \delta_p = 1 - \sigma_\varepsilon^2 / \sigma_p^2$. The latter identity implies that we indeed have $0 \leq \rho_{pm}^2 \leq 1$ (since it can be easily verified that $0 \leq \sigma_\varepsilon^2 \leq \sigma_p^2$ from equation (6)). Combining this inequality and equation (7) we obtain $E[s_{pt} \mid \Phi] \leq E[s_{mt} \mid \Phi]$. Thus, the Sharpe ratio of any portfolio p cannot be higher than the Sharpe ratio of the market (under the CAPM), and the proof is complete. Q.E.D.■

Corollary 1 will be important latter in the exposition.

IV. TESTABLE HYPOTHESES

In an efficient market, prices fully reflect all available information. If we define by Φ^m the set of information used by the market to determine security prices (Φ^m being a subset of Φ), then the EMH can formally be stated as (Fama (1976, p.134)):

$$\Phi^m = \Phi . \tag{12}$$

This identity means that the information used by the market to determine security prices includes all the relevant information. Using Proposition 1, the EMH (equation (12)) implies the following theoretical restriction on the portfolio residual performance:

$$E[e_{pt} \mid \Phi^m] = 0 , \tag{13}$$

where $e_{pt} = s_{pt} - \sum_{k=1}^{K} \rho_{pk} s_{kt}$ is the portfolio residual performance at time t, which measures the discrepancy between the actual performance and the equilibrium performance. The EMH implies that there is no way to profit from available information. The test of market efficiency thus consists in examining the behavior of e_{pt}.

If a market is inefficient, then there will be some outstanding information (from $\Phi^c \equiv \Phi - \Phi^m$, the complement of Φ^m in Φ) that can be used to generate abnormal performances. In other words, one can effectively use information from Φ^c to build a trading rule (represented by portfolio p) that achieves a non-trivial abnormal performance $\tau_p = E[e_{pt} \mid \Phi] \neq 0$. The hypothesis can be directly tested from the following regression model:

$$s_{pt} = \tau_p + \sum_{k=1}^{K} \rho_{pk} s_{kt} + u_{pt}, \tag{14}$$

where $E[u_{pt} \mid \Phi] = E[e_{pt} - \tau_p \mid \Phi] = 0$. The idea is summarized by the following proposition.

PROPOSITION 3: *If the equilibrium model holds, then the efficient market hypothesis is formally rejected when the null hypothesis* H_0^{EMH}: $\tau_p = 0$ *is rejected in favor of the alternative hypothesis* H_A^{EMH}: $\tau_p \neq 0$.

Proof: The proof follows directly from the previous discussion. Indeed, the rejection of the null hypothesis implies an abnormal performance, which is inconsistent with the EMH. Q.E.D.■

The critical advantage of our approach is that the testing takes into account the effect of the postulated equilibrium model (see Proposition 2). Hence, a direct falsification test for the EMH is to find an investment strategy that yields a significant abnormal performance ($\tau_p \neq 0$) while the equilibrium model used is not flawed ($\sum_k \rho_{pk}^2 = 1$). Due to the scale-independence property underlying the Sharpe ratios, we can easily identify the theoretical restrictions on the factor loadings (the ρ's) in our approach. The amplitude of the factor loadings is directly related to the goodness-of-fit of the model, so that the validity of the equilibrium model can be tested. This unique feature of our approach is the key solution to the joint hypothesis problem. [16]

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[16] The traditional test of market efficiency is however not problematic when the postulated equilibrium model actually holds. To illustrate, assume for convenience that the CAPM is the equilibrium model. Using equation (2), the portfolio abnormal return can be written as a function of squared Sharpe ratios as follows: $\alpha_p = \pi\{E[s_{pt} \mid \Phi]^2 - E[s_{mt} \mid \Phi]^2 - (\rho_{pm}^2 - 1)E[s_{mt} \mid \Phi]^2\}$, where π is defined as $\pi = \sigma_p^2 / (\alpha_p + 2\rho_{pm}E[s_{mt} \mid \Phi])$. This relation implies that, when the CAPM holds ($\rho_{pm}^2 = 1$), testing the presence of an abnormal return will be equivalent to assessing the scope of the squared Sharpe ratio difference, and then the traditional test will effectively measure market efficiency. [Note that the squared Sharpe ratio difference is closely related to the test of GRS (1989), J_{GRS}. Indeed, using equation (5.3.42) from Campbell, Lo, and MacKinlay (1997, p.196), the squared Sharpe ratio difference can be written as $E[s_{pt} \mid \Phi]^2 - E[s_{mt} \mid \Phi]^2 = \frac{T-N-1}{N}(1 + E[s_{mt} \mid \Phi]^2)J_{GRS}$.] However, if the CAPM does not hold ($\rho_{pm}^2 < 1$), then the traditional test will not necessarily detect market inefficiency.

V. DATA AND DESCRIPTIVE STATISTICS

In a series of studies, FF (1992, 1993, 1996) reexamine the literature on anomalies in detail. In their well-known 1992 work, the authors confirm that firm size and BEME are particularly significant in explaining the cross-section of stock returns. Moreover, FF (1993, 1996) argue that the TFPM—consisting of the market excess return, SMB: a portfolio that is long in small firms and short in big firms, and HML: a portfolio that is long in value firms and short in growth firms—captures the abnormal returns related to anomalies. Therefore, we focus on the size and value effects to test the efficiency of the international markets. We also consider the momentum effect (Jegadeesh and Titman (1993)). We first present the data and then report the stylized facts on each of these three effects.

A. Data

We focus on the three largest capital markets (the United States, Japan, and the United Kingdom) over the period January 1981 to December 2000. We obtain our data from DATASTREAM.[17] Panel A of Table II reports summary descriptive statistics. Our sample includes 7,079 firms. The United States dominates with 4,620 firms, followed by Japan (1,567) and the United Kingdom (892). For the 20-year sample period, the average market excess return is significantly positive in the United States and the United Kingdom (respectively 0.78% per month (t = 2.81) and 0.73% per month (t = 2.08)) but insignificant for Japan (0.46% per month; t = 1.04). Of the

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[17] Appendix A describes the data.

three, the United States market was the least volatile while the Japanese market was the most volatile.

TABLE II ABOUT HERE

B. Size, Value, and Momentum effects

Before examining the implications of size, value, and momentum effects for the efficiency of international markets, it is important to determine whether the patterns of returns documented in prior empirical studies are robust for our sample.

For each month *t* from July of year *y*-1 to June of year *y*, we rank the stocks within each country based on their size in June *y*-1, their BEME ratio in June *y*-1, and their return between *t*-12 and *t*-2. We then use these three rankings to calculate the quintile breakpoints (20%, 40%, 60%, and 80%) for size, BEME, and momentum. The stocks are subsequently sorted into five size groups, five BEME groups, and five momentum groups based on these breakpoints. The portfolio returns are computed by value-weighting the stock returns within the portfolios. The stocks above the 80% size, BEME, and momentum breakpoints are respectively designated *B* (for big), *V* (for value), and *W* (for winner) while those below the 20% size, BEME, and momentum breakpoints are designated *S* (for small), *G* (for growth), and *L* (for loser). *S* - *B* is the spread between the small and big portfolios, *V* - *G* is the spread between the value and growth portfolios, and *W* - *L* is the spread between the winner and loser portfolios.

Panel B of Table II shows the results. Across the three countries covered, the small portfolio commands a higher mean excess returns than the other groups of firms.

However, the size-spread portfolio is never significant at the 5% level. For the United States, the average S - B return is 0.41% per month (t = 1.15), consistent with the results of Davis, Fama, and French (2000). Similar results are observed for the other countries; the average S - B return is 0.43% per month for Japan (t = 1.09) and 0.41% for the United Kingdom (t = 1.31).

Consistent with the evidence reported in prior empirical studies, the value portfolio appears to have the highest average excess return in all the countries. However, in contrast with the results obtained in the literature, the value effect does not seem to be strong in the United States for our sample period. The average V - G return is only 0.38% per month and is less than two standard errors from zero (t = 1.62). Nonetheless, the value effect is strong in Japan and the United Kingdom. The value-spread portfolio realizes on average 0.74% per month for the United Kingdom and about 1.06% per month in Japan; both figures are reliably significant at the 1% level, consistent with those reported by FF (1998) and Liew and Vassalou (2000).

The momentum effect is weak in Japan with the excess return of the winner portfolio only 0.28% per month higher than that of the loser portfolio (t = 0.58). Still, the momentum effect is strong in both the United States (t = 1.94) and the United Kingdom (t = 2.85), consistent with the evidence of Jegadeesh and Titman (1993) and Rouwenhorst (1998).

VI. TESTING MARKET EFFICIENCY

We first examine the case of the CAPM and then discuss the results obtained with the FFPM.

A. Tests of the EMH and the CAPM

To test the efficiency of international markets with the CAPM, we apply (14) and estimate the following system of equations:

$$s_{pt} = \tau_p + \rho_{pm} s_{mt} + u_{pt}, \qquad p = 1...5. \tag{15}$$

In system (15), the abnormal performance is given by $\tau_p = E[s_{pt}] - \rho_{pm} E[s_{mt}]$. The model implicitly builds on the implication of the CAPM that the expected return should be commensurate to the portfolio systematic risk. Therefore, the traditional approach (implicitly) implies the use of $\rho_{pm} E[s_{mt}]$ as a benchmark for the portfolio performance $E[s_{pt}]$. The difficulty is that—as every standardized return has a unit variance by construction—the portfolio and the benchmark will be comparable (have the same variance) only if $\rho_{pm}^2 = 1$; that is, when the CAPM holds. In contrast, when the model is false, then the abnormal performance (or the abnormal return) obtained from the test of market efficiency could be misleading because of deficient comparability.

We estimate system (15) on size-, BEME-, or momentum-ranked portfolios while testing two hypotheses. First, we test the EMH; that is whether the portfolios realize significant abnormal performances. Next, we test the CAPM: $\delta_p = \rho_{pm}^2 = 1$. We solve system (15) using *generalized least squares* (GLS) with Zellner's (1962) SUR effects.[18]

SIZE—Panel A of Table III shows the results from the GLS estimation of (15) on the size-ranked quintile portfolios for 1981-2000. We first discuss the results obtained for the United States. The point estimate of the slope associated with the market standardized excess return is $\rho_{pm} = 0.50$ for the small portfolio, which corresponds to a goodness-of-fit of roughly $\delta_p = 25.33\%$. This relatively low explanatory power of the market factor leads to the rejection of the CAPM ($p = 0.00$). Because the estimated market correlation is about one half, the traditional test will consider that the benchmark for the performance of the small portfolio is only the half of the market performance, to the consequence that the small portfolio exhibits a significant abnormal performance ($t = 1.89$).

In contrast, the estimated market correlation and goodness-of-fit parameters for the big portfolio are very close to one, to the consequence that the CAPM is not rejected ($p = 0.69$). Given the validity of the CAPM for the big portfolio, the test of market efficiency is of good quality and is acceptable. However, the null of market efficiency cannot be rejected even at the 10% level ($t = 0.90$) for the big portfolio. Due to the *illusive* situation of the small portfolio in the CAPM system, however, the size-spread portfolio now triggers a statistically significant abnormal performance ($t =$

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[18] See also Pindyck and Rubinfeld (1981, p.331-333) for a detailed discussion of SUR effects and Green (2000, p.615-616) for the efficiency gain accruing to GLS relative to OLS.

1.77) albeit there is no size effect (see Table 2). This situation illustrates the problem of the traditional test of market efficiency, which cannot distinguish whether the test is of good quality or not.

TABLE III ABOUT HERE

A similar pattern emerges from the results obtained for the United Kingdom. Because the size-spread portfolio loads negatively on the market factor, the portfolio obtains an abnormal performance that is actually higher than its Sharpe ratio. This situation would be acceptable if, as implied by the CAPM, the market factor fully explains the variation of this portfolio. However, this is not the case because the model explains only 16.90% of the variation of the size-spread portfolio. Here again, the size effect suggested by the abnormal performance is triggered by the failure of the postulated equilibrium model.

For Japan, things are somewhat different, as the size effect is still insignificant. The reason is that the performance of the CAPM regarding the small portfolio is dramatically improved (compare 61.83% to the 25.33% obtained for the United States). The consequence of the improved fit is that the benchmark for Japan is of better quality than that obtained for the other countries, to the extent that the bias in the estimation of the abnormal performance is lower.

BEME—Panel B of Table III reports the results obtained on the BEME-ranked quintile portfolios for 1981-2000. The value portfolio realized a significantly higher abnormal performance than the growth portfolio in all the three countries.

Nonetheless, we cannot confirm the presence of an anomaly because the CAPM is reliably rejected. In particular, the CAPM explains less than 3% of the variation of the value-spread portfolio for all the countries. (This implies a near-zero benchmark.) The inability of the CAPM to explain the difference in behavior between the value and growth portfolios could explain the high abnormal performance observed for the value-spread portfolio. This situation will be illustrated later in the exposition.

MOMENTUM—Panel C of Table III summarizes the results obtained on the momentum-ranked quintile portfolios for 1981-2000. The abnormal performance of the momentum-spread portfolio is indistinguishable from zero for Japan, but is significantly positive for both the United States and the United Kingdom. The latter result seems to suggest the presence of an anomaly. However, the rejection of market efficiency from this inference would be premature because not only the CAPM is rejected across the momentum-ranked quintile portfolios but also the model has no explanatory power in the variation of the momentum-spread portfolios. This lack of explanatory power of the CAPM is consistent with zero benchmarks and so could easily be the cause of the momentum effect.

In summary, the results obtained on the CAPM systems confirm the worse scenario for the traditional tests of market efficiency: it is the rule rather than the exception that the CAPM—the most widely used model to test the EMH—is conclusively rejected by the data. The failure of the CAPM generally triggers a premature rejection of market efficiency because of the deficient benchmark involved. In other words, because the CAPM dramatically fails to explain the behavior of some portfolios, the abnormal performance could suggest the existence of important free lunches when in fact there is none. As an alternative to the CAPM, the next section uses the four-factor pricing model (FFPM), which adds to the CAPM three factors related to size, BEME, and momentum.

B. Tests of the EMH and the FFPM

To test the efficiency of international markets in a model that incorporates size, BEME, and momentum risk factors, we estimate the following system of equations on the characteristic-sorted portfolios:

$$s_{pt} = \tau_p + \rho_{pm} s_{mt} + \rho_{ps} s_{st} + \rho_{pb} s_{ht} + \rho_{pw} s_{wt} + u_{pt}, \qquad p = 1...5, \tag{16}$$

where s_{st}, s_{ht}, and s_{wt} are respectively the standardized returns of the size, BEME, and momentum factors (SMB, HML, and WML) at time t.[19] Because the factors are orthogonalized, the coefficient associated with the market factor is not changed relative to CAPM. If size, value, and momentum risks matter, the abnormal performance from the FFPM will be different from that obtained from the CAPM (the difference will be equal to the risk premium $\sum_k \rho_{pk} E[s_{kt}]$; where $k = s, b,$ and w). We estimate system (16) for the three sets of portfolios and test two hypotheses. First, we test market efficiency from the estimated abnormal performances: $\tau_p = 0$. Second, we test the FFPM by investigating whether the loadings lie close to a 4-hypersphere of center zero and radius one: $\delta_p = \sum_k \rho_{pk}^2 = 1$; $k = m, s, b,$ and w.

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[19] Appendix B describes the construction of the factors while Appendix C details their orthogonalization.

SIZE—Panel A of Table IV summarizes the results from the GLS estimation of (16) on the size-ranked quintile portfolios for 1981-2000. We first discuss the results obtained for the United States. For the big portfolio, the addition of the three factors has little impact since more than 99% of its variance was already explained by the market factor. (The abnormal performance has changed from 0.00 for the CAPM to 0.01 for the FFPM.) The most striking result from the table is the sizable increase in the goodness-of-fit for the small portfolio, which is now about 88.42% (compared with 25.33% for the CAPM). Interestingly, even with this improved fit, the FFPM is still rejected at the 5% level for the small portfolio. Accordingly, the result from the test of market efficiency obtained is not of good quality.[20]

TABLE IV ABOUT HERE

As expected, the added factors, especially SMB, are priced for Japan and the United Kingdom. More importantly, the signs obtained on the loadings tend to confirm the misspecification story advanced above. Indeed, SMB tends to be positively correlated with the small portfolio and negatively correlated with the big portfolio for all three countries. Because of its exposure (protection) against the low realizations of SMB, the small (big) portfolio should command positive (negative) size risk premium. Therefore, when these size risk premiums are ignored (as the CAPM does), the

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[20] Despite the significant loading on the SMB, the abnormal performance of the size-spread portfolio has little changed in amplitude (0.098 versus 0.104) albeit it has became highly statistically significant ($t = 3.89$). The statistical significance is explained by the improved fit, while the unchanged amplitude is because the SMB has a negative Sharpe ratio during 1981-2000 (see Table V in Appendix B).

benchmark for the small (big) portfolio will be understated (overstated) so that, by the combination of the two biases, the abnormal performance is biased upward. As a confirmation of this hypothesis, we observe that the abnormal performance vanishes in Japan and in the United Kingdom when the SMB is taken into account. Accordingly, we conclude that the size effect observed in the CAPM system is not really an anomaly.

BEME—Panel B of Table IV shows the results obtained on the BEME-ranked quintile portfolios. We first discuss the results obtained for the United States. While we reported a significantly positive abnormal performance for the value-spread portfolio in the CAPM system, the estimated abnormal performance is actually negative (t = -3.29) in the FFPM system. This dramatic change in the abnormal return of the performance of the value-spread portfolio (-0.230) is mainly explained by the introduction of the HML. The contribution of HML on the abnormal performance is about $-\rho_{Vb}E[s_{bt}] = -0.432 * 0.408 = -0.176$ for the value portfolio. For the growth portfolio, the contrary happens because that portfolio is negatively correlated with HML. The growth portfolio implies a marginal change of about $-\rho_{Gb}E[s_{bt}] = 0.185 * 0.408 = 0.075$ in the benchmark relative to the CAPM. The combination of the two effects reported above is the main reason for the shift in the abnormal performance; the contribution of HML in the abnormal performance shift is about -0.176 - (0.075) = -0.251.

Similar results are obtained for Japan and the United Kingdom. The value effect, which seemed strong with the CAPM, vanishes when the FFPM is used to measure the portfolio abnormal performance. The reason is that the CAPM ignores the *extra-security* inherent to the growth portfolio as well as the *extra-risk* supported by the value portfolio relative to HML. In other words, the benchmark used in the CAPM is lower than what it should be for the value portfolio and higher than what it should be for the growth portfolio, to the extent that, by the combination of the two biases obtained when HML is ignored, a spurious value effect is documented.

MOMENTUM—Panel C of Table IV summarizes the results obtained on the momentum-ranked quintile portfolios. The loser portfolio is less risky than the winner portfolio relative to WML. The failure to account for momentum risk implies an understated (overstated) abnormal performance for the loser (winner) portfolio with the consequence that abnormal performance of the momentum-spread portfolio is biased upward. The evidence reported in this table suggests this explanation for the momentum effect. For the United Kingdom, the momentum effect fades away within the FFPM system; the abnormal performance obtained on the momentum-spread portfolio is insignificant at the 10% level (t = -0.52). For Japan, the abnormal performance on the momentum-spread portfolio is now significantly positive ($t = 2.55$). This result cannot be taken as evidence against market efficiency, since the FFPM is reliably rejected for the momentum-spread portfolio ($p = 0.00$).[21]

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[21] Moreover, to be economically viable, the abnormal performance should be sufficiently high to cover the transaction costs (e.g., Lesmond, Schill, and Zhou (2004)) and compensate for the idiosyncratic (arbitrage) risk supported by arbitrageurs (e.g., Ali, Hwang, and Trombley (2003)). While this vision of market efficiency is closer to the definition proposed by Jensen (1978, p.96), accounting for these issues would certainly strengthen our case about the absence of free lunches but is nonetheless well beyond the scope of the current research.

Overall, the evidence in this section demonstrates the main point of the paper that testing market efficiency from the standpoint of the traditional approach is at best hazardous and at worst inappropriate and misleading. Not only it is frequent that the postulated asset-pricing model fails to hold but such a failure is likely to affect adversely the test of market efficiency. If the portfolio used is well diversified, then the traditional approach will make a bias while assuming that the portion of the portfolio total variance left unexplained by the model is not rewarded. For instance, when the CAPM is postulated in the traditional test, then it will be implicitly assumed that the portion of the portfolio total risk left unexplained is trivial while, as shown in this section, much of that unexplained variance is systematically related to SMB, HML, and WML. Because the unexplained variance is disregarded in the traditional test, a substantial abnormal performance can spuriously be inferred when the equilibrium model fails. In the next section, we explore the implications of this issue for event studies.

VII. IMPLICATIONS OF THE RESULTS

A. Event Studies

Event studies have a long history. As best we can tell, Dolley (1933) was the earliest published event-study paper, but the first systematic study was that of Fama, Fisher, Jensen, and Roll (1969). Since then, event studies have become one of the most important topics of economics, especially in accounting and corporate finance.[22] The

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[22] See Fama (1991) for a review of this literature.

main purpose of event studies is to examine the impact of an event on returns and hence serves as a framework to test the EMH. The aim of this section is to explore the implications of this research for event studies.

The most commonly used statistic to gauge the impact of an event on returns during a certain period (T) is the cumulative abnormal return (CAR). To compute the CAR, the approach is to postulate an equilibrium model, generally the CAPM, and use it to compute the abnormal return (AR) at time t:

$$AR_t = r_{pt} - \beta_{pm} r_{mt} \,. \tag{20}$$

Using the definition of systematic risk, $\beta_{pm} = \rho_{pm}\sigma_p / \sigma_m$, the abnormal returns are cumulated to obtain the CAR:

$$CAR = \sum_{t=0}^{T} r_{pt} - (\rho_{pm}\sigma_p / \sigma_m) r_{mt} \,. \tag{21}$$

The drawback with the CAR is that it implicitly assumes that the postulated equilibrium model, for instance the CAPM, holds so that all the portfolio variation comes from the market. However, when CAPM fails ($\rho_{pm}^2 < 1$), then the CAR will be misspecified because the portfolio will be judged based solely on the market systematic risk although the portfolio is partly driven by missing factors. When performance is measured from the traditional approach (Jensen's alpha), then a portfolio manager can always increase *performance* by simply neutralizing the market risk and accepting greater exposures to non-market risk factors.

The CAR would lead to erroneous conclusions when the equilibrium model fails, since the unexplained variance would be driven by missing risk factors. One way to solve this problem is to impose the validity of equilibrium model ($\rho_{pm}^2 = 1$) while computing the CAR. Restricting the portfolio to be risky, the corrected CAR (CCAR) can be written as:

$$CCAR = \sum_{t=0}^{T} r_{pt} - (\sigma_p / \sigma_m) r_{mt} \,. \tag{22}$$

In contrast with the benchmark postulated in the traditional approach, the corrected benchmark, $(\sigma_p / \sigma_m) r_{mt}$, implies a higher level of systematic risk; i.e., $Cov\left[(\sigma_p / \sigma_m) r_{mt}, r_{mt}\right] = \sigma_p \sigma_m > Cov[r_{pt}, r_{mt}] = \rho_{pm} \sigma_p \sigma_m$. This is so because the unexplained variance is now accounted as systematic (with respect to the missing factors) and rewarded at the market rate. The most interesting feature of the corrected benchmark, however, is that it implies that the portfolio is compared to a market benchmark that has the same level of volatility, $Var\left[(\sigma_p / \sigma_m) r_{mt}\right] = \sigma_p^2$, thus ensuring a better comparability. The rational behind the CCAR is similar to Modigliani and Modigliani's (1997) new risk-adjusted performance measure (RAP), which adjusts all portfolios to the level of risk in the market benchmark, as well as to that of Graham and Harvey (1997) (GH1), which lever or unlever the market to match the volatility of the portfolio over the evaluation period.

B. Performance Measurement

The most important and most widely used measure of performance can be traced back to the outset of the asset-pricing literature with the measures proposed by Treynor (1965), Sharpe (1966), and Jensen (1968).[23] However, because these traditional measures of performance assume the validity of the CAPM, they remain subject to the joint-hypothesis problem. Below, we discuss how the framework of this study can be used to correct such metrics.

The main idea of the correction is to restrict the performance measurements so as to verify Proposition 2; that is, so that the asset-pricing model holds. Corollary 2 gives the general form of the corrected measure of performance.

COROLARRY 2: *Any performance measurement that verifies following functional form:* $\alpha_p^* = E[r_{pt}] - \sum_k \varpi_k (\sigma_p / \sigma_k) r_{kt}$, *where the weights are verifies* $\sum_k \varpi_k^2 = 1$ *will not be misspecified.*

Proof: The result follows from Propositions 2 and 3. We see that $\alpha_{pt}^* = \sigma_p \tau_{pt}$ can be seen as a measure of abnormal returns (Proposition 3) while the condition on the weights implies that the benchmark is not flawed (Proposition 2). Q.E.D.■

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[23] See Grinblatt and Titman (1995) for a review of this literature.

An interesting particular case satisfying Corollary 2 is to set $\varpi_k = 1/\sqrt{K}$, where K is the number of factors. When the market is the only risk factor (the CAPM), then $\varpi_m = 1$ and the corrected measure of performance is reduced to $\alpha_p^* = E[r_{pt}] - (\sigma_p / \sigma_m) r_{mt}$. The link between the latter measure (the corrected Jensen's alpha) and classical Jensen's alpha (α_p) is obvious: the corrected measure of performance is equal to Jensen's alpha corrected for the bias resulting for the deficiency of the CAPM (see appendix D), $\alpha_p^* = \alpha_p - (1 - \rho_{pm})(\sigma_p / \sigma_m) r_{mt}$. From, this expression, we clearly see that the classical Jensen's alpha will have tendency to yield higher abnormal returns when the equilibrium model fails, especially when the portfolio is volatile.

The corrected Jensen's alpha is also closely related to Sharpe's measure of performance. In fact, the corrected Jensen's alpha, which is proportional to the difference of the Sharpe ratios between the portfolio and the market ($\alpha_p^* = \sigma_p(E[s_{pt}] - E[s_{mt}])$), can be seen as a measure of the distance between the portfolio and the efficient set. The corrected Jensen's alpha can also be related to the recent measures of performance proposed by Modigliani and Modigliani (1997). For instance, the corrected Jensen's alpha can be written as $\alpha_p^* = (\sigma_p / \sigma_m)(RAP_p - RAP_m)$, where $RAP_p = E[s_{pt}]\sigma_m - E[R_{ft}]$ is the Modigliani-Modigliani risk-adjusted performance measure.

C. *Further Empirical Illustration*

To further investigate the impact of the joint-hypothesis, we compute the CAR and CCAR achieved by investment strategies based on the size, value, and momentum effects during 1981-2000. SIZE—Figure 1 shows the results obtained on the size-spread portfolio of each country. Interestingly, for all the countries the CAR of the investment strategy that sells short the big portfolio to buy the small portfolio has tendency to increase over time, suggesting that the strategy beats the market. However, when we correct the benchmark to match the size-investment-strategy volatility, the abnormal returns sharply decreases relative to the CAR. For all countries, the CCAR hits negative values during the sample period. This clearly demonstrates, again, that the traditional approach to test market efficiency is problematic in that it often suggests the presence of free lunches when there is none.

FIGURES 1 TO 3 ABOUT HERE

BEME—Figure 2 shows the CAR and CCAR that an investor would achieve by investing on the value-spread portfolio over the period January 1981 through December 2000. In the United States, despite the insignificant value effect, the CAR over the total sample period is 134%. The value effect seems even more impressive in the United Kingdom and Japan with respective CARs of 176% and 263%. However, the value effect turns out to be less important when the CARs are corrected to take into account the failure of the CAPM. Indeed, the CCAR of the value strategy is negative in December 2000 (about -67%) in the United States whilst it is usually negative in 2000 in the United Kingdom. Albeit it decreases relative to the CAR, the

CCAR is important (about 168% in December 2000) in Japan. This significant value effect in Japan during 1990s should be interpreted with caution as evidence against market efficiency because it is likely to be sample specific (see the discussion by Fama (1998)).

MOMENTUM—Figure 3 shows the CAR and CCAR obtained on the momentum-spread portfolio over 1981-2000. As it can be clearly seen from the comparison of the two plots, the CAR has tendency to overstate the abnormal returns achieved by the momentum strategy. In both the United States and Japan, the CCAR ends the 1990s with a negative market-adjusted performance once a correction is made to account for the failure of the CAPM to explain the momentum-spread portfolio. For the United Kingdom, the CCAR finishes in the positive side, but is, nonetheless, negative in several episodes during the sample period and well bellow the CAR.

VIII. CONCLUSION

One century after the pioneering study by Bachelier (1900), there is no consensus on the efficient market hypothesis (EMH). This study proposes a new approach that can be used to split the joint hypothesis underlying the extant tests of the EMH. To illustrate the general validity of our approach for all current and future *anomalies*, we examine the efficiency of the United States, Japan, and the United Kingdom over the period 1981-2000. We investigate three of the most significant anomalies reported in the literature, namely the size, value, and momentum effects, using both the CAPM and the four-factor pricing model (Fama-French's TFPM augmented with the momentum factor).

Estimating new specifications that disentangle the well-known joint-hypothesis, we show that not only is it the rule rather than the exception that the assumed equilibrium model fails, but such a failure is likely to trigger a premature rejection of the EMH. We illustrate further this phenomenon by identifying the source of the problem: the traditional approach overlooks the unexplained variance, so that a serious error can be inferred when the equilibrium model fails. We finally illustrate the implications of our results for event studies by identifying the shortcoming inherent in the traditional measure of cumulative abnormal returns and the way to correct it.

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APPENDIX A: SAMPLE CONSTRUCTION

The sample includes total monthly returns, market capitalization (size), and BEME ratios. All data are converted into US dollars and the one-month US Treasury bill rate is used as a risk-free rate of return. To be included in the sample for a given month, a stock had to satisfy the following criteria: its return in the current month and in the 12 preceding months be available; its size at the beginning of the month and in the previous June be available; and its BEME ratio at the beginning of the month and in the previous June be positive. Because the database contains fewer stocks in the 1970s, we decided to start in January 1980 to ensure a relatively large number of stocks in our portfolios. The data in 1980 is used to form the characteristic-sorted portfolios and the test period starts in January 1981.

To verify the quality of our data, we first compared the country indexes constructed from our data set with those provided by Morgan Stanley Capital International (MSCI). The correlations between the constructed and the MSCI country indexes are approximately 98%. We also compared our constructed size- and BEME-ranked portfolios to those computed by Fama and French for the United States. The results show that the two sets of factors are very similar. For example, for the size-ranked quintile portfolios, the lowest correlation between our constructed portfolios and those computed by Fama and French is 84.14% (for the small portfolio). For the BEME-ranked quintile portfolios, the equivalent figure is 91.22% (for the value portfolio).

APPENDIX B: CONSTRUCTION OF THE FACTORS

The construction of the factors is similar to FF (1993, 1996). For each month *t* from July of year *y*-1 to June of year y, we rank the stocks within each country based on their size in June *y*-1, their BEME ratio in June *y*-1, and their return between *t*-12 and *t*-2. We then use these three rankings to calculate a 50% breakpoint for size, 30% and 70% breakpoints for BEME and for prior return. The stocks are subsequently sorted into two size groups, three BEME groups, and three prior return (momentum) groups based on these breakpoints. The stocks above the 50% size breakpoint are designated *B* (for big) and the remaining 50% are designated *S* (for small). The stocks above the 70% BEME breakpoint are designated *H* (for high), the middle 40% are designated *M*, and the firms below the 30% BEME breakpoint are designated *L* (for low). The stocks above the 70% momentum breakpoint are designated *W* (for winner), the middle 40% are designated *N*, and the firms below the 30% momentum breakpoint are designated *Lo* (for loser).

For each country, we form six value-weighted size and BEME portfolios, S/L, S/M, S/H, B/L, B/M, and B/H as the intersection of size and BEME groups. We also form six value-weight size and momentum portfolios, S/Lo, S/N, S/W, B/Lo, B/N, and B/W, as the intersection of size and prior return groups. SMB (small minus big) is the equal-weight average of the returns on the small stock portfolios minus the returns on the big stock portfolios:

$$SMB = ((S/L - B/L) + (S/M - B/M) + (S/H - B/H))/3. \tag{A1}$$

Similarly, HML (high minus low) is the equal-weight average of the returns on the value stock portfolios minus the returns on the growth stock portfolios:

$$HML = ((S / H - S / L) + (B / H - B / L)) / 2. \tag{A2}$$

WML (winner minus loser) is the equal-weight average of the returns on the winner stock portfolios minus the returns on the loser stock portfolios:

$$WML = ((S / W - S / Lo) + (B / W - B / Lo)) / 2 \ . \tag{A3}$$

We also determine the market excess return, which is the capitalization-weighted return of all the securities considered within each country in excess of the risk-free rate, $R_m - R_f$.

APPENDIX C: ORTHOGONALIZATION OF THE FACTORS

Because the factors are correlated (see Table V below), we apply the Gram-Schmidt theorem to orthogonalize them. In short, we made the following transformations:

$$\begin{aligned} r_{SMB} &= SMB - \frac{\operatorname{cov}[SMB, r_m]}{\operatorname{var}[r_m]} r_m \\ r_{HML} &= HML - \frac{\operatorname{cov}[HML, r_m]}{\operatorname{var}[r_m]} r_m - \frac{\operatorname{cov}[HML, r_{SMB}]}{\operatorname{var}[r_{SMB}]} r_{SMB} \\ r_{WML} &= WML - \frac{\operatorname{cov}[WML, r_m]}{\operatorname{var}[r_m]} r_m - \frac{\operatorname{cov}[WML, r_{SMB}]}{\operatorname{var}[r_{SMB}]} r_{SMB} - \frac{\operatorname{cov}[WML, r_{HML}]}{\operatorname{var}[r_{HML}]} r_{HML} \end{aligned} \quad . \text{(A4)}$$

The market factor is not modified. The SMB is the first to enter the orthogonal system and is purged from the common covariation with the market factor. Then follows respectively the HML and WML. The choice of this hierarchical structure between the factors is arbitrary.

TABLE V ABOUT HERE

APPENDIX D: THE BIAS IN PERFORMANCE MEASUREMENT

Using equation (3), the abnormal return obtained from the traditional model is: $\alpha_p = \sigma_p \left(E[s_{pt}] - \rho_{pm} E[s_{mt}] \right) = E[r_{pt}] - \rho_{pm} \frac{\sigma_p}{\sigma_m} E[r_{mt}]$. Assuming a risky portfolio, this abnormal return is reduced to $\alpha_p^* = E[r_{pt}] - \frac{\sigma_p}{\sigma_m} E[r_{mt}]$ when the CAPM holds ($\rho_{pm} = 1$). From this, we see that the bias of the traditional approach when the CAPM does not hold is $\alpha_p^* - \alpha_p^* = (1 - \rho_{pm}) \frac{\sigma_p}{\sigma_m} E[r_{mt}]$. One can easily verify that the bias is indeed higher when, *ceteris paribus*, the portfolio is uncorrelated with the market and/or when the portfolio highly volatile.

APPENDIX E: DIVERSIFICATION AND TEST OF MARKET EFFICIENCY

This section investigates whether and how portfolio diversification affects the test of market efficiency. To that end, we assume that there are N securities with identically expected excess return E[r], identical variance σ^2, and identical correlation with each other $\rho > -\frac{1}{N-1}$. Since there is no reason to prefer one security over the others, the best strategy for a portfolio of n assets is to invest $1/n$ in each. Hence, the portfolio expected excess return is $E[r]$ and the standard deviation is: $\sigma_p^n = \sigma\sqrt{\frac{1}{n}+\rho\frac{n-1}{n}}$.

Assuming that the market factor consists in investing $1/N$ of each of the N securities in the market, the (corrected) abnormal performance can be written as:

$$\tau_p^* = E[s_p] - E[s_m] = \frac{E[r]}{\sigma}\left[\frac{1}{\sqrt{\frac{1}{n}+\rho\frac{n-1}{n}}} - \frac{1}{\sqrt{\frac{1}{N}+\rho\frac{N-1}{N}}}\right]. \tag{A5}$$

Because $-\frac{1}{N-1} < \rho \leq 1$, it follows from (A5) that τ_p^* is negative and converges to zero as n increases towards N (or ρ converges to one). Moreover, because the portfolio variance decreases with n (while the expected excess return remains constant) it ensues that the expected abnormal performance increases with n. This positive effect of diversification on performance can be verified from the partial derivatives of τ_p^* with respect to n: $\frac{\partial\tau_p^*}{\partial n} = \frac{(1-\rho)E[r]}{2n^2\sigma\left(\frac{1}{n}+\rho\frac{n-1}{n}\right)^{\frac{3}{2}}}$ and $\frac{\partial^2\tau_p^*}{(\partial n)^2} = -\frac{(1-\rho)E[r]}{4n^3\sigma}\frac{\frac{1}{n}+\rho\frac{4n-1}{n}}{\left(\frac{1}{n}+\rho\frac{n-1}{n}\right)^{\frac{5}{2}}}$. It follows from the expressions above that the marginal effect of diversification on performance is positive but decreases with the number of securities in the portfolio.

To empirically verify the effect of diversification on abnormal performance, we proceed with the following simulation. For each country, we use all the securities that have complete data to randomly draw 1000 samples of n securities (n = 1 fi 50).[24] Then for each sample, we form a portfolio (by equally-weighing the sampled securities) and compute $s^n_{pt} - s_{mt}$, the difference of standardized excess return between the portfolio and the market. Finally, we average the figures (across time and the samples) to obtain for each n the simulated abnormal performances; $\hat{\tau}^*_p = \sum_{p=1}^{1000} \frac{1}{1000} \sum_{t=1}^{240} \frac{s^n_{pt} - s_{mt}}{240}$.

Figure 4, which plots the average abnormal performance as a function of the number of stocks held in the portfolio, shows that abnormal performance tends to increase (at a decreasing rate) with number of securities n in the portfolio. For the United States, the average abnormal performance changes from -0.124 when n = 1 to -0.005 when n = 50, which corresponds to an increase of 95.97%. The corresponding increases are respectively 88.89% and 93.67% for Japan and the United Kingdom.

FIGURE 4 ABOUT HERE

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[24] Because the number of securities in portfolios is pivotal in the simulations, we consider only the securities that have complete data to keep the portfolio dimension to exactly n. Accordingly, we re-compute the market factor for this experiment by equally-weighting the securities with complete data.

Table I

Descriptive Statistics on Size-Ranked Decile Portfolios, 1927-2000

This Table illustrates the problem related to the traditional test of market efficiency by showing some descriptive statistics on size-ranked deciles portfolios and the market factor. The sample period covers January 1927 through December 2000. The statistics reported are the mean excess return, the sample standard deviation, the Sharpe ratio, the correlation coefficient of each portfolio with the market (ρ_{pm}), the abnormal return (α_p) from the market model, and the corrected abnormal performance (CAP), which is the portfolio Sharpe ratio in excess of that of the market. The data are from the CRSP database. All returns are denominated in US dollars and are measured in percent per month.

Portfolio	Mean excess return	Standard deviation	Sharpe Ratio	Correlation (ρ_{pm})	Abnormal return (α_p)	CAP
Small	1.208	10.598	0.114	0.774	0.202	-0.009
2	1.007	9.219	0.109	0.844	0.052	-0.014
3	0.964	8.241	0.117	0.895	0.007	-0.006
4	0.933	7.702	0.121	0.905	0.010	-0.002
5	0.913	7.432	0.123	0.933	0.008	0.000
6	0.886	7.066	0.125	0.951	0.009	0.003
7	0.847	6.712	0.126	0.961	0.008	0.003
8	0.797	6.306	0.126	0.973	0.007	0.004
9	0.757	6.039	0.125	0.982	0.005	0.003
Big	0.645	5.205	0.124	0.986	0.003	0.001
Market	0.678	5.524	0.123	-	-	-

Table II

Summary Descriptive Statistics, 1981-2000

For each country (the United States, Japan, or the United Kingdom), we report the summary descriptive statistics for 1981:1-2000:12. In Panel A, we report the monthly average and total numbers of firms used, the mean and standard deviation of the market excess return, and the t-statistic related to the null that the mean excess return is zero (in parenthesis). The market excess return is the value-weighted average return in excess of the one-month US Treasury bill rates. In Panel B, we report the summary descriptive statistics for size-, BEME-, and momentum-ranked portfolios. The portfolios are constructed as follows. For each month t from July of year y-1 to June of year y, we rank the stocks within each country based on their size in June y-1, their BEME ratio in June y-1, and their return between t-12 and t-2. We then use these three rankings to calculate the quintile breakpoints (20%, 40%, 60%, and 80%) for size, BEME, and momentum. The stocks are subsequently sorted into five size groups, five BEME groups, and five momentum groups based on these breakpoints. The portfolio returns are computed by value-weighting the stock returns within each portfolio. The stocks above the 80% size, BEME, momentum breakpoints are respectively designated B (for big), V (for value), and W (for winner) while those below the 20% size, BEME, momentum breakpoints are designated S (for small), G (for growth), and L (for loser). The column labeled "Return" report the average excess return of each portfolio, followed by the t-statistic (in parenthesis). S - B is the spread between the small and big portfolios, V - G is the spread between the value and growth portfolios, and W - L is the spread between the winner and loser portfolios. The data are from DATASTREAM. All returns are denominated in US dollars and are measured in percent per month.

Panel A. The sample

Country	Mean number of firms	Total number of firms	Mean excess return	Standard deviation	(t-statistic)
United States	2955	4620	0.784	4.328	(2.806)
Japan	1217	1567	0.455	6.752	(1.043)
United Kingdom	660	892	0.726	5.404	(2.080)

Panel B. Size, BEME, and momentum effects

Portfolio	United States		Japan		United Kingdom	
	Excess return	(t-statistic)	Excess return	(t-statistic)	Excess return	(t-statistic)
			Size-Ranked Portfolios			
S	1.207	(3.078)	0.881	(1.612)	1.142	(3.158)
2	0.501	(1.384)	0.507	(0.949)	0.615	(1.724)
3	0.492	(1.419)	0.401	(0.774)	0.809	(2.275)
4	0.683	(2.110)	0.442	(0.895)	0.707	(1.831)
B	0.796	(2.850)	0.451	(1.034)	0.730	(2.077)
S – B	0.411	(1.147)	0.430	(1.091)	0.412	(1.313)
			BEME-Ranked Portfolios			
G	0.812	(2.528)	-0.074	(-0.155)	0.678	(1.805)
2	0.648	(2.151)	0.384	(0.813)	0.382	(1.022)
3	0.634	(2.253)	0.626	(1.402)	0.675	(1.692)
4	0.918	(3.387)	0.688	(1.449)	1.190	(3.378)
V	1.194	(4.111)	0.988	(2.031)	1.414	(3.321)
V – G	0.382	(1.620)	1.062	(3.090)	0.736	(2.233)
			Momentum-Ranked Portfolios			
W	0.235	(0.555)	0.380	(0.659)	0.086	(0.186)
2	0.660	(2.215)	0.648	(1.326)	0.401	(0.986)
3	0.585	(2.073)	0.368	(0.802)	0.975	(2.579)
4	0.768	(2.679)	0.572	(1.277)	0.651	(1.716)
L	0.962	(2.591)	0.663	(1.358)	1.174	(2.894)
W – L	0.727	(1.937)	0.283	(0.576)	1.088	(2.846)

Table III

Test of the EMH and the CAPM, 1981-2000

The table summarizes the results of the estimation of the following system:

$$s_{pt} = \tau_p + \rho_{pm} s_{mt} + u_{pt}, \qquad p = 1 \ldots 5, \qquad (15)$$

where s_p is the standardized excess return of the size-, BEME-, or momentum-ranked portfolios and s_m is the market excess return. The system is estimated with GLS. The results are presented for each country (the US, Japan, and the UK) for 1981:1-2000:12. Below each estimated parameter is reported t-statistic (in parenthesis). The line labeled "CAPM" reports the Wald statistic of the test of the CAPM ($\delta_p = 1$), where δ_p is the squared correlation coefficient between the portfolio and the market factor. The associated p-value is reported [in brackets]. The data are from DATASTREAM. All returns are denominated in US dollars.

Estimate	The United States			Japan			The United Kingdom		
				Panel A. Size-ranked portfolios					
	S	B	S - B	S	B	S - B	S	B	S - B
τ_p	0.108 (1.893)	0.003 (0.896)	0.104 (1.767)	0.051 (1.274)	0.000 (0.017)	0.051 (1.050)	0.118 (2.352)	-0.000 (-0.048)	0.118 (2.261)
ρ_{pm}	0.503 (8.987)	0.999 (294.582)	-0.495 (-8.497)	0.786 (19.658)	0.989 (101.079)	-0.202 (-4.169)	0.637 (12.741)	0.999 (368.680)	-0.362 (-6.950)
R2 (%)	25.33	99.73	23.28	61.83	97.72	6.80	40.58	99.82	16.90
CAPM	175.40 [0.000]	0.16 [0.687]	170.77 [0.000]	36.73 [0.000]	1.39 [0.239]	2390.30 [0.000]	87.12 [0.000]	0.10 [0.747]	531.23 [0.000]
				Panel B. BEME-ranked portfolios					
	G	V	V - G	G	V	V - G	G	V	V - G
τ_p	-0.009 (-0.476)	0.114 (3.157)	0.123 (2.479)	-0.074 (-3.785)	0.073 (2.236)	0.147 (3.204)	-0.006 (-0.243)	0.105 (2.761)	0.112 (2.123)
ρ_{pm}	0.953 (48.637)	0.836 (23.481)	-0.117 (-2.391)	0.953 (48.634)	0.865 (26.524)	-0.089 (-1.929)	0.915 (35.072)	0.811 (21.399)	-0.104 (-1.992)
R2 (%)	90.88	69.87	2.35	90.87	74.77	1.54	83.79	65.77	1.64
CAPM	5.97 [0.015]	25.65 [0.000]	7330.50 [0.000]	5.98 [0.015]	20.08 [0.000]	14923.00 [0.000]	11.51 [0.001]	30.96 [0.000]	8181.70 [0.000]

Table III – *Continued*

Estimate	The United States			Japan			The United Kingdom		
	Panel C. Momentum-ranked portfolios								
	L	W	W - L	L	W	W - L	L	W	W - L
τ_p	-0.103 (-2.442)	0.010 (0.318)	0.114 (1.849)	-0.014 (-0.415)	0.027 (0.951)	0.042 (0.706)	-0.096 (-2.476)	0.072 (2.128)	0.168 (2.952)
ρ_{pm}	0.767 (18.431)	0.866 (26.635)	0.099 (1.635)	0.845 (24.430)	0.897 (31.240)	0.051 (0.870)	0.805 (20.897)	0.856 (25.542)	0.051 (0.905)
R^2 (%)	58.79	74.93	1.11	71.45	80.39	0.32	64.73	73.21	0.34
CAPM	41.72 [0.000]	19.90 [0.000]	6843.80 [0.000]	23.78 [0.000]	14.52 [0.000]	27164.00 [0.000]	32.42 [0.000]	21.77 [0.000]	29845.00 [0.000]

Table IV

Test of the EMH and the FFPM with Size-Ranked Portfolios, 1981-2000

The table summarizes the results of the estimation of the following system:

$$s_{pt} = \tau_p + \rho_{pm} s_{mt} + \rho_{ps} s_{st} + \rho_{ph} s_{ht} + \rho_{pw} s_{wt} + u_{pt}, \qquad p = 1...5, \qquad (16)$$

where s_p is the standardized excess return of the size-, BEME-, or momentum-ranked portfolios, and s_m, s_b, and s_w are respectively the market factor, SMB, HML, and WML. The system is estimated with GLS. The results are presented for each country (the United States, Japan, and the United Kingdom) for 1981:1-2000:12. Below each estimated parameter is reported *t*-statistic (in parenthesis). The column labeled "FFPM" reports the Wald statistic of the test of the FFPM ($\delta_p = \sum_k \rho_{pk}^2 = 1$; $k = m,s,b,$ and w). The associated *p*-value is reported [in brackets]. The data are from DATASTREAM. All returns are denominated in US dollars.

	The United States			Japan			The United Kingdom		
	Panel A. Size-ranked portfolios								
Portfolio	S	B	S - B	S	B	S - B	S	B	S – B
τ_p	0.106 (4.164)	0.008 (3.644)	0.098 (3.894)	0.024 (1.623)	0.002 (0.451)	0.022 (1.516)	0.018 (0.751)	0.004 (2.664)	0.013 (0.568)
ρ_{pm}	0.503 (22.671)	0.999 (494.373)	-0.495 (-22.722)	0.786 (54.989)	0.989 (263.613)	-0.202 (-14.24)	0.637 (30.335)	0.999 (698.687)	-0.362 (-17.243)
ρ_{ps}	0.790 (35.574)	-0.039 (-19.455)	0.829 (38.030)	0.573 (40.073)	-0.139 (-37.184)	0.712 (50.175)	0.681 (32.408)	-0.034 (-23.860)	0.715 (34.033)
ρ_{ph}	0.061 (2.742)	-0.015 (-7.391)	0.076 (3.477)	0.073 (5.083)	0.004 (1.058)	0.069 (4.839)	0.158 (7.509)	-0.010 (-7.168)	0.168 (7.997)
ρ_{pw}	-0.059 (-2.671)	0.002 (0.822)	-0.061 (-2.796)	0.000 (0.002)	0.002 (0.663)	-0.002 (-0.173)	0.048 (2.276)	0.001 (0.587)	0.047 (2.236)
R^2 (%)	88.42	99.90	89.39	95.20	99.67	92.07	89.61	99.95	86.63
FFPM	7.69 [0.006]	0.06 [0.812]	1.87 [0.172]	2.96 [0.085]	0.19 [0.659]	444.78 [0.000]	6.81 [0.009]	0.03 [0.866]	90.47 [0.000]

Table IV – *Continued*

	The United States			Japan			The United Kingdom		
	Panel B. BEME-ranked portfolios								
	G	V	V - G	G	V	V - G	G	V	V – G
τ_p	0.053 (2.986)	-0.054 (-2.177)	-0.107 (-3.291)	-0.033 (-2.033)	-0.008 (-0.431)	0.024 (0.885)	0.022 (0.885)	-0.032 (-0.995)	-0.054 (-1.318)
ρ_{pm}	0.953 (62.306)	0.836 (38.521)	-0.117 (-4.148)	0.953 (61.499)	0.865 (46.739)	-0.089 (-3.317)	0.915 (40.865)	0.811 (28.759)	-0.104 (-2.868)
ρ_{ps}	0.023 (1.509)	0.059 (2.701)	0.036 (1.255)	0.034 (2.175)	0.106 (5.705)	0.072 (2.690)	-0.004 (-0.183)	0.082 (2.917)	0.086 (2.372)
ρ_{ph}	-0.185 (-12.099)	0.432 (19.894)	0.617 (21.796)	-0.182 (-11.743)	0.379 (20.463)	0.561 (20.995)	-0.196 (-8.742)	0.382 (13.553)	0.578 (15.880)
ρ_{pw}	0.038 (2.515)	-0.021 (-0.975)	-0.060 (-2.107)	-0.017 (-1.085)	-0.131 (-7.072)	-0.114 (-4.270)	0.075 (3.365)	0.044 (1.575)	-0.031 (-0.850)
R^2 (%)	94.50	88.90	68.00	94.32	91.92	66.95	88.20	81.25	53.23
FFPM	3.42 [0.065]	7.34 [0.007]	283.17 [0.000]	3.54 [0.060]	5.16 [0.023]	447.47 [0.000]	7.86 [0.005]	13.55 [0.000]	223.79 [0.000]
	Panel C. Momentum-ranked portfolios								
	L	W	W - L	L	W	W - L	L	W	W – L
τ_p	0.006 (0.184)	-0.029 (-1.240)	-0.034 (-1.010)	-0.016 (-1.222)	0.030 (1.947)	0.046 (2.545)	0.000 (0.009)	-0.018 (-0.662)	-0.018 (-0.522)
ρ_{pm}	0.767 (28.933)	0.866 (43.067)	0.099 (3.353)	0.845 (67.085)	0.897 (60.173)	0.051 (2.949)	0.805 (29.363)	0.856 (35.504)	0.051 (1.643)
ρ_{ps}	0.133 (5.037)	0.203 (10.081)	0.069 (2.345)	0.265 (21.009)	-0.108 (-7.279)	-0.373 (-21.447)	0.074 (2.710)	0.193 (8.010)	0.119 (3.820)
ρ_{ph}	0.132 (4.991)	-0.160 (-7.959)	-0.292 (-9.906)	0.082 (6.480)	-0.085 (-5.699)	-0.167 (-9.572)	0.172 (6.261)	-0.114 (-4.739)	-0.286 (-9.188)
ρ_{pw}	-0.461 (-17.392)	0.299 (14.852)	0.759 (25.743)	-0.414 (-32.88)	0.353 (23.721)	0.768 (44.123)	-0.376 (-13.726)	0.285 (11.808)	0.661 (21.243)
R^2 (%)	83.56	90.51	76.81	96.29	94.77	91.41	82.37	86.34	70.20
FFPM	11.56 [0.001]	6.16 [0.013]	44.43 [0.000]	2.27 [0.132]	3.24 [0.072]	63.02 [0.000]	12.58 [0.000]	9.29 [0.002]	104.69 [0.000]

Table V

Summary Descriptive Statistics on the factors, 1981-2000

For each country, we report the summary descriptive statistics on SMB, HML, and WML for 1981:1-2000:12. Their construction as well as their orthogonalization is described in Appendix B. The statistics presented are the average raw returns with the t-statistic (in parenthesis), average orthogonalized returns with the t-statistic (in parenthesis), and the correlation matrix between the factors. The data are from DATASTREAM. All returns are denominated in US dollars and are measured in percent per month.

Variable	Raw return		Orthogonalized return		Correlation matrix (%)		
	Mean	(*t*-statistic)	Mean	(*t*-statistic)	r_m	SMB	HML
Panel A. The United States							
SMB	-0.090	(-0.344)	-0.013	(-0.052)	-0.104		
HML	0.755	(4.232)	0.916	(6.317)	-0.331	-0.442	
WML	0.764	(3.325)	1.195	(5.450)	0.001	0.010	-0.247
Panel B. Japan							
SMB	0.097	(0.337)	0.083	(0.289)	0.046		
HML	0.568	(3.276)	0.601	(3.523)	-0.178	-0.022	
WML	-0.041	(-0.124)	0.246	(0.826)	-0.110	-0.400	-0.152
Panel C. The United Kingdom							
SMB	-0.003	(-0.010)	0.175	(0.769)	-0.351		
HML	0.838	(4.465)	0.862	(4.667)	0.001	-0.162	
WML	0.943	(4.255)	1.294	(6.248)	-0.059	0.129	-0.347

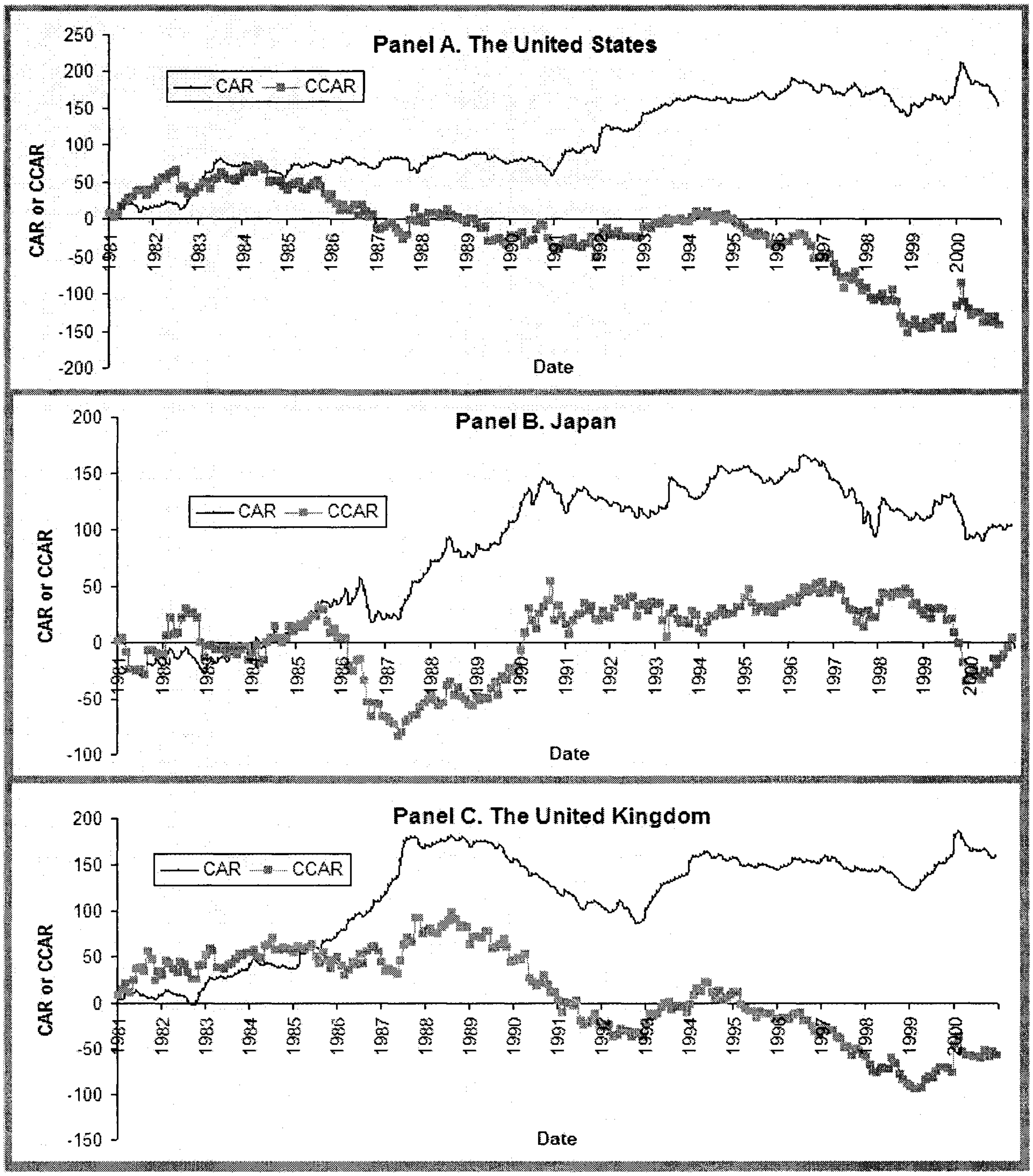

Figure 1. Plots of the CAR and CCAR of the Size Investment Strategy, 1981–2000. The cumulative abnormal return (CAR, equation (21)) is computed by adding the abnormal returns obtained on the auto-financed portfolio that is long on the small portfolio and short on the big portfolio. The corrected CAR (CCAR, equation (22)) is similar to the CAR except that the CAPM is restricted to theoretically hold (i.e., the benchmark is constructed to match the portfolio volatility). The sample period runs from January 1981 to December 2000.

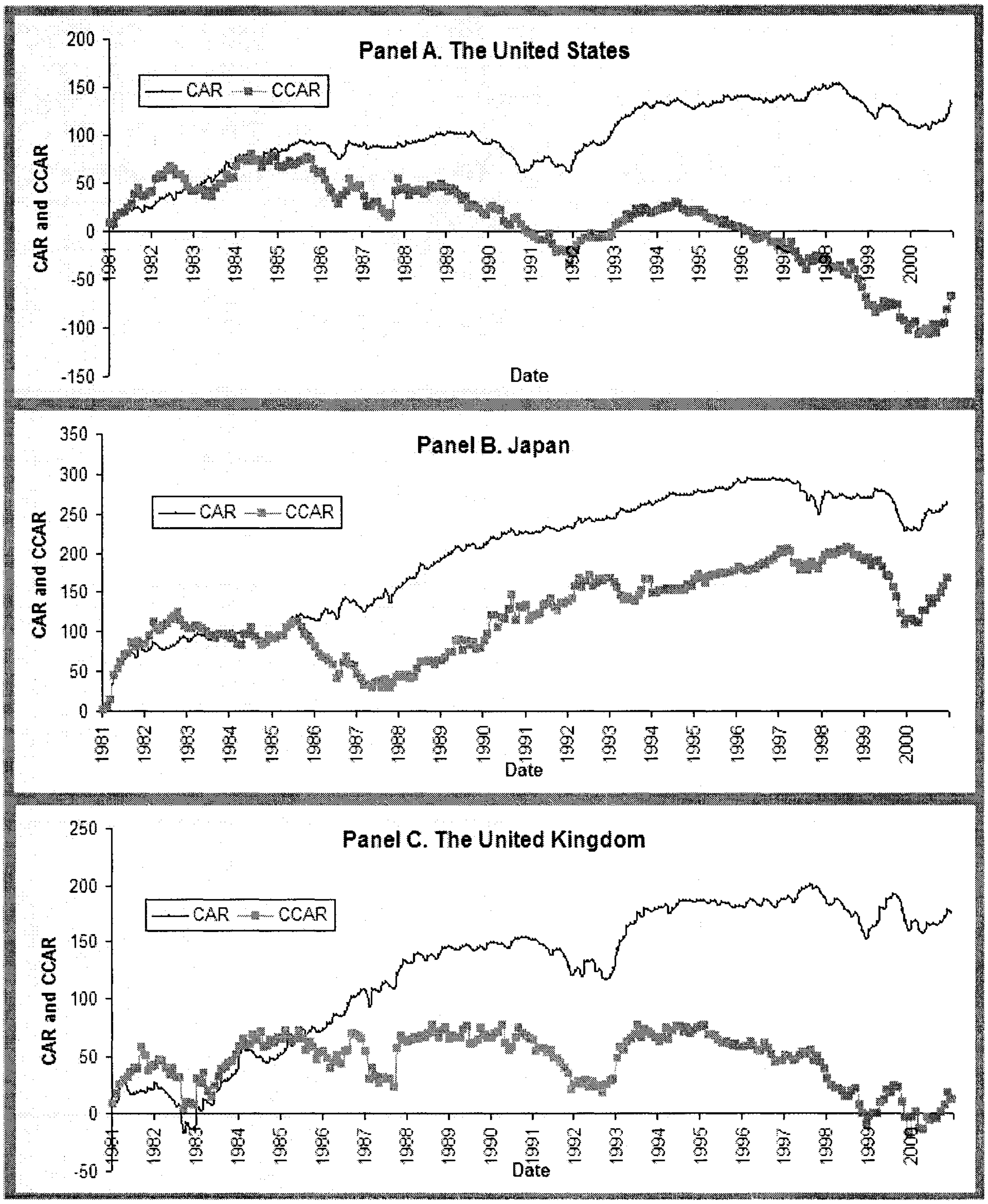

Figure 2. Plots of the CAR and CCAR of the Value Investment Strategy, 1981–2000. The cumulative abnormal return (CAR, equation (21)) is computed by adding the abnormal returns obtained on the portfolio that is long on the value portfolio and short on the growth portfolio. The corrected CAR (CCAR, equation (22)) is similar to the CAR except that the CAPM is restricted to hold. The sample period runs from January 1981 to December 2000.

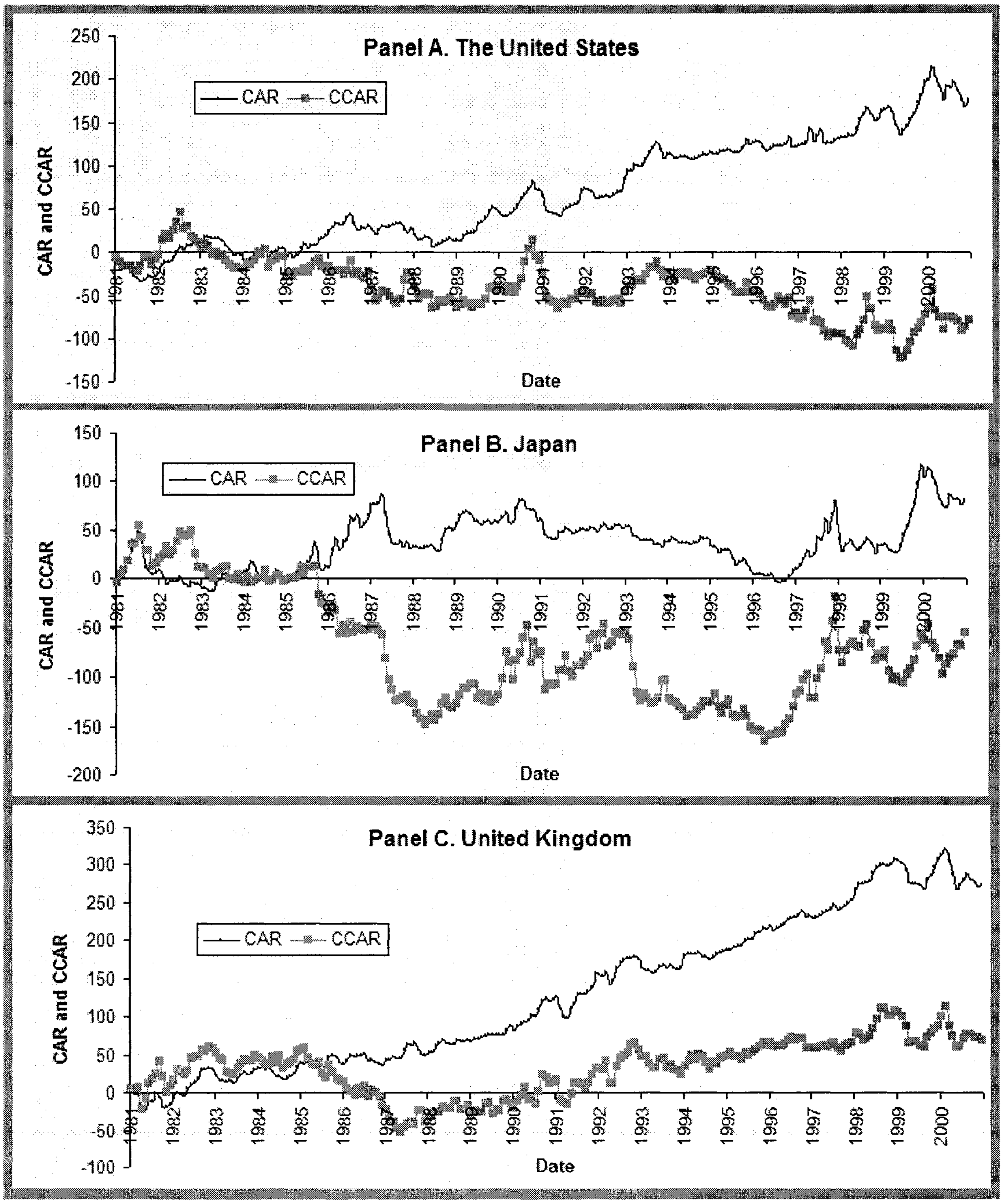

Figure 3. Plots of the CAR and CCAR of the Momentum Investment Strategy, 1981–2000. The cumulative abnormal return (CAR, equation (21)) is computed by adding the abnormal returns obtained on the portfolio that is long on the winner portfolio and short on the loser portfolio. The corrected CAR (CCAR, equation (22)) is similar to the CAR except that the CAPM is restricted to hold. The sample period runs from January 1981 to December 2000.

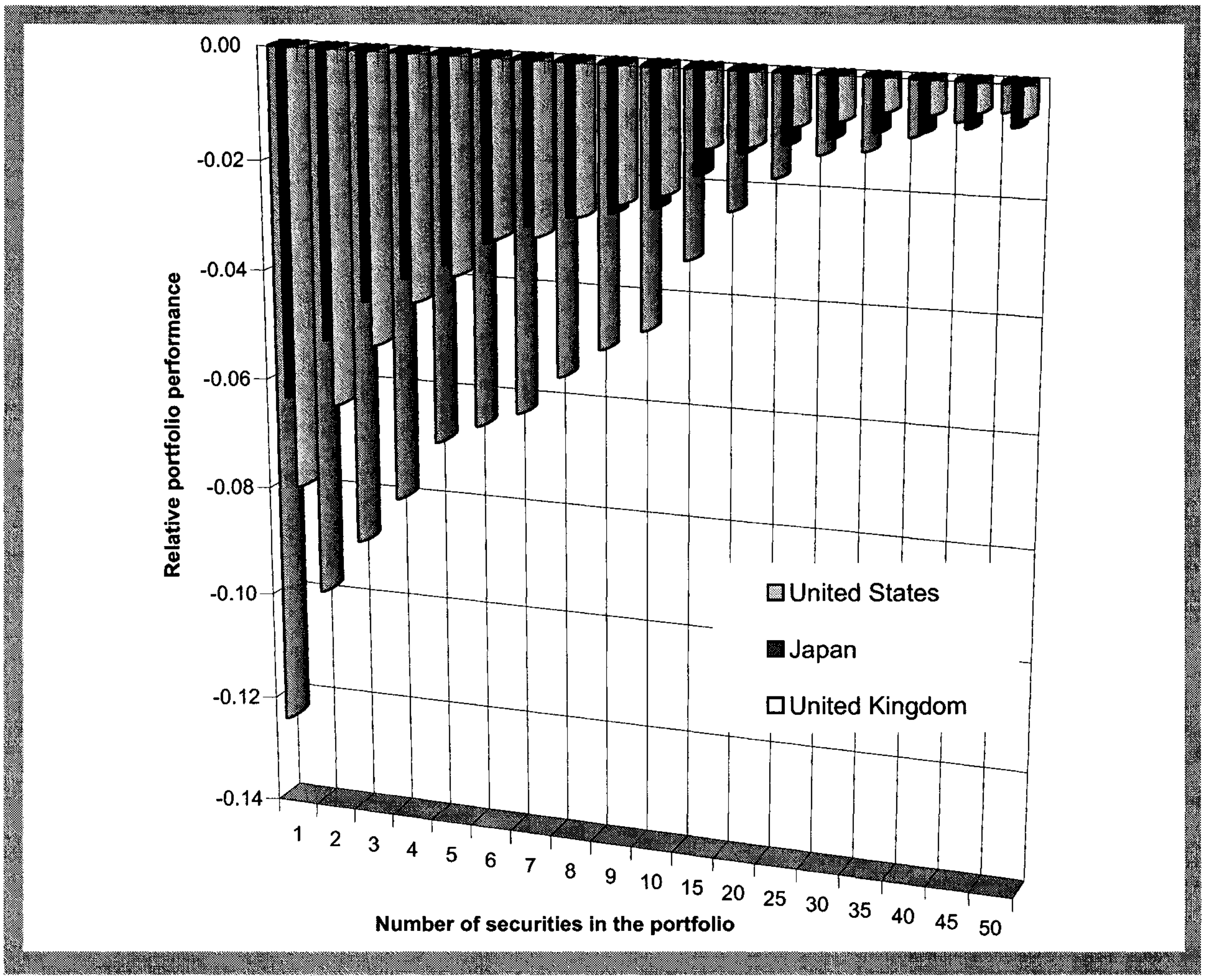

Figure 4. Plots of the Abnormal Performance in Function of the Number of Securities in the Portfolio, 1981–2000. The plot shows for each country (the United States, Japan, or the United Kingdom) the evolution of relative portfolio corrected abnormal performance (τ_p^*) as a function of the number of securities in the portfolio. The data are from DATASTREAM. The sample period runs from January 1981 to December 2000. All returns are denominated in US dollars.

CHAPTER IV:

SUMMARY AND CONCLUSION

This thesis contributes to the important literatures on international asset-pricing and market efficiency. It comprises two essays, of which each proposes a new theory and provides empirical evidence. More specifically, the thesis seeks to answer the following fundamental questions:

(1) What type of risk-return relation should we expect in an integrated global capital market characterized by purchasing power parity deviations when the higher moments of returns are priced?

(2) Do systematic risk terms beyond market and inflation covariance enter the pricing process?

(3) Are these new components of expected return systematically priced in the international markets?

(4) How important are these new components on the variation of risk premia?

(5) What is the consequence of the joint-hypothesis on the tests of market efficiency?

(6) Can the efficiency of international markets be tested without joint-hypothesis?

(7) Are the existing methods of performing event studies and measuring performance misspecified? How?

(8) Can these methods be corrected?

(9) Are the so-called size, value, and momentum anomalies in the international markets real?

Questions 1 to 4 are addressed in the first essay of the thesis (Chapter II). The main contribution is to develop a three-moment international asset-pricing model (TM-IAPM) under full integration and deviations from purchasing power parity (PPP) that prices coskewness. The model is shown to embed the standard IAPMs as special cases when explicit restrictions are imposed (when coskewness is not priced).

We further apply the model to investigate the time-series behavior of market, size, value, and momentum premiums in the United States, Japan, and the United Kingdom equity markets. The evidence demonstrates that the nonlinear model performs quite well and explains most of the variation of these premiums during the 1980s and 1990s. The results also show that coskewness risk is more important than covariance risk in sample or out-of-sample. The implication of these results is that linear IAPMs are misspecified and that investors should use nonlinear models to price international assets.

The second essay of the thesis (Chapter III) addresses questions 5 to 9. The main contribution of the essay is to propose a way to disentangle the test of market efficiency from the test of the equilibrium model, thus resolving the well-known joint-hypothesis problem. This breakthrough comes from the application of the scale-independence property inherent in Sharpe ratios to build a setting in which the efficiency of the market and the validity of the postulated equilibrium models can distinctly be tested.

We apply the new approach to examine the efficiency of the United States, Japan, and the United Kingdom markets over the period 1981-2000 relative to the most robust anomalies, namely the size, value, and momentum effects. The evidence highlights the worse scenario for the traditional tests of market efficiency: it is the rule rather than the exception that the postulated equilibrium model fails, suggesting the presence of important spurious abnormal returns. Given this, we conclude the rejection of the efficient market hypothesis based on the size, value, and momentum effects is premature. We also show how the new framework can be used to correct the existing methods of performing event studies and measuring performance. These corrections take into account the potential failure of the postulated equilibrium model in the test of market efficiency. We finally illustrate the implications of our results for event studies by identifying the shortcoming inherent in the existing measure of cumulative abnormal returns and the way to correct it.

It is hoped that these developments will stimulate further researches on the efficiency of the international markets.