# COMPUTER IDENTIFICATION AND CONTROL OF A HEAT EXCHANGER

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# ABSTRACT

The heat exchanger under study is an experimental model of the shell and tube heat exchanger. The self-tuning regulator algorithm of Åström and Wittenmark [17] is implemented on a digital computer, with the object of controlling the water temperature by manipulating the steam flow rate. The algorithm consists of an on-line least-squares parameter identification of a linear model, combined with a minimum variance control tuned according to the parameter estimates of the model.

Instrumentation and characteristics of the steady-state, dynamics, and noise of the heat exchanger are described. A model of the heat exchanger is obtained from a step response test for the purpose of comparison with the model obtained from the least-squares identification algorithm.

The algorithm does not determine the order of the plant model. This must be decided a priori. Other a priori decisions include the choice of a suitable sampling interval, an estimate of the noise variance, and the starting value of the model parameters. The results of experimental runs with variations in all of these are presented. Convergence difficulties in the parameter identification are encountered in some cases.

# RESUME

L'échangeur de chaleur sous consideration est un modèle expérimental du type vapeur-eau. L'algorithme de contrôle de Åström et Wittenmark [17] est appliqué à un petit ordinateur. L'algorithme consiste en un estimateur des moindres carrés d'un modèle linéaire avec un régulateur à variance minimum calculé sur le modèle estimé. L'objet est de contrôler la température de l'eau par la manipulation du débit de vapeur.

Les caractéristiques en régimes permanent et dynamique sont décrites, ainsi que celles du bruit. Un modèle de l'échangeur est obtenu a partir de la réponse a l'échelon pour fin de comparaison avec les résultats de l'algorithme.

L'algorithme ne determine pas l'ordre du modèle, qu'il faut décider a priori. D'autres décisions a priori sont: le choix d'un intervalle d'échantillonage, la variance du bruit, et les valeurs initiales des paramètres du modèle. Les résultats d'expériences avec variations dans chacun de ces quantités sont présentés. On a observé des difficultés de convergence de l'algorithme en quelques cas.

# ACKNOWLEDGEMENT

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# CHAPTER 1

# INTRODUCTION

## 1.1 Survey of Identification

Adaptive control is the subject of a great deal of current research. As yet, no general framework has been developed for the design of the adaptive controller. At present, the adaptive control system usually consists of two sub-systems: an identifier that determines system parameters, and a controller whose gains depend on the identified parameters.

Several problems arise from the use of this identifier/controller structure. One difficulty is that in order that the identification be accurate and complete, the control signal must excite all modes of the system. Furthermore, an identifier that works in open-loop or with a particular controller in closed-loop may not work with some other controller; with some controllers the identifier may not converge or may produce biased parameter estimates. Still another complaint of the present state of adaptive control, as pointed out by Tse and Athans [1], is that the adaptive controller should use not only the instantaneous estimates of the parameters but also the uncertainty connected with the estimates. Because of the lack of a framework, the approach to these problems is not unified, as is evidenced by the widely differing methods of identification.

The experimental work presented in this paper is connected with one particular identifier/controller combination. Before describing the nature of the experiment, a brief survey of the methods of system identification follows. The emphasis of the review is on the philosophy behind the various methods. The purpose is to put in perspective the particular identification/control scheme chosen here, hence also the experiment, within the adaptive control field.

There are a number of identification methods that have been developed. Among the most popular in the North American literature are the Bayesian, the Correlation methods, the Extended Kalman Filter, the Stochastic Approximation methods, and the Least Squares method.

<u>Bayesian</u>: In Bayesian estimation a probabilistic approach is taken. The solution of the estimation problem consists of the probability density of the unknown. From this density an optimal estimate is made by taking the mean (minimum variance estimate), mode (maximum likelihood estimate), etc., depending on the loss function chosen.

Rather than producing an empirical probability density, the Bayesian estimator fine-tunes an assumed form of the density function. That is, a model of the unknown distribution is assumed, as well as an a priori distribution of the unknown model parameters. Via Bayes' rule, the Bayesian estimator modifies the distribution of the unknown parameters on the basis of observed data.

A feeling for Bayesian estimation can be obtained by looking at Bayes' rule. Suppose a quantity x is observed and a model of the distribution of x is assumed. The model contains an unknown parameter $\theta$. The desired distribution is $f(x|\theta)$ with $\theta$ known. In order to get $\theta$, the problem is turned around via Bayes' rule to get the distribution of $\theta$ based on observed values of x:

$$f(\theta|x) = \frac{f(x|\theta)\,f(\theta)}{\int_{-\infty}^{\infty} f(x|\theta)\,f(\theta)\,d\theta}$$

$f(x|\theta)$, whose form is assumed, is called the likelihood function. $f(\theta)$ is the assumed a priori distribution of $\theta$, whereas $f(\theta|x)$ is the a posteriori distribution. A recursive computation is possible by substituting the a posteriori distribution for the a priori distribution after every observation of x.

The obvious difficulty with the method is the problem of choosing the likelihood function and the a priori distribution (of unknown θ). Convergence requires that the likelihood function be a reasonable approximation of the true distribution of x. Furthermore, the computation is lengthy unless the likelihood function and the a priori distribution are chosen so that the a priori and a posteriori distributions are of the same form.

General presentations of Bayesian estimation can be found in [2] and [3]. In [4] Lainiotis derives a Bayesian recursive algorithm where the unknown distribution is modelled as an expansion of a set of orthogonal functions. The algorithm determines the coefficients of the expansion. The choice of the orthogonal basis is left as an open problem. In [5] Lainiotis presents an estimator that consists of a bank of Kalman filters matched to every possible value of a quantized set of the unknown θ. These filters produce the conditional estimates $f(x|\theta)$, and their outputs are weighted by the a posteriori probabilities. Computer simulations of the estimator were successful. Obviously, however, accuracy requires fine quantization. To meet this problem, Sengbush and Lainiotis [6] have proposed and tested a method that simplifies the parameter quantization.

<u>Correlation methods</u>: The idea behind the correlation methods is to extract the system parameters

from correlations of the output. The autocorrelation of the output is observed. Then equations relating the autocorrelation to the system parameters are solved for the unknown parameters. A more efficient method is obtained by using the autocorrelation of the innovation (one-step-prediction-error) sequence rather than that of the output directly. (The innovation is the difference between the actual output and the output predicted from the previous sample interval according to the estimated system.) The reason for the increased efficiency is that the innovation sequence is less correlated than the output. In fact, as the estimation converges, the innovation becomes more and more white. For both output and innovation correlation methods, the estimates are asymptotically unbiased, and higher efficiency is obtained by using higher order correlations.

In [7] Mehra derives two recursive algorithms, one based on output correlation, the other on innovation correlation. Both algorithms determine the covariance matrices of the process and measurement noises of a system. In [8, 9] Mehra presents an algorithm that identifies a system in canonical form. He also proposes a recursive method for determining the order of the system when it is not known. The method is based on testing the innovation sequence for whiteness.

Extended Kalman Filter: An optimal estimate of the state is given by the Kalman filter, whose design requires exact knowledge of the system parameters and noise statistics. The basis of the Extended Kalman Filter is to adjoin the unknown parameter vector to the state vector and design a Kalman filter for a linearized version of the augmented system, thereby obtaining estimates of both the state and parameters. The nonlinear augmented system is linearized by retaining only the first order terms of a Taylor series expansion about the augmented state vector.

Difficulties of the method lie in the requirement of exact noise statistics, and problems in programming and bias error analysis. Treatment of the Extended Kalman Filter (including filter equations) can be found in Jazwinski [3] and Saridis [10]. Convergent computer simulations are reported in [10].

Stochastic Approximation methods: The stochastic approximation methods essentially are hill-climbing search techniques suitably modified to operate in a stochastic environment. The identification problem is posed as a minimization of a loss function V of the parameters $\beta$. The solution is obtained through an iterative scheme of the form

$$\beta_{k+1} = \beta_k + (\text{gain})_{k+1} * (\text{error correction})_k \quad (1.1)$$

In a deterministic problem, the error correction term may be a negative gradient, and the gain term may be a constant (gradient method), a direction of steepest descent (steepest descent method), an inverse Hessian (Newton's method), etc. The significant modification found in the stochastic approximation is that account is taken of degradation of calculations arising from noisy data. The major modification lies in the gain factor of Equation 1.1. The gain may be a scalar or a matrix, the latter case having faster convergence at the expense of additional computation. The gain may be a prescribed sequence, such as the series 1/k in the scalar case; on the other hand, the algorithm may possess a learning property that maintains a best direction of search.

The main advantage of the stochastic approximation algorithms is their great simplicity and ease of implementation. Hence, although the method is not particularly noted for speed of convergence, it is a popular method for on-line identification. It should be noted, however, that in certain cases, the parameter estimates may be biased unless additional bias correction terms appear in Equation 1.1.

A good survey on stochastic approximation.

methods can be found in a paper by Saridis [11]. Specific open-loop algorithms are presented by Saridis and Stein in [12, 13]. These algorithms are extended by Saridis and Lobbia [14] to the closed-loop case; that is, a controller using estimates of the parameters is involved. In this case, unbiased estimates are produced in spite of the additional bias problems caused by feedback-induced correlations. Successful computer simulation results are presented for both the open-loop and closed-loop algorithms.

Least Squares method: The least squares method is the result of the solution of a regression-type problem. The problem is to fit a linear regression to an input-output history, the regression being, for instance, a system model in canonical form:

$$y(k) + a_1 y(k-1) + \ldots + a_n y(k-n) = b_1 u(k-1) + \ldots + b_n u(k-n) + e(k)$$

The e(k) can be interpreted as noise or simply as residuals (equation errors). As in the stochastic approximation method, the basis of the least squares approach is a minimization; specifically, the minimization of the sum of the squares of the equation errors. The problem formulation and solution, as given in Åström and Eykhoff [15], are briefly reviewed in Chapter 3 of this paper. It is sufficient to say here that a recursion formula for the solution is obtainable and it takes the form of Equation 1.1, and thus appears similar to a stochastic approximation algorithm. The distinction is that the error-correction factor is the one-step-prediction-error, a scalar; and the gain factor is a "learning" vector sequence. If the equation errors are correlated, the parameter estimates will be biased. Several ways of dealing with this problem have been developed. For example, in the method of generalized least squares [15, 16], uncorrelated residuals are obtained from a filter obtained from an autoregression fit of the residuals of an ordinary least squares fit.

Many of the references mentioned thus far provide simulation test results. Direct comparisons of the various on-line identification methods, however, are relatively rare and quite recent. Two such papers are by Saridis [10] and by Isermann et al [16]. The results in [16] lead the authors to the interesting suggestion that all the successful on-line methods that use the same a priori knowledge of the process have about the same performance over a long time period. This leaves the main advantages of the methods to depend on the process; for example, on any special use of a priori knowledge that can be made, on the type of input signal involved, on programming and computational expense, etc.

## 1.2 Statement of the Problem

The experiment presented in this paper tests the particular identifier/controller combination as presented by Åström and Wittenmark [17]. Specifically, this combination consists of an ordinary recursive least squares identification [15] with a minimum variance control computed according to the identified parameters. The result may be viewed as a self-tuning controller. It should be mentioned that although there are quite a number of on-line open-loop identification algorithms, successful closed-loop algorithms are relatively rare. The self-tuning control algorithm in [17] has been tested by extensive simulation studies by the authors of [17], and the algorithm is easily implemented on a mini-computer.

It has been mentioned that a successful open-loop identification algorithm will not necessarily work in closed-loop. This is the case here, where, although open-loop least squares convergence is guaranteed by theory, simulations indicate that the closed-loop self-tuning algorithm does not always converge. Moreover, if the input and noise are correlated, which is the case in closed-loop, the least squares parameter estimates will be biased. Furthermore, for this particular algorithm, it is not always possible to identify all the parameters. This problem, which will be explained in Chapter 3, can be circumvented, however, by fixing one parameter.

A very interesting theoretical result is that if the algorithm converges, it will lead to the optimal minimum variance control even if the parameter estimates are biased [17]. Thus, the bias problem is neatly eliminated here. If the order of the model is too low, however, the algorithm will not lead to a minimum variance control if it converges.

On the whole, the simulation studies show that the algorithm does converge in many cases. Very often, the difference between convergence or divergence depends simply on the initial guess of the parameter values or on the value of the fixed parameter.

In the relatively small number of experimental industrial applications, the algorithm is not without success. In [18] is reported a successful application on a moisture content control loop in a paper mill in Sweden. Models of both third and fourth order were used. In this particular experiment, the algorithm exhibited good start-up and steady-state control, no convergence difficulties, and moderate insensitivity to model order and initial parameter values. A second successful industrial application of the algorithm is reported in [19]. In this case, the plant was an ore crusher.

It is the object of the experiment to further test the self-tuning control algorithm of [17] by applying it to another real-life plant, namely a steam-water heat exchanger. A quantitative performance evaluation is not undertaken. Instead, the questions asked are very basic: Does the algorithm converge? Can conditions be found that predictably affect convergence? The latter question has not yet been satisfactorily answered either by theory or by experiment.

# CHAPTER 2

## The Heat Exchanger System

### 2.1 Introduction

The heat exchanger is a device commonly used to transfer heat from one medium to another. The heat exchanger under study is an experimental model of the shell and tube type. Steam and water are the two media involved in the heat transfer. Within this heat exchanger the water flows through a bundle of seven 1" copper tubes 62" long. This set of tubes is enclosed within a shell of 6" steel pipe and the shell is surrounded by 1" thick insulation. Steam is allowed to flow into the shell and surround the water-carrying copper tubes. As the steam condenses, its latent heat of vaporization is transferred to the water through the copper tube walls. (See [20] for a theoretical study of this heat exchanger.)

In the experiment described in this paper the controlled variable is the temperature of the water flowing out of the heat exchanger (to be called output water temperature). The controlling variable is the steam flow rate. Disturbances exist in the form of water flow rate variations, temperature measurement noise, and temperature set-point changes. The object is to link the steam flow rate to the output water temperature by a sampled-data computer feedback system.

The feedback link is an algorithm that implements the following control law:

$$U(k) = A_1U(k-1) + A_2U(k-2) + A_3U(k-3) + A_4U(k-4)$$
$$+ B_0y(k) + B_1y(k-1) + B_2y(k-2) + B_3y(k-3) + B_4y(k-4)$$

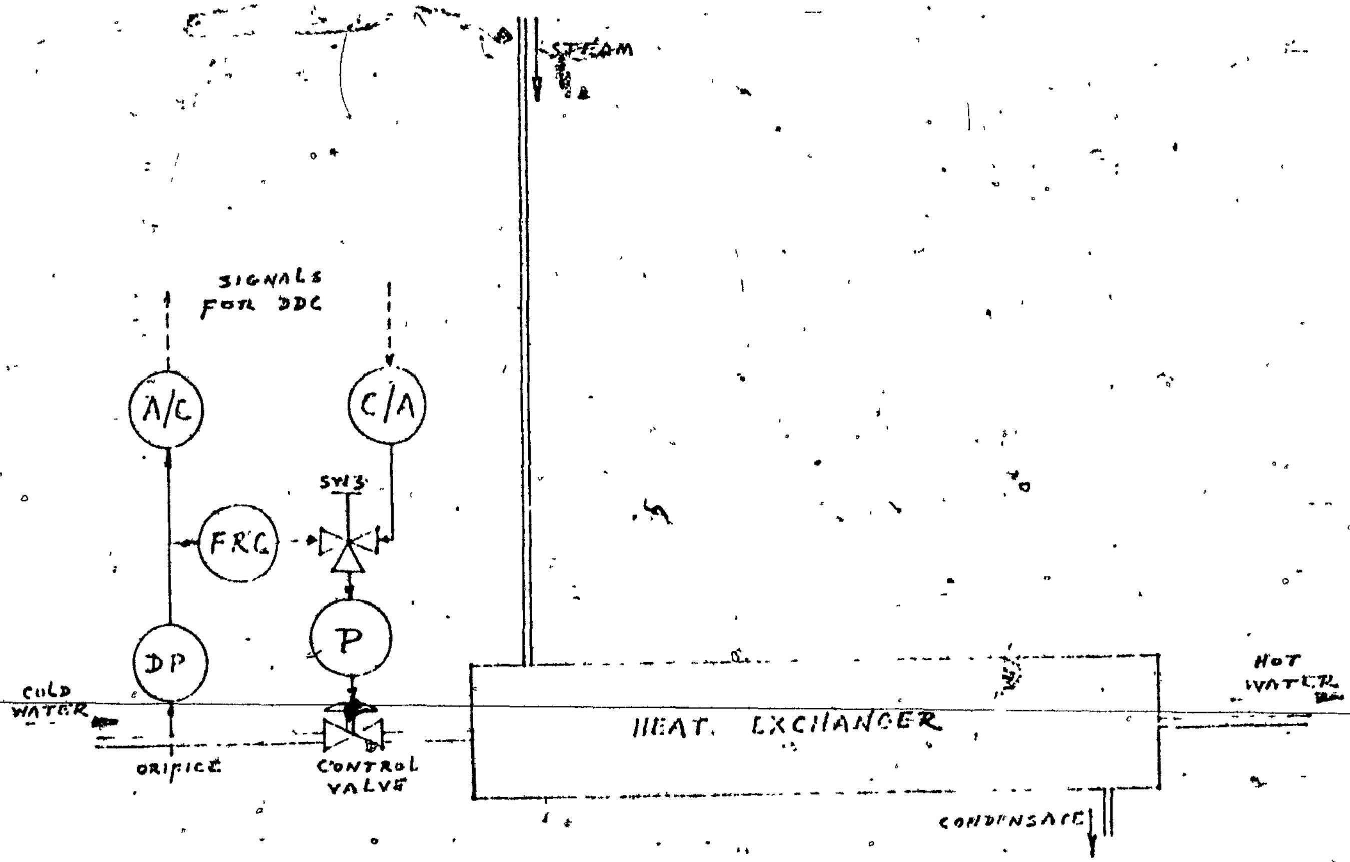
where U(k) is the control signal (steam flow rate), and y(k) is the output error signal (output water temperature minus set-point). The choice of coefficients $A_i$ and $B_i$ depends on the plant (heat exchanger) dynamics and the desired system performance. In this experiment, the parameters of a linear model of the plant are identified on-line by digital computer. These parameters are used to compute the coefficients $A_i$ and $B_i$ of the control law according to a particular control policy. Specifically, the goal here is to minimize the variance of the output error. Since the identified model parameters are used to set the coefficients of the control law, the result is a self-tuning controller.

## 2.2 Water Flow Control

Figure 2.1 shows a block diagram of the water flow control system. The flow measurement takes place at the orifice, which essentially is a constriction in the piping. This constriction causes a pressure drop across the orifice. The pressure difference is proportional to the square of the

FIGURE 2.1

WATER FLOW CONTROL SYSTEM




FRC = Flow Recorder-Controller
DP = Differential Pressure Cell
P = Positioner
A/C = Air-to-Current Converter
C/A = Current-to-Air Converter

flow. This pressure difference signal is applied to a pneumatic differential pressure cell (dp cell), which converts the 0 to 100% differential water pressure to a 3 to 15 psi air pressure signal. This is the input to the pneumatic PID flow controller (FRC). The controller output drives a pneumatic positioner which positions the control valve stem with the aid of mechanical feedback.

The system is equipped with instrumentation that allows direct digital control (DDC) of the water flow. An air-to-current converter, whose output is bridged by a suitable resistor, provides the water flow measurement (see Figure 2.1). The computer control signal is applied to a current-to-air converter which then drives the positioner. The setting of switch-valve SW3 determines whether the positioner and control valve are driven by the pneumatic controller or by DDC.

The pneumatic controller did not produce good water flow control. A persistent limit cycle existed, even at minimum gain settings of the controller and positioner. The magnitude of the limit cycle was smallest at minimum controller gain with the positioner disabled and bypassed, but it was still perceptible. There appeared to be excessive gain in the loop, possibly due to an oversized control valve. Since good water flow regulation is not important to this experiment, the loop was opened by operating the controller in manual mode.

## 2.3 Water Temperature Control

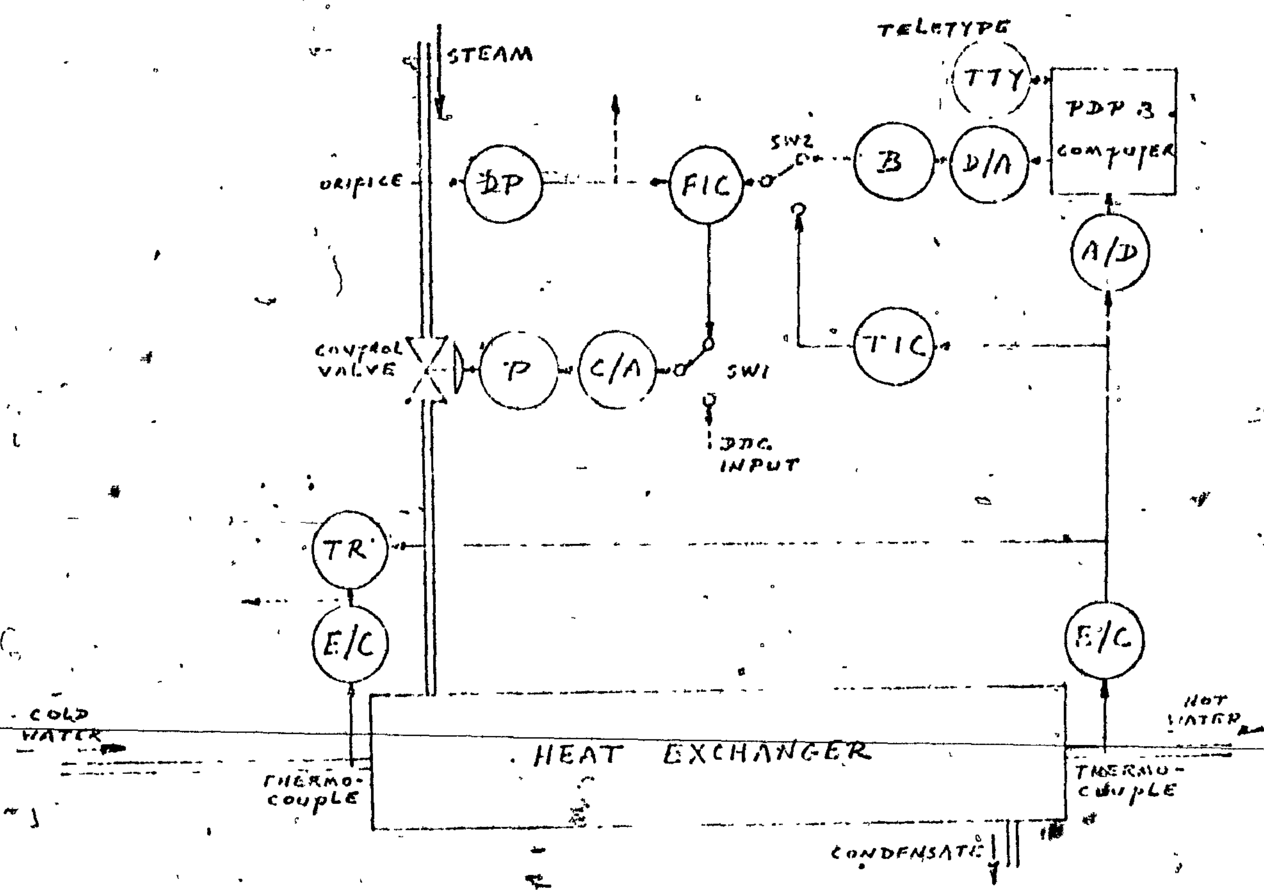
Figure 2.2 shows a block diagram of the water temperature control system. It consists of two loops in cascade. The inner loop is a steam flow control loop. The outer one is a temperature control loop, which determines the set-point of the inner flow loop.

In the inner loop the flow measurement is effected by means of an orifice and an electronic dp cell whose output is a 10 to 50 ma current. This is the input to the electronic PI flow controller (FIC). The controller output feeds a current-to-air converter which in turn drives a positioner and the steam control valve. The flow controller set-point signal is obtained from switch SW2.

At SW2 the outer temperature loop feeds the inner flow loop via the set-point signal. The temperature is measured by a copper-constantan thermocouple. After being converted into a 10 to 50 ma current, the temperature signal feeds two parallel blocks: an electronic PID temperature controller (TIC) and a PDP-8 computer system. Switch SW2 determines which of these temperature control blocks defines the set-point for the steam flow controller. Throughout this

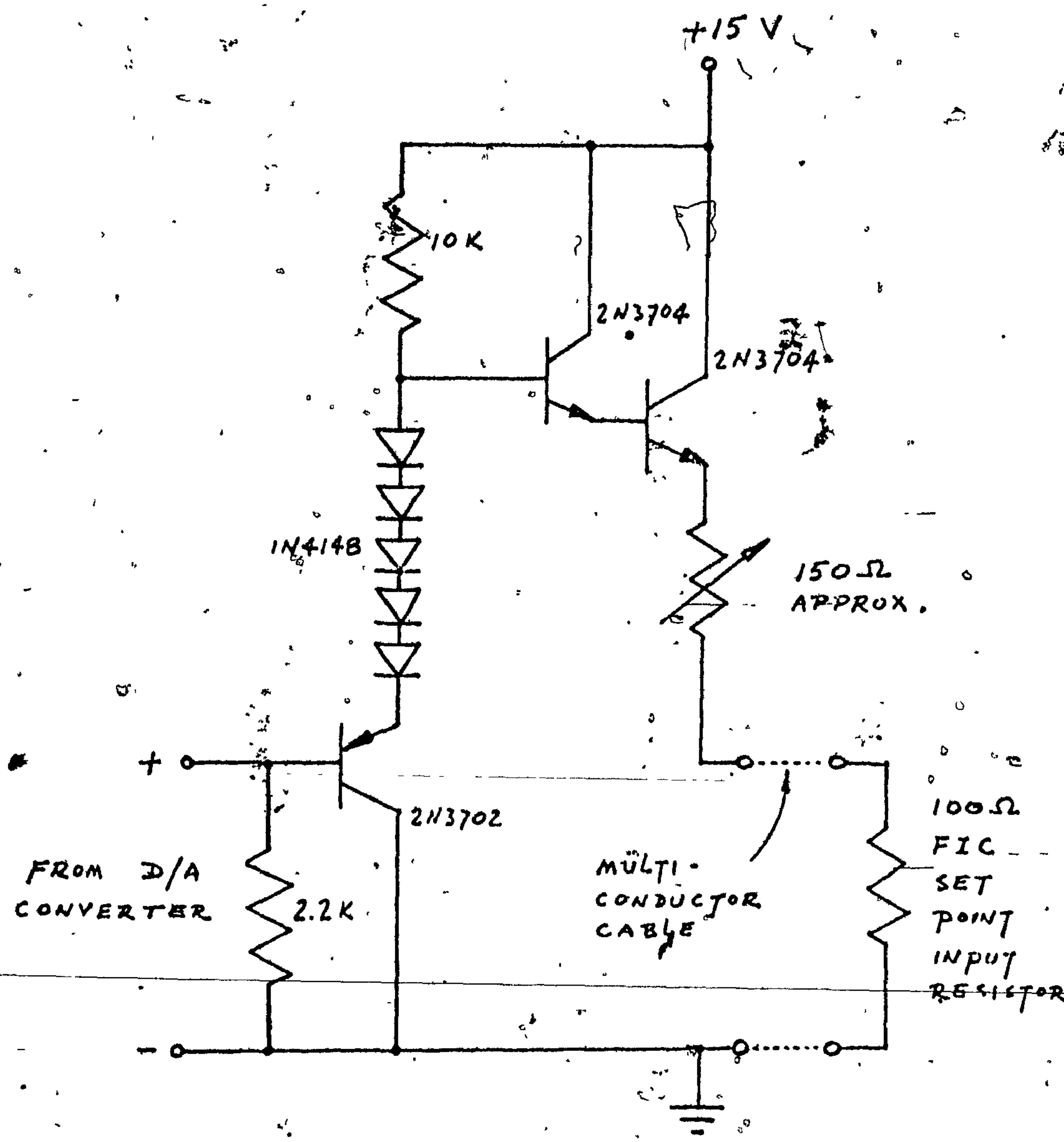
FIGURE 2.2

WATER TEMPERATURE CONTROL SYSTEM




FIC = Flow Indicator-Controller
TIC = Temperature Indicator-Controller
P = Positioner
DP = Differential Pressure Cell
C/A = Current-to-Air Converter
E/C = Emf-to-Current Converter
TR = Temperature Recorder (Dual)
A/D = Analog-to-Digital Converter
D/A = Digital-to-Analog Converter
B = Buffer

FIGURE 2.3




BUFFER CIRCUIT DIAGRAM

experiment, SW2 was in the position shown in Figure 2.2; that is, the computer assumed the role of temperature controller.

In the case of computer control, the 10 to 50 ma current signals are not directly appropriate. At the input, the temperature measurement current from the emf-to-current converter is allowed to flow through a 200Ω resistor. This provides a 2 to 10 volt input to the A/D converter. At the output, the 0 to 10 volt signal from the D/A converter goes through a buffer to provide the 10 to 50 ma flow controller set-point. A diagram of the buffer circuit is shown in Figure 2.3.

## 2.4 The Control Program

The computer used in controlling the heat exchanger is a Digital Equipment Corporation PDP-8/f. This is a general purpose mini-computer having a memory of 4096 12-bit words and a cycle time of 1.5 μs. Peripherals include a teletype, a high speed paper tape reader/punch, an eight-channel analog-to-digital (A/D) converter, and a four-channel digital-to-analog (D/A) converter. Both the A/D and D/A converters operate on -10 to +10 volts and have conversion times of 80 μs and 10 μs respectively.

The computer program consists of two parts. One part is the control program. Its function is to communicate with the operator/user and implement the control law. This program can be used alone if the user himself wishes to tune the controller; that is, specify the coefficients of the control law. The second part of the program, the identification program, was designed to overlay the control program and automatically specify the coefficients according to the plant identification. The resulting controller thus tunes itself. The identification program will be discussed in Chapter 3. The main features of the control program are described in the following paragraphs.

The control program can handle two inputs $U_1$ and $U_2$, and two outputs $y_1$ and $y_2$. The $y_1$ and $y_2$ are actually output error signals of the form $y_i = Z_i - Y_i$, where $Z_i$ are set-points defined by the operator, and $Y_i$ are the actual measured outputs. In the case of this experiment, $U_1$ drives the buffer (B in Figure 2.2) and $Y_1$ is the temperature voltage across the 200Ω resistor at the A/D input.

The complete general control law has the following form:

$$\begin{aligned} U_1(k) = {} & A_1U_1(k-1) + A_2U_1(k-2) + A_3U_1(k-3) + A_4U_1(k-4) \\ & + B_0y_1(k) + B_1y_1(k-1) + B_2y_1(k-2) + B_3y_1(k-3) + B_4y_1(k-4) \\ & + C_1U_2(k-1) + C_2U_2(k-2) + C_3U_2(k-3) + C_4U_2(k-4) \\ & + D_0y_2(k) + D_1y_2(k-1) + D_2y_2(k-2) + D_3y_2(k-3) + D_4y_2(k-4) \end{aligned}$$

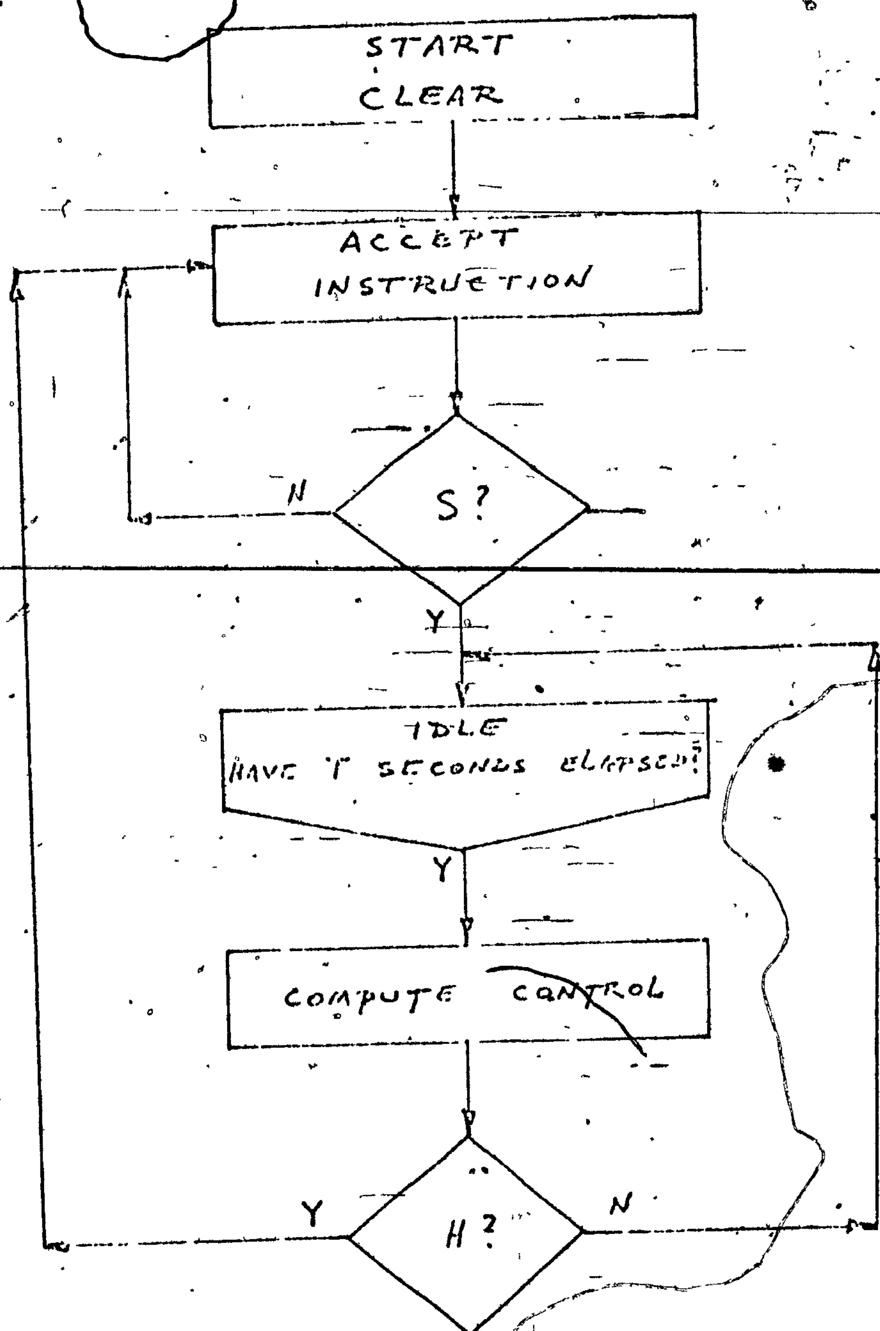
with a similar equation for $U_2$. By appropriate specification of coefficients, the program can be made to control a multivariable process or two independent single-input-single-output processes. The restriction in the latter, however, is that the sampling rates must be identical.

Figure 2.4 shows a simplified flow-chart of the control program. In the instruction mode, an interpreter accepts instructions from the operator via teletype. The operator may specify the coefficients $A_i$, $B_i$, $C_i$, $D_i$ for both $U_1$ and $U_2$. He may also specify the set-points $Z_1$ and $Z_2$, the sample interval T in seconds, and whether he wants the subsequent control history to be printed on teletype or punched on paper tape or both. If the operator types a format error, the program ignores the instruction, types a question mark and awaits the next instruction.

When the operator types the letter S (for Start), the program enters the control mode and implements the control law that the operator just defined. In order to make changes, the operator can type the letter H (for Halt), whereupon the program returns to instruction mode at the end of the current control interval. The operator may then type in the corrected instruction(s) and again type S to continue control.

Since the PDP-8 is not equipped with a real-time clock, the A/D converter with interrupts is used in measuring

FIGURE 2.4




CONTROL PROGRAM FLOWCHART

the sample intervals. During the control mode of the program, A/D conversions are continuously being initiated and counted. When the count reaches a number related to T (specified by the operator), the count is reset to zero and a control computation begins. The control computation is done during the A/D conversion and is interrupted at the end of each conversion. Since the conversion time is relatively short, a large overhead is incurred in A/D interrupt servicing. The resulting computation time is about 200 ms per sample interval.

The computation is done in floating-point using a floating-point software package developed by Digital Equipment Corporation [21,23]. The floating-point number consists of a 12-bit two's complement exponent and a 24-bit two's complement mantissa with the radix point understood to precede the mantissa. The number is stored in three consecutive memory locations. Floating-point computation using this software package is slow but greatly facilitates handling a large range of numbers.

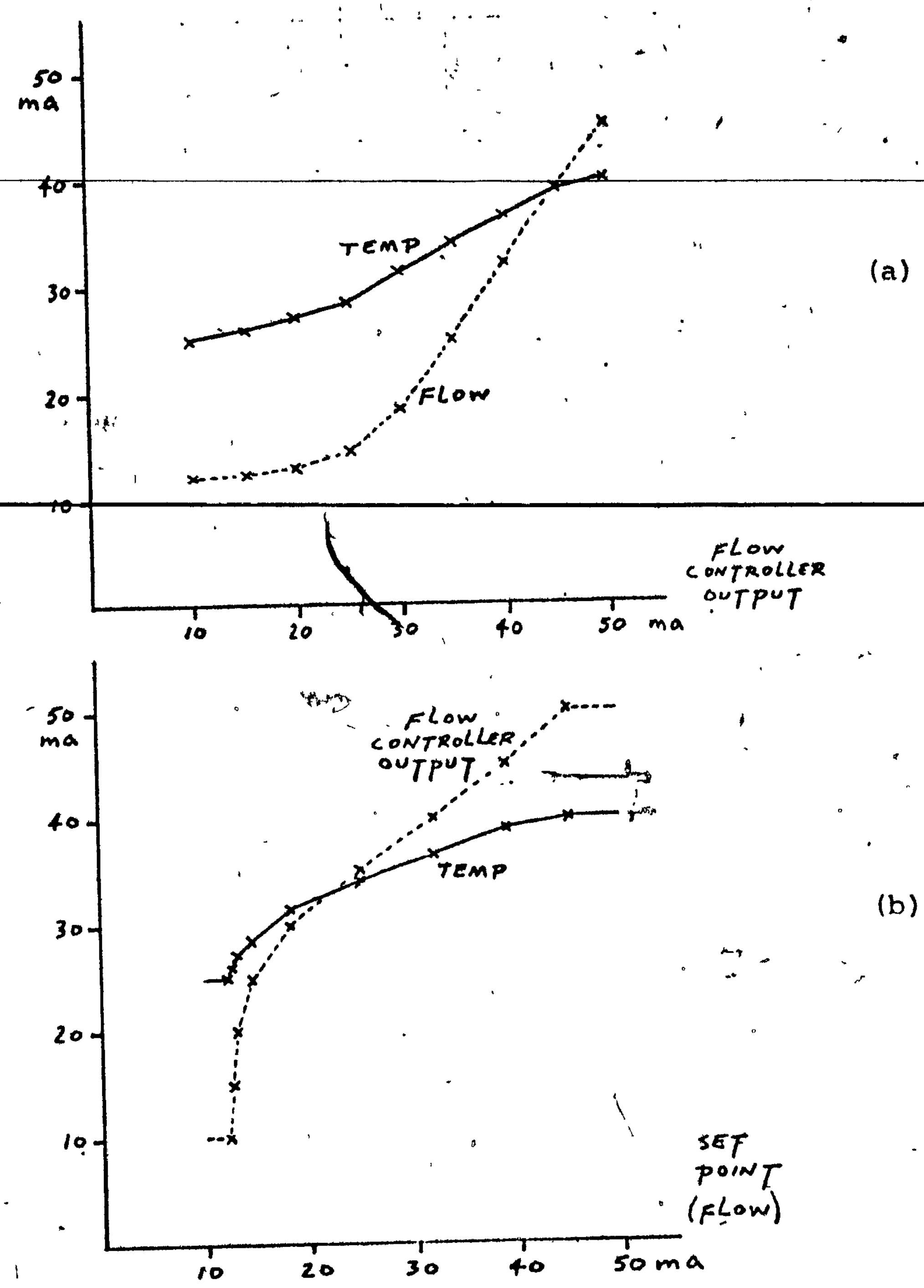
## 2.5 Open Loop Characteristics of the Heat Exchanger

In this section is presented some open loop data for the heat exchanger system. The data consists of the steady-state, the dynamics, and the noise characteristics.

In Figure 2.5a is shown the steady-state curve for both flow control loop and temperature control loop opened. This means simply that the steam flow controller is operated in manual mode. The solid line is the output water temperature current from the emf-to-current converter. The dotted line is the steam flow measurement from the electronic dp cell. The abscissa is the output of the steam flow controller. The flow measurement is not linear with the flow controller output due to the fact that the flow measurement is actually a differential pressure signal which is proportional to the square of the flow, and also due to control valve non-linearity.

Figure 2.5b is the steady-state curve for the flow control loop closed, i.e. the flow controller is in automatic mode, responding to the set-point signal. This in fact is the plant as far as the temperature controller is concerned. The non-linearity of the flow measurement has two significant effects on the closed flow control loop. One is that the output water temperature becomes non-linear, while it was roughly linear in the open loop. A second effect is that although the loop is stable at the lower flow rates, it becomes unstable at higher flow rates due to increased gain in the flow measurement. The resulting limit cycle disappears only at minimum controller gain, which renders the controller rather ineffective over much of the lower flow rate region. A satisfactory control is obtained by disabling and bypassing

FIGURE 2.5




OPEN LOOP STEADY STATE CURVES

the positioner. The controller was then tuned to give an underdamped response with a settling time of about 1 second. A better solution, of course, would be to insert a square-root extractor following the dp cell. This would make all the curves in Figure 2.5 roughly linear and would allow the positioner to be in the loop. The value of the positioner lies in faster valve action and in the reduction of errors due to friction within the valve. The first advantage is not so important here since the 1 second flow response obtained without the positioner is quite short compared with the temperature response.

A step response analysis was done in order to get an estimate of the dynamics of the plant with both flow loop and temperature loop open. The step response is shown in Figure 2.6 with a graphical analysis immediately below it. The step was a downward step such that the flow controller output changed from 75% to 0%. Roughly, the response looks like two lags with a .25 minute delay. Normalizing the response and ignoring the delay, the response can be modelled as $c(t) = 1 - k_1e^{-at} - k_2e^{-bt}$, where a and b are both positive. Assuming that b is significantly greater than a, c(t) approaches $1 - k_1e^{-at}$ as t grows large. From the equality $\ln(1 - c(t)) = \ln(k_1e^{-at}) = \ln(k_1) - at$ which follows, if $\ln(1 - c(t))$ is plotted in time, then $\ln(k_1)$ and $-a$ are readily obtained as the intercept and slope respectively of

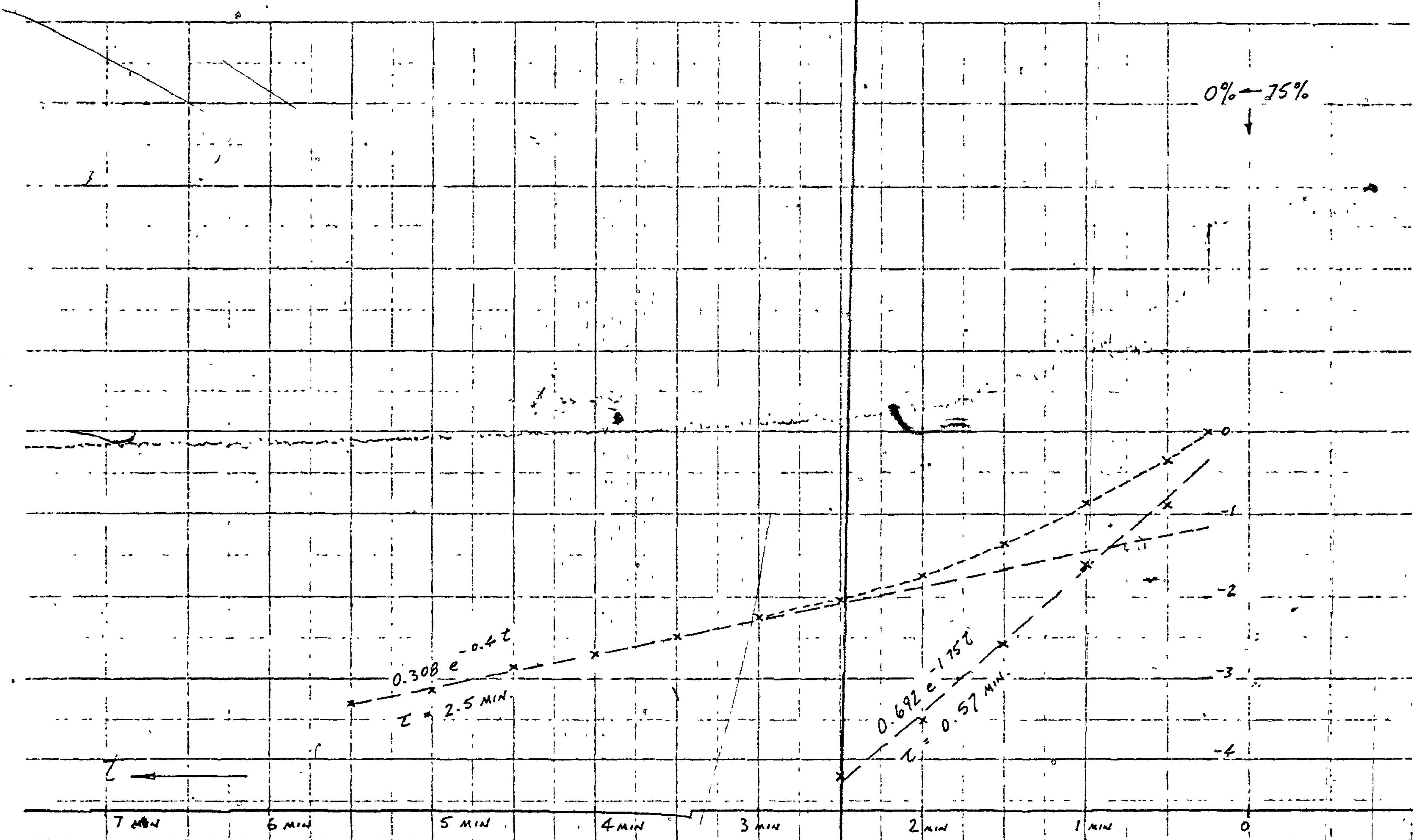



FIGURE 2.6 OPEN LOOP STEP RESPONSE

the $t = \infty$ asymptote. Thus, from the upper left asymptote in Figure 2.6 is obtained $k_1 \doteq .308$ and $a = .4$. Similarly, by plotting $\ln(1 - c(t) - .308e^{-.4t}) = \ln(k_2) - bt$, $k_2$ and b are determined to be .692 and 1.75 respectively. Thus the response is $c(t) = 1 - .308e^{-.4t} - .692e^{-1.75t}$. The corresponding time constants are 2.5 minutes and .57 minutes. From this result, a discrete-time model of the plant will be developed in Chapter 3.

Noise presented two difficulties in this step analysis. One is accuracy, which is especially important in obtaining the $t = \infty$ asymptote. It was for this reason that a downward step change was chosen rather than upward, since it was observed that the noise was greater at higher temperatures. Another problem was that in getting a large enough signal/noise ratio for graphical analysis, it was necessary to use a large step change which would cause a non-linear response if the flow loop were closed. The fact that the temperature is roughly linear over the entire range in open loop is the reason for performing the step-response analysis with the flow loop open. This is not exactly the plant that the computer temperature controller sees, except in the case of small changes in a mid-temperature range. There, the temperature/output gain is about equal to the temperature/set-point gain, these being the open loop and closed loop gains respectively. Furthermore the relatively fast dynamics of the flow controller are practically negligible as far as the heat exchanger model is concerned. Therefore the model obtained from the step-response test is valid as a plant model from the computer's point of view in the mid-temperature range.

An upward step change of the same magnitude (0% to 75% output) produces a noisier but very similarly shaped response, thus substantiating the temperature linearity when the flow loop is open. Near maximum steam flow, the noise becomes severe, steam begins to blow through the steam trap, and the heat exchanger dynamics appear to be different. Because of this and because of the strong non-linearity at the low temperature region, the computer identification and control experiment was carried out in the mid-temperature range.

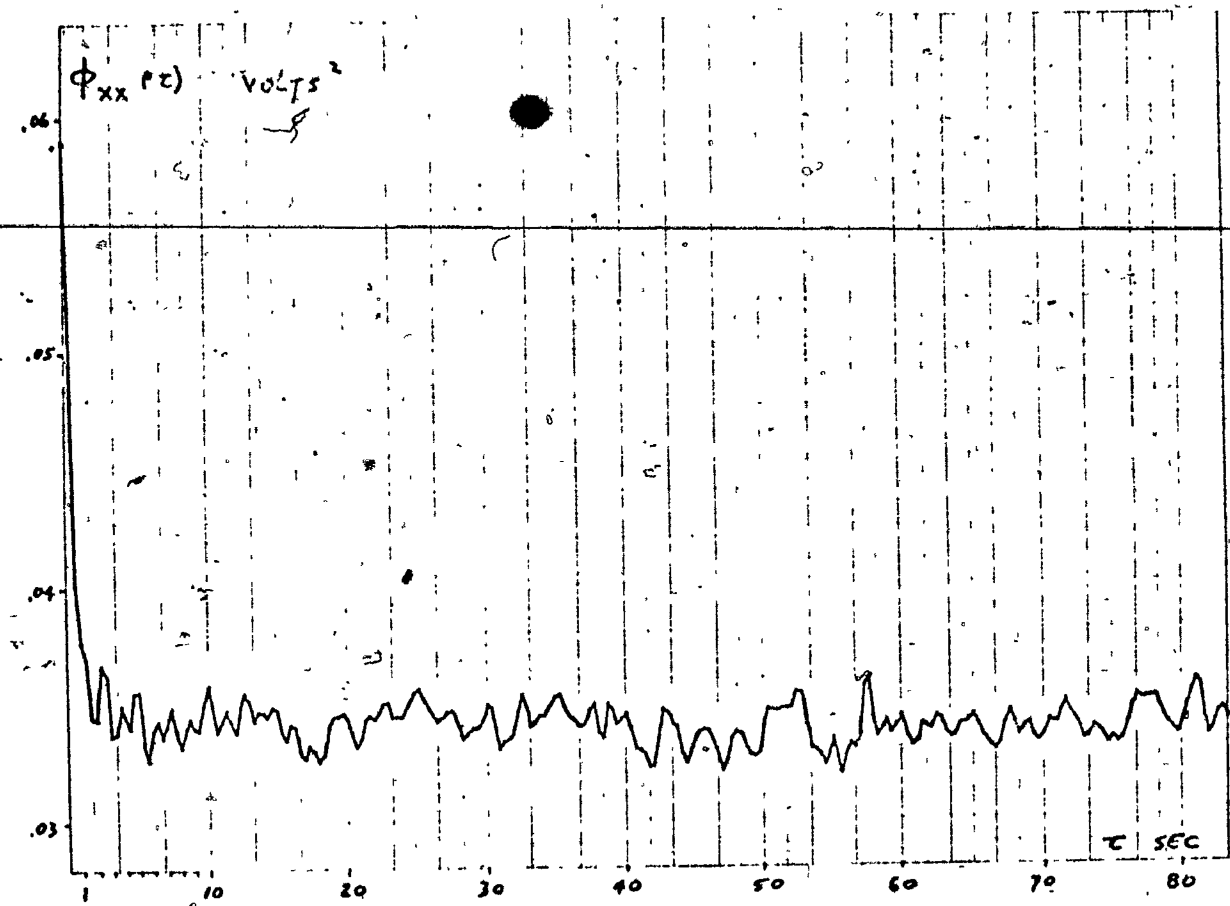
A simple noise analysis was done in the form of an autocorrelation of the mid-temperature steady-state. The noise was evident on a thermometer placed in the outlet water stream, hence it was not entirely measurement noise. Although the water flow recorder registered a steady flow, the flow out of the heat exchanger was not smooth and seemed to be one source of the noise.

The autocorrelation was performed using the PDP-8 computer on a record of 2400 samples obtained at a sampling rate of 3 samples/second. Most of the dc value of the steady-state temperature signal was removed by subtraction in order to minimize round-off errors in the autocorrelation calculation. The resulting autocorrelation function is shown in Figure 2.7. It is evident that to the heat exchanger system, whose dominant time constant is .57 minutes, the noise is essentially white. Assuming that the noise has zero mean, the noise variance is obtained as the difference between $\phi_{xx}(0)$ and $\phi_{xx}(\infty)$. Taking $\phi_{xx}(\infty)$ to be the mid-point of the garbage at the end of the autocorrelation curve of Figure 2.7:

$$\phi_{xx}(0) - \phi_{xx}(\infty) \approx .025 \text{ volt}^2$$

The corresponding rms value is about .16 volt.

FIGURE 2.7




AUTOCORRELATION OF THE STEADY STATE TEMPERATURE

# CHAPTER 3

## The Identification Program

### 3.1 Introduction

The identification program is an implementation of a self-tuning control algorithm presented in a paper by K.J. Åström and B. Wittenmark [17]. The algorithm consists of a least-squares parameter identification of the plant, and the determination of a minimum variance control law from the identified parameters. The least-squares identification algorithm is presented at length in [15].

The plant model has the following form:

$$y(k) + a_1 y(k-1) + \ldots + a_n y(k-n)$$
$$= b_1 u(k-1) + \ldots + b_n u(k-n) + e(k) \qquad (3.1)$$

where u(k) is the control variable, y(k) is the controlled variable (output) and e(k) is a noise variable. ~~The minimum~~ variance performance criterion is $V = E[y^2(k)]$ where E is the expectation operator and V is to be minimized. The optimal control law for this performance criterion is

$$u(k) = \frac{1}{b_1}[a_1 y(k) + \ldots + a_n y(k-n+1) - b_2 u(k-1) - \ldots - b_n u(k-n+1)].$$

Note that the poles of this controller's transfer function exactly cancel the zeros of the plant. If the model parameters $a_i$ and $b_i$ were known, the control law coefficients could immediately be entered to the control program as follows:

$$A_1 = -\frac{b_2}{b_1}, \quad A_2 = -\frac{b_3}{b_1}, \quad \ldots, \quad B_0 = \frac{a_1}{b_1}, \quad B_1 = \frac{a_2}{b_1}, \quad \ldots$$

## 3.2 Least-Squares Parameter Estimation

In order to obtain the parameters $a_i$ and $b_i$, the problem is posed as an optimization. The object is to minimize the function $V = \sum_{k=n}^{N+n} e^2(k)$ where the $e(k)$ are interpreted here as equation errors in the plant model (Equation 3.1). Hence the name least-squares estimation.

Let $y = \begin{bmatrix} y(n+1) \\ \vdots \\ y(n+N) \end{bmatrix}$ , $e = \begin{bmatrix} e(n+1) \\ \vdots \\ e(n+N) \end{bmatrix}$

$$\Phi = \begin{bmatrix} -y(n) & \dots & -y(1) & u(n) & \dots & u(1) \\ \vdots & & \vdots & \vdots & & \vdots \\ -y(n+N-1) & \dots & -y(N) & u(n+N-1) & \dots & u(N) \end{bmatrix}$$

$$\beta^T = [a_1 \; a_2 \; \dots \; a_n \; b_1 \; b_2 \; \dots \; b_n]$$

where $y$, $e$, and $\Phi$ are simply a history of N consecutive sample intervals. Then $e = y - \Phi\beta$ and V is the magnitude squared of vector e. The minimum of V is obtained by setting $\nabla V = 0$, and the solution, if $\Phi^T\Phi$ is not singular, is

$\beta = [\Phi^T\Phi]^{-1}\Phi^T y$ (see [15]). (3.2)

As $N \to \infty$ the covariance of $\beta$ is given by $\sigma^2[\Phi^T\Phi]^{-1}$ where $\sigma^2$ is the variance of $e(k)$.

The determination of $\beta$ from Equation 3.2 is a one-shot computation on an input-output history of N samples. This does not lead to a suitable computer program because excessive computation would be required. In [15] a recursion formula derived from Equation 3.2 is given. Using this formula, the current parameter estimate $\beta$ is updated at each sample interval. The recursion formula is the following:

$$\beta(N+1) = \beta(N) + \Gamma(N)[y(N+1) - \phi(N+1)\beta(N)] \quad , (3.3)$$

where $\Gamma(N) = P(N)\phi^T(N+1)[\alpha + \phi(N+1)P(N)\phi^T(N+1)]^{-1}$

$$P(N+1) = [I - \Gamma(N)\phi(N+1)]P(N)$$

$$\phi(N+1) = [-y(N) \; -y(N-1) \ldots -y(N-n+1) \;\; u(N) \; u(N-1) \ldots u(N-n+1)]$$

$$P(N_0) = \alpha[\Phi^T(N_0)\Phi(N_0)]^{-1}$$

The vector $\Gamma$ and the square matrix P are both of order 2n. The matrix P is the covariance of $\beta$ if the e(k) are independent with variance equal to $\alpha$. If the estimate were perfect and there were no disturbance, then $\phi(N+1)\beta(N)$ would be the value of y(N+1). Thus Equation 3.3 means that at each sample interval $\beta$ is updated by a quantity which is essentially the product of a time varying gain $\Gamma(N)$ and the difference between the actual output and the expected output. In order to start the recursion, $\beta$ and P can be initialized using a priori knowledge, gained possibly from a one-shot least-squares first estimate. Alternatively P(0) can be initialized simply to a constant times the identity matrix, which is the course taken in this experiment.

In brief then, at each control interval the algorithm updates the estimate of the parameters in the model:

$$y(k) + a_1 y(k-1) + \ldots + a_n y(k-n) = b_1 u(k-1) + \ldots + b_n u(k-n)$$

Then a minimum variance control law is defined from the parameters:

$$u(k) = \frac{1}{b_1}[a_1 y(k) + \ldots + a_n y(k-n+1) - b_2 u(k-1) - \ldots - b_n u(k-n+1)]$$

## 3.3 Convergence Properties of the Closed Loop

It can be shown that if the plant is stable, and the e(k) are independent with zero mean, and the input u(k) is independent of e(k) and persistently exciting of order n, then the least-squares estimate of $\beta$ converges to the correct value as $N \to \infty$ [15]. It is to be noted that this convergence is that of an open loop identification, that is the input is generated by an external source and is uncorrelated with e(k). No general convergence theorems have yet been proved for the closed loop, when the input is generated by feedback.

In this experiment, the e(k) are not independent. To illustrate, consider a first order noise-free system: $x(k+1) + ax(k) = bu(k)$. Let the output be a measurement of x(k) corrupted by additive white noise $y(k) = x(k) + n(k)$ (which is essentially the case in this experiment). Then the system seen by the identification algorithm would be

$$y(k+1) + ay(k) = bu(k) + n(k) + an(k-1). \qquad [15]$$

The noise here is not white, but rather a filtered white noise where the filter is the plant. Thus the e(k) in the plant model (Equation 3.1) is not an uncorrelated noise variable. The parameter estimates may still converge, but the estimates will be biased. The estimate $\hat{\beta}$ of $\beta$ will be biased by an amount given by

$$E(\hat{\beta}-\beta) = [E(\Phi^T\Phi)]^{-1}E(\Phi^T e) \qquad [15]$$

where E is the expectation operator. A theorem presented in [17] states that if the parameter estimates converge in closed loop, the resulting controller converges to the minimum variance control. This result holds whether the noise is correlated or not. Thus even if the parameter estimates are biased because of correlated noise, the result will still be a minimum variance controller.

Another theorem in the same paper states that if the parameter estimates converge, certain correlations of the input and output are zero. Specifically, if e(k) is a noise function of order m, then

$$E[y(t+\tau)y(t)] = 0 \qquad \tau = m+1, \ldots, m+n$$

$$E[y(t+\tau)u(t)] = 0 \qquad \tau = m+1, \ldots, m+n \quad .$$

A second convergence difficulty is that since the input is generated by feedback, the input is correlated with the noise. Consider again the first order system $y(k+1) + ay(k) = bu(k)$. The minimum variance control is $u(k) = \frac{a}{b}y(k)$. If the estimates of the parameters a and b are in error by a common multiplicative factor, the resulting control is unchanged and the minimum variance performance criterion $V = E[y^2(k)]$ is unchanged. Furthermore, it can be shown that the least-squares objective function $V = \sum_{k=n}^{N+n} e^2(k)$ will be identical for all a and b with the same ratio a/b [17]. Thus, in general, when the input is generated by feedback, the identification algorithm will determine the parameters only to within a common multiplicative factor. The resulting minimum variance control law will be unique, however, since the multiplicative factor cancels upon division by $b_1$.

In the simulation studies done by Åström and Wittenmark, the value of $b_1$ was fixed. This serves to prevent the algorithm from making large excursions along the linear manifold on which the parameter estimates lie. Åström and Wittenmark report in their paper that the choice of $b_1$ is not crucial in obtaining convergence of the algorithm. In the experimental work done by the author, the value of $b_1$ was not fixed.

To summarize, because of correlated noise the parameter estimates will be biased but will still lead to a minimum variance control. Furthermore, because of feedback, the biased estimates may be out by a multiplicative factor and convergence is not guaranteed. If the parameter estimates converge, however, a unique minimum variance control law can be expected.

## 3.4 The Identification Program

The identification program was written to overlay the control program so that at each sample interval the parameter estimates are updated and the minimum variance control law is determined according to the algorithm just presented. After this tuning step, the control program computes the control variable. The parameter estimates are punched on paper tape at every sample interval, if so requested by the operator.

The identification program performs a fourth order identification and control. Orders greater than four cannot be handled by this program because of insufficient memory in the PDP-8. Additional overlays were written to generate lower order identification and control by changing the order of the matrix and vectors involved.

The computation time for fourth order is about four seconds. Since there are eight parameters, the vectors and the square matrix P are of order 8. Moreover, the computation is hampered by large overheads in the continuous use of the A/D for sample interval measurement, and in the use of software for floating-point arithmetic.

At the start, before asking the operator for instructions, the program sets all coefficients, variables and vectors to zero except for $\beta$. The parameter $b_1$ is initialized to unity while the rest are zero. (The starting value of $b_1$ can be changed however via the switch register on the PDP-8 console.) The value of $b_1$ must not be zero since it appears as a divisor in the determination of the minimum variance control law. The matrix P is initialized to $b_1$ times the identity matrix. The value of $\alpha$ is set (via switch register) to the noise variance obtained from the autocorrelation study discussed in §2.5.

In §2.5 the estimates of two time constants have been obtained as 2.5 minutes and .57 minutes, where the shorter time constant dominates. In order to obtain reasonable values for the parameter estimates, the sample interval T was chosen to be .7 minutes (42 seconds). Reasonable values are such that there is a good pole-zero spread on the z-plane. If T were relatively small the plant would appear essentially as an integrator and the poles would tend towards +1 on the z-plane. In the extreme, the order of the system would decrease as seen by the identifier and the excess poles and zeros may end up anywhere. If T were very large, the plant would look like a delay and the poles would tend towards the origin and again the order of the system would decrease as seen by the identifier.

The identification program has one additional task, that of integrating the control signal u(k). The reason for this is that in a noise-free steady-state, $Y_1(k)$ (output temperature) is a constant, $y_1(k)$ (output error) is zero, and $u_1(k)$ (the control signal) is zero. The steam flow set-point signal $U_1(k)$ however must be a non-zero constant. Thus $u_1(k)$ is not equal to $U_1(k)$ but rather is an additive correction term for $U_1(k)$. Hence $u_1(k)$ is integrated to obtain $U_1(k)$. The resulting overall flow-chart and system diagram are shown in Figures 3.1 and 3.2. In Figure 3.2, $Z_1$ is the temperature set-point defined by teletype. $Y_1$ is the output temperature and $y_1$ is the output error. The $u_1$ signal is the control variable which is integrated to produce $U_1$, which is the steam flow set-point.

## 3.5 Discrete-Time Plant Model

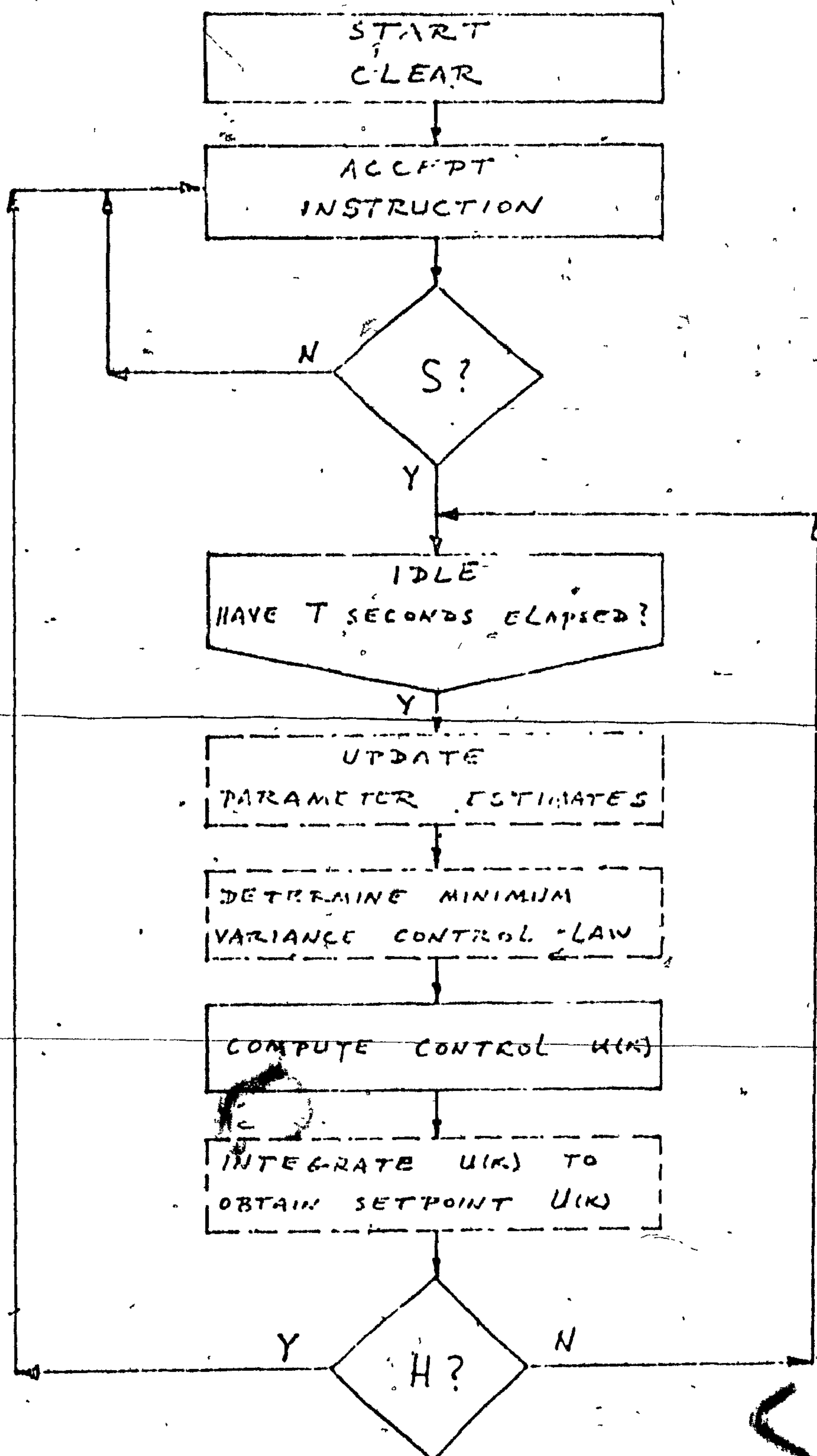
A discrete-time model of the plant was determined from the step response analysis for purposes of comparison with the least-squares identification. As far as the identification algorithm is concerned, the digital integrator in the identification program is part of the plant and is included in the model developed here.

From §2.5 the normalized step response of the heat exchanger (ignoring the delay) is

$$c(t) = 1 - .308e^{-.4t} - .692e^{-1.75t},$$

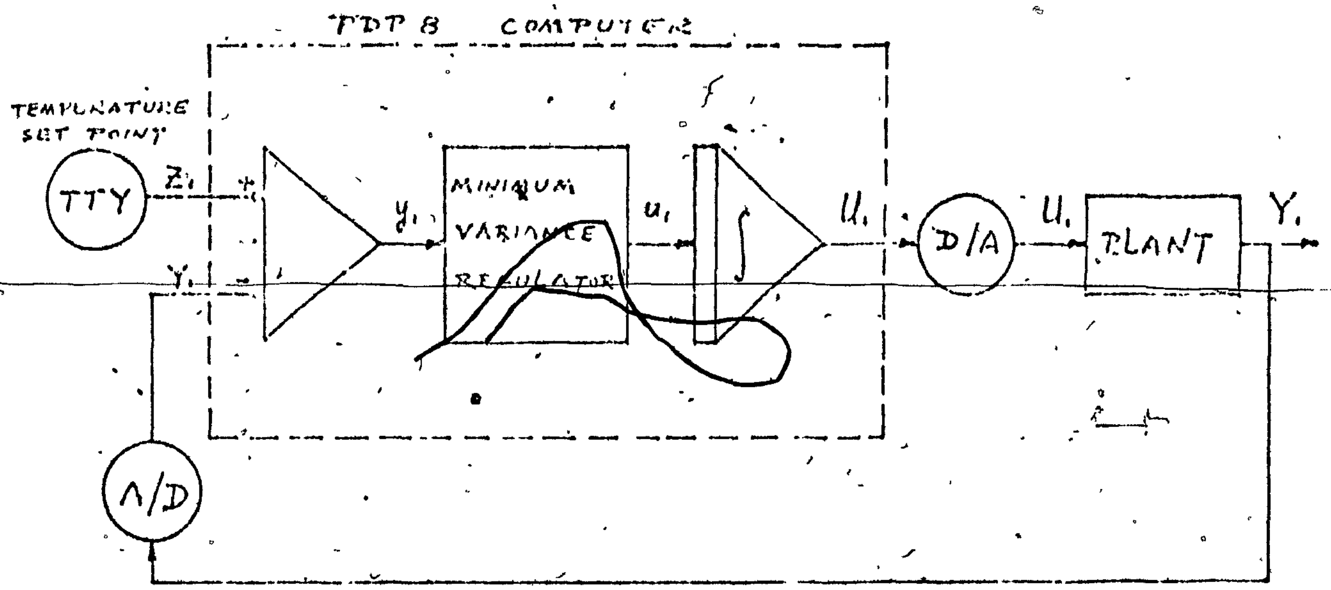
where the unit of time is the minute. For T = .7 minute

FIGURE 3.1




IDENTIFICATION AND CONTROL SYSTEM FLOWCHART

FIGURE 3.2




SELF-TUNING CONTROL SYSTEM

BLOCK DIAGRAM

the z-transform of c(t) is

$$C(z) = \frac{z}{z-1} - \frac{.308z}{z-.756} - \frac{.692z}{z-.294}$$

$$= \frac{.563z(z-.695)}{(z-1)(z-.756)(z-.294)}$$

Since C(z) is a step response, the transfer function is given by $C(z)\left(\frac{z}{z-1}\right)^{-1}$ where $\frac{z}{z-1}$ is the z-transform of the integration operator. However, because of the digital integrator in series with the heat exchanger, the complete plant transfer function for unity gain is

$$C(z)\left(\frac{z}{z-1}\right)^{-1}\left(\frac{z}{z-1}\right) = C(z) \quad .$$

The gain depends on the operating region. From the discussion in §2.5, the analysis above is valid only in a mid-temperature region, where the current gain (from Figure 2.5) is approximately .325. That is, a 1 ma change in set-point produces a .325 ma change in temperature. Since the computer operates on voltages through the A/D and D/A, it is the voltage gain that is required. A .325 ma temperature change corresponds to a .065 volt change. A 1 ma set-point change requires a .250 volt change at the buffer input. Thus the voltage gain is .26 and the overall transfer function is H(z) = .26C(z). When multiplied out, this gives

$$H(z) = \frac{.146z^2 - .102z}{z^3 - 2.05z^2 + 1.272z - .222}$$

In canonical form then, the model is

$$y(k) - 2.05y(k-1) + 1.272y(k-2) - .222y(k-3)$$
$$= .146u(k-1) - .102u(k-2)$$

This is a third order system. The delay (including computation time) is not large relative to T = 42 seconds and has only a minor effect on the order of the zero at the origin. Hence it is ignored for this model.

## 3.6 Program Test

The complete program (identification and control) was tested on a plant consisting of a simple lag set up on a TR-48 analog computer. Nothing more was recorded than the fact that the fourth order identification and control produced fast convergence and good control. Following this test, the experiment proceeded to the heat exchanger. As a result of some convergence difficulties on the heat exchanger, however, further testing of the program using the analog computer was subsequently performed. The results of these tests are presented in Chapter 4.

# CHAPTER 4

## Experimental Results

### 4.1 Preliminary

The aim of the experiments was to determine how the closed loop identification and control performs under various conditions. The conditions that were varied are the order of the model, the sample interval, the starting value of parameter $b_1$, and the value of $\alpha$ (the noise variance estimate). Only the first condition, the order of the model, was exhaustively varied, the other conditions being changed generally only to try to obtain convergence in "difficult" cases. The principal output of each experiment consists of answers to some yes-or-no questions: Do the parameter estimates converge? Is the closed loop system stable?

Convergence of the parameter estimates requires that the control u(k) be persistently exciting. If the system is allowed to remain in a steady-state where y(k) and u(k) are small or zero, not much identification takes place. Hence the system was kept moving by means of set-point changes introduced by the operator via teletype.

A reduction of noise is not expected from the controller. If an accurate model were obtained from the identification, the minimum variance controller would make

-y(k) = e(k) in steady-state. Hence the variance of y will not be less than the noise variance.

Ideally, the plant should be identical in all the experiments. Throughout the experiments, the analog instrumentation settings were constant, and the water flow rate was a constant except for small fluctuations. However, the noise variance was not a constant. On days of cold weather, the additional heating load of the university degraded the quality of the steam available to the heat exchanger. The consequence is reduced noise and lower achievable temperatures. The dynamics did not change significantly. Thus a cold weather run is essentially a low-noise run.

Before proceeding to the experiments proper, some preliminary responses are presented here. Figures 4.1 and 4.2 show open loop step responses for T = 42 and T = 14 seconds. The upper curves are the output temperature voltage $Y_1$. The lower curves are the steam flow set-point signal $U_1$. The purpose of these records is to serve as a comparison with closed loop responses. In closed loop the noise will not be reduced, but the general response shape will be affected.

Another test was to obtain the response for a minimum variance control based on the step response model obtained in §3.5. The control program alone is required since the parameters are given. Since the model has the form

$$y(k) + a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) = b_1 u(k-1) + b_2 u(k-2),$$

the minimum variance control law is

$$u(k) = \frac{a_1}{b_1} y(k) + \frac{a_2}{b_1} y(k-1) + \frac{a_3}{b_1} y(k-2) - \frac{b_2}{b_1} u(k-1) .$$

The steam flow set-point signal $U_1$ is the integral of the control variable: $U_1(k) = U_1(k-1) + u_1(k)$. Therefore by simple algebra,

$$U_1(k) = \frac{a_1}{b_1} y_1(k) + \frac{a_2}{b_1} y_1(k-1) + \frac{a_3}{b_1} y_1(k-2)$$

$$+ \left(\frac{b_1-b_2}{b_1}\right) U_1(k-1) + \left(\frac{b_2}{b_1}\right) U_1(k-2) .$$

Thus in the control law

$$B_0 = \frac{a_1}{b_1} = -14.0$$

$$B_1 = \frac{a_2}{b_1} = 8.72$$

$$B_2 = \frac{a_3}{b_1} = -1.52$$

$$A_1 = \frac{b_1-b_2}{b_1} = 1.69$$

$$A_2 = \frac{b_2}{b_1} = -.69$$

The resulting control is shown in Figure 4.3. Again, the upper curve is the output temperature $Y_1$. The heavy low-frequency rectangular wave superimposed on the output is the set-point defined by the operator. The lower curve is $U_1$.

It is evident that the control is not good. It appears that the gain is excessive. Assuming that $b_1$ and $b_2$ have been underestimated, their values were increased so as to produce a control law with lower gain. Figure 4.4 shows the result for gain reductions by factors of 10 and 2. The control became much less wild but the response slowed considerably at 1/10 gain. In all cases, there were oscillation tendencies, with frequency and settling time decreasing with decreasing gain.

## 4.2 Fourth Order Model

The control algorithm using the fourth order model was successful in the sense that the parameter estimates did converge and the closed loop system was stable. Control histories (response and parameter convergence) for five particular runs are shown in Figures 4.5 to 4.9. Figure 4.10 illustrates the pole-zero locations of the transfer functions of the identified plant and of the corresponding minimum variance controller (the poles and zeros of the plant being on the left and those of the controller on the right). The diagram in Figure 4.10a corresponds to the step response model, obtained in §3.5.

Runs #1 and #2 are each typical of several runs. That is, several runs (using different values of $\alpha$ ) more or less correctly identified the dominant higher frequency pole as in run #1 (see Figure 4.10b), while several other runs placed the pole near the lower frequency as in run #2 (see Figure 4.10c). In both cases, the pole at +1 (due to the digital integrator) was correctly identified. Also in both cases, a complex pole-pair almost cancels a complex zero-pair. This suggests that the algorithm sees the plant essentially as a simple lag in series with an integrator, and that a lower order identification may prove more successful. A further comparison of runs #1 and #2 is obtained by using only the control program and typing in the minimum variance control law based on the identified parameters. The responses shown in Figures 4.11 and 4.12 indicate that run #1, in which the estimated model is closer to the expected model, did in fact produce a better controller.

Run #3 is a low-noise run. The resulting model is similar to that of run #1 but the complex pole-zero pairs are further apart (see Figure 4.10d). Run #4 is also a low-noise run but with $\alpha$ set to a relatively high value of .64 volt$^2$.

Figure 4.10f is the pole-zero pattern for the step response model for T = .14 seconds. Note that because of the smaller sampling interval, the poles have moved closer to +1. Run #5 is for T = 14 seconds. Figure 4.10g indicates that the two significant poles have been identified, but a zero appears outside the unit circle. Thus the identified plant is distinctly non-minimum phase. The transfer function of the corresponding controller is also non-minimum phase since the controller placed a pole directly on the non-minimum phase zero.

In all cases, the parameter estimates converged. The problem of the estimates being in error by an unknown multiplicative factor does not appear to be present, since meaningful pole-zero patterns are obtained from the estimated parameters directly. It is to be noted that in general, the parameter estimates are not to be accepted with confidence for the reasons discussed in §3.3.

## 4.3 Third Order Model

The third order model produced less success than came from the fourth order model. The system was not generally stable and the parameter estimates did not converge. Here, the problem of non-identifiability due to feedback became apparent. The parameter estimates made very large excursions which were not always on a linear path. The response became more sensitive to the value of $\alpha$ and very sensitive to the starting value of $b_1$. For some values of $b_1$ an unstable system was produced. In these cases, incorrect parameter estimates led the system to saturate at minimum or maximum temperature, whereupon no further identification took place.

Responses are shown in Figures 4.13 to 4.17. Figure 4.18 illustrates the pole-zero patterns of the corresponding minimum variance controllers for stable intervals near the end of each run. The parameter estimates are definitely out by a large factor, hence no plant model and corresponding pole-zero pattern is obtained.

An attempt was made to obtain a PID controller by using the model

$$y(k) + a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) = b_1 u(k-1) \quad (4.1)$$

The minimum variance controller is

$$u(k) = \frac{a_1}{b_1} y(k) + \frac{a_2}{b_1} y(k-1) + \frac{a_3}{b_1} y(k-2) .$$

The steam flow set-point signal is the integral of u(k):

$$U(k) = U(k-1) + \frac{a_1}{b_1} y(k) + \frac{a_2}{b_1} y(k-1) + \frac{a_3}{b_1} y(k-2) .$$

The z-transform of the transfer function is

$$\frac{U(z)}{y(z)} = -\left(\frac{a_2}{b_1} + \frac{a_3}{b_1}\right) - \left(\frac{a_3}{b_1}\right)\frac{1}{z} + \left(\frac{a_1}{b_1} + \frac{a_2}{b_1} + \frac{a_3}{b_1}\right)\frac{z}{z-1} \quad (4.2)$$

where the three terms are proportional, derivative, and integral terms respectively.

The performance of the PID controller proved to be even worse and was even more sensitive to the starting value of $b_1$. In run #9 (Figure 4.16) note that after 46 sample intervals, incorrect parameter estimates produced a positive feedback and then a very low gain negative feedback, which then brought the system to the set-point very slowly. One explanation for the poor performance of the PID controller is that the model of Equation 4.1 places both the zeros at the origin. Hence the algorithm is not allowed to identify the zero that nearly cancels the non-dominant low frequency pole whose time constant is 2.5 minutes.

### 4.4 Second Order Model

The second order model also had convergence difficulties. As run #11 (Figure 4.19) shows, although the system generally followed the set-point, it went through periods of high and low gain, positive and negative feedback. Parameter $a_1$ refused to converge, while $a_2$, $b_1$, and $b_2$ either converged or were no longer identified towards the end of the run.

A PI controller was obtained by using the model of Equation 4.1 with $a_3 = 0$. Equation 4.2 then becomes

$$\frac{U(z)}{y(z)} = -\frac{a_2}{b_1} + \left(\frac{a_1}{b_1} + \frac{a_2}{b_1}\right)\frac{z}{z-1}$$

where the terms are proportional and integral terms. As in the third order case, the zero of the plant model transfer function is then fixed at the origin, which is all right here since only two poles will be obtained. If the identification succeeds, these two poles would be the pole at +1 and the higher frequency pole corresponding to the dominant .57 minute time constant. Thus the pole-zero near-cancellation corresponding to the non-dominant pole (2.5 minutes time constant) need not even appear in the identified model, and it may be expected that this model would work. It did not work, however, as Figures 4.20 to 4.26 indicate. Parameter convergence was poor, and there were many cases of very low gain and/or positive feedback (see run #15, Figure 4.24).

The salient problem here is a recurrent situation of control signal saturation followed by a period of very low gain (see Figures 4.24 and 4.25). The control saturation is a consequence of excessive gain or positive feedback. If the system recovers, it emerges with a very low gain. The low gain is a result of an over-estimation of $b_1$ or an under-estimation of the other parameters. This is an over-reaction on the part of the identification algorithm to a previous under(over)-estimation which had led to the control saturation. The saturation/low-gain-recovery phenomenon appears to be a limit cycle of an unstable algorithm (for this model). The reason the phenomenon can be recurrent is that little is learned about the plant during the saturation and the low gain recovery. Since the u(k) are small during this period, their effect is largely buried by noise. The parameter identification during this time is then incorrect or very slow.

### 4.5 First Order Model

A first order model leads to an integral controller. Setting $a_2 = a_3 = 0$, Equation 4.2 becomes $\frac{U(z)}{y(z)} = \frac{a_1}{b_1} \frac{z}{z-1}$ which is the transfer function of an integrator.

The system, using the first order model, was surprisingly more stable than that using the second and third order models. Run #18 (Figure 4.28) was a high-noise run. The parameters converged but yielded a rather high gain. Run #19 (Figure 4.29) was a low-noise run. The parameters also converged and the control was as good as any of the best in these experiments. For T = 14 seconds, however, convergence and stability were difficult to obtain.

### 4.6 Tests on a Simulated Plant

In view of the convergence difficulties apparent in the second and third order identifications, further testing was performed using the analog computer. The heat exchanger was simulated on the analog computer according to the step response model obtained in §2.5. The noise was not included in the simulation, since it was hoped that something may be learned from the deterministic case that may have been previously hidden in the stochastic case. A primary purpose of these tests was to verify that the programming was error-free.

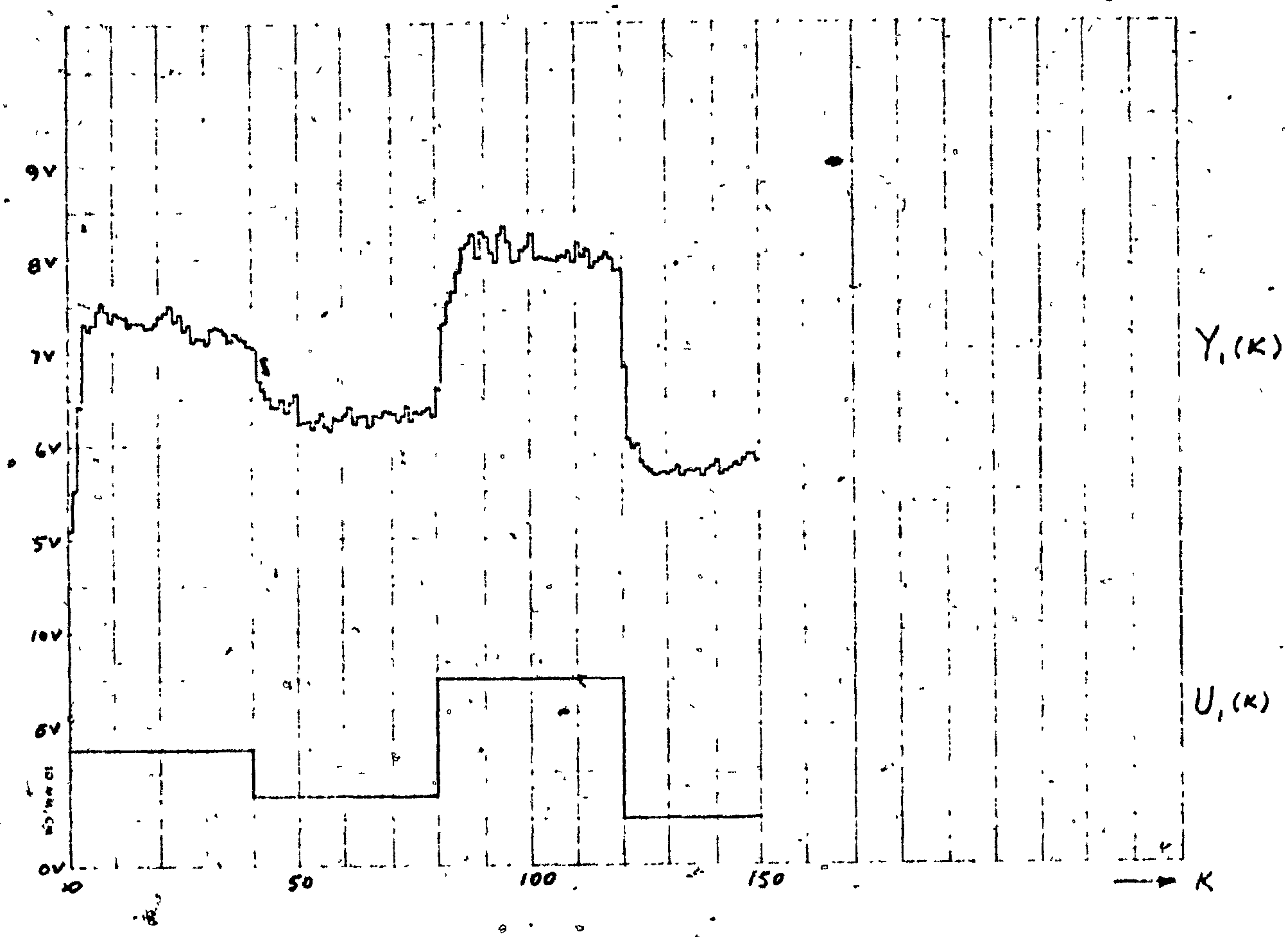
The behaviour of the program on the noise-free simulated system was about the same as that for the noisy real-life system: the first and fourth order identifications converged, and the second and third order identifications did not. The third order case had a strong tendency to saturate and therefore terminate the identification at a very early stage (even before any set-point change was made). The second order case was more stable but still produced poor control and divergent parameters. It was observed, however, that the divergent tendencies in the second order case occurred immediately at the set-point changes. That is, at each set-point change, the parameter values changed drastically but recovered at the next step to an only slightly different set of values. Eventually, however, recovery became less complete and the parameters began to diverge.

This behaviour suggests that the programming is not at fault, and that the difficulty may be eliminated by increasing the value of the noise parameter $\alpha$. This proved to be the case. It was found that smooth consistent convergence could not be obtained in the second order case for α much less than about 6.5 $volt^2$. Similarly, for the third order case, α had to be not less than about 13 $volt^2$. Interestingly, in the third order case, the identification degraded much more rapidly with a reduction of α than in the second order case. In fact there appeared to be a rather sharp border between convergence and divergence, located approximately at $\alpha = 13$ $volt^2$. Decreasing α from this point leads to divergent identification. Increasing α, on the other hand, serves to slow down the identification. This can be expected when noting that α appears in the denominator of the expression for the gain factor Γ in Equation 3.3.

Models of first and fourth order produced no convergence difficulties. Consistent convergence in these cases was obtained for α as low as $2.5 \times 10^{-5}$ $volt^2$. In the fourth order case, this value of α led to convergence on the actual noisy heat exchanger as well as the noiseless simulation. Note that the actual heat exchanger had a noise variance of .025 $volt^2$.

Results are shown in Figures 4.31 to 4.35. The success of these tests validates both the programming and the algorithm. However, it also raises the question of the significance of α. This question is taken up in the next chapter.

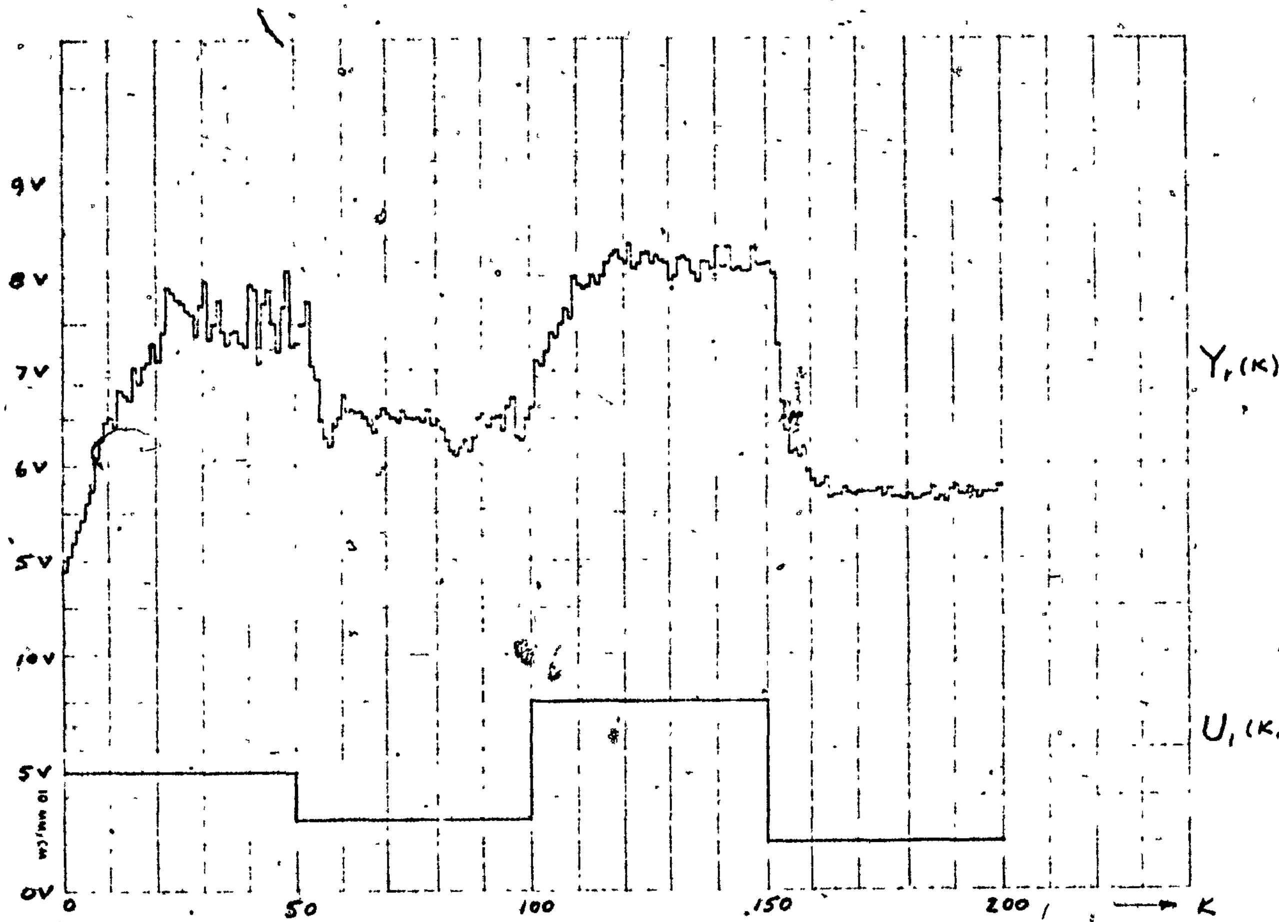
FIGURE 4.1




OPEN LOOP RESPONSE

T = 42 seconds

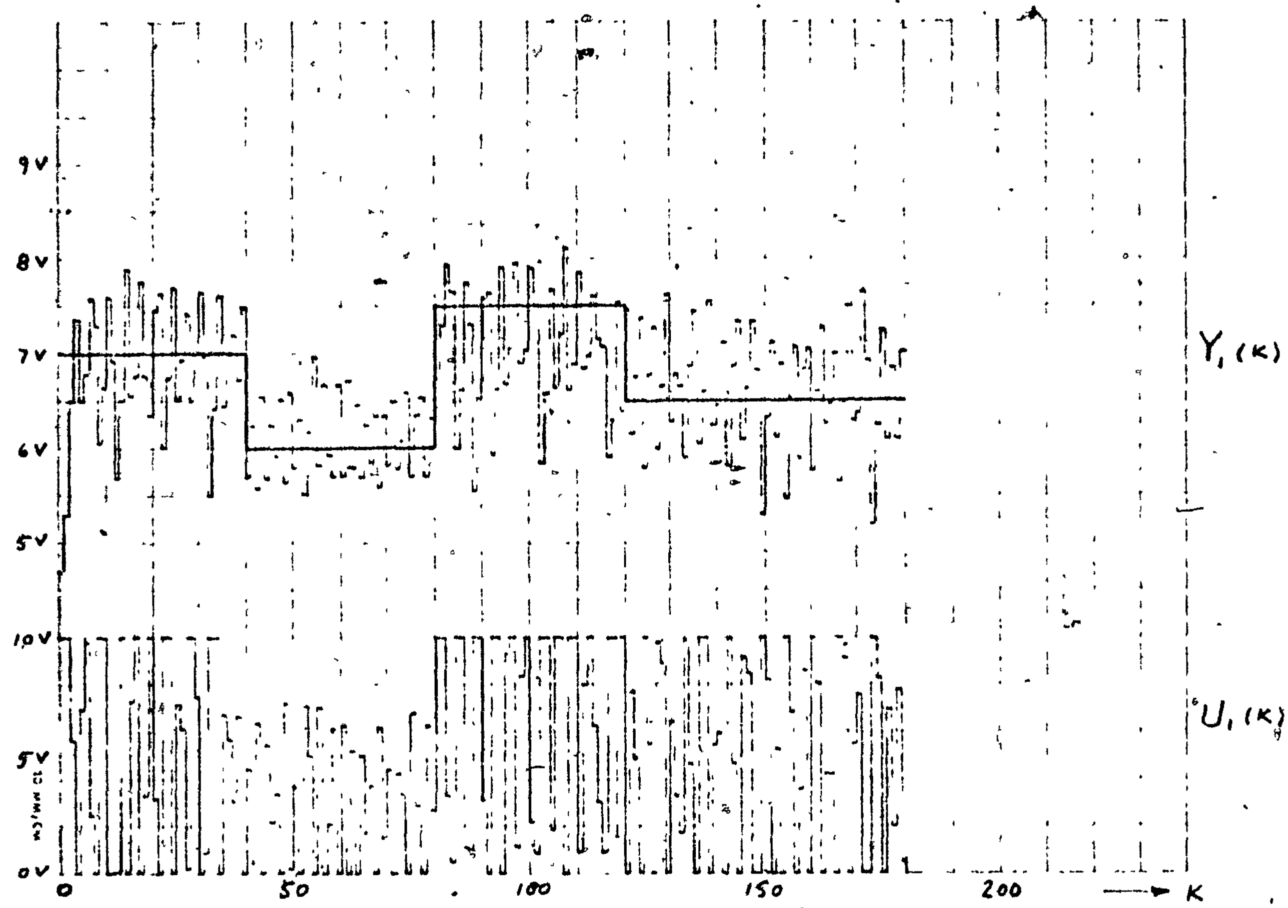
FIGURE 4.2




OPEN LOOP RESPONSE

T = 14 seconds

FIGURE 4.3

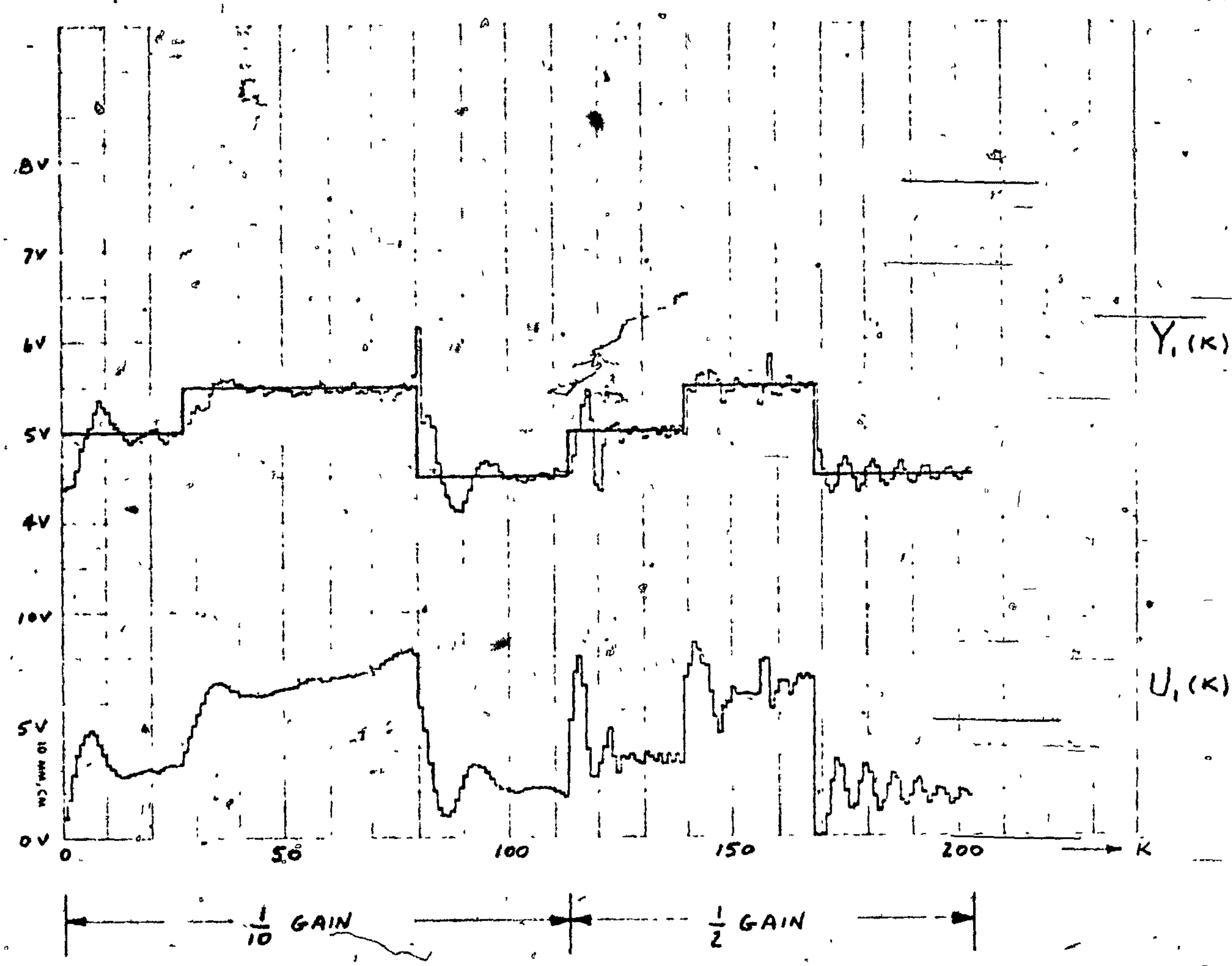



MINIMUM VARIANCE CONTROL

BASED ON STEP RESPONSE MODEL

T = 42 seconds

FIGURE 4.4

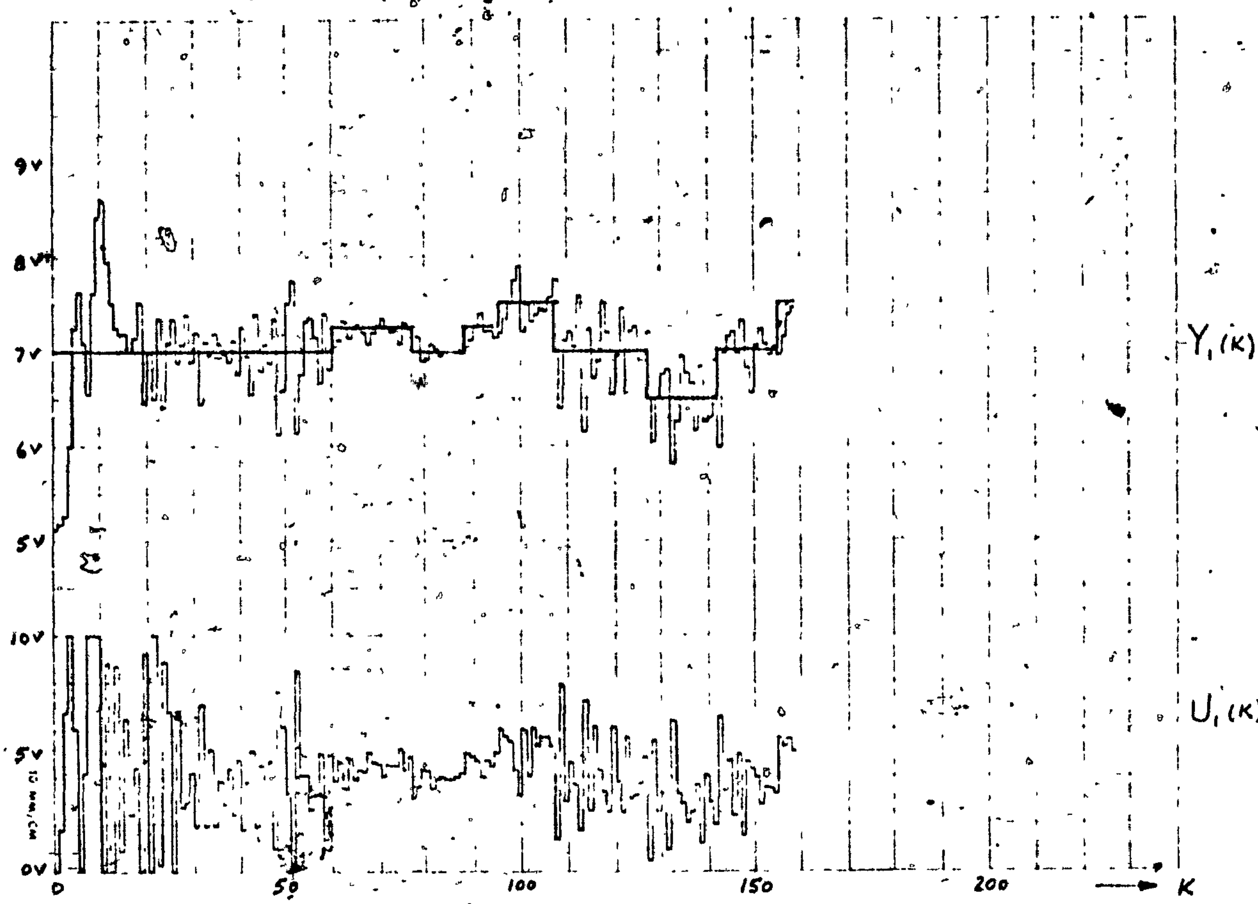



MINIMUM VARIANCE CONTROL

BASED ON STEP RESPONSE MODEL

T = 42 seconds

FIGURE 4.5a



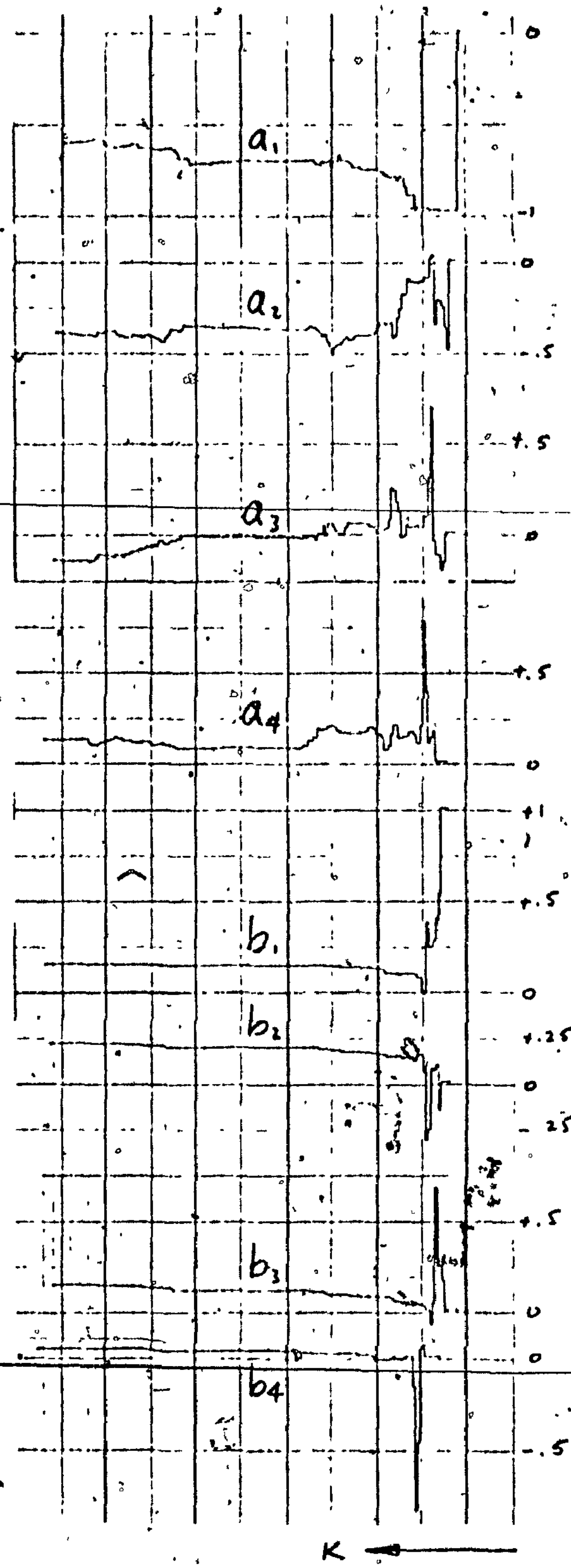

SELF-TUNING CONTROL

FOURTH ORDER　　　　PUN # 1

T = 42 seconds

$b_1(0) = 1$

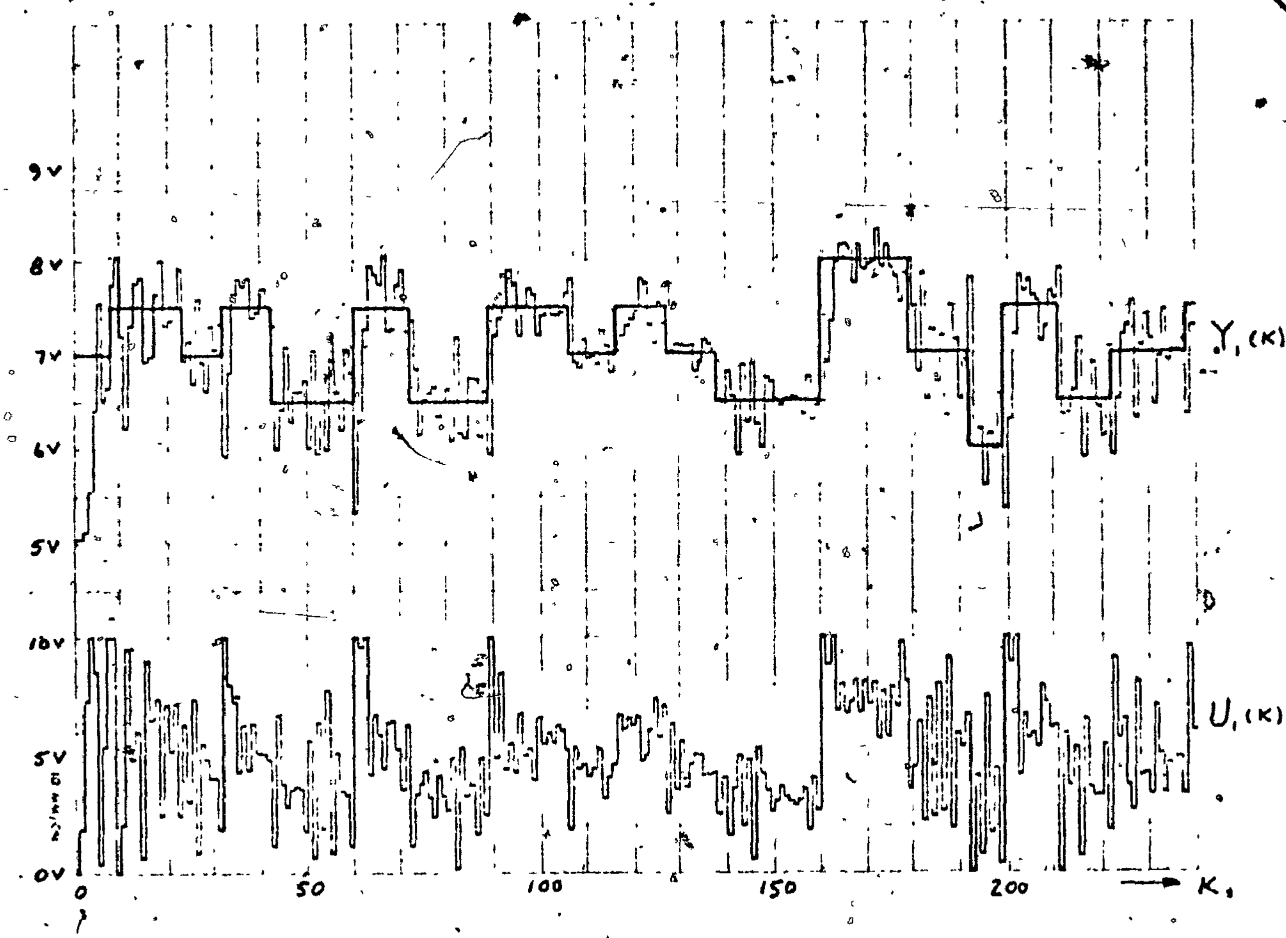
FIGURE 4.5b




PARAMETER ESTIMATES

FOURTH ORDER RUN # 1

FIGURE 4.6a




SELF-TUNING CONTROL

FOURTH ORDER RUN # 2

T = 42 seconds

$b_1(0) = 1$

FIGURE 4.6a (Continued)

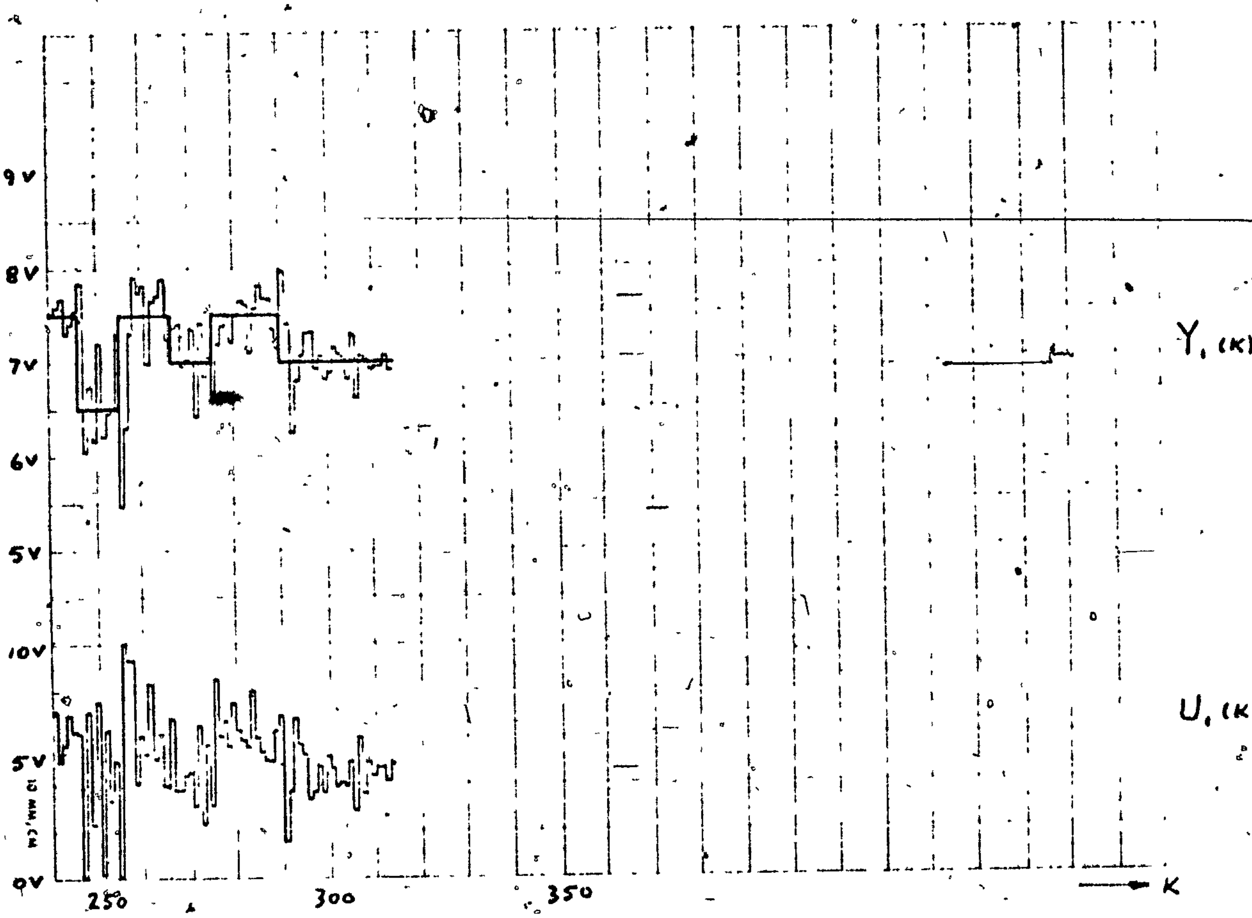
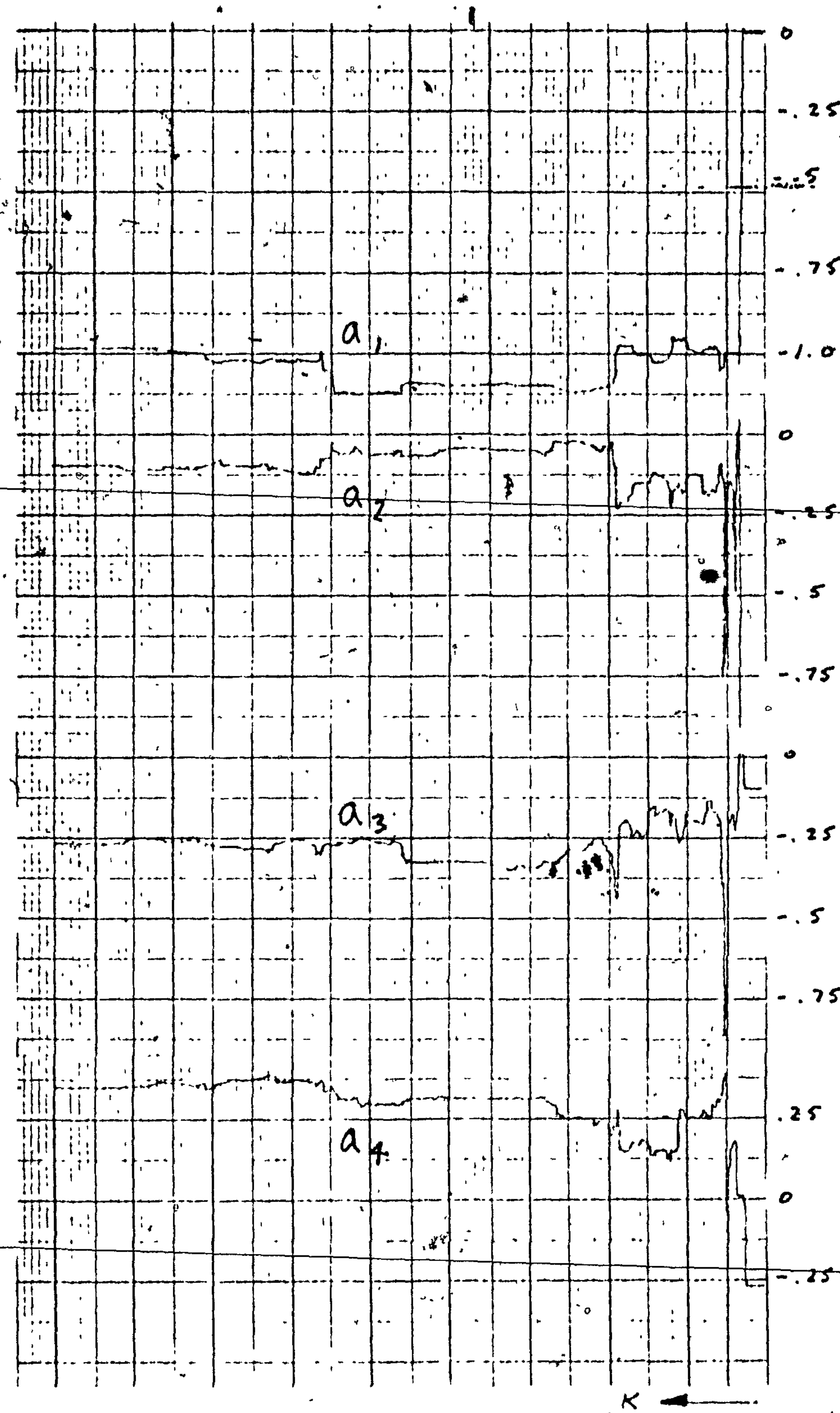


FIGURE 4.6b.




PARAMETER ESTIMATES

FOURTH ORDER

RUN # 2

FIGURE 4.61 (Continued)

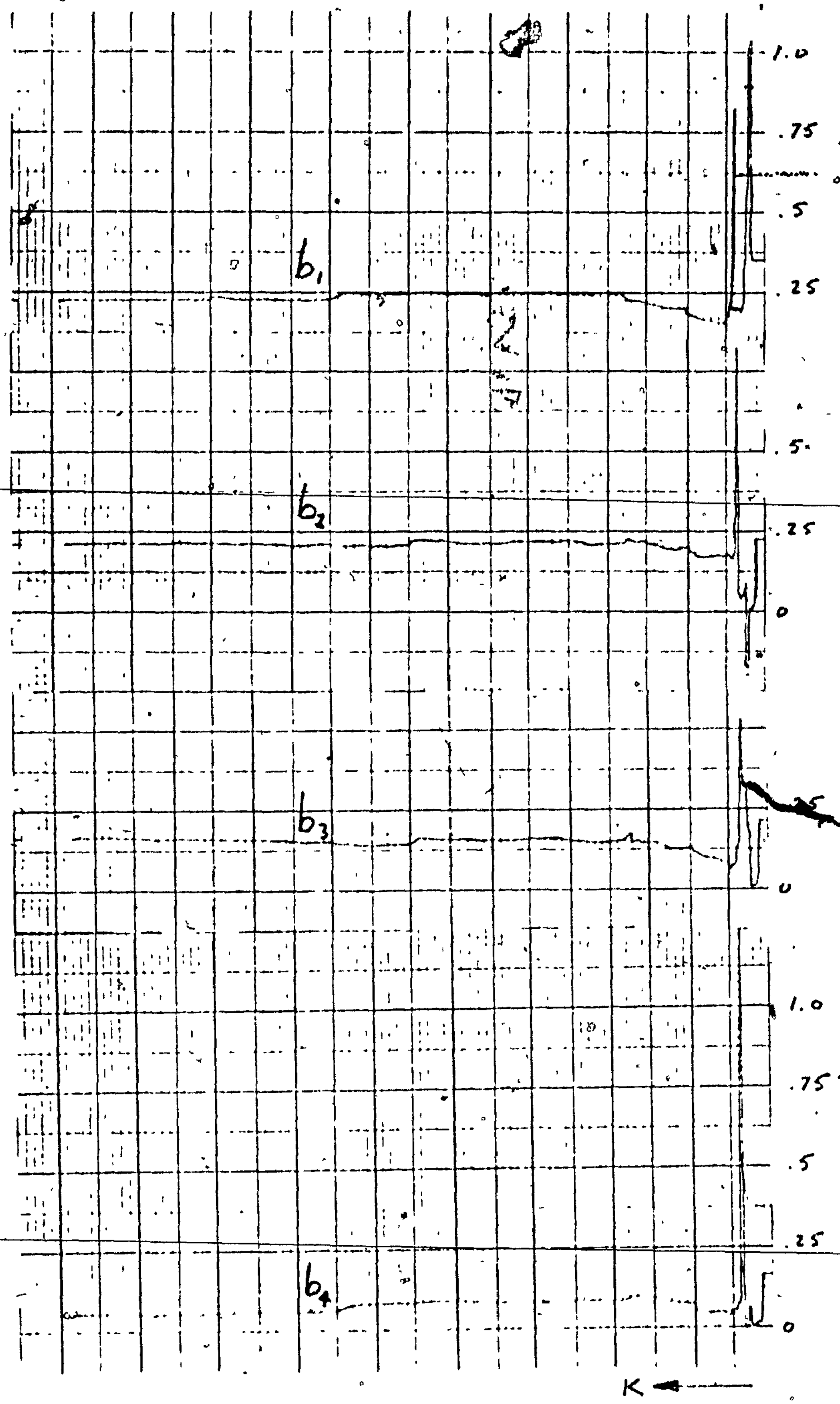
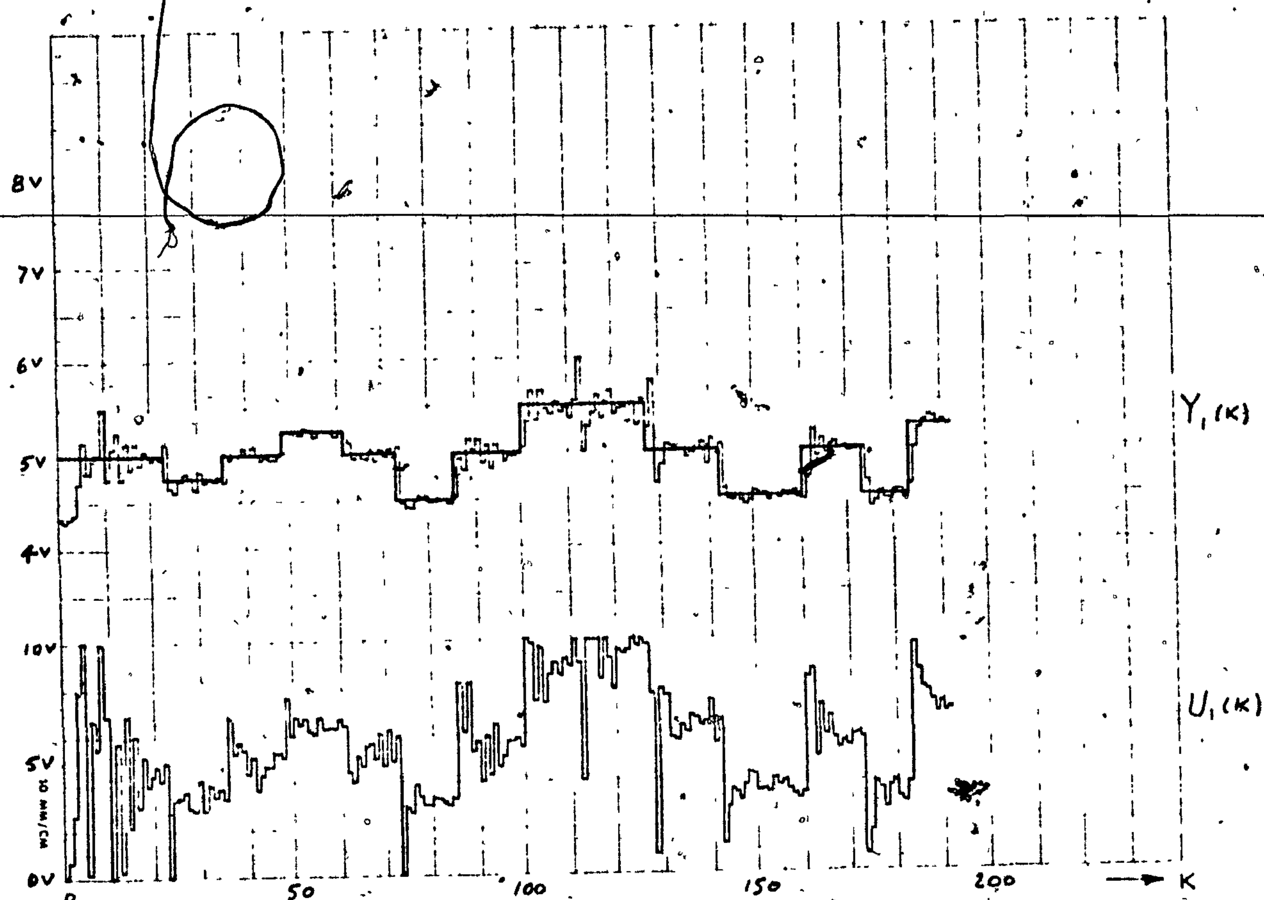


FIGURE 4.7a



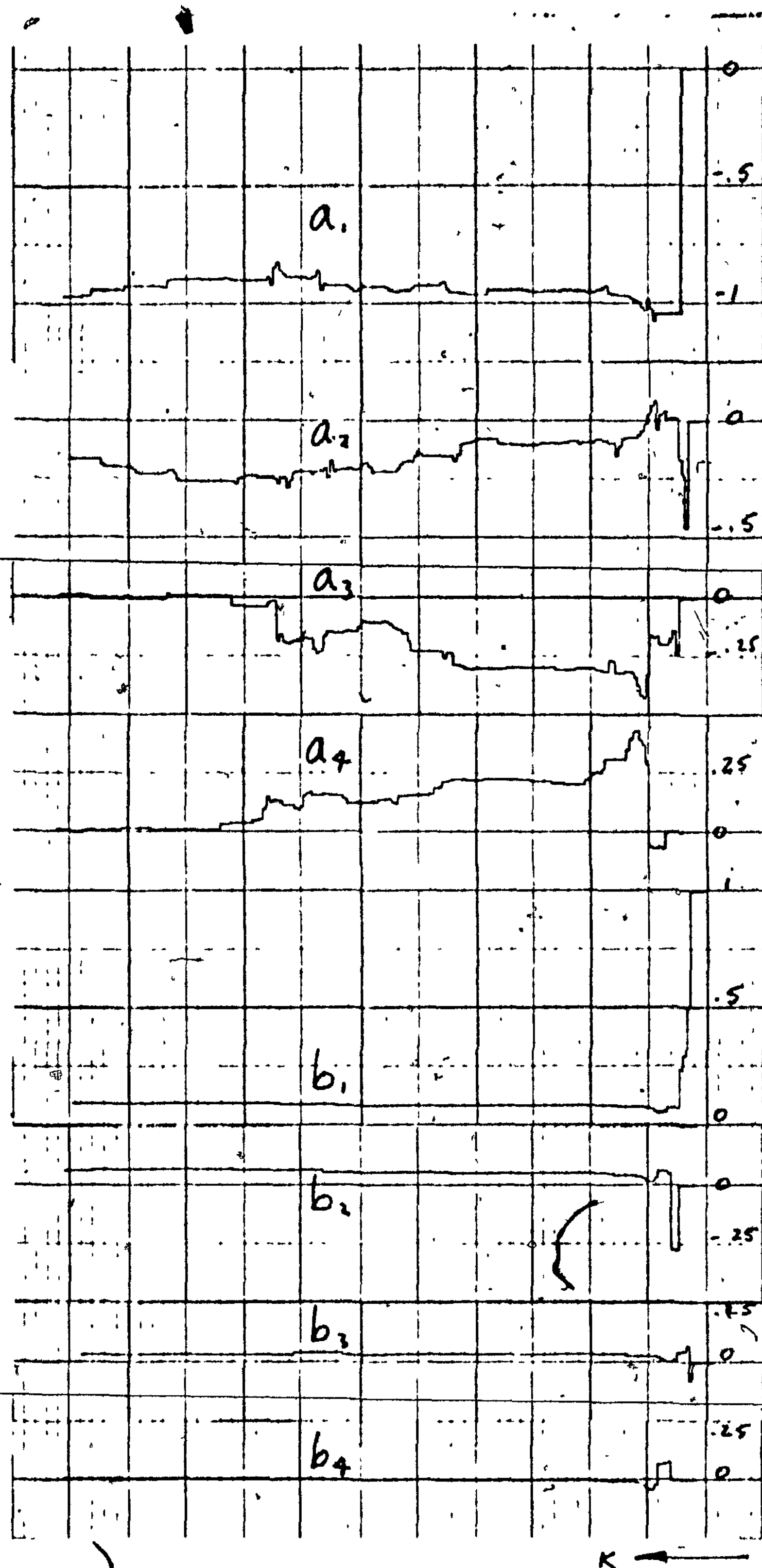

SELF-TUNING CONTROL

FOURTH ORDER RUN # 3

$T = 42$ seconds

$b_1(0) = 1$

FIGURE 4.7b

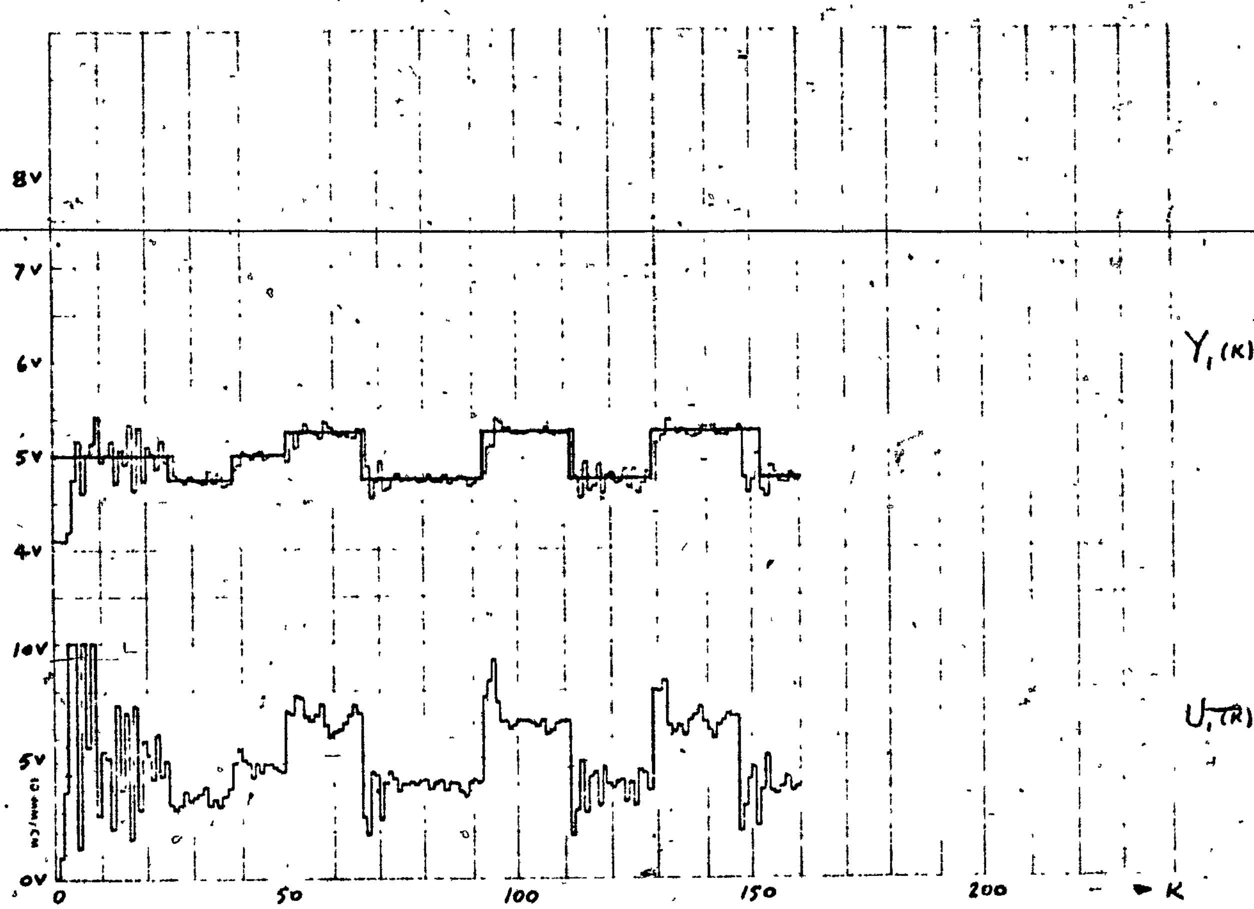



PARAMETER ESTIMATES

FOURTH ORDER

RUN # 3

FIGURE 4.8a




SELF-TUNING CONTROL

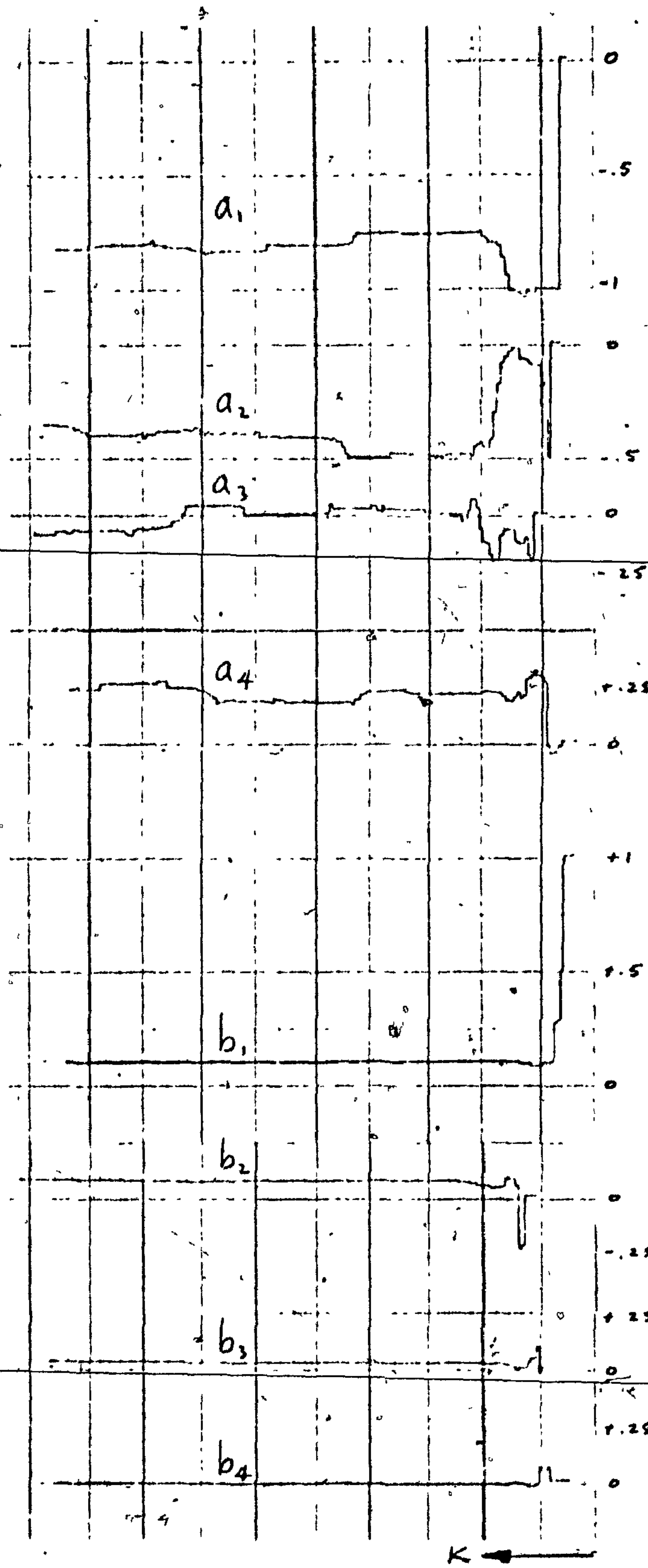
FOURTH ORDER RUN # 4

$T = 42$ seconds

$b_1(0) = 1$

$\alpha = .64$ volts$^2$

FIGURE 4.8b

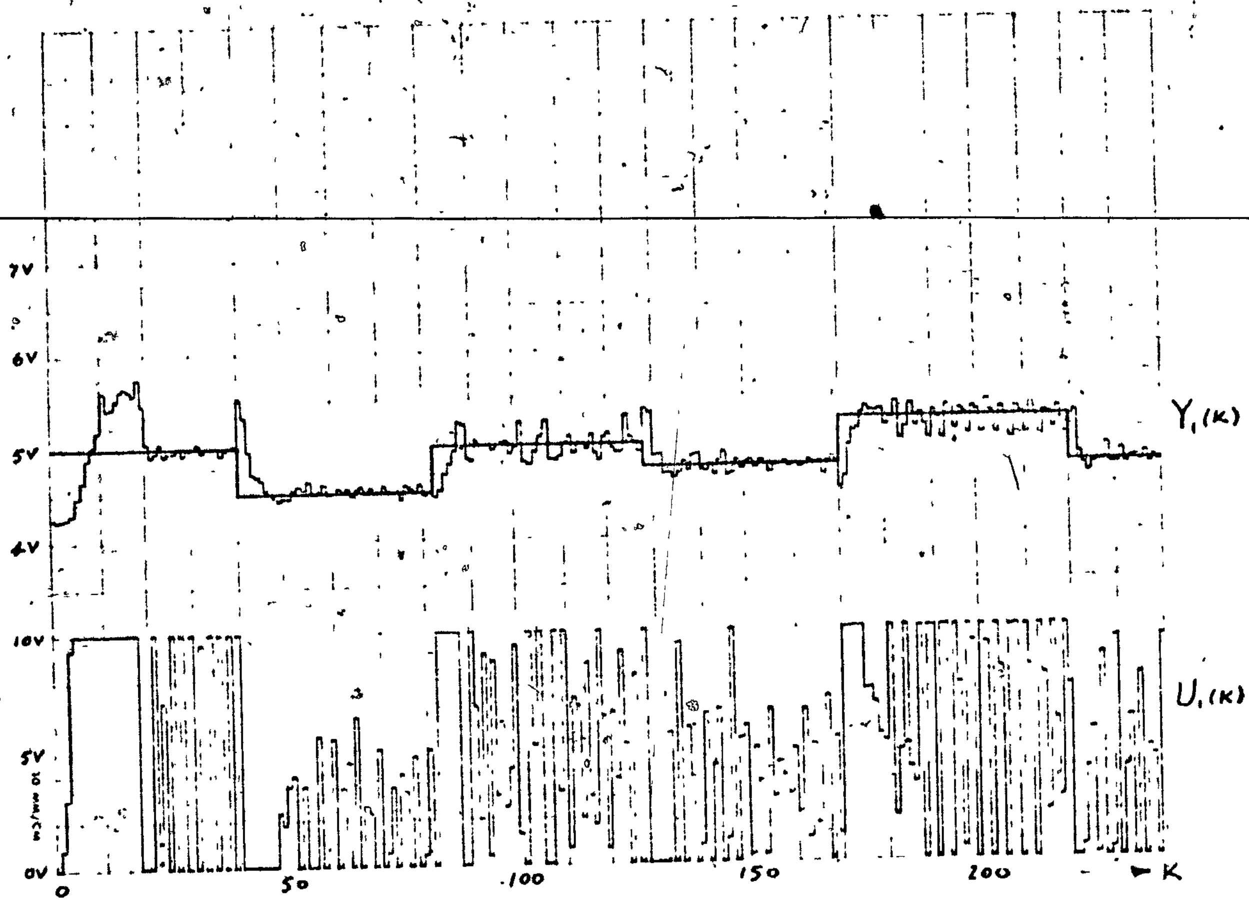



PARAMETER ESTIMATES

FOURTH ORDER

RUN # 4

FIGURE 4.9a



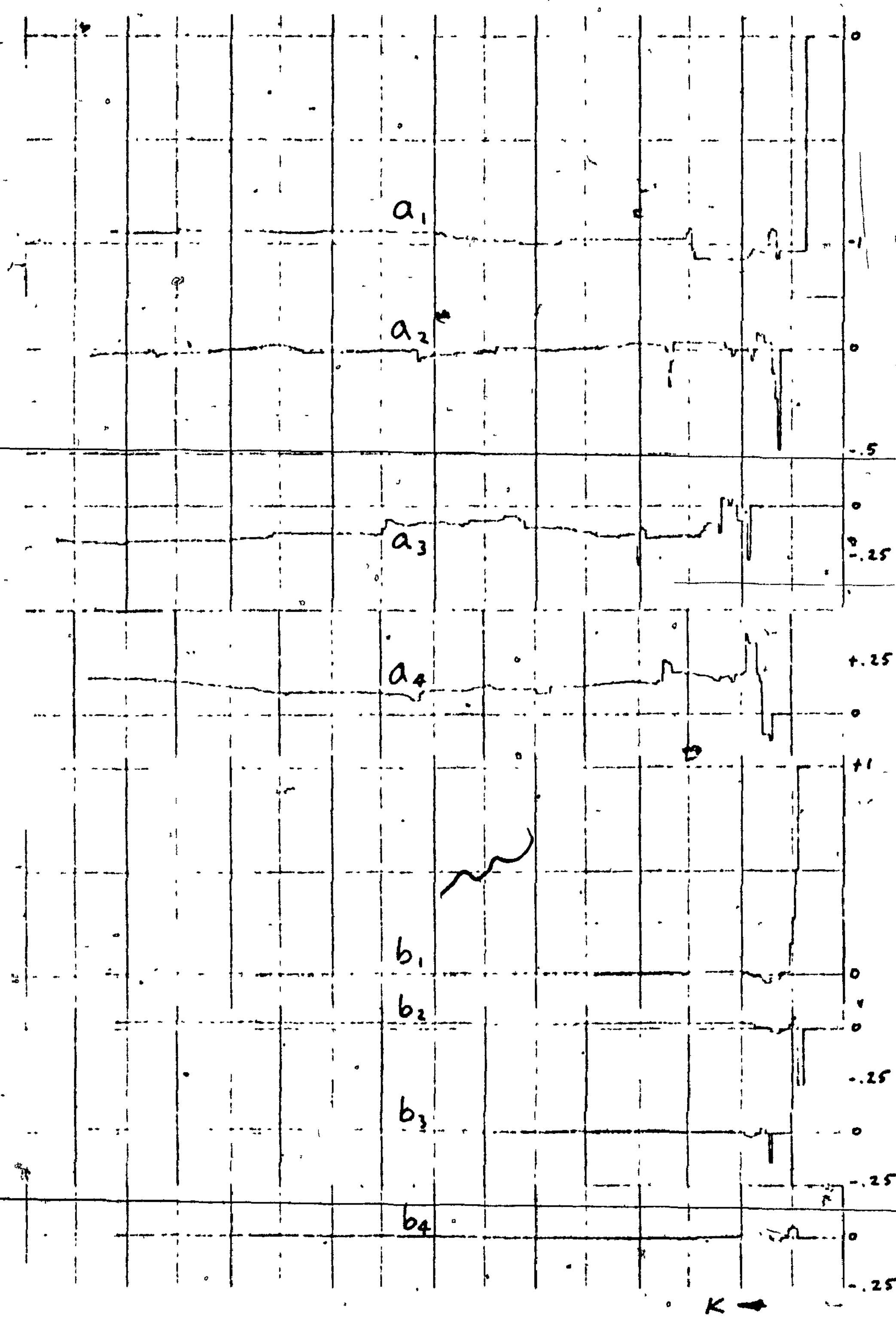

SELF-TUNING CONTROL,

FOURTH ORDER RUN # 5

T = 14 seconds

$b_1(0) = 1$

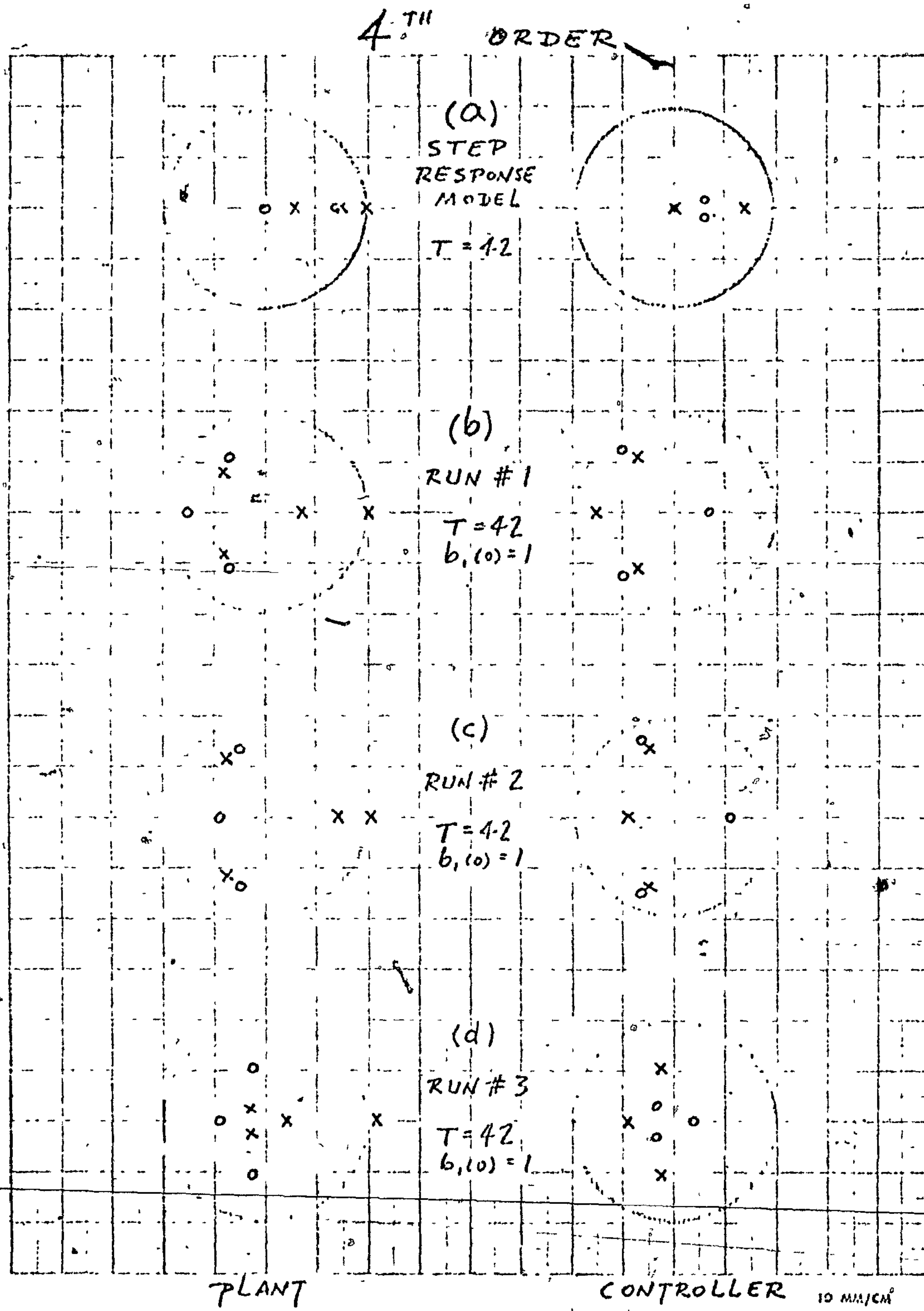
FIGURE 4.9b




PARAMETER ESTIMATES

FOURTH ORDER RUN # 5

FIGURE 4.10




POLE-ZERO PATTERN OF IDENTIFIED PLANT

AND OF CORRESPONDING MINIMUM VARIANCE CONTROLLER

FIGURE 4.10 (Continued)

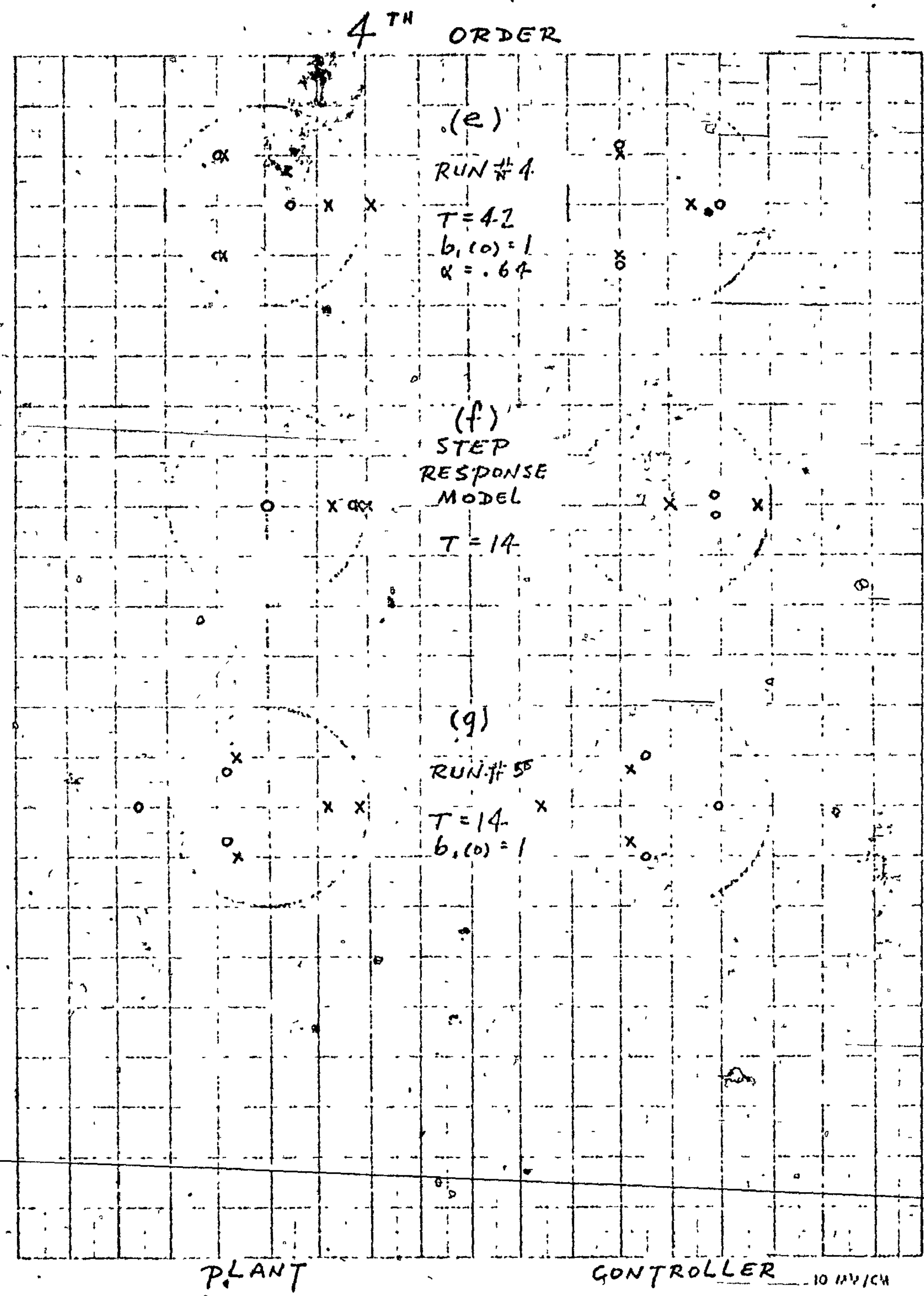
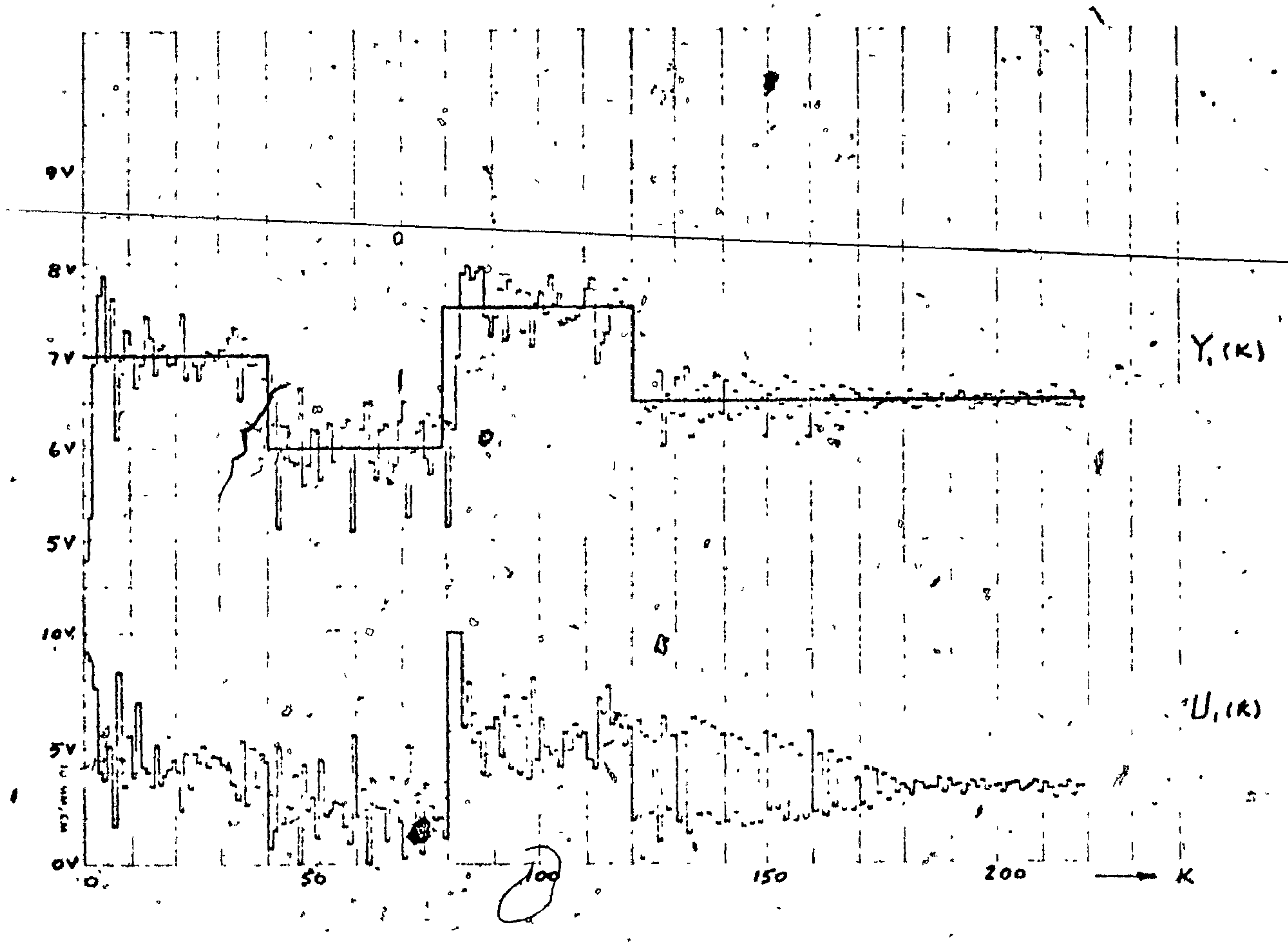


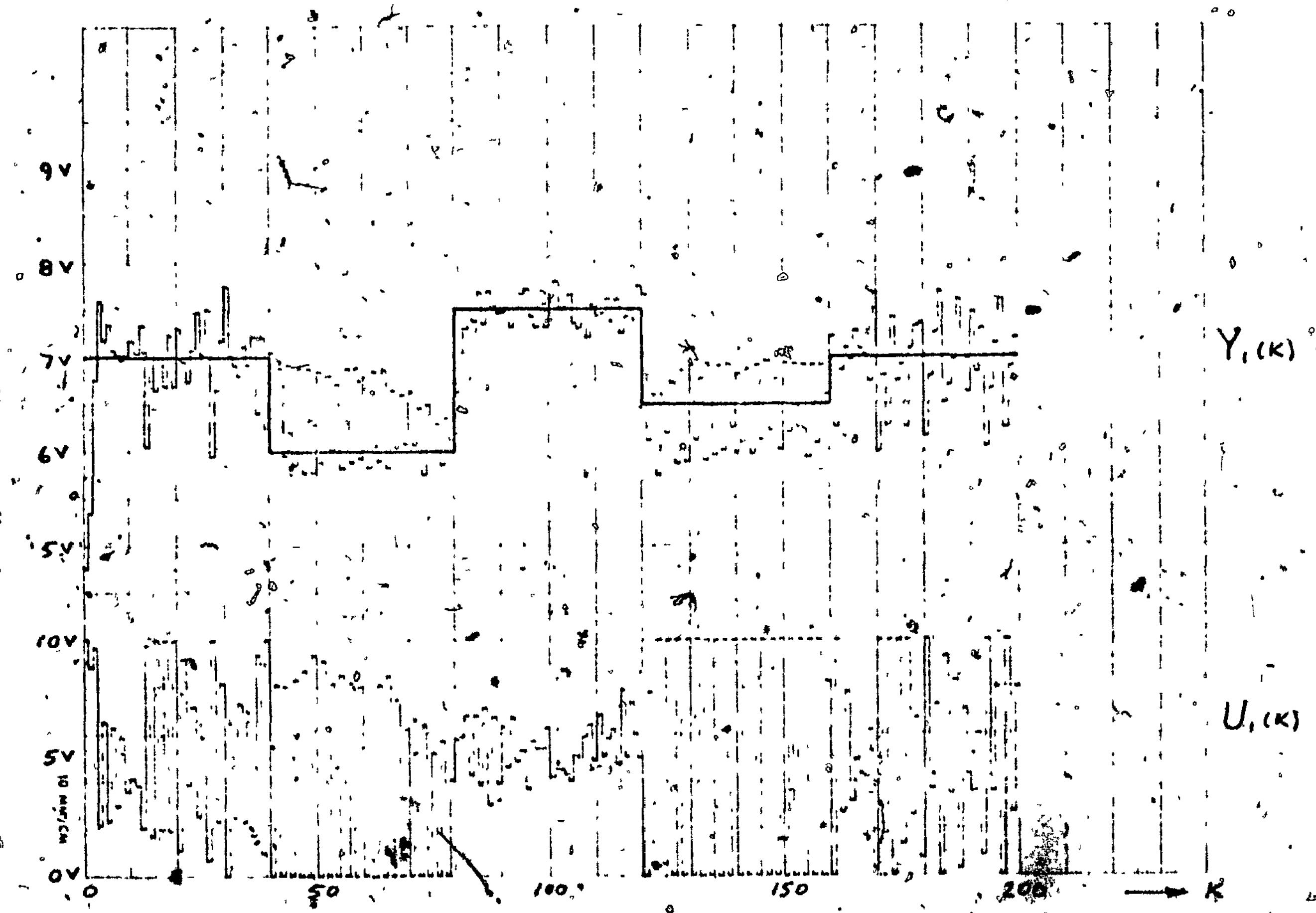
FIGURE 4.11




MINIMUM VARIANCE CONTROL BASED ON RUN #1

T = 42 seconds

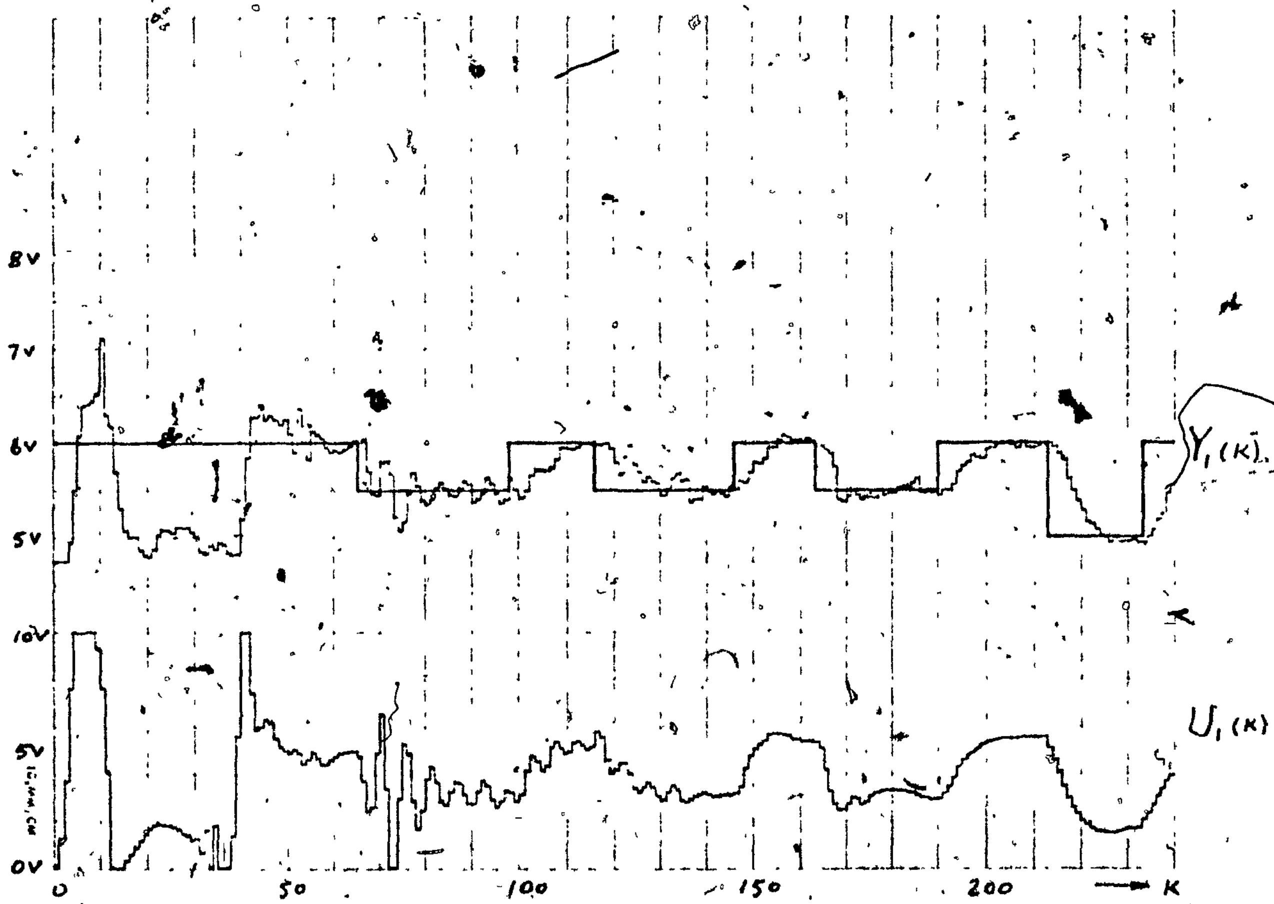
FIGURE 4.12




MINIMUM VARIANCE CONTROL BASED ON RUN # 2

T = 42 seconds

FIGURE 4.13a



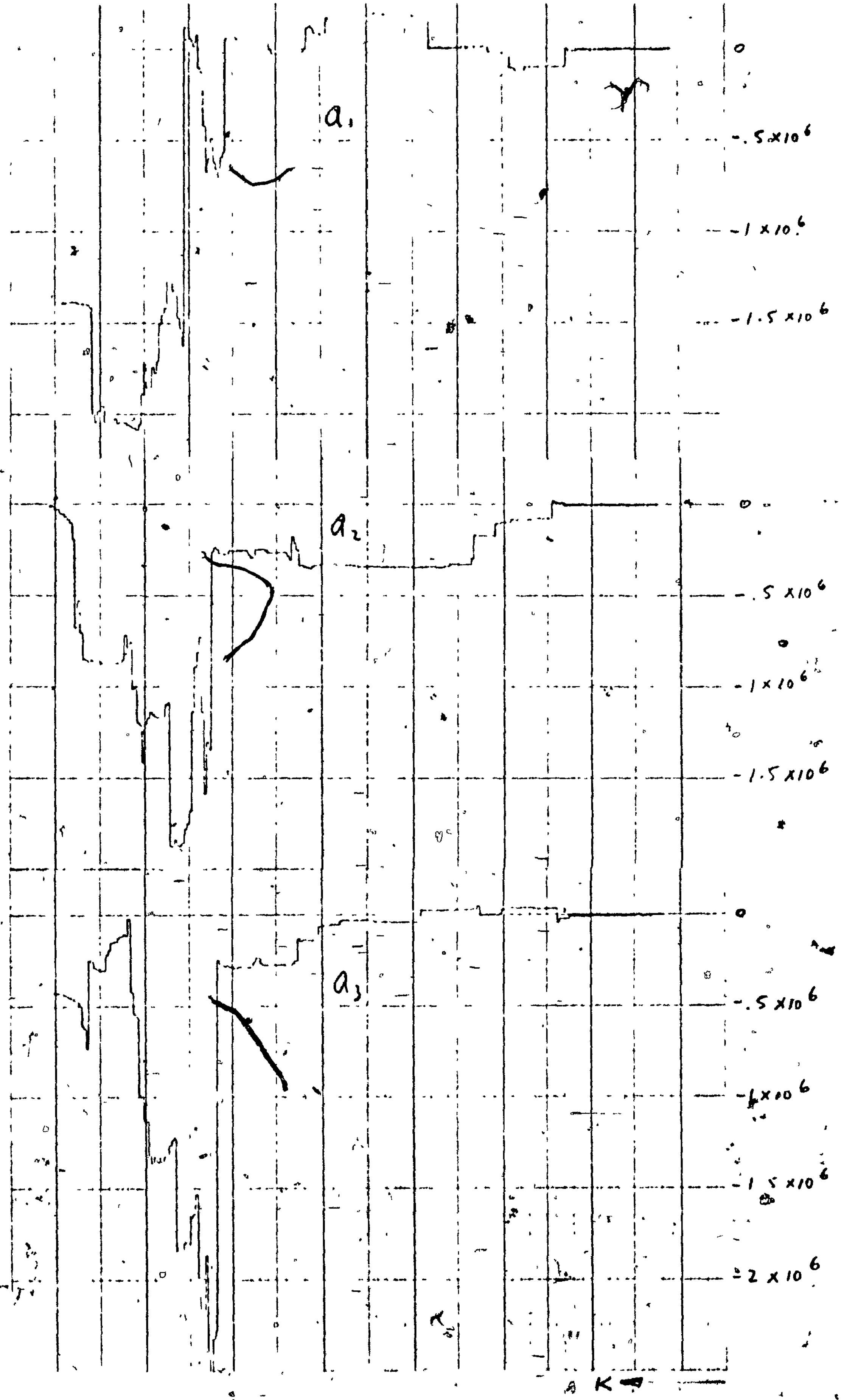

SELF-TUNING CONTROL

THIRD ORDER RUN # 6

T = 42 seconds

$b_1(0) = 1$

FIGURE 4.13b




THIRD ORDER PARAMETER ESTIMATES RUN # 6

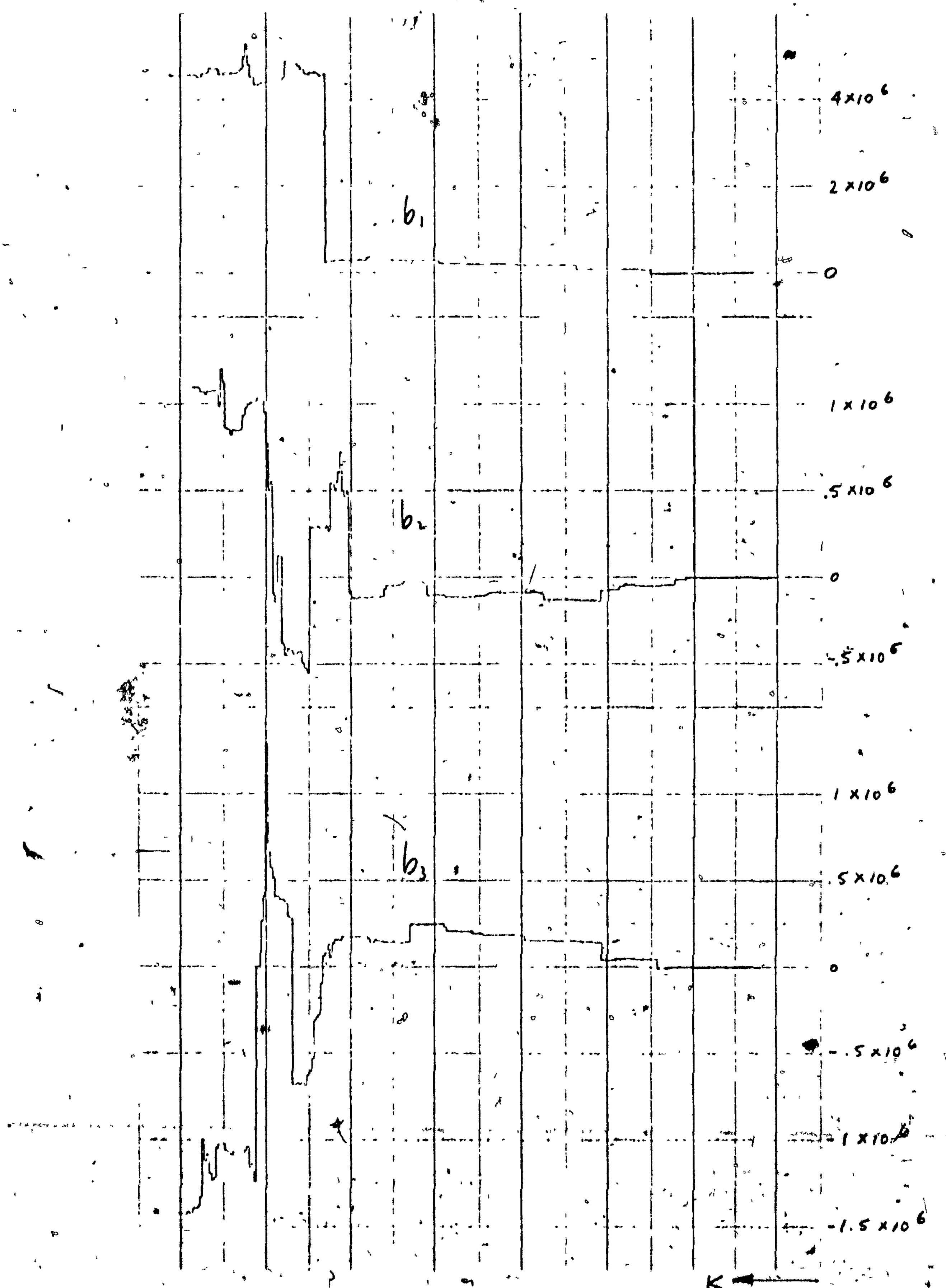
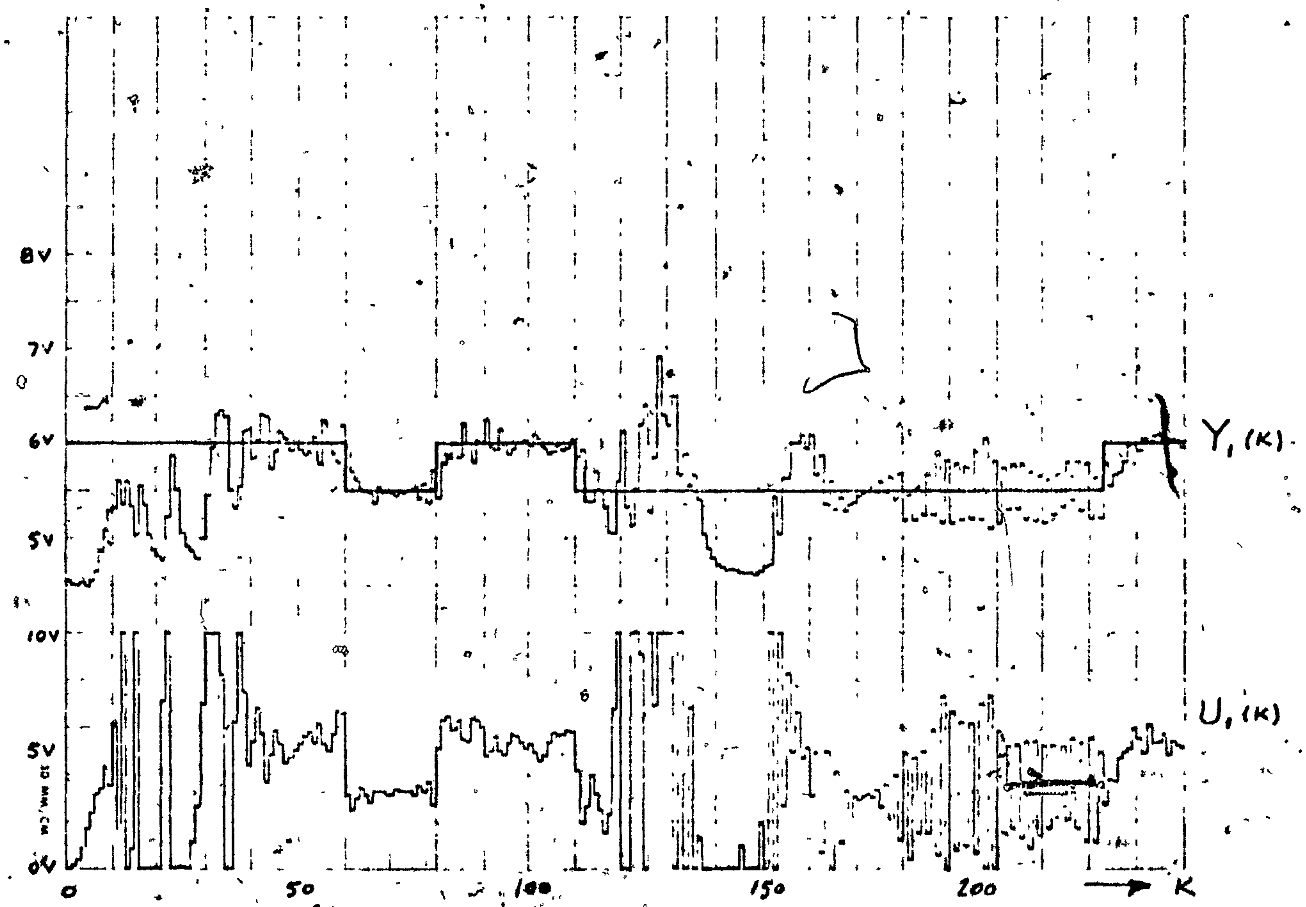

4 x10^6
2 x10^6
0
b1
1 x 10^6
.5 x10^6
0
-.5 x10^6
b2
1 x10^6
.5 x10^6
0
-.5 x10^6
-1 x10^6
-1.5 x10^6
b3
K

FIGURE 4.14a



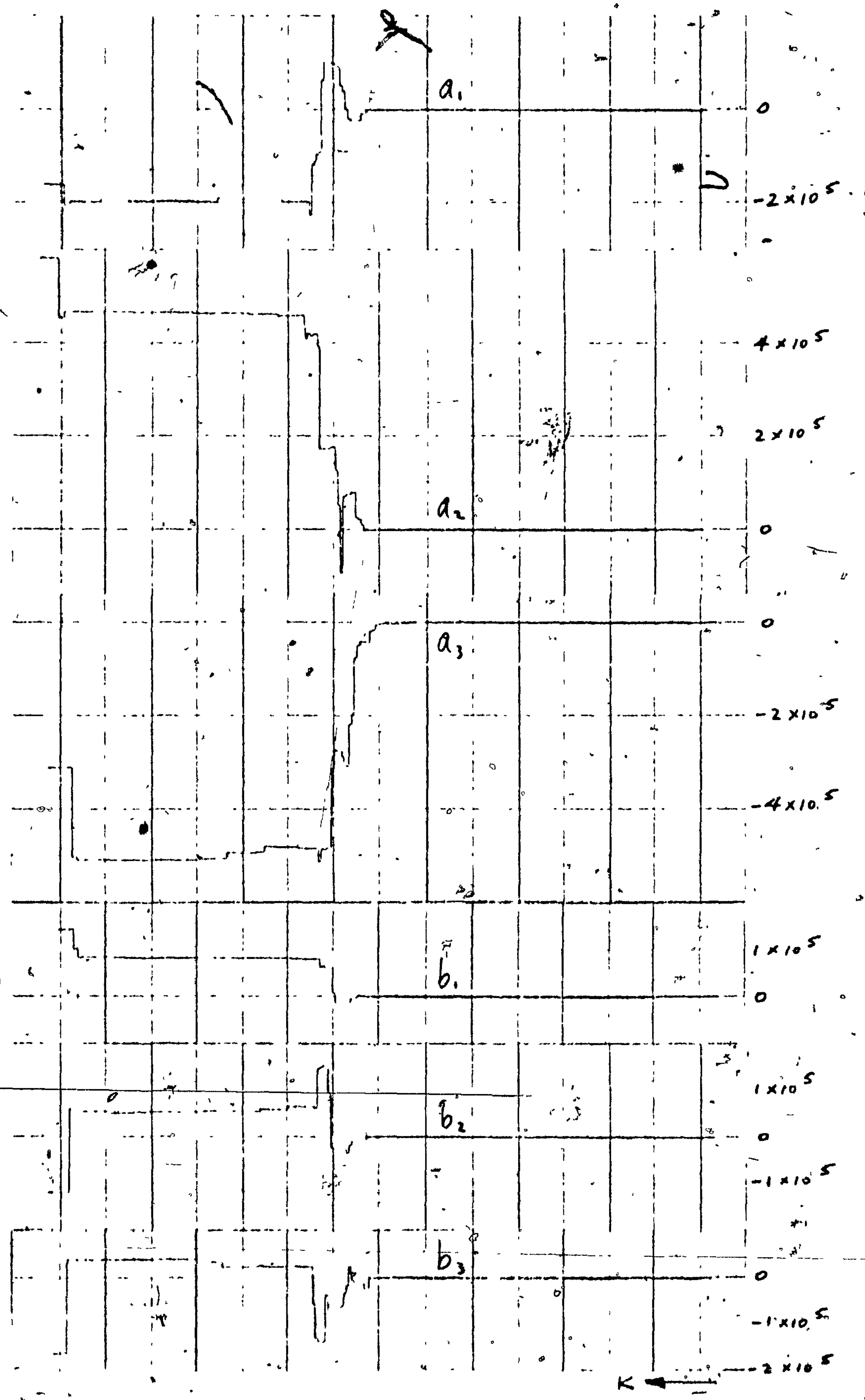

SELF-TUNING CONTROL

THIRD ORDER RUN [illegible]

T = 42 seconds

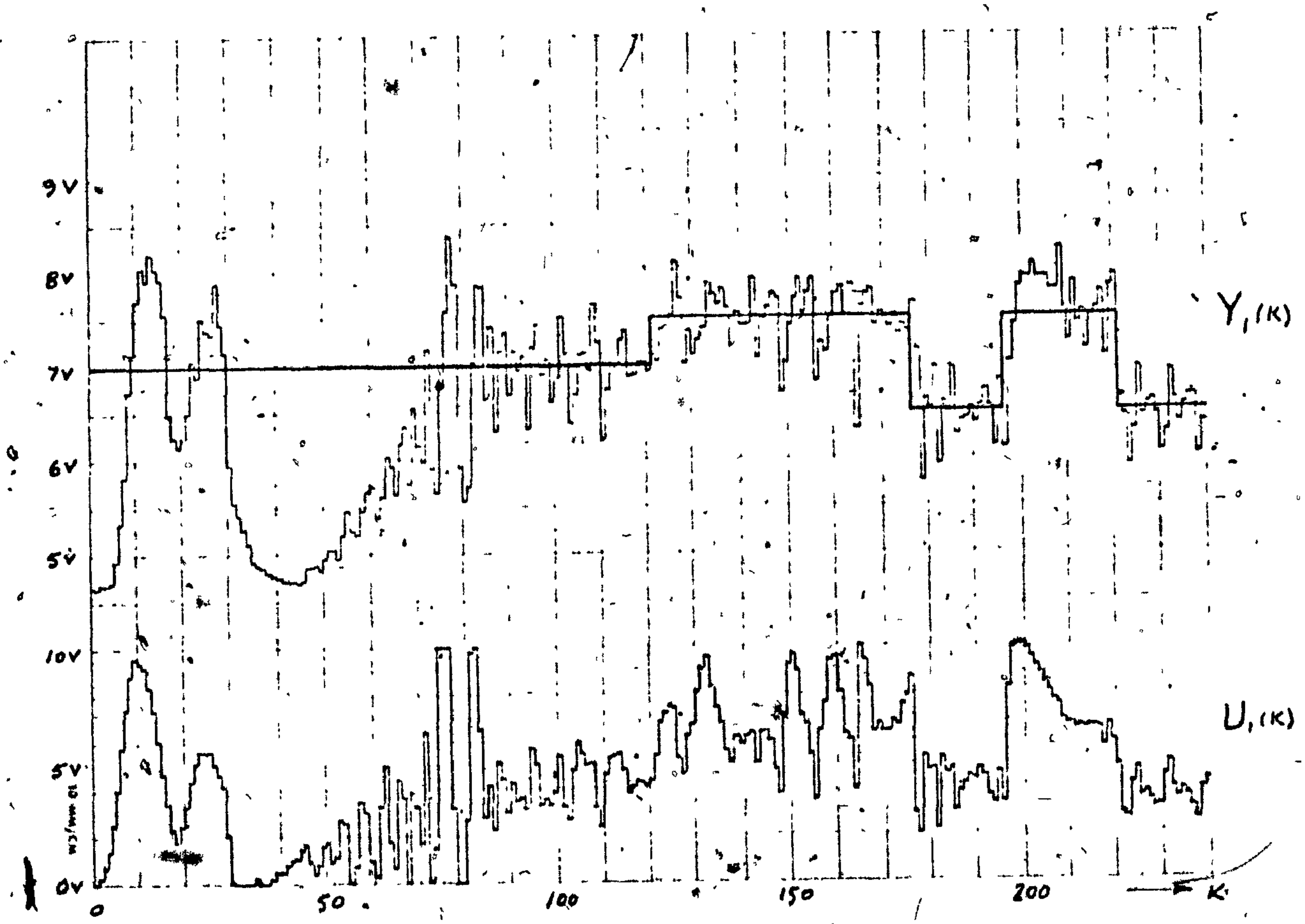
$b_1(0) = 10$

FI'URE 4.14b




THIRD ORDER PARAMETER ESTIMATES RUN # 7

FIGURE 4.15a



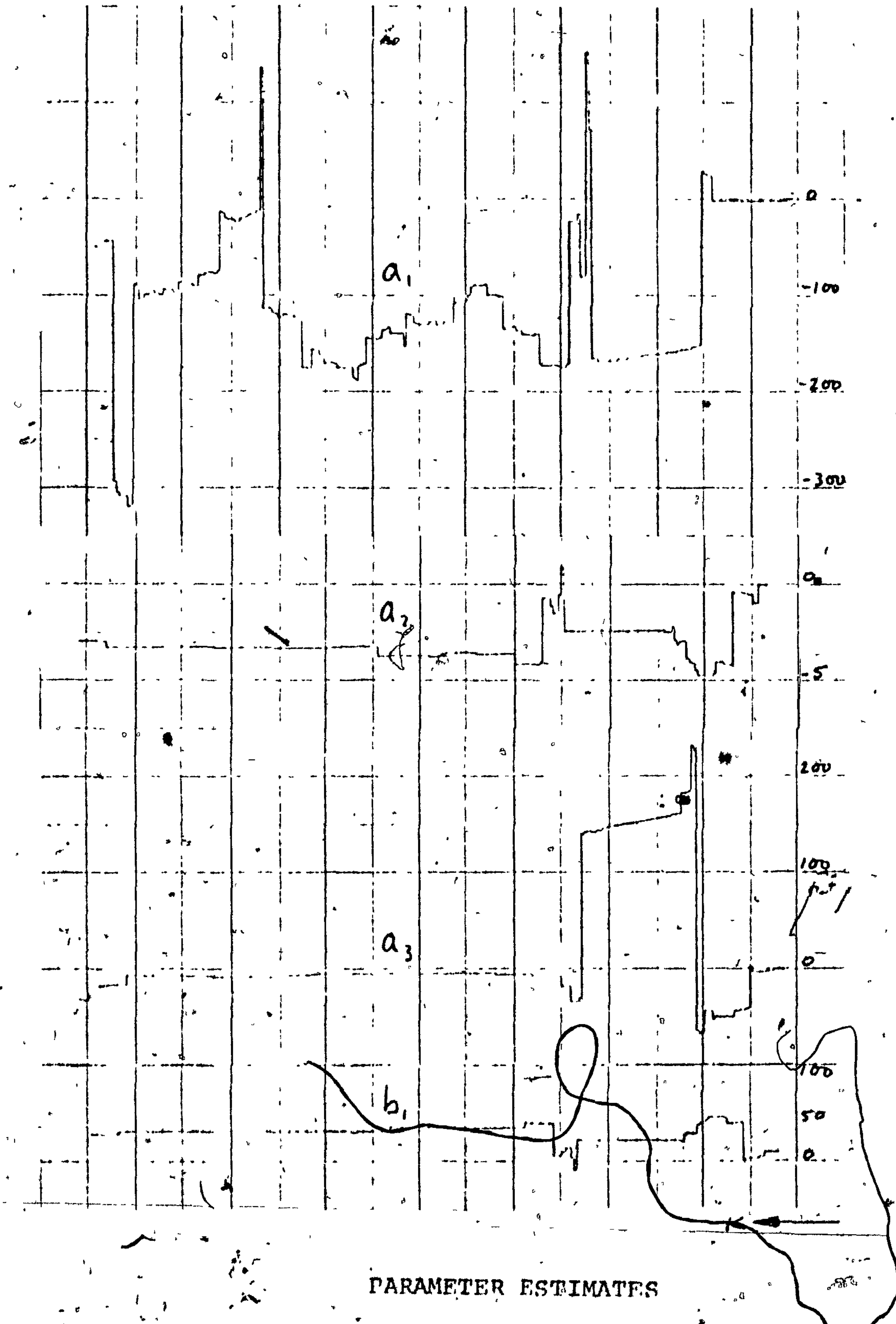

SELF-TUNING CONTROL

THIRD ORDER (PID) RUN # 8

T = 42 seconds

$b_1(0) = 10$

FIGURE 4.15b

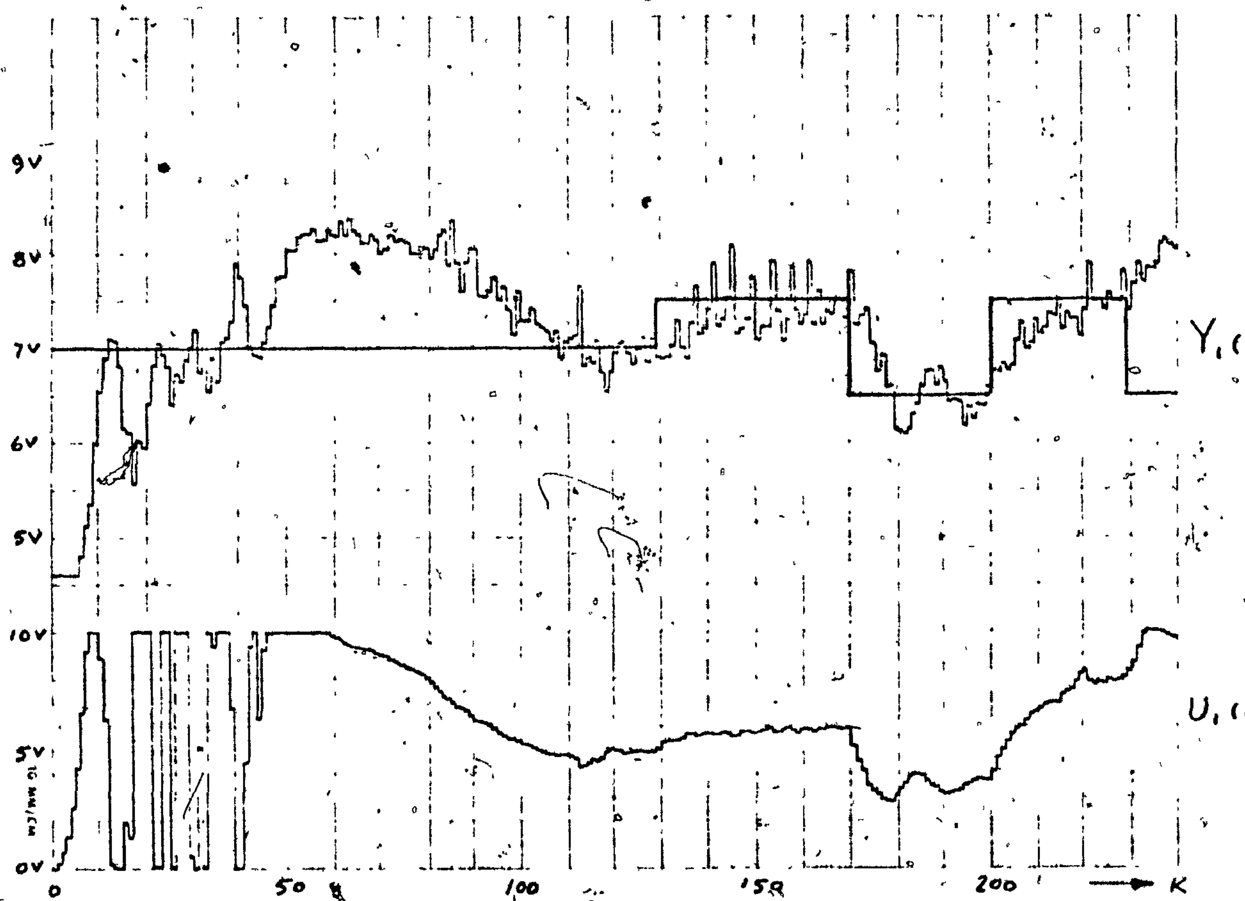



PARAMETER ESTIMATES

THIRD ORDER (PID)

RUN # 8

FIGURE 4.16a



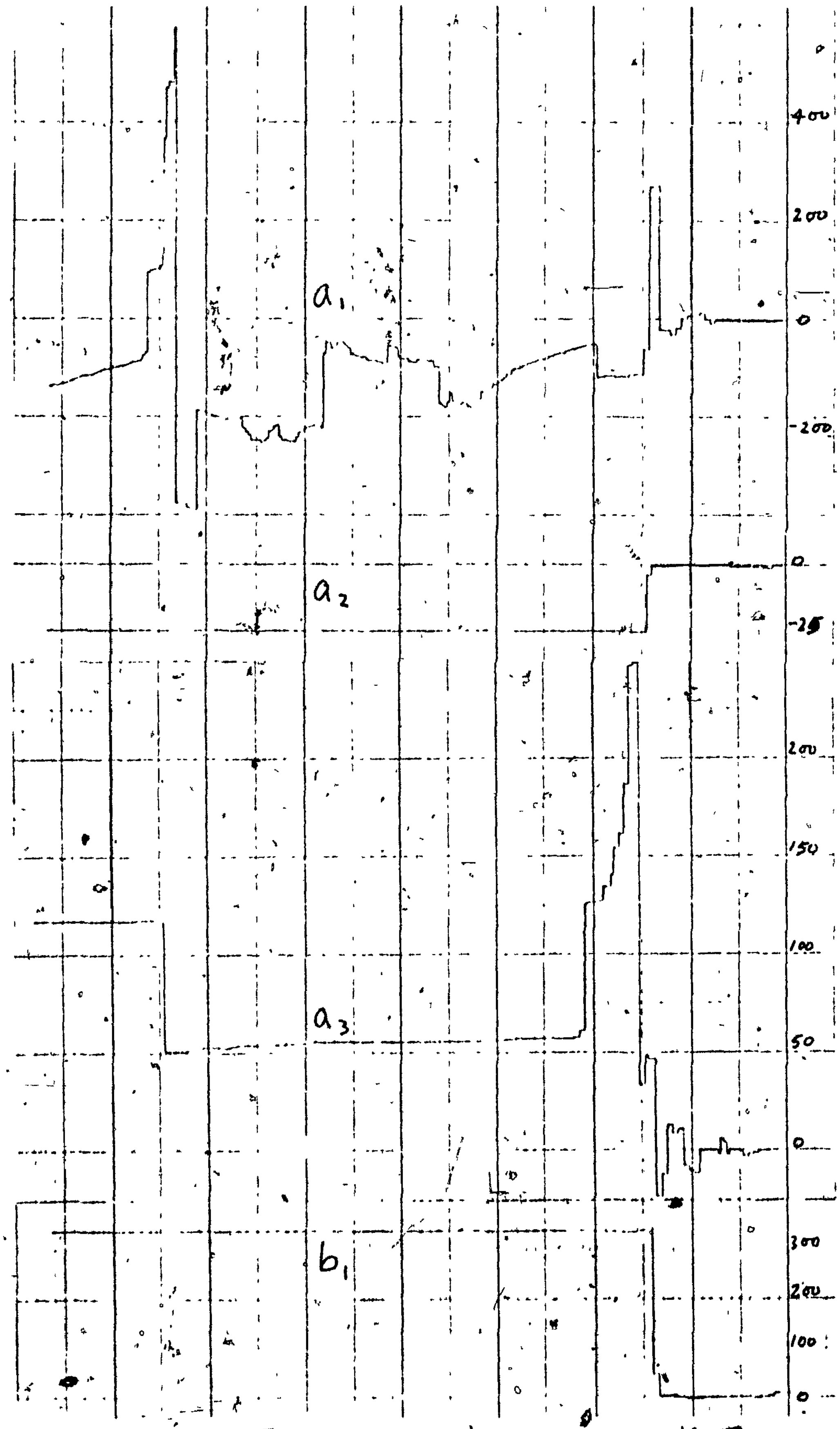

SELF-TUNING CONTROL

THIRD ORDER (PID) RUN # 9

T = 14 seconds

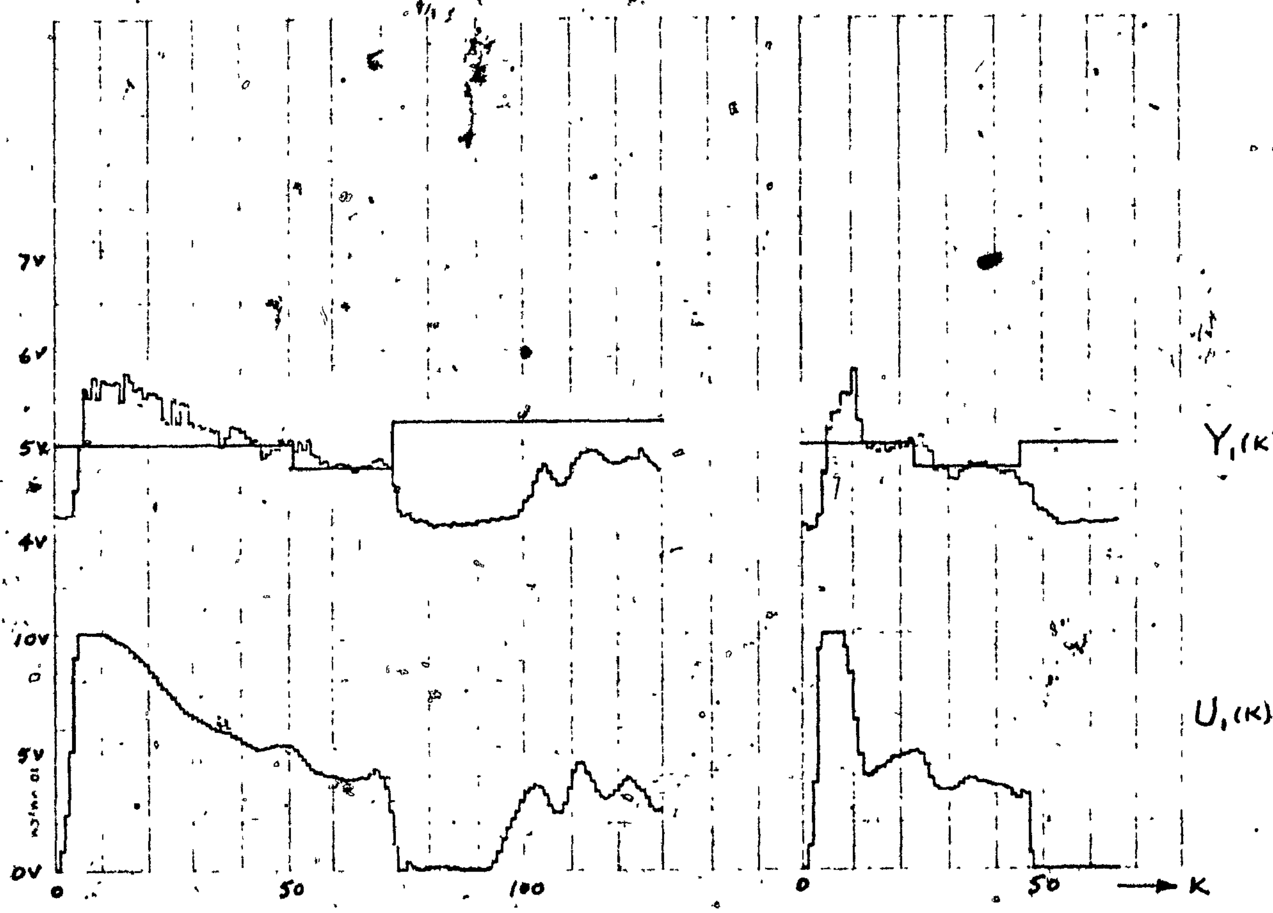
$b_1(0) = 10$

FIGURE 4.16b




THIRD ORDER (PID) PARAMETER ESTIMATES — RUN # 9

FIGURE 4.17a




$3^{rd}$ Order

$3^{rd}$ Order (PID)

SELF-TUNING CONTROL

THIRD ORDER

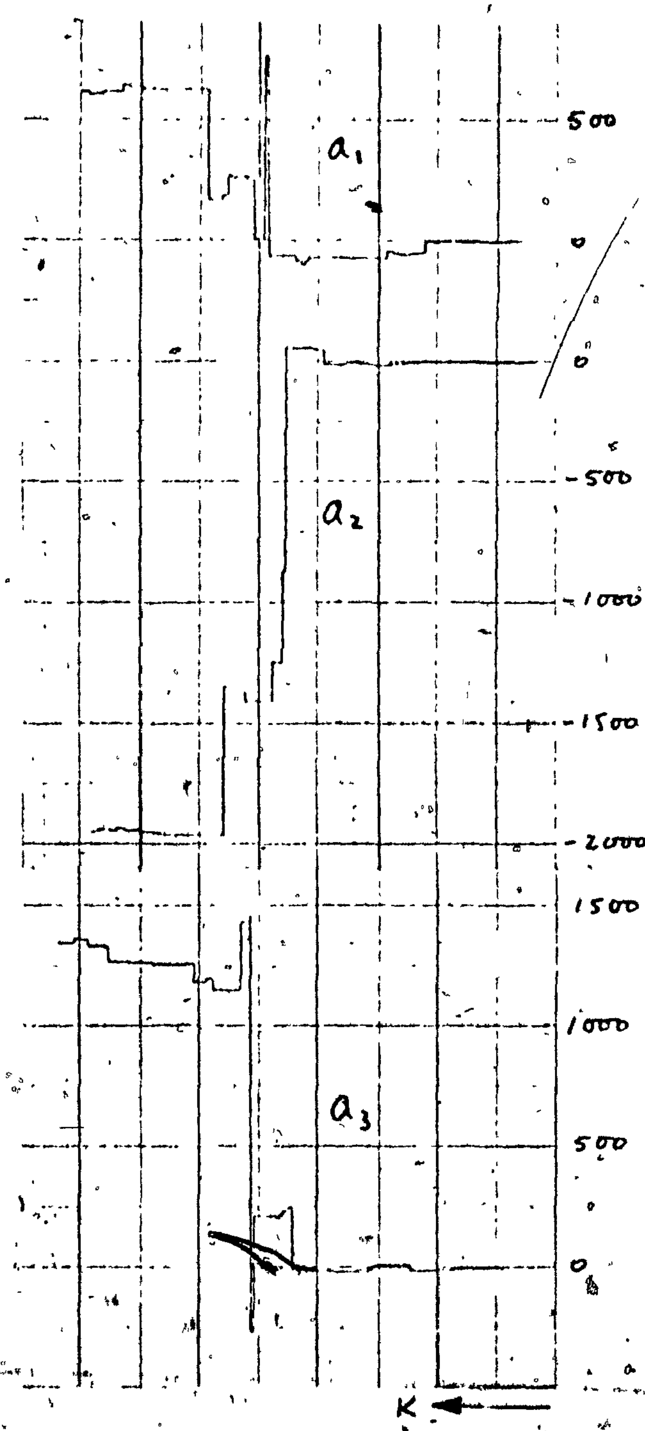
RUN # 10

T = 42 seconds

$b_1(0) = 1$

$\alpha = .64$ volts$^2$

FIGURE 4.17b




PARAMETER ESTIMATES

THIRD ORDER RUN # 10

FIGURE 4.17b (Continued)

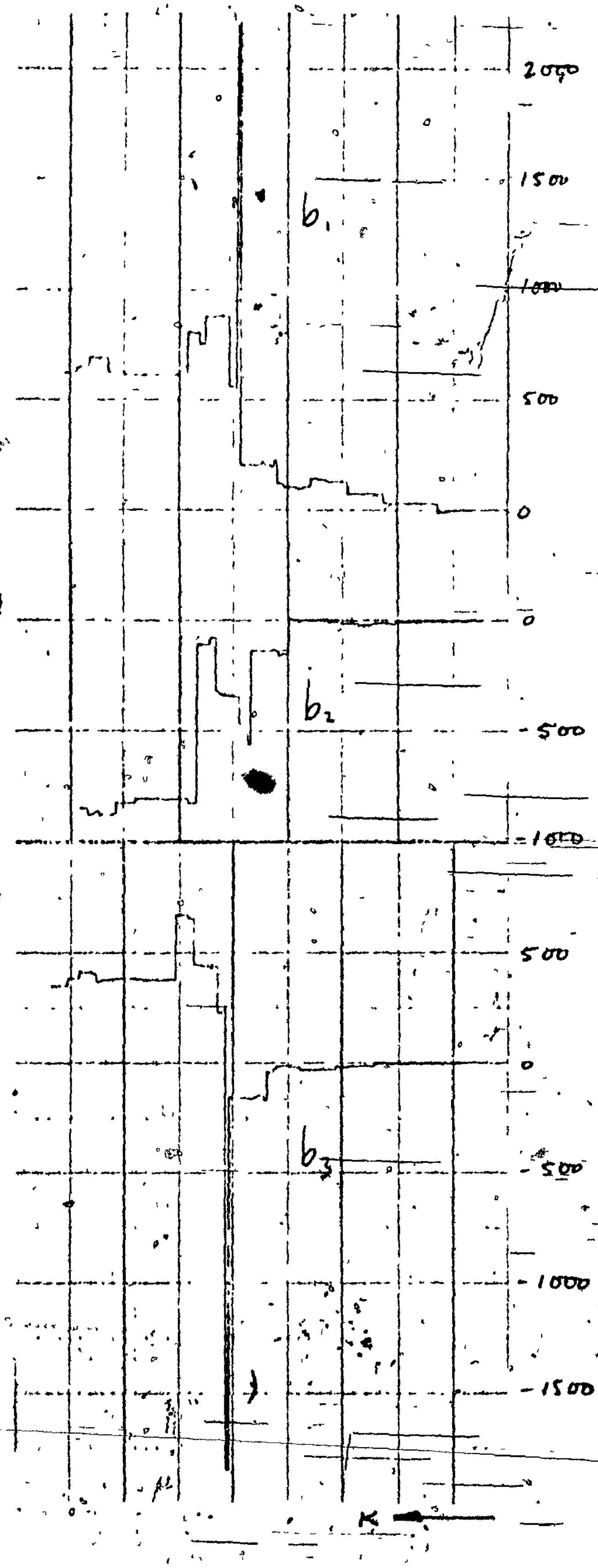
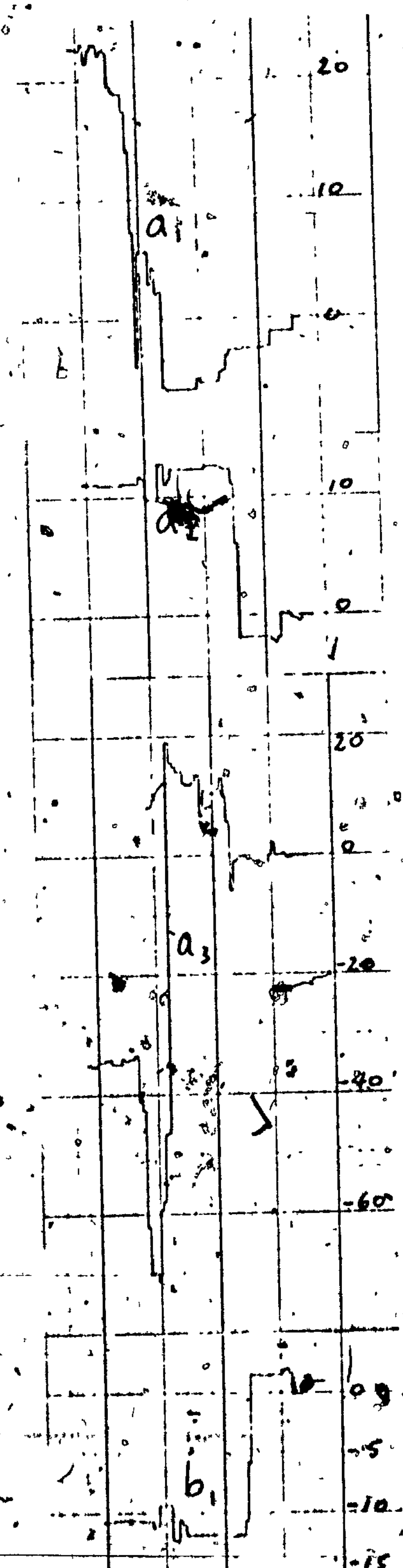


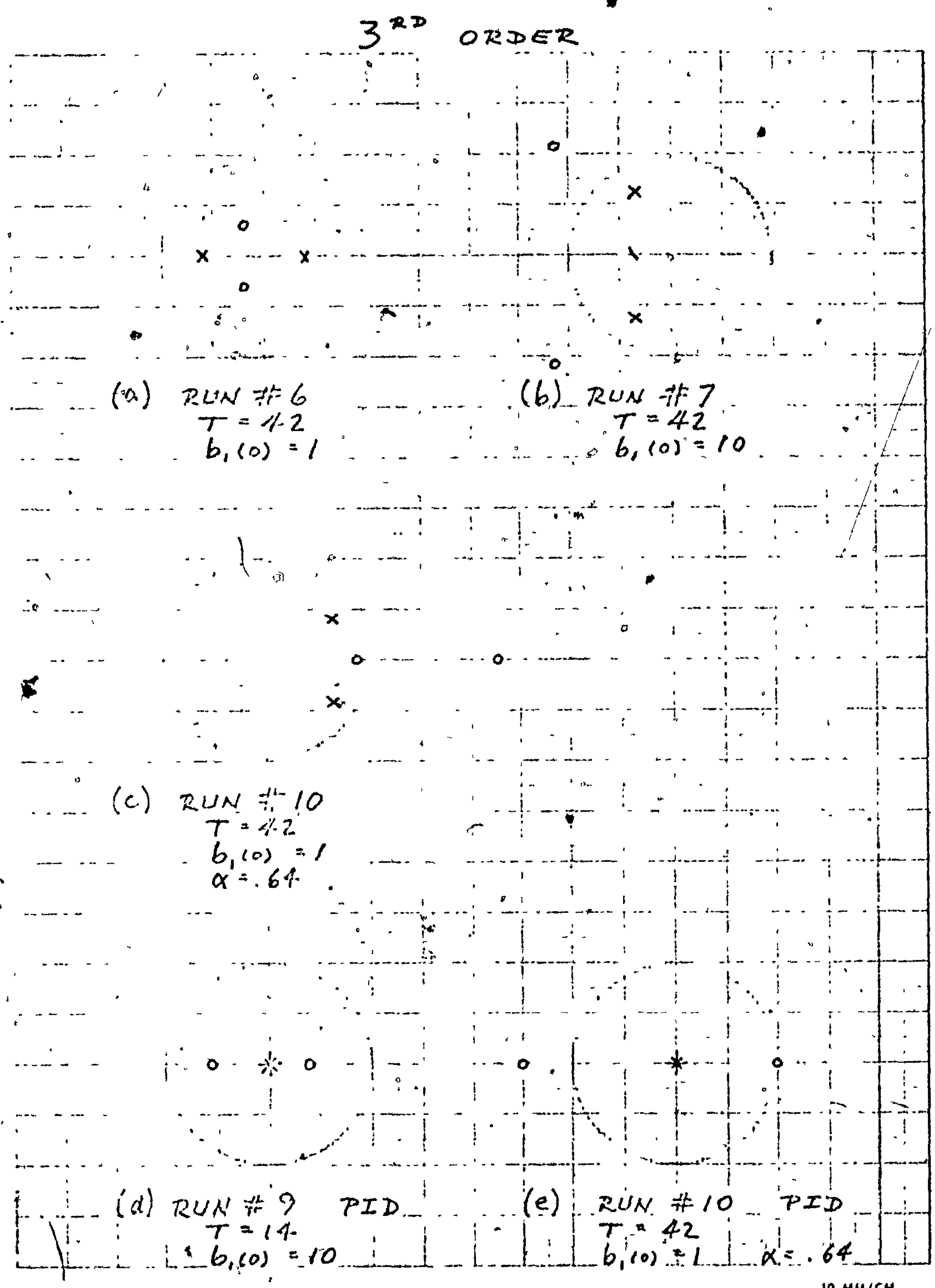
FIGURE 4.17c




PARAMETER ESTIMATES

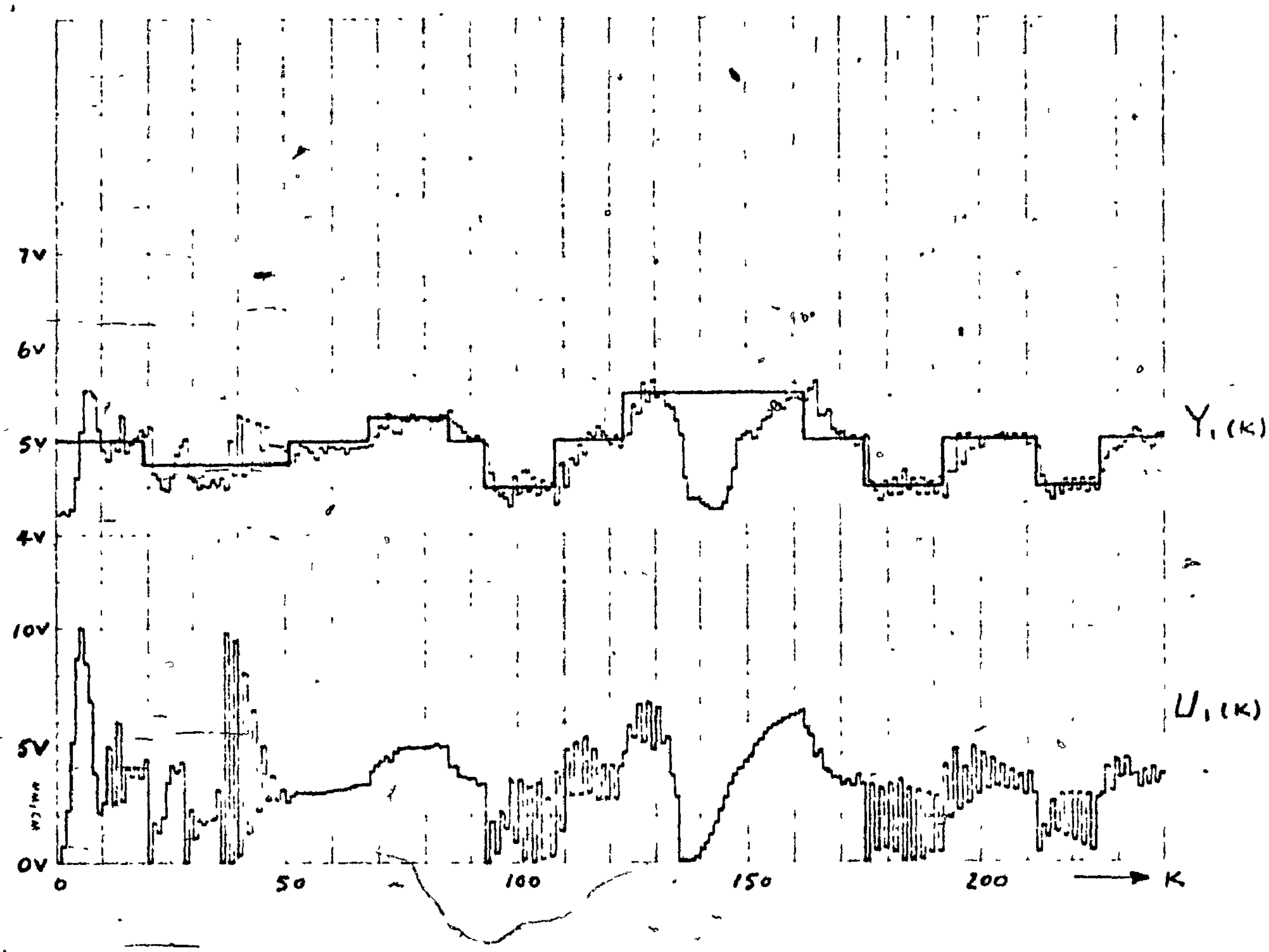
THIRD ORDER (PID) RUN # 10

FIGURE 4.18




POLE-ZERO PATTERN OF MINIMUM VARIANCE CONTROLLER OBTAINED FROM IDENTIFICATION ALGORITHM

FIGURE 4.19a



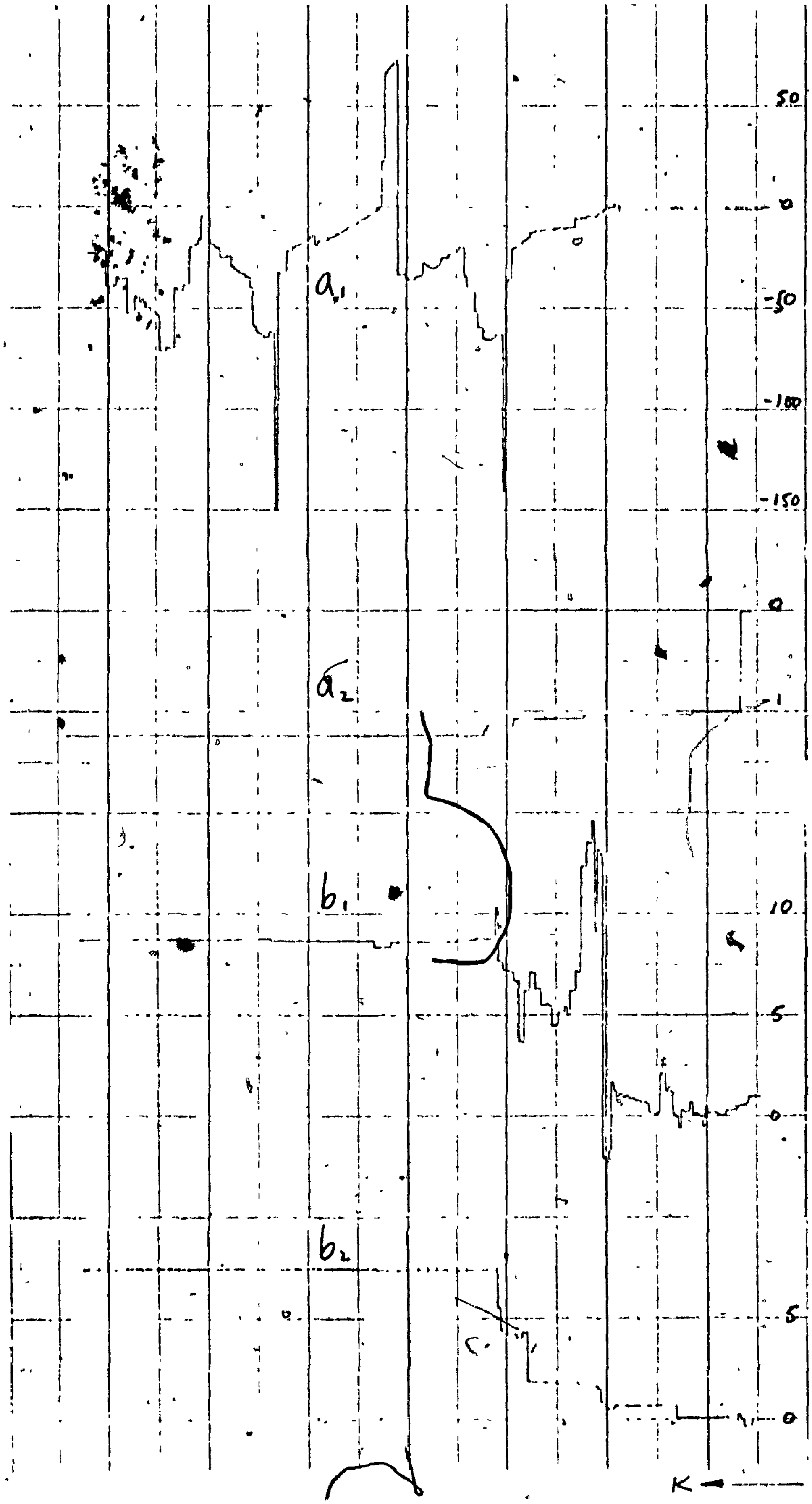

SELF-TUNING CONTROL

SECOND ORDER

RUN # 11

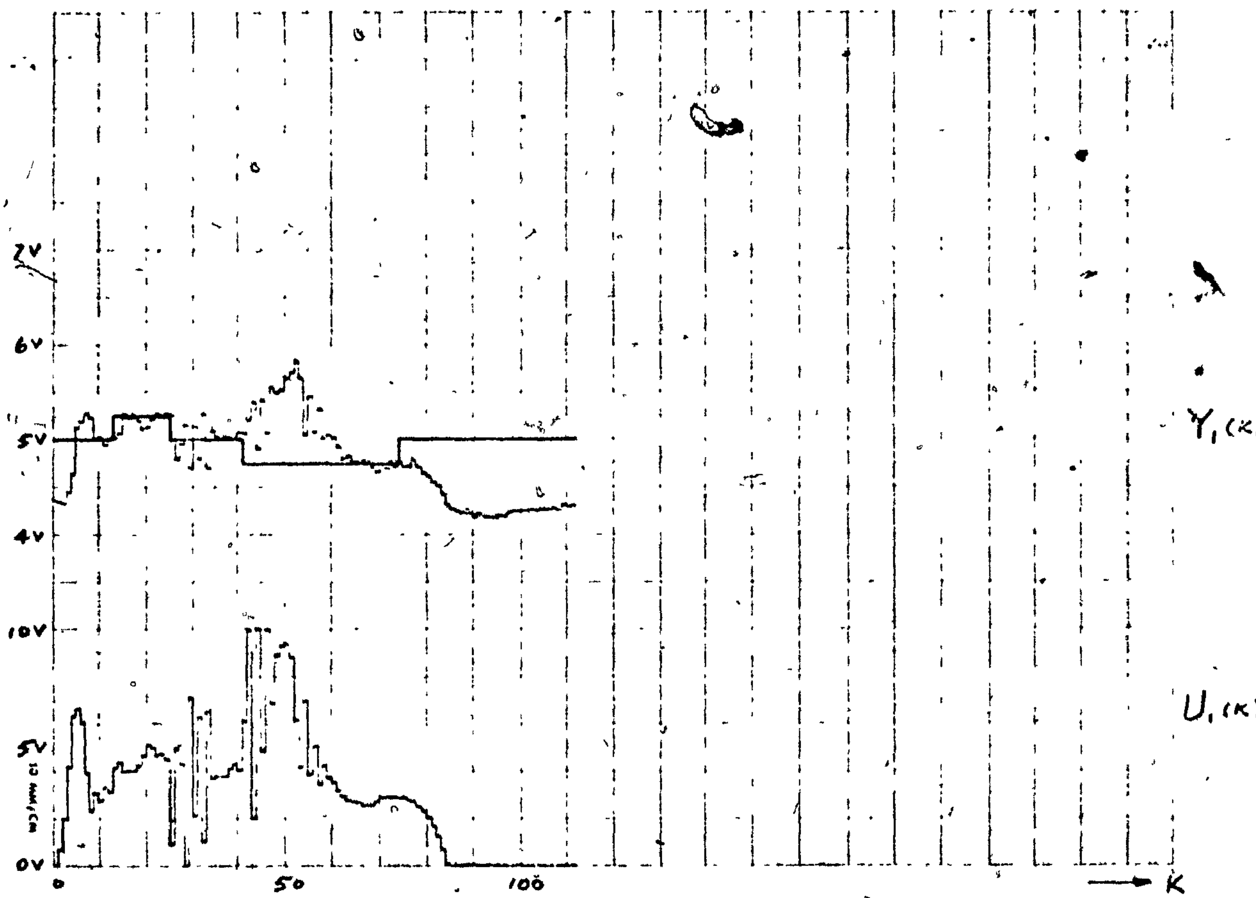
$T = 42$ seconds

$b_1(0) = 1$




SECOND ORDER PARAMETER ESTIMATES RUN # 11

FIGURE 4.20a

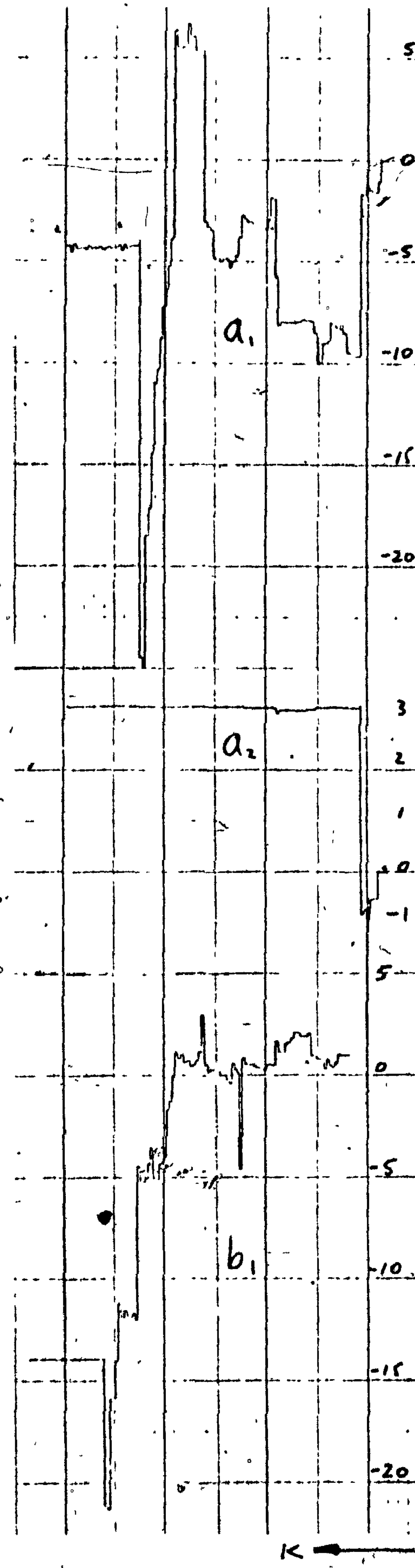



SELF-TUNING CONTROL

SECOND ORDER (PI) RUN # 12

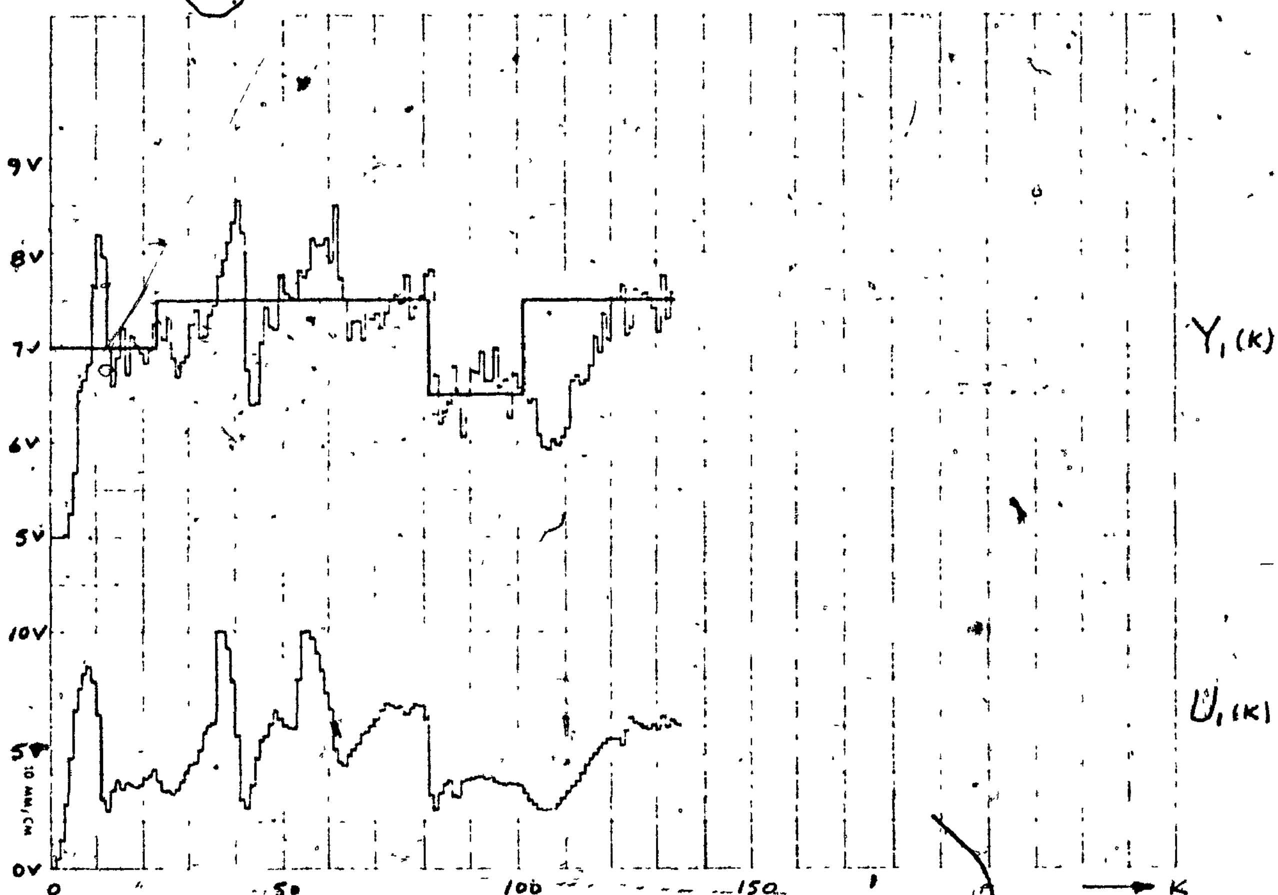
$T = 42$ seconds

$b_1(0) = 1$




SECOND ORDER (PI) PARAMETER ESTIMATES RUN # 12

FIGURE 4.21a




SELF-TUNING CONTROL

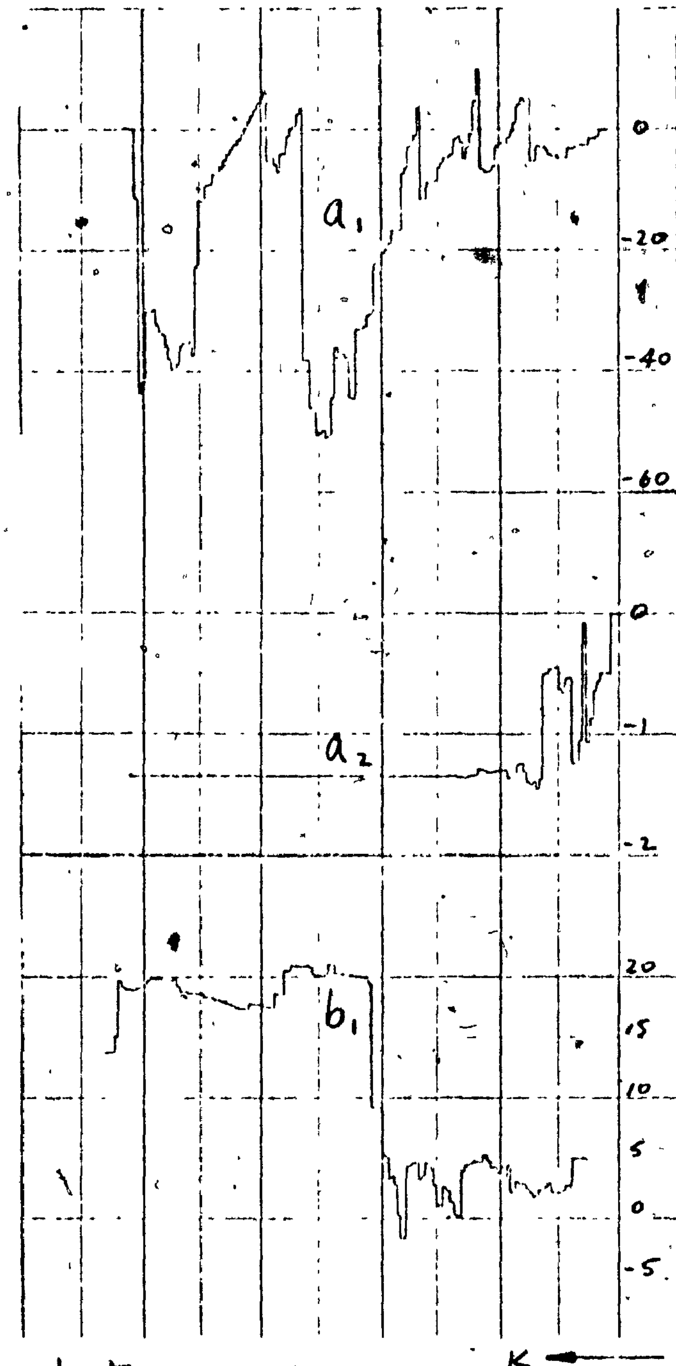
SECOND ORDER (PI)

RUN # 13

T = 42 seconds

$b_1(0) = 5$

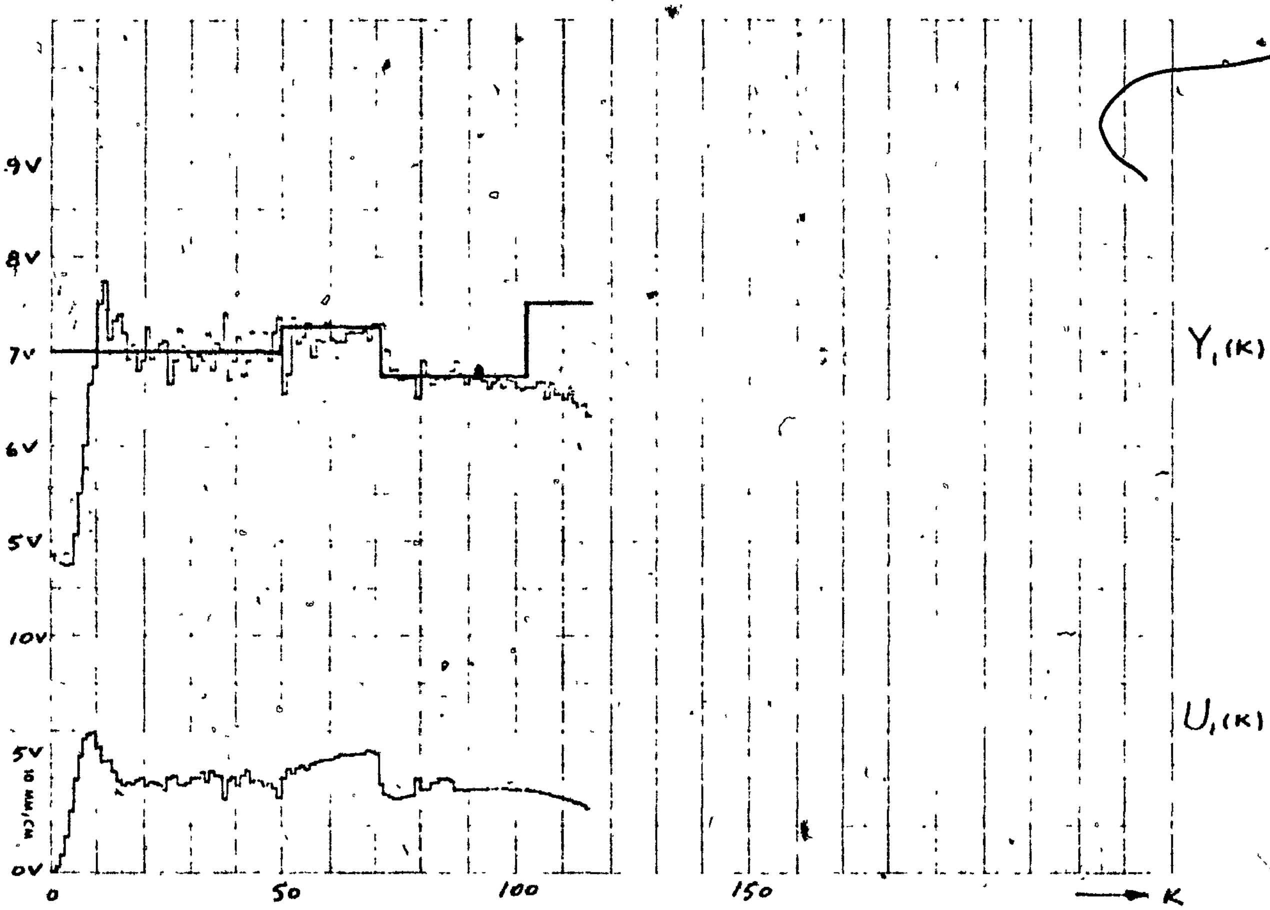
FIGURE 4.21b




PARAMETER ESTIMATES

SECOND ORDER (PI) RUN # 13

FIGURE 4.22a



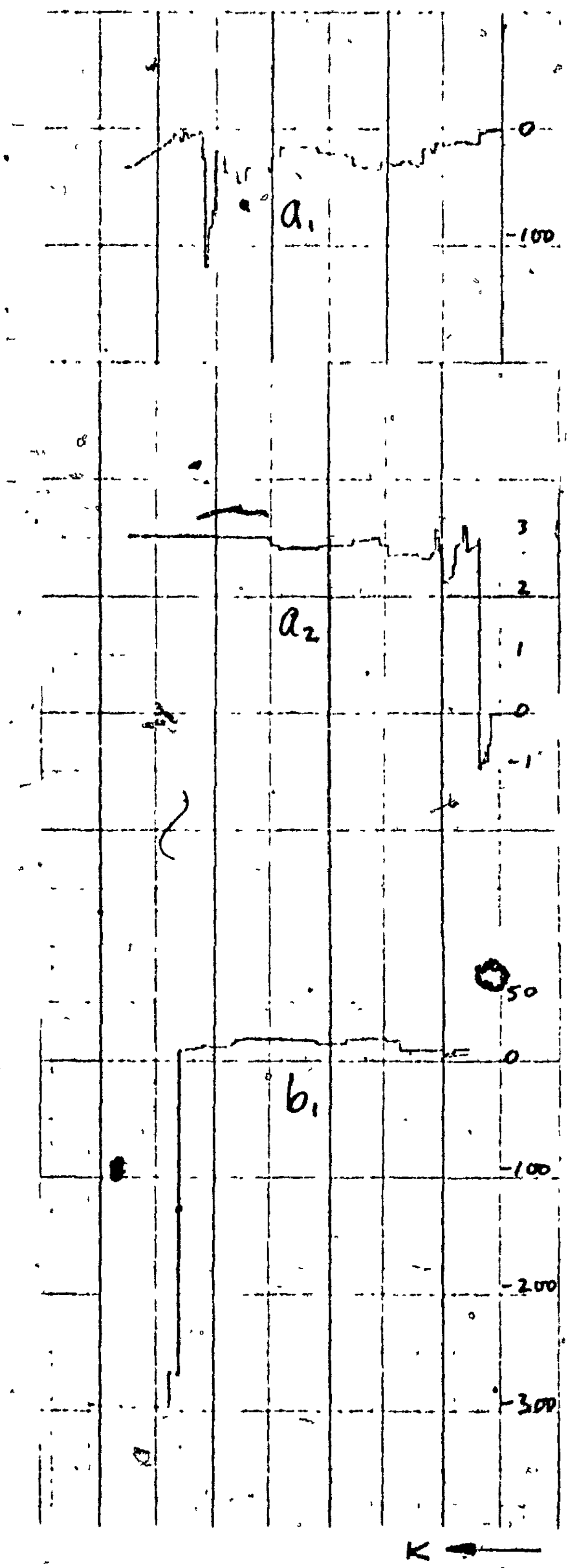

SELF-TUNING CONTROL

SECOND ORDER (PI) RUN # 14

T = 42 seconds

$b_1(0) = 10$

FIGURE 4.22b

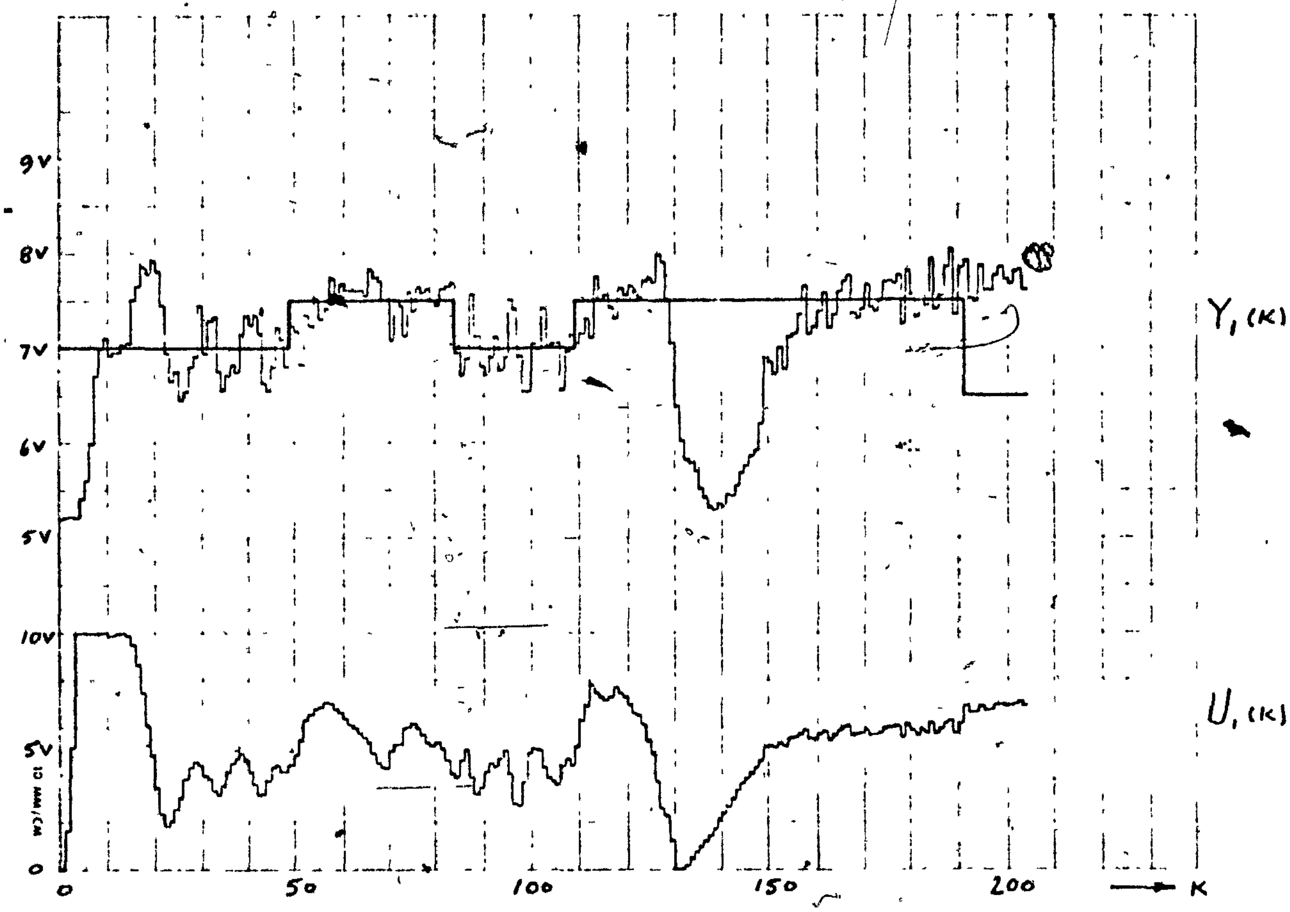



PARAMETER ESTIMATES

SECOND ORDER (PI)

RUN # 14

FIGURE 4.23a



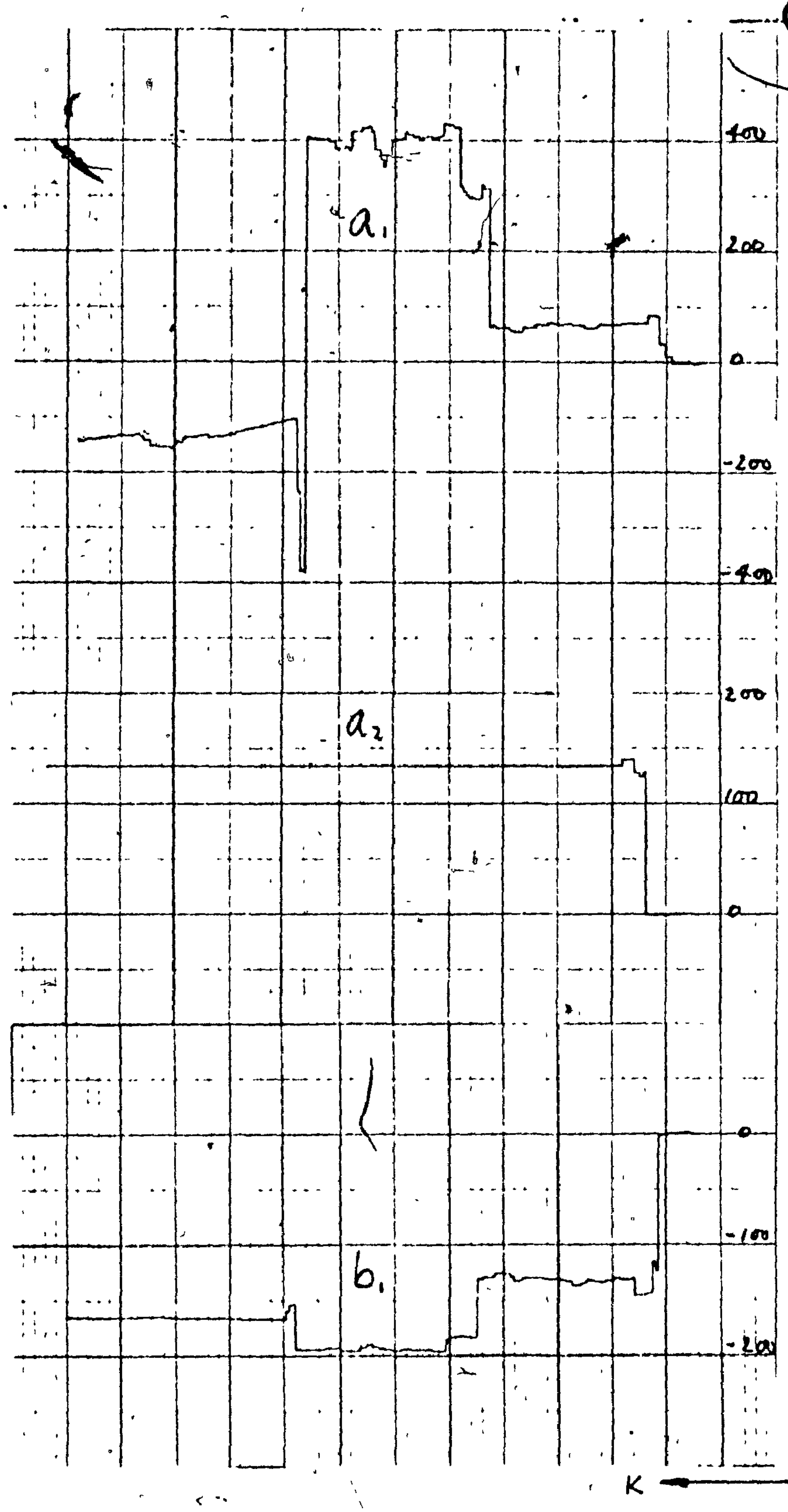

SELF-TUNING CONTROL

SECOND ORDER (PI) RUN # 15

T = 14 seconds

$b_1(0) = 1$

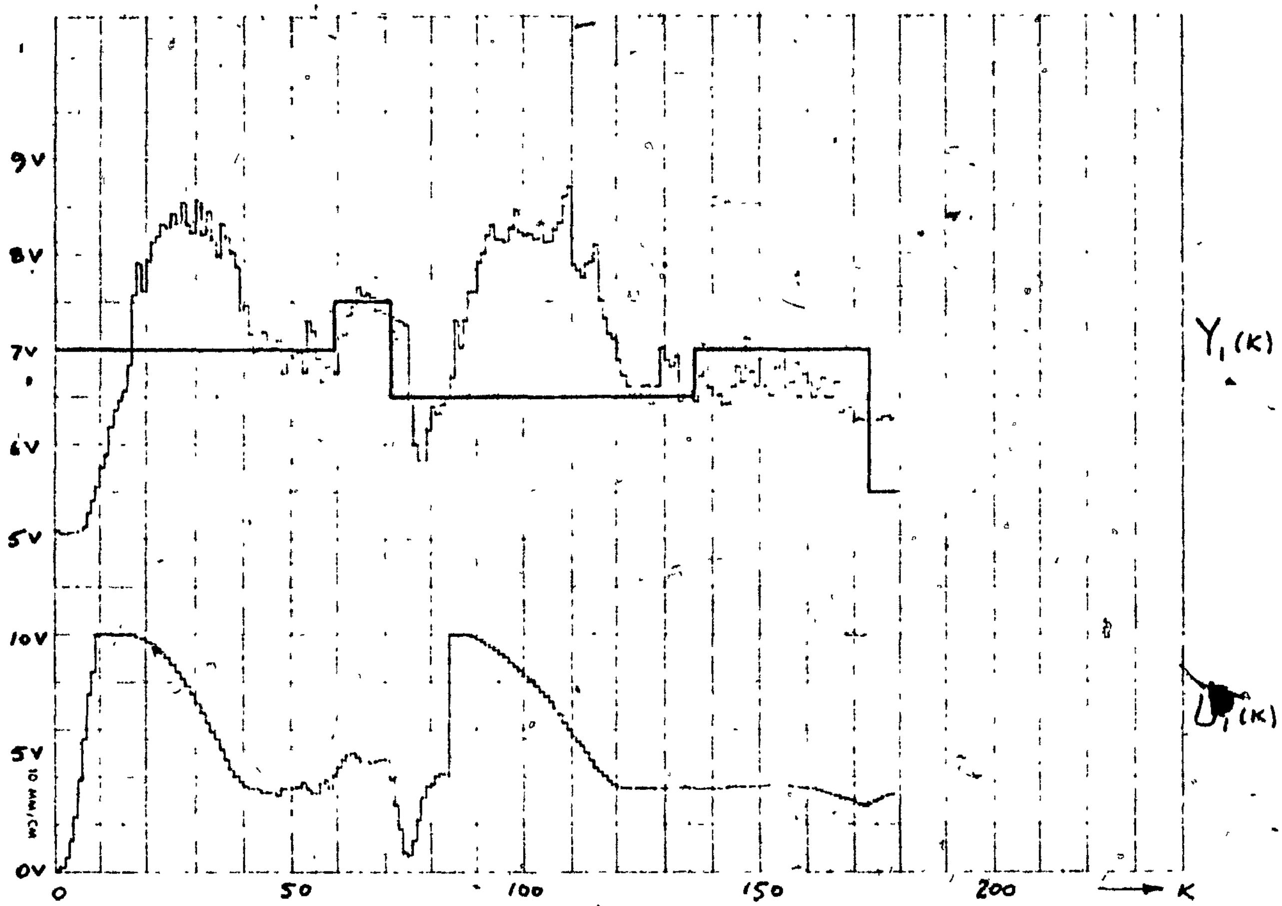
FIGURE 4.23b




PARAMETER ESTIMATES

SECOND ORDER (PI) RUN # 15

FIGURE 4.24a




SELF-TUNING CONTROL

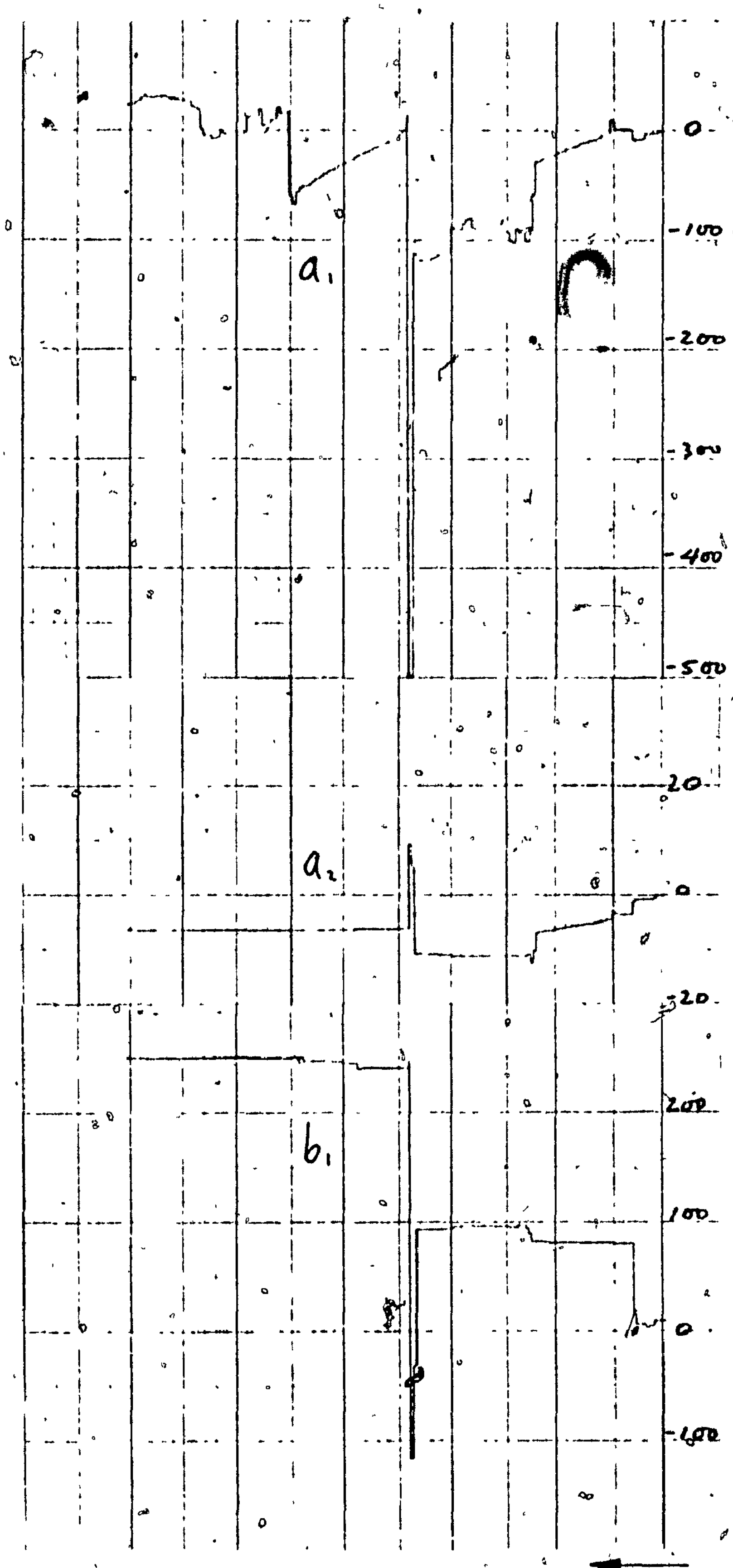
SECOND ORDER (PI)

RUN # 16

T = 14 seconds

$b_1(0) = 10$

FIGURE 4.24b

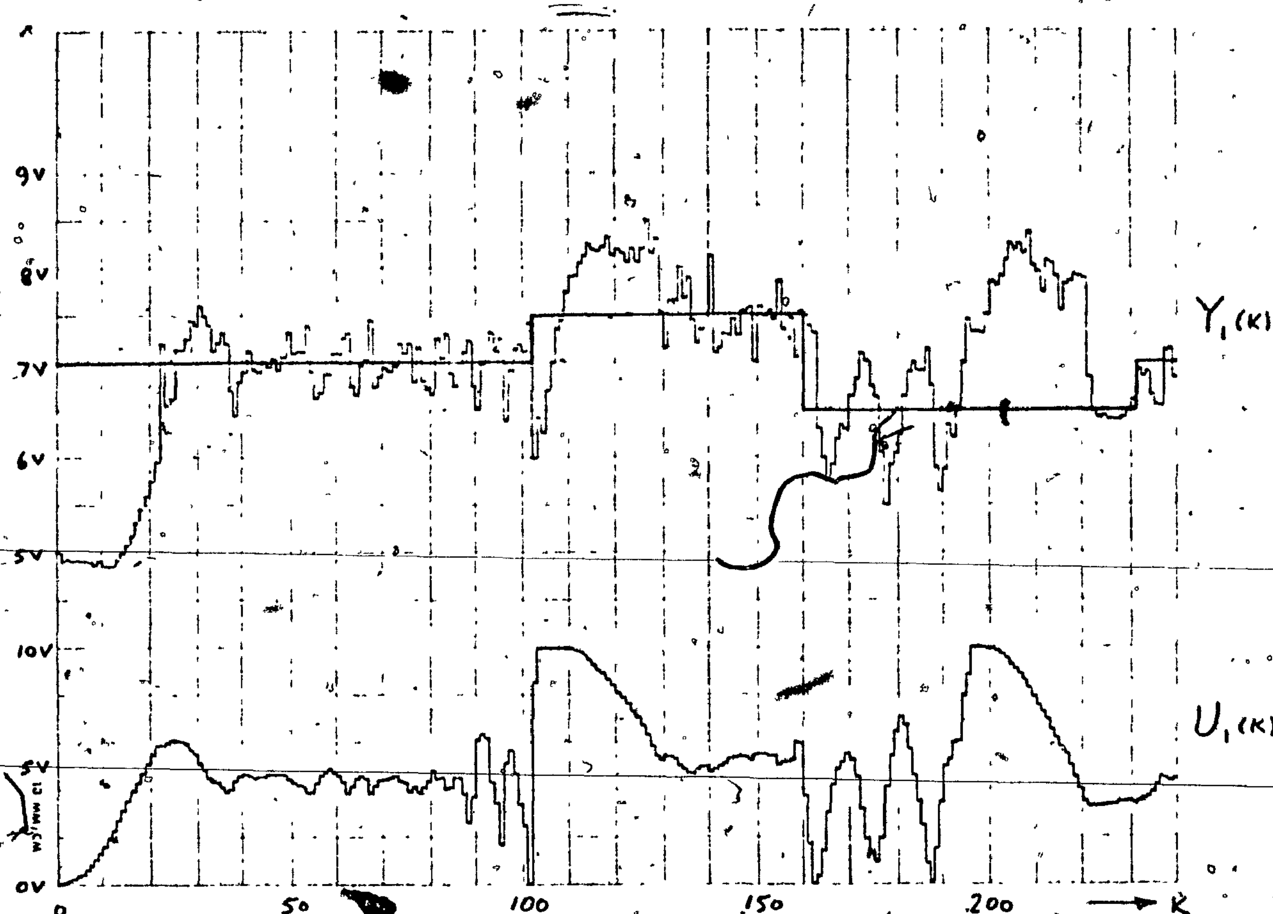



PARAMETER ESTIMATES

SECOND ORDER (PI)

RUN # 16

FIGURE 4.25a




SELF-TUNING CONTROL

SECOND ORDER (PI)

RUN # 17

T = 14 seconds

$b_1(0) = 64$

FIGURE 4.25a (Continued)

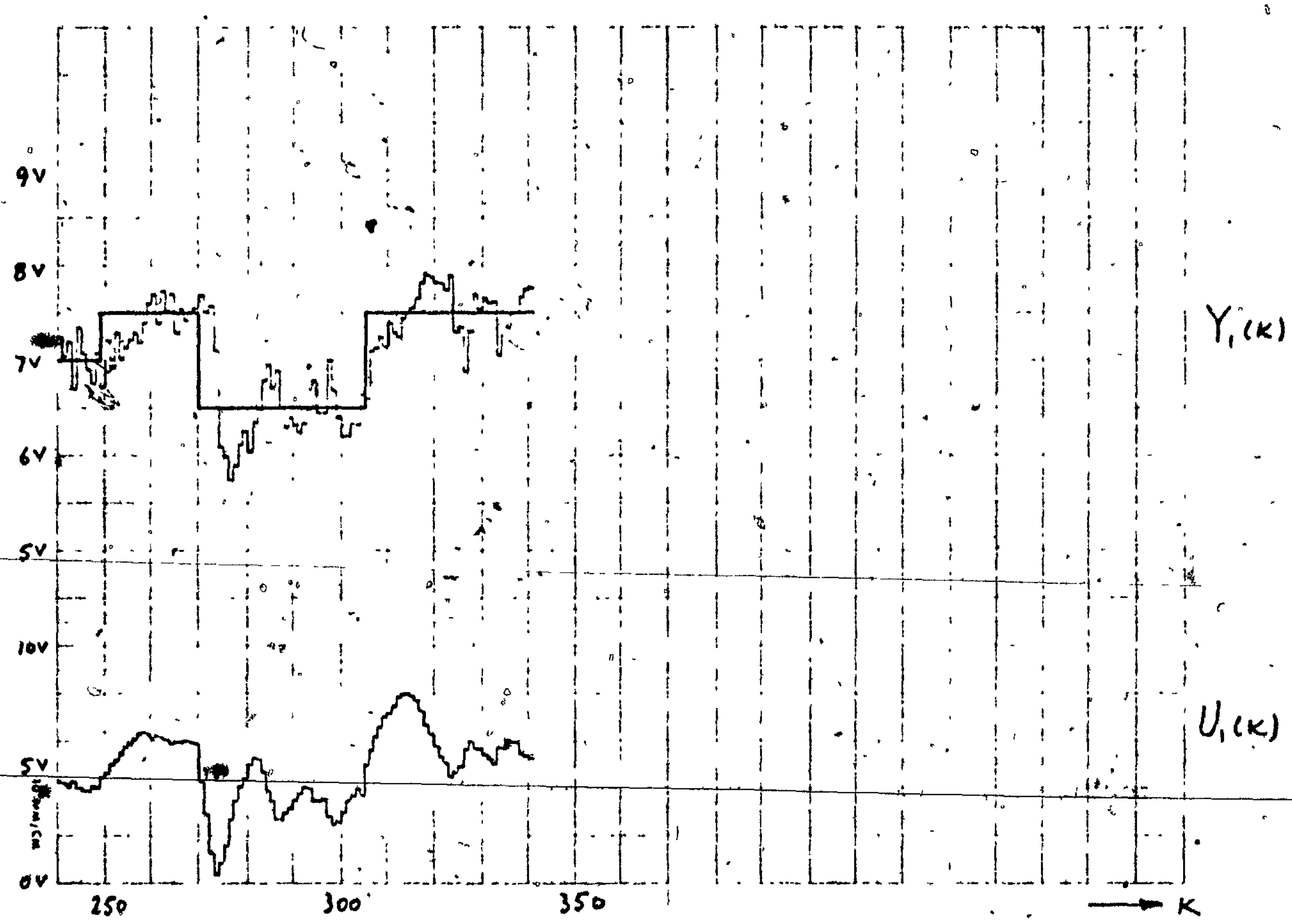
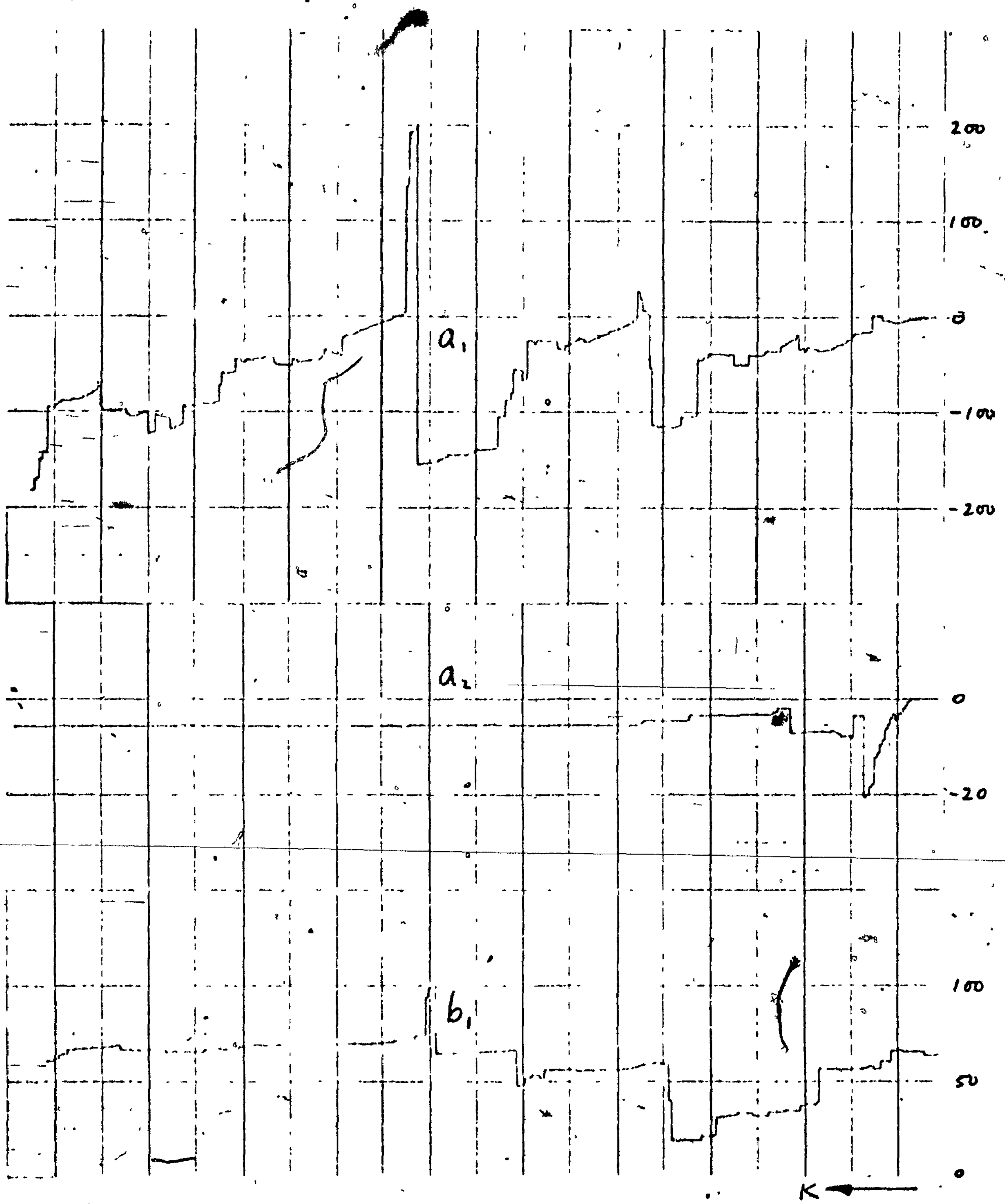


FIGURE 4.25b

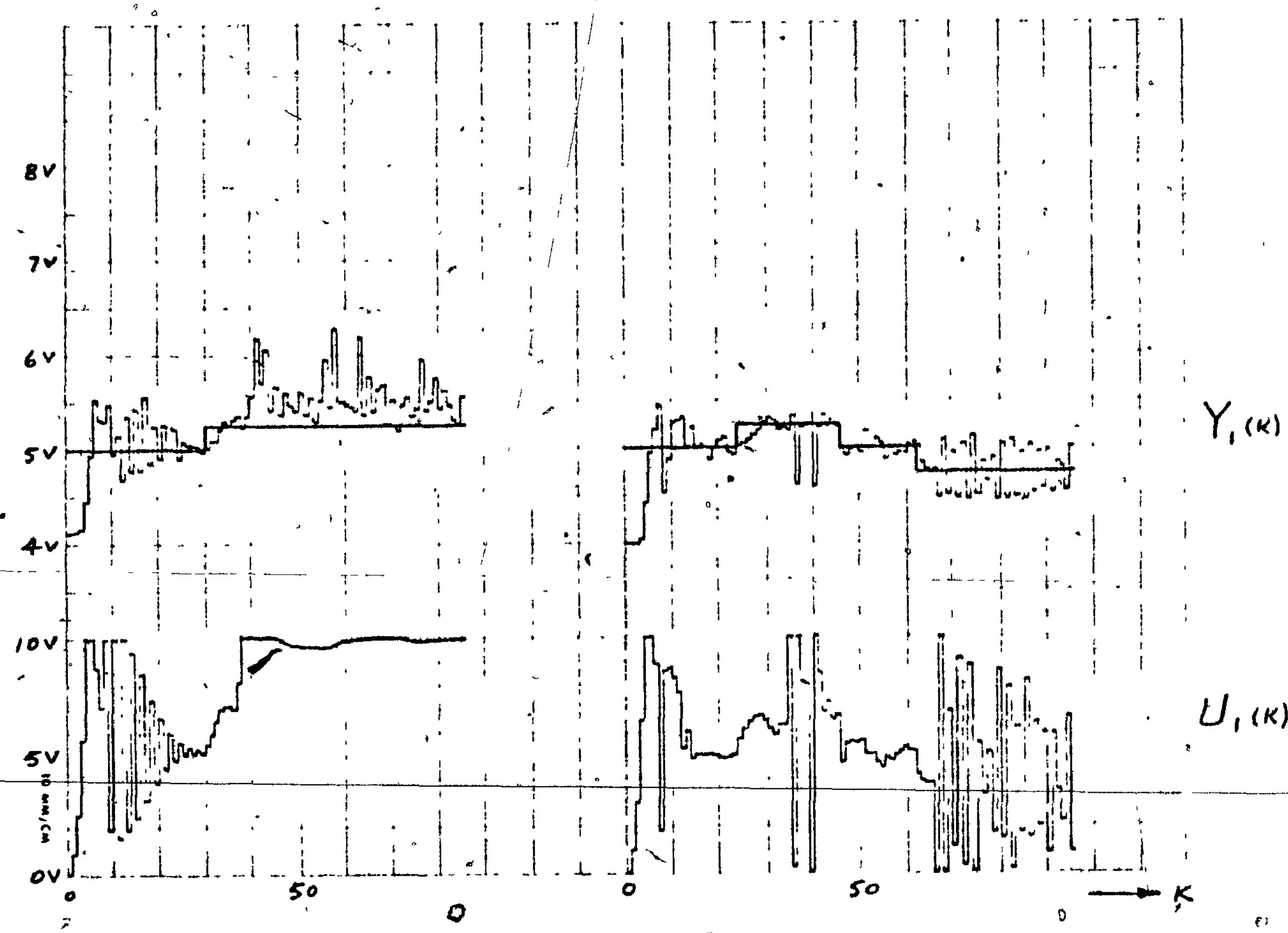



PARAMETER ESTIMATES

SECOND ORDER (PI)

RUN # 17

FIGURE 4.26a




$2^{nd}$ Order

$2^{nd}$ Order (PI)

SELF-TUNING CONTROL

SECOND ORDER

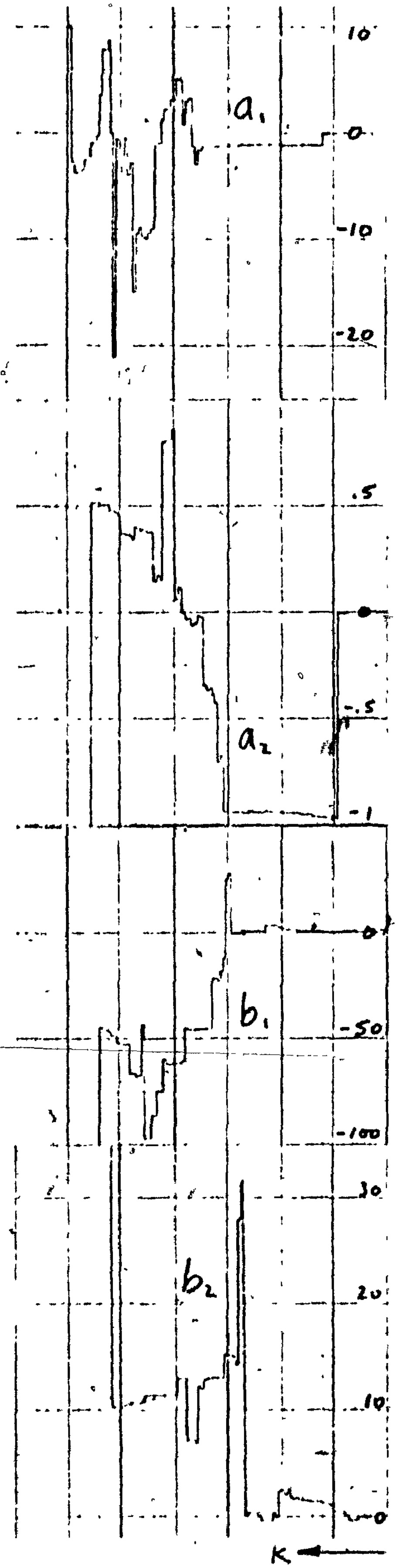
RUN # 18

T = 42 seconds

$b_1(0) = 1$

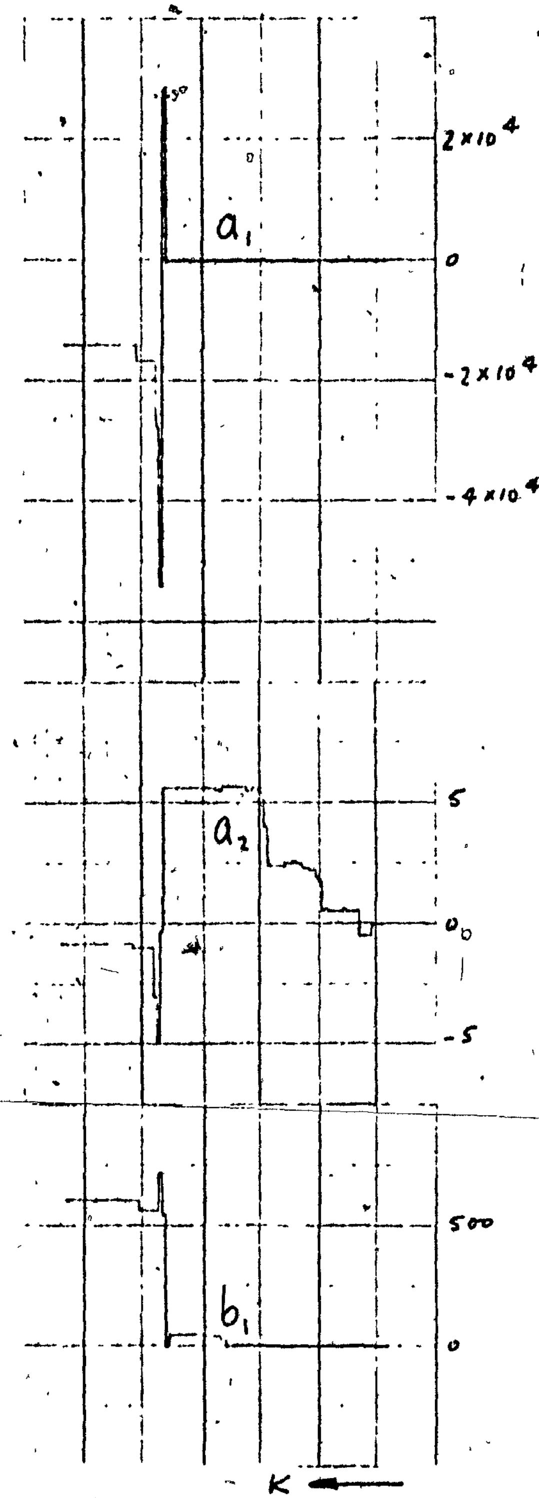
$\alpha = .64$ volts$^2$

FIGURE 4.26b




SECOND ORDER PARAMETER ESTIMATES RUN # 18

FIGURE 4.26c

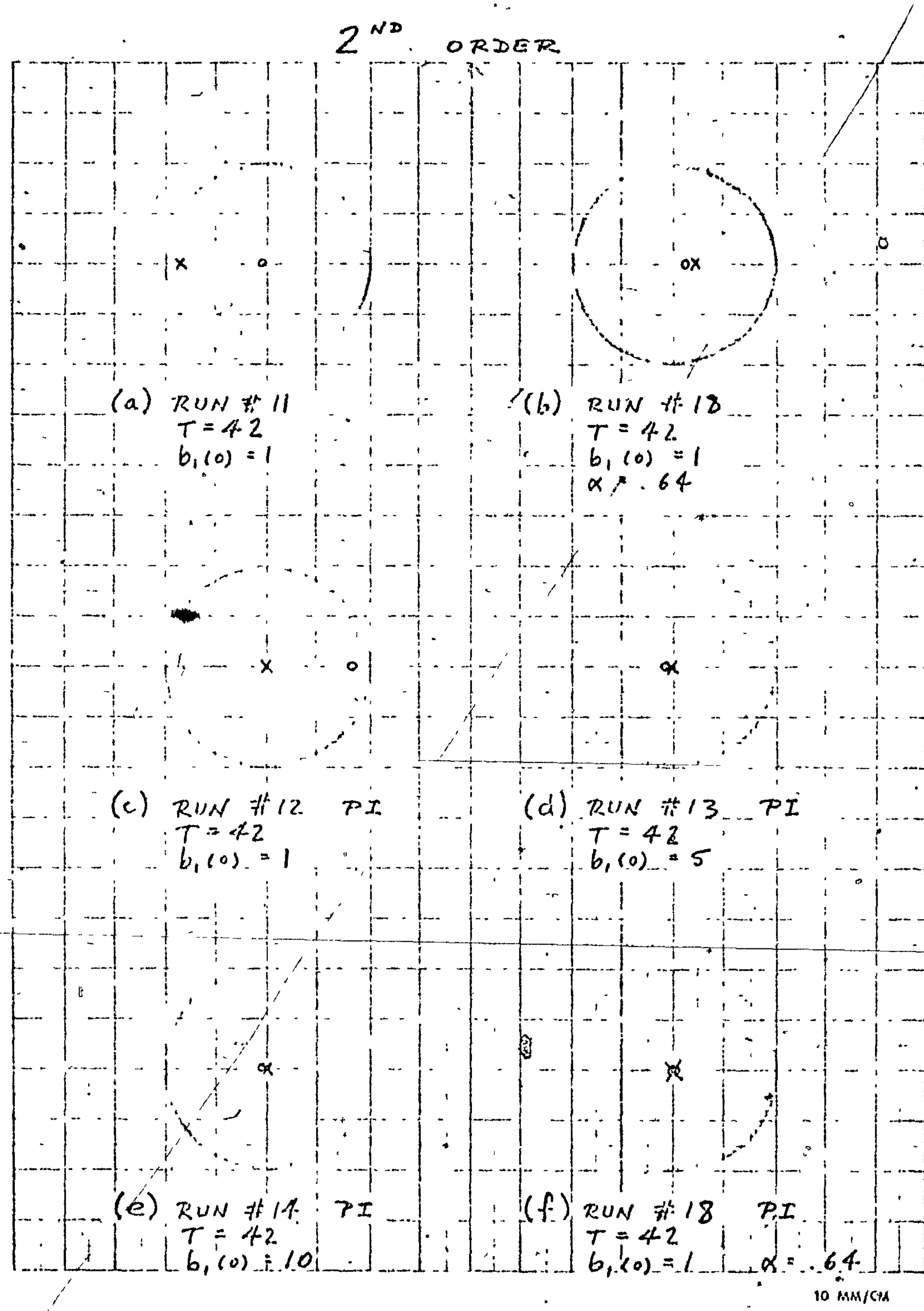



PARAMETER ESTIMATES

SECOND ORDER (PI)

RUN # 18

FIGURE 4.27




POLE-ZERO PATTERN OF MINIMUM VARIANCE CONTROLLER

OBTAINED FROM IDENTIFICATION ALGORITHM

FIGURE 4.27 (Continued)

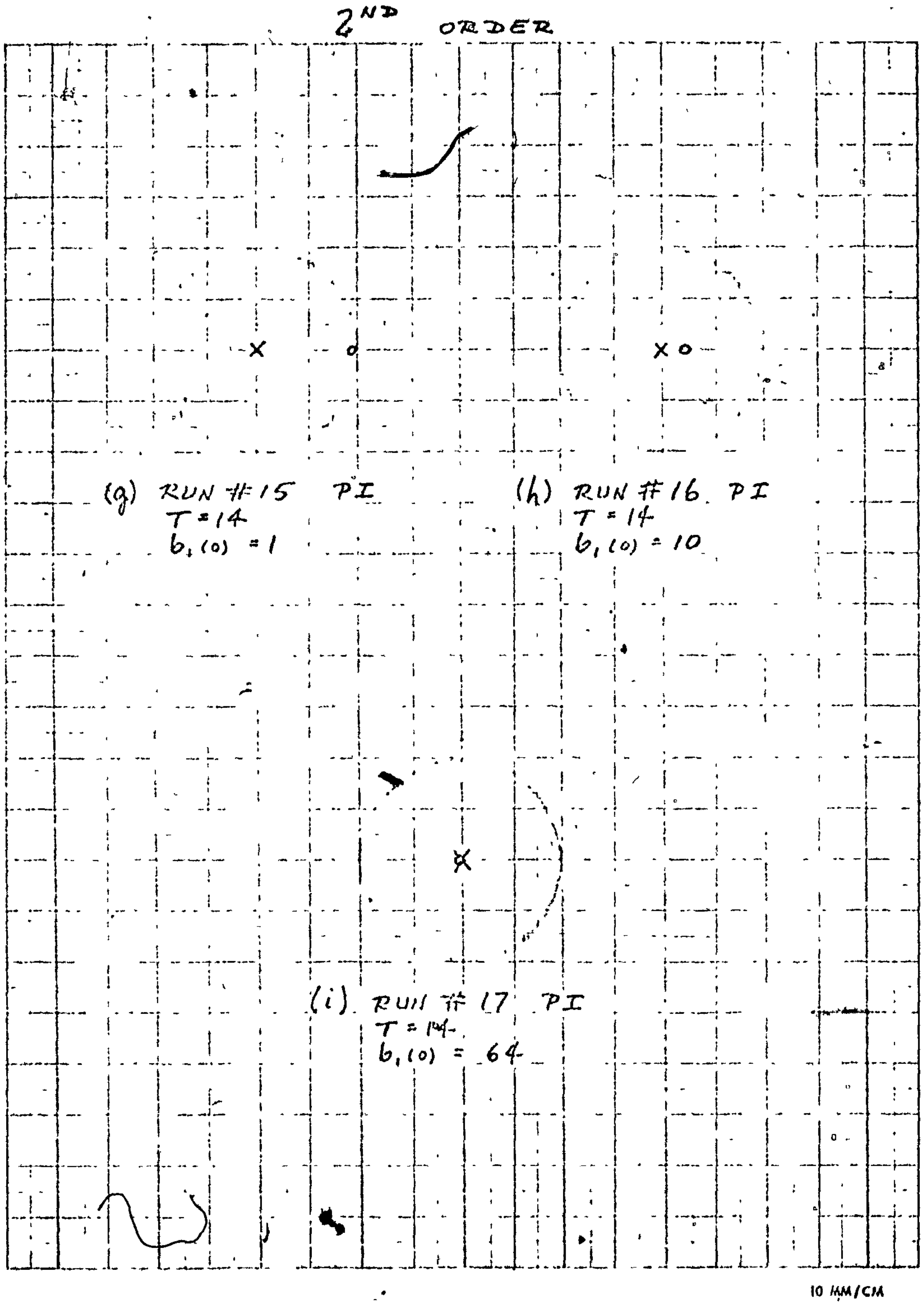
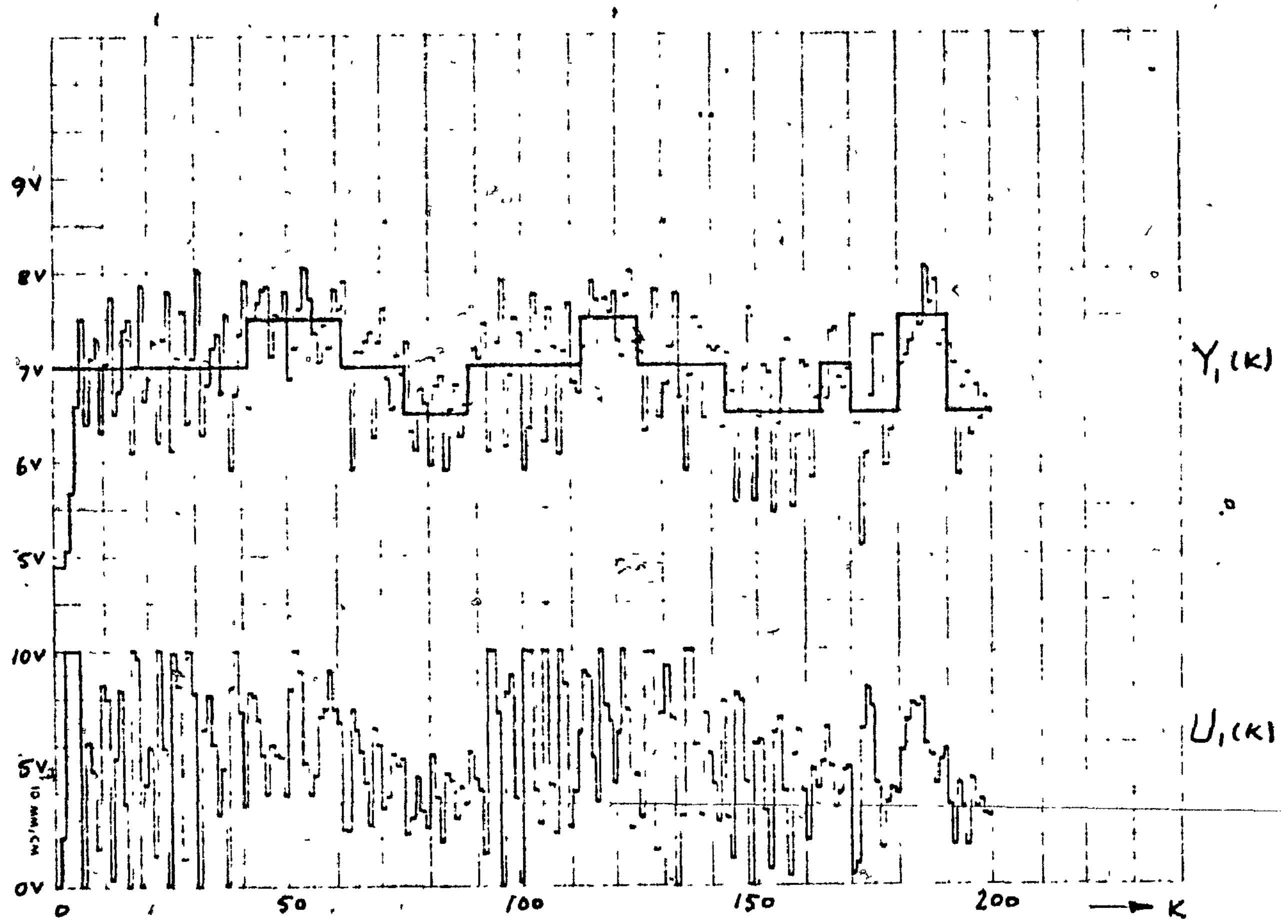


FIGURE 4.28a




SELF-TUNING CONTROLLER

FIRST ORDER

RUN # 19

T = 42 seconds

$b_1(0) = 1$

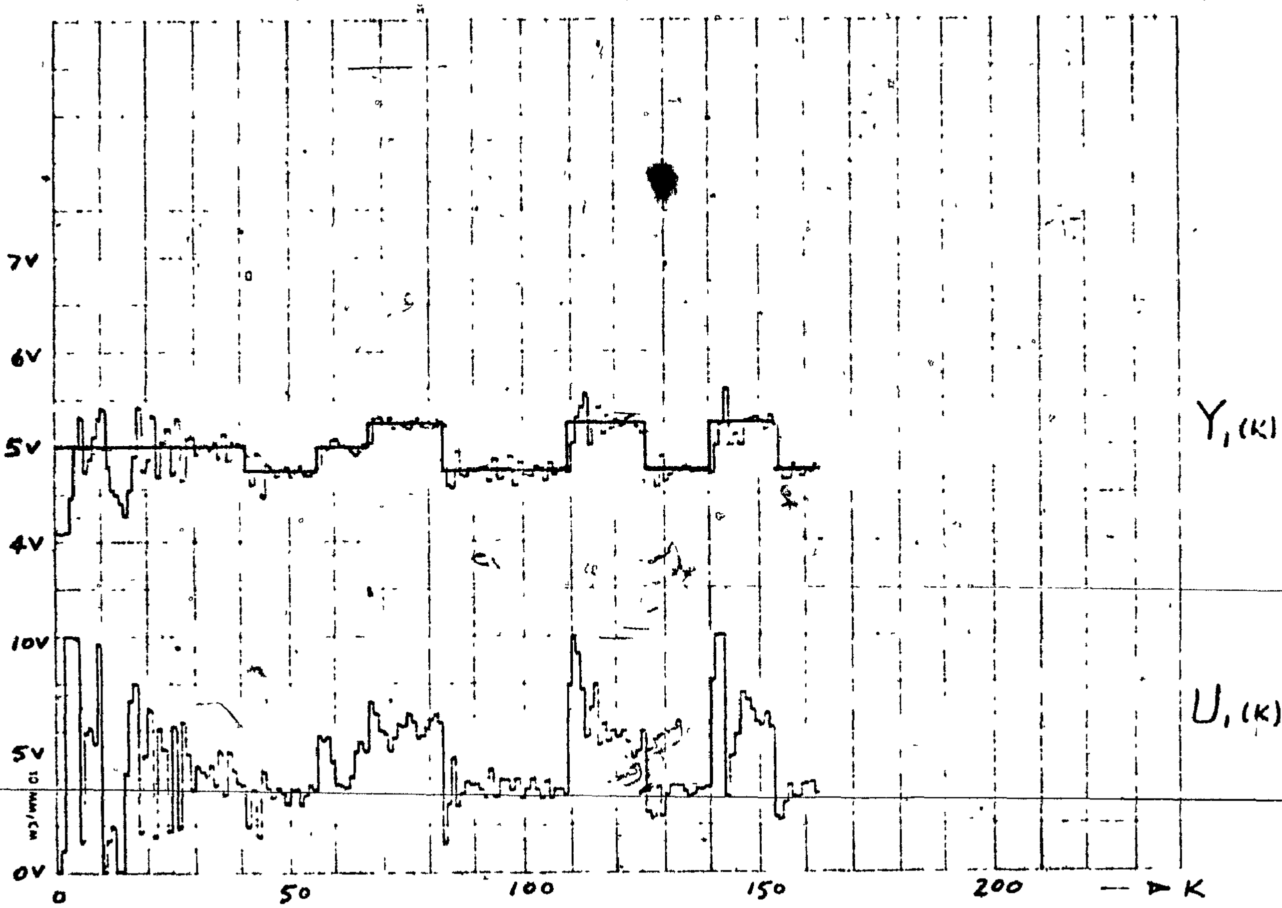
FIGURE 4.28b


PARAMETER ESTIMATES

FIRST ORDER

RUN # 19

FIGURE 4.29a




SELF-TUNING CONTROL

FIRST ORDER

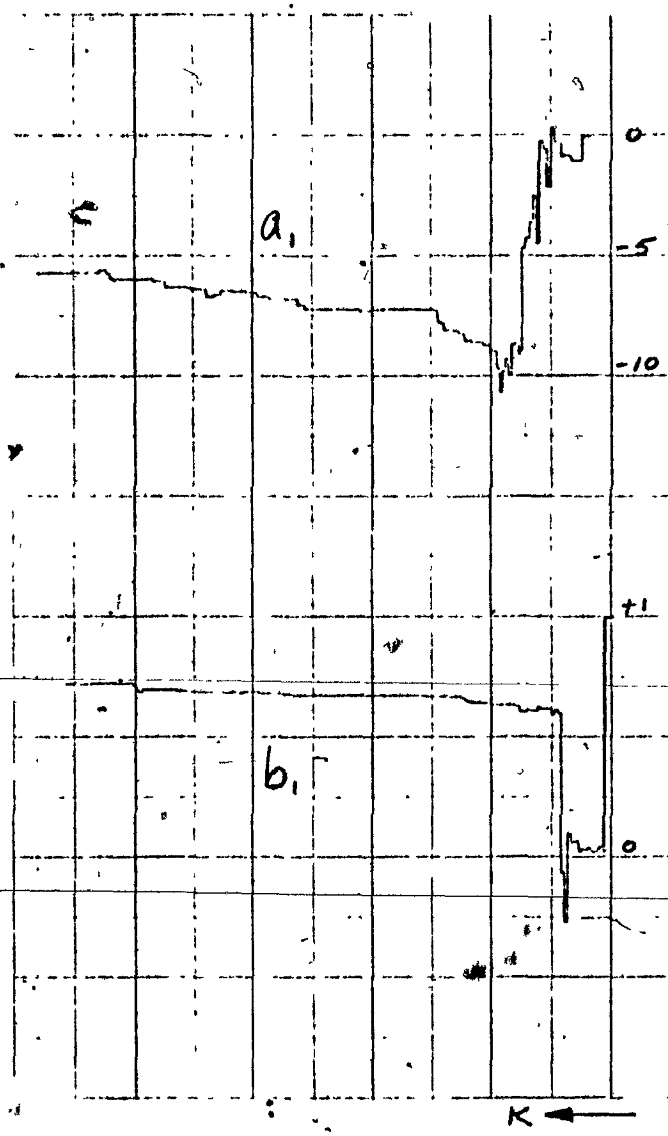
RUN # 20

T = 42 seconds

$b_1(0) = 1$

$\alpha = .64 \text{ volts}^2$

FIGURE 4.29b

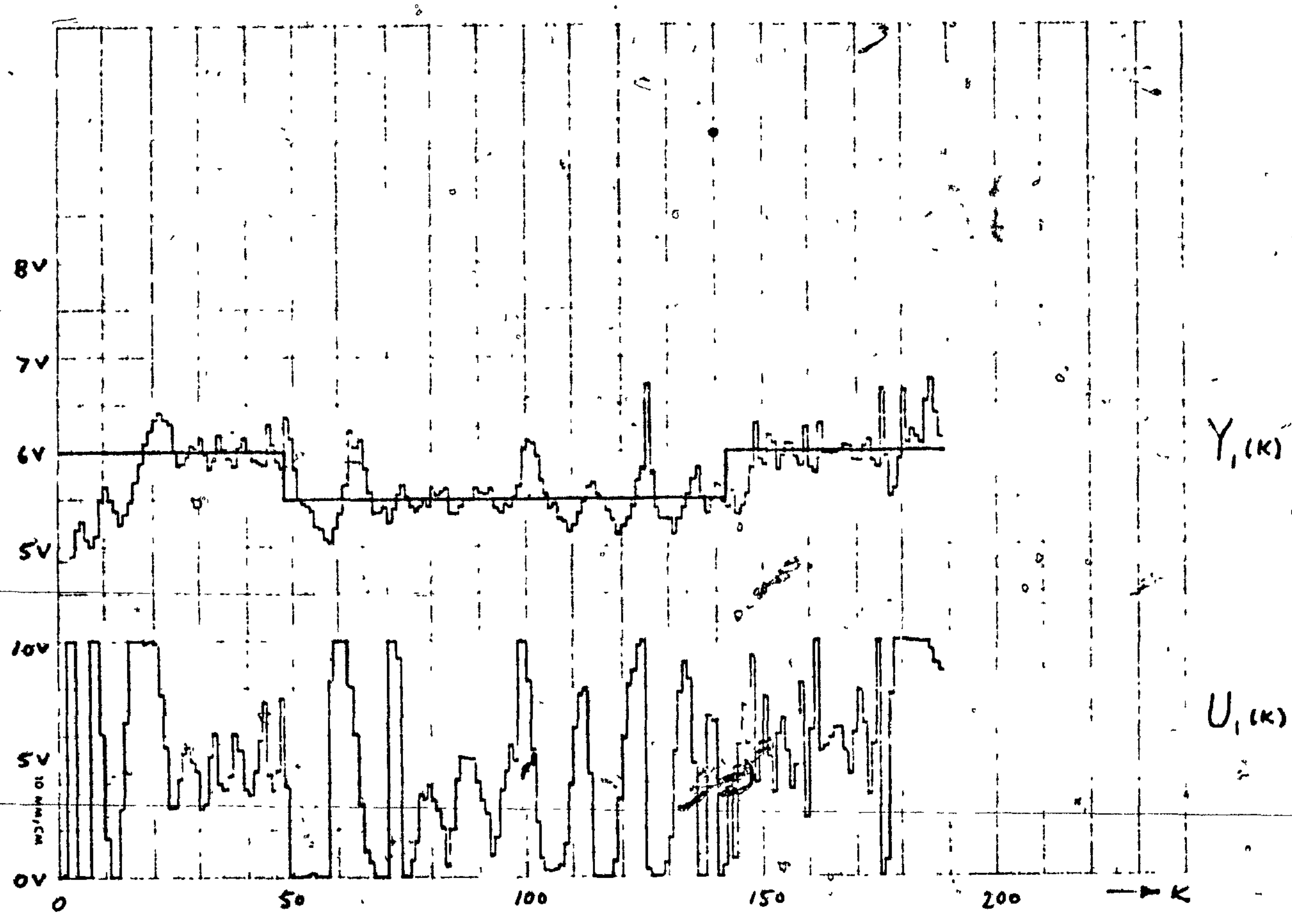



PARAMETER ESTIMATES

FIRST ORDER

RUN # 20

FIGURE 4.30a



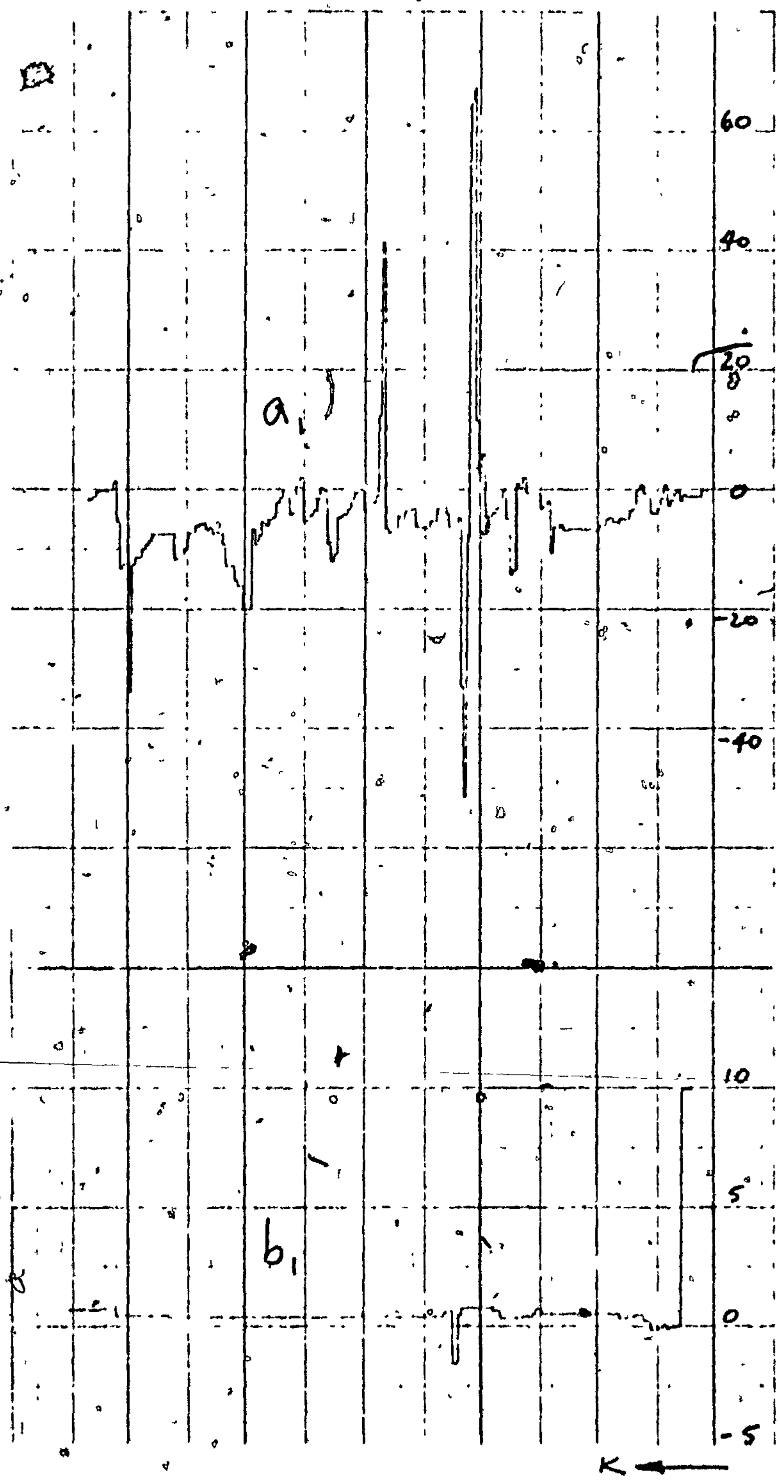

SELF-TUNING CONTROL

FIRST ORDER RUN # 21

T = 14 seconds

$b_1(0) = 10$

FIGURE 4.30b

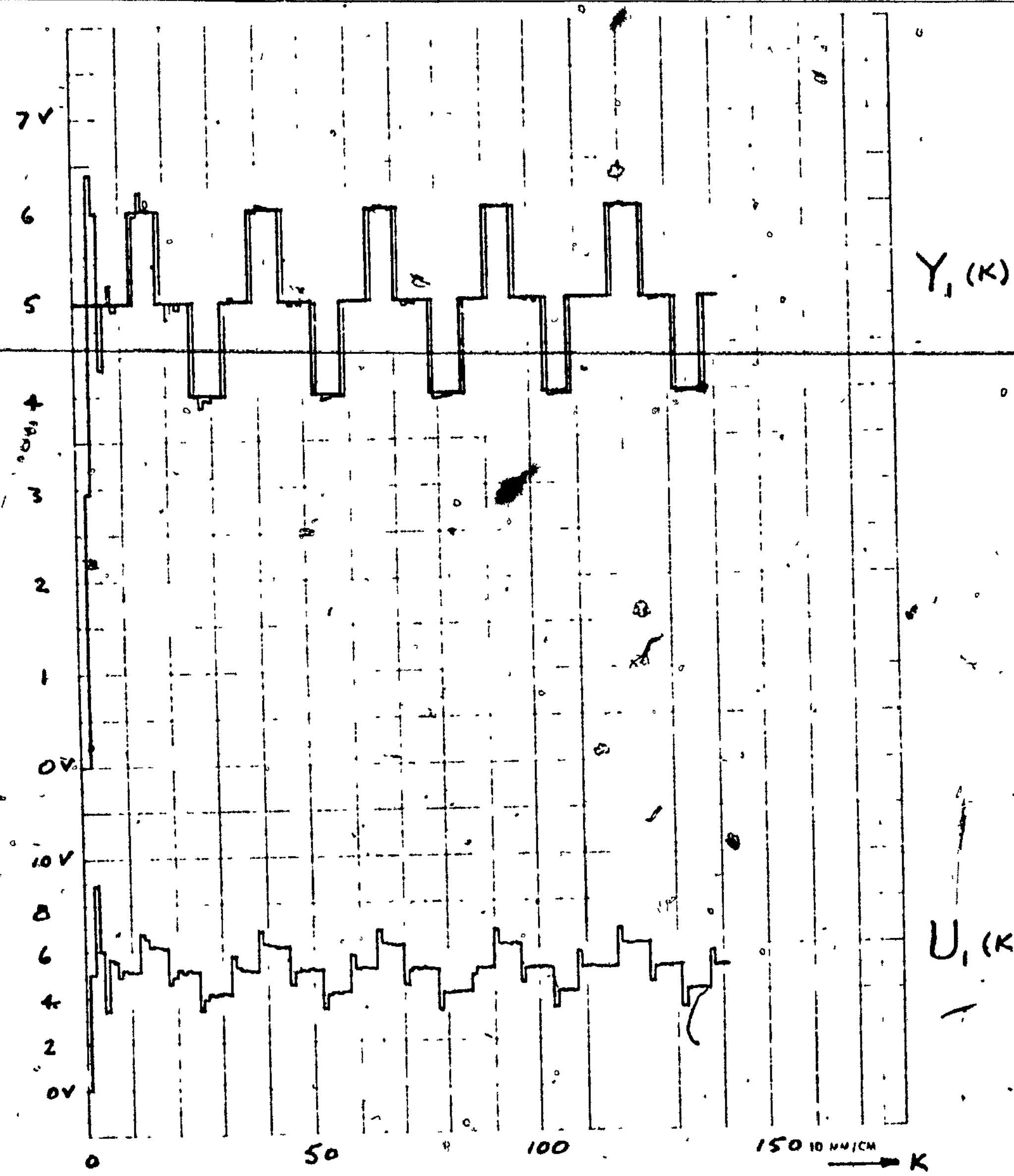



PARAMETER ESTIMATES

FIRST ORDER

RUN # 21

FIGURE 4.31




SELF-TUNING CONTROL

FOURTH ORDER

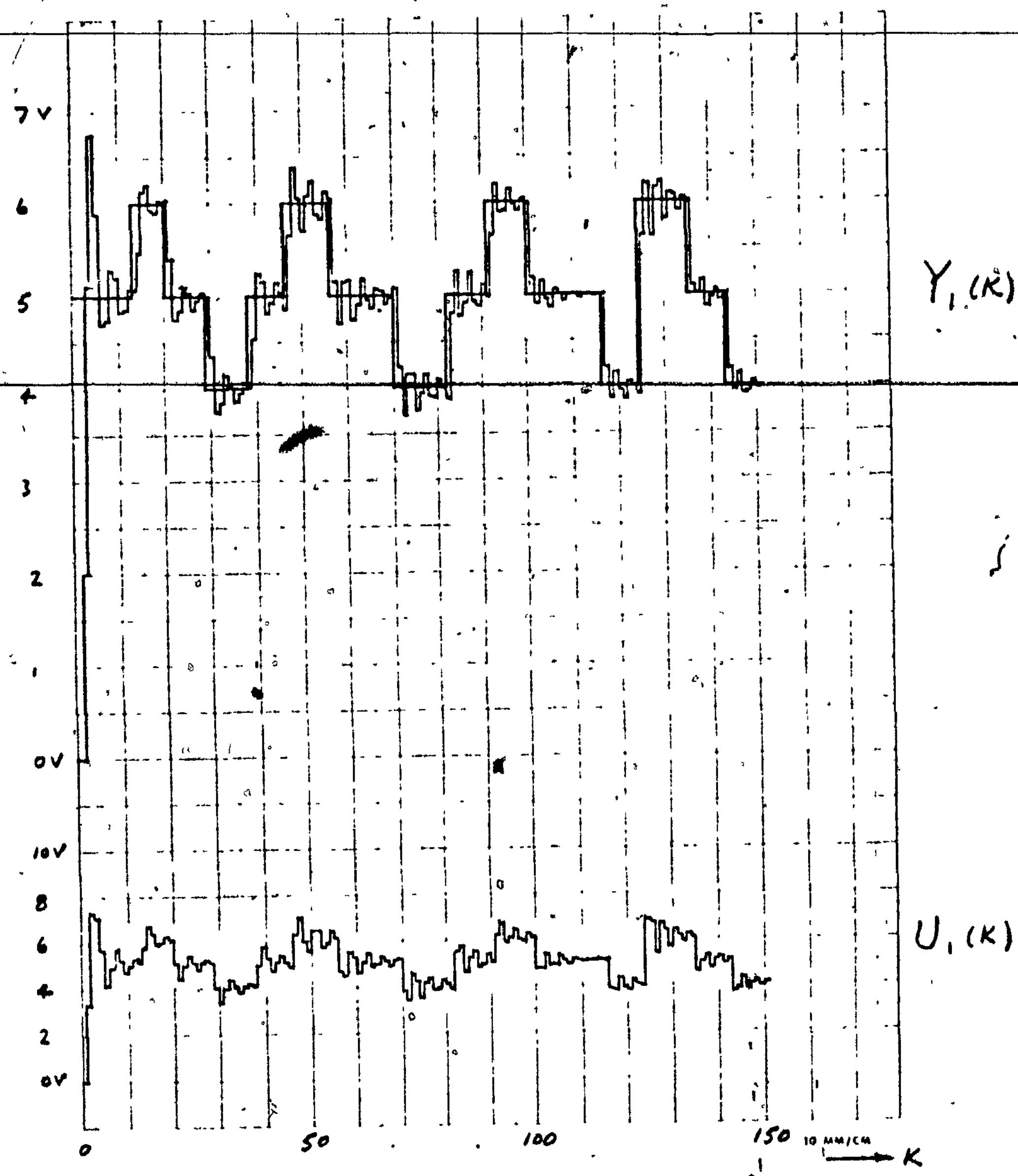
SIMULATED PLANT

T = 42 seconds

$p_1(0) = 1$

$\alpha = 2.5 \times 10^{-5}$ volt$^2$

FIGURE 4.32




SELF-TUNING CONTROL

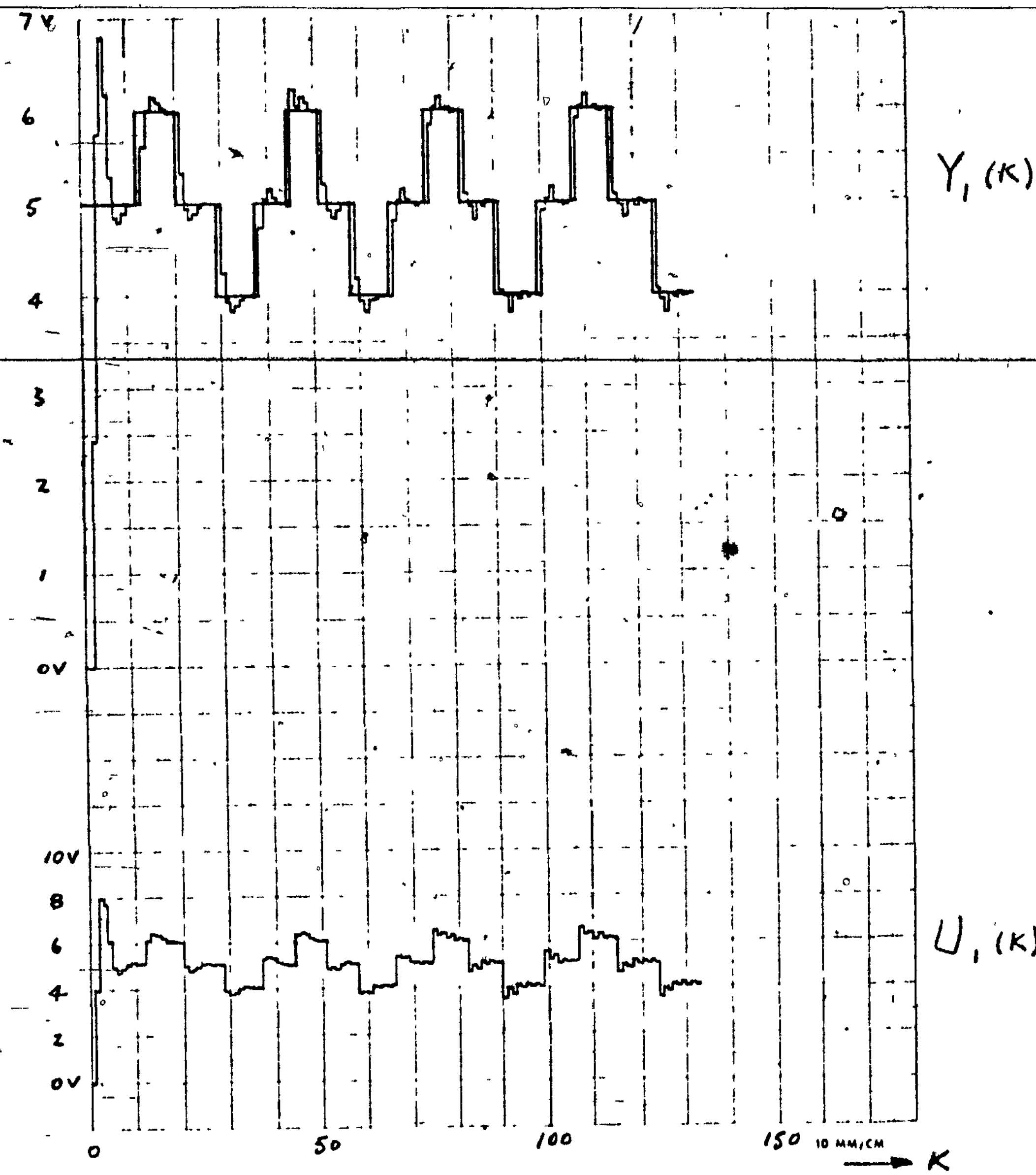
THIRD ORDER SIMULATED PLANT

T = 42 seconds

$b_1(0) = 1$

$\alpha = 13.1$ volt$^2$

FIGURE 4.33




SELF-TUNING CONTROL

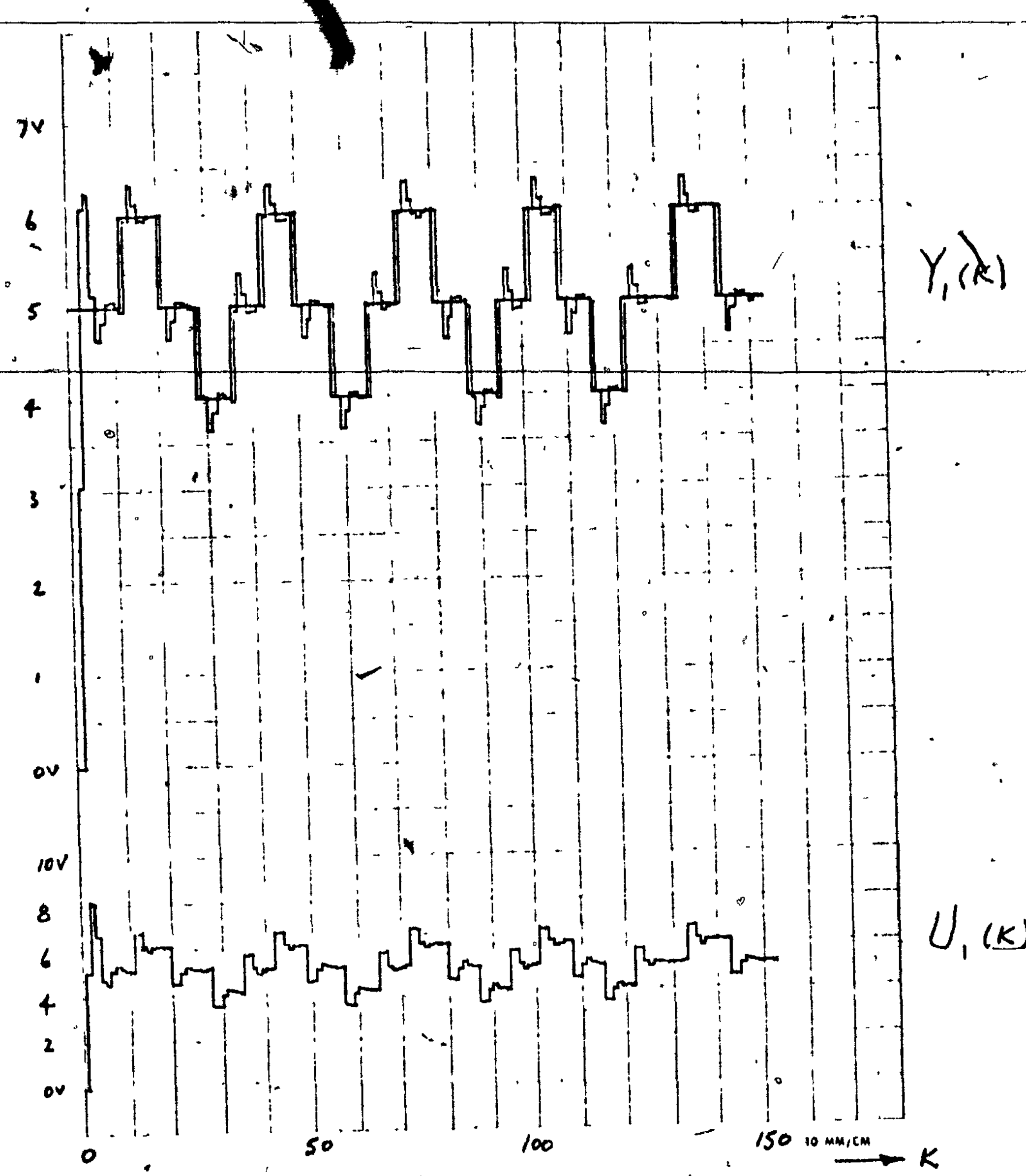
SECOND ORDER SIMULATED PLANT

T = 42 seconds

$b_1(0) = 1$

$\alpha = 6.55$ volt$^2$

FIGURE 4.34




SELF-TUNING CONTROL

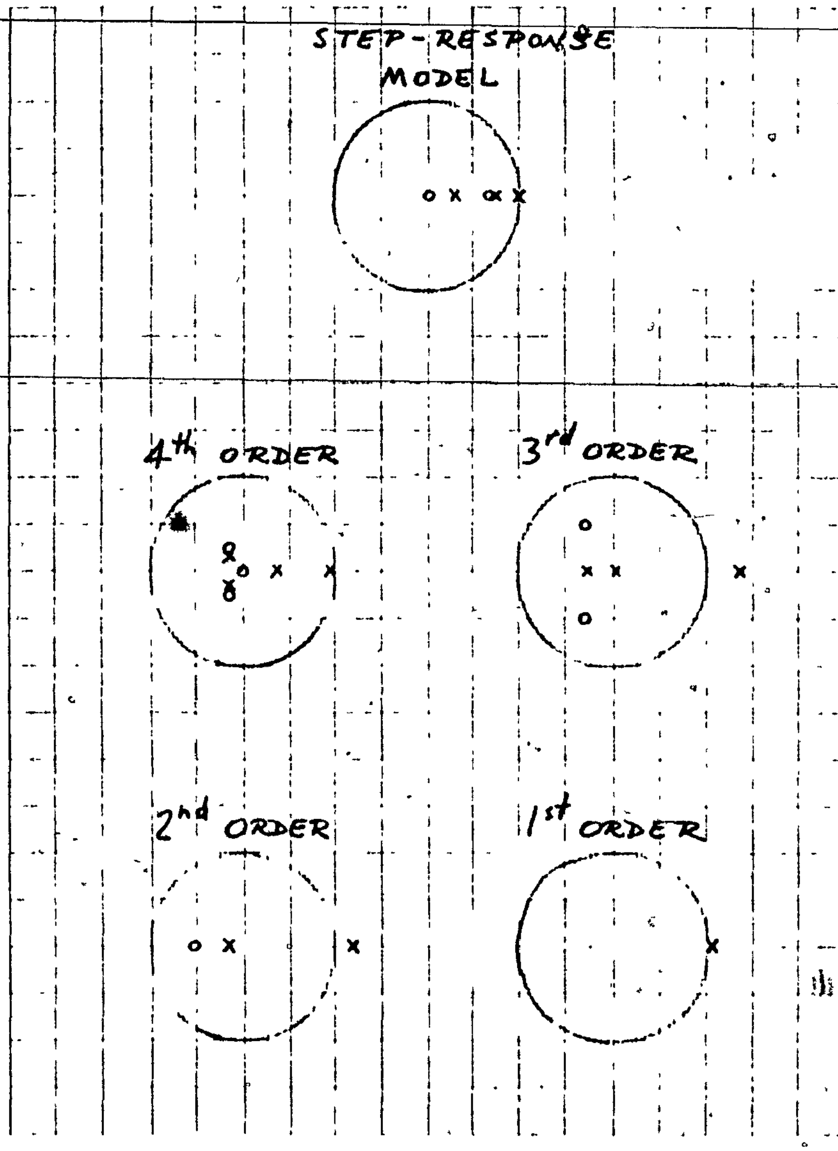
FIRST ORDER SIMULATED PLANT

T = 42 seconds

$b_1(0) = 1$

$\alpha = 2.5 \times 10^{-5}$ volt$^2$

FIGURE 4.35




POLE-ZERO PATTERN OF IDENTIFIED PLANT

# CHAPTER 5

# CONCLUSION

## 5.1 Summary

The heat exchanger was viewed as a single-input-single-output system. The input was a steam-flow set-point signal and the output was the outlet water temperature. The control algorithm used was a combination of a recursive least-squares parameter estimation using a linear model and a minimum variance regulator tuned on-line according to the estimated parameters. In a noise-free steady-state the input and output of the minimum variance regulator are zero. Thus the input to the regulator is required to be a temperature error signal, and the regulator output must be integrated to obtain a set-point signal for the steam flow controller. Hence the plant is augmented within the computer program by an integrator following the output of the regulator. To the identification algorithm, this integrator is part of the plant.

Convergence of the parameter identification cannot be guaranteed because of correlated noise (model equation errors) and because of feedback. Correlated noise leads to biased parameter estimates. The consequence of feedback is non-identifiability beyond specification of a linear manifold on which the parameter estimates lie. Nevertheless, if the parameter estimates converge, a unique minimum variance controller can be expected.

For comparison purposes a model of the plant was obtained from a step response test. The model was valid only in a mid-temperature region. The heat exchanger appeared to be a combination of two lags with a minor delay. The model was augmented to include the digital integrator. A sampling interval of .7 minutes was chosen for most of the experiments so that all the parameters of the correct model would be significant. If not all the model parameters were significant, then the order of the model would be incorrect. An autocorrelation of the steady-state temperature was performed in order to obtain a noise variance estimate required for the identification algorithm.

Using a fourth order model, the least-squares minimum variance control algorithm produced a stable system and converging parameter estimates. The value of the noise variance parameter $\alpha$ did not appear to affect the results. In general, the pole at +1 due to the integrator and the pole corresponding to the dominant time constant of the heat exchanger were approximately correctly identified. Also produced were a pair of complex poles that nearly cancelled a pair of complex zeros. This suggested that a second order model would be more appropriate. (Experiments with second order models did not substantiate this hypothesis however.) In the fourth order experiments, the problem of non-identifiability due to feedback did not make itself apparent.

Experiments with lower-order models met with less success however. The parameter estimates made large excursions, not always along a linear manifold. Stability became sensitive to the value of $\alpha$ and to the starting value of $b_1$. Models leading to PI and PID controllers were even less successful. In the case of the PID controller (third order model) three poles of the plant are to be identified, while two zeros are fixed at the origin. The problem is that only one zero should actually be at the origin and the second cannot be properly adjusted. The first order model produced a simple integral controller. This turned out to have better stability and convergence properties than obtained from second and third order models.

Further tests on an analog computer simulation of the heat exchanger (with no noise) showed that the parameter $\alpha$ affects the algorithm much more than was previously apparent. The first and fourth order identifications converged for a wide range of $\alpha$, including extremely small values. The second and third order identifications also converged, but only for relatively large values of $\alpha$.

Overall, the experiment is a successful

application of the self-tuning control algorithm. The question remains, however, of why α was important in the second and third order identifications, and unimportant in the first and fourth order identifications.

## 5.2 Conclusion

The experiment has been a successful application of the least-squares minimum-variance self-tuning control algorithm. On the whole, the result of the experiment is regarded as a further recommendation of this algorithm for use in industry. It has shown itself to be a robust algorithm, especially if the model order is high enough. In all cases, the algorithm used no a priori knowledge of the parameters. Also, parameter $b_1$ was not fixed. This opened the possibility of non-identifiability. Yet, for proper choices of α, the algorithm converged smoothly for all the model orders from first to fourth. Convergence was not slow either, with moderately good control being obtained within 20 or 30 steps.

A question remains, however, on the surprising effect α has on the algorithm. Parameter α is a noise variance parameter. Considering its location in the recursive formula of Equation 3.3, it can be viewed as a measure of the algorithm's immunity to noise. Giving a large value to $\alpha$ makes the algorithm more immune to noise, but slows down the convergence. Reducing $\alpha$ speeds up convergence (by increasing gain $\Gamma$) but thereby makes the algorithm more susceptible to noise. It could be expected that too small a value for $\alpha$ would lead to diverging identification. Presumably, the best compromise is to set $\alpha$ equal to the actual plant noise variance. The question, then, is why does the fourth order identification converge in the stochastic environment of the heat exchanger with $\alpha$ three orders of magnitude less than the actual measured noise; this while the second and third order identifications did not converge in the deterministic environment of the analog simulation until $\alpha$ corresponded to an rms noise value of about one third of the entire output range.

Since no such peculiarity is reported in the experiences of [18, 19], it is possible that it is a result here of the lack of the stabilizing effect of fixing the value of $b_1$. In any case it is an interesting question whose answer would probably be very instructive about the general behaviour of the algorithm. It is suggested that further study of the algorithm, either experimental or theoretical, in a deterministic environment may illuminate the nature of the effect of $\alpha$. The reason for a deterministic study is that the least-squares identification scheme is essentially the solution of a deterministic optimization problem. If there is anything interesting about α of fundamental nature to be learned, a stochastic environment would tend to mask it.

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# APPENDIX I

## System Wiring Diagrams

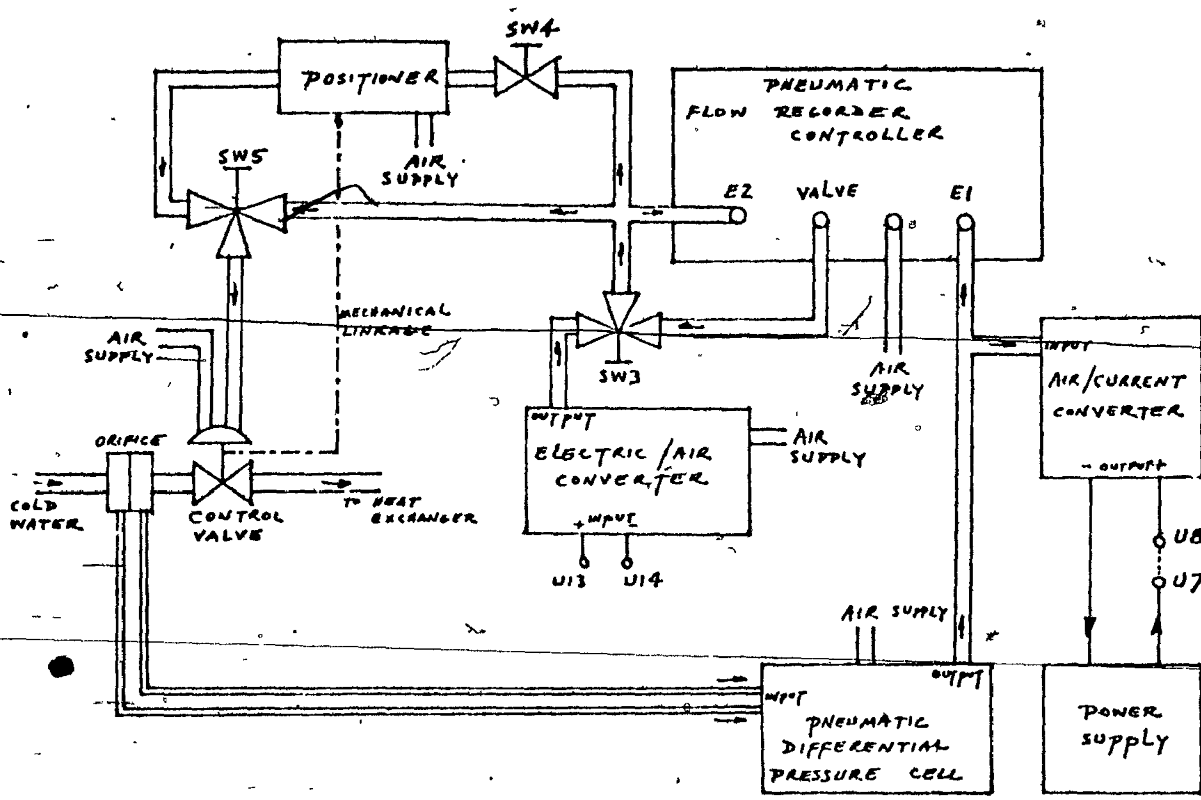
The illustrations in this Appendix are intended to provide more detail of the water flow control and water temperature control systems. The main purpose of this is to record the wiring of the heat-exchanger/computer-station link for purposes of future experimentation.

Figure Ia shows the water flow control system. The arrows indicate the direction of fluid and current flow. Terminals U13, U14 and U7, U8 are the input/output terminals for direct digital control (DDC). These terminals are located on the upper panel of terminals at the heat exchanger. By using the table in Figure Ic, the corresponding terminals at the computer station can be found.

Figure Ib shows the water temperature control system. The cold and hot water temperature signals are available at terminals U1, U2 and U3, U4 respectively. The steam flow signal appears at U11, U12. These signals are 10 to 50 ma currents and hence the terminal pairs must close the circuit. They may be shorted if the signal is not being used, or they may be bridged at the computer station by a resistor to produce a voltage signal. Terminals U15, U16 provide a direct digital control signal which can be engaged by setting switch SW1.

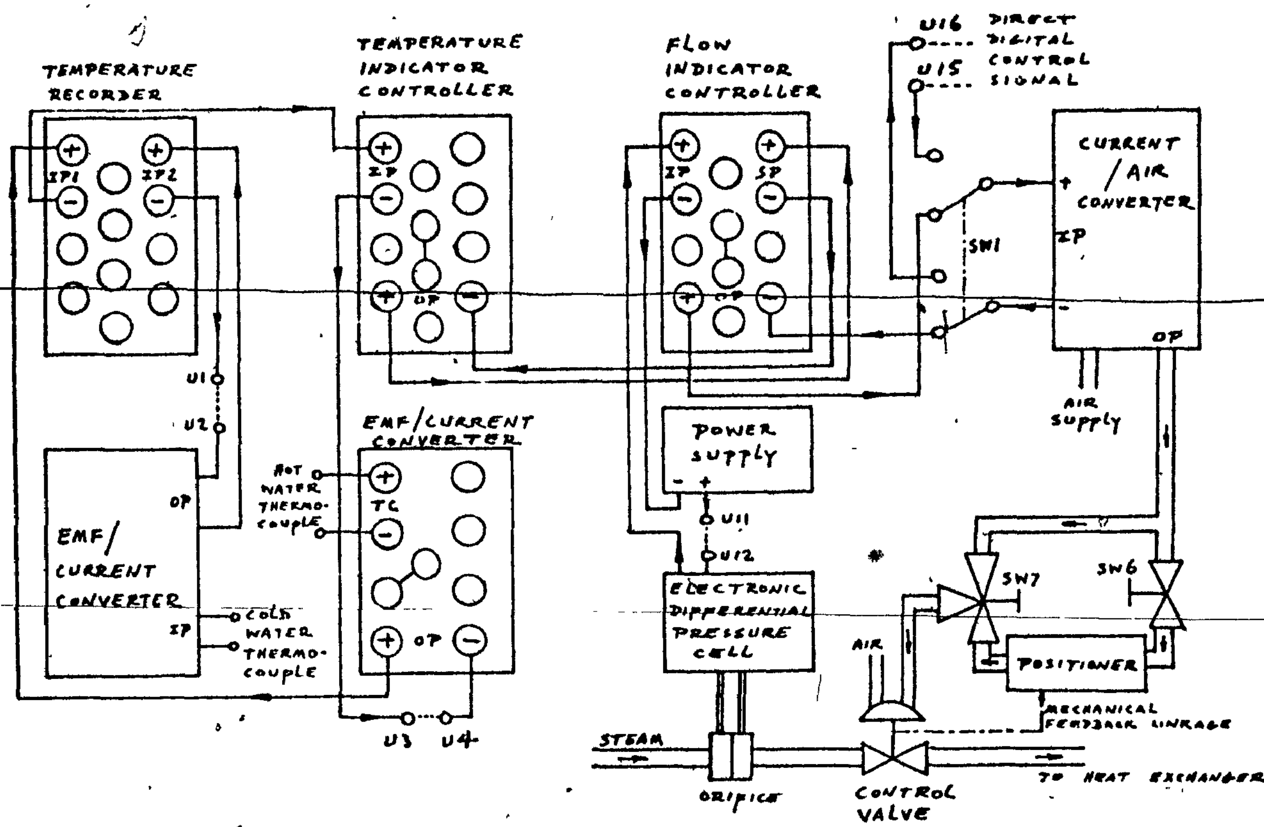
Switch SW2 is not shown here because it does not presently exist as a piece of hardware. The system is shown with the analog temperature controller feeding the set-point signal to the SP terminals of the steam flow controller. To effect the switch to computer set-point control, the temperature controller output is disconnected from the flow controller set-point and terminated by a 100Ω resistor. The set-point terminals of the flow controller are then connected to terminals U5, U6 which carry the required set-point signal from the computer station.

FIGURE Ia




WATER FLOW CONTROL SYSTEM

FIGURE Ib




WATER TEMPERATURE CONTROL SYSTEM

FIGURE I c

TABLE OF HEAT-EXCHANGER/COMPUTER-STATION TERMINAL CONNECTION

HEAT EXCHANGER | | | | | ROOM 502 McCONNELL ENG. BLDG.

| PANEL # | TERMINAL # | WRAPPER COLOUR | WIRE COLOUR | SIGNAL | PANEL # | TERMINAL # |
|---|---|---|---|---|---|---|
| U | 1 | GREEN | BLACK | + COLD WATER | D | 3 |
| | 2 | | WHITE | - TEMPERATURE MEASUREMENT | | 4 |
| U | 3 | GREEN | YELLOW | + HOT WATER | D | 5 |
| | 4 | | BLACK | - TEMPERATURE MEASUREMENT | | 6 |
| U | 5 | RED | RED | + TEMPERATURE CONTROLLER | D | 7 |
| | 6 | | BLACK | - SET-POINT COMMAND | | 8 |
| U | 7 | RED | BLUE | + WATER FLOW | D | 9 |
| | 8 | | BLACK | - MEASUREMENT | | 10 |
| U | 9 | BLUE | YELLOW | NOT USED | D | 11 |
| | 10 | | RED | | | 12 |
| U | 11 | BLUE | RED | + STEAM FLOW | C | 1 |
| | 12 | | GREEN | - MEASUREMENT | | 2 |
| U | 13 | BLUE | WHITE | + WATER FLOW COMMAND | C | 3 |
| | 14 | | RED | - (DIRECT DIGITAL CONTROL) | | 4 |
| U | 15 | BLUE | BLUE | - STEAM FLOW COMMAND | C | 5 |
| | 16 | | RED | + (DDC AT SW1) | | 6 |
| L | 1 | BLUE | GREEN | NOT USED | C | 7 |
| | 2 | | BLACK | | | 8 |
| L | 3 | BLUE | ORANGE | NOT USED | C | 9 |
| | 4 | | BLACK | | | 10 |
| L | 5 | BLUE | BROWN | NOT USED | C | 11 |
| | 6 | | BLACK | | | 12 |

# APPENDIX II

## Control Program Listing

## (In Assembler Language [21,22])

```
                              /CONTROL PROGRAM
                              /
                        ADCL=6530
                        ADLM=6531
                        ADLE=6536
                        ADST=6532
                        ADSK=6534
                        AD[illegible]=6533
                        DADS=6442
                        DALA=6443
                        DALD=6444
                        DAST=6446
                        DASI=6441
                              *1
0001   5402                   JMP I 2
0002   0200                   SEDV
                              *5/
0005   7400                   7400
0006   7200                   7200
0007   5000                   5000
                              *11
0011   0013    M13,           13
0012   0001    M1024,         1
0013   2030                   2030
0014   4400                   4400
                              *63
0063   1000    MTTY,          TTY
0064   1200    MREAD,         READ
0065   1210    MPRINT,        PRINT
0066   1215    MLF,           LF
0067   1222    MCR,           CR
0070   1227    MP,            P
0071   1261    MO,            O
0072   1271    MT,            T
0073   1303    MS,            S
0074   0243    MSET,          SET
0075   0700    MCLEAR,        CLEAR
0076   1400    MCLOCK,        CLOCK
0077   1000    [illegible],   [illegible]
```

```
0100  1644  XAD,     AD
0101  2127  XDA,     DA
0102  0554  XU1,     554
0103  0607  XU2,     607
0104  0570  XY1,     570
0105  0623  XY2,     623
0106  0400  XCOEF,   400
0107  7777  MN1,     -1
0110  7775  MN3,     -3
0111  0000  COUNT,   0
0112  0000  NUM,     0
0113  0013  U1,      13
0114  0000           0
0115  0000           0
0116  0013  U2,      13
0117  0000           0
0120  0000           0
0121  0013  Y1,      13
0122  0000           0
0123  0000           0
0124  0013  Y2,      13
0125  0000           0
0126  0000           0
0127  0013  Z1,      13
0130  0000           0
0131  0000           0
0132  0013  Z2,      13
0133  0000           0
0134  0000           0
0135  0000  E1,      0
0136  0000  E2,      0
0137  0220  XINIT,   INIT
0140  2000  XDAO,    DAO
0141  0226  XERR,    ERROR
0142  0247  XFIX,    FIX
0143  0000  HALT,    0
0144  7433  LSTRT,   -345
0145  0000  LCOUNT,  0
0146  0000  HSTRT,   0
0147  0000  HCOUNT,  0
0150  0003  N3,      3
                     /
                     /INTERRUPT SERVICING
                     /
                     *200
0200  3003  SERV,    DCA AC
0201  7204           GLK
0202  3306           DCA LINK
0203  6031           KSF
0204  7410           SKP
0205  5707           JMP I XINT          /KEYBOARD
0206  6531           ADSF
0207  7410           SKP
0210  5727           JMP I XIA[illegible]          /A/D
```

```
0211  6041            TSF
0212  7410            SKP
0213  5730            JMP I XITP      /PRINTER
0214  6021            RSF
0215  7410            SKP
0216  5731            JMP I XIPP      /PUNCH
0217  6007            CAF
0220  7300   EXIT,    CLA CLL /RETURN FROM INTERRUPT SERVICING
0221  1326            TAD LINK
0222  7010            RAR
0223  1325            TAD AC
0224  6001            ION
0225  5400            JMP I 0
0226  7300   ERROR,   CLA CLL
0227  1045            TAD 45
0230  7510            SPA
0231  5236            JMP.+5
0232  7300            CLA CLL
0233  7040            CMA
0234  7010            RAR
0235  5705            JMP I R1
0236  7300            CLA CLL
0237  7020            CML
0240  7010            RAR
0241  7001            IAC
0242  5705            JMP I R1
0243  4464   SET,     JMS I MREAD
0244  1703            TAD I MM61
0245  7510            SPA
0246  5471            JMP I XO
0247  3111            DCA COUNT
0250  1111            TAD COUNT
0251  1107            TAD MN1
0252  7540            SMA SZA
0253  5471            JMP I XO
0254  7300            CLA CLL
0255  1111            TAD COUNT
0256  7004            RAL
0257  1111            TAD COUNT
0260  1334            TAD ADZ
0261  3111            DCA COUNT
0262  4477            JMS I KEO
0263  4405            JMS I 5
0264  4407            JMS I 7
0265  3326            [illegible]
0266  0000            FEXT
0267  4542            JMS I MFIX
0270  3511            DCA I COUNT
0271  5463            JMP I MTTY
0272  0000   HEADER,  0
0273  7300            CLA CLL
0274  1[illegible]    TAD HEAD
0275  [illegible]     DCA 1[illegible]
0276  7[illegible]    CLA
```

```
0277  1410           TAD I 10
0300  7004           RAL
0301  7430           SZL
0302  5670           JMP I HEADER
0303  7010           RAR
0304  7440           SZA
0305  5314           JMP .+7
0306  1341           TAD KSP
0307  4465           JMS I MPRINT
0310  4465           JMS I MPRINT
0311  4465           JMS I MPRINT
0312  4465           JMS I MPRINT
0313  4465           JMS I MPRINT
0314  4465           JMS I MPRINT
0315  5276           JMP HEADER+4
0316  0000  CLDA,    0
0317  3724           DCA I MDACH
0320  4551           JMS I MDA
0321  2724           ISZ I MDACH
0322  4551           JMS I MDA
0323  5716           JMP I CLDA
0324  0144  MDACH,   DACH
0325  0000  AC,      0
0326  0000  LINK,    0
0327  1600  MIAD,    IAD
0330  1633  MITP,    ITP
0331  1660  MIPP,    IPP
0332  1663  MIMD,    IMD
0333  1162  MMLC1,   MLC1
0334  0130  ADZ,     Z1+1
0335  2075  D1,      DONE+1
0336  0010  M2047,   0
0337  0145           0145
0340  4600           4600
0341  0240  KSP,     240
0342  0342  HEAD,    .
0343  0240           240
0344  0240           240
0345  0332           332
0346  0261           261
0347  0000           0
0350  0331           331
0351  0261           261
0352  0000           0
0353  0325           325
0354  0261           261
0355  0000           0
0356  0325           325
0357  0261           261
0360  0000           0
0361  0332           332
0362  0262           262
0363  0000           0
0364  0331           331
```

```
0365  0262           262
0366  0000           0
0367  0305           305
0370  0262           262
0371  0000           0
0372  0325           325
0373  0262           262
0374  0215           215
0375  0212           212
0376  4000           4000
                     /
                     /SYSTEM INITIALIZATION
                     /
                     *700
0700  0000  CLEAR,   0
0701  7000           NOP
0702  7200           CLA
0703  1326           TAD MM242
0704  3111           DCA COUNT
0705  1106           TAD MCOEF
0706  1107           TAD MM1
0707  3010           DCA 10
0710  3410           DCA I 10
0711  2111           ISZ COUNT
0712  5310           JMP.-2
0713  7200           CLA
0714  1327           TAD MSSG
0715  3010           DCA 10
0716  7200           CLA
0717  1410           TAD I 10
0720  4465           JMS I MPRINT
0721  7440           SZA
0722  5316           JMP.-4
0723  4725           JMS I MCLDA
0724  5700           JMP I CLEAR
0725  0311  MCLDA,   CLDA
0726  7536  MM242,   -242
0727  0727  MSSG,    .
0730  0215           215
0731  0212           212
0732  0212           212
0733  0244           244
0734  0320           320
0735  0322           322
0736  0317           317
0737  0303           303
0740  0305           305
0741  0323           323
0742  0323           323
0743  0240           240
0744  0303           303
0745  0317           317
0746  0316           316
0747  0304           304
```

```
0750   0302          302
0751   0317          317
0752   0314          314
0753   0215          215
0754   0212          212
0755   0212          212
0756   0000          0
                     /
                     /START OF PROGRAM
                     /
                     *1000
1000   6002          IOF
1001   6032          KCC
1002   6046          TLS
1003   4475          JMS I XCLEAR
1004   4467          JMS I XCR
1005   4466          JMS I XLF
                     /ACCEPT INSTRUCTIONS FROM OPERATOR
1006   7200   TTY,   CLA
1007   1355          TAD CAR
1010   4465          JMS I XPRINT
1011   7200          CLA
1012   4464          JMS I XREAD
1013   1353          TAD M132
1014   7450          SNA
1015   5474          JMP I XSET          /DEFINE SET POINTS Z1,Z2
1016   1354          TAD K5
1017   7450          SNA
1020   5326          JMP U    /DEFINE CONTROL LAW FOR U1 OR U2
1021   7001          IAC
1022   7450          SNA
1023   5472          JMP I XT            /DEFINE SAMPLE INTERVAL T
1024   7001          IAC
1025   7450          SNA
1026   5473          JMP I XS            /START COMMAND
1027   1150          TAD K3
1030   7450          SNA
1031   5470          JMP I XP            /REQUEST OUTPUT
1032   1352          TAD M14             /ON PRINTER/PUNCH
1033   7450          SNA
1034   5272          JMP D    /DEFINE COEFFICIENT D
1035   7001          IAC
1036   7450          SNA
1037   5256          JMP C    /DEFINE COEFFICIENT C
1040   7001          IAC
1041   7450          SNA
1042   5262          JMP B    /DEFINE COEFFICIENT B
1043   7001          IAC
1044   7450          SNA
1045   5247          JMP A    /DEFINE COEFFICIENT A
1046   5471          JMP I X[illegible]
1047   1000   A,     TAD [illegible]
1050   3007          DCA [illegible]
1051   1[illegible]  TAD I[illegible]1
```

```
1052   3305          DCA I1
1053   1361          TAD INST2
1054   3277          DCA I2
1055   5276          JMP CONT
1056   1356    C,    TAD UA
1057   1363          TAD K27
1060   3357          DCA UAD
1061   5251          JMP A+2
1062   1356    B,    TAD UA
1063   1364          TAD K12
1064   3357          DCA UAD
1065   1365          TAD INST3
1066   3305          DCA I1
1067   1367          TAD INST4
1070   3277          DCA I2
1071   5276          JMP CONT
1072   1356    D,    TAD UA
1073   1371          TAD K39
1074   3357          DCA UAD
1075   5265          JMP B+3
1076   4464    CONT, JMS I XREAD
1077   0000    I2,   0
1100   7510          SMA
1101   7410          SKP
1102   5471          JMP I X0
1103   3372          DCA ADDR
1104   1372          TAD ADDR
1105   0000    I1,   0
1106   7550          SPA SNA
1107   7410          SKP
1110   5471          JMP I X0
1111   4477          JMS I XE0
1112   1372          TAD ADDR
1113   7100          CLL
1114   7004          RAL
1115   1372          TAD ADDR
1116   1357          TAD UAD
1117   3372          DCA ADDR
1120   6002          NOC
1121   4405          JMS I 5
1122   4407          JMS I 7
1123   6772          FPUT I ADDR
1124   0000          FEXT
1125   5000          JMP TTY
1126   1100    U,    TAD XCOEF
1127   3356          DCA UA
1130   4464          JMS I XREAD
1131   1362          TAD KM61
1132   7500          SMA
1133   7410          SKP
1134   5471          JMP I X0
1135   7450          SNA
1136   5000          JMP .+6
1137   1100          TAD KM1
```

```
1140  7440           SZA
1141  5471           JMP I MC
1142  1373           TAD K466
1143  3356           DCA UA
1144  4464           JMS I MREAD
1145  1374           TAD M15
1146  7440           SZA
1147  5471           JMP I MC
1150  4430           JMS I MLF
1151  5200           JMP TTY
1152  0014  M14,     14
1153  7646  MM132,   -132
1154  0005  M5,      5
1155  0276  SAP,     276
1156  0[illegible]  [illegible]A,  4[illegible]
1157  0000  UAD,     0
1160  1113  INST1,   TAD MM3
1161  1362  INST2,   TAD MM61
1162  7717  MM61,    -61
1163  0033  M37,     33
1164  001[illegible]  M12,  1[illegible]
1165  1[illegible]  INST3,  TAD MM4
1166  7774  MM4,     -4
1167  137[illegible]  INST4,  TAD MM60
1170  77[illegible]  MM60,  -60
1171  [illegible]  M[illegible],  47
1172  0000  ADDR,    0
1173  0466  K466,    466
1174  7763  M15,     -15
                     *1200
1200  0000  READ,    0
1201  7200           CLA
1202  6031           KSF
1203  5202           JMP .-1
1204  6036           KRB
1205  4210           JMS PRINT
1206  0307           AND MASK1
1207  5600           JMP I READ
1210  0000  PRINT,   0
1211  6041           TSF
1212  5211           JMP .-1
1213  6046           TLS
1214  5610           JMP I PRINT
1215  0000  LF,      0
1216  7200           CLA
1217  134[illegible]  TAD M212
1220  4210           JMS PRINT
1221  5615           JMP I LF
1222  0000  CR,      0
1223  7200           CLA
1224  1341           TAD M215
1225  4210           JMS PRINT
1226  5622           JMP I CR
1227  4[illegible]  [illegible],  JMS [illegible]
```

```
1230   4200      JMS READ
1231   1335      TAD MC
1232   7510      SPA
1233   5261      JMP O
1234   3111      DCA COUNT
1235   1111      TAD COUNT
1236   1110      TAD KM3
1237   7540      SMA SZA
1240   5261      JMP O
1241   4200      JMS READ
1242   1336      TAD M15
1243   7440      SZA
1244   5261      JMP O
1245   4215      JMS LF
1246   7300      CLA CLL
1247   3733      DCA I MOPT1
1250   3734      DCA I MOPT2
1251   1111      TAD COUNT
1252   7010      RAR
1253   7430      SZL
1254   2733      ISZ I MOPT1
1255   7010      RAR
1256   7430      SZL
1257   2734      ISZ I MOPT2
1260   5463      JMP I MTTY
1261   4222      JMS CR
1262   4215      JMS LF
1263   7200      CLA
1264   1342      TAD K277
1265   4210      JMS PRINT
1266   4222      JMS CR
1267   4215      JMS LF
1270   5463      JMP I MTTY
1271   4325   T, JMS EO
1272   6032      KCC
1273   4405      JMS I 5
1274   4407      JMS I 7
1275   3345      FMPY TWENTY
1276   0000      FEXT
1277   4542      JMS I MFIX
1300   7041      CIA
1301   3146      DCA MSTRT
1302   5463      JMP I MTTY
1303   4222   S, JMS CR
1304   4215      JMS LF
1305   7200      CLA
1306   1734      TAD I MOPT2
1307   7440      SZA
1310   4744      JMS I MREAD
1311   7200      CLA
1312   1146      TAD MSTRT
1313   3147      DCA MCOUNT
1314   1144      TAD LSTRT
1315   3145      DCA LCOUNT
```

```
1316  3110          DCA NUM
1317  3143          DCA HALT
1320  6042          TCF
1321  6530          ADCL
1322  4500          JMS I XAD
1323  6001          ION
1324  5324          JMP.
1325  0000  EQ,     0
1326  4200          JMS READ
1327  1343          TAD MM75
1330  7440          SZA
1331  5261          JMP 0
1332  5725          JMP I EQ
1333  2151  MOPT1,  OPT1
1334  2152  MOPT2,  OPT2
1335  7720  M60,    -60
1336  7763  M15,    -15
1337  0177  MASK1,  177
1340  0212  K212,   212
1341  0215  K215,   215
1342  0277  K277,   277
1343  7703  MM75,   -75
1344  [illegible]  [illegible],  [illegible]
1345  [illegible]  TWENTY,  5
1346  2400          2400
1347  0000          0
                    /
                    /CONTROL ALGORITHM
                    /IMPLEMENTATION OF CONTROL LAW
                    /
                    *1400
1400  7000  CLOCK,  NOP
1401  7300          CLA CLL
1402  4407          JMS I 7
1403  5127          FGET Z1
1404  [illegible]   FNOR
1405  6504          FPUT I [illegible]1
1406  5121          FGET Y1
1407  [illegible]   FNOR
1410  [illegible]   [illegible] I [illegible]1
1411  6504          FPUT I [illegible]1
1412  0000          FEXT
1413  7300          CLA CLL
1414  1044          TAD 44
1415  1107          TAD [illegible]1
1416  3044          DCA 44
1417  [illegible]   JMS I [illegible]
1420  3135          DCA [illegible]1
1421  4407          JMS I 7
1422  [illegible]   FGET Z2
1423  [illegible]   FNOR
1424  6505          FPUT I [illegible]2
1425  [illegible]   FGET Y2
1426  [illegible]   FNOR
```

```
1427   5565          FGET I MXC
1430   6565          FPUT I MXC
1431   0000          FEXT
1432   7300          CLA CLL
1433   1044          TAD 44
1434   1177          TAD KM1
1435   3044          DCA 44
1436   4542          JMS I XFIX
1437   3136          DCA E2
1440   7200          CLA
1441   1196          TAD XCOEF
1442   3360          DCA COEF
1443   1102          TAD XU1
1444   3361          DCA VAR
1445   4327          JMS CTRL
1446   1366          TAD TEN
1447   3114          DCA U1+1
1450   1102          TAD XU1
1451   3361          DCA VAR
1452   4327          JMS CTRL
1453   1366          TAD TEN
1454   3117          DCA U2+1
1455   7000          NOP
1456   4541          JMS I XDAO
1457   1362          TAD KM10
1460   7001          IAC
1461   3111          DCA COUNT
1462   1102          TAD XU1
1463   1364          TAD X66
1464   1110   UP,    TAD KM3
1465   3361          DCA VAR
1466   1361          TAD VAR
1467   1110          TAD KM3
1470   3365          DCA VARM3
1471   4407          JMS I 7
1472   5765          FGET I VARM3
1473   6761          FPUT I VAR
1474   0000          FEXT
1475   7200          CLA
1476   1361          TAD VAR
1477   2111          ISZ COUNT
1500   5364          JMP UP
1501   7200          CLA
1502   4407          JMS I 7
1503   5113          FGET U1
1504   7000          FNOR
1505   6500          FPUT I XU1
1506   5116          FGET U2
1507   7000          FNOR
1510   6500          FPUT I XU2
1511   0000          FEXT
1512   7000          NOP
1513   7200          CLA
1514   1100          TAD HALT
```

```
1515 7450             SNA
1516 5316             JMP .
1517 7200             CLA
1520 6002             IOF
1521 6040             TLS
1522 1363             TAD MM
1523 4465             JMS I MPRINT
1524 4467             JMS I MCR
1525 4466             JMS I MLF
1526 5463             JMP I XTTY
1527 0000   CTRL,     0
1530 7200             CLA
1531 3366             DCA TEM
1532 3367             DCA TEM+1
1533 3370             DCA TEM+2
1534 1362             TAD MM18
1535 3111             DCA COUNT
1536 4407             JMS I 7
1537 5760             FGET I COEF
1540 3761             FMPY I VAR
1541 1366             FADD TEM
1542 6366             FPUT TEM
1543 0000             FEXT
1544 7200             CLA
1545 1360             TAD COEF
1546 1150             TAD M3
1547 3360             DCA COEF
1550 1361             TAD VAR
1551 1150             TAD M3
1552 3361             DCA VAR
1553 2111             ISZ COUNT
1554 5336             JMP CTRL+7
1555 4542             JMS I XFIX
1556 3366             DCA TEM
1557 5727             JMP I CTRL
1560 0000   COEF,     0
1561 0000   VAR,      0
1562 7756   MM18,     -22
1563 0300   MM,       300
1564 0066   M66,      66
1565 0000   VARM3,    0
1566 0000   TEM,      0
1567 0000             0
1570 0000             0
                      /
                      /INTERRUPT SERVICE ROUTINES
                      /
                      *1600
1600 6530   IAD,      ADCL      /A/D INTERRUPT
1601 7300             CLA CLL
1602 6533             ADRB
1603 7004             RAL
1604 7510             SPA
1605 5[illegible]             JMP .+3
```

```
1606  0273          AND MASKC
1607  7041          CIA
1610  3274          DCA ADREG
1611  1112          TAD MUX
1612  7450          SNA
1613  5221          JMP.+6
1614  7200          CLA
1615  1274          TAD ADREG
1616  3276          DCA BY2
1617  3112          DCA MUX
1620  5224          JMP.+4
1621  1274          TAD ADREG
1622  3275          DCA BY1
1623  2112          ISZ MUX
1624  2145          ISZ LCOUNT
1625  5242          JMP NOCLK
1626  1144          TAD LSTRT
1627  3145          DCA LCOUNT
1630  2147          ISZ MCOUNT
1631  5242          JMP NOCLK
1632  1146          TAD MSTRT
1633  3147          DCA MCOUNT
1634  1173          TAD MCLOCK
1635  3000          DCA 0
1636  1275          TAD BY1
1637  3122          DCA Y1+1
1640  1276          TAD BY2
1641  3125          DCA Y2+1
1642  4244  NOCLK,  JMS AD
1643  5537          JMP I MEXIT
1644  0000  AD,     0
1645  7300          CLA CLL
1646  1112          TAD MUX
1647  6531          ADSM
1650  1112          TAD MUX
1651  1277          TAD ENABLE
1652  6536          ADLE
1653  6532          ADST
1654  5644          JMP I AD
1655  6142  ITP,    TCF        /PRINTER INTERRUPT
1656  2000          ISZ 0
1657  5537          JMP I MEXIT
1660  6122  IPP,    PCF        /PUNCH INTERRUPT
1661  2000          ISZ 0
1662  5537          JMP I MEXIT
1663  6032  IKB,    KCF        /KEYBOARD INTERRUPT
1664  7200          CLA        /DETECT HALT COMMAND
1665  6034          KRS
1666  0300          AND MSK1
1667  1301          TAD KK
1670  7450          SNA
1671  2143          ISZ HALT
1672  5537          JMP I MEXIT
1673  0777  MASKC,  0777
```

```
1674   0000   ADDRC,   0
1675   0000   DY1,     0
1676   0000   DY2,     0
1677   1000   ENABLE,  1000
1700   0177   MSK1,    177
1701   7670   ML,      -110
                       /
                       /OUTPUT ROUTINE
                       /
                       *2000
2000   0000   DAO,     0
2001   7300            CLA CLL
2002   3344            DCA DACH
2003   1114            TAD U1+1
2004   4327            JMS DA
2005   2344            ISZ DACH
2006   1117            TAD U2+1
2007   4327            JMS DA
2010   1130            TAD Z1+1
2011   4241            JMS HALF
2012   4276            JMS OUT
2013   11[illegible]   TAD Y1+1
2014   4241            JMS HALF
2015   4276            JMS OUT
2016   1125            TAD E1
2017   4276            JMS OUT
2020   1114            TAD U1+1
2021   4241            JMS HALF
2022   4276            JMS OUT
2023   1133            TAD Z2+1
2024   4241            JMS HALF
2025   4276            JMS OUT
2026   11[illegible]   TAD Y2+1
2027   4241            JMS HALF
2030   4276            JMS OUT
2031   1136            TAD E2
2032   4276            JMS OUT
2033   1117            TAD U2+1
2034   4241            JMS HALF
2035   4276            JMS OUT
2036   4753            JMS I MRET
2037   7000            NOP
2040   5600            JMP I DAO
2041   0000   HALF,    0
2042   7100            CLL
2043   7510            SPA
2044   7020            CML
2045   7010            RAR
2046   5641            JMP I HALF
2047   0000   FIX,     0         /FLOATING TO FIXED POINT CONVERSIC
2050   7200            CLA
2051   1044            TAD 44
2052   754[illegible]  SZA SMA
[illegible]  [illegible]  JMP .+3
```

```
2054  7200              CLA
2055  5647              JMP I FIX
2056  1047              TAD MM13
2057  7450              SNA
2060  5274              JMP DONE
2061  7500              SMA
2062  5541              JMP I MERR
2063  3044              DCA 44
2064  7100              CLL
2065  1045              TAD 45
2066  7510              SPA
2067  7020              CML
2070  7010              RAR
2071  3045              DCA 45
2072  2044              ISZ 44
2073  5264              JMP.-7
2074  1045  DONE,       TAD 45
2075  5647              JMP I FIX
2076  0000  OUT,        0
2077  3045              DCA BUFFER
2100  1351              TAD/OPT1
2101  7440              SZA
2102  4307              JMS PUNCH
2103  1352              TAD OPT2
2104  7440              SZA
2105  4754              JMS I MPRNT
2106  5676              JMP I OUT
2107  0000  PUNCH,      0           /PAPER TAPE OUTPUT
2110  7200              CLA
2111  1345              TAD BUFFER
2112  7002              BSW
2113  0346              AND MSK77
2114  1350              TAD BITS
2115  6026              PLS
2116  5316              JMP.
2117  7200              CLA
2120  1345              TAD BUFFER
2121  0346              AND MSK77
2122  1350              TAD BITS
2123  6026              PLS
2124  5324              JMP.
2125  7200              CLA
2126  5707              JMP I PUNCH
2127  0000  DA,         0           /D/A OUTPUT
2130  6442              DAPS
2131  7510              SPA
2132  1177              TAD MM1
2133  6444              DALD
2134  7300              CLA CLL
2135  1344              TAD DACH
2136  6443              DALA
2137  6446              DAST
2140  6441              DASK
2141  5340              JMP.-1
```

```
2142  7200            CLA
2143  5707            JMP I DA
2144  0000  DACN,     0
2145  0000  BUFFER,   0
2146  0077  MSK77,    77
2147  7765  MM13,     -13
2150  0200  DITC,     200
2151  0000  OPT1,     0
2152  0000  OPT2,     0
2153  2304  MRET,     RET
2154  2200  MPRNT,    PRNT
                      *2200
2200  0000  PRNT,     0         /PRINTER OUTPUT
2201  7200            CLA
2202  1734            TAD I MBUF
2203  3045            DCA 45
2204  1311            TAD M13
2205  3044            DCA 44
2206  3046            DCA 46
2207  4407            JMS I 7
2210  7000            FNOR
2211  4010            FDIV K1024
2212  0000            FEXT
2213  4542            JMS I MFIX
2214  3734            DCA I MBUF
2215  1734            TAD I MBUF
2216  7500            SMA
2217  5224            JMP.+5
2220  7041            CIA
2221  3734            DCA I MBUF
2222  1347            TAD MINUS
2223  7410            SKP
2224  7200            CLA
2225  1340            TAD K240
2226  6046            TLS
2227  5227            JMP.
2230  7300            CLA CLL
2231  1335            TAD K1000
2232  4251            JMS DIGIT
2233  1336            TAD K100
2234  4251            JMS DIGIT
2235  4276            JMS PERIOD
2236  1337            TAD K10
2237  4251            JMS DIGIT
2240  7001            IAC
2241  4251            JMS DIGIT
2242  1346            TAD K240
2243  6046            TLS
2244  5244            JMP.
2245  6046            TLS
2246  5246            JMP.
2247  7200            CLA
2250  5600            JMP I PRNT
2251  0000  DIGIT,    0
```

```
2252  3344              DCA DEC
2253  1344              TAD DEC
2254  7041              CIA
2255  3345              DCA MDEC
2256  1127              TAD M11
2257  3111              DCA COUNT
2260  1734              TAD I XBUF
2261  2111              ISZ COUNT
2262  7000              NOP
2263  1345              TAD MDEC
2264  7500              SMA
2265  5261              JMP.-4
2266  1344              TAD DEC
2267  3734              DCA I XBUF
2270  1111              TAD COUNT
2271  1343              TAD M260
2272  6046              TLS
2273  5273              JMP.
2274  7200              CLA
2275  5651              JMP I DIGIT
2276  0000  PERIOD,     0
2277  1342              TAD MPRD
2300  6046              TLS
2301  5301              JMP.
2302  7200              CLA
2303  5676              JMP I PERIOD
2304  0000  RET,        0
2305  7200              CLA
2306  1733              TAD I YOPT1
2307  7450              SNA
2310  5314              JMP.+4
2311  7200              CLA
2312  6026              PLS
2313  5313              JMP.
2314  7200              CLA
2315  1732              TAD I YOPT2
2316  7450              SNA
2317  5704              JMP I RET
2320  7200              CLA
2321  1340              TAD MCR
2322  6046              TLS
2323  5323              JMP.
2324  7200              CLA
2325  1341              TAD MLF
2326  6046              TLS
2327  5327              JMP.
2330  7200              CLA
2331  5704              JMP I RET
2332  2152  YOPT2,      OPT2
2333  2151  YOPT1,      OPT1
2334  2145  XBUF,       BUFFER
2335  1750  M1000,      1750
2336  0144  M100,       144
2337  0012  M10,        12
```

| | | | |
|---|---|---|---|
| 2340 | 0015 | M[illegible], | 015 |
| 2341 | 0210 | MLT, | 210 |
| 2342 | 0250 | M[illegible]D, | 250 |
| 2343 | 0260 | M[illegible]S, | 260 |
| 2344 | 0000 | [illegible], | 0 |
| 2345 | 0000 | MDTC, | 0 |
| 2346 | 0240 | M240, | 240 |
| 2347 | 0015 | MINUS, | 15 |

# APPENDIX III

## Identification Program Listing

```
                         /IDENTIFICATION PROGRAM
                         /
                         *20
0020   1710   XCLEAR,    CLEAR
0021   2400   XIDENT,    IDENT
0022   2730   XPCHB,     PCHB
0023   0000   INDIR,     0
0024   3027              DCA TEM
0025   1427              TAD I TEM
0026   5423              JMP I INDIR
0027   0400   TEM,       0
                         *102
0102   0554   UIM1,      554
                         *107
0107   7777   KM1,       -1
                         *111
0111   0000   COUNT,     0
                         *113
0113   0013   U1,        13
0114   0000              0
0115   0000              0
                         *121
0121   0013   Y1,        13
0122   0000              0
0123   0000              0
                         *127
0127   0013   Z1,        13
0130   0000              0
0131   0000              0
                         *150
0150   0003   K3,        3
0151   3400   VAC,       3400
0152   3430   MAC,       3430
0153   3730   MREG,      3730
0154   4230   P,         4230
0155   4530   PHI,       4530
0156   4560   BETA,      4560
0157   4610   GAMMA,     4610
0160   4640   FREG,      4640
```

```
0161   4643   VREG,    4643
0162   0013   SUMUI,   13
0163   0000            0
0164   0000            0
0165   0000   ARG1,    0
0166   0000   ARG2,    0
0167   0000   ADI,     0
0170   0000   KOUNT,   0
0171   7770   KM8,     -10
0172   0573   YIMI,    573
0173   2151   OPTI,    2151
0174   2107   PUNCH,   2107
0175   2145   BUFFER,  2145
0176   7750   KM24,    -30
0177   3777   K3777,   3777
                       *701
0701   4420            JMS I XCLEAR
                       *1400
1400   4421            JMS I XIDENT
                       *1450
1450   4652            JMS I XSAT
1451   7410            SKP
1452   2350   XSAT,    SAT
1453   7000            NOP
1454   7000            NOP
                       /
                       /MATRIX AND VECTOR INITIALIZATION
                       /
                       *1710
1710   0000   CLEAR,   0
1711   7300            CLA CLL
1712   3163            DCA SUMUI+1
1713   3164            DCA SUMUI+2
1714   1151            TAD VAC
1715   1137            TAD K11
1716   3010            DCA 10
1717   1353            TAD M1273
1720   3111            DCA COUNT
1721   3410            DCA I 10
1722   2111            ISZ COUNT
1723   5321            JMP.-2
1724   1171            TAD KM8
1725   3111            DCA COUNT
1726   1154            TAD P
1727   3165            DCA ARG1
1730   4407   R0,      JMS I 7
1731   5352            FGET KSTRT
1732   6565            FPUT I ARG1
1733   0000            FEXT
1734   7300            CLA CLL
1735   1165            TAD ARG1
1736   1355            TAD T27
1737   3165            DCA ARG1
1740   2111            ISZ COUNT
```

```
1741  5330            JMP R0
1742  4437            JMS I 7
1743  5352            FGET XSTRT
1744  5756            FPUT I XBETA5
1745  0000            FEXT
1746  7300            CLA CLL
1747  5710            JMP I CLEAR
1750  6505  M1273,    -1273
1751  3661  YADDI,    ADDI
1752  0001  XSTRT,    1
1753  2000            2000
1754  0000            0
1755  0033  T27,      33
1756  4574  XBETA5,   4574
                      *2003
2003  1163            TAD SUMUI+1
                      *2020
2020  1163            TAD SUMUI+1
                      *2037
2037  4402            JMS I XPCHB
                      /
                      /CONTROL VARIABLE SATURATION HANDLER
                      /
                      *2350
2350  0000  SAT,      0
2351  7300            CLA CLL
2352  1114            TAD UI+1
2353  7510            SPA
2354  5370            JMP NEGU
2355  1163            TAD SUMUI+1
2356  7500            SMA
2357  5366            JMP NEGU-2
2360  7300            CLA CLL
2361  1163            TAD SUMUI+1
2362  7041            CIA
2363  1177            TAD K3777
2364  3114            DCA UI+1
2365  1177            TAD K3777
2366  3163            DCA SUMUI+1
2367  5750            JMP I SAT
2370  1163  NEGU,     TAD SUMUI+1
2371  7500            SMA
2372  5366            JMP NEGU-2
2373  7300            CLA CLL
2374  1163            TAD SUMUI+1
2375  7041            CIA
2376  3114            DCA UI+1
2377  5366            JMP NEGU-2
```

```
                         /
                         /IDENTIFICATION ALGORITHM
                         /UPDATE BETA
                         /
                         *2400
2400   0000   IDENT,     0          /ENTRY FROM CONTROL PROGRAM
2401   4761              JMS I XVXV
2402   0157              GAMMA
2403   0155              PHI
2404   4762              JMS I XADDI
2405   0152              MAC
2406   4763              JMS I XMXM
2407   0152              MAC
2410   0154              P
2411   4317              JMS MTXFR
2412   0153              MREG
2413   0154              P
2414   4757              JMS I XGPHI
2415   4764              JMS I XMXV
2416   0154              P
2417   0155              PHI
2420   4765              JMS I XVDTV
2421   0155              PHI
2422   0151              VAC
2423   4407              JMS I 7
2424   5560              FGET I FREG
2425   1372              FADD ALPHA
2426   6560              FPUT I FREG
2427   4560              FDIV I FREG
2430   4560              FDIV I FREG
2431   6560              FPUT I FREG
2432   0000              FEXT
2433   4766              JMS I XVXS
2434   0155              PHI
2435   0160              FREG
2436   4301              JMS VTXFR
2437   0151              VAC
2440   0161              VREG
2441   4764              JMS I XMXV
2442   0154              P
2443   0161              VREG
2444   4301              JMS VTXFR
2445   0151              VAC
2446   0157              GAMMA
2447   4760              JMS I XVNEG
2450   0155              PHI
2451   4765              JMS I XVDTV
2452   0155              PHI
2453   0156              BETA
2454   4407              JMS I 7
2455   5127              FGET Z1
2456   7000              FNOR
2457   6375              FPUT REG
2460   5121              FGET Y1
```

```
2461  7000            FNOR
2462  2375            FSUB DEG
2463  1560            FADD I FREG
2464  6560            FPUT I FREG
2465  0000            FEXT
2466  4766            JMS I XVXS
2467  0157            GAMMA
2470  0160            FREG
2471  4767            JMS I XVPLSV
2472  0156            BETA
2473  0151            VAC
2474  4301            JMS VTXFR
2475  0151            VAC
2476  0156            BETA
2477  4770            JMS I XPBETA     /COMPUTE MIN VARIANCE CONTROL
2500  5600            JMP I IDENT      /RETURN TO CONTROL PROGRAM
                      /
                      //INPUT-OUTPUT ROUTINES
                      /MATRIX ARITHMETIC ROUTINES
                      /
2501  0000  VTXFR,    0          /VECTOR TRANSFER ROUTINE
2502  7300            CLA CLL
2503  1701            TAD I VTXFR
2504  4023            JMS INDIR
2505  3165            DCA ARG1
2506  2301            ISZ VTXFR
2507  1701            TAD I VTXFR
2510  4023            JMS INDIR
2511  3166            DCA ARG2
2512  2301            ISZ VTXFR
2513  1171            TAD K18
2514  3111            DCA COUNT
2515  4335            JMS TXFR
2516  5701            JMP I VTXFR
2517  0000  MTXFR,    0          /MATRIX TRANSFER ROUTINE
2520  7300            CLA CLL
2521  1717            TAD I MTXFR
2522  4023            JMS INDIR
2523  3165            DCA ARG1
2524  2317            ISZ MTXFR
2525  1717            TAD I MTXFR
2526  4023            JMS INDIR
2527  3166            DCA ARG2
2530  2317            ISZ MTXFR
2531  1371            TAD K164
2532  3111            DCA COUNT
2533  4335            JMS TXFR
2534  5717            JMP I MTXFR
2535  0000  TXFR,     0
2536  4407            JMS I 7
2537  5565            FGET I ARG1
2540  6566            FPUT I ARG2
2541  0000            FEXT
2542  4346            JMS ARG
```

```
2543  2111              ISZ COUNT
2544  5336              JMP TXFR+1
2545  5735              JMP I TXFR
2546  0000   ARG,       0
2547  7300              CLA CLL
2550  1165              TAD ARG1
2551  1150              TAD K3
2552  3165              DCA ARG1
2553  1166              TAD ARG2
2554  1150              TAD K3
2555  3166              DCA ARG2
2556  5746              JMP I ARG
2557  2635   XGPHI,     GPHI
2560  2705   XVNEG,     VNEG
2561  3027   XVXV,      VXV
2562  2661   XADDI,     ADDI
2563  3200   XMXM,      MXM
2564  3237   XMXV,      MXV
2565  3117   XVDTV,     VDTV
2566  3000   XVXS,      VXS
2567  3265   XVPLSV,    VPLSV
2570  2600   XPRETA,    PRETA
2571  7700   M100,      -100
2572  0001   ALPHA,     1
2573  2000              2000
2574  0000              0
2575  0000   REG,       0
2576  0000              0
2577  0000              0
                        /
                        /COMPUTE MINIMUM VARIANCE CONTROL LAW
                        /
                        *2600
2600  0000   PRETA,     0
2601  4407              JMS I 7
2602  5556              FGET I BETA1
2603  4755              FDIV I BETA5
2604  6764              FPUT I B0
2605  5752              FGET I BETA2
2606  4755              FDIV I BETA5
2607  6765              FPUT I B1
2610  5753              FGET I BETA3
2611  4755              FDIV I BETA5
2612  6766              FPUT I B2
2613  5754              FGET I BETA4
2614  4755              FDIV I BETA5
2615  6767              FPUT I B3
2616  5774              FGET I XZERO
2617  2756              FSUB I BETA6
2620  4755              FDIV I BETA5
2621  6761              FPUT I A1
2622  5774              FGET I XZERO
2623  2757              FSUB I BETA7
2624  4755              FDIV I BETA5
```

```
2625  6762          FPUT I A2
2626  5774          FGET I XZERO
2627  2760          FSUB I BETA8
2630  4755          FDIV I BETA5
2631  6763          FPUT I A3
2632  0000          FEXT
2633  7300          CLA CLL
2634  5600          JMP I PBETA
2635  0000  GPHI,   0        /GET VECTOR PHI
2636  4770          JMS I XVTXFR
2637  0172          YIM1
2640  0155          PHI
2641  4305          JMS VNEG
2642  0155          PHI
2643  1102          TAD UIM1
2644  3165          DCA ARG1
2645  1371          TAD PHI5
2646  3166          DCA ARG2
2647  1372          TAD K14
2650  3111          DCA COUNT
2651  4407  R12,    JMS I 7
2652  5565          FGET I ARG1
2653  6566          FPUT I ARG2
2654  0000          FEXT
2655  4773          JMS I XARG
2656  2111          ISZ COUNT
2657  5251          JMP R12
2660  5635          JMP I GPHI
2661  0000  ADDI,   0        /ADD IDENTITY MATRIX
2662  7300          CLA CLL
2663  1661          TAD I ADDI
2664  4023          JMS INDIR
2665  3165          DCA ARG1
2666  2261          ISZ ADDI
2667  1171          TAD K18
2670  3111          DCA COUNT
2671  4407  R9,     JMS I 7
2672  5565          FGET I ARG1
2673  1775          FADD I XONE
2674  6565          FPUT I ARG1
2675  0000          FEXT
2676  7200          CLA
2677  1165          TAD ARG1
2700  1376          TAD K27
2701  3165          DCA ARG1
2702  2111          ISZ COUNT
2703  5271          JMP R9
2704  5661          JMP I ADDI
2705  0000  VNEG,   0        /MULTIPLY VECTOR BY -1
2706  7300          CLA CLL
2707  1705          TAD I VNEG
2710  4023          JMS INDIR
2711  3165          DCA ARG1
2712  2305          ISZ VNEG
```

```
2713  1171            TAD KM8
2714  3111            DCA COUNT
2715  3044  R3,       DCA 44
2716  3045            DCA 45
2717  3046            DCA 46
2720  4407            JMS I 7
2721  2565            FSUB I ARG1
2722  6565            FPUT I ARG1
2723  0000            FEXT
2724  4773            JMS I XARG
2725  2111            ISZ COUNT
2726  5315            JMP R3
2727  5775            JMP I XNEG
2730  0000  PCHB,     0          /PUNCH PARAMETER ESTIMATES ON PAPER TAPE
2731  1573            TAD I OPT1
2732  7450            SNA
2733  5730            JMP I PCHB
2734  7200            CLA
2735  1176            TAD KM24
2736  3111            DCA COUNT
2737  1156            TAD BETA
2740  1107            TAD KM1
2741  3010            DCA 10
2742  1410            TAD I 10
2743  3575            DCA I BUFFER
2744  4574            JMS I PUNCH
2745  2111            ISZ COUNT
2746  5342            JMP .-4
2747  6026            PLS
2750  5350            JMP .
2751  5730            JMP I PCHB
2752  4563  BETA2,    4563
2753  4566  BETA3,    4566
2754  4571  BETA4,    4571
2755  4574  BETA5,    4574
2756  4577  BETA6,    4577
2757  4602  BETA7,    4602
2760  4605  BETA8,    4605
2761  0400  A1,       400
2762  0403  A2,       403
2763  0406  A3,       406
2764  0414  B0,       414
2765  0417  B1,       417
2766  0422  B2,       422
2767  0425  B3,       425
2770  2501  XVTXFR,   VTXFR
2771  4544  PHI5,     4544
2772  7774  KM4,      -4
2773  2546  XARG,     ARG
2774  3166  XZERO,    ZERO
2775  3163  KONE,     ONE
2776  0033  K27,      33
```

```
                        *3000
3000  0000  VXS,        0          /MULTIPLY VECTOR BY SCALAR
3001  7300              CLA CLL
3002  1600              TAD I VXS
3003  4023              JMS INDIR
3004  3165              DCA ARG1
3005  2200              ISZ VXS
3006  1600              TAD I VXS
3007  4023              JMS INDIR
3010  3167              DCA AD1
3011  2200              ISZ VXS
3012  1151              TAD VAC
3013  3166              DCA ARG2
3014  1171              TAD KM18
3015  3111              DCA COUNT
3016  4407  R6,         JMS I 7
3017  5565              FGET I ARG1
3020  3567              FMPY I AD1
3021  6566              FPUT I ARG2
3022  0000              FEXT
3023  4753              JMS I YARG
3024  2111              ISZ COUNT
3025  5216              JMP R6
3026  5600              JMP I VXS
3027  0000  VXV,        0          /VECTOR CROSS PRODUCT
3030  7300              CLA CLL
3031  1627              TAD I VXV
3032  4023              JMS INDIR
3033  3167              DCA AD1
3034  2227              ISZ VXV
3035  1171              TAD KM18
3036  3170              DCA KOUNT
3037  1152              TAD MAC
3040  3166              DCA ARG2
3041  1627  R8,         TAD I VXV
3042  4023              JMS INDIR
3043  3165              DCA ARG1
3044  1171              TAD KM18
3045  3111              DCA COUNT
3046  4407  R7,         JMS I 7
3047  5567              FGET I AD1
3050  3565              FMPY I ARG1
3051  6566              FPUT I ARG2
3052  0000              FEXT
3053  4753              JMS I YARG
3054  2111              ISZ COUNT
3055  5246              JMP R7
3056  1167              TAD AD1
3057  1150              TAD K3
3060  3167              DCA AD1
3061  2170              ISZ KOUNT
3062  5241              JMP R8
3063  2227              ISZ VXV
3064  5627              JMP I VXV
```

```
3065  0000  VPLSV,  0           /ADD TWO VECTORS
3066  7300          CLA CLL
3067  1665          TAD I VPLSV
3070  4023          JMS INDIR
3071  3165          DCA ARG1
3072  2265          ISZ VPLSV
3073  1665          TAD I VPLSV
3074  4023          JMS INDIR
3075  3166          DCA ARG2
3076  2265          ISZ VPLSV
3077  1174          TAD KM18
3100  3111          DCA COUNT
3101  1151          TAD VAC
3102  3167          DCA AD1
3103  4407  R5,     JMS I 7
3104  5565          FGET I ARG1
3105  1566          FADD I ARG2
3106  6567          FPUT I AD1
3107  0000          FEXT
3110  4753          JMS I YARG
3111  1167          TAD AD1
3112  1150          TAD K3
3113  3167          DCA AD1
3114  2111          ISZ COUNT
3115  5303          JMP R5
3116  5665          JMP I VPLSV
3117  0000  VDTV,   0           /VECTOR DOT PRODUCT
3120  7300          CLA CLL
3121  1717          TAD I VDTV
3122  4023          JMS INDIR
3123  3165          DCA ARG1
3124  2317          ISZ VDTV
3125  1717          TAD I VDTV
3126  4023          JMS INDIR
3127  3166          DCA ARG2
3130  1171          TAD KM18
3131  3111          DCA COUNT
3132  3044          DCA 44
3133  3045          DCA 45
3134  3046          DCA 46
3135  4407          JMS I 7
3136  6560          FPUT I FREG
3137  0000          FEXT
3140  4407  R4,     JMS I 7
3141  5565          FGET I ARG1
3142  3566          FMPY I ARG2
3143  1560          FADD I FREG
3144  6560          FPUT I FREG
3145  0000          FEXT
3146  4753          JMS I YARG
3147  2111          ISZ COUNT
3150  5340          JMP R4
3151  2317          ISZ VDTV
3152  5717          JMP I VDTV
```

```
3153  2546  YARG,   ARG
3154  0000  THREE,  0
3155  3362          DCA TEMP
3156  1362          TAD TEMP
3157  7104          CLL RAL
3160  1362          TAD TEMP
3161  5754          JMP I THREE
3162  0000  TEMP,   0
3163  0001  ONE,    1
3164  2000          2000
3165  0000          0
3166  0000  ZERO,   0
3167  0000          0
3170  0000          0
                    *3200
3200  0000  MX1,    0        /MATRIX MULTIPLICATION
3201  7300          CLA CLL
3202  1600          TAD I MX1
3203  4023          JMS INDIR
3204  3370          DCA ARG5
3205  2200          ISZ MX1
3206  1600          TAD I MX1
3207  4023          JMS INDIR
3210  3371          DCA ARG6
3211  2200          ISZ MX1
3212  1171          TAD K13
3213  3360          DCA KCOUNT
3214  1153          TAD YARG
3215  3372          DCA ARG7
3216  4276          JMS MTRANS
3217  3371          ARG6
3220  4237  R2,     JMS MXV
3221  3370          ARG5
3222  3371          ARG6
3223  4751          JMS I YVTXFR
3224  3151          VAC
3225  3372          ARG7
3226  1371          TAD ARG6
3227  1362          TAD K24
3230  3371          DCA ARG6
3231  1372          TAD ARG7
3232  1362          TAD K24
3233  3372          DCA ARG7
3234  2360          ISZ KCOUNT
3235  5220          JMP R2
3236  5600          JMP I MX1
3237  0000  MXV,    0        /MATRIX-VECTOR MULTIPLICATION
3240  7300          CLA CLL
3241  1637          TAD I MXV
3242  4023          JMS INDIR
3243  3366          DCA ARG3
3244  2237          ISZ MXV
3245  1637          TAD I MXV
3246  4023          JMS INDIR
```

```
3247  3367            DCA ARG4
3250  1151            TAD VAC
3251  3167            DCA AD1
3252  2237            ISZ MXV
3253  1171            TAD KM18
3254  3170            DCA KOUNT
3255  4764   R1,      JMS I YVDTV
3256  3366            ARG3
3257  3367            ARG4
3260  4407            JMS I 7
3261  5560            FGET I FREG
3262  6567            FPUT I AD1
3263  0000            FEXT
3264  7200            CLA
3265  1366            TAD ARG3
3266  1362            TAD K24
3267  3366            DCA ARG3
3270  1167            TAD AD1
3271  1150            TAD K3
3272  3167            DCA AD1
3273  2170            ISZ KOUNT
3274  5255            JMP R1
3275  5637            JMP I MXV
3276  0000   MTRANS,  0          /MATRIX TRANSPOSITION
3277  7300            CLA CLL
3300  1676            TAD I MTRANS
3301  4023            JMS MDIR
3302  3167            DCA AD1
3303  2276            ISZ MTRANS
3304  1171            TAD KM18
3305  3111            DCA COUNT
3306  3356            DCA II
3307  3357   R11,     DCA JJ
3310  1171            TAD KM18
3311  3170            DCA KOUNT
3312  1356   R10,     TAD II
3313  7041            CIA
3314  1357            TAD JJ
3315  7550            SPA SNA
3316  5346            JMP MS
3317  7300            CLA CLL
3320  1356            TAD II
3321  7006            RTL
3322  7004            RAL
3323  1357            TAD JJ
3324  4765            JMS I X3
3325  1167            TAD AD1
3326  3165            DCA ARG1
3327  1357            TAD JJ
3330  7006            RTL
3331  7004            RAL
3332  1356            TAD II
3333  4765            JMS I X3
3334  1167            TAD AD1
```

```
3335  3166           DCA ARG2
3336  4407           JMS I 7
3337  5565           FGET I ARG1
3340  6560           FPUT I FREG
3341  5566           FGET I ARG2
3342  6565           FPUT I ARG1
3343  5560           FGET I FREG
3344  6566           FPUT I ARG2
3345  0000           FEXT
3346  7200  MIS,     CLA
3347  2357           ISZ JJ
3350  2170           ISZ KOUNT
3351  5312           JMP R10
3352  2356           ISZ II
3353  2111           ISZ COUNT
3354  5307           JMP R11
3355  5676           JMP I MTRANS
3356  0000  II,      0
3357  0000  JJ,      0
3360  0000  KCOUNT,  0
3361  2501  XMTXFR,  MTXFR
3362  0030  X24,     30
3363  2517  XNTXFR,  NTXFR
3364  3117  XVDTV,   VDTV
3365  3154  X3,      THREE
3366  0000  ARG3,    0
3367  0000  ARG4,    0
3370  0000  ARG5,    0
3371  0000  ARG6,    0
3372  0000  ARG7,    0
```