Failure of laminated composite pinned connections

by

Mahmood M. Shokrieh

Department of Mechanical Engineering
McGill University
Montreal, Canada.

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the Degree of Master of Engineering

July, 1991

Abstract

In this investigation the behavior of pin-loaded composite plates is studied analytically. A progressive damage model is presented which is capable of predicting the three different mechanisms of failure: bearing, shearout, and net tension. The model consists of three major parts: stress analysis, failure analysis, and material property degradation rules.

Based on the model a computer code is developed. The computer code is capable of assessing damage, evaluating residual strength, and predicting ultimate strength of pin-loaded composite plates. Predicted results are compared with available experimental data. Excellent agreement between the predicted and the experimental data was found.

The computer code is used to study geometric parameters that influence joint strength. Such studies are useful in designing mechanical fastened joints using advanced composites.

Résumé

Cette recherche fait l'object de l'analyse mathématique des plaques de matériaux composites ayant des goupilles. Une méthode de modelage du dommage progressif causé par ces goupilles est présentée. Cette méthode prédit les trois différents mécanismes de rupture pouvant être observés: rupture par flambage, cisaillement et tension. Le modèle comprend trois parties: analyse des contraintes, analyse de la rupture et dégradation des propriétés du matériel.

Un logiciel d'ordinateur est développé, basé sur ce modèle. Le logiciel détermine les ruptures locales, évalue la résistance résiduelle et prédit la résistance ultime des plaques composites. Les résultats prédits par ce modèle se compare de façon excellente aux données expérimentales.

Le logiciel est utilisé afin d'étudier les paramètres géométriques qui peuvent influencer la résistance des joints. Une telle étude est utile pour prédire le comportement des matériaux composites lorsque des joints mécaniques sont utilisés.

Acknowledgments

The author wishes to express his deepest sincere gratitude and appreciation to Dr. Larry B. Lessard, for his guidance, insight, and support during the investigation.

The author is indebted to his wife and child, for their patience and support. The author appreciates them for their comprehension and tolerance.

To the memory of my father

Contents

List of Figures

List of Tables

List of Symbols

Symbol	Description
D	Hole Diameter
E	Edge Distance
E_x	Longitudinal modulus
E_y	Transverse modulus
E_s	Shear modulus
e_{M+}	Matrix tension failure mode parameter
e_{M-}	Matrix compression failure mode parameter
e_{F+}	Fiber tension failure mode parameter
e_{F-}	Fiber compression failure mode parameter
e_{FM-}	Fiber-matrix shearing failure mode parameter
L	Length of the plate
P	Applied load
$[Q]_{x-y}$	On-axis modulus of unidirectional composites
$[Q]_{1-2}$	Off-axis modulus of unidirectional composites
t	Thickness of the plate
$[T]_\varepsilon$	Strain transformation matrix
$[T]_\sigma$	Stress transformation matrix

W	Width of the plate
X_t	Longitudinal tensile strength
X_c	Longitudinal compressive strength
Y_t	Transverse tensile strength
Y_c	Transverse compressive strength
S	Shear strength
ε_i	On-axis strains (i=x,y,s)
ε_j	Off-axis strains (j=1,2,6)
θ	Measuring angle around hole
ν_x	Longitudinal Poisson's ratio
ν_y	Transverse Poisson's ratio
σ_i	On-axis stresses (i=x,y,s)
σ_j	Off-axis stresses (j=1,2,6)
ϕ	Angle of ply

Chapter 1

Introduction

Increasing application of composite materials in highly stressed light weight constructions necessitates more accurate knowledge of mechanical behavior of such materials. Mechanically fastened joints are used for joining composite material components, hence access to a model for predicting the strength of such joints is very important for optimum design.

There are numerous studies on the behavior of pin-loaded composite plates. These works can be categorized into two basic groups, stress analyses and failure analyses. In the first category, no attempt is made to study strength of joints, and only the stresses around hole are studied. In the second category, the strength of pin-loaded joints is studied.

1.1 A Review of Stress Analysis of Pinned and Bolted Joints

There are numerous investigations related to stress analysis of pinned or bolted joints in composite materials. However, it must be mentioned that, there is no survey paper for stress analysis of pin-loaded composite plates.

The investigations can be categorized into four groups, analytical, experimental, numerical, and combined numerical-experimental techniques. It must be noted that no attempt was made in these investigations for failure analysis of pinned or bolted joint composite plates.

Analytical Techniques

In this category, investigators used analytical techniques for stress analysis of pin-joint composite plates.

De Jong [1] investigated the stress distribution around a pin-loaded hole in an elasticity orthotropic or isotropic plate. De Jong [2] calculated stresses for infinite orthotropoic plates with a circular hole loaded by a perfectly fitting rigid pin with arbitrary load direction. He also evaluated the effect of friction at the interface between pin and plate material. Zhang and Ueng [3] obtained a compact analytical solution for stresses around a pin-loaded hole in an orthotropic plate by complex stress functions which satisfied the displacement boundary conditions along the hole. Mangalgiri [4] developed a method for partial contact of pin-loaded holes in composites. Hyer and Klang [5] studied the effects of pin elasticity, friction, and clearance on the stresses near the hole in a pin-loaded orthotropic plate. They showed that pin elasticity was not as important as clearance, friction, or the elastic properties of the plate in determining contact stresses. Mangalgiri and Dattaguru [6] developed a method for solving contact problems for an orthotropic plate with a smooth misfit pin under arbitrary orientated biaxial loading. Hyer *et al.* [7] studied the effects of pin elasticity, clearance, and friction on the stresses around the hole edge in a pin-loaded orthotropic plate.

Experimental Techniques

In this category, investigators used experimental techniques for stress analysis of pinned and bolted joint composite plates.

Prabhakaran [8] applied the methods of photo-orthotropic elasticity to the study of bolted joints in composites. Hyer and Liu [9] discussed the design, fabrication, and testing of photoelastic models of double-lap, multiple-pin connectors. Hyer and Liu [10] determined the stresses around the hole in pin-loaded fiber-reinforced glass-epoxy plates by using transmission photoelasticity. Hyer and Liu [11] experimentally determined the stresses in transparent glass-epoxy plates loaded by a steel pin through a hole. Prabhakaran and Naik [12] described a fiber-optic technique for measuring the angle of contact in a clearance-fit bolt-loaded hole. Also they [13] studied the influence of interfacial friction on the contact angle by using a fiber optic technique.

Numerical Techniques

In this category, investigators used numerical techniques for stress analysis of pin-joint composite plates.

Rao [14] presented an elastic analysis of pin joints. Wong and Matthews [15] presented the results of a two-dimensional finite element analysis of bolted joints in fiber reinforced plastic. Crews *et al.* [16] calculated stresses for finite-size orthotropic laminates loaded by a frictionless steel pin in a circular hole of the same diameter. Wilkinson and Rowlands [17] developed an iterative finite-element method for determining stresses and strains near pin-loaded holes in finite orthotropic plates. They studied, friction, and clearance between rigid pin and hole. Matthews *et al.* [18] presented a three-dimensional finite element analysis of bolt- and pin-loaded fiber-reinforced laminates. Rahman *et al.* [19] described an iterative finite element technique for solving frictional contact problems. Mangalgiri *et al.* [20] developed a simple finite element technique to handle the moving contact problem at the pin-hole interface in a fastener. Naik and Crews [21] developed a simple method for the stress analysis of a clearance-fit bolt under bearing loads. They used a finite element method with an inverse formulation. Ericksson [22] calculated contact stresses and stresses in the vicinity of the hole boundary with account taken of the contact problem. He also studied the effects of laminate elastic properties, clearance, friction, load magnitude, and bolt stiffness. Tsujimoto and Wilson [23] investigated an elasto-plastic finite element analysis of pin-loaded joints in laminated composites. Yogeswaren and Reddy [24] used a mixed finite element scheme with a dynamic as well as static coefficient of friction in the evaluation of contact stresses in pin-loaded plates. Marshall *et al.* [25] performed a three dimensional finite element analysis of pin-loaded and bolted holes in composite laminates. Ramamurthy [26] presented a stress analysis of pin loaded lugs accounting for proper interface conditions in a misfit case. He also [27] studied the behavior of the interference fit pins in a composite plate subjected to both pull and push type of loads.

Combined Numerical-Experimental Techniques

In this category, investigators used numerical and experimental techniques for stress analysis of pin-joint composite plates.

Wilkinson *et al.* [28] determined stresses and strains associated with single-fastener mechanical joints in wood, numerically and experimentally. Rowlands *et al.* [29] analyzed stresses of single- and double-bolted mechanical fasteners in orthotropic materials numerically and experimentally. They studied the effects of variations in friction, material properties, load distribution among the bolts, end distance, bolt clearance, and bolt spacing. Hyer and Liu [30] used birefringent glass-epoxy and a numerical stress-separation scheme to compute the stresses in the vicinity of a pin-loaded hole.

1.2 A Review of Failure Analysis of Pinned and Bolted Joints

Also there are numerous investigations related to failure analysis of pinned joints in composite materials. There are incomplete surveys [31-34] of some investigations on failure analysis of pin-loaded composite plates.

For failure analysis of pin-loaded composite plates, investigators used different methods. Experimental, analytical, numerical, combined analytical-experimental, and combined numerical-experimental methods have been used by authors.

Experimental Methods

In this category, investigators used experimental techniques for failure analysis of pin-joint composite plates.

Stockdale and Matthews [35] showed experimentally that, in GRP [1] with a 0/90° lay-up, the clamping effect of the bolt prevents the delamination with consequent increase in failure load. Quinn and Matthews [36] measured the pin-bearing strength of glass fiber reinforced plastic plates. Matthews and Hirst [37] measured the bolt bearing strength of single-hole specimens in CFRP [2], GFRP[3], and GRP as the direction of bolt load was changed. Johnson and Matthews [38] indicated experimentally that, for the fiber reinforced plastics, significant damage occurs when the hole elongates by about 0.4 % of the original diameter. Hyer and lightfoot [39] presented the experimental result of a series tests designed to determined the load carrying capacity of composite bolted joints as a function of joint width, bolt diameter, joint thickness, and the number of bolts. Godwin and Matthews [40] reviewed published work, mostly experimental, relating to all aspects of screwed, riveted and bolt joints in glass and carbon fiber-reinforced epoxy and polyester resin. Tang [41] proposed a tension failure factor to determine the failure mechanisms of composite laminates subjected to combined tension and bolt load. Wichorek [42] presented the results of an experimental program to determine the bolted-joint strength and failure modes of graphite/polyimide laminates. Matthews *et al.* [43] showed experimentally that, for single bolt joints in 0/90° hybrid laminates, the bearing behavior is similar to that of other types of fiber reinforced plastic. Collings [44] derived equations for predicting

[1] Glass fiber reinforced polyester

[2] Carbon fiber reinforced plastics

[3] Glass fiber reinforced plastics

the ultimate bearing strengths of constrained pin-loaded holes using a semi-empirical approach. Godwin *et al.* [45] presented the results of an experimental study of multi-bolt joints in GRP. Collings and Beauchamp [46] described the bearing deflection behavior of a loaded, torque-tightened bolt in CFRP laminates. Kretsis and Matthews [47] carried out tests on single-hole bolted joints in a variety of lay-ups with two resin systems. Theuer and Arendts [48] studied bolt bearing strength and notch sensitivity of carbon fiber reinforced carbon. Akay [49] examined pin bearing behavior over a range of laminate system consisting of unidirectional and woven carbon fiber reinforced epoxy matrices under static and dynamic loading. Eriksson [50] presented the results of an experimental program which measured the bearing strengths of two different types of graphite/epoxy specimens. He also outlined a new approach for predicting bearing failure.

Analytical Methods

In this category, investigators used analytical techniques for failure analysis of pin-joint composite plates.

Ueng and Zhang [51] used a compact analytical solution for computing the stresses along a characteristic curve. They used the Yamada-Sun failure criterion to evaluate the ultimate load and the failure mode. Smith *et al.* [52] presented a simple three dimensional approach for predicting bearing stress at failure in a composite bolted joint.

Numerical Methods

In this category, investigators used numerical techniques for failure analysis of pin-joint composite plates.

Waszczak and Cruse [53] used a finite element approach and a distorsional energy failure criterion for predicting failure mechanism and ultimate load. Humphris [54] investigated the strength of a laminated composite lug structure by using a finite element method. Agarwal [55] used a two dimensional finite element method and average stress criterion for predicting the various mechanisms of failure. Soni [56] used two dimensional finite element method and tensor polynomial failure criterion for predicting the strength of pin-loaded composite laminates. Chang *et al.* [57] used two dimensional linear elastic finite element method and Yamada's failure criterion for predicting the failure strength and failure mechanisms of mechanically fastened fiber reinforced composite laminates. Chang *et al.* [58, 59] extended an existing model to laminates containing two or more pin-loaded holes. Chang *et al.* [60] presented a method for calculating the failure strength and failure mechanisms of composite laminates containing a pin-loaded hole for materials exhibiting nonlinearly elastic behavior. They considered net tension and shearout mechanisms, but a bearing mechanism was not considered in their model. Chang [61] performed an analysis to evaluate the effect of the assumed pin load distribution on the calculated strength and predicted the failure mechanism of pin loaded holes in laminated composites. Chang and Chang [62] developed a progressive damage model for bolted joints in laminated composites. Again they considered only net tension and shearout mechanisms. Arnold *et al.* [63] used a two dimensional finite element technique and an extension of the Whitney-Nuismer point stress approach to predict the strength of pinned and bolted composite joints. Lessard [64] performed an analytical investigation to study the damage in a composite plate containing a pinned joint and subjected to in-plane loading.

Analytical-Experimental Methods

In this category, investigators used analytical and experimental techniques for failure analysis of pin-joint composite plates.

Klang and De Jong [65] presented an elasticity solution and experimental results for a finite width joint with one connector. Oplinger [66] summarized the results of analytical and experimental studies on pinned or bolted joints. Hart-Smith [67] discussed the various factors affecting the strength of bolted or riveted joints in advanced composites. Smith and Pascoe [68] measured the pin-bearing strengths of different stacking sequences of quasi-isotropic CFRP laminates. Smith *et al.* [69] presented results for the strengths of single-lap bolted joints as functions of width and edge distance. Naik and Crews [70] presented a combined experimental and analytical study conducted to investigate and predict the damage-onset failure mechanisms of a graphite/epoxy laminate subjected to combined bearing and bypass loading.

Numerical-experimental Methods

In this category, investigators used numerical and experimental techniques for failure analysis of pin-joint composite plates.

Tsiang and Mandell [71] investigated the buildup of damage in bolt-loaded composite laminates. Serabian and Oplinger [72] studied pin-loaded composite plates by a linear and nonlinear elastic, orthotropic, plane stress finite element approximation, using laminate mechanical properties found from mechanical testing and the application of laminate plate theory for a 0/90 pin-loaded laminate. Crews and Naik [73] conducted a combined numerical and experimental study to determine the behavior of a graphite/epoxy laminate subjected to combined bearing and bypass loading. Conti [74] investigated the influence of geometric parameters on the stress field generated in a laminate with a pin-loaded hole. He also used Azzi-Tsai failure criterion for predicting the laminate strength. Jurf and Vinson [75] presented an investigation into the bolted joint strength of kevlar/epoxy and graphite/epoxy composite laminates. Tsai and Morton [76] investigated a quasi-isotropic graphite/epoxy plate loaded through an aluminum pin. They used a hybrid experimental/numerical analysis to determine the displacement and stress fields.

1.3 Objective

Progressive damage modelling is a qualitative method for studying the behavior of pin-loaded composite plates. This method has been used by investigators for studying the behavior of bolted joints in laminated composites which may fail in either shearout or net tension mechanisms [62]. But no model exists which consider the bearing mechanism.

The objective of this investigation is to study the behavior of pin-loaded composite plates. The response of the plates due to local damage as a result of stress concentrations is the primary concern. For this purpose a progressive damage model is developed. The model is capable of predicting the three different mechanisms of failure, bearing, shearout, and net tension.

Based on the model a computer code was developed. The computer code is capable of assessing damage, evaluating residual strength, and predicting ultimate strength of pin-loaded composite plates. Validation of the model and the subsequent computer code is examined by predicting some experimental results.

By using the computer code, the effects of ply orientation, type of loading, and geometric parameters on the strength and response of the plates are also studied.

Chapter 2

Description of the Problem

Consider a composite plate with a pin-loaded hole, i.e., the plate has a circular hole filled with a rigid pin. Load is applied at one end of the plate and is resisted by the rigid pin. The coordinate axis, dimensions, and nomenclature are shown in Fig. 2.1.

The plate is a laminated composite (Fig. 2.2) made of layers of continuous fibers embedded in an organic matrix. Each layer of the laminate is called a "ply" or "unidirectional layer". The ply orientation of laminate can be selected arbitrarily, but it must be symmetric [1] with respect to the midplane of the plate .

2.1 Explanation of the Process

The composite plate is loaded with an in-plane load "P" (Fig. 2.1). By increasing the load to a certain value, failure will start at a position near the edge of the hole. This load is called *first ply failure load.* If after failure initiation the load is increased, failure

[1] Ply orientation must be symmetric in order to prevent out-of-plane deflection.

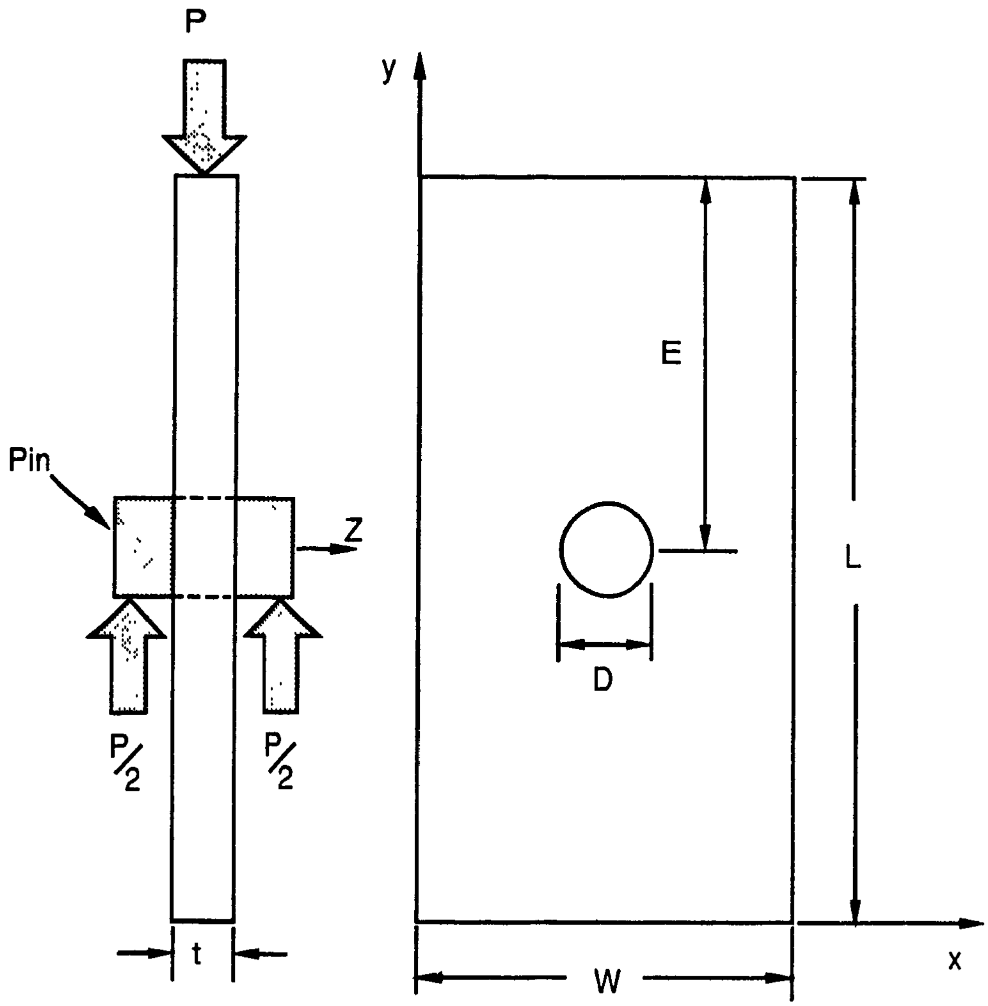


Figure 2.1: Description of the problem.

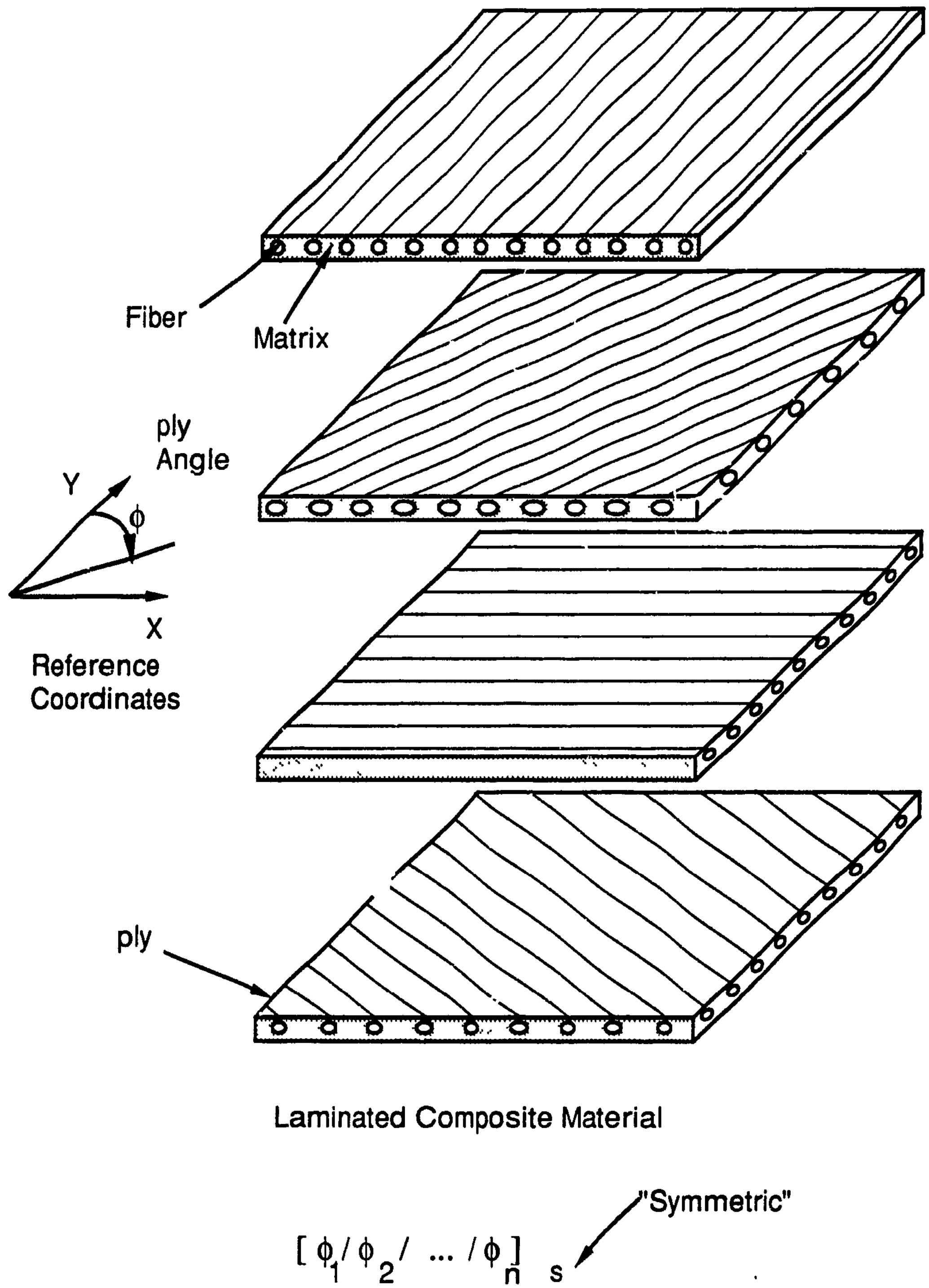


Figure 2.2: Sketch of a laminated composite.

will propagate in different directions. Finally at higher load, damage will propagate to an extent that the plate can not tolerate any additional load. This value is called *ultimate strength*. It has been observed experimentally that mechanically fastened joints fail under three basic mechanisms. These mechanisms are *net tension*, *shearout*, and *bearing*. Typical damage due to each mechanism is shown in Fig. 2.3. The magnitude of the first ply failure load, the position of the initial failure, the direction of the failure propagation (or mechanism of failure), and the ultimate strength depend upon the material properties, dimensions, laminate configurations, and many other parameters.

2.2 Requirements

In this research it is desired to develop a *progressive damage model*[2] for finding:

- First ply failure load.
- Direction of failure propagation (failure mechanisms).
- Residual strength.
- Ultimate strength.

in a pin-loaded composite plate.

In order to establish such a model, a certain procedure must be followed. First, the pin-loaded composite plate stresses must be analyzed, and in this research the

[2] A model that is capable of assessing damage with arbitrary ply orientations and of predicting the ultimate tensile strength of the laminates. The model determines the state of propagation of damage at any load level, from failure initiation, to ultimate failure.

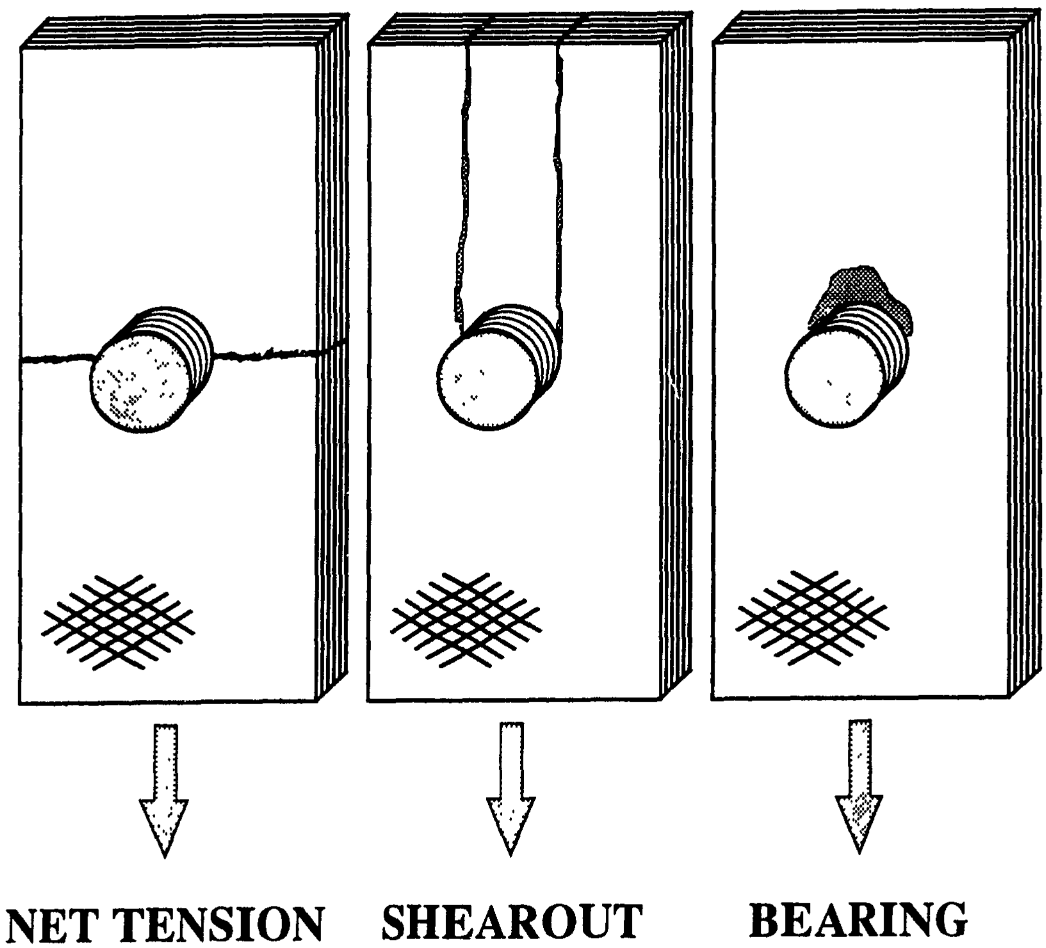


Figure 2.3: Different mechanisms of failure in pin-loaded composite plate.

finite element method [3] has been used. Afterwards, for predicting failure, the stresses must be checked by a set of failure criteria [4]. Then, on the condition that failure exists, material properties of the failed region must be changed and the whole process must be repeated until the plate can not tolerate any more load.

For this purpose a user-friendly computer code was developed. It has been called "PDPIN" [5]. This code can be used to analyze and design laminated composites containing pin-loaded hole. Details of the model and the computer code are explained in chapter 6.

2.3 Summary

A description of the problem is presented. It is desired to develop a progressive damage model for pin-loaded composite plates. This model must be capable of assessing damage, evaluating residual strength, and predicting ultimate strength of pin-loaded composite plates.

[3]Refer to chapter 3 for more details.

[4]Refer to chapter 4 for more details.

[5]Abbreviation of progressive damage modeling of pin-loaded composite plates.

Chapter 3

Stress Analysis

So far there is no exact closed form solution for a pin-loaded composite plate. Furthermore, after failure, due to changing of material properties of failed regions, closed form solutions are not valid. For this reason and because of the nature of *progressive damage modeling* [1], the finite element technique has been utilized for stress analysis in this research. The stress analysis was carried out using I-DEAS [77] software.

For analyzing the stresses in a pin-loaded composite plate, two possible choices are two dimensional and three dimensional stress analysis. Failure of a mechanically fastened joint is a three dimensional phenomenon. Edge effects, delamination, out-of-plane buckling, and stacking sequence are examples showing that for accurate analysis of a pin-loaded composite plate, three dimensional analysis is needed. But computational costs are a severe restriction for using three dimensional finite element analysis. On the other hand, two dimensional finite element analysis has been used by many authors, such as [15,51,55-62,64] successfully. Therefore by considering some

[1]For more details refer to chapter 6.

assumptions two dimensional stress analysis has been used in this research.

3.1 Assumptions

Two dimensional Stress Analysis

By considering the following restrictions, two dimensional stress analysis can be used successfully:

- The thickness of the laminate must be small compared with its length and width (plane stress condition).
- The applied load must be in-plane. [58]
- The applied load must be symmetric with respect to the mid-plane. [2] [58]
- Out-of plane displacement in the Z direction (Fig. 2.1) must be ignored (i.e. global or geometric buckling is not considered). [15]
- The influence of the stacking sequence must be ignored (e.g. the difference between $[0/\pm 45/90]_s$ and $[90/\pm 45/0]_s$ is not considered. It has 10 to 20 percent error in failure strength in some cases). [15, 36]

Friction Between Pin and Hole

The effect of friction between pin and hole on the stresses around the hole has been investigated by many authors [2,5,7,13,17,19,22,24,29]. This parameter is studied in this research but is not considered in the failure analysis. For more details refer to section 3.4.2.

[2] For preventing the creation of moment about the axes.

Clearance

The effect of clearance between the diameter of the pin and the hole on the stresses around the hole is investigated by [21, 22]. In this research it is assumed that there is no clearance between pin and hole.

Pin Elasticity

The effect of pin elasticity on the stresses around the hole is investigated by [5, 7]. It has been found that this parameter has not a great influence on the stresses around the hole. Therefore pin has been assumed rigid.

Material Properties

Material properties of the composite plate is assumed linearly elastic. It has been found [57, 58, 60] that this assumption provides reasonable results for laminates containing pin-loaded hole except for ply orientations $[\pm 45]_s$ and $[0/90]_s$, where differences up to 40% were noted.

3.2 Boundary Conditions

Because of symmetry with respect to the Y-axis (Fig. 3.1) half of the plate can be considered for analysis. Thus all of the nodes on the edges AB and CD are fixed in the X-axis direction and are free in the Y-axis direction.

For modeling the pin load there are two methods.

Cosine Load Distribution

In this method the pin load can be modeled by applying a cosine load distribution around the hole (arc BE) and applying displacement boundary conditions on the edge

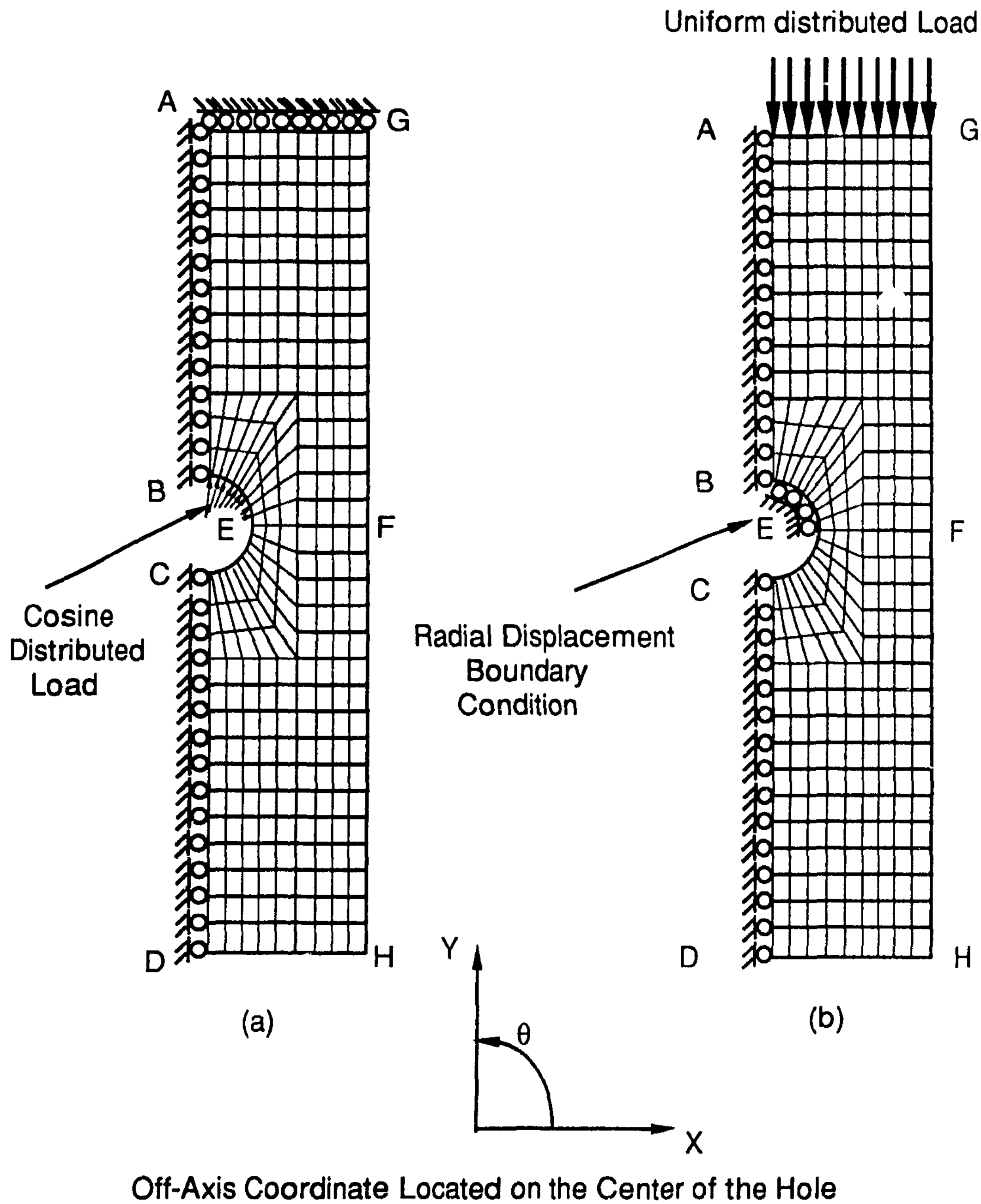


Figure 3.1: Finite element model of a pin-loaded composite plate.

AG (Fig. 3.1 (a)). Nodes on this edge must be fixed in the Y-axis direction and be free in the X-axis direction). This method has been used by [45,54,57,58,61,71], but the stresses obtained around the hole by this method are not exact. Although this method has been used by many authors, it has been shown [1] that this assumption is less valid as the degree of orthotropy increases. Thus when the failure criterion is sensitive to the stresses near the hole, this method is not reliable. Also this method will be highly inaccurate in the post-failure domain, when hole edge stresses may be changed.

Radial Displacement Boundary Conditions

In this method pin load can be modeled by applying displacement boundary conditions around the hole (nodes on arc BE must be fixed radially and be free tangentially) and applying uniform distributed compressive load over the edge AG or uniform distributed tensile load over the edge DH (Fig. 3.1 (b)). This method has been used by [22,25,34,55,56,74]. The obtained stresses around the hole by this method are more reliable. Thus for modeling the pin load in this research this method has been used.

3.3 Types of Element

There are two types of suitable elements, in the structural element library of I-DEAS [77], that can be used for this study. Those are the linear quadrilateral thin shell element, and the parabolic quadrilateral thin shell element. Both these elements have been used for stress analysis of the model. After checking the stresses extracted from these elements it was concluded that the second element is more accurate than the first for this problem, therefore the parabolic quadrilateral thin shell element has been used for stress and failure analysis of the problem.

Linear Quadrilateral Thin Shell Element

This is an element with four nodes, and each node has three translational degrees of freedom and three rotational degrees of freedom. (Fig. 3.2 a)

Parabolic Quadrilateral Thin Shell Element

This is an element with eight nodes, and each node has three translational degrees of freedom and three rotational degrees of freedom. (Fig. 3.2 b)

3.4 Results

In this section the results of the stress analysis of a pin-loaded composite plate, by using linear and parabolic elements are presented. The material and the physical properties of the plate, the magnitude of the pin load, and the laminate configuration are listed in table 3.1. On-axis [3] stresses (σ_x) along the line AB, the arc BE, and the line EF (see Fig. 3.1) are computed by using the two different kinds of elements. After selecting parabolic quadrilateral elements, because of accuracy, the effect of the friction between the pin and edge of the hole on the stresses around the hole is studied.

[3]Stresses in material axis directions. For more details refer to appendix A.

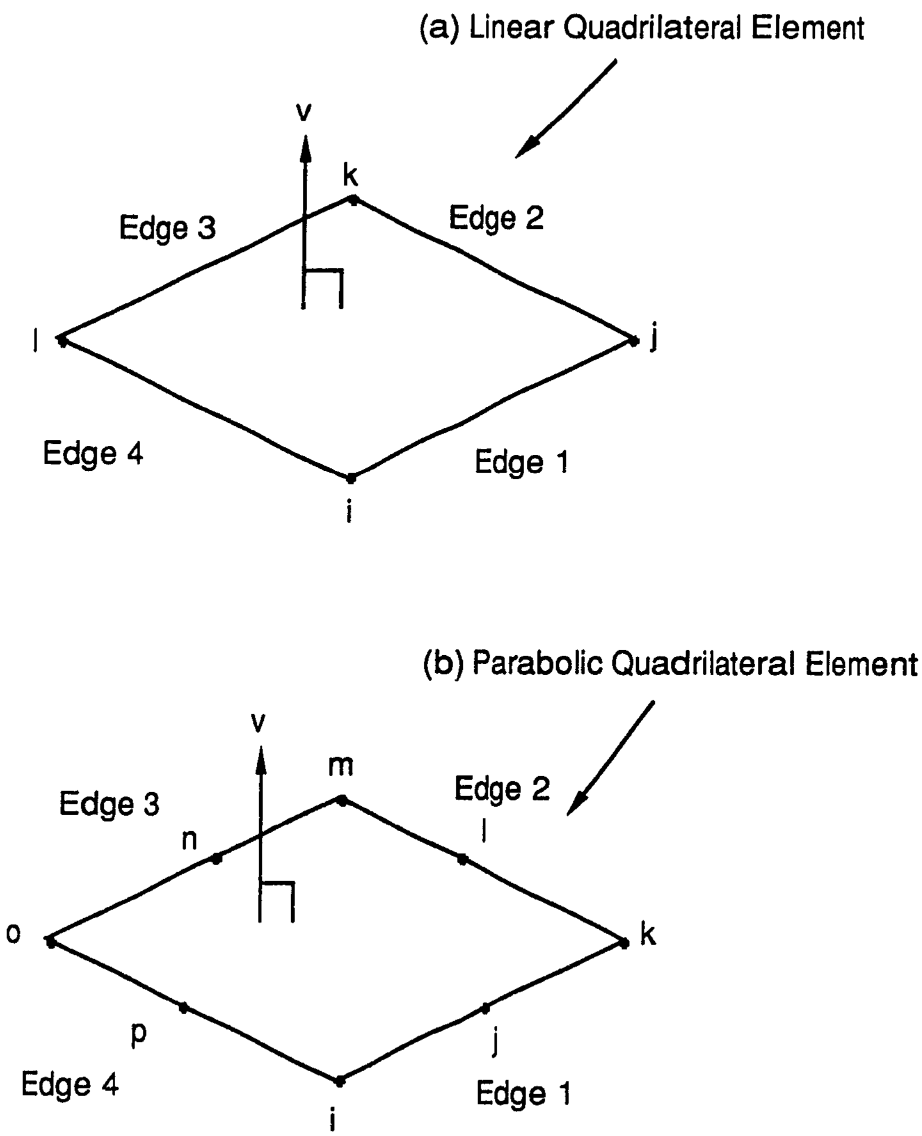


Figure 3.2: Two types of element.

Material Properties	**Symbol (units)**	
Longitudinal Modulus	E_x (GPa)	156
Transverse Modulus	E_y (GPa)	13
Shear Modulus	E_s (GPa)	7
Poisson's Ratio	ν_x	.23
Longitudinal Tensile Strength	X_t (MPa)	1517
Longitudinal Compressive Strength	X_c (MPa)	1593
Transverse Tensile Strength	Y_t (MPa)	46
Transverse Compressive Strength	Y_c (MPa)	253
Shear Strength	S (MPa)	107

Physical Properties	**Symbol (units)**	
Length	L (mm)	50.8
Width	W (mm)	25.4
Diameter	D (mm)	6.35
Edge Distance	E (mm)	25.4
Thickness	t (mm)	3.429

Laminate Config.	**Loading**	**Symbol (units)**	
$[0/\pm 45/90]_s$	Compressive Load	F (N)	2225

Table 3.1: Material and physical properties of a pin-loaded composite plate (T300/976 Graphite/Epoxy).

3.4.1 Comparison of Two Element Types

Linear Elements

On-axis stresses (σ_x) in the center of each element, along the line AB for each ply are shown in Fig. 3.3.

On-axis stresses (σ_x) in the center of each element, along the line EF for each ply are shown in Fig. 3.4. As shown in this figure the stresses tend to zero in the region far from the edge of the hole.

On-axis stresses (σ_x), in the center of each element along the arc BE for each ply are shown in Fig. 3.5. As shown in this figure the stresses between 70° and 90° seem slightly inaccurate (the curves are not smooth).

Parabolic Elements

On-axis stresses (σ_x) in the center of each element, along the line AB for each plies are shown in Fig. 3.6.

On-axis stresses (σ_x), in the center of each element along the line EF for each ply are shown in Fig. 3.7. As shown in this figure the stresses tend to zero in the region far from the edge of the hole.

On-axis stresses (σ_x), in the center of each element along the arc BE for each ply are shown in Fig. 3.8. As shown in this figure the stresses between 70° and 90° are smooth.

Comparison and Selection

In Fig. 3.9, on-axis stresses in the center of each element along the edge AB of ply number 1, computed by linear and parabolic quadrilateral elements, are compared. After comparison between the extracted stresses from the linear and the parabolic

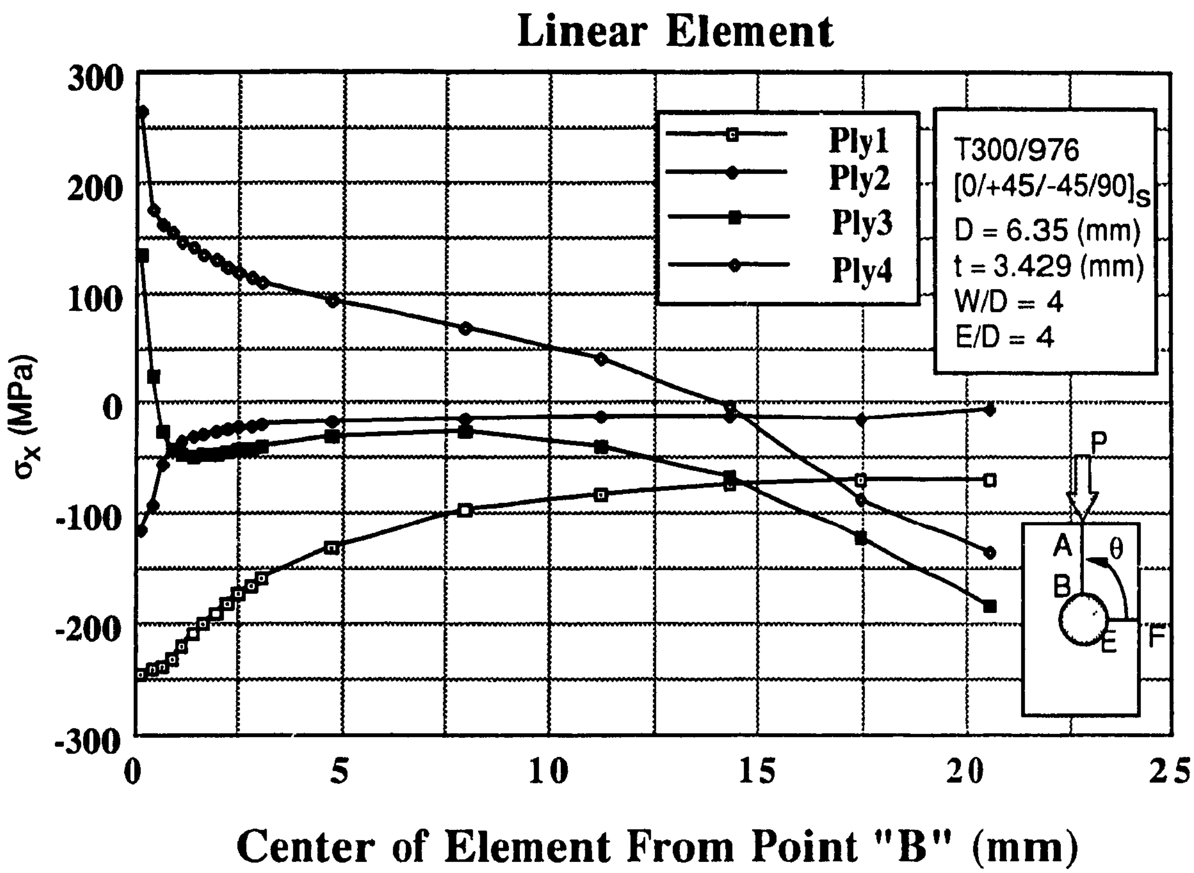


Figure 3.3: On-axis stresses along the line AB.

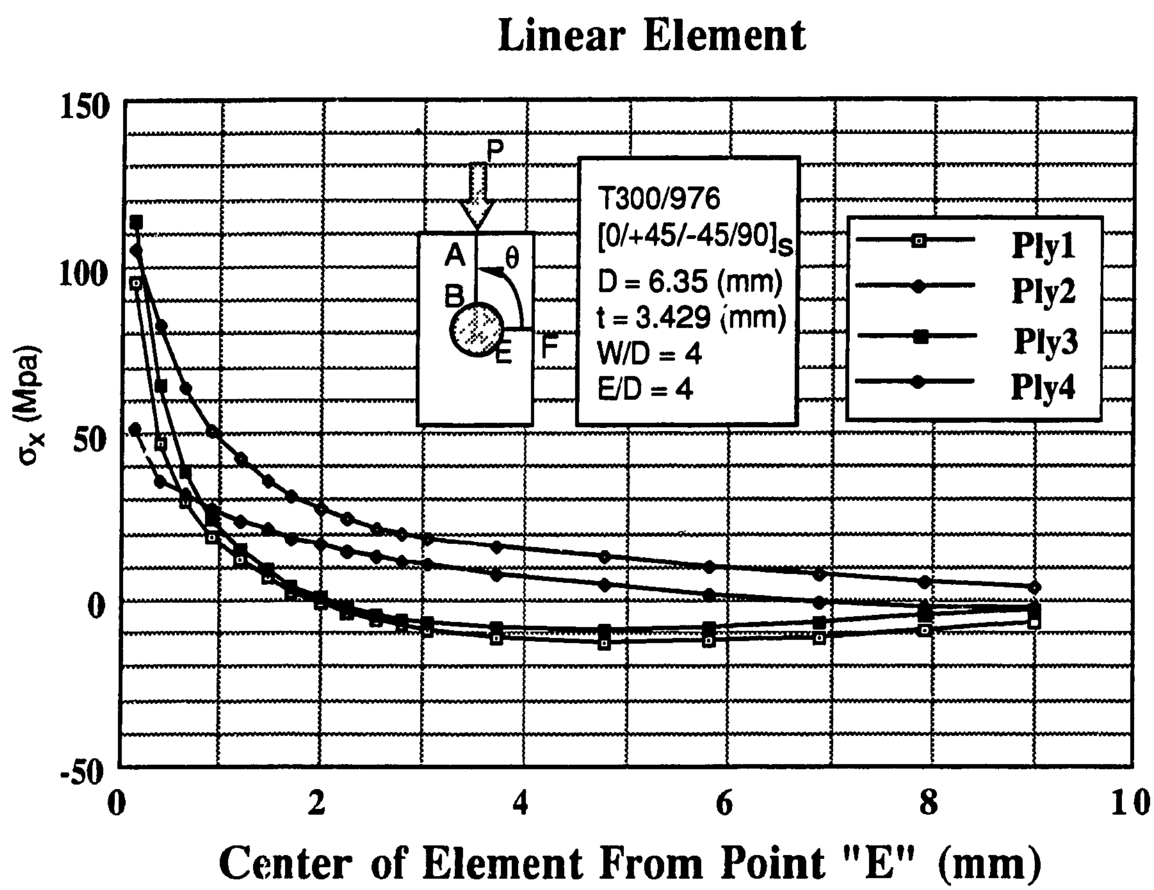


Figure 3.4: On-axis stresses along the line EF.

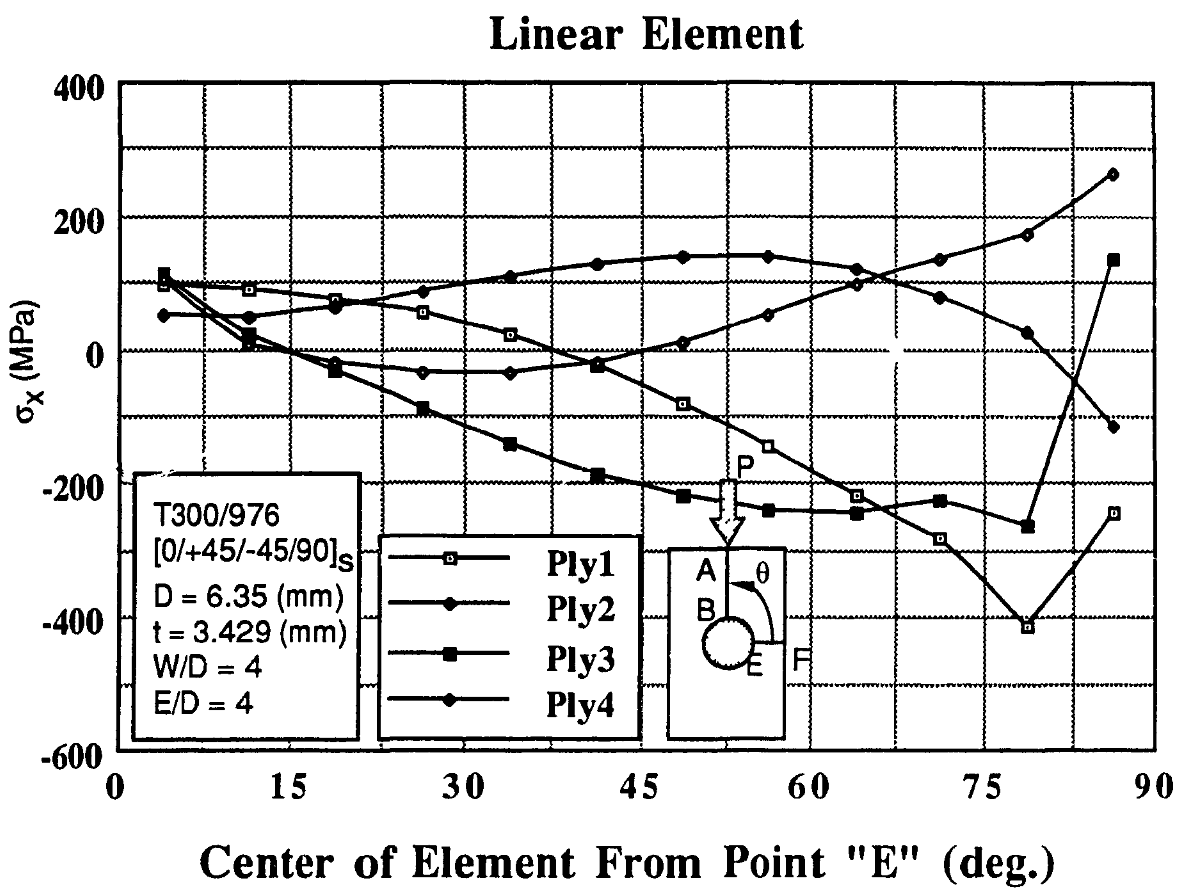


Figure 3.5: On-axis stresses along the arc BE.

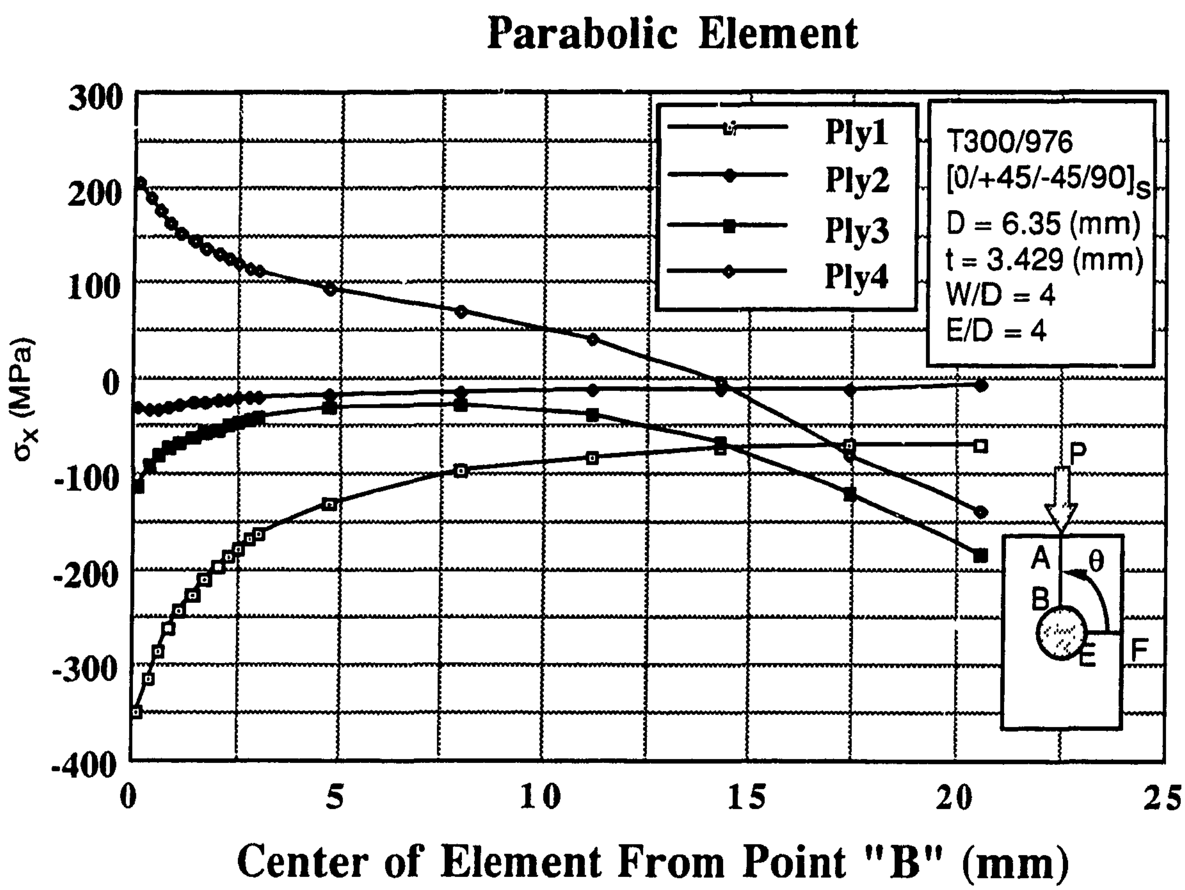


Figure 3.6: On-axis stresses along the line AB.

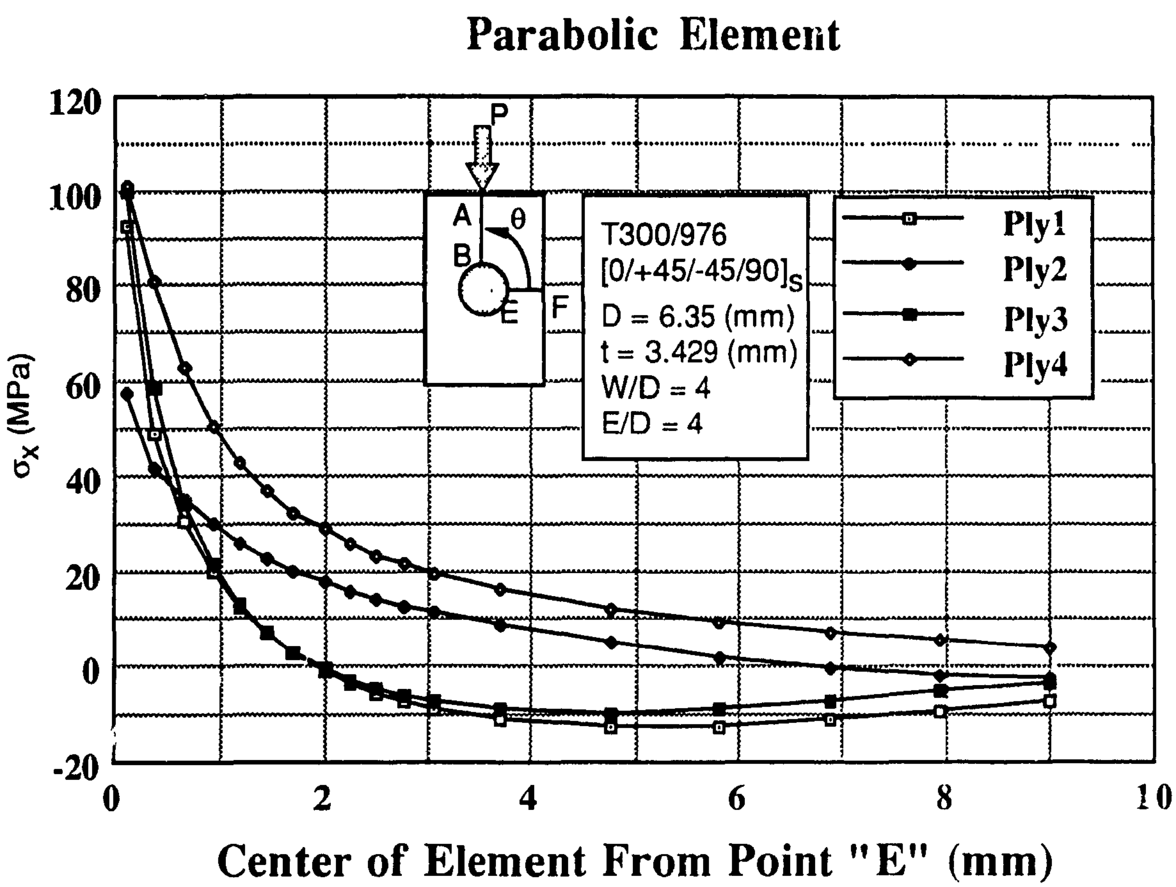


Figure 3.7: On-axis stresses along the line EF.

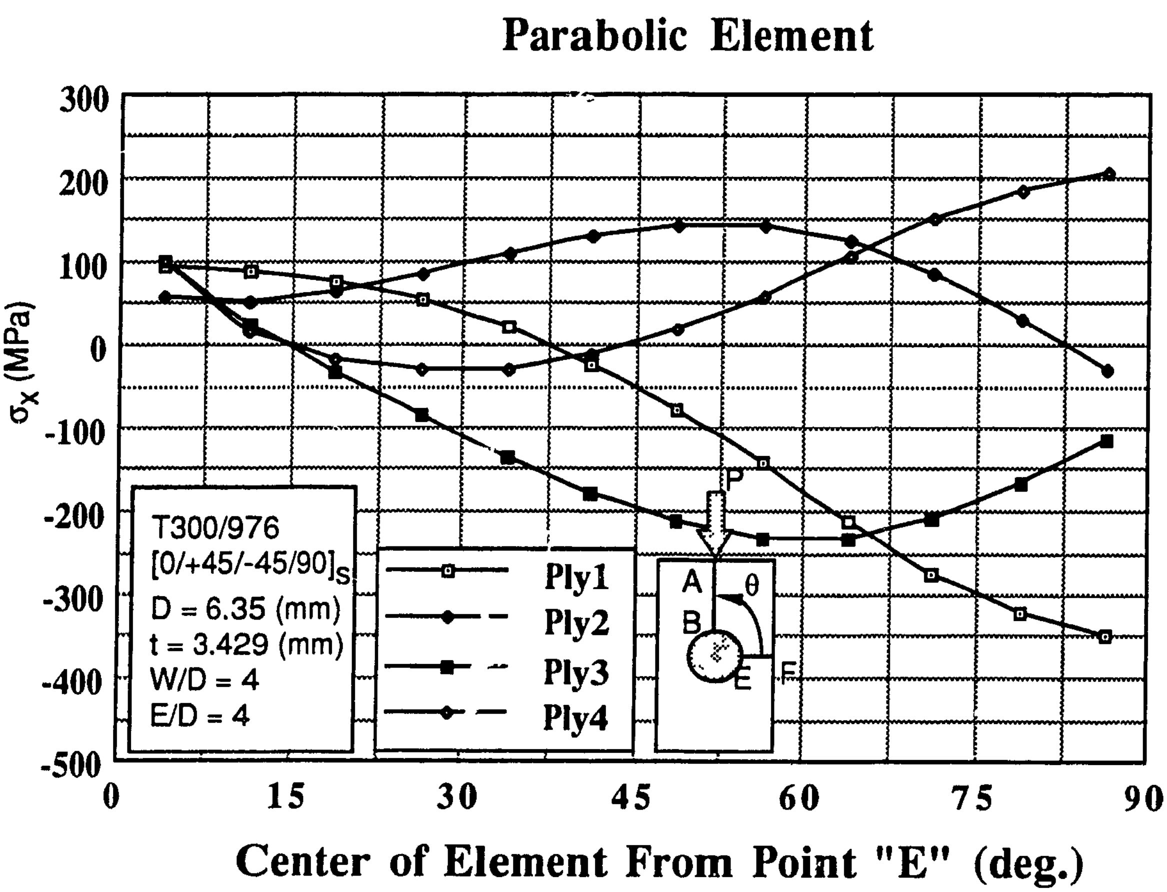


Figure 3.8: On-axis stresses along the arc BE.

elements, it is obvious that the parabolic element is more accurate for this problem. Hence this type of element is used for failure analysis.

3.4.2 Study the Effect of Friction

For modeling the friction between the pin and the edge of the hole, all of the nodes around the arc BE are fixed tangentially. By this assumption coefficient of friction between the pin and the edge of the hole is assumed to be infinite [25]. By using parabolic quadrilateral elements, stresses in the center of each element along the line AB, the line EF, and the arc BE are computed. The results are shown in figures 3.10, 3.11, and 3.12. As shown in Fig 3.10 and in Fig 3.11 the stresses for ply number 1 seem inaccurate (The stress on the edge of the hole is less than the stresses far from it). Because of this problem it would appear that the model in this situation is slightly overconstrained.

3.5 Summary

After comparison between extracted stresses from the linear and the parabolic elements, the latter element is used for stress and failure analysis of the problem. It is assumed that there is no clearance between pin and hole, and pin is assumed rigid. Material properties of the composite plate are assumed linear elastic. As it was mentioned in 3.4.2 applying tangential restraints around the hole is not a good assumption for modeling the friction between pin and edge of the hole. Therefore the effect of friction is not considered in the failure analysis of the problem.

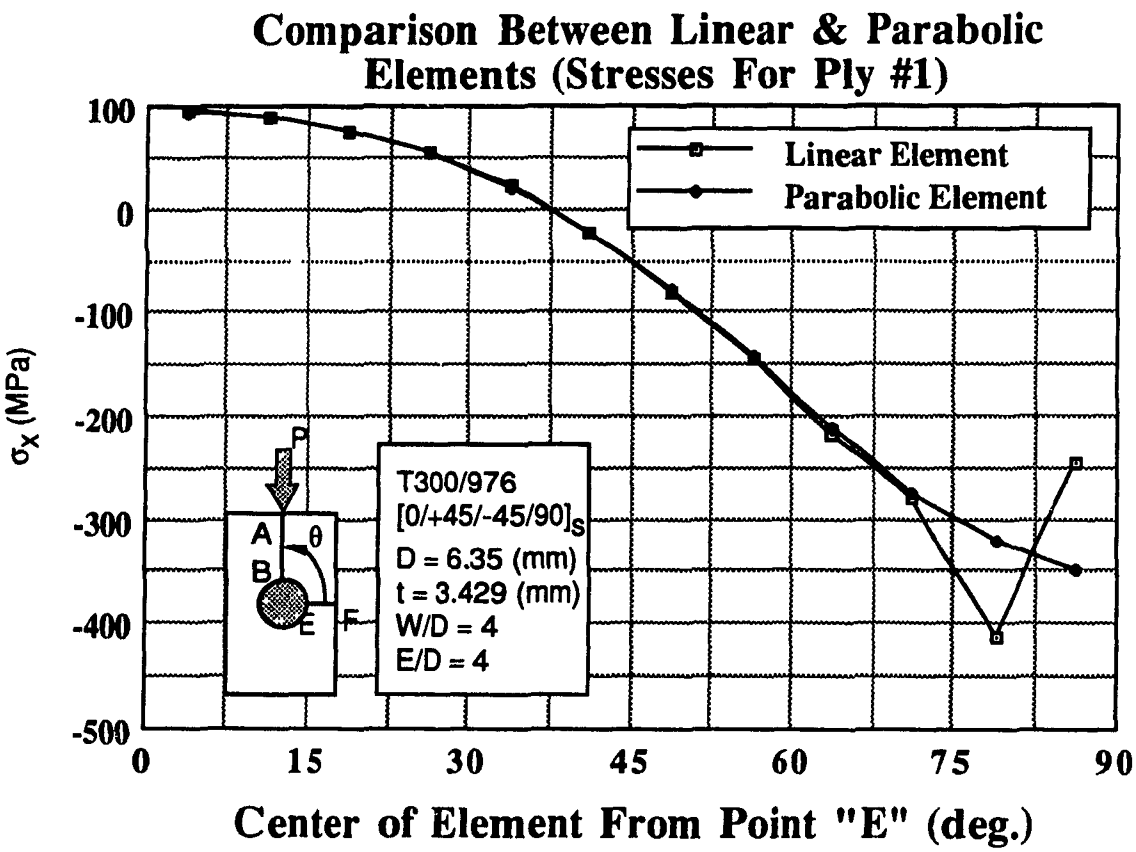


Figure 3.9: Comparison between linear and parabolic elements.

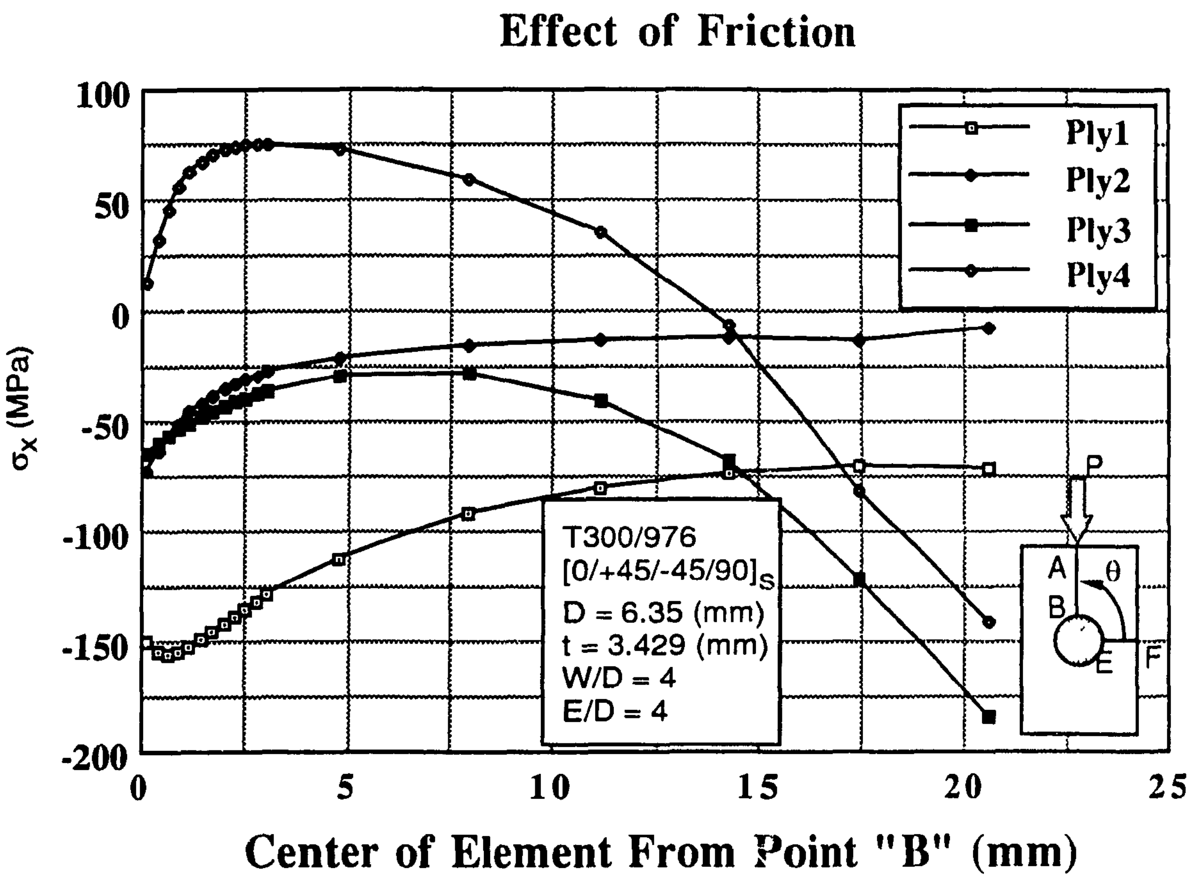


Figure 3.10: On-axis stresses along the line AB.

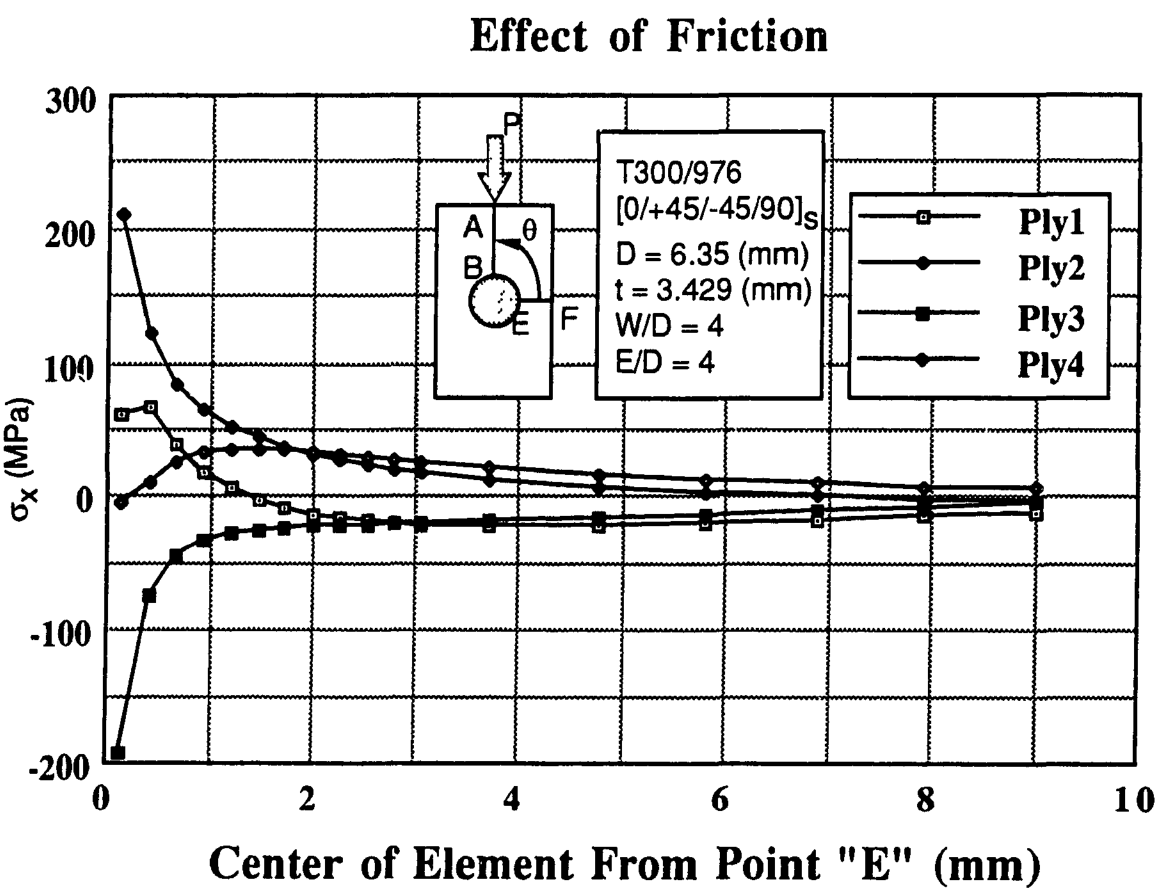


Figure 3.11: On-axis stresses along the line EF.

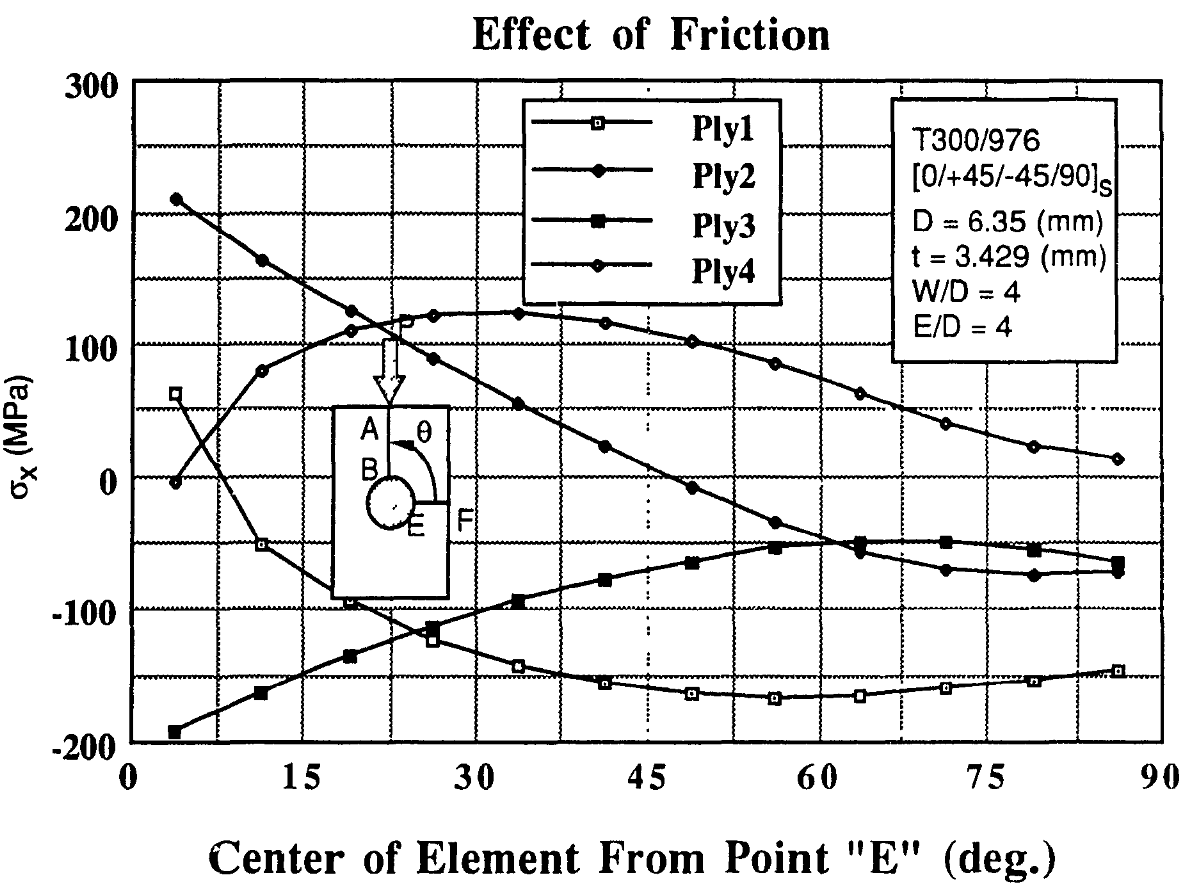


Figure 3.12: On-axis stresses along the arc BE.

Chapter 4

Failure Analysis

To analyze the strength of a laminated composite, it would be extremely costly and time consuming to perform static tests for all possible stacking sequences. Thus for this purpose, strength theories are required. Failure analysis is a tool for predicting the strength of materials under complex loading conditions using strength data obtained from uniaxial tests. There are numerous failure criteria for composite materials [78], each with some advantages and disadvantages.

For finding the strength of pin-loaded composite plates, different failure criteria have been used by authors. Waszczak and Cruse [53] used a distorsional energy failure criterion. Oplinger [34] and Humphris [54] used the Tsai-Hill criterion. Agarwal [55] applied an average stress criterion over a characteristic distance. Soni [56] applied the Tsai-Wu failure criterion. Chang *et al.* [57-61], and Ueng and Zhang [51] used the Yamada-Sun failure criterion over a characteristic distance. Tsiang and Mandell [71] applied the Tsai-Hill and maximum stress failure criteria. Conti [74] applied the Azzi-Tsai failure criterion. Chang and Chang [62] used a modified Hashin failure criteria.

Serabian and Oplinger [72] applied Hoffman failure criterion. Recently Lessard [64] used another modified Hashin failure criteria.

Most of the existing failure criteria are not able to determine the *modes of failure*[1]. Among them, Hashin failure criteria are suitable for distinguishing between the different modes of failure. In this research Hashin failure criteria [79] modified by Chang and Chang [62], and Lessard [64] have been used. This set of failure criteria is able to distinguish between the modes of failure and to find all of the *mechanisms of failure*[2].

4.1 Failure Criteria

A set of failure criteria have been used in this research. Each of these failure criteria corresponds to a certain failure mode. The failure modes are, *matrix tension failure mode, matrix compression failure mode, fiber tension failure mode, fiber compression failure mode*, and *fiber-matrix shearing failure mode* (Fig. 4.1).

Matrix Tension Failure Mode

For predicting matrix tension failure mode ($\sigma_y > 0$), the failure criterion has the form

$$\left(\frac{\sigma_y}{Y_t}\right)^2 + \left(\frac{\sigma_s}{S_c}\right)^2 = e_{M+}^2 \tag{4.1}$$

This failure criterion states that when, in any one of the plies in a laminate, the stresses σ_y and σ_s satisfy Eq. (4.1) (with $e_{M+} > 1$), the layer fails by matrix tension.

[1]For more details about the modes of failure refer to section 4.1

[2]For more details about the mechanisms of failure refer to section 2.1

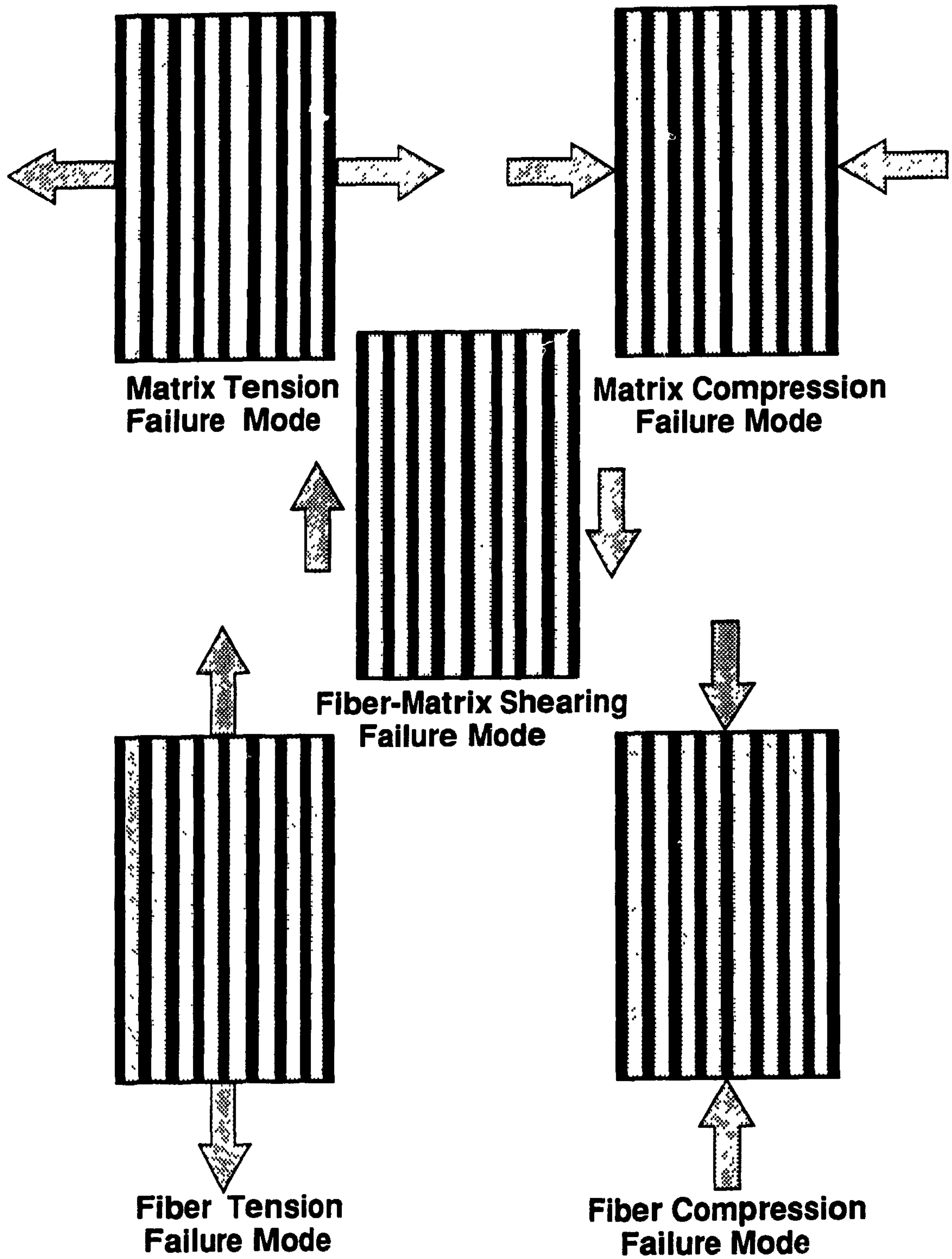


Figure 4.1: The five failure modes, viewed at the ply level.

Matrix Compression Failure Mode

For predicting matrix compression failure mode ($\sigma_y < 0$), the failure criterion has the form

$$\left(\frac{\sigma_y}{Y_c}\right)^2 + \left(\frac{\sigma_s}{S_c}\right)^2 = e_{M-}^2 \tag{4.2}$$

This failure criterion states that when, in any one of the plies in a laminate, the stresses σ_y and σ_s satisfy Eq. (4.2) (with $e_{M-} > 1$), the layer fails by matrix compression.

Fiber Tension Failure Mode

For predicting fiber tension failure mode ($\sigma_x > 0$), the failure criterion has the form

$$\left(\frac{\sigma_x}{X_t}\right)^2 + \left(\frac{\sigma_s}{S_c}\right)^2 = e_{F+}^2 \tag{4.3}$$

This failure criterion states that when, in any one of the plies in a laminate, the stresses σ_x and σ_s satisfy Eq. (4.3) (with $e_{F+} > 1$), the layer fails by fiber tension.

Fiber Compression Failure Mode

For predicting fiber compression failure mode ($\sigma_x < 0$), the failure criterion has the form

$$\left(\frac{\sigma_x}{X_c}\right)^2 = e_{F-}^2 \tag{4.4}$$

This failure criterion states that when, in any one of the plies in a laminate, the stresses σ_x satisfy Eq. (4.4) (with $e_{F-} > 1$), the layer fails by fiber compression.

This failure criterion was developed and used by Chang and Lessard [80] for analyzing compression failure in laminated composites containing an open hole. Fiber compression failure mode corresponds to the bearing mechanism, therefore by this failure criterion (Eq. 4.4) the bearing mechanism can be modeled.

Fiber-Matrix Shearing Failure Mode

For predicting fiber-matrix shearing failure mode ($\sigma_x < 0$), the failure criterion has the form

$$\left(\frac{\sigma_x}{X_c}\right)^2 + \left(\frac{\sigma_s}{S_c}\right)^2 = e_{FM^-}^2 \tag{4.5}$$

This failure criterion states that when, in any one of the plies in a laminate, the stresses σ_x and σ_s satisfy Eq. (4.5) (with $e_{FM^-} > 1$), the layer fails by fiber-matrix shearing.

4.2 Summary

It is believed that the accurate study and design of pin-loaded joints in laminated composites cannot be achieved without using a progressive damage model, [3] and separating the different modes is a required feature for establishing such a model. This set of failure criteria (Eqs. 4.1 to 4.5), is a suitable form for modeling the different modes of failure and therefore finding the different mechanisms of failure.

[3]This model is explained in chapter 6.

Chapter 5

Material Property Degradation

Conventional stress analysis like laminated plate theory is by definition limited to an intact continuum. Thus after failure occurrence in a ply, laminated plate theory is not applicable. But, once failure occurs, a failed ply can be replaced with an intact ply of lower material properties (see Fig. 5.1). Therefore in this case conventional stress analysis can be applied.

Decreasing the material properties of failed plies is not arbitrary. It depends upon the modes of failure explained in the previous chapter. Each mode of failure

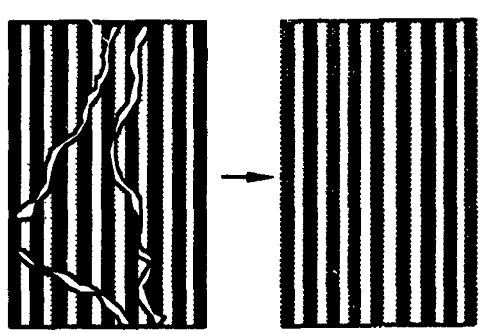


Figure 5.1: Degraded ply is modeled by an intact ply of lower material properties

corresponds to a special material property degradation rule [62, 80]. In the following sections material property degradation due to each mode of failure is explained.

5.1 Material Property Degradation Rules

Material Property Degradation Due to Matrix Tension Failure Mode

For matrix tension failure mode (Eq. 4.1) in a ply, the transverse modulus E_y and Poisson's ratio ν_y are reduced to zero, i.e., the matrix can no longer carry any load in tension. However, the longitudinal modulus E_x and shear modulus E_s are unchanged, i.e., in the failed ply, the in plane material properties are reduced as follows:

For $\sigma_y > 0$ and $e_{M^+} > 1$

$$\left\{ \begin{matrix} E_x \\ E_y \\ E_s \\ \nu_x \\ \nu_y \end{matrix} \right\} \rightarrow \left\{ \begin{matrix} E_x \\ 0 \\ E_s \\ \nu_x \\ 0 \end{matrix} \right\} \tag{5.1}$$

or in the form of stiffness matrix

$$\begin{bmatrix} \frac{E_x}{1-\nu_x\nu_y} & \frac{E_y\nu_x}{1-\nu_x\nu_y} & 0 \\ \frac{E_x\nu_y}{1-\nu_x\nu_y} & \frac{E_y}{1-\nu_x\nu_y} & 0 \\ 0 & 0 & E_s \end{bmatrix} \rightarrow \begin{bmatrix} E_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & E_s \end{bmatrix}$$

Material Property Degradation Due to Matrix Compression Failure Mode

The matrix compression failure mode (Eq. 4.2) results in the same type of damage to the composite ply as the matrix tension failure mode. Thus the transverse modulus

E_y and Poisson's ratio ν_y are reduced to zero while the longitudinal modulus E_x and shear modulus E_s are unchanged.

For $\sigma_y < 0$ and $e_{M-} > 1$

$$\begin{Bmatrix} E_x \\ E_y \\ E_s \\ \nu_x \\ \nu_y \end{Bmatrix} \rightarrow \begin{Bmatrix} E_x \\ 0 \\ E_s \\ \nu_x \\ 0 \end{Bmatrix} \qquad (5.2)$$

or in the form of stiffness matrix

$$\begin{bmatrix} \frac{E_x}{1-\nu_x\nu_y} & \frac{E_y\nu_x}{1-\nu_x\nu_y} & 0 \\ \frac{E_x\nu_y}{1-\nu_x\nu_y} & \frac{E_y}{1-\nu_x\nu_y} & 0 \\ 0 & 0 & E_s \end{bmatrix} \rightarrow \begin{bmatrix} E_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & E_s \end{bmatrix}$$

Material Property Degradation Due to Fiber Tension Failure Mode

Fiber tension failure mode (Eq. 4.3) in a ply is a catastrophic mode of failure and when it occurs, the material in that region cannot sustain any additional load. Thus the material properties for the failed ply and all other plies are reduced to zero.

For $\sigma_x > 0$ and $e_{F+} > 1$

$$\begin{Bmatrix} E_x \\ E_y \\ E_s \\ \nu_x \\ \nu_y \end{Bmatrix} \rightarrow \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (5.3)$$

or in the form of stiffness matrix

$$\begin{bmatrix} \frac{E_x}{1-\nu_x\nu_y} & \frac{E_y\nu_x}{1-\nu_x\nu_y} & 0 \\ \frac{E_x\nu_y}{1-\nu_x\nu_y} & \frac{E_y}{1-\nu_x\nu_y} & 0 \\ 0 & 0 & E_s \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Material Property Degradation Due to Fiber Compression Failure Mode

The fiber compression failure mode (Eq. 4.4) is a catastrophic mode of failure and when it occurs, the material in that region cannot sustain any more load. Thus the material properties for the failed ply and all other plies are reduced to zero.

For $\sigma_x < 0$ and $e_{F^-} > 1$

$$\begin{Bmatrix} E_x \\ E_y \\ E_s \\ \nu_x \\ \nu_y \end{Bmatrix} \rightarrow \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{5.4}$$

or in the form of stiffness matrix

$$\begin{bmatrix} \frac{E_x}{1-\nu_x\nu_y} & \frac{E_y\nu_x}{1-\nu_x\nu_y} & 0 \\ \frac{E_x\nu_y}{1-\nu_x\nu_y} & \frac{E_y}{1-\nu_x\nu_y} & 0 \\ 0 & 0 & E_s \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Material Property Degradation Due to Fiber-Matrix Shearing Failure Mode

In fiber-matrix shearing failure mode (Eq. 4.5), the material can still carry load in the fiber direction and in the matrix direction, but shear loads can no longer be carried. This is modeled by reducing the shear property and the Poisson's ratios, ν_x and ν_y, to zero.

For $\sigma_x < 0$ and $e_{FM^-} > 1$

$$\begin{Bmatrix} E_x \\ E_y \\ E_s \\ \nu_x \\ \nu_y \end{Bmatrix} \rightarrow \begin{Bmatrix} E_x \\ E_y \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{5.5}$$

or in the form of stiffness matrix

$$\begin{bmatrix} \frac{E_x}{1-\nu_x\nu_y} & \frac{E_y\nu_x}{1-\nu_x\nu_y} & 0 \\ \frac{E_x\nu_y}{1-\nu_x\nu_y} & \frac{E_y}{1-\nu_x\nu_y} & 0 \\ 0 & 0 & E_s \end{bmatrix} \rightarrow \begin{bmatrix} E_x & 0 & 0 \\ 0 & E_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5.2 Summary

Each mode of failure corresponds to a suitable material property degradation rule. Thus for each mode of failure, predicted by the set of failure criteria (Eqs. 4.1 to 4.5), a suitable equation for material property degradation (Eqs. 5.1 to 5.5) is associated. These equations are required for establishing a progressive damage model.

Chapter 6

Progressive Damage Modeling

The idea of progressive damage modeling was proposed by Chou *et al.* [81]. The idea of using discrete failure criteria (fiber and matrix failure) was used by Sandhu *et al.* [82]. In 1987 Chang and Chang [62] presented a model, consisting of discrete failure criteria coupled with material property degradation rules, for analysis of pinned composite joints in net tension or shearout failure mechanism.

The objective of this chapter is to establish a model which can simulate damage progression from initial failure (*first ply failure*) to catastrophic failure (*ultimate strength*) for pin-loaded composite plates failing in net tension, shear-out, and bearing mechanisms. A computer code is developed in IDEAL (I-DEAS language [77]) for this purpose.

The model consists of three major parts, namely, stress analysis, failure analysis, and material property degradation. These parts were discussed in chapters 3, 4, and 5 respectively. In the following section, the model is described in detail.

6.1 Modeling

A simplified demonstration of the progressive damage model is given in Fig. 6.1. A pin-loaded composite plate is modeled by two dimensional finite element meshes and suitable boundary conditions. Because of the symmetry, only half of the plate is considered (Fig. 6.1a).

Before *first ply failure* (failure initiation), the composite plate behaves linearly elastic (Fig. 6.1a). As the plate is loaded further, damage [1] will begin to appear, usually near the stress concentration at the hole boundary (Fig. 6.1b).

At this stage, material properties of the failed region must be changed with suitable material property degradation rules [2]. By increasing the load at this step, damage will propagate and will eventually reach the edge of the plate (Fig. 6.1c). At this point, composite plate cannot tolerate any more load, therefore the point of *ultimate strength* has been reached.

6.1.1 Details of The Model

As was mentioned already, the progressive damage model is an integration of stress analysis, failure analysis, and material property degradation rules. The details of the model can be best described with the help of the flow chart shown in Fig. 6.2.

The following procedure is used to model the damage progression process:

1) First, the finite element model must be prepared. In this step the physical

[1] Damage can be detected by failure criteria (explained in chapter 4.)

[2] Material property degradation rules are explained in chapter 5.

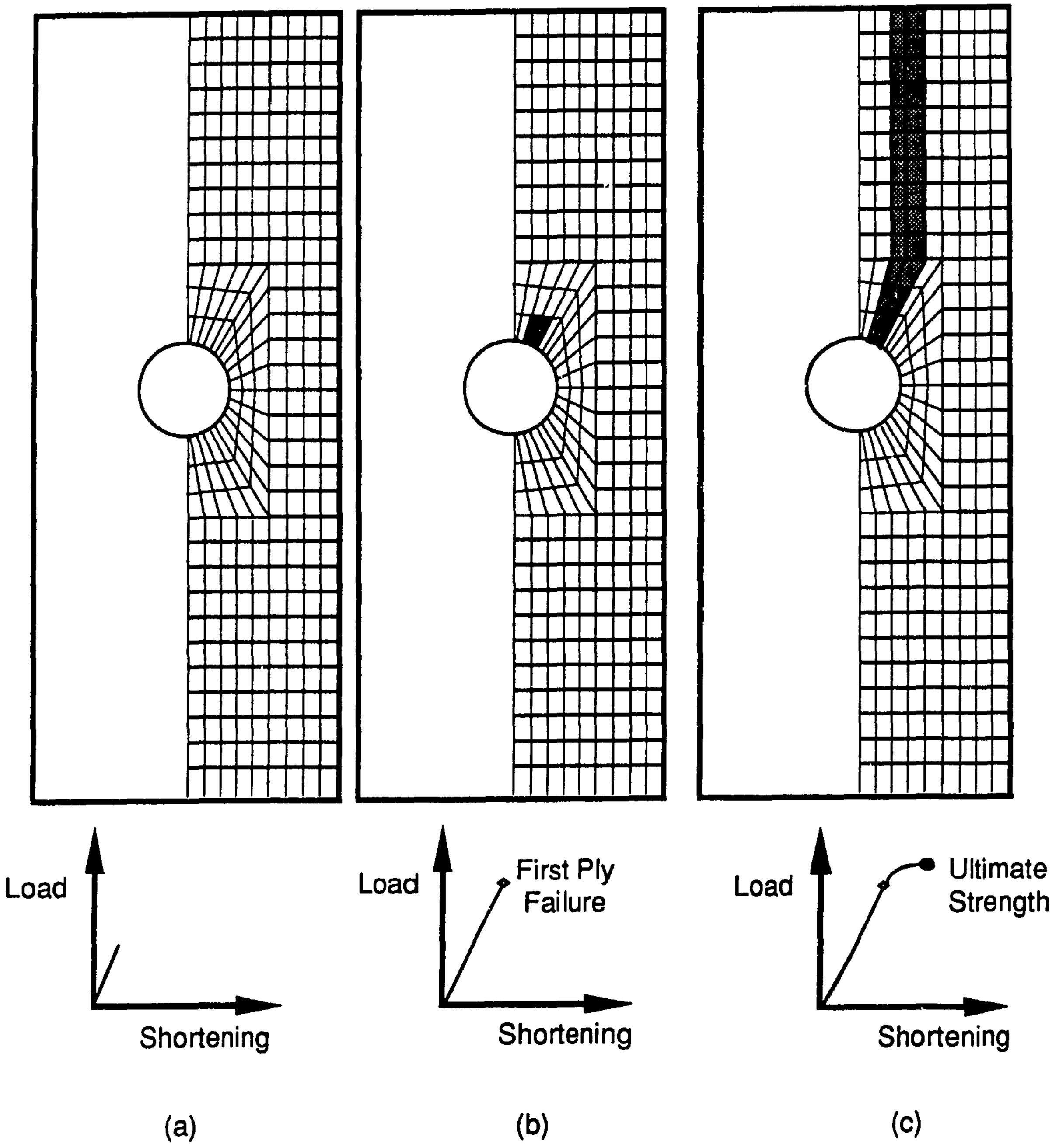


Figure 6.1: A simplified demonstration of the progressive damage model.

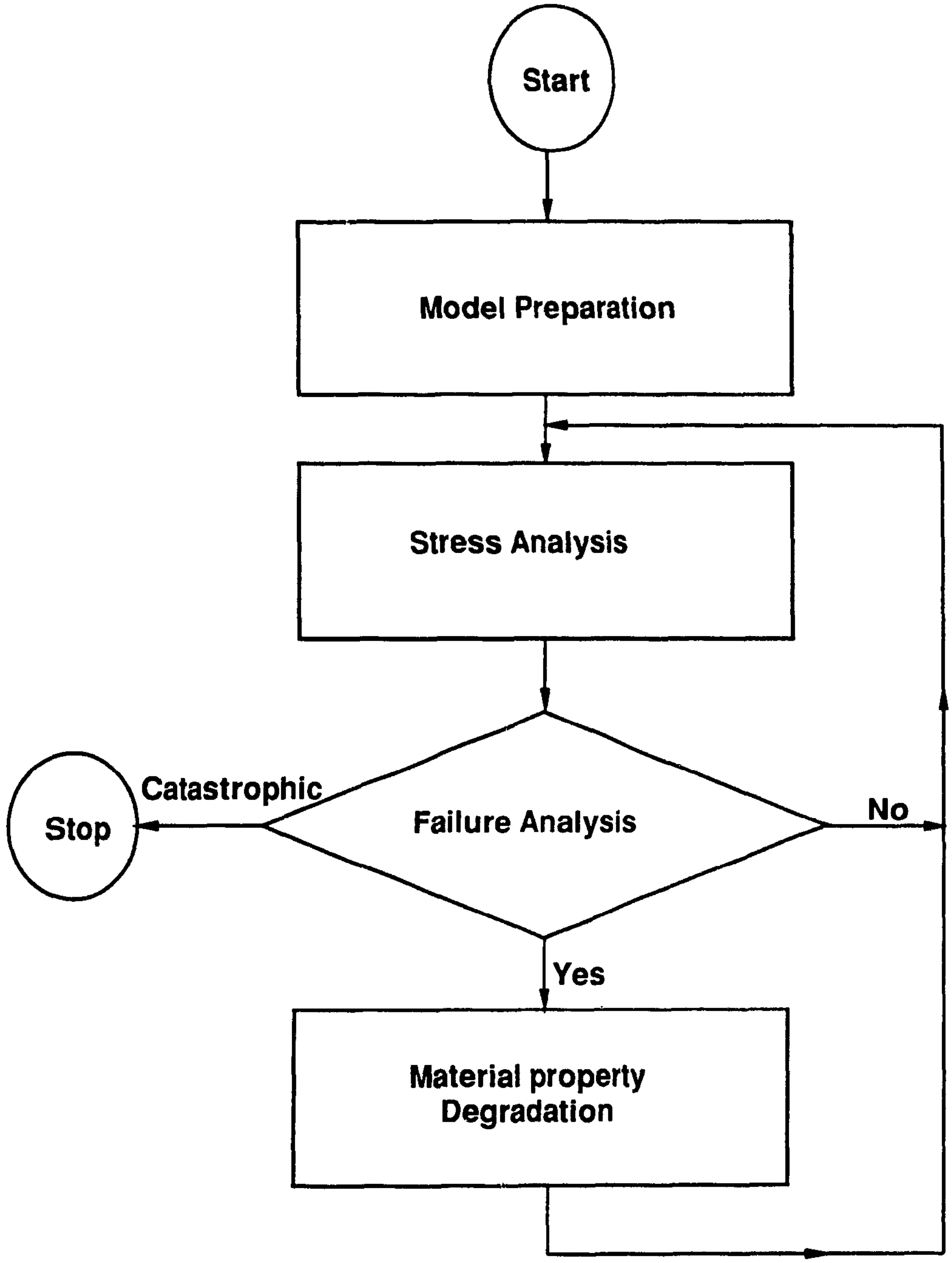


Figure 6.2: Flow chart of the progressive damage model.

properties, material properties, and appropriate boundary conditions of the problem must be provided. Then a load increment must be selected. In table 3.1 an example of the required parameters is given.

2) In the next step, a finite element stress analysis is performed at a selected load increment. In this step, on-axis stresses, [3] in the center of each ply of each element, are obtained. These stresses will be used for failure analysis in the following step.

3) By using the failure criteria (Eqs. 4.1 to 4.5) failure analysis is performed for all elements. If the failure criteria are not met, there is no failure at this load increment. At this stage the program returns to the stress analysis step to calculate the on-axis stresses at the selected load increment. These new on-axis stresses must be added to the previously obtained on-axis stresses. If there are failures in certain plies of certain elements, then the program must continue to the next step.

4) By using Eqs. 5.1 to 5.5, material property degradation is performed for failed plies. In this step a new stiffness matrix is assigned to the model. Now the program must return to the stress analysis step, and this loop must be continued.

As the load level increases, more and more elements will fail until ultimate failure has been reached. This situation (catastrophic failure) can be detected in one of three ways: a) Incremental displacement in a load step is much greater than incremental displacement in the first load step. b) The failure reaches to the edge of the plate, which indicates that the model has completely failed. c) Singularity in stiffness matrix. At this stage the program must be stopped.

The above mentioned model has been implemented into a user-friendly computer

[3] Definition of the on-axis stress is presented in appendix A.

code, designated "PDPIN". The code has been written in IDEAL (I-DEAS language) [77]. The flow chart of the computer program, and the computer code are presented in appendix B and C, respectively.

6.2 Explanation of the Output of the Computer Code

By running the computer code "PDPIN", an output file "TABLE2.DAT" is created. This file contains the information of the failure mode of each ply of each failed element. This file is processed by another file "PLOT.PRG" [4]. After processing, the result is a graphical representation of the damage propagation for each ply of each element of composite plate. It means that the computer code simulates the initiation and propagation of damage from first ply failure to catastrophic failure.

As an example of the output of the computer code, the graphical results of the simulated behavior of a $[0/90]_s$ pin-loaded composite plate is presented in this section.

The specifications of the plate, material type, loading condition, and graphical result of the damage propagation of ply number one (0^o ply) are shown in Fig. 6.3. Damage is initiated on the hole boundary about 45^o from the loading direction at a load of 3338 N. By increasing the load, damage propagates around the hole. At a load of 4450 N, multiple failure modes occur near the hole boundary. Finally, at a load of 5563 N, failure reaches the edge, and the program is stopped. As shown in Fig. 6.3, the composite plate failed by the shearout mechanism.

[4]Flow chart and computer code of PLOT.PRG is presented in appendices D and E, respectively.

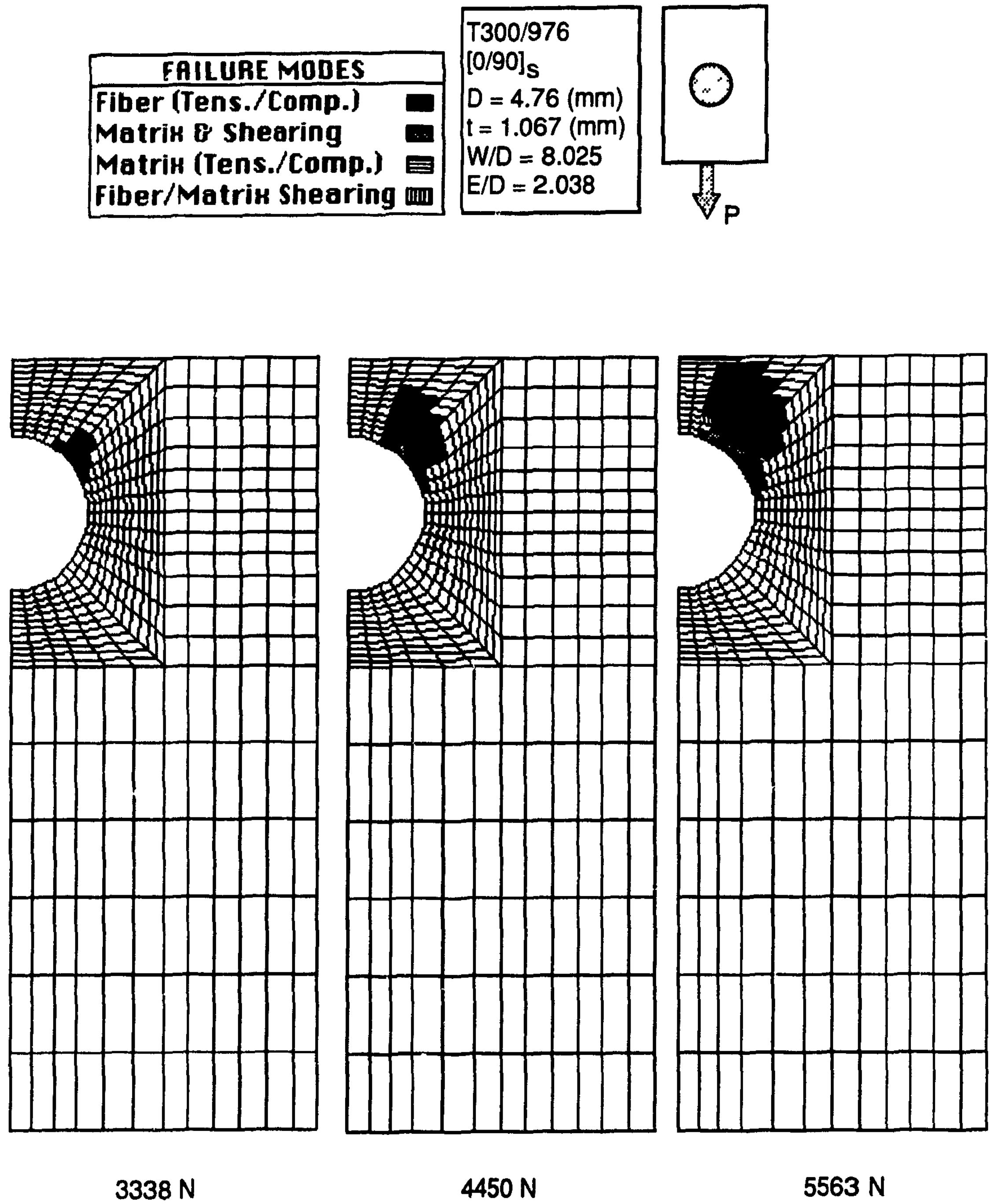


Figure 6.3: Graphical representation of damage propagation of $[0/90]_s$ at different load steps.

6.3 Summary

A progressive damage model based on the three major parts, stress analysis, failure analysis, and material property degradation, is established. A computer code based on the model is developed. The model is capable of assessing damage, evaluating residual strength, and predicting ultimate strength of pin-loaded composite plates. The progressive damage model and the corresponding computer code are capable of assessing all three different mechanisms of failure, namely, net tension, shearout, and bearing. Therefore the model and the computer code can be used for optimum design of pin-loaded composite plates.

Chapter 7

Results and Discussions

In the previous chapter, a progressive damage model was established by implementing three major parts, namely, stress analysis, failure analysis, and material property degradation in a user friendly computer code "PDPIN". The model can now be used to study the behavior of pin-loaded composite plates.

In the following sections, experimental validation of the model is discussed, and a parametric study of different laminate configurations ($[0/90]_s$ and $[0/\pm 45/90]_s$), different plate dimensions (W/D, E/D), and different loading conditions (tension or compression) is presented.

7.1 Experimental validation of the model

For the purpose of comparison between the extracted numerical results from the model and the experimental results, the experimental data measured by [83] have been used. The experimental data are presented in table 7.1.

Case	D (mm)	W/D	E/D	t/D	Lam. Config.
a	4.76	5.336	2.983	.224	$[0/\pm 45/90]_s$
b	4.76	8.025	4.013	.224	$[0/\pm 45/90]_s$
c	4.76	8.025	2.038	.224	$[0/\pm 45/90]_s$

Table 7.1: Experimental data.

Also the material properties of T300/SP286, used by [83] are presented in table 7.2. The results of three case studies are presented in table 7.3. The experimental results from Ref. [83] have been presented for the purpose of comparison.

In case (a), the predicted failure load by the model is 4450 N, and the experimental failure load is 4982 N, therefore the absolute error is 10.7%, showing a good agreement between model and experiment. The experimental mechanism of failure is net tension. The predicted failure mechanism is combined shearout and net tension.

In case (b), the predicteu failure load is 5340 N, and the experimental failure load is 5137 N. Absolute error is 3.9%, showing a very good agreement between model and experiment. The experimental mechanism of failure is bearing. The predicted failure mechanism is combined bearing and shearout.

In case (c), the predicted failure load by the model is 4450 N, and the experimental failure load is 4226 N. Absolute error is 5.3%, showing a very good agreement between the model and experiment. The experimental mechanism of failure is shearout. The predicted failure mechanism is also shearout.

As an example of this set of data, the predicted mechanism of failure of case (c),

Material Properties	Symbol (unit)	
Longitudinal Modulus	E_x (GPa)	130
Transverse Modulus	E_y (GPa)	8
Shear Modulus	E_s (GPa)	5
Poisson's Ratio	ν_x	.3
Longitudinal Tensile Strength	X_t (MPa)	1231
Longitudinal Compressive Strength	X_c (MPa)	1083
Transverse Tensile Strength	Y_t (MPa)	50
Transverse Compressive Strength	Y_c (MPa)	193
Shear Strength	S (MPa)	50

Table 7.2: Material properties of T300/SP286.

Case	Predicted Failure Load (N)	Average Experimental Failure Load (N) [83]	% Error $\left\|\left(1-\frac{F_{pre}}{F_{exp.}}\right)*100\right\|$
a	4450	4982	10.7
b	5340	5137	3.9
c	4450	4226	5.3

Table 7.3: Comparison between predicted and experimental data.

for ply 4 at final failure load is shown in Fig. 7.1.

By comparing of the results of the model and the experimental results, it can be concluded that the model is capable of assessing damage, evaluating the mechanism of failure, predicting the residual strength, and finding the maximum strength of laminated composites containing a pin-loaded hole. Therefore a reliable parametric study of different laminate configurations ($[0/90]_s$ and $[0/\pm 45/90]_s$), different plate dimensions (W/D, E/D), and different loading conditions (tension or compression) can be performed.

7.2 Parametric Study

In this section the results of parametric study of a cross ply laminate $[0/90]_s$ and a quasi-isotropic laminate $[0/\pm 45/90]_s$ are presented. For each of these configurations under tension or compression pin load, mechanism of failure, and variation of ultimate strength with respect to different W/D and E/D are studied.

7.2.1 Cross Ply Laminate

By considering a constant width to diameter ratio (W/D), and changing edge distance to diameter ratio (E/D), the behavior of a pin-loaded cross ply laminate $[0/90]_s$ is studied. Variation of strength of the cross ply laminate with variation of E/D, under tension and compression, is demonstrated in Fig. 7.2 and Fig. 7.3, respectively. It is well known from experimental data [40] that, for a fixed large value of E/D (or W/D), bearing strength increases with W/D (or E/D) and reaches a plateau. The

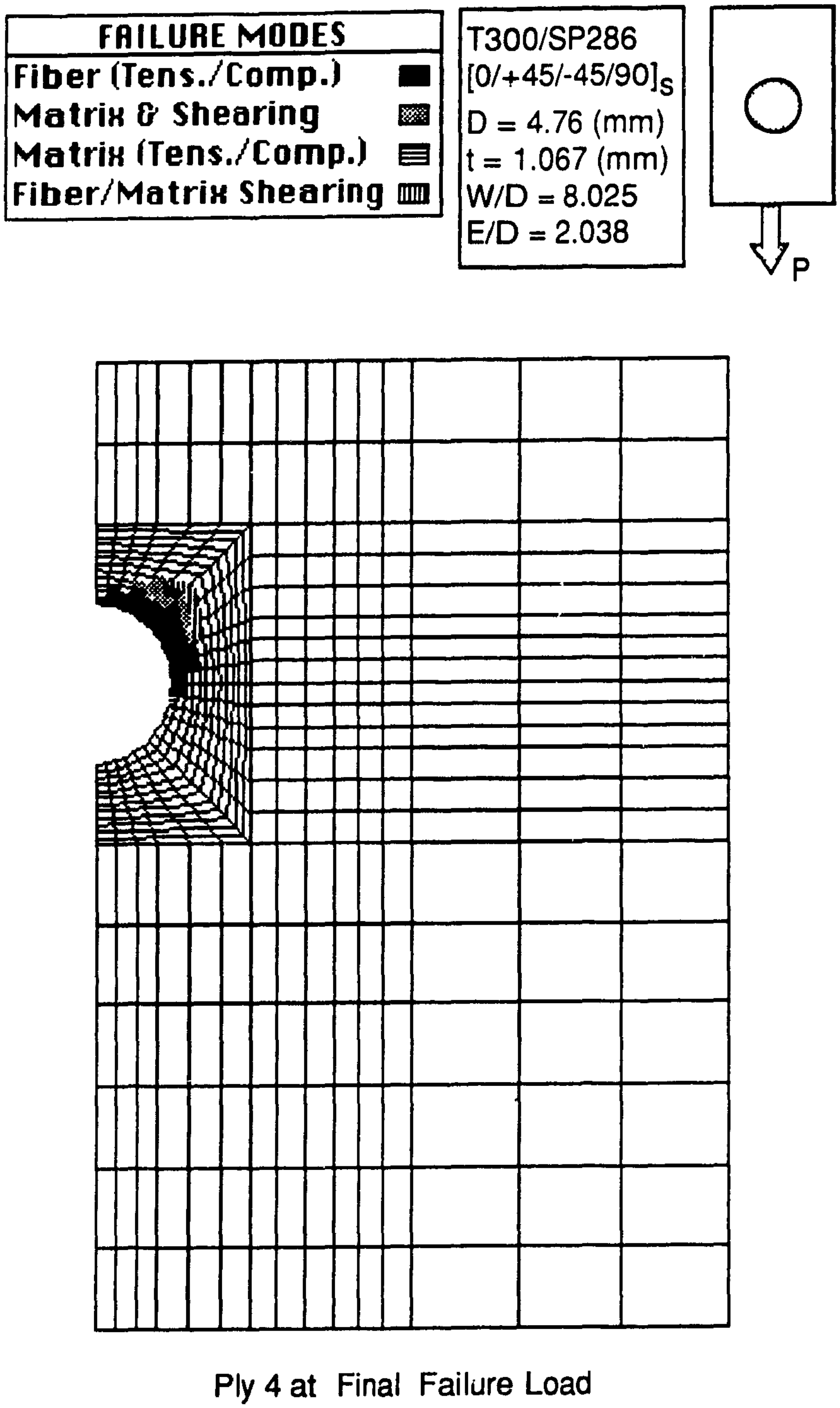


Figure 7.1: Predicted mechanism of failure for case (c) of experimental data.

behavior of curves in Fig. 7.2 and Fig. 7.3 corresponded completely to experimental evidence [40, 43, 47].

As an example of this set of data, predicted mechanisms of failure of the cross ply laminate, with constant W/D and different E/D, under tension, are shown in Fig. 7.4. By increasing E/D, the mechanism of failure is changed from shearout to bearing. This behavior corresponds to experimental evidence.

Also by considering a constant edge distance to diameter ratio (E/D), and changing width to diameter ratio (W/D), the behavior of a pin-loaded cross ply laminate is studied. Variation of strength of the cross ply laminate with variation of W/D, under tension and compression, is demonstrated in Fig. 7.5 and Fig. 7.6, respectively. The behavior of curves in these figures corresponded completely to experimental evidence.

As an example of this set of data, propagation of failure of the cross ply laminate, with W/D=4 and E/D=4, under compression, at different load levels, is shown in Fig. 7.7. Damage is initiated on the hole boundary about 30^o from the loading direction at a load of 5563 N. By increasing load, damage propagates around the hole. At a load of 8900 N, multiple failure modes occur near the hole boundary. Finally, at a load of 13350 N, because of large deflection, the computer program is stopped. As shown in Fig. 7.7, the composite plate failed by the shearout mechanism.

7.2.2 Quasi-Isotropic Laminate

By considering a constant width to diameter ratio (W/D), and changing edge distance to diameter ratio (E/D), the behavior of a pin-loaded quasi-isotropic laminate $[0/\pm 45/90]_s$ is studied. Variation of strength of this quasi-isotropic laminate with

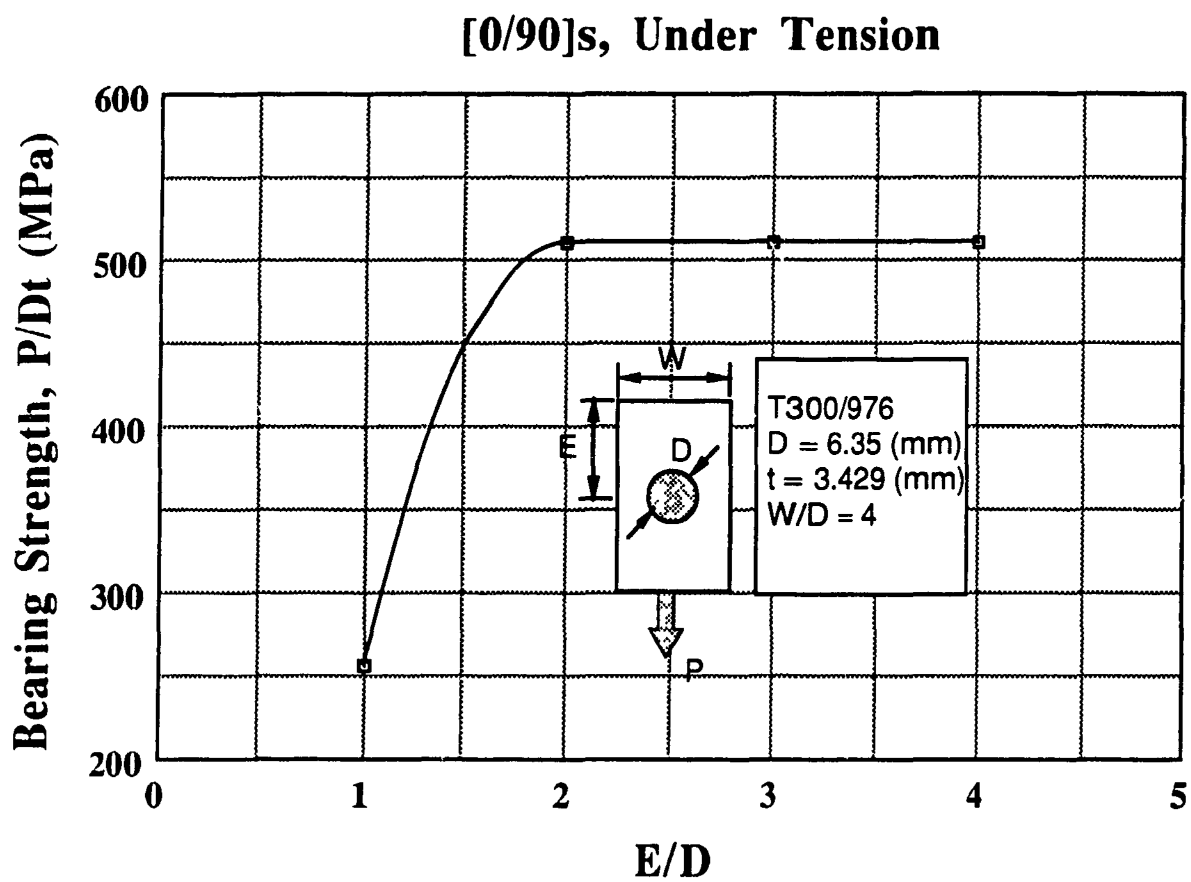


Figure 7.2: Variation of ultimate strength of a cross ply laminate $[0/90]_s$, under tension pin load, with variation of E/D.

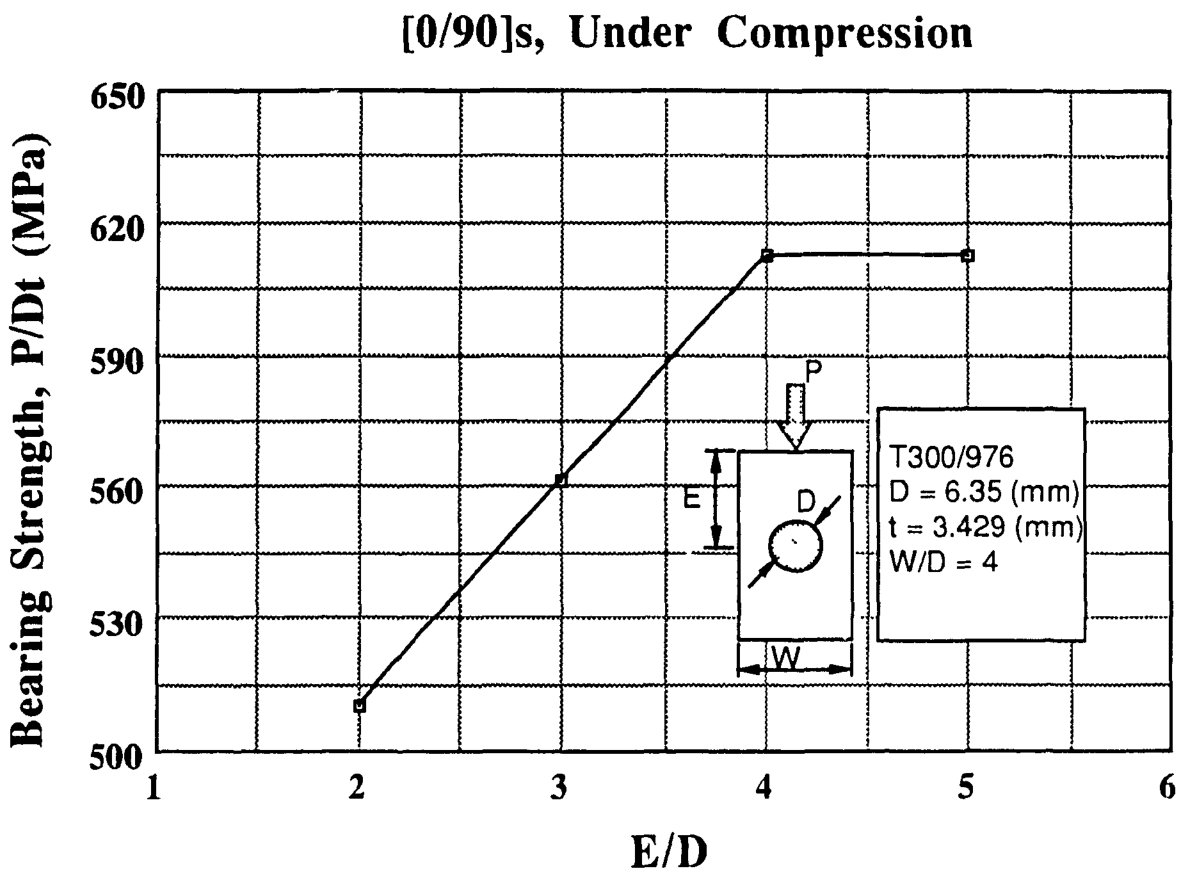


Figure 7.3: Variation of ultimate strength of a cross ply laminate $[0/90]_s$, under compression pin load, with variation of E/D.

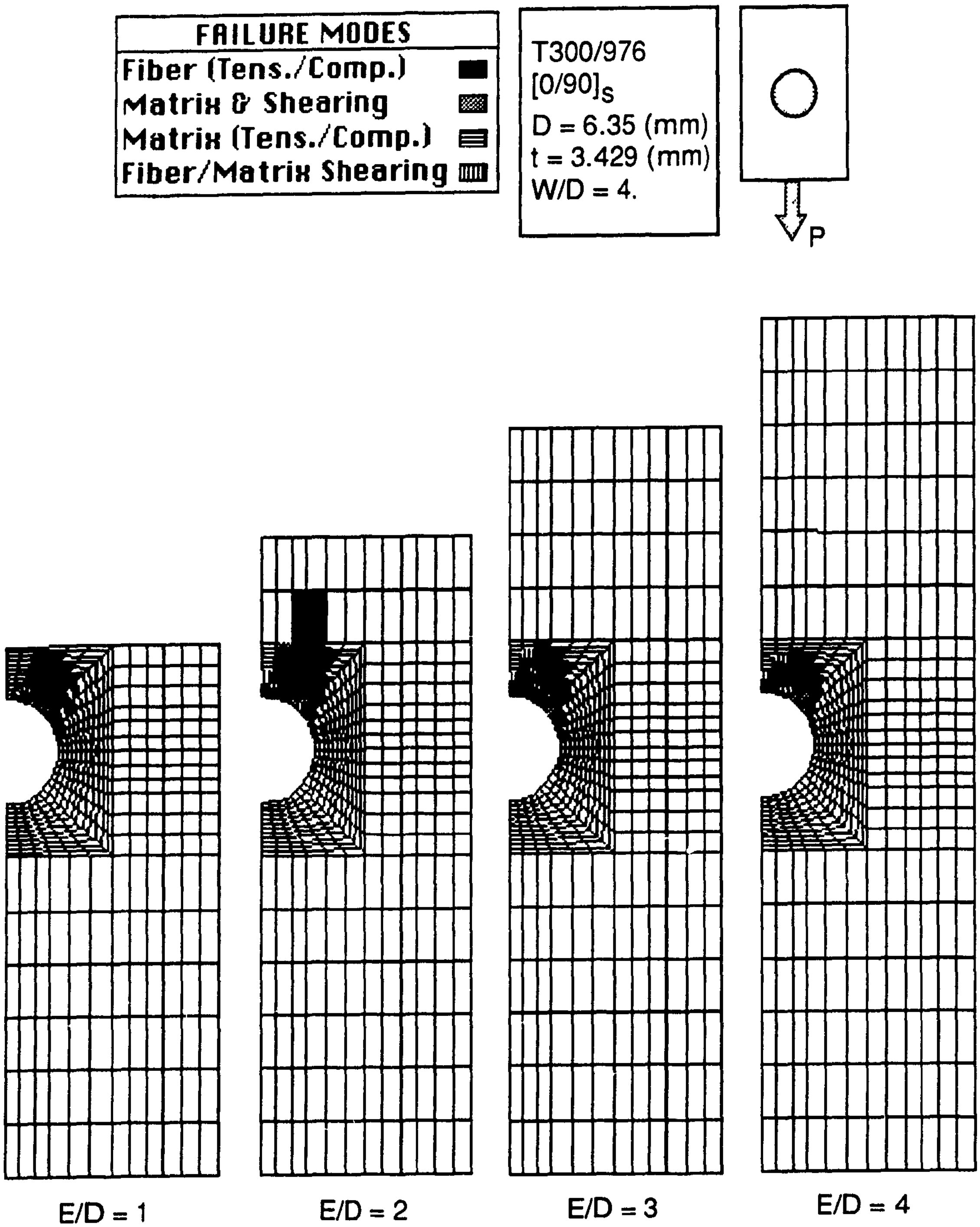


Figure 7.4: Mechanisms of failure of a cross ply laminate $[0/90]_s$ with different E/D, for ply 1, under tension pin load.

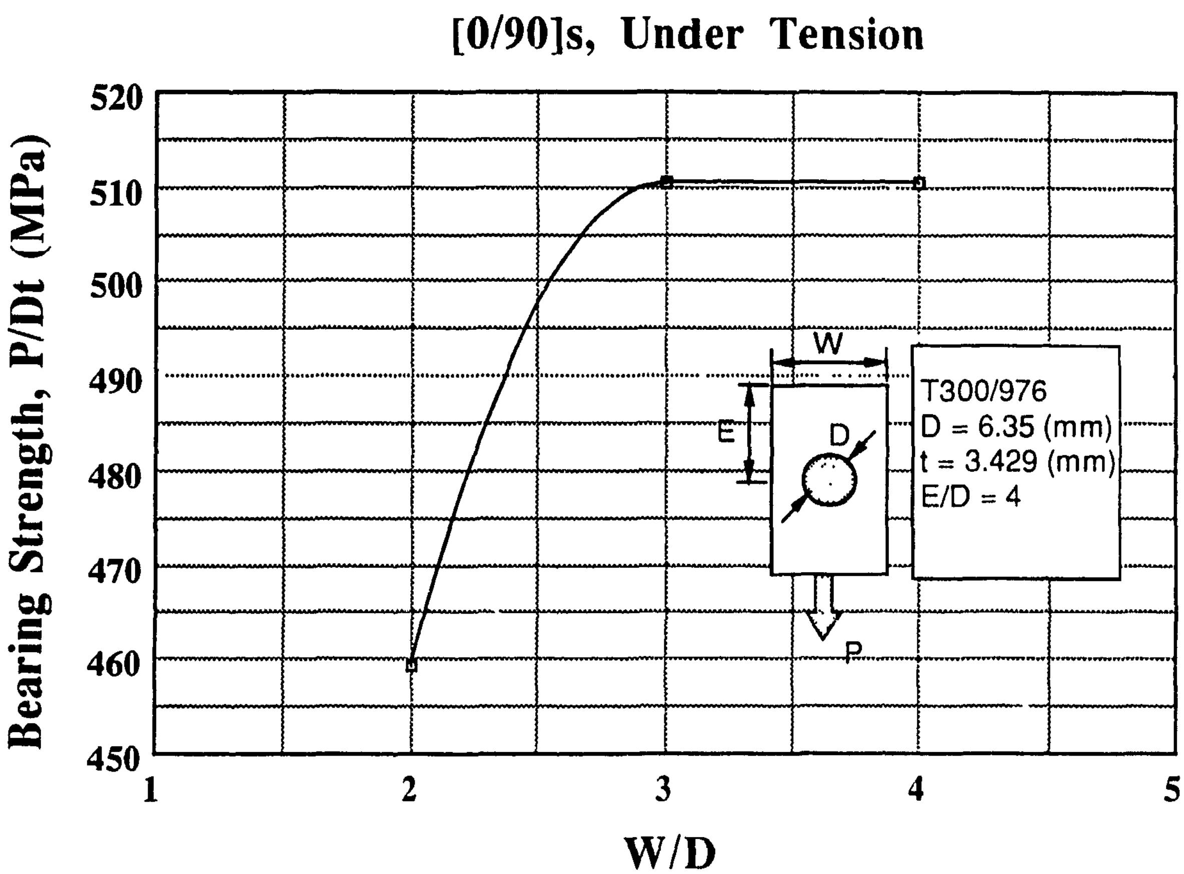


Figure 7.5: Variation of ultimate strength of a cross ply laminate $[0/90]_s$, under tension pin load, with variation of W/D.

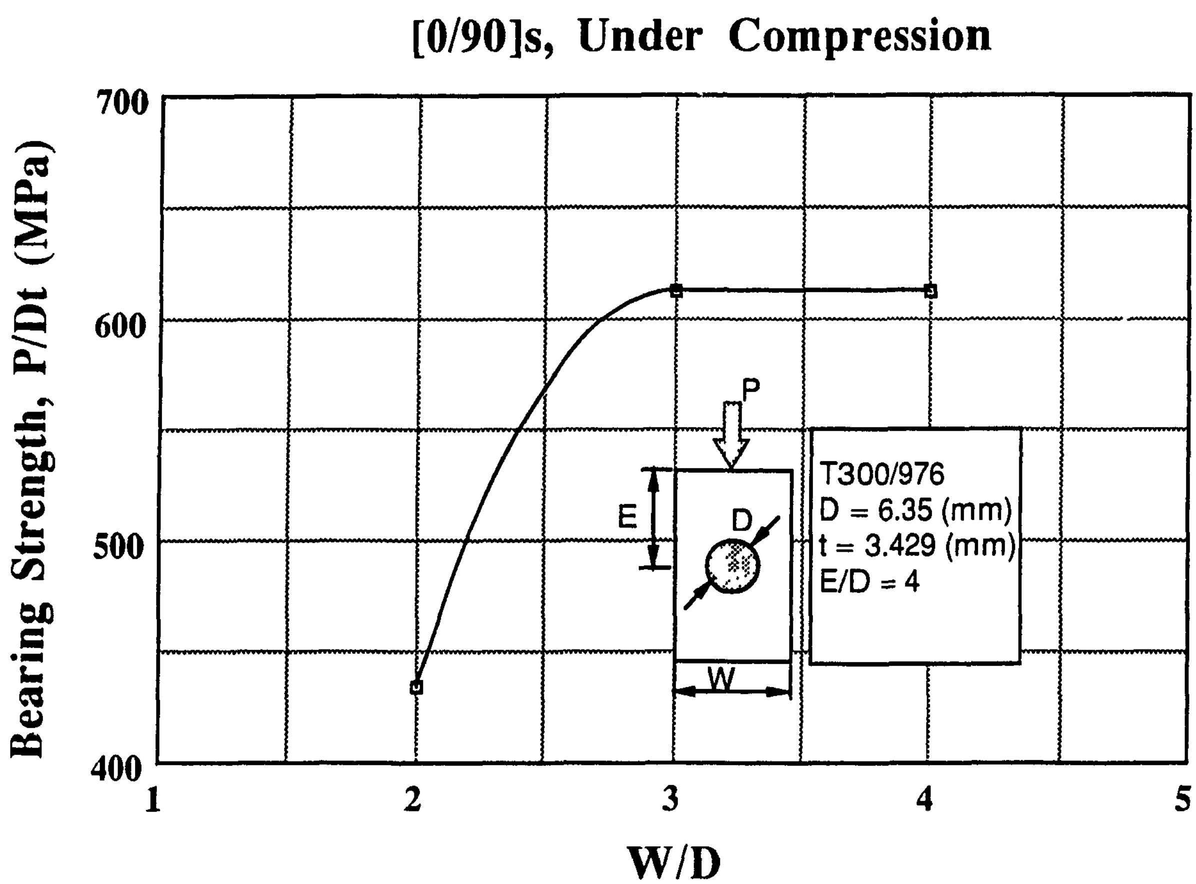


Figure 7.6: Variation of ultimate strength of a cross ply laminate $[0/90]_s$, under compression pin load, with variation of W/D.

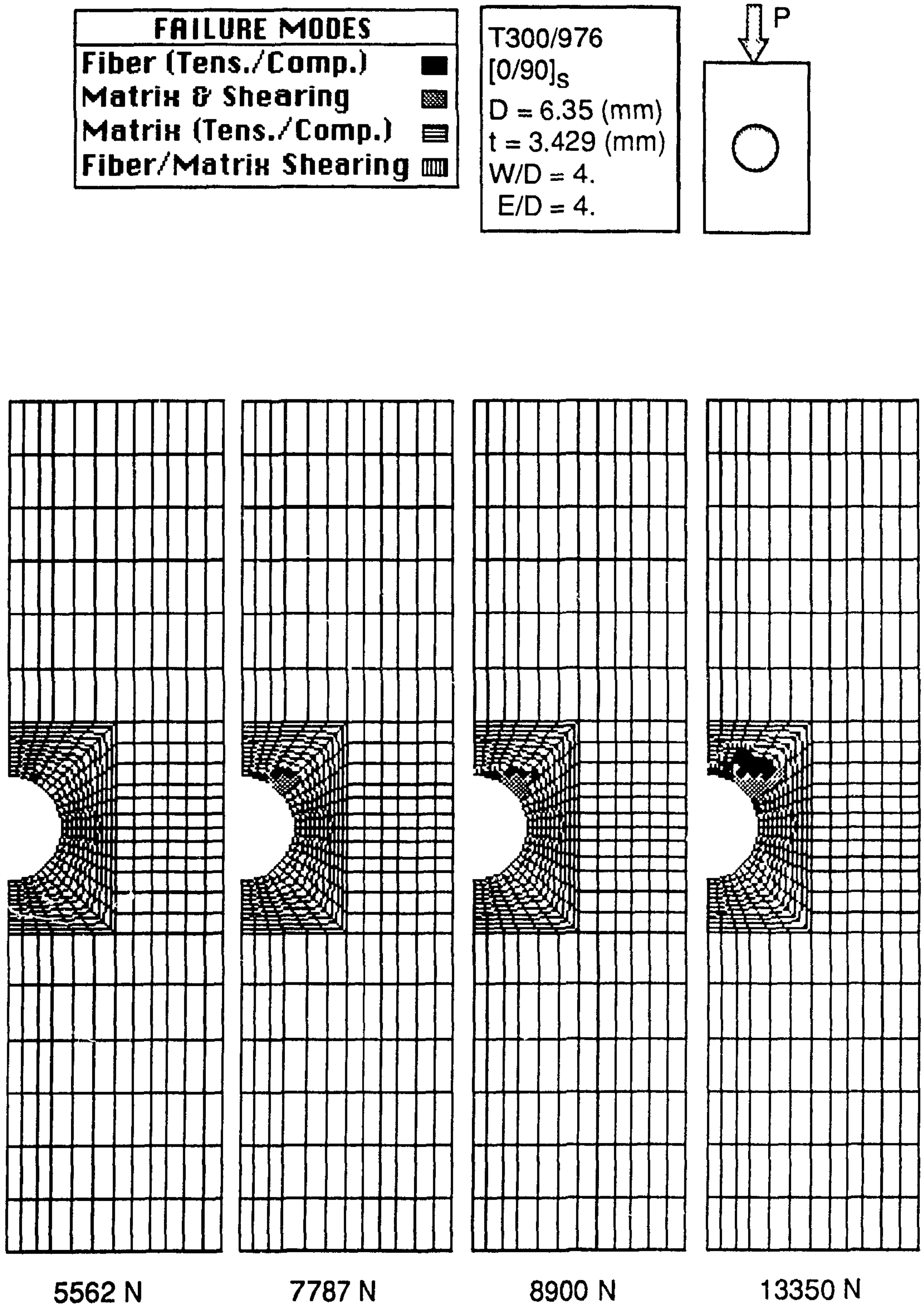


Figure 7.7: Graphical representation of damage propagation of $[0/90]_s$, in ply 1, under compression, at different load levels.

variation of E/D, under tension and compression, is demonstrated in Fig. 7.8 and Fig. 7.9, respectively. The behavior of curves in these figures corresponded completely to experimental evidence.

As an example of this set of data, propagation of failure of the quasi-isotropic laminate, with W/D=4 and E/D=4, under compression, at different load levels, is shown in Fig. 7.10. Damage is initiated on the hole boundary about 0^o from the loading direction at a load of 11125 N. By increasing the load, damage propagates around the hole. At a load of 13350 N, multiple failure modes occur near the hole boundary. Finally, at a load of 20025 N, because of large deflection, the computer program is stopped. As shown in Fig. 7.10, the composite plate failed by a combined bearing and shearout mechanism.

Also by considering a constant edge distance to diameter ratio (E/D), and changing width to diameter ratio (W/D), the behavior of a pin-loaded quasi-isotropic laminate is studied. Variation of strength of the quasi-isotropic laminate with variation of W/D, under tension and compression, is demonstrated in Fig. 7.11 and Fig. 7.12, respectively. The behavior of the curves in these figures corresponded completely to experimental evidence.

As an example of this set of data, propagation of failure of a quasi-isotropic laminate, with W/D=2 and E/D=4, under tension, at different load levels, is shown in Fig. 7.13. Damage is initiated on the hole boundary about 90^o from the loading direction at a load of 6675 N. By increasing the load to 8900 N, damage propagates around the hole. At a load of 11125 N, multiple failure modes occur near the hole boundary. Finally, at a load of 13350 N, because of large deflection, the computer

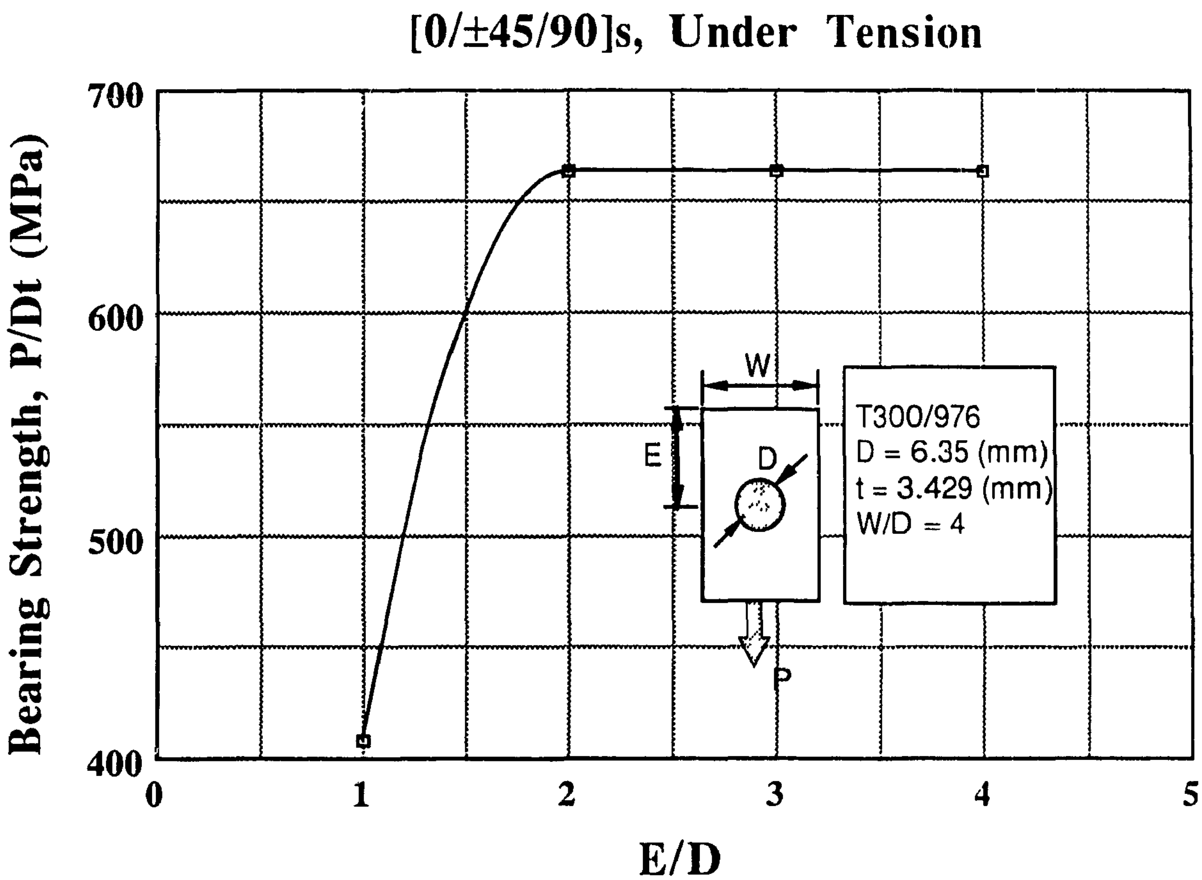


Figure 7.8: Variation of ultimate strength of a quasi-isotropic laminate $[0/\pm45/90]_s$, under tension pin load, with variation of E/D.

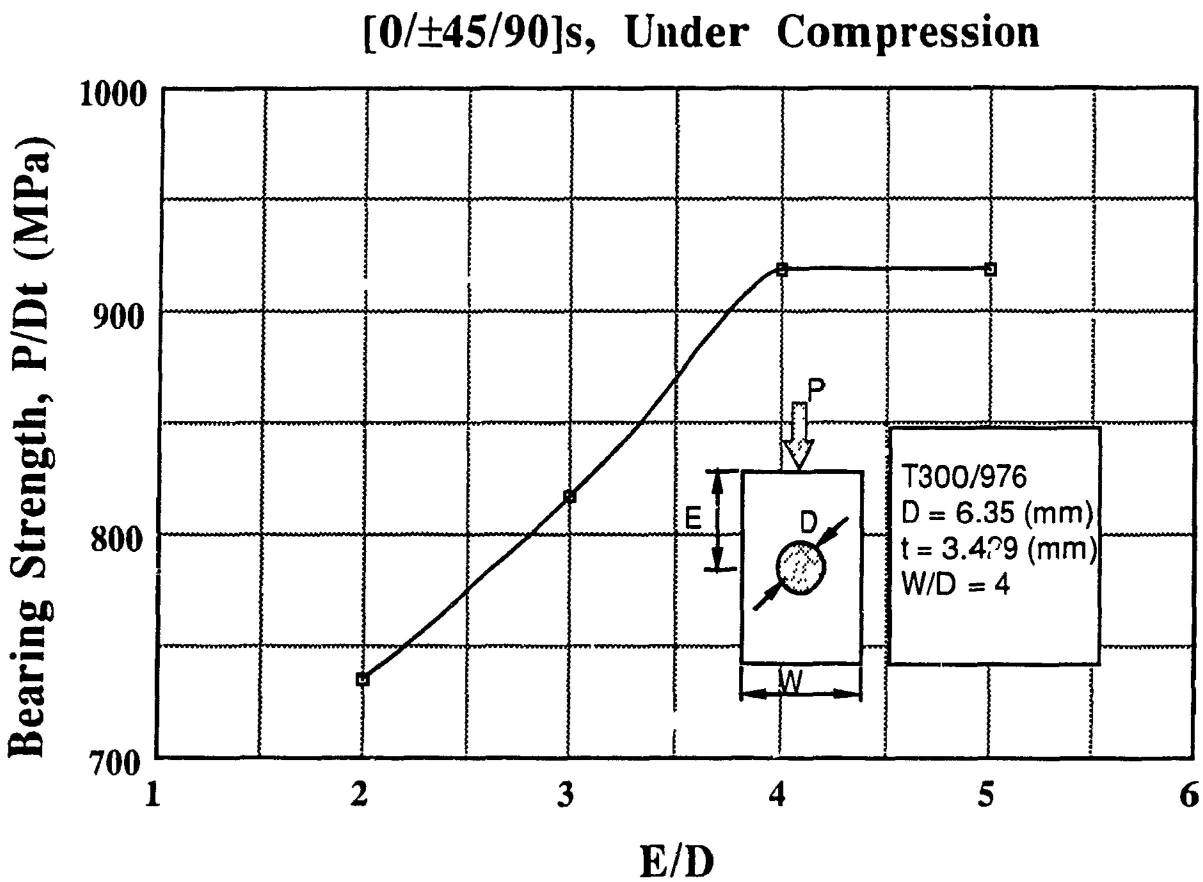


Figure 7.9: Variation of ultimate strength of a quasi-isotropic laminate $[0/\pm 45/90]_s$, under compression pin load, with variation of E/D.

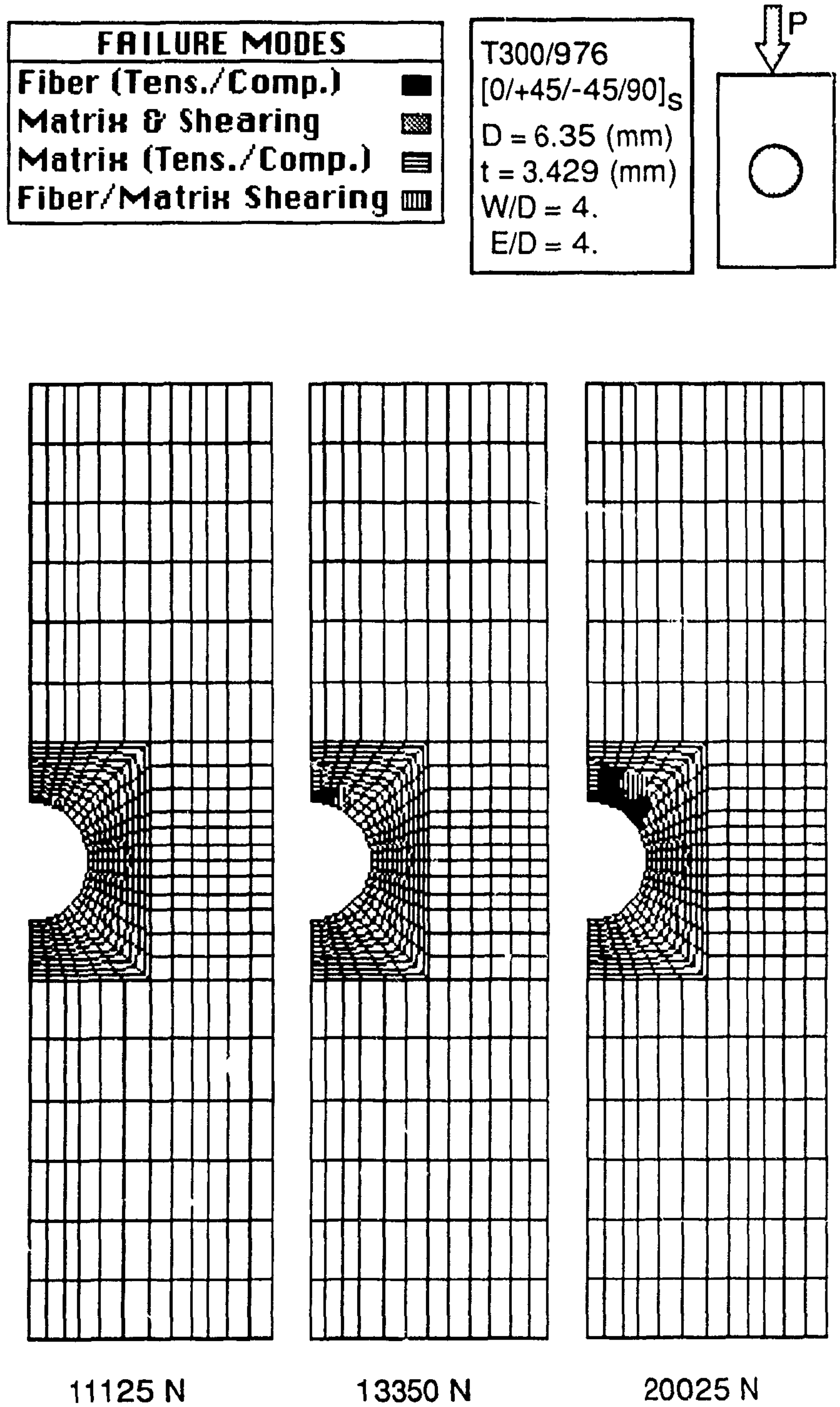


Figure 7.10: Graphical representation of damage propagation of $[0/\pm 45/90]_s$, in ply 1, under compression, at different load levels.

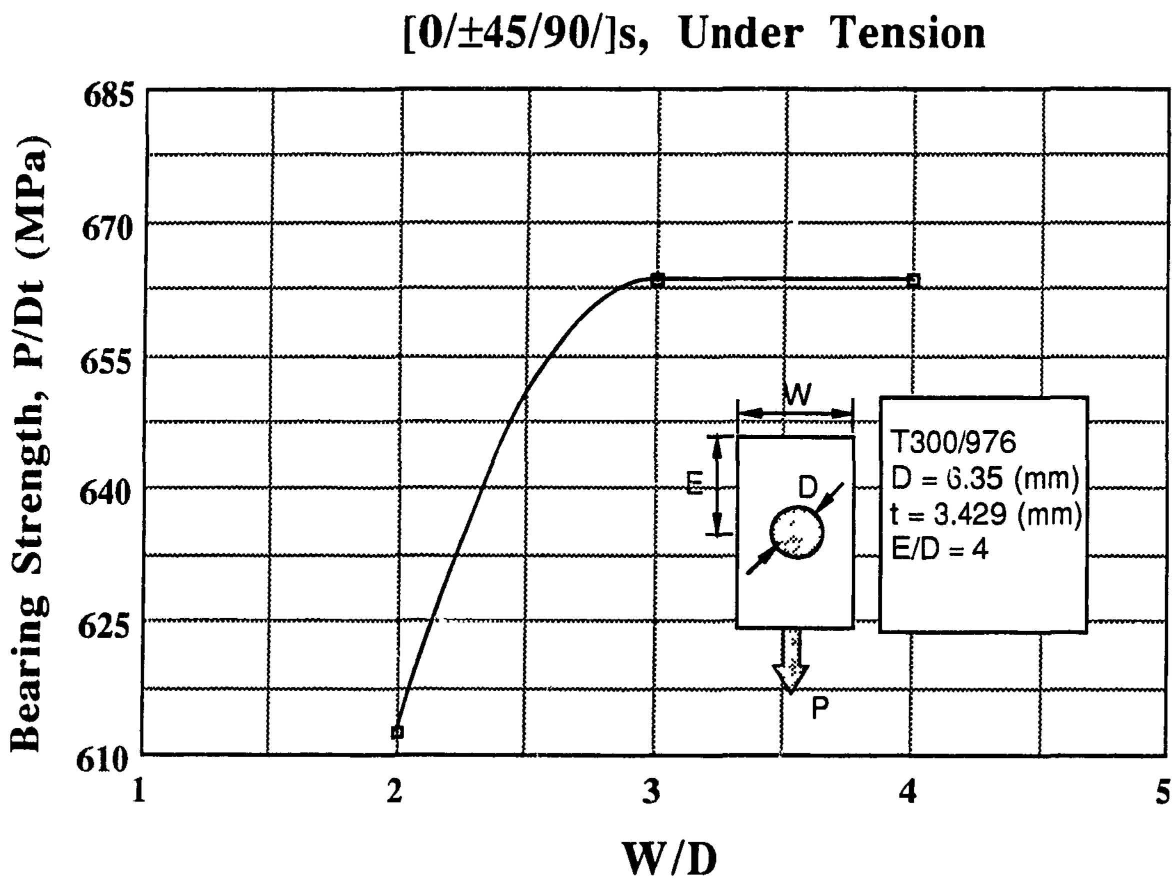


Figure 7.11: Variation of ultimate strength of a quasi-isotropic laminate $[0/\pm45/90]_s$, under tension pin load, with variation of W/D.

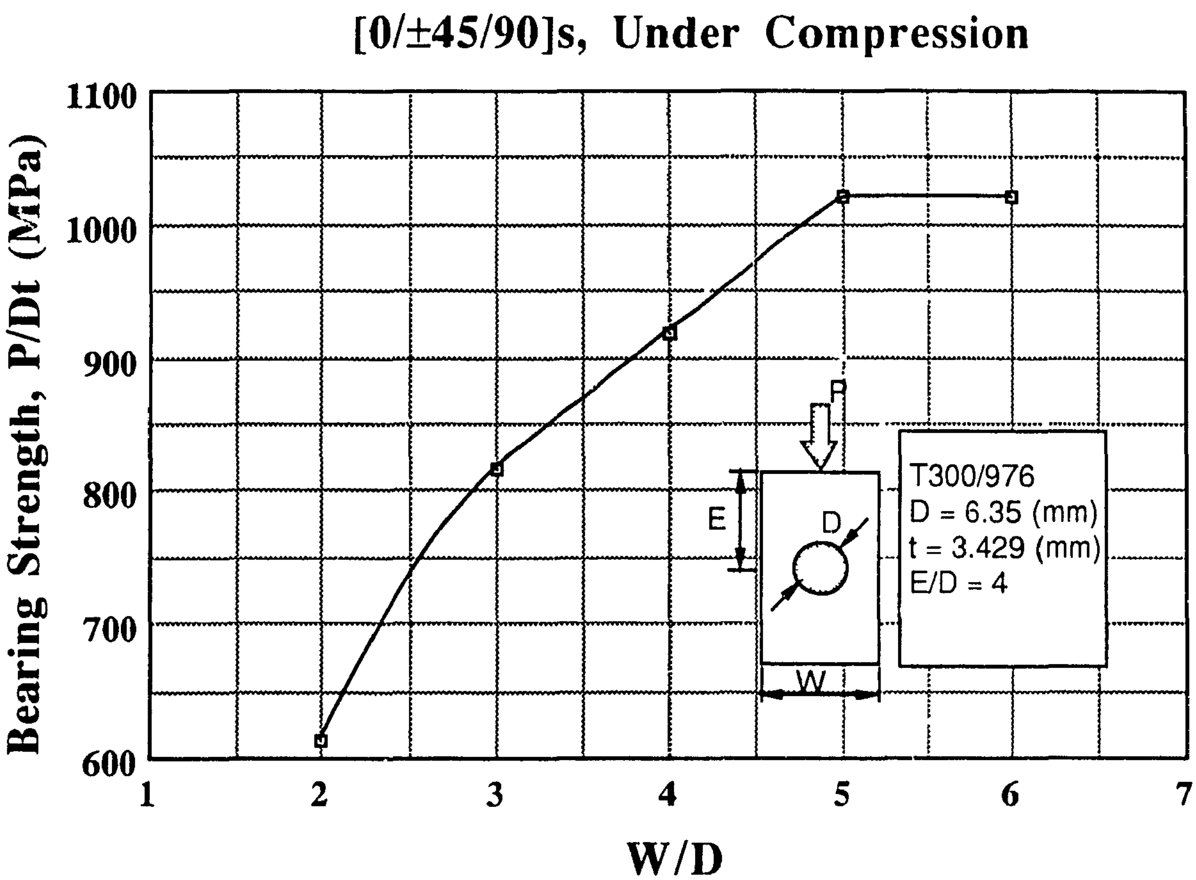


Figure 7.12: Variation of ultimate strength of a quasi-isotropic laminate $[0/\pm45/90]_s$, under compression pin load, with variation of W/D.

program is stopped. As shown in Fig. 7.13, the composite plate failed by the net tension mechanism.

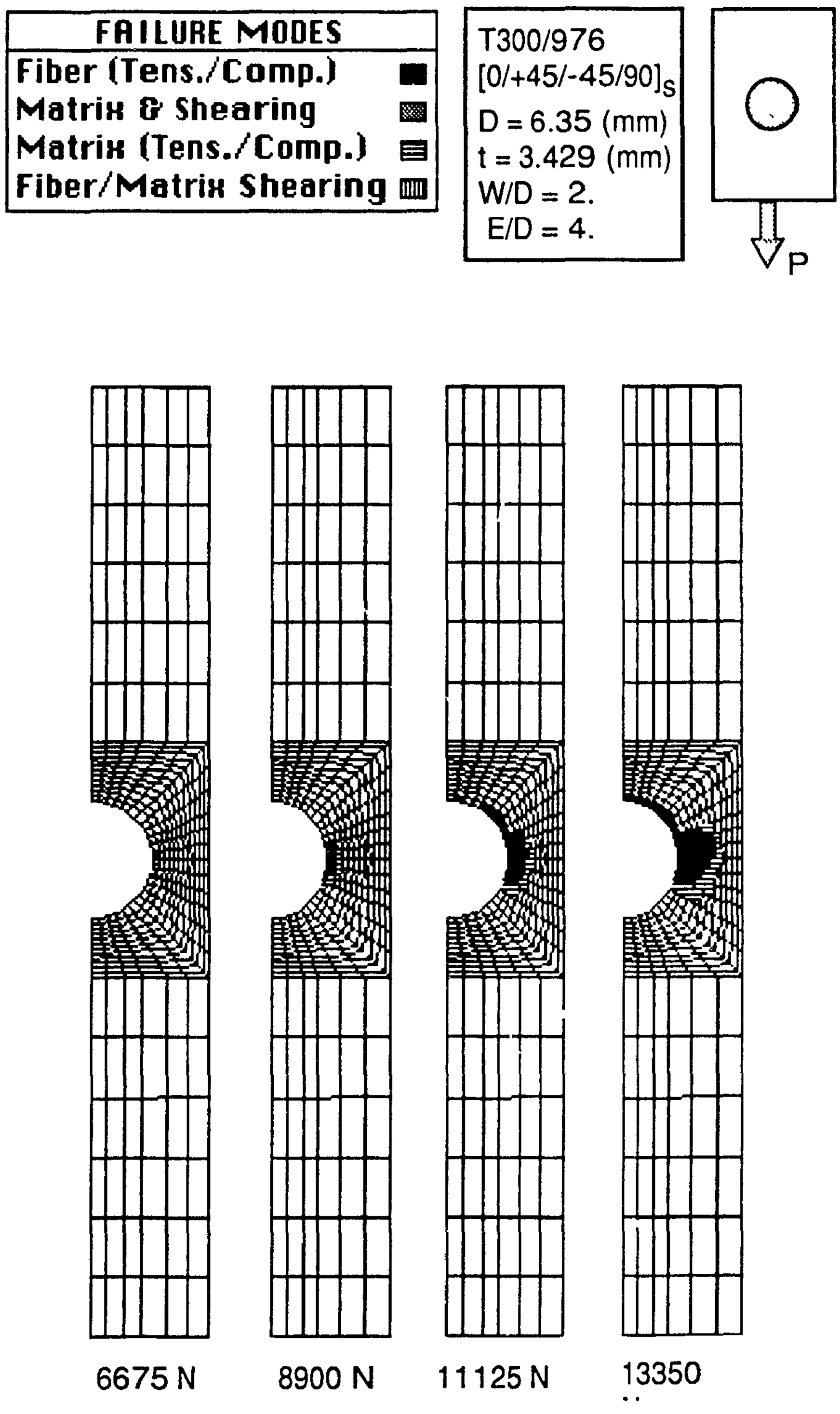


Figure 7.13: Graphical representation of damage propagation of $[0/\pm 45/90]_s$, in ply 4, under tension, at different load levels.

Chapter 8

Conclusions and Recommendations

8.1 Conclusions

This research examines the behavior of pin-loaded composite plates. For this purpose a progressive damage model is developed based on three major parts: stress analysis, failure analysis, and material property degradation rules. The model is capable of assessing damage, evaluating residual strength, and predicting ultimate strength of pin-loaded composite plates. The model is implemented in a user friendly computer code. The progressive damage model and the corresponding computer code are capable of assessing all three different mechanisms of failure, namely, net tension, shearout, and bearing. The model is evaluated by available experimental data. An excellent agreement between predicted and experimental data shows the accuracy of the model.

8.2 Recommendations

In order to extend the capability of the progressive damage model developed in this research, the following recommendations should be considered:

- Failure of pin and bolted joint is a three dimensional phenomenon. Hence for the purpose of accurate study of this phenomenon, an analysis must include through thickness stresses. By this extension, *delamination*, an important problem in mechanically fastened joints, can be taken into account.

- The constitutive equation for shear stress and shear strain of composite materials is highly nonlinear. By implementing this nonlinearity in the stress analysis part of the model, accuracy of the results for special laminate configurations will be improved.

- There are several possible ways to model the bolt/hole contact problem. One way is to assume a cosine load distributed around the hole. It has been proven that this method is not very accurate. Another way is to apply displacement boundary conditions around the hole, which is more accurate than the first one. Performing a complete contact analysis is a third possibility, which is certainly the most accurate method.

- Friction between pin and hole is an important problem to be considered.In a real situation, the coefficient of friction can vary for the different plies of a laminate.

References

[1] De Jong, Th., "Stresses Around Pin-Loaded Holes in Elastically Orthotropic or Isotropic Plates," Journal of Composite Materials, Vol. 11, (July 1977), pp. 313–331.

[2] De Jong, Th., "Stresses Around Pin-Loaded Holes in Composite Materials," Mechanics of Composite Materials, Recent Advances, Pergamon Press, (1982), pp. 339-353.

[3] Zhang, K. D., and Ueng, C. E. S., "Stresses Around a Pin-Loaded Hole in Orthotopic Plates," Journal of Composite Materials, Vol. 18, (Sept. 1984), pp. 432–446.

[4] Mangalgiri, P. D., "Pin-Loaded Holes in Large Orthotropic Plates," AIAA Journal, Vol. 22, No. 10, (Oct. 1984), pp. 1478–1484.

[5] Hyer, M. W., and Klang, E. C., "Contact Stresses in Pin-Loaded Orthotropic Plates," International Journal of Solids & Structures, Vol. 21, No. 9, (1985), pp. 957–975.

[6] Mangalgiri, P.D., and Dattaguru, B., "A Large Orthotropic Plate With Misfit Pin Under Arbitrarily Oriented Biaxial Loading," Composite Structures, Vol. 6, (1986), pp. 271–281.

[7] Hyer, M. W., Klang, E. C., and Cooper, D. E., "The Effects of Pin Elasticity, Clearance, and Friction on the Stresses in a Pin-Loaded Orthotropic Plate," Journal of Composite Materials, Vol. 21, (March 1987), pp. 190–206.

[8] Prabhakaran, R. "Photoelastic Investigation of Bolted Joints in Composites," Composites, Vol. 13, (July 1982), pp. 253–256.

[9] Hyer, M. W., and Liu, D. H., "Photoelastic Determination of Stresses in Multiple-Pin Connectors," Experimental Mechanics, Vol. 23, (Sept. 1983), pp. 249–256.

[10] Hyer, M. W., and Liu, D. H., "Stresses in Pin-Loaded Plates," Mechanical Characterization of Load Bearing Fiber Composite Laminates, Proceedings of the European Mechanics Colloquium 182, Brussels, Belg., (Aug. 1984), pp.179–190.

[11] Hyer, M. W., and Liu, D. H., "Stresses in Pin-Loaded Plates: Photoelastic Results," Journal of Composite Materials, Vol. 19, (March 1985), pp. 138–153.

[12] Prabhakaran, R., and Naik, R. A., "Investigation of Non-Linear Contact for a Clearance-Fit Bolt in a Graphite/Epoxy Laminate," Composite Structures, Vol. 6, (1986), pp. 77–85.

[13] Prabhakaran, R., and Naik, R. A., "A Fiber Optic Technique to Assess the Influence of Interfacial Infraction in a Pin-Loaded Hole," ISA Transactions, Vol. 28, No. 2, (1989), pp. 35-39.

[14] Rao, A. K., "Elastic Analysis of Pin Joints," Computers & Structures, Vol. 9, (1978), pp. 125–144.

[15] Wong, C. M. S., and Matthews, F. L., "A Finite Element Analysis of Single and Two-Hole Bolted Joints in Fibre Reinforced Plastic," Journal of Composite Materials, Vol. 15, (Sept. 1981), pp. 481–491.

[16] Crews, J. H. Jr. and Hong, C. S. "Stress-Concentration Factors for Finite Orthotropic Laminates With a Pin-Loaded Hole,"NASA TP-1862, National Aeronautics and Space Administration, USA, (May 1981).

[17] Wilkinson, T. L. and Rowlands, R. E., "Analysis of Mechanical Joints in Wood," Experimental Mechanics, Vol. 21, (Nov. 1981), pp. 408–414.

[18] Matthews, F. L., Wong, C. M., and Chryssafitis, S., "Stress Distribution Around a Single Bolt in Fiber-Reinforced Plastic," Composites, Vol. 13, (July 1982), pp. 316–322.

[19] Rahman, M.U., Rowlands, R. E., Cook, R. D., and Wilkinson, T. L., "An Iterative Procedure for Finite-Element Stress Analysis of Frictional Contact Problems," Computers & Structures, Vol. 18, No. 6, (1984), pp. 947–954.

[20] Mangalgiri, P. D., Dattaguru, B., Rao, A. K., "Finite Element Analysis of Moving Contact in Mechanically Fastened Joints," Nuclear Engineering and Design, Vol.78, (1984), pp. 303–311.

[21] Naik, R. A., and Crews, J. H. Jr., "Stress Analysis Method for a Clearance-Fit Bolt Under Bearing Loads," AIAA Journal, Vol. 24, No. 8 (Aug. 1986), pp. 1348–1353.

[22] Ericksson, L. I., "Contact Stresses in Bolted Joints of Composite Laminates," Composite Structures, Vol. 6, (1986), pp. 57–75.

[23] Tsujimoto, Y., and Wilson, D., "Elasto-Plastic Failure Analysis of Composite Bolted Joints," Journal of Composite Materials, Vol. 20, (May 1986), pp. 236–252.

[24] Yogeswaren, E. K., and Reddy, J. N., "A Study of Contact Stresses in Pin-Loaded Orthotropic Plates," Computers & Structures, Vol. 30, No. 5, (1988), pp. 1067–1077.

[25] Marshall, I. H., Arnold, W. S., Wood, J., and, Mousley, R. F., "Observations on Bolted Connections in Composite Structures," Composite Structures, Vol. 13, (1989), pp. 133–151.

[26] Ramamurthy, T. S., "Recent Studies on the Behaviour of Interference Fit Pins in Composite Plates," Composite Structures, Vol. 13, (1989), pp. 81-99.

[27] Ramamurthy, T. S., "New Studies on the Effect of Bearing Loads in Lugs With Clearance Fit Pins," Composite Structures, Vol. 11, (1989), pp. 135-150.

[28] Wilkinson, T. L., Rowlands, R. E., and Cook, R. D., "An Incremental Finite-Element Determination of Stresses Around Loaded Holes in Wood Plates," Computers & Structures, Vol. 14, NO. 1- 2, (1981), PP. 123–128.

[29] Rowlands, R. E., Rahman, M.U., Wilkinson, T. L., and Chiang, Y. I., "Single- and Multiple-Bolted Joints in Orthotropic Materials," Composites, Vol. 13, (July 1982), pp. 273–279.

[30] Hyer, M. W., and Liu, D. H., "Stresses in a Quasi- Isotropic Pin-Loaded Connector Using Photoelasticity," Experimental Mechanics, Vol. 21, (March 1984), pp. 48–53.

[31] Vinson, J. R., and Sierakowski, R. L., "Joining of Composite Material Structures," The Behavior of Structures Composed of Composite Materials, by Martinus Nijhoff Publishers, Dordrecht, 1986, pp. 239–283.

[32] Vinson, J. R., "Mechanical Fastening of Polymer Composites," Polymer Engineering and Science, Vol. 29, No. 19, (Oct. 1989), pp 1332–1339.

[33] Ojalvo, U. I., 'Survey of Mechanically fastened Splice-Joint Analyses," Advances in Joining Technology, Ed. by Burk, J. J., Groun, A. E , and Tarpinian A , Chestnut Hill, Army Material Technology Conference (1976), pp. 379-403.

[34] Oplinger, D. W., "Stress Analysis of Composite Joints," Advances in Joining Technology, Ed. by Burk, J. J., Groun, A. E., and Tarpinian A., Chestnut Hill, Army Material Technology Conference (1976), pp. 405–451.

[35] Stockdale, J. H., and Matthews, F. L, "The Effect of Clamping Pressure on Bolt Bearing Loads in Glass Fiber-Reinforced Plastics," Composites, (Jan. 1976) PP. 34–38.

[36] Quinn, W. J., and, Matthews, F. L, "The Effect of Stacking Sequence on the Pin-Bearing Strength in Glass Fibre Reinforced Plastic," Journal of Composite Materials, Vol. 11, (Oct. 1977), pp. 139–145.

[37] Matthews, F. L., Hirst, I. R., "The Variation of Bearing Strength With Load Direction," Symposium: Joining in Fibre Reinforced Plastics, (Sept. 1978), Imperial College, London, pp. 126– 132.

[38] Johnson, M., and Matthews, F. L., "Determination of Safety Factors for Use When Designing Bolted Joints in GRP," Composites, Vol. 10, (July 1979), pp. 73–76.

[39] Hyer, M. W., and Lightfoot, M. C., "Ultimate Strength of high-load-capacity Composite Bolted Joints," Composite Materials: Testing and Design (Fifth Volume), ASTM STP 674, Tsai, S. W., Ed., American Society for Testing and Materials, (1979), pp. 118–136.

[40] Godwin, E. W., and Matthews, F. L., "A Review of the Strength of Joints in Fibre-Reinforced Plastics," Composites, Vol. 11, (July 1980), pp. 155–160.

[41] Tang, S., "Failure of Composite Joints Under Combined Tension and Bolt Loads," Journal of Composite Materials, Vol. 15, (July 1981), pp. 329–335.

[42] Wichorek, G. R., "Experimental Data on Single-Bolt Joints in Quasi-Isotropic Graphite/Polyimide Laminates," NASA Technical Paper 2015, (May 1982).

[43] Matthews, F. L., Roshan, A. A., Phillips, L. N., "The Bolt Bearing Strength of Glass/Carbon Hybrid Composites," Composites, Vol. 13, (July 1982), pp. 225–227.

[44] Collings, T. A., "On the Bearing Strengths of CFRP Laminates," Composites, Vol. 13, (July 1982), pp. 241–252.

[45] Godwin, E. W., "Strength of Multi-Bolt Joints in GRP," Composites, Vol. 13, (July 1982), pp. 268–272.

[46] Collings, T. A., Beauchamp, M. J., "Bearing Deflection Behavior of a Loaded Hole in CFRP," Composites, Vol. 15, (July 1984), pp. 33–38.

[47] Kretsis, G., and Matthews, F. L., "The Strength of Bolted Joints in Glass Fibre/Epoxy Laminates," Composites, Vol. 16, (April 1985), pp. 92–102.

[48] Theuer, A., and Arendts, F. J., "Bolt Bearing Strength and Notch Sensitiveness of Carbon-Carbon," Fourth European Conference on Composite materials, Stuttgart, (1990), pp. 487-492.

[49] Akay, M., "Bearing Strength of Carbon Fibre/Epoxy Composites Under Static and Dynamic Loading," Fourth European Conference on Composite materials, Stuttgart, (1990), pp. 883-889.

[50] Eriksson, I., "On the Bearing Strength of Bolted Graphite/Epoxy Laminates," Journal of Composite Materials, Vol. 24, (Dec. 1990), pp. 1246–1269.

[51] Ueng, C. E. S. and Zhang, K. D., "Strength Prediction of a Mechanically Fastened Joint in Laminated Composites," AIAA Journal, Vol. 23, No. 11, (1985), pp. 1832–1834.

[52] Smith, P. A., Ashby, M. F., and Pascoe, K. J., "Modelling Clamp-Up Effects in Composite Bolted Joints," Journal of Composite Materials, Vol. 21, (Oct. 1987), pp. 878–897.

[53] Waszczak, J. P., and Cruse, T. A., "Failure Mode and Strength Prediction of Anisotropic Bolt Bearing Specimens" Journal of Composite Materials, Vol. 5, (July. 1971), pp. 421–425.

[54] Humphris, N. P., "Migration of the Point of Maximum Stress in a Laminated Composite Lug Structure - A Stepwise Approach," Symposium: Joining in Fibre Reinforced Plastics, (Sept. 1978), Imperial College, London, pp. 79–86.

[55] Agarwal, B. L., "Static Strength Prediction of Bolted Joints in Composite Material," AIAA Journal, Vol. 18, No. 11, (Nov. 1980), pp. 1371–1375.

[56] Soni, S. R., "Failure Analysis of Composite Laminated With a Fastener Hole," Joining of Composite Materials, ASTM STP 749, Kedward, K. T., Ed., Am. Soc. for Testing and Materials, (1981) pp. 145–164.

[57] Chang, F. K., Scott, R. A., and Springer G. S., "Strength of Mechanically Fastened Composite Joints," Journal of Composite Materials, Vol. 16, (Sept. 1982), pp. 470–494.

[58] Chang, F. K., Scott, R. A., and Springer G. S., "Failure of Composite Laminates Containing Pin Loaded Holes – Method of Solution," Journal of Composite Materials, Vol. 18, (Sept. 1984), pp. 255–278.

[59] Chang, F. K., Scott, R. A., and Springer G. S., "Design of Composite Laminates Containing Pin Loaded Holes," Journal of Composite Materials, Vol. 18, (Sept 1984), pp. 279–239.

[60] Chang, F. K., Scott, R. A., and Springer G. S., "Failure Strength of Nonlinearly Elastic Composite Laminates Containing a Pin Loaded Hole," Journal of Composite Materials, Vol. 18, (Sept. 1984), pp. 464–477.

[61] Chang, F. K., "The Effect of Pin Load Distribution on the Strength of Pin Loaded Holes in Laminated Composites," Journal of Composite Materials, Vol. 20, (Oct. 1986), pp. 401–408.

[62] Chang, F. K., and , Chang, K. Y., "Post-Failure Analysis of Bolted Composite Joints in Tension or Shear-Out Mode Failure," Journal of Composite Materials, Vol. 21, (Oct. 1987), pp. 809–833.

[63] Arnold, W. S., Marshall, I. H., and Wood, J., "Optimum Design Considerations for Mechanically Fastened Composite Joints," Composite Structures, Vol 16, (1990), pp. 85–101.

[64] Lessard, L. B., "Bearing Failure of Composite Pinned Joints Using Progressive Damage Modeling," ICCM/8, International conference on composite materials, Honolulu Hawaii, (July 15–19, 1991).

[65] Klang, E. D., and De Jong Th., "Elastic Response and Damage Accumulation of Composite Materials Used in Pinned-Connections," Mechanical Characterization of Load Bearing Fiber Composite Laminates, Proceedings of the European Mechanics Colloquium 182, Brussels, Belg., (Aug. 1984), pp.132–138.

[66] Oplinger, D. W., "On the Structural Behavior of Mechanically Fastened Joints in Composite Structures," in Proceedings of Fourth Conference on Advanced Composites in Structural Design, (1978), pp. 575–602.

[67] Hart-Smith , L. J., "Mechanically Fastened Joints for Advanced Composite - Phenomenological Considerations and Simple Analysis," in Proceedings of Fourth Conference on Advanced Composites in Structural Design, (1978), pp. 543–574.

[68] Smith, P. A., and Pascoe, K. J., "The Effect of Stacking Sequence on the Bearing Strengths of Quasi-Isotropic Composite Laminates," Composite Structures, Vol. 6, (1986), pp. 1–20.

[69] Smith, P. A., Pascoe, K. J., Polak, C., and Stroud, D. O., "The Behaviour of Single-Lap Bolted Joints in CFRP Laminates," Composite Structures, Vol. 6, (1986), pp. 41–55.

[70] Naik, R. A., and Crews, J. H. Jr., "Ply-Level Failure Analysis of a Graphite/Epoxy Laminate Under Bearing-Bypass Loading," Composite Materials: Testing and Design (Ninth Volume), ASTM STP 1059, Garbo, S. P., Ed., American Society for Testing and Materials, Philadelphia, (1990), pp. 191–211.

[71] Tsiang, T. H., and Mandell, J. F., "Damage Development in Bolt Bearing of Composite Laminates," AIAA Journal, Vol. 23, No. 10, (1985), pp 1570 1577

[72] Serabian, S. M., and Oplinger, D. W., "An Experimental and Finite Element Investigation into the Mechanical Response of 0/90 Pin-Loaded Laminates," Journal of Composite Materials, Vol. 21, (Oct. 1987), pp 631 649.

[73] Crews, J. H. Jr., and Naik, R. A., "Combined Bearing and Bypass Loading on a Graphite/Epoxy Laminate," Composite Structures, Vol. 6, (1986), pp. 21 40.

[74] Conti, P., "Influence of Geometric Parameters on the Stress Distribution Around a Pin-Loaded Hole in a Composite Laminate," Composite Science and Technology, Vol. 25, (1986), 83–101.

[75] Jurf, R. A., and Vinson, J. R., "Failure Analysis of Bolted Joints in Composite Laminates," Composite Materials. Testing and Design (Ninth Volume), ASTM STP 1059, Garbo, S. P., Ed., American Society for Testing and Materials, Philadelphia, (1990), pp. 165–190.

[76] Tsai, M. Y., and Morton, J., "Stress and Failure Analysis of a Pin-Loaded Composite Plate: An Experimental Study," Journal of Composite Materials, Vol. 24, (Oct. 1990), pp. 1101-1120.

[77] I-DEAS, Integrated Design Engineering Analysis Software, User Manual, Structural Dynamics Research Corporation, 1988.

[78] Nahas, M. N., "Survey of Failure and Post-Failure Theories of Laminated Fiber-Reinforced Composites," Journal of Composites Technology & Research, Vol. 8, No. 4, (1986), pp. 138–153.

[79] Hashin, Z., "Failure Criteria for Unidirectional Fiber Composites," Journal of Applied Mechanics, Vol. 47, (June 1980), pp. 329–334.

[80] Chang, F. K., and Lessard, L. B., "Damage Tolerance of Laminated Composites Containing an Open Hole and Subjected to Compressive Loading: Part I–Analysis," Journal of Composite Materials, Vol. 25, (1991), pp. 44–46.

[81] Chou, S. C., Orringer, O., and Rainey, J. H., "Post Failure Behavior of Laminates: II - Stress Concentration," Journal of Composite Materials, Vol. 11, (Jan. 1977), pp. 71–78.

[82] Sandhu, R. S., Sendeckyj, G. P., and Gallo, R. L., "Modeling of the Failure process in Notched Laminates," Mechanics of Composite Materials: Recent Advances, Hashin, Z., and Herakovitch, C. T., Eds., IUTAM Symposium on Mechanics of Composite Materials, Blacksburg, VA, (Aug. 1982), pp. 179–189.

[83] Van Siclen, R. C., "Evaluation of Bolted Joints in Graphite/Epoxy," Proceedings of the Army Symposium on Solid Mechanics: Role of Mechanics in the Design of Structural Joints, (Sept. 1974), pp. 120–138.

Appendix A

Transformation Equations

In isotropic materials, physical properties do not change with reference coordinates. Principal axes are useful for such materials and maximum principal strains and stresses are applicable in failure theories.

In composite materials, properties are not isotropic, thus they change with reference coordinates. Transformation from one coordinate system to another is a useful tool for stress and failure analysis of composite materials. For a laminate there are two coordinate systems:

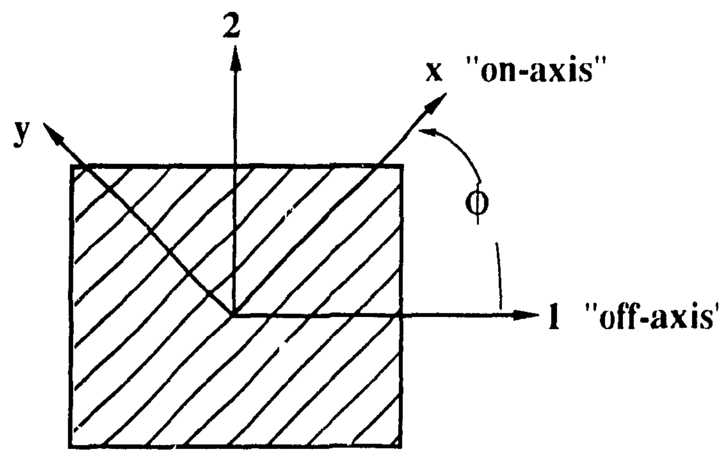


Figure A.1: On and off-axis coordinate systems.

- *Off-axis coordinate system.* This is a common coordinate system for all plies of a laminate (Fig. A.1). The symbols "1" and "2" are used as labels for this coordinate system. In this thesis, load is applied in the 2-axis direction.

- *On-axis coordinate system.* For each ply this coordinate system is unique and coincides with the fiber direction. Thus the on-axis coordinate system is not a common coordinate system for all plies. The symbols "x" and "y" are used as labels for this coordinate system. (Fig. A.1)

To transform strains and stresses from off-axis to on-axis coordinate system, the formulations are respectively

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{Bmatrix} = [T]_\varepsilon \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} \tag{A.1}$$

Where

$$[T]_\varepsilon = \begin{bmatrix} cos^2\phi & sin^2\phi & cos\phi sin\phi \\ sin^2\phi & cos^2\phi & -cos\phi sin\phi \\ -2cos\phi sin\phi & 2cos\phi sin\phi & cos^2\phi - sin^2\phi \end{bmatrix} \tag{A.2}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = [T]_\sigma \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} \tag{A.3}$$

where

$$[T]_\sigma = \begin{bmatrix} cos^2\phi & sin^2\phi & 2cos\phi sin\phi \\ sin^2\phi & cos^2\phi & -2cos\phi sin\phi \\ -cos\phi sin\phi & cos\phi sin\phi & cos^2\phi - sin^2\phi \end{bmatrix} \tag{A.4}$$

Composite laminates are normally made of off-axis in addition to on-axis plies, thus the stiffness of unidirectional composites with arbitrary ply orientation is important. To derive the off-axis modulus of unidirectional composites, start with the on-axis constitutive relation:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = [Q]_{x-y} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{Bmatrix} \quad (A.5)$$

Where

$$[Q]_{x-y} = \begin{bmatrix} \frac{E_x}{1-\nu_x\nu_y} & \frac{E_y\nu_x}{1-\nu_x\nu_y} & 0 \\ \frac{E_x\nu_y}{1-\nu_x\nu_y} & \frac{E_y}{1-\nu_x\nu_y} & 0 \\ 0 & 0 & E_s \end{bmatrix} \quad (A.6)$$

By substituting Eqs. A.1 and A.3 into Eq. A.5, the latter equation becomes

$$[T]_\sigma \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = [Q]_{x-y}[T]_\varepsilon \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} \quad (A.7)$$

By pre-multiplying Eq. A.7 on both sides by $[T]_\sigma^{-1}$, the following is obtained

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = [T]_\sigma^{-1}[Q]_{x-y}[T]_\varepsilon \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} \quad (A.8)$$

By comparing Eqs. A.8 and A.5 the off-axis stiffness of unidirectional composites is extracted

$$[Q]_{1-2} = [T]_\sigma^{-1}[Q]_{x-y}[T]_\varepsilon \quad (A.9)$$

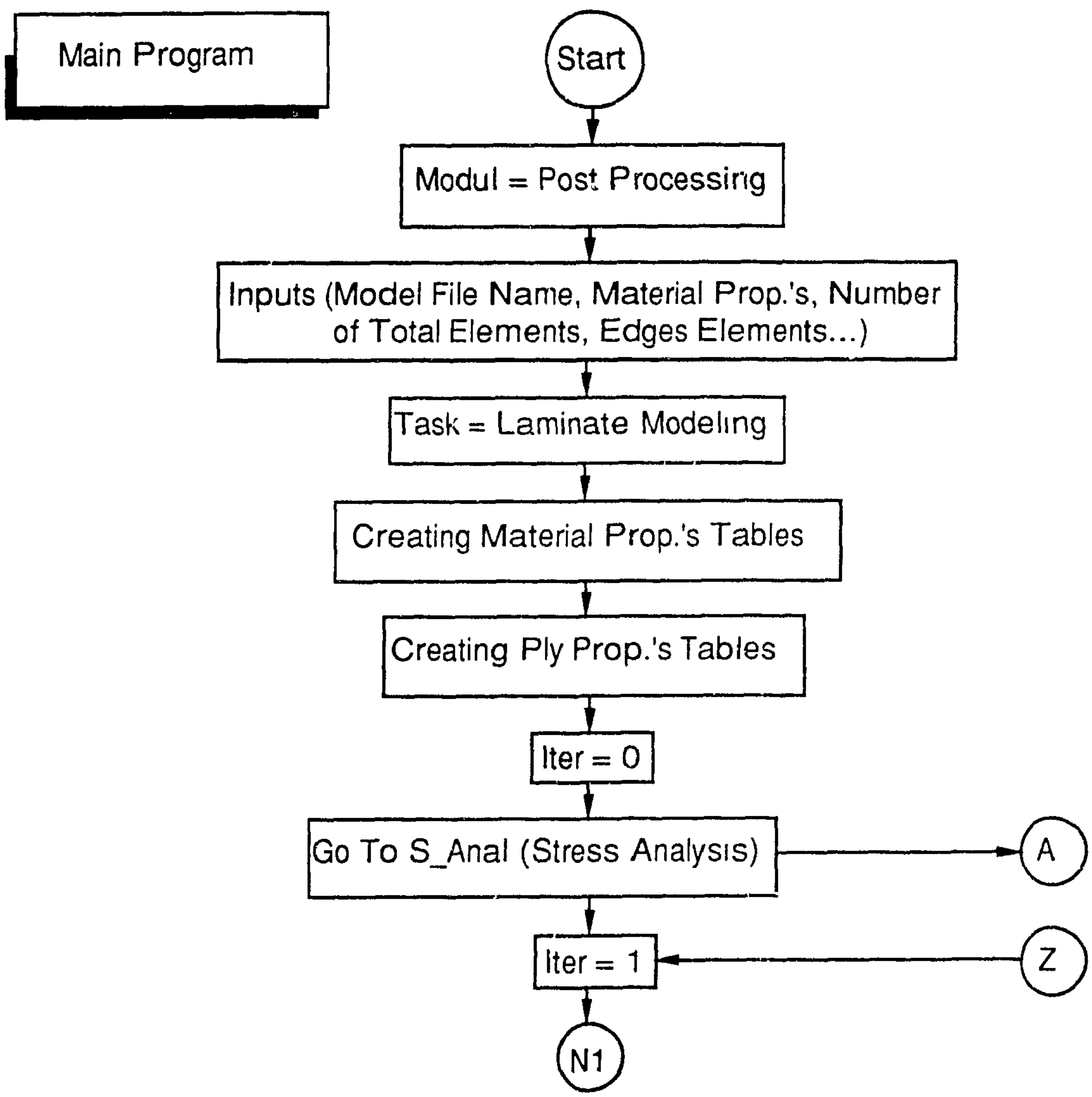
After simplifying Eq. A.9 we have

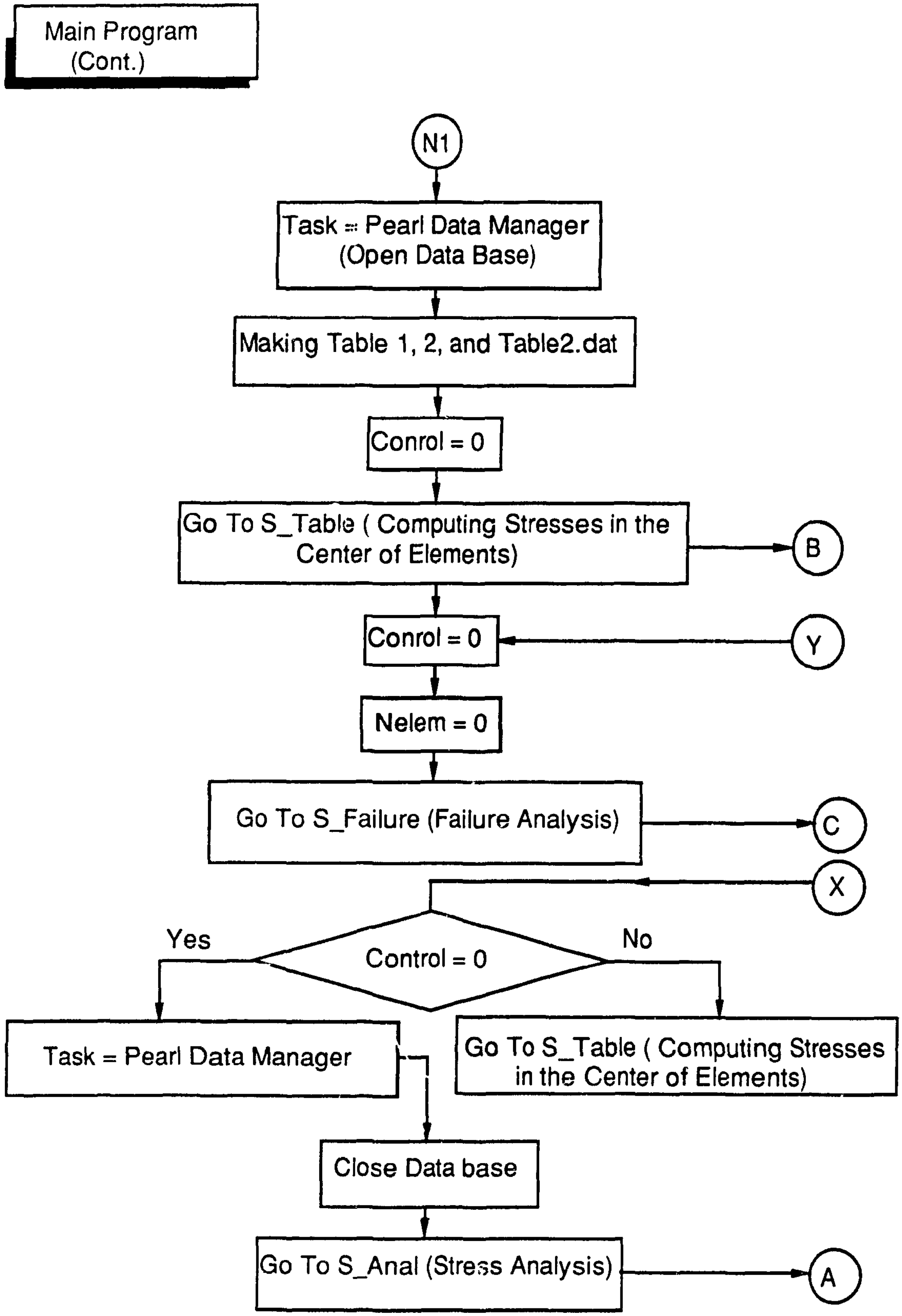
$$\begin{Bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \\ Q_{16} \\ Q_{16} \end{Bmatrix} = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & 4m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2-n^2)^2 \\ m^2n^2 & m^2n^2 & m^4+n^4 & -4m^2n^2 \\ m^3n & -mn^3 & mn^3-m^3n & 2\left(mn^3-m^3n\right) \\ mn^3 & -m^3n & m^3n-mn^3 & 2\left(m^3n-mn^3\right) \end{bmatrix} \begin{Bmatrix} Q_{xx} \\ Q_{yy} \\ Q_{xy} \\ Q_{ss} \end{Bmatrix} \tag{A.10}$$

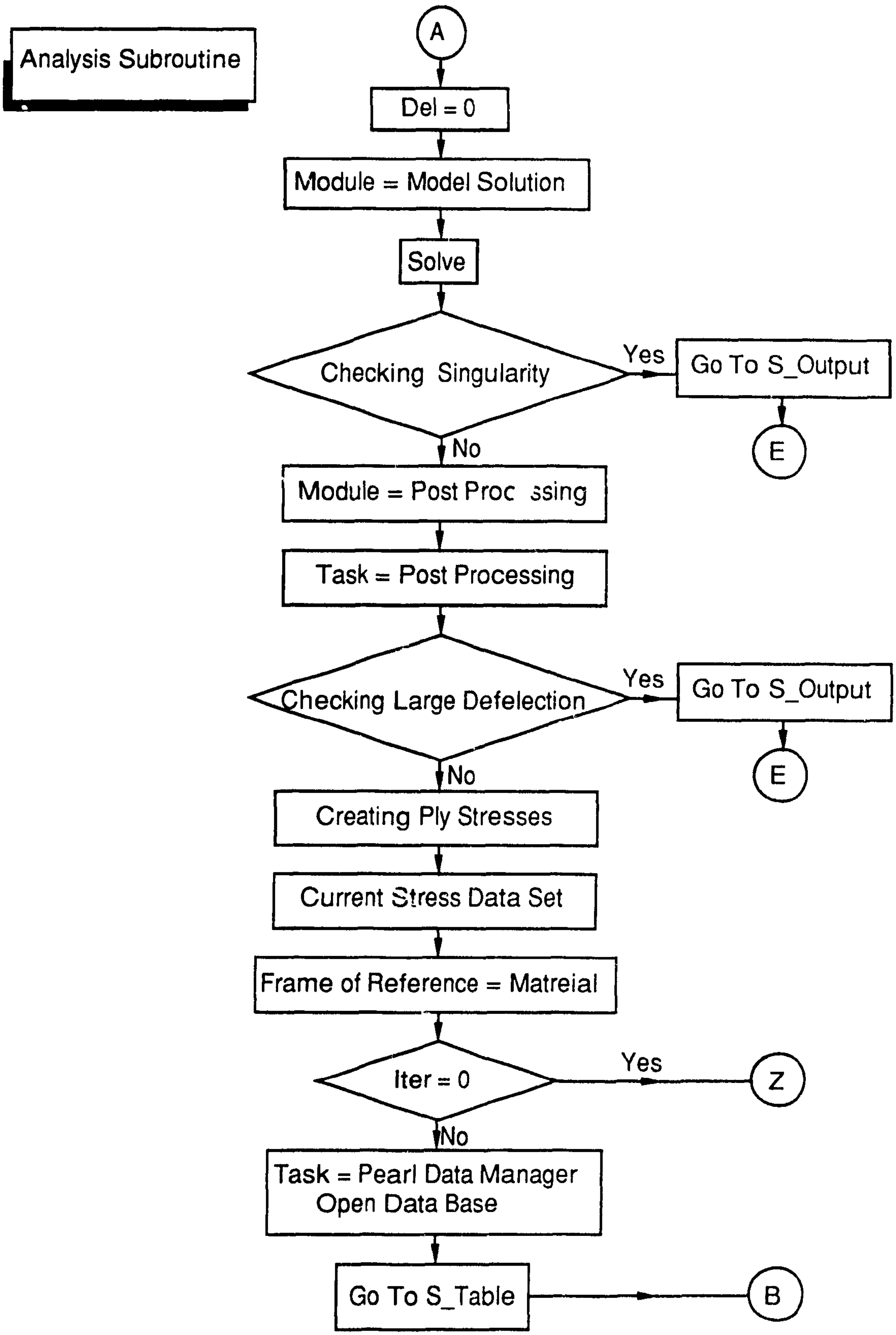
Where $m = cos\phi$ and $n = sin\phi$.

Appendix B

Flow Chart of PDPIN




Main Program
(Cont.)
N1
Task = Pearl Data Manager
(Open Data Base)
Making Table 1, 2, and Table2.dat
Conrol = 0
Go To S_Table (Computing Stresses in the
Center of Elements)
B
Conrol = 0
Y
Nelem = 0
Go To S_Failure (Failure Analysis)
C
X
Yes
Control = 0
No
Task = Pearl Data Manager
Go To S_Table (Computing Stresses
in the Center of Elements)
Close Data base
Go To S_Anal (Stress Analysis)
A


Analysis Subroutine
A
Del = 0
Module = Model Solution
Solve
Checking Singularity
Yes
Go To S_Output
E
No
Module = Post Proc ssing
Task = Post Processing
Checking Large Defelection
Yes
Go To S_Output
E
No
Creating Ply Stresses
Current Stress Data Set
Frame of Reference = Matreial
Iter = 0
Yes
Z
No
Task = Pearl Data Manager
Open Data Base
Go To S_Table
B

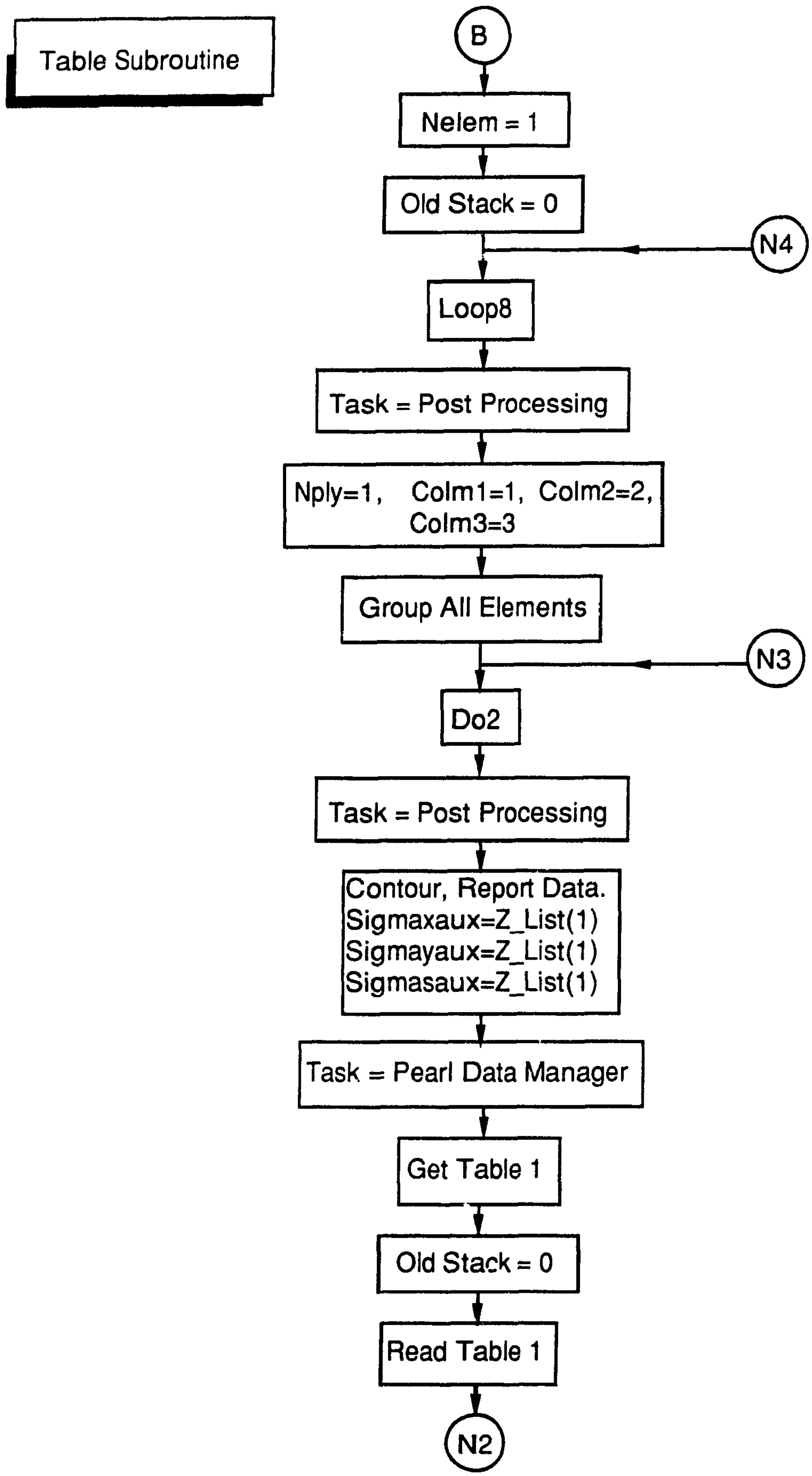


Table Subroutine
B
Nelem = 1
Old Stack = 0
N4
Loop8
Task = Post Processing
Nply=1, Colm1=1, Colm2=2, Colm3=3
Group All Elements
N3
Do2
Task = Post Processing
Contour, Report Data.
Sigmaxaux=Z_List(1)
Sigmayaux=Z_List(1)
Sigmasaux=Z_List(1)
Task = Pearl Data Manager
Get Table 1
Old Stack = 0
Read Table 1
N2

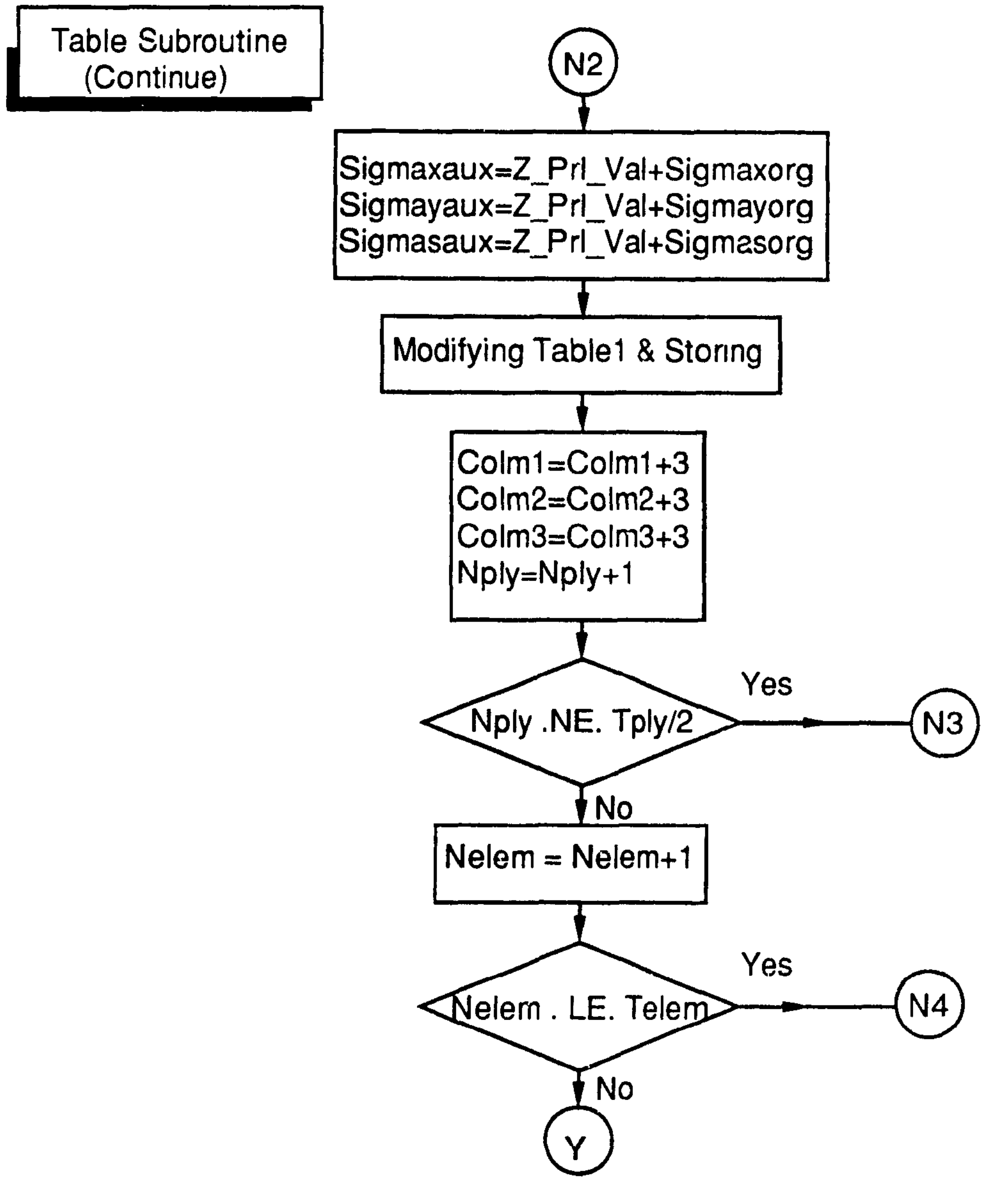
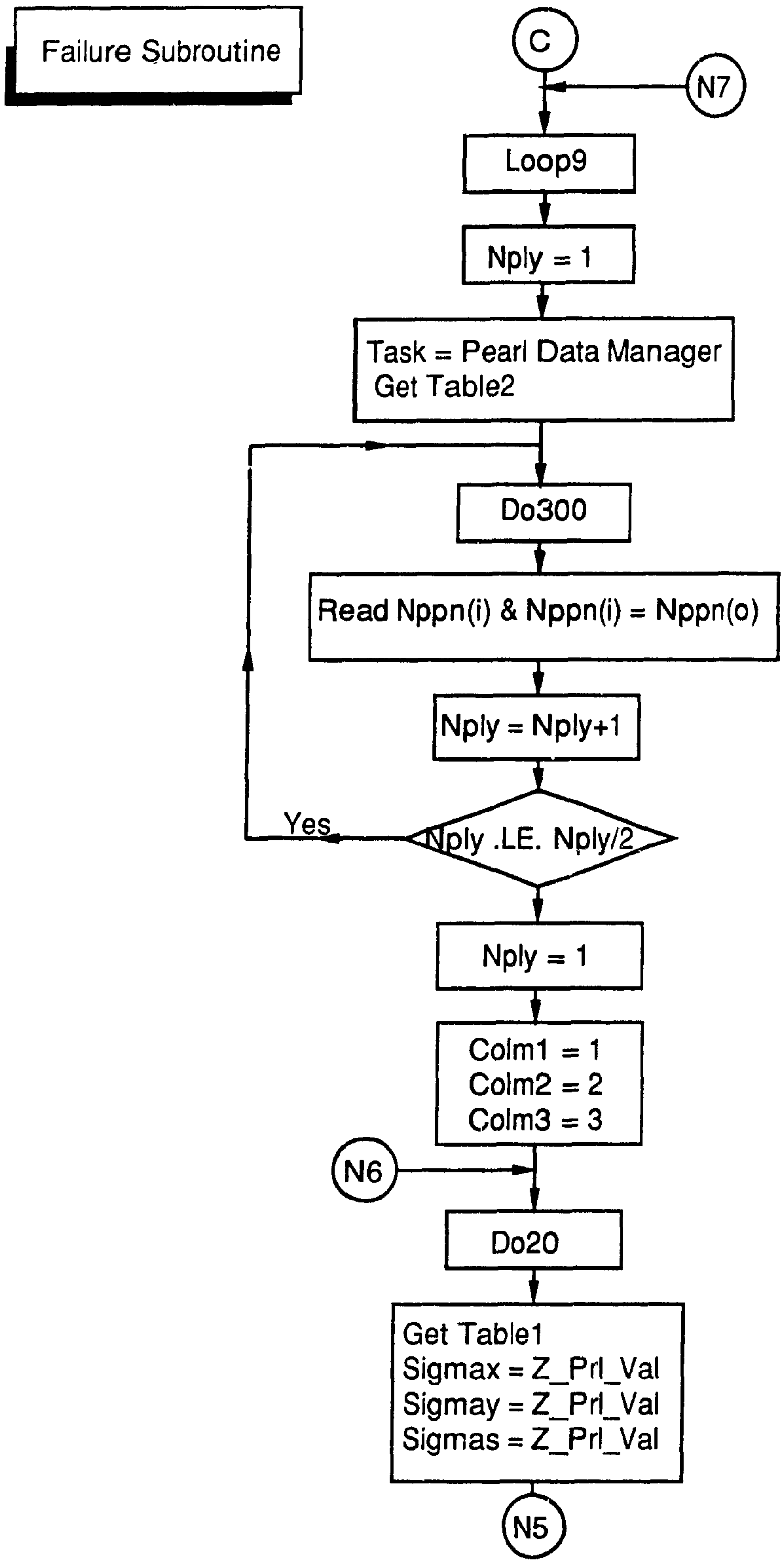
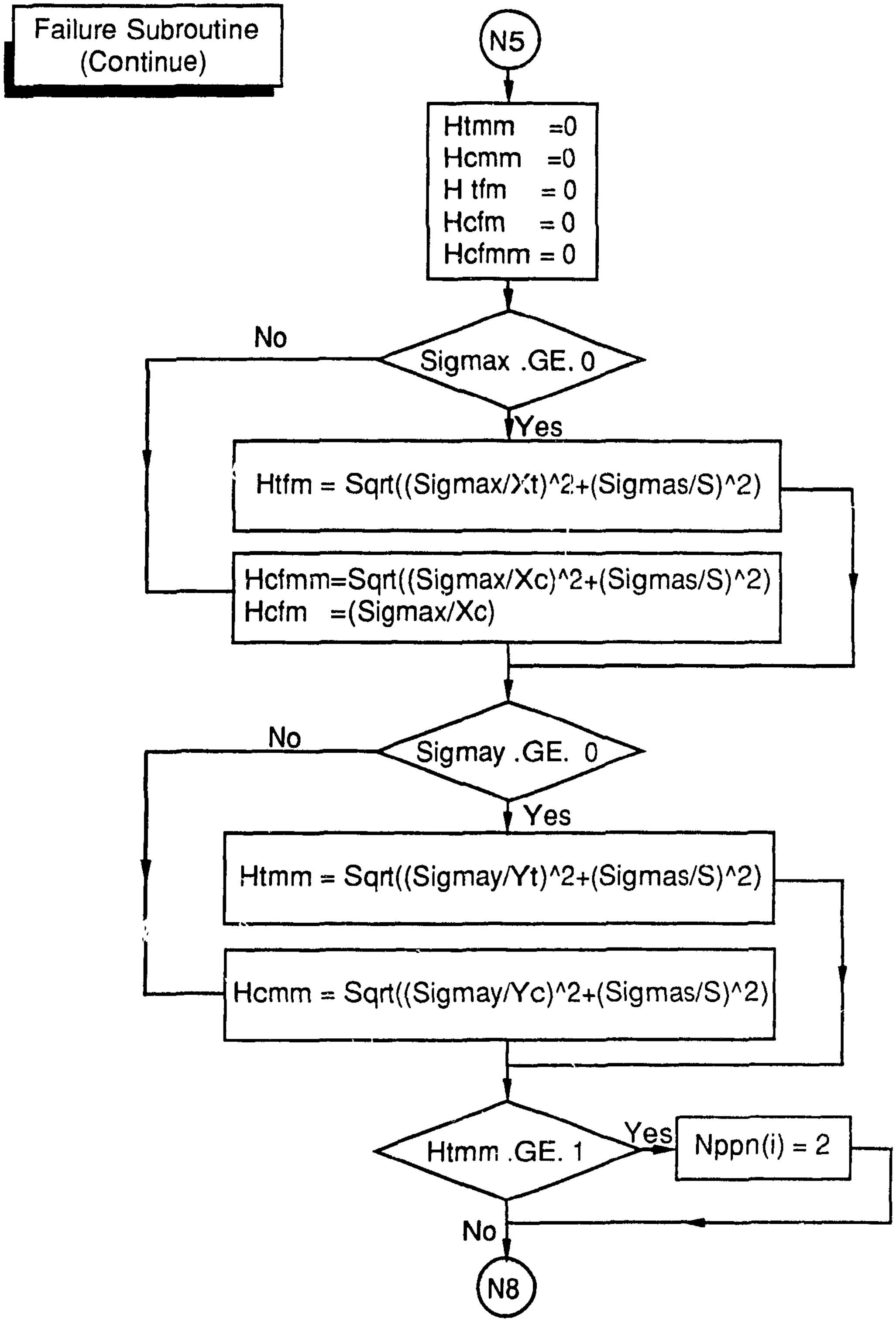
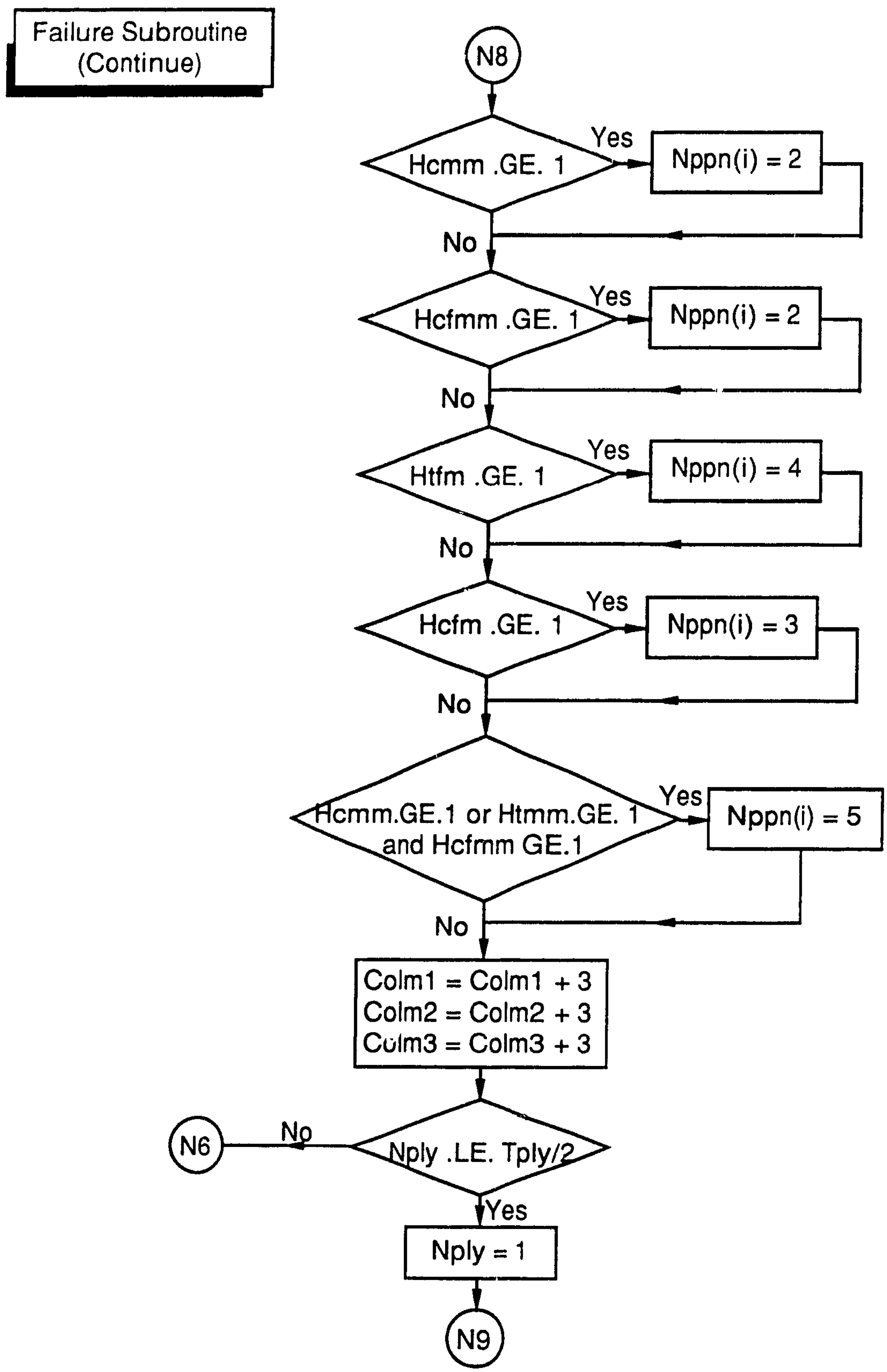
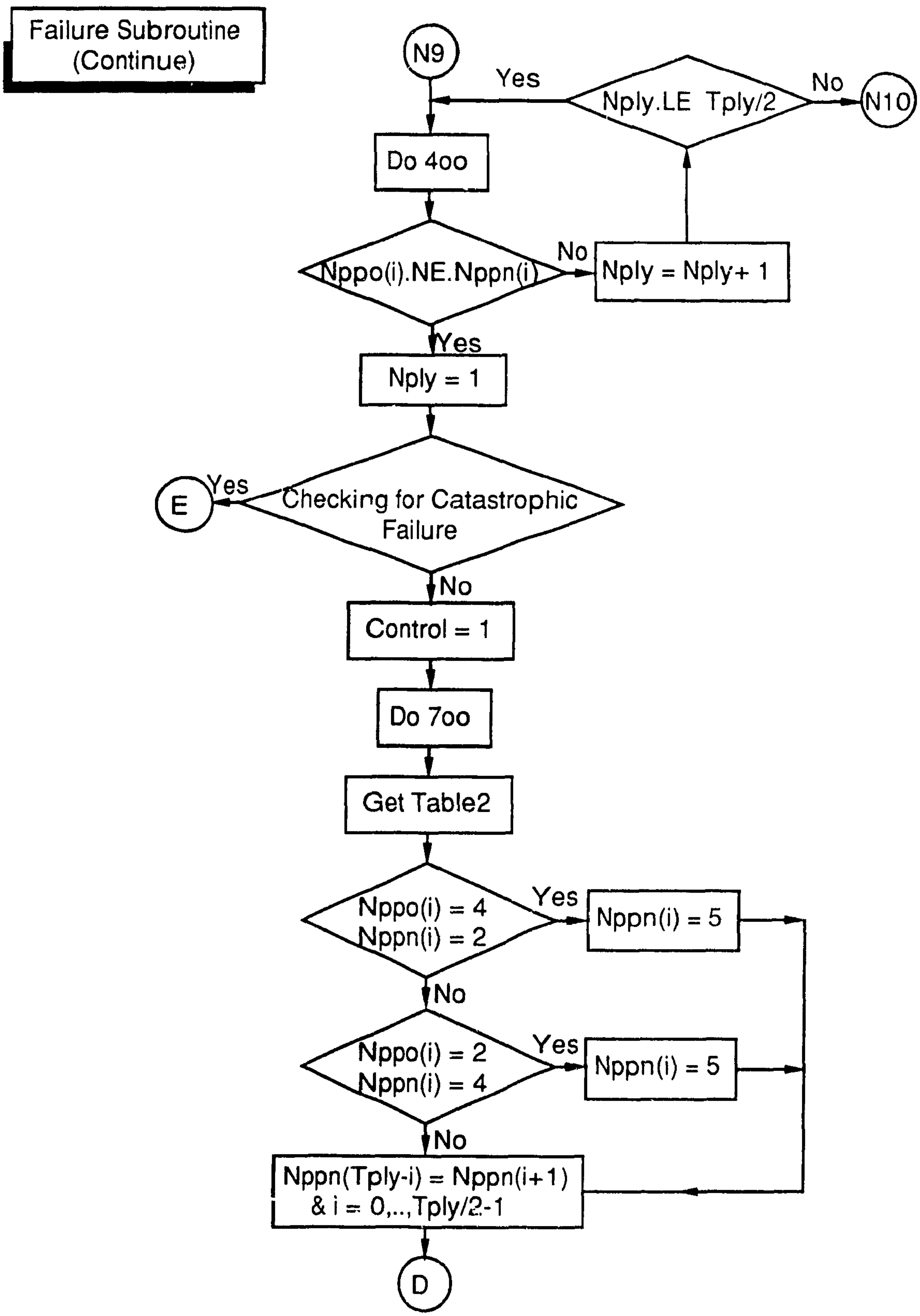


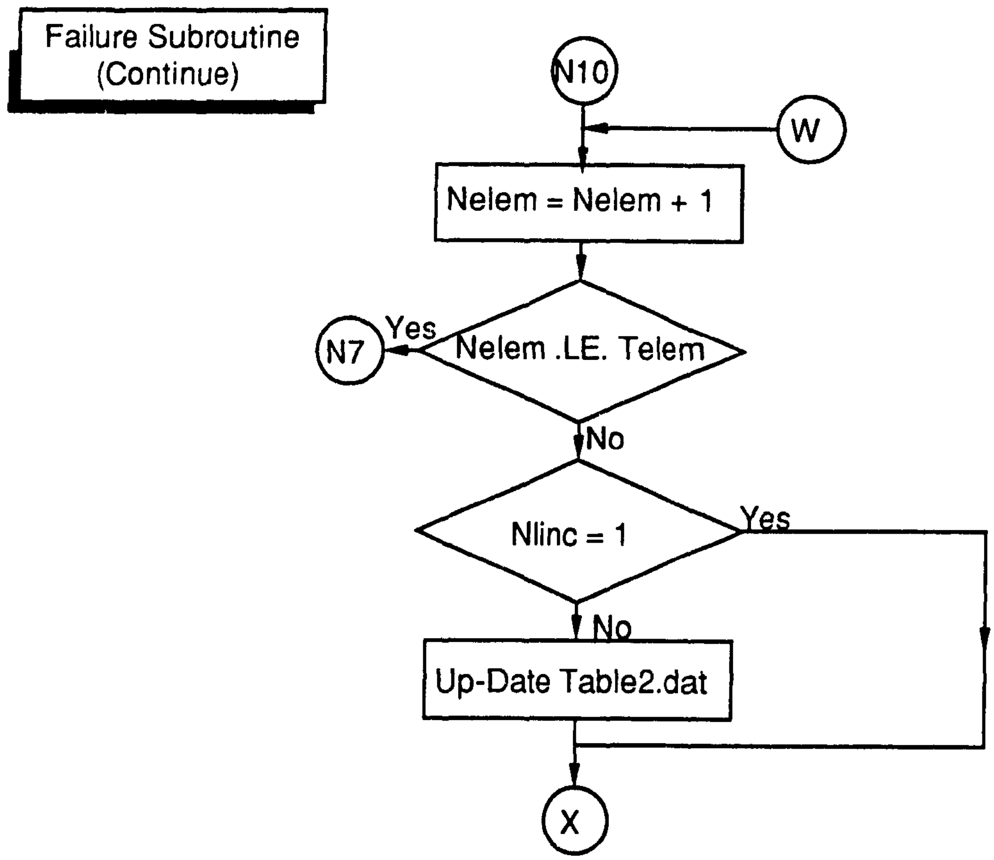
Table Subroutine
(Continue)
N2
Sigmaxaux=Z_Prl_Val+Sigmaxorg
Sigmayaux=Z_Prl_Val+Sigmayorg
Sigmasaux=Z_Prl_Val+Sigmasorg
Modifying Table1 & Storing
Colm1=Colm1+3
Colm2=Colm2+3
Colm3=Colm3+3
Nply=Nply+1
Nply .NE. Tply/2
Yes
N3
No
Nelem = Nelem+1
Nelem . LE. Telem
Yes
N4
No
Y


Failure Subroutine
C
N7
Loop9
Nply = 1
Task = Pearl Data Manager
Get Table2
Do300
Read Nppn(i) & Nppn(i) = Nppn(o)
Nply = Nply+1
Yes
Nply .LE. Nply/2
Nply = 1
Colm1 = 1
Colm2 = 2
Colm3 = 3
N6
Do20
Get Table1
Sigmax = Z_Prl_Val
Sigmay = Z_Prl_Val
Sigmas = Z_Prl_Val
N5

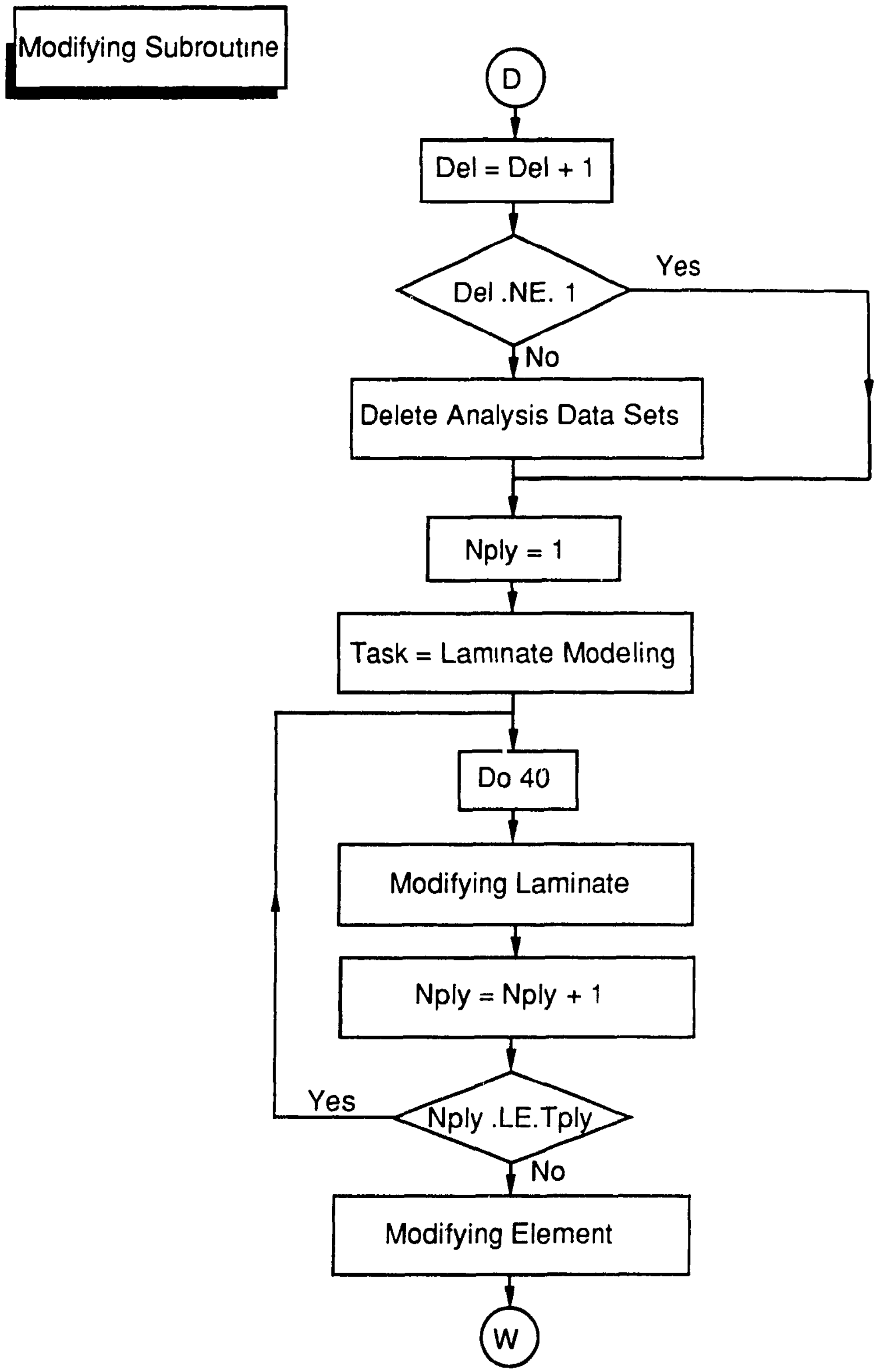

Failure Subroutine
(Continue)
N5
Htmm =0
Hcmm =0
H tfm = 0
Hcfm = 0
Hcfmm = 0
Sigmax .GE. 0
No
Yes
Htfm = Sqrt((Sigmax/Xt)^2+(Sigmas/S)^2)
Hcfmm=Sqrt((Sigmax/Xc)^2+(Sigmas/S)^2)
Hcfm =(Sigmax/Xc)
Sigmay .GE. 0
No
Yes
Htmm = Sqrt((Sigmay/Yt)^2+(Sigmas/S)^2)
Hcmm = Sqrt((Sigmay/Yc)^2+(Sigmas/S)^2)
Htmm .GE. 1
Yes
Nppn(i) = 2
No
N8


Failure Subroutine (Continue)
N8
Hcmm .GE. 1
Yes
Nppn(i) = 2
No
Hcfmm .GE. 1
Yes
Nppn(i) = 2
No
Htfm .GE. 1
Yes
Nppn(i) = 4
No
Hcfm .GE. 1
Yes
Nppn(i) = 3
No
Hcmm.GE.1 or Htmm.GE. 1 and Hcfmm GE.1
Yes
Nppn(i) = 5
No
Colm1 = Colm1 + 3
Colm2 = Colm2 + 3
Colm3 = Colm3 + 3
Nply .LE. Tply/2
No
N6
Yes
Nply = 1
N9

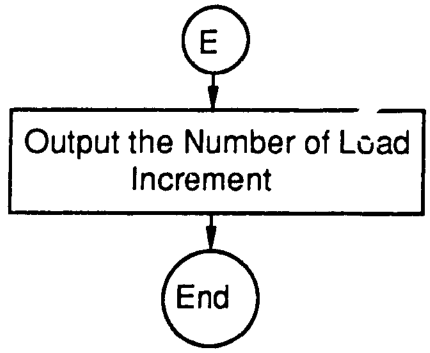

Failure Subroutine (Continue)
N9
Yes
Nply.LE Tply/2
No
N10
Do 4oo
Nppo(i).NE.Nppn(i)
No
Nply = Nply+ 1
Yes
Nply = 1
E
Yes
Checking for Catastrophic Failure
No
Control = 1
Do 7oo
Get Table2
Nppo(i) = 4
Nppn(i) = 2
Yes
Nppn(i) = 5
No
Nppo(i) = 2
Nppn(i) = 4
Yes
Nppn(i) = 5
No
Nppn(Tply-i) = Nppn(i+1)
& i = 0,..,Tply/2-1
D



Failure Subroutine (Continue)
N10
W
Nelem = Nelem + 1
Yes
N7
Nelem .LE. Telem
No
Nlinc = 1
Yes
No
Up-Date Table2.dat
X


Modifying Subroutine
D
Del = Del + 1
Del .NE. 1
Yes
No
Delete Analysis Data Sets
Nply = 1
Task = Laminate Modeling
Do 40
Modifying Laminate
Nply = Nply + 1
Yes
Nply .LE.Tply
No
Modifying Element
W

Output Subroutine



Appendix C

Computer Code (PDPIN)

```
C : ******************************************************************
C : *                          [ PDPIN ]                             *
C : *  [ Progressive Damage Model, Pin-Loaded Composite Plate ]      *
C : *                [ By Mahmood M. Shokrieh ]                      *
C : *                          (1991)                                *
C : *  This  program is developed in  IDEA1 (I_DEAS language) for    *
C : *    predicting the failure load and failure mechanisms of       *
C : *                  pin-loaded composite plates.                  *
C : *                                                                *
C : ******************************************************************
C :
C :                      ***************************
C :                      *     MAIN PROGRAM        *
C :                      ***************************
C :
C :
K : "NM"
K : "P"
C : AUTO CLEANING
K : #OUTPUT "AUTO CLEANING"
K : "/"
K : "MF"
K : "US"
K : "AL"
K : "ON"
K : 0
K : "/"
C :
C :
C :                    ************INPUT**************
C :
K : #DELETE ALL
C :
K : #MFNAME=" "
K : #INPUT "ENTER MODEL FILE NAME" MFNAME
C :
C : LONGITUDINAL TENSILE STRENGTH (force/length**2)
K : #XT=1517E6
C : LONGITUDINAL COMPRESSION STRENGTH (force/length**2)
K : #XC=1593E6
C : TRANSVERSE TENSILE STRENGTH (force/length**2)
K : #YT=46E6
C : TRANSVERSE COMPRESSION STRENGTH (force/length**2)
K : #YC=253E6
C : SHEAR STRENGTH (force/length**2)
```

```
K . #S=107E6
C .
C . NUMBER OF LAMINATE DEFINITION
K : #NLD=1
C : NUMBER OF LOAD INCREMENT
K : #NLINC=1
C :
K : #INPUT "NUMBER OF TOTAL PLIES" TPLY
C . NPPN(i)=NUMBER OF PLY PROPERTY -NEW
K : #DECLARE NPPN(TPLY)
C : NPPO(i)=NUMBER OF PLY PROPERTY -OLD
K : #DECLARE NPPO(TPLY)
C : DISPLACEMENT CRITERION
K : #DECLARE DISPCR(30)
C : MAXIMUM DISPLACEMENT
K : #DISPMAX=1
C : NUMBER OF TOTAL ELEMENTS
K : #TELEM=432
C : SINGULARITY CHECKING
K : #SINCHK=" "
C : OUTPUT CONTROL
K : #OUT=0
C :
C : #INPUT "PLY THICKNESS (length)" PLYTIK
K : #PLYTIK=.85725
C :
K : #ECHO NONE
C :
K : #DECLARE NCELEM(29)
K : #TCELEM=29
C :
K : #NCELEM(1)=1
K : #NCELEM(2)=2
K : #NCELEM(3)=3
K : #NCELEM(4)=4
K : #NCELEM(5)=5
K : #NCELEM(6)=6
K : #NCELEM(7)=12
K : #NCELEM(8)=18
K : #NCELEM(9)=24
K : #NCELEM(10)=30
K : #NCELEM(11)=36
K : #NCELEM(12)=37
K : #NCELEM(13)=43
K : #NCELEM(14)=49
K : #NCELEM(15)=60
K : #NCELEM(16)=66
K . #NCELEM(17)=72
K : #NCELEM(18)=88
K : #NCELEM(19)=89
K : #NCELEM(20)=90
K : #NCELEM(21)=96
K : #NCELEM(22)=102
K : #NCELEM(23)=103
K : #NCELEM(24)=114
K : #NCELEM(25)=120
K : #NCELEM(26)=126
K : #NCELEM(27)=132
K : #NCELEM(28)=138
K : #NCELEM(29)=144
C :
C :
C : ***CREATING NEW MATERIAL TABLES FOR FAILED LAMINATE***
K : #OUTPUT "CREATING NEW MATERIAL TABLES FOR FAILED LAMINATE"
K : "TA"
K : "L"
K : "MAT"
C :
C : material property table for matrix cracking situation
K : #OUTPUT "material property table for matrix cracking situation"
K : "CO"
K : 2
K :
K : "MO"
K : 4
K : 102
```

```
K : 13E4
K : 103
K : 13E4
K : 104
K : 1E-3
K : 105
K : 1E-3
K : 106
K : 1E-3
K : "D"
K : "Y"
C :
C : material property table for fiber breakage situation
K : #OUTPUT "material property table for fiber breakage situation"
K : "CO"
K : 2
K :
K : "MO"
K : 5
K : 101
K : 156E4
K : 102
K : 13E4
K : 103
K : 13E4
K : 104
K : 1E-3
K : 105
K : 1E-3
K : 106
K : 1E-3
K : 108
K : 7E4
K : 109
K : 7E4
K : 110
K : 7E4
K : "D"
K : "Y"
C :
C : material property table for fiber-matrix shearing situation
K : #OUTPUT "material property table for fiber-matrix shearing situation"
K : "CO"
K : 2
K :
K : "MO"
K : 6
K : 104
K : 1E-3
K : 105
K : 1E-3
K : 106
K : 1E-3
K : 108
K : 7E4
K : 109
K : 7E4
K : 110
K : 7E4
K : "D"
K : "Y"
C :
C : mat. property table for matrix cracking & fiber-matrix shearing situation
K : #OUTPUT "mat. prop.table for mat. crak.& fiber-matrix shearing situation"
K : "CO"
K : 2
K :
K : "MO"
K : 7
K : 102
K : 13E4
K : 103
K : 13E4
K : 104
K : 1E-3
```

```
K : 105
K : 1E-3
K : 106
K : 1E-3
K : 108
K : 7E4
K : 109
K : 7E4
K : 110
K : 7E4
K : "D"
K : "Y"
K : "/"
C :
C :            **********CREATING NEW PLY PROPERTY TABLES*********
K : #OUTPUT "CREATING NEW PLY PROPERTY TABLES"
K : #NMPT=4
K : "P"
K : "CR"
K : #LOOP3:
K : "E"
K :
K : NMPT
K : PLYTIK
K :
K : #NMPT=NMPT+1
K : #IF (NMPT LE 7) THEN GOTO LOOP3
K : "/"
C :
C :
C :          *******************FIRST ANALYSIS******************
C :
K : #OUTPUT "FIRST ANALYSIS"
K : #GOTO S_ANAL
K : #M_ANAL1:
C :
C : PEARL DATA MANAGER
K : #OUTPUT "PEARL DATA MANAGER"
K : "/"
K : "TA"
K : "PM"
K : "O"
K : MFNAME
K :
C :
C : OLD STACK LENGTH MUST BE ZERO
K : "MA"
K : "O"
K : 0
C :
C : MAKING TABLE 1 FOR PLY STRESSES
K : #OUTPUT "MAKING TABLE 1 FOR PLY STRESSES"
K : "/"
K : "CR"
K : 1
K : "R"
K :
K : "ME"
K : "FL2"
K :
K : 0
K :
C :
C : add column
K : "MO"
K : #NPLY=1
K : #IA=2
K : #LOOP4:
K : #IB=1
K : #IF (NPLY EQ 1) THEN #IB=2
K : #DO1:
K : "CC"
K : 1
K : IA
K : #IB=IB+1
K : #IA=IA+1
```

```
K : #IF (IB LE 3) THEN GOTO DO1
K : #NPLY=NPLY+1
K : #IF (NPLY LE (TPLY/2)) THEN GOTO LOOP4
C :
C : add row
K : #NELEM=2
K : #LOOP5:
K : "CR"
K : 1
K : #NELEM=NELEM+1
K : #IF (NELEM LE TELEM) THEN GOTO LOOP5
K : "/"
C :
C : save table 1
K : "STO"
K :
C :
C : MAKING TABLE 2 FOR PLY PROPERTIES OF TOTAL ELEMENTS
K : #OUTPUT "MAKING TABLE 2 FOR PLY PROPERTIES OF TOTAL ELEMENTS"
K : "CR"
K : 1
K : "I"
K :
K :
K : 1
K :
C :
C : add column
K : "MO"
K : #NPLY=2
K : #LOOP6:
K : "CC"
K : 1
K : NPLY
K : #NPLY=NPLY+1
K : #IF (NPLY LE (TPLY/2)) THEN GOTO LOOP6
C :
C : add row
K : #NELEM=2
K : #LOOP7:
K : "CR"
K : 1
K : #NELEM=NELEM+1
K : #IF (NELEM LE TELEM) THEN GOTO LOOP7
K : "/"
C :
C : save table 2
K : "STO"
K :
C :
C : MAKING TABLE2.DAT
K : #OUTPUT "MAKING TABLE2.DAT"
K : "TR"
K : "OW"
K : TABLE2
K :
K :
K :
K : "WT"
K :
K :
K :
K : "CW"
K :
C :
C : ***COMPUTING STRESSES IN THE CENTER OF EACH ELEMENT FOR TABLE1***
K : #OUTPUT"COMPUTING STRESSES IN THE CENTER OF EACH ELEMENT FOR TABLE1"
K : #CONTROL=0
C :
C : POST PROCESSING TASK
K : "/"
K : "TA"
K : "P"
C :
K : #GOTO S_TABLE
K : #M_TABLE:
```

```
C :
K : #CONTROL=0
K : #NELEM=1
K : #GOTO S_FAILR
K : #M_FAILR:
C :
K : #IF (CONTROL EQ 0) THEN GOTO S_TABLE
C :
K : "/"
K : "TA"
K : "PM"
K : "CLO"
K :
K : #GOTO S_ANAL
C :
C :
C :             **************END OF MAIN PROGRAM***************
C :
C :
C :
C :
C :
C :
C :                             ************************
C :                             * ANALYSIS SUBROUTINE *
C :                             ************************
C :
K : #S_ANAL:
K : #OUTPUT "ANALYSIS SUBROUTINE"
C :
C : DEL IS A CHARACTER FOR DELETING ANALYSIS DATA SET IN S_MODIF
K : #DEL=0
C :
K : "NM"
K : "MS"
K : "TA"
K : "L"
C :
C : EXECUTION OPTIONS
K : "E"
K : "D"
K : "SC"
K : 0
K : "D"
K : "SR"
K : "OF"
K : "/"
C :
C : CASE SET, USE CASE 1
K : "C"
K : "U"
K : 1
C :
C : METHOD OF SOLUTION
K : "ME"
K : "R"
K : "I"
C :
C : OUTPUT SELECTION
K : "O"
K : "D"
K : "SL"
K : "SN"
K : "SL"
K : "/"
K : "SO"
K :
C :
C : CHECKING SINGULARITY
K : #IF ( Z_LIST(3) EQ 17761 ) THEN #SINCHK="SINGULARITY"
K : #IF ( Z_LIST(3) EQ 17761 ) THEN GOTO S_OUTPUT
C :
C : CHECKING INFINITE DISPLACEMENT
K : "NM"
K : "P"
```

```
K : "TA"
K : "P"
K : "A"
C :
K : "CU"
K : 1
C :
K : "GR"
K : "NE"
K : "E"
K : *
K : "Y"
C :
K : "/"
K : "CO"
K : "REP"
K : "R"
K : "ON"
K : "E"
K : "T"
C :
K : #IF (NLINC EQ 1) THEN #DISPMAX=Z_LIST(11)
K : #DISPCR(NLINC)=Z_LIST(11)/DISPMAX
K : #IF (DISPCR(NLINC) GE 10) THEN GOTO S_OUTPUT
C :
C :
C : CREATING PLY STRESSES BY STRAINS
K : #OUTPUT "CREATING PLY STRESS BY STRAINS"
K : "/"
K : "A"
K : "CR"
K : "L"
K : "PS"
K : 2
K :
K :
K :
K :
K :
K :
K :
C :
C : PLY STRESS DATA SET CURRENT
K : "/"
K : "A"
K : "CU"
K : 3
C :
C : FRAME OF REFERENCE MUST BE MATERIAL AXIS, RAW DATA SWITCH OFF.
K : "CO"
K : "DA"
K : "F"
K : "MA"
K : !
K : !
K : "REP"
K : "R"
K : "OF"
K : "/"
C :
K : #IF (NLINC EQ 1) THEN GOTO M_ANAL1
C :
C : PEARL DATA MANAGER
K : #OUTPUT "PEARL DATA MANAGER"
K : "TA"
K : "PM"
K : "O"
K : MFNAME
K : "C"
C :
K : #GOTO S_TABLE
C :
C :         *************END OF ANALYSIS SUBROUTINE*************
C :
C :
C :
```

```
C :
C :
C :                                 ********************
C :                                 *TABLE  SUBROUTINE*
C :                                 ********************
C :
K : #S_TABLE:
K : #OUTPUT "TABLE SUBROUTINE"
K : #NELEM=1
C :
C : OLD STACK LENGTH MUST BE ZERO FOR GROUP ELEMENT
K : "/"
K : "TA"
K : "P"
K : "GR"
K : "O"
K : 0
C :
K : #LOOP8:
C :
C : POST PROCESSING TASK
K : "/"
K : "TA"
K : "P"
C :
K : #NPLY=1
K : #COLM1=1
K : #COLM2=2
K : #COLM3=3
C :
C : GROPE OF ELEMENTS
K : "GR"
K : "NE"
K : "E"
K : "L"
K : NELEM
K :
K : "Y"
C :
K : #DO2:
K : "/"
K : "TA"
K : "P"
C :
K : "CO"
C :
C : X_STRESS COMPONENT
K : "DA"
K : "S"
K : "L"
K : NPLY
K : "X"
K : "REP"
K : "E"
K : "T"
K : #SIGMAXORG=Z_LIST(1)
K : !
C :
C : Y_STRESS COMPONENT
K : "DA"
K : "S"
K : "L"
K : NPLY
K : "Y"
K : "REP"
K : "E"
K : "T"
K : #SIGMAYORG=Z_LIST(1)
K : !
C :
C : S_STRESS COMPONENT
K : "DA"
K : "S"
K : "L"
K : NPLY
K : "XY"
```

```
K : "REP"
K : "E"
K : "T"
K : #SIGMASORG=Z_LIST(1)
C :
C : PEARL DATA MANAGER
K : "/"
K : "TA"
K : "PM"
C :
K : "G"
K : 1
C :
C : OLD STACK MUST BE ZERO
K : "MA"
K : "O"
K : 0
C :
K : "/"
K : "A"
C :
K : "RV"
K : COLM1
K : NELEM
K : #SIGMAXAUX=Z_PRL_VAL+SIGMAXORG
C :
K : "RV"
K : COLM2
K : NELEM
K : #SIGMASAUX=Z_PRL_VAL+SIGMASORG
C :
K : "RV"
K : COLM3
K : NELEM
K : #SIGMAYAUX=Z_PRL_VAL+SIGMAYORG
C :
K : "/"
K : "MO"
K : "E"
K : COLM1
K : NELEM
K : SIGMAXAUX
C :
K : "E"
K : COLM2
K : NELEM
K : SIGMASAUX
C :
K : "E"
K : COLM3
K : NELEM
K : SIGMAYAUX
K : "/"
K : "STO"
K :
K :
C :
K : #COLM1=COLM1+3
K : #COLM2=COLM2+3
K : #COLM3=COLM3+3
K : #NPLY=NPLY+1
K : #IF (NPLY LE (TPLY/2)) THEN GOTO DO2
C :
K : #NELEM=NELEM+1
K : #IF (NELEM LE TELEM) THEN GOTO LOOP8
C :
K : #GOTO M_TABLE
C :
C :        *************END OF TABLE SUBROUTINE*****************
C :
C :
C :
C :
C :
C :
C :
C :                        ********************************
```

```
C :                     * FAILURE ANALYSIS SUBROUTINE *
C :                     *******************************
C :
K : #S_FAILR:
K : #OUTPUT "FAILURE ANALYSIS SUBROUTINE"
C :
K : #LOOP9:
K : #NPLY=1
C :
C : READING OLD NUMBER OF PLY PROPERTY NPPO(i) FROM TABLE NO. 2 ELEMENT
K : "/"
K : "TA"
K : "PM"
K : "G"
K : 2
K : "A"
K : #DO300:
K : "RV"
K : NPLY
K : NELEM
K : #NPPO(NPLY)=Z_PRL_VAL
K : #NPPN(NPLY)=NPPO(NPLY)
K : #NPLY=NPLY+1
K : #IF (NPLY LE (TPLY/2)) THEN GOTO DO300
K : #NPLY=1
C :
K : #COLM1=1
K : #COLM2=2
K : #COLM3=3
C :
K : #DO20:
K : "/"
K : "G"
K : 1
K : "A"
C :
K : "RV"
K : COLM1
K : NELEM
K : #SIGMAX=Z_PRL_VAL
C :
K : "RV"
K : COLM2
K : NELEM
K : #SIGMAS=Z_PRL_VAL
C :
K : "RV"
K : COLM3
K : NELEM
K : #SIGMAY=Z_PRL_VAL
C :
C : APPLYING FAILURE CRITERION
K : #HTFM=0
K : #HCFMM=0
K : #HCFM=0
K : #HTMM=0
K : #HCMM=0
C :
K : #IF (SIGMAX GT 0) THEN GOTO TFM
K : #IF (SIGMAX LT 0) THEN GOTO CFM
C :
C : tensile fiber mode
K : #TFM:
K : #HTFM=SQRT((SIGMAX/XT)*(SIGMAX/XT)+(SIGMAS/S)*(SIGMAS/S))
K : #GOTO NEXT1
C :
C : compression fiber mode
K : #CFM:
K : #HCFMM=SQRT((SIGMAX/XC)*(SIGMAX/XC)+(SIGMAS/S)*(SIGMAS/S))
K : #HCFM=-SIGMAX/XC
C :
K : #NEXT1:
K : #IF (SIGMAY GT 0) THEN GOTO TMM
K : #IF (SIGMAY LT 0) THEN GOTO CMM
C :
C : tensile matrix mode
```

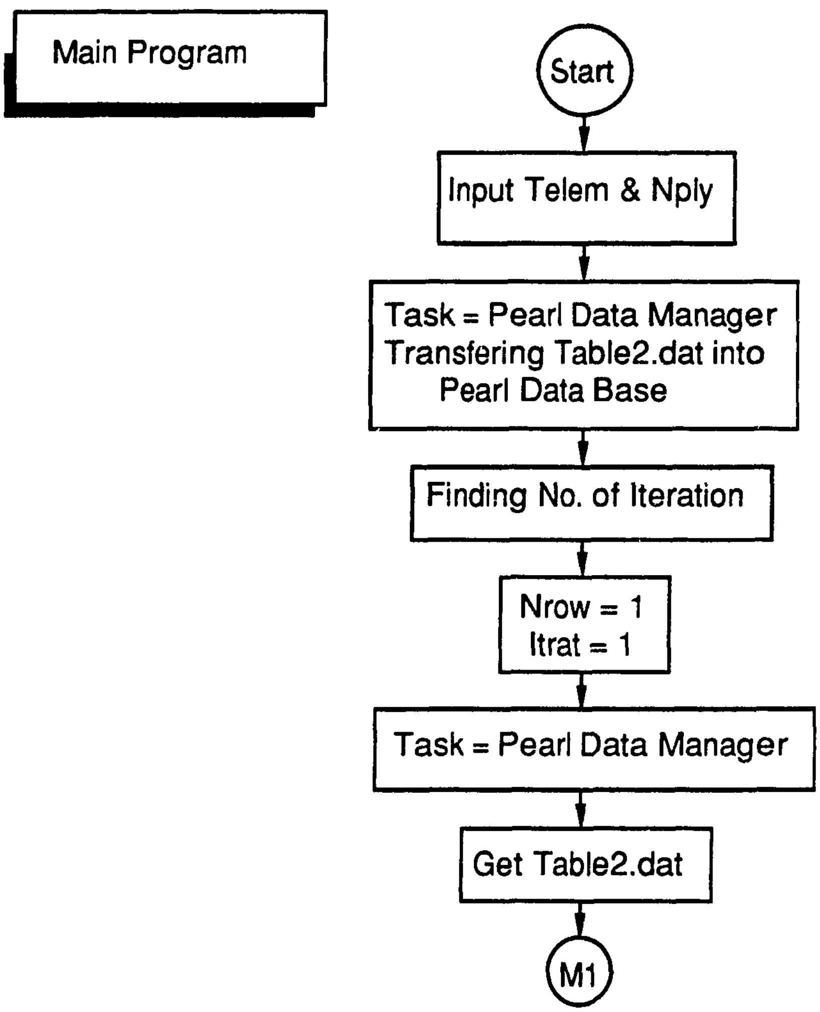
```
K : #TMM:
K : #HTMM=SQRT((SIGMAY/YT)*(SIGMAY/YT)+(SIGMAS/S)*(SIGMAS/S))
K : #GOTO NEXT2
C :
C : compression matrix mode
K : #CMM:
K : #HCMM=SQRT((SIGMAY/YC)*(SIGMAY/YC)+(SIGMAS/S)*(SIGMAS/S))
C :
K : #NEXT2:
K : #IF (HTMM GE 1) THEN #NPPN(NPLY)=2
K : #IF (HCMM GE 1) THEN #NPPN(NPLY)=2
K : #IF (HCFMM GE 1) THEN #NPPN(NPLY)=4
K : #IF (HCMM GE 1 OR HTMM GE 1 AND HCFMM GE 1) THEN #NPPN(NPLY)=5
K : #IF (HTFM GE 1) THEN #NPPN(NPLY)=3
K : #IF (HCFM GE 1) THEN #NPPN(NPLY)=3
C :
C :
K : #COLM1=COLM1+3
K : #COLM2=COLM2+3
K : #COLM3=COLM3+3
K : #NPLY=NPLY+1
K : #IF (NPLY LE (TPLY/2)) THEN GOTO D020
C :
C : CHANGING ALL TO 3
K : #NPLY=1
K : #D0777:
K : #IF (NPPN(NPLY) NE 3) THEN GOTO NEXT777
K : #NPLY=1
K : #D0666:
K : #NPPN(NPLY)=3
K : #NPLY=NPLY+1
K : #IF (NPLY LE (TPLY/2)) THEN GOTO DC 66
K : #GOTO NEXT666
K : #NEXT777:
K : #NPLY=NPLY+1
K : #IF (NPLY LE (TPLY/2)) THEN GOTO D0777
K : #NEXT666:
C :
C : CHECKING FOR FAILURE IN PLIES OF ELEMENT
K : #NPLY=1
K : #D0400:
K : #IF (NPPO(NPLY) NE NPPN(NPLY)) THEN GOTO NEXT500
K : #NPLY=NPLY+1
K : #IF (NPLY LE (TPLY/2)) THEN GOTO D0400
K : #GOTO NEXT600
K : #NEXT500:
K : #NPLY=1
C :
C : CHECKING FOR CATASTROPHIC FAILURE
K : #SS=1
K : #LOOP10:
K : #IF (NELEM EQ NCELEM(SS)) THEN GOTO NEXT1001
K : #SS=SS+1
K : #IF (SS LE TCELEM) THEN GOTO LOOP10
K : #GOTO NEXT1002
K : #NEXT1001:
K : #OUT=1
K : #NEXT1002:
C :
C : MODIFYING TABLE NO. 2
K : #CONTROL=1
K : #D0700:
K : "/"
K : "G"
K : 2
K : "A"
K : "RV"
K : NPLY
K : NELEM
K : #IF (NPPO(NPLY) EQ 4 AND NPPN(NPLY) EQ 2) THEN #NPPN(NPLY)=5
K : #IF (NPPO(NPLY) EQ 2 AND NPPN(NPLY) EQ 4) THEN #NPPN(NPLY)=5
K : "/"
K : "MO"
K : "E"
K : NPLY
K : NELEM
```

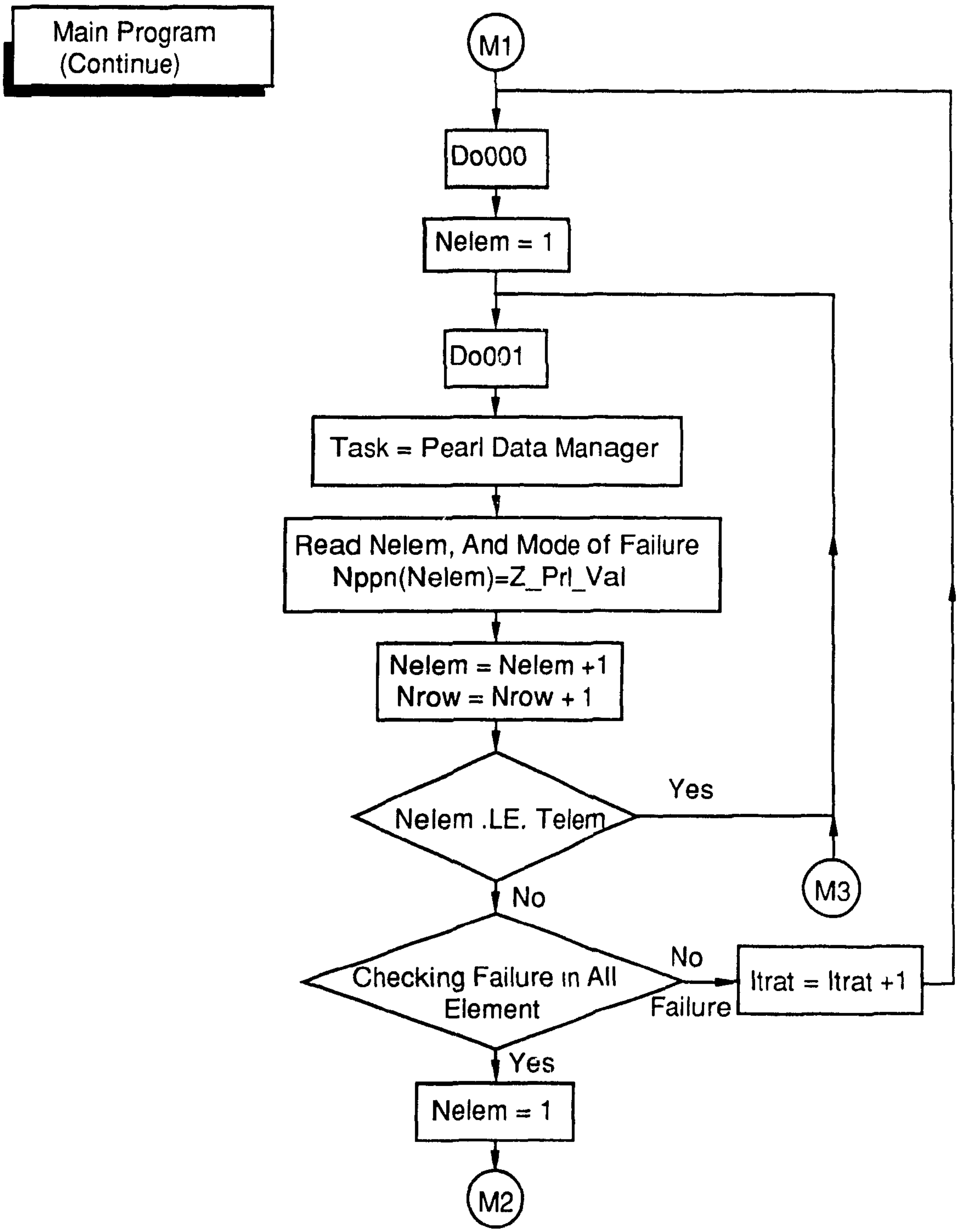
```
K : NPPN(NPLY)
K : "/"
K : "STO"
K :
K :
C :
K : #NPLY=NPLY+1
K : #IF (NPLY LE (TPLY/2)) THEN GOTO DO700
K : #NPPN(3)=NPPN(2)
K : #NPPN(4)=NPPN(1)
C :
K : #GOTO S_MODIF
K : #M_MODIF:
K : #NEXT600:
C :
K : #NELEM=NELEM+1
K : #IF (NELEM LE TELEM) THEN GOTO LOOP9
C :
C : MAKING TABLE2.DAT UP_DATE
K : #IF (NLINC EQ 1) THEN GOTO NEXT333
K : #OUTPUT "MAKING TABLE2.DAT UP_DATE"
K : "/"
K : "TA"
K : "PM"
K : "G"
K : 2
K : "TR"
K : "OW"
K : TABLE2
K : "A"
K :
K :
K : "WT"
K :
K :
K :
K : "CW"
K :
K : #NEXT333:
C :
C : NUMBERS OF LOAD INCREMENT
K : #NLINC=NLINC+1
C :
K : #IF (OUT EQ 1) THEN GOTO S_OUTPUT
K : #GOTO M_FAILR
C :
C :   *************END OF FAILURE ANALYSIS SUBROUTINE*************
C :
C :
C :
C :
C :
C :                         **********************
C :                         *MODIFYING SUBROUTINE*
C :                         **********************
C :
K : #S_MODIF:
K : #OUTPUT "MODIFYING SUBROUTINE"
C :
K : #DEL=DEL+1
K : #IF (DEL NE 1) THEN GOTO NEXT900
C :
C : DELETE ANALYSIS DATA SETS
K : #OUTPUT "DELETE ANALYSIS DATA SETS"
K : "/"
K : "TA"
K : "P"
K : "A"
K : "DE"
K : *
K : "Y"
K : #NEXT900:
C :
K : #NPLY=1
C :
K : "/"
```

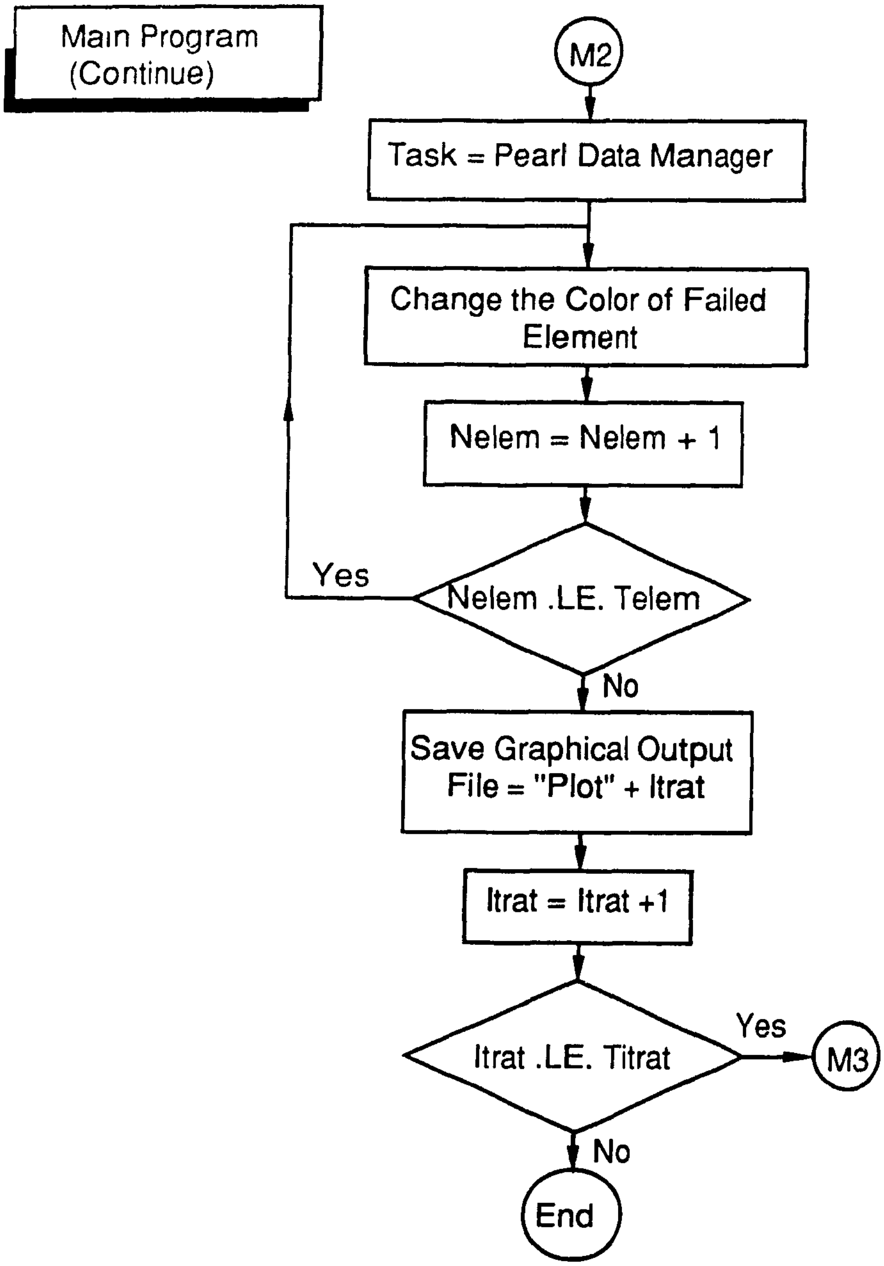
```
K : "TA"
K : "L"
K : "LA"
K : #D040:
K : "MO"
K : "STK"
K : "MOD"
K : NPLY
K : NPPN(NPLY)
K :
K :
K :
K :
K :
K :
C :
K : #NPLY=NPLY+1
K : #IF (NPLY LE TPLY) THEN GOTO D040
K : #NLD=NLD+1
K : "STO"
K : NLD
K :
K :
C :
C : MODIFYING ELEMENT
K : "/"
K : "E"
K : "MO"
K : NELEM
K : "D"
K : "MT"
K :
K : "Y"
K : "/"
C :
K : #GOTO M_MODIF
C :
C :          *********END OF MODIFYING SUBROUTINE*****************
C :
C :
C :
C :
C :
C :
C :
C :                               *********************
C :                               * OUTPUT SUBROUTINE *
C :                               *********************
C :
K : #S_OUTPUT:
K : #OUTPUT "OUTPUT SUBROUTINE"
K : #OUTPUT "NUMBER OF LOAD INCREMENT=",NLINC
K : #GOTO END
C :
C :          ************END OF OUTPUT SUBROUTINE*****************
C :
C :
C :
C :
C :
C :
C :
K : #END:
E :                       **** END OF SESSION ****
```

Appendix D

Flow Chart of PLOT




Main Program
(Continue)
M1
Do000
Nelem = 1
Do001
Task = Pearl Data Manager
Read Nelem, And Mode of Failure
Nppn(Nelem)=Z_Prl_Val
Nelem = Nelem +1
Nrow = Nrow + 1
Nelem .LE. Telem
Yes
No
M3
Checking Failure in All
Element
No
Failure
Itrat = Itrat +1
Yes
Nelem = 1
M2


Main Program
(Continue)
M2
Task = Pearl Data Manager
Change the Color of Failed Element
Nelem = Nelem + 1
Yes
Nelem .LE. Telem
No
Save Graphical Output File = "Plot" + Itrat
Itrat = Itrat +1
Yes
Itrat .LE. Titrat
M3
No
End

Appendix E

Computer Code (PLOT)

```
C : ********************************************************************
C : *                          [ PLOT ]                                *
C : *                  [ By Mahmood M. Shokrieh ]                      *
C : *                           (1991)                                 *
C : *       This  program is developed in  IDEAl (I_DEAS language) for *
C : *              Graphical repr-sentation of damage in pin-loaded    *
C : *                        composite plates.                         *
C : *                                                                  *
C : ********************************************************************
C :
K : #DELETE ALL
K : #INPUT "NUMBER OF TOTAL ELEMENTS" TELEM
K : #DECLARE NPP(TELEM)
K : #INPUT "INPUT PLY NUMBER THAT YOU WANT TO SEE THE RESULT ON IT" NPLY
K : #FILE=" "
C :
K : #ECHO NONE
C :
C : TRANSFERRING TABLE2.DAT INTO PEARL DATA BASE.
C :
K : "/"
K : "TA"
K : "PM"
K : "O"
K : TEST
K :
K : "CR"
K : NPP1
K :
K :
K : NPP2
K :
K :
K :
K :
K : "TR"
K : "OR"
K : TABLE2
K :
K : "R"
K :
K :
K :
K : "/"
K : "STO"
K :
C : FINDING THE NUMBER OF ROW OF TABLE2.DAT
```

```
C :
K : "MA"
K : "L"
K : "1"
K : "N"
K : "N"
K : #TROW=Z_LIST(1)
K : #TITRAT=TROW/TELEM
C :
C : ****************** MAKING PICTURE FILE ****************************
C :
K : #NROW=1
K : #ITRAT=1
K : "/"
K : "TA"
K : "PM"
K : "G"
K : "1"
K : #D0000:
K : #NELEM=1
K : #D0001:
K : "/"
K : "TA"
K : "PM"
K : "A"
K : "RV"
K : NPLY
K : NROW
K : #NPP(NELEM)=Z_PRL_VAL
K : #NROW=NROW+1
K : #NELEM=NELEM+1
K : #IF (NELEM LE TELEM) THEN GOTO D0001
C :
C : ******************** CHECKING FAILURE ******************************
C :
K : #NELEM=1
K : #D0002:
K : #IF (NPP(NELEM) NE 1) THEN GOTO NEXT001
K : #NELEM=NELEM+1
K : #IF (NELEM LE TELEM) THEN GOTO D0002
K : #ITRAT=ITRAT+1
K : #IF (ITRAT LE TITRAT) THEN GOTO D0000
K : #NEXT001:
C :
C : ************** CHANGING COLOR OF FAILED ELEMENTS ******************
C :
K : #NELEM=1
K : "/"
K : "TA"
K : "MP"
K : "E"
K : #D0003:
K : #IF (NPP(NELEM) EQ 1) THEN GOTO NEXT004
C :
C : *********************** SELECTING COLOR ****************************
C :
K : #IF (NPP(NELEM) EQ 2) THEN #COLOR=14
K : #IF (NPP(NELEM) EQ 3) THEN #COLOR=11
K : #IF (NPP(NELEM) EQ 4) THEN #COLOR=1
K : #IF (NPP(NELEM) EQ 5) THEN #COLOR=8
K : "MO"
K : "L"
K : NELEM
K : "D"
K : "CO"
K : COLOR
K :
K :
K : #NEXT004:
K : #NELEM=NELEM+1
K : #IF (NELEM LE TELEM) THEN GOTO D0003
C :
C : SAVING GRAPHICAL OUTPUT
K : #FILE="PLOT"+ITRAT
K : "/"
K : "DO"
```

```
K : "CT"
K : "E"
K : "/"
K : "MF"
K : "PI"
K : "W"
K : "ON"
K : "F"
K : "B"
K : "REDI"
K : FILE
K :
K : "W"
K : "OF"
K : "DR"
K : #ITRAT=ITRAT+1
K : #IF (ITRAT LE TITRAT) THEN GOTO D0000
E : **** END OF SESSION ****
```