HEAT EXCHANGE AND SEA ICE GROWTH IN ARCTIC CANADA

by

D.M. Leahey

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Department of Meteorology,
McGill University,
Montreal.

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Abstract

Stefan's equation for ice accretion has been modified to take into account the effects of snow depth and density. For regions in the Canadian Arctic the modified equation, when applied in the presence of a light snow cover with climatic estimates of heat loss, gave a satisfactory explanation of the observed ice growth (correlation coefficient = .99).

When a heavy snow cover is present it is necessary to calculate actual values of heat loss for use in the modified equation. To allow a quick evaluation of the daily heat loss, an equation has been developed for the net long wave radiation. Proposals have also been presented for quickly estimating the sum of the sensible and latent heat fluxes.

With a heavy snow cover, the correlation between the ice accretion predicted by the equation and the observed ice thicknesses was .94. The corresponding correlation for Zubov's widely employed empirical formula was .785.

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I. INTRODUCTION

The growth of sea ice is dependent upon many meteorological elements such as wind, cloud amount, air temperature, dew point and snow cover. By far the most important of these are air temperature and snow cover.

Certain oceanographic factors also influence the growth of sea ice. Chief among these are currents, water temperature, mixing depth and salinity. While in certain localities oceanographic variables can play a decisive role in ice formation (e.g., in areas influenced by the Gulf Stream) consideration limited to meteorological factors can usually supply a satisfactory explanation for the observed ice accretion.

Many empirical formulas relate ice growth to accumulated negative temperatures alone. There are at least two empirical formulas which take into separate account both accumulated negative temperatures and snow depth.

Of the various theoretical formulas for predicting ice accretion, only Kolesnikov's makes allowance for the presence of snow. It is, however, an unwieldy expression that is impractical for operational use at present.

The present study was started with the hope of developing an equation for predicting ice accretion, which would take into account snow depth and density and be in simple enough form to be of operational use. As will be shown, a modified version of Stefan's equation appears to fulfill these requirements.

II Empirical Formulas for Predicting Sea Ice Growth

Weiprecht (1873-74), quoted after Zubov (1938), is usually credited with the first systematic studies relating ice growth to the sum of negative air temperatures. He measured the ice thickness at three locations on Franz Josef Land and related the observed growth to the sum of daily mean negative air temperatures. The mathematical expression derived by Weiprecht is as follows:

$$I = 1.69\ (\Sigma\theta)^{0.56}$$

Here I is the ice thickness (cm), $\Sigma\theta$ is the sum of daily mean negative air temperatures (°C). This last quantity is often referred to as the accumulated frost degree days.

Many similar formulas have been developed for various locations. D.L. Sokolovsky and V.K. Stabrikov (1935) studied ice growth on the Volga River and found the following dependency:

$$I = 1.35\ (\Sigma\theta)^{0.55}$$

A study was done by D.B. Karelin (1938) on the Kara Sea at Dikson Island. He introduced the following dependency of ice thickness on the sum of mean daily negative air temperatures:

$$I = 2.15\ (\Sigma\theta)^{0.52}$$

Lebedev (1938) used 19 stations in the Kara, East Siberian, Chuckchee, Barents and Laptev Seas for 24 years of observations and produced the following equation:

$$I = 1.33\ (\Sigma\theta)^{0.58}$$

The values obtained from this equation were found by Lebedev to vary no more than 12% from the observed values.

Greyston (Bilello, 1961b) found that the expression $I = 1.53\ (\Sigma\theta)^{0.59}$ fitted data obtained near Churchill, Canada, for snow free ice.

The above five equations of the form $I = a\ (\Sigma\theta)^{b}$ are selected from numerous examples. The constants, a and b, vary from place to place in such a fashion as to take into account the peculiarities of the local climatological and oceanographic factors.

Perhaps the best known and most generally used empirical equation is that of Zubov (1938):

$$I^2 + 50I = 8\Sigma\theta$$

Zubov developed this equation from observations made at Uedinenia Island for the year 1935-36 and at Cape Schmidt for 1936-37, plus observations at certain other stations (Zubov does not specify which). The correlation coefficient for this equation, when applied to observations of ice thickness in all the bordering seas of the Soviet Arctic, was .76. As Zubov's equation is currently employed by the Canadian Sea Ice Forecasting Central in Halifax, its accuracy and that of the equation presented in this study will later be discussed.

All of the above equations incorporate the effect of the snow depth upon ice growth but make no attempt to evaluate its effect separately. This can be a serious shortcoming. A blanket of insulating snow can, under certain circumstances, reduce ice growth by as much as 70-80% (Kolesnikov, 1940).

Lebedev (1938) introduced an empirical formula which recognizes the damping effect snow has on ice growth:

$$I = 1.245 \ (\Sigma\theta)^{0.62} \ (\delta)^{-0.15}$$

where δ is the snow depth in centimeters. This equation was based on data from only one year's observation at Yana River at Kozachye and the Kolyma River at Konzobay.

A semi-empirical formula for evaluating sea ice growth in increments has been proposed by Assur (1956) and can be put in the form

$$\frac{1}{I}\frac{\Delta S}{\Delta I} = a + b\frac{\delta}{I}$$

The terms are defined as follows:

$\Delta S = \int_{t_1}^{t_2} \theta \, dt$, accumulated degree days of frost (below $-1.8^{\circ}C$) during the ice growing period.

t = time in days

$\Delta I = I_2 - I_1$, increase in ice thickness (cm) during the time interval

$I = \frac{I_1 + I_2}{2}$ average thickness of the ice during accretion of ΔI (cm) and

$\delta = \frac{\delta_1 + \delta_2}{2}$ average thickness of the snow cover (cm) during accretion ΔI.

a and b are coefficients that vary in space and time.

Bilello (1961b) did a study of Assur's equation and with ice accretion intervals of 20 cm. solved for a and b at five stations in the Canadian Arctic:

TABLE I: VALUES OF a and b

	a	b	b/a
Alert	.168	1.561	9.3
Eureka	.160	1.348	8.4
Isachsen	.148	.651	4.4
Mould Bay	.164	.976	6.0
Resolute	.156	1.152	7.4

The coefficient a is a function of ice thickness and snow depth. Bilello does not comment upon the accuracy of this increment method.

III. Theoretical Formulas for Predicting Sea Ice Growth

The question of ice growth due to heat transfer was first treated theoretically by Stefan (1891). The heat conducted by a unit area of ice from the water in a period of time dt is

$$k \frac{d\phi}{dz}$$

where $\frac{d\phi}{dz}$ is the vertical temperature gradient at the ice-water interface. k is the coefficient of thermal conductivity of ice. $\frac{d\phi}{dz}$ may be approximated by $\frac{\beta}{I}$ where I is the thickness of the ice (cm) and β is the difference in temperature (^{o}C) between the lower and upper surfaces of the ice.

This heat loss results in the formation of an additional layer of ice. In this manner:

$$\frac{k\beta}{I} dt = \rho L \, dI$$

where L is the heat of fusion and ρ is the density of the ice. Integrating, one obtains:

$$k \int_{o}^{t} \beta \, dt = \rho L \int_{o}^{I} I dI$$

$$\text{or} \qquad k \sum \beta = \frac{\rho L I^{2}}{2}$$

Here the integral has been approximated by the summation over the growing period of the daily mean temperature difference between water and air. If the water is fresh $\beta = \theta$ (because the freezing point is $0^{o}C$) and Stefan's formula takes on the same form as the empirical equations

$$I = \frac{2k}{\rho L} (\Sigma\theta)^{0.50}$$

This equation must be used with caution, because

(1) Here θ refers to the ice surface temperature (which might be approximated by the air temperature in the absence of snow). Ice surface temperatures are not readily obtainable, and attempts to apply the equation with air temperatures in the presence of a snow cover have failed.

(2) The equation makes no allowance for the variable energy content of an ice cover and,

(3) It does not take into account the time needed for a change in surface temperature to alter the gradient at the ice-water interface.

Schwerdtfeger (1964) used ice surface temperatures from Button Bay near Churchill, Manitoba, to derive a two-stage correction to Stefan's equation. His refinements take into account the effect of heat storage in the ice. Even with Schwerdtfeger's correction, Stefan's equation is only approximate and still requires a knowledge of the ice surface temperature. Furthermore, considering the many conditions accompanying sea ice formation, the unmodified Stefan's equation should be satisfactory for general results (Zubov, 1938). (see Appendix)

The only theoretical formula which takes into account the effects of variable snow cover is due to Kolesnikov (1946):

$$\Delta\xi^2 + \left(\frac{1.32\delta}{\xi_0^2} + \frac{90}{1.75V_0^{.656} + 5.23\times10^{-12}T_0^3} + 2\xi_1\right)\Delta\xi$$

$$= \frac{9.73}{1 - \frac{S_1}{S_{T_\lambda}} + \frac{T_\lambda}{80}\left(\frac{C_2\rho_2}{\rho_1} - C_1\right)} \int_{t_1}^{t_2} \frac{(T_\lambda - \delta)\,dt}{1 + \frac{C_1}{2K_1}(T_\lambda - \theta)}$$

where ξ_1 - initial ice thickness in cm.

$\Delta\xi$ - increase in ice thickness in cm.

S_1 - salt content of the ice

S_{T_λ} - salinity of salt water solution at temperature of freezing

ρ_1 - density of the ice

t - time

$t_2 - t_1$ - time interval

$K_1 - L + T_\lambda \left(C_2 \frac{\rho_2}{\rho_1} - C_1\right)$

ρ_2 - density of sea water

δ - snow thickness in cm

ρ_0 - snow density

V_o - wind speed

T_o - air temperature (°C)

C_1 - specific heat of sea ice

T_λ - temperature of freezing

γ - equivalent temperature, described below

C_2 - specific heat of sea water

L - latent heat of fusion for salt water ice:

$$80 \left(1 - \frac{S_1}{S_{T_\lambda}} \right) \text{(cal/gm)}$$

The equivalent temperature γ is a resultant temperature which takes into account in a complex manner the effects of (1) net long wave radiation (2) convection (3) evaporation and condensation (4) wind speed (5) humidity (6) cloud cover and (7) insolation.

For a discussion of the various elements entering into the equation and an evaluation of their effects on the predicted growth, the reader is referred to Callaway (1954). Two approximations entering Kolesnikov's equation should be noted however:

(1) He supposed the Fourier heat conduction equation to be applicable to sea ice.

(2) He assumed that there was no heat storage in the snow.

The Fourier equation assumes that the heat storage is dependent only upon the temperature gradient. But sea ice contains trapped pockets of brine. As the temperature of the ice is lowered the brine solidifies with an attending release of latent heat. Phase transitions between ice, brine, and solid salts in winter ice result in a considerable contribution to its heat storage (Savel'ev 1958) which are not accounted for by Fourier's equation.

Kolesnikov recommends that his equation be solved on a daily basis. Owing to the small coefficient of heat conductivity of snow (10^{-4} - 10^{-3} cal/sec · cm · $^{\circ}$C) his assumption of no heat storage over such a short period of time will generally not be tenable.

Kolesnikov applied his equation for fresh water ice (for which the Fourier equation should be accurate) when there was an average of 30 cm of snow on the ice. He found the correlation between the ice thickness predicted by his equation and the observed values to be "fairly good". In an instance in which the snow depth towards the end of the ice growing season reached 60 cm he found the correlation to be "much worse".

Apart from these considerations, however, Kolesnikov's equation is impractical on account of its intractibility. The time needed to evaluate it on a daily basis is too great to permit its use as an operational tool.

IV Stefan's Equation, Modified to take into account Variable Snow Cover.

If we assume that there is no heat storage in the snow, then:

$$K_s \left(\frac{\theta_i - \theta_s}{\delta} \right) = -Q \qquad \text{--- (1)}$$

where K_s - the thermal conductivity of the snow (cal/sec • cm • ^oC)

θ_i - temperature of the ice surface ^oC

θ_s - temperature of the snow surface ^oC

δ - depth of the snow (cm)

Q - heat absorbed at the snow surface (cal/cm^2 • sec)

Even if there is no heat storage, (1) will only be approximate because (a) It fails to take into account the convective sensible heat transfer that is possible in an unconsolidated snow pack. Yen (1962) calculated theoretically that a vertical draft of air of 1 cm/sec in loose snow would increase the effective thermal conductivity by as much as 30%. In the Arctic, however, where the wind packed snow is more dense ($\sim .4$ g/cm^3) than the snow of southerly latitudes (density $\sim .2$ g/cm^3) this source of error should not be great. (b) The solar radiation component of Q is not absorbed at the snow surface but rather is attenuated as it penetrates the snow in a manner well approximated by Beer's law (Liljequist, 1954). Equation (1) should be better in the Arctic during the polar night than in more southerly latitudes, where insolation is an important component of the heat budget.

Solving equation (1) for θ_i we have

$$\theta_i = \theta_s - \frac{Q\delta}{K_s} \qquad \text{--- (2)}$$

Now Stefan's equation is $I^2 = \frac{2k}{\rho L}\Sigma\beta$

Where β is the mean daily difference between the freezing point of sea water and the ice surface temperature. If we take the freezing point of sea water to be -1.8°C, $\beta = (-1.8 - \theta_i)$. Using this result together with the relation expressed in equation (2) in Stefan's formula, we have:

$$I^2 = -\frac{2k}{\rho L}\Sigma\left(\theta_s + 1.8 - \frac{Q\delta}{K_s}\right) \qquad \text{--- (3)}$$

Here the summation is to be taken over daily mean values of the argument.

While equation (3) is based upon the assumption of no heat storage, it does not hold this restriction to short periods of time, as does Kolesnikov's equation. The period over which it is evaluated can vary to make the assumption of no heat storage in the snow consistent with the snow depth. Thus when there is little snow cover it can be used to predict daily variations in ice thickness. If, however, the snow is deep it can be evaluated at the end of much longer growing periods (10 days, 50 days ...)

V The Values for K_s

A formula has been presented by Abels (1892) which relates the heat conductivity of snow to its density

$$K_s = .0068\rho^2 \quad \text{cal/cm} \cdot \text{sec} \cdot {}^{\circ}\text{C}$$

At southerly latitudes the snow density is close to .2g/cm^3. Substituting this in Abels' equation:

$$K_s = 27.2 \times 10^{-5} \text{ cal/cm} \,.\, \text{sec} \cdot {}^{\circ}\text{C}$$

$$\text{or} \quad K_s = 23.6 \text{ cal/cm} \cdot \text{day} \cdot {}^{\circ}\text{C}$$

Using this in equation (3) one obtains:

$$I^2 = -\frac{2k}{\rho L} \sum (\theta_s + 1.8 - \frac{Q\delta}{23.6}) \quad \text{---} \ (4)$$

This equation is very sensitive to errors in Q . Thus with a snow depth δ of 20 cm ($\sim$8 inches) an error in Q of 10 cal/cm^2• day will introduce an error into the above equation of 8.5°C. The magnitude of this error is comparable in size to θ_s. Later in this study the approximations which enter into the evaluation of Q will be analyzed. Some of the assumptions may result in sizable errors in Q. In the author's opinion, the methods for evaluating Q, on a routine basis, are too approximate to justify using equation (4) with any confidence.

In the Arctic regions the snow density is greater than in the temperate zones. Table II displays the average snow densities at northern stations. These come from weekly values reported by Bilello (1964):

Table II: Snow Densities at Northern Stations

Year	Place	No. of Measurements	Average density
1960-61	Eureka	25	.35
1960-61	Mould Bay	23	.41
1960-61	Resolute	34	.38
1961-62	Isachsen	12	.35
1961-62	Mould Bay	27	.39
1961-62	Resolute	27	.38

Longley (1960), from a study of data collected at Resolute, showed that snow density values cannot be considered accurate beyond $\pm .05 g/cm^3$. The average value of ρ_o from Table II is $.380 g/cm^3$. Substituting this value into Abels' equation and converting from seconds to days we find $K_s = 85$ cal/cm • day • $^\circ$C. In view of Longley's findings no attempt should be made to adjust this K_s value to agree with the density as actually reported by the station.

With this K_s, equation (3) for Arctic regions becomes:

$$I^2 = \frac{2k}{\rho L} \sum (\theta_s + 1.8 - \frac{Q\rho}{85}) \quad \text{--- (5)}$$

Equation (5) is less sensitive to Q than the equivalent equation appropriate for southerly latitudes (equation (4)). For that reason, and because equation (1) is more valid in northern regions, the study of ice growth in this investigation has been restricted to areas within the Arctic circle.

VI. Measurements of Ice Thickness and Snow Depth

Weather permitting, ice thickness and snow depth are measured every week at many weather stations in the Arctic. The time of the first measurement depends upon the type of weather following freeze-up and the courage of the observer.

Measurements of ice thickness are made by means of a special auger kit. A one inch diameter vertical hole is drilled through the ice with an auger operated by a hand brace. A graduated tape with a wire assembly is lowered through the hole. The weight consists of a metal rod six inches long attached to one end of the tape. When the rod clears the bottom of the hole it is swung to a horizontal position against the lower surface of the ice. The actual ice thickness is then read off the tape at the ice surface, to the nearest half inch. Measurements throughout each season are made as closely as possible to the same spot.

No attempt has been made in this study to adjust the ice thickness as reported by the observer. Reports, however, that were clearly non-representative were rejected. Three reasons can be given for these non-representative reports:

(1) there was an error made by the observer in taking the measurement,

(2) there was a sudden growth of "white ice",

(3) there was rafting in the vicinity of the measured ice.

The first reason advanced is a common source of mistakes in the making of most measurements. It is sporadic and easily spotted. The other two causes are more serious as their effect can persist and cause a systematic deviation of the observed from the predicted ice thicknesses.

"White ice" is caused by the freezing of slush. Heavy loads of snow can depress an ice surface below its hydrostatic level and cause flooding of the ice. Flooding may also be caused by tidal effects. Snow wetted in consequence of the flooding will freeze, giving "white ice". A detailed discussion of the white ice problem at Knob Lake has been given by Shaw (1965). Bilello (1964) estimated that the phenomena occurred once or twice a year at 21 locations examined by him.

Rafting is the sliding of one sheet of ice under another as a result of pressure due to currents or wind. It is most commonly observed during autumn freeze-up or during spring thaw.

With ice thickness measurements, the Meteorological Branch, Department of Transport, Canada, publishes snow depth measurements over the ice, together with the nature of the snow surface (smooth, drifted, soiled, etc.). These weekly snow depth measurements represent what the observer considers to be an average over the area involved. In using these snow depths in equation (5), a linear variation was assumed to occur from one weekly observation to the next. The thought of relating the increase (or decrease) of snow thickness over the ice to that measured at the meteorological station was rejected on account of the importance and frequency of blowing snow (Fraser, 1964).

VII The Snow Surface Temperature

There are instrumental problems which make it difficult to measure snow surface temperatures. The problems are caused by strong temperature gradients on each side of the surface and by drifting snow. For Polar regions, observations are available from "Maud" (Sverdrup, 1933) and North Pole 2 (Yakovlev, 1955).

The difference between the snow surface temperature and the air temperature at screen level varies according to wind speed and cloud cover (Sverdrup, 1933) and with the thickness of the snow-ice cover of the ocean. This last dependency arises from the control such a cover exercises over the heat transported upward from the water.

Vowinkel and Orvig (1964b), considering the lack of data, proposed the following monthly corrections to reduce the air temperature at screen level to the temperature of the snow surface:

MONTH	J	F	M	A	M	J	J	A	S	O	N	D	
correction	-2	-1	-1	1	1	1	1	1	1	0	-1	-1	°C

These corrections are based upon data from "Maud" and from North Pole 2.

Later in this study it will be necessary to calculate the heat budget of snow surfaces in the Arctic. It will be important to recognize a temperature difference between the snow surface and screen level. Owing to the paucity of data, Vowinkel and Orvig's results for the polar ocean have been accepted and extended to include the Arctic Archipelago. The snow surface temperature θ_s in future calculations made by the author will be determined by the relation:

θ_s = mean daily air temperature measured at the weather station + appropriate correction factor.

The mean daily air temperatures have been extracted from the Monthly Record and the Arctic Summary. Both of these are publications of the Canadian Meteorological Service.

VIII Values of $\frac{2k}{\rho L}$

The equation that will be studied in this thesis is:

$$I^2 = -\frac{2k}{\rho L}\sum(\theta_s + 1.8 - \frac{Q\delta}{85}) \qquad \text{--- (5)}$$

The value of $\frac{2k}{\rho L}$ used in this study was determined by adjusting equation (5) so that it would best agree (in the least square sense) with observed values of ice thickness. Let I be the value of ice thickness predicted by equation (5) and let I' be the observed ice thickness, then $\frac{2k}{\rho L}$ must satisfy the equation

$$\frac{\partial \sum'}{\partial \frac{2k}{\rho L}} (I - I')^2 = 0 \qquad \text{--- (6)}$$

Here $\sum'$ indicates summation over the number of observations under consideration. Now $I = (-\frac{2k}{\rho L}\sum(\theta_s + 1.8 - \frac{Q\delta}{85}))^{\frac{1}{2}}$ where $\sum$ indicates summation over the number of ice growth days used in the study. Substituting this value for I into equation (6) and performing the indicated operations one finds:

$$\frac{2k}{\rho L} = \left(\frac{\sum' I' \left(\sum -(\theta_s + 1.8 - \frac{Q\delta}{85})\right)^{\frac{1}{2}}}{-\sum'\left(\sum(\theta_s + 1.8 - \frac{Q\delta}{85})\right)}\right)^2 \qquad \text{--- (7)}$$

Ice thickness and snow data from three stations (Holman Island, Clyde and Mould Bay) were used to calculate a value of $\frac{2k}{\rho L}$ from (7). These data are displayed in Tables III - IV. All three sites for the selected years have one thing in common -- low to moderate snow depths. Under this circumstance:

(1) the assumption of no heat storage in the snow should be reasonable

(2) as values of $\frac{\delta}{85}$ will be small, equation (5) should be

insensitive to errors in Q; in consequence one can use the monthly climatic values of Q that have been published (Vowinkel and Orvig, 1964d, 1964b, and Vowinkel and Taylor, 1965). Only the radiation components of Q have been published for the Arctic Archipelago. The sensible and latent heat components which Vowinkel and Taylor calculated for the Polar Ocean were, however, assumed to hold for the Arctic Archipelago.

The pertinent climatic values of Q for Holman Island, Clyde and Mould Bay are given in Table VI. They made possible the calculation of right hand side of equation (5) without difficulty. In the case of Clyde, instances were encountered when $\frac{Q\delta}{85}$ was less than $\theta_s + 1.8$. In this circumstance equation (5) will predict a decrease in ice thickness. To avoid any gross errors between predicted and actual values of ice thickness, data from periods during which $\frac{Q\delta}{85}$ was less than $\theta + 1.8$ were not included under the summation sign in equation (5). In other words, it was assumed that ice does not grow when $\frac{Q\delta}{85}$ is less than $\theta_s + 1.8$.

From equation (7) values of $\frac{2k}{\rho L}$ of 8.4, 8.6 and 8.8 $\mathrm{cm}^2/°\mathrm{c}$ were calculated for Clyde, Holman Island and Mould Bay respectively. The average value of 8.6 was adopted for use by the author. This is also the value calculated for the site, Holman Island, which had the least amount of snow. The equation proposed for use in predicting ice thickness in the Arctic is then

$$I^2 = -8.6 \sum (\theta_s + 1.8 - \frac{Q\delta}{85}) \qquad --- (8)$$

This equation should be applied with the following restrictions:

(1) It should not be applied beyond the first week of May. The term $\frac{Q\delta}{85}$ will act to decrease or increase the amount of predicted ice, according to whether Q is negative or positive. Thus in May, when Q is usually positive, equation (8) can predict substantial ice growth. As ice accretion is virtually completed by the first of May, continued use of equation (8) beyond this period might result in a large error in the final predicted value.

(2) For the reasons previously given, equation (8) should not be used with data collected when $\frac{Q\delta}{85} < \theta_s + 1.8$

TABLE III

Ice Thickness and Snow Depth Observations at Clyde (70.5°N, 68.5°W) for the ice growing season 1960-61. The underlined values of ice thickness were not considered representative.

Date	Ice Thickness (cm)	Snow Depth (cm)	Date	Ice Thickness (cm)	Snow Depth (cm)
Nov. 5	freeze-up commenced		Feb. 10	94	36
Nov. 11	15	0	Feb. 17	102	36
Nov. 18	28	3	Feb. 24	102	36
Nov. 25	38	5	Mar. 3	107	38
Dec. 2	38	5	Mar. 10	117	36
Dec. 9	48	8	Mar. 17	117	38
Dec. 16	46	10	Mar. 24	117	36
Dec. 23	53	18	Mar. 31	122	36
Jan. 6	64	18	Apr. 7	124	36
Jan. 13	74	15	Apr. 14	140	30
Jan. 20	79	25	Apr. 21	142	33
Jan. 27	84	33	Apr. 28	137	33
Feb. 3	89	30	May 5	140	30

TABLE IV

Ice Thickness and Snow Depth Observations at Holman Island (74.7^oN, 95^oW) for the ice growing season 1961-62. The underlined values of ice thickness were not considered representative.

Date	Ice Thickness (cm)	Snow Depth (cm)
Oct. 5	freeze-up commenced	
Oct. 7	6	0
Oct. 13	19	3
Oct. 20	29	3
Oct. 27	38	3
Nov. 3	46	3
Nov. 10	51	5
Nov. 17	62	5
Nov. 24	67	3
Dec. 1	77	5
Dec. 8	83	5
Dec. 15	90	8
Dec. 22	100	8
Jan. 12	119	8
Jan. 19	130	10
Jan. 26	132	10
Feb. 2	135	10
Feb. 9	138	10
Feb. 16	146	10
Feb. 23	154	13
Mar. 2	157	10
Mar. 23	173	8
Mar. 30	178	8
Apr. 6	182	8
Apr. 13	182	8
Apr. 20	185	8
Apr. 27	189	6

TABLE V

Ice Thickness and Snow Depth Observations at Mould Bay 76.2°N, 119.3°W) for the ice growing season 1961-62. The underlined values of ice thickness were not considered representative.

Date	Ice Thickness (cm)	Snow Depth (cm)	Date	Ice Thickness (cm)	Snow Depth (cm)
Sept. 22	freeze-up commenced		Jan. 26	127	15
Oct. 13	43	3	Feb. 2	137	13
Oct. 20	53	4	Feb. 9	137	13
Oct. 27	58	8	Feb. 16	145	10
Nov. 3	64	8	Feb. 23	132	13
Nov. 10	69	5	Mar. 2	150	20
Nov. 17	79	8	Mar. 9	155	20
Nov. 24	84	15	Mar. 19	157	15
Nov. 30	89	18	Mar. 23	163	15
Dec. 8	94	18	Mar. 30	163	25
Dec. 15	97	18	Apr. 6	173	23
Dec. 21	97	18	Apr. 13	168	48
Dec. 29	99	25	Apr. 20	193	23
Jan. 5	112	25	Apr. 27	180	38
Jan. 13	117	20	May 4	198	33
Jan. 18	117	30			

TABLE VI

Pertinent monthly climatic values of Q (cal/cm^{2}· day) for Clyde, Holman Island, Mould Bay, Resolute, and Eureka

	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
Clyde	-35	-105	-145	-100	-85	-100	-75	-45	10
Holman Island	-35	- 75	- 85	- 85	- 80	- 75	-55	-30	30
Mould Bay	-35	- 75	- 95	- 85	-80	- 75	-65	-50	30
Resolute	-40	- 90	-100	-100	-90	- 85	-75	-70	20
Eureka	-40	- 90	- 90	-100	-85	- 90	-75	-90	20

The good agreement between values of I predicted, by equation (8), and observed is shown in Figure 1. The modified Stefan equation (equation (8)) overestimates ice thickness at the beginning of the growing season and underestimates it towards the end of the growing seatson. An increasing value of $\frac{2k}{\rho L}$ with time would explain the systematic variation. Such a trend in the behaviour of $\frac{2k}{\rho L}$ has been noted and analyzed by Bilello (1961a). Working with data from Eureka he found values of $\frac{2k}{\rho L}$ ranging from 8.2 cm^2/ ^{o}C • day to 16 cm^2/ ^{o}C • day. His average value was 11 cm^2 / ^{o}C • day.

Figure 2 shows the success of Zubov's equation in predicting the same ice growth as was shown in Figure 1. Equation (8) predicts the growth more faithfully than Zubov's. The correlation coefficients between the observed and calculated ice thickness for equation (8) and for Zubov's equation were .99 and .91 respectively.

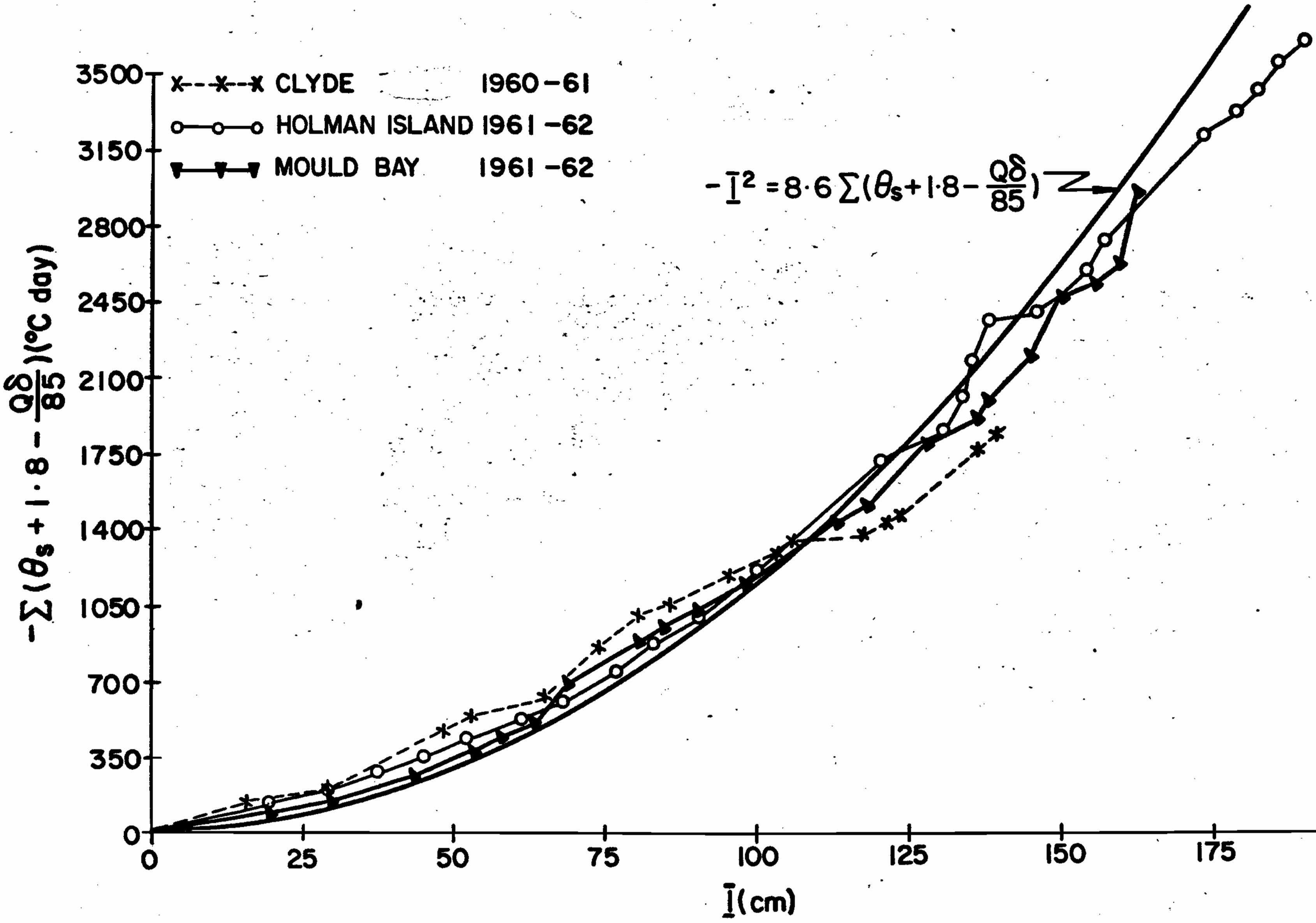

FIG.1 PLOT OF OBSERVED ICE THICKNESS $\underline{I}$, AGAINST $-\sum(\theta_s+1\cdot8-\frac{Q\delta}{85})$

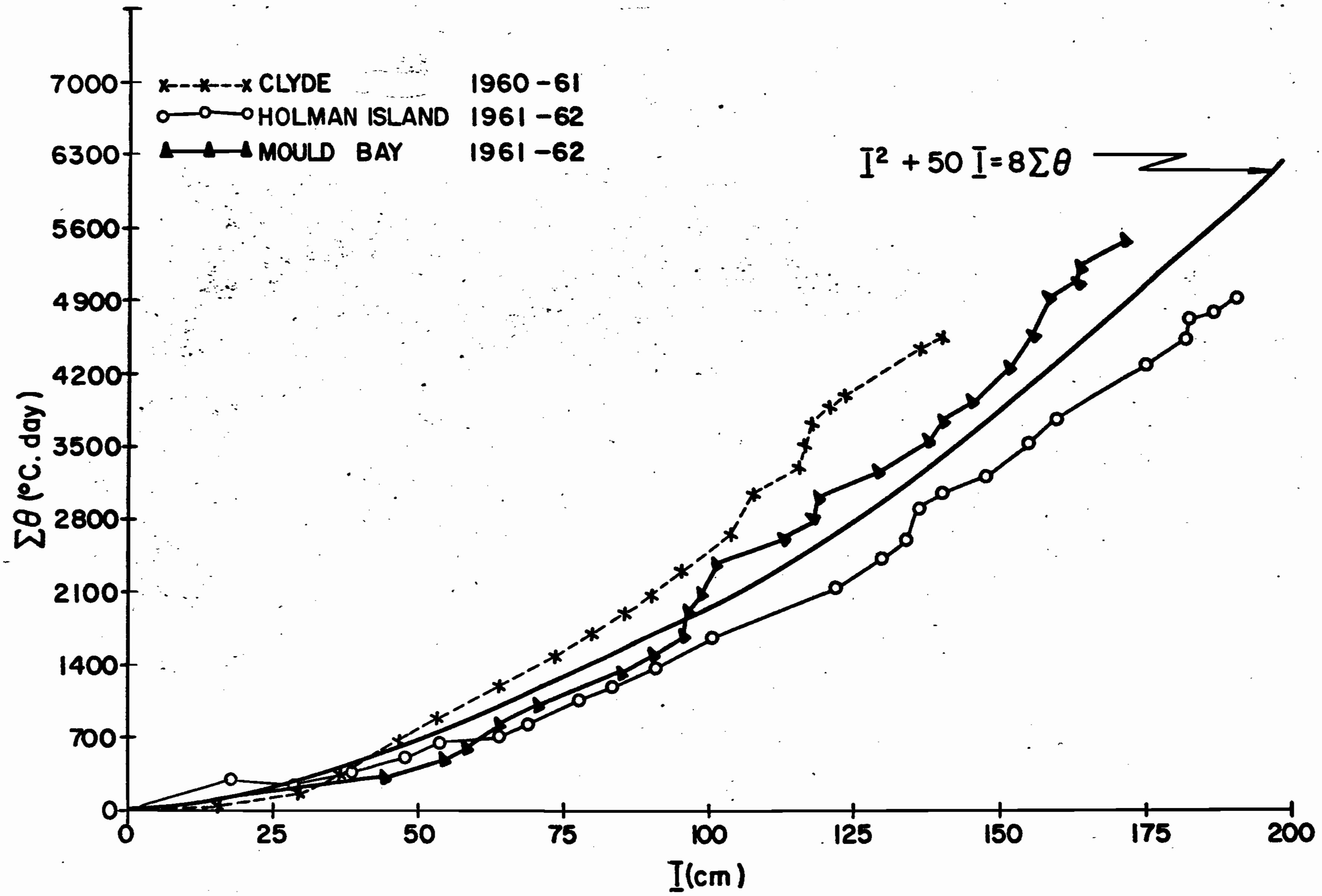

FIG. 2 PLOT OF OBSERVED ICE THICKNESS I, AGAINST $\sum\theta$

IX Application of the Modified Stefan's Equation with Climatic Values of Q When There is a Deep Snow Cover

Equation (8) will be more sensitive to errors in Q when δ is large. The situation is, however, more complicated than this equation would indicate. Difficulties with instances of deep snow will arise due to the inevitable large heat storage. This storage is not accounted for by equation (8).

If the snow depth before the end of March was greater than 40 cm, the ice growing season is considered to have been influenced by heavy snow. This definition, while arbitrary, has been based upon the following considerations.

(1) When $\frac{Q\delta}{85} < \theta_s + 1.8$ the snow is considered thick enough to prevent growth of the ice (see previous section). Now, from Table VI, $\frac{Q}{85} \sim -1$, and snow surface temperatures are generally greater than -38°C. Therefore, periods with snow depths of 40 cm or greater will usually prevent ice accretion.

(2) As the ice thickens, its growth becomes less sensitive to surface conditions. By the end of March an increase in snow depth should not greatly effect ice growth.

Ice and snow depth measurements from September 1959 to May 1964 for eight Arctic stations (Alert, Clyde, Eureka, Holman Island, Isachsen, Mould Bay, Resolute and Sachs Harbour) showed that of 39 cases there were ten instances when the above criteria for heavy snow fall were satisfied.

In this section the degree of agreement between values of I predicted by equation (8) and those actually observed will be considered for four cases when δ was large. At present only climatic values of Q will be considered. Tables VII - X display ice thickness and snow depth measurements at Eureka, Resolute, Clyde and Mould Bay in years when the heavy snow fall criteria were met. Appropriate values of Q have been given in Table VI.

TABLE VII

Ice Thickness and Snow Depth Observations at Eureka (80.0°N, 85.9°W) for the ice growing season 1961-62. The underlined values of ice thickness were not considered representative.

Date	Ice Thickness (cm)	Snow Depth (cm)
Sept. 6	freeze-up commenced	
Sept. 15	18	3
Sept. 22	28	5
Sept. 29	36	5
Oct. 6	46	5
Oct. 13	46	5
Oct. 20	51	5
Oct. 27	58	8
Nov. 3	61	8
Nov. 10	66	8
Nov. 17	71	13
Nov. 24	76	13
Dec. 1	84	13
Dec. 8	94	15
Dec. 15	99	15
Dec. 22	107	15
Dec. 29	114	18
Jan. 5	117	18
Jan. 12	117	23
Jan. 19	122	20
Jan. 26	130	25
Feb. 2	124	36
Feb. 9	127	48
Feb. 16	130	43
Feb. 23	135	43
Mar. 2	140	46
Mar. 9	145	48
Mar. 16	150	56
Mar. 23	155	56
Mar. 30	157	56
Apr. 6	163	53
Apr. 13	170	36
Apr. 20	175	38
Apr. 27	178	20
May 4	188	28

TABLE VIII

Ice Thickness and Snow Depth Observations at Resolute (14.7°N. 95.0°W) for the ice growing season 1961-62. The underlined values of ice thickness were not considered representative.

Date	Ice Thickness (cm)	Snow Depth (cm)	Date	Ice Thickness (cm)	Snow Depth (cm)
Sept. 26	freeze-up occurred		Jan. 19	109	28
Oct. 1	9	3	Jan. 26	107	25
Oct. 8	25	5	Feb. 4	114	36
Oct. 13	20	3	Feb. 9	119	30
Oct. 21	36	5	Feb. 16	124	30
Oct. 29	46	15	Feb. 24	132	33
Nov. 3	51	15	Mar. 2	132	38
Nov. 10	51	15	Mar. 11	137	38
Nov. 18	66	28	Mar. 16	142	41
Nov. 24	74	30	Mar. 23	147	38
Dec. 3	69	15	Mar. 31	147	51
Dec. 8	76	15	Apr. 6	147	53
Dec. 15	84	20	Apr. 13	152	53
Dec. 22	81	28	Apr. 20	160	51
Dec. 29	86	33	Apr. 27	178	43
Jan. 5	91	25	May 4	165	58
Jan. 12	102	28			

TABLE IX

Ice Thickness and Snow Depth Observations at Clyde (70.5°N, 68.5°W) for the ice growing season 1963-64. The underlined values of ice thickness were not considered representative.

Date	Ice Thickness (cm)	Snow Depth (cm)	Date	Ice Thickness (cm)	Snow Depth (cm)
Nov. 16	freeze-up commenced		Feb. 14	99	53
Nov. 22	25	0	Feb. 21	96	56
Nov. 29	43	3	Feb. 28	99	58
Dec. 6	56	3	Mar. 6	112	56
Dec. 13	64	5	Mar. 13	122	56
Dec. 23	68	10	Mar. 20	112	61
Dec. 27	74	13	Mar. 28	117	66
Jan. 3	76	15	Apr. 3	114	66
Jan. 10	79	15	Apr. 10	117	66
Jan. 19	79	30	Apr. 17	117	69
Jan. 24	84	30	Apr. 24	119	62
Jan. 31	89	23	May 1	124	69
Feb. 7	94	30	May 9	130	77

TABLE X

Ice Thickness and Snow Depth Observations at Mould Bay (76.2°N, 119.3°W) for the ice growing season 1963-64. The underlined values of ice thickness were not considered representative.

Date	Ice Thickness (cm)	Snow Depth (cm)	Date	Ice Thickness (cm)	Snow Depth (cm)
Sept. 25	freeze-up commenced		Jan. 24	107	48
Oct. 6	23	0	Jan. 31	115	51
Oct. 13	41	0	Feb. 7	112	58
Oct. 21	48	4	Feb. 14	122	61
Oct. 25	53	9	Feb. 21	127	58
Nov. 1	56	10	Feb. 28	130	58
Nov. 9	56	10	Mar. 6	135	56
Nov. 15	58	15	Mar. 13	143	56
Nov. 22	66	15	Mar. 20	144	66
Nov. 29	73	20	Mar. 27	145	58
Dec. 7	71	25	Apr. 3	153	51
Dec. 14	81	25	Apr. 10	153	61
Dec. 20	86	30	Apr. 17	155	53
Dec. 27	91	30	Apr. 24	155	56
Jan. 3	97	30	May 1	158	58
Jan. 10	102	33	May 8	158	61
Jan. 17	104	46			

Figures 3 and 4 show how the values of I predicted by equation (8) depart from the observed values. The equivalent data for Zubov's equation are given by Figures 5 and 6. The correlation coefficient between the observed and predicted values of I for both equations for all four cases was .785.

The degree of success in predicting the ice thickness is also indicated in Table XI where values of $\left(\frac{\Sigma'(I - I')^2}{n}\right)^{\frac{1}{2}}$ are displayed.

Here I = predicted values of ice thickness

I' = measured ice thickness

n = number of ice thickness measurements.

The summation sign Σ' extends over the number of observations, n. The Table also includes data from the three previously discussed cases of light to moderate snow depths.

TABLE XI

Values of $\left(\frac{\Sigma'(I - I')^2}{n}\right)^{\frac{1}{2}}$ for the Modified Stefan's Equation and for Zubov's Equation

Place	Season	Modified Stefan's Equation	Zubov's Equation
Clyde	1960-61	10 cm	18 cm
Holman Island	1961-62	6	11
Mould Bay	1961-62	6	10
Mould Bay	1963-64	32	31
Resolute	1961-62	29	21
Eureka	1961-62	20	26
Clyde	1963-64	11	20

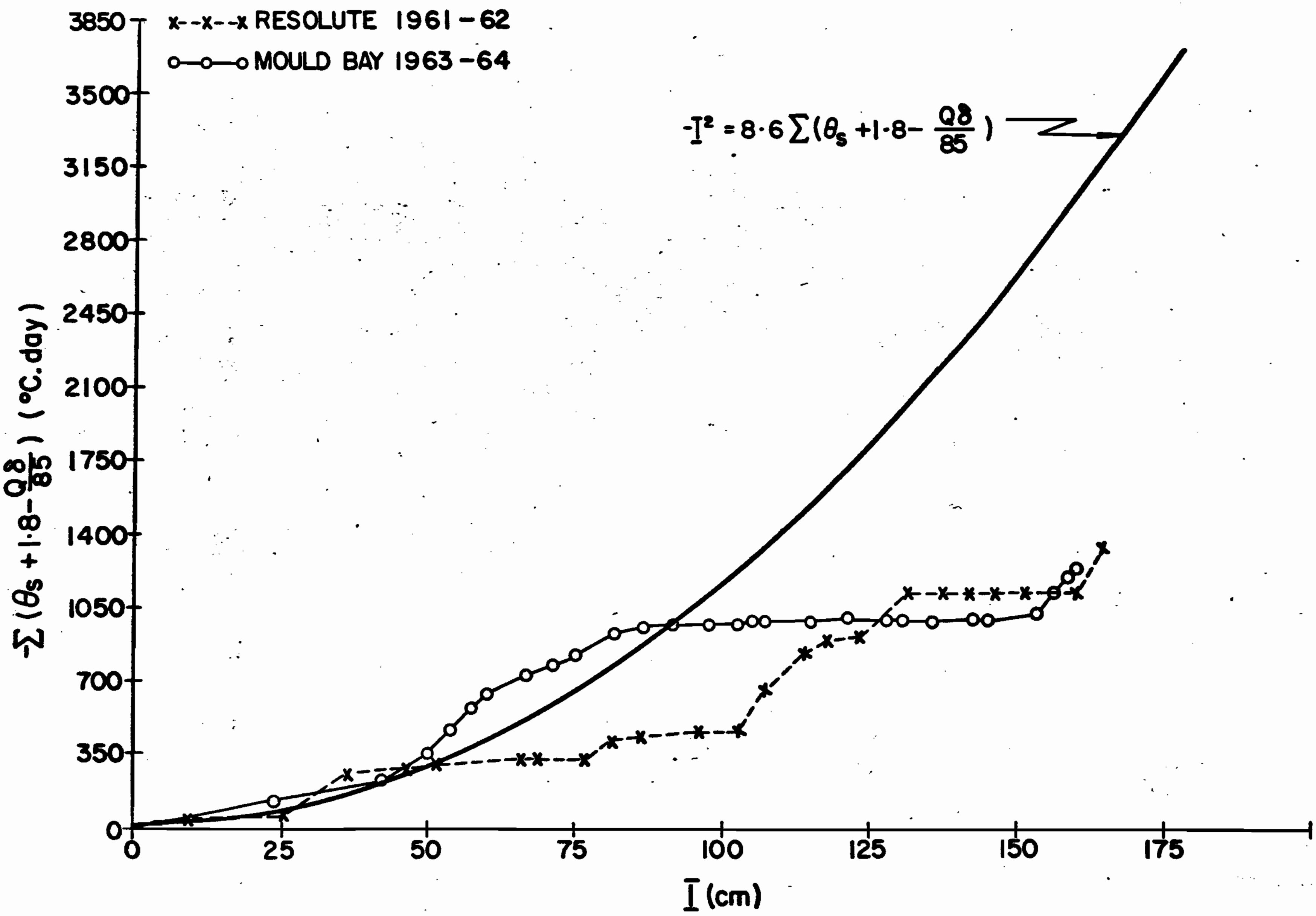

FIG. 3 PLOT OF OBSERVED ICE THICKNESS $\bar{I}$, AGAINST $-\sum(\theta_s + 1\cdot8 - \frac{Q\delta}{85})$ USING CLIMATIC VALUES OF Q (cal/cm.^{2} day)

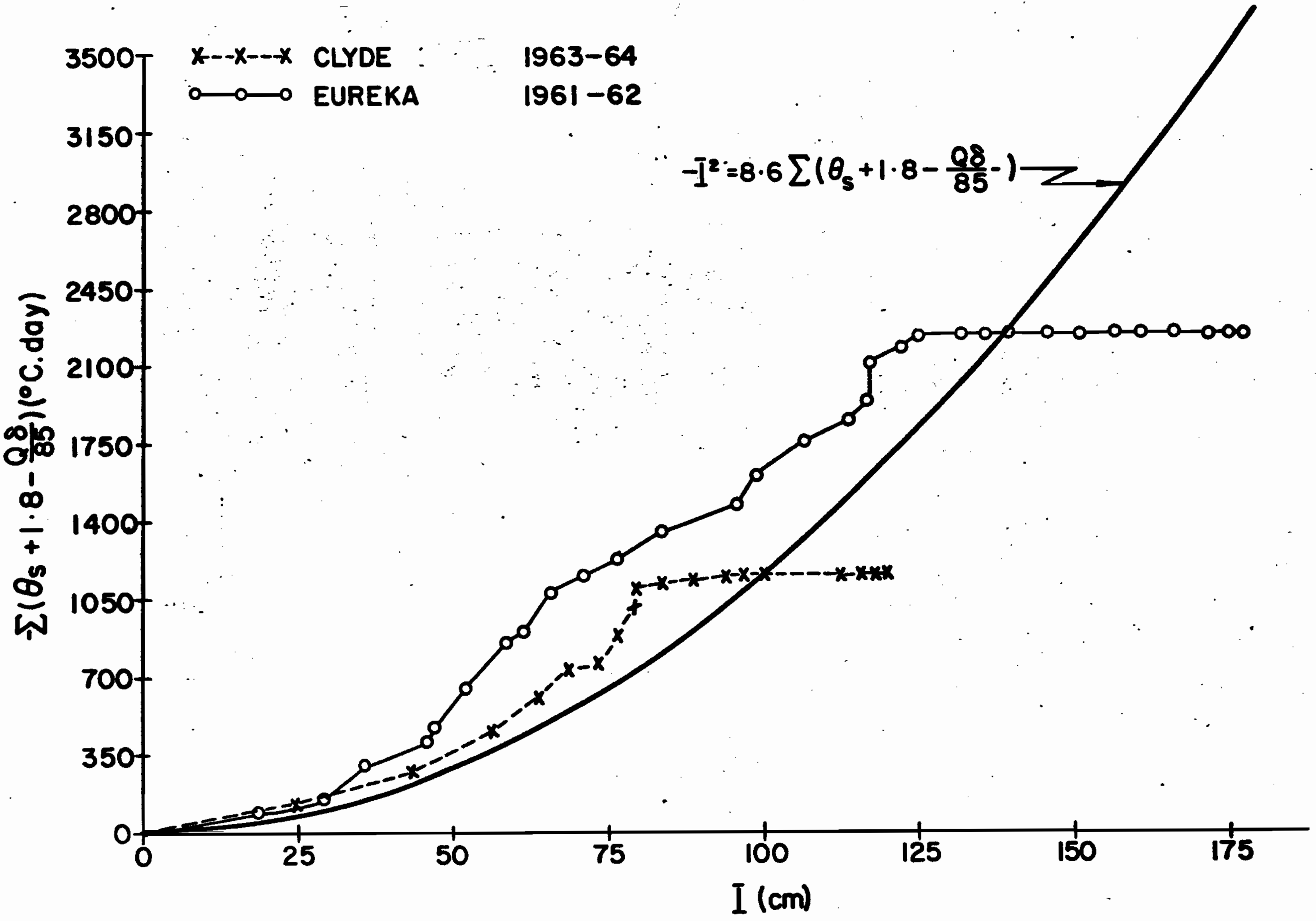

FIG. 4 PLOT OF OBSERVED ICE THICKNESS I, AGAINST $-\sum(\theta_s + 1.8 - \frac{Q\delta}{85})$ USING CLIMATIC VALUES OF Q (cal/cm.^{2} day)

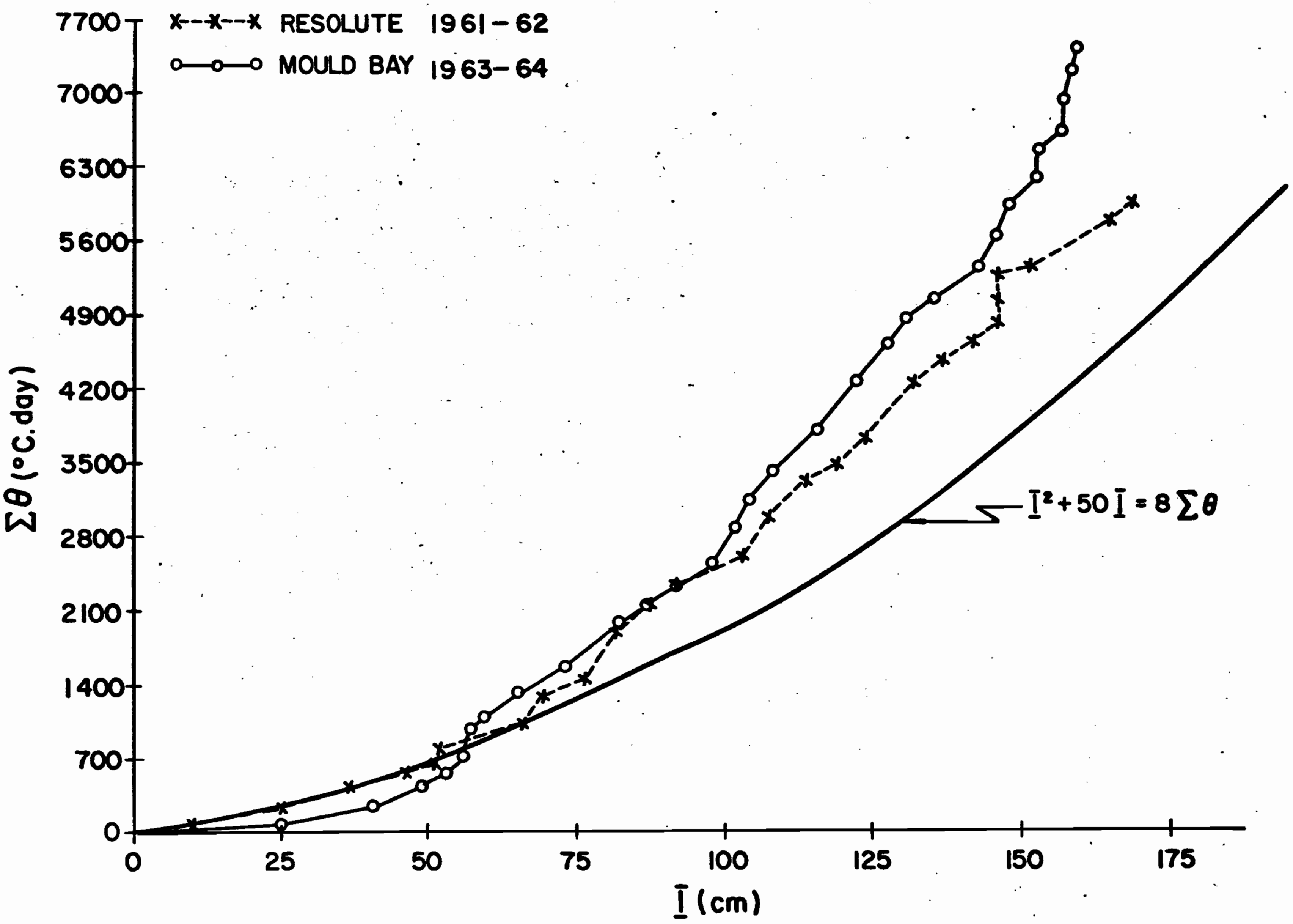

FIG. 5 PLOT OF OBSERVED ICE THICKNESS AGAINST $\sum\theta$

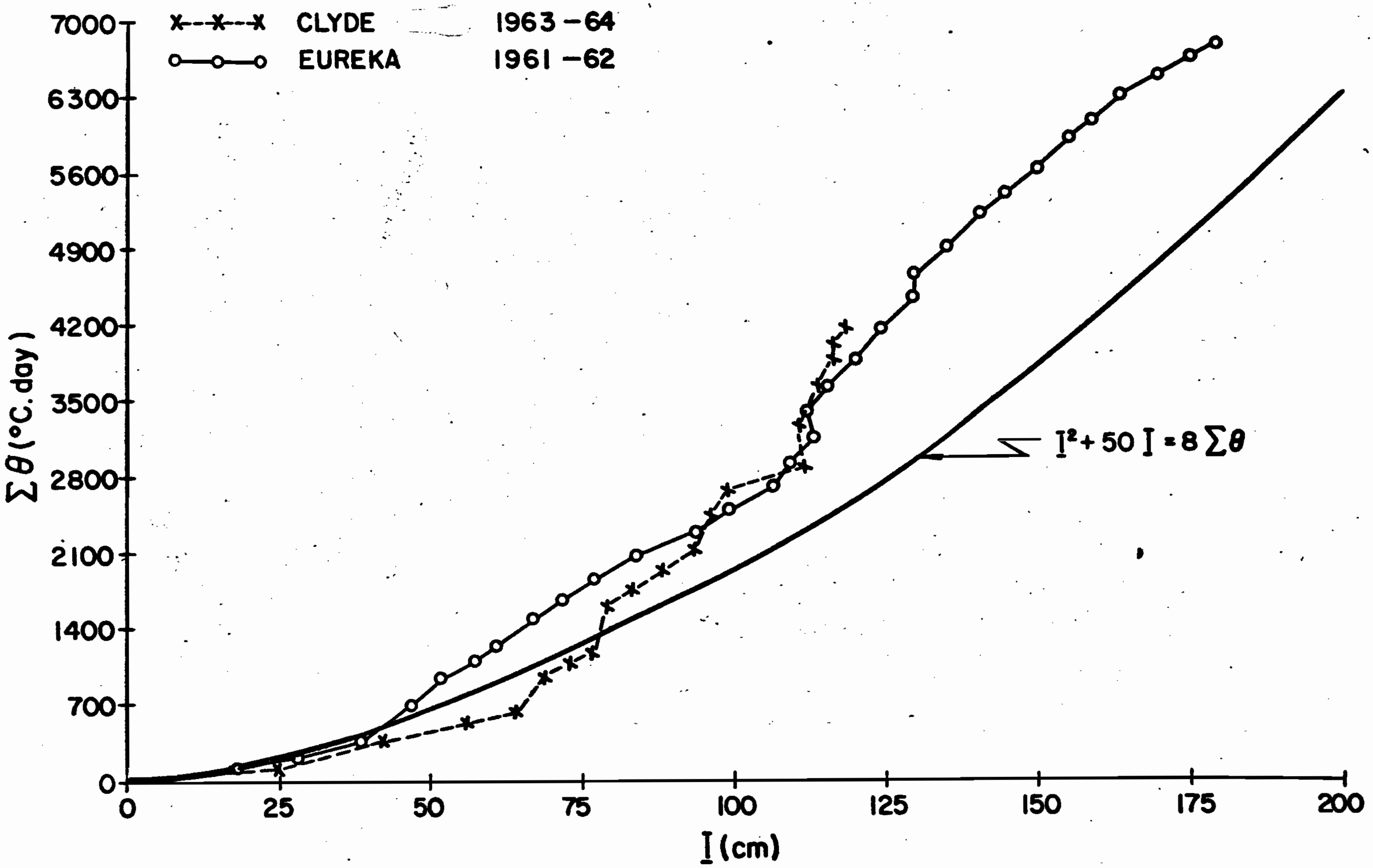

FIG. 6 PLOT OF OBSERVED ICE THICKNESS I, AGAINST $\sum\theta$

Table XI shows that with climatic values of Q, equation (8) was superior to Zubov's equation in predicting ice growth when the snow depth was light to moderate. When, however, the snow cover was heavy the two equations were of comparable accuracy.

X Calculation of Daily Heat Loss at Mould Bay

The modified Stefan's equation for Arctic regions is

$$I^2 = -8.6 \Sigma (\theta + 1.8 - \frac{Q\delta}{85})$$

Let Δ I be the error induced in the calculation of ice thickness by an error in Q of ΔQ. By differentiating equation (8) with respect to Q it can be shown that

$$\Delta I = \frac{+8.6 \delta \Delta Q}{2 \times 85 I}$$

$$= 0.05 \frac{\delta}{I} \Delta Q$$

An error in the calculated value of I due to an error in Q is thus proportional to the ratio $\frac{\delta}{I}$. When this ratio is large it may be necessary to use actual values of Q in place of climatic ones.

In this section the daily heat budget calculations for Mould Bay during the ice growing seasons 1961-62 and 1963-64 will be discussed, and comparison will be made between actual and climatic values of Q. It is hoped that conditions at Mould Bay may be representative of the Arctic. In the winter of 1963-64 there was approximately twice as much snow as during the winter of 1961-62. (see Tables V and X).

The heat gain Q at the snow surface is given by the equation

$$Q = Q\uparrow + Q\downarrow + Q_L + Q_H + Q_s$$

where $Q\uparrow$ - long wave radiation from the ground to the atmosphere

$Q\downarrow$ - long wave back-radiation from the atmosphere to the ground
Q_L - latent heat gain
Q_H - sensible heat gain
Q_S - heat gained from insolation

The sign convention in this equation is such that heat leaving the surface has a negative sign while that reaching the surface is positive. Thus $Q\uparrow$ is always negative while $Q\downarrow$ and Q_s are always positive. The sign of Q_H and Q_L depend upon the temperature and water vapour gradients at the snow surface, respectively.

(i) <u>Long wave radiation emitted by the snow surface</u>.

The daily heat loss from the snow surface by long wave radiation was calculated by the Stefan-Boltzmann formula

$$Q\uparrow = \sigma T^4$$

Here T is the temperature of the snow surface in degrees absolute. Geiger (1961) presented the value of σ: (8.26×10^{-11} cal/cm^2 • min. • K^4) employed in this study. Use of this equation, which assumes that snow will radiate as a black body, has been justified by Vowinkel and Orvig (1964).

A discussion has already been given as to how the snow surface temperature was calculated. In reference to possible errors in this temperature, one should note that a difference of one degree C would change the emitted long wave radiation by about 8 cal/cm^2 • day.

(ii) <u>Long wave radiation received at the surface</u>.

Bolz (1949) suggested that long wave radiation received at the ground follows the relation:

$$G_n = G_o\ (1 + kn^2) \qquad \text{--- (9)}$$

where G_n = the long wave radiation received at the ground with n tenths of cloud.

G_o - clear sky radiation
k - a cloud coefficient
n - cloud amount in tenths.

The long wave back radiation, G_o, emitted with clear skies was determined from the Yamamoto chart (Yamamoto G, 1952). The temperature and moisture profiles from the two daily radiosonde flights at Mould Bay were averaged and reduced according to the linear pressure reduction (Elsasser 1960). Tables of the daily temperature and corresponding optical depths at seven levels (surface, 900 mb, 800 mb, 700 mb, 600 mb, 500 mb, and 400 mb levels) were constructed and the data plotted on the charts. Five hundred charts were prepared and planimetered to yield the clear sky back radiation for each day of the two years from mid September to the end of May.

Clouds are also assumed to obey the Stefan-Boltzmann law. From Mould Bay synoptic data it was possible to estimate the height of the clouds. With this estimation, on consulting the radiosonde data, one could find the cloud base temperature. It follows then that with an overcast sky the long wave back radiation is

$$G_{10} = \sigma T_c^4$$

where T_c is the cloud bottom temperature (^oK). Knowing this, one can solve (9) for k

$$k = (\frac{\sigma T_c^4}{G_o} - 1)$$

Long wave radiation from a sky with n tenths of cloud is then given the following equation:

$$G_n = G_o \; (1 + (\frac{\sigma T_c^4}{G_o} - 1) \; n^2) \qquad \text{---(10)}$$

Difficulty arises with using this equation in the determination of T_c. Visual observations of cloud base heights are not accurate. In consequence the T_c's estimated from the radiosonde data can be in error by more than 5 degrees. Fortunately, estimates of cloud height can be expected to improve with increased cloud cover. An examination of equation (10) will show that an error in T_c with only one tenth of cloud cover has only 1/100 the influence on G_n of a similar error with overcast cloud. Values of n were taken from the synoptic reports published by the Canadian Meteorological Service.

(iii) Solar radiation absorbed at the ground.

Short wave radiation absorbed at the earth's surface is a function of latitude, atmospheric ozone, time of year, particulate matter suspended in the air, cloud amount, precipitable water and albedo. If the insolation from clear sky, S_o, is known, then the absorbed radiation Q_s, may be given by the relation:

$$Q_s = S_o \; (1 - k_1 n) \; (1 - A)$$

where k_1 = cloud factor

n = cloud amount in tenths

A = surface albedo

Tables of S_o, for Arctic regions, have been presented by Vowinkel and Orvig (1964a) as have values of k_1 for different cloud types (Vowinkel and Orvig, 1962). Values for n were obtained from the Arctic Summaries referred to earlier.

The albedo is the ratio of reflected to incoming radiation. Snow albedo is a function of the sun's altitude, of the roughness of the snow surface, and of the age of the snow. Sverdup, in working up data from the Maud (1933), used a value of .76 for the months of March and April. He used a value of .62 for May. Kolesnikov (1946) employed an albedo of .65. In this study a constant value of .70 has been assumed. Angstrom (1925) found this to be an average value for several days' old snow.

(iv) Sensible heat flux at the ground.

Following Vowinkel and Taylor (1965), a formula devised by Shuleikin (1953) was used to calculate sensible heat flux from the snow surface.

For surfaces warmer than air $Q_H = 30.24\ (\theta_s - \theta_a)$

For surfaces colder than air $Q_H = .226\ (\theta_s - \theta_a) \cdot V$

where Q_H = sensible heat flux ($cal/cm^2 \cdot day$)

θ_s = temperature of the snow surface ^{o}C

θ_a = temperature of the air ^{o}C

V = wind speed in m/sec.

(v) Latent heat flux

The latent heat flux Q_L over ice at below freezing temperatures is given by the formula:

$$Q_L = RE$$

where R = the latent heat of sublimation

and E = the amount of sublimated ice (in mm/day.)

A widely accepted formula for E is the one introduced by Sverdrup:

$$E = K(e_s - e_a) \cdot V$$

where K = exchange coefficient

e_s = water vapour pressure at the snow surface

e_a = water vapour pressure at screen level

V = wind speed (m/sec.)

There have been controversies over the appropriate values of K. It is known to vary with wind speed. The present author has accepted Jacobs and Clark's (1943) value K = .144 for his studies at Mould Bay.

(vi) Total heat loss at the snow surface.

Figure 7 shows the daily heat loss from the surface at Mould Bay for the ice growing seasons 1961-62 and 1963-64. In only one month (January) did the heat loss in 1963-64 exceed the heat loss in 1961-62. A summary of the monthly heat losses is given in Table XII. Included for the sake of comparison are the climatological values given by Vowinkel et al.

TABLE XII

Average values of Q at Mould Bay ($cal/cm^2 \cdot day$)

Year	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
1961-62	-140	-69	-111	- 83	-95	-67	-93	-42
1963-64	- 80	-52	- 86	-101	-83	-48	-88	-32
climatological	- 64	-77	- 85	- 89	-73	-63	-64	38

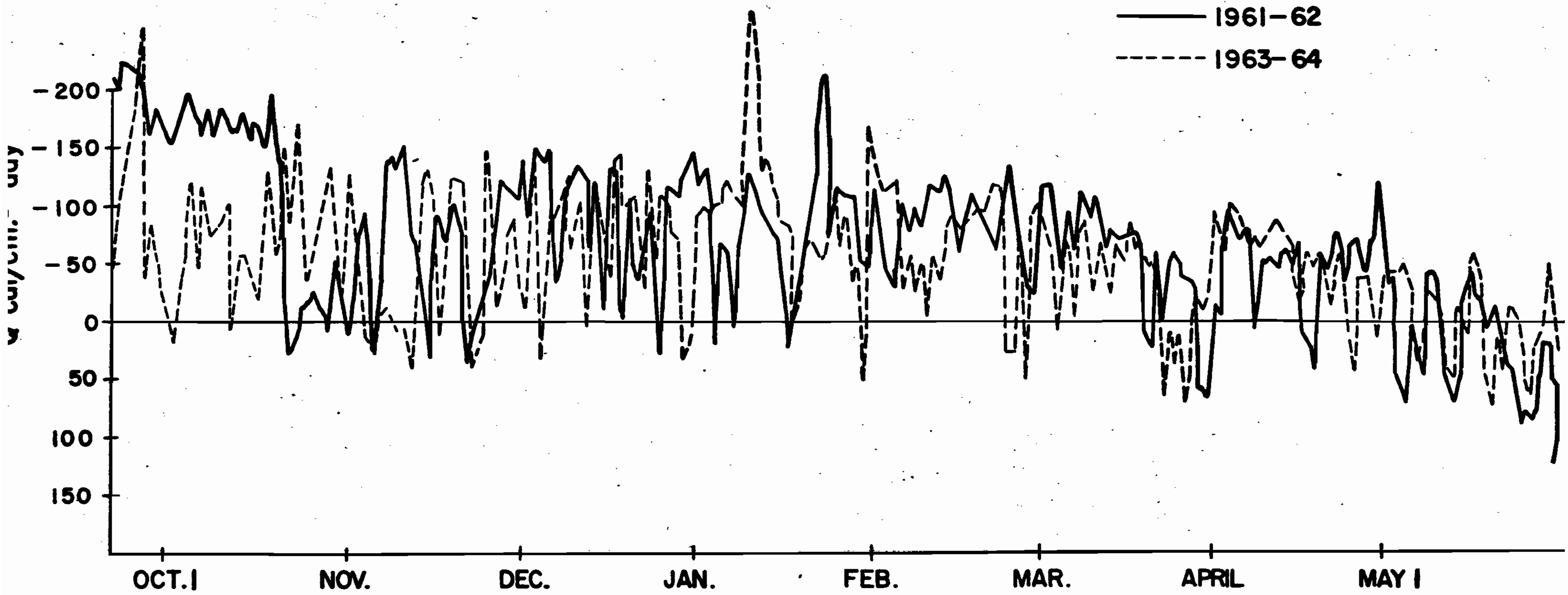

FIG. 7 DAILY HEAT LOSS FROM THE SNOW SURFACE AT MOULD BAY.

A break up of the heat loss into its components is given in Table XIII.

TABLE XIII
Components of Q at Mould Bay in cal./cm^2 • day

	Year	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
Q_H	1961-62	0	4	5	8	4	4	- 30	- 30
	1963-64	0	3	4	6	3	4	- 30	- 30
	climato-logical	0	6	5	4	2	2	- 30	- 30
Q_E	1961-62	- 21	- 5	- 6	- 2	- 2	- 2	- 8	- 37
	1963-64	- 15	- 8	- 6	- 6	- 5	- 6	- 11	- 21
	climato-logical	0	- 3	- 0	- 3	0	0	1	13
Q_s	1961-62	5	-	-	-	-	25	87	119
	1963-64	5	-	-	-	-	26	86	145
	climato-logical	1	-	-	-	-	21	80	170
$Q\downarrow$	1961-62	374	377	280	297	285	336	336	462
	1963-64	420	416	348	257	284	306	287	439
$Q\uparrow$	1961-62	-503	-445	-390	-386	-382	-430	-478	-556
	1963-64	-495	-463	-433	-358	-365	-378	-420	-565
$Q_s + Q\uparrow + Q\downarrow$									
	1961-62	-124	- 68	-110	- 89	- 97	- 69	- 55	25
	1963-64	- 70	- 47	- 85	-101	- 81	- 46	- 47	19
	climato-logical	- 65	- 80	- 90	- 90	- 75	- 65	- 35	55

The climatological values of Q are in reasonably good agreement with the calculated values. A net average heat gain given by Vowinkel et al. as opposed to a net loss found by the present author for the month of May might be ascribed to:

1. <u>differences in albedo</u>: Regional studies of the heat balances would take into consideration land areas in the vicinity of Mould Bay. These lose their snow in May and in consequence acquire a lower albedo. If, instead of .70, a value for the albedo of .60 had been used in this study there would have been an average Q in May of -2 cal/cm^2 • day and 17 cal/cm^2 • day in the years 1961-62 and 1963-64 respectively.

2. <u>the latent heat loss</u>: The climatological values of Q_E shown in Table XIII were calculated for the Polar Ocean. In May the difference in moisture content of the air over continental areas and over the ice fields is apparently great enough to make a direct comparison of the Q_E's unrealistic.

On a seasonal basis (Oct.-May) the average heat loss was as follows:

1961-62	87 cal/cm^2 • day
1963-64	71 cal/cm^2 • day
climatological	60 cal/cm^2 • day

Ice growth at Mould Bay for both growing seasons 1961-62 and 1963-64 commenced on September 22. For that reason, heat budget calculations were done for the last nine days of September. Following are the results of that calculation, together with the pertinent monthly climatological data.

TABLE XIV

Components of Q at Mould Bay for the last 9 days of September, together with the climatological values for that month (cal/cm^2 • day).

	Q_H	Q_E	Q_s	$Q\downarrow$	$Q\uparrow$	$Q\downarrow+Q\uparrow+Q_s$
1961-62	-30	-19	17	356	-553	-180
1963-64	-30	-40	22	505	-585	- 58
climato-logical	-30	-20	47	-	-	- 25

This discussion of the actual values of Q versus the climatological values has served to high-light the following points:

(1) $(\theta_s + 1.8 - \frac{Q\delta}{85})$ from Oct. to May should be approximately the same whether one uses actual or climatological values of Q, providing δ is small. An illustrated example of this is given in Figure 8. It shows little difference in the accuracy of equation (8) when applied with actual or climatological quantities for Q. It is based upon data at Mould Bay for 1961-62, a year when the average seasonal heat loss was 45% greater than the climatological heat loss.

(2) Daily values of Q can vary by more than 100 cal/cm^2 • day (Figure 7). Such a variation can be highly significant when δ is large. Suppose, for example, that the climatological value of Q for February is -85 cal/cm^2 • day and δ is 60 cm. Then $\frac{Q\delta}{85} = -60^\circ C$ and, according to the theory, no ice will grow during February unless $1.8 + \theta_s$ is less than $-60^\circ C$. Equation (8) will, however, predict ice accretion if $Q > \frac{85}{60}(\theta_s + 1.8)$. Given

the broad daily variation in Q it is likely that if one used actual values, as opposed to climatic ones, one would predict accretion.

Figure 9 shows the great difference in the accuracy of equation (8) when it is applied with actual and climatological values of Q in the presence of a deep snow cover. It is based upon data at Mould Bay for 1963-64, a year when the average seasonal heat loss was only 18% greater than the climatological heat loss.

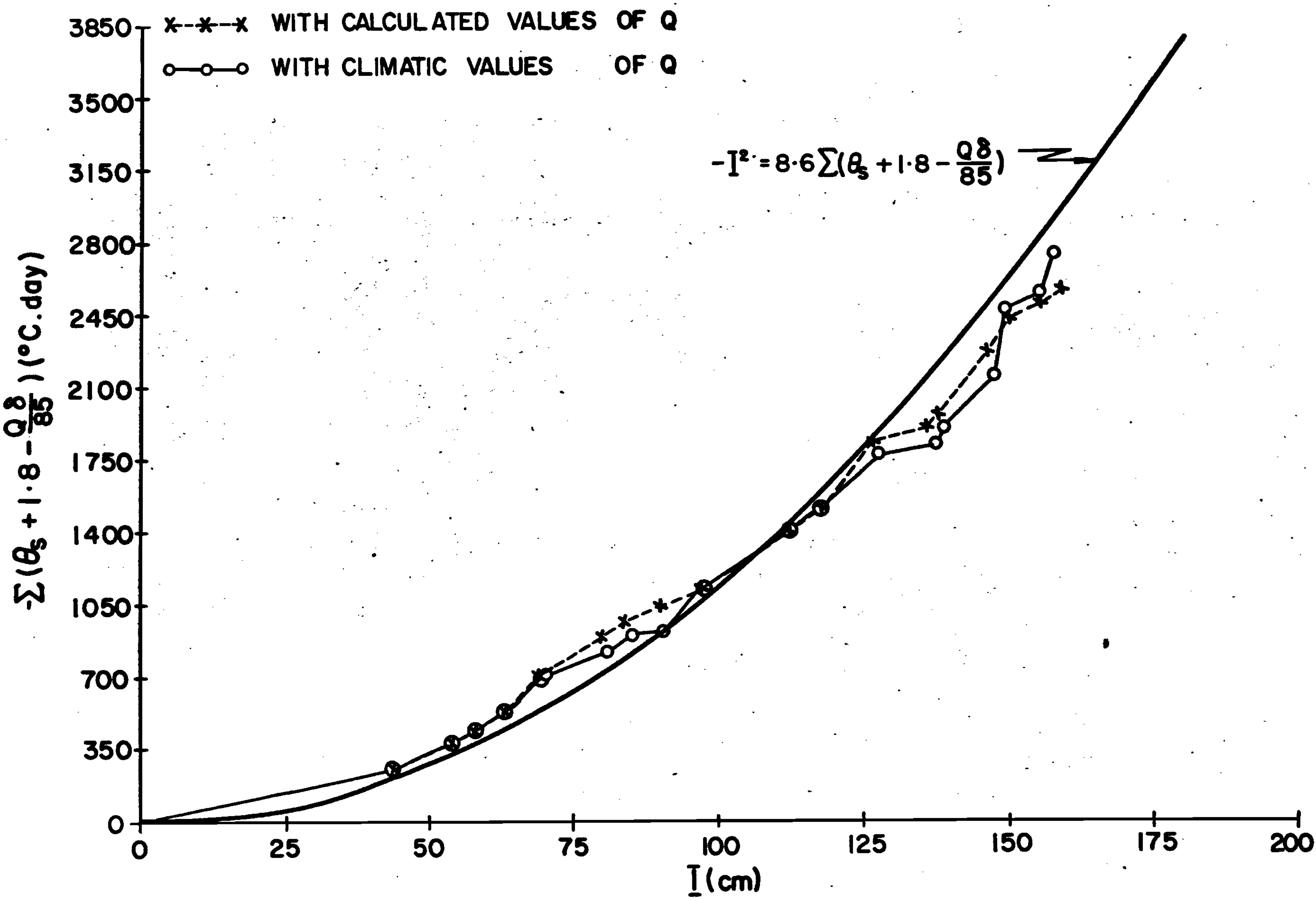

FIG. 8 PLOT OF OBSERVED ICE THICKNESS $\underline{I}$, AT MOULD BAY (1961-62) AGAINST $-\sum(\theta_s + 1{\cdot}8 - \frac{Q\delta}{85})$

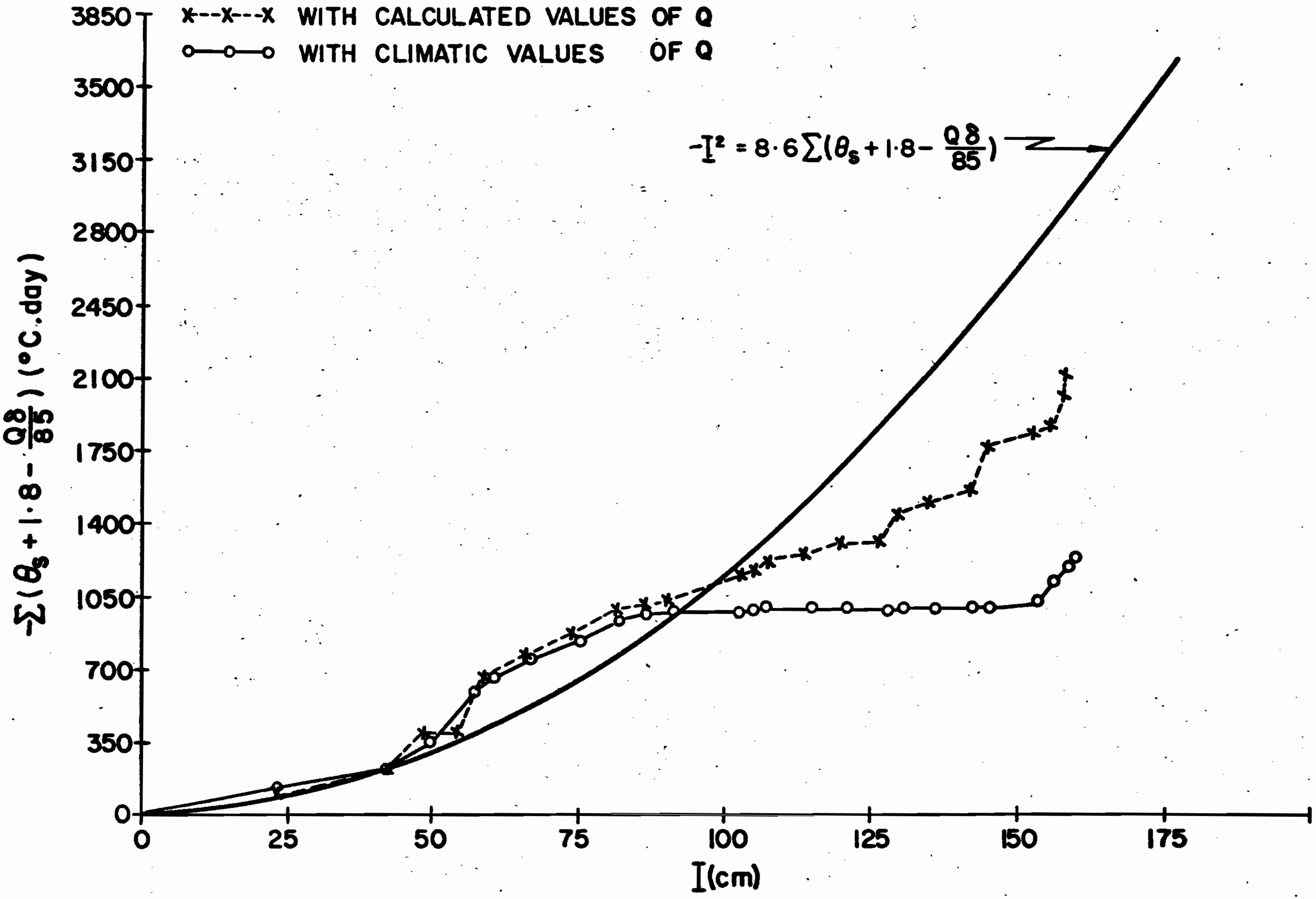

FIG. 9 PLOT OF OBSERVED ICE THICKNESS I, AT MOULD BAY (1963-64) AGAINST $-\sum(\theta_s + 1 \cdot 8 - \frac{Q\delta}{85})$

XI Estimated Daily Values of Q

One of the aims of this study was to produce an equation for forecasting ice thickness which was based upon physical principles and yet simple enough to allow its application on a routine basis. When δ is small, equation (8) with climatological values of Q fulfills these requirements. When, however, one wishes to use equation (8) in the presence of a deep snow cover with actual values of Q, the task of evaluating equation (8) will be time-consuming.

A means must be found of eliminating some of the labour involved in calculating Q. The heat budget equation is

$$Q = Q_E + Q_H + Q\uparrow + Q\downarrow + Q_s$$

Each of the above symbols has been previously defined. Following are suggestions for estimating the components of the heat budget from generalizations on the findings of the previous section.

(i) $Q_E + Q_H$:

Q_E and Q_H are the same order of magnitude. For the months of September, April and May they have the same sign. For these periods it is recommended that the value of $Q_E + Q_H$ be taken as -50 cal/cm^2 • day.

For the month of October, $Q_H = 0$ and a value of $Q_E = -10$ cal/cm^2 • day, seems acceptable. From November to March inclusive, Q_E and Q_H have opposite signs and $Q_E + Q_H \sim 0$. The following values of $Q_E + Q_H$ are then recommended:

-50 cal/cm^2 • day for the months of September, April and May.

-10 cal/cm^2 • day for the month of October.

0 for the months of November to March inclusive.

(ii) $Q\downarrow + Q\uparrow$:

The greatest amount of labour expended in calculating the Mould Bay heat budget was spent on the long wave back radiation. More than 250 hours was spent in evaluating G_o from Yamamoto's radiation chart. The labour involved in such a calculation will be excessive.

For temperate latitudes there are empirical equations which relate G_o to screen level temperature and vapor pressure (Brunt, 1932; Swinbank, 1963; Martin and Palmer, 1964). These cannot, however, be applied 'a priori' to the Arctic in the winter season for two reasons:

1. The temperature profile at this time in the Arctic is characterized by a strong inversion (Vowinkel, 1965) that is not found in more southerly latitudes.
2. Values of vapour pressure during the polar night in the Arctic range from 0-3 mbs. In more temperate regions the range is more typically 3-22 mbs.

For these reasons, clear sky radiation in the Arctic should behave in a fashion different from that predicted by existing equations. There was a total of 505 values of G_o calculated for Mould Bay. These quantities can be used to develop an equation for G_o which should be suitable to Arctic regions.

A multivariate regression analysis between the values of G_o, σT^4 and $\sqrt{e}$ was undertaken. Here:

σT^4 = black body radiation from the snow surface (cal/cm^2 • day)

e = vapour pressure at screen level.

In other words, the constants in the following equation were found

$$G_o = \overline{G_o} + b_{12.3}\,(\sigma T^4 - \overline{\sigma T^4}) + b_{13.2}\,(\sqrt{e} - \overline{\sqrt{e}}) \quad \text{-- (11)}$$

where the bar over the symbol represents the average value of that quantity and

$$b_{12.3} = r_{12.3}\,\frac{\sigma_1}{\sigma_2}\sqrt{\frac{1 - r_{13}^2}{1 - r_{23}^2}}$$

$$b_{13.2} = r_{13.2}\,\frac{\sigma_1}{\sigma_3}\sqrt{\frac{1 - r_{12}^2}{1 - r_{23}^2}}$$

Where σ_1 = standard deviation of G_o
σ_2 = standard deviation of σT^4
σ_3 = standard deviation of $\sqrt{e}$
r_{12} = correlation coefficient between G_o and σT^4
r_{13} = correlation coefficient between G_o and $\sqrt{e}$
r_{23} = correlation coefficient between σT^4 and $\sqrt{e}$
$r_{12.3}$ = partial correlation coefficient between G_o and σT^4 with the effect of $\sqrt{e}$ removed.
$r_{13.2}$ = partial correlation coefficient between G_o and $\sqrt{e}$ with the effect of σT^4 removed.

Values of all these statistical quantities for the full period (1961-62, 1963-64) are given in Table XV together with the same quantities for each season.

TABLE XV

Statistical quantities for the total period of study and for each individual season. For explanation of the symbols see the text.

		Seasons 1961-62; 1963-64	Season 1961-62	Season 1963-64
$\overline{G_o}$	(cal/cm^2 · day)	290	292	288
$\overline{\sigma T^4}$	(cal/cm^2 · day)	444	449	439
$\sqrt{e'}$	((mb)$^{\frac{1}{2}}$)	0.70	0.73	0.68
σ_1	(cal/cm^2 · day)	59	55	64
σ_2	(cal/cm^2 · day)	76	71	81
σ_3	((mb)$^{\frac{1}{2}}$)	0.42	0.40	0.45
r_{12}		.898	.876	.893
r_{13}		.892	.855	.882
r_{23}		.965	.937	.960
$r_{12.3}$		.313	.415	.340
$r_{13.2}$		.221	.203	.193
$b_{12.3}$		.409	.478	.450
$b_{13.2}$		52.0	39.0	44.0

With the data from the combined ice growing season equation (11) becomes

$$G_o = 72 + .409\ \sigma T^4 + 52\ \sqrt{e'} \qquad \text{---(12)}$$

If this equation is divided by σT^4 and one substitutes $\sigma T^4 = \overline{\sigma T^4} = 444$ cal/cm^2 · day wherever σT^4 appears in the denominator one gets

$$G_o = \sigma T^4\ (.571 + .117\ \sqrt{e'}\ ) \qquad \text{---(13)}$$

Brunt's equation, derived from data collected at Bensen, England is

$$G_o = \sigma T^4 (.52 + .065 \sqrt{e})$$

Martin and Palmer's equation, which was derived from data collected at weather ship P, is $G_o = \sigma T^4 (.526 + .077 \sqrt{e})$ Martin and Palmer concluded that their constants (.526, .077) were compatible with Brunt's (.520, .065). The constants from equation (13) are larger than either of those of the other investigators. A tentative explanation can be given for the larger size of each of these coefficients.

(1) The size of the first coefficient (.571) reflects the temperature inversion over the station.

(2) The size of the second coefficient (.117) reflects the greater sensitivity of clear sky radiation on vapor pressure at low values of e than at high values of e.

The correlation between the observed values of G_o and the ones calculated from equation (12) was .91. While this is a high value, it is nonetheless smaller than the one obtained by Brunt (.97) and by Martin and Palmer (.950). The lesser accuracy might be accounted for by the following:

(1) Brunt used actual measurements of G_o. The calculation of G_o in this study from Yamamoto charts doubtless introduced errors.

(2) Martin and Palmer used the Elsasser radiation charts to evaluate G_o. They restricted their attention, however, to days when the sky was cloudless. By using average daily data collected at Mould Bay during cloudy conditions for evaluating clear sky back radiation, the present author would presum-

ably introduce "noise" into the calculations.

Equation (12) can be simplified still further by noting in Table XV the very high correlation between σT^4 and $\sqrt{e'}$ (r_{23} = .97). Regressing $\sqrt{e'}$ on σT^4 one gets:

$$\sqrt{e'} = \overline{\sqrt{e'}} + \frac{\sigma_3}{\sigma_2} r_{23} (\sigma T^4 - \overline{\sigma T^4})$$

or $\sqrt{e'} = 5.35 \times 10^{-3} \sigma T^4 - 1.67$

If this is now introduced into equation (12) the resulting expression for G_o is

$$G_o = .687 \sigma T^4 - 15 \quad \text{cal/cm}^2 \cdot \text{day} \qquad ---(14)$$

Equation (14) could have been derived more directly by simply regressing G_o onto σT^4:

$$G_o - G_o = b_{12}(\sigma T^4 - \overline{\sigma T^4})$$

$$b_{12} = r_{12} \frac{\sigma_1}{\sigma_2} = .699$$

$$G_o = 290 + .699(\sigma T^4 - 444)$$

$$= .699 \sigma T^4 - 20 \qquad ---(15)$$

Equation (14) and (15) are essentially identical over the range of σT^4 from 300 cal/cm^2 • day to 600 cal/cm^2 • day. The correlation coefficient between the observed values of G_o and the values predicted from equation (14) or (equation 15) is .90.

The substantial success of equation (14) can be judged from Figure 10 which shows a plot of G_o against σT^4 for the ice growing season 1961-62. The equivalent graph for 1963-64 is very similar.

Knowing G_o one can calculate the back radiation from cloudy skies from Bolz's formula:

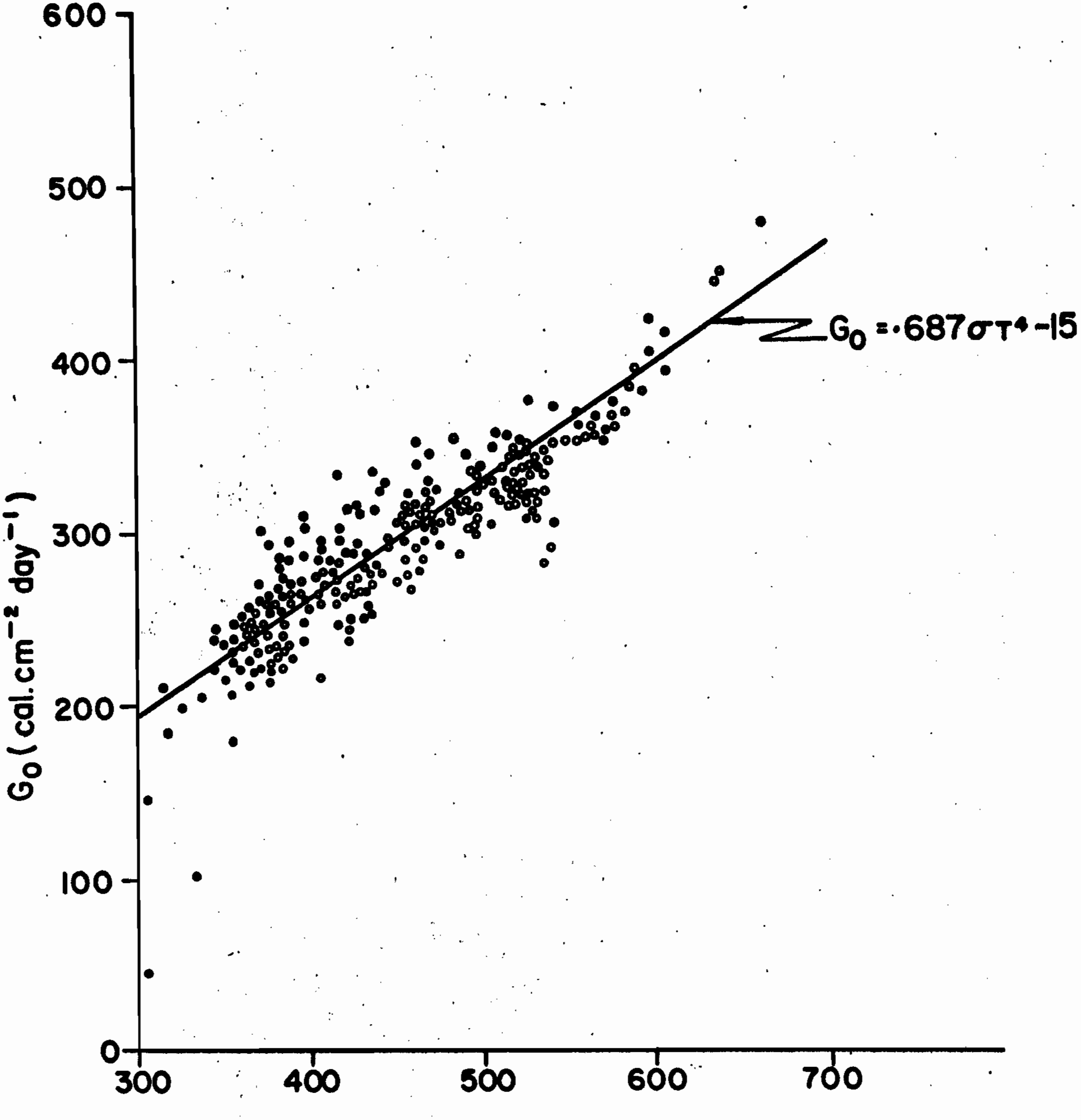

FIG.10 PLOT OF G_0 AGAINST σT^4 FOR MOULD BAY-1961-62

$$G_n = G_o (1 + kn^2)$$

where n is the cloud amount in tenths. Putting n = 10 one gets

$$k = \left(\frac{G_{10}}{G_o} - 1 \right)$$

G_{10} is the radiation from an overcast sky. If the cloud base temperature is T_c (^oK), G_{10} is usually assumed to equal σT_c^4. Estimations of T_c can be made from synoptic and radiosonde data. In the course of the Mould Bay studies, k was evaluated 379 times.

The k's were divided into two classes; all k's calculated for low clouds (height of cloud base $<$ 1000m.) and for medium clouds (height of cloud base $>$ 1000m.). No k's were calculated for Cirrus clouds. There were two reasons for ignoring the effects of Cirrus on the back radiation:

(1) Cirrus radiation is not well known

(2) If one assumes that Cirrus radiates with an emissivity of $\frac{1}{2}$ a black body, the appropriate G_{10} at Mould Bay will be less than G_o. This means that Cirrus clouds will reduce back radiation. Such a conclusion is hard to credit.

Table XVI presents the mean values (M_L, M_M) and standard deviations (σL, σM) for each class. The subscripts L, M refer to low and medium clouds respectively. Also in this table is shown the size of the population from which each statistic is drawn.

TABLES XVI

The cloud factor k

Season	M_L	σL	no. of cases	M_M	σM	no. of cases
1961-62	.59	.14	108	.63	.16	95
1963-64	.62	.12	96	.65	.16	80

Two things in Table XVI are noteworthy:

(1) M_M is larger than M_L. This is to be expected, for, as a result of the temperature inversion, clouds in the medium height class should radiate at higher temperatures than lower clouds.

(2) The difference between M_L and M_M is small. When the large values of the standard deviations are considered, this difference is probably not too significant. One value (k = .62) should be adequate for both low and medium clouds.

In conclusion, the following formula is recommended for the long wave back radiation in the Arctic:

$$G_n = (.687\ \sigma T^4 - 15)(1 + .62n^2)$$

The net long wave radiation G_n, is then only a function of temperature and cloud cover.

$$Q\uparrow + Q\downarrow = \sigma T^4 + G_n = \sigma T^4\ (-.313 + .425n^2) - 15(1 + .62n^2)$$

For the aid of future studies this function is tabulated in Table XVII.

TABLE XVII

The net long wave radiation (cal/cm^2 • day) as a function of cloudiness (n) and snow surface temperature (θ_s)

θ_s °C / n	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	-221	-219	-211	-197	-177	-155	-124	-89	-48	0	52
- 2	-216	-214	-207	-193	-173	-151	-121	-88	-47	0	50
- 4	-210	-208	-201	-188	-168	-147	-118	-86	-47	- 1	48
- 6	-204	-202	-195	-182	163	-143	-115	-84	-46	- 2	46
- 8	-199	-197	-190	-178	-159	-140	-112	-82	-45	- 2	44
-10	-193	-191	-185	-173	-155	-136	-109	-80	-44	- 3	42
-12	-188	-186	-180	-168	-150	-132	-106	-78	-44	- 3	40
-14	-103	-181	-175	-163	-146	-129	-104	-76	-43	- 4	38
-16	-178	-176	-170	-159	-142	-126	-101	-75	-42	- 4	36
-18	-173	-171	-165	-155	-139	-123	- 99	-73	-42	- 5	35
-20	-168	-166	-160	-150	-135	-119	- 96	-71	-41	- 5	33
-22	-163	-161	-156	-146	-131	-116	- 94	-70	-40	- 6	31
-24	-158	-157	-152	-142	-127	-113	- 91	-68	-40	- 6	29
-26	-154	-152	-148	-138	-124	-110	- 89	-67	-37	- 7	28
-28	-150	-148	-143	-134	-120	-107	- 87	-65	-37	- 7	26
-30	-145	-143	-139	-130	-117	-104	- 84	-64	-36	- 8	24
-32	-141	-139	-135	-127	-113	-101	- 82	-62	-35	- 8	23
-34	-136	-135	-131	-123	-110	- 98	- 80	-61	-35	- 9	21
-36	-132	-131	-127	-119	-107	- 95	- 78	-59	-34	- 9	20
-38	-128	-127	-123	-115	-104	- 93	- 76	-58	-34	- 9	19
-40	-124	-123	-119	-112	-100	- 90	- 74	-57	-33	-10	18

(iii) Q_s

The short wave component is of great importance to the heat budget during the ice growing season in the months of April and May. The previously cited formula

$$Q_s = S_o(1 - k_1 n)(1 - A)$$

is already in simple form. Values for S_o and k_1 may be obtained from Vowinkel and Orvig (1964a).

XII Application of the Modified Stefan's Equation with Actual Values of Q when there is a Heavy Snow Cover.

The recommendations proposed in section XI for calculating daily values of Q were used in predicting the ice growth for the previously treated instances of heavy snow cover. (Mould Bay 1963-64, Resolute 1961-62, Eureka 1961-62, Clyde 1963-64).

Figures 11 and 12 indicate the degree of success achieved in predicting the observed values of ice thickness. The over all correlation between I' and I was .94.

Table XVIII displays the values of $(\frac{\Sigma'(I - I')^2}{n})^{\frac{1}{2}}$ corresponding to Figures 11 and 12 . Included for purposes of comparison are some of the values given in Table XI.

TABLE XVIII

Values of $(\frac{\Sigma'(I - I')^2}{n})^{\frac{1}{2}}$ for the Modified Stefan's Equation with Climatic and Estimated Daily Values of Q.

Place	Season	With Climatic Values of Q	With Estimated Daily Values of Q
Mould Bay	1963-64	32cm	15cm
Resolute	1961-62	29	11
Eureka	1961-62	20	17
Clyde	1963-64	11	9

The greatest improvements in prediction accomplished by using calculated instead of climatic values of Q were made at Resolute and Mould Bay. At these stations the root mean square

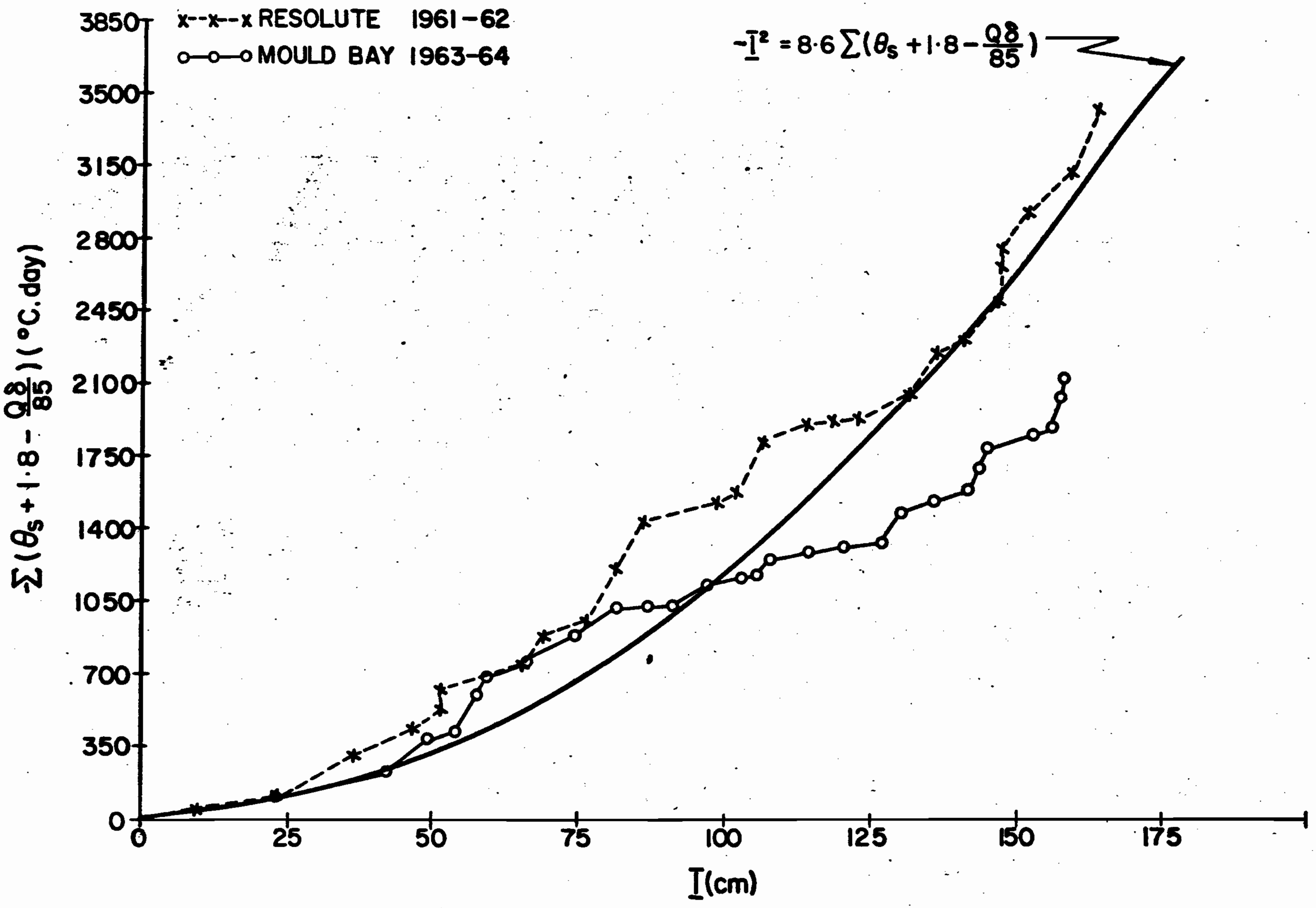

FIG. 11 PLOT OF OBSERVED ICE THICKNESS $\underline{I}$, AGAINST $-\sum(\theta_s + 1{\cdot}8 - \frac{Q\delta}{85})$ USING CALCULATED VALUES OF Q (cal/cm.² day)

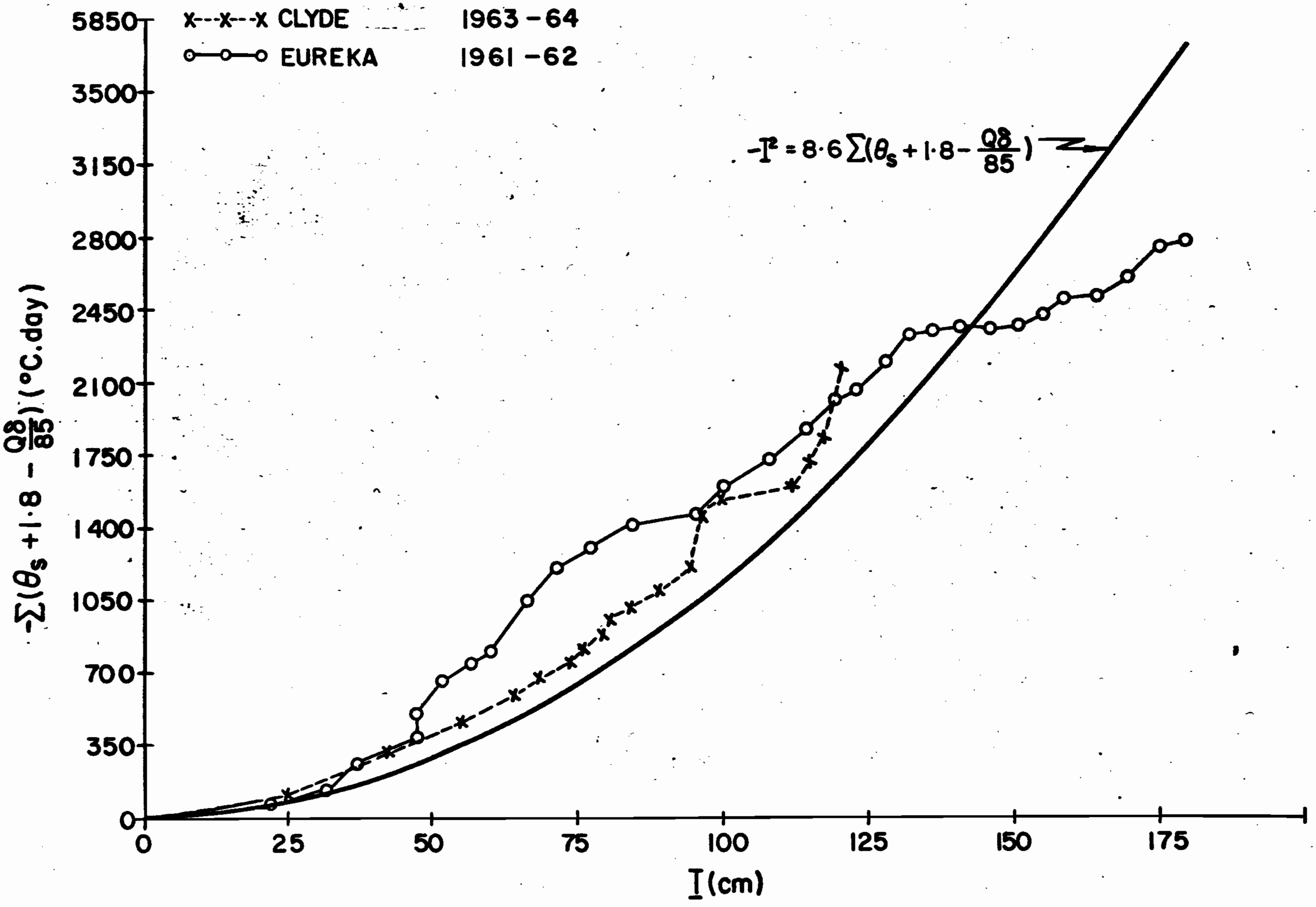

FIG.12 PLOT OF OBSERVED ICE THICKNESS I, AGAINST $-\sum(\theta_s + 1\cdot8 - \frac{Q\delta}{85})$ USING CALCULATED VALUES OF Q (cal/cm.^{2} day)

of (I - I') was reduced by more than 50%. At the other two stations the success attained with climatic values of Q had been more satisfactory than at Resolute and Mould Bay. Consequently the achievement of the modified equation when used with actual values of Q was less spectacular.

XIII Conclusion:

The purpose of this study was to develop an ice-growth equation based upon physical principles that would give a satisfactory explanation of observed ice growth and be of simple enough form to be of operational use. The modified Stefan's equation

$$I^2 = - \frac{2k}{\rho L} \sum (\theta_s + 1.8 - \frac{Q\rho}{K_s})$$

will satisfy these conditions. For use in Arctic regions, evidence has been presented to show that this equation may be applied with $\frac{2k}{\rho L} = 8.6$ cm^2/°C · day and $K_s = 85$ cal/cm^2 . °C day.

In Arctic regions the modified Stefan equation is harder to evaluate than Zubov's empirical formula. It should be noted, though, that it is only in instances of deep snow cover (about one case in four) that one needs to determine values of Q. In these cases the better correlation found between predicted and observed ice thicknesses with this equation (.94) than with Zubov's (.78) would justify the extra labour.

When the snow cover is light or moderate, the difference between these correlations (.98, .91) is not as significant; but with light to moderate snow conditions one can use climatic values of Q. This allows the evaluation of the modified equation in a time only slightly longer than the time needed for the use of Zubov's equation.

Appendix

Stefan's ice growth formula was introduced in section III of this study: $I^2 = \frac{2k}{\rho L} \Sigma \beta$ where β is the difference between the ice surface temperature and the freezing temperature of sea water. If one accepts the value of 8.6 cm^2/ C • day for $\frac{2k}{\rho L}$, which was proposed in section VIII and takes the freezing point of sea water to be -1.8°C., Stefan's equation becomes $I^2 = -8.6 \Sigma (\theta_i + 1.8)$ where θ_i is the sea ice surface temperature.

Sea ice surface temperatures are not measured on a routine basis; however, they are available at Mould Bay for the two years (1961-62; 1963-64) which have been analyzed in this investigation. These surface temperatures were made by resistance thermometers which were installed flush with the surface of the ice, after the ice had reached a thickness of about 50 cm. Temperature readings were usually made once a day at 1400 LST, weather permitting. Occasionally, as during December 1961, personnel shortages limited the readings to once a week. In what follows the temperatures were assumed to vary linearly over periods when no observations were made.

As the ice thickness already had an initial value I_o when the thermometers were installed, the following relation was used in testing Stefan's equation:

$$I^2 = I_o^2 - 8.6 \Sigma (\theta_i + 1.8)$$

where the summation sign extends over the sampling period beginning with the installation of the thermometer. Figure

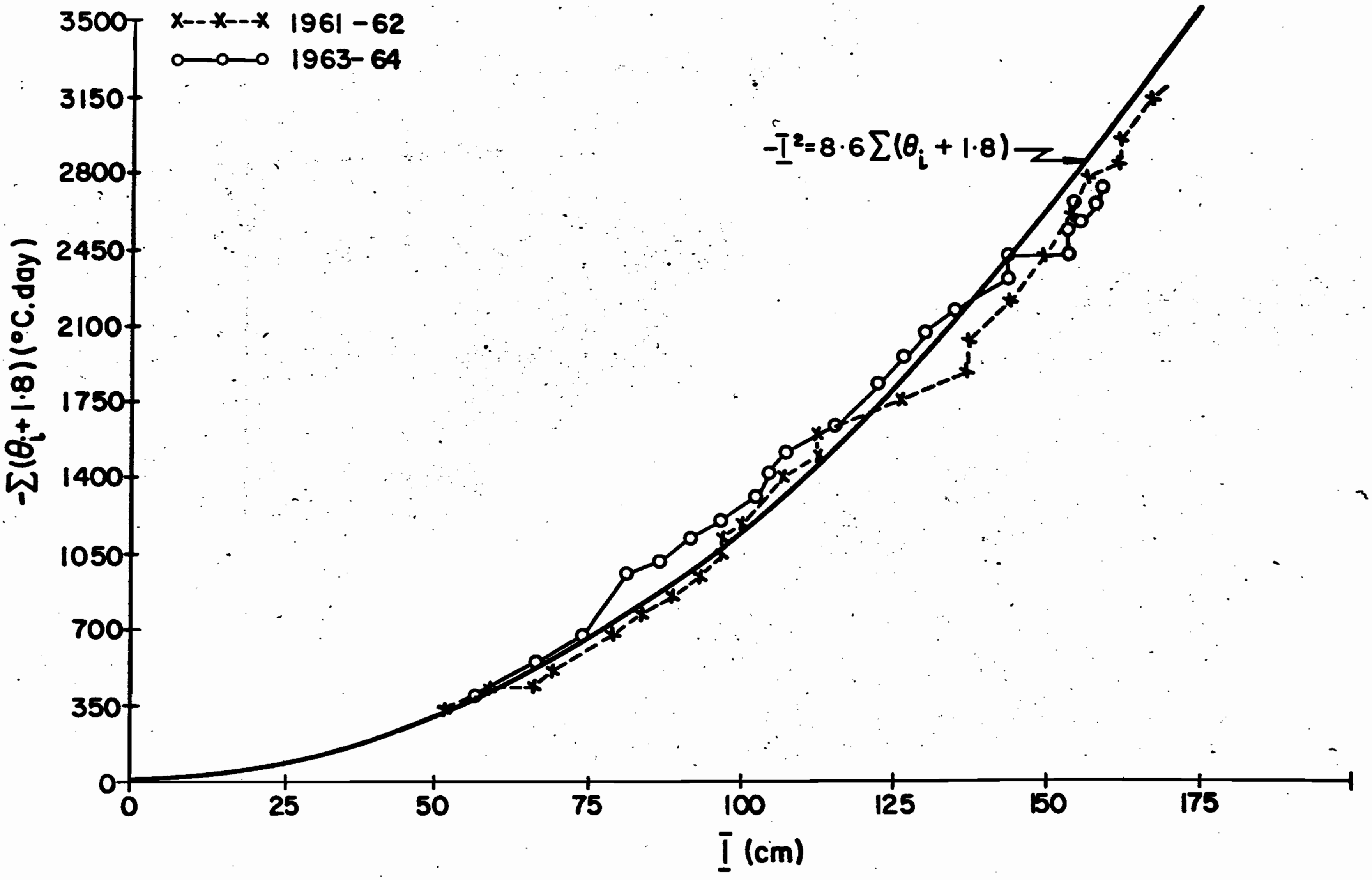

FIG. A PLOT OF OBSERVED ICE THICKNESS AT MOULD BAY AGAINST $-\sum(\theta_i + 1 \cdot 8)$

A shows the degree of success achieved by Stefan's formula. The r. m. s. value of (I - I') for 1961-62 and 1963-64 was 3.7 and 5 cms. respectively. The correlation between the observed and predicted values of ice thickness was .992.

The very close agreement between observed and predicted values of ice growth is remarkable in view of the following considerations:

(1) The ice surface temperatures and the ice thickness were not measured at the same site. Variations in snow depth and density can result in spatial differences in ice thickness.

(2) The values of $\frac{2k}{\rho L}$ (8.6) was not determined solely from Mould Bay data. All three quantities ρ, L, k are known to vary with time and location.

(3) For the growing season 1963-64 sea ice surface temperatures were made only once a week during the month of December. Avoidable errors would have been introduced by the consequent interpolation.

For the information of ice breakers and engineering concerns, Stefan's equation appears wholly adequate.

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