# Environmental Conditions Favouring Ice Pellet Aggregation

by

Hannah Carmichael

Department of Atmospheric and Oceanic Sciences
McGill University, Montreal

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# ABSTRACT

Winter precipitation is an important issue in Canada because of its common occurrence and associated destructive consequences. Prediction of the precipitation type when temperatures are near 0°C is often difficult because so many types can occur. This study examines the microphysics of ice pellet formation, in particular the ability of these to form aggregates and the consequences of these aggregates. This issue was examined by modelling the freezing of a distribution of precipitation particles as they fall through the atmosphere and interact through collisions. Three mechanisms for aggregation were examined, collisions among the particles involved in these mechanisms were modelled and the relative importance of each mechanism was determined. It is shown that, for the conditions considered, aggregates are often able to collect freezing rain drops and that aggregation can sometimes be very effective at eliminating freezing rain but the conditions need to be precise for this to occur.

# RÉSUMÉ

Les divers types de précipitations observées durant les tempêtes hivernales sont souvent la cause d'inconvénients durant cette période au Canada. Il est difficile de prédire ces divers types de précipitations du fait de leur sensibilité à certaines conditions atmosphériques, en particulier à des températures près de 0°C. Cette étude examine la microphysique de la formation des granules de glace. Plus précisément, la capacité de ces granules de former des agrégats et les conséquences de ces agrégats sur les autres types de précipitation présents. Cette recherche repose sur l'étude du regel d'une distribution de pluie verglaçante dans une atmosphère sous le point de congélation à l'aide de simulations incluant les interactions entre particules. Une attention particulière a été prêtée sur trois principaux aspects. Premièrement, trois mécanismes formant des agrégats de particules ont été étudiés. Deuxièmement, les collisions parmi les particules entraînées dans ces mécanismes ont été modélisées. Finalement, l'importance relative de chaque mécanisme a été déterminée. Les résultats illustrent que pour les conditions atmosphériques considérées, la pluie verglaçante est souvent collectée par les agrégats de particules formés durant leur descente dans l'atmosphère. De plus, l'agrégation de particules s'avère efficace à l'élimination complète de la pluie verglaçante dans des conditions atmosphériques précises et favorables.

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| | |
|---|---|
| $C_d$ | Drag coefficient |
| $c_w$ | Specific heat of water (J/ gK) |
| **D** | Diameter of particle (m) |
| $D_v$ | Diffusivity of water vapour in air ($m^2$/s) |
| **E** | Interfacial distance of ice shell (cm) |
| $f$ | Ventilation coefficient |
| **g** | Acceleration due to gravity (m /$s^2$) |
| **IP** | Ice pellets |
| $k_a$ | Heat conductivity of air (J/ mKs) |
| **L** | Characteristic length of particle (m) |
| $L_m$ | Latent heat of fusion of water at 0°C (J/ g) |
| $L_s$ | Latent heat of sublimation of water (J/ g) |
| **M** | Mass of particle minus mass of air displaced by the particle (g) |
| $N_{Sc,v}$ | Schmidt number for vapour in air |
| **N(D)** | Number of drops of a certain diameter per unit volume (1/$cm^4$) |
| $\rho_a$ | Density of air (g/ $cm^3$) |
| $\rho_v$ | Density of water vapour (g/ $cm^3$) |
| $\rho_w$ | Density of liquid water (g/ $cm^3$) |
| **r** | Radius of a drop (m) |
| $R_e$ | Reynolds Number |
| **t** | Time (s) |
| $t_f$ | Freezing time of a drop (time from the instant of nucleation until the drop is completely frozen) |
| $T_{min}$ | Minimum temperature of the refreezing layer |
| **ZR** | Freezing rain |

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# Chapter 1

## Introduction

The study of winter precipitation is particularly important in the scope of atmospheric science because of its hazardous consequences. Hazardous conditions, such as icing and snowfall accumulation, can have detrimental effects on many ground-based activities and industries. Freezing precipitation at the surface is also often an indication of freezing precipitation aloft, which can be very dangerous for aircraft.

Various types of winter precipitation can occur when the temperature is near 0°C. Such types include snow, freezing rain, ice pellets and wet snow. These types can occur at once, or in close succession, especially within the transition regions of storms. In transition regions, the area between all rain and all snow, liquid, frozen and semi-frozen precipitation can all occur simultaneously. When the temperature is near 0°C it is difficult to correctly forecast the type of precipitation and the different types can have significantly different consequences.

Currently the quantitative ability to forecast winter precipitation (forecasting amount of precipitation) exceeds that of warm-seasonal forecasting (Ralph et al., 2005). However, in winter, qualitative forecasts (the type of precipitation) are more poorly forecast than in summer, but can be as or more important than quantitative ones (Ralph et al., 2005). Small errors in prediction of winter events, when precipitation can be frozen or liquid, can have more disastrous effects than those caused by small errors in predicting summer precipitation, since most precipitation is liquid. At a recent US workshop on cool-season quantitative precipitation forecasting (Ralph et al., 2005), the modelling working group agreed that "the most serious problem (in quantitative forecasting)…is the accurate determination of precipitation type…". In attempting to address the problem of determination of precipitation type it was agreed that the highest priority processes to be examined were those of cloud microphysics (Ralph et al., 2005).

Research has been conducted on all types of precipitation, but the least amount of attention has been paid to the detailed study of ice pellets (Gibson and Stewart, 2007). Ice pellets are a type of cold season precipitation consisting of transparent or translucent pellets of ice. They commonly occur with freezing rain, sometimes with snow and often during the passage of a warm front. A considerable amount is known about the production of ice pellets, such as the environmental conditions necessary for their formation and the freezing of these semi-frozen particles upon entry into the refreezing layer. However, while the environmental conditions necessary for ice pellet formation are generally known, detailed knowledge about the individual particles is not.

This relative deficiency in ice pellet research is likely due to the fact that ice pellet events are not as common as other winter precipitation events. They are not directly linked with hazardous conditions as other types of winter precipitation are. Also, they occur over small spatial scales and small temporal scales. This smallness of scale further leads to difficulties in studying them, because it makes observing ice pellet events difficult to achieve. In order to observe ice pellets, one must rely on forecasts, which are created for a large geographical area, whereas ice pellet events are small scale, occurring over distances as small as a few kilometres (Gibson and Stewart, 2007).

Freezing rain is often closely linked with ice pellets, either occurring simultaneously or in close succession in the transition regions of storms. Freezing rain can be very dangerous when it reaches the surface and freezes; it freezes on the surface of objects and can create a dangerously slippery surface. If nucleation of freezing rain drops is achieved aloft, the former freezing rain particle is frozen by the time it reaches the surface, or is at least composed of a frozen ice shell surrounding a liquid water core. When nucleation occurs aloft instead of at the surface, the icing hazards that occur with freezing rain are decreased. A frozen pellet at the surface is less hazardous than a freezing rain drop that coats surface objects with ice.

This nucleation of freezing rain drops aloft can be accomplished through collisions with ice pellets, resulting in ice pellet aggregates. When ice pellets collide with freezing rain, the supercooled drops freeze to the surface of the ice pellets and create aggregates. Ice pellet aggregates are particles that are composed of more than one component particle. In the storm studied by Gibson and Stewart (2007), 9% of the observed particles were ice pellet aggregates. Of these aggregates, some appeared to be created through collisions of freezing rain drops with ice pellets, and others by collisions between ice pellets or an ice pellet and a semi-frozen particle. Freezing rain and ice pellets often occur together, and collisions between the two precipitation types can reduce the amount of freezing rain that reaches the surface. These collisions also increase the number of ice pellet aggregates at the surface.

Given the importance of improving forecasting of winter precipitation and the lack of information on ice pellets, the objective of this study is to investigate the physics of ice pellet formation with particular attention paid to the conditions favouring aggregation. Three proposed mechanisms of aggregation will be examined and the atmospheric conditions favouring each mechanism investigated. Conclusions will be drawn about the importance of each mechanism of aggregation and what environmental conditions are the most favourable to aggregate production by any of the three aggregation mechanisms.

The thesis will be organized according to the following outline. Chapter 2 starts with a detailed overview of winter precipitation formation, including a review of the current literature on this subject. Special attention will be paid to the formation of ice pellets and aggregates of ice pellets created through three different mechanisms. Chapter 3 contains a description of the model and experimental techniques used in examining the freezing of semi-frozen particles upon entry into the refreezing layer. Also included is a description of the environmental conditions used in the simulations and of the techniques used to predict the aggregation efficiency by the three different mechanisms. Chapter 4 consists of a review of the results obtained by modelling the freezing of particles and creation of ice pellet aggregates by the three formation mechanisms. Chapter 5 discusses the results obtained, with a more detailed analysis of the results of the fourth chapter. Chapter 6 summarizes the study and conclusions are drawn from this body of work.

# Chapter 2

## Winter Precipitation Formation

This chapter contains an overview of different types of winter precipitation. Section 2.1 summarizes the formation mechanisms of various winter precipitation types. A description of the physics of the freezing of ice is discussed (Section 2.2), as well as the surface characteristics of ice (Section 2.3). A more detailed examination of the formation of ice pellets and of ice pellet aggregates follows (Section 2.4), as well as a description of the three proposed formation mechanisms (Sections 2.4.1- 2.4.3) and a summary of the current knowledge on ice pellet aggregates is presented in section 2.4.4.

### 2.1 Precipitation Type Formation

A warm front is an area of strong temperature gradient with temperature at the surface increasing rapidly as the front passes. During the passage of a warm front, many types of precipitation occur and this is the environment in which ice pellets are most commonly observed. A time evolution of vertical temperature profiles during the passage of a warm front, along with the associated precipitation is shown in Figure 2-1. Ice pellets usually fall in the transition zone between snow and freezing rain/ drizzle. From the cold to the warm side of the transition zone, snow falls through a melting zone that increases in depth, so the precipitation process aloft evolves from experiencing no melting to incomplete melting to complete melting (Cortinas et al., 2004). Of the 34 cases of freezing rain and ice pellet events studied by Zerr (1997), 29 occurred during the passage of a warm front.

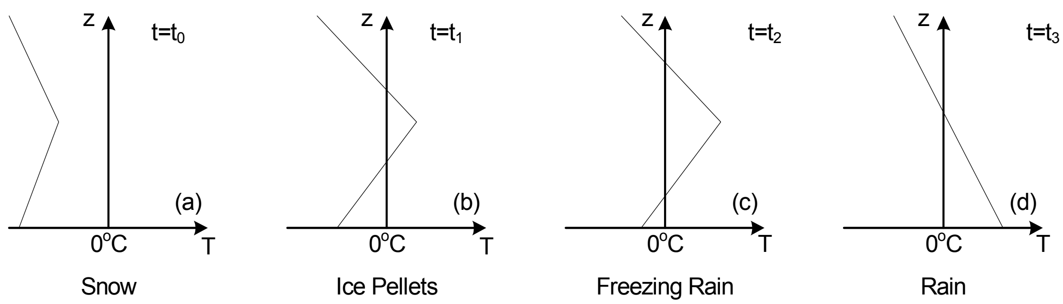


**Figure 2-1:** A typical time evolution of vertical temperature profiles during the passage of a warm front. The precipitation produced starts as snow (a), changes to ice pellets (b), then freezing rain (c) and ends as rain (d).

Ice pellets and freezing rain usually need an elevated melting layer (T > 0°C) and a refreezing layer above the surface (T < 0°C). A schematic of this typical environmental temperature profile is shown in Figure 2-2.

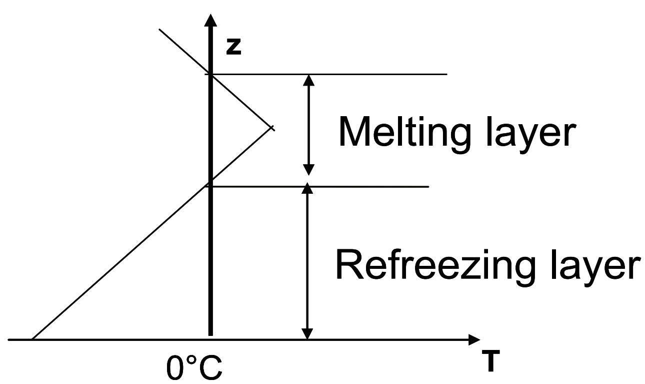


**Figure 2-2 :** Typical environmental temperature profile for the formation of freezing rain and ice pellets. The profile is characterized by a melting layer aloft and a refreezing layer above the surface.

Freezing rain and ice pellets are formed under similar conditions and often occur at the same time. Both are formed when frozen precipitation falls through a melting layer aloft and the precipitation particles melt partially, producing ice pellets, or melt completely, producing freezing rain. The depth and strength of the melting layer aloft has a significant impact on whether the precipitation particles will be partially frozen upon entry into the refreezing layer, and therefore on the type of precipitation that will be produced at the surface.

Upon entry into the refreezing layer, all particles may be entirely melted, all particles may be partially frozen, or the distribution may be somewhere in between, with some completely melted particles and some partially frozen particles. If the melting layer is sufficiently deep, all particles will melt completely and freezing rain will be produced. Upon entry into the refreezing layer, the liquid drops become supercooled. These supercooled drops fall through the depth of the refreezing layer remaining in liquid form until they reach the surface. They are unlikely to refreeze before contact with the surface and upon contact at the surface, refreeze. This is the mechanism for formation of freezing rain (Zerr, 1997).

Similar to freezing rain, ice pellets are most likely formed when snowflakes falling through a melting layer are partially melted, then, upon entry into the refreezing layer, because some fraction of ice remains in them, they begin to refreeze and fall to the surface as ice pellets (Zerr, 1997). According to Cortinas et al. (2004), the main factor determining whether the precipitation produced is ice pellets rather than freezing rain was incomplete melting of snowflakes in the melting layer, and secondary factors were the strength and depth of the refreezing layer. When a very shallow melting layer exists, the particles undergo slight melting, but upon entry into the refreezing layer, they all contain some fraction of ice, so that they all refreeze within the layer, before contact with the surface. Depending on the degree of melting, and therefore collapse of the crystal structure, ice pellets, irregular ice particles or refrozen wet snow are created (Stewart and Crawford, 1995). For the case when ice pellets and freezing rain occur together, the melting layer is of moderate depth, causing complete melting of the smallest particles, and incomplete melting of the larger particles.

Upon entry into the refreezing layer, the completely melted particles become supercooled and fall as freezing rain drops. These drops do not refreeze, because the degree of supercooling is not sufficient to produce ice nucleation. The partially melted particles begin to refreeze and have the potential to completely refreeze before reaching the surface and fall as ice pellets, or liquid-core pellets. Zerr (1997) found that the storms that contained both freezing rain and ice pellets had a minimum temperature in the refreezing layer 4°C warmer than the minimum temperature for ice pellets alone. The difference in the depth of the refreezing layer was not significantly different for the two scenarios. This signifies that the minimum temperature of the refreezing layer, rather than the depth of the refreezing layer is the more important factor in the refreezing of partially frozen precipitation particles.

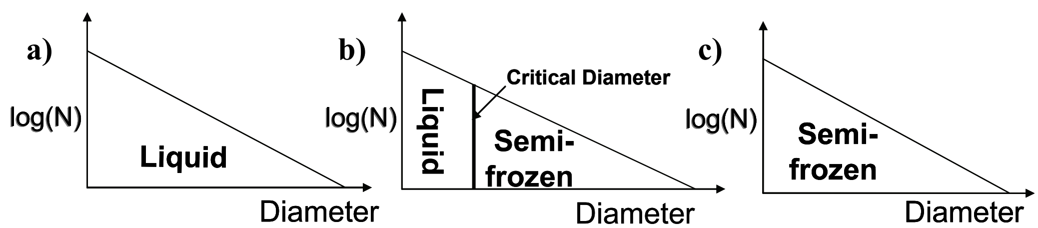


**Figure 2-3:** The variation of phase in the distribution of particles upon entry into the refreezing layer. This ranges from a) complete melting of all particles, through b) complete melting of some particles, while others still contain ice, to c) incomplete melting of all particles.

Three different phase distributions of particles are produced in the different melting layer scenarios described and these are summarized graphically in Figure 2-3. For the phase distribution in which some particles are completely liquid and others are mixed phase, a critical diameter exists below which all precipitation is entirely liquid. All particles above this diameter contain some ice, because larger particles need more time to melt completely than smaller particles. All precipitation particles larger than this diameter contain some ice and all below this diameter are entirely melted. This critical diameter divides the size distribution into particles that will become freezing rain and those that will become ice pellets. From this it is straightforward to understand how and why ice pellets and freezing rain often occur together.

## 2.2 Freezing of Falling Precipitation

When water drops freeze, they are normally supercooled before they undergo ice nucleation. When ice nucleation occurs, dendrites grow quickly through the drop and continue until the latent heat released due to the phase change warms the droplet to 0°C. A fraction of the droplet is frozen during this rapid warming and that fraction is ΔT/80, where ΔT is the degree of supercooling. It is assumed that the heat transfer in a droplet is spherically symmetric, so that a frozen shell forms on the outside and grows inward. A schematic illustration of the freezing of a semi-frozen drop is shown in Figure 2-4. The outward movement of the outer boundary of the ice pellet is small compared to the inward movement of the inner boundary as the droplet undergoes complete freezing. This causes the pressure inside the pellet to rise and the shell sometimes cracks: spicules, and other protuberances are created (Gibson and Stewart, 2007).


**Figure 2-4 :** Evolution of a semi-frozen drop to an ice pellet during passage through the refreezing layer.

Freezing rain drops (supercooled water drops) do not freeze until they come into contact with the surface or another object that will initiate ice nucleation. Upon contact with an ice nucleus, the freezing rain drops will begin freezing immediately, because they have been supercooled during their passage through the refreezing layer.

In this study it was assumed that no ice nucleation of freezing rain occurred unless the freezing rain came into contact with a frozen particle. If a particle were completely liquid upon entry into the refreezing layer, it would remain liquid throughout its passage through the refreezing layer unless it collided with a frozen particle. Also, no ice nucleation occurs in the formation of an ice pellet in the refreezing layer. Upon entry into the refreezing layer, the particles that will become ice pellets already have within them some ice, which will begin to freeze immediately upon entry into the refreezing layer.

According to the American Meteorological Society (AMS) Glossary of Meteorology, ice pellets are defined as "*A type of precipitation consisting of transparent or translucent pellets of ice, 5 mm or less in diameter. They may be spherical, irregular, or (rarely) conical in shape. Ice pellets usually bounce when hitting hard ground and make a sound upon impact. Now internationally recognized, ice pellets includes two basically different types of precipitation, known in the United States as 1) sleet and 2) small hail. Thus a two-part definition is given: 1) sleet or grains of ice, generally transparent, globular, solid grains of ice that have formed from the freezing of raindrops or the refreezing of largely melted snowflakes when falling through a below-freezing layer of air near the earth's surface; 2) small hail, generally translucent particles, consisting of snow pellets encased in a thin layer of ice. The ice layer may form either by the accretion of droplets upon the snow pellet or by the melting and refreezing of the surface of the snow pellet.*" (Glickman, 2000)

Due to the way in which water drops freeze, from the outside in, a semi-frozen drop consists of a frozen shell surrounding a liquid interior. As the frozen shell advances toward the middle of the pellet, the liquid fraction in the pellet decreases and the solid fraction increases. The definition of ice pellets by the AMS states that ice pellets are completely frozen, so these pellets with some liquid in the centre can not technically be defined as ice pellets. The existence of these semi-frozen pellets has been examined (Thériault et al. 2006) and such particles were referred to as liquid-core ice pellets. For different environmental temperature profiles, the amount of liquid-core pellets present at the surface differs. In some storms all the pellets are fully frozen ice pellets upon reaching the ground, whereas in others, liquid-core pellets exist in large numbers at the surface.

## 2.3 Water Films on the Surface of Ice

A liquid-like layer exists on the surface of ice, and most other solids, even at temperatures up to tens of degrees Celsius below the solid's bulk melting point. Even if the temperature of a solid is uniform throughout and below the melting point of the bulk solid, a thin film of liquid exists on its surface and is referred to as surface melting or pre-melting (Wettlaufer and Dash, 2000). Pre-melting of ice begins at -35°C, which is well below the melting point of ice (Rosenberg, 2005).

In the mid 1800s, Michael Faraday studied the freezing together of two ice cubes. He suggested that a thin film of water on the surface of ice freezes when positioned between two pieces of ice, while it remains in liquid form on the ice's surface. Faraday's experiments were the first to examine the process of pre-melting. In response to criticism that the cause of freezing together of the ice pieces was due to pressure exerted on them, Faraday developed a more thorough experiment, submerging 2 pieces of ice in a water bath at 0°C. The ice pieces were each attached to lead weights, so that when they were displaced laterally they would tend to return to their initial positions. He found that if the ice pieces came into contact with each other, they would stick, even though the force from the lead weight was pulling them toward their initial positions (Rosenberg, 2005).

Continuing the study of the freezing of water films between two pieces of ice, the force of adhesion between two ice spheres was tested and observed to increase with increasing temperature up to -4°C. Nuclear Magnetic Resonance (NMR) imaging techniques provide evidence for a liquid layer on the surface of ice; below the melting point there is a narrow adsorption line, unlike the broad line that would be expected from a solid. The molecules on the surface of ice rotate about $10^5$ times faster than those in the bulk of the ice, which is about 1/25 times the rotation rate of molecules in liquid water. Perhaps the best evidence for the liquid-like layer on the surface of ice comes from X-ray diffraction, which shows intermolecular distance on the surface of ice is only slightly different from that of liquid water. The maximum thickness of the surface layer on ice was found to vary from 12 nm at -24°C to 70 nm at -0.7°C and increases in thickness when salt is present (Rosenberg, 2005).

Another experiment involving the sticking together of two spheres of ice was conducted by Hosler et al. (1957) in which two spheres of ice (each with a radius of 0.74 cm) were each suspended by a string in a chamber in which temperature and humidity were controlled. The ice spheres were forced to touch with minimal force (≤ 0.5 dynes) and not disturbed for one minute. After the minute, the strings by which the spheres were suspended were pulled apart until the spheres separated. The normal force required for the separation of the ice spheres was calculated for varying temperatures (ranging from -80°C to 0°C) in three different vapour pressure conditions.

Experiments were also carried out to test the force to separate ice spheres that were left to touch for times from five seconds to two minutes. The results did not show any appreciable difference in force required to separate the spheres (Hosler et al., 1957). These experiments were conducted in conditions of supersaturation with respect to ice, saturation with respect to ice and in a dry environment.

The results of the experiment (Figure 2-5) revealed that for both environmental conditions. The force required to separate the ice spheres increases with increasing temperature, until both environments require the same amount of force at 0°C. In conditions of ice saturation, measurable sticking began to occur at -25°C, and in a dry environment there was no sticking below -3°C. These results suggest that, as temperature increases, the contact area between the ice spheres increases, thereby increasing the amount of freezing between the spheres and the force required to separate them once frozen together. The fact that there are two curves suggests that environmental vapour pressure as well as temperature has an effect on the contact area and the minimum sticking temperature (Hosler et al., 1957).

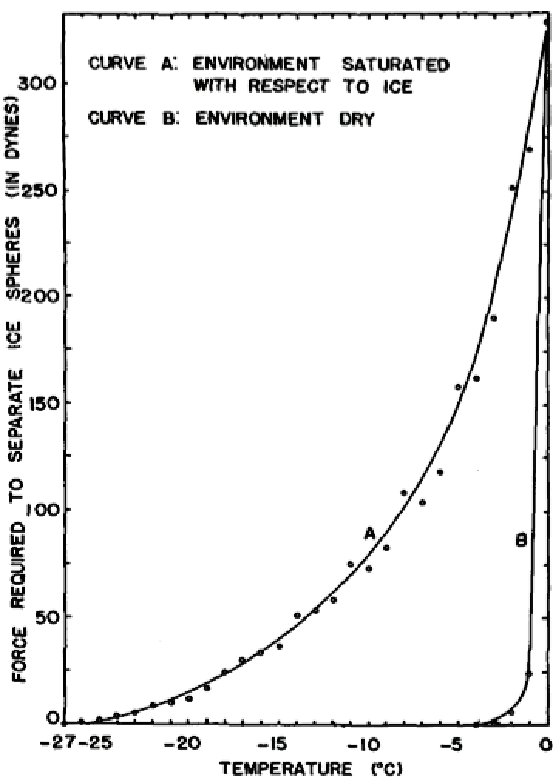


**Figure 2-5:** Mean force required to separate ice spheres as a function of temperature for two different levels of environmental water vapour. Figure adapted from Hosler et al. (1957).

The ice sphere experiment indicates that as temperature and vapour pressure increase, aggregation should increase. If ice is evaporating, sticking will not occur below -3°C, implying that the state of the surface of the ice as well as its temperature is important in the freezing together of two bodies of ice. The explanation for the freezing together of the ice spheres is assumed to be due to the layer of liquid water on the surface of ice that varies in thickness directly with temperature. This is the reason why two ice spheres can stick together without force being applied. The film on the surface remains liquid while it is on the surface, but upon contact with the other ice sphere the area of the water film that is sandwiched between the two pieces of ice freezes and freezes the two spheres together (Hosler et al., 1957).

On further investigation into the water film on ice, Chong and Chen (1974) detailed a study that employed a numerical model to investigate the water films that exist on ice pellets or hailstones. In this study, they used a simple hydrodynamic model for the determination of the maximum thickness of a water film that can be supported on an ice core. It was assumed that the hydrometeor was in two parts: the ice core, consisting of solid ice and the outer water shell, which is subject to deformation. It was also assumed that the particle falls under the influence of gravity without tumbling or rotation. The volume of the particle was also assumed to be conserved (Chong and Chen, 1974).

Water was assumed initially to be evenly distributed on the surface of the ice core. Calculations were carried out for twenty initial water film thicknesses ranging from 0.1 to 2 mm on ice cores of radius 1 to 5 mm. The shapes for small hydrometeors were similar to free falling water drops. As the size of the hydrometeor increases, the deformation of the water film is more pronounced. The results of this study show that water may exist on the surface of a hydrometeor with a radius less than 4.5 mm (Chong and Chen, 1974).

## 2.4 Ice Pellet Aggregates

Ice pellets come in many shapes and sizes. Gibson and Stewart (2007) observed that some had spikes or spicules, others bulges, and many had other irregularities. Gibson and Stewart (2007) also noted that in the storm studied, there were ice pellets that consisted of more than one component particle and these were referred to as ice pellet aggregates.

Aggregates can vary greatly in appearance and structure from one to the next and their structure depends on their formation mechanism. Some aggregates are composed of component pellets of significantly different sizes, or of components of almost the same size. Within some aggregate particles the borders between the component particles is readily apparent, whereas in others the boundaries are so blurred that the number of component particles cannot definitively be determined. Gibson and Stewart (2007) classified particles in which a clearly defined ice neck is present between the component particles as aggregates. Those in which the boundaries between the original component particles are less clear were classified as fused particles (Gibson and Stewart, 2007). In this study, all particles consisting of more than one component particle are referred to as aggregates. This term encompasses both aggregates and fused particles as defined by Gibson and Stewart (2007).

Although the existence of aggregates of ice pellets has been documented by others (for example, Stewart and Crawford, 1995), little attention has been paid to this type of particle. The fact that it is made up of more than one original particle implies that it results from particle interactions. These interactions can include those involving supercooled drops (freezing rain). In such instances, the aggregational process must decrease the concentration of such drops and therefore decrease the amount of freezing rain at the surface.

The occurrence of ice pellets differs from storm to storm, some producing mostly aggregated particles and some in which very few aggregates are observed (Gibson, 2007, personal communication). The typical number of component particles present in an aggregate can also vary widely from storm to storm. In a winter storm near Montreal, Quebec on November 4$^{th}$, 2003, Gibson and Stewart (2007) noted that 9% of the observed ice pellets were aggregates consisting of two to five component particles. The observed aggregate with the largest number of components was composed of 5 individual particles and is shown in Figure 2-6.

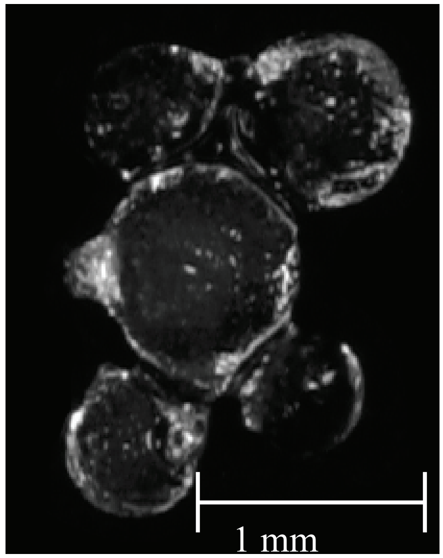


**Figure 2-6 :** An aggregate composed of five component particles – the largest of the aggregates observed near Montreal in the storm of November 4th, 2003 (Gibson and Stewart, 2007).

Ice pellet aggregates were observed in five of the eight winter storm transition regions studied in the second Canadian Atlantic Storms Program (CASP II) experiment in 1992 near St. John's Newfoundland, and in two of the six transitions studied by CASP in 1986 near Halifax, Nova Scotia (Stewart and Crawford, 1995). Most aggregates observed in CASP II were composed of two to three component particles of roughly the same size. The maximum sizes of the components were up to 2 mm in diameter. However, in one of the storms studied (February 9th, 1992) aggregates were observed that consisted of 20 to 30 component particles. In this storm, the component particles of aggregates were about 0.75 mm diameter - smaller than in the other storms (Stewart and Crawford, 1995).

Several possible mechanisms for the formation of ice pellet aggregates have been proposed. Through examination of photographs, it appears that some aggregates and fused particles are formed by a collision of a pellet with a freezing rain drop.

Others appear to be formed by collisions in which the two component pellets collide with enough force to crack the shells on liquid-core ice pellets, freezing them together. Others appear to have aggregated due the surface liquid water film on the ice pellets freezing them together upon contact (Gibson and Stewart, 2007). These three mechanisms are discussed below and summarized in Table 2-1.

**Table 2-1 :** A summary of ice pellet aggregate types and their formation mechanisms.

| Aggregate Type | Formation Mechanism |
|---|---|
| Ice pellet and liquid drop | Rapid freezing of a supercooled drop occurs upon collision with an ice pellet. |
| Ice pellet and liquid-core pellet | Partial destruction of the liquid-core ice shell occurs upon collision, causing rapid freezing of internal water. |
| Ice pellet and ice pellet | Freezing of the surface water film occurs upon collision. |

The presence of aggregates of ice pellets is expected to depend on precipitation rate. As precipitation rate increases, the probability of collisions between particles increases as well. With more collisions occurring, there is a greater chance that some of those collisions will result in aggregation. To determine the probable number of collisions that will occur within a distribution of particles, the mean free path between collisions is calculated for a given size of collector particle. The collector particle is the larger of the two particles involved in any collision. The mean free path between collisions is affected by precipitation rate and it is the average distance that a collector particle of a given size will fall before colliding with any smaller particle in the population.

### 2.4.1 Aggregation by Collisions with Supercooled Liquid Drops

Ice pellets and freezing rain often occur together and collisions between these two types of precipitation are likely to occur. As freezing rain drops fall through the refreezing layer, they become supercooled, but lack ice nuclei to initiate ice nucleation. Upon a collision of a supercooled liquid drop (freezing rain drop) and any particle that may act as an ice nucleus, the supercooled drop will undergo rapid freezing. If a supercooled drop were to collide with a frozen particle (ice pellet or liquid-core pellet) it could undergo rapid freezing while still in contact with the ice pellet and create an ice pellet aggregate. An illustration of the formation of an aggregate by a collision with a freezing rain drop is shown in Figure 2-7.

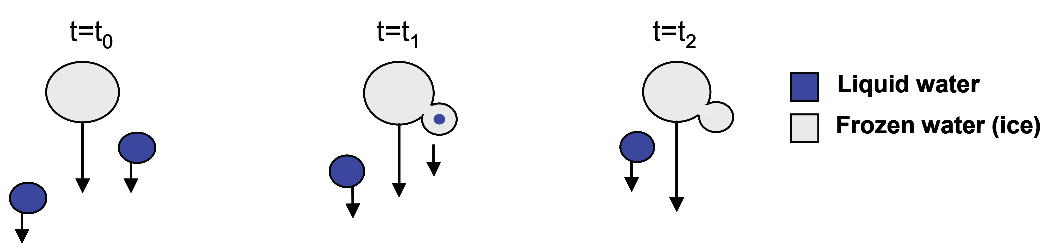


**Figure 2-7 :** Formation of an aggregate by a collision with an ice pellet and a freezing rain drop as they fall through the refreezing layer.

This mechanism of aggregate formation is favourable for improving surface conditions during a storm; ice pellets collide with freezing rain drops, which freeze to the pellet before they reach the surface, thereby reducing the amount of surface icing will occur. Freezing rain, since it is supercooled and requires contact with an ice nucleus before freezing is initiated, exists throughout the depth of the refreezing layer. Therefore, this aggregation mechanism can occur throughout the entire depth of the refreezing layer.

### 2.4.2 Aggregation by Collisions Cracking the Ice Shells on Liquid-Core Ice Pellets

The second proposed formation mechanism for ice pellet aggregates examined is that of collisions of pellets with liquid-core pellets. A diagram of the formation of an aggregate by this mechanism is shown in Figure 2-8. The theory behind this aggregational process is that a collision between an ice pellet (or liquid-core pellet) and a liquid-core pellet results in cracking or breaking of the ice shell of the thinner shelled pellet. Then, because the structure of the pellet is such that there is a shell of ice surrounding a core of water, some internal water escapes through the crack. Upon exposure to the outside air, the internal water freezes quickly, binding the two pellets together.

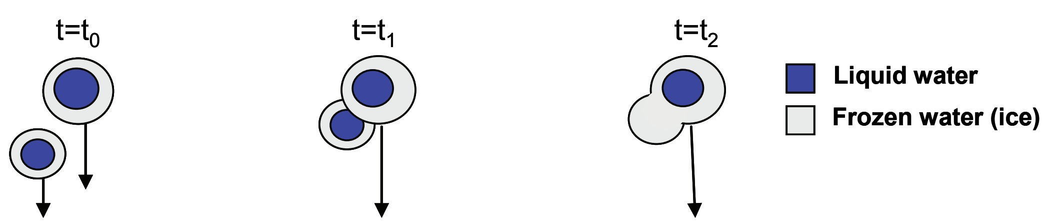


**Figure 2-8 :** Formation of an ice pellet aggregate by collision with a thin-shelled liquid-core ice pellet as they both fall.

For this aggregational mechanism to occur, the shell of one of the colliding pellets must be thin enough that the force of collision will cause it to crack or break. Therefore, this aggregation mechanism only occurs over the depth of the refreezing layer in which liquid-core pellets with shells thin enough to crack or break exist. Aggregates formed by the cracking of the ice shells on liquid-core ice pellets will not have very well defined boundaries between component particles.

### 2.4.3 Aggregation by the Freezing of the Surface Water Films on Ice Pellets

Even when both component particles of a collision are entirely frozen before coming into contact with each other, it is still possible for an aggregate to be created from the collision. All ice has a thin layer of liquid water on its surface, even below its melting point, and the freezing together of the water films on two component pellets is the manner in which such a collision can result in an aggregate. If the collision between two completely frozen particles is not violent (very large differences in terminal velocities) enough that the pellets bounce apart, aggregates could be formed by water film freezing. If the contact between the pellets is gentle, the lengthened time of contact will enable the water films on the surface of the pellets to freeze together while they are in contact. This type of aggregation produces aggregates in which the boundaries of the component particles are readily discernable, with a small area of contact between the two pellets, as shown in Figure 2-9.

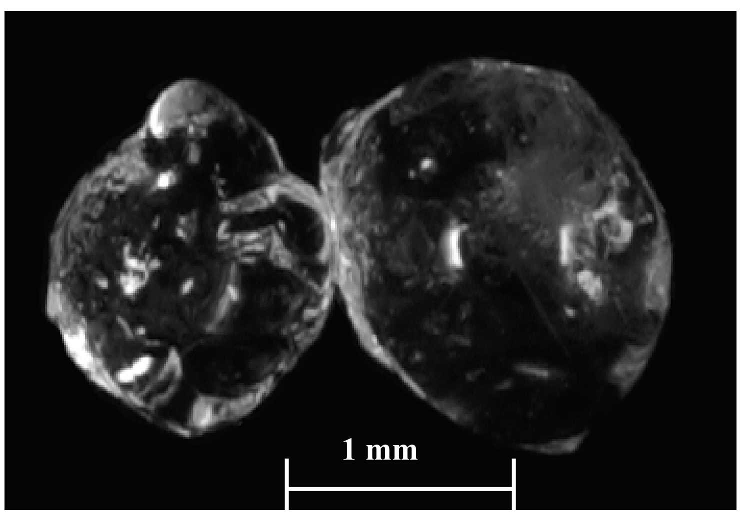


**Figure 2-9 :** Photograph of an ice pellet aggregate most likely formed by the freezing together of the liquid water films on the surface of two ice pellets. This photograph was taken by Steve Gibson on November 4th, 2003.

In the CASP II field experiment it was observed that many of the aggregates that occurred would only have been possible had the pellets had a water film on their surface that froze on contact with another pellet, creating the aggregate. Also, it was noted that the individual particles that composed the aggregate were generally of the same size. This was hypothesized to be the case because particles of the same size would have similar terminal velocities. This would mean a smaller force of collision, which would allow their water films to freeze together upon impact instead of the pellets bouncing apart (Stewart and Crawford, 1995).

Once a semi-frozen particle begins to freeze, it forms a frozen shell upon which a water film can exist. Surface water films exist on ice pellets throughout the depth of the refreezing layer, so this aggregation mechanism can happen through the entire depth of the refreezing layer.

### 2.4.4 Aggregation Summary

Atmospheric conditions determine the nature of precipitation that reaches the surface and the potential for aggregate production in a given ice pellet storm. If the minimum temperature of the refreezing layer is very low, refreezing will be rapid and there will not be many collisions with liquid-core pellets that create aggregates. If the melting layer is very shallow, or has a maximum temperature near 0°C, so that none of the frozen particles falling through it melt completely, collisions with freezing rain drops will not occur, which decreases the occurrence of aggregates. All the methods of aggregation require a collision between a pellet and another particle. The likelihood of collisions, and therefore aggregation, depends on the precipitation rate in the surrounding environment, the terminal velocities of the component particles and therefore the size distribution of falling particles.

# Chapter 3

## Model Description

Many types of winter precipitation are formed in an environment with a melting layer aloft and a refreezing layer below. Frozen precipitation falling through this elevated melting layer is partially or completely melted when it enters the refreezing layer. Upon passage into the refreezing layer, this partially melted precipitation begins to freeze and may be partially or completely refrozen upon reaching the surface.

This chapter describes the experimental method used to examine the behaviour of precipitation of different phases in this environment – specifically, on precipitation in the refreezing layer above the surface. First the environmental conditions used in the simulations are described (Section 3.1) and the equations used for calculations of particle terminal velocity discussed (Section 3.2). Then the study of the transition of a particle from liquid to solid is discussed, through examination of the freezing of a semi-frozen particle (Section 3.3). Collisions between particles were examined and through this, the possibility and relative importance of three proposed methods of aggregation were examined (Section 3.4).

### 3.1 Environmental Conditions

The environment in which the simulation occurred was saturated with respect to water, with temperature and pressure varying with height. The temperature profile decreased linearly from 0°C at the top of the layer to a specified minimum temperature ($T_{min}$) at the surface. A schematic diagram of this profile is shown in

Figure 3-1. Minimum temperatures used for the simulation of the freezing of semi-frozen particles were -0.5°C, -2°C, -4°C and -6°C. These temperatures were chosen based on the minimum temperatures of ice pellet events recorded by Zerr (1997). Zerr (1997) found that the mean minimum temperature of the environment during events containing ice pellets along with other precipitation types was -6.7°C. The mean minimum temperature for events when ice pellets occur alone is -8.8°C. These minimum temperatures are lower than some of the minimum temperatures used in these simulations, however, these numbers were chosen in order to investigate conditions when the minimum temperature is near 0°C as well as at lower temperatures. While there were no ice pellet events in Zerr's study with minimum temperatures near 0°C, this does not mean that they do not occur at these temperatures. Observations of precipitation type can be incorrect and when more than one type of precipitation occurs at once, the less dominant type may not be listed in observations of conditions. It was decided that a wider range of minimum temperatures than those presented by Zerr (1997) would be used in simulations to obtain a broader knowledge of mixed phase precipitation and ice pellet events.

The pressure profile used in the simulations also varied with height and the pressure values at the top and bottom of the layer are also shown in Figure 3-1. The pressure profile decreased linearly from 880 hPa at the top of the layer, to 1000 hPa at the surface. These values were obtained from the approximation that pressure will decrease 100 hPa with every 1000 m increase in altitude and a standard surface pressure of 1000 hPa. The depth of the refreezing layer used was 1200 m, which is consistent with the depths measured in the observational study by Zerr (1997); in the 34 storms studied (of which 13 contained ice pellets) this was the average depth of the refreezing layer in which ice pellets were observed. All simulations were carried out using Matlab Student v. 7.1.

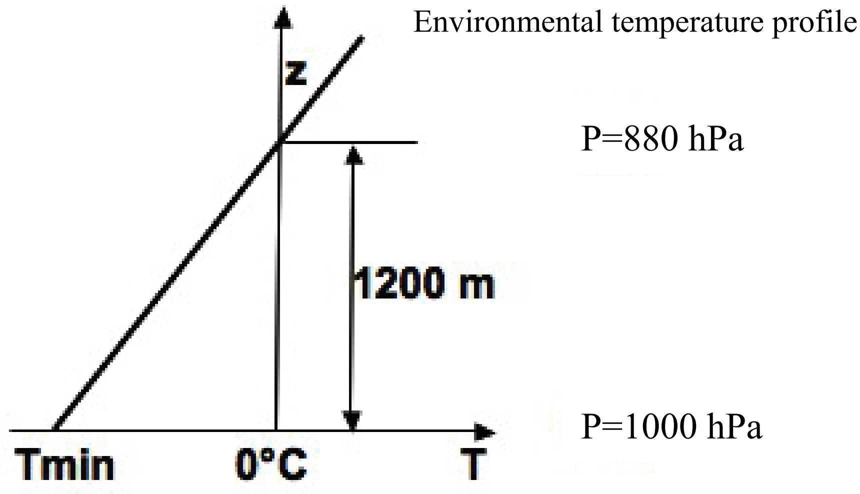


**Figure 3-1 :** Basic atmospheric temperature and pressure profile used in simulations.

The diameters of the precipitation particles modelled in these simulations ranged from 0.2 mm to 4 mm. This diameter range was chosen based on the size distribution of the ice pellets observed by Gibson and Stewart (2007). The initial size distribution (upon entry into the refreezing layer) of the particles in this study was assumed to be a Marshall-Palmer distribution. Simulations were carried out for distributions with precipitation rates of 1 mm/h, 2 mm/h, 5 mm/h, 10 mm/h and 25 mm/h. The highest precipitation rates used here are higher than are generally recorded during ice pellet events. These high rates were chosen because aggregation is hypothesized to increase as precipitation increases. Simulations for high precipitation rates were performed to force conditions to produce maximum aggregation and to examine the theoretical limit of complete elimination of freezing rain through collisions with ice pellets.

## 3.2 Falling of Particles

In this study it is assumed that during passage through the melting layer, all particles have melted sufficiently so that collapse of the crystal structure has occurred. This means that the semi-frozen particles fall with the terminal velocity of a solid particle, such as a hailstone, rather than that of refrozen wet snow. Refrozen wet snow, due to the preservation of its original crystalline structure has a significantly lower terminal velocity than that of a solid particle. This assumption simplifies the calculation of terminal velocities of individual particles during their passage through the refreezing layer so that velocity is only dependent on the size of the particle, not the degree of melting that the particle underwent in the melting layer.

When calculating distance fallen as a function of time since passage into the refreezing layer, the terminal velocity of the semi-frozen particles was estimated. In the case of ice pellets, but especially aggregates, sizeable irregularities exist on the surface of the pellets. These surface irregularities cause pellets to differ significantly in shape from a smooth sphere (Gibson and Stewart, 2007). These irregular particles would fall slower than smooth spheres of the same equivalent diameter.

To account for this diminished terminal velocity of irregular particles, the terminal velocity of ice pellets was estimated using an equation developed to model the terminal velocity of falling hailstones (Equation 3-1). Though Equation 3-1 was developed from observations of hailstones larger than the pellets examined in this study, this equation was chosen because it accounts for surface irregularities.

$$V_t = 11.45 D_e^{0.50} \qquad \textbf{(3-1)}$$

$D_e$ is the equivalent volume diameter of a spherical hailstone in centimetres and $V_t$ is given in m/s (Matson and Huggins, 1980).

For smooth spheres, the terminal velocity is calculated using the following equation (Stewart, 1977):

$$V_t = \sqrt{\frac{8Mg}{C_d \rho_a \pi L^2}} \qquad \textbf{(3-2)}$$

For a falling smooth sphere, the drag coefficient, $C_d$, has a value of 0.47. Here M is the mass of particle minus the mass of air displaced by that particle and is measured in grams. The characteristic length (L) used for calculations is the diameter of the drop and the terminal velocity calculated using Equation 3-2, gives $V_t$ in cm/s.

At small diameters the terminal velocity calculated with Equation 3-1 is similar, though slower, to that of a smooth sphere of the same size, but the difference in terminal velocity increases as diameter increases. This equation may be underestimating $V_t$ if the pellets have very smooth surfaces and are nearly perfect spheres with little surface irregularities, but will be a more accurate approximation for those that are not.

For freezing rain, the terminal velocity relation as outlined by Uplinger (1981) was used. In this equation, D is the diameter of the drop in mm, giving terminal velocity, $V_t(D)$, in m/s.

$$V_t(D) = 4.874De^{-0.195D} \qquad \textbf{(3-3)}$$

## 3.3 Freezing of Particles

Using the methods of Pruppacher and Klett (1997) freezing times were calculated for the range of drop sizes (diameters from 0.2 mm to 4 mm) in environments with varying degrees of supercooling (ΔT). Using the terminal velocity relations previously discussed (Section 3.2), freezing times were calculated as functions of both distance fallen and time elapsed since entry into the refreezing layer. The equation used to calculate the time for a drop to completely freeze ($t_f$ from equation 16-36 and approximation 16-39 in Pruppacher and Klett, 1997) was

$$t_f = \frac{\rho_w L_m r^2 \left[1 - \Delta T c_w / L_m\right]}{3 f \Delta T \left[k_a + L_s D_v \dfrac{\delta \rho_v}{\delta T}\right]} \qquad \textbf{(3-4)}$$

The ventilation coefficient used in this equation (Equation 13-61 from Pruppacher and Klett, 1997) is

$$f = 0.78 + 0.308\left(N_{Sc,v}^{1/3} R_e^{1/2}\right) \qquad \textbf{(3-5)}$$

By fitting a curve to the experimentally determined ventilation coefficients shown in Figure 13-25 of Pruppacher and Klett (1997), a good simplification to this ventilation coefficient was found to be $f = r^{0.7}$, which was the equation used to calculate the ventilation coefficient in these simulations. Pruppacher and Klett (1997) give samples of ventilation coefficients calculated when using given values of all other variables in the ventilation coefficient equation. The approximated ventilation coefficient was tested, using the specified values for the other variables, and the approximation was very close to the true ventilation coefficient. For definitions of the symbols used in these equations refer to the List of Symbols (page xiii).

Freezing times of semi-frozen particles after entry into the refreezing layer were calculated. This was done assuming a minute initial ice fraction in all pellets. Using the freezing times calculated, the methods of King (1975) were used to plot interfacial distance over time. Interfacial distance is the distance from the centre of a semi-frozen pellet to the internal boundary of its ice shell.

A graphic representation of this distance is shown in Figure 3-2. If a minute initial fraction of ice in a particle is assumed, the interfacial distance decreases from a value equal to the radius of the drop upon entering the refreezing layer to zero when the pellet is completely frozen.

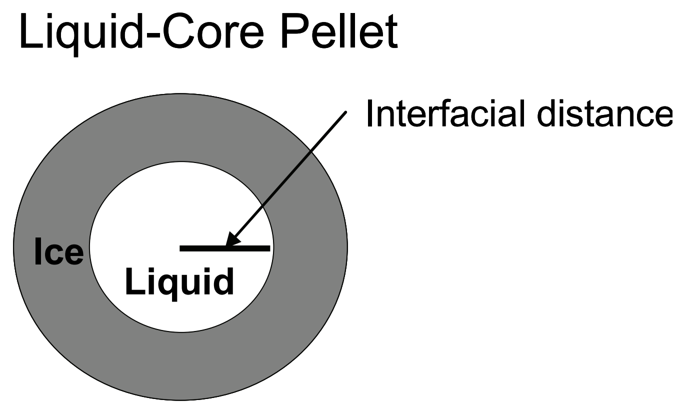


**Figure 3-2 :** Illustration of the interfacial distance of a liquid-core pellet's ice shell.

According to King (1975), for cases when the ratio of effective thermal conductivity of the environment to that of ice, is much less than one, the interfacial distance of the ice shell over time can be approximated by

$$E = r[1 - t/t_f]^{1/3} \qquad \textbf{(3-6)}$$

This ratio accounts for the enhancement of heat and mass transfer due to ventilation around the particle. At 20°C, the effective thermal conductivity of air is 0.025 W/mK and that of ice is 2.1W/mK (Huskeflux, 2007), and therefore the ratio between the two is much less than one. Using the interfacial distance calculated at each time step, both the thickness of the liquid-core shell and the mass fraction of ice present in the pellets were calculated over time. These results were then calculated as functions of distance fallen since entry into the refreezing layer.

## 3.4 Particle Collisions Resulting in Aggregate Production

The size distribution of particles in this simulation was assumed to follow the Marshall-Palmer equation (Marshall and Palmer, 1948).

$$N(D) = N_0 e^{-\lambda D} \qquad \textbf{(3-7)}$$

The number of particles of each size (diameter interval) is the integral of N(D) over that diameter interval ($dN/dD$). The slope of the exponential size distribution, λ, is defined as $\lambda(R) = 41R^{-0.21}$ where R is the precipitation rate in mm/h. A graphical representation of this size distribution is shown in Figure 3-3. The number of particles decreases exponentially as particle size increases. In Marshall-Palmer distributions the y-intercept, $N_0$, is a constant with a value of 0.08 $cm^{-4}$. It is not dependent on precipitation rate, so the intercept in this diagram will not change for varying precipitation rates. However, the slope of the distribution, λ, will decrease with increasing precipitation rate, sloping toward larger maximum diameters for higher precipitation rates.

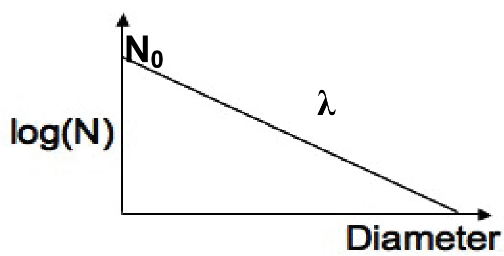


**Figure 3-3 :** Marshall-Palmer distribution of particle sizes, with y-intercept, $N_0$, and slope, **λ**.

To determine characteristic distances between collisions and therefore predict the probability of collisions between particles, the mean free path for collisions between collector particles and all smaller particles was calculated following the methods of McFarquhar and List (1991). In this case the mean free path, Λ, is the average distance that a collector particle, $D_L$, will fall before colliding with a smaller particle, D, of size down to a minimum diameter, $D_S$ ($D_S \leq D \leq D_L$). The equation for mean free path is shown below (McFarquhar and List, 1991).

$$\Lambda\left(D_L, D : D_S \leq D \leq D_L\right) = \frac{4V_L}{\int_{D_S}^{D_L} N(D)\pi\left(V_L - V(D)\right)\left(D_L + D\right)^2 dD} \qquad \textbf{(3-8)}$$

If a particle with a certain diameter, $D_L$, falls at a terminal velocity, $V_L$, through a volume of still air that is populated with drops of smaller diameters, D, of sizes down to $D_S$, falling at terminal velocity, $V_S$, the mean free path of a collector drop is the average distance that the drop will fall before colliding with any smaller drop.

The mean free path for collisions was calculated for collector drops with diameters of 0.5 mm, 1.5 mm, 2.5 mm and 3.5 mm colliding with all smaller drops larger than $D_S$ and is shown in Figure 3-4. These collector drop diameters were chosen to give a good representation of the mean free paths for collector pellets over the entire range of precipitation particle sizes examined in this study (0.2 mm - 4 mm). For initial simulations, a rain rate of 5 mm/h was used. In later calculations the mean free path for collector drops in distributions with different rain rates was calculated as well.

The mean free path for collisions increases markedly as the size of the collector pellet decreases. This is as expected, because as the collector drop diameter approaches that of the smaller drops their terminal velocities also approach each other. This decreases the chance of a collector drop overtaking a smaller drop. Figure 3-4 shows that a collector drop with a diameter of 1.5 mm has a mean free path for collisions that is smaller than the average depth of the of the refreezing layer with all drops smaller than 0.5 mm with a precipitation rate of 5 mm/h. So, a collector drop of 1.5 mm diameter should undergo at least one collision with a drop of diameter ≤ 0.5 mm in this depth. A collector pellet with a diameter of 3.5 mm has a mean free path for collisions less than the typical depth of the refreezing layer with all drops smaller than 1 mm diameter with a precipitation rate of 5 mm/h, so should also experience at least one collision with a drop of this size or smaller during its passage through the refreezing layer. For a collector drop with a diameter less than 1.5 mm, the mean free path for collisions with all drops is greater than the depth of the refreezing layer, so it may not experience any collisions during its passage through the refreezing layer.

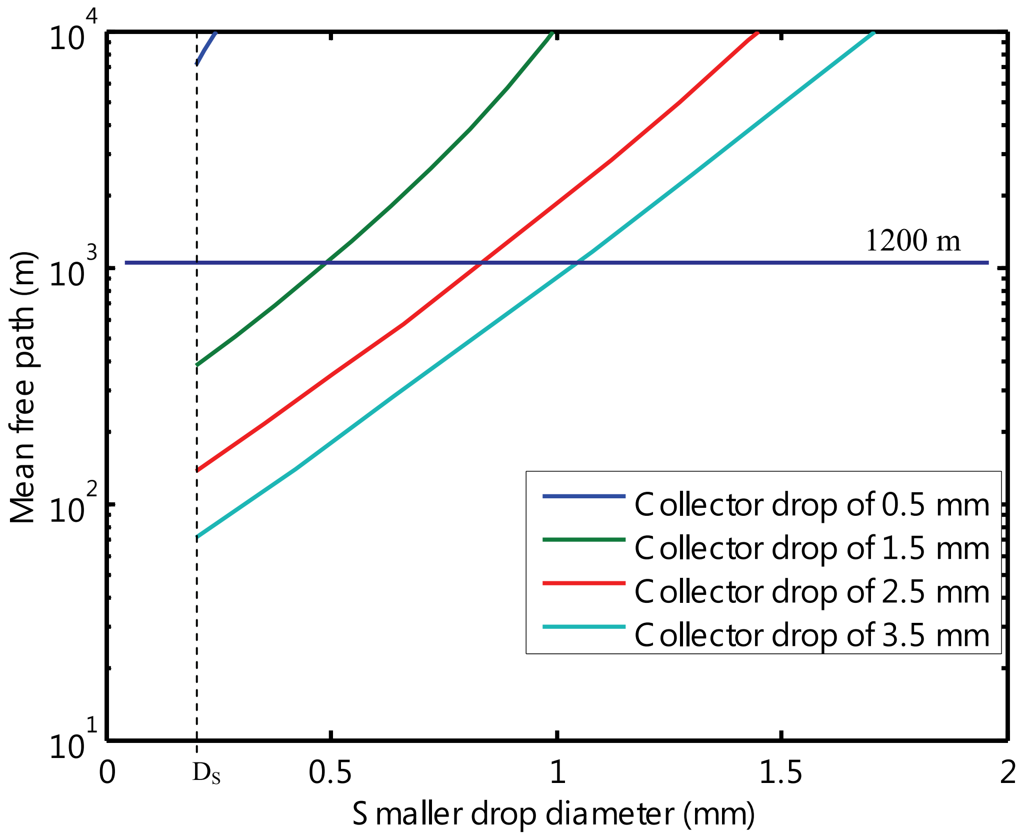


**Figure 3-4 :** The mean free path for collisions between collector drops of four diameters and all smaller drops in an environment with a precipitation rate of 5 mm/h. The blue horizontal line is the average height of the refreezing layer (1200 m) as observed by Zerr (1997). The black vertical dashed line is the minimum particle diameter considered ($D_S$ = 0.2 mm).

When studying collisions between particles, certain assumptions must be made about what happens when these collisions occur. Collision efficiency is the fraction of drops that are in the path swept out by the collector drop that actually do collide with it (Rogers and Yau, 1989). The path swept out by the collector drop is the area below the falling drop where smaller drops may be collected by it.

It is also called the sweep-out area and is equal to the cross-sectional area of the collector drop. Collision efficiency is dependent on the relative importance of the inertial and aerodynamic forces. The minimum diameter of the collected precipitation particles examined herein was such that aerodynamic effects would not overcome inertial forces. The smallest particles in question should collide with the collector drop and not be swept aside by the stream flow. In this experiment, the collision efficiency was assumed to be one for all collisions. Therefore, when investigating the probability of collisions, it is assumed that a collision will occur every time a smaller particle is overtaken by a collector drop (one collision every mean free path for collisions).

Coalescence efficiency characterizes what happens to the two particles during and after the collision. It is the ratio of the number of coalescences to that of the number of collisions (Rogers and Yau, 1989). If the particles all bounce apart upon collision, coalescence efficiency is zero. If all collisions result in complete joining of the initial particles with no break-up, coalescence efficiency is one. In the simulations of collisions of ice pellets with supercooled drops, break-up was neglected, so the coalescence efficiency was assumed to be one. In collisions between ice pellets and liquid-core pellets, aggregation will occur if the mean free path for collisions is less than the depth in which thin-shelled liquid-core ice pellet aggregates occur and if the diameters of the pellets are within the threshold values for aggregation. When these conditions are both satisfied, coalescence efficiency is assumed to be one. However, when either of these conditions is not satisfied, coalescence efficiency is zero. For collisions between ice pellets, coalescence efficiency will be one if the difference in kinetic energy between the two pellets is less than the threshold value. Coalescence efficiency will be zero otherwise and the pellets will bounce apart instead of aggregating.

The last efficiency to be discussed is collection efficiency. Collection efficiency is the product of collision and coalescence efficiencies (Rogers and Yau, 1989).

When coalescence efficiency and collision efficiency are both assumed to be one, the collection efficiency is one as well. For collisions where the coalescence efficiency is zero, collection efficiency is zero as well.

### 3.4.1 Aggregation by Collisions with Supercooled Liquid Drops

When examining collisions between an ice pellet and a freezing rain drop, a critical diameter of the population of precipitation particles was chosen. This is the diameter above which all particles have some ice in them upon entry into the refreezing layer. All particles larger than the critical diameter evolve into ice pellets as they fall through the refreezing layer. All particles below this diameter are completely melted upon entry and become freezing rain drops in the refreezing layer (see Figure 2-3).

#### 3.4.1.1 Calculating Collisions Throughout the Depth of the Refreezing Layer

An average ice pellet diameter was determined for each critical diameter by calculating a weighted mean diameter of all the particles ranging from the critical diameter to the maximum diameter. The weighting method was a simple weighting by number density of particles. Using this average diameter as the collector pellet diameter, the mean free path was calculated for a collector pellet of average diameter colliding with all freezing rain drops smaller than the critical diameter. The size and phase distribution of particles is shown in Figure 3-5 along with the average ice pellet diameter.

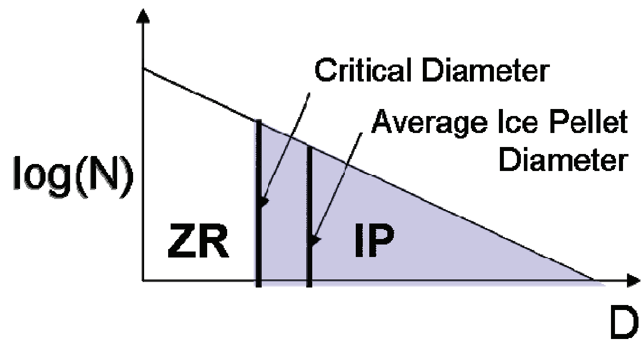


**Figure 3-5 :** Illustration of the distribution of ice pellets (IP) and freezing rain (ZR) depending on critical diameter. All particles greater than the critical diameter are ice pellets (or liquid-core ice pellets) and all smaller are freezing rain drops. The average ice pellet diameter is shown as well.

Calculations of the mean free path for collisions were carried out for critical diameters ranging from 0.5 mm to 4 mm. The number of probable collisions expected through the depth of the refreezing layer was then determined, assuming that once every mean free path there was a collision. The number of semi-frozen particles as well as the number of completely melted particles in one unit volume was calculated for each critical diameter in five precipitation rates.

The number of ice pellets (collector pellets) in the distribution bounds the number of collisions; there cannot be more collisions than there are collector pellets. However, each collector pellet can collide with and collect more than one freezing rain drop. A collector pellet can collect as many drops as it will probabilistically collide with during its passage through the refreezing layer. Through these collisions, the number of collector pellets is conserved, but upon reaching the surface these pellets may be composed of single pellets, or may be aggregated particles through collisions with one or more freezing rain drops.

In these preliminary calculations, the collisions are calculated through the depth of the refreezing layer. Therefore the probable number of collisions that one ice pellet will experience during its passage through the layer was assumed to be the simple result obtained by dividing the average depth of the layer by the mean free path for collisions. The number of collisions that will occur (All Collisions) was computed by multiplying the number of collisions for one pellet by the number of ice pellets (IP) for a given critical diameter. These calculations are shown in Equation 3-9.

$$\text{All Collisions} = \frac{\text{Average depth of refreezing layer}}{\text{Mean free path of collisions}}\left(\text{Number of IP}\right) \quad \textbf{(3-9)}$$

The final number of freezing rain drops (ZR) after collisions with ice pellets was calculated by subtracting the number of collisions from the initial number of freezing rain drops. The percent decrease in freezing rain through collisions with ice pellets was then calculated by dividing the final number of freezing rain drops by the initial number of freezing rain drops as shown in Equation 3-10.

$$\text{Decrease in freezing rain (\%)} = \frac{\text{Initial number of ZR - All Collisions}}{\text{Initial number of ZR}}\left(100\right) \quad \textbf{(3-10)}$$

To be able to compare the efficiency of this method of aggregation to the two other proposed mechanisms, the fraction of all particles that are aggregates was calculated. This is the percent of all original particles (both ice pellets and freezing rain drops) that are aggregates at the surface, formed by collisions between ice pellets and freezing rain drops aloft.

$$\text{Fraction of aggregates (\%)} = \frac{\text{Initial number of ZR - All Collisions}}{\text{Initial number of all particles}}\left(100\right) \quad \textbf{(3-11)}$$

This method of calculation of decrease of freezing rain through collisions with ice pellets is an approximation. It is most likely an over-estimation of collisions and therefore aggregate production, because as one freezing rain particle is scavenged, the population of freezing rain decreases, decreasing the number of freezing rain drops available for subsequent collisions. However, this method may also under-estimate collisions. Collisions are under-estimated because when an aggregate is created, the dimensions of the aggregate are larger than those of an individual ice pellet and its sweep-out area is correspondingly larger. As ice pellets become larger through aggregation and their sweep-out area increases, their potential to collect more drops increases and they continue increasing in size at a faster rate. Due to the decrease in freezing rain drop concentration, as well as the increase in average ice pellet size, another method was attempted to more accurately calculate the decrease of freezing rain through collisions with ice pellets during the passage through the refreezing layer.

#### 3.4.1.2 Recalculating Freezing Rain Amount and Average Ice Pellet Diameter at Height Intervals Through the Refreezing Layer

In the second method of calculating the decrease in freezing rain amount, the number of freezing rain drops available for collisions as well as the average ice pellet diameter, were recalculated at height intervals through the refreezing layer. The height interval used was 300 m, giving four recalculations through the refreezing layer. At each height level, the number of freezing rain drops that had been scavenged and the number remaining were recalculated. The average ice pellet diameter (weighted mean) was recalculated at height intervals as well to account for the mass added to the ice pellet size distribution by scavenged freezing rain drops.

The average ice pellet diameter was recalculated by adding the product of the number of freezing rain drops scavenged (# ZR scavenged) by the average freezing rain drop volume (ave $V_{ZR}$) to the product of the mean ice pellet volume (ave $V_{IP}$) and the number of ice pellets (# IP). This was then divided by the number of ice pellet in the population to get an average ice pellet volume after collisions with freezing rain drops. The new average ice pellet diameter (New $D_{IP}$) was then calculated from the volume relation. This calculation is summarized below.

$$\text{New } D_{IP} = 2\sqrt[3]{\frac{3}{4\pi}\left(\frac{(\text{ave } V_{ZR})(\#\ \text{ZR scavenged}) + (\text{ave} V_{ip})(\#\ \text{IP})}{\#\ \text{IP}}\right)} \tag{3-12}$$

Using these values recalculated at each height level, the probable number of all collisions, as well as the decrease in freezing rain and the fraction of all particles that are aggregates at the surface was calculated for this method following the same procedure outlined in Section 3.4.1.1.

A problem with this analysis is that for an accurate estimation of the number of probable collisions, the mean free path would also have to be recalculated at each height level. As the number of freezing rain particles decreases and the average size of the ice pellets increases, the exponential size distribution of the particles changes, which affects the mean free path. This suggests that the exponential size distribution would have to be re-adjusted at each height level as well to improve the accuracy of this calculation.

#### 3.4.1.3 Recalculating $N_0$, Freezing Rain Amount and Average Ice Pellet Diameter at Height Intervals Through the Refreezing Layer

A third method of calculating the decrease in freezing rain through collisions with ice pellets was carried out. In this method, average ice pellet diameter, concentration of freezing rain drops, and the y-intercept of the exponential size distribution, $N_0$, of the distribution were recalculated. The recalculation of $N_0$ thereby readjusted the size distribution at each height level. A height interval of 300 m was again used, meaning that the parameters were calculated four times through the depth of the refreezing layer.

At each height level, the number of freezing rain drops that had been scavenged and the number remaining were calculated. The average ice pellet diameter (weighted mean) was also readjusted at height intervals to account for the freezing rain drops scavenged by the ice pellets. Using the total remaining number of freezing rain drops, $N_T$, the y-intercept of the exponential size distribution, $N_0$, was recalculated at each height level. Hence, the total number of freezing rain drops for a given critical diameter is defined as

$$N_T = \int_{d_0}^{d^*} N_0 \exp(-\lambda D) dD \qquad \textbf{(3-13)}$$

Where λ is the slope of the distribution, $N_T$ is the total concentration of freezing rain drops (for a given critical diameter), $d_0$ is the minimum diameter of freezing rain drops counted and d* is the critical diameter of the population. Thus, based on Equation 3-7, the new y-intercept of the freezing rain size distribution is

$$N_0 = \frac{N_T \lambda}{\Gamma(1, \lambda d^*) - \Gamma(1, \lambda d_0)} \qquad \textbf{(3-14)}$$

The incomplete gamma is used to account only for the particles with sizes between 0.2 mm and the critical diameter (refer to Figure 3-6 for an illustration of the diameter interval for freezing rain drops).

It was decided to recalculate $N_0$ (the y-intercept of the distribution) in the exponential size distribution while maintaining the value of λ (the slope of the distribution). As the population of freezing rain drops is decreased through collisions, it is assumed that drops of all sizes are scavenged in equal proportions, so the slope of the size distribution does not change. Drops of one size are not are scavenged more than others, so on a logarithmic scale, the number of drops of one size should exhibit the same relation to the numbers of drops of other sizes (same slope, λ). The number of particles of any given size decreases, but the number of drops of one size does not change in relation to the number of drops of the other sizes. A diagram of the evolution of the size distribution of freezing rain drops when recalculated at height levels is shown in Figure 3-6.

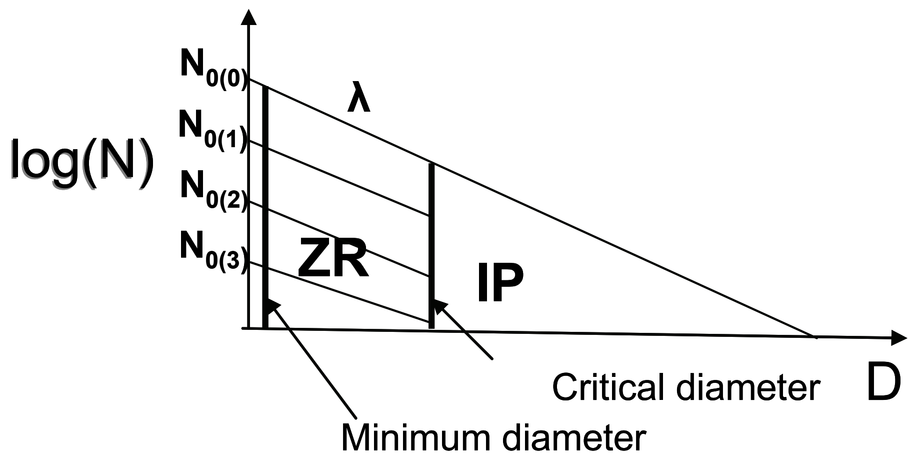


**Figure 3-6 :** Schematic diagram of the evolution of the exponential size distribution of freezing rain drops (ZR) as they are scavenged through collisions with ice pellets (IP). $N_{0(0)}$ is the y-intercept of the size distribution at the top of the layer (time = $t_0$). $N_{0(1)} - N_{0(3)}$ are the intercepts recalculated at subsequent height levels in the refreezing layer (time = $t_1 - t_3$).

In this figure, the critical diameter is shown on the size distribution as the dividing line between freezing rain drops and ice pellets. The slope of the exponential size distribution (λ) does not change, but as the number of freezing rain drops are decreased, $N_0$ decreases. The diagram shows (not to scale) the initial $N_0$ value as well as those recalculated at height levels through the depth of the refreezing layer. $N_{0(0)}$ is the initial intercept with a value of 0.08, $N_{0(1)}$ is the first recalculated value of $N_0$, and continuing on in the same fashion recalculating $N_{0(2)}$ and $N_{0(3)}$ at subsequent height levels. The evolution of the ice pellet size distribution during passage through refreezing layer is illustrated in Figure 3-7.

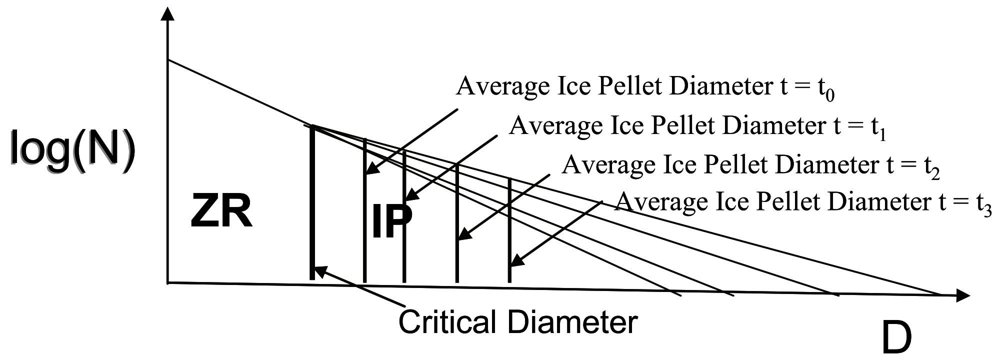


**Figure 3-7 :** Schematic diagram of the evolution of the exponential size distribution of ice pellets (IP) as they grow by collecting freezing rain drops (ZR). The time evolution of the average ice pellet diameters recalculated at height intervals is also shown.

In Figure 3-7, the critical diameter is the dividing line between freezing rain drops and ice pellets (as in Figure 3-6). The slope of the exponential size distribution ($\lambda$) changes, but because the number of ice pellets does not change, the minimum diameter of ice pellets does not change either. As the ice pellets undergo collisions with freezing rain drops and increase in size, the maximum diameter increases, and so the average ice pellet diameter increases as well (not to scale).

The probable number of all collisions was calculated for each height interval as in Section 3.4.1.1. Using the concentration of freezing rain drops at the surface calculated using this method, the decrease in freezing rain at the surface and the fraction of all particles that are aggregates at the surface were also calculated following the procedure outlined in Section 3.4.1.1.

### 3.4.2 Aggregation by Collisions Cracking the Ice Shells on Liquid-Core Ice Pellets

To begin the investigation of aggregates formed through the cracking of thin-shelled liquid-core ice pellets, the zones in which these aggregates can be formed must be defined. To examine the zones in which this aggregation mechanism may occur, an investigation was conducted into the threshold thickness for an ice shell on a liquid-core pellet to crack upon impact. Shells below this threshold thickness break upon impact with another particle and shells above this threshold thickness remain intact upon collision. Through a review of photographs taken by Steven Gibson in ice pellet storms occurring from 2003-2006 (Gibson and Stewart, 2007), a threshold thickness was estimated.

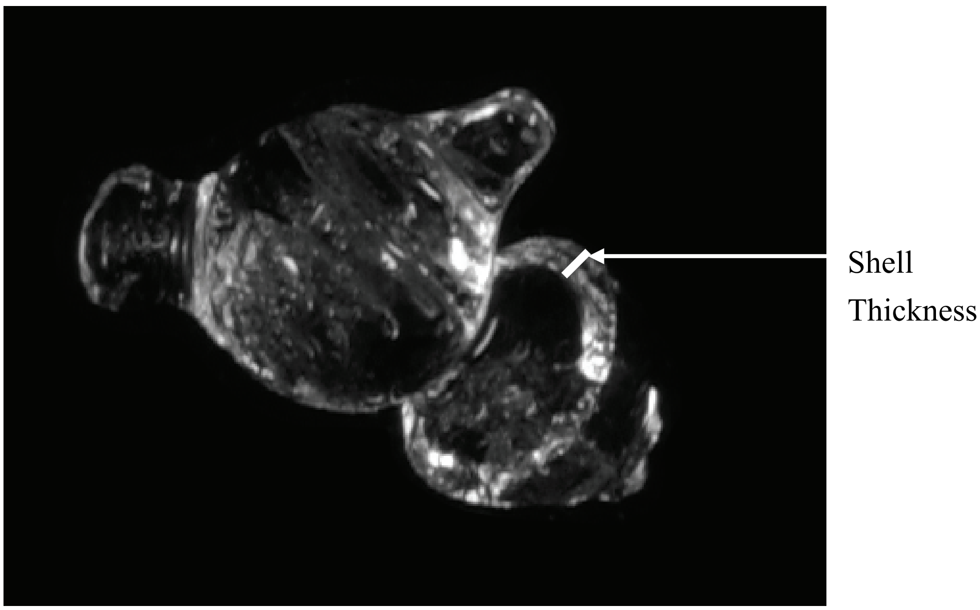


**Figure 3-8 :** Photograph of an ice pellet aggregate assumed to have been formed by collision of a liquid-core pellet with an ice pellet. Photograph was taken by Steve Gibson on November 4th, 2003.

From the examination of photographs of ice pellet aggregates that appeared to have been formed by collisions between an ice pellet and a liquid-core pellet, a shell thickness of 10% of the diameter was decided upon as the threshold breaking thickness. An illustrative aggregate exhibiting a broken shell most likely created by a liquid-core pellet colliding with an ice pellet is shown in Figure 3-8. In this picture the thickness of the ice shell can be examined in comparison to its diameter. The thickness of the ice shell in this figure is ~11% of the diameter of the pellet.

In the simulations, coalescence efficiency for collisions with liquid-core pellets is zero if the ice shell is thicker than 10% of diameter at the time of collision. This will be an upper limit of aggregate production. In the case of the aggregate pictured in Figure 3-8, the shell is undoubtedly thicker than it was at the time of collision. This is a photograph of the particle at the surface, but the break of the shell happened aloft, not at the surface. Upon collision and breaking of the shell, some of the internal water would have frozen to the ice shell, making it thicker than it was at the time of collision. The shell could have continued to freeze water in contact with it during its fall, increasing its thickness from what it was at the moment of impact. It is expected that this maximum threshold thickness for aggregation may lead to a higher calculated production of aggregates by this mechanism than are actually produced.

The depth over which liquid-core ice pellets exist with shells thinner than 10% of their diameter was calculated as a function of particle size in each of the environmental temperature profiles described earlier. These are the depths in which collisions between an ice pellet and a thin-shelled liquid-core pellet will be assumed to produce an aggregate by this mechanism.

In this experiment it was assumed that it is the larger pellet that will crack upon collision with a smaller pellet. This assumption was made to maximize calculated aggregation by this mechanism. This maximizes aggregation because if all particles enter the refreezing layer with a minute initial ice fraction, it will take longer for the thickness of the shell of the larger pellet to reach 10% of its diameter than it will take a smaller pellet. If it takes longer for the shell of the larger pellet to reach 10% thickness, there will be a larger distance in which thin-shelled liquid-core pellets exist so this should maximize collisions in the depth with thin-shelled pellets and therefore maximize aggregation by this mechanism.

If the larger pellet is the pellet that will break in the collision, there must also be a minimum diameter threshold for aggregation. This threshold diameter for collisions to create an aggregate will be referred to as $D_{min}$ (for a given collector pellet diameter). All pellets below $D_{min}$ will have insufficient force to break the shell on the collector pellet. As the size of the smaller pellet decreases, the probability that it will have enough force to break the shell decreases as well. The smallest pellets will not be able to break the shells on large liquid-core ice pellets.

The number of expected collisions was calculated in a similar manner to the number of expected collisions with freezing rain drops. However, in this case, the depth over which aggregate producing collisions can occur is only the depth in which thin-shelled liquid-core pellets exist. The mean free path for collisions resulting in aggregation increases significantly when the minimum diameter restriction is placed on the component particles in the collisions.

The mean free path for collisions, the depths in which thin-shelled liquid-core pellets exist and the minimum diameter threshold were used to find aggregation efficiency of this mechanism. The mean free path for collisions was calculated for collector pellets of diameters 0.5 mm – 4 mm, colliding with all smaller drops of diameters $D_{min}$ up to the collector pellet diameter. This was done for environments with five precipitation rates (1 mm/h, 2 mm/h, 5 mm/h, 10 mm/h and 25 mm/h). These mean free paths were then used to calculate the probable number of collisions that one collector pellet will have within the depths in which the collector pellets will have sufficiently thin shells.

For the number of collisions between ice pellets, the number of collector pellets of each size needed to be calculated. This was achieved by integrating the Marshall-Palmer equation over the diameter interval from D1 to D2, where D1 is the diameter of the collector pellet – 0.05 mm and D2 is the diameter of the collector pellet + 0.05 mm. In this manner, the number of particles that had diameters equal to that of the collector pellet diameter +/- 0.05 mm was obtained for collector pellets at 0.1 mm diameter intervals through the range of collector pellet sizes. This calculation is shown in Equation 3-15, in which all variables are dependent on collector pellet diameter (D) and Collisions (D) is the total number of collisions that the pellets in one collector pellet bin (0.1 mm width) experience.

$$\text{Collisions (D)} = \frac{\text{Depth with thin-shelled pellets (D)}}{\text{Mean free path of collisions (D)}}\left(\text{Number of IP (D)}\right) \tag{3-15}$$

To calculate the total number of collisions in the distribution, the number of collisions for each diameter bin (collector pellet diameter) was summed over the entire range of diameters (0.5 mm – 4 mm).

$$\text{All Collisions} = \sum_{D=0.5\,mm}^{4\,mm} \text{Collisions } (D) \tag{3-16}$$

The total fraction of aggregates produced by this mechanism was then calculated as follows:

$$\text{Fraction of aggregates (\%)} = \frac{\text{All Collisions}}{\text{Initial number of IP}}(100) \tag{3-17}$$

Using this method, the fraction of aggregates produced by the cracking of the shells on liquid-core ice pellets was calculated in environments with four different temperature profiles as explained earlier ($T_{min}$= -0.5°C, -2°C, -4°C and -6°C).

### 3.4.3 Aggregation by the Freezing of the Surface Water Films on Ice Pellets

To examine aggregates created by the freezing together of the surface water films on ice pellets, a threshold kinetic energy difference between the two pellets had to be chosen. Because of the possibility of pellets bouncing apart upon impact instead of sticking together, a threshold difference in kinetic energies of the two colliding pellets must exist. The threshold difference must be less than the kinetic energy required to cause the smaller pellet to be deflected significantly. Above this threshold pellets will bounce apart upon collision, and below it they will form an aggregate.

The threshold difference in kinetic energy between the component pellets in a collision was decided to be the kinetic energy of the smaller pellet. If the kinetic energy difference of the collision is less than the threshold kinetic energy, an aggregate will be produced. If the collision is relatively gentle (small KE of collision), the pellets have enough time for their water films to freeze together and create an aggregate. If the kinetic energy difference of the collision is greater than the threshold energy, the pellets bounce apart upon collision and do not create an aggregate. It is assumed that all collisions that have a difference in kinetic energy between the pellets less than the kinetic energy of the smaller pellet will result in an aggregate and all those with a greater kinetic energy difference will not. No information was found in the current literature upon which to base this threshold. Collisions and sticking of ice crystals has been studied, but no information on the sticking of ice spheres was found.

The kinetic energy was calculated for collector pellets with diameters ranging from 0.5 mm – 4.0 mm at 0.1 mm intervals. Then the kinetic energy difference was calculated for smaller pellets of all diameters over the size range of 0.2 mm – 4 mm. The kinetic energy of collisions was calculated for the collector pellets colliding with all smaller pellets. From the kinetic energy threshold of collisions, a minimum diameter was found to correspond to each size of collector pellet for successful collision (aggregation). The minimum diameter is the minimum diameter a smaller pellet may have for a collector pellet to collide with it and form an aggregate. This is the diameter range within which the kinetic energy of collisions is not more than the threshold kinetic energy.

For another method to estimate a reasonable kinetic energy threshold for aggregate production by this mechanism, raindrop coalescence efficiency was examined. Using the theory of coalescence efficiency by Brazier-Smith et al. (Rogers and Yau, 1989), a best fit line was fitted to the bottom half of the curve where coalescence is one. The equation of the line is y = 0.84x, where x is the collector pellet diameter (in mm) and y is the minimum possible diameter of the smaller pellet that will form an aggregate by this mechanism. The minimum diameters using these two thresholds were calculated over the range of diameters use in this study and are plotted for comparison in Figure 3-9.

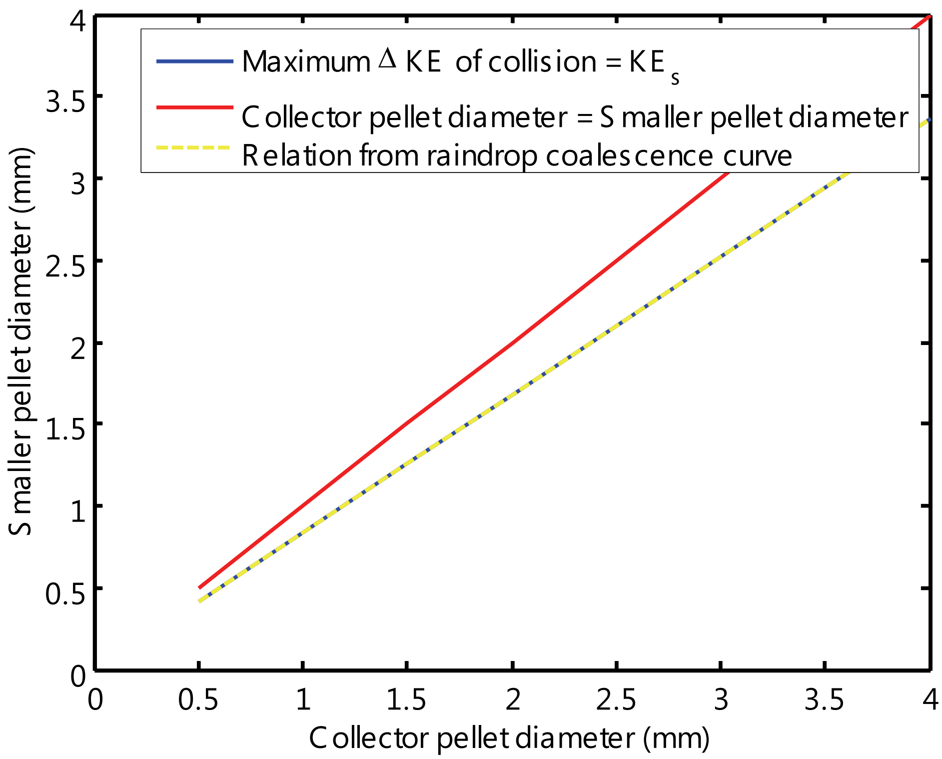


**Figure 3-9 :** The minimum smaller pellet diameter for which collisions between pellets will create an aggregate by liquid water film freezing, given the collector pellet diameter. The blue line indicates the minimum diameter calculated through kinetic energy comparisons and the yellow dashed line indicates that minimum diameter calculated to follow the curve when raindrop coalescence efficiency is one. The red line indicates when the two diameters are equal.

The two minimum diameter approximation lines in Figure 3-9 are very similar. In fact, they had to be shown as a solid and a dashed line to be able to view both at once (in Figure 3-9). The approximation from the raindrop coalescence curve with coalescence efficiency of one was used as the minimum diameter in the calculation of mean free path between collisions. On this plot, the zone of possible diameter combinations that will produce an aggregate upon collision are those that fall between the red line (Collector pellet diameter = Smaller pellet diameter) and the two overlapping lines.

The mean free path for collisions was calculated for the range of collector pellet sizes colliding with all pellets with diameters from the minimum diameter up to that of the collector pellet. The mean free paths calculated were then used to calculate the probable number of collisions. To calculate the number of collisions between ice pellets, the number of collector pellets of each size was calculated. This was achieved by integrating the Marshall-Palmer equation over the diameter interval from D1 to D2, where D1 is the diameter of the collector pellet – 0.05 mm and D2 is the diameter of the collector pellet + 0.05 mm. In this manner, the number of particles that had diameters equal to that of the collector pellet diameter +/- 0.05 mm was obtained for collector pellets at 0.1 mm diameter intervals through the range of collector pellet sizes.

$$\text{Collisions (D)} = \frac{\text{Depth of refreezing layer}}{\text{Mean free path of collisions (D)}}\left(\text{Number of IP (D)}\right)$$

**(3-18)**

To calculate the total number of collisions in the distribution, the number of collisions for each diameter bin (collector pellet diameter) was summed over the entire range of diameters (0.5 mm – 4 mm).

$$\text{All Collisions} = \sum_{D=0.5\,mm}^{4\,mm} \text{Collisions } (D) \tag{3-19}$$

From the estimate of the number of collisions, the number of aggregates created by this formation mechanism was determined and the percentage of aggregates in a given distribution was calculated over all diameters (0.5 mm to 4 mm), depending on precipitation rate. The total fraction of aggregates produced by this mechanism was calculated as follows, using the same procedure as that used in Section 3.4.2.

$$\text{Fraction of aggregates (\%)} = \frac{\text{All Collisions}}{\text{Initial number of IP}}\left(100\right) \tag{3-20}$$

Since the choice of kinetic energy difference was somewhat arbitrary, given the lack of information on such collisions, the same procedure was carried out, finding the percent of aggregates when the kinetic energy threshold was twice the kinetic energy of the smaller pellet. When the kinetic energy threshold is twice that of the smaller pellet, an equation of y = 0.76x relating collector pellet and minimum smaller pellet diameters can be fitted to the kinetic energy relations. x is collector pellet diameter and y is the smaller pellet diameter. The minimum smaller pellet diameters for the two kinetic energy thresholds are shown in Figure 3-10 as well as the line where the diameter of the collector pellet equals the diameter of the collector pellet.

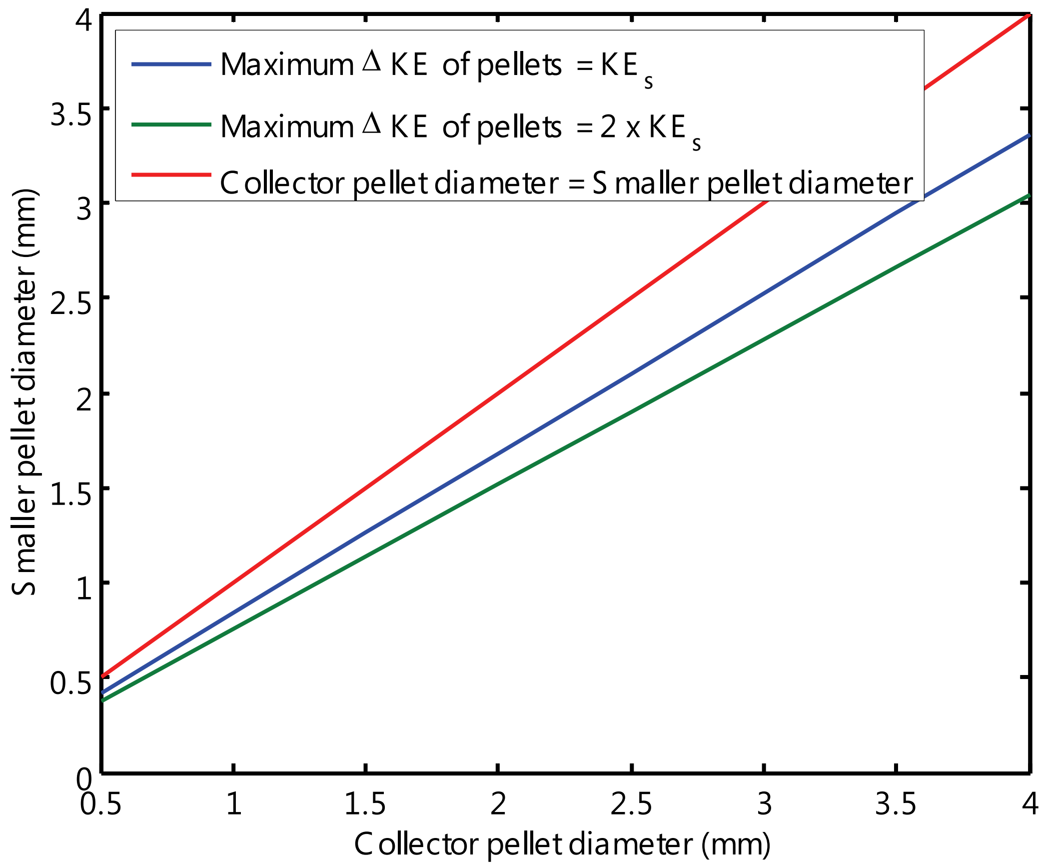


**Figure 3-10 :** Minimum smaller pellet diameter as a function of collector pellet diameter. The blue line is for the case when the maximum difference in kinetic energies of the two colliding pellets is equal to that of the smaller pellet ($KE_s$). The green line is for the case when the maximum difference of the kinetic energies is less than twice the kinetic energy of the smaller pellet. The red line is the line when the diameter of the collector pellet equals that of the smaller pellet.

In Figure 3-10 the zone of possible diameter combinations falls between the red line (where the diameters are equal) to one or the other line depending on the threshold chosen. When the maximum kinetic energy threshold for aggregation is larger, the zone of possible diameter combinations is larger as well. In the next chapter, the results for collisions between pellets using these two kinetic energy limits will be presented.

# Chapter 4

## Results

Using the methods outlined in the previous chapter, the freezing, falling and collisions in a distribution of frozen, semi-frozen and melted precipitation particles was carried out. The results of these simulations are presented here. This chapter is organized into three main sections: Falling of Particles (Section 4.1), Freezing of Particles (Section 4.2) and Particle Collisions Resulting in Aggregation (Section 4.3). In the first section (Section 4.1), the terminal velocity relations of the different particles are presented and discussed. The second section (Section 4.2) outlines data on the freezing of semi-frozen particles, focusing on the thickness of liquid-core ice pellet shells and the fraction of ice in particles as they evolve into ice pellets during their passage through the refreezing layer. The third section (Section 4.3) focuses on collisions between particles and the ability of these collisions to create aggregates by each of the three proposed mechanisms.

### 4.1 Falling of Particles

Terminal velocities of falling raindrops, terminal velocities of ice pellets using a relation developed for falling hailstones, and terminal velocities computed for a smooth sphere with the density of ice were calculated following the methods outlines in Section 3.2. To examine the differences between these three terminal velocities, all three were plotted over the range of particle sizes used in this study (0.2 mm to 4 mm) in Figure 4-1. Terminal velocities were calculated in environments with an atmospheric pressure of 1000 hPa and environmental temperature of 271 K.

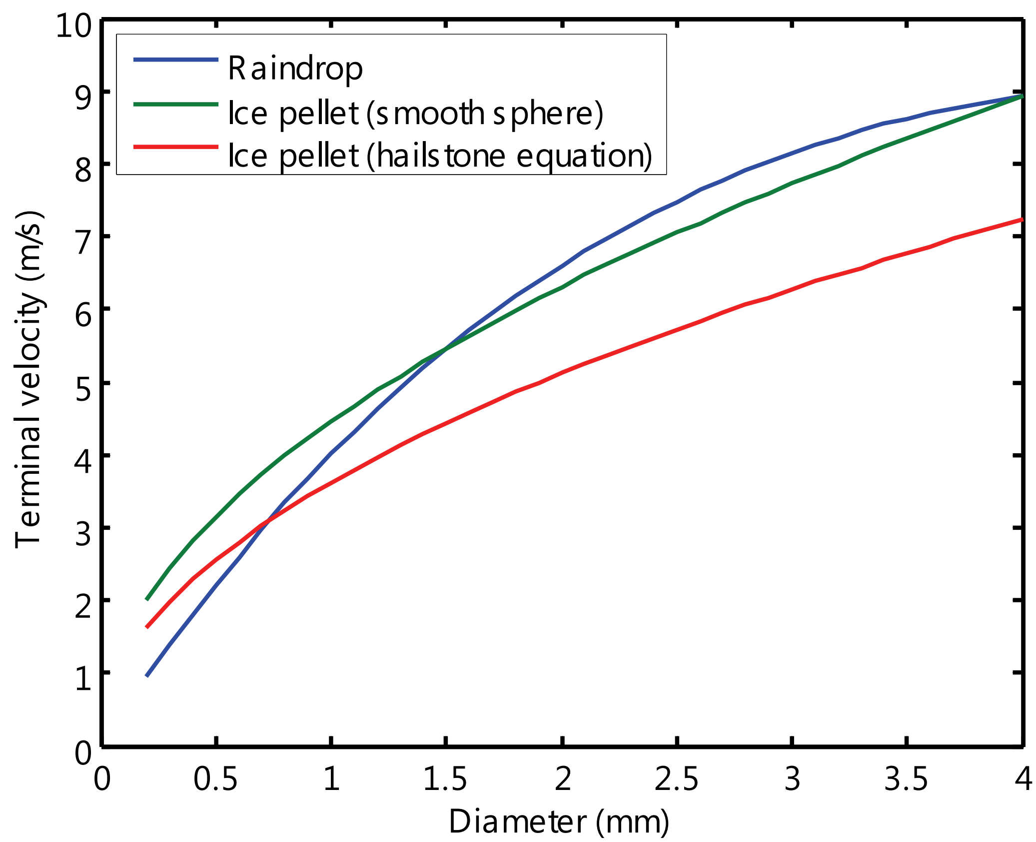


**Figure 4-1 :** The terminal velocity of ice pellets modelled as smooth spheres or as hailstones, as well as the terminal velocity of raindrops as functions of particle diameter.

The ice pellet terminal velocity estimated using the hailstone velocity equation is slower for all sizes than an ice pellet falling as a smooth sphere with the same equivalent volume diameter. For all sizes, the hailstone equation terminal velocity results are ~20% slower than the terminal velocity of a smooth sphere. For small particles, the terminal velocity difference is small, but as the diameter (or equivalent volume diameter) increases, the difference in terminal velocities of the two methods increases. As ice pellet diameter increases, the effect of possible irregularities increases as well, increasing drag and decreasing terminal velocity.

Comparing the terminal velocities of raindrops and ice pellets, Figure 4-1 shows that below ~ 0.8 mm, ice pellets modeled as hailstones fall faster than rain drops, but above this size, raindrops fall faster. The difference in terminal velocity increases as size of the particles increases. The raindrop terminal velocity curve plateaus around 4 mm. This is because as the size of raindrops increases, deformation increases. From this comparison of the terminal velocities of different particles with size, an ice pellet should be able to overtake smaller freezing rain drops or smaller ice pellets as they all fall through the layer.

## 4.2 Freezing of Particles

To calculate the freezing times of falling particles and the distance that a particle will fall in that time, the equation for freezing times (Equation 3-4) was used in conjunction with that for interfacial distance (Equation 3-6) to calculate the mass fraction of ice in pellets over time. Using the hailstone terminal velocity equation, the time of fall was calculated at each height step, enabling calculation of the mass fraction of ice in pellets during their passage through the refreezing layer. The ice fraction in semi-frozen particles was calculated every 10 m during their fall, for diameters ranging from 0.5 mm to 4 mm in the previously described environmental temperature and pressure profile (Figure 3-1), with a minimum temperature in the layer of -6°C. The initial ice fraction was assumed to be minute – just enough to initiate freezing upon entry into the refreezing layer.

Figure 4-2 shows that in an environment with a minimum temperature of -6°C, only the smallest particles (< 1.5 mm) are completely frozen upon reaching the surface. The largest of the particles (> 3.0 mm) are composed of less than 50% ice upon reaching the surface. From Figure 4-2, it is clear that in these environmental conditions, liquid-core pellets exist at the surface. If the minimum temperature were warmer, freezing would be even slower, so there would be a greater possibility for liquid-core pellets. In order to see the difference in the ice fraction in pellets and therefore freezing times for pellets in environments with different temperature profiles, the ice fraction is plotted as a function of height for four different minimum temperatures in Figure 4-3.

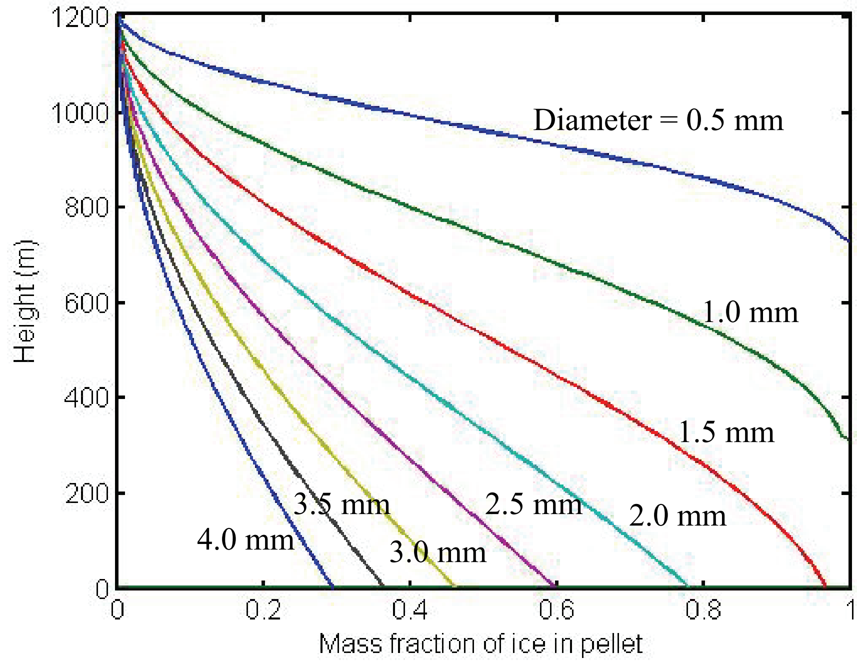


**Figure 4-2 :** Mass fraction of ice in freezing pellets of different diameters as a function of height. These fractions were calculated in an environmental temperature profile linearly decreasing from 0°C at the top of the refreezing layer to -6°C at the surface.

The freezing of semi-frozen particles during their passage through the refreezing layer is dependent on their size, the ambient temperature and the depth of the layer through which they fall. In these comparative plots, the time for a semi-frozen particle to completely freeze is significantly different. With a minimum temperature of -0.5°C no particles freeze completely during their passage through the refreezing layer and they all reach the surface as liquid-core pellets. When the minimum temperature is lower, only the smallest particles freeze completely during their passage through the refreezing layer and all larger particles reach the surface as liquid-core pellets. Therefore, if a minute initial fraction of ice is present in all pellets upon entry into the refreezing layer and the depth of the refreezing layer is near 1200 m, there should be liquid-core pellets at the surface in all of the temperature profiles examined in this study. This calculation of ice fraction assumes only a minute initial fraction of ice in all particles upon entry into the refreezing layer. If the initial ice fraction varied with particle size, the results may be different.

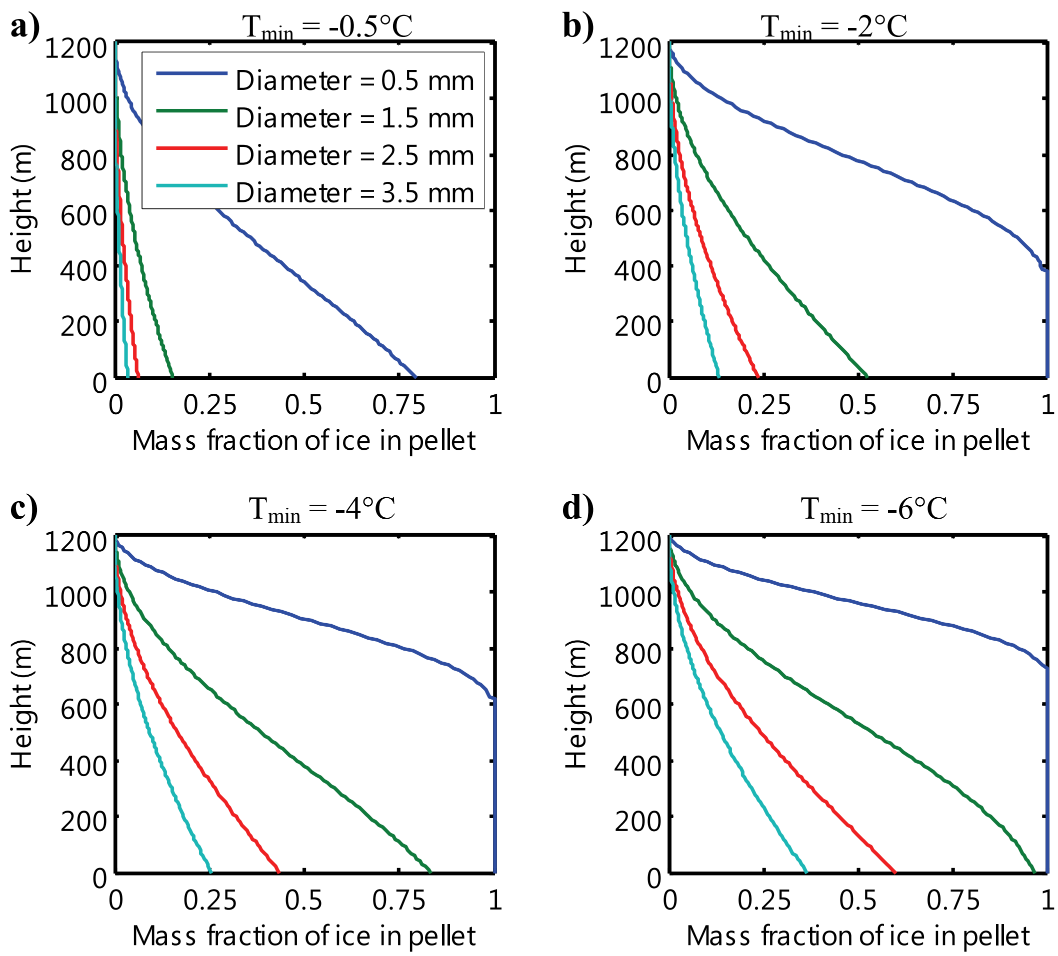


**Figure 4-3 :** The mass fraction of ice in freezing pellets of four diameters as a function of height. The four panels are for environmental temperature profiles as shown in Figure 3-1, with minimum temperatures of a) -0.5°C, b) -2°C, c) -4°C and d) -6°C.

## 4.3 Particle Collisions Resulting in Aggregate Production

This section is an overview of the results from the simulations of collisions between particles and the conditions in which aggregation occurs as a result of these collisions. This is organized similar to Section 3.4, with individual sections outlining results from the investigation of each of the three proposed aggregation mechanisms.

### 4.3.1 Aggregation by Collisions with Supercooled Liquid Drops

The mean free paths of collisions between a collector pellet of 3 mm diameter and raindrops of all diameters less than 3 mm was calculated for five different precipitation rates and is shown in Figure 4-4. The mean free path increases as the size of the smaller particle approaches that of the collector pellet and decreases with increasing precipitation rates. As the size of the drop approaches that of the collector pellet, the difference in terminal velocity decreases, so fewer collisions occur in a given distance. As precipitation rate increases, the concentration of collector pellets increases. The number of available freezing rain drops to collide with increases as well, so there are more collisions in a given depth.

The average height of the refreezing layer is shown in Figure 4-4 to compare the mean free paths calculated to the average depth of the refreezing layer an ice pellet event. In all precipitation rates the mean free path of some collisions is less than this depth. There will be at least one collision between a collector pellet of 3 mm and a smaller drop within the layer.

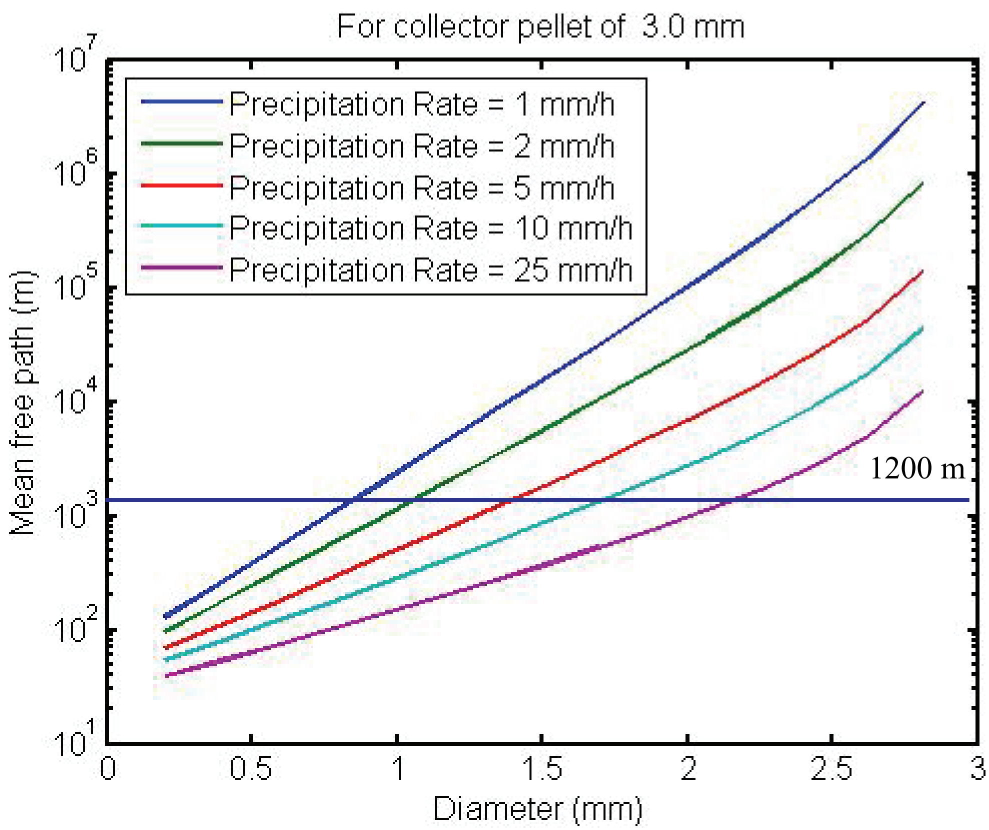


**Figure 4-4:** Mean free path for collisions between an ice pellet of 3 mm and all smaller freezing rain drops, depending on precipitation rate. The blue horizontal line is the average height of the refreezing layer (1200 m) as observed by Zerr (1997).

The mean free paths calculated for this collector pellet diameter range from a minimum of 40 m for the largest precipitation rate and smallest collected drop to $10^6$ m for the lowest precipitation rate and the largest collected drop diameter. When the precipitation rate is 1 mm/h, there should be at least one collision between the collector pellet and a drop smaller than 0.8 mm. For the case when the precipitation rate is 25 mm/h there should be at least one collision in the refreezing layer between the collector pellet and any drop smaller than 2.2 mm. For the smallest drops and the highest precipitation rate, the mean free path for collisions is 40 m, so there will be 30 collisions with drops of 0.2 mm during the collector pellet's passage through the refreezing layer.

The results presented in Figure 4-4 show that collisions between ice pellets and freezing rain drops will occur during their passage through the refreezing layer. More extensive investigation into these collisions follows. The mean free path for collisions between ice pellets and freezing rain drops was calculated for distributions with varying critical diameters. This was done to examine collisions between ice pellets and freezing rain drops when the relative abundance and the size of the particles involved vary. The critical diameter of the distribution affects the average size of the collector pellets. All particles with diameters larger than the critical diameter evolve into ice pellets in the refreezing layer and all with diameters lower than the critical diameter become freezing rain drops. The average size of a collector pellet is a weighted mean of all particles larger than the critical diameter up to the maximum diameter. The weighted mean ice pellet diameters for five precipitation rates and four critical diameters are shown in Table 4-1.

**Table 4-1 :** Weighted mean diameters of collector pellets as functions of critical diameter and precipitation rate.

| Precipitation rate | 1 mm/h | 2 mm/h | 5 mm/h | 10 mm/h | 25 mm/h |
|---|---|---|---|---|---|
| Critical Diameter (mm) | Weighted mean diameter of semi-frozen pellets (mm) | | | | |
| 0.5 | 0.8 | 0.8 | 0.8 | 0.9 | 0.9 |
| 1.5 | 1.8 | 1.8 | 1.9 | 1.9 | 2.0 |
| 2.5 | 2.8 | 2.8 | 2.9 | 2.9 | 3.0 |
| 3.5 | 3.6 | 3.7 | 3.7 | 3.7 | 3.7 |

The weighted mean diameters are all slightly larger than the critical diameter, and as the critical diameter increases, the weighted mean diameters are closer to the value of the critical diameter. This follows from the understanding of the size distribution of particles. The abundance of particles decreases sharply with size (number distribution shown in Figure 2-3), so the weighted mean pellet diameter is closer to the critical diameter than the maximum diameter. If the mean pellet diameter were used, rather than the weighted mean, the mean for a critical diameter of 1 mm would be 2.5 mm, but due to a larger abundance of smaller pellets, the weighted mean is lower, varying from 1.8 – 2 mm depending on precipitation rate. As the precipitation rate increases, the weighted mean diameters increase in size because as precipitation rate increases, the abundance of larger particles increases. As the precipitation rate increases, the weighted mean diameter increases its spread from the critical diameter and approaches the un-weighted mean.

Collisions between frozen (or partially-frozen) pellets and liquid drops for critical diameters of 0.5 mm, 1.5 mm, 2.5 mm and 3.5 mm were simulated. The weighted mean diameters for each critical diameter were used to find the mean free path between collector pellets of the mean diameter and all rain drops smaller than the critical diameter. Mean free paths were calculated using the average ice pellet diameter calculated for each critical diameter and precipitation rate. The results of the mean free path calculations are presented in Table 4-2. These results are also represented graphically in Figure 4-5.

The mean free path for collisions decreases as the critical diameter of the distribution increases. Also, as the precipitation rate increases, the mean free path decreases. The depth of the refreezing layer is shown in Figure 4-5 for comparison between the calculated mean free path and the average depth of the refreezing layer.

**Table 4-2 :** Mean free path for collisions between a collector pellet and all freezing rain drops smaller than the critical diameter. These are shown as functions of precipitation rate and critical diameter of the distribution.

| Precipitation rate | 1 mm/h | 2 mm/h | 5 mm/h | 10 mm/h | 25 mm/h |
|---|---|---|---|---|---|
| Critical diameter (mm) | Mean Free Path (m) | | | | |
| 0.5 | 3600 | 2800 | 2100 | 1700 | 1300 |
| 1.5 | 520 | 400 | 290 | 230 | 170 |
| 2.5 | 210 | 170 | 120 | 97 | 75 |
| 3.5 | 130 | 98 | 73 | 60 | 47 |

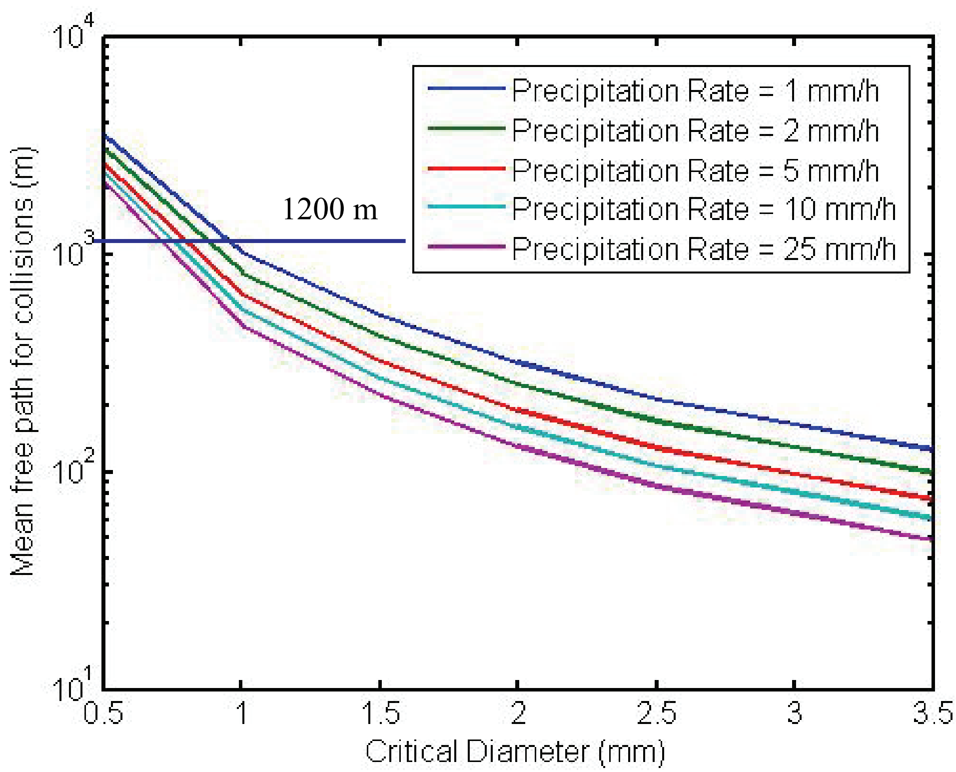


**Figure 4-5 :** Mean free path for collisions of ice pellets with freezing rain drops, depending on the critical diameter of the distribution and the precipitation rate. The blue horizontal line is the average height of the refreezing layer (1200 m) as observed by Zerr (1997).

The mean free paths for all precipitation rates are less than the depth of the refreezing layer if the critical diameter is ≤ 0.9 mm. As the critical diameter of the distribution increases, the number of semi-frozen particles for a given precipitation rate decreases and the number of freezing rain drops increases. The concentrations of ice pellets and freezing rain drops are shown in Figure 4-6 and Figure 4-7 as functions of both precipitation rate and critical diameter.

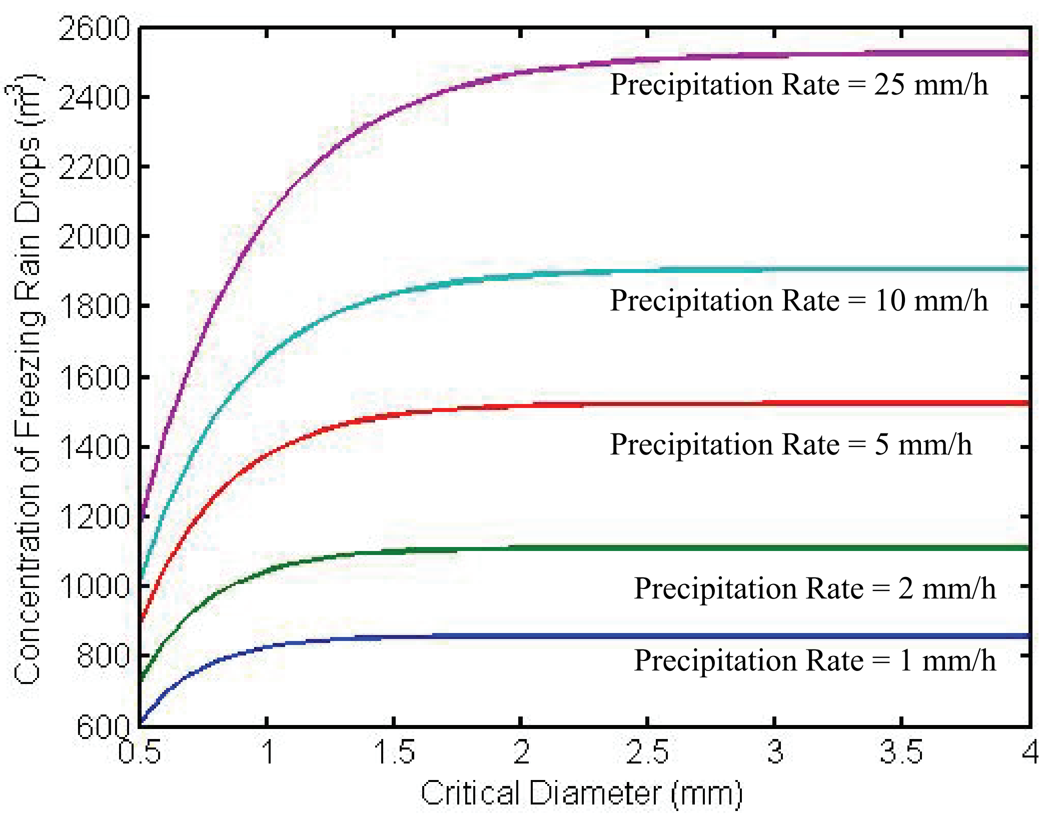


**Figure 4-6 :** The concentration of freezing rain drops per unit volume for five precipitation rates, and as a function of the critical diameter of the distribution.

The concentration of freezing rain drops increases sharply as critical diameter increases from its minimum value, but then plateaus as critical diameter continues to increase. The behaviour of the concentration of freezing rain drops is similar in varying precipitation rates, but plateaus at lower critical diameters in lower precipitation rates. High precipitation rates have greater concentrations of freezing rain drops than low precipitation rates. Since the size distribution in this study is exponential, with abundance sharply decreasing with size, the concentration of freezing rain drops increases as the critical diameter increases, but eventually plateaus. As the critical diameter increases, the number of additional freezing rain drops present for an incremental increase in critical diameter is very small.

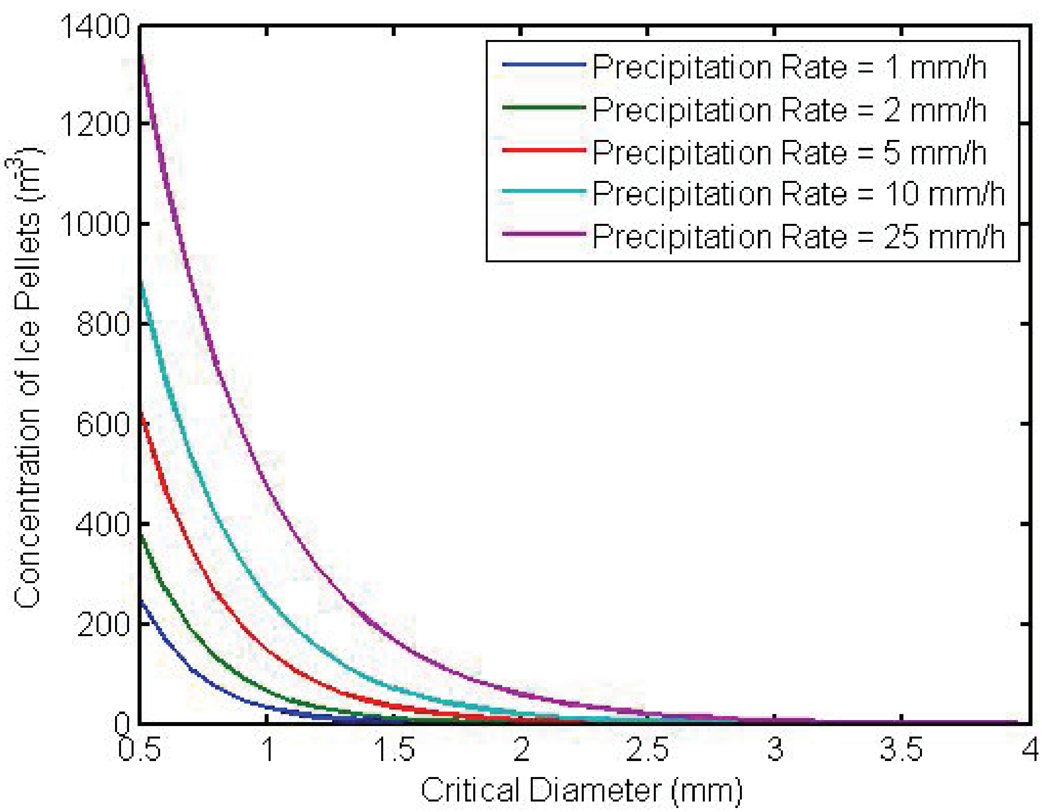


**Figure 4-7 :** The number of ice pellets (per unit volume) as a function of the critical diameter of the distribution for five precipitation rates.

The concentration of ice pellets sharply decreases as critical diameter increases, to almost no ice pellets present for distributions with large critical diameters. The concentration of ice pellets as a function of the critical diameter and precipitation rate has an opposite relation to the concentration of freezing rain drops. There are a certain number of particles in a unit volume for a given precipitation rate, and these are either categorized as ice pellets or freezing rain drops. Therefore the concentration of ice pellets and the concentration of freezing rain drops are inversely proportional; as one increases, the other must decrease by the same amount.

#### 4.3.1.1 Calculating Collisions Throughout the Depth of the Refreezing Layer

The number of collisions and therefore aggregate production for distributions with varying critical diameter depends on the mean free path for collisions and the depth of the refreezing layer. The total number of collisions throughout the depth, the decrease in freezing rain amount through collisions with ice pellets, and the total fraction of aggregates produced through these collisions were calculated. These calculations were carried out following the procedure outlined in Section 3.4.1.1. The decrease in freezing rain drops through collisions with ice pellets is presented in Figure 4-8 for critical diameters ranging from 0.5 mm - 4 mm.

Figure 4-8 shows that the decrease in freezing rain through collisions with ice pellets is a significant process in the atmosphere and can completely eliminate freezing rain in specific conditions. For distributions with a low critical diameter, in very large precipitation rates, freezing rain can be completely eliminated through collisions with ice pellets. As the precipitation rate decreases, for a given critical diameter, the potential for elimination of freezing rain decreases as well. As the critical diameter of the distribution increases, the decrease in freezing rain through collisions with ice pellets approaches zero.

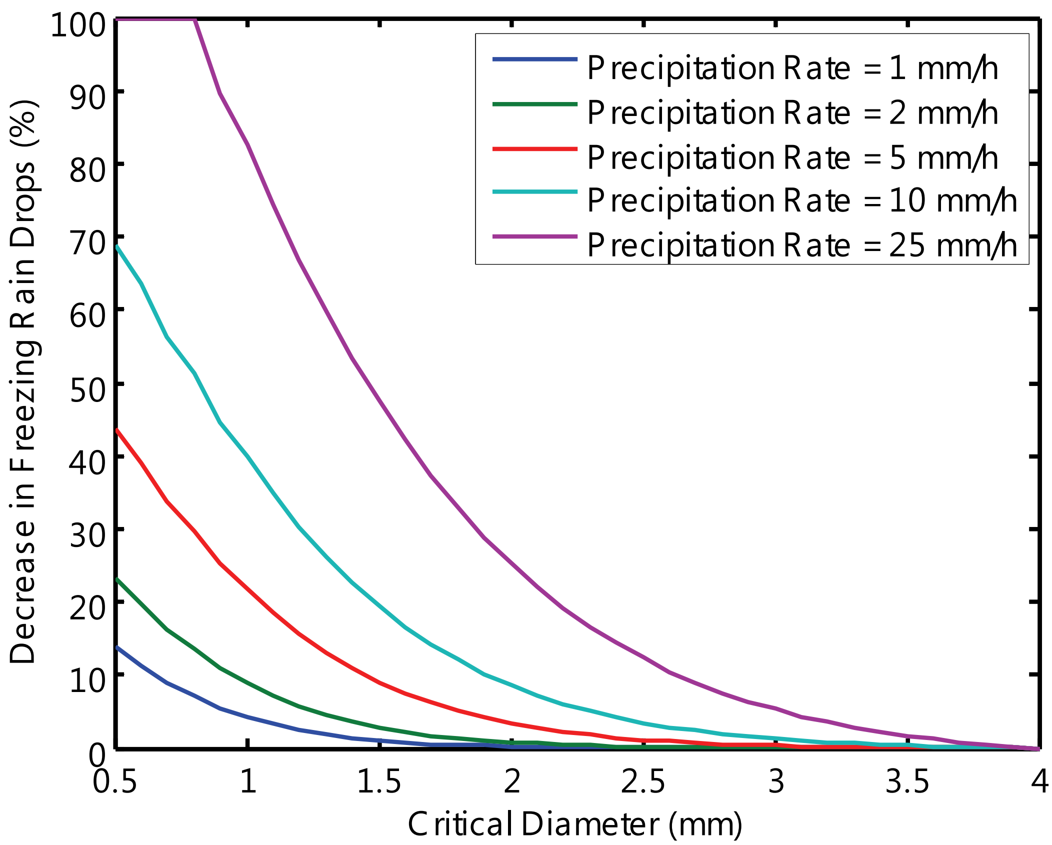


**Figure 4-8 :** The percent decrease in freezing rain for five precipitation rates, as a function of the critical diameter of the distribution of particles.

From Figure 4-8 it appears that the limiting parameter for the elimination of freezing rain is not the mean free path for collisions, but the number of ice pellets available for the scavenging of freezing rain drops. If the limiting parameter were the mean free path for collisions, there would be more scavenging of freezing rain drops in distributions with larger critical diameter, as mean free path decreases with increasing critical diameter. The determining factor for the scavenging of freezing rain drops is the relation between the number of ice pellets available to scavenge the freezing rain and the number of freezing rain drops available to be scavenged, not the mean free path for collisions. Scavenging of freezing rain by ice pellets is most effective when there is a large concentration of ice pellets to collide with the freezing rain drops and minimal numbers of freezing rain drops available to be scavenged. As critical diameter increases, the number of ice pellets decreases and the number of freezing rain drops increases. This is why as critical diameter reaches its maximum value, the decrease in freezing rain reaches its minimum value (approaches zero).

While the decrease in freezing rain is a very important value to examine, when comparing this mechanism with the other two proposed mechanisms for aggregate production, another measure must be used. In order to compare the efficiency of this mechanism with the others, when there is no freezing rain, the total aggregate production must be examined. The total number of particles that are aggregates upon reaching the ground was calculated and is shown in Figure 4-9.

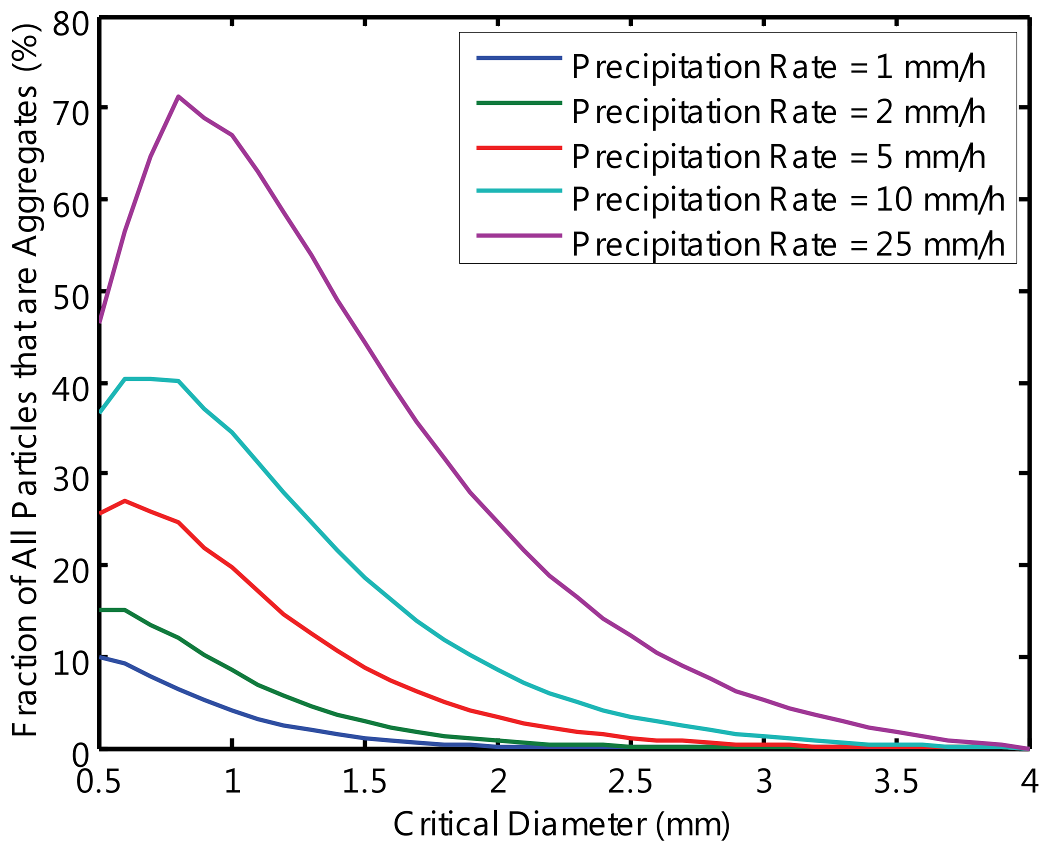


**Figure 4-9 :** The fraction of aggregates at the surface produced through collisions with freezing rain drops, depending on precipitation rate and critical diameter.

The results presented in Figure 4-9 show an interesting curve. The general trend is a decrease in aggregate production as critical diameter increases and precipitation rate decreases. However, the fraction of aggregates produced peaks at low critical diameters and the peak occurs at larger critical diameters as precipitation rate increases. The peak at low critical diameters is most likely due to the larger concentration of ice pellets as small critical diameters. Even after the ice pellets have scavenged all the rain the fraction of all particles that are aggregates is not as large as it would be at a slightly larger critical diameter. Because there are so many ice pellets for a critical diameter of 0.5 mm and a precipitation rate of 25 mm/h, even after all the freezing rain drops are scavenged, only ~50% of all particles are aggregates.

Aggregate production is strongly dependent on the precipitation rate, with much higher rates of aggregate production in larger precipitation rates. The fraction of aggregates produced ranges from a maximum production of 70% in a precipitation rate of 25 mm/h, to a maximum production of 10% aggregation in a precipitation rate of 1 mm/h. Aggregation efficiency decreases as critical diameter increases, similar to the calculations of the decrease in freezing rain shown earlier (Figure 4-8).

This outcome may occur because the mean free path between collisions is so large for small critical diameters. As the critical diameter of the distribution increases, the mean free path rapidly decreases, while the number of collector pellets decreases at a slower rate. Once the decrease in mean free path with increasing critical diameter is closer to a linear relationship on a log plot (when the critical diameter is larger than 1 mm) the effects balance out and then for larger critical diameters, the number of ice pellets present is the most important factor in aggregation efficiency. The percent of aggregates produced decreases to nearly zero for very large critical diameters.

#### 4.3.1.2 Recalculating Freezing Rain Amount and Average Ice Pellet Diameter at Height Intervals Through the Refreezing Layer

To account for the over- and under-estimations of the previous method, the number of freezing rain drops available for collisions and the average ice pellet diameter were recalculated at height intervals through the layer, not just calculated over the entire depth of the refreezing layer. Here the height intervals used are 300 m, so the concentration and average diameters are recalculated at elevations of 900 m, 600 m, 300 m and at the surface.

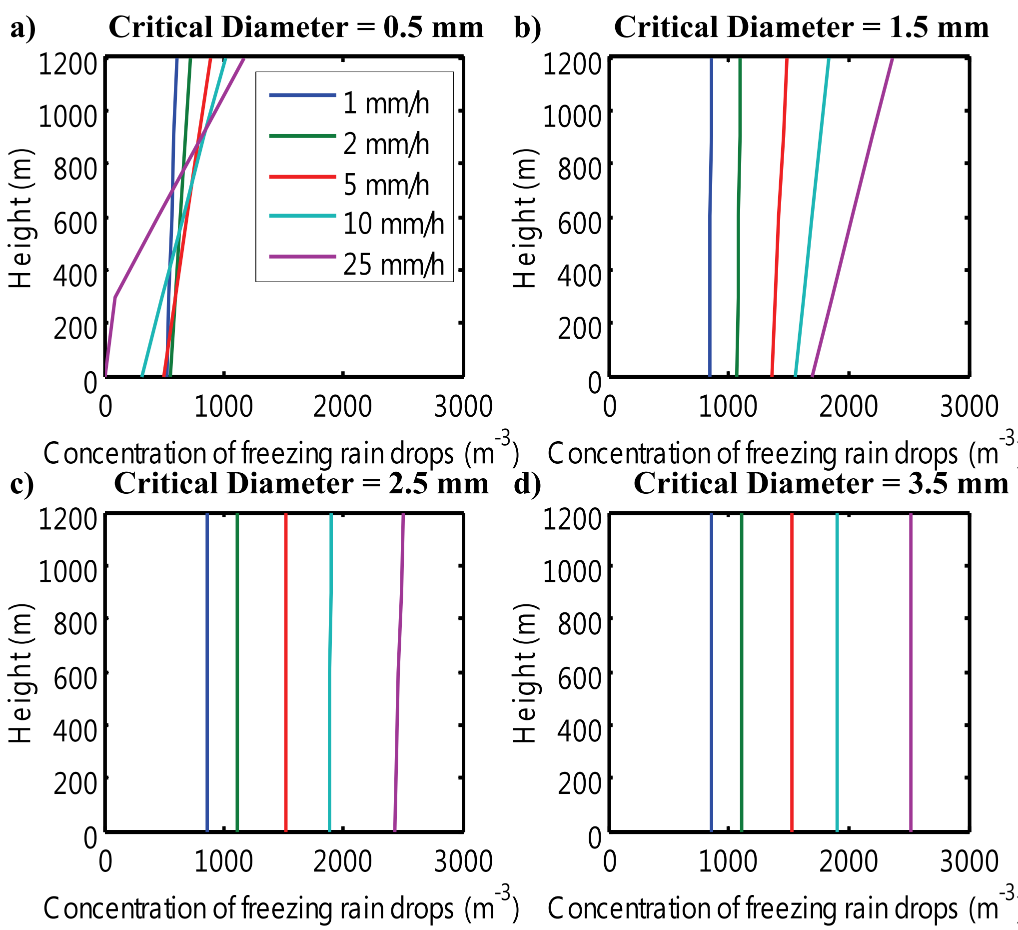


**Figure 4-10:** Concentration of freezing rain drops, as a function of precipitation rate, recalculated at 300 m height intervals through the refreezing layer. The individual plots are for distributions with critical diameters of a) 0.5 mm, b) 1.5 mm, c) 2.5 mm and d) 3.5 mm.

To determine how the concentration of freezing rain drops evolves during passage through the refreezing layer, the concentration of freezing rain drops was calculated at each height lever and is plotted as a function of height in Figure 4-10.

When the critical diameter of the distribution is small, the concentration of freezing rain drops decreases more notably with height than that for large critical diameters. As precipitation rate decreases, the decrease in freezing rain drops with height is smaller as well. For the largest critical diameters there is minimal reduction in freezing rain, which agrees with the results of the calculations of the decrease in freezing rain presented in Figure 4-8 and Figure 4-12.

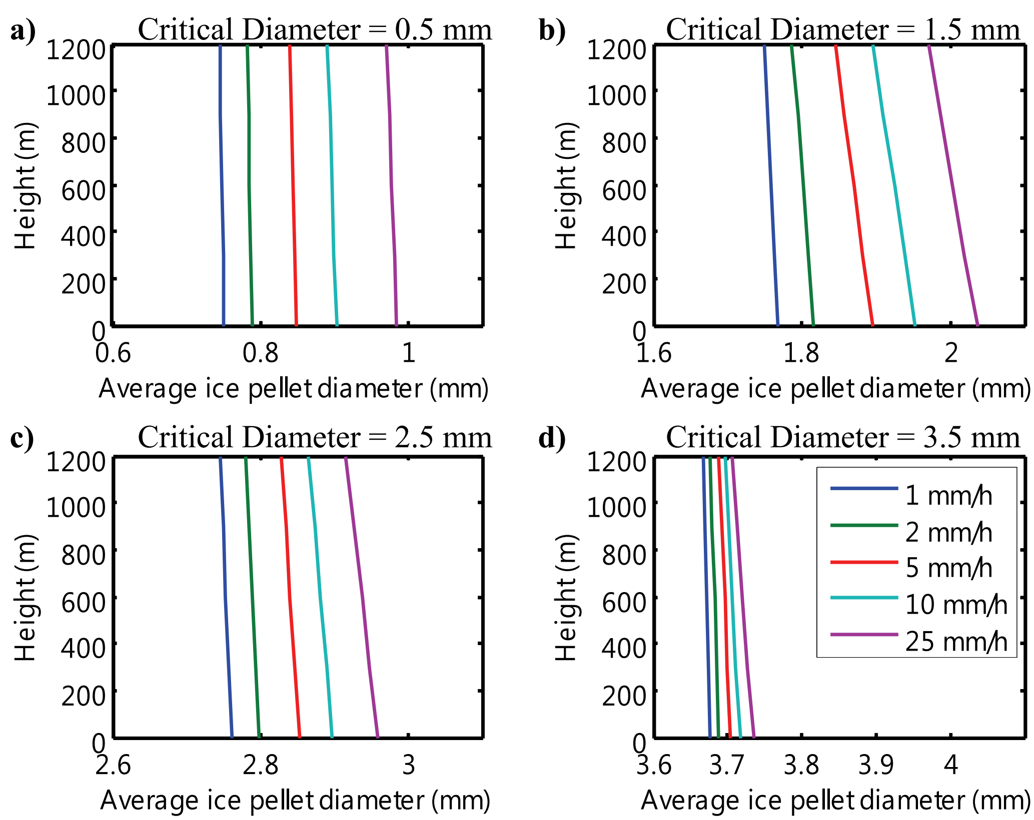


**Figure 4-11 :** Average ice pellet diameters recalculated through the depth of the refreezing layer as collisions occur with freezing rain drops. Average diameters vary depending on the precipitation rate and are shown for critical diameters of a) 0.5 mm, b) 1.5 mm, c) 2.5 mm and d) 3.5 mm.

In a distribution with a critical diameter of 0.5 mm the results are significantly different from cases with larger critical diameters. Figure 4-10 a) is the only plot in which the lines of freezing rain drop concentration for the different precipitation rates cross each other. The number of freezing rain drops at upper levels in precipitation rates higher than 10 mm/h fall off to concentrations at lower levels less than those for smaller precipitation rates. Ice pellets in environments with higher precipitation rates are more efficient at scavenging freezing rain drops than environments with lower precipitation rates. This is especially true when the distribution has a small critical diameter. As critical diameter increases, higher precipitation rates still show higher levels of freezing rain decrease, but the difference between precipitation rates is not as marked.

To examine the second modification that was made to the initial method for the determination of the decrease in freezing rain, the average diameter of ice pellets recalculated at height levels is presented in Figure 4-11.

Analogous to the plot of freezing rain concentration as a function of height fallen through the refreezing layer (Figure 4-10), the average ice pellet diameter increases more markedly in larger precipitation rates than in smaller precipitation rates. As precipitation rate increases, the number of freezing rain drops available for scavenging increases. However, unlike the behaviour of freezing rain concentration with height and critical diameter, the average ice pellet diameter increases most over the depth of the layer for distributions with a critical diameter of 1.5 mm. As critical diameter increases, the number of freezing rain drops that are available for collisions increases, while the number of collector pellets decreases. Ice pellets are able to scavenge more drops in distributions with low critical diameter (0.5 mm), because there are the most ice pellets at that critical diameter. However, the mass of scavenged freezing rain drops is distributed over a large number of ice pellets, so the average ice pellet diameter does not increase very much. As critical diameter increases, the number of freezing rain drops that are scavenged decreases, but the number of ice pellets among which the scavenged mass is distributed decreases as well. The same number of scavenged rain drops will have a larger effect on the average ice pellet diameter than it would in distributions with smaller critical diameter.

The results for the decrease in freezing rain amount when recalculating the number of freezing rain drops and average ice pellet diameter at 300 m height intervals through the refreezing layer are presented in Figure 4-12.

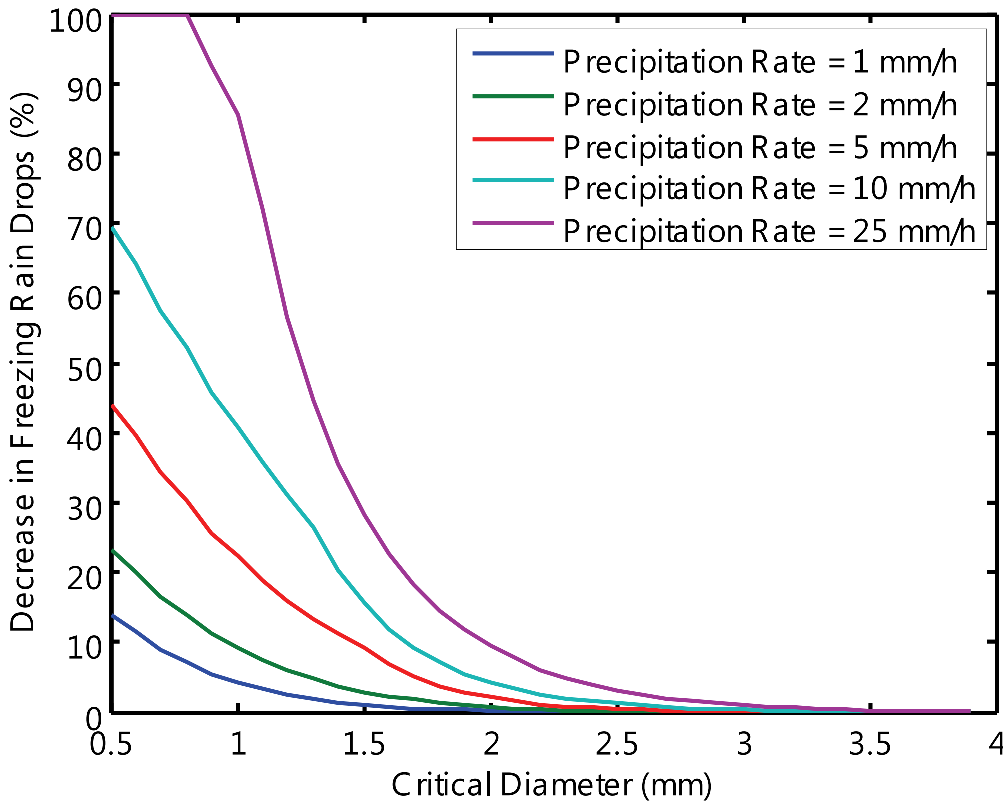


**Figure 4-12 :** The decrease in freezing rain at the surface through collisions with ice pellets as a function of critical diameter and precipitation rate. In this case, the number of freezing rain drops and average ice pellet diameter were recalculated at height intervals of 300 m.

The decrease in freezing rain is still significant even when the number of freezing rain drops is recalculated at height levels through the refreezing layer. The results obtained by recalculating parameters at height intervals show similar results to those calculated only once through the depth of the refreezing layer. To more closely examine the differences between the results obtained by the different methods, the two are plotted together in Figure 4-13.

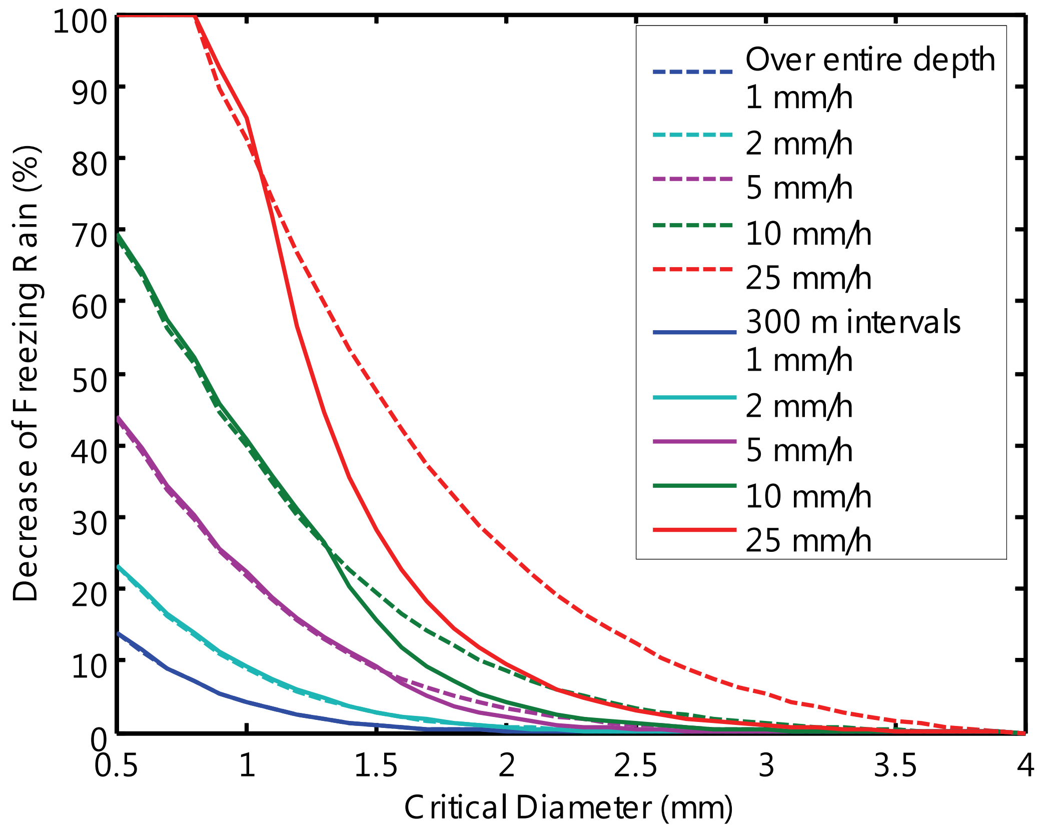


**Figure 4-13 :** The percent decrease in freezing rain at the surface through collisions of ice pellets with freezing rain drops aloft as a function of critical diameter and precipitation rate. The dashed lines show the decrease when freezing rain amount is only calculated once over the depth of the refreezing layer (results of Figure 4-8). The solid lines show the decrease obtained when the number of freezing rain drops and average ice pellet diameter are recalculated at height intervals of 300 m (results of Figure 4-12).

As critical diameter increases, the spread in results for the two calculation methods increases. When decrease is calculated only once over the entire depth, the decrease in ZR is generally greater than when parameters are recalculated at height intervals. The difference in the results is greater for large critical diameters. For small precipitation rates the percent decrease calculated by the two methods is very similar. As the precipitation rate increases the spread in the values calculated by using the two methods increases. The percent decrease in freezing rain approaches zero with increasing critical diameter faster when the number of freezing rain drops and average ice pellet diameter is recalculated at height intervals than when the decrease is calculated through the entire depth of the layer.

In order to compare the results obtained in this section with those that will be presented in later sections focussing on aggregation by the other two mechanisms, the percent of aggregates needed to be calculated, not just the percent decrease in freezing rain. Here, the percent of aggregates is the number of aggregates at the bottom of the layer divided by the total number of original particles in the distribution (both ice pellets and freezing rain drops). The results of this calculation are shown in Figure 4-14.

The data show a curve similar to that obtained by the previous method. The general trend is a decrease in aggregate production as critical diameter increases and precipitation rate decreases. In a precipitation rate of 25 mm/h in a distribution with a critical diameter of 0.5 mm, ~ 50% of particles will be aggregates. As the critical diameter increases, the percent of aggregates produced increases to ~ 70% at a critical diameter of 0.8 mm. As critical diameter continues to increase aggregate production rapidly decreases.

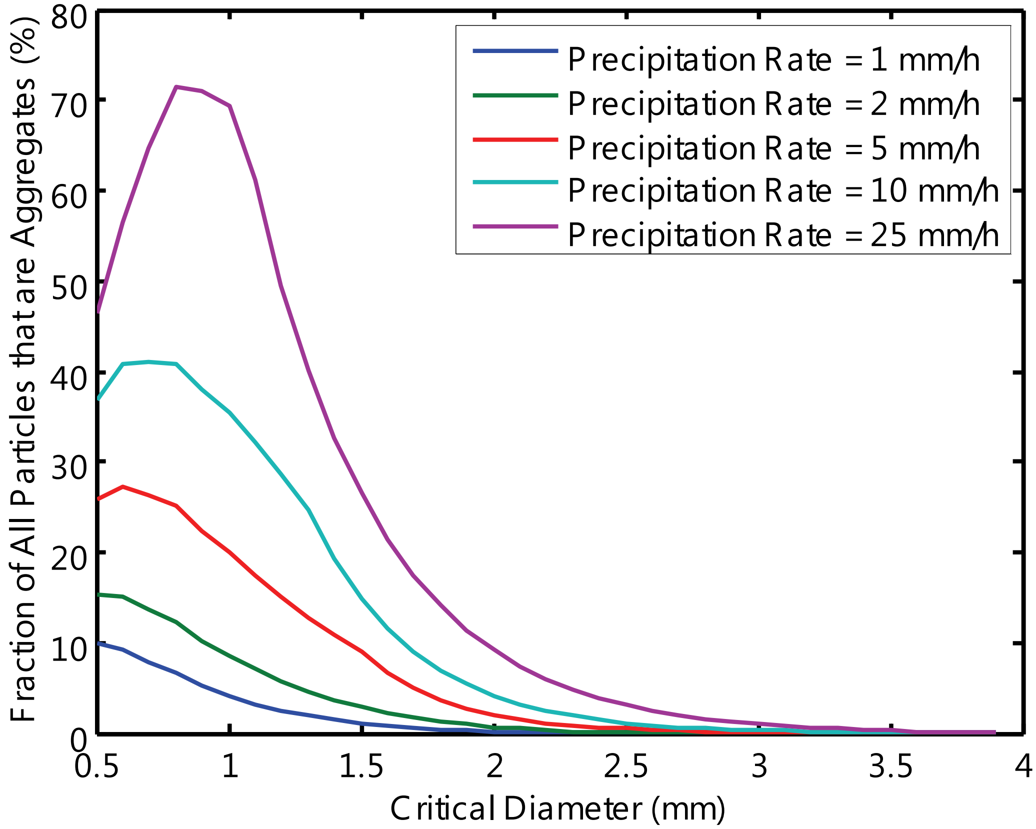


**Figure 4-14 :** The fraction of aggregates produced at the surface through collisions with freezing rain drops aloft as a function of critical diameter and precipitation rate. In this case, the number of freezing rain drops and average ice pellet diameter were recalculated at height intervals of 300 m.

These results are also similar to those calculating the percent of aggregates at the surface if only calculated once through the depth of the refreezing layer, but like the percent decrease in freezing rain results show some differences. To better examine the differences between the results obtained by the different methods, the two are plotted (as in Figure 4-13) together in Figure 4-15.

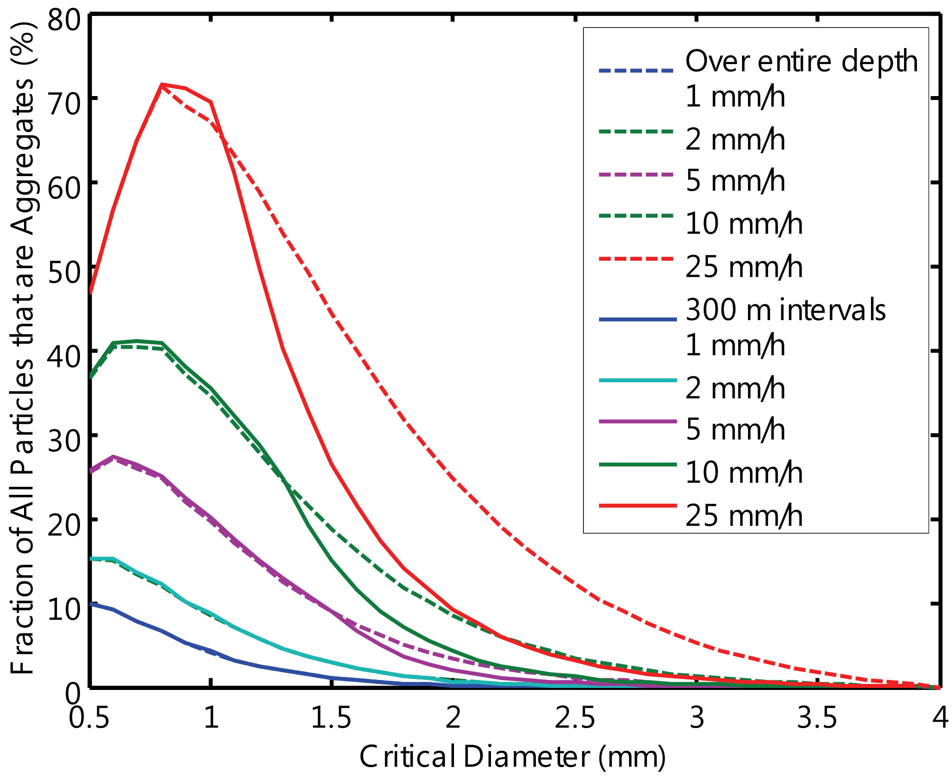


**Figure 4-15 :** The fraction of aggregates at the surface produced through collisions of ice pellets with freezing rain drops aloft as a function of critical diameter and precipitation rate. The dashed lines show the decrease when freezing rain amount is only calculated once over the depth of the refreezing layer (results of Figure 4-9). The solid lines show the decrease obtained when the number of freezing rain drops and average ice pellet diameter are recalculated at height intervals of 300 m.

As in the comparison figure for decrease in freezing rain (Figure 4-13), the results of the two methods are similar for small critical diameters and small precipitation rates and their differences increase as both parameters increase. The percent of aggregates produced decreases more sharply with increasing critical diameter when the number of freezing rain drops and average ice pellet diameter is recalculated at height intervals than when it is calculated over the entire depth.

#### 4.3.1.3 Recalculating $N_0$, Freezing Rain Amount and Average Ice Pellet Diameter at Height Intervals Through the Refreezing Layer

The results from the third and final method of calculating the percent decrease of freezing rain and also the percent of aggregates produced will now be presented. This is the method in which not only the average ice pellet diameter and number of freezing rain drops was recalculated at height intervals through the refreezing layer. The y-intercept, $N_0$, of the exponential size distribution was recalculated as well.

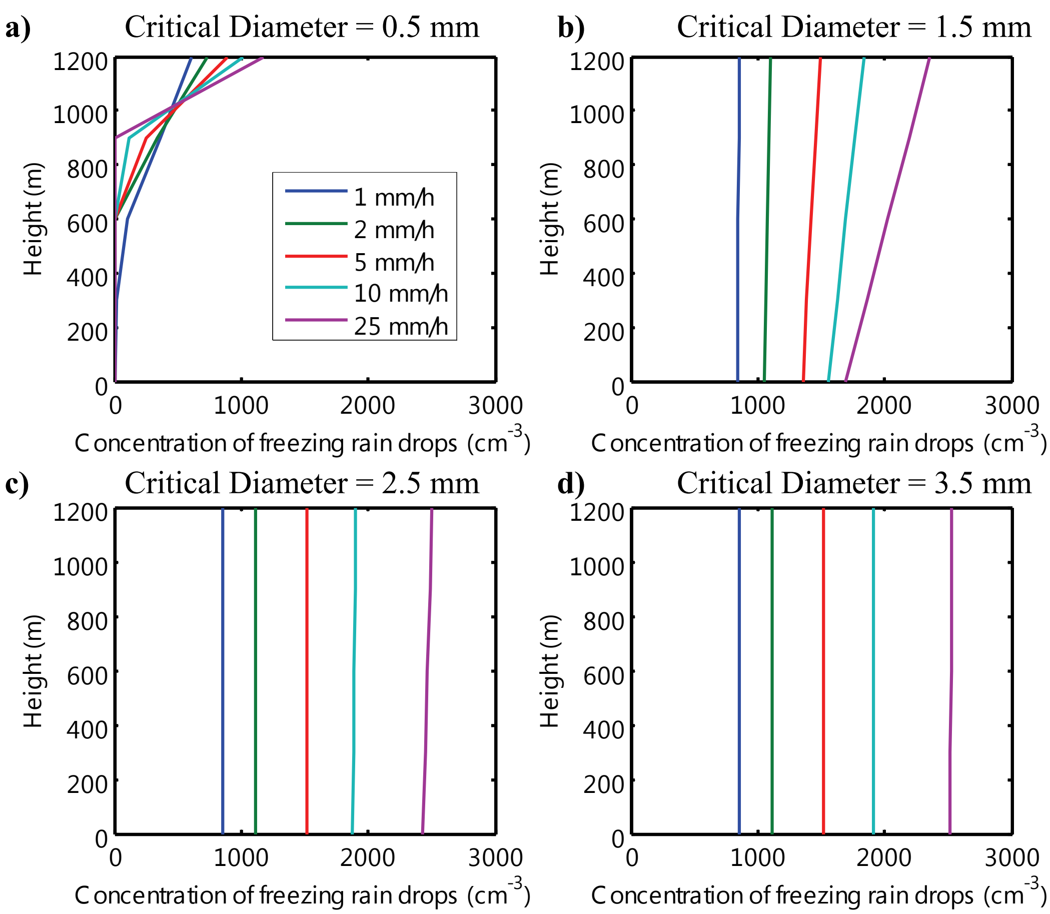


**Figure 4-16 :** The concentration of freezing rain drops with height when $N_0$ is recalculated at 300 m height intervals through the refreezing layer. The concentrations depend on precipitation rate and are shown for distributions with critical diameters of a) 0.5 mm, b) 1.5 mm, c) 2.5 mm and d) 3.5 mm.

When $N_0$ is recalculated at each height level, the mean free path for collisions increases, because the number of available particles for collisions is reduced. The number of freezing rain particles at each height level was calculated (as in Section 4.3.1.2) and is shown in Figure 4-16. Similar to the results of the previous section, decrease in freezing rain amount is most pronounced in distributions with small critical diameters. There are more ice pellets available to scavenge the freezing rain drops at low critical diameters, and fewer freezing rain drops to be scavenged.

When the critical diameter is 0.5 mm, the results are quite different from those when $N_0$ was not recalculated at each level. However, when the critical diameter is larger than 0.5 mm, the differences are not as significant. The number of freezing rain drops decreases much faster (when the critical diameter is 0.5 mm) when $N_0$ is recalculated at intervals through the depth. For distributions with a critical diameter of 0.5 mm, freezing rain is completely eliminated before reaching the surface for all precipitation rates.

The average ice pellet diameter as a function of height was also plotted (as in Section 4.3.1.2) and these are shown in Figure 4-20. The average ice pellet diameters increase more with distance fallen for distributions with a large critical diameter than they increase in distributions with a low critical diameter. This is because in distributions with large critical diameters, there are fewer ice pellets, so their average diameter is affected more by collisions with freezing rain drops than it would be for lower critical diameters. At low critical diameters there are more ice pellets to collide with freezing rain. Though more freezing rain drops are collected for low critical diameters, the average ice pellet diameter is not as strongly affected because the collected mass of freezing rain is spread over more pellets.

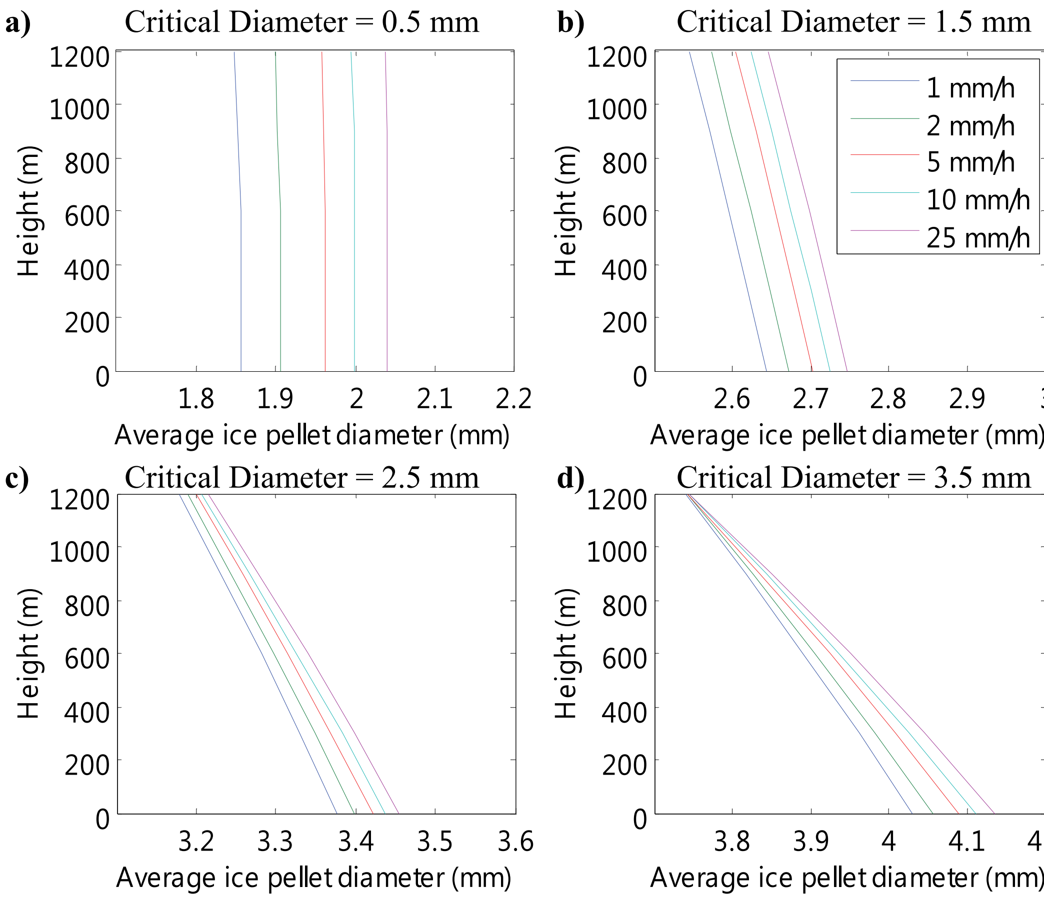


**Figure 4-17 :** Average ice pellet diameters with height when $N_0$ is recalculated at 300 m height intervals through the refreezing layer. Average diameters depend on precipitation rate and are shown for distributions with critical diameters of a) 0.5 mm, b) 1.5 mm, c) 2.5 mm and d) 3.5 mm.

$N_0$ values at each height level were also plotted to observe their change with height in the refreezing layer and this is shown in Figure 4-18. Values of $N_0$ were calculated at each height level for each of the four critical diameters in the same manner as freezing rain amount and average ice pellet diameters.

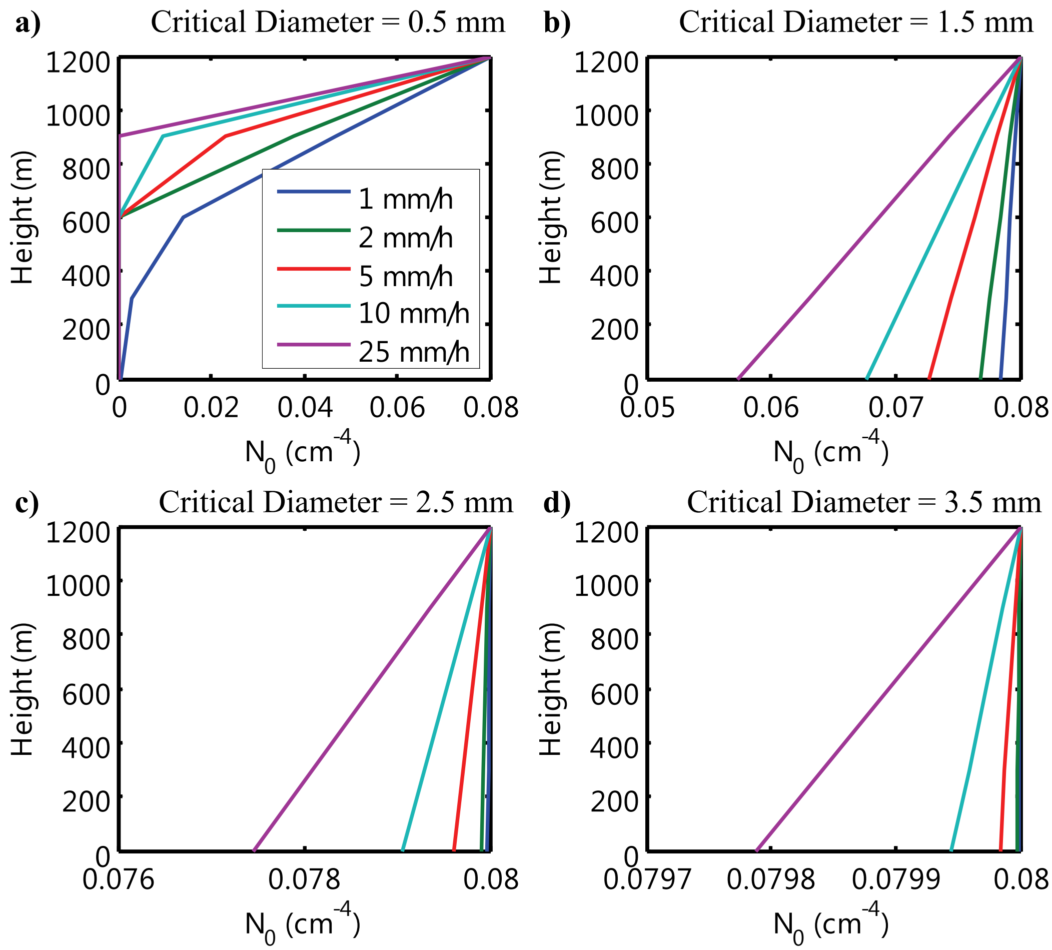


**Figure 4-18 :** $N_0$ values for the evolving exponential size distribution of freezing rain drops as functions of height and precipitation rate as they are scavenged by collisions with ice pellets. $N_0$ values are shown for distributions with critical diameters of a) 0.5 mm, b) 1.5 mm, c) 2.5 mm and d) 3.5 mm.

The recalculation of $N_0$ at each height level shows that $N_0$ does decrease significantly and, at low critical diameter, reaches zero before the bottom of the refreezing layer in all precipitation rates. The decrease in $N_0$ with height is more drastic for low critical diameters. For a distribution with the lowest critical diameter (0.5 mm) the decrease in $N_0$ values through the depth of the refreezing layer is 100% in all precipitation rates. The maximum change in $N_0$ when the critical diameter is 1.5 mm is ~25% (for a precipitation rate of 25 mm/h). When the critical diameter is 3.5 mm, the maximum difference in $N_0$ vales through the refreezing layer is only ~ 2%.

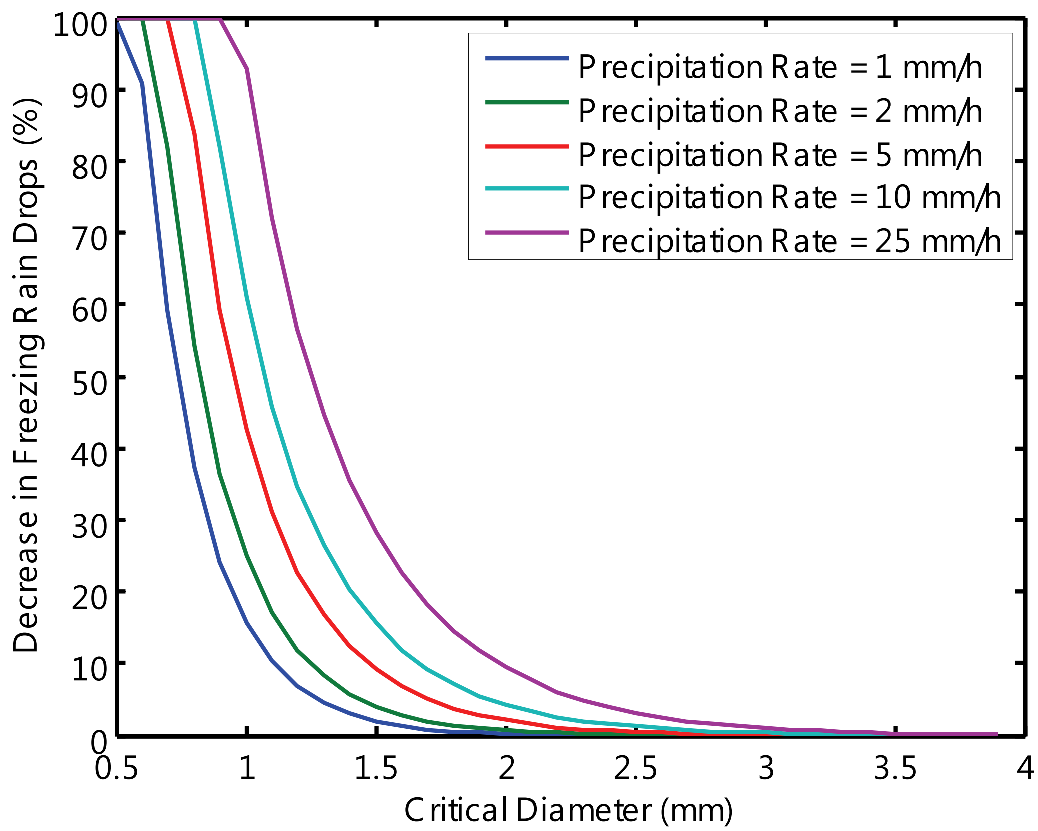


**Figure 4-19 :** The decrease in freezing rain drops at the surface through collisions with ice pellets aloft as a function of critical diameter and precipitation rate. The number of freezing rain drops, average ice pellet diameter and $N_0$ of the exponential size distribution were recalculated at height intervals of 300 m through the depth of the refreezing layer.

As for the other methods, the decrease in freezing rain at the surface was calculated for five precipitation rates as a function of critical diameter and the results of these calculations are shown in Figure 4-19. These results differ significantly from the results obtained by the first two methods of calculating the decrease in freezing rain amount (Figure 4-13). In distributions with low critical diameters when $N_0$ is recalculated at height intervals, complete elimination of freezing rain is achieved for all precipitation rates. As in the other results, scavenging of freezing rain decreases as critical diameter increases and is larger for higher precipitation rates.

As in the previous sections, the fraction of all particles that are aggregates was calculated as well and the results of these calculations are shown in Figure 4-20.

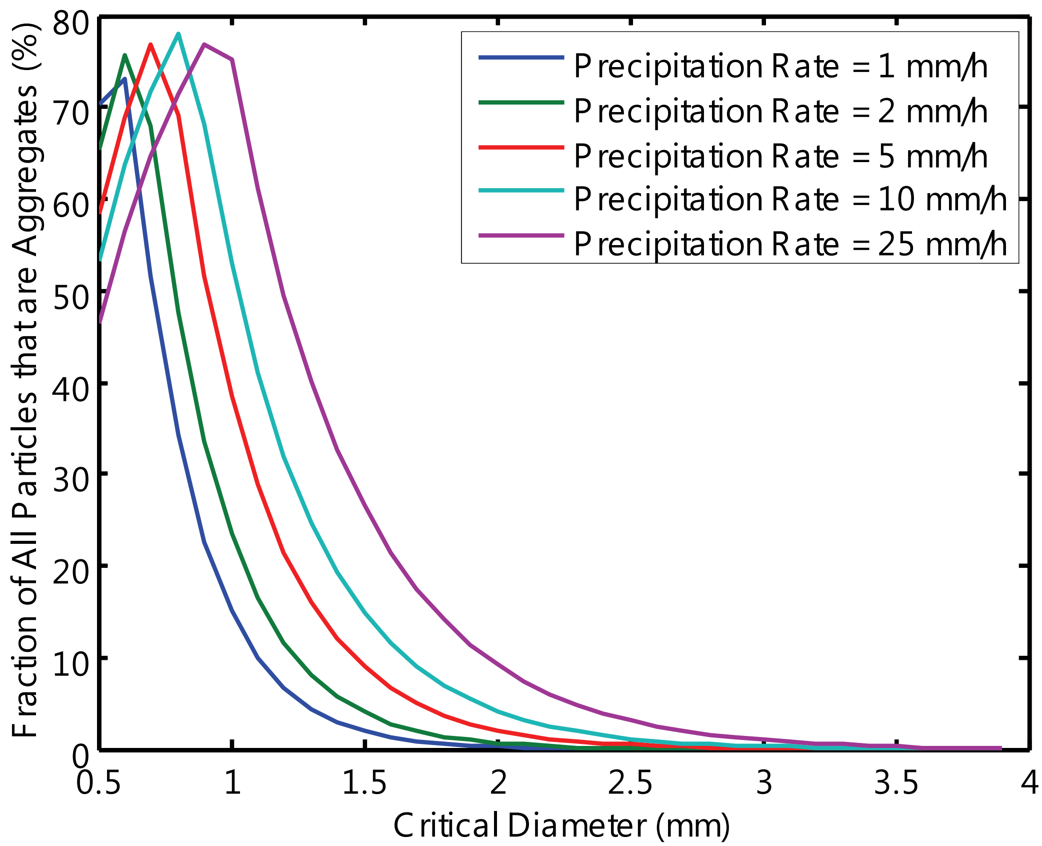


**Figure 4-20 :** The fraction of aggregates produced at the surface through collisions of ice pellets with freezing rain drops aloft as a function of precipitation rate and critical diameter. The number of freezing rain drops, average ice pellet diameter and $N_0$ of the exponential size distribution were recalculated at height intervals of 300 m through the depth of the refreezing layer.

The results presented in Figure 4-20 differ notably from the results from the other two calculation methods when $N_0$ was a constant throughout the refreezing layer (Figure 4-15). The maximum values of aggregate production are roughly the same for all three calculation methods (~70%). However, unlike for the other methods, where this maximum aggregate production is only achieved for precipitation rates of 25 mm/h, this method of calculation achieves this maximum value for all precipitation rates. Similar to the results of the previous sections, the fraction of aggregates produced decreases as the critical diameter of the distribution increases and as the precipitation rate decreases.

### 4.3.2 Aggregation by Collisions Cracking the Ice Shells on Liquid-Core Ice Pellets

In Section 4.2, the existence of liquid-core particles was justified by the freezing times of semi-frozen particles in typical environmental profiles. Liquid-core pellets should commonly occur at the surface in the given environmental conditions. Now that the existence of liquid-core ice pellets has been justified, the zones in which they can occur need to be examined. Since the threshold thickness of an ice pellet shell was assumed to be 10% of its diameter, the zone in which liquid-core ice pellets occur with shells thin enough to crack and create aggregates upon collision is the zone in which liquid-core ice pellets exist with shells thinner than 10% of their diameter. The heights above which liquid-core ice pellets exist with shells thinner than 10% of their diameter are shown in Figure 4-21, dependent on minimum temperature of the layer and size of pellet. Collisions that occur above this height level, for a given $T_{min}$ and diameter of pellet will have the possibility to produce an aggregate.

Since the initial ice fraction used here was minute (just enough to initiate freezing in an otherwise liquid drop), these lines are the minimum height at which liquid-core pellets could exist with shells thin enough to crack upon impact. If initially there were more ice in the particles, the depth over which this aggregate formation mechanism occurs will be even smaller.

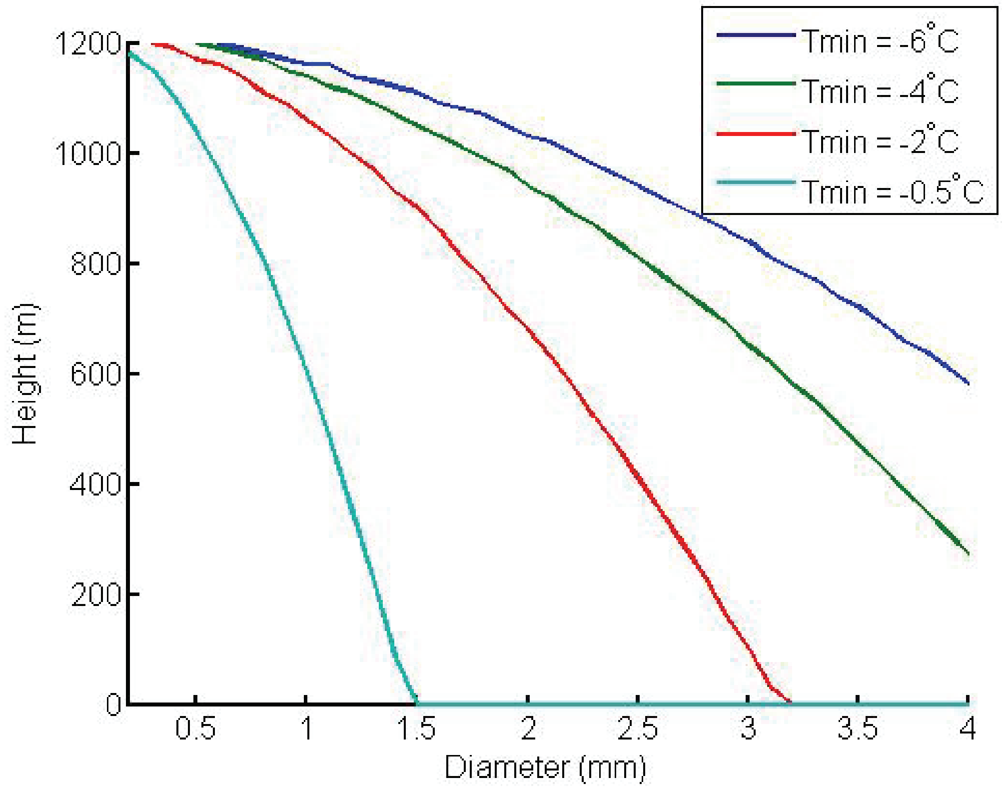


**Figure 4-21 :** The height above which the shells of liquid-core pellets are thin enough to crack upon collision with another pellet. These are calculated as a function of particle diameter in environments with four values of $T_{min}$ (-0.5°C, -2°C, -4°C, -6°C).

If collisions between pellets have the potential to create aggregates, they must occur within the depth in which thin-shelled liquid-core pellets exist. From examination of Figure 4-21, for an environment with a minimum temperature of -0.5°C (slow freezing rates), all particles with diameters above 1.5 mm will have shells sufficiently thin to crack upon collisions through the entire depth of the refreezing layer. The average depth of a refreezing layer during an ice pellet event is consistent with the depths over which we will find liquid-core pellets. The average depth of the refreezing layer is generally larger than the mean free path of collision. Collisions with liquid-core pellets will occur, resulting in aggregation by this mechanism.

For this mechanism of aggregation another limit was placed on the collisions that produce aggregates. Since in these collisions it is assumed that it is the larger pellet that will crack upon impact, there is a minimum diameter ($D_{min}$) threshold for a collision to produce an aggregate by this mechanism. Since there is a deficit in the literature on collisions between ice pellets and little mention even of the existence of liquid-core pellets, the value of this threshold had to be hypothesized. The threshold minimum size of the smaller pellet for the shell of the larger pellet to crack was decided to be 40% of the diameter of the larger pellet. Smaller pellets will not have sufficient force to crack the larger shell. Following the procedure outlined in Section 3.4.2 the fraction of aggregates produced by this mechanism was calculated for environments with four minimum temperatures and five precipitation rates and the results are presented in Table 4-3.

**Table 4-3 :** The fraction of aggregates produced by collisions cracking the shells on liquid-core ice pellets. This is calculated for five precipitation rates in environmental temperature profiles with four values of $T_{min}$.

| $T_{min}$ | Precipitation rate | | | | |
|---|---|---|---|---|---|
| | 1 mm/h | 2 mm/h | 5 mm/h | 10 mm/h | 25 mm/h |
| | Fraction of Aggregates Produced (%) | | | | |
| -6°C | 0.2 | 0.3 | 0.5 | 0.8 | 1.3 |
| -4°C | 0.2 | 0.4 | 0.7 | 1.2 | 2.0 |
| -2°C | 0.4 | 0.7 | 1.4 | 2.2 | 3.8 |
| -0.5°C | 1.7 | 2.8 | 5.2 | 8.0 | 13.3 |

The values in Table 4-3 show that aggregation through collisions cracking the ice shells on thin-shelled liquid-core pellets is not as important in the atmosphere as aggregation by collisions with freezing rain drops. The combination of the small depth available for collisions that will cause aggregation and the long mean free paths of collision because of the minimum diameter restriction, means that this aggregation mechanism is not as effective at creating aggregates as the first mechanism. The maximum fraction of aggregates produced by this mechanism is 13% in an environment with a minimum temperature of -0.5°C and for a precipitation rate of 25 mm/h. This is much smaller than the maximum fraction of aggregates produced by the first mechanism (70% for a precipitation rate of 25 mm/h).

To determine whether the minimum diameter threshold chosen for liquid-core aggregates correlated with observations, photographs were examined of aggregates that appear to have been formed by collisions with liquid-core pellets. The photographs examined were those taken by Steve Gibson on November 4th, 2003 (Gibson and Stewart, 2007). A comparative analysis of these photographs was completed, revealing that the largest diameter difference in component particles of the aggregates was roughly 75%.

If a minimum diameter threshold of 75% were used instead of 40%, the number of aggregates produced by this mechanism would be even smaller. So, given that these results are maximized in various ways and still show much lower aggregation efficiency than the previous mechanism, this mechanism is not considered to be as important a process for aggregation as that by collisions with freezing rain.

### 4.3.3 Aggregation by the Freezing of the Surface Water Films on Ice Pellets

The first step in examining collisions between two completely frozen pellets was to decide on a threshold difference in the size, and therefore kinetic energy, of the two pellets in order for aggregation to occur. If the difference in kinetic energies of the two pellets is larger than this threshold, the smaller pellet will bounce away from the collector pellet upon collision and will not form an aggregate. If the kinetic energy difference is less than the threshold, there will be sufficient time for the liquid-water films to freeze together and an aggregate will be formed. It is assumed that there is no temperature dependence on the stickiness of ice pellets in the range of temperatures in this study. As shown in Figure 2-5, when the environment is saturated with respect to ice, the force required to separate two ice spheres is substantial for all temperatures in the range of this study (0°C to -6°C).

The minimum smaller pellet diameter that will form an aggregate by liquid water film freezing depending on collector pellet diameter was computed using both kinetic energy difference thresholds (as explained in Section 3.4.3 and shown in Figure 3-10). The mean free path between collisions of completely frozen pellets was calculated for the entire range of collector pellet sizes (0.5 mm – 4 mm) colliding with all pellets with diameters between the collector pellet diameter, and the minimum smaller pellet diameter as shown in Figure 3-9. The mean free path for collisions that will result in aggregation was plotted as a function of collector pellet diameter for five precipitation rates. The results obtained for a kinetic energy threshold equal to the kinetic energy of the smaller pellet are shown in Figure 4-22.

The mean free path lines exhibit an upward curve on the semi-logarithmic plot. The distance that a collector pellet will fall before creating an aggregate increases greatly as the size of the collector pellet increases. The values for mean free path in different precipitation rates are similar for small collector pellets. The difference in mean free path in different precipitation rates increases as the size of the collector pellets increases. For cases with larger precipitation rates, the minimum value of mean free path does not occur at the point when the collector pellet is a minimum. In the case for a precipitation rate of 25 mm/h, the minimum mean free path occurs for collector pellets of 1.2 mm diameter, not 0.5 mm.

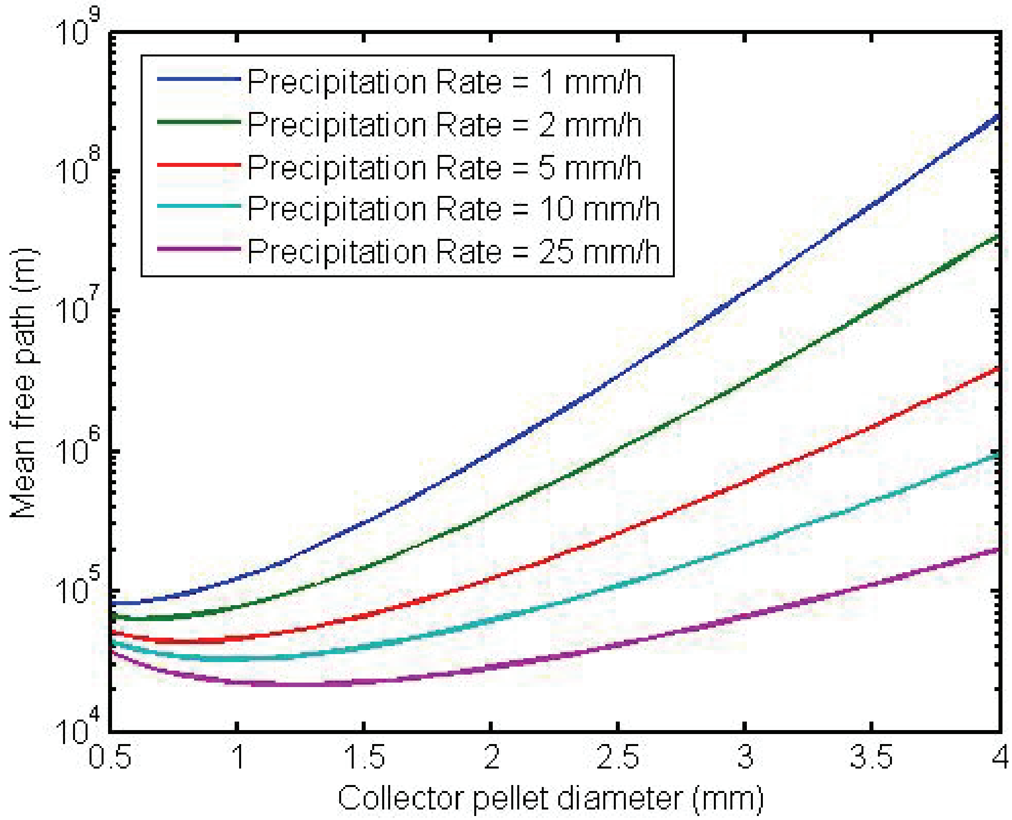


**Figure 4-22 :** The mean free path for collisions between completely frozen pellets that will create an aggregate for five precipitation rates. This is the mean free path for a collector pellet and a smaller pellet with diameter not less than the minimum diameter as shown in Figure 3-9.

Mean free path is not solely affected by differences in the terminal velocities of the component particles, but also on their abundance, so this curve is justified. Although the spread between the collector pellet diameter and the minimum diameter possible for aggregation increases as the size of the collector pellet increases, the abundance of particles decreases exponentially with increasing collector pellet size. The potential decrease in mean free path because of a greater range of possible diameters for collisions is overshadowed by the increase in mean free path that occurs with the decreasing abundance of particles as diameter increases. All of the mean free paths of collisions between pellets are larger than the average depth of the refreezing layer (1200 m), so collisions between pellets within the prescribed diameter range are unlikely within the depth of the refreezing layer. Therefore aggregates produced by this mechanism will be unlikely within the average depth of the refreezing layer.

The aggregation results were recalculated using a higher kinetic energy threshold to determine if the probability of collisions resulting in aggregation would increase. The maximum kinetic energy difference threshold between the components for collisions resulting in aggregation was increased to twice the kinetic energy of the smaller pellet. Using the minimum diameters calculated and shown in Figure 3-10, the mean free paths of collisions that will produce aggregates by this mechanism were calculated and are shown in Figure 4-23.

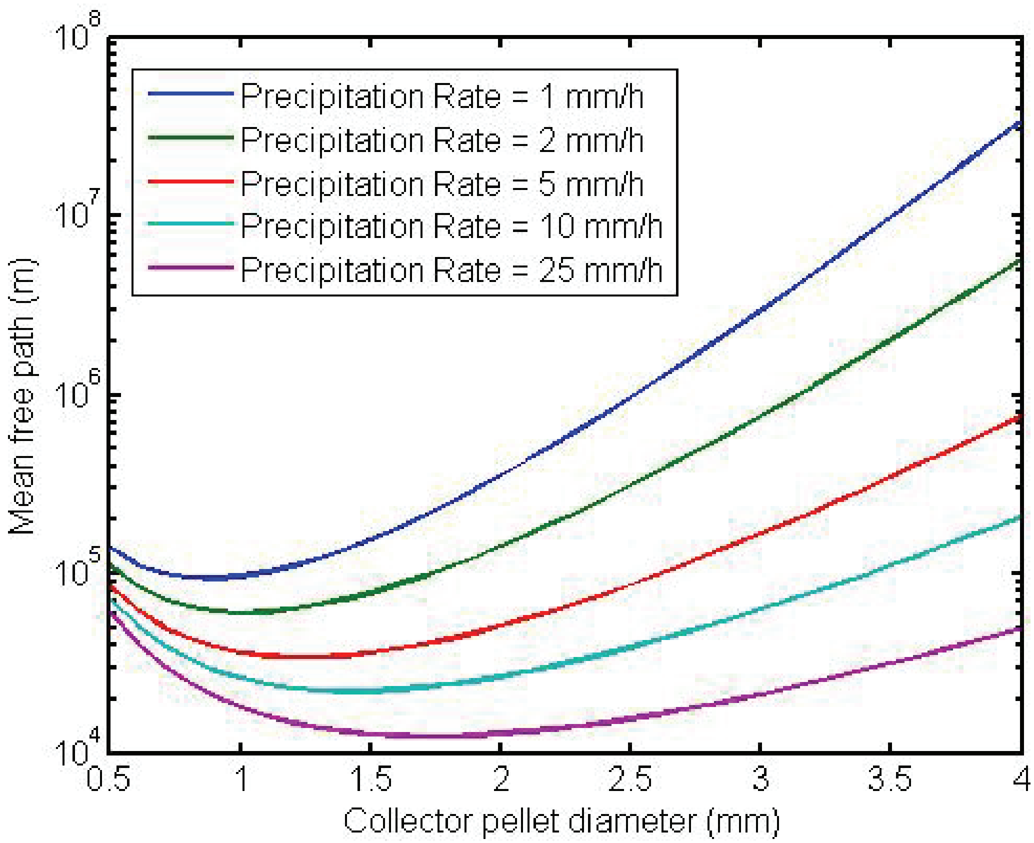


**Figure 4-23 :** The mean free path for collisions between completely frozen pellets that will create an aggregate for five precipitation rates. This is the mean free path for a collector pellet and a smaller pellet with diameter not less than the minimum diameter as shown in Figure 3-9.

The mean free paths calculated using the greater threshold are all still greater than the average depth of the refreezing layer (1200 m). Even though the range of possible diameter combinations is larger for the larger kinetic energy threshold, the mean free paths calculated are all still larger than the average depth of the layer. For this threshold, the lines of mean free path for collisions producing an aggregate have a larger dip, and minimum values of mean free path fall at diameters greater than the minimum collector pellet diameter for all precipitation rates.

Increasing the threshold kinetic energy for collisions to produce aggregates decreased the mean free path for larger collector pellet diameters, but increased the mean free path for collisions for small collector pellet diameters. Using the mean free paths calculated for the two threshold cases, the number of collisions occurring throughout the depth of the refreezing layer was calculated. This is assumed to be the number of aggregated particles that will reach the surface. The percent of ice pellets that reach the surface as aggregates was then calculated following the procedure outlined in Section 3.4.3. The fraction of aggregates produced at the surface for different precipitation rates is shown for both kinetic energy thresholds in Table 4-4.

The maximum fraction of aggregates produced by this mechanism is much less than the maximum aggregation achieved by either of the other two mechanisms. As in the results for aggregation by the other mechanisms, precipitation rate increases aggregate production. Aggregation efficiency is greater when the threshold kinetic energy difference of collisions that will produce an aggregate is raised to twice the kinetic energy of the smaller pellet.

If the kinetic energy threshold is twice that of the smaller pellet, this mechanism produces a larger fraction of aggregates than liquid-core aggregation in environments with $T_{min} < -3°C$ for all precipitation rates. These rates of aggregate production are low, so this mechanism may not be important in the atmosphere. However, Stewart and Crawford (1995) suggested that a number of the ice pellet aggregates they observed were probably formed by this process because of the distinct component particles connected by a very narrow ice neck. Perhaps the kinetic energy thresholds used in these calculations are too high.

**Table 4-4 :** Percentage of aggregates formed by the freezing of surface liquid water films on ice pellets. Results are shown for five precipitation rates with two kinetic energy thresholds for collisions to produce an aggregate.

| Kinetic Energy threshold (Minimum diameter threshold) | Precipitation rate | | | | |
|---|---|---|---|---|---|
| | 1 mm/h | 2 mm/h | 5 mm/h | 10 mm/h | 25 mm/h |
| | Fraction of All Particles that are Aggregates (%) | | | | |
| Max ΔKE of collision = 2xKE of small pellet ($D_{min}$= 0.76$D_L$) | 0.4 | 0.6 | 1.1 | 1.7 | 3.0 |
| Max ΔKE of collision = KE of small pellet ($D_{min}$= 0.84$D_L$) | 0.1 | 0.2 | 0.5 | 0.7 | 1.2 |

Since both of the maximum kinetic energy difference thresholds were chosen without any quantitative knowledge of what actually occurs during a collision between ice spheres, an analysis of photographs of aggregates that appeared to have been formed by this process was undertaken. The photographs taken by Steven Gibson in the storm of November 4$^{th}$, 2003 were examined. The relative diameters of the component particles in aggregates that appear to have been formed by the freezing together of surface water films were measured. The largest difference in component size was a smaller pellet with a diameter ~40% of the larger pellet's diameter. Calculations of mean free path for collisions that would produce aggregates were made using this minimum diameter relation. Mean free paths were calculated for collisions between collector pellets and all pellets with diameters less than the collector pellet, down to a minimum smaller pellet diameter 40% of the collector pellet. As with the other diameter/kinetic energy thresholds, the mean free path was used to calculate the number of favourable collisions in the distribution and the fraction of aggregates that are produced by this mechanism.

**Table 4-5 :** Fraction of aggregates produced by the freezing of surface liquid water films on ice pellets. Results are shown for five precipitation rates with a minimum diameter threshold for collisions to produce an aggregate of 40% of the collector pellet diameter, $D_L$.

| Minimum diameter threshold | Precipitation rate | | | | |
|---|---|---|---|---|---|
| | 1 mm/h | 2 mm/h | 5 mm/h | 10 mm/h | 25 mm/h |
| | Fraction of All Particles that are Aggregates (%) | | | | |
| $D_{min}= 0.4D_L$ | 4.1 | 6.4 | 11.3 | 17.1 | 27.0 |

The aggregate efficiency when the minimum diameter is 40% of the collector pellet is much higher than the results obtained when the threshold minimum diameter is larger. The results in

Table 4-5 show that this aggregation mechanism is indeed important in the atmosphere and produces a larger fraction of aggregates for all precipitation rates than aggregation by liquid-core ice pellet aggregation in any environmental temperature profile.

# Chapter 5

## Discussion and Synthesis of Results

Three predicted mechanisms for aggregation have been investigated here. The first mechanism is aggregation through collisions of ice pellets and freezing rain drops in which the freezing rain drop freezes to the surface of the ice pellet. The second proposed mechanism is an aggregate formed upon collision of a liquid-core pellet and another pellet in which the shell of the liquid-core pellet breaks and the internal water freezes the two pellets together. The third proposed mechanism is aggregation by surface liquid water films freezing together when two pellets collide with a small kinetic energy of collision.

A detailed study of the formation of aggregates of ice pellets as well as the conditions under which these types of particles occur was carried out. Ice pellet aggregates sometimes occur and sometimes do not. This chapter addresses key aspects of the aggregational process by each of the three proposed mechanisms based on the results presented in the previous section. A discussion of the impacts of precipitation rate on all of the three mechanisms is presented in Section 5.1. Section 5.2 examines the theoretical threshold for the elimination of freezing rain through collisions with ice pellets. The effects of varying the initial ice fraction in the distribution of particles upon entry into the refreezing layer are discussed in Section 5.3. Section 5.4 is comprised of a comparison of the aggregation efficiencies of the three mechanisms.

### 5.1 Precipitation Rate Impacts

Precipitation rates in actual ice pellet events are often much lower than most of those used in these simulations. The high precipitation rates (up to 25 mm/h) were used to determine if the mechanisms could be forced to give large values of aggregation. Precipitation rates in freezing rain and ice pellet events are usually on the order of 1 mm/h. In a study of freezing rain ice loads, Jones (1998) noted that the hourly precipitation rate in freezing rain storms varied from less than 0.5 mm/h up to 7 mm/h. However, the average precipitation rate was about 1 mm/h. These results were obtained from 169 storms over 44 years of meteorological data at Springfield, Illinois (Jones, 1998).

The precipitation rate affects the number of particles in a unit volume and thereby affects the efficiency of all three mechanisms of aggregation. The mean free paths and therefore the number of collisions that occur in a distribution of precipitation particles is dependent on precipitation rate. As precipitation rate increases, the mean free path decreases, and this decrease in mean free path is more pronounced as the diameter of the collector pellet increases. This affects collision probability throughout the depth of the layer.

For the first mechanism, the maximum fraction of aggregates produced is ~70% for all precipitation rates. The peaks in aggregate production occur at larger critical diameters for higher precipitation rates. As critical diameter increases, the fraction of aggregates produced is greater for larger precipitation rates. In the other two mechanisms, larger precipitation rates produce significantly more aggregates. For the second mechanism, aggregate production for a precipitation rate of 1 mm/h falls to ~10% of the aggregate production for a precipitation rate of 25 mm/h. For the third mechanism, aggregate production for a precipitation rate of 1 mm/h falls to 15% (using $D_{min}$= 0.4 $D_L$) of its value for a precipitation rate of 25 mm/h.

## 5.2 Threshold for the Elimination of Freezing Rain

There are three factors affecting the overall decrease in freezing rain through collisions with ice pellets: the number of ice pellets present; the number of freezing rain drops present; and the mean free path for collisions between the two. Of these three factors, the number of collector pellets present, rather than the mean free path between collisions is the variable limiting the number of collisions, and therefore also in decreasing the amount of freezing rain.

For a distribution of precipitation particles with a critical diameter of 0.5 mm, freezing rain is completely eliminated through collisions with ice pellets for all precipitation rates. For a population of precipitation particles with a critical diameter of 1 mm, the decrease in freezing rain falls from ~95% for a precipitation rate of 25 mm/h to ~15% for a rate of 1 mm/h. If the critical diameter is 1.5 mm, the decrease in freezing rain through collisions with ice pellets falls from ~30% for a precipitation rate of 25 mm/h to ~2% in a 1 mm/h precipitation rate.

For distributions with a critical diameter of 0.5 mm (with average collector pellet diameter of 0.8 mm) the mean free path for collisions is larger than the depth of the refreezing layer. The mean free path for distributions with small critical diameters is higher than that for distributions with large critical diameters because the terminal velocities of collector and smaller particle are closer to the same value. Also, there will not be as many freezing rain particles to collide with as critical diameter decreases. These factors may to conclusions that there would be more collisions when the pellets are on average larger. However, the number of ice pellet particles decreases so sharply as critical diameter increases, that the small number of ice pellets present in distributions with low critical diameters overwhelms these factors. Decrease in freezing rain amount by ice pellet scavenging is strongly correlated with number of ice pellets present, decreasing markedly as critical diameter increases.

## 5.3 Initial Ice Fraction for Differing Melting Layers Aloft

This study only examines the behaviour of particles in the refreezing layer above the surface. The particles that enter the refreezing layer could have traversed a melting layer of any depth and strength. Based on the critical diameter of the distribution of particles, the liquid water fraction of the particles can be calculated using the following equation (adapted from Szyrmer and Zawadzki, 1999):

$$Water\ Fraction = \left(\frac{d^*}{D}\right)^{1.3} \tag{5-1}$$

Where d* is the critical diameter of the distribution and D is the diameter of the particle. The fraction of water is one when the diameter is equal to the critical diameter; the particle is all water. The water fraction in a semi-frozen particle was calculated for particles over the range of diameters in distributions with seven critical diameters and is shown in Figure 5-1.

Alternately, these data may be presented as the ice fraction in particles, similar to the organization of the results presented in earlier sections (Section 4.2) and the results are presented in Figure 5-2.

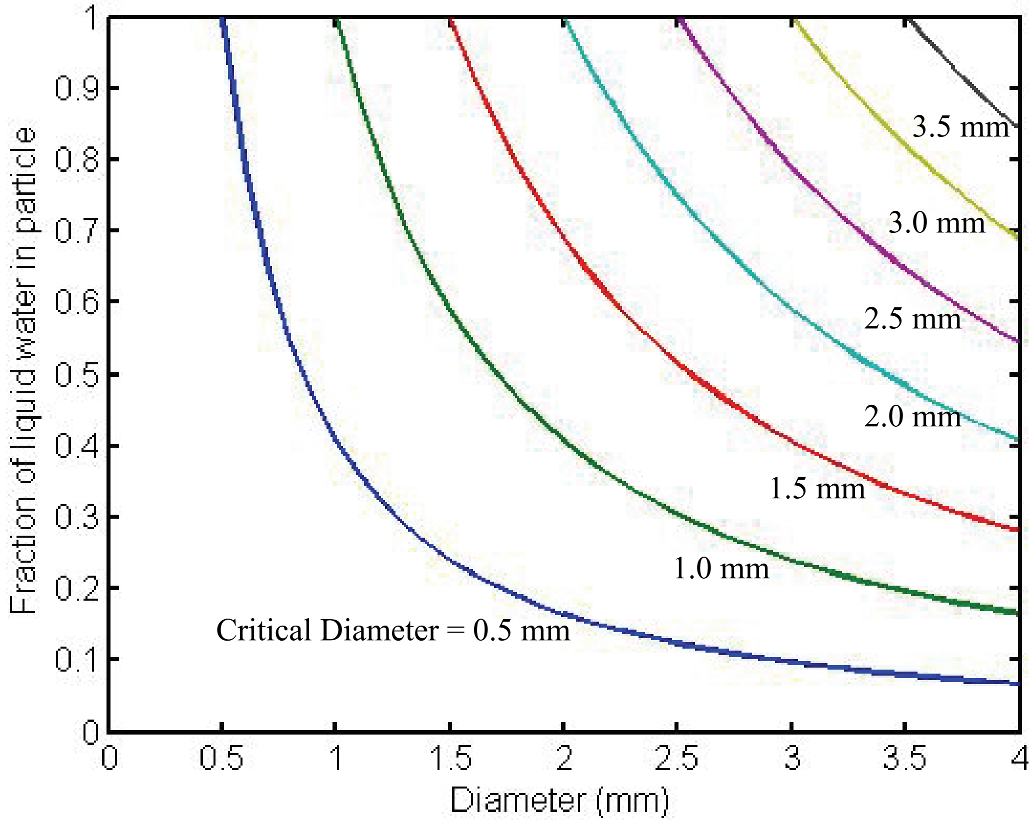


**Figure 5-1 :** Liquid water fraction in particles as a function of particle diameter. The liquid water fraction is shown for distributions with seven critical diameters.

The ice fraction data shows that as critical diameter of the distribution increases, the minimum diameter of a particle that contains ice correspondingly increases. When the diameter is equal to that of the critical diameter, there is a minute fraction of ice in the particle. As the diameter of the particle increases, the fraction of ice in the particle increases as well. For distributions with small critical diameters (≤ 2 mm), the ice fraction increases rapidly then plateaus, showing a resemblance to a power curve. For distributions with larger critical diameters (> 2 mm), the curve is closer to a linear relationship, increasing as diameter of the pellet increases.

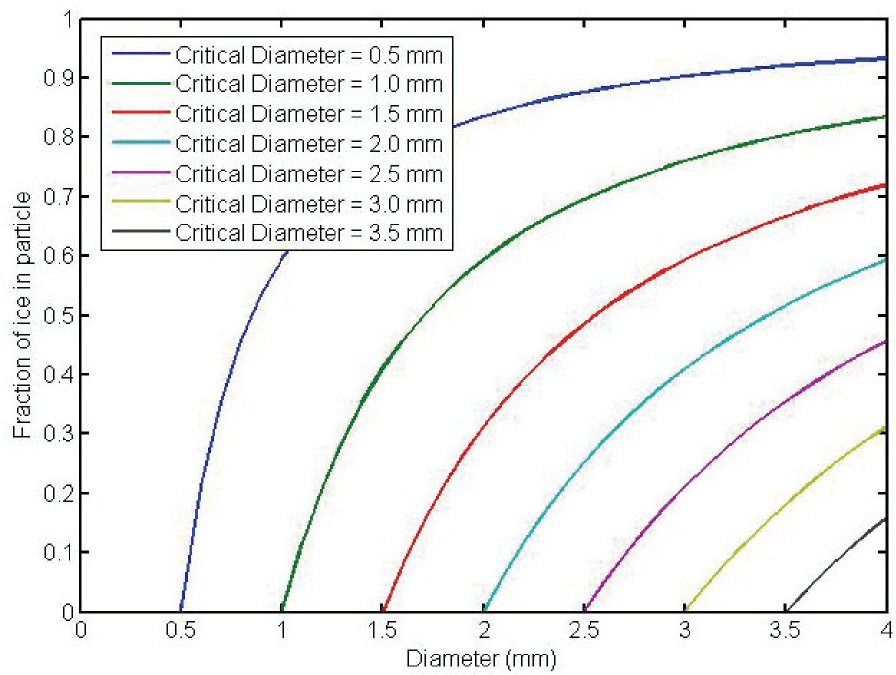


**Figure 5-2**: The fraction of ice in particles as a function of particle diameter. The ice fraction is shown for distributions with seven critical diameters.

Originally, simulations of the freezing and collisions between particles were meant to be re-run using this ice fraction data as the initial ice fraction in liquid-core pellets upon entry into the refreezing layer. The results using a varying initial ice fraction were then meant to be compared with the results obtained using a minute initial ice fraction in all pellets. However, if the initial fraction of ice were increased, the only mechanism of aggregation that would be affected would be aggregates formed by the cracking of liquid-core ice pellets. The efficiency of aggregation by collisions with freezing rain would not be affected because the thickness of the shell or the fraction of ice in the pellet does not matter in this aggregation mechanism, just that the particle is frozen and is falling as an ice pellet (whether its core be liquid or not). Also, the initial fraction of ice in the pellets would not affect aggregation by the freezing together of the surface water films on ice pellets, because liquid water films exist on completely frozen pellets as well as on liquid-core pellets so the fraction of ice in the pellet does not affect the ability of this mechanism to create aggregates.

Since aggregation by the cracking of liquid-core pellets is the only aggregation mechanism that will be affected by varying the initial ice fraction, the simulations were not re-run with varying initial ice fractions. Aggregation by this mechanism is not significant when all factors are maximized (minute initial ice fraction, $T_{min}$ near 0°C, large kinetic energy/minimum diameter threshold). If the initial ice fraction were varied, the fraction of aggregates produced by this mechanism would be even lower. Larger pellets would have more ice in them, meaning that it would take less time to completely freeze, less time for their shells to reach 10% thickness, so less opportunity for aggregation. Also, if the critical diameter were higher, aggregation production would be much lower. As critical diameter increases, the number of ice pellets available for collisions exponentially decreases. For these reasons the simulations were not carried out with a varying initial ice fraction.

## 5.4 Relative Importance of the Three Proposed Mechanisms

Each aggregation mechanism has its limits. Aggregates formed by collisions with freezing rain are without limits on the depth of their occurrence or on the size of the particles involved, but exhibit significant dependence on the amount of ice pellets present, and therefore the critical diameter of the distribution. Also, the lower terminal velocity for a pellet compared to that for a freezing rain drop of the same size makes it more difficult for collisions among similar sized particles to occur. This method of aggregation is more efficient, or occurs with a higher frequency than aggregation occurring through collisions with liquid-core ice pellets. Aggregates formed by collisions with liquid-core pellets can only occur within the depth of the refreezing layer that contains liquid-core pellets with shells thin enough to crack upon collision. Collisions resulting in aggregation by liquid water film freezing can occur throughout the entire depth of the refreezing layer, so they have potential to occur with higher frequency than collisions with liquid-core pellets.

There are no limits imposed on the size of the particles that can form aggregates by the first mechanism, unlike the size restrictions on the component particles of the other two mechanisms. Liquid-core ice pellet aggregates have size restrictions imposed on the component particles that will create an aggregate upon collision. The smaller pellet in the collision must not be below a minimum diameter or the larger shell will not crack. Aggregates formed by liquid water film freezing are also limited by the size of the particles involved, but can occur throughout the entire depth of the refreezing layer, like aggregates through collisions with freezing rain drops. In their case, the force of the collision must not be above a certain threshold or the pellets will bounce apart instead of aggregating.

To compare the differences in the aggregation efficiency of the three mechanisms, the conditions necessary for each mechanism to produce a fraction of aggregates of 5%, 25% and 50% were examined and these results are presented in Table 5-1.

Of the three mechanisms, aggregation through collisions with freezing rain drops has the greatest potential for creating aggregates. However, for this aggregation mechanism to occur, freezing rain must be occurring with the ice pellets. For aggregation by this mechanism to be important (>25%) the critical diameter of the distribution must be $\leq$ 1.5 mm for a precipitation rate of 25 mm/h or $\leq$ 1 mm for a precipitation rate of 2 mm/h.

**Table 5-1 :** The conditions necessary for the three aggregation mechanisms to produce a given fraction of aggregates at the surface. R is the precipitation rate and $T_{min}$ is the minimum temperature in the refreezing layer. The depth of the refreezing layer used in these calculations is 1200 m and the maximum precipitation rate is considered to be 25 mm/h. If the fraction of aggregates for a mechanism does not reach the given value within the range of the study, the conditions box shows N/A.

| **Conditions Necessary to Produce Given Fraction of Aggregates** | | | |
|---|---|---|---|
| **Critical Diameter** | 5% | 25% | 50% |
| | **Mechanism 1: Collisions with Freezing Rain Drops** | | |
| 0 mm | N/A | N/A | N/A |
| 0.5 mm | $R \geq$ 0 mm/h | $R \geq$ 0 mm/h | 0 mm/h $\leq R \leq$ 20 mm/h |
| 1.0 mm | $R \geq$ 0 mm/h | $R \geq$ 2 mm/h | $R \geq$ 9 mm/h |
| 1.5 mm | $R \geq$ 3 mm/h | $R \geq$ 25 mm/h | N/A |
| 2.0 mm | $R \geq$ 15 mm/h | N/A | N/A |
| 2.5 mm | N/A | N/A | N/A |
| **Mechanism 2: Cracking of the Ice Shells on Liquid-Core Ice Pellets** | | | |
| 0 mm | $T_{min} \geq$ -0.5°C<br>$R \geq$ 5 mm/h | N/A | N/A |
| **Mechanism 3: Freezing of the Surface Water Film on Ice Pellets** | | | |
| 0 mm | $R \geq$ 1.5 mm/h | $R \geq$ 21 mm/h | N/A |

# Chapter 6

## Summary and Concluding Remarks

Three aggregation mechanisms were examined. These were aggregates formed by: collisions between ice pellets and freezing rain drops; collisions between ice pellets and liquid-core pellets; and collisions between ice pellets and other ice pellets. In the first mechanism, aggregation is achieved due to rapid freezing of freezing rain drops upon collision with ice pellets. In the second mechanism aggregation occurs when the collision between pellets cracks the shell of a thin-shelled liquid-core pellet, allowing some internal liquid water to escape and freeze the two pellets together. The third mechanism achieves aggregation by the liquid water films on the surface of both the ice pellets freezing the two pellets together upon collision.

A numerical study of ice pellet aggregation has been carried out to model the behaviour of falling particles. Freezing of particles from an initial state with a minute ice fraction was examined, as well as collisions between particles that create ice pellet aggregates. The simulations in this study were carried out in an environment with a refreezing layer 1200 m deep, with linearly varying environmental temperature and pressure profiles. Aggregation of particles was examined in environmental temperature profiles with four different minimum temperatures. The critical diameter of the population of particles varied as well and aggregation was examined for these values. A summary of the research findings will be presented and conclusions drawn from these results. Also, limitations of the study due to significant gaps in current research will be discussed.

## 6.1 Optimal Conditions for Aggregation

The three proposed mechanisms for aggregation of ice pellets occur over different zones. The mechanism involving collisions with supercooled raindrops can occur over the entire depth of the refreezing layer. Aggregation resulting from the collision of ice pellets (liquid-core or entirely frozen ice pellets) with liquid-core pellets occurs over a shorter depth. The depth for this mechanism is that within which liquid-core pellets exist with ice shells thinner than the threshold thickness for cracking. This is the maximum ice shell thickness that will be able to crack upon collision and cause aggregation. Aggregates formed by the third proposed mechanism can occur throughout the entire depth of the refreezing layer, because ice pellets occur throughout the layer. The three aggregation mechanisms examined in this study experience their maximum potential for aggregate production in different conditions and these optimal conditions for aggregation are examined below.

### 6.1.1 Aggregation by Collisions with Supercooled Liquid Drops

Ice pellet aggregation can be an important process in the atmosphere. Reduction of freezing rain through collisions with ice pellets can reach levels of complete elimination of freezing rain. However, the conditions for this to occur are precise. For complete elimination of freezing rain through collisions with ice pellets, the critical diameter of the distribution must be quite low ($\leq$ 1 mm). The critical diameter is the diameter above which all particles contain some ice and below which all particles are completely melted.

This mechanism has the greatest potential to create ice pellet aggregates within the conditions studied. It can, in optimal aggregation conditions, create a distribution of particles at the surface composed of 70% aggregated particles, which is much higher than the potential for aggregation by either of the two other mechanisms. However, for this aggregation mechanism to be important, the critical diameter needs to be small but greater than zero, and therefore there needs to be some freezing rain falling with the ice pellets. This mechanism cannot occur in events that contain only ice pellets and no freezing rain.

### 6.1.2 Aggregation by Collisions Cracking the Ice Shells on Liquid-Core Ice Pellets

Aggregates formed through collisions with thin-shelled liquid-core pellets can only occur in the depth of the refreezing layer in which liquid-core pellets exist with shells thin enough to crack upon impact with another ice pellet. The depth where thin-shelled liquid-core pellets exist depends on the temperature profile of the environment. Liquid-core ice pellet aggregates will therefore be much more common in environments with a minimum temperature in the refreezing layer close to 0°C. Incidence also increases with increasing precipitation rate.

This aggregation mechanism has the smallest potential to create ice pellet aggregates. When conditions for aggregation are optimized and assumptions made also maximize the aggregate production calculated, the maximum fraction of aggregates produced by this mechanism is 13%. This occurs for a precipitation rate of 25 mm/h in an environment with a minimum temperature of -0.5°C in the refreezing layer.

### 6.1.3 Aggregation by the Freezing of the Surface Water Films on Ice Pellets

Aggregates formed by collisions between completely frozen ice pellets can occur throughout the depth of the refreezing layer, but are limited by the size of the component particles that will result in aggregation. This decrease in available component particles because of size restrictions increases the mean free path for collisions that will result in aggregation, thereby decreasing the efficiency of this mechanism of aggregation. If the threshold kinetic energy for collisions was larger than the threshold used here, this aggregation mechanism could be a more important factor in aggregate production. As in the other aggregation mechanisms, increased precipitation rate increases the number of aggregates produced, because there are more particles present for larger precipitation rates, therefore more collisions between particles and more chance for aggregation.

This aggregation mechanism has the second greatest potential of the three mechanisms to create aggregates. The maximum production of aggregates calculated for this mechanism was 27% in a precipitation rate of 25 mm/h, when the diameter of one of the components is at minimum 40% of the diameter of the collector pellet. This can occur in environments with any minimum temperature ($T_{min}$) value in the range of the study.

## 6.2 Aggregation When All Mechanisms Occur Simultaneously

When the distribution being examined has a low critical diameter, aggregates formed by collisions with freezing rain drops will be the dominant aggregate type. As critical diameter increases, the population of particles has less potential to produce aggregates by any of the three mechanisms. As critical diameter increases, the concentration of ice pellets in the population decreases exponentially and aggregation production accordingly decreases. Aggregates formed through collisions with freezing rain produce many more aggregates than the other two mechanisms when the others are calculated with a critical diameter of 0 mm (all particles contain some ice and evolve into ice pellets in the layer). If the critical diameter were to increase, the efficiency of the second two mechanisms to produce aggregates would be even smaller. Therefore, when freezing rain is present with ice pellets, aggregation by collisions with freezing rain will be by far the most important aggregation mechanism.

When there is no freezing rain occurring with ice pellets, there can be no aggregation by collisions with freezing rain drops. If the environmental temperature is low, aggregates formed by the freezing of surface liquid water films will be the dominant mechanism. Aggregates formed by cracking of liquid-core ice pellets will not have the chance to form, because in a cold environment the ice shells will grow more rapidly, reaching their threshold thickness before they undergo a collision. Aggregation by the freezing of surface water films is the most consistent aggregation mechanism. It depends on the fewest factors, can occur throughout the entire depth and has no minimum temperature requirement.

If the temperature is near 0°C liquid-core aggregates as well as water film aggregates will both be important. Liquid-core ice pellet aggregates are only important when the temperatures in the refreezing layer are close to 0°C. The semi-frozen particles will undergo slow freezing and will have ice shells below the threshold thickness for a large portion of the refreezing layer and will experience collisions before the threshold thickness for cracking is exceeded.

The calculations for these mechanisms have been conservative for the most part. Given the conservative assumptions made, the results do show that aggregation is an important process in the atmosphere. The results are significant, and may be more so if less conservative assumptions are made.

## 6.3 Concluding Remarks

This study is the first to examine ice pellet aggregates in detail. It has illustrated some of the means of their formation as well as their importance in some winter storms. It has also shown that, within precise environmental conditions, aggregation can significantly affect the phases of particles reaching the surface. If the temperature is cold enough, the precipitation rate is high enough, and the refreezing layer deep enough for collisions between particles, there will be a significant reduction in the amount of freezing rain that reaches the surface, due to ice pellet aggregation.

Insight gained from this research can be applied to prediction concepts. First, if the critical diameter of the distribution could be predicted from the depth and strength of the melting layer aloft, the fraction of aggregates could be estimated. If the critical diameter of the distribution of particles is low, the amount of freezing rain expected at the surface will be significantly reduced through collisions with ice pellets during their passage through the refreezing layer. As the critical diameter of the distribution increases and precipitation rate decreases, aggregational processes will have less of an effect on the decrease in freezing rain expected at the surface. Second, the depth of the refreezing layer could also give clues about the number of aggregates expected at the surface. If the refreezing layer is very shallow, there will be few aggregates formed by any mechanism, except possibly liquid-core aggregates, and therefore there will be little reduction in freezing rain. A deep refreezing layer will increase the potential for aggregates by collisions with freezing rain and by liquid water film freezing, which can both occur throughout the depth of the refreezing layer. A deeper refreezing layer will result in a larger reduction in freezing rain.

Though this study shed light upon the physics of ice pellet formation and that of ice pellet aggregates, there are substantial gaps in the current body of knowledge on the processes involved in aggregation. The gaps in information available on which to base assumptions have made a thorough study difficult. For example, there needs to be greater attention paid to the study of collisions between particles, specifically collisions between ice pellets and/or liquid-core pellets. There is a wide body of research on collisions between two liquid drops, collisions involving droplets and ice crystals, and collisions of two or more ice crystals but no studies were found on collisions between ice pellets and or liquid-core pellets. As well, the threshold kinetic energy difference used in the simulations of liquid water film aggregation was based upon reasonable assumptions, but it is important that the actual threshold be confirmed experimentally. If the threshold was found to be larger than that used in calculations, this mechanism may be determined to be more important than the calculations show. In addition, the sensitivity of results to different methods of calculating aggregation implies that a more general approach is needed in the future and this would need to include more sophisticated techniques to account for the fall velocities of non-spherical particles and aggregates.

Observations of ice pellet events certainly need to improve in quality. When ice pellets occur, especially with freezing rain, they are often left out of recorded operational observations. For example, in the storm studied by Gibson and Stewart (2007) over three hours of ice pellets were never mentioned in the operational observations even though these observations were noted by a manual observer at a site within 1 km of the photographing site (Gibson, 2005). As well, there is currently no mention at all of ice pellet aggregates or liquid-core pellets in operational observations yet this study has shown how important these particles can be.

In summary, this study has identified some of the key conditions that lead to ice pellet aggregation and has further shown that aggregates can act to significantly reduce more hazardous forms of winter precipitation such as freezing rain.

# References


Chong, S.-L. and C.S. Chen, 1974: Water shells on ice pellets and hailstones. *J. Atmos. Sci.,* **31**(5), 1384-1391.

Cortinas Jr, J.V., B.C. Bernstein, C.C. Robbins and J.W. Strapp, 2004: An analysis of freezing rain, freezing drizzle, and ice pellets across the United States and Canada: 1976-90. *Wea. Forecasting*, **19**(2), 377-390.

Gibson, S., 2005: Detailed observations of ice pellets and an analysis of their characteristics and formation mechanisms, Department of Atmospheric and Oceanic Science, McGill University, 106 pp.

Gibson, S. and R.E. Stewart, 2007: Observations of ice pellets in a winter storm. *Atmos. Res.,* **85**, 64-76.

Glickman, T.S., 2000: *Glossary of Meteorology Second Edition*. Amer. Meteor. Soc., 855.

Hosler, C.L., D.C. Jensen and L. Goldshlak, 1957: On the aggregation of ice crystals to form snow. *J. Atmos. Sci.*, **14**(5), 415-420.

Huskeflux, cited 2007: Hukseflux Thermal Sensors
[Available online at
http://www.hukseflux.com/thermal%20conductivity/thermal.htm]

Jones, K.F., 1998: A simple model for freezing rain ice loads. *Atmos. Res.*, **46**, 87-97.

King, W.D., 1975: Freezing rates of water droplets. *J. Atmos. Sci.,* **32**(2), 403-408.

Marshall, J.S. and W.M. Palmer, 1948: The distribution of raindrops with size. *J. Atmos. Sci.*, **5**(4), 165-166.

Matson, R.J. and A.W. Huggins, 1980: The direct measurement of the sizes, shapes and kinematics of falling hailstones. *J. Atmos. Sci.*, **37**(5), 1107-1125.

McFarquhar, G.M. and R. List, 1991: The raindrop mean free path and collision rate dependence on rainrate for three-peak equilibrium and Marshall-Palmer distributions. *J. Atmos. Sci.*, **48**(17), 1999-2003.

Pruppacher, H.R. and J.D. Klett, 1997: *Microphysics of Clouds and Precipitation*, 2nd Ed.. Kluwer Academic Publishers, 954 pp.

Ralph, F.M., R.M. Rauber, B.F. Jewett, D.E. Kingsmill, P. Pisano, P. Pugner, R.M. Rasmussen, D.W. Reynolds, T.W. Schlatter, R.E. Stewart, S.Tracton and J.S. Waldstreicher, 2005: Improving short-term (0-48 h) cool-season quantitative precipitation forecasting: Recommendations from a USWRP workshop. *Bull. Amer. Meteor. Soc.,* **86**(11), 1619-1632.

Rogers, R.R and M.K.Yau, 1989: *A Short Course in Cloud Physics*, 3rd Ed.. Pergamon, 290 pp.

Rosenberg, R., 2005: Why is ice slippery? *Amer. Inst. Phys.: Physics Today,* 50-55.

Stewart, R.E., 1977: Experimental investigation of the aerodynamics of freely falling disks, Department of Physics, University of Toronto, 244 pp.

Stewart, R.E. and R.W. Crawford, 1995: Some characteristics of the precipitation formed within winter storms over eastern newfoundland. *Atmos. Res.*, **36**(1-2), 17-37

Szyrmer, Wanda and Isztar Zawadzki, 1999: Modeling of the melting layer. Part I: dynamics and microphysics. *J. Atmos. Sci.*, **56,** 3573–3592

Thériault, J. M., R.E. Stewart, J. A. Milbrandt and M.K. Yau, 2006: On the simulation of winter precipitation types. *J. Geophys. Res.*, **111**(D18202).

Uplinger, W.G., 1981: A new formula for raindrop terminal velocity. *20th Conf. on Radar Meteorology*, Boston. 389-391.

Wettlaufer, J.S. and J.G. Dash, 2000: Melting below zero. *Sci. Amer.*, **282,** 50-53.

Zerr, R.J., 1997: Freezing rain: an observational and theoretical study. *J. Appl. Mereor.*, **36**, 1647-1661.