A Framework for the Qualitative Kinematics of Planar Mechanisms

Jiming Liu
B.Sc, M.A.

Department of Electrical Engineering
McGill University

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Abstract

There is a growing interest in knowledge-based qualitative reasoning systems that generate descriptions of physical realities as they appear and as they may effectively be thought about by humans. Yet there is much research work needed in investigating useful knowledge representation and reasoning techniques for various problem domains. This thesis explores the use of qualitative geometric reasoning in the domain of kinematics. It focuses on the position and velocity analysis of contact-surface-invariant (CSI) mechanisms and the determination of interaction between the bodies of contact-surface-varying (CSV) mechanisms.

In the thesis, two central geometric representation and reasoning techniques are formulated for the analysis of CSI mechanisms. The first technique applies qualitative geometric properties of triangles to reason about positions and linear velocities, whereas the second one employs a search procedure to resolve qualitative velocity constraint equations expressed in terms of the relative motion vectors of individual mechanism bodies. The problem of CSV mechanisms is studied by means of deriving kinematic state transitions based upon the description of vertex-contact (VC) configurations and the vertex placements with respect to VC configurations.

Finally, in conclusion, the significant contributions of the present study to the development of applied artificial intelligence (AI) are highlighted. At the same time, the limitations of the qualitative kinematic reasoning techniques and some future research topics are discussed.

Résumé

Il existe un intérêt croissant en sysèmes de raisonnement qualitatif basé sur la connaissance qui génèrent efficacement des descriptions physiques telles qu'elles apparaissent et telles qu'elles peuvent êtres perçues pas les êtres humains. Toutefois, il reste beau coups de liavail de recherches á faire dans le domaine qui a trait à la représentation de connaissances et aux techniques de raisonnements pour de problems differents. Ce mémoire explore l'utilisation du raisonnement qualitatif géométrique dans le domaine de la cinématique. Elle se concentre sur l'analyse de la position et la vélocité des mécanismes á surfaces de contact invariantes (CSI) et sur la détermination de l'interaction entre corps á surfaces de contact variantes (CSV).

Dans cette thèse, deux techniques principales de représentation géométriques sont formulées pour l'analyse des systèmes CSI. La première applique les propriétés qualitatives géométriques des triangles pour raisonner sur les position et vitesse linéaires, alors que la seconde, emploie une procédure de recherche qui qui résoud les équations, de la vitesse qualitative de contrainte, exprimées en fonction du vecteur mouvement relatif des méchanismes individuels des corps. Le problème de méchanisme CSV est étudié en formulant des états de transitions cinématiques basés sur la description de la configuration du vertex de contact (VC) et du vertex placement par rapport à la configuration de VC.

Et pour conclure, les contributions apportées, dans la présente étude, au développement de l'Intelligence artificielle sont citées. Les limitations du raisonnement qualitatif cinématique ainsi que de futurs sujets de recherche sont egalement discutés.

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Chapter 1 Introduction

The problems of kinematic analysis of mechanisms are traditionally solved using either analytic (including constraint methods), graphical, or numerical simulation techniques [Hartenberg and Denavit 64] [Hinkle 53] [Hunt 59] [Uicker, Denavit and Hartenberg 64]. These approaches, although complete, fail to provide much insight into how a mechanism functions kinematically.

The aim of this thesis is to offer an alternative approach to the kinematic analysis, in which kinematic concepts are defined on a far simpler, but nevertheless formal, *symbolic* basis. This approach enables us to develop artificially intelligent computer systems to perform effective reasoning about the function and geometry of a mechanism based upon incomplete specifications, and to communicate the results with users at a functional and qualitative level. The symbolic vocabularies and descriptions used will have direct physical and geometric interpretations with respect to the mechanism under consideration.

In this thesis, a framework for the qualitative kinematic analysis of planar mechanisms is developed, which utilizes qualitative representation and reasoning techniques. The emphasis is on the position, displacement, and velocity analysis of contact-surface-invariant mechanisms and the kinematic state identification of contact-surface-varying mechanisms in particular. The present study of two dimensional qualitative reasoning problems in kinematic analysis will serve as the first step in solving the three dimensional problems.

The organization of the thesis is outlined as follows:

In the next chapter, we present some terminologies and notations which are helpful for clarifying concepts, conventions, and symbols used in the rest of contents.

Chapter 3 gives some background in qualitative reasoning. Major work done in the related areas is reviewed. It also defines the scope of the present study and its relationships with other theories.

Chapter 4 presents fundamental notions, principles, axioms, and theorems, upon which the formalism of qualitative kinematics relies. The first two sections are concerned with the qualitative descriptions of trigonometric properties and arithmetic. The third one gives a formal characterization of the motion of mechanism components.

Chapter 5 introduces a qualitative kinematic analysis method for determining the position as well as the velocity distributions of mechanisms. This method, utilizing qualitative trigonometry and some velocity properties of mechanisms, is computationally quite efficient. However, it is restricted to the analysis of simple contact-surface-invariant mechanisms such as linkages.

A more general method for analyzing contact-surface-invariant mechanism kinematics, the *relative motion method*, is presented in Chapter 6. The relative motion method determines the linear instantaneous velocities of a mechanism by solving qualitative constraint equations. The qualitative constraints are derived by means of kinematic modeling of the mechanism bodies and their relationships.

Chapter 7 deals with the interaction of bodies in contact-surface-varying mechanisms. An algorithm is designed, which derives the qualitative kinematic state transitions based on the extremal configurations of a CSV mechanism.

Finally, in Chapter 8, the significant contributions of this research to the development of applied artificial intelligence are concluded. At the same time, the limitations of the qualitative kinematic reasoning techniques and some future research topics are discussed.

Chapter 2 Terminology and Notation

In this chapter, we will clarify the intended meanings of some classical kinematic concepts in the context of the present research. Also, we will define the new concepts that are being used in the proposed qualitative kinematics.

2.1 Mechanisms

In general, a *mechanism* can be viewed conceptually as a collection of rigid bodies whose relative positions are constrained. The individual parts of a mechanism, regardless of their shapes or the type of connections between them, are called *links*. When each point on every body moves in a plane, and all these planes are parallel, the mechanism is called a *planar mechanism*. Here the term *rigid body*, or simply called a *body*, refers to an assembly of component particles which remain fixed in relation to one another.

2.1.1 Kinematic Pairs

The connections between bodies, which constrain their relative motions and are made by pairs of elements coming into contact, are known as *kinematic pairs*. Considering only planar mechanisms, the kinematic pairs fall into two categories, i.e., *lower pairs* and *higher pairs.* As defined by Franz Reuleaux [Reuleaux 76], lower pairs are those having surface contact, while higher pairs are those having line or point contact.

2.1.2 Kinematic Chains

When a set of bodies are connected to one another by means of pairing to transmit motion, it is called a *kinematic chain.* A kinematic chain is not necessarily a mechanism; it becomes one when so constructed as to allow constrained relative motion among its bodies. We call the type of chains that allows definite relative motion between links *constrained kinematic chains.*

A *driver* (or driving body) is that part of a mechanism which causes motion, such as a crank component. A *follower* (or driven body) is that part of a mechanism whose motion is affected by the motion of the driver, such as a slider component.

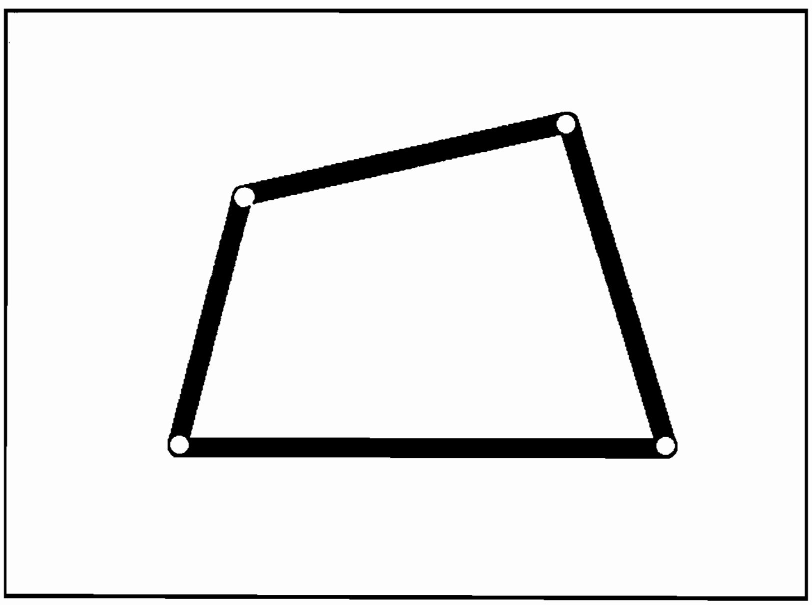

Figure 2.1 An example of CSI mechanisms

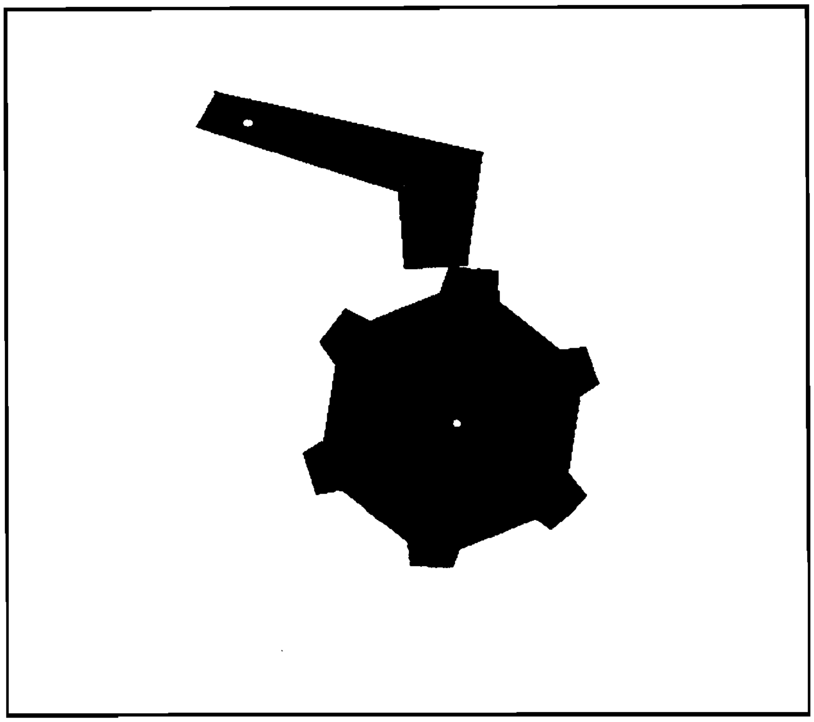

Figure 2.2 An example of CSV mechanisms

2.1.3 Contact-Surface-Invariant (CSI) and Contact-Surface-Varying (CSV) Mechanisms

In this thesis, we shall distinguish two types of planar mechanisms, namely, *contact-surface-invariant* (CSI) and *contact-surface-varying* (CSV). The former is defined as the one in which the point of contact between two bodies lies on a single *continuous* boundary facet (e.g., one continuous boundary segment in a two-dimensional case), whereas the latter changes the contact facets throughout a constrained motion. Examples of CSI and CSV mechanisms are shown in Figures 2.1 and 2.2, respectively.

2.2 Motion Characteristics

Generally speaking, *motion* can be defined as a change of position. It is always a relative term; that is, a body moves with respect to another body. It is usual in kinematic analysis to select a fixed body of the mechanism as its *fixed frame of reference.* Therefore, we speak of the *absolute motion* of a body when we mean its motion relative to this frame. The term *relative motion* refers to the movement of one body relative to another body that may also be moving with respect to the fixed frame.

2.2.1 Coordinate Systems

In kinematic analysis, if the degrees of freedom of a mechanism is n and its configuration is described by only n coordinates, then these coordinates are referred to as the *generalized coordinates.* In the case of a planar *constrained* mechanism, there will be only *one* generalized coordinate. On the other hand, if the selected coordinates define the orientation of each moving body with respect to a non-moving body or with respect to another moving body, these coordinates are referred to as *relative coordinates.* In a four-bar linkage, there will be three relative coordinates. See Figure 2.3.

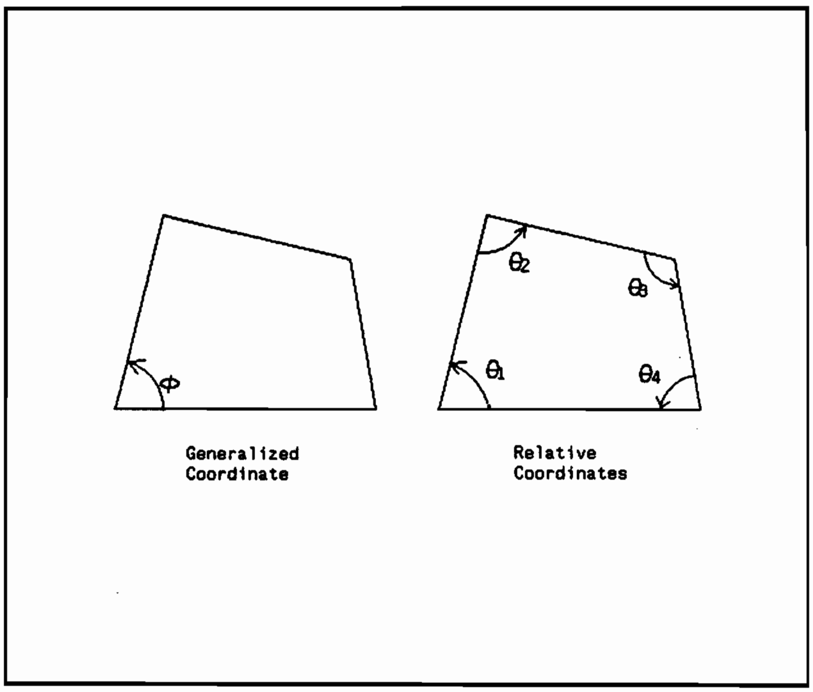

Figure 2.3 Examples of coordinates

2.2.2 Position, Displacement, and Velocity

Instantaneous linear position (x, y, or $s = (x, y)$), which is measured at a given instant, is defined in this study as the two-dimensional Cartesian position of a moving point with respect to some reference frame.

Instantaneous angular position (θ), which is the position found at a given instant, is defined as the angle between the position of a rotating point and a reference frame originated at its rotational axis.

Linear displacement ($\triangle x$, $\triangle y$, or $\triangle s = (\triangle x, \triangle y)$) is the change in linear position of a moving point during some time interval.

Angular displacement ($\triangle\theta$) is the change in angular position (or the amount of rotation) of a rotating point during some time interval.

Velocity is defined as the rate of change in position of a moving point with respect to time. *Instantaneous velocity* is the velocity found at a given instant.

Relative velocity of a moving point A with respect to another point B is defined as

$$\triangle v_{A/B} = \triangle v_A - \triangle v_B \tag{2.1}$$

where $\triangle v_A$ and $\triangle v_B$ denote the velocities of points A and B relative to some reference frame, respectively.

2.2.3 Motion Vector

In qualitative kinematics, the *motion vector* of a given point is defined as a vector that has two qualitatively-valued components (in two-dimensional cases). Unlike the velocity

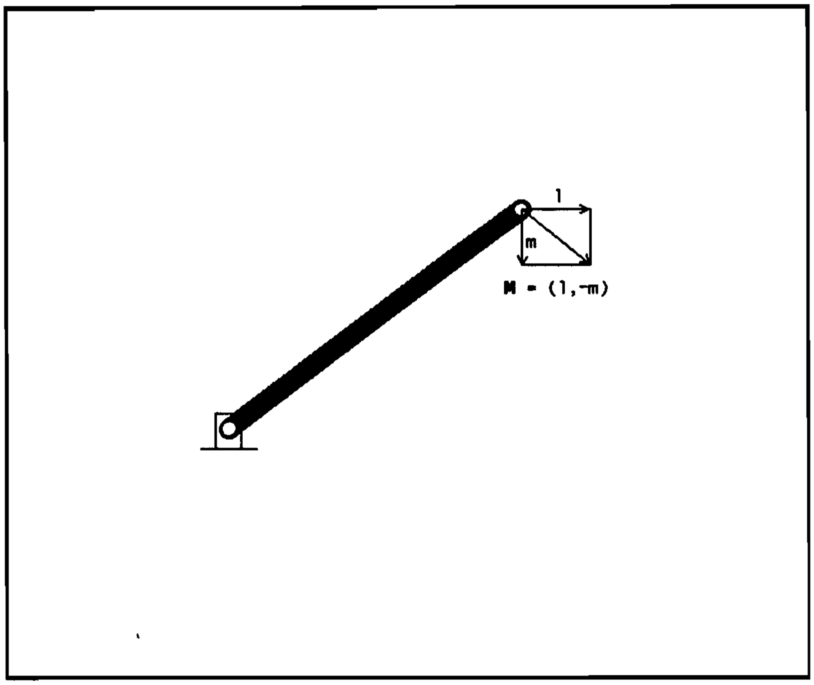

Figure 2.4 The motion vector of a rotational link

vector in classical kinematics, a motion vector indicates only the *approximate direction* of either an actual or a potential motion by specifying the qualitative values of its $x-$ and $y-$ direction motion components (see Figure 2.4). The five qualitative values used are *very small, small, medium, large,* and *very large.* The rationale for choosing these qualitative values and how motion vectors are used in qualitative kinematic analysis will be shown in Chapters 4 and 6, respectively.

2.3 Summary

In this chapter, we have defined some of the kinematic concepts, such as, kinematic chain, CSI and CSV mechanisms, and motion vector etc. As will be shown in the chapters that follow, these terminologies are used to describe mechanism configurations and to characterize qualitative kinematic properties.

Chapter 3 Background

There have been a number of theoretical frameworks developed in the area of artificial intelligence (AI) research, to provide tools and techniques for imparting the capability of reasoning about *naive views of physical phenomena* into computers [Bobrow 85] [Hobbs and Moore 85]. In what follows we shall review some of the theories from which fundamental ideas behind the present study have been inspired.

3.1 Qualitative Reasoning

One of the commonly-shared approaches named *qualitative reasoning*, as originally proposed and contributed by deKleer and Brown, Forbus, and Kuipers, attempts to infer predictions and explanations about physical behaviors [Forbus 84] [Kuipers 84] [deKleer and Brown 85]. It performs reasoning or *envisionment* based upon a qualitative description of the system or mechanism being analyzed [deKleer 77]. Such a description is qualitative and incomplete in nature, however it is still possible to derive some distinguishable states of the system.

Although the general intent of qualitative approaches is to capture the *deep incomplete knowledge* underlying human experts' reasoning in analyzing physical realities (e.g., systems functionality), there have existed some methodological differences among various theories [Bonissone and Valavanis 85].

Qualitative physics formulated by deKleer and Brown suggests that the behavior of a system be determined by interrelating the behaviors of its components, according to connectivity [deKleer and Brown 85]. In this case, the system is first decomposed conceptually into a collection of components and then the behavior of each component is modeled as a set of constraint confluences [Cochin 80] [Williams 84].

On the other hand, Kuipers sets forwards a more efficient approach [Kuipers 84] [Kuipers 86]. In his *qualitative simulation*, the constraint model of a system's structure is directly derived from a set of observable parameters and their mathematical interrelations. In most cases, this approach can yield similar results to deKleer and Brown's, but in a relatively efficient way.

Unlike the above two approaches, Forbus' *qualitative process theory* is more concerned with the active processes underlying physical realities [Forbus 84]. Thus it provides a tool for expressing and reasoning about more *intuitive* notions commonly used by humans [diManzo and Trucco 87] [Weld 85].

3.1.1 Formal Theory of Naive Physics

In qualitative physics research, apart from modeling and reasoning about human deep physical knowledge, attention has also been paid to the development of formal theories to represent human *intuitive knowledge* of physical situations. Thus instead of deriving functions from physical structures as mentioned above,the fundamental methodology used in this approach is to axiomatize commonsense by proposing topological relations, time ontologies, and primitives etc. The representative work done in this area is Hayes' naive physics [Hayes 79] [Hayes 85a] [Hayes 85b].

3.2 Qualitative Kinematics

In the previous section, we have reviewed the key ideas underlying some of the qualitative reasoning theories in general. In this section, we shall focus our attention on the domain of *kinematics* in particular, to briefly discuss what has been done in this area, what the proposed framework of *qualitative kinematics* is about, and how it is formulated.

3.2.1 Qualitative Kinematics: Connectivity Analysis

Forbus, Nielsen, and Faltings have made some significant efforts in proposing a theory of *qualitative kinematics* [Forbus, Nielsen and Faltings 87]. In their theory, the key to qualitative spatial reasoning is computing distinct legal contact configurations, termed *place vocabularies*, from the quantitative *configuration space* representation of a mechanism. The connectivity relationships are the primary constituents of qualitative states. The transitions of kinematic states during pairwise object interactions are derived via envisionment. Examples of this approach have been shown in the cases of planar ratchet and escapement mechanisms [Faltings 87].

In kinematic design, the problem of modifying objects' shapes to obtain certain motion transfer or constraint functions has been addressed by Faltings [Faltings 88], and Joscowicz and Addanki [Joscowicz and Addanki 88]. In their approaches, the design of mechanism shapes is basically viewed as an inverse process of the qualitative kinematic analysis using place vocabularies. In doing so, a goal-directed causal analysis method is used to activate appropriate refinement operators for modifying shapes.

3.2.2 A New Framework for Qualitative Kinematics of Mechanisms

The framework for qualitative kinematics of planar mechanisms, as formulated in this thesis, deals qualitatively and geometrically with the relative motions of the bodies of which they can be composed. It is essentially a combination of *qualitative geometry* with the concept of *time*. The force acting on a mechanism is not considered in this work.

The usefulness of qualitative kinematics lies in that it not only offers a practical means for building artificially intelligent systems that can reason about kinematic behaviors from mechanism configurations or vice verse but also serves as a general framework in applied kinematics for the analysis and design of mechanisms. Although the qualitative description of behaviors does not explicitly tell us how to specifically solve certain problems, it can be used in building or expanding our model of a specific situation [Kuipers 85].

The qualitative kinematics framework developed here provides useful methods for solving the following specific problems:

1. Position and linear velocity analysis of a linkage mechanism (or equivalent) with a single generalized coordinate.

2. Linear velocity analysis of a CSI mechanism with more than one relative Cartesian co-ordinates.

3. Identification of kinematic states and transitions of a CSV mechanism.

In solving the first problem mentioned above, qualitative trigonometric rules are utilized to derive the position of each lower pair as well as the velocity distribution on each link, whereas in solving the second problem qualitative arithmetic is applied. In both the qualitat e trigonometry and arithmetic, *quantity spaces* of qualitative values are constructed [Hayes 79] [Forbus 84]. Hence in qualitative kinematic analysis, continuous parameters such as the angles of a triangle or a linear velocity are represented by a set of distinct qualitative values - symbolic vocabularies, so that below or above these values, radically different kinematic states can be identified.

In [Faltings 87], the qualitative kinematic states are mainly organized around connectivity - the direct contact between mechanism bodies, thus it is not clear how such an approach can be applied to the CSI mechanisms where connectivity relationships are retained during the interaction of bodies.

In the present study, two methods for the velocity analysis of CSI mechanisms are proposed, i.e., (1) qualitative trigonometric reasoning method and (2) relative motion method. The first method is in general quick and accurate enough for many purposes, but it fails to deal with certain complex CSI mechanisms. The second method, although relatively less efficient, can be applied to analyzing any CSI mechanisms.

The proposed method for kinematic state transition analysis is close to that of Forbus, Nielsen, and Faltings'. However the present approach does not require configuration space (C-space) representation [Lozano-Perez and Wesley 79] [Brooks and Lozano-Perez 83]. It utilizes incomplete quantitative information (i.e., some specific configurations and the information about the placement of vertices of one body with respect to the edges of another in such configurations) to derive qualitative spatial representation and avoids the computation of a complete configuration space.

3.3 Summary

In this chapter, some of the classical frameworks for reasoning about physical systems, namely, qualitative physics, qualitative simulation, and qualitative process theory, have been reviewed. These theories have inspired some of the ideas developed in this thesis. Furthermore, the relevant studies on qualitative kinematics have been examined. Finally, the scope of the present study has been outlined.

Chapter 4 Fundamentals of Qualitative Kinematics

This chapter presents some fundamental notions, principles, theorems, and constructions which are necessary for the geometric and motion characterization of mechanisms in qualitative kinematics. It is organized into four sections, namely, qualitative trigonometry, qualitative arithmetic, axioms and theorems in revolute/prismatic-pairing body motion, and mechanism graphs. The detailed applications will be shown in the following chapters.

4.1 Qualitative Trigonometry

In kinematic analysis of mechanisms, the qualitative description of position of one body with respect to another to which it is connected is always desired. In the case of linkage analysis, if the configurations of linkages are represented as polygons which are formed by straight line segments and vertices, the position information can then be determined from the lengths of line segments and the angles in the polygons. In this case, the qualitative information can be immediately derived from qualitative trigonometric properties. The *qualitative trigonometry* to be presented in this section is mainly concerned with the qualitative description of geometric properties of triangles.

4.1.1 The Qualitative Measurement of Angles

In qualitative trigonometry, we use a set of distinct points in the range of $0 \sim \pi$ to partition the quantity space of angles into qualitatively distinct regions, as represented by a set of symbolic vocabularies. The distinct regions defined in this study are shown in Figure 4.1. It is assumed that if an angle is measured counterclockwise from a line segment, it is considered to be a positive angle, otherwise it is a negative one.

The rationale for such a division is mainly that it is consistent with human commonsense. As it may be noted, humans are very good at making qualitative measures with respect to some *symmetric* or *neutral* references. For example, the symbolic vocabularies *small* and *large*, as used in everyday life, are in fact understood with respect not to a certain numerical range but rather to an average or medium reference. The proposed angular measurement resembles the one that humans may use in qualitative analysis.

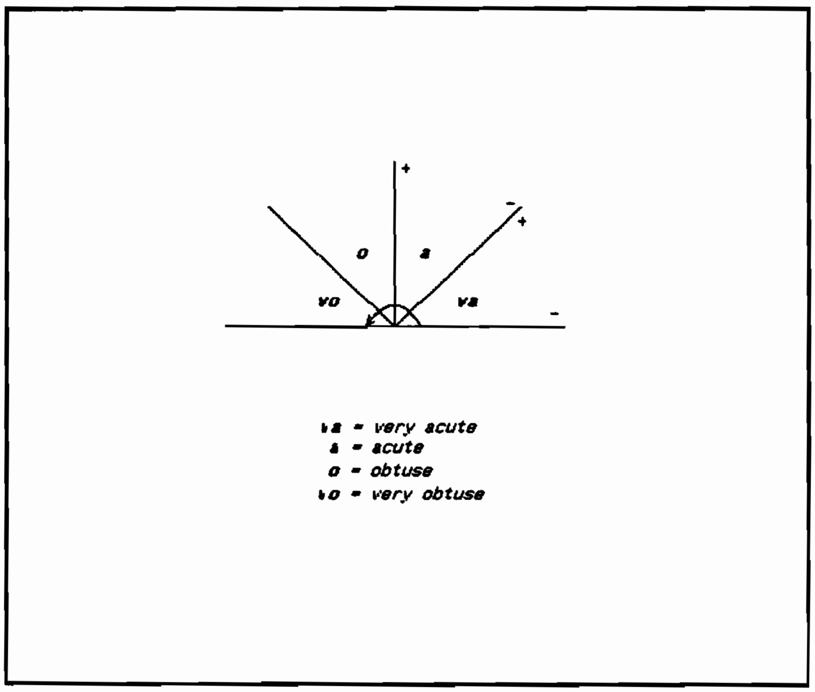

Figure 4.1 The quantity space of angles

4.1.2 Partial Ordering Relationships

Apart from the qualitative description of qualitative angles, the magnitudes of side lengths in a triangle must also be dealt with, if we want to develop a system of qualitative side-angle properties for all triangles. The idea used for the length description is in fact quite similar to the one above, i.e., using relative qualitative measurement to approximate quantities. But, instead of determining a qualitative value by comparing the quantity with some distinct point, we specify the qualitative length with respect to other lengths by using a set of partial ordering relationships.

The partial ordering relations are defined as follows:

$$
\begin{aligned}
1 < \frac{A}{B} < 1+\epsilon &\equiv A >\simeq B & \qquad >\simeq + >\simeq \Rightarrow > \\
1+\epsilon < \frac{A}{B} < 2 &\equiv A > B & \qquad > + > \Rightarrow \gg \\
\frac{A}{B} > 2 &\equiv A \gg B & \qquad < + < \Rightarrow \ll
\end{aligned}
$$

where $0 < \epsilon \ll 1$, $\equiv$ indicates a direct mapping from a quantitative relation to a qualitative relation, and $\Rightarrow$ defines a rewriting rule which means the right hand side is *directly derivable* from the left hand side. These conventions are proposed having examined the side relationships of various triangles in such a way that humans may qualitatively characterize them. The qualitative descriptions of side lengths presented above will enable us to distinguish various types of triangles.

4.1.3 Qualitative Geometric Rules in Triangles

Based upon the previous geometric characterization of angles and side relationships, the qualitative properties in triangles can therefore be derived. The results are shown in Table 4.1. These properties will be used in the next chapter to infer the trigonometric relationships between links as well as joint angles.

With respect to the triangle as shown in Figure 4.2, Table 4.1 can, for example, be read as if $\angle c$ is *very acute* and *close to zero*, and A is *slightly longer* than B, then C will be *much shorter* than A, C will be *much shorter* than B, $\angle a$ will be *obtuse*, and $\angle b$ will be *acute*. As a matter of fact, it can be noted that among all the three side relationships and three angles in a triangle, if any two of these qualitative descriptions are given, the rest can be inferred.

Table 4.1 Geometric rules in triangles

$\angle c =$	va^-	va^+	a^-	a^+	o	vo
$A >\simeq B$	$C \ll A$ $C \ll B$ $\angle a = o$ $\angle b = a$	$C < A$ $C < B$ $\angle a = a$ $\angle b = a$	$C <\simeq B$ $\angle a >\simeq \angle b$ $\angle a = a$	$C >\simeq A$ $\angle a >\simeq \angle b$ $\angle a = a$	$C > A$ $C > B$ $\angle a = va$ $\angle b = va$	$C > A$ $C > B$ $\angle a = va$ $\angle b = va$ $A + B >\simeq C$
$A > B$	$C \ll A$ $C < B$ $\angle a = vo$ $\angle b = va$	$C < A$ $C \sim B$ $\angle a = o$ $\angle b = a$	$C < A$ $C > B$ $\angle a = o$ $\angle b = va$	$C \simeq A$ $C > B$ $\angle a = a$ $\angle b = va$	$C > A$ $C \ll B$ $\angle a = va \sim a$ $\angle b = va$	$C > A$ $C \ll B$ $\angle a = va$ $\angle b = va$ $A + B >\simeq C$
$A \gg B$	$C < A$ $C + B >\simeq A$ $\angle a = vo$ $\angle b = va$	$C < A$ $C \gg B$ $\angle a = o$ $\angle b = va$	$C < A$ $C \gg B$ $\angle a = o$ $\angle b = va$	$C >\simeq A$ $C \gg B$ $\angle a = a$ $\angle b = va$	$C >\simeq A$ $C \gg B$ $\angle a = a$ $\angle b = va$	$C > A$ $C \gg B$ $\angle a = \pi - \angle c$ $\angle b = va$ $A + B >\simeq C$

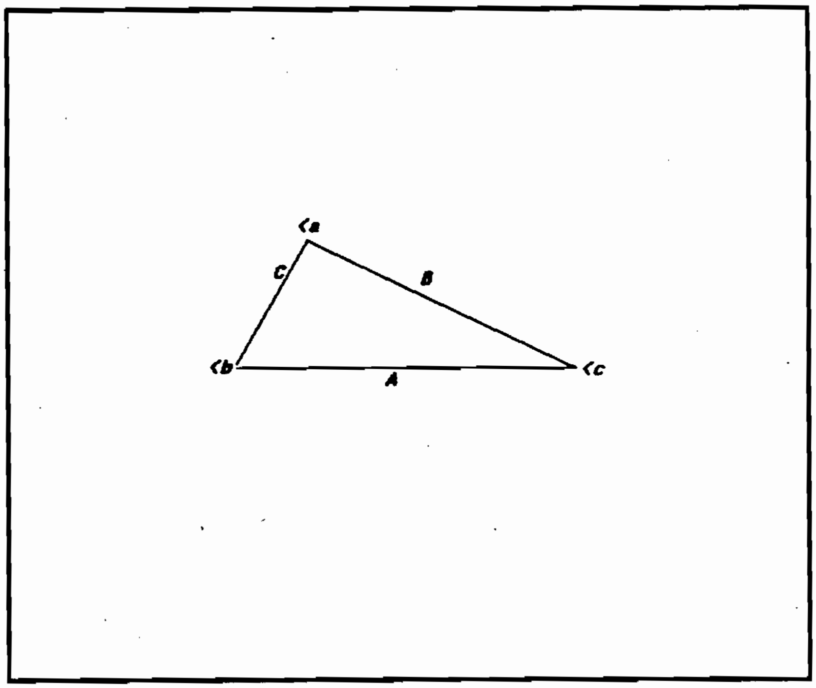

Figure 4.2 The triangle as referred by Table 4.1

4.2 Qualitative Arithmetic

Qualitative arithmetic is essentially composed of a qualitative quantity space and a set of rules and conventions for qualitative addition, subtraction, and vector modification. It is intended to provide a qualitative characterization of the numerical quantities, arithmetic operators, and relations. As will be seen in Chapter 6, this formalism plays a very important role in resolving velocity constraint equations of CSI mechanisms in a Cartesian space.

4.2.1 Quantity Space

In the foregoing discussion, a set of qualitative values will be used in describing the magnitudes of velocity components of a body. It contains the values of: *very small (vs)*, *small(s)*, *medium (m)*, *large (l)*, and *very large (vl)*. The identification of this quantity space is based upon the distinct states of a body's rotational motion with respect to a Cartesi coordinate system. A clearer understanding of it can be gained by looking at Axiom 4.1 in Section 4.3.

4.2.2 Rules

As will be shown in Chapter 6, the rules of qualitative addition, subtraction, and vector modification can be applied to search for the solutions of qualitative velocity constraint equations as well as to compare the relative velocity magnitudes between different bodies. These rules as defined in this study are presented in Tables 4.2 and 4.3.

Table 4.2 Rules for qualitative addition and subtraction

$+/-$	vs	s	m	l	vl
vs	$s/0$				
s	m/vs	$l/0$			
m	l/s	vl/vs	$NIL/0$		
l	vl/m	NIL/s	NIL/vs	$NIL/0$	
vl	NIL/l	NIL/m	NIL/s	NIL/vs	$NIL/0$

Table 4.2 shows the results of adding (or subtracting) the values in the top row to (or from) those in the leftmost column, where NIL means that the result will not be represented by any of those defined qualitative values. From this table, we know, for example, if we add the value *s* to *m*, the result will be *vl*. On the other hand, if we subtract *s* from *m*, it will give us *s*. These two cases can also be represented as rewriting rules, i.e.,

$$m + s \Rightarrow vl$$

$$m - s \Rightarrow s$$

Table 4.3 Rules for qualitative vector modification

	(vl,0)	(l,vs)	(l,s)	(m,s)	(m,m)
Decrease	(l,0)	(l,vs)	(m,vs)	(s,vs)	(s,s)
Increase	(vl,0)	(vl,s)	(vl,m)	(l,m)	(l,l)

Table 4.3 shows the results of applying the functions in the leftmost column to the values in the top row, where *Decrease* and *Increase* are qualitative magnitude decreasing and increasing modifiers respectively. For example, $Decrease(l,s) \Rightarrow (m, vs)$.

It should also be mentioned that in addition to *Decrease* and *Increase*, there are two other useful vector modifiers, namely, *Unitary* and *Inverse*. Both of them return the original values they take, but the difference between them is that *Inverse* assigns its values with opposite directions against the original ones.

Now if we recall the definition of motion vector presented in Subsection 2.2.3, we know that the actual velocity of a given point p in a link i relative to another link j must be proportional to the relative motion vector at p, i.e.,

$$V_{i/j}(p) = \lambda m_{i/j}(p)$$

where λ denotes a series of vector modifiers (that is analogous to a numerical scalar).

4.3 Axioms and Theorems in Revolute/Prismatic-Pairing Body Motion

Of various methods for transmitting motions, revolute and prismatic pairing methods are of most interest in qualitative kinematics. The examples of mechanisms using these methods are linkages. In linkages, the motion of one link relative to another satisfies a certain constraint imposed by their intermediate pairs and the velocity can be determined given the link's relative instantaneous position. In other words, it is possible to describe the constrained motion of the mechanism composed of such links in terms of the sum of individual links' relative motions.

Before we can qualitatively analyze the motions of a CSI mechanism using the relative motion approach, we shall first formulate some fundamental axioms, theorems, and constructions concerning the motion of CSI mechanism components. In Chapter 6, the applications of those axioms and theorems will be discussed in connection with linkages.

4.3.1 Revolute-Pairing Bodies

Suppose that body A is connected to body B by means of a revolute pair. In this case, the motion of A relative to B may be described in terms of the motion of A with respect to a reference frame on B originated at the rotational axis. In the foregoing discussion, we will use Cartesian coordinate systems as the relative reference frames. The relative instantaneous angular position of a given point on body A is defined as the smallest non-negative angle formed by the x-axis and the line segment passing through the point and its rotational axis. Hence, no matter in which quadrant the line segment lies, its relative angular position (θ) is always within the range of $[0, \frac{\pi}{2}]$.

Axiom 4.1 Let a point on the body A be in rotation with respect to a reference frame and the line segment passing through the point and its rotating axis be l. If the rotation is counterclockwise, then when l is in the first quadrant, the motion vectors corresponding to the set of qualitative angles (i.e., ordered angular regions as mentioned in Section 4.1) can be described as in Table 4.4.

Table 4.4 Relative motion vectors of a rotating point in quadrant I

$\theta_I =$	va^-	va^+	a^-	a^+
(m_x, m_y)	(-s, vl)	(-m, l)	(-l, m)	(-vl, s)

Theorem 4.1 (Change of direction) In Axiom 4.1, if the point rotates in an opposite direction, then the corresponding motion vectors will have the same magnitudes as before but with opposite directions.

Theorem 4.2 (Symmetrical property of a circular motion) In Axiom 4.1, if l is in the second quadrant, then the direction of y-coordinate components (m_y) in the corresponding motion vectors will change from positive to negative. If l is in the third quadrant, then both x and y components in the corresponding motion vectors will change directions. If l is in the fourth quadrant, then x components in the corresponding motion vectors will change direction.

In general, it is always possible to determine the constrained motion of a mechanism from its reversed motion.

Theorem 4.3 (Inversion of a constrained motion) Suppose that some constrained relative motion in a constrained closed-loop kinematic chain, A, is given. If one link of A moves over its entire range of motion but with an opposite driving direction at each position, then the motions of all links in A reverses their directions.

4.3.2 Prismatic-Pairing Bodies

The relative motion between two bodies A and B of a prismatic pair can be described in the same way as that of the revolute pair. As shown in Figure 4.3, a reference frame for the motion of A is fixed on B.

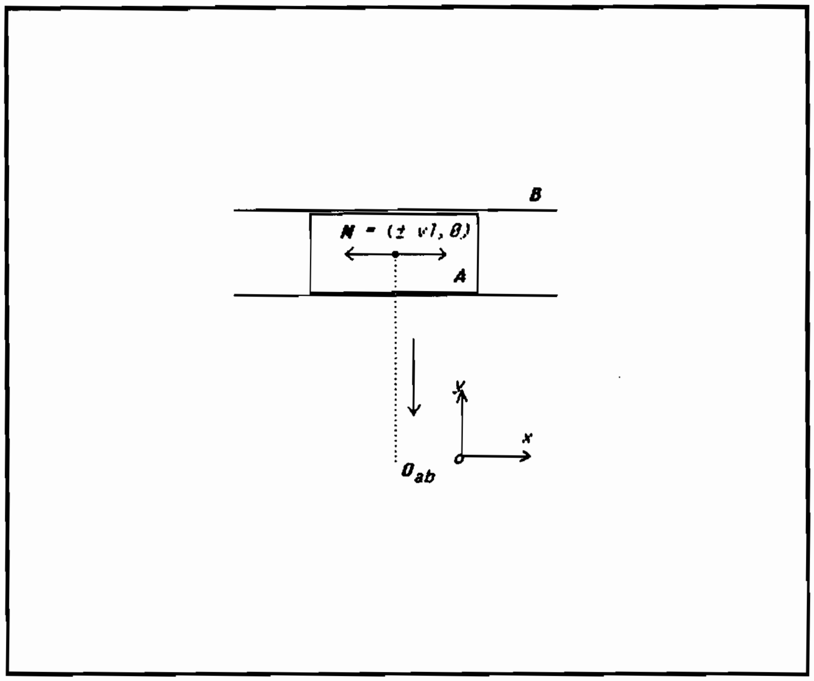

Figure 4.3 The relative motion vector of a prismatic-pairing body

Axiom 4.2 If the x-axis of the Cartesian system is parallel to a common tangent on the contact surface, then the motion of A relative to the frame can be described in terms of A's relative motion vector, $(\pm vl, 0)$.

Axiom 4.3 The prismatic motion of A relative to B is equivalent to the rotation of A relative to B with its center at infinity.

Having understood the relative motion vectors of revolute-pairing and prismatic-pairing mechanism components, we can readily determine the constrained motion of an intermediately connected (e.g., linkage) mechanism, the details of which will be shown in Chapter 6.

4.4 Mechanism Graphs

In this thesis, we use the term *mechanism graph* to refer to the specific connectivity (i.e., graphical) representation of a planar mechanism. Such a graph is intended to ease the process of deriving the structure of the mechanism, from which the qualitative position and velocity analysis is carried out.

In general, the mechanism graph of a constrained kinematic chain is defined as a set $V = \{v_i\}$ of vertices and a set $E = \{e_j\}$ of edges satisfying the following conditions:

1. Every edge in E represents a link (either actual or imaginary) in the kinematic chain.

2. Every vertex in V represents a lower pair in the kinematic chain.

3. Every edge in E contains precisely two vertices of V, and these agree with its end points (i.e., start and end vertices if the pair of end points is ordered).

4. The edges in E have no common points, except for vertices of V.

5. V and E are both finite sets.

A finite sequence e_1, e_2,...,e_n of edges of a mechanism graph is said to constitute a *chain progression* of length n if there exists an appropriate sequence of $n + 1$ vertices v_0,v_1,...,v_n such that

$$e_i \sim (v_{i-1},\ v_i) \quad \text{for i=1,2,...,n}$$

If all elements of a progression represent n distinct edges and $n+1$ distinct vertices except at $v_0 = v_n$, the progression is called a *closed-loop chain progression*.

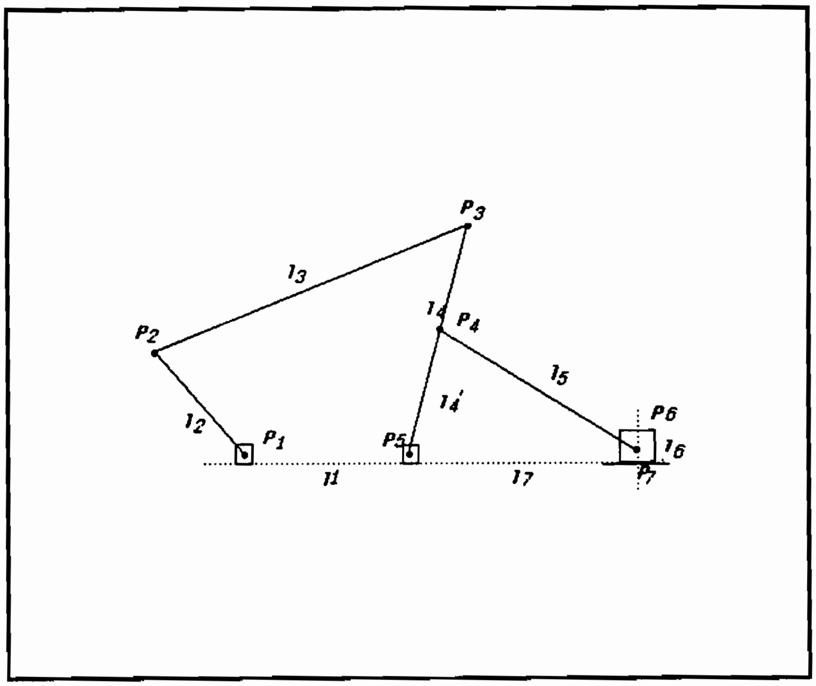

Figure 4.4 A CSI mechanism

4.4.1 Deriving Mechanism Graphs

Since the planar CSI mechanisms being studied in this thesis constitute constrained kinematic chains, their corresponding mechanism graphs, based on the above conventions, can be represented as closed-loop graphs.

An algorithm for constructing a mechanism graph and its independent loops

Input : A CSI mechanism.

Output : A corresponding mechanism graph given as a set of independent closed-loop chains.

Begin

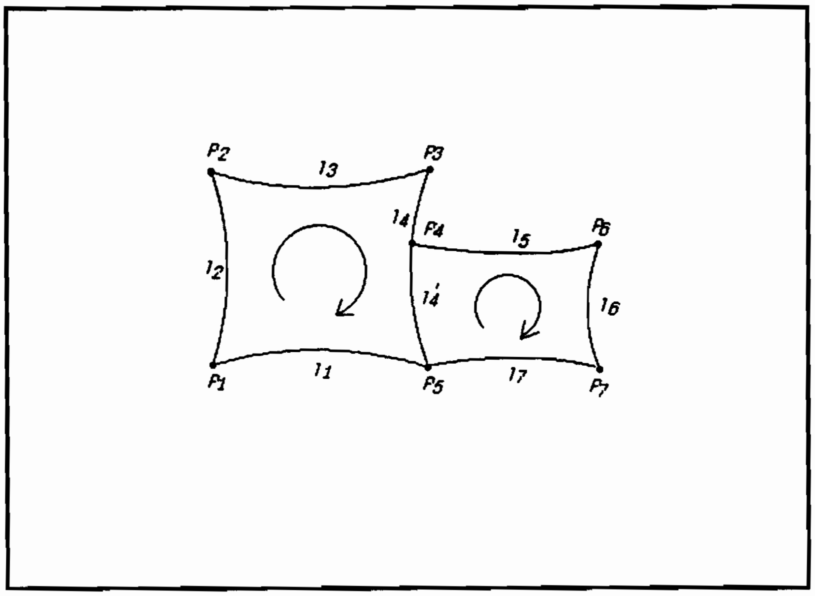

Figure 4.5 The mechanism graph of a CSI mechanism

1. Represent the set of fixed bodies in the mechanism as an open-loop chain progression by associating each vertex with a fixed body and connecting each pair of vertices with an edge.

2. Start with one fixed body, represent each succeeding body that moves relatively to it as a consecutive edge. If the body is in a sliding contact, then add two consecutive edges.

3. If the next succeeding edge meets any other edge(s), then backtrack from the joined vertex each pair of chain progressions to their common root. Each backtracked pair constitutes an independent loop, which corresponds to an independent kinematic subchain in the mechanism. Otherwise go to Step 2.

4. If there still remains any fixed body uncovered, then choose this body as a new starting vertex, go to Step 2.

End

As an example of applying the above algorithm, we have shown in Figure 4.5 the mechanism graph of a CSI mechanism presented in Figure 4.4.

4.5 Summary

This chapter has presented the fundamentals of the qualitative kinematics, including qualitative trigonometry, qualitative arithmetic, axioms and theorems concerning the qualitative motion characteristics of mechanism components, and a mechanism graphical representation technique. These axioms, principles, and constructions will form the foundation for the later discussion of qualitative kinematic analysis.

Chapter 5 Positions and Velocities by Qualitative Trigonometry

In qualitative kinematic analysis and synthesis of mechanisms, it is always desirable to know what motions the various parts of a mechanism undergo and the relationships between these motions. As an example in linkage analysis, one may want to ask the following question: suppose that two links are (either directly or indirectly) connected in a linkage mechanism, how would the movement of one link be constrained by the second?

It is obvious that the determination of motions and their interrelationships relies upon the position and velocity analysis of individual bodies. In classical kinematics, such an analysis usually requires explicitly given *quantitative* information. With analytical methods, for example, in order to solve the constraint equations for all the coordinates values at any given instant, the values of k coordinates must be known (where k is equal to the number of degrees of freedom). And since the equation in general nonlinear, iterative numerical methods, although computationally expensive, may have to be used [Uicker, Denavit and Hartenberg 64]. Furthermore, the numerical solution has to be interpreted by humans if the function of the mechanism is to be understood.

In order to develop computer programs that can automatically generate solutions at a functional and qualitative level, an alternative approach must be sought. Gelsey presents an automated reasoning method for analyzing machine geometry and kinematics, which uses a *constructive solid geometry* representation as its input [Gelsey 87] [Requicha 80]. Functional solutions are thus derived from a library of known relationships imposed by pairs. The limitation of this approach is that it cannot describe the motion relationships in linkages and other complex CSI mechanisms.

In this chapter, we will discuss a method for qualitatively analyzing the configuration, the motion transmission, and the velocity distribution of a linkage mechanism. This method requires only incomplete or *qualitative* dimensional specifications. Here, a configuration is defined as the instantaneous positions of individual links in the linkage. Given any instantaneous configuration of the linkage, the velocity distribution in a certain link is defined as the absolute linear velocities of a set of points in the link. Motion transmission refers to the absolute linear velocity relationships between a driving link and a driven link.

The basic idea behind the present method of analysis is that by applying the qualitative trigonometric properties as shown in Chapter 4, we are able to derive the qualitative geometric descriptions of both the position of any given link and the velocities in a single or different links. Thus, instead of analytically computing the configurations or graphically constructing and visualizing the velocities as in classical kinematics, this method is mainly based on qualitative geometric reasoning, which is computationally more efficient for deriving qualitative solutions to the position/velocity problems.

5.1 Instantaneous Configuration

In order to specify the positions of all bodies in a mechanism, a set of parameters called coordinates must be used. In the case of classical kinematic analysis, if k is the number of degrees of freedom of the mechanism, it is known that the complete description of the mechanism would require minimal k independent coordinates, which are referred in Subsection 2.2.1 to as generalized coordinates. In the example shown in Figure 5.1, the angle ϕ, describing the orientation of the link A with respect to a reference frame, is selected as a generalized coordinate. Hence for any given configuration, i.e., known ϕ, any other information on the position of any point in the linkage can be calculated.

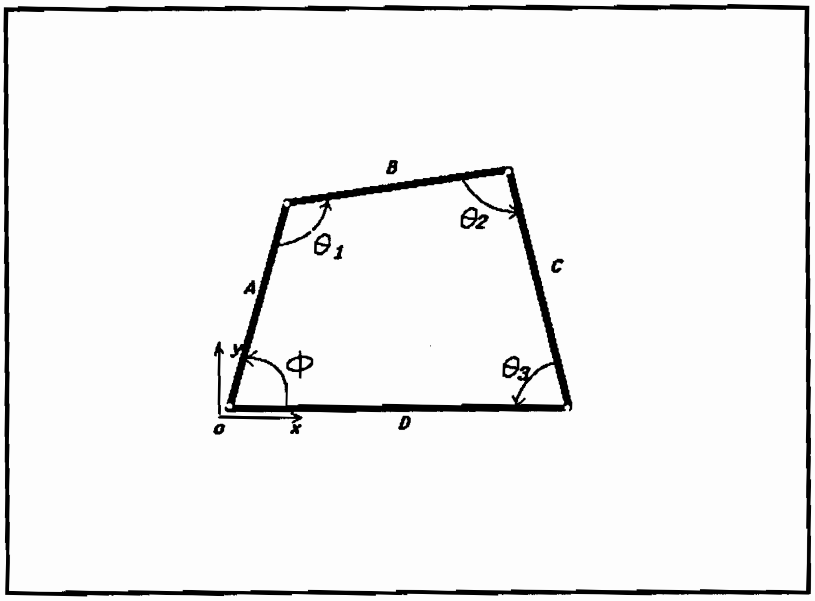

Figure 5.1 A four-bar linkage with generalized coordinate ϕ.

In qualitative position analysis of linkages, we are also interested in describing configurations of a mechanism given a minimum number of independent coordinates. However the method used for the analysis differs significantly from the classical analytical method in that the former relies upon qualitative trigonometric reasoning. The type of problems to be solved in the qualitative analysis may be stated as follows:

Given the partial ordering relationships between the lengths of all links and the qualitative value of one generalized coordinate, i.e., the orientation of a certain link, find the positions of other links in terms of their inner joint angles (see Figure 5.1).

The fundamentals of position analysis with the qualitative trigonometric method can best be illustrated by following the process in a simple example as shown in Figure 5.2.

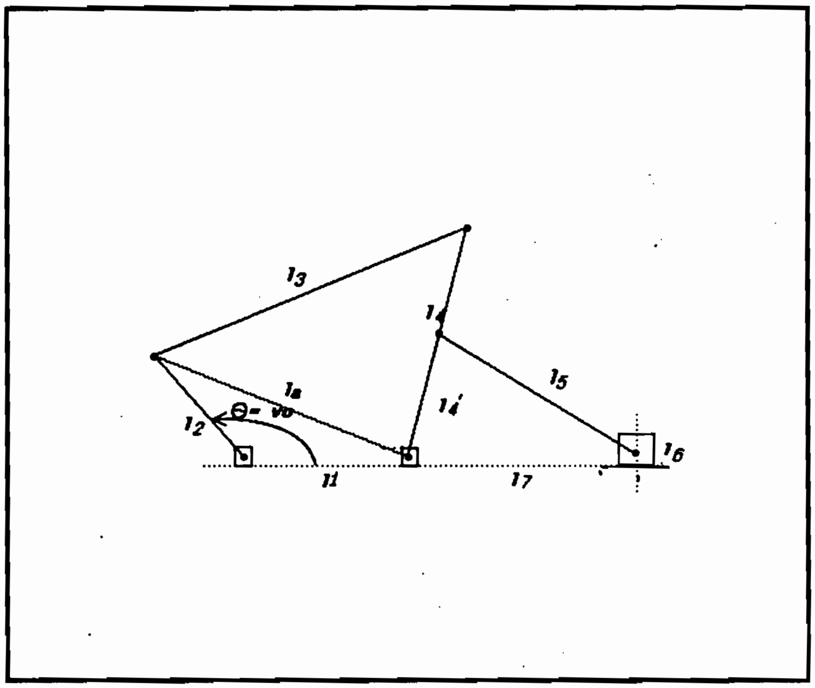

Figure 5.2 An example of qualitative reasoning about configurations.

In this example, we are given the partial ordering of the lengths of links in the linkage (where the slider is by Axiom 4.3 considered as a special case of rotating link) and the qualitative value of the input angle. These qualitative conditions may be stated as follows: $l_3 > l_4 >\simeq l_5 \simeq l_7 > l_2 \simeq l_1$, $l_4 > l'_4$, and $\theta_{12} = vo$. The instantaneous configuration expressed in terms of joint angles θ_{23}, θ_{34}, θ_{41}, θ_{45}, and θ_{57} is desired, where θ_{ij} denotes the joint angle between links i and j.

As the first step of analysis, the linkage is represented as a mechanism graph and accordingly two independent four-bar linkages in the mechanism are detected. How this step is carried out can be found in Section 4.4. The resultant graph has been shown in Figure 4.5. Next we start with the four-bar linkage that contains the known driver link l_2 and add an intermediate link l_a between the end points of l_2 and l_1. Thus by applying the rule given in Table 4.1 (i.e., in the 1st row and the 6th column), we can infer that $l_2 + l_1 >\simeq l_a$, $\theta_{2a} = va$, and $\theta_{1a} = va$. From the given conditions, we further know that $l_3 >\simeq l_a >\simeq l_4$. Consequently, we can obtain the angles $\theta_{3a} = a^-$ and $\theta_{4a} >\simeq \theta_{34} = a$ (by applying the rule determined from the 1st row and 3rd column of Table 4.1). From these results, we are able to derive $\theta_{23} = \theta_{2a} + \theta_{3a} = a$ and $\theta_{14} = \theta_{1a} + \theta_{4a} = o$. Hence the approximate configuration of the first four-bar linkage is determined. To analyze the connected second linkage, we choose the shared link between the two linkages as its new driver link and then repeat above steps. However it should be noted that in the second case the equivalent follower link 6 becomes a single point, i.e., the linkage forms a triangle, therefore no intermediate link is needed. Without going through all the steps, we have give the results of analysis as follows: $\theta_{45} = a$ and $\theta_{57} = va$. Therefore the configuration of the entire mechanism is qualitatively described.

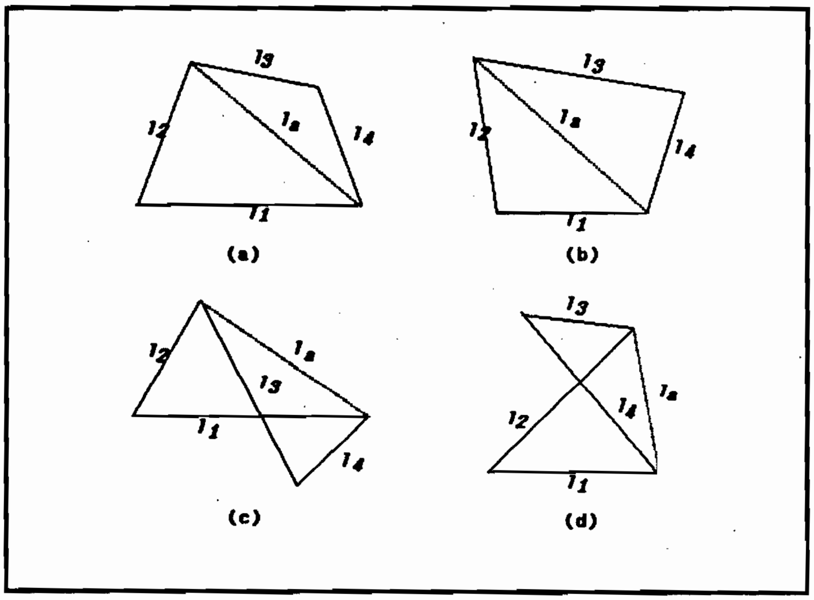

Figure 5.3 An illustration of the position analysis algorithm

An algorithm for the position analysis of linkages, using qualitative trigonometric method, is summarized as follows (see Figure 5.3):

Position analysis algorithm

Input : The partial ordering of lengths of all the links in a linkage and the qualitative value of an input angle.

Output : The qualitative configuration of the linkage expressed in terms of its joint angles.

Begin

1. Represent the linkage into an equivalent mechanism graph.

2. Find the constrained closed-loop that contains the known driver link. Analyze the sublinkage corresponding to this loop as if it is a four-bar linkage.

3. If the four-bar linkage contains one slider, then directly infer its configuration using qualitative trigonometric rules and go to Step 6; else form an intermedium link l_a between the end points of the driver link l_2 and its reference link l_1. Apply trigonometric rules to infer both the length ordering relationships between l_a and other links in the linkage and the joint angles between links l_a and l_1, denoted by θ_{1a}, and between links l_a and l_2, denoted by θ_{2a} (The links consecutive to l_1 and l_2 are denoted by l_4 and l_3, respectively).

4. Infer the joint angle between l_3 and l_4, denoted by θ_{34}, based on the ordering of l_a, l_3, and l_4 and similarly infer the joint angles between l_3 and l_a and between l_4 and l_a, denoted by θ_{3a} and θ_{4a} respectively.

5. Determine the joint angles between l_1 and l_4 and between l_2 and l_3, denoted by θ_{14} and θ_{23} respectively. If the configuration being considered forms a crossing polygon, then $\theta_{14} = |\theta_{1a} - \theta_{4a}|$ and $\theta_{23} = |\theta_{2a} - \theta_{3a}|$ (see Figures 5.3c and 5.3d). Otherwise $\theta_{14} = \theta_{1a} + \theta_{4a}$ and $\theta_{23} = \theta_{2a} + \theta_{3a}$ (see Figures 5.3a and 5.3b).

6. If there can be found another loop which contains either a fixed reference link (maybe the same as previous one) and one analyzed link, or two analyzed links, then consider one analyzed link as a new driver link. Go back to Step 3.

End.

In principle, most of the CSI mechanisms can be analyzed using this method. For instance, if an independent linkage contains a single slider, then the configuration to be described becomes the sliding position with respect to a certain joint. However in the cases where the linkage contains either two sliding-contact pairs or one higher pair, the configuration problems would become quite ill-defined, i.e., there is no general definition of position. Hence, the method described here does not cover such mechanisms.

5.2 Motion Transmission and Velocity Distribution

In this section, we discuss how to reason about the transmission of motion between two links as well as the velocity distribution on a given link using qualitative trigonometric rules. The qualitative analysis of velocities will be illustrated in terms of a linkage mechanism.

Before we proceed, it would be necessary to review one of the important classical kinematic concepts, i.e., the *instantaneous axis of velocity*. By an instantaneous axis of velocity we mean an axis about which a member of the mechanism rotates. It can be either a fixed or a moving axis. At any instant, the moving axis may be viewed as a stationary axis with properties similar to a fixed axis.

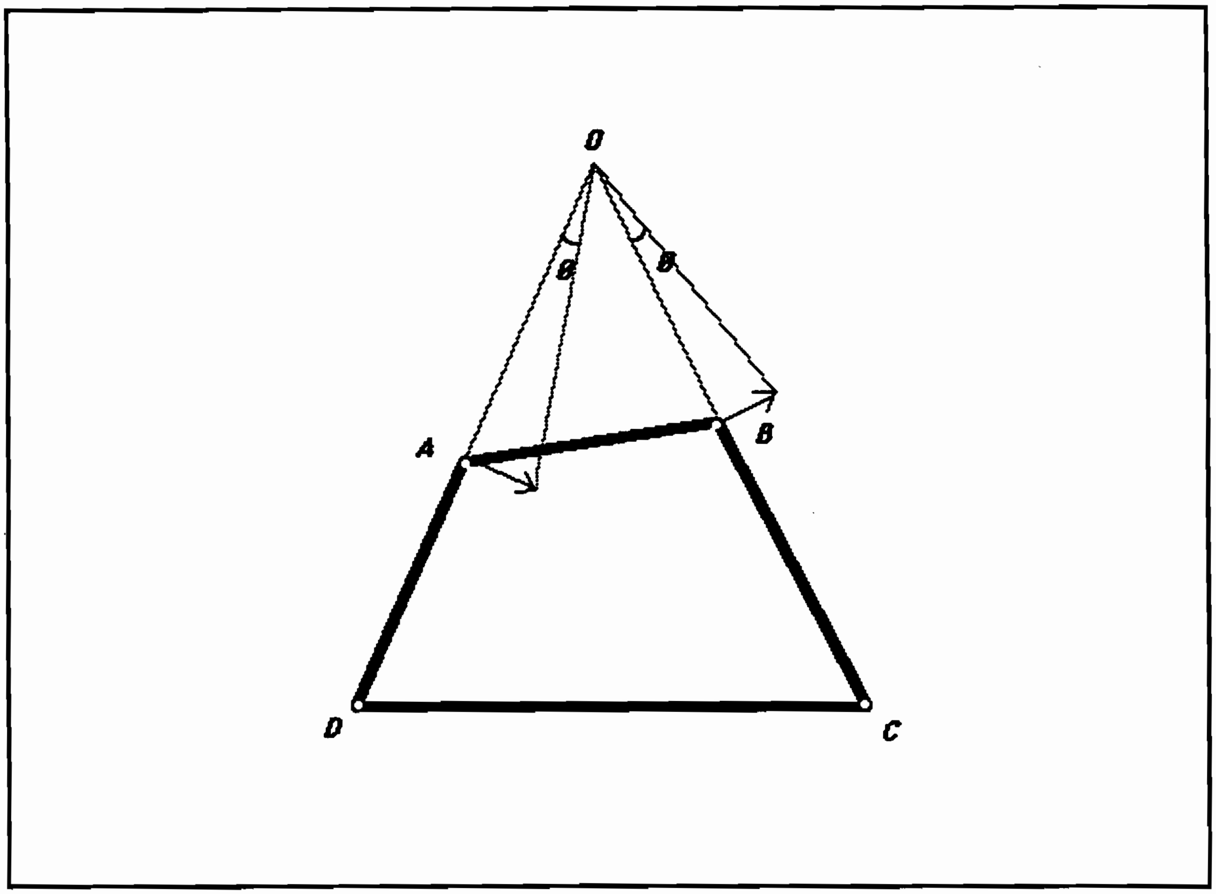

Figure 5.4 The determination of instantaneous axes of a four-bar linkage.

According to the above definition, it is obvious that the absolute instantaneous linear velocities of points on a given link are perpendicular to lines joining the points with the instantaneous axis. And it is based upon this property that we can find the instantaneous axis of each link in a linkage. In the example shown in Figure 5.4, we can immediately observe that the driver and follower links of the linkage rotate or oscillate about their respective fixed axes, D and C. As far as the floating link (i.e., connecting rod) is concerned, it is useful to note that the absolute linear velocity of point A on the driver is known in direction. And another point B has a velocity in the direction perpendicular to the follower. Both A and B are connected to the end points of the floating link and at the instant under consideration these two points are tending to rotate about its instantaneous axis, O. Thus by definition, the instantaneous axis of velocity of the floating link can be located by intersecting the lines perpendicular to the directions of velocities at A and B.

Furthermore, as the absolute instantaneous linear velocities of points on a given link is proportional to the distances of the points from the instantaneous axis, we will be able to determine the *velocity distribution* on each link and to infer from one link to another the *motion transmitted*.

In fact, the idea of instantaneous axis has already been applied in classical kinematics to analyzing velocities. However, such an analysis is mainly based on the graphical construction of all the axes and the visual determination of the distances from a given axis to certain points on the corresponding link. In order to locate the axes, it usually requires to apply Kennedy's Theorem, i.e., the axes of any three planar bodies lie on a straight line, which is quite intuitive.

In the method presented here, we use the same concept of instantaneous axis. But instead of graphically analyzing a linkage and constructing its axes, we apply a generalized analysis algorithm to guide the representation and inference of axes in terms of their qualitative locations. The location of an instantaneous axis is derived from its properties as well as the qualitative trigonometric rules. Similarly, the distances from a given axis to other points of interest are inferred and therefore the qualitative analysis of velocities in the linkage is carried out.

There are primarily two general types of motion transmission problems to be con-

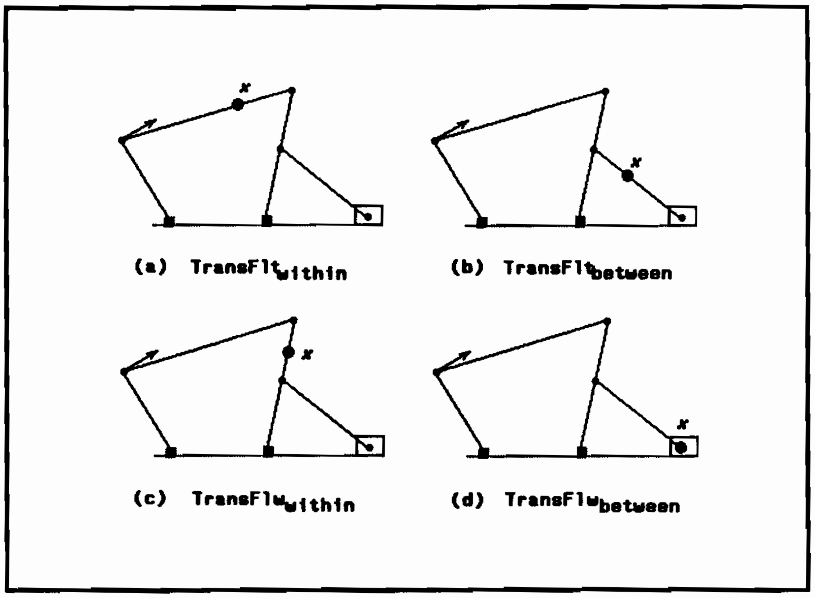

Figure 5.5 Types of velocity analysis problems to be considered

sidered, that is, either the desired velocity is on a *floating* link or on a *follower* link. In what follows we will denote them as $TransFlt$ and $TransFlw$ types, respectively. The only distinction between the follower link and the floating link is that the former has one end point being fixed (i.e., connected to a fixed link where the reference frame is located) whereas the latter doesn't. Under each of these two types, there further exist two situations to be distinguished, depending upon whether the input and the desired velocities are located in a single four-bar (or equivalent) linkage or in two different four-bar linkages. These situations are denoted by subscripts $_{within}$ and $_{between}$, respectively. Therefore, in total the analysis method will deal with four specific types of problems (see Figure 5.5).

The details of the method are given as follows in an algorithm for analyzing motion transmissions. It should be noted that when the velocities of two or more points on a given link are analyzed, the *velocity distribution* of this link is obtained. Therefore, here we think of the velocity distribution problem as a special case of the motion transmission problem.

An algorithm for reasoning about velocities

Input : The qualitative configuration of a linkage mechanism in which the velocity of the driver link and the position of certain point(s) in a link are given.

Output : The qualitative velocity relationship between the driver link and the given point (or among various points in a link).

Begin

1. Represent the linkage mechanism as a closed-loop mechanism graph.

2. Divide the mechanism into a set of independent four-bar linkages (or equivalents) by graph searching.

3. Whenever the location of the linear velocity on an input (or original driver) link is not at an end point, compare the distances from the fixed axis of the link to the location and to another end point and thus derive the qualitative relationship between the two distances (see Figure 5.6).

4. Start with the linkage where the driver is included. Locate the instantaneous axis of the floating link.

5. If the problem is of $TransFlt_{within}$ type, or $Trans**_{between}$ type but the linkage shares the floating link with another four-bar linkage, then

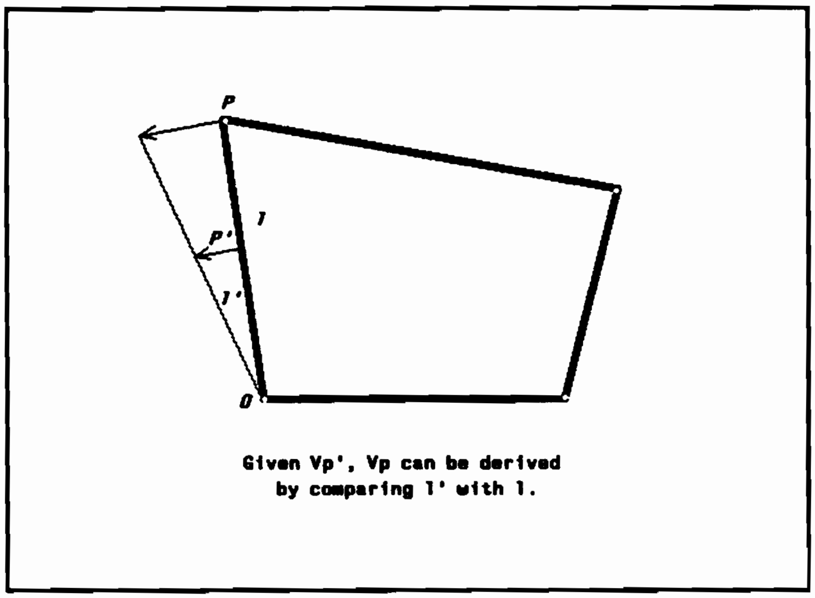

Figure 5.6 Velocity distribution with respect to a fixed axis

5a. Compare the distances from the axis to the end point of the driver link and to the desired location on the floating link.

5b. If the problem is of $TransFlt_{within}$ type, then based on the ordering of the distances obtained so far, derive the velocity relationship between the (original) known driver link and the the desired point and end the analysis. Otherwise go to Step 7.

6. If the problem is of $TransFlw_{within}$ type, or $Trans**_{between}$ type but the linkage shares the follower link with another four-bar linkage, then

6a. Compare the distances from the axis to the end point of the driver link and to that of the follower link. If the desired point or the shared axis is not on the end point of the follower link, then with respect to its fixed axis, further compare the distances from the axis to the end point and to the desired point.

6b. If the problem is $TransFlw_{within}$, then based on the ordering of the distances obtained so far, derive the velocity relationship between the (original) known driver and the follower and end the analysis. Otherwise go on.

7. Consider the shared link as a new driver link in the next connected four-bar linkage and go to Step 3.

End.

Where symbol *** denotes either Flt or Flw.

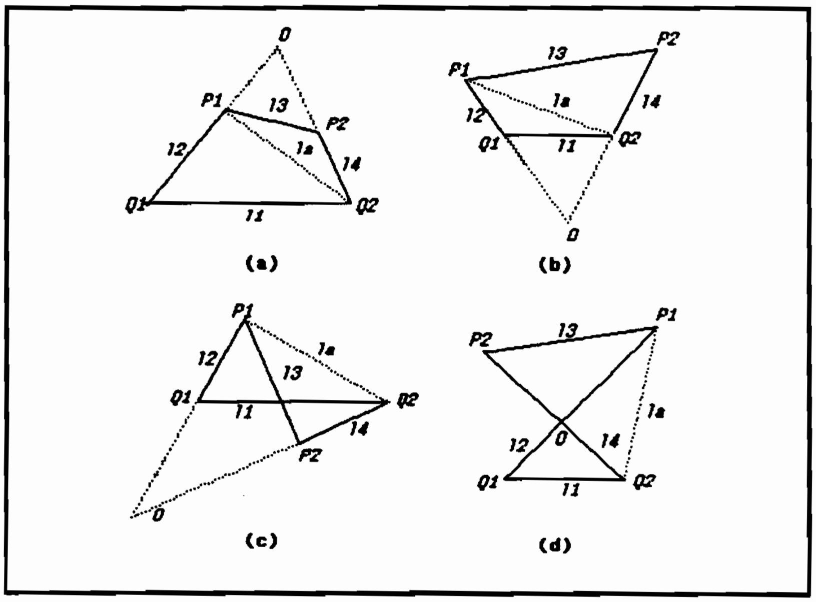

Figure 5.7 The determination of instantaneous axis for a floating link in various configurations.

In the above algorithm, how to locate an instantaneous axis for a floating link and to compare distances with respect to the axis has not been explicitly stated. The following subroutine provides the detail (see Figure 5.7):

A subroutine for determining distances with respect to an instantaneous axis

Input : The qualitative configuration of a four-bar (or equivalent) linkage specifying the partial ordering of lengths of the links and the qualitative values of the joint angles (θ_{ij} denotes the joint angle between links i and j).

Output : The location of instantaneous axis, O, of the floating link with respect to the fixed link and the relationship between two qualitative distances, OP_1 and OX, from the axis to the end point and to the desired point on the floating link respectively.

Begin

1. Determine the joint angles from an intermediate link l_a (connecting l_1 and l_2) to l_2 and l_3.

2. If $max(\theta_{2a}, \theta_{3a}) < \theta_{23}$, then if it satisfies the following condition, that is, either $sum(\theta_{12}, \theta_{14}) < \pi$ or $sum(\theta_{23}, \theta_{34})) > \pi$, then $OQ_1 = OP_1 + l_2$ and $OQ_2 = OP_2 + l_4$ (see Figure 5.7a), where OP_1 and OP_2 with respect to l_3 are inferred from $\pi - \theta_{23}$ and $\pi - \theta_{34}$, else $OQ_1 = OP_1 - l_2$ and $OQ_2 = OP_4 - l_4$ (see Figure 5.7b), where OP_1 and OP_2 with respect to l_3 are inferred from θ_{23} and θ_{34}. If the desired point, X, is not P_2, then further infer the relationship with the distance OX.

3. If $max(\theta_{2a}, \theta_{3a}) > \theta_{23}$, then if $\theta_{2a} > \theta_{3a}$ then $OQ_1 = OP_1 - l_2$ and $OQ_2 = OP_2 + l_4$ (see Figure 5.7c), where OP_1 and OP_2 with respect to l_3 are inferred from $\pi - \theta_{34}$ and θ_{23}, else $OQ_1 = l_2 - OP_1$ and $OQ_2 = l_4 - OP_2$ (see Figure 5.7d), where OP_1 and OP_2 with respect to l_3 are inferred from θ_{34} and θ_{23}. If the desired point, X, is not P_2, then further infer the relationship with the distance OX.

End.

Where $sum(*,*)$, $max(*,*)$, and $min(*,*)$ denote the sum, maximum, and minimum of the two given parameters, respectively. The determination of joint angles has been shown in the position analysis algorithm.

5.2.1 An Illustrative Example

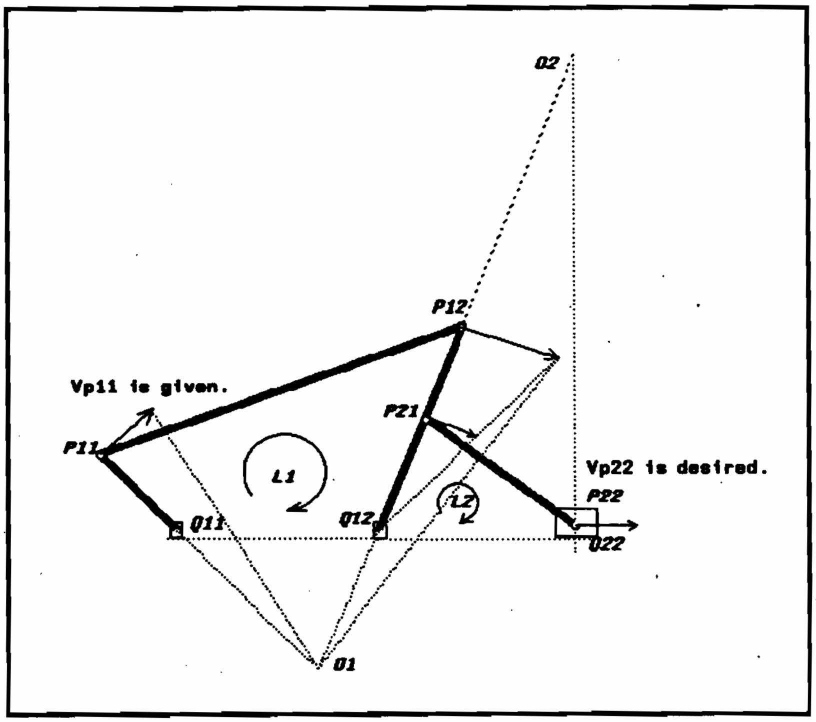

Figure 5.8 An example of qualitative geometric reasoning about velocities

In order to illustrate the qualitative analysis of motion transmission in a linkage mechanism, in what follows we shall give an example. Suppose that the input qualitative configuration in the example is exactly what has been derived in the position analysis example. The transmitted motion at the slider is to be analyzed, as indicated in Figure 5.8. By applying graph searching we can find two independent four-bar linkages, and further by definition we know that the problem is of $TransFlw_{between}$ type.

Now let us start with the linkage containing the driver link. As the location of the linear velocity on the driver link is at its end point, we omit the Step 3 of the algorithm. Next, we determine the instantaneous axis of the floating link in the linkage. In doing so, we follow the above mentioned subroutine to infer O_1Q_{11} and O_1Q_{12}, with respect to l_1, from θ_{12} and θ_{14}, and O_1P_{11} and O_1P_{12}, with respect to l_3, from θ_{23} and θ_{34}. As a result, we have $O_1P_{11} \simeq O_1P_{12}$ and consequently the velocity relationship $V_{P_{11}} \simeq V_{P_{12}}$. Having described the velocity at the joint of l_3 and l_4 with respect to the axis of l_4, we can further analyze the velocity at the shared joint P_{21}. This step will result in $V_{P_{12}} > V_{P_{21}}$.

Next let us consider the second linkage with the shared link as its driver link and repeat the previous steps. Note that the instantaneous axis of the slider is located at infinity (Axiom 4.3). Thus by applying the subroutine, it can be inferred that $O_2P_{22} >\simeq O_2P_{21}$. And the result of velocity analysis for this linkage will become $V_{P_{22}} >\simeq V_{P_{21}}$. If we combine all the velocity relationships obtained so far, we will have the qualitative description of the motion transmission from l_2 to l_6, i.e., $V_{P_{11}} \simeq V_{P_{12}} >\simeq V_{P_{22}} >\simeq V_{P_{21}}$ or $V_{P_{11}} >\simeq V_{P_{22}}$. Hence, from such a result we can conclude that the magnitude of the linear velocity transmitted to the end point P_{22} is *slightly smaller* that of the velocity at P_{11}.

In the above sections, we have discussed only the qualitative analysis of *instantaneous* position and velocity. However, it should also be noted that if the value of the input angle is divided into a set of ordered regions, as shown in Section 4.1, and the motion within each region is considered to be a distinct kinematic state, the possible kinematic *state transitions* of a linkage mechanism can be obtained.

5.3 Summary

In this chapter, we have shown how to analyze the qualitative configurations and velocity distributions of a linkage mechanism, given its qualitative dimensional specifications. An algorithm for the velocity analysis has been developed, which applies the kinematic concept of instantaneous axis. This approach, although computationally efficient, is to some extent limited to the analysis of linkage or simple CSI mechanisms. A more general approach to the qualitative velocity analysis problem will be discussed in the next chapter.

Chapter 6 Relative Motion Method of Analyzing Velocities

In the preceding chapter, we have shown a qualitative geometric reasoning method formulated especially for the analysis of motion relationships in linkage mechanisms. Although the method may further be modified to handle more general CSI mechanisms, the resulting algorithm will conceivablely be complicated and dependent on the mechanisms being analyzed.

In this chapter, we shall discuss a more general approach to deriving the qualitative description of linear velocities in CSI mechanisms based upon individual bodies' *relative motions*. Typically the information required is a description of the mechanism's configuration specifying qualitative positions (e.g., angular positions in the case of four-bar linkages) with respect to a set of *local reference frames* (i.e., relative coordinate systems as defined in Subsection 2.2.1).

From the definition of relative motion, we know that an absolute velocity may be expressed in terms of a sequence of relative velocities to another absolute velocity. In such a case, we say that the absolute velocity satisfies a *velocity constraint equation*. The fundamental idea of qualitative analysis with the relative velocity method is that since we can find a set of motion vectors which qualitatively indicates the direction of relative motion and an actual velocity vector is in fact proportional to its corresponding motion vector, we can write a velocity constraint equation describing the motion of a kinematic chain in terms of relative motion vectors. Further, by evaluating and selecting sets of *vector modifiers*, as defined in Section 4.2, in the equation, we will be able to qualitatively determine both relative and absolute linear velocities.

6.1 Kinematic Modeling

In order to derive a velocity constraint equation, we shall first find the relative motion vectors at the pairs of *links* (here the term link is used in a general sense). In general, corresponding to a specific chain progression in a derived mechanism graph, there exists a directed kinematic chain, $p_0 \rightarrow^{l_1} p_1 \rightarrow^{l_2} p_2 \rightarrow^{l_3} \ldots \rightarrow^{l_n} p_n$, where $p_{k-1} \rightarrow^{l_k} p_k$ denotes that link l_k is directed from lower pair p_{k-1} to p_k. In this chain, the local relative coordinate system for the link l_k, as indicated in Section 4.3, will be centered at the pairing contact p_{k-1} on l_{k-1}. In other words, the determination of positions for local reference coordinate systems in a mechanism will depend on the direction we choose for the chain progression. Given a set of relative reference frames, the derivations of relative motion vectors in relation to specific pairing contacts can be based upon the axioms and theorems presented in Section 4.3.

As we know, an actual velocity vector has the same direction as its corresponding motion vector and their magnitudes are proportional to each other. Therefore having obtained the motion vectors of a set of connected links, we can further find a constraint equation of the actual velocity vectors. In doing so, we may apply the following two theorems of constrained kinematic chains.

Theorem 6.1 (Loop postulate) The algebraic sum of relative actual velocity vectors associated with the consecutive lower pairs of links in a simple closed-loop kinematic chain is zero.

From theorem 6.1, it is possible to further derive the following theorem:

Theorem 6.2 (Vertex postulate) The actual velocity vectors of two links with respect to the same frame are equal at their lower-pairing contact.

6.2 Qualitative Analysis of Relative Velocities

In this section, we discuss how to derive the qualitative description of motion of any specific link, given the kinematic model of a CSI mechanism expressed in terms of an actual velocity constraint equation.

6.2.1 Solving Velocity Constraint Equations

The essence of qualitative reasoning about the motion of a CSI mechanism lies in the use of a heuristic search technique to modify the qualitative values of velocity vectors in a constraint equation initialized by motion vectors. The problem of heuristic search for appropriate velocity values can be stated as follows:

Given an initial representation of a velocity constraint equation as expressed in terms of relative motion vectors, determine, for each motion vector, a sequence of modifiers such that the resultant vectors best satisfy the equation. This set of vectors is considered as a qualitative solution of the velocity equation and therefore gives the absolute and/or relative velocities of links in the mechanism.

Here, the vectors that best satisfy the velocity equation are defined as those which, as compared to others resulting from *further* applying modifiers, yield the smallest error with respect to the equation. In order to obtain the overall best solution, the heuristic search is carried out in such a way that at each iteration all the possible modifiers are evaluated and those that can give a temporary best solution with respect to the previous vectors are selected. During each modification of velocity vectors, the derivations and evaluations of qualitative vectors are constructed from the inference rules of qualitative arithmetic, as presented in Section 4.2. The modified velocity vectors are termed *intermediate velocity vectors* or temporary velocity vectors. The algorithm for velocity analysis utilizing the relative motion representation is given as follows:

An algorithm for determining linear velocities of a CSI mechanism

Input : A representation of the mechanism's configuration in terms of the instantaneous position of each link with respect to some local reference frame at its lower pair.

Output : The desired velocity vector of a given link with respect to a fixed or a moving link.

Begin

1. Derive a mechanism graph representation of the CSI mechanism.

2. Determine the independent loops in the graph which correspond to the constrained kinematic subchains in the mechanism.

3. Find the subchain which contains a link whose relative velocity at a certain pair is given.

4. Divide the subchain into two distinct chain progressions directed from the fixed link to the known pair.

5. For each chain progression, according to the given direction, express the velocity at the known pair in terms of the relative velocities of consecutive links. Connect these two expressions into a velocity constraint equation (Theorem 6.2).

6. For each chain progression, find the relative motion vectors of links utilizing the axioms and theorems presented in Section 4.3.

7. Transform the actual velocity vector terms in the original velocity constraint equation into corresponding modified motion vectors. If a relative velocity term is the given velocity then write its qualitative value.

8. Modify the set of motion (or intermediate) vectors in the new equation by using vector modifiers, until the resultant vectors yield the smallest qualitative error in the original constraint equation. In each step of modification, all the combinations of possible modifiers are evaluated and the temporarily best one is selected and applied.

9. Let the set of resultant intermediate vectors be the qualitative solution of the original velocity constraint equation. If the desired relative velocity between two links is within the current loop, then find its qualitative vector value by adding or subtracting the consecutive relative velocities in the given progression direction, else find the absolute velocity value of the link shared by another independent loop and consider the subchain corresponding to the new loop back to Step 4.

End.

It should be noted that the *temporarily best modifiers* for a set of intermediate

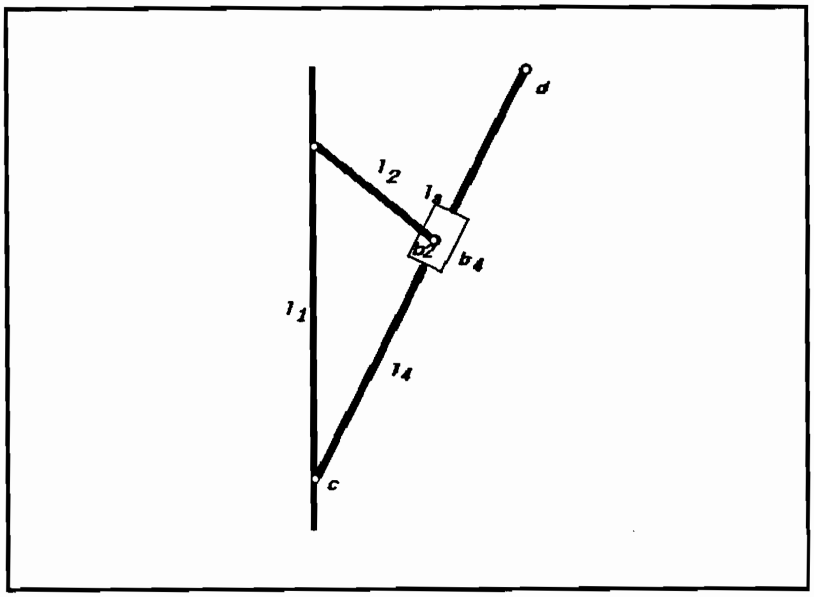

Figure 6.1 A quick-return mechanism

velocity vectors, as mentioned in Step 8, are defined as such that

$$max(|E_{xi}|, |E_{yi}|) = min\{max(|E_{xj}|, |E_{yj}|)\}$$

where E_{xj} and E_{yj} denote the $x-$ and $y-$ velocity-component errors of the constraint equation, respectively, resulting from applying one of the four modifiers, j. E_{xi} and E_{yi} denote the errors resulting from applying temporarily best modifier i.

6.2.2 An Illustrative Example

In this subsection, we present an example of qualitative reasoning about instantaneous linear velocities of a linkage mechanism with the relative motion method. The linkage to be analyzed is a quick-return mechanism as shown in Figure 6.1, where the velocity of point d is desired and V_{b_2} is given as a qualitative row vector (l, l). It can be noted that if the velocity at point b_4 which lies on the link l_4 is known then V_d can be inferred by comparing the distances from the axis c to b_4 and to d. Therefore, the subgoal of the velocity analysis becomes the determination of linear velocity at b_4.

To begin the analysis, we represent the mechanism into an equivalent mechanism graph. Since the component 3 is constrained to slide along link l_4, by using the graph construction algorithm presented in Chapter 2 we can obtain an equivalent linkage mechanism by adding an imaginary link l_a between l_2 and l_4. The corresponding mechanism graph is given in Figure 6.2. It is obvious that the derived graph contains only one independent loop.

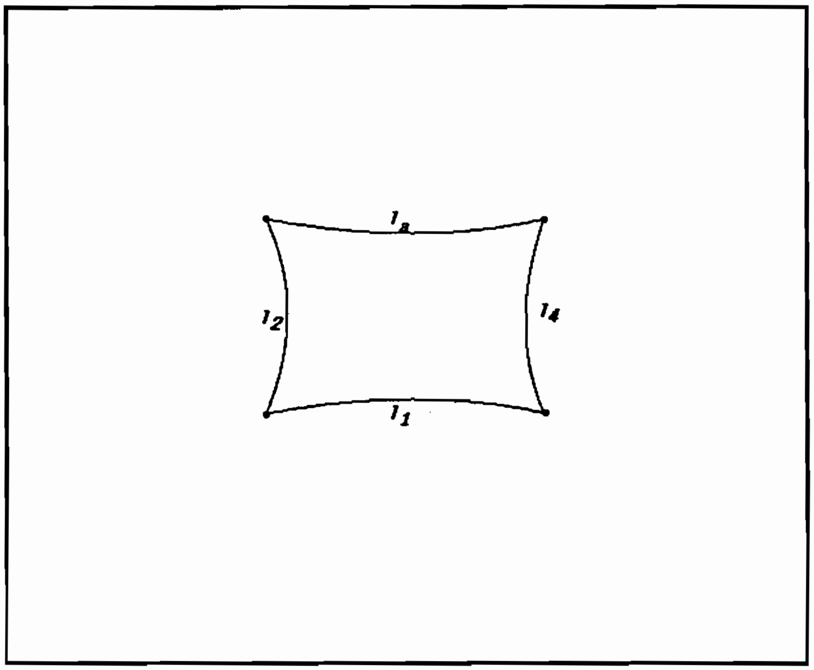

Figure 6.2 The mechanism graph of a quick-return mechanism shown in Figure 6.1

The next step is to construct a velocity constraint equation from the graph. We divide the loop into two distinct chains from one fixed joint to the known joint b_2 and for each of the two chains, write V_{b_2} in terms of the sum of pairwise relative velocities. As the linear velocity at the endpoint of link l_2 relative to the fixed link l_1 is given, $V_{b_2 \leftarrow l_2}$, i.e., the velocity derived from the chain containing l_2, will be written in terms of the known qualitative row vector (l, l). Consequently, by Theorem 6.2 (vertex postulate), we can write a velocity constraint equation for this particular closed-loop mechanism as follows:

$$V_{b_2 \leftarrow l_a, l_4 \dots} = V_{b_2 \leftarrow l_2} \tag{6.1a}$$

or

$$V_{l_4/l_1} + V_{l_a/l_4} = (l, l) \tag{6.1b}$$

Where V_{l_4/l_1} and V_{l_a/l_4} denote the velocities of links l_4 and l_a relative to l_1 and l_4, respectively.

From Section 4.3, we know that the velocities V_{l_4/l_1} and V_{l_a/l_4} are proportional to their corresponding relative motion vectors m_{l_4/l_1} and m_{l_a/l_4}. Therefore the equation 6.1b can further be approximately rewritten as

$$\lambda_1 m_{l_4/l_1} + \lambda_2 m_{l_a/l_4} = (l, l) \tag{6.2}$$

Where λ_1 and λ_2 denote the series of qualitative vector modifiers to be found. The relative motion vectors, corresponding to the given configuration, are shown in Figure 6.3. They are derived straightforwardly based on Axiom 4.1, 4.2 and Theorem 4.1.

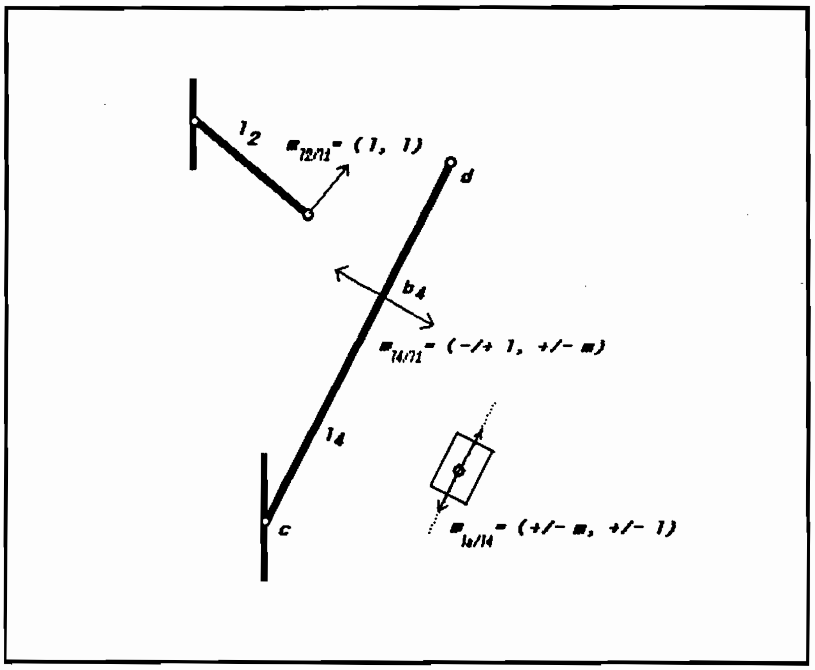

Figure 6.3 The relative motion vectors of links in the mechanism of Figure 6.1

Having obtained equation 6.2, the next step of velocity analysis is to evaluate the possible combinations of the predefined modifiers and assign the best suitable set to the equation. The criterion is that the intermediate vectors resulted from applying modifiers should yield the smallest error in equation 6.1b. This step is repeated until no more error can be reduced. Table 6.1 shows such an iterative process and Table 6.2 gives the details of the qualitative modifier evaluations in Step 3. In the tables, the qualitative inferences involved are based on the rules given in Section 4.3 and the error of Step i is defined as follows:

$$E_i = \lambda_1{}^i v_{l_4/l_1}^{(i-1)} + \lambda_2{}^i v_{l_a/l_4}^{(i-1)} - V_{b-2\leftarrow l_2} \tag{6.3}$$

Where $\lambda_1{}^i$ and $\lambda_2{}^i$ denote the qualitative vector modifiers being applied in Step i and $v_{l_4/l_1}^{(i-1)}$ and $v_{l_a/l_4}^{(i-1)}$ denote the intermediate vectors resulted from the iterative Step $i-1$. Note that in Step 3 the modifier *Inverse* is not evaluated. This is because the directions of velocities have been modified in Step 1 and therefore the remaining steps will deal only with magnitudes.

Table 6.1 The modifications of motion (and intermediate velocity) vectors

Steps	$\lambda_1 m_{l_4/l_1}$	$\lambda_2 m_{l_a/l_4}$	$V_{b_2\leftarrow l_2}$	Errors
0	$(-l,m)$	(m,l)	(l,l)	$(-vl,m)$
1	$Inverse(-l,m)$ $\Rightarrow (l,-m)$	$Identity(m,l)$ $\Rightarrow (m,l)$	(l,l)	$(m,-m)$
2	$Decrease(l,-m)$ $\Rightarrow (m,-s)$	$Identity(m,l)$ $\Rightarrow (m,l)$	(l,l)	$(s,-s)$
3	$Decrease(m,-s)$ $\Rightarrow (s,-vs)$	$Identity(m,l)$ $\Rightarrow (m,l)$	(l,l)	$(vs,-vs)$

From Table 6.1 it may be noticed that since error cannot be reduced further, the value of v_{l_4/l_1}, i.e., $(s,-vs)$, will be considered as the approximate value of V_{l_4/l_1}, that is, the qualitative value of the linear velocity of link l_4 at the instantaneous pairing point b_4 relative to fixed link l_1. Therefore by comparing the distances from the fixed axis c to b_4 and to d, we can determine the qualitative value of the linear velocity at d. As given in the instantaneous configuration of this problem, distance cd is almost twice as long as distance cb_4, that is to say the magnitude of V_d is similarly twice as large as that of V_{b_4}, hence we can derive the qualitative value of V_{b_4} to be $(l, -s)$. This step is readily understood by following the discussion presented in Section 5.2.

Table 6.2 The evaluations of qualitative modifiers in Step 3

Steps	*Increase*	*Decrease*	*Identity*	$v_{l_4/l_1} + v_{l_a/l_4}$	$V_{b_2 \leftarrow l_2}$	Errors
$3a$	${\lambda_1}^3$	${\lambda_2}^3$	-	$(l,-m)+(s,m)$	$(l,-l)$	$(s,-l)$
$3b$	${\lambda_1}^3$	-	${\lambda_2}^3$	$(l,-m)+(m,l)$	$(l,-l)$	$(m,-m)$
$3c$	-	${\lambda_1}^3$	${\lambda_2}^3$	$(s,-vs)+(m,l)$	$(l-,l)$	$*(vs,-vs)$
$3d$	${\lambda_2}^3$	${\lambda_1}^3$	-	$(l,vl)+(s,-vs)$	$(-l,l)$	$(s,0)$
$3e$	${\lambda_2}^3$	-	${\lambda_1}^3$	$(l,vl)+(m,-s)$	$(l-,l)$	$(m,-vs)$
$3f$	-	${\lambda_2}^3$	${\lambda_1}^3$	$(m,l)+(m,-s)$	$(l,-l)$	$(s,-s)$
$3g$	${\lambda_2}^3$,${\lambda_1}^3$	-		$(l,vl)+(l,-m)$	$(-l,l)$	$(l,-s)$
$3h$	-	${\lambda_2}^3$,${\lambda_1}^3$		$(s,m)+(s,-vs)$	$(-l,l)$	$(0,-s)$

6.3 Summary

In this chapter, we have formulated a qualitative representation and reasoning technique for analyzing motions and their relationships constrained by CSI mechanisms. It utilizes the relative motion vector representation of mechanism components and generates solutions by resolving qualitative motion constraint equations. Although this approach appears to be less computationally efficient than the qualitative trigonometric reasoning approach, it is more applicable in solving the general CSI mechanism problems.

The algorithm described in this chapter is designed particularly for solving *instantaneous* velocity problems. Yet it should be noted that this algorithm can also be extended to handle the kinematic state transitions of a moving CSI mechanism. In such a case, the analysis should be preceded by a step consisting of partitioning the value of an input displacement into a quantity space and computing the set of corresponding qualitative configurations, as mentioned in the previous chapters.

Chapter 7 Kinematic State Transitions in CSV Mechanisms

The methods presented in the preceding two chapters are in general limited to the qualitative kinematic analysis of CSI mechanisms. As has been discussed, the first method is primarily based on the instantaneous axis property of CSI mechanisms. The procedure for velocity analysis utilizes qualitative trigonometry, and is particularly designed for linkage mechanisms. On the other hand, the second method is, to some extent, more general, but nevertheless it solves the kinematic state transition problem by partitioning the quantity space of a certain parameter into a finite set of regions (corresponding to a set of discrete kinematic states) and requiring that the qualitative velocity constraint equation for each state be of the *same structure.*

This chapter addresses the problem of identifying kinematic state transitions of contact-surface-varying (CSV) mechanisms, in which the connectivity or contact-surface *varies* with the motion of mechanism bodies.

The function of a CSV mechanism such as a ratchet or an escapement mechanism can best be understood in terms of the interactions between individual bodies, i.e., the change of contact or connectivity. This implies that in the qualitative analysis of a CSV's function, it would be desirable to consider each distinct contact as a discrete kinematic state and describe the transition of the states by deriving a sequence of possible contact changes.

Stanfill has developed a set of algebraic rules to determine where complex surfaces intersect and touch [Stanfill 83]. In his Mack system, shapes are represented in terms of the sum and difference of primitive solids. The underlying objective is to model what seems intuitive and fundamental for humans to reason about the interactions of objects. However since the mechanism parts considered are quite limited, this approach is unable to handle CSV mechanisms of complex shapes.

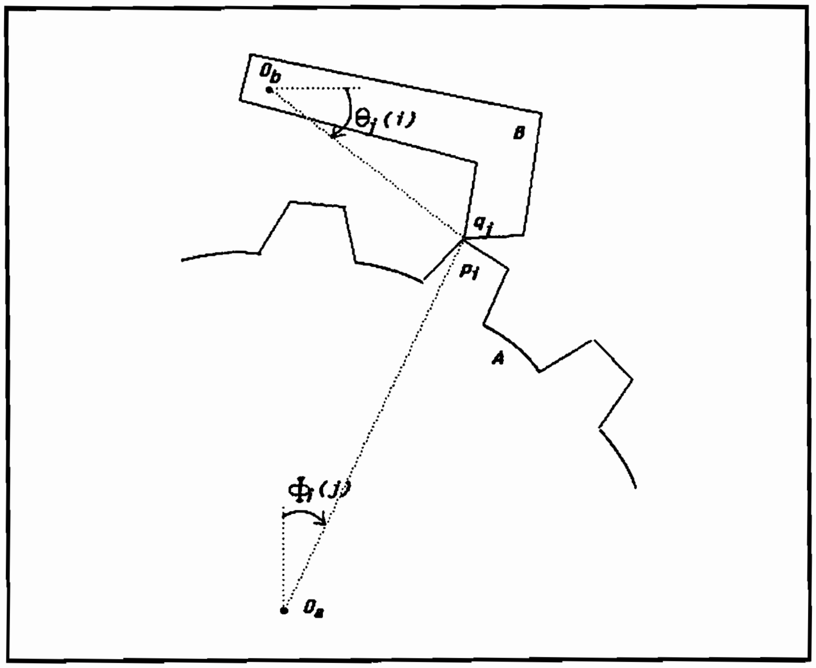

Figure 7.1 A CSV mechanism

Faltings has proposed a method of qualitative spatial reasoning for analyzing kinematic state transitions [Faltings 87]. This method, utilizing configuration space representation, may be described as follows.

In Figure 7.1, ϕ and θ are used to described the positions of bodies A and B, respectively. These two parameters constitute the generalized coordinates (i.e., independent coordinates) of the CSV mechanism. The space spanned by such parameters, which characterizes all positions of mechanism bodies, is called the mechanism's *configuration space*

[Lozano-Perez and Wesley 79] [Brooks and Lozano-Perez 83] [Lozano-Perez 83]. According to Faltings' method, if the configuration space and the constraints for its valid subspace (corresponding to all legal configurations) can be computed, then a set of possible "places", where bodies are in contact can further be derived. Thus by means of envisionment analysis, a sequence of qualitative state transitions can be inferred.

In the method to be described herein, what is required for deriving kinematic states and transitions comprises some specific configurations, i.e., *a subset of CS*, as well as the information about the placement of vertices of one body with respect to the edges of another in those configurations. In other words, the derivations depend only on incomplete configuration information and hence reduce the complexity involved in computing a full configuration space.

In this study, we consider primarily the type of CSV mechanism which is composed of two bodies and assume that (1) the boundary of each body can be described by a simple polygon (i.e., a set of connected non-crossing line segments) and (2) the degree of freedom for each body is equal to one. For other CSV mechanisms, methods of analysis may be obtained by modifying the one for the simpler but similar two-body mechanisms.

Here, a kinematic state is defined as a specific sliding contact between a boundary vertex and a boundary line segment, by which a motion is transmitted. A state transition refers to the change of sliding contact. A vertex-contact (VC) configuration refers to the configuration of a CSV mechanism in which two interacting bodies have a direct contact at their boundary vertices. The typical problem to be solved is stated as follows:

Given, in a two-body CSV mechanism, a set of VC configurations and the vertex placement of the driving body with respect to the boundary of the driven body in correspondence to the mechanism's VC configurations, determine the interactions of bodies, i.e., the *changes of direct contacts as well as sliding motions between two bodies, during a motion of the mechanism. For each new contact point (or within a kinematic state), find the velocity relationship between the bodies.*

7.1 Vertex-Contact Configurations of CSV Mechanisms

Figure 7.1 shows a CSV mechanism. It can be noted that the change of contact during a motion will mainly depend on the geometry of the mechanism bodies as well as their relative positions. Therefore, in order to determine kinematic states, we will require the knowledge of geometric configurations. In this section, we introduce a means of geometric characterization of CSV mechanisms utilizing VC configurations.

7.1.1 The Description of VC Configurations

In Figure 7.1, the series of n vertices of body A are denoted by p_i where $1 \leq i \leq n$ and those of m vertices of body B are denoted by q_j where $1 \leq j \leq m$. Suppose that distances $o_a p_i$, $o_b q_j$, and $o_a o_b$, and the lengths of boundary segments are given. Thus the VC configurations of the mechanism composed of bodies A and B, as in this case vertices p_i and q_j are in a direct contact, can easily be described by deriving (e.g., based on *qualitative trigonometry*) $\phi_i(j)$ and $\theta_j(i)$. To be more general, the description of VC configurations can also be extended to cover prismatic-pairing bodies in CSV mechanisms. In such cases, the angles of each boundary segment relative to a fixed reference frame (rather than $o_a p_i$ and $o_b q_j$) must be given. Hence, the description of VC configurations comprises a set of positions in the sliding motion directions, as shown in Figure 7.2.

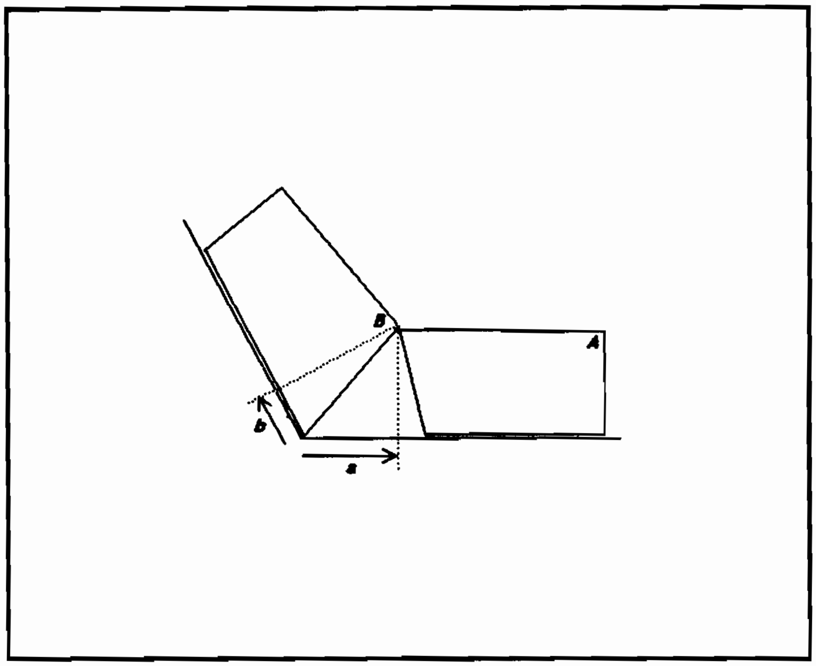

Figure 7.2 The geometric description of prismatic-pairing bodies in a CSV mechanism

7.1.2 Placement of Vertices in VC Configurations

In order to derive the transitions of kinematic states, more information, apart from the description of VC configurations, would be required. Suppose that a boundary vertex is in a direct sliding contact with a line segment and at some point of the line segment the sliding motion *fails* to proceed due to certain geometric constraints. Note that if after the *terminating contact point* the sliding motion was imaginarily continued, an overlap would occur between the boundaries of the bodies. Furthermore, if we let the contact point slide to the end point of the boundary segment, then the overlapping would change accordingly into one of the three possible positions presented in Figure 7.3.

From Figure 7.3, it can be observed that each position corresponds to a change in relative vertex placement as the sliding motion is continued from a given contact point, for instance, an initial contact point or an vertex-contact point, to an end point of the boundary

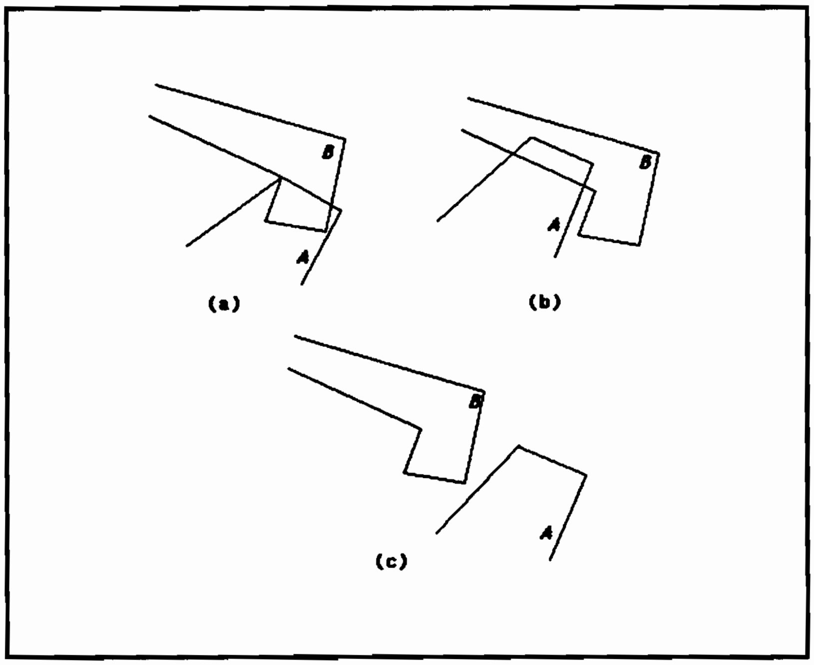

Figure 7.3 Three possible positions into which the overlapping between mechanism bodies may change.

line segment. Here, vertex placement refers to the placement of the vertices of one body relative to the boundary line segments of another in each VC configuration. Such an observation implies that the potential overlap(s) can be determined simply by comparing the placements of vertices in two consecutive VC configurations, i.e., the VC configurations corresponding to two consecutive vertex contacts at which the imaginary sliding motion starts and ends. And consequently, from the information of potential overlap(s), a possible contact point between two bodies can be detected.

Concerning the example in Figure 7.1, the vertex placements may be computed as follows: for a vertex on body A, p_i, in a direct contact with another vertex on body B, q_j, find the placement of the rest of A's vertices with respect to all the B's line segments. Repeat this process for all i's and j's. In order to keep track of the relative vertex placement, the direction of each polygonal curve should be defined. Thus given the direction of a certain boundary line segment, the placement of a vertex can be described in terms of lying on which side of the extended line. This may be done, for instance, by evaluating the vertex in the equation of a line segment. The vertex placement in the case of prismatic-pairing bodies can be derived similarly.

As only convex vertices may form or change sliding contacts, the configurations with *concave-concave* vertex contacts can be ignored during the computation of VCs. Figure 7.4 shows an example of the illegal concave-concave VC configurations.

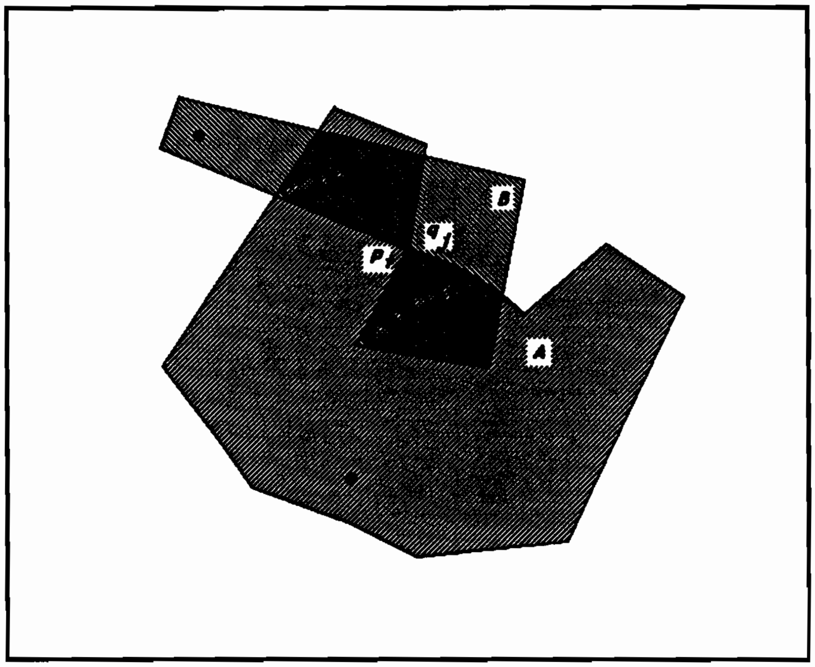

Figure 7.4 An illegal concave-concave VC configuration

7.2 Identification of Kinematic State Transitions

This section describes how the above mentioned representation of VC configurations is used in reasoning about the motions of CSV mechanisms.

7.2.1 Change of Sliding Contacts

The following is an algorithm for identifying a sequence of contact changes in the motion of a CSV mechanism:

An algorithm for determining the change of sliding contacts

Input : The description of VC configurations with respect to a fixed frame and the corresponding vertex placements of a driving body, A, relative to each extended boundary line segment in a driven body, B. Note that in the initial configuration, the contact may exist between a boundary line segment and a boundary vertex.

Output : A sequence of sliding contact changes caused by a given driving motion.

Begin

1. Start with the initial configuration, keep track of the coordinate values and the vertex placement. Assign them to a coordinate vector (a, b) and a placement vector $previous\_vposition$, respectively. Suppose that the sliding vertex moves to one end point of the line segment and find the corresponding VC description.

2. If the direction of A's position change is inconsistent with that of the given motion, then go to Step 1, else keep track of VC description and the vertex placement, and assign them to a coordinate vector (a', b') and a placement vector $current\_vposition$, respectively.

3. Identify the possible overlap(s) during the sliding motion by comparing $previous\_vposition$ with $current\_vposition$:

a. If the position of a vertex of A relative to an extended boundary line of B changes from outside to inside (Note: if we counter-clockwise walk along the boundary of B, the right hand side is defined as the outside), then there exists a possible overlap and the vertex is considered as a potential sliding contact point. And/or,

b. If from $current\_vposition$ it is found that two connected vertices of A have different placements with respect to at least two boundary line segments of B (Note: a contact vertex is considered to be outside of the contact line segment), then there exists a possible overlap. Either one of these two vertices will be the new sliding contact point or the boundary segment formed by the two vertices will be in a direct sliding contact with a vertex in B.

c. If there is no possible overlap region detected, then conclude that the previous sliding motion is valid and go to Step 5.

4. For each possible contact vertex p_i, start with the *furthest* vertex in terms of its position in A, find the line segment to which p_i is an inside vertex.

a. Compute the VC configuration, formed by p_i and one of the line segment end points. The end point is chosen such that p_i has a *larger* position. Update (a', b') and $current\_vposition$. Compare a' with a, if the result is consistent with the direction of the given driving motion and if there cannot be found any possible overlaps in $current\_vposition$, then conclude that the next sliding contact is *between the line segment and* p_i and go to 5, else go to 4c.

b. Compute the VC configuration, formed by p_i and one of the line segment end points. The end point is chosen such that p_i has a *smaller* position. Update

(a', b') and $current\_vposition$. Compare a' with a, if the result is consistent with the direction of the given driving motion and if there cannot be found any possible overlaps in $current\_vposition$, then conclude that the next sliding contact is *between this end point of the line segment and the line segment formed by* p_i *and its succeeding vertex* and go to 5, else go to 4c.

c. If there are *new* possible contact vertices, then recursively run Step 4.

5. Keep track of the sliding contact change. Test whether a blocking configuration has been reached. If not, then consider the VC configuration as an 'initial' configuration (i.e., assign (a', b') to (a, b) and $current\_vposition$ to $previous\_vposition$). With the same sliding vertex, go back to Step 1.

End.

This algorithm can be used to identify a sequence of sliding contact changes between two bodies of a CSV mechanism, if it is given the description of VC configurations and the corresponding vertex placement information. Figure 7.5 depicts the initial configuration of a CSV mechanism. It can be verified that by computing its VC configurations and applying the contact analysis algorithm, we can derive the sliding contact changes (i.e., contact points), as shown in Figure 7.6.

However it may be noticed from the above algorithm that the detail of Step 5 has been left out. Let use now discuss how the test of mechanism movability is performed. First, we consider the following theorem:

Theorem 7.1 (Motion transmission by direct contact) (see Figure 7.7) Suppose that a driving body A is in direct contact with a driven piece B. If with respect to a reference

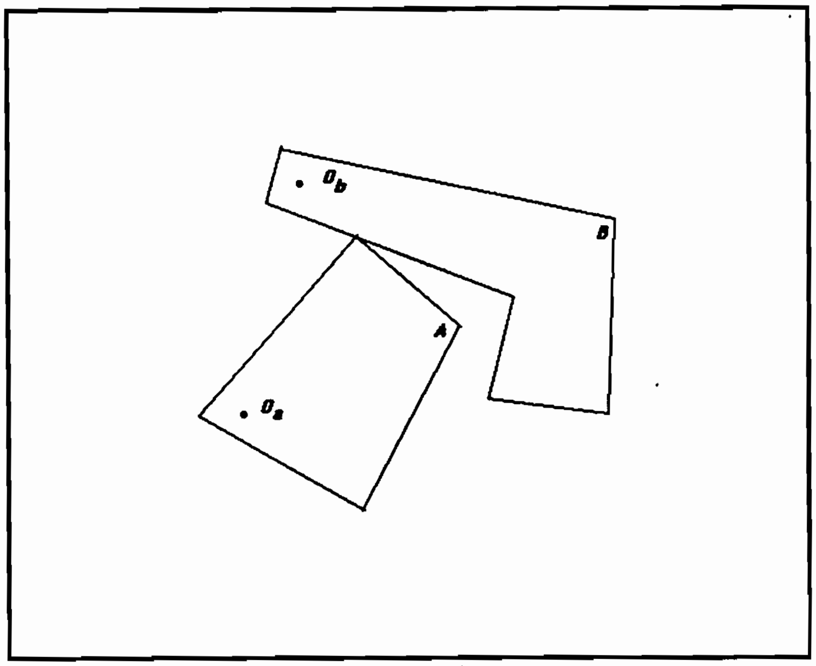

Figure 7.5 The initial configuration of a CSV mechanism

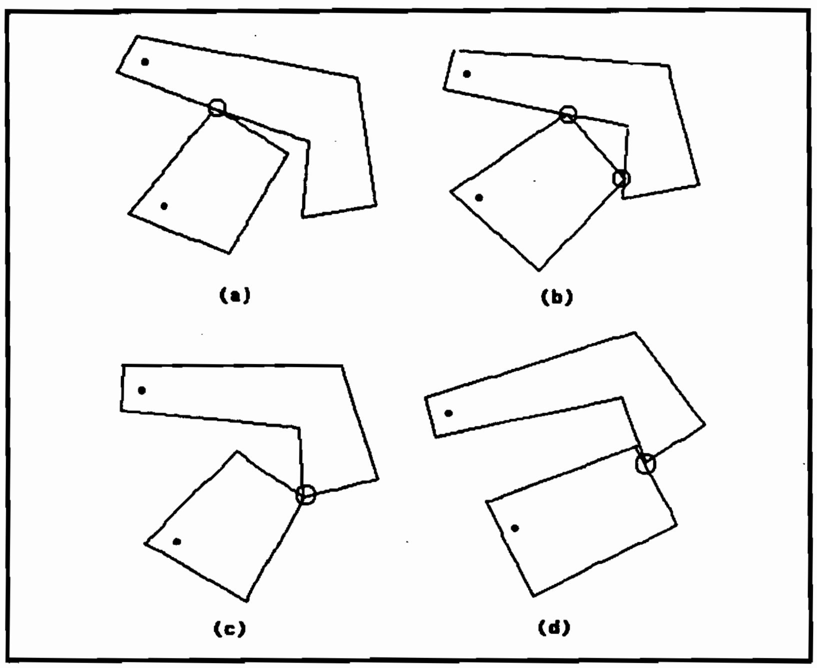

Figure 7.6 The sliding contact changes between two bodies of the CSV mechanism of Figure 7.5

frame the directions of the potential motion vectors of both A and B are not perpendicular to the common normal of the contact surface and both motion vectors are on the same side of the common tangent, then the motion can be transmitted from A to B. The partial ordering relationship between their absolute velocities can be determined from the angles formed by each motion vector and the normal.

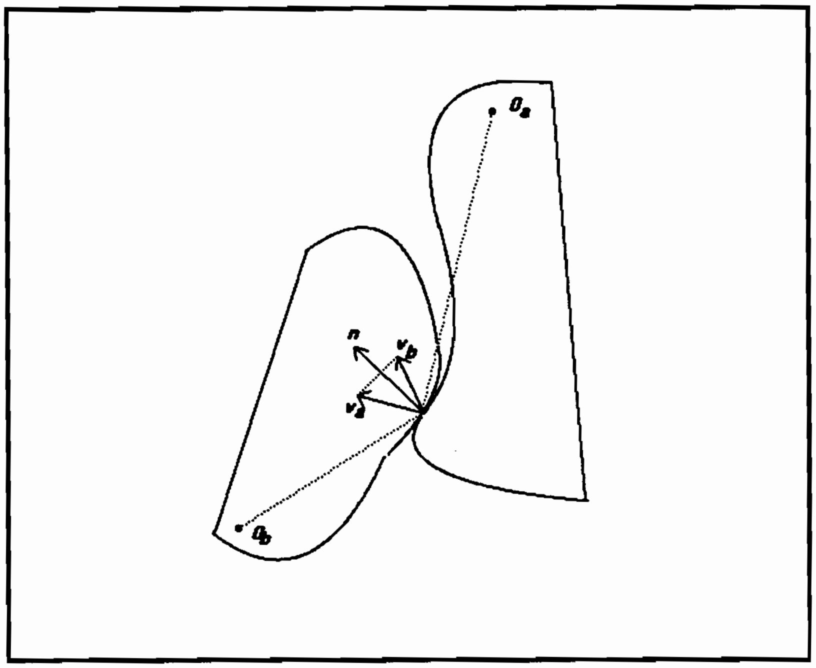

Figure 7.7 Transmission of motion by direct contact

From Theorem 7.1, we can further derive the following corollary:

Corollary 7.1 (see Figure 7.8) Let a driving body A and a driven body B be in direct contact. With respect to a fixed reference frame, if the motion direction of A is the same as that of the common tangent between A and B (i.e., parallel to the current contact boundary line segment), then A will be in sliding motion relative to B and no motion will be transmitted to B, i.e., B remains stationary, as shown in Figure 7.8a. If the motion direction of A is different from that of the common tangent and is toward B, then we have the following two cases. That is, if the direction of B's motion vector (i.e., potential motion direction) is parallel to that of the common tangent, then the CSV mechanism has reached a *blocking configuration* (see Figure7.8b), else the motion of A will be transmitted to B (see Figure7.8c).

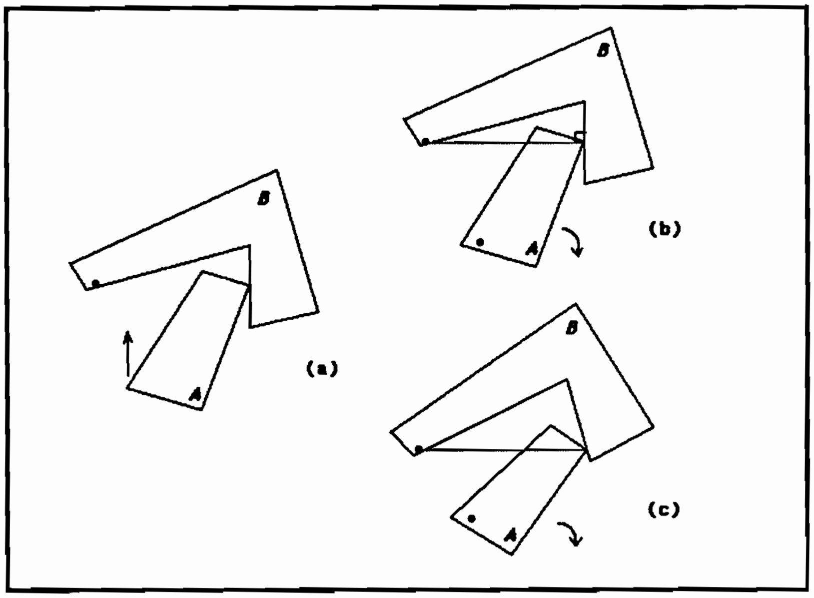

Figure 7.8 Corollary 7.1

The above corollary is very useful in testing the movability of a CSV mechanism, as required in Step 5 of the algorithm. It should be noted that this corollary is generally applicable to not only CSV mechanisms composed of revolute-pairing bodies but also those composed of prismatic-pairing bodies. In applying this corollary, the directions of (potential) motions and common tangent can qualitatively be determined based on the description of the CSV mechanism mentioned in Subsection 7.1.1.

7.2.2 Velocity Relationship between Two Bodies

In this subsection, we discuss how to determine the change of a motion transmission between two bodies of a CSV mechanism. Suppose that at each new sliding contact inferred using the preceding algorithm, the motion transmission relation is approximately constant. Thus, if we can find the *unique* velocity relationship between two bodies within each sliding motion, we will be able to obtain the change of motion transmissions, corresponding to the change of sliding contacts. Hence we will completely solve the problem of deriving kinematic state transitions of a CSV mechanism.

The idea behind velocity relationship analysis is straightforward. From the preceding subsection, we know that given the descriptions of a CSV mechanism and its VC configurations, we can find, by means of simple calculations, the changes in (angular) position of two bodies as a contact vertex slides along a boundary segment from one end point to another. As a result, we can determine the angular velocity relationship between the two bodies during sliding motion. And since the distances from the origin of each coordinate to the vertices of a body are known, we can also derive the corresponding linear velocity relationship. If all these mentioned quantities are given in terms of qualitative values or relations, the qualitative description of the velocity relationship can therefore be obtained.

In order to illustrate the method of velocity relationship analysis, we shall consider an example of sliding contact change as shown in Figure 7.6. It should be noted however that this method is not restricted to revolute-pairing CSV mechanisms.

Let us denote the vertices of bodies A and B as p_i and q_j, respectively, and the angular positions of $o_a p_i$ and $o_b q_j$ as $\phi_i(j)$ and $\theta_j(i)$, with respect to the local reference frames of individual bodies in the configuration where vertices p_i and q_j are in a direct contact (see Figure 7.9). Suppose that the angular velocity of body A is given and we want to find the linear velocity relationship between the two bodies in the *second* sliding contact.

By applying the contact analysis algorithm, we know that the second sliding contact must be formed by vertex p_2 and boundary segment $q_1 q_2$. Therefore we compute the coordinate value differences of $\Delta\phi$ and $\Delta\theta$ as p_2 slides from q_1 to q_2. That is, $\Delta\phi = \phi_2(2) - \phi_2(1)$ and $\Delta\theta = \theta_2(2) - \theta_1(2)$. From these angular differences, we find the angular velocity relationship between the two bodies as the motion transmission relation is here approximated to be

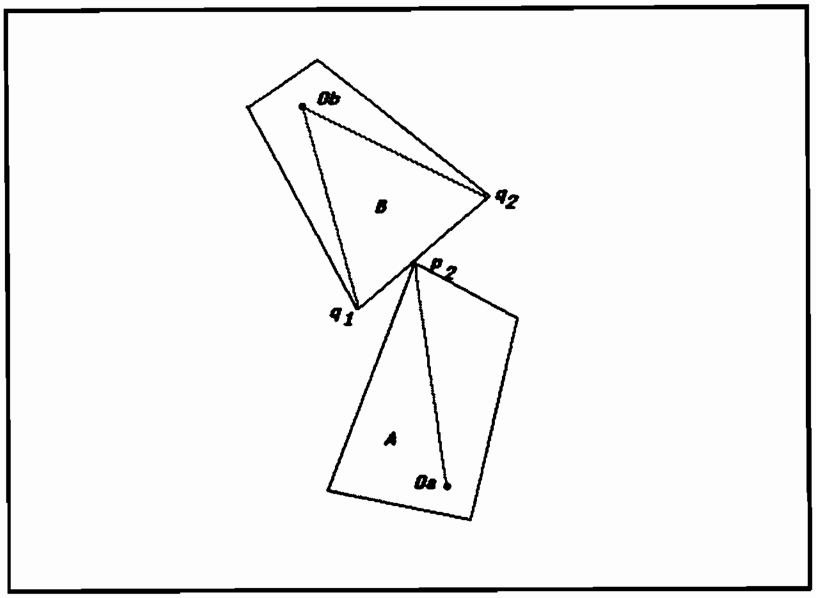

Figure 7.9 The velocity relationship between two interacting bodies

constant. More specifically, we describe the motion transmission relation for the second sliding contact as a ratio between two angular differences, i.e., $\omega_a/\omega_b = \Delta\phi/\Delta\theta$. If the distances from the coordinate origin in body B to vertices q_1 and q_2 are close enough, we can further determine the approximate relationship between the magnitudes of two linear velocities at the sliding contact point, by $v_a/v_b = (\Delta\phi \times o_a p_2)/(\Delta\theta \times o_b q_2)$.

7.3 Summary

In this chapter, we have discussed how to derive kinematic state transitions of CSV mechanisms. The developed method requires only a small set of extremal VC configuration information and is therefore quite efficient. The qualitative kinematic states identified comprise not only potential contact points (i.e., interactions), but also the motions (i.e., velocities) transmitted from one body to another.

The method for qualitative CSV analysis can effectively be used for the qualitative simulation of a mechanism's behavior as well as for the testing and analysis of its functions.

It may also be applied to the area of intelligent robotics for predicting and explaining the potential outcomes of a given manipulation in a physical environment [Coiffet 83] [Will and Grossman 75].

One of the crucial steps involved in such a qualitative analysis is that of determining VC configurations and vertex placements. If this step is carried out quantitatively, it might require a significant amount of computation. However, as mentioned earlier, the derivation of VC configurations and vertex placement information can proceed qualitatively. In such a case, every angle would be described by a qualitative value and every distance would be specified by qualitative partial ordering relations. Thus, the VC configuration description and vertex placement would effectively be inferred based on the qualitative trigonometric rules stated in Table 4.3. In a similar fashion, the test of mechanism movability can be performed. Hence, the computational complexity required for the qualitative kinematic state analysis can be reduced significantly.

Chapter 8 Conclusion

In kinematic analysis, it is always desirable to know what motions the various parts of a mechanism approximately undergo and the relationships between these motions. In classical kinematics, such an analysis usually requires *explicit quantitative information*. The problems associated with quantitative approach are that (1) generating a solution usually constitutes a major computing task and (2) the generated solution has to be carefully interpreted by humans if the function of the mechanism is to be understood.

In order to build intelligent computer systems that can perform effective reasoning about the function of a mechanism based on incomplete specifications and can communicate the results with users at a functional and qualitative level, alternative kinematics frameworks have to be developed.

In this context, the present study has focused on the qualitative approach to kinematics. It offers a set of specific solutions as to how qualitative geometric reasoning can be applied to solve kinematic analysis problems. These solutions will not only serve as a useful framework for kinematics, but will also evolve and necessitate successful application of artificial intelligent (AI) technology in applied kinematic analysis.

8.1 Contributions

The major contributions of this thesis are outlined as follows:

- Several qualitative theory constructs have been developed in the thesis. Among them, the qualitative trigonometry concerns the qualitative description of geometric properties of triangles, which has been utilized in the position and velocity analysis of linkages. The qualitative arithmetic, on the other hand, accounts for a qualitative characterization of the numerical quantities, arithmetic operators, and relations. It plays a very important role in resolving the qualitative velocity constraint equations of CSI mechanisms in a Cartesian space. Also developed in this study are a mechanism graphical representation and a set of motion vector axioms and theorems for revolute/prismatic-pairing bodies. The former enables to capture the connectivity as well as structure (e.g., closed loops) of a mechanism and therefore facilitates the qualitative kinematic analysis using relative motion approach. The latter enables the modeling of the relative motion of each component involved in the mechanism graph.

- An efficient qualitative trigonometry-based method for reasoning about positions and linear velocities in *simple* CSI mechanisms has been described. This method applies the kinematic concept of instantaneous velocity axis to determine the qualitative instantaneous velocity distributions. The location of an instantaneous axis is directly inferred using the qualitative trigonometry.

- A qualitative representation and reasoning method for analyzing motions and their relationships constrained in *general* CSI mechanisms has been provided. This method utilizes a search procedure to resolve qualitative velocity constraint equations expressed in terms of the relative motion vectors of individual mechanism bodies. The qualitativ constraints are derived by means of kinematic modeling of the mechanism bodies and their relationships.

- The problem of CSV mechanisms has been solved by means of identifying kinematic state transitions based on VC configurations and the vertex placements with respect to such configurations.

- The usefulness of the qualitative kinematics framework has been demonstrated with the examples of a complex linkage mechanism, a quick-return mechanism, and two-body CSV mechanisms.

8.2 Discussion

In what follows, we shall discuss the advantages and limitations of the present framework and its relationship to other relevant work in the area of qualitative reasoning.

8.2.1 The Quantity Space

The qualitative quantity space [Forbus 84] is constructed in this study for both the qualitative trigonometry and arithmetic. Consequently, in the kinematic analysis, continuous parameters such as the angles of a triangle and linear velocities are represented by a set of distinct qualitative values - symbolic vocabularies, so that below or above these values, radically different kinematic states can be identified. The qualitative quantity space defined in the present formalism, unlike the +, -, 0 three-valued space as used by Kuipers and deKleer and Brown [Kuipers 84] [deKleer and Brown 85], contains more values and thus has more descriptive power in specifying distinctions and resolving geometric ambiguities. For instance, the conventions on representing angles and side relationships of triangles allow us to effectively distinguish and characterize all types of triangles.

8.2.2 A Component-Based Approach

Conceptually speaking, the relative motion method for CSI mechanisms may be viewed as an application of the classical *qualitative reasoning* formalism which determines the function of a mechanism from its structure. In particular, it shares certain commonalities with deKleer and Brown's *qualitative physics*. For instance, in the relative motion method, the derivation of a global constraint equation from an equivalent mechanism graph requires the identification of lower pairings. This may be thought of being equivalent to deKleer and Brown's connectivity analysis of a physical system. Similarly, the step for determining the relative motion vector of each link would be analogous to deKleer and Brown's component modeling.

8.2.3 Kinematic State Transition of CSI Mechanisms

In this thesis, two methods for the velocity analysis of CSI mechanisms have been proposed, i.e., (1) qualitative trigonometric reasoning method and (2) the relative motion method. Accordingly, algorithms, designed particularly for *instantaneous* velocity problems, have been described. Yet it should be noted that these algorithms can easily be extended to handle the kinematic state transitions of moving CSI mechanisms. In such a case, the analysis should be preceded by the steps of partitioning the quantity space of an input displacement into a finite set of regions (corresponding to a set of discrete kinematic states) and incrementally computing the set of corresponding qualitative configurations.

8.2.4 Comparison of Two Analysis Methods for CSI Mechanisms

In general, both the qualitative trigonometric reasoning method and the relative motion method are applicable only to the qualitative kinematic analysis of CSI mechanisms. The first method is quite quick and accurate enough for many purposes, but it fails to deal with certain complex CSI mechanisms. On the other hand, the second method, although relatively less efficient, can be applied to solving general CSI mechanism problems.

8.2.5 CSV Mechanism Analysis Using Incomplete Information

The present method for CSV mechanism analysis differs from Faltings' place vocabulary method [Faltings 87], in that it is based on incomplete quantitative information. Given some specific configurations and the placement of vertices of one body with respect to the edges of another in such configurations, it derives qualitative geometric characteristics of mechanisms. The computation of a complete configuration space (C-space) is therefore avoided.

8.2.6 Predicate Calculus

In this study, most of the qualitative analysis problems are solved based on the axioms and theorems of qualitative trigonometry, qualitative arithmetic, and pairwise object interaction. Although the reasoning involved is not formalized in terms of *predicate calculus*, such an approach falls into the same category as the axiomatization of naive physics and kinematics [Hayes 79] [Hayes 85a] [Hayes 85b] [Shoham 85].

8.3 Areas of Future Research

This study has highlighted many opportunities for future research. The following are some of the research areas identified.

8.3.1 Empirical Study on Human Mental Representations

The qualitative description of angles and side length in the qualitative trigonometry is developed mainly based on the hypothesis that humans are very good at making qualitative measures with respect to some symmetric or neutral references. This hypothesis might not be firmly justified unless the empirical data could show that humans actually use a similar representation in qualitative analysis. Hence it would be interesting to conduct psychophysical studies on this issue.

8.3.2 Qualitative Analysis of Complex Mechanisms

In this thesis, we have considered primarily the type of CSV mechanism which is composed of two bodies and assumed that (1) the boundary of each body can be described by a simple polygon (i.e., a set of connected non-crossing line segments) and (2) the degree of freedom for each body is one. Therefore, one of the interesting extensions of the present study would be to devise an analysis method for the general CSV mechanisms utilizing not only the VC-based reasoning but also the commonsense axioms and theorems in object motion and interaction (e.g., rolling contacts).

Furthermore, it would also be desirable to examine the possibility of expanding the present two-dimensional geometric reasoning techniques into three dimensional cases.

8.3.3 Qualitative Design of Mechanisms

Kinematic design of mechanisms constitutes a major section of applied kinematics. Traditionally, the design of mechanisms (e.g., type selection and dimensional synthesis) relies on human intuitions [Wilson and Michels 69]. Although well-formulated techniques may be used for some specific mechanisms, the decision on which is the right one to choose and how it should be applied in a given design situation has to be made by human designers [Hartenberg and Denavit 64] [Tao 64]. This has presented a problem in developing *computer-aided design* (CAD) systems that can automatically generate innovative designs and reason about the effect of variations in one part on the other. One of the future studies would be to investigate the useful techniques for qualitative synthesis [Faltings 88] [Dyer and Flowers 84] [Dyer, Flowers and Hodges 86]. Such an effort might eventually lead to an elegant way of combining qualitative analysis and qualitative synthesis into one uniform and coherent theory.

In qualitative design, one of the issues to be explored would be that of linkage (e.g., four-bar linkage) synthesis, based on the linkage dimension- function properties and the qualitative trigonometric reasoning techniques as developed in this thesis. Another issue for future study would concern the design of mechanism shapes. The present study on CSI and CSV mechanism has actually suggested two possible techniques, i.e., theorem-based reasoning and VC configuration-based reasoning. The essence of the theorem-based reasoning approach would be to derive kinematic shape design decisions for a set of bodies based on a backward chaining of the qualitative kinematic axioms and theorems. On the other hand, VC configuration-based reasoning would utilize qualitative geometric synthesis. In such a case, we might first of all compute the generalized coordinate values with respect to the initial as well as the vertex-contact configurations of the mechanism prior to shape modification and the vertex placements corresponding to such configurations, and determine the existing kinematic state transitions. Accordingly, we could decide how to proceed in modifying the boundaries of the mechanism bodies by comparing the desired kinematic state transitions with the existing ones.

8.3.4 Qualitative Kinematic Analysis in Robotics

The qualitative kinematic analysis described in this thesis can find several applications in the area of intelligent robotics. One application would involve the use of the CSV state identification method in simulating the physical behavior of a robotic system to causally predict and explain the potential outcomes of a given manipulation in the environment. Another key application in robotics is the integration of qualitative velocity analysis in synthesizing robot compliant motion strategies, to avoid tedious numerical computations involved in kinematic constraint propagations [Paul and Shimano 76] [Nevins and Whitney 74] [Mason 81]. Future research should explore these avenues of intelligent robotics in detail.

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